Three Essays on the Design of Kidney Exchange and Doctor-Hospital Matching Mechanisms

Yao Cheng

PhD

University of York
Department of Economics
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Abstract

The thesis addresses the problem of the significant shortage of kidneys from donors as well as that of the imbalanced distribution of doctors. In respect to the kidney exchange problem, we propose a general model in which there are a finite number of patient-donor pairs, patients on the waiting list, and single donors. In the first study, patients have general preferences. The kidney of each paired donor is regarded as a private property of the intended kidney recipient, while kidneys from single donors are publicly owned. We propose an appropriate modification of the classic solution of core to the current model and develop a mechanism for finding a core matching that is Pareto optimal and stable against any coalition deviation. The second study focuses on efficient exchange procedures with dichotomous preferences in which only one-way, two-way, three or four-way chains, or cycles of exchange, are used. We derive a tight upper bound of the possible number of feasible kidney transplants in each case of exchange and provide important simulation results. We find that two-way cycles and chains of exchange can substantially increase the number of feasible transplants, that three-way cycles and chains can have a visible effect, and, at most, four-way cycles and chains suffice to capture all the potential gains of exchange. Our results are not only theoretically interesting but also have meaningful policy implications. The third study moves to the doctor-hospital matching problem. This paper studies a general doctor-hospital model under distributional and hierarchical constraints. We find that a matching that satisfies the classic concept of stability does not always exist and hence introduce an appropriate modification of the concept of stability. We furthermore design a doctor-proposing deferred acceptance mechanism with appealing properties in that it is efficient, stable and strategy-proof for doctors.

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Declaration

I declare that this thesis is a presentation of original work. This work has not previously been presented for an award at this, or any other, University. All sources are acknowledged as References.

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Chapter 1

Introduction

This thesis studies two salient issues in matching markets. The first issue is the significant shortage of kidneys from donors for patients who need kidney transplants. The second issue is the problem of the uneven distribution of doctors. Before presenting our formal models, we first provide a brief introduction to each topic.

1.1 Kidney Exchange Problem

Kidney transplantation is the preferred treatment for patients who suffer from kidney failure diseases. Many countries, however, are facing the problem of significant shortages of kidneys from donors. In the United States, in 2014, 98,956 patients waited for kidney transplants, 11,594 candidates received kidneys from deceased donors and 5,082 candidates underwent living donor transplants.[1] In the United Kingdom, 5,816 active or suspended patients were on the waiting list by March, 2013. In the following year, there were 2,804 new registrations but only 2,897 underwent surgery.[2] These shortages also create a waiting time problem. The worst outcome of these long waits is that a patient may die or become too sick to receive a transplant. In the United Kingdom, in 2014, the average waiting time for a patient to receive a kidney transplant was 1,137 days and 551 patients were too sick to receive a transplant and 263 patients died.

[1] The data is obtained from the OPTN/SRTR 2014 annual report, http://onlinelibrary.wiley.com/doi/10.1111/ajt.13666/pdf

[2] The United Kingdom data is obtained from the Organ Donation and Transplantation Activity Report 2013/14, https://nhsbtmediaservices.blob.core.windows.net/organ-donation-assets/pdfs/activity_report_2013_14.pdf

The obstacles to a kidney transplant are mainly caused by biological and moral constraints. The biological constraint revolves around blood-type and tissue-type compatibility. That is, a patient can receive a kidney from a donor only if they are blood-type compatible and tissue-type compatible. In this case, we say the donor is a *compatible* donor for the patient. There are four blood-types, O, A, B and AB, with each type being blood-compatible with itself. In addition, blood-type O is a universal giver and AB is a universal receiver. The tissue-type compatibility is related to the human leukocyte antigens (HLA). Opelz (1997) indicates a negative relationship between the HLA incompatibility and the probability of transplant survival. The second obstacle relates to moral constraint. It is illegal to buy a kidney or compel someone to donate a kidney in all most all countries. The Congress in the United States passed the National Organ Transplant Act (NOTA) in 1984, meaning that organs cannot be priced or treated as a commodity. Therefore, the only source of kidneys is from altruistic donation.

There are many patient-donor pairs, patients on the waiting list and single donors in a kidney exchange pool. Single donors are either altruistic cadavers or altruistic living donors. A patient on the waiting list has no intended donor who is willing to donate their kidney. Normally, patients on the waiting list are waiting for kidneys from single donors. Meanwhile, a patient may come together with a donor as a patient-donor pair. The donor is usually a friend or a family member of the patient and is willing to donate a kidney to the patient. However, even if a paired donor is willing to donate a kidney to the patient they are paired with, the donor is unable to do so if the donor is either blood-incompatible or tissue incompatible with the paired recipient. We refer to such a pair as an incompatible pair. In the past, donors from incompatible pairs usually give up donating their kidneys and their patients go to the waiting list waiting for kidneys from single donors. If we place such incompatible pairs into the kidney exchange pool, paired donors who would otherwise be lost to the exchange pool can thus become available for patients who are compatible.

Paired donors and single donors are potential kidney resources. To take advantage of potential kidney resources, researchers have started to study kidney exchange programmes from medical, ethical and legal perspectives (Rapaport, 1986, Ross et al., 1997, Terasaki et al., 1998, ORGAN, 2000, Montgomery et al., 2006, 2008, Saidman et al., 2006, Roth et al., 2006, Rees et al., 2009). Roth et al. (2004) first transformed the kidney exchange problem to a flourishing area of economic research. They studied a kidney exchange model with a finite number of patient-donor pairs. Patients have general preferences over donors and the waiting list option. They proposed a mechanism called Top Trading Cycles and Chains (TTCC) mechanism, and demonstrate that the TTCC mechanism is personally rational, Pareto optimal and strategy proof. In further work, Roth et al. (2007) went on to consider a more practical kidney exchange model for incompatible patient-donor pairs. Patients are indifferent between compatible kidneys. They proposed efficient sequential matching procedures that maximize the number of transplants through at most two-way, three-way and four-way exchange respectively. They also demonstrate that four-way kidney exchange is sufficient to capture all potential efficiency. More related literature is provided in the introduction section of the first and the second chapters.

The first study explores a general kidney exchange model consisting of incompatible patient-donor pairs, compatible patient-donor pairs, patients on the waiting list, and single donors from cadavers and altruistic living donors. We provide fresh and important insights into the kidney exchange. Firstly, we study a more general model which allows patients on the waiting list and single donors to exchange with patient-donor pairs. The advantages include an improvement in efficiency as well as an increased chance of matches for patients. Secondly, we introduce a new and appropriate modification of the core which takes both private and public resources into account. Thirdly, we demonstrate that the existence of the core is non-empty by constructing a kidney exchange mechanism that always find a matching in the core. The mechanism is a generalization of the celebrated top trading cycle method from Shapley and Scarf (1974). We also find that the intersection of the core and the set of Pareto efficient matchings is non-empty and that the kidney mechanism can always find a matching in that intersection.

The second study considers a more practical model consisting of patient-donor pairs, patients on the waiting list and single donors with the dichotomous preferences of patients. Our study generalises the work of Roth, Sönmez and Ünver (2007). They focus on a kidney exchange model consisting of only incompatible patient-donor pairs and derive the upper bound of possible kidney transplants under two-way, three-way and four-way exchanges, respectively. To account for practical aspects, we apply the real-life environment of the kidney exchange pool in this general model. That is, every member (not only the incompatible pairs) in the kidney exchange pool can be involved in a kidney exchange. We have examined how to design kidney exchange procedures in this practical environment so that a maximum number of patients can receive compatible kidneys. In terms of theoretical aspects, we are able to derive a precise upper bound of the number of patients who can benefit from two-way, three-way, and four-way exchanges, respectively. Furthermore, in this chapter, we apply real-world data from the U.S. in simulations to measure how the mechanism works, and thus provide substantial simulation results. The following findings are discovered. Firstly, the maximum number of transplants in real life is very close to the theoretically predicted number of transplants. Secondly, the involvement of all parties in the kidney exchange pool significantly increases the number of transplants. Thirdly, there is an important policy implication: kidney exchange programmes can actually be decentralised. That is, in countries with a large population, separate kidney exchange programmes can be established in several major regions across the country.

1.2 Doctor-Hospital Matching Problem

The imbalanced distribution of doctors is a common phenomenon in many countries. A resident is medically under-served if he or she lives in an area with insufficient medical personnel. The American Medical Association described this serious problem in the United States: more than 35 million Americans are medically under-serviced and 16,000 doctors are needed urgently to fill that need (Talbott, 2007). China has a similar problem. In urban areas, there are 8.54 health workers per 1000 population, while in rural areas there are just 3.41.[3]

Countries have applied various policies to cope with the problem. The Japanese government started the Japan Residency Matching Programme (JRMP) in 2008 in order to govern the maximum number of doctors in each designated prefecture. The Chinese government has increased the investment in primary health care institutions since 2009, especially in institutions in rural areas. The Chinese government has invested 59,000

[3] The data is obtained from The People's Republic of China health system review, 2015, http://www.wpro.who.int/asia_pacific_observatory/hits/series/china_health_systems_review.pdf

million RMB in rural areas from 2009 to 2011 for the development of county hospitals and village clinics.[4]

Motivated by these real-life policies, we study how to design a mechanism to solve the problem. Gale and Shapley (1962) proposed a famous mechanism called the Deferred Acceptance (DA) algorithm. Because the rural hospital theorem shows that one agent who is unmatched at one stable matching is unmatched in every stable matching, however, it is difficult to apply the deferred acceptance algorithm directly. Some studies have offered solutions to this problem. One method is to set a maximum constraint (regional cap) on a region. The number of doctors in under-serviced areas may increase because the regional cap shunts some doctors from popular areas to rural areas. The second method is to set a floor constraint on a region. The floor constraint guarantees the number of doctors required by that region. Other related literature is provided in the introduction section of the third study.

The third study explores a general doctor-hospital model under distributional and hierarchical constraints. The model encompasses the regional cap constraint as well as the floor constraint at both an institution level and a regional level. Motivated by the real-life health case, this model also considers a hierarchical constraint on hospitals. In detail, every hospital has a grade in the system. Hospitals with a higher grade have priorities to recruit doctors than those with a lower grade. Compared to the existing models, the hierarchical structure of hospitals is more general. We find that the classic solution of stability does not always exist and hence introduce an appropriate definition of stability taking the hierarchical structure and the distributional constraints into account. We also propose a very practical mechanism called the doctor-proposing deferred acceptance mechanism. This decentralises the rights of recruitment of doctors into three parties: hospitals, regional organisers and a national organiser. Each hospital is allocated a quota. As long as the quota is not filled, the hospital can recruit doctors freely. If a hospital wants to recruit more doctors beyond its quota, it needs to compete with other hospitals at a regional level or the national level. Each hospital has its own competitive power that is related to its grade in the hierarchical structure of hospitals. We demonstrate that the mechanism is stable, efficient and strategy-proof for doctors.

[4] The data is obtained from The People's Republic of China health system review, 2015, http://www.wpro.who.int/asia_pacific_observatory/hits/series/china_health_systems_review.pdf

Chapter 2

A General Kidney Exchange Mechanism

2.1 Introduction

Kidney transplantation is the preferred treatment for patients who suffer from diseases with kidney failure. Unfortunately, many countries face a significant shortage of kidneys. For instance, in the United States, 92,885 patients waited for kidney transplants in 2012 but just 16,526 (17.79%) of these patients received transplants in that year (Matas et al., 2015). In the United Kingdom, there were 12,331 patients on the waiting list in 2013 but only 2,897 (23.49%) actually got transplants.[5] This disquieting situation in respect to kidney transplant is largely caused by two factors. Firstly, the purchase of sale of any human organ is illegal in almost all countries, meaning that the only source of kidneys is through altruistic donation. Secondly, a kidney transplant between a patient and a donor needs to meet certain medical requirements. A patient can receive a kidney from a donor only if they are both blood-type and tissue-type compatible.

Altruistic kidney donation comes from two types of donors: single donors and paired donors. Donors whose kidney can be given to any patient are called single donors. They are either altruistic living donors or altruistic deceased donors. Donors who are willing to give one of their kidneys to a designated patient are called paired donors. Every paired

[5] The data in United Kingdom is obtained from the Organ Donation and Transplantation Activity Report 2013/14, https://nhsbtmediaservices.blob.core.windows.net/organ-donation-assets/pdfs/activity_report_2013_14.pdf.

donor comes together with his or her paired patient, usually a friend or a family member of the donor, as a patient-donor pair.

Even if a paired donor is prepared to donate her kidney, however, she may not be able to do so if her kidney is found to be incompatible with her intended recipient on medical grounds. It is therefore not always easy to find a compatible donor for a patient. In this situation, kidney exchange programmes are explored in order to increase the likelihood of patients receiving a suitable kidney. Paired kidney exchange is a typical approach to kidney transplantation. A paired exchange contains two incompatible patient-donor pairs but the patient in each pair is compatible with the donor in the other pair. In this case, both patients can do kidney transplants interchangeably. Another possible approach is a chain exchange starting from a single donor. In a chain exchange, a single donor at the top of the chain gives a kidney to a paired patient, with the last paired donor of the chain giving their kidney to a patient on the waiting list.

To avoid the moral hazard problem, kidney transplants must be carried out simultaneously. To see this, consider two patient-donor pairs (P^1, D^1) and (P^2, D^2) in which patient P^i is compatible with donor D^j but incompatible with D^i, $i, j = 1, 2$, $i \neq j$. Suppose transplants will be done sequentially: First P^1 receives the kidney from D^2 and then P^2 receives the kidney from D^1. Doing so may run a risk of donor D^1's breaking her promise as patient P^1 has already received a kidney. To avoid this problem, transplants for such patient-donor pairs must be done simultaneously. This will not be a problem for transplants from altruistic donors. Simultaneous transplants have to meet logistical and capacity constraints such as the availability of doctors and operation theatres in a hospital or hospitals in close proximity. This paper studies how to alleviate the shortage problem and improve patient welfare under legal, medical and incentive constraints.

The problem of kidney transplantation has been previously studied by medical researchers from the medical, ethical and legal perspectives (Rapaport, 1986, Ross et al., 1997, Ross and Woodle, 2000, Zenios et al., 2001, ORGAN, 2000). Roth et al. (2004) have adopted the mechanism design approach to kidney exchange. They introduce a basic model of kidney exchange with multiple patient-donor pairs. Patients have preferences over donors and the waiting list option. They propose a mechanism called the Top Trading Cycles and Chains (TTCC) mechanism that can find an efficient allocation of kidneys to patients. Their mechanism is closely related to the top trading cycle (TTC) mechanism for housing markets proposed by Shapley and Scarf (1974) and a generalisation of the TTC mechanism for housing allocation on college campuses proposed by Abdulkadiroğlu and Sönmez (1999).

In this paper we consider a general model of kidney exchange. There are a finite number of incompatible patient-donor pairs, compatible patient-donor pairs, patients on the waiting list, referred to as single patients, and altruistic living or deceased donors, referred to as single donors. Each patient has preferences over donors and the waiting option. For a compatible patient-donor pair, the patient can receive a kidney transplant from the donor directly. Allowing compatible patient-donor pairs and single donors to participate in exchange with incompatible pairs and single patients can result in three major benefits. Firstly, it can increase the chance of getting better quality kidneys for patients from compatible patient-donor pairs. Secondly, it can help alleviate the difficulty of finding compatible kidneys for patients from incompatible patient-donor pairs. Thirdly, single donors can be matched with single patients or also with paired patients whose donors can then be given to single patients so that efficiency of exchange can be considerably improved. The involvement of all kinds of patients and donors in these exchanges will create more opportunities to make kidney transplants and thus save more lives and also enhance patients' satisfaction.

In fact, the US National Kidney Registry (NKR) has encouraged compatible pairs and altruistic donors to exchange with incompatible pairs. Table 2.1 illustrates the accumulative total number of chains and loops from NKR in 2016.[6] We use 'cycles' to represent kidney exchanges among patient-donor pairs and 'chains' to represent kidney exchanges starting from non-directed donors (altruistic donors). Observe that each altruistic donor helps save three patients or more on average and each paired patient in cycle can save at least one patient on average. The longest period of waiting time for those patients who can possibly receive kidney transplants deceased from 36 months to 16 months when the number of NKR compatible pairs doubled in 2016.[6] The number of altruistic donor registrations also significantly increased from 1,028 in 2015 to 2,104 in 2016.[7]

[6] The data is obtained from Paired Exchange Results Quarterly Report, as of December 31, 2016, http://www.kidneyregistry.org/pages/p399/NKR_Quarterly_Report_Q4_2016.php.

[7] The data is obtained from Paired Exchange Results Quarterly Report, As of March 31, 2017, http://kidneyregistry.org/pages/p410/NKR_Quarterly_Report_Q1_2017.php.

Table 2.1 Chain and loop statistics of National Kidney Registry in 2016

Category	Count	Mean Length	Transplants
Chains	397	4.49	1783
Cycles	89	2.35	209

Our aim is to develop a mechanism that can match paired donors and single donors compatibly with paired patients and single patients as many and as well as possible. We adopt the classic solution of core to this general model of kidney exchange. Core, as one of the most important economic solution concepts, has been used in various contexts; see Gillies (1959), Debreu and Scarf (1963), Scarf (1967), Shapley (1972), Shapley and Scarf (1974), etc. It is a generalisation of Edgeworth's contract curve and prescribes a set of outcomes that can be attained by the whole group of participants and that are immune to profitable deviation by any group of participants. Our model is closely related to that of Shapley and Scarf (1974). They investigated a basic housing market in which there are several agents each of whom owns an indivisible item, say a house. Each agent has preferences over the houses they want and never wants more than one house, and there is no medium of exchange like money. Agents seek to exchange their houses in order to improve their wellbeing. They show that this market is balanced and has a non-empty core. They also prove that the top trading cycle (TTC) mechanism due to David Gale finds a core allocation.

The conventional definition of core is, however, not suitable for our current model so we need an appropriate modification of this concept. In the model of Shapley and Scarf (1974) every agent has a private property, a house. In our model, we have two types of patients: paired patients and single patients and two types of donors: paired donors and single donors. The kidney of a paired donor can be well regarded as a private property of the intended recipient, the corresponding paired patient, whereas kidneys from single donors (i.e., altruistic donors) are a kind of public property in the sense that every patient is entitled to receive the kidney from any single donor as long as they are compatible. Single patients initially have no endowment at all. Thus our model has a mixed structure of property rights, both private and public, which poses a challenge to the conventional definition of core. To resolve this issue, we propose an appropriate modification of the core as follows. For any given matching μ, we say that a paired patient is endowed with the kidney of her corresponding paired donor and that a single patient is provisionally endowed with the kidney of the single donor if the patient is matched with a single donor at μ. Observe that if a single patient is matched with a paired donor at μ, the patient is not endowed with any kidney. A matching μ is strongly blocked by a coalition S of patients if all patients in that coalition can be made strictly better off than at μ by redistributing the endowed kidneys from paired patients in the coalition, the provisionally endowed kidneys from single patients in the coalition, and the waiting list option among all members in the coalition. A matching μ is in the core if it cannot be strongly blocked by any coalition.

We develop a mechanism that can always find a core matching in our model. As a result, we show that the core of our model is not empty. We also examine a variety of properties of the core and other solutions. For instance, we discuss the relationship between the new concept of core and Pareto optimality. We find that the core may not be Pareto optimal and vice versa. Interestingly, the intersection of the core and the set of Pareto optimal matchings is not empty; in other words, the designed mechanism can always find an allocation in the intersection. We also discuss strict core and other issues.

The rest of this paper is organised as follows. Section 2 introduces the kidney exchange model and basic concepts. Section 3 presents the kidney exchange mechanism. Section 4 gives the main results. Section 5 concludes.

2.2 The Model

In this section, we will introduce a general kidney exchange model. Then, we will introduce the core and Pareto optimality. Last, we will discuss the relationship between core and Pareto optimality.

2.2.1 Kidney Exchange Problem

A kidney exchange pool consists of a finite number of patient-donor pairs, patients on the waiting list and single donors. Let $D^s = \{d_1^s, d_2^s, \ldots, d_k^s\}$ be the set of all single donors. A single donor can donate his or her kidney to any compatible patient. Let $P^w = \{p_1^w, p_2^w, \ldots, p_m^w\}$ be the set of patients on the waiting list. Each patient on the waiting list has no intended donor. We call a patient on waiting list a *single* patient. Let $P^p = \{p_1^p, p_2^p, \ldots, p_n^p\}$ be the set of paired patients with the set of corresponding paired donors $D^p = \{d_i^p, d_2^p, \ldots, d_n^p\}$. Denote $D = D^s \cup D^p$ as the set of all donors and $P = P^p \cup P^w$ the set of all patients.

Every patient has a strict preference over the set of donors and the waiting list option w. The waiting list option w for a paired patient means that his paired donor gives a kidney to a patient and the paired patient waits for a compatible donor on the waiting list. In practice, if the patient agrees his donor to donate a kidney to a patient, then in return the patient obtains a priority on the waiting list. It is easy to see that the preferences of every patient can be very general. However, it is reasonable to assume that every paired patient prefers compatible donors to incompatible ones and prefers the waiting option to any incompatible donor possibly except her own paired donor, and every single patient prefers compatible donors to incompatible ones and that prefers the waiting option to incompatible donors. Note that it is possible for a patient prefers the waiting list option w than a donor whose kidney is compatible with the patient when the patient does not satisfy the quality of the compatible donor.

If a paired patient p^p prefers her paired donor d^p to the waiting list option w, a strict preference of the paired patient p^p in a pair (p^p, d^p) can be presented as,

$$\succ_{p^p}: \; d_1^p, d_3^p, d_1^s, d^p, w, d_2^p$$

If a paired patient p^p prefers the waiting list option w to her paired donor d^p, a strict preference of the paired patient p^p in a pair (p^p, d^p) can be presented as,

$$\succ_{p^p}: \; d_1^p, d_3^p, d_1^s, w, d^p, d_2^p$$

A strict preference of a patient on waiting list p^w can be presented as,

$$\succ_{p^w}: \; d_1^p, d_2^p, w, d_2^s, d_3^p.$$

Let $\succ = (\succ_p)_{p \in P}$ be the preference profile for all patients. Therefore, a kidney exchange problem can be presented as $(P, D, w, \succ)$. Let $\mathscr{P}$ denote the profile of all possible preferences for all patients.

Every patient wants to find a donor as well as possible for herself. A donor d is *acceptable* to a paired patient p^p if $d \succeq_{p^p} d^p$ and is *strongly acceptable* to a paired patient p^p if $d \succeq_{p^p} d^p$ and $d \succeq_{p^p} w$. In other words, if a paired patient weakly prefers a donor to her paired donor, then that donor is acceptable to the paired patient. If a paired patient likes a donor at least as well as her paired donor and the waiting list option w, then that donor is strongly acceptable to the paired patient. An acceptable donor to a paired patient may not be strongly acceptable to that patient. If a paired patient prefers her paired donor to the waiting list w, as long as a donor is acceptable to the patient, then that donor is strongly acceptable to the patient. The waiting list option w is *acceptable* to a paired patient p^p if $w \succeq_{p^p} d^p$. This means that if a paired patient weakly prefers the waiting list option w to her paired donor, then the waiting list option w is acceptable to the paired patient. A donor d is *acceptable* to a single patient p^w if $d \succeq_{p^w} w$. It means that if a single patient likes a donor at least as well as the waiting list option, then that donor is acceptable to the single patient. Furthermore, the waiting list option is acceptable for every single patient.

It is in the interest of each patient to participate in the kidney exchange because there is a chance to receive a desirable donor. A paired patient can obtain a desirable compatible kidney either from her donor or from other donor by exchanging her own donor's kidney. If a paired patient fails to find a strongly acceptable donor, the patient can either give up her own donor to choose the waiting list option or stay with her paired donor to wait for exchange in the next time. Although it is feasible to do a transplant between the patient and the donor in a compatible pair, one incentive for the patient participating in exchange is to find a better compatible donor than her own paired donor.

An outcome of the kidney exchange problem is a matching μ such that (i) each paired patient p^p is either assigned a donor $\mu(p^p) = d$ where $d \in D \setminus \{d^p\}$ or stays with her paired donor $\mu(p^p) = d^p$ or chooses the waiting list option $\mu(p^p) = w$; (ii) each single patient p^w is either assigned a donor $\mu(p^w) = d$ where $d \in D$ or stays on the waiting list $\mu(p^w) = w$; (iii) no kidney can be assigned to more than one patient, that is, $d = \mu(\mu(d))$ for all $d \in D$. Note that the waiting list option w can be assigned to several patients. A matching is said to be *weakly individually rational* if $\mu(p^p) \succeq_{p^p} d^p$ for every paired patient $p \in P^p$ and $\mu(p^w) \succeq_{p^w} w$ for every single patient $p^w \in P^w$. A matching is *individually rational* if $\mu(p^p) \succeq_{p^p} d^p$ and further $\mu(p^p) \succeq_{p^p} w$ for every paired patient

$p \in P^p$, and $\mu(p^w) \succeq_{p^w} w$ for every single patient $p^w \in P^w$. In other words, a matching μ is weakly individually rational if every patient receives an acceptable donor or an acceptable waiting list option w. A matching μ is individually rational if it is weakly individually rational and every paired patient is assigned with either a strongly acceptable donor or the acceptable waiting list option.

2.2.2 The Core

In our model, for each patient-donor pair, the donor's kidney can be regarded as an endowment of the patient, while the kidney of each single donor can be seen as a public property. Given a matching μ, we say that a single patient p^w is *provisionally endowed with the kidney from a single donor* $d \in D$ if the donor is assigned to the patient at μ, i.e., $\mu(p^w) = d$. Both paired patients and provisionally endowed single patients are said to be *resourceful*. A single patient p^w is *unendowed* in a matching μ if the patient is either assigned with the waiting list option $\mu(p^w) = w$ or receives a paired donor $\mu(p^w) = d \in D^p$. Let P^w_μ denote the set of resourceful single patients at matching μ.

We will adopt the widely used notion of core to the current model as its solution. The core of an economic model or any multilateral competitive and cooperative situation consists of those outcomes that cannot be profitably deviated by any coalition of participants through any collusive action of the coalition; seeGillies (1959), Debreu and Scarf (1963), Scarf (1967), Shapley and Scarf (1974), etc. In the current model, the family of all patients is called *the grand coalition*. A set of patients is called *a coalition*. A matching μ is blocked by a coalition if no member in the coalition can become worse off and at least one member of the coalition becomes strictly better off than at matching μ by redistributing the endowed kidneys from paired patients in the coalition, the provisionally endowed kidneys from single patients in the coalition, and the waiting list option among all members in the coalition. A matching μ is strictly blocked by a coalition if every member in the coalition becomes strictly better off than at matching μ by redistributing the endowed kidneys from paired patients in the coalition, the provisionally endowed kidneys from single patients in the coalition, and the waiting list option among all members in the coalition. A matching is in the core if it cannot be strictly blocked by any coalition. A matching is in the strict core if it cannot be blocked by any coalition. Observe that in the current framework, the notion of core has been modified in that those kidneys from altruistic living and cadaver donors (public property) are treated as a kind of private property provisionally owned by single patients.

Given a coalition S and a matching μ, we use $S^p(\mu)$ to denote the family of paired patients in S and $S^e(\mu)$ to denote the family of resourceful single patients in S at μ.

Definition 1 *A matching μ is blocked by a coalition S of patients, if there exists a redistribution ν^S of kidneys from all donors of paired patients in $S^p(\mu)$, those provisionally owned by single patients in $S^e(\mu)$, and the waiting list option w among all patients in S such that $\nu^S(i) \succeq_i \mu(i)$ for all $i \in S$ and $\nu^S(i) \succ_i \mu(i)$ for some $i \in S$. A matching μ is strongly blocked by the coalition S if there exists a redistribution ν^S such that $\nu^S(i) \succ_i \mu(i)$ for all $i \in S$.*

Definition 2 *A matching is in the strict core and is called a strict core matching if it is not blocked by any coalition. It is in the core and is a core matching if it cannot be strongly blocked by any coalition.*

We use an example to explain the endowed kidneys from patients in a coalition. Suppose there are two incompatible pairs (p_1^p, d_1^p), (p_2^p, d_2^p), two patients on the waiting list p_1^w, p_2^w, and two single donors d_1^s, d_2^s in a kidney exchange pool. Give a matching μ such that $\mu(p_1^p) = d_2^p$, $\mu(p_2^p) = d_1^s$, $\mu(p_1^w) = d_1^p$ and $\mu(p_2^w) = d_2^s$. Consider a coalition $S = \{p_1^p, p_2^p, p_2^w\}$. Since d_2^s is assigned to p_2^w, p_2^w is endowed with d_2^s. Therefore, the set of donors who can be redistributed among patients in coalition S is $D^{S^p} = \{d_1^p, d_2^p\}$ and $D_\mu^{S^w} = \{d_2^s\}$. The coalition S blocks the matching μ if there exists a redistribution of kidneys from donors $\{d_1^p, d_2^p, d_2^s\}$ and the waiting list option w such that at least one patient $p \in S$ would be better off than that he received from matching μ without hurting other patients in this coalition. Now we consider another coalition $S' = \{p_1^p, p_2^p, p_1^w\}$. Since $\mu(p_1^w) = d_1^p$, p_1^w has no endowment. The set of donors who can be redistributed among patients in coalition S' is $D^{S'^p} = \{d_1^p, d_2^p\}$ and $D_\mu^{S'^w} = \emptyset$. The coalition S' blocks the matching μ if there exists a redistribution of kidneys from donors $\{d_1^p, d_2^p\}$ and the waiting list option w such that at least one patient $p \in S'$ would be better off than that at matching μ without hurting other patients in this coalition.

Unfortunately in this general model a strict core may not exist. We give an example to illustrate this point.

Example 1 *Consider a kidney exchange pool of one incompatible patient-donor pair* (p_1^p, d_1^p) *and two patients on the waiting list* p_1^w *and* p_2^w*. The preference of patients are*

$$\succ_{p_1^p}: \quad w, \quad d_1^p \qquad\qquad \succ_{p_1^w}: \quad d_1^p, \quad w \qquad\qquad \succ_{p_2^w}: \quad d_1^p, \quad w$$

$$\mu^1 = \begin{pmatrix} p_1^p & p_1^w & p_2^w \\ w & d_1^p & w \end{pmatrix} \qquad \text{and} \qquad \mu^2 = \begin{pmatrix} p_1^p & p_1^w & p_2^w \\ w & w & d_1^p \end{pmatrix}$$

Example 1 has two core matchings, but neither of them is in the strict core. The matching μ^1 is blocked by a coalition $S = \{p_1^p, p_2^w\}$ by constructing the redistribution ν^S such that $\nu^S(p_1^p) = w$ and $\nu^S(p_2^w) = d_1^p$ which makes no patients worse off and p_2^w better off. Similarly, the matching μ^2 is blocked by a coalition $S = \{p_1^p, p_1^w\}$ by constructing the redistribution ν^S such that $\nu^S(p_1^p) = w$ and $\nu^S(p_1^w) = d_1^p$.

Therefore, we explore whether there always exists a matching in the core. Based on the definition of the core, we have the following Lemma 1. It is immediate to see that if a matching is in the core, the matching is individually rational. Otherwise, one patient who receives an unacceptable or weakly acceptable assignment can block the matching easily.

Lemma 1 *Any matching* μ *in the core is individually rational.*

A matching μ is *strongly improved by a cycle*, if there exists a sequence of an even number of distinct patients and their endowed donors $(d_1, p_1, ..., d_M, p_M)$ $(M \geq 2)$ such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1, 2, ..., M$; (ii) $d_{m+1} \succ_{p_m} \mu(p_m)$ for all $m = 1, 2, ..., M$, where $d_{M+1} = d_1$.

Observe that every patient in a cycle is resourceful. A cycle $(d_1, p_1, ..., d_M, p_M)$ strongly improving upon matching μ means that matching p_1 to d_2, p_2 to d_3,..., and p_{M-1} to d_M and p_M to d_1 will make every member in the coalition $p_1, p_2, ..., p_M$ strictly better off than they are at μ . Figure 2.1 shows an example of cycles such that every endowed donor is connected with her resourceful patient.

A matching μ is *strongly improved by a chain*,

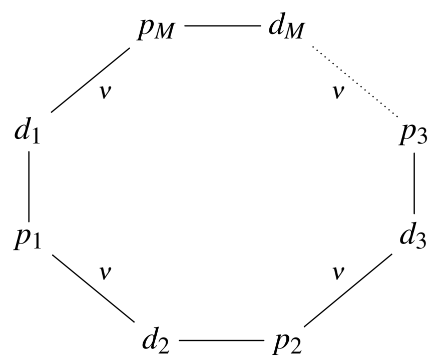
(i) if there is a sequence of an even number of distinct patients and their endowed donors $(d_1, p_1, ..., d_M, p_M, w)$ $(M \geq 1)$ such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1, 2, ..., M$; (ii) $d_{m+1} \succ_{p_m} \mu(p_m)$ for all $m = 1, 2, ..., M$, where $d_{M+1} = w$; or

(ii) if there is a sequence of an odd number of distinct patients and their endowed donors $(p_1, d_1, p_2, ..., d_{M-1}, p_M, w)$ $(M \geq 2)$ such that (i) $p_1 \in P^w$ and d_{m-1} is the endowed donor of the resourceful patient p_m for all $m = 2, ..., M$; (ii) $d_m \succ_{p_m} \mu(p_m)$ for all $m = 1, 2, ..., M$, where $d_M = w$.

Figure 2.2 illustrates an example of improvements by two types of chains in a matching. Observe that if the length of the chain is even, every patient in the chain is resourceful, and that if the length of the chain is odd, every patient in the chain is resourceful except the first member of the chain (see p_1 in Figure 2.2). A resourceful patient is either a paired patient or a single patient who is assigned a single donor at matching μ. The redistribution of the tail member of the chain (see p_M in Figure 2.2) is the waiting list option.

A matching μ is strongly improved by an even chain $(d_1, p_1, ..., d_M, p_M, w)$ if matching p_1 to d_2, p_2 to d_3,..., and p_{M-1} to d_M and p_M to w will make all members in the coalition $\{p_1, p_2, ..., p_M\}$ strictly better off than they are at μ. Similarly, a matching μ is strongly improved by an odd chain $(p_1, d_1, p_2, ..., d_{M-1}, p_M, w)$ if matching p_1 to d_1, p_2 to d_2,..., and p_{M-1} to d_{M-1} and p_M to w will make all members in the coalition $\{p_1, p_2, ..., p_M\}$ strictly better off than they are at μ.

Fig. 2.1 An illustration of improvements by a cycle in a matching.



Lemma 2 *If a weakly individually rational matching μ is strongly improved upon by a coalition S, it must be strongly improved upon by a chain or by a cycle.*

Lemma 2 means that if a weakly individually rational matching is improved upon by a coalition, the matching can be either strongly improved by a cycle or by a chain. Based on the Lemma 2, we can directly get the following Lemma 3.

Fig. 2.2 An illustration of improvements by a chain in a matching.

$$d_1 \text{ —— } p_1 \overset{v}{\text{——}} d_2 \text{ —— } p_2 \overset{v}{\text{——}} \cdots\cdots \overset{v}{\text{——}} d_M \text{ —— } p_M \overset{v}{\text{——}} w$$

$$p_1 \overset{v}{\text{——}} d_1 \text{ —— } p_2 \overset{v}{\text{——}} d_2 \text{ —— } \cdots\cdots \overset{v}{\text{——}} d_{M-1} \text{ —— } p_M \overset{v}{\text{——}} w$$

Lemma 3 *A weakly individually rational matching μ is in the core if it cannot be strongly improved by any cycle or any chain.*

If a matching is individually rational, every paired patient receives a strongly acceptable assignment and every single patient receives an acceptable assignment. When we consider individually rational matching, we have the following lemma.

Lemma 4 *Given a kidney exchange problem $(P, D, w, \succ)$, an individually rational matching μ cannot be strongly improved upon by any chain.*

Lemma 4 means that if an individually rational matching is strongly improved by a coalition, the matching is strongly improved by cycles. In other words, given an individually rational matching, if there exists a coalition such that exchanging kidneys among patients in the coalition makes every patient in the coalition better off , then those patients in the coalition and their donors construct a cycle. By Lemma 4, we have the following result.

Lemma 5 *Given a kidney exchange problem $(P, D, w, \succ)$, an individually rational matching μ is in the core if it cannot be strongly improved by any cycle.*

2.2.3 Pareto Optimality

Pareto optimality is another important solution concept. It stands for the grand set of patients. We say an allocation is Pareto optimal if there does not exist a redistribution such that makes at least one patient better off without hurting any other patient in the pool. Formally,

Definition 3 *Given a matching μ of a kidney exchange model $(P, D, \succ)$, let $\mu(p)$ denote the assignment of patient p at μ. A matching μ is* Pareto optimal *if there is no* *redistribution of ν such that $\nu(p) \succeq_p \mu(p)$ for all $p \in P$ and $\nu(p) \succ_p \mu(p)$ for some $p \in P$.*

Definition 4 *A kidney exchange mechanism χ is* efficient *if it always selects a Pareto optimal matching given a kidney exchange model $(P, D, \succ)$.*

In the work of Shapley and Scarf (1974), a core matching can prevent any deviation including the grand set itself from any coalition and therefore also achieves Pareto optimality. However, in this general model, a matching in the core may not be Pareto optimal and vice versa. We use two examples to demonstrate the point.

Example 2 *(A matching in the core may not be Pareto optimal.) Consider a kidney exchange model with two patient-donor pairs (d_1^p, p_1^p), (d_2^p, p_2^p), two single donor d_1^s, d_2^s and two patients on the waiting list p_1^w, p_2^w. Their preferences are given by*

$$\succ_{p_1^p}: \; d_2^p, \; d_1^p \qquad\qquad \succ_{p_2^p}: \; d_1^s, \; d_2^p$$
$$\succ_{p_1^w}: \; d_1^p, \; w \qquad\qquad \succ_{p_2^w}: \; d_2^s, \; w$$

$$\mu^1 = \begin{pmatrix} p_1^p & p_2^p & p_1^w & p_2^w \\ d_2^p & d_1^s & d_1^p & w \end{pmatrix} \quad \text{and} \quad \mu^2 = \begin{pmatrix} p_1^p & p_2^p & p_1^w & p_2^w \\ d_2^p & d_1^s & d_1^p & d_2^s \end{pmatrix}.$$

There are two matchings in the core. In matching μ^1, patient p_1^p is endowed with her paired donor d_1^p; patient p_2^p is endowed with her paired donor d_2^p; and single patients have no endowment. Consider any coalition, we cannot find a coalition which can strongly improve the matching. Therefore, matching μ^1 is in the core. However, it is not Pareto optimal because μ^2 can improves p_2^w without hurting other patients. In matching μ^2, patient p_1^p is endowed with her paired donor d_1^p; patient p_2^p is endowed with her paired donor d_2^p; single patient p_2^w is endowed with single donor d_2^s; and single patient p_2^w has no endowment. Consider any coalition, we cannot find a coalition which can strongly improve the matching μ^2 so that matching μ is in the core. Furthermore, matching μ^2 is also Pareto optimal because there exists no redistribution which can make at least one patient better off without hurting other patients.

Example 3 *(A Pareto optimal matching may not in the core.) Consider a kidney exchange model with three patient-donor pairs* (d_1^p, p_1^p), (d_2^p, p_2^p), (d_3^p, p_3^p), *two single donors* d_1^s, d_2^s *and two patients on waiting list* p_1^w, p_2^w. *Their preferences are given by*

$$\succ_{p_1^p}: \ d_2^p, \ d_1^s, \ d_1^p \qquad \succ_{p_2^p}: \ d_1^p, \ d_2^s, \ d_2^p$$
$$\succ_{p_3^p}: \ d_2^p, \ d_2^s, \ d_3^p \qquad \succ_{p_1^w}: \ d_3^p, \ w$$
$$\succ_{p_2^w}: \ d_1^p, \ d_1^s, \ w$$

$$\mu^3 = \begin{pmatrix} p_1^p & p_2^p & p_3^p & p_1^w & p_2^w \\ d_1^s & d_2^s & d_2^p & d_3^p & d_1^p \end{pmatrix} \quad \text{and} \quad \mu^4 = \begin{pmatrix} p_1^p & p_2^p & p_3^p & p_1^w & p_2^w \\ d_2^p & d_1^p & d_2^s & d_3^p & d_1^s \end{pmatrix}.$$

There are two Pareto optimal matchings. Matching μ^3 is Pareto optimal but not in the core. In matching μ^3, patient p_1^p is endowed with her paired donor d_1^p; patient p_2^p is endowed with her paired donor d_2^p; patient p_3^p is endowed with her paired donor d_3^p; and patients on waiting list have no endowments. A coalition $S = \{p_1^p, p_2^p\}$ can strongly improve the matching μ^3 through the redistribution $\nu^S(p_1^p) = d_2^p$ and $\nu^S(p_2^p) = d_1^p$. Meanwhile, matching μ^4 is both in the core and Pareto optimal.

Let M^{PO} denote the set of Pareto optimal matchings and M^C denote the set of matchings in the core of a kidney exchange model $(P, D, w, \succ)$. We prove that $M^{PO} \bigcap M^C \neq \emptyset$ by constructing a mechanism, which can always find the intersection of M^{PO} and M^C.

2.3 The Mechanism

In this section, we will propose a kidney exchange mechanism for finding a core matching, and discuss several selection rules in the mechanism.

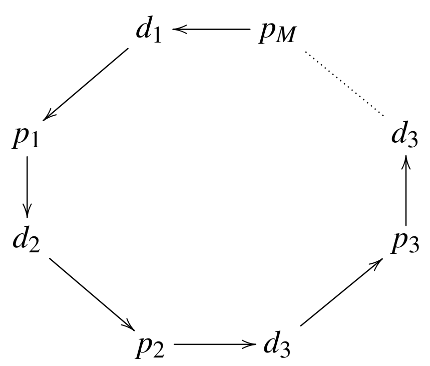
Now we consider a graph such that

(1) Each patient points to either a donor or the waiting list option w, presented by an arrow from the patient to the donor or the waiting list option w.

(2) Each paired donor points to her paired patient, presented by an arrow from the donor to the patient.

A single patient is called an *unaltered* single patient when the patient points to the waiting list option w. Each unaltered single patient keeps waiting on the list. Similarly, a paired patient is called an *unaltered* pair when the paired patient points to her paired donor. Each paired patient from unaltered pairs stays with her paired donor. A paired patient can choose the waiting list option in exchange for the kidney donation from her donor to a patient in the pool. When this happens, the paired patient becomes a new patient on the waiting list and her paired donor acts like a single donor. We say a paired patient turns into a *newcome* single patient and her paired donor turns into a *newcome* single donor when the paired patient chooses the waiting list option w. The corresponding assignment for a newcome single patient is that the patient goes to the waiting list. As long as a paired patient becomes a newcome single patient, her paired patient becomes a newcome single donor.

Fig. 2.3 An example of a PP-TTC cycle in graph.



Definition 5 *A PP-TTC (Patient-proposing TTC) cycle is an ordered list of an even number of paired donors and paired patients* $(d_1, p_1, d_2, p_2, ..., d_M, p_M)$ *(*$M \geq 2$*) such that the paired donor* d_1 *points to her paired patient* p_1*, paired patient* p_1 *points to a donor* d_2*, ..., paired patient* p_m *points to the donor* d_1*.*

Figure 2.3 illustrates an example of a cycle. When a PP-TTC cycle is formed, the corresponding kidney exchange transplants can be carried out

$$\text{Patient } p_1 \text{ is assigned kidney from } d_2,$$
$$\text{Patient } p_2 \text{ is assigned kidney from } d_3,$$
$$\cdots$$
$$\text{Patient } p_m \text{ is assigned kidney from } d_1.$$

Note that each pair can be a part of at most one cycle. A cycle never intersects with other cycles because each patient points to one donor at a time and each paired donor points to her paired patient.

Now we introduce the definitions of PP-TTC chains. We have two types of chains, which are Completed PP-TTC (CPP-TTC) chains and Uncompleted PP-TTC (UPP-TTC) chains.

Fig. 2.4 An example of a CPP-TTC chain and an UPP-TTC chain in graph.

$$p^w \longrightarrow d_1 \longrightarrow p_1 \longrightarrow d_2 \longrightarrow \cdots\cdots \longrightarrow d_M \longrightarrow p_M \longrightarrow d^s$$

$$d_1 \longrightarrow p_1 \longrightarrow d_2 \longrightarrow \cdots\cdots \longrightarrow d_M \longrightarrow p_M \longrightarrow d^s$$

Definition 6 *A CPP-TTC chain is an ordered list of an even number of patients and donors* $(p^w, d_1, p_1, d_2, p_2, ..., d_M, p_M, d^s)$ *($M \geq 0$) such that it begins with a single patient* p^w *who points to paired donor* d_1*, the donor* d_1 *points to her paired patient* p_1*, ..., paired patient* p_M *points to a single donor* d^s*.*

Figure 2.4 shows an example of a CPP-TTC chain. In a CPP-TTC chain $(p^w, d_1, p_1, d_2, p_2, ..., d_M, p_M, d^s)$, every patient receives a kidney and every donor donates a kidney. The single donor p^w is the initial patient of the chain and the single donor d^s is the trailer donor. A CPP-TTC chain can form the following corresponding kidney exchange transplants

Patient p^w is assigned kidney from d_1,
Patient p_1 is assigned kidney from d_2,
...
Patient p_M is assigned kidney from d^s.

Definition 7 *An UPP-TTC chain is an ordered list of an odd number of paired patients and donors* $(d_1, p_1, d_2, p_2, ..., d_M, p_M, d^s)$ *($M \geq 1$) such that paired donor* d_1 *points to her paired patient* p_1*, paired patient* p_1 *points to paired donor* d_2*, ..., paired patient* p_M *points to a single donor* d^s*.*

Figure 2.4 shows an example of a UPP-TTC chain. In an UPP-TTC chain $(d_1, p_1, d_2, p_2, ..., d_M, p_M, d^s)$, every patient receives a kidney but not every donor can donate a kidney. Observe that an UPP-TTC chain only includes paired patients. A donor is *under-demanded* if the donor would like to donate a kidney but (currently) no patient points to her. In figure 2.4, paired donor d_1 is an under-demanded donor. Every UPP-TTC chain has an under-demanded paired donor. Note that if a patient points to an under-demanded donor latter, the donor is not under-demanded any more. An UPP-TTC chain can form the following corresponding kidney exchange transplants

Donor d_1 does not donate kidney,
Patient p^1 is assigned kidney from d_2,
Patient p_2 is assigned kidney from d_3,
...
Patient p_M is assigned kidney from d^s.

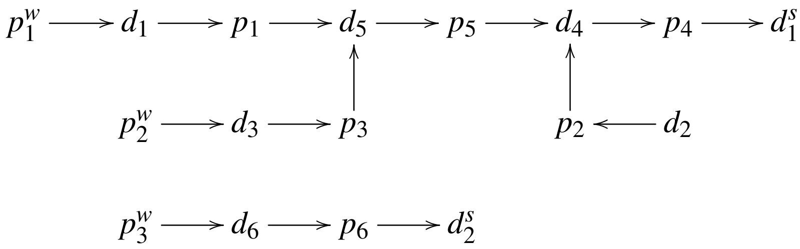
A cycle never intersects with chains while chains can intersect with each other. A donor is *over-demanded* if more than one patient points to that donor and is *demanded* if one patient points to that donor. Two chains are said to be *separate* if there is no over-demanded donor between them and to be *intersectant* if there exists one over-demanded donor. Note that two intersectant chains have only one over-demanded donor. If several chains intersect with each other, there exists at least one over-demanded donor.

Figure 2.5 shows an example of intersectant chains and separate chains. Two CPP-TTC chains $(p^w_1, d_1, p_1, d_5, p_5, d_4, p_4, d^s_1)$ and $(p^w_2, d_3, p_3, d_5, p_5, d_4, p_4, d^s_1)$ intersect with each other and the UPP-TTC chain $(d_2, p_2, d_4, p_4, d^s_1)$ intersects with the CPP-TTC chain $(p^w_1, d_1, p_1, d_5, p_5, d_4, p_4, d^s_1)$. Donors d_5 and d_4 are over-demanded donors because two patients p_1, p_3 point to d_2 and two patients p_2, p_5 points to d_4. Chains $(p^w_1, d_1, p_1, d_5, p_5, d_4, p_4, d^s_1)$ and (p^w_3, d_6, p_6, d^s_2) are separate chains.

Lemma 6 *Consider a graph in which each patient and donor are distinct nodes. Suppose each patient points towards either a donor or the waiting list option w and each paired donor points towards her paired patient. Then there exists at least one of the following results:*

(i) a PP-TTC cycle;

Fig. 2.5 An example of intersectant chains and separate chains.



(ii) a CPP-TTC chain or an UPP-TTC chain;

(iii) an unaltered pair or an unaltered single patient;

(iv) a newcome single patient and a newcome single donor.

Now we introduce the designed kidney exchange mechanism for finding a core matching. In the process of the mechanism, a patient is *active* if the patient has not received an assignment and turns to be *inactive* as long as the patient receives one. A chain is *active* if at least one patient in the chain is active, and otherwise is *inactive*. At the beginning of the mechanism, every patient is active.

The Kidney Exchange Mechanism for Finding a Core Matching

Step 0. Consider a kidney exchange problem $(P, D, w, \succ)$ in which there are many patient-donor pairs, patients on the waiting list and single donors.

Step 1. Each remaining patient points to the best remaining choice from donors who has not been assigned to a patient so far, newcome single donors and the waiting list option w. Each remaining paired donor points to her paired patient.

By Lemma 6, there exists at least one of the following results: a PP-TTC cycle, a CPP-TTC chain, an UPP-TTC chain, an unaltered pair or an unaltered single patient, or a newcome single patient and a newcome single donor.

Step 2. If there does not exist a PP-TTC cycle, an unaltered pair, an unaltered single patient or a newcome single patient, go to Step 3. Otherwise, remove all unaltered pairs and unaltered single patients out of matching with corresponding assignments, if there exists any newcome single patient, remove all newcome single patients out of matching with corresponding assignments and all their paired donors become newcome single donors, and if there exists any PP-TTC cycle, remove one cycle out randomly with the corresponding assignments. If all patients have gone, go to Step 4. Otherwise, go to Step 1.

Step 3. Let M^C denote the set of active CPP-TTC chains and M^U denote the set of active UPP-TTC chains in this step. If no active chain is found, remove all remaining chains with the corresponding assignments and go to Step 4. Otherwise, select one active chain based on *the selection rule*.

(i) If the selected chain is a CPP-TTC chain, remove the chain with corresponding assignments. If all patients have gone, go to Step 4. Otherwise, go to Step 1.

(ii) If the selected chain is an UPP-TTC chain, keep the chain and each patient is finalized with the corresponding assignment and the under-demanded donor is available in the pool, and go to Step 1. Note that except the under-demanded donor, every donor in the chain is assigned to a patient and is *unavailable* in the pool.

Step 4. Stop.

When no active chain remains in Step 3, all remaining chains (if any) are UPP-TTC chains because any inactive CPP-TTC chain is removed out of matching with the corresponding assignments. Moreover, an UPP-TTC chain may turn into a CPP-TTC chain in a latter round. For example, consider a kidney exchange pool of one patient on the waiting list p^w, one pair (d_1, p_1) and one single donor d^s. The preferences of patients are as follows: p_1 prefers d^s to d_1 and p^w prefers d^s to d_1 to w. In first round, there exists one active CPP-TTC chain (p^w, d^s) and one active UPP-TTC chain (d_1, p_1, d^s). If the UPP-TTC chain is selected and kept, donor d^s is assigned to patient p_1 and donor d_1 is still available in the pool. In the second round, the UPP-TTC chain (d_1, p_1, d^s) turns into an active CPP-TTC chain (p^w, d_1, p_1, d^s) in which p^w is active and p_1 is inactive.

When all chains are separate, we can remove all separate chains with their corresponding assignments. When several chains intersect with each other, the selection rules guide on how to allocate over-demanded donors in chains. The selection rules can be in accordance with policy orientations planned by the kidney exchange institutions. Now we discuss several selection rules. To encourage more patient-donor pairs involving in kidney exchanges, one practicable way is to give paired patients higher priorities than single patients when there exist over-demanded donors. Formally,

(i) Given a priority list over the set of patients under the rule that paired patients are prior to single patients. For each over-demanded donor, we pick the patient with highest priority among patients who point to that donor. Let M^P be the set of picked chains from M^C and M^U. Select one picked chain randomly from M^P.

The priority list in selection rule (i) gives paired patients advantages. That is, when a paired patient and a single patient want the same donor, the paired patient has a higher priority to obtain the donor than the single patient. If two paired patients want the same donor, the paired patient who has a higher priority get the donor. The rule is the same for two single patients. The priority list among the same type of patients depends on patient's type, health status, waiting time and so on. In a round of the mechanism, a chain is a *picked* chain if it is a separate chain or is picked based on the priority list. For example, given the priority list ℓ such that $p_1 \succ_\ell p_2 \succ_\ell p_3 \succ_\ell p_4 \succ_\ell p_1^w \succ_\ell p_2^w \succ_\ell p_3^w$, we have $M^P = \{(d_2, p_2, d_4, p_4, d_1^s), (p_3^w, d_6, p_6, d_2^s)\}$ in figure 2.5.

One potential problem of the selection rule (i) is that, in order to obtain a high priority, a paired patient has incentives to bring a donor with a very low quality kidney (such that no patient in the pool wants to accept). To prevent this case, we can further detail the selection (i) as follows:

($i^{'}$) Given a priority list over the set of patients under the rule that paired patients are prior to single patients. If M^C is not empty, pick the patient with highest priority for each over-demanded donor among patients who point to that donor in M^C. Let M_P^C be the set of picked chains from M^C and select one CPP-TTC chain from M_P^C randomly. Otherwise, pick the patient with highest priority for each over-demanded donor among patients who point to that donor in M^U. Let M_P^U be the set of picked chains from M^U and select one UPP-TTC chain from M_P^U randomly.

In selection rule ($i^{'}$), CPP-TTC chains are prior to UPP-TCC chains. In the previous example, we have $M_P^C = \{(p_1^w, d_1, p_1, d_5, p_5, d_4, p_4, d_1^s), (p_3^w, d_6, p_6, d_2^s)\}$ and select one chain from M_P^C randomly. Alternatively, we can apply a tie breaking rule on how to select chains from M_P^C. Since every CPP-TTC chain contains a single patient, we can choose a CPP-TTC chain with a highest priority single patient from M_P^C. Formally,

($i^{''}$) Given a priority list over the set of patients under the rule that paired patients are prior to single patients. If M^C is not empty, pick the patient with highest priority for each over-demanded donor among patients who point to that donor in M^C. Let M_P^C be the set of picked chains from M^C and select one CPP-TTC chain with the highest priority single patient from M_P^C. Otherwise, pick the patient with highest priority for each over-demanded donor among patients who point to that donor in M^U. Let M_P^U be the set of picked chains from M^U and select one UPP-TTC chain from M_P^U randomly.

In the previous example, the CPP-TTC chain $(p_1^w, d_1, p_1, d_5, p_5, d_4, p_4, d_1^s)$ is selected because p_1^w is prior to p_3^w.

We use an example to illustrate how the mechanism works.

Example 4 *There are eight paired patients $\{p_1^p, p_2^p, p_3^p, p_4^p, p_5^p, p_6^p, p_7^p, p_8^p\}$ with paired donors $\{d_1^p, d_2^p, d_3^p, d_4^p, d_5^p, d_6^p, d_7^p, d_8^p\}$, eight single patients $\{p_1^w, p_2^w, p_3^w, p_4^w, p_5^w, p_6^w, p_7^w, p_8^w\}$, and 5 single donors $\{d_1^s, d_2^s, d_3^s, d_4^s, d_5^s\}$. A priority list ℓ is given such that p_1^p is prior to p_2^p, p_2^p is prior to p_3^p, ..., p_7^w is prior to p_8^w. We use selection rule ($i^{'}$) in this example. Patients' preferences are give by the following table.*

Table 2.2 The preferences of patients in Example 4.

Preferences of Patients	Preferences of Patients
$\succ_{p_1^p}: d_5^p, d_6^p, d_3^s, d_1^s, d_5^s\ d_1^p$	$\succ_{p_1^w}: d_3^s, d_5^s$, w
$\succ_{p_2^p}: d_5^p, d_3^s, d_5^s, d_1^p, d_1^p\ d_2^p$	$\succ_{p_2^w}: d_5^p, d_1^p, d_5^s, d_3^s, d_2^s$, w
$\succ_{p_3^p}: d_6^p, d_3^s, d_5^s, d_2^p, d_1^s\ d_3^p$	$\succ_{p_3^w}: d_6^p, d_1^p, d_2^s, d_5^s, d_3^s$, w
$\succ_{p_4^p}: d_5^p, d_3^s, d_5^s, d_2^s, d_1^p\ d_4^p$	$\succ_{p_4^w}: d_4^p, d_3^p, d_3^s, d_5^s, d_4^s$, w
$\succ_{p_5^p}: d_2^p, d_6^p, d_3^s, d_5^s, d_1^s\ d_5^p$	$\succ_{p_5^w}: d_3^s, d_5^s$, w
$\succ_{p_6^p}: d_3^s, d_5^s\ d_6^p$	$\succ_{p_6^w}: d_5^p, d_1^p, d_2^s$, w
$\succ_{p_7^p}: d_1^s, d_6^p, d_3^s, d_8^p\ d_7^p$	$\succ_{p_7^w}: d_2^p, d_6^p, d_3^s, d_5^s$, w
$\succ_{p_8^p}: d_1^s, d_7^p\ d_8^p$	$\succ_{p_8^w}: d_4^p, d_4^s, d_5^s, d_3^s$, w

Fig. 2.6 The first round of Example 4.

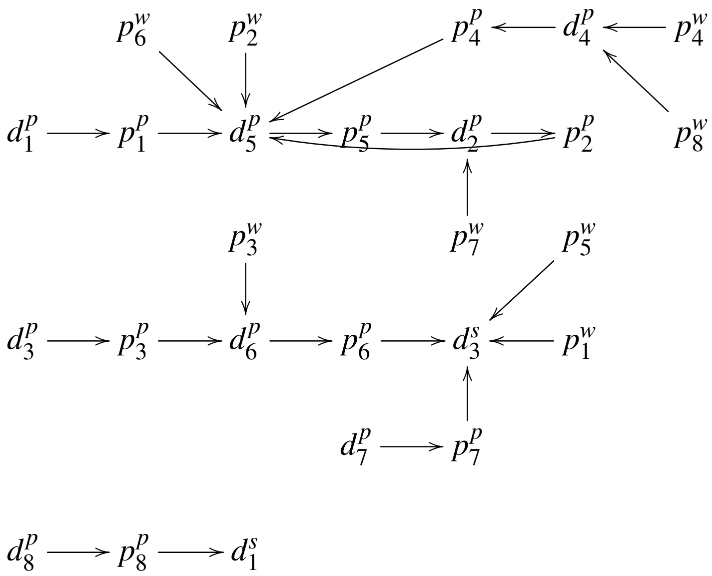
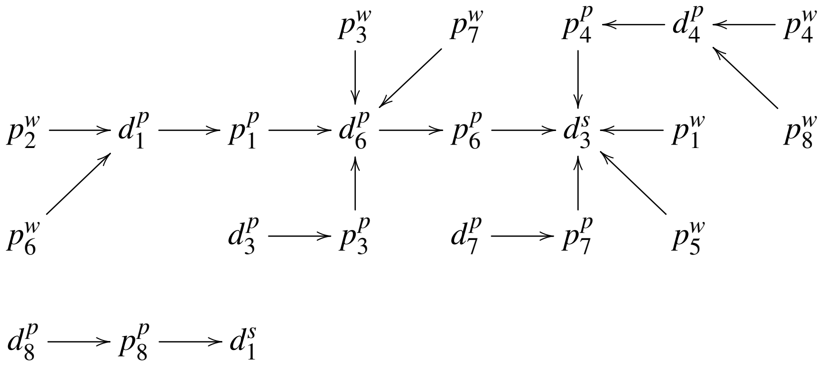


Fig. 2.7 The second round of Example 4.



Round 1: Figure 2.6 shows the first round. There exists one cycle $(d_5^p, p_5^p, d_2^p, p_2^p)$ and we remove the cycle out of matching corresponding assignments such that $\mu(p_5^p) = d_2^p$ and $\mu(p_2^p) = d_5^p$.

Round 2: Figure 2.7 illustrates the second round. There exist several CPP-TTC chains and no cycle. The set of CPP-TTC chains in this round $M^C = \{(p_2^w, d_1^p, p_1^p, d_6^p, p_6^p, d_3^s), (p_6^w, d_1^p, p_1^p, d_6^p, p_6^p, d_3^s), (p_4^w, d_4^p, p_4^p, d_3^s), (p_8^w, d_4^p, p_4^p, d_3^s), (p_3^w, d_6^p, p_6^p, d_3^s), (p_7^w, d_6^p, p_6^p, d_3^s), (p_1^w, d_3^s), (p_5^w, d_1^s)\}$. Based on the selection rule, d_1^p accepts p_2^w; d_6^p accepts p_1^p; d_3^s accepts p_4^p; d_4^p accepts p_4^w. Hence, the set of picked CPP-TTC chains is $M_P^C = \{(p_4^w, d_4^p, p_4^p, d_3^s)\}$ and the CPP-TTC chain is removed out of matching such that $\mu(p_4^w) = d_4^p$, $\mu(p_4^p) = d_3^s$.

Fig. 2.8 The third round of Example 4.

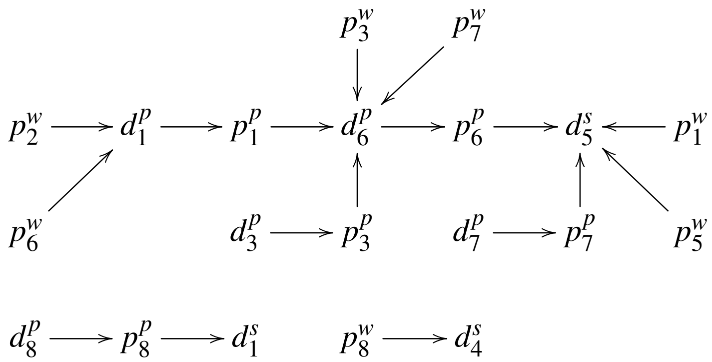
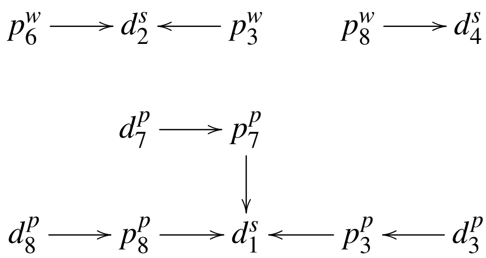


Fig. 2.9 The fifth round of Example 4.



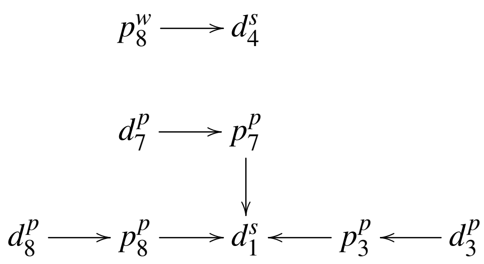
Round 3: Figure 2.8 shows the third round. p_8^w points to d_4^s; p_1^w, p_5^w point to d_5^s. The set of CPP-TTC chains in this round $M^C = \{(p_2^w, d_1^p, p_1^p, d_6^p, p_6^p, d_5^s), (p_6^w, d_1^p, p_1^p, d_6^p, p_6^p, d_5^s), (p_3^w, d_6^p), (p_7^w, d_6^p), (p_1^w, d_5^s), (p_5^w, d_5^s), (p_8^w, d_4^s)\}$. Based on the selection rule, d_1^p accepts p_2^w; d_6^p accepts p_1^p; d_5^s accepts p_6^p. Hence, $M_P^C = \{(p_2^w, d_1^p, p_1^p, d_6^p, p_6^p, d_5^s), (p_8^w, d_4^s)\}$ and pick a CPP-TTC chain randomly like $(p_2^w, d_1^p, p_1^p, d_6^p, p_6^p, d_5^s)$ out of matching such that $\mu(p_2^w) = d_1^p$, $\mu(p_1^p) = d_6^p$, $\mu(p_6^p) = d_5^s$.

Round 4: p_1^w, p_5^w, p_7^w point to the waiting list option w; p_6^w, p_3^w point to d_2^s; p_7^p, p_3^p point to d_1^s. p_1^w, p_5^w, p_7^w turn into unaltered single patients and move them out of matching such that $\mu(p_1^w) = \mu(p_5^w) = \mu(p_7^w) = w$.

Round 5: Figure 2.9 illustrates the fifth round. The set of CPP-TTC chains in this round $M^C = \{(p_3^w, d_2^s), (p_6^w, d_2^s), (p_8^w, d_4^s)\}$. d_2^s accepts p_3^w. Hence, $M_P^C = \{(p_3^w, d_2^s), (p_8^w, d_4^s)\}$ and select a picked CPP-TTC chain randomly like (p_3^w, d_2^s) out of matching such that $\mu(p_3^w) = d_2^s$.

Round 6: p_6^w points to the waiting list option w. p_6^w becomes an unaltered single patient and move patient p_6^w out of matching such that $\mu(p_6^w) = w$.

Fig. 2.10 The seventh round of Example 4.



Round 7: Figure 2.10 shows the seventh round. We have $M^C = M_P^C = \{(p_8^w, d_4^s)\}$ and move the CPP-TTC chain (p_8^w, d_4^s) out of matching such that $\mu(p_8^w) = d_4^s$.

Round 8: Neither a PP-TTC cycle nor an isolated patient nor an isolated pair nor a CPP-TTC chain is found. We go to Step 4. Based on the selection rule, d_1^s accepts p_3^p. The set of picked UPP-TTC chains is $M_P^U = \{(d_3^p, p_3^p, d_1^s)\}$. The UPP-TTC chain (d_3^p, p_3^p, d_1^s) is selected and the assignment $\mu(p_3^p) = d_1^s$ is finalized.

Round 9: p_7^p points to d_8^p; p_8^p points to d_7^p. There exists a cycle $(d_7^p, p_7^p, d_8^p, p_8^p)$ and is moved out of matching such that $\mu(p_7^p) = d_8^p$ and $\mu(p_8^p) = d_7^p$.

Round 10: Neither a PP-TTC cycle nor an isolated patient nor an isolated pair nor a CPP-TTC chain is found and go to Step 4. There does not exist an active chain. Remove the inactive UPP-TTC chain (d_3^p, p_3^p, d_1^s) out of matching with the assignments. That is, $\mu(p_3^p) = d_1^s$ and donor d_3^p do not need to donate a kidney, and stop.

Therefore, the matching μ produced by the mechanism is

$$\mu = \begin{pmatrix} p_1^p & p_2^p & p_3^p & p_4^p & p_5^p & p_6^p & p_7^p & p_8^p \\ d_6^p & d_5^p & d_1^s & d_3^s & d_2^p & d_5^s & d_8^p & d_7^p \\ p_1^w & p_2^w & p_3^w & p_4^w & p_5^w & p_6^w & p_7^w & p_8^w \\ w & d_1^p & d_2^s & d_4^p & w & w & w & d_4^s \end{pmatrix}$$

2.4 Main Results

In this section, we will explore the properties of the designed mechanism. The first property (in Lemma 7) indicates that the matching produced by the mechanism is individually rational. That is, every paired patient receives either an strongly acceptable donor or the acceptable waiting list option and every patient on the waiting list receives an acceptable donor.

Lemma 7 *The kidney exchange mechanism always finds an individually rational matching of a kidney exchange model* $(P, D, w, \succ)$.

Proposition 1 *Let* μ *denote the matching produced by the kidney exchange mechanism of a kidney exchange model* $(P, D, w, \succ)$. *Let* r_p *denote the round in which the patient* p *receives her assignment* $\mu(p)$ *and let* D^r *denote the set of remaining donors who have not been assigned to a patient at the beginning of round* r. *Then* $\mu(p) \succeq_p d$ *for all donors* $d \in D^r$ *where* $r \geq r_p$.

Proposition 1 illustrates an important implication. Consider any two patents p and p' with their assignments $\mu(p)$ and $\mu(p')$ in the matching μ produced by the kidney exchange mechanism. Let r_p be the round in which assigns $\mu(p)$ to patient p and $r_{p'}$ be the round in which assigns $\mu(p')$ to patient p'. If $\mu(p') \succ_p \mu(p)$, then $r_p > r_{p'}$. Further, if patient p has a strict preference and $\mu(p') \sim_p \mu(p)$, then we have $p = p'$ and hence $r_p = r_{p'}$. Therefore, under strict preference, if $\mu(p') \succeq_p \mu(p)$, then $r_p \geq r_{p'}$.

Proposition 2 *Let* μ *denote the matching produced by the kidney exchange mechanism of a kidney exchange model* $(P, D, w, \succ)$. *Consider any resourceful patient* d *in matching* μ *and her endowed donor* d. *Denote* r_p *as the round in which the patient* p *is assigned her assignment* $\mu(p)$ *and* r^d *as the round in which the endowed donor* d *is assigned. Then, we have* $r_p \leq r_d$.

The Proposition 2 implies that in the matching μ produced by the kidney exchange mechanism, the round of each resourceful patient receiving the assignment is no latter than that of assigning her endowed donor.

One of the most important properties of the designed mechanism is that the matching produced by the mechanism is in the core. In other words, the matching produced by the mechanism can not be strongly improved by any coalition.

Theorem 1 *The kidney exchange mechanism always finds a core matching of a kidney exchange model* $(P, D, w, \succ)$.

Recall that a core matching may not be Pareto optimal and vice versa. We demonstrate that the matching produced by the mechanism is also Pareto optimal (see Theorem 2). In other words, the mechanism always finds the intersection of the set of core matchings and the set of Pareto optimal matchings.

Theorem 2 *The kidney exchange mechanism is efficient.*

We say a matching is a *Pareto optimal core allocation* if no other core matching ν can Pareto dominants matching μ. Recall that in Example 2, the core matching μ^2 Pareto dominants the core matching μ^1 and cannot find any other core matching which Pareto dominants the core matching μ^2. Therefore matching μ^2 is a Pareto optimal core allocation. We have an immediate consequence of the Theorem 1 and the Theorem 2 as follows.

Corollary 1 *Given a kidney exchange problem* $(P, D, w, \succ)$*, the matching produced the kidney exchange mechanism is a Pareto optimal core allocation.*

2.4.1 Patients with Indifferent Preferences

In this section, we consider patients with indifferent preferences. Due to the lack of medical knowledge, a patient may feel difficult to tell the difference between donors with almost the same quality, but clearly know which group of donors is more preferred. From the example 1, there may not exist a strict core even in a strict preference profile of patients. When patients have indifferent preferences, we use an example to reiterate the above point.

Example 5 *(There may not exist a strict core matching under indifferent preferences) Consider a kidney exchange model with three patient-donor pairs* $\{d_1^p, p_1^p\}$*,* $\{d_2^p, p_2^p\}$*,* $\{d_3^p, p_3^p\}$*. Their preferences are given by*

$$\succ_{p_1^p}: \ d_2^p, \ d_1^p \qquad \succ_{p_2^p}: \ [d_1^p, d_3^p] \ \ d_2^p \qquad \succ_{p_3^p}: \ d_2^p, \ d_3^p$$

$$\mu^1 = \begin{pmatrix} p_1^p & p_2^p & p_3^p \\ d_2^p & d_1^p & d_3^p \end{pmatrix} \qquad \text{and} \qquad \mu^2 = \begin{pmatrix} p_1^p & p_2^p & p_3^p \\ d_1^p & d_3^p & d_2^p \end{pmatrix}.$$

In this example, there are two core matchings μ^1 and μ^2. None of the core matchings is in the strict core. For instance, the coalition $\{p_2^p, p_3^p\}$ can block the matching μ^1. When patients have indifferent preferences, we demonstrate that the matching produced by the designed mechanism is in the core (see Theorem 3).

Theorem 3 *Given a kidney exchange problem $(P, D, w, \succeq)$, the kidney exchange mechanism always finds a core matching.*

2.4.2 The Existence of Strict Core

In this section, we explore the situations in which there always exists the strict core. We find that if each paired patient prefers her paired donor to the waiting list option w under the strict preference profile of patients, the strict core is always non-empty. We demonstrate that the mechanism always finds a strict core and Pareto optimal matching in the situation.

Theorem 4 *Given a kidney exchange problem $(P, D, w, \succ)$ in which each paired patient prefers her paired donor to the waiting list option, the kidney exchange mechanism always finds a matching in the strict core as well as Pareto optimal.*

It is easy to prove that when every paired patient prefers her paired donor to the waiting list option w, the matching produced by the mechanism is Pareto optimal. Since the situation is a special case of preferences in our general model, we can directly get the result by Theorem 2. The proof of the strict core is in appendix.

2.5 Conclusion

In this paper we have presented a general kidney exchange model. It comprises a finite number of patient-donor pairs, single patients and single donors. Every patient has general preferences over the donors and a waiting list option. The kidney of a paired donor is treated as the endowment of the intended recipient, while kidneys from single donors are treated as public goods. We proposed a new and appropriate modification of (strict) core by taking public resources into account and developed a mechanism for finding a core matching. Thus we proved in a constructive manner that our kidney exchange model has a non-empty core. We further discussed the strict core, core, and Pareto optimal allocations and their relationship.

Chapter 3

Efficient Kidney Allocation with Dichotomous Preferences

3.1 Introduction

Every year in the world hundreds and thousands of patients with severe kidney disease need a kidney transplant. The difficulty of achieving suitable kidney transplants arises in three major aspects. Firstly, there is a significant shortage of kidneys from deceased donors. For instance, in the United States in 2005 more than 60,000 patients were waiting for kidney transplants but only about 9,900 received transplants from deceased donors and 6,563 received transplants from living donors. While over 4,000 patients passed away and about 1,000 had become too sick to have a transplant and were therefore removed from the waiting list (see Roth, Sönmez and Ünver 2007, p.828). In the United Kingdom during the period 2013-2014, which was the best year during the previous ten years, 5,881 active patients were on the waiting list, with 2,142 getting transplants from deceased donors and 1,114 receiving transplants from living donors. Secondly, a patient may receive a kidney from a living donor who can be a family member, a relative, or a friend of the patient. In this case the patient and the donor are called a patient-donor pair, and the patient is a paired patient and the donor a paired donor. However the patient may not be compatible with the donor, and therefore is unable to use the kidney directly because of blood or tissue incompatibility. Thirdly, although most people have one more kidney than they need, it is almost universally illegal to buy or sell a kidney. A central issue here is how to design an effective mechanism to enable as many patients as possible to receive a suitable kidney transplant, within existing medical, legal and social constraints.

The operation of a suitable kidney transplant must satisfy several essential constraints as follows. The first is medical constraint: the patient must be both blood-compatible and tissue-compatible with the donor. The second is the incentive constraint. This will not be a problem when a patient receives a kidney from an altruistic deceased or living donor, but arises in the context of patient-donor pairs. If a paired patient is incompatible with her paired donor, she needs to exchange one kidney for another. Then, the order of implementing the kidney transplants becomes crucial to incentive-compatible exchange. We illustrate this point by an example. Suppose there are two patient-donor pairs. The first paired patient is compatible with the second paired donor while the second paired patient is compatible with the first paired donor. If the first patient first receives kidney transplant from the second donor, there is a possibility that the first donor may regret and renege on her promise because one cannot force her to donate her kidney to the second patient. To avoid this moral hazard, exchanges between the two pairs must be carried out simultaneously. The third is the capacity constraint which is caused by the second constraint. Because transplants need to be performed simultaneously, it means that such operations must take place in the same hospital, or hospitals in close proximity to each other. Even a two-way exchange or two pair exchange already requires four simultaneous surgical treatments. Obviously, in practice, there is a limit to the number of possible kidney transplants in each hospital. It is therefore desirable to have short chains or cycles of exchange.

Kidney exchange has previously been studied by a number of medical researchers (see Rapaport 1986, Ross et al. 1997, Ross and Woodle 2000, Zenios, Woodle, and Ross 2001, etc). Roth, Sönmez and Ünver (2004) initiated the economic analysis of kidney exchange and transformed it into a fertile area of economic research. They examined a model of kidney exchange in which there are many patient-donor pairs. Each patient has strict preferences over compatible kidneys, her paired kidney and the waiting list option. They proposed an exchange mechanism -the top trading cycles and chains (TTCC) mechanism- a generalisation of the top trading cycle procedure from Shapley and Scarf (1974) for a housing market model that achieves efficiency and incentive compatibility; see also Abdulkadiroğlu and Sönmez (1999) for a related mechanism. In this case, cycles and chains could be long. Roth, Sönmez and Ünver (2007) considered a simpler but more practical model where patients are indifferent between compatible kidneys and prefer compatible kidneys to incompatible ones. Their model consists of many incompatible patient-donor pairs. They demonstrate that allowing three-way as well as two-way exchanges could significantly increase the total number of possible exchanges, and that four-way exchanges are sufficient to capture all potential gains arising from exchange. Roth, Sönmez and Ünver (2007) and Saidman et al. (2006) provide computational results on real and simulated patient data to show significant efficiency gains from two-way and three-way exchanges.

In this paper we consider a very general and practical model of kidney exchange. The model consists of compatible patient-donor pairs, incompatible patient-donor pairs, (altruistic) single donors (decreased or living), and patients on the waiting list. Our aim is to explore how many kidney transplants it is possible to arrange within the same medical, incentive, and capacity constraints as those used in Roth, Sönmez and Ünver (2007). In their model, if a patient is compatible with her donor, then a transplant will take place just between the pair. So in their model, exchanges are carried out among only incompatible patient-donor pairs. In contrast, in the current paper we also allow compatible patient-donor pairs to participate in exchange with incompatible patient-donor pairs, if necessary, in order to enable more patients to receive transplants and thus save more lives. Let us show a case in point. Suppose there are three blood-incompatible patient-donor pairs (O,AB), two compatible patient-donor pairs $(AB,O)^c$, and one tissue-incompatible pair $(AB,O)^i$. For each pair, the first component in the notation indicates the patient's blood type, the second is the donor's blood type, and the superscripts c and i stand for tissue-compatible and tissue-incompatible, respectively. If compatible patient-donor pairs do not exchange with incompatible patient-donor pairs, only four patients will receive kidney transplants, i.e. two $(AB,O)^c$ and one two-way exchange $(O,AB)-(AB,O)^i$. In contrast, if we allow compatible pairs to exchange with incompatible pairs, six patients will receive kidney transplants, i.e., three two-way exchanges $(O,AB)-(AB,O)^i$, $(O,AB)-(AB,O)^c$ and $(O,AB)-(AB,O)^c$. In this way, two more patients will get kidney transplants and be saved. Ross and Woodle (2000) suggest the inclusion of compatible pairs in kidney exchange with incompatible ones and Roth, Sönmez and Ünver (2005b, p. 377) also indicate this potential.

We will establish several basic results for this general and practical model, going beyond and improving upon considerably those of Roth, Sönmez and Ünver (2007). Briefly, in each case of k-way exchange, $k = 2, 3, 4$, we derive a tight upper bound (in fact an explicit formula) of the possible number of feasible kidney transplants, and propose a sequential matching procedure to achieve this upper bound. We find that two or three-way cycles and chains of exchange can substantially increase the number of feasible transplants, and at most four-way cycles and chains are sufficient to achieve the full potential gains of exchange. In particular, allowing compatible patient-donor pairs to participate in exchange with incompatible pairs will considerably enhance the efficiency of kidney exchange, which means many more patients can receive transplants thereby potentially saving their lives. It will be shown in Section 3.1 that this benefit becomes very obvious and significant even with just two-way cycles and chains of exchange. This benefit becomes even more substantial as the pool of patients and donors becomes large, or when single donors are also allowed to participate in exchange with all compatible or incompatible pairs. We prove that in every-way (2-way, 3-way or 4-way) of exchange, each cycle contains at most two blood-type compatible pairs and each chain comprises at most one blood-type compatible pair. Moreover, we discuss a more general model of type-compatible exchanges with patient-donor pairs, single donors and patients on the waiting list, and demonstrate that the maximum size of exchange to achieve efficiency equals the number of total types.

As our basic model is quite realistic and general, our analysis will become inevitably much more involved and more difficult due to the large number of combinatorial cases caused by the presence of compatible or incompatible patient-donor pairs, single donors and patients on the waiting list.

To test the theory and explore its policy implications, we provide substantial simulation results. Simulations are carried out based on two real life data sets from the US national patient and donor characteristics, from 1993 to 2002 and from 1995 to 2016, respectively. The first period from 1993 to 2002 is the same as that used by Roth, Sönmez and Ünver (2007), and by Saidman et al. (2006), except that in our new data set we add more relevant information including the distribution of compatible patient-donor pairs and single donors, which is not used in their models. Compared with the first time slot data, the second time slot data from 1995 to 2016 contains more accurate information on tissue-type incompatibility. We run Monte-Carlo simulations of 5000 random population constructions for 25, 50, 100, 150 and 200 incompatible patient-donor pairs, and also run Monte-Carlo simulations of 500 random population constructions for 300 and 400 pairs, respectively, with their corresponding compatible patient-donor pairs and single donors (and patients on the waiting list who need no simulation as they are populous) based on the 1993-2002 data set and the 1995-2016 data set. By comparison, Roth, Sönmez and Ünver (2007) conducted Monte-Carlo simulations of 500 random population constructions for 25, 50, and 100 incompatible patient-donor pairs based on the 1993-2002 data set, by using two-, three-, higher- and unrestricted-way of exchange, and Saidman et al. (2006) tested the cases of 25 and 100 incompatible patient-donor pairs.

In our simulations, we use only two-way chains or cycles of exchange. For the same population, in comparison with Roth, Sönmez and Ünver (2007) whose mechanism will be simply called *the exclusive (exchange) mechanism*, our mechanism (called *the first degree inclusive (exchange) mechanism*) of allowing compatible pairs to exchange with incompatible pairs can result in at least a 10% net increase in feasible kidney transplants, and our mechanism (called *the second degree inclusive (exchange) mechanism*) of allowing compatible pairs and single donors to exchange with incompatible pairs can result in at least a 30% net increase in feasible kidney transplants. For instance, for the 1993-2002 data set, if a population has 100 incompatible patient-donor pairs, the population will have 22 compatible patient-donor pairs and 39 single donors, and the exclusive mechanism will enable 49 incompatible paired patients to get feasible transplants whereas the first degree inclusive mechanism will increase this number to 64 and the second degree inclusive mechanism will raise it to 89. For the 1995-2016 data set, if a population has 100 incompatible patient-donor pairs, the population will have 20 compatible patient-donor pairs and 36 single donors, the exclusive mechanism will enable 34 incompatible paired patients to get feasible transplants whereas the first degree inclusive mechanism will increase this number to 46 and the second degree inclusive mechanism will raise it to 69.

Major findings from our simulations are briefly stated here. Firstly, our simulations clearly indicate that as the number of incompatible patient-donor pairs in the population reaches 100, the slope of the matching rates (in percentage) of incompatible paired patients getting transplants becomes almost flat, albeit upwards, (which implies that the efficiency of exchange becomes asymptotically constant). This is surprising and has an important and novel policy implication: *Kidney exchange can be decentralised* in the sense that in a country with a relatively large population, separate kidney exchange programmes can be established in several major regions, not just one centralised programme for the entire country. Secondly, we find that the actual maximum number of kidney exchanges is surprisingly close to the predicted number given by our derived formulae. Thirdly, we find that as the size of the population gets larger, the predictive power of our theory becomes increasingly better. Fourthly, our results show that because the 1995-2016 data set contains more precise information on the physical characteristics of the population, this will improve the quality of feasible kidney transplants but at the same time reduce the number of feasible transplants by roughly 20%. Fifthly, we find that two-way exchange can reap most benefits of exchange and will play an even more important role in achieving the benefits of exchange as the size of the population increases.

We conclude this introductory section by briefly reviewing several other related papers. Roth, Sönmez and Ünver (2005a) consider a kidney exchange model in which the size of kidney exchanges is restricted to two patient-donor pairs and patients are indifferent compatible kidneys. They propose both deterministic and stochastic efficient and strategy-proof mechanisms. The deterministic ones can accommodate a certain priority structure while the stochastic ones exhibit a distributive justice property. Yilmaz (2011) proposes an egalitarian mechanism that uses two-way exchanges and list exchanges. A list exchange means that an incompatible paired donor gives a kidney to a patient on the waiting list and in return the incompatible paired patient gets a priority on the waiting list. Sönmez and Ünver (2014) study a model consisting of compatible pairs and incompatible pairs under two-way exchange. They examine the structure of Pareto-efficient matchings and show that all such matchings have the same number of kidney transplants for patients. They find a novel application of the well-known Gallai-Edmonds decomposition in kidney exchange. Ausubel and Morrill (2014) observe that incentive compatibility for kidney exchange requires kidney donation to occur no later than the receipt of the associated kidney. They show that sequential exchanges can also increase the number of beneficial exchanges. Andersson and Kratz (2016) examine efficient kidney exchanges under a refined structure of blood type compatibility. In their model, every patient prefers a fully acceptable donor to any donor who is not fully acceptable, and yet prefers an acceptable donor to any unacceptable donor. In a related development, Ünver (2010) studies efficient kidney exchanges in a dynamic environment in which agents arrive according to a stochastic Poisson process. We refer to Sönmez and Ünver (2013) for a survey on the subject and references therein contained.

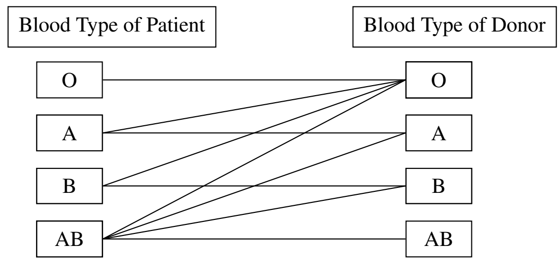
This paper is organized as follows. The model and basic concepts are introduced in Section 2. Maximal numbers of transplants from two, three and four-way exchanges are derived in Section 3. A general n-way exchange model is discussed in Section 4. Simulations are presented in Section 5 and conclusion is given in Section 6. Most of proofs are deferred to the appendix.

3.2 The Model

Kidney exchanges involve patients and donors. A kidney can be transplanted from a willing donor to a patient if the donor's kidney is compatible to the patient both in blood type and tissue type. There are four blood types, A, B, AB, and O. A patient of O type can receive a kidney only from a donor of O type, a patient of A type can receive a kidney from a donor of A or O type, a patient of B type can receive a kidney from a donor of B or O type, while a patient of AB type can receive a kidney from a donor of any blood type. Blood-compatibility is shown in Figure 3.1. Another medical test concerns tissue. Tissue-compatibility is determined by six HLA (human leukocyte antigen) proteins (three from the father and another three from the mother). If the potential recipient shows antibodies against HLA in the donor kidney called a positive crossmatch, then the donor kidney cannot be transplanted to the patient. Unlike blood-compatibility, tissue-compatibility does not require exact HLA match between a patient and a donor. Moreover, in reality, the percentage of tissue-incompatibility is also very low; see Zenios, Woodle and Ross (2001).

Formally our kidney exchange model consists of a set D^S of single donors, a set of patients P^W on the waiting list (on TWL in short) and a set PD of patient-donor pairs. Single donors could be altruistic cadavers or living people. Patients on TWL are also called single patients. A patient-donor pair describes a designated patient and a living donor who is willing to give a kidney to the patient or to exchange a kidney with another kidney for the designated patient. A patient (donor) in a patient-donor pair will be called a paired patient (donor). Patients are indifferent between compatible kidneys, indifferent

Fig. 3.1 Blood-type compatibility between patients and donors.



between incompatible kidneys, and prefer compatible kidneys to incompatible ones. In reality there is always a large pool of patients on the waiting list so that such patients can be found to match compatibly with any given kidney. This will be a part of our model. Our primary objective is to enable as many patients as possible to receive compatible kidneys, i.e., to achieve a maximal number of feasible kidney transplants between patients and donors.

It is natural to bring compatible patient-donor pairs and single donors into exchange with incompatible pairs as more patients can be benefited from their involvement. In practice single donors play a significant role. For instance, the Organ Donation and Transplantation Activity Report from NHS in 2014 shows that the number of living donors in UK from 2013 to 2014 is 1114, meanwhile the number of total kidney donors in USA is 16,526 including 11,195 deceased donors and 5,331 living donors according to OPTN/SRTR 2012 Annual Data Report.

In our paper, the symbol (X,Y) indicates a pair of a patient with blood type X and a donor with blood type Y, and $(X,Y)^i$ ($(X,Y)^c$) means a pair of patient and donor who are tissue-incompatible (tissue-compatible). Furthermore, we use $\#X^d$ to denote the number of single donors with blood-type X, $\#Y^p$ the number of patients on the waiting list with blood-type Y, and $\#(X,Y)$ the number of patient-donor pairs with blood-type X for patients and blood-type Y for donors. For any real number k, $\lfloor k \rfloor$ stands for the largest integer no bigger than k.

An outcome of the kidney exchange problem is a *matching* of kidneys (i.e., donors)/the waiting list option to patients such that each paired patient is either assigned a compatible kidney (i.e., donor) or stays with his paired donor, each patient on the waiting list is either assigned a compatible kidney (i.e., donor) or stays put, and no kidney (i.e., donor) is assigned to more than one patient. A matching μ is *efficient or maximal* if there exists no other matching ν such that $|\nu| > |\mu|$ where $|\mu|$ is the number of possible kidney transplants for the matching μ.

A matching can be made through several ways of exchange between patients and donors. A *two-way cycle exchange* involves two patient-donor pairs in which each patient is compatible with the other patient's donor. For instance, we have two patient-donor pairs (A,B) and (B,A) and use $(A,B)-(B,A)$ to indicate a two-way cycle exchange in which blood-type A patient in first pair receives the kidney from blood-type A paired donor in second pair and blood-type B patient in second pair can receive the kidney from blood-type B paired donor in first pair. A *three-way cycle exchange* involves three patient-donor pairs in which the patient in the first pair is compatible with the donor in the second pair, the patient in the second pair is compatible with the donor in the third pair, and the patient in the third pair is compatible with the donor in the first pair. An example consists of three pairs (X,Z), (Z,Y), and (Y,X), and the three-way cycle exchange is given by $(X,Z)-(Z,Y)-(Y,X)$ in which each patient receives a compatible kidney. Similarly we can define a four-way cycle exchange.

We also need to use chain exchanges. A *one-way chain exchange* involves a single donor, denoted by X^d, and a compatible patient, denoted by Y^p, on the waiting list. We write this exchange as $X^d - Y^p$. A *two-way chain exchange* is a chain $X^d-(X,Y)-Y^p$ in which the patient of blood-type X in the pair receives the kidney from the single donor X^d, and the patient Y^p on the waiting list receives the kidney from the donor in the pair. A *three-way chain exchange* is a chain $X^d-(X,Y)-(Y,Z)-Z^p$ in which the single donor X^d gives her compatible kidney to the patient X in the first pair, the donor Y in the first pair gives hers to the patient Y in the second pair, and the donor Z in the second pair gives hers to the patient Z^p in waiting. Four-chain exchanges can be defined analogously. For a given positive integer k, we say that a matching μ is *k-efficient* if there exists no other matching ν such that $|\nu| > |\mu|$ when the maximum size of kidney exchanges is no more than k-way cycles or chains of exchange. In the following *when we say a k-way exchange, it can be an l-way cycle or chain of exchange for any* $1 \leq l \leq k$.

To derive an analytical expression for the maximum number of feasible transplants among the whole kidney exchange pool, we impose the following three basic assumptions.

Assumption 1 (Upper Bound Assumption): *Every patient on the waiting list is tissue-compatible with every blood-type compatible donor and every paired patient is tissue-compatible with a blood-type compatible single donor or paired donor of any other paired patient.*

This assumption can be seen as a generalization of Assumption 1 of Roth, Sönmez and Ünver (2007, p. 831). With evolving clinical practice, the significance of HLA matching has diminished (Su et al. 2004). To decide whether a person can donate a kidney or not, the HLA level does not play a central role. This is consistent with the practical evidence from OPTN & SRTR annual data report in 2012 that most of transplanted patients have HLA mistakes with donors.

Assumption 2 $\#(A,B) > \#(B,A)$.

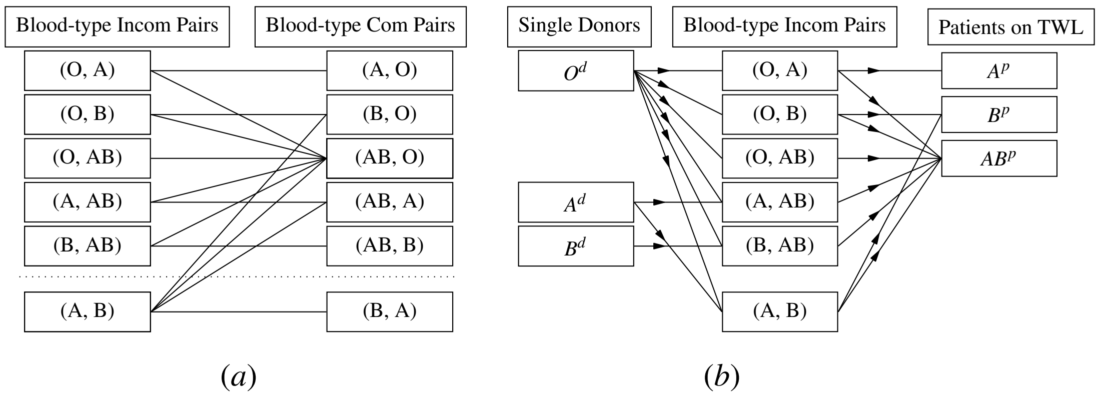
Terasaki, Gjertson, and Cecka (1998) and OPTN & SRTR annual data report in 2012 have provided statistical evidence for this assumption that the number of pairs (A,B) is greater than the number of pairs (B,A). This assumption is used as Assumption 3 in Roth, Sönmez and Ünver (2007, p. 834).

Assumption 3 *Let* (X,Y) *denote a blood-compatible type from* (A,A), (B,B), (AB,AB), (O,O), (A,O), (B,O), (AB,O), (AB,A) *and* (AB,B). *There exists either no pair of type* (X,Y) *or at least one tissue-compatible pair of type* (X,Y).

This assumption can be easily satisfied for a relatively large population and generalizes Assumption 4 of Roth, Sönmez and Ünver (2007, p. 834).

For a relatively large population, due to blood-compatibility constraints, there will be likely higher demand for kidneys of type O than type A or B, and higher demand for kidneys of type A or B than type AB. As a result, pairs of type (O,A), (O,B), (O,AB), (A,AB), or (B,AB) are on the long side of the exchange and will have to wait longer for a feasible exchange than pairs of other types. Their opposite blood-type compatible but tissue-type incompatible pairs are on the short side. This is used as their Assumption 2 of Roth, Sönmez and Ünver (2007, p. 832). Our model will dispense with this assumption and can handle cases that violate or satisfy this assumption.

Fig. 3.2 Two-way cycles (a) and chains (b) of exchange.



3.3 Efficient Kidney Exchange

In this section we will derive a maximum number of feasible kidney transplants, when one-way, two-way or three-way cycles or chains of exchange are used.

3.3.1 Two-Way Exchange

Recall that to distinguish blood-type compatible but tissue-type incompatible pairs and compatible pairs, we use $(X,Y)^i$ to denote the first group and $(X,Y)^c$ to denote the second group. Obviously $\#(X,Y) = \#(X,Y)^i + \#(X,Y)^c$. In the following, the notation $(A,B) - (C,D)/(X,Y)$ means that $(A,B) - (C,D)$ and/or $(A,B) - (X,Y)$, and $(A,B)/(C,D) - (X,Y)$ means that $(A,B) - (X,Y)$ and/or $(C,D) - (X,Y)$.

Figure 3.2 shows several basic two-way cycles and chains of exchange but do not include pairs (X,X). In Figure 3.2(a) the right column above the dot-line represents blood-type compatible pairs while the left column above the dot line stands for the blood-type incompatible pairs. By Assumption 3 all tissue incompatible pairs of type $(X,Y)^i$ on the right side can be matched by two-way cycle $(X,Y)^i - (X,Y)^i$ or two-way cycle $(X,Y)^i - (X,Y)^c$. The problem becomes how to take full advantage of blood-type compatible pairs and single donors to match a maximum number of blood-type incompatible pairs because blood-type incompatible pairs cannot match with each other in two-way cycles.

A cell in the left column linking a cell in the right column means a two-way cycle, for instance, $(O,A) - (A,O)$ and $(O,A) - (AB,O)$. In Figure 3.2(b) a cell in the left column linking a cell in the middle column linking a cell in the right column implies a two-way chain, for instance, $O^d - (O,A) - A^p$, $O^d - (O,B) - B^p$ and $O^d - (O,B) - AB^p$. Using this idea we propose a sequential matching procedure to find a maximal number of (feasible) transplants when at most two-way cycles or chains of exchange will be used. We call it a *sequential 2-way matching procedure*. In the following two-way, three-way, or four-way matching procedures, whenever cycles or chains of exchange are going to be made, priority is given to incompatible pairs.

A Sequential Two-Way Matching Procedure

Step 1: Make a maximum number of two-way cycles of exchange $(A,A)^i - (A,A)^i$. Then make a maximum number of two-way cycles of exchange $(A,A)^i - (A,A)^c$ if any. Carry out transplants for the remaining pairs $(A,A)^c$. Repeat the same process for each type (B,B), (O,O), (AB,AB), respectively.

Step 2: Make a maximum number of two-way cycles of exchange $(O,A) - (A,O)^i$, $(O,B) - (B,O)^i$, $(O,AB) - (AB,O)^i$, $(A,AB) - (AB,A)^i$, $(B,AB) - (AB,B)^i$, and $(A,B) - (B,A)$, respectively.

Step 3: Make a maximum number of two-way cycles or chains of exchange $(O,A) - (A,O)^c$, $(O,B) - (B,O)^c$, $(A,AB) - (AB,A)^c$, $(B,AB) - (AB,B)^c$, $A^d - (A,B) - AB^p/B^p$, $A^d - (A,AB) - AB^p$, and $B^d - (B,AB) - AB^p$, respectively. Match a maximum number of two-way cycles $(B,O)^c - (A,B)$, $(AB,A)^c - (A,B)$, $(B,O)^i - (A,B)$, $(AB,A)^i - (A,B)$ and two-way chain $A^d - (A,B) - Y^p$.

Step 4: Make a maximal number of two-way cycles of exchange

$$(AB,O)^c/(AB,O)^i - (O,A)/(O,B)/(O,AB)/(A,AB)/(B,AB)/(A,B),$$

respectively. And then match a maximum number of single donors O^d with the remaining pairs $(O,A)/(O,B)/(O,AB)/(A,AB)/(B,AB)/(A,B)$, respectively.

Step 5: Match a maximum number of the remaining single donors O^d, A^d, B^d, AB^d with any remaining single patients O^p, A^p, B^p, AB^p. Match a maximum number of two-way cycles of exchange $(A,O)^i - (A,O)^i$. Then make a maximum number of two-way cycles of exchange $(A,O)^i - (A,O)^c$ if any. Repeat the same process for each type $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$, $(AB,B)^i$. Match any remaining paired patients from compatible patient-donor pairs with their own paired donors.

The following example will be used to show how each matching procedure assigns compatible kidneys to patients and how efficiency will be improved as more ways of exchange are permitted.

Example 6 *There are 32 incompatible patient-donor pairs consisting of three incompatible pairs of type $(AB,AB)^i$, five pairs of type (O,A), one pairs of type (O,B), one pair of type (O,AB), two pairs of type (A,AB), seven pairs of type (B,AB), seven pairs of type (A,B), one incompatible pair of each type of $(A,O)^i$, $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$, $(AB,B)^i$ and (B,A); three compatible patient-donor pairs consisting of one compatible pair of each type $(AB,AB)^c$, $(AB,O)^c$ and $(A,O)^c$; and five single donors consisting of three single donors of type A^d, one single donor of type B^d and one single donor of type AB^d, and a large number of single patients.*

Table 3.1 The illustration of the sequential two-way matching procedure.

Steps	Number of Cycles or Chains	Cycles or Chains	Number of Remaining Pairs and Donors
Step 1	2	$(AB,AB)^i - (AB,AB)^i$ $(AB,AB)^i - (AB,AB)^c$	
Step 2	1	$(O,A) - (A,O)^i$	4 (O,A)
	1	$(O,B) - (B,O)^i$	
	1	$(O,AB) - (AB,O)^i$	
	1	$(A,AB) - (AB,A)^i$	(A,AB)
	1	$(B,AB) - (AB,B)^i$	6 (B,AB)
	1	$(A,B) - (B,A)$	6 (A,B)
Step 3	1	$(O,A) - (A,O)^c$	3 (O,A)
	1	$A^d - (A,AB) - AB^p$	2 A^d
	1	$B^d - (B,AB) - AB^p$	5 (B,AB)
	2	$A^d - (A,B) - B^p/AB^p$	4 (A,B)
Step 4	1	$(AB,O)^c - (B,AB)$	4 (B,AB)
Step 5 (End)	1	$AB^d - AB^p$	

Note that we can randomly pick kidney exchanges from cycles $(AB,O)^c - (O,A)/(O,B)/(B,AB)$ and chains $O^d - (O,A)/(O,B)/(B,AB) - Y^p$ in Step 4.

Observe that in the example there are in total 35 patient-donor pairs including 32 incompatible pairs and three compatible ones and many single patients. Table 3.1 shows that when the sequential two-way kidney exchange procedure is implemented, 24 paired patients and 5 single patients can receive kidney transplants and all three compatible pairs are involved in kidney exchange with incompatible pairs. Four pairs of type (B,AB), three pairs of type (O,A) and four pairs of type $(A,B)/(A,AB)$ stay put. In Table 3.1, Step 1 has two cycles, i.e., $(AB,AB)^i-(AB,AB)^i$ and $(AB,AB)^i-(AB,AB)^c$.

We have the following easy observation.

Lemma 8 *Assume that the kidney exchange model satisfies the Assumptions 1 and 3. Let μ be a 2-efficient matching. Then in μ every cycle contains at most two blood-type compatible pairs and every chain contains at most one blood-type compatible pair.*

Proof: It follows immediately from Figure 3.2 and the description of the above matching procedure. □

Proposition 3 *Assume that the kidney exchange model obeys the Assumptions 1, 2, and 3. Then the matching μ obtained from the above mechanism is 2-efficient and the maximum number of transplants through two-way exchanges is*

$$
\begin{aligned}
&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)+\#(B,A)\\
&+\#(A,A)+\#(B,B)+\#(O,O)+\#(AB,AB)\\
&+\#A^d+\#B^d+\#AB^d+\#O^d\\
&+\min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11},N_{12},N_{13},N_{14},N_{15},N_{16},N_{17}\}
\end{aligned}
$$

where

$$
\begin{aligned}
N_1 &= \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_2 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#(B,AB)\\
&\quad +\#(A,B)\\
N_3 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_4 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#B^d\\
&\quad +\#(AB,B)+\#(A,B)\\
N_5 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_6 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)\\
&\quad +\#(B,AB)\ +\#(A,B)
\end{aligned}
$$

$$
\begin{aligned}
N_7 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_8 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#B^d\\
&\quad +\#(AB,B)+\#(A,B)\\
N_9 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_{10} &= \#(O,A)+\#(B,O)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)\\
&\quad +\#(B,AB)+\#(B,A)\\
N_{11} &= \#(O,A)+\#(B,O)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)\\
&\quad +\#(B,AB)\\
N_{12} &= \#(O,A)+\#(B,O)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#B^d\\
&\quad +\#(AB,B)+\#(B,A)\\
N_{13} &= \#(O,A)+\#(B,O)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_{14} &= \#(A,O)+\#(B,O)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)\\
&\quad +\#(B,AB)+\#(B,A)\\
N_{15} &= \#(A,O)+\#(B,O)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_{16} &= \#(A,O)+\#(B,O)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)\\
&\quad +\#B^d+\#(AB,B)+\#(B,A)\\
N_{17} &= \#(A,O)+\#(B,O)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)
\end{aligned}
$$

Proof: Under Assumptions 1 to 3, all blood-type compatible but tissue-type incompatible pairs and pairs of type (B,A) can be matched through two-way cycles. All compatible pairs can be matched because even if paired patients from compatible pairs are not involved into two-way cycles, they can receive their own donors. All pairs of types (A,A), (B,B), (O,O), (AB,AB) can be also matched in two-way cycles. As long as a kidney can be allocated to a patient in waiting, we can always find a compatible patient in waiting because of the large population of patients in waiting. Hence, the maximal number of transplantations for patients in waiting, paired patients from blood-type compatible pairs and paired patients from pairs of type (B,A) is

$$
\begin{aligned}
&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)+\#(B,A)+\#(A,A)+\#(B,B)\\
&+\#(AB,AB)+\#(O,O)+\#A^d+\#B^d+\#AB^d+\#O^d
\end{aligned}
$$

Next, let N be the maximum number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B). The number of two-way cycles $(A,B)-(B,A)$ is bounded by $\#(B,A)$ by Assumption 2. The number of two-way cycles $(O,A)-(A,O)$ is bounded by $\min\{\#(O,A),\#(A,O)\}$. Similarly, the number of two-way cycles $(O,B)-(B,O)$ is bounded by $\min\{\#(O,B),\#(B,O)\}$; the number of two-way cycles and chains $(AB,A)-(A,AB)$, $A^d-(A,AB)-Y^p$ is bounded by $\min\{\#A^d+\#(AB,A),\#(A,AB)\}$; the number of two-way cycles and chains $(AB,A)-(A,B)$, $A^d-(A,B)-Y^p$, $(B,O)-(A,B)$ is bounded by $\min\{\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}+\#(B,O)-\min\{\#(O,B),\#(B,O)\},\#(A,B)-\#(B,A)\}$; the number of two-way cycles and chains $(AB,B)-(B,AB)$, $B^d-(B,AB)-AB^p$ is bounded by $\min\{\#B^d+\#(AB,B),\#(B,AB)\}$; and the number of two-way cycles and chains

$$(AB,O)-(O,A)/(O,B)/(O,AB)/(A,B)/(A,AB)/(B,AB), \text{ and,}$$

$$O^d-(O,A)/(O,B)/(O,AB)/(A,B)/(A,AB)/(B,AB)-Y^w$$

is bounded either by $\#O^d+\#(AB,O)$ or all blood-type incompatible paired patients are matched. Therefore, we have either

$$\begin{aligned} N \leq\ & \#(B,A)+\min\{\#(O,A),\#(A,O)\}+\min\{\#(O,B),\#(B,O)\}+ \\ & \min\{\#A^d+\#(AB,A),\#(A,AB)\}+ \\ & \min\{\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\} \\ & +\#(B,O)-\min\{\#(O,B),\#(B,O)\},\#(A,B)-\#(B,A)\}+ \\ & \min\{\#B^d+\#(AB,B),\#(B,AB)\}+ \\ & \min\{\#B^d+\#(AB,B),\#(B,AB)\}+\#O^d+\#(AB,O) \end{aligned}$$

or

$$N \leq \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB)+\#(A,B)+\#(B,AB).$$

The expressions can be rewritten as follows

$N \leq \min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11},N_{12},N_{13},N_{14},N_{15},N_{16},N_{17}\}$ and hence the maximum number of transplants can be reached is:

$N = \min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11},N_{12},N_{13},N_{14},N_{15},N_{16},N_{17}\}$.

We now prove that the sequential matching procedure achieves the maximum number of kidney transplants.

Since for every one/two-way chains, we can always find a compatible single patient, the number of transplantations for single patients equals $\#A^d + \#B^d + \#AB^d + \#O^d$.

By Assumption 3, all pairs of type $(A,A)^i$, $(B,B)^i$, $(O,O)^i$, $(AB,AB)^i$ can be matched through two-way cycles in Step 1. By Assumption 3, all remaining blood-type compatible but tissue-type incompatible pairs $(A,O)^i$, $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$, $(AB,B)^i$ can be matched through two-way cycles in Step 5. All compatible pairs $(A,O)^c$, $(B,O)^c$, $(AB,O)^c$, $(AB,A)^c$, $(AB,B)^c$, $(A,A)^c$, $(B,B)^c$, $(O,O)^c$, $(AB,AB)^c$ can be matched either through two-way cycles or doing transplantations with their own donors. Moreover, by Assumption 2, all pairs of type (B,A) can be matched through two-way cycle $(A,B)-(B,A)$ in Step 2 so that the remaining number of pairs of type (A,B) is $\#(A,B)-\#(B,A)$. Hence, the number of transplants for compatible pairs, blood-type compatible pairs and pairs of type (B,A) in the procedure is

$$\begin{aligned}&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)\\&+\#(B,A)+\#(A,A)+\#(B,B)+\#(AB,AB)+\#(O,O)\end{aligned}$$

Next, we prove that the maximum number of transplants for blood-type incompatible pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) can be achieved in the procedure.

Denote X_1 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 2 so that

$$X_1 = \#(B,A) + e_1 + e_2 + e_3 + e_4 + e_5$$

where

$$\begin{aligned}e_1 &= \min\{\#(O,A),\#(A,O)^i\}\\e_2 &= \min\{\#(O,B),\#(B,O)^i\}\\e_3 &= \min\{\#(O,AB),\#(AB,O)^i\}\\e_4 &= \min\{\#(A,AB),\#(AB,A)^i\}\\e_5 &= \min\{\#(B,AB),\#(AB,B)^i\}\end{aligned}$$

Denote X_2 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 3 so that

$$X_2 = \quad a_1 + a_2 + b_1 + b_2 + b_3$$

where

$$\begin{aligned}
a_1 = &\quad \min\{\#(O,A) - e_1, \#(A,O)^c\} \\
a_2 = &\quad \min\{\#(O,B) - e_2, \#(B,O)^c\} \\
b_1 = &\quad \min\{\#A^d + \#(AB,A)^c, \#(A,AB) - e_4\} \\
b_2 = &\quad \min\{\#B^d + \#(AB,B)^c, \#(B,AB) - e_5\} \\
b_3 = &\quad \min\{\#A^d + \#(AB,A)^c + \#(AB,A)^i - e_4 - b_1 + \#(B,O)^c \\
&\quad + \#(B,O)^i - e_2 - b_2, \#(A,B) - \#(B,A)\}
\end{aligned}$$

Denote X_3 as the number of blood-type incompatible paired patients from pairs of types (O,A),(O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 4 so that

$$\begin{aligned}
X_3 = &\quad \min\{\#O^d + \#(AB,O)^c + \#(AB,O)^i - e_3, \#(O,A) - e_1 - a_1 \\
&\quad + \#(O,B) - e_2 - a_2 + \#(O,AB) - e_3 + \#(A,AB) - e_4 - b_1 \\
&\quad + \#(B,AB) - e_5 - b_2 + \#(A,B) - \#(B,A) - b_3\}
\end{aligned}$$

Therefore, the total number of transplants for paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) in the procedure is $X = X_1 + X_2 + X_3$; one may refer to Tables from A1 to A15 in Supplement A of Cheng and Yang (2017b) for detail. Then the equation can be rewritten as follows:

$X = \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}\}$.

Therefore, the total number of transplants can be achieved in the mechanism is that

$$\begin{aligned}
&\#(A,O) + \#(B,O) + \#(AB,O) + \#(AB,A) + \#(AB,B) \\
&+ \#(B,A) + \#(A,A) + \#(B,B) + \#(AB,AB) + \#(O,O) \\
&+ \#A^d + \#B^d + \#AB^d + \#O^d \\
&+ \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}\}
\end{aligned}$$

We proved that every matching produced by the procedure achieves the maximum number of transplants in the pool and hence the procedure is 2-efficient. □

Now we compare the lower bound of the number in Proposition 3 with the case in which incompatible patient-donor pairs, compatible patient-donor pairs, and patients in waiting and single donors are treated separately under two-way exchange. We consider the most common situation that the number of blood-type incompatible pairs of each type: $\#(O,A)$, $\#(O,B)$, $\#(O,AB)$, $\#(A,AB)$, and $\#(B,AB)$, is at least as large as the number of its opposite blood-type compatible but tissue-type incompatible pairs: $\#(A,O)^i$, $\#(B,O)^i$, $\#(AB,O)^i$, $\#(AB,A)^i$, and $\#(AB,B)^i$ respectively. We can do similar comparison for other situations. Hence, the maximum number of feasible transplants for the group of incompatible patient-donor pairs under two-way cycles is

$$
\begin{aligned}
&2(\#(A,O)^i + \#(B,O)^i + \#(AB,O)^i + \#(AB,A)^i + \#(AB,B)^i) \\
&+2\#(B,A) + 2(\lfloor \tfrac{\#(A,A)^i}{2} \rfloor + \lfloor \tfrac{\#(B,B)^i}{2} \rfloor + \lfloor \tfrac{\#(AB,AB)^i}{2} \rfloor + \lfloor \tfrac{\#(O,O)^i}{2} \rfloor)
\end{aligned}
$$

The maximum number of transplants for patients on the waiting list under one/two-way chains equals $(\#A^d + \#B^d + \#AB^d + \#O^d)$ because the number of patients on the waiting list exceeds the number of single donors so that a single donor can always find a compatible patient on the waiting list to donate. The maximum number of transplants for the group of compatible patient-donor pairs equals $\#(A,O)^c + \#(B,O)^c + \#(AB,O)^c + \#(AB,A)^c + \#(AB,B)^c + \#(A,A)^c + \#(B,B)^c + \#(O,O)^c + \#(AB,AB)^c$ because every patient in a compatible pair can receive the kidney from its own paired donor.

Since for any blood-type compatible pair of type (X,Y), we have $\#(X,Y) = \#(X,Y)^i + \#(X,Y)^c$, the maximum number of transplants in the whole pool becomes

$$
\begin{aligned}
&\#(A,O) + \#(B,O) + \#(AB,O) + \#(AB,A) + \#(AB,B) \\
&+\#(A,O)^i + \#(B,O)^i + \#(AB,O)^i + \#(AB,A)^i + \#(AB,B)^i \\
&+2\#(B,A) + 2(\lfloor \tfrac{\#(A,A)^i}{2} \rfloor + \lfloor \tfrac{\#(B,B)^i}{2} \rfloor + \lfloor \tfrac{\#(AB,AB)^i}{2} \rfloor + \lfloor \tfrac{\#(O,O)^i}{2} \rfloor) \\
&+\#(A,A)^c + \#(B,B)^c + \#(AB,AB)^c + \#(O,O)^c \\
&+\#A^d + \#B^d + \#AB^d + \#O^d
\end{aligned}
$$

We compare the above number with the lower bound of the number in Proposition 3 and obtain

$$\begin{aligned}
&\min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11},N_{12},N_{13},N_{14},N_{15},N_{16},N_{17}\}\\
&-(\#(A,O)^i+\#(B,O)^i+\#(AB,O)^i+\#(AB,A)^i+\#(AB,B)^i+\#(B,A))\\
&+\#(A,A)+\#(B,B)+\#(AB,AB)+\#(O,O)\\
&-(\#(A,A)^c+\#(B,B)^c+\#(AB,AB)^c+\#(O,O)^c)\\
&-2(\lfloor\tfrac{\#(A,A)^i}{2}\rfloor+\lfloor\tfrac{\#(B,B)^i}{2}\rfloor+\lfloor\tfrac{\#(AB,AB)^i}{2}\rfloor+\lfloor\tfrac{\#(O,O)^i}{2}\rfloor)\\
&\geq 0
\end{aligned}$$

This shows the benefits of allowing compatible patient-donor pairs to join incompatible pairs for exchange and adding two-way chain exchange.

3.3.2 Three-Way Exchange

To improve potential gains of exchange, three-way cycles and three-way chains of exchange can be explored.

Figures 3.3 and 3.4 show all possible three-way cycles and chains under Assumptions 1, 2 and 3. Note that these figures do not include two-way exchanges. Recall that blood-compatible pairs can always be matched by Assumption 3. To have more transplants we can make the best use of every blood-compatible pair to match with a blood-incompatible pair. As a result, three-way cycles can be formed.

We first consider some beneficial three-way cycles or chains with two blood-incompatible pairs. Under three-way exchanges, blood-compatible pair (AB,O) (the right column) can involve not one but two blood-type incompatible pairs through 4 three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$, $(AB,O)-(A,B)-(B,AB)$ and $(AB,O)-(O,B)-(B,AB)$. For blood-compatible pair (B,O), we have just one three-way cycle $(B,O)-(O,A)-(A,B)$. For blood-compatible pair (AB,A), we have also just one three-way cycle $(AB,A)-(A,B)-(B,AB)$. Similarly, we can use single donors to match with two blood-incompatible pairs and patients on the waiting list. Consequently, three-way chains can be generated. With one-way and two-way chains, each single donor can trade with at most one blood-incompatible pair. If three-way chains are allowed, single donor O^d can trade with two blood-incompatible pairs through three-way chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-B^p$, $O^d-(A,B)-(B,AB)-AB^p$ and $O^d-(O,B)-(B,AB)-AB^p$. Moreover, if there is any (A,B) left, type (A,B) can bring an extra blood-incompatible pair into chains through three-way chains $A^d-(A,B)-(B,AB)-AB^p$.

We now consider some beneficial three-way cycles or chains with one pair (B,A) or with one blood-incompatible pair. Observe that (B,A) pairs are on the short side by Assumption 2. These pairs can be very beneficial in the following situations: Firstly, there are pairs or singles, (A,O), (O,B), $A^d/(AB,A)$, and (B,AB). In this case, we cannot match blood-incompatible pairs (O,B) and (B,AB) in a two-way cycle. But if we break two-way cycle $(A,B)-(B,A)$, we can make three-way cycles $(A,O)-(O,B)-(B,A)$ and $(AB,A)-(A,B)-(B,AB)$ and thus increase the number of transplants. Also three-way cycles $(A,O)-(O,B)-(B,A)$ and chains $A^d-(A,B)-(B,AB)-AB^p$ can yield more transplants. Secondly, there are pairs or singles, (A,AB), (B,O), $B^d/(AB,B)$, and (O,A). In this case, we cannot match blood-incompatible pairs (O,A) and (A,AB) in a two-way cycle, but we can make three-way cycles $(B,O)-(O,A)-(A,B)$ and $(AB,B)-(B,A)-(A,AB)$ and increase the number of transplants. Also three-way cycles $(B,O)-(O,A)-(A,B)$ and three-way chains $B^d-(B,A)-(A,AB)-AB^p$ can bring more transplants.

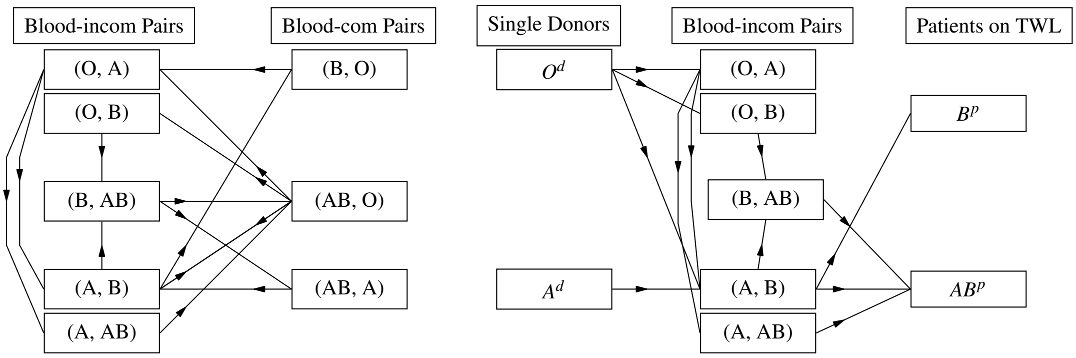
Furthermore, it is easy to see that $(AB,A)-(A,O)$, or $(AB,B)-(B,O)$ can make a three-way cycle of exchange with any pair (X,Y), and that $A^d-(A,O)$ or $B^d-(B,O)$ can yield a three-way chain with any pair (X,Y). In particular, when there are pairs (B,AB), (O,B) and (O,AB), it is impossible to use them in two-way exchange but it is easy to combine them with $(AB,A)-(A,O)$ to yield three-way exchange $(AB,A)-(A,O)-(B,AB)/(O,B)/(O,AB)$. Similarly, we can make three-way exchanges $A^d-(A,O)-(B,AB)/(O,B)/(O,AB)-Y^p$, $(AB,B)-(B,O)-(A,AB)/(O,A)/(O,AB)$ and $B^d-(B,O)-(A,AB)/(O,A)/(O,AB)-Y^p$.

An efficient sequential three-way matching procedure is introduced below.

A Sequential Three-Way Matching Procedure

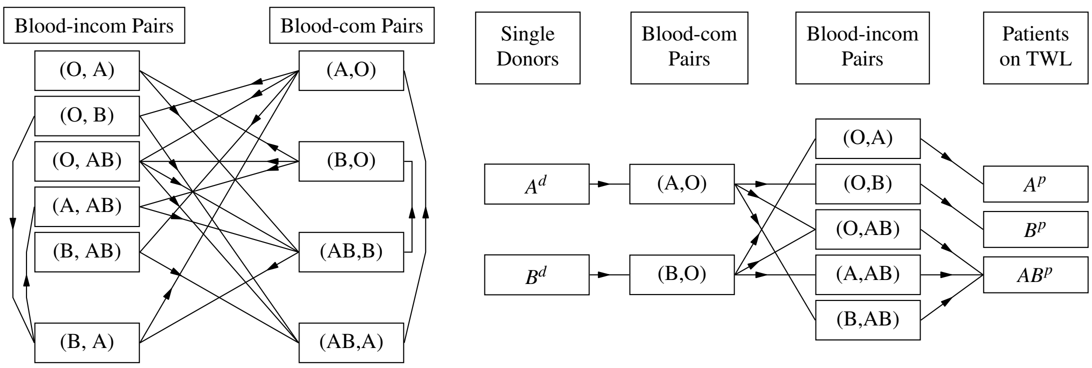
Step 1: Match a maximum number of pairs (A,A), (B,B), (O,O), (AB,AB) through two-way.

Fig. 3.3 Three-way cycles (a) and chains (b) of exchange with two blood-incompatible pairs.



(a) (b)

Fig. 3.4 Three-way cycles (a) and chains (b) of exchange with (B,A) and three-way cycles of exchange with one blood-incompatible pair.



(a) (b)

Step 2: (Take full advantage of three-way cycles and chains starting with A^d, (AB,A) and (B,O)) The number of pairs (A,B) should not exceed $\#(A,B) - \#(B,A)$ in this step.

- Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, where the available number of pairs (AB,A) and single donors A^d in this step is $\#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$.
- Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, where the available number of pairs (O,A) in this step is $\#(O,A) - \min\{\#(O,A), \#(A,O)\}$.
- Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$.
- Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$.

Step 3: (Take full advantage of pairs (B,A)) Denote $\#(X,Y)^r$ as the number of all currently available pairs of any type (O,A), (O,B), (A,AB), (B,AB), (A,O), (B,O), (AB,A) and (AB,B) and $\#X^{dr}$ as the number of currently available single donors of any type A^d and B^d. Match the following three-way cycles and chains.

Step 3.1: Make a maximum number of three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(A,O)-(O,B)-(B,A)$ should equal the total number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs $(O,B)/(A,O)$ used in this step should not exceed $\#(X,Y)^r - \min\{\#(X,Y)^r, \#(Y,X)^r\}$, the number of pairs (AB,A) and single donors A^d used in this step should not exceed $\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r - \min\{B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$.

Step 3.2: Make a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(B,O)-(O,A)-(A,B)$ should equal the total number of three-way cycles $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs $(B,O)/(O,A)$ used in this step should not exceed $\#(X,Y)^r-\min\{\#(X,Y)^r,\#(Y,X)^r\}$, the number of pairs (AB,B) and single donors B^d used in this step should not exceed $\#B^{dr}+\#(AB,B)^r-\min\{\#B^{dr}+\#(AB,B)^r,\#(B,AB)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r-\min\{\#A^{dr}+\#(AB,A)^r,\#(A,AB)^r\}$.

Step 3.3: If there is at least one pair of each type (A,O), (O,B) and (B,AB) which are left from the previous Step 3.1. Then, make a maximum number of three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(A,O)-(O,B)-(B,A)$ should equal the total number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs (O,B) used in this step should not exceed $\#(O,B)^r-\min\{\#(O,B)^r,\#(B,O)^r\}$, the number of pairs (A,O) used in this step should not exceed $\min\{\#(A,O)^r,\#(O,A)^r\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r-\min\{B^{dr}+\#(AB,B)^r,\#(B,AB)^r\}$.

Step 3.4: If there is at least one pair of each type (O,A), (A,AB) and $B^d/(AB,B)$ which are left from the previous Step 3.2. Then, make a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(B,O)-(O,A)-(A,B)$ should equal the total number of three-way cycles $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs (O,A) used in this step should not exceed $\#(O,A)^r-\min\{\#(O,A)^r,\#(A,O)^r\}$, the number of pairs (AB,B) and single donors B^d

used in this step should not exceed $\#B^{dr} + \#(AB,B)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$.

Step 3.5: If there exists at least one pair of each type (B,AB), (O,B) and $A^d/(AB,A)$ which are left from the previous Step 3.1. Then, make a maximum number of three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(A,O)-(O,B)-(B,A)$ should equal the total number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs (O,B) used in this step should not exceed $\#(O,B)^r - \min\{\#(O,B)^r, \#(B,O)^r\}$, the number of pairs (AB,A) and single donors A^d used in this step should not exceed $\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r - \min\{B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$.

Step 3.6: If there exists at least one pair of each type (B,O), (O,A) and (A,AB) which are left form the previous Step 3.2. Then, make a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, subject to the following constraints: the number of three-way cycles $(B,O)-(O,A)-(A,B)$ should equal the total number of three-way cycles $(AB,B)-(B,A)-(A,AB)$ and chains $B^d-(B,A)-(A,AB)-AB^p$, and the number of pairs (A,B) used in this step should not exceed the number of currently available pairs (B,A), the number of pairs $(B,O)/(O,A)$ used in this step should not exceed $\#(X,Y)^r - \min\{\#(X,Y)^r, \#(Y,X)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$.

Step 4: Match the following two-way cycles and two-way chains:

- Match a maximum number of the remaining pairs (A,O) with pairs (O,A). Match a maximum number of the remaining pairs (B,O) with pairs (O,B).

Match a maximum number of the remaining pairs (A,B) with pairs (B,A). Match a maximum number of the remaining pairs (AB,A) and single donors A^d with pairs (A,AB), and match a maximum number of the remaining pairs (AB,B) and single donors B^d with pairs (B,AB).

- Match a maximum number of the remaining pairs (AB,A), (B,O) and single donors A^d with the remaining pairs (A,B), where the available number of pairs (B,O) in this step is $\#(B,O)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,O)^r\}$ and the available number of pairs (AB,A) and single donors A^d is $\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,O)^r\}$.

Step 5: Match a maximum number of the following three-way cycles and chains:

- Three-way cycles $(AB,O)-(O,A)-(A,AB)$ and chains $O^d-(O,A)-(A,AB)-AB^p$.
- Three-way cycles $(AB,O)-(O,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$.
- Three-way cycles $(AB,O)-(O,A)-(A,B)$ and chains $O^d-(O,A)-(A,B)-Y^p$.
- Three-way cycles $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(A,B)-(B,AB)-AB^p$.

Step 6: Match a maximum number of the remaining single donors O^d and pairs (AB,O) with the remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB) and (A,B). Match a maximum number of the combinations of $(AB,A)-(A,O)$ and $(AB,B)-(B,O)$ with remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB) and (A,B). Match a maximum number of the combinations of $A^d-(A,O)$ and $B^d-(B,O)$ with remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB) and (A,B) and patients on TWL.

Step 7: Match a maximum number of remaining blood-compatible but tissue-incompatible pair $(A,O)^i$ through two-way cycles $(A,O)^i-(A,O)^i$. If there is one remaining pair $(A,O)^i$, match the pair $(A,O)^i$ with $(A,O)^c$. Apply the same procedure to any remaining pair $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$ and $(AB,B)^i$. Match a maximum number of remaining single donors O^d, A^d, B^d, AB^d with any remaining single patients O^p, A^p, B^p, AB^p; match any paired patients from compatible pairs with their own paired donors.

We use Example 6 to demonstrate the sequential three-way matching procedure and compare it with the previous procedure.

Table 3.2 The illustration of the sequential three-way matching procedure.

Steps	Number of Cycles or Chains	Cycles or Chains	Number of Remaining Pairs and Donors
Step 1	1	$(AB,AB)^i-(AB,AB)^i$	
	1	$(AB,AB)^i-(AB,AB)^c$	
Step 2	1	$(AB,A)^i-(A,B)-(B,AB)$	6 (B,AB), 6 (A,B)
	1	$A^d-(A,B)-(B,AB)-AB^p$	2 A^d, 5 (B,AB), 5 (A,B)
	1	$(B,O)^i-(O,A)-(A,B)$	4 (O,A), 4 (A,B)
	2	$A^d-(A,B)-(B,AB)-AB^p$	3 (B,AB), 2 (A,B)
Step 4	1	$(O,A)-(A,O)^i$	3 (O,A)
	1	$(O,A)-(A,O)^c$	2 (O,A)
	1	$(A,B)-(B,A)$	(A,B)
	1	$(B,AB)-(AB,B)^i$	2 (B,AB)
	1	$B^d-(B,AB)-AB^p$	(B,AB)
Step 5	1	$(AB,O)^i-(O,A)-(A,AB)$	(O,A), (A,AB)
	1	$(AB,O)^c-(O,A)-(A,AB)$	
Step 6 (End)	1	AB^d-AB^p	

Table 3.2 shows that if we use the sequential three-way matching procedure, 31 paired patients and five single patients will receive kidney transplants and four pairs of type (B,AB), (A,B), (O,B) and (O,AB) stay put. Compared with the previous two-way matching procedures, the three-way procedure increases the maximum number of kidney transplants by seven.

Lemma 9 *Assume that the kidney exchange model satisfies the Assumptions 1 and 3. Then every* 3*-efficient matching* μ *can be transformed to another* 3*-efficient matching in which every cycle contains at most two blood-type compatible pairs and every chain contains at most one blood-type compatible pair.*

Proof: Consider any given 3-efficient matching μ as stated in the lemma. If μ consists only of cycles with no more than two blood-type compatible pairs and chains with no more than one blood-type compatible pair, we are done. Suppose to the contrary that μ contains a cycle with more than two blood-type compatible pairs or a chain with more than one blood-type compatible pair. We only need to consider the case of three-way cycles or chains. We will show that a three-way cycle with three blood-type compatible pairs can be decomposed into three single blood-compatible pairs and a three-way chain with two blood-compatible pairs can be decomposed into two single blood-compatible pairs and a one-way chain in which the single donor donates its kidney to a patient on the waiting list. Then, we will show that the all pairs which are decomposed from cycles and chains can be matched.

Because a blood-type compatible and tissue-type compatible pair can directly do transplant, all blood-type compatible and tissue-type compatible pairs can do transplants separately. Let $\mathscr{D}$ be the set of all blood-type compatible but tissue-type incompatible pairs in a three-way cycle or chain under consideration. Let $(X,Y)^i$ present the type of a blood-type compatible but tissue-type incompatible pair. If there exists two or more pairs of type $(X,Y)^i$, we can have a two-way cycle among them $(X,Y)^i-(X,Y)^i$. Therefore, at most one pair of type $(X,Y)^i$ left after the process. By Assumption 3, there exists at least one blood-type and tissue-type compatible pair of type $(X,Y)^c$. If the compatible pair $(X,Y)^c$ does not involve in any cycle or chain, then we can match the remaining pair $(X,Y)^i$ with pair $(X,Y)^c$. Otherwise, the compatible pair $(X,Y)^c$ involves in a cycle consisting of no more than two blood-type compatible pairs or a chain consisting of no more than one blood-type compatible pair. Then we can use pair $(X,Y)^i$ instead of $(X,Y)^c$ based on Assumption 1 and pair $(X,Y)^c$ do transplant directly. Therefore, all remaining pairs of type $(X,Y)^i$ can be matched.

□

Proposition 4 *Assume that the kidney exchange model satisfies the Assumptions 1, 2, and 3. Then the matching μ generated by the above procedure is* 3*-efficient and the* *maximum number of transplants through at most three-way exchanges is*

$$\begin{aligned}
&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)+\#(B,A)\\
&+\#(A,A)+\#(B,B)+\#(O,O)+\#(AB,AB)\\
&+\#A^d+\#B^d+\#AB^d+\#O^d\\
&+\min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11},N_{12},N_{13},N_{14},N_{15},N_{16},N_{17}\}
\end{aligned}$$

where

$$\begin{aligned}
N_1 &= \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_2 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
&\quad +\#B^d+\#(AB,B)+\#(A,O)\\
N_3 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_4 &= \#(A,O)+\#(O,B)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+2\#(A,B)\\
&\quad +2\#B^d+2\#(AB,B)-\#(B,A)\\
N_5 &= \#(A,O)+\#(O,B)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+2\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_6 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)+\#(A,AB)+\#(B,O)\\
N_7 &= \#(A,O)+\#(O,B)+\#O^d+\#(AB,O)+\#(A,AB)+2\#(A,B)\\
&\quad +\#B^d+\#(AB,B)-\#(B,A)\\
N_8 &= \#(O,A)+\#(B,O)+2\#O^d+2\#(AB,O)+2\#A^d+2\#(AB,A)\\
&\quad +\#B^d+\#(AB,B)+\#(B,A)\\
N_9 &= \#(O,A)+\#(B,O)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)+\#(B,A)\\
N_{10} &= \#(O,A)+\#(B,O)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#(B,AB)\\
&\quad +\#(B,A)\\
N_{11} &= \#(A,O)+2\#(B,O)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+\#(B,AB)\\
&\quad +\#(B,A)\\
N_{12} &= \#(A,O)+2\#(B,O)+2\#O^d+2\#(AB,O)+2\#A^d+2\#(AB,A)+\#(AB,B)\\
&\quad +\#B^d+\#(B,A)
\end{aligned}$$

$$
\begin{aligned}
N_{13} &= \#(A,O)+\#(B,O)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#(B,AB)+\#(B,A)\\
N_{14} &= 2\#(A,O)+\#(B,O)+2\#O^d+2\#(AB,O)+\#(A,AB)+2\#(A,B)\\
&\quad +\#(AB,B)+\#B^d-\#(B,A)\\
N_{15} &= \#(A,O)+\#(B,O)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_{16} &= \#(A,O)+\#(B,O)+2\#O^d+2\#(AB,O)+\#(A,AB)+2\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_{17} &= \#(A,O)+\#(B,O)+\#O^d+\#(AB,O)+\#(A,AB)+\#(A,B)+\#(B,AB)
\end{aligned}
$$

The proof is deferred to the appendix.

3.3.3 Four-Way Exchange

If four-way cycles and chains of exchange can be used, more kidney transplants will be made possible. Figures 3.5 and 3.6 show all four-way cycles and chains of exchange but do not include two- or three-way exchange.

In this case we have a four-way cycle with three blood-incompatible pairs $(AB,O)-(O,A)-(A,B)-(B,AB)$, a four-way chain with three blood-incompatible pairs $O^d-(O,A)-(A,B)-(B,AB)-AB^p$, two four-way cycles with two blood-compatible pairs $(AB,A)-(A,O)-(O,B)-(B,AB)$ and $(AB,B)-(B,O)-(O,A)-(A,AB)$, two four-way chains with one blood-compatible pair $A^d-(A,O)-(O,B)-(B,AB)$ and $B^d-(B,O)-(O,A)-(A,AB)$, one four-way cycle $(AB,A)-(A,B)-(B,O)-(X,Y)$ and one four-way chain $A^d-(A,B)-(B,O)-(X,Y)-Z^p$, where (X,Y) is any pair and Z^p is any single patient.

An efficient sequential matching procedure under four-way exchange is proposed and described as follows.

A Sequential Four-Way Matching Procedure

Step 1: Match a maximum number of pairs (A,A), (B,B), (O,O), (AB,AB) through two-way exchange.

Fig. 3.5 Four-way cycles (a) and chains (b) of exchange with three blood-incompatible pairs.

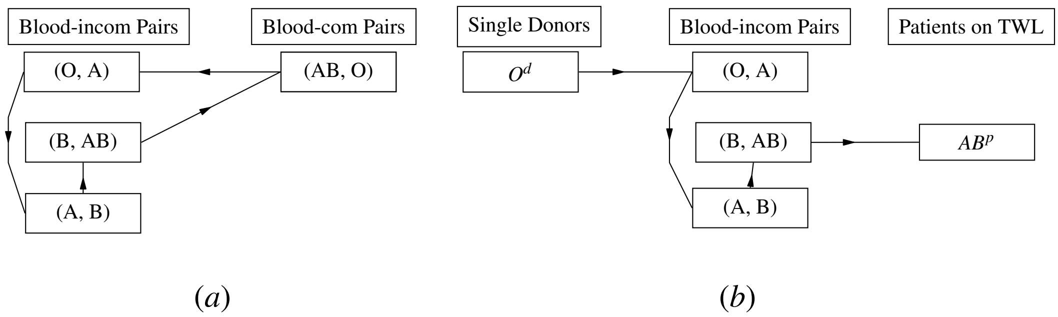
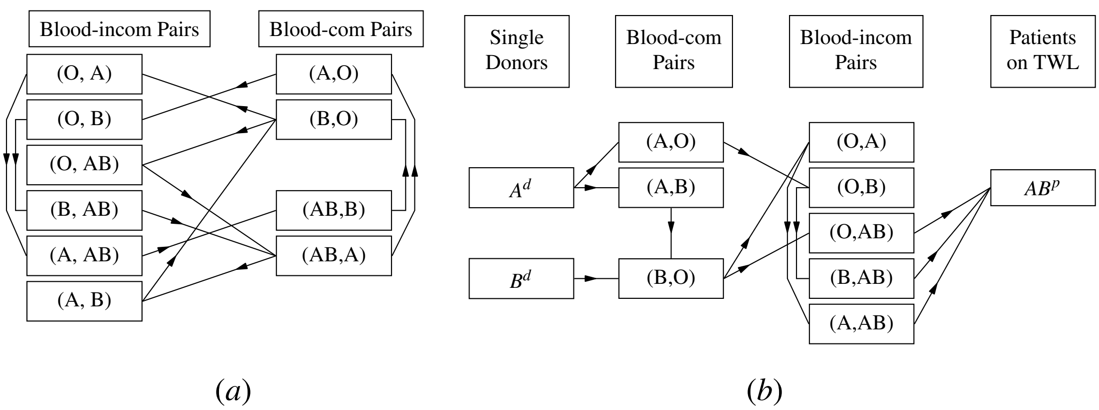


Fig. 3.6 Four-way cycles (a) and chains (b) of exchange with two blood-incompatible pairs.



Step 2: (Take full advantage of three-way cycles and chains starting with A^d, (AB,A) and (B,O)) The number of pairs (A,B) should not exceed $\#(A,B)-\#(B,A)$ in this step.

- Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, where the available number of pair (AB,A) and single donors A^d in this step is $\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$.
- Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, where the available number of pairs (O,A) in this step is $\#(O,A)-\min\{\#(O,A),\#(A,O)\}$.
- Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$.
- Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$.

Step 3: (Take full advantage of four-way cycles and chains with the combinations) Denote $\#(X,Y)^r$ as the number of all currently available pairs of any type (O,A), (O,B), (A,AB), (B,AB), (A,O), (B,O), (AB,A), (AB,B) and (A,B). Denote $\#Y^{dr}$ as the number of all currently available single donors of any type A^d and B^d. Match the following four-way cycles and chains.

Step 3.1: Make a maximum number of four-way cycles $(AB,A)-(A,O)-(O,B)-(B,AB)$ and four-way chains $A^d-(A,O)-(O,B)-(B,AB)-AB^p$, subject to the following constraints: the number of pairs $(O,B)/(A,O)$ used in this step should not exceed $\#(X,Y)^r-\min\{\#(X,Y)^r,\#(Y,X)^r\}$, the number of pairs (AB,A) and single donors A^d used in this step should not exceed $\#A^{dr}+\#(AB,A)^r-\min\{\#A^{dr}+\#(AB,A)^r,\#(A,AB)^r\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r-\min\{B^{dr}+\#(AB,B)^r,\#(B,AB)^r\}$.

Step 3.2: Make a maximum number of four-way cycles $(AB,B)-(B,O)-(O,A)-(A,AB)$ and four-way chains $B^d-(B,O)-(O,A)-(A,AB)-AB^p$, subject to the following constraints: the number of pairs $(B,O)/(O,A)$ used in this step should not exceed $\#(X,Y)^r - \min\{\#(X,Y)^r, \#(Y,X)^r\}$, the number of pairs (AB,B) and single donors B^d used in this step should not exceed $\#B^{dr} + \#(AB,B)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$.

Step 3.3: If there exists at least one pair of each type (A,O), (O,B) and (B,AB) which are left from the previous Step 3.1. Then, make a maximum number of four-way cycles $(AB,A)-(A,O)-(O,B)-(B,AB)$ and four-way chains $A^d-(A,O)-(O,B)-(B,AB)-AB^p$, subject to the following constraints: the number of pairs $(O,B)/(A,O)$ used in this step should not exceed $\#(X,Y)^r - \min\{\#(X,Y)^r, \#(Y,X)^r\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r - \min\{B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$.

Step 3.4: If there exists at least one pair of each type (O,A), (A,AB) and $B^d/(AB,B)$ which are left from the previous Step 3.2. Then, make a maximum number of four-way cycles $(AB,B)-(B,O)-(O,A)-(A,AB)$ and four-way chains $B^d-(B,O)-(O,A)-(A,AB)-AB^p$, subject to the following constraints: the number of pairs (O,A) used in this step should not exceed $\#(O,A)^r - \min\{\#(O,A)^r, \#(A,O)^r\}$, the number of pairs (AB,B) and single donors B^d used in this step should not exceed $\#B^{dr} + \#(AB,B)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$.

Step 3.5: If there exists at least one remaining pair of each type (B,AB), (O,B) and $A^d/(AB,A)$ which are left from the previous Step 3.1. Then, make a maximum number of four-way cycles $(AB,A)-(A,O)-(O,B)-(B,AB)$ and four-way chains $A^d-(A,O)-(O,B)-(B,AB)-AB^p$, subject to the following constraints: the number of pairs (O,B) used in this step should not exceed $\#(O,B)^r - \min\{\#(O,B)^r, \#(B,O)^r\}$, the number of pairs (AB,A) and single donors A^d used in this step should not exceed $\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)\}$, and the number of pairs (B,AB) used in this step should not exceed $\#(B,AB)^r - \min\{B^{dr} + \#(AB,B)^r, \#(B,AB)^r\}$.

Step 3.6: If there exists at least one pair of each type (B,O), (O,A) and (A,AB) which are left from the previous Step 3.2. Then, make a maximum number of four-way cycles $(AB,B)-(B,O)-(O,A)-(A,AB)$ and four-way chains $B^d-(B,O)-(O,A)-(A,AB)-AB^p$, subject to the following constraints: the number of pairs $(B,O)/(O,A)$ used in this step should not exceed $\#(X,Y)^r - \min\{\#(X,Y)^r, \#(Y,X)^r\}$, and the number of pairs (A,AB) used in this step should not exceed $\#(A,AB)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,AB)^r\}$.

Step 4: Do the following two-way cycles and two-way chains:

- Match a maximum number of the remaining pairs (A,O) with pairs (O,A). Match a maximum number of the remaining pairs (B,O) with pairs (O,B). Match a maximum number of the remaining pairs (A,B) with pairs (B,A). Match a maximum number of the remaining pairs (AB,A) and single donors A^d with pairs (A,AB), and match a maximum number of the remaining pairs (AB,B) and single donors B^d with pairs (B,AB).
- Match a maximum number of the remaining pairs (AB,A), (B,O) and single donors A^d with the remaining pairs (A,B), where the available number of pairs (B,O) in this step is

$$\begin{aligned}&\#(B,O)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,O)^r\}\\ &- \min\{\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,O)^r\},\\ &\#(B,O)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,O)^r\}, \#(A,B)^r\}\end{aligned}$$

  and the available number of pairs (AB,A) and single donors A^d is

$$\begin{aligned}&\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,O)^r\}\\ &- \min\{\#A^{dr} + \#(AB,A)^r - \min\{\#A^{dr} + \#(AB,A)^r, \#(A,O)^r\},\\ &\#(B,O)^r - \min\{\#B^{dr} + \#(AB,B)^r, \#(B,O)^r\}, \#(A,B)^r\}\end{aligned}$$

Step 5: Match a maximum number of the following cycles and chains:

- Four-way cycles $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chains $O^d-(O,A)-(A,B)-(B,AB)$.

- Three-way cycles $(AB,O)-(O,A)-(A,AB)$ and chains $O^d-(O,A)-(A,AB)-AB^p$.
- Three-way cycles $(AB,O)-(O,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$.
- Three-way cycles $(AB,O)-(O,A)-(A,B)$ and chains $O^d-(O,A)-(A,B)-Y^p$.
- Three-way cycles $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(A,B)-(B,AB)-AB^p$.

Step 6: Match a maximum number of the remaining single donors O^d and pairs (AB,O) with the remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB) and (A,B). Match a maximum number of the combinations of $(AB,A)-(A,O)$, $(AB,B)-(B,O)$ and $(AB,A)-(A,B)-(B,O)$ with remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB) and (A,B). Match a maximum number of the combinations of $A^d-(A,O)$, $B^d-(B,O)$ and $A^d-(A,B)-(B,O)$ with remaining pairs (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) and patients on TWL.

Step 7: Match a maximum number of remaining blood-compatible but tissue-incompatible pairs $(A,O)^i$ through two-way cycles $(A,O)^i-(A,O)^i$. If there is one remaining pair $(A,O)^i$, match the pair $(A,O)^i$ with $(A,O)^c$. Apply the same procedure to any remaining pair $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$ and $(AB,B)^i$. Match a maximum number of remaining single donors O^d, A^d, B^d, AB^d with any remaining single patients O^p, A^p, B^p, AB^p; match any paired patients from compatible pairs with their own paired donors.

Example 6 will be used to show the performance of the four-way matching procedure. The process and outcome generated by the procedure is shown in Table 3.3. One can see that if the sequential four-way matching procedure is used, 32 paired patients and five single patients will receive kidney transplants and three pairs of type (A,AB), (O,B) and (O,AB) will be left. In comparison with the three-way procedure, the four-way procedure increases the maximum number of kidney transplants by one.

Lemma 10 *Assume that the kidney exchange model satisfies the Assumptions 1 and 3. Then every* 4*-efficient matching* μ *can be transformed to another* 4*-efficient matching* *in which every cycle contains at most two blood-type compatible pairs and every chain contains at most one blood-type compatible pair.*

Its proof is given in the appendix.

Table 3.3 The illustration of the sequential four-way matching procedure

Steps	Number of Cycles or Chains	Cycles or Chains	Number of Remaining Pairs and Donors
Step 1	1	$(AB,AB)^i-(AB,AB)^i$	
	1	$(AB,AB)^i-(AB,AB)^c$	
Step 2	1	$(AB,A)^i-(A,B)-(B,AB)$	6 (B,AB), 6 (A,B)
	1	$A^d-(A,B)-(B,AB)-AB^p$	2 A^d, 5 (B,AB), 5 (A,B)
	1	$(B,O)^i-(O,A)-(A,B)$	4 (O,A), 4 (A,B)
	2	$A^d-(A,B)-(B,AB)-AB^p$	3 (B,AB), 2 (A,B)
Step 4	1	$(O,A)-(A,O)^i$	3 (O,A)
	1	$(O,A)-(A,O)^c$	2 (O,A)
	1	$(A,B)-(B,A)$	(A,B)
	1	$(B,AB)-(AB,B)^i$	2 (B,AB)
	1	$B^d-(B,AB)-AB^p$	(B,AB)
Step 5	1	$(AB,O)^i-(O,A)-(A,B)-(B,AB)$	(O,A)
	1	$(AB,O)^c-(O,A)-(A,AB)$	(A,AB)
Step 6 (End)	1	AB^d-AB^p	

Proposition 5 *Assume that the kidney exchange model obeys the Assumptions 1, 2, and 3. Then the matching μ from the above procedure is 4-efficient and the maximum number of transplants through four-way exchanges equals*

$$\begin{aligned}
&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)\\
&+\#(B,A)+\#(A,A)+\#(B,B)+\#(O,O)+\#(AB,AB)\\
&+\#A^d+\#B^d+\#AB^d+\#O^d\\
&+\min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11}\}
\end{aligned}$$

where

$$\begin{aligned}
N_1 &= \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB)+\#(A,B)+\#(B,AB)\\
N_2 &= \#(O,A)+\#(O,B)+\#O^d+\#(AB,O)+\#A^d+\#(AB,A)+\#(A,B)\\
&\quad +\#B^d+\#(AB,B)\\
N_3 &= \#(A,O)+\#(O,B)+2\#O^d+2\#(AB,O)+\#A^d+\#(AB,A)+2\#(A,B)\\
&\quad +2\#B^d+2\#(AB,B)-\#(B,A)
\end{aligned}$$

$$
\begin{aligned}
N_4 \quad &= \#(A,O) + \#(O,B) + \#O^d + \#(AB,O) + \#(A,AB) + 2\#(A,B) \\
&\quad + \#B^d + \#(AB,B) - \#(B,A) \\
N_5 \quad &= \#(O,A) + \#(B,O) + 2\#O^d + 2\#(AB,O) + 2\#A^d + 2\#(AB,A) \\
&\quad + \#B^d + \#(AB,B) + \#(B,A) \\
N_6 \quad &= \#(O,A) + \#(B,O) + \#O^d + \#(AB,O) + \#A^d + \#(AB,A) + \#(B,AB) \\
&\quad + \#(B,A) \\
N_7 \quad &= \#(A,O) + 2\#(B,O) + 3\#O^d + 3\#(AB,O) + 2\#A^d + 2\#(AB,A) \\
&\quad + \#(AB,B) + \#B^d + \#(B,A) \\
N_8 \quad &= \#(A,O) + 2\#(B,O) + 2\#O^d + 2\#(AB,O) + \#A^d + \#(AB,A) + \#(B,AB) \\
&\quad + \#(B,A) \\
N_9 \quad &= 2\#(A,O) + \#(B,O) + 2\#O^d + 2\#(AB,O) + \#(A,AB) + 2\#(A,B) \\
&\quad + \#(AB,B) + \#B^d - \#(B,A) \\
N_{10} \quad &= \#(A,O) + \#(B,O) + 2\#O^d + 2\#(AB,O) + \#A^d + \#(AB,A) + \#(A,B) \\
&\quad + \#B^d + \#(AB,B) \\
N_{11} \quad &= \#(A,O) + \#(B,O) + \#O^d + \#(AB,O) + \#(A,AB) + \#(A,B) + \#(B,AB)
\end{aligned}
$$

The proof is given in the appendix.

3.4 Multi-Way Cycles and Chains of Exchange

In the previous sections we have focused on two-way, three-way, and four-way cycles and chains of exchange and derived the upper bounds of the possible number of kidney transplants under those given assumptions. In the current section, we consider a more general model of kidney exchange and show that under similar conditions, five or higher-way cycles and chains of exchange even if available will not further increase the number of feasible kidney transplants. In other words, four or less-way exchanges are sufficient to capture all the potential gains of kidney exchange.

Our general model consists of pairs, single donors and patients on the waiting list. We also call a patient on the waiting list *a single patient.* Each pair i has a patient P_i^p and a donor D_i^p. Each single patient is denoted by P_i^s and each single (decreased or altruistic) donor is denoted as D_i^s.

Let $\mathscr{B}$ be the family of primary types such as blood shared by patients and donors with $|\mathscr{B}| = n > 2$. In other words, all patients and donors have their types X in $\in \mathscr{B}$. For any given two primary types $X, Y \in \mathscr{B}$, $X \succeq Y$ means that agent of type X is primary type compatible with agent of type Y. In the context of kidney exchange, a patient of type Y is blood-type compatible with a donor of type X. Following Roth, Sönmez and Ünver (2007), we assume that the compatibility relation $\succeq$ for primary types satisfies reflexivity, asymmetry and transitivity properties:

1. (Reflexivity) $X \succeq X$ for any $X \in \mathscr{B}$,

2. (Asymmetry) $X \succeq Y$ and $X \neq Y \Rightarrow Y \not\succeq X$ for any $X, Y \in \mathscr{B}$, and

3. (Transitivity) $X \succeq Y$ and $Y \succeq Z \Rightarrow X \succeq Z$ for any $X, Y \in \mathscr{B}$.

Blood-type compatibility possess the properties of reflexivity, asymmetry and transitivity.

Let $\mathscr{C}$ be the family of secondary types such as tissue shared by patients and donors with $|\mathscr{C}| = n \geq 2$. For any given two secondary types $Z, W \in \mathscr{C}$, $Z \sim W$ means that agent of type Z is secondary type compatible with agent of type W. In the context of kidney exchange, a patient of type Z is tissue-type compatible with a donor of type W. We assume that the compatibility relation $\sim$ for secondary types satisfies symmetry and intransitivity properties:

I. (Symmetry) $Z \sim W \Rightarrow W \sim Z$ for any $Z, W \in \mathscr{C}$, and

II. (Intransitivity) $Z \sim W$ and $W \sim L \not\Rightarrow Z \sim L$ for any $Z, W, L \in \mathscr{C}$.

Tissue-type compatibility possess the properties of symmetry and intransitivity.

An agent of primary type $X \in \mathscr{B}$ and secondary type $Z \in \mathscr{C}$ is compatible with an agent of primary type $Y \in \mathscr{B}$ and secondary type $W \in \mathscr{C}$ if and only if $X \succeq Y$ and $Z \sim W$. In the context of kidney exchange, a patient of type $Y \in \mathscr{B}$ and $W \in \mathscr{C}$ can accept a kidney from a donor of type $X \in \mathscr{B}$ and $Z \in \mathscr{C}$.

Because the compatibility of secondary types is symmetric and intransitive, we use symbol i to stand for $\nsim$ and symbol c to stand for $\sim$. Let $(X,Y)^t$ describe a pair which has a patient of primary type $X \in \mathscr{B}$ and a donor of primary type $Y \in \mathscr{B}$ and the compatibility relation of secondary types between the patient and the doctor is $t \in \{i, c\}$. Therefore, we have four categories for pairs:

1. $(X,Y)^i$ for any $X,Y \in \mathscr{B}$, and $Y \not\succeq X$,

2. $(X,Y)^c$ for any $X,Y \in \mathscr{B}$, and $Y \not\succeq X$,

3. $(X,Y)^i$ for any $X,Y \in \mathscr{B}$, and $Y \succeq X$,

4. $(X,Y)^c$ for any $X,Y \in \mathscr{B}$, and $Y \succeq X$.

In this model, category 4 demonstrates compatible pairs and the other three categories cover incompatible pairs. To simplify the notation, we write incompatible pairs from categories 1 and 2 as (X,Y) for which donors are primary type incompatible with patients, i.e., $Y \not\succeq X$.

We can describe a three-way cycle as

$$E = ((P_1^p, D_1^p), (P_2^p, D_2^p), (P_3^p, D_3^p)),$$

which means that the paired donor D_1^p is matched with the paired patient P_2^p, the paired donor D_2^p is matched with the paired patient P_3^p, and the paired donor D_3^p is matched with the paired patient P_1^p. Any size cycle can be defined similarly. A cycle E is *feasible* if the type of each donor in E is compatible with the type of patient who is matched with the donor. Also, we can describe a three-way chain as

$$C = (D_1^s, (P_1^p, D_1^p), (P_2^p, D_2^p), P_1^s),$$

in which the single donor D_1^s is matched with the paired patient P_1^p, the paired donor D_1^p is matched with the paired patient P_2^p, and the paired donor D_2^p is matched with the single patient P_1^s. Any size chain can be defined in a similar way. A chain C is *feasible* if the type of every donor in C is compatible with the type of patient who is matched with the donor.

We can recast the Assumptions 1 and 3 into the present model, respectively.

Assumption 4 *Every single agent of primary type $X \in \mathscr{B}$ and secondary type $Z \in \mathscr{C}$ is $Z \sim W$ with every agent of type $Y \in \mathscr{B}$ and $W \in \mathscr{C}$ who is $Y \succeq X$. Every agent in a pair of type $X \in \mathscr{B}$ and $Z \in \mathscr{C}$ is $Z \sim W$ with every agent other than agents in the pair of type $Y \in \mathscr{B}$ and $W \in \mathscr{C}$ who is $Y \succeq X$.*

Assumption 5 *Let $X, Y \in \mathscr{B}$ be such that $Y \succeq X$. There exists either no pair of type (X, Y) or at least one pair of type $(X, Y)^c$.*

When the compatibility relation of primary type satisfies reflexivity, asymmetry and transitivity, the compatibility relation of secondary type satisfies symmetry and intransitivity, and the Assumption 4 for all agents, the Assumptions 5 for paired agents are satisfied, a maximal size exchange in the model can be achieved through no more than n-way cycles and n-way chains. The next two results generalize those of Roth, Sönmez and Ünver (2007, p. 837) to the setting which allows patients on the waiting list and single donors and need to use both cycles and chains of exchange.

Theorem 5 (n-way exchange suffices): *Assume that the Assumption 4 and 5 hold. Let μ be any maximal matching in the sense that any size of kidney exchanges is permitted in the matching. Then there exists a maximal matching ν which contains at most n-way cycles and chains of exchange but has the same number of patients matched with compatible donors as in the matching μ.*

The proof of this theorem is given in the appendix. The following is an immediate consequence of the theorem.

Corollary 2 (Four-way exchange suffices in kidney exchange): *Consider a kidney exchange model under the Assumptions 1 and 3. Let μ be any maximal matching without any restriction on the size of exchange. Then there exists a maximal matching ν which contains at most four-way exchanges but has the same number of patients who can benefit from exchanges as in the matching μ.*

3.5 Simulations Based on the USA Data

In this section, we use two data sets from the U.S. Organ Procurement and Transplantation Network (OPTN) and the Scientific Registry of Transplant Recipients (SRTR) from 1993 to 2002 and from 1995 to 2016, respectively,[8] to generate simulated data reflecting the characteristics of the population involved and to test how well our theoretical results can predict. Although the simulated population which is almost identical or very close to the

[8] They are retrieved from http://optn.transplant.hrsa.gov/data/view-data-reports/national-data.

real life situation may not fully meet the simplifying assumptions made for the model, we find that the predicted maximum number of transplants given by our derived formulas is surprisingly close to the number of transplants that can be actually realized.

3.5.1 Data Construction

Data is collected for two time slots. The first time slot data is from 1993 to 2002 and is shown in Table 3.5, and the second time slot data is from 1995 to 2016 and is shown in Table 3.6. These data sets illustrate the national characteristics of the USA population involved in kidney exchanges. The first period data from 1993 to 2002 is largely similar to those used by Roth, Sönmez and Ünver (2007), and Saidman et al. (2006), except that in our new data set we include more relevant information like the distribution of compatible patient-donor pairs and single donors, which are not used in Roth, Sönmez and Ünver (2007), and Saidman et al. (2006).

Patient-Donor Pairs and Single Donors Construction

Following Roth, Sönmez and Ünver (2007), to avoid the complications of possible impact of genetics on immunological incompatibilities we exclude all blood-related incompatible patient-donor pairs in all our samples.

In the first time slot from 1993 to 2002, we use the same characteristics of incompatible pairs as that of Roth, Sönmez and Ünver (2007) but add the blood-type characteristics for compatible patients, compatible donors and single donors; see Table 3.5. The second time slot data from 1995 to 2016 contains more detailed information about characteristics of the population. Compared to three levels of PRA (Percent Reactive Antibody) of patients from the data of the first time slot, five levels of PRA called CPRA (Calculated Percent Reactive Antibody) are provided in the data of the second time slot. The second time slot data contains also the information of compatible paired patient gender, compatible paired patient CPRA types and the blood-type information of incompatible paired donor; see Table 3.6.

It is important to point out that in the OPTN/SRTR annual report there is no clear information about the number of incompatible patient-donor pairs. Following Roth, Sönmez and Ünver (2007) we use newly-added patients on the waiting list every year as approximately incompatible paired patients and the blood-type distribution of donors whose kidneys have been transplanted as the blood-type distribution of incompatible paired donors. The information on single donors is collected from the data of deceased donors in each year.

Because there exist a large number of patients on the waiting list, we can always find a patient who is compatible with any given kidney. Hence, we do not need to simulate any data for patients on the waiting list.

Tissue-type Incompatibility

Tissue-type compatibility is the second condition for kidney transplants. In our simulations of the first time slot from 1993 to 2002, we adopt the same method as used by Roth, Sönmez and Ünver (2007) such that patients are divided into three groups based on the difficult level of tissue-type compatible with a random donor. In the first group called Low PRA group, patients are tissue-type incompatible with less than 10 percent of the population. The second group called Medium PRA contains patients who are tissue-type incompatible with 10-80 percent of the population. And, the third one called High PRA has patients who have a tissue-type incompatibility problem with more than 80 percent of the population. We use the following categories as used by Roth, Sönmez and Ünver (2007):

1. In Low PRA group, each patient is tissue-type incompatible with 5 percent of the population,
2. In Medium PRA group, each patient is tissue-type incompatible with 45 percent of the population, and
3. In High PRA group, each patient is tissue-type incompatible with 90 percent of the population.

In our simulations for the second time slot from 1995 to 2016, CPRA index is used to check whether a patient is sensitive or not according to OPTN/SRTR database. Five levels are calculated in CPRA index, which are 0, 1-19, 20-79, 80-97, and 98-100. If a patient CPRA equals 0, it means the patient has no PRA problem with potential donors; 1-19 means the patient has 1 percent to 19 percent to have problem with potential donors and so on. In this simulation, we divide patients into five groups based on the difficult levels of tissue-type compatibility with a random donor. Based on the CPRA data, we use the following five groups:

1. In 0 CPRA group, each paired patient is tissue-type incompatible with 0 percent of the population;

2. In 1-19 CPRA group, each paired patient is tissue-type incompatible with 9.5 percent of the population;

3. In 20-79 CPRA group, each paired patient is tissue-type incompatible with 50 percent of the population;

4. In 80-97 CPRA group, each paired patient is tissue-type incompatible with 88 percent of the population;

5. In 98-100 CPRA group, each paired patient is tissue-type incompatible with 99 percent of the population;

Because the data from 1995 to 2016 contains more detailed information on the tissue-type compatibility of patients and donors, it provides more accurate information than the first time slot data does. This has important implications: it will yield better results as shown in the subsequent section.

According to Zenios, Woodle and Ross (2001), a female patient is more likely to have a positive corssmatch with her husband. For instance, when positive crossmatch probability is 11.1 percent between random pairs, it becomes 33.3 percent between female patients and their donor husbands. Hence, when a patient is female and her potential donor is her husband, we adjust the probability of tissue-type incompatibility between them by using the formulas

$$PRA^{*} = 100 - 0.75(100 - PRA) \quad and \quad CPRA^{*} = 100 - 0.75(100 - CPRA).$$

3.5.2 Simulations

We generate a Monte-Carlo simulation size of 5,000 random population constructions for five population sizes of 25, 50, 100, 150 and 200 incompatible patient-donor pairs together with the corresponding population sizes of compatible patient-donor pairs and single donors according to the population distributions given by Table 3.5 for the period of 1993 to 2002 and by Table 3.6 for the period of 1995 to 2016, respectively. In addition we do a Monte-Carlo simulation size of 500 random population constructions for two big population sizes of 300 and 400 incompatible patient-donor pairs. Note that for these big population sizes we only generate 500 instead of 5,000 random population constructions in order to save time as it involves a relatively large and computationally difficult integer programming problem.

In our simulations we use the Algorithm by Edmonds (1965) to find the maximal number of incompatible paired patients who can actually receive a compatible kidney when the exclusive exchange mechanism, the first degree inclusive mechanism and the second degree inclusive mechanism are applied respectively. This maximal number will be simply called *simulation*. We compare these numbers with those predicted by the formula given by Proposition 3 to see how close or far the actual maximal number of kidney transplants can be from the predicted number based on the formula in Proposition 3. As said earlier, we only use two-way exchanges in all simulations. Following Roth, Sönmez and Ünver (2007), we make use of two types of upper bounds:

Upper Bound 1. This is the number given by the formula in Proposition 3 for the simulated population sample of 25, 50, 100, 150, 200, 300, and 400 incompatible patient-donor pairs.

Upper Bound 2. For each simulated population sample, there may exist some patients who cannot find a compatible donor in the simulated population. We exclude those hopeless patients from the sample and compute the number given by the formula in Proposition 3 for the remaining population. This number is called the Upper Bound 2 and clearly gives a more accurate upper bound for the number of feasible transplants that can be realized.

For each population size of 25, 50, 100, 150, and 200 incompatible patient-donor pairs, we generate 5000 random samples and calculate the average of all 5000 simulations, upperbound 1's and upperbound 2's. For each population size of 300 and 400 incompatible patient-donor pairs, we generate 500 random samples and calculate the average of all 500 simulations, upperbound 1's and upperbound 2's. All results are collected in Tables 3.7 and 3.8 for the period of 1993-2002 and the period of 1995-2016, respectively.

3.5.3 Discussion of the Simulation Results

The simulation results from Tables 3.7, 3.8, 3.9, and 3.10 indicate that

1. the simulation results are very close to the theoretical bounds predicted by the formula in Proposition 3. Note that all our simulated population samples contain tissue-type incompatibilities, whereas Proposition 3 basically assumes away the issue of tissue-type incompatibility.

2. when both compatible patient-donor pairs and single donors participate in kidney exchanges, efficiency of exchange increases significantly.

3. increasing the size of the population can help the theory predict better.

4. two-way exchanges can achieve most of the potential gains from exchange. Even more so if the size of population gets bigger.

5. when the number of incompatible patient-donor pairs exceeds a certain threshold, say, 100, efficiency of exchange becomes almost a constant. This strongly suggests that it is possible to decentralise kidney exchanges in a number of places with a relatively large size of population.

6. more accurate information can improve the quality of transplants and at the same time reduce the matching rate. This will be explained in the following subsection.

Before explaining the above points in detail, we introduce two performance measures. We first define the deviation of each simulation with upper bound 1 and upper bound 2 by

$$\frac{\text{upper bound i} - \text{simulation}}{\text{upper bound i}}, \quad \text{i=1, 2}$$

All deviations are given in Table 3.9. It is clear that as the size of the population increases, the deviation becomes smaller.

We next define the matching rate for each case of feasible transplants for incompatible paired patients over the number of incompatible patient-donor pairs under each exchange mechanism by

$$\frac{\text{the number of feasible transplants for incompatible paired patients}}{\text{the number of all incompatible paired patients}}$$

All matching rates are collected in Table 3.10 and shown in Figure 3.7. It is clear that as the size of the population increases, the matching rate increases.

Points 1 and 3 can be seen from Table 3.9. For the two data sets, the table indicates that the deviation becomes smaller as the size of the population increases, and that the 2nd degree inclusive mechanism performs better than the 1st degree inclusive mechanism which outperforms the exclusive exchange mechanism. Look at the case of the 1993-2002 data set. For 25 incompatible pairs, the deviations for upper bound 1 under the exclusive exchange mechanism, the 1st degree inclusive mechanism and the 2nd degree inclusive mechanism are 27%, 19% and 10%, respectively; and the corresponding deviations for upper bound 2 are 7%, 8% and 6%, respectively. For 100 incompatible pairs, the deviations for upper bound 1 under the exclusive exchange mechanism, the 1st degree inclusive mechanism and the 2nd degree inclusive mechanism are 12%, 6% and 2%, respectively; and the corresponding deviations for upper bound 2 are 6%, 4% and 2%, respectively. For 200 incompatible pairs, the deviations for upper bound 1 under the exclusive exchange mechanism, the 1st degree inclusive mechanism and the 2nd degree inclusive mechanism are 6%, 2% and 0.7%, respectively; and the corresponding deviations for upper bound 2 are 4%, 2% and 0.7%, respectively. This suggests that increasing the size of the population can make the theory predict better. These observations hold true also for the 1995-2016 data set.

Points 2 and 4 become quite obvious if we compare our Table 3.7 with Table 2 of Roth, Sönmez and Ünver (2007, p.841) for the data of the same period of 1993-2002. For instance, for a population of 25 incompatible patient-donor pairs, in their Table 2 under two-, three-, ..., unlimited-way exchange, their mechanism gives 11.992 feasible transplants, whereas our Table 3.7 shows that under two-way exchange, our 1st degree inclusive mechanism gives 12.838 feasible transplants and the 2nd degree inclusive mechanism yields 19.59 feasible transplants.

Finally we turn to Point 5. Figure 3.7 demonstrates that overall the slope of matching rate is upward and when the number of incompatible patient-donor pairs is below 100-a kind of threshold, the slope is relatively steep, and after 100, the slope becomes almost flat albeit upward, i.e., efficiency of exchange is nearly a constant. This may have important policy implications: Kidney exchanges could be *decentralised.* Any country with a large population like USA can have several separate kidney exchange programmes spread across the country where each programme covers a sufficient number of patients and donors, say, no less than 100 of incompatible patient-donor pairs. This can be very important in practice, as the life of kidneys from decreased donors is short and shortening travelling time can be extremely helpful.

An Explanation of the Matching Rate on the Second Dataset

In this subsection we explain why the matching rate in the 1995-2016 dataset (the second time slot) is lower than in the 1993-2002 dataset (the first time slot). In our simulations, we first draw a population of n incompatible pairs from the pool. Each incompatible pair is either blood-type incompatible or tissue-type compatible or both. When a compatible pair is drawn, we put the compatible pair back to the pool and keep drawing pairs from the pool until the population of n incompatible pairs is generated.

From the information given in Tables 3.5 and 3.6, we can calculate the percentage of incompatible pairs in the pool. The percentage of blood-type incompatible pairs for the first time slot and the second time slot are 0.3163 [9] and 0.30767 [10], respectively.

We give an example of the calculation by using the first group of each time slot. 89.24 percent of patients have no tissue type problem (CPRA=0) in the second time slot while 70.19 percent of patients have a low PRA value of 5 percent in the first time slot. Therefore, the percentages of drawing incompatible pairs from this group in the first and second time slots are given as follows, respectively:

$$\text{(Low PRA): } 5\% + 95\% * 0.3163 = 0.05 + 0.300485 = 0.350485$$

$$\text{(0): } 0\% + 100\% * 0.30767 = 0.30767.$$

When an incompatible paired patient is tissue-type compatible with a paired donor, the patient is blood-type incompatible with the donor. We have seven types of blood-type incompatible pairs (O,A), (O,B), (O,AB), (A,B), (B,A), (A,AB) and (B,AB). From the theoretical part, we can see that the blood-type incompatible pairs are difficult to find compatible pairs because they cannot match with each other except $(A,B)-(B,A)$, especially among incompatible pairs.

[9] $48.14\% * (1-48.14\%) + (33.73\% * 14.28\%) * 2 + 33.73\% * 3.85\% + 3.85\% * 14.28\% = 0.3163.$

[10] $48.46\% * (1-55.3\%) + 33.22\% * 9.9\% + 14.48\% * 32.46\% + 33.22\% * 2.34\% + 2.34\% * 14.48\% = 0.30767.$

Table 3.4 The percentage of incompatible pairs in the pool

Groups from 1992-2003	The rate of tissue type incompatible pairs (%)	The rate of blood-type incompatible but tissue type compatible pairs (%)	The rate of incompatible pairs (%)
Low PRA	5	30.0485	35.0485
Medium PRA	45	17.3965	62.3965
High PRA	9	3.163	93.163
Average	21.3385	24.88	46.2185
Groups from 1995-2016	The rate of tissue type incompatible pairs (%)	The rate of blood-type incompatible but tissue type compatible pairs (%)	The rate of incompatible pairs (%)
0	0	30.767	30.767
1-19	9.5	27.844	37.344
20-79	50	15.3835	65.3835
80-97	88	3.692	91.692
98-100	99	0.30767	99.30767
Average	5.658	29.026	34.68445

We can see that blood-type incompatible pairs account for 53.89 (0.213385*0.3163+ 0.2488/0.462185) percent of the total incompatible pairs in the first time slot. While blood-type incompatible pairs account for 83.69 (0.05658*0.30767 + 0.29026/0.3468) percent of the total incompatible pairs in the second time slot, which is 29.8 percent higher than that of the first time slot. This means that the number of blood-type incompatible pairs from the second time slot is larger than those from the first time slot.

On the other hand, the number of blood-type compatible but tissue type incompatible pairs $(0.213385*(1-0.3163)=0.146)$ in the first time slot is larger than that in the second one $(0.05658*(1-0.30767)=0.039)$. Since blood-type incompatible pairs cannot be matched except $(A,B)-(B,A)$ with each other, it will be more difficult for incompatible paired patients to be matched in the second time slot than in the first time slot. This shows why the matching rate in the second time slot is lower than that in the second time slot.

3.6 Conclusion

The current study has been motivated by two major issues concerning kidney exchange. The first issue is very practical and concerns the engineering aspect of conducting efficient kidney exchanges in a real-life environment. In this environment, there are many compatible patient-donor pairs, incompatible patient-donor pairs, patients on the waiting list, and single donors who are altruistic living or cadaver donors, and kidney exchanges can be done mostly by two-way, occasionally by three-way, and rarely by four-way exchange. We have examined how to design kidney exchange procedures in this practical environment so that a maximum number of patients can receive compatible kidneys. The second issue is more theoretical and concerns the derivation of a precise upper bound of a possible number of patients who can benefit from two-way, three-way, and four-way exchanges, respectively.

Our model is very practical and general, as it reflects a typical real-life kidney exchange environment by including incompatible patient-donor pairs, compatible patient-donor pairs, patients on the waiting list, and single donors, who can be altruistic living donors or decreased donors. A salient feature of the current model is to allow compatible patient-donor pairs and single donors to participate in kidney exchange with incompatible patient-donor pairs. In this way, the number of incompatible paired patients who can receive compatible kidneys will be increased considerably and is directly proportional to the number of compatible paired donors and single donors, meaning that more lives can be saved.

For this general model we have derived a precise maximum number of patients who can possibly receive compatible kidneys under two-way, three-way and four-way exchanges respectively, although the analysis becomes more difficult and more complicated. In each case (two-, three-, or four-way exchange), we develop a procedure by which kidney exchange should be conducted to enable the maximum number of patients to receive compatible kidneys. It is shown that, even for this general model, at most four-way cycles or chains will be sufficient to accomplish all the potential gains of kidney exchange, and that in every efficient exchange, each cycle contains at most two blood-type compatible pairs and each chain contains at most one blood-type compatible pair. We have also provided substantial simulation results based on the USA national patient data for the period 1993-2002 and the period 1995-2016. Our results shed new insights into the kidney exchange problem and are stated as follows.

Our results are fully consistent with those found in Roth, Sönmez and Ünver (2007), in which kidney exchanges are carried out only among incompatible pairs. In our model, however, when compatible patient-donor pairs are allowed to exchange with incompatible patient-donor pairs, the number of incompatible paired patients who can receive compatible kidneys increases considerably; and this number will increase further when both compatible patient-donor pairs and single donors participate in exchange with incompatible pairs. Our theory can predict surprisingly well in the sense that the actual maximum number of feasible kidney transplants is very close to the number predicted by our derived formula. As the size of the population increases, the predictive power of our theory in fact becomes stronger; and two-way exchange can accomplish most of the potential gains of exchange. Indeed if the population is large enough, it is sufficient to use two-way exchange to clear all incompatible pairs. Our results have a novel and significant policy implication: kidney exchange can be decentralised in the sense that in a country with a large population, several separate kidney exchange programmes can be established across the country, not just one centralised programme for the entire country. In the course of our study it has become clear to us that at the current stage it is very difficult to conduct simulations with a population size of 500 incompatible patient-donor pairs, as it involves a quite large and difficult integer programming problem. We expect to report simulation results in the near future by also making use of three-, four- or higher-way cycles and chains of exchange.

We hope that the current study will be useful in helping design practical kidney exchange programmes and in stimulating further research.

Table 3.5 Patient-donor pair and single donor distributions used in simulations based on OPTN/SRTR database from 1993 to 2002.

Incompatible paired patient blood type	**Percent**
O	48.14
A	33.73
B	14.28
AB	3.85
Patient gender	**Percent**
Female	40.9
Male	59.1
Relationship of patient-donor pair	**Percent**
Spouse	48.97
Other	51.03
PRA types	**Percent**
Low PRA	70.19
Medium PRA	20.00
High PRA	9.81
Compatible paired patient blood type	**Percent**
O	45.12
A	38.54
B	12.64
AB	3.7
Compatible paired donor blood type	**Percent**
O	63.74
A	27.12
B	8.08
AB	1.06
Single donor blood type	**Percent**
O	47.31
A	38.14
B	11.16
AB	3.39
Transplant ratio by donor types	**Percent**
Single Donors	39.83
Paired Donors	22.77

The OPTN/SRTR database from 1993 to 2002 can be retrieved from https://optn.transplant.hrsa.gov/data/view-data-reports/national-data.

Table 3.6 Patient-donor pair and single donor distributions used in simulations based on OPTN/SRTR database from 1995 to 2016.

Incompatible paired patient blood type	**Percent**	**S.D.**
O	48.46	0.0032
A	33.22	0.0047
B	14.48	0.0028
AB	3.84	0.0011
Incompatible paired patient gender	**Percent**	**S.D.**
Female	40.1	0.0117
Male	59.9	0.0117
Incompatible paired patient CPRA type	**Percent**	**S.D.**
0	89.24	0.0145
1-19	2.79	0.0071
20-79	4.64	0.005
80-97	2.03	0.001
98-100	1.3	0.002
Compatible paired patient blood type	**Percent**	**S.D.**
O	44.71	0.0092
A	38.47	0.0075
B	12.99	0.0044
AB	3.83	0.0029
Compatible paired patient gender	**Percent**	**S.D.**
Female	39.95	0.0204
Male	60.05	0.0204
Compatible paired patient CPRA type	**Percent**	**S.D.**
0	73.11	0.0241
1-19	9.43	0.0154
20-79	12.82	0.0084
80-97	3.38	0.0041
98-100	1.26	0.0025
Relationship of patient-donor pair	**Percent**	**S.D.**
Spouse	35.8	0.1201
Other	64.2	0.1201
Incompatible paired donor blood type	**Percent**	**S.D.**
O	55.3	0.0122
A	32.46	0.0081
B	9.9	0.0041
AB	2.34	0.0022
Compatible paired donor blood type	**Percent**	**S.D.**
O	64.66	0.011
A	26.45	0.0074
B	7.91	0.0044
AB	0.98	0.0021
Single donor blood type	**Percent**	**S.D.**
O	47.59	0.0068
A	37.41	0.0084
B	11.57	0.0055
AB	3.43	0.0026
Transplant ratio by donor type	**Percent**	**S.D.**
Single Donors	36.02	0.0398
Paired Donors	19.9	0.039

The OPTN/SRTR database from 1995 to 2016 can be retrieved from https://optn.transplant.hrsa.gov/data/view-data-reports/national-data.

Table 3.7 Simulation results about average maximal number of incompatible paired patients actually receiving transplants and average predicted number by the formula based on the 1993-2002 data.

Population Size of Incompatible Pairs	Method	Number of incompatible paired patients getting transplants		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Simulation	8.9992 (3.3465)	12.8388 (3.36736)	19.5904 (3.1966)
	Upper Bound 1	12.4444 (3.62319)	15.8782 (3.55402)	21.919 (3.0039)
	Upper Bound 2	9.7012 (3.69614)	14.0782 (3.59381)	20.964 (3.02684)
n=50	Simulation	21.7872 (5.04759)	29.599 (5.17304)	42.8134 (4.77275)
	Upper Bound 1	27.0408 (5.16082)	33.5676 (5.31818)	45.413 (4.45821)
	Upper Bound 2	23.7656 (5.47378)	31.9192 (5.4182)	44.8486 (4.41678)
n=100	Simulation	49.8772 (7.36965)	64.2164 (7.4473)	89.8862 (6.9542)
	Upper Bound 1	56.7104 (7.36069)	68.614 (7.58903)	92.2014 (6.59551)
	Upper Bound 2	53.4844 (7.70327)	67.4584 (7.6945)	92.0746 (6.57535)
n=150	Simulation	78.9256 (9.29992)	100.014 (9.42842)	137.567 (8.63815)
	Upper Bound 1	86.692 (9.1035)	104.442 (9.48898)	139.417 (8.33299)
	Upper Bound 2	83.6704 (9.54597)	103.647 (9.58259)	139.383 (8.31955)
n=200	Simulation	108.716 (10.7764)	135.571 (10.9588)	184.819 (10.3357)
	Upper Bound 1	116.799 (10.5688)	139.742 (11.0306)	186.254 (10.1569)
	Upper Bound 2	114.232 (10.9591)	139.168 (11.0965)	186.245 (10.1546)
n=300	Simulation	170.54 (13.8317)	208.974 (13.8698)	280.91 (13.4347)
	Upper Bound 1	178.668 (13.6163)	212.676 (14.0197)	281.688 (13.4062)
	Upper Bound 2	176.948 (13.9028)	212.404 (14.0379)	281.688 (13.4062)
n=400	Simulation	231.628 (15.1099)	281.492 (15.1398)	375.198 (15.2474)
	Upper Bound 1	239.524 (14.592)	284.636 (15.0674)	375.65 (15.2176)
	Upper Bound 2	238.36 (14.8267)	284.466 (15.1155)	375.65 (15.2176)

Table 3.8 Simulation results about average maximal number of incompatible paired patients actually receiving transplants and average predicted number by the formula based on the 1995-2016 data.

Population Size of Incompatible Pairs	Method	Number of incompatible paired patients getting transplants		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Simulation	6.6844 (3.02308)	9.6722 (3.16884)	16.1756 (3.39085)
	Upper Bound 1	8.3772 (3.29944)	11.3094 (3.4135)	17.6964 (3.55437)
	Upper Bound 2	6.832 (3.12092)	10.0494 (3.30825)	16.7298 (3.50851)
n=50	Simulation	15.008 (4.5394)	21.5734 (4.71549)	33.8482 (4.9819)
	Upper Bound 1	18.5984 (4.79534)	24.1956 (4.9522)	36.1456 (5.1907)
	Upper Bound 2	16.0188 (4.75009)	22.3364 (4.88135)	34.7956 (5.1391)
n=100	Simulation	34.496 (6.8107)	46.3272 (7.05924)	69.7068 (7.42242)
	Upper Bound 1	39.6832 (6.96165)	50.2572 (7.27533)	73.0118 (7.62722)
	Upper Bound 2	35.8428 (7.01817)	47.6532 (7.22253)	71.1594 (7.55584)
n=150	Simulation	54.2632 (8.65407)	71.5348 (8.9778)	105.994 (9.24828)
	Upper Bound 1	60.934 (8.82225)	76.3784 (9.16827)	110.046 (9.4449)
	Upper Bound 2	56.2608 (8.89426)	73.313 (9.17326)	107.991 (9.44535)
n=200	Simulation	74.134 (10.0771)	96.6472 (10.4245)	142.411 (10.6297)
	Upper Bound 1	81.8596 (10.1768)	102.143 (10.691)	146.966 (10.8525)
	Upper Bound 2	76.5832 (10.2132)	98.7708 (10.659)	144.941 (10.8549)
n=300	Simulation	114.904 (11.8696)	147.89 (12.4127)	215.976 (13.0876)
	Upper Bound 1	124.292 (11.9257)	154.37 (12.6063)	221.19 (13.282)
	Upper Bound 2	118.272 (12.0396)	150.724 (112.6819)	219.358 (13.3162)
n=400	Simulation	155.572 (13.846)	198.49 (14.1654)	288.54 (14.4048)
	Upper Bound 1	166.024 (13.8123)	205.776 (14.4168)	294.304 (14.7397)
	Upper Bound 2	159.384 (13.8554)	202.008 (14.3976)	292.802 (14.7343)

Table 3.9 Deviation from upper bounds 1 and 2 in simulation based on the 1993-2002 data and 1995-2016 data.

Data from 1993-2002				
Population Size of Incompatible Pairs	Method	Deviation Value		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Upper Bound 1	0.2768	0.1914	0.1062
	Upper Bound 2	0.0724	0.088	0.0655
n=50	Upper Bound 1	0.1943	0.1182	0.0572
	Upper Bound 2	0.0832	0.0727	0.04538
n=100	Upper Bound 1	0.1205	0.0641	0.0251
	Upper Bound 2	0.0674	0.0481	0.0238
n=150	Upper Bound 1	0.0896	0.0424	0.0133
	Upper Bound 2	0.0567	0.0351	0.013
n=200	Upper Bound 1	0.0692	0.0299	0.0077
	Upper Bound 2	0.0483	0.0258	0.00766
n=300	Upper Bound 1	0.04549	0.0174	0.00276
	Upper Bound 2	0.03621	0.01615	0.00276
n=400	Upper Bound 1	0.03296	0.011	0.0012
	Upper Bound 2	0.02824	0.0104	0.0012

Data from 1995-2016				
Population Size of Incompatible Pairs	Method	Deviation Value		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Upper Bound 1	0.2021	0.1448	0.0859
	Upper Bound 2	0.0216	0.0375	0.03312
n=50	Upper Bound 1	0.1665	0.108375	0.06356
	Upper Bound 2	0.032337	0.03416	0.02723
n=100	Upper Bound 1	0.1307	0.07819	0.04526
	Upper Bound 2	0.037575	0.028622	0.02041
n=150	Upper Bound 1	0.1094	0.063415	0.03682
	Upper Bound 2	0.0355	0.02425	0.01849
n=200	Upper Bound 1	0.09437	0.0538	0.03099
	Upper Bound 2	0.03198	0.0215	0.01745
n=300	Upper Bound 1	0.075532	0.041977	0.02357
	Upper Bound 2	0.0284767	0.0188	0.015418
n=400	Upper Bound 1	0.062954	0.03541	0.01958
	Upper Bound 2	0.02391	0.017415	0.01455

Table 3.10 Matching rates of incompatible paired patients in simulation based on the 1993-2002 data and 1995-2016 data.

Data from 1993-2002				
Population Size of Incompatible Pairs	Method	Matching Rate		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Simulation	0.35997	0.51355	0.78362
	Upper Bound 1	0.49778	0.63513	0.87676
	Upper Bound 2	0.38805	0.56313	0.83856
n=50	Simulation	0.43574	0.59198	0.85627
	Upper Bound 1	0.54096	0.67135	0.90826
	Upper Bound 2	0.47531	0.63838	0.89697
n=100	Simulation	0.49877	0.64216	0.89886
	Upper Bound 1	0.5671	0.68614	0.92201
	Upper Bound 2	0.53484	0.67458	0.92075
n=150	Simulation	0.52617	0.66676	0.91711
	Upper Bound 1	0.57795	0.69628	0.92945
	Upper Bound 2	0.5578	0.69098	0.92945
n=200	Simulation	0.54358	0.67786	0.9241
	Upper Bound 1	0.584	0.69871	0.93127
	Upper Bound 2	0.57116	0.69584	0.93122
n=300	Simulation	0.5685	0.69658	0.936
	Upper Bound 1	0.59556	0.70892	0.93896
	Upper Bound 2	0.5898	0.708	0.93896
n=400	Simulation	0.57907	0.70373	0.93799
	Upper Bound 1	0.59881	0.71159	0.9391
	Upper Bound 2	0.5959	0.7116	0.9391

Data from 1995-2016				
Population Size of Incompatible Pairs	Method	Matching Rate		
		The Exclusive Exchange Mechanism	The First Degree Inclusive Exchange Mechanism	The Second Degree Inclusive Exchange Mechanism
n=25	Simulation	0.267376	0.386888	0.647024
	Upper Bound 1	0.335088	0.452376	0.707856
	Upper Bound 2	0.27328	0.401976	0.669192
n=50	Simulation	0.310016	0.431468	0.676964
	Upper Bound 1	0.371968	0.483912	0.722912
	Upper Bound 2	0.320376	0.446728	0.695912
n=100	Simulation	0.34496	0.463272	0.697068
	Upper Bound 1	0.396832	0.502572	0.730118
	Upper Bound 2	0.358428	0.476532	0.711594
n=150	Simulation	0.36175	0.476869	0.7066
	Upper Bound 1	0.406226	0.509189	0.73364
	Upper Bound 2	0.375072	0.48875	0.71994
n=200	Simulation	0.37067	0.483236	0.712055
	Upper Bound 1	0.409298	0.510715	0.73483
	Upper Bound 2	0.382916	0.493854	0.724705
n=300	Simulation	0.383	0.49296	0.71992
	Upper Bound 1	0.4143	0.51456	0.7373
	Upper Bound 2	0.39424	0.5024	0.73119
n=400	Simulation	0.38893	0.496225	0.72135
	Upper Bound 1	0.41506	0.51444	0.73576
	Upper Bound 2	0.39846	0.50502	0.7320

Fig. 3.7 Matching rates of incompatible paired patients based on the 1993-2002 data (a) and based on the 1995-2016 data (b).

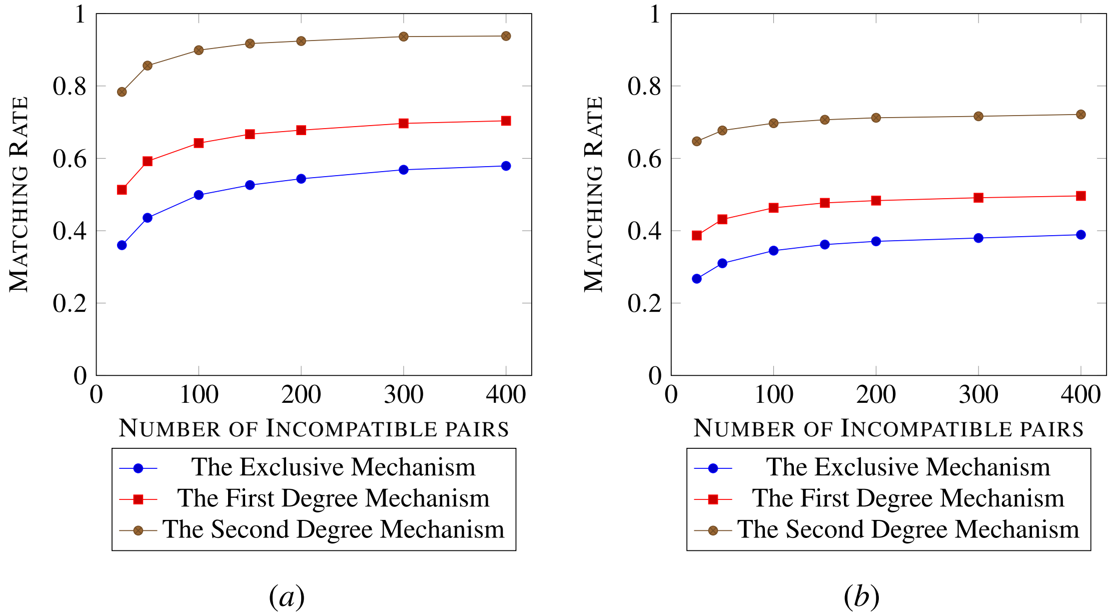


Table 3.11 Running time in simulation.

Population size	Time in sec. for the 1995-2016 data	Time in sec. for the 1993-2002 data
25	4021.05	3716.65
50	15543.8	16232.1
100	91568.9	104266
150	341273	383060
200	875914	965278
300	408724	460416
400	$1.23238 * 10^6$	$1.4043 * 10^6$

Computing Facility Information (ADACX Cluster):

Facilities for computationally intensive tasks are provided within RCSS by a cluster of 25 application servers, each having 2 oct-core Intel E5-2690 processors (2.9GHz, 16 physical processor cores per server, 32 including hyperthreading). Each server has 192GB 1600MHz RAM and a 10Gbit network connection to a local file server. All are running 64 bit Windows Server 2008 R2 Enterprise and Citrix XenApp 6. The system is accessible via a web interface and Citrix client software from all machines on campus and globally from any machine that can establish a VPN network connection to York. For more information, please visit https://www.york.ac.uk/economics/resources/computing/cluster/

Chapter 4

A Stable Hospital-Doctor Matching Mechanism under Distributional and Hierarchical Constraints

4.1 Introduction

The imbalanced distribution of doctors is matter of concern all over the world. The problem is that urban areas attract an excessive number of doctors while rural districts suffer from a shortage of doctors. Talbott (2007) described this serious problem in the United States, whereby 16,000 doctors were needed immediately to meet the needs for health care of 35 million Americans who live in underserved areas. A similar situation can be found in China where urban hospitals attract most of the health professionals (Anand et al., 2008, Hou and Ke, 2015). Evidence shows that, in 2012, the average number of doctors per 1,000 population in urban areas was 3.19 while that in rural areas was only 1.4.[11] Moreover, the OECD states that the density of doctors in urban regions is consistently larger than that in rural areas in many countries including France, Australia and Canada, although the definition of urban and rural regions varies across countries.

Many countries have implemented different types of policies to address this problem. The Japanese government started a programme called the 'Japan Residency Matching Programme' (JRMP) in 2008. In this programme, the Japanese government sets an upper

[11]The data comes from China health and family planning statistical yearbook 2013 P147, http://www.nhfpc.gov.cn/ewebeditor/uploadfile/2014/04/20140430131845405.pdf

bound on a region (called a regional cap) to restrict the total number of doctors in that region. They apply the deferred acceptance algorithm with respect to the regional caps in the programme. In detail, every hospital is assigned an artificial capacity according to the regional caps and then the programme applies the deferred acceptance algorithm with respect to the artificial capacities. The Chinese government, meanwhile, started a three-year plan in 2010 to provide financial support to train health professionals for rural areas. In addition, they give educational institutions special quotas to enroll medical students with rural backgrounds. Meanwhile, they offer students three or five years' free medical education in exchange for obligatory rural service (Hou and Ke, 2015). In Germany, the number of practice permits for new ambulatory care physicians in each region is regulated based on a national service delivery quota (OECD).

In the aspect of market design, it is worth studying whether a systematic mechanism to address the problem exists. Although the Deferred Acceptance (DA) algorithm proposed by Gale and Shapley (1962) is one of the most famous mechanisms in two-sided matching, it is difficult to place more doctors in remote areas by applying the algorithm directly. McVitie and Wilson (1970) have proved that in the marriage problem, a person who is unmarried in one stable solution will be unmarried in any other stable solution. The theorem is extended variously by Gale and Sotomayor (1985a,b), Roth (1984, 1986), Hatfield and Milgrom (2005). One extension is the well-known rural hospital theorem that any hospital that does not fill its quota at some stable matching is assigned precisely the same set of students at every stable matching (Roth, 1986).

Two types of distributional constraints have been considered to cope with this problem. The first type is the maximum constraint (also called the upper bound constraint). One example is the regional cap applied in JRMP by the Japanese government. The number of doctors in rural areas may increase because the regional cap shunts some doctors from popular areas to rural areas. Kamada and Kojima (2014), however, show that the JRMP mechanism is inefficient and propose a more efficient mechanism called the Flexible Deferred Acceptance mechanism (FDA). The FDA mechanism enables a hospital that has reached its artificial cap to recruit doctors if the regional cap is not binding. The second type of constraint that can be applied is the floor constraint (also called the lower bound constraint). The floor constraint guarantees a certain number of doctors allocated to a subset of hospitals. There is a lack of literature studying the floor constraints in doctor-hospital markets.

Distributional constraints are applied in many other matching markets. In the student-school matching problem, public schools are often required to balance the composition of students to create a diverse environment for students. For instance, the Educational Option schools in New York City restrict different ratio constraints to accept students with a range of different abilities (Abdulkadiroğlu et al., 2005). In the programme allocation problem, the total number of students in several programmes are bounded when those programmes share one building (Abdulkadiroglu and Sönmez, 2003). In the cadet-branch matching market, some constraints are implemented by a programme called 'Officer Career Satisfaction Programme' (OCSP) to increase retention rates among junior officers (Sönmez and Switzer, 2013).

This paper studies a general framework of the doctor-hospital market under distributional and hierarchical constraints. There are a finite number of hospitals, doctors, and regions in the market. Each region has a subset of hospitals. Each hospital has a strict preference over doctors and belongs to one region. Each doctor has a strict preference over hospitals. Due to the limited resources such as equipment, funding and office space, every hospital has a capacity that bounds recruiting quotas in that hospital. Besides its capacity, each hospital also has a minimum constraint which indicates the fewest number of recruited doctors required at the hospital. When a hospital plans to build a new department or to expand an existing department, it needs to recruit a certain number of doctors. When the hospital has no such demand, its minimum constraint is equal to zero.

Each region has a maximum constraint, namely the regional cap, and a minimum constraint, namely the regional floor constraint. Everyone has the right to receive medical treatment. Therefore, it is reasonable to require a minimum number of doctors in a certain area to meet local medical demands. The regional cap for a region is the maximum number of recruited doctors in the region, and is effective if the regional cap is smaller than the sum of capacities of the hospitals belonging to that region. Although the regional cap for a popular area can shunt some doctors to other areas, it is hard to guarantee a certain number of doctors in unpopular areas. The floor constraint for a region, however, guarantees the minimum number of doctors required in the designated region. It is effective if the floor constraint for a region is larger than the sum of the minimum constraints for the hospitals in that region. That is, when the minimum number of doctors required by all the hospitals in a region cannot meet the medical demands in that region, the floor constraint can ensure that the remaining demands are met.

Motivated by real-life cases, this paper also takes a hierarchical constraint on hospitals into account. In China, hospitals have a hierarchical structure and are comprised of three grades I, II, and III. Grades I and II individually have three levels A, B and C. Grade III has four levels E, A, B and C. The hierarchical constraint incorporates various kinds of grading structures, including one in which all hospitals have the same grade. In this paper, every hospital has its own grade. A hospital with a higher level normally has an advantage in recruiting doctors over one with a lower level because of their advanced equipment, adequate funding, better reputations, etc. In addition, the college and university admission system in China also has a hierarchical structure. Colleges and universities have three grades and those with a higher grade have priorities in terms of enrolling students.

By considering the distributional constraints and the hierarchical constraint, the structural properties of the stable matching are different than before. We therefore introduce an appropriate modification of the stability to this new model by considering both the distributional and the hierarchical constraints. The distributional constraints are related to the feasibility, and the hierarchical constraint is related to the market rules. The market rules include two cases. Firstly, hospitals with a higher grade have more competitive power to recruit doctors than those with a lower grade. Secondly, hospitals with the same grade have the same competitive power to recruit doctors.

We propose a doctor-proposing deferred acceptance mechanism to show how to allocate doctors to hospitals in the described market. The right to recruit doctors is decentralised into three parties: hospitals, regional organisers, and the national organiser. Each hospital receives a recruiting quota. As long as the quota is not filled, the hospital can recruit doctors freely. If the hospital wants to recruit more doctors, it needs to compete with other hospitals at a regional level or at the national level. Each hospital has its competitive strength which is related to its grade. The mechanism has very positive properties. Firstly, the mechanism always finds a matching that respects all distributional constraints. Secondly, the matching produced by the mechanism cannot be blocked by any doctor-hospital pair. Thirdly, the matching produced by the mechanism is efficient such that no other feasible matching exists that makes at least one agent (doctor or hospital) better off without hurting other agents. Furthermore, the dominant strategy for every doctor in the mechanism is to report his or her true preference.

We now briefly review several other related literatures. Abdulkadiroglu and Sönmez (2003) explored the shortcomings of several current school choice plans in Boston Columbus, Minneapolis, and Seattle, and proposed two practical mechanisms for both standard school choice model and that with type specific ceilings. Biró et al. (2010) considered an extension of the college admission problem, which was to set a common quota on certain bounded subsets of colleges.They proved that stable matching might not exist. Goto et al. (2015) studied a student-school problem under the regional cap constraints and proposed the Adaptive Deferred Acceptance mechanism (ADA). Ehlers et al. (2014) studied a school choice model with controlled choice constraints and proposed a student exchange mechanism to find a constrained efficient assignment by interpreting control constraints as soft bounds-flexible. Fragiadakis et al. (2016) studied matching markets in which institutions may have minimum and maximum quotas. They proposed two strategy-proof mechanisms: extended-seat DA (ESDA) and multi-stage DA algorithm (MSDA). Both mechanisms are strategy-proof and ESDA produces a fair matching while MSDA produces a non-wasteful matching. Due to the incompatibility of fairness and non-wastefulness in the existence of minimum quotas, they also introduce new second-best definitions of fairness and non-wastefulness, which were satisfied by their mechanisms. Goto et al. (2016) study student-school matching problems under regional constraints and a hierarchical structure of regions. Two strategy-proof mechanisms, PLDA-RQ and RSDA-RQ, are proposed based on the framework of matching with contracts by Hatfield and Milgrom (2005). We refer to Kojima and Troyan (2011) for a survey on the subject and the references contained therein.

The structure of this paper is as follows. The second section illustrates the model, the appropriate definition of stability, and the definitions of efficiency and strategy-proofness for doctors. The third section introduces the doctor-proposing deferred acceptance mechanism. The fourth section shows the main results of the mechanism. The last part is the conclusion.

4.2 The Model

Let $D = \{d_1, d_2, \ldots, d_n\}$ denote a set of n doctors and $H = \{h_1, h_2, \ldots, h_m\}$ a set of m hospitals. There are k regions in the market denoted by $R = (r_1, r_2, \ldots, r_k)$, where $k \geq 1$. Each hospital h belongs to one region denoted by $r(h)$ and the set of hospitals H is divided into hospitals in different regions, that is, $H_r \cap H_{r'} = \emptyset$ if $r \neq r'$ and $H = \cup_{r \in R} H_r$, where H_r denotes the set of hospitals in region $r \in R$. Each hospital h has a minimum quota p_h and its capacity q_h, where $q_h \geq p_h \geq 0$ for all $h \in H$. Denote $P^h = (p_{h_1}, p_{h_2}, \cdots, p_{h_m})$ as the vector of minimum quotas of all hospitals and $Q^h = (q_{h_1}, q_{h_2}, \cdots, q_{h_m})$ as the vector of capacity quotas of all hospitals. Each region has a regional minimum quota denoted as p_r and a regional maximum quota denoted as q_r, where $q_r \geq p_r \geq 0$ for all $r \in R$. Denote $P^r = (p_{r_1}, p_{r_2}, \ldots p_{r_k})$ as the vector of regional minimum quotas of all regions and $Q^r = (q_{r_1}, q_{r_2}, \ldots q_{r_k})$ as the vector of regional maximum quotas of all regions. Let $Q = P^h \cup Q^h \cup P^r \cup Q^r$ denote the profile of quotas of hospitals and regions. To consistent with regional quotas and hospital quotas, we assume that $\sum_{h \in H_r} p_h \leq p_r \leq q_r \leq \sum_{h \in H_r} q_h$ for all $r \in R$. To ensure a feasible matching, we have $n \geq \sum_{r \in R} p_r$.

There is a hierarchical constraint on hospitals denoted as $HS : H \rightarrow \{1, ..., l\}$, where $l \geq 1$. Each hospital $h \in H$ belongs to one grade and denote $hs(h) = i$ where $i \in \{1, ..., l\}$ as the grade for hospital h. For instance, $hs(h) = 1$ means hospital h is at the grade 1. A hospital $h \in H$ has a higher grade than another hospital $h' \in H \setminus \{h\}$ if and only if $hs(h) < hs(h')$. In this case, a hospital h with $hs(h) = 1$ has the highest grade. Let $HS(h) = \{hs(h_1), hs(h_2), \ldots, hs(h_m)\}$ be the grade vector for all hospitals. In addition, denote H_i be the set of hospitals having grade i and H_i^r as the set of hospitals having grade i in region $r \in R$, where $|H_i^r| \geq 0$. Note that existing models can be regarded as a special case in this setting in which $l = 1$, that is, all hospitals have the same grade.

Each doctor $d \in D$ has a strict preference relation $\succ_d$ over the set of hospitals H. Note that a doctor's preference over regions can be reflected by her preference over hospitals. For instance, when a doctor prefers region A to region B, then the doctor can present her preference in which all hospitals in region A are preferred to any hospital in region B. For any $h, h' \in H$, $h \succeq_d h'$ if and only if $h \succ_d h'$ or $h = h'$. Let $\succ_D = (\succ_d)_{d \in D}$ denote the preference profile of all doctors. Each hospital $h \in H$ has a strict preference relation $\succ_h$ over the set of doctors D. For any $d, d' \in D$, $d \succ_h d'$ if and only if $d \succ_h d'$ or $d = d'$.

Let $\succ_H = (\succ_h)_{\in H}$ denote the preference profile of all hospitals. Each doctor has a strict preference relation over all hospitals and vice versa. Furthermore, we assume that $\succ_h$ is responsive with q_h for all $h \in H$ (Roth, 1985). That is, it is unacceptable for hospital $h \in H$ to accept any set of doctors exceeding its capacity, and the hospitals' preference of a doctor does not depend on other doctors. Formally, the preference $\succ_h$ is responsive with capacity q_h if (i) $\emptyset \succ_h D'$ for any $D' \subseteq D$ with $|D'| > q_h$, (ii) For any $D' \in D$ with $|D'| \leq q_h$, $d \in D \setminus D'$ and $d' \in D'$, $(D' \cup d) \setminus d' \succeq_h D'$ if and only if $d \succeq_h d'$, and (iii)For any $D' \in D$ with $|D'| \leq q_h$ and $d' \in D'$, $D' \succeq_h D' \setminus d'$ if and only if $d' \succeq_h \emptyset$.

To sum up, a doctor-hospital matching problem is presented as $(D, H, R, Q, HS, \succ)$. Every hospital wants to recruit its best-preferred doctors and every doctor wants to find their best-preferred hospital. An outcome of the model is a mapping $\mu : D \cup H \to 2^{D \cup H}$ such that (i) $\mu(d) \in H \cup \{\emptyset\}$ for all $d \in D$, (ii) $\mu(h) \subseteq D$ for all $h \in H$, and (iii) for any $d \in D$ and $h \in H$, $\mu(d) = h$ if and only if $d \in \mu(h)$. A matching μ is *feasible* if $p_h \leq |\mu(h)| \leq q_h$ for all $h \in H$ and $p_r \leq |\mu(r)| \leq q_r$ for all $r \in R$, where $\mu(r) = \cup_{h \in H_r} \mu(h)$. That is, a feasible matching respects all distributional constraints. A matching μ is *individually rational* if (i) $\mu(d) \succeq_d \emptyset$ for all $d \in D$ and (ii) $d \succeq_h \emptyset$ for all $d \in \mu(h)$.

4.2.1 Stability

One of the most important solution concepts in a two-side matching is stability. Gale and Shapley (1962) introduced a standard definition of stability. In the context of a doctor-hospital market, a matching is *stable* if it is individually rational and can not be blocked by any doctor-hospital pair. A doctor-hospital pair (d, h) blocks a matching μ if (i) $h \succ_d \mu(d)$; and (ii) either $|\mu(h)| < q_h$ or $d \succ_h d'$ for some $d' \in \mu(h)$. However, there may not exist a stable matching under distributional constraints (Fragiadakis et al., 2016, Kamada and Kojima, 2014). An example is provided as follows.

Example 7 *(Impossibility of a stable matching): Consider the following market of two doctors and three hospitals. Let* $D = \{d_1, d_2\}$*,* $H = \{h_1, h_2, h_3\}$ *and* $R = \{r_1, r_2\}$*, where* $r_1 = \{h_1\}$ *and* $r_2 = \{h_2, h_3\}$*. The maximum and minimum quotas for hospitals are given as* $q_{h_1} = q_{h_2} = q_{h_3} = 1$ *and* $p_{h_1} = p_{h_2} = p_{h_3} = 0$*; the regional maximum and minimum quotas are given as* $q_{r_1} = 1$*,* $q_{r_2} = 2$*,* $p_{r_1} = 1$ *and* $p_{r_2} = 0$ *respectively. Their preferences are given*

$\succ_{h_1}: \quad d_2, \quad d_1 \qquad \succ_{h_2}: \quad d_2, \quad d_1 \qquad \succ_{h_3}: \quad d_1, \quad d_2$

$\succ_{d_1}: \quad h_2, \quad h_3, \quad h_1 \qquad \succ_{d_2}: \quad h_3, \quad h_2, \quad h_1$

Table 4.1 The feasible matching outcomes of Example 7

Matching	h_1	h_2	h_3
μ_1	d_1	d_2	$\emptyset$
μ_2	d_1	$\emptyset$	d_2
μ_3	d_2	$\emptyset$	d_1
μ_4	d_2	d_1	$\emptyset$

Table 4.1 shows the four feasible matchings in this example. None of the matchings is stable. According to the definition, blocking pairs have two situations. In the first situation, we say a doctor d claims an occupied position at a hospital h in a matching μ if $h \succ_d \mu(d)$ and $d \succ_h d'$ for some $d' \in \mu(h)$. In matching μ_2, pair (d_1, h_3) can form such blocking pair because d_1 prefers h_3 to their current allocation $\mu(d_1) = h_1$ and h_3 prefers d_1 to d_2 who is assigned to h_3. Similarly, the blocking pair (d_2, h_2) can be found in matching μ_4. In the second situation, we say a doctor d claims an empty position at a hospital h if $h \succ_d \mu(d)$ and $|\mu(h)| < q_h$. In matching μ_1, doctor-hospital pair (d_2, h_3) can form such blocking pair because doctor d_2 prefers h_3 to their allocation $\mu(d_2) = h_2$ and h_3 has an empty position to accept doctor d_2. Similarly, the blocking pair (d_1, h_2) can be found in matching μ_3.

Furthermore, we use Example 7 to show that the stability concept with only regional caps is no longer suitable in this general model. We use the stability concept from Kamada and Kojima (see P.15, Kamada and Kojima, 2014). Considering their model with only regional caps and hospital capacity in this example, there is a stable allocation μ_5 such that $\mu_5(d_1) = h_2$ and $\mu_5(d_2) = h_3$. No doctor is assigned to the region r_1, which needs (may be urgently) at least one doctor. In our model, μ_5 is obviously no longer feasible.

The matching can be easily broken and becomes unstable when a doctor claims an occupied position at a hospital. In this case, the hospital has incentives to break the matching by accepting their preferred doctor instead of the less preferred one. Once the less preferred doctor is rejected, the doctor has to apply to other hospitals and may become a new blocking doctor, and so on. In this paper, such kind of blocking pair is impermissible. However, whether a matching can be broken when a doctor claims an empty position at a hospital depends on the distributional constraints. Normally, as long as the capacity of a hospital is not filled and the doctor is acceptable for the hospital, then the hospital would prefer to accept the doctor because the hospital's preference is responsive with its capacity. Consider the situation in which one of the distributional constraints would be violated if a doctor claims an empty place at a hospital successfully. In this case, the hospital would not accept the doctor because of the restriction from the market. It is reasonable to tolerate a blocking doctor-hospital pair (d,h) if it is infeasible for the hospital h to recruit the doctor d.

Formally, given a matching μ, a blocking pair (d,h) is *tolerable* if $h \succ_d \mu(d)$, $|\mu(h)| < q_h$ and it satisfies one of the following conditions:

(i) $|\mu_{\mu(d)}| = p_{\mu(d)}$, or

(ii) $\mu(d) \notin H_{r(h)}$ and $|\sum_{h \in H_{r(\mu(d))}} \mu(h)| = p_{r(\mu(d))}$, or

(iii) $\mu(d) \notin H_{r(h)}$ and $|\mu(r(h))| = q_{r(h)}$.

Condition (i) shows the case that the minimum constraint of the hospital $\mu(d)$ is filled. If d claims an empty position at h successfully, the minimum constraint of $\mu(d)$ is violated. Condition (ii) describes the situation that h and $\mu(d)$ are located at different regions and the minimum constraint of the region $r(\mu(d))$ is filled. If d claims an empty position at h successfully, the minimum constraint of $r(\mu(d))$ is violated. Condition (iii) considers the situation that h and $\mu(d)$ are locate at different regions and the regional cap of $r(h)$ is binding. If d claims an empty position at h successfully, the regional cap is violated.

Therefore, a new appropriate definition of strongly stability under distributional constraints is proposed.

Definition 8 *A matching μ is blocked by a doctor-hospital pair (d,h) if either (i) $h \succ_d \mu(d)$ and $d \succ_h d'$ for some $d' \in \mu(h)$; or (ii) $h \succ_d \mu(d)$, $|\mu(h)| < q_h$ and μ' is feasible, where $\mu^{'}$ is the matching such that $\mu^{'}(d) = h$ and $\mu^{'}(d) = \mu(d')$ for all $d' \neq d$. A matching μ is* ***strongly stable*** *if it is feasible, individually rational and is not blocked by any pair.*

However, the strongly stable matching does not always exist. We demonstrate this point in the following example.

Example 8 *(Strongly stable matching does not always exist): Consider the following market of three doctors and three hospitals of the same grade. Let $D = \{d_1, d_2, d_3\}$,*

$H = \{h_1, h_2, h_3\}$ and $R = \{r_1, r_2\}$, where $r_1 = \{h_1\}$ and $r_2 = \{h_2, h_3\}$. The maximum and minimum quotas for hospitals are given as $q_{h_1} = q_{h_2} = q_{h_3} = 2$ and $p_{h_1} = p_{h_2} = p_{h_3} = 0$; the regional maximum and minimum quotas are given as $q_{r_1} = 2$, $q_{r_2} = 1$, $p_{r_1} = 1$ and $p_{r_2} = 0$ respectively. Their preferences are given

$$\succ_{h_1}: d_1, d_2, d_3 \qquad \succ_{h_2}: d_2, d_3, d_1 \qquad \succ_{h_3}: d_1, d_3, d_2$$
$$\succ_{d_1}: h_2, h_3, h_1 \qquad \succ_{d_2}: h_3, h_2, h_1 \qquad \succ_{d_3}: h_2, h_3, h_1$$

Table 4.2 The feasible matching outcomes of Example 8

Matching	h_1	h_2	h_3
μ_1	d_1, d_2	d_3	$\emptyset$
μ_2	d_1, d_3	$\emptyset$	d_2
μ_3	d_2, d_3	d_1	$\emptyset$
μ_4	d_1, d_2	$\emptyset$	d_3
μ_5	d_1, d_3	d_2	$\emptyset$
μ_6	d_2, d_3	$\emptyset$	d_1

Table 4.2 shows the six feasible matchings. None of the matchings is strongly stable. In matching μ_1, doctor-hospital (d_2, h_2) forms an intolerable blocking pair because d_2 claims an occupied position at hospital h_2. Similarly, intolerable blocking pairs can be found in matching μ_2, μ_3 and μ_4. In matching μ_5, a pair (d_2, h_3) forms an intolerable blocking pair because d_2 claims an empty seat at hospital h_3 without violating any distributional constraints. Similarly, a matching μ_6 can be blocked by pair (d_1, h_2).

Observe that when the number of doctors cannot meet the total number of positions provided by hospitals, it may not be able to avoid violating the property of strongly stability. Let *flexible quotas* denote the gap between the number of doctors and the sum of minimum quotas of all hospitals. Hospitals need to compete with each other to obtain a flexible quota to recruit a doctor. The competitive power of a hospital is related to its grade in the hierarchical system. In this paper we consider the following market rules such that: (i) hospitals with the same grade have the same competitive power, and (ii) hospitals with a high grade have more competitive power than those with lower grades. When hospitals with the same grade compete with each other for limited flexible quotas, a reasonable way is to equalize the chances for those hospitals to obtain flexible quotas. Therefore, a weaker definition of stability is proposed by taking the market rules into account. Formally,

Definition 9 *A matching μ is blocked by a doctor-hospital pair (d,h) if either (i) $h \succ_d \mu(d)$ and $d \succ_h d'$ for some $d' \in \mu(h)$; or (ii) $\mu(d) = \emptyset$, $h \succ_d \mu(d)$, $|\mu(h)| < q_h$ and μ' is feasible; or (iii) if $\mu(d) \in H$, $h \succ_d \mu(d)$, $|\mu(h)| < q_h$ and μ' is feasible, then either $hs(\mu(d)) > hs(h)$ when $hs(\mu(d)) \neq hs(h)$ or $|\mu^{'}(h)| - p_h \leq |\mu^{'}(\mu(d))| - p_{\mu(d)}$ when $hs(\mu(d)) = hs(h)$, where $\mu^{'}$ is the matching such that $\mu^{'}(d) = h$ and $\mu^{'}(d') = \mu(d')$ for all $d' \neq d$. A matching μ is **stable** if it is feasible, individually rational and is not blocked by any pair.*

We now discuss the situations of blocking pairs. In situation (i), any pair (d,h) in which doctor d and h prefer each other can block the matching. In situation (ii), an unmatched doctor d can claim an empty position at hospital h successfully if moving doctor d to hospital h is feasible. In situation (iii), when a matched doctor d wants to claim an empty position at hospital h, either of two cases need to be satisfied. The first case is that if hospital h and hospital $\mu(d)$ have different grades, then hospital h has a higher grade. This follows the market rule (i). It indicates that hospital h has more competitive power to recruit doctor d. In contrast, if hospital $\mu(d)$ has a higher grade than hospital h, then the pair (d,h) is inadequate to block the matching. The second case is that if hospital h and hospital $\mu(d)$ have the same grade, then

$$|\mu^{'}(h)| - p_h \leq |\mu^{'}(\mu(d))| - p_{\mu(d)}$$

This follows the market rule (ii). An intuitive approach is to think that, since both hospitals have the same competitive power, they should have the same chance to recruit doctors. Hence, if moving the doctor to the hospital will cause an imbalance allocation of allocation between those hospitals, such a movement should not be approved. That is, the pair (d,h) is inadequate to block the matching. The similar idea can be found in Kamada and Kojima (2014). Otherwise, if the new matching μ' balance the allocation of flexible quotas, the pair (d,h) forms a blocking pair.

We now use the following two examples to illustrate how the market rules (i) and (ii) work. Example 9 describes the role of market rule (i) and Example 10 illustrates the role of market rule (ii).

Example 9 *Consider the following market of two doctors and three hospitals in one region r. Let* $D = \{d_1, d_2\}$*,* $H = \{h_1, h_2, h_3\}$*. The maximum and minimum quotas for hospitals are given as* $q_{h_1} = q_{h_2} = q_{h_3} = 1$ *and* $p_{h_1} = 1$*,* $p_{h_2} = p_{h_3} = 0$*; the regional maximum and minimum quotas are given as* $q_r = 3$ *and* $p_r = 1$ *respectively. Moreover, there are two hierarchical positions in the example. Hospitals* h_1 *and* h_2 *have the highest hierarchical position* hs_1 *while hospital* h_3 *has the second hierarchical position* hs_2*. Their preferences are given*

$$\succ_{h_1}: \; d_2, \; d_1, \qquad \succ_{h_2}: \; d_2, \; d_1 \qquad \succ_{h_3}: \; d_1, \; d_2$$
$$\succ_{d_1}: \; h_2, \; h_3, \; h_1 \qquad \succ_{d_2}: \; h_3, \; h_2, \; h_1$$

Table 4.3 The feasible matching outcomes of Example 9

Matching	h_1	h_2	h_3
μ_1	d_1	d_2	$\emptyset$
μ_2	d_1	$\emptyset$	d_2
μ_3	d_2	$\emptyset$	d_1
μ_4	d_2	d_1	$\emptyset$

In example 9, there is one flexible quota.[12] Hospitals need to compete with each other to obtain the flexible quota. Table 4.3 shows all feasible matchings. None of the matchings is strongly stable, but matching μ_1 is stable. In matching μ_1, d_2 prefers h_3 and h_3 has empty position. However, h_3 has less competitive power than h_2 and hence the flexible quota is assigned to h_2 rather than h_3. Therefore pair (d_2, h_3) is inadequate to block the matching.

Example 10 *Consider the following market of three doctors* $D = \{d_1, d_2, d_3\}$ *and three hospitals* $H = \{h_1, h_2, h_3\}$ *in two regions* $r_1 = \{h_1\}$ *and* $r_2 = \{h_2, h_3\}$*. The maximum and minimum quotas for hospitals are given as* $q_{h_1} = q_{h_2} = q_{h_3} = 2$ *and* $p_{h_1} = 1$*,* $p_{h_2} = p_{h_3} = 0$*; the regional maximum and minimum quotas are given as* $q_{r_1} = 2$*,* $q_{r_2} = 4$ *and* $p_{r_1} = 1$*,* $p_{r_2} = 0$ *respectively. All hospitals have the same hierarchical position. Their preferences are given*

$$\succ_{h_1}: \; d_3, \; d_2, \; d_1, \qquad \succ_{h_2}: \; d_2, \; d_3, \; d_1 \qquad \succ_{h_3}: \; d_1, \; d_2 \; d_3$$
$$\succ_{d_1}: \; h_2, \; h_3, \; h_1 \qquad \succ_{d_2}: \; h_3, \; h_2, \; h_1 \qquad \succ_{d_3}: \; h_2, \; h_3, \; h_1$$

[12] The calculation is that $|D| - \sum_{h \in H} p_h = 1$.

Table 4.4 Three matching outcomes of Example 10

μ	h_1	h_2	h_3
μ_1	d_1	d_2, d_3	$\emptyset$
μ_2	d_3	d_2	d_1
μ_3	d_3	$\emptyset$	d_1, d_2

In example 10, there are twelve feasible matchings and none of the matchings is strongly stable. Since any blocking pair (d,h) such that d claims an occupied position at h is impermissible, only three feasible matchings satisfy this requirement. Table 4.4 shows the three feasible matchings. We show that matching μ_2 is stable. In matching μ_1, (d_2,h_3) blocks the matching. In detail, d_2 claims an empty position at h_3 and it is feasible to move d_2 to h_3 without violating any constraints. Meanwhile, h_2 and h_3 have the same competitive power. That is, if there exist two flexible quotas and both hospitals apply, we allocate each hospital with one flexible quota. Hence, the movement of pair d_2 should be approved and the matching is blocked. A similar situation can be found in μ_3. However, the situation is different in μ_2 which has two doctors d_1 and d_2 claiming empty position at h_2 and h_3, respectively. Firstly, if we move either doctor, then it will cause an imbalance of assignments among hospitals with the same grade based on the market rule (ii). Such movement is not approved. Secondly, if we move both doctors, then a new intolerable blocking pair (d_3,h_2) blocks the matching. Such movement should not be approved. Therefore, all doctors will remain at the stage in the matching μ_2.

4.2.2 Efficiency

Another important solution concept is efficiency. An efficient feasible matching means that we cannot find any other feasible matching that can make at least one agent (doctor or hospital) better off without hurting any other agents.

Definition 10 *A matching μ is efficient if it is feasible and there is no feasible matching μ' such that $\mu'(i) \succeq \mu(i)$ for all $i \in D \cup H$ and $\mu'(i) \succ \mu(i)$ for some $i \in D \cup H$.*

In a standard model of matching, a stable matching is efficient. Fortunately, the following theorem shows that the property of efficiency still holds under the new definition of stable.

Theorem 6 *Any stable matching is efficient.*

Theorem 6 indicates that as long as a matching is stable, the matching is efficient such that there exists no other feasible matching which can make at least one agent better off without hurting other agents. The Proof work can be seen in the Appendix C.

4.2.3 Strategy proofness

As preferences are private information, a major concern is the incentive compatibility. Denote $\chi(\succ)$ as the matching produced by mechanism χ at preference profile $\succ$ and $\chi_i(\succ)$ as the allocation for agent $i \in D \cup H$. We say a mechanism χ is *strategy-proof* if there does not exist a preference profile $\succ$, an agent $i \in D \cup H$, and preferences $\succ'_i$ such that $\chi_i(\succ'_i, \succ_{-i}) \succ_i \chi_i(\succ)$.

It stands to reason that mechanism designers would like to use a mechanism in which truthful reports are a dominant strategy for any agent under the mechanism. However, it has been proved that no mechanism can be both strategy-proof and stable for all possible preference profiles (Roth, 1982a, Roth and Sotomayor, 1990). Therefore, we focus on the incentive compatibility for doctors. A mechanism is incentive compatible for doctors if no doctor has an incentive to misreport their preferences in the mechanism. Let $\succ_{-d} := (\succ_{d'})_{d' \in D \setminus d}$.

Definition 11 *A mechanism χ is said to be strategy-proof for doctors if $\chi_d(\succ_D) \succeq_d \chi_d(\succ'_d, \succ_{-d})$ for all $d \in D$.*

4.3 The Mechanism

In this section, we will introduce the doctor-proposing deferred acceptance mechanism which includes three levels of institutions: hospitals, regional organisers, and the national organiser. They work in the following way.

Each hospital h is allocated p_h quotas so that the hospital can recruit as many as doctors freely but no more than the number of the allocated quota p_h. We call the allocated quotas of hospitals *hospital quotas*. Each region r is allocated $\bar{p}_r$ number of flexible quotas, where $\bar{p}_r = p_r - \sum_{h \in H_r} p_h$. We call the flexible quotas from regions *regional quotas*. The regional organiser of region r can allocate doctors freely to any hospitals in region r with respect to its regional quota. When a hospital receives a quota from the regional organiser, the hospital can recruit one more doctor than its hospital quota. The national organiser has e number of flexible quotas, where $e = n - \sum_{r \in R} p_r$. The e number of doctors can be freely allocated to any hospital. We call the flexible quotas from the nation *national quotas*. When a national quota is allocated to one hospital, one more doctor can be recruited in that hospital. As long as a hospital receives more applications than its quota, the hospital can apply for flexible quotas from its region or the nation.

The allocation of regional and national quotas is related to the market rules. That is, hospitals with a higher grade have a stronger competitive power to claim a flexible quota than those with lower grades, and hospitals with the same grade have the same competitive power to claim a flexible quota. In the second case, we aim to equalize the opportunities for hospitals with the same grade. The method is to give an order over all hospitals with the same grade i denoted as PL_i. Each hospital that claims flexible quotas obtains one flexible quota, starting from the highest ordered hospital and proceeding to the lowest ordered hospital, and then going back to the highest ordered hospital after the lowest ordered hospital according to PL_i until all flexible quotas are allocated. Denote $PL = \{PL_1, PL_2, ..., PL_l\}$ the priority list profile.

A Doctor-proposing Deferred Acceptance Mechanism

Step 0. Given any doctor-hospital model $(D, H, R, Q, HS, \succ)$ and the priority list profile PL, begin with an empty matching μ such that $\mu_i = \emptyset$ for all $i \in D \cup H$.

Step 1. Each doctor d who is currently not matched to any hospital applies to his/her best preferred remaining hospital among the hospitals that have not rejected d so far. When doctor d has no remaining hospital to propose, we remove doctor d out of matching with $\mu(d) = \emptyset$.

Step 2. Each hospital h keeps most preferred doctors with respect to the capacity based on its preference and rejects the remaining doctors. If there is any rejection, go to Step 1. Otherwise, let $D_h^{'}$ be the entire set of doctors who have applied to but not be rejected by hospital h so far. Each hospital h temporarily choose $\min\{p_h, |D_h^{'}|\}$ best preferred doctors according to preference $\succ_h$. If there is no doctor left, each remaining doctor is assigned to his/her temporarily assignment and go to Step 4. Otherwise go to Step 3.

Step 3. Begin with $e = n - \sum_{r \in R} p_r$ and $\bar{p}_r = p_r - \sum_{h \in H_r} p_h$ for all $r \in R$. Do the following processes from the set of first grade hospitals H_1 to the set of last grade hospitals H_l.

Given $PL_i = (h_1, h_2, ..., h_{|H_i|})$ for the set of hospitals with grade i, starting with h_1, proceeding to h_2,..., to $h_{|H_i|}$ and go back to h_1, each hospital $h \in H_r$ temporarily accepts one best preferred doctor from the remaining doctors who have not been temporarily accepted by the hospital h, as long as there remains either a regional quota from region r or an national quota and the modified regional maximum quota $\bar{q}_r = q_r - \sum_{h \in H_r} p_h$ is not filled. Repeat the procedure until either all flexible quotas are allocated; all regional caps are filled; or no doctor remains to be matched, and then doctors who have not been temporarily accepted by any hospital are rejected.

If there exists no rejection, each remaining doctor is assigned to his/her temporarily assignment and go to Step 4. Otherwise, go back to Step 1.

Step 4. Stop.

The doctor-proposing mechanism provides a solution to handle the minimum requirements and the maximum requirements from both hospitals and regions. To illustrate this point, we now demonstrate how the system works. At the beginning of matching, when all doctors make their applications, the national organiser receives the information of all the available number of quotas which equals to n. Each regional organiser receives p_r number of regional quotas and delivers part of these quotas to each hospital in the region. Each hospital h in the region receives p_h number of quotas from the regional organiser. Each hospital h freely chooses p_h doctors among the applications in step 2. When a hospital wants to recruit more doctors, it turns out to be step 3 . Regional organisers and the national organiser allocate their quotas based on the market rules. Example 11 shows the processes of the mechanism.

Example 11 *There are fourteen doctors $D = \{d_1, d_2, d_3, \ldots, d_{14}\}$ and seven hospitals $H = \{h_1, h_2, h_3, \ldots, h_7\}$ in the matching model. There are three regions in the market $R = \{r_1, r_2, r_3\}$, where $r_1 = \{h_1, h_2\}$, $r_2 = \{h_3, h_4\}$ and $r_3 = \{h_5, h_6, h_7\}$. Moreover, the regional minimum quota of each region is that $p_{r_1} = 3$, $p_{r_2} = 2$ and $p_{r_3} = 4$; and the regional maximum quota of each region is that $q_{r_1} = 4$, $q_{r_2} = 3$ and $q_{r_3} = 7$. We specify an order list among hospitals with the same position hs_i such that $h_i \succ h_j$ for all $i < j$. Tables 4.5 and 4.6 illustrate the preference profile, hierarchical position, and capacity for each hospital and the preference profile for each doctor respectively.*

Table 4.5 The preference profile for each hospital

Preference of h	p_h	q_h	$hs(h)$
$\succ_{h_1}: d_1, d_3, d_4, d_2, d_5, d_9, d_{11}, d_6, d_7, d_{10}, d_8, d_{12}, d_{13}, d_{14}$	1	3	1
$\succ_{h_2}: d_3, d_9, d_2, d_1, d_4, d_5, d_8, d_7, d_{10}, d_6, d_{11}, d_{12}, d_{13}, d_{14}$	1	2	2
$\succ_{h_3}: d_4, d_7, d_{11}, d_{10}, d_9, d_8, d_3, d_1, d_2, d_5, d_6, d_{12}, d_{14}, d_{13}$	1	4	1
$\succ_{h_4}: d_1, d_3, d_5, d_7, d_2, d_4, d_8, d_9, d_{10}, d_{11}, d_6, d_{12}, d_{13}, d_{14}$	1	2	2
$\succ_{h_5}: d_6, d_1, d_2, d_4, d_3, d_5, d_7, d_{11}, d_{10}, d_8, d_9, d_{12}, d_{13}, d_{14}$	1	3	1
$\succ_{h_6}: d_2, d_6, d_3, d_1, d_{10}, d_7, d_8, d_4, d_5, d_9, d_{11}, d_{12}, d_{13}, d_{14}$	1	3	2
$\succ_{h_7}: d_7, d_5, d_1, d_3, d_2, d_6, d_8, d_{11}, d_9, d_6, d_{10}, d_{12}, d_{13}, d_{14}$	1	3	1

Table 4.6 The preference profile for each doctor

Preference of d	Preference of d
$\succ_{d_1}: h_1, h_2, h_3, h_4, h_5, h_6, h_7$	$\succ_{d_8}: h_6, h_1, h_2, h_3, h_4, h_7, h_5$
$\succ_{d_2}: h_2, h_4, h_1, h_3, h_5, h_7, h_6$	$\succ_{d_9}: h_5, h_4, h_3, h_1, h_2, h_6, h_7$
$\succ_{d_3}: h_4, h_1, h_3, h_2, h_7, h_6, h_5$	$\succ_{d_{10}}: h_2, h_6, h_4, h_1, h_3, h_7, h_5$
$\succ_{d_4}: h_5, h_1, h_4, h_3, h_2, h_7, h_6$	$\succ_{d_{11}}: h_7, h_5, h_6, h_1, h_2, h_3, h_4$
$\succ_{d_5}: h_1, h_3, h_4, h_7, h_2, h_5, h_6$	$\succ_{d_{12}}: h_2, h_4, h_3, h_1, h_5, h_7, h_6$
$\succ_{d_6}: h_2, h_3, h_1, h_5, h_7, h_6, h_4$	$\succ_{d_{13}}: h_4, h_5, h_7, h_3, h_1, h_2, h_6$
$\succ_{d_7}: h_1, h_2, h_3, h_6, h_7, h_4, h_5$	$\succ_{d_{14}}: h_6, h_3, h_1, h_2, h_5, h_4, h_7$

National organiser has $e = n - \sum_{r \in R} p_r = 5$ number of flexible quotas and the number of flexible quotas from each regional organiser is that $\bar{p}_{r_1} = 1$, $\bar{p}_{r_2} = 0$, $\bar{p}_{r_3} = 1$. The capacity of flexible quotas for each regional organiser is $\bar{q}_{r_1} = 2$, $\bar{q}_{r_2} = 1$, and $\bar{q}_{r_3} = 4$. Each hospital h receive p_h number of quotas.

Round 1: Doctors d_1, d_5 and d_7 apply to hospital h_1; doctors d_2, d_6, d_{10} and d_{12} apply to hospital h_2; doctors d_3 and d_{13} apply to hospital h_4; doctors d_4 and d_9 apply to hospital h_5; doctors d_8 and d_{14} apply to hospital h_6; and doctor d_{11} applies to hospital h_7. In Step

2, h_2 rejects d_6 and d_{12} based on the preference due to the excess of its capacity and go back to Step 1.

Round 2: Doctor d_6 applies to hospital h_3; doctor d_{12} applies to hospital h_4. In Step 2, h_4 first rejects d_{13} based on the preference, due to the excess of its capacity and go back to Step 1.

Round 3: Doctor d_{13} applies to hospital h_5. In Step 2, there is no rejection. Then, each hospital tentatively accepts p_h best preferred doctors if any and go to Step 3. That is, h_1 accepts d_1, h_2 accepts d_2, h_3 accepts d_6, h_4 accepts d_3, h_5 accepts d_4, h_6 accepts d_8, and h_7 accepts d_{11}. In Step 3, we start with the set of first grade hospitals $H_1 = \{h_1, h_3, h_5, h_7\}$ and $PL_1 = (h_1, h_3, h_5, h_7)$. Thus, h_1 accepts d_5 with $e = 5$, $\bar{p_{r_1}} = 0$; h_5 accepts d_9 with $e = 5$, $\bar{p_{r_3}} = 0$; h_1 accepts d_7 with $e = 4$, $\bar{p_{r_1}} = 0$; h_5 accepts d_{13} with $e = 3$, $\bar{p_{r_1}} = 0$. Then, we consider the set of second grade hospitals $H_2 = \{h_2, h_4, h_6\}$ and $PL_2 = (h_2, h_4, h_6)$. Thus, h_2 rejects d_{10} because the capacity of flexible quotas is filled in region r_1; h_4 accepts d_{12} with $e = 2$, $\bar{p_{r_2}} = 0$; h_6 accepts d_{14} with $e = 1$, $\bar{p_{r_3}} = 0$.

Round 4: Doctor d_{10} applies to hospital h_6. In Step 2, there is no rejection. Then, each hospital tentatively accepts p_h best preferred doctors if any and go to Step 3. That is, h_1 accepts d_1, h_2 accepts d_2, h_3 accepts d_6, h_4 accepts d_3, h_5 accepts d_4, h_6 accepts d_8, and h_7 accepts d_{11}. In Step 3, we start with the set of first grade hospitals $H_1 = \{h_1, h_3, h_5, h_7\}$ and $PL_1 = (h_1, h_3, h_5, h_7)$. Thus, h_1 accepts d_5 with $e = 5$, $\bar{p_{r_1}} = 0$; h_5 accepts d_9 with $e = 5$, $\bar{p_{r_3}} = 0$; h_1 accepts d_7 with $e = 4$, $\bar{p_{r_1}} = 0$; h_5 accepts d_{13} with $e = 3$, $\bar{p_{r_1}} = 0$. Then, we consider the set of second grade hospitals $H_2 = \{h_2, h_4, h_6\}$ and $PL_2 = (h_2, h_4, h_6)$. Thus, h_2 rejects d_{10} because the capacity of flexible quotas is filled in region r_1; h_4 accepts d_{12} with $e = 2$, $\bar{p_{r_2}} = 0$; h_6 accepts d_{10} with $e = 1$, $\bar{p_{r_3}} = 0$; h_6 accepts d_{14} with $e = 0$, $\bar{p_{r_3}} = 0$. There is no rejection and the process ends.

We obtain the following matching μ

$$\mu = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 \\ d_1, d_5, d_7 & d_2 & d_6 & d_3, d_{12} & d_4, d_9, d_{13} & d_8, d_{10}, d_{14} & d_{11} \end{pmatrix}$$

4.4 Main Results

In this section, we will evaluate how better the doctor-proposing mechanism could work. We first show that the matching proceed by the mechanism is always feasible given a doctor-hospital matching problem $(D,H,R,Q,HS,\succ)$.

Proposition 6 *The Doctor-proposing Deferred Acceptance mechanism can always find a feasible matching.*

We say a doctor d claims an occupied position at hospital h in a matching μ if $h \succ_d \mu(d)$ and $d \succ_h d'$ for some doctor $d' \in \mu(h)$. Proposition 7 indicates that in the matching produced by the mechanism, none of the doctors can claim an occupied position at any hospital. We say the mechanism *treats doctor equally* if no doctor can claim an occupied position in the matching produced by the mechanism.

Proposition 7 *The Doctor-proposing Deferred Acceptance mechanism treats doctor equally.*

One of central targets in the matching theory is the stability. Empirical studies show that a stable mechanism often succeeds whereas unstable ones often fail (Kojima, 2011). However, the stability before does not work in this new model because of the distributional and the hierarchical constraints. This paper proposes a new concept of stability with respects to all quota constraints and the rules for hierarchy of hospitals. One of the main results is that the matching produced by the Doctor-proposing Deferred Acceptance mechanism is stable.

Theorem 7 *The Doctor-proposing Deferred Acceptance mechanism can always find a stable matching.*

The intuition for the stability is that the matching produced by the mechanism cannot be blocked by any doctor-hospital pair. In the mechanism, a hospital will reject a doctor when either enough better doctors have been accepted by that hospital; or it will violate either quota constrains or the market rules. The proof work is in the Appendix C. Based on Theorem 6 and Theorem 7, we have the following Corollary 3.

Corollary 3 *The matching produced by the Doctor-proposing Deferred Acceptance mechanism is efficient.*

An efficient matching means that there exists no other feasible matching which makes at least one agent better off without hurting other agents.

Whether a designed mechanism is strategy-proof for players is important, because it provides players the incentives to play the 'game' truthfully. In this paper, we focus on the doctor side and design a mechanism that is strategy-proof for doctors.

Theorem 8 *The Doctor-proposing Deferred Acceptance is strategy-proof for doctors.*

The intuition of strategy-proofness for doctors is that every doctor finds that it is non-profitable for her to misrepresent her preference in the mechanism. That is, the doctor who presents a fake preference instead of their true preference will not obtain a better assignment than the assignment given under their true preference.

4.5 Conclusion

This study has explored how to match doctors to hospitals practically in the described market and has provided a solution to handle the minimum requirements and the maximum requirements from both hospitals and regions. Our current study provides important insights into the practical design of the doctor-hospital allocation problem. Theoretically, it includes both maximum constraints and floor constraints for a subset of hospitals in the market. It provides more flexible choices on both types of constraints for hospitals, regional organisers, and the national organiser. Because of the floor constraints, it is not easy to find a feasible matching in this general model, but we demonstrate that the designed mechanism can always find a matching that respects all distributional constraints. Practically, our model reflects the real-life situation in the doctor-hospital market by taking the hierarchical constraint on hospitals into account. It generalises the structure of hospitals into the current research. Moreover, a practical doctor-proposing deferred acceptance mechanism is designed, in which the rights of recruiting doctors are decentralised into three parties: hospitals, regional organisers, and the national organiser. We have proved that the designed mechanism has several positive properties. In details, the mechanism always finds a stable matching such that no doctor and hospital can form a permissible blocking pair. It is also efficient in the sense that no agent can make themselves better off without hurting any other agents. Furthermore, stating their true preferences is a dominant strategy for doctors in the mechanism.

Appendix A

Appendices for Chapter 2

A.1 Appendix for the Existence of Core

Proof of Lemma 1: Let μ denote a matching in the core. Suppose that the matching μ is not individually rational. We have the following two cases.

Case 1. Suppose a paired patient $p^p \in P^p$ receives an assignment at matching μ that is not strongly acceptable to him. There are two situations. If we have $d^p \succ_{p^p} \mu(p^p)$. Then, a coalition $S = \{p^p\}$ can block the matching μ by constructing a redistribution ν^S such that $\nu^S(p^p) = d^p$, which is contradicting to the fact that μ is in the core. If we have $\mu(p^p) \succ_{p^p} d^p$ but $w \succ_{p^p} \mu(p^p)$. Then, a coalition $S = \{p^p\}$ can block the matching μ by constructing a redistribution ν^S such that $\nu^S(p^p) = w$, which is contradicting to the fact that μ is in the core.

Case 2. Suppose a patient on the waiting list $p^w \in P^w$ receives an unacceptable donor at matching μ. That is, $w \succ_{p^w} \mu(p^w)$. Then, a coalition $S = \{p^p\}$ can block the matching μ by constructing a redistribution ν^S such that $\nu^S(p^w) = w$, which is contradicting to the fact that μ is in the core.

□

Proof of Lemma 2: By definition, there exists a weakly individually rational matching ν amongst patients in the coalition S such that $\nu(p) \succ_p \mu(p)$ for all patient $p \in S$. Since the matching is weakly individually rational, we have $\nu(p) \neq d^p$ for each paired patient p^p in S, where d^p is the endowed donor of p^p, and $\nu(p) \neq w$ for each single patient in S. Let D^S_μ denote the set of endowed donors of resourceful patients from the coalition S. We consider a bipartite graph $G = (S, D_\mu^S \cup \{w\}, E)$ such that every patient $p \in S$ constitutes the set of vertices in S, and every donor $d \in D_\mu^S$ or the waiting list option w constitutes the set of vertices in $D_\mu^S \cup \{w\}$. There exists a set of edges E linking one vertex $p \in S$ with one vertex in $D_\mu^S \cup \{w\}$ according to an incidence function φ_G that (i) either d is the endowed donor of patient p or $\nu(p) = d$ or $\nu(p) = w$. Choose any patient $p \in S$ such that $\nu(p) \succ \mu(p)$ and let G' be maximal connected subgraph of G which contains p. Since the degree of each vertex in graph G is at most two, G' must be a chain or a cycle. Therefore, the matching μ is either improved by a chain or by a cycle.

□

Proof of Lemma 4: Let μ denote an individually rational matching of a kidney exchange problem $(P, D, \succ)$. Two types of chains are considered.

Case 1. Suppose that the matching μ can be strongly improved by a chain $C^1 = (d_1, p_1, ..., d_M, p_M)$ $(M \geq 1)$ such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1, 2, ..., M$; (ii) $d_{m+1} \succ_{p_m} \mu(p_m)$ for all $m = 1, 2, ..., M$, where $d_{M+1} = w$. Let ν denote the new matching for patients in the chain such that $\nu(p_m) = d_{m+1}$ for $m = 1, 2, ..., M-1$ and $\nu(p_M) = w$. If patient p_M is a patient on the waiting list $p_M \in P^w$, we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since μ is individually rational, $\mu(p_M) \succeq_{p_M} w$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting with the fact that p_M is resourceful. If patient p_M is a paired patient $p_M \in P^p$, we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since matching μ is individually rational, we have $\mu(p_M) \succeq_{p_M} w$ and also $\mu(p_M) \succeq_{p_M} d_M$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting to the assumption that the matching μ is strongly proved by the chain.

Case 2. Suppose that the matching μ can be strongly improved by chain $C^2 = (p_1, d_1, p_2, ..., d_{M-1}, p_M)$ $(M \geq 2)$ such that (i) $p_1 \in P^w$ and d_{m-1} is the endowed donor of the resourceful patient p_m for all $m = 2, ..., M$; (ii) $d_m \succ_{p_m} \mu(p_m)$ for all $m = 1, 2, ..., M$, where $d_M = w$. Let ν denote the new matching for patients in the chain such that $\nu(p_m) = d_m$ for $m = 1, 2, ..., M-1$ and $\nu(p_M) = w$. If patient p_M is a patient on waiting list $p_M \in P^w$, we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since μ is individually rational, $\mu(p_M) \succeq_{p_M} w$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting with the fact that p_M is resourceful. If patient p_M is a paired patient $p_M \in P^p$, we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since matching μ is individually rational, we have $\mu(p_M) \succeq_{p_M} w$

and $\mu(p_M) \succeq_{p_M} d_M$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting to the assumption that the matching μ is strongly proved by the chain.

□

Proof of Lemma 7: Consider any kidney exchange problem $(P, D, \succ)$ and let μ denote the matching produced by the kidney exchange mechanism and $\mu(p)$ be the allocation for patient p at matching μ. Suppose that the matching μ is not individually rational. There exits either a paired patient $p^p \in P^p$ with his paired donor d^p whose assignment is either $d^p \succ_{p^p} \mu(p^p)$ or $w \succ_{p^p} \mu(p^p)$ or both; or a patient on the waiting list $p^w \in P^w$ whose assignment is $w \succ_{p^w} \mu(p^w)$. Therefore, two cases are considered.

Case 1. Suppose a paired patient $p^p \in P^p$ with his paired donor d^p has either $d^p \succ_{p^p} \mu(p^p)$ or $w \succ_{p^p} \mu(p^p)$ or both. The patient p^p must be rejected from all his strongly acceptable donor and then p^p points to a better choice between his paired donor d^p and the waiting list option w. If $d^p \succ_{p^p} w$, paired patient p^p points to d^p and the patient-donor pair (p^p, d^p) is an unaltered pair. In the mechanism, if any unaltered pair (p^p, d^p) is found, the unaltered pair will be removed out of the matching with the assignment that $\mu(p^p) = d^p \succeq_{p^p} w$, which is contradicting with the assumption. If $w \succ_{p^p} d^p$, paired patient p^p points to w and turns into a newcome patient. In the mechanism, if any newcome single patient is found, the newcome single patient will be removed out of the matching with the assignment that $\mu(p^p) = w \succeq_{p^p} d^p$, which is contradicting with the assumption.

Case 2. Suppose a patient on the waiting list $p^w \in P^w$ has $w \succ_{p^w} \mu(p^w)$. The patient p^w must be rejected from all his acceptable donor and then p^w points to the waiting list option w. Therefore, the patient p^w is an unaltered single patient. In the mechanism, if any unaltered single patient is found, the unaltered single patient will be removed out of the matching with the assignment that $\mu(p^w) = w$, which is contradicting with the assumption.

□

Proof of Proposition 1: Let μ denote the matching produced by the kidney exchange mechanism of a kidney exchange model $(P, D, \succ)$. Suppose there exists a patient p who receives his assignment $\mu(p)$ at round r_p and a donor $d \in D^r$ where $r \geq r_p$ such that $d \succ_p \mu(p)$. Since the matching μ is individually rational, if p is a paired patient and his paired donor is d', we have $d \succ_p \mu(p) \succeq_p d'$ and $d \succ_p \mu(p) \succeq_p w$; and if p is a patient on the waiting list, we have $d \succ_p \mu(p) \succeq_p w$.

(i) If $r = r_p$, at the beginning of round r_p in the mechanism, p points to the best available choice. We have $\mu(p) \succeq_p d$ where $d \in D^{r_p}$ because p points to his assignment $\mu(p)$ in this round, which is contradicting to the assumption $d \succ_p \mu(p)$ for some $d \in D^{r_p}$.

(ii) If $r > r_p$. p must point to d at some round r' before round r_p and be rejected. Therefore, d is assigned to a patient $p' \neq p$ and hence $d \notin D^{r'+1}$, which is contradicting with the assumption $d \in D^r$, where $r > r_p$.

□

Proof of Proposition 2: Let μ denote the matching produced by the kidney exchange mechanism of a kidney exchange model $(P, D, \succ)$. There are two types of patients: paired patients and patients on the waiting list. Every paired patient is resourceful and a patient on the waiting list p^w is resourceful in the matching μ if $\mu(p^w) = d^s$ where $d^s \in D^s$.

Case 1. Consider any resourceful patient on the waiting list p^w in the matching μ. A patient on the waiting list p^w is resourceful if the patient is assigned a single donor $\mu(p^w) = d^s$ where $d^s \in D^s$. Therefore, we have d^s is the endowed donor of the patient on the waiting list p^w. Let r_{p^w} be the round in which the patient p^w is assigned his assignment $\mu(p^w)$ and r_{d^s} be the round in which the single donor d^s is assigned. It is obvious that $r_{p^w} = r_{d^s}$ because $\mu(p^w) = d^s$.

Case 2. Consider any paired patient p^p with his endowed donor d^p. Let r_{p^p} be the round in which the patient p^p is assigned his assignment $\mu(p^p)$ and r_{d^p} be the round in which the paired donor d^p is assigned. In round r_{p^p}, a paired patient is involved in one of the following results: an unaltered pair, a PP-TTC cycle, a CPP-TTC chain or an UPP-TTC chain or a newcome single patient.

2(a). Suppose the paired patient p^p is unaltered. In the mechanism, as long as a patient-donor pair becomes an unaltered pair, the unaltered paired patient is removed together with his paired donor. That is, $\mu(p^p) = d^p$. Therefore, we have $r_{p^p} = r_{d^p}$.

2(b). Suppose the paired patient p^p is involved in a PP-TTC cycle. A cycle is made up of paired patients and their paired donors. When a PP-TTC cycle is removed in the mechanism, all paired patients and their paired donors are assigned. Therefore, we have $r_{p^p} = r_{d^p}$.

2(c). Suppose the paired patient p^p is involved in a CPP-TTC chain. A CPP-TTC chain starts with a patient on waiting list, links with paired patients and their donors if any, and ends with a single donor. When a CPP-TTC chain is removed in the mechanism, all paired patients and their paired donors in the chain are assigned. Let r_c be the round that the CPP-TTC chains is moved out of the mechanism. Then the CPP-TTC chain may be an UPP-TTC chain at some round before the round r_c. Two situations are considered. If the paired donor d^p is not an under-demanded donor at any round $r \leq r_{d^p}$, then we have $r_{p^p} = r_{d^p}$. If paired donor d^p is an under-demanded donor at some round $r \leq r_{d^p}$, then the paired patient p^p is assigned a donor and the paired donor d^p is still available in the latter stage because p^p is involved in the CPP-TTC chain. Therefore, we have $r_{p^p} < r_{d^p}$.

2(d). Suppose the paired patient p^p is involved in an UPP-TTC chain. In an UPP-TTC chain, there exists one under-demanded donor in the chain. If the paired donor d^p is not the under-demanded donor of the UPP-TTC chain, two situations are considered. If the paired donor d^p is not an under-demanded donor at any round $r \leq r_{d^p}$, then we have $r_{p^p} = r_{d^p}$. If paired donor d^p is an under-demanded donor at some round $r \leq r_{d^p}$, then we have $r_{p^p} < r_{d^p}$. If the paired donor d^p is the under-demanded donor of the UPP-TTC chain, then the paired patient p^p is assigned a donor and the paired donor d^p is still available in the latter stage. The case is that no patient wants the paired donor and hence $\mu(d^p) = d^p$. In both cases, we have $r_{p^p} \leq r_{d^p}$.

2(e). Suppose the paired patient p^p becomes a newcome single patient and points to the waiting list w in round r_{p^p}. When a newcome single patient is found in the mechanism, we assign the newcome single patient the wating list option w and his paired donor d^p becomes a newcome single donor. If the mechanism is not stop, d^p is available in the next round. Therefore, we have $r_{p^p} < r_{d^p}$. If the mechanism stops in r_{p^p}, we have $r_{p^p} = r_{d^p}$.

Combing cases 1 and 2, we prove the proposition.

□

Proof of Theorem 1: Consider a kidney exchange problem $(P, D, \succ)$ and let μ denote the matching produced by the kidney exchange mechanism. By Lemma 7, the matching μ is individually rational. Suppose that the matching μ is not in the core. Then, the matching μ can be blocked by a coalition $S = \{p_1, p_2, ..., p_M\} \subseteq P$ ($M \geq 2$) such that there exists a redistribution ν^S of their endowed donors such that $\nu^S(p) \succ_p \mu(p)$ for every patient $p \in S$.

By Lemma 5, the coalition S constitutes a cycle which can strongly improve the matching μ. Therefore, every patient $p \in S$ is resourceful. Let r_p denote the round in which patient $p \in S$ receives his assignment $\mu(p)$ in the kidney exchange mechanism procedure.

Case 1. If $r_{p_1} = r_{p_2} = ... = r_{p_M} = r$, we have $\mu(p) \in D^r$ for every patient $p \in S$. By Proposition 1, we have $\mu(p) \succeq_p d$ for all $d \in D^r$ for every patient $p \in S$. Suppose patient $p \in S$ has $\nu^S(p) \succ_p \mu(p)$, where $\nu^S(p)$ is the endowed donor of some patient $p' \in S$. By Proposition 2, we have $\nu^S(p) \in D^r$ and hence $\mu(p) \succeq_p \nu^S(p)$, which is contradicting to the assumption that $\nu^S(p) \succ_p \mu(p)$.

Case 2. If there exists at least two patients $p, p' \in S$ such that $r_p \neq r_{p'}$. let round r be the latest round amongst $r_{p_1}, r_{p_2}, \ldots, r_{p_M}$. Label the cycle as $c^S = (d_1, p_1, d_2, p_2, ..., d_M, p_M)$ for the coalition S in which p_1 is assigned its assignment in round r such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1, 2, ..., M$; (ii) $\nu^S(p_m) = d_{m+1}$ for all $m = 1, 2, ..., M$, where $d_{M+1} = d_1$. By Propositions 1 and 2, we have $r_{p_2} \leq r_{d_2} < r$ because $\nu^S(p_1) = d_2 \succ_{p_1} \mu(p_1)$. Similarly, we have $r_{p_{n+1}} < r_{p_n}$ for $n = 1, 2, ..., M$, where $p_{M+1} = p_1$. Therefore, we have $r < r_{p_M} < r_{p_{M-1}} <, ..., < r_{p_2} < r_{p_1} = r$, where $r_{p_{M+1}} = r_{p_1} = r$, which is impossible.

Therefore, the matching μ produced by the kidney exchange mechanism cannot be improved by any cycle. By Lemma 5, the individually rational matching μ is in the core.

□

Proof of Theorem 2: Consider a kidney exchange problem $(P, D, \succ)$ and let μ denote the matching produced by the kidney exchange mechanism. Suppose the matching μ is not Pareto efficient such that there exists a redistribution ν such that $\nu(p) \succeq_p \mu(p)$ for all $p \in P$ and $\nu(p) \succ_p \mu(p)$ for some $p \in P$. Let U_μ be the set of any donor $d \in D$ who is not assigned to a patient such that $\mu(d) = d$. Suppose there exists a patient p who is $\nu(p) = d \succ_p \mu(p)$ for some $d \in U_\mu$. The matching μ is individually rational so that we have p must point to d and be rejected. Thus, $\mu(d) \neq d$, which is contradicting with the assumption. Therefore, the redistribution of any patient $p \in P$ has $\nu(p) \neq d$ for all $d \in U_\mu$.

The matching μ is individually rational and hence the redistribution ν is individually rational. In other words, we have $\nu(p^p) \succeq_{p^p} \mu(p^p) \succeq_{p^p} d^p$ and $\nu(p^p) \succeq_{p^p} \mu(p^p) \succeq_{p^p} w$ for each paired patient $p^p \in P^p$ with his paired donor d^p, and $\nu(p^w) \succeq_{p^w} \mu(p^w) \succeq_{p^w} w$ for each patient on waiting list $p^w \in P^w$.

Let $p_1 \in P$ be the patient who has $\nu(p_1) \succ_{p_1} \mu(p_1)$. Since the matchings μ and ν are individually rational, $\nu(p_1) \notin U_\mu$. If p_1 is a paired patient and d_1 is his paired donor, we have $\nu(p_1) \succ_{p_1} \mu(p_1) \succeq_{p_1} d_1$ and $\nu(p_1) \succ_{p_1} \mu(p_1) \succeq_{p_1} w$. Therefore, $\nu(p_1) \in D \setminus U_\mu \cup \{d_1\}$. If p_1 is a patient on the waiting list, we have $\nu(p_1) \succ_{p_1} \mu(p_1) \succeq_{p_1} w$. Therefore, $\nu(p_1) \in D \setminus U_\mu$ and hence $\nu(p_1) = \mu(p_2)$ for some patient p_2. That is, we have the fact that $\nu(p_1)$ is a donor not the waiting list option w. Similarly, under the strict preference profile, we have $\nu(p_2) = \mu(p_3) \succ_{p_2} \mu(p_2)$;...; $\nu(p_{M-1}) = \mu(p_M) \succ_{p_{M-1}} \mu(p_{M-1})$; $\nu(p_M) = \mu(p_1) \succ_{p_M} \mu(p_M)$, where $\nu(p_2),...,\nu(p_M) \in D \setminus U_\mu$. Since each $\mu(p_i)$ where $i = \{1,2,...,M\}$ is a donor, $(\mu(p_1), p_1, \mu(p_2), p_2, ..., \mu(p_M), p_M)$ constructs a cycle such that $\nu(p_m) = \mu(p_{m+1}) \succ_{p_m} \mu(p_m)$ for $m = 1,2,...,M$, where $\mu(p_{M+1}) = \mu(p_1)$. Let r_p denote the round in which patient $p \in S$ is assigned with his or her assignment $\mu(p)$ at matching μ in the kidney exchange mechanism procedure. Let round r be the latest round amongst $r_{p_1}, r_{p_2},..., r_{p_M}$.

Reorder the cycle $c = (\mu(p^1), p^1, \mu(p^2), p^2, ..., \mu(p^M), p^M)$, in which p^1 is assigned his or her assignment $\mu(p^1)$ in round r. By Proposition 1, we have $r_{p^2} < r$ because $\mu(p^2) \succ_{p^1} \mu(p^1)$; Similarly, we have $r_{p^{n+1}} < r_{p^n}$ for $n = 1,2,...,M$, where $p^{M+1} = p^1$. Therefore, we have $r < r_{p^M} < r_{p^{M-1}} <,...,< r_{p^{n+1}} < r_{p^n} <,...,< r_{p^1} = r$ and $r_{p^{M+1}} = r_{p^1} = r$, which is impossible.

□

Proof of Theorem 3: Consider a kidney exchange model $(P, D, \succeq)$ in which there exists some patient who has indifferent preference over donors. We can construct a new model $(P, D, \succ)$ in the following way. For every patient who has a indifferent preference, we use a tie-breaking rule with respects to the part of the patient's strict preference. Based on Theorem 2, the kidney exchange mechanism find a matching μ in the core. We will prove that the matching μ is in the core of the model $(P, D, \succeq)$. Suppose that μ is not in the core. Then μ must be strictly improved upon by a coalition S such that there exists a redistribution ν^S by exchanging resources from patients in the coalition that makes all patient in the coalition better off. That is, $\nu^S(p) \succ_p \mu(p)$ for all $p \in S$. We have $\nu^S(p) \succ_p \mu(p)$ for all $p \in S$ in both kidney exchange problems $(P,D,\succeq)$ and $(P,D,\succ)$. Therefore, μ must be strongly improved by the same coalition with respects to the constructed preference $\succ$, which is contradicting to the fact that μ is in the core of model $(P,D,\succ)$.

□

A.2 Appendix for the Existence of Strict Core

Lemma 11 *Given a kidney exchange problem $(P,D,w,\succ)$ and no paired patient prefers the waiting list option w to his paired donor, an individually rational matching μ cannot be improved upon by any chain.*

Proof of Lemma 11: Let μ denote an individually rational matching of a kidney exchange problem $(P,D,\succ)$. Two types of chains are considered.

Case 1. Suppose that the matching μ can be improved by a chain $C^1 = (d_1, p_1, ..., d_M, p_M)$ $(M \geq 1)$ such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1,2,...,M$; (ii) $d_{m+1} \succeq_{p_m} \mu(p_m)$ for all $m = 1,2,...,M$ and $d_{m+1} \succ_{p_m} \mu(p_m)$ for some $m = 1,2,...,M$, where $d_{M+1} = w$. Let ν denote the new matching for patients in the chain such that $\nu(p_m) = d_{m+1}$ for $m = 1,2,...,M-1$ and $\nu(p_M) = w$. In the case that no paired patient prefers the waiting list option w to his paired donor, patient p_M is a patient on waiting list $p_M \in P^w$ and we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since μ is individually rational, $\mu(p_M) \succeq_{p_M} w$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting with the fact that p_M is resourceful.

Case 2. Suppose that the matching μ can be improved by chain $C^2 = (p_1, d_1, p_2, ..., d_{M-1}, p_M)$ $(M \geq 2)$ such that (i) $p_1 \in P^w$ and d_{m-1} is the endowed donor of the resourceful patient p_m for all $m = 2,...,M$; (ii) $d_m \succeq_{p_m} \mu(p_m)$ for all $m = 1,2,...,M$ and $d_m \succeq_{p_m} \mu(p_m)$ for some $m = 1,2,...,M$, where $d_M = w$. Let ν denote the new matching for patients in the chain such that $\nu(p_m) = d_m$ for $m = 1,2,...,M-1$ and $\nu(p_M) = w$. Similarly, patient p_M is a patient on the waiting list $p_M \in P^w$ and we have $\nu(p_M) = w \succeq_{p_M} \mu(p_M)$. Since μ is individually rational, $\mu(p_M) \succeq_{p_M} w$. Under the strict preference $\succ$, we have $\mu(p_M) = w$, which is contradicting with the fact that p_M is resourceful.

□

By Lemma 11, we have the following result.

Lemma 12 *Given a kidney exchange problem $(P, D, w, \succ)$ and no paired patient prefers the waiting list option w to his paired donor, an individually rational matching μ is in the strict core if it cannot be improved by any cycle.*

Proof of Theorem 4: Consider a kidney exchange problem $(P, D, w, \succ)$ and let μ denote the matching produced by the kidney exchange mechanism. By Lemma 7, the matching μ is individually rational. Suppose that the matching μ is not in the strict core. Then, the matching μ can be blocked by a coalition $S = \{p_1, p_2, ..., p_M\} \subseteq P$ ($M \geq 2$) such that there exists a redistribution ν^S of their endowed donors such that $\nu^S(p) \succeq_p \mu(p)$ for every patient $p \in S$ and $\nu^S(p) \succ_p \mu(p)$ for some patient $p \in S$.

By Lemma 12, the coalition S constitutes a cycle which can improve the matching μ. Therefore, every patient $p \in S$ is resourceful. Let r_p denote the round in which patient $p \in S$ is assigned with his or her assignment $\mu(p)$ in the kidney exchange mechanism procedure.

Case 1. If $r_{p_1} = r_{p_2} = ... = r_{p_M} = r$, we have $\mu(p) \in D^r$ for every patient $p \in S$. By Proposition 1, we have $\mu(p) \succeq_p d$ for all $d \in D^r$ for every patient $p \in S$. Suppose patient $p \in S$ has $\nu^S(p) \succ_p \mu(p)$, where $\nu^S(p)$ is the endowed donor of some patient $p' \in S$. By Proposition 2, we have $\nu^S(p) \in D^r$ and hence $\mu(p) \succeq_p \nu^S(p)$, which is contradicting to the assumption that $\nu^S(p) \succ_p \mu(p)$.

Case 2. If there exists at least two patients $p, p' \in S$ such that $r_p \neq r_{p'}$. let round r be the latest round amongst r_{p_1}, r_{p_2},..., r_{p_M}. Label the cycle as $c^S = (d_1, p_1, d_2, p_2, ..., d_M, p_M)$ for the coalition S in which p_1 is assigned his or her assignment in round r such that (i) d_m is the endowed donor of the resourceful patient p_m for all $m = 1, 2, ..., M$; (ii) $\nu^S(p_m) = d_{m+1}$ for all $m = 1, 2, ..., M$, where $d_{M+1} = d_1$. By Propositions 1 and 2, we have $r_{p_2} \leq r$ because $\nu^S(p_1) = d_2 \succeq_{p_1} \mu(p_1)$ under strict preference profile $\succ$. Similarly, we have $r_{p_{n+1}} \leq r_{p_n}$ for $n = 1, 2, ..., M$, where $p_{M+1} = p_1$. Let patients p_m and p_{m+1} be the two patients such that $r_{p_m} \neq r_{p_{m+1}}$. Therefore, we have $r \leq r_{p_M} \leq r_{p_{M-1}} \leq, ..., \leq r_{p_{m+1}} < r_{p_m} \leq, ..., \leq r_{p_1} = r$, where $r_{p_{M+1}} = r_{p_1} = r$, which is impossible.

Therefore, the matching μ produced by the kidney exchange mechanism cannot be improved by any cycle. By Lemma 12, the individually rational matching μ is in the strict core.

□

Appendix B

Appendices for Chapter 3

Proof of Lemma 9: Consider any given 3-efficient matching μ as stated in the lemma. If μ consists only of cycles with no more than two blood-type compatible pairs and chains with no more than one blood-type compatible pair, we are done. Suppose to the contrary that μ contains a cycle with more than two blood-type compatible pairs or a chain with more than one blood-type compatible pair. We only need to consider the case of three-way cycles or chains. We will show that a three-way cycle with three blood-type compatible pairs can be decomposed into three single blood-compatible pairs and a three-way chain with two blood-compatible pairs can be decomposed into two single blood-compatible pairs and a one-way chain in which the single donor donates its kidney to a patient on the waiting list. Then, we will show that the all pairs which are decomposed from cycles and chains can be matched.

Because a blood-type compatible and tissue-type compatible pair can directly do transplant, all blood-type compatible and tissue-type compatible pairs can do transplants separately. Let $\mathscr{D}$ be the set of all blood-type compatible but tissue-type incompatible pairs in a three-way cycle or chain under consideration. Let $(X,Y)^i$ present the type of a blood-type compatible but tissue-type incompatible pair. If there exists two or more pairs of type $(X,Y)^i$, we can have a two-way cycle among them $(X,Y)^i - (X,Y)^i$. Therefore, at most one pair of type $(X,Y)^i$ left after the process. By Assumption 3, there exists at least one blood-type and tissue-type compatible pair of type $(X,Y)^c$. If the compatible pair $(X,Y)^c$ does not involve in any cycle or chain, then we can match the remaining pair $(X,Y)^i$ with pair $(X,Y)^c$. Otherwise, the compatible pair $(X,Y)^c$ involves in a cycle consisting of no more than two blood-type compatible pairs or a chain consisting of no more than one blood-type compatible pair. Then we can use pair $(X,Y)^i$ instead of $(X,Y)^c$ based on Assumption 1 and pair $(X,Y)^c$ do transplant directly. Therefore, all remaining pairs of type $(X,Y)^i$ can be matched. □

Proof of Proposition 4: Under Assumption 1 and 3, all blood-type compatible pairs but tissue-type incompatible pairs (A,A), (B,B), (AB,AB), (O,O), (A,O), (B,O), (AB,O), (AB,A), (AB,B) can be matched through two-way and three-way cycles. Under Assumptions 1 and 2, all pairs of type (B,A) can be matched through two-way cycles. All compatible pairs can be matched because even if paired patients from compatible pairs are not involved into two-way cycles, they can receive their own donors. As long as a kidney can be allocated to the waiting list, we can always find a compatible patient in waiting list because of the large population of patients on the waiting list. Hence, the maximal number of transplantations for patients on the waiting list, paired patients from blood-type compatible pairs and paired patients from pairs of type (B,A) is:

$$\begin{aligned}&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)\\&+\#(B,A)+\#(A,A)+\#(B,B)+\#(AB,AB)+\#(O,O)\\&+\#A^d+\#B^d+\#AB^d+\#O^d\end{aligned}$$

Let N be the maximum number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B). We first consider three-way cycles with two blood-incompatible pairs. We can match three-way cycles starting with pairs (O,AB) and three-way chains starting with single donors O^d latter because a pair (O,AB) or a single donor O^d can match with any patient. We now consider three-way cycles $(AB,A)-(A,B)-(B,AB)$, $(B,O)-(O,A)-(A,B)$ and chains $A^d-(A,B)-(B,AB)-AB^p$. By Assumption 2, all pairs (B,A) can be matched and hence the number of remaining pairs (A,B) is $\#(A,B)-\#(B,A)$. To make full advantage of those three-way cycles and chains, we need avoid the over-match problems. Consider the process that match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and then match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$. If all pair (A,B) are matched in three-way cycles $(AB,A)-(A,B)-(B,AB)$, we will lose efficiency when there are sufficient pairs (B,O) to make two-way cycles $(B,O)-(O,B)$ but insufficient pairs (AB,A) to make two-way cycles $(AB,A)-(A,AB)$. The method is to restrict the number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ by $\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$ and then release. After matching three-way cycle $(AB,A)-(A,B)-(B,AB)$ under restriction, we match three-way cycle $(B,O)-(O,A)-(A,B)$. When there are remaining pair (A,O), pair (B,AB) and pair (O,B), we will lose efficiency because one more pair can be matched by separating a three-way cycle $(B,O)-(O,A)-(A,B)$ and a two-way cycle $(AB,A)-(A,AB)$ into a two-way cycle $(B,O)-(O,B)$, a two-way cycle $(A,O)-(O,A)$ and a three-way cycle $(AB,A)-(A,B)-(B,AB)$ with remaining pairs. The method is to restrict the number of cycle $(B,O)-(O,A)-(A,B)$ by $\#(O,A)-\min\{\#(A,O),\#(O,A)\}$ and then release. Therefore, the procedure of taking full advantage of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and $(B,O)-(O,A)-(A,B)$ is as follows:

> Process 1: The number of pairs (A,B) in this process should not exceed $\#(A,B)-\#(B,A)$. Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$, where the available number of (AB,A) and A^d is $\min\{\#A^d+\#(AB,A),\#(A,AB)\}$. Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$, where the available number of (O,A) is $\#(O,A)-\min\{\#(A,O),\#(O,A)\}$.
>
> Process 2: Match a maximum number of three-way cycles $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)-AB^p$. Match a maximum number of three-way cycles $(B,O)-(O,A)-(A,B)$.

The number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) in the procedure is $2*g_1+2*g_2+2*g_3+2*g_4$, where

$$\begin{aligned}
g_1 = &\ \min\{\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\},\#(A,B)-\#(B,A),\\
&\ \#(B,AB)\}\\
g_2 = &\ \min\{\#(B,O),\#(O,A)-\min\{\#(A,O),\#(O,A)\},\#(A,B)-\#(B,A)-g_1\}\\
g_3 = &\ \min\{\#A^d+\#(AB,A)-g_1,\#(A,B)-\#(B,A)-g_1-g_2,\#(B,AB)-b_1\}\\
g_4 = &\ \min\{\#(B,O)-g_2,\#(O,A)-g_2,\#(A,B)-\#(B,A)-g_1-g_2-g_3\}
\end{aligned}$$

After the procedure, sixteen situations occur when we match remaining (O,A) with (A,O), (O,B) with remaining (B,O), (A,AB) with remaining pair (AB,A) and single donor A^d, and remaining (B,AB) with pair (AB,B) and single donor B^d.

(1) When (O,A), (O,B), (A,AB), (B,AB) remaining, we have $\min\{\#(A,O),\#(O,A)\} = \#(A,O)$, $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A),\#(A,AB)\}$, $g_2 = \min\{\#(B,O), \#(A,B) - \#(B,A) - g_1\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2\}$ and $g_4 = 0$. There is no potential gains from three-way cycles and chains with one pair (B,A) or with one blood-incompatible side because all blood-type compatible pairs are matched. Therefore, we first take full of advantage of three-way cycles and chains starting from single donor O^d and pairs of type (AB,O) and then match remaining pairs with single donor O^d and pairs of type (AB,O). The maximum number of transplants in situation (1) is:

$$
\begin{aligned}
N = \quad & 2*g_1 + 2*g_2 + 2*g_3 + w_1 + w_2 + w_3 + w_4 + 2*g_5 + 2*g_6 + 2*g_7 + 2*g_8 + w_5 \\
where & \\
w_1 = \quad & \#(A,O) \\
w_2 = \quad & \#B^d + \#(AB,B) \\
w_3 = \quad & \#A^d + \#(AB,A) - g_1 - g_3 \\
w_4 = \quad & \#(B,O) - g_2 \\
g_5 = \quad & \min\{\#O^d + \#(AB,O), \#(O,A) - g_2 - w_1, \#(A,AB) - w_3\} \\
g_6 = \quad & \min\{\#O^d + \#(AB,O) - g_5, \#(O,B) - w_4, \#(B,AB) - g_1 - g_3 - w_2\} \\
g_7 = \quad & \min\{\#O^d + \#(AB,O) - g_5 - g_6, \#(O,A) - g_2 - w_1 - g_5, \\
& \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\} \\
g_8 = \quad & \min\{\#O^d + \#(AB,O) - g_5 - g_6 - g_7, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3 - g_7, \\
& \#(B,AB) - g_1 - g_3 - w_2 - g_6\} \\
w_5 = \quad & \min\{\#O^d + \#(AB,O) - g_5 - g_6 - g_7 - g_8, \#(O,A) + \#(O,B) + \#(O,AB) \\
& + \#(A,AB) + \#(A,B) - \#(B,A) + \#(B,AB) - 2*g_1 - 2*g_2 - 2*g_3 \\
& - w_1 - w_2 - w_3 - w_4 - g_5 - g_6 - g_7 - g_8\}
\end{aligned}
$$

The maximum number of feasible transplants can be rewritten as:

$$
\begin{aligned}
N = \quad & 2*g_1 + 2*g_2 + 2*g_3 + w_1 + w_2 + w_3 + w_4 + 2*g_5 + 2*g_6 + 2*g_7 + 2*g_8 + w_5 \\
= \quad & \min\{N_1, N_3, N_6, N_7, N_{10}, N_{12}, N_{15}, N_{17}\}.
\end{aligned}
$$

One may refer to Tables from B1 to B4 in Supplement B of Cheng and Yang (2017b).

(2) When (A,O), (B,O), (A,AB), (B,AB) remaining, we have $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A),\#(A,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O),\#(O,A)\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2\}$ and $g_4 = \min\{\#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$

because there is no pair (O,B), pair (AB,B) and single donor B^d left. Moreover, there is no potential gains from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$ and $B^d-(B,O)$ because there is no pair (AB,A), pair (AB,B), single donor A^d and single donor B^d left. Since there are remaining pair (B,O), we can match remaining pair (A,B) with (B,O). Then, do the same matching process as situation (1). Because there is no remaining pair (O,A) and (O,B), we have $g_5=g_6=g_7=0$. The maximum number of transplants is:

$$
\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+s_1+2*g_8+w_5 \\
\textit{where} & \\
s_1 = \quad & \min\{\#(B,O)-g_2-g_4-w_4, \#(A,B)-\#(B,A)-g_1-g_2-g_3-g_4\} \\
g_8 = \quad & \min\{\#O^d+\#(AB,O), \#(A,B)-\#(B,A)-g_1-g_2-g_3-g_4-s_1, \\
& \#(B,AB)-g_1-g_3-w_2\} \\
w_5 = \quad & \min\{\#O^d+\#(AB,O)-g_8, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3-2*g_4-w_1 \\
& -w_2-w_3-w_4-s_1-g_8\}
\end{aligned}
$$

The maximum number of transplants N can be rewritten as

$$
\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+s_1+2*g_8+w_5 \\
= \quad & \min\{N_1, N_3, N_8, N_{10}\}.
\end{aligned}
$$

One may refer to Tables from B5 to B6 in Supplement B for detail.

(3) When (O,A), (O,B), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d+\#(AB,A), \#(A,AB)\}=\#(A,AB)\}$, $\min\{\#(A,O), \#(O,A)\}=\#(A,O)$, $g_1=\min\{\#(A,B)-\#(B,A), \#(B,AB)\}\}$, $g_2=\min\{\#(B,O), \#(A,B)-\#(B,A)-g_1\}$, $g_3=0$ and $g_4=0$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no pair (A,O) and pair (A,AB) left. Moreover, there is no potential gains from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$ and $B^d-(B,O)$ because there is no pair (A,O) and pair (B,O) left. Because there is no remaining pair (A,AB) and (B,AB), we have $g_5=g_6=g_8=0$. Because we have pair (AB,A) and single donor A^d remaining, we can first match remaining (A,B) with remaining pair (AB,A) and single donor A^d and then process the same procedure as situation (1) and the number of

transplants N is

$$\begin{aligned}
N = &\quad 2*g_1+2*g_2+w_1+w_2+w_3+w_4+s_1+2*g_7+w_5 \\
where & \\
s_1 = &\quad \min\{\#A^d+\#(AB,A)-g_1-w_3, \#(A,B)-\#(B,A)-g_1-g_2\} \\
g_7 = &\quad \min\{\#O^d+\#(AB,O), \#(O,A)-g_2-w_1, \#(A,B)-\#(B,A)-g_1-g_2-s_1\} \\
w_5 = &\quad \min\{\#O^d+\#(AB,O)-g_7, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
&\quad +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-w_1-w_2-w_3 \\
&\quad -w_4-g_7-s_1\}
\end{aligned}$$

The maximum number of transplants can be rewritten as:

$$N = 2*g_1+2*g_2+w_1+w_2+w_3+w_4+s_1+2*g_7+w_5 = \min\{N_1, N_{10}, N_{11}, N_{17}\}$$

One may refer to Table B7 in Supplement B for detail.

(4) When (A,O), (B,O), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d+\#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $g_1 = \min\{\#(A,B)-\#(B,A), \#(B,AB)\}$, $g_2 = \#(O,A)-\min\{\#(A,O), \#(O,A)\}$, $g_3 = 0$ and $g_4 = \min\{\#(O,A)-g_2, \#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no (O,B) and (A,AB) left. Because there is no remaining (O,A), (O,B), (A,AB) and (B,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O). That is, $g_5 = g_6 = g_7 = g_8 = 0$. Since there is remaining pair (B,O), pair (A,O), pair (AB,A), pair (AB,B), single donor A^d and B^d, we can match the combinations of $(AB,A)-(A,O)$, $A^d-(A,O)$, $(AB,B)-(B,O)$ and $B^d-(B,O)$ to any pair, and match pairs (AB,A) and (B,O) with remaining pair (A,B). To take full advantage of the combinations, we first reserve the maximum number of the combinations and then match remaining pairs (AB,A), (B,O) and single donor A^d with pair (A,B). Then, match remaining pairs with the combinations, single donor O^d and pairs of type (AB,O). The maximum number of transplants in situation (4) is:

$$\begin{aligned}
N = &\quad 2*g_1+2*g_2+2*g_4+w_1+w_2+w_3+w_4+s_1+w_5 \\
where & \\
c_2 = &\quad \min\{\#A^d+\#(AB,A)-g_1-w_3, \#(A,O)-w_1\} \\
c_3 = &\quad \min\{\#B^d+\#(AB,B)-w_2, \#(B,O)-g_2-g_4-w_4\}
\end{aligned}$$

$$
\begin{aligned}
s_1 = \quad & \min\{\#A^d + \#(AB,A) - g_1 - w_3 - c_2 + \#(B,O) - g_2 - g_4 - w_4 - c_3, \\
& \#(A,B) - \#(B,A) - g_1 - g_2 - g_4\} \\
w_5 = \quad & \min\{\#O^d + \#(AB,O) + c_2 + c_3, \#(O,A) + \#(O,B) + \#(O,AB) + \#(A,AB) \\
& + \#(A,B) - \#(B,A) + \#(B,AB) - 2 * g_1 - 2 * g_2 - 2 * g_4 - w_1 \\
& - w_2 - w_3 - w_4 - s_1\}
\end{aligned}
$$

The maximum number of transplants can be rewritten as:

$$
N = 2*g_1 + 2*g_2 + 2*g_4 + w_1 + w_2 + w_3 + w_4 + s_1 + w_5 = \min\{N_1, N_2, N_3, N_7, N_{10}, N_{17}\}
$$

One may refer to Tables from B8 to B11 in Supplement B for detail.

(5) When (A,O), (B,O), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $\min\{\#(A,O), \#(O,A)\} = \#(O,A)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no pair (O,B) and pair (A,AB) left. Since we have taken full advantage of three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,A)-(A,B)-(B,AB)$, $A^d-(A,B)-(B,AB)-AB^p$, all surplus pair (A,B) ($\#(A,B)-\#(B,A)$) are matched in the procedure. Therefore, there is no potential gains for $(AB,A)-(A,B)-(B,AB)$ by breaking up a two-way cycle $(A,B)-(B,A)$. Because there is no remaining (O,A), (O,B) and (A,B), there is no potential gains from three-way cycles and chains starting with single donor O^d and pairs of type (AB,O). That is, $g_5 = g_6 = g_7 = g_8 = 0$. Because all pair (A,B) are matched, we have $s_1 = 0$. Since there is remaining pair (A,O), pair (B,O), pair (AB,A) and single donor A^d, we can match the combinations $(AB,A)-(A,O)$ and $A^d-(A,O)$ to any pair. To take full advantage of pair (B,O), pair (AB,A) and single donor A^d, we do the same process as situation (4). The maximum number of transplants is

$$
N = 2*g_1 + w_1 + w_2 + w_3 + w_4 + w_5 = \quad \min\{N_1, N_3, N_7\}
$$

One may refer to Table B12 in Supplement B for detail.

(6) When (A,O), (B,O), (A,AB), $B^d/(AB,B)$ remaining, we have $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2, \#(B,AB) - b_1\}$ and $g_4 = \min\{\#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. By Assumption 2, all pairs (B,A) can be matched

in two-way cycles $(A,B)-(B,A)$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no pair (O,A), (O,B) left. There is also no potential gains from three-way cycle $(AB,B)-(B,A)-(A,AB)$ and/or three-way chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking up two-way cycle $(A,B)-(B,A)$ because there is no pair (O,A) left. Because there is no remaining (O,A), (O,B) and (B,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O). That is, $g_5=g_6=g_7=g_8=0$. There is potential gains from the combinations $(AB,B)-(B,O)$, $B^d-(B,O)$ and two-way cycles $(B,O)-(A,B)$. To take full advantage of the combinations, we do the same process as situation (4). The maximum number of transplants is

$$N=2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+s_1+w_5=\min\{N_1,N_3,N_{10}\}$$

One may refer to Tables from B13 to B18 in Supplement B for detail.

(7) When (A,O), (O,B), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d+\#(AB,A),\#(A,AB)\}=\#(A,AB)\}$, $g_1=\min\{\#(A,B)-\#(B,A),\#(B,AB)\}$, $g_2=\#(O,A)-\min\{\#(A,O),\#(O,A)\}$, $g_3=0$ and $g_4=\min\{\#(B,O)-g_2,\#(O,A)-g_2,\#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no pair (A,AB) left. Based on Assumption 2, all pairs (B,A) can be matched by two-way cycle $(A,B)-(B,A)$. There is no potential gains for $(A,O)-(O,B)-(B,A)$ by breaking up two-way cycle $(A,B)-(B,A)$. Because there is no remaining (O,A), (A,AB) and (B,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O). That is, $g_5=g_6=g_7=g_8=0$. Since there is remaining pair (A,O), pair (AB,A) and single donor A^d, we can match the combinations of $(AB,A)-(A,O)$ and $A^d-(A,O)$ to any pair and match remaining pair (AB,A) with remaining pair (A,B). To take full advantage of pair (A,O), pair (AB,A) and single donor A^d, we do the same process as situation (4). The maximum number of transplants is

$$N=2*g_1+2*g_2+2*g_4+w_1+w_2+w_3+w_4+s_1+w_5=\min\{N_1,N_{10},N_{17}\}$$

One may refer to Tables B19 and B20 in Supplement B for detail.

(8) When (O,A), (B,O), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \min\{\#(A,B) - \#(B,A), \#(B,AB)\}$, $g_2 = \#(A,B) - \#(B,A) - g_1$ and $g_3 = g_4 = 0$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ because there is no pair (O,B) and pair (A,AB) left. Since all surplus pair (A,B) $(\#(A,B)-\#(B,A))$ are matched in the procedure, there is no potential gains for $(B,O)-(O,A)-(A,B)$ by breaking up a two-way cycle $(A,B)-(B,A)$. Because there is no remaining pair (O,B), pair (A,AB), pair (A,B) and pair (B,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O), and two-way cycles $(AB,A)-(A,B)$, $(A^d-(A,B)-Y^p$ and $(B,O)-(A,B)$. That is, $s_1 = 0$ and $g_5 = g_6 = g_7 = g_8 = 0$. Since there is remaining pair (B,O), pair (AB,B) and single donor B^d, we can match the combinations $(AB,B)-(B,O)$ and $B^d-(B,O)$ to any pair. To take full advantage of the combinations, we do the same process as situation (4). The maximum number of transplants is

$$N = 2*g_1 + 2*g_2 + w_1 + w_2 + w_3 + w_4 + w_5 = \min\{N_1, N_7, N_{17}\}$$

One may refer to Table B21 in Supplement B for detail.

(9) When (A,O), (O,B), (A,AB), $B^d/(AB,B)$ remaining, we have $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2, \#(B,AB) - b_1\}$ and $g_4 = \min\{\#(B,O) - g_2, \#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. Based on Assumption 2, all (B,A) can be matched by two-way cycles $(A,B)-(B,A)$. There is no potential gains from three-way cycles and chains with one pair (B,A), $(A,O)-(O,B)-(B,A)$, $(AB,B)-(B,A)-(A,AB)$, $B^d-(B,A)-(A,AB)-AB^p$ by breaking up two-way cycle $(A,B)-(B,A)$. Because there is no remaining pair (O,A) and pair (B,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O). That is, $g_5 = g_6 = g_7 = g_8 = 0$. Since there is no remaining pair (B,O), pair (AB,A) and single donor A^d, there is no beneficial from the combinations and two-way cycles $(B,O)-(A,B)$, $(AB,A)-(A,B)$ and chain $A^d-(A,B)-Y^p$. We do the same process as situation (4) with

$c_2 = c_3 = s_1 = 0$. The maximum number of transplants is

$$N = 2*g_1 + 2*g_2 + 2*g_3 + 2*g_4 + w_1 + w_2 + w_3 + w_4 + w_5 = \min\{N_1, N_{10}\}$$

One may refer to Tables from B22 to B25 in Supplement B for detail.

(10) When (O,A), (B,O), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$. Since all surplus pair (A,B) ($\#(A,B) - \#(B,A)$) are matched in the procedure, there is no potential gains from three-way cycle $(B,O)-(O,A)-(A,B)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ by breaking up a two-way cycle $(A,B)-(B,A)$. Because there is no remaining pair (O,B), pair (A,B) and pair (A,AB), there is no potential gains from three-way cycles and chains starting from single donor O^d and pairs of type (AB,O). That is, $g_5 = g_6 = g_7 = g_8 = 0$. Because all pair (A,B) are matched, we have $s_1 = 0$. We can do the same process as situation (4). The maximum number of transplants is

$$N = 2*g_1 + w_1 + w_2 + w_3 + w_4 + w_5 = \min\{N_1, N_7\}$$

One may refer to Table B26 in Supplement B for detail.

(11) When (A,O), (O,B), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(O,A)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$.

There is potential gains from three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chains $A^d-(A,B)-(B,AB)$ by breaking two-way cycle $(A,B)-(B,A)$ because two more blood-type incompatible pairs of types (O,B) and (B,AB) can be matched in this case. Since all surplus pairs (A,B) ($\#(A,B)-\#(B,A)$) are matched in Step 1, the number of remaining (A,B) equals to $\#(B,A)$. Therefore, to take full advantage of pairs (B,A) and (A,B), we match the maximum number of $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ bounded by the number of remaining pairs (A,O), (O,B), $A^d/(AB,A)$, (B,AB) and (B,A). If all remaining pairs (AB,A) and single donors A^d are matched, there is potential gains from three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$

by breaking two-way cycle $(AB,A)-(A,AB)$ and chain $A^d-(A,AB)-AB^p$ because one more pair can be matched in this case. Similarly, if all remaining pairs (A,O) are matched, there is potential gains from three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ by breaking two-way cycle $(A,O)-(O,A)$. Because there is either no remaining pair (A,B) and pair (O,A) or no remaining pair (O,A) and pair (A,AB), there is no potential gains from three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$, $O^d-(A,B)-(B,AB)-AB^p$. That is, $g_5=g_7=g_8=0$. Since there is remaining pair (A,O), pair (AB,A) and single donor A^d, we can match the combinations $(AB,A)-(A,O)$ and $A^d-(A,O)$ to any pair. Since there is no reaming (A,B), there is no potential gains by matching remaining pair (AB,A) and single donor A^d with remaining pair (A,B). Therefore, the maximum number of transplants in situation (11) is

$$
\begin{aligned}
N = \quad & 2*g_1+w_1+w_2+w_3+w_4+2*u_1+v_1+v_2+2*g_6+w_5 \\
where \quad & \\
u_1 = \quad & \min\{\#(A,O)-w_1,\#(O,B)-w_4,\#A^d+\#(AB,A)-g_1-w_3, \\
& \#(B,AB)-g_1-w_2,\#(B,A)\} \\
v_1 = \quad & \min\{\#A^d+\#(AB,A)-g_1-u_1,\#(A,O)-w_1-u_1,\#(O,B)-w_4-u_1, \\
& \#(B,AB)-g_1-w_2-u_1,\#(B,A)-u_1\} \\
v_2 = \quad & \min\{\#(A,O)-u_1,\#A^d+\#(AB,A)-g_1-u_1,\#(O,B)-w_4-u_1, \\
& \#(B,AB)-g_1-w_2-u_1,\#(B,A)-u_1\} \\
c_2 = \quad & \min\{\#A^d+\#(AB,A)-w_3-u_1-v_2,\#(A,O)-w_1-u_1-v_1\} \\
g_6 = \quad & \min\{\#O^d+\#(AB,O),\#(O,B)-w_4-u_1-v_1-v_2,\#(B,AB)-g_1-u_1-w_2 \\
& -v_1-v_2\} \\
w_5 = \quad & \min\{\#O^d+\#(AB,O)+c_2-g_6,\#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-w_1-w_2-w_3-w_4-2*u_1 \\
& -v_1-v_2-2*g_6\}
\end{aligned}
$$

The maximum number of transplants is

$$
\begin{aligned}
N = \quad & 2*g_1+w_1+w_2+w_3+w_4+2*u_1+v_1+v_2+2*g_6+w_5 \\
= \quad & \min\{N_1,N_3,N_7,N_8,N_9,N_{10},N_{14},N_{15},N_{16},N_{17}\}
\end{aligned}
$$

One may refer to Tables from B27 to B30 in Supplement B for detail.

(12) When (O,A), (O,B), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$. Because all pairs (A,O) and pairs(B,O) are matched, there is no potential gains from the combinations. Since there is no reaming (A,B), there is no potential gains by matching remaining pair (AB,A) and single donor A^d with remaining pair (A,B). There is potential gains from three-way cycle $(A,O)-(O,B)-(B,A)$, three-way cycle $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ by breaking two-way cycle $(A,O)-(O,A)$ because one more pair can be matched in this case. That is, $v_2 \neq 0$. Because there is either no remaining pair (O,A) and pair (A,B) or no remaining pair (O,A) and pair (B,AB), there is no potential gains from three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$, $O^d-(A,B)-(B,AB)-AB^p$. That is, $g_5 = g_7 = g_8 = 0$. We can do the same process in situation (11). The maximum number of transplants is

$$N = 2*g_1 + w_1 + w_2 + w_3 + w_4 + v_2 + 2*g_6 + w_5 = \min\{N_1, N_7, N_{14}, N_{15}, N_{16}, N_{17}\}$$

One may refer to Table B31 in Supplement B for detail.

(13) When (A,O), (O,B), (A,AB), (B,AB) remaining, we have $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2\}$ and $g_4 = \min\{\#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. Because no remaining pair (O,A) is left, there is no potential gains from three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$ and chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$. That is, $g_5 = g_7 = 0$. Because all pairs (AB,A), (AB,B) and single donors A^d, B^d are matched, there is no potential gains from the combinations. Since no reaming pair (B,O), pair (AB,B) and single donor A^d is left, there is no potential gains by matching remaining pair (AB,A), pair (B,O) and single donor A^d with remaining pair (A,B). There is potential gains from three-way cycle $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ by breaking two-way cycle $(AB,A)-(A,AB)$ and chain $A^d-(A,AB)-AB^p$ because one more pair can be matched in this case. That is, $v_1 \neq 0$. To take full advantage of three-way cycles and chains, we first match $(A,O)-(O,B)-(B,A)$,

$(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$, and match three-way cycles $(AB,O)-(O,B)-(B,AB)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$, $O^d-(A,B)-(B,AB)-AB^p$ if any. Then, we match remaining pairs with pair (AB,O) and single donor O^d. Therefore, the maximum number of transplants in situation (13) is

$$\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+v_1+2*g_6+2*g_8+w_5 \\
\textit{where} \quad & \\
g_8 = \quad & \min\{\#O^d+\#(AB,O)-g_6, \#(A,B)-\#(B,A)-g_1-g_2-g_3-g_4, \\
& \#(B,AB)-g_1-g_3-w_2-v_1-g_6\} \\
w_5 = \quad & \min\{\#O^d+\#(AB,O)-g_6-g_8, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3-2*g_4 \\
& -w_1-w_2-w_3-w_4-v_1-2*g_6-2*g_8\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+v_1+2*g_6+2*g_8+w_5 \\
= \quad & \min\{N_1, N_3, N_8, N_9, N_{10}, N_{15}\}
\end{aligned}$$

One may refer to Tables from B32 to B37 in Supplement B for detail.

(14) When (O,A), (B,O), (A,AB), $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $g_1 = \#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A), \#(A,AB)\}$, $g_2 = \#(A,B)-\#(B,A)-g_1$ and $g_3 = g_4 = 0$. Because no remaining pair (O,B), (A,B), (B,AB) is left, there is no potential gains from three-way cycles $(AB,O)-(O,B)-(B,AB)$, $(AB,O)-(O,A)-(A,B)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$, $O^d-(A,B)-(B,AB)-AB^p$. That is, $g_6 = g_7 = g_8 = 0$. Since no remaining pair $A^d/(AB,A)$, there is no potential gains from two-way cycle $(AB,A)-(A,B)$ and chain $A^d-(A,B)-Y^p$. There is potential gains from three-way cycle $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking two-way cycle $(A,B)-(B,A)$ because two more blood-type incompatible pairs of types (O,A) and (A,AB) can be matched in this case. Since all surplus pairs (A,B) ($\#(A,B)-\#(B,A)$) are matched in Step 1, the number of remaining (A,B) equals to $\#(B,A)$. Therefore, we take full advantage of (B,A), (A,B) and match the maximum number of $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ bounded by the number of remaining pairs (O,A), (B,O),

(A,AB), $B^d/(AB,B)$ and (B,A). If all remaining pairs (AB,B) and single donors B^d are matched, there is potential gains from three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking two-way cycle $(AB,B)-(B,AB)$ and chain $B^d-(B,AB)-AB^p$ because one more pair can be matched in this case. Similarly, if all remaining pairs (B,O) are matched, there is potential gains from three-way cycles $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking two-way cycle $(B,O)-(O,B)$.

Since there is remaining pair (B,O), pair (AB,B) and single donor B^d, we can match the combinations of $(AB,B)-(B,O)$ and $B^d-(B,O)$ to any pair. Therefore, the maximum number of transplants in situation (14) is

$$\begin{aligned}
N = \quad & 2*g_1+2*g_2+w_1+w_2+w_3+w_4+2*u_2+v_3+v_4+2*g_5+w_5 \\
where \quad & \\
u_2 = \quad & \min\{\#(B,O)-g_2-w_4,\#(O,A)-g_2-w_1,\#(A,AB)-w_3, \\
& \#B^d+\#(AB,B)-w_2,\#(B,A)\} \\
v_3 = \quad & \min\{\#B^d+\#(AB,B)-u_2,\#(B,O)-g_2-w_4-u_2,\#(O,A)-g_2-w_1-u_2, \\
& \#(A,AB)-w_3-u_2,\#(B,A)-u_2\} \\
v_4 = \quad & \min\{\#(B,O)-g_2-u_2,\#B^d+\#(AB,B)-w_2-u_2,\#(O,A)-g_2-w_1-u_2, \\
& \#(A,AB)-w_3-u_2,\#(B,A)-u_2\} \\
c_3 = \quad & \min\{\#B^d+\#(AB,B)-w_2-u_2-v_4,\#(B,O)-w_4-u_2-v_3\} \\
g_5 = \quad & \min\{\#O^d+\#(AB,O),\#(O,A)-g_2-w_1-u_2-v_3-v_4,\#(A,AB)-w_3-u_2 \\
& -v_3-v_4\} \\
w_5 = \quad & \min\{\#O^d+\#(AB,O)+c_3-g_5,\#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-w_1-w_2-w_3-w_4 \\
& -2*u_2-v_3-v_4-2*g_5\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned}
N = \quad & 2*g_1+2*g_2+w_1+w_2+w_3+w_4+2*u_2+v_3+v_4+2*g_5+w_5 \\
= \quad & \min\{N_1,N_3,N_4,N_5,N_7,N_{10},N_{11},N_{13},N_{15},N_{17}\}
\end{aligned}$$

One may refer to Tables from B38 to B41 in Supplement B for detail.

(15) When (O,A), (B,O), (A,AB), (B,AB) remaining, we have $\min\{\#(A,O),\#(O,A)\} = \#(A,O)$, $g_1 = \#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$, $g_2 = \#(A,B)-\#(B,A)-g_1$ and $g_3 = g_4 = 0$. Because no remaining pair (O,B), (A,B) is left, there is no potential gains from three-way cycles $(AB,O)-(O,B)-(B,AB)$, $(AB,O)-(O,A)-$

(A,B), $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$, $O^d-(A,B)-(B,AB)-AB^p$. That is, $g_6=g_7=g_8=0$. Because all pairs (AB,A), (AB,B) and single donors A^d, B^d are matched, there is no potential gains from the combinations. Since no remaining pair $A^d/(AB,A)$, there is no potential gains from two-way cycle $(AB,A)-(A,B)$ and chain $A^d-(A,B)-Y^p$. There is potential gains from three-way cycle $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking two-way cycle $(AB,B)-(B,AB)$ and chain $B^d-(B,AB)-AB^p$ because one more pair can be matched in this case. That is, $v_3\neq 0$. We can do the same process in situation (14). The maximum number of transplants is

$$N=2*g_1+2*g_2+w_1+w_2+w_3+w_4+v_3+2*g_5+w_5=\min\{N_1,N_3,N_4,N_5,N_7,N_{15}\}$$

One may refer to Table B42 in Supplement B for detail.

(16) When (O,A), (O,B), (A,AB), $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O),\#(O,A)\}=\#(A,O)$, $g_1=\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$, $g_2=\min\{\#(B,O),\#(A,B)-\#(B,A)-g_1\}$, $g_3=\min\{\#A^d+\#(AB,A)-g_1,\#(A,B)-\#(B,A)-g_1-g_2,\#(B,AB)-b_1\}$ and $g_4=0$. Because no remaining pair (B,AB) is left, there is no potential gains from three-way cycles $(AB,O)-(O,B)-(B,AB)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$, $O^d-(A,B)-(B,AB)-AB^p$. That is, $g_6=g_8=0$. Because all pairs (B,O) and (A,O) are matched, there is no potential gains from the combinations. Since all pair $A^d/(AB,A)$ and (B,O) are matched, there is no potential gains from two-way cycles $(AB,A)-(A,B)$, $(B,O)-(A,B)$ and chain $A^d-(A,B)-Y^p$. There is potential gains from three-way cycle $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$ by breaking two-way cycle $(B,O)-(O,B)$ because one more pair can be matched in this case. That is, $v_4\neq 0$. To take full advantage of three-way cycles and chains, we first match $(B,O)-(O,A)-(A,B)$, $(AB,B)-(B,A)-(A,AB)$ and chain $B^d-(B,A)-(A,AB)-AB^p$, and match three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$ and chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$ if any. Then, we match remaining pairs with pair (AB,O) and single donor O^d. Therefore, the maximum number of transplants in

situation (16) is

$$
\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+v_4+2*g_5+2*g_7+w_5 \\
\textit{where} & \\
g_7 = \quad & \min\{\#O^d+\#(AB,O)-g_5, \#(O,A)-g_2-w_1-v_4-g_5, \\
& \#(A,B)-\#(B,A)-g_1-g_2-g_3\} \\
w_5 = \quad & \min\{\#O^d+\#(AB,O)-g_5-g_7, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3-w_1-w_2-w_3 \\
& -w_4-v_4-2*g_5-2*g_7\}
\end{aligned}
$$

The maximum number of transplants is

$$
\begin{aligned}
N = \quad & 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+v_4+2*g_5+2*g_7+w_5 \\
= \quad & \min\{N_1, N_{10}, N_{11}, N_{13}, N_{15}, N_{17}\}
\end{aligned}
$$

One may refer to Tables from B43 to B46 in Supplement B for detail. Combining cases (1) to (16), we have proved that the maximum number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) is

$$
N = \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}\}
$$

We now prove that the sequential mechanism is 3-efficient which achieves the maximum number of transplants in the pool by exploring every possible route. Figures 3.3 and 3.4 show all possible three-way cycles and chains can occur in the mechanism.

Since for every one/two-way chains, we can always find a compatible patient on the waiting list in the mechanism, the number of transplantations for patients on the waiting list through one/two/three-way chains in the mechanism equals to $\#A^d+\#B^d+\#AB^d+\#O^d$. Based on Assumption 3, all pairs of type $(A,A)^i$, $(B,B)^i$, $(O,O)^i$, $(AB,AB)^i$ can be matched through two-way in Step 1. Based on Assumption 3, all pairs of type $(A,O)^i$, $(B,O)^i$, $(AB,O)^i$, $(AB,A)^i$, $(AB,B)^i$ can be matched through two-way and three-way cycles from Step 2 to Step 7 in the mechanism. All compatible pairs $(A,O)^c$, $(B,O)^c$, $(AB,O)^c$, $(AB,A)^c$, $(AB,B)^c$, $(A,A)^c$, $(B,B)^c$, $(O,O)^c$, $(AB,AB)^c$ can be matched either through two-way/three-way cycles or doing transplantations with their own donors. Moreover, under Assumption 2, all pairs of type (B,A) can be matched through two-way cycle $(A,B)-(B,A)$ or three-way cycle $(AB,B)-(B,A)-(A,AB)$, $(A,O)-(O,B)-$

(B,A) or three-way chain $B^d-(B,A)-(A,AB)-AB^p$ in Step 3 and Step 4. Hence, the number of transplantations for compatible pairs, blood-type compatible pairs and pairs of type (B,A) in the mechanism is

$$\begin{aligned}&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)\\&+\#(B,A)+\#(A,A)+\#(B,B)+\#(AB,AB)+\#(O,O)\end{aligned}$$

Next, we prove that the maximum number of transplants for blood-type incompatible pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) can be achieved in the mechanism.

Denote X_2 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 2 so that

$$\begin{aligned}X_2 = \quad & 2*b_1+2*b_2+2*b_3+2*b_{21}\\ where & \\ b_1 = \quad & \min\{\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\},\\ & \#(A,B)-\#(B,A),\#(B,AB)\}\\ b_2 = \quad & \min\{\#(B,O),\#(O,A)-\min\{\#(A,O),\#(O,A)\},\#(A,B)-\#(B,A)-b_1\}\\ b_3 = \quad & \min\{\#A^d+\#(AB,A)-b_1,\#(A,B)-\#(B,A)-b_1-b_2,\#(B,AB)-b_1\}\\ b_{21} = \quad & \min\{\#(B,O)-b_2,\#(O,A)-b_2,\#(A,B)-\#(B,A)-b_1-b_2-b_3\}\end{aligned}$$

Denote X_3 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 3 so that

$$\begin{aligned}X_3 = \quad & 2*e_1+2*e_2+2*f_2+2*f_3+2*f_4+2*f_5\\ where & \\ e_1 = \quad & \min\{\#(A,O)-a_1,\#(B,A),\#A^d+\#(AB,A)-b_1-b_3-a_1,\\ & \#(O,B)-a_4,\#(B,AB)-b_1-b_3-a_2\}\\ e_2 = \quad & \min\{\#B^d+\#(AB,B)-a_2,\#(A,AB)-a_3,\#(B,O)-b_2-a_4-b_{21},\\ & \#(O,A)-b_2-b_{21}-a_1,\#(B,A)\}\\ f_2 = \quad & \min\{\#(A,O)-a_1-e_1,\#(B,A)-e_1-e_2,\#A^d+\#(AB,A)-b_1-b_3-e_1,\\ & \#(O,B)-a_4-e_1,\#(B,AB)-b_1-b_3-a_2-e_1\}\\ f_3 = \quad & \min\{\#B^d+\#(AB,B)-a_2-e_2,\#(A,AB)-a_3-e_2,\#(B,O)-b_2-e_2-b_{21},\\ & \#(O,A)-b_2-b_{21}-a_1-e_2,\#(B,A)-e_1-e_2\}\\ f_4 = \quad & \min\{\#(A,O)-e_1,\#(B,A)-e_1-e_2,\#A^d+\#(AB,A)-b_1-b_3-e_1-a_3,\\ & \#(O,B)-a_4-e_1,\#(B,AB)-b_1-b_3-a_2-e_1\}\\ f_5 = \quad & \min\{\#B^d+\#(AB,B)-e_2,\#(A,AB)-a_3-e_2,\#(B,O)-b_2-e_2-a_4-b_{21},\\ & \#(O,A)-b_2-b_{21}-a_1-e_2,\#(B,A)-e_1-e_2\}\end{aligned}$$

$$
\begin{aligned}
&where \\
a_1 = &\quad \min\{\#(A,O), \#(O,A) - b_2 - b_{21}\} \\
a_2 = &\quad \min\{\#B^d + \#(AB,B), \#(B,AB) - b_1 - b_3\} \\
a_3 = &\quad \min\{\#A^d + \#(AB,A) - b_1 - b_3, \#(A,AB)\} \\
a_4 = &\quad \min\{\#(B,O) - b_2 - b_{21}, \#(O,B)\}
\end{aligned}
$$

Denote X_4 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 4 so that

$$
\begin{aligned}
X_4 = &\quad \#(B,A) + p_1 + p_2 + p_3 + p_4 + a_8 \\
&where \\
p_1 = &\quad a_1 - f_4 \\
p_2 = &\quad a_2 - f_5 \\
p_3 = &\quad a_3 - f_2 \\
p_4 = &\quad a_4 - f_3 \\
a_8 = &\quad \min\{\#A^d + \#(AB,A) - b_1 - b_3 - e_1 - f_2 - f_4 - p_3 - c_2 + \#(B,O) - b_2 - b_{21} \\
&\quad - e_2 - f_3 - f_5 - p_4 - c_3, \#(A,B) - \#(B,A) - b_1 - b_2 - b_3 - b_{21}\} \\
&where
\end{aligned}
$$

$$
\begin{aligned}
c_2 = &\quad \min\{\#A^d + \#(AB,A) - b_1 - b_3 - e_1 - f_2 - f_4 - p_3, \#(A,O) - e_1 - f_2 - f_4 - p_1\} \\
c_3 = &\quad \min\{\#(B,O) - b_2 - b_{21} - e_2 - f_3 - f_5 - p_4, \#B^d + \#(AB,B) - e_2 - f_3 - f_5 - p_2\}
\end{aligned}
$$

Denote X_5 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 5 so that

$$
\begin{aligned}
X_5 = &\quad 2 * b_4 + 2 * b_5 + 2 * b_6 + 2 * b_7 \\
&where \\
b_4 = &\quad \min\{\#O^d + \#(AB,O), \#(O,A) - b_2 - b_{21} - e_2 - f_3 - f_5 - p_1, \\
&\quad \#(A,AB) - e_2 - f_3 - f_5 - p_3\} \\
b_5 = &\quad \min\{\#O^d + \#(AB,O) - b_4, \#(O,B) - e_1 - f_2 - f_4 - p_4, \\
&\quad \#(B,AB) - b_1 - b_3 - e_1 - f_2 - f_4 - p_2\} \\
b_6 = &\quad \min\{\#O^d + \#(AB,O) - b_4 - b_5, \#(O,A) - b_2 - b_{21} - e_2 - f_3 - f_5 - p_1 - b_4, \\
&\quad \#(A,B) - \#(B,A) - b_1 - b_2 - b_3 - b_{21} - a_8\} \\
b_7 = &\quad \min\{\#O^d + \#(AB,O) - b_4 - b_5 - b_6, \#(A,B) - \#(B,A) - b_1 - b_2 \\
&\quad - b_3 - b_{21} - a_8 - b_6, \#(B,AB) - b_1 - b_3 - e_1 - f_2 - f_4 - p_2 - b_5\}
\end{aligned}
$$

Denote X_6 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 6 so that

$$\begin{aligned} X_6 = a_9 = \quad & \min\{\#O^d + \#(AB,O) - b_4 - b_5 - b_6 - b_7 + c_2 + c_3, \#(O,A) \\ & + \#(O,B) + \#(O,AB) + \#(A,AB) + \#(A,B) + \#(B,AB) - 2*b_1 \\ & -2*b_2 - 2*b_3 - 2*b_{21} - 2*e_1 - 2*e_2 - 2*f_2 - 2*f_3 - 2*f_4 \\ & -2*f_5 - p_1 - p_2 - p_3 - p_4 - a_8 - 2*b_4 - 2*b_5 - 2*b_6 - 2*b_7\} \end{aligned}$$

Therefore, the total number of transplants for paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) in the mechanism is $X = X_2 + X_3 + X_4 + X_5 + X_6$. The equation can be rewritten as follows:

$$X = \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}\}$$

One may refer to Tables from B47 to B82 in Supplement B for detail. Therefore, the total number of transplants can be achieved in the mechanism is that

$$\begin{aligned} & \#(A,O) + \#(B,O) + \#(AB,O) + \#(AB,A) + \#(AB,B) \\ & + \#(B,A) + \#(A,A) + \#(B,B) + \#(AB,AB) + \#(O,O) \\ & + \#A^d + \#B^d + \#AB^d + \#O^d \\ & + \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}\} \end{aligned}$$

Therefore, we proved that every matching produced by the mechanism achieves the maximum number of transplants in the pool and hence the mechanism is 3-efficient.

□

Proof of Lemma 10: Consider any given 4-efficient matching μ as stated in the lemma. If μ consists only of cycles with no more than two blood-type compatible pairs and chains with no more than one blood-type compatible pair, we are done. Suppose to the contrary that μ contains a cycle with more than two blood-type compatible pairs or a chain with more than one blood-type compatible pair. We only need to consider the case of four-way cycles or chains, as the case of three-way cycles or chains is reduced to that of Lemma 9.

A four-way cycle may consist of either four or three blood-type compatible pairs and a four-way chain may consist of either three or two blood-type compatible pairs. Such a cycle or chain can be decomposed into small cycles or chains as follows:

a. In a four-way cycle with four blood-compatible pairs, we can decompose it into four single blood-compatible pairs.

b. In a four-way chain with three blood-type compatible pairs, we can decompose it into three single blood-compatible pairs and a one-way chain in which the single donor gives its kidney to a patient on the waiting list.

c. A four-way cycle with three blood-compatible pairs can be split into a three-way cycle with two blood-compatible pairs and one blood-compatible pair.

Let $(P_1, D_1) - (P_2, D_2) - (P_3, D_3) - (P_4, D_4)$ denote a four-way cycle with three blood-compatible pairs. If there exists pair (X, X) from types (A,A), (B,B), (O,O) and $AB,AB)$, then the four-way cycle can be separated into a three-way cycle and a blood-type compatible pair (X, X). If there exists pair of type (AB, O) in the cycle, then the four-way cycle can be separated into a three-way cycle and a blood-type compatible pair because pair (AB, O) is compatible with any pair by Assumption 1. If there exists pairs of types (AB,A) and (A, O) in the cycle, then the cycle can be separated into a three-way starting with $(AB,A) - (A, O)$ and a blood-type compatible pair because the combination of $(AB,A) - (A, O)$ is compatible with any pair. Similar to pairs of types (AB, B) and (B, O).

Now we consider other four-way cycles without pairs (AB, O), (A,A), (B,B), (O,O), $AB,AB)$ and the combinations of $(AB,A) - (A, O)$ and $(AB, B) - (B, O)$. In a four-way cycle $(A, O) - (P_2, D_2) - (P_3, D_3) - (P_4, D_4)$, two cases occur. In the first case, there exists one (B, O) and hence there is no (AB, O), (AB,A), (AB, B) in the cycle. Therefore, the cycle have either two pairs (A, O) or two pairs (B, O), then the four-way can be separated into a three-way cycle and one blood-type compatible pair. In the second case, there exists pair (AB, B) and hence there is no (AB, O), (AB,A), (B, O) in the cycle. Therefore, the cycle have either two pairs (A, O) or two pairs (AB, B), then the four-way can be separated into a three-way cycle and one blood-type compatible pair. The similar proof can be applied to four-way cycle $(B, O) - (P_2, D_2) - (P_3, D_3) - (P_4, D_4)$, $(AB,A) - (P_2, D_2) - (P_3, D_3) - (P_4, D_4)$ and $(AB, B) - (P_2, D_2) - (P_3, D_3) - (P_4, D_4)$.

d. A four-way chain with two blood-type compatible pair can be decomposed into two-way or three-way cycles with at most two blood-compatible pairs or chains with at most one blood-compatible pair.

Let $X^d - (P_1, D_1) - (P_2, D_2) - (P_3, D_3) - Y^p$ denote a four-way chain with two blood-compatible pairs. Firstly, from the previous proof, if there exists either pair (AB, O), or pairs (AB,A) and (A,O), or pairs (AB,B) and (B,O), the chain can be separated into a three-way cycle and a one-way chain such that the single donor donates its kidney to a patient on the waiting list. Secondly, if there exist a single donor O^d in the chain, the the four-way chain can be separated into a three-way chain and a blood-type compatible pair because single donor O^d is compatible with any patient by Assumption 1. If there exists single donor A^d and pair (A,O) in the chain, then the chain can be separated into a three-way starting with $A^d-(A,O)$ and a blood-type compatible pair because the combination of $A^d-(A,O)$ is compatible with any pair. Similar to single donor B^d and pair (B,O). Thirdly, if there exists pair (X,X) from types (A,A), (B,B), (O,O) and $AB,AB)$, then the four-way chain can be separated into a three-way chain and a blood-type compatible pair (X,X). Fourthly, we consider the situation that there exists a blood-type compatible pair (AB,D) in the four-way chain. If pair (P_1,D_1) is type (AB,D), then the chain can be divided into a three-way cycle $(P_1,D_1)-(P_2,D_2)-(P_3,D_3)$ and a one-way chain X^d-Y^p because patient of type AB can receive kidney from any donor by Assumption 1. Similarly, if pair (P_2,D_2) is type (AB,D), then the chain can be divided into either a three-way chain $X^d-(P_1,D_1)-(P_2,D_2)-Y^p$ and a blood-type compatible pair (P_3,D_3) if pair (P_1,D_1) is blood-type incompatible pair; or a two-way cycle $(P_2,D_2)-(P_3,D_3)$ and two-way chain $X^d-(P_1,D_1)-Y^p$ if pair (P_3,D_3) is blood-type incompatible pair. If pair (P_3,D_3) is type (AB,D), the chain can be separated into a three-way chain $X^d-(P_1,D_1)-(P_2,D_2)-Y^p$ and a blood-type compatible pair (P_3,D_3).

Now we consider other four-way chains without single donor O^d, pairs (AB,O), (AB,A), (AB,B), (A,A), (B,B), (O,O), $AB,AB)$ and the combinations of $A^d/(AB,A)-(A,O)$, $B^d/(AB,B)-(B,O)$. Hence, a four-way chain $A^d-(P_2,D_2)-(P_3,D_3)-(P_4,D_4)$ has two pairs of type (B,O) and can be separated into a three-way chain and a blood-type compatible pair (B,O). A four-way chain $B^d-(P_2,D_2)-(P_3,D_3)-(P_4,D_4)$ has two pairs of type (A,O) and can be separated into a three-way chain and a blood-type compatible pair (A,O). In a four-way chain $AB^d-(P_2,D_2)-(P_3,D_3)-(P_4,D_4)$, we have pair (P_2,D_2) of type (AB,D_2) because donor AB^d can only donate to patient of type AB. Hence, the chain can be separated into a three-way cycle starting with (P_2,D_2) and a one-way chain.

Therefore, every cycle and chain under consideration can be decomposed into a smaller cycle or chain or a blood-type compatible pair. Then, we will show that the same set of pairs which are decomposed from cycles and chains is matched. Because a blood-type compatible and tissue-type compatible pair can directly do the transplant, all blood-type compatible and tissue-type compatible pairs can do the transplants separately. Let $\mathscr{D}$ be the set of all blood-type compatible but tissue-type incompatible pairs in a cycle or chain under consideration. Let $(X,Y)^i$ present the type of a blood-type compatible but tissue-type incompatible pair. If there exists two or more pairs of type $(X,Y)^i$, we can have a two-way cycle among them $(X,Y)^i-(X,Y)^i$. Therefore, at most one pair of type $(X,Y)^i$ left after the process. By Assumption 3, there exists at least one blood-type and tissue-type compatible pair of type $(X,Y)^c$. If the compatible pair $(X,Y)^c$ does not involve in any cycle or chain, then we can match the remaining pair $(X,Y)^i$ with pair $(X,Y)^c$. Otherwise, the compatible pair $(X,Y)^c$ involves in a cycle consisting of no more than two blood-type compatible pairs or a chain consisting of no more than one blood-type compatible pair. Then we can use pair $(X,Y)^i$ instead of $(X,Y)^c$ based on Assumption 1 and pair $(X,Y)^c$ do transplant directly. Therefore, all remaining pairs of type $(X,Y)^i$ can be matched.

□

Proof of Proposition 5: The proof is similar to one in the case of three-way cycles and chains. Let N be the maximum number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) when four-way cycles and chains are considered. We will prove that

$$N = \min\{N_1,N_2,N_3,N_4,N_5,N_6,N_7,N_8,N_9,N_{10},N_{11}\}$$

Sixteen situations are considered. We only discuss the potential gains from four-way cycles and chains because the analysis of cycles and chains other than four-way exchange in each situation is similar to one in the case of three-way cycles and chains.

(1) When (O,A), (O,B), (A,AB), (B,AB) remaining, we have $\min\{\#(A,O),\#(O,A)\} = \#(A,O)$, $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \min\{\#(B,O), \#(A,B) - \#(B,A) - g_1\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2\}$ and $g_4 = 0$. There is potential gains from four-way cycle $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ if any pair (A,B) remains. Because

all remaining blood type compatible pairs are matched, there is no potential benefit from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$. Therefore, we first take full of advantage of four-way cycle$(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ and the maximum number of transplants in situation (1) is:

$$
\begin{aligned}
N = \; & 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+3*d_1+2*g_5+2*g_6+2*g_7 \\
& +2*g_8+w_5 \\
where & \\
d_1 = \; & \min\{\#O^d+\#(AB,O),\#(O,A)-g_2-w_1,\#(A,B)-\#(B,A)-g_1-g_2-g_3, \\
& \#(B,AB)-g_1-g_3-w_2\} \\
g_5 = \; & \min\{\#O^d+\#(AB,O)-d_1,\#(O,A)-d_1-g_2-w_1,\#(A,AB)-w_3\} \\
g_6 = \; & \min\{\#O^d+\#(AB,O)-d_1-g_5,\#(O,B)-w_4,\#(B,AB)-g_1-g_3-w_2-d_1\} \\
g_7 = \; & \min\{\#O^d+\#(AB,O)-d_1-g_5-g_6,\#(O,A)-d_1-g_2-w_1-g_5, \\
& \#(A,B)-\#(B,A)-g_1-g_2-g_3-d_1\} \\
g_8 = \; & \min\{\#O^d+\#(AB,O)-d_1-g_5-g_6-g_7,\#(A,B)-\#(B,A)-g_1-g_2-g_3 \\
& -d_1-g_7,\#(B,AB)-g_1-g_3-w_2-g_6-d_1\} \\
w_5 = \; & \min\{\#O^d+\#(AB,O)-d_1-g_5-g_6-g_7-g_8,\#(O,A)+\#(O,B)+\#(O,AB) \\
& +\#(A,AB)+\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3 \\
& -w_1-w_2-w_3-w_4-3*d_1-2*g_5-2*g_6-2*g_7-2*g_8\}
\end{aligned}
$$

The maximum number of transplants can be rewritten as:

$$
\begin{aligned}
N = \; & 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+3*d_1+2*g_5+2*g_6+2*g_7 \\
& +2*g_8+w_5 = \min\{N_1,N_2,N_4,N_5,N_6,N_7,,N_8,N_{10},N_{11}\}.
\end{aligned}
$$

One may refer to Tables from C1 to C4 in Supplement C of Cheng and Yang (2017) for detail.

(2) When (A,O), (B,O), (A,AB), (B,AB) remaining, we have $g_1=\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$, $g_2=\#(O,A)-\min\{\#(A,O),\#(O,A)\}$, $g_3=\min\{\#A^d+\#(AB,A)-g_1,\#(A,B)-\#(B,A)-g_1-g_2\}$ and $g_4=\min\{\#(O,A)-g_2,\#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (O,A) and pair (O,B) are matched. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (AB,A) and pair (AB,B), single donor A^d and single

donor B^d are matched. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (2):

$$N = \min\{N_5, N_2, N_1, N_6\}.$$

(3) When (O,A), (O,B), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $g_1 = \min\{\#(A,B) - \#(B,A), \#(B,AB)\}\}$, $g_2 = \min\{\#(B,O), \#(A,B) - \#(B,A) - g_1\}$, $g_3 = 0$ and $g_4 = 0$. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (A,AB) and pair (B,AB) are matched. There is no potential benefits from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (B,O) are matched. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (B,AB) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (3):

$$N = \min\{N_1, N_6, N_8, N_{11}\}$$

(4) When (A,O), (B,O), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $g_1 = \min\{\#(A,B) - \#(B,A), \#(B,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = 0$ and $g_4 = \min\{\#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (B,AB) are matched. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (A,AB) and pair (B,AB) are matched. Because there are remaining pair (AB,A), pair (B,O) and single donor A^d, there is potential gains from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$. To take full advantage of the combinations, based on the three-way process in situation (4), we first reserve the maximum number of

the combinations and then match two-way cycles $(AB,A)-(A,B)$, $(B,O)-(A,B)$ and chain $A^d-(A,B)-Y^p$. The maximum number of transplants in situation (4) is:

$$\begin{aligned}
N = &\quad 2*g_1+2*g_2+2*g_4+w_1+w_2+w_3+w_4+c_4+s_1+w_5 \\
\textit{where} & \\
c_2 = &\quad \min\{\#A^d+\#(AB,A)-g_1-w_3, \#(A,O)-w_1\} \\
c_3 = &\quad \min\{\#B^d+\#(AB,B)-w_2, \#(B,O)-g_2-g_4-w_4\} \\
c_4 = &\quad \min\{\#A^d+\#(AB,A)-g_1-w_3-c_2, \#(B,O)-g_2-g_4-w_4-c_3, \\
&\quad \#(A,B)-\#(B,A)-g_1-g_2-g_4\} \\
s_1 = &\quad \min\{\#A^d+\#(AB,A)-g_1-w_3-c_2-c_4+\#(B,O)-g_2-g_4-w_4-c_3-c_4, \\
&\quad \#(A,B)-\#(B,A)-g_1-g_2-g_4-c_4\} \\
w_5 = &\quad \min\{\#O^d+\#(AB,O)+c_2+c_3+c_4, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
&\quad +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_4-w_1 \\
&\quad -w_2-w_3-w_4-s_1-c_4\}
\end{aligned}$$

The maximum number of transplants can be rewritten as:

$$N = 2*g_1+2*g_2+2*g_4+w_1+w_2+w_3+w_4+c_4+s_1+w_5 = \min\{N_1,N_2,N_4,N_6,N_{11}\}$$

One may refer to Tables from C5 to C8 in Supplement C for detail.

(5) When (A,O), (B,O), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#A^d+\#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $\min\{\#(A,O), \#(O,A)\} = \#(O,A)$, $g_1 = \#(A,B)-\#(B,A)$ and $g_2 = g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (A,AB), pair (O,A) and pair (O,B) are matched. There is no potential gains from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (A,B) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (5):

$$N = \quad \min\{N_1,N_2,N_4\}$$

(6) When (A,O), (B,O), (A,AB), $B^d/(AB,B)$ remaining, we have $g_1 = \#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A), \#(A,AB)\}$, $g_2 = \#(O,A)-\min\{\#(A,O), \#(O,A)\}$, $g_3 = \min\{\#A^d+$

$\#(AB,A)-g_1, \#(A,B)-\#(B,A)-g_1-g_2, \#(B,AB)-b_1\}$ and $g_4 = \min\{\#(O,A)-g_2, \#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (O,A) and pair (O,B) are matched. There is no potential gains from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (AB,A) and single donor A^d are matched. Therefore, there is no potential gains from four-way cycles and chains and the maximum number of transplants is the same as that of three-way cycles and chains in situation (6):

$$N = \min\{N_1, N_2, N_6\}$$

(7) When (A,O), (O,B), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)\}$, $g_1 = \min\{\#(A,B)-\#(B,A), \#(B,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = 0$ and $g_4 = \min\{\#(B,O)-g_2, \#(O,A)-g_2, \#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. There is no potential gains from four-way cycles and chains starting with the combinations $(AB,A)-(A,O)$ and $A^d-(A,O)$ because all pair (A,AB) and pair (B,AB) are matched. There is no potential gains from four-way cycles and chains starting with the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (B,O) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (7)

$$N = \min\{N_1, N_6, N_{11}\}$$

(8) When (O,A), (B,O), $A^d/(AB,A)$, $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \min\{\#(A,B) - \#(B,A), \#(B,AB)\}$, $g_2 = \#(A,B)-\#(B,A)-g_1$ and $g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (B,AB) are matched. There is

no potential gains from four-way cycles and chains starting with the combinations $(AB,B)-(B,O)$ and $B^d-(B,O)$ because all pair (A,AB) and pair (B,AB) are matched. There is no potential gains from four-way cycles and chains starting with the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (A,B) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (8)

$$N = \min\{N_1, N_4, N_{11}\}$$

(9) When (A,O), (O,B), (A,AB), $B^d/(AB,B)$ remaining, we have $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(O,A) - \min\{\#(A,O), \#(O,A)\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2, \#(B,AB) - b_1\}$ and $g_4 = \min\{\#(B,O) - g_2, \#(O,A) - g_2, \#(A,B) - \#(B,A) - g_1 - g_2 - g_3\}$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. There is no potential benefits from four-way cycle and chain starting from the combinations $(AB,A)-(A,O)$, $(AB,B)-(B,O)$, $A^d-(A,O)$, $B^d-(B,O)$ because all pair (A,AB), pair (O,A) and pair (B,AB) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (9)

$$N = \min\{N_1, N_6\}$$

(10) When (O,A), (B,O), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (A,B) are matched. There is no potential gains from four-way cycles and chains starting from the combinations $(AB,B)-(B,O)$, $B^d-(B,O)$, $(AB,A)-(A,O)$ and $A^d-(A,O)$ because all pair (A,O), pair (AB,B) and single donor B^d are matched. There is no potential gains from four-way cycles and chains starting from the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (A,B) are matched. Therefore, the maximum number of transplants is the same as that of three-way cycles and chains in situation (10)

$$N = \min\{N_1, N_4\}$$

(11) When (A,O), (O,B), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(O,A)$, $\min\{\#A^d + \#(AB,A), \#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B) - \#(B,A)$ and $g_2 = g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) and pair (A,B) are matched. There is no potential gains from four-way cycles and chains starting by the combinations $(AB,A)-(A,B)-(B,O)$ and $A^d-(A,B)-(B,O)$ because all pair (A,B) and pair (B,O) are matched. There is potential gains from four-way cycle $(AB,A)-(A,O)-(O,B)-(B,AB)$ and chain $A^d-(A,O)-(O,B)-(B,AB)-AB^p$. To take full advantages, we match the maximum number of four-way cycle $(AB,A)-(A,O)-(O,B)-(B,AB)$ and chain $A^d-(A,O)-(O,B)-(B,AB)-AB^p$ bounded by the number of remaining pairs (A,O), (O,B), $A^d/(AB,A)$ and (B,AB). If all remaining pairs (AB,A) and single donors A^d are matched, there is potential gains from four-way cycle $(AB,A)-(A,O)-(O,B)-(B,AB)$ and chain $A^d-(A,O)-(O,B)-(B,AB)$ by breaking two-way cycle $(AB,A)-(A,AB)$ and chain $A^d-(A,AB)$ because one more pair can be matched in this case. Similarly, if all remaining pairs (A,O) are matched, there is potential gains from three-way cycles $(A,O)-(O,B)-(B,A)$, $(AB,A)-(A,B)-(B,AB)$ and chain $A^d-(A,B)-(B,AB)-AB^p$ by breaking two-way cycle $(A,O)-(O,A)$.

Therefore, the maximum number of transplants in situation (11) is

$$N = 2*g_1 + w_1 + w_2 + w_3 + w_4 + 2*u_1 + v_1 + v_2 + 2*g_6 + w_5$$

where

$$\begin{aligned}
u_1 = {} & \min\{\#(A,O) - w_1, \#(O,B) - w_4, \#A^d + \#(AB,A) - g_1 - w_3, \\
& \#(B,AB) - g_1 - w_2\} \\
v_1 = {} & \min\{\#A^d + \#(AB,A) - g_1 - u_1, \#(A,O) - w_1 - u_1, \#(O,B) - w_4 - u_1, \\
& \#(B,AB) - g_1 - w_2 - u_1\} \\
v_2 = {} & \min\{\#(A,O) - u_1, \#A^d + \#(AB,A) - g_1 - u_1, \#(O,B) - w_4 - u_1, \\
& \#(B,AB) - g_1 - w_2 - u_1\} \\
c_2 = {} & \min\{\#A^d + \#(AB,A) - w_3 - u_1 - v_2, \#(A,O) - w_1 - u_1 - v_1\} \\
g_6 = {} & \min\{\#O^d + \#(AB,O), \#(O,B) - w_4 - u_1 - v_1 - v_2, \#(B,AB) - g_1 - u_1 - w_2 \\
& -v_1 - v_2\} \\
w_5 = {} & \min\{\#O^d + \#(AB,O) + c_2 - g_6, \#(O,A) + \#(O,B) + \#(O,AB) + \#(A,AB) \\
& +\#(A,B) - \#(B,A) + \#(B,AB) - 2*g_1 - w_1 - w_2 - w_3 - w_4 - 2*u_1 \\
& -v_1 - v_2 - 2*g_6\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned} N = &\ 2*g_1+w_1+w_2+w_3+w_4+2*u_1+v_1+v_2+2*g_6+w_5 \\ = &\ \min\{N_1,N_2,N_4,N_5,N_6,N_9,N_{10},N_{11}\} \end{aligned}$$

One may refer to Tables C9 and C10 in Supplement C for detail.

(12) When (O,A), (O,B), $A^d/(AB,A)$, (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $\min\{\#A^d+\#(AB,A),\#(A,AB)\} = \#(A,AB)$, $g_1 = \#(A,B)-\#(B,A)$ and $g_2 = g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (A,B) are matched. There is potential gains from four-way cycle $(AB,A)-(A,O)-(O,B)-(B,AB)$ and chain $A^d-(A,O)-(O,B)-(B,AB)-AB^p$ by breaking two-way cycle $(A,O)-(O,A)$ because one more pair can be matched in this case. That is, $v_2 \neq 0$. We can do the same process in situation (11). The maximum number of transplants is

$$N = 2*g_1+w_1+w_2+w_3+w_4+v_2+2*g_6+w_5 = \min\{N_1,N_4,N_9,N_{10},N_{11}\}$$

One may refer to Table C11 in Supplement C for detail.

(13) When (A,O), (O,B), (A,AB), (B,AB) remaining, we have $g_1 = \#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\}$, $g_2 = \#(O,A)-\min\{\#(A,O),\#(O,A)\}$, $g_3 = \min\{\#A^d+\#(AB,A)-g_1,\#(A,B)-\#(B,A)-g_1-g_2\}$ and $g_4 = \min\{\#(O,A)-g_2,\#(A,B)-\#(B,A)-g_1-g_2-g_3\}$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (O,A) are matched. There is potential gains from four-way cycle $(AB,A)-(A,O)-(O,B)-(B,AB)$ and chain $A^d-(A,O)-(O,B)-(B,AB)-AB^p$ by breaking two-way cycle $(AB,A)-(A,AB)$ and chain $A^d-(A,AB)-AB^p$ because one more pair can be matched in this case. That is, $v_1 \neq 0$. To take full advantage of four-way cycles and chains, we can first do the same process in situation (11) and then match three-way cycles $(AB,O)-(O,B)-(B,AB)$, $(AB,O)-(A,B)-(B,AB)$ and chains $O^d-(O,B)-(B,AB)-AB^p$, $O^d-(A,B)-(B,AB)-AB^p$ if any before matching remaining pairs with pair (AB,O) and single donor O^d. Therefore, the maximum number of transplants in

situation (13) is

$$\begin{aligned}
N = \;& 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+v_1+2*g_6+2*g_8+w_5 \\
\textit{where} & \\
g_8 = \;& \min\{\#O^d+\#(AB,O)-g_6,\#(A,B)-\#(B,A)-g_1-g_2-g_3-g_4, \\
& \#(B,AB)-g_1-g_3-w_2-v_1-g_6\} \\
w_5 = \;& \min\{\#O^d+\#(AB,O)-g_6-g_8,\#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3-2*g_4 \\
& -w_1-w_2-w_3-w_4-v_1-2*g_6-2*g_8\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned}
N = \;& 2*g_1+2*g_2+2*g_3+2*g_4+w_1+w_2+w_3+w_4+v_1+2*g_6+2*g_8+w_5 \\
= \;& \min\{N_1,N_2,N_5,N_6,N_{10}\}
\end{aligned}$$

One may refer to Tables from C12 to C17 in Supplement C for detail.

(14) When (O,A), (B,O), (A,AB), $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(A,B) - \#(B,A) - g_1$ and $g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (A,B) are matched. There is potential gains from four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ because two more blood-type incompatible pairs of types (O,A) and (A,AB) can be matched in this case. Therefore, we take full advantage of (B,A), (A,B) and match the maximum number of four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ bounded by the number of remaining pairs (O,A), (B,O), (A,AB) and $B^d/(AB,B)$. If all remaining pairs (AB,B) and single donors B^d are matched, there is potential gains from four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ by breaking two-way cycle $(AB,B)-(B,AB)$ and chain $B^d-(B,AB)-AB^p$ because one more pair can be matched in this case. Similarly, if all remaining pairs (B,O) are matched, there is potential gains from four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ by breaking two-way cycle $(B,O)-(O,B)$. Therefore, the maximum number of transplants

in situation (14) is

$$\begin{aligned}
N = & \quad 2*g_1+2*g_2+w_1+w_2+w_3+w_4+2*u_2+v_3+v_4+2*g_5+w_5 \\
where & \\
u_2 = & \quad \min\{\#(B,O)-g_2-w_4, \#(O,A)-g_2-w_1, \#(A,AB)-w_3, \\
& \quad \#B^d+\#(AB,B)-w_2\} \\
v_3 = & \quad \min\{\#B^d+\#(AB,B)-u_2, \#(B,O)-g_2-w_4-u_2, \#(O,A)-g_2-w_1-u_2, \\
& \quad \#(A,AB)-w_3-u_2\} \\
v_4 = & \quad \min\{\#(B,O)-g_2-u_2, \#B^d+\#(AB,B)-w_2-u_2, \#(O,A)-g_2-w_1-u_2, \\
& \quad \#(A,AB)-w_3-u_2\} \\
c_3 = & \quad \min\{\#B^d+\#(AB,B)-w_2-u_2-v_4, \#(B,O)-w_4-u_2-v_3\} \\
g_5 = & \quad \min\{\#O^d+\#(AB,O), \#(O,A)-g_2-w_1-u_2-v_3-v_4, \#(A,AB)-w_3-u_2 \\
& \quad -v_3-v_4\} \\
w_5 = & \quad \min\{\#O^d+\#(AB,O)+c_3-g_5, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
& \quad +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-w_1-w_2-w_3-w_4 \\
& \quad -2*u_2-v_3-v_4-2*g_5\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned}
N = & \quad 2*g_1+2*g_2+w_1+w_2+w_3+w_4+2*u_2+v_3+v_4+2*g_5+w_5 \\
= & \quad \min\{N_1, N_2, N_3, N_4, N_6, N_8, N_{10}, N_{11}\}
\end{aligned}$$

One may refer to Tables C18 and C19 in Supplement C for detail.

(15) When (O,A), (B,O), (A,AB), (B,AB) remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \#(A,B) - \#(B,A) - g_1$ and $g_3 = g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ since all pair (A,B) are matched. There is potential gains from four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ by breaking two-way cycle $(AB,B)-(B,AB)$ and chain $B^d-(B,AB)-AB^p$ because one more pair can be matched in this case. That is, $v_3 \neq 0$. We can do the same process in situation (14). The maximum number of transplants is

$$\begin{aligned}
N = & \quad 2*g_1+2*g_2+w_1+w_2+w_3+w_4+v_3+2*g_5+w_5 \\
= & \quad \min\{N_1, N_2, N_3, N_4, N_{10}\}
\end{aligned}$$

One may refer to Table C20 in Supplement C for detail.

(16) When (O,A), (O,B), (A,AB), $B^d/(AB,B)$ remaining, we have $\min\{\#(A,O), \#(O,A)\} = \#(A,O)$, $g_1 = \#A^d + \#(AB,A) - \min\{\#A^d + \#(AB,A), \#(A,AB)\}$, $g_2 = \min\{ \#(B,O), \#(A,B) - \#(B,A) - g_1\}$, $g_3 = \min\{\#A^d + \#(AB,A) - g_1, \#(A,B) - \#(B,A) - g_1 - g_2, \#(B,AB) - b_1\}$ and $g_4 = 0$. There is no potential gains from four-way cycles and chains $(AB,O)-(O,A)-(A,B)-(B,AB)$ and chain $O^d-(O,A)-(A,B)-(B,AB)-AB^p$ because all pair (B,AB) are matched. There is potential gains from four-way cycle $(AB,B)-(B,O)-(O,A)-(A,AB)$ and chain $B^d-(B,O)-(O,A)-(A,AB)-AB^p$ by breaking two-way cycle $(B,O)-(O,B)$ because one more pair can be matched in this case. That is, $v_4 \neq 0$. To take full advantage of three-way cycles and chains, we can first do the same process in situation (14) and then match three-way cycles $(AB,O)-(O,A)-(A,AB)$, $(AB,O)-(O,A)-(A,B)$ and chains $O^d-(O,A)-(A,AB)-AB^p$, $O^d-(O,A)-(A,B)-Y^p$ if any before matching remaining pairs with pair (AB,O) and single donor O^d. Therefore, the maximum number of transplants in situation (16) is

$$\begin{aligned}
N = &\quad 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+v_4+2*g_5+2*g_7+w_5 \\
\textit{where} & \\
g_7 = &\quad \min\{\#O^d+\#(AB,O)-g_5, \#(O,A)-g_2-w_1-v_4-g_5, \\
&\quad \#(A,B)-\#(B,A)-g_1-g_2-g_3\} \\
w_5 = &\quad \min\{\#O^d+\#(AB,O)-g_5-g_7, \#(O,A)+\#(O,B)+\#(O,AB)+\#(A,AB) \\
&\quad +\#(A,B)-\#(B,A)+\#(B,AB)-2*g_1-2*g_2-2*g_3-w_1-w_2-w_3 \\
&\quad -w_4-v_4-2*g_5-2*g_7\}
\end{aligned}$$

The maximum number of transplants is

$$\begin{aligned}
N = &\quad 2*g_1+2*g_2+2*g_3+w_1+w_2+w_3+w_4+v_4+2*g_5+2*g_7+w_5 \\
= &\quad \min\{N_1, N_6, N_8, N_{10}, N_{11}\}
\end{aligned}$$

One may refer to Tables from C21 to C24 in Supplement C for detail. Combining cases (1) to (16), we have proved that the maximum number of transplants for blood-type incompatible paired patients of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) is

$$N = \quad \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}\}$$

We now prove that the sequential mechanism is 4-efficient which achieves the maximum number of transplants in the pool. The same as the mechanism when three-way cycles and chains are allowed, the number of transplantations for compatible pairs, blood-type compatible pairs and pairs of type $(B-A)$ in the mechanism is

$$\begin{aligned}&\#(A,O)+\#(B,O)+\#(AB,O)+\#(AB,A)+\#(AB,B)\\&+\#(B,A)+\#(A,A)+\#(B,B)+\#(AB,AB)+\#(O,O)\end{aligned}$$

Next, we prove that the maximum number of transplants for blood-type incompatible pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) can be achieved in the mechanism by exploring every possible route.

Denote X_2 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 2 so that

$$\begin{aligned}
X_2 =\ & 2*b_0+2*b_1+2*b_2+2*b_{21}\\
\textit{where}\ & \\
b_1 =\ & \min\{\#A^d+\#(AB,A)-\min\{\#A^d+\#(AB,A),\#(A,AB)\},\\
& \#(A,B)-\#(B,A),\#(B,AB)\}\\
b_2 =\ & \min\{\#(B,O),\#(O,A)-\min\{\#(A,O),\#(O,A)\},\#(A,B)-\#(B,A)-b_1\}\\
b_3 =\ & \min\{\#A^d+\#(AB,A)-b_1,\#(A,B)-\#(B,A)-b_1-b_2,\#(B,AB)-b_1\}\\
b_{21} =\ & \min\{\#(B,O)-b_2,\#(O,A)-b_2,\#(A,B)-\#(B,A)-b_1-b_2-b_3\}
\end{aligned}$$

Denote X_3 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 3 so that

$$\begin{aligned}
X_3 =\ & 2*e_1+2*e_2+2*f_2+2*f_3+2*f_4+2*f_5\\
\textit{where}\ & \\
e_1 =\ & \min\{\#(A,O)-a_1,\#A^d+\#(AB,A)-b_1-b_3-a_1,\#(O,B)-a_4,\\
& \#(B,AB)-b_1-b_3-a_2\}\\
e_2 =\ & \min\{\#B^d+\#(AB,B)-a_2,\#(A,AB)-a_3,\#(B,O)-b_2-a_4,\\
& \#(O,A)-b_2-a_1\}\\
f_2 =\ & \min\{\#(A,O)-a_1-e_1,\#A^d+\#(AB,A)-b_1-b_3-e_1,\\
& \#(O,B)-a_4-e_1,\#(B,AB)-b_1-b_3-a_2-e_1\}\\
f_3 =\ & \min\{\#B^d+\#(AB,B)-a_2-e_2,\#(A,AB)-a_3-e_2,\#(B,O)-b_2-e_2,\\
& \#(O,A)-b_2-a_1-e_2\}\\
f_4 =\ & \min\{\#(A,O)-e_1,\#A^d+\#(AB,A)-b_1-b_3-e_1-a_3,\\
& \#(O,B)-a_4-e_1,\#(B,AB)-b_1-b_3-a_2-e_1\}\\
f_5 =\ & \min\{\#B^d+\#(AB,B)-e_2,\#(A,AB)-a_3-e_2,\#(B,O)-b_2-b_{21}-e_2\\
& -a_4,\#(O,A)-b_2-b_{21}-a_1-e_2\}
\end{aligned}$$

$$
\begin{aligned}
&\textit{where}\\
a_1 = &\quad \min\{\#(A,O),\#(O,A)-b_2-b_{21}\}\\
a_2 = &\quad \min\{\#B^d+\#(AB,B),\#(B,AB)-b_1-b_3\}\\
a_3 = &\quad \min\{\#A^d+\#(AB,A)-b_1-b_3,\#(A,AB)\}\\
a_4 = &\quad \min\{\#(B,O)-b_2-b_{21},\#(O,B)\}
\end{aligned}
$$

In Step 4, all remaining pairs (B,A) are matched with pair (A,B). Denote X_4 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 4 so that

$$
\begin{aligned}
X_4 = &\quad \#(B,A))+p_1+p_2+p_3+p_4+a_8+c_4\\
&\textit{where}\\
p_1 = &\quad a_1-f_4\\
p_2 = &\quad a_2-f_5\\
p_3 = &\quad a_3-f_2\\
p_4 = &\quad a4-f_3\\
a_8 = &\quad \min\{\#A^d+\#(AB,A)-b_1-b_3-e_1-f_2-f_4-p_3-c_2-c_4\\
&\quad +\#(B,O)-b_2-e_2-f_3-f_5-p_4-c_3-c_4,\#(A,B)-\#(B,A)\\
&\quad -b_1-b_2-b_3-b_{21}-c_4\}\\
&\textit{where}\\
c_2 = &\quad \min\{\#A^d+\#(AB,A)-b_1-b_3-e_1-f_2-f_4-p_3,\#(A,O)-e_1-f_2\\
&\quad -f_4-p_1\}\\
c_3 = &\quad \min\{\#(B,O)-b_2-b_{21}-e_2-f_3-f_5-p_4,\#B^d+\#(AB,B)-e_2-f_3\\
&\quad -f_5-p_2\}\\
c_4 = &\quad \min\{\#A^d+\#(AB,A)-b_1-b_3-e_1-f_2-f_4-p_3-c_2,\#(B,O)-b_2\\
&\quad -b_{21}-e_2-f_3-f_5-p_4-c_3,\#(A,B)-\#(B,A)-b_1-b_2-b_3-b_{21}\}
\end{aligned}
$$

Denote X_5 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 5 so that

$$
\begin{aligned}
X_5 = &\quad 3*d_1+2*b_4+2*b_5+2*b_6+2*b_7\\
&\textit{where}\\
d_1 = &\quad \min\{\#O^d+\#(AB,O),\#(O,A)-b_2-b_{21}-e_2-f_3-f_5-p_1,\#(A,B)\\
&\quad -\#(B,A)-b_1-b_2-b_3-b_{21}-a_8,\#(A,AB)-e_2-f_3-f_5-p_3\}\\
b_4 = &\quad \min\{\#O^d+\#(AB,O)-d_1,\#(O,A)-b_2-b_{21}-e_2-f_3-f_5-p_1-d_1,\\
&\quad \#(A,AB)-e_2-f_3-f_5-p_3-d_1\}\\
b_5 = &\quad \min\{\#O^d+\#(AB,O)-b_4-d_1,\#(O,B)-e_1-f_2-f_4-p_4,\\
&\quad \#(B,AB)-b_1-b_3-e_1-f_2-f_4-p_2\}
\end{aligned}
$$

$$
\begin{aligned}
b_6 = \ & \min\{\#O^d + \#(AB,O) - b_4 - b_5 - d_1, \#(O,A) - b_2 - b_{21} - e_2 - f_3 - f_5 \\
& -p_1 - b_4 - d_1, \#(A,B) - \#(B,A) - b_1 - b_2 - b_3 - b_{21} - a_8 - d_1\} \\
b_7 = \ & \min\{\#O^d + \#(AB,O) - b_4 - b_5 - b_6 - d_1, \#(A,B) - \#(B,A) - b_1 - b_2 \\
& -b_3 - b_{21} - a_8 - b_6 - d_1, \#(B,AB) - b_1 - b_3 - e_1 - f_2 - f_4 - p_2 - b_5\}
\end{aligned}
$$

Denote X_6 as the number of blood-type incompatible paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) involved in Step 6 so that

$$
\begin{aligned}
X_6 = a_9 = \ & \min\{\#O^d + \#(AB,O) - b_4 - b_5 - b_6 - b_7 + c_2 + c_3 + c_4, \#(O,A) \\
& +\#(O,B) + \#(O,AB) + \#(A,AB) + \#(A,B) + \#(B,AB) - 2*b_1 \\
& -2*b_2 - 2*b_3 - 2*e_1 - 2*e_2 - 2*f_2 - 2*f_3 - 2*f_4 - 2*f_5 \\
& -p_1 - p_2 - p_3 - p_4 - a_8 - 3*d_1 - 2*b_4 - 2*b_5 - 2*b_6 - 2*b_7 - b_{21} - b_{21}\}
\end{aligned}
$$

Therefore, the total number of transplants for paired patients from pairs of types (O,A), (O,B), (O,AB), (A,AB), (B,AB), (A,B) in the mechanism is $X = X_2 + X_3 + X_4 + X_5 + X_6$. The equation can be rewritten as follows:

$$
X = \min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}\}
$$

One may refer to Tables from C25 to C60 in Supplement C for detail. Therefore, the total number of transplants can be achieved in the mechanism is that

$$
\begin{aligned}
& \#(A,O) + \#(B,O) + \#(AB,O) + \#(AB,A) + \#(AB,B) \\
& +\#(B,A) + \#(A,A) + \#(B,B) + \#(AB,AB) + \#(O,O) \\
& +\#A^d + \#B^d + \#AB^d + \#O^d \\
& +\min\{N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}\}
\end{aligned}
$$

Therefore, we proved that every matching produced by the mechanism achieves the maximum number of transplants in the pool and hence the mechanism is 4-efficient.

□

Proof of Corollary 2: Consider an efficient matching μ for a population stated in the theorem. If the maximal matching μ only consists of n-way cycles and chains, or smaller cycles and chains, we are done. Otherwise, we will prove that there exists a matching ν which consists of at most n-way cycles or chains can match the same set of receiving agents as matching μ.

We will prove the theorem for the case in which the largest exchanges (cycles or chains) in matching μ is (n+1)-way. The same proof can be applied to show that for any maximal matching in which the largest exchanges is m-way where $m > (n+1)$, there exists another matching which matches the same set of receiving agent through (m-1)-way or smaller exchanges. Then, repeat the same proof process to obtain the desired result.

Three cases may occur in the matching μ, which are both cycle and chain have (n+1)-way or either cycle or chain has (n+1)-way. We will prove the most complicated case such that matching μ consisting of both (n+1)-way cycles and (n+1)-way chains. Then, other two situations can be automatically proved.

Let

$$E^0 = ((P_1^p, D_1^p), (P_2^p, D_2^p), (P_3^p, D_3^p), ..., (P_n^p, D_n^p), (P_{n+1}^p, D_{n+1}^p))$$
$$C^0 = (D_1^s, (P_1^p, D_1^p), (P_2^p, D_2^p), ..., (P_n^p, D_n^p), P_1^s)$$

be any (n+1)-way cycle and chain respectively in matching μ. We will prove that all receiving agents in these two exchanges can be matched via smaller exchanges without changing the set of pairs that are matched.

Since we have only n types, there are at least two receiving agents in cycle E^0 who have the same type. Pick any two such receiving agent. We have two cases to consider.

Case 1. The two receiving agents are not matched together.

Suppose these receiving agents are P_1^p and P_n^p in cycle E^0. The receiving agent P_1^p is matched with donating agent D_{n+1}^p and the receiving agent P_n^p is matched with donating agent D_{n-1}^p. Since agents P_1^p and P_n^p have the same type, donating agents D_{n-1}^p and D_{n+1}^p are compatible with the two receiving agent P_1^p and P_n^p. Hence, the (n+1)-way cycle can be divided into two smaller cycles as follows.

$$E_1^1 = ((P_1^p, D_1^p), (P_2^p, D_2^p), (P_3^p, D_3^p), ..., (P_{n-1}^p, D_{n-1}^p)), E_2^1 = ((P_n^p, D_n^p), (P_{n+1}^p, D_{n+1}^p))$$

Suppose these receiving agents are P_1^p and P_1^s in chain C^0. The receiving agent P_1^p is matched with donating agent D_1^s and the receiving agent P_1^s is matched with donating agent D_n^p. Since agents P_1^p and P_1^s have the same type, donating agents D_1^s and D_n^p are compatible with both the two agents. Hence, the (n+1)-way chain C^0 can be divided into one cycle and one chain as follows.

$$C_2^1 = (D_1^s, P_1^s),\ E_2^1 = ((P_1^p, D_1^p), (P_2^p, D_2^p), ..., (P_n^p, D_n^p))$$

Case 2. The two receiving agents are matched together. Suppose agents P_1^p and P_2^p have the same type.

Under cycle E^0, since agent P_1^p is matched with donating agent D_{n+1}, donating agent D_{n+1} is compatible with the receiving agent P_2^p. Hence, the following n-way exchange is feasible.

$$E_1^2 = ((P_2^p, D_2^p), (P_3^p, D_3^p), ..., (P_{n-1}^p, D_{n-1}^p))$$

Under chain C^0, since agent P_1^p is matched with donating agent D_1^s, donating agent D_1^s is compatible with receiving agent P_2^p. Hence, the following n-way chain is feasible.

$$C_2^2 = (D_1^s, (P_2^p, D_2^p), ..., (P_{n-1}^p, D_{n-1}^p), (P_n^p, D_n^p), P_1^s)$$

Now, we will prove that the remaining pair (P_1^p, D_1^p) can be matched in an exchange without affecting pairs that are matched under μ. Because of the Assumption 1, we directly use "type" to present the primary type. Let pair (P_1^p, D_1^p) be of type $(X,Y)^t$ where $t \in \{i,c\}$, and hence receiving agent P_2^p is type X. Since donating agent D_1^p of type Y is compatible with receiving agent P_2^p, we have $Y \succeq X$. Therefore, pair of type (X,Y) is primary type compatible pair.

Let $\mathscr{A}$ be the set of $n+1$-way cycles and $n+1$-way chains in case 2. From previous proof, every cycle can be separated into a n-way cycle and one remaining primary type compatible pair and every chain can be separated into a n-way chain and one remaining primary type compatible pair. Let $\mathscr{D}$ be the set of remaining primary type compatible pairs in $\mathscr{A}$. Then, we have $Y \succeq X$. If remaining pairs are compatible, we can do transplants directly. Otherwise, let $(X,Y)^i$ present the type of a primary type compatible but secondary type incompatible pair. If there exists two or more pairs of type $(X,Y)^i$, we can match them by two-way cycles $(X,Y)^i - (X,Y)^i$. Therefore, at most one pair of type $(X,Y)^i$ left. By Assumption 5, there exists at least one pair of type $(X,Y)^c$. If the pair $(X,Y)^c$ does not involve in a cycle or a chain, we can match the remaining pair $(X,Y)^i$ with pair $(X,Y)^c$. Otherwise, pair $(X,Y)^c$ involves in a cycle or chain no larger than n-way, then the remaining pair $(X,Y)^i$ can replace the position of pair $(X,Y)^c$ and pair $(X,Y)^c$ can do the transplant straightforwardly.

□

There is a long list of tables in the Appendix D (Supplementary A, B and C) which are used to help the reader understand various cases in the proof of our results.

Appendix C

Appendices for Chapter 4

Proof of Theorem 6: Let μ denote a stable matching of a doctor-hospital matching problem $(A,R,Q,HS,\succ)$. We therefore have $|\mu| = \min\{|D|, \sum_{r\in R} q_r\}$. The proof is as follows. Suppose that there exists a stable matching μ that $|\mu| > \min\{|D|, \sum_{r\in R} q_r\}$. Then, we have $\min\{|D|, \sum_{r\in R} q_r\} \neq |D|$ because the maximum number of matching doctors equals to the total number of doctors. If $\min\{|D|, \sum_{r\in R} q_r\} = \sum_{r\in R} q_r$, the matching μ is infeasible because at least one of regional caps is violated, which is contradicting to the stability. Suppose that there exists a stable matching μ that $|\mu| < \min\{|D|, \sum_{r\in R} q_r\}$. There exists a hospital h has $|\mu^{'}(h)| < q_h$ and a doctor d with $\mu(d) = \emptyset$. Then a pair (d,h) can block the matching μ, which is contradicting to the stability.

Now we prove the Theorem 6. Suppose that μ is a stable but inefficient matching so that there exists a feasible matching $\mu^{'}$ such that $\mu^{'}(i) \succeq_i \mu(i)$ for all $i \in D \cup H$ and $\mu^{'}(i) \succ_i \mu(i)$ for some $i \in D \cup H$. It implies that $|\mu'| = \min\{|D|, \sum_{r\in R} q_r\}$. (Otherwise, at least one doctor d becomes worse off in matching μ', contradicting with the assumption.) Since the matching is bilateral, there exists a doctor $d \in D$ who has $\mu^{'}(d) \succ_d \mu(d)$ under the strict preference. Since μ is a stable matching, we have $\mu(d) \succeq_d \emptyset$ and hence $\mu^{'}(d) \neq \emptyset$ and $\mu^{'}(d) \in H$.

Denote $h = \mu^{'}(d)$. Since μ is a stable matching, $h \succ_d \mu(d)$ implies one of the following cases:

(1) $\mu(d) = \emptyset$.

(2) $|\mu(h)| = q_h$ and $d' \succ_h d$ for all $d' \in \mu(h)$.

(3) $|\mu(\mu(d))| = p_{\mu(d)}$ and $d' \succ_h d$ for all $d' \in \mu(h)$.

(4) $|\mu(r(h))| \leq q_{r(h)}$, $|\mu(r(\mu(d)))| \geq p_{r(\mu(d))}$ and $d' \succ_h d$ for all $d' \in \mu(h)$.

Case (1). Suppose $\mu(d) = \emptyset$. There exists a doctor $d^{'} \in D$ such that $\mu(d^{'}) \neq \emptyset$ and $\mu^{'}(d^{'}) = \emptyset$ because $|\mu| = |\mu'| = \min\{|D|, \sum_{r\in R} q_r\}$. It is contradicting to the assumption that $\mu^{'}$ Pareto dominates μ.

Case (2). Suppose $|\mu(h)| = q_h$ and $d' \succ_h d$ for all $d' \in \mu(h)$. Then there exists a doctor $d^{''} \in \mu^{'}(h) \setminus \mu(h)$ such that $d^{''} \succ_h d'$ for some $d' \in \mu_h$. Otherwise, $\mu(h) \succ_h \mu^{'}(h)$ because of the responsiveness of the preference of h. Since matching μ is stable, $\mu(d^{''}) \succ_{d''} h$, contradicting with the assumption.

Case (3). Suppose $|\mu(\mu(d))| = p_{\mu(d)}$ and $d' \succ_h d$ for all $d' \in \mu(h)$. There exists a doctor $d^{''} \in \mu^{'}(\mu(d)) \setminus \mu(\mu(d))$ such that $d^{''} \succ_{\mu(d)} d$. Otherwise, $\mu(\mu(d)) \succ_{\mu(d)} \mu^{'}(\mu(d))$ because of the responsiveness of the preference of $\mu(d)$. Since matching μ is stable, $\mu(d^{''}) \succ_{d''} \mu(d)$, contradicting with the assumption.

Case (4). Suppose $|\mu(r(h))| \leq q_{r(h)}$, $|\mu(r(\mu(d)))| \geq p_{r(\mu(d))}$ and $d' \succ_h d$ for all $d' \in \mu(h)$. If $|\mu^{'}(h)| \leq |\mu(h)|$, then there exists a doctor $d^{''} \in \mu^{'}(h) \setminus \mu(h)$ such that $d^{''} \succ_h d'$ for some $d' \in \mu(h)$. Otherwise, $\mu(h) \succ_h \mu^{'}(h)$ because of the responsiveness of the preference of h. Since μ is stable, $\mu(d^{''}) \succ_{d''} h$, contradicting the assumption. If $|\mu^{'}(h)| > |\mu(h)|$, since $|\mu| = |\mu'| = \min\{|D|, \sum_{r\in R} q_r\}$, there exists a hospital $h' \in H_{r(h)}$ with $|\mu^{'}(h')| < |\mu(h')|$. Since $\mu^{'}$ Pareto dominates μ, there exists a doctor $d^{''} \in \mu^{'}(h') \setminus \mu(h')$ such that $d^{''} \succ_{h'} d'$ for some $d' \in \mu(h')$. Otherwise, $\mu(h') \succ_{h'} \mu^{'}(h')$ because of the responsiveness of the preference of h. Since μ is stable, $\mu(d^{''}) \succ_{d''} h'$, contradicting the assumption.

□

Let μ denote the matching produced by the Doctor-proposing Deferred Acceptance mechanism of doctor-hospital matching problem $(D, H, R, Q, HS, \succ)$.

Denote μ^k the set of doctors tentatively accepted by hospitals at round k, A^k the set of doctors tentatively accepted by hospitals in step 2, B^k the set of doctors tentatively accepted by hospitals in step 3, $A^k(h)$ and $B^k(h)$ the set of doctors tentatively accepted by hospital h at step 2 and 3 respectively, $A^k(r)$ and $B^k(r)$ the set of doctors tentatively accepted by hospitals in region r at step 2 and 3 respectively. Denote D_h^k as the set of doctors who are not tentatively accepted by hospital h at the beginning of step 3 in round k and $D^k = \cup_{h\in H} D_h^k$ the set of doctors who are not tentatively accepted at the beginning of step 3 in round k. Formally, we have $\mu^k = A^k \cup B^k$, $A^k = \cup_{r\in R} A^k(r)$, $B^k = \cup_{r\in R} B^k(r)$, $A^k(r) = \cup_{h\in r} A^k(h)$, and $B^k(r) = \cup_{h\in r} B^k(h)$. Let K denote the last round in which the mechanism terminates such that $\mu = \mu^K = A^K \cup B^K$.

Given a set of doctors D_h who propose to a hospital $h \in H$, label d_h^i as a doctor d who is the i^{th} highest preferred doctor in hospital h among all doctors $d \in D_h$. We have the following picking order rule s such that

(i) $d_h^i \succ_s d$ for every doctor $d \in D_{h'}$ if $hs(h) < hs(h')$ for $h, h' \in H$; and

(ii) $d_h^i \succ_s d_{h'}^j$ for all $i < j$ and $d_h^i \succ_s d_{h'}^i$ for all $h \succ_{PL_{hs(h)}} h'$ if $hs(h) = hs(h')$ for $h, h' \in H$.

Denote s^k as the picking order in round k. Then, the Step 3 of the mechanism in round k can be represented as follows.

Step 1 (Respect to regional maximum quota): Based on the picking order s^k on doctors $d \in \cup_{h\in H} D_h^k$, each region choose $\min\{\bar{q}_r, \sum_{h\in H_r} |D_h^k|\}$ number of doctors. Let D_1^k be the set of doctors chosen at step 1;

Step 2 (Respect to the flexible quotas): Pick doctors from D_1^k one by one based on the picking order s^k. A doctor d is accepted by $\mu^k(d)$ if we have either $\sum_{r\in R}\{|T_r| - \min\{|T_r|, \bar{p}_r\}\} < e$ or $|T_r| < \bar{p}_r$, where $|T_r| = \cup_{h\in H_r}\{|T_h| - \min\{|T_h|, p_h\}\}$ and T_h is the set of doctor accepted by h so far.

Therefore, we have the following facts at each round $k \le K$ in the mechanism

Fact 1. $|A^k(h)| \le |A^{k'}(h)|$ for all $k' \ge k$.

Fact 2. $|\sum_{r\in R} B^k(r)| \le \min\{e + \sum_{r\in R} \bar{p}_r, \sum_{r\in R} q_r - \sum_{h\in H} p_h\}$.

Fact 3. If hospital h rejects some doctor d in round k, then $p_h \le |\mu^k(h)| \le q_h$.

Fact 4. If $|A^k(h)| < p_h$, then $|B^k(h)| = 0$; If $|A^k(h)| = p_h$, then $|A^{k'}(h)| = p_h$ for all $k' \ge k$.

Fact 5. If $\sum_{r\in R}\{|B^k(r)| - \min\{|B^k(r)|, \bar{p}_r\}\} = e$, then $\sum_{r\in R}\{|B^{k'}(r)| - \min\{|B^{k'}(r)|, \bar{p}_r\}\} = e$ for all $k' \ge k$; If region r has $|B^k(r)| \ge \bar{p}_r$, then $|B^{k'}(r)| \ge \bar{p}_r$ for all $k' \ge k$.

Fact 6. $|D_h^k| \ge |B_h^k|$ for all $h \in H$.

Fact 7. If hospital h rejects some doctor d at Step 3 in round k, then

(i) either $\sum_{r\in R}(|B^k(r)| - \min\{|B^k(r)|, \bar{p}_r\}) = e$ and $|B^k(r(h))| \ge \bar{p}_{r(h)}$ or $|B^k(r(h))| = \bar{q}_{r(h)}$ is satisfied; and

(ii) $d_{h'}^{|B^k(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H_{r(h)}$ with $|B^k(h')| > 0$; and

(iii) if $|B^k(r(h))| < \bar{q}_{r(h)}$, $d_{h'}^{|B^k(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H \setminus H_{r(h)}$ with $\sum_{h \in H_{r(h')}} |B^k(h)| > \bar{p}_{r(h')}$ and $|B^k(h')| > 0$.

We have the following Lemma for the mechanism.

Lemma 13 *If hospital h rejects some doctor d at Step 3 in round k, then*

(i) $d_{h'}^{|B^{k'}(h')|} \succ_s d_h^{|B^k(h)|+1}$ *for all* $h' \in H_{r(h)}$ *with* $|B^{k'}(h')| > 0$ *in all rounds* $k' \geq k$*; and*

(ii) if $|B^k(r(h))| < \bar{q}_{r(h)}$, $d_{h'}^{|B^{k'}(h')|} \succ_s d_h^{|B^k(h)|+1}$ *for all* $h' \in H \setminus H_{r(h)}$ *with* $\sum_{h \in H_{r(h')}} |B^{k'}(h)| > \bar{p}_{r(h')}$ *and* $|B^{k'}(h')| > 0$ *in all rounds* $k' \geq k$.

Proof of Lemma 13: Since hospital h rejects some doctor at Step 3 in round k (we have $k < K$), either all flexible quotas or regional cap of $r(h)$ are filled in round k. Based on Fact 7, we have the case in round k. We will prove by induction. The inductive part of the proof is to show that, if when hospital h rejects some doctor d at Step 3 in round k, $d_{h'}^{|B^l(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H_{r(h)}$ with $|B^l(h')| > 0$; and if $|B^k(r(h))| < \bar{q}_{r(h)}$, $d_{h'}^{|B^l(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H \setminus H_{r(h)}$ with $\sum_{h \in H_{r(h')}} |B^l(h)| > \bar{p}_{r(h')}$ and $|B^l(h')| > 0$ in all rounds l where $k \leq l \leq m$, then $d_{h'}^{|B^{m+1}(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H_{r(h)}$ with $|B^{m+1}(h')| > 0$; and if $|B^k(r(h))| < \bar{q}_{r(h)}$, $d_{h'}^{|B^{m+1}(h')|} \succ_s d_h^{|B^k(h)|+1}$ for all $h' \in H \setminus H_{r(h)}$ with $\sum_{h \in H_{r(h')}} |B^{m+1}(h)| > \bar{p}_{r(h')}$ and $|B^{m+1}(h')| > 0$ in round $m+1$, where $k \leq m < K$.

Suppose that there exists some hospital $h' \in H$ has one of the two following cases. Case A. $h' \in H_{r(h)}$ and $d_h^{|B^k(h)|+1} \succ_s d_{h'}^{|B^{m+1}(h')|}$ with $|B^{m+1}(h')| > 0$ in round $m+1$. Case B. $h' \in H \setminus H_{r(h)}$, $|B^k(r(h))| < \bar{q}_{r(h)}$ and $d_h^{|B^k(h)|+1} \succ_s d_{h'}^{|B^{m+1}(h')|}$ with $\sum_{h \in H_{r(h')}} |B^{m+1}(h)| > \bar{p}_{r(h')}$ and $|B^{m+1}(h')| > 0$ in round $m+1$. Based on the picking order rules, we have $hs(h) \leq hs(h')$ in both cases. Either Case A or Case B has the following situations.

Situation (1). If $|B^m(h')| \geq |B^{m+1}(h')|$. According to the inductive assumption, we have $d_{h'}^{|B^m(h')|} \succ_s d_h^{|B^k(h)|+1}$ in round m. Based on the picking order rules, we have $hs(h') \leq hs(h)$ and hence $hs(h) = hs(h')$. Therefore, based on the picking order rules, we have $|B^m(h')| \leq |B^k(h)|+1$ and $|B^k(h)|+1 \leq |B^{m+1}(h')|$ and hence $|B^m(h')| \leq |B^{m+1}(h')|$. Therefore, $|B^m(h')| = |B^{m+1}(h')|$. We have $|B^m(h')| = |B^{m+1}(h')| = |B^k(h)|+1$. In round m, we have $h' \succ_{PL_{hs(h)}} h$ based on the picking order rules. Similarly, in round $m+1$, we have $h \succ_{PL_{hs(h)}} h'$, which is contradicting to the previous result.

Situation (2). If $|B^m(h')| < |B^{m+1}(h')|$. h' obtains at least a new flexible quota in round $m+1$. Since hospital h rejects some doctor at Step 3 in round k, either all flexible quotas or regional cap of $r(h)$ are filled. B1. If $|B^k(r(h))| = \bar{q}_{r(h)}$, there exists some hospital $h^{''} \in r(h)$ losing one flexible quota. B2. If $|B^k(r(h))| < \bar{q}_{r(h)}$, there exists some hospital $h^{''}$ losing one flexible quota. If $h^{''} \notin r(h)$, we have $\sum_{h \in H_{r(h'')}} |B^m(h)| > \bar{p}_{r(h'')}$ and $|B^m(h^{''})| > 0$ in round m. Either case B1 or case B2 has the following results.

Based on Fact 7, we have $d_{h'}^{|B^{m+1}(h')|} \succ_s d_{h''}^{|B^{m+1}(h^{''})|+1}$ in round $m+1$. Based on the picking order rules, we $hs(h') \leq hs(h^{''})$. According to the inductive assumption, $d_{h''}^{|B^m(h^{''})|} \succ_s d_h^{|B^k(h)|+1}$. Based on the picking order rules, we have $hs(h^{''}) \leq hs(h)$. Therefore, we have $hs(h) = hs(h') = hs(h^{''})$. Hence $|B^{m+1}(h')| \leq |B^{m+1}(h^{''})| + 1$, $|B^m(h^{''})| \leq |B^k(h)| + 1$ and $|B^k(h)| + 1 \leq |B^{m+1}(h')|$. Since $|B^m(h^{''})| > |B^{m+1}(h^{''})|$, we have $|B^{m+1}(h')| \leq |B^{m+1}(h^{''})| + 1 \leq |B^m(h^{''})| \leq |B^k(h)| + 1 \leq |B^{m+1}(h')|$ and hence $|B^{m+1}(h')| = |B^{m+1}(h^{''})| + 1 = |B^m(h^{''})| = |B^k(h)| + 1$. In round m, we have $h' \succ_{PL_{hs(h)}} h$ based on the picking order rules. Similarly, in round $m+1$, we have $h \succ_{PL_{hs(h)}} h'$, which is contradicting to the previous result.

Therefore, we have proved the Lemma 13 by induction.

□

Consider the situation that hospital h rejects some doctor d at Step 3 in round k. According to the picking order rules, if a hospital h' has $d_{h'}^{|B^k(h')|} \succ_s d_h^{|B^k(h)|+1}$, we have $hs(h) \geq hs(h')$ and if $hs(h') = hs(h)$, we have $|B^k(h')| \leq |B^k(h)| + 1$. Based on Lemma 13, we have the following Corollary 4 and Corollary 5.

Corollary 4 *If hospital h rejects some doctor d at Step 3 in round k, we have*

(i) $hs(h) \geq hs(h')$ for all $h' \in H_{r(h)}$ with $|B^{k'}(h')| > 0$ in any round $k^{'} \geq k$; and

(ii) If $|B^k(r(h))| < \bar{q}_{r(h)}$, $hs(h) \geq hs(h')$ for all $h' \in H \setminus H_{r(h)}$ with $|B^{k'}(h')| > 0$ and $|B^{k'}(r(h'))| > \bar{p}_{r(h')}$ in any round $k^{'} \geq k$.

Corollary 5 *If hospital h rejects some doctor d at Step 3 in round k, then we have*

(i) $|B^{k^{'}}(h)| + 1 \geq |B^{k^{'}}(h')|$ for all $h' \in H_{r(h)}$ with $hs(h') = hs(h)$ in any round $k^{'} \geq k$; and

(ii) If $|B^k(r(h))| < \bar{q}_{r(h)}$, $|B^{k^{'}}(h)| + 1 \geq |B^{k^{'}}(h')|$ for all $h' \in H \setminus H_{r(h)}$ with $hs(h') = hs(h)$ and $|B^{k'}(r(h'))| > \bar{p}_{r(h')}$ in any round $k^{'} \geq k$.

Lemma 14 *If hospital h rejects some doctor d at Step 3 in round k, then $p_h \leq |\mu^{k^{'}}(h)| \leq |\mu^k(h)|$ for all $k' \geq k$.*

Proof of Lemma 14: According to Fact 1 and 3, we have $|\mu^{k'}(h)| \geq p_h$ for all $k' \geq k$ when hospital h rejects some doctor d at Step 3 in round k. We now prove that $|\mu^{k'}(h)| \leq |\mu^{k}(h)|$ for all $k' \geq k$. Clearly, we have $|\mu^{k'}(h)| \leq |\mu^{k}(h)|$ when $k' = k$.

Suppose there exists $|\mu^{m}(h)| > |\mu^{k}(h)|$ in round $m > k$. That is, h obtains one flexible quota in round m. Since hospital h rejects some doctor d at Step 3 in round k (we have $k < K$), either all flexible quotas or regional cap $r(h)$ are filled. Case A. If $|B^k(r(h))| = \bar{q}_{r(h)}$, there exists some hospital $h' \in r(h)$ losing one flexible quota in round m. Case B. If $|B^k(r(h))| < \bar{q}_{r(h)}$, there exists some hospital h' losing one flexible quota. If $h'' \notin r(h)$ so that h' loses an national quota, we have $\sum_{h \in H_{r(h'')}} |B^{m-1}(h)| > \bar{p}_{r(h'')}$ and $|B^{m-1}(h'')| > 0$ in round $m-1$. Either Case A or Case B has the following results.

Based on Corollary 4, we have $hs(h) \leq hs(h')$ and $hs(h) \geq hs(h')$ and hence $hs(h) = hs(h')$. Based on Fact 7, we have $d_h^{|B^m(h)|} \succ_s d_{h'}^{|B^m(h')|+1}$ and hence $|B^m(h)| \leq |B^m(h')|+1$. We have $d_{h'}^{|B^{m-1}(h')|} \succ_s d_h^{|B^k(h)|+1}$ based on Lemma 13 and hence $|B^{m-1}(h')| \leq |B^k(h)| + 1$. Since $|B^{m-1}(h')| > |B^m(h')|$ and $|B^k(h)| < |B^m(h)|$, we have $|B^m(h)| \leq |B^m(h')| + 1 \leq |B^{m-1}(h')| \leq |B^k(h)| + 1 \leq |B^m(h)|$. Therefore, we have $|B^m(h)| = |B^m(h')| + 1 = |B^{m-1}(h')| = |B^k(h)| + 1$. In round $m-1$, since we have $|B^{m-1}(h')| = |B^k(h)| + 1$, $d_{h'}^{|B^{m-1}(h')|} \succ_s d_h^{|B^k(h)|+1}$ and $hs(h) = hs(h')$, $h' \succ_{PL_{hs(h)}} h$ based on the picking rule. In round m, since we have $|B^m(h')| + 1 = |B^m(h)|$, $d_h^{|B^m(h)|} \succ_s d_{h'}^{|B^m(h')|+1}$ and $hs(h) = hs(h')$, $h \succ_{PL_{hs(h)}} h'$ based on the picking rule, which is contradicting to the previous result.

□

Lemma 14 implicates that a hospital would reject a doctor at Step 3 only if the number of doctors proposing to the hospital is larger than its minimal constraint. Let b^k be the number of flexible quotas assigned to the hospital in round k. If the hospital rejects some doctor at Step 3 in round k, then no more than b^k number of flexible quotas will be assigned to the hospital in latter rounds.

Proof of Proposition 6: In a doctor-hospital market $(D, H, R, Q, HS, \succ)$, let μ denote the matching produced by the Doctor-proposing Deferred Acceptance mechanism. Suppose that matching μ is not feasible so that at least one distributional constraint is not satisfied. It is obvious $n \geq |\mu|$, where n is the total number of doctors.

Case a: Violation of the capacity of hospital. Suppose that there exists a hospital h which is $|\mu(h)| > q_h$. At each round k, every hospital h rejects doctors with respect to its capacity so that we have $|A^k(h)| + |B^k(h)| \leq q_h$ for all rounds $1 \leq k \leq K$, which is contradicting with the assumption.

Case b: Violation of the minimum quota of hospital. Suppose there exists a hospital h which is $|\mu(h)| < p_h$. (i) If $n > |\mu|$, there exists some doctor d who is not assigned to any hospital $\mu(d) = d$. Doctor d must apply to hospital h at some round k and be rejected. Based on Fact 3, we have either $|\mu^k(h)| = q_h$ or $|A^k(h)| \geq p_h$, which are contradicting with the assumption. (ii) If $n = |\mu|$, we have $|\sum_{r\in R} B^k(r)| \leq e + \sum_{r\in R} \bar{p}_r$ at any round k and $|\mu^K(h)| = |A^K(h)| + |B^K(h)|$. If $|B^K(h)| > 0$, there must be $|A^K(h)| = p_h$, which is contradicting with the assumption. Therefore, we have $|B^K(h)| = 0$ and $|\mu^K(h)| = |A^K(h)|$. Based on Fact 1, the maximum number of tentatively accepted doctors in step 2 at any round k is $|A^k| = \sum_{h'\in H\setminus\{h\}} \min\{|p_{h'}|, |A^k(h')|\} + |A^k(h)| < \sum_{h\in H} p_h$. Meanwhile, based on Fact 2, the maximum number of tentatively accepted doctors in step 3 at any round k is $|B^k| = e + \sum_{r\in R} \bar{p}_r = n - \sum_{r\in R} p_r + \sum_{r\in R}(p_r - \sum_{h\in H_r} p_h) = n - \sum_{h\in H} p_h$. Hence, we have $|\mu^K| = |A^K| + |B^K| = n - \sum_{h\in H} p_h + \sum_{h'\in H\setminus\{h\}} \min\{|p_{h'}|, |A^K(h')|\} + |A^K(h)| < n$, which is contradicting to $n = |\mu| = |\mu^K|$.

Case c: Violation of minimum regional constraint. Suppose there exists one region $r \in R$ with $|\mu(r)| < p_r$. (i) If $n > |\mu|$, there exists some doctor d who is not assigned to any hospital $\mu(d) = d$. Doctor d must have applied to every hospital belonging to region r. Since $|\mu(r)| < p_r \leq \sum_{h\in H_r} q_h$, there must exists at least one hospital h whose capacity is not filled when doctor d applies to the hospital h at some round k and be rejected. That is, all first type flexible quotas e are filled and all second type flexible quotas in the region $\bar{p}_r$ are filled, which is contradicting to the assumption. (ii) If $n = |\mu|$, we have $|\sum_{r\in R} B^k(r)| \leq e + \sum_{r\in R} \bar{p}_r$ at any round k. Since $|\mu(r)| < p_r$, the region r does not consume any first type flexible quotas e. Considering the terminate round K, the maximum number of accepted doctors in all regions except region r is $\sum_{r'\in R\setminus\{r\}} |\mu^K(r')| = \sum_{r'\in R\setminus\{r\}}(|A^K(r')| + |B^K(r')|) = \sum_{h\in H\setminus\{H_r\}} \min\{|p_h|, |A^K(h)|\} + e + \sum_{r'\in R\setminus\{r\}} \bar{p}_{r'} \leq e + \sum_{r'\in R\setminus\{r\}} p_{r'} (= n - p_r)$. Hence, we have $\sum_{r\in R} |\mu^K(r)| = n - p_r + |\mu^K(r)| < n$, which is contradicting to $n = |\mu|$.

Case d: Violation of regional cap. At each round k, we have $\sum_{h\in H_r} |T_h| = |A^k(r)| + |B^k(r)| \leq q_r$. Hence, considering the terminate round K, we have $|\mu^K(r)| = |A^K(r)| +$

$|B^K(r)| \leq q_r$ for all $r \in R$, which means that there exists no region r that violates the maximum regional quota.

Combing cases (a), (b), (c) and (d), the mechanism always produce a feasible matching.

□

Proof of Proposition 7: Suppose there exists a doctor-hospital pair (d,h) such that $h \succ_d \mu(d)$ and $d \succ_h d'$ for some $d' \in \mu(h)$ in matching μ. Doctor d must apply to hospital h before applying to hospital $\mu(d)$ and be rejected in some round k .

Suppose doctor d is rejected by hospital h at step 2. According to the mechanism rule, each hospital rejects doctors with respect to its capacity based on its preference and hence $d' \succ_h d$ for all doctors d' has not been rejected in step 2 in round k. If h rejects no doctor at Step 3 in latter rounds, it is impossible for hospital h to accept doctor d' who is $d \succ_h d'$ because each hospital at each round will consider the new proposals and the tentatively accepted proposals in previous round together and select the set of best preferred doctors. If h rejects at least one doctor at Step 3 in latter rounds, based on Lemma 14 hospital h will not obtain extra flexible position in latter rounds. Hence, hospital h has no chance to accept any doctor d' who is $d \succ_h d'$ in latter rounds because each hospital at each round will consider the new proposals and the tentatively accepted proposals in previous round together and select the set of best preferred doctors.

Suppose doctor d is rejected by hospital h at step 3 in round k, based on Lemma 14 hospital h will not obtain extra flexible position in latter rounds. Hence, hospital h has no chance to accept any doctor d' who is $d \succ_h d'$ in latter rounds because each hospital at each round will consider the new proposals and the tentatively accepted proposals in previous round together and select the set of best preferred doctors.

□

Proof of Theorem 7: In a doctor-hospital market $(D,H,R,Q,HS,\succ)$, let μ denote the matching produced by the Doctor-proposing Deferred Acceptance mechanism.

We first show that the matching μ produced by the mechanism is individually rational. No agent will be assigned to an unacceptable agent because each doctor has preference over all hospitals and each hospital has preference over all students.

Based on Proposition 7, if a pair (d,h) blocks the matching μ, then d claims an empty position at h. Suppose a doctor-hospital pair (d,h) blocks the matching μ. Then, d must have applied to h and was rejected by h.

Situation (1). Suppose $\mu(d) = \emptyset$. We have $|\mu| < n$, which means the number of doctors is larger than the sum of regional caps. Since d claims an empty position at h, h rejects some doctor at Step 3 and hence either all regional caps or flexible quotas are filled. Since doctor d is unmatched $\mu(d) = \emptyset$, we have $|\mu| = \sum_{r\in R} q_r$ by Proposition 6. The new matching $\mu^{'}$ such that $\mu^{'}(d) = h$ and $\mu^{'}(d') = \mu(d')$ for all $d' \neq d$ is feasible, which is contradicting to the fact $|\mu^{'}| \leq \min\{n, \sum_{r\in R} q_r\}$.

Situation (2). Suppose $\mu(d) \in H$. Two cases are considered.

Case A. Suppose $hs(\mu(d)) > hs(h)$. A1. If $r(\mu(d)) = r(h)$, we have $|\mu(h)| < q_h$ and $|\mu(\mu(d))| > p_{\mu(d)}$ because the blocking pair is feasible to be moved together. Since $|\mu(h)| < q_h$, there exists some round $k < K$ such that h rejects some doctor $d' \succeq_h d$ at Step 3. Based on Corollary 4(i) and $|\mu(\mu(d))| > p_{\mu(d)}$, we have $hs(\mu(d)) \leq hs(h)$, which is contradicting to $hs(\mu(d)) > hs(h)$. A2. If $r(\mu(d)) \neq r(h)$, we have $|\mu(\mu_d)| > p_{\mu(d)}$, $|\mu(h)| < q_h$, $|\mu(r(h))| < q_{r(h)}$, and $|\mu(r(\mu(d)))| > p_{r(\mu(d))}$ because the blocking pair is feasible to be moved together. Since $|\mu_h| < q_h$, $|\mu(r(h))| < q_{r(h)}$, there exists some round $k < K$ such that h rejects some doctor $d^* \succeq_h d$ at step 3 and $|\mu^k(r(h))| < q_{r(h)}$. Since $|\mu(\mu(d))| > p_{\mu(d)}$ and $|\mu(r(\mu(d)))| > p_{r(\mu(d))}$, we have $|B^K(\mu(d))| > 0$ and $\sum_{h\in H_{r(\mu(d))}} |B^K(h)| > \bar{p}_{r(\mu(d))}$. According to Corollary 4(ii), we have $hs(\mu(d)) \leq hs(h)$, which is contradicting to $hs(\mu(d)) > hs(h)$.

Case B. Suppose $hs(h) = hs(\mu(d))$ and $|\mu^{'}(h)| - \bar{p}_h \leq |\mu^{'}(\mu(d))| - \bar{p}_{\mu(d)}$. Equivalently we need prove $|B^K(h)| + 1 \leq |B^K(\mu(d))| - 1$. B1. If $r(\mu(d)) = r(h)$, we have $|\mu(\mu(d))| > p_{\mu(d)}$, and $|\mu(h)| < q_h$ because the blocking pair is feasible to be moved together. Since $|\mu(h)| < q_h$, hospital h must rejects some doctor d' at step 3 in round $k^{'} \geq k$. Hence, we have $|B^K(h)| + 1 \geq |B^K(\mu(d))| > |B^K(\mu(d))| - 1$ because $|\mu(\mu(d))| > p_{\mu(d)}$, which is contradicting with the assumption. B2. If $r(\mu(d)) \neq r(h)$, we have $|\mu(\mu(d))| > p_{\mu(d)}$, $|\mu(h)| < q_h$, $|\mu(r(h))| < q_{r(h)}$, and $|\mu(r(\mu(d)))| > p_{\mu(d)}$ because the blocking pair is feasible to be moved together. There must exist some round $k < K$ such that h rejects some doctor $d^* \succeq_h d$ at step 3 because $|\mu(h)| < q_h$. We have $|B^K(h)| + 1 \geq |B^K(\mu(d))| > |B^K(\mu(d))| - 1$ based on Lemma 5(ii), which is contradicting with the assumption.

Combing Situation (1) and (2), we have proved that the matching produced by the doctor-proposing mechanism is stable.

□

Proof of Theorem 8: Denote $\succ_D$ be the set of true preference for all doctors in the model. We wish to consider the set preference $\succ_D^{'}$ such that $\succ_d^{'} \neq \succ_d$ for some doctor $d \in D$ and $\succ_{d'}^{'} = \succ_{d'}$ for all doctor $d' \neq d$. That is, $\succ_D^{'} = (\succ_{d'}, \succ_{-d})$. Let μ be the matching produced by mechanism under true preference $\succ_D$ while $\mu^{'}$ be the matching produced by mechanism under preference $\succ_D^{'}$.

We first show that we only need to consider a specific kind of misrepresentation. A simple equivalent misrepresentations $\succ_D^{''}$ is considered that the manipulator d_m simply put his match in $\mu^{'}(d_m)$ to the top of his list. That is, d_m prefers $\mu^{'}(d_m)$ to all other hospitals under his preference $\succ_{d_m}^{''}$. Let $\mu^{''}$ be the matching produced by the mechanism under preference profile $\succ_D^{''}$. We have the following Lemma 15. The similar idea is proposed by Roth in 1982.

Lemma 15 *If* $\mu^{'} = \chi(\succ_D^{'})$ *and* $\mu^{''} = \chi(\succ_D^{''})$, *then* $\mu^{'}(d_m) = \mu^{''}(d_m)$.

Proof of Lemma 15: Matching $\mu^{'}$ is a stable matching under preference $\succ_D^{'}$. Since the only party whose preference has changed is d_m and she has moved $\mu^{'}(d_m)$ to the top of his list in preference $\succ_{d_m}^{''}$, matching $\mu^{'}$ is also a stable matching under preference $\succ_D^{''}$. Remember that $\mu^{''}$ is stable matching with respect to preference $\succ_D^{''}$.

Suppose $\mu^{''}(d_m) \neq \mu^{'}(d_m)$. Since $\mu^{'}(d_m)$ is the top choice of $\succ_{d_m}^{''}$, we have $\mu^{'}(d_m) \succ_{d_m}^{''} \mu^{''}(d_m)$ and hence $\mu^{'}(d_m) \in H$. Let $\mu^{'}(d_m) = h$. Since $\mu^{''}$ is stable, we have $d' \succ_h^{''} d_m$ for all $d' \in \mu^{''}(h)$. Hence, for each doctor $d' \in D \setminus \{d_m\}$ who has $d' \in \mu^{''}(h)$, we have either $\mu^{'}(d') = h$ or $\mu^{'}(d') \succ_{d'}^{'} h$. Note that $\succ_h^{'} = \succ_h^{''}$. (Otherwise, if $h \succ_{d'}^{'} \mu^{'}(d')$, (d', h) can form a blocking pair that d' claims an occupied position at h, which is contradicting to the stability of $\mu^{'}$.)

Let M denote a set of doctors who prefer $\mu^{'}(d)$ to $\mu^{''}(d)$ under preference profile $\succ^{''}$. We have $\mu^{'}(d) \in H$ for all $d \in M$. Denote round l be the earliest round in which some $d \in M$ is rejected by $\mu^{'}(d)$ in matching $\mu^{''}$ of preferences $\succ^{''}$. Denote $h' = \mu^{'}(d)$. Suppose that there exists a doctor $d^{'}$, who has $\mu^{'}(d^{'}) \succ_{d'}^{'} h'$, applying to h' and be tentatively accepted by h' in round l of matching $\mu^{''}$. If $d' = d_m$, we have $\mu^{'}(d_m) \succ_{d_m}^{''} h' \succeq_{d_m}^{''} \mu^{''}(d_m)$.

If $d' \neq d_m$, since $\mu^{'}(d^{'}) \succ^{'}_{d'} h'$, we have $\mu^{'}(d^{'}) \succ^{''}_{d'} \mu^{''}(d^{'})$. Hence $d^{'} \in M$ and $d^{'}$ is rejected by $\mu^{'}(d')$ earlier than round l, which is contradicting to the assumption. Therefore, no doctor $d^{'}$ who has $\mu^{'}(d^{'}) \succ^{'}_{d'} h'$ applies to h' and is tentatively accepted by h' in round l.

Since $\mu^{'}$ is stable (of $\succ^{'}$ and $\succ^{''}$), we have $d \succ^{'}_{h'} d'$ for all $d' \in D$ with $h' \succ^{'}_{d'} \mu^{'}(d')$ and $h' \succ^{''}_{d'} \mu^{''}(d')$. Therefore, we have $|\mu^{''l}(h')| < |\mu^{'l}(h')|$ in round l based on Lemma 14. Hence d is rejected by h' at Step 3 and hence h' loses a flexible quota in round l of matching $\mu^{''}$. There exits some hospital $h^{''}$ obtaining the flexible quota. Hence there exists some doctor $d' \in \mu^{''}(h^{''}) \setminus \mu^{'}(h^{''})$, who is accepted by $h^{''}$ in or latter than round l, consumes such flexible quota. If $\mu^{'}(d^{'}) \succ^{''}_{d'} h^{''}$, we have $d^{'} \in M$ and be rejected by $\mu^{'}(d^{'})$ before round l, which is contradicting to the assumption. If $h^{''} \succ^{''}_{d'} \mu^{'}(d^{'})$, we have $d' \neq d_m$ and $(d^{'}, h^{''})$ can form a blocking pair in matching $\mu^{'}$ (of $\succ^{''}$), which is contradicting to the stability of $\mu^{'}$. Consequently, no doctor d has $\mu^{'}(d) \succ_d \mu^{''}(d)$. Therefore, we have $\mu^{''}(d_m) = \mu^{'}(d_m)$.

□

Therefore, based on Lemma 15, Theorem 8 can be proved by proving that no simple misrepresentation $\succ^{'}_{D} = (\succ^{'}_{d_m}, \succ_{-d_m})$ can be successful, where $\succ^{'}_{d_m}$ means a simple representation by manipulator d_m. Then, we prove that the following Lemma 16 works in our mechanism.

Lemma 16 *If $\succ^{'}_{d_m}$ is a simple misrepresentation such that $\mu^{'} = \chi(\succ^{'}_{D})$ and either $\mu^{'}(d_m) \succ_{d_m} \mu(d_m)$ or $\mu^{'}(d_m) = \mu(d_m)$, then for each $d \in D$, either $\mu^{'}(d) \succ_d \mu(d)$ or $\mu^{'}(d) = \mu(d)$.*

Proof of Lemma 16: Based on Theorem 6, we have matching μ is stable for the preference profile $\succ_D$ as well as matching $\mu^{'}$ for the preference profile $\succ^{'}_{D}$. Suppose we have some doctor $d \in D \setminus \{d_m\}$ with $\mu(d) \succ_d \mu^{'}(d)$. Since every doctor other than d_m states the same preferences in profiles $\succ^{'}$ and $\succ$, d must be rejected by $\mu(d)$ at some step of matching $\mu^{'}$. Let M denote the set of doctor d with $\mu(d) \succ_d \mu^{'}(d)$.

Denote round l be the earliest round in which some doctor $d \in M$ is rejected by $\mu(d)$ of matching $\mu^{'}$. Note that $d \neq d_m$. Let $h = \mu(d)$. Suppose that there exists a doctor d', who has $\mu(d') \succ_{d'} h$, applying to h and be tentatively accepted by h in round l of matching $\mu^{'}$. Since we have either $\mu^{'}(d_m) \succ_{d_m} \mu(d_m)$ or $\mu^{'}(d_m) = \mu(d_m)$, $d' \neq d_m$. Therefore, we have $\mu(d') \succ'_{d'} \mu'(d')$ because $\mu(d') \succ_{d'} h$. Hence d' is rejected by $\mu(d')$ earlier than round l, which is contradicting to the assumption.

Since μ is stable of $\succ$, we have $d \succ_h d'$ for all $d' \in D$ with $h \succ_{d'} \mu(d')$. If $h \succ_{d_m} \mu(d_m)$, d is preferred by h to d_m. If $\mu(d_m) \succ_{d_m} h$, d_m will not propose to h because we have either $\mu'(d_m) \succ_{d_m} \mu(d_m)$ or $\mu'(d_m) = \mu(d_m)$. Therefore, $|\mu'^l(h)| < |\mu^l(h)|$ in round l based on Lemma 14. Hence d is rejected by h at Step 3 and hence h loses a flexible quota in round l of matching μ'. There exits some hospital h' obtaining the flexible quota. Hence there exists some doctor $d' \in \mu'(h') \setminus \mu(h')$, who is accepted by h' in or latter than round l, consumes such flexible quota with higher priority. If $\mu(d') \succ'_{d'} h'$, d' is rejected by $\mu(d')$ before round l, which is contradicting to the assumption. Otherwise, $h' \succ'_{d'} \mu(d')$. If $d' = d_m$, we have $h' \succ_{d_m} \mu(d_m)$ and d_m must apply to h' and be rejected in matching μ. Since h' has a higher priority to obtain a flexible quota than (d,h), (d_m,h') can form a blocking pair in matching μ, which is contradicting to the stability of μ. If $d' \neq d_m$, d' must apply to h' and be rejected of matching μ. Therefore, (d',h') can form a blocking pair in matching μ, which is contradicting to the stability of μ. Therefore, d cannot be rejected by $\mu(d)$. Consequently, we have $M = \emptyset$, which completes the proof.

□

Now, we are ready to prove Theorem 8. Let $\mu = \chi(\succ_D)$ be the matching under true preference profile and $\mu' = \chi(\succ'_D)$ be the matching under preference profile where doctor d_m makes a simple misrepresentation $\succ'_{d_m}$ and other doctors present their true preference, that is $\succ'_D = (\succ'_{d_m}, \succ_{-d_m})$.

We want to show that d_m's simple misrepresentation cannot be successful, and we will proceed either $\mu'(d_m) \succ_{d_m} \mu(d_m)$ or $\mu'(d_m) = \mu(d_m)$ based on Lemma 16 and only the latter alternative can occur. Let round l be the round that the manipulator d_m proposes to her match $\mu(d_m)$ of $\chi(\succ_D)$ and we will prove that every doctor $d \in D$ who propose to her match $\mu(d)$ at round l or latter has $\mu'(d) = \mu(d)$.

We first prove that $\mu'(d) = \mu(d)$ for all $\mu(d) = d$. Suppose there exists $\mu(d) = d$ and $\mu'(d) \neq \mu(d)$. We have $\mu'(d) \in H$. Since μ is stable, either all flexible quotas are filled or regional cap are filled. Moreover, for any $h \in H$, $d' \succ_h d$ for all $d' \in \mu(h)$. Hence there exists that one doctor d' with $\mu(d') \in H$ has $\mu'(d') = d'$. Therefore, $(d', \mu(d'))$ forms a blocking pair in μ', which is contradicting to the stability of μ'.

Then look at a doctor d who propose to her match $\mu(d) \in H$ at the last round L of $\chi(\succ_D)$. Suppose we have $\mu^{'}(d) \neq \mu(d)$ so that $\mu^{'}(d) \succ_d \mu(d)$ based on Lemma 16. Then, every doctor $d' \notin \mu(\mu(d))$ in matching μ has $\mu(d') \succ_{d'} \mu(d)$. Otherwise, there exists one doctor d' who applies to $\mu(d)$ and is rejected. According to Lemma 14, some doctor must be rejected by $\mu(d)$ when d apply to $\mu(d)$ in round L. That is, round L will not be the last round of $\chi(\succ_D)$. Therefore, $\mu(d)$ rejects no doctor. Since $\mu^{'}(d) \succ_d \mu(d)$, we have $|\mu^{'}(\mu(d))| < |\mu(\mu(d))|$. In matching μ, $\mu(d)$ must consume at least one flexible quota because of the feasibility of matching $\mu^{'}$. It means that there at least exits one flexible quota available in the last round. Two cases are considered.

Case A. If the flexible quota is a regional quota, every hospital in region $r(\mu(d))$ rejects no doctor at Step 3 because there are enough quotas are available until last round. There exists one hospital $h' \in r(\mu(d))$ which has $|\mu^{'}(h')| > |\mu(h')|$. Hence there exists $d' \in \mu^{'}(h') \setminus \mu(h')$ such that $h' \succ_{d'} \mu(d')$ based on Lemma 16, which is contradicting with the fact that h' rejects no doctor.

Case B. If the flexible quota is a national quota, there are enough quotas available until last round. There exists one hospital h' which has $|\mu^{'}(h')| > |\mu(h')|$ and $\sum_{h \in H_{r(h')}} |\mu(h)| < q_{r(h')}$ receiving the quota. Since $|\mu(h')| < q_{h'}$ and $\sum_{h \in H_{r(h')}} |\mu(h)| < q_{r(h')}$, h' rejects no doctor because of enough quotas. There exists some doctor $d' \in \mu^{'}(h') \setminus \mu(h')$ such that $\mu^{'}(d') \succ_{d'} \mu(d')$ based on Lemma 16, which is contradicting with the fact that h' rejects no doctor.

Therefore, we have any doctor d who propose her match at last round has $\mu^{'}(d) = \mu(d)$. If the manipulator d_m proposes her match $\mu(d_m)$ at last round ($l = L$), then we proved the theorem.

Suppose the manipulator d_m proposes his match $\mu(d_m) \in H$ at round $l < L$, we will prove by induction that for every d (including d_m) who propose his match $\mu(d)$ at round $l \leq r < L$, $\mu^{'}(d) = \mu(d)$. We have proved that $\mu^{'}(d') = \mu(d')$ for any d' who makes his match in last round. The inductive part of the proof is to show that, if $\mu^{'}(d') = \mu(d')$ for every d who proposes his match $\mu(d')$ in round $r+1$ or latter, then $\mu^{'}(d) = \mu(d)$ for every d who proposes his match $\mu(d)$ in round r. Look at a doctor d_q who propose his match $\mu(d_q)$ at round $l \leq r < L$. Let $D' = \{d \in D | \mu(d_q) \succ_d \mu(d)\}$ be the subset of doctors who prefer $\mu(d_q)$ to their matches. Suppose that $\mu^{'}(d_q) \neq \mu(d_q)$. Based on Lemma 16, we have $\mu^{'}(d_q) \succ_d \mu(d_q)$. Two situations are considered.

Situation (1). Suppose D' is empty. That is, no doctor $d' \notin \mu(\mu(d_q))$ will propose to $\mu(d)$ in matching $\mu^{'}$ based on Lemma 16. Hence we have $p_{\mu(d_q)} \leq |\mu^{'}(\mu(d_q))| < |\mu(\mu(d_q))|$ so that $\mu(d_q)$ obtains at least one flexible quota in matching μ because of feasibility of $\mu^{'}$. There exists some hospital h' which has $|\mu(h')| < |\mu^{'}(h')| \leq q_{h'}$. According to the inductive assumption, we have $\mu^{'}(d') = \mu(d')$ for any doctor d' who is rejected by h' in round r or latter round. Based on Lemma 14, h' rejects no doctor in round r or latter round and rejects some doctor at Step 3 because of flexible quota in some round $s < r$. Since $\mu(d_q)$ does not reject any doctor and accepts d_q in round $r \geq l$, there exists some doctor $d^{''} \neq d_m$ is rejected by some hospital $h^{''}$ because of some doctor who is accepted by $\mu(d_q)$ in round r or latter in round r. According to the inductive assumption, we have $\mu^{'}(d^{''}) = \mu(d^{''})$. Since $d^{''}$ is kept until round r and h' rejects some doctor at Step 3 before round r, $(d^{''}, h^{''})$ forms a blocking pair in matching $\mu^{'}$, which is contradicting to the stability of $\mu^{'}$.

Situation (2). Suppose D' is not empty. Let d' be the doctor who is best preferred by $\mu(d_q)$ among doctors in D'. Because d_q is accepted by $\mu(d_q)$ at round r, $\mu(d')$ makes his match after round r based on Lemma 14. Therefore, we have $\mu^{'}(d') = \mu(d')$ and $d' \neq d_m$. If $|\mu^{'}(\mu(d_q))| < |\mu(\mu(d_q))|$, we have $\mu^{'}(d_q) = \mu(d_q)$ from previous proof in situation (1). If $|\mu^{'}(\mu(d_q))| \geq |\mu(\mu(d_q))|$, there exists one doctor $d \in D' \setminus \{d'\}$ has $\mu^{'}(d) = \mu(d_q)$. Since $\mu^{'}(d') = \mu(d')$ and d' is the best preferred by $\mu(d_q)$ among doctors in D', $(d', \mu(d_q))$ forms a blocking pair in matching $\mu^{'}$, which is contradicting to the stability of $\mu^{'}$.

Therefore, we prove that $\mu^{'}(d_m) = \mu(d_m)$ and thus telling the true preference is a dominant strategy for doctors in our mechanism.

□

Appendix D

Supplementaries for Chapter 3

To help reader understand various cases in the proof of the third chapter, we provide some instructions on how to read tables in this section.

The notations in the first line of each table are one-to-one correspondence with those in proof formulas. For example, the notations in the first line of Table A1 are one-to-one correspondence with the proof formulas in the sequential two-way matching procedure (see p82 to p83).

We provide each case a serial number. For example, we use Table B1 and Table B2 to present the first 45 cases labelled from A1 to A42.

D.1 Supplementary A

Table A1 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
1	BA	OA	OB	OAB	AAB	BAB	0	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
2	BA	OA	OB	OAB	AAB	BAB	0	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
3	BA	OA	OB	OAB	AAB	BAB	0	0	0	0	AB - BA	0	N1
4	BA	OA	OB	OAB	AAB	ABBi	0	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
5	BA	OA	OB	OAB	AAB	ABBi	0	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
6	BA	OA	OB	OAB	AAB	ABBi	0	0	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
7	BA	OA	OB	OAB	AAB	ABBi	0	0	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
8	BA	OA	OB	OAB	AAB	ABBi	0	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
9	BA	OA	OB	OAB	AAB	ABBi	0	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
10	BA	OA	OB	OAB	AAB	ABBi	0	0	0	-ABBi + BAB	AB - BA	0	N1
11	BA	OA	OB	OAB	ABAi	BAB	0	0	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
12	BA	OA	OB	OAB	ABAi	BAB	0	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
13	BA	OA	OB	OAB	ABAi	BAB	0	0	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N2
14	BA	OA	OB	OAB	ABAi	BAB	0	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi	N1
15	BA	OA	OB	OAB	ABAi	BAB	0	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
16	BA	OA	OB	OAB	ABAi	BAB	0	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
17	BA	OA	OB	OAB	ABAi	BAB	0	0	AAB - ABAi	0	AB - BA	0	N1
18	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
19	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
20	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N4
21	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - B + BAB	N1
22	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
23	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
24	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N2
25	BA	OA	OB	OAB	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi	N1
26	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
27	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
28	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
29	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
30	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
31	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
32	BA	OA	OB	OAB	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	AB - BA	0	N1
33	BA	OA	OB	ABOi	AAB	BAB	0	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
34	BA	OA	OB	ABOi	AAB	BAB	0	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
35	BA	OA	OB	ABOi	AAB	BAB	0	0	0	0	AB - BA	ABOc + O	N3
36	BA	OA	OB	ABOi	AAB	BAB	0	0	0	0	AB - BA	-ABOi + OAB	N1
37	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
38	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
39	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	ABBc + B	AB - BA	ABOc + O	N5
40	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
41	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
42	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
43	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	-ABBi + BAB	AB - BA	ABOc + O	N3
44	BA	OA	OB	ABOi	AAB	ABBi	0	0	0	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
45	BA	OA	OB	ABOi	ABAi	BAB	0	0	A + ABAc	0	BOc + BOi - OB	ABOc + O	N10

Table A2 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
46	BA	OA	OB	ABOi	ABAi	BAB	0	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
47	BA	OA	OB	ABOi	ABAi	BAB	0	0	A + ABAc	0	AB - BA	ABOc + O	N2
48	BA	OA	OB	ABOi	ABAi	BAB	0	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
49	BA	OA	OB	ABOi	ABAi	BAB	0	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
50	BA	OA	OB	ABOi	ABAi	BAB	0	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
51	BA	OA	OB	ABOi	ABAi	BAB	0	0	AAB - ABAi	0	AB - BA	ABOc + O	N3
52	BA	OA	OB	ABOi	ABAi	BAB	0	0	AAB - ABAi	0	AB - BA	-ABOi + OAB	N1
53	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N12
54	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
55	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	ABBc + B	AB - BA	ABOc + O	N4
56	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - B + BAB + OAB	N1
57	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N10
58	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
59	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N2
60	BA	OA	OB	ABOi	ABAi	ABBi	0	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
61	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
62	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
63	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N5
64	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
65	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
66	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
67	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N3
68	BA	OA	OB	ABOi	ABAi	ABBi	0	0	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
69	BA	OA	BOi	OAB	AAB	BAB	0	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
70	BA	OA	BOi	OAB	AAB	BAB	0	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
71	BA	OA	BOi	OAB	AAB	BAB	0	-BOi + OB	0	0	AB - BA	0	N1
72	BA	OA	BOi	OAB	AAB	BAB	0	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
73	BA	OA	BOi	OAB	AAB	BAB	0	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
74	BA	OA	BOi	OAB	AAB	BAB	0	BOc	0	0	AB - BA	ABOc + ABOi + O - OAB	N11
75	BA	OA	BOi	OAB	AAB	BAB	0	BOc	0	0	AB - BA	-BOc - BOi + OB	N1
76	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
77	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
78	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
79	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
80	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
81	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
82	BA	OA	BOi	OAB	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	AB - BA	0	N1
83	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N12
84	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
85	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N13
86	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB - BOc - BOi + OB	N1
87	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
88	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
89	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N11
90	BA	OA	BOi	OAB	AAB	ABBi	0	BOc	0	-ABBi + BAB	AB - BA	-BOc - BOi + OB	N1

Table A3 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
91	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
92	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
93	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N2
94	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi	N1
95	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
96	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
97	BA	OA	BOi	OAB	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	AB - BA	0	N1
98	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	A + ABAc	0	0	ABOc + ABOi + O - OAB	N10
99	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
100	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
101	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
102	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	AAB - ABAi	0	AB - BA	ABOc + ABOi + O - OAB	N11
103	BA	OA	BOi	OAB	ABAi	BAB	0	BOc	AAB - ABAi	0	AB - BA	-BOc - BOi + OB	N1
104	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
105	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
106	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N4
107	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - B + BAB	N1
108	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
109	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
110	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N2
111	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi	N1
112	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
113	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
114	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
115	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
116	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
117	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
118	BA	OA	BOi	OAB	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	0	N1
119	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	A + ABAc	ABBc + B	0	ABOc + ABOi + O - OAB	N12
120	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
121	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	A + ABAc	-ABBi + BAB	0	ABOc + ABOi + O - OAB	N10
122	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
123	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N12
124	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
125	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N13
126	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB - BOc - BOi + OB	N1
127	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
128	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
129	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N11
130	BA	OA	BOi	OAB	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-BOc - BOi + OB	N1
131	BA	OA	BOi	ABOi	AAB	BAB	0	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
132	BA	OA	BOi	ABOi	AAB	BAB	0	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
133	BA	OA	BOi	ABOi	AAB	BAB	0	-BOi + OB	0	0	AB - BA	ABOc + O	N3
134	BA	OA	BOi	ABOi	AAB	BAB	0	-BOi + OB	0	0	AB - BA	-ABOi + OAB	N1
135	BA	OA	BOi	ABOi	AAB	BAB	0	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + O	N10

Table A4 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
136	BA	OA	BOi	ABOi	AAB	BAB	0	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
137	BA	OA	BOi	ABOi	AAB	BAB	0	BOc	0	0	AB - BA	ABOc + O	N11
138	BA	OA	BOi	ABOi	AAB	BAB	0	BOc	0	0	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
139	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
140	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
141	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	ABBc + B	AB - BA	ABOc + O	N5
142	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
143	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
144	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
145	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	AB - BA	ABOc + O	N3
146	BA	OA	BOi	ABOi	AAB	ABBi	0	-BOi + OB	0	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
147	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N12
148	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
149	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	ABBc + B	AB - BA	ABOc + O	N13
150	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB - BOc - BOi + OAB + OB	N1
151	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N10
152	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
153	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	-ABBi + BAB	AB - BA	ABOc + O	N11
154	BA	OA	BOi	ABOi	AAB	ABBi	0	BOc	0	-ABBi + BAB	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
155	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + O	N10
156	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
157	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	A + ABAc	0	AB - BA	ABOc + O	N2
158	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
159	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
160	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
161	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	AB - BA	ABOc + O	N3
162	BA	OA	BOi	ABOi	ABAi	BAB	0	-BOi + OB	AAB - ABAi	0	AB - BA	-ABOi + OAB	N1
163	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	A + ABAc	0	0	ABOc + O	N10
164	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
165	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + O	N10
166	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
167	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	AAB - ABAi	0	AB - BA	ABOc + O	N11
168	BA	OA	BOi	ABOi	ABAi	BAB	0	BOc	AAB - ABAi	0	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
169	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N12
170	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
171	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + O	N4
172	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - B + BAB + OAB	N1
173	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N10
174	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
175	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N2
176	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
177	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12

Table A5 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
178	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
179	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N5
180	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
181	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
182	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
183	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N3
184	BA	OA	BOi	ABOi	ABAi	ABBi	0	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
185	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	A + ABAc	ABBc + B	0	ABOc + O	N12
186	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
187	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	A + ABAc	-ABBi + BAB	0	ABOc + O	N10
188	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
189	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N12
190	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
191	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N13
192	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB - BOc - BOi + OAB + OB	N1
193	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N10
194	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
195	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N11
196	BA	OA	BOi	ABOi	ABAi	ABBi	0	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
197	BA	AOi	OB	OAB	AAB	BAB	-AOi + OA	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
198	BA	AOi	OB	OAB	AAB	BAB	-AOi + OA	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
199	BA	AOi	OB	OAB	AAB	BAB	-AOi + OA	0	0	0	AB - BA	0	N1
200	BA	AOi	OB	OAB	AAB	BAB	AOc	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
201	BA	AOi	OB	OAB	AAB	BAB	AOc	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
202	BA	AOi	OB	OAB	AAB	BAB	AOc	0	0	0	AB - BA	ABOc + ABOi + O - OAB	N7
203	BA	AOi	OB	OAB	AAB	BAB	AOc	0	0	0	AB - BA	-AOc - AOi + OA	N1
204	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
205	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
206	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
207	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
208	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
209	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
210	BA	AOi	OB	OAB	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	AB - BA	0	N1
211	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
212	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
213	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N9
214	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
215	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
216	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
217	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N7

Table A6 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
218	BA	AOi	OB	OAB	AAB	ABBi	AOc	0	0	-ABBi + BAB	AB - BA	-AOc - AOi + OA	N1
219	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
220	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
221	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N2
222	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi	N1
223	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
224	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
225	BA	AOi	OB	OAB	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	AB - BA	0	N1
226	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
227	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
228	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N6
229	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - AOc - AOi + OA	N1
230	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
231	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
232	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	AAB - ABAi	0	AB - BA	ABOc + ABOi + O - OAB	N7
233	BA	AOi	OB	OAB	ABAi	BAB	AOc	0	AAB - ABAi	0	AB - BA	-AOc - AOi + OA	N1
234	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
235	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
236	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N4
237	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - B + BAB	N1
238	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
239	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
240	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N2
241	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi	N1
242	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
243	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
244	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
245	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
246	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
247	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
248	BA	AOi	OB	OAB	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	AB - BA	0	N1
249	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
250	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
251	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N8
252	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
253	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
254	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
255	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N6
256	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - AOc - AOi + OA	N1
257	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
258	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1

Table A7 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
259	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N9
260	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
261	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
262	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
263	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N7
264	BA	AOi	OB	OAB	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	AB - BA	-AOc - AOi + OA	N1
265	BA	AOi	OB	ABOi	AAB	BAB	-AOi + OA	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
266	BA	AOi	OB	ABOi	AAB	BAB	-AOi + OA	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
267	BA	AOi	OB	ABOi	AAB	BAB	-AOi + OA	0	0	0	AB - BA	ABOc + O	N3
268	BA	AOi	OB	ABOi	AAB	BAB	-AOi + OA	0	0	0	AB - BA	-ABOi + OAB	N1
269	BA	AOi	OB	ABOi	AAB	BAB	AOc	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
270	BA	AOi	OB	ABOi	AAB	BAB	AOc	0	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
271	BA	AOi	OB	ABOi	AAB	BAB	AOc	0	0	0	AB - BA	ABOc + O	N7
272	BA	AOi	OB	ABOi	AAB	BAB	AOc	0	0	0	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
273	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
274	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
275	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	ABBc + B	AB - BA	ABOc + O	N5
276	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
277	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
278	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
279	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	AB - BA	ABOc + O	N3
280	BA	AOi	OB	ABOi	AAB	ABBi	-AOi + OA	0	0	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
281	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N16
282	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
283	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	ABBc + B	AB - BA	ABOc + O	N9
284	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
285	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
286	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
287	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	-ABBi + BAB	AB - BA	ABOc + O	N7
288	BA	AOi	OB	ABOi	AAB	ABBi	AOc	0	0	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
289	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	A + ABAc	0	BOc + BOi - OB	ABOc + O	N10
290	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
291	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	A + ABAc	0	AB - BA	ABOc + O	N2
292	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
293	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
294	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
295	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	AB - BA	ABOc + O	N3
296	BA	AOi	OB	ABOi	ABAi	BAB	-AOi + OA	0	AAB - ABAi	0	AB - BA	-ABOi + OAB	N1
297	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	A + ABAc	0	BOc + BOi - OB	ABOc + O	N14
298	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
299	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	A + ABAc	0	AB - BA	ABOc + O	N6

Table A8 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
300	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi - AOc - AOi + OA + OAB	N1
301	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
302	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
303	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	AAB - ABAi	0	AB - BA	ABOc + O	N7
304	BA	AOi	OB	ABOi	ABAi	BAB	AOc	0	AAB - ABAi	0	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
305	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N12
306	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
307	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	AB - BA	ABOc + O	N4
308	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - B + BAB + OAB	N1
309	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N10
310	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
311	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N2
312	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
313	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
314	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
315	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N5
316	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
317	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
318	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
319	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N3
320	BA	AOi	OB	ABOi	ABAi	ABBi	-AOi + OA	0	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
321	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N16
322	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
323	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	AB - BA	ABOc + O	N8
324	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
325	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N14
326	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
327	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N6
328	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi - AOc - AOi + OA + OAB	N1
329	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N16
330	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
331	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N9
332	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
333	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
334	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
335	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N7
336	BA	AOi	OB	ABOi	ABAi	ABBi	AOc	0	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1

Table A9 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
337	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
338	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
339	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	-BOi + OB	0	0	AB - BA	0	N1
340	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
341	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
342	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	BOc	0	0	AB - BA	ABOc + ABOi + O - OAB	N11
343	BA	AOi	BOi	OAB	AAB	BAB	-AOi + OA	BOc	0	0	AB - BA	-BOc - BOi + OB	N1
344	BA	AOi	BOi	OAB	AAB	BAB	AOc	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
345	BA	AOi	BOi	OAB	AAB	BAB	AOc	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
346	BA	AOi	BOi	OAB	AAB	BAB	AOc	-BOi + OB	0	0	AB - BA	ABOc + ABOi + O - OAB	N7
347	BA	AOi	BOi	OAB	AAB	BAB	AOc	-BOi + OB	0	0	AB - BA	-AOc - AOi + OA	N1
348	BA	AOi	BOi	OAB	AAB	BAB	AOc	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N14
349	BA	AOi	BOi	OAB	AAB	BAB	AOc	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
350	BA	AOi	BOi	OAB	AAB	BAB	AOc	BOc	0	0	AB - BA	ABOc + ABOi + O - OAB	N15
351	BA	AOi	BOi	OAB	AAB	BAB	AOc	BOc	0	0	AB - BA	-AOc - AOi - BOc - BOi + OA + OB	N1
352	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
353	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
354	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
355	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
356	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
357	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
358	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	AB - BA	0	N1
359	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N12
360	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
361	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N13
362	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB - BOc - BOi + OB	N1
363	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
364	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
365	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N11
366	BA	AOi	BOi	OAB	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	AB - BA	-BOc - BOi + OB	N1
367	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
368	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
369	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N9
370	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
371	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
372	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
373	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N7
374	BA	AOi	BOi	OAB	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	AB - BA	-AOc - AOi + OA	N1
375	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N16
376	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1

Table A10 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
377	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N17
378	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB - BOc - BOi + OA + OB	N1
379	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N14
380	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
381	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N15
382	BA	AOi	BOi	OAB	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	AB - BA	-AOc - AOi - BOc - BOi + OA + OB	N1
383	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
384	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
385	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N2
386	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi	N1
387	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
388	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
389	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	AB - BA	0	N1
390	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	A + ABAc	0	0	ABOc + ABOi + O - OAB	N10
391	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
392	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
393	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
394	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	AB - BA	ABOc + ABOi + O - OAB	N11
395	BA	AOi	BOi	OAB	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	AB - BA	-BOc - BOi + OB	N1
396	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
397	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
398	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	AB - BA	ABOc + ABOi + O - OAB	N6
399	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - AOc - AOi + OA	N1
400	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
401	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
402	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	AB - BA	ABOc + ABOi + O - OAB	N7
403	BA	AOi	BOi	OAB	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	AB - BA	-AOc - AOi + OA	N1
404	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	A + ABAc	0	0	ABOc + ABOi + O - OAB	N14
405	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
406	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N14
407	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
408	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	AAB - ABAi	0	AB - BA	ABOc + ABOi + O - OAB	N15
409	BA	AOi	BOi	OAB	ABAi	BAB	AOc	BOc	AAB - ABAi	0	AB - BA	-AOc - AOi - BOc - BOi + OA + OB	N1
410	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
411	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
412	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N4
413	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - B + BAB	N1
414	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
415	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
416	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N2
417	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi	N1

Table A11 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
418	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N12
419	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
420	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N5
421	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB	N1
422	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N10
423	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
424	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	0	N1
425	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	ABBc + B	0	ABOc + ABOi + O - OAB	N12
426	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
427	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	-ABBi + BAB	0	ABOc + ABOi + O - OAB	N10
428	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
429	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N12
430	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - B - BA + BAB - BOc - BOi + OB	N1
431	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N13
432	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - B + BAB - BOc - BOi + OB	N1
433	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N10
434	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - BA - BOc - BOi + OB	N1
435	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N11
436	BA	AOi	BOi	OAB	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-BOc - BOi + OB	N1
437	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
438	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
439	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N8
440	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
441	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
442	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
443	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N6
444	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - AOc - AOi + OA	N1
445	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N16
446	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
447	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N9
448	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB + OA	N1
449	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + ABOi + O - OAB	N14
450	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
451	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N7
452	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	-AOc - AOi + OA	N1
453	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	A + ABAc	ABBc + B	0	ABOc + ABOi + O - OAB	N16
454	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
455	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	A + ABAc	-ABBi + BAB	0	ABOc + ABOi + O - OAB	N14
456	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
457	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N16

Table A12 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
458	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OB	N1
459	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + ABOi + O - OAB	N17
460	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - AOc - AOi - B + BAB - BOc - BOi + OA + OB	N1
461	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + ABOi + O - OAB	N14
462	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - AOc - AOi - BA - BOc - BOi + OA + OB	N1
463	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + ABOi + O - OAB	N15
464	BA	AOi	BOi	OAB	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-AOc - AOi - BOc - BOi + OA + OB	N1
465	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
466	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
467	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	-BOi + OB	0	0	AB - BA	ABOc + O	N3
468	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	-BOi + OB	0	0	AB - BA	-ABOi + OAB	N1
469	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + O	N10
470	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
471	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	BOc	0	0	AB - BA	ABOc + O	N11
472	BA	AOi	BOi	ABOi	AAB	BAB	-AOi + OA	BOc	0	0	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
473	BA	AOi	BOi	ABOi	AAB	BAB	AOc	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
474	BA	AOi	BOi	ABOi	AAB	BAB	AOc	-BOi + OB	0	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
475	BA	AOi	BOi	ABOi	AAB	BAB	AOc	-BOi + OB	0	0	AB - BA	ABOc + O	N7
476	BA	AOi	BOi	ABOi	AAB	BAB	AOc	-BOi + OB	0	0	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
477	BA	AOi	BOi	ABOi	AAB	BAB	AOc	BOc	0	0	A - AAB + ABAc + ABAi	ABOc + O	N14
478	BA	AOi	BOi	ABOi	AAB	BAB	AOc	BOc	0	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
479	BA	AOi	BOi	ABOi	AAB	BAB	AOc	BOc	0	0	AB - BA	ABOc + O	N15
480	BA	AOi	BOi	ABOi	AAB	BAB	AOc	BOc	0	0	AB - BA	-ABOi - AOc - AOi - BOc - BOi + OA + OAB + OB	N1
481	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
482	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
483	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	AB - BA	ABOc + O	N5
484	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
485	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
486	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
487	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	AB - BA	ABOc + O	N3
488	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	-BOi + OB	0	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
489	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N12
490	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
491	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	AB - BA	ABOc + O	N13
492	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB - BOc - BOi + OAB + OB	N1
493	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N10
494	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
495	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	AB - BA	ABOc + O	N11
496	BA	AOi	BOi	ABOi	AAB	ABBi	-AOi + OA	BOc	0	-ABBi + BAB	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1

Table A13 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
497	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N16
498	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
499	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	AB - BA	ABOc + O	N9
500	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
501	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
502	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
503	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	AB - BA	ABOc + O	N7
504	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	-BOi + OB	0	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
505	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N16
506	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
507	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	ABBc + B	AB - BA	ABOc + O	N17
508	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB - BOc - BOi + OA + OAB + OB	N1
509	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N14
510	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
511	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	AB - BA	ABOc + O	N15
512	BA	AOi	BOi	ABOi	AAB	ABBi	AOc	BOc	0	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi - BOc - BOi + OA + OAB + OB	N1
513	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + O	N10
514	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
515	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	AB - BA	ABOc + O	N2
516	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
517	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
518	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
519	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	AB - BA	ABOc + O	N3
520	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	-BOi + OB	AAB - ABAi	0	AB - BA	-ABOi + OAB	N1
521	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	A + ABAc	0	0	ABOc + O	N10
522	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
523	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + O	N10
524	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
525	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	AB - BA	ABOc + O	N11
526	BA	AOi	BOi	ABOi	ABAi	BAB	-AOi + OA	BOc	AAB - ABAi	0	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
527	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	BOc + BOi - OB	ABOc + O	N14
528	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
529	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	AB - BA	ABOc + O	N6
530	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	A + ABAc	0	AB - BA	-A + AAB - ABAc - ABAi - ABOi - AOc - AOi + OA + OAB	N1
531	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
532	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
533	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	AB - BA	ABOc + O	N7
534	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	-BOi + OB	AAB - ABAi	0	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
535	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	A + ABAc	0	0	ABOc + O	N14

Table A14 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
536	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	A + ABAc	0	0	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
537	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	ABOc + O	N14
538	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	AAB - ABAi	0	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
539	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	AAB - ABAi	0	AB - BA	ABOc + O	N15
540	BA	AOi	BOi	ABOi	ABAi	BAB	AOc	BOc	AAB - ABAi	0	AB - BA	-ABOi - AOc - AOi - BOc - BOi + OA + OAB + OB	N1
541	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N12
542	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
543	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + O	N4
544	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - B + BAB + OAB	N1
545	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N10
546	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
547	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N2
548	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi + OAB	N1
549	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N12
550	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
551	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N5
552	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB + OAB	N1
553	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N10
554	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
555	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N3
556	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi + OAB	N1
557	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	ABBc + B	0	ABOc + O	N12
558	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
559	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	-ABBi + BAB	0	ABOc + O	N10
560	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
561	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N12
562	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - B - BA + BAB - BOc - BOi + OAB + OB	N1
563	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N13
564	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - B + BAB - BOc - BOi + OAB + OB	N1
565	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N10
566	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - BA - BOc - BOi + OAB + OB	N1
567	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N11
568	BA	AOi	BOi	ABOi	ABAi	ABBi	-AOi + OA	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi - BOc - BOi + OAB + OB	N1
569	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	ABOc + O	N16
570	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1

Table A15 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ under two-way mechanism.

Serial	e_0	e_1	e_2	e_3	e_4	e_5	a_1	a_2	b_1	b_2	b_3	X_3	Result
571	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	AB - BA	ABOc + O	N8
572	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	ABBc + B	AB - BA	-A + AAB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
573	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	ABOc + O	N14
574	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
575	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	ABOc + O	N6
576	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	A + ABAc	-ABBi + BAB	AB - BA	-A + AAB - ABAc - ABAi - ABOi - AOc - AOi + OA + OAB	N1
577	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N16
578	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
579	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N9
580	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB + OA + OAB	N1
581	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	ABOc + O	N14
582	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi + BOc + BOi - OB	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
583	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N7
584	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	-BOi + OB	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi + OA + OAB	N1
585	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	A + ABAc	ABBc + B	0	ABOc + O	N16
586	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	A + ABAc	ABBc + B	0	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
587	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	A + ABAc	-ABBi + BAB	0	ABOc + O	N14
588	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	A + ABAc	-ABBi + BAB	0	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
589	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	ABOc + O	N16
590	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABBc - ABBi - ABOi - AOc - AOi - B - BA + BAB - BOc - BOi + OA + OAB + OB	N1
591	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	AB - BA	ABOc + O	N17
592	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	ABBc + B	AB - BA	-ABBc - ABBi - ABOi - AOc - AOi - B + BAB - BOc - BOi + OA + OAB + OB	N1
593	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	ABOc + O	N14
594	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	A - AAB + ABAc + ABAi	-A + AAB + AB - ABAc - ABAi - ABOi - AOc - AOi - BA - BOc - BOi + OA + OAB + OB	N1
595	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	ABOc + O	N15
596	BA	AOi	BOi	ABOi	ABAi	ABBi	AOc	BOc	AAB - ABAi	-ABBi + BAB	AB - BA	-ABOi - AOc - AOi - BOc - BOi + OA + OAB + OB	N1

D.2 Supplementary B

Table B1 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	$2*g_5$	Serial
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(ABO + O)	A1
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A2
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A3
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A4
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A5
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A6
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A7
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A8
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*(-AO - BO + OA)	A9
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A10
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A11
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A12
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A13
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A14
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A15
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A16
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A17
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A18
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A19
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A20
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A21
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A22
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	2*AAB	A23
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(ABO + O)	A24
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A25
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A26
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A27
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A28
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A29
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A30
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A31
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A32
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A33
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A34
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	A35
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	A36
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	A37
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	A38
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	A39
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(ABO + O)	A40
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A41
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A42

Table B2 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under three-way exchanges.

Serial	$2*g_6$	$2*g_7$	$2*g_8$	w_5	Result
A1	0	0	0	0	N12
A2	2*(ABO + AO + BO + O - OA)	0	0	0	N12
A3	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N12
A4	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + AO + BA + 2*BO + O - OA - OB	N3
A5	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB + AO - B + BA + BAB + 2*BO - OA + OAB - OB	N1
A6	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A7	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A8	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A9	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A10	2*(-AAB + ABO + O)	0	0	0	N12
A11	2*OB	2*(-AAB + ABO + O - OB)	0	0	N12
A12	2*OB	2*(-AAB - AO - BO + OA)	2*(ABO + AO + BO + O - OA - OB)	0	N12
A13	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
A14	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
A15	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A16	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A17	2*OB	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
A18	2*OB	2*(-A + AB - ABA - BA - BO)	0	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
A19	2*(-A - ABA - ABB - B + BAB)	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N12
A20	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A21	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A22	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB + ABO + B + BA - BAB + BO + O	N17
A23	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB - AO + B + BA - BAB + OA + OAB + OB	N1
A24	0	0	0	0	N15
A25	2*(ABO + AO + BO + O - OA)	0	0	0	N15
A26	2*OB	0	0	ABO + AO + BO + O - OA - OB	N3
A27	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
A28	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
A29	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
A30	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N15
A31	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
A32	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
A33	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
A34	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
A35	2*(ABO + O)	0	0	0	N15
A36	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
A37	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
A38	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
A39	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
A40	0	0	0	0	N12
A41	2*(ABO + AO + BO + O - OA)	0	0	0	N12
A42	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N12

Table B3 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	$2*g_5$	Serial
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A43
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A44
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A45
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A46
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A47
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*(-AO - BO + OA)	A48
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A49
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A50
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A51
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A52
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A53
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A54
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A55
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A56
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A57
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A58
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A59
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A60
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A61
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	2*AAB	A62
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(ABO + O)	A63
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A64
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A65
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A66
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A67
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-AO - BO + OA)	A68
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A69
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A70
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A71
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A72
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	2*(-A + AAB + AB - ABA - BA - BO)	A73
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(ABO + O)	A74
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-AB - AO + BA + OA)	A75
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-AB - AO + BA + OA)	A76
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-AB - AO + BA + OA)	A77
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-AB - AO + BA + OA)	A78
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-AB - AO + BA + OA)	A79
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-A + AAB - ABA)	A80
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-A + AAB - ABA)	A81
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-A + AAB - ABA)	A82
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-A + AAB - ABA)	A83
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	2*(-A + AAB - ABA)	A84

Table B4 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under three-way exchanges.

Serial	$2*g_6$	$2*g_7$	$2*g_8$	w_5	Result
A43	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + AO + BA + 2*BO + O - OA - OB	N3
A44	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB + AO - B + BA + BAB + 2*BO - OA + OAB - OB	N1
A45	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A46	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A47	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A48	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A49	2*(-AAB + ABO + O)	0	0	0	N12
A50	2*OB	2*(-AAB + ABO + O - OB)	0	0	N12
A51	2*OB	2*(-AAB - AO - BO + OA)	2*(ABO + AO + BO + O - OA - OB)	0	N12
A52	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
A53	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
A54	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A55	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A56	2*OB	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
A57	2*OB	2*(-A + AB - ABA - BA - BO)	0	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
A58	2*(-A - ABA - ABB - B + BAB)	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N12
A59	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
A60	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
A61	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB + ABO + B + BA - BAB + BO + O	N17
A62	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB - AO + B + BA - BAB + OA + OAB + OB	N1
A63	0	0	0	0	N15
A64	2*(ABO + AO + BO + O - OA)	0	0	0	N15
A65	2*OB	0	0	ABO + AO + BO + O - OA - OB	N3
A66	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
A67	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
A68	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
A69	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N15
A70	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
A71	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
A72	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
A73	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
A74	0	0	0	0	N15
A75	2*(AB + ABO + AO - BA + O - OA)	0	0	0	N15
A76	2*(AB - BA - BO + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
A77	2*(AB - BA - BO + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
A78	2*(-ABB - B + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
A79	2*(-ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
A80	2*(A - AAB + ABA + ABO + O)	0	0	0	N15
A81	2*(AB - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
A82	2*(AB - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
A83	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
A84	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1

Table B5 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (2) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	Serial
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B1
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B2
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B3
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B4
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B5
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B6
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	B7
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	B8
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	B9
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	B10
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	B11
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B12
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B13
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B14
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B15
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B16
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B17
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	B18
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	B19
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	B20
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	B21
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	B22
0	0	2*(A + ABA)	2*OA	0	ABB + B	B23
0	0	2*(A + ABA)	2*OA	0	ABB + B	B24
0	0	2*(A + ABA)	2*OA	0	ABB + B	B25
0	0	2*(A + ABA)	2*OA	0	ABB + B	B26
0	0	2*(A + ABA)	2*OA	0	ABB + B	B27
0	0	2*(A + ABA)	2*OA	0	ABB + B	B28
0	0	2*(A + ABA)	2*OA	0	ABB + B	B29
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	B30
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	B31
0	0	2*(AB - BA)	0	OA	ABB + B	B32
0	0	2*(AB - BA)	0	OA	ABB + B	B33
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B34
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B35
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B36
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B37
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B38
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B39
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	B40
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	B41
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	B42
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	B43
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	B44

Table B6 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (3) under three-way exchanges.

Serial	w_3	w_4	s_1	$2*g_8$	w_5	Result
B1	0	OB	BO - OA - OB	2*(ABO + O)	0	N8
B2	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
B3	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
B4	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
B5	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
B6	0	OB	-A + AB - ABA - BA - OA	0	ABO + O	N3
B7	0	OB	-A + AB - ABA - BA - OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B8	0	OB	0	0	ABO + O	N3
B9	0	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B10	A - AB + ABA - AO + BA + OA	OB	0	0	ABO + O	N3
B11	A - AB + ABA - AO + BA + OA	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B12	0	OB	BO - OA - OB	2*(ABO + O)	0	N8
B13	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
B14	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
B15	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
B16	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
B17	0	OB	-A + AB - ABA - BA - OA	0	ABO + O	N3
B18	0	OB	-A + AB - ABA - BA - OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B19	0	OB	0	0	ABO + O	N3
B20	0	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B21	A - AB + ABA - AO + BA + OA	OB	0	0	ABO + O	N3
B22	A - AB + ABA - AO + BA + OA	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B23	0	OB	BO - OA - OB	2*(ABO + O)	0	N8
B24	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
B25	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
B26	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
B27	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
B28	0	OB	-A + AB - ABA - BA - OA	0	ABO + O	N3
B29	0	OB	-A + AB - ABA - BA - OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B30	0	OB	0	0	ABO + O	N3
B31	0	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B32	A - AB + ABA + BA	OB	0	0	ABO + O	N3
B33	A - AB + ABA + BA	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B34	0	OB	BO - OA - OB	2*(ABO + O)	0	N8
B35	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
B36	0	OB	BO - OA - OB	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
B37	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
B38	0	OB	BO - OA - OB	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
B39	0	OB	-A + AB - ABA - BA - OA	0	ABO + O	N3
B40	0	OB	-A + AB - ABA - BA - OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B41	0	OB	0	0	ABO + O	N3
B42	0	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
B43	A - AB + ABA + BA	OB	0	0	ABO + O	N3
B44	A - AB + ABA + BA	OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table B7 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (3) under three-way exchanges.

Serial	$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	s_1	$2*g_7$	w_5	Result
C1	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	BO	0	0	ABO + O	N17
C2	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	BO	0	0	-AO - BO + OA + OAB + OB	N1
C3	2*BAB	2*BO	AO	0	AAB	0	A - AAB + ABA - BAB	2*(ABO + O)	0	N11
C4	2*BAB	2*BO	AO	0	AAB	0	A - AAB + ABA - BAB	2*(-AO - BO + OA)	ABO + AO + BO + O - OA	N10
C5	2*BAB	2*BO	AO	0	AAB	0	A - AAB + ABA - BAB	2*(-AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
C6	2*BAB	2*BO	AO	0	AAB	0	A - AAB + ABA - BAB	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17
C7	2*BAB	2*BO	AO	0	AAB	0	A - AAB + ABA - BAB	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
C8	2*BAB	2*BO	AO	0	AAB	0	AB - BA - BAB - BO	0	ABO + O	N17
C9	2*BAB	2*BO	AO	0	AAB	0	AB - BA - BAB - BO	0	-AO - BO + OA + OAB + OB	N1
C10	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	-AB + BA + BAB + BO	0	0	ABO + O	N17
C11	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	-AB + BA + BAB + BO	0	0	-AO - BO + OA + OAB + OB	N1

Table B8 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_4$	w_1	w_2	w_3	w_4	Serial
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D1
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D2
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D3
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D4
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D5
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D6
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D7
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	D8
2*BAB	0	2*OA	0	0	AAB	OB	D9
2*BAB	0	2*OA	0	0	AAB	OB	D10
2*BAB	0	2*OA	0	0	AAB	OB	D11
2*BAB	0	2*OA	0	0	AAB	OB	D12
2*BAB	0	2*OA	0	0	AAB	OB	D13
2*BAB	0	2*OA	0	0	AAB	OB	D14
2*BAB	0	2*OA	0	0	AAB	OB	D15
2*BAB	0	2*OA	0	0	AAB	OB	D16
2*BAB	0	2*OA	0	0	AAB	OB	D17
2*BAB	0	2*OA	0	0	AAB	OB	D18
2*BAB	0	2*OA	0	0	AAB	OB	D19
2*BAB	0	2*OA	0	0	AAB	OB	D20
2*BAB	0	2*OA	0	0	AAB	OB	D21
2*BAB	0	2*OA	0	0	AAB	OB	D22
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D23
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D24
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D25
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D26

Table B9 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under three-way exchanges.

Serial	c_2	c_3	s_1	w_5	Result
D1	A - AAB - AB + ABA + BA	BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
D2	A - AAB - AB + ABA + BA	BO - OB	0	OAB	N1
D3	A - AAB - AB + ABA + BA	AB + ABB + B - BA - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
D4	A - AAB - AB + ABA + BA	AB + ABB + B - BA - BAB	0	OAB	N1
D5	AO - OA	BO - OB	0	ABO + AO + BO + O - OA - OB	N17
D6	AO - OA	BO - OB	0	OAB	N1
D7	AO - OA	AB + ABB + B - BA - BAB	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
D8	AO - OA	AB + ABB + B - BA - BAB	0	OAB	N1
D9	A - AAB + ABA - BAB	BO - OA - OB	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N10
D10	A - AAB + ABA - BAB	BO - OA - OB	0	AB - BA - BAB - OA + OAB	N1
D11	A - AAB + ABA - BAB	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
D12	A - AAB + ABA - BAB	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
D13	A - AAB + ABA - BAB	ABB + B	AB - BA - BAB - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
D14	A - AAB + ABA - BAB	ABB + B	AB - BA - BAB - OA	OAB	N1
D15	AO	BO - OA - OB	A - AAB + ABA - AO - BAB	ABO + AO + BO + O - OA - OB	N10
D16	AO	BO - OA - OB	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
D17	AO	BO - OA - OB	AB - BA - BAB - OA	ABO + AO + BO + O - OA - OB	N17
D18	AO	BO - OA - OB	AB - BA - BAB - OA	OAB	N1
D19	AO	ABB + B	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	ABB + ABO + AO + B + O	N10
D20	AO	ABB + B	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	-A + AAB + AB - ABA + ABB + AO + B - BA - BO + OAB + OB	N1
D21	AO	ABB + B	AB - BA - BAB - OA	ABB + ABO + AO + B + O	N2
D22	AO	ABB + B	AB - BA - BAB - OA	OAB	N1
D23	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
D24	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	OAB	N1
D25	A - AAB + ABA - BAB	ABB + B	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
D26	A - AAB + ABA - BAB	ABB + B	0	OAB	N1

Table B10 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_4$	w_1	w_2	w_3	w_4	Serial
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D27
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D28
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D29
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	D30
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D31
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D32
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D33
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D34
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D35
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D36
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D37
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D38
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D39
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D40
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D41
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D42
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D43
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	D44
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D45
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D46
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D47
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D48
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D49
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D50
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D51
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	D52

Table B11 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under three-way exchanges.

Serial	c_2	c_3	s_1	w_5	Result
D27	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	ABO + AO + BO + O - OA - OB	N17
D28	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	OAB	N1
D29	AB + AO - BA - BAB - OA	ABB + B	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
D30	AB + AO - BA - BAB - OA	ABB + B	0	OAB	N1
D31	A - AAB + ABA - BAB	BO - OA - OB	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N10
D32	A - AAB + ABA - BAB	BO - OA - OB	0	AB - BA - BAB - OA + OAB	N1
D33	A - AAB + ABA - BAB	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
D34	A - AAB + ABA - BAB	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
D35	A - AAB + ABA - BAB	ABB + B	AB - BA - BAB - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
D36	A - AAB + ABA - BAB	ABB + B	AB - BA - BAB - OA	OAB	N1
D37	AO	BO - OA - OB	A - AAB + ABA - AO - BAB	ABO + AO + BO + O - OA - OB	N10
D38	AO	BO - OA - OB	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
D39	AO	BO - OA - OB	AB - BA - BAB - OA	ABO + AO + BO + O - OA - OB	N17
D40	AO	BO - OA - OB	AB - BA - BAB - OA	OAB	N1
D41	AO	ABB + B	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	ABB + ABO + AO + B + O	N10
D42	AO	ABB + B	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	-A + AAB + AB - ABA + ABB + AO + B - BA - BO + OAB + OB	N1
D43	AO	ABB + B	AB - BA - BAB - OA	ABB + ABO + AO + B + O	N2
D44	AO	ABB + B	AB - BA - BAB - OA	OAB	N1
D45	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
D46	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	OAB	N1
D47	A - AAB + ABA - BAB	ABB + B	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
D48	A - AAB + ABA - BAB	ABB + B	0	OAB	N1
D49	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	ABO + AO + BO + O - OA - OB	N17
D50	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	OAB	N1
D51	AB + AO - BA - BAB - OA	ABB + B	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
D52	AB + AO - BA - BAB - OA	ABB + B	0	OAB	N1

Table B12 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (5) under three-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	c_2	w_5	Result
E1	2*(AB - BA)	OA	ABB + B	AAB	OB	A - AAB - AB + ABA + BA	A - AAB - AB + ABA + ABO + BA + O	N3
E2	2*(AB - BA)	OA	ABB + B	AAB	OB	A - AAB - AB + ABA + BA	-AB - ABB - B + BA + BAB + OAB	N1
E3	2*(AB - BA)	OA	ABB + B	AAB	OB	AO - OA	ABO + AO + O - OA	N7
E4	2*(AB - BA)	OA	ABB + B	AAB	OB	AO - OA	-AB - ABB - B + BA + BAB + OAB	N1

Table B13 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	Serial
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F1
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F2
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F3
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F4
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F5
0	0	2*(A + ABA)	2*OA	0	-A - ABA + BAB	F6
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F7
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F8
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F9
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F10
0	0	2*(AB - BA)	0	OA	-AB + BA + BAB	F11
0	0	2*(AB - BA)	0	OA	-AB + BA + BAB	F12
0	0	2*(AB - BA)	0	OA	-AB + BA + BAB	F13
0	0	2*(AB - BA)	0	OA	-AB + BA + BAB	F14
0	0	2*BAB	2*OA	0	0	F15
0	0	2*BAB	2*OA	0	0	F16
0	0	2*BAB	2*OA	0	0	F17
0	0	2*BAB	2*OA	0	0	F18
0	0	2*BAB	2*OA	0	0	F19
0	0	2*BAB	2*OA	0	0	F20
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F21
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F22
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F23
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F24
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F25
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F26
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F27
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F28
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F29
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	-A - ABA + BAB	F30
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F31
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F32
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F33

Table B14 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

Serial	w_3	w_4	c_3	s_1	w_5	Result
F1	0	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F2	0	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F3	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	A + ABA + ABB + ABO + B - BAB + O	N10
F4	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
F5	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	A + ABA + ABB + ABO + B - BAB + O	N3
F6	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	AAB + OAB	N1
F7	0	OB	A - AB + ABA + BA + BO - OB	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
F8	0	OB	A - AB + ABA + BA + BO - OB	0	AAB + OAB	N1
F9	0	OB	A + ABA + ABB + B - BAB	0	A + ABA + ABB + ABO + B - BAB + O	N3
F10	0	OB	A + ABA + ABB + B - BAB	0	AAB + OAB	N1
F11	A - AB + ABA + BA	OB	BO - OB	0	ABO + BO + O - OB	N10
F12	A - AB + ABA + BA	OB	BO - OB	0	-A + AAB + AB - ABA - BA + OAB	N1
F13	A - AB + ABA + BA	OB	AB + ABB + B - BA - BAB	0	AB + ABB + ABO + B - BA - BAB + O	N3
F14	A - AB + ABA + BA	OB	AB + ABB + B - BA - BAB	0	-A + AAB + AB - ABA - BA + OAB	N1
F15	A + ABA - BAB	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F16	A + ABA - BAB	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F17	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	ABB + ABO + B + O	N10
F18	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
F19	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	ABB + ABO + B + O	N3
F20	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	-A + AAB - ABA + BAB + OAB	N1
F21	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-AB + ABO + BA + BAB + BO + O - OB	N10
F22	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-A + AAB - ABA + BAB + OAB	N1
F23	A + ABA - BAB	OB	ABB + B	0	ABB + ABO + B + O	N3
F24	A + ABA - BAB	OB	ABB + B	0	-A + AAB - ABA + BAB + OAB	N1
F25	0	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F26	0	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F27	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	A + ABA + ABB + ABO + B - BAB + O	N10
F28	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
F29	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	A + ABA + ABB + ABO + B - BAB + O	N3
F30	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	AAB + OAB	N1
F31	0	OB	A - AB + ABA + BA + BO - OB	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
F32	0	OB	A - AB + ABA + BA + BO - OB	0	AAB + OAB	N1
F33	0	OB	A + ABA + ABB + B - BAB	0	A + ABA + ABB + ABO + B - BAB + O	N3

Table B15 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	Serial
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F34
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	-AB + BA + BAB	F35
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	-AB + BA + BAB	F36
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	-AB + BA + BAB	F37
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	-AB + BA + BAB	F38
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F39
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F40
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F41
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F42
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F43
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	F44
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F45
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F46
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F47
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	F48
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F49
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F50
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F51
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F52
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F53
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	-A - ABA + BAB	F54
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F55
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F56
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F57
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F58
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F59
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F60
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F61
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F62
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F63
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F64
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F65
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F66

Table B16 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

Serial	w_3	w_4	c_3	s_1	w_5	Result
F34	0	OB	A + ABA + ABB + B - BAB	0	AAB + OAB	N1
F35	A - AB + ABA + BA	OB	BO - OB	0	ABO + BO + O - OB	N10
F36	A - AB + ABA + BA	OB	BO - OB	0	-A + AAB + AB - ABA - BA + OAB	N1
F37	A - AB + ABA + BA	OB	AB + ABB + B - BA - BAB	0	AB + ABB + ABO + B - BA - BAB + O	N3
F38	A - AB + ABA + BA	OB	AB + ABB + B - BA - BAB	0	-A + AAB + AB - ABA - BA + OAB	N1
F39	A + ABA - BAB	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F40	A + ABA - BAB	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F41	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	ABB + ABO + B + O	N10
F42	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
F43	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	ABB + ABO + B + O	N3
F44	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	-A + AAB - ABA + BAB + OAB	N1
F45	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-AB + ABO + BA + BAB + BO + O - OB	N10
F46	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-A + AAB - ABA + BAB + OAB	N1
F47	A + ABA - BAB	OB	ABB + B	0	ABB + ABO + B + O	N3
F48	A + ABA - BAB	OB	ABB + B	0	-A + AAB - ABA + BAB + OAB	N1
F49	0	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F50	0	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F51	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	A + ABA + ABB + ABO + B - BAB + O	N10
F52	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
F53	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	A + ABA + ABB + ABO + B - BAB + O	N3
F54	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	AAB + OAB	N1
F55	0	OB	A - AB + ABA + BA + BO - OB	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
F56	0	OB	A - AB + ABA + BA + BO - OB	0	AAB + OAB	N1
F57	0	OB	A + ABA + ABB + B - BAB	0	A + ABA + ABB + ABO + B - BAB + O	N3
F58	0	OB	A + ABA + ABB + B - BAB	0	AAB + OAB	N1
F59	A - AB + ABA - AO + BA + OA	OB	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N10
F60	A - AB + ABA - AO + BA + OA	OB	AO + BO - OA - OB	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
F61	A - AB + ABA - AO + BA + OA	OB	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
F62	A - AB + ABA - AO + BA + OA	OB	AB + ABB + AO + B - BA - BAB - OA	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
F63	A + ABA - BAB	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F64	A + ABA - BAB	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F65	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	ABB + ABO + B + O	N10
F66	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1

Table B17 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	Serial
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F67
0	2*(-AO + OA)	2*BAB	2*AO	0	0	F68
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F69
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F70
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F71
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F72
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F73
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F74
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F75
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F76
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F77
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	-A - ABA + BAB	F78
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F79
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F80
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F81
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	-A - ABA + BAB	F82
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F83
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F84
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F85
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	-AB - AO + BA + BAB + OA	F86
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F87
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F88
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F89
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F90
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F91
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	F92
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F93
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F94
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F95
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	F96

Table B18 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (6) under three-way exchanges.

Serial	w_3	w_4	c_3	s_1	w_5	Result
F67	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	ABB + ABO + B + O	N3
F68	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	-A + AAB - ABA + BAB + OAB	N1
F69	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-AB + ABO + BA + BAB + BO + O - OB	N10
F70	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-A + AAB - ABA + BAB + OAB	N1
F71	A + ABA - BAB	OB	ABB + B	0	ABB + ABO + B + O	N3
F72	A + ABA - BAB	OB	ABB + B	0	-A + AAB - ABA + BAB + OAB	N1
F73	0	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F74	0	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F75	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	A + ABA + ABB + ABO + B - BAB + O	N10
F76	0	OB	A + ABA + ABB + B - BAB	-A - ABA - ABB - B + BAB + BO - OA - OB	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
F77	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	A + ABA + ABB + ABO + B - BAB + O	N3
F78	0	OB	A + ABA + ABB + B - BAB	-A + AB - ABA - BA - OA	AAB + OAB	N1
F79	0	OB	A - AB + ABA + BA + BO - OB	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
F80	0	OB	A - AB + ABA + BA + BO - OB	0	AAB + OAB	N1
F81	0	OB	A + ABA + ABB + B - BAB	0	A + ABA + ABB + ABO + B - BAB + O	N3
F82	0	OB	A + ABA + ABB + B - BAB	0	AAB + OAB	N1
F83	A - AB + ABA - AO + BA + OA	OB	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N10
F84	A - AB + ABA - AO + BA + OA	OB	AO + BO - OA - OB	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
F85	A - AB + ABA - AO + BA + OA	OB	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
F86	A - AB + ABA - AO + BA + OA	OB	AB + ABB + AO + B - BA - BAB - OA	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
F87	A + ABA - BAB	OB	BO - OA - OB	0	ABO + BO + O - OA - OB	N10
F88	A + ABA - BAB	OB	BO - OA - OB	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
F89	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	ABB + ABO + B + O	N10
F90	A + ABA - BAB	OB	ABB + B	-ABB - B + BO - OA - OB	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
F91	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	ABB + ABO + B + O	N3
F92	A + ABA - BAB	OB	ABB + B	AB - BA - BAB - OA	-A + AAB - ABA + BAB + OAB	N1
F93	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-AB + ABO + BA + BAB + BO + O - OB	N10
F94	A + ABA - BAB	OB	-AB + BA + BAB + BO - OB	0	-A + AAB - ABA + BAB + OAB	N1
F95	A + ABA - BAB	OB	ABB + B	0	ABB + ABO + B + O	N3
F96	A + ABA - BAB	OB	ABB + B	0	-A + AAB - ABA + BAB + OAB	N1

Table B19 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (7) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_4$	w_1	w_2	Serial
2*(AB - BA)	0	0	OA	-AB + BA + BAB	G1
2*(AB - BA)	0	0	OA	-AB + BA + BAB	G2
2*(AB - BA)	0	0	OA	-AB + BA + BAB	G3
2*(AB - BA)	0	0	OA	-AB + BA + BAB	G4
2*BAB	0	2*BO	-BO + OA	0	G5
2*BAB	0	2*BO	-BO + OA	0	G6
2*BAB	0	2*BO	-BO + OA	0	G7
2*BAB	0	2*BO	-BO + OA	0	G8
2*BAB	0	2*BO	-BO + OA	0	G9
2*BAB	0	2*BO	-BO + OA	0	G10
2*BAB	0	2*OA	0	0	G11
2*BAB	0	2*OA	0	0	G12
2*BAB	0	2*OA	0	0	G13
2*BAB	0	2*OA	0	0	G14
2*BAB	0	2*OA	0	0	G15
2*BAB	0	2*OA	0	0	G16
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	G17
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	G18
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	G19
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	G20
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G21
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G22
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G23
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G24
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G25
2*BAB	2*(-AO + OA)	2*(AO + BO - OA)	-BO + OA	0	G26
2*BAB	2*(-AO + OA)	2*AO	0	0	G27
2*BAB	2*(-AO + OA)	2*AO	0	0	G28
2*BAB	2*(-AO + OA)	2*AO	0	0	G29
2*BAB	2*(-AO + OA)	2*AO	0	0	G30
2*BAB	2*(-AO + OA)	2*AO	0	0	G31
2*BAB	2*(-AO + OA)	2*AO	0	0	G32
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	G33
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	G34
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	G35
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	G36

Table B20 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (7) under three-way exchanges.

Serial	w_3	w_4	c_2	s_1	w_5	Result
G1	AAB	BO	A - AAB - AB + ABA + BA	0	A - AAB - AB + ABA + ABO + BA + O	N10
G2	AAB	BO	A - AAB - AB + ABA + BA	0	-BO + OAB + OB	N1
G3	AAB	BO	AO - OA	0	ABO + AO + O - OA	N17
G4	AAB	BO	AO - OA	0	-BO + OAB + OB	N1
G5	AAB	0	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G6	AAB	0	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G7	AAB	0	AO + BO - OA	A - AAB + ABA - AO - BAB - BO + OA	ABO + AO + BO + O - OA	N10
G8	AAB	0	AO + BO - OA	A - AAB + ABA - AO - BAB - BO + OA	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
G9	AAB	0	AO + BO - OA	AB - BA - BAB - BO	ABO + AO + BO + O - OA	N17
G10	AAB	0	AO + BO - OA	AB - BA - BAB - BO	OAB + OB	N1
G11	AAB	BO - OA	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G12	AAB	BO - OA	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G13	AAB	BO - OA	AO	A - AAB + ABA - AO - BAB	ABO + AO + O	N10
G14	AAB	BO - OA	AO	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
G15	AAB	BO - OA	AO	AB - BA - BAB - OA	ABO + AO + O	N17
G16	AAB	BO - OA	AO	AB - BA - BAB - OA	-BO + OA + OAB + OB	N1
G17	AAB	-AB + BA + BAB + BO	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G18	AAB	-AB + BA + BAB + BO	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G19	AAB	-AB + BA + BAB + BO	AB + AO - BA - BAB - OA	0	AB + ABO + AO - BA - BAB + O - OA	N17
G20	AAB	-AB + BA + BAB + BO	AB + AO - BA - BAB - OA	0	AB - BA - BAB - BO + OAB + OB	N1
G21	AAB	0	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G22	AAB	0	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G23	AAB	0	AO + BO - OA	A - AAB + ABA - AO - BAB - BO + OA	ABO + AO + BO + O - OA	N10
G24	AAB	0	AO + BO - OA	A - AAB + ABA - AO - BAB - BO + OA	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
G25	AAB	0	AO + BO - OA	AB - BA - BAB - BO	ABO + AO + BO + O - OA	N17
G26	AAB	0	AO + BO - OA	AB - BA - BAB - BO	OAB + OB	N1
G27	AAB	BO - OA	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G28	AAB	BO - OA	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G29	AAB	BO - OA	AO	A - AAB + ABA - AO - BAB	ABO + AO + O	N10
G30	AAB	BO - OA	AO	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
G31	AAB	BO - OA	AO	AB - BA - BAB - OA	ABO + AO + O	N17
G32	AAB	BO - OA	AO	AB - BA - BAB - OA	-BO + OA + OAB + OB	N1
G33	AAB	-AB + BA + BAB + BO	A - AAB + ABA - BAB	0	A - AAB + ABA + ABO - BAB + O	N10
G34	AAB	-AB + BA + BAB + BO	A - AAB + ABA - BAB	0	AB - BA - BAB - BO + OAB + OB	N1
G35	AAB	-AB + BA + BAB + BO	AB + AO - BA - BAB - OA	0	AB + ABO + AO - BA - BAB + O - OA	N17
G36	AAB	-AB + BA + BAB + BO	AB + AO - BA - BAB - OA	0	AB - BA - BAB - BO + OAB + OB	N1

Table B21 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (8) under three-way exchanges.

Serial	$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	c_3	w_5	Result
H1	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	OB	BO - OB	ABO + BO + O - OB	N17
H2	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	OB	BO - OB	-AO + OA + OAB	N1
H3	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	OB	AB + ABB + B - BA - BAB	AB + ABB + ABO + B - BA - BAB + O	N7
H4	2*(AB - BA)	0	AO	-AB + BA + BAB	AAB	OB	AB + ABB + B - BA - BAB	-AO + OA + OAB	N1
H5	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	OB	-AB + BA + BAB + BO - OB	-AB + ABO + BA + BAB + BO + O - OB	N17
H6	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	OB	-AB + BA + BAB + BO - OB	-AB - AO + BA + BAB + OA + OAB	N1
H7	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	OB	ABB + B	ABB + ABO + B + O	N7
H8	2*BAB	2*(AB - BA - BAB)	AO	0	AAB	OB	ABB + B	-AB - AO + BA + BAB + OA + OAB	N1

Table B22 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (9) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	Serial
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	-BO + OA	I1
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	-BO + OA	I2
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	I3
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	I4
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	I5
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	I6
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	I7
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	I8
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	-BO + OA	I9
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	-BO + OA	I10
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	I11
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	I12
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	I13
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	I14
0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	-BO + OA	I15
0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	-BO + OA	I16
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	I17
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	I18
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	I19
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	I20
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	I21
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	I22
0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	-BO + OA	I23
0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	-BO + OA	I24
0	2*(-AO + OA)	2*BAB	2*AO	0	I25
0	2*(-AO + OA)	2*BAB	2*AO	0	I26
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	I27
0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	I28

Table B23 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (9) under three-way exchanges.

Serial	w_2	w_3	w_4	w_5	Result
I1	-A - ABA + BAB	0	0	ABO + O	N10
I2	-A - ABA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I3	-A - ABA + BAB	0	BO - OA	ABO + O	N10
I4	-A - ABA + BAB	0	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I5	-A - ABA + BAB	0	A - AB + ABA + BA + BO	ABO + O	N10
I6	-A - ABA + BAB	0	A - AB + ABA + BA + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I7	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	ABO + O	N10
I8	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I9	0	A + ABA - BAB	0	ABO + O	N10
I10	0	A + ABA - BAB	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I11	0	A + ABA - BAB	BO - OA	ABO + O	N10
I12	0	A + ABA - BAB	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I13	0	A + ABA - BAB	-AB + BA + BAB + BO	ABO + O	N10
I14	0	A + ABA - BAB	-AB + BA + BAB + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I15	-A - ABA + BAB	0	0	ABO + O	N10
I16	-A - ABA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I17	-A - ABA + BAB	0	BO - OA	ABO + O	N10
I18	-A - ABA + BAB	0	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I19	-A - ABA + BAB	0	A - AB + ABA + BA + BO	ABO + O	N10
I20	-A - ABA + BAB	0	A - AB + ABA + BA + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I21	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	ABO + O	N10
I22	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I23	0	A + ABA - BAB	0	ABO + O	N10
I24	0	A + ABA - BAB	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I25	0	A + ABA - BAB	BO - OA	ABO + O	N10
I26	0	A + ABA - BAB	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I27	0	A + ABA - BAB	-AB + BA + BAB + BO	ABO + O	N10
I28	0	A + ABA - BAB	-AB + BA + BAB + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1

Table B24 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (9) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	Serial
2*(A - AAB + ABA)	0	2*AAB	2*BO	-BO + OA	I29
2*(A - AAB + ABA)	0	2*AAB	2*BO	-BO + OA	I30
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	I31
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	I32
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	I33
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	I34
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	I35
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	I36
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	-BO + OA	I37
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	-BO + OA	I38
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	I39
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	I40
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	I41
2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	-AB + BA + BAB + OA	I42
0	0	2*(A + ABA)	2*BO	-BO + OA	I43
0	0	2*(A + ABA)	2*BO	-BO + OA	I44
0	0	2*(A + ABA)	2*OA	0	I45
0	0	2*(A + ABA)	2*OA	0	I46
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	I47
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	I48
0	0	2*(AB - BA)	0	OA	I49
0	0	2*(AB - BA)	0	OA	I50
0	0	2*BAB	2*BO	-BO + OA	I51
0	0	2*BAB	2*BO	-BO + OA	I52
0	0	2*BAB	2*OA	0	I53
0	0	2*BAB	2*OA	0	I54
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	I55
0	0	2*BAB	2*(AB - BA - BAB)	-AB + BA + BAB + OA	I56

Table B25 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (9) under three-way exchanges.

Serial	w_2	w_3	w_4	w_5	Result
I29	-A - ABA + BAB	0	0	ABO + O	N10
I30	-A - ABA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I31	-A - ABA + BAB	0	BO - OA	ABO + O	N10
I32	-A - ABA + BAB	0	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I33	-A - ABA + BAB	0	A - AB + ABA + BA + BO	ABO + O	N10
I34	-A - ABA + BAB	0	A - AB + ABA + BA + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I35	-AB + BA + BAB	A - AB + ABA + BA	BO	ABO + O	N10
I36	-AB + BA + BAB	A - AB + ABA + BA	BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I37	0	A + ABA - BAB	0	ABO + O	N10
I38	0	A + ABA - BAB	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I39	0	A + ABA - BAB	BO - OA	ABO + O	N10
I40	0	A + ABA - BAB	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I41	0	A + ABA - BAB	-AB + BA + BAB + BO	ABO + O	N10
I42	0	A + ABA - BAB	-AB + BA + BAB + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I43	-A - ABA + BAB	0	0	ABO + O	N10
I44	-A - ABA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I45	-A - ABA + BAB	0	BO - OA	ABO + O	N10
I46	-A - ABA + BAB	0	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I47	-A - ABA + BAB	0	A - AB + ABA + BA + BO	ABO + O	N10
I48	-A - ABA + BAB	0	A - AB + ABA + BA + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I49	-AB + BA + BAB	A - AB + ABA + BA	BO	ABO + O	N10
I50	-AB + BA + BAB	A - AB + ABA + BA	BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I51	0	A + ABA - BAB	0	ABO + O	N10
I52	0	A + ABA - BAB	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I53	0	A + ABA - BAB	BO - OA	ABO + O	N10
I54	0	A + ABA - BAB	BO - OA	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
I55	0	A + ABA - BAB	-AB + BA + BAB + BO	ABO + O	N10
I56	0	A + ABA - BAB	-AB + BA + BAB + BO	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1

Table B26 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (10) under three-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	w_5	Result
J1	2*(AB - BA)	AO	ABB + B	AAB	OB	ABO + O	N7
J2	2*(AB - BA)	AO	ABB + B	AAB	OB	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1

Table B27 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under three-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	$2*u_1$	v_1	v_2
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	K1
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	K2
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	K3
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	K4
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	K5
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO + BA + OA	K6
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO + BA + OA	K7
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO + BA + OA	K8
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO + BA + OA	K9
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO + BA + OA	K10
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO - BO + OA + OB	K11
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO - BO + OA + OB	K12
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	K13
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	K14
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	K15
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	K16
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	K17
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AB - ABB - AO - B + BA + BAB + OA	K18
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AB - ABB - AO - B + BA + BAB + OA	K19
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K20
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K21
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K22
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K23
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K24
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K25
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K26
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K27
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K28
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K29

Table B28 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under three-way exchanges.

Serial	c_2	$2*g_6$	w_5	Result
K1	0	2*(ABO + O)	0	N14
K2	0	2*(-AO - BO + OB)	ABO + AO + BO + O - OB	N7
K3	0	2*(-AO - BO + OB)	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
K4	0	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + ABO + AO + B - BA - BAB + O	N17
K5	0	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
K6	0	2*(ABO + O)	0	N16
K7	0	2*(-BA - BO + OB)	ABO + BA + BO + O - OB	N7
K8	0	2*(-BA - BO + OB)	-AB - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
K9	0	2*(-AB - ABB - B + BAB)	AB + ABB + ABO + B - BAB + O	N17
K10	0	2*(-AB - ABB - B + BAB)	AB + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
K11	0	0	ABO + O	N7
K12	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
K13	0	2*(ABO + O)	0	N15
K14	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
K15	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
K16	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
K17	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
K18	0	0	ABO + O	N17
K19	0	0	-AO - BO + OA + OAB + OB	N1
K20	A - AAB - AB + ABA	2*(ABO + O)	A - AAB - AB + ABA	N9
K21	A - AAB - AB + ABA	2*(ABO + O)	-AB - ABB - 2*ABO - B - BA + BAB - BO - 2*O + OAB + OB	N1
K22	A - AAB - AB + ABA	2*(-BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N3
K23	A - AAB - AB + ABA	2*(-BA - BO + OB)	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
K24	A - AAB - AB + ABA	2*(-AB - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
K25	A - AAB - AB + ABA	2*(-AB - ABB - B + BAB)	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
K26	AO - BA - OA	2*(ABO + O)	AO - BA - OA	N16
K27	AO - BA - OA	2*(ABO + O)	-AB - ABB - 2*ABO - B - BA + BAB - BO - 2*O + OAB + OB	N1
K28	AO - BA - OA	2*(-BA - BO + OB)	ABO + AO + BO + O - OA - OB	N7
K29	AO - BA - OA	2*(-BA - BO + OB)	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1

Table B29 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under three-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	$2*u_1$	v_1	v_2
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K30
2*(AB - BA)	OA	ABB + B	AAB	BO	2*BA	0	0	K31
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	K32
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	K33
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	K34
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	K35
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	K36
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA	0	K37
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA	0	K38
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA	0	K39
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA	0	K40
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA	0	K41
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB - ABA - ABB - B + BAB	0	K42
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB - ABA - ABB - B + BAB	0	K43
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	K44
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	K45
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	K46
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	K47
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	K48
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA - BA - BO + OB	0	K49
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA - BA - BO + OB	0	K50
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	K51
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	K52
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	K53
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	K54
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	K55
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	K56
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	K57
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	K58

Table B30 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under three-way exchanges.

Serial	c_2	$2*g_6$	w_5	Result
K30	AO - BA - OA	2*(-AB - ABB - B + BAB)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N17
K31	AO - BA - OA	2*(-AB - ABB - B + BAB)	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
K32	0	2*(ABO + O)	0	N8
K33	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
K34	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
K35	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
K36	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
K37	0	2*(ABO + O)	0	N9
K38	0	2*(-BA - BO + OB)	ABO + BA + BO + O - OB	N3
K39	0	2*(-BA - BO + OB)	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
K40	0	2*(-AB - ABB - B + BAB)	AB + ABB + ABO + B - BAB + O	N10
K41	0	2*(-AB - ABB - B + BAB)	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
K42	0	0	ABO + O	N10
K43	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
K44	0	2*(ABO + O)	0	N15
K45	0	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N3
K46	0	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
K47	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
K48	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
K49	0	0	ABO + O	N3
K50	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
K51	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N3
K52	A - AAB - AB + ABA + BA + BO - OB	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
K53	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N7
K54	AO + BO - OA - OB	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
K55	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
K56	A - AAB + ABA + ABB + B - BAB	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
K57	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N17
K58	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1

Table B31 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (12) under three-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	v_2	$2*g_6$	w_5	Result
L1	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(ABO + O)	0	N14
L2	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AO - BO + OB)	ABO + AO + BO + O - OB	N7
L3	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AO - BO + OB)	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
L4	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + ABO + AO + B - BA - BAB + O	N17
L5	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
L6	2*(AB - BA)	AO	ABB + B	AAB	BO	BA	2*(ABO + O)	0	N16
L7	2*(AB - BA)	AO	ABB + B	AAB	BO	BA	2*(-BA - BO + OB)	ABO + BA + BO + O - OB	N7
L8	2*(AB - BA)	AO	ABB + B	AAB	BO	BA	2*(-BA - BO + OB)	-AB - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
L9	2*(AB - BA)	AO	ABB + B	AAB	BO	BA	2*(-AB - ABB - B + BAB)	AB + ABB + ABO + B - BAB + O	N17
L10	2*(AB - BA)	AO	ABB + B	AAB	BO	BA	2*(-AB - ABB - B + BAB)	AB + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
L11	2*(AB - BA)	AO	ABB + B	AAB	BO	-BO + OB	0	ABO + O	N7
L12	2*(AB - BA)	AO	ABB + B	AAB	BO	-BO + OB	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
L13	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(ABO + O)	0	N15
L14	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
L15	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
L16	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
L17	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
L18	2*(AB - BA)	AO	ABB + B	AAB	BO	-AB - ABB - B + BA + BAB	0	ABO + O	N17
L19	2*(AB - BA)	AO	ABB + B	AAB	BO	-AB - ABB - B + BA + BAB	0	-AO - BO + OA + OAB + OB	N1

Table B32 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	Serial
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M1
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M2
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M3
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M4
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M5
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M6
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M7
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	M8
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M9
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M10
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M11
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M12
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M13
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M14
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M15
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M16
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M17
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M18
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M19
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M20
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M21
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M22
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M23
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M24
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M25
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	M26
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M27
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M28
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M29
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M30
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	M31
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M32
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M33
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M34
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M35
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	M36
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	M37
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	M38
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	M39

Table B33 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

Serial	w_4	v_1	$2*g_6$	$2*g_8$	w_5	Result
M1	BO - OA	0	2*(ABO + O)	0	0	N8
M2	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N8
M3	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
M4	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M5	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
M6	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M7	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M8	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M9	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N8
M10	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M11	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M12	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M13	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M14	AO + BO - OA	0	2*(ABO + O)	0	0	N15
M15	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N3
M16	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
M17	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
M18	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
M19	BO - OA	0	2*(ABO + O)	0	0	N8
M20	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N8
M21	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
M22	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M23	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
M24	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M25	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M26	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M27	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N8
M28	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M29	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M30	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M31	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M32	AO + BO - OA	0	2*(ABO + O)	0	0	N15
M33	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N3
M34	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
M35	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
M36	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
M37	BO - OA	0	2*(ABO + O)	0	0	N8
M38	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N8
M39	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3

Table B34 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	w_4	Serial
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	BO - OA	M40
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	BO - OA	M41
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	BO - OA	M42
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	BO - OA	M43
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	BO - OA	M44
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	M45
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	M46
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	M47
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	M48
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	M49
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M50
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M51
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M52
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M53
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M54
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M55
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M56
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M57
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M58
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M59
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M60
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M61
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M62
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M63
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M64
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M65
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M66
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M67
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	M68
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	M69
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	M70

Table B35 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

Serial	v_1	$2*g_6$	$2*g_8$	w_5	Result
M40	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M41	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
M42	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M43	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M44	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M45	0	2*(ABO + O)	0	0	N8
M46	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M47	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M48	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M49	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M50	A - AB + ABA + BA	2*(ABO + O)	0	0	N8
M51	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M52	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M53	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M54	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M55	BA	2*(ABO + O)	0	0	N9
M56	BA	2*(-BA - BO + OB)	0	ABO + BA + BO + O - OB	N3
M57	BA	2*(-BA - BO + OB)	0	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
M58	BA	2*(-AB - ABB - B + BAB)	0	AB + ABB + ABO + B - BAB + O	N10
M59	BA	2*(-AB - ABB - B + BAB)	0	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
M60	-AB - ABB - B + BA + BAB	0	0	ABO + O	N10
M61	-AB - ABB - B + BA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
M62	AO - OA	2*(ABO + O)	0	0	N15
M63	AO - OA	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N3
M64	AO - OA	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
M65	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
M66	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
M67	-BO + OB	0	0	ABO + O	N3
M68	-BO + OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
M69	0	2*(ABO + O)	0	0	N8
M70	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N8

Table B36 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	w_4	v_1	Serial
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M71
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M72
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M73
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M74
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M75
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	BO - OA	0	M76
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	M77
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	M78
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	M79
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	M80
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	M81
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	A - AB + ABA + BA	M82
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	A - AB + ABA + BA	M83
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	A - AB + ABA + BA	M84
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	A - AB + ABA + BA	M85
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	A - AB + ABA + BA	M86
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	BA	M87
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	BA	M88
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	BA	M89
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	BA	M90
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	BA	M91
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	-AB - ABB - B + BA + BAB	M92
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	-AB - ABB - B + BA + BAB	M93
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	AO - OA	M94
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	AO - OA	M95
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	AO - OA	M96
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	AO - OA	M97
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	AO - OA	M98
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	-BO + OB	M99
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	BO	-BO + OB	M100

Table B37 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under three-way exchanges.

Serial	$2*g_6$	$2*g_8$	w_5	Result
M71	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
M72	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M73	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
M74	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M75	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M76	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M77	2*(ABO + O)	0	0	N8
M78	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M79	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M80	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M81	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M82	2*(ABO + O)	0	0	N8
M83	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
M84	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
M85	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N10
M86	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
M87	2*(ABO + O)	0	0	N9
M88	2*(-BA - BO + OB)	0	ABO + BA + BO + O - OB	N3
M89	2*(-BA - BO + OB)	0	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
M90	2*(-AB - ABB - B + BAB)	0	AB + ABB + ABO + B - BAB + O	N10
M91	2*(-AB - ABB - B + BAB)	0	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
M92	0	0	ABO + O	N10
M93	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
M94	2*(ABO + O)	0	0	N15
M95	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N3
M96	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
M97	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
M98	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
M99	0	0	ABO + O	N3
M100	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table B38 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under three-way exchanges.

$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	$2*u_2$	v_3	v_4	Serial
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	0	NN1
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	0	NN2
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	0	NN3
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	0	NN4
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	0	NN5
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	0	NN6
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	0	NN7
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	0	NN8
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	0	NN9
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-ABB - B + BA + BAB	0	NN10
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-ABB - B + BA + BAB	0	NN11
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-ABB - B + BA + BAB	0	NN12
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-ABB - B + BA + BAB	0	NN13
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-ABB - B + BA + BAB	0	NN14
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-A + AAB - ABA - ABB - B + BAB	0	NN15
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-A + AAB - ABA - ABB - B + BAB	0	NN16
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	0	NN17
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	0	NN18
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	0	NN19
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	0	NN20
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	0	NN21
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - AO - B + BA + BAB + OA	0	NN22
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - AO - B + BA + BAB + OA	0	NN23
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	0	NN24
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	0	NN25
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	0	NN26
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	0	NN27
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	OB	NN28
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	OB	NN29
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	OB	NN30
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	OB	NN31

Table B39 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under three-way exchanges.

Serial	c_3	$2*g_5$	w_5	Result
NN1	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
NN2	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
NN3	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
NN4	A - AAB + ABA + ABB + B - BAB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
NN5	0	2*(ABO + O)	0	N4
NN6	0	2*(-AB - ABB - AO - B + BA + OA)	AB + ABB + ABO + AO + B - BA + O - OA	N3
NN7	0	2*(-AB - ABB - AO - B + BA + OA)	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
NN8	0	2*(-A + AAB - ABA - ABB - B)	A - AAB + ABA + ABB + ABO + B + O	N7
NN9	0	2*(-A + AAB - ABA - ABB - B)	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
NN10	0	2*(ABO + O)	0	N5
NN11	0	2*(-AB - AO + OA)	AB + ABO + AO + O - OA	N3
NN12	0	2*(-AB - AO + OA)	-A + AAB + AB - ABA - ABB + AO - B + BAB - OA + OAB	N1
NN13	0	2*(-A + AAB - ABA - BA)	A - AAB + ABA + ABO + BA + O	N7
NN14	0	2*(-A + AAB - ABA - BA)	A - AAB - AB + ABA - ABB - AO - B + 2*BA + BAB + OA + OAB	N1
NN15	0	0	ABO + O	N7
NN16	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
NN17	0	2*(ABO + O)	0	N15
NN18	0	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N3
NN19	0	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
NN20	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
NN21	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
NN22	0	0	ABO + O	N3
NN23	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
NN24	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
NN25	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
NN26	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
NN27	A - AAB + ABA + ABB + B - BAB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
NN28	0	2*(ABO + O)	0	N11
NN29	0	2*(-AO - BO + OA)	ABO + AO + BO + O - OA	N10
NN30	0	2*(-AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
NN31	0	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17

Table B40 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under three-way exchanges.

$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	$2*u_2$	v_3	v_4	Serial
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	OB	NN32
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB - BO + OB	NN33
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB - BO + OB	NN34
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB - BO + OB	NN35
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB - BO + OB	NN36
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB - BO + OB	NN37
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	NN38
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	NN39
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	NN40
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	NN41
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	NN42
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	NN43
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	NN44
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	NN45
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	NN46
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	0	NN47
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	0	NN48
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	0	NN49
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	0	NN50
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN51
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN52
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN53
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN54
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN55
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN56
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN57
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN58
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN59
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN60
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN61
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*BA	0	0	NN62

Table B41 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under three-way exchanges.

Serial	c_3	$2*g_5$	w_5	Result
NN32	0	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
NN33	0	2*(ABO + O)	0	N13
NN34	0	2*(-AB - AO + OA)	AB + ABO + AO + O - OA	N10
NN35	0	2*(-AB - AO + OA)	-A + AAB + 2*AB - ABA + AO - BA - BO - OA + OAB + OB	N1
NN36	0	2*(-A + AAB - ABA - BA)	A - AAB + ABA + ABO + BA + O	N17
NN37	0	2*(-A + AAB - ABA - BA)	A - AAB + ABA - AO + BA - BO + OA + OAB + OB	N1
NN38	0	0	ABO + O	N10
NN39	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
NN40	0	2*(ABO + O)	0	N15
NN41	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
NN42	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
NN43	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
NN44	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
NN45	0	0	ABO + O	N17
NN46	0	0	-AO - BO + OA + OAB + OB	N1
NN47	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N10
NN48	AO + BO - OA - OB	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
NN49	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
NN50	AB + ABB + AO + B - BA - BAB - OA	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
NN51	-AB + BO - OB	2*(ABO + O)	-AB + BO - OB	N13
NN52	-AB + BO - OB	2*(ABO + O)	-A + AAB - AB - ABA - 2*ABO - AO - BA - 2*O + OA + OAB	N1
NN53	-AB + BO - OB	2*(-AB - AO + OA)	ABO + AO + BO + O - OA - OB	N10
NN54	-AB + BO - OB	2*(-AB - AO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
NN55	-AB + BO - OB	2*(-A + AAB - ABA - BA)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
NN56	-AB + BO - OB	2*(-A + AAB - ABA - BA)	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
NN57	ABB + B - BA - BAB	2*(ABO + O)	ABB + B - BA - BAB	N5
NN58	ABB + B - BA - BAB	2*(ABO + O)	-A + AAB - AB - ABA - 2*ABO - AO - BA - 2*O + OA + OAB	N1
NN59	ABB + B - BA - BAB	2*(-AB - AO + OA)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
NN60	ABB + B - BA - BAB	2*(-AB - AO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
NN61	ABB + B - BA - BAB	2*(-A + AAB - ABA - BA)	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
NN62	ABB + B - BA - BAB	2*(-A + AAB - ABA - BA)	A - AAB - AB + ABA - AO + BA + OA + OAB	N1

Table B42 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (15) under three-way exchanges.

Serial	$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	v_3	$2*g_5$	w_5	Result
O1	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	ABB + B	AAB	OB	0	0	ABO + O	N7
O2	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	ABB + B	AAB	OB	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
O3	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(ABO + O)	0	N4
O4	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-AB - ABB - AO - B + BA + OA)	AB + ABB + ABO + AO + B - BA + O - OA	N3
O5	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-AB - ABB - AO - B + BA + OA)	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
O6	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-A + AAB - ABA - ABB - B)	A - AAB + ABA + ABB + ABO + B + O	N7
O7	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-A + AAB - ABA - ABB - B)	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
O8	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	BA	2*(ABO + O)	0	N5
O9	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	BA	2*(-AB - AO + OA)	AB + ABO + AO + O - OA	N3
O10	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	BA	2*(-AB - AO + OA)	-A + AAB + AB - ABA - ABB + AO - B + BAB - OA + OAB	N1
O11	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	BA	2*(-A + AAB - ABA - BA)	A - AAB + ABA + ABO + BA + O	N7
O12	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	BA	2*(-A + AAB - ABA - BA)	A - AAB - AB + ABA - ABB - AO - B + 2*BA + BAB + OA + OAB	N1
O13	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-A + AAB - ABA	0	ABO + O	N7
O14	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-A + AAB - ABA	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
O15	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(ABO + O)	0	N15
O16	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N3
O17	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
O18	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
O19	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
O20	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB - AO + BA + OA	0	ABO + O	N3
O21	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB - AO + BA + OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table B43 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	Serial
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P1
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P2
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P3
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P4
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P5
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P6
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P7
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	P8
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P9
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P10
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P11
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P12
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P13
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P14
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P15
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P16
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P17
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P18
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P19
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P20
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	P21
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	-A + AAB - ABA + BAB	AAB	P22
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	-A + AAB - ABA + BAB	AAB	P23
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P24
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P25
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P26
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P27
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P28
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P29
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P30
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	P31
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	P32

Table B44 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under three-way exchanges.

Serial	w_4	v_4	$2*g_5$	$2*g_7$	w_5	Result
P1	0	0	2*(ABO + O)	0	0	N11
P2	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P3	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P4	0	0	2*AAB	2*(-AAB + ABO + O)	0	N11
P5	0	0	2*AAB	2*(-AAB - AO - BO + OA)	ABO + AO + BO + O - OA	N10
P6	0	0	2*AAB	2*(-AAB - AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P7	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P8	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P9	0	0	2*(ABO + O)	0	0	N11
P10	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P11	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P12	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P13	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P14	0	0	2*(ABO + O)	0	0	N11
P15	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P16	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P17	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA + ABO - BAB + O)	0	N11
P18	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	ABO + AO + BO + O - OA	N10
P19	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P20	0	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P21	0	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P22	A - AAB - AB + ABA + BA + BO	0	0	0	ABO + O	N17
P23	A - AAB - AB + ABA + BA + BO	0	0	0	-AO - BO + OA + OAB + OB	N1
P24	0	0	2*(ABO + O)	0	0	N11
P25	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P26	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P27	0	0	2*AAB	2*(-AAB + ABO + O)	0	N11
P28	0	0	2*AAB	2*(-AAB - AO - BO + OA)	ABO + AO + BO + O - OA	N10
P29	0	0	2*AAB	2*(-AAB - AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P30	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P31	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P32	0	0	2*(ABO + O)	0	0	N11

Table B45 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under three-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	v_4	Serial
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	P33
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	P34
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	P35
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	P36
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P37
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P38
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P39
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P40
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P41
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P42
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P43
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	P44
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	P45
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	P46
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	P47
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	P48
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	P49
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	BA	P50
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	BA	P51
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	BA	P52
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	BA	P53
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	BA	P54
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB - AO + BA + OA	P55
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB - AO + BA + OA	P56
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	P57
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	P58
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	P59
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	P60
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	P61
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-A + AAB - ABA	P62
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-A + AAB - ABA	P63

Table B46 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under three-way exchanges.

Serial	$2*g_5$	$2*g_7$	w_5	Result
P33	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P34	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P35	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P36	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P37	2*(ABO + O)	0	0	N11
P38	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P39	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P40	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA + ABO - BAB + O)	0	N11
P41	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	ABO + AO + BO + O - OA	N10
P42	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P43	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P44	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P45	2*(ABO + O)	0	0	N11
P46	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
P47	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
P48	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
P49	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
P50	2*(ABO + O)	0	0	N13
P51	2*(-AB - AO + OA)	0	AB + ABO + AO + O - OA	N10
P52	2*(-AB - AO + OA)	0	-A + AAB + 2*AB - ABA + AO - BA - BO - OA + OAB + OB	N1
P53	2*(-A + AAB - ABA - BA)	0	A - AAB + ABA + ABO + BA + O	N17
P54	2*(-A + AAB - ABA - BA)	0	A - AAB + ABA - AO + BA - BO + OA + OAB + OB	N1
P55	0	0	ABO + O	N10
P56	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
P57	2*(ABO + O)	0	0	N15
P58	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
P59	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
P60	2*(-A + AAB - ABA - ABB - B + BAB)	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
P61	2*(-A + AAB - ABA - ABB - B + BAB)	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
P62	0	0	ABO + O	N17
P63	0	0	-AO - BO + OA + OAB + OB	N1

Table B47 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	$2*b_4$	Serial
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(ABO + O)	Y1
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y2
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y3
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y4
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y5
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y6
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y7
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y8
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*(-AO - BO + OA)	Y9
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y10
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y11
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y12
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y13
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y14
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y15
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y16
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y17
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y18
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y19
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y20
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y21
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y22
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	2*AAB	Y23
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*(ABO + O)	Y24
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*(-AO - BO + OA)	Y25
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*(-AO - BO + OA)	Y26
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*AAB	Y27
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*AAB	Y28
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*AAB	Y29
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*AAB	Y30
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	2*AAB	Y31
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	Y32
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y33
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y34
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y35
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y36
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y37
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	Y38
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	Y39
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	Y40
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	Y41
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	Y42
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	Y43
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y44
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	Y45

Table B48 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y1	0	0	0	0	N12
Y2	2*(ABO + AO + BO + O - OA)	0	0	0	N12
Y3	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N12
Y4	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + AO + BA + 2*BO + O - OA - OB	N3
Y5	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB + AO - B + BA + BAB + 2*BO - OA + OAB - OB	N1
Y6	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y7	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y8	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y9	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y10	2*(-AAB + ABO + O)	0	0	0	N12
Y11	2*OB	2*(-AAB + ABO + O - OB)	0	0	N12
Y12	2*OB	2*(-AAB - AO - BO + OA)	2*(ABO + AO + BO + O - OA - OB)	0	N12
Y13	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
Y14	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
Y15	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y16	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y17	2*OB	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
Y18	2*OB	2*(-A + AB - ABA - BA - BO)	0	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
Y19	2*(-A - ABA - ABB - B + BAB)	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N12
Y20	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y21	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y22	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB + ABO + B + BA - BAB + BO + O	N17
Y23	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB - AO + B + BA - BAB + OA + OAB + OB	N1
Y24	0	0	0	0	N11
Y25	0	0	0	ABO + AO + BO + O - OA	N10
Y26	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y27	0	2*(-AAB + ABO + O)	0	0	N11
Y28	0	2*(-AAB - AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
Y29	0	2*(-AAB - AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y30	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y31	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y32	0	0	0	0	N15
Y33	2*(ABO + AO + BO + O - OA)	0	0	0	N15
Y34	2*OB	0	0	ABO + AO + BO + O - OA - OB	N3
Y35	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y36	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y37	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y38	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N15
Y39	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y40	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y41	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y42	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y43	0	0	0	0	N11
Y44	0	0	0	ABO + AO + BO + O - OA	N10
Y45	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1

Table B49 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	Serial
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	Y46
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	Y47
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y48
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y49
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y50
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y51
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y52
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y53
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y54
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	Y55
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y56
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y57
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y58
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y59
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y60
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y61
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y62
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	Y63
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	Y64
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	Y65
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y66
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y67
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y68
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y69
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y70
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y71
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y72
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	Y73
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	Y74
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	Y75
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	Y76
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	Y77
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	Y78
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	-A + AB - ABA - BA - OA	Y79
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	-A + AB - ABA - BA - OA	Y80
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	0	Y81
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	0	Y82
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	0	Y83
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	0	Y84
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	Y85
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	Y86
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	-A + AB - ABA - BA - OA	Y87
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	-A + AB - ABA - BA - OA	Y88
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	Y89
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	0	Y90

Table B50 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y46	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y47	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y48	2*(ABO + O)	0	0	0	0	N11
Y49	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y50	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y51	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA + ABO - BAB + O)	0	0	N11
Y52	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N10
Y53	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y54	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y55	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y56	0	2*(ABO + O)	0	0	0	N8
Y57	0	2*OB	0	2*(ABO + O - OB)	0	N8
Y58	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y59	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y60	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y61	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y62	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y63	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y64	0	0	0	0	ABO + O	N10
Y65	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y66	0	2*(ABO + O)	0	0	0	N8
Y67	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N8
Y68	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y69	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y70	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y71	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y72	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y73	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y74	0	0	0	2*(ABO + O)	0	N8
Y75	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y76	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y77	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y78	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y79	0	0	0	0	ABO + O	N3
Y80	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y81	0	0	0	0	ABO + O	N10
Y82	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y83	0	0	0	0	ABO + BO + O - OA - OB	N10
Y84	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y85	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y86	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y87	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y88	0	0	0	0	AAB + OAB	N1
Y89	0	2*(ABO + O)	0	0	0	N8
Y90	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3

Table B51 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y91
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y92
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y93
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	OB	Y94
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	OB	Y95
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	A - AB + ABA + BA + BO	Y96
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	A - AB + ABA + BA + BO	Y97
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	Y98
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	Y99
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	Y100
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	Y101
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y102
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y103
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y104
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y105
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y106
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	OB	Y107
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	OB	Y108
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y109
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	Y110
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	Y111
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	Y112
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	Y113
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	Y114
BA	0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	A + ABA - BAB	0	Y115
BA	0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	A + ABA - BAB	0	Y116
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	BO - OA	Y117
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	BO - OA	Y118
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y119
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y120
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y121
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y122
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y123
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	Y124
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	-AB + BA + BAB + BO	Y125
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	-AB + BA + BAB + BO	Y126
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	Y127
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	Y128
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	Y129
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	Y130

Table B52 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y91	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y92	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y93	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y94	0	0	0	0	0	ABO + O	N3
Y95	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y96	0	0	0	0	0	ABO + O	N10
Y97	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y98	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
Y99	0	0	0	0	0	AAB + OAB	N1
Y100	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y101	0	0	0	0	0	AAB + OAB	N1
Y102	0	0	2*(ABO + O)	0	0	0	N15
Y103	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y104	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y105	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y106	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y107	0	0	0	0	0	ABO + O	N3
Y108	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y109	0	0	0	0	0	ABO + O	N10
Y110	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y111	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y112	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y113	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
Y114	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y115	0	0	0	0	0	ABO + O	N10
Y116	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y117	0	0	0	0	0	ABO + O	N10
Y118	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y119	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y120	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y121	-ABB - B + BO - OA - OB	0	0	0	0	ABB + ABO + B + O	N10
Y122	-ABB - B + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
Y123	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + B + O	N3
Y124	AB - BA - BAB - OA	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y125	0	0	0	0	0	ABO + O	N10
Y126	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y127	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N10
Y128	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y129	0	0	0	0	0	ABB + ABO + B + O	N3
Y130	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1

Table B53 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y131
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y132
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y133
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y134
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y135
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y136
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y137
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y138
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y139
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y140
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	Y141
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	Y142
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	Y143
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	Y144
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	Y145
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	Y146
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	BA	AO	ABB + B - BA	A + ABA	OB	Y147
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	BA	AO	ABB + B - BA	A + ABA	OB	Y148
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	BA	AO	ABB + B - BA	A + ABA	OB	Y149
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	BA	AO	ABB + B - BA	A + ABA	OB	Y150
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	BA	AO	ABB + B - BA	A + ABA	OB	Y151
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-A + AAB - ABA	AO	A - AAB + ABA + ABB + B	A + ABA	OB	Y152
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-A + AAB - ABA	AO	A - AAB + ABA + ABB + B	A + ABA	OB	Y153
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y154
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y155
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y156
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y157
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y158
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB - AO + BA + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	Y159
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB - AO + BA + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	Y160
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	0	Y161
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	0	Y162
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	0	Y163
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	0	Y164
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	0	Y165
BA	0	2*(AB - BA)	0	0	0	0	0	BA	0	0	AO	BAB	A + ABA	-AB + BO	Y166
BA	0	2*(AB - BA)	0	0	0	0	0	BA	0	0	AO	BAB	A + ABA	-AB + BO	Y167
BA	0	2*(AB - BA)	0	0	0	0	0	BA	0	0	AO	BAB	A + ABA	-AB + BO	Y168
BA	0	2*(AB - BA)	0	0	0	0	0	BA	0	0	AO	BAB	A + ABA	-AB + BO	Y169
BA	0	2*(AB - BA)	0	0	0	0	0	BA	0	0	AO	BAB	A + ABA	-AB + BO	Y170

Table B54 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y131	0	2*(ABO + O)	0	0	0	0	N15
Y132	0	2*(-AB - AO + BA + OA)	2*(AB + ABO + AO - BA + O - OA)	0	0	0	N15
Y133	0	2*(-AB - AO + BA + OA)	2*(AB - BA - BO + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y134	0	2*(-AB - AO + BA + OA)	2*(AB - BA - BO + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y135	0	2*(-AB - AO + BA + OA)	2*(-ABB - B + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y136	0	2*(-AB - AO + BA + OA)	2*(-ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y137	0	2*(-A + AAB - ABA)	2*(A - AAB + ABA + ABO + O)	0	0	0	N15
Y138	0	2*(-A + AAB - ABA)	2*(AB - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y139	0	2*(-A + AAB - ABA)	2*(AB - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y140	0	2*(-A + AAB - ABA)	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y141	0	2*(-A + AAB - ABA)	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y142	0	2*(ABO + O)	0	0	0	0	N4
Y143	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	AB + ABB + ABO + AO + B - BA + O - OA	N3
Y144	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
Y145	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB + ABA + ABB + ABO + B + O	N7
Y146	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
Y147	0	2*(ABO + O)	0	0	0	0	N5
Y148	0	2*(-AB - AO + OA)	0	0	0	AB + ABO + AO + O - OA	N3
Y149	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + AB - ABA - ABB + AO - B + BAB - OA + OAB	N1
Y150	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA + ABO + BA + O	N7
Y151	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB - AB + ABA - ABB - AO - B + 2*BA + BAB + OA + OAB	N1
Y152	0	0	0	0	0	ABO + O	N7
Y153	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y154	0	2*(ABO + O)	0	0	0	0	N15
Y155	0	2*(-AO - BO + OA + OB)	0	0	0	ABO + AO + BO + O - OA - OB	N3
Y156	0	2*(-AO - BO + OA + OB)	0	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y157	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y158	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y159	0	0	0	0	0	ABO + O	N3
Y160	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y161	0	2*(ABO + O)	0	0	0	0	N11
Y162	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y163	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y164	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y165	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y166	0	2*(ABO + O)	0	0	0	0	N13
Y167	0	2*(-AB - AO + OA)	0	0	0	AB + ABO + AO + O - OA	N10
Y168	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + 2*AB - ABA + AO - BA - BO - OA + OAB + OB	N1
Y169	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA + ABO + BA + O	N17
Y170	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA - AO + BA - BO + OA + OAB + OB	N1

Table B55 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(AB - BA)	0	0	0	0	0	-AB - AO + BA + OA	0	0	AO	BAB	A + ABA	AO + BO - OA	Y171
BA	0	2*(AB - BA)	0	0	0	0	0	-AB - AO + BA + OA	0	0	AO	BAB	A + ABA	AO + BO - OA	Y172
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y173
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y174
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y175
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y176
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y177
BA	0	2*(AB - BA)	0	0	0	0	0	-A + AAB - ABA	0	0	AO	BAB	A + ABA	A - AAB - AB + ABA + BA + BO	Y178
BA	0	2*(AB - BA)	0	0	0	0	0	-A + AAB - ABA	0	0	AO	BAB	A + ABA	A - AAB - AB + ABA + BA + BO	Y179
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	OB	Y180
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	OB	Y181
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	OB	Y182
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	OB	Y183
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	OB	Y184
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-ABB - B + BA + BAB	AO	ABB + B - BA	A + ABA	OB	Y185
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-ABB - B + BA + BAB	AO	ABB + B - BA	A + ABA	OB	Y186
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-ABB - B + BA + BAB	AO	ABB + B - BA	A + ABA	OB	Y187
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-ABB - B + BA + BAB	AO	ABB + B - BA	A + ABA	OB	Y188
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-ABB - B + BA + BAB	AO	ABB + B - BA	A + ABA	OB	Y189
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-A + AAB - ABA - ABB - B + BAB	AO	A - AAB + ABA + ABB + B	A + ABA	OB	Y190
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-A + AAB - ABA - ABB - B + BAB	AO	A - AAB + ABA + ABB + B	A + ABA	OB	Y191
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y192
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y193
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y194
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y195
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	Y196
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	Y197
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	Y198
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	OB	Y199
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	OB	Y200
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	OB	Y201
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	OB	Y202
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	Y203
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	Y204
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	Y205
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	Y206
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	Y207
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB - BO + OB	0	0	AO	BAB	A + ABA	-AB + BO	Y208
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB - BO + OB	0	0	AO	BAB	A + ABA	-AB + BO	Y209
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB - BO + OB	0	0	AO	BAB	A + ABA	-AB + BO	Y210

Table B56 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y171	0	0	0	0	0	ABO + O	N10
Y172	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y173	0	2*(ABO + O)	0	0	0	0	N15
Y174	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y175	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y176	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y177	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y178	0	0	0	0	0	ABO + O	N17
Y179	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y180	0	2*(ABO + O)	0	0	0	0	N4
Y181	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	AB + ABB + ABO + AO + B - BA + O - OA	N3
Y182	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
Y183	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB + ABA + ABB + ABO + B + O	N7
Y184	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
Y185	0	2*(ABO + O)	0	0	0	0	N5
Y186	0	2*(-AB - AO + OA)	0	0	0	AB + ABO + AO + O - OA	N3
Y187	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + AB - ABA - ABB + AO - B + BAB - OA + OAB	N1
Y188	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA + ABO + BA + O	N7
Y189	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB - AB + ABA - ABB - AO - B + 2*BA + BAB + OA + OAB	N1
Y190	0	0	0	0	0	ABO + O	N7
Y191	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y192	0	2*(ABO + O)	0	0	0	0	N15
Y193	0	2*(-AO - BO + OA + OB)	0	0	0	ABO + AO + BO + O - OA - OB	N3
Y194	0	2*(-AO - BO + OA + OB)	0	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y195	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y196	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y197	0	0	0	0	0	ABO + O	N3
Y198	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y199	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
Y200	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y201	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
Y202	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y203	0	2*(ABO + O)	0	0	0	0	N11
Y204	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y205	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y206	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y207	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y208	0	2*(ABO + O)	0	0	0	0	N13
Y209	0	2*(-AB - AO + OA)	0	0	0	AB + ABO + AO + O - OA	N10
Y210	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + 2*AB - ABA + AO - BA - BO - OA + OAB + OB	N1

Table B57 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB - BO + OB	0	0	AO	BAB	A + ABA	-AB + BO	Y211
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB - BO + OB	0	0	AO	BAB	A + ABA	-AB + BO	Y212
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	0	0	AO	BAB	A + ABA	AO + BO - OA	Y213
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	0	0	AO	BAB	A + ABA	AO + BO - OA	Y214
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y215
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y216
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y217
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y218
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	-AB - ABB - B + BA + BAB + BO	Y219
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	AO	BAB	A + ABA	A - AAB - AB + ABA + BA + BO	Y220
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	AO	BAB	A + ABA	A - AAB - AB + ABA + BA + BO	Y221
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	Y222
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	Y223
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	Y224
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	Y225
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y226
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y227
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y228
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y229
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y230
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y231
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y232
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y233
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y234
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y235
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y236
BA	0	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	AO	BAB	A + ABA	OB	Y237
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y238
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y239
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y240
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y241
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y242
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y243
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y244
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y245
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y246
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y247
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y248
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y249
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y250

Table B58 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y211	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA + ABO + BA + O	N17
Y212	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA - AO + BA - BO + OA + OAB + OB	N1
Y213	0	0	0	0	0	ABO + O	N10
Y214	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y215	0	2*(ABO + O)	0	0	0	0	N15
Y216	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y217	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y218	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y219	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y220	0	0	0	0	0	ABO + O	N17
Y221	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y222	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y223	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y224	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
Y225	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y226	0	2*(ABO + O)	0	0	0	-AB + BO - OB	N13
Y227	0	2*(ABO + O)	0	0	0	-A + AAB - AB - ABA - 2*ABO - AO - BA - 2*O + OA + OAB	N1
Y228	0	2*(-AB - AO + OA)	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y229	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y230	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
Y231	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y232	0	2*(ABO + O)	0	0	0	ABB + B - BA - BAB	N5
Y233	0	2*(ABO + O)	0	0	0	-A + AAB - AB - ABA - 2*ABO - AO - BA - 2*O + OA + OAB	N1
Y234	0	2*(-AB - AO + OA)	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
Y235	0	2*(-AB - AO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y236	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
Y237	0	2*(-A + AAB - ABA - BA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y238	0	2*(ABO + O)	0	0	0	0	N12
Y239	0	2*(-AO - BO + OA)	2*(ABO + AO + BO + O - OA)	0	0	0	N12
Y240	0	2*(-AO - BO + OA)	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N12
Y241	0	2*(-AO - BO + OA)	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + AO + BA + 2*BO + O - OA - OB	N3
Y242	0	2*(-AO - BO + OA)	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB + AO - B + BA + BAB + 2*BO - OA + OAB - OB	N1
Y243	0	2*(-AO - BO + OA)	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y244	0	2*(-AO - BO + OA)	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y245	0	2*(-AO - BO + OA)	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y246	0	2*(-AO - BO + OA)	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y247	0	2*AAB	2*(-AAB + ABO + O)	0	0	0	N12
Y248	0	2*AAB	2*OB	2*(-AAB + ABO + O - OB)	0	0	N12
Y249	0	2*AAB	2*OB	2*(-AAB - AO - BO + OA)	2*(ABO + AO + BO + O - OA - OB)	0	N12
Y250	0	2*AAB	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6

Table B59 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y251
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y252
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y253
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y254
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y255
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y256
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y257
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y258
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y259
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	Y260
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y261
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y262
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y263
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y264
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y265
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y266
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y267
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	Y268
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y269
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y270
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y271
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y272
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y273
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y274
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y275
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y276
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y277
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y278
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	Y279
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	Y280
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	Y281
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	Y282
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	Y283
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	Y284
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y285
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y286
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y287
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y288
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y289
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y290

Table B60 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y251	0	2*AAB	2*OB	2*(-AAB - AO - BO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
Y252	0	2*AAB	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y253	0	2*AAB	2*OB	2*(-AAB - AO - BO + OA)	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y254	0	2*AAB	2*OB	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
Y255	0	2*AAB	2*OB	2*(-A + AB - ABA - BA - BO)	0	-AAB - AB - ABB - AO - B + BA + BAB + OA + OAB - OB	N1
Y256	0	2*AAB	2*(-A - ABA - ABB - B + BAB)	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N12
Y257	0	2*AAB	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	A + ABA + ABB + ABO + AO + B - BAB + BO + O - OA	N10
Y258	0	2*AAB	2*(-A - ABA - ABB - B + BAB)	2*(-AAB - AO - BO + OA)	0	AAB + AB + ABB + AO + B - BA - BAB - OA + OAB + OB	N1
Y259	0	2*AAB	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB + ABO + B + BA - BAB + BO + O	N17
Y260	0	2*AAB	2*(-A - ABA - ABB - B + BAB)	2*(-A + AB - ABA - BA - BO)	0	2*A - AAB - AB + 2*ABA + ABB - AO + B + BA - BAB + OA + OAB + OB	N1
Y261	0	2*(ABO + O)	0	0	0	0	N11
Y262	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y263	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y264	0	2*AAB	0	2*(-AAB + ABO + O)	0	0	N11
Y265	0	2*AAB	0	2*(-AAB - AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
Y266	0	2*AAB	0	2*(-AAB - AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y267	0	2*AAB	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y268	0	2*AAB	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y269	0	2*(ABO + O)	0	0	0	0	N15
Y270	0	2*(-AO - BO + OA)	2*(ABO + AO + BO + O - OA)	0	0	0	N15
Y271	0	2*(-AO - BO + OA)	2*OB	0	0	ABO + AO + BO + O - OA - OB	N3
Y272	0	2*(-AO - BO + OA)	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y273	0	2*(-AO - BO + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y274	0	2*(-AO - BO + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y275	0	2*(-A + AAB + AB - ABA - BA - BO)	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N15
Y276	0	2*(-A + AAB + AB - ABA - BA - BO)	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y277	0	2*(-A + AAB + AB - ABA - BA - BO)	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y278	0	2*(-A + AAB + AB - ABA - BA - BO)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y279	0	2*(-A + AAB + AB - ABA - BA - BO)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y280	0	2*(ABO + O)	0	0	0	0	N11
Y281	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y282	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y283	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y284	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y285	0	2*(ABO + O)	0	0	0	0	N11
Y286	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N10
Y287	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y288	0	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA + ABO - BAB + O)	0	0	N11
Y289	0	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N10
Y290	0	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1

Table B61 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y291
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	Y292
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y293
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y294
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y295
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y296
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y297
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y298
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y299
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	Y300
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	Y301
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	Y302
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y303
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y304
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y305
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y306
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y307
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y308
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y309
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	Y310
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y311
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y312
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y313
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y314
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y315
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y316
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	Y317
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	Y318
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	Y319
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y320
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y321
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y322
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y323
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y324
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	Y325
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y326
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y327
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y328
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y329
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	Y330

Table B62 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y291	0	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y292	0	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y293	0	0	2*(ABO + O)	0	0	0	N8
Y294	0	0	2*OB	0	2*(ABO + O - OB)	0	N8
Y295	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y296	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y297	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y298	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y299	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y300	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y301	0	0	0	0	0	ABO + O	N10
Y302	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y303	0	0	2*(ABO + O)	0	0	0	N8
Y304	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N8
Y305	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y306	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y307	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y308	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y309	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y310	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y311	BO - OA - OB	0	0	0	2*(ABO + O)	0	N8
Y312	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y313	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y314	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y315	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y316	-A + AB - ABA - BA - OA	0	0	0	0	ABO + O	N3
Y317	-A + AB - ABA - BA - OA	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y318	0	0	0	0	0	ABO + O	N10
Y319	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y320	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y321	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y322	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y323	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y324	-A + AB - ABA - BA - OA	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y325	-A + AB - ABA - BA - OA	0	0	0	0	AAB + OAB	N1
Y326	0	0	2*(ABO + O)	0	0	0	N8
Y327	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y328	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y329	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y330	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1

Table B63 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y331
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y332
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y333
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y334
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y335
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y336
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y337
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y338
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y339
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y340
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y341
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y342
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y343
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y344
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	Y345
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y346
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y347
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y348
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y349
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y350
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	Y351
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y352
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y353
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y354
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y355
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y356
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y357
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y358
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y359
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y360
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	Y361
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y362
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y363
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y364
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y365
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y366
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y367
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y368
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y369
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y370

Table B64 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y331	0	OB	0	0	0	0	0	ABO + O	N3
Y332	0	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y333	0	A - AB + ABA + BA + BO	0	0	0	0	0	ABO + O	N10
Y334	0	A - AB + ABA + BA + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y335	0	OB	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
Y336	0	OB	0	0	0	0	0	AAB + OAB	N1
Y337	0	OB	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y338	0	OB	0	0	0	0	0	AAB + OAB	N1
Y339	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	2*(ABO + O)	0	0	0	N15
Y340	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y341	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y342	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y343	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y344	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	ABO + O	N3
Y345	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y346	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	0	0	0	ABO + O	N10
Y347	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y348	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y349	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y350	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N3
Y351	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y352	A + ABA - BAB	0	0	0	0	0	0	ABO + O	N10
Y353	A + ABA - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y354	A + ABA - BAB	BO - OA	0	0	0	0	0	ABO + O	N10
Y355	A + ABA - BAB	BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y356	A + ABA - BAB	OB	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y357	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y358	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	ABB + ABO + B + O	N10
Y359	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
Y360	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + B + O	N3
Y361	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y362	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	ABO + O	N10
Y363	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y364	A + ABA - BAB	OB	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N10
Y365	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y366	A + ABA - BAB	OB	0	0	0	0	0	ABB + ABO + B + O	N3
Y367	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y368	AAB	A - AAB - AB + ABA + BA + BO	0	0	2*(ABO + O)	0	0	0	N15
Y369	AAB	A - AAB - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y370	AAB	A - AAB - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1

Table B65 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y371
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y372
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y373
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	Y374
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y375
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y376
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y377
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y378
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y379
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	Y380
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	Y381
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	Y382
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	Y383
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	Y384
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	Y385
BA	2*(AB - BA)	0	0	0	0	0	0	0	BA	0	AO - BA	ABB + B	Y386
BA	2*(AB - BA)	0	0	0	0	0	0	0	BA	0	AO - BA	ABB + B	Y387
BA	2*(AB - BA)	0	0	0	0	0	0	0	BA	0	AO - BA	ABB + B	Y388
BA	2*(AB - BA)	0	0	0	0	0	0	0	BA	0	AO - BA	ABB + B	Y389
BA	2*(AB - BA)	0	0	0	0	0	0	0	BA	0	AO - BA	ABB + B	Y390
BA	2*(AB - BA)	0	0	0	0	0	0	0	-BO + OB	0	AO + BO - OB	ABB + B	Y391
BA	2*(AB - BA)	0	0	0	0	0	0	0	-BO + OB	0	AO + BO - OB	ABB + B	Y392
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y393
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y394
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y395
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y396
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y397
BA	2*(AB - BA)	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB	0	AB + ABB + AO + B - BA - BAB	ABB + B	Y398
BA	2*(AB - BA)	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB	0	AB + ABB + AO + B - BA - BAB	ABB + B	Y399
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	ABB + B	Y400
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	ABB + B	Y401
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y402
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y403
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y404
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y405
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y406
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	Y407
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y408
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y409
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y410

Table B66 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y371	AAB	A - AAB - AB + ABA + BA + BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y372	AAB	A - AAB - AB + ABA + BA + BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y373	AAB	OB	0	0	0	0	0	ABO + O	N7
Y374	AAB	OB	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y375	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	ABO + O	N17
Y376	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y377	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N17
Y378	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y379	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N7
Y380	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
Y381	AAB	BO	0	0	2*(ABO + O)	0	0	0	N14
Y382	AAB	BO	0	0	2*(-AO - BO + OB)	0	0	ABO + AO + BO + O - OB	N7
Y383	AAB	BO	0	0	2*(-AO - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
Y384	AAB	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O	N17
Y385	AAB	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
Y386	AAB	BO	0	0	2*(ABO + O)	0	0	0	N16
Y387	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + BA + BO + O - OB	N7
Y388	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	-AB - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y389	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + B - BAB + O	N17
Y390	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y391	AAB	BO	0	0	0	0	0	ABO + O	N7
Y392	AAB	BO	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y393	AAB	BO	0	0	2*(ABO + O)	0	0	0	N15
Y394	AAB	BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y395	AAB	BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y396	AAB	BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y397	AAB	BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y398	AAB	BO	0	0	0	0	0	ABO + O	N17
Y399	AAB	BO	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y400	AAB	OB	0	0	0	0	0	ABO + O	N7
Y401	AAB	OB	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y402	AAB	BO	0	0	0	0	0	ABO + O	N17
Y403	AAB	BO	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y404	AAB	OB	0	0	0	0	0	ABO + BO + O - OB	N17
Y405	AAB	OB	0	0	0	0	0	-AO + OA + OAB	N1
Y406	AAB	OB	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N7
Y407	AAB	OB	0	0	0	0	0	-AO + OA + OAB	N1
Y408	AAB	0	A - AAB + ABA - BAB	0	0	2*(ABO + O)	0	0	N11
Y409	AAB	0	A - AAB + ABA - BAB	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N10
Y410	AAB	0	A - AAB + ABA - BAB	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1

Table B67 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y411
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y412
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y413
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	Y414
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y415
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y416
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y417
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y418
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y419
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	Y420
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y421
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y422
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y423
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y424
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y425
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y426
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y427
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y428
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y429
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y430
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y431
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y432
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y433
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y434
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y435
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y436
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y437
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y438
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y439
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	Y440
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y441
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y442
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y443
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y444
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y445
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y446
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y447
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y448
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y449
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y450

Table B68 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y411	AAB	0	A - AAB + ABA - BAB	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N17
Y412	AAB	0	A - AAB + ABA - BAB	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
Y413	AAB	0	AB - BA - BAB - BO	0	0	0	0	ABO + O	N17
Y414	AAB	0	AB - BA - BAB - BO	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y415	AAB	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y416	AAB	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y417	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	ABO + AO + BO + O - OA	N10
Y418	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y419	AAB	0	AB - BA - BAB - BO	0	0	0	0	ABO + AO + BO + O - OA	N17
Y420	AAB	0	AB - BA - BAB - BO	0	0	0	0	OAB + OB	N1
Y421	AAB	BO - OA	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y422	AAB	BO - OA	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y423	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	ABO + AO + O	N10
Y424	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
Y425	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	ABO + AO + O	N17
Y426	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	-BO + OA + OAB + OB	N1
Y427	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N10
Y428	AAB	OB	0	0	0	0	0	AB - BA - BAB - OA + OAB	N1
Y429	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
Y430	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y431	AAB	OB	AB - BA - BAB - OA	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
Y432	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y433	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y434	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y435	AAB	OB	AB - BA - BAB - OA	0	0	0	0	ABO + AO + BO + O - OA - OB	N17
Y436	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y437	AAB	OB	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	0	0	0	0	ABB + ABO + AO + B + O	N10
Y438	AAB	OB	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + AO + B - BA - BO + OAB + OB	N1
Y439	AAB	OB	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + AO + B + O	N2
Y440	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y441	AAB	-AB + BA + BAB + BO	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y442	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y443	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB + ABO + AO - BA - BAB + O - OA	N17
Y444	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y445	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
Y446	AAB	OB	0	0	0	0	0	OAB	N1
Y447	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
Y448	AAB	OB	0	0	0	0	0	OAB	N1
Y449	AAB	OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N17
Y450	AAB	OB	0	0	0	0	0	OAB	N1

Table B69 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y451
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y452
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y453
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y454
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y455
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y456
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y457
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	Y458
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y459
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y460
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y461
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y462
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y463
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y464
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y465
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y466
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	Y467
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	Y468
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y469
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y470
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y471
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y472
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y473
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y474
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y475
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y476
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y477
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y478
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y479
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y480
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y481
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y482
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	Y483
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y484
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y485
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y486
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y487
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y488
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y489
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y490
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y491

Table B70 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y451	AAB	OB	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
Y452	AAB	OB	0	0	0	0	0	OAB	N1
Y453	AAB	-AB + BA + BAB + BO	0	0	0	0	0	ABO + O	N17
Y454	AAB	-AB + BA + BAB + BO	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
Y455	AAB	OB	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N17
Y456	AAB	OB	0	0	0	0	0	-AB - AO + BA + BAB + OA + OAB	N1
Y457	AAB	OB	0	0	0	0	0	ABB + ABO + B + O	N7
Y458	AAB	OB	0	0	0	0	0	-AB - AO + BA + BAB + OA + OAB	N1
Y459	0	0	0	0	2*(ABO + O)	0	0	0	N8
Y460	0	0	0	0	2*OB	0	2*(ABO + O - OB)	0	N8
Y461	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y462	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y463	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y464	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y465	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y466	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y467	0	0	0	0	0	0	0	ABO + O	N10
Y468	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y469	0	BO - OA	0	0	2*(ABO + O)	0	0	0	N8
Y470	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N8
Y471	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y472	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y473	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y474	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y475	0	BO - OA	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y476	0	BO - OA	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y477	0	OB	BO - OA - OB	0	0	0	2*(ABO + O)	0	N8
Y478	0	OB	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y479	0	OB	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y480	0	OB	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y481	0	OB	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y482	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	ABO + O	N3
Y483	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y484	0	BO - OA	0	0	0	0	0	ABO + O	N10
Y485	0	BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y486	0	OB	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y487	0	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y488	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y489	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y490	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y491	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	AAB + OAB	N1

Table B71 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y492
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y493
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y494
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y495
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y496
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y497
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y498
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y499
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y500
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y501
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y502
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y503
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y504
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y505
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y506
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y507
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y508
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y509
BA	0	0	2*(AB - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y510
BA	0	0	2*(AB - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y511
BA	0	0	2*(AB - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y512
BA	0	0	2*(AB - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y513
BA	0	0	2*(AB - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y514
BA	0	0	2*(AB - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	Y515
BA	0	0	2*(AB - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	Y516
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y517
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y518
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y519
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y520
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y521
BA	0	0	2*(AB - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	Y522
BA	0	0	2*(AB - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	Y523
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	ABB + B	Y524
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	ABB + B	Y525
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y526
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y527
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y528
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y529
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y530
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y531
BA	0	0	2*BAB	2*BO	0	0	0	0	0	0	-BO + OA	0	Y532

Table B72 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y492	0	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	0	0	0	N8
Y493	0	A - AB + ABA + BA + BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y494	0	A - AB + ABA + BA + BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y495	0	A - AB + ABA + BA + BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y496	0	A - AB + ABA + BA + BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y497	0	OB	0	0	0	0	0	ABO + O	N3
Y498	0	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y499	0	A - AB + ABA + BA + BO	0	0	0	0	0	ABO + O	N10
Y500	0	A - AB + ABA + BA + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y501	0	OB	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
Y502	0	OB	0	0	0	0	0	AAB + OAB	N1
Y503	0	OB	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y504	0	OB	0	0	0	0	0	AAB + OAB	N1
Y505	0	BO	0	0	2*(ABO + O)	0	0	0	N8
Y506	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y507	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y508	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y509	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y510	A - AB + ABA	BO	0	0	2*(ABO + O)	0	0	0	N9
Y511	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + BA + BO + O - OB	N3
Y512	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
Y513	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + B - BAB + O	N10
Y514	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
Y515	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	ABO + O	N10
Y516	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y517	A - AB + ABA - AO + BA + OA	BO	0	0	2*(ABO + O)	0	0	0	N15
Y518	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y519	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y520	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y521	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y522	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	ABO + O	N3
Y523	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y524	A - AB + ABA + BA	OB	0	0	0	0	0	ABO + O	N3
Y525	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y526	A - AB + ABA + BA	BO	0	0	0	0	0	ABO + O	N10
Y527	A - AB + ABA + BA	BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y528	A - AB + ABA + BA	OB	0	0	0	0	0	ABO + BO + O - OB	N10
Y529	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
Y530	A - AB + ABA + BA	OB	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N3
Y531	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
Y532	A + ABA - BAB	0	0	0	0	0	0	ABO + O	N10

Table B73 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	0	0	2*BAB	2*BO	0	0	0	0	0	0	-BO + OA	0	Y533
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y534
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y535
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y536
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y537
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y538
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y539
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y540
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	Y541
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y542
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y543
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y544
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y545
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y546
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y547
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y548
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y549
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y550
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y551
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y552
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y553
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y554
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	Y555
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	Y556
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	Y557
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y558
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y559
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y560
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y561
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y562
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y563
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y564
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y565
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y566
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y567
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y568
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y569
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y570
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y571
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	Y572
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y573

Table B74 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	$2*a_9$	Result
Y533	A + ABA - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y534	A + ABA - BAB	BO - OA	0	0	0	0	0	ABO + O	N10
Y535	A + ABA - BAB	BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y536	A + ABA - BAB	OB	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y537	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y538	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	ABB + ABO + B + O	N10
Y539	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
Y540	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + B + O	N3
Y541	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y542	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	ABO + O	N10
Y543	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y544	A + ABA - BAB	OB	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N10
Y545	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y546	A + ABA - BAB	OB	0	0	0	0	0	ABB + ABO + B + O	N3
Y547	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y548	0	0	0	0	2*(ABO + O)	0	0	0	N8
Y549	0	0	0	0	2*OB	0	2*(ABO + O - OB)	0	N8
Y550	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y551	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y552	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y553	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y554	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y555	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y556	0	0	0	0	0	0	0	ABO + O	N10
Y557	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y558	0	BO - OA	0	0	2*(ABO + O)	0	0	0	N8
Y559	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N8
Y560	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y561	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y562	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y563	0	BO - OA	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y564	0	BO - OA	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y565	0	BO - OA	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y566	0	OB	BO - OA - OB	0	0	0	2*(ABO + O)	0	N8
Y567	0	OB	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y568	0	OB	BO - OA - OB	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y569	0	OB	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N10
Y570	0	OB	BO - OA - OB	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y571	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	ABO + O	N3
Y572	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y573	0	BO - OA	0	0	0	0	0	ABO + O	N10

Table B75 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y574
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y575
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y576
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y577
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y578
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y579
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	Y580
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y581
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y582
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y583
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y584
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y585
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y586
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	Y587
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y588
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y589
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y590
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y591
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y592
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	Y593
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y594
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y595
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y596
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y597
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	Y598
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y599
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y600
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y601
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y602
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	BA	0	0	0	OA	ABB + B	Y603
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	Y604
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	Y605
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y606
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y607
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y608
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y609
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	Y610
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	Y611
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	Y612
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	ABB + B	Y613
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	ABB + B	Y614

Table B76 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y574	0	BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y575	0	OB	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y576	0	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y577	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y578	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y579	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y580	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	AAB + OAB	N1
Y581	0	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	0	0	0	N8
Y582	0	A - AB + ABA + BA + BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y583	0	A - AB + ABA + BA + BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y584	0	A - AB + ABA + BA + BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y585	0	A - AB + ABA + BA + BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y586	0	OB	0	0	0	0	0	ABO + O	N3
Y587	0	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y588	0	A - AB + ABA + BA + BO	0	0	0	0	0	ABO + O	N10
Y589	0	A - AB + ABA + BA + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y590	0	OB	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N10
Y591	0	OB	0	0	0	0	0	AAB + OAB	N1
Y592	0	OB	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N3
Y593	0	OB	0	0	0	0	0	AAB + OAB	N1
Y594	0	BO	0	0	2*(ABO + O)	0	0	0	N8
Y595	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y596	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y597	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y598	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y599	A - AB + ABA	BO	0	0	2*(ABO + O)	0	0	0	N9
Y600	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + BA + BO + O - OB	N3
Y601	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
Y602	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + B - BAB + O	N10
Y603	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
Y604	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	ABO + O	N10
Y605	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y606	A - AB + ABA - AO + BA + OA	BO	0	0	2*(ABO + O)	0	0	0	N15
Y607	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y608	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y609	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y610	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y611	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	ABO + O	N3
Y612	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y613	A - AB + ABA + BA	OB	0	0	0	0	0	ABO + O	N3
Y614	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table B77 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y615
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y616
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y617
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y618
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y619
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y620
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	0	0	0	0	0	0	-BO + OA	0	Y621
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	0	0	0	0	0	0	-BO + OA	0	Y622
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y623
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y624
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y625
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y626
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y627
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y628
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y629
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	Y630
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y631
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y632
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y633
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y634
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y635
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y636
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	0	ABB + B	Y637
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	0	ABB + B	Y638
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	0	ABB + B	Y639
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	0	ABB + B	Y640
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	0	ABB + B	Y641
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO + BA + OA	0	AO - BA	ABB + B	Y642
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO + BA + OA	0	AO - BA	ABB + B	Y643
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO + BA + OA	0	AO - BA	ABB + B	Y644
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO + BA + OA	0	AO - BA	ABB + B	Y645
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO + BA + OA	0	AO - BA	ABB + B	Y646
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO - BO + OA + OB	0	AO + BO - OB	ABB + B	Y647
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO - BO + OA + OB	0	AO + BO - OB	ABB + B	Y648
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y649
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y650
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y651
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y652
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	Y653
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	0	AB + ABB + AO + B - BA - BAB	ABB + B	Y654
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	0	AB + ABB + AO + B - BA - BAB	ABB + B	Y655

Table B78 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y615	A - AB + ABA + BA	BO	0	0	0	0	0	ABO + O	N10
Y616	A - AB + ABA + BA	BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y617	A - AB + ABA + BA	OB	0	0	0	0	0	ABO + BO + O - OB	N10
Y618	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
Y619	A - AB + ABA + BA	OB	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N3
Y620	A - AB + ABA + BA	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
Y621	A + ABA - BAB	0	0	0	0	0	0	ABO + O	N10
Y622	A + ABA - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y623	A + ABA - BAB	BO - OA	0	0	0	0	0	ABO + O	N10
Y624	A + ABA - BAB	BO - OA	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y625	A + ABA - BAB	OB	0	0	0	0	0	ABO + BO + O - OA - OB	N10
Y626	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
Y627	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	ABB + ABO + B + O	N10
Y628	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
Y629	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + B + O	N3
Y630	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y631	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	ABO + O	N10
Y632	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y633	A + ABA - BAB	OB	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N10
Y634	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y635	A + ABA - BAB	OB	0	0	0	0	0	ABB + ABO + B + O	N3
Y636	A + ABA - BAB	OB	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
Y637	AAB	BO	0	0	2*(ABO + O)	0	0	0	N14
Y638	AAB	BO	0	0	2*(-AO - BO + OB)	0	0	ABO + AO + BO + O - OB	N7
Y639	AAB	BO	0	0	2*(-AO - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
Y640	AAB	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O	N17
Y641	AAB	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
Y642	AAB	BO	0	0	2*(ABO + O)	0	0	0	N16
Y643	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + BA + BO + O - OB	N7
Y644	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	-AB - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y645	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + B - BAB + O	N17
Y646	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y647	AAB	BO	0	0	0	0	0	ABO + O	N7
Y648	AAB	BO	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
Y649	AAB	BO	0	0	2*(ABO + O)	0	0	0	N15
Y650	AAB	BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N7
Y651	AAB	BO	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
Y652	AAB	BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N17
Y653	AAB	BO	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
Y654	AAB	BO	0	0	0	0	0	ABO + O	N17
Y655	AAB	BO	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1

Table B79 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y656
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y657
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y658
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y659
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y660
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y661
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y662
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y663
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y664
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y665
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y666
BA	2*(AB - BA)	0	0	0	2*BA	0	0	0	0	0	OA	ABB + B	Y667
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	OA	ABB + B	Y668
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	OA	ABB + B	Y669
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	OA	ABB + B	Y670
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	OA	ABB + B	Y671
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	OA	ABB + B	Y672
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA	0	0	0	OA	ABB + B	Y673
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA	0	0	0	OA	ABB + B	Y674
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA	0	0	0	OA	ABB + B	Y675
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA	0	0	0	OA	ABB + B	Y676
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA	0	0	0	OA	ABB + B	Y677
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB - ABA - ABB - B + BAB	0	0	0	OA	ABB + B	Y678
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB - ABA - ABB - B + BAB	0	0	0	OA	ABB + B	Y679
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	OA	ABB + B	Y680
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	OA	ABB + B	Y681
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	OA	ABB + B	Y682
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	OA	ABB + B	Y683
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	OA	ABB + B	Y684
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	0	OA	ABB + B	Y685
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	0	OA	ABB + B	Y686
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	OA	ABB + B	Y687
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	OA	ABB + B	Y688
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	OA	ABB + B	Y689
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	OA	ABB + B	Y690
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	OA	ABB + B	Y691
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	OA	ABB + B	Y692
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	OA	ABB + B	Y693
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	OA	ABB + B	Y694
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	Y695
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	Y696
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	Y697
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	Y698
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y699
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y700
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y701
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y702

Table B80 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y656	AAB	BO	0	0	2*(ABO + O)	0	0	A - AAB - AB + ABA	N9
Y657	AAB	BO	0	0	2*(ABO + O)	0	0	-AB - ABB - 2*ABO - B - BA + BAB - BO - 2*O + OAB + OB	N1
Y658	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N3
Y659	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y660	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
Y661	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y662	AAB	BO	0	0	2*(ABO + O)	0	0	AO - BA - OA	N16
Y663	AAB	BO	0	0	2*(ABO + O)	0	0	-AB - ABB - 2*ABO - B - BA + BAB - BO - 2*O + OAB + OB	N1
Y664	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + AO + BO + O - OA - OB	N7
Y665	AAB	BO	0	0	2*(-BA - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y666	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N17
Y667	AAB	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y668	0	BO	0	0	2*(ABO + O)	0	0	0	N8
Y669	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N3
Y670	0	BO	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y671	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N10
Y672	0	BO	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y673	A - AB + ABA	BO	0	0	2*(ABO + O)	0	0	0	N9
Y674	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	ABO + BA + BO + O - OB	N3
Y675	A - AB + ABA	BO	0	0	2*(-BA - BO + OB)	0	0	-A + AAB - ABA - ABB - B + BA + BAB + BO + OAB - OB	N1
Y676	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	AB + ABB + ABO + B - BAB + O	N10
Y677	A - AB + ABA	BO	0	0	2*(-AB - ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + B - BA - BAB - BO + OAB + OB	N1
Y678	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	ABO + O	N10
Y679	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
Y680	A - AB + ABA - AO + BA + OA	BO	0	0	2*(ABO + O)	0	0	0	N15
Y681	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N3
Y682	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
Y683	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N10
Y684	A - AB + ABA - AO + BA + OA	BO	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
Y685	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	ABO + O	N3
Y686	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
Y687	AAB	BO	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N3
Y688	AAB	BO	0	0	0	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y689	AAB	BO	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N7
Y690	AAB	BO	0	0	0	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
Y691	AAB	BO	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
Y692	AAB	BO	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y693	AAB	BO	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N17
Y694	AAB	BO	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y695	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + O	N3
Y696	AAB	OB	0	0	0	0	0	-AB - ABB - B + BA + BAB + OAB	N1
Y697	AAB	OB	0	0	0	0	0	ABO + AO + O - OA	N7
Y698	AAB	OB	0	0	0	0	0	-AB - ABB - B + BA + BAB + OAB	N1
Y699	AAB	BO	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + O	N10
Y700	AAB	BO	0	0	0	0	0	-BO + OAB + OB	N1
Y701	AAB	BO	0	0	0	0	0	ABO + AO + O - OA	N17
Y702	AAB	BO	0	0	0	0	0	-BO + OAB + OB	N1

Table B81 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y703
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y704
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y705
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y706
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y707
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y708
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y709
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	Y710
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y711
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y712
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y713
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y714
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y715
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	Y716
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y717
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y718
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y719
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y720
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y721
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y722
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y723
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y724
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y725
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y726
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y727
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y728
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y729
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y730
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y731
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y732
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y733
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y734
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y735
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	Y736
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y737
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y738
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y739
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y740
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y741
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y742
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y743
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y744
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y745
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y746
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y747
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	Y748

Table B82 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in three-way mechanism.

Serial	p_3	p_4	a_8	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
Y703	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
Y704	AAB	OB	0	0	0	0	0	OAB	N1
Y705	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
Y706	AAB	OB	0	0	0	0	0	OAB	N1
Y707	AAB	OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N17
Y708	AAB	OB	0	0	0	0	0	OAB	N1
Y709	AAB	OB	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
Y710	AAB	OB	0	0	0	0	0	OAB	N1
Y711	AAB	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y712	AAB	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y713	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	ABO + AO + BO + O - OA	N10
Y714	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
Y715	AAB	0	AB - BA - BAB - BO	0	0	0	0	ABO + AO + BO + O - OA	N17
Y716	AAB	0	AB - BA - BAB - BO	0	0	0	0	OAB + OB	N1
Y717	AAB	BO - OA	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y718	AAB	BO - OA	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y719	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	ABO + AO + O	N10
Y720	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
Y721	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	ABO + AO + O	N17
Y722	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	-BO + OA + OAB + OB	N1
Y723	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N10
Y724	AAB	OB	0	0	0	0	0	AB - BA - BAB - OA + OAB	N1
Y725	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N10
Y726	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
Y727	AAB	OB	AB - BA - BAB - OA	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
Y728	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y729	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	ABO + AO + BO + O - OA - OB	N10
Y730	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
Y731	AAB	OB	AB - BA - BAB - OA	0	0	0	0	ABO + AO + BO + O - OA - OB	N17
Y732	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y733	AAB	OB	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	0	0	0	0	ABB + ABO + AO + B + O	N10
Y734	AAB	OB	A - AAB + ABA - ABB - AO - B - BAB + BO - OA - OB	0	0	0	0	-A + AAB + AB - ABA + ABB + AO + B - BA - BO + OAB + OB	N1
Y735	AAB	OB	AB - BA - BAB - OA	0	0	0	0	ABB + ABO + AO + B + O	N2
Y736	AAB	OB	AB - BA - BAB - OA	0	0	0	0	OAB	N1
Y737	AAB	-AB + BA + BAB + BO	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N10
Y738	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y739	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB + ABO + AO - BA - BAB + O - OA	N17
Y740	AAB	-AB + BA + BAB + BO	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
Y741	AAB	OB	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N10
Y742	AAB	OB	0	0	0	0	0	OAB	N1
Y743	AAB	OB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N3
Y744	AAB	OB	0	0	0	0	0	OAB	N1
Y745	AAB	OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N17
Y746	AAB	OB	0	0	0	0	0	OAB	N1
Y747	AAB	OB	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N7
Y748	AAB	OB	0	0	0	0	0	OAB	N1

D.3 Supplementary C

Table C1 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	$3*d_1$	$2*g_5$	Serial
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(ABO + O)	0	R1
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R2
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R3
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R4
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R5
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R6
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R7
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R8
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R9
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA + ABO + BA + BO + O)	R10
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R11
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R12
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R13
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R14
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R15
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R16
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R17
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R18
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R19
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R20
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB + ABO + B - BAB + O)	R21
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	R22
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	R23
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R24
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R25
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R26
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R27
0	2*BO	2*(A + ABA)	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R28
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	R29
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R30
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R31
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R32
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R33
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R34
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R35
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R36
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R37
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R38
0	2*BO	2*(AB - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R39
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(ABO + O)	R40
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-AB - AO + BA + OA)	R41
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-AB - AO + BA + OA)	R42
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-AB - AO + BA + OA)	R43
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-AB - AO + BA + OA)	R44
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-AB - AO + BA + OA)	R45
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-A + AAB - ABA)	R46
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-A + AAB - ABA)	R47

Table C2 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under four-way exchanges.

Serial	$2*g_6$	$2*g_7$	$2*g_8$	w_5	Result
R1	0	0	0	0	N7
R2	2*(ABO + AO + BO + O - OA)	0	0	0	N5
R3	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N5
R4	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
R5	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
R6	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
R7	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
R8	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
R9	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
R10	0	0	0	0	N10
R11	2*(ABO + AO + BO + O - OA)	0	0	0	N10
R12	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
R13	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
R14	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
R15	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
R16	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
R17	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
R18	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R19	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R20	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
R21	0	0	0	0	N8
R22	0	0	0	ABO + AO + BO + O - OA	N6
R23	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
R24	0	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N8
R25	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
R26	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
R27	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
R28	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
R29	0	0	0	0	N10
R30	2*(ABO + AO + BO + O - OA)	0	0	0	N10
R31	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
R32	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
R33	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
R34	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
R35	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
R36	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
R37	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R38	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R39	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
R40	0	0	0	0	N10
R41	2*(AB + ABO + AO - BA + O - OA)	0	0	0	N10
R42	2*(AB - BA - BO + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
R43	2*(AB - BA - BO + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
R44	2*(-ABB - B + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
R45	2*(-ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
R46	2*(A - AAB + ABA + ABO + O)	0	0	0	N10
R47	2*(AB - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4

Table C3 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	$3*d_1$	$2*g_5$	Serial
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-A + AAB - ABA)	R48
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-A + AAB - ABA)	R49
0	2*(AB - BA)	0	AO	ABB + B	A + ABA	-AB + BA + BO	0	2*(-A + AAB - ABA)	R50
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(ABO + O)	0	R51
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R52
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R53
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R54
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R55
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R56
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R57
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R58
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-AO - BO + OA)	0	R59
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA + ABO + BA + BO + O)	R60
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R61
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R62
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R63
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R64
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	R65
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R66
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R67
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R68
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R69
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	R70
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB + ABO + B - BAB + O)	R71
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	R72
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	R73
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R74
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R75
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R76
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R77
2*(A - AAB + ABA)	2*BO	2*AAB	AO	ABB + B	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	R78
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(ABO + O)	R79
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R80
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R81
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R82
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R83
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-AO - BO + OA)	R84
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R85
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R86
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R87
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R88
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	ABB + B	A - AB + ABA + BA + BO	0	0	2*(-A + AAB + AB - ABA - BA - BO)	R89
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	0	R90
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	0	R91
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	0	R92
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	0	R93
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	ABB + B	AAB	A - AAB - AB + ABA + BA + BO	0	0	R94

Table C4 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (1) under four-way exchanges.

Serial	$2*g_6$	$2*g_7$	$2*g_8$	w_5	Result
R48	2*(AB - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R49	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R50	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
R51	0	0	0	0	N7
R52	2*(ABO + AO + BO + O - OA)	0	0	0	N5
R53	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N5
R54	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
R55	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
R56	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
R57	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
R58	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
R59	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
R60	0	0	0	0	N10
R61	2*(ABO + AO + BO + O - OA)	0	0	0	N10
R62	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
R63	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
R64	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
R65	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
R66	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
R67	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
R68	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R69	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R70	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
R71	0	0	0	0	N8
R72	0	0	0	ABO + AO + BO + O - OA	N6
R73	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
R74	0	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N8
R75	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
R76	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
R77	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
R78	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
R79	0	0	0	0	N10
R80	2*(ABO + AO + BO + O - OA)	0	0	0	N10
R81	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
R82	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
R83	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
R84	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
R85	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
R86	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
R87	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R88	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R89	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
R90	2*(ABO + O)	0	0	0	N10
R91	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
R92	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
R93	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
R94	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1

Table C5 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_4$	w_1	w_2	w_3	w_4	Serial
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S1
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S2
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S3
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S4
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S5
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S6
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S7
2*(AB - BA)	0	0	OA	-AB + BA + BAB	AAB	OB	S8
2*BAB	0	2*OA	0	0	AAB	OB	S9
2*BAB	0	2*OA	0	0	AAB	OB	S10
2*BAB	0	2*OA	0	0	AAB	OB	S11
2*BAB	0	2*OA	0	0	AAB	OB	S12
2*BAB	0	2*OA	0	0	AAB	OB	S13
2*BAB	0	2*OA	0	0	AAB	OB	S14
2*BAB	0	2*OA	0	0	AAB	OB	S15
2*BAB	0	2*OA	0	0	AAB	OB	S16
2*BAB	0	2*OA	0	0	AAB	OB	S17
2*BAB	0	2*OA	0	0	AAB	OB	S18
2*BAB	0	2*OA	0	0	AAB	OB	S19
2*BAB	0	2*OA	0	0	AAB	OB	S20
2*BAB	0	2*OA	0	0	AAB	OB	S21
2*BAB	0	2*OA	0	0	AAB	OB	S22
2*BAB	0	2*OA	0	0	AAB	OB	S23
2*BAB	0	2*OA	0	0	AAB	OB	S24
2*BAB	0	2*OA	0	0	AAB	OB	S25
2*BAB	0	2*OA	0	0	AAB	OB	S26
2*BAB	0	2*OA	0	0	AAB	OB	S27
2*BAB	0	2*OA	0	0	AAB	OB	S28
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S29
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S30
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S31
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S32

Table C6 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under four-way exchanges.

Serial	c_2	c_3	c_4	s_1	w_5	Result
S1	A - AAB - AB + ABA + BA	BO - OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
S2	A - AAB - AB + ABA + BA	BO - OB	0	0	OAB	N1
S3	A - AAB - AB + ABA + BA	AB + ABB + B - BA - BAB	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S4	A - AAB - AB + ABA + BA	AB + ABB + B - BA - BAB	0	0	OAB	N1
S5	AO - OA	BO - OB	0	0	ABO + AO + BO + O - OA - OB	N11
S6	AO - OA	BO - OB	0	0	OAB	N1
S7	AO - OA	AB + ABB + B - BA - BAB	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
S8	AO - OA	AB + ABB + B - BA - BAB	0	0	OAB	N1
S9	A - AAB + ABA - BAB	BO - OA - OB	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N6
S10	A - AAB + ABA - BAB	BO - OA - OB	0	0	AB - BA - BAB - OA + OAB	N1
S11	A - AAB + ABA - BAB	ABB + B	0	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
S12	A - AAB + ABA - BAB	ABB + B	0	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
S13	A - AAB + ABA - BAB	ABB + B	0	AB - BA - BAB - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S14	A - AAB + ABA - BAB	ABB + B	0	AB - BA - BAB - OA	OAB	N1
S15	AO	BO - OA - OB	0	A - AAB + ABA - AO - BAB	ABO + AO + BO + O - OA - OB	N6
S16	AO	BO - OA - OB	0	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
S17	AO	BO - OA - OB	0	AB - BA - BAB - OA	ABO + AO + BO + O - OA - OB	N11
S18	AO	BO - OA - OB	0	AB - BA - BAB - OA	OAB	N1
S19	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
S20	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
S21	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S22	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA	OAB	N1
S23	AO	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	ABO + AO + BO + O - OA - OB	N6
S24	AO	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
S25	AO	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OB	ABO + AO + BO + O - OA - OB	N11
S26	AO	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OB	OAB	N1
S27	AO	ABB + B	AB - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
S28	AO	ABB + B	AB - BA - BAB - OA	0	OAB	N1
S29	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
S30	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	0	OAB	N1
S31	A - AAB + ABA - BAB	ABB + B	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S32	A - AAB + ABA - BAB	ABB + B	0	0	OAB	N1

Table C7 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_4$	w_1	w_2	w_3	w_4	Serial
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S33
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S34
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S35
2*BAB	0	2*(AB - BA - BAB)	-AB + BA + BAB + OA	0	AAB	OB	S36
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S37
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S38
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S39
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S40
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S41
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S42
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S43
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S44
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S45
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S46
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S47
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S48
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S49
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S50
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S51
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S52
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S53
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S54
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S55
2*BAB	2*(-AO + OA)	2*AO	0	0	AAB	OB	S56
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S57
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S58
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S59
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S60
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S61
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S62
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S63
2*BAB	2*(-AO + OA)	2*(AB + AO - BA - BAB - OA)	-AB + BA + BAB + OA	0	AAB	OB	S64

Table C8 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (4) under four-way exchanges.

Serial	c_2	c_3	c_4	s_1	w_5	Result
S33	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	0	ABO + AO + BO + O - OA - OB	N11
S34	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	0	OAB	N1
S35	AB + AO - BA - BAB - OA	ABB + B	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
S36	AB + AO - BA - BAB - OA	ABB + B	0	0	OAB	N1
S37	A - AAB + ABA - BAB	BO - OA - OB	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N6
S38	A - AAB + ABA - BAB	BO - OA - OB	0	0	AB - BA - BAB - OA + OAB	N1
S39	A - AAB + ABA - BAB	ABB + B	0	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
S40	A - AAB + ABA - BAB	ABB + B	0	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
S41	A - AAB + ABA - BAB	ABB + B	0	AB - BA - BAB - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S42	A - AAB + ABA - BAB	ABB + B	0	AB - BA - BAB - OA	OAB	N1
S43	AO	BO - OA - OB	0	A - AAB + ABA - AO - BAB	ABO + AO + BO + O - OA - OB	N6
S44	AO	BO - OA - OB	0	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
S45	AO	BO - OA - OB	0	AB - BA - BAB - OA	ABO + AO + BO + O - OA - OB	N11
S46	AO	BO - OA - OB	0	AB - BA - BAB - OA	OAB	N1
S47	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
S48	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
S49	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S50	AO	ABB + B	A - AAB + ABA - AO - BAB	-A + AAB + AB - ABA + AO - BA - OA	OAB	N1
S51	AO	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	ABO + AO + BO + O - OA - OB	N6
S52	AO	ABB + B	-ABB - B + BO - OA - OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
S53	AO	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OB	ABO + AO + BO + O - OA - OB	N11
S54	AO	ABB + B	-ABB - B + BO - OA - OB	AB + ABB + B - BA - BAB - BO + OB	OAB	N1
S55	AO	ABB + B	AB - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
S56	AO	ABB + B	AB - BA - BAB - OA	0	OAB	N1
S57	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
S58	A - AAB + ABA - BAB	-AB + BA + BAB + BO - OB	0	0	OAB	N1
S59	A - AAB + ABA - BAB	ABB + B	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
S60	A - AAB + ABA - BAB	ABB + B	0	0	OAB	N1
S61	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	0	ABO + AO + BO + O - OA - OB	N11
S62	AB + AO - BA - BAB - OA	-AB + BA + BAB + BO - OB	0	0	OAB	N1
S63	AB + AO - BA - BAB - OA	ABB + B	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
S64	AB + AO - BA - BAB - OA	ABB + B	0	0	OAB	N1

Table C9 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under four-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	$2*u_1$	v_1	v_2
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	T1
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	T2
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	T3
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	T4
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	OA	T5
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO - BO + OA + OB	T6
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AO - BO + OA + OB	T7
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	T8
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	T9
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	T10
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	T11
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	A - AAB - AB + ABA - AO + BA + OA	T12
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AB - ABB - AO - B + BA + BAB + OA	T13
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(AO - OA)	0	-AB - ABB - AO - B + BA + BAB + OA	T14
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	T15
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	T16
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	T17
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	T18
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA + AO - BA - OA	0	T19
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA - BA - BO + OB	0	T20
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB + AB - ABA - BA - BO + OB	0	T21
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	T22
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	T23
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	T24
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	T25
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	AAB	0	T26
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB - ABA - ABB - B + BAB	0	T27
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(A - AAB - AB + ABA + BA)	-A + AAB - ABA - ABB - B + BAB	0	T28
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	T29
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	T30
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	T31
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-BO + OB)	0	0	T32
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	T33
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	T34
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	T35
2*(AB - BA)	OA	ABB + B	AAB	BO	2*(-AB - ABB - B + BA + BAB)	0	0	T36

Table C10 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (11) under four-way exchanges.

Serial	c_2	$2*g_6$	w_5	Result
T1	0	2*(ABO + O)	0	N9
T2	0	2*(-AO - BO + OB)	ABO + AO + BO + O - OB	N4
T3	0	2*(-AO - BO + OB)	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
T4	0	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + ABO + AO + B - BA - BAB + O	N11
T5	0	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
T6	0	0	ABO + O	N4
T7	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
T8	0	2*(ABO + O)	0	N10
T9	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
T10	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
T11	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
T12	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
T13	0	0	ABO + O	N11
T14	0	0	-AO - BO + OA + OAB + OB	N1
T15	0	2*(ABO + O)	0	N10
T16	0	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N2
T17	0	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
T18	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
T19	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
T20	0	0	ABO + O	N2
T21	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
T22	0	2*(ABO + O)	0	N5
T23	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N2
T24	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
T25	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N6
T26	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
T27	0	0	ABO + O	N6
T28	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
T29	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N2
T30	A - AAB - AB + ABA + BA + BO - OB	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
T31	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N4
T32	AO + BO - OA - OB	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
T33	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
T34	A - AAB + ABA + ABB + B - BAB	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
T35	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N11
T36	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1

Table C11 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (12) under four-way exchanges.

Serial	$2*g_1$	w_1	w_2	w_3	w_4	v_2	$2*g_6$	w_5	Result
U1	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(ABO + O)	0	N9
U2	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AO - BO + OB)	ABO + AO + BO + O - OB	N4
U3	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AO - BO + OB)	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
U4	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + ABO + AO + B - BA - BAB + O	N11
U5	2*(AB - BA)	AO	ABB + B	AAB	BO	AO	2*(-AB - ABB - AO - B + BA + BAB)	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
U6	2*(AB - BA)	AO	ABB + B	AAB	BO	-BO + OB	0	ABO + O	N4
U7	2*(AB - BA)	AO	ABB + B	AAB	BO	-BO + OB	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
U8	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(ABO + O)	0	N10
U9	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
U10	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
U11	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
U12	2*(AB - BA)	AO	ABB + B	AAB	BO	A - AAB - AB + ABA + BA	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
U13	2*(AB - BA)	AO	ABB + B	AAB	BO	-AB - ABB - B + BA + BAB	0	ABO + O	N11
U14	2*(AB - BA)	AO	ABB + B	AAB	BO	-AB - ABB - B + BA + BAB	0	-AO - BO + OA + OAB + OB	N1

Table C12 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	Serial
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V1
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V2
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V3
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V4
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V5
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V6
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V7
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	ABB + B	0	V8
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V9
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V10
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V11
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V12
2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V13
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V14
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V15
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V16
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V17
2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V18
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V19
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V20
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V21
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V22
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V23
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V24
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V25
0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	ABB + B	0	V26
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V27
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V28
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V29
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V30
0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + BA + OA	ABB + B	0	V31
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V32
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V33
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V34
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V35

Table C13 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

Serial	w_4	v_1	$2*g_6$	$2*g_8$	w_5	Result
V1	BO - OA	0	2*(ABO + O)	0	0	N5
V2	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N5
V3	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
V4	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V5	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
V6	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V7	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V8	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V9	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N5
V10	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V11	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V12	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V13	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V14	AO + BO - OA	0	2*(ABO + O)	0	0	N10
V15	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N2
V16	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
V17	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
V18	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
V19	BO - OA	0	2*(ABO + O)	0	0	N5
V20	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N5
V21	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
V22	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V23	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
V24	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V25	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V26	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V27	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N5
V28	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V29	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V30	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V31	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V32	AO + BO - OA	0	2*(ABO + O)	0	0	N10
V33	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N2
V34	AO + BO - OA	0	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
V35	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6

Table C14 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	Serial
0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	V36
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V37
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V38
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V39
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V40
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V41
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V42
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V43
2*(A - AAB + ABA)	0	2*AAB	2*OA	0	ABB + B	0	V44
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V45
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V46
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V47
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V48
2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V49
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V50
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V51
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V52
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V53
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V54
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V55
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V56
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V57
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V58
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V59
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V60
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V61
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V62
2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	OA	ABB + B	A - AB + ABA + BA	V63
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V64
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V65

Table C15 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

Serial	w_4	v_1	$2*g_6$	$2*g_8$	w_5	Result
V36	AO + BO - OA	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
V37	BO - OA	0	2*(ABO + O)	0	0	N5
V38	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N5
V39	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
V40	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V41	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
V42	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V43	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V44	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V45	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N5
V46	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V47	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V48	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V49	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V50	BO	AO - OA	2*(ABO + O)	0	0	N10
V51	BO	AO - OA	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N2
V52	BO	AO - OA	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
V53	BO	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
V54	BO	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
V55	BO	-BO + OB	0	0	ABO + O	N2
V56	BO	-BO + OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
V57	BO	A - AB + ABA + BA	2*(ABO + O)	0	0	N5
V58	BO	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V59	BO	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V60	BO	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V61	BO	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V62	BO	-AB - ABB - B + BA + BAB	0	0	ABO + O	N6
V63	BO	-AB - ABB - B + BA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
V64	BO - OA	0	2*(ABO + O)	0	0	N5
V65	BO - OA	0	2*(-BO + OA + OB)	2*(ABO + BO + O - OA - OB)	0	N5

Table C16 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	$2*g_4$	w_1	w_2	w_3	Serial
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V66
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V67
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V68
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V69
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V70
0	0	2*(A + ABA)	2*OA	0	ABB + B	0	V71
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V72
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V73
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V74
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V75
0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	A - AB + ABA + BA + OA	ABB + B	0	V76
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V77
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V78
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V79
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V80
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V81
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V82
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V83
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V84
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V85
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V86
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V87
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V88
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V89
0	0	2*(AB - BA)	0	OA	ABB + B	A - AB + ABA + BA	V90

Table C17 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (13) under four-way exchanges.

Serial	w_4	v_1	$2*g_6$	$2*g_8$	w_5	Result
V66	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
V67	BO - OA	0	2*(-BO + OA + OB)	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V68	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
V69	BO - OA	0	2*(-BO + OA + OB)	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V70	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V71	BO - OA	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V72	A - AB + ABA + BA + BO	0	2*(ABO + O)	0	0	N5
V73	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V74	A - AB + ABA + BA + BO	0	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V75	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V76	A - AB + ABA + BA + BO	0	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V77	BO	AO - OA	2*(ABO + O)	0	0	N10
V78	BO	AO - OA	2*(-AO - BO + OA + OB)	0	ABO + AO + BO + O - OA - OB	N2
V79	BO	AO - OA	2*(-AO - BO + OA + OB)	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
V80	BO	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
V81	BO	AO - OA	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
V82	BO	-BO + OB	0	0	ABO + O	N2
V83	BO	-BO + OB	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
V84	BO	A - AB + ABA + BA	2*(ABO + O)	0	0	N5
V85	BO	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
V86	BO	A - AB + ABA + BA	2*(-A + AB - ABA - BA - BO + OB)	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
V87	BO	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	A + ABA + ABB + ABO + B - BAB + O	N6
V88	BO	A - AB + ABA + BA	2*(-A - ABA - ABB - B + BAB)	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
V89	BO	-AB - ABB - B + BA + BAB	0	0	ABO + O	N6
V90	BO	-AB - ABB - B + BA + BAB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1

Table C18 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under four-way exchanges.

$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	$2*u_2$	v_3	Serial
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	W1
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	W2
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	W3
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	-A + AAB - ABA + BAB	AAB	OB	0	0	W4
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	W5
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	W6
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	W7
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	W8
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	BAB	W9
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-A + AAB - ABA - ABB - B + BAB	W10
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-A + AAB - ABA - ABB - B + BAB	W11
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	W12
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	W13
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	W14
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	W15
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - B + BA + BAB + BO - OB	W16
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - AO - B + BA + BAB + OA	W17
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(ABB + B - BAB)	-AB - ABB - AO - B + BA + BAB + OA	W18
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	W19
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	W20
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	W21
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-A + AAB - ABA)	0	W22
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W23
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W24
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W25
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W26
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W27
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W28
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W29
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W30
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W31
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W32
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W33
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W34
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W35
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB + BA + BO - OB)	0	W36
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	W37
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	W38
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	W39
0	2*(AB - BA)	AO	BAB	A + ABA	OB	2*(-AB - AO + BA + OA)	0	W40

Table C19 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (14) under four-way exchanges.

Serial	v_4	c_3	$2*g_5$	w_5	Result
W1	0	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N11
W2	0	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
W3	0	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N4
W4	0	A - AAB + ABA + ABB + B - BAB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
W5	0	0	2*(ABO + O)	0	N3
W6	0	0	2*(-AB - ABB - AO - B + BA + OA)	AB + ABB + ABO + AO + B - BA + O - OA	N2
W7	0	0	2*(-AB - ABB - AO - B + BA + OA)	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
W8	0	0	2*(-A + AAB - ABA - ABB - B)	A - AAB + ABA + ABB + ABO + B + O	N4
W9	0	0	2*(-A + AAB - ABA - ABB - B)	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
W10	0	0	0	ABO + O	N4
W11	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
W12	0	0	2*(ABO + O)	0	N10
W13	0	0	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N2
W14	0	0	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
W15	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
W16	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
W17	0	0	0	ABO + O	N2
W18	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
W19	0	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N11
W20	0	A - AAB - AB + ABA + BA + BO - OB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
W21	0	A - AAB + ABA + ABB + B - BAB	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N4
W22	0	A - AAB + ABA + ABB + B - BAB	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
W23	AB + ABB + B - BA - BAB - BO + OB	0	2*(ABO + O)	0	N10
W24	AB + ABB + B - BA - BAB - BO + OB	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
W25	AB + ABB + B - BA - BAB - BO + OB	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
W26	AB + ABB + B - BA - BAB - BO + OB	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
W27	AB + ABB + B - BA - BAB - BO + OB	0	2*(-A + AAB - ABA - ABB - B + BAB)	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
W28	-A + AAB + AB - ABA - BA - BO + OB	0	0	ABO + O	N11
W29	-A + AAB + AB - ABA - BA - BO + OB	0	0	-AO - BO + OA + OAB + OB	N1
W30	OB	0	2*(ABO + O)	0	N8
W31	OB	0	2*(-AO - BO + OA)	ABO + AO + BO + O - OA	N6
W32	OB	0	2*(-AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
W33	OB	0	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N11
W34	OB	0	2*(-A + AAB + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
W35	-AO - BO + OA + OB	0	0	ABO + O	N6
W36	-AO - BO + OA + OB	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
W37	0	AO + BO - OA - OB	0	ABO + AO + BO + O - OA - OB	N6
W38	0	AO + BO - OA - OB	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
W39	0	AB + ABB + AO + B - BA - BAB - OA	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N2
W40	0	AB + ABB + AO + B - BA - BAB - OA	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1

Table C20 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (15) under four-way exchanges.

Serial	$2*g_1$	$2*g_2$	w_1	w_2	w_3	w_4	v_3	$2*g_5$	w_5	Result
AA1	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	ABB + B	AAB	OB	0	0	ABO + O	N4
AA2	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	AO	ABB + B	AAB	OB	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
AA3	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(ABO + O)	0	N3
AA4	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-AB - ABB - AO - B + BA + OA)	AB + ABB + ABO + AO + B - BA + O - OA	N2
AA5	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-AB - ABB - AO - B + BA + OA)	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
AA6	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-A + AAB - ABA - ABB - B)	A - AAB + ABA + ABB + ABO + B + O	N4
AA7	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	ABB + B	2*(-A + AAB - ABA - ABB - B)	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
AA8	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-A + AAB - ABA	0	ABO + O	N4
AA9	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-A + AAB - ABA	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
AA10	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(ABO + O)	0	N10
AA11	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-AO - BO + OA + OB)	ABO + AO + BO + O - OA - OB	N2
AA12	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-AO - BO + OA + OB)	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
AA13	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
AA14	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB + BA + BO - OB	2*(-A + AAB + AB - ABA - BA - BO + OB)	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
AA15	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB - AO + BA + OA	0	ABO + O	N2
AA16	0	2*(AB - BA)	AO	ABB + B	A + ABA	OB	-AB - AO + BA + OA	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table C21 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	Serial
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB1
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB2
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB3
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB4
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB5
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB6
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB7
2*(A - AAB + ABA)	2*BO	2*AAB	AO	-A - ABA + BAB	0	AB8
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	AB9
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	AB10
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	AB11
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	AB12
2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	AB13
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB14
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB15
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB16
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB17
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB18
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB19
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB20
2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	AO	0	A + ABA - BAB	AB21
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	-A + AAB - ABA + BAB	AAB	AB22
2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	AO	-A + AAB - ABA + BAB	AAB	AB23
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB24
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB25
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB26
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB27
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB28
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	AB29

Table C22 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under four-way exchanges.

Serial	w_4	v_4	$2*g_5$	$2*g_7$	w_5	Result
AB1	0	0	2*(ABO + O)	0	0	N8
AB2	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB3	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB4	0	0	2*AAB	2*(-AAB + ABO + O)	0	N8
AB5	0	0	2*AAB	2*(-AAB - AO - BO + OA)	ABO + AO + BO + O - OA	N6
AB6	0	0	2*AAB	2*(-AAB - AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB7	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB8	0	0	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB9	0	0	2*(ABO + O)	0	0	N8
AB10	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB11	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB12	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB13	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB14	0	0	2*(ABO + O)	0	0	N8
AB15	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB16	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB17	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA + ABO - BAB + O)	0	N8
AB18	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	ABO + AO + BO + O - OA	N6
AB19	0	0	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB20	0	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB21	0	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB22	A - AAB - AB + ABA + BA + BO	0	0	0	ABO + O	N11
AB23	A - AAB - AB + ABA + BA + BO	0	0	0	-AO - BO + OA + OAB + OB	N1
AB24	0	0	2*(ABO + O)	0	0	N8
AB25	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB26	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB27	0	0	2*AAB	2*(-AAB + ABO + O)	0	N8
AB28	0	0	2*AAB	2*(-AAB - AO - BO + OA)	ABO + AO + BO + O - OA	N6
AB29	0	0	2*AAB	2*(-AAB - AO - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1

Table C23 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under four-way exchanges.

$2*g_1$	$2*g_2$	$2*g_3$	w_1	w_2	w_3	w_4	v_4	Serial
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	0	0	AB30
0	2*BO	2*(A + ABA)	AO	-A - ABA + BAB	0	0	0	AB31
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	AB32
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	AB33
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	AB34
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	AB35
0	2*BO	2*(AB - BA - BO)	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	AB36
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB37
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB38
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB39
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB40
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB41
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB42
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB43
0	2*BO	2*BAB	AO	0	A + ABA - BAB	0	0	AB44
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	AB45
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	AB46
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	AB47
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	AB48
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	ABB + B - BAB	AB49
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-A + AAB - ABA	AB50
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-A + AAB - ABA	AB51
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	AB52
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	AB53
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	AB54
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	AB55
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB + BA + BO	AB56
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB - AO + BA + OA	AB57
0	2*(AB - BA)	0	AO	BAB	A + ABA	-AB + BA + BO	-AB - AO + BA + OA	AB58

Table C24 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in situation (16) under four-way exchanges.

Serial	$2*g_5$	$2*g_7$	w_5	Result
AB30	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB31	2*AAB	2*(-A + AB - ABA - BA - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB32	2*(ABO + O)	0	0	N8
AB33	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB34	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB35	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB36	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB37	2*(ABO + O)	0	0	N8
AB38	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB39	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB40	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA + ABO - BAB + O)	0	N8
AB41	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	ABO + AO + BO + O - OA	N6
AB42	2*(-A + AAB - ABA + BAB)	2*(A - AAB + ABA - AO - BAB - BO + OA)	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB43	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB44	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB - BO)	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB45	2*(ABO + O)	0	0	N10
AB46	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
AB47	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
AB48	2*(-A + AAB - ABA - ABB - B + BAB)	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
AB49	2*(-A + AAB - ABA - ABB - B + BAB)	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
AB50	0	0	ABO + O	N11
AB51	0	0	-AO - BO + OA + OAB + OB	N1
AB52	2*(ABO + O)	0	0	N8
AB53	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
AB54	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
AB55	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
AB56	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
AB57	0	0	ABO + O	N6
AB58	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1

Table C25 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	Serial
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(ABO + O)	0	X1
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X2
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X3
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X4
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X5
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X6
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X7
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X8
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	0	X9
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA + ABO + BA + BO + O)	X10
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	X11
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	X12
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	X13
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	X14
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*(A - AB + ABA - AO + BA + OA)	X15
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	X16
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	X17
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	X18
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	X19
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	2*AAB	X20
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB + ABO + B - BAB + O)	X21
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	X22
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	X23
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	X24
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	X25
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	X26
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	X27
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	2*AAB	X28
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*(ABO + O)	X29
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*(-AO - BO + OA)	X30
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*(-AO - BO + OA)	X31
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*AAB	X32
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*AAB	X33
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*AAB	X34
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*AAB	X35
BA	0	2*BO	2*(A + ABA)	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	2*AAB	X36
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	X37
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X38
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X39
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X40

Table C26 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X1	0	0	0	0	N7
X2	2*(ABO + AO + BO + O - OA)	0	0	0	N5
X3	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N5
X4	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X5	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X6	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X7	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X8	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X9	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X10	0	0	0	0	N10
X11	2*(ABO + AO + BO + O - OA)	0	0	0	N10
X12	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
X13	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X14	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X15	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X16	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
X17	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X18	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X19	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X20	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X21	0	0	0	0	N8
X22	0	0	0	ABO + AO + BO + O - OA	N6
X23	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X24	0	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N8
X25	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
X26	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X27	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X28	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X29	0	0	0	0	N8
X30	0	0	0	ABO + AO + BO + O - OA	N6
X31	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X32	0	2*(-AAB + ABO + O)	0	0	N8
X33	0	2*(-AAB - AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
X34	0	2*(-AAB - AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X35	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X36	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X37	0	0	0	0	N10
X38	2*(ABO + AO + BO + O - OA)	0	0	0	N10
X39	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
X40	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1

Table C27 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	Serial
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X41
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X42
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X43
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X44
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X45
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X46
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X47
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	X48
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X49
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	2*(-AO - BO + OA)	X50
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X51
BA	0	2*BO	2*(AB - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	X52
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(ABO + O)	X53
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-AO - BO + OA)	X54
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-AO - BO + OA)	X55
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-A + AAB - ABA + BAB)	X56
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-A + AAB - ABA + BAB)	X57
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-A + AAB - ABA + BAB)	X58
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-A + AAB - ABA + BAB)	X59
BA	0	2*BO	2*BAB	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	2*(-A + AAB - ABA + BAB)	X60
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X61
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X62
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X63
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X64
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X65
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X66
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X67
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	0	X68
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	0	0	0	X69
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	0	0	0	X70
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X71
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X72
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X73
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X74
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X75
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X76
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X77
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	BO - OA	0	0	0	0	X78
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	0	0	0	X79
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	BO - OA - OB	0	0	0	X80

Table C28 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X41	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X42	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X43	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
X44	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X45	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X46	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X47	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X48	0	0	0	0	N8
X49	0	0	0	ABO + AO + BO + O - OA	N6
X50	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X51	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X52	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X53	0	0	0	0	N8
X54	0	0	0	ABO + AO + BO + O - OA	N6
X55	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X56	0	2*(A - AAB + ABA + ABO - BAB + O)	0	0	N8
X57	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
X58	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X59	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X60	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X61	2*(ABO + O)	0	0	0	N5
X62	2*OB	0	2*(ABO + O - OB)	0	N5
X63	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X64	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X65	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X66	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X67	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X68	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X69	0	0	0	ABO + O	N6
X70	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X71	2*(ABO + O)	0	0	0	N5
X72	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N5
X73	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X74	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X75	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X76	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X77	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X78	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X79	0	0	2*(ABO + O)	0	N5
X80	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N2

Table C29 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	X81
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	X82
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	X83
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	X84
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	ABB + B	0	OB	X85
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	X86
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	BO - OA	X87
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X88
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X89
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X90
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X91
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X92
BA	0	2*(-AO + OA)	2*(A + ABA)	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	OB	X93
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	X94
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	X95
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	X96
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	X97
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	A - AB + ABA + BA + BO	X98
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	OB	X99
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	OB	X100
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	A - AB + ABA + BA + BO	X101
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	A - AB + ABA + BA + BO	X102
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	X103
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	X104
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	X105
BA	0	2*(-AO + OA)	2*(A + ABA)	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	OB	X106
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	X107
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	X108
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	X109
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	X110
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	AO + BO - OA	X111
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	OB	X112
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	OB	X113
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	X114
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	AO + BO - OA	X115
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	X116
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	X117
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	X118
BA	0	2*(-AO + OA)	2*(AB + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	A - AB + ABA - AO + BA + OA	OB	X119
BA	0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	A + ABA - BAB	0	X120

Table C30 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X81	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X82	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N6
X83	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X84	-A + AB - ABA - BA - OA	0	0	0	0	0	0	ABO + O	N2
X85	-A + AB - ABA - BA - OA	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X86	0	0	0	0	0	0	0	ABO + O	N6
X87	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X88	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X89	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X90	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X91	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X92	-A + AB - ABA - BA - OA	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X93	-A + AB - ABA - BA - OA	0	0	0	0	0	0	AAB + OAB	N1
X94	0	0	0	0	2*(ABO + O)	0	0	0	N5
X95	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X96	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X97	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X98	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X99	0	0	0	0	0	0	0	ABO + O	N2
X100	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X101	0	0	0	0	0	0	0	ABO + O	N6
X102	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X103	0	0	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N6
X104	0	0	0	0	0	0	0	AAB + OAB	N1
X105	0	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X106	0	0	0	0	0	0	0	AAB + OAB	N1
X107	0	0	0	0	2*(ABO + O)	0	0	0	N10
X108	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X109	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X110	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X111	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X112	0	0	0	0	0	0	0	ABO + O	N2
X113	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X114	0	0	0	0	0	0	0	ABO + O	N6
X115	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X116	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X117	0	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X118	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N2
X119	0	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X120	0	0	0	0	0	0	0	ABO + O	N6

Table C31 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	0	2*(-AO + OA)	2*BAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	A + ABA - BAB	0	X121
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	BO - OA	X122
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	BO - OA	X123
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X124
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X125
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X126
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X127
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X128
BA	0	2*(-AO + OA)	2*BAB	2*AO	0	0	0	0	0	0	0	0	A + ABA - BAB	OB	X129
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	-AB + BA + BAB + BO	X130
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	-AB + BA + BAB + BO	X131
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	X132
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	X133
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	X134
BA	0	2*(-AO + OA)	2*BAB	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	X135
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X136
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X137
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X138
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X139
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X140
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X141
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X142
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X143
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X144
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X145
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	A + ABA	-AB + BA + BO	X146
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	X147
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	X148
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	X149
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	X150
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	ABB + B	AO	0	A + ABA	OB	X151
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-A + AAB - ABA	AO	A - AAB + ABA + ABB + B	A + ABA	OB	X152
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-A + AAB - ABA	AO	A - AAB + ABA + ABB + B	A + ABA	OB	X153
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	X154
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	X155
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	X156
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	X157
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB + BA + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	OB	X158
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB - AO + BA + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	X159
BA	0	2*(AB - BA)	0	0	0	0	0	0	0	-AB - AO + BA + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	OB	X160

Table C32 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X121	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X122	0	0	0	0	0	0	0	ABO + O	N6
X123	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X124	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X125	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X126	-ABB - B + BO - OA - OB	0	0	0	0	0	0	ABB + ABO + B + O	N6
X127	-ABB - B + BO - OA - OB	0	0	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
X128	AB - BA - BAB - OA	0	0	0	0	0	0	ABB + ABO + B + O	N2
X129	AB - BA - BAB - OA	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X130	0	0	0	0	0	0	0	ABO + O	N6
X131	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X132	0	0	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N6
X133	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X134	0	0	0	0	0	0	0	ABB + ABO + B + O	N2
X135	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X136	0	0	0	2*(ABO + O)	0	0	0	0	N10
X137	0	0	0	2*(-AB - AO + BA + OA)	2*(AB + ABO + AO - BA + O - OA)	0	0	0	N10
X138	0	0	0	2*(-AB - AO + BA + OA)	2*(AB - BA - BO + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X139	0	0	0	2*(-AB - AO + BA + OA)	2*(AB - BA - BO + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X140	0	0	0	2*(-AB - AO + BA + OA)	2*(-ABB - B + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X141	0	0	0	2*(-AB - AO + BA + OA)	2*(-ABB - B + BAB)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X142	0	0	0	2*(-A + AAB - ABA)	2*(A - AAB + ABA + ABO + O)	0	0	0	N10
X143	0	0	0	2*(-A + AAB - ABA)	2*(AB - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X144	0	0	0	2*(-A + AAB - ABA)	2*(AB - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X145	0	0	0	2*(-A + AAB - ABA)	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X146	0	0	0	2*(-A + AAB - ABA)	2*(-ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X147	0	0	0	2*(ABO + O)	0	0	0	0	N3
X148	0	0	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	AB + ABB + ABO + AO + B - BA + O - OA	N2
X149	0	0	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
X150	0	0	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB + ABA + ABB + ABO + B + O	N4
X151	0	0	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
X152	0	0	0	0	0	0	0	ABO + O	N4
X153	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X154	0	0	0	2*(ABO + O)	0	0	0	0	N10
X155	0	0	0	2*(-AO - BO + OA + OB)	0	0	0	ABO + AO + BO + O - OA - OB	N2
X156	0	0	0	2*(-AO - BO + OA + OB)	0	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X157	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X158	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X159	0	0	0	0	0	0	0	ABO + O	N2
X160	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1

Table C33 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	Serial
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	X161
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	X162
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	X163
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	X164
BA	0	2*(AB - BA)	0	0	0	0	0	ABB + B - BAB	0	0	AO	BAB	A + ABA	X165
BA	0	2*(AB - BA)	0	0	0	0	0	-A + AAB - ABA	0	0	AO	BAB	A + ABA	X166
BA	0	2*(AB - BA)	0	0	0	0	0	-A + AAB - ABA	0	0	AO	BAB	A + ABA	X167
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	X168
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	X169
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	X170
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	X171
BA	0	2*(AB - BA)	0	0	0	0	0	-AB + BA + BO	0	0	AO	BAB	A + ABA	X172
BA	0	2*(AB - BA)	0	0	0	0	0	-AB - AO + BA + OA	0	0	AO	BAB	A + ABA	X173
BA	0	2*(AB - BA)	0	0	0	0	0	-AB - AO + BA + OA	0	0	AO	BAB	A + ABA	X174
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	X175
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	X176
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	X177
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	X178
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	BAB	AO	0	A + ABA	X179
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-A + AAB - ABA - ABB - B + BAB	AO	A - AAB + ABA + ABB + B	A + ABA	X180
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-A + AAB - ABA - ABB - B + BAB	AO	A - AAB + ABA + ABB + B	A + ABA	X181
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	X182
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	X183
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	X184
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	X185
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - B + BA + BAB + BO - OB	AO	AB + ABB + B - BA - BO + OB	A + ABA	X186
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	X187
BA	0	2*(AB - BA)	0	0	0	2*(ABB + B - BAB)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	AO	AB + ABB + AO + B - BA - OA	A + ABA	X188
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	X189
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	X190
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	X191
BA	0	2*(AB - BA)	0	0	0	2*(-A + AAB - ABA)	0	0	0	0	AO	BAB	A + ABA	X192
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	X193
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	X194
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	X195
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	X196
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	AB + ABB + B - BA - BAB - BO + OB	0	0	AO	BAB	A + ABA	X197
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	AO	BAB	A + ABA	X198
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	AO	BAB	A + ABA	X199
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	X200

Table C34 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X161	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(ABO + O)	0	0	0	0	N10
X162	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X163	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X164	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X165	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X166	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N11
X167	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X168	0	0	0	0	2*(ABO + O)	0	0	0	0	N8
X169	0	0	0	0	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6
X170	0	0	0	0	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X171	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X172	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X173	AO + BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X174	AO + BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X175	OB	0	0	0	2*(ABO + O)	0	0	0	0	N3
X176	OB	0	0	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	AB + ABB + ABO + AO + B - BA + O - OA	N2
X177	OB	0	0	0	2*(-AB - ABB - AO - B + BA + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA + BAB - OA + OAB	N1
X178	OB	0	0	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB + ABA + ABB + ABO + B + O	N4
X179	OB	0	0	0	2*(-A + AAB - ABA - ABB - B)	0	0	0	A - AAB - AB + ABA - AO + BA + BAB + OA + OAB	N1
X180	OB	0	0	0	0	0	0	0	ABO + O	N4
X181	OB	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X182	OB	0	0	0	2*(ABO + O)	0	0	0	0	N10
X183	OB	0	0	0	2*(-AO - BO + OA + OB)	0	0	0	ABO + AO + BO + O - OA - OB	N2
X184	OB	0	0	0	2*(-AO - BO + OA + OB)	0	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X185	OB	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X186	OB	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X187	OB	0	0	0	0	0	0	0	ABO + O	N2
X188	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X189	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N11
X190	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
X191	OB	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N4
X192	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
X193	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(ABO + O)	0	0	0	0	N10
X194	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X195	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X196	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X197	-AB - ABB - B + BA + BAB + BO	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X198	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N11
X199	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X200	0	0	0	0	2*(ABO + O)	0	0	0	0	N8

Table C35 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	c_4	$3*d_1$	Serial
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	0	0	0	X201
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	0	0	0	X202
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	0	0	0	X203
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	OB	0	0	AO	BAB	A + ABA	0	0	0	0	X204
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	0	0	AO	BAB	A + ABA	AO + BO - OA	0	0	0	X205
BA	0	2*(AB - BA)	0	0	0	2*(-AB + BA + BO - OB)	0	-AO - BO + OA + OB	0	0	AO	BAB	A + ABA	AO + BO - OA	0	0	0	X206
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	0	0	0	X207
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	0	0	0	X208
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	0	0	0	X209
BA	0	2*(AB - BA)	0	0	0	2*(-AB - AO + BA + OA)	0	0	0	0	AO	BAB	A + ABA	OB	0	0	0	X210
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(ABO + O)	X211
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X212
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X213
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X214
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X215
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X216
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X217
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X218
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-AO - BO + OA)	X219
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X220
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X221
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X222
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X223
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X224
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X225
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X226
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X227
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X228
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X229
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A + AB - ABA - BA - BO)	X230
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X231
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X232
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X233
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X234
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X235
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X236
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X237
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	ABB + B	0	0	0	0	3*(-A - ABA - ABB - B + BAB)	X238
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X239
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X240

Table C36 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X201	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6
X202	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X203	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X204	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X205	0	0	0	0	ABO + O	N6
X206	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X207	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X208	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X209	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N2
X210	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X211	0	0	0	0	0	N7
X212	0	2*(ABO + AO + BO + O - OA)	0	0	0	N5
X213	0	2*OB	0	2*(ABO + AO + BO + O - OA - OB)	0	N5
X214	0	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X215	0	2*OB	0	2*(-A + AB - ABA + AO - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X216	0	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X217	0	2*OB	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X218	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X219	0	2*(-A - ABA - ABB + AO - B + BAB + BO - OA)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X220	2*(A - AB + ABA + ABO + BA + BO + O)	0	0	0	0	N10
X221	2*(A - AB + ABA - AO + BA + OA)	2*(ABO + AO + BO + O - OA)	0	0	0	N10
X222	2*(A - AB + ABA - AO + BA + OA)	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
X223	2*(A - AB + ABA - AO + BA + OA)	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X224	2*(A - AB + ABA - AO + BA + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X225	2*(A - AB + ABA - AO + BA + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X226	2*AAB	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
X227	2*AAB	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X228	2*AAB	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X229	2*AAB	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X230	2*AAB	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X231	2*(A + ABA + ABB + ABO + B - BAB + O)	0	0	0	0	N8
X232	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6
X233	2*(A + ABA + ABB - AO + B - BAB - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X234	2*AAB	0	2*(A - AAB + ABA + ABB + ABO + B - BAB + O)	0	0	N8
X235	2*AAB	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
X236	2*AAB	0	2*(A - AAB + ABA + ABB - AO + B - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X237	2*AAB	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X238	2*AAB	0	2*(AB + ABB + B - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X239	2*(ABO + O)	0	0	0	0	N8
X240	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6

Table C37 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	a_8	c_4	$3*d_1$	Serial
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X241
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X242
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X243
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X244
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X245
BA	2*(A - AAB + ABA)	2*BO	2*AAB	0	0	0	0	0	0	0	AO	-A - ABA + BAB	0	0	0	0	0	X246
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X247
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X248
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X249
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X250
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X251
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X252
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X253
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X254
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X255
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X256
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA + BA + BO	0	0	0	0	X257
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	X258
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	X259
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	X260
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	X261
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	0	0	0	0	AO	-AB + BA + BAB + BO	A - AB + ABA + BA + BO	0	0	0	0	X262
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X263
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X264
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X265
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X266
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X267
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X268
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X269
BA	2*(A - AAB + ABA)	2*BO	2*(-A + AAB - ABA + BAB)	0	0	0	0	0	0	0	AO	0	A + ABA - BAB	0	0	0	0	X270
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X271
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X272
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X273
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X274
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X275
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X276
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X277
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	ABB + B	0	0	0	0	0	X278
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	0	0	X279
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	0	0	0	0	0	X280

Table C38 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X241	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X242	2*AAB	0	2*(-AAB + ABO + O)	0	0	N8
X243	2*AAB	0	2*(-AAB - AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
X244	2*AAB	0	2*(-AAB - AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X245	2*AAB	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X246	2*AAB	0	2*(-A + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X247	2*(ABO + O)	0	0	0	0	N10
X248	2*(-AO - BO + OA)	2*(ABO + AO + BO + O - OA)	0	0	0	N10
X249	2*(-AO - BO + OA)	2*OB	0	0	ABO + AO + BO + O - OA - OB	N2
X250	2*(-AO - BO + OA)	2*OB	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X251	2*(-AO - BO + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X252	2*(-AO - BO + OA)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X253	2*(-A + AAB + AB - ABA - BA - BO)	2*(A - AAB - AB + ABA + ABO + BA + BO + O)	0	0	0	N10
X254	2*(-A + AAB + AB - ABA - BA - BO)	2*OB	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X255	2*(-A + AAB + AB - ABA - BA - BO)	2*OB	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X256	2*(-A + AAB + AB - ABA - BA - BO)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X257	2*(-A + AAB + AB - ABA - BA - BO)	2*(-AB - ABB - B + BA + BAB + BO)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X258	2*(ABO + O)	0	0	0	0	N8
X259	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6
X260	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X261	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X262	2*(-A + AAB + AB - ABA - BA - BO)	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X263	2*(ABO + O)	0	0	0	0	N8
X264	2*(-AO - BO + OA)	0	0	0	ABO + AO + BO + O - OA	N6
X265	2*(-AO - BO + OA)	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X266	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA + ABO - BAB + O)	0	0	N8
X267	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	ABO + AO + BO + O - OA	N6
X268	2*(-A + AAB - ABA + BAB)	0	2*(A - AAB + ABA - AO - BAB - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X269	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X270	2*(-A + AAB - ABA + BAB)	0	2*(AB - BA - BAB - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X271	0	2*(ABO + O)	0	0	0	N5
X272	0	2*OB	0	2*(ABO + O - OB)	0	N5
X273	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X274	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X275	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X276	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X277	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X278	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X279	0	0	0	0	ABO + O	N6
X280	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1

Table C39 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	Serial
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X281
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X282
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X283
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X284
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X285
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X286
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X287
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X288
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X289
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X290
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X291
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X292
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X293
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X294
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	ABB + B	0	X295
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X296
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X297
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X298
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X299
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X300
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X301
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X302
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*AO	0	0	0	0	0	0	0	-A - ABA + BAB	0	X303
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X304
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X305
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X306
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X307
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X308
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X309
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	0	X310
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X311
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X312
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X313
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X314
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X315
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*AAB	2*(-A + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	0	X316
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	X317
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	X318
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	X319
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	A - AB + ABA - AO + BA + OA	X320

Table C40 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X281	BO - OA	0	0	0	0	2*(ABO + O)	0	0	0	N5
X282	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N5
X283	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X284	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X285	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X286	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X287	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X288	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X289	OB	BO - OA - OB	0	0	0	0	0	2*(ABO + O)	0	N5
X290	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X291	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X292	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N6
X293	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X294	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	ABO + O	N2
X295	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X296	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X297	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X298	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X299	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X300	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X301	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X302	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X303	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	AAB + OAB	N1
X304	A - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X305	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X306	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X307	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X308	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X309	OB	0	0	0	0	0	0	0	ABO + O	N2
X310	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X311	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N6
X312	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X313	OB	0	0	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N6
X314	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X315	OB	0	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X316	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X317	AO + BO - OA	0	0	0	0	2*(ABO + O)	0	0	0	N10
X318	AO + BO - OA	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X319	AO + BO - OA	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X320	AO + BO - OA	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6

Table C41 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	X321
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	X322
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	ABB + B	X323
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X324
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X325
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X326
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X327
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X328
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB + AB - ABA + AO - BA - OA)	0	0	0	0	0	0	0	AO	-AB - AO + BA + BAB + OA	X329
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X330
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X331
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X332
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X333
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X334
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X335
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X336
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X337
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X338
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*AO	0	0	0	0	0	0	0	0	X339
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X340
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X341
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X342
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X343
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X344
BA	2*(A - AAB + ABA)	2*(-AO + OA)	2*(-A + AAB - ABA + BAB)	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X345
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X346
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X347
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X348
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X349
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X350
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X351
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	ABB + B	X352
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X353
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X354
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X355
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X356
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X357
BA	2*(A - AAB + ABA)	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	0	AO	-A + AAB - ABA + BAB	X358
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	X359
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	X360

Table C42 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X321	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X322	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	ABO + O	N2
X323	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X324	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X325	A - AB + ABA - AO + BA + OA	AO + BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X326	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X327	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X328	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N2
X329	A - AB + ABA - AO + BA + OA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X330	A + ABA - BAB	0	0	0	0	0	0	0	0	ABO + O	N6
X331	A + ABA - BAB	0	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X332	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X333	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X334	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X335	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X336	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	ABB + ABO + B + O	N6
X337	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
X338	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	ABB + ABO + B + O	N2
X339	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X340	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	ABO + O	N6
X341	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X342	A + ABA - BAB	OB	0	0	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N6
X343	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X344	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABB + ABO + B + O	N2
X345	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X346	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X347	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X348	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X349	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X350	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X351	AAB	OB	0	0	0	0	0	0	0	ABO + O	N4
X352	AAB	OB	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X353	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N11
X354	AAB	A - AAB - AB + ABA + BA + BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X355	AAB	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N11
X356	AAB	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
X357	AAB	OB	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N4
X358	AAB	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA - AO + BA + OA + OAB	N1
X359	AAB	BO	0	0	0	0	2*(ABO + O)	0	0	0	N9
X360	AAB	BO	0	0	0	0	2*(-AO - BO + OB)	0	0	ABO + AO + BO + O - OB	N4

Table C43 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	X361
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	X362
BA	2*(AB - BA)	0	0	0	0	0	0	0	AO	0	0	ABB + B	X363
BA	2*(AB - BA)	0	0	0	0	0	0	0	-BO + OB	0	AO + BO - OB	ABB + B	X364
BA	2*(AB - BA)	0	0	0	0	0	0	0	-BO + OB	0	AO + BO - OB	ABB + B	X365
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	X366
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	X367
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	X368
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	X369
BA	2*(AB - BA)	0	0	0	0	0	0	0	A - AAB - AB + ABA + BA	0	-A + AAB + AB - ABA + AO - BA	ABB + B	X370
BA	2*(AB - BA)	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB	0	AB + ABB + AO + B - BA - BAB	ABB + B	X371
BA	2*(AB - BA)	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB	0	AB + ABB + AO + B - BA - BAB	ABB + B	X372
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	ABB + B	X373
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	ABB + B	X374
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X375
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X376
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X377
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X378
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X379
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	AO	-AB + BA + BAB	X380
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X381
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X382
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X383
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X384
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X385
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X386
BA	2*BAB	2*BO	0	0	0	0	0	0	0	0	AO	0	X387
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X388
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X389
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X390
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X391
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X392
BA	2*BAB	2*(-AO + OA)	0	2*(AO + BO - OA)	0	0	0	0	0	0	-BO + OA	0	X393
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X394
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X395
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X396
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X397
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X398
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X399
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X400

Table C44 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X361	AAB	BO	0	0	0	0	2*(-AO - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
X362	AAB	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O	N11
X363	AAB	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
X364	AAB	BO	0	0	0	0	0	0	0	ABO + O	N4
X365	AAB	BO	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X366	AAB	BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X367	AAB	BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X368	AAB	BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X369	AAB	BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X370	AAB	BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X371	AAB	BO	0	0	0	0	0	0	0	ABO + O	N11
X372	AAB	BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X373	AAB	OB	0	0	0	0	0	0	0	ABO + O	N4
X374	AAB	OB	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X375	AAB	BO	0	0	0	0	0	0	0	ABO + O	N11
X376	AAB	BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X377	AAB	OB	0	0	0	0	0	0	0	ABO + BO + O - OB	N11
X378	AAB	OB	0	0	0	0	0	0	0	-AO + OA + OAB	N1
X379	AAB	OB	0	0	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N4
X380	AAB	OB	0	0	0	0	0	0	0	-AO + OA + OAB	N1
X381	AAB	0	A - AAB + ABA - BAB	0	0	0	0	2*(ABO + O)	0	0	N8
X382	AAB	0	A - AAB + ABA - BAB	0	0	0	0	2*(-AO - BO + OA)	0	ABO + AO + BO + O - OA	N6
X383	AAB	0	A - AAB + ABA - BAB	0	0	0	0	2*(-AO - BO + OA)	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X384	AAB	0	A - AAB + ABA - BAB	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA + ABO + BA + BO + O	N11
X385	AAB	0	A - AAB + ABA - BAB	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO)	0	A - AAB - AB + ABA - AO + BA + OA + OAB + OB	N1
X386	AAB	0	AB - BA - BAB - BO	0	0	0	0	0	0	ABO + O	N11
X387	AAB	0	AB - BA - BAB - BO	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X388	AAB	0	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X389	AAB	0	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X390	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	0	0	ABO + AO + BO + O - OA	N6
X391	AAB	0	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X392	AAB	0	AB - BA - BAB - BO	0	0	0	0	0	0	ABO + AO + BO + O - OA	N11
X393	AAB	0	AB - BA - BAB - BO	0	0	0	0	0	0	OAB + OB	N1
X394	AAB	BO - OA	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X395	AAB	BO - OA	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X396	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	ABO + AO + O	N6
X397	AAB	BO - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
X398	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	0	0	ABO + AO + O	N11
X399	AAB	BO - OA	AB - BA - BAB - OA	0	0	0	0	0	0	-BO + OA + OAB + OB	N1
X400	AAB	OB	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N6

Table C45 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X401
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X402
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X403
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X404
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X405
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X406
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X407
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X408
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X409
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X410
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X411
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X412
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X413
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X414
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X415
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X416
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X417
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X418
BA	2*BAB	2*(-AO + OA)	0	2*AO	0	0	0	0	0	0	0	0	X419
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X420
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X421
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X422
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X423
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X424
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X425
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X426
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X427
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X428
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X429
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X430
BA	2*BAB	2*(-AO + OA)	0	2*(AB + AO - BA - BAB - OA)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X431
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X432
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X433
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X434
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X435
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X436
BA	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	0	0	AO	0	X437
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X438
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X439
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X440

Table C46 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X401	AAB	OB	0	0	0	0	0	0	0	AB - BA - BAB - OA + OAB	N1
X402	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
X403	AAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X404	AAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X405	AAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	OAB	N1
X406	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X407	AAB	OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X408	AAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X409	AAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	OAB	N1
X410	AAB	OB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
X411	AAB	OB	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X412	AAB	OB	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X413	AAB	OB	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	OAB	N1
X414	AAB	OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X415	AAB	OB	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X416	AAB	OB	AB + ABB + B - BA - BAB - BO + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X417	AAB	OB	AB + ABB + B - BA - BAB - BO + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	OAB	N1
X418	AAB	OB	0	AB - BA - BAB - OA	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
X419	AAB	OB	0	AB - BA - BAB - OA	0	0	0	0	0	OAB	N1
X420	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X421	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X422	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	AB + ABO + AO - BA - BAB + O - OA	N11
X423	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X424	AAB	OB	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
X425	AAB	OB	0	0	0	0	0	0	0	OAB	N1
X426	AAB	OB	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X427	AAB	OB	0	0	0	0	0	0	0	OAB	N1
X428	AAB	OB	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X429	AAB	OB	0	0	0	0	0	0	0	OAB	N1
X430	AAB	OB	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
X431	AAB	OB	0	0	0	0	0	0	0	OAB	N1
X432	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	ABO + O	N11
X433	AAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X434	AAB	OB	0	0	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N11
X435	AAB	OB	0	0	0	0	0	0	0	-AB - AO + BA + BAB + OA + OAB	N1
X436	AAB	OB	0	0	0	0	0	0	0	ABB + ABO + B + O	N4
X437	AAB	OB	0	0	0	0	0	0	0	-AB - AO + BA + BAB + OA + OAB	N1
X438	0	0	0	0	0	0	2*(ABO + O)	0	0	0	N5
X439	0	0	0	0	0	0	2*OB	0	2*(ABO + O - OB)	0	N5
X440	0	0	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N2

Table C47 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X441
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X442
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X443
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X444
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X445
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	X446
BA	0	0	2*(A + ABA)	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	X447
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X448
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X449
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X450
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X451
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X452
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X453
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X454
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X455
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X456
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X457
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X458
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X459
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X460
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X461
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	ABB + B	X462
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X463
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X464
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X465
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X466
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X467
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X468
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X469
BA	0	0	2*(A + ABA)	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X470
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X471
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X472
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X473
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X474
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X475
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X476
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X477
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X478
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X479
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X480

Table C48 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X441	0	0	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X442	0	0	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X443	0	0	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X444	0	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X445	0	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X446	0	0	0	0	0	0	0	0	0	ABO + O	N6
X447	0	0	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X448	0	BO - OA	0	0	0	0	2*(ABO + O)	0	0	0	N5
X449	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N5
X450	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X451	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X452	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X453	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X454	0	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X455	0	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X456	0	OB	BO - OA - OB	0	0	0	0	0	2*(ABO + O)	0	N5
X457	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X458	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X459	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N6
X460	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X461	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	ABO + O	N2
X462	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X463	0	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X464	0	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X465	0	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X466	0	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X467	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X468	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X469	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X470	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	AAB + OAB	N1
X471	0	A - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X472	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X473	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X474	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X475	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X476	0	OB	0	0	0	0	0	0	0	ABO + O	N2
X477	0	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X478	0	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N6
X479	0	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X480	0	OB	0	0	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N6

Table C49 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X481
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X482
BA	0	0	2*(A + ABA)	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X483
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X484
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X485
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X486
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X487
BA	0	0	2*(AB - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X488
BA	0	0	2*(AB - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	X489
BA	0	0	2*(AB - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	X490
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X491
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X492
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X493
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X494
BA	0	0	2*(AB - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X495
BA	0	0	2*(AB - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	X496
BA	0	0	2*(AB - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	X497
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	ABB + B	X498
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	ABB + B	X499
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X500
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X501
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X502
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X503
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X504
BA	0	0	2*(AB - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X505
BA	0	0	2*BAB	2*BO	0	0	0	0	0	0	-BO + OA	0	X506
BA	0	0	2*BAB	2*BO	0	0	0	0	0	0	-BO + OA	0	X507
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X508
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X509
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X510
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X511
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X512
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X513
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X514
BA	0	0	2*BAB	2*OA	0	0	0	0	0	0	0	0	X515
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X516
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X517
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X518
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X519
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X520

Table C50 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X481	0	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X482	0	OB	0	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X483	0	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X484	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X485	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X486	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X487	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X488	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X489	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	ABO + O	N2
X490	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X491	0	BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X492	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X493	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X494	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X495	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X496	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	ABO + O	N6
X497	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X498	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	ABO + O	N2
X499	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X500	A - AB + ABA + BA	BO	0	0	0	0	0	0	0	ABO + O	N6
X501	A - AB + ABA + BA	BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X502	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	ABO + BO + O - OB	N6
X503	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
X504	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N2
X505	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
X506	A + ABA - BAB	0	0	0	0	0	0	0	0	ABO + O	N6
X507	A + ABA - BAB	0	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X508	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X509	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X510	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X511	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X512	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	ABB + ABO + B + O	N6
X513	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
X514	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	ABB + ABO + B + O	N2
X515	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X516	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	ABO + O	N6
X517	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X518	A + ABA - BAB	OB	0	0	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N6
X519	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X520	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABB + ABO + B + O	N2

Table C51 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	0	0	2*BAB	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X521
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X522
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X523
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X524
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X525
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X526
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X527
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X528
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	ABB + B	X529
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	X530
BA	2*(A - AAB + ABA)	0	2*AAB	2*BO	0	0	0	0	0	0	-BO + OA	-A - ABA + BAB	X531
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X532
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X533
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X534
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X535
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X536
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X537
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X538
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X539
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X540
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X541
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X542
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X543
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X544
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X545
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	ABB + B	X546
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X547
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X548
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X549
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X550
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X551
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X552
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X553
BA	2*(A - AAB + ABA)	0	2*AAB	2*OA	0	0	0	0	0	0	0	-A - ABA + BAB	X554
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X555
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X556
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X557
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X558
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X559
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X560

Table C52 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X521	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X522	0	0	0	0	0	0	2*(ABO + O)	0	0	0	N5
X523	0	0	0	0	0	0	2*OB	0	2*(ABO + O - OB)	0	N5
X524	0	0	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X525	0	0	0	0	0	0	2*OB	0	2*(-A + AB - ABA - BA - BO)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X526	0	0	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X527	0	0	0	0	0	0	2*OB	0	2*(-A - ABA - ABB - B + BAB - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X528	0	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X529	0	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X530	0	0	0	0	0	0	0	0	0	ABO + O	N6
X531	0	0	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X532	0	BO - OA	0	0	0	0	2*(ABO + O)	0	0	0	N5
X533	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(ABO + BO + O - OA - OB)	0	N5
X534	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X535	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A + AB - ABA - BA - OA)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X536	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	A + ABA + ABB + ABO + B - BAB + O	N6
X537	0	BO - OA	0	0	0	0	2*(-BO + OA + OB)	0	2*(-A - ABA - ABB - B + BAB + BO - OA - OB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X538	0	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X539	0	BO - OA	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X540	0	OB	BO - OA - OB	0	0	0	0	0	2*(ABO + O)	0	N5
X541	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	A - AB + ABA + ABO + BA + BO + O - OB	N2
X542	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X543	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	A + ABA + ABB + ABO + B - BAB + O	N6
X544	0	OB	BO - OA - OB	0	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X545	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	ABO + O	N2
X546	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X547	0	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X548	0	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X549	0	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X550	0	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X551	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X552	0	OB	-A - ABA - ABB - B + BAB + BO - OA - OB	0	0	0	0	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X553	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X554	0	OB	-A + AB - ABA - BA - OA	0	0	0	0	0	0	AAB + OAB	N1
X555	0	A - AB + ABA + BA + BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X556	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X557	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X558	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X559	0	A - AB + ABA + BA + BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X560	0	OB	0	0	0	0	0	0	0	ABO + O	N2

Table C53 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	Serial
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	ABB + B	X561
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X562
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X563
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X564
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X565
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X566
BA	2*(A - AAB + ABA)	0	2*AAB	2*(-A + AB - ABA - BA)	0	0	0	0	0	0	A - AB + ABA + BA + OA	-A - ABA + BAB	X567
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X568
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X569
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X570
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X571
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	AO - OA	0	0	0	OA	ABB + B	X572
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	X573
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-BO + OB	0	0	0	OA	ABB + B	X574
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X575
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X576
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X577
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X578
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	A - AB + ABA + BA	0	0	0	OA	ABB + B	X579
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	X580
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	-AB - ABB - B + BA + BAB	0	0	0	OA	ABB + B	X581
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	ABB + B	X582
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	ABB + B	X583
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X584
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X585
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X586
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X587
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X588
BA	2*(A - AAB + ABA)	0	2*(-A + AAB + AB - ABA - BA)	0	0	0	0	0	0	0	OA	-AB + BA + BAB	X589
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	0	0	0	0	0	0	-BO + OA	0	X590
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*BO	0	0	0	0	0	0	-BO + OA	0	X591
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X592
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X593
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X594
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X595
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X596
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X597
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X598
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*OA	0	0	0	0	0	0	0	0	X599
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	X600

Table C54 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X561	0	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X562	0	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	ABO + O	N6
X563	0	A - AB + ABA + BA + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X564	0	OB	0	0	0	0	0	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N6
X565	0	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X566	0	OB	0	0	0	0	0	0	0	A + ABA + ABB + ABO + B - BAB + O	N2
X567	0	OB	0	0	0	0	0	0	0	AAB + OAB	N1
X568	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X569	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X570	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X571	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X572	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X573	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	ABO + O	N2
X574	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X575	0	BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X576	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X577	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X578	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X579	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X580	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	ABO + O	N6
X581	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X582	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	ABO + O	N2
X583	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X584	A - AB + ABA + BA	BO	0	0	0	0	0	0	0	ABO + O	N6
X585	A - AB + ABA + BA	BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X586	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	ABO + BO + O - OB	N6
X587	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
X588	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	AB + ABB + ABO + B - BA - BAB + O	N2
X589	A - AB + ABA + BA	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA + OAB	N1
X590	A + ABA - BAB	0	0	0	0	0	0	0	0	ABO + O	N6
X591	A + ABA - BAB	0	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X592	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	ABO + O	N6
X593	A + ABA - BAB	BO - OA	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X594	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABO + BO + O - OA - OB	N6
X595	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - OA + OAB	N1
X596	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	ABB + ABO + B + O	N6
X597	A + ABA - BAB	OB	-ABB - B + BO - OA - OB	0	0	0	0	0	0	-A + AAB + AB - ABA + ABB + B - BA - BO + OAB + OB	N1
X598	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	ABB + ABO + B + O	N2
X599	A + ABA - BAB	OB	AB - BA - BAB - OA	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X600	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	ABO + O	N6

Table C55 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	Serial
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	X601
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	X602
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	X603
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	X604
BA	2*(A - AAB + ABA)	0	2*(-A + AAB - ABA + BAB)	2*(AB - BA - BAB)	0	0	0	0	0	0	X605
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	X606
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	X607
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	X608
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	X609
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	OA	0	X610
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO - BO + OA + OB	0	X611
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AO - BO + OA + OB	0	X612
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	X613
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	X614
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	X615
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	X616
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	A - AAB - AB + ABA - AO + BA + OA	0	X617
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	0	X618
BA	2*(AB - BA)	0	0	0	2*(AO - OA)	0	0	0	-AB - ABB - AO - B + BA + BAB + OA	0	X619
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	X620
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	X621
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	X622
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	X623
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA + AO - BA - OA	0	0	0	X624
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	0	X625
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB + AB - ABA - BA - BO + OB	0	0	0	X626
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	X627
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	X628
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	X629
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	X630
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	AAB	0	0	0	X631
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB - ABA - ABB - B + BAB	0	0	0	X632
BA	2*(AB - BA)	0	0	0	2*(A - AAB - AB + ABA + BA)	0	-A + AAB - ABA - ABB - B + BAB	0	0	0	X633
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	X634
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	X635
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	X636
BA	2*(AB - BA)	0	0	0	2*(-BO + OB)	0	0	0	0	0	X637
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	X638
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	X639
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	X640

Table C56 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	p_1	p_2	p_3	p_4	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X601	-AB + BA + BAB + OA	0	A + ABA - BAB	-AB + BA + BAB + BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X602	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	0	0	0	0	0	0	0	-AB + ABO + BA + BAB + BO + O - OB	N6
X603	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X604	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	0	0	0	0	0	0	0	ABB + ABO + B + O	N2
X605	-AB + BA + BAB + OA	0	A + ABA - BAB	OB	0	0	0	0	0	0	0	-A + AAB - ABA + BAB + OAB	N1
X606	0	ABB + B	AAB	BO	0	0	0	0	2*(ABO + O)	0	0	0	N9
X607	0	ABB + B	AAB	BO	0	0	0	0	2*(-AO - BO + OB)	0	0	ABO + AO + BO + O - OB	N4
X608	0	ABB + B	AAB	BO	0	0	0	0	2*(-AO - BO + OB)	0	0	-AB - ABB - B + BA + BAB + BO + OA + OAB - OB	N1
X609	0	ABB + B	AAB	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O	N11
X610	0	ABB + B	AAB	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB)	0	0	AB + ABB + B - BA - BAB - BO + OA + OAB + OB	N1
X611	AO + BO - OB	ABB + B	AAB	BO	0	0	0	0	0	0	0	ABO + O	N4
X612	AO + BO - OB	ABB + B	AAB	BO	0	0	0	0	0	0	0	-AB - ABB - AO - B + BA + BAB + OA + OAB	N1
X613	-A + AAB + AB - ABA + AO - BA	ABB + B	AAB	BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X614	-A + AAB + AB - ABA + AO - BA	ABB + B	AAB	BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N4
X615	-A + AAB + AB - ABA + AO - BA	ABB + B	AAB	BO	0	0	0	0	2*(-A + AAB + AB - ABA - BA - BO + OB)	0	0	A - AAB - 2*AB + ABA - ABB - AO - B + 2*BA + BAB + BO + OA + OAB - OB	N1
X616	-A + AAB + AB - ABA + AO - BA	ABB + B	AAB	BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N11
X617	-A + AAB + AB - ABA + AO - BA	ABB + B	AAB	BO	0	0	0	0	2*(-A + AAB - ABA - ABB - B + BAB)	0	0	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OAB + OB	N1
X618	AB + ABB + AO + B - BA - BAB	ABB + B	AAB	BO	0	0	0	0	0	0	0	ABO + O	N11
X619	AB + ABB + AO + B - BA - BAB	ABB + B	AAB	BO	0	0	0	0	0	0	0	-AO - BO + OA + OAB + OB	N1
X620	OA	ABB + B	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(ABO + O)	0	0	0	N10
X621	OA	ABB + B	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	ABO + AO + BO + O - OA - OB	N2
X622	OA	ABB + B	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AO - BO + OA + OB)	0	0	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA + OAB - OB	N1
X623	OA	ABB + B	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N6
X624	OA	ABB + B	A - AB + ABA - AO + BA + OA	BO	0	0	0	0	2*(-AB - ABB - AO - B + BA + BAB + OA)	0	0	-A + AAB + 2*AB - ABA + ABB + AO + B - 2*BA - BAB - BO - OA + OAB + OB	N1
X625	OA	ABB + B	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	ABO + O	N2
X626	OA	ABB + B	A - AB + ABA + BA + BO - OB	BO	0	0	0	0	0	0	0	-A + AAB - ABA - ABB - B + BAB + OAB	N1
X627	OA	ABB + B	0	BO	0	0	0	0	2*(ABO + O)	0	0	0	N5
X628	OA	ABB + B	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	A - AB + ABA + ABO + BA + BO + O - OB	N2
X629	OA	ABB + B	0	BO	0	0	0	0	2*(-A + AB - ABA - BA - BO + OB)	0	0	AAB - AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X630	OA	ABB + B	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	A + ABA + ABB + ABO + B - BAB + O	N6
X631	OA	ABB + B	0	BO	0	0	0	0	2*(-A - ABA - ABB - B + BAB)	0	0	AAB + AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X632	OA	ABB + B	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	ABO + O	N6
X633	OA	ABB + B	A + ABA + ABB + B - BAB	BO	0	0	0	0	0	0	0	-A + AAB + AB - ABA - BA - BO + OAB + OB	N1
X634	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N2
X635	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X636	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N4
X637	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB + BO + OAB - OB	N1
X638	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
X639	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X640	OA	ABB + B	AAB	BO	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N11

Table C57 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	2*(AB - BA)	0	0	0	2*(-AB - ABB - B + BA + BAB)	0	0	0	0	0	OA	ABB + B	AAB	BO	X641
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	AAB	OB	X642
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	AAB	OB	X643
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	AAB	OB	X644
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	ABB + B	AAB	OB	X645
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	BO	X646
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	BO	X647
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	BO	X648
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	BO	X649
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X650
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X651
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X652
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X653
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X654
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X655
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X656
BA	2*(AB - BA)	0	0	0	0	0	0	0	0	0	OA	-AB + BA + BAB	AAB	OB	X657
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X658
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X659
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X660
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X661
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X662
BA	2*BAB	0	0	2*BO	0	0	0	0	0	0	-BO + OA	0	AAB	0	X663
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X664
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X665
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X666
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X667
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X668
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	BO - OA	X669
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X670

Table C58 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X641	0	0	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X642	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + O	N2
X643	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB + OAB	N1
X644	0	0	0	0	0	0	0	ABO + AO + O - OA	N4
X645	0	0	0	0	0	0	0	-AB - ABB - B + BA + BAB + OAB	N1
X646	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + O	N6
X647	0	0	0	0	0	0	0	-BO + OAB + OB	N1
X648	0	0	0	0	0	0	0	ABO + AO + O - OA	N11
X649	0	0	0	0	0	0	0	-BO + OAB + OB	N1
X650	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
X651	0	0	0	0	0	0	0	OAB	N1
X652	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X653	0	0	0	0	0	0	0	OAB	N1
X654	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X655	0	0	0	0	0	0	0	OAB	N1
X656	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
X657	0	0	0	0	0	0	0	OAB	N1
X658	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X659	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X660	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	0	0	ABO + AO + BO + O - OA	N6
X661	A - AAB + ABA - AO - BAB - BO + OA	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB + OB	N1
X662	AB - BA - BAB - BO	0	0	0	0	0	0	ABO + AO + BO + O - OA	N11
X663	AB - BA - BAB - BO	0	0	0	0	0	0	OAB + OB	N1
X664	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X665	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X666	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	ABO + AO + O	N6
X667	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - BO + OAB + OB	N1
X668	AB - BA - BAB - OA	0	0	0	0	0	0	ABO + AO + O	N11
X669	AB - BA - BAB - OA	0	0	0	0	0	0	-BO + OA + OAB + OB	N1
X670	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + BO + O - OA - OB	N6

Table C59 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

b_0	$2*b_1$	$2*b_2$	$2*b_3$	$2*b_{21}$	$2*e_1$	$2*e_2$	$2*f_2$	$2*f_3$	$2*f_4$	$2*f_5$	p_1	p_2	p_3	p_4	Serial
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X671
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X672
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X673
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X674
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X675
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X676
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X677
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X678
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X679
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X680
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X681
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X682
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X683
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X684
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X685
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X686
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X687
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X688
BA	2*BAB	0	0	2*OA	0	0	0	0	0	0	0	0	AAB	OB	X689
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	-AB + BA + BAB + BO	X690
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	-AB + BA + BAB + BO	X691
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	-AB + BA + BAB + BO	X692
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	-AB + BA + BAB + BO	X693
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X694
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X695
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X696
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X697
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X698
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X699
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X700
BA	2*BAB	0	0	2*(AB - BA - BAB)	0	0	0	0	0	0	-AB + BA + BAB + OA	0	AAB	OB	X701

Table C60 The maximum number of paired patients from pairs of types $(O-A)$, $(O-B)$, $(O-AB)$, $(A-AB)$, $(B-AB)$, $(A-B)$ in four-way mechanism.

Serial	a_8	c_4	$3*d_1$	$2*b_4$	$2*b_5$	$2*b_6$	$2*b_7$	a_9	Result
X671	0	0	0	0	0	0	0	AB - BA - BAB - OA + OAB	N1
X672	-ABB - B + BO - OA - OB	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
X673	-ABB - B + BO - OA - OB	0	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X674	AB - BA - BAB - OA	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X675	AB - BA - BAB - OA	0	0	0	0	0	0	OAB	N1
X676	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X677	A - AAB + ABA - AO - BAB	0	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X678	AB - BA - BAB - OA	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X679	AB - BA - BAB - OA	0	0	0	0	0	0	OAB	N1
X680	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N6
X681	-A + AAB - ABA - ABB + AO - B + BAB + BO - OA - OB	A - AAB + ABA - AO - BAB	0	0	0	0	0	AB + ABB + B - BA - BAB - BO + OAB + OB	N1
X682	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X683	-A + AAB + AB - ABA + AO - BA - OA	A - AAB + ABA - AO - BAB	0	0	0	0	0	OAB	N1
X684	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N6
X685	A - AAB + ABA + ABB - AO + B - BAB - BO + OA + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	-A + AAB + AB - ABA + AO - BA - OA + OAB	N1
X686	AB + ABB + B - BA - BAB - BO + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X687	AB + ABB + B - BA - BAB - BO + OB	-ABB - B + BO - OA - OB	0	0	0	0	0	OAB	N1
X688	0	AB - BA - BAB - OA	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
X689	0	AB - BA - BAB - OA	0	0	0	0	0	OAB	N1
X690	0	0	0	0	0	0	0	A - AAB + ABA + ABO - BAB + O	N6
X691	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X692	0	0	0	0	0	0	0	AB + ABO + AO - BA - BAB + O - OA	N11
X693	0	0	0	0	0	0	0	AB - BA - BAB - BO + OAB + OB	N1
X694	0	0	0	0	0	0	0	A - AAB - AB + ABA + ABO + BA + BO + O - OB	N6
X695	0	0	0	0	0	0	0	OAB	N1
X696	0	0	0	0	0	0	0	A - AAB + ABA + ABB + ABO + B - BAB + O	N2
X697	0	0	0	0	0	0	0	OAB	N1
X698	0	0	0	0	0	0	0	ABO + AO + BO + O - OA - OB	N11
X699	0	0	0	0	0	0	0	OAB	N1
X700	0	0	0	0	0	0	0	AB + ABB + ABO + AO + B - BA - BAB + O - OA	N4
X701	0	0	0	0	0	0	0	OAB	N1

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