Sui Generis-ness, Parsimony and Innocence

The (Meta)^{2}physics of Parthood

Fabio Ceravolo

Submitted in accordance with the requirements for the degree of Doctor of Philosophy

The University of Leeds
School of Philosophy, Religion and the History of Science

July, 2018

The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others.

Acknowledgements

No matter how valuable, none of what follows would have been remotely conceivable without the enduring support of Steven French and J. Robert G. Williams, who found me an enthusiastic neophyte and turned me into a (more) careful arguer. As the outcome of innumerable and unquantifiably fruitful discussions, this dissertation owes to them any substantial claim it manages to establish. For trivialities and outright mistakes, on the contrary, I deem myself entirely responsible.

Very special thanks deserve also my examiners, Darren Bradley and Katherine Hawley, for being among the first readers of the final manuscript. It goes without saying that this work will only have a chance at making broader impact thanks to Darren's and Katherine's challenging feedback. Further, having kept in touch with them from an early stage of writing contributed greatly to shape my ideas. Darren organised frequent workshops on the topic of theory choice in metaphysics, which kept me updated on the forthcoming literature and provided chances to discuss it with some of the most influential writers in the field. Katherine supported my application for a four-months visiting scholarship at St. Andrews' *Arché* centre – one of the most formative experiences of my academic life – and even after I returned to Leeds she was always happy to spend time discussing my latest writings.

This possibility of frequent interaction was especially motivating for two reasons. First, as the reader will notice, my strategy in the second part of the thesis (Chapters 6 – 10) owes much to "Ontological Innocence", a paper Katherine published in 2014 shortly before I begun studying for my PhD. Second, I am indebted to ideas Katherine presented in a panel discussion at the Ninth International Conference of the *German Society for Analytic Philosophy* (GAP 9) in 2015. This time the subject is the relationship between metaphysics and science and particularly how to approach metaphysical questions with methods constrained and informed by reputable scientific knowledge. The question I approach asks about the structure of part-whole relations (in a sense we will have time to clarify) and an early, guiding idea I found in Katherine's talk was that answering this question requires a detour through meta-metaphysical topics. We need to ask, for example, whether relations retrieved in advanced science, whose properties seem to reveal surprising new features of the way a part relates to its whole, count as relations of part-whole in a literal and non-metaphoric sense (Chapters 2, 3). These meta-metaphysical preliminaries to using science in theorising about the part-whole relations are so important as to informing my thesis' title, which reveals that what follows is a work in *(Meta)*^{2}*physics*. An exponent follows 'Meta' to indicate our aim of invoking science to uncover surprising features of part-whole relations, which metaphysics alone cannot discover. Moreover, the second power of (Meta) is 'metameta-', which indicates the importance of the meta-metaphysical route for pursuing the former aim.

To continue, I am indebted to a number of exceptional philosophers, whose comments greatly enhanced the outlook of this work. To my eyes, these figures boldly lead the discipline "where no one has gone before" or most certainly will in the very near future. Thanks, therefore, to Kenneth Aizawa, Claudio Calosi, Christina 'Squared Cee' Conroy, Javier Cumpa, Ilaria Canavotto, Jonathan Diettrich, Robert DiSalle, John Divers, Mauro Dorato, Joaquim Giannotti, Carl Gillett, Brigitte Falkenburg, Kit Fine, Suki Finn, Akiko Frischhut, Micheal Huemer, Naoaki Kitamura, James Ladyman, Joseph Melia, D. Hugh Mellor, Kohei Morita, Alyssa Ney, Andrea Raimondi, Juha Saatsi, Thomas Sattig, Theodore Sider, Jonathan Schaffer, Jeroen Smid, Alistair Wilson and Jessica Wilson.

To a similar extent I am grateful to the entire graduate community at Leeds and St. Andrews, with a honourable mention to my most frequent philosophical interlocutors: Callum Duguid, Douglas Earl, Jade Fletcher, James Fraser, Will Gamester, Lewis Hickley, Alice Murphy, Simon Newey, Robert Pezet, Nahuel Sznajderhaus and Nick Tusker.

For its financial support through three of my four years at Leeds, my highest gratitude goes to The University of Leeds, whose 110th Anniversary Scholarship I was awarded in 2014. I am also thankful, more specifically, to the

University's School of Philosophy, Religion and the History of Science for a number of travel grants, which I could avail myself of to attend exceptionally formative events in the UK, USA, Europe and Canada.

A final set of acknowledgements relates less to my daily academic surroundings but is by no means less important. I am grateful to *The University of Leeds' Japanese Society* and the *European Network of Japanese Philosophy* for quenching my thirst for symbols with more symbols. To them I say, vowing perpetual friendship: 私たちは誰でも他者との一体感を切望する何かをうちに秘めている (*deep within us lies a will for oneness with others*). As it happens, example sentences from online dictionaries can conceal great wisdom[1].

I owe much professionally and as a person to my friends at *EF - Education First*: Martina Argine, Rob Arnsmar, Karlie Bertram, Kat Insull, Nora Johnsson, Bryan and Mickey Lao, Lea Le Nabec, Laura Salonen, Joshua Webster, Alice Zentilomo and many others. They all taught me to 'sparkle' and not to compromise on my personality in return for monetary rewards. This is the same lesson I learned from some brilliant pieces of song writing, particularly by my friend Ichiko Aoba and The Jezabels.

Despite seven years spent away from my hometown, my closest friends never failed to support me. For this I wish to thank you, members of the Οἶδα Council of Milan, who luckily came into existence at closeby places and times: Pietro Barbini, Guido Beduschi, Dario and Bianca Bonetti, Arianna Cardella, Vincenzo Cirillo, Alessandro De Cia, Lorenzo Foletto, Bianca Giacobone, Gianluca Meneghel, Elia Nigris, Alesia Preite and Ayurzana Purevdorj.

Last, far from the kind of acknowledgement that demands justification, I would like to *dedicate* this thesis to Francesca and Enzo, whose parental patience exceeds the limits of pure reason, and to Yuqing, who clears away the darkness like moonlight from the skies.

[1] "他者" (*tasha*), In: JapanDict (© 2016), available at: https://www.japandict.com/他者.

Abstract

A metaphysical naturalist could find the following combination of claims attractive. First, part-whole and composition in physics are *sui generis* and lack some of the 'core' features we ascribe to these concepts and their worldly satisfiers in first-order metaphysics. Second, having agreed that some physical objects of interest satisfy *sui generis* concepts and/or relate by *sui generis* relations, none among these objects satisfies a classical concept or relate by a classical part-whole relation (e.g. the concept or relation of mereological part). The first claim I read as one of 'appropriation': the structural relations between physical objects of interest are *sui generis* and yet they pertain to the mereological kind. The second I read as one of 'elimination': metaphysically abstracted part-whole (or composition) has no instances in well-regarded physical domains. The dissertation argues for appropriation and against elimination. For appropriation, because current physics sanctions *relata* of part-whole relations (or at least satisfiers of part-whole concepts) that clash with intuitive, seemingly analytic principles for part-whole, e.g. the Antisymmetry postulate (x and y are mutual parts only if identical). Against elimination, because whether these objects of interest to physics also relate by 'canonical' part-whole (with the intuitive principles) is largely a question of parsimony. One removes instances of the canonical relations because these are not needed to account for the composition of objects that already relate by the non-canonical ones. But some of these relations at least (such as mereological part-whole) resist the pressure from parsimony, for they come at no cost once the objects already relate non-canonically (e.g. in opposition to the Antisymmetry postulate). The latter we can argue for in (at least) two ways: 1. canonical and non-canonical part-whole are members of a single kind, 2. canonical part-whole is of a kind with identity. Given either view and a preference for theories with minimal kinds, instances of the canonical relation do not increase a theory's profligacy, because their kind is already instanced in a theory of objects that relate non-canonically. My preference is for the latter view.

Table of Contents

Introduction

We take very much for granted about things that relate as part to whole, such as a pizza's slice and the whole pizza, a page and a book, a wheel and my bicycle. We would not think of a slice of pizza, for example, as is 'its own' part and so we would not say, correspondingly, that the slice is part of the slice. For though giving up your entire slice when I (shamelessly) request a 'part' of it would be very kind, it would also be, in a sense, inappropriate. My demand for a part of the slice was actually for some smaller piece of it. Or at least, the demand is construed naturally in this way in most contexts. Situations, in which I demand a 'part' of something planning to have the whole thing, are rare.

Here is one such situation. As a student of gastronomic science I might sit in an exam and be asked to indicate which 'portion' of the slice contains the highest amount of calories – by shading it, perhaps, or anyway by selecting it among some options, which include the entire slice. It does not seem inappropriate, this time, to suppose that the correct answer is the slice itself. The slice is the slice's portion with the highest amount of calories, for I get the most calories by eating the entire slice.

A common way of explaining the difference between these cases is that ordinary language and thought deploy two distinct concepts, one primitive and one defined. The primitive concept, *part-whole*, is universally reflexive: every thing is part of itself in the sense specified by this concept. The defined concept, *proper part*, holds just in case a part (in the primitive sense of *part-whole*) is distinct from its whole[1]. Most frequently, ordinary language deploys a predicate underpinned by the defined concept. Thus in my request for a 'part' of the slice, only the slice's proper parts satisfy '...is a part of...'. In rarer occasions, such as the gastronomy exam, the deployed predicate is also satisfied by the entire slice.

This reflection on the concepts underpinning our ordinary utterances illuminates one sense in which we 'take for granted' that no thing is ever its own part. 'Taking for granted' here means failing to pay attention to other areas of ordinary language and

[1] Alternatively, one can reverse the order of definition, taking proper part as the primitive and 'x is a part of y' $=_{df}$ x is a proper part of y or $x = y$.

thought, i.e. failing to recognise that, though rarely, concepts like part-whole underpin ordinary utterances. This 'light' sense of 'taking it for granted' is *not* the one under focus in this dissertation. For a contrast, the focus is on a 'heavier' cousin, which the light sense helps us introduce.

Sometimes we may pay the utmost attention to the concept underpinning our ordinary utterances and yet continue to assume – i.e. 'take it for granted' – that parts lack certain structural properties, because these clash with what is appropriate to claim in these ordinary contexts. For example, we make no appropriate use of 'portion' if we answer the gastronomic question so that *the entire pizza* is the slice's portion with the highest caloric intake. For no ordinary way of using 'portion' makes it legitimate to classify the pizza as one of the slice's portions (or, equally, to classify the slice as a portion of any of its sub-portion, e.g. the slice's bottom, half, tip, circular sections, etc.).

It is clear, rather, that in classifying the pizza as a portion of the slice, we can only be talking metaphors. We could say, for example, that this slice is so tasty that the entire pizza is a 'part' of it; and by this mean only that eating any sub-portion of the slice (none of which is the entire pizza) is as intense an experience as it is to eat the entire pizza. And in fact there are cases of ordinary language where '…is part of…' holds metaphorically in this way. Juliet loves Romeo and knows Romeo loves her back, so in a leap of romanticism she whispers: "I am part of you and you part of me". Juliet's intended meaning is not the sharpest – perhaps it is something along the lines of: Romeo and I would not feel all right if taken apart. But no matter these specifics, what seems clear is that by no means the meaning will be that – in a literal sense – the two lovers are mutual parts.

In fact, that no distinct objects are mutual parts is a tenet we 'take for granted' not because we ignore concepts that underpin our ordinary utterances (e.g. utterances we have not yet considered), but because, apparently, utterances underpinned by a concept that allows distinct mutual parts traffic in metaphors. Under focus in the rest of this dissertation is this stronger sense of 'taking for granted'[2].

[2] What we take for granted in this stronger sense includes more than the claim that no distinct entities are mutual parts. We will discuss this in due time (Chapters 1, 4).

Appropriation: *sui generis* part-whole and science

Now for the aim: the reason for choosing this focus and not another. We should not, I think, accept all claims we usually take for granted in this stronger sense. For arguments against some of these claims (including the claim that no distinct entities have mutual parts) we look not at ordinary language, where the statements that contradict the claims are often metaphoric, but at advanced science. Indeed, from scientifically eminent accounts of the part-whole relation we collect evidence of two kinds against the claims: 1. 'worldly' evidence that some *entities* of scientific interest (posited by advanced theories) relate as part to whole in opposition to the claims – e.g. each being a part of the (distinct) other; and 2. 'conceptual' evidence that some concepts of scientific interest (deployed by advanced theories) belong to the family of *part-whole* and yet the claims fail to be true in virtue of them.

The first part of the dissertation (Chapter 1 – 5) gathers evidence of these types: conceptual and worldly. The evidence is taken from the algebra of vectors and its areas of application in physics, for indeed it is these sectors that supply entities and concepts clashing with the principles taken for granted. The supplied 'deviant' concept is the concept of a linear vector and the supplied 'deviant' entities (relating as part to whole in opposition to the claims) are its satisfiers, i.e. the things that at the actual world have the properties and structure of a linear vector.

To be sure, it will not be enough for collecting the worldly and conceptual evidence that we point at this sector of advanced science, and specifically at the entities and concepts it supplies in opposition to the claims. Beforehand, we need to ensure that the supplied entities relate as part to whole in a literal sense and that the deployed concepts partake of the mereological family, i.e. are genuine concepts of part-whole. For this reason, while we illustrate these concepts and entities in Chapters 4 and 5, a preliminary discussion occupies Chapters 2 and 3, where we argue it is possible in principle that the advanced science posits parts in a literal sense and uses genuine concepts of part-whole.

The relation by which the posited entities relate as part to whole and the employed concepts will be *sui generis*, but not in the usual sense of 'being of their own kind'. These concepts and relations, indeed, are of a kind with their mereological cousins, for they both apply to parts and wholes in a literal sense. For them to be *sui generis*, accordingly, will be to partake of this kind while challenging or breaking its 'canon'. The canon are the properties of the part-whole relation or concept that we 'take for granted' from the analysis of ordinary language and thought – for example, the property that no two distinct objects relate by part-whole 'mutually', each being a part of which the other is a whole. The dissertation's first half claims that, in advanced science like the sectors of application of linear algebra, one of the following holds: either it supplies canon-breaking concepts of part-whole, or it supplies entities that relate as part to whole (in a literal sense) in opposition to the canon.

Elimination: is scientific part-whole only *sui generis*?

That some objects of interest to advanced science relate as part to whole in canon-breaking ways or satisfy a canon-breaking concept of part-whole is our starting point in Chapter 6, which marks the beginning of the second half. There is an attractive view for those who would like their metaphysics of part-whole to be constrained by the advanced science: namely that these objects of interest relate as part to whole *only* in canon-breaking ways and/or satisfy *only* canon-breaking concepts. For example, one could argue that every vector is only a linear component of vectors, which in turn are linear components of it. For the vectors resulting out of linear combination can always be combined with new vectors to return the original one (to this example we return at several stages).

This view departs from that defended in the first half. That a vector is 'part' of another in a literal sense when it is a linear component of it is a claim of 'appropriation', according to which the relation between vector components and linear resultants (and/or the concept that resultant and components satisfy) deserves a place in the mereological family despite the fact that vectors relate to other vectors in opposition to the canon. On the contrary, that vectors relate as part to whole only in this canon-breaking way (or satisfy only canon-breaking concepts) is a claim of 'elimination', according to which the canonical part-whole relation (concept) has no instances (satisfiers) in these domains of interest to science.

The second half of the dissertation argues against this eliminative doctrine, so that extending the mereological family to canon-breaking relations (concepts) falls short of proving that scientific ontology includes only entities relating as part to whole in opposition to the canon. A gap lies, that is, between appropriation and elimination. That scientific objects of interest (e.g. linear vectors) break the canon (by relating as part to whole in opposition to it, or satisfying non-canonical concepts of part-whole) is compatible with them figuring, in addition, in ordinary part-whole relations (satisfying ordinary concepts of part-whole). To accept the former while denying the latter requires an additional argument.

The argument for closing the gap that we consider (and reject) is admittedly only one of many which a friend of the elimination doctrine could deploy. It states that the objects we focus on – the linear vectors – relate as part to whole only in opposition to the canon (e.g. by being parts of distinct vectors that are in turn part of them) because having them relate in canonical ways would add to our ontology or ideology, and these additions should not be granted 'without necessity'. To put it differently, one removes instances of the canonical relations because these are not needed to account for the composition of objects that already relate by the scientifically eminent, but deviant ones.

One can, however, deflect this use of arguments from profligacy by noting that some canonical relations come at no cost on the side of ontology and ideology. Once the objects of scientific interest relate deviantly – e.g. by being parts of distinct things that are parts of them – it contributes no 'additional' entity and no 'additional' concept to suppose that they are also relate canonically, so that (e.g.) they are parts of some things without the latter being in turn parts of them. The sense in which this supposition contributes no 'additional' entities or concepts demands clarification – a task that I leave to three concluding chapters (7, 8 and 9).

The resulting picture, which hopefully I will have provided by the end of the thesis, is twofold. On the one hand, we should not refrain from pointing at scientifically advanced 'structural' relations (e.g. the vector component to resultant relation) for examples of non-canonical part-whole relations. On the other hand, we should not move on to conclude that objects entering these relations (e.g. linear vectors) are only parts in ways that clash with the canon. By also thinking of them as canonical parts (say, parts of things that are not parts of them in turn) we make no addition to the initial ontology (the related objects, i.e. the linear vectors) or ideology (the deployed concepts of part-whole).

Hopefully, we will be able to look at this picture as answering the question of what we can learn about relating as part to whole from reputable scientific sources. A part of the question ('appropriation') asks whether the scientific subject matter is really the same; that is, whether the sources deploy concepts in the family of part-whole and/or posit entities that relate as part to whole in a literal sense. Here the right conditions hold for a positive answer. Therefore, we can advocate a naturalistic policy according to which current science informs us that some things in the world relate as part to whole in opposition to canonical principles[3]. Another part of the question ('elimination') asks whether learning this much from science is enough to prevent the deviantly related entities from entering part-whole relations that comply with the canon. This is an inference we should resist, for a reasonable way of arguing for it assumes that entering the additional relations comes at a cost in ontology or ideology. But this premise – so I will argue – fails if the additional relations are of the mereological kind; that is, if the scientific objects of interest enter additional *part-whole* relations. Appropriating mereological vocabulary for some non-canonical but scientifically eminent relations is legitimate, but not so inferring from here that scientific domains of interest exemplify only non-canonical relations. For the latter claim we need more than simply 'observing' canon-breaking features in the scientifically informed accounts of part-whole. We need, rather, a thesis to the effect that entering additional standard relations contributes further ontology or ideology and that we should resist these additions. This is not a thesis we easily win if the additional relations are of the mereological kind.

[3] Or at least satisfy a concept of part-whole that lacks the principles as conceptual consequences, see Chapter 5, Section 2.

I

General Composition Questions and Core Principles

1. Composition and part-whole

Let us begin by bringing our concepts in sync with the concepts many writers employ when it comes to debating about parts and wholes. By *part-whole* I mean a relation between two singular things (x, y), such that x is a part of y. By *composition* I mean a relation between a plurality of things and a singular thing (xs, y), such that the latter (y) is a whole (or 'sum') that has as parts all of the former (xs)[1]. Interchangeably (unless otherwise specified), by composition I also mean a relation between a variable number of singular things x_1, x_2, ... and a singular thing y, such that y is a whole (or 'sum') that has as parts each of x_1, x_2, ... Accordingly, we say of particular things taken

[1] Throughout the entire work, I assume that no composite entity y is – in a literal sense – many-one identical to its composers (xs). The thesis I deny goes by the name of strong composition as identity (S-CAI). This preference for the falsity of S-CAI is supported by reputed arguments (Sider 2014 and more recently Calosi 2016) – which time prevents me from inspecting appropriately. As things currently stand, however, it would be unwise to claim that these popular arguments entirely dispose S-CAI. Contemporary discussion has seen a resurgence of S-CAI and mounted forceful defences. The latter – too – hinge on claims we cannot fully lay justice to, such as the claim that it is possible to count objects (as one or as many) relative to concepts (Wallace 2011, Bohn 2016), and that we should reconceive of the primitives involved in the logic of identity in view of a tighter analogy with the composition relation (Cotnoir 2013; see Hawley 2013 for a reply and Smid (2017a: ch. 2) for further criticism).

individually that they are part of a whole, and only of many things considered together that they compose a whole. A page is part of my book, while all pages and the cover compose the book. A room is part of a flat, while all rooms compose the flat. Perhaps, if the notions of part-whole and composition extend to groups and their members, then I am part of Leeds University's Japanese society, while all society members, myself included, compose the society.

The subjects of this chapter are methodology and the properties of part-whole and composition, the moral being that only by spelling out our questions and theoretical aims clearly can we individuate the properties we do and do not *need* to posit: the bare necessities for our theorisation to come and the features we can ultimately remain neutral on.

2. General composition and part-whole questions

We have agreed on composition being a relation between a plurality of things (*xs*) and some singular thing (*y*) the *xs* 'compose' or 'make up'. Can we say more as to what it is to make up a whole? How is it, exactly, that the plural *xs* relate to their whole (the *y*) when the *xs* compose *y*? Call this question as to what kind of relation composition *is* the *general composition question* (GCQ) [2].

Perhaps *xs* and *y* relate spatiotemporally: for *xs* to relate to *y* as parts to sum, *y* has to occupy the region of space that the *xs* also (jointly) occupy. Or perhaps *y* must occupy a physical state that results out of linearly combining the states occupied by each of the *xs*. Or finally, perhaps there is no (finite) answer to this question that doesn't invoke concepts closely related to composition itself: mereological concepts. For example, one could gloss '*xs* compose *y*' as: for the *xs* to relate to *y* as parts to

[2] The question as to which among some entities are composers of some whole goes under the name of *special composition question* (SCQ). The special composition question is not a demand for the nature of the composition, but rather a question about the way the *xs* ought to relate to each other in order for there to be a sum of which the *xs* are parts. What conditions must the *xs* satisfy for them to participate in a case of composition with some *y*? The distinction between general and special composition questions is a pioneering aspect of a pioneering book: Peter Van Inwagen's *Material Beings* (1990).

whole, *y* has each of the *xs* as a part and every thing that has any part in common with *y* also has that part in common with some of the *xs*[3].

Notice that we can raise a 'general' question not only about composition, but also about *part-whole*, which we have agreed to be a relation between some (singular) thing *x* and another (singular) thing *y*, such that the former is part of the latter. Could we specify this mode of relating between *x* and *y*? How is it exactly, that *x* relates to *y* when *x* is part of *y*? Perhaps they relate spatiotemporally: every time that *x* is part of *y*, *y* occupies a certain region of space(time) and *x* occupies a sub-region of it. Perhaps they relate by sub-state occupation: every time that *x* is part of *y*, *y* occupies a physical state that has *x*'s state as a linear component. Or finally, perhaps there is no answer to this question that doesn't invoke concepts closely related to part-whole itself: mereological concepts. In parallel to its composition relative, answers to this general question about part-whole single out conditions that (a) are met by some relation holding between *x* and *y*; and (b) are jointly sufficient and necessary for that relation to be a part-whole relation, i.e. for the *y* to relate to the *x* as whole to part.

3. Partial and full answers

Let us focus for illustration on the general composition question (everything we establish will hold, however, also for its part-whole cousin). As it is usually formulated, the question requires jointly necessary and sufficient conditions on 'the *xs* compose *y*' (for all *xs* and *y*). To answer it in this traditional formulation, what we need is an instance of the following schema, labelled GCA for *G*eneral *C*omposition *A*nswer:

(GCA) For all *xs* and *y*, the *xs* compose *y* if, and only if, Γ.

[3] This gloss dates back to Tarski (1935), see Varzi (2014). A popular alternative (Simons 1987, Lewis 1991) states that the *xs* relate to *y* as composers to sum just in case every thing that shares a part with *y* also shares a part with at least one of the *xs*.

whereby Γ contains the specifics of the relation that binds *xs* to *y* just in case the *xs* compose *y*.

Not every theoretical aim one might decide to pursue requires giving or considering answers to the general question in its traditional formulation[4]. We may rest content, for particular purposes, to offer or consider merely sufficient or merely necessary conditions on 'the *xs* compose *y*' (for all *xs* and *y*). Answers that provide such conditions Γ' – i.e. only sufficient or only necessary – we can call *partial*. For a contrast, one answers the question 'fully' if one provides jointly necessary and sufficient conditions as demanded by the traditional formulation. The schemata needed for partial answers are, accordingly:

$(GCA)_{PN}$ For all *xs* and *y*, the *xs* compose *y* only if Γ'.

$(GCA)_{PS}$ For all *xs* and *y*, if Γ', then the *xs* compose *y*.

The aims I pursue in the first part of this work requires that we *consider* and *challenge* various partial answers of the form of $(GCA)_{PN}$, i.e. answers giving *necessary* conditions. On the right-hand side of these answers stand features of composition and part-whole, which many are tempted to see as inalienable to the ordinary, common-sensical conception of these relations. We begin describing these features in greater detail in Section 6, but for the time being, as an illustrative case, we may consider:

The Antisymmetry postulate: If *x* is a part of *y*, then *y* is not a part of *x* unless $y = x$.

[4] The idea that one may accept different answers to the general question compatibly with different theorisation aims (Section 3, 4) is a lesson from Hawley (2006).

To features like the Antisymmetry postulate – arguably engrained in the ordinary part-whole concepts[5] – I would like to object. In particular, on my view these features may well pertain to ordinary concepts, but do not pertain to all concepts in the *family* of part-whole. There are concepts – notably, concepts of advanced science – that we legitimately classify as 'part-whole' but which fail to display the familiar features. Moreover, some branches of the sciences not only construe a concept of part-whole without the familiar features, but also claim that the concept has actual satisfiers; that is, that there exist real parts and wholes without the familiar features. My view of these entities is the same as my view of the concepts: it is legitimate to classify them as genuine parts and genuine wholes, in spite of lacking the familiar features we expect in light of the ordinary concept. This is admittedly still rough, but to reach closer approximations, we can help ourselves to the above framework of $(GCA)_{PN}$. On the view I defend, one needs to challenge instances of these principles that report the conceptually familiar features on the right-hand side. But there is more to the principles to be challenged. For these need, namely, not only report the familiar features, but also be presented as claims *about* concepts (if we argue that the features do not pertain to all concepts of composition and part-whole); and as claims about real composites and real parts and wholes (if we argue that the features do not pertain to all composites, parts and wholes in the ontology of advanced physics). In other words, to understand the exact content of the view I would like to defend, we can start by getting clear on what the rejected principles exactly state.

4. The conceptual and empirical tasks

Following Dowe (2000) – who discusses an analogous general question about causation – let us distinguish between two 'tasks' that we can set ourselves before offering an instance of $(GCA)_{PN}$. In giving the instance, we can either decide to analyse the properties of some or all concepts of composition, or to address how

[5] Not everybody accepts that Antisymmetry is essential to ordinary concepts, but to this topic we will return in more detail in Section 6.2.

metaphysics, science, or a combination of the two jointly shed light on the concept's worldly satisfiers: the real composite and composers. Performing conceptual analysis alone produces – in Dowe's words – the 'conceptual task'; seeking worldly correlates – the concepts' satisfiers – produces 'the empirical task'[6].

Both the conceptual and the empirical task aim at answering general questions while making them precise[7]. For stated only as a request for conditions Γ (partial or full) on 'the *xs* compose *y* (for all *xs* and *y*)' – as we presented it above – the general question is still open to interpretation[8]. It could be asking for conditions drawn from the analysis of composition concepts; and specifically, what must hold of composite and composers by virtue of some concept of composition in particular (e.g. the ordinary concept), or what must hold of them by virtue of some concept of composition or other. On the former interpretation, the question is about the features we expect to hold given what we know about a specific concept; on the latter, it is about the 'limits' of composition concepts: the features we expect to hold by virtue of any such concept[9].

Faced with instances of $(GCA)_{PN}$ that report this different information, the attitude of a critic (like myself) will also change. To deny that a certain feature Γ holds of a particular concept of composition (part-whole), we need to embark in the analysis of that concept and show that it does not deliver Γ. On the contrary, to deny that Γ holds of some concept of composition (part-whole) or other, it suffices to find one such concept whose analysis does not deliver Γ. As anticipated in the last section, my focus

---

[6] The distinction between what holds of a concept (e.g. the concept 'part' or 'composition') and what holds of the concept's worldly satisfiers (the real parts and wholes) owes much to a Canberra-style methodology. See in particular the classics Lewis (1994) and Jackson (1994), as well as the helpful Braddon-Mitchell and Nola (2009), Braddon-Mitchell and Miller (2016) Papineau (2014, 2015). Writers who also invoke the conceptual/empirical task distinction in the context of theorising about causation include Salmon (1984), Bigelow and Pargetter (1990).

[7] For remarks in this spirit and an 'empirical task' aimed at composition, see Aizawa and Gillett (2017: 23): "We cannot emphasise too strongly how important it is to disambiguate the project that a V-framework under discussion is intended to pursue both with regard to the intended object phenomena, scope and generality […]". A 'V-framework' is an account of the worldly composition relation or other relations that track successful 'bottom up' explanations in science. See also Wilson (2013, 2017) for a classification of these tracking relations.

[8] And so is the corresponding general question about causation, says Dowe.

[9] In his early discussion of the general composition question (1990: Ch. 4), Van Inwagen seems to presuppose 'high standards' (Cf. Hawley 2006) for counterexampling an answer, offered as an instance of (GCA). In particular, to reject the instance a critic should present *conceptual* scenarios that falsify it. This suggests that Van Inwagen thinks of the question as one about the composition concept, in line with the conceptual task.

is on this second task, whereby the conditions Γ that we show not to pertain to every genuine concepts of composition (part-whole) are some of those most familiarly related to the ordinary concept (see Section 6).

Parallel reasoning holds for the instances of $(GCA)_{PN}$ that answer the general question within the empirical task, i.e. reading it as a question not about what holds of part-whole concepts, but what holds of these concepts' satisfiers: the real parts and wholes, composers and composites. In the traditional formulation, the question could be asking for various types of conditions (partial or full). First, depending on the scope of the prefixing quantifiers ('for all *xs* and *y*') it could ask for what holds of composites and composers drawn from a specific domain of entities: say, what conditions hold of any material composites and composers, or of any composite and composing properties, events, etc. The critic deflects an answer given on this line by showing that the conditions do not hold for composite and composers from that very domain.

Second, the question could ask for what holds of composites and composers at some particular classes of (metaphysically) possible worlds, or at all such worlds. In the latter case, the focus is on the features of composites and composers that hold of metaphysical necessity and the critic denies an answer along these lines by showing that the features Γ are metaphysically contingent; in the former, the focus is on features Γ of which we know for certain that they hold of composites and composers at particular worlds (e.g. the actual) or particular classes of worlds (e.g. the nomically possible ones). The critic rebuts an answer along these lines by showing that composites and composers defy Γ in at least one world of the class.

These factors of variation in instances of $(GCA)_{PN}$ – subject domain and modal force – show that offering one such instance is really the last step in the process of answering the general question, as this is traditionally formulated. Before, one needs to get clear on the scope of one's project and the sought features of composition (part-whole)[10]. This sensitivity to the theoriser's aims bears emphasis. It might not be necessary – given a set of theoretical purposes – to meet all of the potential tasks associated with answering the general question. As just seen, for example, answering the question could require singling out necessary and sufficient conditions Γ that hold of metaphysical necessity. But if one's task is to illustrate which *actual* features

[10] I owe these observations largely to Hawley (2006), and, more recently, Kovacs (2014).

hold of composites and composers (parts and wholes), then one could suspend one's interest in what holds of them of metaphysical necessity.

What is the strategy from the critic's point of view? Like before, the counterexamples we provide vary with the kind of question the opponent intends to answer: If the opponent seeks relations R that track part-whole at the actual world, then we will not respond with possible but non-actual cases of *x* and *y* related by part-whole but not by R, or with possible but non-actual cases of *x* and *y* related by R but not by part-whole. Similarly, if the opponent's focus is on relations R that track part-whole between material objects, then we do not offer counterexamples of non-material *x* and *y* related as part to whole by relations other than R, or counterexamples of non-material *x* and *y* related by R, but not as parts to whole.

As for myself, what instances of $(\text{GCA})_{\text{PN}}$ offered within the empirical task do I intend to criticise? As anticipated (Section 3), the conditions Γ I plan to reject are some among those familiar from the analysis of ordinary concepts of composition and part-whole (including the Antisymmetry postulate and those we discuss in Section 6). Therefore, even within the empirical task, where the opponent characterises real parts and wholes and not concepts thereof, the focus is on showing that some among these parts and wholes fail conditions we would expect them to have based on the analysis of ordinary notions.

Further, the instances of $(\text{GCA})_{\text{PN}}$ I plan to reject ascribe these familiar features Γ to composites and composers (parts and wholes) picked from a particular domain and at the actual world. The task of describing this domain is non-trivial and, at this stage, would be premature (it will get the larger coverage it deserves in Chapter 5). But for a helpful approximation, we can think of it as containing *some* among all entities of interest to advanced physics that we can represent as linear algebraic vectors. It is among the actual features of composites and composers from this confined domain that I retrieve no familiar Γ. I do not – accordingly – make the same claim about familiar, medium-sized objects. The latter's relationship to the objects of physical interest we keep under focus is far from straightforward. Besides, as far as this work is concerned, nothing of interest about the mereology of this familiar, mid-scale domain could follow from the fact that the physical objects in our scope defy Γ.

5. The conceptual in the empirical 1: undeserving satisfiers

When I say – pursuing the empirical task – that the composites' and composers' (parts' and wholes') features of interest to advanced physics are "unexpected" or "surprising" (relative to the familiar Γ), I mean that they deviate significantly from those we would expect[11] real composites and composers (parts and wholes) to satisfy by virtue of possessing *ordinary* composition- and part-whole concepts.

For this claim to have plausibility, the empirical and conceptual task ought to run in parallel and – to some extent – 'overlap'[12]. Specifically, before making the claim we need to face the following question in the conceptual task: *can* advanced science truly evidence that, at the actual world, parts and wholes offend against the familiar properties Γ, which follow from ordinary concepts and are largely taken for granted? An opponent, indeed, could resist the thought of science offending in such radical ways against features that ordinary concepts expect parts and wholes to satisfy. Instead, when the offence is radical (as it is when it challenges Γ) we should say that the subject matter of the offending science is not literally a relation of part-whole (or composition), but rather a *sui generis* relation that vaguely resembles it.

David Lewis (1994) figures among those who worry along this line[13], suggesting that no satisfier *x* of an ordinary concept *C* (e.g. part-whole) can offend to the ordinary conceptual role of *C* too radically. All entities that would offend radically to the features of the satisfied concepts (say, by satisfying the part-whole concepts but lacking Γ) are best seen as not worthy of satisfying the concepts in the first place. Instead, they will satisfy vaguely similar, yet *sui generis* concepts.

At this juncture, my view departs from Lewis'. The objects of physical interest that relate as part to whole in offence to Γ do indeed satisfy a concept in the *literal family* of part-whole. This need not be the (or one of the) ordinary concept(s), but it needs ensure that when we label the satisfiers 'parts' or 'wholes' we thereby speak literally, or to put it with Lewis, that the satisfiers "deserve a name ... drawn from our

[11] Why would we expect in the first place that real parts and wholes (composers and composites) satisfy features pertaining to the analysis of the ordinary notions? The reasonable answer would seem to be that the analysis of ordinary concept C (e.g. part-whole, composition) provides an initial grip on the worldly properties of satisfiers of C or concepts in C's family (i.e. concepts whose satisfiers we can label 'C' without a shift to non-literal, figurative language (see below)). For a view on this line, see Sattig (2015: 2.3).

[12] This section follows Dowe (2000: 8-9).

[13] Cf. Dowe (2000: 9).

traditional vocabulary" (Ib. 415). We may still – as I will do from the next chapter onwards – depict these satisfiers as '*sui generis*'[14]. Yet this phrase's meaning changes accordingly, coming to indicate that the objects of physical interest relate as part to whole not in a figurative, non-literal sense but rather in a perfectly literal and yet 'genre-breaking' one. Relating as part to whole without presenting Γ opposes the canon, but this falls short of making a metaphor out of our talk of the *relata* as parts and wholes.

I entrust the defence of this non-Lewisian approach to the next two chapters, where I fend off various arguments that articulate Lewis' concern and conclude – as he does – that no satisfier of concepts in the part-whole family deviates significantly from the ordinary notions. For the time being, notice only a dialectical point. I tackle these worries *before* portraying the objects of scientific interest, whose features deviate from those expected by the analysis of ordinary concepts. This is because when we argue in reaction to the Lewisian line – as in Chapters 2 and 3 – the focus is on questions of philosophical method that hold interest no matter if there currently is or isn't a branch of science evidencing 'genre-breaking' part-whole and composition relations. The issue is whether scientific evidence of this sort can ensue in principle, or whether all putative evidence one may offer speaks instead for non-genuine, *sui generis* relations, which deserve the label 'part-whole' only by extension and figuratively. The analysis of the relevant science, which pertains entirely to the empirical task, begins in Chapter 4.

6. The conceptual in the empirical II: unfamiliar features

We set out two aims for the coming chapters: to portray the branch of physics that promises instances of part-whole and composition without the familiar features Γ (Chapters 4, 5); and to argue that entities relating as parts to wholes (composers to composites) without Γ remain 'parts' and 'wholes' ('composers' and 'composites') in a literal sense so that it is possible for a branch of science to bring about these non-canonical instances in the first place (Chapters 2, 3).

[14] Following Ladyman and Ross (2007: 21).

Let us now get clear on the nature of the unfamiliar and 'genre-breaking' features Γ. In so doing, we continue to support the empirical task (seeking the worldly features of things that relate as part to whole, or composers to composite) with the conceptual one (analysing ordinary part-whole and composition concepts). For, indeed, portraying the features requires that we first survey our concepts for an account of what they are and then invoke the relevant science to attest cases of part-whole without them.

6.1 The target concepts

More precisely, even before analysing our concepts to pin down the familiar features we intend to challenge, we need to decide on *which* concepts to analyse. The family of ordinary part-whole and composition concepts extends widely, with different analytic features following from each member. Fortunately, however, the features challenged[15] by advanced science are quite 'stable' across the family, so that no matter what member we decide to analyse, these follow in only slightly different formulations Γ, Γ', … . Besides, only minor details change in the ways Γ, Γ', … suffer from the scientific evidence: for to raise a challenge to each of Γ, Γ', … we only need to present the same evidence in slightly different ways (Chapter 4). Accordingly, for the time being we may stipulate the following as our target concepts, returning to the alternatives in due course:

> **Part-whole**: The target concept of part-whole holds of two arguments, *x* and *y*, such that *x* is part of *y*[16]. It satisfies *at least* one order property: general Reflexivity (every *x* is part of *x*)[17].

[15] Recall that the challenge is twofold: lacking these features Γ are – according to the challenge – both scientific concepts *C'* that fall in the family of part-whole (i.e. have satisfiers of which the labels 'part' and 'whole' hold literally) and the real parts and wholes that satisfy *C'*.

[16] There is no third argument relativising 'is part of…' to a particular time: thus the focus is on 'a-temporal' parthood. As far as I can see, the evidence from advanced science I present in Chapters 4 and 5 challenges features of the a-temporal notion.

[17] I also assume that *part-whole* defines *overlap* (*x* overlaps *y* $=_{df}$ one part of *x* is a part of *y*) and *disjointness* (*x* disjoins *y* $=_{df}$ no part of *x* is a part of *y*). It matters for my aims that the definition of *overlap* and *disjointness* in terms of part-whole is given in these exact terms. But I allow for the possibility that *overlap* or *disjointness*, rather than *part-whole*, work as the main primitive. In this case – if *part-whole* is defined by *overlap* or *disjointness* – it matters for my purposes that (i) what I state as a definition remains true as a bi-conditional (*x* overlaps *y just*

I emphasise 'at least'. For, as the informed reader knows, popular theories like classical extensional mereology (see Varzi 2015: S. 2) ascribe part-whole not just (general) Antisymmetry and Reflexivity, but also Transitivity: for all *x*, *y* and *z*, if *x* is part of *y* and *y* is part of *z*, then *x* is part of *z*. Securing Transitivity is crucial to the theoretical aims of mereology, but not so much for ours. Indeed – strictly speaking – none of the challenges we pose to the familiar features Γ of this *part-whole* concept demands that the concept is transitive. Or to put it differently, the scientific evidence we keep under focus strikes against features Γ of concepts of *part-whole* with general Reflexivity, whether or not they are also transitive. On Transitivity, accordingly, we can afford a convenient neutrality.

> **Proper part**: The target ordinary concept of part-whole holds of two arguments, *x* and *y*, such that *x* is a proper part of *y*. It satisfies *at least* general Irreflexivity (no *x* is a proper part of *x*) and Asymmetry (for all *x* and *y* if *x* is a proper part of *y* then *y* is not a proper part of *x*). It is defined via 'part-whole' and non-identity: '*x* is a proper part of *y*' $=_{df}$ '*x* is a part of *y* distinct from *y*'[18].

Similar remarks hold in respect to 'at least': *proper part* needs only these minimal order properties in order for its familiar consequences Γ to face challenges in a naturalistic setting. Further, worthy of emphasis is *proper part*'s defined status. While it matters for my aims that the definition is given in the above terms (part-whole and identity) rather than others (see Fn. 17), I allow for the possibility that *proper part* plays the role of the main primitive, with *part-whole* defined in terms of it. However, even if one construes *proper part* in this way, I require for my purposes that what I stated as a definition remains true as a bi-conditional: for all *x* and *y*, *x* is proper part of *y* just in case *x* is a part of *y* distinct from *y*. Similarly, I require that *part-whole* – now

in case one part of *x* is a part of *y;* and analogously for *disjointness*); and (ii) that *part-whole* – now construed as a defined concept – maintains the order properties I appoint it with (Antisymmetry, Reflexivity).

[18] At the price of rejecting the Antisymmetry postulate, one can re-define *proper part* as: *x* is a proper part of *y* $=_{df}$ *x* is part of *y* and y is not part of *x* (Thomson 1998, Cotnoir 2010). This is one of the 'alternatives' I alluded to in the opening of this section: by depicting the concept *proper part* in this way, rather than as I have stipulated, we can object to its familiar consequences Γ' simply by presenting the scientific evidence in a slightly different way. Keeping this in mind (Chapter 4), let us persist for now with the usual understanding of proper part as 'part of… distinct from…'.

construed as a defined concept – maintains the order properties I portrayed: Antisymmetry and Reflexivity.

> **Composition:** The target ordinary notion of composition can be construed either binarily with singular-singular (x, y) and plural-singular arguments (xs, y); or with a variable number n of singular arguments ((x_1, ..., x_n, y))[19]. In either case, 'xs' and 'x_1, ..., x_n' are the 'composers' of y, while y is the 'composite' of the xs (or x_1, ..., x_n). General Asymmetry holds: if the xs compose y, then y does not compose the xs (for no single thing composes many). Moreover, either composition satisfies Irreflexivity in the singular arguments (for all x, x composes x), or it is *defined* in such a way that this order property follows[20].

Prima facie, this account of *part-whole*, *proper part* and *composition* might look suspiciously bare: hardly any theory of these three concepts or of the corresponding relations holding between their worldly satisfiers[21] gets under way with as small a number of constraints. But this impression is misleading and the worries of minimalism unfair. For the account is neither an exhaustive analysis of the three concepts, nor a complete theory of the corresponding relations. It is, on the contrary, a list of principles that we must assume true of the concepts of *part-whole*, *proper part* and *composition* in order for their other attractive familiar properties (Γ) to face challenges in a naturalistic setting. But, ultimately, what are these properties?

---

[19] Including the case of n = 1.

[20] Notably, singular Irreflexivity follows from Leibniz' Law together with Needham's (1981) definition (the *xs* compose $y =_{df}$ every thing that overlaps z overlaps one of the *xs*); or alternatively from the Reflexivity postulate for part-whole together with Tarski's (1935) definition (the *xs* compose $y =_{df}$ every one of the *xs* is part of y and every thing that is part of y overlaps one of the *xs*). See Varzi (2015: S. 4.2) and Chapter 6 for more discussion. Lewis (1991) endorses Tarski's definition, Simons (1987) and Casati and Varzi (1999) prefer Needham's.

[21] Neither classical mereology, nor Simons' (1987: Ch. 1) logically weaker variants (Cf. Koslicki 2007). Not even, moreover, the non-classical mereologies that reject some of the principles I assume, e.g. the irreflexivity and asymmetry of proper part (Cotnoir and Bacon 2012), or the Antisymmetry postulate of part-whole (Cotnoir 2010).

6.2 Core principles

We can approach the features in Γ aided by some new terminology. Let us call a certain principle $\gamma \in \Gamma$ "core" if **γ** meets at least one of three conditions:

1. **γ** holds conceptually of *part-whole*, *proper part* or *composition*, when the latter are construed with the last section's properties.

2. **γ** makes a statement that has been largely and usually perceived of as holding conceptually of the notions so construed[22].

3. **γ** does not hold conceptually of one of the three concepts, but reports an attractive 'intuition'[23] about the concepts' satisfiers (real parts and wholes, composers and composites).

For a contrast, if a principle $\gamma' \notin \Gamma$ belongs not to the "core", but to the "periphery", then it lacks all of these features: it does not hold conceptually of one of the three notions, it has not been largely and usually perceived to so hold and it reports no intuition satisfied by a conceptual truth about one of the notions.

While this is – clearly – not the only way of dividing up principles about the three concepts *part-whole*, *proper part* and *composition* (Section 7), it is a helpful division to gather (in the 'core') principles that face challenges in a naturalistic context. There is some agreement (with *provisos*, see Section 7.3) that principles with core status include[24]:

The Antisymmetry postulate: If x is a part of y, then y is a part of x only if $x = y$

[22] Because the core includes principles of type (b), membership in the core does not suffice for conceptual status. There might be, indeed, core principles merely *perceived* by the community to be true as a matter of concepts.

[23] In the rest of this work, I use this term in a wide sense, compatible with various accounts of what it is for a principle to report an intuition (Section 8.2, see Fn. 44 for discussion). 'Intuition assessment' is the corresponding *method* of formulating a principle **γ** starting from intuitions.

[24] But perhaps are not limited to, see Bennett (2015) for a comment on Lewis' (1991) assessment of analytic principles in classical mereology. According to Bennett, Lewis also assigns conceptual status to *Uniqueness*, that is, the principle that any two x, y that compose a whole z compose no other whole distinct from z.

> *Weak Supplementation* (WS): If x is a proper part of y, then there is some part z of y distinct and disjoint from x.
>
> *Idempotence*: The composite of x alone exists and is x.

It is against these three principles that I will turn my twofold objection (Section 4). Thus, some scientific concept in the literal family of part-whole (proper part, composition) fails the Antisymmetry postulate (Weak Supplementation, Idempotence). And at the actual world, some objects of scientific interest relate (literally) as parts to wholes (proper parts to wholes, composers to composites) while failing Antisymmetry (WS, Idempotence).

We cast this objection in the way we already anticipated (Section 5). Focus for illustration on *part-whole* and the Antisymmetry postulate. We begin (Chapters 2, 3) by arguing in general terms that (1) scientific concepts belong to the literal family of *part-whole* even if they fail the postulate; and that (2), similarly, satisfiers of these scientific concepts can relate as parts to wholes in ways that offend to the postulate. Following (Chapter 4, 5), we descend to details and argue (3) that advanced science evidences actual objects of interest that satisfy the concepts, relate as parts to wholes and yet fail the postulate; or at least (4) that it deploys concepts in the part-whole family without Antisymmetry (even if the concepts' satisfiers do not truly relate as parts to wholes in opposition to it). Similar reasoning holds for *proper part* and Weak Supplementation, as well as for *composition* and Idempotence.

But for now we had better pause for more details about the core and the core principles *per se*, before looking at the ways we can challenge them in a naturalistic context.

7. Alternative core-periphery distinctions

Antisymmetry, Weak Supplementation (WS) and Idempotence are the 'core' principles we keep under focus in the rest of this work. The reader might worry over my claim that each of these principles (a) enjoys conceptual status; or (b) has been largely perceived to enjoy it; or (c) voices an intuition that we expect a conceptual truth to voice. Let us put off these worries until the next section and – for the moment – work under this helpful assumption. I would like to add more about the general nature of the 'core', which unites the three principles.

Principles in the 'core' include, as agreed, principles that satisfy one of (a), (b) or (c). We draw the core/periphery distinction along these conceptual/intuitive lines, but concurrently we notice that 'core' and 'periphery' are suggestive terms and that certainly various policies ascribe these labels other than by assessing the principles' putatively conceptual or intuitive status. For example, we might think that a principle **γ** is 'core' if it partakes of the axiom system of classical extensional mereology (being either an axiom or a consequence thereof), while peripheral principles are neither axioms nor consequences of axioms. Or alternatively, we might deem **γ** core (peripheral) if **γ** is metaphysically necessary (contingent).

What is the usefulness of drawing the core/periphery distinction according to the conceptual/intuitive criterion? At a first look, and in a way we will articulate more carefully down the line (Chapter 2, Sections 1, 2), construing the core with these conceptual and intuitive instruments helps with describing the impact of scientific evidence on principles that regulate part-whole and composition. There is an elusive sense in which evidence of a 'superficial' type should be less reason to worry than evidence prompting revision of the core. Articulating policies for core and periphery status helps spell out this evasive idea.

In more detail, suppose that some evidence challenges a principle, which some way of drawing the core/peripheral distinction classifies as peripheral. In this case, there is room for accommodating this evidence without thereby questioning the *methods* that guide the classification. For example, suppose that as a result of taking in some evidence from the biological account of the part-whole relation (as per Rescher 1955) we come to doubt Transitivity (for all x, y and z, if x is a part of y and y a part of z, then x is a part of z), allowing for intransitive instances of part-whole[25]. Also suppose (for

[25] Varzi's (standard) response (2014: 1.3) to this biological evidence is that the part-whole relation in the subject matter of biology is intransitive but distinct from the relation in the scope of the Transitivity postulate. The latter is a *general* (transitive) part-whole relation, i.e. a relation that any two entities instantiate insofar as they stand in some specific part-whole relation or other (Cf. Fine 2010: 565). We return in due course to general part-whole relations (Chapter 3: Section 2). For the moment, let us suppose (for the argument's sake) that the biological evidence forces us to drop the Transitivity postulate and so that the part-

illustration) that Transitivity *resists* ascription of conceptual or intuitive status, so that by our policy it belongs outside the core. Accordingly, accommodating non-transitive cases of part-whole does not interfere with the methods (conceptual analysis, assessment of intuitions) that separate core from periphery.

But as it happens, these two are also methods one could invoke to theorise about worldly part-whole. Some deliverances of conceptual analysis or principles largely thought to be conceptual constitute attractive worldly hypotheses (e.g. that no two things are ever mutual parts, that no single thing composes anything other than itself) and when it comes to part-whole, the same holds of principles that report intuitions, e.g. the intuition that every proper part comes with a 'remainder', as reported by Weak Supplementation or similar principles (see next section). Evidence against the periphery – scientifically motivated as it may be – constitutes no threat whatsoever against using these methods to yield worldly products.

Call a 'metaphysics of part-whole' a choice of principles held as worldly truths about part-whole and composition. When the methods associated with some way of drawing the core/periphery distinction survive the evidence unchallenged, then we can say that our metaphysics of part-whole has been 'informed' by evidence from science. Information is intentionally a 'friendly' term. Informed metaphysics implements evidence, but without renouncing methodological prerogatives.

Of course – as just seen – one may draw the core/periphery distinction in various ways. Thus, evidence that we accommodate relative to one such way may come to challenge core principles and the associated methodologies relative to another. For example, the Transitivity postulate would belong to the core if we took this to consist of all axioms and consequences of classical mereology.

This variation of the evidence's seriousness along with the chosen policy for dividing core and periphery is just what we should expect. Presumably in bringing in some evidence rather than some other we already aim at challenging some and not other metaphysical methods for seeking the worldly principles that regulate part-whole. And my goal in bringing in my evidence of interest down the line (Chapter 3, 4) is explicitly in challenging conceptual and intuitive methods.

---

whole relation admits of at least some non-transitive instances (whereby this relation is not a species of a general relation but rather the same holding outside of biology, e.g. between a table-leg and a table).

7.1 Evidence against the core

Worthy of emphasis in the last section is the claim that we present the scientific evidence to *challenge* the conceptual and intuitive methods that cluster mereological principles like Antisymmetry, WS and Idempotence into a single core. But why just challenging and not entirely defusing? What do I expect to happen when a package of scientifically informed evidence impacts a core principle?

Recall that I understand this evidence as twofold (Section 4): conceptual and empirical. That is, either it is evidence that advanced science uses concepts opposing the principles (e.g. a concept of *part-whole* without Antisymmetry, a concept of *proper part* without Weak Supplementation); or it is evidence that this science posits instances ('cases') of the corresponding relation (part-whole, proper part, composition) that defy the principles. We can begin to approximate the effect of core-threatening evidence by outlining three possible and mutually alternative outcomes, with the promise of contributing further detail in the chapters to come (2, 3):

> (a) the methods used to tell apart core principles (conceptual analysis, assessment of intuitions) deliver principles unsuitable for worldly truth. We can carry on attributing Antisymmetry a conceptual status, but concurrently we will have to accept that worldly part-whole is not antisymmetric (taking up this first option requires evidence of the 'empirical' kind, e.g. actual cases of the part-whole relation that defy Antisymmetry).
>
> (b) the methods deliver principles suitable for worldly truth, but our *prima facie* perception of the deliverances has so far been inaccurate. Thus Antisymmetry has been largely perceived as a conceptual principle, but something has gone amiss in the assessment of what follows from the concept. The scientific evidence works as a prompt for reconsidering these conceptual consequences (taking up this second option requires evidence of the 'empirical' or the 'conceptual' kind, e.g. non-antisymmetric concepts of part-whole or actual cases of the part-whole relation that defy Antisymmetry).
>
> (c) the methods deliver core principles suitable for actual truth and the perception of these deliverances is accurate. However, the evidence leads us to

admit some physical objects related in opposition to the core principles or concepts whose properties oppose the principles. The relation holding between the objects is not a part-whole relation and the added concept is not a concept in the family of *part-whole*. Rather, both are merely similar (though to a high degree) to genuine part-whole relations or concepts. (For example: Antisymmetry remains a conceptual principle true of worldly part-whole, while the evidence speaks for a *sui generis* non-antisymmetric relation which resembles part-whole) [26]. Compared to (a) and (b), this is a retreat position. I will deal with it in more detail in the next chapter.

Taking up one of these three options amounts not just to *informing* one's metaphysics of part-whole with accredited evidence. It amounts, we could say, to an *amendment* of the metaphysics. The amendment may proceed as per options (a) and (b), that is: by changing the methods appropriate for generating worldly truths about part-whole or by re-assessing our estimate of which principles the methods deliver.

There actually is – I hold – evidence from physically interpreted mathematics prompting revision of the conceptual/intuitive "core" in this way – or more specifically, of the three members of the conceptual core listed above: *Antisymmetry*, *Weak Supplementation* and *Idempotence*. Having illustrated this evidence, I will argue in the forthcoming chapters (Chapter 2, 3, 4) that it deserves handling in terms of strategy (a) or (b). As for (c), it is a defensive option that I see no reason to endorse, despite its popularity (Chapters 2, 3).

Accordingly, the ideal response to the availability of this evidence is a disjunction: the evidence either shows (1) that analysing concepts is not an acceptable method to engender worldly truths about part-whole and composition; or (2) that we have misjudged the deliverances of procedures for analysing concepts[27]. Following down one of these two paths exemplifies a type of naturalism – a way of letting one's metaphysics be informed by science – that is stronger than choosing one's superficial

[26] Here '*sui generis*' has the original meaning of 'of its own kind', not the transposed meaning of 'pertaining to the same kind but breaking canonical (e.g. core) principles'

[27] For those who would like to go down the defensive path (c), the evidence shows that concepts have been analysed correctly and the analysis delivers true principles, but in addition to these principles we need to acknowledge 'deviant' scientific concepts (sufficiently similar to *part-whole* with the conceptually established features) and 'deviant' cases of non-antisymmetric relations between objects of scientific interest (sufficiently similar to cases of part-whole with the conceptually established features).

principles according to the available evidence. This stronger policy is metaphysical amendment.

8. Do we need *naturalistic* evidence against the core?

Now that we gained a preliminary idea of the naturalistic evidence we are after (more in Chapters 4, 5) and the ways in which we expect it to impact on the core principles (Chapters 2, 3), we are left with one last question for this chapter: the core contains Antisymmetry, Weak Supplementation and Idempotence. Is it so clear that each of these meets the conditions for membership in the core?

8.1. Antisymmetry and Idempotence

It suffices for some principle γ to fall in the core that it has conceptual status or at least that it is 'largely perceived' to have this status. However, whether Antisymmetry and Idempotence meet one of these two disjuncts is far from clear. For consider Idempotence. The principle states that the sum of every single thing x, if it exists, is identical to x. It seems natural, for an example of a concept of composition that defies the principle, to think of singleton-formation[28]: if any singleton set $\{a\}$ of a exists, then it is distinct from its sole 'former' a. Although not everybody agrees that singletons and members satisfy a concept of *composition* (Lewis is a notable objector, see 1991: 40), it seems at least suspicious to grant Idempotence a place in the core without discussing these putative counterexamples.

My response appeals to the dialectical situation. I grant that it is not straightforward to classify Idempotence as a conceptual truth, yet in spite of these difficulties the assumption remains a good one to explore, because it is not in my favour.

My aim is to challenge Idempotence with naturalistic evidence, not to endorse it. If we agree on construing this principle as a conceptual truth, then one of the

[28] See e.g. Fine (2010) and Chapter 3: Section 3.

challenges' possible outputs, i.e. (a), is that we should not rely[29] on its analytic content as a guide to formulate worldly principles about proper composers and composites. For in the 'empirical evidence' scenario, science posits composers and composites that dispense with Idempotence. Another potential effect (b) is that we misconceived of *composition*'s analytic properties in the first place. For in the 'conceptual evidence' scenario, science advances composition concepts without Idempotence.

Both of these outputs, however, are in agreement with the critic who denies Idempotence conceptual status. In fact, this construal of the principle is incompatible with the scenario in (b) (present concepts of composition deployed without Idempotence), but this means that the sceptic and friends of the scenario in (b) argue for just the same conclusion, i.e. that Idempotence has no conceptual status. Starting off as a sceptic, one may exploit the scientific evidence presented here as further support for the desired conclusion. Indeed, the sceptic wins an interesting argument to the effect the conceptual construal should be avoided for a naturalistic reason.

Further, suppose that the evidence proved to support scenario (a), rather than (b). Even in this case we do not enter on a collision course with the sceptic. Scenario (a) states that advanced science posits composers and composites related in opposition to the core principles. While this is compatible with the core principles preserving their conceptual status, it is also compatible with the sceptic's rejection of this status.

Parallel considerations hold for the Antisymmetry postulate. The principle states that only identical things are mutual parts. But it seems natural, for an example of a concept of part-whole that offends the principle, to think of *exact co-location*[30]: if any two distinct objects x and y occupy the same region of space r, then x occupies an (improper) sub-region of y and y an (improper) sub-region of x. Not everybody will agree that any such x and y satisfy a concept of part-whole (the satisfied concept might be one of spatial inclusion without part), but it seems at least suspicious to grant Idempotence a place in the core without discussing these putative counterexamples.

We return, therefore, to the dialectical situation. By construing Antisymmetry as a truth of concept (or largely believed to be such), I do not oppose those who claim against this construal that *exact co-location* is a concept of part-whole without

[29] If not defeasibly.
[30] See Thomson (1998), Cotnoir (2010).

Antisymmetry. For my aim is to challenge Antisymmetry, not to endorse it. If we agree in the first place on construing this principle as a conceptual truth, then two of the challenges' possible outputs – (a) and (b) – are compatible with the critic who rejects this construal. According to (a), we should not rely on *part-whole*'s analytic content[31] to formulate worldly principles about real parts and wholes (for science parts and wholes that dispense with Antisymmetry)[32]. According to (b), we misjudge *part-whole*'s analytic properties (because advanced science evidences concepts that dispense with the principle). Thus we should avoid the construal for naturalistic reasons, which makes it already for a point of interest, even if the construal is implausible in the first place.

8.2. Weak supplementation (a primer)

The principle of *Weak Supplementation* (WS) states that if x is a proper part of y, then some z distinct and disjoint from x is a part of y. It falls under a family of 'decomposition' principles[33], which try to distinguish composites from atoms[34] more fine-grainedly than we manage by saying that atoms are things without proper parts and composites are non-atoms. On the account I assume in this work, WS reports an intuition about the properties of proper parts and wholes at the actual world[35]. Let us call it the 'remainder' intuition: that no thing with a *single* proper part can be a composite, and that for every putative proper part of a whole we also need to posit a 'remainder'[36].

---

31 If not defeasibly.

32 The posited cases of part-whole without Antisymmetry hold between *relata* that do not exactly co-occupy the same region of space r (Chapter 5). Thus we gain a new case scenario for mutual parts at the actual world, which does not require that there be coincident objects and that occupation of another object's improper region suffices for parthood.

33 The labelling is standard in formal approaches to mereology. See Varzi (2015: 3.2) for a recent introduction.

34 Assumption: 'x is an atom' $=_{df}$ 'no thing z is a proper part of x'.

35 For the view that WS voices an intuition, see Bennett (2013: S. 5) and Lando (2017: 27-8). See the end of this section for the view that WS analyses the concept *proper part*.

36 As a noteworthy exception, some authors admit of composites without reminders, thus differentiate composites from atoms more superficially based on the latter (but not the former) being entities without proper parts. This exceptional view moves explicitly against the intuition that a reminder-less relation would

On WS's own perspective, the remainder demanded by the intuition is some second part *z* of the composite distinct and disjoint from the first proper part *x*. But WS's point of view is not the only one available to voice the remainder intuition. One weaker possibility views the remainder as just a distinct proper part, additional to the first (Company). For another, it is a proper part that is not part of the first (Strong Company). And yet some others (this being the classical mereological option) argue that if some object fails to be a proper part of another, then the latter has a part disjoint from the former (Strong Supplementation).

For most of this work, the focus remains on Weak Supplementation, as this will be the particular mode of articulating the 'remainder' intuition that we present against the physical evidence (Chapter 4, 5) with the aim of undermining the intuition as a guide in theorising about physical part-whole. This is a choice that owes mostly to the popularity of WS as an appropriate 'reporter'[37] of the intuition (following Simons 1987: 26-7; See Chapter 2, Section 3)[38]. However, another important gain of seeking evidence against WS - not other decomposition principles – comes in illustrative power. Many who agree on articulating the 'remainder' intuition by principles other than WS – notably, Strong Supplementation (STR) – also accept WS[39]. So, thanks to the evidence against WS a chance arises for challenging not just one, but multiple popular choices for articulating the intuition. All of this is – notice – an anticipation: We return more carefully in Chapter 2 (Sections 3, 4) to this issue of which decomposition principles we should present against the evidence in order to undermine the value of the 'remainder' intuition in the appropriate way.

For the time being, let us close this primer section about decomposition principles by asking what it is for WS to voice an intuition (specifically: the 'remainder' intuition) and what it is for the naturalistic evidence to undermine its value. I give the term 'intuition' a wide acception. On one reading, for WS to voice the intuition that proper parts come with remainders could just be for it to make an analytic statement

---

not be part-whole. It has its roots in Aristotle and Brentano, but one finds it more recently defended in Fine (1982). For commentary and other references, see Simons (1987: Ch. 1) and Varzi (2015: S. 3.1).

[37] I use 'articulate', 'voice' and 'report' synonymously.

[38] There is a sophisticated landscape of decomposition principles claimed to articulate the 'remainder' intuition. Hovda (2009), Gilmore (2009) and Donnelly (2011). We will briefly notice when a certain bit of presented evidence also impacts on these other principles.

[39] An exception is Baron and Cotnoir (2012), see Chapter 2, Fn. 29.

about *proper part* (it holds in virtue of the concept of *proper part* that every *y* has a proper part *z* distinct and disjoint from each of its proper parts *x*)[40]. The reason to include this construal of WS as an analytic statement is that often it goes hand in hand with the construal of it as an intuition. For instance, Varzi ascribes WS analytic status in (2009) and intuitive status in (2015: 3.1); while Koslicki ascribes both in the same work (2008: 183, Fn 24; and 2008: 167)[41].

On this reading, the empirical and conceptual challenge to WS give the same results as the challenges to Idempotence and Antisymmetry: namely, we had better not rely[42] on WS's analytic content in formulating a worldly principle (as per (a)); or we have misassessed what holds conceptually of *proper part* (as per (b)). We would apply – here – the dialectical strategy of the last section. We assume in our disfavour that WS's analytic statement is true and reveal naturalistic arguments for pursuing one of (a) and (b).

I wish to grant, however, other senses of 'intuition' in which WS can be said to articulate the remainder intuition. Rather than portraying WS as an analytic statement about *proper part*, we can depict it as making a statement about real proper parts; that is, that at the actual world they carry remainders in such a way that every *y* has a proper part *z* distinct and disjoint from each of its proper parts *x*. The content of these statements – the particular way proper parts carry remainders at the actual world – is also the subject matter of an 'intuition' *stricto sensu*, i.e. something that many will find appealing and 'seemingly true' of worldly proper parts[43].

The possibility that WS conveys an intuition in this strict sense adds variety in the estimated impact of the naturalistic evidence, i.e. evidence from concepts of proper part that dispense with WS and evidence from genuine proper parts that fail WS at

[40] Among those who view WS as a (true) analytic statement, see Simons (1987: 116), Varzi (2008, 2009).

[41] On one interpretation of Varzi's and Koslicki's view, WS makes an analytic statement that is accessible to us via intuition. The subject matter of the intuition could be, directly, *proper part*'s conceptual content (i.e. we intuit that *proper part* – the concept – satisfies WS), or worldly state of affairs (we intuit that real proper parts always come with remainders in the way specified by WS). Cf. Dowe (2000: 8-9)).

[42] If not, perhaps, defeasibly.

[43] There is much disagreement on what it is for statements to 'seem true' in an intuitive way. For our purposes, we can use this label as a placeholder for (at least some of) the various available options (see Pust 2017 for an overview). These range from deflationary (Dorr 2010) to substantive (Bealer 1998, 2002). Thus, for Dorr a statement (p) seems true if it reports a belief of unclear origin whose consequences we simply wish to explore (also see Ladyman (unpublished) for criticism). For Bealer, (p) seems true iff it reports a *sui generis* intellectual seeming.

the actual world. Present evidence of the latter sort (the 'empirical evidence' scenario), we conclude that the voiced intuition should not be used to formulate worldly principles about proper parts; or at least that we misperceived WS as the decomposition principle that voices the remainder intuition.

These conclusions correspond, respectively, to (a) and a variant of (b) specific to the case of decomposition principles. Suppose that advanced science unveils concepts of *proper part* that dispense with WS. According to the original variety of (b), evidence of this 'conceptual' type is supposed to correct our initial assessment of analytic properties: if advanced science deploys a new concept of part-whole without Antisymmetry, then we can concede we were wrong on Antisymmetry pertaining to the analytic properties of all concepts of part-whole.

However, this will not be the appropriate lesson if WS reports an intuition, rather than making an analytic statement, whence the need for a variety of scenario (b). In response to advanced scientific concepts without WS we learn that WS's own way of voicing the 'remainder' intuition should not be used[44] to formulate analytic properties for all concepts of proper part. For at least the concepts of proper part deployed in advanced science, we will rather let the science itself decide. Decomposition principles might follow from the advanced concepts, which voice the 'remainder' intuition differently from WS (setting other conditions on what counts as a 'remainder'), or even avoid voicing the intuition altogether (licencing proper parts with no remainders). In fact, the concepts I focus on in later chapters (4, 5) are of the latter and more radical variety.

9. Conclusion

The above should suffice for a presentation of the core principles and the results we expect from challenging them in a naturalistic context. The core principles – Antisymmetry, Weak Supplementation and Idempotence – state conditions

[44] If not, perhaps, defeasibly.

necessary for certain *x* and *y* (*xs* and *y*) to relate as part to whole (composers to composite). We can take these conditions to describe either what holds by virtue of certain mereological concepts (i.e. *part-whole, proper part*, *composition*), or what holds of the *x* and *y* (*xs* and *y*) that relate as part to whole (composers to composite). Construed in either of these two ways, the principles face a challenge when advanced science deploys concepts that oppose them or posits instances of mereological relations that fail them. The next chapters are entrusted with setting up these challenges. To start with (Chapters 2, 3), we will need to ensure that advanced science can bring about the challenges in principle, by showing the concepts it deploys to be genuinely mereological (not *sui generis* concepts similar to the mereological ones) and the instances it posits to be instances of genuinely mereological relations (not of *sui generis* relations similar to the mereological ones). Following (Chapter 4, 5), we will need to portray the branch of science which, in fact, deploys advanced concepts and/or posits instances of mereological relations that fail the core principles.

II

Part-Whole: A *sui generis* affair

In the previous chapter I ascribed only a small handful of properties to the concepts of part-whole, proper part and composition. These included the definition of proper part as distinct part (with 'part' used as a primitive); the definitions of overlap and disjointness as, respectively, presence and lack of shared parts; the Reflexivity postulate of part; the existence of lone composites (composites of a single thing). Having appointed the concepts these features, we could shortlist other principles ('core' principles), which look attractive if viewed as analytic consequences of the concepts so characterised or as principles that voice intuitions about what is required for some thing to be part of another.

We were left, however, with a question as to how it is possible for well-supported physics to 'object' to the core principles, either showing that they are not true of all concepts of part-whole (composition, proper part), or evidencing particular parts and wholes (composers and composites) that, at the actual world, fail the principles. It is now time to consider this question more thoroughly.

Here is the plan: Section 1 likens the objection from physics to the worries naturalist philosophers raise against accounts of part-whole that do not traffic in the corresponding physical notion, which is highly *sui generis*. Sections 2 and 3 argue that objections of this kind against the core principles (Antisymmetry, Weak Supplementation and Idempotence) are more serious than other naturalistically guided objections against principles for part-whole and composition. For the former objections – unlike the latter – question not just particular principles but popular methods to develop and endorse core principles. Section 4 responds to arguments by Kit Fine (2010), Giorgio Lando (2017) and Karen Bennett (2017) that would make principles like those we target (e.g. Antisymmetry) invulnerable to objections from physics. Section 5 finally assesses the naturalistic evidence's impact. On the most promising way of articulating this impact, the evidence leads us (a) to deny worldly guidance for the methods delivering core principles like Antisymmetry (e.g. conceptual analysis or assessment of intuitions); or (b) to claim that we misperceived the deliverances of these methods (thus, say, that Antisymmetry really isn't intuitive or true as a matter of concepts); or, finally, (c) to claim that the methods are truth-guiding and the perceptions well-placed, but the evidence speaks for cases of relations that offend the core principles and yet are conspicuously similar to genuine part-whole. I notice that the latter offers a nice compromise for strenuous defenders of the core principles. However, it is also a retreat option that demands elaborate articulation. And overall, owing to the failure of Fine's, Lando's and Bennett's arguments in S. 4, more revisionary rivals such as (a) and (b) gain advantage.

1. Three varieties of naturalist scepticism

The main risk when we bring in metaphysical resources to theorise about part-whole is to pay too little attention to the way this is more or less directly characterised in physics or the special sciences, and hence, presumably, theorise about a relation that applies nowhere in nature[1]. The worry is put fiercely – but fierceness enlivens the sense of challenge – by Ladyman and Ross (2007: 21):

> Why suppose that there is any such thing [as part-whole]? It is supposed to be the

[1] In Ladyman's (2011) understanding, metaphysics has naturalistic value if it produces a framework of concepts potentially exploitable for *future* physics (similarly, see French and McKenzie 2012). Thus even if a theory of the part-whole relation were artificial, on this view it would still be valuable as a prospectively exploitable framework of concepts. See Chakravarrty (2017: 61-2) for a critique.

> relation that obtains between parts of any whole, but the wholes mentioned above are hugely disparate and the composition relations studied by the special sciences are *sui generis*. We have no reason to believe that an abstract composition relation is anything other than an entrenched philosophical fetish.

This passage illustrates the attitude of surprisingly many philosophers who understand their theorising about reality as constrained by the natural and/or special sciences[2] [3]. But what is this scepticism's exact dictate?

Ladyman and Ross oppose features of part-whole that abstract away from genuinely scientific part-whole (or composition), because they are devised with the resources and aims of *a priori* metaphysical enquiry. Their quote brings up two broad-scope claims: one is the thesis that appropriately informed accounts of part-whole (or composition) are *sui generis* relative to 'abstracted away' *a priori* accounts. Call this the *sui generis*-ness thesis – it is encoded in the lines: "[i]t [composition] is supposed to be the relation that obtains between parts of any wholes, but the wholes [of science] are hugely disparate and the composition studied by the special sciences *sui generis*". The other is an ontological claim: there is no (or there are no instances of) metaphysically abstracted or idealised part-whole. This is encoded in the initial rhetorical question and in the (in)famous aggressive close.

Lining up these two bigger themes immediately makes us wonder about their connection; and particularly, whether *sui generis*-ness entails elimination, whether it is because of its deviance from scientific canon that the abstract metaphysical dummy deserves no ontological dignity. As it happens, I deem this question an important one to assess in the context of the naturalist program: what does the abstracted-ness of metaphysical part-whole have to do with its non-existence? Or, as I have put it in the Introduction, can we move from applying the part-whole vocabulary to *sui generis* physical relations to disposing of canonical part-whole? We

---

[2] Just to survey the battery of sceptics: Ladyman and Ross (2007), Smith (2007), Wilson (2008), Healey (2013).

[3] This debate about naturalistically optimal features for part-whole and composition intersects with a growing debate about naturalistically optimal features for relations of *metaphysical dependence* ('vertical relations' in a recent labelling by Aizawa and Gillett (2017)). Standardly, composition is considered part of this 'verticality' family, as it is a natural thought that composites metaphysically depend on their components (Wilson (2013), Aizawa and Gillett (2017), Bennett (2017)).

will devote our energies to this question in the second part of this work. For the time being, we stick to the *sui generis*-ness thesis. How is it to be understood?

Let us start from what it is not. The thesis should not be mistaken for the claim that using predicates like '...is a part of...' or '...compose...' (and mereological cognates) in physics and the special sciences is entirely illegitimate. For indeed this claim is extraordinarily difficult to defend. That part-whole vocabulary has *some kind* of legitimacy is evident from its predominance within different areas of physical science: from theoretical to experimental. Even advanced varieties of relativistic mechanics admit of distinctions between simple and complex bodies. Quantum mechanics admits of the distinction between single-body and many-body systems. Quantum chromodynamics licenses elementary (quarks) and composite particles. And finally, quantum field theory admits of composite fields (although perhaps it does not admit of simple fields, see McKenzie 2012).

Granted the legitimacy of labelling many relations between objects of physical interest 'part-whole', one can question the semantic significance of these legitimate uses. There are, it seems to me, at least three possibilities, all of which resonate well with the idea that scientifically informed part-whole is *sui generis*:

(i) Those who apply 'part-whole' vocabulary to the physical objects of interest thereby latch onto cases of non-standard and deviant relations, which together with the abstract metaphysical relation count as genuine varieties of *part-whole*. Instances of these varieties will differ in various respects, perhaps even with regard to 'core' respects like Antisymmetry, Weak Supplementation, Idempotence, etc. (See Chapter 1: Section 7). Yet in spite of these differences, the label 'part-whole' holds for all varieties literally. This possibility is friendly to mereological pluralism, i.e. the view that not one but many distinct relations are relations of part-whole. Some of these are part-whole relations of a *sui generis* scientific variety, while some others are part-whole relations of a dummy and metaphysically abstracted variety[4].

[4] Mereological pluralists include include Husserl (1900-1/1913), McDaniel (2004), Fine (2010).

(ii) Users of the predicate latch onto cases of a single, genuinely part-whole relation whose features vary with its *relata*'s scientific category. Some instances of the unique relation holding between objects of physical interest might be non-standard and deviant, clashing with 'core' principles like Antisymmetry, etc. Even so, the label 'part-whole' is legitimate and holds of all cases literally. This view guarantees mereological monism (only one relation is a part-whole relation), but accounts for the *sui generis*-ness of some cases by appeal to types of *relata*[5]*:* instances of (say) the sub-field to field relation would still be instances of the only part-whole relation, but exemplify *sui generis* features owing to their field-like *relata*.

(iii) Users of the predicate latch onto relations that are *not* genuine part-whole, hence the predicate is used relaxedly and non-literally. The predicate, however, can detach from literal standards to various degrees, which depend on the degree of similarity between cases of the deviant relation latched onto (e.g. the subfield-field relation) and cases of genuine canonical part-whole. Indeed, if deviant and genuine relations are substantively similar, then part-whole vocabulary applied to the former relations departs only moderately from literal standards and, accordingly, we win a view that resonates well with the *sui generis*-ness theme. On this view, cases of scientifically informed part-whole are *sui generis* in that there is a class of conspicuously similar cases (the cases of genuine metaphysical part-whole) to which mereological vocabulary applies perfectly literally (compare: one can use the predicate '…is a fish' literally to pick out a taxonomically appropriate non-mammalian vertebrate, or one can use the predicate *highly* non-literally to pick out a seemingly apathetical person – a 'cold fish' – or, finally, one can use it *slightly* non-literally to pick out a whale. What decides between these possibilities, it would seem, is the conspicuousness and/or the relevance of similarities between satisfiers of the predicate).

---

[5] A metaphysical analogue is arguably Simons (1987: Ch. 5).

> This third possibility will occupy us towards the end of this chapter, where I notice that it compromises on the assessment of evidence for part-whole without core principles (e.g. Antisymmetry). Indeed, (iii) gives a moderate outlook of the evidence's effect against the principles and the metaphysical methods that sustain them (conceptual analysis, intuition assessment). Despite this, difficulties in articulating (iii) and the possibility to disarm arguments that downplay the evidence's impact suggests that we are better off with one of the less compromising varieties: (i) or (ii).

Each of (i), (ii) and (iii) accommodates the prominence of part-whole vocabulary in contemporary science, hence none entails that the latter is entirely illegitimate. Accordingly, these three options stand out as attractive interpretations of Ladyman and Ross' *sui generis* thesis. So construed, the naturalist worry conveyed by the thesis becomes a worry with admitting, in the case of (i), excessively abstract varieties of part-whole and, in the case of (ii), excessively abstract features of the unique part-whole relation. At least some items of physical interest – so goes the worry – would not stand to each other as part to whole and display these abstract features (ii) or instantiate abstract varieties of the part-whole relation (i). In the case of (iii), on the other hand, the worry is that we prefer an abstract but literal relation in place of conspicuously similar and scientifically informed rivals. At least some objects of physical interest would stand to each other sufficiently analogously to genuine parts and wholes, but lack the latter's abstracted metaphysical features.

2. The arch-rival of *sui generis*-ness

To shed more light on this tripartite interpretation of the *sui generis*-ness thesis, let us consider a common rival; that is, a view opposed most certainly by (i) and (ii), and potentially by (iii). I have in mind a doctrine most commonly associated with David

Lewis (1986a), as reported by Koslicki (2009: 21) in her excellent discussion of the *sui generis*-ness theme in *The Structure of Objects*:

> My objection [to the idea that there are several, non-standard senses of composition] is that I do not see by what right the operations are called *combining* operations. [...] If what goes on is unmereological, in what sense is the new one *composed* of the old ones? In what unmereological sense are they present in it? After all, not just any operation that makes new things from old is a form of composition! There is no sense in which my parents are parts of me, and no sense in which two numbers are parts of their greatest common factor; and I doubt that there is any sense in which Bruce is part of his unit set. [. . .]. [If the friend of "*sui generis* composition"] does insist that his unmereological composition is nevertheless composition, in a perfectly literal sense, then I need to be told why. Saying so doesn't make it so. What is the *general* notion of composition, of which the mereological form is supposed to be only a special case? I would have thought that mereology already describes composition in full generality (Lewis 1986a: 38-9).

On Lewis' view, entities combine as parts to wholes (or as composers to composites) in no way other than by customary 'mereology', by which I assume he means the classical extensional theory of mereology (CEM) he defends in a number of other works (notably *Parts of Classes*). Classical mereology (as Chapter 1 prefigured) portrays part-whole as a partial order and defines a composition relation via the principle: *y* is a sum of some *xs* $=_{df}$ every thing *x* that overlaps *y* overlaps some of the *xs*[6]. Besides, the proper part relation is understood in non-identity terms (*x* is a proper part of *y* $=_{df}$ *x* is a part of *y* distinct from *y*) and equipped with the Strong Supplementation axiom (STR).

Why is Lewis' position in the quote incompatible with (i), (ii), and potentially (iii)? As for (i) and (ii), it is because the part-whole relation of CEM is 'canonical (or 'standard'), in the last chapter's sense of satisfying the Antisymmetry postulate, defining a proper part relation that meets Weak Supplementation (WS) and a composition relation that meets Idempotence. Suppose I am right that putative cases

[6] For cognate definitions, see Varzi (2015: S. 4.1).

of part-whole holding between objects of physical interest clash with these canonical conditions. On view (i), we classify the latter as cases of genuine part-whole relations, distinct from canonical ones. Therefore, we resist Lewis' view by granting relations beyond the reach of CEM the status of part-whole relations: mereological pluralism.

On view (ii), we classify the physical cases of interest as cases of the one and only part-whole relation, whose governing postulates change in accordance with the *relata*'s scientific type. Standardly, however, CEM admits of no variation in the postulates governing its relations (part-whole, composition and proper part) in response to variation in *relata*-type. Rather, these postulates – including the strict order properties, STR and Idempotence – hold universally and not conditionally to types of *relata*. The part-whole relation of CEM is not allowed to contravene the Antisymmetry postulate, not even if its *relata* are physical vectors or fields that decompose in opposition to Antisymmetry (Chapter 3). The defined part-whole relation is not allowed to contravene Strong Supplementation, not even if it takes as *relata* a vector **a** that is apparently the only proper part of **2a** (Chapter 4). Accordingly, on view (ii) we maintain a monist standpoint on the number of allowed part-whole relations, but we still resist Lewis' doctrine as this single relation is not the relation of CEM.

What about (iii) – does it act in offence to Lewis' doctrine? The verdict depends on whether Lewis accepts or denies that certain relations can be more significantly analogous than others to CEM's part-whole; in fact, analogous enough to support a *slightly* non-literal use of the mereological vocabulary. The above passage does too little to decide on this matter. In fact, while Lewis rejects a "perfectly literal sense" of "unmereological" composition, he does not question the degree to which "unmereological" composition detaches from the literal standard.

Now, there is at least one case of a *sui generis* relation whose interpretation as a non-literal form of part-whole Lewis strongly opposes: the relation between a structural universal (e.g. *methane*) and its constituent universals (*carbon, hydrogen*)[7]. According to Lewis, the non-literalist about this universal-constituent relation makes co-instantiation between universals a brute necessity, no matter the *degree* of 'non-literalness' invoked[8]. Whence the derisory choice of vocabulary: the conception of

---

[7] Lewis' target is – notoriously – Armstrong's theory of structural universals in Armstrong (1978) and (1986). See (1997) for a later account.

[8] On the contrary, on a literally mereological ('pictorial') conception of the universal-constituent relation,

the universal-constituent relation as 'non-literally' mereological is a "magical" conception:

> Involving [i.e. having a constituent universal] [...] is a matter of necessary connection between the instantiating of one universal and the instantiating of another; and on the magical conception, the universals so connected are wholly distinct atomic individuals. Therein lies the magic. Why *must* it be that if something instantiates *methane,* then part of it must instantiate *carbon?* [9]

Should we accept the Lewisian scepticism on (iii)? Not straightforwardly. For strictly speaking my *sui generis* relation of interest is not the relation between constituent and structural universal but rather the relation between component vector and vector sums, together with the relation between the physical objects that vector components and sums represent in certain interpretations of advanced physics (we return to this selection of the evidence for *sui generis* part-whole in greater detail in Chapter 4, Section 5).

More importantly, even granted Lewis' reasons to oppose (iii), recall that (iii) suffers from separate, characteristic problems, which make it less preferable than (i) and (ii) to start with. The main attraction with (iii) is its friendliness to compromise. For on this view advocates of the Lewisian 'CEM only' doctrine can make sense of naturalistic evidence and the use of mereological labels in science without renouncing the elite position of CEM. Similarly, we could make friends of this evidence and the labels without renouncing the elite position of part-whole relations with features driven by conceptual analysis and intuition assessment.

However, the compromise's price might be too high. To gain any plausibility, (iii) must come with a distinction between *slightly* and *deeply* non-literal applications of

---

necessary co-instantiation is not brute "because *carbon* is part of *methane,* and the whole cannot be wholly present without its part" (Ib. 41). One could reply, however, that even wholes and parts partake of mysterious necessary connection (if composition is not identity), see e.g. Saucedo (2011), Cameron (2014).

[9] Hawley (2010) defends structural universals by construing their relation to the constituents as non-unique, non-extensional composition and then denying that brutely necessary co-instantiation occurs. On her view, non-unique and non-extensional composition is a literal type of composition (Cf. Bird and Hawley 2011: S. 5). This response denies the Lewisian doctrine, i.e. that entities relate by literal composition just in case they relate by CEM-composition (which is unique).

mereological vocabulary to the *sui generis* relation. This distinction also captures the sense in which a friend described as a 'cold fish' makes a *looser* metaphor than whales described as 'fish'. It takes much effort to articulate this distinction and difficulties abound (Yablo 1991), but without it (iii) risks triviality. For without resources to distinguish between highly and moderately non-literal talk, (iii) risks classifying as part-whole any relation whose instances we describe with highly figurative mereological vocabulary[10]. One such risky case is the relation of mutual loving holding between Juliet and Romeo, which we can couch in highly non-literal, even poetic terms by claiming that "Juliet is part of Romeo and Romeo part of Juliet" (Lando 2017: 23-5). The aim of the *sui generis*-ness thesis is to 'de-insulate' the part-whole relation of CEM and the canonical part-whole relations embedding them in a larger rubric, which features *sui generis* relations with naturalistic credentials[11]. We cannot afford, however, to extend the rubric to any relation supporting highly figurative mereological vocabulary without trivialising the project. But this is exactly the risk we run by accepting (iii).

Accordingly, (iii) faces not just a potential threat from 'magical' necessary connections, but also other complications, and notably complications in articulating the distinction between strong and loose metaphors. Combined with the possibility to resist the main objections to (i) and (ii) (Cf. Chapter 3) this observation makes the latter preferable articulations the *sui generis*-ness thesis. I would like the naturalistically eminent evidence for *sui generis* part-whole to disprove the Lewisian doctrine, showing that entities relate as parts to whole by means other than the CEM relation. And indeed I would also like the evidence to disprove a stronger doctrine, which states that entities relate as part to whole only by canonical part-whole relations[12]. However, in order to use the evidence for these purposes we had better interpret it in the most secure and least objectionable way: which is to say along the lines of (i) or (ii).

Having laid out this framework for understanding the *sui generis*-ness thesis, we can begin to address it. Our first question is not which *specific* evidence can be brought to

---

[10] See Chapter 3: S.3 for more discussion.

[11] Cf. Aizawa and Gillett (2017: 3-4).

[12] Once again, canonical (viz. standard) part-whole relations are relations that satisfy the Antisymmetry postulate, define proper part relations that meet WS and define composition relations that meet Idempotence.

the fore from science in favour of the thesis (understood as (i) or (ii)). Indeed, we will deal with this aspect in due course (Chapter 4). Rather, we ask what *type* of evidence sustains the thesis. Some evidence is significantly less momentous than some other and while in the hunt for evidence we should aim as much as possible at the desired prey (the momentous evidence), we will first quickly examine these lesser targets.

3. More on peripheral and core evidence

To distinguish the more from the less momentous evidence we are about to elaborate on our discussion of core and peripheral principles from Chapter 1 (Section 6). We agreed on labelling 'core' the principles delivered by the analysis of concepts and the assessment of intuitions. Further, the exact concepts we analyse and the exact resources we deploy to report the intuitions are fixed by certain independent assumptions: part-whole is reflexive, composition has 'lone' instances, and 'proper part of *x*' is defined as 'part of *x* and distinct from *x*'.

Accordingly, it holds in virtue of the reflexive concept of part – not another concept – that no two things are mutual parts unless identical (Antisymmetry); and it holds in virtue of the concept of composition with 'lone' instances – not another concept – that the sum of a single thing is that very thing and nothing else (Idempotence). Finally, we articulate the intuition that every proper part comes with a 'remainder' with the resources offered by the proper part predicate defined via non-identity[13].

This combination of assumptions fixes the analysed concepts and the resources for articulating the intuitions. It leaves us with a list of clear candidate principles which represent the output of conceptual and intuitive methods for theorising about part-

[13] We set aside the attempts to articulate the 'remainder' intuition with *proper part* differently defined. In particular, one could reject the Antisymmetry postulate for part-whole and embrace the 'mutual parts' definition of proper part (x is a proper part of $y =_{df} x$ is part of y and y is not part of x), as in Thomson (1998) and Cotnoir (2010). Donnelly (2011: 236-7) duly notes that this definition feeds a new decomposition principle, weaker than Weak Supplementation (WS), which articulates the 'remainder' intuition. The principle is identical to WS in syntax but contains a different primitive for *proper part*. For reasons of brevity, I keep my focus on WS and maintain that this principle, not the alternative, canonically conveys the 'remainder' intuition (see Section 6 for more about this exclusion). I suspect, however, that the naturalistic evidence presented in opposition to WS (Chapter 3, 4) also challenges the alternative.

whole: Antisymmetry, Weak Supplementation, Idempotence[14]. By setting *these* principles as the target of the naturalistic evidence, we also appoint the evidence with a clear aim: casting into doubt the foundational methods, i.e. conceptual analysis and intuition assessment, which deliver the candidate principles.

This 'casting into doubt', which we called 'amendment', opposes another effect – 'accommodation' – occurring when the evidence's target is not core but rather peripheral (Section 7). By *accommodating* some evidence, we drop peripheral principles and, concurrently, we keep pursuing the methods that gather other principles into the core. For a contrast, by using the evidence to *amend*, we raise trouble not just for the threatened principles but also for the methods that produce them. In particular, on the liberal approach to amendment I defended in Chapter 1 we can envisage three possible effects: (a) the amending evidence shows that the methods (conceptual analysis and intuition assessment) fail to reveal principles true at the actual world; or (b) it shows that practitioners of the methods misperceived the threatened core principles as their deliverances; or finally (c) it shows that, although the methods output principles true at the actual world, in the subject matter of advanced science figure not genuine part-whole relations but rather only idiosyncratic relations very similar to part-whole.

We now make a further step towards isolating the principles that, if targeted by the evidence, threaten one of the amending effects (a), (b) and (c). Notice that it *suffices* for entrance in the 'core' that a principle analyses one of the concepts *part-whole* or *composition*[15] or articulates an intuition (e.g. the 'remainder' intuition) with the resources we have selected ('proper part' defined via non-identity). Accordingly, perhaps one may imagine principles that manage to enter the core, are threatened by some physical evidence, and yet their being threatened by the evidence fails to challenge the conceptual and intuitive methods according to (a), (b) and (c). Present any such principles, our aims of amendment will only be met by narrowing the focus: that is, finding evidence in opposition not just to any principle that manages to enter

[14] These are far from exhausting the list of principles representative of such methods, but they give us enough to elaborate on.

[15] Whereby we assume (1) that it holds conceptually of *part-whole* that every x is part of x (the Reflexivity principle) and (2) that it holds conceptually of composition that there are composites of 'lone' (i.e. singular) things.

the core, but to those among them that we 'irrevocably' need for pursuing the conceptual and intuitive methods.

An example will illustrate. Among many other case studies, in "Physical Composition" (2013, S. 18, 19) Healey worries about articulating the part-whole relation in the way preferred by *some* versions of classical mereology. For on these articulations, the part-whole predicate cannot be interpreted in terms of the system/sub-system relation of orthodox quantum mechanics[16]. In fact – so goes the argument – it suffices to look at the Schrödinger equation for a hydrogen atom to challenge the decomposition principle that goes by the name of Complementation:

> If *x* is not a proper part of *y*, then there is some *z* whose proper parts are all the proper parts of *x* disjoint from *y* (Varzi 2014: S. 3).

We cannot expect to give full justice to Healey's detailed case, so we will keep the presentation constrained to highlight our methodological point of interest. Very briefly, fuelling the argument against Complementation is the fact that aspects of the Schrödinger equation recommend different decompositions for the atom: multiple decompositions. The equation's *linear decomposition* features wave-functions for the electron and proton that we would normally take to exhaustively compose the atom. The equations' *solutions* feature wave-functions for two composite systems of an electron and a proton, one described by relative position coordinates and another described by centre of mass position coordinates. For Healey (for reasons I will not delve into), all four subsystems are distinct and disjoint[17]. Hence it follows contra

[16] Healey's thesis *differs* from Maudlin's in (1998, 2007). Maudlin occasionally charges classical mereological part-whole as a poor interpretation of the orthodox system/sub-system relation. However, at a closer look (Calosi, Fano and Tarozzi 2011) his target is the thesis of mereological supervenience, which states that all whole-properties supervene on some property of the whole's composers (or on some property of one of the whole's parts). The non-supervening properties are (expectably) the whole's non-factorisable states. However, absent composition as identity (S-CAI) mereology *per se* is not committed to the modal relations between whole- and parts-properties and does not entail mereological supervenience. Thus challenging the latter is not challenging mereology as a theory of the system/sub-system relation. For the falsity of composition as identity in an orthodox quantum context, see Calosi and Morganti (2016). Finally, notice that one need not construe the occupation of non-factorisable states as properties of *composites*, see Teller (1998).

[17] Moreover, each sustains distinct explanatory purposes. It is not clear whether Healey considers both decompositions real parts of the atom, or, more radically, considers the existence of a certain decomposition

Complementation that "[t]he hydrogen atom is not a subsystem of the proton, but there is no subsystem of the hydrogen atom whose subsystems are just the electron, the centre-of-mass position subsystem and the electron-proton relative position subsystem" (Ib. 56-7)[18].

Having briefly examined the evidence's nature and its impact against Complementation, let us keep these in mind in returning to our point of method, i.e. that evidence of this kind is insufficient for the naturalistic aim of amendment. As a decomposition principle, Complementation voices (in its own way) the intuition that proper parts come with remainders[19], which suffices for making it a core principle. However, even if we were to reject Complementation owing to Healey's evidence, arguably the principle's failure would not provide for a deep challenge (of the amendment type) against the method of intuition assessment *per se*.

Let me explain. The evidence would – if successful – manage to undermine Complementation in relation to the method. It would show, in accordance with the conditions for amendment, that (a) Complementation is a product of the method with no worldly status; or that (b) Complementation has been misperceived as a product of the method; or finally (c) that the orthodox state/sub-state relation[20] is *sui generis* and defeats understanding in terms of literal part-whole, which obeys Complementation. However, the evidence would not suffice for any of the following amendments:

> (a)' Intuition assessment *per se* is incapable of delivering *any* decomposition principle that complies with the 'remainder' intuition and states a worldly truth;

relative to a certain present explanatory purpose. The latter alternative resonates well with his renowned pragmatist approach to scientific ontology, which he invokes elsewhere in his paper (1, 7-8, 26).

[18] Incidentally, notice Healey's assumption that nothing but one of the deliverances of the Schrödinger's equation (one of the four sub-systems) is a remainder of the atom relative to the proton, from which he concludes that there is no such remainder. This seems to me another case of the naturalist jumping from *sui generis*-ness to eliminative conclusions. It is one thing to argue that examining the equation reveals four subsystems rather than the usual two, quite another that this entails that *there is* no sum of three out of the four subsystems. At this stage, I ask my reader to notice a meta-ontological 'gap' between the two theses: questions as to the values of parsimonious ontologies, or of the ontological innocence of such sums may come in between to revert the verdict. These questions will concern us in the second part of the thesis.

[19] Cf. Simons (1987: 88).

[20] More precisely: the 'proper' version of the state/sub-state relation, which we define via non-identity as 'substate of *x* distinct from *x*'.

(b)' The doctrine that proper parts come with remainders was never a product of intuition assessment. Rather, the assessment's output has been misperceived.

(c)': The orthodox state/sub-state relation is idiosyncratic and resists understanding in terms of literal part-whole. For only the latter, but not the former, satisfies principles capable of expressing the 'remainder' intuition.

Thus the naturalistic evidence acts 'locally' against Complementation, but not 'globally' against intuition assessment. Accordingly, it misses an opportunity for revealing that it was initially a bad idea to capitalise on this intuition for worldly truths; or for revealing that the intuition's content was misperceived; or finally for revealing that idiosyncratic relations of physical interest – highly similar to proper part – obtain without remainders.

Why does the evidence fail to act globally in this way? The reason was shortly anticipated in Chapter 1 (Section 6) and it is that, usually, principles other than Complementation are held accountable for articulating – to a satisfying degree – the intuition that proper parts come with remainders.

If the weaker principle (STR) succeeded in articulating the intuition to a satisfying degree, then the failure of its stronger relative (Complementation) would bear no global impact in the above sense[21]. Strategies would still be in place for metaphysicians who traffic in intuitive methods to capitalise on the 'remainder' intuition for worldly truth while rejecting Complementation[22].

Similar remarks hold for the principle of Weak Supplementation (WS: if x is a proper part of y, then y has a proper part z distinct and disjoint from x), which

---

[21] Though it would leave us without the consequence that all pairwise disjoint parts x, z of some whole y have a sum, see Varzi (2014: S. 3.3).

[22] For a picture of the state/sub-state relation faithful to strong supplementation, but not complementation, see Calosi, Tarozzi and Fano (2011). Their aim is to construe orthodox quantum systems as sums that respect the principle of Extensionality (two sums are identical iff they have the same parts), which follows from Strong Supplementation (Ib. 1748 and Varzi 2014: 3.2). Their critical targets are Maudlin's (1998) no go arguments for Extensionality of orthodox quantum systems. As already seen (Fn. 7), Calosi, Tarozzi and Fano's complain that Maudlin's argument is best construed as attacking mereological supervenience, not Extensionality.

(granted the Antisymmetry postulate) Complementation and Strong Supplementation (separately) entail. Again, if this weaker principle (WS) reported the 'remainder' intuition to an acceptable degree, then strategies would still be in place to capitalise on the intuition for worldly truths, even granted naturalistic evidence against Strong Supplementation[23].

When it comes to decomposition principles with some claim to advance the 'remainder' intuition, Weak and Strong supplementation are far from exhaustive samples[24]. In fact – and admittedly – my main reason for putting them forward as obvious alternatives to Complementation is simply that they enjoy continued popularity[25]. Strong Supplementation features in a canonical axiomatisation of classical mereology (CEM)[26], while Weak Supplementation was influentially defended by Peter Simons (1987) as the *minimal* rendition of the remainder intuition (we will return to this claim in a moment)[27].

Having hinted at this vast decomposition landscape, we may now return to our naturalistic methodology of 'amendment' and, in the next section, ask for principle(s) absent which we fail to articulate the remainder intuition acceptably. Identifying these principles matters, for, as just seen, only evidence opposing *them*, as opposed to their weaker relatives, guarantees the expected 'global' amendment.

---

[23] In this case, the evidence would leave us without a canonical formulation for classical mereology (CEM, see Varzi 2014: S. 2.4) which includes Strong Supplementation as an axiom.

[24] See Hovda (2009), Gilmore (2009) and Donnelly (2011), among others.

[25] See Donnelly (2011: Fn. 1) for further references.

[26] This appears in Simons (1987), goes by the name of axiomatisation 'from below' and features STR together with the Antisymmetry, Reflexivity and Transitivity postulates. The (equivalent) axiomatisation 'from above' (as given e.g. in Lewis 1991) has only Transitivity, Unique Composition (If any xs compose some y, then they compose no other z distinct from y) and Unrestricted Composition (for all *xs*, there is a sum *y* of the *xs*). For this helpful distinction between axiomatisations from 'above' and 'below', see Lando (2017: 35-7).

[27] Though Donnelly (2011; Fn. 12) observes that Simons occasionally holds onto a weaker principle in (1987: 177-95). The principle's syntactic form is just the same as WS', but a new definition of 'proper part' is concealed: *x* being the proper part of *y* is now defined as '*x* is part of *y* and *y* is not part of *x*'.

4. The ideal decomposition principles

Landmark pages from Peter Simons' *Parts: A Study in Ontology* (1987: Section 1.1) offer a standard response to the question of which decomposition principles are minimally necessary to articulate the 'remainder' intuition ('every proper part has a remainder'). These minimal principles (p) articulate the intuition in such a way that, by virtue of satisfying (p), the *proper part* relation becomes formally *unlike* other relations, which evidently do not count as literal varieties of proper part[28]. In other words, (p) filters out interpretations of the proper part relation into domains ordered by relations evidently different from it.

To cast this argument, we first need to assume that proper part is *asymmetric* (if x is a proper part of y, then y is not a proper part of x) and *transitive* (if x is a proper part of y and y a proper part of z, then x is a proper part of z), so that it forms a strict partial order[29]. Simons argues that by adding WS to these order properties we disable interpretations of *proper part* into domains of strictly ordered objects, which resist classification as literal proper parts. In fact, these unwanted interpretations feature objects ordered by a relation R, which fails a principle corresponding to WS:

(WSR) If $x\mathrm{R}y$, then for some z, $z\mathrm{R}y$ and $z \mid_{\mathrm{R}} y$.

where '$\mid_{\mathrm{R}}$' is the discreteness relation defined in terms of R ('$x \mid_{\mathrm{R}} y =_{\mathrm{df}}$ there is no z, such that $z\mathrm{R}x$ and $z\mathrm{R}y$)[30].

All relations R that fail (WSR) can order domains in which at least some particular x bears R to y without a distinct and discrete z bearing R to y. For one such domain, Simons invokes the natural field (the set of all natural numbers) and for the particular relations R that intuitively fail to interpret *proper part* in the natural field, he considers the comparative *strictly smaller than*, and the algebraic *proper divisor of*.

---

[28] Cf. Koslicki (2008: S. 2.7.1) for endorsement.

[29] Irreflexivity (no x is a proper part of x) follows from the definition of proper part, which we still assume to be in terms of non-identity: proper parts are parts distinct from the whole.

[30] We shall use 'discrete' for the correspondent of the 'disjoint' relation defined on R, see Donnelly (2011: S. 2)

As defined on the naturals, these relations both fail (WSR). As for the comparative relation (*strictly smaller than*), one natural number *x* (e.g. 2) can be strictly smaller than another *y* (e.g. 4) without there being a natural *z* distinct from the first, strictly smaller than the second and such that no number *w* is smaller than or equal to both. Indeed, any natural *z* we may consider for the role of weakly supplemented remainder fails the condition for disjointness. Some number, and namely 1, is invariably smaller than or equal to every natural, including z and its purported remainder. Similarly, no pair of distinct naturals is discrete in respect to the algebraic relation (*proper divisor of-*), for any naturals pairwise taken have a common divisor in 1.

Next – says Simons – if part-whole lacked (WS) in addition to the three strict partial order properties, then nothing would prevent us from thinking of the naturals as relating – literally – by proper part. In this interpretation, 1 could be a proper part of every other number, and every number has as proper parts all numbers strictly smaller than it. This he finds offensive to the 'remainder' intuition (Ib. 27-8). Putting this together, it suffices for a certain decomposition principle (p) to articulate the intuition legitimately that (p) figures in the order properties of *proper part* but its correspondent fails for *strictly smaller than* and *proper divisor of*. All principles sustaining (p) as an addition to their order properties express the intuition acceptably, to some degree[31].

Looking back at our dialectic, let us secure a take home message out of Simons' argument. As just seen, the argument gives a sufficient condition on principles that articulate the intuition acceptably. These are, namely, the principles without correspondents in the order properties of *strictly smaller than* and *proper divisor of-*. The latter relations are clearly not mereological and, according to Simons, the ideal construal of the reminder intuition should not allow to portray them as such. Thus the argument also works as a filter for principles that *fail* to articulate the intuition acceptably. These are – of course – the principles that have a correspondent in the order properties of the target unmereological relations[32].

---

[31] In fact, Gilmore (2009) observes that to cross out Simons' unintended interpretation we need – strictly speaking – a principle weaker than WS. This goes by the name of Quasi-Supplementation (QS): 'if *x* is a proper part of *y*, then some two disjoint *z* and *w* are proper parts of *y*'.

[32] Such as the principles of Company and Strong Company, mentioned in Chapter 1 (S. 8.2). See Simons (1987: 26-7) and Varzi (2014: S. 3.1).

Now, suppose we accepted with Simons that all relations formally analogous to *proper divisor of* or *strictly smaller than* (with no order properties to make a structural difference) fails to articulate the intuition. We would then know what kind of evidence best challenges *any* intuition-friendly decomposition principle, in accordance with the project of 'global' amendment. This would be, namely, evidence for literal proper part relations in science, which are formally analogous to *proper divisor of* or *strictly smaller than*. This method is attractive and comes at the cost of pinning down the content of the 'remainder' intuition in Simons' preferred way. By accepting so much and holding fixed Simons' understanding of what it takes to articulate the intuition acceptably, we earn great advantages in regard to identifying the deeply challenging evidence. In our quest for the 'impactful' evidence, we would know where to go.

So much seems plausible to me and – indeed – in picking out the 'deeply challenging' evidence (Chapters 3, 4) I will by and large hold fixed Simons' understanding of the intuition. Having said this, the 'remainder' intuition remains just that, an intuition, suggesting that it cannot be a fully uncontroversial matter to divide the decomposition principles exactly on Simons' lines. Perhaps some could uphold the intuition while putting forward models of proper part analogous to models of *proper divisor of*[33]; or even *strictly smaller than*[34]. Accordingly, although we will select the challenging evidence with Simons' division in mind, we need an emergency strategy to deal with the possibility that Simons misassessed the intuition and erred in pinning down the principles that articulate it.

Simons could err by rounding up or down. If he errs by rounding up, then the acceptable articulators (the principles that articulate the intuition acceptably to some degree) incorporate more features than the three strict order properties (Irreflexivity, Transitivity, Asymmetry) and WS[35], because there are models with WS formally analogous to non part-like relations. In this case, I insist that picking up evidence

---

[33] One such model is a universe in which all objects have at least two proper parts (hence there are parts all the way down) and any parts of an object, taken pairwise, share a part, see Simons (1987: 27).

[34] Here the model is a universe all of whose objects have one single proper part: say, the universe, its only proper part, the only proper part of this only proper part, and so on indefinitely. See Simons (Ib.). As *proper part* is transitive (by assumption) and in this model chains of proper part do not terminate, every object has at least two proper parts.

[35] Or (QS), in Gilmore's (2009) rendition, see Fn. 23.

against WS still makes for a good degree of challenge to the intuitive method, even if WS is now ruled out from the class of acceptable articulators.

This is because *most* upholders of principles 'stronger' than WS also accept WS itself (here I understand a principle's 'strength' as inversely proportional to the number of models it is compatible with: the stronger principles admit fewer models than WS, for there are models that only WS, but not the stronger principles admit). For example, all subscribers to CEM[36] rule in Strong Supplementation (STR) together with the Antisymmetry postulate, which jointly entail WS. It is natural to presume that these subscribers will face the same 'global' damage to intuition-friendly principles (STR) present informed evidence against WS. The presumption, which Chapter 4 confirms, is that the challenge against WS transfers to all principles that entail it.

Besides, STR delivers WS granted only the addition of Antisymmetry and, as Chapter 1 remarked, Antisymmetry remains a popular candidate for a conceptual truth (about the concept of part-whole with reflexive instances)[37]. Accordingly, we can expect evidence against WS to impact on a larger group of subscribers to STR – beyond those who subscribe to the full-force extensional package. Owing to the evidence's wider target, presenting this as opposed to other evidence remains a reasonable policy, even if WS fails to articulate the 'remainder' intuition appropriately.

For a contrast, we need to be more concessive if Simons errs by 'rounding down'. In this case, a principle (p) can acceptably articulate the 'remainder' intuition even if the proper part relation that figures in it is formally analogous to the two 'filter' mathematical relations: *strictly smaller than* and *proper divisor of*. Accordingly, all

[36] That is, CEM, whose postulates are in Fn. 18.

[37] Bacon and Cotnoir (2012) deny the Antisymmetry postulate and maintain Strong Supplementation (at the price of taking a transitive, reflexive and symmetric *proper part* as the main primitive). On this view – a 'non-wellfounded mereology' that admits of proper part 'loops' – we can hold onto STR without worrying about counterexamples to WS. Indeed, one cannot derive the latter from the former principle without the Antisymmetry postulate. This view already buys into a great deal of conceptual revisionism by dropping Antisymmetry, as well as the irreflexivity and asymmetry of *proper part*. Accordingly, a friend of 'global' amendment could argue as follows. If theirs were one of the proper part relations portrayed in advanced science, then we would achieve amending effects 'elsewhere', i.e. on the Antisymmetry postulate and the order properties of proper part, albeit not on the principles that articulate the remainder intuition. This conditional claim is correct: the membership of Cotnoir's and Baron's proper part relation in the deviant relations of advanced science certainly guarantees some amendment. The antecedent, however, is false. Some proper part relations evidenced in advanced science, as we will see (Chapter 3, 4), also offend against Baron's and Cotnoir's version of STR.

evidence of physical proper parts with the bare structure of *smaller than* or *of proper divisor of* (as these are exemplified by the naturals) would still prompt us to drop stronger principles like WS. However, the evidence would not achieve global amendment, for now the replacing decomposition principles[38] would articulate the intuition appropriately.

We need, therefore, a concessive move. We grant that the intuition may be articulated by principles whose correspondents are satisfied by the 'filter' relations (this is a genuine possibility, given difficulties with pinning down the intuition's content). We also grant that in this scenario evidence opposing WS would not amend the method of intuition assessment according to one of the above (a), (b), or (c), i.e. (a) show that its deliverances fail to qualify for worldly truths; (b) show that the intuition's content has been misperceived; (c) or speak for *sui generis* physical relations in opposition to the intuition.

However, granted the concession, there remains some rationale to keep pursuing evidence in opposition to WS, rather than evidence in opposition to the new acceptable articulators of the 'remainder' intuition. In particular, by indicating physically interesting proper parts that behave in offence to WS, then we still earn an effect close enough to (b): i.e. showing that the content of the 'remainder' intuition has been misperceived. For WS is perceived *often enough* as a principle appropriate for articulating the intuition[39]. Accordingly, our evidence will impact these frequent perceivers prompting a change of resources (away from WS). It still holds as a matter of intuition that proper parts come with some 'remainder', and this intuition still has unchallenged worldly value, but what we thought to be an adequate expression of the intuition (WS) we now reconsider.

---

[38] E.g. Company and Strong Company (Fn. 24).

[39] Even authors who appoint it a conceptual status (e.g. Varzi 2007, Koslicki 2008: 167) also, occasionally, endow it with intuitive pull (Varzi 2015: S. 3.1; Koslicki 2008: 183, Fn. 24). See Chapter 1 (S. 6.3) for more discussion.

5. Peripheral and core evidence: a summary

Having now distinguished between the kinds of evidence that hold against mereological principles, we can refer back to Ladyman and Ross' *sui generis*-ness thesis to check our progress in articulating it.

I have first outlined (Section 1) what I think the thesis should be taken to entail: i.e. that capitalising on the evidence that opposes popular mereological principles we choose one of the following. First (i), we may separate out various relations, all literally classified as 'part-whole', some of which are *sui generis* and offend against the popular principles. Second (ii), we may stay with a unique part-whole relation, whose formal properties vary according to the *relata*'s scientific category and, for certain categories (see Chapter 5), clash with the popular principles. Third (iii), we may introduce various idiosyncratic relations, which clash with the principles and sufficiently resemble genuine varieties of part-whole. Among these options (iii) faces difficulties in articulation, so I have suggested we capitalise on the evidence for *sui generis* part-whole along the lines of (i) and (ii), setting (iii) aside.

Following this, I have clarified what *type* of evidence that we should expect to guide us towards one of these two consequences: evidence of the 'globally amending' type. In light of this evidence, one can challenge the methods that deliver the target popular principles in one of three ways (Section 3): by denying that the methods' deliverances (the principles) are worldly truths; by reconsidering what counts as a deliverance of the methods; or finally by holding onto the deliverances and their worldly status and let the evidence speak 'only' for principles of idiosyncratic relations closely resembling part-whole.

Following onto our progress so far, the question we now turn to is whether there can actually *be* evidence of this globally amending type, capable of sustaining the *sui generis*-ness thesis construed as one of (i) or (ii).

The answer is not as trivial as one may initially expect. In particular, it does not suffice to dig into accredited science for accounts of interesting objects that *at first glance* relate by part-whole in opposition to the canonical principles, e.g. with instances of symmetric part-whole, or of proper part without WS. These 'first glance' impressions could still underachieve spectacularly and fall short of proving that the interesting objects relate by genuine part-whole relations in accordance with (i) or (ii). To put it differently, perhaps there are reasons to interpret much (even all) of the evidence putatively against the core principles as insufficiently strong to produce an impact as strong as (i) or (ii). At the time of writing, it seems to me that this topic has received comparatively little attention, with only some arguments for this negative conclusion being adaptable from Fine (2010) and, more recently, Lando (2017) and Bennett (2017)[40]. The need for more arguments is urgent, for as I proceed to show, none of these adaptable proposals manages to meet the goal.

[40] Clarification. Certainly there has been wide discussion of which principles of part-whole and composition one could sacrifice in response to scientific evidence. Notably, but among other cases, one can read Armstrong's theory of structural universals in (1986, 1997) as an attempt to motivate non-canonical cases of part-whole from biochemistry. Where C and H are respectively the universals *carbon* and *hydrogen*, Hawley (2009) argues that the composition of C and H fails Uniqueness, so that as Armstrong expects, it delivers both CH and CH_4. Bennett (2013) maintains Uniqueness but argues that we should replace part-whole with a previously unknown mereological primitive to allow that H can be part of some whole 'multiple times over' (see Chapter 4 for more detail).
Here is how my present aims differ: Hawley and Bennett ask which principles (alternative to full-force mereological rivals) guarantee that CH and CH_4 are both composites of C and H. What we do here is assess *how* renouncing or changing principles impacts the part-whole relation they govern (is it still genuine part-whole? Is it a variety of part-whole?) and the methods we putatively follow to posit such principles (are they truth-guiding, do we perceive their deliverances correctly?) These questions are vividly revived by Fine (2010) and followed through more recently by Sattig (2016: Ch. 1) and Lando (2017).

III

Can there be evidence against the core? Yes

1. Introduction

By disarming some major objections, this chapter shows that it is *possible* for scientific practice to evidence highly deviant cases of part-whole – particularly, cases deviating from core conceptual and intuitive constraints on the very notion of part. For illustration, I will keep the focus on the Antisymmetry postulate (if x is part of y, then y is part of x only if $x = y$); and therefore ask whether it is possible to scrutinise scientific practice for objects of interest, which combine to clash with Antisymmetry by *genuine* part-whole relations.

The sceptics agree that no matter how many ways there are for entities to stand as parts to wholes[1], no entity can so relate to some distinct thing, which in turn relates as part to whole to it. In the following, we keep various sources of scepticism under focus, each articulating reasons for blocking the allocation of the *sui generis* cases to

[1] Drawing from the last chapter, we can now give the phrase 'ways of relating as part to whole' two interpretations: (1) pluralist; and (2) monist *cum* variation in governing principles. On a pluralist interpretation, for two entities to relate as part to whole in a particular way is for them to relate by a distinctive relation of part-whole (a *variety* of part-whole). Two other entities relating as part to whole in a different way also instantiate a different relation. On a monist interpretation, two entities that relate as part to whole in a particular way instantiate the only part-whole relation, but the relation's governing principles are sensitive to the related entities' kinds (see Chapter 2, Section 1).

the rubric of part-whole. Before differentiating their various challenges, it will be helpful to notice what none of the sceptics claim. When asked why we should not appoint the *sui generis* cases part-whole status, the sceptic we consider never replies that doing so evidences a 'misunderstanding' of the very notion of part-whole. More precisely, this objector concedes that solely misunderstanding the notion does not originate a sceptic challenge.

Diversifying the objections away from simple 'misunderstanding' is fortunate for the sceptic. For considered *per se*, misunderstanding the part notion counts little against the worldly claim that some objects of interest relate by deviant (e.g. symmetric) part-whole. As I have suggested in Chapter 1 – endorsing Dowe's insights – one capitalises on 'non-misunderstood', canonical notions of 'part' only for defeasible hypotheses circa the relation's worldly profile. On the contrary, if in forming our deviant notion we surveyed eminent scientific practices, then the notion's deviant character ceases to matter. The defeasible anti-symmetric hypotheses are dropped and the relations with worldly instances include symmetric part-whole[2].

So how do the sceptics distinguish their objections from first-pass claims of 'misunderstanding'? Most often, the sceptic's challenge capitalises on the negative consequences we face if we *concede* that the *sui generis* physical relations are genuine part-whole relations. For the sceptic, this concession comes with bullets hard to bite, as by endowing these *sui generis* cases with genuine part-whole character:

1. We renounce a general relation with cohesive governing principles and, accordingly, an explanation given in terms of the relation's cohesiveness of why familiar judgements about what is part of what never offend against the principles (Fine 2010).

2. We create a formal mismatch with paradigmatic cases of part-whole (Lando 2017).

[2] Further, having agreed that eminent naturalistic evidence disarms the defeasible hypotheses, the next step is deciding on the evidence's impact on conceptually-driven 'core' principles (Chapter 1: S. 7.2).

3. We make it "impermissible to deny" that any *sui generis* case is a case of part-whole (Bennett 2017).

These refined challenges keep us occupied throughout the chapter. They obtain independently of the degree to which we understand the part-notion when we classify the *sui generis* cases as instances of it. In particular, they concede that we can base our understanding directly on the *sui generis* cases[3] (including physically advanced cases) rather than on the canonical, conceptual and intuitive take on part-whole.

The focus is on the price to pay, which according to the sceptic is a deficient application of attractive *virtues*: diminished explanatory power (Fine), diminished unification under paradigmatic cases (Lando) and allowance for *ad hoc*-ness (Bennett).

It is important to stress this virtue-theoretic nature of the sceptics' challenges. For guiding my allocation of the *sui generis* cases under part-whole is itself a virtue: namely, the naturalistic virtue that promotes metaphysical theories (of the nature of part-whole) which incorporate scientifically eminent contents (*sui generis* part-whole cases in opposition to core principles). My disagreement with the sceptic – accordingly – can be construed as a competition of virtues: a naturalistic predisposition presses us to construe part-whole flexibly enough[4] to incorporate the *sui generis* cases. The sceptics emphasise additional factors, which counterbalance the naturalistic drive. Thus, for the sceptic we may be able to incorporate the scientific content and construe part-whole 'flexibly', but we should not give in to *ad hoc*-ness or give up wanted explanations and unifying aspects.

My preference for the naturalistic inclination leads me to incorporate the *sui generis* cases in the rubric of part-whole. However, I deny that the naturalistic inclination alone prevails just by virtue of being 'naturalistic' and I accept that one can occasionally deny authority to scientific evidence (here: speaking for *sui generis* part-

[3] See Bennett (2017: 48).

[4] Again (see Fn. 2), we can understand 'flexible' construals of part-whole either as pluralistic construal (there are multiple relations of part-whole, some of which relate the objects in the *sui generis* cases); or as monistic construal *cum* postulate-variation (there is only one relation of part-whole, but the principles governing it are sensitive to the types of related objects).

whole) in exchange for better scores on additional virtues[5]. Accordingly, the sceptics' case should be taken to heart and given an articulated response.

To put it differently, we can only motivate our alignment with the naturalistic injunction (accommodate the *sui generis* cases in a theory of part-whole) by first defeating the sceptics on their own ground. In this chapter – accordingly – we present each sceptic with one of two challenges: either (1) the virtues they promise to enhance deserve no enhancement[6] (e.g. there is no need for additional explanatory power, coherence with paradigmatic cases, etc.); or (2), contrary to the sceptics' own prediction, these virtues can still be enhanced within a naturalistic framework, which classifies *sui generis* cases as cases of part-whole.

2. An argument from best explanation

Our first candidate for an argument against genuine evidence in conflict with Antisymmetry is best illustrated by Fine in the following passage (2010: 581):

> It [anti-symmetry] provides a key test for our having a coherent conception of part in the first place. For under the pluralist approach, we have wished to maintain that there are many different ways in which one object can be part of another (through membership, subset, mere part, and so on). But what assurance can we have that our judgements in all of these cases are informed by a single coherent conception of part? A key – perhaps the key test of coherence is that the resulting general relation of part should be anti-symmetric. For it would be too much of a coincidence, so to speak, if anti-symmetry held even though there was no single coherent conception of part in virtue of which it could be seen to be hold.

[5] This flexible understanding of metaphysical naturalism is a dialectically useful assumption, because it is not to my advantage.

[6] I operate with an epistemic understanding of virtues. On this view, the sceptic expects certain virtues to be enhanced (explanatory power, unification, etc.) because s/he believes that a metaphysical theory (of the nature of part-whole) is more likely to be true if equipped with these virtues. Again, this is a useful assumption, because it is not to my own advantage.

Suppose that, like Fine, you admit a number of distinct part-whole relations (this would make you a mereological pluralist). You might wonder about the features to ascribe to a relation, *general part-whole*, that applies every time two *relata* stand in some variety or other of part-whole. Following Fine, call general part-whole *coherent* just in case, for at least some formal property F, all cases of general part-whole instantiate the formal property. If the formal property in question is Antisymmetry, as Fine believes, this condition reads that general part-whole is coherent just in case all of its instances are antisymmetric. Admit even only one non-antisymmetric case of a part-whole variety and general part-whole will shift to incoherence.

Fine's argument puts constraints on the defensible *sui generis*-ness theses and on the impact of putative scientific evidence against Antisymmetry. As for *sui generis*-ness, if general part-whole ought to be coherent, then the evidence cannot prompt introduction of a variety without antisymmetry or introduction of non-antisymmetric cases for the one genuine variety of part-whole. As for evidential impact, the coherence of the general relation entails that no variety of part-whole has antisymmetric instances. Accordingly, there is no way of using the evidence to threaten the status of Antisymmetry as a core principle. It is perfectly legitimate to maintain that Antisymmetry is a well-perceived deliverance of truth-guiding metaphysical methods (conceptual analysis, intuition assessment, etc.).

Is the incoherence of general part-whole *by itself* a reason to refrain from symmetric cases? Presumably not. It is far from evident that coherence gains in plausibility by just being defined as uniformity of formal properties across instances. There are, indeed, general relations whose incoherence we quietly accept. The binary *looking at* obtains every time somebody looks at some thing in one or another distinctive way (angrily, compassionately, etc.). Clearly some instances of *looking at* are symmetric: instances that obtain when the thing looked at is capable of looking back in the same distinctive way and so does. Moreover, cases of looking where the looker stands by a mirror are also reflexive, and cases where this mirror reflects the image of a second looker looking at a third are transitive. Thus there would seem to be viable instances of *looking at* with and without various formal properties (antisymmetry, transitivity, reflexivity).

Similarly, the binary *loving* is a general relation that obtains every time somebody loves somebody else in one or another distinctive way (passionately, Platonically, etc.). General *loving* admits of symmetric and antisymmetric instances: all cases such that the loved one loves the lover back in the same way. It admits of reflexive and non-reflexive instances: cases such that the lover loves or deplores (doesn't love) him- or herself. And it obviously admits of transitive and intransitive instances: those such that the lover also loves the loved's loved ones. In sum, when it comes to *looking at* and *loving*, incoherence (unsystematicity of formal properties across instances) is no source of concern. Why would it be when it comes to general part-whole?

Perhaps we should not value coherence *per se*, but rather think of it as the best explanation for the fact that all *familiar* instances are antisymmetric[7]. This proposal makes better sense of Fine's tone in the quote. He urges us to look for a collection of features of general part-whole to illuminate our "judgements" about particular cases. These particular cases include (but are not limited to) the relation between a pint of beer and a quarter of it, a set and its sub-sets and also (for Fine) a singleton and its unique member (Cf. Ib. 562), all of which (according to Fine) we familiarly deem cases of part-whole[8]. But no familiar judgement – Fine could add – ever deems two distinct objects mutual parts. Rather, the available judgements form a coherent 'rubric', a well-defined class of instances of part-whole (between the set and the subset, the pint and the quarter, etc.) none of whose members offends against Antisymmetry. Putting this together, the best explanation of why familiar judgements converge on this rubric, never sanctioning a case of mutual parts between distinct objects, is that the general relation and all varieties of part-whole are indeed coherent in respect to Antisymmetry.

The efficacy of best explanations in metaphysics raises issues we cannot expect to cover in full depth. Let us stay, therefore, with the argument's non-abductive premise that our rubric of familiar judgements is coherent; that is, that we classify a large array of familiar cases as cases of regular, canonical part-whole, without ever classifying a familiar case as one of deviant (e.g. symmetric) part-whole.

The truth of this premise hinges on the interpretation of 'familiar'. For example, the premise fails on an interpretation, according to which the assessed cases are 'familiar'

[7] Koslicki (2009: 82) has additional reasons, not based on explanatory power, to value coherent part-whole. To these we return in the next section.

[8] With Lando we can add (2017: 18): (1) the relation between a parliamentary faction (the Kemalists) and the Parliament; (2) the relation between the word 'Christmas' and the sentence 'I wish you a merry Christmas'; (3) the relation between Luxembourg and the European Union. Whether these objects relate by part-whole in a worldly sense is of course a substantive question. But our claim of interest here is that (1) – (3) feature among our familiar judgements as to what is part of what and confirm that no such judgement ever offends against Antisymmetry.

in the sense of 'commonplace for members of the scientific community'. Scientists indeed extend the part-whole vocabulary to capture *sui generis*, deviant cases[9]; thus judgements describing the latter as cases of genuine part-whole are certainly familiar to them. Now, this interpretation of 'familiar' as 'commonplace in the scientific community' is clearly not Fine's intended interpretation. So what is?

While there are a number of potentially interesting interpretations to work with, I recommend that we work with the instruments Fine himself indicates elsewhere in the article (2010: 560-1). On his view, the familiar judgements are those amenable to two 'semi-reflective conditions'[10]: (a) one thing is 'in' the other (the quarter 'in' the pint, the member 'in' the singleton, the sub-set 'in' the set); and (b) the existence of one thing makes a difference to the identity of the other (without the quarter the pint is not a pint, without the member the singleton is not the singleton of that member, without a sub-set a set is not the same set). Conditions (a) and (b) are marks of the cases we are prone to deem part-whole, and as it happens, these cases are all canonical. Thus the argument becomes: the cases we are prone to judge cases of part-whole based on their meeting (a) and (b) are all canonical, and the best explanation of this systematic canonicity is that there are indeed only canoincal cases to assess; that is, that general part-whole is coherent.

Now, even so construed, the first premise threatens falsity. It is not clear that all instances meeting the semi-reflective conditions and deemed cases of part-whole are truly canonical. We can notice this by running exotic thought experiments to test our judgement on exceptional satisfiers of the conditions[11]. Imagine a wall reducing in size to become a single brick. Suppose that the wall survives the shrinkage and is diachronically identical throughout. Now, carrying the brick with you, you enter a time-machine and go back to the time and place when the wall stands tall in front of you. You add the brick you carried – or perhaps it would be better to say, you add the

---

[9] See Chapter 4 for more discussion.

[10] See (Ib. 561). I say *semi*-reflective because condition (a), that one thing is 'in' the other, relies on a primitive: *being 'in'*. It remains quite obscure on what grounds Fine deems one thing 'in' the other. My suspect is that, as elsewhere (1999: 62-5, 2003, see also Sattig 2016: S. 1.1.3 and Lando 2017: 28-9) he is guided by semantic intuition. For intuitive reasons, the predicate applies truthfully to the objects. As I elaborate below, I doubt that intuitive constraints on the truthful applicability of the predicate can be so fine-grained to rule out applicability to the scientifically interesting deviant cases.

[11] For this and other exotic scenarios, see Effingham and Robson (2007). See also Kleinschmidt (2014) and Varzi (2014: S. 2.1). Effingham (2010) and Robson and Effingham (2010) make of the exotic scenarios an argument for perdurantism, but see Donnelly (2011) for a response.

wall itself reduced to brick-size to the wall standing tall in front of you. The newly composed wall – it would seem – has itself as a (sub-located) part. The latter is 'in' the former and (arguably) changes what it is – yet it is identical with it!

There are of course questions pertaining to what this and similar thought experiments really manage to show[12]: perhaps not that cases of symmetric part-whole are physically possible, because we might not be imagining a physically possible world when we imagine ourselves travelling backwards in time. Perhaps not even that symmetric part-whole is metaphysically possible, for we might not be imagining a metaphysically possible world when we imagine the wall (as we have done) as an object with diachronic identity. Finally, perhaps the thought experiment does not even show that symmetric part-whole is conceptually legitimate, because we might not succeed in imagining genuine part-whole when at the end of the story we imagine that the wall is part of the wall. To the contrary, we might have been mistaken in assessing the contents of our imagination[13].

These questions, however, need not presently matter. Suppose that the thought-experiment portrayed the wall-wall case as part of a metaphysically or conceptually impossible story. In either case, it is amenable to (a) and (b) and we are prone to classify it as part-whole (if this weren't the assessment's output, there would be no point in making use of the stories to illustrate something of significance about part-whole). This could be enough to block Fine's abduction as we construed it, for just as the argument sets no interpretation of 'familiar', it also sets no constraint on semi-reflective assessment. The *sui generis* cases might well be assessed, indeed, by considering them within a metaphysically and even conceptually impossible story.

Fine could reply that for the abductive conclusion to follow, the assessed cases need to be conceptually or metaphysically possible – not presented as part of an exotic story. Not even this, however, will help. I might have used an exotic, perhaps conceptually impossible illustration to trick the semi-reflective assessment into classifying a deviant case as part-whole. But I will soon capitalise on naturalistic resources for conceptually possible actual instances of part-whole that fail the

---

12 Cotnoir and Bacon (2011: S. 2).

13 I owe this point to Williams (2006), who on the same grounds denies the conceivability of mereological gunk.

Antisymmetry postulate and – I would say[14] – comply with the semi-reflective conditions (Chapter 5, Section 3).

To anticipate, my favourite example involves a component vector and the vector resultant it is a component of, both of which we can think of (for the time being) as purely mathematical objects: members of a vector space[15]. Of component vectors we could say semi-reflectively, but legitimately, that they are 'in' the resultant and that if one component vector were not to exist, i.e. were removed from the vector space it is a member of, then the resultant vector would not be what it is (for it would not be identical to the superposition of its components). Yet among the components of a vector's component we can find the vector itself (see next chapter). These second-generation components are still 'in' the original vector (if the first-generation components were, I would not see why these would not be), and removing each of them (including the original vector itself) from vector space changes what the vector is. Accordingly, it would seem that some actual, deviant cases can be found that satisfy the semi-reflective conditions. These are cases of the linear algebraic relation of component-to-sum, whose *relata* are purely mathematical vectors[16].

At this stage Fine could try to argue that these cases from the algebra of vectors fail the semi-reflective conditions, so that (say) the intended, semi-reflective sense of being 'in' is not the same in which a component vector is 'in' the resultant. But of course this sophistication of the conditions' legitimate application borders on *ad hoc*-ness, and is yet to be defended (we reject some possible defences in Section 5). In sum, it is at best controversial that semi-reflective assessments based on the Finean conditions classify as cases of part-whole only canonical (e.g. antisymmetric) cases. This challenges our attempt to make sense of Fine's argument abductively. The abductive argument's conclusion was that general part-whole is coherent under Antisymmetry, which would have entailed that no two distinct things relate symmetrically by a genuine part-whole relation (each being a part of the other). Now, however, we lack the abductive premise, which is that familiar judgements (about

---

[14] See below, Section 5.

[15] It does not matter, for the time being, whether vectors are themselves membered (i.e. sets) or member-less.

[16] They are – to be sure – not the only cases: additional ones involve the physical objects that interpret purely mathematical vectors, thus those who doubt the existence of purely mathematical vectors earn their supply of deviant cases. For these additional cases, however, I defer to our larger-scale discussion of vectors and vector-parts in the next two chapters.

what is a part of what) never sanction distinct mutual parts. Consequently, Fine's abduction fails as an attempt to dissuade us from deeming deviant cases genuine cases of part-whole.

3. An argument from paradigms

For other reasons to discard impactful evidence against core principles like Antisymmetry one can consult a recent approach by Giorgio Lando (2017), which claims that any part-whole relation comes with the formal features of 'paradigmatic' cases. A paradigmatic case, for Lando, is a case featuring two material objects *x* and *y*, one of which occupies a spatial (or spatiotemporal, proper or improper) sub-region of the other. Paradigmatic cases are thus cases of the *sub-location* relation.

Being paradigmatic is neither sufficient nor necessary for being an instance of a part-whole relation (Ib. 25). It is not necessary because objects with no location in space (or spacetime) stand to each other as parts to whole[17]. And it is not sufficient because some objects sub-locate material whole without being parts of them[18]. Rather, Lando views paradigmatic cases as a *heuristic* – a *ceteris paribus* aid to identify the core features of all part-whole relations in a "principled and projectable" way (Ib. 27). These will be – according to him – the formal features of the sub-location relation: antisymmetry, reflexivity and transitivity[19]; common to any pair of things,

---

17 Lando intends this as a metaphysical possibility, which also obtains actually if at the actual world there are non-located objects (e.g. abstract objects) that relate as part to whole.

18 Though see Markosian (2014). The view that every sub-located object is part of the object it sub-locates follows from another view, which is more often discussed: the 'doctrine of arbitrary undetached parts' (DAUP: Every object located at region *r* has a proper part located at every proper sub-region of *r*). The label and influential criticism owe to Van Inwagen (1981).

19 Notice that attempts have been made to reject the antisymmetry of sub-location. Notably, Thomson (1998) argues as follows: the Statue and the Clay that makes it up are distinct and coincident. Statue sub-locates Clay, because it occupies a sub-region of the region Clay exactly occupies. This sub-region is just the entire region occupied by Clay; that is, an improper sub-region of Clay's region. But so does Clay sub-locate Statue, because it occupies a sub-region of the region Statue exactly occupies. This sub-region is just the entire region occupied by Statue; that is, an improper sub-region of Statue's region. Like Lando, Thomson takes these facts of sub-location to be indicative of facts of part-whole, *yet* she concludes that Statue and Clay are mutual parts. For Lando's response, see (2017: S. 9.4). Cotnoir's (2010) formalism for part-whole without the Antisymmetry postulate models Thomson's coincident mutual parts. The formalism includes the non-standard definition of proper part "x is proper part of $y =_{df} x$ is part of y and y is not part of x", which becomes available without Antisymmetry (see Chapter 4: S. 3.3 for more discussion).

one of which occupies a sub-region of the other. Having extracted the formal features from paradigmatic instances, Lando argues that we can use them to test what does and doesn't count as a case of part-whole[20]. For two objects to relate as parts to whole, it is necessary[21] that, formally, they relate in just the same way as the *relata* of sub-location.

Prior to critical assessment, I should emphasise a truly likeable feature of this view, which – I think – brings great methodological improvement. Imagine some *x* and *y* standing in a relation-token *R*, which impresses us as a potential case of part-whole (this could well be one of the cases we will deal with in the next chapter, thus in which *x* is a component vector and *y* a linear vector sum). To determine whether a theory of part-whole ought to accommodate this case, we should test it against nothing but the paradigmatic formal properties (let me anticipate that if *x* and *y* are a component and a vector sum, and the paradigm is indeed Lando's material sub-location, then the test fails, because cases of component-sum are symmetric).

For a contrast, on a rival but clearly less transparent 'proposal' we decide whether to accommodate a particular case of *R* based on whether the English predicate '… is part of' applies literally or metaphorically of *x* and *y*. Many (e.g. Yablo 1999) note a number of difficulties with capturing the literal/metaphorical distinction in precise terms[22]. Accordingly, absent secure criteria, stalemate arises provided just that an opponent *insists* that she means '…is part of' to apply literally to *x* and *y*. For imprecision in the literal/non-literal distinction leaves us little option to meet these opponents and as a result, like Lando I feel the urge for different policies to filter out deviant cases, leaving appeal to the literal/non-literal distinction as last resort.

Thus I find myself in agreement with Lando that, overall, appeal to paradigms should be preferred. Indeed, the method of extracting formal features from paradigms has undeniably greater projectibility. Faced with unscrupulous debate on whether '…is a part of' applies literally or metaphorically to some curiously related *x* and *y*, we might have the chance to *rule out* the *x*-*y* case from the list of cases a theory

---

[20] Lando is a mereological monist: he believes in only one relation of part-whole. Monism, however, does not follow from the claim that every genuinely part-whole relation has paradigmatic formal features (Ib. 27).

[21] Not sufficient. There might be cases with the paradigmatic formal features that escape classification as part-whole. Lando holds that to count these cases in, we need an appeal to the Finean semi-reflective condition (a): one relatum is 'in' or 'contained' in the other. As before, the boundaries of these predicates remain obscure.

[22] See also Chapter 2: S. 2.

of part-whole ought to accommodate simply by pointing at its non-paradigmatic formal properties.

What remains to be checked, of course, is whether material sub-location is a plausible choice for a paradigm. Lando's method avoids the appeal to the literal/non-literal distinction when it comes to assigning deviant cases the status of part-whole. However, this promise of methodological superiority alone reveals nothing about *whether* part-whole is confined to formal similarities with sub-location in the first place.

In fact, some relations admit of no paradigmatic confining, leaving us no chance other than to deal with deviant cases via the literal/non-literal distinction. Our previous examples suffice as a case in point: *looking at* and *loving*. Notice that whatever the paradigm for *looking at* is, it cannot manage to filter out cases in the way Lando envisages; that is, by demanding that all cases of *looking at* have the formal features of the paradigmatic cases. For as seen in the previous section, there are genuine cases of *looking at* and *loving* with transitive and intransitive, symmetric and antisymmetric, reflexive and irreflexive features. Accordingly, no candidate for a paradigm case can sensibly filter out these cases by imposing a specific package of formal features.

As a lesson, we may be able to take in the conditional claim that we would gain considerable benefit *if* part-whole were confined to formal similarity to some paradigm. However, we ought not view these benefits as themselves supporting the confining, especially with naturalistic evidence questioning it. Rather, if the confining is not forthcoming, we will have to renounce the benefits. A genuine issue will remain, absent paradigms, as to the methods for filtering out the curious cases.

Let us stay with *looking at* and *loving* for illustration. There are relation-tokens whose classification as instances of *looking at* or *loving* unavoidably gives us pause. We may say, for example, that 'these walls look at the enemy fearlessly', or that 'nature loves mankind'. Absent sensible formal constraints arising from the paradigm, we are left with a question as to how to assess these curious cases. Guidance from the paradigm *would* indeed prevent the appeal to the clouded literal/non-literal distinction, but as already discussed, and unfortunately, these two relations lack formally coherent paradigms, capable of offering the expected guidance. Now, with naturalistic evidence threatening to disclose the same situation in respect to part-whole, the safest attitude would seem to be the following. We should conclude that filtering out the curious cases, whose classification as part-whole gives us pause, can be a difficult process, backed by neither paradigms nor the clouded literal/non-literal distinction.

We can forestall a critic. Having used the naturalistic evidence to deny that part-whole has formally coherent paradigmatic cases (say, paradigmatic cases equipped with Antisymmetry), would it not become too difficult, perhaps even impossible to filter out any case, whose classification as part-whole gives us pause? Although she does not discuss the paradigm method, Karen Bennett (2017: 36) raises essentially the same concern: If we concede part-whole[23] status to the pause-giving cases (as we could do absent paradigms), would it not become "impermissible" to deny part-whole status to any such case?

My answer is in the negative. Some relations (*looking at*, *loving* and, for me, part-whole) may well lack paradigmatic cases, and absent these cases we may decide to classify some pause-giving cases as cases of these relations. This, however, does not yet make it impermissible, or even demanding, to filter out other pause-giving cases. For while it is true that filtering out becomes *more* challenging absent paradigms and a clear grasp on the literal/non-literal distinction, I find it sensible to entrust the

[23] To be sure, Bennett raises this worry slightly differently and in a different context. She would not object to classifying certain exotic cases as cases *of part-whole*. However, she would object against classifying exotic cases as cases of *proper part*. For Bennett, all relations of proper part belong to a class of comparative fundamentality relations (x is more fundamental than y) such that one of two *relata* 'builds' the other, in a sense of 'building' picked out by a mix of conceptual analysis and theoretical usefulness (see 2017: 39). All building relations are asymmetric (for so is comparative fundamentality) and have *relata*, which exemplify the 'builder-built' relationship.

Two observations. 1. The argument from impermissibility is more general than Bennett's theory, thus we can assess it in our own context as an argument against exotic cases of part-whole: if we sanction cases where part-whole is in opposition to core principles, then we make it impermissible to deny other exotic cases part-whole status. 2. More specifically on Bennett's account, one can ask whether there are exotic cases opposing a property of building, e.g. asymmetry, which I classify as cases of *proper part*. Any such classification would set me on a collision course with Bennett's account, for it would entail that at least one proper part relation is not a relation of building. The answer anticipates Chapter 3: whether there are any such cases depends on the adopted definition of *proper part*. My favourite definition is in terms of non-identity (i.e. x is a part of y distinct from y) and indeed models some pairs of distinct physical objects as mutual proper parts. However, one can adopt Cotnoir's (2010) alternative: x is a proper part of $y =_{df} x$ is part of y and y is not part of x – at the price of dropping the Antisymmetry postulate for part-whole. Of the objects of scientific interest I keep under focus, Cotnoir's definition models no pair of distinct ones as mutual proper parts, in accordance with Bennett's account. For brevity, I stick to my favourite definition – and accept entering on a collision course with Bennett. But I do not oppose those who are attracted by Bennett's classification of *proper part* among the building relations and wish to switch to Cotnoir's definition. Particularly, the switch preserves my claim that the part-whole relation holding between these objects of interest is deviant, i.e. fails the Antisymmetry postulate, defines a proper part relation that fails intuitively forceful supplementation principles, and defines a composition relation that fails Idempotence. I will return to this point in the next chapter (Section 4).

naturalistic evidence *itself* with at least some initial authority to decide on the filtering.

We know, for example, that putative cases of part-whole with symmetric features (such as the component-to-vector sum case, see next chapter) involve *sui generis* entities familiar only to scientific practitioners: mathematical vectors and the items of revisionary field-ontologies that interpret them, whose putative 'decompositon' into additional fields mirrors the 'decomposition' of a vector (see McKenzie 2012). Suppose that as a filtering constraint we adopted the *relata*'s membership in an advanced scientific subject matter. While counting in cases with a naturalistically eminent source, this new constraint filters a good class of deviant cases that we correctly expect *not* to fall under the rubric of part-whole. Lando's most frequent example (2017: 19, 23, 28) portrays a lover's statement: 'You are part of me and I am part of you' (similarly, we can imagine a writer of faltering Shakespearean inspiration writing 'Romeo was part of Juliet and Juliet part of Romeo'). Suppose that the referents of 'I' and 'you' (and of 'Romeo' and 'Juliet') are different persons. Surely the lack of formal guidance removes a helpful filter and tempts us to go down a dangerous route, that is, to count this case out invoking only the figurativeness of the statement's vocabulary. Before giving in to the temptation, however, we may also notice that advanced physical theories rarely (if at all) make claims about people, hence that they are not likely to indicate people as the bearers of symmetric part-whole[24]. This provides an independent reason to filter the case out, meeting our initial expectations.

4. Which *sui generis*-ness thesis?

No restraints arise from the above arguments against accommodating deviantly related scientific objects of interest as objects that relate as part to whole in a perfectly literal sense. As was anticipated earlier (Chapter 2, Section 2), this possibility of accommodating the deviant cases within the rubric of literal part-whole makes an

---

[24] Of course, other sciences like psychology make such claims, but they do not handle part-whole in the same revisionary way as physics. So they would not offer grounds to filter in the lovers case even if we granted them authority to do so.

impact on the *methods* that inform the principles they offend (e.g. Antisymmetry, informed by conceptual analysis). Having now deflected the reasons against accommodating the cases, we are in a better position to articulate the *kind* of exerted impact.

I have distinguished, recall, between three types of accommodation and three types of impact that the accommodation bears on the methods that sustain the canonical principles challenged by the scientific cases. The evidence can be accommodated (i) by admitting additional varieties of part-whole with deviant governing principles; (ii) by letting the governing principles of the one part-whole relation vary relative to type of *relata*; or finally (iii) by admitting additional relations that are no literal varieties of part-whole but resemble the latter in conspicuous or crucial respects. On the other hand, the types of impact include (iv) denying that conceptual and intuitive methods delivering the offended principles (e.g. Antisymmetry) guide us to the truth; (v) denying that we correctly perceive the methods' deliverances (e.g. denying that Antisymmetry is a product of conceptual analysis or intuitions assessment); (vi) maintaining truth-guidance and correct perception, but taking up option (iii) from the accommodation options, and so admit deviant cases only sufficiently similar to part-whole.

Options (iii) and the correlated (vi) are interesting compromise options. In fact, unlike (v) and (iv) they manage to retain the informing methods (conceptual analysis, assessment of intuitions) and unlike (i) and (ii) they are compatible with part-whole having only one category-neutral variety. Yet quite unfortunately, these views suffer from difficulties in articulation. In particular, they risk trivialising the rubric, extending it to include *any* deviant case with some claim to be sufficiently similar to genuine part-whole, e.g. cases of the symmetric component-sum relation, by which we label a vector a 'component' of a sum, as well as cases of the symmetric mutual love relation, by which we label Juliet 'part of' Romeo.

Perhaps the advocate of view (iii) could seek help from certain distinctions, such as the distinction between slight and deep non-literalness (of the mereological vocabulary applied to deviant cases)[25]; or a distinction between the deviant cases more and less similar to cases of genuine part-whole. Tuning these distinctions into (iii) could give the view a chance of filtering some deviant cases out of the rubric (those which satisfy mereological vocabulary deeply non-literally and/or are similar

[25] See Chapter 2, Section 2.

to the genuine cases to a poor degree), while filtering in others (those which satisfy mereological vocabulary slightly non-literally and are similar to the genuine cases to a high degree).

However, these distinctions look at best in need of careful articulation. This is why, on balance, I take the rebuttal of Fine's and Lando's arguments in the previous section to support one of the stronger varieties (i) and (ii) over the compromising variety (iii). As a mereological pluralist, Fine already admits varieties of part-whole, with general part-whole being the relation that holds between any two *relata* that stand in some variety or other (Section 2). As the constraint that general part-whole has only anti-symmetric instances ultimately fades, we can supplement Fine's pluralist view with yet another variety – this time deviant.

Lando's paradigm account is neutral with respect to pluralism (2017: 27) – what matters is not how many varieties there are but simply that no variety has instances offensive to the paradigmatic formal properties. As the support from paradigms fades, we can either add one more variety with deviant instances, or, as per option (ii), we can maintain a monist stance and have it that the one and only part-whole relation admits of deviant cases when it takes on scientifically interesting *relata* (fields, vectors, etc.).

Options (i) and (ii) exert a stronger impact on the methods sustaining the problematic principles. Indeed, both options classify the deviant cases as cases of genuine part-whole varieties. Suppose (for illustration) that these cases clashed with Antisymmetry. As a result of the impact, we cannot maintain that Antisymmetry is a product of *a priori* methods (conceptual analysis, intuition assessment) that are both truth-guiding and well-practiced. Indeed, by one of (i) or (ii) we agree that some variety of part-whole has instances offensive to Antisymmetry. So either the methods generally guide us to the truth but Antisymmetry has been falsely perceived as one of their deliverances, or we are perfectly right in counting Antisymmetry among the deliverances, but the methods are not a systematic guide to the truth.

5. Any more anxiety on naturalistically guided, yet deviant part-whole?

By responding to Fine's and Lando's arguments, we removed two particular obstacles to accommodating the naturalistic evidence in an impactful way. Now we need to address two remaining questions. The first asks whether any additional obstacles are forthcoming, and the second whether there are general reasons not to expect additional obstacles.

5.1 Additional obstacles

There certainly might be obstacles I do not yet currently see, and careful backtracking may help us to identify at least two I have treated somewhat superficially.

1. Recall the Finean semi-reflective condition ((a) in Section 4.1), according to which a part *x* of *y* should be 'in' or 'contained' in *y*. An opponent might grant this condition filtering power to rule out deviant cases, arguing that some or all of these fail to satisfy it.

The criteria for predicating '...is in' or '...is contained' appropriately of some particular *x* and *y* remain uncertain: both predicates are primitive for Fine and applied or rejected largely on the basis of intuitions of their semantic properties; that is, intuitions that the statement resulting out of applying them to some particular objects is true (Sattig 2016: S. 1.1.3, Lando 2017: 26). Time prevents us from considering a growing literature concerning justification by intuition in metaphysics[26], so suffice it instead to raise a question of boundaries. Lando claims that '... is in' needs no informative or non-circular characterisation, as "[n]o serious debate in mereology hinges on the intuitive constraint, because in each case it is clear what respects the constraint, and what does not" (2017: 28).

On the contrary, it seems to me that the *sui generis* cases of advanced science leave the intuitions' outputs unsettled – and with the outputs unsettled, one can not filter these cases out by straightforward evidence that intuitions incline one way or the

[26] Suffice it to say that the possibility of justification by intuition is far from generating agreement.

other. I do not find myself able to say clearly whether a component vector is 'in' the sum. I know for certain that the preposition 'in' does not collocate in English with nouns for component and sum vectors, but the same holds of the preposition together with nouns for sets and their members (of the latter we say that they are 'members of…', not 'in' the sets). Yet set-membership is considered one of the cases favourable to the intuition (Fine 2010), so (I take it) we should not use valid English collocations as a source for assessing the intuition's response.

Perhaps the intuition ultimately leans one way or another because the way a certain whole is represented makes it clear that it is a *collection* or a *grouping* of certain other things[27]. This is the rationale behind the Cantorian (1936/2012) first-pass articulation of the set-concept[28]. One may demand that some comparable principle holds of every satisfier of the semi-reflective condition; that is, that the entity representing the condition-satisfier in a formal theory obviously displays it as a group or a collection of things[29]. Even this appeal to formal representation of groups and collections, however, fails to filter out vectors. As we will better see in the next chapter, a vector **v** is represented by an (1 x *n*) (or (*n* x 1)) matrix, with each entry in the *n* columns (or rows) representing in turn another vector, a linear component of **v** that plays the role of basis vector in the vector space of which **v** is a member. Linear components are the source of deviant structure that will interest us in the next chapters; and yet (it would seem), canonical linear algebraic representers (matrices) classify vectors as groups/collections of their linear components, seemingly including them among the things that would satisfy the pre-reflective condition glossed with the Cantorian first-pass principle.

---

27 See Lewis (1991: 29) for some variations on the principle in textbooks on set theory.

28 Cantor supplies the first-pass principle with the condition that one must be able to gather the grouped elements in thought. Lewis (1991: 29-30) objects to this psychological addition on the basis (1) that there are sets whose elements are so numerous not to be collectable in thought; and (2) that the principle seems to licence naïve principles of comprehension.

29 As is known, Lewis (1991: 31) objects to this account on the basis that the 'grouping' terminology falters when it comes to singleton sets: "[singleton sets are] [a] cause for student protest [to the first-pass account] if ever there was one […]. [H]e [a student] has no elements or objects – I stress the plural – to be 'combined' or 'collected or 'gathered together' into one". On the contrary, for Fine, reflective condition (a) applies just as well to singletons. Here we are assuming that the condition applies only if we can predicate of its satisfiers that a formal theory of them represent them as a 'group'. On this assumption, we read Fine as disagreeing with Lewis. For Fine, 'stressing the plural' is mistaken: singletons are successfully represented as groups in set theory.

These sketchy, but hopefully efficient, remarks show that filtering vector components and sums out of the containment intuition is more difficult than expected. Most likely, this is because we were not meant to filter them out in the first place.

2. There is also a chance of challenging the naturalist by direct appeal to the literal/non-literal distinction. This would have it that talk of vector/vector (or sub-field/field) part-whole is metaphorical. We resist this in two ways:

2a. By appeal to the unclear borders of the literal/non-literal distinction, setting the burden on the opponent to offer reasons to count scientific part-whole talk as a definite case of non-literality;

2b. By appeal to charitable interpretation of scientists. In fact, scientists would seem to abstain from certain metaphors, aware of their lack of truth-guiding value. While one may occasionally describe hadrons as 'bags' of quarks (Nambu 1985), proficient physicists systematically renounce this visual aid and formulate all of the same regularities and empirical consequences of quark confinement (e.g. the unobservability of free quarks) in quantum-field theoretic language.

The significance of episodes like this is twofold. On the one hand, they suggest that metaphors are consciously avoided[30]; on the other, they motivate a charitable assumption. According to this assumption, the examined scientific language does not indulge too frequently in metaphors, and certainly not as frequently as it gives way to mereological predicates to describe deviantly related items, such as mathematical vectors and (components and resultants) their putative physical interpreters (fields and sub-fields). Having settled these, it will be time to focus on the specifics of these deviant cases, which I have often invoked up to this point.

[30] Yablo (1998) argues that scientists unconsciously sanction frequent metaphors (and concludes that we ought not read off ontological commitments from scientific language, see also Quine (1960: 250). However, the frequent parts of scientific language he deems non-literal are all cases of scientific idealisation (e.g. 'the ideal plane'). The latter are arguably unlike descriptions of vector components (or their physical interpreters) as 'parts', so we cannot conclude from the latter's frequency to their non-literalness. Rather, their frequency continues to speak for their literalness because (modulo the case of idealisations), metaphors are used only infrequently and are often consciously avoided.

5.2 Can there be a general argument against additional obstacles?

This concludes our defence of the claim that evidence striking against core principles demands accommodation. Absent obvious additional objections, the sensible conclusion is that this evidence speaks for extending the rubric of part-whole to include deviant varieties (i), or deviant principles, which govern the unique part-whole relation when this applies to particular categories of *relata* (ii). These extensions threaten the methods that accompany the offended core principles, in the ways already discussed (most recently in Section 4).

Let me note, in conclusion, an advantage and a disadvantage of the defence I have so far provided. The disadvantage arises in that I have only taken the *via negationis*: support for my conclusion comes from the lack of obvious remaining rivals. This dialectical fact makes my position sensitive, in principle, to all the rivals I have failed to discuss. Having now disarmed a significant number of rivals, however, I find it reasonable to expect my opponents to make the next move.

As for the advantage, unlike other naturalistic approaches[31] I have taken to heart the discussion of *a priori* methods against strongly impactful scientific evidence (Section 1). I have embraced a policy for letting the contents of a metaphysical theory (here: a theory about the worldly features of part-whole) be constrained and informed by evidence from science. The policy is not particularly sophisticated: the frequency of mereological or near-mereological expressions used in connection with highly *sui generis* entities supports extending the part-whole rubric in accordance with (i) or (ii). Yet, though unsophisticated, the policy is liberal: it does not state, for example, that for knowledge of which principles govern the worldly part-whole relations we should *only* turn to the systems of beliefs produced by the scientific communities, which host practitioners of the deviant cases without Antisymmetry, Weak Supplementation, etc[32]. Instead, the policy requests that all of us committed to naturalism broadly construed consider the evidence as putting *some* pressure on a view of worldly part-whole without (i) and (ii)[33].

---

31 See the list of Ladyman and Ross' devotees in (Chapter 2, Fn. 2).

32 For naturalism in this unpermissive sense, see Ney (2012) and Ladyman (unpublished).

33 Or at least, everybody who is committed to naturalism in a permissive sense, which I interpret as the following claim (cf. Chakravarrty 2017: Ch. 2): a metaphysical theory (here, a theory of part-whole's worldly principles) should aim at a *balance* of scientifically informed contents, methods and aims. Relations of part-

This pressure comes with no corresponding obligation to embrace (i) and (ii). Before doing so, one should consult the potential comebacks, which balance the evidence's pull by stressing other virtues for a worldly theory of part-whole. This chapter discussed and rejected some attractive comebacks. In two cases – Lando's and Bennett's – I argued that the emphasised virtues are not (fully) precluded to those who accept the evidence's pull. Thus one can set filters on the rubric of exotic cases classified as cases of part-whole even absent paradigms for the part-whole relation and absent a general part-whole relation coherent under the features the exotic cases offend against.

In one more case – Fine's – I argued that we are better off promoting the evidence's pull towards (i) and (ii) over the competing virtues. We are better off sanctioning cases of part-whole that clash with Antisymmetry, because contra Fine's expectation, a general relation of part-whole coherent under Antisymmetry grants no additional explanatory power[34].

---

whole without core principles (or relations that sacrifice the core principles when they apply to *relata* of certain scientific categories) pertain to the theory's scientifically informed contents. However, as the permissive policy demands a *balance* of contents with methods and aims, one can renounce contents to gain on the side of method. In practice, one can renounce deviant governing principles for part-whole (renouncing the content suggested by the evidence) and gain better methods, as I explain in the main text.

[34] On a final note, there is another case in which we are better off promoting the evidence's pull towards (i) and (ii) over a competing virtue. Koslicki (2009: 4.4) worries that by sanctioning cases in offence to Antisymmetry – thereby making part-whole incoherent in respect to this property – we 'handwrite' the principles that govern part-whole relations in a 'piecemeal' manner, i.e. sensitively to the part-whole structure of local and restricted classes of material objects. This counts as a form of complexity, which all else being equal we had better avoid. The reply here is that even granted this advantage of a coherent general relation, still we would need to handwrite distinctive principles for the relation that obtains between the physical objects of interest, even if this relation has now been denied genuine part-whole status to preserve the coherence of the general relation. We will not escape the 'piecemeal' handwriting work, thus we might as well pursue the pull of the naturalistic evidence towards (i) and (ii) and deny the coherence of the general relation.

IV

Vectors and *sui generis* part-whole

We tend to believe that no whole is a part of any of its proper parts. If you ask me, as Lando (2017: 22) did, to give you a *piece* of my cake, then there are really only two kinds of pieces I can give you: either (as a limiting case) the cake itself, or a part of the cake distinct from it, none of whose parts is the cake. The 'or' is exclusive: there is no third option; and particularly, among the proper parts of the cake I can give you if I decide not to give you the entire cake, none has the entire cake as a part. The only case in which I manage to give you a piece of the cake that has the cake as a part is the limiting case: that is, when I give out my whole cake. These ideas converge in the Antisymmetry postulate:

Antisymmetry: if x is a part of y, then y is a part of x just in case $y = x$.

Antisymmetry is a serious candidate for entry into our core principles: there is some appeal in construing it as a conceptual principle and only a few philosophers reject this construal[1]. Evidence against antisymmetric parts would certainly challenge conceptual methods for construing one's worldly theorisation about part-whole (see Chapter 1, Section 8.1).

Responding to various sceptics in the previous chapter, I have suggested that such evidence is available in principle, showing that *a priori* arguments for limiting the evidence's impact on the methods fail. This enquiry sustained Ladyman and Ross' suggestion that leading approaches in the literature on part-whole overlook *sui generis*, but scientifically motivated accounts. The verdict is that they do so because they fail to encompass deviant cases that offend against core principles – like, indeed, Antisymmetry. Similarly, motivated accounts of *proper part* and *composition* should aim at including deviant cases that clash, respectively, with Weak Supplementation and Idempotence.

This chapter focuses on one such deviance: the case of vectors that stand to each other as component to resultant. We start with illustrating the basic algebraic properties of vectors (Section 2). Then we outline the vector-vector relations that threaten to qualify as part-whole, proper part and composition in offence to the core principles (Sections 3, 4). Finally, the next chapter discusses these relations' role in an argument against the core principles.

1. Algebraic properties

Basic linear algebra comes with a distinction between vectors ($\mathbf{v}_1$, $\mathbf{v}_2$ ...,) and scalars (c_1, c_2, ...,, d_1, d_2, ...). On a standard use of terminology, multiplying a vector by a scalar yields a vector and so does summing two or more vectors. The sum of any two or more vectors, each multiplied by some scalar, goes by the name of *linear superposition*. The scalar each vector is multiplied by in a linear superposition could of course be the algebraic identity; that is, the scalar that returns the same vector if multiplied by it[2]. This special case of linear superposition – in which the identity scalar multiplies each superposing vector – we can call *vector sum* or *vector addition*.

[1] Notably, Cotnoir (2010), Bacon and Cotnoir (2012). Thomson (1998), Tillman and Fowler (2012) and occasionally Simons (1987: 177-80) reject the construal of Antisymmetry as a worldly principle.

[2] This is of course 1 if as the relevant scalar we consider natural or real numbers.

Further, the result of linearly superposing two or more vectors is called the *resultant vector* (or simply the *resultant*). The superposed vectors that output the resultant are the resultant's *components*. Finally, we refer to a vector $\mathbf{v}_1$ as the *linear combination* of some (finite) distinct vectors $\mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n$ if there are some scalars that, multiplied by any of $\mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n$ deliver vectors whose vector sum is $\mathbf{v}_1$. For (possibly infinitely many) vectors $\mathbf{v}_1, \mathbf{v}_2 \ldots$ to be *linearly independent* means that none of them is a linear superposition of any combination of the others. Any group of linearly independent vectors is called a *basis*.

For some basis of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$, a vector space is the set of all scalar products of $\mathbf{v}_1, \ldots, \mathbf{v}_n$ as well as of all linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_n$; or differently put: the closure of some linearly independent vectors under scalar product and linear combination. All elements of a vector space are of course vectors. Moreover, all vectors of a vector space satisfy:

The vector space axioms:

$\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$	$(\mathbf{x}+\mathbf{y})+\mathbf{z} = \mathbf{x}+(\mathbf{y}+\mathbf{z})$	$\mathbf{0}+\mathbf{x} = \mathbf{x}+\mathbf{0} = \mathbf{x}$
$(-\mathbf{x}) + \mathbf{x} = \mathbf{x} + (-\mathbf{x}) = \mathbf{0}$	$0\mathbf{x} = \mathbf{0}$	$1\mathbf{x} = \mathbf{x}$
$(cd)\mathbf{x} = c(d\mathbf{x})$	$c(\mathbf{x}+\mathbf{y}) = c\mathbf{x} + c\mathbf{y}$.	$(c+d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$.

Whereby variables in bold ($\mathbf{x}$, $\mathbf{y}$) stand for vectors, variables in italics (c, d) stand for scalars, $\mathbf{0}$ is the 'zero' vector[3] and 0 the null scalar (the scalar that turns into 0 every scalar it multiplies and leaves unchanged every scalar it adds to).

Two comments on this basic terminology will help us orient the coming discussion. First, I should emphasise that up to this point I have omitted all talk of vectors as quantities 'with magnitude'[4]. For a contrast, this talk is indispensible for presenting vectors in physics and geometry. If we decide to represent vectors as ($1 \times n$) or ($n \times 1$)

---

[3] In matrix representation, where n is the number of vectors in the space's basis, the zero vector is a ($1 \times n$) or ($n \times 1$) matrix, all of whose entries are 0.

[4] As well as with direction and sense. Though characteristic to the vector quantities of physics and geometry, these will play no significant role in the following. But see Massin (2009) and Wolff (2016) for useful illustrations.

matrices (as is commonplace), then the magnitude of a vector **v** is computed as a function – the 'norm' – of the *n* values in the (1 x *n*) or (*n* x 1) matrix that represents **v**[5]. Given as the image of the norm[6], magnitudes allow us (in physics) to represent a system's momentary state not as an array of values of independent quantities (the 'characteristic state' representation) but as a single (positive) quantity value that depends functionally on the values of the independent quantities. Similarly, given as the image of the norm, magnitudes represent the vectors' geometrical length and are required to calculate (via the inner product operation) vector-vector distances and angles (provided that the vector *operanda* are members of the same space).

Advanced physics avails itself without fail of vectors rich enough to support magnitudes and the more advanced geometric properties (distances, angles) that require them[7]. So why do we omit magnitudes? In short, it is because the challenge to the core principles 'already' arises from vectors equipped with 'bare' vector space structure, i.e. membership in a vector space *V* and axioms for linear combinations and scalar products. The challenge ensues at the level of vector spaces, i.e. when we observe the algebraic relations that obtain between vectors equipped with the axioms and devoid of geometrical characterisation. Vectors understood pre-geometrically, therefore, are all that is needed to make my point that the core principles fail in well-regarded scientific areas.

Second, and relatedly, one may legitimately ask what *kind* of challenge to the principles we can raise by noticing the properties of the algebraic relations (e.g. linear combination) that obtain between vectors with bare vector space structure. On my view – whose details follow in the next chapter – the challenge is (primarily) of a conceptual kind (cf. Chapter 1, Section 4): when we predicate of vectors with bare structure that they relate as component to sum we introduce a concept which belongs to the family of part-whole but fails the core principles. The failing principles are construed conceptually: they state that it pertains to all *concepts* of part-whole, proper part and composition (not to the concept's satisfiers, the real parts, wholes and

---

5 There is a great variety of norm functions, see Shilov (1978) and Blyth and Robertson (2002: Ch. 5) for an accessible introduction. For the purposes of this work we are only interested in this general claim: vector magnitude is represented by the image of the relevant norm.

6 Usually the norm function is $|\boldsymbol{x}| = \sqrt{< x, x >}$, where <**x**, **x**> is the inner product of *x* times *x* (returning a scalar).

7 And actually even richer: Hilbert-space vectors, for example, also support Cauchy-completeness, which is a topological property.

composers) that no two things are mutual parts unless identical (Antisymmetry), that all proper parts come with distinct and disjoint remainders (WS), etc.

Viewing the raised challenge in this way justifies the omission of magnitude and the more advanced geometric properties that require it. Richer vectors with magnitude and these properties still satisfy the purely algebraic concept: they are members of vector spaces equipped with the vector space axioms[8]. Satisfying the richer vector concept entails satisfying the linear algebraic one: every vector with 'bare' vector space structure *and* magnitude is *a fortiori* a vector with 'bare' vector space structure. Accordingly, theories that include in their resources the richer concepts (say, to define physical states) also include the linear algebraic concept – hence deploy a concept of part-whole that opposes the core principles[9]. As the linear algebraic concept features in the advanced resources, we can limit ourselves to studying its own properties, rather than the properties of the richer concepts[10].

2. Deviant superposition 1

2.1 Antisymmetry

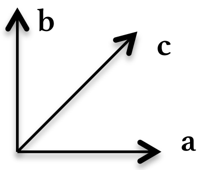
Vectors that linearly superpose into another (distinct) vector can sometimes be themselves the product of linear superpositions from the vectors they superpose into. This applies insofar as the space the vectors live in admits of sufficient vectors to generate the symmetry, no matter their magnitude and the physical state or

[8] The converse does not hold: it does not follow from vectors having 'bare' vector space structure that they also have magnitudes and the geometric properties that require them.

[9] What if one wanted, in addition, to argue from the algebraic relations of 'bare' vectors to the claim that *there are* objects of scientific interest that, at the actual world, relate (literally) as parts to wholes but fail the core principles? (In Chapter 1, Section 4, we called this the 'empirical task'). A (longer) route exists and the next chapter explores it.

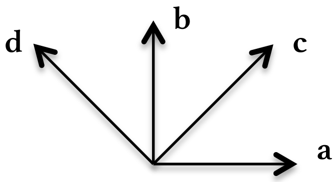
[10] Provided that we construe them as statements about concepts (i.e. the concepts *part-whole*, *proper part* and *composition*).

geometrical properties they represent. Start with a familiar graphical representation of the parallelogram rule[11]:



And distinguish two facts: one is that **c** is the vector sum of **a** and **b** (**c** = **a** + **b**). Another is that in the above diagram **c** represents a certain geometric length: the length of a parallelogram's diagonal (the lengths of whose sides are represented by **a** and **b**). Let us not be misguided by the fact that my representation depicts vectors as if they had these geometric properties. The appearance of having geometric properties aids us visually to make a point that only requires us to think of components and resultants as members of a vector space, that is, vectors with the sole mathematical properties that they multiply scalars or add to vectors, delivering other vectors in both cases[12].

So **c** is the vector sum of **a** and **b**; and **b** is a component of **c**. Now, to show that **c** is also a component of **b**, we add another vector **d**, linearly dependent on **c** and **b** such that **b** is the vector sum of **d** and **c**.



---

[11] Kit Fine first suggested using this example to argue against the core principles in personal communication (2013), and of course I should be held fully responsible for any mistake in the setup. At the time, and consistently with his position in (2010), Fine rejected both (a) that the relation between the component and the vector sum is a genuine part-whole relation; and (b) that components and vector sums satisfy a genuine part-whole concept. To my eyes his denials conceal inappropriate methods for assessing what counts as a part-whole relation or concept, as discussed in Chapters 2 and 3. When it comes to components and vector sums, my view is that *linear superposition* is a genuine part-whole concept, of which some (but not all) satisfiers also stand in a genuine part-whole relation (Chapter 5 returns in more detail to this point). As a genuine part-whole concept, with a place in the advanced scientific resources (Section 2), *linear composition* challenges Antisymmetry, understood as a conceptual truths about all concepts of part-whole.

[12] With the above norm function (Fn. 6), the norm of **c** is $|a+b| = < a + b, a + b >$.

Here is our putative deviance: **c** results from summing **a** with a remainder **b** and, vice versa, **b** results from summing **c** with a distinct remainder **d**. Excluding the remainder and considering only the binary relations with singular arguments holding between a vector sum and one of its components, we find that this relation of 'component-sum' admits of symmetric instances[13]. These instances are couched in mereological vocabulary: one *relatum* is, indeed, a 'component' and the other a 'sum'. Further, as we shall see in the next section, the formal properties of component-sum other than symmetry correspond to those of ordinary part-whole, as component-sum is reflexive and transitive. On a naturalistic policy and absent arguments that exploit other policies (Chapter 3), the symmetric cases of component-sum challenge the Antisymmetry postulate in at least one of two ways. Either the principles is false if construed as a statement about all concepts of part-whole, because linear algebra introduces an advanced concept without the postulate; or it is false construed as a statement about the features that parts and wholes have at the actual world, because linear algebra posits some objects of interest (vectors with 'bare' vector space structure like **a**, **b**, **c** and **d**), which relate as parts to wholes in denial of the postulate. As anticipated, I prefer the conceptual option and I do not find it easy to argue from the symmetric cases of component-sum to there being objects of interest that, at the actual world, relate as parts to whole in opposition to the Antisymmetry postulate. But this is a topic for the next chapter, where I will return to what can be done for so arguing. For the moment, let us continue our survey of the principles that face a challenge given the properties of component-sum.

2.2 Idempotence

According to the Idempotence postulate, if a single thing x composes any thing, then it composes just x. Our candidates for the vectors' composites are – predictably – the vector's linear sums (e.g. **c** in the previous section), which have all of the composing vectors (**a**, **b**) as linear components. That linear summation ('+' in the vector space

[13] A clarification: when I say that linear combination is all we need to illustrate symmetric instances of component-sum, I do not mean that these instances appear in all ensembles of linearly independent vectors closed under scalar multiplication and linear combination. The focus, rather, is only on sufficiently rich vector spaces, that is: spaces with enough vectors to work as *relata* of the symmetric instances.

axioms) invalidates Idempotence is to be expected. For indeed one of the axioms ($\mathbf{0} + \mathbf{x} = \mathbf{x}$) states that the identity element of the vector sum operation is the zero vector. When we add a non-zero vector to itself, the result is not *guaranteed* to be the same vector: $\mathbf{a} + \mathbf{a} + \mathbf{a} \ldots \neq \mathbf{a}$. Rather, adding some vector to itself either returns a different vector, or in some cases has an undefined result (whether the result will be distinct or undefined depends on the available scalars: if these include, say, only 0, 1 and the even integers, then $\mathbf{a} + \mathbf{a} = 2\mathbf{a}$, but $\mathbf{a} + \mathbf{a} + \mathbf{a}$ is undefined).

We need to deal more carefully with the piece of terminology 'self-adding a vector' and 'summing a vector to itself'. Notice that Idempotence – as we understand it here – entails that composition is reflexive: if some y is the sum of x alone (C(x, y)), then $y = x$. The key observation in respect to vectors is that not just one but many vectors are sums of **a** alone; and namely, just as many as the number of times we can iterate the linear sum operation on **a** without obtaining an undefined result.

To represent this more explicitly, we can use our assumption that the composition predicate with singular arguments is variably polyadic (Chapter 1, Section 1), and so reformulate Idempotence as a conjunction of statements:

> If some y is the composite of x and x (C(x, x, y)), then $y = x$ and;
> If some z is the composite of x, x and x (C(x, x, x, z)), then $z = x$ and;
> …

If true, the new formulation ensures that x is the only sum of x, no matter how many 'times' x is added to x. If C is '…is the linear sum of', some conjuncts will be false (and so will be the whole conjunction). For depending on the number of iterations, the linear sum of **a** and **a** returns either a distinct vector **c** or is undefined. When it is undefined a conjunct of Idempotence remains (trivially) true, because its antecedent will be false: for a certain number of iterations of linear sum on **a**, there is no sum of **a**. But when it is defined and distinct from **a**, then a conjunct fails: for a certain number of iterations, a vector **c** other than **a** is the sum of **a**. And so does the whole conjunction: at least some composite of **a** alone is distinct from **a**[14].

[14] In Chapter 6, we also notice that Idempotence follows from popular definitions of 'sum' adopted by classical mereologists.

The remarks I offered in regard to Antisymmetry continue to hold for Idempotence. Up to this point we have noticed that vectors with 'bare' structure relate by '... is the linear sum of' in opposition to Idempotence, because occasionally (depending on the number of iterations), the linear sum of **a** alone is distinct from **a**. We may still ask, however, whether the challenge we raise to Idempotence is conceptual or worldly in character. In particular, the challenge could hold that the concept *linear sum* allows for distinct sums of lone vectors (hence falsify Idempotence construed as a truth of concept); or it could hold that, at the actual world, some object of interest sums linearly into things distinct from it (hence falsify Idempotence construed as a worldly principle). As anticipated, I prefer the conceptual option, but this is a topic better discussed separately (Chapter 5).

2.3 Weak Supplementation

Up to this point, we understood '...is a proper part of *y*' as '...is a part of *y* distinct from *y*'. Accordingly, it is natural (though not compulsory, see below) to assume:

1. '...is a proper component of resultant **b**' $=_{df}$ '...is a component of **b** distinct from **b**';

2. '**a** overlaps (disjoins) **b**' $=_{df}$: 'some (no) **c** is both a component of **a** and a component of **b**' (notice that for some **c** to be a joint component of **a** and **b** it suffices that some two linear combinations featuring **c** in the addends deliver **a** and **b**, respectively).

Weak Supplementation (WS) states that if *x* is a proper part of *y*, then there is some other part *z* of *y* disjoint from *x*. If the above definitions (1., 2.) are sensible, component-sum satisfies WS only if, for distinct vectors **a** and **b**, if **a** is a component of **b** then there is a component **c** of **b** distinct and disjoint from **a**.

There are two difficulties for WS. First, to act in accordance with the principle, the linear combinations that deliver **b** must feature a proper component other than **a**, which disjoins **a**. It seems clear, however, that the chances of granting **b** one such depend on the basic features of our vector space. The greater the number of basis vectors whose linear closure generates the space, the higher the chances that, contra WS, all linear components of **b** overlap **a**.

Second, as Varzi (2014: S. 3.2) points out, WS entails Antisymmetry if *part* is reflexive and transitive, so (contrapositively) the negation of WS follows if *part* is reflexive, transitive and not antisymmetric. This shows that we need not consult the features of our vector space in too much depth to show that component-sum (the vector space correspondent of *part*) fails WS. Indeed, we have just noted that component-sum admits of cases in offence to Antisymmetry, so to complete our argument against WS, it remains to check that component-sum is transitive and reflexive.

Component-sum is most certainly transitive: for mutually distinct **a**, **b** and **c**, if **b** is the resultant of some linear combination from **a**, and **c** the resultant from of some linear combination from **b**, then there is a linear combination that features **a** and delivers **c**. With **x** and **y** being variables for vectors, this reads as a statement of transitivity: If $\mathbf{a} + \mathbf{x} = \mathbf{b}$ and $\mathbf{b} + \mathbf{y} = \mathbf{c}$, then $\mathbf{a} + \mathbf{x} + \mathbf{y} = \mathbf{c}$. That is, if **a** is a component of **b** and **b** a component of **c**, then **a** is a component of **c**.

One can make a case that component-sum is also reflexive, i.e. that every **a** is its own component (though this is admittedly more controversial, see below). Reflexivity holds thanks to the property of **0** (the zero vector) of returning any vector unchanged if vector-summed to it. Indeed, for arbitrary vectors **a**: $\mathbf{a} + \mathbf{0} = \mathbf{a}$. It is natural to think of the arguments of '+' (the linear combination operator) as the components of its image[15]. Thus, we obtain that **a** is a component of **a** because in at least one case of '+', **a** figures as both argument and image: $\mathbf{a} + 0 = \mathbf{a}$.

It would seem, accordingly, that WS is bound to fail for component-sum, because the latter is systematically reflexive and transitive, but occasionally symmetric. However, and unfortunately, we cannot trust *prima facie* impressions. The argument we have given lends itself to three objections which support the defenders of WS in this context.

1. First, as anticipated, the argument for Reflexivity is controversial. One may want to grant that the image of '+' in '$\mathbf{a} + \mathbf{0}$' – that is, **a** – is the *linear sum* of the arguments

[15] Equally, it is plausible that the arguments of canonical 'sum' operators are parts of their images, see Fine 2010: 567.

(taken together), but not that every argument is a component of the image. It strikes me as the correct result that **a** is a *sum* of **a** and **0**, for summing **0** to **a** should indeed 'contribute' nothing to the latter[16]. However, it is more controversial that **0** is a *component* of **a** – as it would be if we allowed arguments of '+', singularly taken, to be components of the image. A critic could deny this on Finean (2010) lines (Chapter 3): granting **0** a place among the components of **a** would go against the semi-reflective conditions that distinguish *part-whole* from other, merely analogous relations. Parts make a difference to what the whole is: Socrates is literally a part of its singleton, but **0** is not literally a part of **a**, for while the singleton would not be *Socrates'* singleton had it not had Socrates as a part, **a** would just be **a** even if it hadn't had **0** as a part[17].

I find the objection fair, because – as already argued (Chapter 3) – we should grant the semi-reflective methods leverage in filtering genuine from merely analogous part-whole relations. While relations like component-sum come couched in mereological vocabulary (one *relatum* is a 'component' and the other a 'sum'), this gives us only a clue for taking them as literal part-whole relations. To capitalise on this clue, we need first reject the arguments that ignore it. Some of these arguments ignore the clue by requiring for a relation's literal part-whole status that its *relata* satisfy the Finean semi-reflective conditions. To reject these arguments, I suggested that we show them to be innocuous, i.e. that the *relata* of component-sum indeed satisfy the conditions. To ensure that component-sum is a literal part-whole relation, therefore, we had better deny that it sanctions **0** as a component of non-zero vectors. Accordingly, we had better reject our premise in the argument for the reflexivity of component/sum, that is, that arguments of '+' in '**a** + **0**' are components of the image **a**.

---

16 For example, in physics **0** is not representative of any state, hence its addition to a vector that is such a representative should not be regarded as delivering anything new. See the next chapter (Section 3) for more discussion.

17 Though one could argue as follows for the claim that **0** makes indeed a difference to what **a** is: (1) **a** is a vector with 'bare' linear structure; (2) the linear algebraic definition of '**x** is a vector' is '**x** is a member of a vector space', i.e. a member of a set *V*, all members of which satisfy the vector space axioms in relation to all members *y* of *V* and all members of a scalar field $c_1, \ldots, c_n$ (for this definition, see Chapter 5, Section 1). But that **0** is a component of **a** (for any **a** ∈ V) is one of the vector axioms (**a** + **0** = **a**). Hence **a** would not be 'what it is', i.e. the member of a set of axiom-satisfiers, were **0** not a component of it. Now, I will not capitalise on this suggestion. As we noticed in the previous chapter (Section 3), due to unclarity in the semantics of '…is in' and, now, 'what it is', it remains partly unclear which curiously related objects we can ban from the list of genuine parts and wholes by appeal to the Finean conditions. For the sake of argument, we assume in the main text that **0** makes no difference to what **a** is, hence that the conditions clearly ban it from the list.

2. We also know that WS is one among *many* decomposition principles, each articulating in a different way the intuition that proper parts come with remainders (Chapter 1, Section 8.2; Chapter 2, Section 3). A prominent alternative is the principle of Strong Supplementation (STR), which rendered in component-sum language states: if some vector **a** is not a proper component of a vector **b**, then **b** has a component **c** that shares no components with **a**. Now, arguing against WS from the order properties of component-sum does not *per se* allow us to argue against STR. For absent the Antisymmetry postulate, STR does not entail WS, hence not all arguments that conclude against WS also conclude against STR (Varzi 2014: S. 3.2).

This fact, however, undermines our rationale for arguing against WS in the first place. For though in a way different from WS, STR voices the intuition that proper parts come with remainders. And if STR remains true of component-sum, then we cannot 'amend' the intuition as a method (Chapter 2), i.e. advise against using it for guidance to conceptual truths about proper parts[18]. Rather, the concept of proper part linear algebra deploys still complies with the intuition, provided we articulate it via STR rather than WS[19].

3. Not only could a critic point out that my argument against WS fails its rationale (as it does not advise against the remainder intuition as a method). They could also claim that the argument fails to conclude against WS in the first place. For recall that in formulating WS we assumed the standard definition of 'proper part' in terms of non-identity (x is proper part of $y =_{df} x$ is a part of y distinct from y). But following Thomson (1998), Cotnoir (2010) suggests an alternative:

Proper part$_2$: x is a proper part$_2$ of y just in case x is part of y and y is not part of x.

[18] Also see Chapter 1, Section 8.2.

[19] Nor would we be able to advise against the intuition as a guide to principles that give the properties of real parts and wholes at the actual world. For even if linear algebra posited vectors that relate as proper parts to wholes (in a literal sense), the relation would not be in opposition to principles (STR) that articulate the intuition (though in a way different from WS).

This definition is equivalent to the standard one if part-whole ('part') satisfies the Antisymmetry postulate[20]. But if the postulate fails, as it does for component-sum, then Proper $part_2$ can fail to classify a part *x* of some distinct *y* as a proper part of *y*. This will happen, specifically, when *y* is also part of *x*[21]. As we have just seen (Section 3.1), distinct vectors can be mutual components in just this way. Provided the vector space has enough members, the resultant **c** of a linear combination from **a** and some reminder **b** features in combinations with distinct remainders **d** that return one of its components (say, **b**). The new definition (Proper $part_2$) predicts that vectors **b** relating to **c** in this mutual way are not proper parts of **c**. Consequently, on the new definition we cannot 'test' WS on **b**, i.e. show that **b** is a proper part of **c** without appropriate reminders – because **b** is not a proper part of **c** to begin with.

If a vector space is sufficiently rich, i.e. contains enough linear sums and scalar products, then all of its members could be like **b**: linearly combine (with some remainder **a**) only into distinct vectors **c** that, in turn, linearly combine into **b** (together with some remainder **d**). Under Proper $part_2$, the component-sum relation that relates vectors of these richer spaces satisfies WS. No vector whatsoever is a proper part that lacks distinct and disjoint remainders, for no vector is a proper part to start with.

In three ways, therefore, one can resist my argument that component-sum is incompatible with WS: 1. concede that the argument concludes against WS but deny that component-sum is a literal part-whole relation; 2. concede that the argument concludes against WS but deny that it amends the 'remainder' intuition; 3. adjust the definition of 'proper part' and deny that the argument concludes against WS.

To answer these three objections it will be helpful to study another algebraic relation of interest: the special case of component-sum in which all of a vector **c**'s components are vectors **a** that can return **c** as the linear sum of themselves only: **a** + **a** + … + **a** = **c**[22].

[20] The two definitions agree that *x* is not a proper part of *y* if *x* = *y*. Moreover, given Antisymmetry, there are no distinct mutual parts: if any distinct *x* and *y* relate as part to whole, then the relation is not mutual. Either *x* is part of *y* or *y* is part of *x*, but not both. Therefore, Proper $part_2$ cannot fail to classify *x* as a proper part of *y* if *y* is distinct from *x* and has the latter as a part.

[21] Cf. Donnelly (2011: 233).

[22] As linear sum ('+') is not idempotent (Section 3.2), **c** ≠ **a**.

3. I-splits

Let us introduce an operation whose domain and co-domain is the entire vector space. The operation picks one vector (**a**) and outputs a sequence of vectors ($\mathbf{b}_1, \ldots, \mathbf{b}_n$). Each output vector ($\mathbf{b}_i$, for $1 < i < n$) I name an *idempotent split* ('i-split') of **a** and define it as a vector such that for some positive integer *n*, *n* iterations of vector summation of **b** to **b** itself delivers **a**. Thus if $n = 1$, then one iteration of self-addition to **b** will re-deliver **a**: $\mathbf{b} + \mathbf{b} = \mathbf{a}$. If $n = 2$, then two iterations will re-deliver **a**: $\mathbf{b} + (\mathbf{b} + \mathbf{b}) = \mathbf{a}$; If $n = 3$, then $\mathbf{b} + (\mathbf{b} + (\mathbf{b} + \mathbf{b})) = \mathbf{a}$, and so on (notice that *n* need *not* be a member of the scalar field used to identify the vector space). We also admit the special case of $n = 0$. With $n = 0$, the i-split of **a** is the vector **b** that delivers **a** if added to the zero vector. This special case guarantees that every vector has itself as one of its *i*-splits: for every vector is such that adding itself to the zero vector delivers itself[23] (on the contrary, not all vectors have i-splits distinct from themselves: there might no pair <**b**, *n*> such that **b** is a vector, *n* is a positive integer and *n* iterations of vector addition of **b** to **b** delivers **a**)[24].

Consider now the relation (binary, and with singular argument) *i-split/vector*: the set of ordered pairs <**i(a)**, **a**>, such that **a** is a vector and **i(a)** is an *i*-split of **a**. It is – as just seen – a reflexive relation, for every vector is its own i-split[25]. It is also transitive – as one may see from the fact that *i*-splits re-deliver the original vectors as addends of vector summation. Suppose that **b** is an *i*-split of some **a** (with $n = j$) and that **c** is an i-split of **b** (with $n = k$). This delivers $\mathbf{b} + \mathbf{b} + (\ldots) + \mathbf{b} = \mathbf{a}$ and $\mathbf{c} + \mathbf{c} + (\ldots) + \mathbf{c} = \mathbf{b}$, which entail, respectively, that **b** is an i-split of **a** and that **c** is an i-split of **b**. By the transitivity of identity: $\mathbf{a} = \mathbf{c} + \mathbf{c} + (\ldots) + \mathbf{c}$, which entails that **c** is an i-split of **a**.

---

[23] As for **0** itself, the definition entails that it is no i-split of any non-zero vector. It is, however, its own i-split.

[24] Moreover, some vectors might have multiple i-splits, each of which added to itself *n* times re-delivers the original vector – whence the allowance for multiple images of the i-split operation.

[25] That i-split/vector is reflexive and '+' is not idempotent jointly preclude us from defining 'vector sum' in a popular mereological way: '**c** is the sum of $\mathbf{a}_1, \ldots, \mathbf{a}_n$' $=_{df}$ each $\mathbf{a}_j \in \mathbf{a}_1, \ldots, \mathbf{a}_n$ is an i-split of **b** and every i-split $\mathbf{a}_i$ of **c** shares an i-split with at least one of $\mathbf{a}_1, \ldots, \mathbf{a}_n$ (the Tarski-Lewis definition). Under this definition, '+' is idempotent if i-split/vector is reflexive. Suppose that **b** were not the sum of **b** ($\mathbf{b} + \mathbf{b} + \ldots + \mathbf{b} \neq \mathbf{b}$). By reflexivity, any **b** is an i-split of **b**. Moreover, every i-split $\mathbf{a}_j$ of **b** (including **b**) shares an i-split with **b**, namely $\mathbf{a}_i$ itself. Thus by the Tarski-Lewis definition, **b** is a sum of **b**. A contradiction, which forces us to deny a premise. Reflexivity holds of i-split/vector by definition and one cannot negotiate the non-Idempotence of '+'. Thus the only way out of the *reductio* is dropping the Tarski-Lewis style definition for 'vector sum'.

Granted certain conditions[26], i-split/vector is also anti-symmetric: if **b** is an *i*-split of **a** and **a** ≠ **b**, then **a** is not an i-split of **b** (or equivalently: if **b** is an i-split of **a**, then **a** is an i-split of **b** only if **a** = **b**). Antisymmetry holds because the value of *n* indicates how many times we need to add **b** (the i-split) to *itself*. Suppose (this being the special condition I alluded to) that the scalar field we define the vector space on is the set of real numbers, and assume that *x* is an arbitrary (non-zero) real. Suppose there exists i-splits **y** of *x***a** with *n* = 1, such that **y** = ½ *x***a** and that **y** + **y** = ½ *x***a** + ½ *x***a** = *x***a**. Now, no matter what real number *x* is, *x***a** is not an i-split of ½ x***a***, because no sequence of self-additions of *x***a** delivers ½ *x***a**. In fact, there is no *positive integer n* such that *n* (*x***a**) = ½ *x***a**. It follows that although **y** (i.e. ½ *x***a**) is an *i*-split of *x***a**, the converse does not hold: *x***a** is not an i-split of **y**. The argument applies to arbitrary non-zero reals *x* and arbitrary vectors **y** and **a**, such that $\mathbf{y} = \frac{1}{n}x\mathbf{a}$, that is: such that **y** is an i-split of *x***a**. We can thus generalise to: for any two vectors **y** and **a**, if **y** is an i-split of **a**, then **a** is not an i-split of **y**, unless **a** = **y** (in which case **a** would be an i-split of **y**, as every vector is its own i-split) and unless *x* = 0.

The latter is *almost* a statement of Antisymmetry for the relation i-split/vector. To turn it into one such statement, we need to check that with *x* = 0, **a** is not an *i*-split of **y** unless **a** = **y**. And indeed we can check this. By the vector space axioms, 0**a** = **0** (that is, a vector multiplied by the null scalar gives the zero vector). Is **0** an i-split of **y**? Not if **y** is distinct from **0**, for no self-addition of **0** delivers **y**. And if **y** is itself **0**, then certainly **0** is an i-split of **0**, because all vectors are their own i-split. However, this is perfectly compatible with Antisymmetry.

It is attractive to think of i-split/vector as a member of the part-whole rubric, i.e. a part-whole relation in a literal sense. Indeed, all cases of i-split/vector are cases of component-sum and all cases of component-sum come couched in mereological vocabulary in the sectors of application of linear algebra. Absent arguments that ignore this linguistic clue (Chapter 2, 3) we can capitalise on this vocabulary to include i-split/vector in the rubric.

Including i-split/vector in the rubric allows us to re-gain at least one core principle lost with component-sum: the Antisymmetry postulate. No challenge arises against this postulate – construed conceptually – from the fact that linear algebra deploys a concept of part-whole with the properties of i-split/vector. Nor does a challenge arise

---

[26] Notably (see below) that we define the vector field on the field of real numbers. I leave it a task for another day to find a general proof, indepedent of choosing a particular scalar field.

against the postulate – construed worldly – from linear algebra positing objects that relate literally as part to whole by relating as i-split to vector.

Having said this, however, i-split/vector deviates from the core principles exactly where component-sum failed to do so: namely, in respect to WS. Let us see why. Transposed into the framework of i-splits, and under the standard definition of 'proper part', WS states: if **y** is an i-split of **x**, then there is some other i-split **z** of **x**, distinct from **y**, that has no i-splits in common with **y**. Let us retain the assumption that the field defining our vector space is the set of all reals. For arbitrary reals *x* and positive integers *n*, $\frac{1}{n}\, x\boldsymbol{a}$ indicates an arbitrary i-split of (*x***a**), for self-adding $\frac{1}{n}\, x\boldsymbol{a}$ *n* times delivers *x***a**. Consider two positive integers *j*, *k*. $\frac{1}{j}\, x\boldsymbol{a}$ and $\frac{1}{k}\, x\boldsymbol{a}$ are also i-splits of x*a*, and do have i-splits in common, no matter what *j* and *k* are. In fact, $\frac{1}{jk}\, x\boldsymbol{a}$ is an i-split of $\frac{1}{j}\, x\boldsymbol{a}$: summing the former to itself *k* times delivers just the latter. And vice versa, $\frac{1}{jk}\, x\boldsymbol{a}$ is an i-split of $\frac{1}{k}\, x\boldsymbol{a}$: suffice it to add to former to itself *j* times to deliver the latter. The procedure generalises for all positive integers *j* and *k*, showing that any two distinct i-splits of (*x***a**) have i-splits in common, contra Weak Supplementation.

So i-split/vector fails WS under the standard definition of 'proper part' (given in terms of identity). Notably, it also does so under Thomson's and Cotnoir's definition, Proper part$_2$. For present Antisymmetry this definition agrees with the standard on what counts as a proper part of what. Proper part$_2$, in particular, cannot fail to classify as a proper part of *y* a part of *y* distinct from it (Section 3.3). Therefore, under both definitions the i-splits $\frac{1}{j}\, x\boldsymbol{a}$ and $\frac{1}{k}\, x\boldsymbol{a}$ of *x***a** count as proper parts of the latter, failing WS in respect to it. Indeed, $\frac{1}{j}\, x\boldsymbol{a}$ has no appropriate reminder, for another (arbitrarily chosen) proper part of *x***a** (i.e. $\frac{1}{k}\, x\boldsymbol{a}$) shares with the former at least an i-split (i.e. $\frac{1}{jk}\, x\boldsymbol{a}$).

That WS does not fail under Proper part$_2$ was one of the objections we could raise to the view that component-sum offends against WS. Another was that even if WS failed for component-sum, this failure would not amend the 'remainder' intuition, i.e. advise against using it for guidance to conceptual truths about proper parts. Indeed, as component-sum also fails Antisymmetry, lacking WS is compatible with satisfying STR, where STR is also a principle that voices the intuition. This second objection, too, ceases to apply for i-split/vector, which has the Antisymmetry postulate. As STR and the postulate jointly entail WS, all arguments against WS also conclude against STR[27]. So does, for example, the argument given in this section, i.e. that arbitrarily chosen i-splits of xa, $\frac{1}{j}\ x\boldsymbol{a}$ and $\frac{1}{k}\ x\boldsymbol{a}$, always overlap in $\frac{1}{jk}\ x\boldsymbol{a}$.

The situation is therefore as follows: no single relation of interest to linear algebra challenges both the Antisymmetry postulate and WS. One should, rather, invoke a distinctive relation to challenge each principle. Component-sum lacks the Antisymmetry postulate, so we can advise against viewing the latter as (a) true of all part-whole concepts (if linear algebra deploys deviant part-whole concepts) or (b) true at the actual world of all parts and wholes (if linear algebra posits some objects of interest that relate as parts to whole in offence to the postulate, e.g. vectors with 'bare' vector space structure).

Having said this, one can deny that component-sum fails WS (by adjusting the definition of 'proper part'); or concede that it fails WS but deny that the failure raises enough of a challenge, i.e. advises against using the intuition underpinning WS (the 'remainder' intuition) for conceptual or worldly principles about proper parts.

For a relation that fails WS and raises a challenge of this type, we must turn to i-split/vector. Coming without remainders, or at least remainders understood via WS and STR, i-splits offend to the intuition underpinning these two principles that proper parts are never unaccompanied. Now, WS and STR do not (admittedly) exhaust our options for articulating the intuition (see next section)[28]. However, as I have already argued in Section 4 of Chapter 2, their failure for i-split/vector makes it more plausible to approach the intuition with an amending attitude, that is, to advise

---

27 For an example of i-split/vector without STR, consider a vector space whose scalar field is the set of all integers (positive and negative). For any member **a** of this space, 4**a** is also a member of it, and has **a** and 2**a** as i-splits. According to STR, every **y** that does not have **x** as a proper i-split has at least one i-split disjoint from **x**. But while 4**a** is not an i-split of 2**a**, satisfying the antecedent of STR, none of 2**a**'s i-splits disjoins 4**a**. For one, 2**a** itself is an i-split of 4**a**, and so is **a**.

28 Chances are that i-splits offend against more articulating principles. These include the principles of Company and Strong Company (Chapter 1, S. 8.2), as well as Gilmore's (2009) Quasi-supplementation (QS): If *y* has any proper parts, then some two disjoint *x* and *z* are proper parts of *y*. The two company principles require that every proper part *x* of *y* comes with a distinct proper part *z* of *y*, and the 'strong' version adds that *z* ought not be a part of *x*. They both fail because some vectors have single i-splits. For example, if the scalar field includes only the integers, then **a** is the only i-split of 2**a** (and 3**a**). As for QS, it fails for the same reason as WS. If a vector *x***a** has any i-split $\frac{1}{j}\ x\boldsymbol{a}$ (where *x* is a real), then any *i*-split $\frac{1}{k}\ x\boldsymbol{a}$ of *x***a**, arbitrarily chosen, overlaps $\frac{1}{j}\ x\boldsymbol{a}$ in $\frac{1}{jk}\ x\boldsymbol{a}$.

against understanding it as a guide to what holds of proper parts conceptually or worldly.

Whether we should block the worldly or conceptual version of WS and STR (as well as of the Antisymmetry postulate and Idempotence) is a question for the next chapter. To conclude the present one, I would like to discuss an alternative take on the 'remainder' intuition, which has it that not even i-split/vector offends against it.

4. Bennett's 'slotted' part-whole

All i-splits are linear components of their vectors, for to be an i-split **i(a)** of **a** is to return **a** via a linear summation all of whose arguments are **i(a)**: **i(a)** + **i(a)** + **i(a)** ... + **i(a)** = **a**. Therefore, i-splits also relate to their vectors by component-sum. But cases of component-sum are couched in mereological vocabulary and, absent arguments that ignore this clue (Section 3.1), we can present them as literal (though deviant) cases of part-whole. This literal status prompts a suggestion: that i-split/vector is a special part-whole relation, which according to Bennett (2013) we can describe as holding between some *x* and *y* not just once, but 'multiple times over'.

By way of analogy, Bennett (2013: 83) considers the relation *being three feet from*, which "can hold multiple times between the same two entities, [e.g.] two antipodal points on a sphere, such that the shortest distance between them along the surface is three feet". Infinitely many arcs on the sphere connect the antipodal points in such a way that their distance on the surface is three feet. Accordingly, the relation of being at one such distance holds 'several times' of one point relative to the other. And similarly with part-whole: one thing can be part of another not just once but 'repeatedly'.

Of course multiply distant antipodal points are supposed to offer only an initial, analogical grip on this notion, which Bennett intends to elucidate[29]. For Bennett, the

[29] Parts 'twice over' have a main selling point: they are intended to handle Armstrong's theory of structural universals (1997) (this theory interests us here only marginally, because our main case-study for science-driven deviance in composition comes from vector spaces). For Armstrong, the universal methane (CH_4) has constituent universals *carbon* (C) and *hydrogen* (H), but this case of constituency cannot adhere strictly to classical standards, that is: classical extensional mereology (as Lewis (1986a) famously points out). If it did, CH_4 would be identical to the universal *hydrocarbon* (CH) by virtue of having the same constituents of the latter (C and H). One possibility is to deny the classical extensionality principle, but there remains an

thesis that objects can be parts of a whole 'twice over' is compatible with 'core' mereological principles: notably, versions of WS and STR (see below). But there is a price to pay for the compatibility: we need to accept that there is more to the part-whole relation than standing as a part to a whole in a primitive, not further illustrated way. Rather, so says Bennett, if some *x* is part of *y*, then *x* is an occupier of one of *y*'s 'slots'. Occupying a whole's slots is akin to occupying a role, say – as she has it – being in pain or being the President of the United States. The analogy runs as follows (Ib. 86): Parts can overlap distinct objects at the same time, and so can role-occupiers simultaneously occupy distinct roles. Besides, roles may have different occupants at different times, and so can things be parts of distinct things at distinct times[30]. Finally, roles have only one momentary occupier, and so every mereological 'role' or 'slot' associated with an object features only one part at a time.

Granted these similarities, a natural suggestion for Bennett is that to be part of a whole is like being the occupier of a role. To occupy a role is to be entrusted with the causal powers associated with the role (e.g. the prerogatives of the US President), that is: to be causally and nomologically related to other things (people, institutions, etc.) capable of being causally acted upon. Similarly, to occupy a part-slot, or an object's 'mereological role', is to take part in a relational net: not the net of things we may act causally on by virtue of occupying a role, but rather the net of things we relate to mereologically by virtue of being some thing's part. This net includes the whole we are a part of and the parts that we in turn have if we are a part of that whole.

Accordingly, rather than with '...is a part of', we then start with the primitives 'occupies' and 'has a slot', each of which is irreflexive, asymmetrical and transitive[31]. Part-whole becomes defined as:

---

explanatory demand as to how the universals can differ if their difference is not in constituents (Bennett 2013: 96, but see Hawley 2010: 24-5 for a reply and her S. 5 for a view that claims additional modal advantages to dropping extensionality). The answer, for Bennett, is that CH_4 has H as a constituent four times over. It bears emphasis that, according to Bennett, a theory that allows for parts multiple times over is compatible with extensionality. The compatibility, however, comes at Bennett's own terms. In particular, the theory is only compatible with the principle if we re-conceptualise '*x* is a part of *y*' as 'x occupies a mereological role within *y*', in the way illustrated in the main text.

[30] These aspects of the analogy quite evidently assume that the compared part-whole relation is time-indexed.

[31] The formal properties of 'is a slot' and 'occupies' are not introduced *ex novo*, but follow from three axioms (p. 92), which state, respectively: (1) that a *y* occupied by some *x* is a part-slot of some *z* ("only slots are filled"); (2) that an *x* occupying some *y* is not a part-slot of anything ("slots cannot fill"); and (3) that a slot *x* of *y* does not itself have slots ("slots don't have slots").

Part as slot-occupation: *x* is part of *y* $=_{df}$ *y* has a slot *z* and *x* occupies *z*.

which immediately makes sense of entities being parts *n* times over. For the latter it will suffice, indeed, that some whole *y* has *n* slots $s_1, \ldots, s_n$ and a single thing occupies all such slots.

In the interest of brevity, we need to introduce only parts of Bennett's system that bear upon our present interests. Particularly:

(1) Proper part$_3$: *x* is a *proper part* of *y* not (as we normally take it) just in case *x* is a part of *y* distinct from *y*, but rather just in case *x* is a slot-occupant of *y* and *y* is not a slot-occupant of *x*.

(2) If *x* occupies a slot z_1 of *y* and *y* occupies a slot z_2 of *x*, then $x = y$. For Bennett, this principle works as an axiom (*Mutual occupancy as identity*, see p. 93) and entails that part-whole is antisymmetric. Indeed, by *Part as slot occupation*, the antecedent of this conditional is equivalent to *x* is part of *y* and *y* is part of *x*, delivering the antisymmetry statement.

Distinguishing between a whole's slot-structure and occupants sanctions the view that some things are parts 'twice over' while concurrently recovering certain core principles. For simplicity and coherence with the previous sections, we look only at some simple cases. First, the principle in (2) illustrates the recovery of part-whole Antisymmetry. Second, Bennett considers[32]:

Slot-WS: If *x* is a proper part of *y*, then some *z* is a slot of *y* but not a slot of *x*.

[32] We are simplifying Bennett's point to some extent. She does not simply 'consider' Slot-WS. In parallel to the classical mereologist, she subscribes (Ib. 96) to the doctrine that wholes cannot be mutually distinct without their distinctness being reflected by their mereological structure. This thought motivates classical theorists to use Strong Supplementation as an axiom. But we can equally understand the doctrine as a claim about slot-structure: all distinct wholes have mereological differences broadly construed, i.e. intended as differences in their slot-structure. Hence we take as an axiom not the classical Strong Supplementation, but its slot variant: 'If some *z* is a slot of *x* and *y* is not a part of *x*, then some slot of *y* is not a slot of *x*'. In turn, *this* principle *entails* the slot variant of WS.

that is, the slot-equivalent of canonical Weak Supplementation. The reason to take the slot-equivalent rather than the canonical principle is, expectably, that *Part as slot occupation* allows for parts multiple times over, and this in turn is incompatible with canonical WS: a whole with only some proper part *x*, no matter how many times over, has no proper part distinct and disjoint from *x*.

Let it be clear that these remarks illustrate only the surface of Bennett's proposal[33]. The surface, however, is enough for our limited focus on the core principles and the challenges sourcing from i-split/vector. For suppose that i-splits were parts of their vectors 'multiple times over', according to (the surface of) Bennett's scheme. On this assumption, every proper i-split **i(a)** comes with remainders, because it occupies multiple slots of the vector **a** it sums into (in particular, **i(a)** occupies as many slots of **a** as the factor *n* that multiplies **i(a)** to deliver **a**). Slot-WS, says Bennett, "recaptures some of the *intuitive force* behind the original [i.e. canonical WS]" (Ib. 85)[34] and "captures at least some of what drives the original [principle]. ... [It] require[s] that there be a remainder between a whole and each of its proper parts" (Ib. 97-8).

Latching onto the previous sections, we could translate this as follows: Slot-WS articulates in a particular way an intuition as to what it takes for an object to be a proper part of some other, i.e. that it never comes unaccompanied. Canonical WS and STR also articulate this intuition (in their own specific ways), but fail for i-split vectors. Absent WS and STR (and other popular decomposition principles, see Fn. 27), I find it legitimate to deny the intuition guidance to conceptual or worldly truths about proper parts: conceptual if in describing i-splits linear algebra only introduces a new concept of proper part; worldly if linear algebra posits objects of interest which, at the actual world, relate as i-splits to linear sum.

However, given that i-splits satisfy Slot-WS, I might be wrong in drawing these large-scale conclusions against the intuition. For just like its canonical cousins Slot-WS voices the intuition in a distinctive way. We might find, therefore, that evidence from i-splits speaks not so much against the intuition *per se* (i.e. its guidance role for conceptual or worldly truths about proper parts), but rather only against the

---

[33] Bennett herself deems her proposal incomplete owing to lack of composition principles, see (Ib. S. 7 and Cotnoir (2013)).

[34] My emphasis.

canonical articulations. Accordingly, this pushes us back to one of the objections[35] in Section 3.3: i-splits fail only the canonical decomposition principles (WS and STR), not all principles (Slot-WS) with some claim of articulating the 'remainder' intuition.

Having spotted the familiar objection, how should we respond? My inclinations are concessive: I see no major opposition against i-splits being parts of their vectors multiple times over[36]. Besides, I am ready to grant that i-splits would satisfy Slot-WS if they were parts of their vectors 'multiple times over', and also that Slot-WS articulates the 'remainder' intuition, blocking my use of i-splits to argue against the intuition's guidance. Thus in responding to the objection I deny none of these claims. Rather, my response is that i-splits understood as parts 'multiple times over' continue to challenge WS and STR. The challenge they pose, however, has nothing to do with amending the intuition that proper parts come with remainders.

Let us develop this point in more detail. When we ask whether i-splits are parts 'multiple times over', or whether Slot-WS voices the 'remainder' intuition, we already understand 'part-whole' and 'proper part' in Bennett's own terms, connecting part-whole to slot occupations: a part of *y* is a slot-occupant of *y* and a proper part of *y* is a part of *y* such that *y* is not part of it. So, prior to asking about the intuitive legitimacy of Slot-WS, or about i-splits as proper parts 'multiple times over', we need to agree that the subject matter of our questions is 'proper part' understood as non-mutual slot occupation (as per Proper $Part_3$ above).

Now, a principle is 'core' if it holds conceptually of parts and wholes, if it has been largely perceived of as holding conceptually, or if it voices intuitions as to what it takes for some thing to be part of another (Chapter 1). To 'amend' a core principle with naturalistic evidence (e.g. the properties of i-splits) one needs to show that the principle holds conceptually but not of worldly parts and wholes (say, because linear algebra posits objects that relate as part to whole in offence to the conceptual principles), or that its conceptual or intuitive status has been misperceived (say, because linear algebra deploys a concept of part-whole, or proper part, that lacks the principle).

Coming now to i-splits, the price for having them satisfy a remainder principle (Slot-WS) is that we view them as parts 'multiple times over', i.e. occupiers of their

---

[35] Though in Section 3.3 we raised this objection to component-sum, not i-split/vector.

[36] Not, at least, opposition specific to i-splits partaking of Bennett's framework. For general objections to the framework, see Cotnoir (2013).

vector's slots. Yet one can ask whether it is conceptually or intuitively legitimate for part-whole to be a relation akin to slot-occupation, that is: whether it respects concepts or intuitions to think of parts as standing to wholes in a way similar to how occupants stand to the roles they occupy (notice that if a role is – as it is sometimes taken to be – a second-order functional property, then this amounts to comparing an object-object relation to an object-property relation). For Bennett (Ib. 102-3), there is no offense to concepts or intuitions:

> So, facing the question head on, do I honestly think that objects can have a part twice over? No. But – and this is the crucial point – I do not think that I am entitled to that conclusion on the basis of the nature of parthood. That is, I do not think it is either obvious or analytic [i.e., presumably, intuitive or conceptual] that objects cannot have parts twice over.

Should we disagree? Should we say that the violation of conceptual or intuitive canon kicks in at this early stage? That seems to me a bold point, which assumes an inflexible view of what holds conceptually or intuitively of part-whole. So I suggest we make a concession. We allow that it causes no offence to concepts of part-whole to view parts in analogy to role-occupiers (according to Bennett's own points of analogy). Even so, viewing parts in this analogical way offends what many *perceive* to be constraints on part-whole's conceptual content or constraints on what holds intuitively of objects that are parts of some whole.

The evidence from the algebraic account of i-split/vector does not oppose the view that we use our intuitions for guidance to remainder principles, for i-splits construed as parts 'multiple times over' satisfy Slot-WS, that is, a principle which voices the remainder intuition in a distinctive way. The evidence is strong enough, however, to change our perception of what holds conceptually or intuitively of objects that stand as part to whole. *Prima facie* it would seem to hold conceptually that parts are unlike role-occupiers, and it would seem to hold intuitively that the remainders proper parts come with are additional parts (as they are according to WS and STR), not additional slots. Yet i-splits construed as parts multiple times over are akin to occupiers and supply slot-like remainders, for in accordance with Slot-WS the whole vector $\mathbf{a} = n \cdot \mathbf{i(a)}$ has distinct and disjoint slots, not distinct and disjoint i-splits. So understood, the evidence is correctly rendered as evidence 'amending' core principles, and namely all principles that like WS and STR posit part-like remainders and are incompatible with *Part as slot occupation*.

5. Conclusion

The linear algebraic relations holding between vectors with 'bare' vector space structure raise a number of challenges to the core principles. Most straightforwardly, the seemingly part-like relation of component-sum fails the Antisymmetry postulate and the composition-like relation of linear sum fails Idempotence. Thus a vector can function as component of one of its distinct components (S. 3.1) and the linear sum **b** of a single summand **a** can be distinct from **a** (no matter the number of iterations of linear sum on **a**: **a** + **a** ... + **a** = **b**) (S. 3.2).

These cases lead us one step closer to denying that Antisymmetry (Idempotence) holds conceptually of parts and wholes (composers and composites). The remaining step, which we attempt in the next chapter, consists in an argument that linear algebra's deployed concepts (component-sum, linear sum) are no *sui generis* algebraic concepts, but rather mereological concepts *tout court*, that is, concepts of part-whole. Similarly, the case-studies of this chapter lead us one step closer to denying that Antisymmetry (Idempotence) holds *worldly* of all things that relate as part to whole (composers to composite). The remaining step, which we attempt in the next chapter, is an argument that linear algebra not only deploys mereological concepts *tout court*, but also posits objects (vectors with 'bare' structure) that relate as parts to whole (composers to composite) in offence to Antisymmetry (Idempotence).

I found more difficulties in arguing from the linear algebraic relations against 'remainder' principles. One can resist arguments, according to which component-sum fails WS; and alternatively, one can argue that component-sum fails WS but satisfies STR, so that at least some principle voicing the 'remainder' intuition continues to guide us in theorising about concepts of proper parts and the features of entities that relate as proper part to whole. The same situation ensues when we consider i-split/vector as the algebraic relation that provides evidence against the supplementation principles. It would seem, initially, that i-splits (**i(a)**) relate to the (distinct) vector they sum into by a relation with no remainders – of either WS's or STR's kind. Though this is correct, one can reinstate the point that some principle true of i-split/vector – Slot-WS – voices the 'remainder' intuition in a distinctive way, hence that the intuition continues to guide us in picking out conceptual or worldly properties of proper parts. Slot-WS, however, comes in exchange of construing i-splits as parts 'multiple times over'. While saving the guidance of the 'remainder' intuition, this construal impacts other attractive principles, which one would think of as guides to conceptual or worldly properties of proper parts. Specifically, these principles state that parts are unlike role occupiers and that the remainders parts come with (if at all distinct from the whole) are themselves additional parts.

V

Vectors: the concept and the entity

The previous chapter remarked that algebraic relations like component-sum and i-split-vector can contribute evidence of two kinds – conceptual and worldly – against the canonical principles of Antisymmetry, Idempotence and Weak Supplementation. The evidence is conceptual if in describing vectors as *relata* of these relations we deploy a concept of part-whole (or composition) that fails the principles, i.e. a concept such that it does not follow from '*x* is part of *y*' that *x* and *y* are mutual parts only if identical, that *x* is a proper part of *y* only if *y* comes with a distinct and disjoint remainder, etc. The evidence is empirical, on the contrary, if we not only describe vectors with this deviant conceptual resource, but also posit some entities of interest that satisfy the deviant concept and relate as parts to whole (or composers to composite) in opposition to the principles (e.g. one being a part of another, which is in turn part of it, or a proper part without remainder).

In the scope of this chapter is the question of whether evidence from the linear algebraic relations is conceptual or empirical. Section 1 presents an illustrative worry: for at least some entities, relating as component to sum does not suffice for relating as part to whole (in a literal sense). For although we couch these *relata* in mereological vocabulary (one is a 'component' and the other a 'sum'), additional intuitive conditions required for part-whole fail, so we cannot capitalise on the vocabulary for classifying the two *relata* as part and whole[1] (cf. Chapter 3).

For an answer to this worry, we appeal to a Lewisian (1994) thesis (Section 2.1). It is not required for an entity *x* to satisfy a concept (here: the concept of vector component) that, at the actual world, *x* has all the features that hold analytically of the concept. An *x* that lacks some of these properties is an unconventional, but still legitimate satisfier, which, in Lewis' words, "deserves" to satisfy the concept in spite of departing from ideal profile we expect satisfiers to display.

This, I argue, is an attractive way of organising our thinking about *relata* of component-sum (Section 2.3). The concept of vector component is a concept of part-whole, which all *relata* of component-sum satisfy. For a contrast, it is the world that (at times) provides non-ideal satisfiers: entities with less than all the features delivered by the concept's analysis or the assessment of intuitions about what it takes for some thing to be part of another. If successful, this strategy leaves us with conceptual evidence: for in satisfying 'vector component', some entities that do not relate as parts to whole (failing the connected intuitions), still satisfy a mereological concept, of which the core principles are not analytic consequences.

The strategy is attractive but – let it be clear – not compulsory. It states that two or more entities satisfy a mereological concept ('vector component') without relating in the world as part to whole. But an opponent might find these two claims incompatible granted certain 'unfriendly' assumptions on what it takes for a concept to be mereological (Section 2.2). We will be more inclined to accept both claims, therefore, only on friendlier assumptions.

Deciding on our assumptions – friendly or unfriendly to the view that components and sum satisfy a mereological concept without relating as part to whole – is a task I find particularly difficult (Section 3). So I try to go around it: at least some satisfiers of the linear algebraic concept ('vector component') have all of the concept's analytic features and satisfy the relevant intuitions. If a vector sum decomposes to reveal that it is itself a component of one of its (distinct) components, or if it decomposes to reveal distinct components with no remainders, then so do certain physical systems that satisfy the corresponding concept. This is (my understanding of) a point emphasised in a gradually growing literature on composition in physics, championed

[1] We discussed a similar worry in the previous chapter (S. 3.3), whereby **0** and any arbitrary vector **a** were the *relata* of component-sum that do not qualify for literal part-whole.

by Wilson (2008), Healey (2013) (and earlier on Redhead (1987)), who all, unsurprisingly, subscribe to the '*sui generis*-ness' thesis (Chapter 2) that, at the actual world, physical *objects* relate as parts to wholes in literal but deviant ways (i.e. in opposition to canonical, but metaphysically abstracted principles). Towards the end of this chapter, we polish this observation and exploit it for evidence of the worldly kind.

1. An illustrative worry

Recollecting from the previous chapter, a vector with 'bare' vector space structure is an object **x** that satisfies the 'vector space' axioms (below). Vectors with 'bare' structure are members of vector spaces (*V*) but have no magnitude or direction. They only acquire the latter when in addition to the 'bare' structure (membership in spaces and axioms) we equip them with a norm function and the operation of inner product.

The vector space axioms:

$\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$	$(\mathbf{x}+\mathbf{y})+\mathbf{z} = \mathbf{x}+(\mathbf{y}+\mathbf{z})$	$0+x = x+0 = x$
$(-\mathbf{x}) + \mathbf{x} = \mathbf{x} + (-\mathbf{x}) = \mathbf{0}$	$0\mathbf{x} = \mathbf{0}$	$1\mathbf{x} = \mathbf{x}$
$(cd)\mathbf{x} = c(d\mathbf{x})$	$c(\mathbf{x}+\mathbf{y}) = c\mathbf{x} + c\mathbf{y}$.	$(c+d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$.

For an illustrative worry about thinking of vectors with bare structure as parts, consider the set of negative and positive integers. It is closed under operations of sum and scalar multiplication which respect the vector space axioms. Further, iterating linear sum (+) twice leads to having the same integer on both sides of the operation, as a summand and as a sum: 2+3 = 5 and 5 + (-3) = 2. Thus some cases of component-sum holding between negative and positive integers violate Antisymmetry: 2 is a component of 5 and 5 a component of 2. However, it would seem obvious that this instance of component-sum cannot be one of part-whole: 5 looks nothing like a part of 2.

Inspiring this objection are early observations by Simons (1987: 13-4), who worries extensively about the unintended interpretations of axiom systems meant to describe part-whole. A certain set of principles, spelling out the formal properties and the class of *relata* for some relation *R* might be too bare to prevent that some *x* and *y*, such that *x* is *not* a part of *y*, come to stand in *R*. Now, one may avoid this concern for unintended interpretations by thinking of the principles as necessary, but insufficient conditions for applying R to any *x* and *y*. For example, the core principles of Chapter 1 (Antisymmetry, Reflexivity, Weak Supplementation, etc.) were not meant as sufficient conditions ensuring that some *relata x* and *y* of a relation with such features are one a part of the other (Chapter 1). Similarly, compliance to the vector axioms could be necessary, but insufficient to ensure that elements of a vector space relate as parts to whole.

This move granted, the challenge remains for us to make it a non-*ad hoc* claim that the integers meet the two algebraic conditions (are members of vector spaces and satisfy the space axioms) without 5 being a part of 2. If we are to deny that 5 is a part of 2, while being a linear component of it[2], the denial need not be *ad hoc*. It may not be Simons' exact concern, but it is certainly the concern of a close relative: Simons*'s.

For a first attempt to answer Simons*'s concern, we may try to concede that we have been too liberal in counting several mathematical items as vectors. Indeed, we have required (a) that vectors be represented by a (1 x *n*) or an (*n* x 1) matrix (Chapter 4, Section 1) and (b) that they obey the vector space axioms. But positive and negative integers certainly meet these conditions: each integer *m* satisfies the axioms and is represented by a (1 x 1) matrix [*m*]. As a way of de-liberalising our attitude towards vectors, we could strengthen condition (a) (representation by matrices), arguing that vectors (satisfiers of the vector space axioms) are not just represented by but are *identical* to (1 x *n*) and (*n* x 1) matrices. This would align us with a definition for vectors standard in linear algebra and would be sufficient to count integers out. In fact, an integer is not identical to a (1 x 1) matrix: multiplying an integer by an (*n* x *n*) matrix (for *n* > 1) yields an (*n* x *n*) matrix, but multiplying a (1 x 1) matrix by an (*n* x *n*) matrix is undefined.

[2] Or similarly, deny that 0 (the scalar) is a part of every positive and negative integer, while being a mutual component of it.

One reason not to fully like this reply is that it entails that only matrices count as vectors with 'bare' structure. But this limitation is unnecessary. Various objects of interest to areas of application of linear algebra satisfy the vector space axioms (Leuenberger and Keller 2009: 371; see Section 3 below) and pure algebraists usually define vectors as *members* of a vector space[3], setting no constraints on the type of objects that bear the membership relation (other than the constraint of behaving in respect to '+' in accordance with the space axioms).

Accordingly, let us hold our assumption that vectors are only represented by (and not identical to) matrices. Even so, Simons*'s worry admits of a reply. There is nothing unexpected in denying that certain satisfiers of the vector space axioms (e.g. certain integers) do not relate as part to whole. Even if we dress the instance of component-sum between 5 and 2 in mereological language (say, we name 5 a 'component' of 2), this provides only a clue, not a full argument that 5 relates to 2 as part to whole. Other factors can still block the conclusion – notably, failure to comply to the Finean (2010) pre-reflective condition of containment (Chapter 3), for it is evident (i.e. accessible by intuition) that 5 is not 'in' 2.

That not all things relating as component to sums also relate as part to whole is a view we can accept[4], for it is compatible with gathering evidence of the worldly and conceptual sort from the linear algebraic relations against the core principles. In particular, evidence of either type, worldly and empirical, remains available even granted that 5 is no part of 2. We could still have the former evidence (worldly) because while an integer summand does not relate to its sum as part to whole, vector space members of various areas of application of linear algebra could still do so (Section 3)[5]. Besides, we could still have the latter evidence (conceptual). For it is true that the satisfiers of the concept 'vector component' (e.g. integers) fail conditions we expect to obtain of parts and wholes (e.g. the containment intuition). But this is a fact about the satisfiers, not the concept. There is room to argue, indeed, that while

[3] See Axler (1997), Blyth and Robertson (2002) and, notably, Shilov (1971: 31-32), who writes: "The concept of a linear space generalises that of the set of all vectors. The generalization consists first in getting away from the concrete nature of the objects involved (directed line segments) without changing the properties of the operations on the objects, and second in getting away from the concrete nature of the admissible numerical factors (real numbers) […] The elements of a linear space will be called *vectors*, regardless of the fact that their concrete nature may be quite unlike the more familiar directed line segments."

[4] In the previous chapter, for example, we accepted that while **0** is a component of every vector it is part of none.

[5] Indeed, Simons*'s claim is that some, not all, satisfiers of the vector axioms fail to relate as part to whole.

'vector component' is a mereological concept, the world fails to supply entirely appropriate satisfiers. We explore these two routes in the rest of this chapter, arguing first (Section 2) that the concept 'vector component' has imperfect satisfiers and then (Section 3) that satisfiers other than positive integers relate as part to whole.

2. Less than ideal satisfiers

Suppose that some thing *x* satisfies the concept of 'vector component' but lacks a feature needed for it to be part of its vector sum *y*. From this situation we conclude only that the concept's satisfiers (*x* and *y*) are not 'ideal', not that 'vector component' is no mereological concept.

2.1 A quasi-Lewisian approach

Let us get clear on what it is for 'vector component' to be a mereological concept and for its satisfiers to be less than ideal. On a natural understanding[6], the concept is mereological if from '**x** is vector component of **y**' it follows conceptually that '**x** is part of **y**'. In the consequent of this conceptual consequence, 'part' picks out a relation of part-whole, which lacks the core principles component-sum clashes with, e.g. the Antisymmetry postulate. Looking back at Chapter 2 (Section 1), i.e. our discussion of what it could mean that scientific relations of part-whole are *sui generis*, the picked out relation can be either one of many part-whole relations, varying in the satisfied order properties, or the only part-whole relation, whose order properties change according to the type of *relata*[7].

The satisfiers (*x*, *y*) are less than ideal, on the other hand, if they fail some consequences ('*x* is part of *y*') of the concept they satisfy ('vector component'). This is supposed to be a coherent position: *x* and *y* do not relate as part to whole and yet

---

[6] Though see the next section (2.2).

[7] In neither case the picked out relation is the part-whole relation of classical mereology (CEM), see Chapter 2, Section 2.

satisfy a mereological concept. But how can it be? One explanation originates from Lewis (1994: 489), who invokes less than ideal satisfiers for various concepts (though not for 'vector component'). An example is the concept '…is simultaneous with'. Of simultaneous events – according to Lewis – it holds analytically that they occur at identical times independently of the observer. But there are no events at the actual world with identical times for all observers. So, if the concept has any satisfiers, none of these come with the property (occurrence at the same time for all observers) expected from the concept's analytic structure.

But does the concept have satisfiers at all? Yes, for that some events are simultaneous is a proposition of common sense (a Moorean proposition) which Lewis thinks there is little reason to discard[8]. Further, the proposition is only true if some actual events satisfy the concept. Accordingly, on Lewis' explanation less than ideal satisfiers are needed to validate the common sense proposition without committing to events that occur at identical times across frames.

Lewis' explanation combines two philosophical methods[9]: one prescribes that we first analyse concepts and only later assign them satisfiers from the actual world[10]. The second method prescribes that we maintain at least some Moorean propositions as non-negotiable 'starting points' for metaphysical enquiry (Nolan 2016: 27-8). In our case of interest, satisfiers (**x**, **y**) of 'vector component' fail '**x** is a part of **y**'. To explain how this is coherent we need to construe the latter ('**x** is a part of **y**') as a consequence of the former ('**x** is a vector component of **y**', more on this below), but we need not invoke the Moorean method. We will not say, indeed, that 'vector component' has satisfiers because it is a firm truth of common sense that there are vector components (it is obviously not). The concept has satisfiers, rather, because at the actual world some entities **x** and **y** (e.g. 5 and 2) satisfy the vector space axioms and are such that **y** is the image of a linear combination of which **x** is an argument. Satisfying the axioms

---

[8] Standardly, from a Moorean point of view (1939/1993: 166) we have little reason to discard commonsensical propositions because the latter are more plausible than any argument against them. On Lewis' more specific reasons for holding onto the Moorean propositions, see Nolan (2016: 30-1).

[9] See Nolan (2016: S. 4.2) for this helpful distinction.

[10] What makes actual entities better candidates for satisfying the concept is the number of analytic features they have: the more analytic properties satisfied, the better the candidate. I will not attempt to offer a general recipe for telling apart less than ideal, but still appropriate satisfiers from straightforward non-satisfiers. Suffice it to say that there is a genuine question as to how much a satisfier can depart from the ideal analytic profile of its concept before ceasing to be considered a satisfier of that concept.

is enough to be members of a vector space (Section 1); that is – on the algebraic definition – vectors with 'bare' structure. Therefore, all **x** and **y** that satisfy the axioms also satisfy the concept 'vector'. Further, being such that **y** is the image of a linear combination of which **x** is an argument is enough for **x** to be a component of **y**. Therefore, all **x** and **y** that satisfy this condition also satisfy the concept 'vector component'.

As there are **x** and **y** with these characteristics – for example, the integers 5 and 2 – little reason remains to doubt that 'vector component' has indeed some satisfiers. These satisfiers do not satisfy, however, '**x** is a part of **y**', meaning that if the satisfied concept ('vector component') is in the first place mereological, i.e. entails '**x** is a part of **y**' analytically, then **x** and **y** are less than ideal satisfiers. But is the concept mereological in the first place?

2.2 Evidence of the conceptual kind

As anticipated earlier on in this section, it is natural to call 'vector component' a mereological concept if '**x** is a part of **y**' follows analytically from '**x** is a vector component of **y**'[11].

But it is not trivial that 'vector component' meets this construal. There is a difference, to start with, between the claim that a concept *C* is satisfied by entities that fail some of its consequences γ and the claim that γ is a consequence of *C*. As Dowe (2000: 10-11) remarks, the former claims of entities failing γ at the actual world that they still "ha[ve] a right to the word" '*C*'; for example, of entities that relate in opposition to Antisymmetry that still have a right to the word 'part'. The latter, for a contrast, means that γ is part of the *C*'s analysis.

My arguments in Chapters 2 and 3 play in favour of the first claim. For there I observed that components and sum, which relate in opposition to Antisymmetry, are couched in mereological terms: indeed, one is the 'component' and the other the 'sum'. I took this clue to suggest, on a naturalistic policy, that a component is part of a

---

[11] Whereby 'part' picks out a part-whole relation without one or more core principles (e.g. the Antisymmetry postulate) or at least a relation that fails the principles when it applies to relate of certain types, e.g. satisfiers of the vector space axioms.

vector sum in a literal sense (Chapter 3, Section 1). However, this suggested claim could mean only that a satisfier of 'vector component' has a right to the word 'part' (provided it also satisfies the connected intuition of containment), not that it holds conceptually of '**x** is a component of **y**' that '**x** is a part of **y**'. This latter claim, indeed, is far from trivial – which is why I said in the opening of this chapter that thinking of vectors like 5 and 2 as less than ideal satisfiers of a mereological concept is an 'attractive', but not compulsory view.

Consider, in particular, a position on which the conditions γ that hold conceptually of *C* need to hold of every satisfier of *C*. On this conception we will deny that '**x** is part of **y**' is analytic to *C*. For only features of all satisfiers are analytic to the concept; or in Dowe's words (Ib.), *C*'s 'conceptual structure' includes only its 'empirical analysis', i.e. the features had at the actual world by the entities to which the concept applies literally. Given that 5 relates to 2 as component to sum but not as part to whole, on this view there is no chance of making '**x** is a part of **y**' a conceptual consequence of '**x** is a component of **y**'. What holds conceptually of '**x** is a component of **y**' is, rather, at most a set-theoretic statement: **x** is a member of a vector space *V*, which also has **y** as a member, and **x** delivers **y** if combined linearly with one or more members of *V*[12].

This view entails that 'vector component' is no mereological concept, hence that uses of the concept in advanced science contribute no evidence of the conceptual kind. A concept used in science offers evidence of this kind only if, in addition to failing the principles, it is also mereological. Yet now the only concepts that fail (e.g.) the Antisymmetry postulate are advanced concepts of linear algebra, which on this view we need to classify as *sui generis*, not mereological. Admittedly, for friends of this view the available evidence against Antisymmetry (and the other core principles) can only be of the worldly kind; that is, evidence of satisfiers relating, at the actual world, as part to whole in opposition to the principles.

But what is the alternative? How can we classify 5 and 2, which relate as component to sum but not as part to whole, as satisfiers of a mereological concept? If we deny that the consequences γ of *C* are conditions that need to hold of *all* satisfiers, we could still accept the following: while holding of only some satisfiers (or perhaps no satisfiers at all), γ describes how we are prone to think of satisfiers when we have incomplete information about what kind of things they are.

---

[12] In particular, if **x** combines with **0**, then **x** delivers **x** (and **x** = **y**). If it combines with **x**, then it delivers a vector of which **x** is a proper i-split.

For example, we are prone to think of events that satisfy '… is simultaneous with…' as events that occur at identical times for all observers insofar as we ignore that measures of time elapsing vary with the observers' relative velocity (hence can only be identical if each event's observer moves with the same velocity relative to the other). Similarly, we are prone to infer that '**x** is a part of **y**' when all we know of vector components **x** is that they relate by membership to a vector space, and are such that **x** together with one or more elements from the space (possibly **0** or **x** itself) return **y** as the image of '+' (where '+' has the property listed in the vector space axioms).

One can suggest that absent more accurate information about which entities satisfy 'vector component', the inference to '**x** is part of **y**' is legitimate for two reasons, which parallel those given in Chapters 2 and 3. The first is a linguistic clue: linear algebra omits claims about which objects satisfy 'vector component', as well as claims about the properties of any *particular* satisfier. In fact, all of its claims have the status of the vector space axioms, i.e. generalisations true of all satisfiers (insofar as they are members of vector spaces). But even so, in linear algebra **y** and **x** are couched in mereological terms, so that if **x** and **y** are respectively the image and an argument of '+', then **y** is a 'sum' of which **x** is the 'component'.

This choice of terms gives enough of a clue to suspect it is legitimate (all else equal) to infer to '**x** is part of **y**' (whereby 'part' picks out a part-whole relations with deviant features). As we should now realise, the 'legitimacy' of the inference will not consist in the consequent ('**x** is part of **y**') being true of arbitrary satisfiers **x** and **y** if the antecedent is. Rather, the consequent is true of 'ideal', but false of 'less than ideal' satisfiers. *A fortiori*, it is false of arbitrary satisfiers, because (as seen) these include less than ideal ones.

The second reason for it being legitimate in this sense to infer '**x** is part of **y**' from '**x** is a vector component to **y**' is that typical blocks to exploiting these linguistic clues from linear algebra are absent. Consider the Finean 'containment' intuition, i.e. that it holds of all parts *x* of *y* that *x* is 'in' *y* (in an intuitive and not further explicable sense of 'in'). That 5 is not 'in' 2 holds of particular satisfiers of 'vector component', i.e. positive integers. No corresponding statement '**x** is not in **y**' holds (like the vector space axioms) of arbitrary vectors, which are members of the same vector space and such that '+' delivers **y** when the arguments are **x** and other members of *V*. Nor does the negation of this statement ('**x** is in **y**') hold of arbitrary vectors that relate as component to sum, for there are cases of component and sum (like 5 being a component of 2), which defy the pre-reflective containment terminology. Accordingly, when the focus is limited to generalisations true of all vectors (which belong to the same space *V* and relate as component to sum) we find no indication of true sentences that affirm or deny the containment intuition. The containment terminology ('…is in…') is insufficiently precise to filter out component vectors **x** from the list of parts of their sums **y**.

If this is on the right line, accordingly, one can exploit the above linguistic clues (that arguments of '+' are couched in mereological vocabulary) to legitimise the inference that components are parts of their vector sums without facing the usual obstacles connected to the containment intuition. We agreed (Section 2) that having this consequence is all that is needed for 'vector component' to be a mereological concept, which evidences that not all concepts of this type satisfy the core principles.

Let us wrap up. Suppose we agreed that 'vector component' is a mereological concept if '**x** is part of **y**' is a conceptual consequence of '**x** is a vector component of **y**'. We can then argue that 'vector component' is mereological and fails the core principles (e.g. the Antisymmetry postulate) at two costs: first (i), distinguishing between ideal and less than ideal satisfiers; and second (ii), arguing that a conceptual consequence can be true only of the former, but not of the latter. While the initial agreement seems natural, there is no denying that (i) and (ii) are genuine costs. Consider, in particular, (ii). Whether 'vector component' is mereological depends on a controversial view as to what it takes to assign it conceptual consequences. The concept will not be mereological if in selecting its consequences we help ourselves to its "empirical analysis" (Dowe 2000: 10); in particular, to properties had at the actual world by all satisfiers. It will be mereological, on the contrary, if we help ourselves to linear algebra's choice of terms (i.e. 'component', 'sum') for arbitrary vectors that function as the arguments and image of '+'.

Now, on the view that conceptual consequences hold of all satisfiers, we also gather no evidence of the conceptual kind against the core principles: 'vector component' is not a mereological concept that fails the principles (e.g. Antisymmetry), because it is not a mereological concept in the first place. On the contrary, we gather the evidence if 'vector component' has its conceptual consequences determined by the linear algebraic choice of terms ('component', 'sum'). For on this view the concept is indeed mereological – has '**x** is a part of **y**' as a conceptual consequence – in spite of having less than ideal satisfiers that do not relate as part to whole.

A natural question, at this stage, is whether (and how) we can argue for the latter and against the former view, so that, indeed, 'vector component' challenges the core principles with evidence of the conceptual kind. It is a question that could lead us far afield, and namely into discussing what guidance we should accept for assigning conceptual consequences: guidance from the concept's empirical analysis or from linguistic usage?

As a proper consideration of how best to respond lies outside of the present work's scope, I prefer to go around this question. The aim I outlined in the Introduction and previous chapters (Chapter 1, 4) is to show that evidence against the core principles is of a conceptual *or* worldly kind; that is, either evidence of mereological concepts that fail the principles or evidence that some entities, at the actual world, relate as part to whole in opposition to them. We now realise that for conceptual evidence we need a specific (attractive, but not compulsory) view about what counts as a conceptual consequence of '**x** is a vector component of **y**'. To convince of our main point those who accept a rival view (i.e. that from '**x** is a vector component of **y**' follow only conditions true of all satisfiers), we can at least hope to disclose evidence of the worldly kind.

3. Evidence of the worldly kind

We agreed on appointing (1 x *n*) and (*n* x 1) matrices (where *n* is a positive integer) as mathematical representers of vectors with 'bare' vector space structure (i.e. all things that satisfy the vector space axioms). When we consider the physical interpretation of vectors, it is standard to assume that the *n* integers in the matrices represent values of independent 'characteristic' quantities. A quantity is characteristic if its values are members of the set of quantity values that constitute a system's momentary state[13]

[13] We need not accept that all vectors represent quantities of states viable for some systems. Of all Hilbert-space (H) vectors, for example, only those inhabiting finite-dimensional sub-spaces of H represent ('pure') quantum-mechanical states.

(which values figure in the set is a function of a reference theory – thus we say that each theory comes associated with its own characterstic state: the classical state, the relativistic state, the quantum state, etc.)[14]. Further, together with the *n* values in a vector's (1 x *n*) or (*n* x 1) matrix, a vector's magnitude[15] also comes to represent features of characteristic states[16]. Computed as the *n* values' norm function, the magnitude represents the characteristic momentary state of a system not as an array of values of independent quantities but as a single (positive) value functionally dependent on the values of the independent quantities.

Consider now two vectors **x** and **y** that relate as component to sum in opposition to one or more core principles, e.g. so that **x** and **y** are distinct mutual components. Also suppose that each of **x** and **y** represents, respectively, distinct states of a single system *s* or distinct states of distinct systems *s'*, *s''*. Could these states represented by vectors that relate as component to sum, or perhaps the systems that occupy them, relate as part to whole?

To answer this question positively, it will be helpful to examine an ill-suited case, which initially suggests a negative answer. Studying the decomposition of a vector, at times, reveals no deviant sub-systems of the system with the characteristic features represented by the vector sum; that is, no sub-systems that relate to it by some relation *R* (which could be argued to be part-whole) in opposition to the core principles. We can profitably rely on classical physics for one of the most familiar cases: the description of a material string's regular oscillations. We assume that a string satisfies ideal constraints: it is a system of point-particles with uniform mass

[14] See Thalos (1999) and Wilson (2010).

[15] On vector magnitudes, cf. Chapter 4, Section 2.

[16] And so do a vector's orientation and sense (cf. Chapter 4, Section 2, Fn. 4). These features will play a marginal role in what follows, so we can discuss them briefly and by means of familiar examples (see Massin 2009, Wolff 2016). In 3-dimensional Galilean space, a vector's orientation is usually the angle included between the vector and an appropriate reference frame (which intersects the vector at the origin). It represents the direction in 3-space of a system's characteristic quantity, for example, a classical system's velocity or momentum. Let it be understood, however, that orientation is by no means only a feature of vectors whose matrices and magnitudes take values for 3-space position (quantum spin vectors are legitimately oriented and yet take values from higher-dimensional quantity spaces, see Wolff (2016)). A vector's sense is an ordered pair of points on a line parallel to the vector in Galilean 3-space. For classical quantities, the order of the pair determines the bearer of those quantities' momentary values. Thus according to the appointed sense, a vector may represent the instantaneous velocity $v(s_1)$ of a system s_1 relative to another system s_2 non-inertial to s_1, or the velocity $v(s_2)$ of the latter relative to the former. The generalizability of these examples from classical states to the entire domain of characteristic states is a delicate issue which (fortunately) we need not touch upon.

density (ρ) and tension (T), perturbed at one of its extremities with constant frequency. Wave-vectors oriented in the direction of propagation (away from the perturbed extremity) describe the oscillation's wavelength (λ) and frequency (ω).

Two are the key factors to notice. First: there is actually a sequence of wave-vectors, all oriented in the direction of propagation, which differ by the magnitudes of wavelength and frequency. One such vector, called the *fundamental mode*, comes with maximal wavelengths and minimal frequencies: $\omega = \frac{1}{2} L \sqrt{\frac{T}{\varrho}}$ and $\lambda = \frac{1}{\omega}$. Additional vectors, called the *regular harmonics*, come with increasing frequencies and decreasing wavelengths: the first harmonic doubles the frequency of the fundamental mode and halves its wavelength; the second harmonic comes with quadruple frequency and a fourth of the original wavelength, and so on (in the ideal string case, the harmonic sequence is unbounded if continuum-many point-particles make up the string).

Second: all wave-vectors with harmonic-like magnitudes for frequency and wavelength combine linearly into the wave-vector with values for the fundamental mode and, similarly, various combinations of vectors in the sequence superpose into some other vector in the sequence. Thus, for example, not only the fundamental mode is the vector sum of all harmonics including the second, but the second harmonic is also the vector sum of some vectors in the sequence, including the fundamental mode. On the face of it, therefore, vectors in the fundamental-harmonics sequence seem to be related by component-sum in opposition to Antisymmetry (as is characteristic of linear combination, see Chapter 4, Section 3).

Now, the crucial question is whether this symmetric linear structure of the wave-vectors entails a symmetric structure for the target system (i.e. the ideal string) and its sub-systems, or at least of the target system's characteristic state (a pair of wavelength and frequency values) and its sub-states. It does not – presumably – if the string is an ensemble of *classical* point-particles (as assumed). Describing different frequencies and wavelengths, each harmonic describes different positions for the string. However, each of some equally massive point-particles can only occupy one momentary position[17], so, collectively, they can only exemplify one oscillatory motion – and exemplifying one motion excludes all others. The wave-vectors describing all

[17] Assuming (as is plausible) that at the actual world none of the string's constituting point-particles has travelled backwards in time and locates at distinct points throughout the string's oscillation period.

other motions stand, accordingly, for actually unoccupied but nomologically possible states of the string. As the states represented by the harmonic vectors are unoccupied (at the actual world), they cannot be taken to inform us of some internal structure of the string, which relates to it in the same deviant way as the harmonic wave-vector relates to the fundamental. On the contrary, the string's internal structure features only the constituting point-particles, which relate to it in compliance with the core principles.

Similarly, as only the string's fundamental and not its harmonic states is occupied at the actual world, there will be no token instances of the string's harmonic state, which relate to a token instance of the fundamental state in the same deviant way as the harmonic vector relates to the fundamental vector. The instance of the string's harmonic state, rather, has no other 'internal' structure as the string's particular values of wavelength and frequency, which relate to the whole state (wavelength + frequency) in compliance to the core-principles[18].

Now, notice that, if anything, this familiar case illustrates the contingency of our worry that studying the decomposition of a vector into its linear components reveals nothing deviant about the way sub-systems are parts of the systems described by the vector whose decomposition we study. In the ideal string case we seem to be guided by an eminently classical description of the system, which complements the description given by its wave-vector. Thus the string is an ensemble of point-particles, which can but occupy one position at a time, hence exemplify one wavelength and frequency of oscillation. This is why we refuse to take in information about the string's internal structure (or the internal structure of the string's fundamental state) from the components of the fundamental mode vector.

But it is no mystery – as Wilson (2008) and Healey (2013) emphasise – that wave vectors decomposable into linear modes also describe highly *sui generis* objects:

[18] Though one could argue as follows. Because harmonic frequency states are unoccupied, the fact that their representing vectors combine linearly into the fundamental wave-vector shows neither that a sub-system of the string relates to the string by a mutual part-whole relation (following the pattern of linear combination), nor that an instance of the string's harmonic state so relates to an instance of its fundamental state. However, it shows that this symmetric relation clashing with the core principles holds between a *type* harmonic state, which is not instanced by any of the string's sub-systems and a type fundamental state, which is instanced by the string. On this view, therefore, the deviant part-whole relation clashing with the core principles holds between type states. The obtaining of this relation offers the worldly evidence that some posits of advanced physics, i.e. the harmonic and fundamental states, intended as types, relate as part to whole in opposition to the core principles. As this possibility is clearly in my favour, I will continue to ignore it, focussing only on the states' occupiers and the states' instances (tokens).

lightwaves and fields (plus, decomposition into linear modes carries over to quantum field theory, with Fock-vector decompositions to describe the energetic states of the various quantum fields)[19]. It is – admittedly – difficult to give metaphysical underpinning to each of these *sui generis* entities, but exactly because of this difficulty my sense is that it is plausible to help ourselves to the harmonic vectors to describe these entities' internal structure. The wave vectors coupled with these entities lift the block posed by the ideal string model.

Given the degree of ontological revisionism usually embedded in these hypotheses, the idea that deviantly decomposing vectors instruct us about deviant decompositions of the corresponding system cannot be quietly classified as lack of philosophical training. If the entities under focus are *sui generis* systems with so far unsettled metaphysical underpinning, it seems plausible to use the resources we already possess (linear vector decomposition) as a means of information about the system's structure[20]. The alternative would be to keep what we learned from the ideal

---

[19] Redhead (1982) and Auyang (1995) use the ideal string case to illustrate the closest classical analogue of a quantum field equation (at least, it must be pointed out, for fields that admit of neat linear mode decomposition – I am grateful to Douglas Earl for repeatedly illustrating this point to me). I suggest that given the degree of ontological revision associated with field ontologies, we should *at least* deem it an open possibility that deviant vector-vector part-whole, as illustrated by Fourier analysis, corresponds to deviant subfield-field part-whole, using as support for this claim the fact that the classical 'containment barriers' (material point-particles) of classical physics have been lifted. See McKenzie (2012: Ch. 5, 6) for a similar point.

[20] Let us consider an objection. To guarantee *worldly* evidence of part-whole obtaining in opposition to the canonical principles, as we attempt to do in this section, we need to ensure not only that at the actual world some components and resultants relate as part to whole in a literal sense, but also that, in the first place, both components and resultants *exist* at the actual world. Now, influential arguments from Cartwright (1983), and later Wilson (2009), motivate anti-realism about components, concluding that at least components of a particular kind fail to co-exist with the associated resultants. These components include Newtonian and electrostatic forces, the claim being that no (token) components of (token) resultant forces exist in conjoined circumstances, such as when a Newtonian force heading North-East results out of two sub-forces heading North and East, or when two oppositely charged particles accelerate away from each other pushed by the resultant of Newtonian and electrostatic forces (Cartwright 1983: 79). Cartwright and Wilson motivate their anti-realism with distinct arguments. For Cartwright, no components exist (contra Mill 1844) because systems manifesting the kinematic effects of the resultants fail to manifest the kinematic effects of the components (see Creary 1981 for a response). For Wilson, no components exist because each of the resultant and the components is a sufficient cause of the same kinematic effects and the best resolution of the ensuing overdetermination of causes eliminates the components. Now, though Cartwright's and Wilson's anti-realism is limited to (component) classical forces, one may suspect that their arguments threaten the *sui generis* components I indicate as the main worldly sources of deviant part-whole: quantum fields and the (token) characteristic states of classical systems that manifest wave-like phenomena (pairs of wavelength and frequency values). Having noted this risk, I reply concessively. The prior task I take on in this work is to show that if any such objects exist, then they relate as part to whole in a literal sense. This is in itself a claim of interest, even if conditionalised on the failure of component anti-realism. Besides, the claim offers the beginning of a response to Wilson's argument, which deserves further exploration. For, traditionally, one resolves overdeterminations of causes not just by barring one of these causes from existence (eliminativism), but also by arguing that the two causes stand in an 'intimate' relation, such as, indeed, part-whole (Wilson 2009: 542).

string case about harmonic wave vectors, i.e. that they offer no information on the part-whole structure of material systems. But the consequences would be surprising. Without a device like wave vector decomposition, which pins down these *sui generis* entities' internal structure, the relevant physics would too easily imply that the *sui generis* entities have *no* internal structure: neither structure of the neat mereological kind (for the metaphysical underpinning of these *sui generis* entities remains unclear), nor structure of the deviant algebraic kind (for we retain the lesson from the ideal string model that harmonic vectors disclose no internal structure). This is surprising, as said, because it precludes a possibility clearly suggested – on a naturalistic policy – by the fact that the *sui generis* entities are naturally couched in mereological vocabulary: they are 'complex' fields of which harmonic components offer information on 'component' fields. We are after all used to this faltering of classical guidance, so making our preparations for more faltering seems a meticulous policy[21].

[21] This suggestion (unfortunately) is not strong enough to develop into an argument by induction. We know of certain propositions (p), (p)', each of which we presumed justified prior to a event of 'faltering' (that simultaneity is Lorentz-invariant, that systems only occupy states with definite (probability-1) values, that the number of point-particles is Lorentz-invariant, etc.). While this could yield an induction basis to expect some new proposition (p)* to falter, there is no guarantee that the faltering proposition asserts that part-whole as applied to material systems has only antisymmetric instances.

VI

Sui generis-ness and ideological profligacy

The naturalist's aim of 'appropriating' cases of component-sum seemed ultimately successful, for of vectors that relate as component to sum (and belong to the same vector space) one of two claims hold: either they have a right to mereological vocabulary and relate as a part to a whole in a literal sense; or at least, while failing to so relate, satisfy a genuine mereological concept (Chapter 5). From this chapter onward, we explore and assess the other face of the naturalist's program (Cf. Introduction): the aim of letting evidence for entities that relate as component to sum – such as the evidence we gather from the sectors of application of linear algebra – speak against the occurrence of additional, canonical part-whole relations. For example, if a vector combines linearly with another to return further elements of the same vector space, is it also part of a fully classical mereological sum? Or, for that matter, is it a part or composer of any object in ways faithful to the core principles: Antisymmetry, Weak Supplementation and Idempotence? In addition, more generally, once we agree that some elements of a vector space exemplify relations of component-sum, should we also agree that *none* of these elements relate to another as a part to whole while complying with the principles?

This chapter backs up the following claim: considerations against hypotheses that posit vectors (and linear sums thereof-) *together* with cases of ordinary part-whole – holding between these vectors – are, if anything, considerations of parsimony: i.e., motivated by principles that advise (*ceteris paribus*) against positing additional complexity (in a sense to be clarified, first superficially and then more in depth in the next chapters). In a word, the richer hypotheses bear the symptomatic marks of profligate hypotheses: familiar part-whole cases come at a price of profligacy.

This result reveals a meta-ontological deficit we can invoke in reaction to the naturalist's program: eliminative conclusions against familiar part-whole depend mostly on the stance we take on principles that prescribe a preference for parsimonious over profligate hypotheses, not directly on the evidence that certain objects of physical interest (e.g. vectors) are deviant in respect to the part-whole relationship.

1. Type-1 Compatibility

For a first attempt, we could try to argue for compatibility of the familiar part-whole relationship with vector spaces as follows: we consider no objects additional to our vector space members (hence, we stick with basis vectors and all their scalar products and linear combinations), but view some of these vectors as themselves familiar parts. Perhaps in spite of relating symmetrically to many vectors via component-sum, they also relate antisymmetrically to some vectors by familiar part-whole. We will call this type of compatibility *Type-1 compatibility*, as it demands no additions to the original vector space. If successfully established, Type-1 compatibility challenges the naturalist's eliminative ambitions, in that some of the scientifically supported but deviantly related objects (vector space elements) also exemplify canonical part-whole relations 'among themselves'.

David Lewis (1991, 1993) put forward the most illustrious and ambitious example of Type-1 compatibility project when he argued that sets and classes are mereological sums in disguise. We cannot linger too long on the Lewisian approach, but suffice it to notice a relevant characteristic (1993: 204-5). Imagine that we decided to identify cases of part-whole holding between sets and elements via the transitive ancestral of the membership relation: so that some x is part of y just in case either x is a member of y or x is a member of a member of y. By our policy, we count any thing as part of its singleton (if it has one), but this is incompatible with full-force mereology, which admits of no wholes with single parts. As a mereological monist, for Lewis there is no part-whole relation other than the mereological relation, and so it is the mereological relation that we ought to accommodate among classes and members (Cf. Chapter 2). As a result, Lewis argues that we should look for a set theoretical relation other than the membership ancestral all of whose cases 'track'[1] cases of part-whole. The most suitable relation for this purpose is the subclass relation: if x is a subclass of y, then x is a part of y (a proper part of x is a proper subclass and an improper part if x is an improper subclass)[2].

Transposed into our terms, for Lewis set theory is Type-1 compatible in that every class relates to its subclasses by the mereological part-whole relation. So licensing subject-specific deviant relations like membership (which violates Weak Supplementation in the singleton case) does not entail that we should do away with cases of standard part-whole: indeed, every case of the sub-class relation tracks one such case.

Notice that less is required for Type-1 compatibility than having cases of *subclass* track cases of part-whole. Had Lewis been interested only in Type-1 compatibility, he could have rested content with only some, but not all cases of the membership ancestral tracking cases of part-whole[3]. He would not have needed to invoke *subclass* for an additional relation, all of whose cases are capable of doing the tracking.

Call *Generality* the premise that all cases of some one relation S track cases of standard/canonical part-whole. Generality is not required for Type-1 compatibility. Set theory can still be Type-1 compatible in that *some* cases of the membership ancestral track cases of part-whole without there being a relation S (the subclass relation) all of whose cases manage to track. Similarly, perhaps linear algebra could be Type-1 compatible in that *some* cases of some linear algebraic relation L (component-sum, i-split, etc.) track cases of standard part-whole, without there being some relation L', all of whose cases manage to track. Therefore, in considering whether cases of some linear algebraic relation L track cases of standard part-whole,

---

[1] Henceforth, by saying that a case of a set theoretical or linear algebraic relation S (e.g. the transitive ancestral, component-sum, *etc.*) *tracks* a case of a mereological relation M I mean that if some x bears S to some y, then x also bears M to y.

[2] Fine (2010) denies monism and accepts the transitive ancestral as a genuine part-whole variety. As a result, he also concedes that x is a part of $\{x\}$.

[3] For discussion of Lewis' complete aims in (1991) and (1993), see (1993: 3-4).

we can envisage failures of Generality. We can consider candidate relations only some of whose cases track cases of part-whole. *Considering* failures of Generality will not automatically make it plausible that linear algebra is Type-1 compatible. We need to survey the linear algebraic relations, which are candidates for tracking part-whole while avoiding substantial problems.

1.1 Option (1): component-sum

When it comes to vector spaces, what options do we have for the tracking relation? Not many. It would be an immediately unviable proposal, for example, to suggest that parts of a vector are its linear components, that is (recall), any vectors that deliver the former vector by some linear combination. *Some* linear components have in turn linear components, which at times happen to include the vector sum itself, *contra* Antisymmetry. We have illustrated this case in the previous chapter through the parallelogram rule: vector-sums **v** whose components include **c** relate in turn by component-sum to **c**, while a standard sum D whose standard parts include *x* does not relate in turn by standard part-whole to *x*.

Under our conditions, Lewis would scrutinise linear algebra for an additional tracking relation, based on the fact that component-sum spawns deviant symmetric cases that cannot track part-whole. However, having now noticed the Generality assumption, we may try to drop it and resist the search for additional candidates. Without Generality, we simply say that only some cases of component-sum track cases of part-whole.

A first, immediate result of denying Generality is that we face a choice as to which component-sum cases to filter out from the tracking. Some choices will be arbitrary: for example, we may practice a policy according to which, if two (distinct) vectors relate by component-sum symmetrically, we let only one of the two be part of the other, but not vice versa. In the parallelogram rule case, this move predicts that although a component vector **b** (together with remainder **a**) is a linear component of a linear sum **c** and the linear sum **c** (together with remainder **d**) is in turn a component of **b**, only one of **b** and **c** is part of the other. However, seen as candidates for tracking part-whole, both the case of component-sum from **b** to **c** and the case of component-sum from **c** to **b** seem equally strong candidates. Arbitrariness is difficult to sustain. The cases of component-sum we cross out have equal candidacy to support part-whole, and this equal candidacy leaves us in a stall – we have no reason to prefer one over the other.

As another possibility, we could sanction tracking only for (distinct) components and sums that do not behave symmetrically with respect to linear combination. We may count in – that is – only pairs of vectors such that one is a linear component of the other and there is no linear component combined with which the latter returns the former. This proposal deals with the above case of arbitrariness by taking it by the horns: it entails that *no* cases of component-sum with symmetric correspondents (e.g. the case of **b** being a component of **c** and the case of **c** being a component of **b**) track cases of part-whole. It seems plausible to manage the arbitrariness in just this way, because licensing as cases of part-whole both a case of component-sum and its symmetric correspondent leaves us with deviance, and licensing only one of the two is no more motivated than licensing the other. Unfortunately, granted the correctness of its policy for handling arbitrariness, the view that only cases of component-sum without symmetric correspondents track part-whole faces difficulties with the highly deviant vector-space structure. There are no vectors whose (distinct) linear sums fail to be linear components of them and this is due to the presence of additive inverses and of the zero vector. Every arbitrary vector **a** is a component of **0** if added to its additive inverse: **a** + (-**a**) = **0**. Besides, zero is a component of every thing: **0** + **a** = **a**. Therefore, all cases of (distinct) components and sums are such that the sum, together with some remainder, returns the initial component.

Let us not give up the anti-Generality suggestion that less than all cases of component-sum can track cases of part-whole. A version of this proposal holds that the only successfully tracked cases are cases of reflexive component-sum. Every vector is its own component – thanks again to the zero vector: **a** + **0** = **a** (Chapter 4, Section 3.2). Perhaps the tracking could stop just there: only cases of component-sum between a vector and itself tracks cases of part-whole, every vector is only part of itself, like a sum or an atom that is not in turn a proper part of anything. I find this possibility promising, and will return to it in due course.

1.2 Option (2): i-split

A second category of attempts deputes the tracking not to component-sum but to one of its special cases: i-split/vector. In this proposal, vectors relate mereologically not to any but to some particular components: their i-splits, that is (recall), the vectors that return the original vectors if self-added a positive number *n* of times. This saves us from loss of Antisymmetry. In fact, no case of i-split/vector of has a symmetric correspondent (other than the identity case: every **a** is its own i-split). No vector **a** has a (distinct) i-split **i(a)**, whose i-splits **i(i(a))** include that very vector[4].

The proposal that vectors relate as wholes to parts to their i-splits must be dealt with carefully, as there are tensions with Weak Supplementation and Idempotence. Starting with the former, depending on the features of the vector space, there may be vectors that relate by i-split/vector to a single i-split. Suppose, for illustration, that our available scalars include only positive and negative integers. A vector 2**a** has only **a** as an i-split. Of course, owing to Weak Supplementation, nothing has a proper part without also relating to a distinct (and disjoint) proper part, but this is just the way in which 2**a** relates to **a**.

We may think of addressing this bug in two ways: by increasing the available scalars so that *no* vector has really only one (distinct) i-split, or (once again) by sidestepping Generality, so that even if there are cases of i-split/vector without remainder, these do not track part-whole relations. But not even this will be particularly helpful. For suppose that we deem a vector with proper parts not just any vector with i-splits distinct from it, but only vectors with at least two such i-splits - thus **4a** would have proper parts but **2a** wouldn't, because only **4a** has at least two i-splits: **2a** and **a**. This will not be enough for Weak Supplementation. Focus only on the vectors that relate by i-split/vector to two or more i-splits. Chances are that taken pairwise, these *i*-splits in turn have some i-split in common: for instance, **4a** has **2a** and **a**, but **2a** and **a** have **a** in common; and so do the i-splits of 6**a**, i.e. 3**a**, 2**a** and **a**. In fact, the argument I have given in Chapter 3 (S. 4) shows for arbitrary reals *x* and vectors **a** that any two distinct i-splits of (*x***a**), $\frac{1}{n}$ (x**a**) and $\frac{1}{j}$ (*x***a**), share common i-splits, that is: $\frac{1}{nj}$ (xa). If this common i-split is also a common part, then any two distinct i-splits also have parts in common, contra Weak Supplementation.

[4] The argument, with the scalar field being the set of all reals, is in Chapter 4, Section 4.

We now know, however, that we can try to indulge in denial of Generality, so as to take only $\frac{1}{n}$(x**a**) and $\frac{1}{j}$ (x**a**) as proper parts of x**a**, but not the former's common i-split $\frac{1}{nj}$(x**a**) as a common part of these. Unfortunately, this leads again to arbitrariness, though in a way subtler than before. There is a first level of arbitrariness in that denying that the common i-split is a common part could mean that we take it to be a part of only one of the vectors it is common to. This is quite straightforwardly arbitrary, for both vectors are equal candidates for having the common i-split as a part. But there is also a second level of arbitrariness, which we face if we take the first level by the horns and decide that common i-splits are part of *neither* vector they are common to. This move is arbitrary because we have initially decided to pick $\frac{1}{n}$(x**a**) and $\frac{1}{j}$ (x**a**) as parts of **a**, but an equal candidate vector for having i-splits as parts is $\frac{1}{n}$ (x**a**) itself. According to our policy, this reinstates $\frac{1}{nj}$(x**a**), which is an i-split of $\frac{1}{n}$(x**a**), as a part of the latter. Therefore, which common i-split fails to be a common part depends on an arbitrary decision: the decision to supply i-split-like parts to **a**, rather than to $\frac{1}{n}$(x**a**).

As a result, arbitrariness arises if we cross out common i-splits as common parts, no matter whether we think that common i-splits should be part of only one of the vectors they are common to, or that they should be part of neither. The remaining option is that they should be part of both, which as just seen creates conflict with Weak Supplementation. These considerations should suffice to illustrate dim prospects for Weak Supplementation if we were to think of the i-splits of a vector distinct from it as the latter's standard parts. We now move to Idempotence, for which the situation is equally unfavourable.

1.3 Option (3): linear summation

Idempotence (Chapter 4) is a property not of part-whole but of composition. Consequently, while so far we asked for linear algebraic relations capable of tracking standard part-whole, now we ask for algebraic relations capable of tracking standard composition. Granted Idempotence, the composite of a single composer is the composer itself, no matter how many times we add: x + x … + x = x (or equivalently, sum(x, x, …) = x, for some idempotent operator 'sum'). On the contrary, without Idempotence the composite differs from the single composer for at least some number of iterations of '+'.

Linear summation (*l-sum*) is not idempotent, for summing **a** to itself any number of times does not return **a**: l-sum(**a**, **a**, **a**, …) ≠ **a**. Contrarily, full-scale mereological summation is paradigmatically idempotent. In fact, one can check Idempotence for mereological sums directly from classical way(s) of defining the operator (Varzi 2014: S. 4.2):

> *Tarski-sums*[5]: Some *z* is the sum of *y* and *x*, just in case *y* and *x* are parts of *z* and everything that is part of *z* overlaps one of *x* or *y*.
>
> *Needham-sums*[6]: Some *z* is a sum of *y* and x $=_{df}$ everything that overlaps *z* overlaps one of *x* or *y*.

Needham-sums entails Idempotence as a matter of Leibniz' Law: for any *x*, clearly any thing that overlaps *x* overlaps *x*, so *x* is a sum of *x*. Tarski-sums validates Idempotence as a matter of mereology: by the reflexivity of part-whole, for any *x*, *x* is part of *x* and every part *y* of *x* shares a part with *x*, namely *y* itself.

Here is the conflict between linear and idempotent summation. To illustrate it, we start once again from Lewis (1991: 29-30; 1993: 208), who uses Idempotence failures as a reason to deny mereological sum status to singleton sets[7]. Consider singleton-formation, the operation that sends any set or non-set *a* into its singleton $\{a\}$ (if the latter exists). Singletons differ from their unique elements, so $\{a\} \neq a$ (for all *a*)[8]. Suppose that sum(*a*, *a*, …) = $\{a\}$, where 'sum' is some idempotent sum operator. By

---

[5] See Tarski (1935), and later on Lewis (1991). As Varzi notes, this definition deems some entity a non-sum if it violates strong supplementation.

[6] See Needham (1981), and later on Simons (1987) and Casati and Varzi (1999).

[7] See also Fine (2010: 567) for an analogous argument that uses multisets instead of singleton sets.

[8] This is guaranteed by the axiom of regularity, which states that every set X has an element that does not intersect it. If X is a singleton $\{x\}$, that element has to be *x*. But empty intersection ($\bigcap\{x\}, x = \emptyset$) entails $\{x\}$ $\neq$ x.

Idempotence, sum(a, a, ...) is also identical to a, but identity is Euclidean, so a = $\{a\}$, contradicting set-theory.

Let us transpose the argument onto linear sums. We know that linear summation is not idempotent: l-sum(**a**, **a**, ...) $\neq$ **a**. Summing **a** to itself, no matter how many times, always returns something distinct from **a** (for **a** $\neq$ **0**). Moreover, for some vector **a**, adding **a** to itself n or m times (for $m \neq n$) returns distinct vectors: $n \cdot \mathbf{a}$ and $m \cdot \mathbf{a}$. Now, for idempotent sums, sum(**a**, **a**, ...) = **a**; so, if l-sum(**a**, **a**, ...) = sum(**a**, **a**, ...), the Euclidean properties of identity predict that l-sum(**a**, **a**, ...) = **a**, contradicting the algebra of linear summation.

We should be careful in assessing what exactly this Lewis-style argument entails. The conclusion is that for some positive integer n ($n > 1$), we cannot have $n \cdot \mathbf{a}$ = sum(**a**, **a**, ...), that is, that vectors cannot be sums of some *one* i-split distinct from them. On the contrary, we can maintain compatibly with the argument that vectors are sums of themselves only (improper sums), so: $n \cdot \mathbf{a}$ = sum($n \cdot \mathbf{a}$, $n \cdot \mathbf{a}$, ...). If vectors had this feature, they would share it with mereological atoms. And as I anticipated (S. 2.1), the view that vectors are mereologically atomic will be of interest in the coming sections.

There are 'last resource' responses to the Lewis-style argument, which include (1) using the argument as a *reductio* of vector-space algebra (rather than as a proof that some vectors are not idempotent sums); and (2) changing the logical properties of identity. However, if these were the only moves left, then the radicalness of each would be a sign that the argument is decisive. Changing the logic of identity brings no advantage in the present dialectic: we would re-gain familiarity of 'sum' only at the price of unfamiliarity in identity. Besides, arguing by *reductio* that $n \cdot \mathbf{a}$ is not identical to l-sum(**a**, **a**, ...), contra the predictions of linear algebra, contravenes all sensible naturalistic policies, by which we should not interfere with successful and largely accepted frameworks[9].

As I have said, 'last resource' replies would demonstrate the strength of the argument *if* left as the only replies. Yet what about denying Generality? We could have it, in principle, that not all but only some cases of linear summation are cases of idempotent summation. Particularly, a vector sum $n \cdot \mathbf{a}$ of i-splits **a** may not be an idempotent sum of **a**, yet perhaps $n \cdot \mathbf{a}$ could relate as an idempotent sum to vectors other than its i-splits.

---

[9] Cf. Lewis' famous remarks in (1991: 87).

At this stage, denial of Generality becomes unhelpful. Surely we may envisage in principle that $n \cdot \mathbf{a}$ is the output of some idempotent operator (sum) applied to vectors other than its i-splits **a**. However, we must remember that whatever candidates **b** and **c** we pick for this purpose, **b** and **c** will automatically be parts of $n \cdot \mathbf{a}$ (if we want $n \cdot \mathbf{a}$ to relate to **b** and **c** as a sum in the *canonical* sense, then certainly we cannot deny that **b** and **c** are parts of $n \cdot \mathbf{a}$ if the latter is a sum of them). Having now discussed several candidates for parts of vectors, we seem to be short on options. Taking **b** and **c** to be any two linear components of $n \cdot \mathbf{a}$ will not do: for licensing **b** and **c** as parts of $n \cdot \mathbf{a}$ also licenses $n \cdot \mathbf{a}$ as part of **c** or of **b** (Section 2.1). On the other hand, taking distinct i-splits $\mathbf{i(n \cdot a)}$ and $\mathbf{i'(n \cdot a)}$ licenses part-whole without Weak Supplementation (Section 2.2). Besides, in either case, further interfering with Generality leads to arbitrariness: blocking '$n \cdot \mathbf{a}$ is part of **b**' is arbitrary because we could just as well block the symmetric correspondent '**b** is part of $n \cdot \mathbf{a}$', and so is blocking cases of common i-splits as cases of common parts (following S. 2.2). Therefore, denying Generality in the Idempotence case, having only some cases of linear sum track cases of standard sum, would seem to leave us with either deviant part-whole, or without plausible options.

2. The atoms-only view

We labelled a theory 'Type-1 compatible' if it posits cases of standard part-whole holding only between vector space elements. Pursuing Type-1 compatibility is one way to admit entities that relate as part to whole in opposition to the core principles[10] while also accepting that these entities partake in canonical part-whole relations. Yet Type-1 compatibility looked unpromising so far, because using cases of various vector-vector algebraic relations as 'trackers' for standard part-whole relations threatened arbitrariness. Once we have labelled a certain linear component (or an *i*-split) a part, immediately we come across other cases of the same algebraic relations, which on pain of arbitrariness our policy for the former case classifies as cases of

[10] Or satisfy mereological concepts that fail the principles, see Chapter 5. For simplicity, henceforth I will omit this disjunct.

part-whole. As a result, this move restores the deviance we initially wanted to avoid, challenging core principles like Antisymmetry and Weak Supplementation.

To be sure, there is a Type-1 compatible view we alluded to which manages to avoid all observed cases of arbitrariness, although it does so quite drastically. The view – let us label it the 'atoms-only' view – states that all vector space elements, with no exception, are mereological atoms. By 'mereological atoms', we mean entities with three necessary characteristics: (a) having no proper parts; (b) having only themselves as parts; and (c) being their own sums (sum(x, x, …) = x, for some idempotent 'sum' operator).

Before commenting on each of these features, let us confirm quickly that this view does away with the above cases of arbitrariness:

(1) I have argued that it is arbitrary to pick as parts of a vector **v** its distinct linear components **c**. In fact, if **c** is part of **v**, then given the symmetry of component-sum, **v** is an equal candidate for part-whole with **c**. Yet equal candidacy makes the choice of one over the other case arbitrary. The atoms-only view, according to which all vectors are atoms, undercuts this arbitrariness, because it entails that neither **c** is part of **v**, nor **v** is part of **c**. Rather, **v** and **c** have no parts distinct from themselves.

(2) I have also argued that it is arbitrary to pick as parts of a vector **v** the i-splits **i(v)**, **i'(v)** distinct from it. In fact, if these are parts of **v** and are arbitrarily chosen among all i-splits of **v**, then there is an i-split common to both which, for the more subtle reasons given in Section 1.2 above, it is arbitrary not to treat as a common part – *contra* Weak Supplementation. The atoms-only view undercuts this arbitrariness again, because it entails that no i-split of **v** aside from **v** itself is a part of **v**.

Having ascertained that the atoms-only proposal dodges the observed threats of arbitrariness, we can proceed to clarify its properties. In conclusion we will ascertain its merits.

Let us start from the composition aspect. To be atoms, all vectors **a** must satisfy sum(**a**, **a**, …) = **a** and hence be sums of themselves according to some idempotent operator 'sum'. Looking back at Section 1.3, we have observed that this case of Idempotence is compatible with the Lewis-style argument given there, which predicts for idempotent operators 'sum' that no vector **a** is a sum of its i-splits distinct from it (at most, the vector is a linear sum of those i-splits, but the linear sum operation is not idempotent).

Which idempotent sum operator do we take? The choice is not too relevant, for no significant point in the following depends on finer-grained differences between idempotent operators. For illustration, we can opt for *Needham-sums*, which, once again (Section 1.3), defines z as a sum of x and y just in case everything that shares a part with z shares a part with x or y. Needham-sums checks Idempotence very quickly: every x is a sum of x, because by Leibniz's Law every thing that overlaps x overlaps x.

Besides the choice of operator, there is an obvious question pertaining to composition. The atoms-only view claims that all vectors are atoms, so that there are improper sums of vectors. But are there proper sums of vectors, that is: sums with the atomic vector as proper parts[11]? If we took in not only Needham-sums but also full-force classical mereology along with it, then we would be forced to posit one such sum for *every* two atomic vectors, and in this case the sum wouldn't be a vector, because we just agreed that all vectors are atomic. Notice that we limit ourselves explicitly to Type-1 compatibility, which demands that all canonical part-whole relations obtain 'among' vectors. If all vectors are atoms, then mereological sums of vectors would be non-vectors, and hence the resulting position would not be Type-1 compatible.

Moving now to part-whole, the atoms-only view comes with two conditions: that vectors should have no proper parts and that they should be parts of themselves. The first condition, self-parthood, is straightforward: every vector **a** is its own component, because l-sum(**a**, **0**) = **a**. Yet while the condition is quickly secured, it matters to dissipate a potential confusion. By invoking the expression 'l-sum(**a**, **0**) = **a**', we are tracking a reflexive case of part-whole from the case of component-sum holding between **a** and **a**, but we are not tracking a case of part-whole from the case of component-sum holding between **0** and **a**. In fact, as discussed in Chapter 4 (Section 3.3), one can accept that **0** is a component of **a** while rejecting that **0** is part of **a**. For in

[11] As in previous chapters, we maintain the canonical definition of 'x is a proper part of y' as 'x is a part of y distinct from it'.

contrast with Fine's intuitive conditions for relating as part to whole, **o** fails to make a difference to 'what **a** is': **a** would still be the entity it is had it not had **o** as a part[12]. The atoms-only view requires only that **o** is a part of **o**, not that it is a part of other vectors. Accordingly, we can accept that the case of component-sum between **a** and **a** tracks a case of part-whole while denying that so does the case of component-sum between **o** and **a**.

On the second condition attached to the atoms-only view, i.e. that vectors lack proper parts, we need a concession. I have stated that choosing candidates for proper parts from the available linear components and the available *i*-splits leads ultimately to taking arbitrariness on board. However, I have given only 'negative' reasons to doubt the ascription of proper parts to vectors and, as a result, I should concede openness to more viable candidates for this role. Ideal candidates, which I do not yet see, will not give rise to arbitrariness or re-introduce deviance in the way linear components and *i*-splits did.

Admittedly, more candidates of this type become available as soon as we consider the entities that satisfy the vector axioms according to specific disciplines. The relativistic approach, for example, defines (bound) vectors as functions from Minkowski spacetime points to sets of coordinates invariant under co-variant and contra-variant transformations (Chapter 5, Section 3). If the relativistic vectors are functions, then they are *sets* of ordered pairs (at least according to a widespread account of functions as relations between sets). And contra the atoms-only view, perhaps as set-theoretical entities the vectors might possess standard proper parts: namely, their subsets (Lewis 1991) or the things they relate to by the ancestral of membership (Fine 2010).

Here is how I think we should read this situation: some mathematical objects that play a foundational role in general relativity and satisfy the vector space axioms are functions, i.e. items with set-like nature. We should concede that in principle, these foundationally relevant objects could possess a standard structure, relating canonically (e.g.) to their subsets or to things they bear the ancestral of membership to[13]. Even so, notice that linear algebra proper defines as sets only vector spaces, not

---

[12] Notice, however, that what is required for **a** to be the entity it is remains unclear. Arguably, on some way of understanding this clause, arguably, **0** indeed makes a difference to what **a** is. See Chapter 4, Fn. 17.

[13] There remains an open question as to whether the resulting position would be Type-1 or Type-2 compatible. The position would be Type-1 compatible if the items the relativistic functions relate to

the vectors that generate them[14]. The generating vectors, rather, are defined only as *members* of the generated spaces, which leaves it open whether or not they are themselves membered (that is, non-empty sets)[15].

The atoms only-view, which states that a vector's only standard parts are its improper parts (i.e. the vector itself), is admittedly more appropriate for non set-like vectors. For set-like vectors, on the contrary, the odds are on some different method for ascribing standard structure (perhaps letting the vectors' standard parts be their subsets or the things they relate to by the ancestral of membership). This granted, there are dialectical reasons to keep the focus entirely on the atoms-only view. The aim of this and the remaining chapters is to show that we do not make our theories too profligate if we concede that they posit vector space members with both deviant (linear) and canonical structure (more precisely, we argue that the type of profligacy these theories incur into does not suffice for discarding them and preferring the rivals, according to which vectors have only linear structure)[16]. As I will make clear in due course, the arguments given for this terminal conclusion apply not just if the hypotheses to be protected from discard posit vectors whose only standard parts are their improper parts (in line with the atoms-only view), but also if they posit vectors with standard proper parts, such as subsets or ancestral members.

Accordingly, while the atoms-only view fails for some entities that satisfy the vector space axioms (e.g. the relativistic vectors), it still offers a useful framework to state the terminal conclusion: we can hold onto hypotheses H_1 with vectors whose only standard structure consists of their improper parts, even if a rival hypothesis H_2, which posits the same vectors but sacrifices their standard structure, is less profligate than H_1. Having stated the conclusion in such a way that H_1 (the hypothesis we want to protect from discard) satisfies the atoms-only view, we will later be able to extend

canonically are themselves members of the same vector space of the functions. It would be Type-2 compatible if, on the contrary, the things the functions relate to canonically were not members of the functions' vector space, but objects that come in addition to these members.

[14] See Chapter 5, Fn. 3 for references.

[15] This algebraic characterisation (featuring spaces as sets/classes and vectors as members) licenses foundationally relevant mathematical items that (a) satisfy the vector space axioms and yet (b) are not sets. Foundationally relevant vectors with these characteristics include quantum-mechanical Hilbert-space vectors and quantum field-theoretic Fock-space vectors, none of which are directly treated as set-like entities by the corresponding theories.

[16] See Interlude and Chapter 10.

it to hold onto hypotheses H_3, whose vectors have more than their improper parts as standard structure (say, they also have subsets and ancestral members). Like H_1, these new hypotheses H_3 are not made more profligate than H_2 (the hypothesis whose vectors have only component-sum structure) by the fact that their vectors have structure additional to components and sum – even if the structure they have in addition is not, as it is in the atoms-only view, just the vectors' improper parts.

3. Sider's pure nihilism

In addition to claiming that vectors are indeed complex, i.e. have standard proper parts, there is another possibility for denying that vectors are mereological atoms. For this new objector, while vectors satisfy the predicate '…is an atom', the predicate is not any more defined as 'bearer of no proper parts', as we have so far taken it to be[17]. Rather, '…is an atom' is either a primitive or it is defined in terms of some non-mereological predicate. For our purposes, we can let the defining predicate be conveniently informed by linear algebra: '…is a vector space element'. Predicating atomicity of vectors is just to predicate of them that they are elements of a vector space.

By itself alone, this view is neutral as to whether vectors have standard proper parts, a negative answer not being any more guaranteed by satisfaction of '…is an atom'. However, Sider (2013) makes a clever use of atomicity neutrally understood. He argues like before that '…is an atom' should be pinpointed to some non-mereological predicate (or left primitive, see Smid (2017: 2371)), but to this he adds (1) the nihilist claim that there are only atoms (every thing is an atom) and (2) the claim that theories having as their subject matter some or all of the atoms ought to be formulated without mereological predicates, hence without '…is part of', '…overlaps…', '…disjoins…', '…is a sum of', *etc.* (Ib. 4)[18]. The forfeit extends to the items of

[17] And as it is taken to be in full force classical mereology.

[18] The theory should also be formulated in a fundamental language, equipped with joint-carving ideological resources (Ib. S. 3 and Chapter 7, Section 2.2). Particularly, to preserve the truth of 'There is an X' (where X is a composite object) in English, the nihilist's existential quantifier is a quantifier of Ontologese, not of English.

mereological vocabulary that the atoms-only view accepts in its formulation; that is: vocabulary that ascribes reflexive instances of the part-whole relation linking atoms to themselves, vocabulary that ascribes no proper parts to atoms, and vocabulary that predicates of an atom that it is its own sum.

Smid (2015) labels Sider's position – quite vividly – *pure nihilism*: it is, indeed, a form of nihilism because it states that every thing is an atom, and it is 'pure' because no item in the vocabulary of a (fundamental)[19] theory of the atoms is mereological, not even the predicate '...is an atom'. Now suppose that a theory of some atoms can be formulated in the way the pure nihilist envisages, that is, as a theory stating that only atoms exist, defining '...is an atom' in terms of some non-mereological predicate and concurrently featuring no mereological vocabulary whatsoever. Sider (2013: S. 1) stresses repeatedly that theories formulated in the pure nihilist's way are motivated by their greater ideological parsimony, for by renouncing '...is part of' and its cognates, they save on the overall number of undefined predicates[20]. Particularly, they feature fewer undefined predicates than rival theories which maintain the atoms as their subject matter but supply additional mereological vocabulary, for instance describing the atoms as bearers of no proper parts, bearers of the part-whole relation to themselves, etc.

Transposing this into our case of interest, suppose that the predicate '...is an atom' is defined by a non-mereological predicate that vectors satisfy, i.e. '...is a vector space element'. Further, imagine a theory of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of a certain space V, stating of each $\mathbf{v}_j$ that it is a member of V and that it obeys the vector space axioms. Membership in V is a predicate accepted even by rivals who hold theories of $v_1, \ldots, v_n$ laden with mereological vocabulary (e.g. theories that following onto the atoms-only view predicate of these vectors that they have no proper parts, or that they are parts only of themselves, or that each is its own sum). For these rivals, the mereological predicates come as a surplus, contributing to overall ideological profligacy.

So goes, at least, the general idea. We will return in due time to ideological parsimony (Chapter 7, Section 2), resting content for the time being with two observations. First, the pure nihilist position resonates well with the eliminativist

---

[19] See Chapter 7, Section 2.2 for a qualification of Sider's need for a *fundamental* theory here.

[20] For this understanding of ideological parsimony, which seems operative in Sider, see Cowling (2014) and our Chapter 7, Section 2.

naturalist's punch-line that present physically motivated but deviant cases, such as cases of component-sum, leaving no room for canonical ones. Indeed, with mereological vocabulary pressed out of the picture by constraints on ideological parsimony, we will not even be allowed to state that each vector bears the part-whole relation to itself or that each is its own sum.

Second, so far we have observed that theories that forfeit mereological vocabulary edge out their rivals (including rivals who apply the atoms-only view) in regard to ideological parsimony, but we have not yet granted that these higher scores grant the theories an advantage over the rivals[21]. This latter claim requires two additional arguments: one to the effect that greater ideological parsimony indeed bolsters preference over the profligate rivals, and another to the effect that no additional arguments support the rivals just as much as greater parsimony supports the parsimonious theories.

I will discuss the link between ideological parsimony and preference more extensively in the remaining chapters. As for the lack of support for the rivals, the present dialectic allows us to remain neutral. My aim, in fact, is not to defend the parsimonious hypotheses (in so doing I would need, as Sider (2013) notes, a defence against arguments for the profligate rivals that outbalance the push of ideological parsimony for the hypothesis I defend). Rather, my aim is the thesis that licensing entities with deviant structure, such as components and sums, does not per se stop these entities from having additional, canonical structure, i.e. partake of part-whole relations with core features. It is all the better for my position if additional arguments balance out the effect of ideological parsimony and sustain profligate hypotheses that licence canonical part-whole. On the contrary, the worst-case scenario is that in which there are no such additional arguments and the fate of the profligate hypotheses hinges entirely on the success of greater parsimony in establishing preference and on the success of other arguments for the parsimonious hypotheses. My attention will be fully on this worst case scenario: I will question, that is, the force of the motivations for the parsimonious hypotheses, and most prominently whether their greater parsimony is significant for preference.

Sider stresses repeatedly (Ib. 7-8) that bolstering pure nihilist theories over rivals with additional mereological primitives is *only* parsimony-based preference, but can

---

[21] To be sure, we have also been brief in understanding ideological parsimony and what it is to achieve a higher score. Chapter 7 (Section 1) returns to details.

there be additional motivations for theories of vectors that forfeit mereological vocabulary? If there could be, then questioning the link between parsimony and preference becomes insufficient to prove my *non sequitur* thesis, i.e. that licencing entities with deviant linear components does not force us to deny them canonical parts.

On this question I need to be concessive. Although there could be arguments in favour of the parsimonious hypothesis other than arguments moving from greater parsimony to preference[22], it would not be purposeless to focus on and dismiss only the latter. The dismissal would, granted, fail to secure the *non sequitur* thesis, because securing the thesis also demands a dismissal of the outstanding argument. Yet, even so, the dismissal would still show that nothing less than the outstanding argument is required to hold onto the Sider-style hypothesis, its trademark greater parsimony will not do.

4. Conclusion

For the naturalist who follows the eliminativist part of the program, once we have counted in entities that relate as component to sum (pushed by the naturalistic evidence) we should also deny that these entities partake of familiar part-whole relations, which comply with the core principles.

The above sections make an important observation for our assessment of the eliminative naturalist. Some eliminativist hypotheses draw support over their 'conciliatory' rivals because the latter are more complex than the former. In particular, a hypothesis H_1, which posits vector space elements $\mathbf{v}_1, .., \mathbf{v}_n$ and states of some of these that they relate as component to sum, is ideologically simpler than a rival hypothesis H_2, which posits $\mathbf{v}_1, ..., \mathbf{v}_n$, states of some $\mathbf{v}_i, \mathbf{v}_j \in \mathbf{v}_1, ..., \mathbf{v}_n$ that they relate as component to sum and of each $\mathbf{v}_1, ..., \mathbf{v}_n$ that it is a mereological atom, i.e. a part of itself only with no other parts than itself.

[22] The additional motivations sustain the Sider-style hypothesis from 'outside' the ballpark of parsimony. In this case the Sider-style parsimonious hypothesis would benefit from an outstanding argument distinct from arguments from greater parsimony to preference.

This observation and the accompanying example disclose what I have so far alluded to (Introduction) as a 'gap' in the naturalist's position. It is one thing to appropriate the mereologist's language and claim that components relate to sums as parts to whole (in a literal sense), and quite another to argue that components and sum partake in no canonical relations, contrary to the atoms-only view and its cognates (e.g. the view that they have their subsets or ancestral members as parts). The gap has 'meta-ontological' character because whether it will be filled depends ultimately on eminent meta-ontological themes: whether hypotheses, in which vectors have only linear structure, are effectively parsimonious and whether greater parsimony is a guide to preferring these hypotheses over their rivals (in which vectors have canonical parts). Starting from the next chapter, we inspect how these questions relate mutually in the abstract and how they play out more specifically within our dispute about vector composition. To prefigure – unfortunately for the eliminativist naturalist – prospects in our case of interest are not too promising.

VII

Arguments from profligacy: Ontology, ideology and beyond

Concluding the last chapter, we noticed that some hypotheses which depict a certain subject matter of physical interest (vector space elements) as taking part in deviant and standard part-whole relations offend against ideological parsimony if compared to rival hypotheses that only depict the very same subject matter as taking part only in deviant part-whole relations. Our handling of parsimony and profligacy was useful but unrefined, and the possibility of discarding a theory based on its greater profligacy is one we only briefly alluded to. In this new chapter, which plays the role of interlude, we look in greater detail at (1) measures of parsimony and profligacy; (2) the form of arguments that move against profligate hypotheses; and finally (3) the range of responses available for blocking these arguments. Our focus will be coarse-grained and not limited to rivalries between hypotheses with subject matters of physical interest. However, we will return to the contents of this interlude many times in future places to assess how these rivalries play out.

1. Assessing arguments from profligacy

Arguments from profligacy – or equivalently: profligacy charges – are easily described in a couple of lines: for two or more hypotheses, if one of these is measurably more parsimonious than all others, then we conclude (defeasibly, and all things being equal) that we should prefer the former to the latter. Having noted this self-contained description, however, the demand for articulation ensues just as quickly. For 'measurably' and 'preference' may be efficacious from an intuitive point of view, but each clearly needs refinement.

1.1 Preference

For ease of illustration, I find it convenient to start from preference; so, to look at what the charge's conclusion exactly states. 'Preferring' one over a rival hypothesis means to believe the former and disbelieve the latter; or at least to believe the former to a higher degree than the latter. The charge thus prescribes that in response to a hypothesis' H greater parsimony relative to competitors H_1, ..., H_n, unless additional factors interfere, we choose to believe H or at least to believe H to a greater degree than any more profligate rival H_1, ..., H_n.

An important question is why, according to the argument's advocates, we should aim at beliefs[1] in the parsimonious hypotheses rather than at beliefs in the profligate competitors. It is because, we assume, choosing which hypothesis to believe in aims at obtaining a true belief, and a hypothesis' greater parsimony functions as a (defeasible) guide to the theory's truth. On a common way of putting it[2], this assumption endows parsimony with an 'epistemic' role. Defeasibly and all things equal, we have good reason to suppose that picking up a belief in a parsimonious hypothesis and rejecting one in a profligate competitor is to pick up a true belief and reject a false one[3].

Having agreed on ascribing parsimony an epistemic role, we sum up the charge's conclusion and its justification as: we ought to pick up beliefs in parsimonious hypotheses, because the believed hypotheses' parsimony suggests (defeasibly and *ceteris paribus*) that the picked belief is a true one.

---

[1] Or higher degrees thereof.

[2] Originating from Sober (2001, 2006) and reproduced in Baker's (2016: S. 2) useful introduction.

[3] Standardly, the view that a theory's parsimony informs choice of true beliefs opposes a weaker view, according to which parsimony guides choice aimed not at truth but only at pragmatic and heuristic virtues ("being more perspicuous, … easier to use and manipulate, and so on", Baker (2016: S. 2); see also Melnyk (2003: 249)). For some episodes of the long rivalry between these two views in philosophy of science, see French (2014: Chapter 2).

Accordingly, it suffices for rejecting the charge that in response to H's greater parsimony we do not pick up for truth-guidance beliefs in H. This includes two possibilities[4]: Firstly (1), a theory's parsimony relative to a rival is detrimental to its truth, in which case we best honour the aim of gaining true belief by choosing the rival. Secondly (2), the theory's parsimony over the rival is irrelevant for truth, in which case there are two further options for best honouring the aim of gaining true beliefs. One (2.1) is by suspending preference, that is, by believing neither H nor any of its rivals (or believe each to an equal degree)[5]. Another (2.2) is by preserving whatever belief we had prior to considering H's greater parsimony. Sober (2014: 2, 12, 61)[6] labels responses of this 'agnostic' kind (suspensions of preference or preservation of prior belief ascription) 'the razor of silence'.

To anticipate a later point (Chapter 9, Section 2), my focus in this second part of the dissertation lies only on (1) and not on these agnostic rebuttals (2.1) and (2.2). This choice of focus owes to the present dialectic. My terminal conclusion, as we will later discuss, is that we should not discard profligate hypotheses H_2, which posit vectors that not only relate as component to sum but also enter standard part-whole relations. However, only (1) and not (2.1) or (2.2) threatens this conclusion. For consider less profligate, rival hypotheses with vectors that only relate as components to sums and enter no additional standard relations. The terminal conclusion would be easily met if the policy for coping with H_2 and this rival were (2.1) or (2.2), as according to these policies H_2 should not be discarded owing to its greater profligacy. Only the first policy (1) raises trouble for H_2, because it capitalises on H_2's profligacy to discard it. Accordingly, only (1) will be our critical focus in protecting H_2 from discard.

1.2 Measures of profligacy

The second question to the charge's concise formulation asks for the meaning of 'measurably': we should pick up beliefs in parsimonious hypotheses rather than in

---

[4] See the introduction to Sober (2014).

[5] Or believe each to an equal degree.

[6] See also his (2006, 2009).

their profligate rivals only if the parsimonious hypotheses are *measurably* more parsimonious than the rivals. What does measurability consist in?

For a hypothesis to be measurable is for it to be more or less parsimonious than its competitors, relative to some standard. As we will observe in greater detail below, the nature of the standards depends on the intended justification for the charge's conclusion. Suppose that the conclusion is justified because by accepting beliefs in parsimonious hypotheses we follow a reputable scientific practice, which honours the aim of gaining true beliefs and discarding false ones (this is a common way of justifying the conclusion, see Chapter 8). For any rivalry we wish to address in this way we ought to make sure that the measuring standards are *exactly* those that inform scientific choices, not *sui generis* or abstracted standards[7].

As a result of this dependence from justification, it may take long to come to classify the standards entirely. Yet among the many that one may envisage, four standards are particularly … standard: they appear frequently in the literature. Here they are:

The 'standard' profligacy standards

Type of standard		*Standard*	*Hypothesis H_1 is more parsimonious than H_2 just in case…*
Ontological	Plain	1. Quantitative	H_1 posits fewer entities than H_2
	Selective	2. Qualitative	H_1 posits tokens of fewer distinct kinds than H_2
		3. Fundamental	H_1 posits fewer

[7] It might be very difficult to say whether two collections of hypotheses (particularly: one containing scientific hypotheses and the other containing first-order metaphysical hypotheses) are ordered for parsimony by the exact same standard. It is typical for those who are sceptic of parsimony having an epistemic role in metaphysics to argue for subtle differences in standards. For discussion of this argument strategy, see Saatsi (2017).

			fundamental entities than H_2
		…	…
Ideological	Selective	4. Primitive ideology	H_1's ideology features fewer primitives than H_2's.
		…	…
…	…	…	…

Even though the 'standard' standards are familiar subjects of discussion, the above presentation has some distinctive traits. First, it is commonplace to divide standards into ontological and ideological, the first penalising a hypothesis H for abundance in the entities it is committed to (or henceforth: abundance in its 'posits'); and the latter penalising it for abundance in its representational resources. Further to this distinction, standards divide into 'plain' and 'selective'. A plain standard penalises H for *any* ontological or ideological addition (any additional entity or item of ideology), a selective standard penalises it for additions of specific types: entities that add to the number of instantiated kinds, fundamental entities[8], ideological primitives.

1.3 Conditions on measurability (1)

Second, the ellipses '…' under the ontological and ideological headings emphasise that the four standards are not the only candidates for sustaining arguments from ontological and ideological profligacy. As seen at the beginning of the last section (1.2), one can motivate additional standards by envisaging a particular justification for

[8] This latter measure is associated with Jonathan Schaffer's (2003, 2009 and elsewhere) use of '… is fundamental' as a predicate that attaches to entities (See Sider 2013: S. 3 for objections). In (2016), Schaffer also takes in the definition of '…is fundamental' as '… is not metaphysically grounded in anything'. Accordingly, here the fundamental standard penalises ungrounded entities.

the charge's conclusion. If the reason for looking at parsimonious hypothesis for candidate true beliefs is that parsimony guides scientists in the choice of hypotheses, then the standards we use in choosing ought to be the exact same standards that apply in science. And it is often argued that these add to the basic four[9].

Besides, sustaining the argument could be none of the four basic standards taken by itself alone, but rather a *combination* thereof. We could (for example) measure two competitors H_1 and H_2 not with a function of only their overall posits (or only their kind tokens, fundamentals, etc.), but rather with a composed function of their posits *and* overall kind-tokens, or kind-tokens and fundamentals, or overall posits and overall fundamentals, and so forth.

We remain neutral on the number and nature of candidate combinations. But having said this, we need to mention a feature common to any candidate standard (basic or not), which helps us think of two or more hypotheses' parsimony comparatively, as the charge demands us to. To set up comparisons between differently parsimonious hypotheses, each of the latter must be assigned a *degree* of parsimony[10]. The set of all degrees is partially ordered, so as to support comparative relations between degrees: *greater (smaller) than-*, *greater or equal to-*, *smaller or equal to-, etc.* Hypotheses occupy degrees smaller than the degrees of their competitors just in case they are more parsimonious than them, and conversely, they occupy greater degrees if they are more profligate. Finally, equally profligate hypotheses occupy the same degrees.

For the four customary standards (quantitative, qualitative, fundamental and ideological), we can conveniently identify degrees with positive integers. The set of all integers (and *a fortiori* that of all positive integers) is totally ordered, and hence supports the needed inequalities. Moreover, each positive integer conveniently represents the number of posits and primitives associated with each theory, so the 'location' of each hypothesis on the degree scale is straightforward: in fact, it is identical to the number of the hypothesis' posits (or primitives).

For standards other than the customary four it might not be straightforward to determine how a hypothesis' number of posits (or of posits of a particular type, or of items of primitive ideology) maps into a totally ordered set that allows comparisons.

---

[9] For ontological parsimony, Baker (2003) is a classic example.

[10] The labels 'degree of parsimony' and 'degree of profligacy' are used interchangeably.

For illustration, imagine a standard S that measures a theory more parsimonious than another – not just by virtue of its sparse overall posits, or just by virtue of its sparse tokens of distinct kinds, but rather as a combined function of these two factors. According to S, some hypotheses A will outrank a competitor B by virtue of having less overall posits (but more kind-tokens), while some other hypotheses C will outrank a competitor D by virtue of having less kind-tokens (but more overall posits). Here the measuring degrees cannot be identical to the overall number of posits, otherwise (contrary to expectations) D would count as more parsimonious than C. Nor can the degrees be identical to the number of kind-tokens, otherwise B would be more parsimonious than A. More likely, the degrees will be a function of the two combined parameters. Similarly, Schaffer's (2016) multi-factor approach predicts that hypotheses H_1 are preferable[11] to competitors H_2 just in case either H_1 has fewer fundamentals than H_2, or, if the fundamentals are equal, H_1 has more non-fundamentals ('derivatives') than H_2. Like before we cannot map hypotheses into totally ordered degrees with the identity function from the number of fundamentals: this would deliver (contrary to expectations) that all hypotheses with the same fundamentals are equally preferable, even if they differ in the number of derivatives. What we need instead is a function f that maps each hypothesis to a number n on the ordered set, such that (for any H_1, H_2) $f(H_1) > f(H_2)$ if H_1 has more fundamentals than H_2 and such that $f(H_1) > f(H_2)$ if H_1 and H_2 have the same fundamentals but H_1 has more derivatives than H_2.

Some multi-factor standards are notable for the simplicity of their mapping (Schaffer's being a good specimen), but the point deserving the greatest emphasis, at this stage, is only that all functions mapping hypotheses to degrees meet the conditions for successful comparisons. Equally parsimonious hypotheses are mapped to the same degree, while differently parsimonious hypotheses are mapped to degrees that reflect their difference. In particular, parsimonious hypotheses will come with degrees smaller than those of profligate rivals.

[11] Here we cannot say 'more parsimonious', for according to Schaffer this theory measures not the competitors' fundamental parsimony, but the combination of their fundamental parsimony and their 'strength', i.e. their ability to 'generate' derivative objects from their supply of fundamentals.

2. Ideological profligacy

The last chapter made fruitful use of ideological profligacy, but it also relied on two implicit assumptions. First, the standard is of the 'selective' kind. Second, there is a background approach to the role of ideology in metaphysics and the relationship of ideology to metaphysical naturalism we need to agree on.

2.1 Selectivity

The standard measures profligacy based on abundance in primitives, not abundance in overall items of ideology[12]. A theory's primitives are those among its representational resources that the theory does not define. Thus the membership, but not the subset predicate is a primitive of set theory; for to be a subset *x* of some set *y* is to be a set all of whose members are members of it. And of course many take as a primitive the predicate for mereological part-whole, which defines those for proper part, overlap, disjointness, and sum[13].

A question relevant to our purpose in the second half of this dissertation is whether '...is a vector' is a primitive of linear algebra. The purpose, recall (Chapter 6), is to shield from worries of profligacy hypotheses H_2, which posit vector components and sums as well as vectors with canonical parts, e.g. vectors with the properties of mereological atoms. The more parsimonious rival H_1, from which we would like to protect H_2, posits components and sums equipped with linear structure, but not with any additional, canonical parts. Now, the project of 'protecting' H_2 from H_1 makes sense only if H_2 is indeed more profligate than H_1. On the side of ideology, this means that H_2 deploys more primitives than H_1. But what are H_1's and H_2's deployed

---

[12] It is commonplace to pick a selective measure for ideological profligacy rather than a plain one. But why so? Cowling (2014: 3893) suggests that it is because defined ideology "admits of definition in terms of primitive ideology and therefore 'comes for free' granted the *analysans*". Fiddaman and Rodriguez-Pereyra (2018) argue that we should use the selective standard to avoid penalising theories whose abundant defined ideologies add to the theories' intelligibility.

[13] Notice that although my focus is exclusively on predicates, I do not confine primitive ideology to predicates. Rather, the upper limit is Sider's (2011) conception, which also incorporates modal operators, quantifiers and Boolean connectors.

primitives, and which among these make up for the difference between H_1 and H_2? A good *prima facie* assessment of the situation is, indeed, that both H_1 and H_2 deploy the predicate '...is a vector', while H_2 also includes '...is part of', as it claims of some vectors that they are self-parts. Yet is the vector-predicate a primitive? Not according to linear algebra (cf. Chapter 5, Section 1)[14], where the usual definition states that some mathematical object is a vector if it is a *member* of a vector space; that is, a member of a set any two or more members of which (**x**, **y**) satisfy the vector space axioms, e.g. $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$; $\mathbf{x} + \mathbf{0} = \mathbf{x}$, etc. (Chapter 4, Section 2).

This observation helps us get more precise on the resources at stake in our rivalry of interest, i.e. the rivalry between H_1 and H_2. We can agree that H_2 exceeds H_1 due to its using the mereological predicate ('...is part of') to state that some vectors enter canonical part-whole relations, e.g. by being their own parts. Further – this being where the observation is relevant – both H_1 and H_2 agree on the deployed set-theoretic and algebraic resources: the predicate for membership and that for being a linear component. Membership is deployed because both hypotheses posit vectors $\mathbf{v}_1$, ..., $\mathbf{v}_n$, which by the algebraic definition are members of vector spaces. As for the predicate for being a linear component, it is deployed because both hypotheses posit vector space members and any such members relate as component to sum to other vectors in the same space as illustrated by the vector space axioms[15].

---

[14] Cf. Leuenberger and Keller (2009: 371) for the opposite view that '…is a vector' is "a rich and fruitful concept that transcends particular definitions that may be offered". This view (as I understand it here) states that '…is a vector' is a primitive predicate. It is friendly to my purpose of protecting H_2 from charges of excessive profligacy, because it explains straightforwardly what primitive ideology H_2 has in common with H_1 (namely, the predicate '…is a vector'). Accordingly, we can safely ignore it, because in so doing we make an assumption that is not in our favour.

[15] More precisely, the axioms claim of (arbitrarily selected) members of the same vector space that they relate not as component to sum, but as vector sum to composer vectors; that is, by a composition-like and not a part-like relation. This is because (arguably) the operator '+' figuring in the axioms is a composition-like operator: if **y** is the image of '+' under arguments $\mathbf{x}_1, \ldots, \mathbf{x}_n$, then $\mathbf{x}_1, \ldots, \mathbf{x}_n$ compose **y**. There is a question, accordingly, as to whether among H_2's algebraic primitives we should include '…is a vector sum of…', '…is a vector component of', or both. The first is variably polyadic (or has a singular and plural argument, cf. Chapter 1, Section 1), the latter is binary (and has only singular arguments). In the following, I will mostly ignore this complication and assume that the part-like predicate is a primitive (Chapter 1, Section 6), which either defines the composition-like predicate or figures together with it in the primitives of H_1 and H_2. Thus if **y** is the image of '+' under arguments $\mathbf{x}_1, \ldots, \mathbf{x}_n$, as in the vector space axioms, then each $\mathbf{x}_j$ of the $\mathbf{x}_1, \ldots, \mathbf{x}_n$ is a component of **y**, whereby '…is a component of…' is either the primitive of H_2 that defines '… is the vector sum of…' or at least it is a primitive of H_2 together with the latter. A growingly influential approach by Fine (2010), which we do not pursue here, uses as primitive neither the part-like, nor the composition-like predicate. Instead – so Fine – the primitive is the composition *operator*, which in our case of interest figures directly in the vector space axioms ('+'). On this view, it holds as a matter of definition that **x** is a linear component of **y** just in case **y** is the image of '+' when '+' takes as argument either **x** or a vector **x'**, which is

2.2 Ideology and metaphysics

For an argument from profligacy to be based on the ideological standard, it must be possible to prefer a theory on the basis of its ideology (whereby preference, recall, is choice guided by the aim of obtaining true belief). So it must be possible to select a theory and obtain a true belief in virtue of its ideology.

Imagine surveying various hypotheses H_1, ..., H_n with the aim of choosing one H_j based on features of its ideology $I(H_j)$, such as its sparseness relative to the ideology of the competitors. Further, suppose that the choice is made with the aim of obtaining a true belief: so that it is not just choice but *preference*. It seems clear that to justify a preference based on $I(H_i)$ rather than on other features of H_i, something in the nature of $I(H_j)$ must ensure that the obtained belief is true.

What is needed for the sparseness and other features of $I(H_i)$ to work for preference is a theory about the nature of ideology with two features[16]: objectivity and independence from ontology. $I(H_j)$ is objective if the choice as to which primitives it contains is not legitimated by *any* aim of the theoriser, but rather by one aim in particular: namely, conveying information about the kind of facts that would obtain if the theory were true. Further, $I(H_i)$ is independent from ontology if this conveyed information is not about the theory's ontological commitments. Rather, it is a *sui generis* kind of information about the ideological side of reality's structure.

Agreeing in spirit with Sider (2011), Cowling (2014: 3893) illustrates this point through the dispute between modalists and modal eliminativists[17]. The former believe that we should state all modal truths in a non-modal language (e.g. quantification over possible worlds), the latter admit of at least some irreducible modal machinery (e.g. boxes and diamonds). What explains this difference in resources? Not a divergence in the theorisers' preferences and pragmatic aims, and not even a divergence in ontology, for modalists and modal eliminativists agree on

---

identical to some $\mathbf{x_i}$ in a sequence $\mathbf{x_1}$, $\mathbf{x_2}$, … $\mathbf{x_n}$, whereby the sequence is such that $\mathbf{x_n} = \mathbf{y}$, $\mathbf{x_1} = \mathbf{x}$ and every $\mathbf{x_i}$ is an argument of '+', which delivers $\mathbf{x_{(i+1)}}$ as the image (cf. Fine 2010: 567).

[16] Here I follow Cowling's presentation (2014: S. 2), which provides as much detail as we need (also see Sider 2011: 12-14). Sider (2011: Ch. 3) contains the argument for why ideology latches onto real goings on, which is the same as in Williams (2005, 2007). Moving backwards, Lewis (1983) introduces the objective view of ideology, while Quine (1951, 1953a) allows for choosing a theory's ideology following non-epistemic aims.

[17] Quine (1953b) and Lewis (1986: Ch. 1) exemplify eliminativism. Melia (1991) defends modalism.

what exists and on the facts that obtain. Rather, explaining the difference is the difference in the *kind* of facts that have to obtain given the truth of all modal sentences: for the modalist it will be irreducibly modal facts, while for the eliminativist it will be facts devoid of any modal character.

Retrospectively, this illuminates our discussion of Type-1 compatible views in the last chapter. The 'atoms-only' view (Section 3) states that every element $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of a vector space *V* is an atom (has no proper parts). Its competitor for ideology (Section 4) states (with Sider's 'pure nihilism') that $\mathbf{v}_1, \ldots, \mathbf{v}_n$ satisfy only physical or mathematical predicates (e.g. being an element of a vector space), which the atoms-only view agrees on. The competitors agree on the furniture of reality, which consists of vector space elements and vector spaces (i.e. sets). So, explaining the difference is the kind of facts expected to obtain if each view is true: for the competitor it is membership facts (facts of each vector being a member of the vector space) and linear algebraic facts (facts of some vectors being components, i-splits, linear sums of other vectors), while for the atoms-only view it is membership and linear algebraic facts in addition to mereological facts (facts of every vector having no proper parts, being sums of themselves only and parts only of themselves).

2.3 Relationship to naturalism (1): Contents and profligacy

Let us consider more closely this description of the atoms-only view as a supplier of ideology for mereological facts. The naturalist who follows the eliminative path aims at doing away with cases of 'structural' relations other than those privileged by scientific support – for example, cases of vectors $\mathbf{v}_j$ being self-parts and self-sums (cf. Introduction and Chapter 6). If we now understand 'doing away' as removal of ideology, we find that the argument from ideological profligacy starts to play in the naturalist's favour[18]. If we are ready to accept that the true theories' ideology conveys content about the kinds of facts that obtain, then a successful argument from

[18] North (2009) leads a (realist) line in philosophy of physics, according to which ideology latches onto and commits to aspects of reality's structure; and sparse ideology works as an epistemic instrument of preference (Ib., 65-6). However, not everybody agrees on the epistemic role of sparse ideology, see e.g. French (2014: 30-1). These criticisms (see also Curiel 2014) certainly undercut some motivation for seeking sparse ideologies in metaphysics.

ideological profligacy delivers what the naturalist is after: an hypothesis without ideology if not for scientifically privileged facts.

To illustrate why arguments from ideological parsimony exemplify this naturalist program in an attractive way, consider Chakravartty's understanding of naturalism in metaphysics as importation of "content" (or "subject matter"), "methods" and "aims" (2017: 79-80). A helpful way of visualising the importation of "subject matter" is as copying content of scientific interest 'somewhere' into one's metaphysical theory[19] – for example, positing a supply of vectors that includes some elements of a basis together with their linear combinations and scalar products, and then claiming that some of these vectors relate as component to sum. While this seems sufficiently clear, our interest now is on two other aspects of the "importation" imagery.

(1) First is the 'place' in the metaphysical theory's structure in which the content is imported. The options here include the ontology (the theory's information about what exists), the ideology, understood as the theory's information about the kind of facts that obtain, and perhaps a third dimension (Cf. next section).

(2) Second is the 'self-contained' character of importation. Having imported a certain element of scientific interest somewhere in the structure of one's metaphysical theory, still not all is decided as to what importing this structure should entail. To reconnect with our sample hypotheses of Section 1 and Chapter 4, having imported the primitive '...is a vector' in the ideology and vector themselves in the ontology, it remains an open question whether the ideology can host further primitives for mereological facts about the vectors, i.e. facts of self-parthood and self-composition. The importation of content alone is not enough to reach a decision. We need, rather, consider the importation of methods, and an immediately relevant method, threatening to decide against the additional ideology for mereological facts about the vectors, is to prefer a theory based on ideological sparseness.

[19] Whether the copying process outputs just the same element of scientific interest we begun with is controversial; indeed varieties of naturalism differ greatly in respect to possibilities left to the copier to interpret the imported element. See Morganti and Tahko (2017) and Ney (2012) for contrasting example of how vast these differences can get.

2.4 Relationship to naturalism (2): Localising the threat

This provides for the urgency of assessing the argument from ideological profligacy. Now, does the fate of the naturalist's project hinge *entirely* on this argument? Probably not.

Suppose that the argument from ideological parsimony failed so that the greater profligacy of the atoms-only view does not count against preference. Even so, the naturalist can rest content with an argument from ontological profligacy. If this type of profligacy is a factor for preference, then we can at least prefer the atoms-only view and its rival (without ideology for '…is part of…') to a hypothesis H_3 that posits vectors, vector spaces and full-force mereological sums of vectors, distinct from the spaces and the vectors they are sums of. Resisting hypotheses of this new type, and more generally all hypotheses more profligate than the atoms-only view in the ontological respect, makes it for another possible objective of the naturalist's project.

Which, if any, of these two objectives is the naturalist's pick is hard to say. Arguing for hypotheses without canonical structure from removal of profligate ontology (e.g. mereological sums of vectors) and from removal of profligate ideology (e.g. ideology for facts of self-parthood between vectors) seem both plausible ways of proceeding. To me, this difficulty with localising the content threatened by worries of profligacy signals an urgency to discuss arguments of both types. Concurrently, the little space that remains only allows setting the focus on ideological parsimony. By so doing, and by omitting discussion of the ontological standards, the following chapters (8, 9 and 10) effectively fulfil only a part and not the entirety of our project. We will manage to protect hypotheses with mereological predicates, such as the atoms-only view, from the threat posed by rivals that renounce such resources. However, we will not yet manage to defend another class of hypotheses. This comprises the hypotheses that posit not only linear vectors, understood as mereologically atomic, but also non-vector like entities that work as canonical parts of vectors, or sums of which the vectors are canonical parts.

3. Beyond ontology and ideology?

Does the naturalist's success hinge entirely on arguments from ideological and ontological profligacy? Let us consider this question from within the realm of profligacy: aren't there arguments sustained by *additional measures* of profligacy? For an answer, we need to consider one last feature of the 'four standards' chart (Section 1.2), which might have passed behind the scenes. The ellipses on the bottom row, outside of the 'ideological' heading, indicate the possibility of measures that penalise neither abundance of posits, nor abundance of ideology, but some additional feature of the competing hypotheses instead.

Under this class fall growingly popular techniques, which opt for penalising the hypotheses' score for statistical model selection[20]. In a nutshell, the preferable hypotheses present information of higher quality. All usually welcome by statisticians as formulations of Ockham's razor (hence candidates for parsimony standards), such penalties still vary widely[21], and the area is characterised by a degree of foundational uncertainty. Having said this, some of these proposals are very relevant to interpreting the naturalist's strategy for elimination of non-*sui generis*, scientifically unmotivated composites. For one, it is parsimony based on one such strategy for model selection that Ladyman and Ross (2007: Chapter 4, S. 4.4) invoke, engaging directly with first-order hypotheses that make these uninformed posits.

Model selection approaches deserve methodological remarks, which grant them a place in the current dialectic. Cowling (2014: 3896) invokes the vitality of this research program to argue for an 'egalitarian' view of the measures that sustain viable arguments from parsimony:

> Although I am content to follow Lewis and others in assuming this Quinean conception of theories here, I do take it that our best account of parsimony ought not ride roughshod over alternative proposals [i.e. model selection techniques] for articulating implementable accounts of simplicity and complexity. We therefore have another reason to deny an inegalitarian view of parsimony, which finds no support from other approaches to understanding simplicity and complexity.

---

[20] On model selection, see Claeskens and Hjort (2008). Romeijin (2014: S. 5) is a philosopher-friendly introduction.

[21] There are, to start with, the algorithmic information approaches of the next section (Li and Vitányi 1997). Besides, approaches based on the Bayesian Information Criterion (BIC) and Akaike's theorem (AT) penalise information quality as a function of the hypotheses' number of independent parameters and distance between their presented data and a sample of the real data distribution (Forster and Sober 1994).

I agree that the vitality of the model selection program should lead us to an egalitarian position, although my view differs slightly from Cowling's. If among the *candidate* measures for sustaining an argument from profligacy we already consider ideological measures, then the vitality of the model selection program leads us to consider as additional candidates the ontological measures; and vice versa, if we already consider the ontological measures, then the vitality leads us to candidate their ideological relatives. This is because – as Cowling correctly says – as soon as one concedes the candidacy of model selection techniques, it becomes unmotivated to hold onto measures of only *one* preferred type: ontological or ideological. If two types of standard, one 'traditional' and one from the model selection domain, are already among your candidates for running an argument from profligacy, why not consider a third?

Therefore, with Cowling we react to the vitality of the model selection program with a concession. *Some* selection techniques are presumably good candidates for running arguments from profligacy (though the question of which technique exactly remains unsettled due to the discipline's foundational unrest). Accordingly, to make a fully informed decision on our rivals of interest (H_1, H_2), we should admit guidance from features other than their ideological structure: namely, their ontological and information structure. That we assess the rivalry as we will do in the remaining chapters – that is, only looking at ideological profligacy – becomes a temporary way of proceeding, which concedes that an important element is missing.

To put it differently, dismissing a challenge from ideological profligacy – as we proceed to do – is only the first step towards protecting a hypothesis like H_2 (which states of some vectors $\mathbf{v_1}$, ..., $\mathbf{v_n}$ that they relate as component to sum and as part to whole) from a hypothesis like H_1 (which states of $\mathbf{v_1}$, .., $\mathbf{v_n}$ only that they relate as component to sum). To ensure that we will not be forced into preferring H_1, in spite of H_2 resisting the challenge from profligate ideology, one must also argue that H_2 is no more complex than H_1 on the side of ontological or information structure (or, alternatively, that though H_2 has such additional complexity, the latter does not count towards preferring the rival).

To restate what I already revealed, the next chapters' focus is exclusively on resisting the challenge from ideological profligacy: we first classify and articulate strategies for *responding* to these challenges (Chapter 8), then we argue for our particular rivalry of interest that greater profligacy in the ideological respect does not count towards preference (Chapters 9, 10, 11). That more than this effort is needed to protect H_2 from its rival is a point I fully concede, yet defer until another day.

4. Resisting arguments from profligacy

Emerging from the last sections is an urgency to reject arguments from profligacy to start (though not complete, see Section 3) our defence of hypotheses that contribute familiar mereological complexity to vector spaces. Now what does it take – in our rivalry of interest – for a reply to these arguments?

4.1 Anti-profligacy responses (aka. levelling down)

For some help on the side of variables, let us continue to indicate by H_2 and H_1 – respectively – a profligate hypothesis and (one of) its parsimonious competitor(s), whereby the competition is relative to the standards of profligacy that matter for discussion. According to this last section, responding to a profligacy challenge will take the following claim: The challenger correctly observes that a hypothesis (H_2) is profligate under some measure M (for us: a 'plain' ideological measure), but mistakenly thinks that profligacy in this measure legitimises discarding the hypothesis. The inference is mistaken because what legitimises discarding the hypothesis, instead, is profligacy in a different measure M', in respect to which H_2 is not any more profligate than its rival.

To put it differently, this strategy concedes that H_2 offends against some measuring standard, having more posits or ideological primitives of the penalised kind than H_1. It denies, however, that the offended standard succeeds in fuelling a successful argument from profligacy, i.e. in licensing a defeasible inference against H_2. At most, it might initially appear as if the violated standards suffice for concluding defeasibly against the hypothesis, but this appearance owes to insufficient reflection about what type of standard it takes to sustain a successful charge.

This response exemplifies what Hawley (2014) calls a 'levelling-down' approach. Imagine a subscriber to the classical mereological doctrine that any two objects have a sum (Unrestricted Composition, UC) who, accordingly, accepts a hypothesis H_2 for which any two of *n* objects compose. Particularly, on this view, even spatially scattered and insignificantly related objects make up a whole: the left half of a trout and right half of a turkey (Lewis 1991: 7); the tip of my nose and a supernova, and so forth. Compared to a rival H_1 without UC and without the sums of these insignificantly related objects, H_2 offends against plain quantitative profligacy. Yet – so say the levelling-downers – "these additional objects do not 'count' for the purposes of assessing relative parsimony" (Hawley 2014: 81), because plain differences in number of overall posits do not legitimise successful profligacy charges. Perhaps only offending against qualitative or fundamental standards sanctions the charge's conclusion against H_2, or perhaps only offence against a refined quantitative parameter, stating that one should only minimise the additional commitments whose existence brings about phenomena that demand explanation[22]. Yet it is not clear – or so the levelling-downer should argue (Hawley 2014: 83-4) – that wholes made out of insignificantly related objects supply additional kind-tokens, fundamentals, or explanatory demands.

Instructively, this model indicates that two arguments are required for casting the levelling-down response. One is to the effect that some, but not other measures of profligacy legitimise discard; and another is to the effect that the posits or the ideology specific to a profligate hypothesis violate only a 'harmless' standard, incapable of legitimising discard. Moreover, a common guide for identifying legitimising standards – to cast the first argument – is scientific significance: if reputed scientific cases of parsimony-based preference penalise only posits with such-and-such features, then we should act consistently with this policy and accept that posits without these features come at no ontological cost. We will employ just this naturalistic method for identifying the legitimising standards in the next chapters, in which we run the levelling down strategy to assess our rivalry of interest.

Two observations will be helpful before we get to that stage. First, there is no reason of principle why levelling-down type responses would not apply to arguments from

[22] This being Hawley's take on Baker's (2003) famous revision of the quantitative standard based on its appearance in particle physics. See also Jansson and Tallant (2017) for an updated reprise of Baker's standard.

ideological profligacy. True, of the last chapter's four canonical standards (Section 1.2) only one pertains to ideology (and measures plain addition of primitives). But as seen (Ib.) candidate standards extend beyond the canonical four[23]. That the ideological class conceals selective standards, which penalise not all but just some ideological primitives, is something we can realistically expect. Indeed, Cowling (2014) suggests in this spirit that ideological profligacy, like its ontological relative, admits of a qualitative/quantitative distinction. Quantitative ideological profligacy penalises addition of any primitive, while its qualitative cousin penalises only addition of distinct kinds of primitives – primitives, that is, of the same ideological kind.

Combining this proposal with the levelling-down approach, we obtain the following response: For some profligate hypothesis H_2, a survey of the ideological standards reveals that H_2's extra-primitives offend only against plain quantitative ideology, but not against selective qualitative ideology, for these are primitives that share an ideological kind with some primitive in H_1. If the qualitative but not the quantitative standard legitimises arguments from profligacy, then H_2's profligacy in the quantitative respect is no reason for preferring H_1 in its place. On the contrary, H_2 and H_1 come at the same ideological cost.

Second observation: How should friends of profligacy respond to levelling-down strategies? A natural reply has it that it is too demanding to carry out the strategy in full. In particular, to protect H_2 against all possible arguments from profligacy, levelling-downers need survey all measures H_2 offends against and for each of these deny that it legitimises an argument in favour of discarding H_2. This process could prove laborious, for, once again, measures of profligacy extend beyond the canonical four and, in principle, many measures could remain, which H_2 offends against.

For a counter-reply, advocates of levelling-down seem to be in a position to indicate an appropriate point to pass on the burden of naming measures of profligacy H_2 remains unprotected against. A plausible recipe for setting this point ensues by limiting the practice of levelling down to defending H_2 against ideological profligacy. Indeed, the ideological class has only one largely accepted measure, which penalises plain addition of primitives. By arguing that H_2 could always offend against so far unseen measures of ideology, the opponent shifts the burden illicitly; for what is this

---

[23] Or anyway, they do so in the ontological class, counting in all sophisticated standards that come from the analysis of scientific practice. This gives us some reason to also expect proliferation in the ideological class.

far unseen measure, when writers on ideological profligacy largely contemplate only one candidate[24]? Accordingly, if we limit our use of levelling down to defend against pressure from abundant ideology, then only one measure is left for us to de-legitimise before we can pass on the burden.

4.2 Other responses

Before descending into the details of how we apply levelling down to our rivalry of interest (Chapters 8, 9, 10), I should emphasise that this strategy is not alone in the ballpark of responses we can offer to challenges from profligacy[25]. On another influential approach, for example, the challenges fail because the arguer from profligacy thinks of some hypothesis H_2 that it makes strictly more posits (or posits of a certain type: fundamentals, kind-tokens, etc.) than its rival H_1, but she fails to realise that the posits the two hypotheses differ in are "nothing over and above" or "ontological free lunch" relative to the posits they have in common[26].

Like those of the former type, these new responses deny that the profligate hypotheses (H_2) offend against measures that legitimise arguments from profligacy. Yet, this time, the denial ensues not from distinguishing legitimising from non-legitimising measures (with the profligate hypotheses violating only the latter). It owes to H_2 making no posits 'additional' to those made by its rivals.

Yet what is it to make a posit that does not add to another? The recent literature has offered many ways to understand the non-additionality of posits, all hoping to capture the rhetoric behind the obscure phrase "nothing over and above". Virtually every stance in the ontology and metaphysics of composition has its own distinctive

[24] This is not a comparative claim. It is no part of it that restricting levelling down to ideological measures is more efficient a strategy than restricting it to ontological measures, or even more efficient than using it to defend against all measures.

[25] The distinction between these two strategies owes to Hawley (2014), and so do the labels 'levelling down' and 'levelling up', which she uses respectively for the former and latter strategy.

[26] The phrase "ontological free lunch" originates from Armstrong (1997). As for "nothing over and above", it appears frequently and with a variety of intended meanings (as Smid (2016) helpfully notices). The present focus is limited to one (see Section. 2); and namely, a hypothesis' posits are "nothing over and above" some other posits (of the same or of a different hypothesis) just in case they do not add to the ontological commitments we incur into when we commit to the latter posits.

take. Thus nihilists have claimed that a composite *y* is nothing over and above its parts in that it is nothing at all (Merricks (2001: 13-15); French (2014: ch. 7)), while for friends of strong composition as identity (S-CAI), *y* being nothing over and above its parts *xs* boils down to its being many-one identical to the *xs*[27]. Finally, as Sider (2015: 191) notes[28], discarding the S-CAI and the nihilist interpretations makes the question of how we understand Innocence if anything more urgent. For composites being numerically distinct from and yet nothing over and above the things that compose them is "arguably central to our ordinary conception", while apparently "mak[ing] no sense"[29].

Now, if our present task were to argue for our profligate hypothesis of interest (H_2) by appeal to "nothing over and above", then the next step would be to untie this dialectical tangle; that is, decide how to go about explaining what it is not to add to some things if not being identical to them. This enquiry, which would bring us far afield, is luckily not our present enquiry. For one, relieving pressure from profligacy by appeal to "nothing over and above" is a strategy limited to hypotheses that compete for ontological, not ideological profligacy[30]. Accordingly, at least as the strategy is usually understood (see Sider 2013: 240), a hypothesis ceases to be profligate in respect to a rival if its *posits*, not its primitives, come to satisfy the phrase relative to the posits (not the primitives) shared with the rival (whatever it is to satisfy the phrase if not being identical to the shared posits).

Second, the appeal to "nothing over and above" is only one among many strategies found in the ballpark of responses to charges from profligacy. It suffices for achieving

---

[27] Armstrong (1997: 12) is among those who accept this connotation of the phrase (see Smid 2016: 106). On the contrary, the connotation we assume here (that x is nothing over and above y if x posits no additional commitment relative to y) corresponds more closely to Armstrong's "ontological free lunch" (1997: 13).

[28] Cf. 2007: S. 1.

[29] How do we go about answering this challenge? An appropriate answer should go in two steps. First (1), we explain what it is for a posit not to add to another. Second (2), we explain how it is that a posit not adding to another in this way allows to escape pressure from profligacy. Let us focus for brevity only on Step (1). On a *primitivist* approach, usually associated with Lewis (1991), we say nothing more on what it is to be a non-additional posit *y* other than it is to be a posit that bears a relation (R) sufficiently similar to identity to the thing *x* it does not add to. The notion of 'not adding to' is familiar from identity: for if *x* is identical to *y*, then *y* does not add to *x*. So we can infer defeasibly and from the points of analogy between identity and R that y does not add to *x*. On a *mereological* approach (owing to (Varzi 2000, 2014)), for *y* not to add to *x* is not for *y* to stand to *x* in a relation analogous to identity (whose *relata* do not add to each other), but rather for *x* to completely overlap *y*, whereby complete overlap is sharing of all parts: *x* completely overlaps *y* iff all parts of *x* are parts of *y*.

[30] See Sider (2013: 240).

our aim of protecting H_2 from the consequences of ideological profligacy[31] that we succeed in presenting one of these responses. What I take to be the ideal candidate, as already revealed, is levelling-down.

In addition to levelling down and strategies that appeal to "nothing over and above" there is another possibility for protecting H_2's from the consequences of ideological profligacy. Here the user of profligacy arguments singles out a particular hypothesis H_j (possibly H_2) as profligate in some respect M and proceeds to discard it in favour of a competitor H_i. However, no justification for discarding this particular H_j comes from (i) importation of parsimony-based methods from science; or (ii) epistemic principles that legitimise belief in H_i by showing that H_i's probability is higher than H_j's. Absent this justification[32] – so we conclude – discarding M is illegitimate.

Though promising[33], these responses from failed scientific or probabilistic justification remain a last resource in our enquiry. For one, the request for a probabilistic justifier is not universally accepted[34] [35]. Thus when Sider (2013) argues from ideological parsimony to his nihilist hypotheses devoid of mereological primitives (cf. Chapter 6, Section 4; Chapter 7, Section 2), he confesses that "principles of parsimony [including principles that promote sparse primitives] cannot be derived from more fundamental epistemic principles" (2013: Fn. 7). I am not convinced that pointing at some dismissible epistemic principles would change his mind so easily.

Besides, for these responses to have plausibility, they must presuppose that the target arguments from profligacy exemplify (i) and (ii) in particular ways; e.g. via a particular argument for importing the justification from science or a particular

---

[31] Again, H_2 is the hypothesis that ascribes $\mathbf{v_1}, \ldots, \mathbf{v_n}$ both linear and canonical structure, for example claiming with the atoms-only view that some of $\mathbf{v_1}, \ldots, \mathbf{v_n}$ are parts of themselves only and have no additional part.

[32] This classification of the justification sources owes to Huemer (2009).

[33] Huemer (2009) defends (ii) and rejects (i), but denies that (ii) justifies all uses of profligacy arguments in philosophy. Particularly, it does not justify popular uses in the philosophy of mind and mathematics. Bradley (2017) disagrees.

[34] Though it is to a good extent. Among those who expect probabilistic justification, see Fiddaman and Rodriguez-Pereyra (forthcoming), Rodriguez-Pereyra (2002: 208-9), Sober (1985, 2009, 2014), Schaffer (2015), Jansson and Tallant (2016), Bradley (2017).

[35] Commentators (e.g. Nolan 2015) name David Lewis as the example of a philosopher who leaves implicit all justification for employing arguments from profligacy (that is, justification for choosing a particular measure of profligacy, as well as justification for concluding as the arguments prescribe). However, for Cowling (2014: 3898), even Lewis seems to rely on (i), as he "proceeds by fixing upon our best theories [e.g. set theory] and holding whatever features support these theories to be reasonably viewed as theoretical virtues".

technique that assigns more parsimonious hypotheses higher probabilities. For example, Huemer's (2009) outline of (i) and (ii) (especially (ii)) has been influential and seems to capture the intents of many users of profligacy challenges. Yet suppose – for a dialectical point – that his argument for (i) failed, that the techniques for ascribing parsimonious theories lower probabilities in (ii) were unmotivated, or that they failed to apply to our rivalry of interest[36], so that H_2 does not come out less likely than its rival H_1 by virtue of them. If viable, this possibility works in our favour, as it casts into doubt the motivation for profligacy arguments in our area of interest. Yet to obtain the latter result I still prefer an argument by levelling down. For rejecting some particular arguments for (i) and (ii) (say, Huemer's) invites the reply by the friends of profligacy challenges that not all sources of motivation from probability or science have been assessed (cf. again Sider 2013: Fn. 7)[37]. Perhaps more could be said in support of (i) and (ii), e.g. more than Huemer's particular arguments. This situation – unfavourable for meeting our aim of protecting H_2 – makes it difficult to assign the burden of the proof; that is, to decide whether it is up to us, who reject some particular ways of articulating (i) and (ii), to find and reject more articulations, or whether it is up to friends of profligacy arguments to find more defensible ones. Fortunately (like before) we need not untie this dialectical tangle today. There is a more decisive way of resisting the arguments in our area of interest, which we begin to pursue starting from the next chapter. This is, namely, levelling down.

[36] Bradley (unpublished) argues that the rivalry between (pure) compositionalism and nihilism (stating and denying, respectively, that at least one composite entity exists) escape Huemer's techniques for assigning probabilities in such a way that parsimonious hypotheses come out more likely.

[37] For a probabilistic argument that putatively favours ontological parsimony, which Huemer omits, see Rodriguez-Pereyra (2002: 205), and Fiddeman and Rodriguez-Pereyra (S. 8).

VIII

Levelling down ideological parsimony

Many measures gauge hypotheses for their profligacy. Thus one hypothesis can outscore a rival simply by making more posits (plain quantitative profligacy), by positing more tokens of distinct kinds (qualitative profligacy), or even by positing more fundamental entities (fundamental profligacy). For advocates of the 'levelling-down' approach, it does not suffice for it being (all things equal) rational to discard the hypothesis that this offends against any of these standards. Rather, the standards offended against should be those in terms of which it is legitimate to blame the hypothesis for profligacy. Thus to level down some hypothesis H is to show that it offends *only* against innocuous measures of profligacy – measures, that is, in terms of which greater profligacy does not "matter for theory-choice" (Hawley 2014: 80).

Hawley advises (2014: 83) that we level down hypotheses with arbitrary sums of insignificantly related material objects: the sum of my laptop and the Eiffel Tower, of the left-half of a trout and the right-half of a turkey, of all the cats[1]. These sums – she argues – surely offend against the plain quantitative standard, adding to the overall count of posits, but they do not offend against the qualitative standard, penalising addition of new kinds, or against scientifically refined quantitative standards, penalising addition of entities that contribute additional explanatory complexity.

---

[1] The levelling down strategy is explicit in Lewis' (1973: 87) attitude towards hypotheses with arbitrary sums, as he accepts pressure from qualitative, but not plain quantitative profligacy. What remains implicit, as Cowling (2013: 3896) observes, is Lewis' justification for preferring one standard to the other. Nolan (2014) comments on many other implicit uses of theoretical virtues in Lewis.

In this and the coming chapter, the target sums are not those of insignificantly related mesoscopic objects, but rather those of unfamiliar objects of physical interest which relate as vector component to resultant sum (Chapter 6)[2]. My aim is constructive: to extend the levelling down strategy to these objects and relieve one of their putative violations in matters of profligacy. The target violation sources not (as in Hawley) from addition of posits (or posits of a specific type, like kind-tokens or fundamentals), but rather from addition of primitive ideology[3]. Hypotheses deploy ideology in excess ('part') if besides positing vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ that relate as component to sum they also state that some $\mathbf{v}_j$ of the $\mathbf{v}_1, \ldots, \mathbf{v}_n$ enters a canonical part-whole relation – for example, as in the atoms-only view of Chapter 6, by having itself ($\mathbf{v}_j$) as a part.

1. A new measure of ideology

Can we level down *ideologically* profligate hypotheses with mereological vocabulary[4] to hypotheses that do away with it? Cowling (2014: Section 8) suggests we can, provided we discard the canonical 'plain' ideological standard (minimise addition of new primitives) in favour of:

> *Primitive Kind Ideology* (PKI): H_1 is more parsimonious than H_2 just in case H_1 features less distinct *kinds of primitives*.

Primitive Kind Ideology measures addition of specific *types* of primitives: those that increase the number of distinct ideological kinds. The newly introduced notion of

[2] Though I do not rule out that some of my points could transfer to the familiar mesoscopic domain.

[3] See Chapter 6, Section 4 for the claim that the structure of hypotheses with sums feature not just suspiciously profligate existential content, but also suspiciously profligate representational resources.

[4] These hypotheses include the 'atoms-only' view of Chapter 6 (Section 3) and the other variants of 'pure nihilism' Sider discusses in (2013, see Chapter 6, Section 4).

'ideological kind' naturally grabs the critic's attention, but we will return to it in later chapters (9, 10). What matters for now is that, at least in prospect, Primitive Kind Ideology promises to level down hypotheses (H_2) that supply mereological primitives. Suppose that H_1 is a hypothesis *without* such primitives, such as the hypothesis with solely set-theoretic and linear algebraic ideology we examined in Chapter 4. Levelling down succeeds at two conditions: (1) mereological primitives and some primitive that H_1 also accepts partake of the same ideological kind; (2) preference between H_1 and the rival equipped with the mereological predicates depends not on plain ideological profligacy but on kind ideological profligacy.

Therefore, arguments from membership in a common ideological kind pave the way for shielding rich hypotheses like H_2, but they make up only half of required story. Indeed, the arguments ascribe H_2 primitives of the same kind as H_1, yet before concluding that H_2 comes at no cost additional to H_1, we also need to take a delicate decision as to which measure best adjudicates between the hypotheses: does only Primitive Kind Ideology measure the costs?

This division of work required for levelling down allows us to fix a strategy. I find it difficult to argue for (2) directly, so, instead, in Chapter 9 I will attack the most popular rationale to oppose it, which owes to Sider (2013). While this dialectic does not establish (2), it leaves us with two interesting claims. First, there is a genuine possibility that our rivalries of interest will be solved by kind ideological parsimony, because no pressure against this standard arises from the main argument supporting the rival 'plain' policy. Second, it is interesting to explore whether this possibility is viable. On the selective standard, mereological predicates in H_2 resist the shaving only if they are of a kind with some primitive of H_1. Yet the question of which among H_1's primitives shares a kind with mereological primitives in this way is not trivial. Cowling suggests sketchily that the key kind-sharer is *identity*. In Chapter 10, I will provide details.

2. Part-whole or composition?

We make a preliminary observation before starting the levelling-down process. So far we have been assuming that the ideologically profligate hypotheses of interest –

those with linear-algebraic, set-theoretic and mereological ideology, which we currently try to level down – use part-whole as a primitive. Yet for Cowling it is identity and *composition* that share a common ideological kind, not identity and part-whole (2014: 3906). Therefore, even if we could get Cowling's suggestion running, we would still need an argument to prevent our hypotheses from offending against (PKI) with their part-whole primitive.

An intriguing response is to reformulate the hypotheses so that they feature primitive composition and part-whole defined in terms of it. In this guise, for example, the atoms-only view of Chapter 6 (Section 3) would state not that every member $\mathbf{v}_1, \ldots, \mathbf{v}_n$ of a vector space V is part of itself only and has no proper parts, but rather that every such member composes into itself only. Statements about the parts of $\mathbf{v}_1, \ldots, \mathbf{v}_n$ would then follow from the definition of part-whole given in terms of composition.

As at least one influential attempt has been made (Fine 2010) to break through the study of formal mereology with composition as a primitive, one might suggest that this is enough of a promising start to accept primitive composition in the hypotheses. I would agree, but not to the point of focusing my attention *only* on levelling down hypotheses with primitive composition. Rather, hypotheses with primitive part-whole remain first priority in the levelling-down agenda, and it is so much the better if the story for levelling down these hypotheses also ensures the levelling down of primitive composition. From Cowling's suggestion we will learn as much as we can – notably, (PKI) – in the process of our quite different task of levelling down primitive part-whole.

Now, there is a rival procedure towards the same goal. Smid (2017) wishes to level *up* primitive part-whole, dropping (PKI) and showing that one of the part-whole and identity primitives is "nothing over and above" the other, because it is defined in terms of it. He sets off from two famous passages from Armstrong (1997: 37-8) and Lewis (1991: 84-5), both of which present the idea that composition, part-whole, overlap and identity are all members of the same kind. For example, for Lewis: "[s]o striking this analogy is [between mereological relations and identity] that it is appropriate to mark it by speaking of mereological relations – the many-one relation of composition, the one-one relations of part to whole and overlap – as kinds of identity. Ordinary identity is the special, limiting case of identity in the broadened sense" (Ib.)[5].

Smid dislikes the allusion to kind-membership, which he finds imprecise, and prefers talk of 'limiting cases'. He observes correctly that Armstrong (Ib.) valued this idea substantively, and used it to describe classical mereology (with the principle of extensionality)[6] as "a development of the logic of identity" (1997: 38). The thought is that two completely overlapping objects (i.e. sharing all parts with each other) are identical: if x completely overlaps y, y completely overlaps x and mereology is extensional, then $y = x$[7].

Having said this, one needs to get clear on why this characteristic of complete overlap (obtaining in an identity way in the symmetric cases) contributes to mereology 'developing' the logic of identity. For Smid, the contribution is that mereology has the logic of identity as a sub-class[8]. In this situation – he says – it will suffice to adopt the primitives of the more general theory, i.e. mereology. But crucially for levelling-up, the classical extensional system for mereology admits a definition for identity in terms of part-whole: '$x = y =_{df} x$ is part of y and y is part of x'[9].

The 'levelling-up' effects are evident: all hypotheses with identity, which we previously thought to have no mereological ideology, revert to hypotheses with part-whole and cease to compete for profligacy. It is not even necessary to challenge Sider on the legitimate measures of profligacy: the penalised hypotheses remain those that contribute 'plainly' to the number of primitives[10].

---

[5] Quoted in Smid (2017: 2382).

[6] In the version of Leonard and Goodman (1940).

[7] Armstrong also deems a 'limiting case' of identity the relation of *partial* overlap, i.e. sharing *some* parts with another object. This relation has only symmetric cases, e.g. two neighbouring offices sharing a wall, and when the shared parts are all the parts of the sharing objects, it also applies in an identity way, see (1997: 38).

[8] Here is Smid (2017: 2382): "If one thinks indeed that mereology is an extension of the logic of identity, one could just as well reverse the order and hold that mereology is the logic of (general) identity, whereas the logic of pure identity (the notion expressed by '=') is only a part of this general logic. In that case, it makes sense to define '=' in terms of the only primitive ('is part of') of your 'general logic of identity' (i.e. mereology)".

[9] Which is also from Leonard and Goodman (1940: 46).

[10] Sider has its own objections to defining identity in the mereological way, see (2013: Fn. 10), Smid responds in (2017: S. 2).

As these conclusions favour our dialectic – saving the hypotheses with primitive part-whole - Smid's approach remains a rival at home. The policy towards these internal debates (Chapter 6) is to come back at them at the end of our priority task, which is that of challenging the naturalist-friendly uses of Ockham's razor to shave off canonical mereological structure from domains of physical interest. Having said this, however, at least one *en passant* observation on the in-house rival will be helpful.

I find a different reaction to Armstrong's observation at least as plausible as Smid's. The idea that complete overlap can obtain 'in an identity way' (that its symmetric cases are also cases of identity) suggests not that the logic of identity is a sub-theory of mereology, but rather that complete overlap should be defined as the disjunction of identity and proper complete overlap (all parts of *x* are parts of *y* and at least one part of *y* is not a part of *x*).

In this picture, rather than eliminating identity for complete overlap we eliminate symmetric complete overlap for identity[11]. *Prima facie*, Armstrong's remark that some cases of overlap behave in an identity way motivates this reaction (fundamentally there are no symmetric cases of complete overlap, only cases of identity) as much as it motivates Smid's (there are genuine cases of complete overlap, and they define identity). Besides, my reaction would not seem to drop the intuition that identity is a 'limiting case' of complete overlap: when there are no more parts of an object *y* that we can supply to an object *x*, then we have reached the case of *y* being identical to *x.* This interpretation seems to me to conform to the wording 'limiting case' just as much as the interpretation that in the limit of supplying parts of *y* to *x*, *x* and y completely overlap, which defines $x = y$.

3. Ideological and other kinds

[11] A similar policy applies to partial overlap, i.e. sharing some parts with another object. This relation has only symmetric instances, some of which obtain in an identity way. Instead of redefining it in terms of a non-symmetric relation, we redefine it as identity or *proper* partial overlap, whereby the latter is sharing of some, but not all parts.

Let us return to the project of levelling down hypotheses rich in mereological predicates by showing that these partake of an ideological kind with identity. When it comes to what it is in virtue of which composition and identity partake of the same kind, Cowling gives us enough to elaborate on:

> More generally, composition has a strong claim to being viewed as a broadly logical relation. Like identity, it contributes nothing to the non-structural, qualitative character of the world, and, like identity, facts about its general nature seem to be a non-contingent matter. Furthermore, regardless of whether one endorses nihilism, classical extensional mereology demands certain conceptual ties between these relations. Most notably, the uniqueness of composition precludes distinct entities being composed of the very same objects. In light of these connections, I take it that a plausible conception of ideological kindhood holds identity and composition to be of a common ideological kind (2013: 3906).

On a natural construal, this passage maintains that composition and identity are similar in various respects, all of which can be used to infer (defeasibly) that composition partakes of an ideological kind that has identity as a member (henceforth: K). The respects of similarity are (1) *Uniqueness* (every *xs* compose only one sum; every *x* is identical to only one thing); (2) *Necessity* (If some *xs* compose into a sum *y*, then it is necessarily the case that *xs* compose into *y*; If *x* is identical to *y*, then it is necessarily identical to it)[12]; and (3) *Anti-quality* (Composition, like identity, "contributes nothing to the non-structural, qualitative character of the world").

Further, common to all three principles is their viability for nihilists. For suppose that there were no instances of composition (not even instances of 'lone' composition, i.e. entities that are sums of themselves). In this case, Necessity and Uniqueness would be trivially true because their antecedents are false for all *xs* and *y*, and Anti-quality would be true for two reasons. First, because there are no proper composites

[12] I take it that this is what Cowling has in mind when he claims: "like identity, facts about [composition's] nature seem to be a non-contingent matter" (2013: 3906). Indeed, Cowling invokes Cameron (2007) as an objector, and the principle Cameron objects to in (2007) is *Necessity*.

and composites that do not exist cannot contribute to the qualitative aspect of the world. Second, because improper composites are identical to the things that compose them, and we have already assumed[13] in setting up the argument that identicals do not contribute qualitatively.

It is worth asking why Cowling's preferred list of similarities does not extend further. For example, consider Unrestrictedness: every *xs* have a sum *y*, which in the logic of identity corresponds to: every *x* is identical to some *y*[14]. Unrestrictedness is clearly unavailable for nihilists, because it states that it suffices for there to *be* a sum of the *xs* that the *xs* exist, while the nihilist argues that no condition on the *xs* (let alone their bare existence) suffices for the existence of their sum.

Presumably Cowling's reason for omitting Unrestrictedness from the points of resemblance is that nihilists would accept the argument's conclusion – composition and identity partake of a common ideological kind K – only if all premises were principles they accept. In fact, the argument dialectical role changes if Unrestrictedness enters the reasoning. If the argument is successful *without* Unrestrictedness, then the effect is to remove one of the nihilist's alleged points of advantage, i.e. greater parsimony, no matter which theory of composition one ought to accept for doing so. On the other hand, if the argument is successful but requires Unrestrictedness, then the effect is to offer a means of resisting pressure from profligacy to those who already deny nihilist hypotheses.

We focus on the argument with the former, more powerful dialectical role: blocking a point of advantage for the nihilist without requiring a non-nihilist theory of composition for so doing. Accordingly, we exclude Unrestrictedness from the premises of the required reasoning and we assume that the composition predicate that will partake of identity's ideological kind satisfies only principles available to the nihilist.

3.1 Initial difficulties with turning the suggestion into an argument

[13] The assumption is not entirely straightforward. Recently, exotic time travel cases have been brought to bear to argue that identity (or at least identity at a time) can make contributions to the qualitative aspect of the world. For discussion, see Kleinschmidt (2014).

[14] Unrestrictedness appears in Lewis' (1991) list of points of analogy between composition and identity, see below.

There are eye-catching affinities between Cowling's suggestion and a famous one attributed to Lewis' *Parts of Classes* (1991)[15], according to which points of analogy between identity and composition confirm (defeasibly) that the *relata* of composition, like those of identity, are ontologically innocent (recalling from Chapter 8, one of two *relata* is innocent if it does not add to the ontological commitments one incurs into by positing the other).

However, impressions aside, the affinities have tight limits. For one, Lewis' preferred points of analogy differ from Cowling's, including Unrestrictedness (see above) and Location Inheritance (every sum *y* of the *xs* is located where the *xs* are located; every *y* identical to *x* is located where *x* is located)[16]. For another, Cowling's quote is too bare to decide on its exact argument structure, and *a fortiori* too bare to decide whether the intended structure matches Lewis'[17]. According to Cowling, the similarities make it 'plausible' that composition partakes of identity's ideological kind, but the mechanism that guarantees plausibility remains concealed[18]. Adding this to the differences in conclusions (the *relata* of composition are innocent; composition partakes of K), I find it legitimate to develop Cowling's suggestion in directions unconstrained by the Lewisian text[19].

Having said this, a remarkable affinity remains between Lewis and Cowling. In both cases we infer (defeasibly) that composition has one of identity's features starting from a list of respects in which the two are similar. Moreover, in both cases we expect

---

[15] We need exegetical good sense. This attribution owes to Yi (1997). See Bohn (2011), Bennett (2015: 256-7), Smid (2014: 3266) for a contrasting opinion.

[16] Lewis also includes *Ease of Description*, i.e., that all descriptions applicable to parts are *ipso facto* applicable to wholes. But see Berto & Carrara (2009: 349-50) for convincing criticism.

[17] According to Yi (1997), Lewis casts an argument by analogy. These arguments start by attributing some base features $A_1, \ldots, A_n$ and an analogical feature B to a term of comparison N_1. Second, they identify a target N_2 (the thing that will be analogous to N_1). Third, they note that N_2 also has $A_1, \ldots, A_n$. Putting this together, they conclude that N_2 also has feature B. Let's now apply the model to the case of identity and composition. These two relations will feature as terms of comparison (N_1 and N_2 respectively). The base properties that make the two analogous are uniqueness, unrestrictedness and location inheritance[17]. The analogically transmitted property B is of course Innocence.

[18] This difference is not to Cowling's detriment. For if Yi is right that Lewis argues by analogy (as in Fn. 20), then one could question the force of these argument types in philosophy by invoking their overlooked, but sophisticated success conditions in science, see e.g. Bartha (2010).

[19] Yi (1997: Section 2) objects forcefully to Lewis' argument, which he interprets as in Fn. 19 (see Varzi 2014: 50-1) for an endorsement).

to win this feature thanks *entirely* to the respects of similarity, not thanks to a *sui generis* feature unique to composition. How is this possible? For Cowling's conclusion – the only subject of this chapter – what seems required is a story of how composition comes to partake of K (a kind that has identity as a member) in virtue of these points of resemblance.

Clearly Cowling's quote *per se* provides only the early beginnings of this story[20]. Strictly speaking, the function of his quote is to isolate a resemblance class of which (primitives for) identity and composition are members. This is, namely, the class of all primitives that resemble identity in the key respects of Necessity, Anti-quality and Uniqueness. However, it is one thing to highlight this class and the associated membership conditions, and quite another to show that one of its members – composition – partakes of a kind that has identity as a member (K). Surely this latter claim cannot be justified to a critic without answering *some* questions about the properties of ideological kinds. But how many questions?

3.2 Cardinal questions on kind structure (1)

We divide questions about the properties of ideological kinds between those that deserve an urgent answer and those that can (arguably) be postponed for later enquiry. In either case, all answers that we should give for a development of Cowling's suggestion are bound to remain controversial and subject to substantive debate. Highlighting them in this section will help us get clear on what we demand of ideological kinds prior to claiming their benefits on the side of parsimony arguments.

1. For a start, we certainly need to affirm that divisions into ideological kinds are objective, or at least not too wildly subjective. The risk with 'wild' subjectivity is that critics could reject our preference for mereologically rich hypotheses (which we take at no cost) based on it being a subjective matter that mereological predicates like part-whole and composition are of a kind with identity.

To be clear, it is neither easy nor uncontroversial to explain what the objectivity of kinds consists of. The literature on *natural* kinds, which we will often consult when

[20] Not that this can be imputed to Cowling, who makes his fruitful remark at the end of his paper.

such difficulties arise, offers various friendly options (but unavoidably even the choice within this camp of friends will fail to satisfy many). On some metaphysically inclined ('naturalist' or 'weakly realist')[21] approaches, the world is responsible for the objective kind-groupings[22]. These approaches are attractive in our context[23], because we have already accepted (Chapter 7) that plain items of ideology make commitments to aspects of reality's structure, with the items in true theories making commitments that match the real aspects. Accordingly, it would seem natural to assign the same role to items of true theories that are also tokens of ideological kinds. These items would latch onto further aspects which constitute an objective classification of the former. However, we cannot pretend that the move is uncontroversial. Even if we accepted that reality is responsible for the first layer of aspects, their classification into a second layer could be entirely up to us[24].

2. Second, we need to deny that too many members from the various classes of things that resemble identity in some respect count as one in kind with it. Otherwise

[21] For the label 'weak realism', see Bird and Tobin (2017).

[22] Typically, real divisions between natural kinds are considered to be timeless. But Boyd (1991, 1999), Millikan (1999) and more recently Magnus (2012) argue influentially for allowing change over time. Their view accounts most fruitfully for divisions between biological kinds, but if the fundamental laws of nature change over time, then perhaps even physical kinds like *electron* and *quark* have time-evolving boundaries. What about ideological kinds? Answering this question will be hard until we have a first-pass idea of the groupings of primitives that underpin the kinds (Section 4). So I suggest that we leave time evolution an open possibility, at least for *some* ideological kinds. On Boyd's, Millikan's and Magnus' views, the change in the kind divisions is explained by natural processes (natural selection, perhaps change of the physical laws, etc.), which vary the respects of similarity necessary to enter the kinds (e.g. evolution pushes genes out of a species' gene pool, thus, over time, requirements on being a member of that species vary, see Bird and Hawley (2011: 216)). This feature seems to block kinds with identity and composition as members, for what *natural* processes impact on their similarities? Having said this, the view is not inconsistent with there being non-evolving kinds. And as I argue in the next section, there is a rationale for expecting ideological kinds not to be *entirely like* natural kinds.

[23] But Dupré (1993) observes that although classifications are grounded in real distinctions, various sciences (and common sense) disagree on the distinctions salient to the taxonomy. Moreover, the disagreement has no privileged resolution: there are legitimate, incompatible policies for classifying the same item as member of a natural kind. I cannot do full justice to this position, but I am not so sure that it offends our aim of motivating (PKI). The choice for salient respects seems to be discipline-specific and our rivalry between hypotheses with and without mereological predicates takes place within a single discipline (metaphysics). Accordingly, as our opponent is a metaphysician, she is prevented from rejecting our claim that composition comes at no cost based on picking other salient distinctions.

[24] There is a plethora of positions that posit objective kind divisions by appeal not to real world differences, but to universally shared beliefs or conceptions. Even beginning to discuss these positions would lead us far afield, but see Hacking (1995, 1999) for a classic view of this stripe on social kinds, and Khalidi (2010) for an extension to chemical kinds.

put, conditions for co-membership in a kind K with identity are to some extent selective. Fixing restrictive entrance conditions is another controversial decision[25], but missing out on it would be end game for (PKI). For again, if the conditions for entering K are too liberal, then the principle proves too powerful: too many different primitives would come at no cost additional to the cost of identity.

A critic could press us on this second concession. This talk of liberal and restrictive entrance conditions is too vague[26]: for how many, exactly, are the few primitives that enter a kind K with identity as a member?

I agree on the question's importance, for without this piece of information (what respects of similarity link identity with its kind co-members) we risk one among (i) proliferation or (ii) excessive restriction of primitives that partake of K. As we observed, (i) makes ideological kinds implausible instruments of parsimony, which legitimise too many primitives at no cost. On the contrary, (ii) risks excluding composition from the primitives that successfully come at no cost. The conditions for entering K could be so strict as to admit only primitives similar to identity not just in Cowling's respects, but also (say) in respect of *Symmetry* (if $x = y$, then $y = x$).

In fact, it is lacking this piece of information that leads us to reject Cowling's conclusion – that composition is one of a kind with identity – even granted the other claims he makes in the quoted passage (2013: 3906). For, as said, the quote highlights a resemblance class with composition and identity as members, but ensures neither that this class is the only one to supply members for identity's kind (opening the door to proliferating kind-partakers); nor that it is the right class for supplying these kind-members (for the right class could collect only primitives that resemble identity in respect to Symmetry, in addition to Cowling's respects).

Having said this, I remain more optimistic than the envisaged critic. To pin down the members $P_1, \ldots, P_N$ of an ideological kind that already includes some primitive P we can look directly at the cases of scientific practice that, by shaving off P, have been taken to motivate plain ideological profligacy (Chapter 9: Sections 1, 2). An initial

[25] Quine (1969) argues that co-membership in a natural kind requires natural similarity in any respect, but Mill (1884) already noticed that natural similarity in respect of (e.g.) being white does not make it for a *kind* of white things.

[26] This critic's question goes by the name of *kind-hood question* (Bird and Hawley 2011: 205-6). The question asks which among some natural similarities 'underpin' kinds. Moreover, for Boyd, Millikan and Magnus (see Fn. 22), requirements for co-membership can require similarity in extrinsic properties (e.g. a common causal origin) and underpin the kinds only at particular times.

grouping of primitives, which includes P, is visibly suggested by these practices, which make clear what types of conceptual resources are being sacrificed by enabling new sacrifices of resources $P_1, \ldots, P_n \neq P$ of the same type. It is attractive to exploit this consistency for information about the kind-members of P[27], but I will begin articulating this proposal from Section 4.

3.3. Cardinal questions on kind structure (2)

We have answered two questions about the properties of ideological kinds, assuming objective world-based divisions and strict entrance conditions (with information on 'local' groupings coming from the analysis of scientific practice).

These answers are, admittedly, not only controversial but far from covering the entire range of issues that pertain to an account of ideological kinds. However, we can *argue* that they are enough for us to start a piecemeal enquiry into groupings of primitives with a claim to be kinds. I say 'argue', for I do not think that deflecting the need for an analysis of ideological kinds is a straightforward matter. We need to demonstrate, not assume, that generating a full account is a second priority.

Looking back at the more developed literature in this area, consider another cluster[28] of questions commonly answered by a full account of *natural* kinds: (1) Which types of classes collect members of natural kinds (resemblance classes, genus-species classes, determinable-determinate classes)? (2) Do kinds and the similarities underpinning them figure in natural laws? (and if so, what laws: fundamental or non

[27] What we cannot get out of the analysis of scientific practice are conditions for co-membership in same kind of X, where X is an *arbitrary* primitive. Fortunately, we do not need this depth of analysis. If we can extract information as to the primitives that enter a kind with =, then that will be enough for assessing the status of composition and part-whole. Cowling claims: "certain diagnostics are useful for discerning ideological kinds – e.g., whether the concepts in question are interdefinable – but fixing upon the particular ideological kinds is (and should be) a matter of careful, case-by-case metaphysical examination" (2013: 3898). I agree on the piecemeal character of the examination.
As for its nature, by deeming it "metaphysical" Cowling means that accepting common kind-hood for certain primitives should be oriented by the problems we can solve in metaphysics. For example, if we accept that primitives for monadic and comparative naturalness ('…is natural', '…is more natural than…') come in one kind, then we can accept both at no cost and solve objections to theories that posit one but lack the other (2013: 3900). On this point I diverge. My acceptance of common kind-hood is oriented not by the fact that attractive metaphysical hypotheses come at no cost, but by the fact that plausible kind-groupings for ideology are visible in scientific practice.

[28] For other key questions, see Bird and Tobin (2017: 1.1.1).

fundamental?)[29]; (3) By what mechanism do kinds and the kind-underpinning similarities allow induction inferences between each other[30]?

Cowling thinks that we need not answer these and other questions about natural kinds prior to using (PKI) proficiently. For him, we already penalise hypotheses with profligate *ontological* kinds, even if we do not possess a full account of the penalised and retained kinds along the questions' lines. For example, there is no full analysis of the kind 'set'[31], yet (according to a familiar story) mathematicians systematically renounce kinds other than 'set' by holding onto set theory in face of alternatives[32]. Therefore, the point is that no analysis of ideological kinds is demanded prior to using (PKI), because no analysis of ontological kinds is demanded prior to using arguments from qualitative profligacy.

Let us try to look at this point as one of interpretation of the mathematical practice. Take indeed the case of (pure) set theory and assume that 'set' is an ontological kind. Set theory is paradigmatically parsimonious, thus it seems that the mathematicians' practice of holding onto it in the face of richer alternatives is interpreted *correctly* as one of renouncing additional kind-tokens. Mathematicians can penalise for qualitative profligacy without an analysis of ontological kinds.

Having said this, it seems possible to interpret the mathematical practice in this way – as a case of applied qualitative parsimony – only thanks to an available 'contrast class'. We know that what we would have to do to renounce entities 'plainly' (as opposed to 'selectively' by renouncing kind-tokens) is to minimise the number of pure sets. While we realise this, we also observe that mathematicians do not minimise the number of pure sets at all. Rather, from the point of view of plain

---

[29] E.J. Lowe (2006) argues for kind-featuring laws of nature. Hawley and Bird (2011) observe that kinds figure only in non-fundamental laws and argue that the fundamental ones feature only the kind-underpinning similarities.

[30] Once we have settled these questions, we can also provide a metaphysical underpinning for the mechanisms that allow kinds to partake in laws, support induction, have essential properties, etc. Here the debate is between universalist, particularist and *sui generis* positions. For this blueprint of the debate, see Bird and Hawley (2011: S. 2), who also opt for kinds as structural universals (without unique composition, see Chapter 4: Fn. 29). Lowe (2006) is the paradigmatic approach to *sui generis* positions, with kinds as an irreducible ontological category.

[31] The assumption here is that 'set' is no natural kind.

[32] For Cowling, the success of set theory in mathematics is also the reason why Lewis values qualitative parsimony.

quantity pure set theory is tremendously profligate. Therefore, the only interpretation that makes sense of the mathematicians' practice as a case of applied parsimony is one in which the sacrificed posits are tokens of new kinds. This is why Cowling is correct on there being proficient users of parsimony principles for kinds: If the mathematical practice is interpreted as a case of applied parsimony at all, then it can only be interpreted as a case of applied qualitative parsimony.

However, I doubt that we can produce an analogous story for ideological kinds. Suppose that we looked at the mathematical practice from the ideological point of view; that is, trying to figure out what representational resources are renounced systematically by holding onto pure set theory. The evidence now is that mathematicians cling to theories replete with set-theoretic resources (e.g. membership) and reject richer alternatives. For an example of these alternatives, consider a theory with primitive membership and a primitive *P* for the relation 'being a linear component of…' [33]. The practice of eliminating *P* makes sense primarily not as removal of a *kind* of primitive, but rather as 'plain' removal, i.e. minimisation of the overall number of primitives. Thus, in contrast with the ontological case, one can make sense of this practice as a case of applied parsimony without invoking removal of kinds. Even though Cowling is right on proficient uses of qualitative parsimony without analysis of the involved kinds, his evidence cannot support the analogous point that (PKI) is used proficiently without analysis of the associated kinds.

3.4 Second priority for analyses of ideological kinds

So, do we need an analysis of ideological kinds prior to using (PKI)? It could be seen a consequence of my arguments that we do, *pace* Cowling. However, and for two reasons, this too would be an exaggeration.

1. We start with gathering evidence about groupings for some primitives P from cases of scientific practice, in which the preferred stock of representational resources renounce P (Chapter 9, Section 1). As a first-pass policy, suppose that we inferred that these preliminary groupings with P, P_1, …, P_N as members are ideological kinds. This

[33] This theory holds that linear components **a** of vector sums **b** are neither members nor subsets of **b**. One way of achieving this is by denying that vectors are sets, as in Chapter 3.

can put us at odds with some answers to the questions in (1), (2), and (3). For example, it could turn out that the similarities S_1, ..., S_n underpinning the kind of P, P_1, ..., P_N play no role in fundamental laws of nature, or that these similarities are not amenable to determinate-determinable, or genus-species structure (because they contain no similarity S_j in respect to a common determinable or similarities S_K in respect to a common genus accompanied by differences D_k separating out the genus' species).

These consequences could force us into a specific position in respect to questions (1)–(3). By reaching this position, granted, we cease to be neutral on the questions. However, the answers we end up giving could still be plausible. Perhaps the groupings of primitives we infer from our first-pass assessment of the evidence meet appropriate structural constraints, defensible for at least those particular groupings.

On the contrary, suppose that the reached position in respect to 1., 2., 3., *etc.* were untenable, for example because ideological kinds deserve a determinable-determinate structure, or deserve underpinning-similarities that partake in fundamental laws. Here we will confess to have read the scientific evidence inappropriately and return to it with a more structured blueprint of the groupings we are looking for.

Either way, this policy of combining the analysis of evident scientific groupings with philosophical accounts of the kinds' structure has point of attractiveness. Namely, it sets the groundwork – reading groupings off scientific practice – on a par with building an account of kinds. If the injunction is plausible, neither task is prior to the latter.

2. There is another reason why we had better not build the account prior to running arguments based on (PKI). The subject matter of (PKI) is ideological kinds, not natural kinds, thus it is not clear what we should accept as an objection to inferring that our first-pass groupings are ideological kinds. Differently put: what we accept as 'untenable' positions on questions like (1)–(3) depends on a delicate issue: how many structural *desiderata* ideological kinds have in common with natural kinds.

The issue's delicacy could play in favour of sceptics, who doubt that we can move from groupings to kinds without getting clear on the constraints on groupings that claim to be kinds. However, to turn the issue in our favour, we add the following observation. Some of the inferences we take from groupings to kinds will resist the sceptic's objection, for we can gauge our expectations about these constraints *partly* based on the features of groupings that would clearly come out as ideological kinds if there were any ideological kinds.

Consider the case of 'part' and 'proper part'. The primitives are inter-definable[34], which (I assume) makes for "strong evidence that [they] are of a common ideological kind" (2013: 3899)[35]. Moreover, they are similar in at least some logical properties; for example, according to mereology, they are both transitive. But this similarity in respect to transitivity is neither a similarity in respect to a common determinable, nor a similarity that features in laws of nature or strengthens inductive inferences in an appreciable sense (we do not infer *inductively* from the two primitives being members of K' to their transitivity properties)[36].

Now suppose that 'part' and 'proper part' were one in ideological kind. The supposition is plausible, for if there is one indubitable case of common ideological kind-hood, this is that of inter-definable primitives. Moreover, it follows from it that at least some ideological kinds escape – without objection – requirements that are attractive to natural kind theorists. On this basis, I propose we derive the following sensible policy. Assessing the evidence from scientific practice, we may begin producing a first-pass scheme for groups of primitives with a claim to be ideological

---

[34] For the inter-definability of 'part' and 'proper part' (x is a part of $y =_{df} x$ is a proper part of y or $x = y$) we need no assumption other than reflexivity for 'part' and Leibniz' Law (Lejewski 1957, Varzi 2014: S. 1). On the contrary, the interdefinability of 'part' in terms of 'overlap' (x is part of $y =_{df}$ every z that overlaps x overlaps y, as in Goodman (1951)) is achieved only in stronger systems, such as extensional and general mereology. This observation owes to Parsons (2014).

[35] This figures as the premise of an argument (2013: 5.1), which concludes to (PKI). According to the argument, the choice between some theories formulated with alternative, but inter-definable primitives <P_1, P_2> is arbitrary unless (PKI) is true. But we should reject arbitrariness in theory choice, hence (PKI) is true. Examples might be modalist theories that differ only in whether the box is defined by the diamond operator, or *vice versa* (Ib. 3899). If (PKI) is true and if P_1 and P_2 are of a common ideological kind, then theories with *both* P_1 and P_2 come non-arbitrarily and at no cost additional to the cost of P_1 and P_2. Hence (PKI) is motivated by its ability to remove instances of arbitrariness. For reasons outlined in Fn. 30, I reject these 'teleological' arguments for (PKI).

[36] What about it being in the essence of 'part' and 'proper part' that they are transitive? For the sake of brevity, I have largely omitted the area of essentialism and will continue to do so, maintaining neutrality. Suffice it to note that my discussion in Chapters 2 and 3, resulting in algebraic relations (component-sum, i-split/vector) being members of the 'part-whole rubric' is compatible with it being essential of part-whole to have logical properties lacked by these algebraic relations, e.g. Antisymmetry. On *one* way of articulating the claim that the algebraic relations partake of the rubric, the relations are genuine varieties of part-whole. This position (mereological pluralism, which I am attracted to) is *compatible* with essentialism about Antisymmetry. Just, the logical property will be essential to some of the many available part-whole relations (the members of the rubric).

kinds. We will not suffer objections from having missed structural constraints on these groupings (e.g. underpinning similarities in respect to common determinables, or featuring in natural laws, etc.) *if* cases of inter-definable primitives, which paradigmatically belong to the same kind, feature none of the constraints that the objector holds missing[37].

The hope is for this policy to grant us leverage in classifying *some* groups of primitives as kinds without a full analysis of the ideological kind notion. The policy is not as effective as to grant that all ideological kinds lack plausible constraints on natural kinds. Presumably, the issue of whether a particular grouping of primitives P_1, ..., P_n has a claim for kind-hood without the constraints deserves piecemeal discussion and separate justification for each particular grouping.

4. Conclusion

In this chapter we introduced the possibility of protecting ideologically profligate hypotheses by showing that their items of ideology in excess do not penalise the hypothesis against a rival which sacrifices them. In fact, according to this strategy, penalties apply only if the exceeding items partake of exceeding ideological kinds; that is, kind, none of whose members figure in the ideology of the rival. Applied to our case of interest, this strategy holds that theories positing vectors $\mathbf{v}_1$, ..., $\mathbf{v}_n$ with linear structure[38] deploy just the same kinds of primitives as rivals, which portray some $\mathbf{v}_j$ of $\mathbf{v}_1$, ..., $\mathbf{v}_n$ as entering a canonical part-whole relation. It takes additional ideology ('part'), but not ideology of additional kinds, for example, to state as in the atoms-only view of Chapter 6 that some $\mathbf{v}_j$ has itself as a part.

Up to this point we only considered the bare guidelines of this strategy, telling it apart from 'in-house' competitors (Section 2) and shedding more light on its key

[37] Notice that I am not using 'paradigms' as in Lando (2017), whose approach I criticised in Chapter 2. He thinks that the relation of spatial inclusion is 'paradigmatic' in that its logical properties (Reflexivity, Antisymmetry and Transitivity) are necessary conditions on all part-whole relations. On the contrary, I suggest that absent further analysis of ideological kinds, we can infer defeasibly that kinds K'' have the properties of paradigmatic kinds K' with interdefinable members. These properties are not necessary conditions on all ideological kinds.

[38] That is, satisfiers of the concept 'vector component', see Chapter 5.

notion, i.e. ideological kind, in relation to the more familiar one of natural kind (Section 3).

To fulfil the strategy we need two further theses, which conveniently occupy the last two chapters. First, it must be appropriate to resolve our rivalry of interest counting ideological kinds, not plain items of ideology. Second, hypothesis that posit vector space elements with linear structure and attribute them canonical structure, e.g. the atoms only view, ought to deploy just the same ideological kinds as their rivals; that is, hypotheses that posit vector space elements with solely linear, component-sum structure.

IX

Levelling down ideological parsimony (II)

Ideological kinds in scientific practice

In this and the following chapter, we continue the process for levelling down ideological parsimony, which amounts to showing[1] (1) that not (just) parsimony in 'plain' ideology, but (also) parsimony in ideological kinds is a factor for theory-choice; and (2) that the primitives of mereologically rich hypotheses (here: the part-whole primitive) are of a common ideological kind with some primitives of the hypotheses' rivals (here: identity). Each of these two steps occupies a dedicated chapter. Here I argue that the episodes of scientific practice usually taken to motivate parsimony in 'plain' ideology equally motivate parsimony in ideological kinds. So, if we accept these episodes as motivators for the former type of parsimony, little reason remains not to accept them as motivators for the latter.

Second (Sections 2, 3), drawing on the last chapter's observations (Chapter 9, Section 3), I argue that the scientific episodes offer clear and reliable information about members of the kinds they suggest we give up (in switching to more

[1] Cf. Chapter 8, Section 4.1.

parsimonious hypotheses) and preserve (when holding onto parsimonious hypotheses present richer alternatives). The displayed information, specifying which resource is a member of which kind, depends on the theory's exact 'taxonomising' devices and the purposes for which they are introduced.

From these cases of scientific practice we will learn as much as we possibly can for a naturalised approach to ideological kinds. Particularly, bundling resources into kinds – rather than mere groups – will only be allowed granted scientifically significant conditions of entrance in the group, imposed for scientifically significant purposes. This is still in line with Chakravarrty's (2017) tripartite account of naturalised metaphysics, which I availed myself of in Chapter 7 (Section 2.3), and according to which a successfully informed metaphysical theory imports *contents*, *methods* and *aims* from advanced science[2].

Putting this together and moving into the next chapter, our theory of interest is a theory of ideological kinds that ascribes resources for part-whole, composition and identity to the same kind. The imported 'contents' are the kind's entrance conditions, which should reflect a scientifically reputable way of collecting certain resources into a group and excluding others from it. The imported 'aims' are the purposes for building the taxonomy in this, rather than another way. For success in metaphysics, these purposes too should be endowed with scientific significance. Meeting this challenge, we will win an argument that identity partakes of a single ideological kind with the resources for part-whole and composition.

1. Physical justifiers for sparse ideology

Having discussed ideological kinds to some length already, we came to consider them a live option among the set of resources that fuel arguments from parsimony:

Type of measure	*Name*	H_1 is more parsimonious than H_2 just in case…

[2] In some optimally balanced mutual proportion. On the debated interpretation of 'import' see Chapter 7: Fn. 19.

Ideology	Plain	Ideological quantitative	H_1 features less primitives than H_2.
	Selective	Ideological qualitative	H_1 features less kinds of primitives than H_2.

Sider's "Against Parthood" (2013) is the *locus classicus* for arguing from profligate ideology against hypotheses rich in mereological predicates. To summarise what we already observed (Chapters 6), Sider accepts 'plain' ideology as the measure of profligacy that guides theory-choice and argues for 'deleting' (Ib. 243) mereological resources, like the part-whole predicate, leaving only set-theoretic membership and primitives of fundamental physics. The latter are resources we deploy no matter whether or not we couch some objects of interest $x_1, \ldots, x_n$ in mereological terms, stating e.g. that some x_j is part of x_k, that some $x_1, x_2, \ldots, x_i$ compose some x_k, or, finally, that each $x_1, \ldots, x_n$ is an atom (in the mereological sense, i.e. an object with no part other than itself). Accordingly, all hypotheses that couch $x_1, \ldots, x_n$ in these ways (including the nihilist hypothesis claiming that each $x_1, \ldots, x_n$ is an atom) deploy more ideology than a rival, which posits the same objects ($x_1, \ldots, x_n$) but replaces all occurrences of the mereological vocabulary with occurrences of the membership or of some relevant physical primitive. These primitives figure not just in the rivals but also in the abundant hypotheses with mereological vocabulary. Therefore, the 'plain' principle that promotes sparseness of primitives (all things equal) favours the former hypotheses over the latter.

Sider deals only marginally with the motivations for the principles that instruct us (all things equal) to prefer fewer primitives. When he does address this question, however, it is clear that he expects the motivation to come from science (Ib.: 244-5)[3].

[3] In a footnote (2013: Fn. 7), he confesses a "suspect that principles of parsimony cannot be derived from more fundamental epistemic principles". He has in mind principles for assigning higher probabilities (priors and posteriors) to parsimonious hypotheses, such as those discussed in Huemer (2009). Huemer argues forcefully that these policies for probability assignments do not settle all rivalries between differently profligate *philosophical* hypotheses in favour of the parsimonious rival. So, one could envisage invoking Huemer's result to deflate Sider's preference for philosophical theories with sparse ideologies. Yet Sider refuses to motivate his preference via probability assignments (or at least, via Huemer's assignments). Thus I

Particularly, for Sider the practice of setting preference on sparse ideologies makes best sense of certain episodes of scientific theory-choice, which should be interpreted, accordingly, as successful applications of the parsimony principle. Finally, having agreed that preference for sparse ideologies is responsible for rivalry resolution in science, we transfer the principle to the rivalries that interest us in philosophy[4].

1.1 Newtonian spacetime and absolute velocity

For an exemplar episode, which takes up just this role, Sider turns to the elimination, or 'deletion' of ideology for defining absolute velocity in terms of the relations between points of a Newtonian spacetime structure[5]. He starts by observing (correctly) that physicists and philosophers of physics agree[6] on a conditional claim:

---

am afraid that the best strategy to engage with Sider is to challenge him on the kind of motivation he expects: that is, preference for sparse ideologies makes best sense of scientific practice.

[4] Strictly speaking, that preference for sparse ideologies resolves scientific rivalries, such as those discussed below, does not *entail* that it resolves philosophical rivalries, such as those we keep under focus throughout this work. Huemer (2009) thinks that the two claims are connected by an abductive premise: the best explanation for the use of parsimony principles in science is that *in general*, i.e. in both science and philosophy, they track hypotheses more likely to be true. Having set out this reasoning, he offers a criticism in line with Swinburne (1997: 47): the connecting premise is justified circularly, for the explanation that parsimony principles track true hypotheses in science is 'best' only because itself parsimonious (for endorsement, see Bradley (2017)). Alternatively, to contrast Huemer's reconstruction of the connecting premise, one could argue that the ability of the principles to resolve rivalries (tracking the hypotheses most likely to be true) extend only to rivalries with *sui generis* features lacked in the philosophical cases and not to all hypotheses, scientific and philosophical alike (see Saatsi (2017) for an analogous point in the context of justifying inference to the best explanation in metaphysics).
Be it as it may, we need not worry about finding a defensible way of motivating parsimony principles in philosophy from science. For if this type of motivation fails, then it is all the better for our terminal conclusion that ideological profligacy poses no threat to hypotheses with familiar mereological structure. We thus assume, for the sake of argument, that there is some available connection between successful uses of the principles in science and motivated use in philosophy, moving on to argue that if the connection holds for principles that promote sparse 'plain' ideology, then it also holds for principles that promote sparse ideological kinds.

[5] For a presentation in terms of 'deletion', see Balashov (2010: Chapter 3). Dasgupta (2009) discusses the case of absolute velocity influentially, taking it to motivate highly deflationary metaphysical conclusions: fundamentally, there are no individuals.

[6] Some philosophers of physics argue for (truth-guiding) deletion of excess resources elsewhere in physics. Notably, North (2009) defends the Hamiltonian over the Lagrangian formulation of classical mechanics based on the former's space of solutions requiring less representational resources than the latter's. However, as this argument generated substantial debate (see e.g. Curiel (2014) and French (2014: S. 2.5) for responses), we cannot straightforwardly appeal to it for a justification of (truth-guiding) sparse ideology in philosophy (assuming here that North's elimination of resources from the space of solution corresponds to elimination

that if the laws of Newtonian mechanics[7] were true, then "it would have been more reasonable" (Ib. 6) to accept a theory of spacetime without resources for defining a physical system's absolute velocity. Particularly, the theory should omit all instances of binary predicates $P_j(x, y)$ to express the spatiotemporal distances Δr between two non-simultaneous[8] points x and y (for simplicity, and without significant loss of accuracy, we shall talk as if the deleted element were Δr, rather than the binary predicates).

Without resources for stating that spacetime points lie at this distance Δr, we lose the possibility of defining (instantaneous) absolute velocities for systems moving along any (accelerated or unaccelerated) path[9]. Indeed, the definition requires just such distances (Balashov 2010: 45): The magnitude of a system's absolute velocity is the ratio of displacement along the path (Δr) to elapsed time (Δt), in the limit of Δt tending to 0[10]. But displacements along paths are distances between non-simultaneous points, which we have decided to cross out from our basic resources.

Now, Sider's 'point of agreement' among physicists and philosophers of physics – that "it is more rational" to accept a theory of spacetime without resources Δr to define absolute velocity – remains conditional: Sacrificing the resources is a legitimate move only granted evidence that the laws of Newtonian mechanics are true (at the actual world). But why so? Why is the deletion conditional to evidence of the laws' actual truth? Given the laws, the state occupied by any arbitrary system *s* evolves insensitively to differences in the absolute velocity of *s* as well as differences

---

of ideology, see Sider (Ib)). We will stay, accordingly, with the wisely chosen rivalry between theories of Newtonian spacetime, which enjoys wider agreement.

[7] More precisely, the Newtonian laws formulated entirely in terms of Galilean-invariant quantities (absolute acceleration) and state-independent quantities (Newtonian mass). See Dasgupta (2009: Fn. 4).

[8] Both Newtonian spacetime (with Δr) and Neo-Newtonian (Galilean) spacetime (without Δr), which here we interpret as competing for ideological profligacy, feature well-defined point-point temporal distances Δt. Also, these distances (Δr Δt) are Euclidean, corresponding to the L_2-norm of the displacement vectors oriented from one point to the other. So is the distance Δx between two mutually simultaneous points, which again is well-defined in both spacetimes.

[9] As the competing spacetime theories (Newtonian and Neo-Newtonian) agree on well-defined temporal distances Δt, we understand paths semi-intuitively as 2-dimensional lines or curves, any two points of which are non-simultaneous. We also assume that the two competing spacetimes agree on the paths' topological features.

[10] This quantity $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\partial x}{\partial t}$ is of course known as the first derivative of position relative to time.

in the absolute velocities of systems *s'* ≠ *s*[11]. Suppose that a system *s* has a momentary value of absolute velocity A(*s*). From this value, the Newtonian laws allow to infer neither subsequent values of absolute velocity or of *s*' other quantities, nor subsequent values of absolute velocity or other quantities of distinct systems *s'* ≠ *s*.

Suppose further that there are no other laws conveying information[12] about the change in *s*'s state over time. If so, then it makes no difference to the state's expected evolution at subsequent times $t_1 > t_2 > ... > t_n$ that at some time $t > t_1$ the system bears some value of absolute velocity A(s). Dasgupta (2009: 40), who discusses the case of absolute velocity on comparable lines, calls this feature of absolute velocity "physical redundancy". Relative to a system of laws (such as the laws of Newtonian mechanics), a quantity Q is physically redundant if a system having or lacking some value of Q at any time makes no difference to the expected subsequent evolution of the system's state given the laws.

In summary, the evolution of systems according to Newtonian mechanics traffics in quantities, none of which is defined by distances between non-simultaneous spacetime points. Moreover, the Newtonian laws do not allow to infer[13] from a system having a momentary value A(*s*) of absolute velocity A to it having successive values of A, or to it having successive values of other quantities, or even to *other systems* having successive values of A or other quantities. Under these conditions, according to Sider, there is agreement among physicists and philosophers of physics that primitive ideology for certain spatiotemporal relations – notably, distances Δr between non-simultaneous points – can be deleted.

---

[11] As states of an arbitrary system *s* we consider combinations of momentary values for *s*' position and absolute acceleration.

[12] To cast this argument, the relation between laws and systems need not be connoted in terms of "governance" (in the sense of Beebee 2000: S. 3); that is (roughly), with facts about what the laws are providing metaphysical ground to facts about the system's physical states at future times. Instead, on the above account one can perfectly accept that the only function of laws in relations to systems is to describe their actual behaviour, represented by the change of their classical state over time, without providing ground for it.

[13] Again this 'infer' *can* be read in a metaphysically thin sense, cf. Beebee (2000: 578) and Fn. 7.

1.2 Interpretation and transfer

Let us pin down two additional points of interest in Sider's reasoning. First, the agreement among physicists on it being "more reasonable" to accept hypotheses without spatiotemporal resources for defining absolute velocity (Δr) is best interpreted epistemically, rather than pragmatically. It is meant to be, that is, an agreement on it being *preferable* to accept these hypotheses, in the sense of Chapter 7 (Section 1): The sparser ideology of hypotheses (H_1) without Δr makes them better candidates for being the contents of a true belief about the structure of spacetime. Choosing these hypotheses over their rivals H_2 (richer in spacetime structure) is, accordingly, the best available procedure for pursuing the aim of gaining true beliefs.

It is – admittedly – far from clear that we should interpret physicists as agreeing on the sparser hypothesis in this *epistemic* sense. The alternative, as we know (Chapter 7, Section 1.1) is a view on which sparser ideologies make hypotheses not better candidates for figuring in true beliefs, but rather better conveyors of pragmatic virtues: manipulability, perspicuity, *etc*. And in the task of interpreting the physicists' choice, the pragmatic approach is far from underrepresented. For example, from a constructive empiricist point of view[14] one could deal with all epistemic theory-choice by rejecting the appeal to choice factors other than differences in empirical adequacy. Among these is indeed the above connection between greater parsimony (measured in some standard M) and preference[15].

Having said this, clearly Sider starts from a different foothold on the status of parsimony-based methods in science. At least in respect to the Newtonian rivalry (opposing spacetimes with and without Δr) he ascribes greater parsimony a role in setting preferences, hence in guiding us to the best choice compatible with the aim of obtaining true beliefs. We follow him – setting aside the pragmatic perspective – with a dialectical aim in mind. Sider's assumption is a substantive one: episodes like the

---

[14] As classically conveyed in Van Fraassen (1980), especially p. 87-90. For discussion, see French (2014: 23-4, 27-8); Mohler and Monton (2017: 2.4).

[15] There is also opposition coming from non-empiricist quarters. For Janssen (2002) and Balashov (2010: 47) who follows him, the shift to the sparser hypothesis aims not at parsimony *per se*, as it does for Sider, but at producing unifying explanations of the motion phenomena ("in the same sense in which Ptolemaic "explanations" of planetary motions are inferior to Copernican explanations", see Balashov (Ib.). On this view, scientific practice evidences at most unificatory virtues, not parsimony virtues. We can safely omit discussion of this view, because our aim is *not* to defend Sider's. Our terminal conclusion that ideological profligacy poses no threat to mereologically rich hypotheses would only benefit from the lack of scientific evidence for discarding hypotheses in virtue of their profligacy. Cf. Fn. 3 above.

agreement on sparser spacetime resources evidence policies for setting preference, not policies for applying pragmatic virtues. However, he uses this assumption to infer that the policies transfer from their domain of origin (the Newtonian dispute) onto our rivalry of interest, resulting in preference for the sparser hypotheses without mereological resources.

I deny neither the interpretative assumption, nor the 'transferral' doctrine that the evidenced policies resolve rivalries outside their domain of origin. I contest, however, the inference. As Section 2 illustrates, the transferable policy need not be of Sider's preferred sort, i.e. a policy that penalises hypotheses rich in mereological resources. It could be, rather, a policy that eliminates kinds of resources, promising to preserve the mereologically rich hypotheses at no cost.

On the contrary, suppose we denied the interpretative assumption or the 'transferral' doctrine. This denial leaves us without clear instruments for preferring metaphysical hypotheses based on their ideologies' sparseness, as Sider desires[16]. Yet this result is compatible with my terminal conclusion that abundance of ideology raises no challenge to the metaphysical hypotheses so far assessed: the atoms-only view of Chapter 6 and in general all hypotheses with vectors portrayed as entering canonical part-whole relations. Accordingly, we can see that my agreement with Sider on the interpretative assumption and the 'transferral' doctrine serves only the purpose of entering his own dialectical game. Without these assumptions there would be no clear challenge from profligacy against the abundant hypotheses I espouse. And even granted these assumptions, the 'transferred' policy might well be one that delivers the hypothesis at no cost.

1.3 Sider's and Dasgupta's razors

Now for a second observation on Sider's reasoning. Having agreed that H_1 owes its advantage over H_2 to the sacrifice of Δr, there remains a question about H_2. What exactly about H_2's additional resources (Δr) legitimises penalising H_2 when assessing the preferable hypothesis? Sider's answer, as we have seen, is that H_2's resources are in excess relative to the resources that figure in the Newtonian laws or define the

---

[16] Cf. Fn. 4.

quantities that figure therein. Indeed, "Newton's laws, as optimally formulated in the context of Newtonian spacetime, do not mention the notion of being at the same absolute position [...][17]. [A]nd so [...] [a theory of Newtonian spacetime with Δr] is less choiceworthy for that reason" (2013: 245)[18].

However, there are alternatives for indicating why H_2's ideology receives a penalty. On one of these, associated with Dasgupta (2009: 43-4), we penalise richer hypotheses like H_2 not because their resources fail to appear in the natural laws or to define quantities that feature therein. Rather, we penalise these hypotheses for not putting their additional resources (Δr) to the service of explaining empirically detectable facts.

Return to the Newtonian dispute for illustration. For Dasgupta, one explains facts of a system *s* having specific momentary values for position and absolute acceleration by citing the Newtonian laws and the system's precedent values of the same quantities (Ib. 44). However, neither the laws nor the said quantities (momentary position, momentary absolute acceleration) are formulated or defined in terms of distances Δr between non-simultaneous points. Thus, resources to express the distances fail to underpin explanations of facts of this kind.

But there are other facts one may want to explain. Specifically, on Dasgupta's view, one explains facts of certain systems *s* having momentary absolute velocity A(*s*) by citing *s*'s preceding absolute velocity A'(*s*), which indeed this time is defined in terms of distances Δr. Now, this possibility to explain later absolute velocities by citing earlier ones is not enough to save the Newtonian spacetime element Δr from the penalty. For the facts that Δr contributes to explain (systems having momentary absolute velocity A(s)) remain undetectable. In particular, given the Newtonian laws, the state occupied by any system (s or $s_1 \neq s$) remains invariant at all subsequent times, whether or not *s* has any momentary absolute velocity. This invariance of the

---

[17] A system has an absolute position if the path that describes its career in spacetime comprises only points at the same spatial position (these paths go under the name of 'absolute position lines', see Balashov 2011: 52). As we discuss below (Section 3), the property of comprising only same-position points, which grants paths the ability to represent absolute positions, is another of the resources renounced in the transition from Newtonian to Neo-Newtonian spacetime (see Balashov 2011: 52).

[18] In her discussion of Neo-Newtonian spacetime, North stands on a similar footing: "[We] tend to infer that there is no more structure to the world than what the fundamental laws indicate there is. Physics adheres to the methodological principle that the symmetries in the laws match the symmetries in the structure of the world. This is a principle informed by Ockham's razor; though it is not just that, other things being equal, it is best to go with the ontologically minimal theory. It is not that, other things being equal, we should go with the fewest entities, but that we should go with the *least structure*. We should not posit structure beyond that which is indicated by the fundamental dynamical laws" (2009: 8).

system's future state suffices for making facts of absolute velocity incapable of producing distinctive detectable evidence. Even if *s* had any momentary value of absolute velocity, the state occupied by an ideal measuring device $s_I \neq s$ would not vary at subsequent times, not allowing *s*'s absolute velocity to be detected by detecting a change in the device's configuration (such as a shift on the position of its pointers)[19].

Summarising, on analysis of the scientific practice there would seem to be at least two policies for penalising H_2's Newtonian resources Δr: Sider penalises H_2 because the resources do not feature in the Newtonian laws or define quantities that feature therein. Dasgupta penalises it because the extra-resources, or the quantities they define (i.e. absolute velocity), partake in no explanation of distinctively detectable facts, such as facts of devices s_I changing their configuration in response to facts of systems $s \neq s_I$ having momentary absolute velocity.

On this construal of the dialectic, Sider and Dasgupta disagree on which penalty has been put into practice in the scientific dispute and, accordingly, one could try to resolve their ensuing debate in favour of one of the two options[20]. In the interest of brevity, I suggest we leave this internal disagreement open and accept that at least one of Sider's and Dasgupta's policies for penalising Δr interprets the practice of preferring sparser hypotheses without Δr legitimately. Thus, the 'agreement' on preferring the sparser hypotheses is agreement on enacting one of two penalties: either a penalty on resources that do not feature in the Newtonian laws (or define quantities that feature therein); or a penalty on resources that do not partake in

---

[19] Dasgupta assumes a view of ideal measurement as detectable shifts in the configuration of mesoscopic systems. Redhead (51-52) is a canonical formulation.

[20] Indeed, so does Sider. He rejects Dasgupta's penalty because it legitimises discarding H_2 too easily. For him, the discard of H_2 obtains on Dasgupta's view simple because H_2's extra-resources (Δr) define quantities that make no distinctively detectable difference on the configuration of ideal devices. He quickly labels the method "verificationist" and wonders "why this epistemic fact [of undetectability] would itself count against the theory [with Newtonian resources]" (Ib. 243). But to be sure, Sider's criticism equivocates: the Dasguptian reason for penalising H_2, which contains the extra-resources Δr, is not that facts about systems *s* having momentary absolute velocity are not distinctively detectable. Rather, the reason is that these facts, while themselves not distinctively detectable, partake only of explanations of non-distinctively detectable facts and of no explanation of distinctively detectable facts. This difference shields Dasgupta's policy from Sider's (rather quick) accusation. Crossing out facts of systems having momentary absolute velocity due to their being undetectable would (perhaps) approximate verificationism and be objectionable on this ground. Crossing them out for their lack of explanatory power is more plausible, for one can accept with Sider that "epistemic fact[s]" of undetectability do not "count against the theory" (Ib.), while maintaining that a basic aim of science remains explaining detectable phenomena.

explaining distinctively detectable facts. If there is evidence for any policy on abundant ideology coming from this episode of scientific practice, then it is evidence for *at least* one of these two: remove ideology that fails to partake in natural laws and/or remove ideology that fails to explain detectable phenomena. On both policies we agree that a sufficient reason for crossing out Δr is that Δr *itself* is redundant, not Δr and everything in the same ideological kind. As our target is this latter claim, which both Sider and Dasgupta endorse, we shall not be interested in the differences between the particular policies they use to motivate it. Rather, the next two sections argue that if we take the Newtonian episode to motivate one or both of these policies (and the associated penalties), then it is *just* as attractive to take it to motivate a policy that minimises tokens of new ideological kinds.

2. What measure of ideological profligacy?

Having highlighted these two uses of preference and penalties in scientific practice, we are two steps away from reaching Sider's conclusion that we should prefer hypotheses that renounce the part-whole primitive. First (i), Sider needs to argue that the two penalties are used legitimately not just to ascribe preference to one of the rivals over spacetime structure. Rather, one can also use the policies legitimately to resolve our philosophical rivalries of interest[21]. Second (ii), granted that the policies apply legitimately beyond the Newtonian case, Sider needs to argue that they are *effective* in assigning preference to the sparse hypothesis without part-whole resources, as he hopes they will. Particularly, the richer hypothesis' resources (the part-whole predicate) should not resist the penalties by having features the penalties pass over, i.e. contributing in explaining detectable facts and figuring in the laws of nature or defining quantities that figure in physical laws.

[21] As noted above (S. 1 and Fn. 3), it is common practice to 'export' penalties from scientific practice and put them to service in resolving metaphysical disputes (see Sider 2009 for an influential analysis of this method). But the motivations for extending their use in this way remain mostly implicit, and for those who try to articulate them (e.g. Huemer 2009) objections are often available.

Though there could be complications with enacting (i) and (ii)[22], I will largely assume their availability to Sider's argument. The picture is therefore that just as the two policies support preference for the sparser hypotheses (without Δr) in scientific practice, so they support preference for the sparser hypotheses without the part-whole predicate in the metaphysical disputes. Particularly, they support these sparser hypotheses in the dispute that we keep under focus, which opposes hypotheses about the same domain of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$, which disagree on whether any particular vector enters a canonical relation complying with the core principles (e.g. by being part of itself only).

Granted the point that scientific practice does motivate parsimony principles, one could think that the natural reaction is conceding to the Siderian that we should drop the abundant hypotheses, in which the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ satisfy the part-whole primitive. But this, too, would be an exaggeration. For just as we examine scientific practice to accord preference in metaphysics through one of the two policies (Sider's or Dasgupta's), we could try to examine further to accord preference via minimisation of ideological kinds. In fact, as I proceed to suggest (Section 3), the very episode we examined so far – the dispute over spacetime structure – makes good sense as an application not just of Sider's and Dasgupta's penalties but also of a penalty on additional kinds. On this proposal, by favouring the sparser hypothesis without Δr scientists should be seen as renouncing all tokens of a kind of resources of which Δr is a member.

While the next section articulates this suggestion in detail, let us anticipate its dialectical impact. Suppose that – as I will suggest – the Newtonian episode evidences a penalty on additional kinds as much as it evidences Sider's and Dasgupta's penalties. And suppose we agreed on using the episode to sanction Sider's or Dasgupta's policies outside of their domain of origin, i.e. on our rivalry of interest. In principle we should also agree on using the episode to sanction a policy of kind minimisation, so that hypotheses rich in mereological primitives face discard only if their extra-resources (the part-whole predicate) are not of a kind with resources already present in the rivals.

[22] For example, one could try to argue that the part-whole predicate escapes Sider's penalty in that it defines quantities that feature in physical laws, in a similar way in which spacetime relations like $P(x, y)$ define absolute velocity. Perry (2015) and Perry (2016) are attempts along these lines.

But wouldn't this strategy leave unsettled which penalty among Sider's, Dasgupta's and the new one from kind-minimisation we should legitimise on our rivalry of interest? And if it does leave the legitimised penalties unsettled, what ensures that the new policy, rather than Sider's or Dasgupta's, solves our rivalry of interest?

Taking on each question in turn: 1. The strategy does not leave unsettled which policy applies legitimately to our rivalry of interest. What is unsettled, rather, is the interpretation of our scientific episode (the choice in favour of sparser spacetime theories) as the application of one policy in particular. Suppose that indeed Δr partakes of an ideological kind K', no member of which appears in the sparser H_1 (anticipating from Section 3). A survey of the episode is unable to evidence one in particular among the three candidate policies, showing them distinctively 'in action'. In fact, as Sider presents it (2013: 242), the episode consists entirely of physicists and philosophers of physics 'agreeing on' or finding it "more rational' to favour the sparser H_1 over the richer H_2 equipped with Newtonian resources (Δr). But surely this expression of favour alone does little to highlight the specific penalties applied to H_2; whether it is, namely, in terms of kind-minimisation or in terms of Dasgupta's or Sider's 'selective' quantitative policies. This indecision in the evidenced penalty suggests that it will be arbitrary to interpret the scenario as the application of one policy in particular: e.g. Sider's but not Dasgupta's, or Dasgupta's but not the new policy by kind-minimisation.

This being said, we are still one step away from capitalising on the scenario to sanction kind-minimisation in our rivalry of interest. If it is arbitrary to interpret the episode as enactment of one penalty in particular, then *a fortiori* it is arbitrary to interpret it as enactment of our penalty on additional kinds. This penalty, when exported from the Newtonian domain onto our rivalry of interest, promises to charge no cost on the richer hypotheses with mereological resources. However, we earn no chance to export the penalty until we use the scenario to evidence it. The above point about arbitrariness confines us to the early, evidencing stage of the process.

However – tackling now question 2. – there is an attractive way out. Given our current dialectic, we need not argue directly for exporting the kind minimisation penalty and against exporting other penalties (i.e. Sider's, Dasgupta's) compatible with the analysis of the Newtonian scenario. For indeed, our terminal conclusion is that in some particular cases of interest one should not discard hypotheses H_2 owing to their abundant primitives. Now, an influential way of denying this conclusion is via Sider's argument that one should prefer theories with overall fewer primitives because such is the verdict of scientific rivalries. Accordingly, to make the conclusion plausible it will be enough to show that Sider's argument has a lacuna. It is far from clear that the scientific verdict includes a policy of preference for fewer resources. Rather, the evidence that scientists agree on the redundancy of Δr could just as well arise out of a preference for minimising kinds: Δr is redundant because it contributes a token of an additional kind – a kind no member of which appears in Newtonian Mechanics formulated on Neo-Newtonian spacetime.

For Sider's argument to conceal this lacuna, it must be plausible that Δr is the member of one such ideological kind – again, a kind all of whose members are redundant in Newtonian mechanics appropriately formulated. This is the claim we make in the next section.

3. Galilean transforms as specifiers of kinds

For the Newtonian scenario to highlight a policy by minimisation of kinds, it must be plausible that Δr is the member of an ideological kind, whose members figure in the abundant Newtonian hypothesis H_2 but not in the sparser H_1. The obvious risk in affirming this (Chapter 8: Sections 3.3, 3.4) is to group Δr with other primitives that figure in H_2 in an *ad hoc* manner; that is, picking and choosing the groups of primitives that qualify for kind-hood on grounds no other than convenience in making our point that members of a kind have been shaved off in the process of favouring H_1[23].

This risk notwithstanding, I argued in the last chapter (and will continue to argue in Section 3.2) that we can safely approach the groupings of resources with a claim to be ideological kinds by surveying significant scientific taxonomies. Well, one attractive and taxonomically significant grouping in spacetime physics collects all resources for

[23] This is reminiscent of Bird's and Hawley's (2011) Mill-style 'kind-hood question': which groupings under natural similarities underpin natural kinds? I argued in Chapter 8 that groups of primitives can constitute ideological kinds without meeting the 'structural' conditions on group members for making a group of naturally similar things a natural kind (Section 3.2 further articulates this argument). But having said this, in neither domain – natural or ideological – we expect an *ad hoc* answer to the 'kind-hood' question.

geometrical quantities, whose values vary under the action of some coordinate transformations. Let us examine this suggestion closely.

3.1 Non-invariants as an ideological kind

In its barest essentials, this proposal runs as follows. One starts with a large group of unclassified geometrical quantities, all well-defined on some spacetime manifold S (say, all Euclidean distances well-defined on Newtonian spacetime, including Δr)[24]. Following, one refers back to transformations T to tell apart the quantities whose values vary under shifts of coordinates. This move creates a first-pass divide between geometrical invariants and non-invariants (henceforth: variants) under the coordinate-shifts operated by T. The transformations specify the entrance conditions in each group. Particularly, a quantity[25] enters the group of variants (invariants) if it follows from the specifics for shifting coordinates given by T that its values vary (remain invariant) under the shifts.

Our target resource Δr is a Euclidean distance between non-simultaneous points. Accordingly, it enters the first-pass group of variants if we set the Galilean boosts ($x' = x - \mathbf{v_x}t$; $t' = t$)[26] as the transformations whose specifics give the entrance conditions. For any two coordinate frames F_1, F_2 that agree on temporal coordinates ($t = t'$), the boosts map F_1's spatial coordinates (xyz) into F_2's ($x'y'z'$) by 'shifting' each coordinate by a factor of velocity oriented in the direction *d* in which F_2 is moving relative to F_1 (if we imagine this orientation as F_1's *x*-direction[27], then the boosts preserve the *y*- and *z*-coordinates: $y = y'$ and $z = z'$). Standardly, this factor is interpreted as the velocity of an

---

[24] The quantities one starts with depend on the properties accorded to S's displacement vectors. The vectors could have only affine structure (roughly: divide in equivalent classes of parallel vectors) or richer Euclidean or non-Euclidean structure (including e.g. vector norms and inner products that contribute lengths, distances and angles, cf. Chapter 4, Section 2). As we keep our focus on the dispute over Newtonian and Galilean spacetime, we keep assuming that the initial, unclassified group of geometrical quantities includes affine and Euclidean quantities.

[25] More presicely, the ideological resource that corresponds to it.

[26] For $\mathbf{v_x}$ being (standardly) the velocity oriented along *x* of the primed coordinate frame relative to the unprimed one. Also, we imagine that the two frames agree on the origin of the time axis ($t = t' = 0$)

[27] As is canonical in textbook presentations, see e.g. Gregory (2006: 40-2); Mould (2002: 1.1), Taylor (2005: 15-17).

observer travelling in direction d relative to the observer that measures quantities with F's coordinates.

If F_1 measures a distance Δr between two non-simultaneous points a and b, then some viable boosts of F_1's coordinates for a and b output coordinates of F_2, which measure $\Delta' r \neq \Delta r$ between a and b. But distances Δr are not the only spatiotemporal quantity whose values change under the action of Galilean boosts. So are the orientation of the lines that connect points across non-simultaneous times (the absolute position lines) as well as the angles that these lines form with all the hyperplanes of mutually simultaneous points (Balashov 2011: 52-3)[28]. And of course so are the instantaneous absolute velocities, defined as the limit of our distances Δr over Δt for $\Delta t \rightarrow 0$. As Δr figures in the calculation, its variance under Galilean boosts transmits to the calculated absolute velocities.

For a contrast, among the quantities that *survive* Galilean boosts unvaried, no matter the relative velocity of any observing frame, are distances Δt between temporal points and spatial distances Δx between temporally simultaneous points. The standard interpretation is that no disagreement occurs between observers in relative motion on the time elapse and spatial distance between simultaneous events.

Finally, among the surviving quantities is instantaneous acceleration, defined on any path as the variation in (average) relative velocity ($\Delta \boldsymbol{v}$) along the path in the limit of Δt tending to 0[29]. This is perhaps the most significant invariance of all. For instantaneous acceleration figures explicitly in two out of three Newtonian laws[30]: the first, which states that "a body unaffected by force retains [...] constant velocity" (i.e.

---

[28] Perhaps the best aid to visualise the effect of these transformations on spacetime quantities is Geroch's (1978) "beveling-the-deck" metaphor (accompanied by Balashov's 2011: 52-3 excellently clear discussion). Here we start by imagining Galilean spacetime as a deck of cards, each of which stands for a plane of mutually simultaneous points. An absolute position line, connecting non-simultaneous points, works like a (thin) needle stapled perpendicularly through the deck. Euclidean distances measured between any two points on the pin have value Δr. Now, Galilean boosts are imagined as actions that bevel the entire deck in the same direction, oriented perpendicularly to the absolute position lines. Imagining that the 'needle' follows through the bevelling, we notice immediately the change in orientation, angle formed with each card and distance between any two points on the pin.

[29] Or equivalently, the first derivative of relative velocity with respect to time: $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{\partial v}{\partial t}$.

[30] Of course, it also figures implicitly in the third law, which is a statement about systems with oppositely directed *forces*, hence also oppositely directed accelerations. We will ignore the third law, however, in the interest of brevity.

null instantaneous acceleration)[31], and the second, $\mathbf{F} = m\mathbf{a}$, which calculates values (orientations and senses) of forces based on mass and instantaneous accelerations. Before setting up our argument that the variant resources form an ideological kind, it is worth pausing for a second on this significance of the acceleration's invariance.

In respect to the second law, we can use the invariance of acceleration to derive the invariance of forces. It followed from the definition of acceleration and the Galilean boosts' specifics ($x = x' - \mathbf{v}_x t$) that values of acceleration (orientations and sense) are invariant under the boosts. Now, in just the same way, the invariance under boosts of values (orientations and senses) of force follows from the boosts' specifics, which leave acceleration invariant, the constancy of Newtonian mass and Newton's second law.

Thus one sees immediately that all resources figuring in the Newtonian laws (absolute acceleration) or defining resources that figure therein (relative velocity, temporal intervals and intervals between simultaneous events), are resources for Galilean-invariant quantities. This bears emphasis in our framework of interest[32], for these resources resist not only the penalty to additional kinds we currently outline, but also Sider's penalty (Section 2), which shaves off all resources without a role in the laws or in defining the quantities therein[33]. Indeed, as all resources of Sider's type are variant, Sider's penalty leaves us with only invariant resources, in accordance with the penalty to additional kinds. This observation reinforces our point in the previous section that we cannot decide on the Newtonian episode being resolved by Sider's policy or a policy by kind minimisation (whereby the latter shaves off all resources of the variant group). Indeed, the end result of Sider's policy is a pool of

---

[31] See Balashov (2011: 52).

[32] Another usually emphasised point of significance of the instantaneous acceleration's invariance is the following. The standard interpretation of the invariance is that no disagreement occurs between observers in relative motions over facts of systems having instantaneous acceleration. Accordingly, in predicting an unaffected system's *s* subsequent accelerations with Newton's first law, we predict facts (i.e. *s* has null acceleration) whose holding is independent of facts about the observers' relative motion. And a similar point holds in respect to the second law: the predictable facts (facts of systems being affected by a force directed along their acceleration's direction with value $|ma|$) hold regardlessly of facts about the relative motions of observers.

[33] They also resist Dasgupta's penalty, as for Dasgupta one way for a resource to explain a distinctively detectable fact (in the intended sense of making a difference to the configuration of ideal device) is by figuring in the laws, see his (2009: 44). In the main text, we keep the focus on Sider's policy for brevity.

resources with a role in the laws or in defining the quantities therein, which the policy by kind minimisation could itself have delivered[34].

3.2 Collecting kinds from taxonomies

Having grouped various (resources for) variant quantities, our question is now whether the group has some claim to be an ideological kind. For our argument in the previous sections (2, 3.1) to go through, the answer has to be positive. For only if Δr forms a kind with resources for the other variant quantities we can argue as we did in the last section that the scientists' agreement on sparser theories of spacetime can be interpreted as the result of a kind minimisation policy.

As I was saying at the beginning of this section, the thought that Δr forms a kind with these resources is somewhat attractive given the resources' surface 'taxonomical significance'. For it is customary in spacetime physics to find quantities classified as invariant or variant relative to sets of coordinate transformations[35]. This section, accordingly, is about the role of taxonomical significance in turning this group into a kind.

We start with an illustrative worry. One may wonder whether it is enough for the variant to form a kind that they appear in the straightforward taxonomy of resources of spacetime physics. To answer positively, i.e. that the resources' appearance in the taxonomy is as decisive a sign as we need of their forming a kind, we return to a point made in the last chapter. There we agreed that it would be desirable to tell apart groups of resources with a serious claim to be ideological kinds by looking at their features and checking them against features of groupings with a serious claim to be natural kinds (Cowling 2014: 3897-8).

---

[34] True, there remains the possibility that Sider's policy shaves off more resources than the kind minimisation policy – leading to a different resolution of the Newtonian dispute. In particular, Sider could eliminate some Galilean-*invariants* present in the richer hypothesis H_2, which have no role in the Newtonian laws or in defining the quantities therein. This granted, whether H_2 contains any such resource depends on the hypothesis' exact formulation, and more generally on the details of its rivalry with H_1. Sider's (Id.) discussion gives no clue that he expects his formulation of the rivalry to favour his policy in this way.

[35] It matters to stress that we divide the quantities in the invariant and variant group relative to a particular set of transformations. For example, the line between invariants and non-invariants changes, obviously, when we draw the distinction based on the Lorentz boosts (with temporal distances joining the non-invariants). See Balashov (2010: Section 3.5)

Yet this was only one of two *desiderata*. While conforming to reputable analyses gives us reasons for thinking of groups of resources as kinds, we also expected some ideological kinds to evade constraints on natural kinds – because so do paradigmatic cases, e.g. kinds of inter-definable predicates like '…is a part of…' and '…is a proper part of…'. Accordingly, we agreed on the overall best strategy for picking the groups with a viable claim for ideological kind-hood: Expect a balance between groups with properties pertaining to natural kinds and groups that, though lacking these properties, still reflect evident taxonomies of scientific interest. The group of variant spacetime resources, collected relative to the Galilean boosts, are likely[36] to fall on the latter side of the spectrum, as they are clearly embedded in the taxonomy of resources conveyed by spacetime physics.

The point is, as it was in the last chapter, that one can legitimately assemble groups with a claim to ideological kind-hood based on evident and significant scientific taxonomies, but without holding precise analyses of what it takes for some grouped resources to constitute an ideological kind. For improvement on this point, I would now like to add more on what it takes for a group of resources (such as the geometrical variants under the Galilean boosts) to be evidently and significantly included in a theory's taxonomy.

To do so, we can return to Cowling's useful comparison with ontological kinds (Chapter 8, Section 3.3). He originally argued that practitioners are proficient in preferring hypotheses with sparse ontological kinds, (e.g. pure set theory, which has only the kind *set*)[37] in spite of lacking an analysis of what it takes for a group of things to constitute an ontological kind. He then inferred that, similarly, we can expect to be ourselves proficient in preferring hypotheses sparse in ideological kinds, while lacking an analysis of what it takes for a group of resources to constitute an ideological kind.

I objected to the argument so construed (Ib.), which says that we can operate a principle of ideological qualitative parsimony (PKI) without analysis if without analysis we already operate a principle of ontological qualitative parsimony. In fact, to operate (PKI) we are now looking for motivation from uses of (PKI) in science. But

---

[36] I say 'likely to' because I have assumed and not argued for the claim that they lack natural kind structure. This is a reasonable assumption, because it is not in my favour.

[37] The assumption, recall, is that *set* is not a natural kind (Chapter 8: Fn. 32).

having rejected Cowling's version of the argument, I do not object to a close cousin: that is, that we can indicate groups of resources with a claim for ideological kind-hood if we proficiently indicate groups of entities with a claim for ontological kind-hood.

Enabling to indicate such groups in both cases, i.e. ontological and ideological, are exactly evident and significant taxonomies. Cowling's prime sample of an ontological kind, *pure set*, will help us illustrate. Suppose (as is reasonable) that *pure set* is not a natural kind. Few would deny pure sets a place in an ontological kind based on difficulties with analysing this notion. The reason for not doubting that they form a kind is the pure sets' appearance in the taxonomy of pure set theory, or I should say, more accurately, its appearance in this taxonomy with a significant purpose.

In fact, pure set theory assembles pure sets *evidently* and *purposefully*. Evidently, because pure sets are the subject matter of the theory's axioms[38], which state generalisations over sets and thereby outline features valid as conditions for entrance in the group. Purposefully, because pure set theory not only uses the axioms to regulate entrance in the group and filter out 'intruders', but it also regulates the entrance in just this way, rather than more flexibly or restrictively, with the aim[39] of giving the foundations of mathematics in its language[40]. The purpose is, namely, to construe various other objects as sets or members of sets, capable of satisfying the axioms.

The point of interest is that this often recognised foundational purpose, which informs the entrance conditions in the group of pure sets, is not remotely similar to

---

[38] I have in mind in particular the Zermelo-Fraenkel (ZF) axioms, as given in Kunen (1980: 1-5; 10-11) and Bagaria (2014: supplement). Here all axioms are generalisations over pure sets (with one exception, the axiom of infinity, which states: There is a set S with the empty set as a member, such that for all members x of I, the union of x and $\{x\}$ is a member of S).

[39] At least according to a standard view, voiced at various places in philosophy and notably by Quine (1960) and Lewis (1991, 1993: 203-4). If Lewis is correct in (1991), we need much less for a foundational language for mathematics: the axioms of classical extensional mereology and a Von Neumann-style theory of singleton functions.

[40] It does not matter whether the aim is satisfied or remains unfulfilled (in fact, a number of non-classical set-theorists disagree on which mathematical objects we ought to pin down to sets. For an outlook see Blizard 1991: 319-21; Aczel 1988: 103-12; Holmes, Forster and Liebert 2012: 1.1, 4.2; Barwise and Moss 1996). My claim here is simply that pure set theory uses the axioms to regulate entrance in the group of sets armed with this aim, and therefore the classification is 'purposeful'.

the purposes we *usually*[41] pursue in gathering entities that partake of the same natural kind, such as grouping entities whose natural similarities and kind-properties are linked by safe inductive connections or constitute the subject matter of the natural laws, and so on. But now the hope is that we said enough to make the following persuasive. Grouping entities into natural kinds might well be associated with some 'elite' goals, different from the goals we pursue in creating other groups, be they groups of entities or of primitives. However, a difference in the pursued goals should not block the attempt to make kinds out of these groups. More likely, we are better off recognising multiple goals for putting together groups with a legitimate claim for kind-hood.

Guidance as to which operating goal legitimises turning a group into a kind comes – now expectably – from a 'copycat' naturalistic policy. We shall turn into kinds the groups that we put together while seeking advanced and reputed scientific purposes[42]. For the foundations of mathematics, one of these goals is to cash in – via the axioms – just the right resources for pinning down enough mathematical objects to the notion of set. For spacetime physics, one of the goals is to cash in – via the coordinate transformations – just enough geometric resources to validate the statements of the Newtonian laws (optimally formulated) in an objective and non-invariant way.

I conclude that we should not overlook the possibility that resources for distances Δr come in an ideological kind with the other resources for quantities that vary under the Galilean boosts. Sometimes groups of resources with a legitimate claim for kind-hood are visible from the relevant scientific taxonomies, when the devices that tell them apart from the entirety of unclassified resources (e.g. the Galilean transformations) are endowed with significant and scientifically respectful goals.

---

[41] At least on a weak realist approach. We know already that the list of aims constraining the groupings suitable for natural kind-hood is long and debated; see the past chapter (Section 3).

[42] Notice that I am still in line with Chakravarrty's (2017) tripartite account of naturalised metaphysics, which prescribes metaphysical theorising equipped with scientific contents, methods and aims (whether 'equipping' means 'overwriting' original contents, methods and aims is the subject of intense debate, with positions ranging on both sides of the spectrum: Morganti and Tahko (2017) on the one side; Ney (2011), Ladyman (unpublished) on the other, see Chapter 7: Fn. 19). Here the theory in question is a theory of the entrance conditions in some ideological kinds as well as of the properties of some kinds' members. The theory is equipped with *contents* and *aims*: contents, because the grouped resources that gain kind status figure directly in spacetime physics; and aims, because underpinning the concession of kind status to the group are the aims of spacetime physics in putting the taxonomy at the forefront.

4. Taking Stock

The gist of the last section is that there are good reasons for taking (resources for) distances Δr to be of a kind with the other quantities that vary under the Galilean transformations. Previously (Section 2), we have also argued that the scientists' agreement on sparser hypotheses (without Δr) could be evidence not just of policies that minimise new resources (Sider, Dasgupta), but also evidence of policies that minimise tokens of new ideological kinds. On the latter interpretation of the evidence, scientists favour hypotheses with fewer tokens of the kind of geometrical variants (under coordinate transformation).

We are now only two steps away from reaching our terminal conclusion that metaphysical theories rich in mereological resources come at no cost. First (i), we need to argue that the penalty on additional kinds is used legitimately not just to ascribe preference to one of the rivals over spacetime structure. Rather, one can also invoke the policy legitimately to resolve our philosophical rivalries of interest. Second (ii), granted that the policies apply legitimately beyond the Newtonian case, we also need to argue that they are *effective* in protecting the richer hypothesis with mereological resources. Particularly, these resources (the predicates of part-whole and composition) should resist the penalties by being of a kind with resources already present in the sparser rival.

Step (ii) is the subject of our next (and last) chapter, where we can finally complete our levelling-down task by showing that part-whole and composition form an ideological kind with the identity predicate. As for step (i), I have already made my point that our dialectic allows assuming it for the sake of argument (Section 1.2 and Fn. 4; Section 2 and Fn. 21). It might well be controversial that the evidenced policies sanction principles of parsimony outside of their domain of origin. Yet, if anything, these potential controversies favour my terminal conclusion, which again is that considerations of plain ideological profligacy do not undermine hypotheses rich in mereological structure.

Accordingly, when I take (i) on board I do not make an assumption in my favour. Instead, I grant friends of plain ideological profligacy the resources they need before showing that even these fall short of proving the conclusion they crave. Even if, generally speaking, we are legitimised in exporting principles of parsimony from reputable episodes of scientific practice, the analysis of these episodes blocks the claim that the exportable principles penalise all excess resources. Rather, given the evidence, the exportable principles might just as well penalise tokens of new ideological kinds, promising to retain the mereologically rich hypotheses.

This dialectic discloses a lacuna in a popular strategy to legitimise ideological parsimony in philosophy, as we do not win this principle on simple analysis of eminent practice. On the negative side, the dialectic does not definitely promote kind ideological parsimony as the principle we win on the analysis of practice. It only releases the principle from the pressure of a popular argument opposing it. However, this is enough to make the principle a little more plausible and, moving into the next chapter, to motivate an enquiry of its prospects in assessing our rivalry of interest.

X

Levelling down ideological parsimony (III)

Identity's kind

In the past two chapters we outlined a method for grouping resources with a claim to being of the same ideological kinds. To do so, we first indicate a theoretical device that collects initially unclassified resources into groups, specifying the groups' entrance conditions and the similarities between each group's members. The Galilean boosts exemplified one such device, grouping initially unclassified geometric quantities according to their variance or invariance relative to coordinate shifts. Second, we argue that using the device to classify resources in this way serves a reputable scientific purpose.

We now replicate these guidelines in arguing for the common kind-hood of part-whole and identity: we isolate a device that gathers them in the same group, we illustrate the mechanism by which it manages to do so, and we argue that the isolated group has a claim to be an ideological kind because the purpose for using the device to taxonomise in this way is scientifically eminent. The task of outlining the device's specifics and how they classify these resources together takes up most of this final chapter (Sections 2, 3); while towards the end (Section 4, 5) we return to the task of arguing for its legitimate kind status.

Along the way we will take the chance to latch onto claims from the previous chapters, using them in the argument that concludes to the oneness in kind of part-whole and identity. The hope is to illustrate how these components come together to resist pressures from ideological profligacy and show that hypotheses rich in mereological resources come at no cost owing to their kind resources.

1. A friendly rival

As we have frequently done in previous chapters, we start from other, attractive ways of reaching the terminal conclusion that hypotheses rich in mereological structure resist pressure from profligacy.

One of these requires us to look back at early chapters (2) for ways of accommodating the thesis that science presents us with *sui generis* cases of part-whole, such as cases of the relation between each component vector and its resultant. One of the accommodation options was pluralist in spirit: every time we are presented with such scientific cases and there is no objection to coaching them in mereological vocabulary in a literal sense, we add one item to the rubric of 'varieties' of part-whole. Thus, a component vector relates to its resultant by a genuine part-whole relation distinct from other part-whole relations, all of whose *relata* satisfy core principles like the Antisymmetry postulate and Weak Supplementation.

Let us now think of these varieties in the rubric as a collection of ideological resources: plausibly, to each variety R corresponds a predicate 'R' satisfied by the entities that relate by R – for example, to the relation of component-sum corresponds a predicate '…is a component of…' satisfied by all entities that relate as component to sum. The key to this friendly proposal is that all resources in the rubric constitute an ideological kind, from the deviant resources of linear algebraic combination to the familiar ones equipped with the Antisymmetry postulate and supplementation principles. The effects of arguing that the part-whole rubric is an ideological kind on the profligacy challenge is obvious. The actual physical ontology contains satisfiers of deviant notions, because it contains entities of scientific interest that relate as component to sum[1]. We now understand this notion as the member of a kind, whose other members include familiar notions[2] (e.g. the part-whole notion of classical mereology or another one equipped with the core principles). Therefore, it turns out that every such familiar member comes at no ideological cost in all hypotheses that already deploy the linear algebraic resources.

If defensible, this position would be attractive and in line with our terminal conclusion that no challenge from ideological profligacy holds against theories with satisfiers of deviant *and* canonical part-whole notions. My policy towards these in-house rivals remains (as in Chapter 8, Section 2) to prioritise reaching the conclusion over settling the internal disputes as to how it is best to reach it. Accordingly, I will only briefly motivate my own procedure over the view discussed in this section that canonical resources come at no ideological cost because of a kind with the deviant linear algebraic resources.

Linear algebraic and canonical resources partake of a group of resources (a 'rubric') each member of which has satisfiers that relate as part to whole in a literal sense. It does not follow, however, that the group these resources partake of has a claim to be an ideological kind. In fact, one can think of the resources in a way similar to the initially unclassified geometric predicates used in the definition of absolute velocity: Δx (Δr) for distances between simultaneous (non-simultaneous) events and Δt for temporal elapse. To promote this group to an ideological kind – so I have argued – the members must hold together 'evidently' and 'purposefully'. Evidently, in that some theoretical device (e.g. the Galilean tranformations) tells them apart from other resources that lack a particular characteristic P (e.g. invariance under Galilean boosts). Purposefully, because one uses the device and operates the distinction in order to pursue a scientifically eminent aim – for geometrical quantities, to tell apart quantities whose value varies relative to facts about the observers' motion.

It is not a straightforward or trivial claim that some device analogous to the Galilean transformations isolates the resources in the rubric by attributing some characteristic feature P, tells them apart from non-members and works as a condition of entrance

---

[1] Though all these satisfiers relate as component to sum, not all – recall from Chapter 5 – relate as part to whole in a literal sense. Each of the positive integers -2 and 5, for example, is a vector component of the other, yet intuitively not a part of it (Ib. Section 2).

[2] Say, the part-whole notion of classical mereology or another one equipped with the core principles (Antisymmetry, Weak Supplementation).

in a kind. This lack of an obvious candidate for the device suggests that we can freely explore other options, leaving it a task for the friendly rivals of this section to argue for the oneness in kind of all members of the rubric. Specifically, instead of accommodating canonical and deviant rubric members in the same kind, we shall accommodate canonical members in the same kind as the identity predicate.

2. Operations and Principles of Identity

To isolate the device that collects part-whole together with identity, we appeal once again to some Finean insights from "Towards a Theory of Part" (2010). Among Fine's multiple[3] aims in this paper is that of *classifying* resources for representing composition relations, some possessing regular ('canonical') features[4], some others possessing highly *sui generis* features. It can be argued – this being the core of this chapter – (1) that Fine's machinery classifies resources for standard part-whole

[3] Which include (but are not limited to): (1) setting intuition-driven filters to the class of *sui generis* relations that count as genuine varieties of part-whole (See Chapter 2 and 3 above); (2) classifying these *sui generis* relations by general principles (this chapter); (3) changing the foundations of mereology (intended as the study of formal systems for parts and wholes) by (3.1) expressing composition as an operation, rather than a predicate, and (3.2) taking composition, rather than part-whole, as the main primitive; (4) embedding composition operations in the explanation of the identity of composed objects (so that some objects can be the objects they are by virtue of being the image of a particular composition operation); (5) defining 'depending for identity' as 'being in the image of an operation with identity-explaining features' (also see Fine 1995: 269-90); (6) classifying the operations' inverses (decomposition) via general principles; (7) putting decomposition operation to the service of identity dependence (as in (4) and (5)), so that parts delivered by decomposition depend for their identity on the decomposed wholes; (8) arguing that there arise no cases of symmetric identity dependence (with particular wholes depending for their identity on their parts and vice versa) even though composition and decomposition operations provide for part-to-whole and whole-to-part dependence. The vastness of this list and the depth of the many topics involved justifies a piecemeal approach, on which we excavate a single area of the Finean proposal, turning it to our own purposes and isolating it from the larger picture (see point (2) in the main text).

[4] Up to this point we have only applied the terms 'canonical' and 'standard' to (predicates for) relations of part-whole, not composition, but extending this terminology is very natural. A part-whole relation is canonical (standard) iff it validates the Antisymmetry postulate and the proper part relation defined in terms of it validates, respectively, Weak Supplementation. In parallel, we can label a composition relation 'canonical' just in case it satisfies Idempotence, the part-whole relation that defines it satisfies the Antisymmetry postulate and the proper part relation defined in terms of this part-whole relation satisfies Weak Supplementation. Notice that in this way we still adhere to the initial choice of using part-whole as the main primitive (Chapter 1). This is in contrast with aim (3) in "Towards a Theory of Part" (cf. Fn. 3), that is, re-founding mereology on the primitive of composition (expressed as an operation, rather than a predicate). The main text explains more about how we can pursue the Finean aim of classifying resources for composition while retaining part-whole as the main primitive.

together with the identity predicate; (2) that it can be decoupled from a number of additional assumptions (all of which are so unique to Fine as to make the account unattractive to a number of other theorists); and finally (3) that the classification based on the Finean machinery allocates resources not to mere taxonomic groups (one of which contains part-whole and identity), but to ideological kinds.

For an initial approach to the Finean devices, we can start by thinking about set formation. The set formed by some $x_1, \ldots, x_n$ is the set A = $\{x_1, \ldots, x_n\}$ with $x_1, \ldots x_n$ as members. Normally, and especially if we took set formation in analogy to composition, we would be tempted to think of set formation as a relation, whose *relata* are the formed set A and the forming $x_1, \ldots, x_n$ (as one can form a set out of various numbers of things, we will take the relation to be variably polyadic). However, Fine wants us to resist the temptation. Instead, we will think of set formation in operational terms. One forms a set A out of some things $x_1, \ldots, x_n$ by applying an operation $\{\quad\}$ to $x_1, \ldots, x_n$ whose *image* for this sequence of *operanda* is A[5].

The operational terminology is *per se* intuitive, but we will return to it in a while to examine the connections to the relational terminology we have used up to this point. For now, to appreciate this intuitiveness, let us notice how we could express in operational language two interesting set-theoretical facts:

(i) every singleton set is distinct from its unique member ($\{a\} \neq a$, for arbitrary a)[6]; and

(ii) sets formed out of members a, b, c... as well as of sets $\{a, b\}$, $\{a, c\}$, ... of members a, b, c... are different from sets formed only out of members a, b, c... For example, for arbitrary a and b, $\{a, b\} \neq \{a, \{a, b\}\}$; $\{a, b\} \neq \{b, \{a, b\}\}$; and $\{a, b\} \neq \{a, \{a, b\}\}$.

[5] The operation's image need not always be defined. In fact, notoriously, we cannot form sets out of any collection of things.

[6] The distinctness of singletons and their unique members follows in ZF from the axiom of foundation, which states that every non-empty set x has at least an element y with no elements in common with x. If y is the only element of x, then given the extensionality of sets, lack of common elements suffices for $y \neq x$.

For (i), we will say that the image of $\{\quad\}$ on arbitrary single *operandum* a in the operation's domain is defined and distinct from *a*; that is, that $\{a\}$ is defined and that $\{a\} \neq a$. For (ii), we will say that the image of $\{\quad\}$ on a and $\{a, b\}$ is defined (as $\{a, \{a, b\}\}$) and distinct from the (defined) image of $\{\quad\}$ applied to *a* and *b*. For (ii) we will also say that *iterating* set formation ($\{\quad\}$) makes a difference to the identity of the formed sets. This means that the set obtained by forming *a* together with the set of *a* and *b* is not the same as the set obtained by forming *a* and *b*.

But what other features of the *operanda* matter for the identity of sets? In canonical Zermelo-Fraenkel (ZF) set theory, among the other irrelevant factors we find the *operanda*'s order and their repetition, thus (for arbitrary *a* and *b*):

$$\{a, b\} = \{b, a\}$$
$$\{a, a, b\} = \{a, b\}$$

Borrowing Fine's terminology, canonical set formation (conceived via the ZF axioms) is 'blind' to the order and repetition of *operanda*, but it is 'sensitive' to iterations, as the set formed out of some *a*, *b*, *c*, ... is not the same as the set formed not just by *a*, *b*, *c*... but also by sets whose elements are some of *a*, *b*, *c*... . Moreover, canonical set formation does not 'collapse', because the image of an arbitrary *operandum* is distinct from it.

By analogy, we could also borrow the physical terminology of invariance: repeating *operanda* and changing their order makes no difference to the image of set formation, hence set formation is 'invariant' in respect to the *operanda*'s order and repetition. On the contrary, the images change if some *operandum x* is set formation's only argument ($\{x\} \neq$ x) or if the arguments contain further iterations of set formation ($\{a, \{a, b\}\} \neq \{a, a, b\}$). Hence, set formation is not invariant in respect to iterative and 'lone' applications.

In outlining these structural features of canonical set formation, we have begun to generalise on arbitrary *operanda*: for any *a*, *b* in $\{\quad\}$'s domain, the result of applying $\{\quad\}$ to a and b is different from the result of applying $\{\quad\}$ to a and $\{a, b\}$, but identical to the result of applying $\{\quad\}$ to *b* and *a* (in inverted order) and to *a*, *a* and *b* (with repetition of *a*). Now, it is a natural step to extend this ability to generalise from the domain of *operanda* of a fixed operation ({ }) to the domain of *operanda* of *arbitrary* operations, which, in Fine's wake, we indicate by a variable Σ. By sanctioning these additional generalisations, we can state the above invariances (relative to order, repetition, further iteration and lone application) in respect not just to set formation ({ }) but to any operation whatsoever (cf. Fine 2010: S. 5):

Collapse: $\Sigma(x) = x$

Absorption: $\Sigma(\ldots, x, \ldots, x, \ldots, y, \ldots) = \Sigma(\ldots x, \ldots, y, \ldots)$

Leveling: $\Sigma(\ldots, x, y, \ldots \Sigma(u, v), \ldots, \Sigma(z, w), \ldots) = \Sigma(\ldots, x, y, \ldots u, v, \ldots z, w, \ldots)$

Permutation: $\Sigma(\ldots, x, \ldots, y, \ldots) = \Sigma(\ldots, y, \ldots, x, \ldots)$

An operation Σ satisfies Collapse if applying it to a single arbitrary *operandum* returns that very *operandum*; it satisfies Absorption (Permutation) if the repetition (order) of any argument does not alter the image and, finally, it satisfies Levelling if the image does not change once one allows "nested" arguments that consist of further applications of Σ.

The 'invariances' stated by Collapse, Absorption, Levelling and Permutation are the anticipated device, which taxonomise our resources in the part-whole rubric. By virtue of the division they operate, identity and ordinary part-whole (complying with the canonical principles) come out as members of a single group of resources with a claim to ideological kind-hood. Or, at least, so I will argue in the coming sections. It will be helpful, before giving this argument, to observe two general features of the invariances.

To begin with, our variable Σ ranges over operations and it is these operational resources that the Finean invariances taxonomise. They include canonical operations like mereological summation (Σ_M) as well as *sui generis* operations like set formation

(Σ_S) and linear summation (Σ_L). The former, Σ_M, takes as arguments any one (x) or more things (x_1, ..., x_n) and returns their mereological sum. The latter, Σ_S and Σ_L take as argument, respectively, one (x) or more (x_1, ..., x_n) entities and one ($\mathbf{x}$) or more vectors ($\mathbf{x_1}$, ..., $\mathbf{x_n}$), and return as images, respectively, the set that has these entities as members and the resultant vector that has these vectors as components[7].

Second, the Finean transformations taxonomise the operations according to the invariances they satisfy. In particular, two operations partake of the same group if they satisfy all the same invariances and to different groups if they differ in at least one. Consider, for reference, mereological summation (Σ_M) and set formation (Σ_S). Both operations take one (x_1) or many things (x_1, ..., x_n) as arguments, but while the former returns their mereological sum, the latter returns the set that has x (or x_1, ..., x_n) as members. On Fine's taxonomy, the two operations fall into separate groups: for while they both satisfy Permutation and Absorption, only mereological summation and not set formation satisfies Collapse and Levelling.

To illustrate further, the same holds of linear summation (Σ_L) in respect to mereological summation (Σ_M) and set formation (Σ_S). While the linear operation satisfies only Permutation, Σ_S also satisfies Absorption (in addition to Permutation) and Σ_M satisfies all four invariances. On the Finean taxonomy, accordingly, each of these three operations partakes of a separate group. The rationale for grouping operations, which underlies the taxonomy, is to bring together all operations that comply with the same invariances and separate those that disagree on at least one.

Having made these preliminary observations, we are still three steps away from arguing – as we wish to – that the Finean invariances classify resources for canonical part-whole in a way helpful to protect us from profligacy charges. Each of these steps occupies a dedicated section in the remainder of this chapter.

1. First, we have just observed that the Finean invariances taxonomise composition operations, not predicates for part-whole relations. Yet up to this point we pursued the aim of resisting profligacy charges assuming that the hypotheses H_2 to be protected from the charges deploy predicate and not operational resources. In fact,

[7] Assumption: all operations Σ taxonomised by the invariances, including Σ_M and Σ_S, are variably polyadic (cf. Fine (2010: 567)). This means that, while remaining defined, they can take one or more arguments ($\Sigma(x)$, $\Sigma(\ldots, x, y, \ldots)$) and possibly the same argument multiple times over ($\Sigma(x, x, \ldots)$). To put it differently, we can make a Finean operation (e.g. Σ_S) undefined by picking particular arguments (e.g. entities which no set has as members), but never by taking arguments in particular numbers.

they *predicate* of some vectors $\mathbf{v}_j, \mathbf{v}_k \in \mathbf{v}_1, \ldots, \mathbf{v}_n$ that they relate as component to vector sum and they *predicate* of some $\mathbf{v}_i \in \mathbf{v}_1, \ldots, \mathbf{v}_n$ that it relates by standard part-whole to some thing *x* (possibly $\mathbf{v}_i$ itself). To be able to argue from the Finean taxonomy to our terminal conclusion against challenges from profligacy that would lead us to discard H_2, we need – in some sense – to 'extend' the taxonomy to predicate-like resources for the part-whole relations.

2. Second, we observed that the Finean invariances allocate operations to the same group if they satisfy all the same invariances but to different groups if they differ in at least one (say, Permutation or Absorption). To use the taxonomy for resisting parsimony charges against H_2, we need to argue that resources for canonical part-whole group together with some other resources that H_2's more parsimonious rival deploys – specifically, the predicate for identity.

3. Third, we observed that the Finean taxonomy allocates resources that satisfy the same invariances (operations and, if my arguments in 1. and 2. are successful, also predicates) to the same *groups*. This is still one step away from arguing that the allocated resources partake of the same ideological kind, as we require for protecting H_2 from the consequences of its greater profligacy. To achieve this, the resources should be allocated not to any group, but rather to a group with a claim to be an ideological kind. As we know, and articulate in the dedicated section (5), this depends on showing that the invariances taxonomise resources purposefully – that is, pursue a scientifically eminent aim.

3. From composition operations to part-whole predicates

A characteristic of the Finean proposal is that composition operations figure as the main primitive concept. The present space does not allow me to defend the Finean choice of primitives, which, while unconventional, promises a number of advantages (see Fn. 3 for an overview). Let us therefore maintain the more standard approach we have so far relied on, according to which the primitive concepts for theorising about part-whole and composition are not operational but relational in character.

Even under this assumption, we can capitalise on the operational machinery to state interesting 'invariance' facts about relations, which allow us to categorise them just as we would if they had an operational nature. To see how, let us start by observing the following about the notion of an arbitrary n-place operation Σ: If y is the image of Σ when Σ's arguments are $x_1, \ldots, x_n$, then y is a *relatum* of an (n+1)-place relation S, such that at least some tuple in the (actual) extension of S is $<x_1, \ldots x_n, y>$. To put it differently, if our main primitives have relational character, then the machinery of operations still states facts about these relations' extensions. For example, by stating that some n-ary operation Σ delivers y when $x_1, \ldots, x_n$ are the arguments we mean (in a language that uses only our relational primitives) that the extension of some (n+1)-ary relation S has $<x_1, \ldots, x_n, y>$ among its ordered tuples.

The next step is to check which interesting facts about relations' extensions correspond to the Finean invariances (Absorption, Leveling, Collapse and Permutation) and how these facts about extensions allow us to group relations 'as if' they were operations, helping us in our project of protecting H_2 from discard. Before moving to this part of the argument, however, it will be helpful to compare the above statement about images of n-ary operations, that they are the last elements of some (n+1)-tuples in the extension of some (n+1)-ary relation, to an importantly different one by Lando (2017: 61-2).

To make his point, Lando invokes arithmetical subtraction: if y (the difference) is the image of subtracting x_1 (the minuend) to x_2 (the subtrahend) then y is the third *relatum* of a three-place relation (*being the difference … given by subtracting … and …*), whose first two *relata* are x_1 and x_2. This is essentially the same as our claim above: if y is the image of subtraction applied to x_1 and x_2, then some triple in the (actual) extension of a ternary relation S has x_1, x_2 and y as the first, second and third member, respectively.

Now, adding to this claim, Lando also argues that operations are a "special type" of relation, and namely, relations whose first n *relata* "*determine* the last *relatum*" (Ib. 61)[8], whereby on his understanding 'determines' means that all (n+1)-tuples in the relation's actual extension whose first n members are $x_1, \ldots, x_n$ can only have y as their $(n+1)^{\text{th}}$ member. Thus, a relation is granted the status of an operation only if its

[8] The emphasis on 'determine' is mine.

extension features no tuple with x_1, …, x_n as the first n members and some thing distinct from y as the last.

To reject this additional claim by Lando, we argue that it faces counterexample. There would seem to be genuine operations, like $\sqrt{\ }$, which deliver multiple images (e.g. i and $-i$) when fed the same arguments (-1). It may be plausible, therefore, that an operation's image is the $(n+1)^{th}$ element of *at least one* $(n+1)$-tuple in the extension of a relation. But we need not accept, concurrently, that y is the $(n+1)^{th}$ argument of *all* $(n+1)$-tuples in the extension with x_1, …, x_n as the first n members.

Let us now check what interesting facts about relations' extensions correspond to the Finean invariances. We can disclose these facts by using the principle just discussed; that is, that the image of an n-ary operation's is the last argument of some $(n+1)$-tuple in the (actual) extension of some relation S. With operations so understood, we obtain the following:

1. If some operation Σ collapses, then some relation S has at least one reflexive instance: a pair $\langle x, x \rangle$ in its extension whose first and second elements are both x.

2. If some operation Σ applied to n arguments x_1, …, x_n absorbs (permutes), then in the extension of some relation S there is a (n+1)-tuple $\langle x_1, \ldots, x_n, y \rangle$, such that y is the last element of every tuple in the extension of S, which contains x_1, …, x_n, some of which multiple times (some of which in different order).

3. If some operation Σ applied to n arguments x_1, …, x_n 'levels', then in the extension of some relation S there is an $(n+1)$-tuple $\langle x_1, \ldots, x_n, y \rangle$, such that y is the last element of every other tuple in the extension of S, which contains as elements some of the x_1, …, x_n and instead of the remaining $x_{j+1}, x_{j+2}, \ldots, x_m \in x_1, \ldots, x_n$ the images of Σ applied to $x_{j+1}, x_{j+2}, \ldots, x_m$.

For additional clarity, we can rephrase these facts in terms not of arbitrary, but of composition operations. If some such operation satisfies one or more of the Finean principles, then the extension at the actual world of a corresponding composition *relation* C satisfies certain correlated facts. If the operation collapses, then the relation has at least one reflexive instance: some thing x composes itself (as opposed to being one of the composers of a distinct thing y). If it levels, then the composite of some things $x_1, \ldots, x_n$ is the same as the composite of *some* of the $x_1, \ldots, x_n$ together with composites of the remaining ones. Finally, if the operation absorbs or permutes, then the composite of $x_1, \ldots, x_n$ does not change, no matter whether the C relates some of the $x_1, \ldots, x_n$ in a particular order or multiple times[9].

It seems clear, therefore, that the Finean principle can guide us in classifying primitive resources for composition, no matter whether we consider these to be operational or relational in character. In particular, nothing about the primitives' character prevents us from accepting the classifying principle (Section 2) that two resources partake of the same group only if they satisfy all the same invariances. The key thought is now that when we use this principle to taxonomise relations, as opposed to operations, deciding on membership in the same or separate groups are facts about these relations' (actual) extensions. These facts, as just seen, correspond to the invariance principles formulated with the operational machinery.

Now, we need one final step to be able to argue that the Finean principles classify mereological resources in a way helpful to resisting challenges from profligacy. As noted at several stages, the hypotheses H_2, which we would like to protect from discard, exceed the ideology of their rivals by deploying additional predicates for (canonical) *part-whole*, not composition relations. The argument we intend to use for showing that H_2's exceeding ideology doesn't prompt discard is that this ideology is of a kind with ideology deployed by the rivals – specifically, the predicate of identity[10]. Accordingly, having shown that the Finean principles can be appealed to for classifying composition relations, in addition to operations, the missing tile is now an argument to the effect that the principles also classify relations of part-whole, in addition to composition.

---

[9] For further illustration, imagine a ternary relation *being the arithmetic sum of*, such that the third element z of every triple $<x, y, z>$ in its actual extension is the image of the arithmetic summation *operation* applied to x and y. As the operation 'permutes', z is also the third element of all triples in the relation's extension whose first two elements are y and x (i.e. $<y, x, z>$). Further, as the operation 'absorbs', z is also the last element of all tuples in the extension of a relation of higher adicity (greater than 3), none of whose elements is other than x or y.

[10] In the terminology of Hawley (2014) and Chapter 7, this counts as an argument by 'levelling down'.

For this additional claim, we can help ourselves to the following fact: that in the framework of primitives we have adopted throughout this thesis (Chapter 1), predicates for part-whole relations invariably define predicates for composition relations (as is standard, for example, in the classical mereological framework). Having recalled this, we can naturally suggest a principle of classification for the defining part-whole relations, which invokes the Finean invariances: If two composition relations satisfy the same invariances, then not only these relations get allocated to a single group, but so do also the part-whole relations that define them. In parallel, if two composition relations differ in at least one satisfied invariance, then not only these relations but also the part-whole relations that define them get allocated to different groups[11].

Two examples will clarify. First, consider set formation and mereological summation understood, respectively, as relations between some things x_1, ..., x_n and the set having x_1, ..., x_n as members and between some things x_1, ..., x_n and their mereological sum. To relate by these relations – to be a set formed of x_1, ..., x_n and to be a sum of the x_1, ..., x_n – is defined, respectively, in terms of set-theoretical membership and mereological part-whole[12]. Besides, the two relations differ in respect to some invariance principles, for set formation, but not mereological summation, fails Collapse and Levelling. According to the Finean taxonomy, this difference in the satisfied principles indicates difference in the allocated groups: set formation and mereological summation will not be grouped together. Now, the new principle predicts that part-whole relations defining composition relations allocated to different groups are similarly allocated to different groups. Should we accept the new principle, mereological part-whole and membership would also belong to distinct taxonomic groups.

Second, consider mereological summation and linear summation. While the former is understood as before, the latter is a relation between n vectors $\mathbf{v_1}$, ..., $\mathbf{v_n}$ and the vector $\mathbf{y} = \mathbf{v_1} + ... + \mathbf{v_n}$ these are linear components of. Like before, these relations

[11] An exception should hold for the case in which one and the same part-whole relation P defines two composition relations C, C', which in turn differ in the satisfied invariances. When this happens, we shall say that the defining composition relation, P, is allocated to two distinct groups of part-whole relations: those that define relations that satisfy all the same invariances of C and those define composition relations that satisfy all the same invariances of C'.

[12] Recall (Chapter 2, Section 2) that the mereological predicate for part-whole satisfies the partial order axioms (Antisymmetry, Reflexivity, Transitivity) together with Strong Supplementation.

differ in respect to the satisfied invariances, as only mereological, but not linear summation satisfies Collapse and Absorption. In mereology, every singular thing is its own sum and repetition of constituents makes no difference to the sum's identity. But for a contrast, combining a single vector yields a distinct vector – a scalar multiple of it – and summing it multiple times in a linear combination makes indeed a difference to the sum (Cf. Chapter 4). This difference in the satisfied principles indicates difference in the allocated groups: linear and mereological summation will be collected separately. Correspondingly, the new taxonomic principle allocates to distinct groups also the part-whole relations that define linear and mereological summation. For mereological summation, this *definiens* is a canonical part-whole relation equipped with partial order axioms and the principle of Strong Supplenetation (Cf. Chapter 1). For linear summation, it is the relation of component-sum, which violates the canon in respect to Antisymmetry and various supplementation principles (Cf. Chapter 4).

As the two examples clarify, this new classification technique allocates members of the part-whole rubric (such as mereological part-whole and component-sum) to different groups. Accordingly, it is not a suitable principle for an argument that canonical rubric members (e.g. mereological part-whole) come for free in a hypothesis that 'already' deploys deviant members (e.g. component-sum). For this argument, we require that the member coming at no cost and the member already deployed partake of the same ideological kind. Yet no two predicates can partake of one kind if, according to the relevant classification technique, they do not even partake of one group.

Having said this, the technique can be used to argue that canonical members, like mereological part-whole, are one in kind with the identity predicate. This obtains for the reasons we proceed to illustrate in the final two sections: namely, (1) that the identity predicate satisfies the same invariances as mereological composition and (2) that the principle co-allocating satisfiers of the same invariances suffices for allocating mereological part-whole and identity not just to groups, but to kinds.

4. Identity and the Invariance Principles

Let us start from the argument that the identity predicate satisfies the same invariances as mereological composition, hence is co-allocated with the latter.

Mereological composition (henceforth: Σ_{CEM}) paradigmatically ticks all of the Finean invariances (cf. Lando 2017: 80-1). It is a standard claim of classical mereology, indeed, that every thing is its own mereological sum ($\Sigma_{CEM}(x) = x$), hence that the relevant composition relation satisfies Collapse. Moreover, it is irrelevant to a sum's identity conditions whether the composers come in a particular order ($\Sigma_{CEM}(a, b) = \Sigma_{CEM}(b, a)$) or with repetition ($\Sigma_{CEM}(a, b) = \Sigma_{CEM}(a, a, b)$), hence the relevant composition relation also satisfies Absorption and Permutation. Finally, it makes no difference to the sum's identity whether the composers include further 'nested' sums or just their composers, in agreement with Leveling[13].

Now, on the Finean taxonomy Σ_{CEM} groups together with identity only if the latter satisfies all four invariances: Collapse, Leveling, Permutation and Absorption. Collapse and Leveling are easy to obtain. Every thing is identical to itself, hence the extension of the identity predicate has all and only reflexive pairs, in accordance to Collapse. Further, Leveling is also confirmed: if some *x* is identical to some *y*, then so is the thing *z* that *x* is identical to. We can read the latter as a statement of Leveling also in operational vocabulary. Suppose that I(*x*) is a (unary) identity operator, which in application to *x* returns the thing *x* is identical to. Notoriously, identity is a Euclidean relation, so that if *x* is identical to *y* and *z*, then *y* is identical to *z*. In operational language, this corresponds to the principle that if I(*x*) returns *y*, then so does the 'nested' I(I(*x*)), which contains an iteration of I(*x*) and has as image the thing *z* identical to the thing *x* is identical to. Given the Euclidean property, this thing *z* is just *y* if *x* is identical to *y* in the first place – in agreement with Leveling.

[13] As Lando puts it (2017: 80): "The principle of *Leveling* obliterates another kind of *stratification*, which happens when some parts of the entities are *grouped* and *encapsulated* in a *subwhole.* In Fine's formulation $(\Sigma(\ldots,x, y, \ldots \Sigma(u, v), \ldots, \Sigma(z, w), \ldots) = \Sigma(\ldots,x, y, \ldots u, v, \ldots z, w, \ldots))$ this grouping is expressed by the two "internal" occurrences of Σ. Sets violate Leveling, which is why, for example, the set {{*a, b*}, {*c, d*}} is different from the set {{*b, c*}, {*a, d*}}. Mereological sums respect Leveling".

While Collapse and Leveling obtain easily for the identity predicate, Permutation and Absorption are more problematic. Understood as a predicate, identity is dyadic[14], its first argument being the thing that is identical to the second. Yet Permutation and Absorption state that order and repetition of the satisfying relation's first n arguments do not make a difference to the last, whereby – crucially – n is greater than 1. For Permutation or Absorption to be satisfied, in other words, there must be *two or more* arguments whose repetition and order does not make a difference to the last.

To work out a solution, we need to refine our claim that the identity predicate satisfies the Finean invariances. Up to this point we assume that the satisfier of the invariances was, indeed, the binary predicate of identity (or, correspondingly, the unary identity operator I). To overcome the problems with Permutation and Absorption we now drop this assumption and argue instead that the invariances' satisfier is a *variably polyadic* predicate I*, different from binary identity but defined by it[15]. This predicate has $n + 1$ arguments, where n is greater than 2. Applied to $n + 1$ entities $x_1, x_2, ..., x_{n+1}$, it states that each among n of them ($x_1, ..., x_n$) is identical to the ($n + 1$)th one (x_{n+1}). Thus I*(a, b, c) reads 'each of a and b is identical to c' (with $n = 2$) and in general, I*($a_1, a_2 ..., a_m, b$) reads 'each of $a_1, a_2 ..., a_m$' is identical to b (with $n = m$). This new predicate I*, as we were saying, is defined by the more common binary predicate for identity in a conjunctive way. 'I*(a, b, c)' is by definition 'a is identical to c and b is identical to c' and more generally, for some number n of arguments, I*($x_1, ..., x_n, y$) is by definition '$x_1 = y$ and $x_2 = y$, ... and $x_n = y$'.

Though different – strictly speaking – from binary identity, the new predicate I* satisfies Permutation and Absorption. If each of some *a, b* is identical to some c, then c is the thing a, a and b are identical to, as well as the thing b and a are identical to. Differently ordering a and b, or listing one of them multiple times make no difference to what each of them is identical to (if anything). In agreement with Absorption, adding another a to the list of entities identical to c makes no difference to what each item in the list is identical to, that is, c. And as for Permutation, reordering the list of entities a, b each of which is identical to c, makes no difference to the fact that each element of the list is identical to c.

---

[14] Correspondingly, I is a unary operator.

[15] Correspondingly, we can also say that the invariances are satisfied by a variably polyadic operator *I#*, which takes n things $x_1, \ldots, x_n$ as arguments and outputs the thing y each of the $x_1, \ldots, x_n$ is identical to.

If the above observations are on the right lines, then I* and Σ_{CEM} satisfy the same invariances, hence the Finean taxonomy groups them together. Now, by the principle introduced in the last section, the resources we allocate to a single group include not only operations or predicates that satisfy the same invariances, but also their *definientes* – that is, the operations or predicates in terms of which the satisfiers of the same invariances are defined. A consequence of this addition to the principles of classification is that we should co-allocate the *definientes* of I* and Σ_{CEM}, which in our framework of primitives means, respectively, the binary predicates for identity and mereological part-whole. Helping ourselves to these maneuvers, therefore, we finally win a view in which resources for a canonical part-whole relation – the mereological relation – group together with the binary identity predicate.

Let us, accordingly, summarise our progress. The identity predicate clearly belongs to the primitives of ideologically sparse hypotheses, which renounce mereological part-whole and retain only deviant relations. One such hypothesis, which we invoked multiple times already, posit vectors $\mathbf{v_1}, \ldots, \mathbf{v_n}$ with 'bare' vector space structure and state only that some of these vectors, taken pairwise, relate as component to vector sum, not that some vectors also enter a canonical part-whole relation, e.g. by being their own mereological parts.

Armed with the new principle of classification we make another step towards arguing that mereological part-whole comes at no additional cost relative to H_1's resources, hence that we should not discard hypotheses like H_2. The new classification principle, indeed, allocates identity and mereological part-whole to the same taxonomic group, i.e. the group of resources that define satisfiers of the same Finean invariances.

Now, the argument for protecting H_2 will be complete only if the group we co-allocate these resources to has a claim to ideological kind-hood. Indeed, this would ensure that H_2's surplus resources do not contribute towards making H_2 renounceable, provided we approach the rivalry between H_1 and H_2 with a policy that minimises the deployed kinds and not the number of overall resources[16]. The

[16] Recall that our terminal conclusion is that H_2 should not be discarded for reasons of ideological profligacy. To reach this conclusion we need use the premise (p) that we should assess the rivalry between H_1 and H_2 should be by minimising the number of deployed kinds, not the overall number of primitives. As I argued in section 4 of the previous chapter, however, while this premise is *needed* for the argument to go through, our current dialectic allows assuming it for the sake of argument and providing no defence of it. For suppose that the premise is false. In this case, the alternative to assessing the rivalry by minimising the deployed kinds is assessing the rivalry 'plainly' by minimising the number of deployed resources. But, as we have seen, a

prospects for claiming that identity's and mereological part-whole's common group is an ideological kind are the subject of the next section.

5. The 'kindhood question' for the Finean taxonomy

To anticipate, I will concede that the claim is available only at a price, which could seem unattractive to some. A key thought in our discussion of ideological kinds up to this point[17] is that for turning a particular group of resources into a kind we require that the resources partake of the group evidently and purposefully. To rehearse once again, 'evidently 'means that some theoretical device (e.g. the Galilean transforms) assigns the resources characteristic features F_1, ..., F_N (invariance or non-invariance under Galilean Boosts), which work as conditions of entrance in the various groups. 'Purposefully' means that when we use the device to assign F_1, ..., F_n and allocate the resources to the various groups we pursue an eminent scientific aim (e.g. distinguishing variant from invariant geometrical quantities).

We shall now put this argument to work in our area of interest. In the role of the categorising device we have, as anticipated, the four Finean invariances: Collapse, Leveling, Absorption and Permutation. The characteristic features, which refer back to the device and work as a condition of entrance in the various taxonomic groups, are the properties of defining a relation that satisfies certain invariances but not others. For example, set-theoretical membership defines a relation – set formation – that satisfies Collapse, Permutation and Absorption, but not Leveling. This property is required for membership to enter a group of resources, all of which are similar in respect to defining relations that satisfy the same three invariances[18]. Besides, it tells

popular argument in favour of this plain assessment (i.e. Sider's) fails, because it requires us to interpret episodes of scientific theory choice as episodes of plain minimisation, while the data allows for interpreting them as episodes of kind minimisation. This leaves us with little reason to discard H_2 even if (p) is false, which is compatible with our terminal conclusion that H_2 should not be discarded on grounds of ideological profligacy.

[17] Cf. Chapter 8, Section 4; Chapter 9, Section 3.2 and the rehearsal in section 1 of this chapter.

[18] As discussed in Fn. 11, I grant that membership (or other predicates) can define multiple relations R', R'', each satisfying different invariances. When this happens, we allocate membership to multiple groups, i.e. the groups of *definientes* of relations that satisfy the same invariances as R' and for relations that satisfy the same

membership apart from all resources that define satisfiers of different invariances – including, for example, mereological part-whole, whose defined relation (mereological composition) also complies with Leveling.

As a result, the first part of our argument for attributing kind-status to the group of part-whole and identity seems plausible: the two resources are allocated to the same group *evidently* via an entrance condition specified by some taxonomic device. Greater controversies – as we will now see – surround the second part of the argument, which has it that the group with part-whole and identity has a claim to ideological kind-hood only if in co-allocating these resources via principles that refer back to the Finean taxonomy we pursue an eminent scientific aim.

5.1 The taxonomy's aim

To confirm the compliance with an eminent scientific aim, the first thing to do is distinguish the question of exactly *what* aim we pursue when we use the above principles to co-allocate part-whole and identity from the question of whether this pursued aim is eminently scientific. We will start from the former.

On a natural understanding, the aim of invoking the above classifying principle (i.e. that predicates enter the same group if they define satisfiers of the same invariances) is to bring together predicates satisfied by objects which, in some sense, have the same type of internal structure. Let me immediately explain. Satisfying the binary identity predicate are objects x, y such that for some things $x_1, x_2, \ldots, x, \ldots, x_n$, each of which is identical to y, no difference to the fact that y is the thing each of $x_1, \ldots, x_n$ is identical to is made by $x_1, \ldots, x_n$ coming in a particular order (Permutation), containing repetitions (Absorption), or containing in place of some $x_j \in x_1, \ldots, x_n$ the

invariances as R". We have largely ignored these cases of overlap, for they are of little relevance to our point that identity groups together with mereological part-whole (these two resources would group together, indeed, no matter how many *other* groups they individually partake of). Yet an opponent could cite the fact that we provide for overlap between our groups as a reason to deny the groups the status of ideological *kinds*. This is once again because in deciding whether a certain group of resources qualifies for kind-hood in the newly introduced ideological sense we might want to help ourselves to standard analyses of natural kinds. Usually, however, the latter make no room for 'partial' overlap: a group is a kind only if it shares members with its subgroups or with itself (but see Khalidi 2010). Now, I already replied to this objection in section 3.3 of Chapter 8. To some extent we can use the properties of natural kinds as a guide to which group of primitives has a claim for ideological kind-hood. More accurately, however, we should balance this factor (conformity with the properties of natural kinds) with another, and namely the fact that the group results out of an evident and purposeful classification.

thing z that x_j is identical to (Leveling). For if each of $x_1, \ldots, x_n$ is identical to some thing y, then this y is also the thing each of $x_n, x_{n-1}, \ldots, x_1$ is identical to, as well as the thing each of $x_1, x_1, x_1, \ldots, x_2, \ldots, x_n$ and each of $x_1, \ldots, z, \ldots, x_n$ is identical to.

Similarly, satisfying the predicate for mereological part-whole are entities x, y such that for some $x_1, x_2, \ldots, x, \ldots, x_n$, all of which jointly compose y, no difference to the fact that y is the sum of the $x_1, \ldots, x_n$ is made by $x_1, \ldots, x_n$ coming in a particular order (the sum of $x_1, \ldots, x_n$ is the same as the sum of $x_n, \ldots, x_1$), by $x_1, \ldots, x_n$ containing repetition (the sum is the same as the sum of $x_1, x_1, \ldots, x_n$), or by the fact that $x_1, \ldots, x_n$ contains the sum of some $x_j, \ldots, x_{j+n}$ in place of $x_j, \ldots, x_{j+n}$.

The claim I will make in either case is that satisfiers of the co-allocated predicates (wholes y with a mereological part x and things y that are identical to some x) have an internal structure similar in respect to 'ignoring' differences in the identity of the structured thing made by order, repetition and nesting of the constituents[19].

For a contrast, the aim in allocating predicates to distinct groups is to emphasise diversity in the internal structure of the satisfiers. For example, satisfiers of component-sum – the linear algebraic structuring relation – are vectors $\mathbf{x}$, $\mathbf{y}$ such that, if $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}, \ldots, \mathbf{x}_n$ are the components of $\mathbf{y}$, then it *does* make a difference to which vector is the vector sum of $\mathbf{x}_1, \ldots, \mathbf{x}_n$ that $\mathbf{x}_1, \ldots, \mathbf{x}_n$ feature repeated elements. The vector sum of $\mathbf{x}_1, \ldots, \mathbf{x}_n$ is not the same vector as the vector sum of (say) $\mathbf{x}_1, \mathbf{x}_1, \ldots, \mathbf{x}_n$. This time, accordingly, we will not say that satisfiers of predicates allocated to distinct groups are similar in respect to 'ignoring' differences in the identity of the structured thing made by order, repetition and nesting of the constituents. Rather, satisfiers of predicates allotted to separate groups differ in that for the satisfier of one predicate, but not the other, differences in at least one aspect of their internal structure (order, repetition and nesting of the constituents) change the identity of the structured thing.

5.2 The aim's scientific eminence

Having clarified exactly what aim we pursue in dividing predicates according to our classification principle, the remainder of this section deals with the question of

[19] Fine (2010: 566-7) describes wholes of this kind as 'flat' (as opposed to 'stratified') and 'insensitive to structure'. On the same line see Sattig (2016: 6-7) and Lando (2017: S. 5.7, in particular p. 78-9).

whether the pursued aim has any scientific 'eminence'. My intent is to show that we can gesture towards a positive answer. I cannot deny, however, that this answer becomes available only at the price of assumptions that are unattractive to some. Accordingly, following our argument that yes, the pursued aim has some kind of scientific eminence, we will terminate this section, chapter and work with some important concessions.

First, however, the argument. The aim, we have said, is to distinguish predicates (for mereological part-whole, identity, component-sum) according to the type of internal structure displayed by their satisfiers. When we ask whether this aim has any kind of scientific 'eminence', it is initially tempting to answer negatively. For compare the aim with its relative discussed in the previous chapter: namely, distinguishing geometrical quantities whose values, respectively, vary and remain invariant under changes in the relative velocity of observers. This latter aim guides the taxonomy of initially unclassified geometrical resources[20] into satisfiers and non-satisfiers of the Galilean transforms (Chapter 9, Section 3). Crucially, and unlike our present one, this aim has a feature that is naturally seen as paradigmatic to 'eminence'. It is, namely, very familiar to practitioners of spacetime physics and pervasive in the history of attempts to classify spacetime resources. Indeed, practitioners divided satisfiers and non-satisfiers first of the Galilean transforms, and later on of the Lorentz transforms. Relative to each set of transformation, the allocation of resources to groups changes, as some Galilean-invariant quantities fail to remain invariant under one or more Lorentzian transforms, hence the corresponding resources partake of the group of invariants in one allocation and of the group of variants in the other. However, although the groups the resources partake of changes in relation to each set of transformations, the purpose of allotting each resource to either group remains the same: separating quantities whose value varies and remains the same following changes in the observers' relative velocity.

Let us put this point differently and closer to our rendition of it in section 2 of the previous chapter. Helping us to a set of transformations, in post-Galilean spacetime physics it is common practice to classify resources for spacetime quantities into invariant and variant with the aim of telling apart the quantities whose value is not affected by the observers' relative velocity. Now, this stability and familiarity are

---

[20] These resources include Δx (Δr) for distances between simultaneous (non-simultaneous) events and Δt for temporal elapse, cf. section 1 of this chapter.

simply not available when it comes to our current aim of interest, which is that of distinguishing the predicates whose satisfiers do not have their identity affected by changes in internal structure (i.e. changes in order, repetition and leveling). Admittedly, invoking the Finean invariances and pursuing this aim is not commonplace among practitioners of any branch of physical science.

How can we argue, therefore, that in applying our principles of classification based on the invariances we pursue 'eminently scientific' aims? The answer I would like to offer is the element of this section that most requires us to concede the viability of alternatives. Before we introduce these concessions, however, let us take a look at the details of the answer.

It is true – we admit – that our aim lacks the familiarity of distinguishing invariant from variant spacetime quantities. Even so, we can deny that this familiarity is necessary for ascribing our aim scientific eminence, and more precisely, for ascribing the *type* of eminence we need for turning the predicates' taxonomic group into an ideological kind. On this suggestion, accordingly, when we pursue our aim and co-allocate part-whole and identity, the aim has *enough* eminence to ensure that the two predicates partake of the same ideological kind, not just the same taxonomic group.

To understand how this can be, we take a better look at our strategy for answering the question of which taxonomic groups qualify for ideological kind-hood (Chapter 8, Section 3.3 and Chapter 9, Section 3.2). In favourable circumstances we can argue that a group of ideological resources is an ideological kind because it relates to its members and to other groups just as a natural kind does (according to standard analyses of the notion which I will not rehearse here, cf. Chapter 8, Section 2). In unfavourable circumstances, i.e. when the target group lacks the structure of a natural kind, we appeal instead to a naturalistic view, whose key claim is that our list of groups that qualify for ideological kind-hood should be informed and constrained by the content, methods and aims of advanced scientific theorising[21].

Now, "informed and constrained" is intentionally a lax choice of words. On one way of being informed and constrained, for example, the list of candidates for kind-hood features only the groups whose members we collect while pursuing *familiar* scientific

---

[21] This holds as a special case of a wider naturalistic policy, according to which the entirety of metaphysical theorising should be informed and constrained by the content, aims and methods of advanced science. The subject matter that we presently inform and constrain in this way is the world's division of ideological resources into kinds. I owe this formulation of metaphysical naturalism in general terms to Chrakravarrty (2017: 81-2). See Chapter 8, Fn. 41 for further discussion.

aims. Thus, it contains the group of invariant and that of variant spacetime quantities, but not our groups of predicates whose satisfiers have the same type of internal structure (part-whole and identity, recall, jointly enter one among these groups). Differently put, here 'informing and constraining' the list with advanced scientific aims simply means 'copying' the aims familiar to practitioners into the requirements for theorising about the world's ideological kind structure. The only taxonomic groups with a claim to ideological kind-hood are those formed while pursuing a *familiar* aim with a well-established place in scientific practice.

Another way of being informed and constrained, however, offers more leeway to write down the list of candidates for kind-hood. This time the aims we need to pursue in putting forward a classification of resources may outreach those familiar to scientific practitioners and include, in addition, aims specific to metaphysical enquiry. In turn, while metaphysical, these aims should be *continuous* with the scientifically familiar aims and arise out of them.

This policy exemplifies a popular[22] approach when it comes to interpreting the motto that metaphysical theorising (here: theorising about the world's ideological kind structure) should be inspired and constrained by advanced scientific contents, methods and aims (here: familiar scientific aims)[23]. Scientific enquiry fixes not the particular contents, methods and aims importable into metaphysical theorising, but rather general 'guidelines' for limiting the viable contents, methods and aims, which in turns scientific enquiry follows in specific ways.

As for us metaphysicians, we should have enough of a naturalistic justification to hold onto our hypotheses insofar as their subject matters, methods and aims comply with the guidelines, even if they do not follow them in the particular ways exemplified by scientific enquiry. Consider our current case of interest. On this

---

[22] Though not compulsory, see below.

[23] For discussion of this approach, including differences we cannot linger on, see Morganti (2013: 4-5, 11, 20), Morganti and Tahko (2017: S. 2). The approach sophisticates the view of earlier writers, such as Sider (2011) and Paul (2012), who claim that science and metaphysics have the same methodology (not methodology similar in respect to general guidelines). The earlier claim is dubitable, as for example it can be shown that the scientists' appeal to theoretical virtues in theory choice differs from the metaphysicians' appeal. The scientists' use of inference to the best explanation includes auxiliary conditions generally ignored in metaphysics (Saatsi 2017), and the scientists use of parsimony (Ladyman and Ross 2007: Ch. 3, Sober 2014: Ch. 2) leans towards model-selection methods, rather than methods based on minimising ontology or ideology (see Chapter 7, Section 3 for further discussion). While these observations pertain to scientific and metaphysical methods, in this section we have pursued a similar point about aims. Scientific and metaphysical aims for classifying ideological resources differ and this is why we can suggest a view on which the differences need not matter for using this aim in theorising, insofar as common guidelines underlie them.

liberalised proposal, the list of candidates for ideological kind-hood can contain our groups of predicates if the aim of distinguishing satisfiers with the same type of internal structure complies with the guidelines of some familiar scientific aim.

We need not look far to find a scientifically familiar aim with the same guidelines. Indeed, taxonomising predicates based on the differences in internal structure of their satisfiers is similar to the aim of taxonomising particular physical systems based on their constituents' properties. For us the various properties securing the membership of a predicate in a group or another are the satisfiers' order and repetition, as well as the presence of 'nested' satisfiers in accordance, respectively, with Permutation, Absorption and Levelling. For the more familiar scientific aim, the properties securing the membership of a system in a group include features of the system's constituents like their state-dependent properties. Thus a system belongs to the group of hydrogen atoms only if one of its two constituents has the quantum numbers of a free electron and the other constituent has those of the free proton. Similarly, an entity belongs to the group of protons only if two of its three constituents have the quantum numbers of a top quark (e.g. 2/3 of the free electron charge) and the remaining constituent has those of a bottom quark (1/3 of the free electron charge). Finally, to draw an example from the frequently discussed algebra of vectors (Chapter 4, Section 2), a vector belongs to the group of vectors with bare 'linear' structure only if its components have no property other than compliance with the vector space axioms; and it belongs to the group of vectors with geometric structure if its linear components also have magnitudes which work as the domain of the relevant norm function.

What unifies all these cases is that they display an evident scientific aim: to taxonomise systems (hydrogen atoms, protons) or mathematical objects (vectors) by using aspects of their internal structure (their constituents' state-dependent or algebraic properties) to decide membership in one or another group. I label the deciding properties an aspect of the classified entities' 'internal structure' to emphasise the analogy with our classification. The two classifications resemble each other in that deciding on membership in a group or another are aspects of internal structure: state-dependent and algebraic properties on one side, display of order, repetition and nesting on the other.

This similarity, I say, supports a tempting claim, and namely, that an aim similar enough in respect to general guidelines to some familiar scientific aim underlies our classification of predicates. If on the right lines, this claim combines with the previous one that we taxonomise our predicates into kinds if in dividing them into groups we pursue an aim with the same guidelines as some familiar scientific aim. The result of this combination is, indeed, that our taxonomised predicates partake not just of taxonomic groups but also of ideological kinds.

5.3 Concessions

I cannot emphasise enough that each of the two claims in this combination is "tempting" but, at the same time, far from being entirely defensible in the little space that remains. Accordingly, it will be helpful to conclude this section by making two major concessions, which reveal two viable angles of attack for our opponents.

Firstly, I have assumed but not defended the view that hypotheses about the division of resources into ideological kinds can be "informed and constrained" by familiar scientific aims in a *weak* sense. On this view, the aims we pursue in compiling our list of groups with a claim to ideological kind-hood can differ from the familiar aims of scientific enquiry (e.g. classifying systems or mathematical objects based on properties of their constituents) provided they satisfy the familiar aims' general guidelines (deciding for membership in one group or the other based on aspects of a complex entity's internal structure).

On a rival view, which I have not argued against, only familiar aims are allowed to inform and constrain our list of groups with a claim to ideological kind-hood. The clash between the advocated liberal position and this rival concerns the correct understanding of metaphysical naturalism. For the rival, we develop viable metaphysical theories (including theories of how reality's ideological structure divides into kinds) at least by pursuing the same aims of advanced science[24]. For the supporter of my liberal view, on the contrary, the aims of advanced science can inform and constrain the viable metaphysical theories without changing the theories' own aims, which are distinctively metaphysical. My concession, in turn, is that I have made no steps to address this disagreement concerning the correct interpretation of

[24] And at most by also employing the same methodology and subject matter.

metaphysical naturalism. The disagreement seems too large in scale for us to do justice to it appropriately in the remaining space.

Secondly, suppose we are right in advocating the liberal reading of "informed and constrained", so that in giving our list of groups with a claim to ideological kind-hood we can pursue not familiar scientific aims but only aims *similar* to these in respect to complying to the same general guidelines. Even so, it remains an open question *how* similar the pursued aims have to be for the list to count as 'informed and constrained' by the familiar aims.

A familiar aim, as seen, is to classify physical systems (hydrogen atoms, protons, etc.) and mathematical objects (vectors) according, respectively, to their constituents' state-dependent and algebraic properties. Surely it is tempting to compare this aim to the pursued one of classifying predicates (for part-whole, identity, component-sum, etc.) according to their satisfiers' sensitivity to differences in internal structure. But tempting similarities should not immediately convince us that pursuing the latter aim will lead us to a taxonomy of resources appropriately informed and constrained by the aims that guide scientific taxonomies. Indeed, some differences between the aims guiding the two classifications could still play against us. For example, it could be required for our list to be appropriately informed and constrained that the aspects of internal structure determining a predicate's membership in one or another group are not the satisfiers' resistance to changes in the constituents' order, repetition and nesting, but rather the particular state-dependent or algebraic properties of the satisfiers' constituents. Similarly, it could be required for our list to be appropriately informed and constrained that the taxonomised entities are not in the first place items of ideology but rather *entities* of scientific interest like physical systems or mathematical vectors.

My concession, in turn, is that I have not yet addressed this question concerning how similar the pursued aims have to be for our list to count as 'informed and constrained' by aims familiar to scientific enquiry. As before, the question seems large in scale and difficult to cope with appropriately in these final stages.

We can conclude this section by observing what impact the two concessions have on our desired conclusion, i.e. that predicates grouped according to their satisfiers' internal structure partake of ideological kinds, rather than mere taxonomic groups. The need to make some concessions will not undermine the conclusion entirely, but it implies that the conclusion only ensues at some costs.

Let us see which costs exactly. As we have argued[25], the predicates we co-allocate would share a kind if in co-allocating them we pursued a purpose informed and constrained by familiar scientific aims. Now, a key point of this section was that we cannot argue for the pursued aim being so 'informed and constrained' without taking a perspective on what the latter exactly means – that is, on what it takes to relate to familiar aims in this informing and constraining way.

Crucially, the perspective I have taken is based on two undefended tenets. First, the aim we pursue in grouping the predicates is similar to but not exactly the same as an aim familiar to scientific enquiry. Second, the pursued aim is similar *enough* to a familiar one for it to count as informed and constrained by the latter. We will be in a position to claim that the resulting taxonomy is one of ideological kinds, rather than mere groups, only if we help ourselves to these tenets. Yet we cannot pretend that the claims stand unrivalled. In fact, that there are other perspectives from which to understand the meaning of 'informed and constrained' is exactly what the concessions concede. In a line, the viability of alternatives indicates that for our desired conclusion we need to accept one and reject another understanding of what it takes to pursue aims informed and constrained by scientific enquiry.

6. Conclusion

I have applied a three-step method for arguing that that (predicates for) canonical part-whole and identity partake of the same ideological kind. The first step consists in isolating a device that gathers the two relations in the same group: the Finean invariance principles of Collapse, Absorption, Permutation and Levelling. While originally developed to classify composition operations, the principles can be used for grouping composition relations. Moreover, by a natural extension, they also group these relations' *definientes*; that is, part-whole relations. Accordingly, two part-whole relations partake of a single group just in case each of them defines some composition relation and these defined relations satisfy the same invariance principles.

[25] Cf. Chapter 8, Section 4 and section 5.2 of this chapter.

The second step consists in showing that, according to the taxonomic principle just introduced, part-whole and identity partake of a single group. This includes two sub-steps. First, identity can be used to define a relation (I*) which satisfies each of the four Finean invariances. Second, we can co-allocate identity with the mereological relation of part-whole, for the latter also defines a relation, mereological composition, which satisfies each of the four invariances.

Finally, in the third step we argue that the co-allocated part-whole and identity partake not just of a taxonomic group but also of an ideological kind. In accordance with the previous chapters, this promotion of the group into a kind requires that the purpose of co-allocating the two relations is informed and constrained by a purpose familiar to scientific enquiry. The purpose we follow is that of grouping predicates according to the type of internal structure had by the things x and y that satisfy them, where by 'internal structure' we now mean the features appearing in the invariance principles: the order, repetition and nesting of y's constituents (which include x), as well as the fact that x alone is identical to the complex thing made out of it. Granted some important concessions on what it is to pursue aims 'informed and constrained' by scientific enquiry, it is attractive to view this aim as informed and constrained by the scientific one of classifying complex systems or complex mathematical objects of interest based on, respectively, the state-dependent and algebraic properties of their constituents.

Let us now zoom out. Where does this leave us in relation to our project of protecting hypotheses with canonical mereological resources (e.g. the predicate for mereological part-whole) against discard owing to abundance of ideology? In Chapter 8 I addressed a popular argument for discarding these hypotheses. The argument had it that some episodes of scientific theory choice favour hypotheses with overall less ideology, hence that we can 'export' the preference for sparser ideologies and apply it to our philosophical rivalry of interest. We replied that the scientific episodes underdetermine the reason to discard the ideologically rich hypotheses: it could be because they feature too many items of ideology or because they deploy too many ideological kinds. This reply shows that we cannot simply discard hypotheses owing to their exceeding ideology. We may still preserve ideologically abundant hypotheses if these, while abundant, minimise the number of deployed kinds.

This possibility led us into this final chapter. Here we claimed that hypotheses with abundant resources (i.e. the mereological part-whole predicate) will not be discarded if the policy to assess them is based on how many kinds they deploy. Their abundant resources, indeed, share a kind with a resource that rivals deploy: the identity predicate.

I had to be concessive on the argument that (resources for) mereological part-whole and identity partake of a single kind. Section 5.3 and this conclusion's summary illustrate the prices we should pay. However, these costs granted, the hope remains to exploit classification principles based on the Finean invariances for co-allocating part-whole and identity in such a way that their common group is also a common kind.

A forward-looking conclusion

It is time to take a bird-eye view and summarise our aim, the progress we have made so far and the open questions that either time or the dialectic we have followed prevented us to address.

Throughout this work we have assessed an attractive naturalistic project, consisting of two sections: 'appropriation' and an 'elimination' part. Advanced science can 'appropriate' the notion of part-whole from metaphysics in two ways. Firstly, it can deploy genuine part-whole concepts whose conceptual consequences differ from those canonically admitted in metaphysics (such as the Antisymmetry postulate). Secondly, it can posit entities of interest that relate as part to whole in a literal sense but in opposition to the same canonical principles (e.g. relating as a part to a distinct whole, which in turn relates to them as a part).

Appropriation can thus occur at the conceptual or at the worldly level. Either way, a successful appropriation guarantees that scientific part-whole is *sui generis* in a sense close to Ladyman and Ross': it partakes of the same family as the canonical notion we invoke in metaphysics, but it breaks the associated canon.

These observations prompted the search for candidate concepts and entities of scientific interest which break the canon in the above way. Respectively, our attention fell on the concept of component-sum and on its satisfiers: that is, vectors that comply with the vector space axioms and relate to other vectors as linear component to resultant. Having focused on this particular concept and these entities of interest (the concept's satisfiers), we argued that the concept is mereological and that the entities relate as part to whole in a literal, non-metaphoric sense.

We evidence this latter claim by observing that scientific practitioners couch component and resultant vectors in mereological terms (one is the 'component', the other the 'sum'). Yet why does the linguistic observation not evidence the *opposite* claim that this talk deploys a concept merely similar to a mereological one or posits entities that relate as part to whole only in a metaphoric sense? My proposal was that the observation supports the appropriation line granted a naturalistic premise about what concepts count as mereological and what entities relate as part to whole in a literal sense. According to this premise, *all things being equal* these theories should be informed and constrained by scientific practice.

Our use of the linguistic observation becomes more plausible once we do away with arguments aimed at showing that not all things are equal. These include (1) the argument that by classifying component-sum as literal part-whole we renounce a coherent theory of the part-whole relation and associated explanatory advantages; (2) the argument that we over-liberalise part-whole and potentially classify as such any relation clashing with the canon; and, finally (3), the argument that we create a mismatch in formal properties with paradigmatic cases of part-whole.

The arguments in (1)–(3) all found replies in earlier chapters, further supporting our use of the linguistic observation. For the purposes of this 'forward-looking' conclusion, however, let us emphasise an uncovered spot in the dialectic, which indicates the need for further research. The strategy to advocate our use of the linguistic evidence suffers from two types of responses: those that invoke 'blocker' arguments other than (1)–(3) and those that reject our naturalistic policy for handling the evidence. That we should be open to further blockers was conceded in section 5.2 of Chapter 3. As for the naturalistic policy, like elsewhere in this work (Chapter 9, Section 5), I rested content with describing it as an attractive method, which we should not expect to stand unrivalled. That scientific practitioners couch their component and resultant vectors in mereological terms seems to support (*ceteris paribus*) the appropriation claim that mereological concepts are deployed and that the posited entities relate as part to whole in a literal sense. However, one of this work's important concessions – a claim we have not explicitly argued against – is that opponents could still demand more than this surface linguistic evidence when it comes to figuring out (a) conditions under which concepts deployed in advanced science are the canon-breaking version of concepts deployed in metaphysics, and (b) conditions under which entities of scientific interest with deviant structure relate as part to whole in a literal sense.

Though not entirely unrivalled, the strategy we have used in arguing for appropriation deserves serious recognition, to the point that the naturalist can hope to embrace it to support a further step, and namely the second, 'eliminative' section of the program. This comes as the conjunction of two sub-claims: (1) advanced science deploys canon-breaking concepts but no concepts that comply with the canon; and (2) advanced science posits satisfiers of canon-breaking concepts but no satisfiers of concepts that comply to the canon.

To illustrate this we helped ourselves to our main canon-breaking concept: component-sum. Of the two above sub-claims, (2) obtains if advanced science posits entities that relate as component to sum but no things that relate as part to whole in agreement with the canon, e.g. no vectors understood as mereological atoms or no mereological sums of vectors. Claim (1), on the other hand, obtains if advanced science deploys the component-sum concept but no canonical concept of part-whole in addition to it, e.g. the classical mereological concept equipped with strict partial order axioms and Strong Supplementation.

Throughout the second part of the thesis, we have focused entirely on (1) and developed a strategy to resist it, thus making at least *some* of the needed steps to argue against elimination. We made, in particular, two such steps. First, we isolated a prominent way of arguing for (1), which should attract the advocates of 'elimination'. This argument has it that we should hold onto advanced hypotheses with only canon-breaking concepts owing to the sparseness of their ideology. In particular, the ideology's sparseness makes the hypotheses preferable to rivals that deploy the same concepts (component-sum) but in addition supply concepts that comply with the canon.

A simple case of one such ideologically profligate rival, which we labelled the 'atoms-only' view, posits vectors $\mathbf{v}_1$, ..., $\mathbf{v}_n$, states that some $\mathbf{v}_i$, $\mathbf{v}_j$ of these, taken pairwise, relate as component to sum and then, crucially, also states that each of the $\mathbf{v}_1$, ..., $\mathbf{v}_n$ is a mereological atom: an entity with no parts that is only part of itself ('part', here, expresses the canonical concept). Now, a successful argument from ideological profligacy, which we could use to defend (1), has it that we should discard the atoms-only view in favour of its ideologically sparse rival, which posits $\mathbf{v_1}, \ldots, \mathbf{v_n}$, states that some of the $\mathbf{v_1}, \ldots, \mathbf{v_n}$ taken pairwise relate as component to sum but *omits* any other statement to the effect that some of the $\mathbf{v_1}, \ldots, \mathbf{v_n}$ satisfies a canonical part-whole concept (including the statement that each $\mathbf{v_1}, \ldots, \mathbf{v_n}$ is a mereological atom).

Yet are arguments of this type successful? Not so easily. The second step we made against (1) has it that, under some circumstances, we can protect hypotheses with both canonical and deviant resources from the arguments that capitalise on their surplus canonical resources to discard the hypotheses and accept their rivals. I will omit the details of this safeguarding plan for profligate hypotheses, as three chapters already made them explicit (Chapter 8, 9 and 10). To refresh the overall picture, the hypotheses survive Sider's argument against surplus ideology *tout court*, because this suffers from a lacuna (Chapter 9, Section 2). Moreover, the hypotheses also survive an argument for penalising abundant ideological kinds, because the additional canonical concepts they come with are of a kind with concepts the rival already deploys (notably, identity).

Having summarised our progress, it will be helpful for the aims of this forward-looking conclusion to acknowledge rival strategies for achieving the 'elimination' plan, which owing to time or the pursued dialectics we had no chance to argue against.

In conclusion of the last chapter, we agreed that my co-allocation of identity and the surplus resources rests on attractive but undefended assumptions of method. In particular, the two concepts partake of the same ideological kind (not just the same taxonomic group) if the purpose of allocating them to the same group is informed and constrained by advanced scientific aims. Admittedly, I have not gone a long way towards addressing alternatives to this naturalistic method or towards defending my claim that the purpose of co-allocating the extra-resources and identity is 'informed and constrained' by some eminent aim of advanced science, rather than merely similar to it.

Second, I have argued that abundant hypotheses like the atoms-only view resist Sider's argument against additional ideology *tout court* because this contains a lacuna. I should concede, however, that resisting a particular argument is not enough for resisting discard owing to abundant ideology *tout court*, no matter how popular the argument. Admittedly, we should be open to the viability of arguments other than

Sider's, which conclude to discarding the atoms-only view and other ideologically abundant hypotheses. A place to retrieve these further arguments could be the area of statistical model selection (as conceded in Chapter 7, Section 3). Indeed, while usually invoked to prefer theories with sparser ontologies (not ideologies), e.g. less posits *tout court* or less instances of different kinds, it is not too difficult to expect this lively research program to yield an argument that ascribes the atoms-only view lower probability – even if the atoms-only view has just the same ontology as its ideologically sparser rival[1]. In light of these concessions, we shall make our claim that the atoms-only view resists pressure from ideological profligacy provisionally and with an eye to the areas counter-replies might come from, which deserve further investigation.

Third, there is one final respect in which our resistance to the 'elimination' program is just in its beginnings and demands further investigation. To approach this we recall that the program is the conjunction of two claims: (1) that advanced science deploys no canonical concept of part-whole and (2) that it posits no entities that relate as part to whole in agreement with the canonical principles (e.g. a vector **v** and a non-vector *s* that is the mereological sum of **v** and other vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$). Suppose that by applying the above strategies we managed to deny (1). Absent a parallel argument against (2), hypotheses whose posits relate as part to whole in agreement with the canon would still suffer discard[2]. Therefore, for a complete attack to the 'elimination' program we require a parallel argument against (2).

This argument could start with the premise that hypotheses with posits that relate as part to whole in agreement with the canon (e.g. $\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_n$ and *s*) contain additional posits compared to rivals whose posits ($\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_n$) relate only as component to sum

[1] For it to work against the atoms-only view and other ideologically rich hypotheses, this argument needs be accompanied by the claim that (all things equal) our preference should be for hypotheses with higher probability (prior or posterior). Throughout the thesis, I have assumed an epistemic connotation for preference (cf. Chapter 7, Section 1): preferring a hypothesis to a rival means to believe it to a greater extent then we believe the rival. Besides (Ib.), I have also assumed an epistemic role for parsimony. This means that according to the objectors who capitalise on profligacy arguments, preferring parsimonious rivals is all things equal a viable way of pursuing the aim of gaining true beliefs. Putting this together, for the envisaged argument from model selection to work against the atoms-only view and other ideologically rich hypotheses, it should be accompanied by the claim that (all things equal) we best practice the aim of gaining true beliefs by preferring hypotheses with higher (prior or posterior) probabilities.

[2] In section 2 of Chapter 6 I have argued that the atoms-only view is no such hypothesis, because it posits only vector space members ($\mathbf{v}_1, \ldots, \mathbf{v}_n$) and none of these, taken pairwise, relate as part to whole in agreement with the canonical principles. This means that even without a parallel argument for (2) we can still protect the atoms-only view from discard.

(for a defence of this premise, cf. Chapter 6, Section 1). Further, we will state that a prominent way of arguing for the latter hypotheses – hence satisfy (2) – is again Ockhamian and has it that we should discard hypotheses with surplus posits (*s*), because these increase either the overall number of posits, or the numbers of instanced kinds or fundamentals (cf. Chapter 7, Section 1). Finally, we respond to this Ockhamian challenge in one of the available ways (cf. Ib., Section 4.2): that is, either by showing that the extra posits (*s*) are "nothing over and above" the rivals' posits (**v**, $\mathbf{v_1}$, ..., $\mathbf{v_n}$) and do not make their hypotheses more profligate in the respect relevant for discard; or by showing that the extra posits make the hypotheses profligate in *some* respect (say, adding to the number of posits *tout court*), but not in the respect the challenge capitalises on.

I will limit myself to these rough guidelines, since the aim of this forward-looking conclusion is to indicate the directions we should further investigate to fulfil our defence of appropriation and denial of elimination[3]. As we have noted when it came to protecting the atoms-only view from the consequences of ideological abundance, the details of the safeguarding argument could take as long as three chapters and yet still leave us with methodological assumptions objectionable to some. My hope is, accordingly, that readers will agree on considering a more thorough defence of these assumptions a matter for another day.

[3] Similarly, the argument we have given against (2) and the argument we have sketched against (1) assume that the best way of arguing for elimination is by Ockhamian arguments. Thus, for example, in the arguers' intentions the atoms-only view fails because it is ideologically profligate and, similarly, hypotheses with posits that relate as part to whole in agreement with the canon fail because they are ontologically profligate. This means that, as things stand, we have only argued against one particular path to elimination (cf. Introduction and Chapter 8, Section 2). We should be open, as always, to the possibility that other paths will be viable. Even in this respect, there might be more steps yet to be made to reach our desired conclusion against elimination.

References

Aczel, P. (1988), *Non-Well-Founded Sets*, CSLI Publications.

Armstrong, D. (1978), *Universals and Scientific Realism*, vol I and II, Cambridge University Press.

- (1997), *A World of State of Affairs*, Cambridge University Press.

Aizawa, K., Gillett, C. (2017), “Introduction: Vertical Relations in Science, Philosophy and the World: Understanding the New Debated About Verticality”, In: Aizawa, K., Gillett, C. (eds), *Scientific Composition and Metaphysical Ground*, Springer, pp. 1-38.

Auyang, S. (1995), *How is Quantum Field Theory Possible?*, Oxford University Press

Axler, S. (1997), *Linear Algebra Done Right*, ed. 2014, Springer.

Bagaria, J. (2017), “Zermelo-Fraenkel Set Theory”, supplementary to Bagaria, J. (2017), “Set Theory”, In: Zalta, E. (ed), The Stanford Encyclopedia of Philosophy, ed. winter 2017, available at: <http://plato.stanford.edu/entires/set-theory/ZF.htm>

Bacon, A., Cotnoir, A. (2013), “Non-Wellfounded Mereology”, The Review of Symbolic Logic, vol. 5, n. 2, pp. 187 – 204.

Baker, A. (2003), “Quantitative Parsimony and Explanation”, British Journal for the Philosophy of Science, vol. 54, pp. 245 – 259.

Baker, A. (2017), “Simplicity”, In: Zalta, E. (ed) The Stanford Encyclopedia of Philosophy, ed. 2016 (Winter), available at: <https://plato.stanford.edu/archives/win2016/entries/simplicity>.

Balashov, Y. (2010), *Persistence and Spacetime*, Oxford University Press.

Barwise, J., Moss, L. (1996), *Vicious Circles: On the Mathematics of Non-Wellfounded Phenomena*, CSLI Publications.

Bealer, G. (1998), “Intuition and the Autonomy of Philosophy”, in De Paul, ., Ramsey, ., DePaul, Michael Raymond and William M. Ramsey (eds.), 1998, *Rethinking Intuition: The Psychology of Intuition and Its Role in Philosophical Inquiry*, Lanham, MD: Rowman and Littlefield.

Bealer, G. (2002), “Modal Epistemology and the Rationalist Renaissance”, in *Conceivability and Possibility*, Tamar Szabo Gendler and John Hawthorne (eds.), New York: Oxford University Press.

Beebee, H. (2000), “The Non-Governing Conception of Laws of Nature”, Philosophy and Phenomenological Research, vol. 61, n. 3, vol. 571 – 594.

Beisbart, C. (2009), “How to Fix Directions, Or: Are Assignments of Vector Characteristics Attributions of Intrinsic Properties?”, Dialectica, vol. 63, pp. 503 – 4.

Bennett, K. (2013), “Having a Part Twice Over”, Australasian Journal of Philosophy”, Vol. 91, n. 1., pp. 83 – 103.

Bennett, K. (2015), “Perfectly Understood, Unproblematic and Certain: Lewis on Mereology”, In: Loewer, B., Schaffer, J. (2015), *A Companion to David Lewis*, Wiley.

Bennett, K. (2017), *Making Things Up*, Oxford University Press.

Bigelow, J., Pargetter, R. (1990), *Science and Necessity*, Cambridge University Press.

Bird, A., Hawley, K. (2011), “What Are Natural Kinds?”, Philosophical Perspectives, vol. 25, n. 1, pp. 205 – 221.

Bird, A.,Tobin, E. (2017), “Natural Kinds”, In: Zalta, E. (ed), The Stanford Encyclopedia of Philosophy, ed. spring 2018, available at: <http://plato.stanford.edu/archives/spr2018/entries/natural-kinds>.

Blizard, W. (1991), “The Development of Multiset Theory”, Modern Logic, vol. 1, n. 4, pp. 319 – 352.

Blyth, T. S., Robertson, E. F. (2002), *Basic Linear Algebra*, Springer Undergraduate Mathematics Series, 2nd edition (2005), Springer.

Bohn, A. (2011), “Commentary on ‘Parts of Classes’”, Humana Mente: Journal of Philosophical Studies, vol. 19, pp. 151 – 158.

Bokulich, A. (2014), “Metaphysical Indeterminacy, Properties and Quantum Theory”, Res Philosophica, vol. 91, n. 3, pp. 449 – 475.

Boyd, R. (1999), “Homeostasis, Species and Higher Taxa”, in R. Wilson, (ed), Species: New Interdisciplinary Essays, MIT Press, pp. 141 – 186.

Braddon-Mitchell, D., Miller, K. (2016), “On Metaphysical Analysis”, In: Loewer, B., Schaffer, J. (eds), *A Companion to David Lewis*, Blackwell, pp. 40 – 59.

Braddon-Mitchell, D., Nora, R. (eds) (2009), *Conceptual Analysis and Philosophical Naturalism*, MIT Press.

Bradley, D. (2017), “Philosophers Should Prefer Simpler Theories”, Philosophical Studies, vol. 174, n. 1, pp. 1- 19.

Bradley, D. (unpublished), “Should We Favour Parsimonious Ontologies?”.

Calosi, C., Fano, V., Tarozzi, G. (2011), “Quantum Ontology and Extensional Mereology”, Foundations of Physics, vol. 41, n. 11, pp. 1740-1755.

Calosi, C., Morganti, M. (2016), “Humean Supervenience, Composition as Identity and Quantum Wholes”, Erkenntnis, vol. 81, n. 6, pp. 1173 – 1194.

Cantor, G. (1936), *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, Springer Collected Works in Mathematics, ed. 2012.

Cartwright, N. (1983), *How the Laws of Physics Lie*, Oxford University Press.

Casati, R., Varzi, A. (1999), *Parts and Places: The Structures of Spatial Representation*, Cambridge (MA): MIT Press.

Claeskens, G., Hjort, N. L. (2008), *Model Selection and Model Averaging*, Cambridge University Press.

Chakravartty, A (2017), *Scientific Ontology: Integrating Naturalized Metaphysics and Voluntarist Epistemology*, Oxford University Press.

Cotnoir, A. (2010), “Anti-symmetry and Non-Extensional Mereology”, Philosophical Quarterly, vol. 60, n. 239, pp. 396 – 405.

Cotnoir, A. (2013), “Composition as General Identity”, In: Bennett K., Zimmerman, D. (eds), Oxford Studies in Metaphysics, vol. 8, Oxford University Press, pp. 295 – 322.

Cowling, S. (2013), “Ideological Parsimony”, Synthese, vol. 190, pp. 3889 – 3908.

Creary, L. (1981), “Causal Explanation and the Reality of Natural Component Forces”, Pacific Philosophical Quarterly, vol. 62, pp. 148-157.

Curiel, E. (2014), “Classical Mechanics is Lagrangian; It is not Hamiltonian”, British Journal for the Philosophy of Science, vol. 65, n. 2, pp. 269 – 321.

Dasgupta, S. (2009), “Individuals: An Essay in Revisionary Metaphysics”, Philosophical Studies, vol. 135, n. 1, pp. 35 – 67.

Donnelly, D. (2011), “Using Mereological Principles to Support Metaphysics”, The Philosophical Quarterly, Vol. 61: n. 243, pp. 225 – 246.

Dowe, P. (2000), *Physical Causation*, Cambridge University Press.

Dupré, J. (1993), *The Disorder of Things: Metaphysical Foundations of the Disunity of Science*, Harvard University Press.

Effingham, N. (2010), “Mereological Explanation and Time Travel”, Australasian Journal of Philosophy, vol. 88, n. 2, pp. 333 – 345.

Effingham, N., Robson, J. (2007), “A Mereological Challenge to Endurantism”, Australasian Journal of Philosophy, vol. 85, vol. 4, pp. 633 – 640.

Gilmore, C. (2009), ‘Why Parthood Might Be a Four-Place Relation, and How It Behaves If It Is’, in L. Honnefelder *et al.* (eds.), *Unity and Time in Metaphysics*, Berlin: de Gruyter, pp. 83–133.

Fiddaman, M., Rodriguez-Pereyra, G. (2018), “The Razor and the Laser”, Analytic Philosophy, vol. 59, n. 2, pp. 1-18.

Fine, K. (1982) ‘Acts, Events, and Things’, in W. Leinfellner *et al.* (eds.), *Language and Ontology. Proceedings of the 6th International Wittgenstein Symposium*, Vienna: Hölder-Pichler-Tempsky, pp. 97–105.

Fine, K. (1999), “Things and Their Parts”, *Midwest Studies in Philosophy*, 23: 61–74.

Fine, K. (2003), “The Non-Identity of a Material Thing and Its Matter”, Mind, vol. 112, pp- 195 - 234.

Fine, K. (2010), ‘Towards a Theory of Part’, *Journal of Philosophy*, 107: 559–589.

French, S. (2014), The Structure of the World: Metaphysics and Representation, Oxford University Press.

French, S., McKenzie, K. (2012), “Thinking Outside the Toolbox: Towards a More Productive Engagement Between Metaphysics and Philosophy of Physics”, European Journal of Analytic Philosophy, vol. 1, pp. 42 – 59.

Forster, M., Sober, E. (1994), “How to Tell when Simpler, More Unified or Less Ad Hoc Theories Will Provide More Accurate Predictions”, British Journal for the Philosophy of Science, vol. 45, pp. 1 – 35.

Geroch, R. (1978), *General Relativity from A to B*, University of Chicago Press.

Goodman, N. (1951), *The Structure of Appearance*, Harvard University Press.

Gregory, R. (2006), *Classical Mehcanics: An Undergraduate Text*, Cambridge University Press.

Hacking, I. (1995), “The Looping Effects of Human Kinds”, In: Sperber, D., Premack, D., Premack, A. (eds), *Causal Cognition: A Multidisciplinary Debate*, Clarendon Press, pp. 35 – 94.

Hacking, I. (1999), *The Social Construction of What?*, Harvard University Press.

Hawley, K. (2006), ‘Principles of Composition and Criteria of Identity’, *Australasian Journal of Philosophy*, 84: 481–493.

Hawley, K. (2010), “Mereology, Modality and Magic”, Australasian Journal of Philosophy, vol. 88, n. 1, pp. 117 – 133.

Hawley, K. (2013), “Cut the Pie Any Way You Like? Cotnoir on General Identity”, *Oxford Studies in Metaphysics*, 8: 323–330.

Hawley, K., (2014), “Ontological Innocence”, In: Baxter, D., Cotnoir, A., Composition as Identity, *Oxford University Press*.

Healey, R. (2013), “Physical Composition”, Studies in History and Philosophy of Modern Physics, vol. 44, n. 1, pp. 48 – 62.

Healey, R. (2016), “Holism and Nonseparability in Physics”, In: Zalta, E. (ed), The Stanford Encyclopedia of Philosophy, ed. 2016 (Spring), available at <http://plato.stanford.edu/archives/spr2016/entries/physics-holism>.

Holmes, T., Forster, T., Liebert, T. (2012), “Alternative Set Theories”, In: Gabbay, D., Kanamori, A., Woods, J. (ed), *Handbook of the History of Logic: Sets and Extensions in the Twentieth Century*, Elsevier.

Hovda, P. (2009), ‘What Is Classical Mereology?’, *Journal of Philosophical Logic*, 38: 55–82.

Huemer, M. (2009), “When is Parsimony a Virtue?”, Philosophical Quarterly, vol. 59, n. 235, pp. 216 – 236.

Husserl, E. (1900-1/1913), *Logische Untersuchungen, II: Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, 2nd edition (1913).

Jackson, F. (1994), “Armchair Metaphysics”, In: O’Leary-Hawthorne, J., Michaelis, M. (eds), *Philosophy in Mind*, Kluwer Academic Publishers, pp- 23 – 42.

Janssen, M. (2002), “Reconsidering a Scientific Revolution: The Case of Einstein versus Lorentz”, Physics in Perspective, vol. 4., pp. 421 – 446.

Jansson, L., Tallant, J. (2017), “Quantitative Parsimony: Probably For the Better”, British Journal for the Philosophy of Science, vol. 68, n. 3, pp. 781 – 803.

Khalidi, S. (2010), “Interactive Kinds”, British Journal for the Philosophy of Science, vol. 61, pp. 335 – 360.

Kleinschmidt, S. (2014), “Introduction”, In: Kleinschdmidt, S., Mereology and Location, Oxford University Press, pp. xiii- xxxiii

Koslicki, K. (2007), *The Structure of Objects*, Oxford University Press.

Kovacs, D. (2014), “What Do We Want to Know When We Ask the Simple Question”, The Philosophical Quarterly, vol. 64, n. 255, pp. 254 – 266.

Kronz, F. M., Tiehen, J. T. (2002), “Emergence and Quantum Mechanics”, Philosophy of Science, vol. 69, n. 2, pp. 324 – 347.

Kunen, K. (1980), *Set Theory*: *An Introduction to Independence Proofs*, North-Holland, reprinted in 2011 as *Set Theory*, Studies in Logic: Mathematical Logic and Foundations, College Publications.

Ladyman, J. (2011), "The Scientistic Stance: The Empirical and Materialist Stances Reconciled", Synthese 178, vol. 1, pp. 87 – 98.

Ladyman, J. (unpublished), "An Apology of Naturalized Metaphysics", unpublished manuscript.

Ladyman, J., Ross, D. (et. al), *Every Thing Must Go: Metaphysics Naturalized*, Oxford University Press.

Lando, G. (2017), *Mereology: A Philosophical Introduction*, Bloomsbury.

Lejewski, C. (1951), "Review of Rescher (1955)", Journal of Symbolic Logic, vol. 22, pp. 213 – 214.

Leonard, H. S., Goodman, N. (1950), "The Calculus of Individuals and Its Uses", Journal of Symbolic Logic, vol. 5, pp. 45 – 55.

Lewis, D. (1973), *Counterfactuals*, Blackwell.

Lewis, D. (1986a), "Against Structural Universals", Australasian Journal of Philosophy, vol. 61, n. 1, pp- 25 – 46.

Lewis, D. (1986b), *On The Plurality of Worlds*, Blackwell.

Lewis, D. (1991), *Parts of Classes*, Blackwell.

Lewid, D. (1993), "Mathematics is Megethology", Philosophia Mathematica, vol. 1, n. 1, pp. 3 – 23.

Lewis, D. (1994), "Humean Supervenience Debugged", Mind, Vol. 103, n. 412, pp. 473 – 490.

Li, M., Vitanyi, P. (1997), *An Introduction to Kolmogorov Komplexity and its Applications*, Springer.

Lowe, J. (2006), *The Four-Category Ontology: A Metaphysical Foundation for Natural Science*, Clarendon Press.

Magnus, P. D. (2012), *Scientific Enquiry and Natural Kinds: From Planets to Mallards*, Palgrave MacMillan (New Directions in the Philosophy of Science).

Markosian, N. (2014), "A Spatial Approach to Mereology", In: Kleinschmidt, S. (eds), Mereology and Location, Oxford University Press, pp. 69 – 90.

Massin, O. (2009), "The Metaphysics of Forces", Dialectica, vol. 63, n. 4, pp. 555 – 589.

Maudlin, T. (1998), "Part and Whole in Quantum Mechanics", In: Castellani, E., *Interpreting Bodies*, Princeton University Press.

Maudlin, T. (2007), “Why Be Humean”, In: Maudlin, T., *The Metaphysics Within Physics*, Oxford University Press, pp. 50-77.

McDaniel, K. (2014), “Compositional Pluralism and Composition as Identity”, In: Baxter, A., Cotnoir, D. (eds), *Composition as Identity*, Oxford University Press.

McKenzie, K. (2012), *Physics Without Fudnamentality*, PhD Thesis, University of Leeds.

Melnyk, A. (2003), *A Physicalist Manifesto: Thoroughly Modern Materialism*, Cambridge University Press.

Mill, J. S. (1884), *A System of Logic*, Longman (London).

Millikan, R. G. (1999), “Historical and the Special Sciences”, Philosophical Studies, vol, 95, pp. 45 – 65.

Monton, B., Mohler, C. (2017), “Constructive Empiricism”, In: Zalta, E. (ed), The Stanford Encyclopedia of Philosophy, ed. summer 2017, available at: <https://plato.stanford.edu/archives/sum2017/entries/constructive-empiricism>.

Moore, G. E. (1939), “Proof of an External World”, Proceedings of the British Academy, vol. 25, pp. 273 – 300, reprint in Baldwin, T. (1993, ed.), *G. E. Moore: Selected Writings*, Routledge, pp. 147-70.

Morganti, M. (2013), *Combining Science and Metaphysics: Contemporary Physics, Conceptual Revision and Common Sense*, Palgrave McMillan.

Morganti, M., Tahko, T. (2017), “Moderately Naturalistic Metaphysics”, Synthese, vol. 194, n. 7, 2557 – 2580.

Mould, R. (2002), *Basic Relativity*, 1st. ed. 1994, Springer

Nambu, Y. (1985), *Quarks: Frontiers in Elementary Particle Physics*, World Scientific (en. tr. R. Yoshida).

Needham, P. (1981), “Temporal Intervals and Temporal Order”, Logique and Analyse, vol. 24, pp. 49 -. 64.

Ney, A. (2011), “Neo-Positivist Metaphysics”, Philosophical Studies, Vol. 160, n. 1, pp. 53 – 78.

North, J. (2009), “The “Structure” of Physics”, Journal of Philosophy, vol. 106, n. 2, pp- 57-88.

Papineau, D. (2014), “The Poverty of Conceptual Analysis”, In: Haug, M. (ed), Philosophical Methodology, London: Routledge.

Papineau, D. (2015), "Naturalism", In: Zalta, E. (ed), The Stanford Encyclopedia of Philosophy, ed. 2016 (Winter), available at: <https://plato.stanford.edu/archives/win2016/entries/naturalism/>.

Parsons, J. (2014), "The Many Primitives of Mereology", In: Kleinschmidt, S. (2014), Mereology and Location, Oxford University Press, pp. 3 -12.

Paul, L. (2012), "Metaphysics as Modelling: The Handmaiden's Tale", Philosophical Studies, vol. 160, pp. 1 – 29.

Perry, Z. (2015), "Properly Extensive Quantities", Philosophy of Science, vol. 82, n. 5, pp. 833 – 844.

Perry, Z. (2016), "Extensive and Intensive Quantities", In: Perry, Z. *Physical Quantities: Mereology and Dynamics*, PhD Thesis, New York University.

Quine, W. V. O. (1951), "Ontology and Ideology", Philosophical Studies, vol. 1, pp. 11-15.

Quine, W. V. O. (1953a), "On What there Is", In: Quine, W.V. O., *From a Logical Point of View*, ed. 1980, Harvard University Press.

Quine, W. V. O. (1953b), "Reference and Modality", In: Quine, W.V. O., *From a Logical Point of View*, ed. 1980, Harvard University Press.

Quine, W. V. O. (1960), *Word and Object*, MIT Press.

Quine, W. V. O. (1969), *Ontological Relativity and Other Essays*, Columbia University Press.

Redhead, M. (1982), "Quantum Field Theory for Philosophers", Proceedings of the Biennial Meeting of the Philosophy of Science Association, 1982, pp. 57 – 99.

Redhead, M. (1987), *Incompleteness, Locality, Realism: A Prolegomenon to the Philosophy of Quantum Mechanics*, Oxford University Press.

Rodriguez-Pereyra, G. (2002), *Resemblance Nominalism: A Solution to the Problem of Universals*, Oxford University Press.

Romeijin, J. (2014), "Philosophy of Statistics", The Standard Encyclopedia of Philosophy, ed. spring 2017, available at: <http://plato.stanford.edu/archives/spr2017/entries/statistics>.

Saatsi, J. (2017), "Explanation and Explanationism in Science and Metaphysics", In: Slater, M., Yudell, Z. (eds.), *Metaphysics and The Philosophy of Science*: New Essays, Oxford University Press.

Salmon, W. (1984), *Scientific Explanation and the Causal Structure of the World*, Princeton University Press.

Sattig, T. (2015), *The Double Lives of Objects: An Essay in the Metaphysics of the Ordinary World*, Oxford University Press.

Saucedo, R. (2011), "Parthood and Location", In: Bennett, K., Zimmermann, D. (eds.), Oxford Studies In Metaphysics, Volume 6: 223 – 284.

Schaffer, J. (2003), "Is There a Fundamental Level?", Nous, Vol. 37, pp. 498 – 517.

Schaffer, J. (2009), "On What Grounds What", In Chalmers, D., Manley, D., Wasserman, R. (eds), *Metametaphysics: New Essays on the Foundation of Ontology*, Oxford University Press, pp. 347 – 383.

Schaffer, J. (2015), "What Not to Multiply Without Necessity", Australasian Journal of Philosophy, vol. 93, pp. 644 – 64.

Schaffer, J. (2016), "Ground Rules: Lessons from Wilson", In Aizawa, K., Gillett, C. (eds), *Scientific Composition and Metaphysical Ground*, Palgrave Macmillan, pp. 143 – 170.

Sider, T. (2007), "Parthood", Philosophical Review, vol. 116, pp. 51 – 91.

Sider, T. (2009), "Ontological Realism", In: Chalmers, D., Manley, D., Wasserman, R. (eds), *Metametaphysics: New Essays on the Foundations of Ontology*, Oxford University Press, pp. 384 – 423.

Sider, T. (2011), *Writing the Book of the World*, Oxford University Press.

Sider, T. (2013), "Nihilism", In: Bennett, K., Zimmerman, D. (eds), Oxford Studies in Metaphysics, Volume 8, Oxford University Press, pp. 237 – 293.

Sider, T. (2015), "Nothing Over and Above", Grazer Philosophische Studien, vol. 91, pp. 191 – 216.

Simons, P. (1987), *Parts: A Study on Ontology*, Oxford University Press.

Simons, P. (2009), "Vectors and Beyond: Geometric Algebra and its Philosophical Significance", Dialectica, vol. 63, n. 4, pp. 381 – 395.

Shilov, G. E. (1977), *Linear Algebra*, English Translation by Richard Silverman, Dover.

Smid, J. (2015), "The Ontological Parsimony of Mereology", Philosophical Studies, vol. 172, vol. 12, pp. 3253 – 3271.

Smid, J. (2016), "What Does "Nothing Over And Above Its Parts" Actually Mean?", Philosophy Compass, vol. 12, n. 1.

Smid, J. (2017), "'Identity' as a Mereological Term", Synthese, Vol. 194, n. 7, pp. 2367 – 2385.

Sober, E. (2001), "What is the Problem of Simplicity?", In Zellner, A., Keuzenkamp, H., McAleer, M. (eds*), Simplicity, Inference and Modelling: Keeping it Sophisticatedly Simple*, Cambridge University Press.

Sober, E. (2006), "Parsimony", in Sarkar, S., Pfeifer, J. (eds), *The Philosophy of Science: An Encyclopedia*, Routledge.

Sober, E. (2009), "Parsimony Arguments In Science and Philosophy: A Test Case for Naturalism P", Proceedings and Addresses of the American Philosophical Association, vol. 83, n. 2, pp. 117 – 155.

Sober, E. (2015), *Ockham's Razors: A User's Manual*, Cambridge University Press.

Swinburne, R. (1997), *Simplicity As Evidence of Truth*, Marquette University Press.

Taylor, J. (2005), *Classical Mechanics*, University Science Books.

Tillman, C., Fowler, G. (2012), "Propositions and Parthood: The Universe and Antisymmetry", Australasian Journal of Philosophy, vol. 90, n. 3, pp. 525 – 539.

Teller, P. (1986), "Relational Holism and Quantum Mecahnics", British Journal for the Philosophy of Science, vol. 37, n. 1, pp. 71-81.

Thalos, M. (1999), "Degrees of Freedom: An Essay On Competitions Between Micro and Macro in Mechanics", Philosophy and Phenomenological Research, vol. 59, pp. 1 – 39.

Thomson, J. J. (1998), "The Statue and the Clay", Noûs, Vol. 32, n. 2 pp. 149 – 173.

van Inwagen, P. (1981), "The Doctrine of Arbitrary Undetached Parts", Pacific Philosophical Quarterly, vol. 62, p. 123 – 137.

van Inwagen, P. (1990), *Material Beings*, Cornell University Press.

Van Fraassen, B. (1980), *The Scientific Image*, Oxford University Press.

Varzi, A. (2007), Spatial Reasoning and Ontology: Parts, Wholes, and Locations', in M. Aiello *et al.*(eds.), *Handbook of Spatial Logics*, Berlin: Springer, pp. 945–1038.

Varzi, A. (2008), 'The Extensionality of Parthood and Composition', *The Philosophical Quarterly*, 58: 108–133.

Varzi, A. (2014), "Counting and Countenancing", In: Baxter, D., Cotnoir, A., *Composition as Identity*, Oxford University Press.

Varzi, A. (2015), "Mereology", In: Zalta, E. (ed.), The Stanford Encyclopedia of Philosophy, ed. 2016 (Winter), available at: <https://plato.stanford.edu/archives/win2016/entries/mereology/>.

Williams, G. R. J. (2004), *The Inscrutability of Reference*, PhD Thesis, University of St. Andrews.

Williams, G. R. J. (2006), "Illusions of Gunk", Philosophical Perspectives, vol. 20, n. 1, pp. 493 – 513.

Williams, G. R. J. (2007), "Eligibility and Inscrutability", Philosophical Review, vol. 116, pp. 361 – 399.

Wilson, M. (2008), “Beware of the Blob: Cautions for Would-Be Metaphysicians”, Zimmerman, D. (ed), Oxford Studies in Metaphysics, pp. 42 – 75 (2008).

Wilson, J. (2009), “The Causal Argument Against Component Forces”, Dialectica, vol. 63, n. 4, pp. 525-524.

Wilson, J. (2010), “Nonreductive Physicalism and Degrees of Freedom”, The British Journal for the Philosophy of Science, vol. 61, pp. 279 – 311.

Wilson, J. (2014), “No Work for a Theory of Grounding”, Inquiry: An Interidsciplinary Journal of Philosophy, vol. 57, n. 5-6, pp. 535 – 579.

Wilson J. (2017), “The Unity and Priority Arguments for Grounding”, In: Aizawa, K., Gillett, C. (eds), *Scientific Composition and Metaphysical Ground*, Springer, pp. 171-204.

Wolff, J. (2016), “Spin as a Determinable”, Topoi, vol. 34, n. 2, pp- 379 – 386.

Yablo, S. (1998), “Does Ontology Rest on a Mistake?”, Aristotelian Society Supplementary Volume, vol. 72, n. 1, pp. 229 – 283.

Yi, B. (1999), “Is Mereology Ontologically Innocent?”, Philosophical Studies, vol. 93, n. 2, pp. 141 – 160.