GEOMETRICAL MODELLING AND GRAPHICS DISPLAY OF STRATIGRAPHIC OREBODIES

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TO MY MOTHER, BROTHER and SISTER

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Table of Contents

Abstract iii

Chapter One 1
1.1 Introduction 1
1.2 Review of previous methods 4
1.2.1 Polygonal methods 4
1.2.2 Moving average 5
1.2.3 Weighted moving average 5
1.2.4 Directional search algorithms 6
1.2.5 Trend surface analysis 6
1.2.6 Local polynomial methods 6
1.3 Geostatistical methods 7
1.3.1 Theory of regionalized variables 8
1.3.2 Variogram 11
1.3.3 Models of the variogram 13
1.3.3.1 The spherical model 14
1.3.3.2 The De Wijsian or logarithmic Model 16
1.3.3.3 The exponential model 16
1.3.3.4 The linear model 18
1.3.3.5 The hole effect model 18
1.3.3.6 Kriging 18
1.4 Scope of the present work 23
1.5 Summary 29

Chapter Two 30
2.1 Introduction 30
2.2 Morphological Features 33
2.3 Data Acquisition 38
2.4 Data configuration 40
2.5 Geostatistical Estimates 46
2.6 Summary 60

Chapter Three **61**
3.1 Introduction 61
3.2 Surface Modelling Techniques 63
3.2.1 Introduction 63
3.2.2 Global versus local interpolation techniques. 63
3.2.3 Spline interpolation techniques 65
3.2.3.1 Introduction 65
3.2.3.2 Triangulation 66
3.2.3.3 Bivariate interpolation over triangular patches 70

3.2.4 Adopted modelling techniques 79
3.2.4.1 Introduction 79
3.2.4.2 Bilinear interpolation 79
3.2.4.3 Bivariate interpolation "Quintic scheme" 80
3.3 Application 83
3.3.1 Introduction 83
3.3.2 Data 83
3.3.3 Results 84
3.4 Summary 99

Chapter Four 100
4.1 Introduction 100
4.2 Graphics facilities 102
4.3 Graphics Display Software 105
4.3.1 GINO-F Graphics Routines 109
4.3.2 Applications Program 111
4.3.2.1 DAT_2D Module 112
4.3.2.2 DAT_3D Module 124
4.3.2.3 GRA_2D Module 127
4.3.2.4 GRA_23D Module 133
4.3.2.5 GRAMOD Module 143
4.3.2.6 GRACON Module 144
4.3.2.7 GRATRI Module 144
4.4 Summary 149

Chapter Five 150
5.1 Summary 150
5.2 Research results 154
5.3 Scope for future work 155

Appendix 158

1 Introduction 158

2 Basic Assumptions 158

3 Coordinate system associated with the triangle 160

4 Implementation of the third assumption 164

5 Determination of the coefficients of the polynomial 167

6 Step by step description of the procedure 171

Bibliography and references 172

GEOMETRICAL MODELLING AND GRAPHICS DISPLAY OF STRATIGRAPHIC OREBODIES

Abstract

In this research project the author introduces the use of geometrical modelling techniques alongside geostatistical methods to model a stratigraphic orebody and to present a graphics display system developed as a first step towards a general integrated system for computer aided design and planning in mining.

Geometrical modelling techniques and geostatistical methods are combined to carry out the process of modelling a stratigraphic orebody. From a mining point of view, there are two main features of interest in a stratigraphic orebody:

a- The modelling of the geometry of the orebody.

b- The modelling (estimation) of the physical properties (grades, etc...) of the orebody.

The first feature is the subject of this research project. Modelling methods and techniques developed elsewhere and for different applications, such as Computer Aided Design, have been applied successfully to model the geometry of stratigraphic orebodies. The modelling process consists of the applications of surface modelling techniques to represent the hangingwall and the footwall of the stratigraphic orebody and thereby to produce the space where the physical

properties are geostatistically to be estimated.

The graphics display system is presented to highlight the use of computer graphics techniques to communicate graphically all sorts of information concerning the modelling of stratigraphic orebodies and also to display the end product of the modelling process, such as cross-sections, plane-sections, wireframe and solid models of the orebody. The graphics system itself is part of a computer based system for mine design and planning similar to computer aided design systems used mainly in the manufacturing industry.

The presentation of the research project in this thesis started by the review of the literature of some existing ore reserves estimation methods in the mineral industry, particularly geostatistical methods. Then an overview and the scope of this research project have been given. The second chapter describes the type of data which could be encountered while building a geometrical model of a stratigraphic orebody and a description of data from a nickel vein deposit used as a case study for this research project. The accumulations have been estimated geostatistically subject to geometrical control. The geometrical control concept and surface modelling techniques are presented in chapter three together with the numerical application of modelling a nickel vein deposit using two different surface modelling techniques. Chapter four describes the graphics display system developed to display several geometric features of stratigraphic orebodies in two and three dimensions. The summary of this research project and some concluding remarks are given in chapter five.

CHAPTER ONE

Chapter One

Literature Review

1.1 Introduction

The purpose of this chapter is to review some existing methods in terrain and surface modelling techniques which can be used along-side geostatistical methods to study and to model spatial data and to evaluate natural resources particularly in the mining industry. A brief review of geostatistical methods and the scope of the present research project are presented.

This chapter is also an attempt to highlight the importance of using information technology techniques in the mining industry and in particular the application of computers and mathematics in different stages of evaluation, planning and mining operations of an orebody.

The driving force behind the implementation of these new tools and approaches is that the world's mineral industry is today operating in an international economy in which minerals and mineral products will tend to be produced where they can be produced the cheapest. Productivity and cost requirements are not only crucial but often a matter of survival. By applying new technologies mineral producers are adapting to this competitive world economy. The mining industry was long considered a tailer among the industrial users of information systems. It is reasonable to say that this is no longer the

case and that the use of information technology in mining has now reached a level of sophistication comparable to that found in other manufacturing companies of comparable size (Rendu 1989). In the mineral industry, computer technology has been applied for over 30 years, scores of successes and failures have been reported in conferences and in the literature (Weiss 1986). However, future advances in computer technology will take place at an even faster rate than in the past, concentrating on three major areas of interest: storage, computational power and graphics display. Storage technology has improved continuously throughout the years. In the coming years the laser disk technology will reduce by at least one hundred times the cost of storing fixed or archived data for high speed retrieval. Such a change will make it possible to store the geological, mining and land management information for a project on a single active disk (Khosrow 1984). Computational power will definitely move towards the user of transputer chips for parallel processing. Display of numerical results and graphics pictures has taken a great step forward, graphics terminals have improved in resolution, speed and price. New terminals are becoming available at very low prices which will make them affordable to small and large companies. However, despite their power, computers still are limited by the software that drives them. This is why the development of such software is of prime importance to the success of information technology otherwise a sophisticated computer system will be nothing more than a lifeless toy. Unlike other areas, mining software is specialized, and generalized software written by non-mining specialists cannot solve all the problems of mining. It used to be the case that mining software was developed by individuals such as mining engineers and

geologists but the large growth of computer power and applications has necessitated the team approach to developing software with adequate mining professionals to guide the applications and the evaluation of the results. In addition, while implementing these tools (computer technology) to solve the complex problems of the mining industry, it is of great importance that the mining industry takes advantage of the experience of other industries and sectors involved in using information technology. The mining industry has to develop its own software systems ranging from database systems for its huge amount of information to expert systems for the complexity and multidimensional aspect of its problems. It should avoid ready-made systems (black boxes) designed in most cases by non-specialists in mining and acquire the basics of these new approaches and adapt them to the reality of its problems. Mathematical methods and their implementation to solve mining problems will always be the main source of building solid tools using computer technology (i.e, Geostatistics, Operations Research, ...).

Geostatistics, since its appearance in the sixties, has contributed significantly to better understanding of spatial phenomena and is regarded as the best method applied to estimate natural reserves. It is concerned with the study of the distribution (location) in space of useful values for mining engineers and geologists such as grade, thicknesses and other mineral properties including a most important practical application to the problems arising in ore deposits evaluation. Other methods dealing with spatial data analysis have also been used as mathematical tools for natural reserves estimation. Operations research methods are playing a major role in formulating solutions to mine planning and operational problems.

These methods and means have been applied successfully to mining and processing operations for reserve estimations, orebody modelling, mining design and operation, control of production rates (quality and quantity). However, the rapid rate of advancement associated with these methods and means is imposing on users a great burden of keeping in touch with new products and methods and of being able to implement them for better results.

1.2 Review of previous methods

One of the problems which has faced geologists and other scientists in spatial data analysis (modelling and evaluation) in related disciplines for many years is that of defining quantitatively the forms of surfaces and volumes which have been sampled only at discrete points. Many techniques have been devised in attempts to solve this general problem, or to solve restricted subsets of it. (Henley 1981). Some of these methods, known as modelling techniques, are mentioned in chapter three, however, the purpose of this section is to review the existing methods used mainly for reserve calculations in mining, such as polygonal methods, moving average, trend surface analysis, weighted moving average, etc.... Some of these methods have a theoretical basis, some are developed only on empirical grounds. A brief survey of these methods is given in Henley 1981. Some of these methods are quoted here is this section.

1.2.1 Polygonal methods

The method of calculation by polygons is, in most cases, used with drill-hole data, where polygons may be constructed on planes, cross-sections or longitudinal sections. This method is mainly applied to grade estimates rather than surface representation because of the discontinuity between the adjacent

polygons. In this method, the average grade of mineralisation defined by the sample point within the area contained by the polygon is considered to be accurately representing the grade of the entire volume of material within the polygon. The method assumes that the area of influence of any sample extends halfway to the adjacent sample point is a specific area controlled by the set of adjacent sample points set to one and all others set to zero. This method produces highly biased estimates (Royle 1978).

1.2.2 Moving average

This method is based on moving a search area across the data point locations and the value of the central location is estimated by the simple arithmetic mean of the values of all the points lying within this search area. Thus the weights of the values of all the points contained in the search area are equal to 1/n (n: number of sample points in the search area) and the weights at all other data points are set to zero. This method also gives a discontinuous estimated surface and gives biased estimates (Royle 1978). It has, however, been extensively used in the mining industry for ore reserves estimation and is still in use in some applications such as for spatial analysis of geochemical exploration data (Henley 1981).

1.2.3 Weighted moving average

This method is another version of the moving average method. The weights given to the values at data points in the search area are a function of the distance between the centre point location (point location at which estimate is to be made) and each data point within the search area. Weighting functions which are commonly used include inverse distance ($\frac{1}{d}$), inverse square distance ($\frac{1}{d^2}$), other inverse powers of distance ($\frac{1}{d^c}$), linear, decreasing to zero at the

edge of the search area and others. The technique is simple and cheap to compute and can be effective as it takes into account partial influence (distance) of the sample point in the estimation of the value at the central point. Its drawbacks are mainly its sensitivity to changes in sample spacing and to any anomalies in the data values (Henley 1981).

1.2.4 Directional search algorithms

These methods also belong to the class of weighted average techniques. They are attempts to overcome the problem of clustered and irregularly distributed data points. Methods like these impose directional control and limit the number of data points used in different directions away from the point to be estimated. However, they do not escape the bias associated with weighted average methods (Henley 1981).

1.2.5 Trend surface analysis

These types of methods are used as global methods in surface modelling techniques (see chapter 3). They consist mainly of providing a best fit to the data points by chosing an optimum set of coefficients to produce the lowest sum of squared deviations of the data points from the estimated surface. The method is more suitable for known trends and phenomena represented by the data but it is very sensitive to anomalous values in the data set.

1.2.6 Local polynomial methods

Some of these techniques, based on local procedures, are mentioned in chapter three with the application of two of them for geometrical control of geostatistical estimates. They perform very well with well-behaved data points but some of them are sensitive to noisy data point configurations. Some of

these techniques can produce smooth surfaces and they are widely used in commercial packages of geometrical modelling and computer graphics; they are a subject of intensive research in many disciplines involved mainly with control, object design and presentation and graphics display.

A widely used method for some time, in mining is the triangle method which consists of constructing a series of triangles using the drillhole intersections with the surface as vertices. This method has the advantage in that the three points of each triangle are considered to fit a plane and the estimated points lie on that plane. The area is easily calculated and therefore tonnage (volume) is easily obtained. These types of methods, based on local procedures, are more suitable for geometrical modelling of orebodies. They perform best when the data are uniformly distributed over the whole surface to be modelled and they do not take account of any trend in that surface.

1.3 Geostatistical methods

Geostatistics as an ore reserve estimation method emerged to be one of the more robust techniques for evaluation of mining reserves and later to be adopted in many applications outside mining. It was developed in the early sixties based on the original studies by Krige in South Africa. The technique is actually part of an entirely independent scientific field concerned with the study of continuous domains. The methodology is based on the theory of regionalized variables and employs the estimation technique known as kriging. It has become widely applied throughout the world as an accepted method not only for the assessment of ore deposits but for many applications in other fields such as cartography, civil engineering, forestry, meteorology, ..., in other words it is a method which could be used for the estimation and modelling of many phenom-

ena found in continuous domains.

It is not possible within this discussion to give a comprehensive treatment of the theory. The objective is to develop an overview of the techniques and to provide some preliminary understanding of them. Conventional methods, mentioned above, and classical statistics have not in general given good results when applied to the estimation of ore reserves. Geostatistical techniques, with careful attention paid to the data at hand, will provide not only a good total reserve estimate, but also a more reliable block by block reserve inventory. Geostatistical methods do not increase the quality of basic sample information available, nor they can improve the quality or the accuracy of the basic assay value. However, geostatistics, properly applied, does derive from the raw data the best possible estimates of orebody parameters. Unlike classical statistical approaches, geostatistics recognizes that samples in ore deposits are usually spatially correlated with one another and that closer samples will not, in most cases, be independent.

1.3.1 Theory of regionalized variables

The difference between an ordinary random variable and a variable associated with its spatial location and distribution is the basis of the theory of regionalized variables. The regionalized variable is a function taking a definite value in each point in space. In general, such a function has properties too complex to be studied easily through common methods of mathematical analysis. However, from a geological point of view this function is associated with a number of characteristics (Matheron 1963).

a - The first characteristic is that the regionalized variable is localized: it is

taken or observed in a mineralized space or other spaces of interest. This variable is usually defined on a geometrical support. In the case of ore-grade the support is the volume of the sample with its geometrical shape and size.

b - The second characteristic of a regionalized variable is the apparent continuity in its spatial variation as seen as the discrepancy between the grades of two neighbouring samples. The degree of continuity varies from one type of variable to another for example the degree of variability between thicknesses and grade. In most cases, thicknesses are associated with higher degrees of continuity than grades or accumulations.

c - The third characteristic of a regionalized variable is anisotropy in the fact that the degree of variation between sample values in the space occupied by the variable may depend on direction.

These characteristics of a regionalized variable indicate clearly the interdependence between the localized values of such variables. It is thus likely that values recorded at close locations will be more similar than values recorded at widely spaced locations. If two observations z_1 and z_2 recorded at two locations x_1 and x_2 ($x_2 = x_1 + h$), are separated by a distance h and if the total domain which contains z_1 and z_2 has the same mean values everywhere $E[Z(x)] = m$ (m= mean) then the following expression holds

$$E[Z(x) - Z(x+h)] = 0$$

This equation defines stationarity of the first order showing that the variable has the same mean value m regardless of the location of the variable values.

The expected value of the cross product of the deviation of two observations from that mean is the covariance between these two observations. This may expressed as

$$E[\{Z(x)-m\}\{Z(x+h)-m\}] = K(h)$$

where

K(h) is the covariance.

If it is assumed that for every h, this covariance is independent of the locations x_1 and x_2, then second order stationarity holds. The covariance K(h), as h tends to zero, then approaches the variance of the variable Z:

$$K(0) = E[\{Z(x)-m\}\{Z(x)-m\}] = VAR[Z(x)]$$

In practice the mean value m is always unknown, and may not even be constant. Thus the covariance and the variance cannot be computed and an alternative definition is required which does not require the mean value m. Although the expected value of (Z(x) - Z(x+h)) may be zero, the expected value of the square of this difference is not necessarily zero. The assumption required for overcoming the problem of the unknown mean of the variable can be found in many text books on geostatistics. The simplest formulation is quoted here from Henley (1981) and the assumptions are known as the intrinsic hypothesis:

$$E[Z(x)-Z(x+h)] = 0$$

$$E[\{Z(x)-Z(x+h)\}^2] = VAR[Z(x)-Z(x+h)] = 2\gamma(h)$$

Where $\gamma(h)$ is known as the semivariogram. However, a considerable number of authors call it the variogram and this is the term which will be used here.

1.3.2 Variogram

The computation of the experimental variogram of a regionalized variable is the first step in any geostatistical estimation of this variable. It is defined as one-half of the variance of the differences in sample grades between points separated by a distance h. Variogram functions are used in all subsequent phases of estimation, including kriging, and they are also of valuable assistance in quantitatively defining the traditional concept of area of influence of a sample within a particular deposit. Moreover, they can be used in determining the optimum drillhole spacing to define the reserves at a particular level of confidence. The mathematical formulation of an experimental variogram function is as follows:

$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n} [Z(x) - Z(x+h)]^2$$

where Z(x) is the value of the regionalized variable at a point x, Z(x+h) is the value at another location at a distance h from the point x and n is the number of sample pairs. The sample pairs are each oriented in the same direction and are of equal volume (support). It is possible to compute γ for other spacings by taking alternate sample values along one direction (e.g., h_i = i|h|) and $\gamma(hi)$ is computed in exactly the same way using the n_i possible pairs which are separated by the distance of h_i. The process is repeated for (i=1,k) and the $\gamma(hi)$ are then plotted against the corresponding distances h_i. The resulting graph is known as the experimental variogram, because it is based only on sample values; it is

assumed to represent the true underlying variogram of the deposit. It is possible also to compute $\gamma(h)$ independently for different geographic directions if it is suspected that the data are anisotropic, but the same computational method is used. However, if the data locations are randomly distributed in space occupied by the mineralisation, it may not be possible to find a sufficient number of pairs with exactly the same spacing to compute $\gamma(h)$. To overcome this situation a tolerance or a range of h values must be introduced such as Δh then the distance is expressed as $(h \pm \Delta h)$; all pairs of sample values separated by more than $(h - \Delta h)$ or less than $(h + \Delta h)$ are used to compute the variogram. The same idea of tolerance may also be used in computing directional γ values by using all pairs of samples separated by $(h \pm \Delta h)$ and within a specified angular range. In practice, to obtain a reliable variogram representing the variography of certain regionalized variables, it should be computed with sufficient data points and within homogeneous zones and avoiding, whenever possible, the computation of a single variogram for discontinuous zones.

Commonly, values of the experimental variogram increase with increasing distance of the lag interval h and reach a certain limit at which the variogram becomes more stable and fluctuates around a value known as the sill. The distance at which the variogram reaches the sill is called the range (range of influence). This reflects the classical geological concept of an area of influence. Beyond this distance, sample pairs no longer correlate with one another and become independent. The sill is theoretically equal to the variance of all samples used to compute the variogram. The difference between the sill value and the value of the variogram at any given distance h can be seen as the degree of dependence between samples separated by the distance h. As h approaches the range " a", the degree of correlation between sample values

becomes weaker so that the increase of independence between samples separated by such relative large distance ($h \geq a$).

Another characteristic of the variogram is its value at the origin (h = 0) called the nugget variance. It expresses the local random nature of the mineralisation. Theoretically this value must be zero. However, the variogram may attain significant values for very small distances h thus producing an apparent discontinuity at the origin. High nugget variance values, relative to the sill, can indicate either that the mineralisation is poorly disseminated, or the zone on which the variogram was calculated does not take account of structural discontinuities which exist in the deposit, or that poor sample preparation and assaying procedures were used.

1.3.3 Models of the variogram

The experimental variogram computed from the data tends to be quite erratic especially for noisy data. There are many numerical models which can be fitted to the experimental variogram. These models are defined as simple mathematical functions which relate variogram values γ to distances h, commonly known as the lag intervals. These models are used in subsequent estimation methods in place of the experimental variogram calculated directly from the data.

The process of fitting a theoretical variogram model to an experimental variogram is one of the most important steps in any geostatistical study. Most experimental variograms can be described by one of very few well-known theoretical models, and there is not usually any need for devising new models. There are three features of the experimental variogram which guide practitioners in the fitting of a theoretical variogram model:

a - The presence or absence of the sill in the experimental variogram.

b - The behaviour of the experimental variogram at or near the origin, referred as the nugget effect.

c - The existence or otherwise of a range.

Four theoretical variogram models often used in geostatistical studies are the spherical model, the exponential model, the linear model and the logarithmic (or DeWijsian) model. The spherical model is the one most often used.

1.3.3.1 The spherical model

This model applies in most deposits in which the values of the regionalized variable become independent of each other beyond a certain distance (a = range). It is the most common model in most sedimentary deposits and also in porphyry copper deposits (David 1977). The spherical model is shown in figure 1.1.

The equation of the spherical model is:

$$\gamma(h) = C\left[\frac{3h}{2a} - \frac{h^3}{2a^3}\right] + C_0 \quad for \quad h \leq a$$

$$\gamma(h) = C + C_0 \quad for \quad h > a$$

where

C_0 : is nugget variance

$C_0 + C$: is sill

a : is range

Spherical model

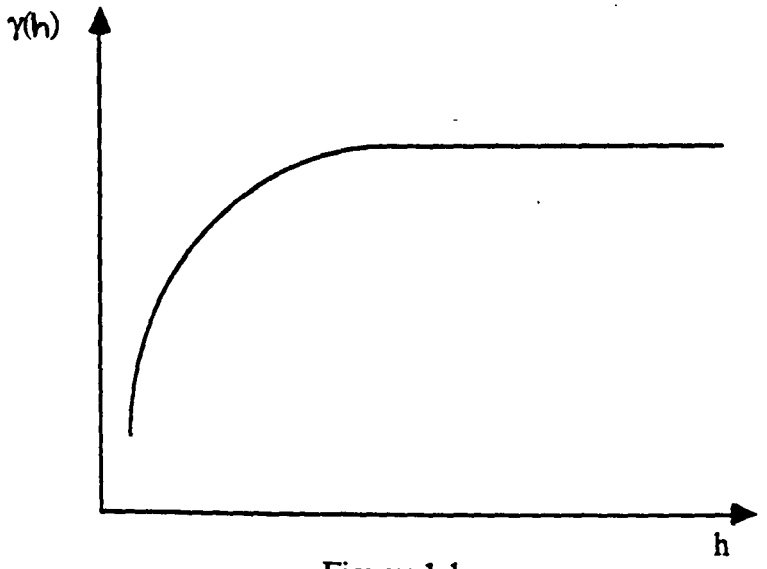


Figure: 1.1

De Wijsian model

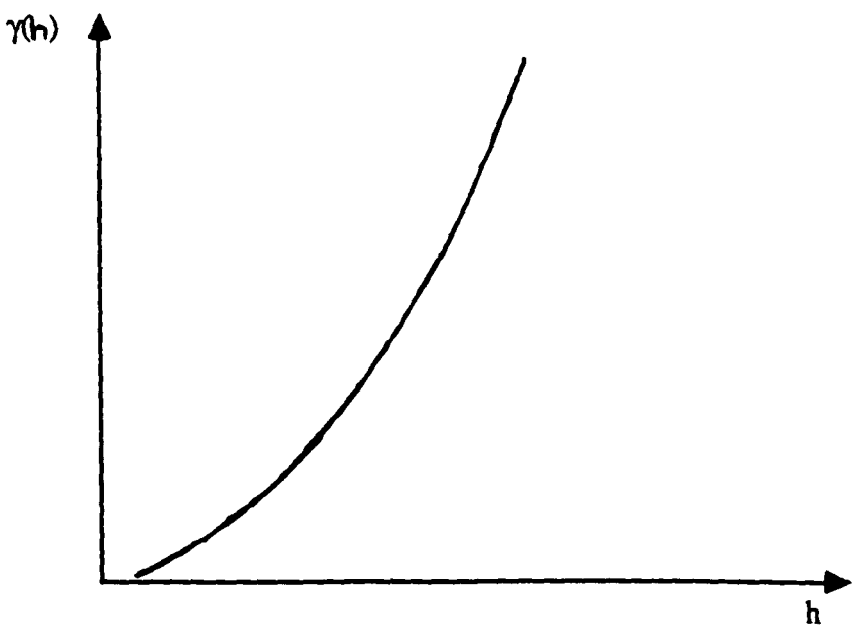


Figure: 1.2

1.3.3.2 The De Wijsian or logarithmic Model

This type of model is encountered in cases where there is no apparent sill and therefore an undefined range of influence: the variogram values increase without limit as the separation (h) between pair values increases. In most cases this model provides a poor fit to the data and has been largely replaced by several other models now available (Henley 1981). These models are widely used in earlier geostatistical works and they were very popular as the first geostatistical applications were carried out on deposits for which variograms of variables such as gold, uranium had no apparent sill ($C + C_0$) (Journel 1978). This model is shown in figure 1.2 and its equation is:

$$\gamma(h) = A\ln(h) + B$$

The De Wijsian variogram model is linear if plotted against the logarithm values of distances (h).

1.3.3.3 The exponential model

The exponential model in not encountered very often in mining practice. It rises more rapidly at the origin but reaches the sill ($C + C_0$) more gradually than the spherical model (David 1977). This model is shown in figure 1.3 and its equation is of the form:

$$\gamma(h) = C\left[1 - \exp^{\left(\frac{-|h|}{a}\right)}\right] + C_0$$

Exponential model

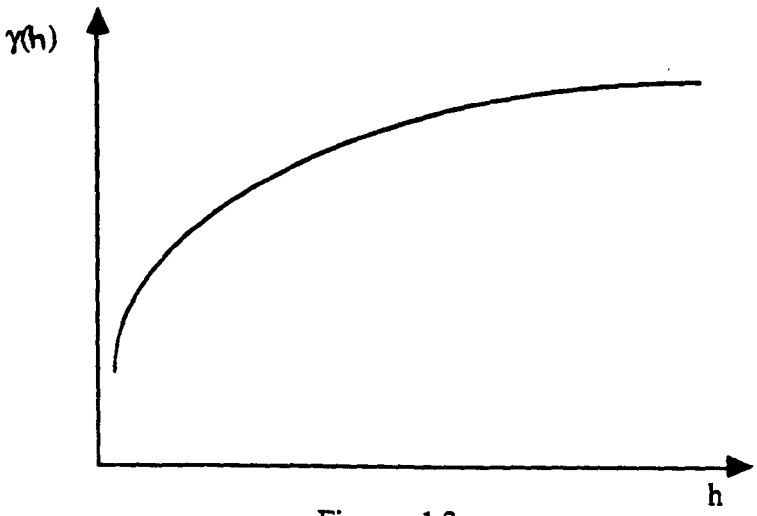


Figure: 1.3

Hole effect model

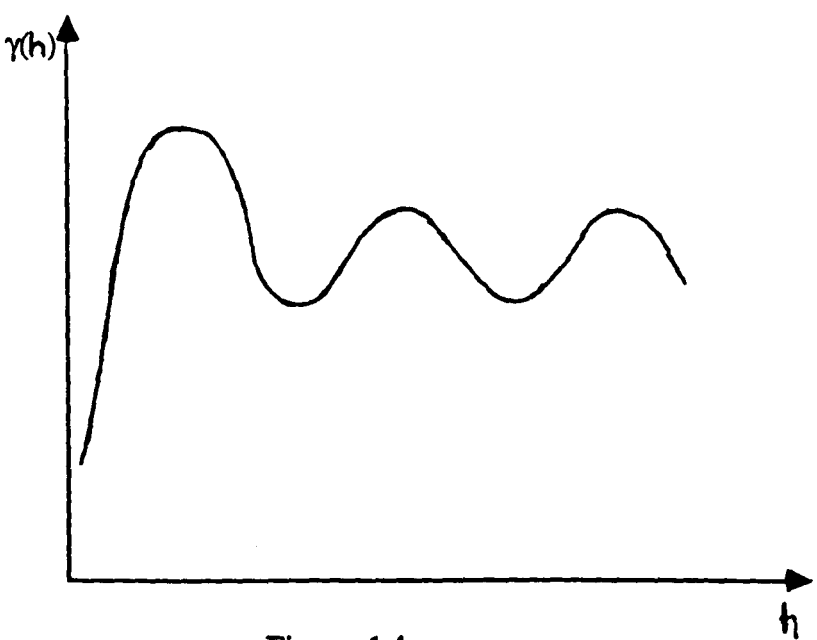


Figure: 1.4

1.3.3.4 The linear model

This is the simplest model to use if the experimental variogram shows a strong linearity in its behaviour. The equation of this model is:

$$\gamma(h) = Ah + B$$

It is quite often possible to use the linear model within a limited neighbourhood (David 1977).

1.3.3.5 The hole effect model

In this model the variogram does not increase monotonically with distance but oscillates according to the following equation:

$$\gamma(h) = C\left[1 - \frac{\sin(ah)}{ah}\right]$$

The model can be used to represent fairly continuous processes; the periodic behaviour is often encountered where there is a succession of rich and poor zones (David 1977). Usually the model reflects a violation of one or more assumptions, such as zonal anisotropy, or a trend or a drift in the mineralisation (Henley 1981). This model is shown in figure 1.4.

1.3.3.6 Kriging

Kriging is a geostatistical interpolation method which calculates the estimated values of the regionalized variable as a linear combination of the nearest values. The coefficients of such a linear combination are obtained from the

variogram model and the method will yield the linear estimates with minimum estimation variance. Unlike other methods, kriging also gives a confidence level on such estimates characterizing the quality of the estimation. However, in the simplest forms of kriging, extreme care should be taken in interpolation of such levels in all cases other than those in which data values are normally distributed. The regionalized variable under study has values $Z_i = Z(x_i)$, each representing the value at a point (location) x_i. The various kriging techniques are all based on the simple weighted linear combination of these values as refined versions of the weighted moving average techniques.

$$Z_k^* = \lambda_1 Z_1 + \lambda_2 Z_2 + \lambda_3 Z_3 + \ldots + \lambda_n Z_n$$

where Z_k^* is the estimator of the value Z_k at location k and the λ_i are the weights allocated to each value of the regionalized variable in the neighbourhood surrounding the location k. In the simplest forms of kriging, these weights λ_i are calculated according to the following criteria:

1 - The estimate is unbiased.

2 - The estimation variance is minimized.

The first criterion is satisfied by requiring the sum of the weights to be equal to one.

$$\sum_{i=1}^{n} \lambda_i = 1$$

and

$$E[Z_k^*] = E\left[\sum_{i=1}^{n} \lambda_i Z_i\right] = E[Z_k]$$

therefore

$$E[Z_k - Z_k^*] = 0$$

The second criterion is that the estimation variance $E\{[Z_k - Z_k^*]^2\}$ is to be minimized.

$$E\{[Z_K - Z_k^*]^2\} = E[Z_k^2] - 2E[Z_v Z_k^*] + E[Z_k^{*2}]$$

The estimation variance σ_e^2 is:

$$\sigma_e^2 = \sigma_v^2 - 2\sum_{i=1}^{n} \lambda_i \sigma_{vx_i} + \sum_i \sum_j \lambda_i \lambda_j \sigma_{x_i x_j}$$

σ_v^2 = variance of the value (grade) of blocks of volume v.

σ_{vx_i} = covariance of the value (grade) of block v and sample value at x_i.

$\sigma_{x_i x_j}$ = covariance of the sample values at locations x_i and x_j.

To find the λ_i weights which minimize the estimation variance under the condition $\Sigma\lambda_i = 1$, the partial derivatives, with respect to all unknowns λ_i and μ the Lagrange multiplier, of $\sigma_e^2 + 2\mu(\Sigma\lambda_i - 1)$ should be equal to zero.

$$\frac{\delta}{\delta\lambda_i} = -2\sigma_{vx_i} + 2\sum_{j=1}^{n} \lambda_j \sigma_{x_j x_i} + 2\mu = 0 \quad for \quad (i = 1, n)$$

$$\frac{\delta}{\delta\mu} = \sum_{i=1}^{n} - 1 = 0$$

then

$$\sigma_{vx_i} - \sum_{j=1}^{n} \lambda_j \sigma_{x_j x_i} - \mu = 0 \quad for \quad (i = 1, n)$$

$$\sum_{i=1}^{n} \lambda_i - 1 = 0$$

from the first equation

$$\sum_{j=1}^{n} \lambda_j \sigma_{x_j x_i} + \mu = \sigma_{vx_i}$$

multiplying both terms by $\sum_{i=1}^{n} \lambda_i$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{x_i x_j} + \mu \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \lambda_i \sigma_{vx_i}$$

and

$$\sum_{i=1}^{n} \lambda_i = 1$$

then

$$\sigma_k^2 = \sigma_v^2 - \sum_{i=1}^{n} \lambda_i \sigma_{vx_i} - \mu$$

is the kriging variance.

All covariances in the kriging system are computed from the variogram model and the distances between samples and the point or the block to be estimated. From the above analysis it can be seen that simple kriging techniques are no more than an unbiased weighted average method using an optimum set of weights ensuring the minimum estimation variance for linear estimates. Linear estimates are however only optimum in the case of normally distributed data.

For non-normal data estimates obtained by simple kriging may not be conditionally unbiased, lower estimation variance may be possible and other optimum criteria may be preferable. Simple kriging will produce the best linear unbiased estimator if all the assumptions outlined above (stationarity, normal distribution, ...) are satisfied. However, in mining it is not always the case that these assumptions are met because of the complexity of the data and also the presence of geological phenomena distorting the uniformity of the data set. Other kriging techniques have been developed to overcome the absence of these assumptions, lognormal kriging based on the transformation of the regionalized variable values to normally distributed values; disjunctive kriging based on the knowledge of the distribution of the regionalized variable values then using best fit approximation to convert the data into the form which will approximate the normal distribution; universal kriging was developed to take account of the presence of certain trends in the data such as the presence of drift where the mean of the regionalized variable value is not constant over the domain occupied by these variable.

Although the assumptions made to introduce the theory of the regionalized variables are very difficult to satisfy, geostatistics is a technique which has provided at least globally unbiased estimates, produced criteria on which the quality of the estimates is judged (minimum variance) and sampling and valuation programs can be designed economically where geostatistics can help to optimize sampling paterns (Royle 1980).

1.4 Scope of the present work

The purpose of the present research project is to provide geometrical control of geostatistical estimates in stratigraphic orebodies together with interactive graphics displays of information concerning the raw data and the numerical model of the orebody. The idea of imposing some sort of geological control, whenever possible, while performing the geostatistical estimation process is an approach adopted by many authors such as Rendu (1982, 1984), Dowd (1983, 1984, 1986, 1989), Royle (1979), Armstrong (1983), Dagbert et al (1984) , Barnes (1982) , etc. The purpose of these developments was to avoid a purely automatic, numerical application of geostatistical methods without any geological constraints or understanding. Rendu pointed out that the link between geology and geostatistics is well known but not always understood and if geostatistics is to give improved reserve estimates, two conditions must be satisfied:

a - geologists must be aware of the methods available to them to control the quality of the geostatistical study,

b - geostatisticians must not ignore geological input, if required, to obtain satisfactory results.

Geological control could also mean the introduction of no quantifiable geological features in the geostatistical modelling process. The detection of these features is of prime importance to the application of an estimation method. Among these features are the geometry of the deposit and the distribution of the mineralisation. The last feature imposes, in most cases, the choice of the kriging technique; the first one has much to do with structural analysis, by imposing geological control over the calculation of the variogram to make it more representative of the variography of the variable under study. However, the introduction of geological control and related information could be done at each

step of a geostatistical study and depends very much on the amount of information available and on the complexity of the space occupied by the mineralisation (deposit). Moreover, there are several geological features where control could be applied to geostatistical estimates such as faulting, folding, rock types, rock characteristics (fracture density, fracture orientation, porosity, ...), geneses of the mineralisation and others (Rendu 1984).

Among several applications of geological control in geostatistics are examples of distinct populations where the variogram is calculated in separate zones (Champigny and Armstrong 1988), (Rendu and Readdy 1982) and others. Variogram calculation techniques for faulted strata-controlled deposits have been discussed, where control is applied on the vector distance between sample values to allow variogram calculation to follow the natural axes of the mineralisation. It consists of transforming the coordinates of the data values on natural axes, or these distances are measured along digitized profiles. This approach requires good knowledge of the stratigraphic position of every data value (Dagbert et Al 1984). Similar approaches have been adopted emphasising that variogram calculation, as the definition of the spatial continuity of the mineralisation, should take account of geological continuity. A good illustration of this approach is found in Barnes (1982) where a paraboloidal reference system is used to compute variograms in a porphyry molybdenum deposit.

Other methods take into account the structure (geometry) and its complexity mainly in stratigraphic and tabular orebodies by using unfolding or unrolling techniques of the curved surfaces delineating the orebody or the mineralized space (Royle 1979) before continuing to estimate the other quality parameters (grade, ...). Geological control could also be applied at the estimation stage (kriging) where block grade estimates are guarded by geological boundaries

(Benest and Winter 1984). The same type of control could be applied in selecting block size to conform to the smallest geological units (rock types, structure of mineralogical domain, ...) avoiding the loss of these geological characteristics. Control of geostatistical estimates of thicknesses in stratigraphic orebodies is another approach which consists of fitting a plane through the mid-points of the stratigraphic unit using spline techniques allowing the control of the estimated thicknesses to be localized correctly (Dowd 1983, 1984, 1986). However, the apparent differences between geometrical and physical (grade) variables, in terms of distribution and homogeneity, have drawn the attention of many authors to treat these two variables using different methods Mallet (1988), Dowd (1989) and Abbachi (1989). The essential of this approach is the construction of the geometry of the orebody using deterministic methods and to estimate, using geostatistical methods, the quality parameters (grades, ...) subject to geometrical (volumetric) control. In the light of this last approach the present research project has been developed to create geometrical models of stratigraphic orebodies using deterministic methods developed elsewhere (e.g., CAD systems in the manufacturing industries) as the main element in computer systems for designing and representing geometric features of different bodies. This project has a number of objectives, It consists in emphasizing the importance of structural control in estimating stratigraphic orebodies, exploring other fields where spatial data are dealt with by different methods and also, as in the last stage, the making use of graphics techniques for the presentation of geometrical features of stratigraphic orebodies. Ultimately, this research project is an attempt to prepare one of the main elements in the process of mine planning and operation which is the exact location and shape of the orebody to be mined out. Figure 1.5 shows the main outline of this evaluation system.

Although sources of information for geometrical modelling of stratigraphic orebodies are of different sorts, the drillhole intersections have been, in this case, the only type of data used, both to characterize the geometry and the quality of the mineralisation. The data have to be sorted out to represent the existing surfaces delimiting the orebody or the stratigraphic unit, as these surfaces will be modelled separately then combined to form the numerical model of the geometry of the orebody. Surface modelling techniques are applied, as a deterministic method. Although they have not been a subject of an extensive investigation, these techniques have demonstrated their usefulness in the geometrical modelling fields. The methods adopted for this research project are based on local procedures of interpolation. They perform on triangulated surfaces fitted, in most cases, to scattered data point locations. These methods are the bilinear interpolation method based on fitting a first degree polynomial (plane) over each triangular patch and a bivariate interpolation method based on fitting a fifth degree polynomial also on every triangular patch and known as Akima's method of interpolation. The first method, unlike the second one, does not produce a smooth surface with a high degree of continuity, it is more suitable for quick modelling operations and it does not require a large computing time. The second method takes a relatively large computing time, especially for large data sets, and it is more suitable for obtaining the final numerical model of the orebody. The thicknesses are then deduced from the geometrical model and they will be used to calculate the grade values from the accumulations estimated using geostatistical methods. The graphics display system is used in many stages of this evaluation process. It displays the preliminary information such as the locations of drillhole intersections, the drillhole intersections with the orebody in

three-dimensional space and the triangulated surfaces (e.g., Hangingwall, footwall). The system also displays information concerning geostatistical structural analysis of the regionalized variable under study such as variograms, histograms, etc. However, the main objective of the graphics display system is to display geometrical features of the orebody in two and three dimension such as cross-sections, plane-sections, wireframe and solid models of the orebody and contour lines of values in a plane. One of the characteristics of this graphics display system is that it displays many types of figures in one single frame in the form of windows.

Most of the software of this evaluation system (geometrical modelling and graphics display system) was developed on the Amdahl VM580 Mainframe computer system and was later restructured and modified to be run on Personal Computers. The number of basic graphics routines has been kept to a minimum so as to allow the portability of the graphics display system between machines and different packages of basic graphics routines (GINO-F, UNIRAS, GHOST, SIMPLEPLOT, ...).

Finally, some remarks have been made in the concluding chapter to conclude this research project, stressing the need for further work in introducing the latest techniques used in the domain of information technology, to adopt techniques developed in other fields in order to ease the complexity of the problems associated with mining (evaluation, planning, operation). Database systems, expert systems, numerical methods, including geostatistics, and graphics display systems will be the cornerstone of future research works if the mining industry is willing to come to terms with one of its major problems: a better processing of huge amounts of data in some areas and the lack of information in others.

Computer evaluation system for stratigraphic orebodies

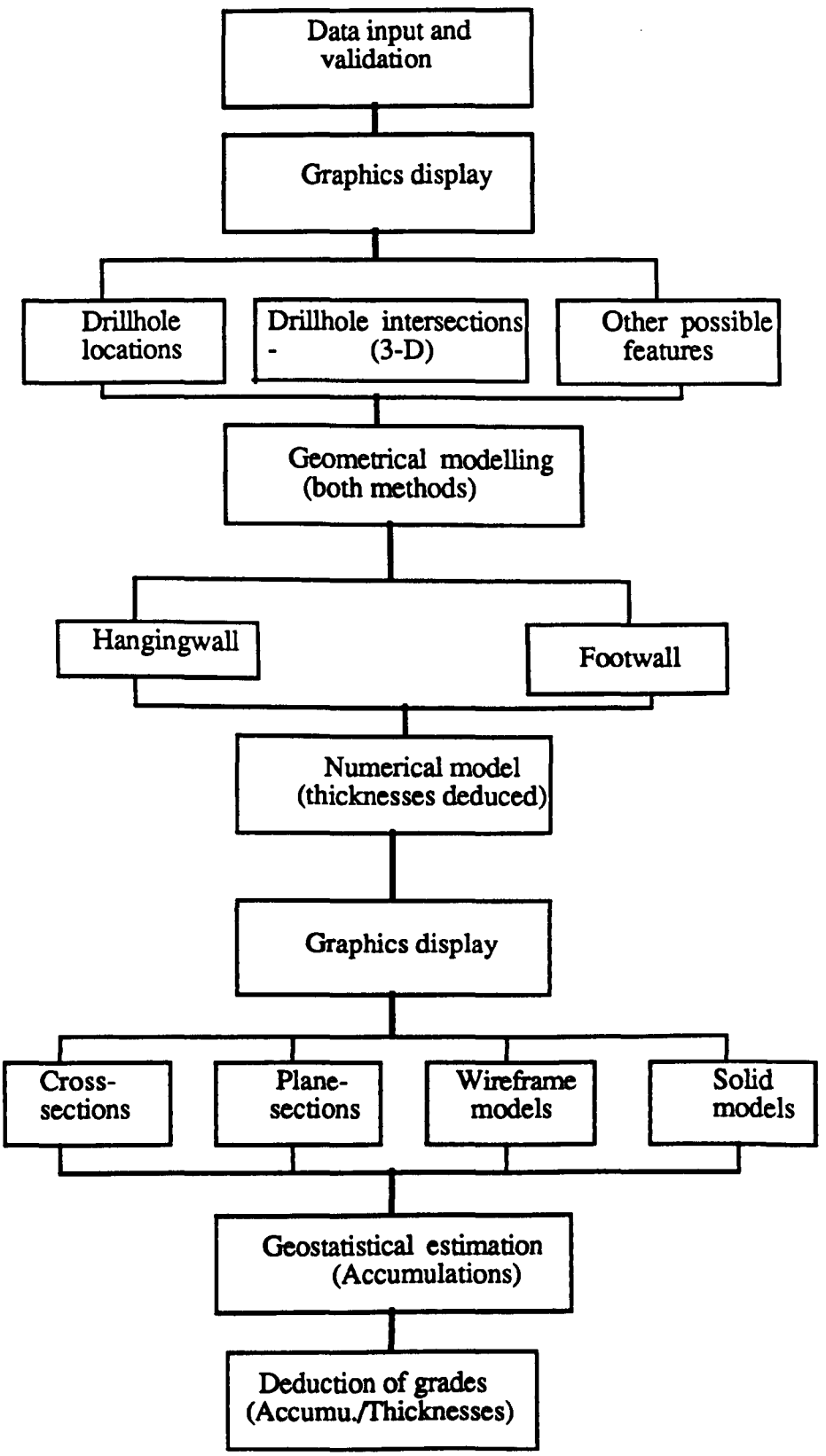


Figure: 1.5

1.5 Summary

This research project has shown the complementary aspect between geostatistical modelling methods and deterministic methods based on surface modelling techniques in order to build more robust orebody modelling systems. The review of the existing numerical methods, at least for evaluation of reserves, shows also that some mathematical and statistical tools are available to the mining industry enabling it to benefit and to participate in the development of methods dealing with spatial data analysis. However, mining reserves estimation methods will always be assessed on the degree to which geological parameters have been taken into account and on the better understanding of the role of these control parameters. The difficulty will also remain in the introduction of geological control parameters as each orebody or mineralized space is a unique case requiring a great deal of expertise and information. This is why information technology techniques are indispensable to cater for the complexity of mining reserves estimation and geometrical modelling of orebodies in particular.

CHAPTER TWO

Chapter Two

DATA

2.1 Introduction

As an important element in the geometrical modelling of an orebody the data available must be as representative of the orebody and as accurate as possible. Information concerning the modelling of an ore deposit could be extended beyond even the limits of the mineralized zone to include geological maps of the region containing all geological units and general features, geo-technical maps representing results from a systematic prospecting campaign, tectonic maps allowing the identification of sub-zones; the most important of all are the locations of known mineralisation and the existing mining activities.

In this chapter drillhole intersections with the mineralization will be the main source of the data for the geometrical modelling system. The integration of all information will be a more complicated task and it is beyond the scope of this research project which is very much concerned with the delineation of a single stratigraphic unit or a succession of stratigraphic units, cf. figure (2.1[a,b]).

Drillhole data are gathered mainly from two drilling campaigns: exploratory drilling and development drilling. The exploratory drilling campaign is the

Drillhole intersections with a stratigraphic orebody (multistrata)

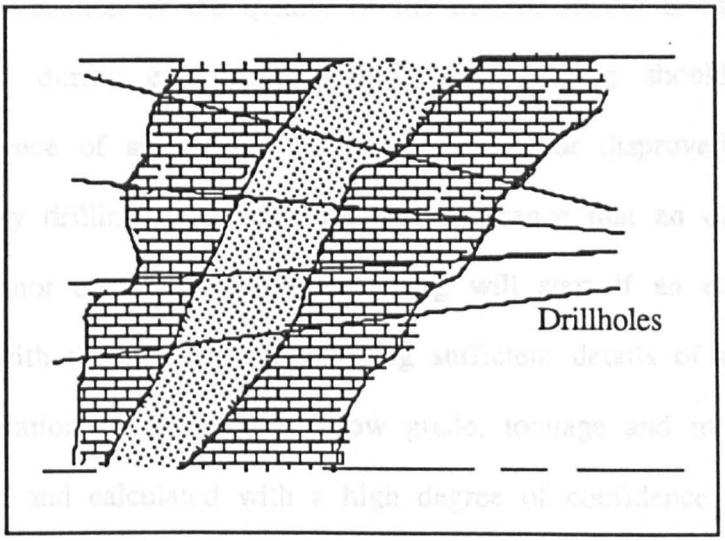


figure: 2.1 (a)

Drillhole intersections with a single stratum

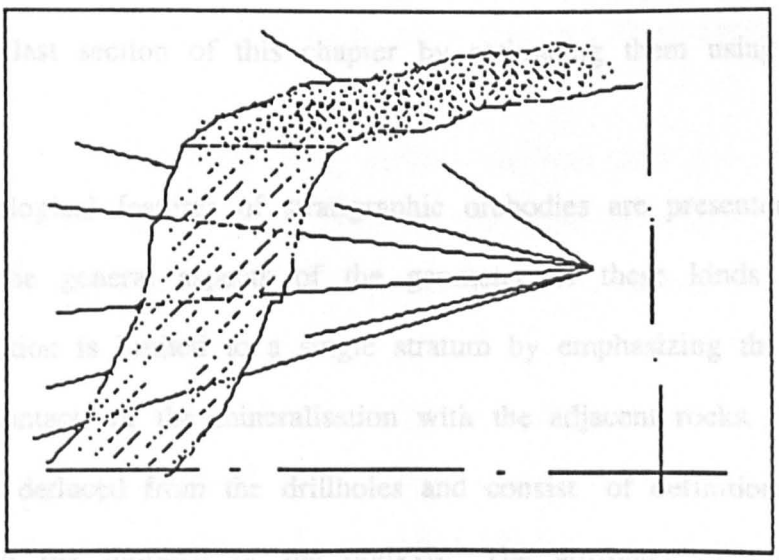


Figure: 2.1 (b)

preliminary stage of getting hard evidence of the existence of an orebody and will give an indication of the quality of the mineralisation. If interesting values are intersected during exploratory drilling then drilling should be continued until the presence of an orebody has been proved or disproved. As a general rule exploratory drilling ends when there is assurance that an economic deposit does or does not exist. Development drilling will start if an economic deposit is indicated, with the purpose of providing sufficient details of the three-dimensional configuration of the area to allow grade, tonnage and mineability factors to be deduced and calculated with a high degree of confidence.

In the following sections, data from stratigraphic orebodies, with drillhole intersections with the orebody as the main source, will be presented. The most important aspect of these data is concerned with the geometry of the orebody. The presentation of attributes of these geometric data (eg, Grade) will be dealt with in the last section of this chapter by estimating them using geostatistical methods.

Morphological features of stratigraphic orebodies are presented in order to understand the general aspects of the geometry of these kinds of orebodies. The presentation is limited to a single stratum by emphasizing the definition of geological contacts of the mineralisation with the adjacent rocks. The geometrical data are deduced from the drillholes and consist of definitions of the hangingwall and the footwall of the orebody. The attributes of these geometric variables are obtained by assaying at least a portion of the core intersecting the mineralisation. But the main feature of interest of these data is the configuration of their locations, as it plays a major role in choosing the most

appropriate surface modelling technique and also in the reliability of the results of the modelling process. In the last section geometrically controlled geostatistical estimates of the attributes are calculated using the GEOSTAT2 software package developed at the Mining and Mineral Engineering Department of the University of Leeds by Dr. P. A. Dowd; a sample of some results is presented.

2.2 Morphological Features

The data collected will be specific to surfaces delineating the stratigraphic orebody (hangingwall and footwall). These surfaces which define the type of geological contacts of the strata with the adjacent rocks classify stratigraphic orebodies in four types cf. Dowd (1986).

1- Structurally well defined footwall and hangingwall within which all attributes are located. It consists of sharp, non-complex geological contacts. These kinds of surfaces are easily modelled even when they are represented by few data points, cf. figure (2.2).

2- Complex but structurally well defined footwall and hangingwall within which all attributes are located. However, the complex shape of the surfaces (walls) will require more data and a robust surface modelling technique cf. figure (2.3).

3- One or both of the footwall and hangingwall defined by a cut-off grade, cf. figure (2.4). This is a special case of category 1. The limit defined by a cut-off grade should be applied only to the estimated thicknesses rather than to the actual data values.

4- Stratigraphic units consisting of two or more sub-units separated by barren or poorly mineralized zones cf. figure (2.5). In this case it is preferable to

model each sub-unit separately, this approach is of course subject to the availability of the data for each sub-unit. It also depends on economical and technical constraints if the sub-units are to be mined separately.

This is a very simple geometrical classification of stratigraphic orebodies; there are in addition some geological features which are often associated with stratigraphic orebodies such as folds and faults. The presence of faults may require strata to be split into two sections and modelled separately, cf. figure (2.6). Folds will not cause a problem of overlapping cf. figure (2.7) if the data are well stored (coding, rearranging, ...) or coordinates of the data points are transformed (rotation); this will not affect the overall geometric model because the interpolation methods to be used are based on local procedures rather than global ones.

Sharp geological contacts

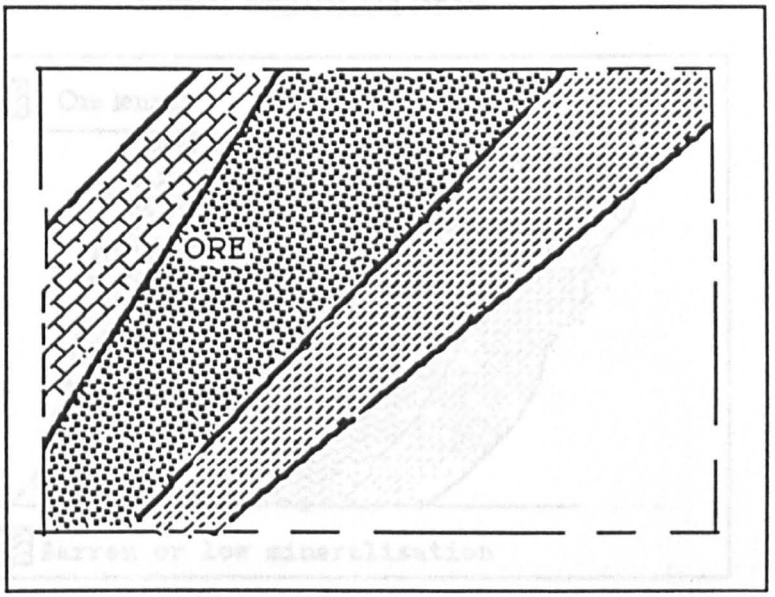


Figure: 2.2

Limit based on cut-off grade

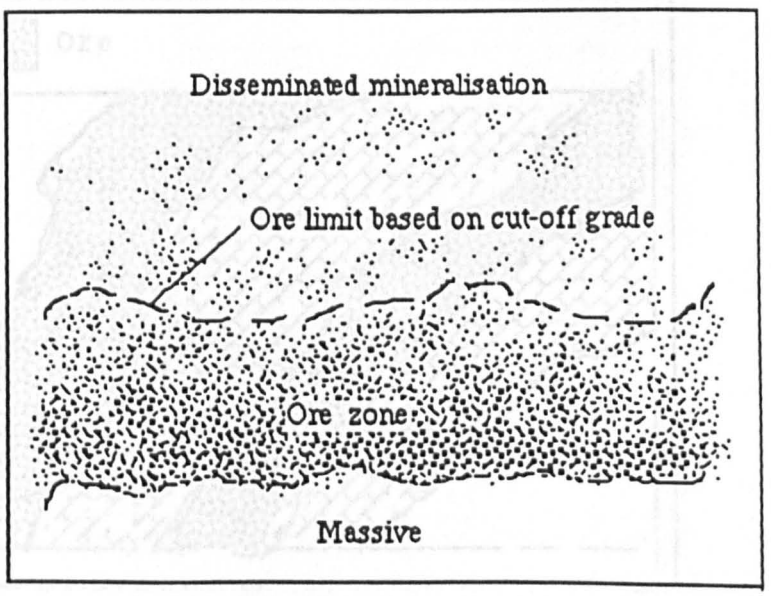


Figure: 2.4

Several well-defined lenses

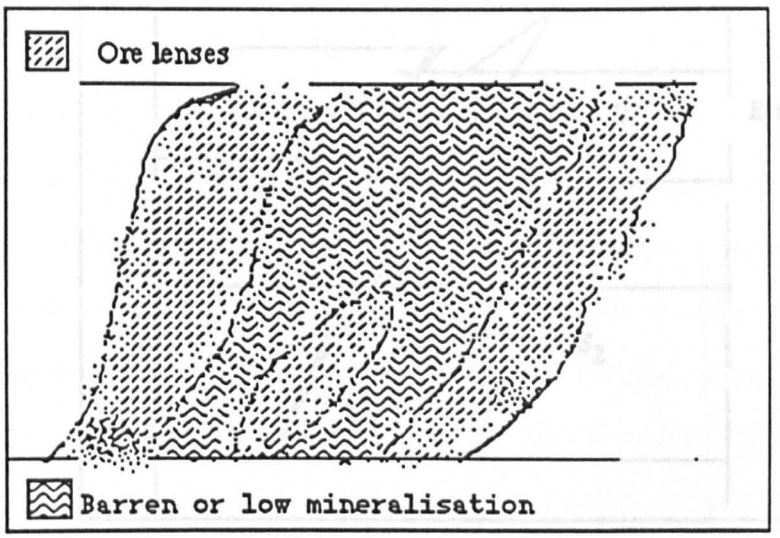


Figure: 2.5

Sharp complex geological contacts

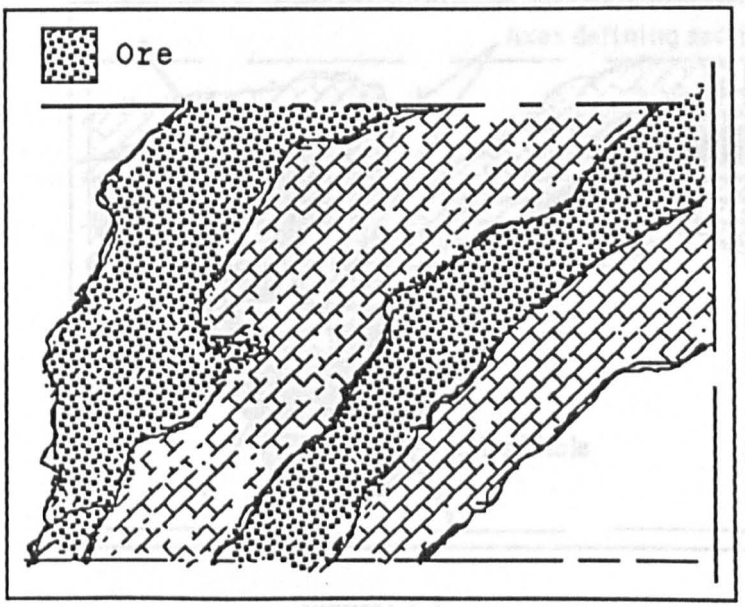


Figure: 2.3

Faulted stratigraphic unit

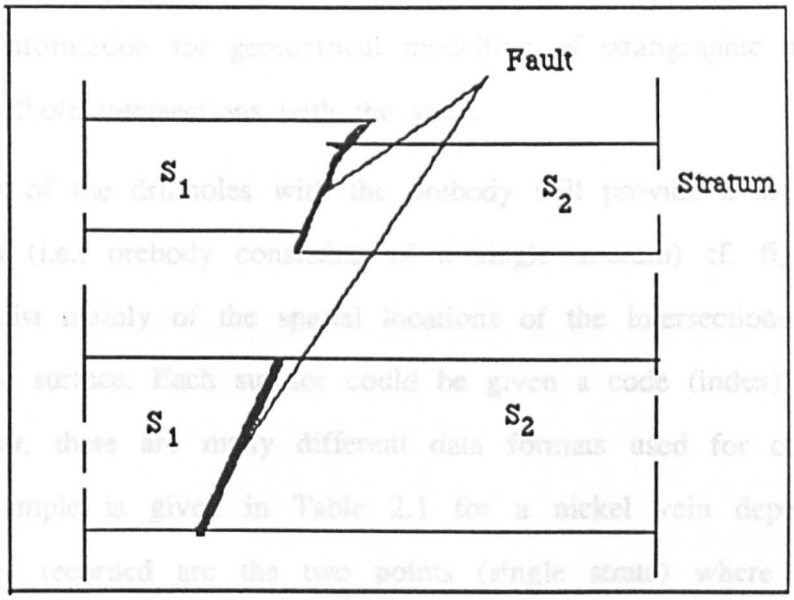


Figure: 2.6

Folded stratigraphic unit
(intersected more than once by one drillhole)

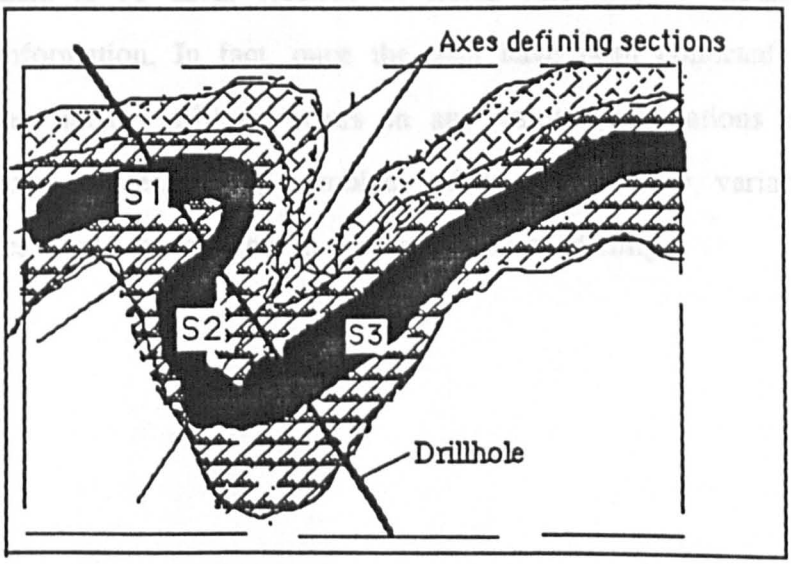


Figure: 2.7

2.3 Data Acquisition

Most available information for geometrical modelling of stratigraphic orebodies come from drillhole intersections with the strata.

The intersections of the drillholes with the orebody will provide a set of at least two surfaces (i.e., orebody consisting of a single stratum) cf. figure (2.1b). The data consist mainly of the spatial locations of the intersections of the drillholes with the surface. Each surface could be given a code (index) for identification. However, there are many different data formats used for computer storage; an example is given in Table 2.1 for a nickel vein deposit, where the coordinates recorded are the two points (single strata) where the drillhole intersects the hangingwall and the footwall together with the average value of the attribute (i.e, %Ni). The ability to make calculations on the input data is an important step in creating a geometrical model; this can involve specification of the data to be used, creation of fields (sections) or modification of the original information. In fact, once the data have been collected and stored in the computer, ideally without errors in any values, calculations may be performed to change scales, units, combine values, create new variables, etc... and separate files could be created in readiness for modelling.

Table: 2.1 Raw data: drillhole intersection coordinates with hangingwall and the footwall.

	hangingwall data			footwall data			
DH. Num	$(X)_h$ North- -19000	$(Z)_h$ East- -16000	$(Y)_h$ R.L. -1300	$(X)_f$ North- -19000	$(Z)_f$ East- -16000	$(Y)_f$ R.L. -1300	%Ni
464	237.40	206.26	80.00	236.85	209.23	80.00	6.10
485	166.41	191.00	80.00	165.84	193.21	80.00	6.00
484	170.27	192.93	80.00	169.96	193.75	80.00	5.90
395	218.50	200.71	84.00	218.31	201.45	84.00	6.70
426	370.01	242.56	84.00	368.64	245.82	84.00	7.30
427	371.89	244.04	84.00	370.60	246.80	84.00	5.70
350	205.27	198.38	87.00	205.11	199.54	87.00	6.90
351	213.41	200.14	87.00	212.92	203.22	87.00	6.60
3d6	261.55	210.11	87.00	261.30	214.74	87.00	7.00
368	269.61	212.53	87.00	269.49	215.57	87.00	7.50
290	207.50	202.19	90.00	207.15	204.31	90.00	5.80
291	214.68	204.63	90.00	214.70	206.39	90.00	5.90
...	.....	.....	.....	.....	.....	.....	....

2.4 Data configuration

There are almost as many configurations of the data point locations as there are modelling applications. In this section a number of common grid types are quoted here, cf. SABIN (1985).

1- Regular grid.

This special case is in some ways regarded as ideal, and there are modelling methods which exploit the known regularity to give good and reliable representations of the data. In fact, there are variants of regular grid configurations which have many advantages for modelling methods.

The cases worth noting are:

a) regular rectangular grid, figure(2.8)

b) Tartan grid, figure(2.9)

c) Regular triangular grid, figure(2.10)

2- Almost regular grid.

This can arise in an experimental situation, where control of the position of the independent variables is not precise but where they can be measured accurately. For example, a regular grid will then give the pattern of data, illustrated in figure(2.11). Each of the regular grids has its corresponding almost regular grid version.

3- Even data.

Here there is no pattern visible at all, the data is randomly spread, but with a uniform density over all the surface to be represented. This kind of data pattern is somewhat artificial, as it rarely occurs in practice.

4- Data with voids.

A void is a region of diameter substantially greater than the mean distance

between data points, containing no data. Voids may arise in surveying or during drilling campaigns because of the physical inaccessibility of certain regions. They also arise where the surveyor or the drilling campaign manager can see that there are no features of interest, cf. figure(2.12).

5- Data with clusters.

A cluster is a region containing a significantly higher than average density of data points. They arise either because it is cheaper, for some reason, to take a large number of measurements in one locality, or because there is a local concentration of information needing many points to capture its fine detail cf. figure(2.13).

6- Tracked data.

A particular kind of cluster is the linear cluster or 'track' which occurs when measuring equipment is being moved and can cheaply take any readings along its path, cf. figure(2.14).

For the application of surface modelling techniques to stratigraphic orebodies, the data points configuration is of great importance in choosing an interpolation method. A more generally occurring case is the randomly distributed data points locations scheme usually referred to as scattered data.

Regular grid

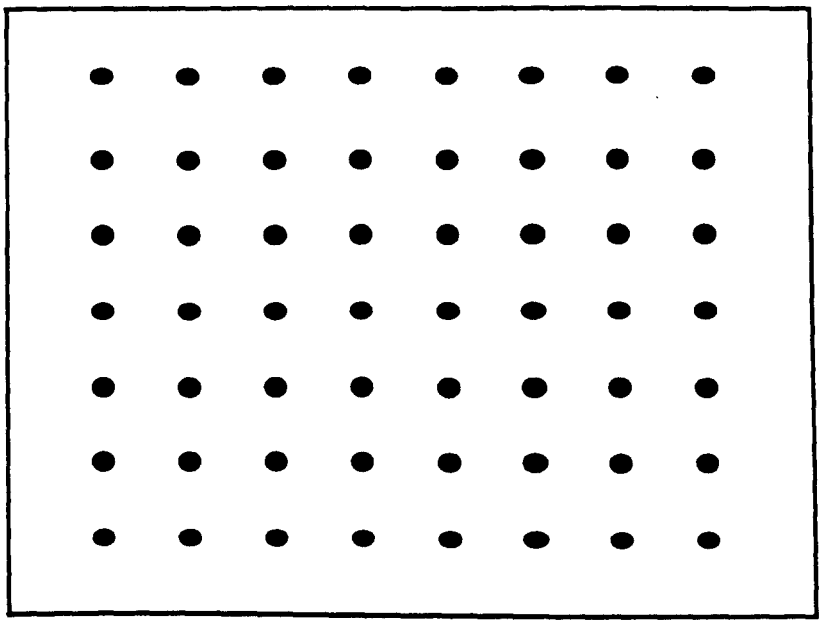


Figure: 2.8

Tartan grid

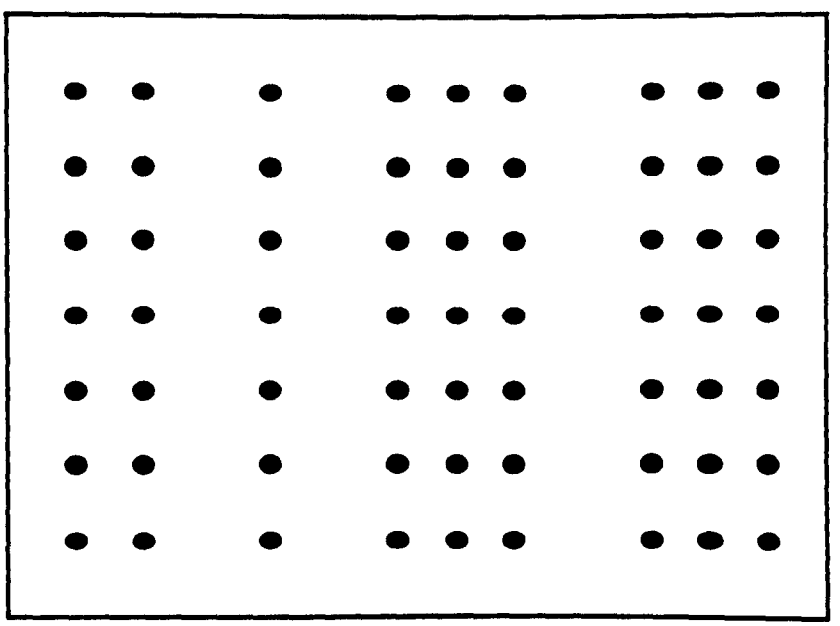


Figure: 2.9 .

Regular triangular grid

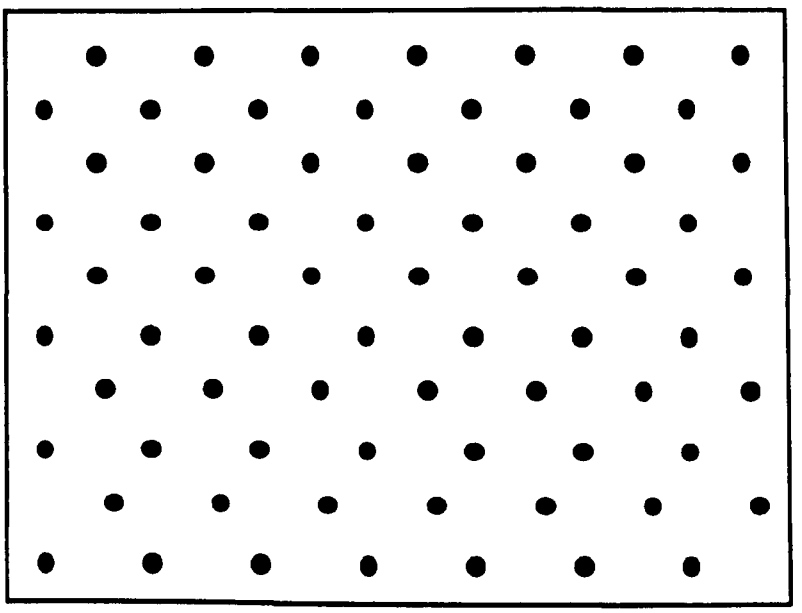


Figure: 2.10

Almost regular

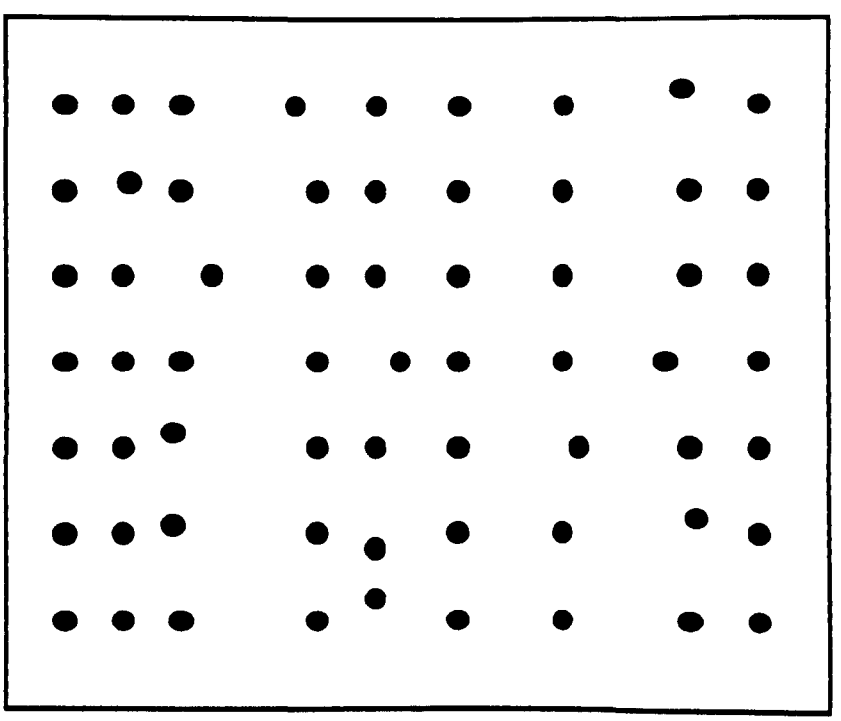


Figure: 2.11

Data with voids

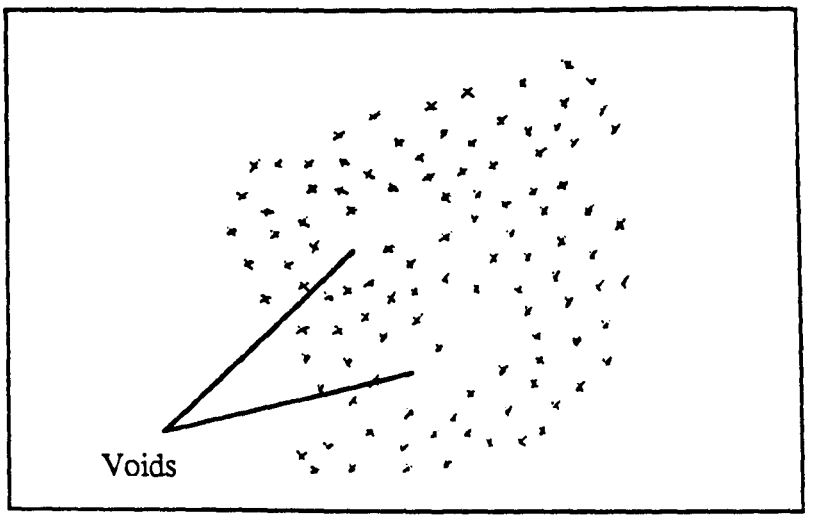


Figure: 2.12

Data with cluster

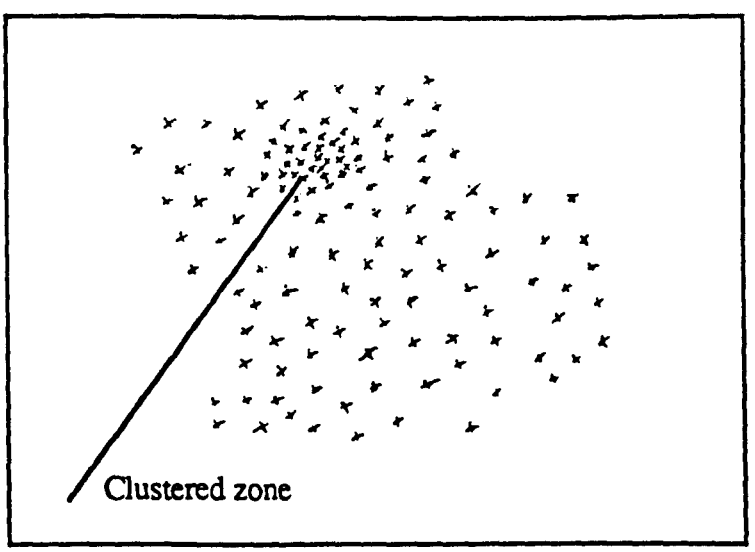


Figure: 2.13

Tracked data

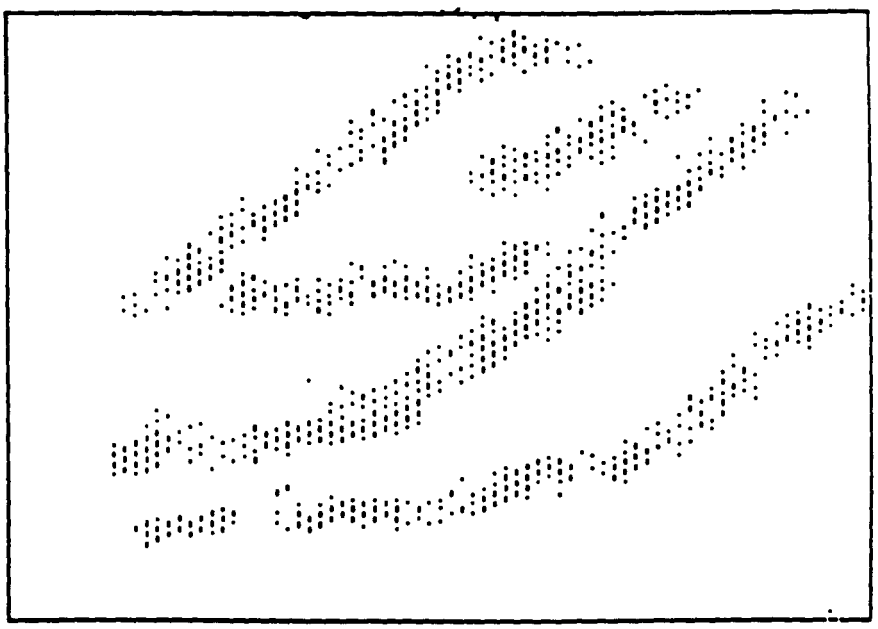


Figure: 2.14

2.5 Geostatistical Estimates

The standard geostatistical approach to the estimation of the shape and location of stratigraphic orebodies is based on accumulations and thicknesses. Thicknesses and locations are measured by drillhole intersections with the orebody (or with each constituent unit of the orebody); these thicknesses are then converted to a common orientation (eg, east-west horizontal thickness, true thickness, etc.) by considering the orientation of the hole and the local or global strike, dip and plunge of the orebody. An example of such conversions is shown in figure 2.15 in which intersections with the orebody are converted to east-west horizontal thicknesses.

Variograms of the converted thicknesses and accumulations are then used, together with the data, to provide estimates of the thickness of the orebody, and thereby the locations of the hangingwall and footwall, at specified locations.

There are two serious problems with this approach. The first is that the necessity of converting the intersections to a common orientation means that important points on the boundaries of the orebody, or its constituent units, are lost. For example, in figure 2.15 when the intersection AB is converted to the east-west horizontal thickness CD, the boundary control points A and B are lost to the subsequent estimation. This loss of control points can lead to serious discrepancies in the modelling of the orebody boundaries, in particular known data points on the boundaries will not be honoured by the estimation.

The second problem is the determination of the location at which an estimate is to be made as illustrated in figure 2.16 which shows intersections on an east-west, vertical cross-section in a stratigraphic silver/lead/zinc orebody.

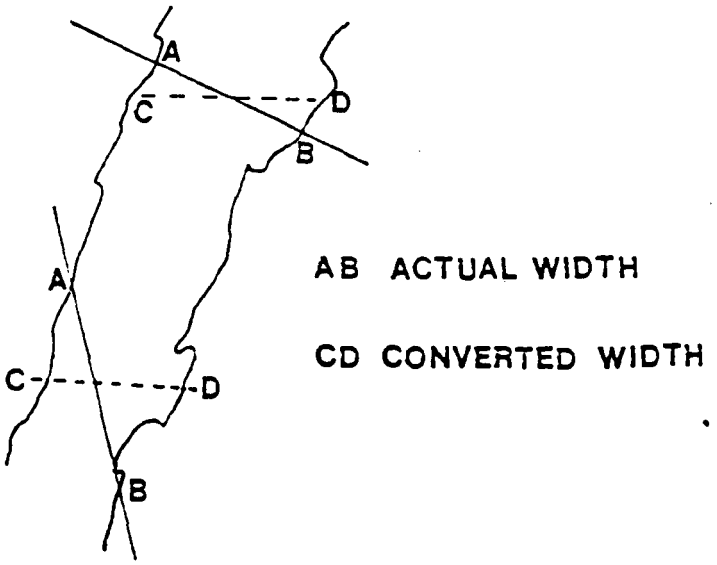


Figure: 2.15 Example of actual and converted intersection widths

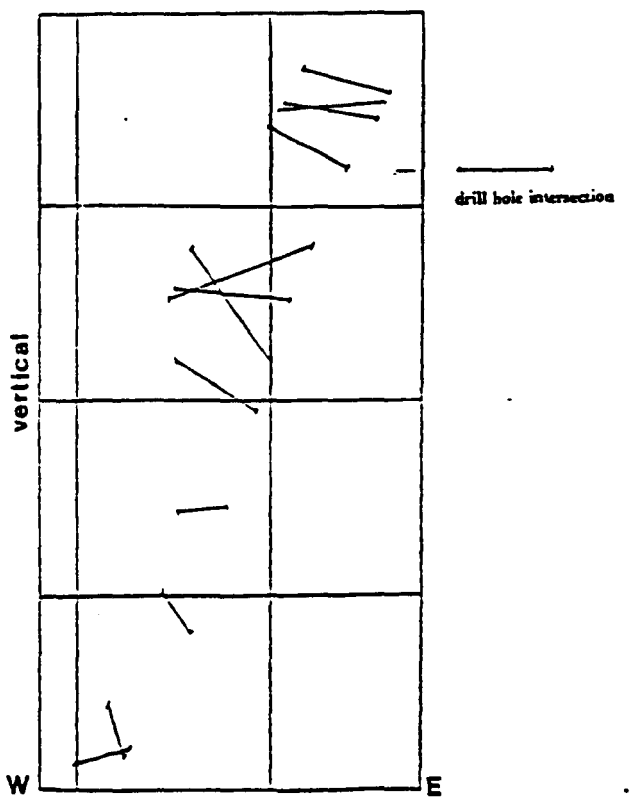


Figure: 2.16 Example of drillhole intersections used to estimate a cross-section

Suppose that the boundaries of this orebody on this particular cross-section are to be estimated from the thickness data (after conversion to a common orientation) available on the cross-section and from some given volume either side of the cross-section. Suppose further that point estimates of the boundary locations at 2m vertical intervals will provide a satisfactory resolution. Thus, for each estimate , the y coordinate (i.e., north south or the cross-section) and the z (vertical 2m interval) are known but the x coordinate (east-west) is unknown and must be estimated along with the thickness itself. This is really the unrolling problem in a different guise. These problems have been discussed at length in Dowd and Scott (1984 and 1986).

Several attempts have been made to solve these problems. In particular, Royle (1979) proposed a method for the least squares fitting of local strike-dip planes. Dowd and Scott (1984) modified this approach to impose piecewise continuity of the local planes. Dowd and Scott (1984 and 1986) fitted spline surface to the mid-points of orebody intersections and then kriged thicknesses on this surface. Dowd (1989) compares spline surface modelling of footwall and hangingwall with kriged thickness estimates of the same boundaries in a multi unit silver/lead/zinc orebody.

There is, in addition, the more general problem of geological control of geostatistical grade estimates. For example, consider the sequence of silver/lead/zinc orebodies in figure 2.17. All units in the sequence have a well-defined hangingwall and footwall with some much more regular than others; some units are continuous and others are discontinuous; some units are of very high grade with others relatively low in grade. Grades are not wholly confined to the stratigraphic units and some of the inter-unit "waste" material contains low grade ore which may have a bearing on the mining method

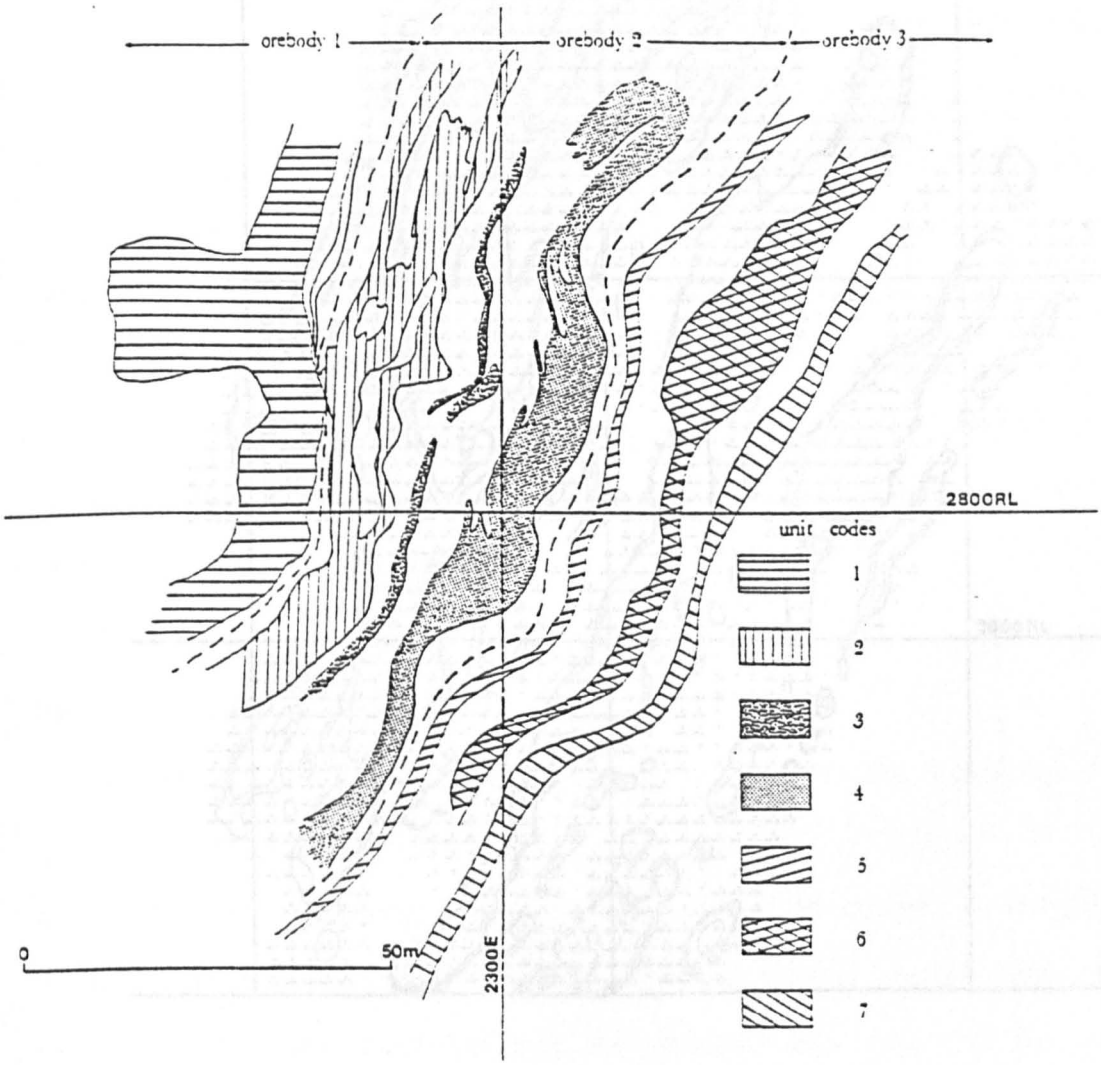


Figure: 2.17 Geological interpretation of cross-section based on all drillhole information

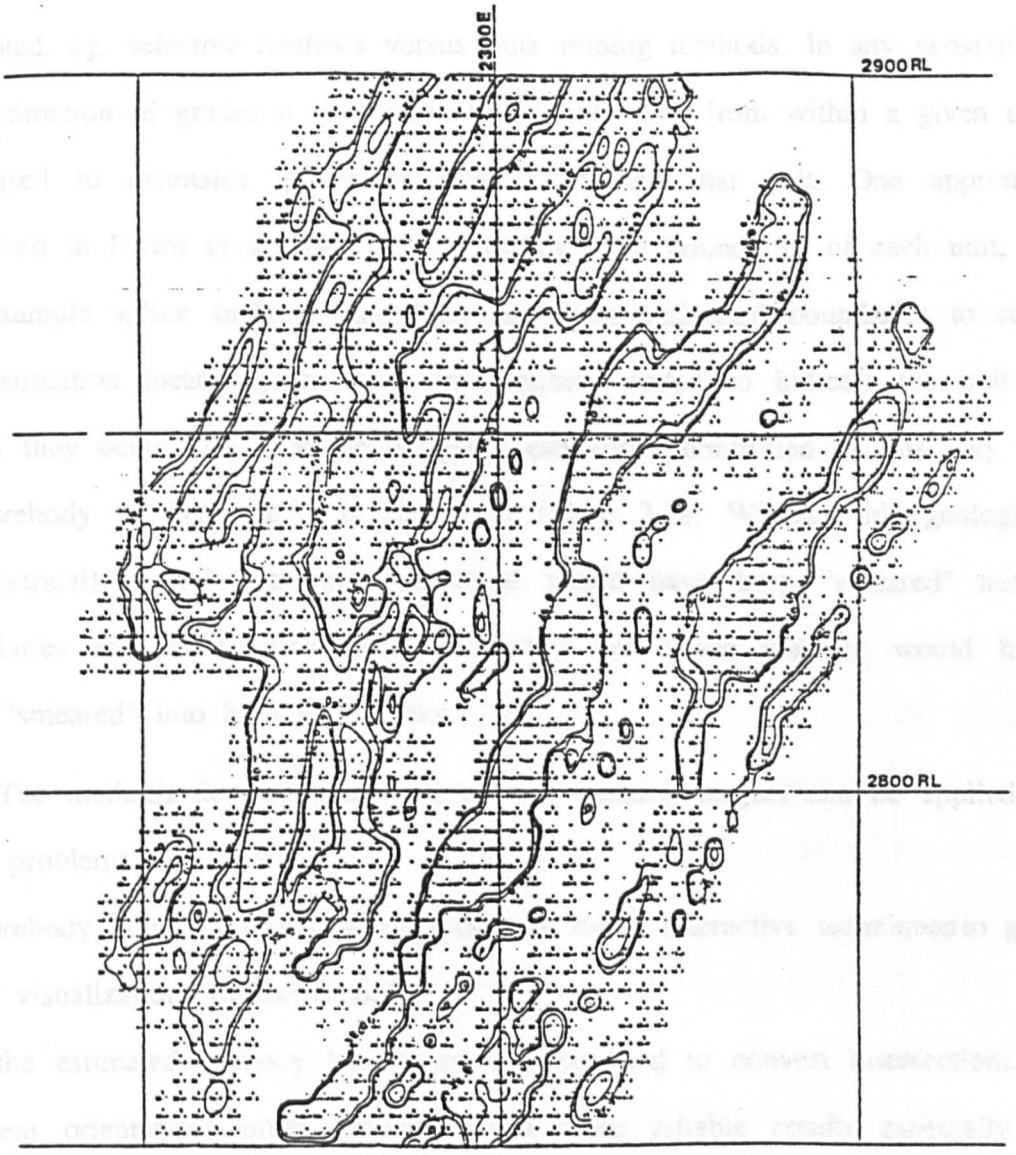


Figure: 2.18 Contours of kriged ore grades with kriging controlled by spline surface estimates of orebody boundaries.

adopted, eg. selective methods versus bulk mining methods. In any geostatistical estimation of grades it is important that only data from within a given unit are used to estimates grades at locations within that unit. One approach, described in Dowd et al (1989), is to estimate the boundaries of each unit, by for example spline surfaces, and then use these estimated boundaries to code the estimation locations; the data are similarly coded to indicate the unit in which they occur. An example of grade estimation controlled in this way for the orebody in figure 2.17 is shown in figure 2.18. Without this geological (geometrical) control high grade material would have been "smeared" across boundaries into waste areas and conversely low grade material would have been "smeared" into high grade units.

The methods developed as part of this research project can be applied to these problems as follows:

a - orebody boundaries can be estimated by rapid, interactive techniques to give quick visualizations of the orebody

b - the estimated orebody boundaries can be used to convert intersections to different orientations often giving much more reliable results especially in erratic orebodies; this may overcome the problems associated with the loss of boundary control points if a complete geostatistical approach is pursued.

c - the estimated boundaries can be used as geological (geometrical) controls on the geostatistical estimation of grades.

As an example, the method referred to in (b) has been used in a vein nickel deposit to convert thicknesses and accumulations to a common orientation; orebody grades were then estimated by kriging the converted accumulations within the estimated orebody outline.

The orebody is the Perseverance nickel deposit which consists of two styles of nickel sulphide mineralisation associated with Archaean ultramafic rocks, contained within a steeply west-dipping sequence of amphibolite gneiss, metamorphosed sediments and volcanic rocks. A sheet-like, vein style, massive sulphide, con formable with the metasediments extends several miles North from the steeply south-plunging, disseminated mineralisation as shown in figure 2.19. This disseminated mineralisation lies on the north-west flank of the locally swollen, regionally extensive, ultramafic unit (Dowd & Milton 1987). The study area is part of the No 3 stope district of the 1A massive sulphide cf. figure 2.20. The variables used are orebody width (thickness) and grade accumulations at recorded locations cf. figure 2.21 and figure 2.22. Thicknesses are obtained using deterministic methods (surface modelling techniques) and grade estimates are obtained in the approximate manner by dividing the estimated accumulation by the thickness under some restrictive conditions.

Among the restrictive conditions which the relationship for estimated grade can be accepted are:

- thicknesses are sufficiently accurate (sufficient amount of data and appropriate configuration)
- accumulation estimates and thicknesses are made from the same data configuration.

More details on geostatistical estimation of this orebody can be found in (Dowd & Milton 1987).

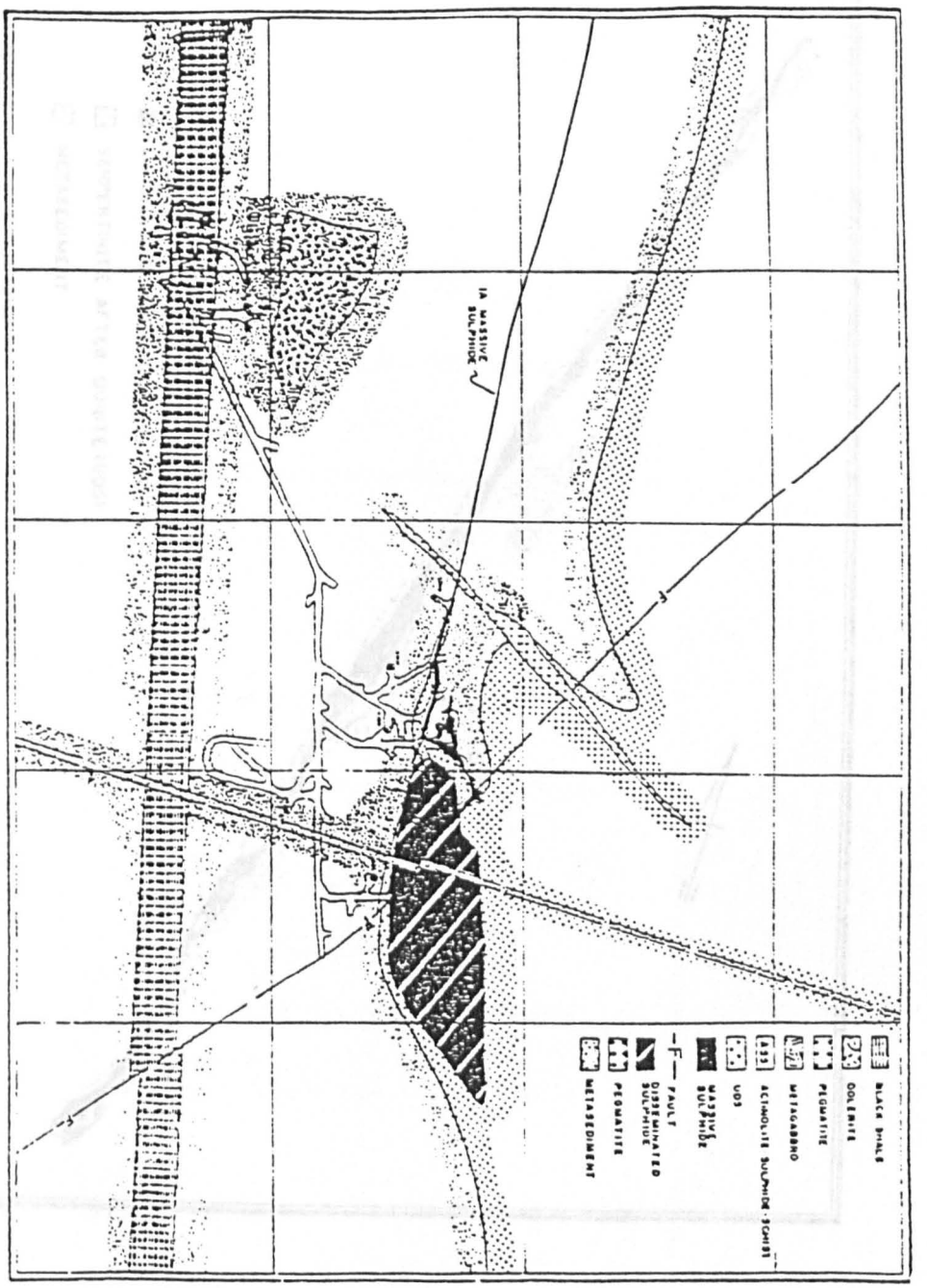

1A MASSIVE SULPHIDE
BLACK SHALE
DOLERITE
PEGMATITE
METAGABBRO
ACTINOLITE SULPHIDE SCHIST
UDS
MASSIVE SULPHIDE
FAULT
DISSEMINATED SULPHIDE
PEGMATITE
METASEDIMENT

Figure: 2.19

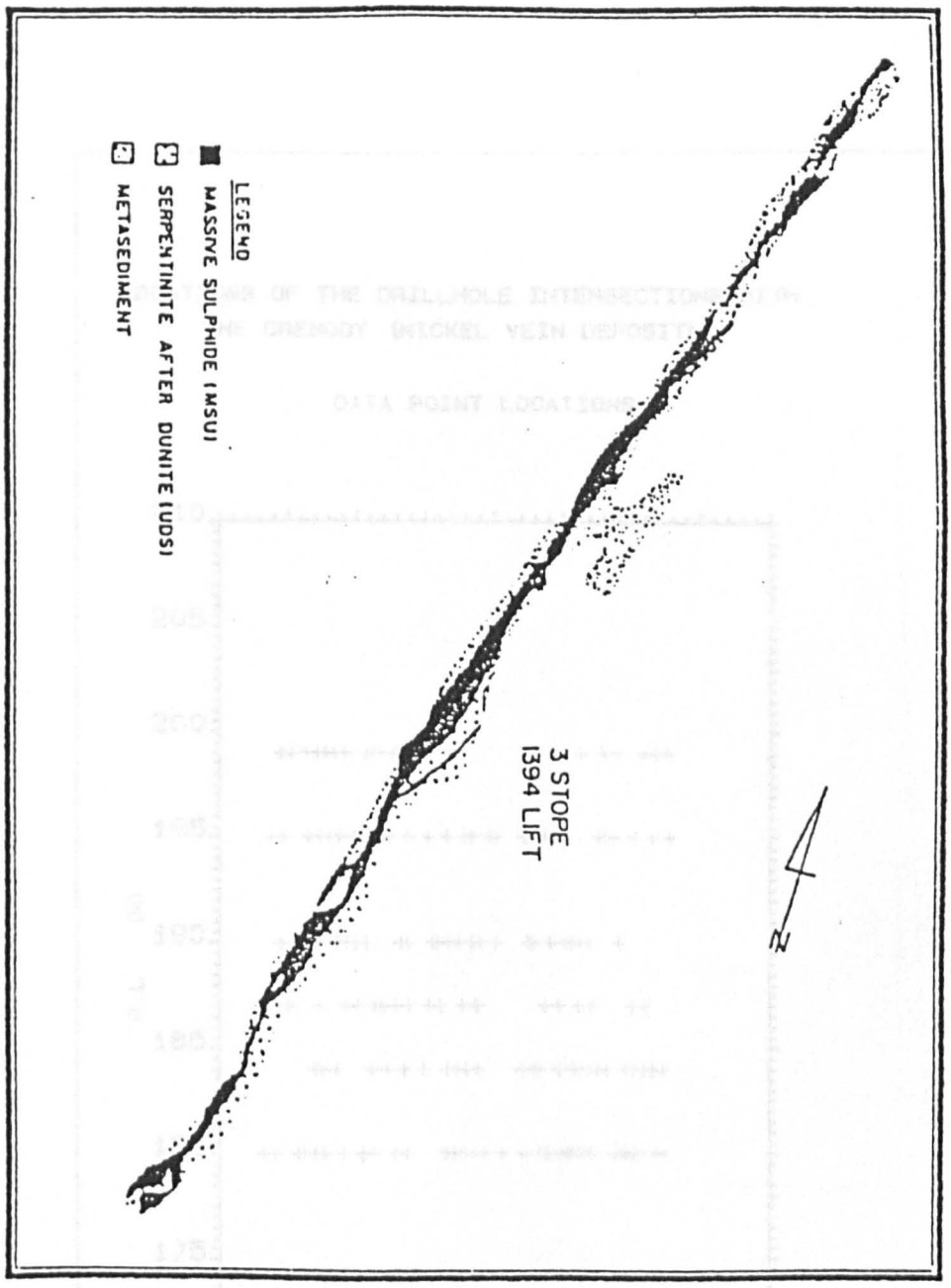

LEGEND
MASSIVE SULPHIDE (MSU)
SERPENTINITE AFTER DUNITE (UDS)
METASEDIMENT
3 STOPE
1394 LIFT
N

Figure: 2.20

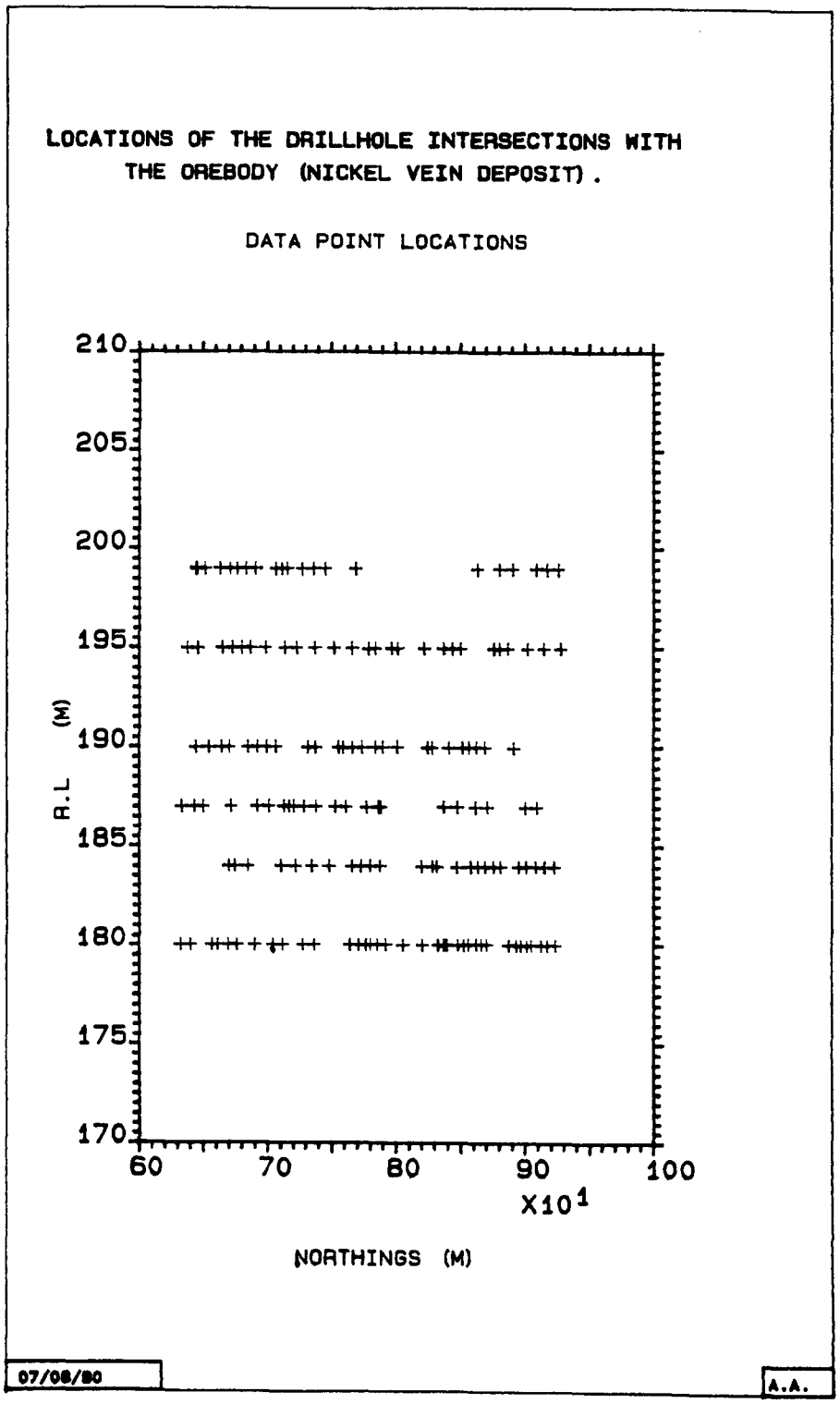
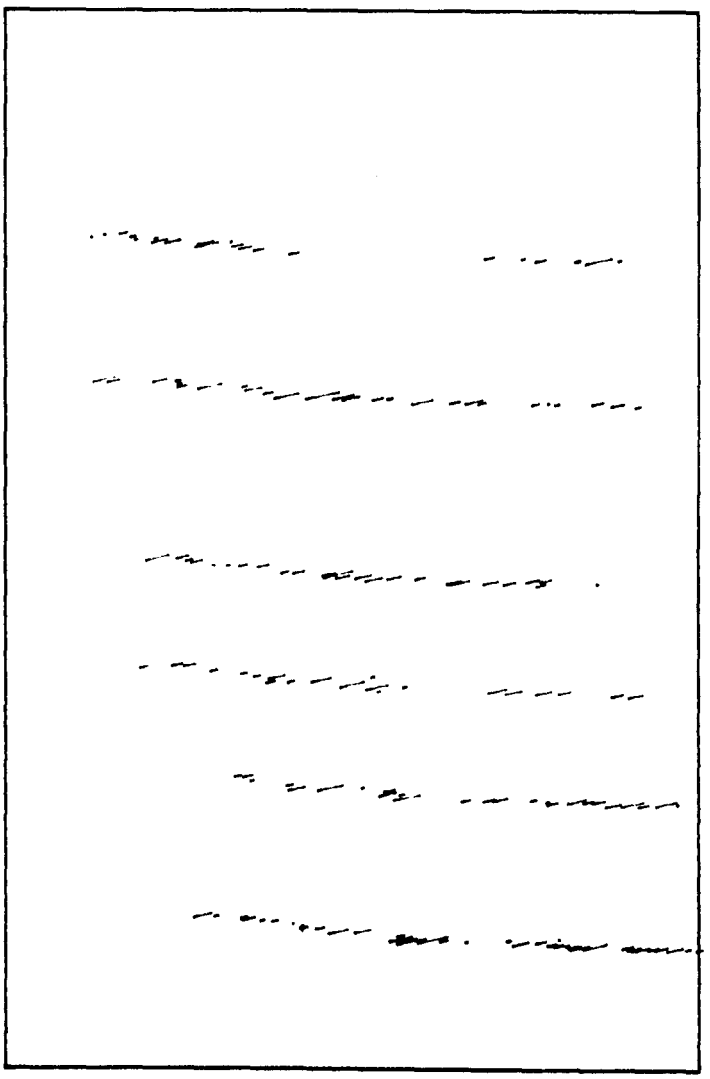

LOCATIONS OF THE DRILLHOLE INTERSECTIONS WITH
THE OREBODY (NICKEL VEIN DEPOSIT).
DATA POINT LOCATIONS
210
205
200
195
190
185
180
175
170
R.L (M)
60
70
80
90
100
X10¹
NORTHINGS (M)
07/08/80
A.A.

Figure: 2.21

DRILLHOLE INTERSECTIONS WITH THE OREBODY (NICKEL VEIN) IN 3-D



DRILLHOLE INTERSECTIONS (3-D)

07/08/90

A.A.

Figure: 2.22

Table: 2.2 Statistics of east-west horizontal accumulations

Total number of samples	156
Mean	15.16
Median	10.45
Variance	103.70
Range of values	0.0 - 45.76

The apparent difference between geometric data (drillhole intersections with the orebody) and the attributes (grade assays) in terms of distribution, homogeneity and importance is one of the reasons for suggesting the treatment of these two kinds of data by different methods. However, the single geostatistical method is not to be excluded as long as the results obtained are satisfactory.

The nickel grade accumulation data are obtained by multiplying the nickel grades by the thicknesses at the mid-points of the intersections of the drillholes with the orebody. Statistics of accumulations are given in Table 2.2.

The variogram of accumulations was calculated in various strike-dip planes; the best results were obtained with a strike of 15° east of north and a dip of 80° west (Dowd & Scott 1987). The results of variogram calculation are summarised in Table 2.3 and figure 2.23.

Table: 2.3 Parameters of spherical model fitted to the accumulations variogram

Model	Spherical
Co	22.75
C	95.0
Range (stike-plunge)	30.0 m

Co = nugget variance

C = structural variance, total variance is (Co + C)

A = range of influence (strike-plunge direction).

Using the KTHAC module in the GEOSTAT2 geostatistical package and the parameters deduced from the structural analysis of the accumulation variable, the estimated values of this variable are shown in Table 2.4.

Table: 2.4 deduction of grade values from estimated accumulations and thicknesses.

Northings	**Elevations**	**Eastings**	**Thickn--esses**	**E* Accum**	**Ni % grade**
19132.50	1380.00	16184.04	2.45	12.74	5.20
19132.50	1382.11	16183.16	2.06	12.83	6.23
19132.50	1384.22	16182.28	1.23	7.07	5.75
19132.50	1386.66	16181.60	1.78	8.70	4.89
19132.50	1388.88	16185.45	2.25	15.79	7.02
.....	.....	.....	....	....	....

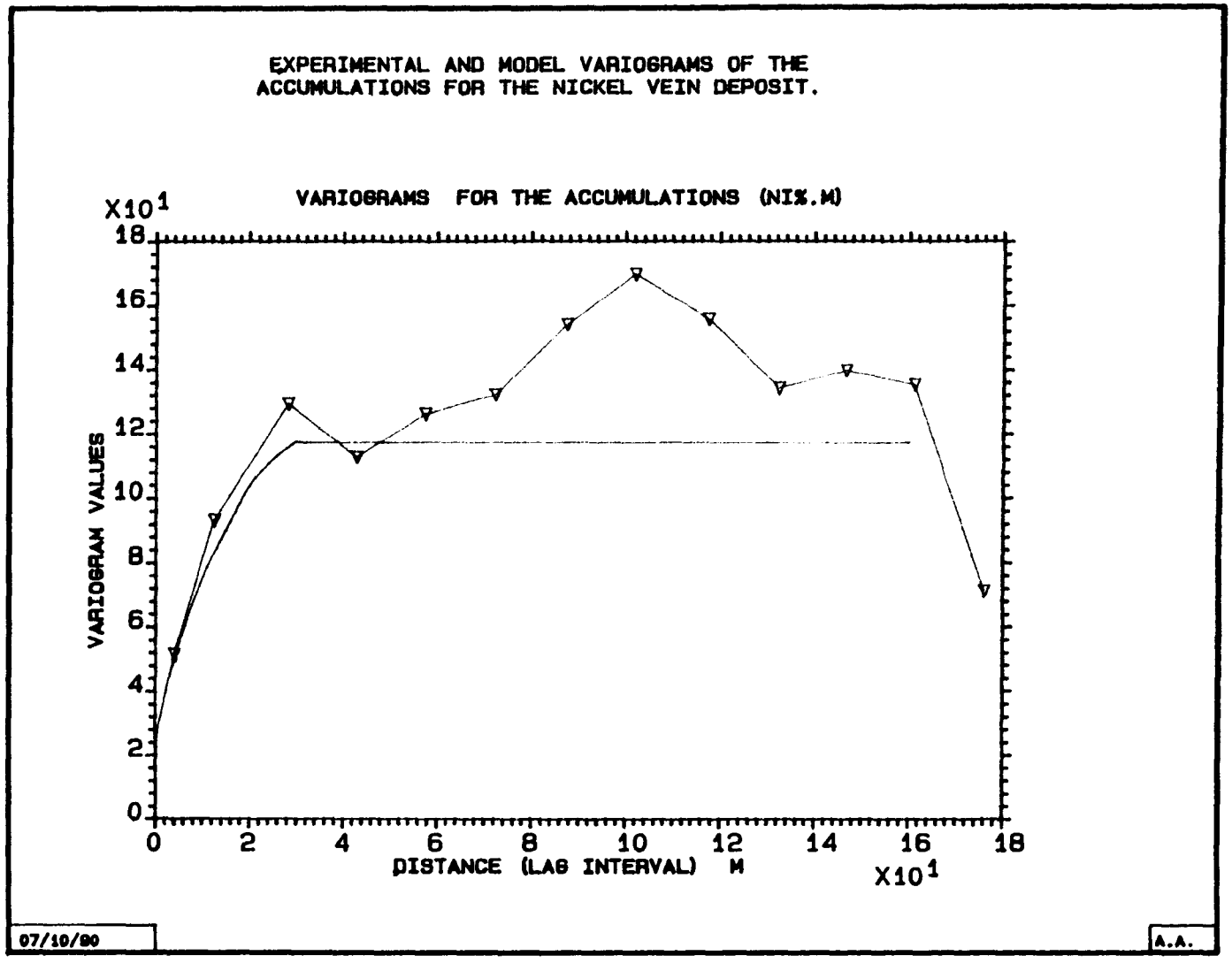

EXPERIMENTAL AND MODEL VARIOGRAMS OF THE
ACCUMULATIONS FOR THE NICKEL VEIN DEPOSIT.
VARIOGRAMS FOR THE ACCUMULATIONS (NI%.M)
X10 1
VARIOGRAM VALUES
0
2
4
6
8
10
12
14
16
18
DISTANCE (LAG INTERVAL) M
X10 1
07/10/90
A.A.

Figure: 2.23

2.6 Summary

To investigate the validity and the representativeness of the raw data is itself a part of the modelling process applied to estimate an orebody. For geometrical modelling applications, the amount of data and its configuration are very important characteristics. When evaluating stratigraphic orebodies the main objective is to determine the geometry of the deposit and therefore it is advisable to relate the raw data to general geometrical features of these types of orebodies such as their morphology and formation. The configuration and the density of data locations are also decisive in determining more robust results especially when using deterministic methods based on local procedures.

Finally, as the attributes (grades) to be deduced from a two step modelling process (deterministic and geostatistical), the accumulation estimates are presented in order to understand the deduction method of the grade values.

CHAPTER THREE

Chapter Three

Geometrical Modelling

3.1 Introduction

The concept of geometrical control as a representation or a design of the shape of a certain domain in which a dynamic or static phenomenon is taking place is an important step towards a better understanding of the phenomenon itself.

Many of such approaches have been adopted extensively in the last two decades, mainly to design physical objects or environments to contain simulated processes and also to design objects to be manufactured (i.e., CAD/CAM). The geometrical control approach is used to control relative movements of dynamic systems (i.e; robotics, flying control, ...) by providing the instant position of the moving object relative to the reference domain.

Geometrical control is to be understood as a modelling operation implemented to control a process. Geometrical modelling, as a combination of mathematical tools and the computational power of a computer, is rapidly increasing in many fields; it is a primary ingredient in computer aided design (CAD) and computer aided manufacturing (CAM) systems, computer graphics, simulation, computer vision, robotics, etc...

The implementation of geometrical modelling techniques differs from one application to another. These applications are of two types:

1) representing realistic, geometric models of things that already exist,

2) design of geometric models of non_existing things and objects. In the case of geometrical control of geostatistical estimates in stratigraphic orebodies, which

is the subject of this research, the first type is of concern to us because of the idea of controlling these estimates by representing the geometry of the containing strata.

The geometrical model representing a stratigraphic orebody or series of stratigraphic units will allow the quick and automatic derivation of any geometric property and control any attribute that the strata are likely to possess. To model the geometry of the strata is mainly to represent the strata from existing information, such as drillhole intersections with the hangingwall and the footwall or intersections with a succession of stratigraphic units cf. figure 2.1[a,b].

After defining the concept of geometrical control in stratigraphic orebodies, this chapter will illustrate, step by step, the key points involved in constructing a geometrical model of a single stratum. These steps are as follows:

1 - Data acquisition and configuration. Primary information should be well understood, checked, validated and, if possible, enhanced without distorting the information cf. chapter 2.

2 - Surface modelling techniques, as mathematical tools to obtain the desired shape: a review of some well known techniques will be given.

3 - The adopted methods. Two methods of surface modelling techniques have been considered for this application.

4 - Numerical application (case study). A Nickel vein deposit has been chosen to test the applicability of this geometrical modelling system, with the objective of controlling the geostatistical estimation of grade values and of course the deduction of the shape of the vein.

3.2 Surface Modelling Techniques

3.2.1 Introduction

Very many problems of form description are a particular case of the following general problem: how can one construct a mapping from the images of a given set of points? There is no unique solution to such a problem, the characteristics of the chosen method of representation (interpolation or approximation) are determined by the context of the particular problem. Furthermore, the technique chosen should depend on the nature of the data and the nature of the phenomenon (as far as it is known).

Many authors have reviewed the existing surface modelling techniques in general and interpolation of scattered data in particular Franke(1982), Barnhill(1983), Sabin(1985) and others.

To simplify the review of some of the interpolation methods developed to deal with the representation of scattered data in two or more independent variables, a simple classification is required. There are two major methods for interpolating a function f(x,y)=Z known at some data points in R^2:

a) Interpolation methods based on global procedures.

b) Interpolation methods based on local procedures.

3.2.2 Global versus local interpolation techniques.

Global refers to modelling procedures based on taking account of all data points to interpolate at any point on the modelled surface.

Local refers to modelling procedures based on taking account only of the data points forming a specific local domain around the point to be interpolated.

The key points in this comparison between the two techniques of interpolation are mainly to do with:

-data

-physical phenomenon

-computational effort.

The density of data points is a significant factor in selecting an interpolation method as global methods are very expensive for large sets of data points, which is the case in geological studies. Although voids and missing information represent a big problem for both methods, the global methods, in most cases, impose the global trend over these regions (voids), the local methods are more sensitive especially over larger voids as shown in figure 2.12. In some cases global techniques require data filtering which in a sense implies the removal of "extraneous" maxima and minima to allow the identification of the underlying trend with some degree of confidence, for example removing selected high frequency components in the data to allow the fitting of a sinusoidal function to represent the actual trend. However, the global procedure of interpolation is widely accepted and adopted in cases of modelling a physical phenomenon where the trends are either apparent or, for physical reasons, known to be present. For these problems, a trend in the data is usually identified, either by "eye-balling" or by analysis. The trend may be polynomial, exponential, oscillatory, etc..., and when identified, a global least squares fit is made of the appropriate simple surface. Local interpolation procedures, on the other hand, depend only on the local data points with a decided advantage in that additional data points can be added with the effect of this addition confined to some small neighbourhood of the new points, which means it is not necessary to recalculate the whole of the interpolated surface.

The computational effort, which is an easily understood criterion, will depend on the magnitude of the computation involved. It is very significant with surface fitting problems using a very large number of data points. However, precise comparisons are made more difficult between local and global methods of interpolation because of the variability of performance with different computer systems, different programming languages, different algorithms and levels of programming skills. So, one or two general statements can be made. It is clear that, for either local or global techniques, the computational effort increases rapidly with the degree of the fitted surface. Fitting techniques that are local will generally offer a significant computational advantage over global techniques.

Taking into consideration the above comparison and the nature of the physical phenomenon which will be under study (stratigraphic orebodies), the choice of local methods looks more advantageous. In addition it is perhaps more common that the functional form of such natural phenomena is not specified in advance to allow the user to adopt the global procedure of interpolation.

3.2.3 Spline interpolation techniques

3.2.3.1 Introduction

The earlier developments in spline theory used for surface modelling, either for representation or design, have concentrated on one method of generating these splines which is on regular patches, mainly rectangular. However, the common case encountered in modelling surfaces defining the stratigraphy of orebodies are those of triangular patches constructed from irregularly spaced data points. Thus, the following investigations and choice of interpolation methods will be concentrated on methods using irregularly distributed data points.

A first step to a piecewise surface modelling technique is the obvious possibility of explicitly dividing the surface into regions along some deliberately chosen boundaries, and fitting some known equations over those regions with a known continuity across all boundaries. Triangles are the most convenient such regions to be considered.

3.2.3.2 Triangulation

Many interpolation problems, whether on the line or in the plane, require that a curve or a surface be assigned function values at nodes that are irregularly spaced. When the interpolation is on the line this presents no great difficulties, but if the data points are scattered on the plane, the situation is not so clear. One useful way to handle this problem is to construct a triangulation of the area formed by the data points in such a way that the set of triangle vertices is precisely the set of data points. In the beginning, triangulation had to be imposed manually, now it may be calculated automatically by applying some optimality criterion.

Many triangulation algorithms have been developed, cf. Sibson (1978), Green and Sibson (1978), Bowyer (1981), Watson (1981), etc..., using different approaches but the most well-known method is the Delaunay triangulation method which will be considered next. In spite of their potential for modelling complex parts, the construction of triangular patches has not found its way into any significant number of CAD/CAM systems for instance. They are still good candidates for complex surfaces and therefore a potential candidate for further development and use.

Quoting from Petrie (1987) the Delaunay triangulation method is described here based on Theissen polygons cf. figure 3.1 defining geometrically the region of influence of a point on an areal basis. This is done by constructing a series

of perpendicular bisectors on the triangles formed around that specific point. These intersect are the Thiessen vertices. The polygons so defined are the Theissen polygons. Furthermore, the data points surrounding a specific data point are known as its Theissen neighbours. Very often, a preliminary step before triangulation begins is to define a set of dummy boundary points to form a perimeter around the edges of the data set area cf. Petrie(1987). This is necessary to create a frame around the modelled surface and a set of boundary triangles which allow points outside the sampled region to be extrapolated. Whether or not these dummy points are added, triangulation of the whole area is usually started with a pair of dummy points known as the initial known neighbours (A,B) located in the bottom left hand corner of the area cf. figure 3.3 and 3.4.

The search for the next neighbour is then made by constructing a circle with the base AB as diameter and then searching to the right (i.e., clockwise) to find if any point falls within this circle. This search can be carried out quite quickly using a computer. If no data point lies within the circle, it is increased in size to perhaps twice the area of the original circle with AB now as a chord in the larger circle cf. figure 3.2. Any data points lying within the new circle are tested to discover which point meets the criteria set for the nearest Theissen neighbour.

Once this has been achieved, the search for the next neighbours then continues to the right then to the next neighbour and so on till the next boundary point is reached. The triangles so formed constitute the so called shell, cf. Petrie (1987).

Theissen polygon

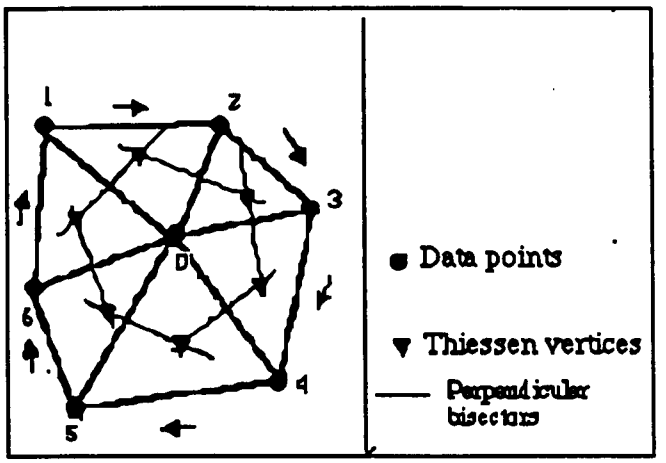


Figure: 3.1

Expending circle searching for the nearst neighbour

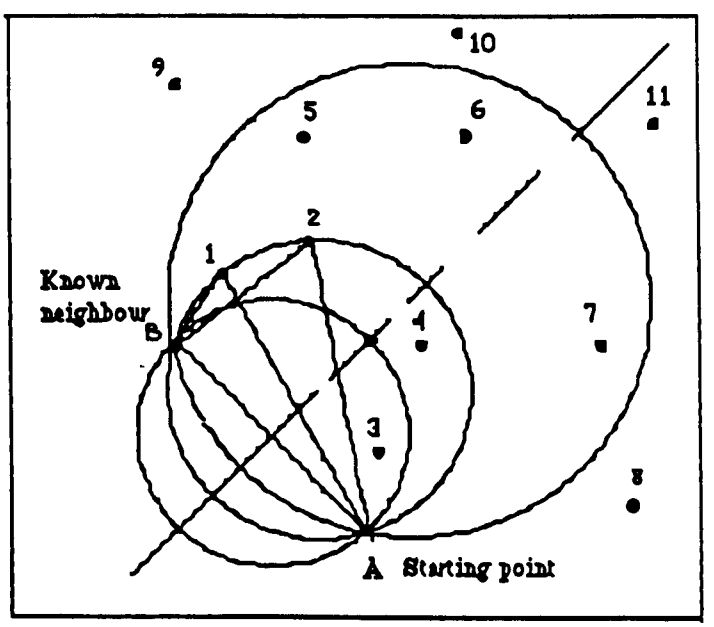


Figure: 3.2

Delaunay Triangulation

Data point location

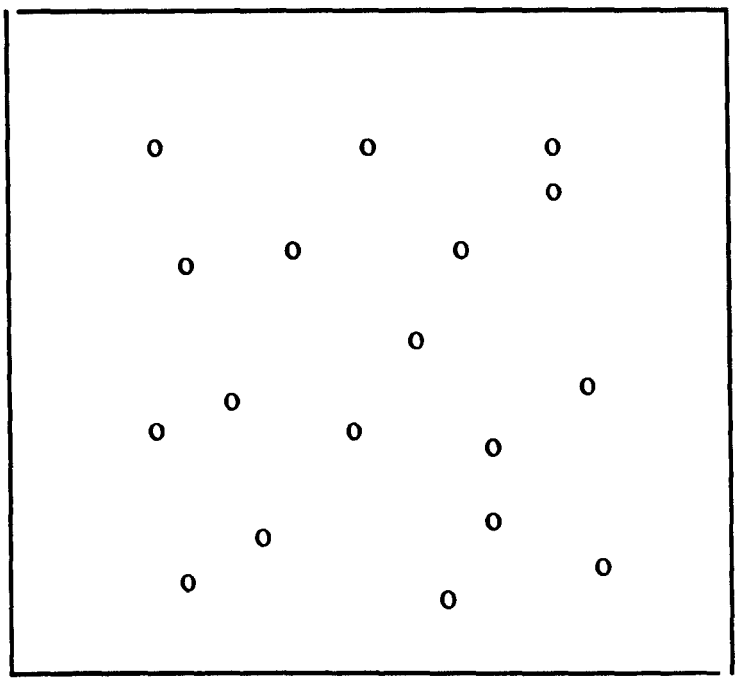


Figure: 3.3

D

Triangulated surface

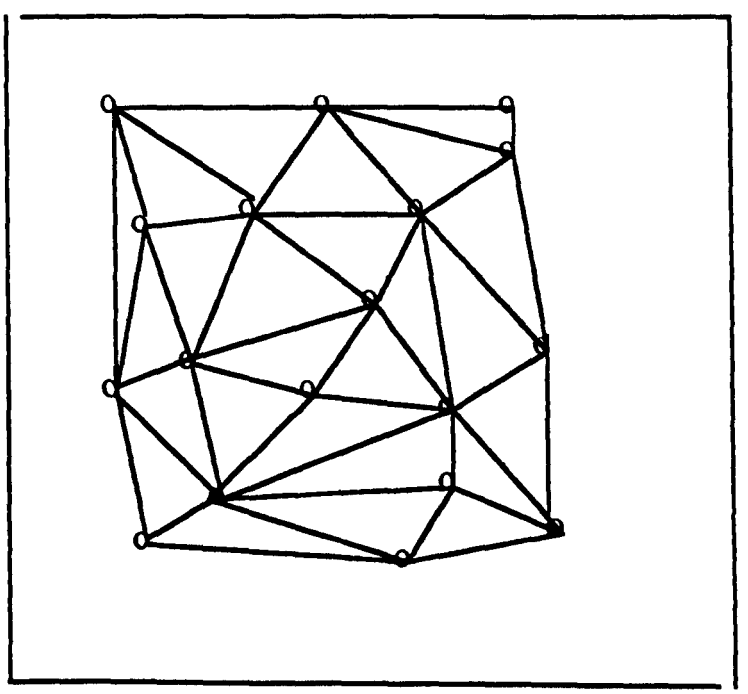


Figure: 3.4

The process of triangulation then continues with each point in the shell being used in turn as the starting point for the search for the next set of Theissen neighbours; this continues in a systematic manner until the neighbours for all the points existing in the data set have been found and the corresponding triangles formed.

Now that it is better known and understood, the Delaunay method is the one used to perform triangulation in the majority of surface modelling packages based on the triangulation method.

3.2.3.3 Bivariate interpolation over triangular patches

Polynomials in two variables as basis functions play the key role in surface representation. A few known basis functions (monomials) are listed in table (3.1), where Z_i is, for instance, the height of an individual point i,xi and yi are the rectangular coordinates of the point i and a_0, a_1, a_2,... are the coefficients of the polynomials. One of such equations will be generated for each individual point i with coordinates xi, yi and zi occurring in each patch.

Table: 3.1 Basis functions of different polynomials forms

individual terms	order of terms	descri--ptive terms	Num of terms	Num of coeff
$Z_i = a_0$	Zero/ P_0	planar	1	1
$+ a_1x + a_2y$	First/ P_1	linear	2	3
$+ a_3x^2 + a_4y^2 + a_5xy$	Snd/ P_2	quadra	3	6
$+a_6x^3 + a_7y^3 + a_8x^2y + a_9xy^2$	Thrd/ P_3	cubic	4	10
$+a_{10}x^4 + a_{11}y^4 + a_{12}x^3y + a_{13}x^2y^2 + a_{14}xy^2$	Frth/ P_4	quarti	5	14
$+ a_{15}x^5 + , \ldots$	Fifth/ P_5	Quinti	6	21

Thus, any function p in the class P_2, for example, must be of the form: $p(x,y)=a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2$ for some real numbers (coefficients) a_0, a_1, a_2, a_3, a_4 and a_5.

More generally, P_n is the class of polynomials containing all functions of the form x^iy^j, where $0 \le (i+j) \le n$ and $i \ge 0; j \ge 0$. In fact, for any n, the number of basis functions in P_n is $1+2+3+4+,\ldots,+(n+1)=\frac{1}{2}(n+1)(n+2)$. If a third rectangular axis Z is introduced along x, y Z= p(x,y) will determine the surface over any domain in x,y.

Because of the irregularly distributed data and the versatility of interpolation methods over triangular patches to represent a surface, the discussion will be limited to these kinds of local methods.

In the following sections some methods will be examined briefly to mention their existence; the adopted methods will be exposed in more detail.

a) Bilinear interpolation over triangular patches

This is a very primitive method. Suppose that there are three given non-collinear points in the plane x,y. It is geometrically obvious that the three function values f_0, f_1 and f_2 assigned to three such points A, B and C respectively, determine a unique triangle in space cf. figure 3.2. The equation of the plane containing this triangle has the following form

$$Z(x,y) = a_1 + a_2x + a_3y \qquad (3.1)$$

for some coefficients a_1, a_2 and a_3. If A, B and C have coordinates $(x_0,y_0),(x_1,y_1)$ and (x_2,y_2) respectively, then the function on the right of this equation is precisely the bivariate linear polynomial $p(x,y)$ for which

$p(x_0, y_0) = f_0$, $p(x_1, y_1) = f_1$ and $p(x_2, y_2) = f_2$. Thus, given A, B and C and f_0, f_1 and f_2, the coefficients a_1, a_2 and a_3 are determined by three simultaneous equations

$$\begin{aligned} a_1 + x_0 a_2 + y_0 a_3 &= f_0 \\ a_1 + x_1 a_2 + y_1 a_3 &= f_1 \\ a_1 + x_2 a_2 + y_2 a_3 &= f_2 \end{aligned} \tag{3.2}$$

These have the explicit solution in terms of determinants

$$a_1 = \Delta^{-1} det \begin{bmatrix} f_0 & x_0 & y_0 \\ f_1 & x_1 & y_1 \\ f_2 & x_2 & y_2 \end{bmatrix}$$

$$a_2 = \Delta^{-1} det \begin{bmatrix} 1 & f_0 & y_0 \\ 1 & f_1 & y_1 \\ 1 & f_2 & y_2 \end{bmatrix}$$

$$a_3 = \Delta^{-1} det \begin{bmatrix} 1 & x_0 & f_0 \\ 1 & x_1 & f_1 \\ 1 & x_2 & f_2 \end{bmatrix}$$

where

$$\Delta = det \begin{bmatrix} 1 & x_0 & y_0 \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix}$$

It can be shown that $\Delta = 0$ precisely when triangle ABC has an area equal to zero, and then there is no longer a unique solution for the equation (3.1). This expresses in mathematical terms the geometrically obvious condition that, for unique linear interpolation, the data points A, B and C must not be collinear. The interpolation can be done on a standard triangle and then transformed to any given triangle T. One such transformation (known as an affine transformation) is:

$$u = a_{11}x + a_{12}y + b_1$$

$$v = a_{21}x + a_{22}y + b_2 \tag{3.3}$$

$$a_{11} = \frac{2y_2 - y_1 - y_0}{\Delta}$$

$$a_{12} = \frac{-2x_2 + x_0 + x_1}{\Delta}$$

$$b_1 = \frac{x_2(y_1 + y_0) - y_2(x_1 + x_0)}{\Delta}$$

$$b_2 = \frac{2}{\sqrt{3}} + \frac{\sqrt{3}(x_2(y_1 - y_0) - y_2(x_1 + x_0))}{\Delta}$$

$$a_{21} = \frac{-\sqrt{3}(y_1 - y_0)}{\Delta}$$

$$a_{22} = \frac{\sqrt{3}(x_1 - x_0)}{\Delta}$$

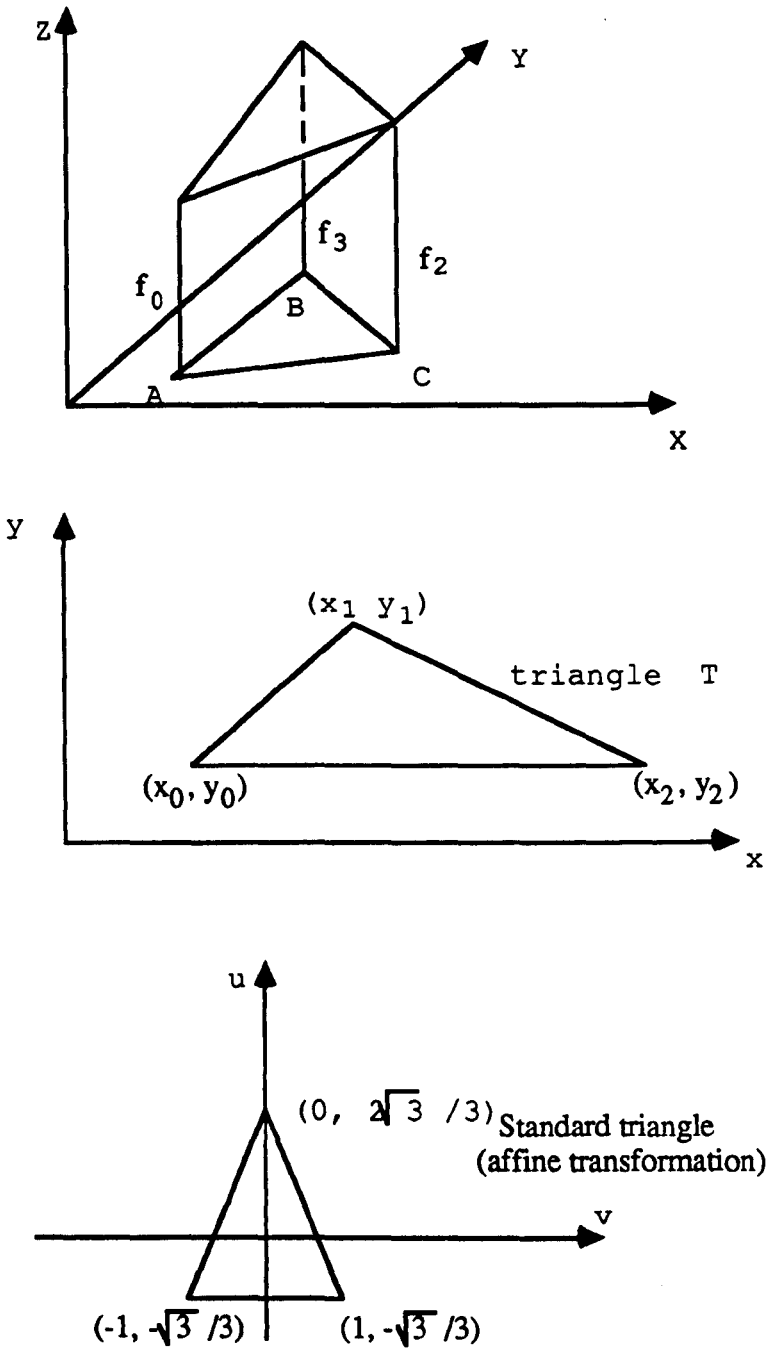

Z
Y
X
f_0
f_2
f_3
A
B
C
y
x
$(x_1\ y_1)$
triangle T
(x_0, y_0)
(x_2, y_2)
u
v
$(0,\ 2\sqrt{3}\ /3)$
Standard triangle
(affine transformation)
$(-1, -\sqrt{3}\ /3)$
$(1, -\sqrt{3}\ /3)$

Figure: 3.5..

This will map the triangle T in the x-y plane cf. figure 3.5 into the standard equilateral triangle in the u-v plane.

b) Higher degree polynomial interpolation over triangles

Implementing higher degree polynomial interpolation schemes means looking for continuity and smoothness of the surface represented (modelled). Because of the lack of continuity, either on vertices or at mid-points on the edges of the triangular patches, in using either linear quadratic or cubic polynomials Lancaster (1986), the quintic scheme seems more appropriately characterized by its continuity in all directions either on the vertices (data points) or at the common edges of the adjacent patches. However, the difficulties encountered in estimating the partial derivatives have always been the problem of interpolation in general. Among many algorithms for implementing this scheme Akima (1975) has proposed a method of bivariate interpolation of scattered data which is based on the fifth degree polynomial fitted over a triangular cell where partial derivatives are estimated using local data surrounding each data point. This method is the second method to be adopted in this research as a surface modelling technique using bilinear methods. It will be examined in more detail later on.

c) Triangle based blending methods

These kinds of methods are conceptually the same as the inverse distance weighting methods except that the weighting functions for these methods are based on triangulation of the data points set Franke (1982). Assume a triangulation of a set of irregularly spaced data points, and suppose that $(x,y) \in T_{ijk}$ where T_{ijk} is the triangle with the vertices (x_i,y_i), (x_j,y_j) and (x_k,y_k).

Then

$$F(x,y) = w_i(x,y)Q_i(x,y) + w_j(x,y)Q_j(x,y) + w_k(x,y)Q_k(x,y) \qquad (3.4)$$

Where the weight functions $w_m(x,y)$, $m=i,j,k$ may be viewed as nine-parameter cubic shape functions with a rational correction to obtain normal derivatives equal to zero to ensure continuity. The weight functions used here are obtained from the minimum norm problem, cf. Nielson (1980). Let b_i, b_j and b_k be barycentric coordinates of (x,y) in T_{ijk} and let l_i, l_j and l_k be lengths of the sides opposite vertices i, j and k respectively. Then the weight function is given by

$$w_k(x,y) = b_k^2(3-2b_k) + 6b_ib_jb_k(\alpha_{kj}-\alpha_{ki}) \tag{3.5}$$

with

$$\alpha_{kj} = \frac{b_kb_j(1+b_i)}{(1-b_i)(1-b_k)}\left[\frac{l_k^2+l_i^2-l_j^2}{2l_i^2}\right]$$

$$\alpha_{ki} = \frac{b_kb_i(1+b_j)}{(1-b_j)(1-b_k)}\left[\frac{l_k^2+l_j^2-l_i^2}{2l_j^2}\right]$$

and other weight functions are obtained by cyclic permutation of the indices. The Q_n in (3.4) can be taken to be any function having the required property (smooth, at least continuous first partial derivatives). Franke (1982) has commented that this method is faster than the same method applied on rectangle based blending methods. However, one of the disadvantages of this method is that the existence of long, slim triangles sometimes yields surfaces which appear to have discontinuities along these triangles because of the very rapid changes in function value across the narrow part.

d) Bezier methods

Bezier methods is the name given to a class of methods of representing piecewise polynomials in terms of a "net" of control points. These surfaces do not interpolate to these points, but rather the shape of the surface is controlled

by them. They are surface representation methods based on rectangular and triangular patches. A major breakthrough was Farin's (1982) creation of Bezier schemes for general triangles. In this section a very passing and short description of the Bernstein-Bezier method, as illustrated by Farin (1982), is given.

Bivariate Bernstein polynomials are defined in terms of barycentric coordinates of some fixed arbitrary triangle T.

Farin has adopted a special notation which is reproduced here to give a better understanding of his presentation of the method. Boldface letters always denote vectors with three coordinates in particular

$$\mathbf{i} = (i_1, i_2, i_3)$$

$$\mathbf{i}_l = (l_1, l_2, l_3)$$

$$\mathbf{j}_1 = (1,0,0); \quad \mathbf{j}_2 = (0,1,0); \quad \mathbf{j}_3 = (0,0,1)$$

Let a triangle in the x,y plane be defined by its three vertices $P_1, P_2, P_3; P_i \in R^2$.

Let P be an arbitrary point in the x y plane ; it has the unique representation:

$$P = u_1 P_1 + u_2 P_2 + u_3 P_3; \quad u_1 + u_2 + u_3 = 1 \tag{3.6}$$

and

$$u_1, u_2, u_3 \geq 0$$

then it is known that P has barycentric coordinates **u** with respect to T. Barycentric coordinates provide a very suitable coordinate system for arbitrary triangles in the plane since they are invariant under affine transformation.

$$B_{\mathbf{i}}^{n}(\mathbf{u}) = \frac{n!}{i_1!i_2!i_3!} u_1^{i_1} u_2^{i_2} u_3^{i_3} \tag{3.7}$$

are the bivariate Bernstein polynomials of degree n defined in terms of barycentric coordinates. $B_{\mathbf{i}}^{n}(\mathbf{u}) = 0$ if at least i_k violates this condition $0 \le i_k \le 1$, $k = 1,2,3$. Since the $B_{\mathbf{i}}^{n}(\mathbf{u})$ are terms of the binomial expression $(u_1 + u_2 + u_3)_n$ therefore

$$\sum_{|\mathbf{i}|=n} B_{\mathbf{i}}^{n}(\mathbf{u}) = \sum_{i_1,i_2,i_3 \ge 0; i_1+i_2+i_3=n} B_{i_1,i_2,i_3}^{n}(u_1,u_2,u_3) = 1 \tag{3.8}$$

$$B_{\mathbf{i}}^{n}(\mathbf{u}) \ge 0 \quad if \quad u_k \ge 0; \quad k = 1,2,3$$

The sum in (3.8) consists of $\frac{1}{2}n(n+1)$ terms. This is the dimension of the linear space of bivariate polynomials of total degree n, and the B_i^n can be shown to provide a basis for this space. Thus, any polynomial p has the unique representation:

$$p(\mathbf{u}) = \sum_{|\mathbf{i}|} B_{\mathbf{i}}^{n}(\mathbf{u}) \tag{3.9}$$

This form is the bivariate Bernstein-Bezier surface, and the evaluation of p(u) can be found in Farin (1983) with more details of the evaluation method of p(u) over adjacent triangles forming the surface to be represented.

It is beyond the scope of this thesis to review all existing methods of interpolating scattered data based on local procedures, however it is worth mentioning some of them, in particular: Inverse distance weighting methods, Clough-Tocher method, Methods based on Coons's patches, Piecewise (cubic

,quadratic, ...) triangular elements. It is also of importance to mention the review of these methods done by Franke (1982) who set up a user guide to a wide range of methods.

3.2.4 Adopted modelling techniques

3.2.4.1 Introduction

Two methods of surface modelling techniques have been adopted for geometrical control of geostatistical estimates in stratigraphic orebodies, both of them are performed on triangular cells as the data dealt with are in most cases irregularly distributed,

i-Bilinear interpolation

ii-Bivariate interpolation based on the Quintic scheme.

These methods are not selected according to the degree of performance but rather to the method of control of the estimates and to the implementation and the availability of such techniques in the form of computer routines to be implemented in the modelling system.

3.2.4.2 Bilinear interpolation

This is based on fitting a bilinear polynomial over a triangular domain. The theory is very simple and it is presented in more detail in section 3.3.3.3.a; in addition, its implementation is very simple and straightforward. It consists of triangulating the surface to be represented so as to obtain a set of triangles which will serve as adjacent domains on which a bilinear function $f(x,y)=a_0+a_1x+a_2y$ is fitted. Then a regular grid is generated all over the surface; each grid point is then checked to determine the triangle face on which it falls and finally, the function $f(x,y)$ is estimated at that grid point using the polynomial fitted over that specific triangle.

Applied to the modelling of hangingwalls and footwalls of stratigraphic orebodies, the technique performs well; it is cheap in run time and is fairly robust to the effects of the large voids which characterize some data sets. However, the major problem which is encountered is the poor smoothing aspect of the surface obtained; in the case of highly erratic orebodies this may be of little significance.

3.2.4.3 Bivariate interpolation "*Quintic scheme*"

This method is known as Akima's method and is based on the previous work of Zenisek and Zlamal, cf. Akima (1978), who introduced the idea of approximation procedures using fifth degree polynomials in the x-y plane over triangular regions. To determine the coefficients of the polynomial to be fitted over each triangle the data point values are defined as function values z=f(x,y) and the estimated first and second order partial derivatives (i.e, $z_x, z_y, z_{xx}, z_{xy}, z_{yy}$) at the three vertices of the triangle are used with the three normal derivatives at the mid-points of the sides of the triangle.
This method will be examined in more detail in this section; step by step calculations of all the coefficients are given in the appendix at the end of this thesis. Triangulation techniques have already been discussed in section 3.2.3.2.

The interpolation of Z values in a triangle is based on the following assumptions, which are quoted here from Akima (1978)

i-the value of the function Z(x,y) at point (x,y) in a triangle is interpolated by a bivariate fifth-degree polynomial in x and y (Quintic Scheme).

$$Z(x,y) = \sum_{j=0}^{5} \sum_{k=0}^{5-j} q_{jk} x^j y^k$$

note there are 21 coefficients (N_{co}) to be determined ($Nco = \frac{1}{2}(n+1)(n+2); \quad n=5, \quad N_{co}=21$).

ii- The values of the function and its first and second order partial derivatives (i.e. $Z, Z_x, Z_y, Z_{xy}, Z_{xx}, Z_{yy}$) are given at each vertex of the triangle. This assumption yields 18 independent conditions.

iii- The partial derivatives of the function are differentiated in the direction perpendicular to each side of the triangle. This assumption yields another three additional conditions. This assumption also assures smoothness of interpolated values, in this case the coordinate system is transformed to another cartesian system, which is called the s-t system, in such a way that the s axis is parallel to each of the sides of the triangle. Since the coordinate transformations between the x-y system and the s-t system are linear, the values $Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}$ at each vertex uniquely determine the values of $Z_s, Z_t, Z_{ss}, Z_{st}, Z_{tt}$ at the same vertex, each of the latter is a linear combination of the former. The Z, Zs and Zss values at two vertices uniquely determine a fifth-degree polynomial in s for Z on the side between these two vertices. Since two fifth-degree polynomials in x and y, representing Z values in two triangles that share the common side are reduced to fifth-degree polynomials in s on the side, these two polynomials in x and y coincide with each other on the common side. This proves continuity of the interpolated Z values along a side of a triangle. Similarly, the values of Z_t and $Z_{st} = (Z_t)_s$ at two vertices uniquely determine a third-degree polynomial in s for Z_t on the side. Since the polynomial representing Z_t is assumed to be third-degree at most with respect to s, two polynomials representing Z_t in two triangles that share the common side also coincide with each other on that side. This proves continuity of Z_t and thus smoothness of Z along the side of the triangle.

The procedures of estimating the five partial derivatives locally at each data point are not unique. Akima's procedure of estimating partial derivatives is performed in two steps, the first-order derivatives in the first step and the

second-order derivatives in the second step. To estimate the first-order partial derivatives at data point P_0, several data points are used $P_i(i=1,2,\ldots,n_c)$ the projections of which are closest to the projection of P_0 selected from all data points other than P_0. Then two data points P_i and P_j out of n_c are taken to construct the vector product of $\overline{P_0P_i}$ and $\overline{P_0P_j}$, a vector that is perpendicular to both vectors with the right-hand rule and has a magnitude equal to the area of the parallelogram formed by $\overline{P_0P_i}$ and $\overline{P_0P_j}$. P_i and P_j are taken in such a way that the resulting vector product always points upward (z component is always positive). Then the vector products of all possible combinations of $\overline{P_0P_i}$ and $\overline{P_0P_j}(i \neq j)$ are constructed and thus the vector sum of all these vectors is obtained. It is then assumed that the first-order partial derivatives Z_x and Z_y at P_0 are estimated as those of the plane that is normal to the resultant vector sum. When $n_c=2$, the estimated Z_x and Z_y are equal to the partial derivatives of a plane that passes through P_0,P_1 and P_2. Also, when $n_c=3$ and the projection of P_0 in the x-y plane lies inside the triangle formed by the projections of P_1,P_2 and P_3, the estimated values of Z_x and Z_y are equal to the partial derivatives of a plane that passes through P_1,P_2 and P_3.

In the second step, partial differentiation described in the preceding paragraph, is applied to the estimated Z_x values at $P_i(i=1,2,\ldots,n_c)$ to obtain estimates of $Z_{xx}=(Z_x)_x$ and $Z_{xy}=(Z_x)_y$ at P_0 and the same procedure is used to obtain $Z_{yy}=(Z_y)_y$ and $Z_{xy}=(Z_y)_x$. Then a simple arithmetic mean of the two values Z_{xy} is obtained to estimate the Z_{xy} at P_0.

The choice of n_c is also not unique; it must not be less than 2 but it must be less than the total number of data points. Akima (1978)has proposed a value between 3 and 5 but Franke (1982) in testing this method suggested the value of 6. However, there are no strict rules to respect in choosing the n_c

value. The choice is mainly to do with the data configuration (i.e, density and homogeneity of distribution of data point locations to cover the whole area). In the case of the nickel vein deposit chosen as a numerical example for this research and after several trials $n_c = 4$ is the best guess.

The determination of the 21 coefficients of the function basis for the bivariate interpolation method (quintic scheme) is reported in the appendix.

3.3 Application

3.3.1 Introduction

As an example of numerical application the vein nickel deposit described in chapter 2 has been chosen to illustrate the application of orebody modelling and possible subsequent geometrical control of geostatistical estimates. To obtain a geometrical model for this specific orebody, two surfaces delineating the orebody, are modelled. The hangingwall and the footwall are then joined together to form the overall model of the vein. This model can then be used to impose geological (geometrical) control on geostatistical estimates of grades or other characteristics.

The results of this application will be presented in the form of cross-sections, plane-sections and wireframe and solid models; characterizing the geometry of the orebody. Both methods, bilinear and bivariate quintic interpolations, have been used.

3.3.2 Data

In this application the variables characterizing the location of the orebody (vein) are the thicknesses indicated by the intersections of the drillholes with the orebody, see table 2.1 for a sample of the data set. The data consist of the coordinates x,y,z of each intersection point of each drillhole with each surface

of the vein and the mean grade value of nickel (% Ni) recorded from the drill-hole core intersecting the vein. These data have been rearranged to form two data sub-sets, one representing the hangingwall the second the footwall. Figure 3.6 shows the data point locations on the major plane of the vein (strike) and figure 3.7 shows the drillhole intersections with the vein in three-dimensional space. The data are irregularly distributed representing effectively the surfaces to be modelled.

3.3.3 Results

Once the files characterizing the hangingwall and the footwall of the vein are created, the modelling process starts by running the modelling system using the bilinear method of interpolation and then the bivariate interpolation based on the quintic scheme. This order of execution is an option for the user to obtain quick results using the first method despite the poor quality of the surface obtained. Then final and better results are obtained by using the second method despite the run time which it takes to produce the modelled surface. Time is, of course, only critical when the system is run on a Personal Computer.

To obtain the numerical model of the vein using the first method the program is run at least twice using the data file for the hangingwall and then the data file for the footwall. Then the two separate surfaces obtained are joined together to form the numerical model of the orebody. The steps in the algorithm for this first method are simple and consist of:

- reading the data points
- defining the grid points where the interpolation is to be performed
- triangulation of the data points figure 3.8
- interpolation at a grid point using the equation fitted over the triangle on which this point falls

- writing and storing the results of each surface
- the numerical model is obtained in the form of a single file by combining the results representing the two surfaces (hangingwall and footwall) table 3.2.

To obtain the numerical model of the vein using the second method the program has to be run at least twice using data files for hangingwall and footwall separately. Then the two surfaces representing the two walls of the vein are joined together to form its numerical model. The steps in the algorithm for this second method consist of:

- reading the data points
- defining the grid points where the interpolation is to be performed
- triangulation of the data points
- estimation of the partial derivatives at each data point
- interpolation at grid point using the equation fitted on the triangle on which this point falls
- writing and storing the results of each surface
- the numerical model of the orebody is obtained in the form of a single file by combining the results representing the two surfaces (hangingwall and footwall).

The results, in the form of a numerical model cf. table 3.1 characterizing the geometrical features of the vein, will be a source of deduction of information needed for display or for use for other purposes eg. mine planning, mine construction, ore evaluation, etc...

These results are also presented graphically in the form of cross-sections figure 3.9 and 3.10, plane-sections figure 3.11 and 3.12, wireframe models of the overall vein figure 3.13 and 3.14 and solid models of the vein figure 3.15

and 3.16. These figures are all taken from the screen of the interactive graphics modelling software developed as part of this research project and is described in chapter 4.

Table 3.1 Results format of the numerical model of the nickel vein deposit.

Xc	Yc	Zf	Zh	Th	%Ni
648.17	196.88	686.20	687.90	1.70	5.62
648.17	198.99	689.06	689.65	0.59	4.34
663.71	180.00	690.72	692.93	2.21	7.22
663.71	182.11	691.01	693.00	1.99	8.33
679.25	186.33	694.99	696.78	1.79	3.56
679.25	188.44	694.88	696.11	1.23	6.21
694.79	198.99	697.76	700.91	3.15	3.89
710.33	180.00	700.12	701.71	1.59	3.17
725.87	182.11	700.74	704.19	3.45	5.87
725.87	184.22	700.75	703.48	2.73	7.22
834.65	198.99	734.79	737.18	2.39	8.32
850.19	180.00	735.52	737.17	1.65	2.44
850.19	182.11	736.50	737.90	1.40	5.77

Xc= northings coordinates of the points situated in the mid-plane

Yc= elevations of the points situated in the mid-plane

Zh = easting coordinates for the hangingwall

Zf = easting coordinates for the footwall

Th= deduced thicknesses

%Ni= estimated (geostatistically) grade values.

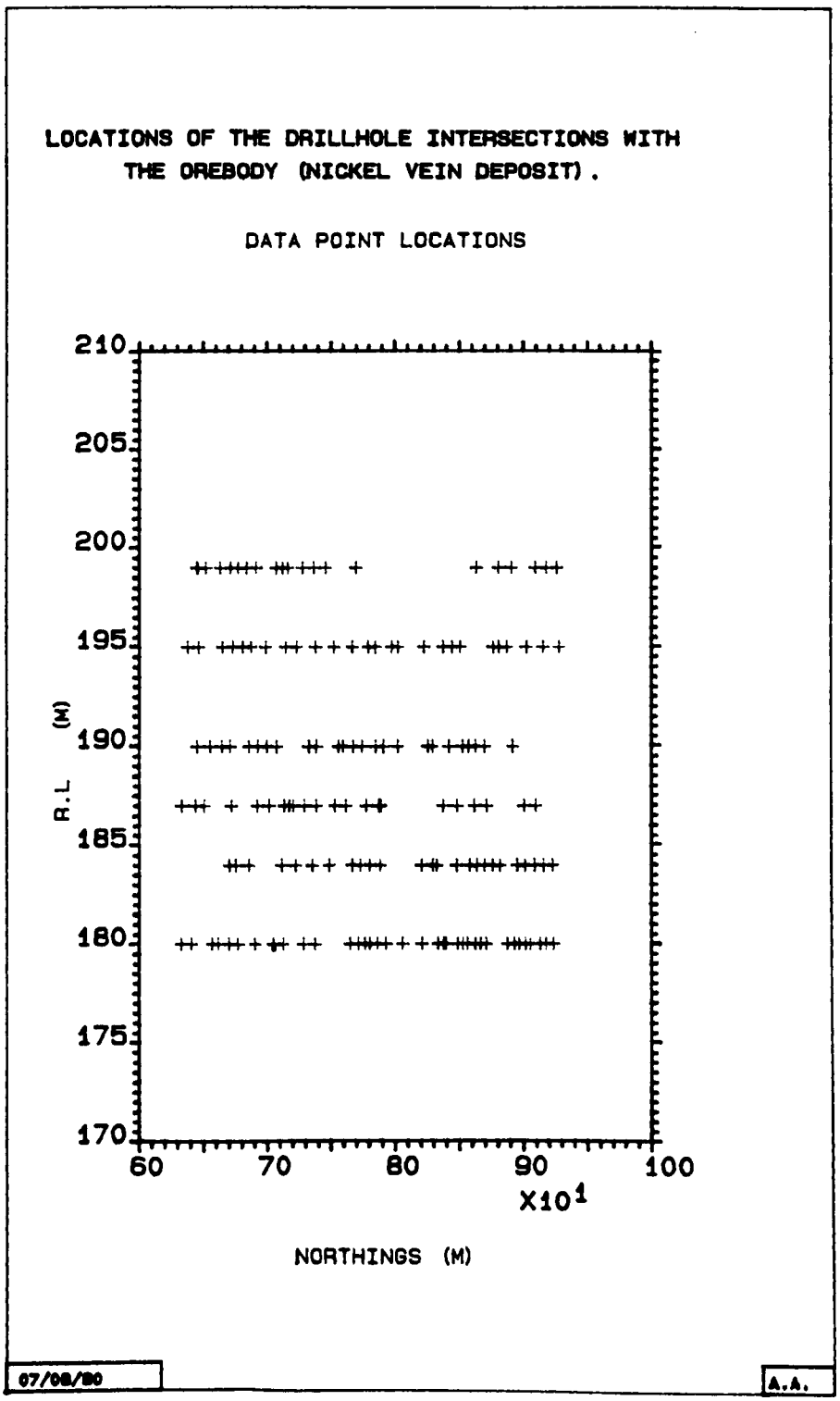
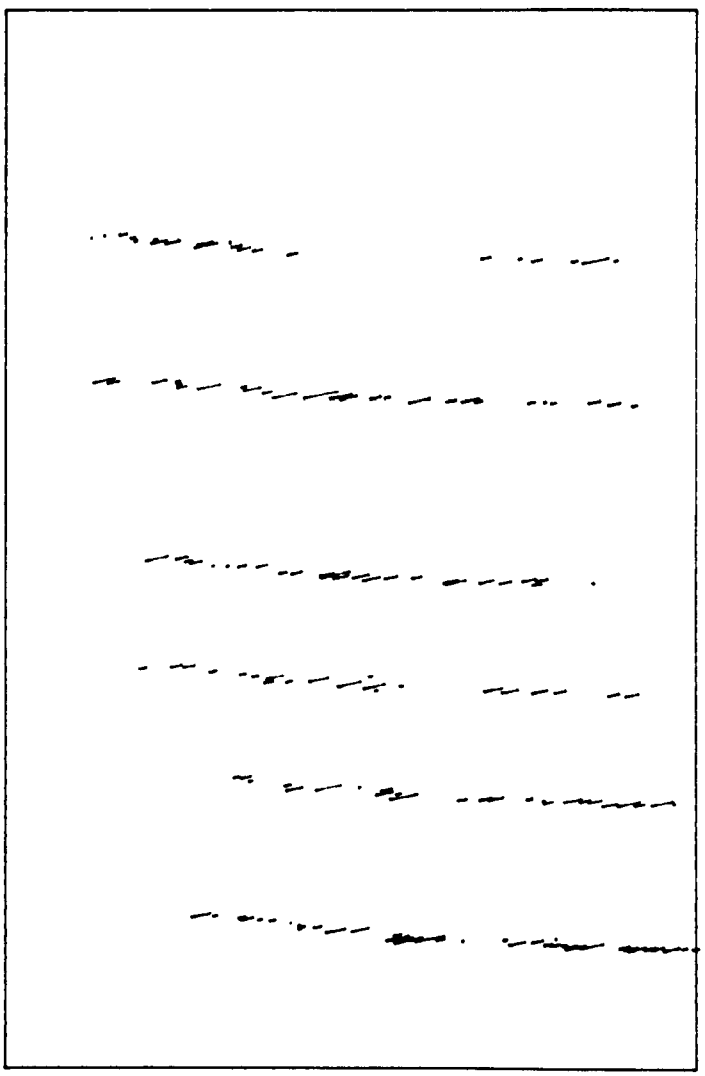

LOCATIONS OF THE DRILLHOLE INTERSECTIONS WITH
THE OREBODY (NICKEL VEIN DEPOSIT).
DATA POINT LOCATIONS
210
205
200
195
190
185
180
175
170
R.L (M)
60
70
80
90
100
X10¹
NORTHINGS (M)
07/08/80
A.A.

Figure: 3.6

DRILLHOLE INTERSECTIONS WITH THE OREBODY (NICKEL VEIN) IN 3-D



DRILLHOLE INTERSECTIONS (3-D)

07/08/90

A.A.

Figure: 3.7

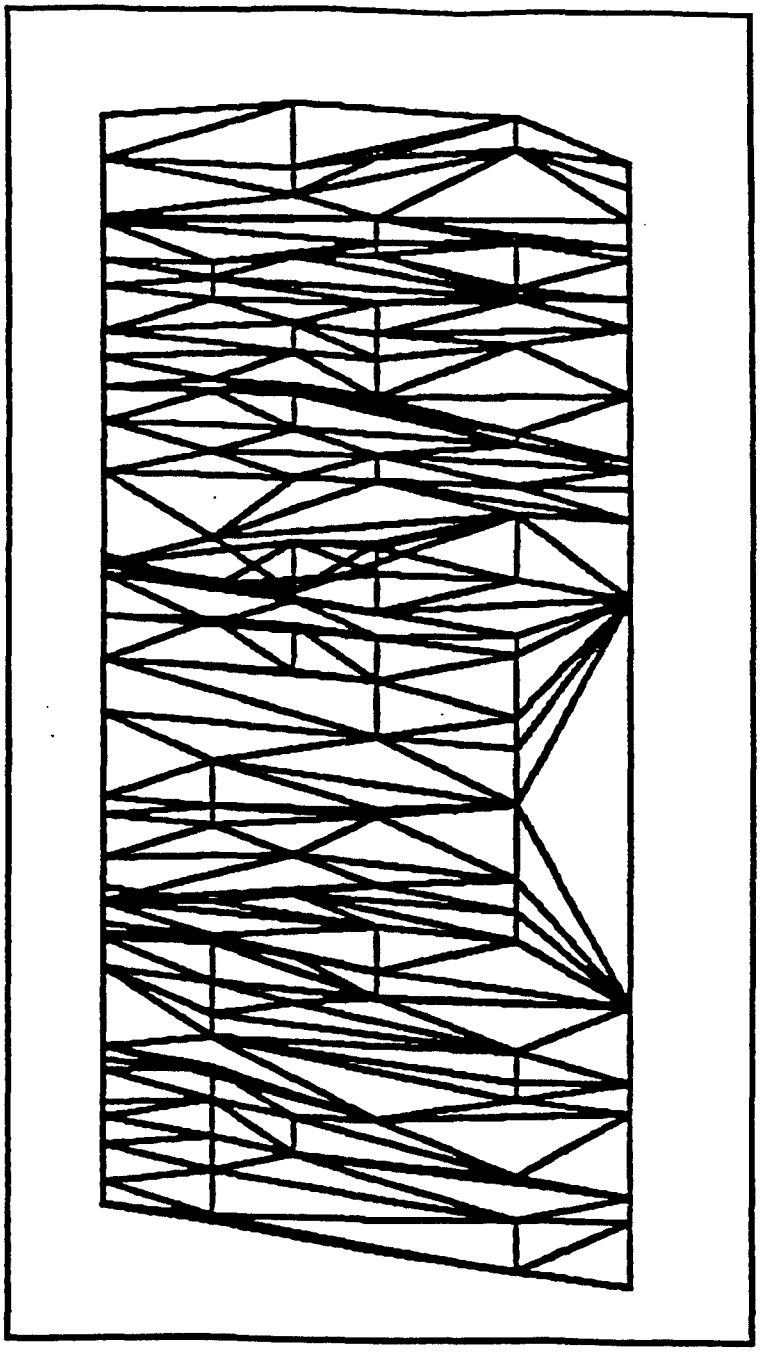


Figure: 3.8

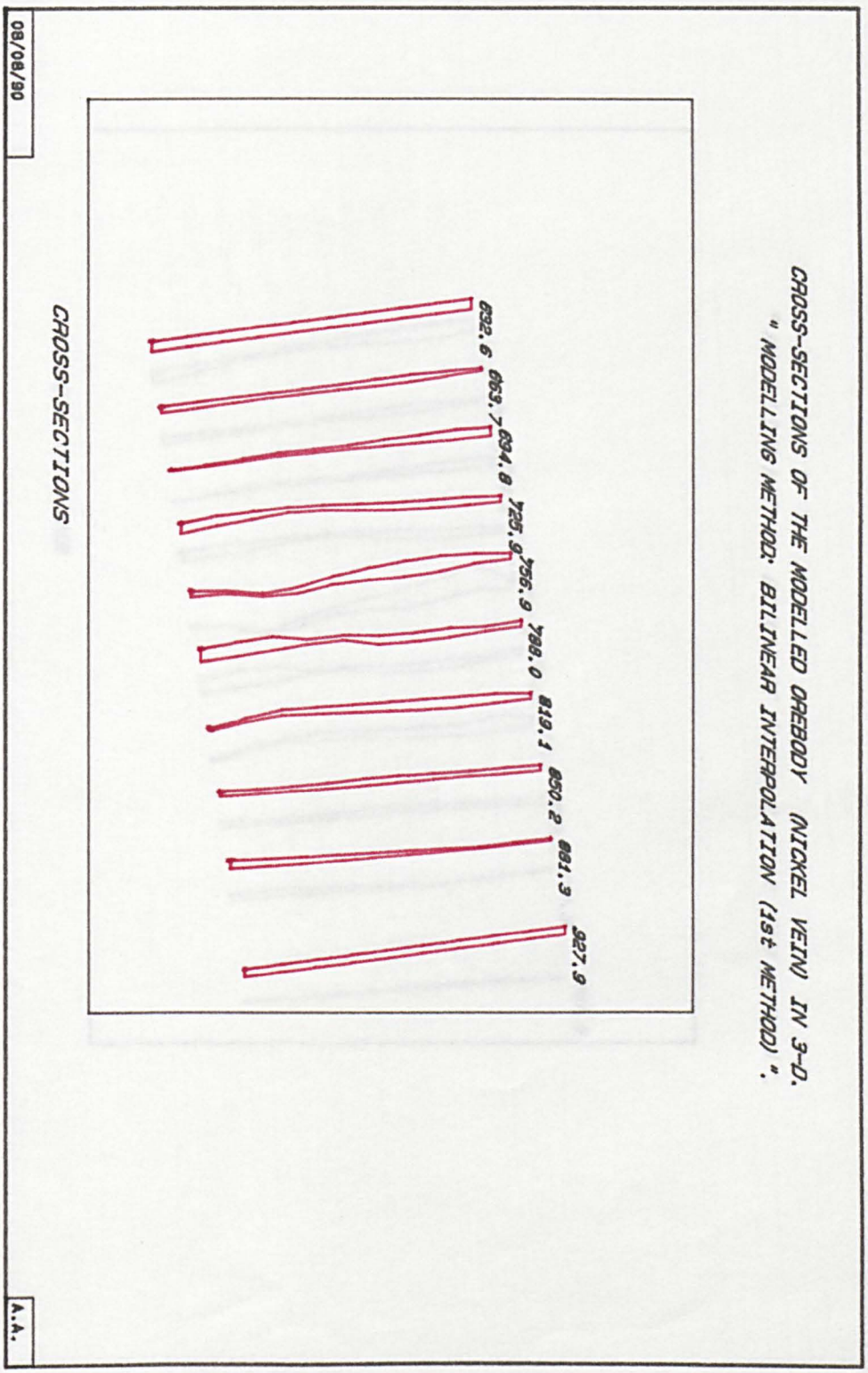

CROSS-SECTIONS OF THE MODELLED OREBODY (NICKEL VEIN) IN 3-D.
" MODELLING METHOD: BILINEAR INTERPOLATION (1st METHOD) ".
08/08/90
CROSS-SECTIONS
632.6
663.7
694.8
725.9
756.9
788.0
819.1
850.2
881.3
912.4
A.A.

Figure: 3.9

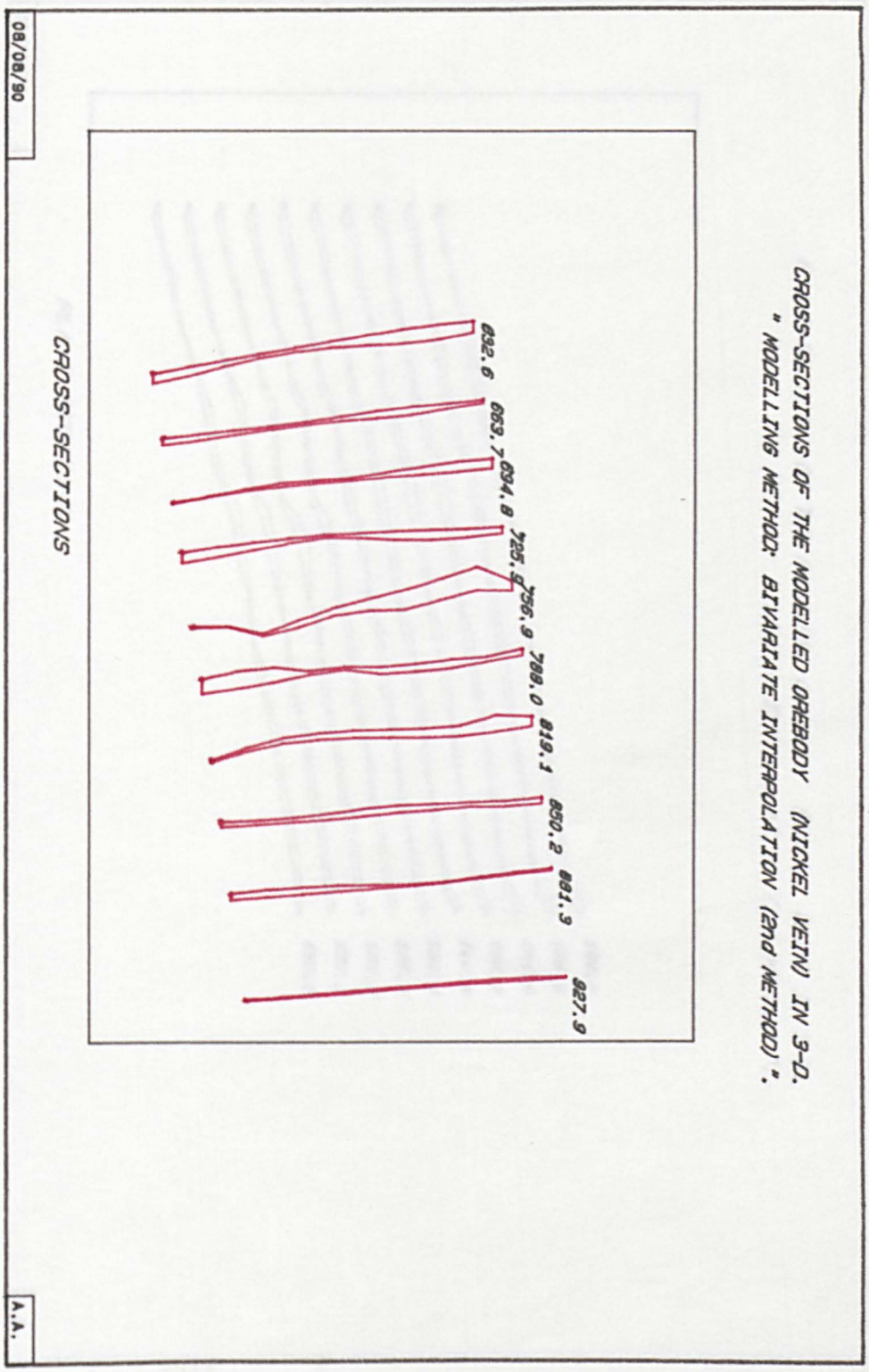

CROSS-SECTIONS OF THE MODELLED OREBODY (NICKEL VEIN) IN 3-D.
" MODELLING METHOD: BIVARIATE INTERPOLATION (2nd METHOD) ".
CROSS-SECTIONS
08/08/90
A.A.
632.6
663.7
694.8
725.9
756.9
788.0
819.1
850.2
881.9
927.9

Figure: 3.10

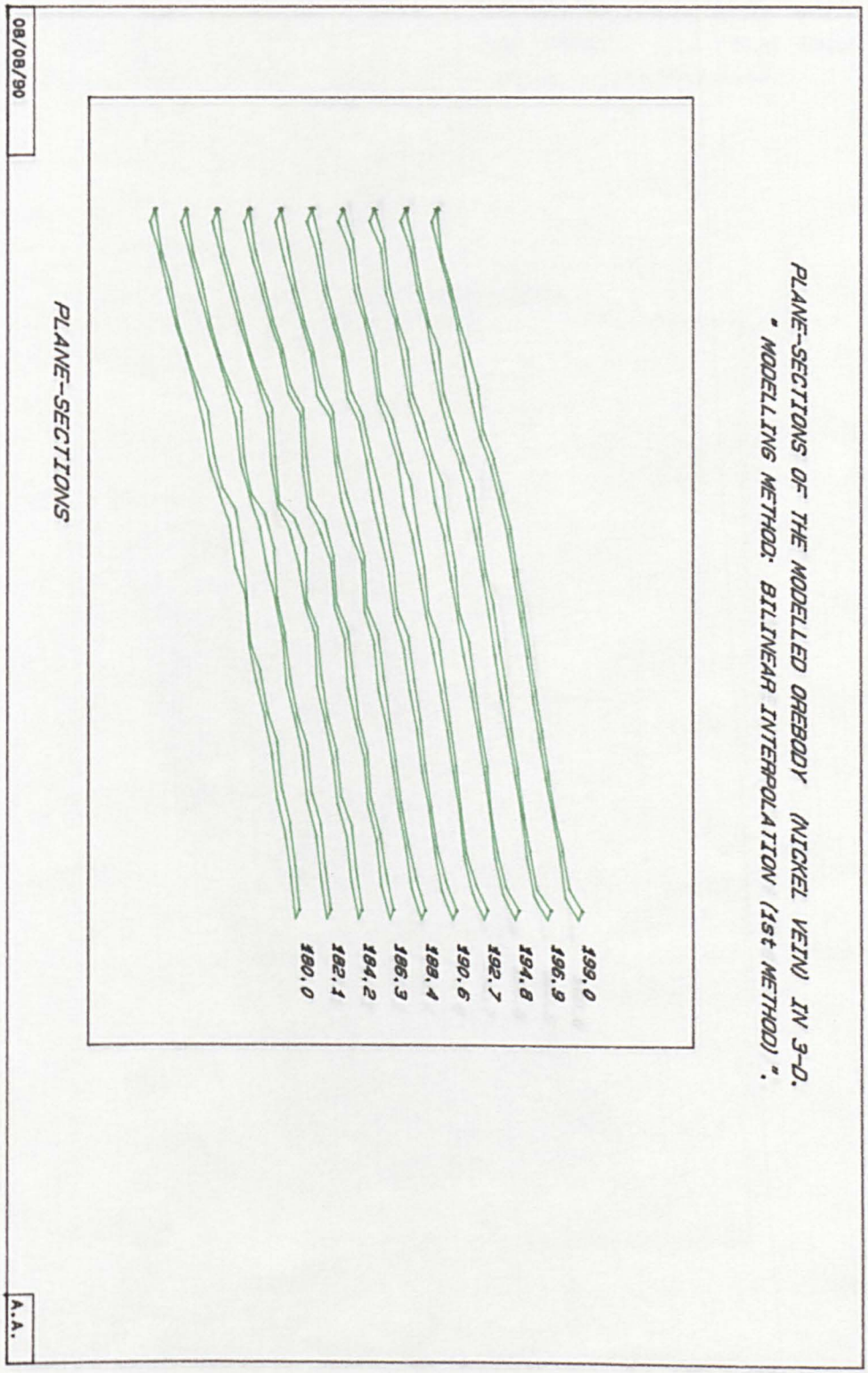

PLANE-SECTIONS OF THE MODELLED OREBODY (NICKEL VEIN) IN 3-D.
" MODELLING METHOD: BILINEAR INTERPOLATION (1st METHOD) ".
199.0
196.9
194.8
192.7
190.6
188.4
186.3
184.2
182.1
180.0
PLANE-SECTIONS
08/08/90
A.A.

Figure: 3.11

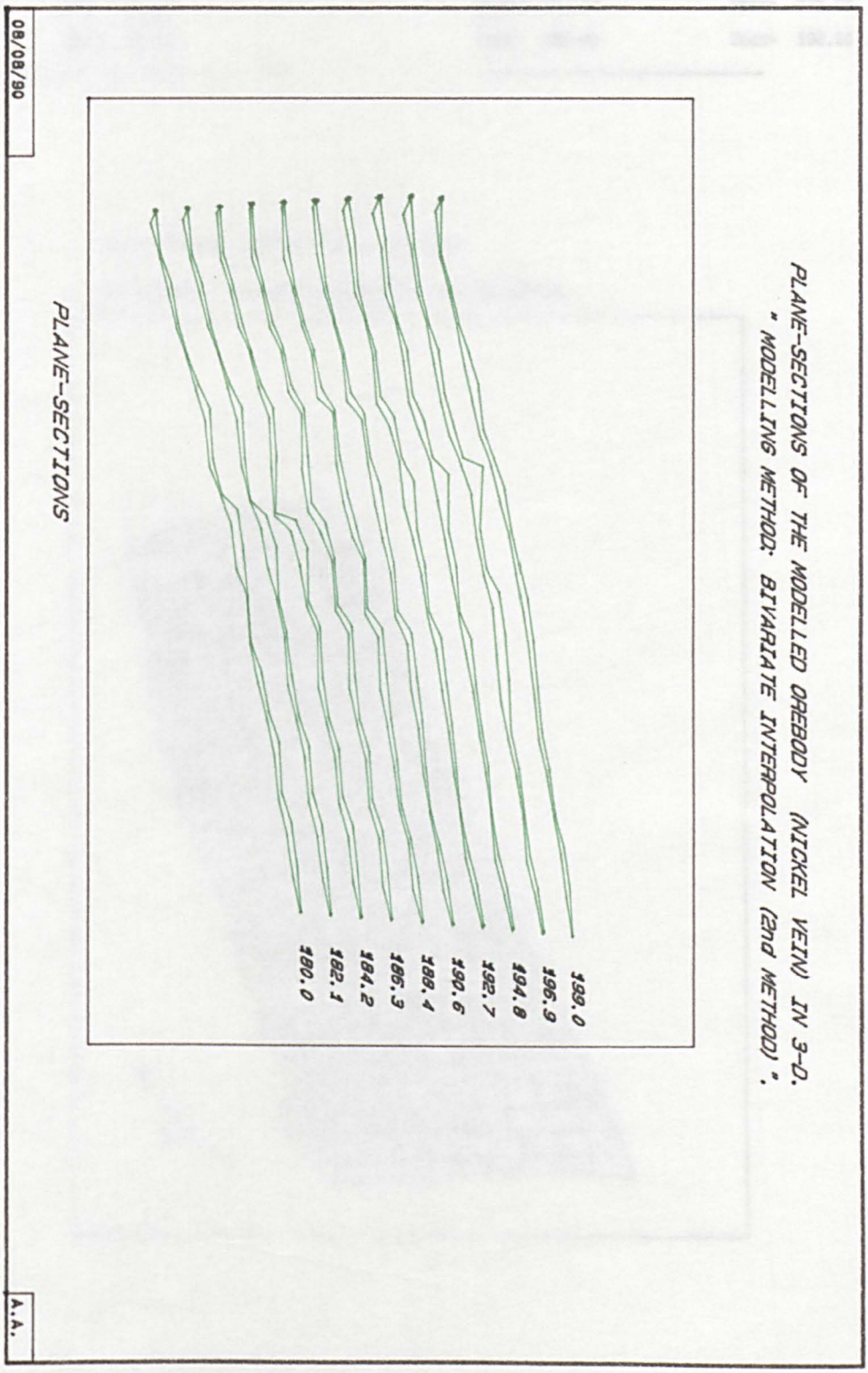

PLANE-SECTIONS OF THE MODELLED OREBODY (NICKEL VEIN) IN 3-D.
" MODELLING METHOD: BIVARIATE INTERPOLATION (2nd METHOD) ".
199.0
196.9
194.8
192.7
190.6
188.4
186.3
184.2
182.1
180.0
PLANE-SECTIONS
08/08/90
A.A.

Figure: 3.12

X-axis for the Northings _____Y-axis for the Eastings_____Z-axis for the elevations `RL`

View Point Coordinates

EX= 70.00	Xmin= 628.50	Xmax= 924.30
EY= -40.00	Ymin= 678.33	Ymax= 776.93
EZ= 10.00	Zmin= 180.00	Zmax= 198.90

Wireframe Model of the Vein

Bilinear interpolation: 1st method

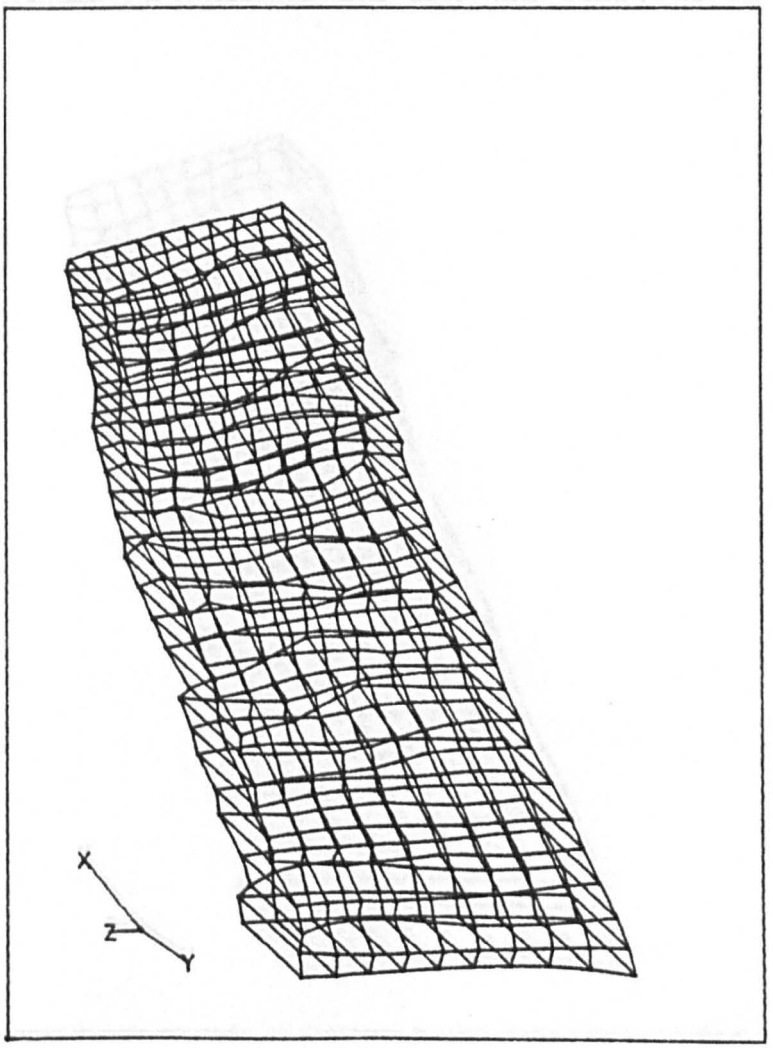


Figure: 3.13

X-axis for the Northings _____Y-axis for the Eastings_____Z-axis for the elevations 'RL'

View Point Coordinates

EX=	70.00	Xmin= 628.50	Xmax= 924.30
EY=	-40.00	Ymin= 678.33	Ymax= 776.93
EZ=	10.00	Zmin= 180.00	Zmax= 198.90

Solid Model of the Vein

Bilinear interpolation: 1st method

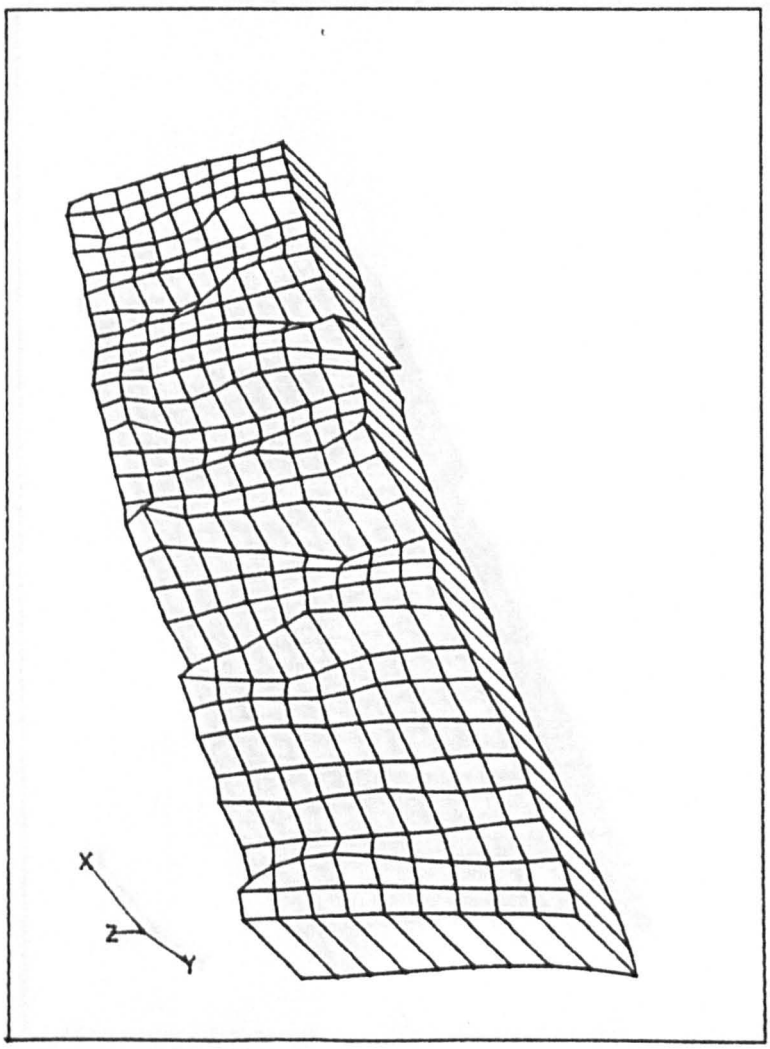


Figure: 3.14

X-axis for the Northings _____Y-axis for the Eastings_____Z-axis for the elevations `RL`

View Point Coordinates

EX=	70.00	Xmin=	628.50	Xmax=	779.30
EY=	-40.00	Ymin=	678.46	Ymax=	734.72
EZ=	10.00	Zmin=	180.00	Zmax=	198.90

Wireframe Model of the Vein

Bvariate interpolation: 2nd method

Figure: 3.15

X-axis for the Northings _____Y-axis for the Eastings_____Z-axis for the elevations `RL`

View Point Coordinates

EX= 70.00	Xmin= 628.50	Xmax= 779.30
EY= -40.00	Ymin= 678.46	Ymax= 734.72
EZ= 10.00	Zmin= 180.00	Zmax= 198.90

Solid Model of the Vein

Bivariate interpolation: 2nd method

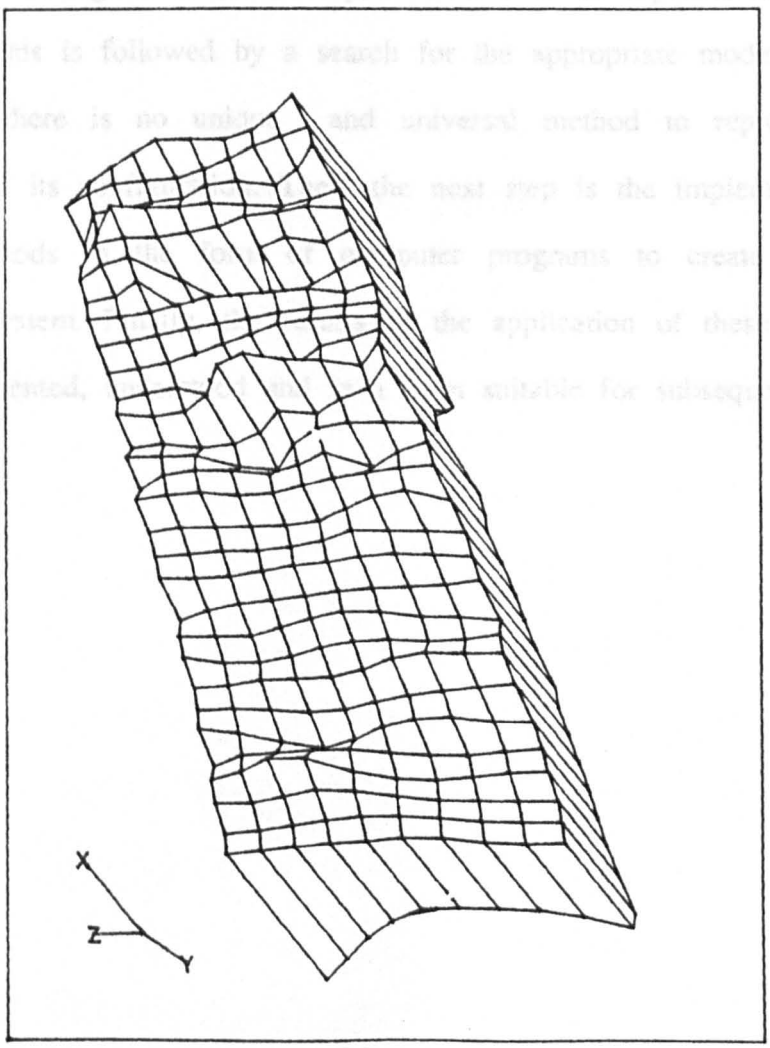


Figure: 3.16

3.4 Summary

Geometrical modelling of stratigraphic orebodies and subsequent geometrical control of geostatistical estimates is a very important application in mine planing and evaluation. As seen in this chapter the method of modelling and control is based mainly on the data at hand (information) and on the modelling techniques available.

A practical geometrical control system will include an understanding of the data and its configuration and the phenomenon to be represented by the available data. This is followed by a search for the appropriate modelling technique, given that there is no unique and universal method to represent any data regardless of its configuration. Then, the next step is the implementation of the chosen methods in the form of computer programs to create a geometrical modelling system. Finally, the results of the application of these methods must be well presented, understood and in a form suitable for subsequent use.

CHAPTER FOUR

Chapter Four

Graphics Display System

4.1 Introduction

The technological developments of the last twenty years have made computer graphics very popular in the field of computer science. It started as a visionary dream in the early sixties when it was seen that computer-generated pictures, produced through the direct interaction of a draftsman or an engineer and a computer, could be a substantial aid in a wide variety of technical tasks. Now computer graphics is one of the most exciting and rapidly growing fields in computer science, used to reduce mountains of data into usable chunks of refined information, such as Computer Aided Design which has allowed the technological community to enter a new era of enhanced creativity and productivity HARRIS (1984).

The general use of graphics techniques either through Computer Aided Design or graphics representation techniques in industry has not taken place overnight, but it has been evolutionary in nature. The universities and the aerospace and automobile companies which are pioneering this area have made significant investments in hardware and software to achieve high resolution graphics capabilities. However, the improved availability of techniques and the lower cost of equipment has enabled smaller companies and those in more conservative industries to follow suit.

Computer graphics has come also at a time when engineers, experts and decision makers are required to understand phenomena with a large amount of information, usually in a numerical format. Computer graphics techniques are to be used to transform this information into a pictorial format for the user to understand and to interact accordingly with it (as commonly said: a picture is worth a thousand words). The progress expected in this field (computer graphics) during the next decade has been linked in magnitude to the progress already achieved by the use of digital computer systems since their introduction and also to the progress of mathematical tools built to model or to represent the real world of certain phenomena.

The evolutionary development of computer graphics in mining engineering follows a pattern quite similar to other areas of computer applications. Prior to the introduction of the computer as a mine design and operation tool, graphics techniques were used in lieu of numerical methods in several steps of the design and operation process. It was originally believed that the introduction of computers in mining the industry would displace the need for graphical representation; however, it was soon recognized that graphical data and information representation was a very expedient means of evaluation. As approximately eighty percent of the data for the design and operation of a mine is available in computer processed data media, computer graphics is enjoying currently a great amount of developmental effort in mining.

In this chapter, computer graphics applications in the mining industry will be presented, and as a case study, these techniques in the form of a computer graphics display system cf. figure 4.1, will be applied to demonstrate the use of graphics to display information and features during the several steps undertaken to estimate a vein nickel deposit (stratigraphic orebody).

The design of this graphics system, after its initial specification, has gone through two main stages. The first stage was the development of the two-dimensional graphics modules specified for more general applications; the second stage was the development of the three-dimensional graphics modules which are more specific to the graphics display of geometrical features of stratigraphic orebodies.

It is of great importance to discuss the facilities needed to develop this graphics display system and how they have been used, either in the Mining and Mineral Engineering Department or at the Computing Service of the University of Leeds. The Computing Service in making these facilities available has enabled us to develop most of the modules of the graphics package on a mainframe machine (Amdahl VM580) before transferring and adapting most of them to run on a Personal Computer based workstation set-up in the Mining and Mineral Engineering Department. Then, as a core element of this chapter, the graphics display system will be presented and described in a more comprehensive manner and examples will be given to illustrate the use of these modules.

Finally, this chapter will be concluded with a few conclusion drawn by the author during the development of this graphics display system.

4.2 Graphics facilities

Because of the rapid change and development occurring in the field of computing hardware and supporting software it may not be of great importance to review the existing computing facilities which are used to develop computer graphics systems and to set up graphics workstations. These changes will automatically require any user of these techniques and means to keep in touch with any new developments. Traditional methods of research, conception, design and

test are simply too time consuming and expensive and these factors have, and are, mounting a tremendous challenge to all of the engineering disciplines to develop sophisticated solutions for difficult problems (performing more work in less time). The complexity of problems in the mining industry in general and orebody modelling and evaluation in particular, is the main ingredient in the mine design and operation process, also require the use of these facilities.

Ideally, the main features and characteristics of an advanced computer graphics workstation are:

- runs under commonly used standard operating systems
- supports high performance graphics
- supports a wide range of input and output devices
- provides an optimizing native code (compilers)
- has a reasonable amount of memory and mass storage
- supports a network interface.

These kinds of workstation offer entirely new capabilities to an engineer, where two- and three-dimensional graphics presentations allow him to work with a computerized model as easy as if it were a physical object. Increases in CPU performance allow him to perform analyses and simulations using the geometry of the object and to obtain real time visual output which is not always possible with a physical model. Other facilities offered by the workstation will give the engineer a more flexible computing environment such as MSDOS, PC-DOS, CMS, etc... and the capability of sharing information and facilities through a network system.

At the Mining Department of the University of Leeds research has been carried out for the application of geometrical modelling techniques and computer graphics display systems for the evaluation of stratigraphic orebodies using two

different types of facilities: mainframe and personal computer based environments. The project was initiated using the computing facilities offered by the Computing Service based on a mainframe computer system which is accessed via a network of terminals installed in many sites on the university campus and the auxiliary facilities, such as plotters and printers, installed in the User Access area of the Computing Service. From the Mining Department the access is via a series of terminals with one terminal the IMLAC SERIES II supporting graphics mode. These facilities, in particular the computing power of the mainframe machine, have contributed significantly to the speed at which such a relatively large scale software system, including the modelling and graphics display systems, was developed. Other facilities were also available such as GINO-F (vers. 2.6) basic graphics routines installed for the general use and a geostatistical package installed by Dr P. A. DOWD. However, in the spirit of portability of the software and for more independence of the user from the expensive and less available computer systems such as mainframe machines, the project has been transferred almost entirely to be run on Personal Computer based systems installed in the Mining and Mineral Engineering Department. The setting up of a personal computer network in the department is a first step towards introducing information technology facilities based on Personal Computer machines. In terms of hardware this network is based on IBM PC's model AT and IBM PS/2 model 30 together with an HP Series II laser printer and a plotter. On the other hand, the main GINO-F basic routines libraries (PC. vers. 2.7c) have been acquired with other supporting software (graphics drivers, compilers, wordprocessors, etc...) installed in a file server machine.

4.3 Graphics Display Software

The previous section in this chapter introduced in brief the hardware side of this graphics system but in order to produce the graphical picture of any information a source code program, known as software, has to be developed to process the numerical information and to command and control the hardware to output graphically this information.

The two- and three- dimensional graphics system developed for this research project is primarily intended for the display, interrogation and manipulation of pictorial information about the geometry of a stratigraphic orebody. As a case study a nickel vein deposit was modelled and displayed, the display ranging from simple curves and figures, displayed in areas alongside with areas containing text and alphanumeric information, in two dimensions (eg, variograms, data point locations, cross-sections, etc...) cf. figure 4.2, to solid and wireframe modelling, cross-sections, plane-sections, etc... in three dimensions, cf. figure 4.3.

This graphics display system is structured in the form of modules set up to carry out specific applications such as data transformation, creating databank, index and text files to produce graphical output. The system uses the general purpose graphics routines from GINO-F. The general structure of this graphics display system is shown in figure 4.1. All significant activities are originated by the application program to handle graphics input and output functions at a high level. However, the computer's operating system plays the role of the infrastructure for the graphics application program, it provides neutral supporting services and it is the software which controls the execution of computer programs and which may provide scheduling, debugging, input/output control, storage assignment, data management and related services.

GRAPHICS DISPLAY DIAGRAM

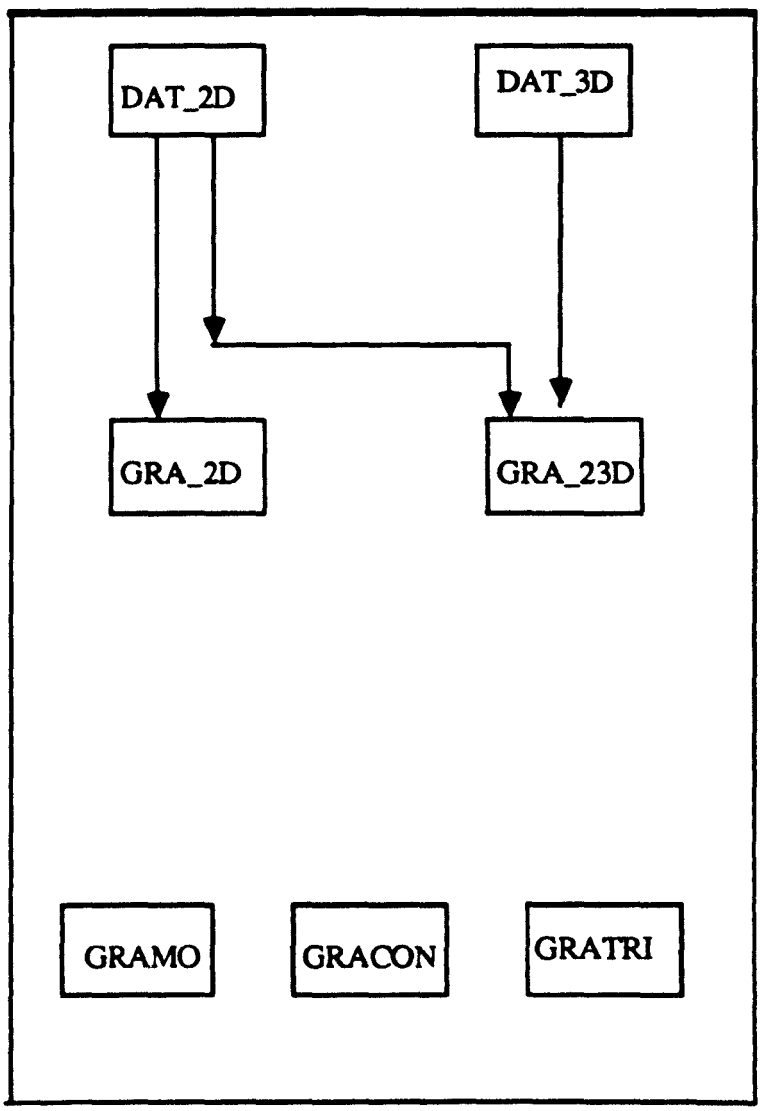


Figure: 4.1

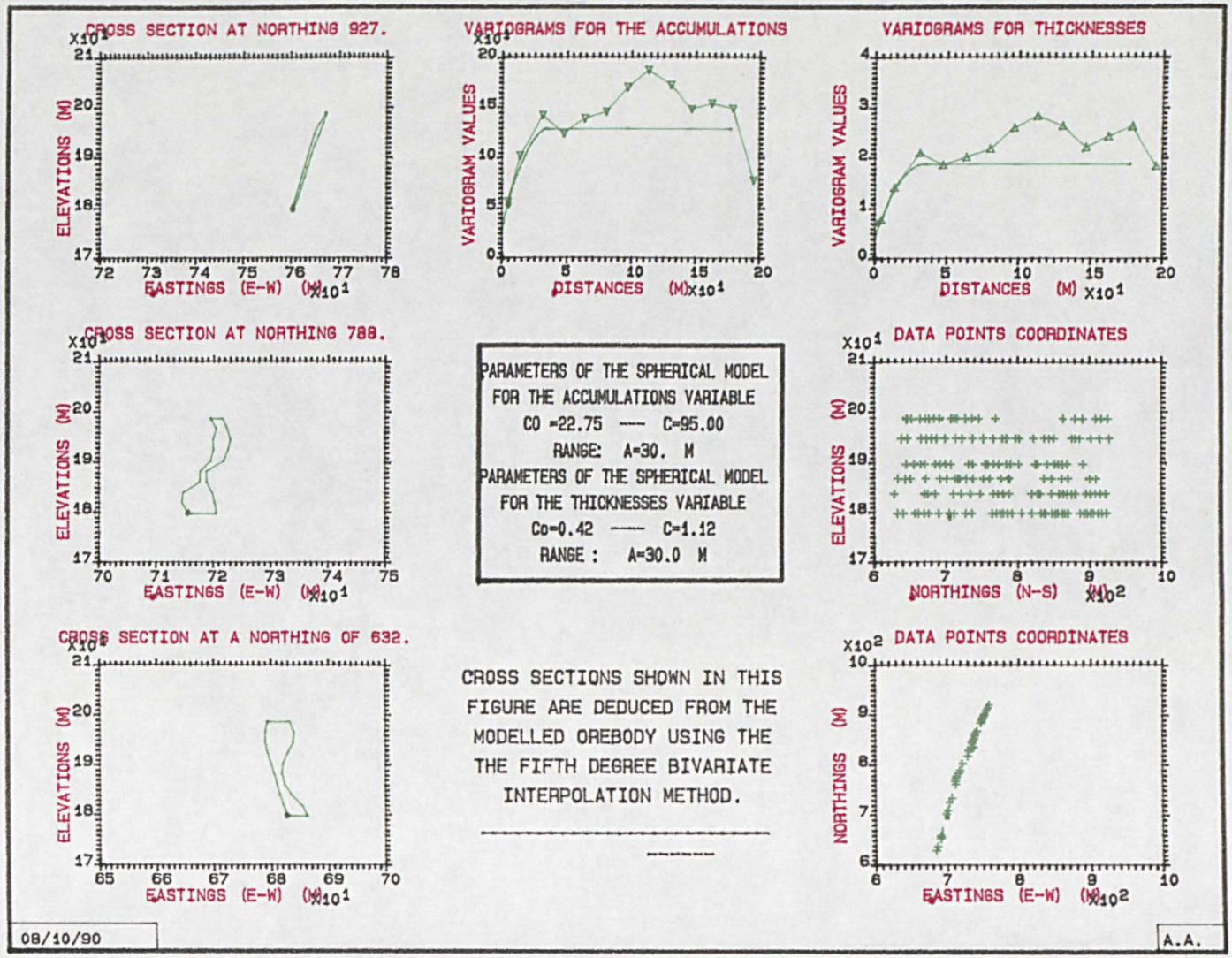

CROSS SECTION AT NORTHING 927.
ELEVATIONS (M)
EASTINGS (E-W) (M)
CROSS SECTION AT NORTHING 788.
ELEVATIONS (M)
EASTINGS (E-W) (M)
CROSS SECTION AT A NORTHING OF 632.
ELEVATIONS (M)
EASTINGS (E-W) (M)
VARIOGRAMS FOR THE ACCUMULATIONS
VARIOGRAM VALUES
DISTANCES (M)
VARIOGRAMS FOR THICKNESSES
VARIOGRAM VALUES
DISTANCES (M)
PARAMETERS OF THE SPHERICAL MODEL
FOR THE ACCUMULATIONS VARIABLE
CO =22.75 ---- C=95.00
RANGE: A=30. M
PARAMETERS OF THE SPHERICAL MODEL
FOR THE THICKNESSES VARIABLE
Co=0.42 ---- C=1.12
RANGE : A=30.0 M
CROSS SECTIONS SHOWN IN THIS
FIGURE ARE DEDUCED FROM THE
MODELLED OREBODY USING THE
THE FIFTH DEGREE BIVARIATE
INTERPOLATION METHOD.
DATA POINTS COORDINATES
ELEVATIONS (M)
NORTHINGS (N-S) (M)
DATA POINTS COORDINATES
NORTHINGS (M)
EASTINGS (E-W) (M)
08/10/90
A.A.

Figure: 4.2

Three-dimensional representation

(output from an HP laser Jet printer)

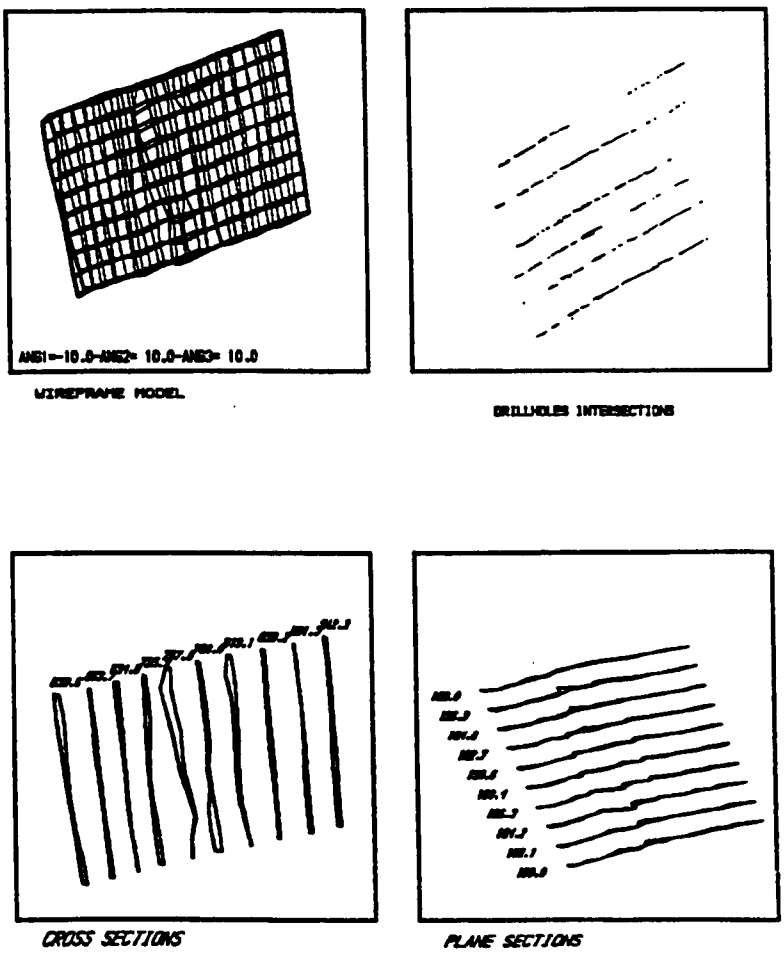


Figure: 4.3

4.3.1 GINO-F Graphics Routines

Gino-F is a software system consisting of a set of subroutines and functions (Gino-F Reference and User manuals) used by an applications program to generate images and pictures on a display device and to receive input data from input devices (eg. Keyboard, mouse, digitizer, etc...). These basic graphics routines free the applications programmer from concern about the details of functions directly related to graphics mode. An applications program (system) can thus be completed in less time and can be written by individuals with limited knowledge of low-level graphics routines and functions.

Historically, these kinds of graphics packages started to emerge in the early 1960's when Calcomp (California Computer Product) developed the Calcomp plotter software package which was the first commercial package. Then TCS (Tektronics Terminal Control System) released in 1971 its Tektronics Plot-10 software package. Calcomp, Tektronics and other early graphics packages were developed by the manufacturers of the graphics display devices to make their product easier to use SCOTT (1982). Equipment manufacturers are still one of the most important sources of graphics packages but universities and independent organizations have also developed a number of graphics including DISSPLA in North America and GINO in the United Kingdom and Europe. They are among the few that have survived and become widely used. More recently, there is GKS (Graphics Kernel Standard) a German designed graphics package which is now emerging as the current international graphics standard; its main characteristics are that it separates graphics applications from device dependencies Scott (1982). In the United Kingdom, at least at the universities, a newly developed graphics package called UNIRAS has been introduced; it

consists of ready made modules for users not willing to develop their own specific applications programs and it consists also of a library of routines and functions enabling the user to develop a graphics application program designed basically on the specification set by the user. These graphics packages and others have been developed and great care was taken in their design, so that they were easy to implement on a large range of computers and capable of driving a wide range of display devices. To facilitate portability between different computer systems such graphics packages were normally written in standard Fortran and relied as little as possible on machine dependant features HARRIS (1984). For this research project the GINO-F graphics package was used because of its availability and popularity among the existing graphics packages. However, a great deal of care has been taken not to limit the applications graphics system by using sophisticated and complex routines and to allow the possibility of adopting another set of graphics routines from a different graphics package.

The GINO-F graphics package which was developed by the computer aided design centre in Cambridge is written in Fortran 77 with an interface section which can be rewritten for particular computer/terminal (device) combinations called drivers. This makes the package relatively machine independent and applications programs using these routines can be run and output on many machines and devices.

The GINO-F graphics package offers a wide variety of facilities which are thoroughly documented (reference and user manuals). These facilities include drawing in both two- and three-dimensional representations, absolute or relative addressing of screen coordinate positions, positioning and displaying alphanumeric data, arc generation, graph plotting and histogram drawing, manipulation

and transformation of figures into two-dimensional viewing planes and routines to refresh the display graphics in use. These routines are easy to access and use, and are compiled in the form of special libraries to enable the user to use only those needed rather than linking that applications program to a single huge library.

These libraries are listed below:

- GINOGRAF : business graphics routines (eg, histograms, graphs, pies charts, etc...) and axis specifications (position, scaling type, tickmarks, etc...)
- GINO_2D : two-dimensional basic graphics routines for drawing lines, curves and symbols and figure transformation (scaling, rotating, translating, etc...) in two dimensions
- GINO_3D : as GINO_2D but in three dimensions
- GINOSURF : graphics routines for contouring and isometric representation of data including automatic and user defined axes setting.

4.3.2 Applications Program

The graphics display system which is developed for this research project exists in two versions written entirely in Fortran 77, the first one was developed on the mainframe and the second version, taking its core component from the first, was developed to run on a Personal Computer. As mentioned above this graphics display system is in the form of modules, the general structure of which is shown in figure 4.1. To avoid the overloading of the Personal Computer system memory the applications program for two- and three-dimensional representation, originated from the mainframe version, has to be restructured in smaller modules.

Because of the insignificant differences between the two versions and because the package is targeted to run on smaller machines (PC's, PS/2's and workstations) the author has decided to overlook the mainframe version and to present the personal computer version. However, some figures have been produced using the first version because of the high quality of the output using the auxiliary facilities linked to the mainframe system.

4.3.2.1 DAT_2D Module

The objective of this module is to create three files to be used by GRA_2D and GRA_23D modules for two-dimensional display of figures, these files are:

- databank file
- index file
- text file.

This module does not require any graphics basic routines from the GINO-F graphics package; its role is to set up and store arguments and specification to be passed to graphics modules through the three files mentioned above. Because of the large amount of information and the editing facilities required the module uses files directly accessed such as the databank file to provide easy access to the databank records. The data stored in the databank file are divided into eight groups, depending on the nature of the data itself such as axis data, area data, label data, etc..., and these groups are indexed and specified in the index file. Next the text file is created to store all text strings entered by the user to describe the figures (eg, labels, titles, text to be output in separate areas).

The DAT_2D module is fully interactive and the user will need this feature of interactivity to make any correction to the data before passing them to

the graphics modules. The editing facilities will help the user avoid the running of the module and the creation of the whole databank from the start if wrong values have been stored. One of the advantages of this module is that it creates databanks and index files to be used by more than one data file; the difference will only be in text files which should, of course, be specific to the data to be displayed.

Before running this module the user already has an idea what the output looks like (eg, number of areas, their positions, their nature (graph or text), the number of curves in each graph, etc...). The module will require the names of the three files to be typed in, then four options appear for the user to chose one of them. These options are:

- C: Create databank
- E: Edit databank
- A: Add one or more areas to existing ones
- F: Exit from this module.

An example will be given to demonstrate how this module is run, mistakes will be intentionally made to show the user how editing facilities can be performed.

Let us consider drawing four areas, three of which contain graphs and the fourth is a text area.

NB: the data to be displayed in different areas are in one file, which is not required at this stage.

The information displayed by the module (questions, comments, etc...) are written in italics, the user's answers are in bold and the author's comment on this example are in normal text fonts.

The start of the example:

It is preferable to have all modules and files in one directory.

Type in DAT_2D

Please insert the databank filename

BANK2D

Please insert the index filename

INDX2D

Please insert the text filename

TEXT2D

Please type in one of the following options

C: Create databank *A: Add one or more areas*

E: Edit databank *F: Exit*

Because no databank has been created before, so the option C is chosen.

C

Basic Data S/1 = Specify *0 = Default*

This option is to give the user the choice of setting up the dimensions and the origin, the scale factor of the area (paper) to contain the graphical output and the type of the device in use. The default values are commonly used.

0

Number of areas desired

4

Now you are defining area number 1 out of 4

Area Data S/1 = Specify *0 = default*

The area data is specified only if it is to contain graphs otherwise the area will contain text.

1

Ibound (0:1) : **1**

Ibound: is a positive or negative integer (0:1) to specify whether a border line is to be drawn around the area or not (0=yes - 1=no).

Bound. Col (1:8) : **1**

This parameter is to specify the colour of the area borders, there are eight colours to chose from.

Axes Col. (1:8) : **1**

This parameter is to specify the colour of the axes.

Text Col. (1:8) : **2**

This parameter is to specify the colour of the labels and title.

Curv Col. (1:8) : **3**

This parameter is to specify the colour of the curve.

Area size Scale (<1.0) : **0.9**

This parameter is to scale the figure inside the area.

Area origin XO. : **10.0**

The origin of the area in the X (horizontal) direction.

Area origin YO. : **10.0**

The origin of the area in the Y (vertical) direction.

Area length : **70.0**

The length of the area (dimension in X direction).

Area width : **50.0**

The width of the area (dimension in Y direction).

Nbr. Graphs (0=text) : **1**

Number of graphs (curves) in one area. 0 for areas containing only text.

If the area has been specified to contain graphs, axes parameters are to be specified.

Itick1 (-2:2) : **2**

This parameter is to control the side (clockwise or anticlockwise) on which the tickmarks on the axis are to be produced (2 for clockwise tickmarks on the X axis).

Itck2 (-2:2) : **-2**

The same as the preceding parameter but on the anticlockwise side of the Y axis.

Ival1 (-1:1) : **1**

This parameter is to control the side (clockwise or anticlockwise) on which the interval values on the X axes are to be produced (1: for clockwise and -1: for anticlockwise values).

Ival2 (-1:1) : **-1**

The same as the preceding parameter but on the anticlockwise side of the Y axes.

Isc1(1:3) : **1**

This parameter is to specify the type of scaling on the X axes:

1 for linear scaling

2 for logarithmic scaling

3 for histogram drawing.

Isc2(1:3) : **1**

The same as the preceding parameter but on the Y axes.

Nints1 : **8**

Number of intervals on the X axes.

Nints2 : **6**

Number of intervals on the Y axes.

Character size data 1/S = Specify 0 = Default

Default values of the character size are: 2.5 and 2.5 respectively the width and the height of the character.

1

This option is to allow the user to specify the size of the characters and numbers used for this area (axis values).

Char. W. : **2.0**

The character width.

Char. H. : **2.0**

The character height.

Label Graph ? 1=yes 0=no

1

X-Axis Name : **.> Northings (m)**

The label of the X axis (horizontal), **.>** will allow the label to be centred otherwise it will be written at the beginning on the axis.

Y-Axis Name : **.> Elevations (m)**

The label for the Y (vertical) axis.

Title : **.> Data point locations**

The title of the figure (area) drawn on the top of area border.

Character size data 1/S = Specify 0 Default

1

Label CW. : **2.5**

The character width of label.

Label CH. : **2.5**

The character height of the label.

Then the specification of each curve if the area contain more than one curve.

Colour graphs (curves) &/or mark them? 1=yes 0=no

1

Symb. CW : **2.0**

Symbol size (the width), the symbol is specified (Sy) in the data file; second parameter on second record representing the data (cf. Table 4.3a).

Symb. CH : **2.0**

Symbol size (the height).

....This curve/line no 1

Specifications are: 0,1,2,3 or 4 for help

the fourth option is chosen to demonstrate theses specifications.

4

0 - the curves are coloured but not marked

1 - user enter mark point and line/curve colour

2 - as 1 + line mode (solid, dashed or chained)

3 - as 2 + dash and dot length are specified for broken lines

3

Ref. point (0=none) : **1**

To specify at which point in the data the symbol is to be displayed. For the value 1 the symbol will be displayed at the end of the curve/line.

Graph Col. (1:8) : **3**

The colour to be given to this curve (eg, 3 is red, 1 is black, ...).

Mode (-2:2) : **2**

The mode specifies the type of curve to be used (solid, dashed or chained)

-2 = chained

0 = solid

2 = dashed

Repeat Sequence : **5**

Number of sequences to be repeated in one curve.

Dash length : **3.0**

The length of the dash on the line.

Dot length : **3.0**

The length of the segment between two dashes.

Now you are defining area number 2 out of 4

This second area is to be specified as a text area.

Area data 1/S = Specify 0 = Default (text)

0

Area size scale (<1.0) : **0.9**

Area origin XO : **10.0**

Area origin YO : **10.0**

NB: The user has made a mistake by giving the area 1 and area 2 the same origin, this mistake will be corrected using the editing facilities rather than starting the whole creation process of the databank.

Area length : **90.0**

Area width : **70.0**

Nbr. Graphs (0=text) : **0**

There are two options offering the possibility to the user to enter the text from the keyboard or using a previously created text file and it must be different from the one specified at the beginning.

0 = input text from the keyboard.

1 = text is stored in a file (different from the specified text file).

0

Number of text lines : **2**

Number of text line in this area.

Line number 1

.> Graphics display of raw data for

Line number 2

.> Geometrical modelling of stratigraphic orebodies

NB: The text line is started by .> sign to indicate that the text is to be drawn (written) on the centre of the line.

Character data 1/S = Specify 0 = Default

1

Char. CW = **2.0**

Char. CH = **2.0**

Next comes the definition of the two remaining areas. Because they contain graphs their parameters are set similarly to those given in the first area except that some parameters such as the position of the area, axis labels and title are different.

Do you want to edit this data bank (Y/N)

(useful if changes are to be made)

The user must remember that a mistake was made in area number two by setting the position of this area to be the same as the first one; the databank thus has to be edited to correct this mistake.

Y

Which information do you want to update

B = Basic data *A = area data*

A

input area number *or 0 to end editing*

2

... which data set do you to update....

T = text data *A = area data*

S = area size *B = basic data*

E = end this level

The mistake was made by giving the wrong position to this area so the wrong value is in area size data set.

S

Mode : V = View record *U = Update record*

V

..... *Ident*	*Name*		*Current value*
16	*area size scale (<1.0)*		*0.9*
17	*area origin*	*XO*	*10.0*
18	*area origin*	*YO*	*10.0*
19	*area length*		*100.0*
20	*area width*		*90.0*
21	*Nbr. Of graphs (0=text)*		*0*

Mode : V= view record *U = update record*

U

which identifier *(0= end this level)*

The record to be updated to change the position of this area is record number 18; then the area will be shifted in the Y direction to a new position (XO=10.0 and YO=90.0 'new value').

18

Current value of area origin YO 10.0

New value:

90.0

which identifier (0= end this level)

0

... which data set do you to update....

T = text data *A = area data*

S = area size *B = basic data*

E = end this level

If no changes are required then

E

input area number or 0 to end editing

Before ending the editing of this databank and as an example the user decided to change the number of intervals on the X axes (Nints1) in area one, then:

1

Which set of data do you want to update

X = axis data *Z = axis scales data*

A = area data *S = area size data*

L = label data *B = basic data*

E = end this level

The number of intervals is specified on the axis data set.

X

Mode : V = View record *U = Update record*

V

.... Ident .... Name ..... Current value

1 Itick1 (-2:2) 2

2 Itick2 (-2:2) -2

3 Ival1(-1:1) 1

4 Ival2(-1:1) -1

5 Isc1(1:3) 1

6 Isc2(1:3) 1

7 Ninits1 10

8 Ninits2 10

9 Char. W 2.0

10 Char. H 2.0

Mode : V = View record U = Update record

U

which identifier (0 = end this level)

7

Current value of Ninits1 10

New value :

8

which identifier (0 = end this level)

0

Which set of data do you want to update

X = axis data Z = axis scales data

A = area data S = area size data

L = label data B = basic data

E = end this level

No more editing is needed for this area.

E

input the area number *or 0 to end editing*

The other areas are supposed to be correctly set up so there is no need to edit any of them.

0

This example of creating and editing the databank has also demonstrated the possibility of creating a number of standard databanks allowing the user to specify only the data to be displayed and the text file which is specific to that data. Although some changes may be needed they are very small. This module can be modified easily to allow specific figures to be displayed such as cross-sections together with grade values, block boundaries alongside with their grade values, etc...

4.3.2.2 DAT_3D Module

This module is for three-dimensional representations and it consists of creating files for GRA_23D module to display three-dimensional figures such as cross-sections, plane-sections, wireframe models, etc.... This module does not require any graphics routines from the GINO-F graphics libraries and it requires only the data file containing the orebody model cf. Table 4.2 to deduce cross-sections, cf. Table 4.5, and plane-sections, cf. Table 4.6, or to draw the wireframe model of the orebody, cf. Table 4.2.

The module creates index, text and drawing data files. The index file (Table 4.4) contains the number of figures and their type, the text file contains mainly titles of figures and drawing data files are those which contain data of cross-sections and plane-sections to be drawn by GRA_23D module. Because of the limited amount of information required by this module it was written without editing facilities; however it does have an option to allow the user either to

use the existing files, created before this run, or to update files for a different picture. The module gives the user the choice of setting up information to draw four types of three-dimensional figures:

1- Cross-sections.

2- Plane-sections.

3- Drillhole intersections with the orebody (raw data).

4- Wireframe model of the orebody.

To set up files to produce these figures, the module starts by asking the user to insert the index file name followed by the name of the file containing the orebody model data, which is in a format shown in Table 4.2, and the name of the text file which will contain the titles of the figures to be drawn. The module then requests the information which is to be written to the index file (Table 4.4). This information is as follows:

A scaling factor which is a factor which will scale the figures to be fitted within a specified window, in most cases the value 0.9 is given.

The module then requests the number of figures to be entered and specification of each figure. The module will display the type of figures to remind the user of the existing options.

As each option (type) is entered the module will ask for the title of the figure to be typed in and to be saved in the text file specified earlier.

The module considers each figure separately because of the specific nature of the data required to draw each figure.

Cross-sections:

The module requests the user to enter the file name which will contain the drawing data points of the cross-sections, then it will request the position of the area to contain this figure. This will be followed by the specification of the

number of cross-sections to be drawn. The module will display the possible coordinates at which cross-sections can be drawn so as to allow the user to take note of the cross-sections needed to be displayed. The user then enters the number of cross-sections and for each cross-section the module will require the fixed coordinate at which the cross-section is to be drawn. Finally, the data point coordinates forming these cross-sections are written in a file to be used by GRA_23D module to draw these cross-sections, cf. Table 4.5.

Pl ane-sections:

The DAT_3D module builds up the data to draw plane-sections of a stratigraphic orebody in the same manner as it does for cross-sections by specifying the name of the file which contains the data point coordinates of these plane-sections, the position and dimensions of the area where the figure is to be displayed, the number of plane-section to be drawn and the fixed coordinates for each plane-section. The file containing all the specifications and data point coordinates for drawing this type of figure is shown in Table 4.6.

Drillhole intersections:

The DAT_3D module will display a message for the user to make sure that the data representing the drillhole intersections with the orebody are in the format shown in Table 4.1. The first record, at the top of the file, specifies the area data: the position and the dimensions of the area to contain the figure. The module also requests the name of the file containing these data.

Wireframe model:

This option is very simple, the module specifies in the index file that this option is exercised to command the GRA_23D module to draw the wireframe model. The area data parameters are specified in the first record of the model data file, cf. Table 4.2.

The data point coordinates of the orebody model should be in fixed format so as to avoid any precision problem in deducing the cross-sections and plane-sections.

4.3.2.3 GRA_2D Module

This module is for two-dimensional representations and is mainly an execution program which uses information and settings obtained by running the DAT_2D module. To run this module the user has only to specify the names of the databank, the index and the text files in addition to the data file containing the information to be drawn.

The GRA_2D module is capable of drawing nine areas (windows) in one run. These areas (windows) may contain graphs, such as curves (up to ten curves in one area), cross-sections and data point locations with different symbols or the area may contain only alphanumeric information cf. Figure 4.2. Together with the DAT_2D module, this module has the capability of incorporating more features to make it a more general-purpose application graphics package. In the present version some useful features, although they are not of great importance to this research project, are missing, such as histograms, pie charts, logarithmic scaling, etc. This is due to the lack of basic graphics routines on the PC version of GINO-F. However, the author is considering these additional features in the future development of this graphics system to serve a large number of users.

The module requires access to the appropriate graphics routines from the GINO-F libraries. To run this module the user must have already run the DAT_2D module to create the necessarily files. The module starts by asking the user to enter the following files:

- Data_file This file contains the data to be drawn and is in the form of individual curves or/and cross-sections. For simple curves, represented by a series of data points to be joined together, the format of the data is shown in Table 4.3b. The first record contains the type of the figure (CUR :means the data is for a simple curve), the second record contain a series of parameters in the following format (lt, sy, ntp axmin, axmax, aymin,aymax):

lt= the type specifies either that the data points are joined by lines (L), a curve fitted through those points (C) or the data point location are presented by symbols (X)

sy= is the symbol code used to represent data point locations (1-8), sy=0 means no symbols are drawn.

ntp= number of data points to be drawn

axmin, axmax = minimum and maximum values given to the X (horizontal) axes; the range of the X axes is wide enough to contain the range of the data to be drawn

aymin, aymax = minimum and maximum values given to the Y (vertical) axes; the range of the Y axes is wide enough to contain the range of the data to be drawn.

For cross-sections the format of the data is shown in Table 4.3a. The format is similar to the one described above, except that in the first record the parameter 'CRX' means that the figure is a cross-section.

- Databank_file This file was created previously by the DAT_2D module and contains all the specifications of the drawings.

- Index_file This file is also created by the DAT-2D module to help the module in accessing the appropriate information in the databank file.

- Text_file This file is created by the user while running the DAT_2D module,

and contains all text and alphanumeric information to be displayed alongside the figures such as labels, titles and descriptive text strings.

The module then gives an option to the user to specify the overall scaling of the figure to be produced. In most cases the user choses the default option by just hitting the return key, unless, for some reason, the changing of the scaling factors is needed, such as obtaining smaller figures.

The module then requests the specification of the output device. At present the PC version supports two devices, an enhanced graphics colour monitor and an HP laser jet series II printer used to obtain hardcopy outputs.

Finally, the figure is displayed on the screen and the user is asked to request the device to continue either by sending the output to the printer to obtain hardcopy or by plotting the figure again on the screen (maybe with different scaling factors) or to enter new files for another figure or to exit the module.

The following is an example of some figures produced by DAT_2D and GRA_2D modules. The example will be related to the data (information) from the nickel vein deposit mentioned earlier.

- Drawing one curve with a text area as in figure 4.4.
- Drawing an area with more than one curve as in figure 4.5.
- Drawing figures to contain simple curves (eg, semivariograms), cross-sections, data point locations and text areas as in figure 4.2.
- Drawing areas to contain data point locations with different symbols as in figure 4.6.

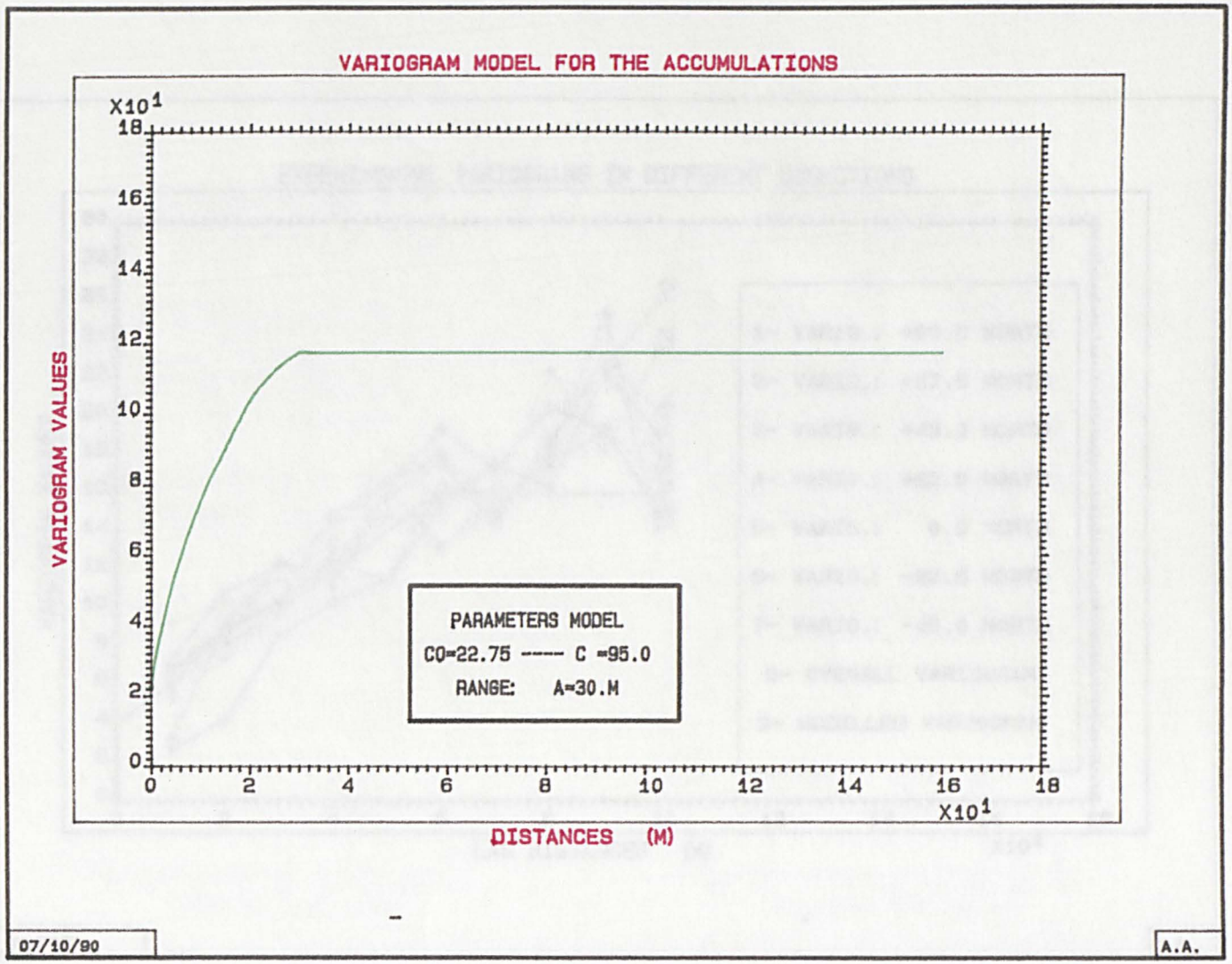

VARIOGRAM MODEL FOR THE ACCUMULATIONS
X10 1
18
16
14
12
10
8
6
4
2
0
VARIOGRAM VALUES
PARAMETERS MODEL
CO=22.75 ----- C =95.0
RANGE: A=30.M
0
2
4
6
8
10
12
14
16
18
X10 1
DISTANCES (M)
07/10/90
A.A.

Figure: 4.4

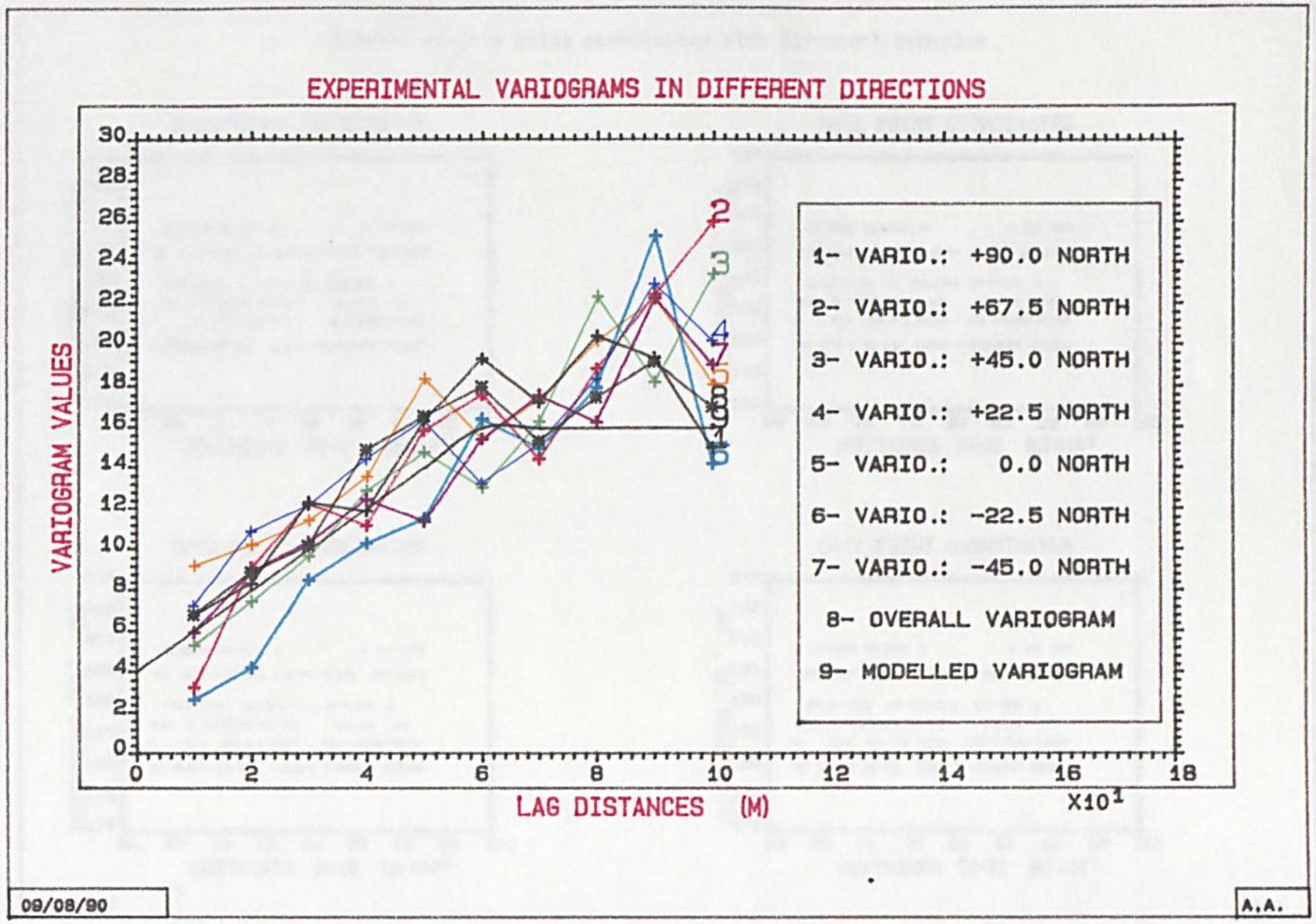

EXPERIMENTAL VARIOGRAMS IN DIFFERENT DIRECTIONS
VARIOGRAM VALUES
LAG DISTANCES (M)
X10¹
1- VARIO.: +90.0 NORTH
2- VARIO.: +67.5 NORTH
3- VARIO.: +45.0 NORTH
4- VARIO.: +22.5 NORTH
5- VARIO.: 0.0 NORTH
6- VARIO.: -22.5 NORTH
7- VARIO.: -45.0 NORTH
8- OVERALL VARIOGRAM
9- MODELLED VARIOGRAM
09/08/90
A.A.

Figure: 4.5

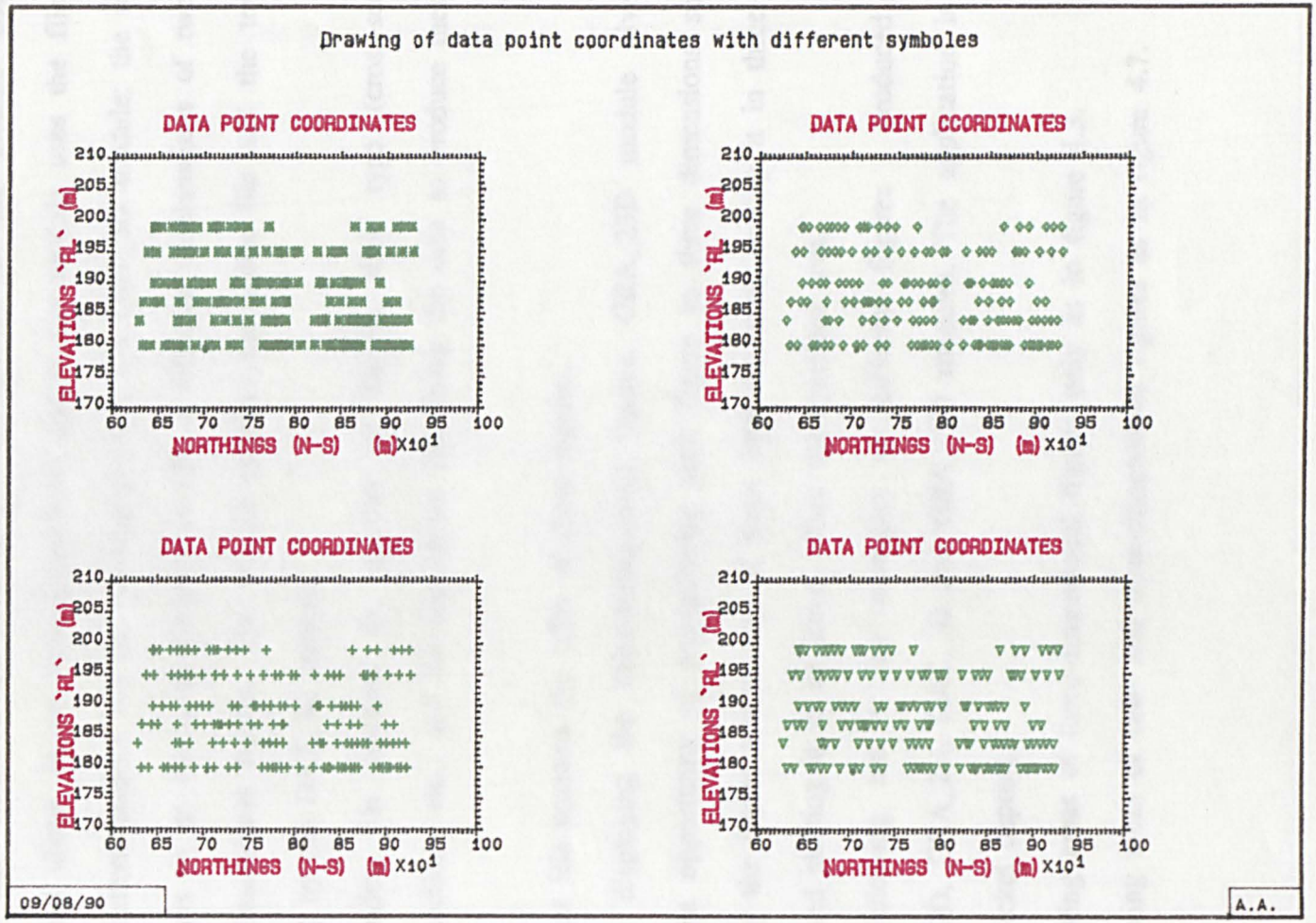

Drawing of data point coordinates with different symboles
DATA POINT COORDINATES
ELEVATIONS `RL` (m)
NORTHINGS (N-S) (m) X10¹
DATA POINT CCORDINATES
ELEVATIONS `RL` (m)
NORTHINGS (N-S) (m) X10¹
DATA POINT COORDINATES
ELEVATIONS `RL` (m)
NORTHINGS (N-S) (m) X10¹
DATA POINT COORDINATES
ELEVATIONS `RL` (m)
NORTHINGS (N-S) (m) X10¹
09/08/90
A.A.

Figure: 4.6

4.3.2.4 GRA_23D Module

This module is a combination of two- and three-dimensional graphics. It is exactly the same in its two-dimensional features as the module GRA_2D described above. In its three-dimensional option the module uses the files and specifications created and set previously using the DAT_3D module; the module then acts as an execution program to obtain different combinations of two- and three-dimensional figures. The module requires the index file and the text file created by the DAT_3D module.

The index file contains the number of figures their type (cross-sections, plane-sections, etc.) and the file names containing the data to produce these figures.

The text file contains the titles of these figures.

Before displaying the three-dimensional figures, GRA_23D module gives the user the opportunity of manipulating each figure in three dimensions such as rotating the figure around any of three cartesian axes, scaling it in three directions and shifting it to be fitted within the specified area.

The following are a few examples of different figures produced using DAT_2D, GRA_2D, DAT_3D and GRA_23D modules. The application is for a vein nickel deposit.

- Drawing areas of three-dimensional figures only as in figure 4.3.
- Drawing areas of two- and three-dimensional figures as in figure 4.7.

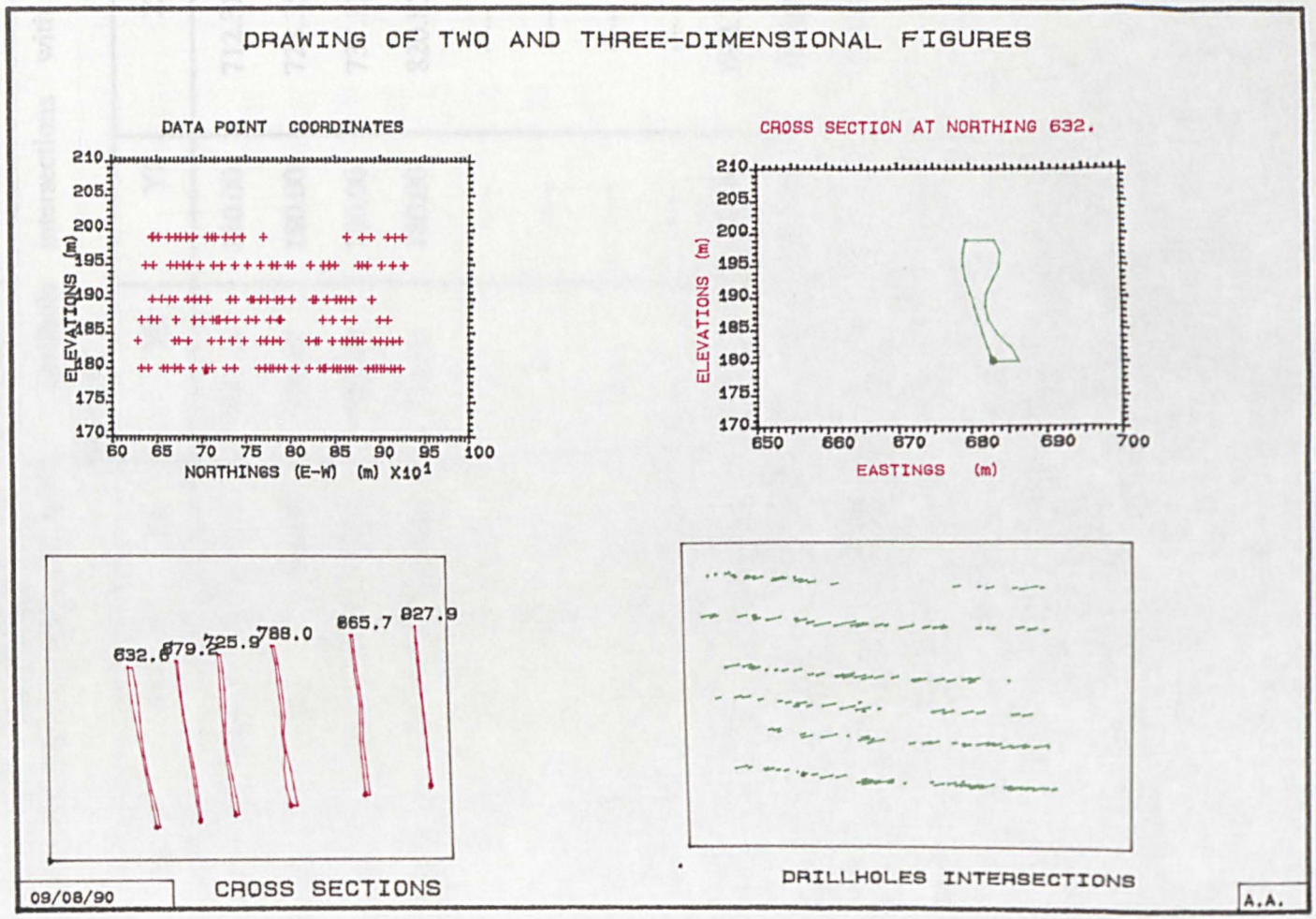

DRAWING OF TWO AND THREE-DIMENSIONAL FIGURES
DATA POINT COORDINATES
ELEVATIONS (m)
NORTHINGS (E-W) (m) X10¹
CROSS SECTION AT NORTHING 632.
ELEVATIONS (m)
EASTINGS (m)
632.6
679.2
725.9
788.0
865.7
927.9
CROSS SECTIONS
DRILLHOLES INTERSECTIONS
09/08/90
A.A.

Figure: 4.7

Table: 4.1 Drillhole intersection locations with the orebody (raw data)

NPTS	XO	YO	XLEN	YLEN
229	10.0	10.0	90.0	90.0

Drillhole intersections with the hangingwall			Drillhole intersections with the footwall		
Xh	Yh	Zh	Xf	Yf	Zf
700.63	180.00	712.31	702.19	180.00	712.31
701.91	180.00	728.35	705.62	180.00	728.35
706.20	180.00	737.12	709.30	180.00	737.12
729.34	180.00	820.82	730.06	180.00	820.82
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....
697.29	190.00	692.75	797.89	190.00	692.75
699.12	190.00	699.69	700.72	190.00	699.69
704.88	195.00	714.41	705.84	195.00	714.41
.....	.....	.....	.....	.....	.....

NPTS: the number data points (drillholes).

XO, YO: Origin of the area to contain the drillhole intersections.

XLEN, YLEN: Dimensions of the area.

X(h/f): Northings.

Y(h/f): Elevations.

Z(h/f): Eastings.

Table: 4.2 The numerical model of the orebody

NX	NY	XO	YO	XLEN	YLEN
20	10	10.0	10.0	270.	180.

NORTHINGS XC	ELEVATIONS YC	EASTINGS (HW)	EASTINGS (FW)
632.63	180.00	682.81	686.33
632.63	182.11	682.81	686.33
632.63	199.00	682.81	686.33
648.17	180.00	688.03	689.07
648.17	182.11	687.99	689.37
648.17	184.22	701.71	705.09
725.87	182.11	700.91	704.23
725.87	184.22	701.01	703.62
725.87	199.00	706.79	708.66
741.41	180.00	707.15	709.98
741.41	182.11	708.23	710.39
.....	.....	.....	.....

NB: The hangingwall and the footwall have been interpolated (modelled) at the same grid points (XC,YC) on the major plane (north-elevation).

NX: Number of grid points in the X direction (North).

NY: Number of grid points in the Y direction (elevation).

XO, YO: Origin of the area to contain the wireframe model.

XLEN, YLEN: Size of the area.

Table: 4.3a Data form for drawing cross-sections in two dimensions

Type
CRX

lt	Sy	Np	Axmin/x	Axmax/x	Axmin/y	Axmax/y
'L'	0	10	600.00	1000.00	170.00	210.00

Xh	Yh	Xf	Yf
682.81	180.00	686.33	180.00
682.19	182.11	685.40	182.11
681.59	184.22	683.62	184.22
.....	.....	.....	.....
.....	.....	.....	.....
.....	.....	.....	.....
.....	.....	.....	.....
.....	.....	.....	.....
.....	.....	.....	.....
679.00	194.77	683.89	194.77
678.99	196.88	684.10	196.88
679.41	198.99	683.25	198.99
.....	.....	.....	.....

Table:4.3b Data form for drawing curves and data point locations

CUR						
'X'	2	155	600.00	1000.00	170.00	210.00

704.62	180.00
.....	.....
710.28	180.00
711.52	180.00
711.04	180.00
714.73	180.00
718.40	180.00
722.37	180.00
729.34	180.00
729.46	180.00
733.50	180.00
737.97	180.00
735.05	180.00
.....	.....
671.50	199.00

Table: 4.3c Data form for drawing data point locations and their attributes Zi.

CUR						
'D'	0	155	600.00	1000.00	170.00	210.00

Xi	Yi	Zi
704.62	180.00	1.24
.....	.....	....
712.33	180.00	3.54
.....	.....	....
671.50	199.00	6.23
.....	.....	....

Type: The type of the drawing; **CRX** for cross-sections and **CUR** for curves and data point locations.

lt: Line type, smooth curves (lt=C), simple line segments (lt=L) and data point locations (lt=X and lt=D if class of attributes are to be shown).

Sy: Symbols to be drawn at data point locations (up to 8 symbols); 0 for no symbols to be drawn.

Np: Number of data points.

Axmin/x & Axmax/x : Minimum and maximum values on the X axis (the range should be wide enough to contain the data to be drawn).

Axmin/y & Axmax/y :Minimum and maximum values on the Y axis.

Xh & Yh : Coordinates of the hangingwall on the cross-section.

Xf & Yf : Coordinates of the footwall on the cross-section.

Table: 4.4 Index file format for three-dimensional figures.

9.00E-001	Scaling factor to allow the figures to be fitted on a specified window (area).
4	Number of figures to be drawn.
1	type 1: cross-sections.
2	type 2: plane-sections.
3	type 3: drillhole intersections.
4	type 4: wireframe model.
632.63 927.90 180.00 199.00 682.81 760.64	Minimum and maximum data values, used for scaling purposes.
CROS3.FIL	Cross-sections drawing file.
PLAN3.FIL	Plane-sections drawing file.
FIL.NID	Drillhole intersections file (raw data).
FILE.NI2	Numerical model of the orebody, specified for wire-frame drawing.

Table: 4.5 Drawing data for cross-sections

10.0	10.0	The origin of the area
90.0	90.0	The dimensions of the area
5	The number of cross-sections	
632.63 694.79 772.49	The fixed coordinates of the cross-sections (in this example the northings)	
10	The number of points on each side of the cross-section	

Northings	Elevations	Eastings (hangingwall)	Eastings (footwall)
632.63	180.00	682.81	686.33
632.63	182.11	682.81	686.33
.....	.....	.....	.....
632.63	196.89	682.81	686.33
632.63	199.00	682.81	686.33
10			
694.79	180.00	697.94	698.33
694.79	182.11	697.89	698.36
.....	.....	.....	.....
694.79	199.00	697.76	700.60
10			

Table: 4.6 Drawing data for plane-sections

10.0	80.0	The origin of the area
90.0	90.0	The dimensions of the area
5	The number of plane-sections	
180.00 184.22 188.44	The fixed coordinates of the plane-sections (in this example the elevations)	
20	The number of points on each side of the plane-section	

Northings	Elevations	Eastings (hangingwall)	Eastings (footwall)
632.63	180.00	682.81	686.33
648.17	180.00	688.62	689.07
.....	.....	.....	.....
912.36	180.00	753.41	756.54
927.90	180.00	759.95	762.55
20			
633.79	184.22	682.81	686.33
710.79	184.22	699.71	701.00
.....	.....	.....	.....
927.79	184.22	759.95	762.57
20			
.....	.....	.....	.....

4.3.2.5 GRAMOD Module

This module is an independent one performing two types of three-dimensional figures of the orebody model:

a- wireframe model

b- solid model, obtained by removing the hidden line from the wireframe.

The module does not require any predefined files for setting-up or manipulating the figures, the only file required is the orebody model file , cf. Table 4.2, and the titles of the figures. The module gives the user the option of specifying the type of figures to be displayed and the areas (windows) to contain them. It also requires a view point coordinate to be specified to allow the user to display the orebody from different directions. In addition to the specification of the overall scaling factor, local scaling factors of individual figures and the translation of these figures to be shifted to the desirable position in the window are requested.

The wireframe model is a straight-forward operation performed by displaying all facets (rectangular) which constitute the external surface of the orebody. The solid model is obtained by removing the hidden lines from the wireframe model. This operation requires a large amount of calculations to detect the visible and the invisible part of the orebody specific to each viewpoint coordinate, then only the visible part is displayed. The algorithm used for performing the removal of hidden lines can be found in ANGEL (1981).

It is worth pointing out that the solid modelling of an orebody represented by a large number of point coordinates takes a relatively large computer processing time which is critical on a PC. To avoid a long wait time before the user sees the solid model appearing on the screen he is advised first to display only the wireframe model before deciding on the solid model to be produced.

The following are examples of different figures produced by GRAMOD module to display a three-dimensional model of a vein nickel deposit:

- Solid and wireframe models produced in one run as in figure 4.8.
- Solid models with different view point coordinates as in figure 4.9.

4.3.2.6 GRACON Module

This module is an independent module which contours values located at regular grid points. For values located on scattered point locations a preliminary interpolation procedure is applied to obtain values on a regular grid. Kriging techniques or deterministic methods are used for this interpolation. The module requests the file containing the coordinates and the values to be contoured and the title of the figure to be produced; the user must also specify the number of contours (classes), and he has the option of drawing data point locations alongside the contour lines. The example in figure 4.10 shows the contours of estimated accumulations (%Ni_m).

4.3.2.7 GRATRI Module

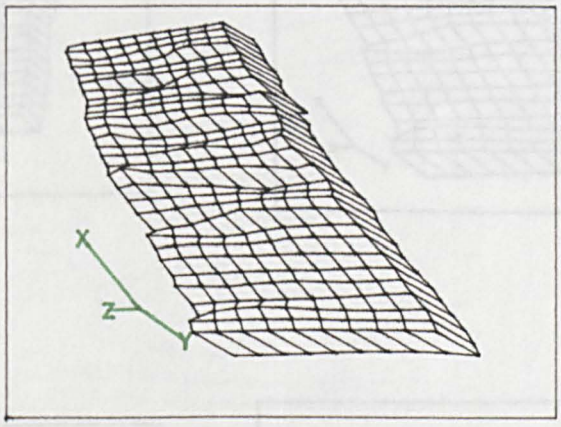
GRATRI is also an independent module; it performs triangulation of surfaces using data point locations representing these surfaces. The module requires only the data file, each data point is specified by its three-dimensional coordinates (X_i, Y_i, Z_i). The triangulation operation is performed using the Delaunay method which is presented in chapter 3 section 3.2.3.2. Whilst the graphical representation of a triangulated surface is trivial it is nevertheless helpful in visualizing the nature of the surface, and anomalies can be detected and taken into account while performing the modelling of the surface. The example in Figure 4.11 shows the triangulation of the hangingwall of the nickel deposit represented by the drillhole intersections with the wall.

X-axis for the Northings _____Y-axis for the Eastings_____Z-axis for the elevations `RL`

View Point Coordinates

EX=	70.00	Xmin=	628.50	Xmax=	924.30
EY=	-40.00	Ymin=	678.33	Ymax=	776.93
EZ=	10.00	Zmin=	180.00	Zmax=	198.90

Solid Model of the Vein
Bilinear interpolation: 1st method



Wireframe Model of the Vein
Bilinear interpolation: 1st method

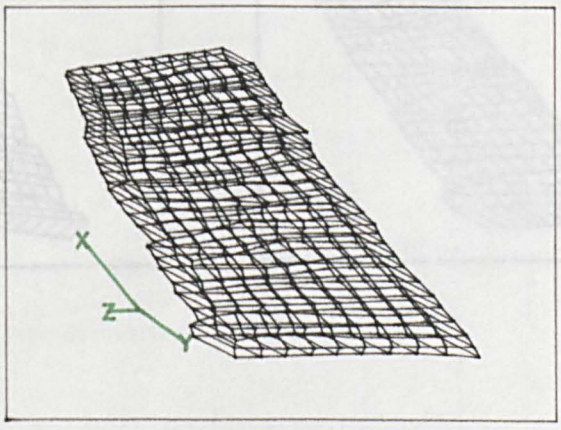


Figure: 4.8

Solid model of the vein nickel deposit

with different viewpoint coordinates

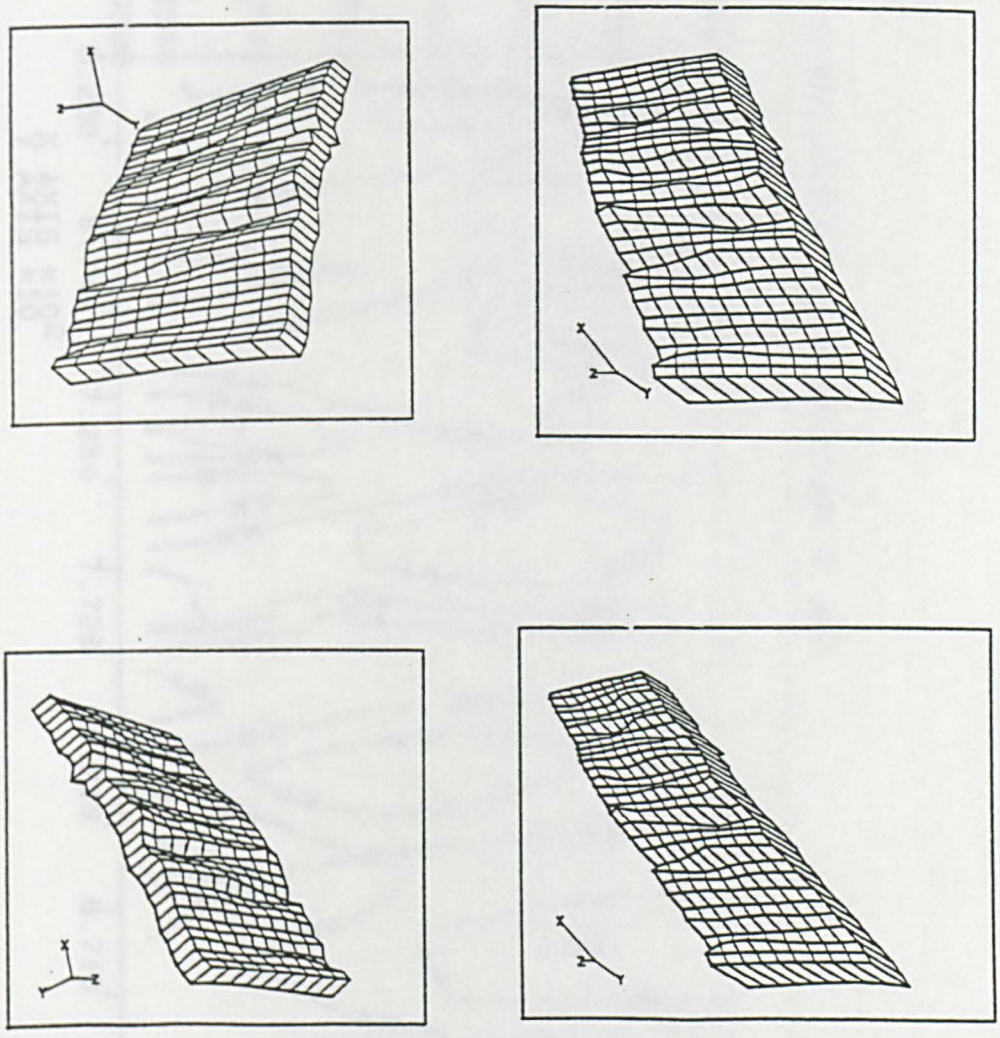


Figure: 4.9

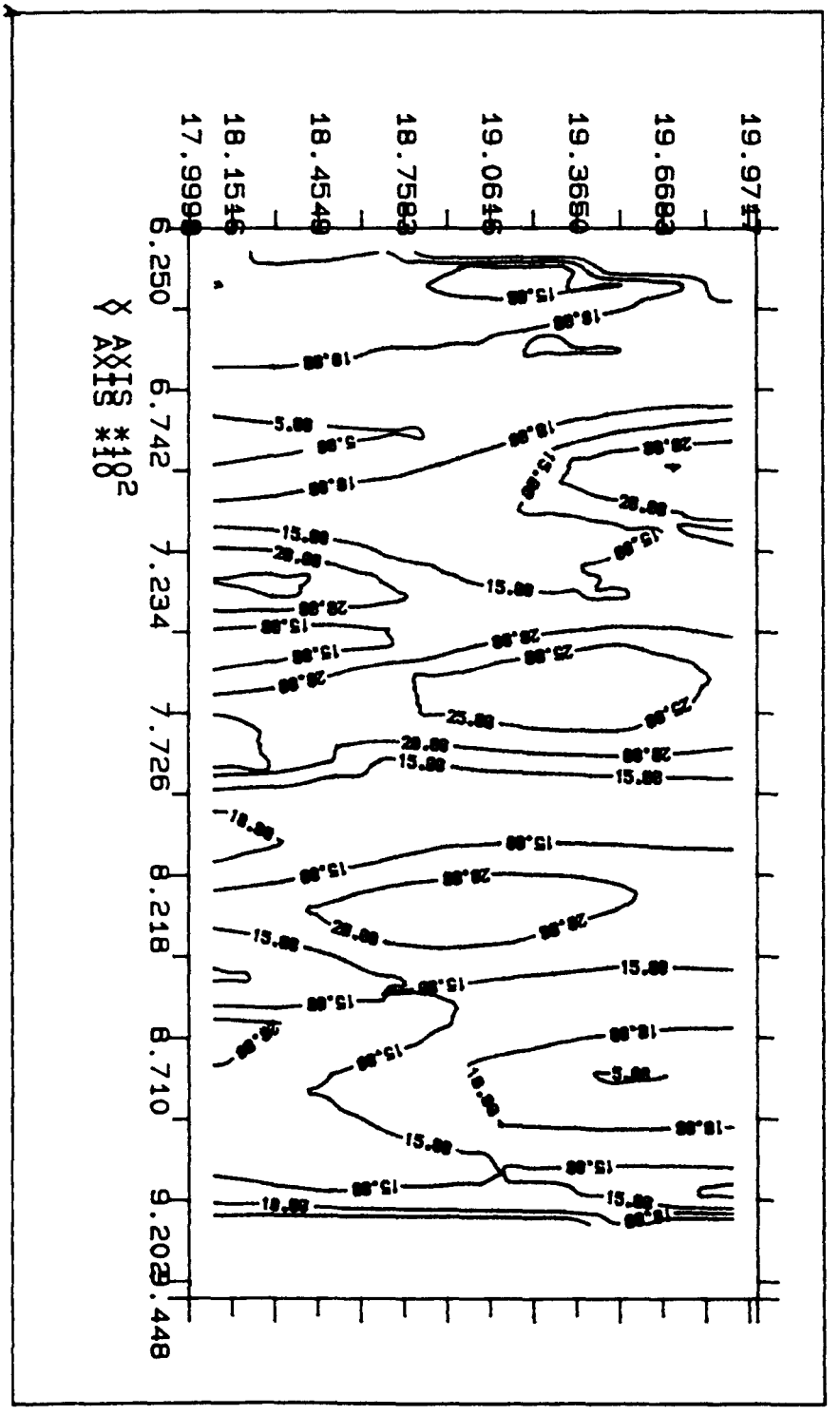

CONTOUR LINES OF THE ACCUMULATIONS (M. SNI)
17.9998
18.1546
18.4549
18.7582
19.0646
19.3650
19.6683
19.9717
6.250
6.742
7.234
7.726
8.218
8.710
9.202
9.448
Y AXIS *10²

Figure: 4.10

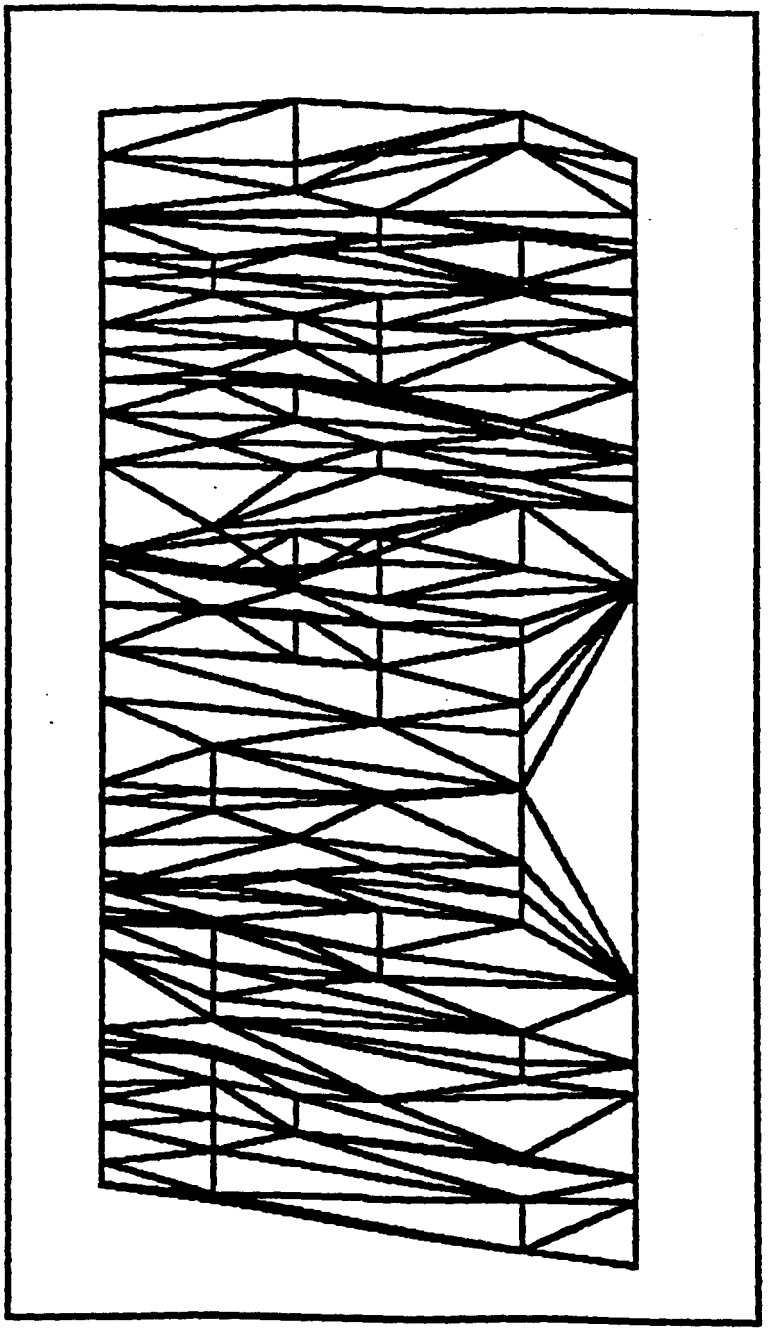


Figure: 4.11

4.4 Summary

The introduction of graphics techniques to display geometrical features of orebodies for estimation and mining purposes has proved to be an excellent technical means by which mining and planning engineers can obtain better and more efficient control over the mining process. It offers the accuracy of displaying the existing numerical information about the orebody together with the possibility of an accurate design of the mine itself. This graphics display system is developed to be more general in its two-dimensional representation and more or less specific to stratigraphic orebodies in its three-dimensional representation. The modular concept allowed the system to be implemented on smaller computers (PC's) and the development of smaller modules is more suitable than developing larger ones. The windowing system allows the display of several features and types of information concerning directly or indirectly the geometrical modelling process of the orebody. The independent modules such as GRAMO, GRATRI and GRACON play the role of complementing the core part of the system which consists of the DAT_2D, DAT_3D, GRA_2D and GRA_23D modules.

Chapter Five

Summary and Conclusions

5.1 Summary

This research project was very much concerned with the introduction of new techniques and methods, developed in other areas, alongside the ones developed in mining for ore reserves estimation and mining planning and operation. The main objective is to increase the awareness among technical staff in the mining industry, including management, of the existing industrial and scientific fields dealing with problems similar to those encountered in mining. These new techniques have much to do with the rapidly increasing use of information technology; at present if geometrical modelling or computer-aided design methods are mentioned they are mainly associated with mechanical engineering and the manufacturing world. However, having said that does not mean that mining technology is out of touch with these applications but the mining community, described as conservative, has not made full use of these new techniques. There is a mounting pressure on the engineering staff in the mining industry to cope with very complicated problems and to plan directly effective mining operations with less abundant mineral resources. Add to this the high risk factor set for investors in this field, and there are obvious advantages to the mining industry in acquiring the relevant information technology and catching up

with other industrial sectors which are benefiting from the introduction of these techniques. The example of the aeronautic and automobile industries which have introduced computer-aided design and computer-aided manufacturing techniques, has shown to what extent these techniques have helped to overcome many problems. One of these problems is that of time: more work is done in less time in making use of these computer based facilities, and there is a corresponding increase in productivity and creativity in those sectors.

In the light of this philosophy, this research project has demonstrated the utility of using geometrical modelling techniques alongside geostatistical methods to build up an evaluation and modelling system to obtain a numerical model of a stratigraphic orebody and the use of graphics techniques to develop a graphics display system to convert the numerical results into a pictorial format which is easier to use. The approach adopted for this research project started with the idea of introducing spline techniques to control geostatistical estimates in stratigraphic orebodies (Dowd, 1983,1984,1986,1989). The search for new methods of applying geometrical control over geostatistical estimates led to the adoption of surface modelling techniques to construct a geometrical model for the control of subsequent geostatistical grade estimates (Abbachi 1989) and as a method of geological (geometrical) modelling in its own right. The need for these surface modelling techniques has led the author into other fields which deal with spatial problems with substantive application of deterministic methods and computer graphics. Although the field used predominantly was that of computer-aided design, there are many fields which could have been explored such as geographical information systems (G.I.S.) or map designers to enhance the modelling systems using geostatistical methods.

It was decided that the geometrical control of geostatistical estimates in stratigraphic orebodies would consist of modelling the hangingwalls and the footwalls of constituent units of the orebody using surface modelling techniques. Chapter two highlights one of the main elements of the modelling system which is the data (raw data) collected to represent these surfaces. The data consist mainly of the intersections of drillholes with the two surfaces which delimit each constituent unit of the orebody. Forms of stratigraphic units and data configurations of the drillhole intersections with the orebody have been presented as these data will be an essential factor in deciding which surface modelling technique will be adopted. It was concluded that the most generally occurring case is the randomly distributed data points locations scheme usually referred to as scattered data.

In chapter three, the author has dealt with the main part of the geometrical control process in stratigraphic orebodies which is surface modelling techniques and their applications. They have been introduced and explained in very general terms to demonstrate the potential of these techniques to model surfaces such as hangingwall and footwall of a stratigraphic unit. Spline techniques were selected for their simplicity and applicability to conditions such as orebody modelling and also their close relationship with kriging estimators. Interpolation methods based on global procedures have been avoided as they are more complicated and are not suitable for erratic, unknown trends, such as the geometrical features of stratigraphic orebodies, and for the long computing time required. As spline interpolation methods are performed over triangular patches a triangulation method is required as a first step to apply these methods. The

Delaunay method of triangulation of surfaces which takes data points (drillhole intersections) as vertices of these triangular patches was chosen because of its availability and its wide applicability in many fields.

Several spline techniques (bivariate interpolation methods) performed over triangular patches have been introduced in an attempt to consider these techniques in more detail. From these possibilities two methods were adopted for the application of geometrical control of geostatistical estimates (Abbachi 1989):

- The Bilinear interpolation method
- The Bivariate, fifth-degree, interpolation method (Quintic Scheme).

These two methods have been adopted because they performed well and both of them have been presented in detail. The first method, which is a bilinear interpolation method based on fitting a bivariate function of degree one on a triangular cell, is a very simple method. It does not require a large amount of computing time even for a surface represented by a large number of data points, however, its disadvantage is the poorly smoothed surface which is obtained. The second method, which is a bivariate interpolation method based on fitting a fifth degree polynomial (Quintic Scheme) over a triangular patch, is known as Akima's method of interpolation of scattered data. This method is more suitable for erratic and complex surfaces as it gives a high degree of continuity and smoothness. However, one of the drawbacks of this method is a relatively large run time, together with the special attention which has to be paid to the data configuration especially when large voids in the data are present. The application of these two surface modelling techniques has been clearly demonstrated in a case study in the last section of chapter three by producing a numerical model of a Nickel vein deposit.

Finally, as graphics techniques are the best medium to communicate the results of this modelling process of an orebody, a graphics display system has been developed and presented in chapter four. This graphics system was built to cater for a wide range of two-dimensional representations of geometric features of stratigraphic orebodies such as cross-sections, profiles, data point locations, etc... in addition to windowing, scaling and typesetting enabling it to be used and perform as a general-purpose graphics system. In its three-dimensional form, this graphics display system displays in three dimensions cross-sections, plane-sections, wireframe and solid models of stratigraphic orebodies.

5.2 Research results

There are two aspects of the results of the research project presented in this thesis: philosophical and practical.

They are philosophical in the sense that the approach adopted is to make use of tools developed in other fields. This still requires more investigative work to be done to provide the full benefit of existing experience in such areas as computer-aided design (C.A.D), geographical information systems (G.I.S) and others and their experience in dealing with spatial problems.

The practical aspect is the successful application of surface modelling techniques alongside geostatistical methods to evaluate stratigraphic orebodies. However, what has been exposed here could be investigated in many research projects because there are many points still to be better understood (cf. Section. 5.3). In this thesis it has been shown that data representing the orebody to be modelled are essential elements in the modelling process; they must to be sufficient and representative. The review of some surface modelling techniques was an attempt at familiarisation with these mathematical tools. The application of two surface

modelling methods has contributed substantially towards building a more reliable numerical model of an orebody. The author does not exclude the existence of better and more suitable methods for these kinds of applications and only a more specific and well targeted research project could provide a good classification of these surface modelling techniques according to their application to mining related problems (eg., Ore reserve estimation, mine design and operation, etc...). One advantage of surface modelling techniques applied to geometrical modelling of orebodies is the immediate interactive response especially if fully integrated with an interactive graphics system.

Finally, the development of a computer graphics display system has been regarded as an attempt at using interactive graphics techniques to display effectively a numerical model of an orebody. It has been designed firstly to represent existing information, secondly to display more information at same time. However, representing only the existing geometrical features of an orebody is not the final aim but a step which could be followed by another one allowing the user to design the elements of a whole mining operation. Then to display maximum information at once on the screen or on any display device, by opening several windows at the same time thus allowing the user to have a full picture of all aspects of a project.

5.3 Scope for future work

The mining industry presently lags behind others in it use of information technology and at the same time is regarded as a future potential user of this technology to deal with its complex problems. Thus, now more than ever before, research on the application of information technology in the mining industry requires a specialized program to be able not only to use or adopt

existing techniques but to contribute to the development of its own specific tools such as geostatistical methods. A special research program for the mining industry would consist of investigating the potentially needed branches of information technology which will boost effectiveness in these areas.

From a mine planning and operational point of view, there are several branches of information technology to be investigated such as:

- Database systems: to allow engineers to access a wide range of information concerning the deposit and the mining operation. This information could be raw data or the result of a modelling process.
- Graphics systems: are very fundamental for any successful integrated computer systems in the mining industry. These graphics display systems should be built to make full use of facilities offered by advanced hardware and graphics techniques available.
- Expert systems (artificial intelligence): Mine planning and operation as a multidimensional process requires the introduction of such systems. Many, if not the majority of the problems in the mining industry are very difficult to describe in a quantitative and exact manner. The need for a computerized problem-solving approach is suited to this type of task as it is capable of arriving at a solution through arguing, reasoning and concluding rather than by counting and calculating.

However, from this particular study of geometrical control of geostatistical estimates and the graphics display of stratigraphic orebodies it is very apparent that surface modelling techniques offer an additional tool to geostatistical methods to develop a more robust ore reserves evaluation system. There is a need for good understanding and treatment of the raw data and the need for combining all sorts of available data; even the so called soft data, seen as an element based

on the expertise, has to be used. A future research project may include an exhaustive investigation of surface modelling techniques and their suitability as deterministic methods to be used for the evaluation and graphical representation of mineral deposits alongside geostatistical methods; this would be a valuable contribution to the advancement of the mining industry. As for graphics techniques, much has to be done to update the existing facilities and to adapt new ones. However, there is a need for integrating any specific graphics display system related to orebody modelling and mine design to most well known computer-aided design packages (eg., Autocad) as it is likely that a standard computer-aided design will be adopted offering a wider range of sophisticated facilities for better display of information.

APPENDIX

Appendix

Interpolation over triangles

Akima's Method

1 Introduction

In this appendix to chapter three, explaining the second interpolation method (Quintic scheme), Akima (1975) gave a step by step procedure, quoted here, of obtaining the values of the interpolated points. It is assumed that the plane is divided into a number of triangles and a procedure for interpolating values of a function in each triangle is described. The primary emphasis is on the smoothness of the interpolation, not only inside the triangle but also along the sides of it; i.e, the interpolation values in a triangle must smoothly connect with the values in an adjacent triangle on the common side of the two triangles.

2 Basic Assumptions

Using a two-dimensional Cartesian coordinate system with x and y axes, the basic assumptions are described as follows.

i- The values of the function at point (x,y) in a triangle are interpolated by bivariate fifth-degree polynomials in x and y (quintic scheme); i.e,

$$Z(x,y) = \sum_{j=0}^{5} \sum_{k=0}^{5-j} q_{jk} x^j y^k \qquad (1)$$

Note that there are 21 coefficients to be determined, (n=5, Nc=1/2(n+1)(n+2)).

ii- The values of the function and its first-order and second-order partial derivatives (i.e, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}) are given at each vertex of the triangle. This assumption yields 18 independent coefficients.

iii- The partial derivatives of the function differentiated in the direction perpendicular (normal) to each of the triangles is a polynomial of degree 3, at most, in the variable measured in the direction of the side of the triangle. When the coordinate system is transformed to another cartesian system, which is called the s-t system, in such a way that the s axis is always parallel to one side of the triangle, the bivariate polynomial in s and t representing the z values must satisfy:

$$Z_{tsss} = 0$$

or

$$\frac{\delta^5 Z_{(s,t)}}{\delta_s^4 \delta_t} = 0 \qquad (2).$$

As a triangle has three sides, this assumption yields three additional conditions.

The purpose of the third assumption is two-fold. This assumption adds three independent conditions to the 18 conditions dictated by the second assumption and thus enables the determination of the 21 coefficients required for the polynomial. It also assures smoothness of interpolated values and therefore smoothness of the resulting surface along the side of the triangle.

As the coordinate transformation between the x-y system and the s-t system is linear, the values of Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy} at each vertex uniquely determine the values Z_s, Z_t, Z_{ss}, Z_{st} and Z_{tt} at the same vertex; each of the latter is a linear combination of the former. Then Z, Z_s and Z_{ss} values at two vertices uniquely determine a fifth-degree polynomial in s for Z on the side between these two vertices. Two fifth-degree polynomials in x-y representing Z values on two triangles that share the common side are reduced to fifth-degree polynomials in s on that side. This proves continuity of the interpolated Z values along a side of a triangle. Similarly, the value Z_t and $Z_{st} = (Z_t)_s$ at vertices uniquely determine a third-degree polynomial in s for Z_t on the side.

As the polynomial representing Z_t is assumed to be a third-degree polynomial at most with respect to s, two triangles that share a common side also coincide with each other on that side. This proves continuity of Z_t and thus the smoothness of Z along the side of the triangle.

3 Coordinate system associated with the triangle

The vertices of the triangle are denoted V_1, V_2 and V_3 in anticlockwise order, and their respective coordinates in the x-y cartesian coordinate system by (x_1 , y_1), (x_2 , y_2) and (x_3 , y_3) as shown in Figure (1a). A new coordinate system is introduced within the triangle, where the vertices are represented by (0,0), (1,0) and (0,1) as shown in Figure (1b). This new coordinate system is called the u-v system.

The coordinate transformation between the x-y system and the u-v system is represented by:

$$x = a_u + b_v + x_0$$

$$y = c_u + d_v + y_0 \tag{3}$$

where

$$a = x_2 - x_1$$

$$b = x_3 - x_1$$

$$c = y_3 - y_1 \tag{4}$$

$$d = y_2 - y_1$$

$$x_0 = x_1$$

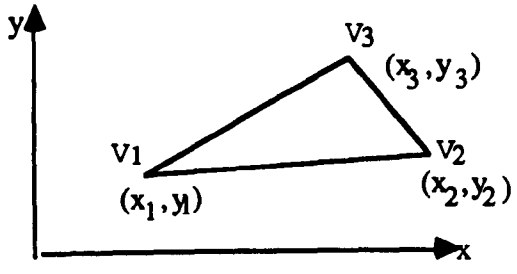
$$y_0 = y_1$$

The inverse relation is:

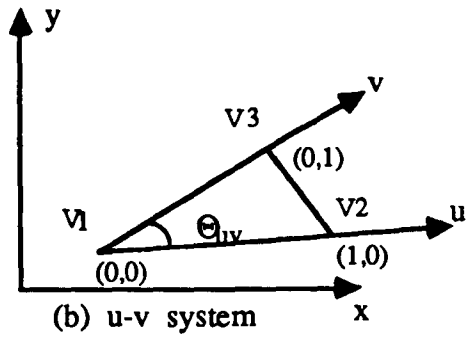
$$u = \frac{[d(x - x_0) - b(y - y_0)]}{(ad - bc)}$$

$$v = \frac{[-c(x - x_0) + a(y - y_0)]}{(ad - bc)} \tag{5}$$

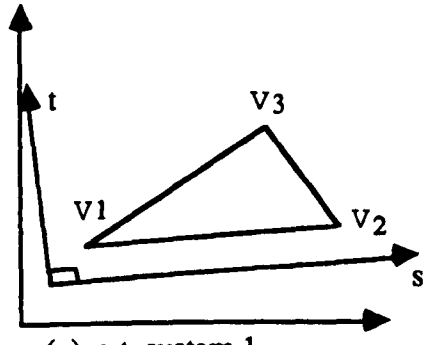
VARIOUS COORDINATE SYSTEMS



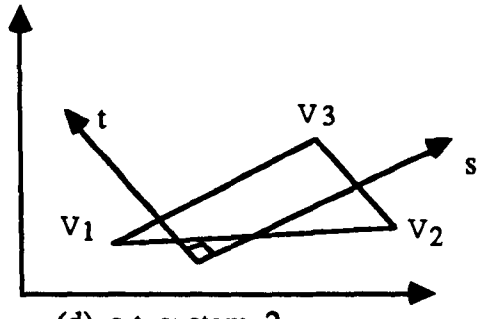
(a) x-y system



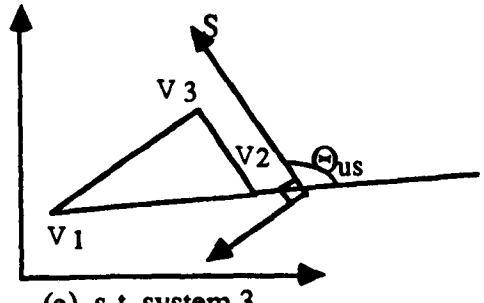
(b) u-v system



(c) s-t system 1



(d) s-t system 2



(e) s-t system 3

FIG. 1

The partial derivatives in the x-y system are transformed to the u-v system by:

$$Z_u = aZ_x + cZ_y$$

$$Z_v = bZ_x + dZ_y \tag{6}$$

$$Z_{uu} = a^2Z_{xx} + 2acZ_{xy} + c^2Z_{yy}$$

$$Z_{vv} = b^2Z_{xx} + 2bdZ_{xy} + d^2Z_{yy}$$

As this coordinate transformation is linear, the interpolating polynomial (1) is transformed to:

$$Z(u,v) = \sum_{j=0}^{5} \sum_{k=0}^{5-j} p_{jk} u^j v^k \tag{7}$$

As it is the p coefficients that are determined directly, as shown later, and are used for interpolating Z values, it is unnecessary to relate the p coefficients to the q coefficients used in (1).

The partial derivatives of Z(u,v) in the u-v system are expressed by:

$$Z_u(u,v) = \sum_{j=1}^{5} \sum_{k=0}^{5-j} j p_{jk} u^{j-1} v^k$$

$$Z_v(u,v) = \sum_{j=0}^{4} \sum_{k=1}^{5-j} k p_{jk} u^j v^{k-1}$$

$$Z_{uu}(u,v) = \sum_{j=2}^{5} \sum_{k=0}^{5-j} j(j-1) p_{jk} u^{j-2} v^k \tag{8}$$

$$Z_{uv}(u,v) = \sum_{j=1}^{4} \sum_{k=1}^{5-j} jk p_{jk} u^{j-1} v^{k-1}$$

$$Z_{vv}(u,v) = \sum_{j=0}^{3} \sum_{k=2}^{5-j} k(k-1) p_{jk} u^j v^{k-2}$$

The length of the unit vectors in the u-v system (i.e, the length of the sides $\overline{V_1V_2}$ and $\overline{V_1V_3}$ by L_u and L_v, respectively, and the angle between u and v by Θ_{uv}, are given b:

$$L_u = a^2 + c^2$$

$$L_v = b^2 + d^2 \tag{9}$$

$$\Theta_{uv} = \tan^{-1}\left(\frac{d}{b}\right) - \tan^{-1}\left(\frac{c}{a}\right)$$

Where a, b, c and d are constant and given in (4).

4 Implementation of the third assumption

The third assumption (2) will be represented in the u-v system and useful equations are derived for determining the coefficients of the polynomial. This will be done for three cases corresponding to the three sides of the triangle. First, consider the case where the s axis is parallel to the side V_1V_2 as shown in Figure (1c) . The coordinate transformation between the u-v system and the s-t system is expressed by:

$$u = \frac{[(\sin\Theta_{uv})(s - s_0) - \cos(\Theta_{uv})(t - t_0)]}{(L_u \sin\Theta_{uv})}$$

(10)

$$v = \frac{(t - t_0)}{(L_v \sin\Theta_{uv})}$$

Where L_u, L_v and Θ_{uv} are constants given in (9).

Partial derivatives with respect to s and t are expressed by:

$$\frac{\delta}{\delta_s} = \frac{1}{L_u}\frac{\delta}{\delta_u}$$

(11)

$$\frac{\delta}{\delta_t} = \frac{\cos\Theta_{uv}}{L_u \sin\Theta_{uv}}\frac{\delta}{\delta_u} + \frac{1}{L_v \sin\Theta_{uv}}\frac{\delta}{\delta_v}$$

respectively from (2), (7) and (11), the following equation is deduced:

$$L_u p_{41} - 5L_v \cos\Theta_{uv} p_{50} = 0 \tag{12}$$

Next, consider the case where the s axis is parallel to side V_1V_3, as shown in Figure (1d). The coordinate transformation is expressed by:

$$u = \frac{-(t - t_0)}{(L_u \sin\Theta_{uv})}$$

(13)

$$v = \frac{[(\sin\Theta_{uv})(s - s_0) + (\cos\Theta_{uv})(t - t_0)]}{(L_v \sin\Theta_{uv})}$$

Partial derivatives are expressed by:

$$\frac{\delta}{\delta_s} = \frac{1}{L_v}\frac{\delta}{\delta_v}$$

(14)

$$\frac{\delta}{\delta_t} = -\frac{1}{L_u \sin\Theta_{uv}}\frac{\delta}{\delta_u} = \frac{\cos\Theta_{uv}}{L_v \sin\Theta_{uv}}\frac{\delta}{\delta_v}$$

Then from (2), (7) and (14) the following equation is obtained:

$$L_v p_{14} - 5L_u \cos\Theta_{uv} p_{05} = 0 \tag{15}$$

Next, consider the third case where the s axis is parallel to side V_2V_3, as shown in figure (1e). The coordinate transformation is expressed by:

$$u = A(ss_0) + B(t - t_0)$$

$$v = C(s - s_0) + D(t - t_0) \tag{16}$$

where

$$A = \frac{\sin(\Theta_{uv} - \Theta_{us})}{(L_u sun \Theta uv)}$$

$$B = \frac{-\cos(\Theta_{uv} - \Theta_{us})}{(L_u \sin \Theta_{uv})}$$

$$C = \frac{\sin \Theta_{us}}{(L_v \sin \Theta_{uv})} \tag{17}$$

$$D = \frac{\cos \Theta_{us}}{(L_v \sin \Theta_{uv})}$$

$$\Theta_{us} = tang^{-1}\left[\frac{(d-c)}{(b-a)}\right] - tang^{-1}\left(\frac{c}{a}\right)$$

Θ_{us} is the angle between the s and u axes. a, b, c and d constants and are given in (4) and L_u, L_v and Θ_{uv} are given in (9).

The partial derivatives with respect to s and t are expressed by:

$$\frac{\delta}{\delta_s} = A\frac{\delta}{\delta_u} + c\frac{\delta}{\delta_v}$$

$$\frac{\delta}{\delta_t} = B\frac{\delta}{\delta_u} + D\frac{\delta}{\delta_v} \tag{18}$$

From (2) and (7) and (18), the following equation is obtained:

$$5A^4Bp_{50} + A^3(4BC + AD)p_{41} + A^2C(3BC + 2AD)p_{32} + AC^2(2BC + 3AD)p_{23} + C^3(BC + 4AD)p_{05} + 5C^4Dp_{05} = 0 \qquad (19)$$

Equations (12), (15), and (19) are the results of the implementation of the third assumption in the u-v coordinate system. They are used for the determining the coefficients of the polynomial (7).

5 Determination of the coefficients of the polynomial

The coefficients of the lower power terms are determined by setting u=0 and v=0 and by inserting the values Z, Z_u, Z_v, Z_{uu}, Z_{uv} and Z_{vv} at V_1 in (7) and (8). The results are:

$$\begin{aligned} p_{00} &= Z(0,0) \\ p_{10} &= Z_u(0,0) \\ p_{01} &= Z_v(0,0) \\ p_{20} &= \frac{Z_{uu}(0,0)}{2} \\ p_{11} &= Z_{uv}(0,0) \\ p_{02} &= \frac{Z_{vv}(0,0)}{2} \end{aligned} \qquad (20)$$

Next, letting u=1 and v=0 and inserting the values of Z, Z_u and Z_{uu} at V_2 in (7) and the first and the third equations in (8), the following three equations are obtained:

$$p_{30} + p_{40} + p_{50} = Z(1,0) - p_{00} - p_{10} - p_{20}$$
$$3p_{30} + 4p_{40} + 5p_{50} = Z_u(1,0) - p_{10} - 2p_{20}$$
$$6p_{30} + 12p_{40} + 20p_{50} = Z_{uu}(1,0) - 2p_{20}$$

Solving these three equations with respect to p_{30}, p_{40} and p_{50} gives:

$$\begin{aligned} p_{30} &= (20Z(1,0) - 8Z_u(1,0) + Z_{uu}(1,0) - 20p_{00} - 12p_{10} - 6p_{20})/2 \\ p_{40} &= -15Z(1,0) + 7Z_u(1,0) - Z_{uu}(1,0) + 15p_{00} + 8p_{10} + 3p_{20} \\ p_{50} &= (12Z(1,0) - 6Z_u(1,0) + Z_{uu}(1,0) - 12p_{00} - 6p_{10} - 2p_{20})/2 \end{aligned} \qquad (21)$$

As p_{00}, p_{10} and p_{20} are already determined by (20), p_{30}, p_{40} and p_{50} can be calculated using (21)

Similarly, using Z, Z_v and Z_{vv} at V_3 (i.e, u=0 and v=1) and working with (7) and the second and the last equations in (8) is to obtain:

$$\begin{aligned} p_{03} &= (20Z(1,0) - Z_v(0,1) + Z_{vv}(0,1) - 20p_{00} - 12p_{01} - 6p_{02})/2 \\ p_{04} &= -15Z(0,1) + 7Z_v(0,1) - Z_{vv}(0,1) + 10p_{00} + 8p_{01} + 3p_{02} \\ p_{05} &= (12Z(0,1) - 6Z_v(0,1) + Z_{vv}(0,1) - 12p_{00} - 6p_{01} - 2p_{02})/2 \end{aligned} \qquad (22)$$

with p_{50} and p_{05} determined p_{41} and p_{14} can be determined from (12) and (15) respectively. The results are:

$$p_{41} = \frac{5L_v \cos\Theta_{uv}}{L_u} p_{50}$$

$$p_{14} = \frac{5L_u \cos\Theta_{uv}}{L_v} p_{05} \qquad (23)$$

Next, the values of Z_v and Z_{uv} at V_2 (i.e, u=1 and v=0) are used with the second and the fourth equations in (8) gives:

$$p_{21} + p_{31} = Z_v(1,0) - p_{01} - p_{11} - p_{41}$$

$$2p_{21} + 3p_{31} = Z_{uv}(1,0) - p_{11} - 4p_{41}$$

Solving these equations gives

$$p_{21} = 3Z_v(1,0) - Z_{uv}(1,0) - 3p_{01} - 2p_{11} + p_{41}$$

$$p_{31} = -2Z_v(1,0) + Z_{uv}(1,0) + 2p_{01} - p_{11} - 2p_{41} \tag{24}$$

Similarly, using the values of Z_u and Z_{uv} at V_3 (i.e, u=0 and v=1) with the first and the fourth equations in (8) gives:

$$p_{12} = 3Z_u(0,1) - Z_{uv}(0,1) - 3p_{10} - 2p_{11} + p_{14}$$

$$p_{13} = -2Z_u(0,1) + Z_{uv}(0,1) + 2p_{10} - p_{10} - p_{11} - 2p_{14} \tag{25}$$

equation (19) is rewritten as:

$$g_1 p_{32} + g_2 p_{23} = h_1 \tag{26}$$

where

$$g_1 = A^2C(3BC + 2AD)$$

$$g_2 = AC^2(2BC + 3AD) \tag{27}$$

$$h_1 = -5A^4B p_{50} - A^3(4BC + AD)p_{41} - C^3(BC + 4AD)p_{14} - 5C^4D p_{05}$$

with A, B, C and D defined by (17).

From the values of Z_{vv} at V_2 and the last equation in (8) the following equations are obtained:

$$p_{22} + p_{32} = h_2 \tag{28}$$

where

$$h_2 = \frac{Z_{vv}(1,0)}{2} - p_{02} - p_{12} \tag{29}$$

Similarly from the values Z_{uu} at V_3 and the third equation in (8) the following equations are obtained

$$p_{30} + p_{23} = h_3 \tag{30}$$

where

$$h_3 = \frac{Z_{uu}(0,1)}{2} - p_{20} - p_{21} \tag{31}$$

Solving (26), (28) and (30) with respect to p_{22}, p_{32} and p_{23} gives the following equations:

$$p_{22} = \frac{g_1 h_2 + g_2 h_3 - h_1}{g_1 + g_2}$$

$$p_{32} = h_2 - p_{22} \tag{32}$$

$$p_{23} = h_3 - p_{22}$$

with g_1, g_2, h_1, h_2, and h_3 given by (27), (29) and (31)

6 Step by step description of the procedure

The coefficients of the polynomial to be fitted over a triangle are obtained by:

i- Determine a, b, c, and d (coefficients of coordinate transformation) from (4).

ii- Calculate the partial derivatives Z_u, Z_v, Z_{uv}, Z_{uu} and Z_{vv} from (6).

iii- Calculate L_u, L_v and Θ_{uv} (constants associated with the u-v coordinate system) from (9).

iv- Calculate A, B, C, and D from (17).

v- Determine 18 coefficients of the polynomial from (20), (21), (22), (23), (24) and (25) in this order.

vi- Calculate g_1, g_2, h_1, h_2 and h_3 from (27), (29) and (31).

vii- Determine the remaining three coefficients from (32).

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