Enhanced Interference Management Techniques for Heterogeneous Cellular Networks

Obinna Samuel Oguejiofor

Submitted in accordance with the requirements for the degree of

Doctor of Philosophy

The University of Leeds

School of Electronic and Electrical Engineering

May 2017

Declaration

The candidate confirms that the work submitted is his own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and the other authors of this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others. Most materials contained in the chapters of this thesis have been previously published in research articles written by the author of this work (Obinna Samuel Oguejiofor), who appears as lead (first) author in all of them.

The research has been supervised and guided by Dr. Li X. Zhang and she appear as a co-author on these articles. All the materials included in this document is the author's entire intellectual ownership.

A) Details of the publications which have been used (e.g. titles, journals, dates, names of authors):

In Chapter 3:
"Global Optimization of Weighted Sum-Rate for Downlink Heterogeneous Cellular Networks", *23rd IEEE International Conference on Telecommunications (ICT), 2016*, Published. Co-authors: Li Zhang. (DOI: 10.1109/ICT.2016.7500345).

In Chapter 4:
"Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks", *European Wireless (EW) 2017*, Accepted. Co-authors: Li Zhang and Naveed Nawaz. (Paper ID: 1570347851).

"UE-Centric Clustering and Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks", *IEEE Access*, Under Review. Co-authors: Li Zhang.

In Chapter 5:
"Decentralized Resource Allocation for Heterogeneous Cellular Networks", *14th IEEE International Symposium on Wireless Communications System (ISWCS)*

2017, Accepted. Co-authors: Li Zhang and Mohanad Alhabo. (Paper ID: 1570355994).

"Distributed Resource Allocation for Two-Tier Heterogeneous Cellular Networks", *IEEE Transactions on Communications*, Under Review. Co-authors: Li Zhang.

In Chapter 6:
"Heuristic Coordinated Beamforming for Heterogeneous Cellular Network", *IEEE Vehicular Technology Conference (VTC) 2016*, Published. Co-authors: Li Zhang. (DOI: 10.1109/VTCSpring.2016.7504269).

B) Details of the work contained within these publications which are directly attributable to Obinna Samuel Oguejiofor:
With the exceptions detailed in Section C, the published work is entirely attributable to Obinna Samuel Oguejiofor: the literature review necessary to construct and originate the ideas behind the published manuscripts, the novel ideas presented in the papers, the design of transmit beamformers for both centralized and distributed implementations for different HetNet scenario and all the work necessary in the editing process of the manuscripts.

C) Details of the contributions of other authors to the work:
Dr. Li X. Zhang is the co-author for all the publications listed above. These publications have been written under her supervision, benefiting from excellent technical advice and editorial, patient guidance and valuable feedback.

Naveed Nawaz and Mohanad Alhabo performed proofreading to the final drafts of the papers to ensure the solidity of the papers.

This thesis is dedicated to God Almighty, my beloved wife; my darling son, parents, and siblings.

Acknowledgements

This thesis would not have been possible without the guidance and help of several individuals who contributed in one way or the other to see that I started and completed my Ph.D. study.

Foremost, my sincere gratitude goes to my supervisor Dr. Li X. Zhang for her persistent support during my Ph.D. study. Furthermore, for her several helpful technical advice, valuable feedbacks, and guidance. Also, for her rigorous reading of my works whenever I send any draft to her and for believing in my capabilities. I could not have imagined having a better supervisor for my Ph.D. study.

I wish to thank professor Jaafar Elmirghani for providing decent labs, offices and conducive atmosphere for learning and doing research in the institute of communication and power networks (ICaPNet). The Labs and Ph.D. offices in ICaPNet have a motivating ambiance which really helps during the course of my Ph.D. study. I am extremely grateful to have such wonderful colleagues at ICaPNet: Mohannad Alhabo, Oghorada Oghorada, Naveed Nawaz, Hayder Al-kashoash, Fadoua Hassen, Miao M. Liu, You You, and many others who make ICaPNet a place to be.

I wish to thank God Almighty for his grace and mercy towards me. Also, I wish to thank my parents, Mr. and Mrs. Edmond Okechukwu Oguejiofor for bringing me up in the fear of God, and for their unconditional support and prayers towards the success of my Ph.D. I am grateful to my beloved wife, Chizoba Victoria Oguejiofor, for her sacrifices, support and being a wonderful role model for our son, Chizitelu Daniel Oguejiofor.

Last but not least, I would like to acknowledge the financial support of the Tertiary Education Trust Fund (TETFund).

Abstract

Interference management is one of the most challenging problems facing wireless communication networks, especially for the cellular wireless communication system that is based on reuse-one deployment. This problem becomes even more noteworthy in the heterogeneous cellular networks (HetNets) where lower power nodes (LPNs) are deployed in the coverage area of the macro base station (MBS). The higher transmit power possessed by the MBS, together with the cell selection procedure employed in HetNet: where a user equipment (UE) may be served by a closer LPN (to enable cell splitting) even though the received power from the MBS could be higher, are some factors that cause interference in HetNet. In the 5th generation mobile networks (5G) when the number of deployed LPNs increases interference will be more serious.

This thesis proposes interference management techniques based on beamforming with different level of cooperation amongst base stations in HetNet. In this thesis, we first designed global beamforming vectors that will maximize the weighted sum-rate of HetNet while fulfilling some power and interference constraints. The interference constraint controls the allowable interference from the MBS to other UEs in the HetNet. The global beamforming vectors were achieved using the Branch and Bound technique which is a global optimization method used in solving non-convex optimization problems. The beamformers that maximize the weighted sum-rate of HetNet are designed jointly by all BSs in the HetNet, hence the implementation is done centrally.

Since each UE in HetNet has peculiar interference situation, we design a UE-centric clustering scheme, which is capable of determining the BSs in the HetNet that interferes each UE the most at a particular time. Afterward, these BSs coordinate interference with the serving BS of this UE and make resource allocation decisions together to

allocate beamforming directions and powers to each UE in the HetNet. This will spatially separate signals sent to each UE, thereby mitigating interference and improving the total data rate achievable in HetNet.

HetNet tends to be distributed, also X2-interface which is the backhaul link that connects BSs in the HetNet has a limited capacity which makes it incapable of withstanding huge burdens in its backhaul. We, therefore, design distributed beamforming directions using only local channel state information available at each transmitter. We also develop optimal power allocation scheme for each UE in each cell to maximize the sum-rate of each cell in the HetNet.

Abbreviations

B&B	Branch and Bound
BS	Base station
BW	Operating bandwidth
CA	Carrier aggregation
CB	Coordinated beamforming
CC	Carrier component
CDF	Cumulative distributed function
CDMA	Code division multiple access
CF	Carrier frequency
CoMP	Coordinated multi-point
CRE	Cell range expansion
CSG	Closed subscriber group
CSI	Channel state information
DoF	Degree of freedom
eICIC	Enhanced inter-cell interference coordination
eNB	Evolved nodeB
FAP	Femtocell access points
FDMA	Frequency division multiple access
FFR	Fractional frequency reuse
HetNet	Heterogeneous cellular network
HPN	High-power node
IC	Interference constraint
ICI	Inter-cell interference
ICIC	Inter-cell interference coordination
ISD	Inter-site distance

JT	Joint transmission
KKT	Karush-Kuhn-Tucker
LPN	Low power node
LTE	Long-term evolution
MatLab	Matrix Laboratory
mBS	Micro base station
MBS	Macrocellular base station
MCP	Multi-cell processing
MIMO	Multi-input multi-output
NP-hard	Non-Deterministic Polynomial-Time hard
OFDMA	Orthogonal frequency division multiple access
OPEX	Operational expenditures
PBS	Pico base station
PFR	Partial frequency reuse
PHY	Physical layer
PL	Penetration loss
PD	Positive definite
PSD	Positive semidefinite
QoE	Quality of experience
QoS	Quality of service
RA	Resource allocation
RAN	Radio Access Network
RN	Relay node
RHS	Right hand side
RRH	Remote radio head
RSRP	Reference signal receive power
SBS	Super base station

SCP	Single cell processing
SDMA	Space division multiple access
SE	Spectral efficiency
SFR	Soft frequency reuse
SINR	Signal to noise and interference ratio
SLNR	Signal to leakage and noise ratio
SNR	Signal to noise ratio
SOC	Second order cone
SOCP	Second-order cone programming
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
UE	User equipment
WLOG	Without loss of generality
ZFBF	Zero-Forcing Beamforming
3GPP	3rd generation partnership project
5G	5th generation mobile networks

Symbols

a	Scalar a.
$\mathbf{a}$	Vector $\mathbf{a}$.
$[\mathbf{a}]_i$	i th component of a vector $\mathbf{a}$.
$\mathbf{A}$	Matrix $\mathbf{A}$.
$\lvert a \rvert$	Magnitude of a.
$[\mathbf{a}]_+$	Obtained from $\mathbf{a}$ by setting negative entries to zero.
$\Re(a)$	Real part of scalar a.
$\Im(a)$	Imaginary part of scalar a.
$\lvert\lvert\mathbf{a}\rvert\rvert_p$	The L_p-norm $\lvert\lvert\mathbf{a}\rvert\rvert_p = (\sum_i \lvert a_i \rvert^p)^{\frac{1}{p}}$ of $\mathbf{a}$
$\mathbf{A}^T$	Transpose of $\mathbf{A}$.
$\mathbf{A}^H$	Complex conjugate transpose of $\mathbf{A}$.
$\mathbf{A}^{-1}$	The inverse of a square matrix $\mathbf{A}$.
$\mathrm{Tr}(\mathbf{A})$	Trace of $\mathbf{A}$.
$\mathbb{E}\{\cdot\}$	Expectation operator.
$\forall a$	The statement holds for all a (in the set that a belongs to).
$a \in \mathcal{S}$	a is a member of the set $\mathcal{S}$.
$a \notin \mathcal{S}$	a is not a member of the set $\mathcal{S}$.
$\{a \in \mathcal{S} : P\}$	The set of all members of $\mathcal{S}$ having a property P.
$\mathcal{S}1 \subseteq \mathcal{S}2$	$\mathcal{S}1$ is included in (is a subset of) $\mathcal{S}2$.
$\mathcal{S}1 \subset \mathcal{S}2$	$\mathcal{S}1$ is included in (is a proper set of) $\mathcal{S}2$.
$\mathbf{card}(\mathcal{S})$	The cardinality (i.e., number of members) of a set.
$f : \mathcal{S}1 \rightarrow \mathcal{S}2$	Function from $\mathcal{S}1$ to $\mathcal{S}2$.
$\mathbf{A} \geq \mathbf{0}$	Means $\mathbf{A}$ is a positive semi-definite matrix.
$\mathbf{a} > 0$	Means all elements of a are positive.
$a \sim \chi(\cdot)$	The random variable a has distribution $\chi(\cdot)$.

$\mathbf{0}$	The vector, whose components are only zeros.
$\mathbf{0}_N$	The N x N matrix of only zeros.
$\Re(a)$	Real part of a scalar a.
$\Im(a)$	Imaginary part of a scalar a.
$\mathbb{R}_+^N$	The set of nonnegative real-valued N x 1 vectors.
$\mathbb{R}_{++}^N$	The set of positive real-valued N x 1 vectors.
$\mathbb{C}^N$	The set of complex-valued N x 1 vectors.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background and Motivation

With the explosive growth of wireless subscribers over the past decade, the wireless industry is faced with an increasing demand for wireless coverage and larger data throughput. Today, the number of mobile subscribers has sky-rocketed to over 3.9 billion, and this trend has been projected to increase to 4.5 billion by 2018 as reported by mobile world congress [3]. This demand is driven by the popularity of powerful devices such as smartphones and tablets which are mobile devices connected with data services. Furthermore, mobile applications have become part and parcels of people's day-to-day life with constant demand on ideal access to social media and video contents. To provide services that will meet these requirements are the goals of mobile network operators. However, these requirements can only be met by mobile networks with adequate capacity and coverage. There are several options that can be considered on how to improve the capacity and coverage challenges [4] in order to meet traffic and data rate demands. These include:

1. **Improving the existing macrocellular networks**

   Improving the existing macro cellular network may be considered as one of the effective solutions for handling the overall capacity and coverage challenges [5]. This can be done by adding more spectrum to the existing sites and using an advanced sectorized antenna which adequately achieves better rates. However, radio frequency spectrum is arguably the most precious resource in wireless communications and is used for many things e.g., radio/television broadcasting, satellite services, and military application, just to mention a few. Spectrum is a scarce commodity and its licenses are very expensive. Therefore, this approach may not be a cost or energy efficient solution to the coverage and capacity challenge.

2. **Densification of the network base station**

   Increasing the number of *macrocellular base station* (MBS) sites in both urban and dense area is a good and popular approach used by network operators to solve the capacity and coverage challenges and also to reduce the site-to-site distance in the macro-network. However, the limitation with this option is the cost involved in setting up new MBS sites in all these areas and how to find the proper location for setting it up, knowing fully well that these cell sites are usually deployed in a regular pattern over an area.

3. **Heterogeneous cellular network** (HetNet)

   One of the optimal solutions in improving the capacity and coverage challenges would be complementing the MBS with small cells known as *low-power nodes* or base stations (LPNs). This would arguably be the most effective option because of the following reasons:

- The low-powered base stations are mostly deployed in an unplanned manner depending on areas where traffic is high. Examples include subways, train stations, supermarkets, airports etc.

- The MBS are usually inadequate to meet the mobile data demands and are mostly deployed in a planned manner, also *user equipment* (UEs) that are located at the cell edges will experience poor throughput due to pathloss, signal attenuation, and interference.

- Where the MBS cannot be deployed due to location requirement, LPN can be easily deployed; also with LPN, site acquisition is easier and cheaper.

- Coordination between the MBS and LPNs will enable close cooperation on cell selection in areas where there is an imbalance in signal strength.

HetNet is more cost-efficient, spectrum-efficient and energy-efficient comparing with the single-tier macro cellular networks.

Therefore, HetNet has been regarded as a key technology in 5G that can be used to increase wireless coverage and meet the explosive but unequal data throughput demand by mobile data subscribers. However, due to the reuse one deployment of small cells in the coverage area of *high-power node* (HPN) also known as MBS, the achievable throughput gain will be significantly limited by severe co-channel interference if not properly managed. Therefore this thesis focuses on the maximization of the spectral efficiency of HetNet by proposing enhanced interference management techniques based on beamforming. The objective is to maximize the sum-rate of the system while satisfying some power and/or interference constraints. Additionally, the following problems are investigated and addressed:

- How to allocate radio resources such as spatial resource (unit beamformers) and power resource to UEs in a distributed HetNet, knowing fully well that resource allocation is important for systems that are limited by co-channel interference rather than noise.
- How to do resource allocation for UEs in a centralized HetNet, if the spatial resource optimization will be determined jointly by every participating BS in the HetNet.
- How to determine the global optimal point using optimization procedure that maximizes the sum-rate of HetNet while fulfilling some power and interference constraints.

1.1.1 Heterogeneous Cellular Network

A HetNet is a network consisting of LPNs deployed in the coverage area of an MBS. The LPN include *remote radio heads* (RRHs), *micro base stations* (mBSs), *pico base stations* (PBSs), *femtocell access points* (FAPs) and *relay nodes* (RN). The major motivation behind LPNs is to exploit the cell-splitting gain, i.e., to augment the number of cells and the number of simultaneous transmissions in each macro cell coverage area in order to boost the spectral efficiency per unit area via spatial reuse. These nodes can be operator deployed or randomly deployed by a user. These nodes can also be distinguished by their power classes, the number of transmit antenna they possess, physical size, coverage area, backhaul, and propagation characteristics. Fig. 1.1 shows Heterogeneous cellular Network comprising of mBS, PBS and MBS. In what follows, we describe each component of HetNet and its characteristics such as transmit power levels, transmission coverage range and the number of UEs they are capable of serving.

1.1.2 Macrocellular Base Station

MBS are usually at the heart of each macrocell and are usually deployed by network operators in a planned manner to cover large areas. Their transmit power levels ranges between 5 W and 40 W and their transmission coverage is in the order of few kilometers. They are very powerful base station which can serve thousands of UEs, it also has a dedicated backhaul and can be regarded as the backbone in the HetNet. It can also coordinate with the LPNs to augment network coverage and throughput capacity in high traffic areas.

1.1.3 Pico Base Station

PBS is a smaller base station compared to the micro base station. They have transmit powers of the order of 250 mW to approximately 2 W. They serve few tens of UEs within the radio range of 300 m or less in the pico cells and are usually deployed by the network operators indoor or outdoor. They can also have multiple transmit antenna. They are usually coordinated with a macro network via the X2 interface in *Evolved Universal Terrestrial Radio Access Network* (E-UTRAN) but in most cases, they usually have the same backhaul and access features as the macrocells.

1.1.4 Femtocell Access Points

Femtocell Access points serve the UEs situated in the femtocells. They are usually smaller than the pico base station and are usually deployed by the consumers. They usually have a backhaul which is facilitated by the owner's wired broadband connection [6]. Their transmit power is 100 mW or less. The femtocells' coverage range is less than 50m. They usually operate in open or restricted (*closed subscriber group* (CSG)).

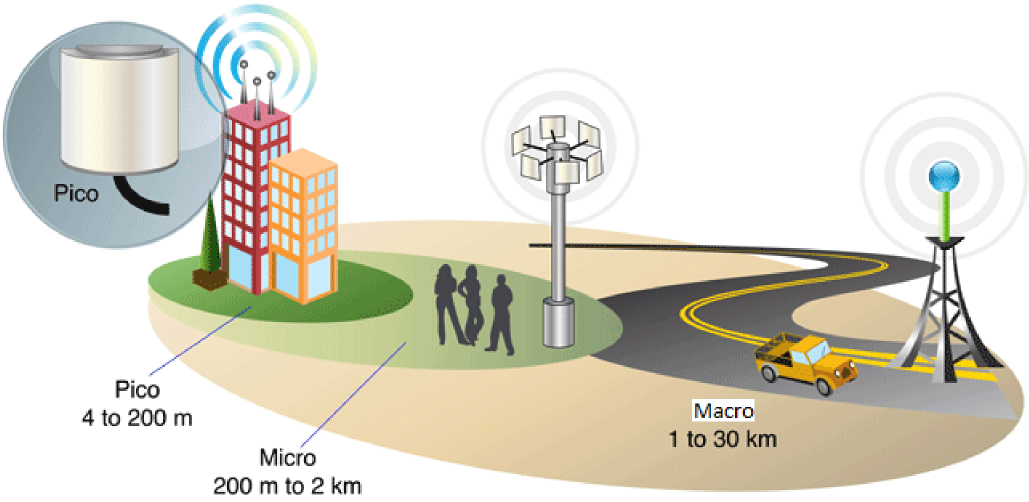

Figure 1.1: Heterogeneous Cellular Network [1]

1.1.5 Remote Radio Heads

RRH is a radio equipment that usually extends the coverage of a macro base station to other areas. They can be used to create distributed base station. They are usually connected to the MBS via a fiber optic cable. They operate at the transmit power that is between the transmit powers operated by micro BS, pico BS, femtocell access points and RN because the transmit power is dependent on the deployment scenario. The MBS is responsible for the control and baseband signal processing while RRH minimizes power losses in the antenna cable and also diminish power consumption.

1.1.6 Relay Nodes

They are full-fledged base station without a wired backhaul but can be deployed having air interface for backhaul connection. They are usually deployed by operators and responsible for the routing of data from macro BS to end users and vice versa. They are usually strategically positioned so as to increase signal strength and improve coverage. Their transmit power usually ranges from 250 mW to 2W

Table 1.1: Categorization and Specification of Nodes in HetNet

Type	**Transmit power**	**Coverage**	**Backhaul**	**Access**	**deployment**
RRHs	40 W	few km	fiber	open to all	indoors/outdoors.
Pico BS	2 W	< 300 m	X2	open to all	indoors/outdoors
Femto BS	100 mW	< 50 m	internet	CSG	indoors user deployed
Relay Nodes	2 W	300 m	wireless	open to all	indoors/outdoors
Micro BS	5-10 W	< 2 km	fiber	open to all	outdoors
Macro BS	40 W	< 40 km	fiber	open to all	outdoors

for outdoors and less than 100 mW for indoor deployment similar to femtocell access points [7]. The categorization and specification of the aforementioned nodes are summarized in Table 1.1.

1.1.7 HetNet Technical Challanges

Despite the known benefits achieved by deploying HetNet over macrocellular single tier networks in terms of improved capacity and coverage. The underlying of LPNs in macrocellular coverage area also poses technical challenges. One of the greatest challenges encountered is the issue of *inter-cell interference* (ICI) which is notorious in reuse one deployment and is capable of degrading the performance of the whole network. This is partly because in HetNet, UEs are not usually served by the most favorable *base station* (BS), radio propagation-wise, in downlink or uplink due to the difference in power class between the MBS and the LPNs [8]. Therefore, such UEs receive severe ICI from other BSs in the network. The issue of inter-cell interference management and coordination will be considered much later in chapter two. Other technical challenges facing LPN deployment include:

- **Access control**

  Access control defines the sets of UEs that can access resources from some LPN. There are three different access modes (open, closed and hybrid access). The open access control mechanism allows all UEs to exclusive rights to the LPN resources but closed and hybrid access allows more access to UEs that belong to the *closed subscriber group* (CSG) and partial access to all other UEs. Consequently, increase in network capacity is also dependent on which access control is used. The restricted access control associated with some LPN e.g., Femto access point may lead to strong interference scenarios in both downlink and uplink transmissions.

- **Mobility and handover**
  Handover brings about seamless uniform service when end UEs move in and out of a cell coverage. They help to balance load traffic by transferring end UEs at the border of overlapping cells from more congested cells to the less congested ones. Mobility usually poses a great challenge in HetNet because quite unlike in homogeneous network the link connection to an LPN usually degrades faster when moving out of the LPN coverage area. Furthermore, because the footprint of the LPN is usually smaller in terms of coverage area it might lead to frequent handovers by highly mobile UEs which might increase both signaling and handover failures. In addition, the probability of handover failure increases the probability of UE outage [9].

- **Energy efficiency**

  HetNet usually brings the network infrastructure closer to the UEs. As a result, the transmit power can be greatly reduced leading to a prolonged battery life and improved *signal to interference and noise ratio* (SINR). But when they are dense and randomly deployed, it will be better to consider the energy efficiency implication of such multi-tier network. The best way to improve the energy efficiency in HetNet is to introduce active/sleep (on/off) modes which will make them consume very little power when idle.

- **Backhauling**

  Because of the scarcity of spectrum, the ongoing development of radio networks to maximize the use of available spectrum put great demands on delay, delay variation and synchronization particularly between the macrocell and small cell. Therefore, HetNets need high performing flexible backhaul so that UEs will not experience a drop in performance when covered by small cells. Backhaul network design will always be a challenging issue because of the complicated topology of the various type of coexisting cells. For cost effective and *quality of service* (QoS) solution to backhauling, network operators need a mixture of

Table 1.2: 3GPP System Simulation Cases

Simulation case	CF(GHz)	ISD(m)	BW(MHz)	PL(dB)	Speed(km/h)
1	2.0	500	10	20	3
2	2.0	500	10	10	30
3	2.0	1732	10	20	3
4	0.9	1000	1.25	10	3

both wired and wireless backhauling technology [10]. In this thesis, we assume that the backhaul has minor latency issues and unlimited backhaul capacity for our centralized HetNet resource allocation implementation.

1.1.8 3GPP Reference System Deployment and Simulaton Baseline Parameters

This subsection describes different system deployments together with the baseline parameters that can be used while performing simulation to evaluate the performance of such system models. We will be utilizing these parameters for most of our simulation in subsequent chapters. The *3rd Generation Partnership Project* (3GPP) defines four reference cases for system-level simulations for both single tier cellular networks and HetNets and this is summarized in Table 1.2.

Table 1.2 shows the 3GPP system simulation cases. Where CF denotes the carrier frequency, BW denotes the operating bandwidth, PL denotes the penetration loss while speed represents the speed at which a UE moves in the simulation. Simulation cases 1 and 3 are commonly used to simulate urban and suburban scenarios, as indicated by their relatively short and large *inter-site distance* (ISD) respectively, and low UE velocities, while case 2 and 4 are less frequently used. It is important to note that these models represent idealized network models, which may differ from realistic ones but are still good approximations.

- **Homogeneous deployment**

Table 1.3: 3GPP System Simulation Baseline Parameters [2]

Parameter	Assumptions
Cellular Layout	Hexagonal grid, 19 cell site, 3 sector per site.
Intersite distance	Depends on Simulation case see Table 1.2
Carrier Frequency and bandwidth	Depends on Simulation case see Table 1.2
Distance dependent pathloss	$L = I + 37.6 \log R$, R in km I = 128.1 if the carrier frequency is 2 GHz. I = 120.9 if the carrier frequency is 900 MHz.
Shadow fading standard deviation	8 dB.
Shadow fading correlation distance	50 m.
Total BS TX power	43 dBm for 1.25 or 5 MHz.
UE power class	23 dBm.
UE speeds of interest	3 km/h, 30 km/h, 120 km/h.

In a homogeneous cellular network, all cells have similar characteristics, such as output power, base station height, antenna patterns, etc. The cell sites are also placed in a regular pattern over an area [11]. Such a homogeneous deployment is also called a macro-only deployment, as only macrocells are present in the deployment. Table 1.3 showcases some baseline simulation parameters for macro-only deployment. The value of the pathloss at a known reference distance usually 1km for an MBS is 128.1 dB at a carrier frequency of 2GHz and 120.9 dB at a carrier frequency of 900 MHz. While the operating bandwidths at the respective carrier frequencies are 10 MHz and 1.25 MHz respectively.

- **Heterogeneous deployment**

Table 1.4: 3GPP Model 1 Simulation Baseline Parameters [2]

Parameter	**Value**
Minimum distance from macro site to pico site	75 m
Minimum distance from macro site to UE	35 m
Minimum distance among pico sites	40 m
Distance dependent path loss, macro to UE	$L(R) = 128.1 + 37.6 \log R$ dB
Distance dependent pathloss, pico to UE	$L(R) = 140.7 + 36.7 \log R$ dB
Carrier Frequency	2 GHz for cases 1 and 3
Base stations per macro cell	1, 2, 4 or 10 nodes
Minimum distance from pico site to UE	outdoor:10 m, Indoor:3 m
UE power class	23 dBm (200 mW)
Total BS TX power	24 or 30 dBm for case 1
UE speeds of interest	3 km/h for cases 1 and 3.

In HetNet, the characteristics of cells can be different, e.g., different output powers and sizes. In a heterogeneous deployment, the typically existing macro-cell is overlaid with LPNs. The LPN can be deployed on the same frequency as the MBS or on a separate frequency. The main differences among these LPNs are summarized in Table 1.1

Regarding backhauling in HetNet, low delay backhauls enable efficient coordination among network nodes. Hence coordination with RRHs is the most efficient, while coordination with Femto access point is the most challenging, especially due to lack of X2 interface. Regarding access method, all base stations in HetNet are usually open access to all UEs with the exception of Femto access points. Closed access modes may create coverage holes when the Femto access point is deployed on the same frequency as the macro cell network, because UEs may not always be able to connect to the cells that provide the strongest signal. Table 1.4 is used to describe the simulation baseline parameters of model 1 for macro cell overlaid with indoor or outdoor picocells. The table showcases the baseline simulation parameters for a macro-pico heterogeneous scenario. The $\boldsymbol{R}$ represent the distance of a user to the base station. The minimum distance of a macro and pico UEs to the macro and pico nodes are 35 m and 10 m respectively in outdoor situations. The pathloss exponent $\boldsymbol{n}$ is an empirical constant which depends on the propagation environment. The pathloss exponent between a macro UE and the MBS is 3.76 while that between a pico UE and the PBS is 3.67.

1.2 Thesis Overview

This thesis comprises of seven chapters. A succinct account of each chapter is given below.

Chapter 1 provides the motivation behind the thesis and the background information necessary for understanding the theories and technical foundation of HetNets. It also includes the contributions of the thesis.

Chapter 2 discusses the sources and causes of interference in HetNet. It also surveys different interference management techniques applicable and inappropriate for HetNet in terms of maximizing its spectral efficiency. The different techniques reviewed can be categorized into the following: frequency-domain techniques, time-domain techniques, power-based techniques and antenna-based techniques. Some of the concept presented in the antenna based techniques are utilized to develop beamforming schemes tailored for HetNet implementation and are discussed in Chapter 3, 4 and 5.

In Chapter 3, we look into how to maximize the weighted sum-rate of HetNet under some power and interference constraints. This problem is generally non-convex and NP-hard. But we are able to solve it using Branch and Bound method in order to get the optimal point that satisfies all the constraints of the non-convex optimization problem.

In Chapter 4, we first determine the optimal significant interfering BSs whose aggregate interference affect each UE the most. Afterward, we demonstrate how each serving BS for these interfered UEs, together with these interfering BSs coordinate and make resource allocation decisions to allocate spatial directions to each UE in the system. Finally, we also show how power resource is allocated to each UE in each cell using convex optimization.

In Chapter 5, we determine how to allocate the spatial resource (unit-beamformer) and power resource in a decentralized way. The resource allocation done in this chapter is distributed, in other words, each BS makes a unilateral decision on how to allocate spatial resource provided that time division duplex based local channel state information is available at each base station.

In Chapter 6 we devised a heuristic distributed algorithm for coordinated beamforming based on the signal-to-leakage-interference-and-noise ratio. This algorithm is implemented in each BS, hence needs only local channel state information for the design of its beamformers.

The closing Chapter 7 deals with conclusions and recommendations for future works.

1.3 Contributions

1. Chapter 3 considers a HetNet of coordinating base stations where each base station is equipped with multiple antennas. The aim is to maximize the weighted sum-rate for the downlink of HetNet. In this chapter, we are able to find the global optimal beamforming vectors that will maximize the weighted sum-rate for this HetNet scenario while fulfilling both the powers and interference constraints of the problem. The devised algorithm that solves this problem is iterative, the starting point of the algorithm which is critical to the performance of the algorithm is obtained by reformulating the optimization problem from non-convex to convex. Afterward, the feasible set of the optimization problem is searched in order to find the optimal solution. Our devised algorithm uses both convex optimization technique and search methods to achieve our aim. This algorithm is implemented using the branch and bound method. These contributions are published in [12].

2. In Chapter 4, we devised a UE-Centric clustering scheme which has the capability to identify the set of base stations that causes the largest aggregate

interference to each UE in the HetNet. These base stations will now coordinate interference with the serving base stations of these interfered UEs and jointly make resource allocation decisions to allocate unit-beamformers to each UE. Afterwards, each base station can now allocate powers to each UE in the HetNet. This technique identifies the most significant interferers and mitigates their interference, thereby maximizing the spectral efficiency of the HetNet. These contributions have been accepted for publication in European Wireless (EW) Conference 2017, and have also been substantially extended and submitted to IEEE Access for possible publication.

3. Chapter 5 considers a decentralized HetNet where the aim is to devise an algorithm that will reduce the total leakage interference leaked to UEs in each cell while transmitting a signal to the desired UE. We are able to devise a distributed resource allocation scheme that can allocate unit-beamformers and powers to UEs in order to maximize the spectral efficiency of each cell in HetNet. These contributions have been accepted for publication in International Symposium on Wireless Communication Systems (ISWCS) 2017, and have been substantially extended and submitted to IEEE Transactions on Communications for possible publication.

4. In Chapter 6, we also considered a decentralized HetNet where the aim is to maximize the sum-rate of the HetNet. We are able to devise a heuristic distributed algorithm based on the signal-to-leakage-interference-and-noise ratio that is capable of maximizing the sum-rate of each cell in HetNet. Our devised distributed algorithm is found to perform better when compared with other distributed algorithms. These contribution has been published in [13].

1.4 List of Publications

The following publications have emanated from the work of this research:

- *Journal Papers*

1. O. Oguejiofor and L. Zhang, "UE-Centric Clustering and Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks", Submitted in *IEEE Access*.
2. O. Oguejiofor, and L. Zhang, "Distributed Resource Allocation for Two-Tier Heterogeneous Cellular Networks", Submitted in *IEEE Transactions on Communications*.

- *Conference Papers*

3. O. Oguejiofor and L. Zhang, "Global Optimization of Weighted Sum-Rate for Downlink Heterogeneous Cellular Networks", Published in *International Conference on Telecommunications (ICT)*. IEEE, 2016. (DOI: 10.1109/ICT.2016.7500345).
4. O. Oguejiofor, L. Zhang, and N. Nawaz "Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks", Accepted in *European Wireless (EW)*. IEEE, 2017.
5. O. Oguejiofor and L. Zhang, "Heuristic Coordinated Beamforming for Heterogeneous Cellular Network", Published in *Vehicular Technology Conference (VTC)*. IEEE, 2016. (DOI: 10.1109/VTCSpring.2016.7504269).
6. O. Oguejiofor, L. Zhang, and M. Alhabo "Decentralized Resource Allocation for Heterogeneous Cellular Networks", Accepted in *International Symposium on Wireless Communications System (ISWCS)*. IEEE, 2017.

- *Co-Authored Publications*

7. M. Alhabo, L. Zhang, and O. Oguejiofor, "Inbound Handover Interference-Based Margin for Load Balancing in Heterogeneous Networks", Accepted in *International Symposium on Wireless Communications System (ISWCS)*. IEEE, 2017.

Chapter 2

Interference Issues and Management Techniques in HetNet

2.1 Introduction

Interference in *Heterogeneous cellular Network* (HetNet) is seen as a major obstacle preventing HetNet from taking its place among emerging technologies that will herald 5G networks. Many excellent works have proposed different interference management schemes or techniques on how interference in HetNet can be managed or mitigated. However, most of these works are utilizing the same interference management scheme applicable to multi-cell single-tier networks for interference management in HetNet. It is important to state that this approach can improve performance to some extent but cannot mitigate dominant interference scenarios in HetNet. This is because when compared to single-tier networks, HetNet has different cell selection procedures, different propagation characteristics, different pathloss environment and heterogeneity of BS power classes. Therefore, interference scenario in HetNet is quite different from that of single-tier homogeneous cellular networks. Furthermore, since the goal for deploying HetNet is to improve spectral efficiency, capacity, and coverage, we found out that most of the interference management techniques available can improve performance when applied to HetNet but will not improve the spectral efficiency of HetNet. In the light of these issues, this chapter will be discussing the major causes of interference in HetNet, different kinds of interference and different interference management techniques appropriate/inappropriate for HetNet in terms of maximizing the spectral efficiency of the HetNet.

2.1.1 Chapter Organization

The rest of this chapter is organized as follows. Section 2.2 presents the major causes of interference in HetNet and different kinds of interference occurring in HetNet. While Section 2.3 presents different interference management techniques that have been proposed in the literature starting from classical methods to the new advanced methods. The summary of the discussions is presented in Section 2.4.

2.2 Interference Issues in HetNet

In this section, we will describe the likely sources of interference in HetNet. The major causes of interference in HetNet can be categorized as follows:

- **Universal frequency reuse deployment**

  Future generation mobile networks demand significant increase of spectral efficiency compared to current 4G systems. Consequently, HetNet can improve the spectral efficiency of the network by utilizing the unplanned reuse-one deployment of small cells in the coverage area of the MBS. This makes the small cells and the macro cell capable of using the same carrier frequency and all available frequency resource because they operate in the same frequency band. However, this can lead to huge co-channel interference. This is because of the broadcast nature of the wireless channel, which makes the desired signal transmitted to a UE, appear as an interference to another UE in the same frequency band. Which if not properly managed will degrade the overall system performance. The unplanned deployment nature of HetNet components by individuals also makes the traditional network planning and optimization become ineffective in contrast to single-tier macro-cellular networks.

- **Cell association and selection methods**

  In single-tier macro-cellular networks, UE(s) are associated with the BS(s) whose signal is received with the largest strength commonly regarded as *reference signal receive power* (RSRP) [14]. However, due to the varying power classes of different transmitters in HetNet, such cell association, and selection policies would cause high traffic load imbalance which can reduce the coverage and rate gain achievable by deploying HetNet [15]. In the light of this, biased-based *cell range expansion* (CRE) [16] in pico cells has been introduced to remedy the problem of load imbalance in downlink HetNet. Its goal is to augment the downlink coverage footprint of LPNs by adding positive bias (cell individual offset) to their RSRP. This bias will allow more UEs to be associated with the LPNs, therefore, achieving an improved cell load balancing. The problem with this approach is that it makes the serving cell selection more uplink relevant, and the UEs in the CRE have the most favorable uplink to their serving LPN. However, these UEs have a more favorable downlink from non-serving MBS, therefore causing huge downlink interference for those UEs. Figure 2.1 illustrate the cell range expansion area of a pico cell.

- **Close subscriber group access**

  CSG access defines set of UEs that can access the resources of a particular BS or access points, in other words, if an UE doesn't belong to this group of UEs, access is denied. A lot of interference situation can be created if an UE that is supposed to connect to a particular BS based on proximity is denied access because it doesn't subscribe for access from it, now connects to a far away BS. It can be easily seen that this UE can cause serious interference (jamming) in uplink to the access points or BS of this CSG where it is denied access while trying to communicate with its serving BS [17].

2.2.1 Different Kinds of Interference

Interference can be simply defined in wireless communication as an unwanted signal that corrupts the desired signal thereby reducing the quality of the desired signal. They operate in the same frequency band and share similar structure/characteristics, hence difficult to terminate. In contrast, interference can be distinguished from thermal noise in its physical and statistical features, because thermal noise is normally distributed whereas interference has the same structure as the desired signal. It is good to note that interference are desired signals in other cells and for other UEs as well. Having said that, in what follows we

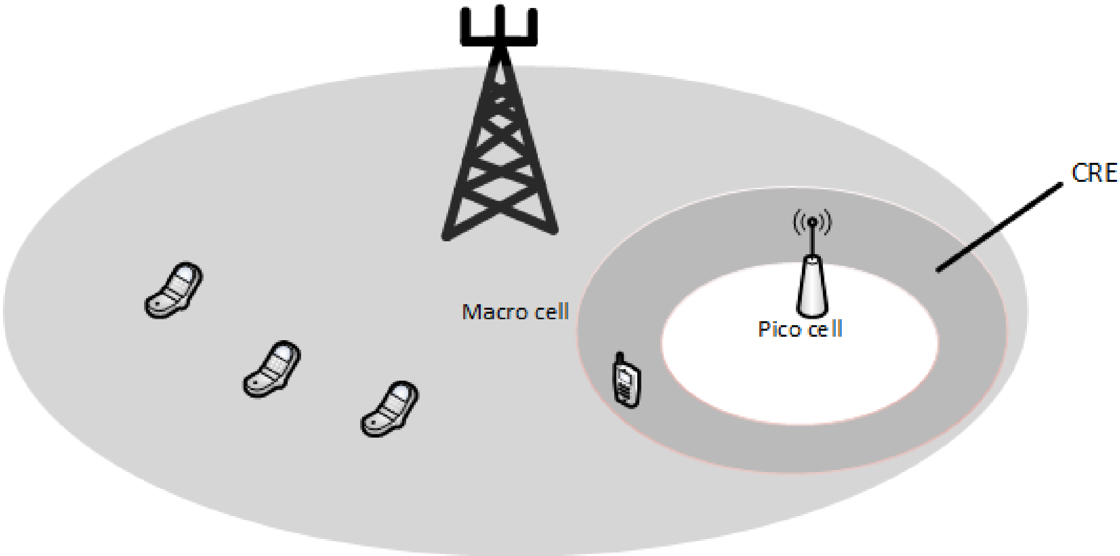

Figure 2.1: Cell Range Expansion Illustration

elaborate more on the different kinds of interference experienced by UEs in the downlink of HetNet. These include intra-cell interference, inter-UE interference, multi-UE interference, inter-cell interference, inter-tier interference, intra-tier interference, cross-tier interference, inter-stream interference, leakage interference and co-channel interference. Some of the aforementioned kinds of interference mean the same thing but are given different names depending on the situation and scenario.

- **Intra-cell interference** (Inter-UE, Multi-UE)

  Intra-cell interference is the kind of interference that occurs within a cell among UEs served by the same BS. Each UE receives its desired signal in addition to other signals intended for other UEs in the same cell. We utilize the intra-cell interference terminology whenever we have multiple cells, like in HetNet and want to distinguish this interference from the one coming from other cells. Inter-UE and Multi-UE interference(s) mean the same thing but this terminology is more appropriate for single cell scenario, where the cell serves more than one UE synchronously in the same time-frequency resources. The desired UE also receive signals intended for other UEs, these other signals are regarded as inter-UE or Multi-UE interference to the desired UE.

- **Inter-cell interference** (Inter-tier, intra-tier, cross-tier)

  Inter-cell interference is the kind of interference that occurs between cells in a multicell scenario. This interference is produced by neighbouring BSs that does not serve the desired UE which belongs to another cell. Inter-tier, intra-tier and cross-tier interferences can be regarded as synonyms to inter-cell interference but they are more specific to the kind of cells involve. Inter-tier and cross-tier interference represent interference between heterogeneous cells while intra-tier interference represents interference between homogeneous cells.

- **Inter-stream interference**

  This kind of interference usually occurs in *multi-input multi-output* (MIMO) systems. In the case of a single-UE MIMO, the signal received by the UE includes a linear combination of multiple streams which causes inter-stream interference.

- **Co-channel interference**

  Co-channel interference is interference that arises in cellular mobile networks owing to the phenomenon of spatial reuse. Thus, besides the intended signal from within the cell, signals at the same frequencies (co-channel signals) are regarded as co-channel interference. It is a more generic way of describing interference received by a UE when the same time-frequency resource is shared.

- **Leakage interference**

  Leakage interference is the interference that is caused to a co-channel UE by a desired signal intended for the desired UE.

2.3 Interference Management Techniques

In this section, we explain how interference has been managed starting from the conventional 2G network up until 4G/4.5G networks. For clarity we also classify

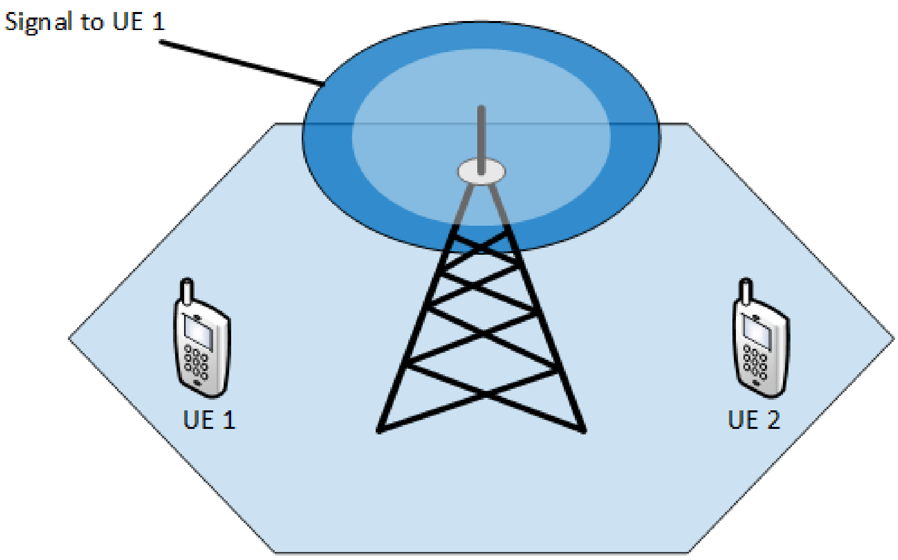

Figure 2.2: Single Antenna Transmission

these schemes into single antenna and multi-antenna technologies for both single cell and multi-cell systems.

- **Single cell system**

  In the traditional single cell, single antenna systems, electromagnetic waves are radiated omnidirectionally as shown in Figure 2.2. Figure 2.2 shows a single antenna transmission, which propagates signal meant for UE 1 omnidirectionally (equally strong in all direction) in a multi-UE system. Therefore creating severe interference in the direction where the intended UE is not located such as UE 2 location. In order to mitigate the inter-UE interference in such situation, many multiple access techniques have been proposed such as *frequency division multiple access* (FDMA) [18], *time division multiple access* (TDMA) [19], *code division multiple access* (CDMA) [20] and recently, *orthogonal frequency division multiple access* (OFDMA) [21]. These techniques divide the total signaling dimension into channels and then assign these channels to different UEs along frequency, time or code axes. Most of these access schemes actually mitigate interference and improve performance however at the cost of sacrificing the spectral efficiency of the system.

In a single cell, multi-antenna system, for instance when the cell is sectorized, each antenna can be used to serve a sector and by applying fixed frequency reuse patterns, adjacent cell sectors will not be utilizing the same frequencies (resources) thereby avoiding interference. This near orthogonal interference avoidance approach is a static way of mitigating *inter-cell interference* (ICI), though it can improve performance, however will not improve the spectral efficiency of the network. A much better way to mitigate interference in a single cell, multi-antenna systems that will improve the spectral efficiency of the cell is by employing *space-division multiple access* (SDMA) [22] in place of TDMA and FDMA. The idea behind SDMA is to use the directivity of the antenna array to reduce the inter-UE interference that will occur when multi-UEs are served simultaneously in the same frequency-time resource. The directivity of the antenna is tightly coupled with the concept of transmit beamforming, that is the ability to transmit signal from multiple antenna arrays using different relative amplitudes and phases such that the components add up constructively in the desired direction and destructive in the undesired direction [23]. Figure 2.3 shows a multi-antenna transmission where the signal transmitted to UE 1 does not interfere with UE 2 and vice versa despite the transmission been done simultaneously in the same time-frequency resource. The ability to direct signals toward only intending UEs will enable global utilization of all spectral resources, thus eliminating the need for fixed frequency reuse patterns and cell sectoring. Note by frequency reuse patterns we mean different frequency planning schemes such as Reuse-3, *fractional frequency reuse* (FFR) [24], *partial frequency reuse* (PFR) [25] and *soft frequency reuse*(SFR) [26]. In Reuse-3, the whole frequency band is divided into three orthogonal but equal sub-bands and allocated to different cells. While in FFR the available frequency resources are divided into two, one is used to serve the cell-edge UEs while the other is used to serve the cell interior UEs. In SFR schemes, the sector uses full power in some frequency sub-bands while reduced power is used in the rest of the frequency band. The SFR scheme may result in under-utilization of available frequency resource due to its strict no-sharing policy.

- **Multi-cell system**

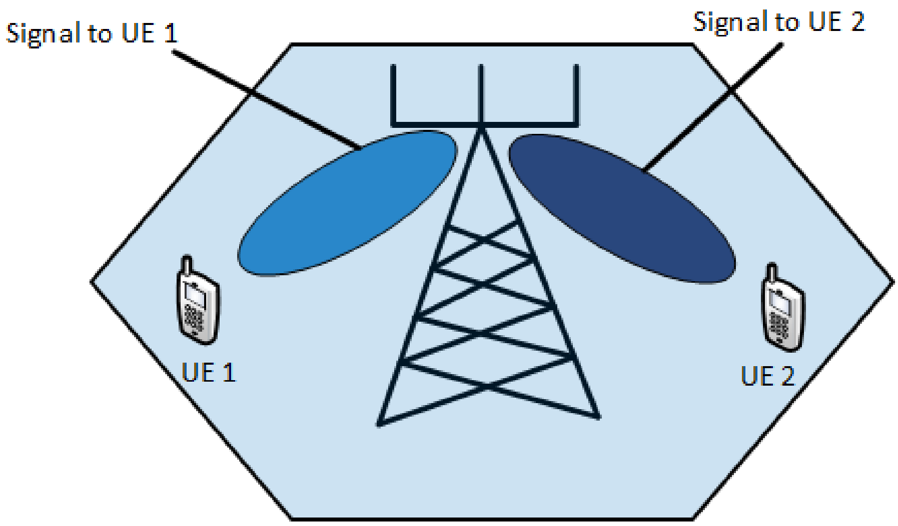

Figure 2.3: Multi-Antenna Transmission

In conventional multi-cell system using single antenna technology, electromagnetic waves are radiated omni-directionally. By utilizing fixed frequency reuse patterns and *single cell processing* (SCP), neighboring cells will be protected from *inter-cell interference* (ICI). This frequency allocation scheme to each cell and UEs are usually computed and evaluated during radio planning process and only long-term readjustment is performed during the operation of the network. However, this approach is statically done and involves a lot of frequency planning to enable successful implementation rollout. Furthermore, static schemes are unsuitable for HetNets because of the unplanned nature of deploying the LPNs close to UEs location hence making prior frequency planning very difficult.

Other dynamic frequency allocation schemes such as frequency-domain *inter-cell interference coordination* (ICIC) [27] have also been proposed which addresses the inter-tier interference by coordinating the use of frequencies among cells.

Coordination here means information exchange among BSs to achieve a common goal (e.g., for interference mitigation). For example, in the *third generation partnership project* (3GPP) *long-term evolution* (LTE), X2 interface [28] can be utilized to connect adjacent *Evolved NodeBs* (eNBs) for the exchange of signalling information. eNBs which is a terminology for BSs in LTE can exploit coordinated shared information to schedule their cell-edge UEs at frequency resources that are used less frequently, thus subjecting them to less interference. Other ICIC techniques which are specified in LTE 3GPP releases 8 and 9 such as power based techniques [29], [30] and time-domain techniques have a limitation when applied to HetNet because when LTE was first conceived, HetNets were not at the forefront of the agenda, thus may not be effective for HetNet dominant interference scenarios. In order to mitigate such dominant interference scenarios, *enhanced inter-cell interference coordination* (eICIC) schemes have been developed and specified in LTE-Advanced releases 10 and 11, enhanced frequency domain and time- domain ICIC [31] is performed through *carrier aggregation* (CA) [32] which is supported by LTE-Advanced (3GPP Release 10) and can be used to avoid co-channel interference in downlink. The aforementioned techniques improve performance by mitigating interferences using either time domain, frequency domain or power-based techniques, however, they do this without fully utilizing system resource leading to scant spectral efficiency of the network.

Other notable ICI mitigation techniques can be categorized under the spectrum management techniques. Where the ICI challenge is addressed by solving spectrum assignment problem [33]. The spectrum resources are allocated in order to maximize the downlink achievable data rate. Furthermore, cognitive radio spectrum aware methods [34], [35] which are based on spectrum sensing, geolocation database lookups or broadcast information via beacon signals is also closely related to spectrum management.

2.3.1 Antenna Based Techniques

Having reviewed how inter-cell interference can be tackled by using time-domain (eICIC), frequency-domain (ICIC), power-based techniques and spectrum management techniques in the preceded subsection. In this subsection, some antenna based techniques used for interference management for both single-tier and multi-tier networks are reviewed. It is worthy to say that there are other interference mitigation techniques which have also been proposed in the literature to remedy ICI such as UE scheduling, and soft handover. Although all these techniques improve system performance, they do not fully utilize system resources. Therefore, this subsection elaborate how multi-cell processing together with MIMO transmits beamforming techniques plus power control can address ICI in order to achieve significant spectral efficiency. This work will be adopting this approach to mitigate inter-cell interference in the heterogeneous cellular network.

- **Transmit beamforming basics**

  Transmit beamforming is a very flexible technique for transmission of signal from multiple antennas to one or multiple UEs [36]. The goal is usually to increase the signal power at the desired UE and reduce interference to non-desired UEs. When the same data signal is transmitted from all antennas using different relative amplitudes and phases such that the signal components add up constructively at the desired UE and destructively at the non-desired UEs it leads to increase in the received signal power (array gain). This corresponds mathematically to designing beamforming vectors/precoding matrix (which describe the amplitude and phases) to have large inner products with vectors describing the desired channels and small inner products with non-desired UE channels [23]. Note, the same principle can also be applied to receive beamforming [37]. The only difference is that transmit beamforming occurs in the downlink while receive beamforming occurs in the uplink.

Figure 2.4 shows a polar plot illustration of receive/uplink beamforming. The beam pattern is adjusted to maximize the gain in the direction of desired signal (main lobe) and to place nulls in the direction of interferers. Null is between the main lobe and side lobes. Figure 2.3 illustrates transmit beamforming. Because of the ability of the transmit beamforming to focus signal energy at certain directions, less energy arrives at other places. This brings about the so-called SDMA, where different spatially separated UEs are served simultaneously in the same frequency-time resource. Making effective use of the available bandwidth, while the interference is controlled spatially. The spectral efficiency increases linearly with the number of antennas in the array if the spatial dimension is used to serve UEs in parallel. In single/multi-tier networks where all BSs are using

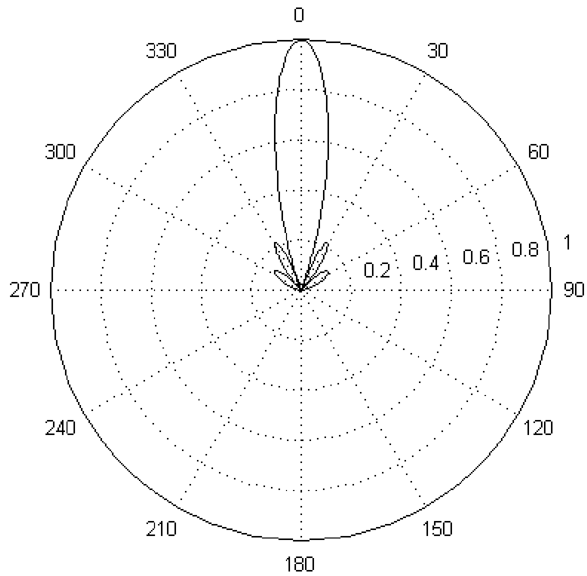

Figure 2.4: Polar Plot Illustration of Receive/Uplink Beamforming

the same frequency resources (in order to maximize the aggregate system-wide spectral efficiency), the counterpart of SDMA in multi-cell system is regarded as *coordinated multi-point* (CoMP). CoMP has been given different names by different authors in different literatures such as co-processing, network MIMO, cooperative processing and multi-cell processing. Whichever name they call it, it is still based on the idea of exploiting the spatial dimension to serve UEs in parallel while controlling interference.

- **Limitation of single cell processing**

  Single cell processing (SCP) is a term in mobile communication networks used for describing BSs that unilaterally serve its own UEs without considering other sources of interference that might affect its UEs but rather treating them as Gaussian noise, as shown in Figure 2.5. Traditional mobile networks use SCP when communicating with its served UEs while interference is managed by using fixed frequency reuse patterns or power control. However, the evolution of new-generation mobile networks demands a significant increase in spectral efficiency compared to current LTE 4G systems. This has enabled the use of more aggressive frequency reuse pattern such as the universal reuse frequency. In

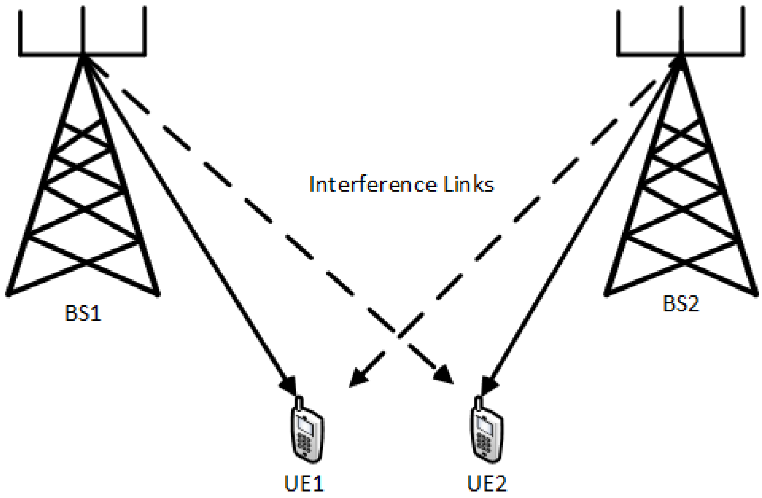

Figure 2.5: Illustration of Single-Cell Processing.

this context, SCP will experience huge ICI among adjacent cells. The system-wide spectral efficiency for single/multi-tier downlink networks can be further significantly improved if the frequency reuse patterns are replaced by cooperation among BSs. The key point here is that BSs will no longer make unilateral decisions but needed joint effort among the cooperating BSs to tackle ICI in the system thereby improving individual BSs performance.

- **Advantages of multi-cell processing**

  Multi-cell processing (MCP) is a good solution and the most advanced way to manage ICI as well as increasing the spectral efficiency of the network. Unlike the SCP where ICI is a limiting factor and difficult to deal with in reuse-one deployment scenario, in MCP, BSs cooperate together in different levels to manage interference and at the same time improving the individual BSs that form the cluster, as illustrated in Figure 2.6. MCP maximizes the spectral efficiency of the system by utilizing radio resources not only in frequency/time domain but also in the spatial domain. The two most significant benefits of MCP are

  1. High capacity gain compared to conventional SCP mobile networks. This comes as a result of having all cooperating BSs share the same bandwidth

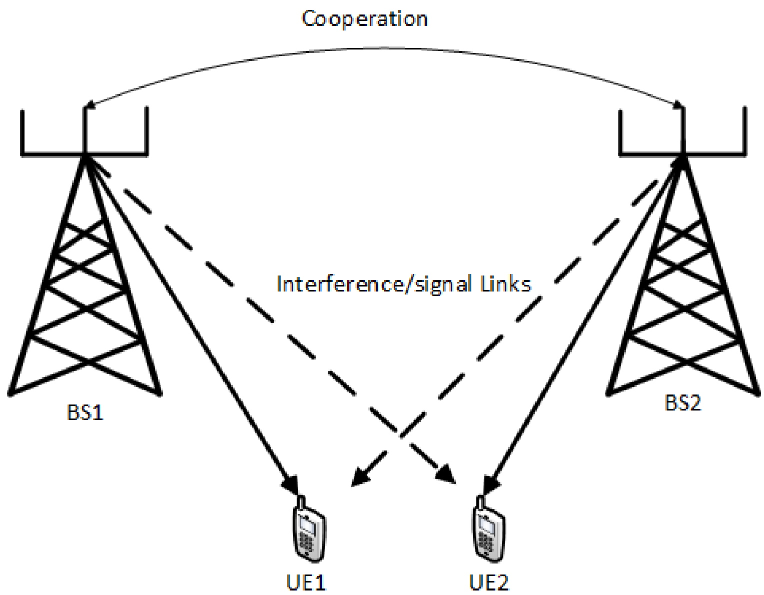

Figure 2.6: Illustration of Multi-Cell Processing.

and therefore avoiding the difficulty for statically frequency planning. This enhances the spectral efficiency which as a result increases the capacity of the system.

2. BSs cooperation can convert harmful ICI into useful signals. This will also improve the capacity of the network.

BS cooperation entails sharing of control signals, transmit data, UE propagation *channel state information* (CSIs) and/or beamformers through high-capacity backhaul links to coordinate transmission. In what follows, we will categorize MCP based on the different levels of cooperation they have among themselves through the backhaul links. Backhaul have a lot in determining the level of cooperation.

- **Control-level cooperation**

  This cooperative strategy when employed by MCP, exchange only control-level signals among BSs leading to small load on the backhaul link. They require some form of joint allocation of available resources to orthogonalize UE transmission in neighbouring cells, by allocating frequency bands and/or timing

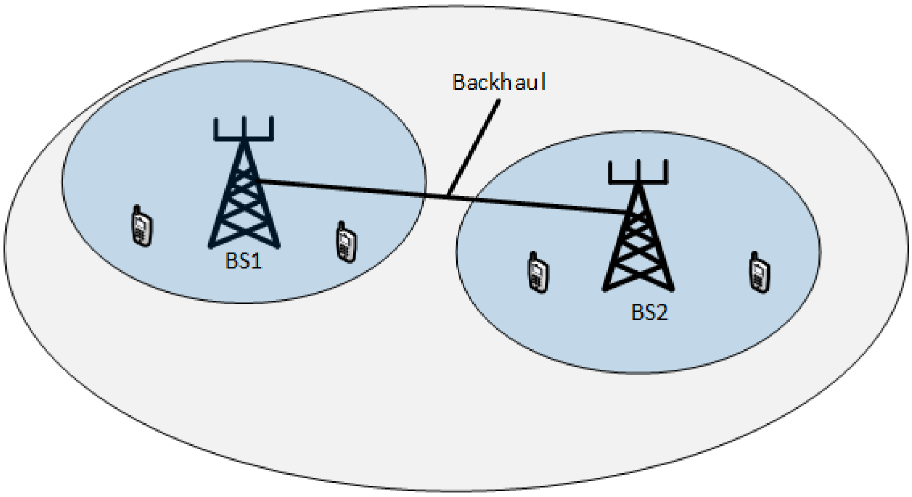

Figure 2.7: Illustration of single-tier scenario of partial cooperation among BSs, where CSI are shared in order to perform CB.

cycles. This is the type of cooperation used by ICIC and eICIC, while these techniques may yield higher sum-rates than static transmission algorithm, they did not utilize all the available frequency and time resources, hence, cannot realize the significant performance gain that is obtainable using MCP.

- **Partial cooperation**

  This cooperative strategy as shown in Figure 2.7 when employed by MCP, exchange only the local CSI of the active UEs among themselves through the limited backhaul link. This will bring about the fair balance between realizing the gains for using MCP and ensuring moderate load on the backhaul links. The shared CSI can be used by each BS to design individual beamformers or precoders for single-stream and multi-stream transmission to its served UEs respectively. This can be described as *coordinated beamforming* (CB) [38–41] in 3GPP LTE-Advanced. In an interference channel, UE1 is served by BS1, while interfering UE2, also, UE2 is served by BS2 while interfering UE1. When CB is applied to it as shown in Figure 2.8, there will not be any interference links from BS1 to UE2 and from BS2 to UE1 respectively because they have been nulled as a result of the designed beamformers.

- **Full cooperation**

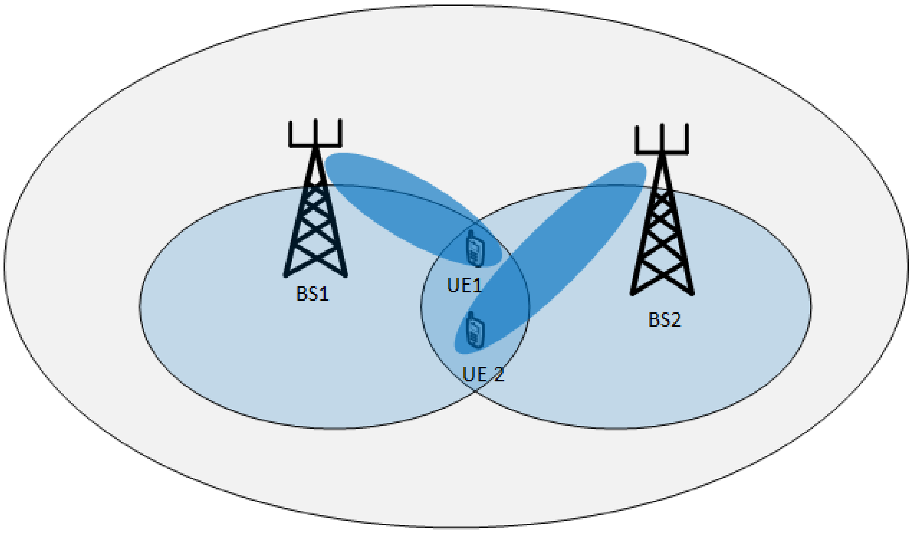

Figure 2.8: Illustration of Coordinated Beamforming in Interference Channels.

Full cooperation as shown in Figure 2.9 requires *super base station* (SBS) that connects all cooperating BSs together so that global information such as data and CSI can be easily collected among cooperating BSs. This cooperative strategy when employed by MCB usually performs joint transmission to multiple UEs making the whole cell that forms part of the cooperating cells operates as a single-cell multi-user multi-antenna system, consequently will yield the highest sum rates at the cost of increased overhead due to the huge exchange of information among the cooperating BSs. This cooperative strategy can be described as *joint transmission* (JT) [42–45] in 3GPP LTE-Advanced, see Figure 2.10. Despite the promising improved performances of JT, some practical implementation questions need to be answered properly such as:

1. How will the huge burden placed on the backhaul links because of the global CSI and data be addressed looking at the distance of the different BSs involved. This will definitely challenge the capability of the X2 interface [28] among BSs.
2. How will the increase in the delay spread because of the need for accurate synchronization between the cooperating BSs be addressed?

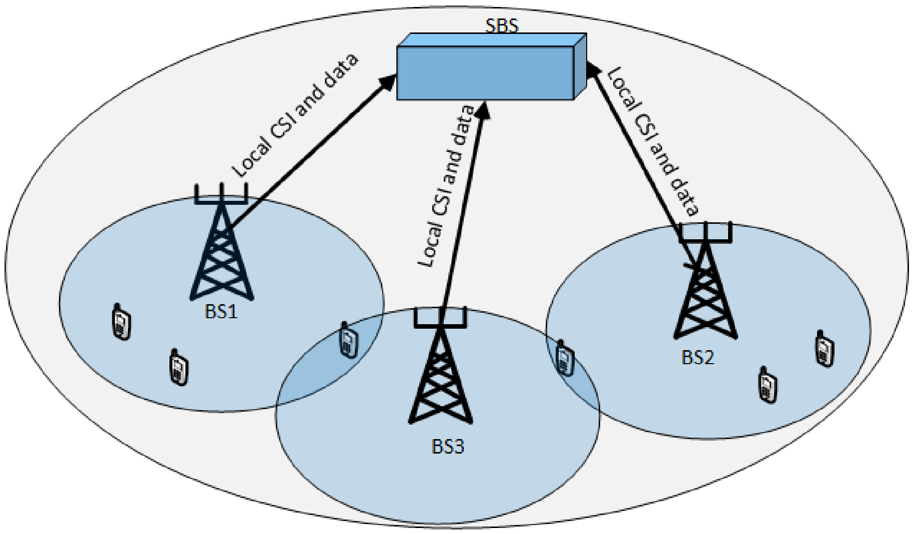

Figure 2.9: Illustration of full cooperation scenario, where all cells send their local CSI and data to the SBS, which will have the global CSI and data needed to perform JT to all UEs in the network.

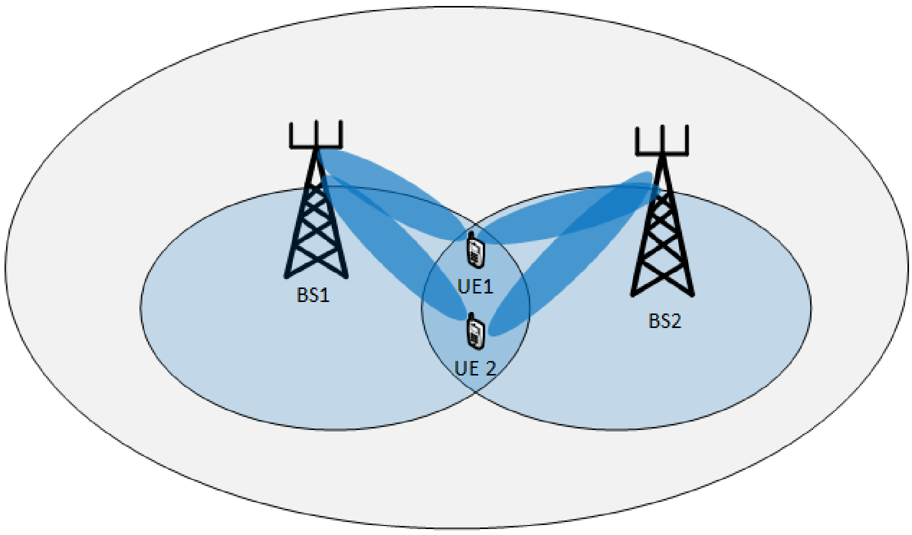

Figure 2.10: Illustration of JT in Interference Channels.

3. Lastly, how will the issue of the central unit (SBS) which is usually used to perform optimization be handled when applied to a complex Heterogeneous cellular network where operations tend to be distributed.

For JT MCP to be implementable in HetNet, the backhaul link must be delay free and have unlimited capacity. Having said that, it is important to note that this is only possible for some HetNet scenarios such as Macro-RRH or Macro-Pico HetNet scenarios, where the macro BS is connected to the LPN through fibre optical link. However, this kind of backhaul link will increase *operating expenditures* (OPEX). If the net gain between OPEX and increased spectral efficiency (performance gain) is small, then the motivation behind increased expenditure for implementing JT cannot be justified. For macro-femto HetNet scenario, the backhaul connection can be user established via the internet, however, it will suffer from extreme delay. Thus the achievable information exchange between macro BS and Femto access points will be minimal. Macro-Femto HetNet can utilize other cooperative strategies such as control level cooperation. Other challenges involved in the implementation of JT or coordinated beamforming MCP in HetNet are:

- **Clustering in HetNet**

  The reasons behind cluster formation in MCP is to achieve a common goal by all participating units, such as interference mitigation and/or improving the received signal quality for UEs at cell edges. As signals decay quickly as distances increase it will not be ideal for long distance BSs to be part of the cluster that will perform JT. If clusters are small, it enables practical channel acquisition, coordination, and synchronization. If clusters are statically formed (Network-centric method), and UEs in the clusters are heterogeneously distributed, then this kind of clusters will provide poor spectral efficiency and will also suffer from out-of-cluster interference. Other ways through which clusters can be formed are UE-centric method [46–48], and combined method [49]. The network-centric approach is less flexible and usually static and is predefined by the network operators on a set of cooperating cells and their cooperating area. In contrast, in UE-centric clustering, each UE chooses a small number of cells that gives the greatest cooperation gain but is however very complex from a scheduling point of view. The hybrid (combined method) produces a trade-off between the performance and complexity of the previously presented approaches.

Table 2.1: Different Optimization Formulations for MCP

	Centralized	**Distributed**
Problem statement	**min** weighted transmitted power **s.t.** per UE QoS constraint	**max** signal to leakage and noise power **s.t.** fixed power constraint.
Problem statement	**min** maximum antenna power **s.t.** per UE QoS constraint	**max** minimum UEs' SINR **s.t.** fixed power constraint.
Problem statement	**max** sum throughput of the system **s.t.** per UE QoS constraint	**max** minimum UEs' throughput **s.t.** fixed power constraint.

- **CSI sharing**

  CSI is essential for transmit beamforming vector or precoding matrix design. In *time division duplex* (TDD) system because of the reciprocity of the channel, the CSI of UEs can always be made available at the transmitter. In *frequency division duplex* (FDD) system the channel is estimated at the receiver using pilots sent from base stations. Then each UE sends the CSI to BS through a return channel (feedback channel). However, the feedback channel is usually rate limited so proper limited feedback mechanism needs to be designed in order to use it effectively.

  Most schemes aim at sharing full CSI at each BS but it is not always necessary depending on the implementation (centralized or distributed). In distributed implementation, only the availability of local CSI rather than global CSI is sufficient for the design of coordinated beamforming. This can be justified for a TDD system where the downlink CSI corresponding to a BS can be directly estimated at the BS by exploiting uplink-downlink reciprocity. On the other hand, one can assume that each BS can estimate the interfering channels to all other UEs in the same cluster. This can be explained by recognizing that a cluster is dynamically set up if a UE can see significant interference from other cells in the same cluster [50]. That is, the channel between the base station and the UEs in other cells and those between the base station and its connected UEs are of similar strength in a cluster.

In MCP centralized implementation, the design objective is to jointly optimize the beamformers and powers for all UEs in the cluster using some design criterion. The difference between centralized implementation and distributed implementation is where the optimization will take place. For centralized implementation, the joint optimization takes place usually at a control unit such as SBS, while for distributed implementation the optimization takes place at each BS. The centralized implementation needs the global CSI of all UEs for it to design good beamformers whereas the distributed MCP implementation needs only the local CSI of the UEs in order to design optimal beamformers. The centralized implementation usually outperforms the distributed implementation but also have overhead issues, in practice, the trade-off between performance and complexity is advised. In Table 2.1, we illustrate different ways the problem statement for centralized and distributed implementation of MCP are formulated. This can also be linked to the subjective decisions of the network designer. Note that not all algorithms that work for single-tier macrocellular networks will also work for multi-tier HetNet.

2.3.2 UE-Side Techniques

UE-side techniques can be described as interference management techniques that can be done by the UE by utilizing its multiple receive antenna to distinguish between desired signal and interference signals. This approach is quite different from the approaches discussed in preceded subsection where interference management techniques are done by the network-side. The proponents argue that putting the responsibility for interference management solely on the network will cause a lot of practical issues and limitations like feedback overhead and backhaul. However, some authors have proposed the synergy between the network side interference management and the UE-side interference management techniques, claiming some significant improvement [51], [52]. To achieve this, the UE receiver architecture needs to improve because the processing of the signals in terms of interference suppression will now be done by it. Traditional UE receivers when receive signal, trie to correctly decode the desired signal and treat the interference signal as noise because they lack the capability to separate the interference from the desired signal. Improving the UE receiver architecture to mitigate interference spatially entails making the UE receiver architecture more complex. Practically, how will the issue of preserving the battery life be solved because this level of processing requires a lot of energy? Furthermore, the more complicated hardware receiver structures also increase cost. However, if this practical concern is put to rest, the benefits that can be drawn from the synergy between the network-side and UE-side implementation will be an enhanced spatial multiplexing gain for the network.

2.4 Summary

This chapter discusses the sources and causes of interference in HetNet, it also reviews different interference management techniques both applicable and inappropriate for HetNet in terms of maximizing the spectral efficiency of HetNet. The interference management techniques as presented in this chapter are categorized into the following: frequency domain techniques, time-domain techniques, power-based techniques and antenna-based techniques which include both the UE-side techniques and the network-side techniques.

The goal of HetNet is to improve spectral efficiency and coverage, in the light of these goals, this thesis will be utilizing multiple antenna techniques or simply called spatial interference coordination schemes to manage interference in HetNet. For the centralized HetNet implementation, we utilize partial cooperation among BSs in HetNet to be able to coordinate interference in HetNet, while for distributed implementation, we don't need any sharing of CSI to design the beamformers needed to spatially separate signals to desired UEs in the HetNet. This approach will enable us maximize the spectral efficiency of the network as well as controlling the interference problem in HetNet.

Chapter 3

Global Optimization of Weighted Sum-Rate for Downlink Heterogeneous Cellular Networks

3.1 Introduction

Heterogeneous cellular Network (HetNet), is a network that is composed of small cells distributed around the coverage area of a conventional macrocellular base station (MBS). HetNet is regarded as a cost-efficient, energy-efficient and spectrum-efficient solutions to improve system coverage and capacity. However, this improved capacity can only be achieved if good interference management scheme is in place for the system under consideration. In this chapter, we aim at maximizing the weighted sum-rate for downlink HetNet while fulfilling some power and interference constraints. Maximizing the sum-rate of HetNet is usually regarded as a difficult problem to solve. However, we provide a solution to this problem using global optimization methods.

Global optimization is a unique type of optimization that is only interested in seeking the optimal optimization variable, which will maximize/minimize the utility/objective over all feasible points. By feasible points, we mean points that satisfy a given constraint set in an optimization problem. Furthermore, an optimization variable is globally optimal or regarded as the solution of the optimization problem if it has the smallest objective or largest utility value among all other feasible points that satisfy the given constraints of the optimization problem. It is quite different from local optimization, where the goal is to seek for a point that is locally optimal, which means that it maximizes/minimizes the utility/objective functions among feasible points that are close to it and not necessarily the one that maximizes/minimizes the utility or objective function. Usually, this point found is not guaranteed to have a higher(lower) utility(objective) value than all other feasible points. So in this chapter, we determine the global solution of the weighted sum-rate maximization problem for a two-tier HetNet, subject to some sets of constraints. Maximizing the weighted sum-rate of a system is generally regarded as an *Non-Deterministic Polynomial-Time hard* (NP-hard) nonconvex problem because there are no known efficient algorithms that can solve it in polynomial time. Usually most authors shy away from this problem because of its non-convex nature, instead, local optimization methods are most widely adopted. In local optimization methods [53,54], the global optimal solution is usually sacrificed for a local optimal solution which can be achieved in polynomial time. However, local optimization methods can only be applied to optimization problems whose objective and constraint functions are differentiable, also they require an initial guess for the starting point of their optimization process which might be critical to the objective value obtained. Similarly, others prefer to solve the reformulated convex version of the non-convex problem, which usually can be solved efficiently and also produces (roughly) global solution. However, the downside to it is that the solution found is not for the exact problem because the search space of the original problem has been reduced, hence is suboptimal too.

3.1.1 Prior Works and Contribution

Many works have been done for maximizing the weighted sum-rate of a system but most of these works are targeted at either single-tier coordinated multi-cell system [55,56] or single-tier single-cell system [57,58] where there are no variations in the power class of the *base stations* (BSs). In our work we consider HetNet and the impact of the significant interfering power generated by the *macro-base station* (MBS) to other co-channel UEs in the multi-tier heterogeneous system together with the interference between small cells.

In this work, we propose a technique which first solves a convex version of the non-convex NP-Hard optimization problem, in order to obtain a good starting point and then performs an exhaustive search within the feasible set of the sum-rate optimization problem to find the global optimum of the non-convex optimization problem. This global optimum is achieved based on *Branch and Bound* (B&B)

framework. B&B algorithms are methods for global optimization for non-convex problems [59]. Implementation of B&B method is peculiar to each problem structure because there is no generic B&B algorithm that can find the global optimum for all non-convex problems. Thus, different authors can develop their algorithm based on B&B and apply it to their problem if possible. In [60, 61], the authors applied B&B algorithm to solve the problem of sum-rate maximization in single-tier multi-cell networks. In [62], the authors utilized B&B method while solving the problem of joint beamforming and user maximization techniques for cognitive radio networks. Furthermore, authors in [63] devised an algorithm based on a B&B framework to maximize the capacity in multihop cognitive radio networks under *signal to interference and noise ratio* (SINR) model. However, in all the aforementioned reviewed works, none have considered developing a B&B algorithm that will maximize the sum-rate for a HetNet scenario. Also, the B&B algorithms developed by these authors if applied to our problem may be inappropriate or may not yield any significant result because of the significant interference situation in HetNet. Therefore in this work, we devised an algorithm based on B&B and adapt it to a two-tier HetNet scenario to be able to solve the weighted sum-rate maximization optimization problem with the associated constraints. In our approach, the feasible solution set that satisfies the constraints of the optimization problem is a subset of a box interval which is assumed to be compact [64] and also a subset of the non-negative orthant $\mathbb{R}_+^{K_r}$, where the optimal solution can be selected from. The B&B algorithm efficiently computes a lower bound and upper bound on the optimal value over this box. The lower bound of this box is initially found by reformulating the non-convex optimization problem into a heuristic convex problem and then solved. While the upper bound is found by assuming each UE achieved the best individual rate using a beamforming scheme that maximizes individual UE rates. This B&B algorithm is iterative and will only terminate if the difference between the upper bound and the lower bound is smaller than a threshold. If not, the initial box is split into two using bisection method where their respective upper and lower bound are determined again, in each iteration the convex feasibility of the point obtained through line search is checked to make sure it satisfies the constraints of the feasible set, otherwise it is discarded. The iterative process continues until a global optimal value is

achieved.
The compromise in achieving the global solution is efficiency. However, in this work, we limit our consideration to a small number of variables and the total number of UEs considered.

3.1.2 Chapter Organization

The rest of the chapter is organized as follows. Section 3.2 describes the system model and problem formulation and it shows how the non-convex problem can be reformulated into a convex heuristic problem which can be easily solved by efficient algorithms. It also shows how to formulate convex feasibility problems. Section 3.3 describes our devised algorithm based on B&B methods while in Section 3.4 we show using simulation results how our proposed method outperforms other existing methods. We summarize our work in section 3.5.
Notations: $(\cdot)^H$ is the transpose conjugate operation, $(\cdot)^T$ is the transpose operation, $\|\cdot\|$ is the norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the expectation over a random variable and $\mathbb{R}_+^K$ denotes the set of real K-vectors with non-negative elements. We use $\mathbb{C}$ to denote the set of complex numbers, while $\mathbb{C}^K$ denotes the set of complex K-vectors. We use uppercase boldface letters for matrices and lower-case boldface for vectors.

3.2 System Model

We consider the downlink of a two-tier HetNet as depicted in Fig. 3.1, which consist of K_p pico cells and a single macrocell making it a total of K_t cells in the system. Each BS has N antennas and communicates with a single active UE which has a single antenna[1] per cell, making the total number of cells to be equal to the total number of UEs in the system. The pico cells are underlaid in the coverage area of the macrocell and all cells use the same carrier frequency. The respective BSs are connected through a limited backhaul link, hence each BS will

[1]We limit each UE to have a single antenna for practical reasons, such as keeping a low cost, reducing the UE hardware complexity and also preserving of battery life. It also reduces the channel state information (CSI) required to be known at the transmitter.

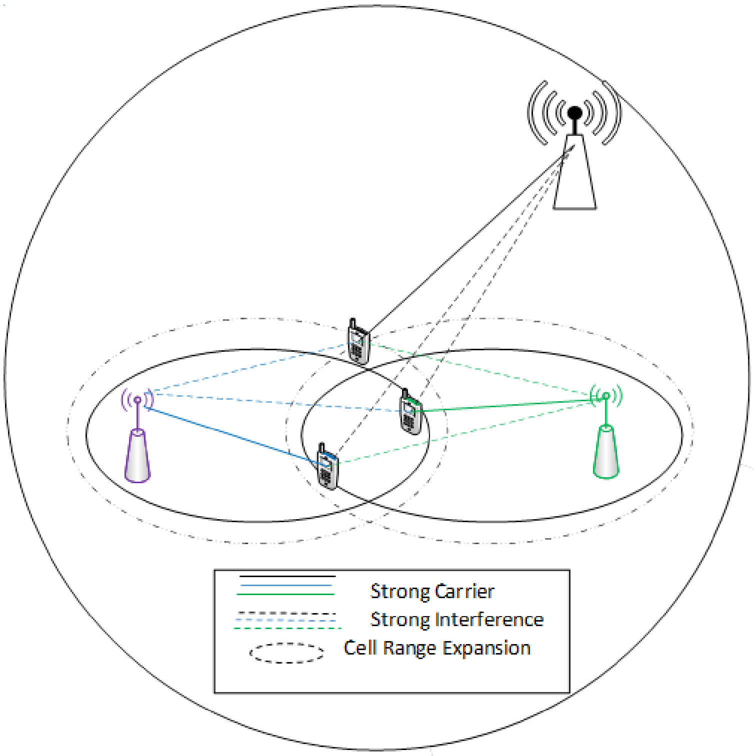

Figure 3.1: Downlink two-tier HetNet model with two pico cells in the coverage area of MBS

only send data to UE belonging to its cell while the beamformers can be jointly optimized by all BSs in the network. We denote the set of BSs in the HetNet by $\mathcal{M} = \{0, 1, \ldots, K_p\}$ where 0 represents the macro BS, also the jth BS is denoted BS_j. The set of UEs served by BS_j is denoted by $\mathcal{S}_j \subset \{1, \ldots, K_r\}$, where K_r denotes the total number of UEs in HetNet. The complex baseband received signal at UE k is $y_k \in \mathbb{C}$ and given by

$$y_k = \sum_{j=1}^{K_t} \sqrt{g_{k,j}} (\mathbf{h}^s_{k,j})^H \mathbf{x}_j + n_k, \tag{3.1}$$

where $\sqrt{g_{k,j}}$ is the large-scale pathloss from the BS_j to UE k. Also $\mathbf{h}^s_{k,j} \in \mathbb{C}^{N\times 1}$ is the small scale (fading) channel vector from the BS_j to UE k while $\mathbf{x}_j$ is the transmitted signal at BS_j. Furthermore, $n_k \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex gaussian, distributed as $n_k \sim \mathbb{C}\mathcal{N}\ (0, \sigma^2)$, where σ^2 is the noise power. Assuming BS$_l$ is the serving BS of UE k, the signal-to-noise-and-interference ratio (SINR) at UE k is

$$SINR_k = \frac{|\mathbf{h}_{k,l}^H \mathbf{x}_l|^2}{\sigma^2 + \sum_{\substack{j \neq l \\ j \in \mathcal{M}}} |\mathbf{h}_{k,j}^H \mathbf{x}_j|^2}, \tag{3.2}$$

where $\mathbf{h}_{k,j} \triangleq \sqrt{g_{k,j}} \mathbf{h}_{k,j}^s$.

3.2.1 Coordinated Beamforming: Problem Formulation

Recall that in coordinated beamforming [38], BS$_l$ transmit a signal to UE k while the beamformers from each BS are jointly optimized by all BSs in the system considered. The transmitted signal by each BS to its served UE is

$$\mathbf{x}_l = \mathbf{w}_k s_k, \quad \forall k \in \mathcal{S}_l, \tag{3.3}$$

where $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$ and $s_k \in \mathbb{C}$ are transmit beamforming vector and information symbol for UE k respectively, s_k is normalized to unit power, $\mathbb{E}[|s_k|^2] = 1$.
Hence the achievable data rate for UE k is

$$r_k = log_2(1 + SINR_k), \quad k = 1, \ldots, K_r, \tag{3.4}$$

which can be expressed in a more detailed form as

$$r_k(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) = log_2 \left(1 + \frac{|\mathbf{h}_{k,l}^H \mathbf{w}_k|^2}{\sigma^2 + \sum_{j \neq l} |\mathbf{h}_{k,j}^H \mathbf{w}_p|^2} \right), \tag{3.5}$$

where $\{\mathbf{w}_k\}_{k=1}^{K_r}$ denotes the set of beamforming vectors of the system.
In this chapter, the target is to select $\{\mathbf{w}_k\}, \quad k = 1, \ldots, K_r$, that will maximize the weighted sum-rate of HetNet, under the given constraints which are:

$$\begin{aligned} \underset{\{\mathbf{w}_k\}_{k=1}^{K_r}}{\text{maximize}} \quad & \sum_{k=1}^{K_r} u_k r_k(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) \\ \text{subject to} \quad & C1 : ||\mathbf{w}_k||_2^2 \leq P_0, \ \forall k \in \mathcal{S}_0, \\ & C2 : ||\mathbf{w}_k||_2^2 \leq P_s, \ \forall k \in \mathcal{S}_s, \ s = 1, \ldots, K_p, \\ & C3 : \mathbf{w}_n^H \mathbf{G}_{k,0} \mathbf{w}_n \leq \tau_k, \ \forall n \in \mathcal{S}_0, \ \forall k \in \mathcal{S}_s, \end{aligned} \tag{3.6}$$

where the utility function represents the weighted sum-rate of the system with the nonnegative factor u_k denoting the individual weights assigned to UE k, chosen to reflect different level of concern about the individual channel gain. A larger gain has larger weight and vice versa, also the second, third and fourth row of (3.6) represent MBS power constraint, *low power node* (LPN) power constraint and interference power constraint (i.e., interference generated from MBS to UE k). Henceforth these constraints will be denoted by $C1 \sim C3$. $\mathbf{G}_{k,0} \triangleq \mathbf{h}_{k,0}\mathbf{h}_{k,0}^H$ is a *positive semidefinite* (PSD) matrix ($\mathbf{G}_{k,0} \succeq \mathbf{0}$), where $\mathbf{h}_{k,0}$ is the channel vector from the MBS to UE k and τ_k is the threshold which controls the allowable level of interference in UE k. These constraints ($C1 \sim C3$) are convex but the utility function is not convex thanks to the $SINR_k$ which are non-convex functions of the beamforming vectors $\{\mathbf{w}_k\}$. Note that an optimization problem is said to be convex if the utility function and the constraints functions are convex[2]. In the light of these conditions, (3.6) can be declared as non-convex. This problem is not tractable because it involves a very large search space where the optimal solution will be selected from. We can reduce the search space of the problem formulation in order to make it convex. The problem is reformulated as

$$\begin{aligned}
&\underset{\{\mathbf{w}_k\}_{k=1}^{K_r}}{\text{minimize}} && -\sum_{i=1}^{K_r} u_k r_k(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) \\
&\text{subject to} && |\mathbf{h}_{k,l}^H \mathbf{w}_k|^2 \geq \gamma_k(\sigma^2 + \sum_{j \neq l} |\mathbf{h}_{k,j}^H \mathbf{w}_p|^2), \\
&\mathit{and} && C1 \sim C3 \ \mathit{in}\ (3.6).
\end{aligned} \tag{3.7}$$

In (3.7) the second row represents the *quality of service* (QoS) constraint expected at each UE k in the system and is generally known as the $SINR_k$ constraint, where $SINR_k \geq \gamma_k \ k = 1, \ldots, K_r$ and $C1 \sim C3$ denotes all the power and interference constraints as in (3.6). In this case γ_k denotes the QoS threshold for each UE in the system while the term $\sum_{j \neq l} |\mathbf{h}_{k,j}^H \mathbf{w}_p|^2$ represents the total interference towards the desired UE k. Note, by including the QoS constraint, we aim to reduce the search space of the optimization problem in order to make it tractable. This formulation is still non-convex but in the next subsection, we show how (3.7), an NP-hard non-convex problem can be made convex.

[2] A function $f_n : \mathbb{R}^k \to \mathbb{R}$ is called convex on $[\mathbf{a}\ \mathbf{b}]$ if for any $\mathbf{x}_1, \mathbf{x}_2 \in [\mathbf{a}\ \mathbf{b}]$ and $r \in [0,1]$, $f_n(r\mathbf{x}_1 + (1-r)\mathbf{x}_2) \leq r f_n(\mathbf{x}_1) + (1-r) f_n(\mathbf{x}_2)$.

3.2.2 Convex Heuristic Reformulation

To solve the non-convex problem, convex heuristics are easily adopted by researchers because of its efficiency. However, it produces a suboptimal solution to the non-convex problem. To reformulate (3.7) into a convex problem, this can be achieved by either fixing the γ_k value at each UE or by fixing the interference term. Note by fixing either of these terms we are actually limiting the search space of the optimization problem in order to achieve convexity. In this subsection, we prefer limiting the interference to a particular fixed threshold Γ_k which is more practical and not equating it to zero as in the case when zero-forcing technique [65,66] is applied, which is seen as an overreaction. Because in practice it is not certain that full *channel state information*(CSI) will be available at the transmitter and it is superfluous to reduce the interference far below the background noise. Hence we obtain our convex reformulation as

$$\begin{aligned} \underset{\{\mathbf{w}_k\}_{k=1}^{K_r}}{\text{minimize}} \quad & -\sum_{k=1}^{K_r} u_k r_k(\mathbf{w}_1, \ldots, \mathbf{w}_{K_r}) \\ \text{subject to} \quad & |\mathbf{h}_{k,l}^H \mathbf{w}_k|^2 \geq \gamma_k(\sigma^2 + \Gamma_k), \\ & \sum_{j \neq l} \mathbf{w}_p^H (\mathbf{h}_{k,j} \mathbf{h}_{k,j}^H) \mathbf{w}_p \leq \Gamma_k, \\ \textit{and} \quad & C1 \sim C3 \ \textit{in} \ (3.6). \end{aligned} \tag{3.8}$$

Then semidefinite relaxation [67] can be applied to the quadratic terms in (3.8) after which it can be efficiently solved by a solver known as SeDuMi [68] or SDPT3 [69] implemented in CVX [70] - a Matlab-based modeling system for convex optimization (see Appendix B.1). We will use the result from here to initialize the proposed B&B procedure which will lead to the global optimal solution of the non-convex problem. We will also use it as a benchmark to compare with our proposed method.

3.2.3 Convex Feasibility Problem

This feasibility problem will help us in our proposed method to always check if a selected solution from a box interval is feasible or not. If not, it can be discarded because it cannot be the optimal solution. Convex feasibility problem is to find any feasible solutions without regard to the utility function. In our case, we seek the set of beamformers $\{\mathbf{w}_k\}_{k=1}^{K_r}$ that satisfy the convex constraints. In this case γ_k value is believed to be known a *priori* but can be computed as $\gamma_k \triangleq 2^{r_k} - 1$ obtainable from (3.4), hence our convex feasibility problem formulation can be formulated as

$$\begin{aligned} &\text{find} && \{\mathbf{w}_k\} \\ &\text{subject to} && |\mathbf{h}_{k,l}^H \mathbf{w}_k|^2 \geq \gamma_k(\sigma^2 + \sum_{j \neq l} |\mathbf{h}_{k,j}^H \mathbf{w}_p|^2), \\ &and && C1 \sim C3 \ in \ (3.6). \end{aligned} \tag{3.9}$$

In order to be easily solved by CVX (a package for specifying and solving convex programs), it demands that the objective function or utility function must be added, even if it represents a dummy, hence the feasibility problem can be formulated as a power control problem such as minimizing some transmitted power in the system subject to QoS constraint, power and interference constraints. If we denote the total power to be minimized as ϑ, which is assumed to be greater than the total power available in the system (e.g., any value > 1). What remain is to replace the upper bound of the MBS power constraint and the LPN power constraints of $C1 \sim C2$ in (3.6) with ϑP_0 and ϑP_s respectively before minimizing over ϑ. The power minimization problem can be formulated as

$$\begin{aligned} &\underset{\{\mathbf{w}_k\}_{k=1}^{K_r}, \vartheta}{\text{minimize}} && \vartheta \\ &\text{subject to} && |\mathbf{h}_{k,l}^H \mathbf{w}_k|^2 \geq \gamma_k(\sigma^2 + \sum_{j \neq l} |\mathbf{h}_{k,j}^H \mathbf{w}_p|^2), \\ &&& ||\mathbf{w}_k||_2^2 \leq \vartheta P_0, \ \forall k \in \mathcal{S}_0, \\ &&& ||\mathbf{w}_k||_2^2 \leq \vartheta P_s, \ \forall k \in \mathcal{S}_s, \ s = 1, \ldots, K_p, \\ &and && C3 \ in \ (3.6). \end{aligned} \tag{3.10}$$

Observe that if the optimal solution $\vartheta^* \geq 1$, then this implies that the feasible solutions selected are infeasible. Note, the optimization problem in (3.10) is convex if the SINR constraint is rewritten as a *second order cone* (SOC) constraint [71]. After we find a feasible solution, we can use other steps in the B&B framework to obtain the global solution.

3.3 Branch and Bound Method for HetNet

Branch and Bound (B&B) method is the method through which we can get the global optimal solution of an NP-hard non-convex weighted sum-rate maximization problem for a two-tier HetNet. It is an iterative method that requires at least two procedures that can efficiently calculate and improve a lower bound (f_{min}) and an upper bound (f_{max}) on the optimal value of the non-convex problem, over a given set or region. In our case, the set or region considered is a subset of a box (K_r-dimensional) interval, $[\mathbf{a}\ \mathbf{b}]$. This set is the feasible set that satisfies our problem formulation in (3.6). Also the utility function in our optimization problem is lipschitz continuous[3] and monotonically increasing over this box interval. The Lipschitz constant will provide limit on how fast the function varies. We denote the initial box as $\mathcal{B} = [\mathbf{a}\ \mathbf{b}] \subseteq \mathbb{R}_+^{K_r}$, this box is assumed to be compact[4] and normal[5] [73] and houses all kind of rates from the worst to the best rates. Furthermore, $\mathbf{a}$ denotes the worst rate vector achievable by UEs in the system thus $\mathbf{a} = \mathbf{0} \in \mathbb{R}_+^{K_r}$ while $\mathbf{b} \in \mathbb{R}_+^{K_r}$ denotes the best rate vector achievable by UEs in the system using egoistic beamforming [74] scheme, such that $\mathbf{a} < \mathbf{b}$. Also $[\mathbf{a}\ \mathbf{b}]$ is defined to be the set of all rates ($\mathbf{r} \in \mathbb{R}_+^{K_r}$) achievable in the system such that $\mathbf{a} \leq \mathbf{r} \leq \mathbf{b}$. Egoistic beamforming is a beamforming scheme where beamformers are designed to maximize the array gain of a single UE in a system. Note this beamforming scheme will always be suboptimal if there are other sources of interference, hence

$$b_k = log_2\left(1 + \frac{p_k|\mathbf{h}_{k,l}^H\tilde{\mathbf{w}}_\mathbf{k}|^2}{\sigma^2}\right) \ k = 1,\ldots,K_r, \tag{3.11}$$

where $\mathbf{b} = [b_1 \ldots b_{K_r}]^T$. p_k is the transmit power constraint at each transmitter. The egoistic beamformers can be obtained using

[3] A function $f : [\mathbf{a}\ \mathbf{b}] \to \mathbb{R}$ is said to be lipschitz continuous with lipschitz constant L_f on $[\mathbf{a}\ \mathbf{b}]$, if there is a nonnegative constant L_f such that $|f(\mathbf{x}) - f(\dot{\mathbf{x}})| \leq L_f||\mathbf{x} - \dot{\mathbf{x}}||_1$, $\forall\ \mathbf{x}, \dot{\mathbf{x}} \in [\mathbf{a}\ \mathbf{b}]$ and $\mathbf{x} \geq \dot{\mathbf{x}}$.

[4] A compact set, intuitively can be described as an interval set, bounded and closed see [72, Theorem 4.14].

[5] See Appendix C.1 for more information. A set $\mathcal{M} \subset \mathbb{R}_+^{K_r}$ is set to be normal if, for any two points $\mathbf{x}, \dot{\mathbf{x}} \in \mathbb{R}_+^{K_r}$ such that $\dot{\mathbf{x}} \leq \mathbf{x}$, if $\mathbf{x} \in \mathcal{M}$, then $\dot{\mathbf{x}}$, too.

$$\tilde{\mathbf{w}}_{\mathbf{k}} = \arg \max_{\substack{\mathbf{w}_k \in \mathbb{C}^{N\times 1} \\ ||\mathbf{w}_k||^2=1}} |\mathbf{h}_{k,l}^H \mathbf{w}_k|^2. \tag{3.12}$$

This best rate vector $\mathbf{b} = [b_1 \dots b_{K_r}]^T$ is not always feasible when co-channel interference is considered in the system while designing the beamformers.
Our feasible set from the original problem formulation for the r_k that optimizes the sum-rate can be denoted as

$$\mathcal{Z} = \left\{ \Big(r_1(\mathbf{w}_1, \dots, \mathbf{w}_{K_r}), \dots, r_{K_r}(\mathbf{w}_1, \dots, \mathbf{w}_{K_r}) \Big) : (\mathbf{w}_1, \dots, \mathbf{w}_{K_r}),\ C1 \sim C3 \ as \ in \ (3.6) \right\}, \tag{3.13}$$

where $\mathcal{Z}$ denotes the set of all feasible solution $(r_1, \dots, r_{K_r})$ for which $(\mathbf{w}_1, \dots, \mathbf{w}_{K_r})$ are feasible and satisfy the $C1 \sim C3$ in (3.6). We can show that this set is compact and normal on $\mathbb{R}_+^{K_r}$, see Appendix D.1. Furthermore, our optimization problem for maximizing the sum-rate of the system in this section is similar to searching for a feasible solution in the box that has the minimum Euclidean distance to $\mathbf{b}$, and this is formulated as

$$\begin{aligned} &\underset{\mathbf{r}}{\text{maximize}} && f(\mathbf{r}) \\ &\text{subject to} && \mathbf{r} \in \mathcal{Z}. \end{aligned} \tag{3.14}$$

Note that our utility function in (3.14) is given as

$$f(\mathbf{r}) = \sum_{k=1}^{K_r} u_k r_k(\mathbf{w}_1, \dots, \mathbf{w}_{K_r}), \tag{3.15}$$

where $\mathbf{r} = [r_1 \dots r_{K_r}]^T$ is the rate vector achievable by UEs in the system. Note that $\mathcal{Z} \subseteq [\mathbf{a}\ \mathbf{b}]$. To achieve the formulation of (3.14) on standard form, see Appendix A.1. The lower bound on the optimal value of the non-convex problem can be found from its convex reformulation, and in this chapter, (3.8) gives the lower bound on the optimal value of (3.14), while (3.9) test for feasible solutions on the optimal solution of the non-convex problem. Let $\grave{\mathbf{r}}$ represents the feasible solution of the box $\mathcal{B}$, that is used to obtain the lower bound on the optimal value. Hence we denote the lower bound on the optimal value of this box as $f_{min}^{\mathcal{B}} = f(\grave{\mathbf{r}})$. Similarly, since $\mathbf{b}$ represents the best rate vector in the system, though might not be feasible, we denote the upper bound on the optimal value of this box as $f^{\mathcal{B}}_{max} = f(\mathbf{b})$. Hence, $f^{\mathcal{B}}_{min} \leq f_{opt} \leq f^{\mathcal{B}}_{max}$, where f_{opt} represents the optimal value of the sum-rate of the system, f_{min} and f_{max} denote lower bound and upper bound on the optimal value of the weighted sum-rate of the system respectively. Similarly, $\grave{\mathbf{r}} \leq \mathbf{r}_{opt} \leq \mathbf{b}$, where $\mathbf{r}_{opt}$ denotes the optimal solution of the system while $\grave{\mathbf{r}}$ and $\mathbf{b}$ denote a local feasible solution and the best solution achievable in the system.

3.3.1 Branching

This is the process of spliting the initial box $\mathcal{B}$ into more than one partitions, usually we start by splitting the box into two partitions. Branching will only be necessary if $f^{\mathcal{B}}_{max} - f^{\mathcal{B}}_{min} > \epsilon$, where $\epsilon > 0$ is the accuracy tolerance of the sum-rate in the B&B method. This expression $f_{max} - f_{min} \leq \epsilon$ can be seen as a termination criteria. The splitting of box $\mathcal{B}$ is done in such a way that they cannot overlap each other, after which the upper and lower bound on the optimal value are calculated for each. After splitting, $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$ where $\mathcal{B}_1$ denotes box 1 and $\mathcal{B}_2$ denotes box 2. Assuming $\mathcal{B}_1 = [\mathbf{a}_1 \ \mathbf{b}_1]$ and $\mathcal{B}_2 = [\mathbf{a}_2 \ \mathbf{b}_2]$ where $\mathbf{a}_1$ and $\mathbf{a}_2$ denote the lower corners of boxes 1 and 2 respectively; $\mathbf{b}_1$ and $\mathbf{b}_2$ denote the upper corners of boxes 1 and 2 respectively. Note that $\mathbf{b}_2 = \mathbf{b}$ and $\mathbf{a}_1 = \mathbf{a}$ of the initial box $\mathcal{B}$ respectively. The feasible solution of the new boxes can be chosen by comparing the feasible solution of the initial box to the lower conner of box 2, if greater than or equal to it, will give rise to a new feasible solution for boxes 1 and 2, which can be computed as

$$\begin{aligned} \mathbf{r}^{\mathcal{B}_1} &= \begin{cases} \grave{\mathbf{r}} - [\grave{\mathbf{r}} - \mathbf{b}_1]_+, & \grave{\mathbf{r}} \geq \mathbf{a}_2, \\ \grave{\mathbf{r}}, & \text{otherwise}, \end{cases} \\ \mathbf{r}^{\mathcal{B}_2} &= \grave{\mathbf{r}}, \end{aligned} \tag{3.16}$$

where the operator $[\cdot]_+$ replaces negative elements with zero. Summarily, the new feasible solution for $\mathcal{B}_1$ and$\mathcal{B}_2$ becomes $\grave{\mathbf{r}}$ and $\grave{\mathbf{r}}$ respectively if for $\mathcal{B}_1$, $\grave{\mathbf{r}} \leq \mathbf{a}_2$ in the first row of (3.16). The local upper bounds on the optimal value for both boxes can also be chosen as

$$
\begin{aligned}
f_{max}^{\mathcal{B}_1} &= min(f(\mathbf{b}_2),\ f(\mathbf{b}_1)),\\
f_{max}^{\mathcal{B}_2} &= f(\mathbf{b}_2)
\end{aligned}
\tag{3.17}
$$

respectively, where the min(·) operator selects the smallest value of its argument. Futhermore, we proceed by removing or prunning parts of the boxes which cannot contain the optimal solution, knowing that $f_{min}^{\mathcal{B}_1} \leq f_{opt} \leq f_{max}^{\mathcal{B}_2}$. This can be done by checking for any part that is less than $f_{min}^{\mathcal{B}_1}$ or greater than $f_{max}^{\mathcal{B}_2}$, these parts cannot contain the optimal solution.

Generally we assume *without loss of generality* (w.l.o.g) that $f_{max}^{\mathcal{B}_t}$ $(for\ t = 1,2)$ is non-increasing while $f_{min}^{\mathcal{B}_t}$ $(for\ t = 1,2)$ is non-decreasing. We assume that the new boxes after prunning will be denoted as $\tilde{\mathcal{B}}_t = [\tilde{\mathbf{a}}_t\ \tilde{\mathbf{b}}_t]\ \forall t = 1,2$. Furthermore, these targeted boxes will be made smaller, because it is important to reduce the search space for the optimal solution in order to minimize complexity. In order to achieve this objective, the lower corners of the new boxes can be computed as [61]

$$
\begin{aligned}
\tilde{a}_1 &= (1-\nu^{\mathcal{B}_1})\mathrm{b}_1^k + \nu^{\mathcal{B}_1}\mathrm{a}_1^k,\\
\tilde{a}_2 &= (1-\nu^{\mathcal{B}_2})\mathrm{b}_2^k + \nu^{\mathcal{B}_2}\mathrm{a}_2^k,
\end{aligned}
\tag{3.18}
$$

where $\tilde{\mathbf{a}} = [\tilde{a}_1 \cdots \tilde{a}_{K_r}]^T$, also, $\tilde{\mathbf{a}}_1$ and $\tilde{\mathbf{a}}_2$ denote the lower corners of the new boxes after prunning, b_1^k and a_1^k denote the k th element of the upper and lower corners of the old boxes respectively before prunning. Furthermore, the parameter $\nu^{\mathcal{B}_t}$ take values between zero and one and can be computed as

$$
\nu^{\mathcal{B}_t} = min\left(\frac{f(\mathbf{b}_t) - f_{min}^{\mathcal{B}_t}}{\mathrm{b}_t^k - \mathrm{a}_t^k}, 1\right)\ t = 1,2. \tag{3.19}
$$

Similarly, the upper conners of the new boxes can be computed as

$$
\begin{aligned}
\tilde{b}_1 &= (1-\mu^{\mathcal{B}_1})\tilde{a}_1 + \mu^{\mathcal{B}_1}\mathrm{b}_1^k,\\
\tilde{b}_2 &= (1-\mu^{\mathcal{B}_2})\tilde{a}_2 + \mu^{\mathcal{B}_2}\mathrm{b}_2^k,
\end{aligned}
\tag{3.20}
$$

where $\tilde{\mathbf{b}} = [\tilde{b}_1 \cdots \tilde{b}_{K_r}]^T$, also, $\tilde{\mathbf{b}}_1$ and $\tilde{\mathbf{b}}_2$ denote the upper corners of the new boxes after prunning. The parameter $\mu^{\mathcal{B}_t}$ can be computed as

$$
\mu^{\mathcal{B}_t} = min\left(\frac{f_{max}^{\mathcal{B}_t} - f(\tilde{\mathbf{a}}_t)}{\mathrm{b}_t^k - \tilde{\mathrm{a}}_t^k}, 1\right)\ t = 1,2, \tag{3.21}
$$

note that the min operator in (3.19) and (3.21) ensure that $\nu^{\mathcal{B}_t}$, $\mu^{\mathcal{B}_t} \leq 1$.

One of these boxes contain the optimal solution, and the most likely one is box 2. This is because maximizing the weighted sum-rate is similar to searching for the feasible point that has the minimum Euclidean distance to the best infeasible individual rate achievable in the system. We check if this box is feasible by solving (3.9) using $\tilde{\mathbf{a}}_2$ to get the QoS constraint. This leads us to bounding procedure in the next subsection.

3.3.2 Bounding Procedure and Line Search

If the box is feasible, bounding procedure involves searching for the best lower and upper bound on the optimal value in each iteration, after which line search technique can be used to locate the optimal solution in that box. This line search corresponds to looking for the best feasible point with the minimum Euclidean length to the best infeasible individual rates in the box. This is achieved by starting with an initial feasible point $\tilde{\mathbf{a}}_2$ which is then added to the product of the step size (positive scalar) and the ascent search direction. The search direction is denoted as $s_d = \frac{(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)}{||(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)||_2}$, and the step size is denoted as $\alpha \in [0,\ ||(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)||_1]$ whose set is searched for the best value. Every value selected when added to $\tilde{\mathbf{a}}_2$ must satisfy the feasibility condition. The line search method can be computed for any selected box using

$$\mathbf{n} = \tilde{\mathbf{a}}_2 + \alpha s_d \tag{3.22}$$

where $\mathbf{n}$ is a feasible point better than $\tilde{\mathbf{a}}_2$, this feasibility point is evaluated by solving (3.9). In each iteration we check to know if the present optimal value of the feasible point is greater than the previous ones, if so we finally update the value to be the best optimal value based on the feasible point. Finally we set $f_{min} = max(f^{\mathcal{B}}(\dot{\mathbf{r}}), f^{\tilde{\mathcal{B}}_t}(\mathbf{n}))$, and $f_{max} = max(f^{\mathcal{B}}(\mathbf{b}), f^{\tilde{\mathcal{B}}_t}(\tilde{\mathbf{b}}))\ \forall t = 2$.
We summarized the B&B method for HetNet using Algorithm 1.

Algorithm 1 Branch and Bound Method for HetNet

Require: B&B accuracy tolerance $\epsilon > 0$

Require: compute best infeasible individual rate $\mathbf{b}$ using (3.11);

Require: compute feasible solution $\grave{\mathbf{r}}$ of initial box using (3.8);

Require: initial box $\mathcal{B} = [\mathbf{a}\ \mathbf{b}]$;

Ensure: $f_{min} = f^{\mathcal{B}}(\grave{\mathbf{r}})$ and $f_{max} = f^{\mathcal{B}}(\mathbf{b})$;

while $f_{max} - f_{min} > \epsilon$ **do**

  split the initial box $\mathcal{B}$ into two, say $\mathcal{B}_1$ and $\mathcal{B}_2$;

  compute and compare the lower and upper bound on the optimal solution for each branch using (3.17);

  prune and reduce the new boxes based on current bounds on optimal value using (3.18)~(3.21) ;

  check feasibility of the outermost box using (3.9);

  If feasible,

  apply line search method using (3.22);

  obtain best feasible point $\mathbf{n}$ and upper bound $f^{\tilde{\mathcal{B}}_t}(\tilde{\mathbf{b}})$;

  set $f_{min} = max(f^{\mathcal{B}}(\grave{\mathbf{r}}), f^{\tilde{\mathcal{B}}_t}(\mathbf{n}))$;

  set $f_{max} = max(f^{\mathcal{B}}(\mathbf{b}), f^{\tilde{\mathcal{B}}_t}(\tilde{\mathbf{b}}))$;

end while

Ensure: final optimal bound $[f_{min}, f_{max}]$;

Ensure: final optimal solution $\mathbf{r}_{opt} = max(\grave{\mathbf{r}}, \mathbf{n})$.

3.4 Simulation Results

In this section, we evaluate the performance of our proposed method, which is based on branch and bound method for HetNet (which combines both convex optimization techniques and search methods) by comparing with the convex optimization method and the egoistic beamforming method, based on average achievable sum-rate, SNR and cumulative distributed function.

3.4.1 Simulation Setting

We considered a simple HetNet system simulation setting with two randomly distributed PBSs deployed in the coverage area of MBS. We assume that the UEs in the HetNet are uniformly distributed and are located at the CRE of the pico cells such that each PBS served UE will receive significant inter-cell interference. The UE served by MBS is located at 240m from the MBS, also the distance between the macrocell UE and the PBS is roughly between 40m and 45m, while the distance between the picocell UE and the MBS is between 230m and 270m. Other simulation parameters are as follows: the transmit powers of the macro and pico BSs are respectively 46dBm and 30dBm. The large-scale path loss model of the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{k,j}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{k,j}}{10^3})$ where $d_{k,j}$ is the distance of UE i to the BS. The channel vectors are generated as uncorrelated rayleigh fading while the large-scale pathloss given by

$$g_{i,k} = \frac{\psi}{d_{k,j}^n}, \tag{3.23}$$

where ψ is a constant which accounts for system losses due to antenna characteristics, etc., it can be determined through the large scale path loss models for both macro and pico cells respectively. n is the path-loss exponent, typically $n > 3$, while $d_{k,j}$ is the distance between BS_j and the kth UE. The fixed system setting for the simulation are as follows; $N = 3$, $K = 3$. 10000 monte carlo runs are used for the channel realizations, while the maximum number of iteration and convex function evaluations for the B&B algorithm are 3000 and 4000 respectively. The B&B accuracy tolerance $\epsilon = 0.01$, while the step size is fixed $\alpha = 0.1$. This settings will be used except otherwise indicated.

Fig. 3.2 shows the average sum-rate achievable as a function of SNR. It compares the average sum-rate achieved in the system using our proposed method, the heuristic convex method and the egoistic beamforming method. Our proposed method outperforms both the heuristic convex method and the egoistic beamforming methods in both low and high SNRs, the lowest performing method is achieved by egoistic beamforming which shows single cell processing without beamforming coordination. It treats all out-of-cell interference as Gaussian noise, hence the reason behind the poor performance.

In Fig. 3.3 the *cumulative distribution function* (CDF) of average sum-rate achieved for the system by different methods are illustrated clearly. The proposed B&B scheme outperforms the heuristic convex and the egoistic schemes.

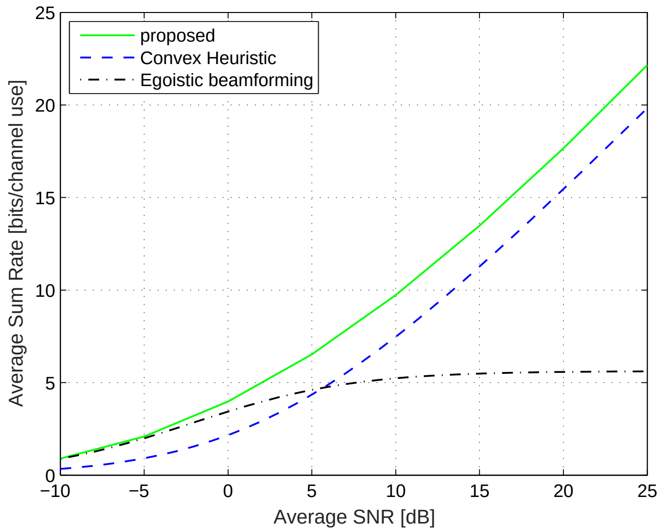

Figure 3.2: Average sum-rate achievable at different SNR for $N = 3$

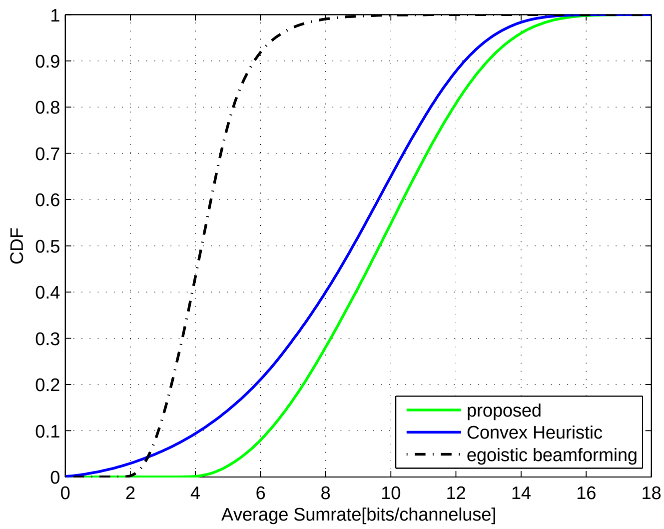

Figure 3.3: The CDF of the average sum-rate achieved by different methods

Fig. 3.4 compares our proposed method with the brute force search method which also gives global optimum solution and is usually a baseline for global convergence of non-convex optimization problems. The result shows that our proposed method only slightly outperforms the brute force search method at

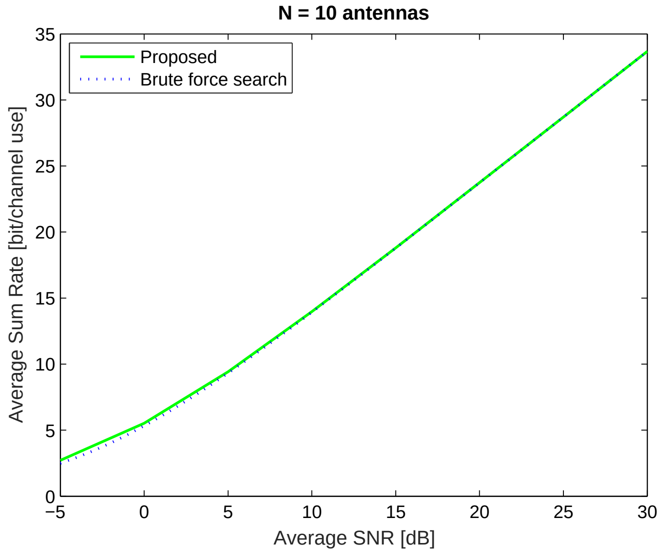

Figure 3.4: Average sum-rate achievable at different SNR for $N = 10$

low SNR between -5dB and 5dB. Nevertheless, the brute force search method is not recommended for implementation in a system setting with more than 6 UEs because of computational complexity involved in each iteration where the utility function is evaluated for each feasible solution in the search space. However, our proposed method involves an intelligent search procedure that searches only parts of the feasible set that contain the optimal solution thereby reducing the computational complexity of our proposed algorithm. The Proposed method is practically feasible for small-scale real-time applications, however, we cannot recommend it for large-scale real-time applications because of its computational complexity.

3.5 Summary

In this chapter, we have shown how the optimal solution of our NP-hard non-convex optimization problem whose target is to maximize the weighted sum-rate of HetNet is achieved. This is done by devising an algorithm based on B&B method. The results obtained show that our method can outperform popular methods using convex optimization for finding the optimal solution to the non-convex NP-hard weighted sum-rate problem in HetNet. The B&B method involves searching of a box interval to get the best feasible solution that maximizes the weighted sum-rate of the system. But this search is not like the brute force search that brings a lot of computational complexity. It is more of an intelligent search because only part of the box that contains the optimal solution is searched, hence reducing computational complexity. Our devised B&B algorithm iteratively improves a lower bound f_{min} and an upper bound f_{max} on the optimal value of (3.14). Furthermore, global convergence to the optimum value is guaranteed when $f_{max} - f_{min} < \epsilon$. The algorithm also discover an ϵ-optimal solution $\mathbf{r}_{\epsilon}^{*}$. The Lipschitz continuity property of our utility function is a sufficient condition for guaranteeing an ϵ-optimal solution in a limited number of iterations.

Chapter 4

UE-Centric Clustering and Resource Allocation for Practical Two-Tier Heterogeneous Cellular Networks

4.1 Introduction

As the demand for mobile data services increases by end-users, operators seek ways to enhance the capacity of their networks. Unfortunately, single-tier networks (macro-cellular networks) could not provide adequate solutions to the problem of capacity and coverage in cellular networks. This prompts ideas like cell splitting which evolves into *Heterogeneous cellular Networks* (HetNets) [75]. HetNet is a network that consists of planned *macro base stations* (MBSs) deployments which transmit signals at higher powers with overlaid smaller cells nodes such as *pico base station* (PBS), *micro base station* (mBS), *femtocell access points* (FAPs), *relay nodes* (RNs) and *remote radio heads* (RRHs). HetNet is one of the key technologies in 5G which can tackle the ever increasing demand for data rate and coverage. However the performance of HetNets depends on *resource allocation* (RA) which is how frequency, time, power and spatial resources are shared among UEs in HetNet in order to maximize the system *spectral efficiency* (SE).

Interference is a limiting factor to the performance of HetNets and if not properly managed will deteriorate the achievable system-wide throughput [76, 77], therefore, RA is very important. There have been different methods proposed in the literature to solve the interference problem. *Multi-cell processing* (MCP) has emerged as an efficient way to suppress interference as well as enhancing the spectral efficiency of the system [13, 78]. In MCP, BSs cooperate together in different levels to manage interference and at the same time improve the individual BSs that forms the cluster. Clustering is very important in multi-cell processing because it can help to group specific BSs together with the goal of mitigating interference and/or improving the received signal quality for UEs at the cell edges. Different clustering schemes have been investigated in literatures and they can be categorized as UE-centric clustering [46–48], network-centric clustering and hybrid clustering [49]. In UE-centric clustering scheme, the UE selects the coordinating BSs based on its point of view, these BSs either serve or reduce interference from it. In contrast, network-centric clustering is performed by the operators on a static or semi-static basis and have been castigated for not fully utilizing the channel variations of UEs present in the network. While hybrid clustering will achieve the trade-off between the performance and complexity of the aforementioned clustering schemes.

Coordinated beamforming (CB) [38] is a type of MCP described in the *3rd generation partnership project* (3GPP) LTE-Advanced which requires partial cooperation between the cooperating BSs. In CB, each BS serves its UE with data while control information is exchanged between BSs with which RA decisions can be made collectively. Compared with *joint transmission* (JT) [79], CB has been shown to be a practical and feasible approach for mitigating interference in downlink of single-tier cellular networks [40, 80–82]. JT has limitation from a practical perspective because it requires global *channel state information* (CSI) and data sharing among all BSs, it also requires a lot of channel estimation and puts huge demands on the backhaul networks. Furthermore, it requires full phase coherence among signals received from different BSs, which is impossible due to difference in propagation delay. Tight synchronization [83], is a very important factor JT needs, to become practically feasible. Some new ideas have emerged on implementing JT using cloud *radio access network* (RAN) technology [84], and using tools from stochastic geometry [85,86]. Though the theories behind it make sense but the practical implementation is where the problem lies. Even if unlimited capacity fibre optical link is utilized for data sharing, it will only increase *operational expenditures* (OPEX). If the net gain between OPEX and increased spectral efficiency is small, then the motivation behind increased expenditure for implementing JT cannot be justified. Although the effectiveness of CB has been well studied in single-tier cellular networks where the multi-cell characteristics and the accompanying inter-cell interference are usually limited to at most three cooperating MBSs, its application in a dense deployed HetNet scenario requires detailed investigation. Therefore in this chapter, we develop a UE-centric clustering scheme that determines the optimal interfering BSs that will coordinate with the serving BS of each interfered UE to allocate resources such as spatial directions and powers to UEs in HetNet, and investigate its performance gains.

4.1.1 Prior Works

Previous works on coordinated beamforming either use the Wyner model [87–89] which is a simplified model where interference only comes from the immediate neighbouring cells, or network-centric model [90–92], which is network with static clusters, these clustering method limits the cooperating area in several fixed BSs thereby cannot flexibly adapt to the changing topology. Furthermore, in [93, 94], BSs are divided into static disjoint cooperation clusters. Each cluster is operated as a single-cell system. However, networks with this kind of clusters usually provide poor spectral efficiency when UE distribution is heterogeneous, also these clusters suffer from out-of-cluster interference and thereby affecting the performance of the system. In [95], UE-centric based clustering is utilized for inter-cell interference nulling. However, this is done for a single-tier small cell network.

RA has attracted a lot of research in the past, however, it is mainly for single-tier networks such as in [38] and references therein. The contributions made in these papers do not address the significant interference problem posed when multi-tier networks are deployed, hence cannot be used in practical realistic multi-tier networks such as HetNet, which have more significant *inter-cell interference* (ICI) situations, different propagation characteristics, different cell selection procedures and different BSs power classes. We affirm that the major difficulty in RA facing HetNet is the issue of co-channel interference which degrades the performance of HetNets when UEs are served in parallel, for HetNet systems using space division multiple access (SDMA) in each cell and cooperation among coordinating BSs. Recently, RA has been investigated for HetNets. In [96–98] the RA utility function is geared towards achieving energy efficiency in HetNet. However, in this work, we differ from the aforementioned reviewed papers in the sense that our RA optimization problem is geared towards achieving spectral efficiency but also constrained the total power at each transmitter to different given values to enable energy efficiency. Furthermore, their RA is done by fixed BSs without considering clustering, which in practice will reduce the improvements they claimed are achievable by their work because of the regular change of the HetNet topology. In contrast, we determine the optimal number of interfering BSs that causes significant interference to each UE based on its point of view. These interfering BSs together with the serving BS of the interfered UE will coordinate and make RA decisions together to mitigate interference and thereby improving the achievable throughput in HetNet.

4.1.2 Contributions

In this chapter, we propose a UE-centric clustering scheme that can determine the optimal interfering BSs that cause significant interference to each UE in HetNet. Afterwards, these interfering BSs coordinate with the serving BSs of the interfered UEs to make resource allocation decisions such as allocating spatial directions and powers to UEs in HetNet to mitigate interference and improve UE performance. The specific methodology for selecting these interfering BSs among all other BSs in the system is as follows. Foremost, each UE measures the interfering signal power from a subset of the interfering BSs, if the interfering signal power sensed by it is less than or equal to the noise power, it will not be considered as significant, hence will be regarded as negligible and modeled as noise. However, if the sensed interfering signal power is greater than the noise power then it informs its serving BS. The serving BS will now select the n-tuple interfering BSs that will cause the aggregate highest interference to this UE based on the information it receives. The serving BS for each of the UEs will now make resource allocation decisions with these interfering BSs to mitigate interference by allocating spatial directions and powers to UEs in the system.

The aim of our RA is to allocate powers and spatial directions to UEs in the system in order to maximize the system sum-rate while satisfying powers, QoS and interference constraints.

4.1.3 Chapter Organization

The rest of this chapter is organized as follows. In Section 4.2 we present the system model while a new UE-centric clustering scheme is presented in Section 4.3. Section 4.4 presents the RA problem formulation, which is readily split into spatial direction allocation and power resource allocation optimization problems respectively and how they are solved. Simulation results are provided in Section 4.5, and the chapter summary is given in the last section. Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $||\cdot||_2$ denotes the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the statistical expectation over a random variable. We use upper-case boldface letters for matrices and lower-case boldface for (column) vectors and either upper-case or lower-case letters without boldface for scalars.

4.2 System Model

We consider the downlink of a two-tier HetNet as depicted in Fig. 4.1[1], which consists of K_p pico cells and K_m macro cells making it a total of K_t cells in the system. All cells in the HetNet use the same carrier frequency, note that this is not the case in orthogonal frequency-division multiplexing (OFDM) systems. The jth BS is denoted BS_j which can be any of the BSs (PBS or MBS) and is assumed to have N antennas with which it communicates with at least one active UE per cell which is assumed to have a single antenna[2]. The set of UEs served by BS_j is denoted by $\mathcal{S}_j \subset \{1, \ldots, K_r\}$, where K_r denotes the total number of UEs in HetNet, also the kth UE is denoted UE k. While the selected n-tuple BSs that interfers UE k is denoted by C_n^k. The main system parameters are listed in Table

[1]Note that the number of pico cells considered for each macro-cell is not limited to one, as suggested by Fig. 4.1 but for clarity we just showed a simplified schematic representation of our considered model. In our simulation, the total number of pico cells considered will be stated.

[2]We limit each UE to have a single antenna for practical reasons, such as, reducing the UE hardware complexity and also preserving of battery life.

1. Note that macro-pico HetNet scenario is preferred in this work to macro-femto HetNet scenario because coordination among BSs will be much easier due to the connecting backhaul link, which uses fibre optical link whereas the macro-femto utilizes internet connection.

The complex-baseband received signal at UE k is $y_k \in \mathbb{C}$ and given by

$$y_k = \sum_{j=1}^{K_t} \sqrt{g_{j,k}} (\mathbf{h}_{j,k}^s)^H \mathbf{x}_j + z_k, \tag{4.1}$$

where $\sqrt{g_{j,k}}$ is the large-scale pathloss from BS_j to UE k. Also $\mathbf{h}_{j,k}^s \in \mathbb{C}^N$ is the small-scale frequency-flat fading channel vector from BS_j to UE k, while $\mathbf{x}_j \in \mathbb{C}^N$ is the data signal vector transmitted at BS_j and intended for it served UEs. Furthermore, $z_k \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex gaussian, distributed as $z_k \sim \mathcal{CN}(0, \sigma^2)$, where σ^2 is the noise power. Assuming BS_l is the serving BS of UE k, the received signal at UE k in (4.1) can be rewritten as

$$y_k = \mathbf{h}_{l,k}^H \mathbf{w}_k s_k + \mathbf{h}_{l,k}^H \sum_{p \in \mathcal{S}_l, p \neq k} \mathbf{w}_p s_p + \sum_{\substack{j \in \mathcal{C}_n^k \\ j \neq l}} \mathbf{h}_{j,k}^H \sum_{\substack{m \in \mathcal{S}_j \\ m \neq k}} \mathbf{w}_m s_m + z_k, \tag{4.2}$$

where $\mathbf{h}_{j,k} \triangleq \sqrt{g_{j,k}} \mathbf{h}_{j,k}^s$, also the transmitted data signal vector is a linear function of the symbols, i.e., $\mathbf{x}_j = \sum_{p \in \mathcal{S}_j} \mathbf{w}_p s_p$, where $\mathbf{w}_p$ denotes the transmit beamformers for each symbol s_p. The first summand of (4.2) is the desired signal transmitted to UE k while the second and third summands represent the intra-cell interference caused by co-channel UE within the same BS and the inter-cell interference caused by co-channel UE in different BSs respectively.

For a HetNet that uses universal frequency reuse one deployment, the important issues that need to be addressed are:

- **issue 1**: how to identify the dominant inter-cell interference from BSs in HetNet to UE k. In other words, which BSs should be selected among the possible n-tuple BSs that interfers UE k the most. Any BS whose interference power towards UE k is less than or equal to the noise power is regarded as negligible interference, hence is not be considered for coordination.

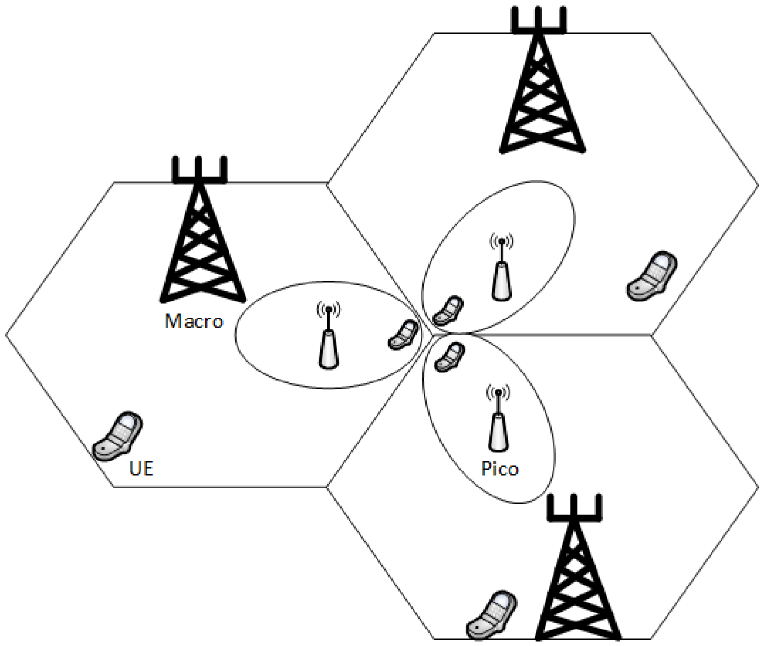

Figure 4.1: Downlink 2-tier HetNet model with overlaid pico cells in the coverage area of MBS.

- **issue 2**: How to jointly design the transmit beamformers that will spatially separate the transmitted signal vector from the interfering BSs in order to avoid interference towards UE k. Note that these interfering BSs are not fixed but selected for UE k by solving issue 1.

4.3 UE-Centric Clustering

In this section we try to resolve issue 1. We provide a solution to it by finding an optimal BS subset that will give the aggregate largest interference to UE k at a given time slot. We now write an abridged expression of (4.2) to show only the summation of inter-cell interference signals,

$$int_{sig} = \sum_{\substack{j \in C_n^k \\ j \neq l}} \mathbf{h}_{j,k}^H \mathbf{x}_j. \tag{4.3}$$

TABLE 1: KEY PARAMETERS

K_p	Total number of PBS in HetNet.
K_m	Total number of MBS in HetNet.
K_t	Total number of BSs in HetNet, $(n \leq K_t)$.
BS_j	The jth BS.
$\mathcal{S}_j$	The set of UEs served by BS_j.
N	Total number of transmit antennas at PBS or MBS.
K	Total number of active served UEs in each cell.
$\sqrt{g_{j,k}}$	The large-scale pathloss from BS_j to UE k.
$\mathbf{h}^s_{j,k}$	The small scale (fading) channel vector from BS_j to UE k.
$\mathbf{x}_j$	The data signal vector transmitted at BS_j and intended for its served UEs.
C^k_n	The selected n-tuple BSs that interfers UE k.
$\mathcal{U}_n$	The collection of all possible n-tuple BS subsets.
$\mathbf{R}_{j,k} \geq \mathbf{0}$	Means $\mathbf{R}_{j,k}$ is a positive semi-definite matrix.
K_r	Total number of UEs in HetNet.
σ^2	Noise Power .
τ_p	Limit of interference power at UE p.
q_j	Power limit at BS_j.

The inter-cell interference power corresponding to (4.3) can be represented by

$$int = \sum_{\substack{j \in C^k_n \\ j \neq l}} |\mathbf{h}^H_{j,k} \mathbf{x}_j|^2. \tag{4.4}$$

Let $\{int^k_n\}_{k \in \mathcal{S}_l}$ denote the set of all aggregate inter-cell interference power calculated from n-tuple BSs interfering UE k with $n \leq K_t$. It is important to note that for a system that comprises of K_t BSs as shown in Fig. 4.1, there are altogether 2^{K_t} possible BS subsets. Let $\mathcal{U}_n$ represent the collection of all possible n-tuple BS subsets in HetNet. The optimal BS subset that will maximize the interference suffered by UE k can be expressed as

$$C_n^{k^*} = \arg\max_{C_n \in \mathcal{U}_n} int_n^k \quad \forall k. \tag{4.5}$$

To be able to find the optimal number of BSs in the optimal BS subsets that will cause the highest interference to UE k, we determine that through the following expression:

$$l_n = \max_{C_n \in \mathcal{U}_n} int_n^k, \tag{4.6}$$

where l_n denote the maximum value of the interference generated to UE k by n-tuple BSs. Accordingly, the serving BS to UE k can choose the optimal number of interfering BSs that it will coordinate with based on l_n. This can be expressed as

$$n_{opt} = \arg\max_{n=1,\ldots,K_t} l_n. \tag{4.7}$$

However, it involves finding $C_n^{k^*}$ using (4.5) and l_n using (4.6) for each n before selecting the optimal one using (4.7).

The optimal interfering BS set for UE k is easily found as $C_{n_{opt}}^{k^*}$ and the optimal number of interfering BSs that needed to coordinate interference with the serving BS of UE k is n_{opt}. Consequently, the signal received by UE k after identifying its dominant inter-cell interferers is given by

$$y_k = \mathbf{h}_{l,k}^H \mathbf{x}_l + \sum_{j=1, j \in C_{n_{opt}}^{k^*}}^{n_{opt}} \mathbf{h}_{j,k}^H \mathbf{x}_j + z_k, \tag{4.8}$$

furthermore, the achievable data rate for UE k in beamforming terms, with s_k normalized to unit power, can also be expressed as

$$r_k = log_2 \left(1 + \frac{|\mathbf{h}_{l,k}^H \mathbf{w}_k|^2}{\sigma^2 + \sum_{p \in \mathcal{S}_l} |\mathbf{h}_{l,k}^H \mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m \in \mathcal{S}_j} |\mathbf{h}_{j,k}^H \mathbf{w}_m|^2} \right). \tag{4.9}$$

For a particular selected BS subset, the received signal y_k in (4.8) suffers from the highest significant inter-cell interference that exist in the system and perculiar to UE k. The corresponding achievable data rate r_k will diminish if these interference sources are not mitigated. Note that if a significant interference source to UE k is not identify and dealt with, it will hinder the performance of UE k. Next Section presents how we resolve issue 2 through RA to make sure that these interference sources are dealt with effectively.

4.4 Resource Allocation

In this section, the serving BS of UE k will make RA decisions together with the selected BS subset that causes interference to UE k. The implementation of this RA needs to be done centrally. Note, RA problems can be formulated in many different ways to suit the desires or objectives of the system designer. For example, if the objective of the system designer or operator is to maximize the throughput for the worst served UE, then max-min based RA optimization will be the right way to tackle that. Furthermore, if the system designer wants to achieve a maximal throughput while ensuring that none of the UEs are starving, proportionality based RA could be good for it. Also, if the aim is to achieve the maximal aggregate throughput of the system, then some of the system resource parameters such as high transmit powers will be allocated to those UEs whose channels have high *signal to noise ratios* (SNRs), while little or no powers will be allocated to UEs with attenuated channel gain. All the aforementioned RA optimization procedures have some advantages and disadvantages in terms of improving system utility and/or individual UE performance. Depending on the RA procedure adopted, there are two major consequences. Firstly, it will define a balance between performance of the system utility and that of each UE in the system. Secondly, it will also determine the extent of computational complexity involved in solving the RA problem. In this chapter, we seek to achieve the fundamental trade-off between maximizing the spectral efficiency of HetNet and achieving a minimum performance level for all UEs in the system. This decision is motivated by the poor individual performance of UEs located at the *cell range expansion* (CRE) [99] area of pico cells in a macro-pico HetNet scenario.

4.4.1 Problem Formulation

Our target is to select $\{\mathbf{w}_k\}_{k=1}^{K_r}$ to maximize the weighted sum-rate, while fulfilling some power, QoS and *interference constraints* (IC) [100], [101]. It is important to note that the individual rate r_k is a function of the signal-to-interference-and-noise-ratio ($SINR_k$). And the optimal interfering BS set $C_{n_{opt}}^{k^*}$ that affects r_k has been used to determine $SINR_k$ as expressed in (4.9). We therefore, formulate the optimization problem as

$$\begin{aligned}
\underset{\{\mathbf{w}_k\}}{\text{maximize}} \quad & \sum_{k=1}^{K_r} u_k r_k(\{\mathbf{w}_k\}) \\
\text{subject to} \quad & C1 : SINR_k \geq \gamma_k \quad k = 1, \ldots, K_r, \\
& C2 : \sum_{k \in \mathcal{S}_s} ||\mathbf{w}_k||_2^2 \leq q_s \quad s = 1, \ldots, K_p, \\
& C3 : \sum_{k \in \mathcal{S}_m} ||\mathbf{w}_k||_2^2 \leq q_m \quad m = 1, \ldots, K_m, \\
& C4 : \sum_{k \in \mathcal{S}_m} \mathbf{w}_k^H \mathbf{R}_{m,p} \mathbf{w}_k \leq \tau_p \quad \forall p \in \mathcal{S}_s,
\end{aligned} \tag{4.10}$$

where the utility function represents the weighted sum-rate of the system with the non-negative factor u_k denoting the individual weight assigned to each UE, chosen to reflect the different level of concern about the individual channel gains. A larger gain has larger weight and vice versa, also constraints ($C1 \sim C4$) represent the desired quality of service constraints, with γ_k denoting the QoS threshold for UE k; PBS power constraint, MBS power constraint and interference power constraint (i.e., interference generated from MBS to UE k) respectively. $\mathbf{R}_{m,p} \triangleq \mathbf{h}_{m,p}\mathbf{h}_{m,p}^H$ is a *positive semidefinite* (PSD) matrix ($\mathbf{R}_{m,p} \geq \mathbf{0}$), where $\mathbf{h}_{m,p}$ is the channel vector from the MBS to UE p and τ_p is the non-negative threshold which controls the allowable level of interference at UE k. Note, that by adding the IC constraint in (4.10), we aim to shape the transmission from the MBS in order to control the significant interference to UEs served by PBS.

Maximizing the weighted sum-rate of HetNet under some given constraints, as expressed in ($C1 \sim C4$) is generally regarded as a non-convex *non-deterministic* *polynomial-time hard* (NP-hard) problem because there are no known efficient algorithms that can solve it in polynomial time. However, this intractable problem can be solved by computer algorithms that run in exponential time such as *branch and bound* (B&B) algorithms [60], which can give global optimal solutions. B&B algorithms can only be considered for small scale problems, i.e. problems with very small problem size because their running times are exponential functions of their problem sizes. Note, the problem size in this paper is regarded to be the number of variables and constraints involved in the optimization problem. To pinpoint the actual cause of non-convexity of the resource allocation optimization problem of (4.10), let's analyze each function that makes up the resource allocation problem: firstly, the utility function in (4.10) is a concave function which can be maximized, though it depends on the SINRs of UEs in the system. The power constraint functions in $C2 \sim C3$ together with the MBS interference power constraint function in $C4$ are all convex functions. The SINR constraint function in $C1$ is a non-convex function of beamforming vectors $\{\mathbf{w}_k\}_{k=1}^{K_r}$, which cannot be classified as a semidefinite constraint or second-order cone constraint. In order to make the constraint convex, $SINR_k \geq \gamma_k$ can be expressed as [102]

$$\frac{1}{\gamma_k}|\mathbf{h}_{l,k}^H \mathbf{w}_k|^2 \geq \sum_{p\in\mathcal{S}_l, p\neq k} |\mathbf{h}_{l,k}^H \mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m\in\mathcal{S}_j} |\mathbf{h}_{j,k}^H \mathbf{w}_m|^2 + \sigma^2, \tag{4.11}$$

we note that the absolute values in (4.11) make $\mathbf{w}_k$ and $e^{j\theta_k}\mathbf{w}_k$ equivalent for any common phase rotation $\theta_k \in \mathbb{R}$, hence we exploit this phase ambiguity to rotate the phase such that $\mathbf{h}_{l,k}^H \mathbf{w}_k$ is real-valued and positive. This insinuate that $\sqrt{|\mathbf{h}_{l,k}^H \mathbf{w}_k|^2} = \mathbf{h}_{l,k}^H \mathbf{w}_k \geq 0$. Therefore, $SINR_k \geq \gamma_k$ can now be rewritten as

$$\frac{1}{\sqrt{\gamma_k}}\Re(\mathbf{h}_{l,k}^H \mathbf{w}_k) \geq \sqrt{\sum_{\substack{p\in\mathcal{S}_l \\ p\neq k}} |\mathbf{h}_{l,k}^H \mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m\in\mathcal{S}_j} |\mathbf{h}_{j,k}^H \mathbf{w}_m|^2 + \sigma^2}, \tag{4.12}$$

where $\Re(\cdot)$ denotes the real part, also, the γ_k value at each UE needs to be fixed and we assume these values to be known a *priori* but can be computed as $\gamma_k \triangleq 2^{r_k} - 1$ obtainable from (4.9). Therefore, the SINR constraint in (4.10) can now be classified as a second-order cone constraint, which is a convex type constraint [103].

We are interested in producing approximate solutions, that are feasible in practice for large scale problems, consequently, we seek to solve the non-convex problem using convex heuristics approach.

Our RA problem in (4.10) is centralized and the optimization variable is the transmit beamformers. Note that the properties of this transmit beamformers include both the spatial characteristic and the corresponding transmission powers. Recall that the aim of our RA is to allocate powers and spatial directions to UEs in the system in order to maximize the system sum-rate while satisfying power, QoS and interference constraints. Having said that, we therefore readily split (4.10) into two sub-problems. The first problem is formulated as a spatial direction allocation problem, while the second is formulated as a power allocation problem. The former needs to be solved centrally while the latter will be solved in a decentralized manner. This technically means that the RA problem in (4.10) is decomposed into two sub problems, giving more freedom to each BS to determine the performance level for each served UE.

4.4.2 Spatial Direction Allocation Problem

The spatial direction allocation problem is expressed as

$$\begin{aligned}
\tilde{\mathbf{w}}_k = & \underset{\{\mathbf{w}_k\}_{k=1}^{K_r}}{\operatorname{argmax}} \quad \sum_{k=1}^{K_r} u_k r_k(\{\mathbf{w}_k\}) \\
& \text{subject to} \quad C1 : \frac{1}{\sqrt{\gamma_k}} \Re(\mathbf{h}_{l,k}^H \mathbf{w}_k) \geq \Gamma_k, \\
& \qquad\qquad\quad C2 \sim C4 \ in \ (4.10), \\
& \qquad\qquad\quad C5 : ||\mathbf{w}_k||^2 = 1 \ \ k = 1, \ldots, K_r,
\end{aligned} \tag{4.13}$$

where $\Gamma_k = \sqrt{\sum_{p \in \mathcal{S}_l, p \neq k} |\mathbf{h}_{l,k}^H \mathbf{w}_p|^2 + \sum_{j=1}^{n_{opt}} \sum_{m \in \mathcal{S}_j} |\mathbf{h}_{j,k}^H \mathbf{w}_m|^2 + \sigma^2}$. To solve (4.13) efficiently we use SeDumi [68], which is a general purpose implementation of interior point method, with CVX [70], providing a Matlab based modelling platform for it. Therefore, the unit-norm beamformers or spatial directions of the system are $\{\tilde{\mathbf{w}}_1, \ldots, \tilde{\mathbf{w}}_{K_r}\}$.

Next Section presents how we design the optimal transmit power allocated to each UE in each cell to improve UE performance and maximize the sum-rate of HetNet.

4.4.3 Power Allocation Problem

Since the major interference problem has been tackled[3] in the previous section by designing unit-norm beamformers $\{\tilde{\mathbf{w}}_1, ..., \tilde{\mathbf{w}}_{K_r}\}$ that will spatially separate data symbols when transmitting to UEs. Any negligible interference in the system will be modelled as part of the background noise. What is left to be done is to select the power allocation coefficient $\{p_k\} \forall k \in \mathcal{S}_j$ which will act as optimum scale factors to each spatial direction $\{\tilde{\mathbf{w}}_k\} \forall k \in \mathcal{S}_j$ in order to maximize the SE of the system as well as satisfying each UE with a minimum performance level. We proceed by formulating our power resource allocation problem as

$$\begin{aligned} \underset{\{p_k\}\forall k \in \mathcal{S}_j}{\text{maximize}} \quad & \sum_{k \in \mathcal{S}_j} log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2} \right), \\ \text{subject to} \quad & \sum_{k \in \mathcal{S}_j} p_k \leq q_j, \\ & log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2} \right) \geq R_k \ \forall k \in \mathcal{S}_j, \\ & p_k \geq 0 \ \ \forall k \in \mathcal{S}_j, \end{aligned} \tag{4.14}$$

where R_k denotes the minimum required data rate for UE k to have good *quality of experience* (QoE). One can easily observe that the power RA problem in (4.14) is a convex optimization problem, because the utility function is a concave function while the constraint functions are: convex function, concave function and concave function respectively. Hence, the global power solution can be obtained efficiently using CVX, a package for specifying and solving convex programs. For fairness in this power RA formulation to be achieved, this constraint $log_2 \left(1 + p_k \frac{|\mathbf{h}_{j,k}^H \tilde{\mathbf{w}}_k|^2}{\sigma^2} \right) \geq R_k$ needs to be active. In some cases it is not but it all depends on how large this threshold R_k is.

We summarized the resource allocation procedure in this chapter using Algorithm 1.

[3]We note that this proposed power allocation scheme will be optimal for transmit strategy utilizing the zeroforcing method. However, we also found out that forcing zeros may also cause a distorted beam pattern with high sidelobes which can lead to increase in the background interference level in the system.

Algorithm 1 Allocation of spatial directions and powers for each UE in two-tier HetNet

---

Input and variables

$\mathcal{S}_j$: set of UEs served by BS_j;

K : total number of UEs in each cell;

procedure

1: **for** UEs $\in \mathcal{S}_j$ i.e. k = 1 to K **do**

2: compute $\mathbf{w}_k$ using (4.13);

3: obtain the unit-norm beamformers $\tilde{\mathbf{w}}_k$ using (4.13);

4: compute $p_k \forall k \in \mathcal{S}_j$ from using (4.14) ;

5: **end for**

BS_j transmits $\mathbf{x}_j = \sum_{k \in \mathcal{S}_j} \sqrt{p_k} \tilde{\mathbf{w}}_k s_k$

4.5 Simulation Results

In this section, we evaluate the performance of our proposed RA methods by comparing with the global optimal method and other existing RA methods based on the average achievable sum-rate, SNR and computational complexity.

4.5.1 Simulation Settings

We consider a simple simulation setting with minimum of five randomly distributed PBSs deployed at hotspot locations in the coverage area of MBS. The minimum distance among pico sites is set to 40m, and we assume that all PBSs are not geometrically separated, hence interference among PBS is possible and therefore considered. The minimum distance from the macro site to the pico sites is 75m. We assume that the UEs in the HetNet are uniformly distributed and are located at the CRE such that each UE will receive significant intercell interference (ICI). Note we concentrate on UEs at the CRE because they suffer both signal attenuation from their serving BS as well as inter-cell interference from neighboring cells. The UEs served by PBS are uniformly distributed between 35m and 55m from the PBS. Similarly, the UEs served by MBS are uniformly distributed between 220m and 260m from the MBS, also, the distance between the macrocell UEs and the PBS is between 40m and 45m, while the distance between the

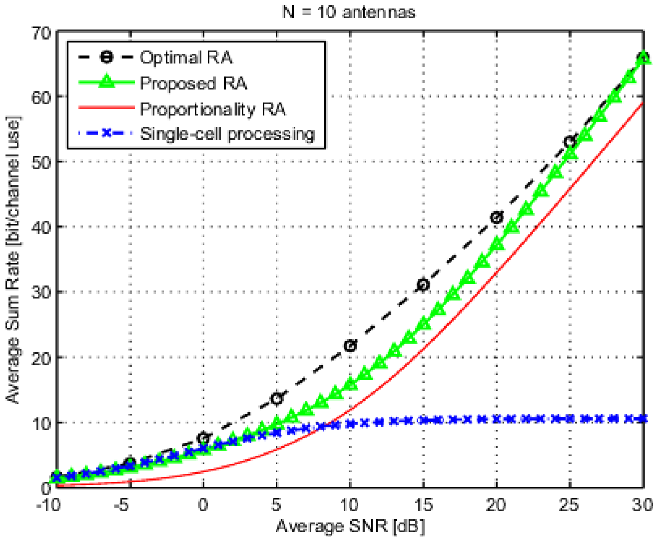

Figure 4.2: Average sum-rate as a function of SNR for different RA implementation.

picocell UEs and the MBS is between 230m and 270m. Other system parameters are also based on the 3GPP simulation baseline parameters and can be found in [2]. The total BS transmit powers for MBS and PBS are 46dBm and 30dBm respectively, assuming a 10MHz bandwidth. The channel vector between BS$_j$ and UE k is modelled as $\mathbf{h}_{j,k} \triangleq \sqrt{g_{j,k}}\mathbf{h}^s_{j,k}$, where $\sqrt{g_{j,k}}$ is the large-scale pathloss from BS$_j$ to UE k, also $\mathbf{h}^s_{j,k} \in \mathbb{C}^N$ is the small scale (fading) channel vector from BS$_j$ to UE k, and the large scale pathloss in linear scale is expressed as

$$g_{j,k} = \frac{\psi}{d^n_{j,k}}, \tag{4.15}$$

where ψ is a constant which accounts for system losses, n is the path-loss exponent, typically $n > 3$, while $d_{j,k}$ is the distance between BS$_j$ and UE k. The large-scale path loss model in dB for the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{j,k}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{j,k}}{10^3})$. This simulation setting will be used except otherwise indicated.

In Fig. 4.2, we show the average sum-rate achievable as a function of SNR. It compares the average sum-rate achieved in the system using our proposed method, the optimal RA method, proportionality RA method and the single-

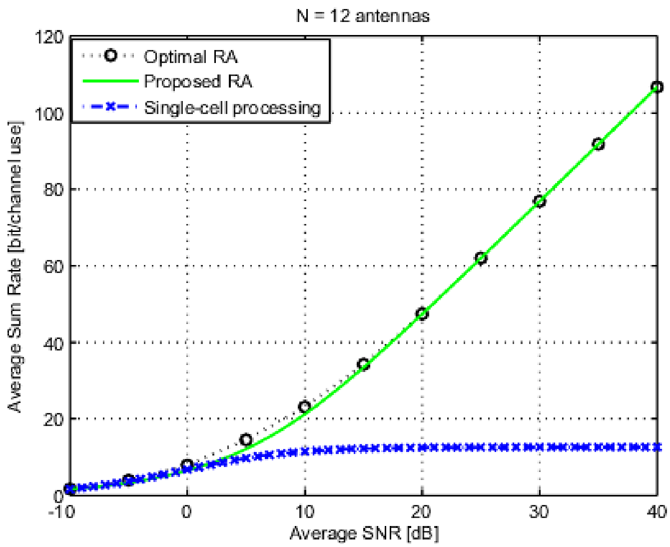

Figure 4.3: Average sum-rate achievable at different SNR for $N = 12$, $K_r = 9$.

cell processing RA method. Note, we implement both our proposed RA method and the optimal RA method using our proposed UE-centric clustering scheme. The optimal RA method utilizes the B&B method. Our proposed method is outperformed by the B&B method whose trade off for such performance is in its complexity. The proportionality RA method performance is inferior to our proposed method because it utilizes the semi-static clustering method proposed in [104] to determine the coordinating BSs that will coordinate interference to each UE. The loss in performance is due to the fact that the BSs that are selected to form cluster are semi-static hence do not always change with the changing topology of HetNet. It fails to identify the strongest inter-cell interfering BSs that affect each UE at a given time. The least performed RA method performs poorly because it only considers its served UEs while designing the beamformers without coordination with other BSs in the system. Furthermore, it models any out-of-cell interference in the system as part of the background noise.

In Fig. 4.3, we show that the performance of our proposed method improves as $N = 12$ transmit antennas while the B&B only slightly outperforms it at low SNR. It goes ahead to prove that our proposed method though suboptimal is asymptotically optimal as N increases. Note, that increase in the number of

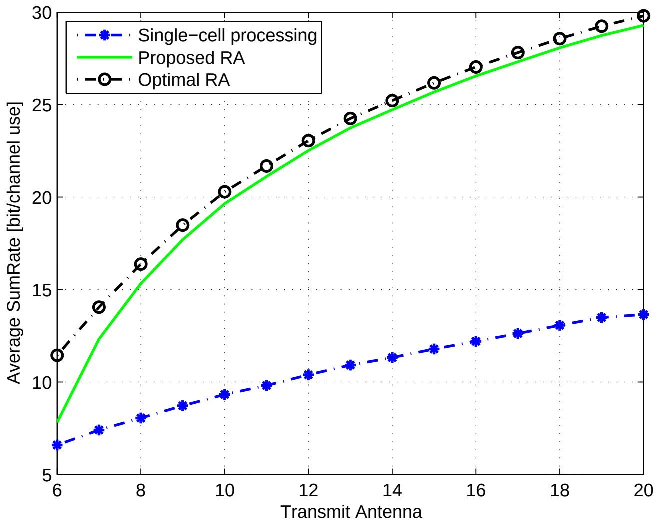

Figure 4.4: Average sum-rate achievable at different transmit antennas for $SNR =$ 10 dB, $K_r = 6$.

transmit antenna is one of the factors that improves the beamforming resolution for our proposed method. It also helps to improve the diminishing signal power due to interference cancellation. Furthermore, Fig. 4.4, shows that as N increases it helps in getting better spatial directions that will improve the performance of the system due to increase in the degree of freedom (DoF).

In Fig. 4.5, we show the effect of the interference threshold $\tau \in \{0, \ldots, 1\}$ on the average sum-rate of HetNet. The performance of our proposed RA method, the optimal RA method, and the single-cell processing RA method are compared when the interference threshold τ is varied. These methods suffer rate loss as τ increases. The proposed method and B&B method perform the best when the allowable interference from the MBS to UEs served by PBS in the system is $\tau = 0.1$. The single-cell processing (no-cooperation) method starts well at $\tau = 0.1$ but suffers consistent rate loss than our proposed method and the global optimal method.

In B&B method, it is well known that in practice the complexity grows ex-

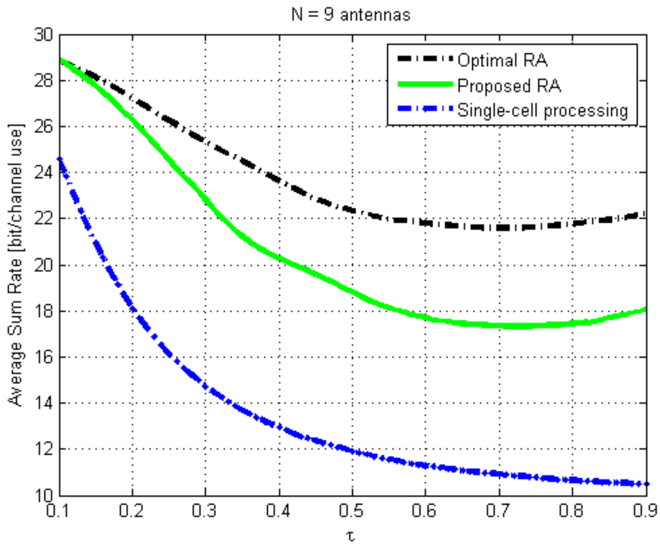

Figure 4.5: Effect of the interference threshold τ on the sum-rate of HetNet for $N = 9$, and $SNR = 15$ dB.

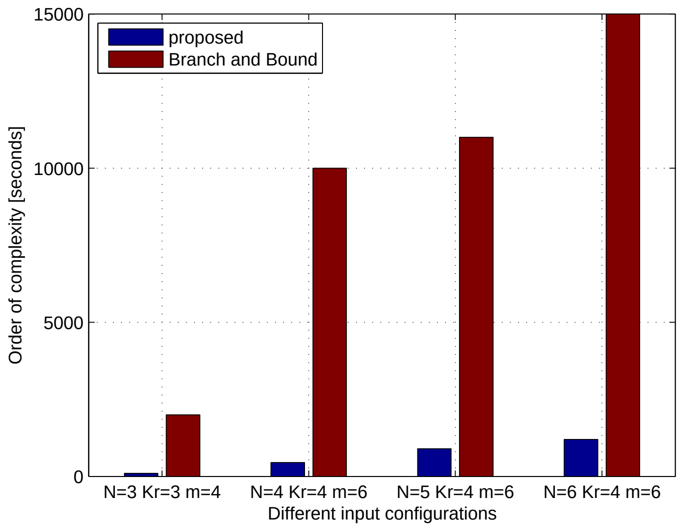

Figure 4.6: Order of complexity as a function of the input size (configurations).

ponentially in order t^n, where n is the problem size (input size) and t is just a constant. In Fig. 4.6, we use a simple scenario to show how different input size configurations give rise to the varying order of complexity for our proposed method and the B&B method. The number of variables, $v_a = NK_r$, where N and K_r have already been used to denote the number of antennas and the total number of UEs in the system. When $K_r = 3$ UEs, $N = 4$ transmit antennas, and $m = 4$ constraints (power and interference constraints), the order of complexity for our proposed method takes roughly 100 seconds to complete a problem size containing NK_r while that of B&B method takes 2000 seconds. Our proposed method computational complexity is polynomial in the number of UEs, transmit antennas, power and interference constraints while that of B&B method has worst case complexity that increases exponentially with the number of UEs. We cannot recommend it to be used for more than $Kr = 6$ UEs, hence should not be used for large-scale real-time application but can be used for small scale applications and for off-line benchmarking.

4.6 Summary

In this chapter, we have developed an UE-centric clustering scheme that can effectively determine the significant interfering BSs that will cause the highest interference to each UE. Afterwards, the serving BSs for these UEs together with these selected interfering BSs will coordinate and make resource allocation decisions to allocate spatial direction to each UE in the system. Our RA strategy can be practically implemented in HetNet. The resources allocated to UEs are the spatial directions (unit-beamformers) and the power resource.

The resource allocation optimization problem for selecting spatial directions is done centrally and is formulated as an NP-hard non-convex problem, which we reformulate to a convex problem for practical implementation purposes and solved using SeDumi, which is a general purpose implementation of interior point method. While our power resource allocation scheme is decentralized and is formulated as maximizing the sum-rate of each cell while achieving a minimum performance level for each UE in the cell. The power RA problem is found to be convex and hence, can be solved efficiently using CVX (a package for specifying and solving convex programs). Results obtained show that our proposed method though suboptimal, when compared to the B&B method, which provides the global optimal solution for the non-convex NP-hard weighted sum-rate maximization problem, improves when the number of transmit antenna increases. Also, our results show that the B&B method has the worst case complexity that increases exponentially with the number of UEs, hence cannot be recommended for large-scale applications but can be used for off-line benchmarking.

Chapter 5

Distributed Resource Allocation for Two-Tier Heterogeneous cellular Networks

5.1 Introduction

Resource allocation (RA) is all about how the best radio resources such as frequency, time, transmit powers, spatial directions (unit norm beamformers), etc., can be properly allocated to *user equipments* (UEs) in a system in order to maximize the system *spectral efficiency* (SE). RA is especially important in system such as *heterogeneous network* (HetNet) [7] which is limited by co-channel interference rather than noise [76]. The basic problem facing RA is the issue of coupling among UEs. UEs are coupled due to interference (inter-UE, inter-cell) and power constraints. This chapter is focused on the optimal distributed resource allocation procedure in two-tier HetNet such that UEs in the *cell range expansion* (CRE) [99], area of the pico cells will experience minimized loss in throughput due to the higher level of interference received from the *macro base station* (MBS). By distributed RA, we mean that the RA algorithm is computed at each BS without exchanging or sharing control variables or channel state information, unlike in a centralized system. In homogeneous cellular network, UE is usually served and connected to the strongest base station in downlink hence interference from other signals are received with a lower power than the desired signal. In contrast and in order to enable cell splitting gain, some UEs in HetNet may be served and connected to the strongest *base station* (BS) in uplink (i.e low powered BS) even though the received power from an MBS could be higher [10]. This method of cell selection in HetNet always causes high level of interference from the MBS to such UEs which are usually located at the CRE of the low powered BS. Apart from this, they also suffer from enormous signal attenuation from their home (serving) BS. These problems therefore cause them to exhibit poorer performance than the interior UEs thereby degrading the system aggregate sum-rate. One of our objectives in this chapter is to find ways to manage interference experience among UEs in HetNet effectively.

Many *Inter-cell Interference Coordination* (ICIC) [8, 105, 106] schemes in *long term evolution* (LTE) have been proposed. In LTE releases 8 and 9, fractional frequency reuse [107] is proposed to deal with interference affecting cell-edge UEs. In LTE-Advanced Releases 10 and 11, multiple *carrier components* (CCs) [108], [32] were introduced and the proposed techniques were categorized into time, frequency and power domain. Traditionally, interference is mitigated by assigning all links orthogonal resources in frequency, time or code. This method decouples all interferences from the links. However, this comes at the expense of the achievable SE of the system.

Because of the high demand for data rate and scarcity of spectrum, universal frequency reuse [109] has been an attractive strategy considered for future generation mobile networks. In this perspective, we introduce an alternative solution to the problem of interference in HetNet based on the use of multi-antenna technology to suppress the leakage powers to other UEs in the system. The inter-UE and inter-cell interferences are reduced using the directivity of the antenna and this approach increases the SE of the network if the spatial dimension is utilized to serve UEs in parallel. Note, that increased radio network capacity can be achieved by improving the SE of the network.

Coordinated multi-point (CoMP) [42, 43, 78, 88] is a multi-antenna inter-cell cooperation technology that mitigates inter-cell interference and increases the rates of UEs at the cell edge by allowing both the UE's serving cell and other cooperating cells to communicate with these UEs simultaneously. We differ a little from this approach which usually requires data to be shared and synchronized among cells in HetNet. In our case all cells in the HetNet will not transmit data to this UE simultaneously. Rather each BS will transmit data to its served UEs but might as well cause interference to other UEs in the network. Also the *physical* (PHY) layer will be based on *space division multiple access* (SDMA) which enables spatial separation of co-channel UE waveforms.

5.1.1 Prior Works and Contributions

Some notable works have considered interference leakage suppression in single cell [110–112], or single-tier multiple cells [13, 113–115]. In these aforementioned works, the spatial resource allocation problem solved by them and some literatures cited therein are different from the one we are solving in this chapter in terms of the objective behind the resource allocation. Furthermore, their unit-norm beamformers are obtained by maximizing the signal-to-leakage-and-noise ratio which is usually formulated as a generalized Rayleigh quotient. Consequently the eigenvector corresponding to the largest eigenvalue gives the optimum solution of the optimization problem. We differ from this method by formulating our spatial RA optimization problem as quadratic optimization problem with non-convex quadratic constraints, which aim to minimize the total leakage caused to other UEs in the system while satisfying a fixed received power for the desired UE when transmitting to it. Also our methods are tailored to underlay HetNets [116] which have more dominant interference scenarios than single-tier networks which are considered by other works. Furthermore, HetNet has different propagation characteristics, deployments and cell selection procedures compared with single-tier cellular networks. We also differ from these authors and others cited in current literatures on how we formulate and obtain the powers that will be allocated to UEs in the system.

HetNet favours coordinated processing but done in a distributed fashion unlike CoMP transmission [43]. Each BS will make RA decisions and be sure that no uncoordinated interference exist from the cell. Our distributed spatial direction allocation problem which is informally formulated as selecting the unit-norm beamformers that will cause the least total leakage power from each transmitter subject to a receive signal power threshold at each UE, can be implemented without the requirement of any exchange among the cells in HetNet provided that *time division duplex* (TDD) based local *channel state information* (CSI) is available at each BS. Our proposed hybrid power resource allocation scheme involves a three step process, starting with two power optimization procedures. These two power optimization procedures are formulated as convex optimization problems and solved by exploiting *Karush-Kuhn-Tucker* (KKT) conditions and

CVX (a Matlab software for discipline convex programming) respectively. The third step involves utilizing results from the aforementioned power optimization procedures to compute average powers allocated to UEs. The resources which we consider as the optimization variables in this chapter are the powers and spatial (beamforming) directions. These are selected and assigned/allocated by each BS to UEs in its coverage in order to satisfy UEs at CRE with the minimum *quality of experience* (QoE)[1] [117] and improve the overall SE of the system.

5.1.2 Chapter Organization

The rest of this chapter is organized as follows. In section 5.2 we present the system model of the considered HetNet. Section 5.3 presents the optimization problem formulation for the spatial resource and power resource allocations and how they are solved. Simulation results and discussions are provided in section 5.4, and the chapter summary is given in the last section.
Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $||\cdot||_2$ denotes the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the statistical expectation over a random variable, $\mathrm{Tr}(\mathbf{X})$ denotes the the trace of a square matrix $\mathbf{X}$ and $\mathbf{card}(\mathcal{D})$ denotes the cardinality of set $\mathcal{D}$. We use upper-case boldface letters for matrices and lower-case boldface for (column) vectors and either upper-case or lower-case letters without boldface for scalars.

5.2 System Model

We consider the downlink of a two-tier HetNet with P pico cells underlaid in a single macro-cellular coverage, making it a total of K_t cells in the system. All cells in the HetNet use the same carrier frequency, note that this is not the case in orthogonal frequency-division multiplexing (OFDM) systems. The jth BS is

[1]QoE is a subjective measure of the quality of service (QoS) provided by the network operator and perceived by end-users. It is related to QoS but differs in the sense that, in QoS, the measure of the service provided for the end-users is solely determined by the network operator or service provider for the overall value of the service provided.

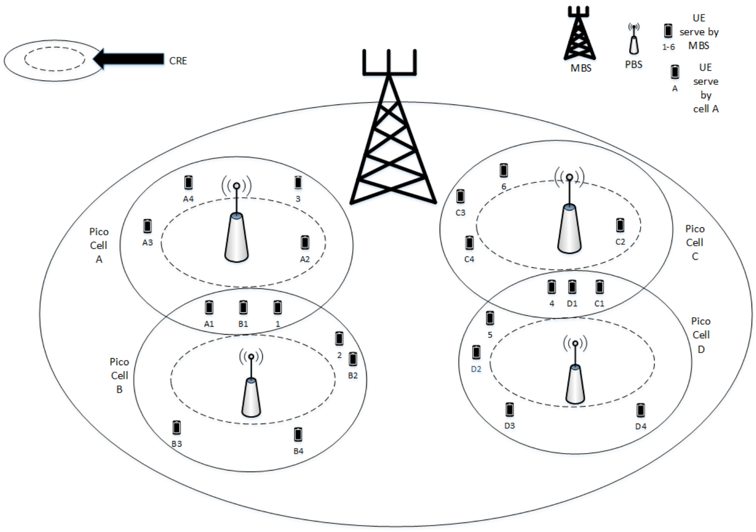

Figure 5.1: Downlink two-tier HetNet model with underlaid hotspot Pico cells in the coverage area of MBS.

denoted BS_j which can be any of the BSs (PBS or MBS) and is assumed to have N antennas with which it serves U UEs with single receive antenna[2] each as depicted in Fig. 1. The set of UEs served by BS_j is denoted by $\mathcal{G}_j \subseteq \{1, \ldots, U\}$ while the set of UEs that BS_j causes interference to in the network is denoted $\mathcal{C}_j \subseteq \{1, \ldots, \bar{U}\}$. We assume that BS_j knows the CSI of all UEs in $\mathcal{C}_j$ while the CSI of any UE $i \notin \mathcal{C}_j$ and interfered by BS_j is assumed to be negligible and need not to be known, rather is treated as Gaussian noise. The complex-baseband received data signal at UE u is $y_u \in \mathbb{C}$ and given by

$$y_u = \sum_{j=1}^{K_t} \sqrt{g_{j,u}} (\mathbf{h}_{j,u}^s)^H \mathbf{x}_j + n_u, \tag{5.1}$$

where $\sqrt{g_{j,u}}$ is the large-scale path-loss from BS_j to UE u. Also $\mathbf{h}_{j,u}^s \in \mathbb{C}^{N \times 1}$ is the small scale frequency-flat fading channel vector from BS_j to UE u. The downlink channel matrix from BS_j to all its served UEs in the same cell is given by

[2]We limit each UE to have a single antenna for practical reasons, such as, reducing the UE hardware complexity, it requires less CSI knowledge at the transmitter and also preserving of battery life.

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{h}_{j,1}^H \\ \vdots \\ \mathbf{h}_{j,U}^H \end{bmatrix} \in \mathbb{C}^{U\times N} \tag{5.2}$$

where $\mathbf{h}_{j,u} \triangleq \sqrt{g_{j,u}}\mathbf{h}_{j,u}^s$, $\mathbf{h}_{j,u}^H$ represent the rows of $\mathbf{H}_j$. Similarly, $\bar{\mathbf{H}}_j \in \mathbb{C}^{\bar{U}\times N}$ represent channel matrix towards UEs $\left\{k : k \in C_j\right\}$ which BS$_j$ interferes. Also, we define this channel matrix $\bar{\mathbf{H}}_{j,u} = [\mathbf{h}_{j,1},\ldots,\mathbf{h}_{j,u-1},\mathbf{h}_{j,u+1},\ldots,\mathbf{h}_{j,U},\bar{\mathbf{H}}_j^T]^T \in \mathbb{C}^{(U-1+\bar{U})\times N}$ as the channel from BS$_j$ to its U-1 served UEs other than UE u as well as the $\bar{U}$ UEs $\in C_j$. While $n_u \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex Gaussian, distributed as $n_u \sim \mathcal{CN}\ (0,\sigma^2)$, where σ^2 is the variance of the noise. $\mathbf{x}_j \in \mathbb{C}^{N\times 1}$ is the transmit signal vector from BS$_j$ in each cell with average power constraint $q_j = \mathbb{E}[\mathrm{Tr}(\mathbf{x}_j\ \mathbf{x}_j^H)]$. To enable spatial separation of data symbols s_u from BS$_j$ to UEs $u \in \mathcal{G}_j$, the transmitted signal vector is represented as a linear function of the symbols or linear combination of the beamforming vectors in the form

$$\mathbf{x}_j = \sum_{u\in\mathcal{G}_j} \mathbf{w}_u s_u, \tag{5.3}$$

where $\mathbf{w}_u \in \mathbb{C}^{N\times 1}$ corresponds to the transmit beamformers for each symbol meant for the UE u.

5.3 Resource Allocation

Resource allocation (RA) involves strategies and algorithm for controlling and sharing radio resource parameters such as frequency, time, transmit powers and spatial directions among UEs in the HetNet to maximize the system SE. The critical problem in RA facing HetNet is the issue of interference (inter-cell interference and inter-UE interference). This chapter aims at allocating powers and spatial (beamforming) directions optimally to UEs such that UEs in CRE will only experience minimized loss in throughput due to the higher level of interference received from the MBS. Traditionally BS$_j$ unilaterally makes resource allocation decisions by allocating spatial directions and powers to its served UEs.

Any resource allocation made without due consideration to UEs $\in \mathcal{C}_j$ will certainly diminish the SE gains in the network. We solve our RA optimization problem for spatial directions and powers for UEs in different steps not jointly.

5.3.1 Problem Formulation

This section aims at designing fixed distributed beamforming directions that will spatially seperate the data symbols sent to UEs in each cell. This will spatially control the inter-UE interference caused in each cell and the interference caused to UEs $\in \mathcal{C}_j$, hence, implicitly solve the problem of inter-cell interference caused in the HetNet.

BS_j serves UEs in $\mathcal{G}_j$, while coordinating interference towards UEs in $\mathcal{C}_j$. By coordinating interference, we mean that the propagation channels from BS_j towards these set of UEs are also considered as input to the beamformer design algorithm in BS_j during the design of its beamformers. We formulate our spatial RA problem informally as selecting the optimal beamformers that will cause the least interference to UEs in the same cell and UEs $\in \mathcal{C}_j$ while fulfilling the desired received power constraint (threshold). This threshold is not constant for every UE but will depend on the propagation characteristics of the cell. Assuming BS_l is the serving BS of UE u, the desired signal received at UE u is

$$y_u^{des} = \mathbf{h}_{l,u}^H \mathbf{w}_u s_u, \tag{5.4}$$

while the leakage signal $\mathbf{y}_u^{leak} \in \mathbb{C}^{(U-1+\bar{U})}$ directed away from this UE is given by

$$\mathbf{y}_u^{leak} = \bar{\mathbf{H}}_{l,u} \mathbf{w}_u s_u. \tag{5.5}$$

Mathematically, the optimization problem can be stated as

$$\mathbf{w}_u^{opt} = \underset{\{\mathbf{w}_u\} \forall u \in \mathcal{G}_l}{\operatorname{argmin}} \sum_{u \in \mathcal{G}_l} ||\mathbf{y}_u^{leak}||^2, \tag{5.6}$$

subject to

$$|y_u^{des}|^2 = \tau_u \quad \forall u \in \mathcal{G}_l. \tag{5.7}$$

To elucidate the optimization problem in beamforming terms, (5.6) and (5.7) will be stated as

$$\mathbf{w}_u^{opt} = \underset{\{\mathbf{w}_u\}}{\operatorname{argmin}} \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u, \tag{5.8}$$

subject to

$$\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u = \tau_u \quad \forall u \in \mathcal{G}_l. \tag{5.9}$$

The constraint for the received signal power for each UE in each cell can be defined as $\tau_u \triangleq \frac{q_l}{U}\ \mathrm{Tr}(\mathbf{R}_{l,u})$. Where $\frac{q_l}{U}$ represent fixed uniform power allocation to all UEs in each cell and $\mathrm{Tr}(\mathbf{R}_{l,u})$ gives the sum of the diagonal of the array covariance matrix of UE u. We assume that the different data symbols are uncorrelated and have normalized power $\mathbb{E}[|s_u|^2] = 1$, also $\mathbf{R}_{l,u} = \mathbf{h}_{l,u}\mathbf{h}_{l,u}^H$ is the array covariance matrix for the desired UE. While $\bar{\mathbf{R}}_{l,u} = \bar{\mathbf{H}}_{l,u}^H \bar{\mathbf{H}}_{l,u}$ is the array covariance matrix for UEs affected by the leakage power. Both $\mathbf{R}_{l,u}$ and $\bar{\mathbf{R}}_{l,u}$ are positive definite (PD) matrices which means that $\mathbf{w}_{l,u}^H \mathbf{R}_{l,u} \mathbf{w}_{l,u} > 0$ and $\mathbf{w}_{l,u}^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_{l,u} > 0$. In what follows, we show detailed analysis on how the optimal beamformers can be obtained. The optimal beamformer solutions can be computed by solving the following non-convex problem

$$\begin{aligned} \underset{\{\mathbf{w}_u\}}{\text{minimize}} \quad & \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u, \\ \text{subject to} \quad & \mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u = \tau_u \ \forall u \in \mathcal{G}_l. \end{aligned} \tag{5.10}$$

It is non-convex because only affine functions[3] are allowed to have equality constraints. But $\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u$ is a quadratic function with a PD matrix $\mathbf{R}_{l,u}$ which makes it a convex function. Therefore, the equality constraint in (5.10) makes the optimization problem non-convex [118].

We obtain the Lagrangian function of (5.10) as

$$\mathcal{L}(\mathbf{w}_u, \beta_u) = \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u - \sum_{u \in \mathcal{G}_l} \beta_u (\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u - \tau_u), \tag{5.11}$$

where $\beta_u \geq 0$ is the Lagrange multiplier associated with τ_u. To solve (5.11), we exploit the stationarity *Karush-Kuhn-Tucker* (KKT) conditions [119] which say that $\partial \mathcal{L} / \partial \mathbf{w}_u = \mathbf{0}$, at the optimal solution. The outcome of this derivative yields the following relationship

$$(\bar{\mathbf{R}}_{l,u} - \mathbf{R}_{l,u}\beta_u)\mathbf{w}_u = \ \mathbf{0} \ \ \forall u, \tag{5.12}$$

[3] A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be affine if its domain is an affine set, and if, for all x, y $\in \mathbb{R}^n$ and $\theta \in \mathbb{R}$, $f(\theta x + (1 - \theta)y) = \theta f(x) + (1 - \theta)f(y)$.

Note, that if $(\bar{\mathbf{R}}_{l,u} - \mathbf{R}_{l,u}\beta_u)$ in (5.12) is not a PD matrix, then it is possible to get a set of $\{\mathbf{w}_u\}$ that will give unbounded direction, which could cause the dual function not to have a finite value but tend towards -∞. By adding $\mathbf{R}_{l,u}\mathbf{w}_u$ to both sides of (5.12) and simplifying further gives us the following relationship

$$\bar{\mathbf{R}}_{l,u}\mathbf{w}_u + \mathbf{R}_{l,u}\mathbf{w}_u = \mathbf{R}_{l,u}\beta_u\mathbf{w}_u + \mathbf{R}_{l,u}\mathbf{w}_u. \tag{5.13}$$

Further regrouping of terms in (5.13) yields

$$(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})\mathbf{w}_u = \beta_u\left(1 + \frac{1}{\beta_u}\right)\mathbf{R}_{l,u}\mathbf{w}_u. \tag{5.14}$$

We decompose parameter "$\mathbf{R}_{l,u}$" in the right hand side (RHS) of (5.14) to get this relationship

$$(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})\mathbf{w}_u = \mathbf{h}_{l,u}\beta_u\left(1 + \frac{1}{\beta_u}\right)\mathbf{h}_{l,u}^H\mathbf{w}_u. \tag{5.15}$$

Finally our beamforming vector can be expressed as

$$\mathbf{w}_u = (\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}\underbrace{\beta_u\left(1 + \frac{1}{\beta_u}\right)\mathbf{h}_{l,u}^H\mathbf{w}_u}_{\text{scalar term}}. \tag{5.16}$$

In (5.16) the scalar term is a single value and can be ignored because it can only contribute to the magnitude but doesn't affect the direction of the beamformer. Therefore, the unit norm beamforming vectors $\{\tilde{\mathbf{w}}_1 \cdots \tilde{\mathbf{w}}_U\}$ are

$$\tilde{\mathbf{w}}_u = \frac{(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}}{||(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}||} \quad \forall u \in \mathcal{G}_l. \tag{5.17}$$

5.3.2 Power Allocation

The major interference problem has been tackled in the previous section by designing unit-norm beamformers $\{\tilde{\mathbf{w}}_u\}\forall u \in \mathcal{G}_j$ that will spatially separate data symbols when transmitting to UEs. Any negligible interference in the system will be modelled as part of the background noise. What is left to be done is to select the power allocation coefficients $\{p_u\}\forall u \in \mathcal{G}_j$ which will act as optimum scale factors to each spatial direction $\{\tilde{\mathbf{w}}_u\}\forall u \in \mathcal{G}_j$ in order to maximize the SE of the system as well as satisfying each UE with a minimum QoE. We propose a hybrid power allocation scheme, which will guarantee the fundamental trade-off between the power scheme that aims to maximize the sum-rate of the system and the power scheme that aims to guarantee fairness for each UE in the cell. It involves a three step process, starting with two power optimization procedures. These two power optimization procedures are formulated as convex optimization problems and solved by exploiting KKT conditions and CVX (a Matlab software for discipline convex programming), respectively. The third step involves utilizing results from the aforementioned power optimization procedures to compute average powers allocated to UEs. In what follows, we will formulate and produce solution for each of the two power optimization procedures, from which we can now develop and allocate the hybrid powers to each UE in each cell.

Note, the relationship between the power allocation coefficients and the beamforming directions is given as

$$\mathbf{w}_u = \sqrt{p_u}\tilde{\mathbf{w}}_u \quad \forall u \in \mathcal{G}_j. \tag{5.18}$$

We proceed by formulating the first power RA problem that will maximize the sum-rate of each cell as

$$\begin{aligned} \underset{\{p_u\}\forall u \in \mathcal{G}_j}{\text{minimize}} \quad & -\sum_{u\in\mathcal{G}_j} log_2\left(1 + p_u \frac{|\mathbf{h}_{j,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma^2}\right) \\ \text{subject to} \quad & \sum_{u\in\mathcal{G}_j} p_u = q_j, \\ & p_u \geq 0 \;\; \forall u \in \mathcal{G}_j. \end{aligned} \tag{5.19}$$

Where the utility function represents the sum-rate achievable by UEs in each cell, q_j is the power limit at BS_j. The power RA problem is convex [119], therefore can be solved efficiently. We obtain the Lagrangian function of (5.19) as

$$\begin{aligned} \mathcal{L}(p_u, \lambda_u, \nu_j) = & -\sum_{u\in\mathcal{G}_j} log_2\left(1 + p_u\rho_u\right) \\ & + \nu_j\left(\sum_{u\in\mathcal{G}_j} p_u - q_j\right) - \sum_{u\in\mathcal{G}_j} \lambda_u p_u. \end{aligned} \tag{5.20}$$

where $\rho_u = \frac{|\mathbf{h}_{j,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma_u^2}$ represents the *signal to noise ratio* (SNR) and $\lambda_u \geq 0$ is the Lagrange multiplier associated with UE u power limit, while ν_j is the Lagrange multiplier associated with BS_j power limit. To solve (5.19), we exploited the KKT optimality conditions which are

$$\sum_{u \in \mathcal{G}_j} p_u = q_j, \tag{5.21a}$$

$$p_u \geq 0 \quad \forall u, \tag{5.21b}$$

$$\lambda_u \geq 0 \quad \forall u, \tag{5.21c}$$

$$\lambda_u p_u = 0 \ \ \forall u, \tag{5.21d}$$

$$-\frac{\rho_u}{(1 + p_u \rho_u) \ln 2} + \nu_j - \lambda_u = 0, \ \forall u, \tag{5.21e}$$

where (5.21a) to (5.21e) represent primal feasibility, primal feasibility, dual feasibility, complementary slackness, and stationarity conditions respectively. We can easily prove that strong duality holds for this problem because the objective and constraint functions are convex and differentiable, also slater's constraint qualification [120] is satisfied. Therefore, KKT conditions are both necessary and sufficient for the optimal solution of this power RA problem. We proceed further by rearranging terms in (5.21e) and noting that λ_u performs as a slack variable which can easily be eliminated. Consequently, we form an equivalent representation of (5.21d) and (5.21e) as

$$p_u(\nu_j - \frac{\rho_u}{(1 + p_u \rho_u) \ln 2}) = 0 \ \forall u \in \mathcal{G}_j \tag{5.22a}$$

$$\nu_j \geq \frac{\rho_u}{(1 + p_u \rho_u) \ln 2} \ \forall u \in \mathcal{G}_j. \tag{5.22b}$$

The inequality in (5.22b) should also hold with equality in order not to violate the complementary slack condition. As a consequence, we establish the following relationships

$$p_u = \frac{1}{\grave{\nu}_j} - \frac{1}{\rho_u} \ \forall u, \tag{5.23}$$

where $\grave{\nu}_j = \nu_j \ln 2$. From (5.23) one can observe that the optimal power coefficients $\{p_u\}$ $\forall u$ is dependent on the SNR $\{\rho_u\}$ $\forall u$ of individual UE channels. If $\grave{\nu}_j < \rho_u$ $\forall u \in \mathcal{G}_j$, positive values of p_u will be allocated to UEs whose channel SNRs are ρ_u $\forall u$ else non-positive values of p_u will be allocated which is not proper. Therefore the power RA problem is solved by

$$p_u = \begin{cases} \frac{1}{\grave{\nu}_j} - \frac{1}{\rho_u}, & \grave{\nu}_j < \rho_u, \\ 0, & \grave{\nu}_j \geq \rho_u. \end{cases} \tag{5.24}$$

We can also find the Lagrange multipliers $\grave{\nu}_j$ by rearranging some terms in (5.23) which will give us this relationship

$$\grave{\nu}_j = \left(\frac{q_j + \sum_{u \in \mathcal{G}_j}^{U} \frac{1}{\rho_u}}{U} \right)^{-1} . \tag{5.25}$$

This power RA leads to water-filling solutions where powers are allocated to UEs based on individual channel gain. At high SNR, the values of $\frac{1}{\rho_u}$ are far less compared to $\grave{\nu}_j$, thus uniform power is allocated to each UE, while at low SNR, the values of $\frac{1}{\rho_u}$ are far more compared to $\grave{\nu}_j$, hence full power is allocated to the UE with the best channel.

In the second power RA problem, a notion of fairness is included into the power RA problem formulation by including one more constraint. Assuming we include the constraint that each UE must have a minimum QoE which might represent the minimum data rate required to be received by UE u in each cell, then the corresponding power RA is formulated as

$$\begin{aligned} \underset{\{p_u\}_{u \in \mathcal{G}_j}}{\text{maximize}} \quad & \sum_{u \in \mathcal{G}_j} log_2 \left(1 + p_u \frac{|\mathbf{h}_{j,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma^2} \right), \\ \text{subject to} \quad & \sum_{u \in \mathcal{G}_j} p_u = q_j \\ & log_2 \left(1 + p_u \frac{|\mathbf{h}_{j,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma^2} \right) \geq R_u \ \forall u, \\ & p_u \geq 0 \ \forall u, \end{aligned} \tag{5.26}$$

where R_u denotes the minimum required data rate for UE u. The optimal solution for $\{p_u\}_{u \in \mathcal{G}_j}$ can also be achieved using a solver known as CVX [121], which is a Matlab software for discipline convex programming. For fairness in this power RA formulation to be achieved, this hard constraint $log_2 \left(1 + p_u \frac{|\mathbf{h}_{j,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma^2} \right) \geq R_u \ \ \forall u$, needs to be active. In some cases it is not but depends on how large the threshold R_u is.

The hybrid power allocation scheme will scan through a set of individual powers gotten through (5.24) and the ones gotten through (5.26). Then it will allocate powers to each UE by computing the average power for each UE based on the aforementioned sets. The goal is to be sure that the trade-off between achieving the QoE of each UE and maximizing the total sum-rate of each cell is attained.

Discussion

Note that the objective of (5.19) is to allocate powers to UEs in each cell in order to maximize the aggregate throughput. Our aim is to achieve the fundamental trade-off between maximizing the aggregate throughput and achieving a minimum fairness condition for each UE in each cell. Consequently, (5.19) will not be appropriate to achieve this goal because it is geared towards achieving large system-rate without much regard to individual performance. However, (5.26) would have done just fine. The drawback of (5.26) is that the threshold R_u, if becomes too large, could make this hard constraint $log_2\left(1+p_u\frac{|\mathbf{h}_{j,u}^H\tilde{\mathbf{w}}_u|^2}{\sigma^2}\right) \geq R_u$ inactive (i.e, not been satisfied with equality at the optimum solution) hence can lead to infeasibility of the solution set, meaning that depending on the threshold R_u, solutions to the optimization problem may not be possible. The purpose to avoid the uncertainty in (5.26) gave rise to the motivation behind hybrid power RA. Which posit the point of view that we can achieve the fundamental trade-off between maximizing the aggregate throughput and maintaining UE fairness, without having to worry about infeasibility problem which may arise in (5.26). Furthermore, in situation where (5.26) is infeasible, the hybrid power allocation scheme will replace infeasibility with zero values for the solution of (5.26), hence leading to uniform power allocation to every UE when the hybrid power is allocated. We summarized the hybrid power allocation scheme and the distributed RA procedure using Algorithm 1 and 2 respectively.

5.3.3 Achievable Rates for UEs in HetNet

We want to calculate the achievable data rate for each UE after allocating the spatial directions and powers accordingly. The data signal received at UE u is given by

$$y_u = y_u^{des} + y_u^{int} + n_u, \tag{5.27}$$

Algorithm 1 Hybrid power allocation scheme for each UE in HetNet in each cell

Input and variables

$\mathbf{P}_w$: set of powers gotten using (5.23);

$\mathbf{P}_f$: set of powers gotten by solving (5.26);

U : total number of UEs in each cell;

u : UE index;

procedure

1: **for** u = 1 to U **do**

2: $\mathbf{P}_{sum}(u) = \mathbf{P}_w(u) + \mathbf{P}_f(u)$;

3: $\mathbf{P}(u) = \mathbf{P}_{sum}(u)/2$;

4: **end for**

Hybrid power allocates $\mathbf{P} = [p_1 \cdots p_U]$

where y_u^{des}, y_u^{int} and n_u represent the desired signal which is obtained by combining (5.4) and (5.18), interference signal and noise respectively. The received interference is given by

$$\begin{aligned} y_u^{int} = & \sum_{k \in \mathcal{G}_l, k \neq u}^{U} \mathbf{h}_{l,u}^H \sqrt{p_k} \tilde{\mathbf{w}}_k s_k \\ & + \sum_{j \neq l}^{K_t} \sum_{m \in \mathcal{G}_j}^{U} \mathbf{h}_{j,u}^H \sqrt{p_m} \tilde{\mathbf{w}}_m s_m, \end{aligned} \tag{5.28}$$

these are signals that are destined for other UEs apart from the desired UE in HetNet. The first term in (5.28) is the inter-UE interference while the second term is the inter-cell interference. The achievable data rate for UE u is given by

$$r_u = log2\left(1 + \frac{G_{y_u^{des}}}{G_{y_u^{int}} + G_{n_u}}\right) \quad \forall u, \tag{5.29}$$

where G denotes the power aspect of the desired signal, interference signal and noise respectively.

5.4 Simulation Results

In this section, we evaluate the performance of our proposed distributed RA methods by comparing with centralized RA method and other existing distributed

Algorithm 2 Distibuted Allocation of spatial directions and powers for each UE in HetNet

Input and variables

$\mathcal{G}_j$: set of UEs served by BS_j;

$\bar{\mathbf{R}}_{l,u}$: array covariance matrix for UEs affected by leakage;

$\bar{\mathbf{R}}_{l,u}$: covariance matrix for the desire UE served by BS_l ;

U : total number of UEs in each cell;

ρ_u : SNR of UE u ;

ν_j : Lagrange multiplier associated with BS_j power limit;

procedure

1: **for** UEs $\in \mathcal{G}_j$ i.e. u = 1 to U **do**

2: compute $\mathbf{w}_u$ from $(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}$ using (5.16);

3: obtain the unit-norm beamformers $\tilde{\mathbf{w}}_u$ using (5.17);

4: compute p_u from $\frac{1}{\nu_j} - \frac{1}{\rho_u}$ using (5.23) and;

5: compute p_u from (5.26);

6: apply hybrid algorithm 1 ;

7: **end for**

BS_j transmits $\mathbf{x}_j = \sum_{u \in \mathcal{G}_j}^{U} \sqrt{p_u}\tilde{\mathbf{w}}_u s_u$

RA methods based on the average achievable rate per cell, SNR, the number of transmit antennas, e.t.c.

5.4.1 Simulation Setting

We consider a simple simulation setting with randomly distributed PBSs deployed at hotspot locations in the coverage area of MBS as illustrated in Fig 5.1. The minimum distance among pico sites is set to 40m, and we assume that all PBSs are not geometrically separated, hence interference among PBS is possible and therefore considered. The minimum distance from the macro site to the pico sites is 75m. We assume that the UEs in the HetNet are randomly distributed and are located at the CRE such that each UE will receive significant *intercell interference* (ICI). The UEs served by PBS are uniformly distributed between 35m and 55m from the PBS. Similarly, the UEs served by MBS are uniformly distributed between 220m and 260m from the MBS, also, the distance between the

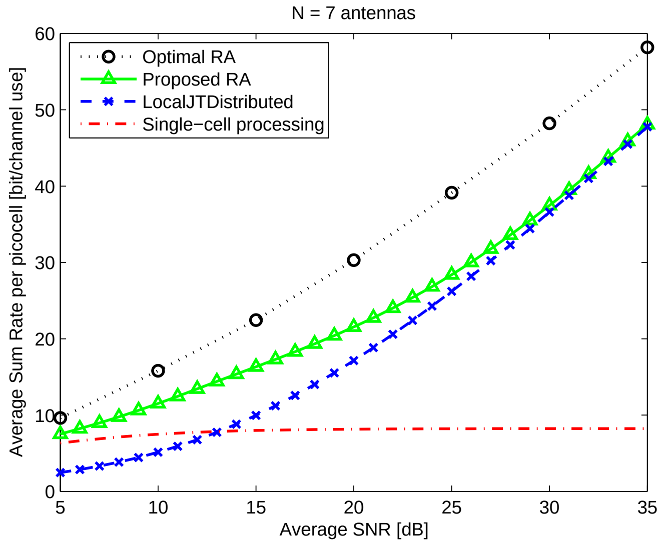

Figure 5.2: Average sum-rate as a function of SNR for different RA strategies, when $N = 7$, $U = 4$ and $\mathbf{card}(\mathcal{C}_j) = 3$ (i.e., 2 macro-cell UEs and one adjacent pico-cell UE).

macrocell UEs and the PBS is roughly between 40m and 45m, while the distance between the picocell UEs and the MBS is between 230m and 270m. Other system parameters are also based on the 3GPP simulation baseline parameters and can be found in [2]. The total BS transmit powers for MBS and PBS are 46dBm and 30dBm respectively, assuming a 10MHz bandwidth. The channel vector between BS_j and UE u is generated by this formulation $\mathbf{h}_{j,u} \triangleq \sqrt{g_{j,u}}\mathbf{h}^s_{j,u}$, where $\sqrt{g_{j,u}}$ is the large-scale pathloss from BS_j to UE u. $\mathbf{h}^s_{j,u} \in \mathbb{C}^N$ is the small scale (fading) channel vector from BS_j to UE u. The large scale pathloss in linear scale is expressed as

$$g_{j,u} = \frac{\psi}{d^n_{j,u}}, \tag{5.30}$$

where ψ is a constant which accounts for system losses, n is the path-loss exponent, typically $n > 3$, while $d_{j,u}$ is the distance between BS_j and UE u. The large-scale path loss model in dB for the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{j,u}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{j,u}}{10^3})$. This simulation settings will be used except otherwise indicated.

5.4.2 Centralized vs Distributed

HetNet favours coordinated processing, but should be done in a distributed fashion to enable practicability and also to avoid computational complexity. Our proposed RA method is computed in a distributed fashion by BS_j using only local CSI whereas the optimal RA depicted in Fig. 5.2 utilizes the B&B method [12] which favours coordinated processing but is implemented in a centralized fashion at a super BS that has the aggregate CSI of all BS in HetNet. B&B method is practically infeasible for large scale networks because of high computational complexity. Considering the trade off between performance and computational complexity and also, possible hardware failures which might lead to coordination failure for a centralized scheme, our distributed RA is hereby recommended. We also compare our proposed RA strategy with the (local) Joint transmission (JT) distributed RA proposed in [122] and we found out that our proposed strategy gives better performance and this is because JT can only maximize its potential if there are exchange of control signaling among BSs. JT needs global channel state information to perform optimally, however in this decentralized scheme, it has only local CSI to work with, this is the reason behind the sub-optimal performance. The least performed RA strategy in Fig. 5.2 is the single-cell processing, this is because it only consider its served UEs while designing the beamformers without coordinating interference to other UEs in the system thereby treating the out-of-cell interference as noise. This improper treatment of interference lead to severe performance loss when compared to other strategies.

5.4.3 Multiple Antenna: Key Component for the Design of 5G

Multiple antenna at BS can help meet high-capacity demands in downlink, also it can help provide fast and reliable transmission without bandwidth expansion or increase in transmit power. Under ideal circumstances, data rate should increase linearly with the number of transmit antenna, i.e., if the spatial dimension is utilized to serve UEs in parallel. Increase in the number of transmit antenna also helps in improving beamforming resolution. Fig. 5.3 shows the average sum-rate

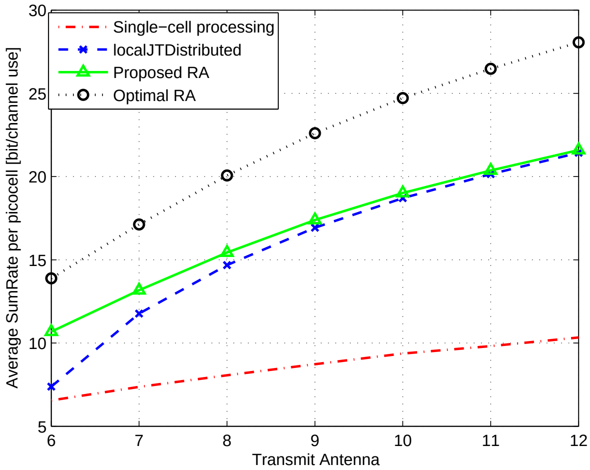

Figure 5.3: Average sum-rate as a function of transmit antenna for different RA strategies, when $SNR = 10dB$, $U = 6$ and $\mathbf{card}(C_j) = 3$ (i.e., 2 macro-cell UEs and one adjacent pico-cell UE).

as a function of the transmit antennas, from this result we observe that the optimal RA strategy has the best performance because it is centralized but is practically infeasible for large scale. We also note that for the distributed strategies, our proposed RA gives the best performance followed by the localJTdistributed and then single-cell processing.

The CDFs of the average sum-rate are shown in Fig. 5.4. The optimal RA gives the best performance because of its centralized nature. Among the distributed strategies compared, our proposed RA outperforms both localJTdistributed and single-cell processing strategies.

5.4.4 Coverage and Capacity Gains

HetNet is a key technology in 5G that enables coverage and capacity gains, but we know that due to interference problem the capacity gains might be stunted. Using our proposed RA we want to evaluate the impact of having more pico-cells in the coverage area of a macro cell will have on the spectral efficiency of HetNet. We were able to get Fig. 5.5 by utilizing the HetNet system model in Fig. 5.1, for some of the simulation setting, i.e., for each pico cell considered, pico UEs served

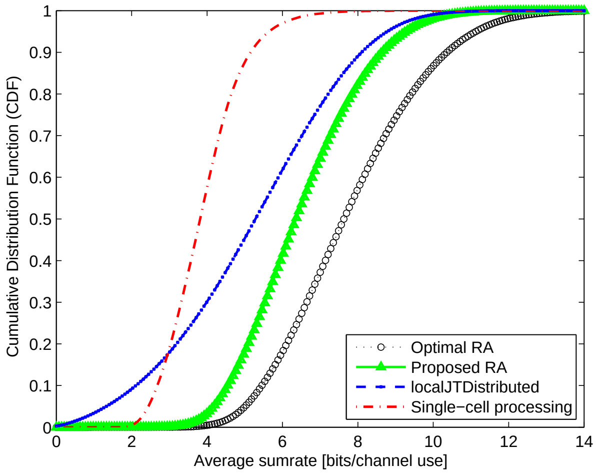

Figure 5.4: Cumulative distribution function (CDF) of average sum-rate for different RA strategies, when $N = 8$, $U = 3$ and $\mathbf{card}(C_j) = 3$ (i.e., 2 macrocell UEs and one adjacent picocell UE).

= 4, co-channel pico UE considered = 1 and co-channel macro UE considered = 2. Fig. 5.5 compares the downlink (DL) cell spectral efficiencies of macro cell against pico cells where the number of pico cell is increasing. The first observation is that the deployment of hotspot pico cells does not affect the performance of the macro cell. Secondly, the second bar in Fig. 5.5 depicts cell-splitting gain provided by deploying pico cell in a hotspot region and lastly, one cannot say that the cell splitting gain is a linear function of the number of pico cells due to the effect of channel gain but can be said to be very close. This shows that our distributed RA strategy helps in managing intercell interference.

5.4.5 Power Allocation: Water-filling vs Fairness

The optimization problem in RA is driven based on the objective of the system designer. Our objective for UEs in the CRE is to achieve the maximal sum-rate for each cell while ensuring a minimum data rate constraint for each UE. In the light of this, the hybrid power RA will enable this. For distributed strategies in Fig. 5.6, we observe that the power RA formulation of (5.19) which is solved by

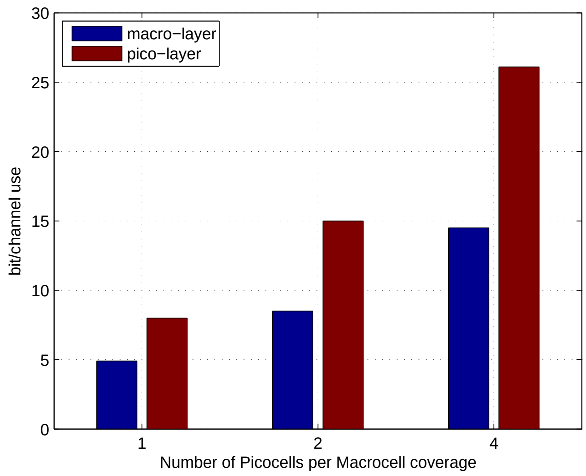

Figure 5.5: Cell spectral efficiencies comprising one, two and four pico cells in the coverage area of a macro cell.

(5.24) will achieve the maximal throughput for each cell but without ensuring fairness to UEs.

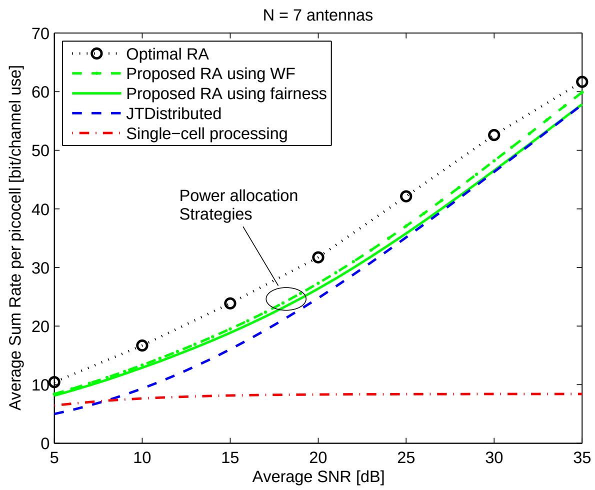

Figure 5.6: Average sum-rate as a function of SNR for different RA strategies, when $N = 7$, $U = 4$ and $\mathbf{card}(\mathcal{C}_j) = 3$ (i.e., 2 macrocell UEs and one adjacent picocell UE).

At high SNR, (5.19) will allocate uniform powers to all UEs in the cell, but at low SNR, little or no power(s) will be allocated to UE(s) with attenuated channel(s) while high or all power will be allocated to UE(s) with strong channel(s). On the contrary, at high or low SNR, (5.26) will make sure that each UE achieves its minimum data rate constraint. This fairness condition makes it to sacrifice maximal sum-rate (providing a lower sum-rate) when compared to (5.19).

5.5 Summary

In this chapter, we have developed a distributed RA strategy for UEs in HetNet such that UEs in the CRE will only experience minimized loss in rate due to higher interference received from the MBS. The resources allocated to UEs are the spatial directions (unit beamformer) and the power resource. We formulate the spatial RA optimization problem as selecting the optimal beamformers that will cause the least interference to UEs in the same cell and UEs $\in \mathcal{C}_j$. Our proposed hybrid power resource allocation scheme involves a three-step process, starting with two power optimization procedures. These two power optimization procedures are formulated as convex optimization problems and solved by exploiting Karush-Kuhn-Tucker (KKT) conditions and CVX (a Matlab software for discipline convex programming) respectively. While the third step involves utilizing results from the aforementioned power optimization procedures to compute average powers allocated to UEs. Results obtained show that our distributed RA strategy outperforms other distributed RA strategies such as localJTdistributed in [122] and the single-cell processing strategy. Our strategy is the closest in performance to the optimal RA strategy which is centralized.

Chapter 6

Heuristic Coordinated Beamforming for Heterogeneous Cellular Networks

6.1 Introduction

Mobile applications have become part and parcel of people's everyday life with the requirement on access to social media, video contents, etc. As the demand for higher data rates increases, operators introduce new techniques and architectures to improve the capacity and coverage of their networks. Heterogeneous cellular network which is a network consisting of *low-powered nodes* (LPNs) in the coverage area of a *macro-cell base station* (MBS) plays an important role to meet future coverage and capacity needs. The dense deployment of LPNs close to mobile subscribers will massively offload macro cell traffic from the MBS with the help of *cell range expansion* (CRE) [99] resulting in an improved spectral efficiency (bits/s/Hz) for the whole network. The problem with HetNet is that due to the unplanned reuse-one deployment of small cells in the coverage area of the MBS, different interference environment will be created. Furthermore, in conventional single-tier networks, a *user equipment*(UE) is associated with the *base station*(BS) whose signal is received with the biggest average strength which can be described as the *reference signal received power* (RSRP) [15]. Whereas in HetNet, some UEs could be associated with a LPN in uplink even though the received power from an MBS could be higher. This enables cell splitting gain however only when the interference from the MBS has been mitigated. Interference is a limiting factor to the performance of dense HetNet with universal frequency reuse and if not properly mitigated will cancel the gain it provides. One of our objectives in this chapter is to find ways to manage interference experience among UEs in HetNet effectively. If this is achieved, it will go a long way to improving the achievable rates for UEs in the system. There have been different methods proposed in the literature to solve the interference problem in HetNet. *Coordinated multi-point* (CoMP) has emerged as an efficient way to substantially suppress interference in cellular networks [78]. CoMP can be broadly divided into two types: *joint transmission* (JT) [43] and *coordinated beamforming* (CB) [38]. The JT CoMP exploits all degrees of freedom provided by the channels hence achieving the highest spectral efficiency. However, based on practical implementation it is more complex and costly comparing with coordinated beamforming because it requires data sharing and tight synchronization. Therefore, this chapter will be considering coordinated beamforming.

In coordinated beamforming, each BS serves its own UEs and together with other cooperating BSs make resource allocation decisions to allocate transmit powers and spatial directions (beamforming directions) to each UE in the network.

6.1.1 Prior Works and Contributions

Many works in the past have considered coordinated beamforming, where the *channel state information* (CSI) of all UEs in a cluster are shared among the BSs that form the cluster. This CSI will now be utilized as part of the input to design beamformers jointly by all the BSs that form the cluster. In [38] the authors are interested in the design criterion, which is to minimize the total power transmitted in the system subject to achieving a fixed *quality of service* (QoS) requirement for all UEs that is part of the cluster. Most techniques proposed by authors [40, 123, 124] using coordinated beamforming assume a high level of cooperation among BSs, making the coordinated beamforming algorithm for the system to be centralized. Hence the optimal beamformers are optimized jointly. We differ from these authors for the following reasons. Firstly, HetNet tends to be distributed unlike macro-only network considered by the aforementioned reviewed papers. Secondly, the X2 interface which is the backhaul link that connects BSs in HetNet doesn't have the capabilities to withstand huge burdens in its backhaul. Therefore, our design criterion aims at maximizing the *signal-to-leakage-and-noise* (SLNR) ratio of each UE in each cell in HetNet. In this approach each transmitter optimizes its beamformers based on local CSI present at each transmitter. This approach will indirectly maximize the average sum-rate achievable in HetNet. Some authors have considered this design criterion in single-cell macro-only networks [110] and in multi-cell macro-only networks [113], However, none has considered it for a HetNet scenario, which have significant interference and different cell association and selection procedure.

Maximizing SLNR is a heuristic way to design beamformers rather than maximizing the *signal-to-interference-and-noise-ratio* (SINR). Because in SINR, beamforming vectors of both the desired UEs and non-intended UEs are coupled together, hence a centralized algorithm is needed to solve them. However, heuristic coordinated beamforming algorithms based on SLNR are usually more efficient and have less complexity compared with global optimal algorithms like *branch and bound* (B&B) and other iterative algorithms which are centralized, and are preferred to be used for off-line benchmarking.

6.1.2 Chapter Organization

The rest of this chapter is organized as follows. In section 6.2 we present the system model considered. Section 6.3 presents the heuristic beamforming optimization problem formulation and how it is solved. Simulation results and discussions are provided in section 6.4, and the summary is given in the last section.

Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $||\cdot||_2$ denotes the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the statistical expectation over a random variable. We use upper-case boldface letters for matrices and lower-case boldface for vectors.

6.2 System Model

Let's consider the downlink of a two-tier HetNet with P pico cells of N_p transmit antennas each, serving U_p UEs with single receive antenna each. These small cells are underlaid in a macro-cellular coverage in the same frequency band where the MBS has N_m transmit antenna with which it serves its U_m UEs as depicted in

Figure 6.1. The set of UEs served by the jth PBS is denoted by $\mathcal{S}_j \subseteq \{1, \ldots, U_p\}$, while the set of UEs served by the MBS is denoted by $\mathcal{M} \subseteq \{1, \ldots, U_m\}$. We assume that each of the U_m UEs has a single receive antenna and is located at the cell edge area of the pico cells. We denote the set of BSs in the HetNet by $\mathcal{D} = \{0, 1, \ldots, P\}$ where 0 represents the macro BS, also we denote the u_pth UE as UE u_p. The received signal at UE u_p in the pth pico cell is $y^p_{u_p} \in \mathbb{C}$, which is a summation of the intended signal, intra-cell interference, and *inter-cell interference* (ICI), and given by

$$\begin{aligned} y^p_{u_p} &= \sqrt{g_{p,u_p}}(\mathbf{h}^s_{p,u_p})^H \mathbf{w}^p_{u_p} x^p_{u_p} + \sum_{\substack{s_p \in \mathcal{S}_p \\ s_p \neq u_p}} \sqrt{g_{p,u_p}}(\mathbf{h}^s_{p,u_p})^H \mathbf{w}^p_{s_p} x^p_{s_p} \\ &+ \sum_{q \neq p}^{P} \sqrt{g_{q,u_p}}(\mathbf{h}^s_{q \longrightarrow p,u_p})^H \sum_{s_q \in \mathcal{S}_q} \mathbf{w}^q_{s_q} x^q_{s_q} \\ &+ \sum_{s_m \in \mathcal{M}} \sqrt{g_{m,u_p}}(\mathbf{h}^s_{m \longrightarrow p,u_p})^H \mathbf{v}^m_{s_m} x^m_{s_m} + z^p_{u_p}, \end{aligned} \tag{6.1}$$

where $\sqrt{g_{p,u_p}}$ and $\mathbf{h}^s_{p,u_p}$ denote the large-scale and small-scale fading gain from the pth pico base station (PBS) to UE u_p. Also, $\sqrt{g_{m,u_p}}$ denotes the large-scale fading gain from the MBS to UE u_p. $\mathbf{w}^p_{u_p}$ and $x^p_{u_p}$ are the transmit beamforming vector and data for pico cell UE u_p respectively. Furthermore, $z^p_{u_p}$ is the complex white gaussian noise with variance $\sigma^2_{u_p}$ at the receiver. The first summand to the fourth summand in (6.1) represent the intended desired signal, intra-cell interference, inter-cell interference from other PBSs and inter-cell interference from MBS respectively. The main system parameters are listed in Table 6.1. $\mathbf{h}_{p,u_p} \in \mathbb{C}^{N_p \times 1}$ is the channel vector from the pth PBS to UE u_p so that $\mathbf{H}_{p \longrightarrow U_p} = [\mathbf{h}_{p,1}\ \mathbf{h}_{p,2} \cdots \mathbf{h}_{p,U_p}]^T \in \mathbb{C}^{U_p \times N_p}$ represent the channel matrix from the pth PBS to all its served intra-cell UEs. Also, $\mathbf{h}_{q \longrightarrow p,u_p} \in \mathbb{C}^{N_p \times 1}$ is the channel vector from the qth pico BS to UE u_p in the pth pico cell so that $\mathbf{H}_{q \longrightarrow U_p} = [\mathbf{h}_{q \longrightarrow p,1}\ \mathbf{h}_{q \longrightarrow p,2} \cdots \mathbf{h}_{q \longrightarrow p,U_p}]^T$ represent interfering channel matrix from the qth PBS to U_p UEs (i.e. all the UEs in the pth pico cell).

Similarly, $\mathbf{h}_{m \longrightarrow p,u_p} \in \mathbb{C}^{N_m \times 1}$ is the channel vector from the MBS to UE u_p in the pth pico cell so that $\mathbf{H}_{m \longrightarrow U_p} = [\mathbf{h}_{m \longrightarrow p,1}\ \mathbf{h}_{m \longrightarrow p,2} \cdots \mathbf{h}_{m \longrightarrow p,U_p}]^T$. Assuming we

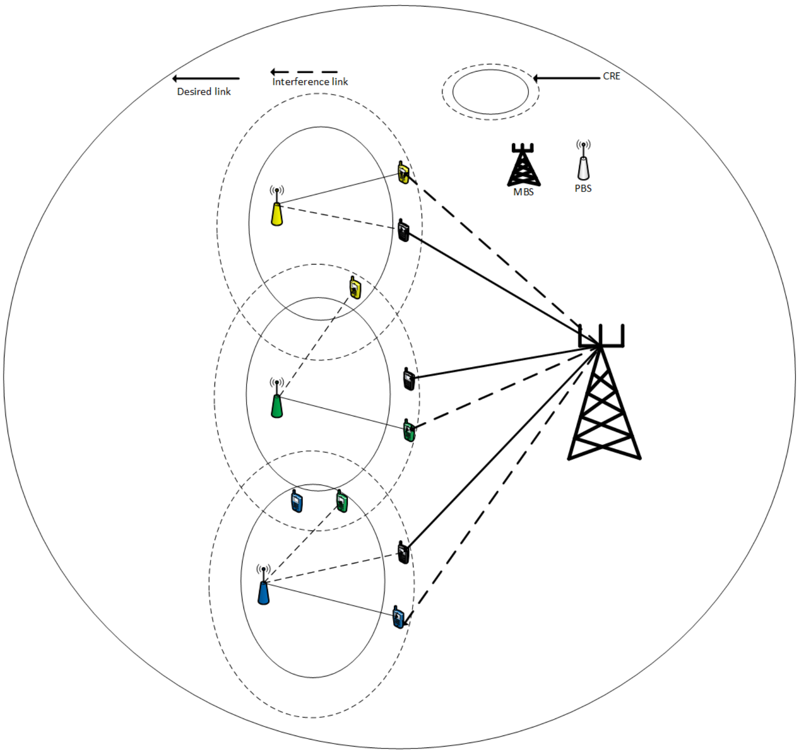

Figure 6.1: Heterogeneous model used in the simulation with one MBS and three PBS, and $U_p = 2, U_m = 3$

have some out-of-cell UEs (including the pico and macro UEs) in HetNet, and we denote the total number of these UEs as $\bar{U}$. Then the channel matrix towards these $\bar{U}$ UEs from the pth PBS is denoted as $\bar{\mathbf{H}}_{p\longrightarrow\bar{U}} \in \mathbb{C}^{\bar{U}\times N_p}$, where $\bar{\mathbf{H}}_{p\longrightarrow\bar{U}} = [\bar{\mathbf{h}}_{p,1}\ \bar{\mathbf{h}}_{p,2}\cdots\bar{\mathbf{h}}_{p,\bar{U}}]^T$ such that $\bar{\mathbf{h}}_{p,\bar{u}}$ represent the rows of $\bar{\mathbf{H}}_{p\longrightarrow\bar{U}}$. Therefore, $\bar{\mathbf{H}}_{p\longrightarrow(\bar{U}+p)} = [\mathbf{h}_{p,1}\ \mathbf{h}_{p,2}\cdots\mathbf{h}_{p,u_{p-1}}\ \mathbf{h}_{p,u_{p+1}}\cdots\mathbf{h}_{p,U_p}\ \bar{\mathbf{h}}_{p,1}\cdots\bar{\mathbf{h}}_{p,\bar{U}}]^T \in \mathbb{C}^{(U_p-1+\bar{U})\times N_p}$ is the extended channel matrix from the pth PBS to all the out-of-cell UEs and the intra-cell UEs but excluding the desired served UE u_p .

For notational convenience henceforth we denote this extended channel matrix $\bar{\mathbf{H}}_{p\longrightarrow(\bar{U}+p)}$ as $\bar{\mathbf{H}}_p$.

The received signal at macrocell UE u_m in the pth pico cell is given by

$$\begin{aligned} y_{u_m}^p &= \sqrt{g_{m\longrightarrow p,u_m}}(\mathbf{h}_{m\longrightarrow p,u_m}^s)^H \mathbf{v}_{u_m}^m x_{u_m}^m \\ &+ \sum_{q\neq m}^{P} \sqrt{g_{q\longrightarrow p,u_m}}(\mathbf{h}_{q\longrightarrow p,u_m}^s)^H \sum_{s_t\in\mathcal{S}_q} \mathbf{w}_{s_t}^p x_{s_t}^p + z_{u_m}^p, \end{aligned} \quad (6.2)$$

Table 6.1: Key Notations and Symbols Used in This Chapter

Symbol	Definition/Explanation	Symbol	Definition/Explanation
P	Number of pico cells covered by the macro cell	$\mathbf{h}^{s}_{p,u_p}$	The small scale (fading) channel vector from pth PBS to UE u_p
N_p or N_m	Total number of transmit antenna at PBSs or MBS	$\mathcal{S}_j$	The set of pico UEs served by the jth PBS
U_p or U_m	Total number of intra-cell UEs served by each PBS or MBS	$\mathcal{M}$	The set of macro UEs served by the MBS
$\bar{U}$	Total number of out-of-cell UEs in HetNet	$y^{p}_{u_p}$	The received signal at UE u_p in the pth pico cell
p, u_p	From the pth PBS to UE u_p	m, u_p	From the MBS to UE u_p
$q \to p, u_p$	From the qth PBS to UE u_p in the pth pico cell	$m \to p, u_p$	From the MBS to UE u_p in the pth pico cell
$q \to U_p$	From the qth PBS to U_p UEs (Interfering link)	$p \to U_p$	From the pth PBS to all its intra-cell served UEs
$m \to U_p$	From the MBS to U_p UEs (Interfering link)	$\sqrt{g_{m,u_p}}$	The large-scale pathloss from the MBS to UE u_p
τ_j	Power limit at BS$_j$	$y^{p}_{u_m}$	The received signal at UE u_m in the pth pico cell

where $\sqrt{g_{m\longrightarrow p,u_m}}$ denotes the large-scale fading gain from MBS to UE u_m in the pth pico cell. $z^{m}_{u_m}$ is the complex white gaussian noise with variance $\sigma^2_{u_m}$ at the receiver. $\mathbf{h}_{m\longrightarrow p,u_m} \in \mathbb{C}^{N_M\times 1}$ is the channel vector from the MBS to UE u_m at the pth pico cell. $\mathbf{h}_{q\longrightarrow p,u_m} \in \mathbb{C}^{N_p\times 1}$ is the channel vector from the qth PBS to UE u_m at the pth pico cell. Suppose we denote the number of other out-of-cell UEs as $\bar{U}$ so that $\bar{\mathbf{H}}_{m\longrightarrow \bar{U}} = [\bar{\mathbf{h}}_{m,1}, \bar{\mathbf{h}}_{m,2} \cdots \bar{\mathbf{h}}_{m,\bar{U}}]^T \in \mathbb{C}^{\bar{U}\times N_M}$ now represent the channel matrix towards these $\bar{U}$ UEs from the MBS, where $\bar{\mathbf{h}}_{m,\bar{u}}$ represent the rows of $\bar{\mathbf{H}}_{m\longrightarrow \bar{U}}$. Therefore,

$\bar{\mathbf{H}}_{m\longrightarrow(\bar{U}+m)} = [\mathbf{h}_{m,1} \cdots \mathbf{h}_{m,u_m-1}\ \mathbf{h}_{m,u_m+1} \cdots \mathbf{h}_{m,U_m}\ \bar{\mathbf{h}}_{m,1} \cdots \bar{\mathbf{h}}_{m,\bar{U}}]^T \in \mathbb{C}^{(U_m-1+\bar{U})\times N_m}$ is the extended channel matrix that include all the out-of-cell and the intra cell UEs but excluding the desired served UE u_m. For notational convenience henceforth we denote this extended channel matrix $\bar{\mathbf{H}}_{m\longrightarrow(\bar{U}+m)}$ as $\bar{\mathbf{H}}_m$.

The HetNet considered has per-base station individual power constraint and we assume that each BS (PBS and MBS) equally allocates total transmit power to its served UEs.

6.3 Beamformer Design

The properties of a beamformer are the spatial characteristics (spatial directions) and the transmission power associated with each spatial direction. Therefore, this relationship can be expressed as

$$\mathbf{w}_u = \sqrt{q_u}\tilde{\mathbf{w}}_u \quad \forall u, \tag{6.3}$$

where u is a UE index which represent any of u_m or u_p. Furthermore, $\tilde{\mathbf{w}}_u$ represent the spatial directions to each UE, while $\sqrt{q_u}$ represent its associated allocated power intended for u. In this section, we are interested in designing the spatial directions of each beamformer heuristically because we are interested in achieving low-complexity algorithms which are practical and more efficient to achieve. For the power allocation, we assume equal power loading from each BS to its served UEs such that the power allocated to UE u_p and UE u_m are $q_{u_p} = \frac{\tau_j}{U_p}$ and $q_{u_m} = \frac{\tau_j}{U_p}$ respectively. The individual power allocated to each UE in each cell can mathematically represented as

$$\sum_{u_p=1}^{U_p} q_{u_p} \leq \tau_j \quad \forall j \in \mathcal{D}, j \neq 0, \tag{6.4}$$

and

$$\sum_{u_m=1}^{U_m} q_{u_m} \leq \tau_j \quad \forall j \in \mathcal{D}, j = 0, \tag{6.5}$$

respectively, where τ_j denotes the power limit at BS_j.

6.3.1 Problem Formulation

SLNR simply can be defined as the ratio between the desired signal power at the intended UE and the noise plus the total interference power leaked to non-intended UEs. For an intended UE u_p, the desired signal received by this UE from the pth pico cell as shown in (6.1) is

$$y_{u_p}^{p(des)} = \mathbf{h}_{p,u_p}^H \mathbf{w}_{u_p}^p x_{u_p}^p, \tag{6.6}$$

where $\mathbf{h}_{p,u_p} \triangleq \sqrt{g_{p,u_p}} \mathbf{h}_{p,u_p}^s$. While the leakage directed away from this UE is

$$\mathbf{y}_{u_p}^{p(leak)} = \bar{\mathbf{H}}_p \mathbf{w}_{u_p}^p x_{u_p}^p. \tag{6.7}$$

Therefore, the SLNR for this pico UE at the pth pico cell is given as

$$\begin{aligned} SLNR_{u_p} &= \frac{|y_{u_p}^{p(des)}|^2}{||\mathbf{y}_{u_p}^{p(leak)}||_2^2 + \sigma^2} \\ &= \frac{\mathbf{w}_{u_p}^{p^H} (\mathbf{h}_{p,u_p} \mathbf{h}_{p,u_p}^H) \mathbf{w}_{u_p}^p}{\mathbf{w}_{u_p}^{p^H} (\bar{\mathbf{H}}_p^H \bar{\mathbf{H}}_p + (\frac{\sigma^2}{q_{u_p}}) \mathbf{I}_{N_p}) \mathbf{w}_{u_p}^p}, \end{aligned} \tag{6.8}$$

where $\mathbf{I}_{N_p}$ denotes $N_p \times N_p$ identity matrix. We assume that x_{u_p} is normalized to unit power, $\mathbb{E}[|x_{u_p}|^2] = 1$. Also the parameter $\frac{1}{q_{u_p}}$ will regularize the noise amplification that usually occur at low SNR.

For an intended UE u_m, the desired signal received by this UE from the MBS as shown in (6.2) is

$$y_{u_m}^{m(des)} = \mathbf{h}_{m \longrightarrow p,u_m}^H \mathbf{v}_{u_m}^m x_{u_m}^m, \tag{6.9}$$

where $\mathbf{h}_{m \longrightarrow p,u_m} \triangleq \sqrt{\alpha_{m \longrightarrow p,u_m}} \mathbf{h}_{m \longrightarrow p,u_m}^s$. While the leakage directed away from this UE is

$$\mathbf{y}_{u_m}^{m(leak)} = \bar{\mathbf{H}}_m \mathbf{v}_{u_m}^m x_{u_m}^m. \tag{6.10}$$

Therefore, the SLNR for UE u_m at the pth pico cell is denoted as

$$\begin{aligned} SLNR_{u_m} &= \frac{|y_{u_m}^{m(des)}|^2}{||\mathbf{y}_{u_m}^{m(leak)}||_2^2 + \sigma^2} \\ &= \frac{\mathbf{v}_{u_m}^{m^H} (\mathbf{h}_{m \longrightarrow p,u_m} \mathbf{h}_{m \longrightarrow p,u_m}^H) \mathbf{v}_{u_m}^m}{\mathbf{v}_{u_m}^{m^H} (\bar{\mathbf{H}}_m^H \bar{\mathbf{H}}_m + (\frac{\sigma^2}{q_{u_m}}) \mathbf{I}_{N_m}) \mathbf{v}_{u_m}^m}. \end{aligned} \tag{6.11}$$

To get the spatial directions for each UE from each BS that will maximize the rate achievable by it, we will optimize the SLNR under a unit-norm constraint. The optimization problem for (6.8) and (6.11) can be stated respectively as

$$\begin{aligned} \mathbf{w}_{u_p}^{opt} &= \underset{\{\mathbf{w}_{u_p}^p\}}{\operatorname{argmax}} \frac{\mathbf{w}_{u_p}^{p^H}(\mathbf{h}_{p,u_p}\mathbf{h}_{p,u_p}^H)\mathbf{w}_{u_p}^p}{\mathbf{w}_{u_p}^{p^H}(\bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + (\frac{\sigma^2}{q_{u_p}})\mathbf{I}_{N_p})\mathbf{w}_{u_p}^p} \\ &\text{subject to} \quad ||\mathbf{w}_{u_p}^p||_2^2 = 1, \end{aligned} \tag{6.12}$$

and

$$\begin{aligned} \mathbf{v}_{u_m}^{opt} &= \underset{\{\mathbf{v}_{u_m}^m\}}{\operatorname{argmax}} \frac{\mathbf{v}_{u_m}^{m^H}(\mathbf{h}_{m\longrightarrow p,u_m}\mathbf{h}_{m\longrightarrow p,u_m}^H)\mathbf{v}_{u_m}^m}{\mathbf{v}_{u_m}^{m^H}(\bar{\mathbf{H}}_m^H\bar{\mathbf{H}}_m + (\frac{\sigma^2}{q_{u_m}})\mathbf{I}_{N_M})\mathbf{v}_{u_m}^m} \\ &\text{subject to} \quad ||\mathbf{v}_{u_m}^m||_2^2 = 1. \end{aligned} \tag{6.13}$$

Note that these optimization problems are shown as generalized quotient problem [125] such that (6.12) and 6.13 are maximized when $\mathbf{w}_{u_p}^p$ and $\mathbf{v}_{u_m}^m$ are the generalized eigenvectors corresponding to the maximum generalized eigenvalue of the following marix pair:

$$(\mathbf{h}_{p,u_p}\mathbf{h}_{p,u_p}^H, \bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + (\frac{\sigma^2}{q_{u_p}})\mathbf{I}_{N_p}), \tag{6.14}$$

and

$$(\mathbf{h}_{m\longrightarrow p,u_m}\mathbf{h}_{m\longrightarrow p,u_m}^H, \bar{\mathbf{H}}_m^H\bar{\mathbf{H}}_m + (\frac{\sigma^2}{q_{u_m}})\mathbf{I}_{N_m}), \tag{6.15}$$

respectively. The optimal spatial directions for each UE in each BS is therefore computed in a closed-form as

$$\tilde{\mathbf{w}}_{u_p}^p = \frac{(\bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + (\frac{\sigma^2}{q_{u_p}})\mathbf{I}_{N_P})^{-1}\mathbf{h}_{p,u_p}}{||(\bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + (\frac{\sigma^2}{q_{u_p}})\mathbf{I}_{N_P})^{-1}\mathbf{h}_{p,u_p}||_2}, \tag{6.16}$$

and

$$\tilde{\mathbf{v}}_{u_m}^m = \frac{(\bar{\mathbf{H}}_m^H\bar{\mathbf{H}}_m + (\frac{\sigma^2}{\eta_m})\mathbf{I}_{N_M})^{-1}\mathbf{h}_{m\longrightarrow p,u_m}}{||(\bar{\mathbf{H}}_m^H\bar{\mathbf{H}}_m + (\frac{\sigma^2}{\eta_{u_m}})\mathbf{I}_{N_M})^{-1}\mathbf{h}_{m\longrightarrow p,u_m}||_2} \tag{6.17}$$

respectively.

6.3.2 Achievable Rates for UEs in HetNet

We want to calculate the achievable data rate for each UE in HetNet. Equal allocation of powers to each UE in HetNet will suit UEs in HetNet such as those at the CRE area of the pico cell. This is because they suffer from attenuation from their serving BSs. Utilizing water-filling strategy for allocating powers to UEs may not be the best for these UEs, since no power may be allocated to UE with attenuated channel gain. The achievable rates for a pico UE and a micro UE are denoted as

$$r^p_{u_p} = log_2(1 + SINR_{u_p}), \tag{6.18}$$

and

$$r^m_{u_m} = log_2(1 + SINR_{u_m}) \tag{6.19}$$

respectively. Where $SINR_{u_p}$ and $SINR_{u_m}$ can be obtained from (6.1) and (6.2) respectively.

6.4 Simulation Results

In this section, we evaluate the performance of our proposed heuristic beamformers by comparing with other heuristic beamformers based on the average achievable sum-rate per cell, SNR and the number of transmit antennas.

6.4.1 Simulation Setting

The Simulation parameters used for our considered HetNet model can be found in [2] while the HetNet model is depicted in Figure 6.1. The transmit powers of the macro and pico BS are respectively 46dBm and 30dBm, assuming a 10MHz bandwidth. We assumme that the UEs in the HetNets are randomly distributed and are located at the CRE, such that each UE will receive significant *inter-cell interference* (ICI). Also, we assume that all PBSs are not geometrically separated, hence interference among PBS is possible and therefore considered. The UEs served by PBS are uniformly distributed between 35m and 55m from the PBS. Similarly, the UEs served by MBS are uniformly distributed between 220m and 260m from the MBS, also, the distance between the macrocell UEs and the PBS is between 40m and 45m, while the distance between the picocell UEs and the MBS is between 230m and 270m. Other system parameters are also based on the 3GPP simulation baseline parameters and can be found in [2]. The channel vector between BS_j which can be any of (MBS or PBS) and UE u (u_m or u_p) is generated by this formulation $\mathbf{h}_{j,u} \triangleq \sqrt{g_{j,u}}\mathbf{h}^s_{j,u}$, where $\sqrt{g_{j,u}}$ is the large-scale pathloss from BS_j to UE u. $\mathbf{h}^s_{j,u} \in \mathbb{C}^N$ is the small scale (fading) channel vector from BS_j to UE u. The large scale pathloss in linear scale is expressed as

$$g_{j,u} = \frac{\psi}{d^n_{j,u}}, \tag{6.20}$$

where ψ is a constant which accounts for system losses, n is the path-loss exponent, typically $n > 3$, while $d_{j,u}$ is the distance between BS_j and UE u. The large-scale path loss model in dB for the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{j,u}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{j,u}}{10^3})$. Other default system setting for the simulation are as follows: $N_m = 8$, $N_p = 8$, for both base stations and $U_p = U_m = \bar{U}$. This simulation settings will be used except otherwise indicated.

6.4.2 Discussion

In our results we compare our proposed method with the *zero-forcing beamforming* (ZFBF) method, together with the single-cell processing method. These strategies are all heuristic ways to select beamformers that will mitigate interference and improve the SNR of each UE in the system. The single-cell processing method selects the optimal beamforming directions when the system only transmits to a single UE and uses the full power available in the system for transmission. Its beamforming direction can be selected as

$$\tilde{\mathbf{w}}_u = \arg \max_{\substack{\mathbf{w}_u \in \mathbb{C}^{N\times 1} \\ ||\mathbf{w}_u||^2=1}} |\mathbf{h}^H_u \mathbf{w}_u|^2. \tag{6.21}$$

The ZFBF method can be used to completely eliminate co-channel interference by selecting beamforming directions that are orthogonal to the channels of non-intending UEs however, the uncertainty in obtaining the CSI at the transmitter makes it impossible in practice.

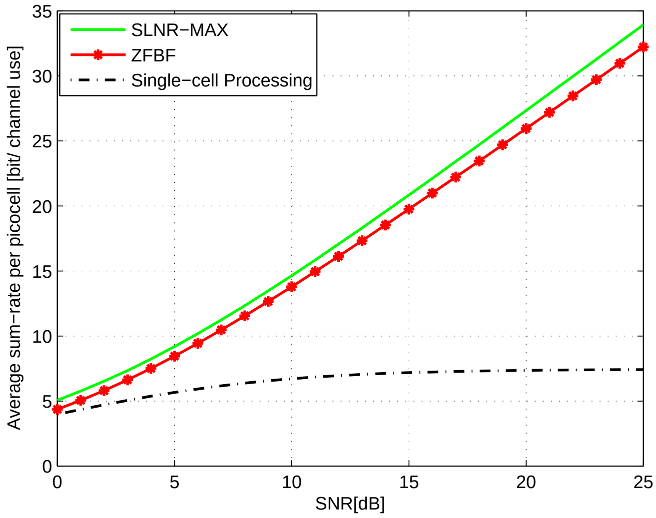

Figure 6.2: Average sum-rate achievable at different SNR for $N_p = 8$, $U_p = \bar{U} = 3$

Figure 6.2 compares the average sum-rate for each pico-cell using our proposed method, the ZFBF method and *single-cell processing* (SCP) method. The scheme with SCP under performs because out-of-cell interference are treated as noise. It will only be optimal for system that transmits to only one UE. Our proposed method out-performs the ZFBF method because it maximizes the SNR of each UE and as well minimizes the interference targeted at it. Whereas the ZFBF approach is only interested in cancelling the interference at the expense of losing some signal gain. Noise as well affects the performance of ZFBF which is not considered during the design of the beamformers. While the noise problem is taken into consideration by our proposed method by multiplying the noise variance with $\frac{1}{q_{u_p}}$, which will regularize the noise amplification at low SNR regime. Note also, that by suppressing the interference in a system below the background noise can cause a distorted beam pattern with high sidelobes that ironically will raise the background interference level in the system.

The CDFs of the average sum-rate of each picocell are shown in Fig. 6.3. The proposed method gives the best performance because it combines the benefit of both SCP and ZFBF at low SNR and high SNR respectively.

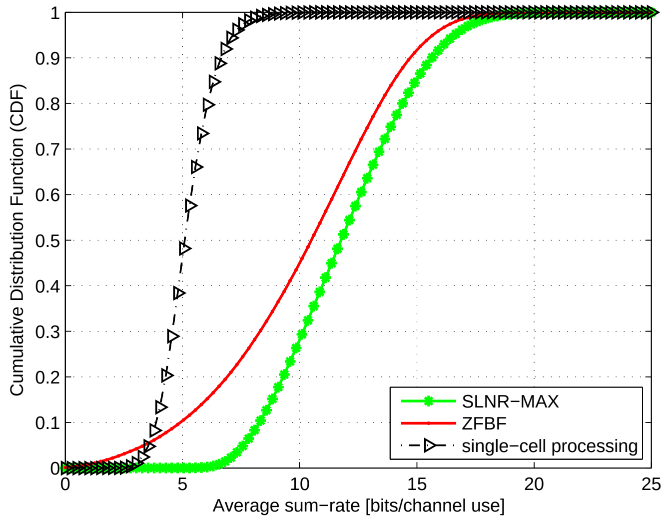

Figure 6.3: The CDF of the average sum-rate rate achieved by different beamforming schemes for $N_p = 8$, $U_p = \bar{U} = 3$

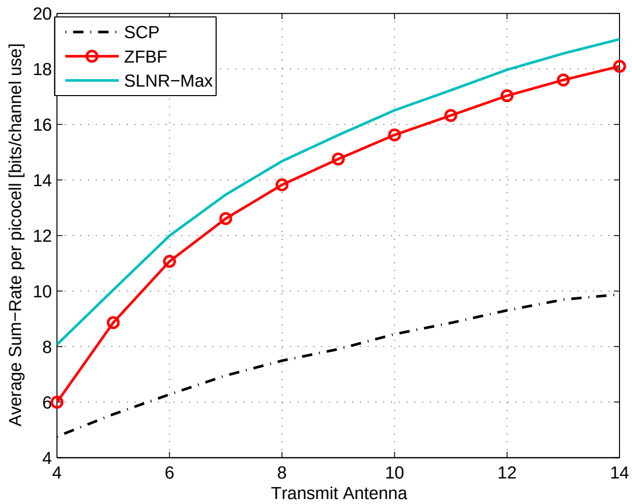

Figure 6.4: Average sum-rate per picocell as a function of transmit antennas for $SNR = 10$ dB, $U_p = \bar{U} = 3$

Multiple antennas at BS can meet high-capacity demand in downlink if utilized to serve many UEs in parallel.

Fig. 6.4 shows different average sum-rate achievable by different heuristic beamforming strategies. Our proposed method shows better performance as the number of transmit antennas increases when compared with the ZFBF method and SCP method.

6.5 Summary

In this chapter, we have developed a heuristic beamforming strategy for a two-tier HetNet. Our design criterion is aimed at maximizing the SLNR of each UE in each cell in order to maximize the aggregate rate achievable in the HetNet. The beamforming directions in each BS are designed by including the channels of those UEs, which this BS cause interference to while communicating with its desired UEs, as an input to the beamforming algorithm. We allocate uniform power to each UE in each cell for fairness in the system most especially for UEs in the CRE area of the pico-cell. Results obtained showed significant improvement over other heuristic beamforming strategies like ZFBF.

Chapter 7

Conclusions and Recommendations

The aim of this thesis is to maximize the spectral efficiency of *heterogeneous cellular networks* (HetNets) while fulfilling some power budget requirement at each *base station* (BS) and some fairness notion. Interference has been identified as an obstacle to be taken care of in order to achieve this goal. To this end, this thesis exploited different level of cooperation amongst BSs in HetNet and proposed several interference management techniques based on transmit beamforming and adapted to different HetNet scenarios. We, therefore, make the following conclusions based on our results and findings.

For the centralized implementation, where we first designed global transmit beamforming vectors among fixed coordinating BSs which controlled the inter-cell interference and improve the spectral efficiency of the considered HetNet scenario. We have shown that this method outperforms other methods that utilize local optimization and convex optimization to design their beamformers. However, we advise that our proposed method will be practically feasible for only small-scale applications.

In Chapter 4 which is another centralized implementation, because of the need for practically feasible low complexity algorithm, and the need to be able to identify BSs dynamically that cause significant interference to each UE in HetNet, we proposed a UE-centric clustering scheme that is capable of identifying the number of significant interfering BSs that will coordinate with the serving BS of each interfered UE. Afterwards, we then formulate the resource allocation optimization problems whose solutions aim to allocate spatial directions and powers respectively from each transmitter to their served UEs in order to mitigate interference, as well improve the spectral efficiency of HetNet. We have shown that our proposed method though suboptimal to the global optimization method proposed in Chapter 3, becomes asymptotically optimal as the number of transmit antennas increases. This is because we noticed that an increase in the number of transmit antennas will enable precise focusing of the transmit beamformers which will improve performance. Furthermore, the diminishing signal power as a result of interference cancellation is improved as the transmit antennas increases.

HetNet tends to be distributed, hence in Chapter 5, we are able to design decentralized beamforming scheme and distributed power resource allocation strategy that can spatially control interference and maximize the sum-rate of each cell, which by extension improves the aggregate sum-rate of the HetNet. Our proposed distributed resource allocation strategy outperforms other distributed RA strategies and is the closest in performance to the optimal RA strategy which is centralized. Considering the trade-off between performance and computational complexity of the centralized implementation, as well as possible hardware failures which might lead to coordination failure for a centralized scheme, our distributed RA is hereby recommended.

In Chapter 6, we devised a heuristic decentralized low complexity algorithm that can spatially control the interference present in HetNet as well as improve the sum-rate of HetNet. Our proposed method, when compared to other low complexity algorithms such as zero-forcing and egoistic beamforming algorithms, show significant improvement.

The remaining of this chapter summarizes the findings of previous chapters and outlines the possible challenging problem and future research directions.

7.1 Thesis Summary

7.1.1 Summary of Chapter 1

The introductory chapter outline the background and motivation of this thesis. The contributions of this thesis were also highlighted.

7.1.2 Summary of Chapter 2

This chapter discusses the sources and causes of interference in HetNet. It also surveys different interference management techniques applicable and inappropriate for HetNet in terms of maximizing its spectral efficiency. The different techniques reviewed can be categorized into the following: frequency-domain techniques, time-domain techniques, power-based techniques and antenna-based techniques.

7.1.3 Summary of Chapter 3

In this chapter, we have shown how the optimal solution of our NP-hard non-convex optimization problem whose target is to maximize the weighted sum-rate of HetNet is achieved. This is done by devising an algorithm based on B&B method. The results obtained show that our method can outperform popular methods using convex optimization for finding the optimal solution to the non-convex NP-hard weighted sum-rate problem in HetNet. The B&B method involves searching of a box interval to get the best feasible solution that maximizes the weighted sum-rate of the system, but this search is not like the brute force search that brings a lot of computational complexity. It is more of an intelligent search because only part of the box that contains the optimal solution is searched, hence reducing computational complexity. Our devised B&B algorithm iteratively improves a lower bound f_{min} and an upper bound f_{max} on the optimal value of (3.14). Furthermore, global convergence to the optimum value is guaranteed when $f_{max} - f_{min} < \epsilon$. The algorithm also discover an ϵ-optimal solution $\mathbf{r}^*_\epsilon$. The Lipschitz continuity property of our utility function is a sufficient condition for guaranteeing an ϵ-optimal solution in a limited number of iterations.

7.1.4 Summary of Chapter 4

In this chapter, we have developed an UE-centric clustering scheme that can effectively determine the significant interfering BSs that will cause the highest interference to each UE. Afterwards, the serving BSs for these UEs together with these selected interfering BSs will coordinate and make resource allocation decisions to allocate spatial direction to each UE in the system. Our RA strategy can be practically implemented in HetNet. The resources allocated to UEs are the spatial directions (unit-beamformers) and the power resource.

The resource allocation optimization problem for selecting spatial directions is done centrally and is formulated as an NP-hard non-convex problem, which we reformulate to a convex problem for practical implementation purposes and solved using SeDumi, which is a general purpose implementation of interior point method. While our power resource allocation scheme is decentralized and is formulated as maximizing the sum-rate of each cell while achieving a minimum performance level for each UE in the cell. The power RA problem is found to be convex and hence, can be solved efficiently using CVX (a package for specifying and solving convex programs). Results obtained show that our proposed method though suboptimal, when compared to the B&B method, which provides the global optimal solution for the non-convex NP-hard weighted sum-rate maximization problem, improves when the number of transmit antenna increases. Also, our results show that the B&B method has the worst case complexity that increases exponentially with the number of UEs, hence cannot be recommended for large-scale applications but can be used for off-line benchmarking.

7.1.5 Summary of Chapter 5

In this chapter, we have developed a distributed RA strategy for UEs in HetNet such that UEs in the CRE will only experience minimized loss in rate due to higher interference received from the MBS. The resources allocated to UEs are the spatial directions (unit beamformer) and the power resource. We formulate the spatial RA optimization problem as selecting the optimal beamformers that will cause the least interference to UEs in the same cell and UEs $\in \mathcal{C}_j$. Our proposed hybrid power resource allocation scheme involves a three-step process, starting with two power optimization procedures. These two power optimization procedures are formulated as convex optimization problems and solved by exploiting *Karush-Kuhn-Tucker* (KKT) conditions and CVX (a Matlab software for discipline convex programming) respectively. While the third step involves utilizing results from the aforementioned power optimization procedures to compute average powers allocated to UEs. Results obtained show that our distributed RA strategy outperforms other distributed RA strategies such as localJTdistributed in [122] and the single-cell processing strategy. Our strategy is the closest in performance to the optimal RA strategy which is centralized.

7.1.6 Summary of Chapter 6

In this chapter, we have developed a heuristic beamforming strategy for a two-tier HetNet. Our design criterion is aimed at maximizing the SLNR of each UE in each cell in order to maximize the aggregate rate achievable in the HetNet. The beamforming directions in each BS are designed by including the channels of those UEs, which this BS cause interference to while communicating with its desired UEs, as an input to the beamforming algorithm. We allocate uniform power to each UE in each cell for fairness in the system most especially for UEs in the CRE area of the pico-cell. Results obtained showed significant improvement over other heuristic beamforming strategies like ZFBF.

7.2 Recommendation for Future Works

The HetNet system model analyzed in this thesis was based on some simplifying assumptions: the unlimited backhaul capacity for the centralized resource allocation implementation, perfect channel state information available at each transmitter. The following future research directions related to beamforming techniques for interference management in heterogeneous cellular networks can be considered.

7.2.1 Joint optimization of Downlink Beamforming and Backhaul Constraints

Backhaul in practical terms is limited and the capacity of a backhaul to BS coordination in HetNet is very important. Note that the aggregate sum-rate achievable by the HetNet is not only dependent on how the optimal beamformers are selected to mitigate the interference in HetNet. Recall, that in coordinated beamforming, all cooperating BSs in HetNet, jointly design their beamforming vectors together, and the channel state information needed for the design of this beamformers are shared via the backhaul. The backhaul rates is a factor needs to be considered in coordinated beamforming. We can affirm that the sum-rate achievable by HetNet is also a function of the backhaul rate. Therefore, joint optimization for both downlink beamforming and backhaul parameters to achieve practical aggregate sum-rate for HetNet should be considered as a challenging problem for future research.

7.2.2 Robustness Beamforming to Channel Uncertainty

Perfect channel state information is needed at the transmitter for the design of accurate and optimal beamformers. Availability of perfect channel state information at the transmitter may not be a reality because of channel uncertainty. The uncertainty originates from a variety of sources. For example, insufficient channel reciprocity, delay in channel state information acquisition on fading channels, faulty channel estimation, and feedback quantization. Robustness here refers to ensuring a certain level of performance. Beamforming design based on corrupt channel state information at the transmitter may not give the desired result. Therefore robust coordinated beamforming schemes should be considered as a challenging problem for future research in HetNet scenarios.

7.2.3 Multi-Carrier Systems

In this thesis, we concentrate on a single sub-carrier for our HetNet system models. However future research can extend it to a multi-carrier system with many sub-carriers. Maximizing the sum-rate of HetNet in a multi-carrier system will be a difficult problem because of the overwhelming multi-carrier complexity involved. How this problem can be solved without dividing the sub-carriers into subsets of manageable sizes is still an open problem for future research.

7.2.4 Multi-Antenna UEs

In this thesis, we have always assumed the user equipment to have a single-antenna, given practical reasons such as: reducing the user equipment hardware complexity and preserving of battery life for such assumptions. When user equipment and base station have multiple antennas, there is more degree of freedom to effectively control interference, however, how to resolve issues like significant increase in complexity and signaling overhead remain unclear. Also, how to save battery life of the user equipment is another issue to resolve. Therefore, practical solutions to the optimal beamforming and how to obtain the trade-off between optimality and complexity are still open problems for future research.

Appendix A

Non-convex optimization problem formulation on standard form

A.1 Standard Form: Problem Formulation

To enable elaborate analysis and numerical computations, it is better to write optimization problem on standard form as

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq b_i,\ i = 1, \ldots, M, \end{aligned}$$

where the M functions $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ are the constraint functions.

To achieve (3.14) formulation on standard form, we denote the concatenation of all the beamforming vectors as $\mathbf{w} = [\mathbf{w}_1^T \cdots \mathbf{w}_K^T]^T \in \mathbb{C}^{NK}$ and let $\mathbf{x} = \begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix}$ be the optimization variable. The utility function is $f_0(\mathbf{x}) = -f(r_1, \ldots, r_K)$, where r_i is obtained from (3.5), also observe that r_i is a function of $\mathbf{x}$. The constraints are given by $f_k(\mathbf{x}) = \mathbf{x}^H\mathbf{x} - p_k$ for $k = 0, \ldots, K_t$, and, $f_m(\mathbf{x}) = \mathbf{x}^H \begin{bmatrix} \Re(\mathbf{G}_m) & -\Im(\mathbf{G}_m) \\ \Im(\mathbf{G}_m) & \Re(\mathbf{G}_m) \end{bmatrix} \mathbf{x} - \tau$ for $m = 0$ (i.e., MBS), where $\mathbf{G}_m = diag(\mathbf{G}_{1m}, \ldots, \mathbf{G}_{K_t m})$. Therefore, (3.14) can be formulated in standard form as

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{x}^H\mathbf{x} \leq p_k \qquad k = 0, \ldots, K_t, \\ & \mathbf{x}^H \begin{bmatrix} \Re(\mathbf{G}_m) & -\Im(\mathbf{G}_m) \\ \Im(\mathbf{G}_m) & \Re(\mathbf{G}_m) \end{bmatrix} \mathbf{x} \leq \tau,\ m = 0. \end{aligned} \tag{A.1}$$

This formulation in standard form provides a compact way of representing optimization problems, but additional information is needed to analyze the problem and thereby devise suitable numerical algorithms. Most optimization problems have no closed-form solutions, but can be solved numerically.

Appendix B

Numerical Solution of Optimization Problem

B.1 Interior Point Method: Using CVX as High-Level Modeling Languages

Depending on the structure of the optimization problem, it can be solved numerically if the objective fuction and constraint functions are linear, or convex.

Interior point methods can be successfully applied to both linear and convex problems. General-purpose implementations of interior point methods are available in SeDuMI and SDPT3. Note, that CVX is a high-level modeling language which can be used to simplify these implementation. In what follows, we will give an example of a linear optimization problem, which we will solve using these implementations.We will also write down the codes and give the output of the CVX after the solution(if any) has been found by it.

Lets consider a simple linear program

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{Ax} \leq \mathbf{b}, \end{aligned} \tag{B.1}$$

with variable $\mathbf{x} \in \mathbb{R}^n$ and arbitrary matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^m$, and $\mathbf{c} \in \mathbb{R}^n$. The following (MatLab/CVX) code generates and solves the random instance of (B.1):

m=16; n=8;

A=randn(m,n); **b**=randn(m,1); **c**=randn(n,1)

cvx_begin

variable **x**(n)

minimize $(\mathbf{c}' * \mathbf{x})$

subject to

$\mathbf{A} * \mathbf{x} \leq \mathbf{b}$
cvx_end.

The expressions below are the output of the linear optimization problem solved by cvx.

Calling SDPT3 4.0: 16 variables, 8 equality constraints For improved efficiency, SDPT3 is solving the dual problem.

---

num. of constraints = 8
dim. of linear var = 16
****
SDPT3: Infeasible path-following algorithms
****
version predcorr gam expon scaleData
NT 1 0.000 1 0
it pstep dstep pinfeas dinfeas gap prim-obj dual-obj cputime

0|0.000|0.000|2.6e+01|7.3e+00|1.6e+03| 1.979587e+01 0.000000e+00| 0:0:00| chol
1|0.709|1.000|7.7e+00|7.4e-02|2.9e+02| 1.758647e+01 1.903119e+00| 0:0:00| chol
2|0.836|1.000|1.3e+00|7.4e-03|4.5e+01| 2.581328e+00 4.668585e+00| 0:0:00| chol
3|0.330|0.584|8.4e-01|3.5e-03|4.0e+01| 3.704831e+00 3.650237e+01| 0:0:00| chol
4|0.286|1.000|6.0e-01|7.4e-05|5.3e+01| 2.118671e+00 3.690856e+02| 0:0:00| chol
5|0.026|0.031|5.8e-01|7.2e-05|1.5e+02| 1.185331e+00 2.320789e+03| 0:0:00| chol
6|0.044|1.000|5.6e-01|7.4e-07|9.4e+03| 1.577089e+00 7.267422e+05| 0:0:00| chol
7|0.011|1.000|5.5e-01|3.3e-07|9.6e+06| 1.188284e+00 4.538406e+09| 0:0:00| chol
8|0.002|1.000|5.5e-01|6.4e-02|1.9e+13| 3.263881e+00 9.881814e+14| 0:0:00| chol
9|0.020|1.000|5.4e-01|2.4e+02|5.7e+15| 1.279364e+00 2.839929e+18| 0:0:00|
sqlp stop: primal problem is suspected of being infeasible

---

number of iterations = 9
residual of primal infeasibility
certificate (y,Z) = 4.50e-16
reldist to infeas. <= 5.38e-17
Total CPU time (secs) = 0.33

CPU time per iteration = 0.04

termination code = 1

DIMACS: 7.8e-01 0.0e+00 4.0e+02 0.0e+00 -1.0e+00 2.0e-03

Status: Unbounded

Optimal value (cvx_optval): -Inf.

In what follows, we give a brief summary of the outcome of the code. cvx_optval contains the value of the objective function, while cvx_status contains a string describing the status of the calculation. The possible values of cvx_status are as follows [126]:

Solved A complementary (primal and dual) solution has been found. The primal and dual variables are replaced with their computed values, and the the optimal value of the problem is placed in cvx_optval (which, by convention, is 0 for feasibility problems).

Unbounded The solver has determined that the problem is unbounded. The value of cvx_optval is set to -Inf for minimizations, and +Inf for maximizations. (Feasibility problems, by construction, never produce an Unbounded status.) The values of any dual variables are replaced with NaN, as the dual problem is in fact infeasible. For unbounded problems, CVX stores an unbounded direction into the problem variables. This is is a direction along which the feasible set is unbounded, and the optimal value approaches $\pm\infty$. It is important to understand that this value is very likely not a feasible point. If a feasible point is required, the problem should be re-solved as a feasibility problem by omitting the objective. Mathematically speaking, given an unbounded direction v and a feasible point x, x + tv is feasible for all $t \geq 0$, and the objective tends to $-\infty$ (for minimizations; $+\infty$ for maximizations) as t $\rightarrow +\infty$ itself.

Failed The solver failed to make sufficient progress towards a solution, even to within the "relaxed" tolerance setting. The values of cvx_optval and primal and dual variables are filled with NaN. This result can occur because of numerical problems within SeDuMi, often because the problem is particularly "nasty" in some way (e.g., a non-zero duality gap).

Appendix C

Monotonic Functions

C.1 Increasing Functions and Normal Set

To enable a proper understanding of the branch and bound method in chapter 3, we elaborate more on the properties of monotonically increasing functions and normal set, which is a concept closely related to monotonicity. Note that by exploiting the monotonicity properties of a function can make some difficult problems tractable.

In what follows, we borrow most of our terminology from multi-criteria optimization [127], for any two vectors $\dot{\mathbf{x}}, \mathbf{x} \in \mathbb{R}^K$ $\Big($meaning that $\dot{\mathbf{x}} = [x_1, \ldots, x_K]^T$ and $\mathbf{x} = [x_1, \ldots, x_K]^T$ where $[\dot{\mathbf{x}}]_i$ and $[\mathbf{x}]_i$ are real numbers for $i = 1, \ldots, K\Big)$ we define the following componentwise relationships:

- $\dot{\mathbf{x}} = \mathbf{x}$ if $[\dot{\mathbf{x}}]_i = [\mathbf{x}]_i$ $\forall i = 1, \ldots, K$.
- $\dot{\mathbf{x}} < \mathbf{x}$ if $[\dot{\mathbf{x}}]_i < [\mathbf{x}]_i$ $\forall i = 1, \ldots, K$.
- $\dot{\mathbf{x}} \leq \mathbf{x}$ if $[\dot{\mathbf{x}}]_i \leq [\mathbf{x}]_i$ $\forall i = 1, \ldots, K$.

Similarly, we can also define componentwise relationships for $\dot{\mathbf{x}} > \mathbf{x}$ and $\dot{\mathbf{x}} \geq \mathbf{x}$ respectively. We denote $\mathbb{R}^K_+ = \left\{\mathbf{x} \in \mathbb{R}^K | \mathbf{x} \geq \mathbf{0}\right\}$ and $\mathbb{R}^K_{++} = \left\{\mathbf{x} \in \mathbb{R}^K | \mathbf{x} > \mathbf{0}\right\}$. For $\mathbf{x} \in \mathbb{R}^K$ let $I(\mathbf{x}) = \{i | [\mathbf{x}]_i = 0\}$ and denote $K_{\mathbf{x}} = \left\{\dot{\mathbf{x}} \in \mathbb{R}^K_+ | [\dot{\mathbf{x}}]_i > [\mathbf{x}]_i \ \forall i \notin I(\mathbf{x})\right\}$, $\mathrm{cl}K_x = \left\{\dot{\mathbf{x}} \in \mathbb{R}^K_+ | [\dot{\mathbf{x}}]_i > [\mathbf{x}]_i \ \forall i \notin I(\mathbf{x})\right\}$.
If $\mathbf{a} \leq \mathbf{b}$, we define the box $[\mathbf{a}, \mathbf{b}]$ to be the set of all rates $\mathbf{x}$ such that $\mathbf{a} \leq \mathbf{x} \leq \mathbf{b}$.

A function $f : \mathbb{R}^K \to \mathbb{R}$ is said to be increasing on $\mathbb{R}^K_+$ if $f(\mathbf{x}) \leq f(\dot{\mathbf{x}})$ whenever $\mathbf{0} \leq \mathbf{x} \leq \dot{\mathbf{x}}$, similar, it is said to be increasing on a box $[\mathbf{a}, \mathbf{b}] \subset \mathbb{R}^K_+$ whenever $\mathbf{a} \leq \mathbf{x} \leq \dot{\mathbf{x}} \leq \mathbf{b}$.

A set $\mathcal{M} \subset \mathbb{R}^K_+$ is called normal if for any two points $\dot{\mathbf{x}}, \mathbf{x} \in \mathbb{R}^K_+$ such that $\dot{\mathbf{x}} \leq \mathbf{x}$, if $\mathbf{x} \in \mathcal{M}$, then $\dot{\mathbf{x}} \in \mathcal{M}$, too. If $\mathcal{M}$ is a normal set, then
$\mathcal{M} \cup \left\{\mathbf{x} \in \mathbb{R}^n_+ | [\mathbf{x}]_i = 0 \text{ for some } i = 1, \ldots, n\right\}$ is still normal. Also note that the

intersection and the union of a family of normal sets are also normal sets. Furthermore, a normal set $\mathcal{M}$ has a nonempty interior if and only if it contains a point $\mathbf{u} \in \mathbb{R}^K_{++}$. The closure of a normal set is normal.

A point $\mathbf{y} \in \mathbb{R}^K_+$ is called an upper boundary point of a bounded normal set $\mathcal{M}$ if $\mathbf{y} \in cl\mathcal{M}$ while $K_{\mathbf{y}} \subset \mathbb{R}^K_+ \setminus \mathcal{M}$. The set of upper boundary points of $\mathcal{M}$ is called the upper boundary of $\mathcal{M}$ and is denoted by $\delta^+\mathcal{G}$. If $\mathcal{M}$ is closed, then obviously $\delta^+\mathcal{G} \subset \mathcal{M}$.

Appendix D

Proof of Set Compactness and Normality

D.1 Compact-Normal Set

In this section, we want to prove that the set $\mathcal{Z}$ is a compact normal set. Which is the feasible set that satisfy the constraint in (3.6), and is defined as

$$\mathcal{Z} = \Big\{\Big(r_1(\mathbf{w}_1,\ldots,\mathbf{w}_K),\ldots,r_K(\mathbf{w}_1,\ldots,\mathbf{w}_K)\Big) : (\mathbf{w}_1,\ldots,\mathbf{w}_K) \in \mathcal{W}\Big\}. \tag{D.1}$$

where $\mathcal{W}$ is the set of feasible transmit beamforming vectors:

$$\mathcal{W} = \Big\{(\mathbf{w}_1,\ldots,\mathbf{w}_K) : \mathbf{w}_i^T\mathbf{w}_i \le P_m,\ \forall m \in \mathcal{M}, m = 0, \mathbf{w}_i^T\mathbf{w}_i \le P_s\ \forall s \in \mathcal{M}, s \ne 0,$$
$$\mathbf{w}_n^H\mathbf{G}_{i,m}\mathbf{w}_n \le \tau_i,\ \forall n,i,\ m = 0\Big\}. \tag{D.2}$$

In what follows, we will prove that set $\mathcal{Z}$ is compact and normal on $\mathbb{R}_+^K$.

Proof. To prove that $\mathcal{Z}$ is a compact set, observe the set of feasible transmit beamforming vectors $\mathcal{W}$ in D.2 is compact. The compactness of $\mathcal{Z}$ can be affirmed by implementing [72, Theorem 4.14], which posit that the continuous mapping of a compact set is also a compact set. Note, that r_i are continuous functions of $\mathbf{w}_1,\ldots,\mathbf{w}_K$ by definition. Since $\mathcal{Z}$ is the image of a continuous mapping from $\mathcal{W}$, it is therefore, compact. It remains to be seen how $\mathcal{Z}$ is a normal set.

For any given $\mathbf{r} = (r_1,\ldots,r_k) \in \mathcal{Z}$, we want to show that any $\grave{\mathbf{r}} = (\grave{r}_1,\ldots,\grave{r}_k) \in \mathbb{R}_+^K$ with $\grave{\mathbf{r}} \le \mathbf{r}$ also belong to $\mathcal{Z}$. In order to achieve this goal, let $\mathbf{w}_1^*,\ldots,\mathbf{w}_K^*$ be a feasible transmit vectors that obtain $\mathbf{r}$, also if we have another transmit vectors $q_1\mathbf{w}_1^*,\ldots,q_K\mathbf{w}_K^*$, where $q_1,\ldots,q_K$ is a set of power allocation coefficient that belong to

$$\mathcal{A} = \Big\{(q_1,\ldots,q_K) : q_i(\mathbf{w}_i^T\mathbf{w}_i) \le P_m,\ \forall m \in \mathcal{M}, m = 0,\ q_i(\mathbf{w}_i^T\mathbf{w}_i) \le P_s\ \forall s \in \mathcal{M}, s \neq 0,$$
$$q_n(\mathbf{w}_n^H\mathbf{G}_{i,m}\mathbf{w}_n) \le \tau_i,\ \forall n,i,\ m = 0\Big\}, \tag{D.3}$$

to make the transmit beamforming vectors feasible. Clearly, the point $\mathbf{r}$ is achieved by choosing $(q_1,\ldots,q_K) = (1,\ldots,1)$. To prove that a given $\grave{\mathbf{r}} \le \mathbf{r}$ belongs to $\mathcal{Z}$, we must find $(q_1,\ldots,q_K) \in \mathcal{A}$ that will give this point. This will corresponds to the conditions $SINR_i = 2^{r_i} - 1\ \forall i$, which can be formulated as K linear equation and solved using the peron-frobenius theorem in [128].

Summarily, we can say that for any $\mathbf{r} \in \mathcal{Z}$, all points that give inferior performance than $\mathbf{r}$ are also contain in $\mathcal{Z}$.

D.2 Maximizing an Increasing Function

In this section, we show how to maximize an increasing function over a compact normal set.

If we are maximizing an increasing function over a compact normal set with non-empty interior, such as

$$max\Big\{f(\mathbf{r})|\ \mathbf{r} \in \mathcal{Z} \cap \mathcal{H}\Big\}, \tag{D.4}$$

where $\mathcal{H}$ is a closed reverse normal set and $f(\mathbf{r})$ is an increasing function on $\mathcal{Z}$. It can be easily proved that the maximum of $f(\mathbf{r})$ over $\mathcal{Z} \cap \mathcal{H}$, if it exists, is attained on $\delta^+\mathcal{Z} \cap \mathcal{H}$, using [73, Proposition 7].

References

[1] P. Smith, P. Dmochowski, P. Teal, and M. Shafi, "Heterogeneous cellular networks," http://www.ecs.victoria.ac.nz/Groups/COMMS/HetNet, [Accessed: 30 June 2014].

[2] TR-36.814, "Further advancements for E-UTRA physical layer aspects: 3GPP technical report," http://www.3gpp.org/ftp//Specs/archive/36_series/36.814/, [Accessed: 12 January 2015].

[3] N. Eddy, "Mobile subscribers to hit 4 billion by 2018: Mwc," http://www.eweek.com/mobile/mobile-subscribers-to-hit-4-billion-by-2018-mwc, [Accessed: 11 January 2016].

[4] F. Yuan, C. Yang, G. Wang, and M. Lei, "Adaptive channel feedback for coordinated beamforming in heterogeneous network," *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 3980–3994, 2013.

[5] H. Beijner, J. Högberg, and H. Marutani, "The smartphone challenge is surmountable," *Ericsson Review*, no. 2, Dec.2011.

[6] J. G. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. C. Reed, "Femtocells: Past, present, and future," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 497–508, 2012.

[7] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, 2011.

[8] I. Guvenc, D. López-Pérez, Y. Yang, and F. Gunnarsson, "Dynamic interference management," in *Heterogeneous Cellular Networks: Theory, Simulation and Deployment*. New York, USA: Cambridge University Press, 2013, ch. 7, pp. 179–216.

[9] H. Claussen, L. T. Ho, and L. G. Samuel, "An overview of the femtocell concept," *Bell Labs Technical Journal*, vol. 13, no. 1, pp. 221–245, 2008.

[10] D. Lopez-Perez, I. Guvenc, G. De la Roche, M. Kountouris, T. Q. Quek, and J. Zhang, "Enhanced intercell interference coordination challenges in heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 22–30, 2011.

[11] M. Folke and D. López-Pérez, "System-level simulation and evaluation models," in *Heterogeneous Cellular Networks: Theory, Simulation and Deployment*. New York, USA: Cambridge University Press, 2013, ch. 3, pp. 179–216.

[12] O. Oguejiofor and L. Zhang, "Global optimization of weighted sum-rate for downlink heterogeneous cellular networks," in *proc. 23rd IEEE Int. Conf. Telecommun, (ICT)*, 2016, pp. 1–6.

[13] O. S. Oguejiofor and L. Zhang, "Heuristic coordinated beamforming for heterogeneous cellular networks," in *proc. IEEE Veh. Technol. conf. (VTC)*, 2016, pp. 1–5.

[14] J. Sangiamwong, Y. Saito, N. Miki, T. Abe, S. Nagata, and Y. Okumura, "Investigation on cell selection methods associated with inter-cell interference coordination in heterogeneous networks for LTE-Advanced downlink," in *11th European Wireless Conf. Sustainable Wireless Technologies (European Wireless)*, 2011, pp. 1–6.

[15] E. Hossain, M. Rasti, H. Tabassum, and A. Abdelnasser, "Evolution toward 5G multi-tier cellular wireless networks: An interference management perspective," *IEEE Wireless Commun.*, vol. 21, no. 3, pp. 118–127, 2014.

[16] I. Guvenc, "Capacity and fairness analysis of heterogeneous networks with range expansion and interference coordination," *IEEE Commun. Lett.*, vol. 15, no. 10, pp. 1084–1087, 2011.

[17] D. López-Pérez, A. Valcarce, G. De La Roche, and J. Zhang, "OFDMA femtocells: A roadmap on interference avoidance," *IEEE Commun. Mag.*, vol. 47, no. 9, 2009.

[18] W. Yu and J. M. Cioffi, "FDMA capacity of gaussian multiple-access channels with ISI," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 102–111, 2002.

[19] D. D. Falconer, F. Adachi, and B. Gudmundson, "Time division multiple access methods for wireless personal communications," *IEEE Commun. Mag.*, vol. 33, no. 1, pp. 50–57, 1995.

[20] J. A. Salehi, "Code division multiple-access techniques in optical fiber networks. part 1: Fundamental principles," *IEEE Trans. Commun.*, vol. 37, no. 8, pp. 824–833, 1989.

[21] H. Sari, Y. Levy, and G. Karam, "An analysis of orthogonal frequency-division multiple access," in *proc. IEEE Global Telecommun. Conf.*, vol. 3, 1997, pp. 1635–1639.

[22] S. V. Bana and P. Varaiya, "Space division multiple access (SDMA) for robust ad hoc vehicle communication networks," in *proc. IEEE Intelligent Transportation Systems*, 2001, pp. 962–967.

[23] E. Björnson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure," *IEEE Signal Process. Mag.*, pp. 142–148, 2014.

[24] H. Lei, L. Zhang, X. Zhang, and D. Yang, "A novel multi-cell OFDMA system structure using fractional frequency reuse," in *proc. IEEE Personal, Indoor and Mobile Radio Commun., PIMRC*, 2007, pp. 1–5.

[25] B. Krasniqi, M. Wrulich, and C. F. Mecklenbrauker, "Network-load dependent partial frequency reuse for LTE," in *proc. IEEE Commun. and Information Technol., (ISCIT)*, 2009, pp. 672–676.

[26] M. Qian, W. Hardjawana, Y. Li, B. Vucetic, J. Shi, and X. Yang, "Inter-cell interference coordination through adaptive soft frequency reuse in LTE networks," in *proc. IEEE Wireless Commun. and Networking Conf.,(WCNC)*, 2012, pp. 1618–1623.

[27] R1-104968, "Summary of the description of candidate eICIC solutions," http://www.3gpp.org/DynaReport/TDocExMtg--R1-62--28032.htm, [Accessed: 12 January 2015].

[28] T. 36.423, "X2 application protocol (X2AP)(Release 9); 3GPP EU-TRA Network," http://www.3gpp.org/ftp//Specs/archive/36_series/36.423/, [Accessed: 12 January 2015].

[29] V. Chandrasekhar, J. G. Andrews, T. Muharemovic, Z. Shen, and A. Gatherer, "Power control in two-tier femtocell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4316–4328.

[30] D. T. Ngo, L. B. Le, and T. Le-Ngoc, "Distributed pareto-optimal power control in femtocell networks," in *proc. IEEE Personal Indoor and Mobile Radio Commun. (PIMRC)*, 2011, pp. 222–226.

[31] M. I. Kamel and K. M. Elsayed, "Performance evaluation of a coordinated time-domain eICIC framework based on absf in heterogeneous LTE-Advanced networks," in *proc. IEEE Global Telecommun. Conf.*, 2012, pp. 5326–5331.

[32] M. Simsek, M. Bennis, and A. Czylwik, "Dynamic inter-cell interference coordination in hetnets: A reinforcement learning approach," in *proc. IEEE Global Commun. Conf.*, 2012, pp. 5446–5450.

[33] C. W. Tan, S. Friedland, and S. H. Low, "Spectrum management in multiuser cognitive wireless networks: Optimality and algorithm," *IEEE J. Sel. Areas Commun*, vol. 29, no. 2, pp. 421–430, 2011.

[34] S. Cheng, S. Lien, F. Chu, and K. Chen, "On exploiting cognitive radio to mitigate interference in macro/femto heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 40–47, 2011.

[35] A. Adhikary, V. Ntranos, and G. Caire, "Cognitive femtocells: Breaking the spatial reuse barrier of cellular systems," in *proc. IEEE Information Theory and Applications Workshop (ITA)*, 2011, pp. 1–10.

[36] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 62–75, 2010.

[37] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in *proc. 49th IEEE Vehicular Technol. Conf.,(VTC)*, vol. 1, 1999, pp. 851–856.

[38] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, 2010.

[39] W. Li, T. Chang, C. Lin, and C. Chi, "Coordinated beamforming for multiuser miso interference channel under rate outage constraints," *IEEE Trans. Signal Process.*, vol. 61, no. 5, pp. 1087–1103, 2013.

[40] Y. Liu, Y. Dai, and Z. Luo, "Coordinated beamforming for miso interference channel: Complexity analysis and efficient algorithms," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1142–1157, 2011.

[41] T. Ren and R. J. La, "Downlink beamforming algorithms with inter-cell interference in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 10, pp. 2814–2823, 2006.

[42] M. Sawahashi, Y. Kishiyama, A. Morimoto, D. Nishikawa, and M. Tanno, "Coordinated multipoint transmission/reception techniques for LTE-Advanced [coordinated and distributed MIMO]," *IEEE Wireless Commun.*, vol. 17, no. 3, pp. 26–34, 2010.

[43] D. Lee, H. Seo, B. Clerckx, E. Hardouin, D. Mazzarese, S. Nagata, and K. Sayana, "Coordinated multipoint transmission and reception in LTE-Advanced: deployment scenarios and operational challenges," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 148–155, 2012.

[44] J. Lee, Y. Kim, H. Lee, B. L. Ng, D. Mazzarese, J. Liu, W. Xiao, and Y. Zhou, "Coordinated multipoint transmission and reception in LTE-Advanced systems," *IEEE Commun. Mag.*, vol. 50, no. 11, pp. 44–50, 2012.

[45] J. Zhao, T. Q. Quek, and Z. Lei, "Coordinated multipoint transmission with limited backhaul data transfer," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2762–2775, 2013.

[46] R1-111282, "Performance evaluation of CoMP JT for scenario 2," http://www.3gpp.org/DynaReport/TDocExMtg--R1-65--28504.htm, [Accessed: 12 January 2015].

[47] R1-111290, "CoMP phase 1 evaluation results," http://www.3gpp.org/DynaReport/TDocExMtg--R1-65--28504.htm, [Accessed: 12 January 2015].

[48] R1-111277, "CoMP JT evaluation for phase 1 homogenous deployment," http://www.3gpp.org/DynaReport/TDocExMtg--R1-65--28504.htm, [Accessed: 12 January 2015].

[49] R1-090140, "Clustering for CoMP transmission," http://www.3gpp.org/DynaReport/TDocExMtg--R1-55b--27322.htm, [Accessed: 12 January 2015].

[50] G. Zhang, Y. Huang, and k. wong, "Network MIMO techniques," in *Heterogeneous Cellular Networks: Theory, Simulation and Deployment*. New York, USA: Cambridge University Press, 2013, ch. 7, pp. 313–351.

[51] M. R. Akdeniz and S. Rangan, "Optimal wireless scheduling with interference cancellation," in *proc. IEEE Int. Symp. Information Theory Proceedings (ISIT)*, 2013, pp. 246–250.

[52] V. Abdrashitov, W. Nam, and D. Bai, "Rate and UE selection algorithms for interference-aware receivers," in *proc. 79th IEEE Veh. Technol. Conf., (VTC Spring)*, 2014, pp. 1–5.

[53] C. H. Papadimitriou and K. Steiglitz, *Combinatorial optimization: algorithms and complexity*. Courier Corporation, 1982.

[54] A. Govan, "Introduction to optimization," in *North Carolina State University, SAMSI NDHS, Undergraduate workshop*, 2006.

[55] H. Zhang, L. Venturino, N. Prasad, P. Li, S. Rangarajan, and X. Wang, “Weighted sum-rate maximization in multi-cell networks via coordinated scheduling and discrete power control,” *IEEE J. Sel. Areas Commun.*, vol. 29, no. 6, pp. 1214–1224, 2011.

[56] S. He, Y. Huang, L. Yang, and B. Ottersten, “Coordinated multicell multiuser precoding for maximizing weighted sum energy efficiency,” *IEEE Trans. Signal Process.*, vol. 62, no. 3, pp. 741–751, 2014.

[57] S. S. Christensen, R. Agarwal, E. De Carvalho, and J. M. Cioffi, “Weighted sum-rate maximization using weighted mmse for MIMO-BC beamforming design,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, 2008.

[58] M. Codreanu, A. Tolli, M. Juntti, and M. Latva-aho, “MIMO downlink weighted sum rate maximization with power constraints per antenna groups,” in *proc. 65th IEEE Veh. Technol. Conf., (VTC)*, 2007, pp. 2048–2052.

[59] S. Boyd and J. Mattingley, “Branch and bound methods,” *Notes for EE364b, Stanford University*, pp. 2006–07, 2007.

[60] S. K. Joshi, P. C. Weeraddana, M. Codreanu, and M. Latva-Aho, “Weighted sum-rate maximization for miso downlink cellular networks via branch and bound,” *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2090–2095, 2012.

[61] E. Björnson, G. Zheng, M. Bengtsson, and B. Ottersten, “Robust monotonic optimization framework for multicell MISO systems,” *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2508–2523, 2012.

[62] K. Cumanan, R. Krishna, L. Musavian, and S. Lambotharan, “Joint beamforming and user maximization techniques for cognitive radio networks based on branch and bound method,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3082–3092, 2010.

[63] Y. Shi, Y. T. Hou, S. Kompella, and H. D. Sherali, "Maximizing capacity in multihop cognitive radio networks under the SINR model," *IEEE Trans. Mobile Comput.*, vol. 10, no. 7, pp. 954–967, 2011.

[64] N. Levine, "When are compact and closed equivalent?" *The American Mathematical Monthly*, vol. 72, no. 1, pp. 41–44, 1965.

[65] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, 2006.

[66] H. Yang and T. L. Marzetta, "Performance of conjugate and zero-forcing beamforming in large-scale antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 172–179, 2013.

[67] F. Alizadeh, "Interior point methods in semidefinite programming with applications to combinatorial optimization," *SIAM J. Optimization*, vol. 5, no. 1, pp. 13–51, 1995.

[68] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization methods and software*, vol. 11, no. 1-4, pp. 625–653, 1999.

[69] R. H. Tütüncü, K.-C. Toh, and M. J. Todd, "Solving semidefinite-quadratic-linear programs using SDPT3," *Mathematical programming*, vol. 95, no. 2, pp. 189–217, 2003.

[70] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.0 beta. 2013," 2015.

[71] S. Kim and M. Kojima, "Second order cone programming relaxation of nonconvex quadratic optimization problems," *Optimization methods and software*, vol. 15, no. 3-4, pp. 201–224, 2001.

[72] W. Rudin, *Principles of Mathematical Analysis (International Series in Pure & Applied Mathematics).* McGraw-Hill Publishing Co., 1976.

[73] H. Tuy, "Monotonic optimization: Problems and solution approaches," *SIAM J. Optimization*, vol. 11, no. 2, pp. 464–494, 2000.

[74] Z. K. M. Ho and D. Gesbert, "Balancing egoism and altruism on interference channel: The MIMO case," in *proc. IEEE Commun. (ICC)*, 2010, pp. 1–5.

[75] X. Chu, D. López-Pérez, Y. Yang, and F. Gunnarsson, *Heterogeneous Cellular Networks: Theory, Simulation and Deployment*. Cambridge University Press, 2013.

[76] W. Nam, D. Bai, J. Lee, and I. Kang, "Advanced interference management for 5G cellular networks," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 52–60, 2014.

[77] W. H. Chin, Z. Fan, and R. Haines, "Emerging technologies and research challenges for 5G wireless networks," *IEEE Wireless Commun.*, vol. 21, no. 2, pp. 106–112, 2014.

[78] R. Irmer, H. Droste, P. Marsch, M. Grieger, G. Fettweis, S. Brueck, H.-P. Mayer, L. Thiele, and V. Jungnickel, "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Commun. Mag.*, vol. 49, no. 2, pp. 102–111, 2011.

[79] M. K. Karakayali, G. J. Foschini, and R. A. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Commun.*, vol. 13, no. 4, pp. 56–61, 2006.

[80] D. Gesbert, S. G. Kiani, A. Gjendemsjo, and G. E. Oien, "Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks," *Proc. IEEE*, vol. 95, no. 12, pp. 2393–2409, 2007.

[81] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, 1981.

[82] X. Shang, B. Chen, and H. V. Poor, "Multiuser MISO interference channels with single-user detection: Optimality of beamforming and the achievable rate region," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4255–4273, 2011.

[83] H.-P. Mayer and H. Schlesinger, "Antenna synchronization for coherent network MIMO," Jul. 12 2016, US Patent 9,391,764.

[84] B. Dai and W. Yu, "Sparse beamforming and user-centric clustering for downlink cloud radio access network," *IEEE Access*, vol. 2, pp. 1326–1339, 2014.

[85] G. Nigam, P. Minero, and M. Haenggi, "Coordinated multipoint joint transmission in heterogeneous networks," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4134–4146, 2014.

[86] R. Tanbourgi, S. Singh, J. G. Andrews, and F. K. Jondral, "Analysis of non-coherent joint-transmission cooperation in heterogeneous cellular networks," in *proc. IEEE Int. Conf. Commun. (ICC)*, 2014, pp. 5160–5165.

[87] A. D. Wyner, "Shannon-theoretic approach to a gaussian cellular multiple-access channel," *IEEE Trans. Inf. Theory*, vol. 40, no. 6, pp. 1713–1727, 1994.

[88] D. Gesbert, S. Hanly, H. Huang, S. S. Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, 2010.

[89] S. Shamai and B. M. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," in *proc. 53rd IEEE Veh. Technol. Conf., (VTC)*, vol. 3, 2001, pp. 1745–1749.

[90] H. Huang, M. Trivellato, A. Hottinen, M. Shafi, P. J. Smith, and R. Valenzuela, "Increasing downlink cellular throughput with limited network MIMO coordination," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2983–2989, 2009.

[91] P. Marsch and G. Fettweis, "On multicell cooperative transmission in backhaul-constrained cellular systems," *Annals of Telecommunications-annales des téléCommunications*, vol. 63, no. 5-6, pp. 253–269, 2008.

[92] P. Marsch, "Static clustering for cooperative multi-point (CoMP) in mobile communications," in *IEEE Int. Conf. Commun., (ICC)*, 2011, pp. 1–6.

[93] H. Wang, X. Zhou, and M. C. Reed, "Coverage and throughput analysis with a non-uniform small cell deployment," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2047–2059, 2014.

[94] S. Akoum and R. W. Heath, "Interference coordination: Random clustering and adaptive limited feedback," *IEEE Trans. Signal Proces.*, vol. 61, no. 7, pp. 1822–1834, 2013.

[95] C. Li, J. Zhang, M. Haenggi, and K. B. Letaief, "User-centric intercell interference nulling for downlink small cell networks," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1419–1431, 2015.

[96] X. Zhang, Z. Su, Z. Yan, and W. Wang, "Energy-efficiency study for two-tier heterogeneous networks (HetNet) under coverage performance constraints," *Mobile Networks and Applications*, vol. 18, no. 4, pp. 567–577, 2013.

[97] Y. Huang, X. Zhang, J. Zhang, J. Tang, Z. Su, and W. Wang, "Energy-efficient design in heterogeneous cellular networks based on large-scale user behavior constraints," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 4746–4757, 2014.

[98] Y. S. Soh, T. Q. Quek, M. Kountouris, and H. Shin, "Energy efficient heterogeneous cellular networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 5, pp. 840–850, 2013.

[99] K. Okino, T. Nakayama, C. Yamazaki, H. Sato, and Y. Kusano, "Pico cell range expansion with interference mitigation toward LTE-Advanced heterogeneous networks," in *proc. IEEE Int. Conf. Commun. Workshops (ICC)*, 2011, pp. 1–5.

[100] G. Scutari, D. P. Palomar, and S. Barbarossa, "Cognitive MIMO radio," *IEEE Signal Process. Mag.*, vol. 25, no. 6, pp. 46–59, 2008.

[101] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, 2010.

[102] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, 2006.

[103] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming." CRC Press, 2001.

[104] J. Park, N. Lee, and R. W. Heath, "Base station cluster patterns for semi-static multi-cell cooperation in irregular network topologies," in *proc. 23rd European Signal Process. Conf. (EUSIPCO), 2015.* IEEE, 2015, pp. 2441–2445.

[105] G. Boudreau, J. Panicker, N. Guo, R. Chang, N. Wang, and S. Vrzic, "Interference coordination and cancellation for 4G networks," *IEEE Commun. Mag.*, vol. 47, no. 4, pp. 74–81, 2009.

[106] R1-104968, "Summary of the description of candidate eICIC solutions," http://www.3gpp.org/DynaReport/TDocExMtg--R1-62--28032.htm, [Accessed: 12 January 2015].

[107] M. Assaad, "Optimal fractional frequency reuse (FFR) in multicellular OFDMA system," in *proc. 68th IEEE Veh. Technol. Conf., (VTC)*, 2008, pp. 1–5.

[108] Z. Shen, A. Papasakellariou, J. Montojo, D. Gerstenberger, and F. Xu, "Overview of 3GPP LTE-Advanced carrier aggregation for 4G wireless communications," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 122–130, 2012.

[109] K. T. Kim and S. K. Oh, "A universal frequency reuse system in a mobile cellular environment," in *proc. 65th IEEE Veh. Technol. Conf., (VTC)*, 2007, pp. 2855–2859.

[110] A. Tarighat, M. Sadek, and A. H. Sayed, "A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios," in *proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing, (ICASSP'05)*, vol. 3, 2005, pp. 1129–1132.

[111] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, 2007.

[112] P. Cheng, M. Tao, and W. Zhang, "A new SLNR-based linear precoding for downlink multi-user multi-stream MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1008–1010, 2010.

[113] W. W. Ho, T. Q. Quek, S. Sun, and R. W. Heath, "Decentralized precoding for multicell MIMO downlink," *IEEE Trans. Wireless Commun.*, vol. 10, no. 6, pp. 1798–1809, 2011.

[114] B. O. Lee, H. W. Je, I. Sohn, O. Shin, and K. B. Lee, "Interference-aware decentralized precoding for multicell MIMO TDD systems," in *proc. IEEE Global Telecommun. Conf.,*, 2008, pp. 1–5.

[115] Y. Huang, G. Zheng, M. Bengtsson, K.-K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming with limited intercell coordination," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 728–738, 2011.

[116] M. Peng, C. Wang, J. Li, H. Xiang, and V. Lau, "Recent advances in underlay heterogeneous networks: Interference control, resource allocation, and self-organization," *IEEE Commun. Surveys & Tuts*, vol. 17, no. 2, pp. 700–729, 2015.

[117] K. U. R. Laghari and K. Connelly, "Toward total quality of experience: A qoe model in a communication ecosystem," *IEEE Commun. Mag.*, vol. 50, no. 4, pp. 58–65, 2012.

[118] A. M. Rubinov and X. Yang, *Lagrange-type functions in constrained nonconvex optimization.* Springer Science & Business Media, 2013, vol. 85.

[119] S. Boyd and L. Vandenberghe, *Convex optimization.* Cambridge university press, 2004.

[120] V. Jeyakumar, N. Dinh, and G. Lee, "A new closed cone constraint qualification for convex optimization," *Applied Mathematics Report AMR*, vol. 4, no. 8, 2004.

[121] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1., march 2014," http://cvxr.com/cvx, [Accessed: 30 June 2014].

[122] E. Bjornson, R. Zakhour, D. Gesbert, and B. Ottersten, "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical CSI," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4298–4310, 2010.

[123] S. He, Y. Huang, S. Jin, and L. Yang, "Coordinated beamforming for energy efficient transmission in multicell multiuser systems," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4961–4971, 2013.

[124] C. Chae, D. Mazzarese, N. Jindal, and R. W. Heath, "Coordinated beamforming with limited feedback in the MIMO broadcast channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1505–1515, 2008.

[125] G. H. Golub and C. F. Van Loan, *Matrix computations.* JHU Press, 2012, vol. 3.

[126] M. C. Grant and S. P. Boyd, "The CVX users guide, release 2.1," http://cvxr.com/cvx/doc/CVX.pdf, [Accessed: 30 June 2014].

[127] M. Ehrgott, *Multicriteria optimization.* Springer Science & Business Media, 2006.

[128] S. U. Pillai, T. Suel, and S. Cha, "The perron-frobenius theorem: some of its applications," *IEEE Signal Processing Magazine*, vol. 22, no. 2, pp. 62–75, 2005.