ESSAYS ON DIGITAL ECONOMICS

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Abstract

This thesis is a collection of four research papers in the field of digital economics. The first two chapters contribute to the literature on digital piracy. While in the past digital piracy was relegated to user activity, it has now turned into a business model based on ads. The first chapter studies the effect of user adoption of ad-blocking technologies aimed at (partially or fully) stopping ads on the strategies set by legal and pirate content providers. It finds that ad-blockers alters the firms' strategies. Depending on how many ads are blocked, the copyright holder, consumers, and society can perceive ad avoidance either as a blessing or as a curse.

The second chapter studies competition between a legal content provider and several pirate alternatives. Facing a pirate ecosystem, the legal provider and the government may find it optimal to tolerate some degree of (commercial) digital piracy.

The third chapter studies the platform's incentives to undertake value-increasing investments when the value generated accrues to different sides of the market (i.e. sellers and buyers). Depending on whether sellers are enabled to join more than one platform, different results and incentives may arise. When some sellers multi-home, prices decrease to buyers and increase to sellers. Moreover, more sellers multi-home and platforms are more likely to invest.

The last chapter explores incentives for a premium provider (Superstar) to offer exclusive deals to competing platforms. When platform competition is intense, more consumers join the platform hosting the Superstar exclusively. This engenders a *ripple effect* on the other side: many other providers enter the market and provide exclusively their product. The welfare analysis suggests that Superstar exclusives may represent a manifestation of the competitive intensity in the market and be the first-best in the industry.

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Author's Declaration

I declare that this thesis is a presentation of original work and I am the main author. I am the sole author of Chapter 2 "Content Providers in a Multi-piracy Ecosystem" and of Chapter 3 "Platform Investments in Two-sided Markets". Chapter 1 "Effects of Ad-blockers Adoption on Digital Piracy: a Blessing or a Curse?" is co-authored with Bipasa Datta. Chapter 4 "Superstars in Two-sided Markets: Exclusives or Not?" is co-authored with Elias Carroni and Shiva Shekhar.

This thesis has not previously been presented for an award at this, or any other, university. All sources are acknowledged as References.

A previous version of the research paper "Effects of Ad-blockers Adoption on Digital Piracy: a Blessing or a Curse?" was awarded the *Sir Alec Cairncross Prize* for the best paper presented by a young researcher at the Annual Conference of the Scottish Economic Society, in Perth (2017).

The research paper "Superstars in Two-sided Markets: Exclusives or Not?" was also nominated for the Antitrust Writing Awards 2019.

Introduction

Digital markets are increasingly becoming relevant for the global economy and sometime pervasive in citizens' life. In 2017, more than 2.2 billion of users were active on Facebook worldwide, 800 million of users were active monthly on Instagram, whereas approximately 117 million and 159 million subscribed to Netflix and Spotify. Facilitated by the diffusion of the Internet and by the access to fast-speed broadband, the digitisation and the advent of social network and applications have changed the way firms and consumers interact in the market. Although it created new opportunities for many, it also posed new challenges for regulators, policy-makers, and ultimately consumers. For instance, ordinary assessments of competition authorities such as the standard "market definition" may not apply in most current digital markets where consumer face no monetary price on their side (see e.g., Evans 2017, Wu 2018) and so new methods need to be identified. These challenges are also exacerbated by the new opportunities for firms to rely on big data, with potentially severe effects on individual privacy (see Acquisti et al. 2016 for a review), and by the development of Artificial Intelligence and machine learning which may foster collusive behaviour (see OECD 2017).

Against this background, this dissertation aims at shedding further light on the functioning of digital markets and is rooted on the following three pillars: the presence of cross-network externalities and the advent of multi-sided markets, the abrupt decline of marginal costs stemming from digitisation, and the role of data. This introduction is organised as follows: it first presents the three pillars and it then guides the reader to the contribution made by each of the four research papers comprehended in this dissertation.

Cross-network externalities and multi-sided markets. In the last two decades, many traditional and digital markets experienced the entry of platforms, that is, matchmakers, enabling interactions between different groups of users and generating more efficient transactions. In the operating system industry, users could now interact with developers and software producers via an operating system and enjoy the possibility to install and use different apps. In the media market and social networks (e.g., Facebook, Instagram, Twitter), advertisers were allowed to place their advertisements (ads, henceforth) on platforms to catch their customer base's eyeballs. In a similar manner, e-commerce platforms (e.g., Amazon and Ebay) have become proper marketplaces not only selling their own products to potential consumers but also offering access to competing merchants in the same way booking portals have rendered possible the match between travellers and hosts. Lending platforms have become popular as well with an increasing number of transactions between borrowers and lenders. In the jargon, markets have become multi-sided (see e.g. the literature pioneered by Caillaud & Jullien 2003, Rochet & Tirole 2003, Armstrong 2006) and users started paying attention not only on their peers' behaviour (*network externality*) but also on the number of users belonging to the group they interact with. The valuable interactions generated by platforms between different groups of users are therefore called *cross-network externalities*. For instance, advertisers care about how many active users are daily available on social media and enjoy a positive interaction when placing an ad, whereas users are more likely to prefer those booking portals offering different options. These interactions may also be less appealing for some users. Advertisements, for instance, are generally reported as a nuisance by users (Anderson & Coate 2005) who may feel disturbed when too many ads are displayed when watching a movie or navigating the Web.

A key feature of these multi-sided business models is that, by working as middlemen between different types of users, platforms can manage their pricing strategies and internalise the cross-network externalities they generate. Due to these externalities, negative prices are not necessarily a sign of predation. In practice, one group of users can get free access or subsidised if belonging to the most elastic side of market, while letting the platform charge more the group of users usually belonging the least elastic side of the market. By the same token, the cost of joining a platform does not necessarily require a monetary compensation: non-monetary prices can take the form of data collection and attention capture. It is a matter of fact that often users face no virtual price on their side but their presence help selling information to other groups of users (e.g., advertisers). This is, for instance, the business model behind successful platforms like Facebook and Instagram, which work as "attention brokers" by making available user attention to advertisers. To have a figure of this remunerative phenomenon, in 2017, digital advertisements alone contributed to the digital economy with US 85 billion and these numbers are likely to increase in the future.[1]

Near-zero marginal costs. A second feature that characterises digital markets is represented by the cost of serving several users. According to Waldfogel (2017), digitisation of content has been associated with declining marginal costs and an increase in the number of content available. To produce a content, platforms (or generally firms) experience some initial costs. Once produced, content can be released to several channels and to different users at *near-zero marginal costs*. In the SVOD (streaming video-on-demand) industry, platforms like Netflix face no extra-cost when an extra subscriber decides to watch a content. What matters are those costs associated with running the infrastructure, maintenance, production and contractual costs (e.g., intellectual property rights) as well as marketing. In the music-on-demand industry, royalties are a form of marginal cost paid any time a content is listened to artists and record labels.[2] As matter of fact, these costs are much lower than those associated with the production, promotion and diffusion of more traditional supports such as CDs and DVDs. Yet, while helping these platforms and content providers to emerge, near-zero marginal costs have also a dark side. As they render reproduction and content sharing much easier, they have facilitated the infringement of intellectual property rights and the emergence of the commercial side of digital piracy adding thus *insult to injury* to the copyright holders.

The role of data. Data are often described as the new oil: they are both extremely valuable and need to be refined before use. These, however, are somewhat different:

[1] See `Adage, December, 20 2017` `http://adage.com/article/digital/iab-record-breaking-year-digital-ad-revenue/311712/`.

[2] The royalty rate changes between platforms. See e.g., `DigitalMusicNews, November, 2 2017:` `https://www.digitalmusicnews.com/2017/11/02/spotify-music-kickup-royalties/`.

data are inherently non-rival (Varian 2018) inasmuch their use does not inhibit other people and companies' use. The increasing number of interactions arising online and the possibility to store larger amount of data at a very low cost have made data collection extremely lucrative. Not only data brokers, which are specialised in this industry, but many other digital companies collect, integrate, manage and monetise data in several ways. Data, for instance, can be used to estimate the willingness to pay of consumers and engage in price discrimination. Platforms can use data to provide more information about their users to advertisers and ease micro-targeting and lower the probability of wasted ads. For instance, in 2017, Google launched a joint project with Mastercard to perfectly link ad viewings and exact online and offline purchases boosting advertisers' revenues.[3] Big data can also induce algorithm learning and contribute to abatement of costs (Belleflamme & Vergote 2018). Having a list of potential customers is also important for marketing purposes and risk mitigation. For instance, content providers can use data about subscribers' preferences to produce new content and identify areas for future investments. Insurance companies can use data on their potential customers to detect potential frauds. Indeed, data can play an important role for firms' strategies. Empirical analyses have, for instance, shown that data-driven decision-making can generate up to 5-6% more profits than expected using similar investments and technologies (Brynjolfsson et al. 2011), whereas price discrimination based on personal browsing data could raise profits by as much as 12.2% (Shiller et al. 2014). On the consumer side, the risk for consumers are not easily identifiable. On the one hand, consumers can benefit from more competition on the market, information sharing and tailored products. On the other hand, especially vulnerable consumers may find themselves in a situation of imperfect and asymmetric information and eventually be penalised when privacy is not enforced (Acquisti et al. 2016).

The above three pillars guide the development of this thesis, which is structured as a collection of four independent yet interrelated research papers, all dealing with digital markets. The four papers follow two main lines. First, I deal with content provision and a new type of digital piracy exploiting the multi-sidedness of the media market and

[3]See `Bloomberg, August, 30 2018`: https://www.bloomberg.com/news/articles/2018-08-30/google-and-mastercard-cut-a-secret-ad-deal-to-track-retail-sales

near-zero marginal costs to study market- and policy-based tools to enforce intellectual property (IP) protection. Second, I study the role played by cross-network externalities in stimulating platforms' investments and data monetisation, and in characterising the contractual relationship between platforms and content providers when some of them are pivotal.

As argued, the digitisation of contents has favoured the entry of new companies working as gatekeepers and has contributed to increasing the number of products that users can access to; meanwhile, the presence of near-zero marginal costs has fostered a new type of digital piracy via peer-to-peer (sharing) networks and for-profits content providers (also called cyberlockers). Something like five million of British use pirated services to watch movies and tv-series[4]. Chapter 1 and 2 delve into the problem of spreading digital piracy in the Web with a new lens by tackling the commercial dimension enabled by pirated services. While in the past, digital piracy was relegated to a user activity, it has now turned into a business model which exploits the demand for free content by displaying somewhat invasive ads to users. Because the price is perceived as a more salient attribute than ads, pirate content providers make freely available content in violation of intellectual property rights and earn illicit money by catering user attention to advertisers. As a result, a double problem emerges for the society: besides infringing copyright protection and displacing the demand (and eventually revenues) of the copyright holder, pirate providers make illicit revenues and cause concerns for national institutions.

In the last decades, scholars and practitioners have assessed the role of digital piracy from several perspectives. In early 2000s, users could rip CDs and DVDs and download music through sites such as Napster. Many scholars have tried to understand the best strategies for content providers and firms to deter, block or even accommodate user-generated piracy (e.g.,, Belleflamme 2003, Bae & Choi 2006). Others instead tried to understand whether digital piracy could ultimately benefit the society by ensuring a competitive threat to the otherwise only available monopolist (e.g., Yoon 2002) and the potential benefit of experimentation for the copyright holder (Peitz & Waelbroeck 2006*b*) and, eventually, artists (Piolatto & Schuett 2012). The commercial side of digital piracy

[4]See `Yougov, April, 20 2017` : https://yougov.co.uk/news/2017/04/20/almost-five-million-britons-use-illegal-tv-streami/

and the new challenges posed by streaming services have not undergone a throughout examination, while increasingly worried international authorities (e.g., European Union, Canada) started to introduce "follow-the-money" policies aimed at tackling this problem and avoiding money going to copyright infringers.[5]

Within this framework, the chapter *"Effects of ad-blockers adoption on digital piracy: a blessing or a curse?"* develops further the current literature by studying a different way to potentially crack down commercial digital piracy and its eventual unintended effects. By considering the streaming video-on-demand market, Chapter 1 studies competition between the legal copyright holder (such as a Netflix) and a pirate content provider illicitly offering the content for free but exposing its users to rather invasive ads.

The paper contributes to the understanding of the strategic interactions arising in this peculiar market and the consequences of user adoption of ad-blocking technologies aimed at (partially or fully) stopping ads. The adoption of ad-blocking technologies has gone up increasingly in recent years and so have the losses for ad-based companies. However, in a market where ads finance illicit revenues, the effect of ad-blockers is a conundrum. On the one hand, if ads are stopped completely, pirate content providers may leave the market or continue to infringe copyright without gaining anything from their activity (e.g., old generation of digital piracy). This case would indirectly accomplish the intention of several authorities to curtail pirates' revenues. On the other hand, if ads are stopped only partially, users may find the piracy alternative more attractive displacing even further the copyright holder's demand. Hence, it would potentially increase pirate revenues and harm the legal provider. This trade-off is analysed in turn. In modelling the activity of the ad-blocker, which is assumed user-oriented, the paper makes also a contribution to the recent yet limited literature investigating the impact of these technologies (Kraemer & Wiewiorra 2016, Ray et al. 2017, Gritckevich et al. 2018). The paper studies a partially user-oriented ad-blocker, which besides consumer welfare, cares also to some extent about the sustainability of ad-sponsored companies. Depending on the above type, different results can prevail. For instance, a full blocking strategy may eventually drive the pirate provider out of the market without improving the condition of the copyright holder,

[5]The European Union published a "follow-the-money" Memorandum of Understanding to dissuade advertisers concerned from placing ads on websites offering pirated material and curtail the revenues of copyright infringers.

whereas partial blocking may spur a pro-piracy effect. An important result is that the adoption of ad-blocking technologies may represent a blessing or a curse for consumers, the copyright holder and the authorities. For instance, authorities need to evaluate carefully the use of this technology when setting up their anti-piracy policy. Moreover, the study shows that, depending on whether the legal copyright holder acts as a leader or a follower (to the pirate content provider), the ad-blocker technology can completely change the strategic relationship between the subscription price set by the legal content provider and the level of ads chosen by the pirate content provider. Indeed, ad-blocking technologies can alter the nature of competition existing in the market and, in doing so, they may create a negative spillover not only to those firms relying on ads (as the pirates are) but also to their competitors following a different business model (as a legal subscription-based firm).

Complementing the previous study, the second chapter, *"Content providers in a multi-piracy ecosystem"* explores the competitive nature of commercial digital piracy by considering a market where a subscription-based legal content provider competes against many pirate content providers. As marginal costs are near-zero, a content (e.g., a movie) can be stolen and reproduced by several (competing) content providers. This phenomenon is common in the streaming video-on-demand market, where users can find a movie either on the subscription-based network or on several pirate websites and entry barriers for copyright infringers are almost null. The economic theory on digital piracy so far lacked in understanding how a subscription-based content provider competes with several pirate alternatives and how the market interactions change when new pirate providers join or exit the market. In a recent paper, Danaher et al. (2018) have shown that the shut-down of a famous pirate website in Brazil, MegafilmesHD, led only some users to subscribe to legal providers while other users simply changed pirate websites.

The paper digs into this issue by providing a comprehensive analysis of the strategic interactions between a subscription-based firm and several pirates. A key feature of a multi-pirate ecosystem, as the one this study introduces, consists on having decreasing searching costs for a user to get on a pirate content provider. When a user looks for an illegal content, she needs to use a search engine to find it. In most cases, this is a labourious activity, which requires effort and time. It follows that the greater the number of pirate alternatives available on the Internet, the lesser will be the time spent by an average user to access it. As search engines often base their algorithm on the basis of the most searched or available content, for a user it gets easier to find the correct link to the pirate websites when all alternatives are shown in the first pages rather than in the following ones. Against this backdrop, the paper finds that when a new pirate provider joins the market, it exerts a competitive pressure on the legal provider who, in turn, reduces its price. This strategic effect generates a non-monotonic effect on its user demand: it spurs a market expansion of the legal provider as long as the entry of new pirates is not excessive (e.g., hump-shaped relation). Hence, there exist a certain number of pirates that make the legal provider endogenously covering the entire market and curtailing pirates' profits. By contrast, in a large ecosystem, with several pirate providers already in the market, the entry of the marginal pirate provider is such that the reduction of searching costs outweighs the legal content provider's price reduction. This ultimately makes any pirate alternative more attractive.

The study also provides an answer to whether there is an optimal number of pirate content providers that the copyright holder and the government would prefer to tolerate. Previous studies, for instance, have shown that the social welfare can increase with digital piracy (Yoon 2002, Minniti & Vergari 2010) but also benefit the legal copyright holder (Inceoglu 2015, Kim et al. 2018) under some circumstances. These studies however do not consider an entire piracy ecosystem where several different providers operate and the eventual optimal number of pirate competitors. This is relevant as, ideally, a government would prefer not to have any pirate provider operating and crack down completely digital piracy. In practice, closing websites may be lengthy and costly: in Canada, for instance, it could take up to 700 days and the litigation cost could hit as much as \$ 338,000.[6] In tackling the issue of what should be an optimal number of pirate providers (when the number cannot be zero), the study finds that the legal provider and the government would unambiguously prefer to tolerate some degree of (commercial) digital piracy rather than having a certain number of pirate providers left in the market without demand. However,

[6] See `Truth on the Market, June, 11 2018.` https://truthonthemarket.com/2018/06/11/canadian-site-blocking-proposal-is-a-good-experiment-in-controlling-piracy/.

a socially optimal level maximising the consumer surplus and the profits in the legal economy would neither be compatible with the number of pirate providers that the legal platform would prefer to let enter the market (i.e., the minimum) nor with the number of pirate providers eradicating all illicit revenues from ads.

The next two research papers follow a different approach and model explicitly the multi-sided dimension of the market and the crucial role played by cross-network externalities, which have been previously introduced. The third chapter, *"Platform investments in two-sided markets"* explores the incentive for platforms to undertake investments and the effect on prices depending on whether (some) users singlehome or multihome. Digital markets are characterised by constant investments directed at improving the user experience while collecting crucial data for the platform. Value-increasing investments are common in digital markets (e.g. better user experience, machine learning, wider compatibility and greater optimization of resources). This is, for instance, what "The Verge" recently suggested about the Amazon Echo's new skill "Look". The tech magazine titled "*Amazon's Echo Look does more for Amazon than it does for your style*" as the platform intended at providing users with comments on personal fashion styles while also gathering important personal data and preferences.[7] Despite the continuous flow of investments in this industry, most of the attention of the literature in Industrial Organisation has been posed on investments undertaken on one side of the market (Belleflamme & Peitz 2010, Lin et al. 2011) or on some specific industries (Njoroge et al. 2013, Casadesus-Masanell & Llanes 2015, Thomes 2015, Li & Zhang 2016) with a little emphasis on how investments impact on homing strategies. This is relevant as investments creating value to users can be heterogeneous depending on how users perceive the interactions with the other side. In filling this gap, the paper studies how investment generating a value for at least one side of the market, where buyers and sellers operate, are passed onto prices and how different network configurations where buyers and sellers can singlehome or multihome can finally affect the incentive to invest. The paper considers the competition between two ex-ante symmetric platforms in two network configurations, considering the case where *(i)* all users singlehome, regardless of the side, and the *(ii)* competitive bottleneck, where

[7] `The Verge, July, 6 2017.` https://www.theverge.com/2017/7/6/15924120/amazon-echo-look-review-camera-clothes-style

buyers singlehome and (some) sellers multihome.

The paper shows that when sellers and buyers singlehome, the platforms fail to pass-through their investments into prices and an *Investment-Prisoner Dilemma* may arise unless the platforms are able to appropriate a sufficiently large component of the investment: platforms engage in an investment war but they finally end up lowering their profits. This happens thus when platforms are not able to monetise their investments through stock consolidation, eventual algorithmic learning, feedback. Furthermore, the paper shows that when (some) sellers can multihome, three results emerge relative to the case with singlehoming sellers. First, because of investments, more sellers decide to multihome relative to the pre-investment case. Second, because more sellers multihome, the platforms can re-organise the pricing structure and increase the price on the multihoming side of the market and a lower price on the singlehoming side of the market. Hence, investments partly restore the bottleneck effect that exists in many but not all competitive bottleneck environments whereby the multihoming side of the market is charged the most (see e.g., Armstrong 2006, Belleflamme & Peitz 2018). In other words, investments impacting on the seller side determine a price change from the buyers to the sellers resembling the pricing strategy set by a monopolist platform. Finally, the paper shows that the incentives to invest are higher for the platforms when sellers multihome and that the *Investment-Prisoner Dilemma* can be averted with a smallest return of investments. For instance, a direct return in the financial market from stock consolidation driven by the investment decision is lower than the one required when all users single-home.

Last but not least, the fourth chapter, *"Superstars in two-sided markets: exclusives or not?"*, studies the usual trade-off between exclusivity and non-exclusivity from the perspective of an agent with market power and a large capture over the other side of the market (i.e., Superstar) and its welfare impact. So far, the literature has provided only a limited understanding of the incentives for a firms and platforms to sign exclusive contracts with producers or providers in the upstream market (e.g., Armstrong & Wright 2007, Hagiu & Lee 2011, Chen & Fu 2017, Weeds 2016, Belleflamme & Peitz 2018), overlooking the role played by agents with market power. Specifically, some aspects have been neglected. First, users belonging to a group are not necessarily homogeneous. Some users may be more important than others. For instance, Superstar artists can be more attracting than niche artists and this is also true in other digital and traditional markets, such as the market for sport broadcasting (Juventus vs. low-ranked team), shopping malls (luxury brands vs. other shops), clouding apps (Red Hat vs. other developers), etc.. Indeed, the study introduces heterogeneity in one side of the market by differentiating, according to the value they cater on the opposite side of the market, between a Superstar and a number of small firms. Second, it is usually assumed that platforms have the power to set prices and offer contracts to users (Armstrong & Wright 2007, Hagiu & Lee 2011). In the paper, the platform(s) can set a price to consumers and, eventually, small firms, whereas the Superstar has the power to make a take-it-or-leave-it offer to one or both platforms. Third, some of the closest papers (Chen & Fu 2017, Weeds 2016) do not deal with a two-sided market framework where different types of users are present and enjoy their two-way interaction. Indeed, they overlooked an important aspect that this paper presents. Specifically, it shows that a Superstar would find it optimal to sign an exclusive contract with one platform whenever the competition in the downstream market for consumers is sufficiently intense. This result is very close to that of Weeds (2016) but this choice is explained by the presence of a novel effect that the paper presents, the ripple effect: a domino effect triggered by an exclusive contract with the Superstar.

The paper indicates that the presence of a pivotal agent (i.e., the Superstar) on one platform attracts more consumers on that platform and this demand effect is stronger the lower the differentiation between the two platforms. Because many consumers move from the rival platform to the one hosting the Superstar, also small firms respond to this market asymmetry. Hence, more firms enter the market and more (small) firms join exclusively the platform with the Superstar rather than multi-homing (as in the case with a Superstar non-exclusive). The paper also discusses the desirability of exclusive contracts with an agent with market power. Contrast to the conventional wisdom, these can be regarded are a manifestation of the competitive forces in the industry as in many cases these are the first-best in the industry. As a result, regulatory authorities (such as the Chinese watchdog SAPPRFT) and policymakers should carefully evaluate any intervention in the market as in most cases exclusivity can represent a first-best for the industry and ban on exclusivity may harm consumers and content providers.

Finally, in the conclusion, I provide a critical discussion of the main findings and a research agenda for the near future.

CHAPTER 1

Effects of ad-blockers adoption on digital piracy: a blessing or a curse?

1.1 Introduction

The growing digitisation of content has been historically pervaded by the presence of illicit behaviours. In the past decades, software companies, major record labels, and blockbuster producers were mainly concerned with consumers ripping CDs and DVDs given their low reproduction costs. With the Internet and the advent of peer-to-peer networks (such as Napster) piracy went online. Users could now share their files with no or little copyright concern.[1] Whilst these forms of user-driven piracy still exist and have been extensively treated by the economic literature (Belleflamme & Peitz 2012, 2014, Gomes et al. 2015, Belleflamme 2016), digital piracy has taken a big change becoming a lucrative business based on revenues from advertisements. In recent years, a large number of platforms, also called cyberlockers, have conquered the web, attracted by surprisingly high profit margins and profitable advertising. While their initial purpose was not to host content in violation of intellectual protection, something about 87.4% of the contents there hosted ended up infringing copyright protection.[2] The most famous example was Megavideo,

[1]See Casadesus-Masanell & Hervas-Drane (2010*a*,*b*) and Klumpp (2014) for the analysis of the evolution of peer-to-peer networks.

[2]See The Digital Citizens Alliance's Report on Cyberlockers, `http://www.digitalcitizensalliances.org/clientuploads/directory/Reports/cyberlockdoor.pdf`.

who made million from advertisements (ads) before its shut down by the US in 2012.[3]

These providers mediate the interaction between advertisers and users by illicitly offering popular content (e.g. movies, shows, tv-series) normally available on legal subscription-based websites. By exploiting the two-sidedness of the media market, they maximise user viewings to boost their advertising revenues and strategically compete with legal providers. In practice, they imitate the functioning of SVOD providers by offering online streaming at a lower quality and with invasive ads which have a little informative content.[4] As the content offered by these websites violates intellectual property rights, it creates a self-selection of advertisers: those advertisers usually concerned with brand safety and which may be damaged from being associated with illicit content tend not to show their ads on these websites, while malware and invasive ads are more likely to be displayed.

In recent years, consumers have found new ways to protect themselves from such nuisance advertising: they can install ad-blocking technologies, that is, plug-ins or software that can potentially block such unwanted ads.[5] The adoption of ad-blockers has gone through a rapid growth in Western countries in recent years, with a growth rate of 30% in 2016 and 615 million devices blocking ads. Inevitably, it poses a threat to the survival of sponsored-business models as it implies a loss of revenue for such firms, e.g. the estimated loss in 2016 accounted for 20 billion of dollars.

The effect of ad-blockers on piracy, however, may not be so straightforward. By operating on the level of ads, an ad-blocker may accomplish the public authorities goal of reducing the money going to the pirate companies while being potentially beneficial for consumers as reducing annoyance costs and the exposition to malware and virus. On the other hand, it may increase the demand for piracy at the expenses of the copyright holder. Motivated by above facts, we develop a game-theoretic model of the price-ads competition between a legal and a pirate firm in the VOD market to examine the impact of

[3]United States vs. Kim Dotcom. Indictment, No. 1:12CR3 (US District Court for the Eastern District of Virginia 2012).

[4]As recently reported by Rafique et al. (2016) about websites offering illicit streaming of sport events, these were "malicious in nature, offering malware (zero-day in one case), showing fake law enforcement messages to collect purported fines, and luring users to install malicious browser extensions" (p. 2).

[5]For instance, security concerns related to the presence of virus and malware represented the main reason to adopt ad-blockers along with unpleasant interruptions. `See e.g. The PageFair, ''The state of the blocked web''`: `https://pagefair.com/blog/2017/adblockreport/`. (Accessed 2017).

ad-blocker adoption on digital piracy. Whilst there is a small number of papers that study the competition between legal and pirate firms (Rasch & Wenzel 2013, Chang & Walter 2015, Madio 2018), most of the literature in that field focuses on deterrence strategies and user-generated piracy. Instead, ad-avoidance has been considered passively (Tåg 2009, Anderson & Gans 2011, Stühmeier & Wenzel 2011, Johnson 2013), with only few papers studying the rules behind ad-blocking technologies (see e.g. Kraemer & Wiewiorra 2016, Ray et al. 2017, Gritckevich et al. 2018).

We set up a simple model of platform competition with vertical and horizontal product differentiation. We consider two markets: (i) a *pre-adoption market*, where a pure price-ads competition takes place between the legal and pirate providers, and (ii) a *post-adoption market*, where the legal firm sets its subscription price and the ad-blocker acts as a gatekeeper controlling the ad flow shown to consumers. In our model, we consider the early generation of ad-blockers, i.e. an user-generated ad-blocker (e.g. uBlock Origin) concerned with the surplus accruing to consumers and protect them from data exploitation, profiling, and invasive advertisements. We do not consider for-profits ad-blockers, such as Ad-Block Plus owned by the German company Eyeo GmBH, which make revenues from whitelisting a fraction of ads for large content providers.[6] The reason is that for-profits ad-blockers normally whitelist those ads respecting some quality standards, which are extremely unlikely to be satisfied on pirate websites.[7] Indeed, a profit-maximising ad-blocker would not be able to whitelist any ad. A *partially user-oriented* ad-blocker suits our economic environment well. The ad-blocker maximises the user surplus, under the condition that profits of the ad-based company (i.e., the pirate website) are non-negative. In other words, while caring mainly about consumers, this type of ad-blocker also pays attention (to some extent) to survival of ad-based firms (as the pirate ones). Furthermore, for each of the above cases, we consider two scenarios where (i) the legal firm leads the game acting as a Stackelberg leader; and where (ii) the pirate firm (or the ad-blocker replacing the pirate firm) leads the game by deciding first on the number of

[6]This approach is instead followed by Kraemer & Wiewiorra (2016), Ray et al. (2017), Gritckevich et al. (2018).

[7]For example, Ad-block Plus, one of the most popular ad-blockers, created the "Acceptable Ads Initiative" to allow advertisers and publishers who have agreed to make ads that abide by user-generated criteria to be whitelisted". The "Acceptable Ads initiative" defines unacceptable pop-ups, pop-unders, rich media ads, pre-roll video ads and other invasive forms, which are all common on pirate content providers. See `https://adblockplus.org/acceptable-ads`.

ads. Considering a sequential structure of the game helps one to understand the strategies followed by the legal content provider, how it reacts or anticipates changes in the decisions undertaken by its competitor, and indeed whether it prefers to be the leader or the follower. For instance, Netflix follows piracy websites when purchasing series[8].

We show that, whilst absent ad-blockers, price and advertisements are strategic complements, the mere adoption of an ad-blocker changes the nature of strategic interactions between the legal and pirate providers (now replaced by ad-blockers) on the basis of who moves first. In the presence of a *partially* user-oriented ad-blocker, when the legal provider moves first it treats price and ads as strategic substitutes: anticipating the reduction in the level of ads shown to pirate customers, the legal firm raises its subscription fees in an attempt to make up for the loss in its demand. This result is due to the fact that, in contrast to the pirate firm who operates on the *elastic* portion of the pirate demand, the ad-blockers chooses the level of ads on the *inelastic portion*. Thus, we find that the legal firm's demand is adversely affected by the presence of ad-blockers - in this sense, the ad-blockers generate a *pro-piracy effect*! In contrast, in the presence of a partially user-oriented ad-blocker, when the legal provider moves second, it treats price and ads as strategic complements (as in the pre-adoption game). In this case, not only that the legal firm enjoys a second movers' advantage in the pre-adoption game, it now charges an even lowered price compared to the pre-adoption scenario, a strategy which is clearly in contrast to the case where it moves as the leader. Regardless of who moves first, the ad-blocker always reduces the pirate's profits down to zero, thus cracking down the commercial dimension of digital piracy.

A rather important result we highlight is that ad-blocking technologies not only impact on the number of ads (and aggregate revenues) of those websites following a ad-based business model but, because of competitive forces, they generate a cascade effect on other firms following a different business model (subscription based). This should clearly taken into account by public authorities.

We further extend our analysis to consider a *fully user-oriented ad-blocker*, which is concerned *only* with the user surplus. For instance, uBlock Origin, a fast-growing ad-blocker, emerged as "pure" alternative for the discontents of the "Acceptable Ads"

[8] See BBC `http://www.bbc.co.uk/news/technology-24108673`.

programme[9], is an open-source software refusing donations to avoid the alteration of its user-oriented goals. We show that this type of ad-blocker pursues a *full blocking strategy* where all ads are blocked. Given that ads are the sources of revenues for the pirate firm, this can make pirate firm exit the market depending upon whether it incurs any fixed costs of production or not. Absent any fixed costs, the pirate firm can passively exist in the market without earning any profits and providing an imperfect alternative of the legal product free of charge and ads. In this case, the ad-blocking software stops piracy from having a commercial nature, redistributing their (profit) surplus from the pirate provider to consumers. The market in this case remains fully covered and the mere adoption of ad-blockers forces the legal provider to reduce its price to mitigate the *pro-piracy effect*. As demand and prices are impacted adversely, this is a curse for the copyright holder! More interestingly, we find that the presence of fixed costs for the pirate firm bestows a monopoly position to the legal firm, there remains a significant portion of unsatisfied consumers who refuse to buy the legal product. Ad-blocker adoption is certainly a curse for the pirate provider and for consumers who no longer have an alternative

The rest of the paper is structured as follows. Section 1.2 provides a review of previous analyses; Section 1.3 presents our model. Section 1.4 considers the game where the legal provider is the Stackelberg leader where we separately analyse equilibrium outcomes for the pre-adoption market and the post-adoption market. In section 1.5, we reverse the order of the move where the legal provider acts as the Stackelberg follower and analyse equilibrium outcomes separately for the pre-adoption and post-adoption markets as in section 1.4. In section 1.6 we extend the model to the case of fully user-oriented ad-blocker. Finally, Section 1.7 concludes the paper. Except for the small proofs, all proofs are relegated to the appendix which also contains derivations of expressions for consumers' surplus for sections 1.4 and 1.5 .

[9]See "The state of ad-blocking" `http://www.upa.it/static/upload/com/comscoreesourcepoint/comscoreesourcepoint.pdf`.

1.2 Related Literature

Digital piracy has been extensively analysed in the economic literature. The early generation of studies dealt mainly with deterrence strategies undertaken by firms and authorities and the effect of digital piracy on the social welfare. For instance, in Belleflamme (2003), a monopolist had to decide whether to block, deter or accommodate user piracy, while in Yoon (2002), digital piracy can increase the social welfare depending on whether there is a problem of underproduction or underutilisation. Among the effects of digital piracy Bae & Choi (2006) show that, by introducing competition in the market, digital piracy can lead to a market expansion for a legal company because of the strategic effect on its price. More recently, Halmenschlager & Waelbroeck (2014) study whether and to what extent online free music streaming services (e.g. Spotify, Deezer) can effectively fight digital piracy. They find that "Freemium" strategies, offering a menu of ad-sponsored music and ad-free music, can help fighting digital piracy and substitute strong copyright protections whenever the number of restrictions imposed on the ad-based service is limited.

The present paper is very close to the literature on competition and digital piracy. Inceoglu (2015) shows that, under some conditions, piracy can benefit an incumbent firm, preventing the entry of other competitors. Herings et al. (2010) consider the case of a physical product competing against a digitalised version available in a peer-to-peer network, in presence of taste heterogeneity, network effects and endogenous pricing strategies. Minniti & Vergari (2010) observe that the effect of digital piracy on the social welfare depends on the market coverage of the peer-to-peer network. On a somewhat different note, in a two-sided market framework, Rasch & Wenzel (2013) study the effect of piracy in a vertically differentiated market with the illegal and legal software, developers and consumers. They find that the effect of piracy on profits and prices can be ambiguous and generate a misalignment of interests depending on whether providers are compatible or not.

One clear limitation of most previous papers is not considering the strategic interactions arising between legal and pirate providers. The literature is limited to a few contributions. Chang & Walter (2015) consider the competition between a legal firm and a for-profits peer-to-peer network making revenues from ads. Differently from ours, they restrict the attention to the pirate choice of quality rather than on the strategic role of ads. They show that, under certain conditions, the legal firm may not be negatively impacted by digital piracy. In a closely related work, Madio (2018) introduces a price-ads competition between a subscription-based legal (dominant) provider and a number of ad-based pirate content providers. The author shows that the legal provider can monopolise the market in a sufficiently large pirate ecosystem.

A number of studies look at digital piracy from an empirical perspective. Danaher et al. (2010) use a quasi-experiment to assess the effect of the NBC's decisions to remove its content from iTunes finding an increase of 11.4% in the illegal downloading of the company's content. Peukert et al. (2017) find that while box offices revenues for popular movies increase after the shut-down of Megaupload, these were impacted negatively for small and mid-range movies. Still on Megaupload, Danaher & Smith (2014) find that the US court order to close the popular cyberlocker led to an increase in digital revenues by 6.5-8.5%, while Danaher et al. (2018) explore the consumer traffic diversion resulting from anti-piracy measures in Brazil. In the latter study is shown that some users simply moved to other pirate websites, while only a small portion turned out subscribing to legal providers such as Netflix. Similarly, Poort et al. (2014) study the effect of The Hague Court's decision of blocking The Pirate Bay, a pirate website using the BitTorrent protocol, on the percentage of other unlicensed platforms. They find that downloading remained constant (a relapse effect), with no blocking effect.

This paper also relates to the recent theoretical and empirical literature on ad-avoidance and ad-blocking technology adoption. Tåg (2009) consider a monopolist deciding to offer an ad-free option to its users at a cost. When this happens, all consumers exposed to ads and not paying for the option experience an increase in the quantity of ads shown to them. As a result, this lowers the consumer surplus and increases profits of the media firm. These results are similar to Anderson & Gans (2011), who provide one of the first analyses on ad-blockers, showing that when consumers deviate from ads by purchasing an ad-blocker at a low price, the content provider reacts by increasing the level of ads for those users not adopting the technology, ultimately generating a circulation spiral and exploiting less ad-sensitive customers. Johnson (2013) model the strategic interaction between consumers and a firm who chooses the level of ads and the ad-avoidance strategy. He shows that, from a social perspective, consumers underutilise ad-blocking. Bounie et al. (2017) study the effect of ad-blockers on publishers' ads strategies. They demonstrate that the number of viewable ads needs to be lowered to deter the consumers' (costly) adoption of ad-blockers. These works, however, assume that ad-blockers are passive, blocking all ads. Only recently some authors started discussing an active role for ad-blocking technologies by considering their business model. Kraemer & Wiewiorra (2016) investigate the profitability of an ad-blocker technology and its welfare consequences by considering two cases: (i) strategic whitelisting, with the ad-blocker available for free to users and which makes revenues from content providers by strategically whitelisting some ads, and (ii) selling to software to the users with a perfect blocking. The authors show that whitelisting some ads increases profits but lower the total surplus relative to a full-blocking ad-blocker who is sold to the users. These two strategies resemble the two equilibrium (partial and full blocking) strategies arising from our analysis, with the difference of considering a user-oriented ad-blocker such as uBlock Origin and a market (digital piracy) where any whitelisting is inhibited by the invasiveness of the ads shown to consumers. Also Ray et al. (2017) study the pricing strategies undertaken by ad-blocker platforms to users and content providers allowing for whitelisting fees, free whitelisting and whitelisting rewards. The authors show that the pricing structure is crucial to enable interactions among users and advertisers and find that paying users for being exposed to ads can be socially optimal. Gritckevich et al. (2018) study the functioning of ad-blockers in a dynamic flavour, showing that the optimal business model depends on the economic value of ads and the heterogeneity of the customer base relative to their sensitivity to the ads. All these papers, however, only consider for-profits ad-blockers, neglecting the impact of commonly used user-generated tools and their strategic decision. In the only empirical paper investing ad-blocking instead, Shiller et al. (2018) assess the effect of ad-blocker adoption on websites' revenues. They find that ad-blocker adoption generates both reduction in revenues as a consequence of blocking ads and a significant reduction in the traffic (with a marginal effect of 0.67%), undermining investments and websites' quality in the long-run. Their conclusion that ad blocking poses substantial threats to the ad-supported web is in harmony with our theoretical results.

1.3 The Model

We consider a fully covered market for piracy competition with vertical and horizontal product differentiation (see also Herings et al. 2010 and Chang & Walter 2015). For the sake of simplicity, we treat the market as one-sided abstracting from network externalities.

1.3.1 Users' Utility

Let v denote the standalone utility of visiting the legal provider (henceforth LP) representing the quality of the information good shown on this provider; and let $v(1-a)$ denote the utility of visiting the pirate provider (henceforth PP) where the scalar a, $a \in (0, 1)$, captures the degradation costs as in Bae & Choi (2006), since the pirate version is an *imperfect* substitute of the original product. For $a \to 0$, the quality of the pirate version approaches the original content, whereas for $a \to 1$ the quality gets poorer. Degradation costs for the pirate providers (also called cyberlockers) can be associated with the absence of subtitles, inferior video-quality, as well as with the difficulties to find movies dubbed or subtitled in languages other than English (American or British).

The potential users are uniformly distributed along a line of unit length as in a traditional Hotelling-style spatial model, where x represents their 'location' along the line. The opportunity cost of 'travelling' is τ, the horizontal product differentiation parameter, and it is linear in distance. The providers are located at the extreme ends of the linear city: the legal provider is located at '0' and the pirate provider is located at '1'. To access the LP, users must pay a subscription price p, while the pirate service is offered free of charge. However, there are certain other frequently encountered non-monetary costs associated with the pirate alternative, e.g.:

1. *Search costs*: There are certain search and learning costs for getting to the (appropriate) pirate provider that we denote by the parameter s. The search cost arises as the link to the provider hosting the pirate content is provided by different (indeed

thousands of) third party websites which create a (sort of) catalogue of all available pirate movies.[10]

2. The *annoyance cost*: Annoyance costs arise due to too many advertisements ('ads" for short) being shown on the PP. Such advertisements, exhibiting porn, dating, betting and online poker websites or exposing users to malware and fake software, while make the business sustainable in the absence of subscription fees, are often invasive for users requiring them to constantly close pop-ups and pop-unders. This creates huge amount of annoyance for the users. To capture such increasing level of annoyance, we assume that the annoyance cost function $\Gamma(A)$ is a quadratic function of the number of ads (denoted by A) given as below:

$$\Gamma(A) = \gamma \frac{A^2}{2}, \quad \gamma > 0$$

where γ, the annoyance cost parameter, is assumed to be the same for the entire population.

When the market is fully covered, the utility of a user is:

$$U_i = \begin{cases} v - p - \tau x, & \text{if going to the LP} \\ v(1-a) - s - \gamma \frac{A^2}{2} - \tau(1-x), & \text{if going to the PP} \end{cases} \tag{1.1}$$

Contrary to Chang & Walter (2015), where the level of advertisements is exogenously given, in our model, the pirate provider can decide on the level of advertisements to be shown on its website without any physical constraint. This is because when cyberlockers stream the movie, users are often exposed to mid-roll ads interrupting their viewing, contrary to Chang & Walter's model where users are exposed to ads only for a limited time whilst downloading a movie. Likewise, such ads usually take the form of pop-ups and pop-unders, thus going beyond the dimension of the browser window.

[10]Third-party websites work as *middlemen* between the user and the illegal content. Sometimes a specific content is present in multiple illegal hosting networks and third party websites gather all these information, even by providing alternatives to link previously blocked by authorities. Thus, the user usually spends time (searching cost) to get to the pirate streaming hosting network. In our model, these are taken as exogenous.

1.3.2 Demand

The market share of each provider is determined by solving the following marginal consumer's problem:

$$v - p - \tau x = (1-a)v - s - \frac{\gamma A^2}{2} - \tau(1-x).$$

Hence, the demands are given by:

$$\text{LP:} \qquad D_{LP} \equiv x = \frac{1}{2} + \frac{av - p + s}{2\tau} + \frac{\gamma A^2}{4\tau} \tag{1.2}$$

$$\text{PP:} \qquad D_{PP} \equiv 1 - x = \frac{1}{2} - \frac{av - p + s}{2\tau} - \frac{\gamma A^2}{4\tau} \tag{1.3}$$

1.3.3 Providers and their payoff functions

The Legal Provider

The legal provider maximises profits by choosing p. We assume that whilst the marginal cost of producing the original information good is zero (e.g., Waldfogel 2017), there is a fixed cost F associated with its production e.g. producing the original content (e.g. Narcos in Netflix), buying temporary or exclusive property rights and such. Thus, the LP's payoff function is:

$$\Pi_{LP} = pD_{LP} - F \tag{1.4}$$

It is easily verified that the above profit function is continuous and concave and hence twice differentiatble.

The Pirate Provider

The pirate provider maximises profits through a sponsored-business model by deciding on the amount of advertisement A to be shown on the consumers' screen. The model works just like a standard price competition model, as the willingness-to-pay for movies of consumers is monetised via ads. The revenue per ad, r, is exogenous to the pirate firm, which has no bargaining power vis-á-vis the advertising industry.

We assume that the PP can incur some fixed costs K for running the pirate provider, $K \geq 0$, although its marginal costs (i.e. the cost of uploading the movie) are set to zero. The profit function of the PP then is given by:

$$\Pi_{PP} = (rA)D_{PP} - K \tag{1.5}$$

The Ad-Blocker

If ad-blocking is adopted, the ad-blocker (henceforth AB) decides on the "acceptable" level of advertisements to be shown on consumers' screen i.e the ad-blocker replaces the PP by making PP take a back-seat position.

We assume that the "partially user-oriented" AB maximises the consumer surplus under the condition that the pirate firm's profits be non-negative. This *constrained* optimisation problem allows us to verify whether partial blocking can be an equilibrium strategy and its consequences on the strategies pursued by the legal content provider. Moreover, this represents a realistic case as some ad-blockers do not always block all ads shown on the user screen. The justification of this assumption is that some ad-blockers can indeed be concerned with the effects of its adoption on ad-sponsored firms, which in our ecosystem are *only* represented by the pirate provider.[11]

It is important to note that the behaviour of the AB in our model is clearly different from that of a typical welfare-maximiser authority who normally considers the payoffs of *all* economic agents in the market[12]. Our approach is consistent with the growing concern arising in the advertisement-based industry regarding the economic consequences of ad-blockers adoption.

1.3.4 Types of Games

We consider a sequential-move game between the LP and the PP (or the AB). The LP moves first deciding on the price p, and the PP (or the AB) moves second choosing A

[11] In our model, we do not endogenously consider the quality of ads. We assume that advertisers and pirate providers cannot secure the 'whitelist' by simply paying a license fee to the AB.

[12] For example, López-Cuñat & Martínez-Sánchez (2015) use a semi-welfare function consisted in the consumer surplus and the profits of the legal firm. However, they show that similar results apply, even when the welfare function also contains the profit surplus of pirate firms.

after observing p. In Section 1.5, we relax this assumption and let the LP act as a follower and deciding on p, after observing the level of A.

Furthermore, for each type of the above game, we consider two different market scenarios:

- **The No-AB game.** This is the *pre-adoption market* where the LP and the PP engage in a sequential-move price-ads competition with no ad-blocker adoption. We call this the "**No ad-blocker adoption**" game (the "No-AB game", in short)
- **The AB game.** This is the *post-adoption market* where the ad-blocker replaces the PP game and takes charge on the level of ads to be shown on consumers' screen. We call this the "**Ad-blocker adoption**" game (or the 'AB game" in short).

The following assumption ensures that the results of our model are real-valued:

Assumption 1. $\tau > av + s$

Assumption 1, other than ensuring tractability of the model, implies that the mere presence of search costs and the quality degradation do not discourage consumers from watching movies on the pirate provider as the opportunity costs of attending a certain provider can be high.

1.4 The Legal Provider as a Leader

1.4.1 The No AB Game

In the second stage of the game, the PP maximises the following function:

$$\max_{A} \Pi_{PP} = \left\{ \frac{1}{2} - \frac{av + s - p}{2\tau} - \frac{\gamma A^2}{4\tau} \right\} rA - K, \tag{1.6}$$

and sets

$$A(p) = \left\{ \frac{2(\tau - (av + s) + p)}{3\gamma} \right\}^{1/2} \tag{1.7}$$

Note that we do not consider the negative value of A.[13]. Hence, we make the following observation:

[13] Assumption 1 suffices to ensure that $A(p)$ is real. Note that the second order condition (SOC) is negative, i.e. $-\frac{3Ar\gamma}{2\tau} < 0$. Hence, concavity requirements are satisfied.

Observation 1. *Absent ad-blocking technology, increases in the parameter values of γ (annoyance cost), s (the searching costs), and a (the quality degradation parameter), lower the (optimal) level of advertisements for a given p.*

The above results are immediate comparative static results. By the same token, we can provide the following lemma:

Lemma 1. *Advertisements and prices are strategic complements.*

The co-movement of prices and advertisements indicate that a reduction in the price charged by the LP forces the PP to strategically reduce the level of advertisements so as to combat the *business stealing* effect coming from the legal provider. Similarly, an increase in the LP's price switches demands from the LP to the PP prompting the PP to place more ads on its platform.[14]

In the first stage of the game, the leader solves the following problem (from equation (1.4)):

$$\begin{aligned} \max_{p} \Pi_{LP} &= \Big\{\frac{1}{2} + \frac{av+s-p}{2\tau} + \frac{\gamma A^2}{4\tau}\Big\}p - F \\ \text{subject to} \quad A(p) &= \Big\{\frac{2(\tau-(av+s)+p)}{3\gamma}\Big\}^{1/2} \end{aligned} \tag{1.8}$$

As a result, the Nash value of p, denoted by p_l^* (where the subscript l denotes the leader) is derived:

$$p_l^* = \tau + \frac{s+av}{2}, \qquad A_f^*|_{noAB} = \Big\{\frac{4\tau-(av+s)}{3\gamma}\Big\}^{1/2}, \tag{1.9}$$

and A_f is determined by appropriate substitution. Note that the subscript 'f' denotes the follower.

Thus the optimal price charged by the LP equals the Hotelling transportation cost (opportunity cost to join the platform) plus a mark-up that is directly proportional to the magnitude of the quality degradation parameter and the searching cost. One implication of this result is that the official anti-piracy measures that attempt to block illegal

[14] Lemma 1 is no longer be valid for analyzing the behavior of the LP (as a Stackelberg leader) when ad-blockers are introduced! See the subsequent analysis.

websites by raising searching costs, actually enable the LP to further increase its price![15] Furthermore, as $a \to 1$ (other things being equal) making pirate movies increasingly poor substitutes of the legal version, the LP exerts its market power to the fullest extent charging the highest possible price: $\lim_{a\to 1} p = \tau + \frac{s+v}{2}$. On the other hand, for $a \to 0$, when both versions of the movie are almost equal qualitatively, the LP is forced to lower its price as the competition gets fierce i.e. $\lim_{a\to 0} p = \tau + \frac{s}{2}$.

Given the optimal price and number of ads, equilibrium demand and profits are as follows

$$D^*_{LP} = \frac{1}{3} + \frac{av+s}{6\tau}, \qquad D^*_{PP} = 1 - D_L = \frac{2}{3} - \frac{av+s}{6\tau} \tag{1.10}$$

$$\Pi^*_{LP} = \frac{(av+s+2\tau)^2}{12\tau} - F, \qquad \Pi^*_{PP} = \frac{2}{3} - \frac{r\Big(4\tau - (av+s)\Big)A^*}{2\tau} - K$$

As in a traditional leader-follower price game, moving first penalises the LP in terms of demand, *i.e.,* $D^*_{LP} < D^*_{PP}$ since

$$D^*_{LP} - D^*_{PP} = \frac{1}{3} + \frac{av+s}{6\tau} - \left(\frac{2}{3} - \frac{av+s}{6\tau}\right) = -\frac{\tau - (av+s)}{3t} < 0 \tag{1.11}$$

as the numerator is positive by Assumption 1.

1.4.2 The AB Game

Consider the scenario where consumers switch on ad-blockers. In the real-world, whilst ad-blockers can block ads (fully or partially), they first need to be downloaded and enabled by users. In our model, we assume *that all consumers* (*who are taken to be homogeneous with respect to all relevant parameter values*) *always switch on the AB whenever this*

[15]Despite the fact that we consider exogenous searching cost, these can be influenced by anti-piracy policies operated by governments. Enforcing the removal of some contents (making the magnet link no longer available) without fully blocking the access to the pirate platform is common in the presence of copyright infringements. In these cases, the pirate content is temporarily removed and uploaded elsewhere, with a different magnet link. However, third party websites linking to the content might not promptly update their pages to the new hosting link, increasing, indeed, the searching time for the users. In some other cases, when the legal hosting platform does not remove illicit materials, some broadband companies might inhibit the access to their customers, inducing pirate users to search for other streaming sources. e.g. Sky Broadband in the UK blocked the access to Rojadirecta (a platform streaming football matches), PopcornTime (providing access to tv-series) and many other websites under the order of the High Court. See: `https://www.sky.com/help/articles/websites-blocked-under-order-of-the-high-court`.

option is available.[16] *as long as the post-adoption level of ads is lower than that in the pre-adoption case.* However, this need not be so - see our discussion in the conclusion.[17]

With the adoption of an AB, the role of the pirate firm is minimised: it is now the AB who is solely responsible for determining the number of ads to be filtered through the website. This makes the PP a completely *passive* player.

The AB maximises aggregate consumer surplus with a non-negativity constraint on the ad-sponsored firm's (the PP in our model) profits:

$$\begin{aligned} \max_{A} \quad & CS = CS_{LP} + CS_{PP} \\ \text{subject to} \quad & \Pi_{PP} \geq 0 \end{aligned} \tag{1.12}$$

where the consumer surplus derived from the legal and pirate providers denoted respectively by CS_{LP} and CS_{PP} are[18]:

$$CS_{LP} = \int_{0}^{D_{LP}} \Big[v - p - \tau x \Big] dx; \qquad CS_{PP} = \int_{D_{LP}}^{1} \Big[v(1-a) - s - \frac{\gamma A^2}{2} - \tau(1-x) \Big] dx;$$

where D_{LP} is given by (1.2) and Π_{PP} by (1.6). The Lagrangean for this problem is

$$\mathcal{L} = CS + \lambda \Pi_{PP}$$

where $\lambda \geq 0$ is the Kuhn-Tucker multiplier. The necessary (K-T) conditions for a maximum are as follows:

$$\begin{aligned} & i) \ \frac{\partial \mathcal{L}}{\partial A} = \frac{\partial CS}{\partial A} + \lambda \frac{\partial \Pi_{PP}}{\partial A} \leq 0, \quad \text{for } A \geq 0 \\ & ii) \ \lambda \Pi_{PP} = 0, \qquad \qquad \qquad \quad \text{for } \lambda \geq 0 \\ & iii) \ \Pi_{PP} = D_{PP} r A - K = \Big\{ \frac{1}{2} - \frac{av + s - p}{2\tau} - \frac{\gamma A^2}{4\tau} \Big\} r A - K \geq 0 \end{aligned} \tag{1.13}$$

[16]Note that this decision does not imply consumers do not see any ads. This means how many ads they have to endure now depends on the AB's decision.

[17] In the piracy ecosystem, it is reasonable that a user with a certain level of familiarity about illegal strategies to find a movie would not find that many problems in installing a simple and often free of charge AB.

[18]See the Appendix for derivations of the consumer surplus.

Let the elasticity of the pirate demand D_{PP} with respect to A be η_A, i.e. $\eta_A = \frac{\partial D_{PP}/D_{PP}}{\partial A/A}$ and note that the sign of η_A is negative since $\partial D_{PP}/\partial A = -\frac{\gamma A}{2\tau}$. Results of the above maximisation problem vary depending on the value of the PP's fixed costs as analysed below.

Case (i) ***No Fixed Costs*** **(K=0)**. Suppose that the fixed costs for running the pirate provider are absent, i.e. $K = 0$. We then have:

Proposition 1. *Whenever* $K = 0$*, the AB always sets* $A^* = 0$ *thereby driving the pirate provider's payoff down to zero.*

Proof. *See Appendix.*

Absent any fixed costs, the AB always drives the pirate's payoff down to zero by following a *full-blocking* strategy. In this case, the PP exists only passively without making any positive profits.

Case (ii) ***Positive Fixed Costs*** **(K > 0)**. First, suppose the cyberlockers encounter positive fixed costs for running the platform. The following proposition not only shows that, in the presence of $K > 0$, the AB chooses a positive level of A, but more importantly, it highlights the optimal *advertising rule* employed by the AB in choosing a particular value of A^*.

Proposition 2. *When* $K > 0$*, AB sets A such that (i)* A^* *is strictly positive; (ii)* $\Pi_{PP} = 0$ *(the constraint binds); and (iii)* A^* *is chosen over the inelastic portion of the pirate demand curve* D_{PP}*.*

Proof. See Appendix.

Whilst the AB does allow some advertisements to go through on the pirate content provider, it chooses just enough to make the pirate payoff go down to zero whilst generating positive demand for the pirate provider (note D_{PP} must be positive in order to satisfy the non-negativity constraint on the pirate payoff). At the same time, the AB chooses $A^* > 0$ in such a way that any change in A is not outweighed by the corresponding change in demand. This is because the optimal level of $A^*|_{AB}$ is chosen within the interval $(0, A^*|_{noAB})$ where $A^*|_{noAB}$ is obtained where $|\eta_A|$ equals 1 (see Figure 1.1). We

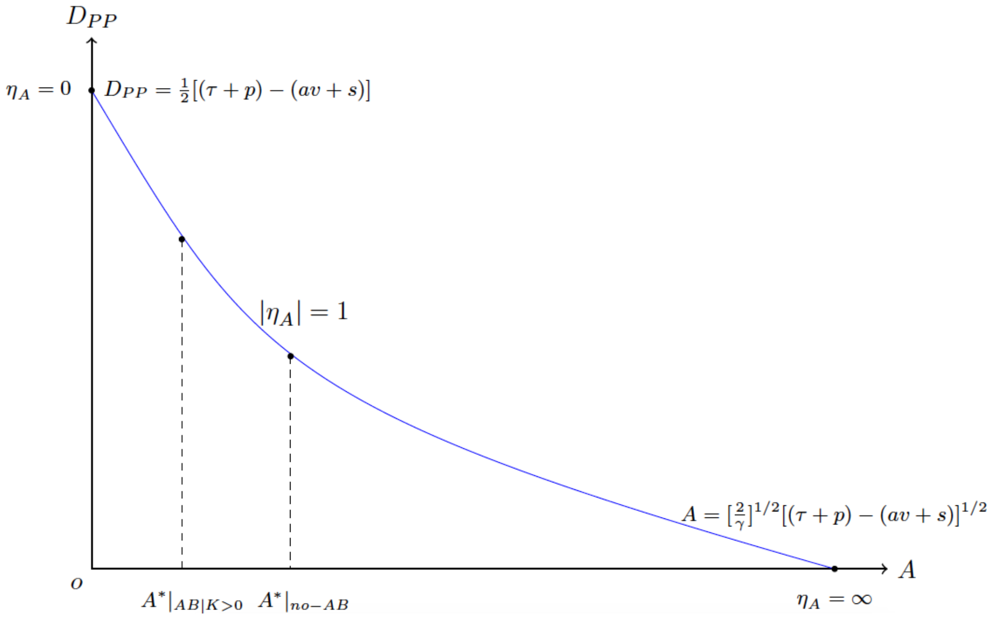


Figure 1.1: Pirate provider's demand

The figure shows how the the AB sets the level of ads over the inelastic portion of the pirate demand curve. Absent AB, the pirate provider instead sets a level of ads such that $|\eta_A| = 1$.

also note that the strategic complementarity between p and A is no longer relevant for the AB implying that the adoption of AB now changes the *nature* of competition between the legal and the pirate providers.

In the first period, given that the optimal level of advertising can differ starkly depending on fixed costs, it is useful to carry out the analysis of this stage for different values of $A^*_{f|AB}$ as below.

Case I: Full Blocking ($A^*_{AB} = 0$). This case prevails if $K = 0$. In the first stage, the LP maximises profits subject to $A^*_{AB} = 0$.

The following proposition shows even with a monopoly position, the LP may not be able to exercise its market power, and in fact ends up making lower profits than in the case when it competes with the PP directly as a follower.

Proposition 3. *When ad-blockers fully shut-down advertisements,*

(i) The price charged to LP users $p^*_l|_{AB} = \frac{\tau+(av+s)}{2}$ is even lower than the pre-adoption case; i.e.

$$p^*|_{AB(A^*=0)} = \frac{t}{2} + \frac{av+s}{2} < p^*|_{noAB} = \tau + \frac{av+s}{2}$$

(ii) The demand faced by the LP, and the profits earned by the LP are lower than that

in the pre-adoption case i.e.

$$D_{LP}|_{AB(A^*=0)} = \frac{1}{4} + \frac{av+s}{4t} < D_{LP}|_{noAB} = \frac{1}{3} + \frac{av+s}{6\tau}$$
$$\Pi^*_{LP|AB} < \Pi^*_{LP|noAB}$$

Proof: See Appendix.

When there are no cost for the pirate providers, full blocking can create unintended effects of the copyright holder. This happens because it creates a pro-piracy effect: the demand for the LP is reduced relative to when AB is not used by consumers. Given full market coverage, this result implies that the pirate platform expands its demand despite not making profits from ads. In this sense, the pirate provider simply survives and divests its commercial nature. In other words, a PP gets closer to the early-generation of peer-to-peer networks, i.e., users were sharing copyrighted materials for purely altruistic reasons. Moreover, the legal provider suffers as forced to reduce its price further, thereby lowering its profits. All in all, AB adoption is a curse for the LP.

Case II: Partial Blocking ($A^*_{AB} > 0$) This case prevails when a "partially user-oriented" AB pursues a constrained maximisation problem in presence of positive fixed costs (Proposition 1). In this case, the LP maximises profits subject to A^*_{AB} is such that $|\eta_A| < 1$ and $\Pi_{PP} = 0$. As a result, we can derive the following Lemma:

Lemma 2. *With ad-blockers adoption, the LP treats p and A as strategic substitutes.*

Proof: See Appendix.

As the AB chooses A in the inelastic portion of the PP demand to maintain $\Pi_{PP} = 0$, this changes the strategic interactions between PP and LP. This is a clear difference with respect to the pre-adoption game case. In the latter scenario, prices and ads were treated as strategic complements. Moreover, the FOC of the LP's problem can be represented as follows:

$$\frac{\partial \Pi_{LP}}{\partial p} = \left\{ \underbrace{\frac{1}{2} + \frac{av - p + s}{2\tau} + \frac{\gamma A^2}{4\tau}}_{D_L} \right\} + p\left\{ -\frac{1}{2\tau} + \frac{\gamma A}{2\tau}\frac{\partial A}{\partial p} \right\} = 0 \tag{1.14}$$

$$\Rightarrow \quad D_{LP} = p\left\{ \frac{1}{2\tau} - \frac{\gamma A}{2\tau}\frac{\partial A}{\partial p} \right\} \quad \Rightarrow \quad p_l^* = \frac{D_{LP}}{\left\{ \frac{1}{2\tau} - \frac{\gamma A}{2\tau}\frac{\partial A}{\partial p} \right\}}$$

The above equation implicitly determines the equilibrium value of $p_l^*|_{AB}$. The following proposition provides a comparison with the pre-adoption market.

Proposition 4. *The equilibrium values of i) $A^*|_{AB}$ is lower than that in the pre-adoption game set by the pirate provider independently: $A^*|_{AB} < A^*|_{noAB}$ ii) $p_l^*|_{AB}$ (chosen by the LP) is higher than that in the pre-adoption game: $p_l^*|_{AB} > p_l^*|_{noAB}$ and iii) the demand faced by the LP is lower than that in the pre-adoption game: $D_{LP}^*|_{AB} < D_{LP}^*|_{noAB}$*

Proof: See Appendix.

Corollary 1. *The Nash value of $A^*|_{AB|K>0} \in \left(0, \left\{\frac{4\tau-(av+s)}{3\gamma}\right\}^{1/2}\right)$.*

Thus, Proposition 4 and Corollary 1 indicate that ad-blocker adoption has a *pro-piracy effect*: it increases the demand for the pirate content provider at the expenses of the legal one. The mere adoption of ad-blockers by consumers amplifies the substitutional effects, shifting part of the pre-adoption demand of the copyright holder to the pirate competitor, which becomes more attractive because of fewer ads. The presence of substitutional effects arising with piracy is not new in the economic literature, with piracy usually displacing the demand for the original product when not working as sampling (Peitz & Waelbroeck 2006*a*).

In this specific case, ads are reduced to the extent that they still preserve the existence of the pirate provider ($\Pi_{PP} = 0$). However, it still generates negative externalities (loss of demand) for the legal incumbent: a curse for the copyright holder and the pirate provider. The actual impact of adblocking adoption on the profitability of the legal platform, however, is ambiguous as the LP can actually increase its price and compensate for lost demand. Given the generic functional form, we are not able to provide a closed-form solution which clarifies the overall effect on profits.

This result has however two rather interesting implications. First, there is a redistribution of surplus from both providers to the (pirate) users: they benefit from fewer ads. Second, ad-blocking can be an effective way to reduce the flow of money going into illegal segments. A blessing for a government concerned willing to reduce pirates' revenues without creating a monopoly in the SVOD market.

1.5 The Legal Provider as a Follower

In this section, we consider the scenario where the pirate provider PP (or the ad-blocker - see below) acts as the leader and the legal content provider, after observing the behaviour of the PP (or the AB), moves as the follower. Previous papers on piracy and on leader-follower game have shown that the incumbent prefers to act as the leader under all different anti-piracy systems (Banerjee 2003), whereas Martınez-Sanchez (2010) has shown that, under price-competition, the government lets the legal incumbent decide on whether to act as a leader or as a follower depending on the magnitude of the piracy monitoring cost. In our model, we do not consider the intervention of the government. Instead, we rely only on the presence of an ad-blocker to reduce the flow of invasive ads.

As with legal provider acting as a leader, we compare pre- and post-adoption of AB. However, as the LP's move is common to both scenarios, we do not need to make any distinction between the pre-adoption and post-adoption strategies of the LP.

Specificaly, by acting as a follwer, the LP solves the following problem

$$\begin{aligned}\max_p \Pi_{LP} &= pD_{LP} - F \\ &= \Big\{\frac{1}{2} + \frac{av+s-p}{2\tau} + \frac{\gamma A^2}{4\tau}\Big\}p - F\end{aligned}$$

The FOC then yields:

$$p(A) = \frac{s+av+\tau}{2} + \frac{A^2\gamma}{4} \tag{1.15}$$

1.5.1 The First Stage

The No-AB Game

In the pre-adoption case, the PP chooses its optimal level of advertisements incorporating $p(A)$ in the first period:

$$\begin{aligned} \max_{A} \Pi_{PP} &= D_P A r - K \qquad s.t. \quad p(A) = \frac{s + av + \tau}{2} + \frac{A^2\gamma}{4} \\ &= \Big\{\frac{6\tau - 2(av+s) - \gamma A^2}{8\tau}\Big\} Ar - K \end{aligned} \tag{1.16}$$

From the FOCs, we obtain

$$A_{1/2} = \pm \Big\{\frac{2(3\tau - (av+s))}{\gamma}\Big\}^{1/2} \tag{1.17}$$

As before, we restrict our analysis only on the positive root. All equilibrium values are summarised below (where the subscripts 'l' or f denote the values for the leader and the follower respectively). All the rest of the Nash equilibrium values are as follows

$$p_f^* = \tau + \frac{(av+s)}{3}, \qquad A^*_{l|noAB} = \Big\{\frac{2(3\tau - (av+s))}{\gamma}\Big\}^{1/2}.$$

The demand shares are

$$D^*_{f|LP} = \frac{1}{2} + \frac{av+s}{6\tau} \qquad D^*_{l|PP} = \frac{1}{2} - \frac{av+s}{6\tau}, \tag{1.18}$$

and equilibrium profits are

$$Pi^*_{f|LP} = \frac{(3\tau + av + s)^2}{18\tau} - F, \qquad \Pi^*_{l|PP} = \Big\{\frac{3\tau - (av+s)}{6\tau}\Big\} rA^*|_{AB} - K. \tag{1.19}$$

Given the above results, we can provide the following observation.

Observation 2. *Absent ad-blocking technology, the PP sets a higher number of ads when moving as a leader than when moving as a follower, i.e.,*

$$\underbrace{A^*_{l|noAB}}_{Game\ B} = \Big\{\frac{2(3\tau - (av+s))}{3\gamma}\Big\}^{1/2} > \underbrace{A^*_{f|noAB}}_{Game\ A} = \Big\{\frac{4\tau - (av+s)}{3\gamma}\Big\}^{1/2}.$$

From the above observation, it immediately follows that the LP obtains a second-mover advantage relative to when it acts as a leader as shown in the following observation.

Observation 3. *Absent ad-blocking technology, the LP charges a lower price and faces higher demand by acting as a follower i.e.* $p_f^* = \tau + \frac{(av+s)}{3} < p_l^* = \tau + \frac{(av+s)}{2}$; $D^*_{f|LP} = \frac{av+s}{6\tau} + \frac{1}{2} > D^*_{l|LP} = \frac{av+s}{6\tau} + \frac{1}{3}$. *Profits are higher when moving second, i.e.* $\Pi^*_{f|LP} = \frac{(av+s+2\tau)^2}{12\tau} > \Pi^*_{l|LP} = \frac{(av+s+3\tau)^2}{18\tau}$ *whenever Assumption 1 holds.*

The AB Game

The ad-blocking technology decides how many ads to show to consumers. In doing so, it maximises consumer surplus paying attention not to shut down the ad-based market. Hence, it solves the following problem

$$\begin{aligned} \max_{A} \quad CS \qquad s.t. \quad & p = \frac{s + av + \tau}{2} + \frac{A^2\gamma}{4} \\ & \Pi_{PP} \geq 0. \end{aligned} \tag{1.20}$$

Again, this is solved by deriving the Kuhn-Tucker conditions for a maximum (see the Appendix). As when the LP acts as a leader, results vary depending on whether $K > 0$ or $K = 0$. What is important to note here is that whilst the way the AB chooses its advertising strategy is *similar* in fashion as in Game A, the optimal level of A^* itself is now different especially when $K > 0$. Interestingly, whilst the AB does not deviate from its *rules* of functioning, regardless of whether it acts as the follower or the leader, the LP's strategies differ depending upon who makes the first move. More specifically, when the LP moves second after observing the level of ads in the pirate segment, it no longer treats p and A as strategic substitutes as it did when LP is a market leader. In fact, it reverts to treating p and A *as strategic complements.* The following proposition gives the results of this game.

Proposition 5. *With* $K > 0$, *the ad-blocker sets* A *such that (i)* A^* *is strictly positive; (ii)* $\Pi_{PP} = 0$ *(the constraint binds); and (iii) the equilibrium value of* $A|^*_{AB,K>0}$ *chosen by the ad-blocker is less than the equilibrium* A *chosen by the PP in the pre-adoption game (i.e. when PP is the leader) i.e.* $A|^*_{AB,K>0} < A|^*_{noAB} \Rightarrow A|^*_{AB,K>0} < \left\{\frac{2(3\tau-(av+s))}{3\gamma}\right\}^{1/2} =$

$A^*_{l|noAB}$. *However, the LP charges a lower price in the presence of an ad-blocker than without i.e.* $p^*_{f|AB} < p^*_{f|noAB} = \tau + \frac{(av+s)}{3}$.

Proof. *See Appendix.*

The above analysis shows that whilst the AB never acts strategically, the LP certainly does. As a result of reduction in the volume of ads to be shown to consumers, the LP reacts by lowering its price in order to combat that attraction of using the pirate platform. More importantly, the LP's strategic behaviour varies depending upon whether it moves first or second where in the former the LP treats p and A as strategic substitutes while in the latter it treats them as strategic complements. This is because when moving as a leader, the LP can anticipate the non-strategic behaviour of the AB who lowers the level of ads regardless. Consequently, the LP attempts to prevent its loss of revenue by raising its price, although the overall effect on the legal platform's profits however remains ambiguous. On the other hand, while acting as a follower, the best the LP can do is to *react* to the level of advertisements chosen by the AB which then gives him a (second-mover's) advantage as it can then set a lower price in response to a lower value of A. However, despite enjoying a second-mover advantage, the LP suffers the most.

1.6 Extension

1.6.1 Fully User-oriented AB

In this section, we consider a "fully user-oriented" ad-blocker pursues an *unconstrained* maximisation problem, whereby it merely maximises the consumers' surplus. The AB here is considered to be *fully* user-oriented as it does not take into consideration the impact of its optimisation strategy on the PP's payoff i.e. such an AB cares *only* about the well-being of the consumers. Our results are presented below.

The legal provider as a leader

Consider a *fully user oriented* AB, who simply maximises the aggregate consumer surplus without paying any attention to the existence of ad-sponsored firms. I.e. the AB simply chooses A to maximise the aggregate consumer surplus without any restrictions on the

PP's payoff. The problem of the AB is simply to solve $\max_A CS$. This leads to the following proposition.

Proposition 6. *Regardless of the value of K, a fully user-oriented AB blocks all ads.*

Proof. See Appendix. □

Proposition 6 illustrates some interesting results: when the AB cares *only* about consumers' welfare, it follows a *full-blocking* strategy and obscure all ads ($A^*_{AB} = 0$).

Absent any fixed costs ($K = 0$), the pirate firm can "theoretically" maintain a passive status where it advertises nothing and earns nothing. In this case, the equilibrium demand faced by the LP is now even *lower* relative to the pre-adoption market despite the price reduction: a pro-piracy effect. Consequently, it earns even *less* payoff compared to the pre-adoption market. As a result, AB adoption is a *curse* for the LP and the pirates, whereas it represents a *blessing* for all consumers. This happens because those consumers who subscribe to the LP benefit from a lower price, whereas those going pirate benefit from the presence of no ads.

On the other hand, the presence of positive fixed costs may generate some unintended effects for a user-oriented AB. Specifically, when not anticipating the effects on the market structure, $A^* = 0$ implies a foreclosure of the pirate provider, who then makes negative profits and exits the market. Anticipating this, the LP may want to monopolise the market by setting its maximum price. Clearly, this translates in a welfare loss for many consumers as those joining the LP face a higher price and the others refrain from joining the LP perhaps because they would rather endure some advertising costs. The well-intended purpose of an ad-blocking technology to maximise consumer surplus then ends up lowering their surplus and changing the market structure. Indeed, a curse for the consumers. However, such a detrimental effect for consumers can be mitigated if the user-oriented AB can anticipate such an effect. In the latter case, for any given K, $A^*(K)$ can be set at the lowest possible level to drive pirate profits to zero making the pirates indifferent between staying in and leaving the market. As a result, a fully user-oriented AB turns out to behave the same way as a partially user-oriented AB (as in the benchmark model): digital piracy is partially boosted, thereby damaging the copyright holder.

The legal provider as a follower

Noteworthy, the above discussion also apply when the LP acts as a follower. In this case, in the first stage of the game, the AB maximises $\max_A CS$ by choosing how many ads to display on pirate consumers' screen. In this case, the first order condition leads to $\frac{\partial CS}{\partial A} \leq 0$ for $A \geq 0$ where the derivation of $\frac{\partial CS}{\partial A}$ is provided in the Appendix. Again, the only equilibrium solution is $A^* = 0$,[19]. As a result, there are two outcomes. When $K = 0$, the PP remains active and generates a pro-piracy effect. In this case, p and D_L are given by $\frac{\tau+(av+s)}{2}$ and $\frac{1}{4} + \frac{av+s}{4\tau}(< 1)$ respectively. A curse of the LP. Else, when $K > 0$, the LP can act as a monopolist after observing the PP going out of the market. Hence, there is partial market coverage and consumers suffer from AB adoption. Again, a user-oriented AB may anticipate the detrimental effects on the market configuration and set a level of ads such that the pirate CP remains active. As a result, this would mitigate the market power of the copyright holder while providing a pirate yet cheaper alternative to final consumers.

The above observation along with the fact that results might be sensitive to the presence of fixed costs for the pirate platforms inspire the following comments. Specifically, one can argue that LP may have some incentives to impact the pirate firm's fixed cost K by undertaking actions that enables the LP to secure a better position in the market. This is because when the PP has some fixed costs, AB adoption makes digital piracy not sustainable. A potential way to increase PP's fixed costs is represented by anti-piracy measures adopted by policy-makers. Hence, the copyright holder may start lobbying national authorities to enforce more effectively copyright protection and seizing pirate cyberlockers. While seizing a website is often difficult in practice, national authorities can implement high sanctions for those cyberlockers hosting copyrighted material without authorisation. The probability of being detected coupled with a sufficiently high sanction can be regarded as a fixed (expected) cost incurred by the PP when maximising its profits. For instance, suppose that fixed costs are such that $K(G\phi)$, where G is the amount of

[19] To see that, note if A is to be positive then $A = \left\{\frac{2(7\tau-(av+s))}{\gamma}\right\}^{1/2}$ which is obtained by setting $\frac{\partial CS}{\partial A} = 0$ which does not satisfy the SOC.

the fine chosen by the government for those infringing property rights and $\phi \in (0, 1)$ the probability of being detected. It follows that, for a strictly positive probability of being detected by the authority and for a strictly positive fine (i.e. $K(.) > 0$), the adoption of fully user-oriented ad-blocking technologies prevents consumers from having a cheaper (although illegal alternative) but does not impact on the strategies followed by the legal platform. As a result, the adoption of fully user-oriented AB can lead to unintended effects for the consumers without improving the condition of the copyright holder.

1.7 Concluding Remarks

This paper provides a novel contribution to the literature on digital piracy consistent with the current evolution of the Internet and the diffusion of the SVOD market. Our paper sheds light on the effects of ad-blocking technologies in an environment characterised by ad-sponsored platforms. Our results build upon some important and growing concerns about the recent trend of ad-blocker adoption. By directly damaging the advertising industry and lowering the viability of sponsored-business models, the adoption of ad-blockers generates indirect effects on other business entities thereby altering the functioning of the entire digital market.

We have shown that, depending on the objective functions of the ad-blockers, very contrasting outcomes can prevail with severe consequences on the market structure. We have also shown that when ad-blockers maximise consumers' surplus subject to the non-negativity constraint on the pirate platform's profit, not only does it lower the pirate platform's profit down to zero (though not necessarily inducing complete exit of the firm), it also generates negative spillovers on the legal competitor by displacing its demand (the *pro-piracy* effect). However, a surprising result here is that the adoption of ad-blocking technologies turns out to changing the way platforms react to the flow of ads. Specifically, our findings show that, when the LP moves first, it anticipates the ad-blocker intervention by increasing its price (strategic substitutability), whereas it treats the ads and the price as strategic complements after observing the ad-blocker decision, when it moves as the follower. On the other hand, when the ad-blockers fully shut down ads, this can lead to different results depending on whether pirate providers face some fixed costs. The presence of fixed costs make the pirate business model unsustainable thereby leading to the exit of the market of the pirate provider. As a result, the legal provider can act as a monopolist and leave some consumers without alternative. A blessing for the copyright holder and a curse for consumers.

Our results are also consistent wth the general concern in the advertising industry regarding the adoption of ad-blocking technologies. In a recent empirical paper Shiller et al. (2018) find that "ad blocking poses a substantial threat to the ad-supported web". Ironically, this turns the duopoly market into a monopoly one, where only the legal platform operates. In this case, as if the legal platform leaves some customers unsatisfied as it can never cover the entire market despite the reduction in its subscription fee to the consumers. Such outcome not just arises from the nature of the price-ads competition but is also due to the externalities arising from the adoption of ad-blockers itself.

On the video-on-demand sector like the one we have analysed in this paper, the presence of an illegal alternative poses a competitive threat to the legal firm, preventing a price hike to the monopolist level. Indeed, implementing policies leading to the subsequent exit of the pirate firm making piracy non-viable may not necessarily be the best policy. From a managerial point of view, however, given that we find that in a price-advertisement competition the adoption of ad-blockers is likely to magnify the substitutional (pro-piracy) effect by displacing the demand for original products, particular attention should be posed on the quality aspect. Specifically, legal platforms should try to differentiate their services relative to cyber-lockers and hosting networks as much as possible by providing better quality and features that are difficult to replicate e.g. user-friendly interfaces, multiple subtitles, wish-lists and such. It seems that Netflix would pursue such strategy by allowing for interactive movies. This strategic differentiation is crucial for those video-on-demand providers for fostering growth other than just using pricing strategies to deter piracy. Alternatively, such strategic differentiation could also take the form of measures devoted to increase consumers' search costs for pirate firms. Overall, 'mild' anti-piracy policies that do not fully shut down piracy, are perhaps preferable.

Our results have important policy implications for *designing* suitable anti-piracy measures. Depending on the *perspective* and the *goal* of anti-piracy measures (i.e. cutting pirate demand or reducing its revenues), ad-blockers can either be regarded as a "blessing" or a "curse". If the anti-piracy measure is targeted towards cutting pirate demand, the government should focus on e.g. increasing search costs for users by seizing websites and hosting network whilst still keeping piracy 'alive' instead of allowing ad-blockers to induce complete shut-down of pirate firms which endows monopoly crown to the legal firm. The latter perspective of ad-blockers adoption can be regarded as a curse! Conversely, if the objective is to block the flow of money into illegal firms, ad-blocker adoption can be seen as a "blessing" as it can lead to zero (or lower when the constraint is not binding) profits earned by pirate firms.

All these findings strongly recommend that both the government and the legal firm should carefully address the presence of this plug-in tool in designing their anti-piracy measures and their managerial strategies as the effect might not be unidirectional. Along with this, limiting the ads flow toward pirates by acting on the advertisers' side of the market, as recently solicited by the Police Intellectual Property Crime Unit (PIPCU) in the UK, might help to disrupt their ads revenues. However, such policies are likely to bring about effects on "reputational" ads (which might pass the ad-blockers' test) but not on those ads aiming to track consumers' behaviours, extort clicks and download malware, which are usually shown on pirate platforms and on which ad-blockers mainly work. Indeed, ad-blocking technologies might accomplish the public purpose of "following" and "stopping" money going to potential criminals behind pirate platforms.

Whilst we think that our model provides novel insights into the functioning of the video-on-demand market by examining the effect of ad-blockers adoption and its relevance in today's anti-piracy policies, there still remain a few limitations that need to be addressed in future research. Firstly, consumers in our model are treated homogeneously in terms of their annoyance costs whereas in reality these can differ. Adopting a heterogeneous consumers approach will be a meaningful extension of our model. Secondly, we have assumed as if there are no costs involved in switching ad-blockers on as consumers in our model always turn on ad-blockers whenever this option is available. However, there may very well be some installations costs involved e.g. looking for the best ad-blockers, installing it or simply being worried about privacy and data extraction, involved with ad-blockers adoption. Hence, it will be important to analyse the implications of our model in the context of *costly* ad-blockers adoption. Thirdly, we mainly considered that ad-blockers are benevolent and care primarily about consumers. Whilst this was indeed the original purpose of many ad-blockers, some others (e.g. Ad-Block Plus) started making some profits by relying on donations and strategic whitelisting (Kraemer & Wiewiorra 2016). The presence of donations proportional to the consumer surplus generated in the economy however would not change our results, but simply scale them. Regarding "for-profits" ad-blockers, because ads in pirate platforms are invasive and less likely to respect any reasonable requirement set by the "Coalition for Better Advertisements" or by the "Acceptable Ads Initiative" launched by AdBlock Plus, it would be unreasonable to consider (illegal) content providers to pay for whitelisting their invasive ads. Fourthly, while this paper contributes to studying the strategies pursued by the ad-blockers, further research need to be carried out to assess how different business models behind ad-blocking technologies are likely to affect the market structure and compete for user adoption. Further research would also be needed to incorporate network externalities and consider competition among catalogues, as suggested by Belleflamme (2016).

Appendix

Derivation of CS and its derivate in the sub-section 4.2.1

The total consumer surplus $CS = CS_{LP} + CS_{PP}$. The surplus of consumers joining the legal firm:

$$\begin{aligned} CS_{LL} &= \int_0^{D_{LP}} \Big[v - p - \tau x\Big] dx = -\frac{x^2\tau}{2} + x(v-p) \\ &= -\frac{(2(av - p + s + \tau) + \gamma A^2)^2}{32\tau} + \frac{2(av - p + s + \tau) + \gamma A^2}{4\tau}(v-p) \end{aligned}$$

Similarly, the surplus of those consumers joining the pirate firm:

$$\begin{aligned} CS_{PP} &= \int_{D_{LP}}^{1} \Big[v(1-a) - s - \frac{\gamma A^2}{2} - \tau(1-x)\Big] dx \\ &= \frac{x^2\tau}{2} + x\Big((1-a)v - s - \frac{\gamma A^2}{2} - \tau\Big)\Big|^{1}_{\frac{1}{2}+\frac{av-p+s}{2\tau}+\frac{\gamma A^2}{4\tau}} \end{aligned}$$

which implies that $CS_{PP} = F(1) - F(D_{LP})$, where

$$\begin{aligned} F(1) &= \frac{\tau}{2} + \Big((1-a)v - s - \frac{\gamma A^2}{2} - \tau\Big) \\ F(D_{LP}) &= \frac{\Big(\frac{1}{2} + \frac{av-p+s}{2\tau} + \frac{\gamma A^2}{4\tau}\Big)^2 \tau}{2} + \Big(\frac{1}{2} + \frac{av-p+s}{2\tau} + \frac{\gamma A^2}{4\tau}\Big)\Big((1-a)v - s - \frac{\gamma A^2}{2} - \tau\Big) \end{aligned}$$

The partial derivatives

(i) The partial derivative of CS_{LP} with respect to A is:

$$\frac{\partial CS_{LP}}{\partial A} = \frac{A\gamma(4v - \gamma A^2 - 2(\tau + p) - 2(av + s))}{8\tau} \tag{A-1}$$

(ii) The related partial derivative of CS_{PP} with respect to A is

$$\frac{\partial CS_{PP}}{\partial A} = \frac{\partial F(1)}{\partial A} - \frac{\partial F(D_{LP})}{\partial A}$$

and subsequently by:

$$\frac{\partial CS_{PP}}{\partial A} = \frac{A\gamma\Big(6(av+s)+3A^2\gamma-2(p+\tau)-4v\Big)}{8\tau} \tag{A-2}$$

Using equations (A-1) and (A-2), the derivative is:

$$\frac{\partial CS}{\partial A} = \frac{\gamma A\Big(A^2\gamma+2(av+s)-2(p+\tau)\Big)}{8\tau} \Leftrightarrow -\frac{\gamma A}{2}D_{PP} \tag{A-3}$$

Proof of Proposition 2.

First, note that with $K > 0$, A^* cannot be zero to ensure $\Pi_{PP} \geq 0$. Hence $A^* > 0$. Second, we cannot have $\lambda = 0$ because, when $\lambda = 0$, the FOC reduces to $\frac{\partial \mathcal{L}}{\partial A} = \frac{\partial CS}{\partial A} = 0$ (since $A > 0$). Also note that $\Pi_{PP} \geq 0$ implies that $D_{PP} > 0$. Since $\frac{\partial CS}{\partial A} = -\frac{A\gamma}{2}D_{PP} < 0 \Leftrightarrow A^*$ must be zero. Hence, a contradiction. It then follows that the constraint must bind, i.e. $\Pi_{PP} = 0$.

Finally, he FOC is $\frac{\partial \mathcal{L}}{\partial A} = \frac{\partial CS}{\partial A} + \lambda\frac{\partial \Pi_{PP}}{\partial A} = 0$. Since $\frac{\partial CS}{\partial A} < 0$, $\frac{\partial \Pi_{PP}}{\partial A}$ must be positive, with $\lambda > 0$. Thus we have,

$$\begin{aligned}
\frac{\partial \Pi_{PP}}{\partial A} &= r\left\{A\frac{\partial D_{PP}}{\partial A} + D_{PP}\right\} > 0 \\
&= rD_{PP}\left\{\underbrace{\frac{\partial D_{PP}/D_{PP}}{\partial A/A}}_{<0} + 1\right\} > 0 \Leftrightarrow |\eta_A| < 1
\end{aligned}$$

It then implies that the AB chooses A^* over the **inelastic** part of the demand curve. ∎

Proof of Proposition 1.

First, note that the PP's profit now is $\Pi_{PP} = rAD_{pp}$. Next, we show that the constraint must bind in this case. Suppose not, suppose that the constraint does not bind i.e. $\lambda = 0$ so that $\Pi_{PP} > 0$ implies that $A > 0$, which in turn means that $\frac{\partial \mathcal{L}}{\partial A} = \frac{\partial CS}{\partial A} = 0$ must hold. Also note that $\Pi_{PP} > 0$ implies $D_{PP} > 0$. But, because $\frac{\partial CS}{\partial A} = -\frac{A\gamma}{2}D_{PP} < 0$, there is a contradiction and A must be zero. Therefore, the constraint must bind i.e. $\lambda > 0 \Rightarrow \Pi_{PP} = 0$. Now, $\Pi_{PP} = 0$ can be satisfied either with $A = 0$ or $A > 0$. Suppose

$A > 0$, it follows that $D_{PP} = 0$. However, because with $A^* > 0$ and $\lambda > 0$, the FOC is

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{\partial CS}{\partial A} + \lambda \frac{\partial \Pi_{PP}}{\partial A} = -\frac{A\gamma}{2} D_{PP} + \lambda \frac{\partial \Pi_{PP}}{\partial A} = 0$$

Now with $D_p = 0$, the above reduces to $\frac{\partial \Pi_{PP}}{\partial A} = 0 \Rightarrow r\left\{A\frac{\partial D_{PP}}{\partial A} + D_{PP}\right\} = 0$ implying $A\frac{\partial D_{PP}}{\partial A} = 0$ which can hold only with $A = 0$. ■

Proof of Proposition 3.

(i) Straightforward differentiation of Π_{LP} with respect to p yields, $p_l^*|_{AB} = \frac{\tau+(av+s)}{2}$ which is clearly less than $p^*|_{noAB} = \tau + \frac{av+s}{2}$. The resulting market share of the LP is $D_{LP|AB} = \frac{1}{4} + \frac{av+s}{4t} < 1$ since $\tau > av + s$ by assumption 1.

(ii) $\Pi^*_{LP|AB,A^*=0} = \frac{(\tau+(av+s))^2}{8\tau} - F$ and $\Pi^*_{LP|noAB} = \frac{(2\tau+(av+s))^2}{12\tau} - F$. Hence, $\Pi^*_{LP|AB,A^*=0} \geq (<) \Pi^*_{LP|noAB}$ according as whether

$$\begin{aligned} 3\left(\tau+(av+s)\right)^2 &\geq (<) 2\left(2\tau+(av+s)\right)^2 \\ \text{or according as whether } \sqrt{3}\left(\tau+(av+s)\right) &\geq (<) \sqrt{2}\left(2\tau+(av+s)\right) \end{aligned}$$

which then further simplies to

$$(av+s)\left(\sqrt{3}-\sqrt{2}\right) \geq (<)\tau\left(2\sqrt{2}-\sqrt{3}\right) \Rightarrow \quad 0.318(av+s) \geq (<) 1.1\tau$$

Given that $\tau > (av+s)$ (Assumption 1), it is then obvious that $\Pi^*_{LP|AB,A^*=0} < \Pi^*_{LP|noAB}$. ■

Proof of Lemma 2.

First, write the optimal A^* as a function of p i.e. $A(p)$. Hence, the pirate's demand $D_{PP} = D_{PP}(p, A(p))$ suppressing other parameters from the demand function. Totally differentiating $\Pi_{PP} = rA(p)D_{PP} - K = 0$ with respect to p and $A(p)$ to obtain:

$$r\left\{D_{PP}\frac{\partial A}{\partial p}dp + A\left(\frac{\partial D_{PP}}{\partial p}dp + \frac{\partial D_{PP}}{\partial A}\frac{\partial A}{\partial p}dp\right)\right\} = 0$$

The above expression can be simplified as

$$
\begin{aligned}
& \underbrace{A\frac{\partial D_{PP}}{\partial p}}_{>0} + \frac{\partial A}{\partial p}\left\{\underbrace{1+\eta_A}_{>0}\right\} D_{PP} = 0 \\
\Leftrightarrow \quad & \frac{\partial A}{\partial p} = -\frac{A\frac{\partial D_{PP}}{\partial p}}{D_{PP}\{1+\eta_A\}} < 0
\end{aligned}
$$

where the fact that $\{1 + \eta_A\} > 0$ follows from the AB's optimisation problem in the second stage - see the proof of Proposition 2. Hence, p and A are treated as strategic substitutes. ■

Proof of Proposition 4.

(i) To show that $A^*|_{AB|K>0} < A^*|_{noAB}$, note that in the pre-adoption game, the PP chooses the level of A, taking p as given, where the elasticity of D_{PP} with respect to A is exactly equal to 1. To see that, re-write, the PP's maximisation problem as: $\max_A \Pi_{PP} = rAD_{PP}(A, p) - K$. The FOC is given by

$$
\begin{aligned}
r\left\{A\frac{\partial D_{PP}}{\partial A} + D_{PP}\right\} &= 0 \Leftrightarrow \left\{1 + \frac{\partial D_{PP}/D_{PP}}{\partial A/A}\right\} = 0 \\
i.e.|\ \eta_A| &= 1
\end{aligned}
$$

Whereas, the AB sets the level of A such that $|\eta_A| < 1$ (see the proof of Proposition 2). Therefore, given the shape of the demand curve, it follows immediately that $A^*|_{AB|K>0} < A^*|_{noAB}$.

(ii) Since the LP treats A and p as strategic substitutes in order to incorporate the AB's behaviour in the 2nd stage of the game, it then follows that LP sets a *higher* price in the post-adoption market (with $K > 0$) than it does in the pre-adoption market.

(iii) Coupled together, these two effects imply that the LP's demand is *lower* when users adopt the AB technology, i.e $D_{LP}|_{AB} < D_{LP}|_{noAB}$ since $\partial D_{LP}/\partial A > 0$ and $\partial D_{LP}/\partial p < 0$. ■

Derivation of CS and its derivatives in the sub-section 5.2.1

$CS = CS_{LP} + CS_{PP}$ where

$$CS_{LP} = -\frac{x^2\tau}{2} + x(v-p)\Big|_0^{\frac{1}{2}+\frac{av-p+s}{2\tau}+\frac{\gamma A^2}{4\tau}}$$

Using $p = \frac{s+av+\tau}{2} + \frac{A^2\gamma}{4}$ in equation (1.17), then

$$\begin{aligned} CS_{LP} &= -\frac{x^2\tau}{2} + x\Big(v - \frac{s+av+\tau}{2} - \frac{A^2\gamma}{4}\Big)\Big|_0^{\frac{2(av+s+\tau)+\gamma A^2}{8\tau}} \\ &= -\frac{\Big(2(av+s+\tau)+\gamma A^2\Big)^2}{128\tau} + \frac{2(av+s+\tau)+\gamma A^2}{8\tau}\Big(v - \frac{s+av+\tau}{2} - \frac{A^2\gamma}{4}\Big) \end{aligned}$$

Next,

$$\begin{aligned} CS_{PP} &= \frac{x^2\tau}{2} + x\Big((1-a)v - s - \frac{\gamma A^2}{2} - \tau\Big)\Big|_{\frac{2(av+s+t)+\gamma A^2}{8\tau}}^{1} \Rightarrow F(1) - F(D_L) \ \text{ where} \\ F(1) &= v(1-a) - s - \frac{\tau}{2} - \frac{\gamma A^2}{2} \quad \text{and} \\ F(D_L) &= \frac{(2(av+s+\tau)+\gamma A^2)^2}{128\tau} + \frac{2(av+s+\tau)+\gamma A^2}{8\tau}\Big(v(1-a) - s - \tau - \frac{\gamma A^2}{2}\Big) \end{aligned}$$

The derivatives

(i) The partial derivative of CS_{LP} with respect to A is:

$$\frac{\partial CS_{LP}}{\partial A} = \frac{\gamma A\Big(8v - 10\tau - 10(av+s) - 5\gamma A^2\Big)}{32\tau} \tag{A-4}$$

(ii) Next, the partial derivative of CS_{PP} with respect to A is:

$$\frac{\partial CS_{PP}}{\partial A} = \frac{\gamma A\Big(7A^2\gamma + 14(av+s) - 18\tau - 8v\Big)}{32\tau} \tag{A-5}$$

Combining equations (A-4) and (A-5), then:

$$\frac{\partial CS}{\partial A} = \frac{\gamma A\Big(\gamma A^2 - 14\tau + 2(av+s)\Big)}{16\tau} \tag{A-6}$$

Proof of Proposition 5.

Part (i) and (ii) of this proof i similar to that provided in the proof of Proposition 2 and hence are not repeated. To prove that $A|^*_{AB,K>0} < A|^*_{noAB}$, note again from the FOC that $\frac{\partial CS}{\partial A} + \lambda\frac{\partial \Pi_{PP}}{\partial A} = 0 \Rightarrow \frac{\partial \Pi_{PP}}{\partial A} > 0$ as $\frac{\partial CS}{\partial A} < 0$. $\frac{\partial \Pi_{PP}}{\partial A} > 0$ then yields $|\widehat{\eta}_A| < 1$, where $\widehat{\eta}_A$ now denotes the elasticity of the "reduced form" demand faced by the AB (i.e. after incorporating the second stage best response function $p(A)$). Since, in the pre-adoption game, the PP chooses $A^*_{leader}(= \left\{\frac{2(3\tau-(av+s))}{3\gamma}\right\}^{1/2})$ where $|\widehat{\eta}_A| = 1$, hence it follows immediately that $A|^*_{AB,K>0} < A|^*_{noAB}$. Given that LP acting as a follower treats p and A as strategic complements, it sets a *lower* value of price compared to the one in the pre-adoption game where $p = \tau + \frac{(av+s)}{3}$. ∎

Proof of Proposition 6.

By maximizing consumer surplus only, there are three candidate solutions: $A_1(p) = 0$, $A_{2/3}(p) = \pm\left\{\frac{2(\tau-(av+s)+p)}{\gamma}\right\}^{1/2}$. Suppose not. Suppose $A^* > 0$. Then,

$$\frac{\partial CS}{\partial A} = \frac{A\gamma}{8\tau}\left\{\gamma A^2 + 2(av+s) - 2(\tau+p)\right\} = 0 \Leftrightarrow A^* = \left\{\frac{2(\tau+p) - 2(av+s)}{\gamma}\right\}^{1/2}.$$

But at this value of A^*, the SOC does not hold as $\frac{\partial^2 CS}{\partial A^2}|_{A^*} = \frac{A^2\gamma^2}{4\tau} > 0$ implying a minimum! Hence, A^*_{AB} must be $= 0$. Similarly, A cannot be negative, therefore $A^*_{AB} = 0$ always. ∎

CHAPTER 2

Content providers in a multi-piracy ecosystem

2.1 Introduction

In the media market, digital piracy is a worldwide and long-standing problem. In 2016 approximately 53% of US citizens used illegal content providers (LaunchLeap 2017). The attractiveness of digital piracy can be easily comprehended: it provides a cheaper yet less perfect alternative to the costly original product. In the streaming video-on-demand (SVOD) market, users not willing to subscribe to a legal content provider can find easy ways to stream a content online. These pirate services (also called cyberlockers) are very common: they passively mimic the functioning of any legal provider and benefit from a network of third-party websites providing a direct link to their content.

Relative to the past, this type of digital piracy presents some novelties and poses new challenges. Firstly, not only does it constitute a problem for the copyright holder by displacing some of its demand, it also raises the problem of how to stop this lucrative activity. The reason why this is problematic is that while a few decades ago digital piracy was mainly user-generated, with users copying or downloading copyrighted content for their personal use, nowadays pirate providers monetise user eyeballs by exposing them to advertisements (ads, henceforth). In practice, pirate websites act as intermediaries between users demanding free content and advertisers with no or limited attention to brand safety when placing an ad on illegal websites (Rafique et al. 2016).[1] Coupled with the growing relevance of the online advertising industry and the popularity of on-demand content, piracy has become utterly profitable. For example, Megaupload, the most famous cyberlocker, made millions of dollars from ads before it shut down in 2012.[2]

Secondly, for each content (e.g., an episode of House of Cards), a legal provider competes against a large number of small cyberlockers (DCA 2017*a*). These cyberlockers attract less traffic than tech giants (e.g., Netflix) and are not able to individually challenge their incumbency. Yet, at the aggregate level, they constitute a multimillion pirate ecosystem based on copyright infringement.[3] In 2016 alone, about 1 billion of movies and tv-shows were pirated, causing pirate websites (including all types, e.g., cyberlockers, torrent, etc.) revenues from advertisements to soar to nearly US$ 209 million and enabling them to make extraordinary profit margins, e.g., 86-93% (Creativity & Entertainment 2017). Nevertheless, the competitive nature of digital piracy has often been overlooked within the economic theory. The early literature on digital piracy has focused mostly on deterrence strategies and the welfare implications of the threat of online piracy[4], whereas only a limited number of studies has dealt with the strategic interactions between the content providers (Chang & Walter 2015, Datta & Madio 2018, Rasch & Wenzel 2013).

Thirdly, pirate providers compete with one another for user attention. On the one hand, these providers compete to attract the legal provider's users. On the other hand, they compete with one another to attract their rivals' turf. The previous literature lacked in understanding the strategic interactions arising in the pirate ecosystem and how these impact on the legal provider's strategies.

Moreover, a non trivial problem is embodied in the searching costs. Pirate providers

[1]In the UK, the City of London Police's Intellectual Property Right Unit provides to copyright owner a list of websites where not to place ads. However, this novel "follow the money" initiative can only work with advertisers concerned about brand safety but not with malvertisers. A similar approach is also followed by the European Union with policies aiming to dissuade large-scale intellectual property rights infringments with some positive effects (Batikas et al. 2018).
For `the EU "follow the money" policy: https://ec.europa.eu/growth/industry/intellectual-property/enforcement_en`. (Accessed 2017).

[2]United States vs. Kim Dotcom. Indictment, No. 1:12CR3 (US District Court for the Eastern District of Virginia 2012).

[3]DCA (2017*b*) collected data regarding 15 streaming cyberlockers, accounting for a total of 103 million monthly visits and estimated average monthly revenues of 103.000 US dollars.

[4]See e.g., Banerjee (2003), Belleflamme (2003), Belleflamme & Picard (2007), Peitz & Waelbroeck (2006*a*,*b*). For a survey, see Belleflamme & Peitz (2012, 2014).

are not easy to find online for the typical everyday user: it requires surfing different pages on a search engine to find the correct magnet link. This is a somewhat annoying yet labourious activity. In a pirate ecosystem, however, searching costs can be dwindled. When the number of pirate alternatives increases, the number of websites associated with a certain keyword (e.g., "streaming movie XYZ") increases and so the number of potential matches in the first pages of a search engine. As finding a pirate website becomes quicker the more pirate websites are in the market, searching costs impact less on users' utility, and the attractiveness of pirate websites increases.

This paper incorporates the above features to study the strategic interactions between legal and pirate providers and how the two types of content providers react to changes in the pirate ecosystem. A pirate ecosystem can be altered in several ways. A new pirate provider can enter the market and offer the same content available on other pirate and legal websites with potentially ambiguous effects. On the one hand, a new entry increases the competition in the market. On the other hand, it reduces searching costs in the pirate ecosystem. A pirate ecosystem can also be altered by the intervention of authorities when seizing websites and enforcing the law. Similarly, the competition in the pirate ecosystem might get fiercer as pirate content providers are more substitute from the user perspective.

We present a simple model where a dominant firm (i.e., the legal provider) competes against several pirate providers. Using the Salop model with a centre (see e.g. Bouckaert 2000), we assume that all pirate providers are uniformly distributed on the perimeter of a Salop circle whereas the legal provider is located at the centre.[5] Both types of providers give access to the same content but with a different quality: the pirate content is of an inferior quality. Users can either subscribe to the legal provider or be exposed to a nuisance from ads displayed on the pirate websites. Pirate users bear two other types of non-monetary costs: the transportation costs, capturing the degree of substitutability among the pirates, and the searching costs. As we discussed above, searching costs are concave and decreasing: the larger the number of pirate content providers in the market, the fewer the time spent by the users to access their content.

We find that searching costs are crucial for this market. An (inverted) U-shaped

[5] Throughout our analysis, we will refer to the SVOD market. The setup can also be applied to other markets where several pirate alternatives have a commercial nature (e.g., music industry).

relation exists between the demand of the pirate (legal) provider and the number of pirate providers in the economy. One may think that more piracy would impact adversely on the legal provider. Instead, because of the searching costs, the legal content provider only suffers in a sufficiently large pirate ecosystem, whereas it displaces pirate demand when the number of pirate rivals is small enough. The mechanism works as follows: when the pirate ecosystem is small, the marginal pirate exerts a competitive pressure over the legal provider which replies by reducing its subscription price. By strategic complementarity, the pirate incumbents react by reducing the flow of ads displayed to consumers. However, the price reduction outweighs the reduction in the number of ads and the legal platform's demand can increase despite the larger pirate ecosystem. This happens as, in a small piracy ecosystem, the marginal pirate provider has a small impact on the attractiveness of piracy. Differently, in a sufficiently large pirate ecosystem, piracy gets more attractive as searching costs are abated significantly with entry: pirate providers can expand their market shares, thereby causing a pro-piracy effect.

An immediate consequence of the above discussion is that the legal content provider can be put in the condition to serve entirely the market for an intermediate dimension of the pirate ecosystem. In other words, it is possible that pirate content providers simply become inactive divesting their commercial nature. This scenario would accomplish the purpose of (endogenous) curtailing pirates' revenues while reducing the market power of the copyright holder. However, it would not be preferred by the legal provider who would instead prefer to serve fewer users by charging a higher price and compete with commercially active pirate providers than monopolising the market at a very low price. A limit pricing strategy would hurt its profits. Interestingly, absent any fixed cost for the pirate providers, this would represent an equilibrium number of pirate providers. This happens as this point, pirate provider would stop entering the market and be in the condition of making zero-profits.

In section 2.6, we expand the model towards the case where searching costs are exogenous. The presence of a pro-piracy effect (expansion of the pirate ecosystem) only depends on the vertical differentiation between the legal and the pirate providers. In this case, both types of content providers are negatively impacted by any entry. In section

2.7, we delve further into the analysis of the interactions among the pirate providers. A user, for instance, may not have strong preferences for a specific pirate provider and be almost indifferent between two of them. In this case, the enhanced competition in the pirate ecosystem will damage all pirate providers and, because of the strategic interactions arising in the economy, also the legal provider will be harmed.

The remainder of the study is structured as follows. In section 2.2, the relevant literature is reviewed. In section 2.3, the preliminaries of the model are presented while in the following section we derive the equilibrium outcomes. In section 2.5, we study the effect of entry in the pirate segment, while the effect when searching costs are exogenous is discussed in the following section. In section 2.7, the analysis of the effect of a fiercer competition among the pirates is presented. Finally, in section 2.8, some concluding remarks are presented.

2.2 Related Literature

This paper stands in parallel to the recent literature on commercial digital piracy and on spatial competition. Regarding the first stream of the literature, few papers are concerned with the new type of digital piracy. Chang & Walter (2015) consider a legal provider competing against a peer-to-peer network, which relies on revenues from ads and chooses the quality of the content. The number of ads, however, is not set by the pirate provider. The authors show that the pirate infrastructure may decrease or even increase the profitability of the legal firms. In a related paper, Datta & Madio (2018) study an economy where a legal provider competes with an ad-based pirate provider. They show that the adoption of an ad-blocker technology can spur digital piracy when ads are only partially blocked. They prove that the adoption of ad-blockers may change the strategic relationship between price and ads from strategic complementarity to strategic substitutability. Rasch & Wenzel (2013) investigate the competition between a legal and a pirate content provider in a two-sided market populated by developers of software and users. Likewise, Kim et al. (2018) study the effect of digital piracy on manufacturers' and retailers' strategies finding that, under some circumstances, tolerating piracy (with a weak copyright enforcement) may lead to a "win-win-win" situation where all economic agents can gain some surplus. In their model, this happens because piracy works as an "invisible hand", fostering competition for the manufacturer and the retailer and reduces the inefficiency caused by double marginalisation. This literature, however, has not dealt with pirate ecosystems, how these impact on legal and pirate providers' strategies, and how much commercial digital piracy a government and the legal content provider can tolerate. These aspects are taken up in this paper.

This paper also relates to the recent literature on spatial competition. Bouckaert (2000) was the first to introduce a Salop circle with a centre to study the choice between opening a retail store or a mail-order business (MOB) located in the centre of the circle. This simple yet general setup has been recently adapted to investigate the competition between an online firm and brick-and-mortar retailers (Madden & Pezzino 2011), R&D investment decisions (Lamantia & Pezzino 2016), and the competition between hospitals (Levaggi & Levaggi 2017).

This considered, Lamantia & Pezzino (2016) is the closest paper to the current study. The entry of a new firm in the local market leads to a drop in the quality and of the level of investment of the local firms, while the effect on the (dominant) firm located in the centre of the circle is ambiguous. They find that irrespective of their position, all types of firms are harmed on their profits.

2.3 The Model

There is a mass of users uniformly located on the circle of unit length. Following Bouckaert (2000), the economy is populated by a legal content provider (CP, hereafter) and a number of pirate providers. The legal CP offers a content for which it has exclusive rights.[6] The CP is located in the centre of the circle while there are N small pirate providers located on the circumference. The pirate CPs provide access to the same content offered by the legal CP in violation of copyright. The pirate CPs are symmetric and horizontally differentiated: τ represents the transportation cost per unit of distance incurred by any consumer; in other words, it measures the degree of substitution among the pirate CPs,

[6] For the sake of simplicity, it is assumed that, over a single content, the legal CP is a monopolist. In other words, there are no strategic interactions with other CPs. For instance, *Narcos* and *Orange is the New Black*, in a given period of time and in a given geographical area, are offered only by Netflix.

e.g., preferences for buffering and a compatibility with the operating system. The legal CP is differentiated from the pirate CPs in two aspects. First, there is *vertical differentiation*, as the content offered by the pirate CPs is of an inferior quality (imperfect substitute).[7] This quality degradation is captured by a. When $a \to 1$, the quality of the pirated content is poor, whereas when $a \to 0$, the quality of the pirated content gets closer to the original. Second, *horizontal differentiation* arises because a user may have strong preferences for the legal CP or the pirate CPs.[8]

The utility of a user i is

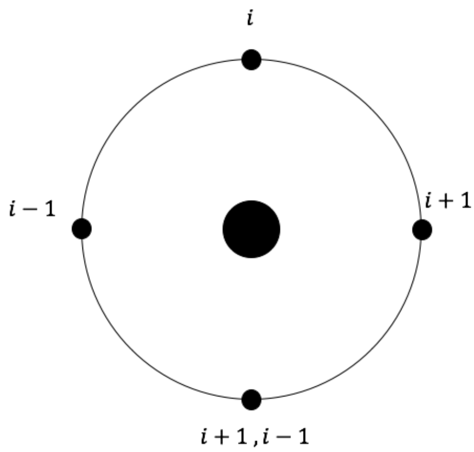
$$U_i = \begin{cases} v - p, & \text{if she subscribes to the legal CP,} \\ v(1-a) - s(N) - \gamma \cdot \frac{A^2}{2}, & \text{if she uses any pirate CP,} \end{cases} \tag{1}$$

where v the intrinsic utility that users obtain by watching the content and p is the subscription price set by the legal CP. It is assumed that the legal CP offers only a content. This is a simplification because legal SVOD platforms usually charge a flat price to have access to the entire catalogue. The user watching the pirate content experiences a quality degradation $a \in [0, 1]$ and bears some searching costs to get to any pirate CP. The searching costs are a concave function, $s(\cdot)$, with the following two properties: $s'(N) < 0$ and $s''(N) < 0$. In other words, the larger the number of pirate CPs, the easier it becomes for consumers to find the movie by just surfing different third-party websites or a search engine. To ensure the concavity of legal CP's profits and of the welfare function, we assume that $15\tau > -2N^3 \cdot s''(N) > 0$. It implies that there should be a certain degree of differentiation between the pirate CPs. Finally, pirate users are exposed to invasive ads, where $\gamma \in [0, 1]$ presents the annoyance parameter for each ad, A, chosen by the pirates. Note that the term is quadratic to capture the increasing invasiveness (Datta & Madio 2018).

[7] Quality degradation is common in the literature on digital piracy. See e.g., Bae & Choi (2006) and, more recently, Kim et al. (2018).

[8] Pirate providers usually contain different types of content, ranging from shows, movies, series, etc, thus offering a wider catalogue with respect to any other legal platform. Users may have some preferences for these CPs also when the legal CP's price is at zero. See e.g., Godinho de Matos et al. (2017).

Figure 2.1: Pirate and Legal Content Providers' Locations



The legal CP lies at the centre. There are $N = 4$ pirate CPs symmetrically distributed on the perimeter of the circle.

The Pirate CPs

To derive the demands of the legal and pirate CPs, Bouckaert (2000) is followed. There are two indifferent users between every two pirate CPs located on the perimeter of the circle. We consider an economy with $N \geq 2$ pirate providers.

- Let $y \leq N$ be the location of the consumer indifferent between two pirate providers located on the circle,

$$v(1-a) - s(N) - \gamma \cdot \frac{A^2}{2} - \tau \cdot y = v(1-a) - s(N) - \gamma \cdot \frac{\hat{A}^2}{2} - \tau \cdot \Big(\frac{1}{N} - y\Big), \quad (2)$$

  with $\hat{A}$ the (average) level of ads chosen by the neighbouring providers and τ representing the transportation cost per unit of distance in the pirate ecosystem. It does capture the degree horizontal differentiation between any two pirate CPs. The indifferent user is located at $y = \gamma \cdot (\hat{A}^2 - A^2)/(4\tau) + 1/(2N)$. Note that $y = 1/(2N)$ when providers are symmetric.

- Let z be the location of the user indifferent between a pirate and the legal CP located at the centre, with ϕ measuring the degree of substitutability between any pirate and the legal provider (i.e., horizontal differentiation parameter). It follows that:

$$v(1-a) - s(N) - \gamma\frac{A^2}{2} - \tau \cdot z = v - p - \phi. \tag{3}$$

The indifferent consumer is located at $z = (\phi + p - a \cdot v - s(N) - \gamma \cdot A^2/2)/\tau$.

It is important to discuss the relationship between ϕ and τ, which captures somewhat similar aspects. τ represents a proxy of the intensity of the competition in the piracy ecosystem and, indeed, the transportation cost per unit of distance incurred by a user to move from its position to the closest CP. Instead, ϕ measures the degree of differentiation between the legal CP and any pirate CPs: its value is constant as considering the distance from the centre to any point on the circumference. As normally preferences against the copyright holder are sufficiently weak, we assume that $s(N) + a \cdot v > \phi$. This implies that the cost for choosing the legal CP is sufficiently low relating to the competitive disadvantage of the pirates, e.g., degradation cost and quality degradation.

Two extreme scenarios arise. First, for $z > y$, it is always too costly for a user to subscribe to the legal CP, so all users choose the pirate CPs. In this case, the model resembles the traditional Salop circle without a centre. However, this scenario is excluded as a market dominated only by pirate CPs is unrealistic: digital piracy exists because of the existence of the legal producer. Hence, the analysis is limited to when $N < N^{max}$ where N^{max} is such that the legal CP gets bankrupt.

Second, when the legal CP sets a sufficiently low subscription price, it is on the condition to serve the entire market. In other words, it drives all the pirate CPs out of the market (i.e., exit) or leaves them temporarily "passive" (i.e., shut down). This case is analysed in turn. An interior and more interesting case arises when $z \leq y$. In this case, digital piracy exists with a positive demand and the legal CP sets an intermediate price. The demand for each pirate CP is $2z$, as shared with the legal provider at the centre and its closest neighbours located on its right-hand and left-hand sides. These case are represented as follows:

$$D^P_= \begin{cases} 0, & \text{if } p < \Theta, \\ 2z & \text{if } \Theta \leq p \leq \frac{\tau}{2N} + \Theta, \\ 2y, & \text{if } p > \frac{\tau}{2N} + \Theta. \end{cases} \tag{4}$$

With a little laxity of notation, we denote $\Theta = \gamma \cdot \frac{A^2}{2} + a \cdot v + s(N) - \phi$.
The profits of each pirate CP $i = 2...N$ are:

$$\Pi_i^P(A, p, N) = D_i^P \cdot A \cdot r - K, \tag{5}$$

where r indicates the exogenous price per impression. We assume that the pirate CPs are relatively small and not able to determine the price per ad (see e.g., Chang & Walter 2015, Datta & Madio 2018, Gritckevich et al. 2018). Without loss of generality, it is assumed that $r = 1$. The pirate CPs set the number of ads displayed on the user screen, where $A^* = \text{argmax } \Pi^P(A, p, N)$. This is in contrast to Chang & Walter (2015), who take the decision on the level of ads as given. On pirate CPs, the number of ads is not constrained by the dimension of the screen as in peer-to-peer networks. First, ads come out in the form of pop-ups and pop-unders, opening several browser windows simultaneously. Second, ads can interrupt at any time the streaming of the content in the form of mid-roll ads. In other words, the pirate CP can strategically decide how to monetise user eyeballs. For the sake of simplicity, throughout the analysis, we assume that $K = 0$. Although fixed costs for running the provider might be relevant (Datta & Madio 2018), their inclusion in the current analysis is meaningless.

The Legal CP

To derive the demand for the legal CP, let us call $\hat{z}$ the user indifferent between watching a content on the legal CP and on one of the many pirate CPs, i.e., $v \cdot (1 - a) - s(N) - \gamma \cdot \frac{A^2}{2} - \tau \cdot \hat{z} = v - p - \phi$. As the pirate CPs are symmetrically distributed on the perimeter of the circle, the distance between any two indifferent users is $\hat{z} = 1/(2N)$. So, the demand for the legal CP is[9]:

$$D^L = \begin{cases} 0, & \text{if } p > \frac{\tau}{2N} + \Theta, \\ 1 + \frac{2N}{\tau} \cdot \left(a \cdot v + s(N) + \gamma \cdot \frac{A^2}{2} - p - \phi \right) & \text{if } \Theta \leq p \leq \frac{\tau}{2N} + \Theta, \\ 1, & \text{if } p < \Theta. \end{cases} \tag{6}$$

Given the above demand function, the profits are expressed as follows

$$\Pi^L(p, A, N) = D^L \cdot p. \tag{7}$$

2.4 Analysis

The timing is as follows. In the first stage, the pirate and legal CPs set the number of ads and the subscription price simultaneously and non-cooperatively. Then, in the second stage users make their decision on the content provider. On the basis of the equilibrium values, we study the effects of changes in the pirate ecosystem (e.g., entry and exit of pirate CPs) and we derive the implications for the optimal number of pirate CPs from the perspective of the legal CP and of the government. By using the demand functions previously derived and rearranging the first-order conditions, the best replies are reported by the following lemma.

Lemma 1. *The best replies of the pirate CP $i =: 1, ..N$ and of the legal CP are*

$$\begin{aligned} A(p, s(N)) &= \left\{ \frac{2(\phi + p - a \cdot v - s(N))}{3\gamma} \right\}^{1/2}, \\ p(A, s(N)) &= \frac{s(N) + a \cdot v - \phi}{2} + \gamma \cdot \frac{A^2}{4} + \frac{\tau}{4N}. \end{aligned} \tag{8}$$

Prices and ads are strategic complements.

Note that the level of ads does not *directly* depend on N, the density in the pirate segment, but it does only through the abatement of searching costs. The annoyance cost γ mediates the reaction of the pirates, which shifts downward when ads get more invasive.

[9] Note that, in equilibrium, we have $\tau > 2N \cdot (a \cdot v - \phi + s(N))$ in order to focus on real levels of ads. When this condition is satisfied, the demand when providers compete cannot be greater than the unit, which represents the highest possible demand. In other words, it ensures that $D^L < 1$.

Consistently with Datta & Madio (2018), prices and ads are strategic complements: there is an upward adjustment in the level of ads, generally occurring when the legal CP raises its price, and a downward change when the legal provider's price is reduced. The following the lemma summarises the equilibrium outcome.

Lemma 2. *In equilibrium, the level of ads set by the each pirate CP and the legal CP's subscription price are*

$$A^* = \left\{\frac{2N(\phi - a \cdot v - s(N)) + \tau}{5N \cdot \gamma}\right\}^{1/2}, \qquad p^* = \frac{2(s(N) + a \cdot v - \phi)}{5} + \frac{3\tau}{10N}. \tag{9}$$

The demands are

$$D_i^{P*} = \frac{1}{5\tau}\Big(4(\phi - a \cdot v - s(N)) + \frac{2\tau}{N}\Big), \qquad D^{L*} = \frac{N}{5\tau}\Big(4(a \cdot v + s(N) - \phi) + \frac{3\tau}{N}\Big). \tag{10}$$

Their profits are

$$\begin{aligned} \Pi_i^{P*} =& \frac{2}{5\tau\sqrt{5\gamma} \cdot N^{3/2}}\Big(2N(\phi - a \cdot v - s(N)) + \tau\Big)^2, \\ \Pi^{L*} =& \frac{1}{50N \cdot \tau}\Big(4(a \cdot v + s(N) - \phi) + \frac{3\tau}{N}\Big)^2. \end{aligned} \tag{11}$$

2.5 Effects of Entry

On the basis of the equilibrium solutions, this section provides an analysis of the changes occurring in the pirate ecosystem when a pirate CP enters or exits the market.

Proposition 1. *Let $\bar{N}$ be such that $-\bar{N}^2 \cdot s'(\bar{N}) = \tau/2$, (i) the relationship between the number of pirates and the level of ads is U-shaped; (ii) the minimum level of ads is at $N = \bar{N}$; and (iii) the price curve is downward sloping.*

Proof. See Appendix. □

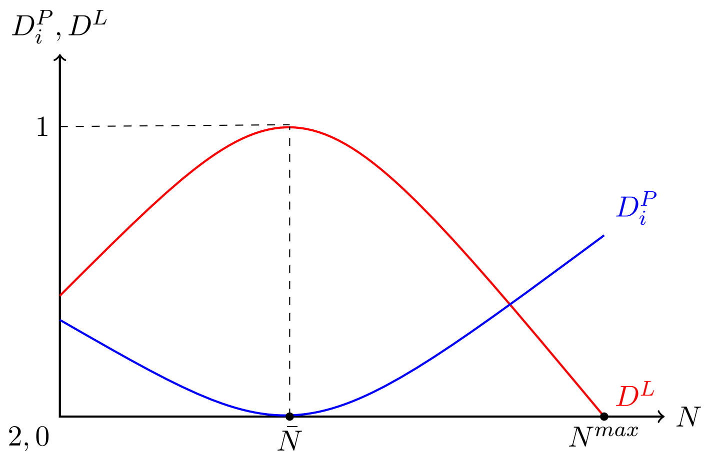
Because of searching costs, a denser piracy ecosystem impacts asymmetrically on the subscription price and on the level of ads. Specifically, the legal CP reacts to a new entry (exit) by reducing (raising) the subscription price. Two effects are instead at the stake for the pirate CPs. On the one hand, because of the strategic complementarity between the level of ads and the price, an entry in the pirate ecosystem leads also the pirate CPs to reduce the number of ads, i.e., $\frac{\partial A(p)}{\partial p}\frac{\partial p}{\partial N} < 0$. On the other hand, because searching costs are reduced when a marginal pirate CP enters the pirate ecosystem, piracy gets more attractive and the competition between the legal and pirate CPs is softened. So, the CPs can increase the number of ads displayed to their users. In a small pirate ecosystem i.e., $N < \bar{N}$, because the reduction of searching costs is narrow, pirate CPs find it optimal to reduce the number of ads. By contrast, in a sufficiently large ecosystem, the searching costs wane significantly, so the pirate CPs inflate the number of ads shown to their users.

Proposition 2. *(i) The demand curve of each pirate CP is U-shaped with N; (ii) The demand curve of the legal CP is hump-shaped with N; (iii) At $N = \bar{N}$, the legal CP serves the entire market, i.e., $D_i^P|_{N=\bar{N}} = 0$ and $D^L|_{N=\bar{N}} = 1$.*

Proof. See Appendix. □

In a small pirate ecosystem, the reduction of the subscription price sustains the demand expansion of the legal CP. Paradoxically, more pirate CPs in the market cause a reduction of the aggregate pirate demand as a consequence of the market interactions. Hence, in the interval $N \in (2, \bar{N}]$, the pirate CPs suffer from an entrant, while the legal CP has rooms for conquering additional market shares. At $N = \bar{N}$, despite the presence of a certain number of pirate CPs, the legal CP serves the entire market and exploits its full market power. This result resembles that of Kim et al. (2018), who find that, in some cases, the presence of digital piracy may turn into a "win-win-win" situation. In other words, the legal CP can disincentivise digital piracy by engaging in a limit pricing strategy, the government can curtail pirates' revenues of the CPs without any copyright enforcement, and consumers can experience a low subscription price for the original product. Importantly, this endogenous monopolisation of the market comes as a consequence of additional entry in the market. However, as we show later, this is neither a private nor a social optima as the the price reduction of the copyright holder does not get compensated by the additional demand: hence profits monotonically go down. Interestingly, this can represent an equilibrium number of pirate providers. As we discuss below, pirate providers would stop entering the market at $N = \bar{N}$ as they would no longer enjoy

Figure 2.2: Effects of N on the demands



The above graph depicts the hump-shaped relation between the legal CP's demand and the number of pirate CPs in the pirate ecosystem and the U-shaped relation of the individual pirate CP's demand and profits.

strictly positive profits. This equilibrium can be stable unless either pirate providers are sufficiently forward looking to coordinate their behaviour and indeed be themselves in the interval $N \in (\bar{N}, N^{max})$ or an invisible hand move them in that scenario. In both these cases, they would therefore benefit from additional entry. Specifically, searching costs would be smaller and smaller and so the pirate CPs would enhance their attractiveness, thereby generating a pro-piracy effect at the expense of the legal CP.

We note that a condition for a maximum in the legal CP's demand is $a \cdot v - \phi = -s(N) - N \cdot s'(N)$. This term can be represented as follows:

$$a \cdot v - \phi = -\frac{\partial}{\partial N}\Big(s(N) \cdot N\Big), \tag{12}$$

where the term $s(N) \cdot N$ indicates the total searching costs. This expression shows that when the quality degradation of the pirate CP net of the cost of going to the legal CP equals the marginal change in total searching costs, the legal CP gets the largest demand. This indicates that as long as vertical differentiation net of user preferences for piracy are larger than the marginal change on total searching costs, the legal CP can increase its attractivess and endogenously fight piracy. Else, the legal CP is in the region where pirate providers are expanding their market benefiting from the reduced searching costs.

These results suggest that, in a static world, having $\bar{N}$ CPs in the pirate ecosystem would induce the legal CP into a limit pricing, serving entirely the market but with a sufficiently low price. Rather paradoxically, the presence of concave searching costs incurred by pirate users generates more demand expansion for the copyright holder as other pirate CPs enter the market. We may speculate about the behaviour of pirate CPs. On the one hand, if pirate CPs were *myopic*, they would stop entering the market at $N = \bar{N}$, and so this would be a potential equilibrium number of pirate providers. On the other hand, if pirate CPs could either coordinate themselves or were sufficiently forward-looking and let other pirates enter the pirate segment, they would again be in a situation of positive profits and on the upward sloping part of their demand-, ad-, and profit-functions: a pro-piracy effect. And, indeed, an equilibrium would emerge just before the legal CP gets bankrupt.[10] Here, it is important to remark that the legal CP may exit the the market for any $N = N^{max}$ such that $p^* = 0$. At this point, we would be back to the traditional Salop model.

An immediate consequence of the above result is presented by the following proposition.

Proposition 3. *At $\bar{N}$, (i) the legal CP set $p^*|_{N=\bar{N}} = a \cdot v + s(\bar{N}) - \phi$; and (ii) pirate CPs shut down, i.e., $A^*|_{N=\bar{N}} = 0$ and $\Pi_i^P|_{N=\bar{N}} = 0$.*

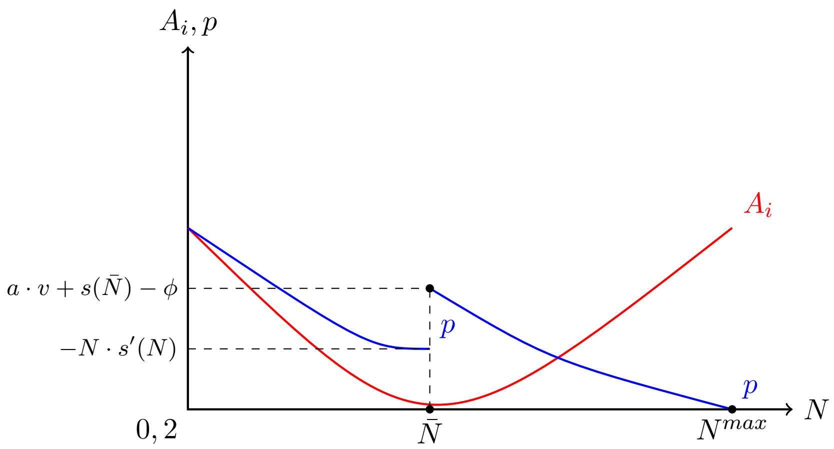
Proof. See Appendix. □

Specifically, the legal CP can be found endogenously in the position to dominate the market as a consequence of additional entry and exploiting the competitive disadvantage of the pirates. In this case, the copyright holder sets the highest possible price to keep its entire demand and this is represented by $p^*|_{N=\bar{N}} = a \cdot v + s(\bar{N}) - \phi > 0$. This price has two important properties. First, it shows that the price equals the vertical differentiation existing in the market and represented by $a \cdot v - \phi$. Second, the price also considers the "remaining" searching costs paid by consumers with $N = \bar{N}$.

Lemma 3. *The profits of the copyright holder monotonically decline with more entry in the market in the interval $N \in [N, \bar{N})$.*

[10]Similar results also apply when positive fixed costs are introduced whenever the pirate CPs can sustain temporary negative profits at $N = \bar{N}$.

Figure 2.3: Effects of N on the level of ads and price



The above graph depicts the effect on the price when N increases, showing a downward sloping curve. It also shows the effect on the ads flow depicting a U-shaped relation with N. note that at $N = \bar{N}$ there is a discontinuity and the legal CP charges $p^* = a \cdot v - \phi + s(\bar{N})$.

Proof. See Appendix. □

The intuition behind this result is rather simple. Any demand expansion effect (whenever present) is dominated by the strategic (price) effect. So, profits of the copyright holder monotonically go down with additional entry in the market in the interval $N \in [N, \bar{N})$. Hence, the legal provider would prefer to restrict the competition to few providers though commercially active rather than engaging in price reduction. Differently, as previously discussed, at $N = \bar{N}$, additional entry in the market would make the copyright holder to serve the entire population and so increase again its profits.

Lemma 3 illustrates a clear misalignment of the interests of the legal CP and of a government aiming at minimising the revenues for those infringing intellectual property rights. When the government, without any court intervention, satisfies its goal of blocking all revenues for the pirate CPs (i.e., $N = \bar{N}$), the legal CP serves the entire market but at a sufficiently low price. However, as shown later, whereas the lucrative activity is curbed, this is neither what a government interested in maximising the social welfare would prefer to have in the market. This stark contrast between the interests of the two economic agents becomes more important when discussing the commercial nature of digital piracy. Because the legal CP finds it optimal to tolerate a certain number of digital

CPs $N^* \in [2, \bar{N})$, it implies that the legal CP would prefer to minimise the presence of piracy even though granting them a commercial dimension (e.g., revenues from ads).

2.6 Exogenous Searching Costs

The searching costs are crucial for the above results. In this section, a robustness check of our analysis is provided by assuming that searching costs are uncorrelated with the number of pirate CPs, i.e., $\partial s(.)/\partial N = 0$. In this case, as in Lamantia & Pezzino (2016), both types of CPs would suffer from the increased competition.

Proposition 4. *When the number of pirate CPs increases and searching costs are independent of it,*

(i) the price and the ads curves are downward sloping;

(ii) the demand of each pirate CP is always reduced;

(iii) the demand of the legal CP always increases with more piracy;

(iv) the profits of legal and pirate content providers are reduced.

Proof. See Appendix. □

All incumbents, regardless of their type, suffer from the competitive pressure exerted by the entrant. On the one hand, both types of CP react by reducing the level of ads and the subscription price. On the other hand, the CPs react in a different manner. While the pirate CPs face a new direct competitor and lose their market shares, the legal CP can attract some users. When the legal CP has a significantly large competitive advantage over the pirate CPs (i.e., $a \cdot v + s > \phi$, that is, quality of the pirate content is very poor), there is a market expansion. However, the profits of the firm located at the centre of the circle (i.e., the legal provider) are harmed. Because the legal CP responds by reducing the subscription price, any demand effect does not compensate for the reduction in the marginal revenues.

2.7 Effects of Competition in the Pirate Ecosystem

So far, changes in the pirate ecosystem have taken the form of changes in the number of pirate CPs. A pirate ecosystem, however, can be affected by a lower degree of product differentiation among the pirate CPs. Here, we study what happens when the competition in the pirate ecosystem gets fiercer and how this impacts on the strategies implemented by the legal CP. A user, for instance, may be almost indifferent to the pirate CPs as long as they provide him with the content at no cost.

Because the pirate CPs react to a more intense competition by reducing the price, also the legal CP reduces the price as well because of the strategic complementarity. The effect on the demands are as followsτ:

$$\begin{aligned} \frac{dD^{L*}}{d\tau} &= -\frac{4N \cdot (a \cdot v + s(N) - \phi)}{5\tau^2} \\ \frac{dD^{P*}}{d\tau} &= \frac{4(a \cdot v + s(N) - \phi)}{5\tau^2} \end{aligned} \tag{13}$$

Hence, the *strategic effect* (e.g. $-dp^*/d\tau$) is accompanied by a demand expansion, which ultimately lowers the impact of digital piracy. Individually, pirate CPs face lost demand. However, as in the above previous cases, the reaction-reduction of the subscription fee does not prevent demand losses. Indeed, a fiercer competition amongst pirate CPs generates a pro-piracy effect.

Proposition 5. *As the competition for pirate consumers becomes stronger ($\tau \to 0$), i) price and ads decrease, ii) the legal (pirate) CP's demand increases (decreases).*

Proof. It follows from the above discussion. □

Overall, irrespective of any demand effect, profits are reduced. The strategic effect of reducing the level of ads and the subscription price always dominates both the pro-piracy effect and the expansion of the legal CP. Indeed, competition in the pirate ecosystem creates (negative) spillovers also affecting the copyright holder.

Proposition 6. *Fiercer competition ($\tau \to 0$) lowers CPs' profits regardless of their type.*

Proof. See Appendix. □

2.8 Concluding Remarks

The recent development of the Internet and of ad-based business models has transformed digital piracy into a mechanism to derive illicit money while damaging the copyright holder. This paper offers a comprehensive analysis of the strategic interactions between legal and pirate content providers in a pirate ecosystem. It delves into the nature of this new type of digital piracy and emphasises the relevance of searching costs in making piracy attractive for many users. We ask the question of whether and to what extent a government may tolerate some degree of digital piracy. Our results show that both the government and the legal CP may find it optimal to tolerate some degree of commercial digital piracy despite the fact that their interests are never aligned. The copyright holder can benefit from a positive number of pirates as the associated price reduction would grant some additional demand and increase its profits provided that the pirate ecosystem is sufficiently low. The fact that the legal CP sets a price depending on how developed pirate CPs are is also consistent with a recent statement of Netflix's CFO "*Piracy is a governor in terms of our price in high piracy markets outside the US [...]. We wouldn't want to come out with a high price because there's a lot of piracy, so we have to compete with that*".[11] This result also suggests that the magnitude of the reaction of the legal CP may be correlated with the wealth of a country, whereby poor countries are those with the greatest number of pirate websites.[12]

Our results also suggest tha the government may be willing to tolerate more commercial piracy than a pirate provider, thereby not intervening when too much entry occurs in the market. Digital piracy can be seen as a way of limiting the market power of the copyright holder by offering users a low-quality alternative. However, our results do not allow to establish directly the socially optimal number of pirates. On the one hand, with $N = 2$, the legal provider can exploit some degree of market power and charge consumers a high price. Similarly, few pirate providers can display a large number of ads to con-

[11] See e.g. Business Insider, 'Netflix has an ingenious, piracy-combating way to set its international pricing', April 17, 2015: `http://uk.businessinsider.com/netflix-piracy-international-pricing-streaming-earnings-2015-4`.

[12] On the development of digital piracy in poor countries, see e.g., The Guardian, 'Why poor countries lead the world in piracy', May 3, 2011: `https://www.theguardian.com/technology/2011/may/03/why-poor-countries-lead-world-piracy`.

sumers. Both strategies reduce welfare of consumers. On the other hand, with $N = \bar{N}$, the legal provider can serve the entire market at a very low cost. Clearly, this is a blessing for consumers but not for the legal provider and for the pirates. Interestingly, an equilbrium with this number of providers would never arise as not strictly profitable for a pirate. From a policy perspective, our conjecture is that a social optimal number of pirates lie in the interval $N \in [2, \bar{N}]$, thereby suggesting that the government may find it optimal to tolerate some degree of digital piracy is not new in the literature (see e.g., Peitz & Waelbroeck 2006*b* find some sampling , with users learning about the product in a pirate environment.

In light of the above results, our analysis offers some implications for policy-makers to develop effective anti-piracy measures devoted not only to fight digital piracy but also to block criminals via "following the money" initiatives. In Spain, the Court recently ordered that Rojadirecta be shut down because of the violation of property rights (held by Movistar+) for the live streaming of sport events; [13] in Italy, the Communications Authority (Agcom) proceeded against Nowvideo for hosting contents under the copyright of Sky Italy; the UK High Court ordered a stoppage of streaming live matches of the Premier League when violating copyrights. If the goal of the government is not to maximise the social welfare but to block the commercial nature of digital piracy, our model surprisingly predicts that a *laissez-faire* policy should be pursued letting some other pirate CP enter the market (up to $N = \bar{N}$) and intervene only when the pirate ecosystem gets sufficiently large. This situation will trigger the reaction of the legal CP, who setting a sufficiently low price (may) leave the pirate CPs without demand and ads. In this sense, market instruments might be more effective in reducing piracy than courts and regulators, unless the enforcement of property rights leads to the simultaneous closure of all pirate websites. In the latter case and in sufficiently large pirate ecosystems instead authorities and courts' interventions would appear beneficial for the copyright holder.

From a managerial perspective, our results suggest that the legal CP may find it optimal to mimic how pirate CPs work and take advantage of that. As some users may prefer a low-quality and ad-based alternative, the legal CP could try to launch a

[13] In Spain, football distributors lost approximately 271 million of euro because of piracy. See e.g., La Coalicion, 'Observatory 2016: Piracy down 4%', April 25, 2017: `http://lacoalicion.es/2017/04/25/piracy-down-4/`.

subsidiary service to serve this market segment or differentiate its services. This is, for instance, what Spotify has tried to do by differentiating its Premium version from the Free and ad-based version. Similar but somewhat different strategies are also pursued by Netflix in the SVOD market, by offering a menu price to its consumers according to the quality of the service and the number of accounts registered. Such a strategy may effectively work in a small pirate ecosystem also benefiting from the zero search costs that the dominant firm incurs.

It is worth mentioning that our framework could also have a more general applicability other than to markets where piracy is relevant. For instance, our setting can be applied to the newspaper market, where, for instance, a dominant subscription-based newspaper (e.g., The Wall Street Journal) with reputable and recognized journalists competes over a single news against a large number of (symmetric) websites simply reporting or slightly re-editing press reports (i.e., quality degradation) and making revenues from annoying ads. In this market configuration, our model predicts that, as long as search costs incurred in order to get to these not-so-famous local websites for having almost the same information (as the one offered by the WSJ) are concave and decreasing, the dominant newspaper would find it optimal to have a positive but sufficiently small number of low-quality competitors. Similarly, our model would predict that, in the case of exogenous search cost, e.g. fast and simple access to these websites on search engines, all websites, regardless of whether subscription- or ad-based, would be harmed by the entry of a new content provider.

Appendix

Proof of Proposition 1

To prove Proposition 1,

(i) Differentiate the equilibrium level of ads with respect to N to obtain

$$\frac{dA^*}{dN} = -\frac{\tau + 2N^2 \cdot s'(N)}{2N^{3/2}\sqrt{5\gamma\{2N(\phi - a \cdot v - s(N)) + \tau\}}} \tag{14}$$

As the denominator is always positive, it shows that $dA^*/dN = 0$ as long as the numerator is equal to 0. The latter term is either a minimum or a maximum for $-\bar{N}^2 \cdot s'(\bar{N}) = \tau/2$. Let $\bar{N} > 2$ be such that $-\bar{N}^2 \cdot s'(\bar{N}) = \tau/2$, then $dA^*/dN > (<)0$ for $-N^2 \cdot s'(N) > (<)\tau/2$. Since by assumption A is continuous and differentiable, the slope is strictly monotonically increasing for $N > \bar{N}$ and strictly monotonically decreasing for $N < \bar{N}$. Indeed, the ads curve is U-shaped with N.

(ii) Since $dA^*/dN > (<)0$ for $N > (<)\bar{N}$, $N = \bar{N}$ is a minimum.

(iii) Consider the effect of N on p^*,

$$\frac{dp^*}{dN} = -\frac{3\tau}{10N^2} + \frac{2s'(N)}{5} < 0 \tag{15}$$

The price curve is convex in N as $\frac{1}{dN}\left(\frac{dp^*}{dN}\right) = \frac{3\tau}{N^3} + \frac{2s''(N)}{5} > 0$ when τ is sufficiently large as we assume, e.g. $15\tau > -2N^3 \cdot s''(N) > 0$.

Proof of Proposition 2

(i) Differentiate (10) with respect to N

$$\frac{dD_i^{P*}}{dN} = -\frac{2}{5N^2} - \frac{4s'(N)}{5\tau} \tag{16}$$

which implies that $dD_i^{P*}/dN > (<)0$ for $-\bar{N}^2 \cdot s'(\bar{N}) > (<)\tau/2$. Using $\bar{N}$ (Proposition 1), it must be that the demand of each pirate CP goes down (up) for $N < (>)\bar{N}$. Given continuity and monotonicity, this suffices to establish that the demand curve is U-shaped in N. Again, it can be checked that the function is convex as $\frac{d}{N}\left(\frac{dD_i^{P*}}{dN}\right) = \frac{4}{5}\left(\frac{1}{N^3} - \frac{s''(N)}{\tau}\right) > 0$.

(ii) Differentiate the demand of the legal CP

$$\frac{dD^{L*}}{dN} = \frac{4}{5\tau}\Big(a \cdot v + s(N) - \phi + N \cdot s'(N)\Big) \tag{17}$$

which is positive for $a \cdot v + s(N) - \phi + N \cdot s'(N) > 0$. Note that the competitive advantage for the legal CP (i.e. $a \cdot v + s(N) > \phi$) is reduced with additional entry in the pirate ecosystem. The function is concave as $\frac{d}{N}\left(\frac{dD^{L*}}{N}\right) = \frac{4}{5}\Big(-\frac{1}{N^3} + \frac{s''(N)}{\tau}\Big) <$

0. Hence, the demand is a hump-shaped, strictly monotonically increasing in the interval $N \in (2, \bar{N}]$ and strictly monotonically decreasing for $N \in [\bar{N}, N^{max})$

(iii) Next, evaluate D^L at $N = \bar{N}$. Using $\tau = -2\bar{N}^2 \cdot s'(\bar{N})$, then

$$D^L|_{N=\bar{N}} = -\frac{1}{10\bar{N} \cdot s'(\bar{N})}\Big(4(a \cdot v + s(\bar{N}) - \phi) - 6\bar{N} \cdot s'(\bar{N})\Big) \tag{18}$$

Consequently, the demand for each pirate CP is

$$D_i^P|_{N=\bar{N}} = -\frac{4}{5\bar{N}^2 \cdot s'(\bar{N})}\Big(\phi - a \cdot v - s(\bar{N}) - \bar{N} \cdot s'(\bar{N})\Big) \tag{19}$$

Suppose at $N = \bar{N}$ both types of CPs are active, i.e., $D_i^P > 0$ and $D^L > 0$. First, suppose $D_i^P|_{N=\bar{N}} > 0$, it then must be that $\phi - a \cdot v > s(\bar{N}) + \bar{N} \cdot s'(\bar{N})$. However, by (17), a condition for D^L to be a maximum is $a \cdot v - \phi = -s(\bar{N}) - \bar{N} \cdot s'(\bar{N})$. Indeed, a contradiction, the individual pirate demand cannot be positive. Substitute it into the demand functions, it follows that $D_i^P|_{N=\bar{N}} = 0$. So, at $N = \bar{N}$ the legal CP serves the entire market.

Proof of Proposition 3

First, note that at $N = \bar{N}$, the legal CP serves the entire market. Hence, by equation (6), $p^* = a \cdot v + s(\bar{N}) - \phi > 0$. Also note that at this price $A^*_{N=\bar{N}} = 0 \Leftrightarrow \Pi_i^P|_{N=\bar{N}} \leq 0$.

Proof of Lemma 3

Differentiate (11) with respect to N:

$$\frac{d\Pi^{L*}}{dN} = \frac{p^*\Big(4N \cdot (a \cdot v - \phi + s(N) + 2N \cdot s'(N)) - 3\tau\Big)}{5N \cdot \tau}, \tag{20}$$

Note that the second term is negative as violating the condition such that $p^* \geq 0$. Hence, profits are always declining with entry. Hence, the optimal number of pirate providers is $N^* = \min[N]$, which in our model is 2. This also implies that profits decline when serving the entire demand. To illustrate it, consider that, from Proposition 2, $dD^L/dN = 0 \Leftrightarrow a \cdot v - \phi = -s(N) - N \cdot s'(N)$, whereas $N = \bar{N}$ is such that $\tau = -2N^2 \cdot s'(N)$ (Proposition

1). Substituting them into the above expression, we obtain

$$\frac{d\Pi^{L*}}{dN}\bigg|_{N=\bar{N}} = -\frac{p^*\Big(4\bar{N}(\bar{N}\cdot s'(\bar{N})) + 6\bar{N}^2\cdot s'(\bar{N})\Big)}{10\bar{N}^3\cdot s'(\bar{N})} = -\frac{p^*}{\bar{N}} = s'(\bar{N}) < 0 \tag{21}$$

So, profits are decreasing at $N = \bar{N} + 1$. From the second derivative at that point, $\frac{d}{dN}\left(\frac{d\Pi^{L*}}{dN}\right) = s''(\bar{N})$, so the function is concave.

Proof of Proposition 4

To prove Proposition 4, it suffices to substitute $s'(N) = 0$ in all the equations. To prove the fourth part (iv), the derivative of $\Pi_P/N < 0$ always. Instead, the derivative $\Pi_L/N < 0$ for any $0 < D_L \leq 1$. Indeed, all profits go down.

Proof of Proposition 6

(i) Consider the effect on the pirate CPs. Differentiate equation (11) with respect to τ

$$-\frac{d\Pi_i^{P*}}{d\tau} = -\frac{A^*\Big(4N(a\cdot v + s(N) - \phi) + \tau\Big)}{N^2\cdot\tau^2} \tag{22}$$

Recall that $A^* \geq 0$. For the sake of simplicity, call $(a\cdot v + s(N) - \phi) = \Delta$. Recall that $\tau > 2N\Delta \equiv E$ to have a real A^*. We now prove that the above expression is negative by contradiction. Suppose $-d\Pi_i^{P*}/d\tau > 0 \Leftrightarrow \tau < -4N\Delta \equiv B$. However, because $B < E \Leftrightarrow -4N\Delta < 2N\Delta$, it follows that $\tau < B$ violates the condition such that $\tau > E$, thus a contradiction. Indeed, the effect on the pirates' profits cannot be positive, therefore $-d\Pi_i^{P*}/d\tau < 0$.

(ii) Consider the effect on the legal CP

$$-\frac{d\Pi^{L*}}{d\tau} = \frac{\Big(4N(a\cdot v + s(N) - \phi) - 3\tau\Big)p^*}{10N\tau} \tag{23}$$

Recall that $p^* \geq 0$. Suppose $-d\Pi^L/d\tau > 0 \Leftrightarrow \tau < -4/3N\Delta \equiv B'$. Again, because $B' < E \Leftrightarrow 4/3N\Delta < 2N\Delta$, it cannot be that $\tau < B'$ while $\tau > E$. Indeed, the effect on the profits cannot be positive, therefore $-d\Pi^{L^*}/d\tau < 0$.

CHAPTER 3

Platform investments in two sided markets

3.1 Introduction

Value-increasing investments are common in digital markets. Digital companies often invest in user experience, better targeting, recommendation systems, algorithmic learning, wider compatibility, and efficiency. In a two-sided market, where platforms compete with one another on different sides of the market (e.g. sellers and buyers in marketplaces, developers and users for operating systems, mobile devices and voice assistants), the effect of investments may be puzzling. On the one hand, investments may directly impact on the utility of some users who appreciate extra features. On the other hand, as sides are interrelated, investments creating some value on one side of the market may generate an effect on the opposite side. Whether and how to engage in investment activities, and their effect on the market configuration, prices and surplus of users, are all issues which have not yet been satisfactorily determined.

The multidimensionality of investment has gained attention with the development of Artificial Intelligence (AI). In 2012, Amazon bought Kiva System (now Amazon Robotics) to automate and optimise the processes in its warehouses (efficiency gain). This acquisition turned out to be beneficial for consumers and merchants because of reduced waiting time. In a similar way, investments on clouding technology (e.g., Amazon, Google, Microsoft) had the two-fold benefit of increasing the opportunities for developers of applications (e.g., reduced build times) and users (e.g., storage capacity) while permitting tech platforms to gain momentum in the stock market.[1] In the market for voice assistants (e.g. Amazon Alexa, Google Home), investments devoted to the user and developer experience (e.g. new skills and large compatibility) were also associated with data collection.[2]

Along with the *possible* multidimensionality of investments arising between sides and also with the platform, any change occurring on one side of the market is likely to have an impact on the opposite side of the market. This propagation mechanism is intrinsic to two-sided markets as the interactions between sellers and buyers are mediated by cross-network externalities.[3] To study how investment decisions shape the relationship between sides depending on who obtains the highest return, the price set by platforms and whether there is an incentive to engage in investment activities, we build a model as in Armstrong (2006). We consider two competing platforms that operate in a fully covered market with users (sellers and buyers) distributed in a Hotelling fashion. Buyers and sellers interact with each other through the platform and obtain positive cross-network externalities. Buyers like the presence of many sellers (i.e., taste for variety) while sellers enjoy a positive benefit when interacting with buyers as final transactions are more likely to take place. Platforms choose their investment effort while setting a price on both sides of the market. When platforms invest they may create a value on both sides of the market and may partly appropriate their investments. Such a setting captures the eventual multidimensionality of investments. For instance, on the one hand, investments can increase the value for users through improved quality, better features and characteristics, greater compatibility

[1] See Investorplace, '4 Tech Stocks That Are Killing It in Cloud Computing' October 24, 2016: `https://investorplace.com/2016/10/4-tech-stocks-killing-it-cloud-computing/`. (Accessed 2018).

[2] *The Verge*, a tech media, recently titled "*Amazon's Echo Look does more for Amazon than it does for your style*" providing an example of how current technologies can transform into data and learning activities the value created for its users. See e.g. `The Verge, July 6, 2017: https://www.theverge.com/2017/7/6/15924120/amazon-echo-look-review-camera-clothes-style`. (Accessed 2018). Similar strategies were also pursued by Apple, with its investments in HealthKit and ResearchKit or by Instagram and Facebook with their *Stories*, which enhance the users' experience while collecting personal information to be monetised via targeted ads. These projects allow consumers to monitor their health conditions, developers and public institutions to access detailed information about patients and to develop new applications (e.g. Medopad), and the company itself by gaining access to patients' lifestyles for its research projects. See e.g. The Financial Time, 'Consumer healthcare platforms on the rise ', February 16, 2015: `https://www.ft.com/content/1147343e-9fd7-11e4-9a74-00144feab7de`. (Accessed 2018).

[3] See Belleflamme & Peitz (2015) for a textbook discussion. See also Caillaud & Jullien (2003), Rochet & Tirole (2003), Parker & Van Alstyne (2005), Armstrong (2006).

and experience. On the other hand, these investments might return to the platform as data collection, stimuli for new ideas, efficiency gains or stock consolidation. However, this does not need to be so as investments may also affect only one type of user (e.g., buyer) without generating any value for the platform and the other type of user (e.g., seller).

Two network configurations are considered. In the first configuration, sellers and buyers only join one platform. In the second configuration (the competitive bottleneck), while buyers join exclusively one platform, some sellers multihome while others singlehome. First, we find that, regardless of the network configuration, profits are always higher with investments. However, this can lead to an Investment-Prisoner Dilemma. When platforms are ex ante symmetric and all users singlehome, platforms are tempted to invest more to steal some users and increase the price level. The ability of users to internalise the investments induces platforms to invest more but, in equilibrium, any demand expansion effect is neutralised by the symmetry of the market. As a result, platforms cannot recoup their investments leading to an Investment-Prisoner Dilemma. This happens unless investments generate some direct returns for the platform.

Second, the presence of multihoming sellers can generate more incentives for investments. Absent investments, sellers face a lower price than buyers. In several cases, platforms prefer to set a negative or zero price on sellers and milk buyers. When investments take place, some (early) singlehomer sellers become multihomers as attracted by investments. As a result, platforms proportionally increase their price to sellers and reduce the price to buyers. That is, investments on the multihoming side of the market create the condition to (partly) restore a bottleneck effect typically present when all sellers multihome (see e.g., Armstrong 2006, Belleflamme & Peitz 2018). To this end, platforms can grab part of the surplus they generate. From a welfare perspective, we show that buyers are ultimately better off as they combine the access to more sellers with the price discount.

The paper also compares the two regimes relative to the level of investment. It shows that letting sellers multihome leads to greater investments when competition in that side of the market is sufficiently intense. When platforms are perceived less differentiated by sellers, these are more prone to multihome, ultimately expanding the market shares of each platform. As a consequence, to attract more sellers, platforms invest more relative to when all users singlehome. For the same reason, when platforms are sufficiently differentiated, investments contribute marginally less to increase market shares. Hence, platforms invest more with singlehoming sellers. These findings are also robust to the cases when investments are either side-specific or follow two separate investment flows, that is when investments returning into a direct benefit for the platform are separate from investments creating value for at least one side of the market.

The paper contributes to the literature on two-sided markets and investment decisions. Although digital platforms are particularly active in investment decisions, previous papers have mainly dealt with investments undertaken on one side of the market. For instance, Belleflamme & Peitz (2010) were the first to study the investment decision from the seller's perspective. They found that multihoming sellers have an incentive to innovate as long as the marginal contribution to the buyers' surplus is larger than their own. In a related study, Lin et al. (2011) introduced endogenous price competition among developers, studying their incentive to innovate. They demonstrated that, for dispersed willingness-to-pay for quality, consumers might be subsidised, thus relaxing the intensity of competition among developers. Some other papers have discussed investments from the perspective of the platform but in specific industries, e.g., internet service providers (Njoroge et al. 2013), operating system (Casadesus-Masanell & Llanes 2015), payment systems (Verdier n.d.), or media (Li & Zhang 2016). Our work shares some similarities with Thomes (2015), who investigated investments into in-house games. He show that when developers multihome and gamers singlehome, a prisoner dilemma arises: both platforms found it optimal to invest but it harms their profits. An innovative enhancement is made by the current study to this literature: by comparing two different network configurations and by studying how prices are shaped by the investments and the surplus accruing to the different agents.

In characterising the investment decisions, this paper offers a more general perspective as it considers the value accruing to one or many economic agents simultaneously. While the model has a direct application for the operating system industry, where developers and consumers interact through a platform, the findings of this study can be generalised to other industries, such as marketplaces dealing with investments in machine learning. For instance, investments in AI promoted by Otto, a German marketplace, generated a value for different economic agents: consumers, for obtaining a product more closely customised to their tastes and reducing searching costs; sellers, for not having the product returned to the warehouse; and the platform, for not losing a transaction in the market and for collecting data.[4] In this regard, our approach is close to the literature on spillover effects and the seminal studies of d'Aspremont & Jacquemin (1988) and Kamien et al. (1992), but without considering forms of cooperation between platforms. In their models, two competing firms invest in a process innovation to reduce their marginal costs and their investments produce positive spillovers for the rival firm.

Finally, by examining how prices can be shaped by investment decisions when sellers multihome, this paper is very close to that of Belleflamme & Peitz (2018). The two authors showed that when some but not all sellers multihome, the platforms, under some circumstances, find it optimal not to set a monopoly price on this side of the market (as instead shown by Armstrong 2006) and the price could be even lower than the one charged when sellers singlehomed. In confirming this view, this paper presents two main results. *First*, it shows that, when only some sellers multihome, the platforms' pricing strategies resemble the strategies set in equilibrium by a platform monopolist with buyers charged a high price than sellers. In most cases, sellers are either subsidised or granted a free entry. Hence, the platforms do not set a monopoly price to provide access to exclusive buyers (bottleneck effect) but rather behave as a monopolist would do in a partially covered market. In the Appendix, the results with a monopolist are reported for the sake of comparison. *Second*, it finds that, because investments generate a demand expansion for both platforms on the multihoming side of the market, each platform's coverage increases and the bottleneck effect is partly restored. Hence, platforms increase the price on the seller side and reduce the price on the buyer side.

The remainder of the paper is structured as follows. Section 3.2 presents the preliminaries of the model while the duopoly case with singlehoming buyers and sellers is

[4]See e.g. The Economist, 'How Germany's Otto uses artificial intelligence', April 12, 2017: `https://www.economist.com/news/business/21720675-firm-using-algorithm-designed-cern-laboratory-how-germanys-otto-uses`. (Accessed 2017).

introduced in Section 3.3. Section 3.4 is a discussion of the results obtained with a competitive bottleneck while a comparison of the results of the two cases is reported in Section 3.5. Section 3.6 considers the case when investments on user experience and platforms monetisation follow two different cost functions, while Section 3.7 concludes the paper.

3.2 The Model

We consider two platforms that compete with one another. The market is fully covered on both sides where a unit mass of sellers s and a unit mass of buyers b make their transactions. To join platform $i = \{1, 2\}$, n_s^i sellers and n_b^i buyers pay a membership fee p_k^i, where $k = \{b, s\}$ identifies the side of the market which the price refers to. The two platforms are horizontally differentiated: platform 1 locates at the coordinate 0, while platform 2 at the coordinate 1 (opposite end). As in Armstrong (2006) and Belleflamme & Peitz (2018), the interaction between sides yield some positive cross-network externalities.[5] A buyer interacting with n_s^i benefit from a cross-group network effect by $\phi_b < 1$. By joining platform i, the buyer enjoys a total cross-group benefit of $\phi_b \cdot n_s^i$, which represents the taste for variety. On the opposite side of the market, a seller interacting with a buyer obtains $\phi_s < 1$, with a total cross-group benefit of $\phi_s \cdot n_b^i$ capturing the idea that sellers care about the number of buyers on the platform (i.e., potential revenues).

As the paper focuses on the strategic role of investments, we assume that when investment effort x^i takes place on platform i, this generates a value to both sids. The value generated on side $k \in \{b, s\}$ is denoted by $\beta_k \in [0, 1)$ (see also Table 1). The main body of the analysis considers a case where investments entail a value to both sides, but these can also be side-specific as considered in the extensions.[6] Hence, investments can be devoted to the seller side (e.g., developers) in the form of a better software development kit and better access to APIs. Alternatively, on the buyer side, these can be associated

[5] The paper assumes that there are no "within-side" network effects. To put it differently, sellers do not compete with other sellers over the same customer and buyers do not care about the decision of their peers. On network effects in two-sided markets, see e.g., Belleflamme & Toulemonde (2016), Belleflamme & Peitz (2019).

[6] When considering marketplaces, where sellers are active, investments undertaken by platforms might be devoted to increasing the probability that the item sold online is not returned to the warehouse, taking the form of better recommendation algorithms or reduced processing and waiting time. Note that investments might also be devoted to strengthening the cross-network externalities, thus increasing the perceived value of the interaction between sides.

with quality improvements, user experience and tailored results.

Finally, let λ_k with $k = \{b, s\}$ denote the position of each seller or buyer uniformly distributed along the interval $[0, 1]$ and $\tau_k > 1$ the transportation cost for unit of distance incurred by a user type-k. The utility of the buyers is:

$$u_b = \begin{cases} v + \beta_b \cdot x^i + \phi_b \cdot n_s^i - p_b^i - \tau_b \cdot \lambda_b, & \text{if joining platform } i \text{ located at } 0, \\ v + \beta_b \cdot x^j + \phi_b \cdot n_s^j - p_b^j - \tau_b(1 - \lambda_b), & \text{if joining platform } j \text{ located at } 1. \end{cases} \tag{1}$$

v is the standalone utility obtained by buyers when joining the platform. On the seller side, the profits are:

$$\pi_s = \begin{cases} n_b^i \cdot \phi_s + \beta_s \cdot x^i - p_s^i - \tau_s \cdot \lambda_s, & \text{if joining platform } i \text{ located at } 0, \\ n_b^j \cdot \phi_s + \beta_s \cdot x^j - p_s^j - \tau_s(1 - \lambda_s), & \text{if joining platform } j \text{ located at } 1. \end{cases} \tag{2}$$

$n_b^i \cdot \phi_s$ can be interpreted as the revenues obtained when interacting with the buyers. Note that there is no standalone utility on the seller side. It implies that sellers are better off only when a transaction is finalised.

By solving for the user indifferent between the two platforms, the following demands can be retrieved:

$$n_b^i \equiv \lambda_b = \frac{1}{2} + \frac{p_b^j - p_b^i}{2\tau_b} + \frac{\phi_b(n_s^i - n_s^j)}{2\tau_b} + \frac{\beta_b(x^i - x^j)}{2\tau_b}, \qquad n_b^j \equiv 1 - \lambda_b = 1 - n_b^i, \tag{3}$$

and

$$n_s^i \equiv \lambda_s = \frac{1}{2} + \frac{p_s^j - p_s^i}{2\tau_s} + \frac{\phi_s(n_b^i - n_b^j)}{2\tau_s} + \frac{\beta_s(x^i - x^j)}{2\tau_s}, \qquad n_s^j \equiv 1 - \lambda_s = 1 - n_s^i. \tag{4}$$

The two competing platforms set the membership fee p_k^i on the side k of the market and decide on the level of investments. Investments x^i may generate some values for the users while also determining a return to the platform. An investment x^i returns into a value $\gamma \cdot x^i$, where $\gamma \in [0, 1)$ captures the ability of the platform to monetise it. This term can be interpreted as the probability that a flow of investment x^i directly returns to the platform. Intuitively, with $\gamma = 0$, a positive investment x^i does not generate any return to the platform. We assume that investment costs are sufficiently convex and represented by $\Omega(x^i)$, with $\Omega'(x^i) > 0$ and $\Omega''(x^i) > 0$. For simplicity, we let $\Omega(x^i) = x_i^2/2$. The profit function Π^i is linear and additive as follows[7]:

$$\Pi^i = n_b^i \cdot p_b^i + n_s^i \cdot p_s^i + \gamma \cdot x^i - \Omega(x^i). \tag{5}$$

When considering a duopoly market configuration, we shall ensure that the profit functions are concave. Hence, we make the following two assumptions.

(A1) $\tau_b \cdot \tau_s > \phi_b \cdot \phi_s$, cross-group externalities are sufficiently low.

(A2) $4\tau_s \cdot \tau_s > (\phi_s + \phi_b)^2 + \beta_s \cdot \beta_b(\phi_s + \phi_b) + \beta_s^2 \cdot \tau_b + \beta_b^2 \cdot \tau_s$, value generation is upward bounded.

Assumption 1 presents a common restriction in the literature. The opportunity cost net of network externalities should be high enough when users benefit from the investments or at least strictly positive when investments are absent. An implication of this assumption is that cross-network externalities should not be too strong or that the sides of the market should be sufficiently differentiated (Economides & Tåg 2012), i.e. $\tau_b \cdot \tau_s > \phi_b \cdot \phi_s$. Assumption 2 requires that, overall, the value accruing to users is not so high such that sellers and buyers exploit them at high enough rates. In other words, there cannot be a situation where investments generate enough surplus for both sides of the market simultaneously. In practice, value-increasing investments might benefit one side more than the other. Alternatively, an investment primarily directed toward own returns cannot entail spillovers which are disproportionately high for both sides.

Competitive Bottleneck We also consider a competitive bottleneck environment. We let one side of the market (endogenously) multihome. In many industries (e.g., marketplace, software industry, video-game), the sellers often join one or more outlets. By eventually multihoming, sellers pay the subscription fee twice (to both platforms), incur total transportation costs τ_s, while benefiting from the investments undertaken by both

[7] Rochet & Tirole (2003) use a multiplicative demand. Parker & Van Alstyne (2005) presents different demand specifications by considering: i) additive; ii) additive recursive; iii) multiplicative; iv) shifted multiplicative; v) multiplicative recursive.

platforms, if any. Indeed, they can obtain up to $\beta_s(x^i + x^j)$.[8] Profits of the sellers are as follows:

$$\pi_s = \begin{cases} n_b^i \cdot \phi_s - p_s^i + \beta_s \cdot x^i - \tau_s \cdot \lambda_s, & \text{if joining platform } i, \\ n_b^j \cdot \phi_s - p_s^j + \beta_s \cdot x^j - \tau_s(1-\lambda_s), & \text{if joining platform } j, \\ (n_b^i + n_b^j)\phi_s - p_s^i - p_s^j + \beta_s(x^i + x^j) - \tau_s, & \text{if multihoming.} \end{cases} \tag{6}$$

Consider the seller indifferent between singlehoming on platform i and multihoming. Assume that her location is at λ_s^i, then $n_b^i \cdot \phi_s - p_s^i + \beta_s \cdot x^i - \tau_s \cdot \lambda_s^i = (n_b^i + n_b^j)\phi_s - p_s^i - p_s^j + \beta_s(x^i + x^j) - \tau_s \Leftrightarrow \lambda_s^i = 1 + \frac{p_s^j - n_b^j \cdot \phi_s - \beta_s \cdot x^j}{\tau_s}$. Next, consider the seller indifferent between multihoming or singlehoming on j, i.e. $\lambda_s^j = 1 + \frac{p_s^i - n_b^i \cdot \phi_s^i - \beta_s \cdot x^i}{\tau_s}$. The total demand for each platform is the sum of those sellers who singlehome and those who multihome. It follows that $n_s^i \equiv \lambda_s^j$ and $n_s^j \equiv 1 - \lambda_s^i$ and the market shares of the two platforms are:

$$n_s^i \equiv \lambda_s^j = \frac{1}{\tau_s}\left[n_b^i \cdot \phi_s - p_b^i + \beta_s \cdot x^i\right] \quad \text{and} \quad n_s^j \equiv 1 - \lambda_s^i = \frac{1}{\tau_s}\left[n_b^j \cdot \phi_s - p_b^j + \beta_s \cdot x^j\right] \tag{7}$$

The demand for the platform increases in the number of consumers the platform can reach (i.e., n_b^i and n_b^j) and in the value generated by the platform on this side of the market, whilst it decreases with the membership fee (i.e., p_b^i and p_s^j). Profits are concave and this is ensured by a slight modification of Assumption 1 (see Appendix).

The timing of the game is as follows:

- **Stage 1:** platforms decide whether or not to invest.
- **Stage 2:** conditional to this decision, platforms choose investments and prices simultaneously.
- **Stage 3:** the market clears. In the end, buyers and sellers decide about the platform to join and the market clears

The analysis is conducted by backward induction. In the next section, the competition between platforms with singlehoming users on both sides of the market is considered, while in Section 3.4 the competitive bottleneck is examined.

[8]This assumption can be relaxed but does not change the results. For instance, by considering decreasing returns when joining the second platform.

Table 3.1: Return(s) to investment

	Parameters		
Type of Investment	γ	β_s	β_b
Mixed case	> 0	≥ 0	≥ 0
Return to the platform	> 0	$= 0$	$= 0$
Value-creation for users	$= 0$	≥ 0	≥ 0

Note: All parameters are bounded between 0 and 1.

3.3 Competition with Singlehoming Users

Consider a market where two platforms compete for singlehoming users on both sides of the market. Once the market shares in the third stage are derived, in the second stage platforms make their decision on the pricing structure and investments to maximise (5). From the pricing strategies (see Appendix), it can be seen that, apart from its direct feedback on the platform (i.e., γ), the level of investments is correlated with the price charged on each side and the value it creates. To see it, consider investments only impacting on the seller side of the market (i.e., $\beta_s > 0$ and $\beta_b = 0$), then investments not only affect the seller price, as one may expect in a traditional one-sided market, but also the buyer side. Such an indirect effect is mediated by the cross-network externalities which govern the pass-through, i.e. $\phi_b \cdot \beta_s \cdot \Delta x$, where $\Delta x = x^i - x^j$.

Note that investments generate a demand expansion on both sides of the market. The total effect on the market shares is as follows:

$$\frac{dn_b^i}{dx^i} = \frac{\phi_b \cdot \beta_s + \tau_s \cdot \beta_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} > 0, \qquad \frac{dn_s^i}{dx^i} = \frac{\phi_s \cdot \beta_b + \tau_b \cdot \beta_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} > 0 \tag{8}$$

Clearly, when platforms behave symmetrically, any investment effect on market shares is neutralised. , From the first-order conditions of equation (5), we obtain the following results:

$$\begin{aligned}
\frac{\partial \Pi^i}{\partial p_s^i} &: -\frac{p_s^i \cdot \tau_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} - \frac{p_b^i \cdot \phi_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} + n_s^i = 0 \\
\frac{\partial \Pi^i}{\partial p_b^i} &: -\frac{p_s^i \cdot \phi_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} - \frac{p_b^i \cdot \tau_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} + n_b^i = 0 \\
\frac{\partial \Pi^i}{\partial x^i} &: -x^i + \gamma + \frac{p_b^i(\beta_s \cdot \phi_s + \beta_b \cdot \tau_s)}{2(\tau_s \cdot \tau_b - (\phi_b + \phi_s))} + \frac{p_s^i(\beta_s \cdot \tau_b + \beta_b \cdot \phi_s)}{2(\tau_s \cdot \tau_b - (\phi_b + \phi_s))} = 0
\end{aligned} \tag{9}$$

Therefore, we can state the following proposition:

Proposition 1. *Regardless of investments, equilibrium prices are* $p_s^* = \tau_s - \phi_b$ *and* $p_b^* = \tau_b - \phi_s$. *When investments are undertaken, their optimal level increases with its return to the users and to the platform, i.e.,* $x^* = \gamma + \frac{1}{2}(\beta_s + \beta_b)$.

Proof. See Appendix. □

Investments always increases with the value generated on the user sides.The greater the value engendered on each side of the market, the greater the investment effort. In equilbirum, platforms split the market equally with $n_s^{1,*} = n_s^{2,*} = 1/2$ and $n_b^{1,*} = n_b^{2,*} = 1/2$ and charge the usual Armstrong (2006)'s price: the Hotelling price discounted by cross-network externalities, such as $p_s^{1,*} = p_s^{2,*} = \tau_s - \phi_b$ and $p_b^{1,*} = p_b^{2,*} = \tau_s - \phi_s$. Due to the symmetry of the industry, platforms fail to pass their investment effort onto prices. These are neither directly nor indirectly paid by sellers and buyers as long as platforms are symmetric. Hence, the competition between symmetric platforms translates into an immediate surplus for buyers and sellers for any $\beta_k > 0$ with $k\{b, s\}$.

Proposition 2. *There is always an incentive to invest to prevent business stealing effects. Hence, in equilibrium both platforms invest and earn*

$$\Pi^{inn} = \Pi^i(x^{i,*}|x^{j,*}) = \Pi^j(x^{j,*}|x^{i,*}) = \frac{1}{8}\Big(4\left(\gamma^2 + \tau_b + \tau_s - (\phi_s + \phi_b)\right) - (\beta_s + \beta_b)^2\Big). \tag{10}$$

Platforms split the demand equally, i.e., $n_s^{i,*} = n_s^{j,*} = 1/2$ *and* $n_b^{i,*} = n_b^{j,*} = 1/2$.

Proof. By plugging $p_s^{inv,*}$ and $p_b^{inv,*}$ into the the profit and demand functions, results are immediate. □

From the above expression, when investments are undertaken, platforms' profits decrease with the square of the investment effort, which instead increases positively with the value accruing to buyers and sellers. The more users evaluate platforms' investments, the more platforms will invest and this would affect their profits adversely, other things equal.

In the next proposition, we provide a comparison of the equilibrium profits.

Proposition 3. *Absent investments, platforms obtain:*

$$\Pi^{no} = \Pi^i(x^i = 0|x^j = 0) = \Pi^j(x^j = 0|x^i = 0) = \frac{1}{2}\Big(\tau_s + \tau_b - (\phi_s + \phi_b)\Big) \tag{11}$$

Profits are higher absent investments (Investment-Prisoner Dilemma) if and only if

$$\gamma \leq \underline{\gamma_{cs}} = \frac{\beta_s + \beta_b}{2}.$$

Proof. See Appendix. □

Investments are ex post profit-enhancing for both platforms when they can ensure a sufficiently large return for themselves. The threshold ensuring greater profits when investments are performed is identified by $\underline{\gamma_{cs}}$ and increases with the value generated on its users. This happens as platforms engage in a sort of investment war in the hope of grasping more users from the rival's turf. In equilibrium, however, the market shares and the prices remain the same and any possibility to increase profits comes from sufficiently direct returns, such as stock consolidation, data monetisation, or efficiency.

Moreover, higher profits with investments do not warrant a platform to be better-off after investment and may be trapped into a Prisoner Dilemma type of situation. Due to business stealing effects, each platform is tempted to invest a bit more of the rival. In equilibrium, they invest symmetrically and they can be worse off when investments do not translate into a direct return. This result closely resembles Thomes (2015): platforms cannot credibly commit to abstaining from investments as there are always investment temptations.

3.4 Competitive Bottleneck with Multihoming Sellers

In this section, we consider the case when (some) sellers multihome. We follow Armstrong (2006) and Belleflamme & Peitz (2018). By anticipating second-period market shares, platforms non-cooperatively and simultaneously set their investment effort.

Platforms maximise their profits by chosing prices and investments. From the first order conditions, we obtain the following results:

$$\begin{aligned}
&\frac{\partial \Pi^i}{\partial p_s^i} : -\frac{p_s^i(2\tau_b \cdot \tau_s - \phi_s \cdot \phi_b)}{2\tau_s(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} - \frac{p_b^i \cdot \phi_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} + n_s^i = 0 \\
&\frac{\partial \Pi^i}{\partial p_b^i} : -\frac{p_s^i \cdot \phi_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} - \frac{p_b^i \cdot \tau_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} + n_b^i = 0 \\
&\frac{\partial \Pi^i}{\partial x^i} : -x^i + \gamma + \frac{p_b^i(\beta_s \cdot \phi_s + \beta_b \cdot \tau_s)}{2(\tau_s \cdot \tau_b - (\phi_b + \phi_s))} + \frac{p_s^i(2\beta_s \cdot \tau_s \cdot \tau_b + \phi_s(\beta_b \cdot \tau_s - \beta_s \cdot \phi_b))}{2\tau_s(\tau_s \cdot \tau_b - (\phi_b + \phi_s))} = 0
\end{aligned} \tag{12}$$

Let the superscript $\{inn, *\}$ indicate the equilibrium results when investments take place and $\{no, *\}$ the equilibrium results when investments are not considered, then:

Lemma 1. *Absent investments, equilibrium prices are:*

$$\begin{aligned}
p_b^{no,*} &= \tau_b \cdot \tau_s - \frac{\phi_s(\phi_s + 3\phi_b)}{4\tau_s} \\
p_s^{no,*} &= \frac{1}{4}\Big(\phi_s - \phi_b\Big)
\end{aligned} \tag{13}$$

When platforms invest, equilibrium prices are:

$$\begin{aligned}
p_b^{inn,*} &= \frac{(4\tau_b \cdot \tau_s - \phi_s(3\phi_b + \phi_s) - \beta_s \cdot \phi_s(\beta_b + 2\gamma))}{2(2\tau_s - \beta_s)} + \frac{\beta_s^2(2\tau_s \cdot \tau_b - \phi_b \cdot \phi_s)}{2\tau_s(2\tau_s - \beta_s)} \\
p_s^{inn,*} &= \frac{\tau_s(\beta_s(\beta_s \cdot \gamma + \beta_b) + (\phi_s - \phi_b)) + \beta_s^2 \cdot \phi_b}{2(2\tau_s - \beta_s^2)}
\end{aligned} \tag{14}$$

and the investment effort is

$$x^* = \frac{2\tau_s(2\gamma + \beta_b) + \beta_s(\phi_s + \phi_b)}{2\,(2\tau_s - \beta_s^2)}. \tag{15}$$

Proof. Immediate from the first-order conditions. □

In theory, with a competitive bottleneck, a platform should set a monopoly price on the multihoming side of the market as providing the only possible access to the buyer side (see e.g. Armstrong 2006). In practice, as recently shown by Belleflamme & Peitz (2018), this is not always the case as letting some sellers multihome induces a price reduction on the seller side relative to when sellers singlehome. This result is also confirmed by our analysis. Consider the equilibrium outcome absent investments. In this case, "the bottleneck effect" does not dominate any other effect and sellers are granted a better treatment relative to buyers. When cross-network externalities equal between sides (i.e., $\phi = \phi_s = \phi_b$), platforms let sellers to entry at no cost and set a higher price on consumers are charged $p_b^{no,*} = \tau_b \cdot \tau_s - \frac{\phi}{\tau_s}$. When buyers benefit more from the interaction with sellers (i.e., $\phi_s < \phi_b$), sellers are subsidised as a higher price can be set to buyers: cross-network externalities are discounted less from $p_b^{no,*}$. The two abovementioned cases resemble the pricing strategies set by Amazon for Alexa developers and Google Play for app programmers. [9] The only case in which sellers experience a higher membership price is when they benefit more from the interaction with buyers. In this case, absent any investment, $p_s^{no,*}$ increases with $\Delta\phi = \phi_s - \phi_b$ and the buyer price decreases.

We note that absent any investment, as long as $\phi_s \leq \phi_b$, letting sellers multihome increases the surplus of sellers relative to a case of singlehoming sellers. In the latter case, sellers always pay a positive price. This result easily emerges when considering $\phi_s \leq \phi_b$, that is, when sellers face any membership fee or are subsidised. When investments take place, $p_s^{inn,*}$ is clearly positive. This is because the multihoming sellers not only have access to the exclusive consumers ("the bottleneck effect") but also to the level of value-increasing investments provided by both platforms. Thus, more sellers prefer to multihome as this homing decision leads to increasing the marginal benefit from investments. A price increase emerges also when $\phi_s > \phi_b$ and the new price set by platforms is as much as larger as $\Delta\phi = \phi_s - \phi_b$ increases. Results clearly resemble those with a monopolist platform (see Appendix).

Formally, absent investments, the market is split as follows:

$$n_b^{no,*} = \frac{1}{2}, \qquad n_s^{no,*} = \frac{\phi_s + \phi_b}{4\tau_s}. \tag{16}$$

[9] See Business Insider, 'Amazon is making a big step with Alexa as the voice wars with Google and Microsoft escalate', May 16, 2017: `http://uk.businessinsider.com/amazon-paying-alexa-developers-2017-5.(Accessed2018).`

With investments, the number of buyers and sellers is going to be

$$n_b^{inn,*} = \frac{1}{2}, \qquad n_s^{inn,*} = \frac{\beta_s(\beta_b + 2\gamma) + (\phi_s + \phi_b)}{2(2\tau_s - \beta_s^2)}, \tag{17}$$

as long as there is some surplus generated by investments on the seller side. Note that more sellers multihome when investments are undertaken. Interestingly, platforms can attract additional sellers and expand their market shares without subtracting them to the rival. Indeed, each platform provides access to exclusive contents to an increased number of sellers and the increase is proportional to the investment effort and the ability to gain from investments. Formally, we have

$$n_b^{inn,*} - n_b^{no,*} = \frac{x^* \cdot \beta_s}{2\tau_s}.$$

In other words, some singlehoming sellers now become multihoming. As a result, there is an incentive for the platforms to set a higher price on this side of the market while reducing the burden for the other side of the market. As compared to the case without investments, the bottleneck is partly restored by the greater market coverage induced by investments. The effect on the price is immediate from the following comparison:

$$\begin{aligned} \triangle p_s = p_s^{inn,*} - p_s^{no,*} &= \frac{\beta_s\Big(\beta_s(\phi_s + \phi_b) + 2\tau_s(2\gamma + \beta_b)\Big)}{4\,(2\tau_s - \beta_s^2)} = \frac{x^* \cdot \beta_s}{2}, \\ \triangle p_b = p_b^{inn,*} - p_b^{no,*} &= -\frac{\beta_s \cdot \phi_s\Big(\beta_s(\phi_s + \phi_b) + 2\tau_s(2\gamma + \beta_b)\Big)}{4\,(2\tau_s - \beta_s^2)} = -\phi_s \cdot \triangle p_s \end{aligned} \tag{18}$$

First, we note that when investments affect only the buyer side of the market without any direct effect on the seller side (i.e., $\beta_s = 0$), the model resembles the results obtained with singlehoming sellers. In other words, as investments do not stimulate additional multihoming, prices remain unaltered on both sides of the market as there is no redistribution of the generated surplus. As long as $\beta_s > 0$, there is an increase in the price on the seller side as each platform moves towards a condition where it monopolises the market. The magnitude of the price change is equal to half of the all value generated on that side of the market. As a result, platforms can lower the price by the same magnitude scaled by the seller cross-network externalities ϕ_s on the buyer side. The larger ϕ_s, the larger the price reduction for the buyers.

Second, by simple comparative statics, we note that any increase in the ability of the buyer side to grab benefit from investments reduces the price differential between the case without and with the investments. For instance, any increase in β_b leads to $\partial\triangle p_b/\partial\beta_b < 0$. As the price differential is negative, β_b forces greater investments and a price change, which turns out reducing $\triangle p_b$, with $p_b^{inn,*}$ converging towards $p_b^{no,*}$. In the same manner, an increase in the ability of the seller side to grasp some value from investments, β_s, is translated into a proportional change in $\triangle p_s$. The next lemma summarises the above discussion.

Proposition 4. *Investments foster more multihoming. The prices are shaped by the intensity of the investment in a way that sellers are charged more while buyers face a lower price. If the investments only affect the buyer side of the market (i.e., $\beta_s = 0$ and $\beta_b > 0$), the results are the same as with singlehoming users on both sides of the market.*

An immediate consequence of the investment activity is that buyers increase their surplus regardless of whether $\beta_b > 0$. The surplus of buyers (may) increase for three effects: *(i)* the price discount (Lemma 4) offered by the platform (which, instead, raises its seller price), *(ii)* the market expansion on the seller side, which is positively welcomed by the buyers because of their taste for variety, and *(iii)* the investment effect whenever buyers directly benefit from $\beta_b > 0$. It is important to note that when buyers do not internalise a direct benefit from investments, the first two effects still generate additional surplus. On the seller side of the market, the surplus creation, in contrast, is less straightforward. We can summarise this as follows:

Corollary 1. *Investments always increase the surplus on the buyer side regardless of whether buyers experience a direct value-creation.*

We have not yet discussed the optimal level of investment undertaken by platforms. This is summarised by the following lemma.

Lemma 2. *The fiercer the competition on the seller side, the greater the level of investment provided by the platforms, i.e. $-\partial x^*/\partial\tau_s > 0$.*

There are some important aspects emerging in this scenario relative to when sellers singlehome (see Lemma 1). First, investments depend on the fierceness of the competition on the multihoming side of the market and on the sum of the cross-network externalities. How would platforms react to a more intense competition on the seller side? Lemma 2 states that as $\tau_s \to 0$, platforms are perceived as more substitute by sellers (i.e., easy portability and compatibility, preference for the brand). As a result, platforms engage in a fiercer competition on the investment dimension. This happens as now sellers can be charged more and determine a feedback on the opposite side of the market in terms of the surplus generated to buyers. Platforms hence try to attract as many sellers as possible by engaging in an investment war. Such a result does not apply to when singlehoming users are considered: in that case, what matters is only how users internalise platforms' investments through β_k for any $k = \{b, s\}$

As in the previous case, platforms are always tempted to increase their investment effort. This is described by the following proposition.

Proposition 5. *Profits are always lower with investments (Investment-Prisoner Dilemma) if*

$$\gamma < \underline{\gamma_{cb}} = \frac{\beta_b}{2} + \frac{\beta_s(\phi_s + \phi_b)}{4\tau_s}.$$

Proof. See Appendix. □

As with singlehoming users, investments may not ensure that platforms are ex post better off. Proposition 5 shows that they are better off investing in value creation if and only if they can generate a sufficiently large feedback effect. In other words, while on the one hand, platforms can now partially transfer their investment effort into prices so as to increase their profits, on the other hand, platforms should find ways to partially internalise what causes an increase in user experience through direct returns. Hence, this needs to be sufficiently large, $\gamma > \underline{\gamma_{cb}}$. Else, unless they are sufficiently forward-looking and able to anticipate such a scenario, platforms could find themselves trapped into a prisoner dilemma type of situation, that is a situation where they invest but lower their profits. From a direct comparison with the singlehoming case, one can easily verify that the minimum level of γ leading to profit-enhancing investments increases with β_b and β_s, but now it also depends on the cross-network externalities and τ_s. Indeed, a greater ability of users to internalise investments makes the competition between platforms harsher in the investment dimension. Investments increase but in parallel platforms should be able to better monetise their investment efforts (greater γ).

Moreover, cross-network externalities play an important role: the term ($\phi_s + \phi_b$ amplifies the interrelation between the sides of the market. Hence, the more buyers and sellers are interrelated, the greater the feedback effect needs to be to avert the prisoner dilemma, that is, the situation for which platforms would rather prefer not to invest but they do so only because of the expected strategic move of the rival. Finally, a third term is crucial to make platforms ex post better off with investments: as platforms move toward monopolistic market shares, namely softening the competition on the seller side $\tau_s \to \infty$, a little effort in monetising investments is required. In other words, when platforms are more differentiated, they find less strategic to investment in value creation as the sellers and buyers will be less prone to switch to the rival platform. As a result, they can invest less and reduce the impact of investment costs on profits.

3.5 A Comparison

So far, we have discussed the two regimes separately. It remains to compare which of the two regimes yields the greatest level of investment, i.e.,

Proposition 6. ***(Total Investment Expenditure).*** *The investment effort is greater (smaller) when sellers multihome provided the existence of enough (low) competition on their side, i.e.* $\tau_s \to 0 (\tau_s \to \infty)$.

Proof. Let x_{cs} be the investment effort with singlehoming sellers and x_{cb} in presence of multihoming sellers (Proposition 5), then

$$\begin{aligned} x_{cb} - x_{cs} &= \frac{2\tau_s(2\gamma + \beta_b) + \beta_s(\phi_s + \phi_b)}{2\left(2\tau_s - \beta_s^2\right)} - \left(\gamma + \frac{\beta_s + \beta_b}{2}\right) \\ &= \frac{\beta_s\Big(\beta_s(2\gamma + \beta_b) + (\phi_s + \phi_b) - 2\tau_s + \beta_s^2\Big)}{2\left(2\tau_s - \beta_s^2\right)} \end{aligned} \tag{19}$$

which ultimately implies that $x_{cb} - x_{cs} > 0 \Leftrightarrow \tau_s < \frac{\beta_s(2\gamma+\beta_b)+(\phi_s+\phi_b)+\beta_s^2}{2}$. The opposite otherwise. □

The above proposition is consistent with the previous discussion. A fiercer competition on the seller side of the market (i.e., reduced transportation costs) fosters more investments from the platforms in the attempt to attract more sellers and, as a result, more buyers. When transportation costs are low, users (of any side) are less attached to their preferred platform and more sensitive to any price and investment change. This implies that more sellers will prefer to multihome by increasing the market coverage of any platform. By contrast, when the market is more differentiated, the incentive for sellers to multihome moving from a situation of exclusivity (singlehoming) is lower and so the returns for the platform. As the marginal return of investment is greater for a fierce enough competition, investments will be higher when sellers multihome relative to when sellers singlehome. In the opposite case, for a sufficiently large differentiation, sellers are less mobile across platforms and so the platforms invest less relative to a purely symmetric market with singlehoming users on the seller and buyer side.

We have also discussed that, relative to the singlehoming case, when sellers multihome, the minimum level of direct return for the platform depends on several parameters. The next proposition compares the two network configurations.

Proposition 7. *Having (some) multihoming sellers requires a lower return to the platform relative to the case with singlehoming sellers, i.e.,* $\underline{\gamma_{cs}} - \underline{\gamma_{cb}} = \frac{\beta_s(2\tau_s - (\phi_s + \phi_b))}{4\tau_s} > 0$.

From a managerial perspective, this result suggests that platforms may find it optimal to soften the competition and maximise the returns from investments by allowing multihoming on one side of the market (i.e., seller side). This is consistent with the conventional wisdom that sees many operating systems allowing large compatibility across platforms without imposing exclusivity restrictions. When considering data extraction, the above proposition suggests that platforms may find it more profitable to allow sellers to multihome as this lowers the minimum amount of data to be collected, integrated and monetised. Indeed, multihoming sellers may mitigate or even avert the prisoner dilemma previously discussed with less effort and data to be monetised.

3.6 Extensions

3.6.1 Separate Investment Flows

Throughout the paper, we have assumed the presence of multidimensional effects arising from the same source of investments. Sometimes investments do not generate any return. To verify how this matters in our framework, this section provides an extension of the main model. It supposes that the investment flows are separate: *(i)* one flow of investments generates some value for at least one side of the market, and *(ii)* another flow of investments return to the platform as a positive feedback. By separating the flows of investments, it does imply that the profit function can be rewritten as $\Pi^i = n_b^i \cdot p_b^i + n_s^i \cdot p_s^i + \gamma \cdot \zeta_i - \frac{z_i^2}{2} - \frac{\zeta_i^2}{2}$, with z_i the amount of investment returning to the platform and ζ_i the level of investments affecting a type-k user of platform i by $\beta_k^i \cdot \zeta_i$. The proposition below summarizes some of the effects arising.

Proposition 8. ***(Separate Investments).*** *When users singlehome, the total level of in-vestment is not affected by the decision of undertaking separate investment programmes. When sellers multihome, the total level of investment, the price on the seller side and platforms' profits decrease, while the price on the buyer side increases.*

Proof. See Appendix. □

Thus, when users are constrained to join only one platform, results remain unaffected by the type of investment flow. Hence, a platform would be indifferent between setting separate investments or allowing multidimensionality between investment creating a value for the users and investments creating a value for the platform itself. The story is somewhat different when sellers multihome. Here, the platforms would rather prefer to differentiate the investments that affect the platform and the investments whose value-creation accrues to the users. This is because the platforms set a lower level of investments on the user side of the market. By setting a lower investment level, fewer sellers will move from exclusivity to non-exclusivity and so the price change will be impacted less. Hence, the bottleneck effect is lowered relative to when investments are multidimensional and so the seller price. Coupled together, a lower investment effort and lower price and demand on the multihoming side of the market generates a reduction in the profits of the platforms. The main findings of our model, however, remain unaltered and a prisoner dilemma could be generated for sufficiently low direct returns.

An interesting question emerging from the above discussion is whether, whenever value creation is not multidimensional, platforms should devote their investment effort only to themselves without impacting on the users. By simple inspection, it can be verified that if platforms can decide which type of investment to pursue, they would always prefer to devote their investment effort towards themselves in the forms of direct return to the platform and not to engage in an investment war with the rival and ex-ante identical platform. To see it formally, we can simply set $z_i = 0$ and see that profits reduce by $\frac{2\beta_b \cdot \tau_s + \beta_s \cdot (\phi_s + \phi_b)}{16\tau_s(2\tau_s - \beta_s^2)}$.

3.6.2 Side-specific investments

One might argue that investments are side-specific. In other words, platforms can only influence the utility, the experience, the profits of one side of the market only. In this subsection, we consider the case in which platforms can engage in investments toward each side. As investments creating a direct value for the platform are always equal to their marginal contribution (γ), we overlook this aspect and we consider only what happens on each side. Call x_k^i investements made on side $k = \{b, s\}$ by platform $i = \{1, 2\}$, platform i obtains the following profits $\Pi^i = n_b^i \cdot p_b^i + n_s^i \cdot p_s^i - \frac{x_s^{i,2}}{2} - \frac{x_b^{i,2}}{2}$. We present the following results.

Proposition 9. ***(Side-specific Investments).*** *When users singlehome, the total level of investment is not affected by the decision of undertaking side-specific investments. Platforms set*

$$x_b^{cs,*} = \frac{\beta_b}{2}, \qquad x_s^{cs,*} = \frac{\beta_s}{2}.$$

When sellers multihome, the total level of investment decreases. Platforms set

$$x_b^{cb,*} = \frac{\beta_b}{2}, \qquad x_b^{cb,*} = \frac{\beta_s(\phi_b + \phi_s)}{2(2\tau_s - \beta_s^2)}.$$

The above proposition presents twofold results. First, with singlehoming users on both sides of the market, results remain identical to those in the benchmark model (Proposition 1). Hence, $x_b^{cs,*} = \frac{\beta_b}{2}$ and $x_s^{cs,*} = \frac{\beta_s}{2}$. Hence, there is no change on aggregate investments and prices and investments are equal to half the value engendered on each side of the market. Second, with multihoming sellers, the investment effort does not change on the buyer side of the market. Platforms set $x_s^{cb,*} = \frac{\beta_s}{2}$. As investments imply the conversion of some singlehoming sellers to multihomers, the investment effort changes and now takes into account the sum of cross-network externalities ($\phi_s + \phi_b$). The larger the interrelation between sides, the larger will be the benefit on this side of the market. This happens because the optimal level of investments can also be written as

$$x_b^{cb,*} = \frac{\beta_s(\phi_b + \phi_s)}{2(2\tau_s - \beta_s^2)} = n_s^{cb,*} \cdot \beta_s.$$

In other words, the investment effort is equal to the the number of sellers in the market multiplied for the value generated by investment on that side of the market. We note that, as any time investments can be separated (see above discussion), the invrestment effort is reduced relative to the comparative case and this decreases by $\frac{\beta_s^2 \cdot \beta_b}{2(2\tau_s - \beta_s^2)}$. This indicates that the multidimensionality of investment matters and induces platforms to over-invest relative to a scenario where side-specific investments can be provided. As a consequence, the number of multihoming sellers is reduced and indeed also prices are less sensitive to the investment effort determining a reduction in total profits.[10]

3.7 Conclusions

This paper provides an analysis of investment decisions in two-sided markets and how the two-sidedness of the market shapes the platform incentives depending on the difference in cross-group externalities between sides and on how investments generate some value for the platform, buyers and sellers.

While, theoretically, platforms can pass their investment efforts onto prices, this may not always be the case when competition exists. When platforms are symmetric and all agents single-home, platforms are always tempted to invest a lit bit more than the rival

[10]The proof is not reported but follows the same mechanism of Proposition 8 by allowing for side-specific investments and $\gamma = 0$ (for the sake of simplicity).

to expand their market share. However, in equilibrium, prices remain unchanged and no market expansion arises. As a result, platforms are trapped in a Prisoner Dilemma type of condition unless able to obtain some direct return.

Results change dramatically when (some) sellers are given the possibility to multihome. First, as long as not all sellers multihome, the bottleneck effect arising in these markets is almost null. As a result, absent investments, platforms charge more buyers than sellers. This result resembles what would happen with a monopolist platform. Second, when investments take place, some singlehoming sellers become multihomers. Hence, platforms can increase the price on this side of the market which is proportional to the level of investment and in turn alleviate the burden on the buyer side of the market. All in all, investments on the sellers side of the market are translated into a surplus for buyers who experience more multihoming sellers (i.e., variety) and a price reduction. This happens as platforms can relax the competition on the multihoming side of the market and compete more for the exclusive buyers offering the latter a price cut. Hence, authorities should persuade platforms not to restrict seller homing to only one platform when these are symmetric before and after investments. Third, the above discussion shows that the bottleneck effect discussed by Belleflamme & Peitz (2018) and which disappears without investments can now be partly restored with investments on the seller side. This would thus induce some singlehoming sellers to multihome. Fourth, a network configuration allowing sellers to multihome can more effectively mitigate or avert the prisoner dilemma typical in a symmetric industry with a lower feedback effect required. It is important to note, however, that this finding does not automatically imply that a competitive bottleneck supports more investments in equilibrium: it does so only when the market competition on the seller side of the market becomes sufficiently intense. Else, increased investments are more likely to be experienced in a fully symmetric setting with sellers and buyers exclusively joining one platform.

To conclude, this paper shows that further research into this area can be conducted. In particular, investments may not only be devoted to generate value for the platform and its users but also to increase or reduce the competition in the market and the cross-network externalities. For instance, platforms in social media may invest in accurate systems to provide sellers or advertisers with better information about the users joining the platform and, hence, increasing their expected utility. Similarly, investments may improve the condition of one side and the expenses of the others. For instance, int he social media industry, investments in moderation of user generated content may increase the willingness to pay of advertisers (who would face a lower brand risk) and reduce the utility of users whose content could be scrutinised.

Appendix - A

Derivation of the Hessian Matrix

To ensure that the platforms' profit function is well-behaved, given the simultaneity of prices and investment decisions, the Hessian matrix is required to be positive semi-definite. The Hessian matrix is

$$\mathbf{H}^i_{\Pi}(x^i, p^i_s, p^i_b) = \begin{bmatrix} -1 & \frac{\beta_s \cdot \tau_b + \beta_b \cdot \phi_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} & \frac{\beta_b \cdot \tau_s + \beta_s \cdot \phi_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} \\ \frac{\beta_s \cdot \tau_b + \beta_b \cdot \phi_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} & -\frac{\tau_b}{(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} & -\frac{\phi_s + \phi_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} \\ \frac{\beta_s \cdot \phi_b + \beta_b \cdot \tau_s}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} & -\frac{\phi_s + \phi_b}{2(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)} & -\frac{\tau_s}{\tau_b \cdot \tau_s - \phi_b \cdot \phi_s}. \end{bmatrix}$$

The three leading principal minors should be alternate in their sign, i.e. $(-1)^r \Delta_r \geq 0$, with $r > 0$ identifying the $r - th$ principal minor. Then,

$$\begin{aligned} \Delta_1 =& -1 \\ \Delta_2 =& \frac{\tau_b^2(4\tau_s - \beta_s^2) - 2\tau_b(\beta_s \cdot \beta_b + 2\phi_b)\phi_s - \beta_b^2 \cdot \phi_s^2}{4(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)^2} \\ \Delta_3 =& \frac{\beta_s^2 \cdot \tau_b + \beta_b^2 \cdot \tau_s - 4\tau_b \cdot \tau_s + \beta_b \cdot \beta_s(\phi_s + \phi_b) + (\phi_b + \phi_s)^2}{4(\tau_b \cdot \tau_s - \phi_b \cdot \phi_s)^2} \end{aligned} \tag{A-1}$$

Hence, we make the following assumptions:

(A1) $\tau_b \cdot \tau_s > \phi_b \cdot \phi_s$.

(A2) $4\tau_s \cdot \tau_s > (\phi_s + \phi_b)^2 + \beta_s \cdot \beta_b(\phi_s + \phi_b) + \beta_s^2 \cdot \tau_b + \beta_b^2 \cdot \tau_s$.

Proof of Proposition 2

To study whether a platform would optimally decide to invest or not and given that a closed-form solution is not computationally feasible due to the number of parameters we use, we study whether there is a tendency for a platform to deviate from the equilibrium where neither platform invests towards an equilibrium where both platforms invest. Suppose an infinitesimal change in the level of investment of platform i from $x^i = \tilde{x^i}$ to $x^i = \tilde{x^i} + \bar{x}$ with $\bar{x} \to 0$, $\tilde{x^i} = 0$ for $x^j = 0$, i.e.

$$\left.\frac{\partial \Pi_i}{\partial x^i}\right|_{x^i=\tilde{x^i};x^j=0} = \gamma + p_b^i \frac{dn_b^i}{dx^i} + \ p_s^i \frac{dn_s^i}{dx^i} - \underbrace{\tilde{x_i}}_{=0} \tag{A-2}$$

First, $\gamma \geq 0$. Second, there is always a demand expansion determined by investments (see equation (8)). Hence, platforms have always incentive to deviate and invest a bit more to gain larger market shares and more revenues.

Proof of Proposition 3

Rearranging equation (10) yields:

$$\Pi^{inn} = \Pi^{no}(x^i = 0; x^j = 0) + \frac{\gamma^2}{2} - \frac{(\beta_s + \beta_b)^2}{8}, \tag{A-3}$$

which implies that $\Pi^{inn} - \Pi^{no} = \frac{\gamma^2}{2} - \frac{(\beta_s+\beta_b)^2}{8}$. As profits $\Pi^{inn} < \Pi^{no}$ when $\gamma \leq \underline{\gamma_{cs}} = \frac{\beta_s+\beta_b}{2}$, then an Investment-Prisoner Dilemma arises. Both platforms would be better by not investing relative to when they engage in investment activities.

Derivation of the Hessian Matrix in Section 3.4

Consider the competitive bottleneck, the Hessian matrix is:

$$\mathbf{H}_{\Pi}^i(x^i, p_s^i, p_b^i) = \begin{bmatrix} -1 & \frac{\beta_s(2\tau_s\cdot\tau_b - \phi_s\cdot\phi_b)+\beta_b\cdot\tau_s\cdot\phi_s}{2\tau_s(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} & \frac{\beta_s\cdot\phi_b+\beta_b\cdot\tau_s}{\tau_b\cdot\tau_s-\phi_b\cdot\phi_s} \\ \frac{\beta_s(2\tau_s\cdot\tau_b - \phi_s\cdot\phi_b)+\beta_b\cdot\tau_s\cdot\phi_s}{2\tau_s(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} & -\frac{1}{\tau_s} - \frac{\tau_b}{(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} & -\frac{\phi_s+\phi_s}{2(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} \\ \frac{\beta_s\cdot\phi_b+\beta_b\cdot\tau_s}{2(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} & -\frac{\phi_s+\phi_b}{2(\tau_b\cdot\tau_s-\phi_b\cdot\phi_s)} & -\frac{\tau_s}{\tau_b\cdot\tau_s-\phi_b\cdot\phi_s}. \end{bmatrix}$$

As the first principal minor is negative ($\Delta^{cb}_{z_i 1} = -1 < 0$), the Hessian Matrix needs a positive second principal minor and a negative third principal minor.

We refer to $\Delta^{cb}_{z_i 3} = \frac{\det \mathbf{H}^{cb}}{4\tau_s(\tau_s \cdot \tau_b - (\phi_s + \phi_b))^2} < 0$, where

$$\det \mathbf{H}^{cb} = 2\beta_s \cdot \beta_b \cdot \tau_s(\phi_s + \phi_b) + \beta_s^2(2\tau_b \cdot \tau_s - (\phi_s + \phi_b) + \tau_s(2\beta_s^2 - + \tau_s \left(2\beta_b^2 \cdot \tau_s + \phi_b^2 + \phi_s^2 + 6\phi_b \cdot \phi_s - 8\tau_b \cdot \tau_s\right))$$

The also implies that, for any $\beta_b = \beta_s = 0$, $8\tau_b \cdot \tau_s > \phi_b^2 + \phi_s^2 + 6\phi_b \cdot \phi_s$.

Proof of Proposition 5

First, we verify whether profits are higher with investments. Consider an infinitesimal change in the level of investment of platform i from $x^i = \tilde{x^i}$ to $x^i = \tilde{x^i} + \bar{x}$ with $\bar{x} \to 0$, $\tilde{x^i} = 0$ for $x^j = 0$, i.e.

$$\begin{aligned} \left.\frac{\partial \Pi_i}{\partial x^i}\right|_{x^i = \tilde{x^i}; x^j = 0} &= \gamma + p_b^i \frac{dn_b^i}{dx^i} + p_s^i \frac{dn_s^i}{dx^i} - \underbrace{\tilde{x}_i}_{=0} \\ &= \underbrace{\gamma}_{\geq 0} + p_b^i \frac{dn_b^i}{dx^i} + p_s^i \frac{dn_s^i}{dx^i} \end{aligned} \tag{A-4}$$

First, $\gamma \geq 0$. Second, there is always a demand expansion determined by investments (see equation (8)). Hence, there is always an incentive to invest. Next, by a simple comparison of equilibrium profits before and after (symmetric) investments, it emerges that

$$\Pi^{inn,*} - \Pi^{no,*} = \frac{\gamma^2 \cdot \tau_s}{2\tau_s - \beta_s^2} - \frac{(\beta_s(\phi_s + \phi_b) + 2\beta_b \cdot \tau_s)^2}{16\tau_s\left(2\tau_s - \beta_s^2\right)}. \tag{A-5}$$

Hence, profits are reduced with investments if

$$\gamma > \underline{\gamma_{cb}} = \frac{\beta_b}{2} + \frac{\beta_s \cdot (\phi_s + \phi_b)}{4\tau_s}. \tag{A-6}$$

Hence, an Investment-Prisoner Dilemma arises: platforms would be better off by not investing relative to when they do engage in investment activities.

Proof of Proposition 8

Consider the profit function when sellers multihome and the symmetric case, by setting the level of the investment, it can be shown

$$\zeta^{cs} = \gamma \qquad \text{and} \qquad z^{cs} = \frac{\beta_b + \beta_s}{2} \tag{A-7}$$

It follows that the total level of investment equals the equilibrium level of investment in Section 3.3, i.e., $x^{cs} = \zeta^{cs} + z^{cs} = \gamma + \frac{\beta_b+\beta_s}{2}$.

In a competitive bottleneck, the story is slightly different. Platforms optimally set the following level of investment

$$\zeta^{cb} = \gamma \qquad \text{and} \qquad z^{cb} = \frac{\beta_s(\phi_s + \phi_b) + 2\tau_s \cdot \beta_b}{2(2\tau_s - \beta_s^2)} \tag{A-8}$$

By a simple comparison, the total level of investment is now smaller than what previously found in the main model. By summing up $\zeta^{cb} + z^{cb}$, the latter is now smaller than the equilibrium level of investment shown in Section 3.4, i.e., $\frac{\beta_s^2 \cdot \gamma}{2\tau_s - \beta_s^2}$.

We can now verify whether different prices are set as a consequence of changes in the level of investments. Clearly, no effect arises when all sellers singlehome as the total investment effort remains the same. Instead, when some sellers multihome, we can see that buyers are charged relatively more when investment flows can separated and this happens because the investment effort is clearly lower. On their side, the price increases by $\frac{\beta_s \cdot \gamma \cdot \phi_s}{2\tau_s - \beta_s^2}$. By contrast, as the level of investments is lower, fewer sellers will start multihoming and therefore the price on their side would decrease by $\frac{\beta_s \cdot \gamma \cdot \tau_s}{2\tau_s - \beta_s^2}$. Let the equilibrium prices when sellers multihome be p_k^{cb} where $k = \{b, s\}$, then

$$\begin{aligned} p_b^{cb} =& \frac{\beta_s(\phi_b - \beta_b \cdot \tau_s) + \tau_s(\phi_s - \phi_b)}{2(2\tau_s - \beta_s^2)}, \\ p_s^{cb} =& \frac{\tau_b \cdot \tau_s - \phi_s(3\phi_b + \phi_s)}{2(2\tau_s - \beta_s^2)} - \frac{\beta_s^2(2\tau_s \cdot \tau_b - \phi_b \cdot \phi_s)}{2(2\tau_s - \beta_s^2)\tau_s} \end{aligned} \tag{A-9}$$

By comparing the effect on the total profits, these now decline by $\frac{\beta_s^2 \cdot \gamma^2}{2(2\tau_s - \beta_s^2)}$ when compared to the main model with multidimensional investments. Finally, a competitive bottleneck environment where both platforms invest will ex post dominate an equilibrium where neither platform investment provided a sufficiently large direct return for the platform. To see it, compare the two profits

$$\Pi_{cb}^{inn} - \Pi_{cb}^{no} = \frac{(4\gamma^2 - \beta_b^2)\tau_s}{4(2\tau_s - \beta_s^2)} - \frac{\beta_s\Big(4\beta_b \cdot \tau_s(\phi_s + \phi_b) - \beta_s(8\gamma^2 \cdot \tau_s + (\phi_s + \phi_b)^2)\Big)}{16\tau_s(2\tau_s - \beta_s^2)} \quad \text{(A-10)}$$

and this is positive for $\gamma \geq \gamma_{cb}$ where $\gamma_{cb} = \frac{\beta_s(\phi_s+\phi_b)+2\beta_b \cdot \tau_s}{2\sqrt{2\tau_s(2\tau_s - \beta_s^2)}}$.

Appendix - B (Monopoly)

The model

Consider a monopolist located at 0, a unit mass of buyers and unit mass of sellers uniformly distributed along the interval $[0,1]$. Buyers and sellers face transportation cost τ_k per unit of distance, with $k = \{b, s\}$. Consider the seller indifferent between joining the platform or staying out of the market, it follows that according to the position λ_s, the demand on this side of the market is

$$n_s \equiv \lambda_s = \begin{cases} 0, & \text{if } \lambda_s = 0 \Leftrightarrow \pi_s < 0 \\ \frac{1}{\tau_s}\Big[n_b \cdot \phi_s - p_s + \beta_s \cdot x\Big], & \text{if } \lambda_s \in [0,1) \Leftrightarrow \pi_s > 0 \\ 1, & \text{if } \lambda_s = 1 \Leftrightarrow \pi_s > 0 \quad \& \quad \tau_s < n_b \cdot \phi \cdot \chi + \beta_s \cdot x - p_s \end{cases} \quad \text{(B-1)}$$

depicting the case of *partial market coverage*, for any marginal developer located at $\lambda_s \in (0,1)$, and *full market coverage*, when transportation costs are not large enough. We do not consider the case with no market share, which arises when the price set by the platform is excessively high, i.e. $p_s > n_b \cdot \phi_s + \beta_s \cdot x \Leftrightarrow \pi_S < 0$.

The utility of buyers is instead $u_b = v + \beta_b \cdot x + \phi_b \cdot n_s - p_b - \tau_c \cdot \lambda_b$, when joining the platform, and $u_s = 0$ otherwise. The demand on the consumer side is kinked:

$$n_b \equiv \lambda_b = \begin{cases} 0, & \text{if } \lambda_b = 0 \Leftrightarrow u < 0 \\ \frac{1}{\tau_b}\Big[v + \beta_b \cdot x + \phi_b \cdot n_s - p_b\Big] & \text{if } \lambda_b \in [0,1) \Leftrightarrow u > 0 \\ 1, & \text{if } \lambda = 1 \Leftrightarrow u_b > 0 \quad \& \quad \tau_b < v + n_s \cdot \phi_b + \beta_b \cdot x - p_b \end{cases} \tag{B-2}$$

Again, we do not consider the case with zero market shares on the consumer side, which arises when the monopolist sets a price $p_b > v + n_s \cdot \phi_b + \beta_s \cdot x$.

The profit function is the same as with competition and so the timing. In the first stage of the game, the platform decides whether to engage in value-increasing investments. Conditional on this decision, the platform decides the level of investments and the pricing strategy on the sides of the market. Finally, the market clears. by solving the game backward, the users make heir decision as follows

$$n_b = \begin{cases} \frac{\tau_s(v-p_b)-p_s\chi}{\varepsilon} + \frac{(\beta_b \cdot \tau_s + \beta_s \cdot \phi_b)x}{\varepsilon} & \text{if } n_s < 1 \quad \& \quad \lambda_s \in [0,1] \\ \frac{v+\beta_b \cdot x + \phi_b - p_c}{\tau_b} & \text{if } n_s = 1 \quad \& \quad \lambda_s \in [0,1] \\ 1, & \text{if } n_s = 1 \quad \& \quad \lambda_s = 1 \end{cases} \tag{B-3}$$

and

$$n_s = \begin{cases} \frac{\chi \cdot \phi(v-p_b)-p_s \cdot \tau_b}{\varepsilon} + \frac{x(\beta_s \cdot \tau_c + \beta_b \cdot \phi_s)}{\varepsilon}, & \text{if } n_b < 1 \quad \& \quad \lambda \in [0,1] \\ \frac{\beta_s \cdot x + \phi_s - p_s}{\tau_s}, & \text{if } n_b = 1 \quad \& \quad \lambda \in [0,1] \\ 1, & \text{if } n_b = 1 \quad \& \quad \lambda = 1 \end{cases} \tag{B-4}$$

From the above expressions, four scenarios arise: partial market covered on both sides of the market ($n_s < 1$ and $n_b < 1$), full market covered ($n_s = 1$ and $n_b = 1$), and two asymmetric scenarios with full market coverage in one side and partial market coverage in the opposite side, e.g. $n_s = 1$ and $n_b < 1$ or $n_s < 1$ and $n_b = 1$. We now discuss the case with partial market coverage on both sides of the market. Then, the other three cases are presented.

Partial market coverage on both sides

Consider a partially covered market with $n_s < 1$ and $n_b < 1$. In the second stage, the monopolist platform makes its strategic decision on the level of investments and on the price on both sides of the market. From the first-order conditions, we can derive the equilibrium values and state the following result: when the platform invests, the equilibrium outcome is

$$\begin{aligned}
x^* &= \Big[\gamma\left((\phi_s+\phi_b)^2 - 4\tau_b\cdot\tau_s\right) - v(\beta_s(\phi_s+\phi_b) + 2\beta_b\cdot\tau_s)\Big](\det\mathbf{H})^{-1}\\
p_s^{inn,*} &= \Big[-v(\beta_b\cdot\beta_s\cdot\tau_s + \beta_s^2\cdot\phi_b + \tau_s\cdot\phi_s) + \gamma(\beta_b\cdot\tau_s(\phi_b-\phi_s) + \beta_s(\phi_b(\phi_s+\beta_b) - 2\tau_b\cdot\tau_s)\Big](2\det\mathbf{H})^{-1}\\
p_b^{inn,*} &= \Big[v(\beta_s^2\cdot\tau_b - 2\tau_b\cdot\tau_s + \phi_s(\phi_s+\phi_b) + \beta_s\cdot\beta_s\cdot\phi_s) + \gamma(\beta_s\cdot\tau(\phi_s-\phi_b) + \beta_b(\phi_s(\phi_s+\phi_b) - 2\tau_b\cdot\tau_s))\Big](2\det\mathbf{H})^{-1}\\
\Pi^{inn,*} &= \frac{\gamma^2\left((\phi_s+\phi_b)^2 - 4\tau_b\cdot\tau_s\right) + v^2\left(\beta_s^2 - 2\tau_s\right) - 2\gamma\cdot v(\beta_s(\phi_s+\phi_b) + 2\beta_b\cdot\tau_s)}{2(\det\mathbf{H})}
\end{aligned} \tag{B-5}$$

where $\det\mathbf{H} = 2\beta_s^2\cdot\tau_b + 2\beta_b^2\cdot\tau_s - 4\tau_b\cdot\tau_s + 2\beta_b\cdot\tau_s(\phi_s+\phi_b) + (\phi_s+\phi_b)^2 < 0$. Absent investments, Nash values are

$$\Pi^{no,*} = \Pi(x=0) = \frac{v^2\cdot\tau_s}{4\tau_b\cdot\tau_s - (\phi_s+\phi_b)^2},$$

$$p_b^{no,*} = p_b(x=0) = \frac{v\left(2\tau_b\cdot\tau_s - \chi(\phi_s+\phi_b)\phi_s\right)}{4\tau_b\cdot\tau_s - (\phi_s+\phi_b)^2}, \qquad p_s^{no,*} = p_s(x=0) = \frac{\tau_s\cdot v(\phi_s-\phi_b)}{4\tau_b\cdot\tau_s - (\phi_s+\phi_b)^2} \tag{B-6}$$

Thus, the price set on each side of the market is sensitive again to the different in the cross-network externalities between sides. It can be seen that $p_s^{no,*} < 0$ for $\phi_s < \phi_b$, $p_s^{no,*} = 0$ for $\phi_s = \phi_b$, and $p_s^{no,*} > 0$ for $\phi_s > \phi_b$. In any case, buyers are always charged a positive price. This pricing structure resembles very closely the pricing structure set by platforms when multihoming sellers are allowed. .

Proposition 10. *A monopolist has always an incentive to invest in value-creation as long as the investment cost function is sufficiently convex. The incentive to invest and the level of investment increase with γ, β_s and β_b.*

Proof. To prove the above proposition (i.e, $\Pi(x^*) > \Pi(x=0)$), the interior solution x^* should be unique. If $\Pi(x)$ is concave and x^* is a local maximiser of the profit function, then x^* is also a global maximiser. Rewrite the profit function as $\Pi = R(x) + \gamma\cdot x - \Omega(x)$, and differentiate with respect to x to obtain $\Pi'(x) = R'(x) + \gamma - \Omega'(x)$. The function is concave if $R''(x) - \Omega''(x) < 0 \Leftrightarrow \Omega''(x) > R''(x)$, i.e. $\Omega''(x)$ should be sufficiently convex such that $\Omega''(x) > R''(x) > 0$ and $R'(x^*) + \gamma = \Omega'(x^*)$ maximises the profit function. Provided that the solution is unique, to prove that $\Pi(x^*) > \Pi(x = 0)$, it suffices that $\Pi'|_{x=0} = R'(x)|_{x=0} + \gamma > 0$. Because $R'(x) > 0$, then $p'_s(x) > 0$ and $p'_c(x) > 0$, therefore, revenues increase, Because investments are profit-enhancing, the profit function is well-behaved, and the investment cost is convex, there must be an interior solution x^* such that $\Pi(x^*) > \Pi(x = 0)$. To see it analytically, let $\Omega = x^2/2$, the incentive to invest is equal to:

$$\triangle\Pi = \Pi(x^*) - \Pi(x = 0) = \frac{\Big(\gamma(4\tau_b \cdot \tau_s - (\phi_s + \phi_b)^2) + v(\beta_s(\phi_s + \phi_b) + 2\beta_b \cdot \tau_s)\Big)^2}{2\left((\phi_s + \phi_b)^2 - 4\tau_c \cdot \tau_s\right)(\det \mathbf{H})} > 0. \tag{B-7}$$

□

Proposition 10 states that the incentive to invest always increases with the ability of the platform to obtain any returns. In the same matter, provided that end-users can benefit from platform's investments (i.e. β_s or β_b), the platform promptly increases its spending and extract from users the resultant surplus. To verify how the platform manages its pricing strategies and how extracts the surplus generated to (some) user(s), let us look at the price differentials.

Proposition 11. *When the market is partially covered and the monopolist engages in value-increasing investments for at least one side of the market, buyers are always charged more for any $\phi_s \leq \phi_b$, while sellers are charged more whenever $\phi_s \leq \phi_b$.*

Formally, call $\triangle p_s = p_s^{inn,*} - p_s^{no,*}$ the price differential for developers and $\triangle p_b = p_b^{inn,*} - p_b^{no,*}$ that one for consumers, then:

$$\begin{aligned} \triangle p_b =& \frac{\xi\Big(\beta_b(2\tau_b \cdot \tau_s - (\phi_s + \phi_b)\phi_s) + \beta_s \cdot \tau_b(\phi_b - \phi_s)\Big)}{(\rho^2 - 4\tau_s \cdot \tau_b)(\det \mathbf{H})}, \\ \triangle p_s =& \frac{\xi\Big(\beta_s(2\tau_b \cdot \tau_s - (\phi_s + \phi_b)\phi_b) - \beta_b \cdot \tau_s(\phi_b - \phi_s)\Big)}{((\phi_s + \phi_b)^2 - 4\tau_s \cdot \tau_b)(\det \mathbf{H})}, \end{aligned} \tag{B-8}$$

where $\xi \equiv \gamma(4\tau_b \cdot \tau_s - (\phi_s + \phi_b)^2) + v(\beta_s(\phi_s + \phi_b) + 2\beta_b \cdot \tau_s) > 0$. First, consider the change in the consumer price. As $\phi_s \leq \phi_b$ and $2\tau_b \cdot \tau_s > (\phi_s + \phi_b)\phi_s$, then $\triangle p_b > 0$ is always positive. Next, consider the price on the seller side. Depending on ϕ different cases can be established: for $\phi_s \rightarrow \phi_b$, it follows $sign(\triangle p_s) = sign(\triangle p_b) > 0$. Suppose not, suppose $\phi_s < \phi_b$, it immediately follows that $\triangle p_b > 0$, whereas $\triangle p_s$ is negatively impacted by β_b.

Thus, cross-network externalities have a crucial relevance when investments are considered. For any $\phi_s \rightarrow \phi_b$, both sides of the market value the opposite side in the same way and as a result, both are charged more when investments affect end-users, e.g. $\beta_s > 0$ and $\beta_b > 0$. This result is rather intuitive as both sides evaluate the interaction with the opposite side of the market in the same manner. Things change when interactions with the other side of the market are valued differently: for $\phi_s < \phi_b$, buyers, who benefit more from cross-network externalities, are charged more relative to the pre-investment case, while the effect on sellers may be ambiguous and increases with ϕ_s.

CHAPTER 4

Superstars in two-sided markets: exclusives or not?

4.1 Introduction

Two-sided platforms enable valuable interactions between different groups of agents. When platforms compete, an agent usually faces a trade-off between single-homing and multi-homing. On the one hand, multi-homing allows an agent to interact with a large mass of agents on the other side. On the other hand, the platform is a bottleneck for single-homing agents. As a result, depending on their relative importance for the other side, this puts them in a better bargaining position vis-à-vis the platform(s).

However, not all agents create the same externalities to the other side of the market. As noted by Biglaiser et al. (2019), some agents are more relevant than others and this can give rise to market power. Examples can be found in several markets. In the music industry, popular artists (e.g., Beyoncé, Taylor Swift) are generally valued more than emerging artists. The same happens in the market for apps (e.g., Whatsapp and Instagram), open source software (e.g., Red Hat), games (e.g, Fortnite), news (i.e., Sean Penn interviewing El Chapo), sport broadcasting (e.g., Real Madrid, Juventus). In retail markets as well, consumers usually value the presence of a branded, luxury or popular retailer differently from the presence of local or not-branded retailers. For simplicity, we call these agents "Superstars".

The aim of the present article is twofold. First, we give a rationale to the choice of a Superstar to sign an exclusive contract with a platform. Second, we identify the impact of such a choice on platform competition, i.e., whether exclusives are pro- or anti-competitive.[1] We consider two platforms acting as intermediaries between consumers and firms (e.g., content providers). Consumers subscribe to a platform to have access to its catalogue. The firm side is composed of a Superstar and a mass of small firms. The Superstar acts as a monopolist supplier of her *premium* product and offers take-it-or-leave-it (TIOLI) contracts to either one or both platforms. The other firms are price-takers and have no bargaining power *vis-à-vis* the platforms.

We find that Superstar exclusivity induces demand asymmetries between platforms. Namely, under non-exclusivity, platforms are symmetric, all small firms multi-home, and the downstream market for consumers is equally split. With Superstar exclusivity, more consumers affiliate with this platform attracted by the exclusive premium product. This generates some positive spillovers on the firm side. First, aggregate variety increases as more firms join the platform hosting the Superstar exclusively than in the case of non-exclusivity. Indeed, some zero-homers become single-homers. Second, some small firms who were previously active on both platforms find it profitable to join only the platform hosting the Superstar. Indeed, some multi-homers become single-homers. All in all, there is a (second-order) feedback effect that we call "ripple effect".

Although this effect emphasizes the gains of exclusivity, the latter would require to give up a large customer base and lose the associated revenues. As a result, exclusives would require the Superstar to extract enough surplus (and revenues) from the platform hosting the premium content. The optimal choice ultimately depends on the fierceness of platform competition, which determines the magnitude of the ripple effect. When competition is sufficiently intense, the ripple effect gets stronger and increases the profits of the Superstar, who then opts for exclusivity. This is because a large mass of consumers would migrate to the platform hosting the Superstar. The Superstar extracts more surplus via

[1] Recent evidences show Superstar exclusive contracts in the music industry changed platform competition, helping Apple and Tidal to gain market shares against Spotify. See e.g., RollingStone, October 5, 2016. 'How Apple Music, Tidal Exclusives Are Reshaping Music Industry': `http://www.rollingstone.com/music/news/inside-the-war-over-album-exclusives-w443385`.

an exclusive deal by exploiting the endogenous asymmetry in the market. The mechanism is reversed when platforms are sufficiently differentiated. In this case, the ripple effect is weakened. Not many consumers would switch from one platform to the other and so the Superstar prefers a wider audience and multi-homes.

This article also sheds some light about potential anti- or pro-competitive implications of exclusive contracts for the small firms and consumers. Typically exclusive contracts and market power ring multiple alarm bells in the policy circles. For instance, in the music industry, the Chinese regulator, SAPPRFT, argued that exclusive contracts "ultimately harm the (music) industry". Similar arguments were made by Spotify in 2016 claiming that Superstar exclusives were bad for artists, consumers, and platforms.[2] Our results offer a different perspective. First, we find that exclusivity always increases the welfare of small firms. This happens because Superstar exclusivity encourages the entry of firms that were not active otherwise. Second, in some cases, consumers benefit from exclusivity because final price do not fully internalize the value added by the Superstar. Indeed, our results suggest that policymakers should not be worried. Contrary to the conventional wisdom that regards exclusivity as potentially dangerous for welfare, it may also represent the first-best outcome in the industry.

In the baseline model we make three simplifying assumptions: *(i)* consumer single-homing, *(ii)* one-sided price competition, and *(iii)* presence of one Superstar. Regarding assumption *(i)*, one could argue that consumers may be damaged by exclusivity when they are allowed to multi-home. The staunch fan of a Superstar with a strong preference for one platform would need to multi-home to access the content of her preferred artist at another platform. Similarly, firms are often subsidized or charged to offer their product. This requires us to relax assumption *(ii)*. Furthermore, there may be more than one Superstar and their decision to go exclusive may restore symmetry between platforms as well as reduce or amplify the ripple effect. We relax these three assumptions in the extensions. We demonstrate that our main results and intuitions remain robust to richer and complex

[2] See e.g., Digitalmusicnews, September 18, 2017. 'The Chinese Government Says Streaming Music Exclusives Suck' `https://www.digitalmusicnews.com/2017/09/18/sapprft-streaming-music-exclusives/`
See e.g., The Verge, August 26, 2016. 'Spotify talent manager: Exclusives are 'bad for the whole industry" `https://www.theverge.com/2016/8/26/12657630/spotify-exclusives-subscriber-numbers-2016-troy-carter`.

scenarios: the Superstar always prefers exclusivity in a sufficiently competitive market as the ripple effect is strong and non-exclusivity otherwise.

The outline of the paper is structured as follows. In Section 4.2, we present some parallels with the existing literature. In Section 4.3, we present the preliminaries of the model. We discuss the main results in Section 4.4 and the implications for the welfare and policy-makers in the subsequent section. Section 4.6 discusses several extensions and shows the generality of the model. Section 4.7 provides a discussion of the main results and their applicability to several industries.

4.2 Related Literature

Our article relates to the stream of the economic literature on two-sided markets (Rochet & Tirole 2003, 2006, Armstrong 2006) and on homing decisions. In a recent article, Belleflamme & Peitz (2018) examine the allocative effects of homing decisions. They show that when platforms prefer to impose exclusivity to both sides of the market, at least one side is likely to be harmed, whereas allowing multi-homing may accomplish the purpose of having all sides of the market and the platforms better off. In a similar and related study, Armstrong & Wright (2007) let platforms offer a contract to sellers. They show that when platforms offer an exclusive contract to some sellers, they do so by charging a prohibitively high price to multi-homing sellers and a discount to single-homers. As a result, there is a partial (complete) foreclosure as all users on this side (both sides) would prefer to single-home. In Hagiu & Lee (2011), platforms bid for content providers on a lump-sum transfer and they distinguish two cases: the outright sale of the content to the platform and the control right for the content providers. More recently, Ishihara & Oki (2017) consider platform competition in a market where a monopolist multi-product content provider decides how many content to provide exclusively to each platform and how this affects its bargaining power relative to the platform(s).

This article takes a different perspective. First, while most of the literature considers markets populated by small agents (see e.g., discussion in Biglaiser et al. (2019)), we explicitly model heterogeneity in market power between agents in one side of the market. Specifically, the Superstar acts as an all-powerful supplier of her product and can exercise market power *vis-à-vis* the platforms. The small firms instead are heterogeneous in their production cost and are price-taker. Second, the Superstar offers a premium product relative to the other firms. This is very similar to the premium content discussed by Armstrong (1999) and more recently by D'Annunzio (2017)[3]. Third, following Rosen (1981) we let the Superstar be more efficient than any other firms. This aspect emerges as the small firms have positive and heterogeneous production costs, whereas production costs of the Superstar are negligible.

To the best of our knowledge, this is the first article incorporating agent's market power in a two-sided market model. The contractual arrangement we use is equivalent to letting the Superstar auctioning her exclusive product and let the platform(s) bid for it. For this, we follow Jehiel & Moldovanu (2000): they implement a second-price sealed bid auction with a fixed fee, where the optimal bid equals the difference between winning and losing the auction. In our case, the contract fee is equal to the difference of the profits obtained by the platform winning the contract and the profits obtained when the rival wins the contract.[4]

Exclusive contracts have recently become topical among scholars. Weeds (2016) studies the incentives of a vertically integrated TV to offer its premium programming to a rival distributor. She finds that when competition is dynamic, exclusivity might be the best solution, thereby contrasting traditional findings in static markets. Because of switching costs, the future market share advantage might outweigh the opportunity cost of renouncing to some current audience. Similar to Weeds (2016), in our model, the emergence of exclusivity is linked to the strength of the downstream competition. However, our result depends on the static ripple effect rather than on the dynamic aspects stemming from switching costs. Moreover, our results also differ from D'Annunzio (2017). She consider two competing platforms and the decision to provide a premium content. She shows that whereas a premium content is always offered exclusively, vertical integration between the provider and one platform may change incentives to invest in quality. In our model,

[3] Armstrong (1999) shows that, in a traditional one-sided market, a premium content is always offered exclusively. Moreover, in comparing different types of contracts, he also shows that, with exclusivity, a lump-sum contract is revenue-maximizing relative to a royalty-based one.

[4] Montes et al. (2018) apply this auction mechanism in a model where an upstream data broker sells data to either one or two downstream firms. They show that, with a contract based on a fixed tariff, the data broker always finds it optimal to sell data exclusively to one firm.

the premium provider faces a trade-off between exclusivity and non-exclusivity and this choice depends on how intense platform competition is. On a somewhat different scenario, Kourandi et al. (2015) study the contractual decision made by Internet Service Providers to content providers. Similar to our results, they show that exclusivity can be welfare enhancing when competition of content providers over informative ads is sufficiently intense. Finally, Chen & Fu (2017) show that an exclusive contract determines a surplus reallocation from firms to consumers. We argue that when considering a two-sided market and the ripple effect, social welfare may increase with exclusivity as determining endogenous entry of additional firms.

Whereas several theoretical articles have dealt with exclusive contracts, the empirical literature is still lacking. Datta et al. (2017) examine music consumption and variety in digital platforms and their impact on discovery and top artists. Ershov (2018) looks at the mobile app market. The latter study is quite relevant for our analysis and concerns the entry of a strong competitor (the Superstar) in the Google Play app store. The author shows that when the Superstar enters in a niche market, she entails a demand-discovery effect and generates additional entry. This result is somewhat similar to our mechanism, for which exclusivity fosters more entry of small firms in the market.

4.3 The model

We consider a two-sided market along the lines of Armstrong (2006) where consumers single-home and firms can either multi-home or single-home. Hereafter, we refer to firms as content providers (CPs). There are two platforms $i = 1, 2$ located at the opposite ends of a unit-length Hotelling line. Platform 1 is located at the coordinate $x_1 = 0$, whereas platform 2 is located at the coordinate $x_2 = 1$. Platforms set prices p_i to consumers, whereas CPs freely access the platform and obtain marginal benefits γ when interacting with consumers (i.e., ancillary revenues such as merchandising).[5] There are two types of CPs: small content providers and the Superstar. These are free to choose to not participate in the market (zero-home), to join one platform (single-home), or to join both platforms (multi-home). A graphical representation of the market is depicted by Figure

[5]This assumption is relaxed in Section 4.6.3.

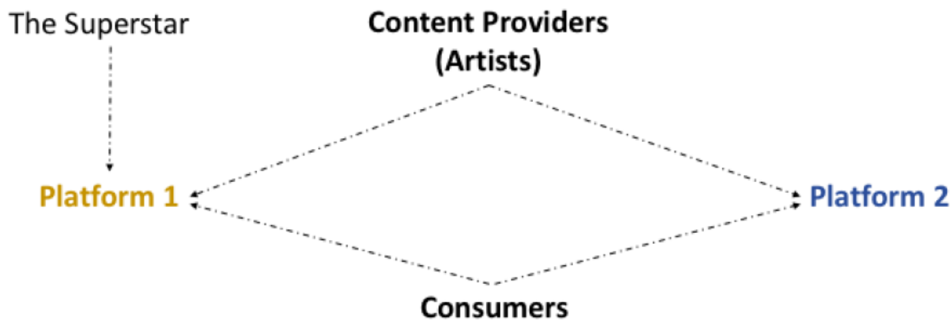


Figure 4.1: Market configuration

4.1.

The Superstar is defined by the following properties. First, she brings to the table an additional value for consumers relative to small CPs. For instance, she offers a premium content with strong capture. Second, she has all the bargaining power over her content and makes a TIOLI offer to the platform(s). The Superstar offers a fixed fee contract from the set $\{\{F_1^E\}, \{F_2^E\}, \{F_1^{NE}, F_2^{NE}\}\}$ to the platforms, where F^E is an exclusive contract and F_i^{NE} is a non-exclusive contract offered to platform i. The profits of the Superstar when offering an exclusive contract to either platform are

$$\pi^S = \gamma \cdot D_i + F^E,$$

when platform i receives the exclusive contract. D_i is the share of consumers subscribing to platform i, while $\gamma > 0$ represents the strength of cross-network externalities. These externalities can be interpreted in terms of final transactions with each consumer joining the platform. For instance, these can identify ancillary revenues such as merchandising, advertising, and any other type of multi-market contact. This assumption captures the idea that CPs care, to some extent, about the size of their potential audience. If the Superstar offers non-exclusive contracts, her profits are

$$\pi^S = \gamma \cdot (D_1 + D_2) + F_1^{NE} + F_2^{NE}.$$

Small CPs have no bargaining power and have heterogeneous production costs denoted by f, which are uniformly distributed on a unit interval and, for each CP, f does not differ across platforms. As the Superstar, CPs obtain the cross-network benefit γ from the participation of an additional consumer at platform $i \in 1, 2$ when affiliating with that platform. A cost-f CP joining platform i obtains the following profits

$$\pi_i^{cp} = \gamma \cdot D_i - f.$$

Small CPs are active on platform i as long as they obtain positive profits i.e., $\pi_i^{cp} \geq 0$ implies $f \leq f_i := \gamma \cdot D_i$. The total mass of small CPs on platform i are

$$n_i = Prob(f \leq \gamma \cdot D_i) = \gamma \cdot D_i. \tag{1}$$

Clearly, a multi-homing CP obtains profits equal to $\pi_i^{cp} = \gamma \cdot (D_1 + D_2) - 2f$.

Consumers are uniformly distributed along the Hotelling line and are identified by their location x. They face a transportation cost τ per unit of distance covered to reach platforms' locations. Consumers identified by the coordinate x closer to platform i tend to prefer platform i all else equal. The utility of a consumer from joining platform i is

$$u_i = v + \phi \cdot g_i + \theta \cdot n_i - p_i - \tau \cdot |x_i - x| \tag{2}$$

where v is the intrinsic utility of joining any of the two platforms, p_i is the subscription price and g_i is a binary function taking the value of 1 when platform i offers the Superstar content and 0 otherwise. The cross-network benefit θ together with ϕ is interpreted as a measure of the aggregate quality of the entire catalogue offered by a platform. Throughout the paper, we assume a sufficiently large v so that consumers always obtain positive utility. We also assume that the transportation cost is sufficiently high (i.e., $\tau > \gamma \cdot \theta + \phi/3$) so as to guarantee concavity in the price of the profit function and positive consumer demands at the two platforms.

The Timing. The timing of the game is as follows. In the first stage, the Superstar decides whether to offer an exclusive or non-exclusive contract. In the second stage, she either offers an exclusive contract (F^E) to platform i or non-exclusive contracts to both the platforms ($\{F_1^{NE}, F_2^{NE}\}$). Platforms accept or reject the offers. In the second stage, price competition takes place and users and CPs make their simultaneous decision on where to join. See also Figure 4.2.

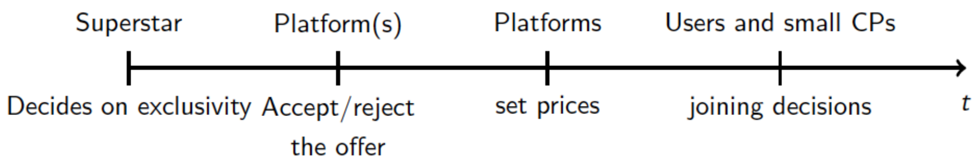


Figure 4.2: Timing of the game

4.4 Analysis

In this section, the model is analyzed by backward induction. We first present the price competition on the consumer side for a given presence of the Superstar in each platform. Then, in the subsequent section, we analyze the optimal contractual choice of the Superstar.

4.4.1 Price competition

Starting from the last stage, for given consumer prices and amount of contents present in each platform, consumers decide which platform to join. Comparing u_1 with u_2, a consumer located at coordinate x will join platform 1 if $x \leq \frac{1}{2} + \frac{\theta \cdot (n_1 - n_2) + (p_2 - p_1) + \phi \cdot (g_1 - g_2)}{2\tau}$, so that the consumer demands on the two platforms are given by

$$D_i(g_i, g_j) = \frac{1}{2} + \frac{\theta \cdot (n_i - n_j) + (p_j - p_i) + \phi \cdot (g_i - g_j)}{2\tau} \qquad D_j(g_j, g_i) = 1 - D_i(g_i, g_j)$$

The mass of small CPs joining platform i is denoted by (1) and given by $n_i = \gamma \cdot D_i$. Since consumers correctly anticipate the number of CPs present in each platform, the consumer demands of platform i and j become

$$D_i(g_i, g_j) = \frac{\tau}{2(\tau - \theta \cdot \gamma)} + \frac{\phi \cdot (g_i - g_j) - (p_i - p_j) - \theta \cdot \gamma}{2(\tau - \theta \cdot \gamma)} \qquad D_j(g_j, g_i) = 1 - D_i(g_i, g_j) \tag{3}$$

Going one step backwards, each platform anticipates the joining decision of consumers and decides the optimal price p_i. Platform $i's$ profits are

$$\Pi_i - g_i \cdot F = p_i \cdot D_i(g_i, g_j) - g_i \cdot F. \tag{4}$$

Notice that, when prices are chosen, platform i has already received (or not) the offer of the Superstar and has already accepted or rejected it. If the Superstar is present exclusively on platform i, i.e., $g_i = 1, g_j = 0$, the platform i has to pay the fixed fee $F = F^E$ in exchange for a rise in consumer demand due to a positive shift equal to ϕ in the consumer utility. If the Superstar is present on both platforms, i.e., $g_i = g_j = 1$, the platform i has to pay the fixed fee $F = F_i^{NE}$ to avoid losing market share to platform j. If the Superstar content is not present on either platform, then $g_i = g_j = 0$.

By differentiating the profits in (4) with respect to p_i, the first-order conditions give the following result.

Lemma 1. *For $i, j \in \{1, 2\}$, with $i \neq j$ firm i's best reply is the following:*

$$p_i(p_j) = \frac{\tau}{2} + \frac{\phi \cdot (g_i - g_j)}{2} + \frac{p_j}{2} - \frac{\theta \cdot \gamma}{2}.$$

As expected, Lemma 1 shows price complementarity and the usual positive effect of the transportation cost on the level of prices. Moreover, as in Armstrong (2006), the last term accounts for the cross-network externalities. The novelty of this paper is the term $\frac{\phi \cdot (g_i - g_j)}{2}$, which captures the impact that the presence of the Superstar has in terms of higher consumer price. Specifically, whenever $g_i = 1 > 0 = g_j$, the Superstar content is exclusive to platform i, which can thus set a higher price in response to rival's price. Differently, if $g_i = g_j$, platforms are symmetric and the model resembles Rasch & Wenzel (2013)'s analysis when content providers' prices are set to zero. Formally, Lemma 1 leads to the following two results.

Lemma 2. *If $g_i = g_j = g \in \{0, 1\}$, the two platforms charge the same price $p^{NE,*} := \tau - \gamma \cdot \theta$ to consumers. The platforms split the market equally ($D^{NE,*} := 1/2$). Content providers with $f < f^{NE} := \gamma/2$ multi-home on both platforms, while content providers with $f > f^{NE}$ zero-home.*

Lemma 2 describes a symmetric scenario where neither platform enjoys the competitive advantage of the Superstar exclusive content. Two cases are comprehended in this scenario. The first one with $g = 0$, in which no platform offers the Superstar content and the second one with $g = 1$, in which both of them offer it. The presence of the Superstar on both platforms makes them symmetric in the eyes of consumers precisely as in the case without any Superstar. Figure 4.3 provides a graphical representation of consumer and content provider demands.

Differently, the equilibrium consumer prices when the Superstar makes an exclusive offer are

Lemma 3. *If $g_i = 1$ and $g_j = 0$, the equilibrium prices are:*

$$p_i^{E,*} = p^{NE,*} + \frac{\phi}{3} \qquad p_j^{E,*} = p^{NE,*} - \frac{\phi}{3}$$

Platform i has a higher consumer demand ($D_i^{E,*} = \frac{1}{2} + \frac{\phi}{6(\tau - \gamma \cdot \theta)} > D_j^{E,*} = 1 - D_i^{E,*}$).

Proof. See Appendix. □

Lemma 3 highlights important differences with the symmetric case described above. First, one can observe that an exclusive contract renders the final prices asymmetric: the price in platform i is always larger than the price in platform j. Then, we also note that

$$p_i^{E,*} = p^{NE,*} + \frac{\phi}{3} > p^{NE,*}, \qquad p_j^{E,*} = p^{NE,*} - \frac{\phi}{3} < p^{NE,*}.$$

So, the price goes up (down) for the platform (not) signing the exclusive contract. Also, the demands of the two platforms are unbalanced in favour of the platform with the Superstar exclusive. We can then conclude that:

Proposition 1. *Superstar exclusivity fosters content variety and induces single-homing of some other content providers. Content providers with $f \leq \gamma \cdot D_j^{E,*}$ multi-home, those content providers with $f \in (\gamma \cdot D_j^{E,*}, \gamma \cdot D_i^{E,*}]$ single-home on platform i, while all content providers with $f > \gamma \cdot D_i^{E,*}$ zero-home.*

The above proposition provides evidence of the mechanism activated by the Superstar. Superstar exclusivity impacts on the homing decision of the other CPs generating additional exclusivity. This is due to an interesting and novel effect which we call *ripple effect*, that is, the feedback generated by exclusivity in one market (the Superstar market) onto another market (the CP one). As depicted by Figure 4.3, when the Superstar offers a non-exclusive contract, the market is equally split and all CPs with low production costs multi-home, while all CPs with high production costs stay out of the market (zero-home). With the Superstar exclusive, more consumers are active on platform i with respect to platform j (first-order effect). Since the number of CPs active on a platform depends on the number of consumers joining that platform, some zero-homers and some multi-homers now become single-homers. Indeed, more CPs enter the market and the mass of CPs active on each platform becomes asymmetric as well.

Figure 4.4 shows this mechanism graphically: all CPs with sufficiently low production costs remain active on both the platforms. Instead, CPs with production costs larger than the utility provided by platform j single-home on platform i, while the others continue to zero-home. In other words, the Superstar's decision triggers a domino effect mediated by cross-network externalities such that (some) high-cost and previously inactive CPs and some CPs previously multi-homing *endogenously* become active exclusively on platform i. Remarkably, this happens without the necessity of an explicit contract as the presence of a larger proportion of consumers on a platform makes it attractive for some high-cost CPs. All in all, Superstar exclusivity generates single-homing of other CPs.

4.4.2 Superstar

Superstar exclusivity

In this subsection, we look at the case when the Superstar offers an exclusive contract to one platform. Borrowing the mechanism from Jehiel & Moldovanu (2000) and Montes et al. (2018), the Superstar offers the contract to platform i under the threat of offering the exclusive content to the rival if platform i rejects the offer. This setting is identical to a setting in which the Superstar let platforms compete in an auction and allocate the exclusive content to the highest bidder. Formally, the Superstar solves the following

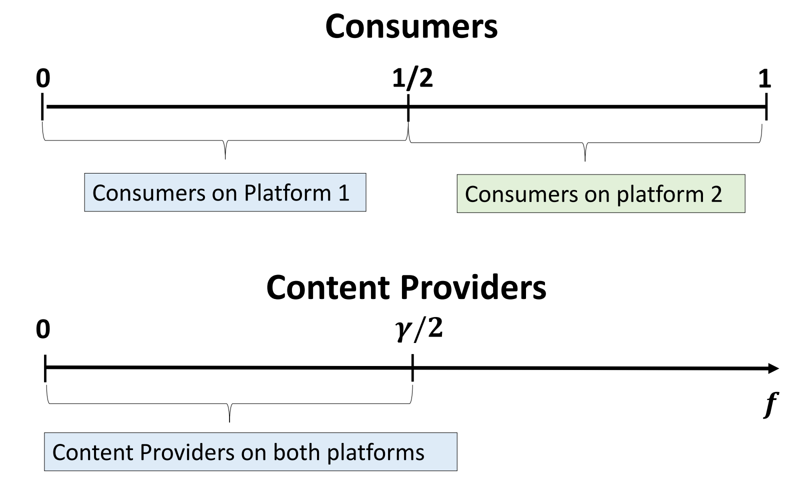


Figure 4.3: Market configuration when the Superstar is absent or offers a non-exclusive contract.

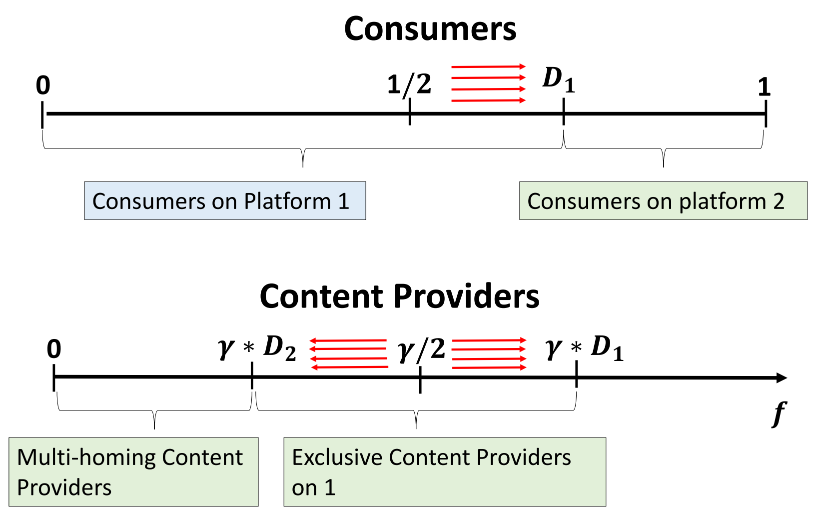


Figure 4.4: Market configuration when the Superstar offers an exclusive contract to platform 1.

problem

$$\max_{F^E} \pi^S = \max \gamma \cdot D_i + F^E$$
$$\text{subject to } \Pi_i^{E,*} - F^E \geq \Pi_i^O$$

where $\Pi_i^O = \Pi_j^{E,*} = \frac{(3(\tau-\gamma\cdot\theta)-\phi)^2}{18(\tau-\gamma\cdot\theta)}$ is the profit of platform i when contractual agreements with the Superstar breaks down and platform j accepts the contract. As a result, the Superstar sets $F^E = \Pi_i^{E,*} - \Pi_i^O$ such that the participation constraint of the platform i is binding.

Lemma 4. *When the Superstar offers an exclusive contract to platform i, she sets a fee equal to* $F^{E,*} = \frac{2\phi}{3}$ *and obtains* $\pi^{S,E*} = \frac{2\phi}{3} + \gamma \cdot \left(\frac{1}{2} + \frac{\phi}{6(\tau-\gamma\cdot\theta)}\right)$.

The above lemma shows that the Superstar appropriates more than two-thirds of the surplus generated by her presence on the consumer side. Given the above contract, demands are $D_i^{E,*} = \frac{1}{2} + \frac{\phi}{6(\tau-\gamma\cdot\theta)}$ and $D_j^{E*} = 1 - D_1^{E,*}$. Notice that, at equilibrium, $1 > D_i^{E,*} > \frac{1}{2} > D_j^{E,*} > 0$. The corresponding total number of CPs on platform 1 is $n_i^{E,*} = \gamma \cdot D_i^{E,*}$ and $n_j^{E,*} = \gamma \cdot D_j^{E,*}$, with $n_i^{E,*} > n_j^{E,*} > 0$. As in Armstrong & Wright (2007), exclusivity induces a demand expansion for the platform getting the contract.

Superstar non-exclusivity

Next, we study an incentive-compatible contract which is accepted by both platforms. To compute the outside option of each platform, we look at the case when the respective platform rejects the contract offered by the Superstar. The profit of a platform i when the Superstar contract is rejected is the one obtained when the offer is accepted only by the rival ($\Pi_j^{E,*} = \frac{(3(\tau-\gamma\cdot\theta)-\phi)^2}{18(\tau-\gamma\cdot\theta)} \equiv \Pi^O$).

Since the Superstar offers a contract to both platforms, she obtains revenues over the entire market. To be incentive compatible, each platform has to prefer profits $\Pi^{NE,*}(g = 1) = (\tau - \gamma \cdot \theta)/2 - F^{NE}$ to the outside option Π^O. Formally, the Superstar solves

$$\max_{F} \pi^S = \max_{F_1^{NE}, F_2^{NE}} \gamma \cdot (D_1 + D_2) + F_1^{NE} + F_2^{NE}$$
$$\text{subject to } \Pi_i^{NE,*}(g=1) - F_i^{NE} \geq \Pi^O \quad \forall i \in 1, 2$$

It follows that the Superstar sets the fixed fees such that the participation constraint of the platforms are binding.

Lemma 5. *When the Superstar offers a non-exclusive contract to both platforms, she sets a fee equal* $F^{NE,*} = \frac{\phi}{3} - \frac{\phi^2}{18(\tau - \gamma \cdot \theta)}$ *and obtains* $\pi^{S,NE*} = \gamma + \frac{2\phi}{3} - \frac{\phi^2}{9(\tau - \gamma \cdot \theta)}$.

Superstar contract choice: Exclusivity or not?

The Superstar's decision is based on the comparison of profits in the two regimes.

Proposition 2. *The Superstar offers an exclusive contract when* $\tau < \bar{\tau} := \frac{9\gamma^2 \cdot \theta + 3\gamma \cdot \phi + 2\phi^2}{9\gamma}$. *Else, she offers a non-exclusive contract.*

Proposition 2 is the result of a trade-off between reaching all consumers (non-exclusivity) and extracting a larger surplus from one platform (exclusivity). The ripple effect elicited by exclusivity results in a larger proportion of consumers joining the platform offering the Superstar content, and so some small CPs single-home along with the Superstar. It is important to note that the ripple effect gets stronger as the degree of differentiation between platforms decreases. This happens because, in a market where platforms are perceived as less differentiated, consumers find themselves less attached to their preferred platform. As a consequence, the exclusive content creates incentives to switch from one platform to another for a larger proportion of consumers as τ decreases. This demand effect is then passed onto the opposite side of the market inducing a larger mass of small CPs active on that platform (ripple effect). This exclusive contract creates a demand expansion on the consumer side resulting in a larger mass of content providers being active, which recursively feeds back onto the consumer side at the platform with exclusive content. The business stealing effect of exclusivity is exacerbated due to two-sidedness of the market. And so, this heightened business stealing effect creates additional revenue which is extracted by the Superstar as $F^{E,*} > 2F^{NE,*}$.

By contrast, when platforms are sufficiently differentiated, the ripple effect is not strong enough as consumers stick to their preferred platform. Indeed, the Superstar prefers a larger audience to the revenues from exclusivity. As in Weeds (2016), it is the intensity of the competition in the market for consumers which makes the difference for a Superstar. However, the mechanism that explains the optimal choice on exclusivity comes from the presence of this complementarity between the two sides which generates the ripple effect.

4.5 Welfare Analysis

To understand the impact of Superstar exclusivity on the welfare of different agents, we first compare Superstar exclusivity with Superstar absence.

Proposition 3. *Superstar exclusivity increases total welfare relative to Superstar absence. Specifically,*

1. *Consumers on the platform with Superstar exclusivity are better-off, whereas those on the rival platform are worse-off. Overall, consumer surplus improves with exclusivity.*

2. *Multi-homing content providers are worse-off whereas those single-homing are better-off. Overall, content provider surplus improves with exclusivity.*

Proof. See Appendix. □

Therefore, there is a positive impact on the total welfare generated by exclusive contracts. In relation to the complete absence of the Superstar, two effects can be highlighted. On the one hand, the consumers and the CPs joining the platform without the Superstar suffer. This is because the presence of the Superstar only on the rival platform results in a shrink in the size of the network. In particular, because of cross-group externalities, the shift of consumers from the platform without a Superstar to the one with the Superstar also reduces the mass of CPs on platform j, with a negative feedback effect on consumers and so on. On the other hand, the platform with the Superstar provides both consumers and single-homing small CPs with a higher surplus compared to the case of no Superstar. In aggregate terms, the second effect always prevails, so consumers and CPs are better off with the Superstar.

The comparison between exclusivity and non-exclusivity becomes now quite straightforward as there are many similarities between the case in which the Superstar multi-homes and the one in which she is absent. This comparison is relevant for two reasons.

First, it allows us to highlight the conditions under which the incentives of the Superstar are aligned/misaligned with the welfare. Second, it challenges the claim made by Spotify in 2016 and by the Chinese regulator that Superstar exclusivity is bad for content providers and consumers. Notice that the surplus of CPs is the same when the Superstar is absent and when she multi-homes, so that we can conclude the following.

Corollary 1. *Overall, content provider surplus improves with exclusivity relative non-exclusivity.*

This result comes as a consequence of the feedback effect inducing some high production cost CPs to single-home and some other CPs to become active in market. While CPs with sufficiently low production cost are not affected by the decision of the Superstar as they continue to multi-home, those who should welcome exclusivity are the CPs with sufficiently high production cost. Interestingly, the presence of an actor with a large market power and with a special treatment creates a positive externality in the market. Indeed, this result suggests that emerging artists who otherwise would have struggled to be active on the market should welcome Superstar exclusivity.

Proposition 4. *Let* $\underline{\gamma} := \frac{5\phi}{3\theta}$ *and* $\tilde{\tau} := \gamma \cdot \theta + \frac{1}{36}\left\{\phi + \sqrt{72\gamma \cdot \theta \cdot \phi + \phi^2}\right\}$. *If competition is sufficiently intense* ($\tau < \tilde{\tau}$) *and content providers' cross-network externalities are sufficiently large* ($\gamma > \underline{\gamma}$), *consumer surplus is higher with Superstar exclusivity relative to non-exclusivity.*

Proof. See Appendix. □

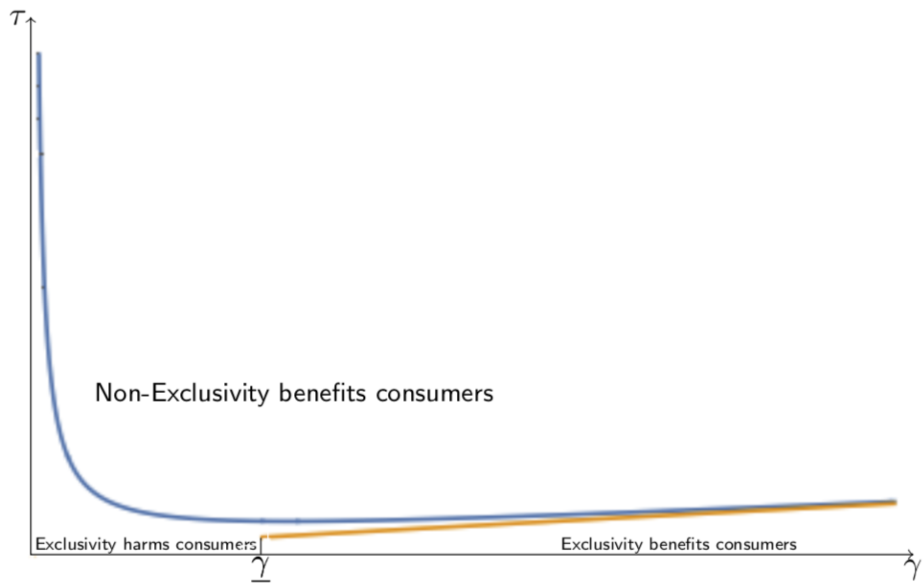
The above proposition suggests that when the market is sufficiently competitive and CPs' ancillary revenues are sufficiently large, the overall effect on consumers is positive. In other words, the positive effect on those consumers joining the platform with the Superstar is strong enough to drive up the total consumer surplus. Instead, when the market is sufficiently differentiated or content providers cross-network externalities (i.e., ancillary revenues) are not large enough, consumers would be better-off with the Superstar offering the contract to both platforms. This result is quite intuitive. If the transportation cost is sufficiently low, more consumers (i.e., consumers located relatively more distant) are willing to join the large platform attracted by the presence of the Superstar. When

CPs' cross-network externalities are sufficiently large, the consumer price (i.e. see Lemma 2) decreases with γ. Indeed, when these two conditions hold, consumers enjoy a larger surplus with exclusivity. Else, consumers prefer non-exclusivity as an exclusive contract will damage consumer welfare.

Putting together the result shown in Proposition 4 with the one in Proposition 2, it is easy to observe that when the Superstar offers non-exclusive contracts, her choice is always welfare-enhancing for consumers and CPs. When the Superstar goes exclusive, it could be detrimental for consumers in a small range of feasible parameters. The following proposition describes this result in detail.

Proposition 5. *If (i) $\gamma < \underline{\gamma}$ and $\tau < \bar{\tau}$ or (ii) $\gamma > \underline{\gamma}$ and $\tilde{\tau} < \tau < \bar{\tau}$, the Superstar incentives are misaligned with those of consumers. In all other cases, the Superstar incentives are aligned with those of consumers. The misalignment occurs only when the Superstar opts for exclusivity.*

Proof. See Appendix. $\square$



The figure shows that in the area on top of the blue line, the Superstar always chooses a non-exclusive contract and benefits the consumers (i.e., high τ). For low τ (under the blue line), the Superstar chooses exclusivity but the effect depends on the parameter γ. When γ is low enough, exclusivity harms consumers, whereas when γ is sufficiently large (under the orange line), exclusivity improves the consumer welfare.

Figure 4.5: Alignment of incentives

Proposition 5 provides a complete picture of the effect of the Superstar's decision on the consumer welfare. It describes situations in which the Superstar's choice between exclusivity and non-exclusivity is not the one that would be preferred by consumers. The Superstar and the consumers both prefer exclusivity when the transportation cost is relatively low and cross-network benefits of CP sufficiently high (below the yellow curve in Figure 3). For sufficiently large transportation costs, the Superstar's choice is aligned with what is beneficial for consumers as well (above the blue curve). In all other cases, the Superstar offers an exclusive deal (because of the ripple effect) but consumers would rather prefer a non-exclusive deal. The eventual convergence of interests has clear implications for policy makers as it does not fully support the claim that exclusivity is bad for consumers.

4.6 Extensions

The above results are robust to several extensions and more complex scenarios. We now consider three alternative model specifications and relax some assumptions which may seem unreasonable at first. Hence, we present a model with multiple Superstars. Then, we introduce the presence of multi-homing consumers. Finally, we formally present a model with a two-sided competition, where platforms can set a price to consumers and CPs.

4.6.1 Two-Superstars

An interesting extension of our model relates to the contracting decisions of multiple Superstars and how these eventually extract the surplus they generate for the platform. Relative to the baseline model with one Superstar, the presence of multiple Superstars lowers the surplus they can grab as the marginal value they create on a platform is now reduced. In turn, this puts platforms in a better bargaining position *vis-à-vis* the Superstar(s). In the following paragraphs, we explore the case of two Superstars.

For simplicity, let us consider $N = 2$ and denote the Superstars by S_k for $k \in \{A, B\}$, each providing consumers with additional value of ϕ_k.[6] These two Superstars simultane-

[6] Our results hold for $N > 2$ Superstars as well.

ously choose their contracts in the first stage. A Superstar S_k's strategy set denoted by $\Omega = \{E_i, E_j, NE\}$ is such that E_i : exclusive contract to platform i, E_j : exclusive contract to platform j, and NE : non-exclusive contracts. The simultaneous choice results in nine possible market outcomes (see Table 4.1).[7] The payoffs of S_k given the strategy choice of the other S_{-k} is given as $\pi_k^{k\omega_k,-k\omega_{-k}}$ where $\omega_k, \omega_{-k} \in \Omega$. The entire game is presented in Appendix.

		S_B		
		E_i	E_j	NE
S_A	E_i	$(\pi_A^{AEi,BEi}, \pi_B^{AEi,BEi})$	$(\pi_A^{AEi,BEj}, \pi_B^{AEi,BEj})$	$(\pi_A^{AEi,BNE}, \pi_B^{AEi,BNE})$
	E_j	$(\pi_A^{AEj,BEi}, \pi_B^{AEj,BEi})$	$(\pi_A^{AEj,BEj}, \pi_B^{AEj,BEj})$	$(\pi_A^{AEj,BNE}, \pi_B^{AEj,BNE})$
	NE	$(\pi_A^{ANE,BEi}, \pi_B^{ANE,BEi})$	$(\pi_A^{ANE,BEj}, \pi_B^{ANE,BEj})$	$(\pi_A^{ANE,BNE}, \pi_B^{ANE,BNE})$

The table depicts the payoffs of a Superstar for a contract strategy given the choice of the other Superstar.

Table 4.1: Payoff Matrix

As mentioned, the presence of another Superstar reduces the marginal value created by a Superstar relative to the benchmark case with a single Superstar. This is because, in case of a contractual breakdown, the platform is now in a better bargaining position. Suppose S_B to offer an exclusive contract to platform i, and S_A opting for exclusivity as well. In case of a contractual breakdown, the threat of S_A exclusively going to its rival does not hurt the platform as much *vis-à-vis* the case of a single Superstar. Hence, the outside option of platform i is larger. The analysis with two Superstars gives us the following result.

Proposition 6. *When there are two Superstars, the equilibrium contract choice is symmetric. Specifically,*

1. *When $\tau < \hat{\tau} := \min\{\tau_A,\ \tau_B\}$, there are multiple Nash Equilibria: (E_i, E_i), (E_j, E_j) and (NE, NE);*
2. *Else, when $\tau \geq \hat{\tau}$, there is a unique Nash equilibrium: (NE, NE);*

where τ_k with $k \in \{A, B\}$ is the transportation cost cut-off to induce exclusivity of Superstar k.

[7] The presence of N Superstars would result in 3^N market outcomes.

Proof. See Appendix. □

Proposition 6 shows that the presence of more than one Superstar always leads to symmetric equilibria. On the one hand, there exist equilibria in which Superstars always find it optimal to offer non-exclusive contracts, regardless of the intensity of competition in the consumer market. On the other hand, when competition is sufficiently intense, Superstars also find it optimal to make an exclusive offer to the same platform.

To grasp the intuition, consider the optimal response of S_B given the choice of S_A. When S_A signs an exclusive contract with platform i, it is first easy to show that S_B would never offer an exclusive contract on platform j. Indeed, this would restore (almost) symmetry between platforms, thereby partially the network effect created by Superstar B. Hence, she faces a choice between offering an exclusive contract platform i and offering a non-exclusive contract to both platform. Namely, she prefers to offer an exclusive contract when she can generate sufficiently large asymmetries in the market and grab the resulting surplus. Else, when S_A offers a non-exclusive contract, the unique best response for the other Superstar is to offer a non-exclusive contract as well. This happens regardless the value of τ. A similar argument can be made for S_A. Hence, we always obtain symmetric contractual choices.

All in all, the analysis of the best responses identifies the presence of different equilibrium strategies. For a sufficiently low τ, Superstars either single-home on the same platform or multi-home. By contrast, for a sufficiently large τ, multi-homing is the only possible equilibrium. These results are consistent with the analysis in the main model and picture a scenario in which Superstars' decisions are self-reinforcing. Superstars follow other Superstars as a lack of coordination would harm them by neutralizing the externalities they create in the market and on the basis on which they exploit their market power. An important and immediate implication is that exclusivity engenders again more exclusivity not only among small CPs (as we previously shown) but also between Superstars.

4.6.2 Multi-homing Consumers

In this section we consider multi-homing consumers. This is important as the value of the interaction between different sides of the market is what engenders our ripple effect, which is crucial for the Superstar's decision. When consumers multi-home, their switching behaviour gets less relevant for CPs. Recent research has highlighted that multiple interactions with the same consumers generate decreasing returns for the opposite side of the market, such as advertisers or content providers (Ambrus et al. 2016, Calvano et al. 2017, D'Annunzio & Russo 2017, Anderson et al. 2018). As our ripple effect depends on how many consumers would switch in response to the Superstar decision, multi-homing is intuitively likely to dampen it. Notwithstanding, we show that for sufficiently low transportation costs, the Superstar prefers to sign an exclusive contract. Else, she prefers non-exclusivity. In the Appendix, we show that our main results and intuitions persist even in a conservative scenario.

4.6.3 Two-sided Pricing

So far, we presented a simple model where platforms charge only consumers and CPs make ancillary revenues. In the real world, it is often the case that platforms also charge the other side of the market. For instance, CPs are remunerated by platforms like Spotify and Tidal whereas Apple developers pay an annual fee to join the Developer Programme. In a two-sided market framework, this implies that platform i sets a duple of prices $\{l_i, p_i\}$ to maximize profits, where p_i is the price set on the consumer side and l_i is the one on the CP side.

Relative to a model *à la* Armstrong (2006), in which the price structure only depends on platform differentiation (τ on both sides) and on cross-group externalities, in this model platforms are differentiated only on the consumer side. This clearly implies that a positive price paid by CPs does not necessarily result in a negative price paid by consumers. Specifically, the consumer price will be positive, whereas the the price for CPs will be always positive when their cross-group externalities $\gamma > \theta$ and negative otherwise. In other words, CPs would be subsidized when ancillary revenues are sufficiently low (see Appendix).

Despite the fact that CPs are now influenced by the price (subsidy) when deciding to join the market, the Superstar's decision again remains influenced only by the fierceness of downstream competition. When ancillary revenues for CPs are sufficiently relatively small, CPs are subsidized ($l_i < 0$) for the externality they create. Under exclusivity, the response of any CP to additional consumer switching from platform j to i is less reactive. So, the platform hosting the Superstar subsidizes CPs even more. In the opposite case, consumers are more important for the other side of the market, so the platform extracts more surplus by charging them a higher price under exclusivity.

As we discussed above, exclusivity entails a direct effect on the consumer side and a feedback on the CPs. By subsidizing or charging CPs, the platform mainly manages the size of the feedback effect. The direct effect on the consumer side of the market (which engenders the feedback effect) instead continues to depend on platform differentiation. So, exclusivity emerges in equilibrium when τ is sufficiently low, and non-exclusivity otherwise. A potential shortcoming may be related to the welfare. As CPs pay a price, it is not straightforward that exclusivity brings about a surplus generation. However, we show that it is the case also in this richer setup (see Appendix).

4.7 Discussion and Concluding Remarks

We rely on a tractable yet intuitive model and provide a framework to understand why exclusive contracts with an agent with market power (i.e., the Superstar) often emerge in several two-sided markets. Specifically, we answer the question "Would a Superstar offer content exclusively on one platform while trading-off a larger audience for a remunerative contract?". This paper presents the mechanism through which the Superstar prefers (not) to offer an exclusive contract to a platform.

In terms of the contractual choice, the Superstar decides to join one platform (exclusivity) rather than all platforms (non-exclusivity) when platforms are not too differentiated from the perspective of final consumers. A ripple effect is associated with the choice of exclusivity, as it leads to asymmetric consumer demand in favor of the platform that hosts the Superstar. As a result, this platform is also more appealing for a larger mass of content providers as some zero-homers and multi-homers become single-homers. This effect becomes less important when platforms are sufficiently differentiated, as consumers will be less likely to switch from their preferred platform. Our model predicts that a Superstar should prefer to be present only on one platform whenever platforms are sufficiently similar in the eyes of consumers.

This result is robust to several extensions analyzed in the paper and it is applicable to multiple industries. For example, in the supply chain industry, an agent offering patent rights for a technology enhancing consumer experience may either sign an exclusive contract or non-exclusive contracts. We conjecture that the platform winning the exclusive right would attract more consumers as well as a larger cluster of small suppliers to that product. This may result in cheaper production costs enhancing further a manufacturing firm's market power *vis-à-vis* the rival. The contractual choice will again depend on the magnitude of the *ripple effect*. Another industry where our results are applicable is the consumer retail mall industry. Popular stores exclusively in one mall can attract a larger footfall of consumers *vis-à-vis* a rival mall. This makes it more attractive for smaller stores to aggregate at that mall and hence more consumers so on and so forth.

This work provides two very clear implications for competition policy. The presence of exclusive deals and market power usually cause concern among policy-makers and regulators. Firstly, we suggest that, in most cases, the contract decision arising in the market is the first best from a policy-maker's perspective. This is because it always generates positive spillovers in the seller side of the market (i.e., firms, content providers, apps, shops) and can benefit consumers in several cases. We thus recommend policy-makers to be circumspect when making market interventionist policies to correct for the apparent harm in the market caused by exclusives. Market intervention in most cases may be detrimental to welfare. Hence, catering to the extant negative view on exclusives is not advisable without a detailed analysis of the specific market. Secondly, we suggest that the *raison d'être* of exclusive deals in the market may not be due to firm anti-competitive strategies but they may be market-determined contracts that turn out to be welfare enhancing.

Appendix

Proof of Lemma 2

When $g_i = g_j = g$, the best replies of the two firms are symmetric, so that $p_j(p_i) = p_i(p_j) = p^{NE,*} = \tau - \gamma \cdot \theta$. Plugging $p^{NE,*}$ and g into the demand and profit functions, we obtain $D^* = 1/2$ and $\Pi^{NE,*} := (\tau - \gamma \cdot \theta)/2 - g \cdot F_i^{NE}$. For the last point, it is just sufficient to notice that $n_i = \gamma \cdot D_i = \gamma/2$. For the sake of notation, we identify these benchmark outcomes with the superscript 'NE'.

Proof of Lemma 3

When $g_i = 1$ and $g_j = 0$, the best replies are retrieved by Lemma 1. The equilibrium prices are simply $p_i^{E,*} = p^{NE,*} + \frac{\phi}{3}$ and $p_j^{E,*} = p^{NE,*} - \frac{\phi}{3}$. Plugging them into the demand functions, the demand for platform i is $D_i^{E,*} = \frac{1}{2} + \frac{\phi}{6(\tau-\gamma\cdot\theta)}$ and the demand for platform j is $D_j^{E,*} = \frac{1}{2} - \frac{\phi}{6(\tau-\gamma\cdot\theta)} = 1 - D_i^{E,*}$. Coupling together equilibrium prices and demands, platforms' profits are $\Pi_i^{E,*} = \frac{(3(\tau-\gamma\cdot\theta)+\phi)^2}{18(\tau-\gamma\cdot\theta)} - F^E$ and $\Pi_j^{E,*} = \frac{(3(\tau-\gamma\cdot\theta)-\phi)^2}{18(\tau-\gamma\cdot\theta)}$ respectively.

Proof of Proposition 3

First, consider the case of no Superstar, i.e., $g_1 = g_2 = 0$. Consumer surplus is

$$CS^b = 2\int_0^{\frac{1}{2}} \Big(v + \theta \cdot \frac{\gamma}{2} - (\tau - \gamma \cdot \theta) - \tau \cdot x\Big)dx = v + \frac{1}{4}\Big(6\gamma \cdot \theta - 5\tau\Big) = v + \frac{1}{4}\Big(\gamma \cdot \theta - 5p^{NE,*}\Big)$$

where the superscript 'b' indicates the consumer surplus in the benchmark regime without the Superstar.[8] This never occurs at equilibrium as the Superstar always finds it profitable to be active in the market. Nevertheless, it is useful to grasp the net effect of the presence of the superstar. In terms of surplus accruing to the CPs, we get:

$$PS^b = \int_0^{\frac{\gamma}{2}} \Big(\gamma - 2x\Big)dx = \frac{\gamma^2}{4}.$$

[8] Note that due to the symmetry of demands under the benchmark regime and the non-exclusive contracts case $p^{b,*} := p^{NE,*}$.

Notably, the reason for the above expression is that, due to the symmetry of the game, on equilibrium, all active CPs multi-home and hence find it profitable to access the total consumer base while incurring double production costs. Indeed, the surplus accruing to these CPs only depends on their cross-network externalities, that is, their transaction with final consumers as there is no price due to affiliating with a platform.

Next, consider the case when the Superstar offers an exclusive contract. The consumer surplus on platform i is:

$$\begin{aligned} CS_i^{E,*} &= \int_0^{D_i^{E,*}} \Big(v + \phi \cdot n_i^{E,*} - p_i^{E,*} + \phi - \tau \cdot x\Big) dx \\ &= \frac{3p_i^{E,*}}{72(p^{NE,*})^2} \Big\{\phi \cdot (7\tau + 6\gamma \cdot \theta) - 3(5\tau - 4v - 6\gamma \cdot \theta) \cdot p^{NE,*}\Big\}. \end{aligned}$$

whereas the consumer surplus on platform j is

$$\begin{aligned} CS_j^{E,*} &= \int_{D_i^{E,*}}^1 \Big(v + \phi \cdot n_j^{E,*} - p_j^{E,*} - \tau(1-x)\Big) dx \\ &= \frac{(3p_j^{E,*})^2}{72(p^{NE,*})^2} \Big\{\phi \cdot (5\tau + 65\gamma \cdot \theta) - 3(5\tau - 4v - 6\gamma \cdot \theta) \cdot p^{NE,*}\Big\}. \end{aligned}$$

The total consumer surplus is

$$CS^{E,*} = CS_i^{E,*} + CS_j^{E,*} = v + \frac{3\gamma \cdot \theta + \phi}{2} + \frac{\tau}{36}\Big\{\frac{\phi^2}{(p^{E,*})^2} - 45\Big\}$$

Consider now the surplus accruing to CPs. For ease of exposition, we distinguish between the surplus generated for those CPs who multi-home (i.e., PS_m^s) and for those who single-home (i.e., PS_s^s) as follows:

$$PS_m^E = \int_0^{n_j} \Big(\gamma - 2x\Big) dx \qquad\qquad PS_s^E = \int_{n_j}^{n_i} \Big(\gamma - x\Big) dx$$

By comparing the consumer surplus on platform i in the two cases, we have

$$CS_i^{E,*} > CS_i^{b*} \Leftrightarrow CS_i^{E,*} - CS_i^{b*} = \frac{\phi \cdot \tau}{12p^{NE,*}} + \frac{(\tau + p^{NE,*}) \cdot \phi^2}{72(p^{NE,*})^2} > 0$$

whereas the effect on the consumer surplus on platform j is

$$CS_j^{E,*} < CS_j^{b,*} \Leftrightarrow CS_j^{E,*} - CS_j^{b,*} = \frac{\phi \cdot (p^{NE,*} - \gamma \cdot \theta)}{12p^{NE,*}} + \frac{(\gamma \cdot \theta - 5p^{NE,*}) \cdot \phi^2}{72(p^{NE,*})^2} < 0.$$

Overall, the consumer surplus increases when there is exclusivity. On the CP side, we have

$$PS^{E,*} - PS^{b,*} = \frac{\gamma^2 \cdot \phi^2}{36(p^{NE,*})^2} > 0$$

where $PS^{E,*} = PS_s^{E,*} + PS_m^{E,*}$. This happens as a larger mass of CPs are now active on platform i

$$\left(PS_s^{E,*} + PS_m^{E,*}/2\right) - PS^b/2 = \frac{\gamma^2 \cdot \phi^2}{24(p^{NE,*})^2} > 0$$

which dominates the surplus loss due to a lower mass of CPs multi-homing and joining platform j

$$PS_m^{E,*}/2 - PS^{b,*}/2 = -\frac{\gamma^2 \cdot \phi^2}{72(p^{NE,*})^2} < 0$$

Proof of Proposition 4

First, we need to calculate the consumer surplus when there is a non-exclusive contract. This is simply equal to $CS^{NE} = CS^b + \phi$. A simple comparison gives

$$\begin{aligned}
CS_i^{E,*} > CS_i^{NE,*} \Leftrightarrow & CS_i^{E,*} - CS_i^{NE,*} = \frac{\phi \cdot (\gamma \cdot \theta - 5p^{NE,*})}{12p^{NE,*}} + \frac{(6p^{NE,*} + \tau) \cdot \phi^2}{72(p^{NE,*})^2} > 0 \\
CS_j^{E,*} < CS_j^{NE,*} \Leftrightarrow & CS_j^{E,*} - CS_j^{NE,*} = -\frac{\phi \cdot p^{b,*} \cdot \tau}{12p^{b,*}} - \frac{(\gamma \cdot \theta - 5p^{b,*}) \cdot \phi^2}{72(p^{b,*})^2} < 0.
\end{aligned}$$

The overall effect on the consumer surplus is such that

$$\begin{aligned}
CS^{E,*} - CS^{NE,*} =& \frac{\phi}{36}\Big\{ -18 + \frac{\tau \cdot \phi}{(p^{b,*})^2}\Big\} > 0 \\
&\Leftrightarrow \tau < \tilde{\tau} := \gamma \cdot \theta + \frac{1}{36}\Big\{\phi + \sqrt{72\gamma \cdot \theta \cdot \phi + \phi^2}\Big\} \quad \text{and} \quad \gamma > \underline{\gamma} := \frac{5\phi}{3\theta}
\end{aligned}$$

Proof of Proposition 5

To prove Proposition 5, compare the two cut-offs of $\tilde{\tau}$ and $\bar{\tau}$. It immediately follows that $\bar{\tau}$ can be larger or smaller than $\tilde{\tau}$ for some parameter ranges. When $\gamma < \underline{\gamma}$, non-exclusivity damages consumers for any level of τ. Hence, when $\gamma < \underline{\gamma}$ and $\tau < \bar{\tau}$, we have a misalignment of incentives: the Superstar chooses an exclusive contract but consumers prefer a non-exclusive one. Suppose $\gamma > \underline{\gamma}$. In this case, if $\tau < \bar{\tau}$ exclusivity is chosen and it increases consumer welfare. It immediately follows that interests of consumers and the Superstar are aligned for $\tau < \min[\tilde{\tau}, \bar{\tau}]$. Else, these are misaligned. Note that misalignment arises only when the Superstar chooses an exclusive contract.

Two-Superstars

To study the effect of more than one Superstar. For simplicity, let there be two Superstars S_k for $i \in \{A, B\}$ generating a consumer benefit ϕ_k in the market. The timing of the game is similar as in the main model. These two Superstars make a simultaneous contract choice in the first stage by choosing among three strategies, $\{E_i, E_j, NE\}$, where E_i (E_j) identifies the case when Superstar S_k offers an exclusive contract to platform i (j) and NE when she offers a non-exclusive contract. Hence, there are nine possible outcomes in the market (Table 4.1). We proceed as follows. First, we present the four main scenarios arising and the outside option each platform faces when a Superstar makes an offer.[9] In particular, we look at the cases when (i) Superstars are exclusive on different platforms, (ii) Superstars are exclusive on the same platform, (iii) one Superstar multi-homes and the other is exclusive, and (iv) both Superstars multi-home. Then, we solve the game and show that the main intuitions of the model remain (almost) unaltered.

Superstars are exclusive on different platforms

Consider the case where Superstar S_A goes exclusively to platform i and Superstar S_B goes exclusively to platform j. The utility of the consumer subscribing to platform i is $u_i = v + \phi_A - p_i - \tau|x_i - x|$, whereas the utility of an agent subscribing to platform j is

[9] It suffices to have only four scenarios to solve the game, as the remaining payoffs can be obtained by appropriate substitution of the notations for those cases.

$u_j = v + \phi_B - p_j - \tau|x_j - x|$. The demands are easily derived. When price competition takes place, platform i sets a price

$$p_i = \tau - \gamma \cdot \theta + \frac{\phi_A}{3} - \frac{\phi_B}{3} \text{ and } p_j = \tau - \gamma \cdot \theta + \frac{\phi_B}{3} - \frac{\phi_A}{3}.$$

It immediately follows that when Superstars are symmetric, $\phi_A = \phi_B$, the prices are the same as in our benchmark regime with superstar multi-homing. Hence, Superstars if they cannot coordinate their behavior will create externalities on each other. This happens because the marginal benefit of having a Superstar on board is reduced as platforms get more symmetric. The resulting platform profits in the second period are

$$\Pi_i^{AEi,BEj} - F_A^E = \frac{(3(\tau - \gamma \cdot \theta) + \phi_A - \phi_B)^2}{18(\tau - \gamma \cdot \theta)} - F_A^E$$

and

$$\Pi_j^{AEi,BEj} - F_B^E = \frac{(3(\tau - \gamma \cdot \theta) - \phi_A + \phi_B)^2}{18(\tau - \gamma \cdot \theta)} - F_B^E$$

where $\Pi_i^{AEi,BEj}$ denotes the profit of platform i and $\Pi_j^{AEi,BEj}$ denotes the profit of platform j.

In the first stage of the game, Superstars make simultaneous TIOLI offers to the platforms. Given that S_B offers an exclusive contract to platform j, S_A offers a fixed tariff such that platform i gets its outside option. The outside option for platform i represents the case in which the exclusive contract is offered to j. Formally, S_A solves

$$\max_{F^{AE}} \pi_A^{AEi,BEj} = \max \gamma \cdot D_i + F_A^E$$
$$\text{subject to } \Pi_i^{AEi,BEj} - F_A^E \geq \Pi_i^O$$

where $\Pi_i^O = \frac{(3(\tau-\gamma\cdot\theta)-(\phi_A+\phi_B))^2}{18(\tau-\gamma\cdot\theta)}$ is the profit of platform i when both Superstars are on platform j. Setting the fixed fees to just satisfy the participation constraint of the platform i, we get $F_k^{AEi,BEj} = \Pi_i^E - \Pi_i^O$. Superstars' profits are then given as

$$\pi_A^{AEi,BEj} = \frac{(3\tau(3\gamma + 4\phi_A) - 4\phi_A \cdot \phi_B - 9\gamma^2 \cdot \theta + 3\gamma(\phi_A - 4\phi_A \cdot \theta - \phi_B)}{18(\tau - \gamma \cdot \theta)}$$
$$\pi_B^{AEi,BEj} = \frac{(3\gamma(3\tau - 3\gamma \cdot \theta - \phi_A) + 3(4\tau + \gamma - 4\gamma \cdot \theta)\phi_B - 4\phi_A \cdot \phi_B)}{18(\tau - \gamma \cdot \theta)}$$

Clearly, similar payoffs are derived when Superstar B (A) offers an exclusive contract to platform i (j).

Superstars are exclusive on the same platform

Consider now the case that both Superstars join the same platform and offer exclusive contracts either on platform i or j. As these two cases are identical, we only present the scenario where both join platform i. As the Superstars' contribution to an agent subscribing to platform i is additive, prices, demands, and profits are identical to those presented in Section 4.4 with replacing $\phi = \phi_A + \phi_B$. The price set by platform i (j) increases (decreases) by $\frac{\phi_A+\phi_B}{3}$ and the corresponding platform profits are

$$\Pi_i^{AEi,BEi} - F_A^E - F_B^E = \frac{(3(\tau - \gamma \cdot \theta) + \phi_A + \phi_B)^2}{18(\tau - \gamma \cdot \theta)} - F_A^E - F_B^E$$

and

$$\Pi_j^{AEi,BEi} = \frac{(3(\tau - \gamma \cdot \theta) - \phi_A - \phi_B)^2}{18(\tau - \gamma \cdot \theta)}.$$

In the first stage of the game, Superstar A (B) makes TIOLI offers. The fixed fees are set to offer the platform i just its outside option. The outside option of platform i when contracting with S_A is the profit i obtains when S_A contracts exclusively with platform j whereas S_B still contracts with platform i.

Hence, the outside option for platform i when rejecting Superstar A and B's offers are

$$\Pi_i^{O,A} = \frac{(3(\tau - \gamma \cdot \theta) - \phi_A + \phi_B)^2}{18(\tau - \gamma \cdot \theta)} \qquad \text{and} \qquad \Pi_i^{O,B} = \frac{(3(\tau - \gamma \cdot \theta) + \phi_A - \phi_B)^2}{18(\tau - \gamma \cdot \theta)}$$

respectively. Similar mechanism works when both Superstars offer a contract to platform j. So the optimal fees are $F_A^E = \Pi_i^{AEi,BEi} - \Pi_i^{O,A}$ for A and $F_B^E = \Pi_i^{AEi,BEi} - \Pi_i^{O,B}$ for B. The resulting Superstar profits are

$$\pi_A^{AEi,BEi} = \frac{(3\tau(3\gamma + 4\phi_A) + 4\phi_A \cdot \phi_B - 9\gamma^2 \cdot \theta + 3\gamma(\phi_A - 4\phi_A \cdot \theta + \phi_B)}{18(\tau - \gamma \cdot \theta)}$$
$$\pi_B^{AEi,BEi} = \frac{(3\gamma(3\tau - 3\gamma \cdot \theta + \phi_A) + 3(4\tau + \gamma - 4\gamma \cdot \theta)\phi_B + 4\phi_A \cdot \phi_B)}{18(\tau - \gamma \cdot \theta)}$$

One Superstar multi-homes and other single-homes

Consider now the case in which one Superstar multi-homes and the other Superstar offers an exclusive content. With appropriate substitution, this case corresponds to four potential scenarios: (i) S_A multi-homes while S_B exclusively goes on platform i, (ii) S_A multi-homes while S_B exclusively goes on platform j, (iii) S_A goes exclusively on platform i and S_B multi-homes, and iii)S_A goes exclusively go on platform j and S_B multi-homes. For the sake of simplicity, let us suppose that S_A multi-homes and S_B offers an exclusive deal to platform i. As A multi-homes, she does not have any impact on prices. Instead, platform i with S_B exclusive charges $p_j^{ANE,BEi} = \tau - \gamma \cdot \theta + \frac{\phi_B}{3}$ while platform j charges $p_i^{ANE,BEi} = \tau - \gamma \cdot \theta - \frac{\phi_B}{3}$. Related profits are

$$\Pi_i^{ANE,BEi} - F_A^{NE} - F_B^{Ei} = \frac{(3(\tau - \gamma \cdot \theta) + \phi_B)^2}{18(\tau - \gamma \cdot \theta)} - F_A^{NE} - F_B^{Ei}$$

and

$$\Pi_j^{ANE,BEi} - F_B^{NE} = \frac{(3(\tau - \gamma \cdot \theta) - \phi_B)^2}{18(\tau - \gamma \cdot \theta)} - F_B^{NE}.$$

In the contracting stage, S_A offers a non-exclusive contract to i (j) under the threat that in case of a contractual breakdown, she would single- home on j (i). S_B offers an exclusive contract under the threat that, in case of a contractual breakdown, she would be exclusive on j. As a result, the outside option for i when an offer is made by S_A is equal to the profit obtained when Superstars are exclusive on different platforms and the fee set by A on i is

$$F_{Ai}^{NE} = \Pi_i^{ANE,BEi} - \frac{(3\tau - 3\gamma \cdot \theta + \phi_B - \phi_A)^2}{18(\tau - \gamma\theta)}.$$

The fee clearly differs on platform j as the outside option is that S_A offers an exclusive contract to platform i who already hosts B. In other words, the outside option for platform j is to be in a situation where both Superstars are on platform i. So,

$$F_{Aj}^{NE} = \Pi_j^{ANE,BEi} - \frac{(3\tau - 3\gamma\theta - \phi_B - \phi_A)^2}{18(\tau - \gamma \cdot \theta)}.$$

Now, consider the threat made by S_B: if i does not the contract, this will be offered to j and the outside option will be $\frac{(3\tau-3\gamma\cdot\theta+\phi_B)^2}{18(\tau-\gamma\theta)}$ so $F_B^{NE} = \Pi_i^{NEi,AEi} - \Pi_j^{ANE,BEj}$. Final profits for S_A and S_B are

$$\pi_A^{ANE,BEi} = \gamma + \frac{\phi_A}{9}(6 - \frac{\phi_A}{\tau-\gamma\cdot\theta}) \qquad \text{and} \qquad \pi_B^{ANE,BEi} = \frac{\gamma}{2} + \frac{\phi_B(4(\tau-\gamma\cdot\theta)+\gamma)}{6(\tau-\gamma\cdot\theta)}$$

Profits in all other scenarios can be easily calculated and they are not reported for the sake of brevity.

Both Superstars multi-home

Finally consider the case that both superstars multi-home. Here, demands are identical to the those in the main model. Platforms are symmetric and set a price equal to $p_i = \tau - \gamma\cdot\theta$ and the corresponding platform profits are

$$\Pi_i^{ANE,BNE} = \frac{\tau-\gamma\cdot\theta}{2} - F_{A,i}^{NE} - F_{B,i}^{NE}.$$

In the first stage, each Superstar makes a TIOLI offer to a platform under the threat of exclusivity on its rival's platform. Formally, a Superstar S_A solves

$$\max_{F_{A,i}^{NE},F_{B,j}^{NE}} \pi_A^{ANE,BNE} = \gamma\cdot 1 + F_{A,i}^{NE} + F_{A,j}^{NE}$$
$$\text{subject to } \Pi_i^{ANE,BNE} - F_{A,i}^{NE} \geq \Pi^O \quad \forall i \in 1,2$$

where $\Pi_i^O = \frac{(3(\tau-\gamma\cdot\theta)-\phi_A)^2}{18(\tau-\gamma\cdot\theta)}$ for $i \in \{1,2\}$ are the profits obtained by the platform when S_A sells exclusively to platform j while S_B multi-homes. Hence, the non-exclusive fees set by Superstar S_A to platform i and j respectively are $F_{A,i}^{NE} = \Pi_i^{ANE,BNE} - \Pi_i^O$ and $F_{A,j}^{NE} = \Pi_j^{ANE,BNE} - \Pi_i^O$. Final profits for S_K are

$$\pi_k^{ANE,BNE} = \tau - \gamma\cdot\theta + \gamma + \frac{\phi_k}{9}(6 - \frac{\phi_k}{\tau-\gamma\cdot\theta}).$$

Simultaneous contract choice of the two Superstars

Suppose that S_{-k} is exclusive on platform i. It is easy to see that S_k would never choose to exclusively offer the contract to platform j. Moreover, the best response of S_k now depends on the transportation cost. If transportation costs are low, then she goes exclusively on platform i else, she multi-homes. Specifically:

$$\pi_k^{AEi,BEi} > \pi_k^{ANE,BEi} \text{ for } \tau < \tau_A := \frac{9\gamma^2 \cdot \theta + 3\gamma \cdot \phi_B + 2\phi_A^2 + 3\gamma \cdot \phi_A + 4\phi_A \cdot \phi_B}{9\gamma}.$$

Similarly, we get the cut-off for S_B when S_A single-homes. Specifically:

$$\pi_B^{AEi,BEi} > \pi_B^{ANE,BEi} \text{ for } \tau < \tau_B := \frac{9\gamma^2 \cdot \theta + 3\gamma \cdot \phi_B + 2\phi_B^2 + 3\gamma \cdot \phi_A + 4\phi_A \cdot \phi_B}{9\gamma}.$$

Here notice that $\tau_A < \tau_B$ when $\phi_A < \phi_B$.[10]

Moreover, for the case that S_B (S_A) multi-homes, the best response of S_A (S_B) is to multi-home as well. Hence, the equilibrium contract choice is given as follows:
For $\tau < \min\{\tau_A,\ \tau_B\}$, the Nash Equilibria are given by (E_i, E_i), (E_j, E_j) and (NE, NE).
For $\tau \geq \min\{\tau_A,\ \tau_B\}$, there is a unique Nash Equilibrium given by (NE, NE).

Multi-homing Consumers

To study multi-homing consumers, we present the most unfavourable scenario for the existence of the ripple effect, that is, when CPs do not weigh differently the value accruing from multi-homing or single-homing consumers. In other words, the value of switching is less relevant.

We begin by considering CP's profits on platform i as equal to $\pi_i^{cp} = \gamma \cdot (D_i^S + D^M) - f$ where D_i^S is the mass of single-homing consumers and D^M the mass of multi-homing consumers. The total mass of small CPs on platform i is:

$$n_i = Prob(f \leq \gamma \cdot (D_i^S + D^M)) = \gamma \cdot (D_i^S + D^M) \tag{5}$$

A multi-homing CP obtains $\pi_i^{cp} = \gamma \cdot (D_1 + D_2) - 2f = \gamma \cdot (1 + D^M) - 2f$. Using the

[10] The difference is given as $\tau_A - \tau_B = \frac{2(\phi_A - \phi_B)(\phi_A + \phi_B)}{9\gamma}$.

same argument, Superstar's profits are:

$$\pi^S = \begin{cases} \gamma \cdot (D_i^S + D^M) + F^E & \text{if exclusive on platform } i \\ \gamma \cdot (D_i^S + D_j^S + 2D^M) + F_1^{NE} + F_2^{NE} & \text{if non-exclusive} \end{cases}$$

Consumers who multi-home obtain the following utility, u^m, such that:

$$u^m = v + \phi \cdot +\theta \cdot \max\{n_1, n_2\} - (p_1 + p_2) - \tau. \tag{6}$$

By comparing the utility in equation (6) with the utility of single-homing in platform $i \in \{1, 2\}$ expressed in equation (2), one can find two cut-offs determining the location of a consumer indifferent between single-homing on each platform and multi-homing:

$$\bar{x}_1 = 1 - \frac{\phi \cdot (1 - g_1) + \theta \cdot \max\{n_1 - n_2, 0\} - p_2}{\tau}, \qquad \bar{x}_2 = \frac{\phi \cdot (1 - g_2) + \theta \cdot \max\{n_2 - n_1, 0\} - p_1}{\tau}. \tag{7}$$

The consumer demand of each platform is the sum of single-homing and multi-homing consumers. Remarkably, consumers multi-home if and only if there is Superstar exclusivity. Else, no consumer would prefer to multi-home as the cut-off $\bar{x}_1$ ($\bar{x}_2$) would be larger (smaller) than 1 (0), hence out of the Hotelling line. As a result, consumers would only single-home. The reference case for non-exclusivity is depicted by the benchmark model where equilibrium results are reported by Lemma 2.

We therefore solve the model only for the case of exclusivity on platform 1. This implies that $g_1 = 1$, $g_2 = 0$, and $n_1 > n_2$. The cut-offs become

$$\bar{x}_1 = 1 - \frac{\theta(n_1 - n_2) - p_2}{\tau}, \qquad \bar{x}_2 = \frac{\phi - p_1}{\tau}. \tag{8}$$

This leads to the following results:

$$D_1 = \frac{\theta - p_1}{\tau}, \qquad D_2 = \frac{\gamma(\theta \cdot \phi - p_1 \cdot \theta) - p_2 \cdot \tau}{\tau(\theta + \tau)} \tag{9}$$

and $n_1 = \gamma \cdot D_1$, whereas $n_2 = \gamma \cdot D_2$. Going one step backwards, each platform anticipates the joining decision of consumers and decides the optimal price p_i. Platform $i's$ profits are $\Pi_i - g_i \cdot F = p_i \cdot D_i - g_i \cdot F$.

By following the same reasoning of the benchmark model, when the Superstar single-homes on platform i, the price set by platform i is $p_i^{E,*} = \frac{\phi}{2}$, whereas the price set by platform j is $p_j^{E,*} = \frac{\phi \cdot \gamma \cdot \theta}{4\tau}$.

Next, to analyze the Superstar's decision, we first consider the case under exclusive contracts. the Superstar makes a TIOLI offer to platform i under the condition that if she rejects, the contract would be offered to the rival platform j:

$$\pi^S = \gamma \cdot (D_i) + F^E$$
$$\text{subject to } \Pi_i^{E,*} - F^{E,*} \geq \Pi_i^O$$

where $\Pi_i^O = \frac{\gamma^3 \cdot \theta^2 \cdot \phi^2}{16\tau^2(\gamma \cdot \theta + \tau)}$ is the profit of firm i when contractual agreements with the Superstar break down and platform j accepts the contracts made by the Superstar. As a result, the Superstar sets $F^{E,*} = \frac{\phi^2}{4\tau}\{1 - \frac{\gamma^2 \cdot \theta^2}{4(\gamma \cdot \theta + \tau)\tau}\}$, and she obtains $\pi^{S,E*} + F^{E,*} = \frac{\phi}{16\tau^2}(8\gamma \cdot \tau + 4\tau \cdot \phi - \frac{\gamma \cdot \theta}{\tau + \gamma \cdot \theta})$.

Then, we solve the model when she offers non-exclusivity. In this case, the Superstar reaches the entire market as in the benchmark model but the outside option is given by the new setting with exclusive contract, i.e., $\Pi_i^O = \Pi_j^{E,*} = \frac{\gamma^3 \cdot \theta^2 \cdot \phi^2}{16\tau^2(\gamma \cdot \theta + \tau)}$.

$$\max_{F_1^{NE}, F_2^{NE}} \pi^S = \max_{F_1^{NE}, F_2^{NE}} \gamma + F_1^{NE} + F_2^{NE}$$
$$\text{subject to } \Pi_i^{NE,*}(g=1) - F_i^{NE} \geq \Pi_i^O \text{ for all } i \in 1, 2,$$

Hence, the Superstar sets $F^{NE,*} = \frac{\gamma^3 \cdot \theta^2 \cdot \phi^2}{16\tau^2(\gamma \cdot \theta + \tau)}$. So, her profits are $\pi^{S,NE*} = \gamma + 2F^{NE,*}$. By comparing Superstar profits in the two regimes, it immediately follows that if

$$\tau < \frac{1}{2}\sqrt{2\phi^2 + 4\gamma \cdot \phi + \gamma^2(2-\theta)^2} - \gamma(1 - \frac{\theta}{2})$$

the Superstar offers an exclusive contract. Else, she offers a non-exclusive contract. Indeed, results follow the same mechanism as in the benchmark model.

Two-sided pricing

A single-homing CP on platform i obtains $\pi_i^{cp} = \gamma \cdot D_i - f - l_i$, where l_i is the price paid by the CP to access the platform. For $l_i < 0$, CPs are subsidized. A multi-homing CP gets $\pi_i^{cp} = \gamma - 2f - l_i - l_j$. Platform $i's$ profits absent the Superstar content are $\Pi_i = p_i \cdot D_i(0, g_j) + l_i \cdot n_i$. When the platform i offers the Superstar content, profits are $\Pi_i + l_i \cdot n_i - F_i = p_i \cdot D_i(1, g_j) + l_i \cdot n_i - F_i$, where $F_i = F^E(F_i^{NE})$ if $g_j = 0(1)$. To ensure a well-behaved profit function, we let v be sufficiently large and we slightly modify the condition on the transportation costs requiring $\tau > \frac{\gamma^2+4\gamma\cdot\theta+\theta^2+2\phi}{6}$. In the third stage, consumer demands become

$$D_i(g_i, g_j) = \frac{\tau + \theta \cdot (l_i - l_j + \gamma) + (p_j - p_i) + \phi \cdot (g_i - g_j)}{2(\tau - \gamma \cdot \theta)} \qquad D_j(g_j, g_i) = 1 - D_i(g_i, g_j)$$

By anticipating future market shares, in the second stage platforms have the following best replies for $i, j \in \{1, 2\}$, with $i \neq j$,

$$\begin{aligned} p_i(p_j, l_j) &= \frac{(4\tau - \gamma(\gamma + 3\theta))(\theta \cdot l_j + p_j + t + \phi(g_i - g_j) - \gamma \cdot \theta)}{8\tau - \gamma^2 - 6\gamma \cdot \theta - \theta^2} \\ l_i(l_j, p_i) &= \frac{(\gamma - \theta)(\theta \cdot l_j + p_j + \tau + \phi(g_i - g_j) - \gamma \cdot \theta)}{8\tau - \gamma^2 - 6\gamma \cdot \theta - \theta^2} \end{aligned}$$

We now identify the equilibrium outcomes when the Superstar multi-homes. Let $g_i = g_j = g = 1$, platforms are symmetric and prices are $\tilde{p}^{NE,*} := \tau - \gamma \cdot (\gamma + 3\theta)/4$ for consumers and $\tilde{l}^{NE,*} := (\gamma - \theta)/4$ for CPs. The demands are given by $\tilde{D}^{NE,*} := 1/2$ and $\tilde{n}^{NE,*} := (\gamma + \theta)/4$.

When the Superstar offers an exclusive contract to platform $i = 1$ ($g_1 = 1$ and $g_2 = 0$), equilibrium prices are

$$\begin{aligned} \tilde{p_1}^{E,*} &= \tilde{p}^{NE,*} \cdot \left(1 + \frac{2\phi}{\eta}\right) & \tilde{p_2}^{E,*} &= \tilde{p}^{NE,*} \cdot \left(1 - \frac{2\phi}{\eta}\right) \\ \tilde{l_1}^{E,*} &= \tilde{l}^{NE,*} \cdot \left(1 + \frac{2\phi}{\eta}\right) & \tilde{l_2}^{E,*} &= \tilde{l}^{NE,*} \cdot \left(1 - \frac{2\phi}{\eta}\right) \end{aligned}$$

where $\eta = 6\tau - \gamma^2 - 4\gamma \cdot \theta - \theta^2 > 0$. It can be easily seen that $\tilde{p_1}^{E,*} > \tilde{p}^{NE,*} > 0$ and $0 < \tilde{p_2}^{E,*} < \tilde{p}^{NE,*}$. When $\gamma > \theta$, the CP price is positive and increases with the Superstar, while $\gamma < \theta$ CPs are subsidized and the subsidy increases with the Superstar. We also note that $\tilde{D_1} < 1$ and $n_1 < 1$, so there is no foreclosure of the rival as a consequence of exclusivity.

Going one step backward, we study the decision of the Superstar following the same reasoning of the previous cases. When the Superstar offers an exclusive contract, her profits are $\pi^{\tilde{S,E*}} = \frac{\gamma}{2} + \frac{\phi\cdot(8\tau+\gamma(2-\gamma)-6\gamma\theta-\theta^2)}{2\eta}$. By contrast, when the Superstar offers a non-exclusive contract, her profits are $\tilde{\pi}^{S,NE*} = \gamma + \tau - (\frac{\gamma^2+6\gamma\cdot\theta+\theta^2}{8} + \frac{(\eta-2\phi)^2(8\tau-\gamma^2-6\gamma-\theta^2)}{\eta^2})$.[11]

By comparing the profits of the Superstar, it follows that the Superstar offers an exclusive contract whenever $\tau < \underline{\tilde{\tau}}$, where

$$\underline{\tilde{\tau}} \equiv \frac{\phi^2}{9\gamma} + \frac{\gamma + 4\gamma\cdot\theta + \theta^2 + \phi}{6} + \frac{\phi}{18\gamma}\Big\{3\gamma\cdot(\gamma\cdot(3+\gamma) - 2\gamma\cdot\theta + \theta^2) + 12\gamma\cdot\phi + 4\phi^2\Big\}^{1/2}.$$

Else, she offers a non-exclusive contract.

In terms of welfare, the gain for CPs due to the presence of the exclusive is denoted by $\delta := PS^{E,*} - PS^{NE,*} = \frac{\phi^2(\gamma+\theta)^2}{4(\gamma^2+4\gamma\theta+\theta^2-6\tau)^2} > 0$. Hence, CPs benefit from exclusivity also with two-sided pricing.

[11] For the sake of completeness, when a platform does not obtain the contract when the rival does, platforms' profits are $\tilde{\Pi^O} = \frac{(\eta-2\phi)^2}{\eta^2}\cdot\tilde{\Pi}_i^{b,*}$.

Conclusions

This thesis contributes to understanding how (some) digital markets work and emphasises on the challenges faced by regulators, companies, and consumers.

The first two chapters look with a new a lens at the commercial evolution of digital pirate. The new commercial dimension let pirate websites react strategically to changes in the video-on-demand market and so do legal subscription-based providers. Such strategic interactions and the related competition have been overlooked in the previous literature, which has mainly dealt with deterrence strategies and the welfare consequences of digital piracy (Belleflamme & Peitz 2012, 2014). So far, only Chang & Walter (2015) has investigated how legal and pirate providers make their decision but they did it for a peer-to-peer network which is limited in the choice of advertisements and does not provide the same user experience as streaming pirate providers (i.e., the cyberlockers). Chapter 1 and 2 overcome their limitations by examining the competition between a legal subscription-based content provider and a pirate provider who monetises users' eyeballs through pop-ups, pop-unders, mid-roll ads.

Chapter 1 provides one of the first analyses on the use of ad-blocking technologies. There are two main limitations of the analysis that remain to be explored in the future. First, it deals with with user-oriented ad-blocking technologies, that is, primarily oriented towards the maximisation of the consumer welfare. Second, it assumes that all users adopt ad-blocking technologies while in the reality users can have different expertise and sensitivities towards advertisements. While these limitations have a minor impact when considering the case of pirate content providers, the results may not be generalised to those markets in which ad-blocking technologies follow different strategies. For instance, some ad-blocking technologies, such as Ad-Block Plus, engage in strategic white-listing funded by accepting monetary compensation to let ads being displayed to users adopting the technology. However, this business model would not be compatible with our framework as a *condicio sine qua non* to have ads displayed is quality compliance. These quality criteria are instead never respected by the invasive ads usually present on pirate websites. Indeed, by looking for a wider applicability, future research should be devoted to deal with strategic white-listing, different business models behind these technologies, its pricing structure and the welfare effects. Along with this, an important result of the paper is that ad-blocking technologies not only impact on those content providers making business via ads but, because of market interactions, it also generates (negative) spillovers for those companies relying on other business models such as a subscription-based system. Further research may be devoted to understanding how ad-blocking affects the incentive structure for content providers, platforms and advertisers to raise or even reduce the quality of their contents, therefore complementing or challenging the few empirical (Shiller et al. 2018) and theoretical results (Ray et al. 2017) so far available.

In addition, Chapter 2 evolves beyond a duopoly towards a market where not only the legal content provider competes against pirate providers for user attention but also where pirate content providers compete with one another. Indeed, by using a modified Salop circle with a centre, the paper provides the first study on how the strategic interactions arising within a piracy ecosystem and between a piracy ecosystem and a subscription-based content provider. Future research may establish what is the optimal degree of digital piracy within an economy as well as how the government could stop ads on websites infringing copyright protection. Another potential avenue for research would be to study how the entry of pirate providers impact on the quality and on the dimension of legal providers' catalogue. For instance, Netflix has recently announced US \$ 8 billion of investment in original product to make its catalogue more appealing and also protect from the competition of new rivals (e.g., Disney, Amazon Prime).

Chapter 3 shows the relevance of data monetisation when investments are, to some extent, multi-dimensional and create the right to incentive for platforms to provide users with more surplus (e.g., user experience, better quality). It also finds that the large presence of competitive bottleneck environments in the tech industry can be explained by the fact that this network structure accommodates better the incentive to invest in value-creation on both sides of the market. An important result relates to how investment decisions shape the pricing structure in the two market environments and that platforms would ideally let more sellers multi-home and charge a price to the monopoly price (although not charged in equilibrium) by investing more. A clear limitation of the above analysis is due to the symmetric structure of the market. In many markets, for instance, some platforms are larger than others and investment may help to restore symmetry. Moreover, it would be interesting to explore investments that increase the surplus of some users while reducing the surplus of others. For instance, in the social media industry, investments in content moderation undertaken by platforms like Facebook, Twitter, Youtube may please advertisers (as this reduces brand safety concerns) but create a disutility for some users (who dislike control on their content). Future research can indeed consider this case.

Furthermore, future research can provide a better sound understanding of the market for data. In recent years, the importance of data and the need for regulation of those brokers collecting personal information has risen the attention of national and supranational authorities. In the United States, the Federal Trade Commission published a report on data brokerage (i.e., Federal Trade Commission 2014) which has uncovered the behaviour of several data brokers. Specifically, companies, such as Acxiom, Corelogic, Experian, collect, integrate, manage, share and sell data from different sources. When selling these data, firms are able to identify their customers and estimate their willingness to pay. Future research could investigate why and when data are traded between companies depending on their information structure and how authorities can intervene to avoid over-extraction of data.

Finally, the fourth paper opens the black box of the contractual relationship between an agent with market power (the Superstar) and platforms. In a clear and elegant manner, the study identifies how the main trade-off faced by the agent with market power when deciding where and how to offer her content. In case of exclusivity, the Superstar can benefit from a ripple effect and extract all the externalities generated by its presence by setting a very large fee. In case of non-exclusivity, the Superstar prefers a larger market coverage. The paper shows that the Superstar offers an exclusive contract as long as the ripple effect is sufficiently strong. This happens when the downstream competition is fierce enough. The paper also presents some important implications for policy-makers and antitrust enforcers. Generally, "market power" and "exclusivity" could be associated to anti-competitive conducts. This paper shows that an exclusive contract signed by the Superstar and a platform is always beneficial for small firms. This happens because of the market expansion and surplus generation engendered by exclusivity. On the consumer side, the decision of the Superstar is mostly aligned with what is optimal for consumers. Only in some cases, consumers can be damaged by Superstar exclusives. Quite remarkably, the main intuitions of this paper are confirmed when relaxing some of the main assumptions.

There are several aspects related to this paper that I am planning to explore in the next future. First, by connecting this work to the literature on vertical restraints, it could be interesting to study different contractual relationship between a Superstar and the platform(s). For instance, these may take the form of non-linear tariffs or royalty-based contracts. The ripple effect is likely to arise as long as the contract is associated with the market expansion on both sides but the threshold may be different. Second, the paper has assumed that the Superstar has all bargaining power vis-á-vis the platform(s). A different set-up may consider a model with Nash bargaining and study how the main results are robust to the relaxation of this assumption. Third, it would be interesting to investigate the incentive for Superstars to launch a proprietary (spin-off) platform and compete with the incumbent on quality, content, and dimension of the catalogue. For instance, a Superstar content producer such as Disney has recently announced the launch of its platform in 2019. As a result, all its content will no longer be distributed online (as they currently are) by Netflix and its users may need to decide whether to switch to the spin-off platform or multi-home.

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