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Master of Philosophy

Novel Techniques for Gas Demand Modeling and Forecasting

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ABSTRACT

The ability to provide accurate forecasts of future gas demand has a major impact on several business processes for Gas Regions in the UK and elsewhere in the world. Long term forecasts provide the guidance for major structural needs, while short term forecasts guide the operations management on requirements of supply purchase, supply storage and delivery. Accurate forecasts guarantee optimum and safe gas supply at the lowest cost.

Currently there is no single technique that produces the perfect forecast, this research will attempt to improve on current methods by applying Non-Linear techniques. The technique to be tested is defined as "Non-Linear Autoregressive Moving Average with eXogeneous Inputs, polynomials, and a Forward Regression with Orthogonal Least Squares estimation procedure".

The goal of the research is is to produce a Mean Average Percentage Error of between 4-6% or better, which was proposed by DNV GL (supplier of software for the Gas Industry), as a valid level of error to make any new methodology of value.

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LIST OF ABBREVIATIONS

ACF Autocorrelation Function

ANN Artificial Neural Networks

ARIMA Autoregressive Integrated Moving Average

ARIMAX Autoregressive Integrated Moving Average with eXogeneous Inputs

ARMA Autoregressive Moving Average

ARMAX Autoregressive Moving Average with eXogeneous Inputs

ARX Autoregressive with eXogeneous Inputs

CCF Cross Correlation Function

FROLS Forward Regression with Orthogonal Least Squares

GA Genetic Algorithm

MAE Mean Absolute Error (also called Mean Absolute Deviation(MAD))

MAPE Mean Average Prediction Error

MLP Multi Layer Perceptron

MPE Mean Prediction Error (also called the Forecast Bias)

MSE Mean Squared Error

NAR Non-Linear Autoregressive

NARIMAX Non-Linear Autoregressive Integrated Moving Average with eXogeneous Inputs

NARMAX Non-Linear Autoregressive Moving Average with eXogeneous Inputs

NARX Non-Linear Autoregressive with eXogeneous Inputs

PACF Partial Autocorrelation Function

RBF Radial Basis Function

RMSE Root Mean Squared Error

SARIMAX Seasonal Autoregressive Integrated Moving Average with eXogeneous Inputs

SNET Seasonal Normal Effective Temperature

SVM Support Vector Machine

SVR Support Vector Regression

Chapter 1

INTRODUCTION TO THE RESEARCH PROJECT

1.1 Hypothesis

The ability to provide accurate forecasts of future gas demand has a major impact on several business processes for Gas Regions in the UK and elsewhere in the world, e.g. Shaikh and Ji (2016); Szoplik (2015); Brabec et al. (2015); Zhu et al. (2015); Khan (2015); Potocnik et al. (2014); Karimi and Dastranj (2014); Taspinar et al. (2013); Pang (2012) and Demirel et al. (2012).

The efficient operation of the gas supply network relies on accurate knowledge of the availability of supply and the characteristics of demand. Assessment of these parameters in the long term enables the planning of grid expansion, together with the investigation of the location and size of storage capabilities which will be needed for supply security. Short term demand analysis facilitates grid control action to follow the demand profile by provision of adequate gas pressures and storage even though the network may be near maximum transportation capacity. Failure to meet a demand will lead to loss of revenue and could result in dangerous operating conditions. A technique which enables the automatic calculation of accurate demand predictions over both long term and short term periods is of considerable assistance and importance in grid control.

This thesis is rather unusual, as the initial research was started in 1971, as a post-graduate student in the Control Engineering Department at Sheffield University, under the supervision of Dr. M.J.H. Sterling. At that time, the research applied the linear modeling techniques Autoregressive Integrated Moving Average (ARIMA) and Autoregressive Integrated Moving Average with eXogeneous Inputs (ARIMAX) (Box and Jenkins, 1970) to short term gas demand modeling and forecasting. The ARIMA/ARIMAX methodology was in its infancy in the 1970s.

A paper written in 1984, (Lyness, 1984), clearly states the reasons and examples, for improved forecasting techniques to help in the different time periods of forecasting for the Gas Industry. These reasons (cost and security of supply), which were relevant then, are still relevant today.

The aim of this thesis is to apply Non Linear techniques specifically Non-Linear Autoregressive Integrated Moving Average with eXogeneous Inputs (NARIMAX), polynomials, and a Forward Regression with Orthogonal Least Squares (FROLS) estimation procedure to data from specific gas regions. The data comes from several gas regions in the UK from the periods 1963-1975 and 2001-2011. The thesis will compare the results of the 2 techniques (ARIMA/ARIMAX and NARIMAX) to data sets from these two time periods. Note: In all cases the data will be transformed to stationarity prior to modeling and forecasting, hence the naming convention for the modeling and forecasting throughout this thesis will be ARMA(X) and NARMA(X), i.e. the "I" for Integration is performed outside the modeling process.

Also over the last 40 years, the UK gas market has gone through major changes. Before 1986, the gas market, in the UK, was owned and managed by British Gas, a government run utility. All the forecasting tools used by the various gas regions were developed in house either in the region or at the British Gas Research and Development Center in London (Note: This was my initial employer and my work focus after leaving Sheffield University in 1973). In 1986, British Gas was privatized and British Gas Research and Development Center later became Advantica in the 1990‘s (supplying the gas regions with Forecasting Tools and Services), before being purchased by GL Denton in 2007 and merging with DNV to become DNV GL in 2013. DNV GL‘s forecasting applications are still supplied to many of the Gas Regions in the UK, as well as to other parts of the world.

Over the years a number of forecasting tools have been developed, from simple linear models (exponential smoothing, simple regression and ARIMA models) to more complex non-linear models (Neural Networks and Fuzzy Logic). Several of these models make up the DNV GL Suite of Forecasting Tools (including ARMA(X)) today.

During the first year of this work, contact was made with DNV GL, who described their history, confirmation of which techniques (over the last 40 years) have been particularly applicable and useful to the subject of short term forecasting, and the tools/methods they have implemented as well as those they researched but did not implement. Their philosophy is that no single tool can provide the exact forecast, and so they have implemented a suite of programs which they aggregate to produce their short term forecasts (within day, daily and weekly). This has been mentioned in two papers, (Armstrong, 2005; Perchard and Whitehand, 2000), as a methodology for Short Term forecasting accuracy.

When discussing the hypothesis, below, with DNV GL (which they have not implemented or researched), they believe it has advantages and possibilities. To this end they supplied ten years of hourly supply and temperatures data for four UK regions. They have also confirmed that forecasting errors in the regions of 4-6% (or better) would make the research of potential commercial value.

Thus the hypothesis for this thesis is: "Non-Linear Modeling of Gas Demand and Temperature using Polynomial Autoregressive Moving Average with eXogeneous Inputs (NARMAX) models, and Forward Regression with Orthogonal Least Squares (FROLS) estimation procedure can produce as good or better forecasts than the traditional linear Autoregressive Moving Average with or without eXogeneous Inputs (ARMAX/ARMA) modeling techniques for short term forecasts in the Gas Domain".

This hypothesis will be measured by:

1. The Mean Average Percentage Error (MAPE) being equivalent or better
2. Models using the NARMAX methodology are similar or simpler
3. Non Linear terms add value to the forecast output

The results will be compared to my original work done using the ARMA/ARMAX models, as well as testing with the new data provided by DNV GL. The goal is to produce forecasts in the region of 4-6% error or better when compared to the actual demands of the comparison year. The main error calculation will be the MAPE as used by DNV GL to compare the accuracy of their forecasts (although other statistics will be used to measure bias and variance).

Note: DNV GL use rules on special days to adapt to variations in demand, e.g. Summer, Weekends, Bank Holidays, Christmas Day etc. They do this using the knowledge of the historical environment, as the modeling techniques do not have sufficient data to model or forecast the results automatically. However, these "knowledge rules" have NOT been included to the work in this thesis, hence the results could potentially be improved if these rules were applied.

Polynomial NARMAX and FROLS optimization is a well established methodology; however applied to the problem of Short Term Gas Demand modeling there are several reasons and potential benefits which make it an interesting research topic:

1. The number of parameter combinations (terms) can be extremely large. An exhaustive search process of selection would be impossible, and hence the methodology provides a very structured approach to the term selection process. The potential parameters are:

   (a) Past demand. Previous research (Antcliffe et al., 1975*a*,*b*,*c*,*d*) and (Perchard and Whitehand, 2000) has shown that the previous four days influence the current day, as well as a week ago, a month ago and possibly a year ago. This was true at the time of the original research and has been confirmed also by DNV GL.

   (b) Temperature. The impact of temperature on demand has been apparent for decades, e.g. Antcliffe et al. (1975*a*,*b*,*c*,*d*); Perchard and Whitehand (2000); Fischer (2010); Geen (2012) and Abiodun (2012). However, the number of temperature combinations is very large, the average (and there are several possible averages – daily average, morning average, evening average etc), the max and min, and temperatures at specific times of the day; as well as the previous days' values of all of the above, hence adding to the total number of terms.

   (c) Wind speed, Precipitation and Cloud Cover. These are secondary factors, which can be added later to improve the model.

(d) Demographic data, Gas Price and Electricity price also affect the demand.

2. Compared to ARMA/ARMAX modeling, the term selection is helped greatly by the FROLS algorithm which selects the most important terms, in order of "value added" to the system output.

3. It is well known in the Gas Industry (Piggott, 2003) that there are interacting effects; hence the methodology of combinations of parameters is an already accepted concept.

4. The relationship between demand and the influencing variables (weather and economic) is non-linear (Geen (2012) and Abiodun (2012)).

5. Few publications covering the combination of NARMAX methodology with polynomials applied to the Gas and Electricity Demand forecasting problem have been written. This will be covered in the detailed literature review Section 2.

6. DNV GL are very interested in the results of the research.

1.2 Document Research Summary

In setting up the document review, the search terms described in Appendix A.1 were used to find articles related to Gas or Electricity Demand Forecasting. Additionally, the journals and sites searched are listed in Appendix A.2.

As a summary of the document search for Gas Demand modeling and forecasting, the following modeling techniques were found: ARMA/ARMAX modeling is still used today (Potocnik et al., 2014; Taspinar et al., 2013; Siddique, 2013; Demirel et al., 2012; Akkurt et al., 2010). Artificial Neural Network (ANN) techniques have had a lot of focus and application (Szoplik, 2015; Yu and Xu, 2014; Karimi and Dastranj, 2014; Taspinar et al., 2013; Siddique, 2013; Demirel et al., 2012) together with Support Vector Machines (Zhu et al., 2015; Zhang et al., 2011; Hanand et al., 2004). Fuzzy Logic/Neuro Fuzzy Systems (Azadeh et al., 2010), Genetic Algorithms (GA) optimization (Yu and Xu, 2014; Karimi and

Dastranj, 2014; Forouzanfar et al., 2010), Expert Systems (Smith et al., 1996), and Case Based Reasoning (Smunek and Pelikan, 2008) have also had some focus in the Demand Forecasting area with more of less success. NARMAX with ANN models have had some focus in the Electricity Load Demand area (Vajk and Hetthessy (2005), and Lee (2002)), but little work has been done using polynomials. From the literature research, Polynomial NARMAX has not been applied, so far, to the Gas Demand modeling and forecasting domain.

A more detailed description of the documents research and their impact on this thesis is covered in the next chapter.

1.3 A Brief Personal History

As mentioned above, this thesis is rather unusual, as the initial work was originally started in 1971/72 when I was registered for a Ph.D. in the Control Engineering Department at Sheffield University. In order to understand the flow of this thesis, I feel a little background is necessary. In 1971, after completing a Masters Degree in the Statistics Department of the University of Sheffield, I was accepted in the Control Engineering Department to complete a Ph.D. in 2 years.

The first year the research involved modeling and predicting Water Demand. The techniques used spectral representation by orthogonal functions. This research was published in the "Journal of the Institution of Water Engineers" in 1974 (Antcliffe and Sterling, 1974). However, although the methodology was valid, and the results successful but there was no industrial uptake because:

1. The data was of poor quality and hence the "rubbish in/rubbish out" principle held.

2. The audience (Water Engineers) could not translate the model to explainable language, which the management required in order to invest in the application of the techniques.

The points above were taken into account for the research work which followed. A new domain was chosen, and a modeling technique was selected which could be explained by engineers to managers. The domain was Gas Demand Modeling and Forecasting, where the data was of a much better quality, and the technique selected was Autoregressive Integrated Moving Average (ARIMA) modeling (Box and Jenkins, 1970), which was in the early stages of application in the early 1970s. The modeling technique had not been applied to Gas Demand Forecasting at that time.

Also mentioned above, the area of Short Term Gas Demand forecasting is very important to the security of supply in the UK (and elsewhere) at a minimum cost (Newton (2010), Milne (2010)). Short term forecasts are used for the daily operations of the system. The UK Gas regions do not own the gas, and hence need to forecast as accurately as possible to keep the supply side and the demand side matched.

In the 1970s the supply storage was covered by large tanks which rose and fell based on gas input/output. Today, these have been replaced by compression techniques in the pipeline. However, since gas travels slowly (can take several days (up to four) to arrive to its final destination), the need for increased forecasting accuracy has become even more important.

Having spent nearly a year on the Water Demand project, there was only one year left to develop the Gas Demand Forecasting models and complete the Ph.D. thesis. This did not happen, but at the end of the year, I was hired by British Gas into their Research and Development Center in London, where my task was to continue to work on Gas Demand Forecasting and develop the ARIMA techniques which were in embryonic form (basically an extension of my research). The results were successful, and after 1 year I was offered a Research Assistants post back in the Control Engineering Department, to continue to develop the techniques, programs and publish the results on behalf of British Gas.

Four papers were published (Antcliffe et al., 1975*a*,*b*,*c*,*d*), covering both Weekly Gas Demand forecasts and Daily Gas Demand forecasts using the ARIMA/ARIMAX techniques. Weekly Demand corrected the demand on a standardized temperature profile and Daily demand was modeled with actual temperatures. A further paper was published in 1975 describing the package of tools developed within the Control Engineering Department, and applied to the research (Batey et al., 1975).

Now to the present, in 2010, I was accepted for as a Part Time Research student in the

Department of Automatic Control and Systems Engineering with the intent of completing my work. Restarting the research after 40 years, required certain changes, although the 2 criticisms above still hold, i.e. accurate data and explainable techniques. The results of the research are described in the Chapters that follow.

1.4 Thesis Structure

The structure of the thesis is as follows:

- Chapter 2 reviews in detail articles and books relevant to Gas Demand Modeling and Forecasting domain, and summarizes those related to Electricity Demand Modeling and Forecasting when relevant to this thesis.

- Chapter 3 describes the theory and methodology for applying ARMAX and Polynomial NARMAX modeling.

- Chapter 4 describes the data used in the thesis.

- Chapter 5 compares ARMAX models and Polynomial NARMAX models applied to Weekly Demand Forecasting

  - Section 5.2 confirms original Ph.D. work for Winter Weekly Demand (correcting for temperature), using ARMA Modeling with Demand data from 1963-1973. This is then compared to Polynomial NARMA modeling using the same data.
  - Section 5.3 repeats the previous section but using actual temperatures (rather than standardizing demand around a temperature profile), thus comparing ARMAX to Polynomial NARMAX, again using Demand and Temperature data from 1963-1973.
  - Section 5.4 models the Yearly Weekly Demand and Temperature data (rather than just the Winter Weekly Demand data) for Gas Demand from 1963 to 1973, comparing again ARMAX to Polynomial NARMAX.
  - Section 5.5 repeats the previous section using the data supplied by DNV GL for Gas Demand from 2001 to 2011.

  - Section 5.6 summarizes the results of the Weekly Demand modeling and forecasting.

- Chapter 6 compares ARMAX models and Polynomial NARMAX models applied to Daily Demand Forecasting

  - Section 6.2 models Eastern Gas Daily Demand and Temperature data from 1971 to 1975.
  - Section 6.3 models X-Gas Daily Demand and Temperature data from 2001 to 2011 supplied by DNV GL.
  - Section 6.4 summarizes the results of the Daily Demand modeling and forecasting.

- Chapter 7 will summarize the overall conclusions and propose future work.

Chapter 2

DETAILED REVIEW OF THE LITERATURE

2.1 Introduction

As described in Chapter 1, accurate forecasting is very important to the Gas Industry. "It is a key process in running the UK Gas Network. An accurate forecast is required to enable system balancing thus ensuring a safe and secure supply at minimum cost", was the way it is described in a paper presented to the Pipeline Simulation Interest Group (PSIG) in 2000 by DNV GL (Perchard and Whitehand, 2000). This was true in the 1970s and is still true today. Improving the forecasts for gas demand using a new technique will have important repercussions to the Gas Industry. This has driven research into new techniques on a continual basis. The reason for this continual focus is also confirmed by the 2013 International Energy Outlook (IEO2013) from the Energy Information Administration - USA (2013). Their forward looking article states that "Natural gas remains an essential resource until 2040, 80% of the global energy production will be supplied by fossil fuels, with natural gas being the fastest-growing, increasing by 1.7% per year".

Energy demand forecasting (electricity and gas) has had a lot of focus over the last 40 years. The period of the 1970s and 1980s saw many papers written, due to the energy crisis of the time. The difference between the two energy sources is the fact that electricity cannot be stored, in large quantities, as cheaply as gas. Some papers are cited in this chapter for electricity as the factors influencing demand, and the techniques applied to the electricity demand forecasting domain, have many similar characteristics with those of gas demand forecasting. It was shown in Dagher (2012), that there were far more papers written, in the 1970s and 1980s, for Electricity Demand Forecasting than for Gas Demand Forecasting, due to the energy crisis at that time. However, in the paper written the same year by Soldo (2012), he wrote that there has been an increase in Gas Load Modeling and Forecasting papers since 2004.

Electricity and gas demand is impacted enormously by weather variables, with temperature having the most significant impact (Fischer, 2010; Sabo et al., 2011; Taspinar et al., 2013; Potocnik et al., 2014). For time scales of a few hours to the longer term of months and years, the factors that influence energy demand (as described by Geen (2012) and Abiodun (2012)) are:

1. Calendar data: the time of the day, the day of the week, public holidays, school holidays and daylight saving time

2. Meteorological data: temperatures, wind speed, solar radiation, rain and snow

3. Economic factors: economic growth, production plans of companies, price (both for Gas and Electricity), population growth

On the time periods of a few hours to a few years, the importance/impact on energy demand of the first two factors (calendar and meteorological data) is equivalent. Economic factors impact longer time scales, as well as exceptional circumstances, for example, an economic crisis (Leguet, 2010; Geen, 2012; Abiodun, 2012).

The two presentations written by the National Grid in February 2012 (Geen, 2012; Abiodun, 2012), explain the different factors and variables (described above) which impact the forecasting process. They write that "the functional relationship is non-linear and there are more or less complex interactions between different data types. Since no simple deterministic laws that relate the predictor variables (calendar data, meteorological data and economic variables) on one side and energy demand as the target variable on the other side seem to exist, it is necessary to use statistical models".

A paper published by Soldo (2012), entitled Forecasting Natural Gas Consumption, reviewed many aspects of modeling and forecasting in the Gas Domain, covering the period 1949 to 2010. In particular the paper starts with a detailed review of papers published during this period. He stated that between 1949 and 2004, 29 papers were published, where as from 2004 to 2010, 47 papers were published. The increase in publication shows the growing need for better and better methods for modeling and forecasting in the gas domain. The paper starts off with a comprehensive literature review from 1949 up to 2010. These are summarized in Tables 1, 2, 3 and 4, in his paper, which list the publications by year, first by author then by applied area, forecast horizon and the gas consumption data used. Table 6 from this paper covered the different techniques used, along with references. Up to 2010 no papers had been published using Polynomial NARMAX. And finally Table 5 listed the input data used for the different techniques, and again listed by published paper and author. The subsections below will cover these techniques for modeling, followed by a detailed review of papers, specifically related to Gas Load Modeling and Forecasting, from 2010 to the present day. The search terms and journals/sites searched are listed in Appendix A.

In support of the subsections below, a paper published in 2006 by Gooijer and Hyndman does an analysis of the 25 years of time series forecasting. Although more oriented to Economic Data, it provides a clear picture of the different techniques available and applied to the modeling and forecasting problem.

2.2 Modeling Techniques

ARMA/ARMAX modeling, although developed by Box and Jenkins in the 1970s is still actively researched and applied today to load demand modeling in the electricity and gas domains. Several papers have been recently published related to Gas Demand Forecasting in Turkey (Akkurt et al., 2010; Demirel et al., 2012; Ervural et al., 2016), USA (Siddique, 2013), Croatia (Potocnik et al., 2014) and Iran (Shakouri and Kazemi, 2016). Similar papers have also been written for Electricity Demand Forecasting in Singapore (Deng and Jirutitijaroen, 2010), Malaysia (Norizan et al., 2010), Kuwait (Almeshaiei and Soltan, 2011) and China (Miao, 2015). Also the technique is part of the package of tools which make up DNV GL's suite of applications that they sell to the UK Gas Regions and other countries today. Hence it can be considered an acceptable base point to compare to the Polynomial NARMAX modeling approach researched in this thesis.

The Neural Networks modeling in the Demand Forecasting Domain became the solution for removing the linear constraints in the 1980s and 1990s. This is probably the second most applied methodology to the Demand modeling and forecasting arena. The disadvantage is that the methodology is a black box formulation and even though it has been proved to provide improved forecasts (Liu, 2011), the inability to describe the black box contents, can be an issue for implementers (Perchard and Whitehand, 2000). Also the risk of over-parameterization and over-fitting has been recognized as an issue (Hippert et al., 2005). Examples of Neural Networks applied to Gas Demand Modeling and Forecasting have been described for the USA (Khotanzad et al., 2000), Serbia (Ivezic, 2006), Iran (Azadeh et al., 2010) and Poland (Szoplik, 2015).

The DNV GL formulation uses a supervised neural network. It is trained with a series of input data (weather and past demand) together with the desired output (actual gas demand). The internal weights of the neural network are adjusted during the training procedure in order to minimize the overall prediction error. The neural networks models used at DNV GL provide within day, day ahead and two day ahead forecasts, and are part of the application suite of multiple prediction methods. The Neural Network is a Multi Layer Perceptron (MLP) network with a single hidden layer and is trained by back propagation of errors with momentum. This methodology is not covered in this research, but a few recent references are included, which show that it is a methodology applied to the Demand Forecasting problem with success for both Electricity and Gas (Sheikh and Unde, 2012; Hooshmand et al., 2013; Yu and Xu, 2014; Karimi and Dastranj, 2014; Szoplik, 2015; Olagoke et al., 2016)

In addition to pure Neural Network applications, mixed techniques have been tested to attempt to improve the modeling and forecast accuracy. Several papers describe Neural Networks in conjunction with Genetic Algorithms applied to the problem of Gas Demand Modeling and Forecasting in China (Yu and Xu, 2014), Iran/Australia (Karimi and Dastranj, 2014) and Turkey (Ervural et al., 2016; Shakouri and Kazemi, 2016).

Several papers applying SVMs to Gas Demand Modeling and Forecasting have also been published recently in China (Zhang et al., 2011) and the UK (Zhu et al., 2015).

Expert Systems have also been used as "add ons" to other forecasting methods, to handle extreme or specific situation adjustments, which would be difficult or even impossible to model mathematically (Smith et al., 1996). DNV GL uses an Expert System for this purpose. The Expert System handles non continuous quantities. They include cloud cover and precipitation, and the rules adjust the forecasts generated by the statistical and Neural Network models.

The underlying theory to NARMAX and FROLS has been documented since the mid 1980s by Professor S.A. Billings et al, (Billings and Voon, 1986; Korenberg et al., 1988; Chen et al., 1989; Billings and Zhu, 1994; Billings and Coca, 2001; Wei et al., 2004) as well as a recent book by Professor S.A. Billings (Billings, 2013).

The search for published papers covering the NARX/NARMAX technique applied to the Energy Modeling and Forecasting domain has turned up few examples. Papers have been published for the Electricity sector in France (Czernichow et al., 1995) , Taiwan (Lee, 2002; Chang, 2009) and Canada (Jazayeri et al., 2007). No papers appear to have been published using NARMAX and Polynomials in the Natural Gas modeling and forecasting domain. The NARMAX methodology has been applied in other domains (motor control, rainfall runoff, flood modeling, traffic flow, space weather to name a few), using different models, specifically, Neural Networks and Wavelets (Ahmed and Jamaluddin, 2001; Prudencio and Ludermir, 2001; Billings and Wei, 2005; Vall and M'hiri, 2008; Zhang et al., 2012).

DNV GL in 2000 (Perchard and Whitehand, 2000) indicated that there was not one specific modeling algorithm which could be used for all situations, and hence have developed a suite of programs, which are then aggregated to provide a short term forecast. This point was also confirmed in a paper covering Gas Demand Forecasting in the USA (Khotanzad et al., 2000).

2.3 Summary of Gas Demand Modeling and Forecasting Papers from 2010 to 2017

Another aspect of energy modeling is the forecast time horizon and data used for modeling and forecasting. The time horizon is often described as Short Term, Medium Term and Long Term Forecasts. Although there is no clear definition for these terms, in the case of Gas Demand Modeling and Forecasting in this thesis, Short Term will refer to time horizons where the focus is on operational information, and hence will cover daily and weekly. DNV GL also include hourly horizons in their Short Term definition. The data used in modeling and forecasting is also varied, covering historical demand data, weather information, and in some cases population size and price indicators. Although these last two inputs are used more for the long term than the short term modeling problems. The description and details of the papers below will cover these aspects in more detail.

The above information is covered in the paper published by Soldo (2012), which described papers published from 1949 to 2010 specifically for Gas Demand modeling and forecasting. The papers discovered during the literature search from 2010 to the present day are listed in Table 2.1 summarized by year. Following the same presentation method as Soldo, the details of each of these papers are then described.

Year	List or Authors by Year
2010	Akkurt et al. (2010)
2011	Zhang et al. (2011), Zhoua et al. (2011), Wadud et al. (2011), Sabo et al. (2011)
2012	Dagher (2012), Geen (2012), Abiodun (2012), Demirel et al. (2012), Pang (2012)
2013	Siddique (2013), Taspinar et al. (2013), Chen et al. (2013)
2014	Karimi and Dastranj (2014), Potocnik et al. (2014), Yu and Xu (2014)
2015	Zhu et al. (2015), Szoplik (2015), Brabec et al. (2015), Khan (2015), Fagianin et al. (2015)
2016	Ghalehkhondabi et al. (2016), Shaikh and Ji (2016), Potocnik and Govekar (2016)
2016 cont.	Zeng and Li (2016), Ervural et al. (2016)
2017	Panapakidis and Dagoumas (2017)

Table 2.1: List of published Gas Modeling/Forecasting papers by year.

2.3.1 2010 Papers

One additional paper from 2010 not covered in Soldo (2012) was found during the literature search.

The paper was written by Akkurt et al. (2010). The focus was on modeling Turkey's gas consumption over two time periods monthly and yearly. For the monthly data, which contained only historical consumption from 1999 to 2007, the authors tested several modeling techniques, and found that Seasonal ARIMA (SARIMA) performed the best when forecasting for the year 2008. For the annual data however, which was from 1987 to 2008, double exponential smoothing (Gardner, 2005) produced the best forecasts (when compared to an ARIMA model) for the years 2007 and 2008.

2.3.2 2011 Papers

Four papers were published in 2011, related to Gas Demand Modeling and Forecasting.

Zhang et al. (2011) modeled daily gas data of a Northern Chinese city from the years 2008 and 2009. Among several input variables (historical consumption, daily temperatures and other weather factors), the authors found the average daily temperature had the most significant influence on the modeling accuracy. The modeling technique they applied to the problem was Support Vector Machine (SVM), made up of 31 terms, covering the 7 previous days of consumption, temperature, weather conditions and date property, together with the actual days, temperature, weather condition and date property. They forecast for 10, 20 and 30 days ahead, and achieved a better than 5% MAPE for each forecast horizon.

Zhoua et al. (2011) used an Output-Input-Hidden Feedback-Elman (OIHF-Elman) neural network (Hrolenok, 2009) to forecast daily gas load. The input variables for the model included the highest temperature for the day and several days prior to the forecast day, the daily gas consumption for the same days as the temperature and the date type (day of week) again for the same days as the temperature. They found that temperature has the biggest impact on consumption, but in certain temperature ranges the impact is almost zero. Also they categorized Monday to Friday differently to Saturday and Sunday. They forecast for 20 days, and compared the OIHF-Elman neural network with a Elman neural network. The OIHF-Elman neural network produced lower forecast errors than the Elman neural network model, except on May 1st. Their conclusion being that more work (and additional inputs) are required to cover special days (like holidays).

Wadud et al. (2011) looked into modeling the gas demand for Bangladesh for the long term up to 2025. The aim of the paper is to improve on the current simplistic forecasting methods used for long term predictions for the country. They included gas price, the countries GDP and population factors in their model. They also included the fact that these variables do not impact the demand immediately, and hence included lag factors for each variable. They modeled annual data for each variable from 1981 to 2008. The modeling technique used was a log-linear Cobb-Douglas functional form (Tan, 2008). Their results showed that there is a large potential for Natural Gas usage in Bangladesh, that is currently untapped, since neither population nor GDP nor price were significant in their model.

Sabo et al. (2011) looked at hourly gas consumption for the city of Osijek (Croatia). They included past consumption and temperature in their model. The data used was from 2008. They showed that a significant change in temperature influences consumption quickly and directly for residential customers and small commercial customers. They also showed that the relationship was linear, using the Gompertz model function (Jukic et al., 2004) and the Fermat - Torricelli - Weber (FTW) method (Bazaraa et al., 2006; Mordukhovich and Nam, 2000) to model the relationships. Finally a comparison between the Gompertz model function, the FTW function and a Linear Function for forecasting future hourly demand, showed that the Linear Function produced the smallest relative percentage error.

2.3.3 2012 Papers

Five papers were published in 2012, related to Gas Demand Modeling and Forecasting.

Dagher (2012) studied the impact of price changes in gas consumption for residential customers for a particular utility company in Colorado. They modeled monthly consumption from 1994 to 2006. Their focus was the length of time between the price change and the return to stability in consumption. The term used for this type of study is called "elasticity". They used an Autoregressive Distributed Lag model (Pesaran, 1999), and found that demand was less sensitive to price and income changes than previously thought. Long run equilibrium is achieved around 18 months after the change, rather than 10 years in previous studies. Additionally, the paper lists the number of energy demand papers written between 1930 and 2007 for both Electricity and Gas. The most papers for both energy sources were written in the 1970s and 1980s due to the energy crisis's at that time.

Geen (2012) authored a report for the National Grid in the UK, which described the methodologies utilized to produce forecasts of peak day gas demand and load duration curves. The report covers the processes used to produce the National Grid's 2011 demand forecasts. It describes in detail how demand forecasts are broken down by region, and the factors that affect the demand forecast. It also states that temperature explains the most variation in demand but including other weather variables improves the forecasts. It then goes on to describe the Seasonal Normal Effective Temperature (SNET) concept, and how this concept is used to correct demand to see underlying patterns. This technique will be used in Chapter 5 for the weekly data modeling. The report then goes on to describe Annual and Daily Demand modeling and forecasting. Several appendices describe in detail the different weather concepts and details about demand differences for weekday/weekend and special days like Christmas.

Abiodun (2012) also authored a presentation for the National Grid in 2012. This covered Short Term Gas Demand Forecasting, describing the factors that drive gas demand, the methods used by the National Grid to forecast Gas demand and factors that impact forecast accuracy. The most important factor influencing demand is temperature, where a 1°C change in temperature can increase demand from 5-6%. Other factors include wind (above 10 Knots), snow, rain, cloud fog and radiation. It also shows however, that the increase in demand caused by temperature is not linear, the largest increase occurs between 7 and 13°C. It goes on to describe user behavior which also impacts demand. These include the transition periods (Autumn and Spring), Bank Holidays (where there can be a variation of demand between 5 and 20%), large customers behavior, interruption to supply, TV weather forecast and special events. The presentation then goes on to describe the times that forecasts are prepared and the data used to develop them. Finally the reasons for discrepancies in the forecast (i.e. forecast accuracy) are listed.

Demirel et al. (2012) compared several modeling methodologies for daily demand and forecasting gas consumption for the largest gas distribution company in Istanbul. The daily data covered the time period 2004 to 2009. Ordinary Least Squares, ARIMAX and three

ANN models were compared. Historical demand, temperature and temperature squared, together with gas price were used as input variables. RMSE, MAE and MAPE were used to compare the models forecasting accuracy for a day ahead prediction. The research found that the neural network model with back-propagation performed better than the other models tested. The variables that had the highest impact on the gas demand were the first lag of demand, temperature and the price of natural gas.

Finally for 2012, Pang (2012) studied the impact of adding additional weather inputs to the modeling of gas demand. His masters thesis analyzed the significance of each additional weather candidate and concluded that with combinations, the forecast of the next one-to-seven days gas demand is improved. The research was carried out in Marquette University's GasDay Laboratory. The GasDay application uses weather data, gas usage data and domain knowledge to forecast natural gas flow. The application currently serves 26 utilities in 22 states, and forecasts around one fifth of the USA's natural gas usage for residential, commercial, and industrial customers in more than 130 operating areas.

2.3.4 2013 Papers

Three papers were published in 2013, related to Gas Demand Modeling and Forecasting.

Siddique (2013) also from Marquette University, focuses his masters thesis on the problem of reducing the manual effort in automating the selection of the specifications of the model type, the model order and the model parameters. The models considered are statistical models (e.g. ARMA, ARMAX) and machine learning models (e.g. ANN, Regression Tree (RT), Support Vector Regression (SVR)).

Taspinar et al. (2013) compared several modeling methods to 1800 days of daily consumption data and associated weather data for two utilities in Turkey. They compared SARIMAX (Hyndman, 2002), ANN-MLP (Stefanowski, 2010), ANN-RBF (Stefanowski, 2010) and Multivariate Ordinary Least Squares (de Chaisemartin, 2011) to the same data. The SARIMAX model produced the best result (using MAPE and RMSE as the forecast accuracy measurements performance) with ambient temperature and cloud cover being the most significant inputs.

Finally, Chen et al. (2013) looked at combining Regression Analysis and Neural Networks. The Regression Analysis modeling was used for trend modeling and the predicted values and errors are calculated by the neural network. To prove the effectiveness of the model, an SVM algorithm (Burges, 1998) compares its results with the result of combination model. The conclusions are that the combination model is both effective and accurate in forecasting short-term gas load and has advantages over other models. The paper looks at data from Shanghai containing historical consumption data, temperature and other weather factors (sunny, cloudy, cloudy to shade, shade to cloudy, shade, shade to rain, rain, snow) from 2005 to 2009. Temperature included highest temperature, minimum temperature and average temperature. To measure the effectiveness of each method on daily predictions, MAE and MSE were used. The conclusions of the paper were that forecasting load demand accurately is a difficult task, hence in order to get accurate results, the combination of linear and nonlinear methods to forecast the load demand, produces improved results.

2.3.5 2014 Papers

Three papers were published in 2014, related to Gas Demand Modeling and Forecasting.

Karimi and Dastranj (2014) developed a hybrid model to predict the natural gas consumption in the city of Yasouj in Iran using daily data from 2006 to 2010. In the model, an ANN-GA algorithm (Mitchell, 1999) integrates weather records (temperatures, humidity, rainfall and wind speed) and actual gas usage to predict daily gas consumption. The neural networks structure and its parameters are optimized by the GA. Their conclusions show that the ANN-GA model compares well with the actual data generating an MAPE of 2.19%.

Potocnik et al. (2014) compared different modeling techniques to two sets of daily consumption data and corresponding weather data. The data, from Croatia, was collected from an individual consumer and from a local distribution company. The data was for the winters of 2011-2012 and 2012-2013. The outside temperature had the most significant effect on demand, as has been found by many other papers described in this thesis (Khotanzad et al., 2000; Timmer and Lamb, 2007; Sabo et al., 2011; Zhang et al., 2011). Twelve different models were tested against both data sets. The results of the forecast period were compared using MAPE. The individual customer consumption was best modeled and forecast using ARX models over different dimensions, whereas the distribution company (serving many customers) was better modeled by Recursive ARX (RARX) models (Filipovic, 2015). It was also found that linear systems (RARX etc), outperformed SVM and ANN. The best model (again measured using MAPE) for both data sets was found to be an RARX model.

Finally, Yu and Xu (2014) proposes a short term load forecasting model based on optimized genetic algorithm and an improved back-propagation (BP) neural network. The daily data for the analysis comes from an area of Shanghai from 2005 to 2008. Minimum, maximum, average temperatures, together with historical data and date type were used to train the model. Using MAE, MAPE and RMSE as forecasting measure, the optimized model performed better than the non-optimized methods. Their conclusion is that the optimized model is very appropriate for short term load forecasting for Shanghai.

2.3.6 2015 Papers

Five papers were published in 2015, related to Gas Demand Modeling and Forecasting.

Zhu et al. (2015) proposed a model based on support vector regression with false neighbours filtered. They applied the model to UK daily data from 2009 to 2012, using the first three years for training the model and the last year for testing the predictions. The effectiveness of the model were measured using MAPE and MAE. The results showed that their methodology outperformed ARMA methods, and ANN methods. Additionally the authors proposed models for each day of the week and showed customer behavior differs daily and hence incorporating this difference of behavior could improve the forecasting performance.

Szoplik (2015) modeled gas consumption for Szczecin (Poland) using an ANN-MLP model. They included calendar data (month, day of month, day of week and hour) and temperature as inputs for the model, as these have an important significance for individual customers and small businesses. They used the model to forecast any day of the year, or hour of the day, using data for modeling from 2009 to 2011 covering roughly 132,000 customers.

Brabec et al. (2015) studied the problem of long term gas meter readings and their disaggregation and the the re-aggregation for shorter time frames (eg. daily or weekly) to allow forecasting demand on these time scales. The problem was analyzed using data from the Czech Republic. Each customer is assigned to a specific segment based on their consumption profile and their type.

Khan (2015) studied both the short and long-term dynamics of natural gas consumption in Pakistan through an econometric model. Data from 1978 to 2011 was used to analyze the impact of specific economic factors (Sector-specific income, price and cross price elasticities) on natural gas demand. They then forecast for the years 2012 through to 2020, developing forecasts for both moderate and extreme situations. Their conclusions were that real GDP per capita exerts a larger impact on gas consumption compared to its price; and that both the price and cross price (prices of other fuel sources) elasticities are relatively low, indicating consumers' indifference, in Pakistan, towards price escalation.

Finally Fagianin et al. (2015) studied modeling methods for both the gas and water domains. A collection of techniques are evaluated using the few publicly available gas datasets. The techniques they used were ANN, Deep Belief Networks, Echo State Networks, Genetic Programming, SVR and Extended Kalman Filter-Genetic Programming. The results show a strong correlation with temperature, producing improved Gas demand forecasts for both long and short-term horizons. The overall best forecast for Gas Demand was achieved with the ANN approach, with a time horizon of 24 h. However, for shorter time horizons of 6 and 12 hours, the SVR has achieved the best results. They also highlighted the high correlation between gas consumption and temperature information.

2.3.7 2016 Papers

Five papers were published in 2016, related to Gas Demand Modeling and Forecasting.

Ghalehkhondabi et al. (2016) studied energy research publications from 2005 to 2015, focusing on forecasting methods, which complemented the work by Soldo (Soldo, 2012). It covered both electricity and gas demand forecasting for multiple time horizons (hourly to yearly). The paper describes the modeling and forecasting as a non-linear problem. The most accurate forecast for one-day ahead were achieved using 6 months input data, and for two day ahead only the last three months of input data were required for accurate forecasts.

Shaikh and Ji (2016) looks at modeling the long term gas consumption needs for China. The paper developed a logistic-population model approach to forecast the medium (2020) and the long term (2035) natural gas consumption for China. The results will assist energy planners and policy makers in developing gas supply and demand side management policies.

Zeng and Li (2016) also studies long term forecasting in China for the period 2011 to 2014. The paper develops a self adapting intelligent grey prediction model. The conclusions of the paper show that China will need to import large quantities of Natural Gas to meet demand.

Potocnik and Govekar (2016) studied hourly forecasting for a Slovenian gas distribution company. They implemented a step-wise regression method, which was used to forecast from 1 to 48 hours in the future. They confirmed that temperature had the most influence on the short term forecasts, and that other variables had little effect on the accuracy of the forecasts. They also confirm that the quality of the temperature date can greatly influence the forecast accuracy.

Finally, Ervural et al. (2016) proposed a forecasting method integrating GA and ARMA methods to take advantages of the unique strength of ARMA and GA models to predict natural gas consumption of Istanbul. The data is monthly and the forecast horizon of several years. Their experimental results show the combined approach is more robust and outperforms classical ARMA models in terms of MAPE values.

2.3.8 2017 Papers

One paper was published in 2017, related to Gas Demand Modeling and Forecasting (up to end of February 2017).

Panapakidis and Dagoumas (2017) describes an automatic specification procedure for models that are based on additivity assumptions and piecewise linear regression. This procedure allowed the analyst to gain insight about the problem by examining automatically selected models, thus easily checking the validity of the forecast. The methodology was applied to one-step ahead daily gas demand forecasting in Spain. Non-Linear models were developed, and empirical results showed that the accuracy of the proposed model is competitive against more complex methods such as neural networks.

2.4 Summary of Electricity Demand Modeling and Forecasting Papers

This subsection will briefly describe papers developed in the Electricity Demand modeling and forecasting area for completeness. Between 2010 and the present day, there were several papers describing forecasting on different time frames (annual, daily and hourly), as well as different methodologies (ARIMA, ANN-GA, ANN-Wavelets and various other techniques). However, several of the Electricity papers have a direct relevance to the current thesis, specifically those related to ARMA(X) and NARMAX and are described below.

Czernichow et al. (1995) developed a NARMAX model based on Simple Recurrent Networks (SRN) using French electricity consumption data. The input variables were historical consumption data (half hourly), weather variables, specifically temperature and cloud cover for the years 1989 to 1992. They also included calendar data (day of the year and day of the week). A NARX model and a NARMAX model were developed and both were compared to an ARMA model. The results of their work showed the NARMAX model produce an MAPE of 1.8% for a forecasting horizon of one day which was comparable to the results of the French Electricity Company (EDF) and 2.2% MAPE for 2 day forecasting horizon which was better than EDF.

Lee (2002) for his masters thesis in Taiwan studied the cluster rule NARMAX method with the Neural Networks for optimization, he found that fewer order terms and more combination terms were possible using this methodology to capture the dynamics of highly non-linear systems. The thesis is in Chinese, and so only a brief summary is available.

Espinoza et al. (2006) considered (N)ARX and AR-NARX models for hourly data modeling and forecasting for a particular substation in Belgium. An AR-NARX model is a (Nonlinear) Auto Regressive model with eXogenous inputs and Auto Regressive residuals. Additionally, partially linear structures with autocorrelated residuals were incorporated into the two model structures. The model's performance was measured using MSE and

MAPE. Least Squares Support Vector Machines (LS-SVM) nonlinear regression formulation was the applied technique for the nonlinear system identification. Based on the LS-SVMs formulation, the linear ARX model, a full black box NARX model and a partially linear structures with autocorrelated residuals have been tested. In all these cases, the solution of each model was characterized by a set of linear equations. By minimization of the MAPE over the daily peaks, and by including an autocorrelation with a time delay of 24 (hours), the results improve substantially for the partially linear structure. This structured model, which is linear on the past values of the load and nonlinear on the calendar and temperature information, shows a final performance on the test set which is comparable to the full black-box NARX model, and yet it retains a linear part which improves its interpret-ability with respect to the fully black-box model.

Jazayeri et al. (2007) developed a NARMAX model using polynomials for a simulated electricity load and then on a Swedish Data set. They developed a multistage algorithm for these Nonlinear Aggregate power systems loads. The first stage attempts to find good initial values of the model parameters by developing discrete equations using the zero-order hold method followed by approximating a 2nd-order polynomial NARMAX model. The initial estimates of the NARMAX parameters are evaluated using an extended least squares approach. Finally, the initial parameter values are used with a Levenberg-Marquardt optimization (Roweis, 2000) routine to compute the optimal parameters.

Chang (2009) developed a methodology for forecasting the hourly load demand of the power provider using a NARMAX model. The proposed method was used to test on hourly data from the Taiwan power supply networks and then forecasting 24 hours ahead. The paper compared the results with an ARMA model for a 7 day time period during August of 2008. The NARMAX model produced lower errors than the ARMA for each day of the week.

Chapter 3

METHODOLOGIES

3.1 Introduction

The two modeling techniques that will be applied in this thesis are ARMA/ARMAX and Polynomial NARMA/NARMAX. This chapter describes in detail these methodologies, together with the work flow, and the measures for analyzing and comparing the results generated by each method.

The two methodologies will be applied to several sets of gas demand data (described in Chapter 4). Gas demand data over a particular period can be characterized by the discrete series $[d_t\ ,\ t = 1, 2, 3,\ \ldots n]$. The objective of demand prediction is thus to establish the values of the series d_t for $t = n + 1, n + 2, \ldots n + m$. Hence each Gas Demand data set will be split into a modeling set and a forecast comparison set, the latter will be used to compare the estimated forecast to the actual data.

3.2 Autoregressive Integrated Moving Average without/with eXogenous inputs (ARIMA/ARIMAX)

The landmark book by Box and Jenkins (1970) produced a structured methodology for linear, non-stationary, multivariate time series analysis, modeling and forecasting. The tools and techniques and the application to Gas Demand Modeling and Forecasting published since then, e.g. Erdogdu (2010); Demirel et al. (2012); Siddique (2013); Taspinar et al. (2013) and Potocnik et al. (2014), have added additional confidence in the results produced by this methodology. The advantage of ARIMA/ARIMAX modeling is its simplicity. The disadvantage is, of course, the assumption that time series in the energy domain are linear in nature.

3.2.1 ARIMA/ARIMAX Software Environment

For the ARIMA/ARIMAX modeling, a general environment was developed, in MATLAB, to facilitate the different time frames of the data sets (weekly and daily), and routines were written to graph and statistically measure the results from both the modeling phase and the forecasting phase. The Econometrics Toolbox in MATLAB was used for modeling. The advantage of this toolbox over the MATLAB System Identification Toolbox is that individual variables may be chosen rather than a range. Code was written for both One-Step Ahead and Multi-Step Ahead forecasts. The One-Step Ahead forecast calculates the time period (next week or day in this thesis) from previous known values of demand and temperature. The Multi-Step Ahead uses known values up to the the start of the forecast period, and calculated forecast values after the start. These are sometimes denoted by OSA (One-Step Ahead) and MPO (Multiple Predicted Output) in this thesis.

3.2.2 ARIMA/ARIMAX Workflow

The workflow for ARIMA/ARIMAX modeling is as follows:

1. Analyze and develop a stationary time series from the original time series, i.e. Transform the original time series
2. Find a best fit model for the resulting transformed series, i.e. Model the transformed Time Series
3. Analyze the modeling results
4. Forecast into the future
5. Inverse-transform to rebuild the original time series
6. Analyze the forecast results
7. Loop back to step 2 if necessary to improve

Each of these steps will be described in detail below.

3.2.2.1 Transforming the Time Series for Stationarity

The ARIMA/ARIMAX method is appropriate only for data that is stationary. Stationarity can be achieved by applying two methods: Power Transformations and Differencing. The aim of these two operations is to produce a zero mean and constant variance across the whole time series.

Power transformations are often required when the time series can only take non-negative values. Frequently, the asymmetry steadily increases in time, and can be attenuated by applying a class of power transformations. Time series transformations were described by Box and Cox (1964). For a time series d_t (where $t = 1, 2, 3, \ldots, n$) possible transformations are :

$$z_t = \begin{cases} {d_t}^{\lambda} & \text{if } \lambda > 0 \\ log(d_t) & \text{if } \lambda = 0 \\ -{d_t}^{\lambda} & \text{if } \lambda < 0 \end{cases} \tag{3.1}$$

where λ is some numeric value.

The reason for changing the sign when λ is negative is to ensure that the transformed values have the same relative ordering as the original values. For time series with non-stationary variance, a natural logarithm transformation is often appropriate (Pankratz, 2009; Senter, 2010).

Power transformations cannot by themselves always stabilize a time varying mean. Non-stationary time series are typically characterized as having increasing (or decreasing) mean levels. A commonly used transformation to remove trends is the difference transformation. Differencing implies calculating the difference among pairs of observations at some time interval. For example, to difference a time series one step apart, subtract the 1st value from the 2nd value, the 2nd value from the 3rd value etc, and if the mean is zero and there is a constant variance then the time series is considered to be stationary. If one step apart differencing does not produce stationarity, additional differencing can be applied. This simple case is defined below:

$$\nabla z_t = z_t - z_{t-1} \tag{3.2}$$

also written as:

$$\nabla z_t = (1 - B)z_t \tag{3.3}$$

i.e. B is the back-shift operator, $Bz_t = z_{t-1}$.

The general form of differencing for time series with growth and periodicity is written as :

$$w_t = \nabla^d \nabla_s^D z_t \tag{3.4}$$

$$for \qquad t = 1, 2, 3, \ldots, n$$

where

w_t is the transformed and differenced series

z_t is the transformed series d_t (Equation 3.1)

$\nabla^d = (1 - B)^d$

$\nabla_s^D = (1 - B^s)^D$

d, D and s are integers and s measures the seasonality of the time series.

To remove a seasonal component and stable growth trend the differencing should have d = D = 1 (Box and Jenkins, 1970). To find the most appropriate values of d, D and s a recursive testing process will often be needed to develop the stationary time series w_t.

The Autocorrelation Function (ACF) helps select the appropriate transformations and differencing operators. The ACF is defined as the Cross-Correlation (CCF) of a signal with itself (Pelgrin, 2011-2012). It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal. The rule of thumb for an ACF is if there are plotted values (lags) that are greater than 2 standard errors away from zero, then these lags indicate statistically significant autocorrelation.

The Partial Autocorrelation Function (PACF) is also used to detect trends and seasonality (Pelgrin, 2011-2012). In general, the PACF is the amount of correlation between a variable and its lag that is not explained by correlations at all lower-order lags. Again significant lags are those with values greater than 2 standard errors away from zero.

The standard error of both the ACF and PACF (for large N) is approximately:

$$SE(r_k) \simeq (\frac{1}{N})^{\frac{1}{2}} \tag{3.5}$$

where r_k is the correlation value at lag k.

The results of the ACF and the PACF on the transformed time series are also used to help identify the Autoregressive (AR) and Moving Average (MA) process orders. For a full ARMA(X) model, the ACF and the PACF tail off either exponentially and/or sine waves. The point at which they tail off indicates the AR and MA model orders (p and q). If there is also a seasonal pattern visible (i.e.spikes in the ACF/PACF equally spaced at ”s” intervals), the point they tail off indicates the seasonal AR and MA model orders (P and Q). A good explanation of the different patterns of ACF and PACF and the corresponding model structure is described in the book by Mills (1990) on page 130, Exhibit 8.4. The Cross Correlation Function (CCF) is also used to detect any relationships between the different variables, and hence suggest possible AR and MA orders.

3.2.2.2 Modeling the transformed Time Series

Once stationarity has been achieved using differencing, the Integrated part of the model is considered achieved, and the modeling process reverts to an ARMAX (or ARMA) activity. The general structure for the ARMAX model is:

$$\phi_p(B)\Phi_P(B^s)w_t = c + \beta x_t + \theta_q(B)\Theta_Q(B^s)a_t \tag{3.6}$$

where

c is a constant

x_t is the Input variable or variables (also called covariate(s))

β is the regression coefficient of the Input variable(s).

$\phi, \Phi, \theta, \Theta$ are polynomials in B such that:

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 \ldots - \phi_p B^p \tag{3.7}$$

and

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} \ldots - \Phi_P B^{Ps} \tag{3.8}$$

are the autoregressive components and

$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 \ldots - \theta_q B^q \tag{3.9}$$

and

$$\Theta_Q(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} \ldots - \Theta_Q B^{Qs} \tag{3.10}$$

are the moving average components. If p, P, q and Q and β are correctly determined, then a_t will be a white noise sequence distributed as N(0,σ_a^2), where σ_a^2 is the variance.

If no input variables are used in the model above, then the ARMAX syntax becomes that of an ARMA model.

3.2.2.3 ARMA/ARMAX notation convention in this thesis

The traditional notation of ARIMA models is ARIMA(p,d,q)(P,D,Q) (Box and Jenkins (1970)). Non-seasonal ARIMA models are denoted as ARIMA(p,d,q) where the parameters "p, d, q" are non-negative integers. The autoregressive model order is denoted by "p", the degree of differencing by "d" and the moving-average model order by "q". Seasonal ARIMA models are denoted ARIMA(p,d,q)(P,D,Q) where the autoregressive, differencing, and moving average variables for the seasonal part of the model are denoted by the uppercase "P", "D" and "Q". Again the parameters "P, D, Q" are non negative integers.

However, using the Econometrics Toolkit allows for specific variables to be chosen, and hence the above notation becomes confusing. For example a model which has Autoregressive variables 1 and 3 and a Moving Average variable at 2, could not be represented by ARMA(3,2), since ARMA(p,q) model means all the Autoregressive variables from 1 to p, and all the Moving Average variables from 1 to q are included in the model.

So for this thesis, after the transformation data process (i.e. "d" and "D" differencing applied to the data), AR(a,b, ... n)/MA(a1,b1, ... n1) is used to represent the models, where a,b ... n and a1,b1, ... n1 represent the specific variables used in the model. Hence the example above becomes AR(1,3)/MA(2) representing the exact variables used in the model. From this point in the application chapters, the models are considered ARMA(X) models and not ARIMA(X) models.

3.2.2.4 Analyze the modeling results

As is often the case, several possible model structures are indicated by the ACF and PACF. Hence the next step is to measure how well each of these structures performs when compared to the actual data. Using MATLAB's functions, each structure is modeled and the residuals (i.e. the difference between the actual data and the modeled data) are calculated.

For each of the models, the residuals are analyzed using the ACF to check that they do actually represent a white noise sequence. The ACF will either confirm an adequate model or suggest possible alterations to provide a better fit. The CCF between the residuals and each input variable also provides evidence of inadequacy of the model (Mills, 1990).

Several additional measure are available to analyze the residuals, as a complement to the ACF. In this thesis the following measures will be used:

1. F = the sum of the squares of the residuals
   - F = $\sum_{t=1}^{n} a_t^2$

2. Akaike Information Criterion (AIC) (Petrov and Csak, 1973; Vrieze, 2012).
   - AIC(p) = $n\ln(\hat{\sigma}_a/n) + 2p$

3. Baysian Information Criterion (BIC) (Vrieze, 2012; Watanabe, 2013).
   - BIC(p) = $n\ln(\hat{\sigma}_a/n) + p + p\ln(n)$,

4. Q Statistic - Ljung Box Q test (Box and Pierce, 1970; Ljung and Box, 1978).
   - Q $= n \sum_{k=1}^{l} r_k^2$,

where

n = the number of effective observations

a = the residuals

p = the number of parameters

$\hat{\sigma}_a$ = the sum of sampled squared residuals.

r_k = the autocorrelation value at lag k.

The aim is to select the model which minimizes F, AIC and BIC and validate that Q is approximately distributed as a χ^2 with $(l-p)$ degrees of freedom, where l is the number after which the autocorrelation values $(r_k) \approx 0$.

3.2.2.5 Forecast future Demand values

If one model stands out from the above activity, then this model will be forecast into the future. The future values of w_t denoted as $\hat{w}_t$, are calculated, for time values of t=$n+1, n+2, \ldots, n+m$, where m equals the forecast time horizon, for each model. These values are written as $\hat{w}_{n+1}, \hat{w}_{n+2}, \ldots, \hat{w}_{n+m}$.
To forecast the future values $\hat{w}_t$, Equation 3.6 may be rewritten as:

$$\hat{w}_t = (\phi_1 B + \phi_2 B^2 + \ldots + \phi_p B^p) w_t + \phi_p(B)\Phi'_P(B^s) w_t + \theta_q(B)\Theta_Q(B^s) a_t \tag{3.11}$$

where:

$$\Phi'_P(B^s) = \Phi_1 B^s + \Phi_2 B^{2s} \ldots + \Phi_P B^{Ps}$$

and the residuals a_t are defined as:

$$a_j = \begin{cases} a_j & \text{if } j < n+1, \\ 0 & \text{if } j \geqslant n+1, \end{cases} \tag{3.12}$$

where $n+1$ is the starting point of the predictions.

Two forecasts will be produced for each model, One-Step Ahead and Multi-Step Ahead. The Multi-Step Ahead forecast sometimes drift away from the actual values as future values of demand are calculated, hence additionally the thesis will calculate shorter term Multi-Step Ahead forecasts to check when the model should be re-calibrated (or re-calculated).

3.2.2.6 Inverse-Transform the Predicted values

The values $\hat{w}_{n+1}, \hat{w}_{n+2}, \ldots, \hat{w}_{n+m}$ are then inverse-transformed to generate the original time series future values $(\hat{d}_{n+1}, \hat{d}_{n+2}, \ldots, \hat{d}_{n+m})$.

For example, using Equation 3.4, with differences 1 and 26 (and s = 1):

$$w_t = \nabla^1 \nabla^{26} z_t \tag{3.13}$$

which can be written as :

$$w_t = (1 - B)(1 - B)^{26} z_t \tag{3.14}$$

which can be expanded as :

$$w_t = z_t - z_{t-1} - z_{t-26} + z_{t-27} \tag{3.15}$$

for t = 1,2,3, ..., n

Future values of w_t (denoted by $\hat{w_t}$) are written as :

$$\hat{w}_{n+i} = \hat{z}_{n+i} - z_{n+i-1} - z_{n+i-26} + z_{n+i-27} \tag{3.16}$$

for i = 1,2,3, ..., m

Future values of z_t (denoted by $\hat{z_t}$) can then be calculated :

$$\hat{z}_{n+i} = \hat{w}_{n+i} + z_{n+i-1} + z_{n+i-26} - z_{n+i-27} \tag{3.17}$$

for i = 1,2,3, ..., m

Finally, the inverse power transformation is applied to $\hat{z}_{n+i}$. For example inverse logarithm is shown in Equation 3.18.

$$\hat{d}_{n+i} = e^{\hat{z}_{n+i}} \tag{3.18}$$

for i = 1,2,3, . . . , m

These forecast values are plotted against the actual values (the forecast comparison data set), and the residuals (e_t) calculated, where

$$e_{n+i} = d_{n+i} - \hat{d}_{n+i} \tag{3.19}$$

for i = 1,2,3, . . . , m

.

However, multiple potential process models with similar F, AIC/BIC or Q values will often be the case, and hence each model will be forecast into the future and compared using specific statistics (Adhikari and Agrawal, 2013), described below.

3.2.2.7 Analyze the forecast results

The model selected should have a balanced set of forecast error measures. A brief description of the measures are given below, and more details can be found in Prestwich et al. (2014). The measures are used throughout the thesis to facilitate the choice of the best model from a forecasting perspective. In the case where several models perform satisfactorily, the best MAPE value is privileged as the final choice.

1. The Maximum Forecast Error - max(e_t) for t=$n+1, n+2, \ldots, n+m$

2. The Minimum Forecast Error - min(e_t) for t=$n+1, n+2, \ldots, n+m$

3. MPE - Mean Prediction Error (also called the Forecast Bias):

$$MPE = (\frac{1}{m})\sum_{t=1}^{m} e_t \tag{3.20}$$

- MPE measures the average deviation of forecast values from the actuals.
- MPE indicates the error direction (the positive and negative values do not cancel out).
- MPE should be close to zero for minimum bias.

4. MAE - Mean Absolute Error (also called Mean Absolute Deviation(MAD)):

$$MAE = (\frac{1}{m})\sum_{t=1}^{m}|e_t| \tag{3.21}$$

- MAE measures the overall error.
- MAE indicates the error direction (the positive and negative values do not cancel out).
- MAE should be close to zero for a good forecast.

5. MSE - Mean Squared Error:

$$MSE = (\frac{1}{m})\sum_{t=1}^{m}e_t^2 \tag{3.22}$$

- MSE measures the average squared deviation of forecast values.
- MSE indicates the impact of large forecast errors.
- MSE should be close to zero, indicating no large forecast errors.

6. MAPE - (Mean Absolute Percentage Error):

$$MAPE = (\frac{1}{m})\sum_{t=1}^{m}(|\frac{e_t}{d_t}|) * 100\% \tag{3.23}$$

- MAPE is the most commonly used measure in the literature for forecast accuracy.
- MAPE values close to zero indicate accurate forecasts, and low over and underestimates.

Additionally, the ACF and CCF will be used to evaluate the residuals (e_t) to confirm uncorrelated values, i.e. no information is still contained in the forecast residuals. If the above measures do not indicate a satisfactory fit, then modifications to the model are made (different transformations, different parameters), and the process is re-executed starting from step 2 in the work-flow for ARIMA/ARIMAX described in the Section 3.2 above.

3.3 Polynomial Non-Linear Autoregressive Integrated Moving Average without/with eXogenous inputs (Polynomial NARIMA/NARIMAX)

As we have seen in the above sections, the ARIMA(X) methodology is relatively simple to apply. The difficulty comes in interpreting the results at the different stages (Transformation, Modeling and Forecasting) to select the most appropriate variables in the equations. This section describes the Polynomial NARIMAX methodology with the Forward Regression Orthogonal Least Squares (FROLS) algorithm. The methodology helps to select and rank significant variables/terms automatically. This methodology was developed by Billings and Voon (1986); Chen et al. (1989); Billings and Zhu (1994), and Billings (2013).

The NARIMAX model can represent a wide class of nonlinear systems (Wei et al., 2004) and is defined as:

$$y(k) = F[y(k), x(k), e(k)] + e(k) \tag{3.24}$$

or

$$\begin{aligned} y(k) =& F[y(k-1), y(k-2), \ldots, y(k-n_y), \\ & u(k-1), u(k-2), \ldots, u(k-n_u), \\ & e(k-1), e(k-2), \ldots, e(k-n_e)] + e(k) \end{aligned} \tag{3.25}$$

where $y(k), x(k)$ and $e(k)$ are the system output, input, and noise sequences respectively; n_y, n_u, and n_e are the maximum lags for the system output, input and noise; and F[•] is some nonlinear function (Billings, 2013).

Essentially, past inputs, outputs and noise terms are used to build the model. The noise is modeled explicitly, hence, unbiased estimates of the system model are calculated in the presence of unobserved highly correlated and nonlinear noise (Billings, 2013). The majority of early applications of NARIMAX models were developed using polynomial expansions. Today, as well as polynomial expansion models, more complex forms based on wavelets and other model forms have been developed to represent highly complex nonlinear systems (Ahmed and Jamaluddin, 2001; Prudencio and Ludermir, 2001; Lee, 2002; Cugliari, 2011; Zhang et al., 2012).

3.3.1 Polynomial NARIMA/NARIMAX Software Environment

Again a general environment was develop specifically for Polynomial NARIMAX modeling and forecasting in MATLAB. For modeling, some specific attributes were developed, including the ability to include or remove specific variables and terms. A cut off selector ($\rho 1$) which stopped the selection of terms if the next term had lower value (contribution to the system output) than a threshold which allowed for an improved term selection process. The value for this threshold was initially set at 0.5%. The code for data analysis, graphing, inverse data transformation was the same as that developed for ARIMA/ARIMAX.

3.3.2 NARIMA/NARIMAX Workflow

The NARIMAX workflow consists of the same steps for ARIMA/ARIMAX described in Section 3.2. Differences, specific to NARIMA/NARIMAX, are described in the sections below.

3.3.2.1 Transforming the Time Series for Stationarity

This activity is identical to that described in Section 3.2.2.1. Once stationarity has been achieved using differencing, the Integrated part of the model is considered achieved, and the modeling process reverts to an NARMAX (or NARMA) activity.

3.3.2.2 Structure Detection and Modeling the Transformed Time Series

The most fundamental part of NARMAX work is to develop the appropriate model structure. The model will be made up of model variables and/or model terms, and it is important to distinguish the difference between them. For Linear models, the model terms and the model variables are equivalent. For Non-Linear models, the model terms are the results of every combination of the model variables. The development of the model structure (or which terms to include in the NARMAX model in Equation 3.25) is vital if a parsimonious system representation is to be found (Billings, 2013). The number of terms depends on the values of n_y, n_u, n_e in 3.25 and the polynomial expansion value l. The maximum number of terms in the NARMAX model (3.25) is given by:

$$n = M + 1 \tag{3.26}$$

where

$$\begin{cases} M = \sum_{i=1}^{l} n_i \\ n_i = [n_{i-1}(n_y + n_u + n_e + i + 1)]/i \\ n_0 = 1 \end{cases} \tag{3.27}$$

For example, a 3 variable model (y_1, y_2, y_3) and 2nd order polynomial produces 10 terms:

$constant$, y_1, y_2, y_3, y_1^2, y_2^2, y_3^2, y_1y_2, y_2y_3 and y_1y_3.

3.3.2.3 Parameter estimation: determine the model coefficients

The model terms that are produced from the different variables do not produce equal value to the system output. The aim is to remove those which add little value and select those terms which make a significant impact on the system output. The process of selecting the high value terms is hence critically important in developing the correct structure of the NARMAX model. These objectives can easily be achieved by using the FROLS algorithm (Wei et al., 2004) which selects the NARMAX model terms one at a time, based on the value the term adds to the system output.

The FROLS algorithm is essentially the basic Orthogonal Least Squares (OLS) algorithm, but at each step a full search of all the, not yet selected, model terms is performed to find the next "best" term (Wei et al., 2004). The selection process is performed by calculating the Error Reduction Ration (ERR). The ERR offers a simple and effective methodology for selecting a subset of significant regressors from a large population of regressors in a forward-regression way. The ERR for a specific term represents the contribution that the specific term makes to the system output.

At step "j", a regressor is chosen if it has the largest value of $[\mathrm{err}]_j$ from all the remaining possible terms. In this way, the model is built up, one term at a time, and as each term is added its significance to the system output is shown through the value of $[\mathrm{err}]_j$.

This choice is independent of how the terms are ordered in the input data. The selection procedure is terminated when either :

$$1 - \sum_{j=1}^{M} [err]_j < \rho \tag{3.28}$$

where M = the number of selected terms and $0 < \rho < 1$ is the chosen tolerance.
OR

$$err_i < \rho 1 \tag{3.29}$$

where $0 < \rho 1 < 1$ is the value under which the selected term's value is no longer significant in adding value to the system output.

The benefits of the FROLS algorithm are clearly explained in Wei et al. (2004); where the author states "This makes the algorithm accessible to, and usable to, both experts and non-experts".

The modeling procedure, in this thesis, starts with a wide range of first order model parameters (indicated by the ACF), letting the FROLS algorithm indicate those model terms which add the most value. The results from this first step will be used to reduce the model parameters (and hence terms) and apply a 2nd order and then a 3rd order etc to the terms, evaluating the ERR profile produced by the selected terms and the order. Initially there are no error variables in the model, hence the model will be a NARX structure. Based on the result of this initial modeling and the prediction process, error variables and terms will be added and the modeling process repeated, to generate a full NARMAX model.

3.3.2.4 Analyze the modeling results

A stated in Billings (2013), "Most studies relating to model validation assume the system under investigation is linear". Hence, if the correct model variables and time delays are chosen for the linear model (with unbiased estimated parameters), then the residuals will be white noise. The autocorrelation function of the residuals and the cross correlation function between the input variables and the residuals can then be used to confirm the model structure is appropriate (Box and Jenkins (1970))." Other linear model tests involving a minimum or maximum value are also of value, for example, the F test, the Akaike Information Criterion (AIC test), and the Q statistic as described in Section 3.2.2.4.

Unfortunately, validation methods developed for linear system models are not sufficient to detect un-modeled nonlinearities. Again as described in Billings (2013), "The core concept in statistical model validation for nonlinear systems in that the residuals should be unpredictable from all linear and non-linear combinations of past inputs, outputs and residuals". However, using ONLY the linear tests can indicate that the model is adequate, whereas non-linear terms are missing which these tests could not evaluate, hence leading to false results.

To overcome this inadequacy, three additional tests were developed (Billings and Voon (1986)), to capture the most nonlinear dynamical effects. The four tests (CCF (Input vs Residuals) and the three Non-Linear Tests) are named as the Linear and Non-Linear Validity tests in this thesis and are described in 3.30 and 3.31.

$$\phi_{u\zeta}(\tau) = 0, \quad \forall\tau \tag{3.30}$$

$$\begin{cases} \phi_{\zeta(\zeta u)}(\tau) = 0, & \tau \geq 0 \\ \phi_{(u^2)'\zeta}(\tau) = 0, & \forall\tau \\ \phi_{(u^2)'\zeta^2}(\tau) = 0, & \forall\tau \end{cases} \tag{3.31}$$

where ϕ represents the cross-correlation, u represents a system input, ζ represents the generated residuals from the modeling process and τ represents the cross-correlation lag.

3.3.2.5 Forecast future Demand values

This activity is identical to that described in Section 3.2.2.5.

3.3.2.6 Analyze the forecast results

This activity is identical to that described in Section 3.2.2.7.

Chapter 4

DATA DESCRIPTION

4.1 Introduction

This chapter describes the data which will be used throughout the remaining chapters. There are three sets of data from two time periods used to test the hypothesis of this thesis. The first two sets of data are from the original work done in the 1970s. They comprises weekly data from the Southern Gas Region and the daily data from the Eastern Gas Region of the UK. The third set of data was provided under Non-Disclosure Agreement (NDA) for a region in the UK for a period of approximately 20 years centered around the year 2000. It is denoted as X-Gas in this thesis. The details of each data set are described below.

4.2 Southern Weekly Gas Data 1963-1973

The data illustrated in Figures 4.1 and 4.2 shows the data provided by Southern Gas. Figure 4.1 shows the total weekly demand (in units of 100,000 therms) for Southern Gas from the weeks starting 1st April 1963 to the week ending 30th March 1973. This demand is the combination of both manufactured gas and natural gas supplies. This data will be used in Chapter 5.

Figure 4.2 shows the weekly effective temperature in degrees Celsius for the same period. The weekly effective temperature was calculated by taking the average of the daily effective temperature for Southern Gas. The daily effective temperature was defined as:

$$T = 2/3Max + 1/3Min \tag{4.1}$$

Figure 4.3 shows the Seasonal Normal Effective Temperature (SNET) cycle for Southern Gas where each point represents the SNET average for the 7 days of that week. This

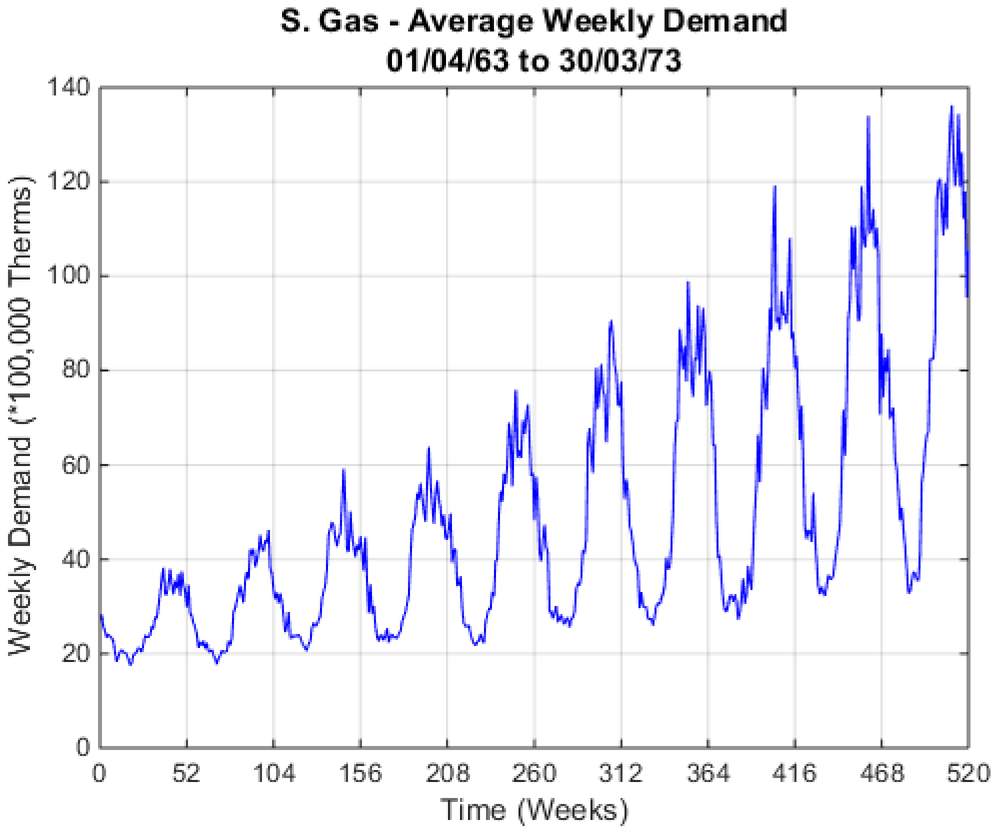

Figure 4.1: S. Gas - Weekly Demand - 1963-1973

Standardized Temperature profile will be used in Section 5.2 to remove the actual temperature from the weekly demand data. The details of the methodology for correcting Demand data to SNET are explained in Appendix B.

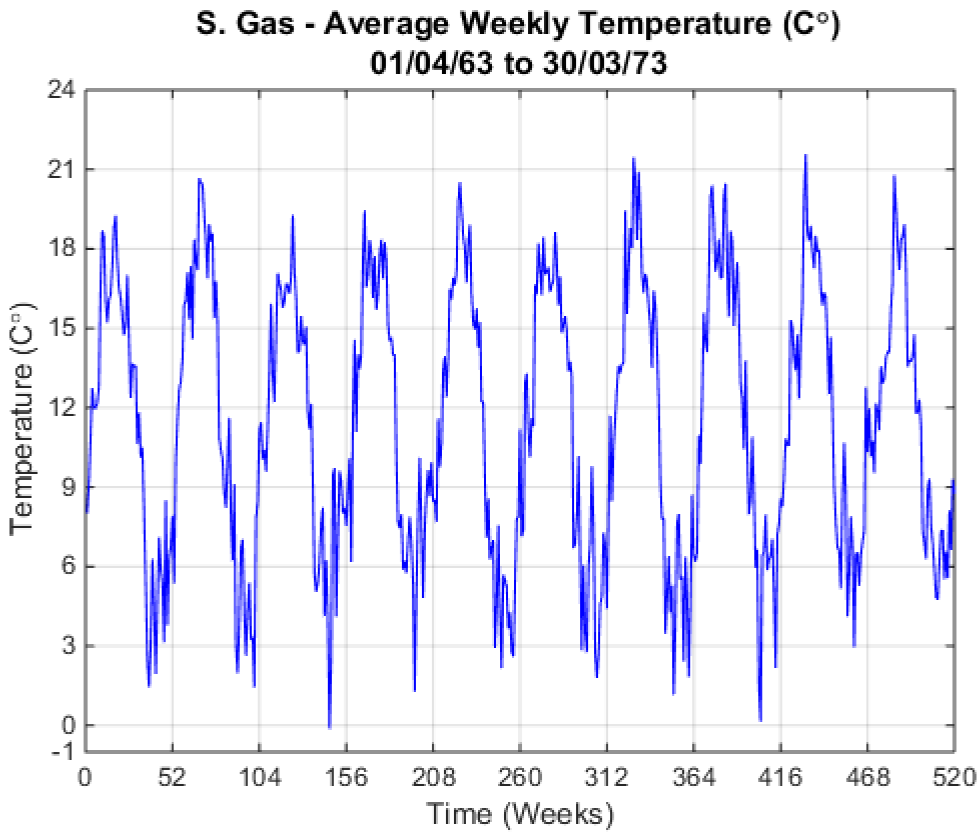

Figure 4.2: S. Gas - Average Weekly Effective Temperature - 1963-1973

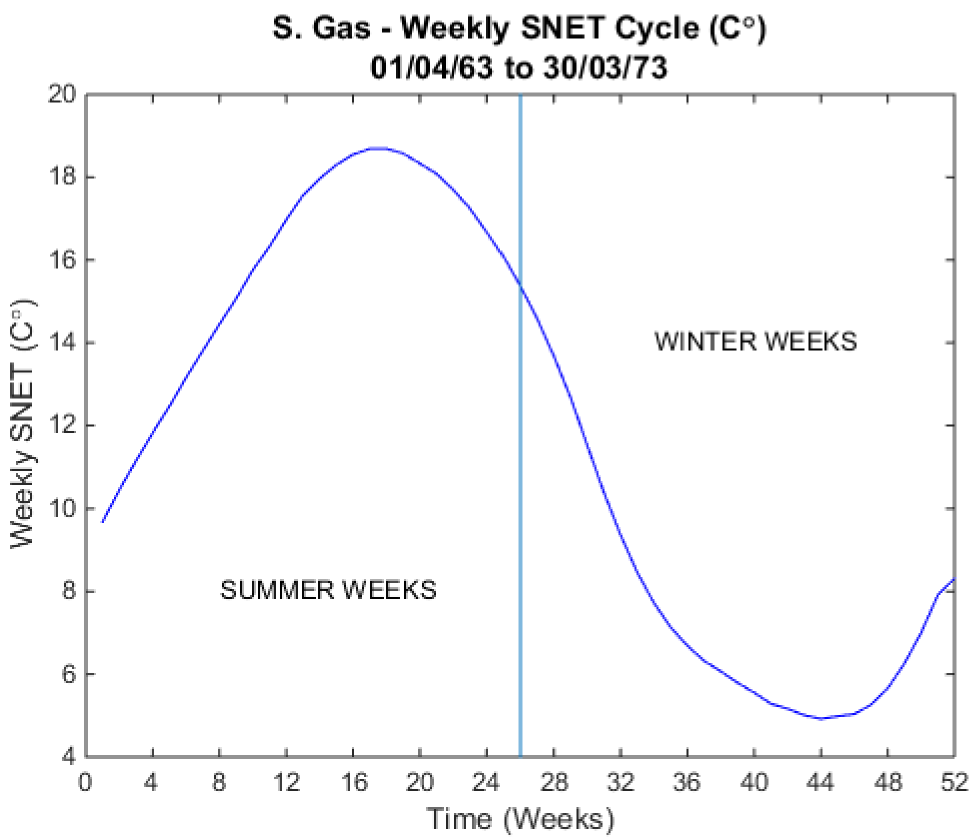

Figure 4.3: S. Gas - Weekly Seasonal Normal Effective Temperature (SNET)

4.3 Eastern Daily Gas Data 1970-1975

The data illustrated in Figures 4.4 and 4.5 shows the data provided by Eastern Gas. Figure 4.4 shows the total daily demand (in units of Million Cubic Meters (MCM)) for Eastern Gas from 1st October 1970 to 20th June 1975. Again this demand is the combination of both manufactured gas and natural gas supplies. This data will be used in Chapter 6.

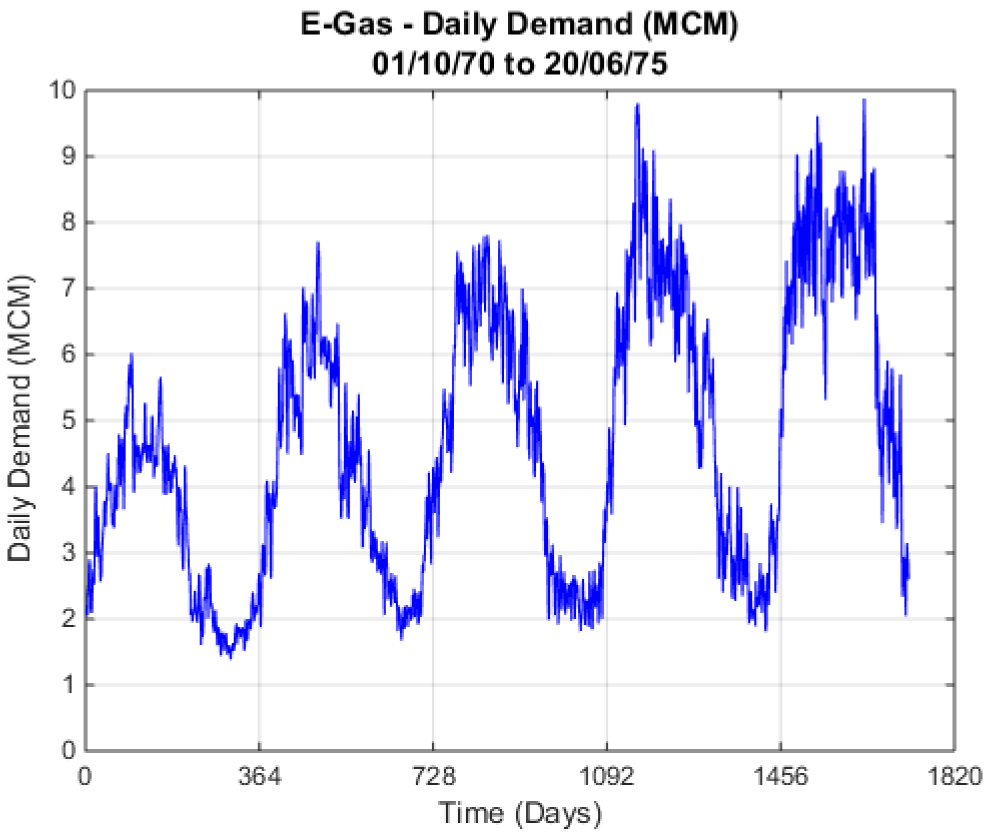

Figure 4.4: E. Gas - Daily Demand - 1970-1975

Figure 4.5 shows the daily effective temperature, described in Equation 4.1, in degrees Celsius for the same period.

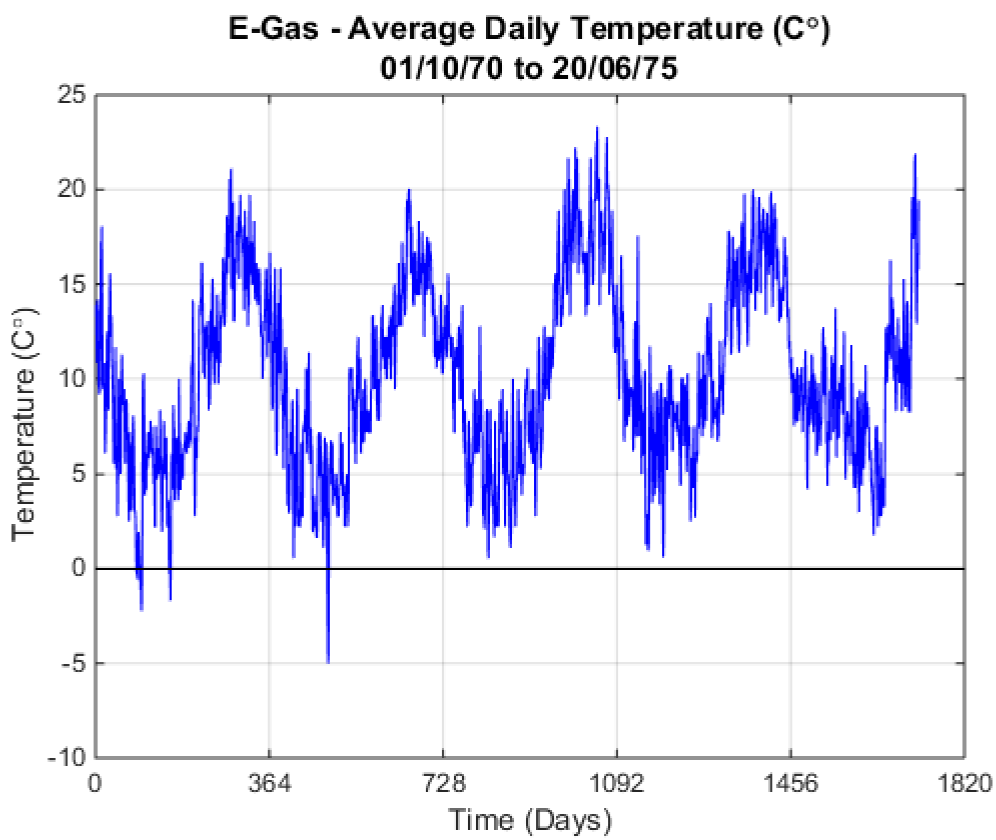

Figure 4.5: E. Gas - Average Daily Effective Temperature - 1970-1975

4.4 X-Gas Daily Gas Data 2001-2011

The data for Chapter 5 and 6 for the years 2001 to 2011, was supplied by DNV GL under NDA. They supplied demand data and temperature data for 5 UK regions of the UK. The demand data was hourly demand from January 1st 2001 (7am) to July 4th 2011 (10am). It is measured in Millions of Cubic Meters (MCM). The demand data set comprises mainly domestic, small commercial and light industrial customers. The temperature data was two-hourly up to November 11th 2007 (7am), and then hourly after that. They also provided wind-speed data for the same periods, which was four-hourly, however, for the thesis, this additional input variable has not been used.

After discussion with DNV GL, the region data was selected, which is called X-Gas in this thesis, and the time frame selected was January 1st 2001 (7am) to July 4th 2011 (6am). The next step was to generate hourly data for temperature between January 1st 2001 and November 11th 2007. This was done by linear interpolation (e.g. temperature at hour 2 was the average of the temperature at hour 1 and hour 3). The next step was to analyze the demand and temperature data to find other missing values. Several situations were found, they included an occasional missing value for a specific hour to several missing values for one or several days. DNV GL advised on how to fix each of these missing value issues, using industry accepted methods for interpolation.

The daily data for demand was then calculated as the sum of the demand from 7am day 1 to 6am on day 2 etc. This is called the gas day. For the daily temperature calculated from the corresponding hourly temperatures, several options were discussed with DNV GL.

- The average of the 24 hourly temperatures (7am to 6am)
- Apply a weighting to the daytime temperature, and calculate an average
- Develop an effective temperature which is a percentage of the previous days temperature added to a percentage of today's temperature

For this thesis, a simple straight 24 hour average is used. Figure 4.6 and Figure 4.7 show the daily demand (MCM) and average temperature(C°) for the period selected above.

The daily data was also transformed into weekly data and used in Chapter 5.

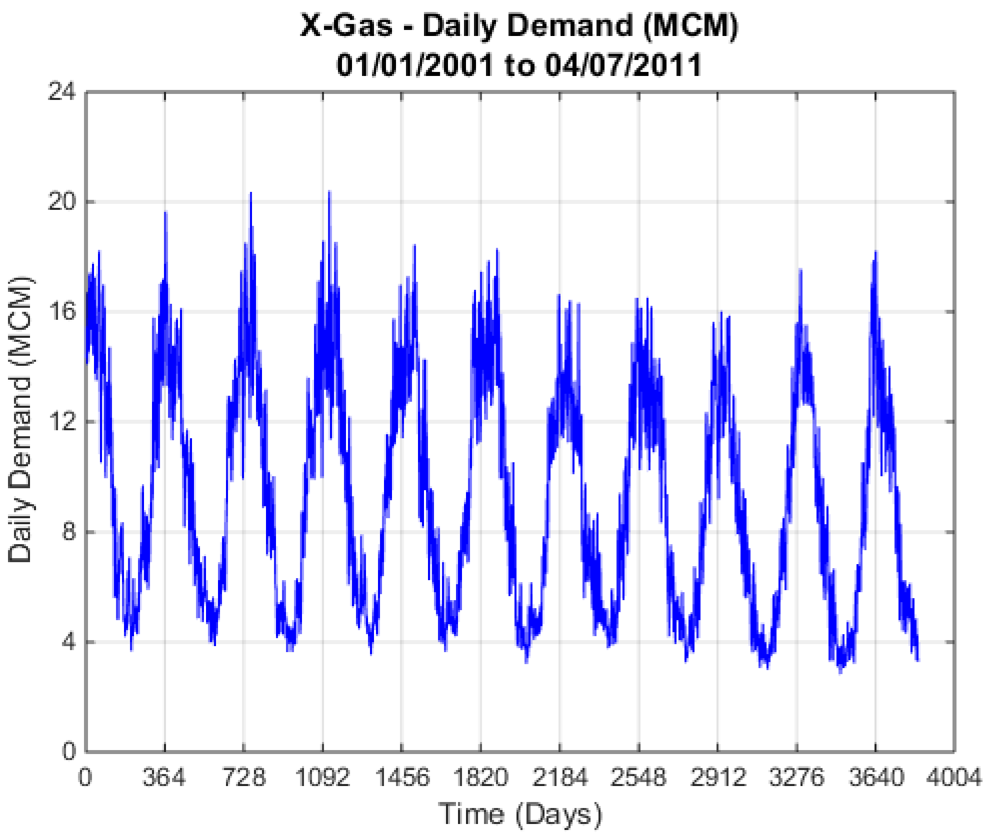

Figure 4.6: X. Gas - Daily Demand - 2001-2011

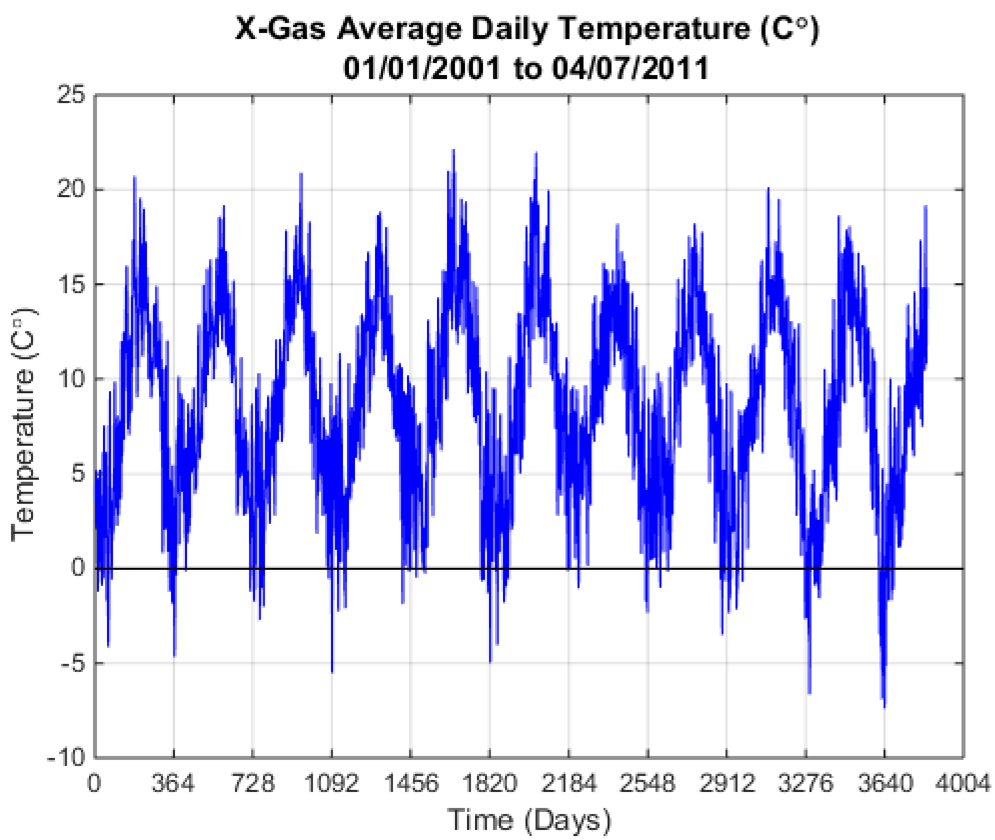

Figure 4.7: X. Gas - Average Daily Temperature - 2001-2011

4.5 Benchmark Data

As mentioned in Chapter 1, the goal is to produce Demand Forecasts of a specified period (e.g. 26 weeks or 14 days) to be within the 4-6% range (MAPE). The figure was specified by DNV GL as a level which would merit usage in an operational environment.

Another measure is the Persistence Model, which takes tomorrow's prediction to be that of today's actual. For the One-Step Ahead forecasts, the MAPE of the Persistence model will be compared to that of the ARMA(X) and the NAR(X)/NARMA(X) models. A smaller MAPE than the Persistence model as well as being within the range 4-6% is the goal of the thesis. The conclusions in each forecasting section will refer back to these benchmark values.

4.5.1 Persistence Model for Weekly Data

The Persistence Model for the 52 Weeks of 1972/73 is shown in Figure 4.8, and the Persistence Model One-Step Ahead statistics for the 26 Winter weeks and for the whole year (52 weeks) for data from both periods (1972/73 and 2010/11) are shown in Table 4.1.

Throughout this thesis, the forecasting statistics will be shown as in Table 4.1. The definitions for MPE, MAE, MSE and MAPE can be found in Section 3.2.2.7, and the last two columns show the largest Over Predicted value and the largest Under Predicted value. The data is shown as 3 values, the value in actual units (i.e. the difference between the predicted value and the actual value for the period being forecast), the location of the Over or Under Predicted value within the period, and the percentage error.

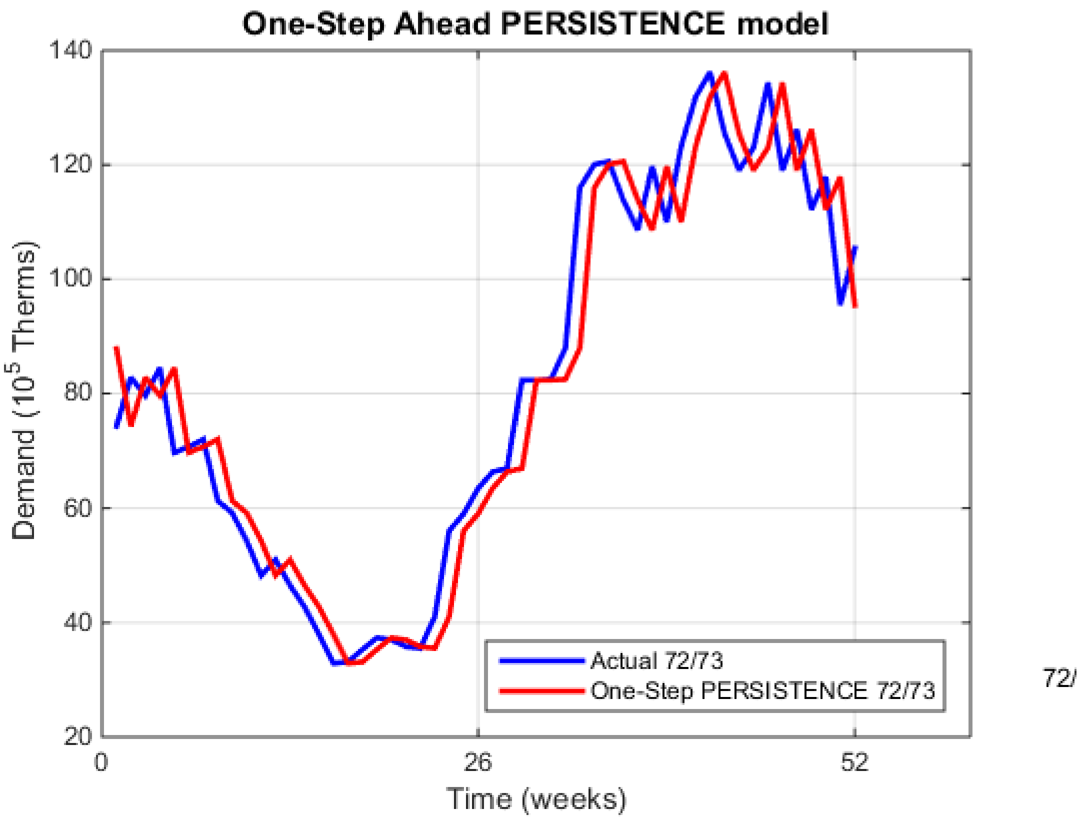

Figure 4.8: Predicted Values for the 1972/73 (Persistence Model)

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
1972/73 Winter-26 weeks	-0.35	6.90	105.96	7.41%	38.12/1/61.75%	-14.94/10/-11.52%
2010/11 Winter-26 weeks	-1.56	8.07	103.36	10.10%	24.03/14/26.28%	-18.13/9/-18.58%
1972/73 Year-52 weeks	-0.34	6.72	79.50	8.25%	22.45/51/23.52%	-28.04/33/-24.17%
2010/11 Year-52 weeks	-0.10	6.16	79.13	11.83%	24.03/40/26.28%	-18.13/35/-18.58%

Table 4.1: One-Step Ahead Statistics for Weekly Demand Forecast (Persistence Model)

4.5.2 Persistence Model for Daily Data

The Persistence Model One-Step Ahead statistics for the 182 Winter days of 1974/75 and 2010/11 are shown in Table 4.2. Figure 4.9 shows the first 56 days of Persistence Model for 2010/11.

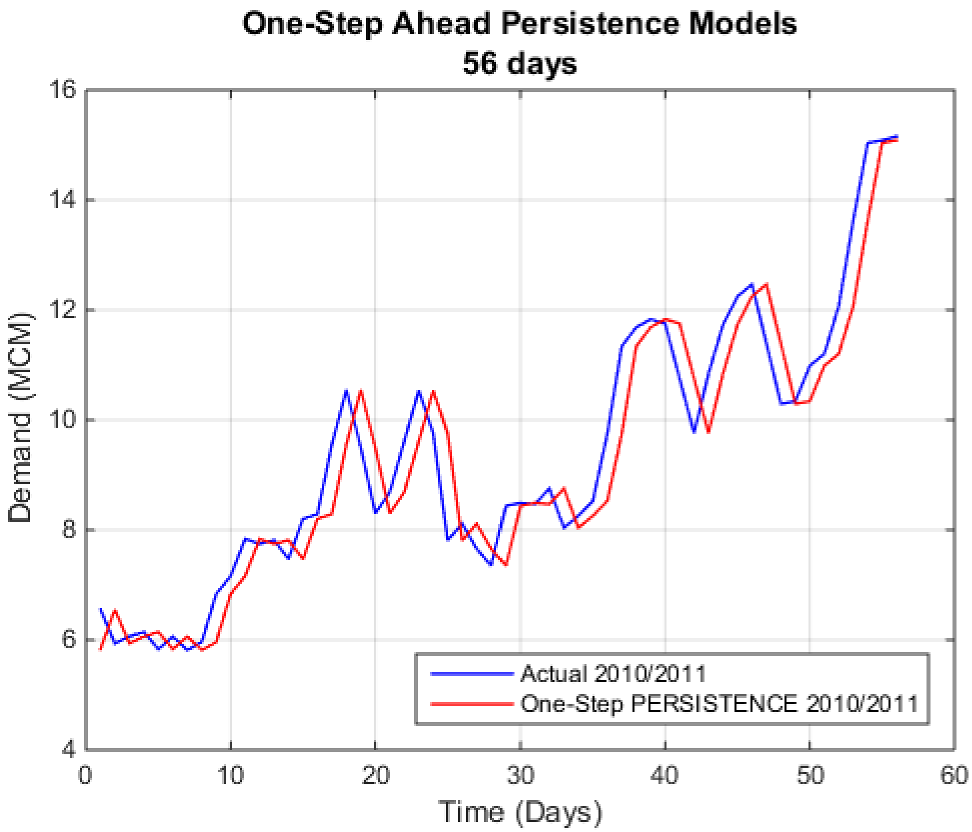

Figure 4.9: Predicted Values for the 2010/11 (Persistence Model)

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
1974/75 Winter-182 Days	-0.01	0.48	0.38	6.37%	1.38/123/20.02%	-1.94/ 92/-23.58%
2010/11 Winter-182 Days	-0.004	0.73	0.82	6.45%	2.29/69/16.43%	-2.05/107/-16.96%

Table 4.2: One-Step Ahead Statistics for Daily Demand Forecast (Persistence Model)

Chapter 5

WEEKLY MODELING AND FORECASTING

5.1 Introduction

This chapter will analyze and produce forecasts for weekly data. ARMAX and NARMAX methodologies (described in Chapter 3) will be applied to the data described in Section 4. A comparison of the forecast results will be shown at the end of this chapter. Since the period of particular interest in the gas industry is from October to March, when the gas demand approaches the system capacity, each method on each set of data will concentrate the effort on producing the most effective forecast (in MAPE terms) for these 26 Winter weeks.

Sections 5.2 to 5.5, below, are focused on presenting the best results produced during the analysis of modeling and forecasting to each data set. However, the details of the flow of the work to produce these results is included in a separate appendix for each of the sections. Finally, Section 5.6 will summarize the conclusions of the ARMAX and NARMAX methodologies applied to each Weekly data set.

This chapter is structured as follows:

Section 5.2 Winter Weekly Data modified to Seasonal Normal Temperature (1963-1973).

Section 5.3 Winter Weekly Data including the Exogenous variable Temperature (1963-1973).

Section 5.4 Yearly Weekly Data including the Exogenous variable Temperature (1963-1973).

Section 5.5 Yearly Weekly Data including the Exogenous variable Temperature (2001-2011).

Section 5.6 Summary of Results and Conclusions.

5.2 *Winter Weekly Modeling and Forecasting with Seasonal Normal Effective Temperature (1963-1973)*

In this section, the modeling and forecasting process will be simplified, by removing the Exogenous variable, temperature, from the equation. The technique to do this, used in the 1970s for weekly demand forecasting by British Gas, and occasionally used today (Geen, 2012), was to modify the demand based on a standardized temperature called Standard Normal Effective Temperature (SNET). This produces a model based on a standardized temperature profile, and the predictions are used for weekly and monthly demand planning. The reason for standardizing on SNET is that for these time frames, the aim is to produce a reasonable forecast for short and medium term planning. Additionally, the weekly and monthly demand forecasts are less temperature variation sensitive, as the temperatures used are already an average for these periods.

5.2.1 ARMA Winter Weekly Modeling and Forecasting with SNET

The ARMA sections, below, are validations of the work done for the original thesis in the 1970s. The work has been revalidated as part of the initial learning process, and several improvements have been made. All the details can be viewed in Appendix C

5.2.1.1 Correcting the data to SNET

Figures 4.1 and 4.2 of Chapter 4 graphed the annual weekly demand data for Southern Gas for the years 1963 though to 1973. Figures 5.1 and 5.2 represent these 2 series with the months April to September removed, thus each consisting of 260 data points i.e. 10 half year cycles (26 weeks), representing the winter period for each of the 10 years under study. Figure 5.3 shows the SNET for Southern Gas for the winter weeks only.

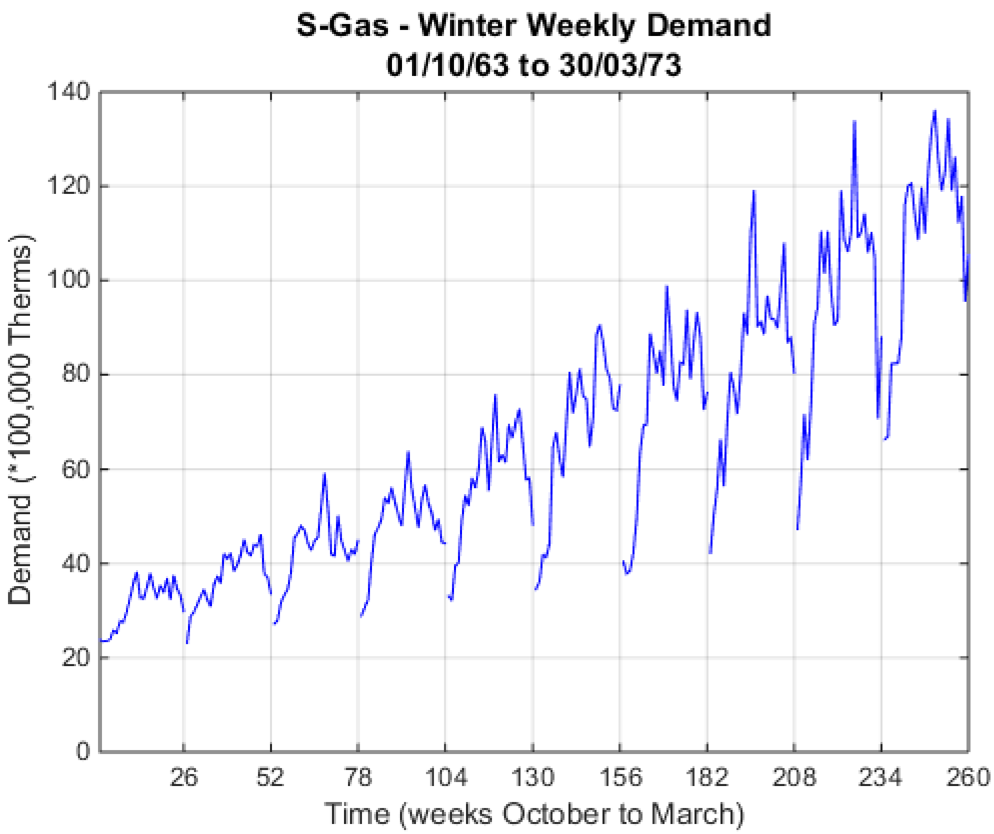

Figure 5.1: S-Gas - Winter Weekly Demand - 1963-1973

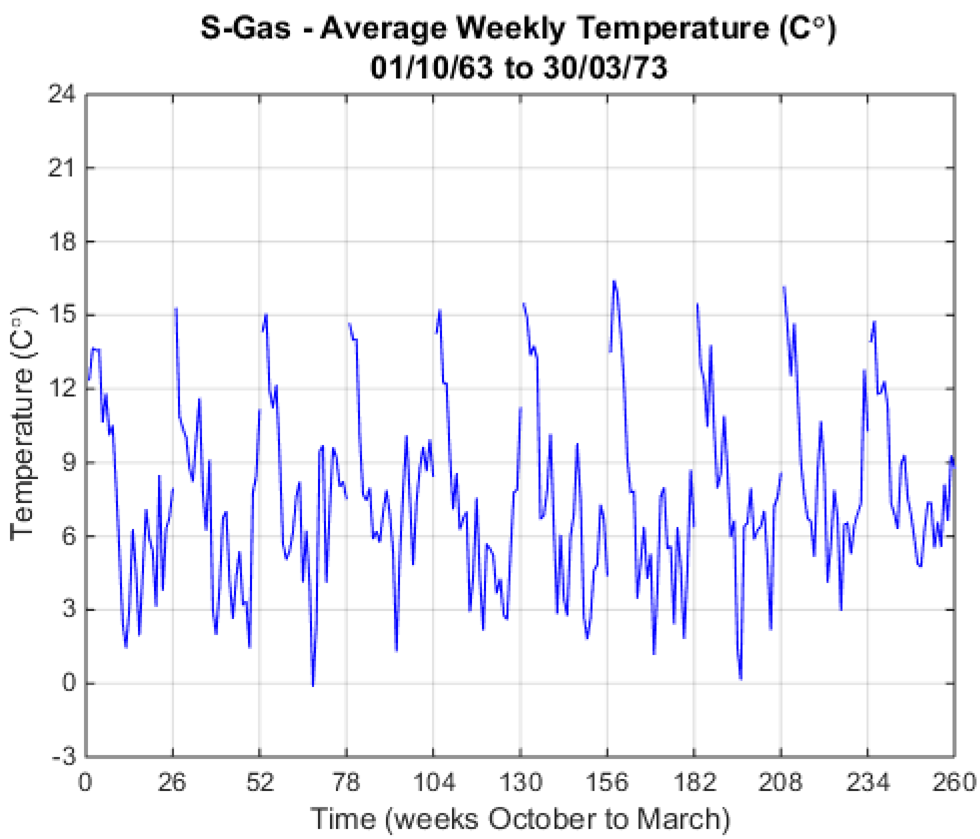

Figure 5.2: S-Gas - Winter Average Weekly Effective Temperature (C°) - 1963-1973

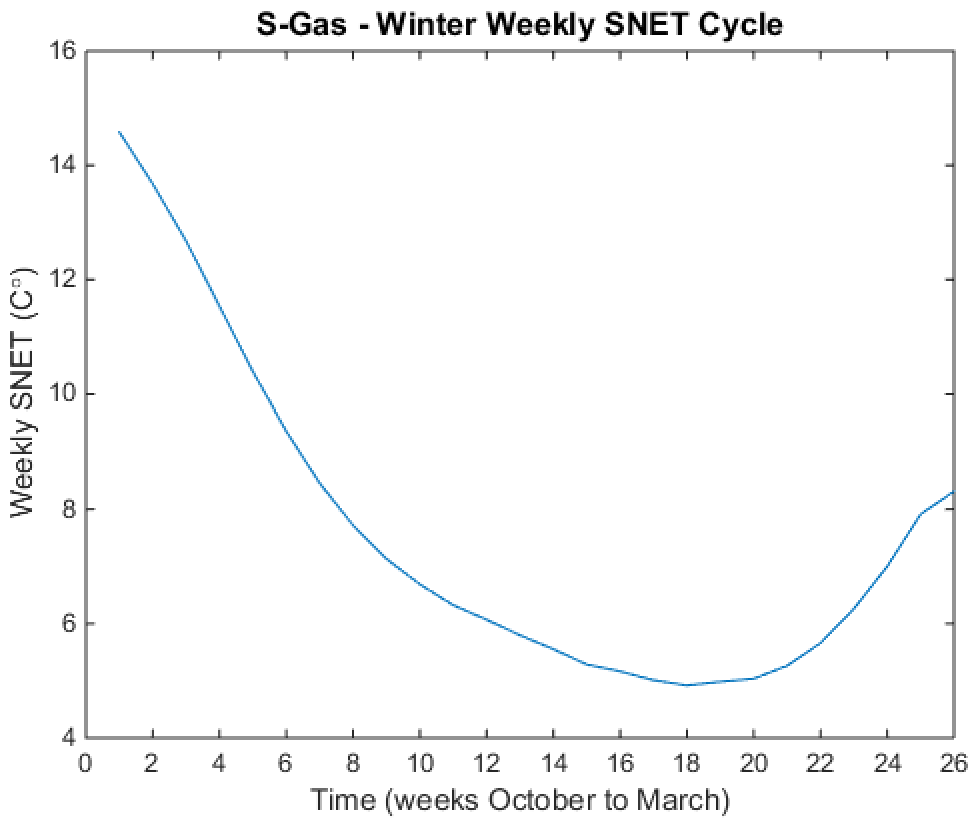

Figure 5.3: S-Gas - Weekly Winter Seasonal Normal Effective Temperature (SNET (C°))

To correct the winter demand in Figure 5.1 to SNET some simple relationship must be found between demand and effective temperature. The formula for correcting the demand-temperature relationship is shown in Equation 5.1. The description of the methodology, the values used in the equation are explained in detail in Appendix B.

$$d_t^i = y_t^i - A_i(T_t^i - SNET_t) \tag{5.1}$$

where

$t = 1, \ldots, 26$ (corresponding to the 1st week in October to the last week in March)

d_t^i = demand corrected to SNET in year i at week t

y_t^i = measured demand in year i at week t

A_i = slope of demand/temperature graph for year i

T_t^i = Average weekly effective temperature for year i at week t

$SNET_t$ = Seasonal Normal Effective Temperature at week t (Figure 5.3)

Finally, each year's data is then combined into a single series d_t, $t = 1, \ldots,$ N, where N = 260, which represents the winter weekly demand corrected to SNET for Southern Gas from 1963 to 1973. This is shown in Figure 5.4.

5.2.1.2 Transforming the data

Figure 5.4 depicting Southern Gas Corrected Winter Weekly Demand clearly shows this time series d_t is non stationary, it has both a growth component and a seasonal component. Chapter 3 (subsection 3.2.2.1), described the transformation process to produce stationarity of a time series, a requirement of the Box and Jenkins modeling process. Several transformations, including logarithmic and powers, were compared to reduce the yearly variance growth. A natural logarithmic transformation was tentatively chosen.

$$z_t = log_e(d_t) \tag{5.2}$$

The Autocorrelation Function (ACF) of the logged data shows that the time series is still not stationary, a seasonal pattern with peaks every 26 weeks exists (showing the

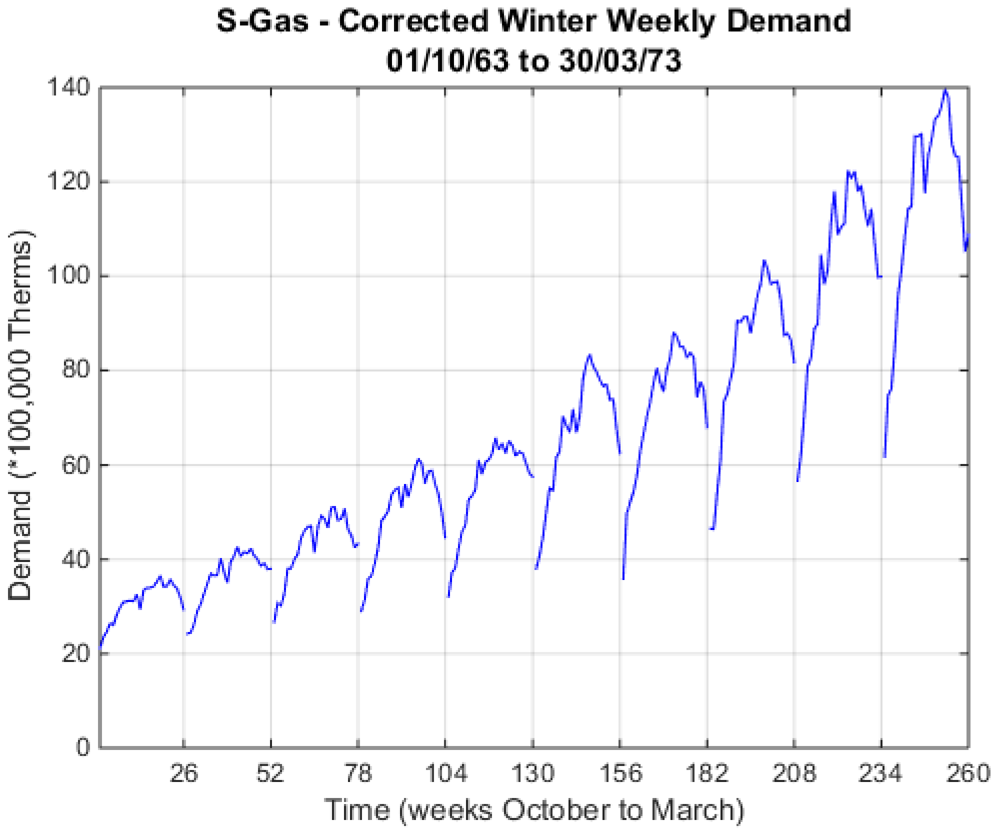

Figure 5.4: S-Gas - Corrected Winter Demand - 1963-1973

relationship winter on winter), and hence difference transformations are also required. Differencing of 1 and 26 are typically appropriate, i.e. $(\nabla\nabla_{26}z_t)$, to produce stationarity.

$$w_t = \nabla\nabla_{26}z_t = (1-B)(1-B^{26})z_t \tag{5.3}$$

This transformed series w_t and is shown in Figure 5.5. Analysis of year on year means and variances confirm that stationarity requirements are met.

The first 9 years of the transformed data (w_t, $t = 1, \ldots, 207$) shown in Figure 5.5, will be used for modeling and the last year will be use to compare with the predicted values of the model selected. The ACF of w_t indicated most lags were not significant (within the +/- limits (2 times the standard error)). However, the lags 1 and 26 are largely outside the 95% confidence limits and are indications of possible Autoregressive and/or Moving Average

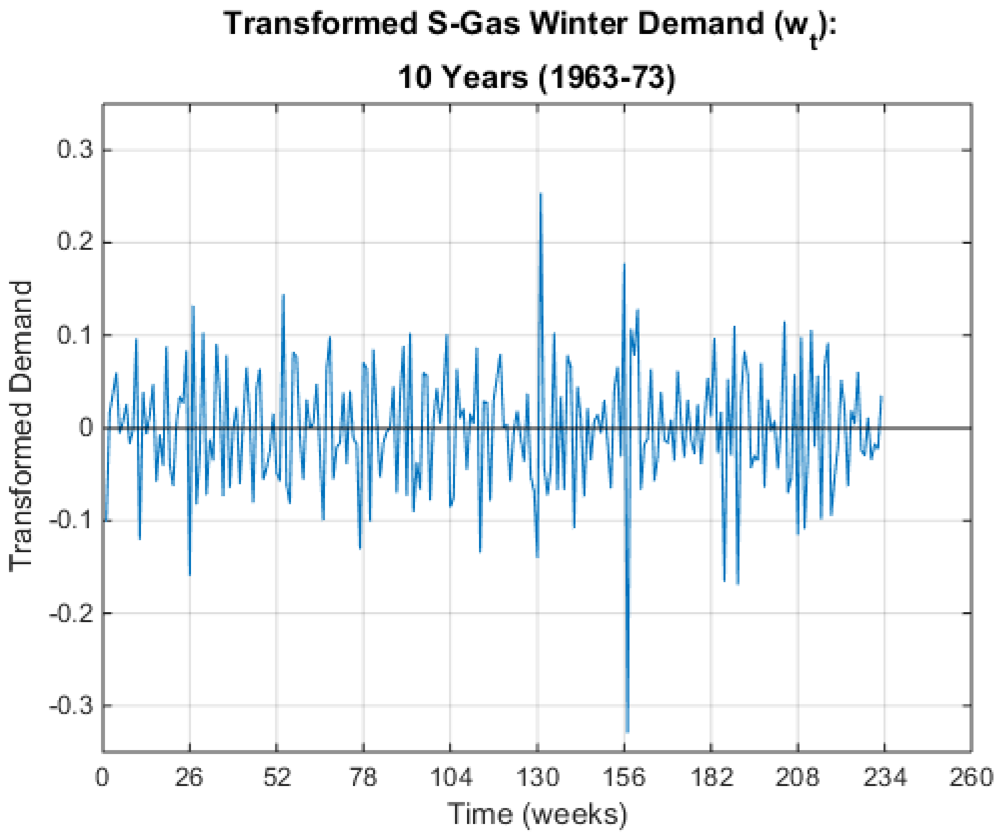

Figure 5.5: Transformed S. Gas Corrected Winter Demand (w_t) - 1963-1973

terms of the ARMA model.

5.2.1.3 ARMA Parameter Identification of the Corrected Winter Weekly Demand

The general form of the Box and Jenkins ARMA model is described in Chapter 3 (Section 3.2.2.2). Since in this case there are no input variables, the general equation becomes :

$$\phi_p(B)\Phi_P(B^s)w_t = c + \theta_q(B)\Theta_Q(B^s)a_t \tag{5.4}$$

If c, p, P, q and Q are correctly determined, then a_t will be a white noise sequence.

The original research selected the following model structure (Equation 5.5). The programs used to estimate the parameters were written in Fortran and run on a UNIVAC 1100 at the British Gas Research Center. The results obtained for ϕ_1, θ_1 and Θ are shown in Equation 5.5. Note that the constant term is zero.

$$(1 - 0.16B)w_t = (1 - 0.72B)(1 - 0.85B^{26})a_t \tag{5.5}$$

For this thesis, MATLAB is used to produce the results of parameter calculations and prediction data for the ARMA models. Although the results in this Section are not exactly the same as the results of the work from the 1970, they are similar, and probably more precise, due to the more powerful computing capabilities of today's environment. When re-evaluating the above data with MATLAB using the Econometrics Toolkit, several other models performed equally well from a modeling perspective as well as for their forecasting capabilities. The list of all these models analyzed and their modeling and forecast statistics can be found in Appendix C. Only the final selected model is described here.

As described in Chapter 3 there are two stages which need to be measured, to select the appropriate model:

1. Measures of how well the model fits the data used for modeling (in this case the first 9 winters) (Section 3.2.2.4)

2. Measures of how good the model is at forecasting (i.e. comparison of the forecast output to the actual output, in this case for the 10th year) (Section 3.2.2.5)

The model which had a balanced set of metrics in both the modeling and forecasting results is shown in Equation 5.6. And using the ARMA notation convention described in Chapter 3.2.2.3, this is an AR(1,2,3,4)/MA(26) model. The AIC value was the smallest at -655, the F value was similar for all the models considered, and 0.483 for this model, and the Q statistic was 37.88 (47 df) which was slightly higher than the other models. The 5% point for a χ^2 with 47 degrees of freedom is 64.00, indicating that there is no reason to believe the model is inadequate. Additionally, the ACF of the residuals from the modeling stage has no significant lags, and hence can be considered white noise.

$$(1 - 0.59B - 0.48B^2 - 0.33B^3 - 0.13B^4)w_t = (1 - 0.59B^{26})a_t \tag{5.6}$$

5.2.1.4 ARMA Forecasting Future One-Step Ahead Demand

The 26 week One-Step ahead forecasts were calculated for the model AR(1,2,3,4)/MA(26) for the winter of 1972 using Equation 5.7. The value of a_t are zero for each of the forecast values (Section 3.2.2.5).

$$\hat{w}_t = 0.59w_{t-1} + 0.48w_{t-2} + 0.33w_{t-3} + 0.13w_{t-4} - 0.59a_{t-26} \tag{5.7}$$

The future values of $\hat{w}_t$ were then inversely transformed (i.e. inverse the differencing (1 and 26) and the logarithm), and are shown in original units in Figure 5.6. The forecast statistics are shown in Table 5.1. The MAPE of 3.71% satisfies the benchmark test (Section

4.5, i.e. below 4% and well below the Persistence Model MAPE of 7.41%.

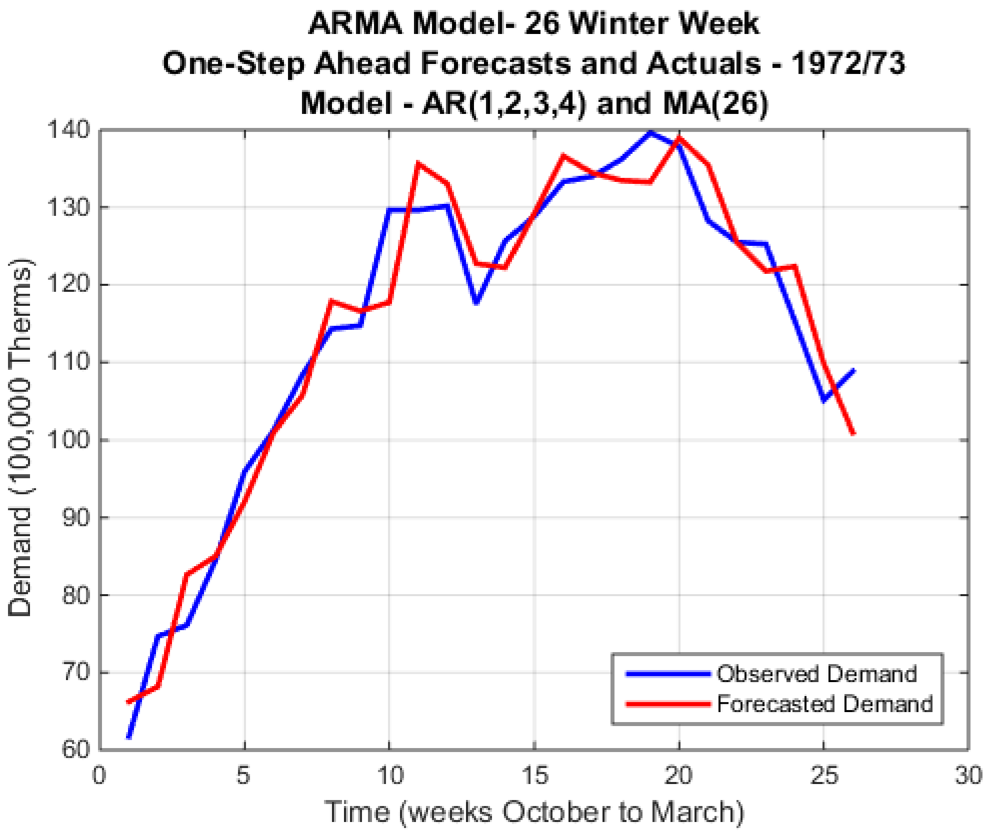

Figure 5.6: 26 week - One-Step Ahead Forecast using AR(1,2,3,4)/MA(26)

Model ARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1,2,3,4)/MA(26)	0.23	4.02	24.15	3.71%	7.22/21/5.63%	-11.94/10/-9.21%

Table 5.1: 26 week - Statistics for One-Step Ahead Weekly Demand Forecast

5.2.1.5 ARMA Forecasting Future Multi-Step Ahead Demand

Long term Multi-Step Ahead forecasts (e.g. for 26 weeks) for this type of data were not actually generated for British Gas. A few weeks ahead would be sufficient to get a rough idea of demand requirements. However, for completeness, the model was run for 26 weeks to see how it would perform. For Multi-Step Ahead forecasts the forecast demand values from the start of the forecast horizon are used. Figure 5.7 shows the result of the 26 week Multi-Step ahead forecast. The values start to drift from the actuals as the forecast horizon advances. The MAPE for this model is 4.53. The statistics are shown in Table 5.2.

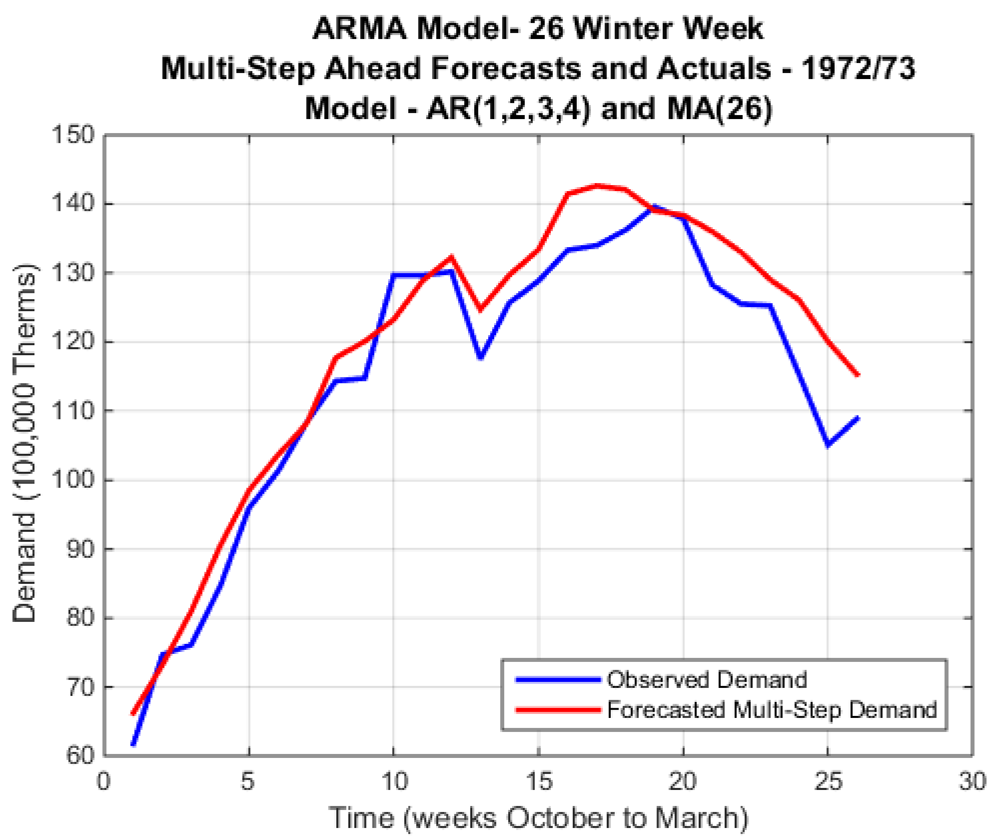

Figure 5.7: 26 week - Multi-Step Ahead Forecast for model AR(1,2,3,4)/MA(26)

Model ARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1,2,3,4)/MA(26)	4.30	5.03	36.78	4.53%	14.93/25/14.2%	-6.46/10/-4.98%

Table 5.2: 26 week - Statistics for Multi-Step Ahead Weekly Demand Forecast

However, performing shorter Multi-Step ahead predictions of 4, 6 and 8 weeks using the AR(1,2,3,4)/MA(26) model produce effective forecasts for the operations managers. The results for the 26 weeks using the different weeks ahead forecasts are shown in Table 5.3. Note that when the weeks ahead number is not a factor of 26, a 2 weeks ahead forecast is performed for the last 2 weeks (i.e. weeks 25 and 26).

Model AR(1,2,3,4)/MA(26)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	0.25	4.80	29.63	4.30%	10.30/24/8.93%	-10.01/10/-7.72%
6 Week Ahead	1.10	3.78	18.40	3.42%	6.43/17/4.80%	-9.29/10/-7.17%
8 Week Ahead	-0.03	3.13	17.48	3.04%	5.90/4/6.98%	-10.01/10/-7.72%
13 Week Ahead	0.47	3.39	18.37	3.13%	8.81/25/7.62%	-8.59/19/-6.15%

Table 5.3: Model Statistics for Various Multi-Step Ahead Weekly Demand Forecasts

The best Multi-Step Ahead period appears to be an 8 week ahead forecast with the 13 week ahead forecast statistics very similar. Figure 5.8 shows the 8 week ahead forecast. Recalibrating against known values every 8 weeks shows the forecasts do not veer away from the actuals. Again the results fall within the limits of the benchmark data.

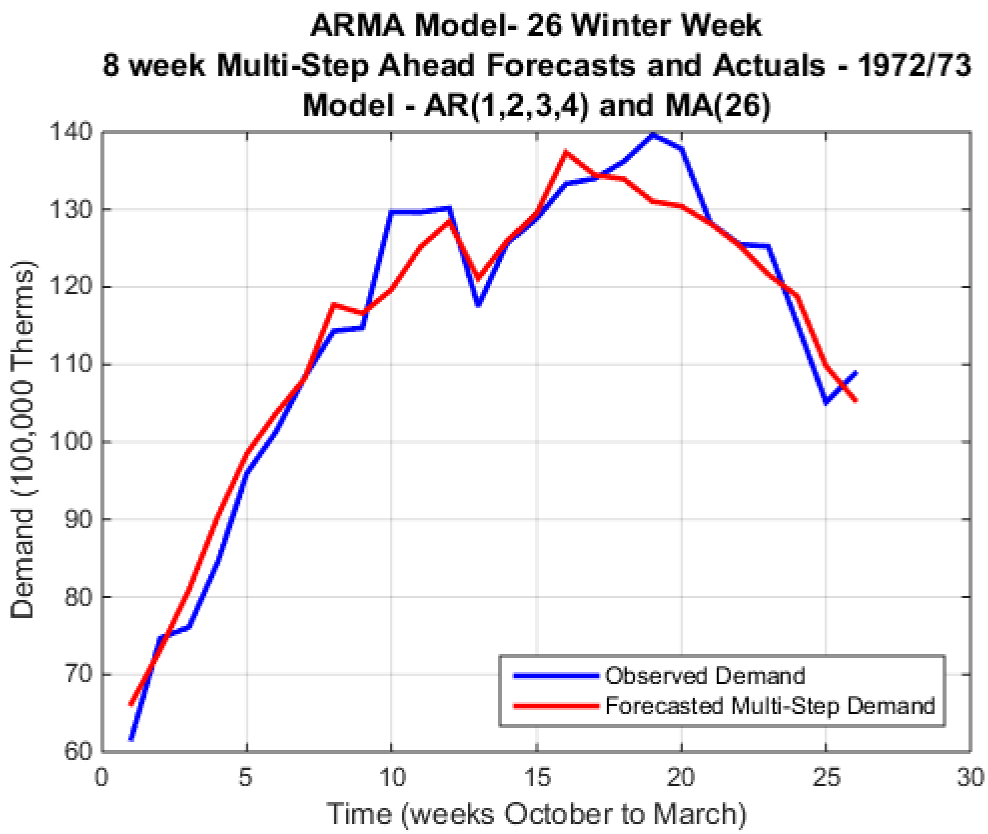

Figure 5.8: 8 week - Multi-Step Ahead Forecast periods for model AR(1,2,3,4)/MA(26)

5.2.2 Summary of ARMA Results

In conclusion, the above work has shown that an Autoregressive Moving Average model can adequately represent and predict weekly demand data for Southern Gas and meets the level of acceptance with a One-Step ahead MAPE of <4%, and all Multi-Step ahead MAPE's of <5%. These results will now be compared to the NARMAX methodology using the same data.

5.2.3 NARMA Winter Weekly Modeling and Forecasting with SNET

As shown in the previous Section, the ARMA methodology is relatively simple to apply. The difficulty comes in interpreting the results at the different stages (Transformation, Modeling and Forecasting) and selecting the most appropriate terms in the models. This section is going to apply the NARMAX methodology with polynomials and the FROLS algorithm helping to select and rank significant terms automatically. The methodology is described in details in Section 3.3.

The Southern Gas Winter Weekly Demand shown in Figure 5.1 is corrected to SNET and shown in Figure 5.4. Since there are no input variables (i.e. no temperature variable in the model), the NARMAX model, as described in Equation 3.25, becomes a NARMA model as follows:

$$y(k) = F[y(k), e(k)] + e(k) \tag{5.8}$$

or

$$\begin{aligned} y(k) =& F[y(k-1), y(k-2), \ldots, y(k-n_y), \\ & e(k-1), e(k-2), \ldots, e(k-n_e)] + e(k) \end{aligned} \tag{5.9}$$

where $y(k)$ and $e(k)$ are the system output and noise sequences respectively; n_y and n_e are the maximum lags for the system output and noise.

5.2.3.1 Transforming the data

As with ARMA modeling, the data should be stationary prior to using the NARMA modeling methodology. The same transformations were performed as for the ARMA modeling of winter weekly demand (section 5.2.1.2), i.e.

1. The actual demand was corrected to SNET, to generate the Corrected Weekly Demand.

2. The Corrected Weekly Demand was then transformed with a logarithmic transformation

3. The logarithm of the Corrected Weekly Demand was then differenced by factors of 1 and 26

Figure 5.5 shows the transformed Southern Gas Corrected Weekly Demand (w_t), which will be treated by the Polynomial NARMA algorithm.

5.2.3.2 NARMA Parameter Identification of the Corrected Winter Weekly Demand

Following the testing of a Linear AR Model, a 2nd Order NAR Model and a 2nd Order NARMA model, it was found that the 2nd Order NARMA model produced the lowest MAPE. A brief description of developing the NARMA Model are shown here, and the details of all the steps to get to this model are shown in Appendix D.

Using the results from the AR and NAR model testing and the knowledge from the ARMA modeling on the same data, The past demand variables $y(k-1)$, $y(k-2)$, $y(k-3)$, $y(k-26)$, $y(k-27)$, $y(k-28)$ and $y(k-29)$ and their associated terms produced the best ERR profile for the 2nd Order NAR model. The ERR total was below 60% (56% to be precise), implying there is information missing from the model. So although the ACF of the residuals, generated from the NAR model, showed no significant lags, MA terms were added to evaluate if the addition of residual terms would improve the modeling statistics and eventually the forecasts. Note : A 2nd Order NAR model with these 7 variables of y , generates 35 terms.

Starting from the residual variables $e(k-1)$ to $e(k-30)$, the FROLS algorithm showed the MA terms $e(k-1)$, $e(k-2)$ and $e(k-26)$ adding value to the system output. Two runs of the algorithm were required to produce no significant lags of the ACF of the generated residuals. The ERR value improved to around 70% (Figure 5.9). The 23 terms selected (from a total of 41 possible terms) are shown in Table 5.4. However, this model still implies there is additional information required to model this data completely. The three Non-Linear tests, described in Section 3.3.2.4, could not be applied as there are no input variables.

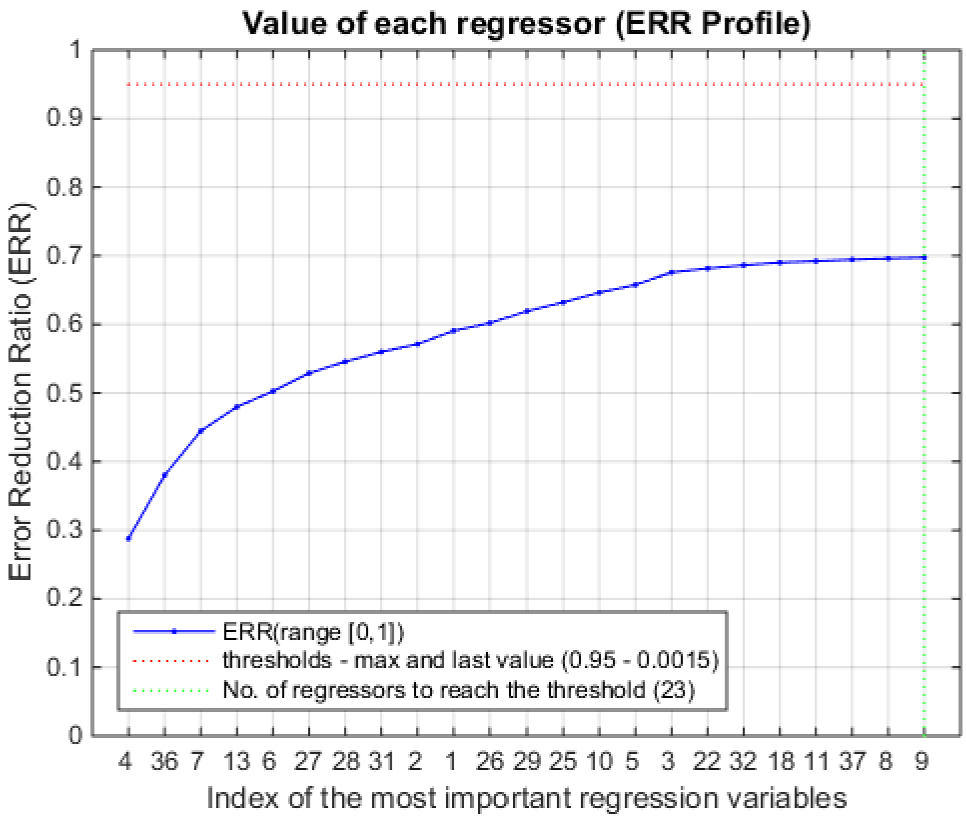

Figure 5.9: ERR Profile for the 2nd Order NARMA Model

The modeling statistics for the 23 term model are:

1. F Statistic with 178 data values = 0.34
2. Q Statistic = 36.98 with 29 degrees of freedom (52-23), which again shows an adequate model (29 df χ^2 value is 42.56 at 5% level)

Index	Model term	Parameter Value	ERR(%)
1	y_{k-26}	-0.58	28.77
2	e_{k-1}	0.03	9.24
3	y_{k-29}	-0.04	6.45
4	$y_{k-1} * y_{k-28}$	-1.23	3.51
5	y_{k-28}	-0.27	2.64
6	$y_{k-26} * y_{k-27}$	4.32	1.67
7	$y_{k-26} * y_{k-28}$	5.93	1.44
•	•	•	•
23	$y_{k-1} * y_{k-2}$	0.85	0.15

Table 5.4: Results of the FROLS algorithm for the 2nd Order NARMA Model

5.2.3.3 NARMA Forecasting Future One-Step Ahead Demand

This model was then run to forecast the winter of 1972/73 using all 23 terms and these forecasts are shown in Figure 5.10.

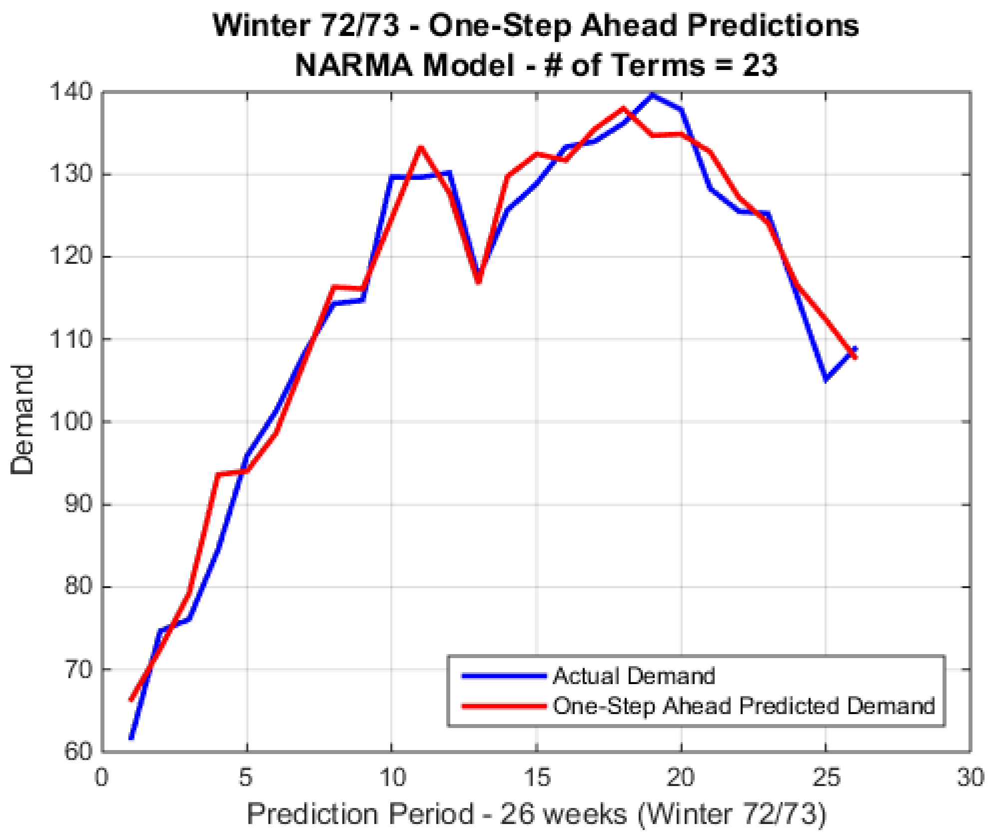

Figure 5.10: 26 week - One-Step Ahead Forecast for the 2nd Order NARMA Model

The forecast statistics are shown in Table 5.5. The MAPE for the NARMA model was 2.81%. which was an improvement over both the ARMA model and the NAR model using the same data.

Model NARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	0.85	2.97	12.76	2.81%	9.08/4/10.74%	-5.03/10/-3.88%

Table 5.5: 26 week - Statistics for One-Step Ahead Weekly Demand Forecast

5.2.3.4 NARMA Forecasting Future Multi-Step Ahead Demand

The NARMA model above was then used to produce Multi-Step Ahead forecasts. The model statistics were better than the AR and NAR models and are shown in Table 5.6. The 26 week forecast is shown in Figure 5.11, and degrades over time, veering away from the actual demand. The ACF of the residuals had significant lags, and rerunning the process several cycle did not completely remove the lags, or improve the modeling or forecast statistics. Hence the results below show the improvements after just the one cycle of including the residuals in the model. However, there is information in the residuals which is not being modeled in the predicted output.

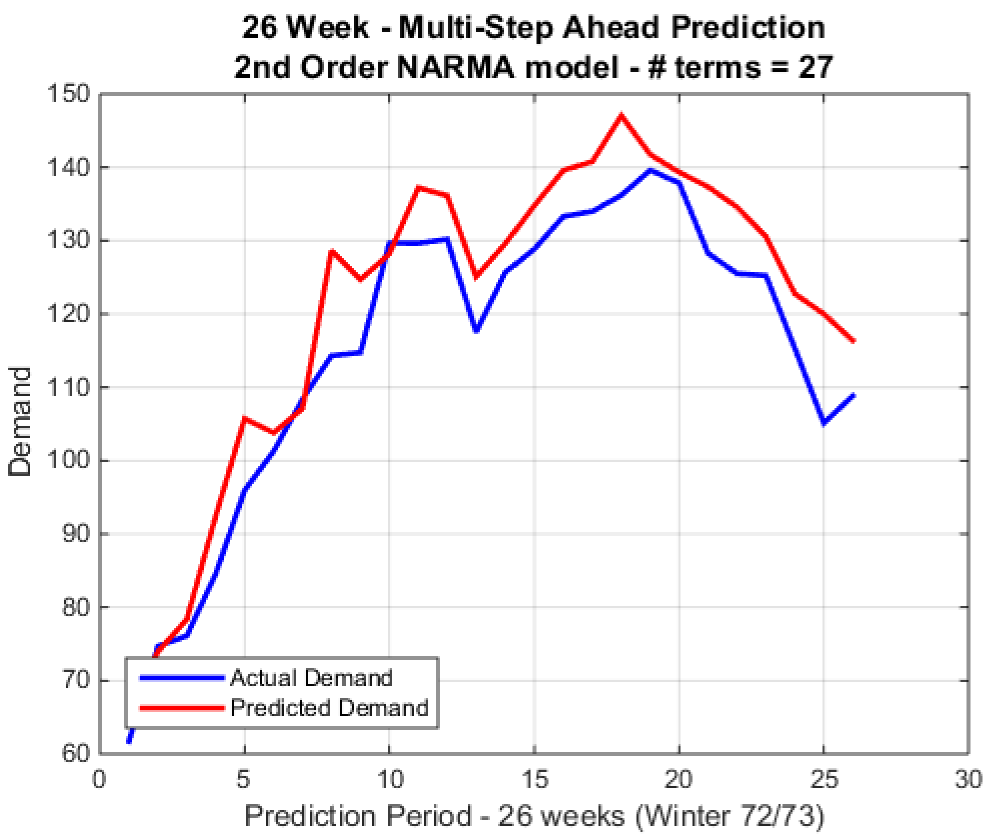

Figure 5.11: 26 week - Multi-Step Ahead Forecast for the 2nd Order NARMA Model

Model NARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	6.15	6.42	55.34	5.71%	14.88/25/14.15%	-1.48/10/-1.14%

Table 5.6: 26 week - Statistics for Multi-Step Ahead Weekly Demand Forecast (2nd Order NARMA Model)

However, performing shorter Multi-Step Ahead forecasts of 4, 6 and 8 weeks using the 2nd Order NARMA model produce improved forecasts. The results for the 26 weeks using the different weeks ahead forecasts are shown in Table 5.7.
Note when the weeks ahead number is not a factor of 26, a 2 weeks ahead forecast is performed for the last 2 weeks (i.e. weeks 25 and 26).

Model NARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	-0.21	4.79	36.75	4.27%	7.88/4/9.32%	-15.77/10/-12.16%
6 Week Ahead	1.22	3.99	26.30	3.68%	11.22/8/9.81%	-8.82/20/-6.40%
8 Week Ahead	-0.82	5.61	48.79	4.93%	14.34/8/12.55%	-15.77/10/-12.16%
13 Week Ahead	2.11	4.38	32.32	4.01%	14.34/8/12.55%	-6.97/20/-5.06%

Table 5.7: Statistics for Various Multi-Step Ahead Weekly Demand Forecasts (2nd Order NARMA Model)

The best Multi-Step Ahead period appears to be an 6 week ahead forecast, but there is very little difference between the 13 week and the 6 week multi-step forecasts, although the over and under estimates of the 6 weeks ahead forecasts are better than the 13 week. The Figure 5.12 graphs this result for the 6 week ahead forecast.

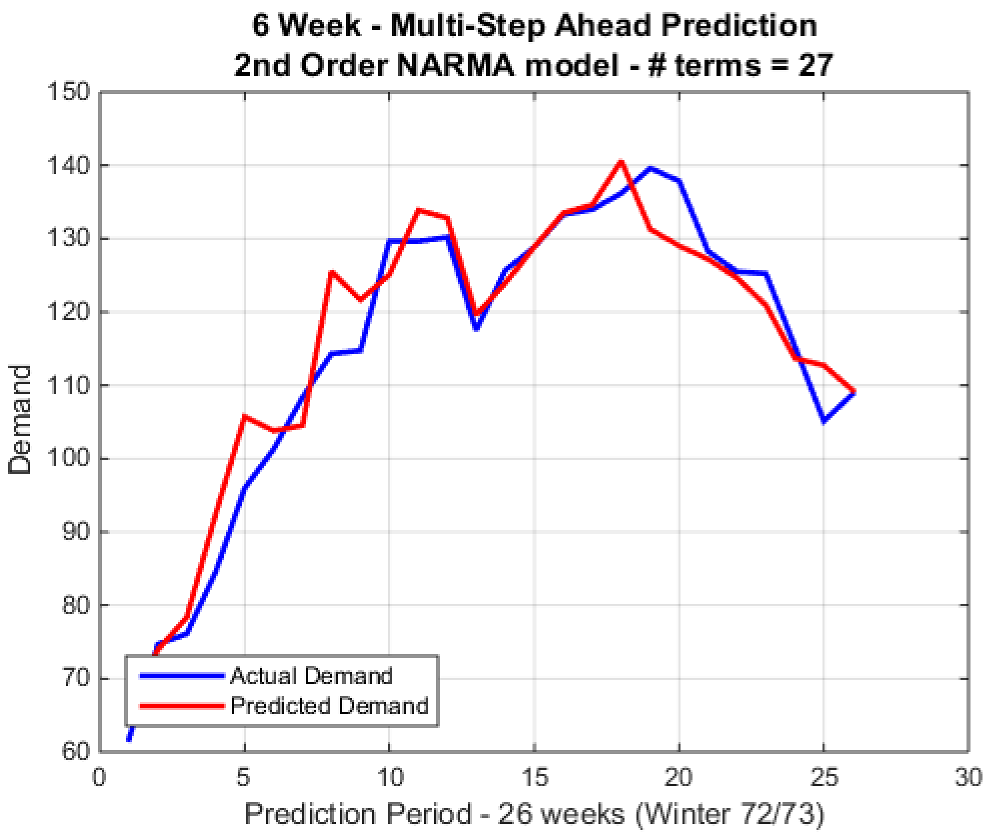

Figure 5.12: 6 Week Ahead Forecast periods for the 2nd Order NARMA Model

5.2.4 Summary of NARMA Results

The 2nd Order NARMA model produced acceptable One-Step Ahead results. However, on the Multi-Step Ahead forecast for 26 weeks, the predicted values veer away from the actuals, and the ACF of the residuals had significant lags. This indicates that there is still information in the residuals that the modeling process cannot accommodate. Also the NAR model had the same failings. Hence for 26 week Multi-Step Ahead forecasts are somewhat dubious. The shorter weeks ahead forecasts appear to be somewhat better.

Comparing the results from Sections 5.2.1 (ARMA) and 5.2.3 (NARMA), the One-Step Ahead forecasts from the 2nd Order NARMA model produced a superior MAPE result (2.81%) over the ARMA model (3.71%). In the case of the Multi-Step Ahead forecast, for 26 week ahead, the ARMA model produced a better forecast with MAPE of 4.53% (over 5.71% for the NARMA model). For shorter Multi-Step Ahead horizons, the ARMA model produced better results all round, with 8 week ahead forecasts producing the best MAPE of 3.04%. The 6 week ahead forecast for the NARMA Model produced an MAPE of 3.68%. The ARMA and NARMA models all produced 4-8 week ahead forecasts with MAPE in the region of 4-5% or better.

Both models (ARMA and NARMA) produce a superior One-Step Ahead MAPE forecast value over the Persistence Model (Section 4.5) and both the One-Step and Multi-Step Ahead forecasts, and were within the 4-6% value set by DNV GL.

5.3 Winter Weekly Modeling and Forecasting with Actual Temperature (1963-1973)

5.3.1 ARMAX Winter Weekly Modeling and Forecasting with Actual Temperature

5.3.1.1 Introduction

The methodology used in the Section 5.2 provided a simple methodology for modeling the winter weekly gas demand using ARMA and NARMA models. In this Section, the actual effective temperature will be included into the equations, thus allowing the full power of both ARMAX and NARMAX to be evaluated. ARMAX is already used extensively in the Gas Forecasting domain, as covered in Chapter 2, whereas NARMAX activity in this domain is limited.

The data in this section is identical to that used in Section 5.2 represented by the Winter Weekly Demand (Figures 5.1) and the Winter Effective Temperature (Figure 5.2). The period under study is again the Winter Weekly Data from October 1963 to the end of March of 1973. The ARMAX methodology which will be used to model this data is described in Chapter 3 with Temperature as the Exogenous variable.

5.3.1.2 Transforming the data

As in the simple ARMA model, the data in an ARMAX model should be stationary, and hence transformation analysis is required for both the Demand and the Temperature. Analysis of the yearly means and variances, suggest a logarithmic transformation for the Demand (as in Section 5.2), and no transformation seems appropriate for Temperature.

Analysis of the Autocorrelation Function of both this initial logarithmic transformation of Demand and the corresponding Temperature show that differencing of 1 and 26 will produce stationary time series for both Demand and Temperature.. The differenced series for Demand and Temperature are shown in Figures 5.13 and 5.14. The data, in blue, represents the transformed data which will be used for modeling and the red data is the comparison data for the forecast. The ACF of the Transformed Winter Demand has significant lags at 1 and 26, and the ACF of the Transformed Temperature has significant lags at 1, 2 and 26.

The Cross correlation Function (CCF) between the Transformed Demand and Temperature showed a significant lag at zero, significant lags at 26 and -26 as well as smaller significant lags at 1, 2, -1 and -2. This indicates that the demand at time t is highly related to the temperature at time t, as well as a 26 lag relationship between demand and temperature.

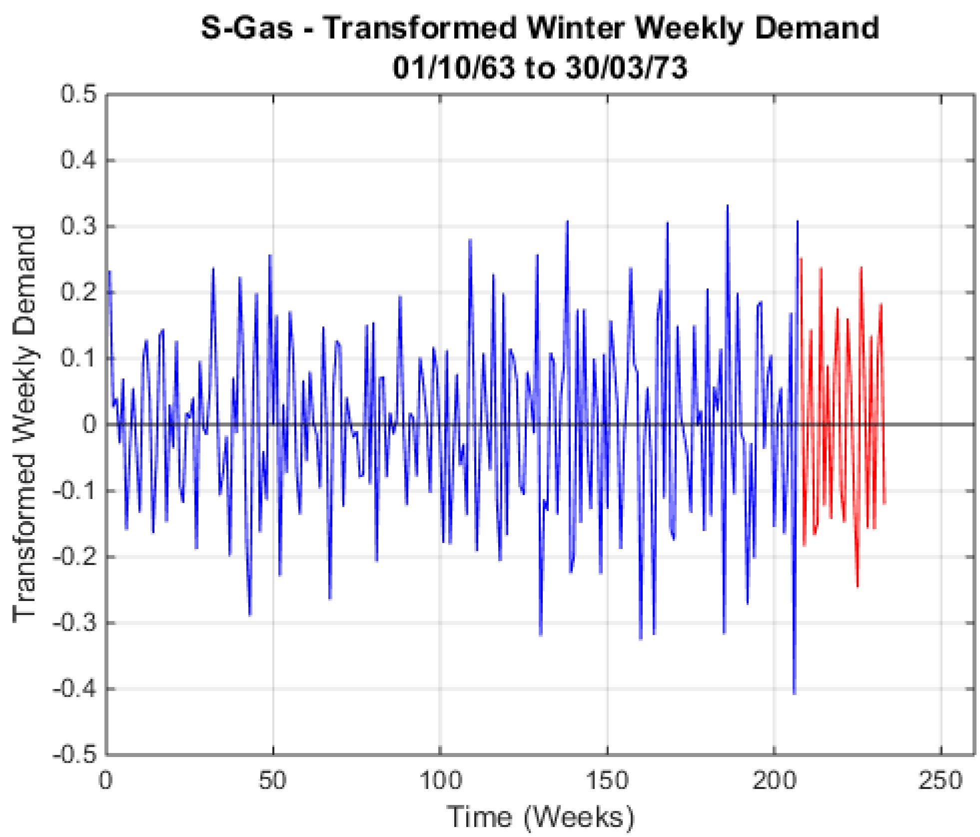

Figure 5.13: Difference of Log of Winter Weekly Demand (w_t)

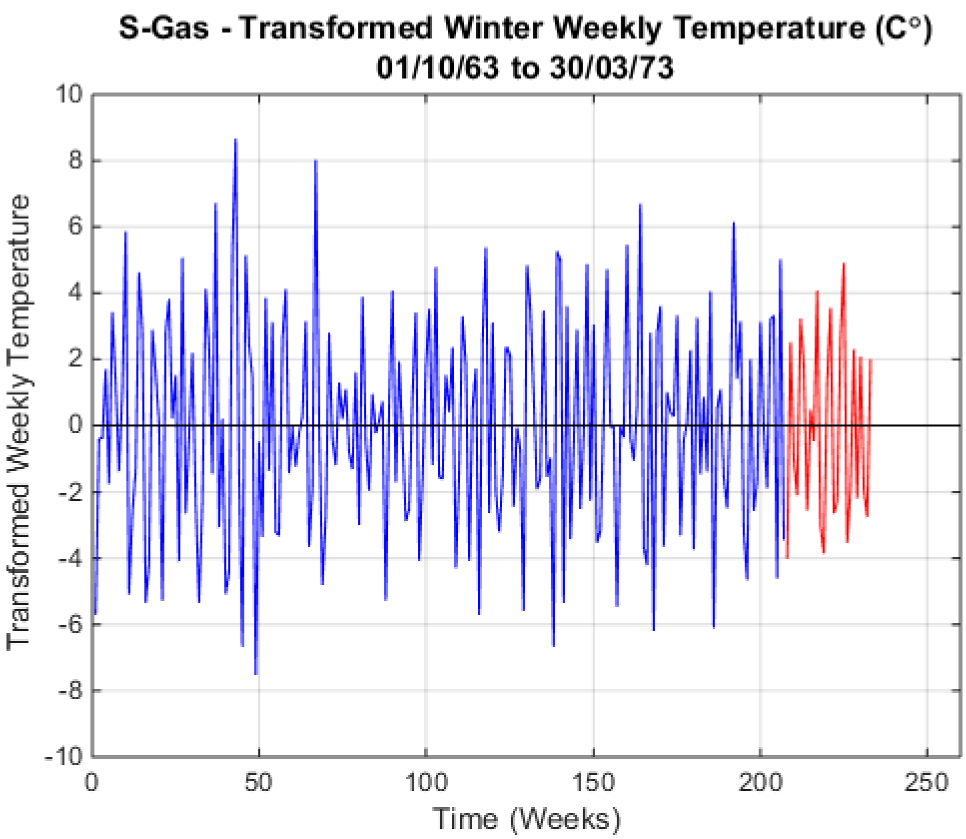

Figure 5.14: Differenced Winter Temperature (x_t)

5.3.1.3 ARMAX Parameter Identification of the Winter Weekly Demand with Temperature

Using the results of the ACF of both the Transformed Winter Demand and Temperature and the CCF results, several models were generated using the MATLAB Economics Toolkit functions for ARMAX. Several delay values were applied to temperature series, but a delay of zero always produced the best results. This implies that future demand at time t_1 is only dependent on the corresponding temperature at t_1; as well as past demand values. The Equation 3.6 from Chapter 3 will be used for the modeling.

Two models generated similar AIC, F and Q statistics and are shown below in Equations 5.10 and 5.11. The constant is zero in both cases. Table 5.8 contains the modeling statistics for each of these two models. Neither of these two models have any lags from the ACF of the residuals outside the 95% confidence limits. As it was impossible to differentiate between the two models at this stage, both models were carried forward to produce both One-Step and Multi-Step Ahead forecast, before choosing the best model. As in Section 5.2, the details of all the models considered and their modeling and forecast statistics are described in Appendix E, and only the best results are detailed in this section.

$$(1+0.40B+0.42B^2)w_t = -0.04x_t + (1-0.97B-0.31B^2+0.28B^3)(1-0.44B^{26})a_t \quad (5.10)$$

$$(1+0.27B)w_t = -0.04x_t + (1-0.77B)(1-0.43B^{26})a_t \quad (5.11)$$

Model ARMAX	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1,2)/MA(1,2,3,26,27,28,29)	-711	0.35	None	29.19	43
AR(1)/MA(1,26,27)	-708	0.38	None	29.35	48

Table 5.8: Model Fit Comparisons for Weekly Demand with Temperature

5.3.1.4 ARMAX Forecasting Future One-Step Ahead Demand

One step ahead forecasts were again calculated for each of the two models in Table 5.8 for the 26 weeks of the winter of 1972/73. Table 5.9 shows the balanced set of metrics for each of the models.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1,2)/MA(1,2,3,26:29)	0.03	4.42	28.71	4.16%	9.97/21/8.38%	-11.88/7/-10.24%
AR(1)/MA(1,26,27)	0.07	4.10	24.82	3.86%	7.87/21/6.13%	-11.13/7/-10.12%

Table 5.9: Model Statistics Comparisons for Weekly Demand with Temperature Forecasts

The two models have very similar forecast statistics, however, model AR(1)/MA(1,26,27) was finally chosen due to the smaller Overestimate and Underestimate predicted demand values. The AR(1)/MA(1,26,27) model is written as:

$$\hat{w}_t = -0.27w_{t-1} - 0.04\hat{x}_t + a_t - 0.77a_{t-1} - 0.43a_{t-26} + 0.43a_{t-27} \tag{5.12}$$

Where $\hat{w}_t$ is the forecast value of demand at time t and $\hat{x}_t$ is the forecast temperature at time t. As the forecast temperature at time t is not available, the actual temperature is used to forecast future demand $\hat{w}_t$. The value of a_t is zero for each of the forecast values (Section 3.2.2.5). The One-Step ahead forecasts are shown in Figure 5.15.

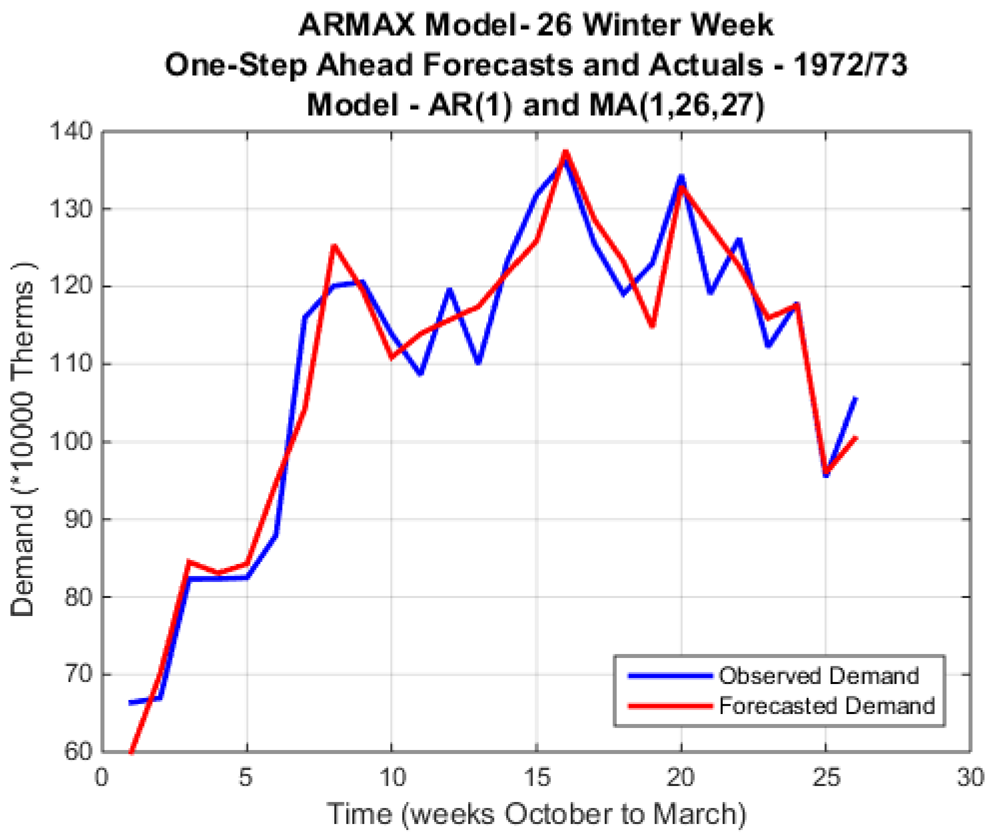

Figure 5.15: One-Step Ahead Forecast for model AR(1)/MA(1,26,27)

5.3.1.5 ARMAX Forecasting Future Multi-Step Ahead Demand

Figure 5.16 shows the Multi-Step Ahead forecast for the AR(1)/MA(1,26,27) model for the full 26 weeks. The MAPE is 3.38% which is an improvement over the One-Step Ahead forecast described above, and and even bigger improvement than the ARMA Multi-Step Ahead forecast from Section 5.2.1.5.

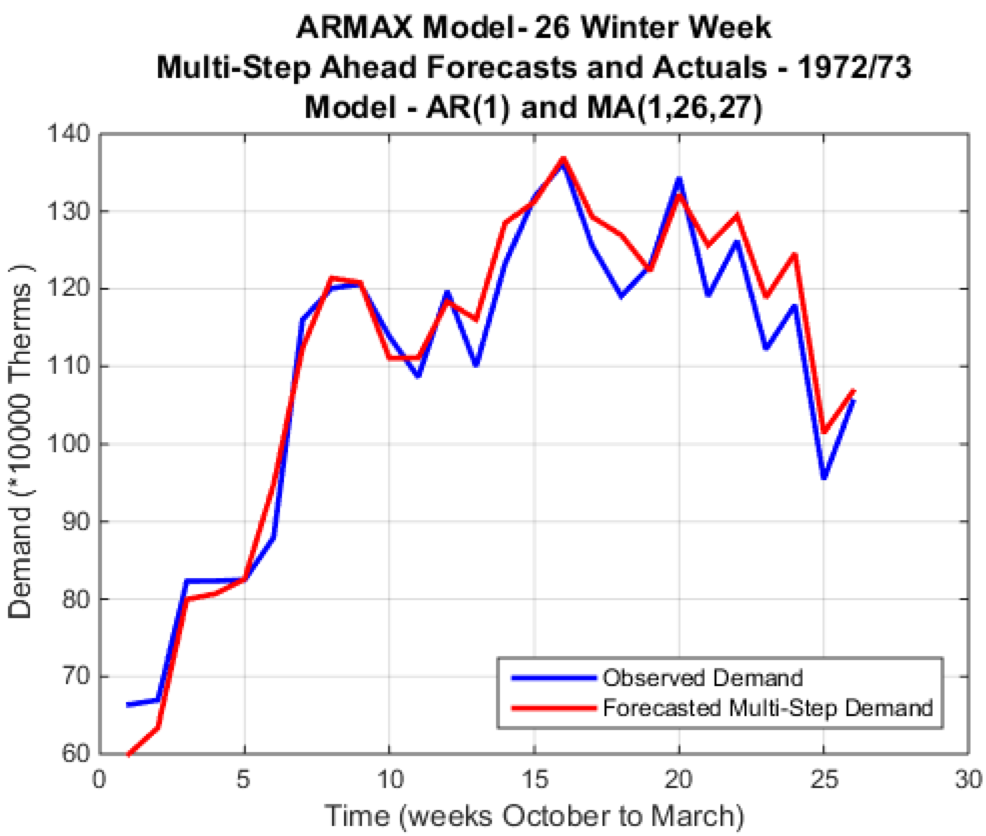

Figure 5.16: Multi-Step Ahead Forecast for model AR(1)/MA(1,26,27)

The statistics associated with this Multi-Step Ahead forecast are shown in Table 5.10.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1)/MA(27)	1.53	3.48	17.97	3.38%	7.88/18/6.62%	-6.36/1/-9.59%

Table 5.10: Model Statistics for 26 Week Multi-Step Weekly Demand Forecast

Again, performing shorter Multi-Step Ahead forecasts of 4, 6, 8 and 13 weeks using the AR(1)/MA(1,26,27) model produces effective forecasts for the operations managers. The results for the 26 weeks using the different weeks ahead forecasts are shown in Table 5.11. Note when the weeks ahead number is not a factor of 26, a 2 weeks ahead forecast is performed for the last 2 weeks (i.e. weeks 25 and 26).

Model AR(1)/MA(1,26,27)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	1.68	4.12	25.49	3.92%	8.78/6/9.99%	-6.36/1/-9.59%
6 Week Ahead	-2.28	5.33	41.12	4.86%	9.36/18/7.86%	-11.79/7/-10.17%
8 Week Ahead	0.49	3.33	15.33	3.21%	7.21/18/6.08%	-6.36/1/-9.59%
13 Week Ahead	-1.72	3.22	17.41	3.05%	6.86/6/7.80%%	-9.14/20/-6.80%

Table 5.11: Model Statistics for Various Multi-Step Ahead Weekly Demand Forecasts

The best Multi-Step Ahead period appears to be a 13 week ahead forecast from an MAPE perspective, with the 8 week ahead similar. Both are close to the 26 week Multi-Step Ahead forecast statistics shown in Tables 5.10. However, the 8 week ahead has smaller overestimate and underestimate predicted demand values and a smaller MPE and MSE. Figures 5.17 graphs this result for the multiple 8 week ahead forecasts.

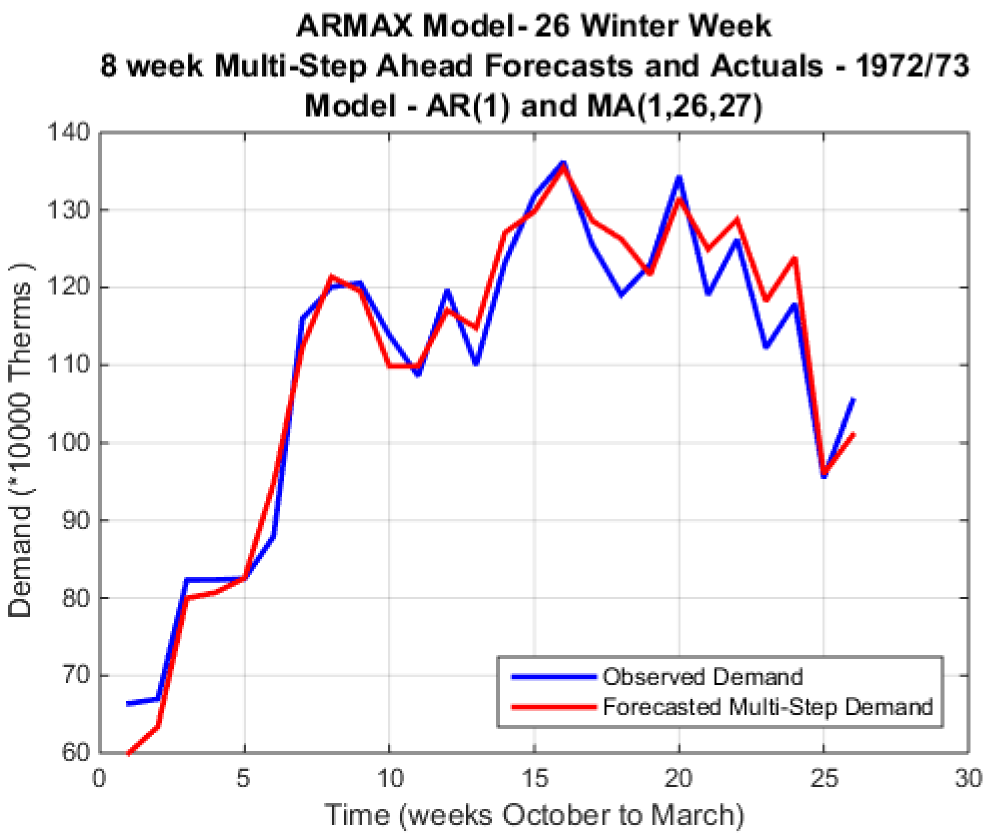

Figure 5.17: 8 week Multi-Step Ahead Forecast periods for model AR(1)/MA(1,26,27)

5.3.2 Summary of ARMAX Results

In conclusion, the above work has shown that an Autoregressive Moving Average model can adequately represent and predict weekly demand data for Southern Gas when temperature is included in the model (thus generating and ARMAX model). This ARMAX model meets the benchmark level of acceptance with a One-Step Ahead MAPE of <4%, and a 26 week

Multi-Step ahead MAPE of <4%. The 13 week, 8 and 4 week multi-step ahead also have an MAPE of <4%, and the MAPE for the 6 week forecast of <5%. All these statistics are satisfy the benchmark criteria (i.e. smaller One-Step Ahead MAPE values than the Persistence Model, and within the 4-6% value set by DNV GL). The next section will apply the NARMAX methodology to the same data.

5.3.3 NARMAX Winter Weekly Modeling and Forecasting with Actual Temperature

As in the previous Section, the Southern Gas Winter Weekly Demand and the Winter Weekly Effective Temperatures described in Chapter 5, and shown in Figures 5.1 and Figure 5.2, are the starting point for the analysis using the NARMAX methodology. The NARMAX model formula for this chapter are described in Section 3.3, and includes both delayed output variables and the input variable temperature.

5.3.3.1 Transforming the data

The same transformations were performed as for the ARMAX modeling of winter weekly demand and temperature (section 5.3.1.2) to produce stationarity, i.e.:

1. The Winter Weekly Demand was transformed with a Natural Logarithmic transformation

2. The Winter Weekly Effective Temperature required no transformation

3. The Log of the Winter Weekly Demand and the Winter Effective Temperature were then differenced by factors of 1 and 26

Figures 5.13 and 5.14 show the transformed Southern Gas Corrected Weekly Demand and Temperature, which will be treated by the NARMAX algorithm. Each series is made up of 233 data points representing the transformed 9 years 1963 to 1972 (blue - 207 weeks) and the forecast comparison year (red - 26 weeks).

5.3.3.2 NARMAX Parameter Identification of the Winter Weekly Demand with Temperature

Using the ACF of both the Transformed Demand and Temperature and the CCF of Demand and Temperature, variables 1 to 30 for both Demand and Temperature were initially included in the NARMAX modeling process.

As in Section 5.2.3, a first step analyzed a Linear model (ARX), followed by inclusion of residuals, thus creating an ARMAX model. Following the linear model analysis, 2nd and 3rd order terms were introduced (both without and with residuals NARX and NARMAX), to find the most appropriate model from a modeling and especially a forecasting perspective. The best results are explained in this section, and the full details of the analysis modeling and forecasting to reach these results are covered in Appendix F.

Running the modeling process without residuals (ARX Model), the terms 1,2 and 26 were selected for both demand and temperature. A 2nd Order NARMAX model was tested, using the same variables as the linear ARX model (i.e. $y(k-1), y(k-2), y(k-26)$ and $x(k), x(k-1), x(k-2), x(k-26)$), giving 35 terms. Incorporating residuals into the model generating a full NARMAX model, with only linear residual terms added, improved the results (after 3 runs) over all the other models (ARX, ARMAX and NARX). The residual term $e(k-26)$ added the most value to the system output, and the ERR total was 91.8%. The terms selected are shown in Table 5.12. The ERR value added by $x(k)$ shows the significance of the temperature at time k on the demand at time k, generating 87% of the ERR total value.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.045	87.09
2	e_{k-26}	-0.348	0.84
3	x_{k-2}	-0.014	0.84
4	y_{k-1}	-0.469	0.41
5	x_{k-1}	-0.020	1.54
6	y_{k-2}	-0.234	0.69
7	y_{k-26}	-0.-040	0.23
8	$x_{k-26} * y_{k-26}$	-0.006	0.79

Table 5.12: Results of the FROLS algorithm for the 2nd Order NARMAX Model

The modeling statistics were:

1. F Statistic with 178 data values = 0.38

2. Q Statistic = 47.26 with 44 degrees of freedom (df) (52-9) which shows an adequate model (45 df χ^2 value is 60.48 at 5% level)

The ACF and the PACF of the residuals showed no significant lags, but the Linear and Nonlinear Validity tests (Figure 5.18) still showed a few lags slightly over the 95% significance levels, but the lags did not represent any logical time factor, and hence have been ignored.

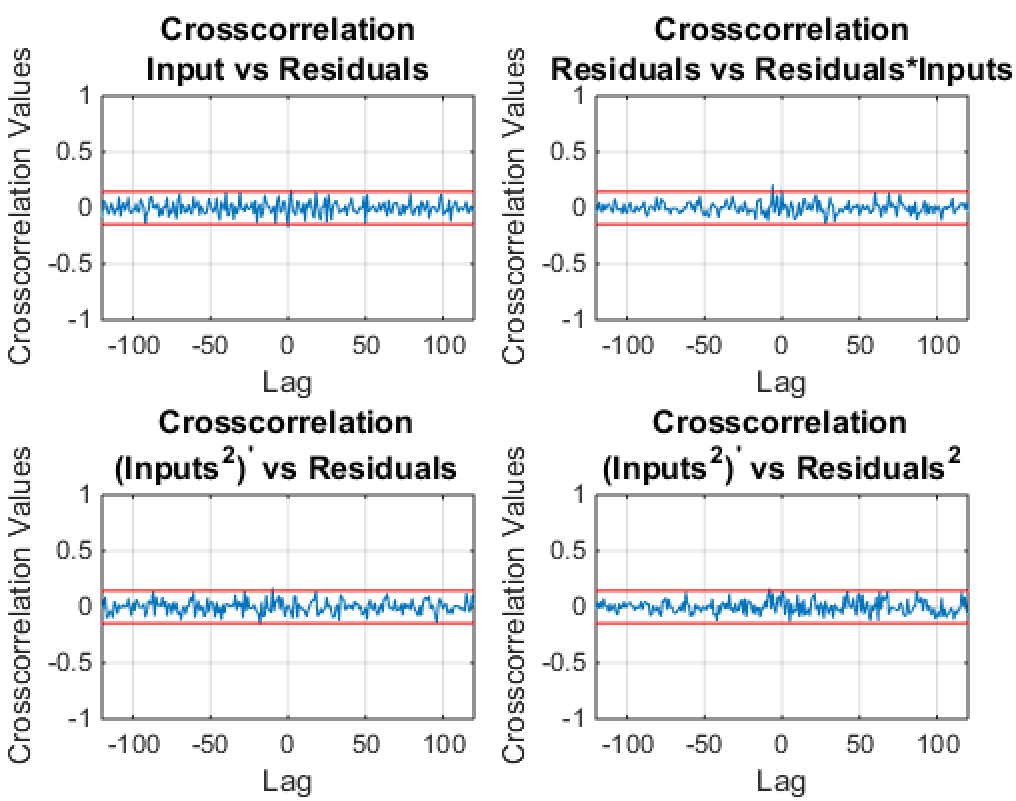

Figure 5.18: Linear and Non-Linear Validity Tests

5.3.3.3 NARMAX Forecasting Future One-Step Ahead Demand

One-Step Ahead Predicted Output for the NARMAX model are shown in Figures 5.19 and the corresponding forecast statistics in Table 5.13.

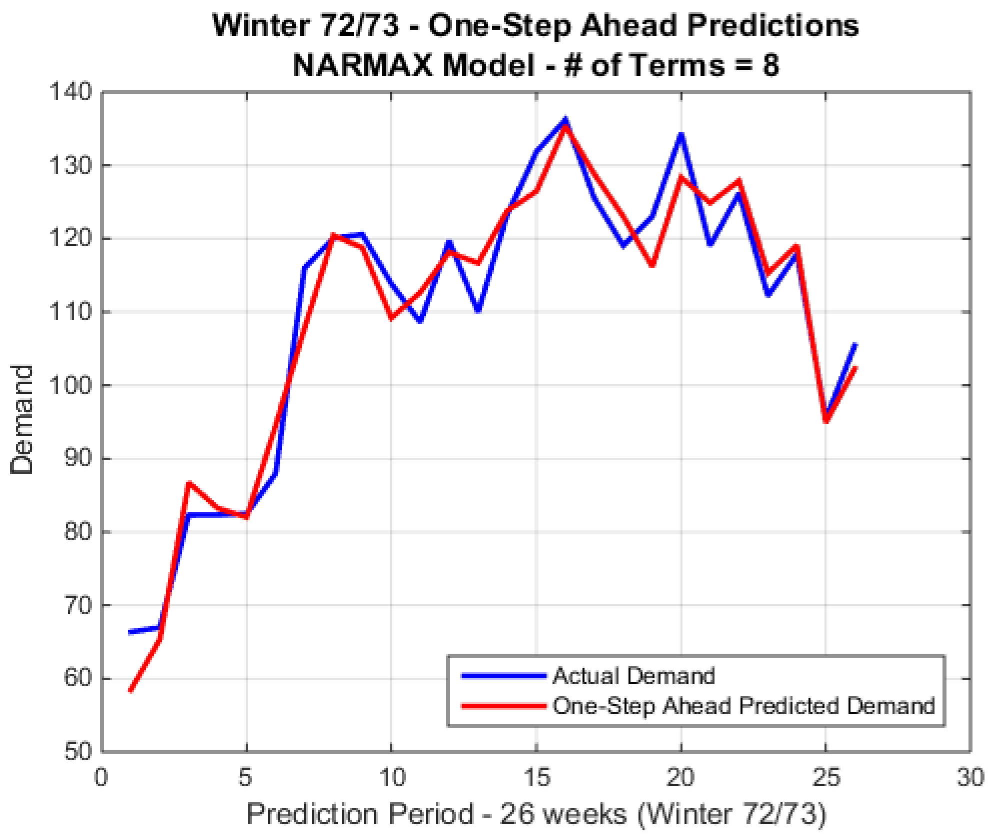

Figure 5.19: 26 week - One-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	0.25	3.53	18.59	3.38%	6.69/13/6.08%	-8.24/7/-7.11%

Table 5.13: NARMAX Model Statistics for One-Step Ahead Weekly Demand Forecast

5.3.3.4 NARMAX Forecasting Future Multi-Step Ahead Demand

Starting from the 2nd Order NARX model (described in Appendix F), the analysis of the Multi-Step residuals (ACF, CCF and the Nonlinear Validity tests) , indicate that the following Moving Average terms $e(k-1), e(k-2), e(k-26)$ and possibly $e(k-52)$ could be relevant to the model. Including these residual terms and running the process to stability produces an ACF for the residuals with no significant lags. The Nonlinear Validity tests showed a small number of significant lags but each close to the significant level. The model terms and values are shown in Table 5.14.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.046	87.14
2	e_{k-1}	0.404	1.32
3	x_{k-2}	-0.006	0.80
4	e_{k-26}	0.368	1.09
5	e_{k-2}	0.263	0.51
6	e_{k-52}	0.120	0.27
7	$x_{k-1} * x_{k-2}$	-0.002	0.14
8	$y_{k-1} * y_{k-2}$	0.561	0.12
9	$x_{k-2} * y_{k-26}$	0.010	0.08

Table 5.14: Results of the FROLS algorithm for the 2nd Order NARMAX Model

The modeling statistics were :

1. F Statistic with 178 data values = 0.27
2. Q Statistic = 25.79 with 43 degrees of freedom (df) (52-9) which shows an adequate model (43 df χ^2 value is 59.30 at 5% level)

This model was then used to forecast future values. The 26 week Multi-Step Ahead forecast is shown in Figure 5.20 and the corresponding forecast statistics are shown in Table 5.15.

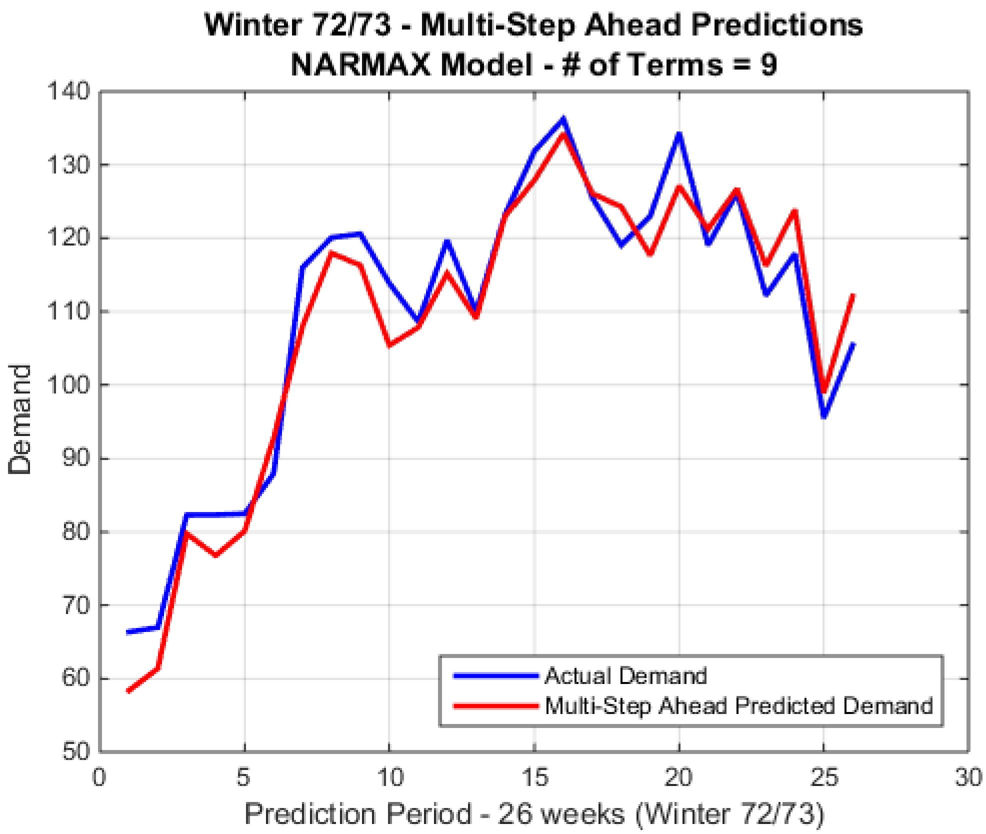

Figure 5.20: 26 week Multi-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	1.48	4.06	22.48	3.99%	8.47/10/7.44%	-6.67/26/-6.33%

Table 5.15: NARMAX Model Statistics for 26 week Multi-Step Ahead Weekly Demand Forecast

Performing shorter Multi-Step Ahead forecasts of 4, 6, 8 and 13 weeks using the 2nd Order NARMAX model produced the following results for the full 26 weeks in Table 5.16.

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	-1.65	4.99	36.54	4.84%	7.99/1/12.03%	-13.08/24/-11.09%
6 Week Ahead	3.28	5.96	51.86	5.53%	13.97/10/12.27%	-10.12/18/-8.50%
8 Week Ahead	0.67	3.75	19.63	3.71%	8.01/7/6.91%	-7.80/24/-6.62%
13 Week Ahead	0.95	4.23	23.71	4.16%	8.47/10/7.44%%	-7.65/26/-7.25%

Table 5.16: NARMAX Model Statistics for Various Multi-Step Ahead Weekly Demand Forecasts (2nd Order NARMAX Model)

The best Multi-Step ahead period appears to be an 8 week ahead forecast, but there is very little difference between it and the 26 week ahead forecast. The higher result for the 6 week ahead is due to the large temperature change during this period, which the model takes time to adjust to. For the 4 week ahead forecast, the high MAPE value is impacted by the first 4 weeks. The AR term $y(k-1)$ has to pick up the last winter week of the previous year, which impacts the calculations heavily. This negative constraint will be removed in the next Section when all 52 weeks of the year will be modeled and forecast. Figure 5.21 graphs this result for the 8 week ahead.

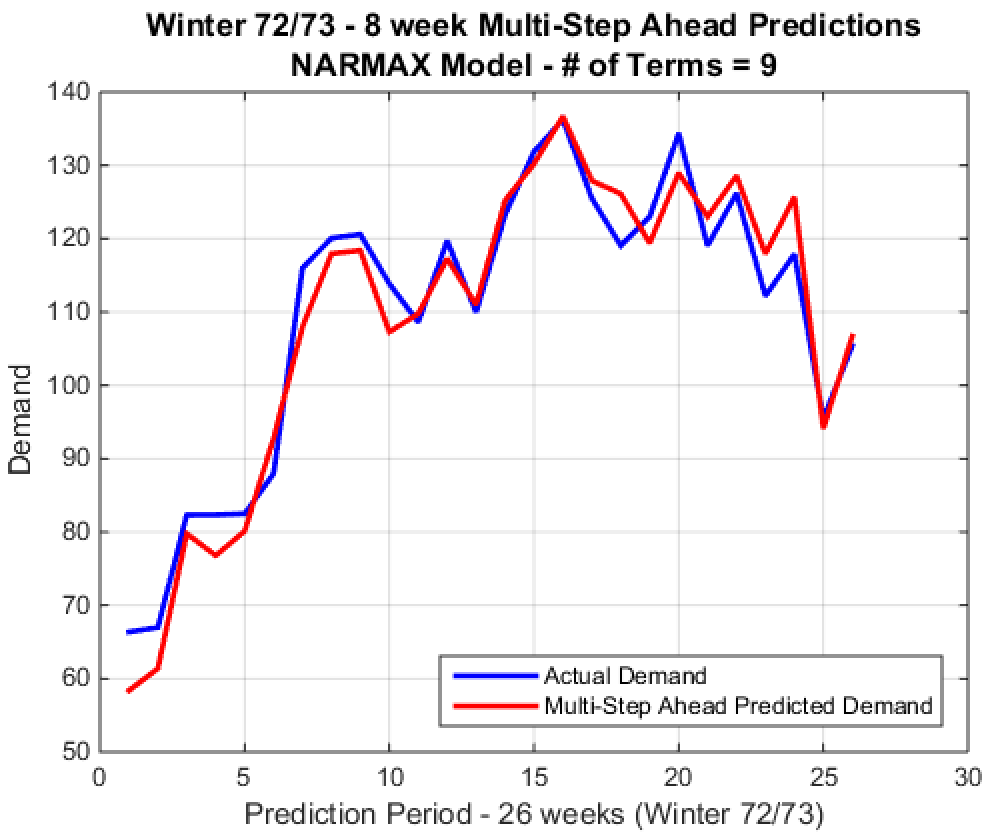

Figure 5.21: 8 Week Ahead Forecast periods for the 2nd Order NARMAX Model

5.3.4 Summary of NARMAX Results

For the One-Step Ahead forecasts, the 2nd Order NARMAX model produced a superior MAPE result (3.38%) over both the NARX model (4.11% MAPE) and the ARMAX model (3.86%). In the case of the Multi-Step Ahead forecast, for 26 week ahead, the ARMAX model produced a better forecast with MAPE of 3.38% (over 3.99% for the 2nd Order NARMAX model). For shorter Multi-Step Ahead horizons, the ARMAX model produced better results all round, with 8 week ahead forecasts producing the best MAPE of 3.21%.

5.4 Yearly Weekly Modeling and Forecasting with Actual Temperature (1963-1973)

5.4.1 ARMAX Yearly Weekly Modeling and Forecasting with Actual Temperature

5.4.1.1 Introduction

Section 5.3 modeled and forecast the Winter weeks only. This has the advantage that users reaction to temperature is consistent across the period of the winter months, once the central heating systems have been switched on. However, it has a major disadvantage in that Autoregressive part of the model will use data that is 26 weeks apart at the start of each Winter period i.e. the low AR values will pick up the end of the previous winter as values for the start of the forecast winter). To overcome this disadvantage, this section will model the full 52 weeks demand with its associated weekly effective temperatures.

As a reminder, Figures 4.1 and 4.2 show the original 10 years of weekly data provided by Southern Gas. In this chapter, the first 9 years (each of 52 weeks) will be used for modeling and the 52 weeks from April 1972 to the end of March 1973 will be used for forecasting comparison. Again temperature will be included in the model, and hence the Equation 3.6 described in Chapter 3 will apply to the following sections.

5.4.1.2 Transforming the data

As in Section 5.3, data stationarity is achieved with a logarithmic transformation for the Demand and no transformation was applied to Temperature. Additionally, the autocorrelation function shows differencing of 1 and 52 were appropriate for both Demand and Temperature. Analysis of the yearly means and variances after transformation and differencing confirm that stationarity is achieved. Figure 5.22 shows the fully transformed Demand and Temperature. The data, in blue, represents the transformed data which will be used for modeling and the red data is the comparison data for the forecast.

The ACF of the Transformed Weekly Demand has significant lags at 1 and 52, and the ACF of the Transformed Temperature has significant lags at 1, 2 and 52. The Cross correlation Function (CCF) between the Transformed Demand and Temperature showed a

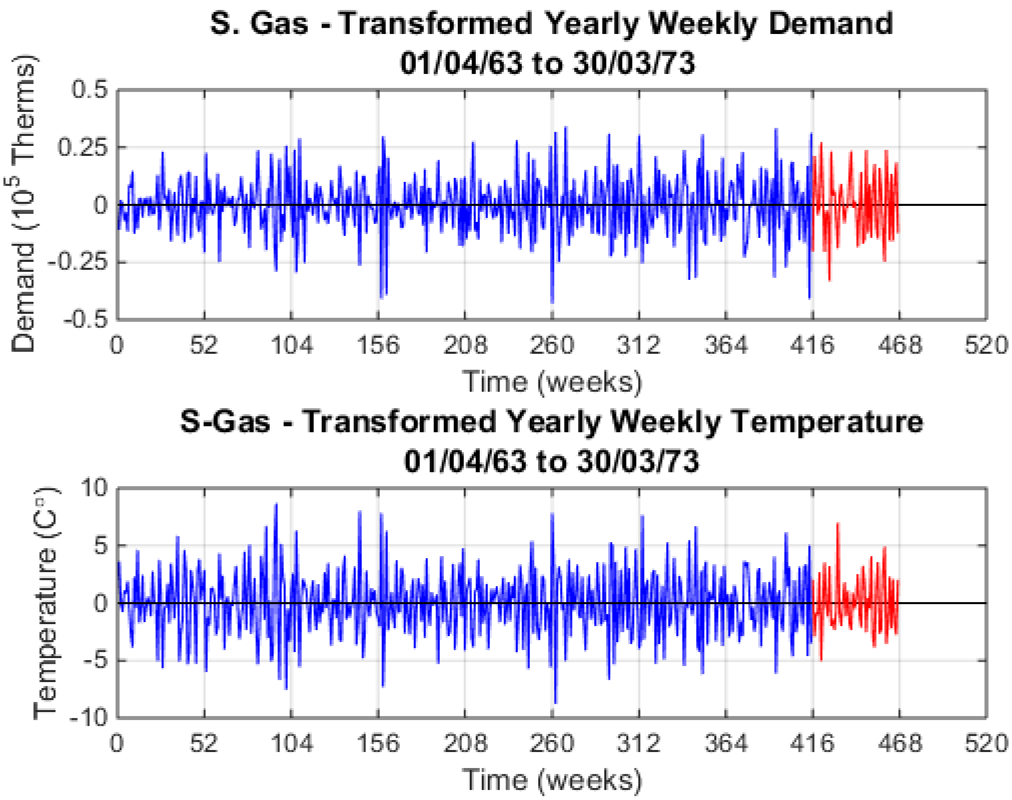

Figure 5.22: Transformed Demand and Temperature

significant lag at zero, significant lags at 52 and -52 as well as smaller significant lags at 1, 2, -1 and -2. This confirms, again, that the demand at time t is highly related to the temperature at time t, as well as a 52 lag relationship between demand and temperature.

5.4.1.3 ARMAX Parameter Identification of the Yearly Weekly Demand with Temperature

Using the results of the ACF of both the Transformed Demand and Temperature, and the CCF results, several models were tested for their AIC and BIC values. The lowest values were produced with AR and MA variables with delays of 1, 2, 3, 51, 52, 53, 54 and 55. The various combinations of these variable lags were then tested to find a balance between the modeling and forecast statistics. Additionally, several delays were applied to temperature series, but a delay of zero always produced the best results, as in Section 5.3 . This implies again that future demand at time t is dependent on the corresponding temperature at t, and past weeks demand values.

Model AR(1,2,51,52,53,54)/MA(52), produces the best results from a modeling perspective. The AIC value was -1435, the F Statistic was 0.73 and the Q Statistic was 41 with 45 (52-7) degrees of freedom.. The χ^2 value is with 45 df is 60.48 (at the 5% level) which shows an adequate model. The model is shown in Equation 5.13. The ACF of the residuals indicated a few lags just outside the +/- 95% confidence limits. Details on all the other models can be found in Appendix G.

$$(1-0.43B-0.20B^2+0.19B^{51}+0.37B^{52}+0.34B^{53}+0.26B^{54})w_t = -0.04x_t+(1-0.84B^{52})a_t \tag{5.13}$$

5.4.1.4 ARMAX Forecasting Future One-Step Ahead Demand

One step ahead forecasts were again calculated for model AR(1,2,51,52,53,54)/MA(52) for the 52 week period April 1972 though to March 1973. Figure 5.23 shows the 52 week One-Step Ahead forecast for the model and Table 5.17 shows the balanced set of metrics for the model (both 52 weeks and 26 Winter weeks).

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	-0.09	3.02	15.05	4.06%	6.95/11/14.40%	-9.96/52/-9.45%
26 Winter Weeks	0.19	3.22	16.25	3.06%	6.49/21/5.45%	-9.96/26/-9.45%

Table 5.17: One-Step Ahead Model Forecast Comparisons for Weekly Demand from the AR(1,2,51,52,53,54)/MA(52) Model

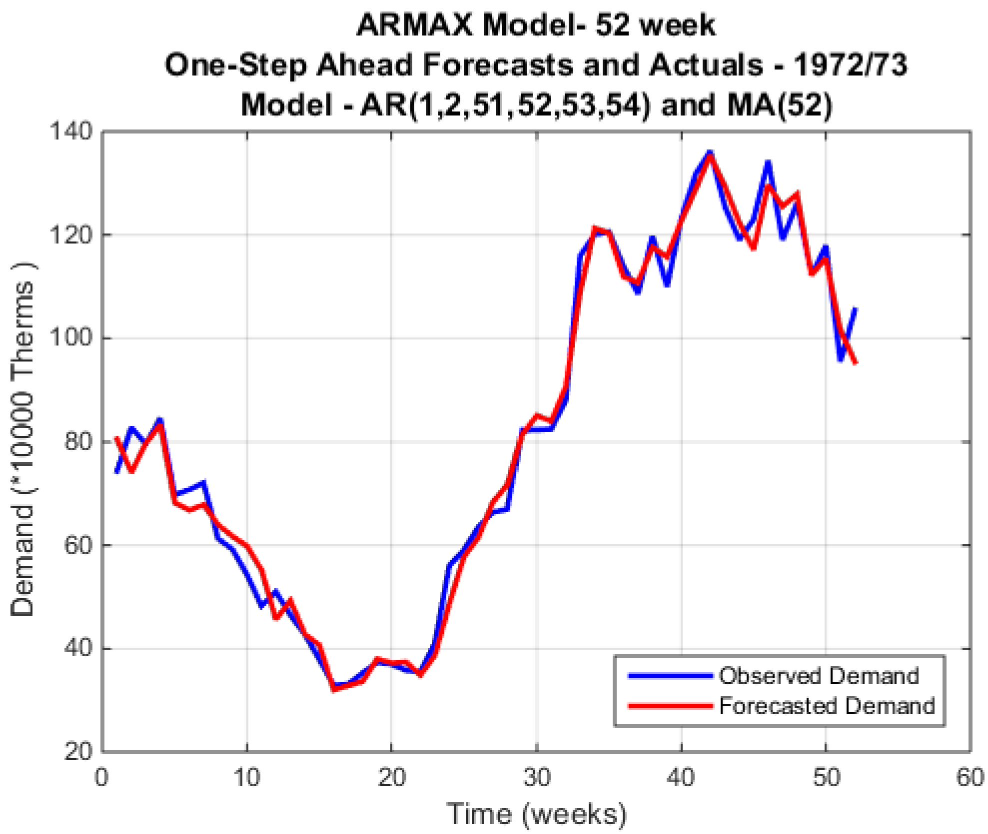

Figure 5.23: 52 Week - One-Step Ahead Forecasts for 1972/73

5.4.1.5 ARMAX Forecasting Future Multi-Step Ahead Demand

Multi step ahead forecasts were again calculated for each of the models in Appendix G for the period April 1972 to March 1973 (i.e. 52 weeks). The model AR(1,2,51,52,53,54)/MA(52) produced the best Multi-Step Ahead forecasts (for both 52 weeks and 26 Winter weeks). Figure 5.24 shows the 52 week Multi-Step Ahead forecast this model and Table 5.18 shows the balanced set of metrics for the model.

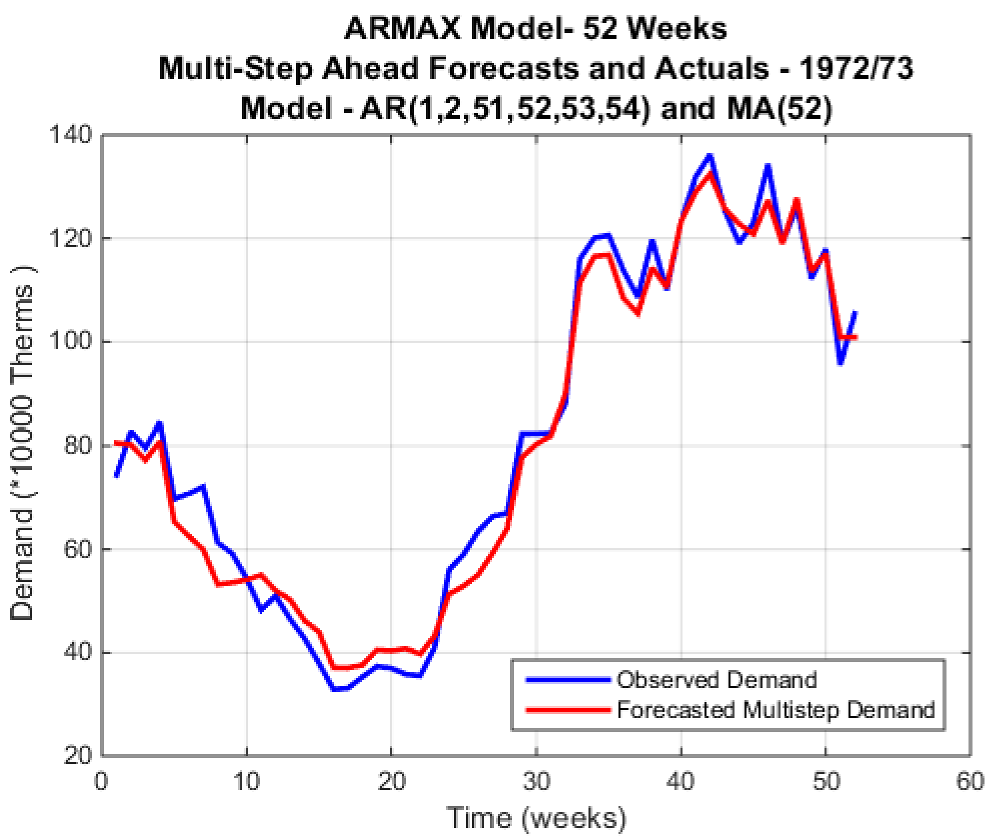

Figure 5.24: 52 Week - Multi-Step Ahead Forecasts for 1972/73

The statistics for various shorter Multi-Step ahead predictions of 4, 6, 8 and 13 weeks using the AR(1,2,51,52,53,54)/MA(52) for the 52 weeks are shown in Table 5.19.
As can be seen, there is no major improvement, however, the MAPE of 5.2% for the 6 week ahead is an improvement over the full 52 week forecast.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 weeks	-1.16	3.88	21.03	6.04%	6.74/11/13.97%	-12.12/7/-16.82%
26 Winter Weeks	-1.90	3.06	13.52	2.98%	5.39/25/5.65%	-7.09/1/-10.68%

Table 5.18: Multi-Step Ahead Model Forecast Comparisons for Weekly Demand from the AR(1,2,51,52,53,54)/MA(52) Model

Model AR(1,2,51,52,53,54)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	-0.46	4.09	26.16	5.73%	15.14/11/31.37%	-10.91/46/-8.12%
6 Week Ahead	1.25	3.52	18.96	5.20%	13.98/11/28.95%	-8.23/6/-11.62%
8 Week Ahead	-0.25	5.00	39.10	7.69%	15.14/11/31.37%	-12.12/7/-16.82%
13 Week Ahead	1.51	5.60	54.96	6.94%	17.37/39/15.80%	-12.53/26/-19.74%

Table 5.19: Model Statistics for Various Multi-Step Ahead Weekly Demand Forecast for the AR(1,2,51,52,53,54)/MA(52) Model

5.4.1.6 ARMAX Modeling and Forecasting the 26 Winter weeks

In this section, all 9 and half years of the weekly data is used for modeling, and this new model will then be used to forecast the 26 winter weeks from October 1972 to March 1973. Starting from the same variables (and delays) for the 9 year model produced good One-Step Ahead forecasts but Multi-Step Ahead forecasts started to drift away from the actuals. The ACF of the residuals showed lags around 104 were significant, and hence combinations for both AR and MA terms of 1,2,50 to 54 and 102 to 106 were tested. Several models produced similar modeling statistics and they are shown in Table 5.20.

Model ARMAX	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(50,51,52,53)/MA(1,52,104)	-1501	0.82	22,25,...	57	45
AR(50,51,52)/MA(1,52,104)	-1503	0.82	22,25,...	59	46
AR(1,50,51,52)/MA(1,52,104)	-1510	0.81	22,25,...	54	45
AR(1,50,51,52)/MA(1,2,52,104)	-1508	0.81	22,25,50,109	54	44
AR(1,2,51,52,53,54)/MA(52)	-1482	0.86	9,22,25,...	47	45

Table 5.20: Model Fit Comparisons for Winter Weekly Demand

Note: All the significant lags in the Table 5.20 are close to the 95% level.

However, the best model from a forecasting perspective was AR(50,51,52,53)/MA(1,52,104) both for One-Step Ahead and Multi-Step Ahead forecasts. The results are shown in Tables 5.21 and 5.22.

The model parameters for AR(50,51,52,53)/MA(1,52,104) are :

$$(1-0.12B^{50}+0.16B^{51}-0.67B^{52}+0.03B^{53})w_t = -0.041x_t+(1-0.23B+0.14B^{52}-0.55B^{104})a_t$$

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(50,51,52,53)/MA(1,52,104)	0.03	3.23	19.97	2.95%	10.22/21/8.59%	-11.05/7/-9.53%

Table 5.21: 26 Week - One-Step Ahead Model Statistics for the Winter Weekly Demand Forecast

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(50,51,52,53)/MA(1,52,104)	2.72	3.42	17.46	3.24%	9.35/6/10.63%	-4.96/20/-3.69%

Table 5.22: 26 Week - Multi-Step Ahead Model Statistics for the Winter Weekly Demand Forecast

Figures 5.25 and 5.26 show the 26 Winter Week One-Step Ahead and 26 Winter Week Multi-Step Ahead for the forecasts based on a the model AR(50,51,52,53)/MA(1,52,104) which was generated using the first 9 and a half years actual data.

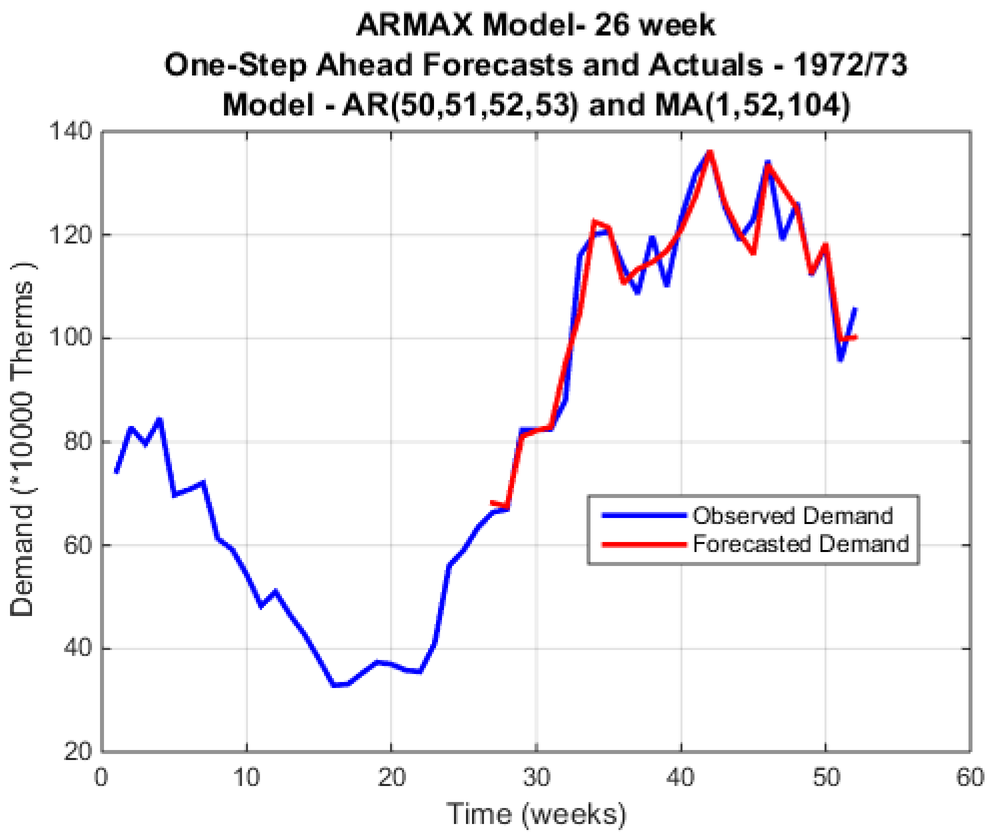

Figure 5.25: 26 Winter Weeks - One-Step Ahead Forecasts for 1972/73

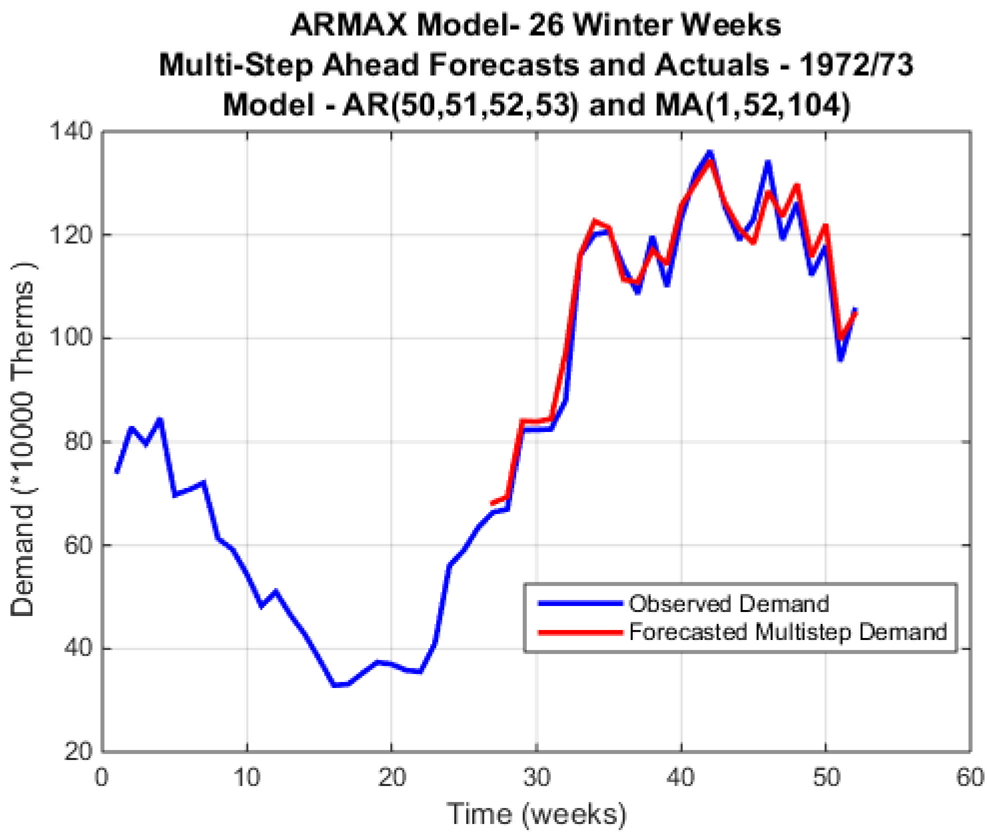

Figure 5.26: 26 Winter Weeks - Multi-Step Ahead Forecasts for 1972/73

5.4.2 Summary of ARMAX Results

In conclusion, the above work has shown that an Autoregressive Moving Average with eXogenous Inputs can again adequately represent and predict weekly demand data for Southern Gas when temperature is included in the model and all 52 weeks are included in the data. It also meets the level of acceptance (Benchmark Data) with a 52 week One-Step Ahead forecast MAPE of 4.06% and a Winter Weeks One-Step ahead MAPE of 3.06%.

For the Multi-Step ahead forecast, the 52 Winter weeks Multi-Step ahead MAPE of 6.04%. This is due to the large errors from the summer weeks. However, when only calculating the statistics for the 26 Winter weeks, the Multi-Step Ahead MAPE is <3%.

When modeling using 9 and a half years demand and temperature data and forecasting the Winter Weeks only for 1972/73, the One Step Ahead Forecast MAPE is again <3% while the 26 week Multi-Step Ahead MAPE is 3.24%. The next section will apply the NARMAX methodology to the same data.

5.4.3 NARMAX Yearly Weekly Modeling and Forecasting with Actual Temperature

As in the previous Section, the Southern Gas Weekly Demand and the Weekly Effective Temperatures described in Figures 4.1 and 4.2 of Chapter 4 are the start point for the analysis using the NARMAX methodology. The NARMAX model formula for this chapter are described in Section 3.3, and includes both delayed output variables and an input variable, temperature.

5.4.3.1 Transforming the data

The same transformations were performed as for the ARMAX modeling of yearly weekly demand and temperature (Section 5.3.1.2) to produce stationarity, i.e.:

1. The Yearly Weekly Demand was transformed with a logarithmic transformation

2. The Yearly Weekly Effective Temperature required no transformation

3. The Log of the Yearly Weekly Demand and the corresponding Effective Temperature were then differenced by factors of 1 and 52

Figure 5.22 shows the transformed Southern Gas Weekly Demand and Temperature, which will be used in this section with NARMAX.

5.4.3.2 NARMAX Parameter Identification of the Yearly Weekly Demand with Temperature

The starting point for the NARMAX modeling was similar to that used in Section 5.4, i.e. identify the range of possible transformed Demand and Temperature variables from the ACF and CCF. The initial range was $y(k-1)$ to $y(k-54)$ and $x(k-1)$ to $x(k-54)$ inclusive.

As in Section 5.3.3, a first step analyzed a Linear model (ARX), followed by inclusion of residuals, thus creating an ARMAX model. Following the linear model analysis, 2nd and 3rd order terms were introduced (both without and with residuals NARX and NARMAX), to find the most appropriate model from a modeling and especially a forecasting perspective. The best results are explained in this section, and the full details of the results from this methodology are covered in Appendix H.

Running the modeling process first without residuals (i.e. an ARX Model), the following variables were selected for demand and temperature i.e. $y(k-1), y(k-2), y(k-52)$ and $x(k), x(k-1), x(k-2), x(k-52)$. However, it was found that $x(k-52)$ did not have any impact on the modeling, and hence was removed. A 2nd Order NARX model was then tested using the same variables as the linear ARX model (i.e. $y(k-1), y(k-2), y(k-52)$ and $x(k), x(k-1), x(k-2)$). This generated 27 terms. Eight terms were selected on reaching the thresholds, with an ERR total of 87.48%. However, the ACF of the residuals showed that there was additional information especially around lags 52 and 104. Hence the Moving Average terms $e(k-52)$, and $e(k-104)$ were added. One run was required to attain no significant values in the ACF and the Validity tests. Nine terms were selected, and the ERR total was 90.62%, and again $x(k)$ produced 85% of this total. The Linear and Non-Linear Validity Tests for this model are shown in Figure 5.27 and the modeling statistics were:

1. F Statistic with 311 data values = 0.67

2. Q Statistic = 45.91 with 43 degrees of freedom (df) (52-9) which shows an adequate model (43 df χ^2 value is 59.30 at 5% level)

The terms selected for the model are shown in Table 5.23.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.045	85.08
2	e_{k-52}	0.40	1.86
3	y_{k-1}	-0.48	0.58
4	x_{k-1}	-0.021	1.60
5	e_{k-104}	0.24	0.40
6	x_{k-2}	-0.013	0.31
7	y_{k-2}	-0.24	0.55
8	$x_{k-2} * y_{k-52}$	-0.013	0.11
9	y_{k-2}^2	0.003	0.08

Table 5.23: Results of the FROLS algorithm for the 2nd Order NARMAX Model

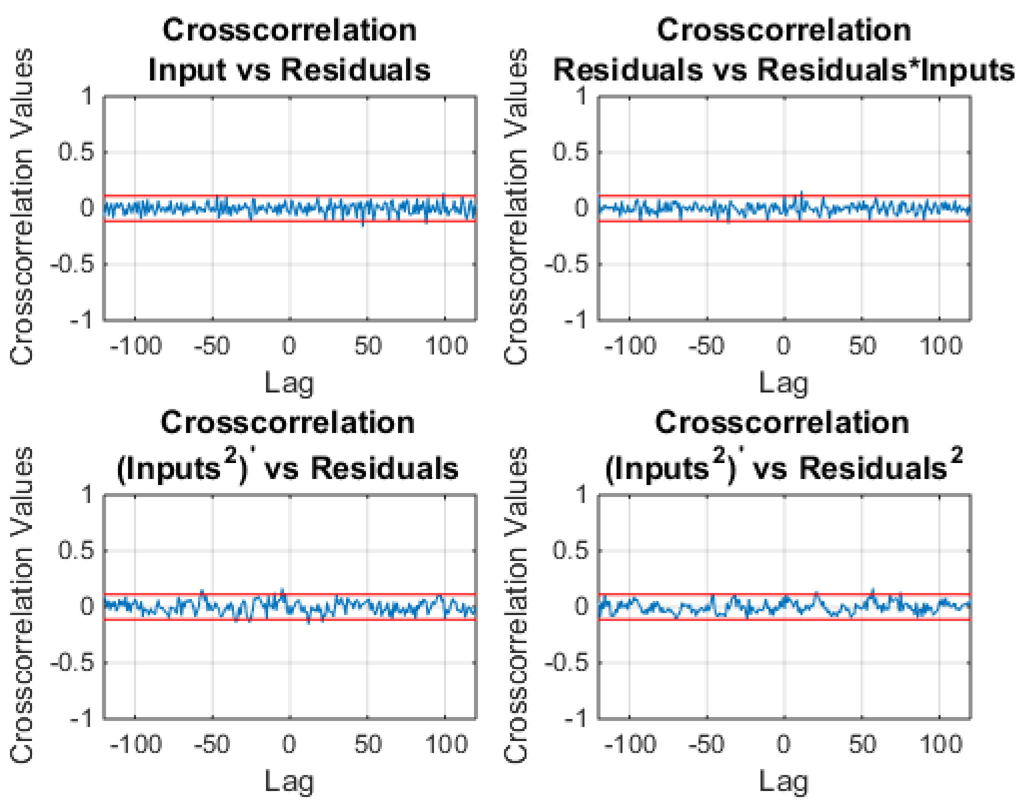

Figure 5.27: Linear and Non-Linear Validity Tests

5.4.3.3 NARMAX Forecasting Future One-Step Ahead Demand

One-Step Ahead Predicted Output for the NARMAX model are shown in Figure 5.28, and the corresponding forecast statistics in Table 5.24 for both the 52 week One-Step Ahead, and the 26 Winter week One-Step Ahead forecasts..

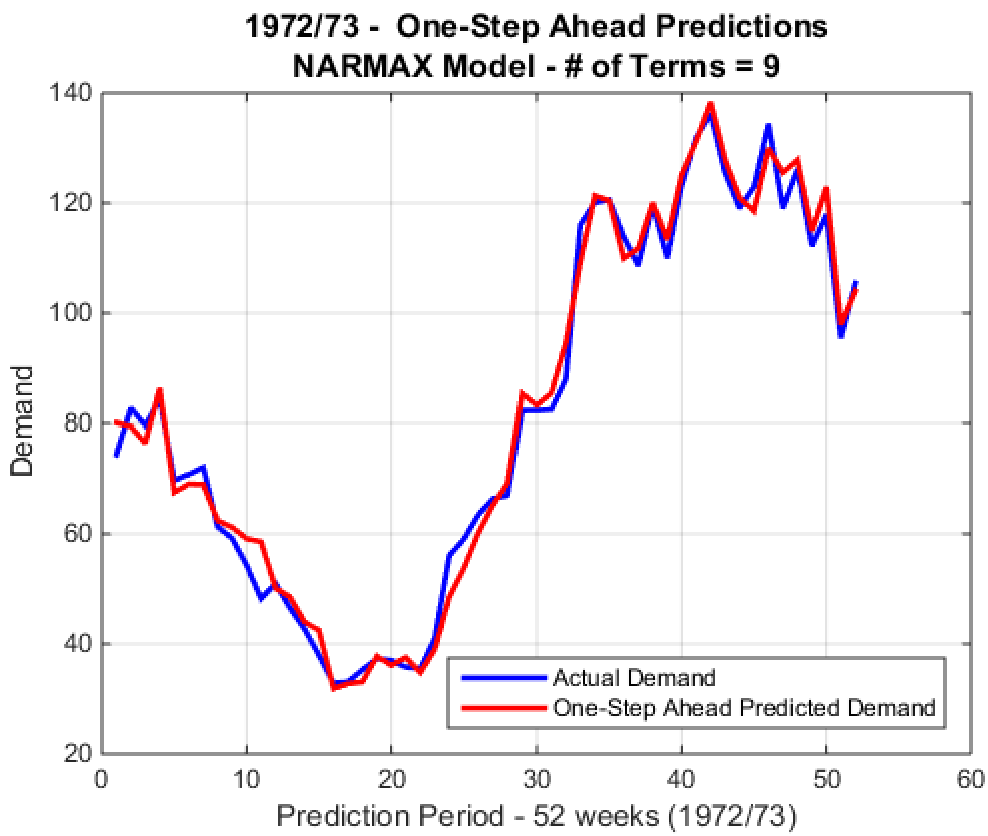

Figure 5.28: 52 week One-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	0.06	2.90	13.68	4.04%	10.77/11/22.31%	-8.36/33/-7.21%
26 Winter Weeks	0.36	2.87	12.86	2.68%	6.85/6/7.78%	-8.36/7/-7.21%

Table 5.24: One-Step Ahead Forecast Statistics for Weekly Demand (2nd Order NARMAX Model)

5.4.3.4 NARMAX Forecasting Future Multi-Step Ahead Demand

Starting from the 2nd Order NARX model (described in Appendix H), the analysis of the Multi-Step Ahead residuals (ACF, CCF and Nonlinear Validity tests) , indicate that following Moving Average terms $e(k-1)$, $e(k-2)$, $e(k-51)$ to $e(k-53)$ and possibly $e(k-104)$ were required. Including these residual terms and running the process to stability found that the $x(k-2)$ and $e(k-104)$ added little value to the results. The ERR total was 89% and the ACF, CCF and the three Nonlinear Validity tests showed a small number of significant lags but each close to the significant level. The model terms and values are shown in Table 5.25.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.044	83.13
2	e_{k-1}	0.44	2.50
3	e_{k-52}	0.41	2.42
4	y_{k-52}	-0.07	0.39
5	e_{k-53}	-0.16	0.21
6	e_{k-51}	0.12	0.18
7	y_{k-2}	0.04	0.14
8	$y_{k-2} * y_{k-52}$	0.23	0.10
9	$x_{k-1} * x_k$	0.0004	0.07

Table 5.25: Results of the FROLS algorithm for the 2nd Order NARMAX Model

The modeling statistics were :

1. F Statistic with 178 data values = 0.91

2. Q Statistic = 59.80 with 43 degrees of freedom (df) (52-9) which is slightly above the 5% χ^2 value (43 df χ^2 value is 59.30 at 5% level)

This model was then used to forecast future values for 52 week Multi-Step Ahead. Analysis of the results of the 52 Week Multi-Step Ahead forecast shows that the highest errors (Over and Under estimates) are during the summer months - week 15 and 7 respectively.

The 52 week Multi-Step Ahead forecast is shown in Figures 5.29 and the corresponding forecast statistics are shown in Table 5.26 for both the 52 and 26 week Multi-Step Ahead values.

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	1.13	3.92	22.12	5.94%	7.39/15/19.47%	-11.35/7/-15.74%
26 Winter Weeks	1.39	3.35	15.39	3.23%	5.14/25/5.39%	-9.18/10/-8.07%

Table 5.26: Multi-Step Ahead Forecast Statistics for Weekly Demand (2nd Order NARMAX Model)

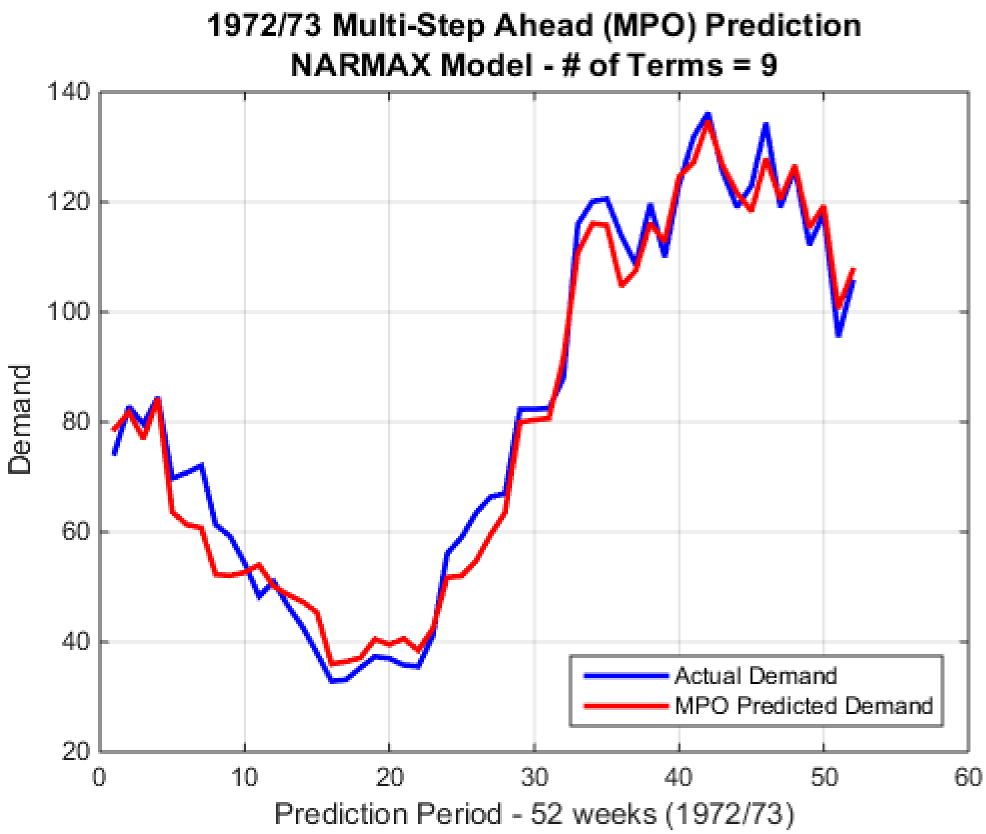

Figure 5.29: 52 week Multi-Step Ahead Forecast for the 2nd Order NARMAX Model

The statistics for various shorter Multi-Step ahead predictions of 4, 6, 8 and 13 weeks using the 2nd Order NARMAX model above for the 52 weeks are shown in Table 5.27. As can be seen, there is no major improvement, however, the MAPE of 5.99% for the 6 week ahead is close to the full 52 week forecast.

Note when the weeks ahead number is not a factor of 52, a 4 weeks ahead forecast is performed for the last 4 weeks (i.e. weeks 49 to 52).

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	0.03	4.96	40.20	6.69%	13.30/36/11.68%	-15.07/11/-31.21%
6 Week Ahead	-1.94	4.22	28.57	5.99%	9.47/6/13.38%	-14.04/11/-29.09%
8 Week Ahead	0.42	5.21	44.93	7.85%	13.30/36/11.68%	-15.25/15/-40.18%
13 Week Ahead	-1.47	5.98	63.41	7.45%	11.35/7/15.74%	-20.85/39/-18.96%

Table 5.27: 52 week - NARMAX Model Statistics for Various Multi-Step Ahead Weekly Demand Forecasts

The best Multi-Step ahead period appears to be an 6 week ahead forecast, but there is very little difference between it and the 52 week ahead forecast. Figure 5.30 graphs this result for the 6 week ahead.

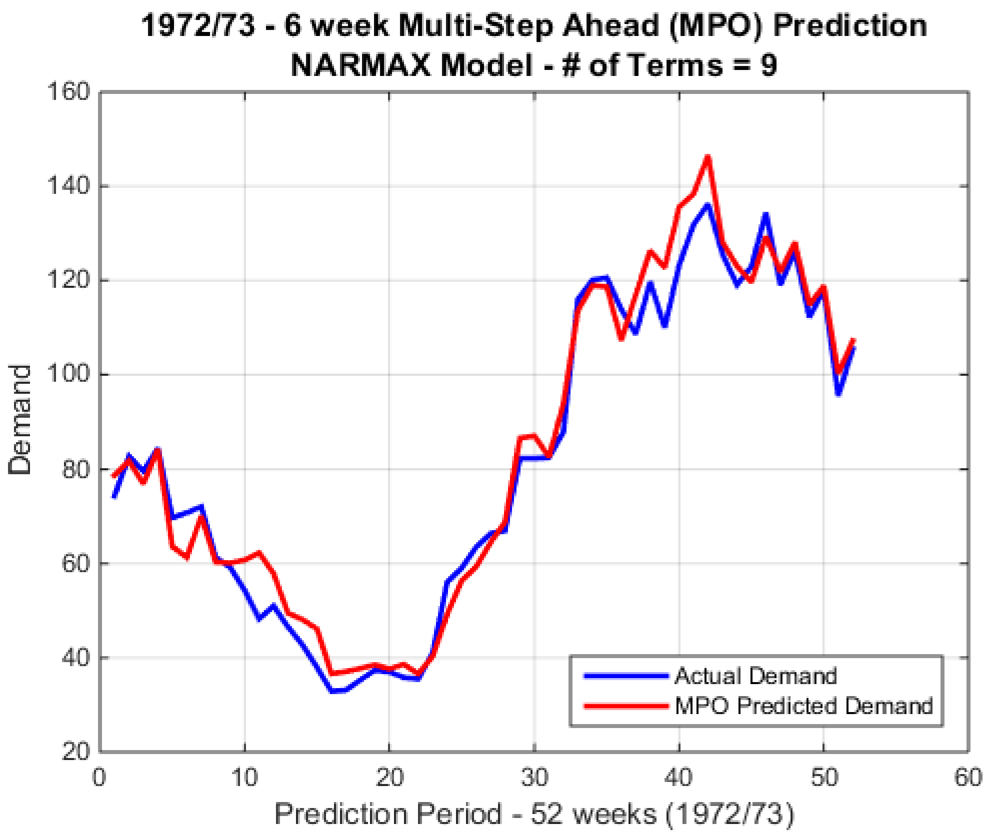

Figure 5.30: 6 Week Ahead Forecast periods for the 2nd Order NARMAX Model

5.4.3.5 NARMAX Modeling and forecasting ONLY the 26 Winter weeks

Repeating the methodology as described in Section 5.5.1.6, all 9 and half years of the weekly data is used for modeling. This new model will then be used to forecast the 26 winter weeks from October 1972 to March 1973. Starting from the same variables (and delays) for the 9 year model produced good One-Step Ahead forecasts but Multi-Step Ahead forecasts started to drift away from the actuals. The ACF of the residuals showed lags around 104 were significant, and hence combinations for both AR and MA terms of 1, 2, 50 to 54 and 102 to 106 were tested. For both One-Step Ahead and Multi-Step Ahead for the 26 Winter weeks, the terms were $y(k-1), y(k-2), y(k-52), x(k), x(k-1), x(k-2), e(k-1), e(k-2), e(k-52)$ and $e(k-104)$.

The model terms and values are shown in Table 5.28.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.047	83.19
2	e_{k-52}	0.56	2.62
3	e_{k-1}	-0.16	1.23
4	x_{k-2}	-0.009	0.74
5	e_{k-104}	0.23	0.49
6	y_{k-1}	-0.46	0.12
7	x_{k-1}	-0.02	0.53
8	y_{k-2}	-0.13	0.22
9	y_{k-1}^2	0.11	0.07

Table 5.28: Results of the FROLS algorithm for the 2nd Order NARMAX Model

Figure 5.31 shows the 26 week One-Step Ahead forecast and the associated statistics for the 2nd Order NARMAX model are shown in Table 5.29.

The Multi-Step Ahead forecast and associated statistical tables for the 2nd Order

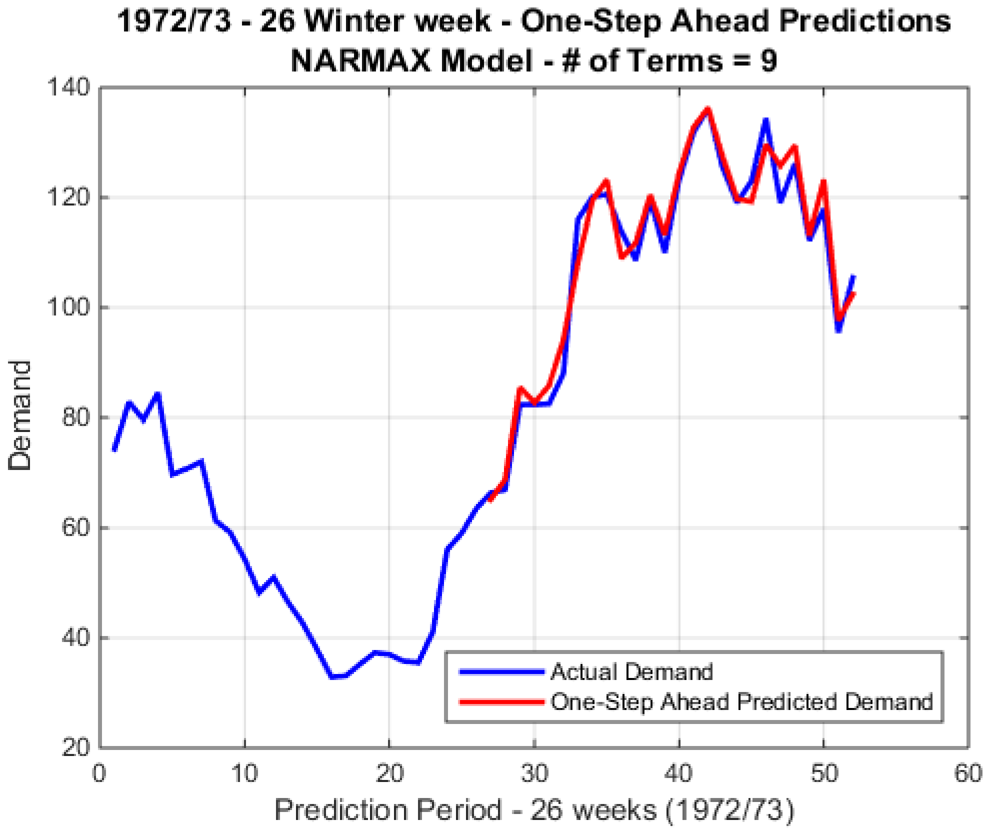

Figure 5.31: 26 Week One-Step Ahead Forecast periods for the 2nd Order NARMAX Model

NARMAX model are shown in Figure 5.32 and Table 5.30.

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
26 Winter weeks	0.83	2.85	12.36	2.67%	6.73/21/5.65%	-7.91/7/-6.82%

Table 5.29: Model Statistics for 26 Winter Weeks - One-Step Ahead Weekly Demand Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
26 Winter weeks	-3.01	3.84	26.99	3.59%	4.04/7/3.48%	-11.22/24/-9.51%

Table 5.30: Model Statistics for 26 Winter Weeks - Multi-Step Ahead Weekly Demand Forecast for the 2nd Order NARMAX Model

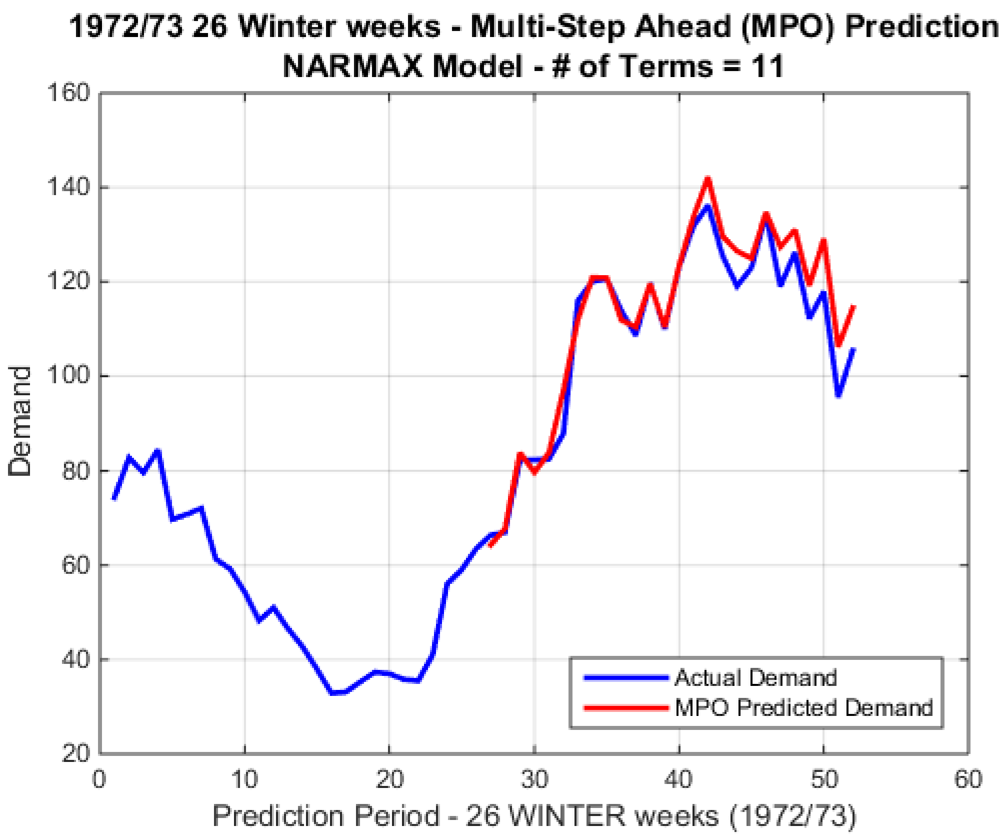

Figure 5.32: 26 Winter Week Multi-Step Ahead Forecast periods for the 2nd Order NARMAX Model

5.4.4 Summary of NARMAX Results

For the 52 week One-Step Ahead forecasts, the ARMAX and NARMAX forecasting results are very similar (MAPE 4.06% vs 4.04%). However, the statistics from the same model for only the 26 Winter weeks, show the 2nd Order NARMAX One-Step Ahead forecast is slightly better (MAPE 2.68% vs 3.06%).

In the case of the Multi-Step Ahead forecast, the two methodologies (ARMAX and NARMAX) produce results very similar statistics for both the 52 week and the 26 week. For shorter term multi-step forecast horizons, the results are also very similar.

Finally, running the methodology on 9 and a half years and forecasting ONLY the 26 Winter weeks, produced similar results for each model. One-Step Ahead MAPE were 2.95% for ARMAX and 2.67% for NARMAX. For the Multi-Step Ahead the MAPE results were 3.24% for ARMAX and 3.59% for NARMAX.

5.5 *Yearly Weekly Modeling and Forecasting with Actual Temperature (2001-2011)*

5.5.1 *ARMAX Yearly Weekly Modeling and Forecasting with Actual Temperature*

5.5.1.1 *Introduction*

Section 5.4 modeled and forecast the Yearly and Winter weeks Demand based on data from the period 1963 to 1973. This Section will apply the same methodology to the data from 2001 to 2011.

As a reminder, Figures 4.6 and 4.7 of Chapter 4 show the original 10 years of daily data provided by DNV GL for the region X-Gas. This data is converted to weekly totals for the demand data and the 7 day average for the Temperature data, generating 520 weekly values. The two series are shown in Figures 5.33 and 5.34.

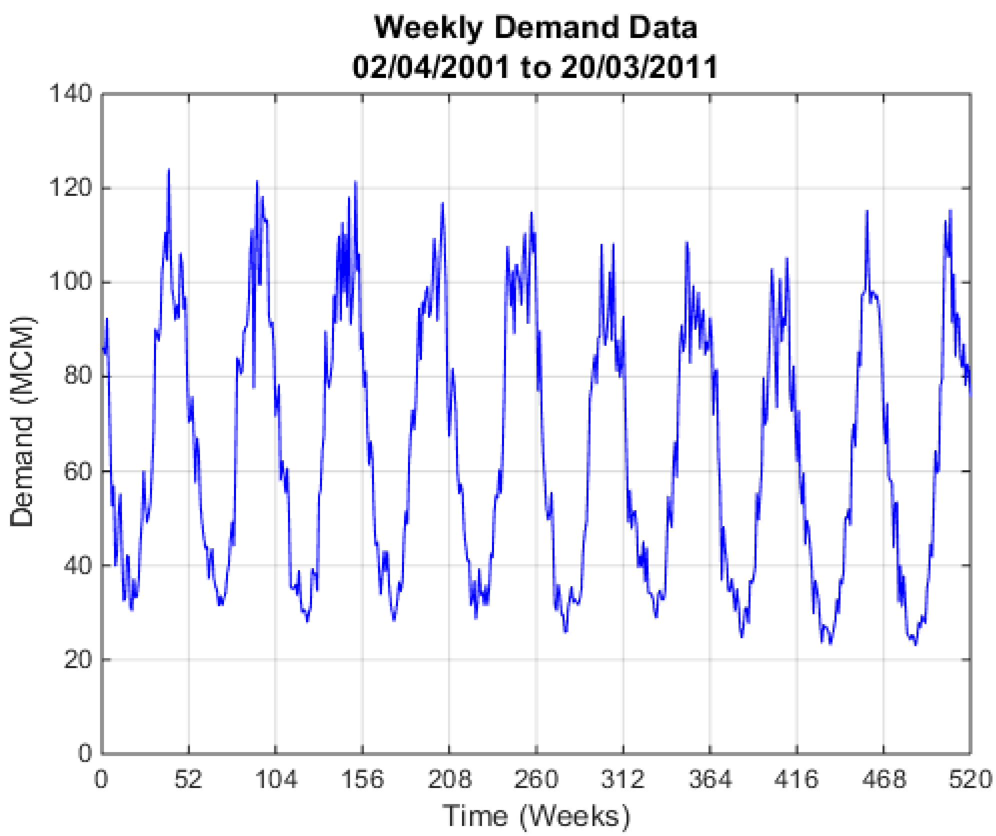

Figure 5.33: Weekly Demand

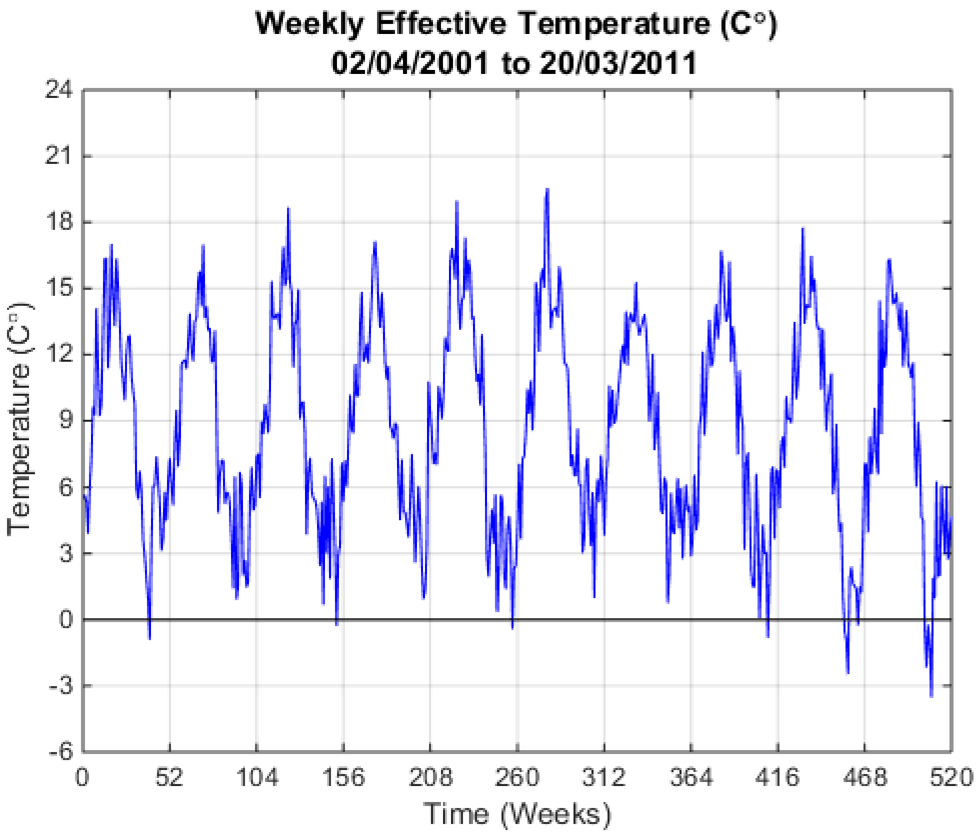

Figure 5.34: Weekly Effective Temperature

The start and end weeks are similar to the 1970s data, i.e. the initial week is early April 2001, and the last week is the end of March 2011. The first 9 years (each of 52 weeks) will be used for modeling and the 52 weeks from April 2010 to the end of March 2011 will be used for forecasting comparison. Again temperature is included in the model, and hence the Equation 3.6 described in Chapter 3 will apply to the following sections. A periodic cycles of 52 weeks is still relevant to the new data, however the slope of growth year on year is much flatter than the data from 1970, this probably due to the stable population in the region under study.

5.5.1.2 Transforming the data

As in Section 5.4, data stationarity is achieved with a logarithmic transformation for the Demand and no transformation applied to Temperature together with differencing of 1 and

52 for both Demand and Temperature. Analysis of the yearly means and variances after transformation and differencing confirm, again, that stationarity is achieved. Figure 5.35 shows the transformed Demand and Temperature data for the 10 years 2001 to 2011. The data, in blue, represents the transformed data which will be used for modeling and the red data is the comparison data for the forecast.

The ACF of the Transformed Weekly Demand has significant lags at 1, 2 and 52, and the ACF of the Transformed Temperature has significant lags at 1, 2, 3, 4, 5 and 52. The Cross correlation Function (CCF) between the Transformed Demand and Temperature showed the same profile as for the 1970s data i.e. a significant lag at zero, significant lags at 52 and -52 as well as smaller significant lags at 1, 2, -1 and -2.

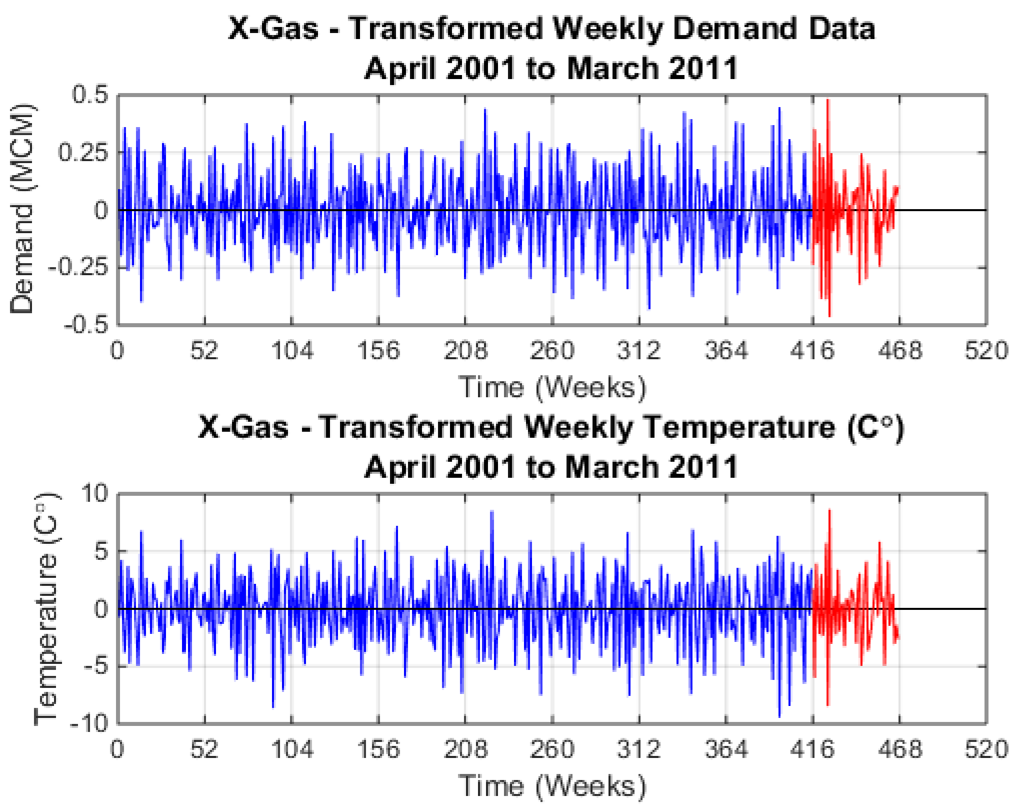

Figure 5.35: Transformed Demand and Temperature

The first test was to see how well the model from Section 5.4, i.e. AR(1,2,51,52,53,54)/MA(52), would perform when applied to the new data. The results are shown in Table 5.31, however, there were significant lags in the ACF of the residuals at lags 3 and 104, and hence alternative models were considered to see how they could improve the results. The best models are described in the sections below.

Model AR(1,2,51:54)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week One-Step Ahead	0.02	4.20	34.94	6.82%	18.18/39/17.27%	-16.01/40/-13.86%
26 week One-Step Ahead	-0.08	5.58	53.30	6.88%	18.18/13/17.27%	-16.01/14/-13.86%
52 week Multi-Step Ahead	1.14	3.30	16.21	6.52%	12.18/39/11.58%	-8.88/46/-10.60%
26 week Multi-Step Ahead	0.35	3.71	20.67	4.61%	12.18/13/11.58%	-8.88/20/-10.60%

Table 5.31: Model Statistics for Various Weekly Demand Forecasts.
Model : AR(1,2,51,52,53,54)/MA(52)

5.5.1.3 ARMAX Parameter Identification of the Yearly Weekly Demand with Temperature

Several models were again tested using the measures of AIC/BIC, F and Q statistics. Models with similar modeling statistics were then used to forecast the 10th year, and the forecast statistics calculated. The best model from a forecasting perspective was found to be AR(1,2,3,4,52,53)/MA(52), and temperature x at time t and $t-1$. For this model, the AIC value was -1099, the F Statistic was 1.63 and the Q Statistic was 47.74 with 45 (52-7) degrees of freedom (the χ^2 value is 60.48 with 45 df at 5% level and hence shows an adequate model). The model is shown in Equation 5.14. The ACF of the residuals found a few lags just outside the +/- 95% confidence limits, specifically lag 5, but adding these lags to the model parameters did not improve the results.

$$(1-0.50B-0.37B^2-0.26B^3-0.15B^4+0.22B^{52}+0.16B^{53})w_t = -0.05x_t-0.01x_{t-1}+(1-0.77B^{52})a_t \tag{5.14}$$

5.5.1.4 ARMAX Forecasting Future One-Step Ahead Demand

One step ahead forecasts were calculated for the 52 week period April 2010 though to March 2011. Figure 5.36 shows the 52 week One-Step Ahead forecasts for the model AR(1,2,3,4,52,53)/MA(52) and Table 5.32 shows the balanced set of metrics for the model for both the 52 and 26 week One-Step Ahead forecasts.

Model AR(1:4,52,53)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	-0.05	2.92	14.69	5.28%	7.82/6/17.88%	-10.82/46/-12.92%
26 Winter weeks	-0.02	3.48	19.46	4.41%	7.65/18/8.19%	-10.82/20/-12.92%

Table 5.32: Model Statistics for One-Step Ahead Weekly Demand Forecasts
Model : AR(1,2,3,4,52,53)/MA(52)

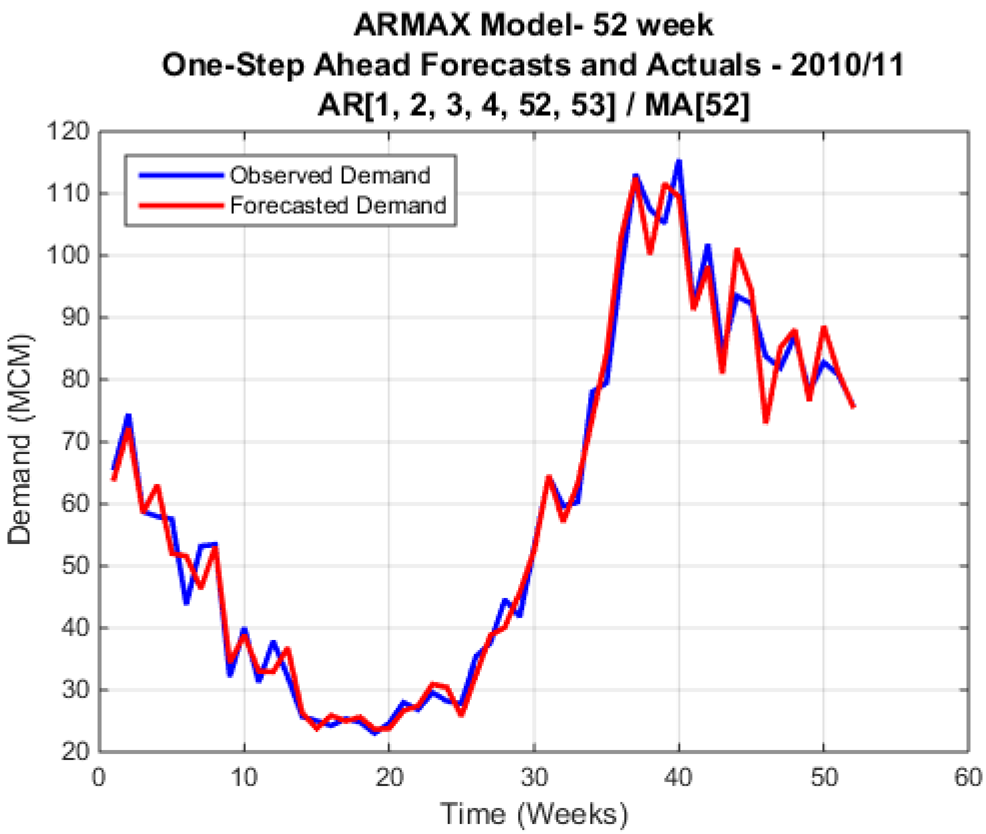

Figure 5.36: 52 Week - One-Step Ahead Forecasts for 2010/2011

5.5.1.5 ARMAX Forecasting Future Multi-Step Ahead Demand

Multi-Step Ahead forecasts were again calculated for model AR(1,2,3,4,52,53)/MA(52). Figure 5.37 shows the 52 week Multi-Step Ahead forecast this model and the statistics are shown in Table 5.33.

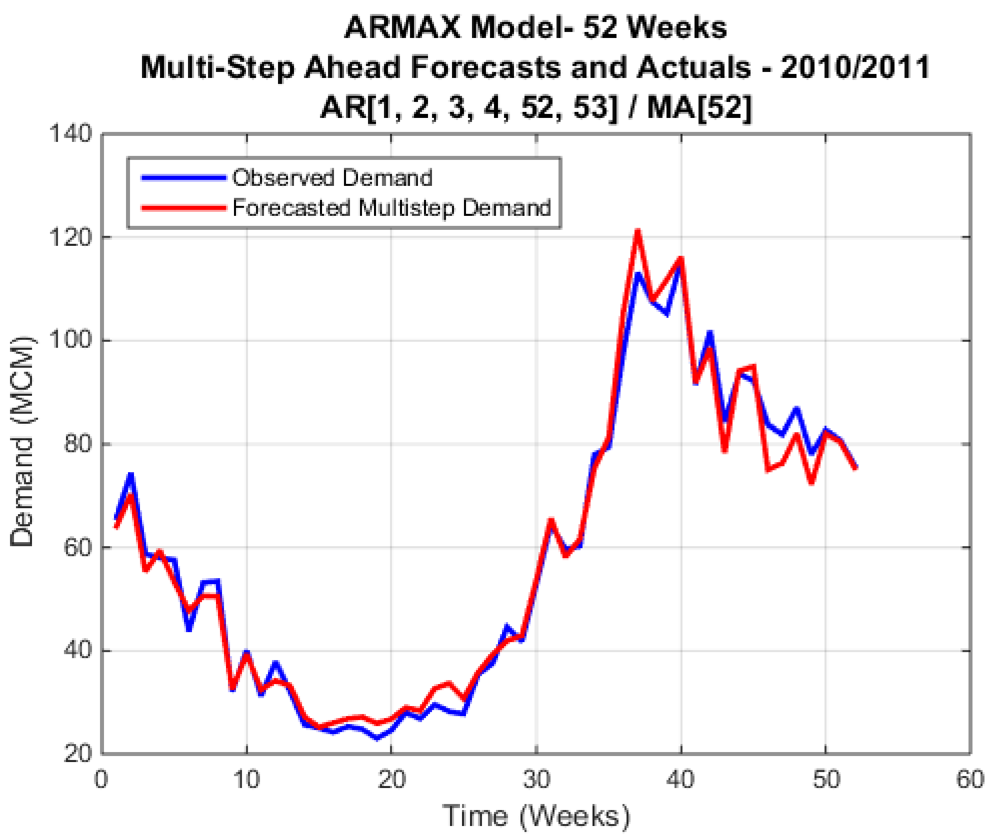

Figure 5.37: 52 Week - Multi-Step Ahead Forecasts for 2010/2011

Model AR(1:4,52,53)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	0.09	2.62	11.52	5.03%	8.42/37/7.44%	-8.65/46/-10.33%
26 Winter weeks	-0.28	2.99	16.17	3.69%	8.42/11/7.44%	-8.65/20/-10.33%

Table 5.33: Model Statistics for Multi-Step Ahead Weekly Demand Forecasts
Model : AR(1,2,3,4,52,53)/MA(52)

Additionally, starting from the actual demand of the last week of September 2010, the 26 Winter Weeks statistics (October 2010 to March 2011) for AR(1,2,3,4,52,53)/MA(52), also produced good results and are shown in Table 5.34. Figure 5.38 shows this 26 week Multi-Step Ahead forecast.

Model AR(1:4,52,53)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
26 Winter weeks	-1.20	3.09	16.84	3.78%	7.02/11/6.20%	-9.52/20/-11.36%

Table 5.34: Model Statistics for Multi-Step Ahead Weekly Demand Forecasts
Model : AR(1,2,3,4,52,53)/MA(52)

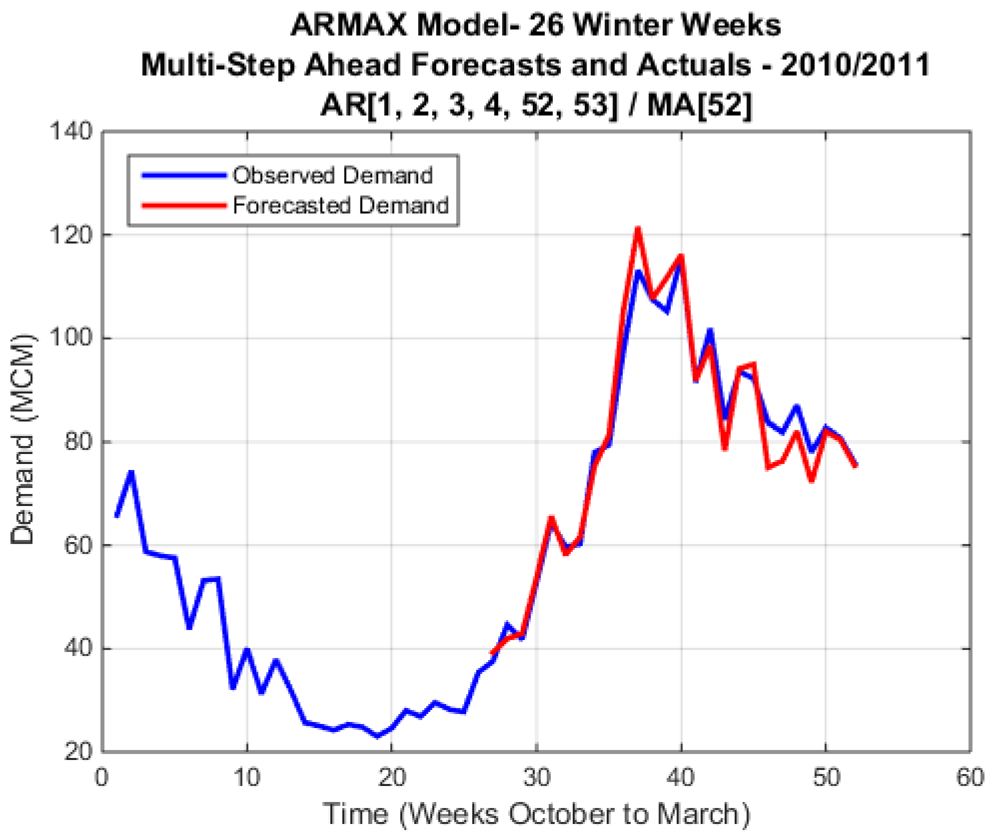

Figure 5.38: 26 Winter Weeks - Multi-Step Ahead Forecasts for 2010/2011

The statistics for various shorter Multi-Step ahead predictions of 4, 6, 8 and 13 weeks using the AR(1,2,3,4,52,53)/MA(52) for the 52 weeks are shown in Table 5.35.
As can be seen, there is no major improvement over the full 52 week forecast.

Model AR(1:4,52,53)/MA(52)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	-0.35	3.48	18.26	6.61%	10.59/36/10.86%	-9.48/28/-21.25%
6 Week Ahead	-1.39	3.89	21.33	7.76%	6.13/36/6.28%	-10.58/42/-10.38%
8 Week Ahead	-0.41	3.87	24.04	6.93%	11.65/37/10.29%	-10.95/32/-18.36%
13 Week Ahead	-1.61	3.43	21.49	5.45%	7.02/37/6.20%	-13.02/46/-15.55%

Table 5.35: Model Statistics for 52 Week - Various Multi-Step Ahead Forecasts
Model : AR(1,2,3,4,52,53)/MA(52)

5.5.1.6 ARMAX Modeling and Forecasting for the 26 Winter weeks

In this section, all 9 and half years of the weekly data is used for modeling, and this new model will then be used to forecast the 26 winter weeks from October 2010 to March 2011. Starting from the same variables (and delays) for the 9 year model produced good One-Step Ahead forecasts but Multi-Step Ahead forecasts started to drift away from the actuals. The ACF of the residuals showed several significant lags, and hence other models with various combinations for AR and MA terms were tested. Several models produced similar modeling statistics and are shown in Table 5.36. The 9 year model statistics are shown at the end of the table for reference.

Model ARMAX	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1:2,52:53,104)/MA(1,52)	-1227	1.52	156	29.77	97
AR(1:2,52:53,104:105)/MA(1)	-1227	1.51	156	30.00	98
AR(1,52:53,104:105)/MA(1)	-1226	1.52	80,156	30.96	99
AR(1:2,52:53,104:105)/MA(1,52)	-1225	1.52	156	29.71	97
AR(1:4,52,53)/MA(52)	-1173	1.71	5,157	45	45

Table 5.36: Model Fit Comparisons for Winter Weekly Demand

Note: Variable 156 was included into the models, both as an AR and MA term, but did not improve the results.

Each of the models, in Table 5.36, produced similar One-Step and Multi-Step ahead forecasts. Model AR(1:2,52:53,104:105)/MA(1) was chosen due to slightly better overall balanced results. These results are shown in Table 5.37.

Model AR(1:2,52:53,104:105)/MA(1)	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
One-Step Ahead	-0.1	4.20	29.54	5.24%	10.78/13/10.24%	-11.57/20/-13.82%
Multi-Step Ahead	-3.76	4.39	28.45	5.41%	3.72/6/3.54%	-11.92/20/-14.24%

Table 5.37: 26 Week - Model Statistics for AR(1:2,52:53,104:105)/MA(1) Model Forecast for the Winter Weekly Demand

The model parameters for AR(1:2,52:53,104:105)/MA(1) are :

$(1 + 0.29B^1 + 0.06B^2 - 0.72B^{52} + 0.16B^{53} - 0.40B^{104} - 0.01B^{105})w_t = -0.05x_t - 0.01x_{t-1} + (1 - 0.93B)a_t$

Figures 5.39 and 5.40 show the 26 Winter Week One-Step Ahead and 26 Winter Week Multi-Step Ahead for the forecasts based on a the model AR(1:2,52:53,104:105)/MA(1) which was generated using the first 9 and a half years actual data.

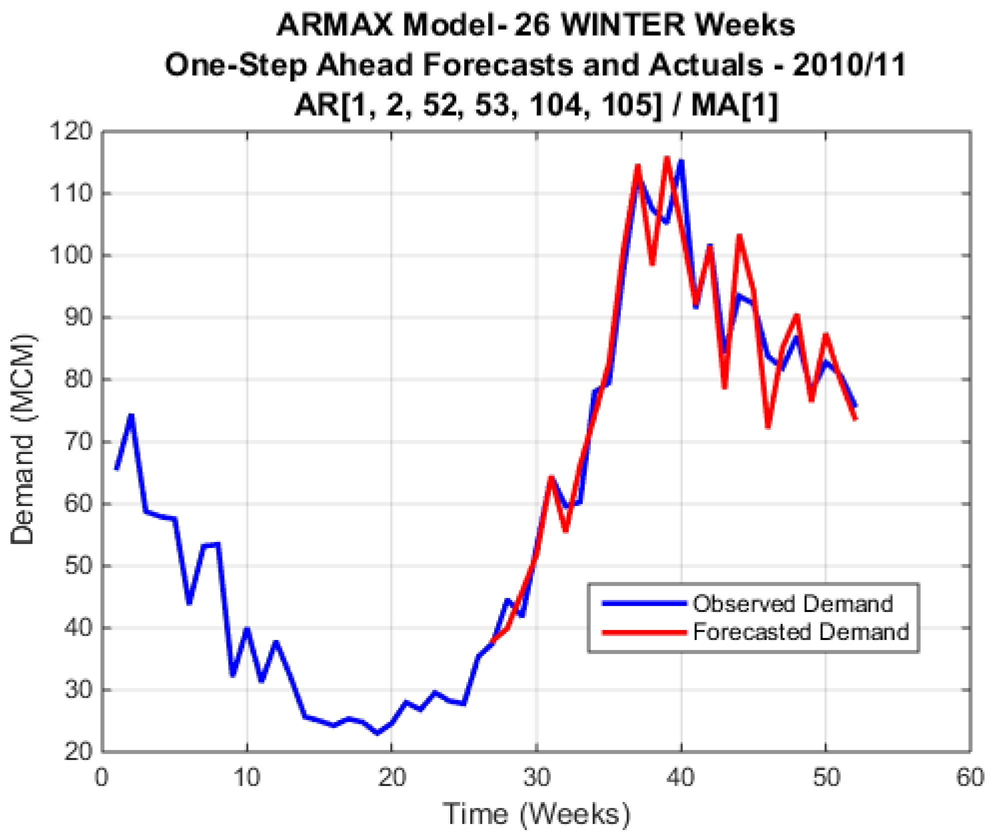

Figure 5.39: 26 Winter Weeks - One-Step Ahead Forecasts for 2010/2011

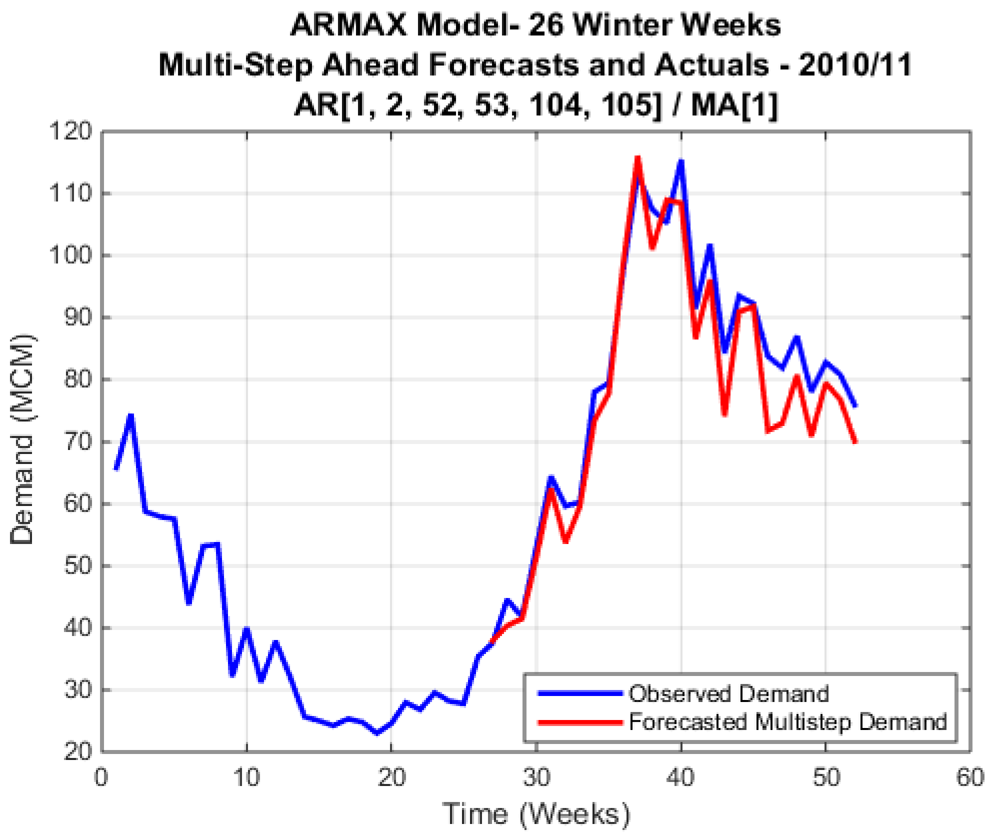

Figure 5.40: 26 Winter Weeks - Multi-Step Ahead Forecasts for 2010/2011

5.5.2 Summary of ARMAX Results

In conclusion, the model selected for the data from 2001 to 2011 was different than that used for the data from 1963 to 1973. This implies that the behavior of consumption changed from the earlier period to today. The new model meets the level of acceptance with a 52 week One-Step Ahead forecast MAPE of 5.28% and a Winter Weeks One-Step ahead MAPE of 4.41%. For the Multi-Step ahead forecast, the 52 Winter weeks Multi-Step ahead MAPE of 5.03% and the 26 Winter weeks MAPE of 3.69%. When modeling using 9 and a half years demand and temperature data and forecasting the Winter Weeks only for 2010/11, the One Step Ahead Forecast MAPE is 5.24% while the 26 week Multi-Step Ahead MAPE is 5.41%. The next section will apply the NARMAX methodology to the same data. All the One-Step Ahead results are better than the Persistence model, and all results are within the 4-6% levels set by DNV GL or better.

5.5.3 NARMAX Yearly Weekly Modeling and Forecasting with Actual Temperature

As in the previous Section, the X-Gas Weekly Demand and the Weekly Effective Temperatures described in Figures 5.33 and 5.34 are the start point for the analysis using the NARMAX methodology. The NARMAX model formula for this chapter are described in Section 3.3, and includes both delayed output variables and an input variable, temperature.

5.5.3.1 Transforming the data

The same transformations were performed as for the ARMAX modeling of yearly weekly demand and temperature (section 5.3.1.2) to produce stationarity, i.e.:

1. The Yearly Weekly Demand was transformed with a logarithmic transformation
2. The Yearly Weekly Effective Temperature required no transformation
3. The Log of the Yearly Weekly Demand and the corresponding Effective Temperature were then differenced by factors of 1 and 52

Figure 5.35 shows the transformed X-Gas Corrected Weekly Demand and Temperature, which will be treated by the NARMAX algorithm. The data, in blue, represents the transformed data which will be used for modeling and the red data is the comparison data for the forecast.

5.5.3.2 NARMAX Forecasting Future One-Step and Multi-Step Ahead Demand using Models from Section 5.4.3

The first test was to see how well the One-Step and Multi-Step Ahead forecast models, using the 1963 to 1973 data, from Section 5.4.3, would perform when applied to the new data from the years 2001 to 2011.

The results for both the 52 week and 26 Winter weeks One-Step and Multi-Step Ahead statistics are shown in Table 5.38. The results shown are acceptable, however a few lags of residuals are just above the 95% confidence level. Additional models were considered but no model was found which produced improved results. Figures 5.41 and 5.42 show the results of the 52 week One-Step and Multi-Step Ahead forecasts with the new data.

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week One-Step Ahead	-0.20	2.43	10.32	4.46%	7.08/6/16.12%	-8.48/46/-13.86%
26 week One-Step Ahead	-0.36	2.95	13.84	3.69%	5.82/18/6.22%	-8.48/20/-10.12%
52 week Multi-Step Ahead	0.14	2.39	8.65	4.79%	6.28/6/14.35%	-7.94/40/-6.88%
26 week Multi-Step Ahead	-0.42	2.25	7.89	2.75%	4.52/19/4.91%	-7.94/14/-6.88%

Table 5.38: Model Statistics for Weekly Demand Forecast. Model : NARMAX from Section 5.4.3

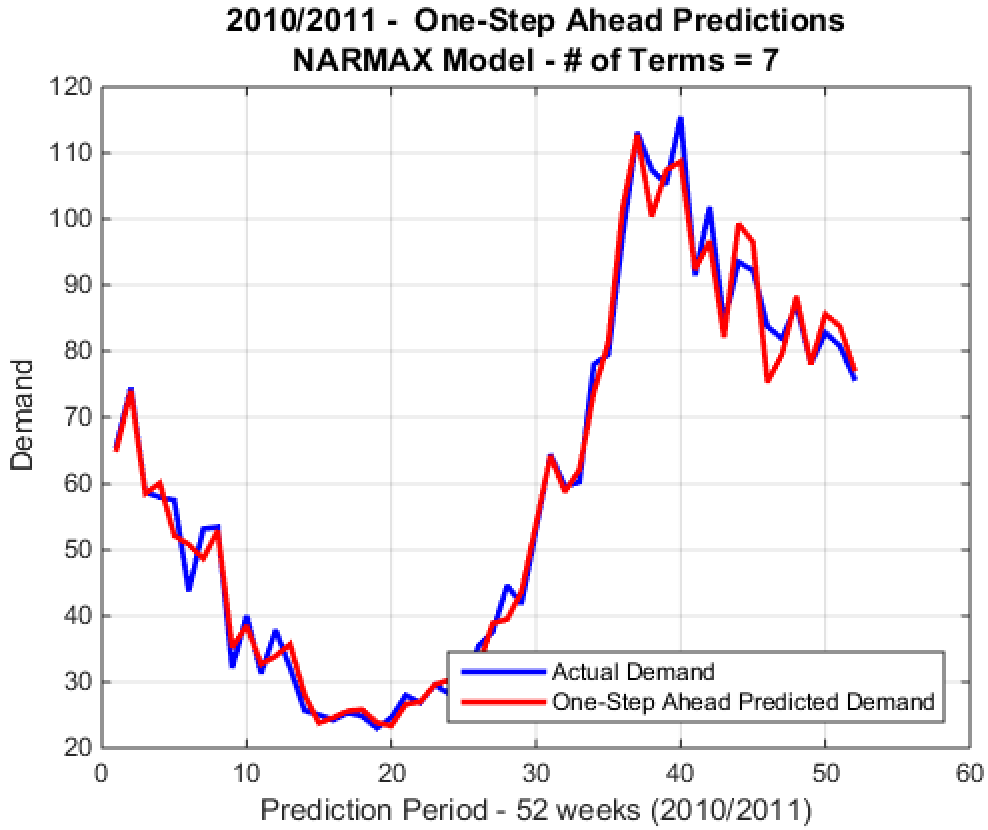

Figure 5.41: 52 week - One-Step Ahead Forecast for the 2nd Order NARMAX Model

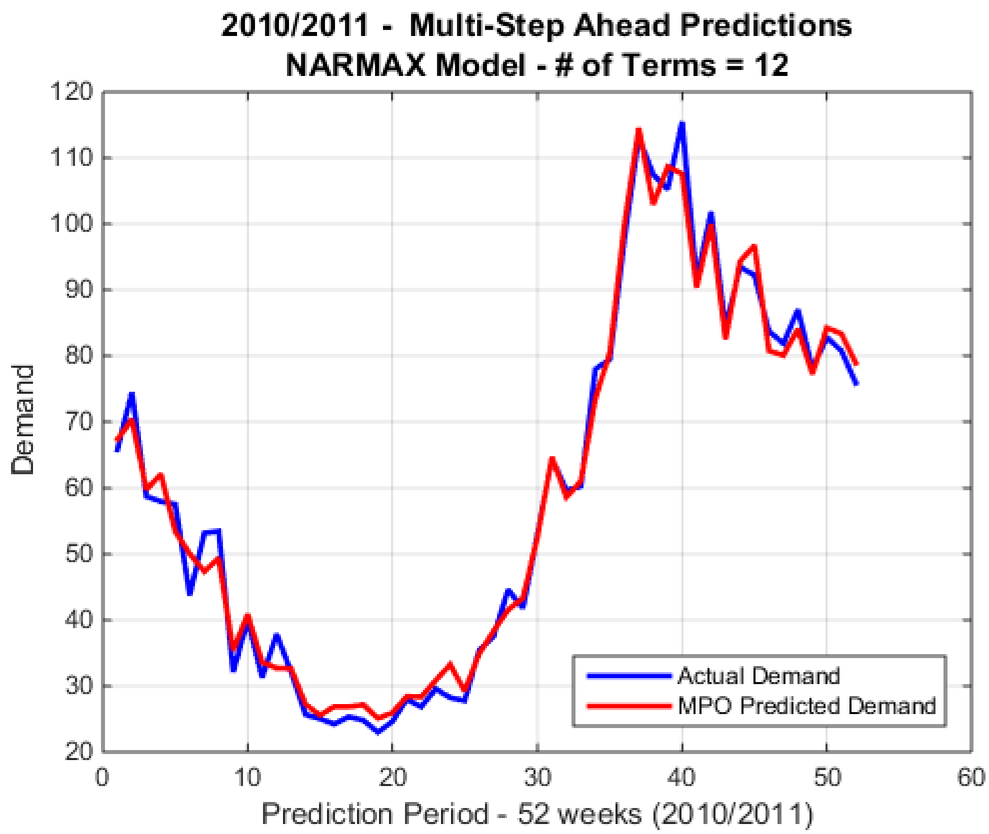

Figure 5.42: 52 week - Multi-Step Ahead Forecast for the 2nd Order NARMAX Model

The statistics for various shorter Multi-Step ahead predictions of 4, 6, 8 and 13 weeks using the 2nd Order NARMAX model above for the 52 weeks are shown in Table 5.39. As can be seen, there is no major improvement, however, the MAPE of 4.87% for the 13 week ahead is the smallest MAPE.

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
4 Week Ahead	0.63	4.06	22.42	8.14%	7.80/44/8.35%	-10.22/40/-8.85%
6 Week Ahead	-0.71	3.97	24.35	8.69%	6.89/16/28.45%	-11.76/7/-22.11%
8 Week Ahead	0.53	4.26	24.32	8.19%	11.67/45/12.66%	-9.90/32/-16.60%
13 Week Ahead	-0.41	2.69	11.70	4.87%	6.28/40/14.35%	-11.38/40/-9.85%

Table 5.39: 52 week - NARMAX Model Statistics for Various Multi-Step Ahead Weekly Demand Forecasts

5.5.3.3 NARMAX Modeling and forecasting ONLY the 26 Winter weeks

Repeating the methodology as described in Section 5.5.1.6, all 9 and half years of the weekly data is used for modeling. This new model will then be used to forecast the 26 Winter weeks from October 2010 to March 2011. Starting from the same variables (and delays) for the 9 year model, good One-Step Ahead forecasts were produced. However, the ACF of the residuals indicated $e(k-2)$ was present. Hence running the process with $y(k-1)$, $y(k-2)$, $y(k-52)$, $x(k)$, $x(k-1)$, $e(k-1)$, $e(k-2)$ and $e(k-52)$ produced an F value of 1.98 and a Q value of 44.58 with 43 df, indicating an adequate model. The model terms and values are shown in Table 5.40. Figure 5.43 shows the 26 week One-Step Ahead forecast and the associated statistics for the 2nd Order NARMAX model are shown in Table 5.41.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.048	73.05
2	e_{k-52}	0.50	6.60
3	e_{k-1}	0.08	3.42
4	x_{k-1}	-0.03	2.47
5	y_{k-52}	-0.06	0.62
6	y_{k-1}	-0.43	0.34
7	e_{k-2}	0.39	0.59
8	y_{k-2}	0.09	0.42
9	$y_{k-1} * y_{k-52}$	-0.15	0.07

Table 5.40: 2nd Order NARMAX Model for One-Step Ahead Winter Forecasts

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	-0.18	3.07	14.64	3.77%	6.00/24/7.25%	-8.58/20/-10.25%

Table 5.41: Model Statistics for 26 Winter Weeks One-Step Ahead Weekly Demand Forecast

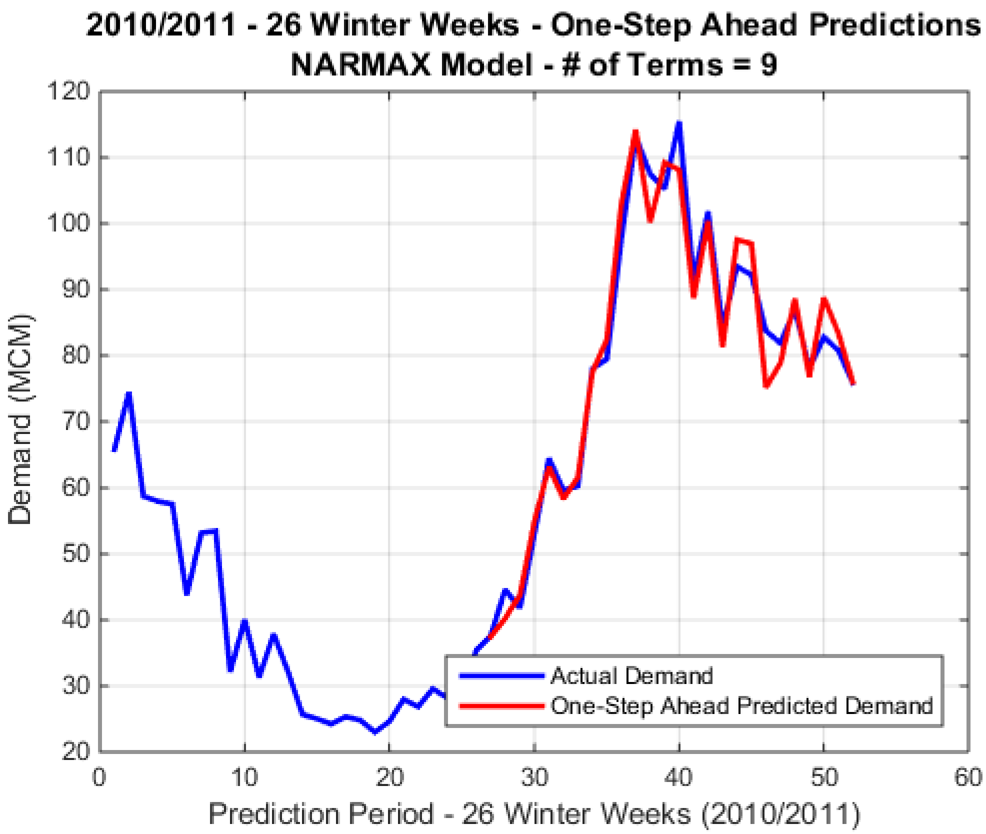

Figure 5.43: 26 Winter Weeks - One-Step Ahead Forecast periods for 2nd Order NARMAX Model

Applying the model in Table 5.40 to a Multi-Step Ahead forecasts, the forecast values started to drift away from the actuals. The ACF of the residuals showed lags 2 and 3 were significant, and hence combinations for both AR and MA terms, including these variables were tested. The new model is shown in Table 5.42. The Multi-Step Ahead forecast and associated statistics for this 2nd Order NARMAX model are shown in Figure 5.44 and Table 5.43.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.047	73.05
2	e_{k-52}	0.41	5.58
3	e_{k-1}	0.39	4.10
4	x_{k-1}	-0.01	2.27
5	y_{k-52}	-0.10	0.71
6	y_{k-1}	-0.13	0.14
7	y_{k-3}	-0.04	0.13
8	$y_{k-1} * y_{k-52}$	-0.17	0.11
9	$y_{k-3} * y_{k-52}$	0.19	0.10

Table 5.42: 2nd Order NARMAX Model for Multi-Step Ahead Winter Forecasts

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
2nd Order	0.32	3.18	16.08	3.77%	6.45/20/7.71%	-8.82/13/-8.38%

Table 5.43: Model Statistics for 26 Winter Weeks Multi-Step Ahead Weekly Demand Forecast

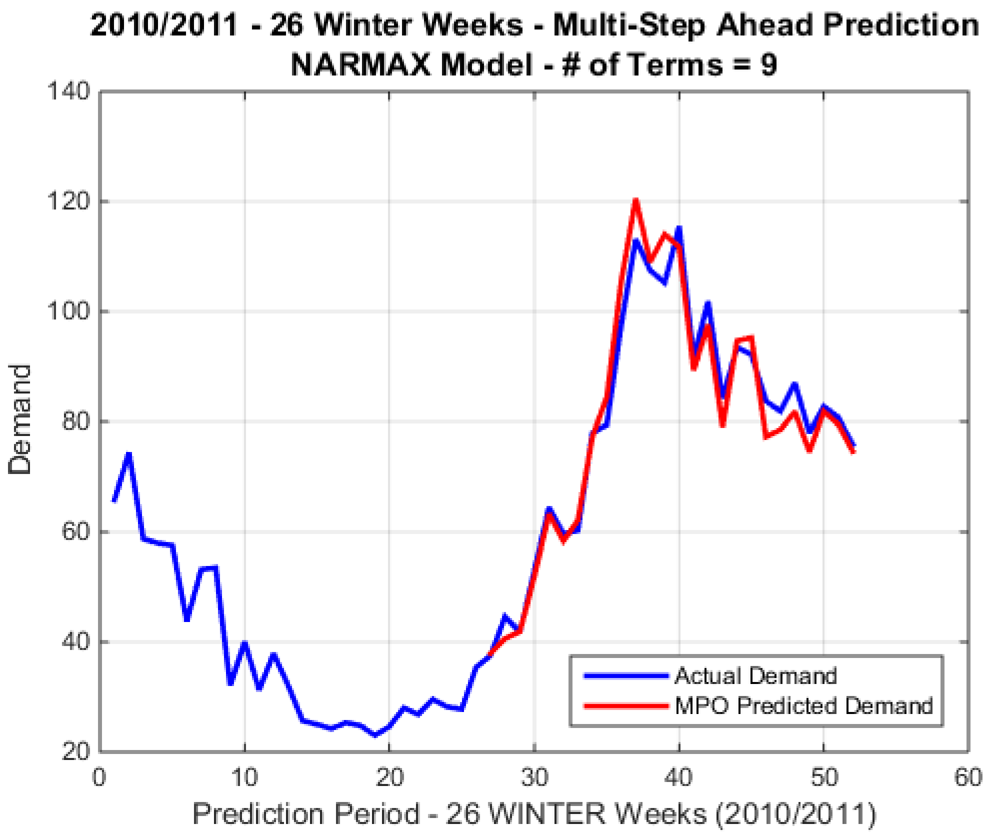

Figure 5.44: 26 Week - Multi-Step Ahead Forecast periods for the 2nd Order NARMAX Model

5.5.4 Summary of NARMAX Results

The NARMAX model developed for the 1970s data performed satisfactorily for the 2000s data. For the 52 week One-Step Ahead forecasts, the 2nd Order NARMAX model results are slightly improved over the ARMAX results (MAPE 4.46% vs 5.28%). Similarly for the statistics from the same model for only the 26 Winter weeks, the 2nd Order NARMAX forecast is slightly better (MAPE 3.69% vs 4.41%).

In the case of the Multi-Step Ahead forecast, the NARMAX 2nd Order Model produced slightly improved results for the 52 week ahead (4.79% vs 5.03%) as well as improved results for the 26 week (2.75% vs 3.69%).

Finally, running the methodology on 9 and a half years and forecasting ONLY the Winter weeks, the NARMAX models again, produced better MAPE results over the ARMAX models for both the One-Step (3.77% vs 5.24%) and Multi-Step Ahead forecasts (3.77% vs 5.41%).

5.6 Weekly Forecasting Summary and Conclusions

5.6.1 Introduction

The 4 previous sections have shown that the Polynomial NARMAX methodology has the capabilities to model Weekly Gas Demand, thus providing another methodology for software developers. Tables 5.44, 5.45 and 5.46 summarize the results of the forecasting for the different data sets and time horizons.

For the Benchmark test, the One-Step Ahead MAPE for the forecast values (for both ARMA(X) and NARMA(X) models) are all smaller than the Persistent Model values in Table 4.1, i.e. Winter Weeks values of 7.41% and 10.10%, and the Yearly Weeks values of 8.25% and 11.83%. Also the results for both the One-Step and the Multi-Step Ahead MAPE forecast values were all within the 4-6% value set by DNV GL.

The 2nd Order NARMAX models provided improved forecasts in all cases over ARX/NARX models produced by the FROLS algorithm. The NARMAX modeling described in these sections, shows that the Demand for week t is heavily dependent on the Temperature at time t. but is not sufficient to produce acceptable forecasts on its own. The NARMAX methodology then has to select terms which were very similar in the percentage they add to the system output. To facilitate the selection, domain knowledge was helpful to remove terms that made little sense to the forecast, in a similar way to the method used for ARMAX modeling. This then helped the FROLS algorithm to select significant and appropriate terms.

Note: Non-linear combinations of the residual terms were also tested in each case, and did not improve the forecasts.

5.6.2 Winter Weekly Forecast Summary

The results of the ARMA/ARMAX and the Polynomial NARMAX models, when only the Winter weeks were modeled, produced acceptable forecasts. However, for the first two sets of results in Table 5.44, the low AR terms, in the model, use the end of the previous year values at the start of the forecast year, which caused large errors to occur on the initial forecast values. This obviously had an impact on the value of the overall statistics. Removing this constraint, by modeling the full 52 weeks of the year, the polynomial NARMAX models produced superior One-Step Ahead forecasts and Multi-Step Ahead forecasts over the ARMAX models. These results are very encouraging, and have the possibility of being improved, as additional weather variables are introduced.

		26 Winter Week Forecast	
Data	Model	One-Step Ahead	Multi-Step Ahead
		MAPE %	MAPE %
1963-1973	ARMA Winter with SNET	3.71	4.53
	NARMA Winter with SNET	2.81	5.71
1963-1973	ARMAX Winter Model	3.86	3.38
	NARMAX Winter Model	3.38	3.99
1963-1973	ARMAX Yearly Model	3.06	2.98
	NARMAX Yearly Model	2.68	3.23
2001-2011	ARMAX Yearly Model	4.41	3.69
	NARMAX Yearly Model	3.69	2.75

Table 5.44: 26 Winter Weeks - Model Forecast MAPE Summary

5.6.3 Winter Weekly Forecast (using 9.5 years data) Summary

The results achieved for both the ARMAX and Polynomial NARMAX method when using 9 and a half years of data, and forecasting the 26 Winter weeks are again within the Benchmark tests. The results are similar for both methodologies for the 1963-73 data, but NARMAX produced improved results for the 2001-2011 data. The values are shown in Table 5.45.

		26 Week Forecast	
Data	Model	One-Step Ahead	Multi-Step Ahead
		MAPE %	MAPE %
1963-1973	ARMAX Yearly Model	2.95	3.24
	NARMAX Yearly Model	2.67	3.59
2001-2011	ARMAX Yearly Model	5.24	5.41
	NARMAX Yearly Model	3.77	3.77

Table 5.45: 26 Winter Weeks (9.5 Years Data) - Model Forecast MAPE Summary

5.6.4 Yearly Weekly Forecast Summary

Forecasting the full 52 weeks, the Polynomial NARMAX methodology produced slightly improved results over the ARMAX methodology for both One-Step Ahead and Multi-Step Ahead forecasts for both the 1963/73 and the 2001/11 data sets (Table 5.46). They are within the Benchmark Data of 4-6%. The large Over and Under Estimates for the 52 week forecasts occur mostly in the summer weeks, hence further improvements are possible by applying corrections to summer temperature data, which DNV GL perform, but which have not been applied to the data used in this thesis.

		52 Week Forecast	
Data	Model	One-Step Ahead	Multi-Step Ahead
		MAPE %	MAPE %
1963-1973	ARMAX Yearly Model	4.06	6.04
	NARMAX Yearly Model	4.04	5.94
2001-2011	ARMAX Yearly Model	5.28	5.03
	NARMAX Yearly Model	4.46	4.79

Table 5.46: 52 Weeks - Model Forecast MAPE Summary

Chapter 6

DAILY MODELING AND FORECASTING

6.1 Introduction

This chapter will analyze and produce forecasts for daily data. ARMAX and NARMAX methodologies (described in Chapter 3) will be applied to the data described in Section 4.3. A comparison of the forecast results will be shown at the end of this chapter. Since the period of particular interest in the gas industry is from October to March, when the gas demand approaches the system capacity, each method on each data set will concentrate the effort on producing the most effective forecast (in MAPE terms) for these Winter days.

This chapter is structured as follows:

Section 6.2 Daily Demand Data from Eastern Gas for the years 1970 to 1975.

Section 6.3 Daily Demand Data from X-Gas for the years 2001 to 2011.

Section 6.4 Summary of Results and Conclusions.

6.2 Daily Modeling and Forecasting with Actual Temperature (1970-1975)

6.2.1 ARMAX Modeling and Forecasting with Actual Temperature

The data in Figures 4.4 and 4.5 of Section 4.3 depicts the daily demand and temperature for the period 1/10/1970 to 20/6/1975. The data covers nearly 5 years of daily demand and temperature for Eastern Gas (1724 days). The first set of tests on this data was to decide how many days data should be used for modeling. Initially, the days from 1/10/1970 to the end of September 1974, were modeled, but it was found that fewer data points produced similar results, and hence a smaller modeling data set was selected. The start point for modeling was selected as Sunday 1/10/1972 and the end point was Saturday 28/9/1974 (728 days - 104 weeks). The forecasting period was then set as starting on Sunday 29/9/1974 and running through to 29/3/1975 (182 days (26 weeks)), covering the Winter period.

The ARMAX sections, below, are not validations of the work done for the original thesis in the 1970s (Antcliffe et al., 1975*d*), because the theory and application of ARMA with eXogenous inputs was in its infancy at that time, and required a complex two step process to model demand with an input variable (temperature). Today, the process has been simplified and improved and is supported in the Systems Identification Toolbox and Econometrics Toolbox in MATLAB, and hence this methodology has been used in the thesis. However, several aspects of the previous work are still relevant, i.e. the amount of data to use (described above), the generation of a stationary time series, and model terms relevant to the final model.

Figure 6.1 shows the daily demand and temperature data for Eastern Gas (noted as E-Gas in the Figures and Tables) for the period described above (i.e. 01/10/1972 to 29/03/1975). The data in blue is that used for modeling, and the data in red is that used for comparison with the forecast values.

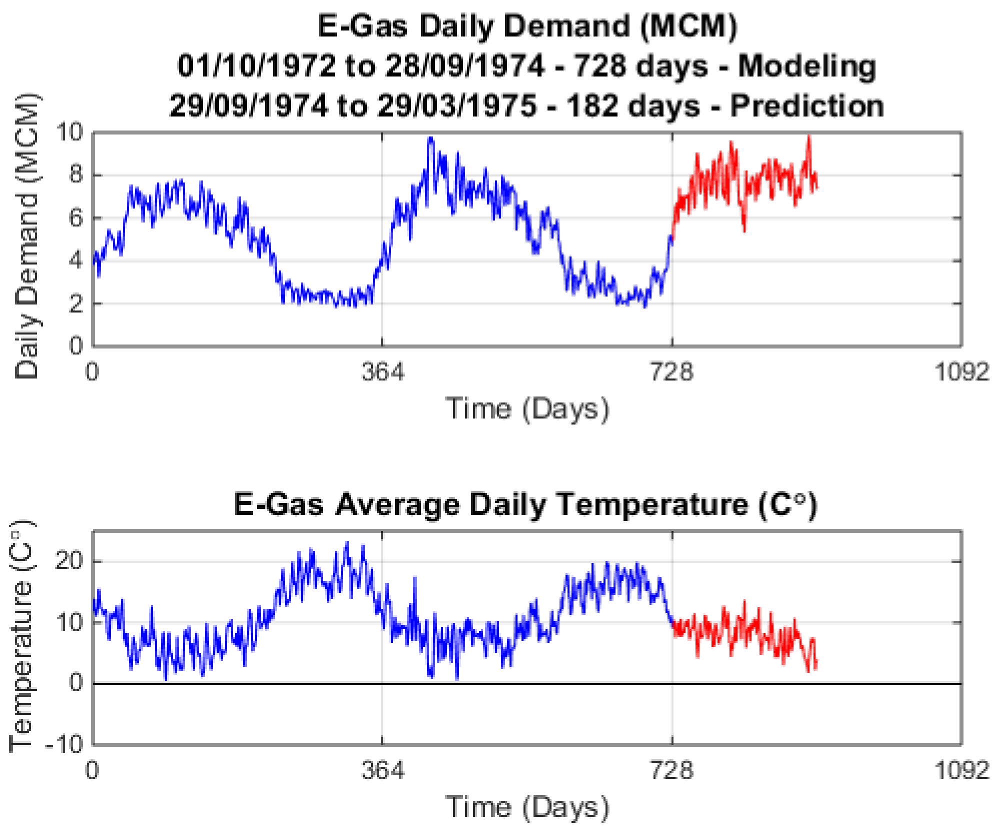

Figure 6.1: E-Gas Daily Demand and Temperature (1972-1975)

6.2.1.1 Transforming the data

As in the previous chapter, the data depicting Eastern Gas Daily Demand clearly shows the time series is non stationary, it has both a small growth component and a seasonal component. Several transformations were tested, including logarithmic and powers, to reduce the yearly variance growth. Again a natural logarithmic transformation was tentatively chosen.

The Autocorrelation Function (ACF) of the initial transformed data shows that the time series is still not stationary, a seasonal pattern with peaks every 7 days exists (showing the relationship week on week), and hence difference transformations are also required. Differencing of 1 and 7 are typically appropriate, i.e. $(\nabla\nabla_7 z_t)$, to produce stationarity.

For the Temperature time series, only a difference transformation of 1 and 7 was applied. The transformed series (Demand and Temperature) are shown in Figure 6.2 for the modeling data only (i.e. 728-8 data elements). Analysis of year on year means and variances confirm that stationarity requirements are met.

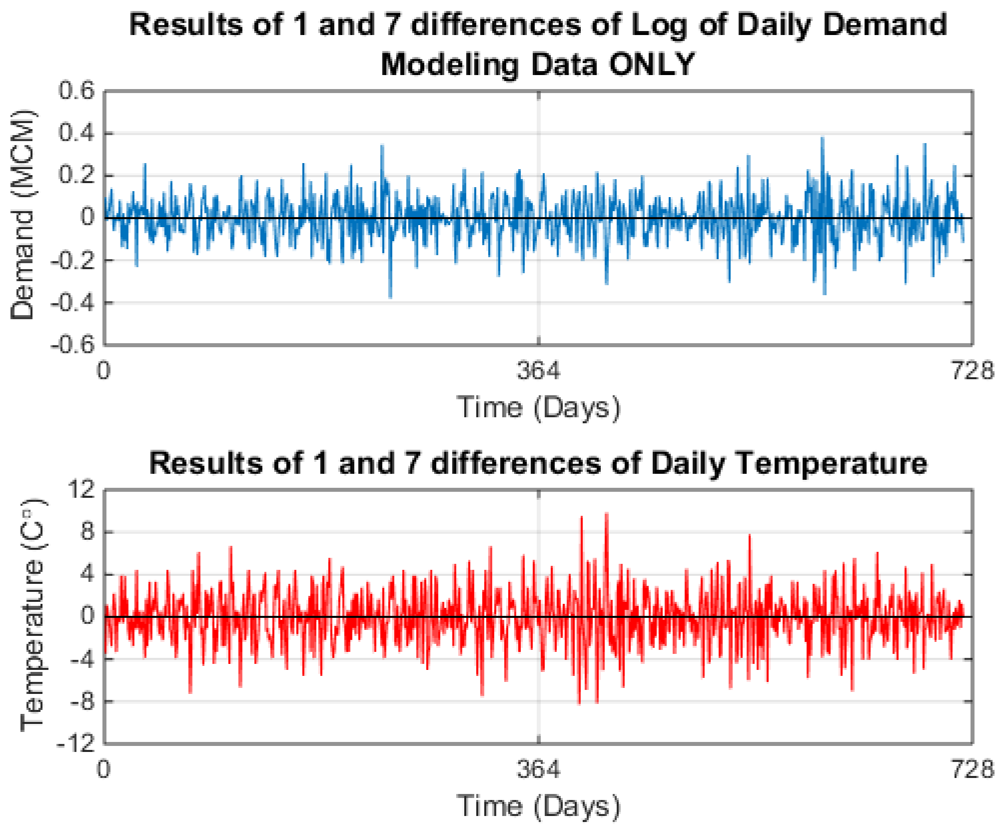

Figure 6.2: Transformed E-Gas Demand and Temperature - Modeling Data ONLY (1972-1974)

6.2.1.2 ARMAX Parameter Identification of the E-Gas Daily Demand with Temperature

Using the results of the ACF of both the transformed Daily Demand and Temperature and the CCF results, the lags 1, 2, 3, 4, 7, 8, 9, 10, 364 and 365 were significant. Several models were generated with AR and MA variables of these values, using the Economics Toolkit functions for ARMAX. Several delay values were applied to temperature series, a delay of 2 always produced the best results. This implies that future demand at time $t+1$ is dependent on the corresponding temperatures at $t+1$, t and $t-1$, as well as past demand values.

The model and parameter values generated, on the transformed data above, is described in Equation 6.1 which represents a AR(1)/MA(1,7,8,364,365) model with 2 time delays on temperature. The modeling statistics, for this model, are shown in Table 6.1.

$$
\begin{aligned}
(1+0.18B)w_t &= -(0.029+0.01B+0.004B^2)x_t \\
+(1-0.49B-0.86B^7+0.42B^8&+0.11B^{364}-0.024B^{365})a_t
\end{aligned}
\tag{6.1}
$$

Model ARMAX	Temperature Delay	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1)/MA(1,7,8,364,365)	2	-2119	2.14	91	389	711

Table 6.1: Model Fit Statistics for Daily Demand Model

6.2.1.3 ARMAX - Forecasting Future One-Step Ahead Demand

One-Step and Multi-Step Ahead forecasts were calculated for the 182 winter days, and the balanced set of metrics are shown in Table 6.2. The One-Step Ahead forecasts are shown in Figure 6.3, and the Multi-Step Ahead Forecasts are shown in Figure 6.4.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
One-Step Ahead	0.01	0.33	0.17	4.33%	1.09/32/12.01%	-1.37/96/-18.04%
Multi-Step Ahead	11.58	11.61	217.92	148.17%	34.96/180/428%	-0.76/9/-10.23%

Table 6.2: Model Statistics for Daily Demand Forecast starting 29/09/1974 for 182 days

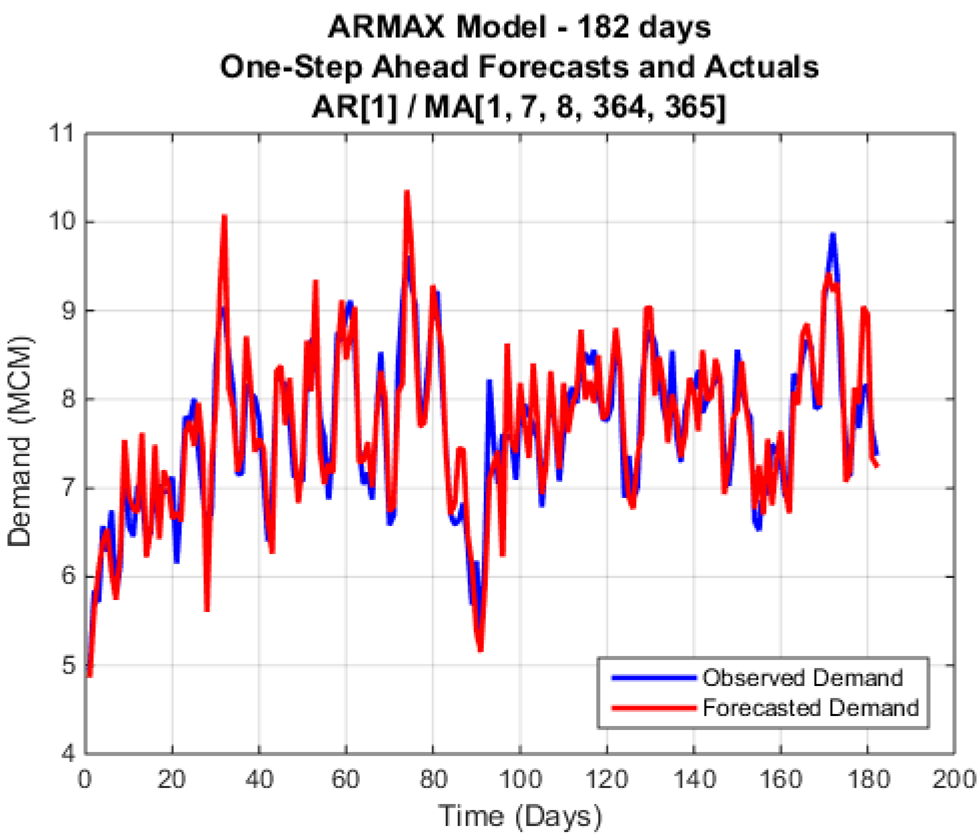

Figure 6.3: One-Step Ahead Forecast for model AR(1)/MA(1,7,8,364,365)

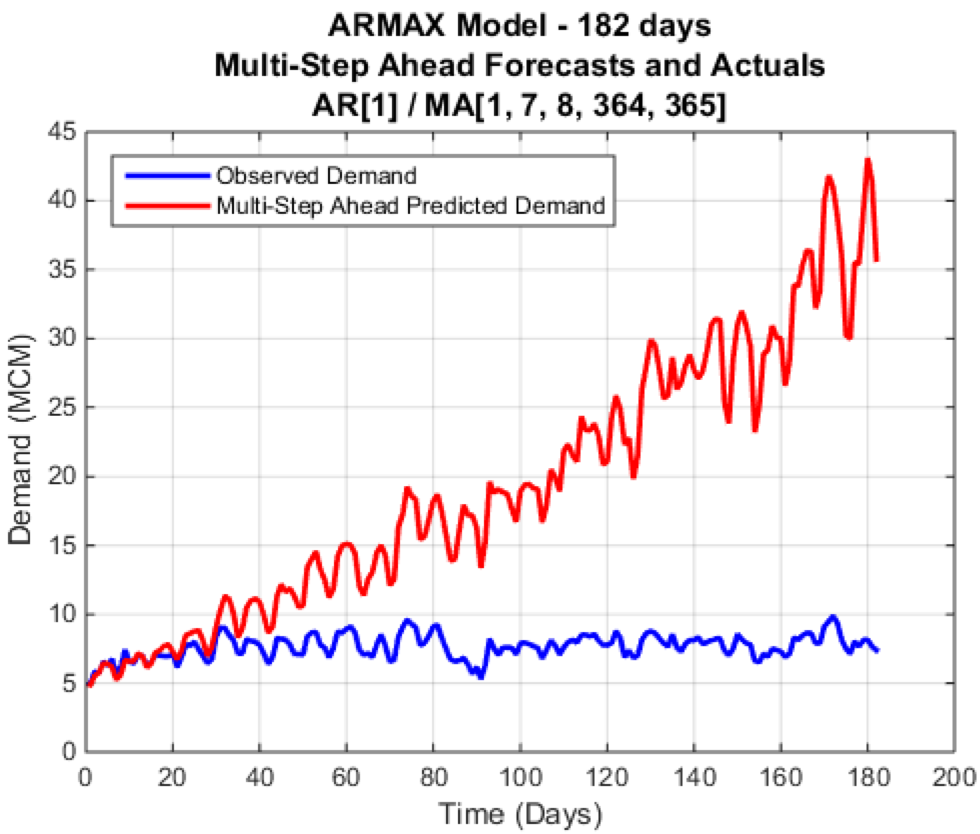

Figure 6.4: Multi-Step Ahead Forecast for model AR(1)/MA(1,7,8,364,365)

As can be seen in Figure 6.4 (the 182 day Multi-Step Ahead graph), the forecast values start to veer away from the actual values between day 14 and day 28. Hence the modeling process was repeated every 14 days through the Winter period, generating 13 two week forecasts. The results of the recalculations are shown in Tables 6.3 and 6.4 for the One-Step Ahead statistics for the different periods. Note that in each of the tables, the Period Date is the start date for the days ahead forecast. Additionally, the first 7-day ahead forecast for each period is also included.

One Step Ahead MAPE (%) Values for the period 29/09/1974 to 29/03/1975							
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	4.10	4.76	4.34	5.17	2.96	2.46	7.61
7	4.13	3.87	4.23	4.86	3.06	2.57	6.62

Table 6.3: One-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

One Step Ahead - MAPE (%) Values for the period 29/09/1974 to 29/03/1975)						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	2.19	3.05	1.91	3.03	2.96	4.11
7	1.42	2.93	1.57	2.95	3.34	3.65

Table 6.4: One-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

6.2.1.4 ARMAX - Forecasting Future Multi-Step Ahead Demand

Table 6.2 above, shows the overall Multi-Step Ahead MAPE is 148.17%. The first test was to see how well the 182 day model would forecast 14 days ahead (i.e. starting from known values every 14 days). The results are shown in Tables 6.5 and 6.6. Figures 6.5 shows the 14 days ahead results for the 182 winter days. Table 6.7 shows the average forecast statistics for these same periods. The aim in recalculating every 14 days is to improve on the results shown in Tables 6.5, 6.6 and 6.7. Note that the first 7 days Multi-Step Ahead forecasts for each period are also included in the tables.

Multi-Step Ahead - MAPE (%) Values for the period 29/09/1974 to 29/03/1975							
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	3.95	3.27	16.08	9.03	8.23	8.38	16.73
7	4.17	3.20	8.11	7.18	7.72	4.40	21.33

Table 6.5: 182 Day - Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 29/09/1974 to 29/03/1975						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	15.50	4.25	3.49	3.53	7.58	4.86
7	7.11	4.78	3.03	2.47	7.29	3.27

Table 6.6: 182 Day - Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
14 days	0.12	0.61	0.66	8.07%	2.58/110/33.84%	-1.73/80/-18.97%
7 days	0.06	0.53	0.51	7.10%	2.39/180/29.33%	-2.46/96/-32.41%

Table 6.7: Average Forecast Statistics for Various ARMAX Multi-Step Ahead Daily Demand starting 29/09/1974 for 182 days

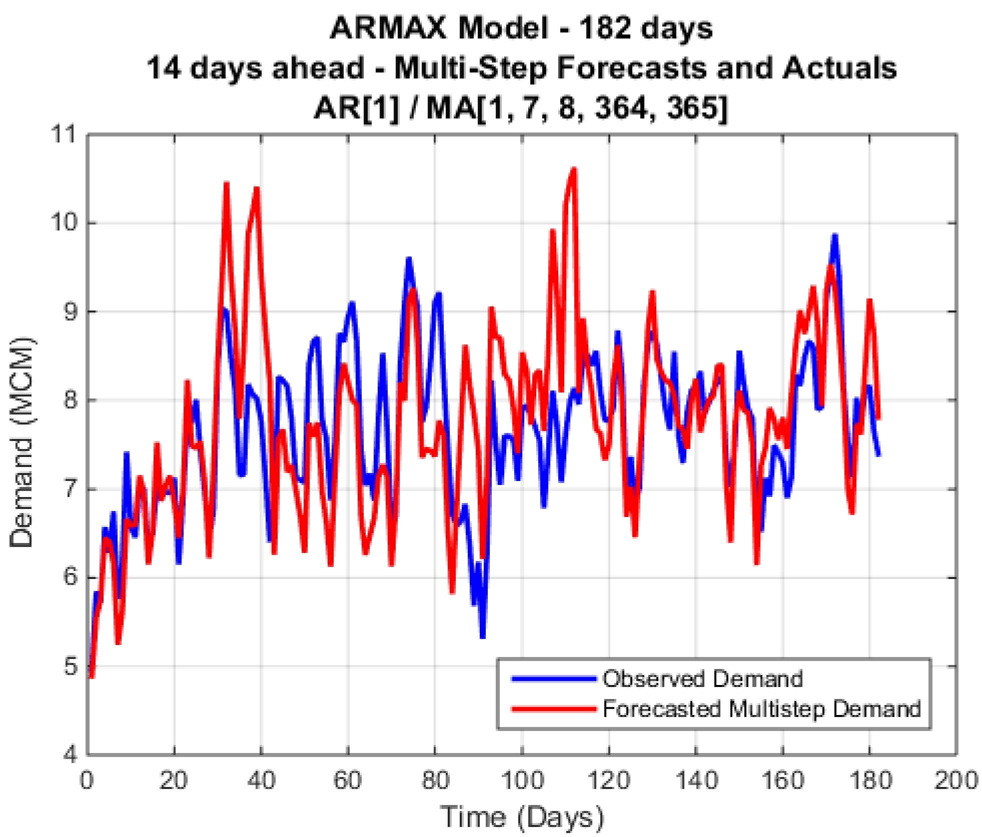

Figure 6.5: 14 day ahead - Multi-Step Forecast for 182 days
Model AR(1)/MA(1,7,8,364,365)

The modeling process was then run for each of the 14 day periods, and the results of the recalculations are shown in the Tables 6.8 and 6.9. In this case the forecasts were calculated for 1 to 7 days ahead as well as the full 14 day interval.

Multi-Step Ahead - MAPE (%) Values for the period 29/09/1974 to 29/03/1975							
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	3.95	10.14	12.05	6.73	3.97	2.04	12.43
7	4.17	7.34	6.18	4.05	4.66	2.48	17.99
6	3.36	6.56	6.55	3.93	4.29	2.39	18.51
5	2.30	7.11	5.77	2.61	3.96	2.38	18.59
4	2.38	5.89	3.92	2.46	2.77	2.74	14.51
3	2.52	4.07	1.67	2.74	2.40	3.10	12.04
2	3.41	3.30	2.15	2.13	3.51	3.20	8.18
1	1.66	0.82	2.66	3.68	4.35	2.43	2.64

Table 6.8: Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 29/09/1974 to 29/03/1975						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	2.92	3.26	4.67	4.04	4.24	4.85
7	3.08	3.65	4.52	2.33	5.67	3.97
6	3.23	3.44	4.47	1.85	6.05	3.68
5	3.45	3.60	4.58	2.21	6.38	4.09
4	2.85	2.96	4.63	2.67	7.51	2.91
3	2.17	2.46	4.09	2.67	8.63	1.21
2	1.25	2.14	4.36	4.30	7.54	1.58
1	0.21	0.02	4.35	6.06	7.81	2.41

Table 6.9: Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

6.2.2 Summary of ARMAX Results

In conclusion, for the One-Step Ahead forecasts, the 14 day ahead MAPE (%) values in Tables 6.3 and 6.4 are all very similar to the full winter forecast value of 4.33% MAPE value in Table 6.2, except for the Christmas fortnight (Period 7). The 7 days ahead forecast is slightly lower than the 4.33% for each of the periods except the Christmas fortnight, again. This may indicate that re-evaluating the model more often (every 7 days, or less) could be a solution for improvement, except for the Christmas week which would need some manual intervention or special rules applied (which is the case for DNV GL).

Comparing the Multi-Step Ahead forecasts for each of the recalculated models in Tables 6.8 and 6.9 with the 14 days ahead forecasts of the original model in Tables 6.5 and 6.6 the recalculated models produce an improved or similar MAPE (%) values for 14 days ahead except for Periods 2, 3 and 7. These periods represent the start of the increased central heating period, and the Christmas fortnight. The remaining results in Tables 6.8 and 6.9 are mostly all lower than the MAPE (%) values in Table 6.7, indicating that re-calculating the model every 14 or 7 days improves the forecasting capabilities of the model. These results will now be compared to the NARMAX methodology using the same data.

6.2.3 NARMAX Modeling and Forecasting with Actual Temperature

As in the previous Section, the Eastern Gas Daily Demand and the Daily Effective Temperatures described in Figures 4.4 and 4.5 of Section 4.3 are the start point again for the analysis using the NARMAX methodology. The first set of tests on this data was to decide how many days data should be used for modeling with polynomial NARMAX. In contrast to the decision made in the ARMAX section, the days from Thursday 1/10/1970 to the end of September 1974, were modeled, and it was found that the results were superior using the complete data set. Hence the start point for modeling was selected as Sunday 4/10/1970 and the end point was Saturday 28/9/1974 (1456 days - 208 weeks). The forecasting period was then set as starting on Sunday 29/9/1974 and running through to 29/3/1975 (182 days (26 weeks)), covering the Winter period.

The NARMAX model formula for this chapter are described in Section 3.3, and includes both delayed output variables and an input variable, temperature.

6.2.3.1 Transforming the data

The same transformations were performed as for the ARMAX modeling of daily demand and temperature (Section 6.2.1.1) to produce stationarity, i.e.:

1. The Daily Demand was transformed with a logarithmic transformation
2. The Daily Effective Temperature required no transformation
3. The Log of the Daily Demand and the corresponding Effective Temperature were then differenced by factors of 1 and 7

Figure 6.6 shows the transformed Eastern Gas Daily Demand and Temperature modeling data, which will be used in this section.

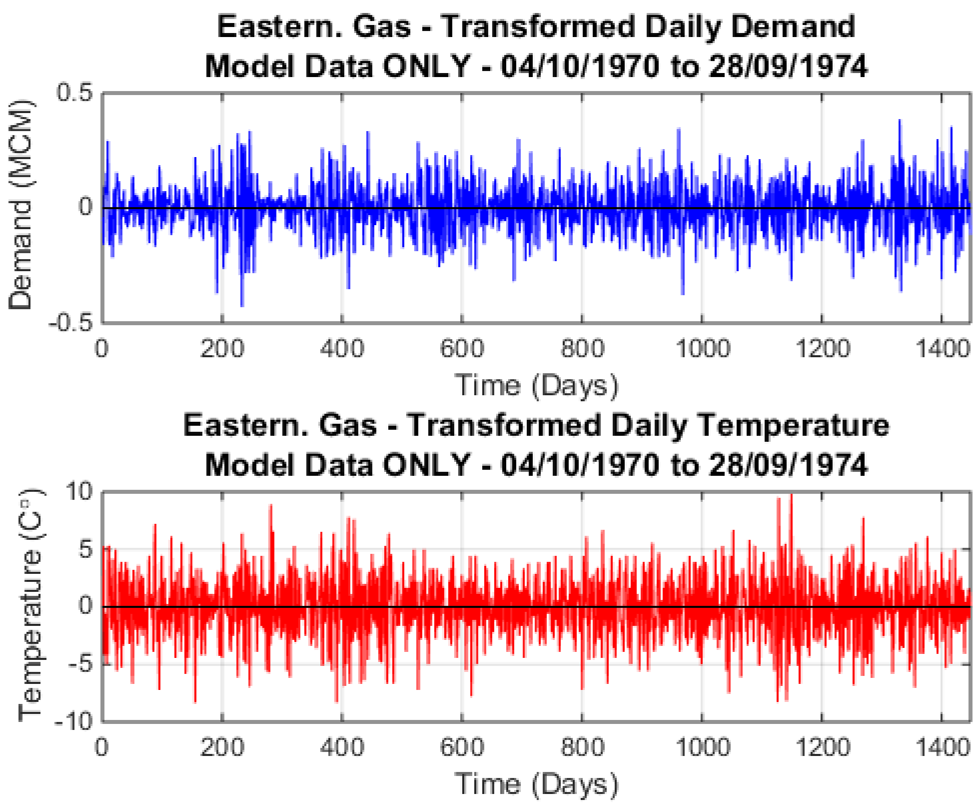

Figure 6.6: Transformed E-Gas Demand and Temperature - Modeling Data ONLY (1970-1974)

6.2.3.2 NARMAX Parameter Identification of the E-Gas Daily Demand with Temperature

The starting point for the NARMAX modeling was similar to that used in Section 5.4, i.e. identify the range of possible transformed Demand and Temperature variables from the ACF and CCF. The initial range selected was $y(k-1)$ to $y(k-28)$ and $x(k-1)$ to $x(k-28)$ inclusive.

As in Section 5.4.3, a first step analyzed a Linear model (ARX), followed by inclusion of residuals, thus creating an ARMAX model. Following the linear model analysis, 2nd and 3rd order terms were introduced (both without and with residuals NARX and NARMAX), to find the most appropriate model from a modeling and especially a forecasting perspective. The final results are explained in this section.

Running the modeling process first without residuals (i.e. an ARX Model), the following variables were selected, by the FROLS algorithm, for demand and temperature; $y(k-1)$ and $y(k-7)$ and $x(k)$, $x(k-1)$ and $x(k-2)$. A 2nd Order NARX model was then tested using the same variables as the linear ARX model which generated 20 terms. Seven terms were selected on reaching the thresholds, with an ERR total of 57%. The ACF of the residuals showed that there was additional information, to be modeled, especially around lags 2, 7, 14 and 364. The CCF of the Input (Temperature) and the Residuals showed significant lags at 7 and 14.

Several combinations of the Moving Average terms $e(k-1)$, $e(k-2)$, $e(k-7)$, $e(k-14)$, and $e(k-364)$ were tested. Two runs were required to attain no significant values in the Validity tests and twelve terms were selected at the threshold with the ERR total of 72.41%. The modeling statistics were:

1. F Statistic with 311 data values = 4.04

2. Q Statistic = 659 with 716 degrees of freedom (df) which shows an adequate model (700 df χ^2 value is 762 at 5% level, and the 750 df is 814 at 5%).

The terms selected for the model are shown in Table 6.10.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.026	44.87
2	e_{k-7}	0.711	16.82
3	x_{k-1}	-0.017	5.35
4	y_{k-1}	-0.287	2.53
5	x_{k-2}	-0.006	1.28
6	e_{k-364}	-0.141	0.76
7	e_{k-2}	0.0791	0.20
8	$y_{k-1} * y_{k-7}$	-0.482	0.13
9	$x_{k-1} * x_k$	-0.001	0.13
10	y^2_{k-7}	0.174	0.12
11	$x_{k-1} * y_{k-1}$	0.021	0.11
12	y^2_{k-1}	0.272	0.10

Table 6.10: Results of the FROLS algorithm for the 2nd Order NARMAX Model

6.2.3.3 NARMAX Forecasting Future One-Step Ahead Demand

One-Step Ahead Predicted Output for the NARMAX model are shown in Figures 6.7 for the 182 days of winter starting from 29/09/1974 through to 29/03/1975 and the corresponding forecast statistics in Table 6.11.

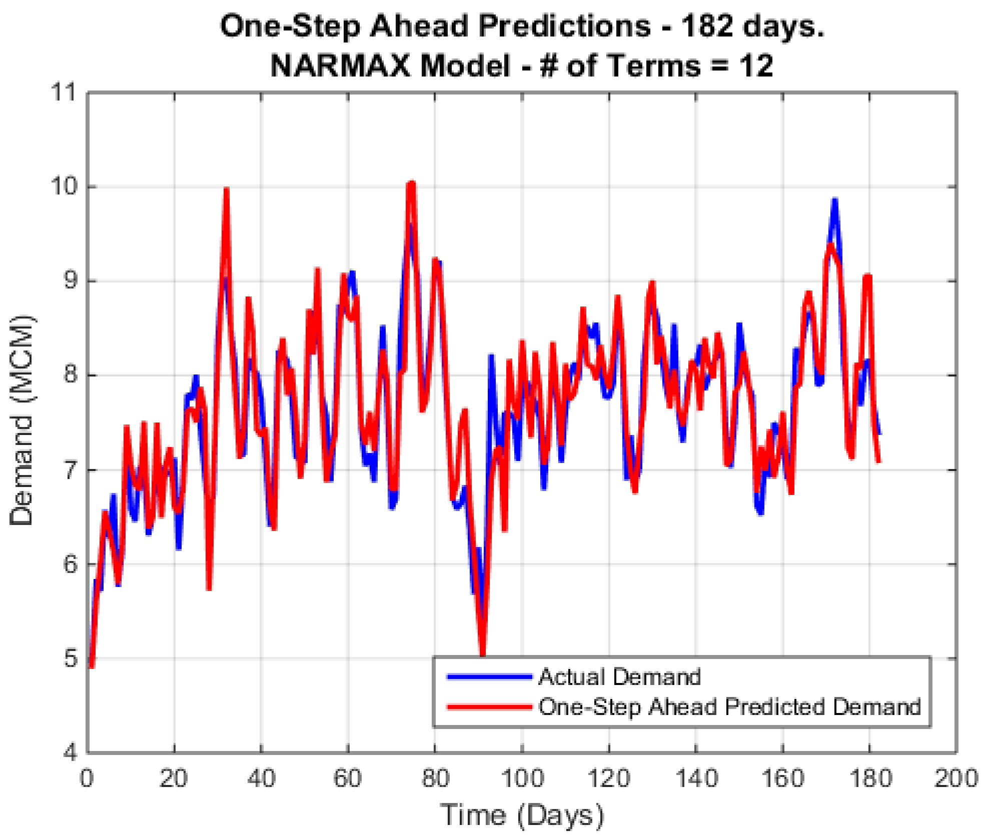

Figure 6.7: 182 Day One-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
182 Day	0.02	0.31	0.15	4.12%	0.99/32/11.00%	-1.34/93/-16.27%

Table 6.11: 182 Day - One-Step Ahead Forecast Statistics for Daily Demand (2nd Order NARMAX Model)

As in Section 6.2.1.2, the choice was made to re-evaluate the model every 14 days, and confirmed later with the Multi-Step Ahead forecast (see Figure 6.8). The MAPE for the 14 and 7 One-Day Ahead forecasts for each of the recalculated models is shown in Tables 6.12 and 6.13.

	One Step Ahead MAPE (%) Values for the period 29/09/74 to 29/03/1975						
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	3.45	4.60	4.36	4.34	2.99	2.17	7.86
7	3.10	3.81	3.49	4.26	2.60	2.43	7.64

Table 6.12: One-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 1

One Step Ahead - MAPE (%) Values for the period 29/09/74 to 29/03/1975						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	2.29	2.97	1.98	2.48	3.45	4.12
7	1.26	2.82	2.33	2.52	3.96	3.42

Table 6.13: One-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 2

6.2.3.4 NARMAX Forecasting Future Multi-Step Ahead Demand

Starting from the 2nd Order NARX model (described above), the analysis of the Multi-Step Ahead residuals (ACF, CCF and Nonlinear Validity tests) , indicate again that the following Moving Average terms $e(k-1)$, $e(k-2)$, $e(k-7)$, $e(k-14)$, $e(k-21)$ and $e(k-364)$ were significant. Including these residual terms and running the process to stability found that the $e(k-1)$, $e(k-7)$ and $e(k-364)$ produced the best forecast results. The ERR total was 72.5% and the ACF, CCF and the three Nonlinear Validity tests showed a small number of significant lags but each close to the significant level. The model terms and values are shown in Table 6.14.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.027	44.87
2	e_{k-7}	0.72	18.25
3	x_{k-1}	-0.014	5.47
4	e_{k-1}	0.10	1.92
5	e_{k-364}	-0.14	0.66
6	x_{k-2}	-0.005	0.43
7	y_{k-1}	-0.16	0.44
8	$y_{k-1} * y_{k-7}$	-0.43	0.15
9	$x_{k-1} * x_k$	-0.001	0.10
10	x_{k-1}^2	-0.001	0.08
11	x_{k-2}^2	0.005	0.09
12	y_{k-7}^2	0.11	0.03

Table 6.14: Results of the FROLS algorithm for the 2nd Order NARMAX Model

This model was then used to forecast future values. The 182 day forecast (Figure 6.8) shows again, that the values start to veer from the actuals between 14 and 28 days. The forecast statistics are shown in Table 6.15.

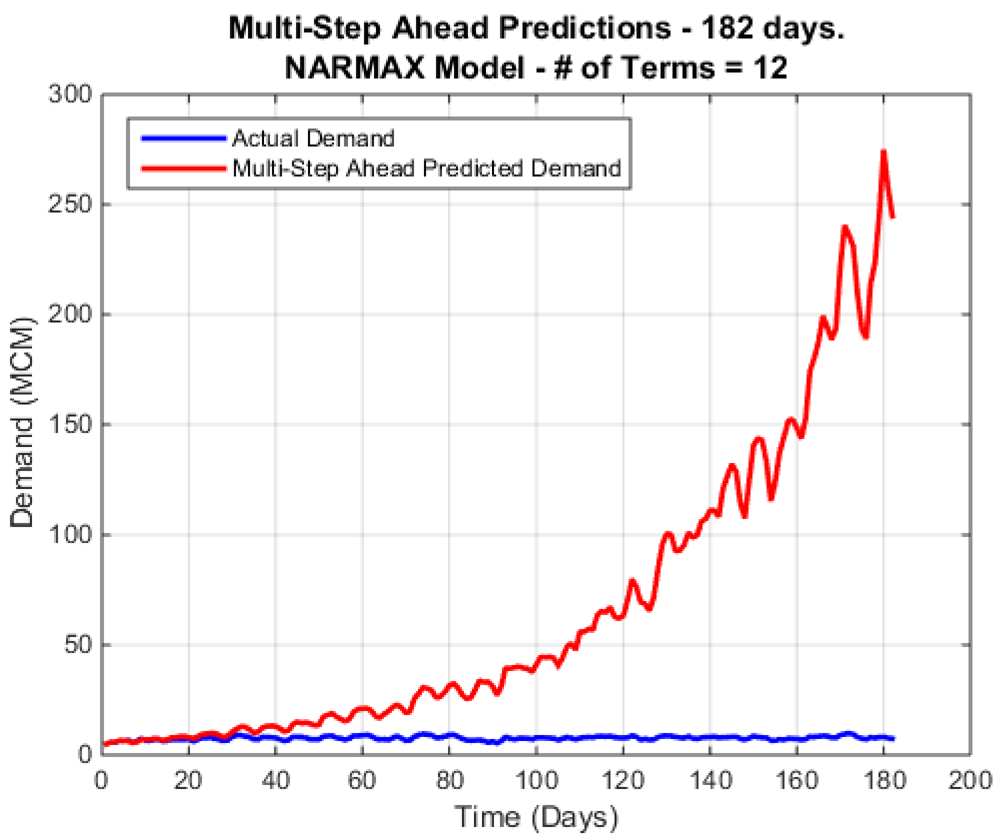

Figure 6.8: 182 Day Multi-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
182 Day	57.71	57.73	7787	731%	266/180/3271%	-045/6/-6.69%

Table 6.15: 182 Day - Multi-Step Ahead Forecast Statistics for Daily Demand (2nd Order NARMAX Model)

14-day Multi-Step Ahead forecasts were then calculated using the model in Table 6.14 and the MAPE results for each period are shown in Tables 6.16 and 6.17. Table 6.18 shows the average forecast statistics for the 14-day Multi-Step Ahead forecast, and Figures 6.9 graphs the results. The aim in recalculating every 14 days is to improve on the results shown in Tables 6.16, 6.17 and 6.18.

Multi-Step Ahead - MAPE (%) Values for the period 29/09/74 to 29/03/1975							
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	4.53	3.22	15.19	7.38	6.60	9.23	18.06
7	3.14	3.04	7.24	6.32	6.76	5.43	22.46

Table 6.16: 182 Day - Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 29/09/74 to 29/03/1975						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	16.49	3.10	3.22	3.85	5.55	4.65
7	7.52	4.12	2.70	2.39	6.31	3.05

Table 6.17: 182 Day - Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 2

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
14 days	0.14	0.59	0.65	7.78%	2.71/110/35.57%	-1.87/80/-20.50%
7 days	0.06	0.52	0.49	6.90%	2.25/ 89/39.53%	-2.51/96/-32.97%

Table 6.18: NARMAX Average Model Statistics for Various Multi-Step Ahead Daily Demand Forecasts starting 29/09/1974 for 182 days

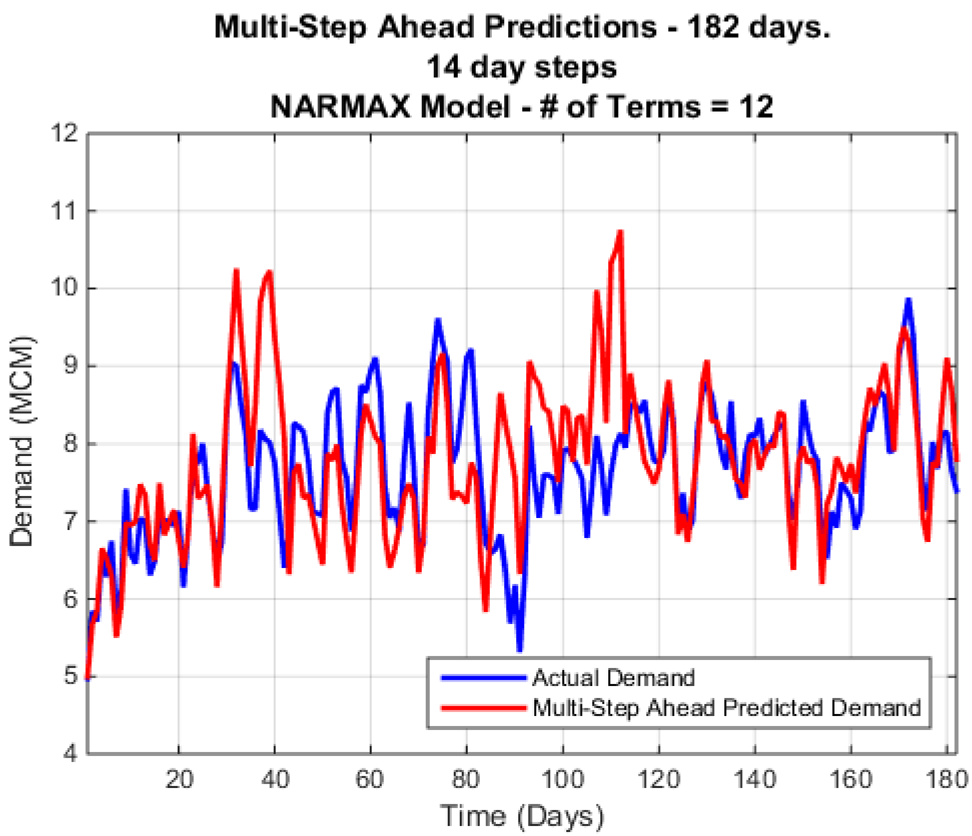

Figure 6.9: 14 day ahead - Multi-Step Forecast for 182 days - 2nd Order NARMAX Model

The model was recalculated every 14 days as in Section 6.2.1.4. The MAPE for the different Multi-Day Ahead forecasts for each of the 14 days ahead models is shown in Tables 6.19 and 6.20. In this case the forecasts were calculated for 1 to 7 days ahead as well as the full 14 day interval. It was noted that the FROLS algorithm modified the model in Table 6.14 slightly each period, adding and removing terms, as well as changing the parameter values, thus adapting the model Period on Period. This will be discussed in Chapter 7.

Multi-Step Ahead - MAPE (%) Values for the period 29/09/74 to 29/03/1975 (182 days)							
Days Ahead	Period 1 29/09/74	Period 2 13/10/74	Period 3 27/10/74	Period 4 10/11/74	Period 5 24/11/74	Period 6 08/12/74	Period 7 22/12/74
14	4.53	6.30	8.49	3.57	4.23	5.01	15.96
7	3.14	4.45	4.66	2.74	4.68	2.78	20.47
6	2.91	3.38	5.08	3.04	4.60	2.87	20.96
5	2.16	4.02	5.15	2.27	3.79	3.09	21.06
4	1.77	3.14	3.69	2.45	2.40	3.42	16.42
3	1.98	2.45	2.14	1.99	1.86	3.28	12.74
2	1.71	2.45	1.40	2.51	2.55	0.74	8.50
1	0.46	0.77	1.28	3.45	1.85	0.79	3.69

Table 6.19: Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 29/09/74 to 29/03/1975 (182 days)						
Days Ahead	Period 8 05/01/75	Period 9 19/01/75	Period 10 02/02/75	Period 11 16/02/75	Period 12 02/03/75	Period 13 16/03/75
14	3.48	2.50	3.56	5.23	3.42	5.07
7	1.99	3.02	3.49	2.96	4.67	4.25
6	1.83	2.96	3.32	2.80	4.86	4.04
5	1.89	3.00	3.43	3.05	5.15	4.34
4	1.53	2.11	3.81	3.74	6.43	3.33
3	1.02	1.62	3.37	4.48	9.93	1.70
2	1.08	2.27	3.59	5.92	7.28	1.45
1	0.61	0.09	5.60	6.78	7.74	1.95

Table 6.20: Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 2

6.2.4 Summary of NARMAX Results

In conclusion, for the One-Step Ahead forecasts, the 14 day ahead MAPE (%) values in Tables 6.12 and 6.13 are all very similar to the full winter forecast value of 4.12% MAPE value in Table 6.11, except for the Christmas fortnight (Period 7). The 7 days ahead forecasts are slightly lower than the 4.12% for each of the periods except the Christmas fortnight, again.

Comparing the Multi-Step Ahead forecasts for each of the recalculated models in Tables 6.19 and 6.20 with the 14 days ahead forecasts of the original model in Tables 6.16 and 6.17; the recalculated models produce an improved or similar MAPE (%) values for all the periods. The MAPE values were all lower than the average 14 days MAPE of 7.78% in Table 6.18 except for Periods 3 and 7, representing the drop in temperatures and the start of the Central Heating increase; and the Christmas fortnight. For the 7 day ahead forecasts, they were all below the 6.90% except the Christmas period.

The comparison of these results with the ARMAX modeling on the same data will be covered in Section 6.4.

6.3 Daily Modeling and Forecasting with Actual Temperature (2001-2011)

6.3.1 ARMAX - Daily Modeling and Forecasting with Actual Temperature

6.3.1.1 Introduction

Section 6.2.1 modeled and forecast the Eastern Gas Daily Demand based on data from the period 1970 to 1975. This Section will apply the same methodology to the data for X-Gas from 2001 to 2011.

As a reminder, Figures 4.6 and 4.7 of Chapter 4 show the original 10 years of daily data (Demand and Temperature) provided by DNV GL for the region X-Gas. Again, the first set of tests on this data was to decide how many days data should be used for modeling. Initially, the days from 29/09/2001 to the start of October 2010, were modeled, but it was found that fewer data points produced similar results, and hence a smaller modeling data set was selected. The start point for modeling was selected as Saturday 04/10/2008 and the end point was Friday 01/10/2010 (728 days - 104 weeks). The forecasting period was then set as starting on Saturday 02/10/2010 and running through to Friday 01/04/2011 (182 days (26 weeks)), covering the Winter period.

The demand data and the associated daily effective temperatures are shown in Figure 6.10. The data, in blue, represents the data which will be used for modeling and the red data is the comparison data for the forecast.

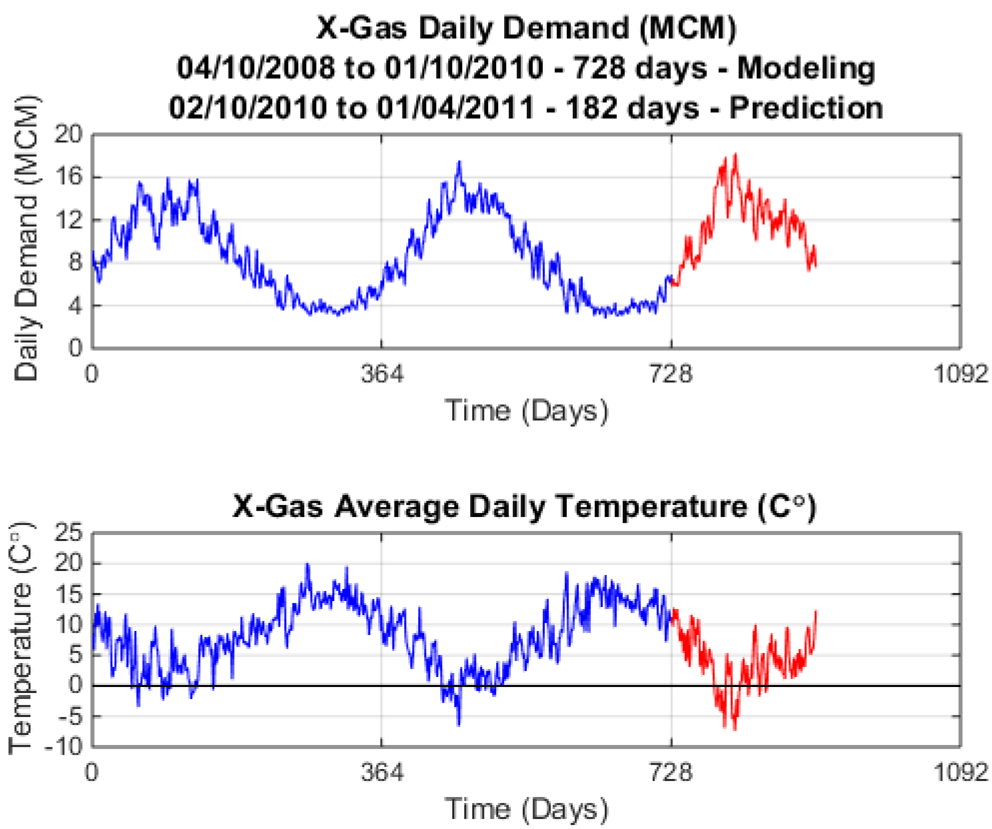

Figure 6.10: X-Gas Daily Demand and Temperature (2008 to 2011)

6.3.1.2 Transforming the data

As in Section 6.2.1, the data depicting X-Gas Daily Demand clearly shows this time series is non stationary, it has both a small growth component and a seasonal component. The growth component, however, for X-Gas is smaller than that of E-Gas, due to a stable population and a stable gas usage in 2000, compared the growth of the 1970s. We will see later that this has an impact on the variables selected in the model. The same transformations were selected as for E-Gas, i.e. a logarithmic transformation of X-Gas Demand and differencing of 1 and 7 for both the demand and temperature. The transformed data, used for modeling, is shown in Figure 6.11.

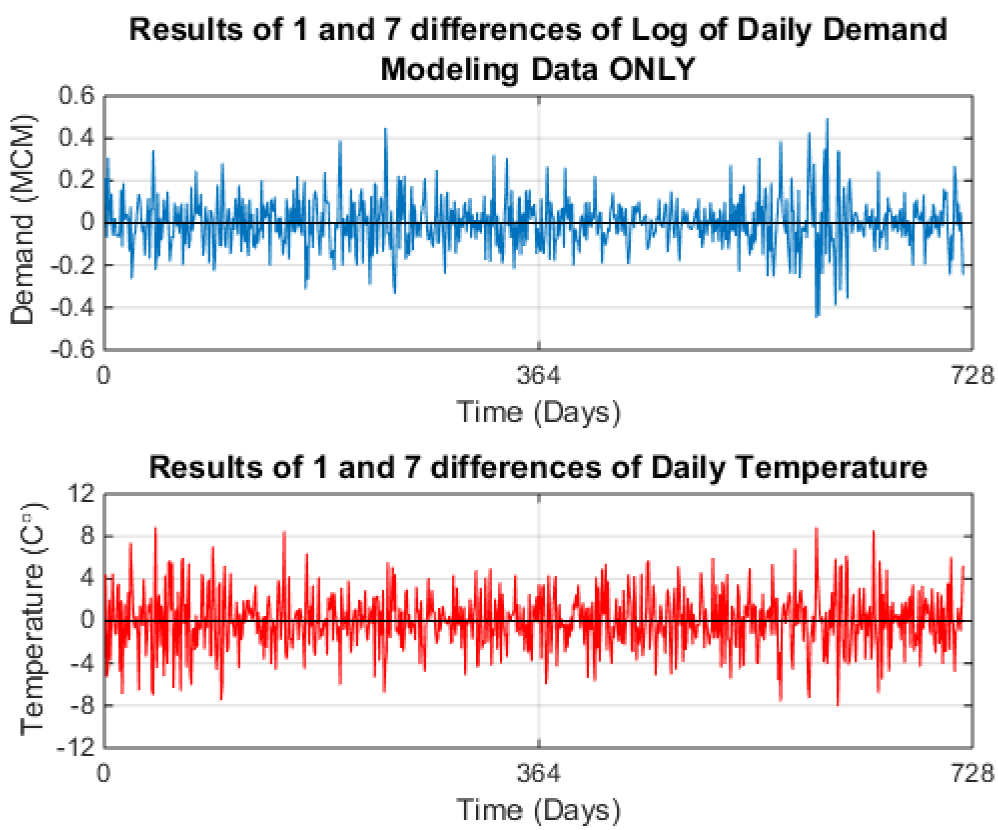

Figure 6.11: Transformed X-Gas Demand and Temperature - Modeling Data ONLY (2008-2010)

6.3.1.3 ARMAX Parameter Identification of the X-Gas Daily Demand with Temperature

The first test was to see how well the ARMAX model from Section 6.2.1.2, i.e. AR(1)/MA(1,7,8,364,365), would perform when applied to the new data. However, the ACF of both the transformed demand and temperature did not show significant lags at 364 or 365. This is probably due to the fact that the growth component of the X-Gas demand data for 2001-2011 is flatter than that of E-Gas for 1970-1975, hence alternative models were considered. The significant lags from the ACF and the PACF for the transformed demand were 1 and 7, and those for transformed Temperature were 2 and 7. Several models were generated with AR and MA variables of these values, combined with different delay values applied to temperature series. Again a delay of 2, for temperature, always produced the best results.

The generated models with similar modeling statistics were then used to forecast the 182 days for the winter period 2010 to 2011, and the forecast statistics calculated. The best model from a forecasting perspective was found to be AR(1,2)/MA(1,2,7,8,9), and temperature delay of 2. The ACF of the residuals found a few lags just outside the +/- 95% confidence limits, specifically lag 51, but adding these lags to the model parameters did not improve the results.

The model and parameter values generated for AR(1,2)/MA(1,2,7,8,9) model with 2 time-delays on temperature, is described in Equation 6.2 , and the modeling statistics, for this model, are shown in Table 6.21.

$$\begin{aligned}(1+0.61B+0.02B^2)w_t &= -(0.024+0.018B+0.005B^2)x_t \\ &+(1-0.97B+0.05B^2-0.86B^7+0.85B^8-0.05B^9)a_t\end{aligned} \tag{6.2}$$

Model ARMAX	Temperature Delay	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1,2)/MA(1,2,7,8,9)	2	-1917	2.92	51	411.94	710

Table 6.21: Model Fit Statistics for Daily Demand Model

6.3.1.4 ARMAX - Forecasting Future One-Step Ahead Demand

One-Step and Multi-Step Ahead forecasts were calculated for the 182 Winter days, and the balanced set of metrics are shown in Table 6.22. The One-Step Ahead forecasts are shown in Figure 6.12, and the Multi-Step Ahead Forecasts are shown in Figure 6.13.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
One-Step Ahead	0.02	0.68	0.69	6.09%	2.28/77/13.65%	-2.40/69/-14.79%
Multi-Step Ahead	17.45	17.46	551.68	154.78%	53.99/178/617%	-0.31/2/-4.72%

Table 6.22: Forecast Statistics for Daily Demand starting 02/10/2010 for 182 days

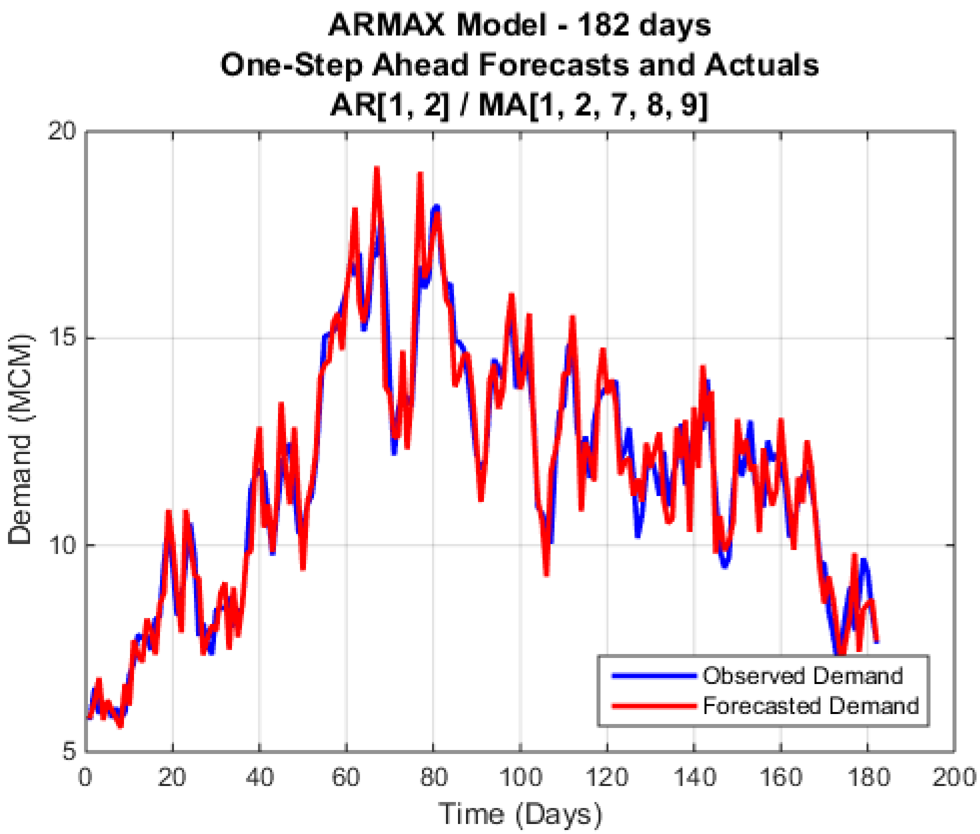

Figure 6.12: One-Step Ahead Forecast for model AR(1,2)/MA(1,2,7,8,9)

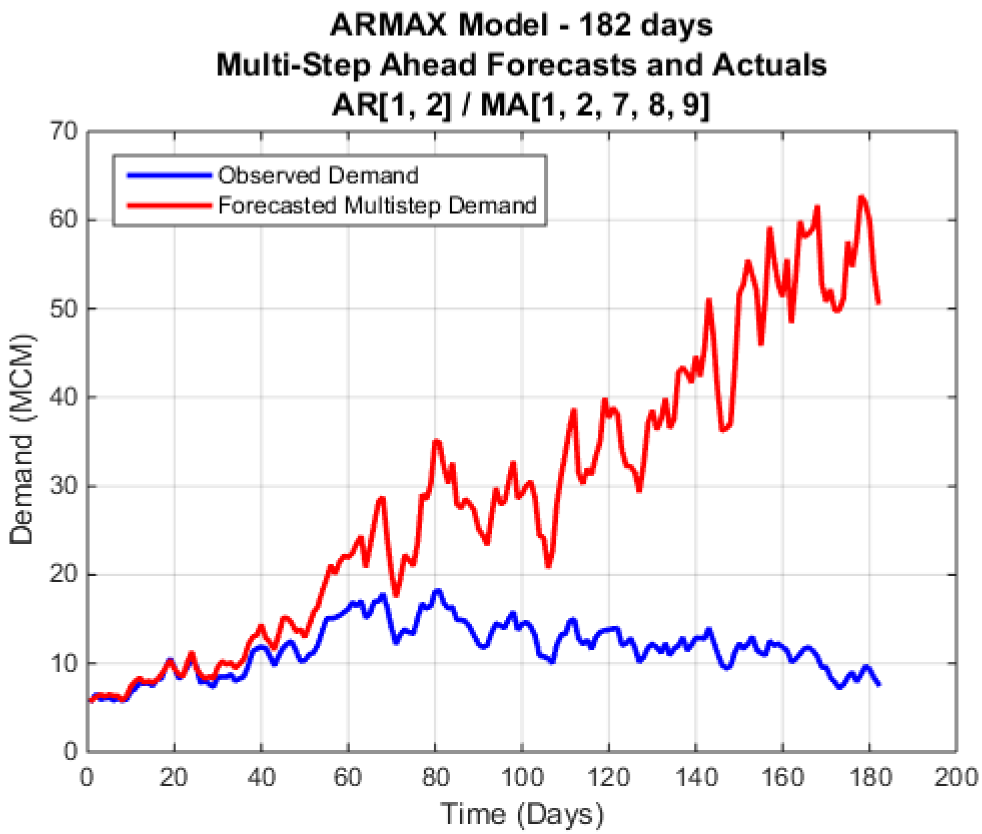

Figure 6.13: Multi-Step Ahead Forecast for model AR(1,2)/MA(1,2,7,8,9)

As with the Multi-Step Ahead forecasts for the 1970-75 period in Section 6.2.1.3, the 182 day Multi-Step Ahead forecast start to veer away from the actual values between day 14 and day 28 (Figure 6.13). Hence the modeling process was again repeated every 14 days through the Winter period, generating 13 two week forecasts. The One-Step Ahead MAPE results of the recalculations are shown in Tables 6.23 and 6.24 for the different periods. Note that in each of the tables, the period date is the start day for the days ahead forecasts and the first 7-day ahead forecast for each period is also included.

One Step Ahead MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	6.16	6.02	3.98	4.06	4.28	2.88	2.84
7	4.84	3.67	2.48	4.54	3.66	2.97	2.81

Table 6.23: One-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

One Step Ahead MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	4.92	3.34	5.60	3.97	5.35	8.08
7	3.13	3.55	3.84	2.88	5.65	5.52

Table 6.24: One-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

6.3.1.5 ARMAX - Forecasting Future Multi-Step Ahead Demand

Table 6.22 above, shows the overall Multi-Step Ahead MAPE is 154.78%. The first test was to see how well the 182 day model would forecast 14 days ahead (i.e. starting from known values every 14 days). The results are shown in Tables 6.25 and 6.26, where, again, the Period Date is the start date for each of the forecasts. Figures 6.14 shows the 14 days ahead results for the 182 winter days. Table 6.27 shows the average forecast statistics for these same periods. The aim in recalculating every 14 days is to improve on the results shown in Tables 6.25, 6.26 and 6.27. Note that the first 7 days Multi-Step Ahead forecasts for each period are also included in the tables.

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	5.34	5.42	6.39	17.88	4.50	14.83	14.33
7	5.23	4.93	7.16	10.43	3.43	6.00	8.96

Table 6.25: 182 Day - Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	11.42	9.78	37.32	6.14	9.24	7.93
7	6.36	5.70	25.40	4.62	9.01	7.57

Table 6.26: 182 Day - Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
14 days	0.58	1.42	4.26	11.58%	7.46/138/65.07%	-3.41/ 98/-21.57%
7 days	0.03	0.83	1.29	7.25%	4.91/132/43.82%	-2.23/159/-18.28%

Table 6.27: Average Forecast Statistics for Various ARMAX Multi-Step Ahead Daily Demand starting 02/10/2010 for 182 days

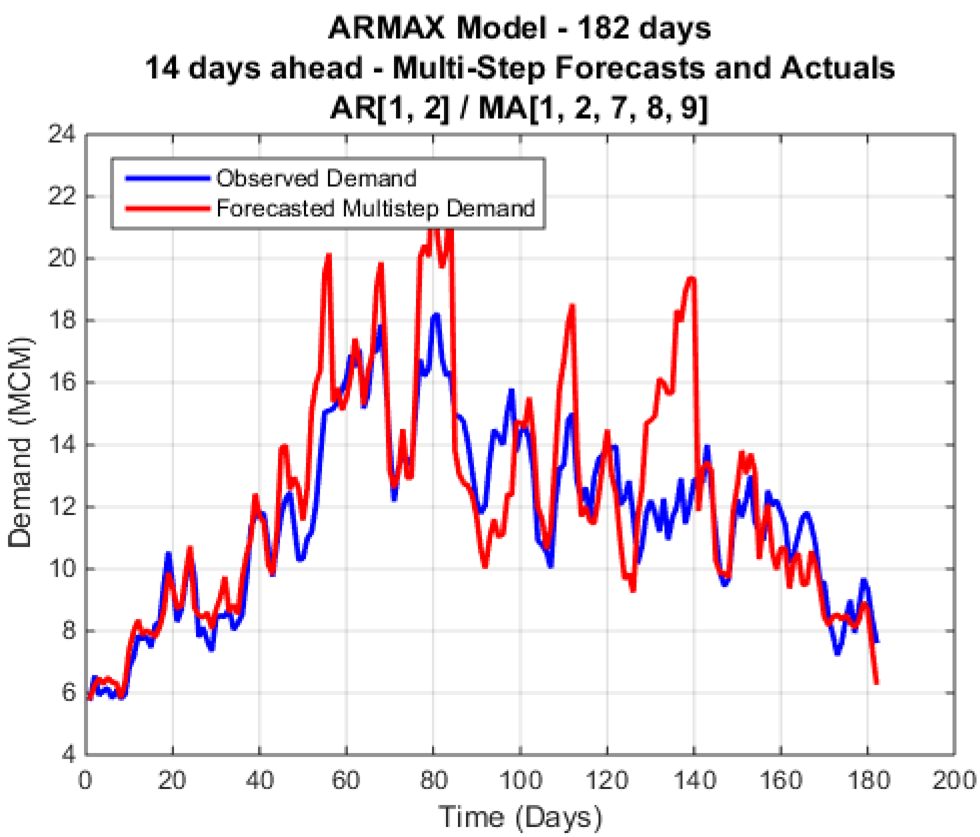

Figure 6.14: 14 day ahead - Multi-Step Forecast for 182 days
Model AR(1,2)/MA(1,2,7,8,9)

The modeling process was then run for each of the 14 day periods, and the results of the recalculations are shown in the Tables 6.28 and 6.29. In this case the forecasts were calculated for 1 to 7 days ahead as well as the full 14 day interval.

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	5.34	4.10	7.80	7.34	4.12	9.10	5.25
7	5.23	3.89	8.11	5.13	2.24	2.71	3.06
6	5.40	4.51	7.76	4.03	2.45	1.73	3.54
5	4.72	4.09	7.29	4.51	2.01	2.05	3.45
4	4.51	3.22	7.54	5.07	1.49	2.52	3.56
3	4.75	2.21	7.05	5.01	1.57	2.53	4.25
2	2.60	2.91	5.41	3.41	2.18	2.73	4.10
1	0.47	1.75	4.53	2.37	2.87	2.09	2.51

Table 6.28: Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	3.81	3.72	10.37	3.22	4.17	8.39
7	3.18	2.47	9.12	1.59	4.13	11.04
6	3.29	1.48	9.39	1.81	4.73	10.63
5	3.73	1.71	8.25	1.96	4.49	9.00
4	4.06	1.80	8.71	1.91	3.55	6.02
3	4.40	0.59	8.84	2.02	4.47	4.52
2	2.71	0.20	8.50	2.33	5.39	3.08
1	0.54	0.20	7.60	4.43	5.05	1.70

Table 6.29: Multi-Step Ahead - MAPE (%) Values - ARMAX Daily Demand Model - Part 2

6.3.2 Summary of ARMAX Results

In conclusion, for the One-Step Ahead forecasts, the 14 day ahead MAPE (%) values in Tables 6.23 and 6.24 are all very similar or improved values when compared to the full winter forecast of 6.09% MAPE value in Table 6.22, except for the last fortnight in March 2011 (Period 13). The high forecast error in Period 13 is due to a very high variability in the temperature during this period associated with the end of Winter and Central Heating being turned off. The 7 days ahead forecasts are all lower than the 6.09% for each of the periods. Hence, again, this may indicate that re-evaluating the model every 7 days, or less, could be a solution for improvement (see overall conclusions in Chapter 7).

Comparing the Multi-Step ahead forecasts, for each of the recalculated models in Tables 6.28 and 6.29 with the 14 days ahead forecasts of the original model in Tables 6.25 and 6.26 the recalculated models produce an improved or similar MAPE (%) values except for the final Period (Period 13). The same is true for the 7 days ahead forecasts. The remaining results in Tables 6.28 and 6.29 are mostly all lower than the MAPE (%) values in Table 6.27, indicating that re-calculating the model every 14 or 7 days improves the forecasting capabilities of the model. These results will now be compared to the NARMAX methodology using the same data.

6.3.3 NARMAX Modeling and Forecasting with Actual Temperature

As in the previous Section, the X-Gas Daily Demand and the Daily Effective Temperatures described in Figures 4.6 and 4.7 are the start point again for the analysis using the Polynomial NARMAX methodology. The same start point was selected for modeling, i.e. Saturday 04/10/2008 and the end point was Friday 01/10/2010 (728 days - 104 weeks). The forecasting period was then set as starting on Saturday 02/10/2010 and running through to Friday 01/04/2011 (182 days (26 weeks)), covering the Winter period. This data is shown in Figure 6.10.

6.3.3.1 Transforming the data

The same transformations were performed as for the ARMAX modeling of daily demand and temperature (Section 6.3.1.2) to produce stationarity, i.e.:

1. The Daily Demand was transformed with a logarithmic transformation

2. The Daily Effective Temperature required no transformation

3. The Log of the Daily Demand and the corresponding Effective Temperature were then differenced by factors of 1 and 7

The transformed data is shown in Figure 6.11.

6.3.3.2 NARMAX Parameter Identification of the X-Gas Daily Demand with Temperature

From the ACF and PACF analysis of Section 6.3.1.3, the variables with lags of 364 and 365 were again omitted from the analysis. Running the modeling process first without residuals (i.e. an ARX Model), the following variables were selected, by the FROLS algorithm, for demand and temperature; $y(k-1)$, $y(k-2)$ and $y(k-7)$ and $x(k)$, $x(k-1)$ and $x(k-2)$. A 2nd Order NARX model was then tested using the same variables as the linear ARX model which generated 27 terms. Twelve terms were selected on reaching the thresholds, with an ERR total of 56%. The ACF of the residuals showed that there was additional information, to be modeled, especially around lags 7 and 14. The CCF of the Input (Temperature) and the Residuals showed significant lags at 7.

Several combinations of the Moving Average terms $e(k-1)$ to $e(k-7)$ were tested and twelve terms were selected by the FROLS algorithm at the threshold with the ERR total of 67.95%.

The modeling statistics were:

1. F Statistic with 713 data values = 3.38

2. Q Statistic = 460 with 701 degrees of freedom (df) which shows an adequate model (700 df χ^2 value is 762 at 5% level, and the 750 df is 814 at 5%).

The terms selected for the model are shown in Table 6.30.

Index	Model term	Parameter	ERR(%)
1	y_{k-7}	-0.082	27.94
2	x_k	-0.023	14.75
3	e_{k-7}	0.707	8.95
4	x_{k-1}	-0.018	10.72
5	e_{k-1}	0.180	2.93
6	x_{k-2}	-0.006	0.87
7	$x_{k-2} * y_{k-2}$	0.014	0.47
8	e_{k-2}	0.137	0.39
9	y_{k-1}	-0.127	0.42
10	y_{k-1}^2	0.322	0.23
11	$x_{k-2} * y_{k-1}$	0.017	0.23
12	$x_{k-2} * x_k$	0.001	0.06

Table 6.30: Results of the FROLS algorithm for the 2nd Order NARMAX Model

6.3.3.3 NARMAX Forecasting Future One-Step Ahead Demand

One-Step Ahead Predicted Output for the NARMAX model are shown in Figures 6.15 for the 182 days of winter starting from 02/10/2010 through to 01/04/2011 and the corresponding forecast statistics in Table 6.31.

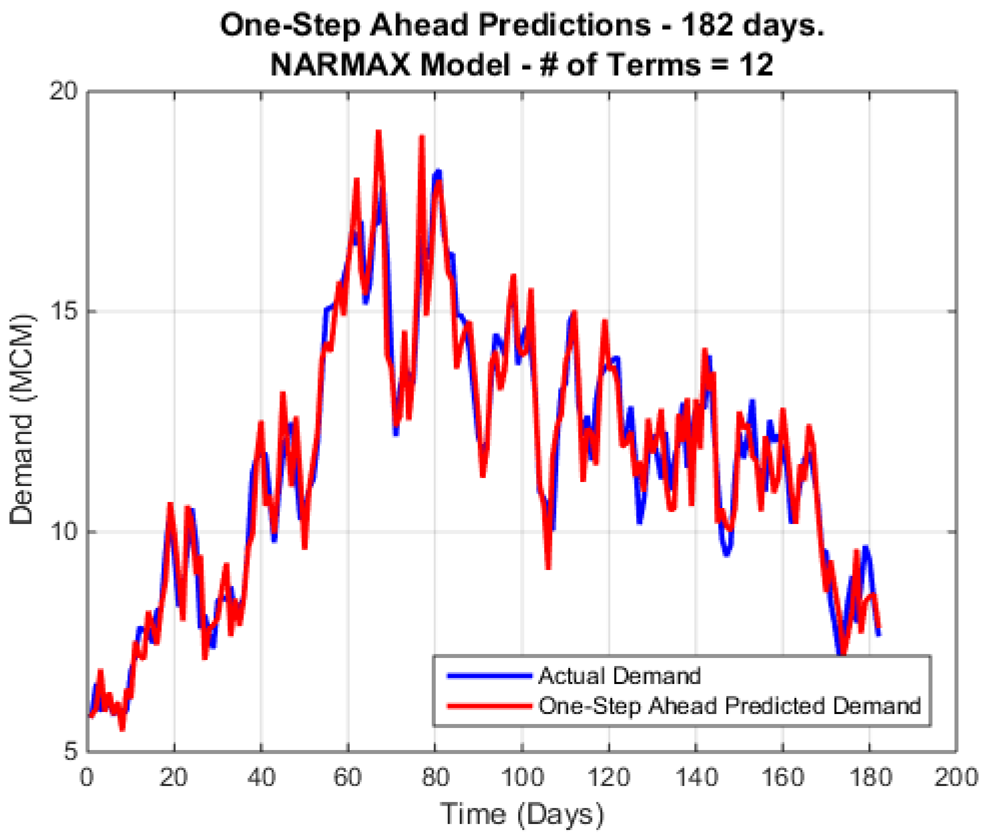

Figure 6.15: 182 Day One-Step Ahead Forecast for the 2nd Order NARMAX Model

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
182 Day	-0.03	0.63	0.79	5.67%	2.28/77/13.60%	-2.22/69/-13.69%

Table 6.31: 182 Day - One-Step Ahead Forecast Statistics for Daily Demand (2nd Order NARMAX Model)

The Multi-Step Ahead forecast for the 182 Winter days, again, veers away from the actuals between the first 14 and 28 days, as in Section 6.3.1.4, and hence the model was re-evaluated every 14 days for both One-Step and Multi-Step Ahead forecasts. The MAPE for the different One-Day Ahead forecasts for each of the 14 days ahead models is shown in Tables 6.32 and 6.33.

One Step Ahead MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	5.77	6.31	3.30	4.26	4.31	3.84	8.53
7	4.52	4.07	2.45	4.63	3.71	4.93	6.84

Table 6.32: One-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 1

One Step Ahead MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	4.96	3.11	6.00	3.62	5.16	8.08
7	3.67	4.07	4.76	3.03	5.83	5.52

Table 6.33: One-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 2

6.3.3.4 NARMAX Forecasting Future Multi-Step Ahead Demand

Starting from the 2nd Order NARX model (described above), the analysis of the Multi-Step Ahead residuals (ACF, CCF and Nonlinear Validity tests), indicate again that the following Moving Average terms $e(k-1)$, $e(k-2)$ and $e(k-7)$ were significant. For the generated model (14 terms), the ERR total was 69.87% and the ACF, CCF and the three Nonlinear Validity tests showed a small number of significant lags but each close to the significant level. The model terms and values are shown in Table 6.34.

Index	Model term	Parameter	ERR(%)
1	y_{k-7}	-0.043	29.34
2	x_k	-0.024	15.32
3	x_{k-1}	-0.018	8.81
4	e_{k-7}	0.770	11.40
5	e_{k-1}	0.209	1.77
6	e_{k-2}	0.197	1.26
7	x_{k-2}	-0.006	1.05
8	$x_{k-2} * y_{k-2}$	0.014	0.23
9	$x_{k-1} * y_{k-1}$	-0.028	0.25
10	x_{k-1}^2	-0.001	0.11
11	x_k^2	-0.001	0.14
12	y_{k-1}	-0.058	0.0.07
13	$x_{k-1} * x_{k-2}$	-0.001	0.07
14	$x_{k-2} * x_k$	-0.008	0.05

Table 6.34: Results of the FROLS algorithm for the 2nd Order NARMAX Model

This model was then used to forecast future values. The 182 day forecast values, again, start to veer from the actuals between 14 and 28 days, the 14-day Multi-Step Ahead forecasts were calculated using the model in Table 6.34 and the MAPE results for each period are shown in Tables 6.35 and 6.36. Table 6.37 shows the average forecast statistics for the 14-day Multi-Step Ahead forecast, and Figures 6.16 graphs the results. The aim in recalculating every 14 days is to improve on the results shown in Tables 6.35, 6.36 and 6.37.

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	4.22	5.46	6.39	20.93	4.67	19.04	7.56
7	4.55	5.57	6.95	11.80	3.30	8.62	5.67

Table 6.35: 182 Day - Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	13.27	10.97	39.93	9.15	8.71	6.78
7	6.16	6.89	27.10	5.41	9.18	8.79

Table 6.36: 182 Day - Multi-Step Ahead - MAPE (%) Values - NARMAX Daily Demand Model - Part 2

Model NARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
14 days	0.80	1.50	5.24	12.08%	8.14/ 84/49.87%	-3.30/125/-25.67%
7 days	-0.08	1.69	1.68	8.18%	5.49/ 77/32.81%	-2.45/ 84/-15.03%

Table 6.37: NARMAX Average Forecast Statistics for Various Multi-Step Ahead Daily Demand
starting 02/10/2010 for 182 days

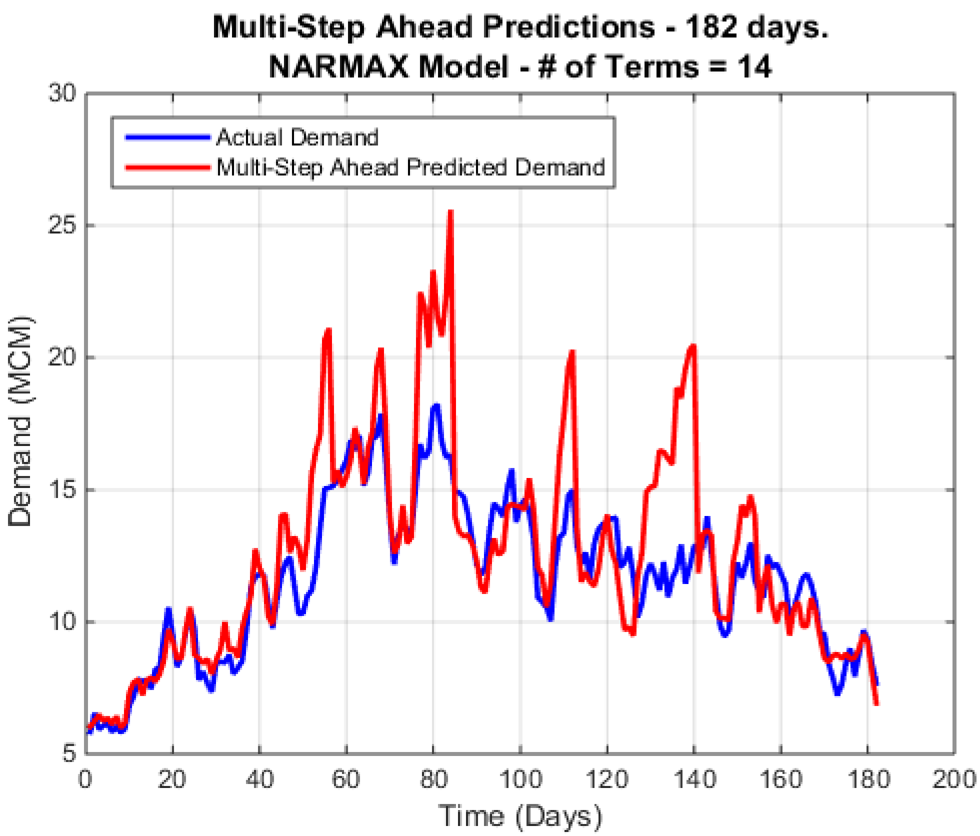

Figure 6.16: 14 day ahead - Multi-Step Forecast for 182 days - 2nd Order NARMAX Model

The model was recalculated every 14 days as in Section 6.3.1.5. The MAPE for the different Multi-Day Ahead forecasts for each of the 14 days ahead models is shown in Tables 6.38 and 6.39. In this case the forecasts were calculated for 1 to 7 days ahead as well as the full 14 day interval. It was noted, again, that the FROLS algorithm modified the model in Table 6.34 slightly each period, adding and removing terms, as well as changing the parameter values, thus adapting the model Period on Period. This will be discussed in Chapter 7.

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011							
Days Ahead	Period 1 02/10/10	Period 2 16/10/10	Period 3 30/10/10	Period 4 13/11/10	Period 5 27/11/10	Period 6 11/12/10	Period 7 25/12/10
14	4.22	4.54	5.58	4.26	5.77	9.57	4.31
7	4.55	4.44	5.29	3.64	2.57	6.28	4.52
6	4.49	5.12	5.33	3.88	2.94	3.94	4.33
5	4.50	4.27	5.73	4.16	1.99	1.78	3.48
4	4.88	3.15	6.06	2.71	2.15	1.54	3.34
3	5.62	2.50	4.31	2.74	2.24	1.90	4.03
2	4.07	2.96	3.79	1.33	2.16	2.84	3.96
1	3.25	5.24	2.83	1.02	3.17	0.82	1.24

Table 6.38: Multi-Step Ahead - MAPE (%) Values - Daily Demand Model - Part 1

Multi-Step Ahead - MAPE (%) Values for the period 02/10/2010 to 01/04/2011						
Days Ahead	Period 8 08/01/11	Period 9 22/01/11	Period 10 05/02/11	Period 11 19/02/11	Period 12 05/03/11	Period 13 19/03/11
14	3.57	5.12	7.23	7.02	5.35	7.83
7	2.15	3.20	8.41	2.93	4.22	10.47
6	2.46	1.75	9.77	2.15	4.50	10.64
5	2.49	2.05	9.13	1.73	4.72	9.16
4	3.02	2.40	10.09	1.90	3.59	5.61
3	3.95	0.40	10.80	1.89	3.81	4.54
2	2.24	0.57	10.51	2.65	5.39	4.23
1	1.31	1.05	11.10	4.98	5.44	1.95

Table 6.39: Multi-Step Ahead - MAPE (%) Values - Daily Demand Model - Part 2

6.3.4 Summary of NARMAX Results

In conclusion, for the One-Step Ahead forecasts, the 14 day ahead MAPE (%) values in Tables 6.32 and 6.33 are all very similar to the full winter forecast values of 5.67% MAPE value in Table 6.31, except for the Christmas fortnight (Period 7) and Period 13 (large temperature variability period). The 7 days ahead forecasts is similar or slightly lower than the 5.67% for each of the periods except the Christmas fortnight, again.

Comparing the Multi-Step Ahead forecasts for each of the recalculated models in Tables 6.38 and 6.39 with the 14 days ahead forecasts of the original model in Tables 6.35 and 6.36; the recalculated models produce an improved or similar MAPE (%) values for all the periods, with the exception of Period 13. The MAPE (%) values were all lower than the average 14 days MAPE of 12.08% in Table 6.37. For the 7 day ahead forecasts, all were below the 8.18% except for the Period 13. The comparison of these results with the ARMAX modeling on the same data will be covered in Section 6.4.

6.4 Daily Forecasting Summary and Conclusions

6.4.1 Introduction

The two previous sections have shown again that the Polynomial NARMAX methodology has the capabilities to model Daily Gas Demand. The focus of the research was on the Winter Days for each set of data, as this is the most sensitive time period for gas demand supply/demand match.

For the Benchmark tests, the One-Step Ahead MAPE for the 182 Winter days shown in Tables 6.41 and 6.42 for both ARMAX (4.33% and 6.09%) and NARMAX (4.12% and 5.67%) are smaller than the Persistence Model values in Table 4.2, i.e. 6.37% and 6.45%. Also the results of the 182 Day One-Step Ahead forecasts are within or close to the 4-6% value set by DNV GL. However, the ERR total in the NARMAX modeling, for both time periods, was around the 75% level, indicating that other information is missing to achieve the 95% level without modeling on 100s of terms. This missing information will be discussed in the Chapter 7 on future work opportunities.

For the Multi-Step Ahead both models (and both periods), produced forecasts for 182 days which veered away from the actuals between the 14 and 28 days horizon. However, using the models to forecast every 14 days did not produce MAPE values within the Benchmark level of 4-6%. Hence the conclusion to recalculate the models every 14 days. Table 6.40 shows the MAPE value forecasting every 14 days with the 182 Day Model, and the recalculated 14-day model forecasts. As can be seen, the two modeling techniques (ARMAX and NARMAX) produce similar results. The 14 day models have a much improved MAPE value and fall within the 4-6% Benchmark criteria.

The Sections 6.4.2 and 6.4.3 summarize the results of the Daily Demand forecasting for the two time periods of 1974-1975 and 2010 to 2011.

ARMAX 1970/75	182 Day Model	14 Day Model
14 Days ahead	8.07	5.81
7 Days ahead	7.10	5.39
NARMAX 1970/75	182 Day Model	14 Day Model
14 Days ahead	7.78	5.49
7 Days ahead	6.90	4.87
ARMAX 2001/11	182 Day Model	14 Day Model
14 Days ahead	11.58	5.90
7 Days ahead	7.25	4.76
NARMAX 2001/11	182 Day Model	14 Day Model
14 Days ahead	12.08	5.73
7 Days ahead	8.18	4.82

Table 6.40: Multi-Step Ahead MAPE (%) Values Comparison

6.4.2 Daily Summary and Conclusions (1970-1975)

For the One-Step Ahead forecasts, both modeling methods produce similar statistics for the 182 day ahead forecasts (Table 6.41). Both models had difficulty with the Christmas Period (Period 7) and the Periods 2, 3 and 12 where the heating systems are turned on and off at the start and end of the lower temperatures of the Winter period. The MAPE results in Table 6.41 meet the benchmark criteria, i.e. less than the Persistence model MAPE (6.38%) and within the 4-6% (DNV GL).

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
ARMAX	0.01	0.33	0.17	4.33%	1.09/32/12.01%	-1.37/96/-18.04%
NARMAX	0.02	0.31	0.15	4.12%	0.99/32/11.00%	-1.34/93/-16.27%

Table 6.41: 182 Day - One-Step Ahead Forecast Statistics Comparison (1970-1975)

For the Multi-Step ahead forecasts, both the ARMAX and NARMAX forecasts veer away from the actuals between 14 and 28 days ahead from the start of the 182 day forecast period. Hence the choice to re-calculate the model every 14 days. The models were very similar (from an MAPE perspective) for the 14-day ahead forecasts (Figure 6.17). Both models had difficulty with the Christmas period (Period 7). Figure 6.18 compares the two models for Period 1 (start point 29/9/1974).

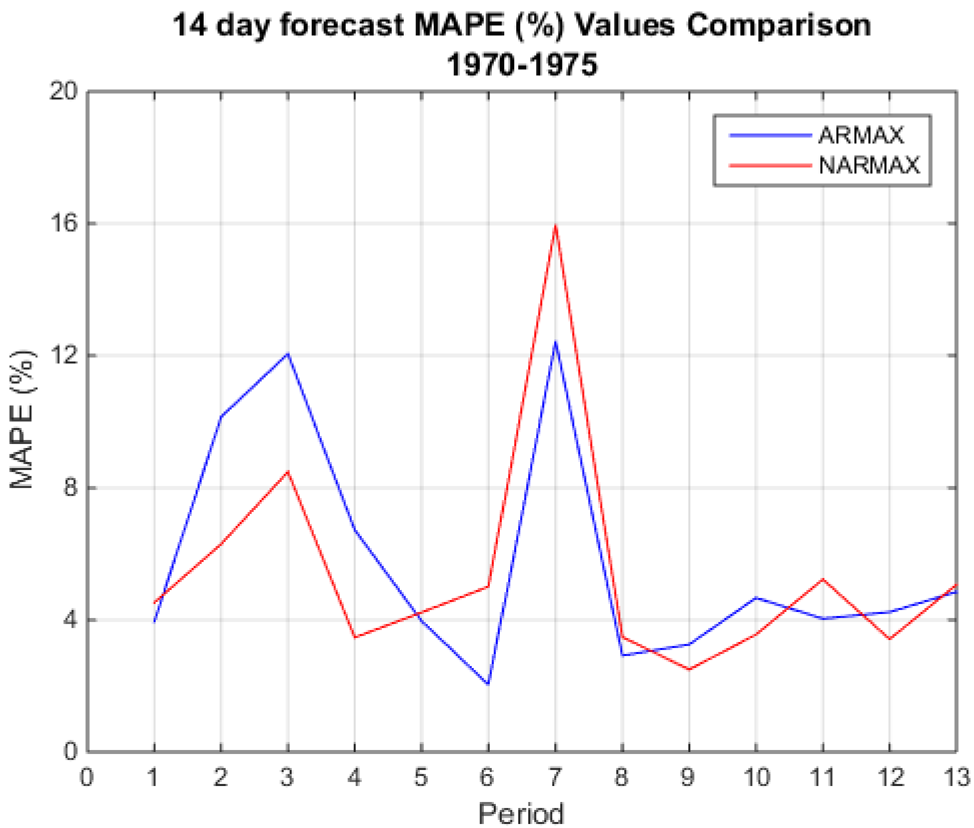

Figure 6.17: 14-day ahead Forecast - MAPE (%) Values Comparison ARMAX vs NARMAX

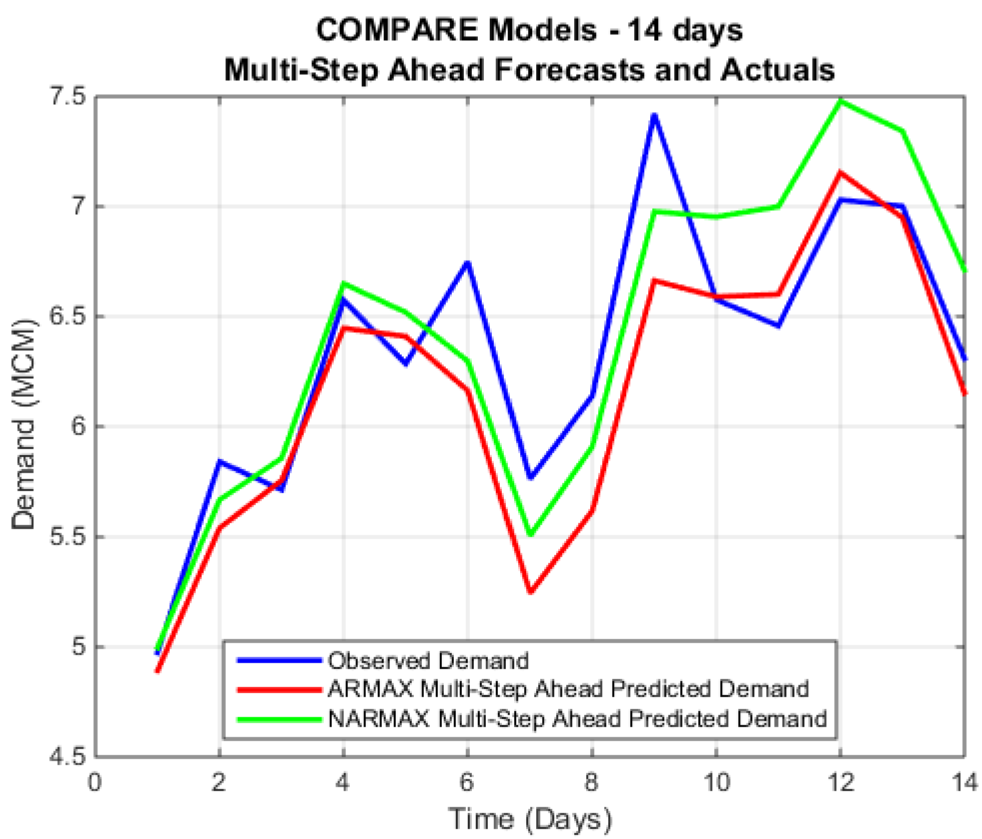

Figure 6.18: Period 1 - 14 Day Multi-Step Ahead Forecast - Comparison ARMAX vs NARMAX

6.4.3 Daily Summary and Conclusions (2001-2011)

For the One-Step Ahead forecasts, both modeling methods produce similar statistics for the 182 day ahead forecasts (Table 6.42). The MAPE results are close to the benchmark criteria, i.e. less than the Persistence model MAPE (6.38%) and close to 4-6% (DNV GL).

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
ARMAX	0.02	0.68	0.69	6.09%	2.28/77/13.65%	-2.40/69/-14.79%
NARMAX	-0.03	0.63	0.79	5.67%	2.28/77/13.60%	-2.22/69/-13.69%

Table 6.42: 182 Day - One-Step Ahead Forecast Statistics Comparison (2001-2011)

For the Multi-Step ahead forecasts, both the ARMAX and NARMAX forecasts veer away again from the actuals between 14 and 28 days ahead from the start of the 182 day forecast period. The models were very similar (from an MAPE perspective) for the 14 day ahead forecasts (Figure 6.19). Figure 6.20 compares the two models for Period 1 (start point 02/10/2010).

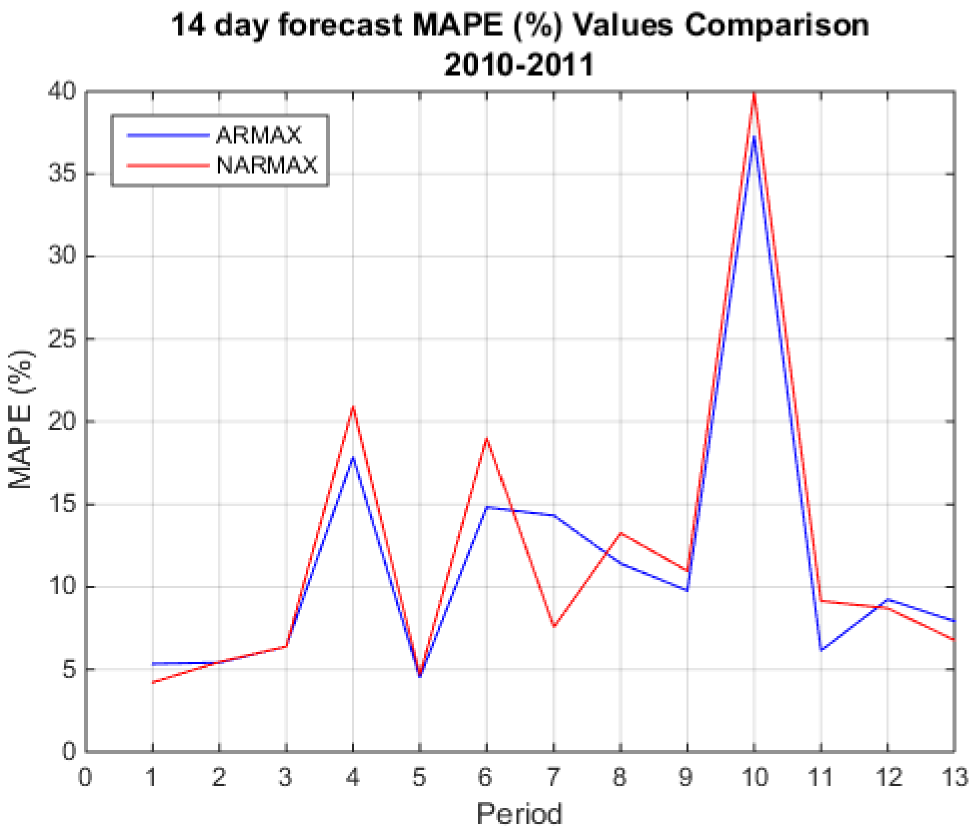

Figure 6.19: 14-day ahead Forecast - MAPE (%) Values Comparison ARMAX vs NARMAX

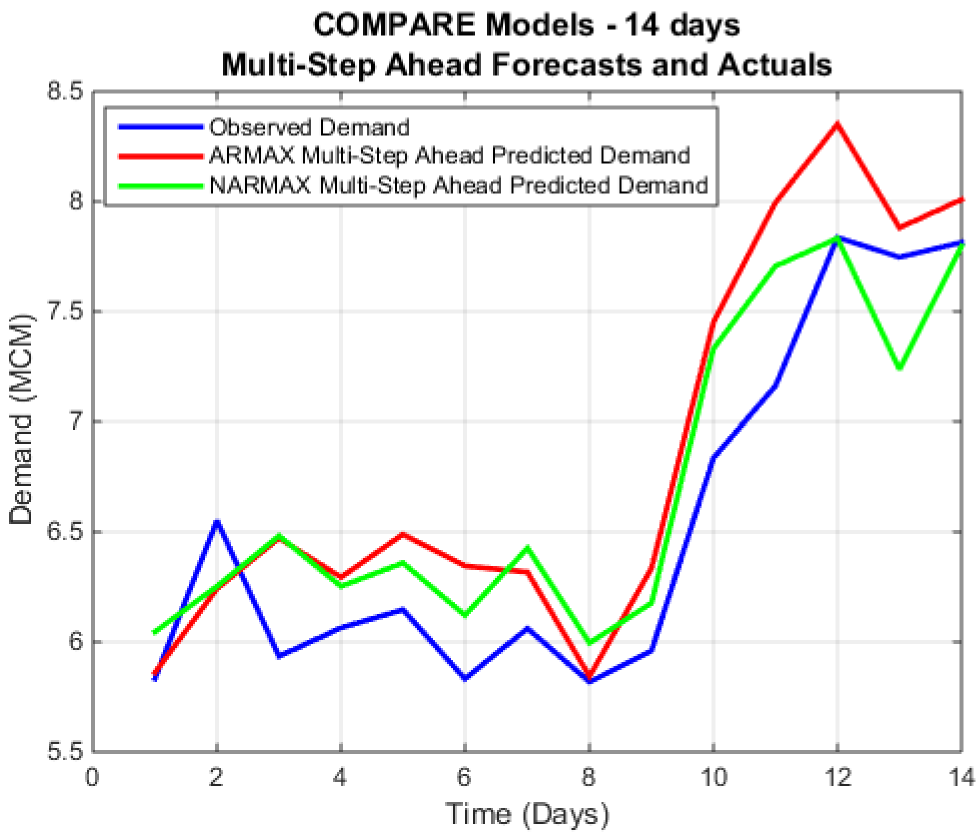

Figure 6.20: Period 1 - 14 Day Multi-Step Ahead Forecast - Comparison ARMAX vs NARMAX

Chapter 7

OVERALL CONCLUSIONS AND FUTURE RESEARCH OPPORTUNITIES

7.1 Overall Conclusions

Gas demand forecasting has significant implications on the costs and security of the energy supply. Accurate forecasting models are needed for secure and reliable energy system operation. In this research traditional Box and Jenkins ARMAX and the Polynomial NARMAX methodologies are applied to the Gas Demand forecasting problem. These methodologies are compared with each other in terms of the superiority in forecasting performance with the hypothesis that "Non-Linear Modeling of Gas Demand and Temperature using Polynomial Autoregressive Moving Average with eXogeneous Inputs (NARMAX) models, and Forward Regression with Orthogonal Least Squares (FROLS) estimation procedure can produce as good or better forecasts than the traditional linear Autoregressive Moving Average with or without eXogeneous Inputs (ARMAX/ARMA) modeling techniques for short term forecasts in the Gas Domain".

The thesis studied weekly and daily forecasting for several Gas Regions in the UK using data from the 1960s-70s and 2000s. The focus of the research was on the Winter periods, as this is the most critical period for gas delivery and security. The summary of the results, the conclusions and future research potential are presented below, and show that the goal of the thesis has been achieved.

Additionally, from an operational point of view the NARMAX methodology associated with the FROLS algorithm offers a major advantage over the ARMAX methodology (even for linear models), in the fact that the FROLS algorithm updates the model over time by including and removing terms automatically to react to the changing conditions over the winter period, thus removing the need for a high level of modeling expertise within the operational environment. This was noted in Chapter 6 with the rolling 14-Day models, where from a set list of variables and a 2nd Order NARMAX model, the terms selected, by the FROLS algorithm for each of the 13 different 14-Day models, were slightly different Period on Period. Using the ARMAX methodology the adaptation to the model(s) would have to be performed manually.

7.1.1 Weekly Conclusions

Weekly Demand forecasting was performed with four different data scenarios. For the models based on the 52 weeks, described in Sections 5.4 and 5.5, the Polynomial NARMAX methodology produced slightly improved results over the ARMAX methodology for both One-Step Ahead and Multi-Step Ahead forecasts for both the 1963/73 and the 2001/11 data sets. These are shown in Table 7.1. The MAPE values are within or close to the Benchmark Data of 4-6%. The One-Step ahead MAPEs are both less than the Persistence model MAPE values of 8.25% and 11.83%. The large Over and Under Estimates for the 52 week forecasts occur mostly in the summer weeks, hence further improvements are possible by applying corrections to summer temperature data. This is described, below, in the Section 7.2 on future research areas.

52 Weeks MAPE			
Data	Model	One-Step Ahead	Multi-Step Ahead
		MAPE %	MAPE %
1963-1973	ARMAX Yearly Model	4.06	6.04
	NARMAX Yearly Model	3.97	5.97
2001-2011	ARMAX Yearly Model	5.28	5.03
	NARMAX Yearly Model	4.46	4.79

Table 7.1: 52 Weeks - Model Forecast MAPE Summary

The first two developed models (Sections 5.2 and 5.3) were based on the winter weeks only. Although this method is applied today, by DNV GL, the disadvantage is that the low

AR terms select the end of Winter data of the previous year, which will impact the forecasts until the AR terms start to select the current winter data. For these models, the One-Step Ahead forecasts were slightly superior for NARMA(X) over ARMA(X), and slightly inferior for the Multi-Step Ahead and are shown in Table 5.44. However, when developing models for the full year and forecasting the Winter period ONLY, the ARMAX and NARMAX models produces similar MAPE results for the 1963/73 data and the 2nd Order NARMAX model produces superior MAPE results for the 2001/11 data as shown in Table7.2. Again the MAPE values fall within the Benchmark Data of 4-6% and are less than the Persistence Model MAPE values of 7.41% and 10.10% (Table 4.1).

26 Winter Weeks MAPE			
Data	Model	One-Step Ahead	Multi-Step Ahead
		MAPE %	MAPE %
1963-1973	ARMAX Yearly Model	3.05	2.98
	NARMAX Yearly Model	2.98	3.23
2001-2011	ARMAX Yearly Model	4.41	3.69
	NARMAX Yearly Model	3.69	2.75

Table 7.2: 26 Winter Weeks - Model Forecast MAPE Summary

The results above are encouraging and warrant further research. The goal of the thesis is met and shows that Polynomial NARMAX offers the capabilities of an additional methodology for Gas Demand Weekly forecasting.

7.1.2 Daily Conclusions

Daily Demand forecasting was performed with two different data scenarios (daily data from 1970-75 and 2001-2011). The focus, again, was on the Winter period, a 182 day forecast horizon was selected (starting the first week in October of each forecast year). For the One-Step Ahead forecasts, the Polynomial NARMAX methodology produced slightly improved results over the ARMAX methodology for both time horizons and the results are shown in Table 7.3. Both modeling techniques fall within the 4-6% Benchmark (although the results with the 2001-2011 data are close to the upper limit), as well as being lower than the Daily Persistence MAPE values of 6.37% and 6.45% (Table 4.2).

182-Day Model - One-Step Ahead MAPE		
Data	Model	One-Step Ahead
		MAPE %
1970-1975	ARMAX Yearly Model	4.33
	NARMAX Yearly Model	4.12
2001-2011	ARMAX Yearly Model	6.09
	NARMAX Yearly Model	4.46

Table 7.3: 182-Day Model - One-Step Ahead Model Forecast MAPE Summary

For the 182-Day Model Multi-Step Ahead forecasts, both methodologies started to veer away from the actuals between the 14 and 28 day forecast horizon. The choice was made to recalculate the model every 14 days. The ERR total in all cases was only 75%, which indicated that information was missing from the modeling process (see Section 7.2 below). The forecast values were calculated for each period for 14, 7, 6, 5, 4, 3, 2 and 1 day ahead and their MAPE values calculated. The average values are shown in Table 7.4.

Average Multi-Step Ahead MAPE (%) Values for 14-Day Models Days									
Data	Model	14	7	6	5	4	3	2	1
1970-1975	ARMAX Daily Model	5.81	5.39	5.25	5.16	4.48	3.83	3.62	3.01
	NARMAX Yearly Model	5.49	4.87	4.82	4.80	4.7	3.74	3.19	2.70
2001-2011	ARMAX Daily Model	5.90	4.76	4.62	4.40	4.15	4.03	3.48	4.98
	NARMAX Daily Model	5.72	4.82	4.72	4.25	3.88	3.83	3.59	3.34

Table 7.4: 14-Day Models - Average MAPE (%) Values Summary

The Multi-Step Ahead results in Table 7.4 are all within the 4-6% Benchmark level, and improve as the days ahead calculations become shorter. This indicates that recalculating the model on a shorter time scale could improve the results (see Section 7.2). Although the methods are very similar in the results achieved, there is a major advantage of the NARMAX methodology over ARMAX. This became clear during the Daily forecasting chapter where the model was recalculated every 14 days, and as the two week models moved across the winter period, the FROLS algorithm modified the selected terms automatically. With the ARMAX methodology, this action would have to be manually performed. This benefit will be described below in possible future research. Both methods adapt the parameter values as the models move across the winter period.

Again, the results above are encouraging and warrant further research. The goal of the thesis is met and shows that Polynomial NARMAX offers the capabilities of an additional methodology for Gas Demand Daily forecasting.

7.2 *Future*

There are many opportunities for future work with the Polynomial NARMAX methodology in the Gas Demand forecasting domain. These future work opportunities fall into two categories - Application Areas and Technical Research Areas, and are described below.

7.2.1 Application Areas

1. Daily forecasting - Include additional weather variables, in the modeling process, to increase the ERR Total (which is around 75% in this thesis for Daily Modeling and Forecasting). Many candidates exist, including different average temperature calculations, maximum and minimum daily temperatures, wind speed, humidity, chill factors etc

2. Optimal number of days ahead model calculation - Table 7.4 above, shows that the MAPE average decreases as the days ahead horizon decreases, hence developing models which recalculate fewer days ahead (7, 6, 5 days etc) to produce the optimal days ahead remodeling horizon. This work would then lead to a model which would re-calculate the parameter values, add or remove terms on each recalculation to match the periods specificity automatically. A major improvement over the manual intervention required

for the ARMAX methodology.

3. Day models - Develop Daily models which represent each day of the week, as the gas consumption profile is potentially different for each day of the week.

4. Data correction - Apply data correction to temperature data over a certain level, which would impact the spring and summer calculations, as well as special days e.g. Christmas and Bank Holidays, to help the models over these periods. This is the technique used by DNV GL, and none were applied in this thesis.

5. In-day forecasting - Develop Models and Forecasts for In-day periods using hourly data. In-Day forecasting is an important operational time horizon for Grid Controllers and a major part of DNV GL's application suite.

6. Number of historical days data for modeling? - Study why the daily models in Chapter 6 used a different number of days for the ARMAX modeling, compared to NARMAX modeling.

7. Summer months modeling and forecasting - Most of the work in this thesis has focused on the Winter period, hence similar application study could be performed on the Summer period.

7.2.2 Research Areas

In addition to the application areas described above, there are some technical research areas worth pursuing.

1. The advantage of the NARMAX methodology of automatically selecting variables and terms was challenged during the Weekly Demand modeling in Chapter 5. The FROLS algorithm had difficulty in selecting relevant terms after the first term due to the fact that the first term, $x(k)$ (the future temperature at time k), represented over 85% of the system output value as measured by ERR. Research into fine tuning the algorithm for situations like this would alleviate the need for manual intervention of removing obvious spurious term selections.

2. There are some interesting areas of new research in the area of time varying parameters and their impact on forecast accuracy. There appears to have been little work published using the above techniques to Demand Load Forecasting and specifically integrating time variant parameters into the polynomial model itself (Liu and Peng (2009) and Huang et al. (2009)). These include:

   (a) The impact of the time change in winter? The day for this time change varies each year. This could benefit Daily forecasting.

   (b) The impact of holidays, which move with time, like Christmas Day and New Year's Day. This could benefit Daily forecasting.

   (c) The impact on demand of temperatures close to daylight and nightfall hours. This could benefit In-Day forecasting.

   (d) The impact of the major temperature changes at the start and end of the Winter period. This could benefit all time frames.

7.2.3 Publications

No papers have been published from this work so far. Also there appears to have been little work published using the Polynomial NARMAX methodology to the Gas Demand Forecasting domain which provides an opportunity for disseminating the work. The references in the Bibliography are published in a variety of journals, hence offering a wide possibility for publications. There are five Journals of interest for publishing the work which stand out:

1. The International Journal of Forecasting

2. The International Journal of Applied Forecasting

3. The Journal of Control

4. The Journal of Applied Energy

5. Pipeline Simulation Interest Group (PSIG)

Finally, the data was provided under NDA for X-Gas (as well as other regions and additional weather variables) and the results have met the Benchmark they indicated, i.e. between 4 and 6% MAPE. The opportunities described above could provide continued relationship between DNV GL and the University of Sheffield to enhance industrial collaboration. The research produced in this thesis has been exciting and rewarding, and I hope it will be of benefit to future researchers.

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Appendix A

SEARCH CRITERIA AND SITES/JOURNALS

This Appendix lists the search terms used and the journals researched for the thesis. The papers reviewed as part of the learning process, or reviewed relative to the content of the thesis are found in the Bibliography.

A.1 Search Criteria

- ARIMA
- ARMA
- ARMAX
- Box and Jenkins
- Electricity Demand Forecasting
- Forecasting
- Gas Demand Forecasting
- Load Forecasting
- NARIMAX
- NARMAX
- NARX
- Non Linear ARMA(X)
- Non Linear Modeling
- Polynomial Modeling
- Short Term Energy Forecasting
- Transfer Functions

A.2 Sites/Journals

- American Institute of Chemical Engineers - AIChE Journal
- Annual Reviews in Control
- Applied Mathematical Modeling
- Asian Journal of Control
- Automatica
- Communications in Nonlinear Science and Numerical Simulation
- Computers in Industry
- Control Engineering Practice
- Electric Power Systems Research
- Engineering Applications of Artificial Intelligence
- Expert Systems with Applications
- Google Scholar
- IEEE Transactions on Power Systems
- IEEE Xplore
- Information Sciences
- International Journal of Applied Simulation and Modeling
- International Journal of Forecasting
- International Journal of Power and Energy Systems
- International Journal of Robust and Nonlinear Control
- Institute of Business Forecasting
- Institute of International Forecasters
- Journal of Forecasting
- Journal of Natural Gas - Science and Engineering
- Journal of Process Control
- JSTOR
- Mathematics and Computers in Simulation
- Nonlinear Analysis
- Nonlinear Dynamics of Production Systems
- Pipeline Simulation Interest Group (PSIG) - simulation and forecasting

- Research Gate
- Science Direct
- SciVerse
- Simulation Modeling Practice and Theory
- Sheffield On-line Library
- Southern Gas Association (SGA) the Forecaster Forum (October each year) is a conference specifically covering forecasting in the US market - dominated by a university / business partnership that do most of the forecasting services in the US
- Systems & Control Letters
- Wiley

Last search : 1/3/2017

Appendix B

TRANSFORMATION OF WEEKLY DEMAND DATA WITH SNET

B.1 Correction to Seasonal Normal Effective Temperature (SNET)

To correct the winter demand in Figure 5.1 to SNET some simple relationship must be found between demand and effective temperature. Figure B.1 shows the relationship of demand against effective temperature for the winter months for each of the years 1963 to 1973. It is clear that the spread is too large to produce a single linear relationship between demand in therms and effective temperature in degrees Celsius, which will be appropriate for a 10 year period.

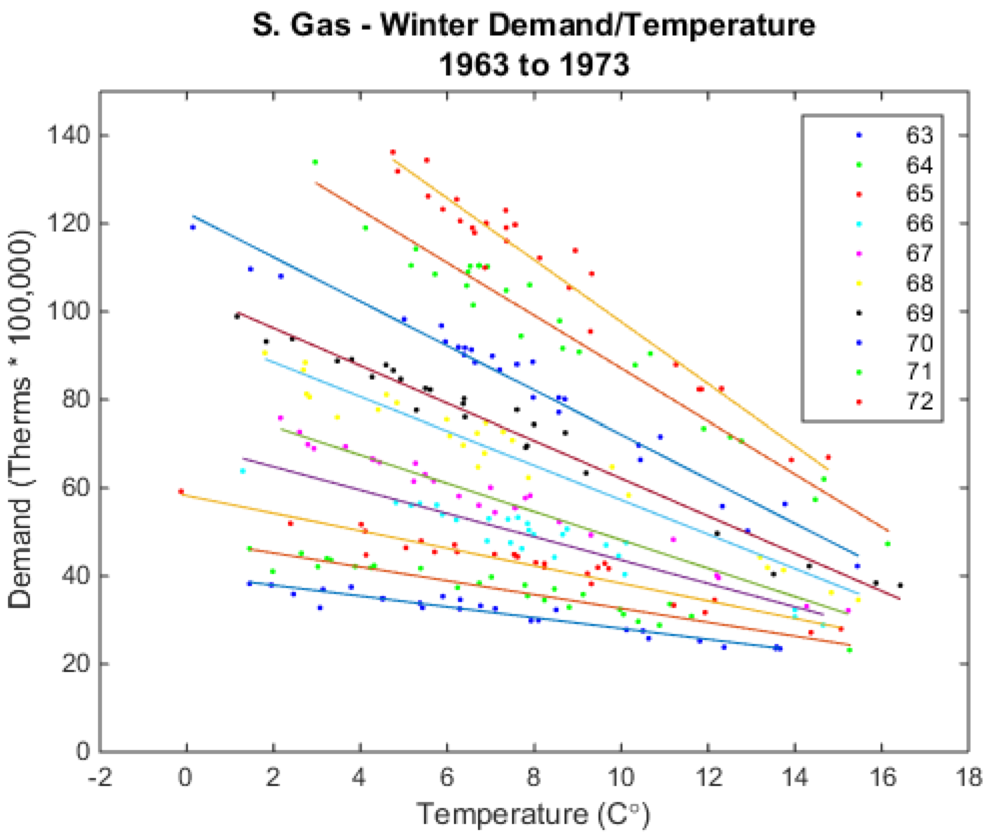

Figure B.1: S. Gas - Winter Demand /Temperature 1963 to 1973

Table B.1 gives the values for gradient and constant for each of the 10 years, and also the correlation coefficient (R) of each line. The correlation coefficients for demand against effective temperature are all over -0.95, thus indicating that there is a strong linear relationship between demand and effective temperature for each of the 10 years.

Year	Gradient A 10^5 Therms/C$^\circ$	Constant B 10^5 Therms	Correlation Coefficient R
63/4	-1.23	40.37	-0.96
64/5	-1.57	48.36	-0.96
65/6	-1.99	58.24	-0.96
66/7	-2.65	70.12	-0.96
67/8	-3.21	80.31	-0.99
68/9	-3.90	96.24	-0.98
69/70	-4.26	104.74	-0.99
70/71	-5.03	122.40	-0.98
71/72	-6.00	147.03	-0.98
72/73	-7.02	167.87	-0.98

Table B.1: Demand/Temperature Parameters

The relationship between demand and effective temperature is shown in equation B.1.

$$y^i = A_i T^i + B_i \qquad i = 63, \ldots, 72 \tag{B.1}$$

where

y^i represents the weekly demand of year i

T^i represents the weekly temperature of year i

A_i and B_i are the regression coefficients for year i

Therefore demand will be corrected, to SNET, by a different relationship each year, using the values in Table B.1. The demand-temperature relationship is shown in equation B.2.

$$d_t^i = y_t^i - A_i(T_t^i - SNET_t) \tag{B.2}$$

where

$t = 1, \ldots, 26$ (corresponding to the 1st week in October to the last week in March)

d_t^i = demand corrected to SNET in year i at week t

y_t^i = measured demand in year i at week t

A_i = slope of demand/temperature graph for year i

T_t^i = Average weekly effective temperature for year i at week t

$SNET_t$ = Seasonal Normal Effective Temperature at week t (Figure 5.3)

Finally, each year's data is then combined into a single series d_t, $t = 1, \ldots,$ N, where N = 260, which represents the winter weekly demand corrected to SNET for Southern Gas from 1963 to 1973. This is shown in Figure B.2.

NOTE:

At the time I worked on the problem for British Gas, they used the SNET profile and did not take into account the relationship year on year. I suggested the model above and applied it to my work to show the weekly demand forecast was better than their simple method. Analyzing the same data during my original thesis work, I found that the relationships for A and B were, in fact, not linear, (A_i's fits a 3rd Order polynomial better, and B_i's fits a 2nd order polynomial) and hence could possible improve the forecasts. The reason for the non linearity, I believe was that the growth on S. Gas demand was starting to flatten off in the early 1970s.

For this thesis, I have used the linear relationship for the values of the A_i's and the B_i's themselves fit to straight lines when plotted against time (t):

$$A_i = -0.63t - 0.23 \qquad t = 1, \ldots, 10 \tag{B.3}$$

$$B_i = +13.89t + 17.61 \qquad t = 1, \ldots, 10 \tag{B.4}$$

and hence it was possible to predict the demand/temperature relationship for future years.

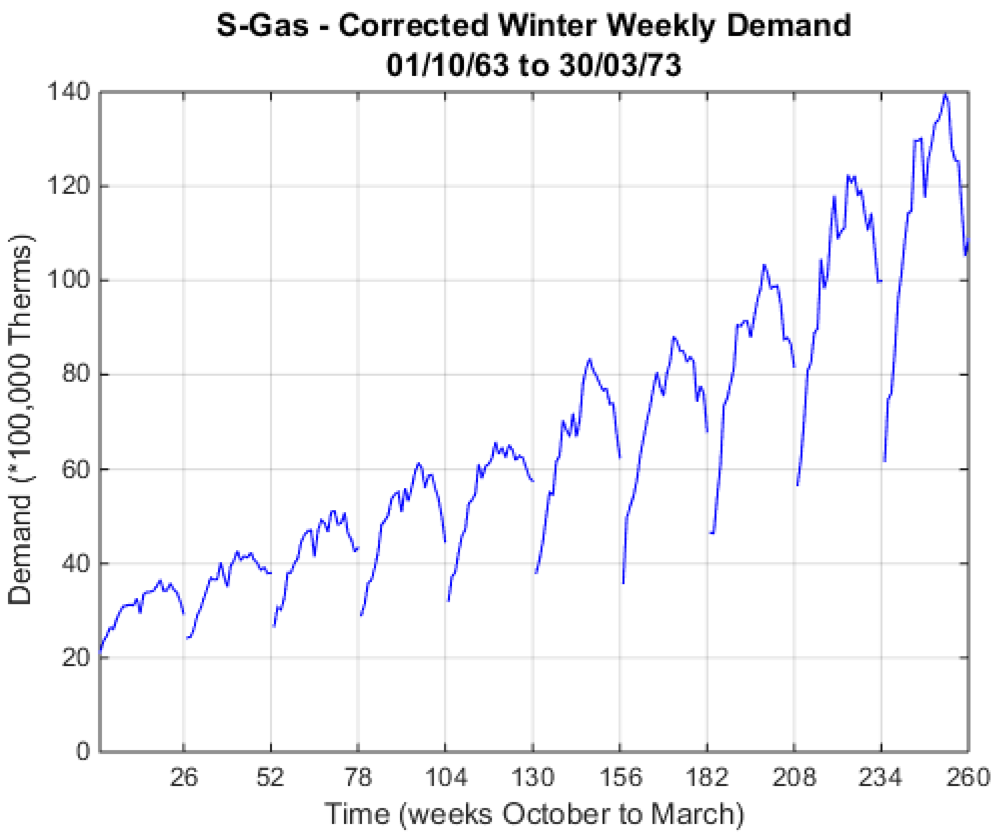

Figure B.2: S. Gas - Winter Corrected Demand 1963 to 1973

B.2 Conclusion

In conclusion, in calculating the relationship between demand and effective temperature, it was found that the gradient increases in absolute value from 1963 to 1973. This indicates that the demand for Southern Gas became less temperature sensitive due to the increase in the domestic consumption component. This change to temperature sensitivity was possibly due to economic factors that came into play in this period; prosperity in the south of England, hence less price conscious, as well as population growth of affluent people into the area.

Appendix C

ARMA WINTER WEEKLY MODELING AND FORECASTING WITH SNET (1963-1973)

C.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.2.

The starting point is the Corrected Winter Weekly Demand time series, which is described in Appendix B. This data is shown in Figure 5.4. The transformation of the data was described in Section 5.2.1.2.

C.2 Parameter Identification to the Corrected Demand data

The Transformed Corrected Demand data $w_t, t = 1, \ldots, 207$ is shown in Figure 5.5. The first 9 years will be used for modeling and the last year will be use to compare with the predicted values of the model selected.

The general form of the Box and Jenkins ARMA model is described in Chapter 3 (subsection 3.2.2.2). The original research selected the following model structure (equation C.1). The model has and Autoregressive term of 1 and Moving Average terms 1, 26 and 27.

$$(1 - \phi_1 B)w_t = (1 - \theta_1 B)(1 - \Theta B^{26})a_t \tag{C.1}$$

where ϕ_1 is an autoregressive parameter of order 1, θ_1, is the moving average parameter of order 1, and Θ is the periodic moving average parameter and a_t is a series of residuals.

The programs used to estimate the parameters were written in Fortran and run on a UNIVAC 1100 at the British Gas Research Center. The results obtained for ϕ_1, θ_1 and Θ were :

$$(1 - 0.16B)w_t = (1 - 0.72B)(1 - 0.85B^{26})a_t \tag{C.2}$$

However, when re-evaluating the above data with MATLAB using the Econometrics Toolkit, several other models performed equally well from a modeling perspective as well as for their forecasting capabilities. The advantage of the Econometrics Toolkit over the Systems Identification Toolkit, for ARMA modeling, is that specific model parameters and terms can be selected. The possible models are shown in List C.2, AR(1)/MA(1,26,27) O is the model from the original work in the 1970s, and AR(1)/MA(1,26,27) N is the revised version using MATLAB. For each of the models, the constant c was almost zero and hence is not included.

AR(1)/MA(1,26,27) O - $(1 - 0.16B)w_t = (1 - 0.72B)(1 - 0.85B^{26})a_t$

AR(1)/MA(1,26,27) N - $(1 + 0.19B)w_t = (1 - 0.80B)(1 - 0.58B^{26})a_t$

AR(1,2,3,4)/MA(26) - $(1 - 0.59B - 0.48B^2 - 0.33B^3 - 0.13B^4)w_t = (1 - 0.59B^{26})a_t$

AR(1,2)/MA(1,2,26,27,28) - $(1-0.67B+0.11B^2)w_t = (1+0.05B-0.64B^2)(1-0.59B^{26})a_t$

AR(1,2)/MA(1,2,3,26,27,28,29) - $(1+0.18B-0.77B^2)w_t = (1-0.83B+0.80B^2-0.51B^3)(1-0.60B^{26})a_t$

AR(0)/MA(1,2,26,27,28) - $w_t = (1 - 0.60B - 0.15B^2)(1 - 0.58B^{26})a_t$

AR(0)/MA(1,26,27) - $w_t = (1 - 0.69B)(1 - 0.57B^{26})a_t$

AR(1)/MA(1,2,26,27,28) - $(1 - 0.75B)w_t = (1 + 0.1B - 0.58B^2)(1 - 0.59B^{26})a_t$

List C.2: List of Possible Models

Table C.1 shows the measurements from the modeling stage for each of the possible models in List C.2. The AIC measure and F statistic are all very similar. The Q Statistic was calculated for each model with m as 52 (the point at which the ACF values can be considered as zero), and the number of parameters of each individual model used to define the degrees of freedom. The 5% point for a χ^2 with 40 degrees of freedom is 55.75 and 50 degrees of freedom is 67.50, indicating again that there is no reason to believe the models in List C.2 are in anyway inadequate. Significant lags (greater than 2 times the standard deviation) are listed for each of the models. The lags 30 and 77 appear in some models, but seem to have little relevance to any specific cycle. In summary, there is little to distinguish between the possible models (although AR(1,2)/MA(1,2,3,26,27,28,29) appears to be the best, statistically) and hence forecast values were calculated for each model before selecting the best model.

Model ARMA	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1)/MA(1,26,27) O		0.423		43.16	48
AR(1)/MA(1,26,27) N	-659	0.478	30,77	35.5	48
AR(1,2,3,4)/MA(26)	-655	0.483		37.88	47
AR(1,2)/MA(1,2,26:28)	-656	0.469	30	33.34	45
AR(1,2)/MA(1,2,3,26:29)	-656	0.461		27.61	43
AR(0)/MA(1,2,26:28)	-657	0.477	30,77	36.21	47
AR(0)/MA(1,26,27)	-657	0.480	77	38.06	49
AR(1)/MA(1,2,26:28)	-658	0.472		34.80	46

Table C.1: Model Fit Comparisons for Weekly Demand

C.3 Forecasting Future One-Step Ahead Demand

One step ahead forecasts were calculated for each of the models in Table C.1 for the winter of 1972. The forecasts values $\hat{w}_{t+i}$ have then to be inverse-transformed to produce actual forecasts in 100,000 therm units (using equations 3.17 and 3.18 in Section 3.2.2.6).

Table C.2 shows the balanced set of forecast metrics for each of the models in Table C.1. Although the model AR(1,2)/MA(1,2,3,26:29) appeared to produce the best modeling results in Table C.1, the model AR(1,2,3,4)/MA(26) produce the best results in Table C.2 from a prediction standpoint. The MAPE was used as the final selection choice. The results of the best model are described in Section 5.2.1.4.

Model ARMA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1)/MA(1,26,27) O	1.88	3.64	22.20	3.33%	13.11/259/12.47%	-8.24/244/-6.36%
AR(1)/MA(1,26,27) N	0.29	4.07	25.03	3.77%	7.38/21/5.76%	-12.27/10/-9.47%
AR(1,2,3,4)/MA(26)	0.23	4.02	24.15	3.71%	7.22/21/5.63%	-11.94/10/-9.21%
AR(1,2)/MA(1,2,26:28)	0.28	4.13	26.43	3.80%	7.87/21/6.13%	-13.13/10/-10.12%
AR(1,2)/MA(1,2,3,26:29)	0.29	4.35	26.72	4.03%	8.94/13/7.61%	-12.14/10/-9.36%
AR(0)/MA(1,2,26:28)	028	4.21	26.57	3.88%	7.82/21/6.10%	-12.88/10/-9.93%
AR(0)/MA(1,26,27)	0.27	4.09	25.33	3.78%	7.52/21/5.86%	-12.57/10/-9.70%
AR(1)/MA(1,2,26:28)	0.27	4.13	26.57	3.80%	7.91/21/6.16%	-13.23/10/-10.20%

Table C.2: 26 week - Statistics for One-Step Ahead Weekly Demand Forecast

Note: In Tables C.1 and C.2, due to space requirements, the terms 26,27,28 have been rewritten as 26:28, etc.

C.4 Forecasting Future Multi-Step ahead Demand

This is described in detail in Section 5.2.1.5 and not duplicated here.

Appendix D

NARMA WINTER WEEKLY MODELING AND FORECASTING WITH SNET (1963-1973)

D.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.2.3.

D.2 Term Selection for the Linear Model

For this set of data there are no input variables, and a linear model will be considered initially. For linear models, the model terms and the model variables are exactly the same. After testing various combinations of terms, 29 terms for output variable y were selected $(y_{k-1}, y_{k-2} \dots y_{k-29})$. As there are no error variables on the first iteration, we will end up with a AR model. Note: The 29 terms were selected after trials of multiples of 26 (the seasonal period). This represents the polynomial:

$$w(t) = (\phi_1 B + \phi_2 B^2 + \ldots + \phi_p B^{29}) w_t + a_t \tag{D.1}$$

or

$$w(t) = \phi_1 w_{t-1} + \phi_2 w_{t-2} + \ldots + \phi_{29} w_{t-29} + a_t \tag{D.2}$$

where $w(t)$ is the transformed demand data.

This linear AR model gave the ERR profile shown in Figure D.1 and the list of terms shown in Table D.1.

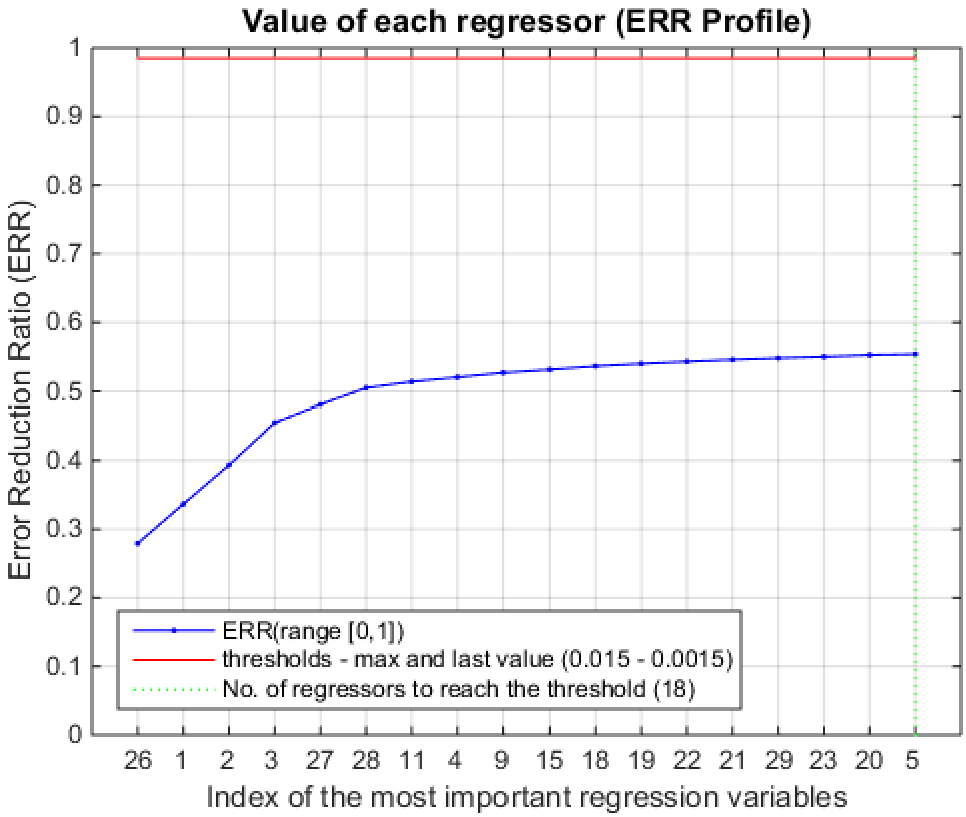

Figure D.1: ERR Profile for Transformed Weekly Demand (linear AR model)

Index	Model term	Parameter	ERR(%)
1	y_{k-26}	-0.505	27.93
2	y_{k-1}	-0.627	5.72
3	y_{k-2}	-0.511	5.68
4	y_{k-3}	-0.312	6.16
5	y_{k-27}	-0.327	2.60
6	y_{k-28}	-0.192	2.41
•	•	•	•
18	y_{k-5}	-0.042	0.12

Table D.1: Results of the FROLS algorithm applied to Linear Model - 18 terms

D.3 Forecasting using the Linear AR Model

From the 18 most significant terms 50% of the value is produced by 6 terms (the remainder added approximately 1% of the value). The 18 term model and the 6 term model were then both used to forecast future demand. The difference between the two models was insignificant, and hence the simpler 6 term model is shown below. Table D.2 shows the new parameter values and their ERR% when only the 6 terms above are included.

Index	Model term	Parameter	ERR(%)
1	y_{k-26}	-0.53	27.99
2	y_{k-1}	-0.57	5.62
3	y_{k-2}	-0.463	5.78
4	y_{k-3}	-0.248	6.00
5	y_{k-27}	-0.306	2.57
6	y_{k-28}	-0.21	2.35

Table D.2: Results of the FROLS algorithm applied to the Linear Model - 6 terms

As in the ARMA modeling, the F and Q statistics were calculated. The Q statistic shows the model is adequate.

1. F Statistic with 178 data values = 0.42

2. Q Statistic = 31.99 with 46 degrees of freedom (df) (52-6) which shows an adequate model (46 df χ^2 value is 62.83 at 5% level)

The ACF of the residuals has no significant lags, hence the residuals can be considered as a white noise sequence. However, the model was rerun a second time with the residuals included in the calculations to test if including MA terms would improve the model from a forecasting perspective. The results produced were worse with an MAPE of >4%. Hence in the linear case no Moving Average (MA) terms exist in the model. Also, the model validity tests (described in chapter 5 of reference Wei et al. (2004)) cannot be run for linear models with no input, but will be seen later Chapters, when actual temperature is included in the modeling process.

One-Step Ahead Predictions for the year 72/73 using the 6 significant terms are shown in Figure D.2. The forecast data statistics are shown in Table D.3.

The MAPE for this linear model was 3.74% which is equivalent to the ARMA model developed in this section above. Note including all 18 terms, the MAPE was 3.63%.

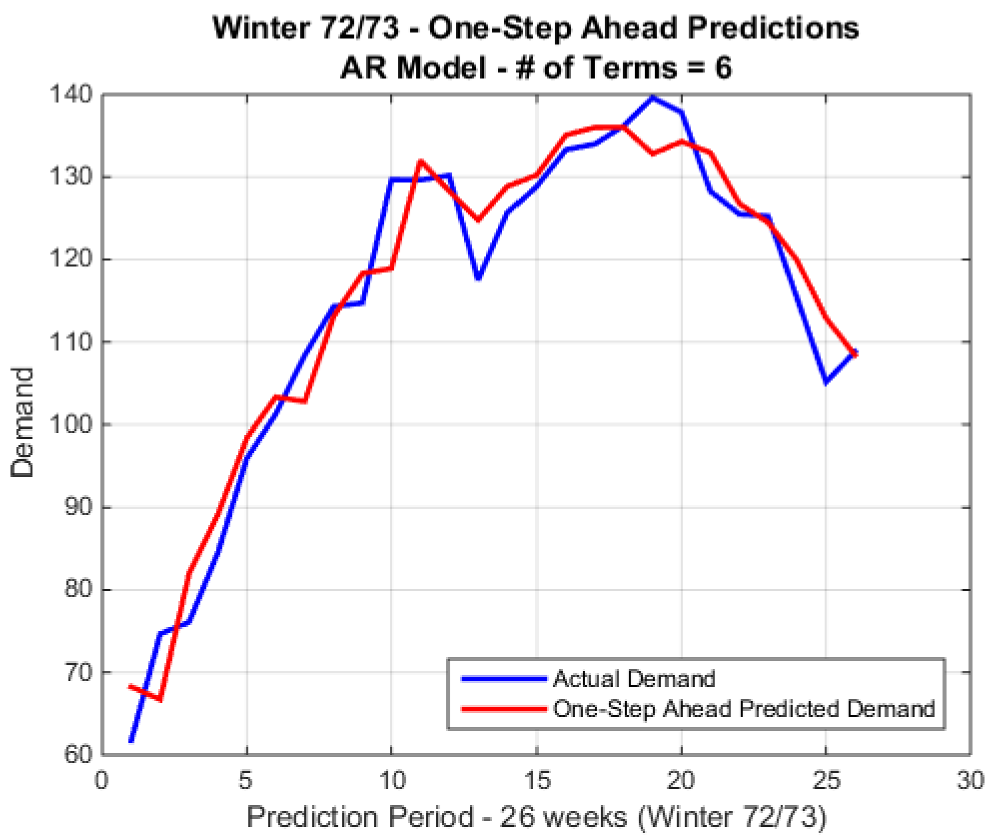

Figure D.2: Predicted vs Actual Demand for the Winter of 72/73 (AR model)

Model AR	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR	0.86	3.86	22.25	3.74%	10.75/10/8.29%	-7.76/25/-7.39%

Table D.3: 26 week - Statistics for One-Step Ahead Weekly Demand Forecast

D.4 2nd Order Model for Winter Weekly Demand

The next step was to look at possible model orders (i.e. non linear components which would generate a NAR or NARMA model), and a variety of variables and terms.

Starting again from parameters y_{k-1} to y_{k-29}, the parameters y_{k-1}, y_{k-2}, y_{k-3}, y_{k-26}, y_{k-27}, y_{k-28} and y_{k-29} and their associated terms produced the most improved ERR profile for the 2nd Order model. A 2nd Order model and these 7 variables of y , generates 35 terms, which when entered into the FROLS algorithm, 27 terms were selected generating an ERR total of 67% (an improvement over the linear AR model and shown in Figure D.3).

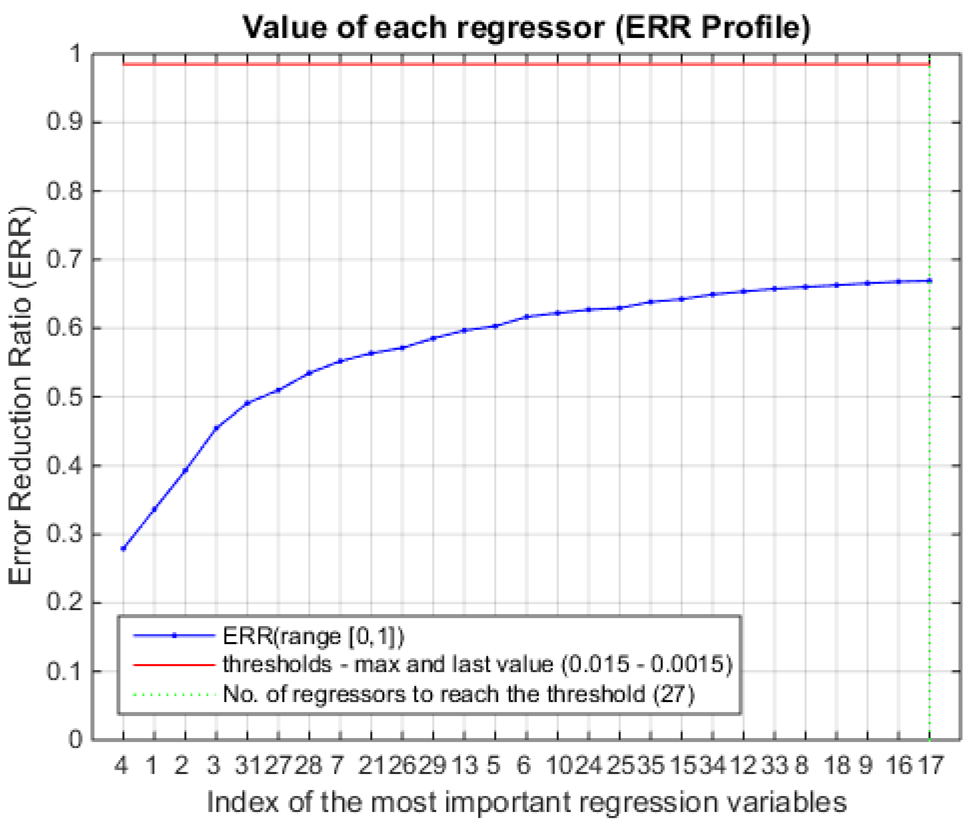

Figure D.3: ERR Profile for Transformed Weekly Demand (2nd Order)

Again, the ERR profile does not attain the 95%, indicating there is still information missing from the model. The terms selected and their values are described in Table D.4. Note that 12 terms generate 60% of the ERR total, but when compared to the 27 term model, it did not improve the modeling statistics or the forecasts for the year 1972/73.

Index	Model term	Parameter Value	ERR(%)
1	y_{k-26}	-0.474	27.93
2	y_{k-1}	-0.530	5.72
3	y_{k-2}	-0.486	5.68
4	y_{k-3}	-0.280	6.16
5	$y_{k-27} * y_{k-28}$	-3.11	3.58
6	$y_{k-26} * y_{k-27}$	5.08	1.90
7	$y_{k-26} * y_{k-28}$	4.36	2.49
8	y_{k-29}	-0.021	1.74
9	y^2_{k-3}	0.47	1.15
10	y^2_{k-26}	2.081	0.8
11	$y_{k-26} * y_{k-28}$	1.65	1.4
12	$y_{k-1} * y_{k-28}$	-2.26	1.15
•	•	•	•
27	$y_{k-2} * y_{k-26}$	-0.95	0.11

Table D.4: Results of the FROLS algorithm applied to the 2nd Order NAR Model

Following the selection of the 27 most significant terms, the model was used to generate one step ahead predicted values for the model data. The modeling statistics for the 27 term model are:

1. F Statistic with 178 data values = 0.28

2. Q Statistic = 24.28 with 25 degrees of freedom (df) (52-27) which shows an adequate model (25 df χ^2 value is 37.65 at 5% level)

The ACF of the residuals, again, showed no values above/below the threshold, which indicates the error part of the model is white noise. The model validity tests again, cannot be run for 2nd Order model with no input, but will be seen in later Chapters.

Finally, predictions for the year 72/73 are performed using all 27 terms and are shown in Figure D.4.

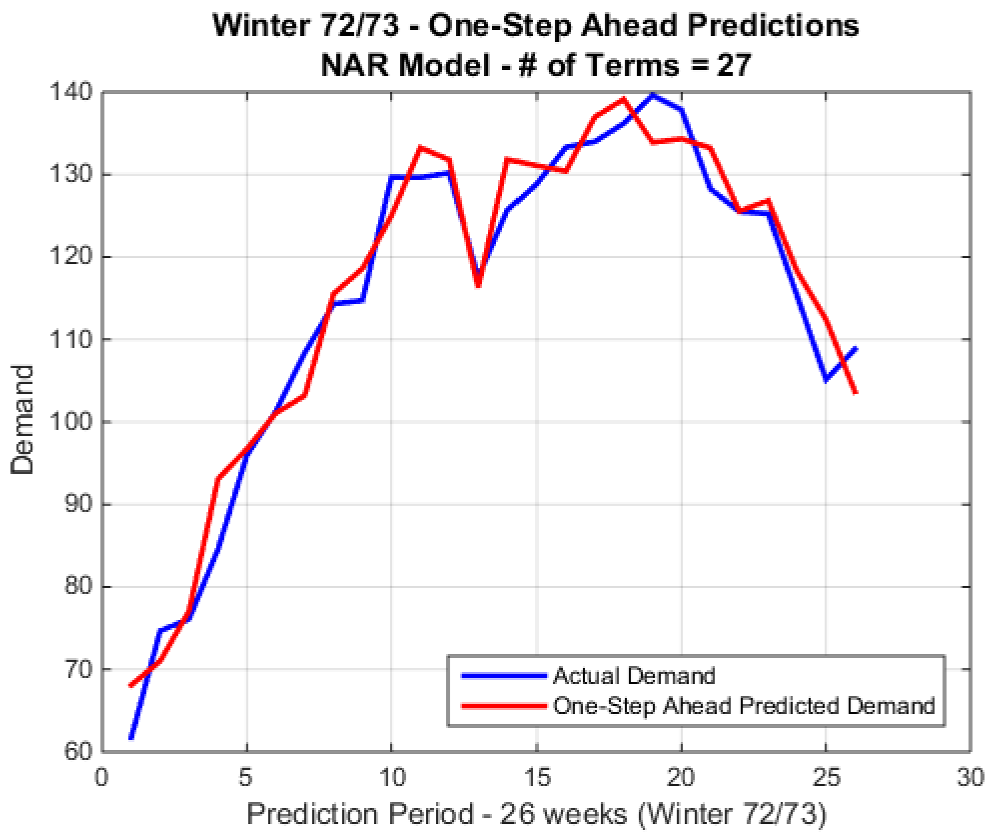

Figure D.4: Predicted Values for the Winter of 72/73 (2nd Order NAR Model)

The forecast statistics are shown in Table D.5. The NARMA methodology, produced a NAR model with improved results over the ARMA model for this data, producing a MAPE of 3.26%.

A second run was performed including the residuals into the modeling process, to evaluate if any MA terms would improve both the modeling statistics and the forecasts. The residual terms initially chosen were e_{k-1} to e_{k-30} inclusive. Also no interaction was set up between the residual terms and the input and delayed output variables. The results are now described in Section 5.2.3.3.

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
NAR	-0.99	3.47	16.90	3.26%	8.52/4/10.08%	-5.72/19/-4.10%

Table D.5: 26 week - Statistics for One-Step Ahead Weekly Demand Forecast

D.5 Forecasting Future Multi-Step ahead Demand

As in the ARMA modeling section of this chapter, the 2nd order NAR model was evaluated for the ability to forecast multiple steps ahead using the calculated forecast values for future values.

Figure D.5 shows the result of the 26 week Multi-Step ahead forecast for the 2nd order NAR model. Again (as with the ARMA model) the values start to drift from the actuals as the forecast horizon advances in time. The statistics for the model are shown in Table D.6.

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
NAR	9.73	8.51	114.51	8.51%	19.90/25/18.93%	-0.60/2/-0.80%

Table D.6: 26 week - Statistics for Multi-Step Ahead Weekly Demand Forecast

The ACF of the residuals from the MPO first cycle, showed significant lags at 1, 2 and 26, indicating that there was information in the residuals to be modeled. The process was rerun including the residual values in the calculations. The model, thus becomes a NARMA model.

Again the results are now described in Section 5.2.3.

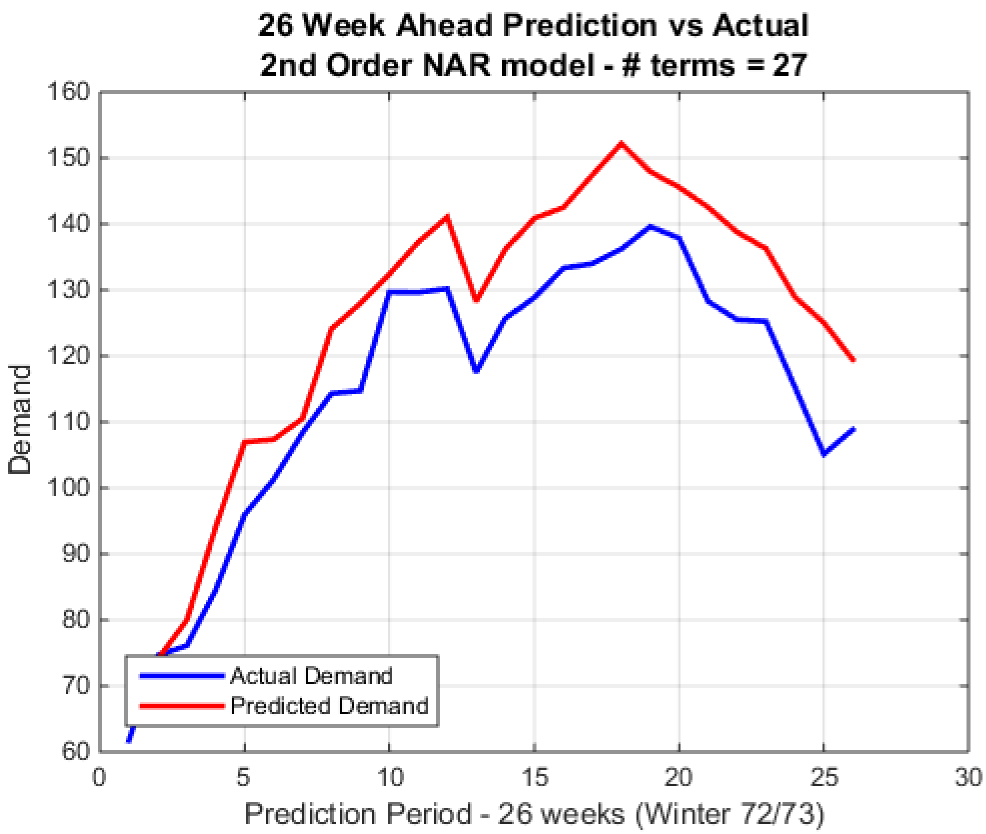

Figure D.5: 26 week - Multi-Step Ahead Forecast for 2nd Order NAR Model

Appendix E

ARMAX WINTER WEEKLY MODELING AND FORECASTING WITH TEMPERATURE (1963-1973)

E.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.3.

The starting point is the Winter Weekly Demand time series, which is described in Chapter 4. This data is shown in Figures 5.13 and 5.14. The transformation of the data was described in Section 5.3.1.2.

E.2 Parameter Identification

Several potential models were initially considered (based on their AIC and BIC values), and these are shown in List E.2. The constant is zero in all cases. Table E.1 contains the modeling statistics for each of these models. Model AR(1,2)/MA(1:3,26:29) and AR(1)/MA(26,27) appear to produce the best results from a modeling perspective. Note that none of the models analyzed have any lags from the analysis of the residuals outside the 95% confidence limits.

AR(1,2)/MA(1:3,26:29) - $(1 + 0.40B + 0.42B^2)w_t = -0.04x_t + (1 - 0.97B - 0.31B^2 + 0.28B^3)(1 - 0.44B^{26})a_t$

AR(1)/MA(26,27) - $(1 + 0.27B)w_t = -0.04x_t + (1 - 0.77B)(1 - 0.43B^{26})a_t$

AR(0)/MA(26,27) - $w_t = -0.04x_t + (1 - 0.56B)(1 - 0.43B^{26})a_t$

AR(1:3)/MA(26) - $(1 - 0.48B - 0.26B^2 - 0.08B^3)w_t = -0.04x_t + (1 - 0.42B^{26})a_t$

AR(1,2)/MA(26) - $(1 - 0.46B - 0.22B^2)w_t = -0.04x_t + (1 - 0.43B^{26})a_t$

E.2: List of Models

Model ARMAX	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
AR(1,2)/MA(1:3,26:29)	-711	0.35		29.19	43
AR(1)/MA(26,27)	-708	0.38		29.35	48
AR(0)/MA(26,27)	-706	0.38		33.79	49
AR(1:3)/MA(26)	-699	0.39		34.46	48
AR(1,2)/MA(26)	-699	0.39		35.62	49

Table E.1: Model Fit Comparisons for Weekly Demand

E.3 Forecasting Future One-Step Ahead Demand

One step ahead forecasts were again calculated for each of the models in Table E.1 for the winter of 1972. Table E.2 shows the balanced set of metrics for each of the models.

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
AR(1,2)/MA(1:3,26:29)	0.03	4.42	28.71	4.16%	9.97/21/8.38%	-11.88/7/-10.24%
AR(1)/MA(26,27)	0.07	4.10	24.82	3.86%	7.87/21/6.13%	-13.13/7/-10.12%
AR(0)/MA(26,27)	-0.01	4.18	25.95	3.96%	8.69/21/7.30%	-12.09/7/-10.42%
AR(1:3)/MA(26)	0.03	4.11	25.79	3.85%	8.61/21/7.24%	-12.21/7/-10.52%
AR(1,2)/MA(26)	0.02	4.20	26.45	3.97%	8.65/21/7.27%	-12.28/7/-10.59%

Table E.2: Model Statistics Comparisons for Weekly Demand Forecasts

The models AR(1,2)/MA(1:3,26:29) and AR(1)/MA(26,27) appeared to produce the best modeling results in Table E.1. The models AR(1:3)/MA(26) and AR(1)/MA(26,27) produce the best results in Table E.2 from a prediction standpoint with similar MAPE values of 3.85% and 3.86% respectively. Model AR(1)/MA(26,27) was finally chosen due to the smaller Overestimate and Underestimate predicted demand values. The One-Step Ahead results for this model is described in detail in Section 5.3.1.4.

E.4 Forecasting Future Multi-Step ahead Demand

This is described in detail in Section 5.3.1.5 and not duplicated here.

Appendix F

NARMAX WINTER WEEKLY MODELING AND FORECASTING WITH TEMPERATURE (1963-1973)

F.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.3.3.

The starting point is the Winter Weekly Demand time series, which is described in Chapter 4. This data is shown in Figures 5.13 and 5.14. The transformation of the data was described in Section 5.3.1.2.

F.2 Model Analysis - ARX

As in Section 5.2.3, a first step analyzed a Linear model (ARX), followed by inclusion of residuals, thus creating an ARMAX model. Following the linear model analysis, 2nd and 3rd order terms will be introduced (both without and with residuals NARX and NARMAX), to find the most appropriate model from a modeling and especially a forecasting perspective. The best results will then we explained (as well as the results from the other models).

Using the cross correlation, variables 1 to 30 for both Demand and Temperature were initially included in the NARMAX model (no residuals in this first run, hence representing a ARX model). The terms 1,2 and 26 were selected for both demand and temperature, and the ERR profile is shown in Figure F.1 and the terms selected generate nearly 92% of the total (Table F.1). The profile shows again the significance of the temperature at time t on the demand at time t, generating 87% of the ERR value.

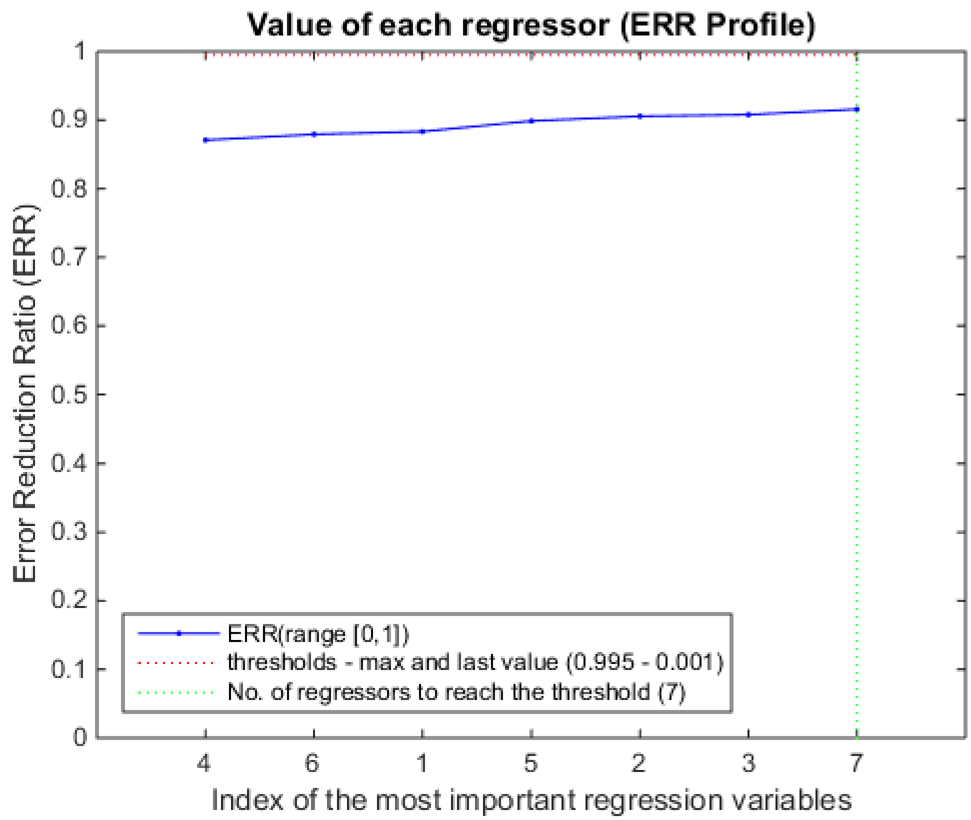

Figure F.1: ERR Profile

The modeling statistics for the 7 term model are:

1. F Statistic with 178 data values = 0.31

2. Q Statistic = 33.48 with 45 degrees of freedom (df) (52-7) which shows an adequate model (45 df χ^2 value is 61.67 at 5% level)

Although the ACF of the residuals , shows no major significant lags, there seems to be some value still in the residuals around the 26th lag. The four Nonlinear Validity Tests for

Index	Model term	Parameter	ERR(%)
1	x_k	-0.042	87.09
2	x_{k-2}	-0.011	0.84
3	y_{k-1}	-0.424	0.41
4	x_{k-1}	-0.017	1.54
5	y_{k-2}	-0.188	0.69
6	y_{k-26}	-0.327	0.23
7	x_{k-26}	-0.012	0.79

Table F.1: Results of the FROLS algorithm applied to Linear ARX Model

this linear ARX model shows a few significant missing terms/variables (Figure F.2), hence, there does appear to be value in the residuals, which will be checked in the next section.

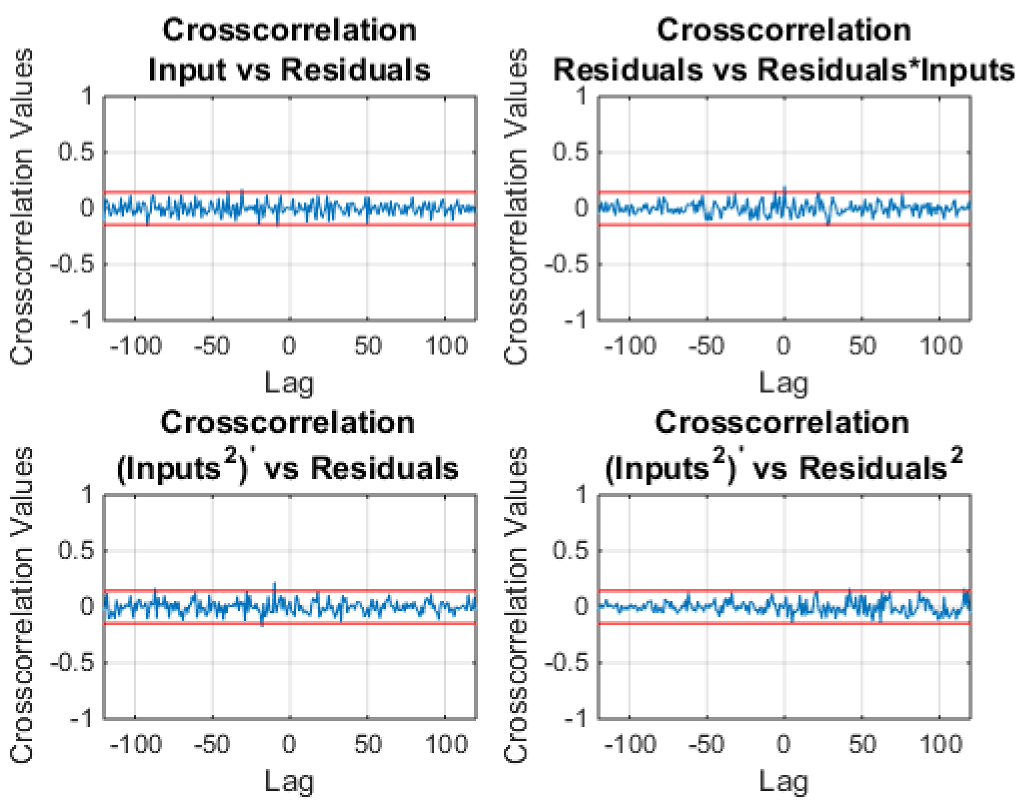

Figure F.2: Linear ARX Model Validity Tests

One-Step Ahead Predicted Output for this linear ARX model are shown in Figure F.3. The corresponding forecast statistics are shown in Table F.2. .

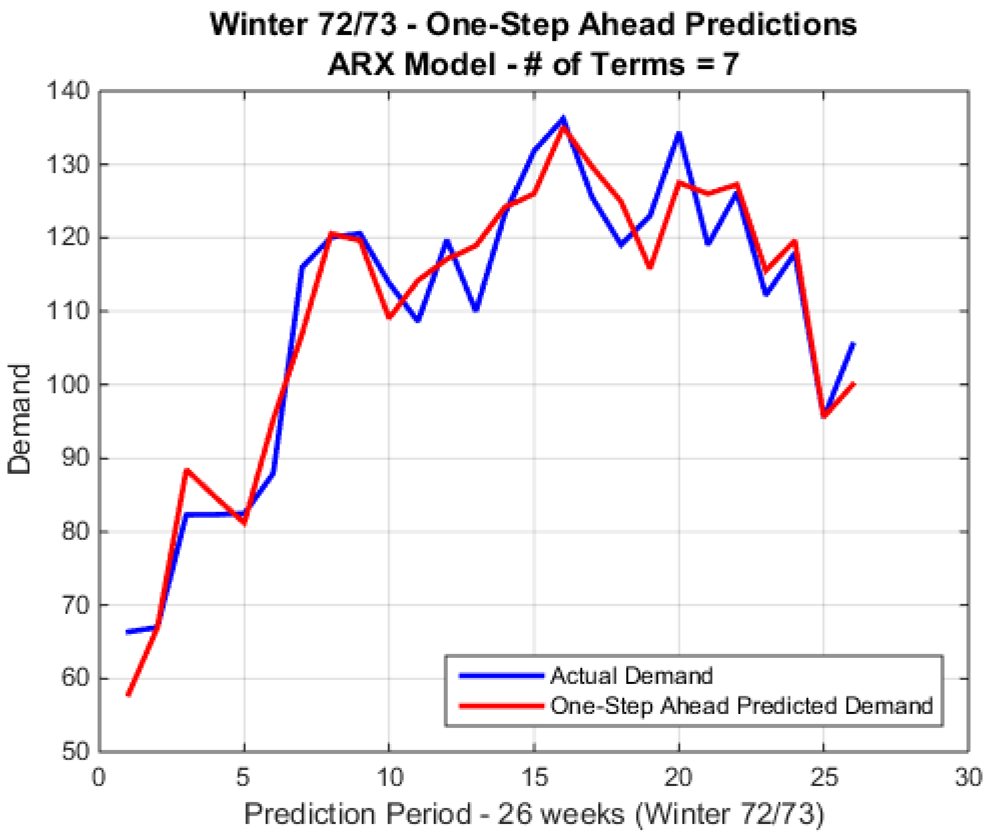

Figure F.3: Predicted vs Actual Demand for the Winter of 72/73 (ARX model)

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
Linear ARX Model	0.05	4.18	25.96	3.98%	8.93/13/8.12%	-9.03/7/-7.79%

Table F.2: Model Statistics Comparisons for Weekly Demand Forecast (ARX Model)

F.3 Model Analysis - ARMAX

As described above, following the results of the Linear ARX model, residual terms were incorporated into the data, thus generating an ARMAX model and the process repeated. The ACF of the residuals indicated some value around the 26th lag, and the Non-Linear validity tests also showed some significant lags, although not apparently relevant to the periodicity of the data, several combinations of e_{k-1} to e_{k-26} were analyzed. The term e_{k-26} was found to add the most value, and two runs with this residual were necessary to stabilize the terms' parameter values. The ERR total was 91.49% and the terms in the model are shown in Table F.3 :

Index	Model term	Parameter	ERR(%)
1	x_k	-0.045	87.14
2	e_{k-26}	-0.168	0.74
3	y_{k-1}	-0.447	0.90
4	x_{k-1}	-0.019	1.32
5	x_{k-2}	-0.013	0.53
6	y_{k-2}	-0.226	0.62
7	y_{k-26}	-0.182	0.18
8	x_{k-26}	-0.006	0.08

Table F.3: Results of the FROLS algorithm applied to Linear ARMAX Model

The modeling statistics are :

1. F Statistic with 178 data values = 0.34

2. Q Statistic = 36.57 with 44 degrees of freedom (df) (52-8) which shows an adequate model (44 df χ^2 value is 60.48 at 5% level)

One Step Ahead Predicted Output for the linear ARMAX model is shown in Figures F.4. The corresponding forecast statistics for this model are shown in Table F.4.

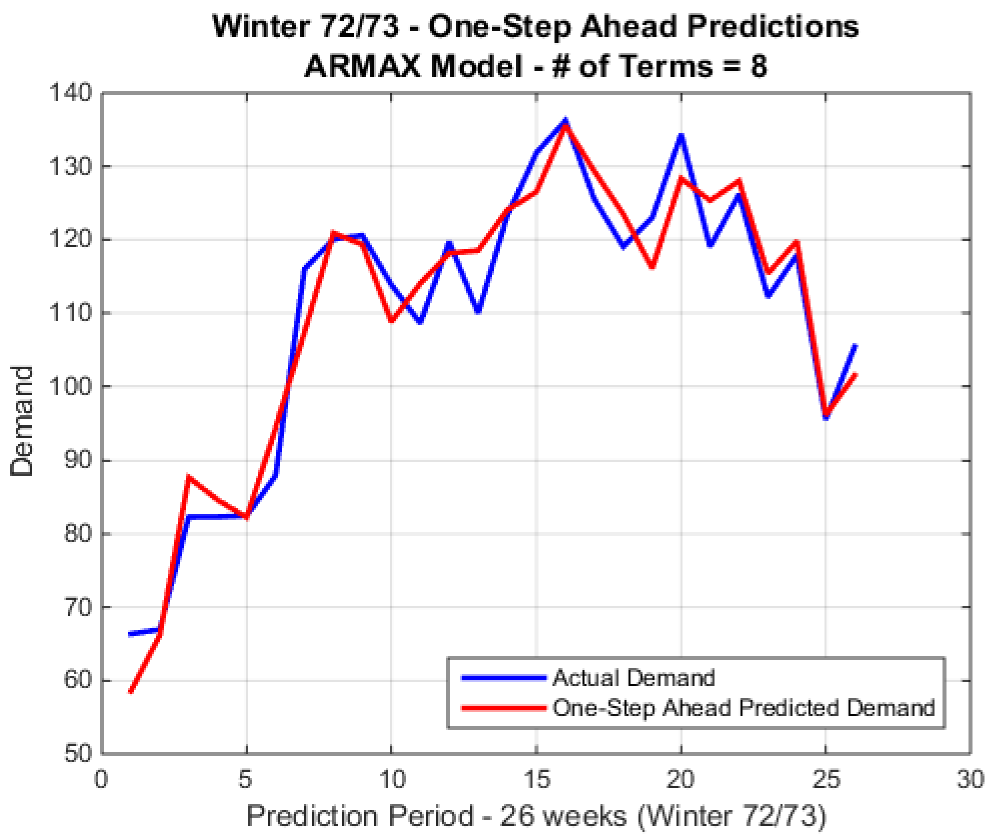

Figure F.4: Predicted vs Actual Demand for the Winter of 72/73 (ARMAX model)

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
ARMAX Model	0.14	3.84	21.72	3.66%	8.54/13/7.77%	-8.60/7/-7.41%

Table F.4: Model Statistics Comparisons for Weekly Demand Forecast (ARMAX Model)

F.4 Model Analysis - NARX

The next step was to test different model orders, thus generating a possible NARX or NARMAX model. A 2nd Order NARX model was tested, initially, using the same variables as the linear ARX model (i.e. $y_{k-1}, y_{k-2}, y_{k-26}$ and $x_k, x_{k-1}, x_{k-2}, x_{k-26}$). This generated 35 terms. Nine terms were selected on reaching the thresholds, with an ERR total of 91.78%. The selected terms are shown in Table F.5. The second order terms, however, appear to add little value to the ERR total.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.042	87.09
2	x_{k-2}	-0.010	0.84
3	y_{k-1}	-0.426	0.41
4	x_{k-1}	-0.017	1.54
5	y_{k-2}	-0.168	0.69
6	y_{k-26}	-0.356	0.23
7	x_{k-26}	-0.014	0.79
8	$x_{k-1} * x_k$	0.001	0.15
9	$y_{k-2} * x_k$	-0.007	0.05

Table F.5: Results of the FROLS algorithm applied to 2nd Order NARX Model

The modeling statistics are :

1. F Statistic with 178 data values = 0.31
2. Q Statistic = 33.76 with 43 degrees of freedom (df) (52-9) which shows an adequate model (45 df χ^2 value is 59.30 at 5% level)

The ACF and the PACF of the residuals showed no significant lags, but the four Nonlinear Validity tests showed a few lags slightly over the 95% significance levels. One-Step Ahead Predicted Output for the 2nd Order NARX model are shown in Figures F.5; and the corresponding forecast statistics for this model are in Table F.6.

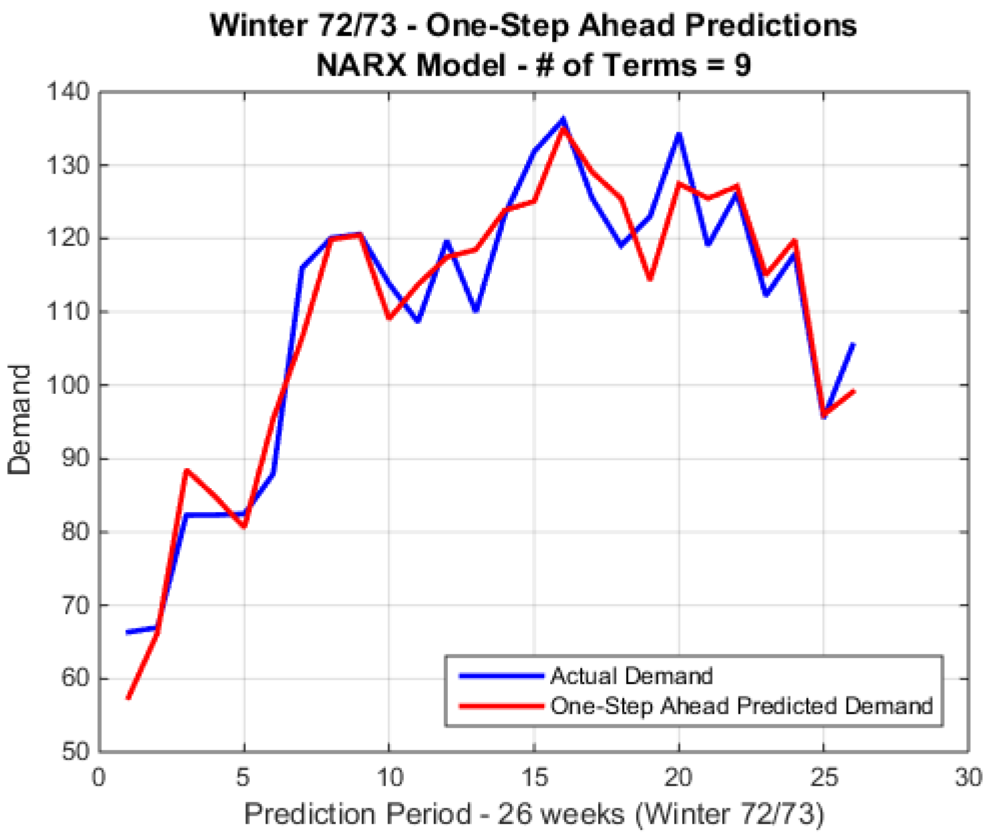

Figure F.5: Predicted vs Actual Demand for the Winter of 72/73 (NARX model)

Incorporating residuals into the model generating a full NARMAX model are described in Section 5.3.3.3.

Model	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
NARX Model	0.21	4.29	27.28	4.11%	8.52/13/7.74%	-9.52/7/-8.21%

Table F.6: Model Statistics Comparisons for Weekly Demand Forecast (NARX Model)

F.5 Forecasting Future Multi-Step ahead Demand

This is described in detail in Section 5.3.3.4 and not duplicated here.

Appendix G

ARMAX YEARLY WEEKLY MODELING AND FORECASTING WITH TEMPERATURE (1963-1973)

G.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.4.

The starting point is the Yearly Weekly Demand time series, which is described in Chapter 4. This data is shown in Figures 4.1 and 4.2. The transformation of the data was described in Section 5.4.1.2.

G.2 Parameter Identification

Using the results of the autocorrelation of both the Transformed Winter Demand and Temperature, several models were tested for their AIC and BIC values. The lowest values were produced with AR and MA variables with delays of 1, 2, 3, 51, 52, 53, 54 and 55. The various combinations of these variable lags were then tested to find a balance between the modeling and forecast statistics. Additionally, several delays were applied to temperature series, but a delay of zero always produced the best results. This implies again that future demand at time t is dependent on the corresponding temperature at t, and past weeks demand values.

The list of models which generated the best balance of statistics are shown below in

List G.2 and Table G.1. Model AR(1,2,51,52,53,54)/MA(52), produces the best results from a modeling perspective. Note that each of the models have a few lags outside the 95% confidence limits when analyzing the ACF of the residuals. However, the value of these lags is only just above or below the + or - 95% limits and do not indicate the models are inadequate in anyway.

- AR(1,2,51,52,53,54)/MA(52) - $(1 - 0.43B - 0.20B^2 + 0.19B^{51} + 0.37B^{52} + 0.34B^{53} + 0.26B^{54})w_t = -0.04x_t + (1 - 0.84B^{52})a_t$

- AR(1,2,52,53,54)/MA(52) - $(1 - 0.40B - 0.18B^2 + 0.35B^{52} + 0.34B^{53} + 0.26B^{54})w_t = -0.04x_t + (1 - 0.85B^{52})a_t$

- AR(1,2)/MA(52) - $(1 - 0.40B - 0.18B^2)w_t = -0.04x_t + (1 - 0.56B^{52})a_t$

- AR(1,2,3)/MA(52) - $(1 - 0.40B - 0.19B^2 - 0.01B^3)w_t = -0.04x_t + (1 - 0.56B^{52})a_t$

List G.2: List of Possible ARMAX Models

Model AR/MA	AIC	F Values	Significant Lags	Q Value	Degrees of Freedom
1,2,51,52,53,54/52	-1435	0.73	4,22,185	41	45
1,2,52,53,54/52	-1425	0.75	22,25,51, ...	48	46
1,2/52	-1406	0.80	22,25,51,53, ...	65	49
1,2,3/52	-1404	0.80	22,25,51,53, ...	66	48

Table G.1: Model Fit Comparisons for Weekly Demand

G.3 Forecasting Future One-Step ahead Demand

One step ahead forecasts were again calculated for each of the models in Table G.1 for the period April 1972 though to March 1973. Table G.2 shows the balanced set of metrics for the models in List G.2 for all of the 52 weeks.

Model AR/MA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
1,2,51,52,53,54/52	-0.09	3.02	15.05	4.06%	6.95/11/14.40%	-9.96/52/-9.45%
1,2,52,53,54/52	-0.06	3.13	16.16	4.15%	8.24/47/6.93%	-9.91/33/-8.55%
1,2/52	-0.01	3.19	16.54	4.20%	8.83/47/7.42%	-11.53/33/-9.94%
1,2,3/52	-0.01	3.20	16.58	4.21%	8.83/47/7.42%	-11.54/33/-9.95%

Table G.2: 52 Week - One-Step Ahead Model Forecast Comparisons for Weekly Demand

The model AR(1,2,51,52,53,54)/MA(52) produced the best 52 week One-Step Ahead forecast statistics. The One-Step Ahead results for this model is described in detail in Section 5.4.1.4.

G.4 Forecasting Future Multi-Step ahead Demand

Multi step ahead forecasts were again calculated for each of the models in Table G.1 for the period April 1972 to March 1973 (i.e. 52 weeks). In this case the last actual demand used in the last week of March 1972. Table G.3 shows the balanced set of metrics for each of the models in List G.2 for all of the 52 weeks.

Model AR/MA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
1,2,51,52,53,54/52	-1.16	3.88	21.03	6.04%	6.74/11/13.97%	-12.12/7/-16.82%
1,2,52,53,54/52	-0.87	4.24	23.53	6.77%	8.07/11/16.71%	-10.79/7/-14.97%
1,2/52	-1.89	4.51	27.44	6.78%	8.69/21/7.30%	-12.09/7/-10.42%
1,2,3/52	-1.84	4.49	27.15	6.77%	8.61/21/7.24%	-12.21/7/-10.52%

Table G.3: 52 Week - Multi-Step Ahead Model Forecast Comparisons for Weekly Demand

Additionally, starting from the actual demand of the last week of September 1972, the 26 Winter Weeks statistics (October 1972 to March 1973) for the models, are shown in Table G.4.

Model AR/MA	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
1,2,51,52,53,54/52	-1.90	3.06	13.52	2.98%	5.39/25/5.65%	-7.09/1/-10.68%
1,2,52,53,54/52	-2.57	3.63	18.06	3.46%	3.93/25/4.12%	-8.01/20/-5.96%
1,2/52	-3.89	4.42	28.40	4.23%	3.18/25/3.33%	-9.54/7/-8.23%
1,2,3/52	-3.82	4.39	27.82	4.19%	3.22/25/3.37%	-9.47/7/-8.17%

Table G.4: 26 Week - Multi-Step Ahead Model Forecast for the Winter Weekly Demand

The model statistics for AR(1,2,51,52,53,54)/MA(52) produced the best 52 and 26 week Multi-Step MAPE and are described in Section 5.4.1.5 and 5.5.1.6.

Appendix H

NARMAX YEARLY WEEKLY MODELING AND FORECASTING WITH TEMPERATURE (1963-1973)

H.1 Introduction

This appendix provides the details which culminated in the results described in Section 5.4.3.

The starting point is the Yearly Weekly Demand time series, which is described in Chapter 4. This data is shown in Figures 4.1 and 4.2. The transformation of the data was described in Section 5.4.1.2.

A first step analyzed a Linear model (ARX), followed by inclusion of residuals, thus creating an ARMAX model. Following the linear model analysis, 2nd and 3rd order terms were introduced (both without and with residuals NARX and NARMAX), to find the most appropriate model from a modeling and especially a forecasting perspective. The results are explained below.

H.2 Model Analysis - ARX/ARMAX

Using the cross correlation, variables 1 to 52 for both Demand and Temperature were initially included in the NARMAX model (no residuals in this first run, hence representing an ARX model). The terms 1, 2 and 52 were selected for both demand and temperature, and the ERR profile is shown in Figure H.1 and the terms selected generate nearly 89% of the total (Table H.1). The profile shows again the significance of the temperature at time t on the demand at time t, generating 83% of the ERR value.

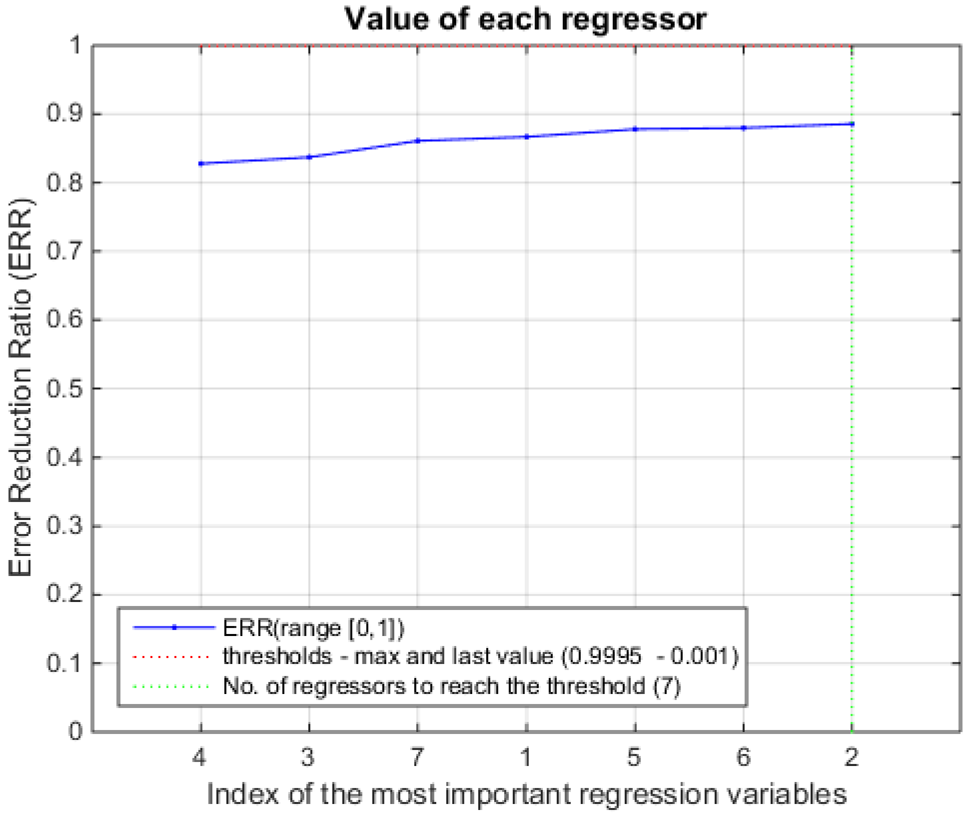

Figure H.1: ERR Profile

The ACF of the residuals of the above ARX model, shows significant lags at 104 and 22, as well as values close to significant around lag 52. Hence Moving Average components will be included, generating an ARMAX model.

Several combinations of e_{k-1} to e_{k-104} were analyzed. The terms e_{k-52} and e_{k-104} were found to add the most value. However, the value added of the term x_{k-52} was so small it was dropped from the input terms. The effect on the ERR total increased slightly to 90.56%. Two runs with the error terms e included were required to stabilize the ACF of the generated residuals. The resulting terms and values in the model are shown in Table H.2.

The modeling statistics are :

Index	Model term	Parameter	ERR(%)
1	x_k	-0.041	82.77
2	y_{k-52}	-0.38	0.94
3	x_{k-52}	-0.014	2.38
4	y_{k-1}	-0.40	0.59
5	x_{k-1}	-0.016	1.11
6	x_{k-2}	-0.011	0.19
7	y_{k-2}	-0.21	0.59

Table H.1: Results of the FROLS algorithm applied to Linear ARX Model

1. F Statistic with 311 data values = 0.76

2. Q Statistic = 47.02 with 44 degrees of freedom (df) (52-8) which shows an adequate model (44 df χ^2 value is 60.48 at 5% level)

The ACF and the four Nonlinear Validity Tests for this linear ARMAX model (shown in Figure H.2) shows few significant values.

The 52 week and 26 Winter weeks One Step Ahead forecast for this ARMAX model is shown, in Figure H.3. The corresponding forecast statistics for this model are shown in Table H.3.

Index	Model term	Parameter	ERR(%)
1	x_k	-0.045	85.08
2	e_{k-52}	0.41	1.32
3	e_{k-104}	0.29	0.68
4	y_{k-1}	-0.48	0.68
5	x_{k-1}	0.21	1.59
6	x_{k-2}	-0.013	0.27
7	y_{k-2}	-0.23	0.64
8	y_{k-52}	-0.01	0.01

Table H.2: Results of the FROLS algorithm applied to Linear ARMAX Model

Model ARMAX	MPE	MAE	MSE	MAPE	Over Prediction Value/Loc/%	Under Prediction Value/Loc/%
52 week	0.08	2.74	12.29	3.82%	9.78/11/20.27%	-7.75/24/-13.82%
26 Winter weeks	0.47	2.80	12.15	2.60%	6.42/13/5.84%	-7.59/7/-6.54%

Table H.3: Model Forecast Comparisons for 52 weeks and 26 Winter Weeks Demand (ARMAX Model)

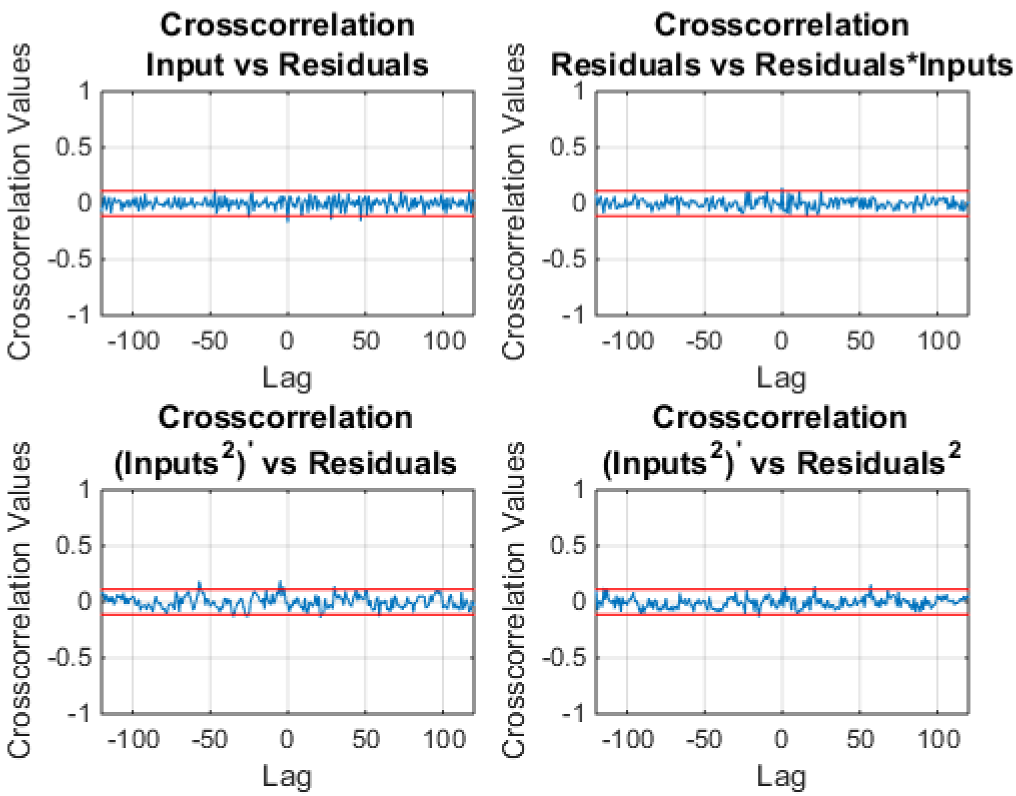

Figure H.2: Linear ARMAX Model Validity Tests

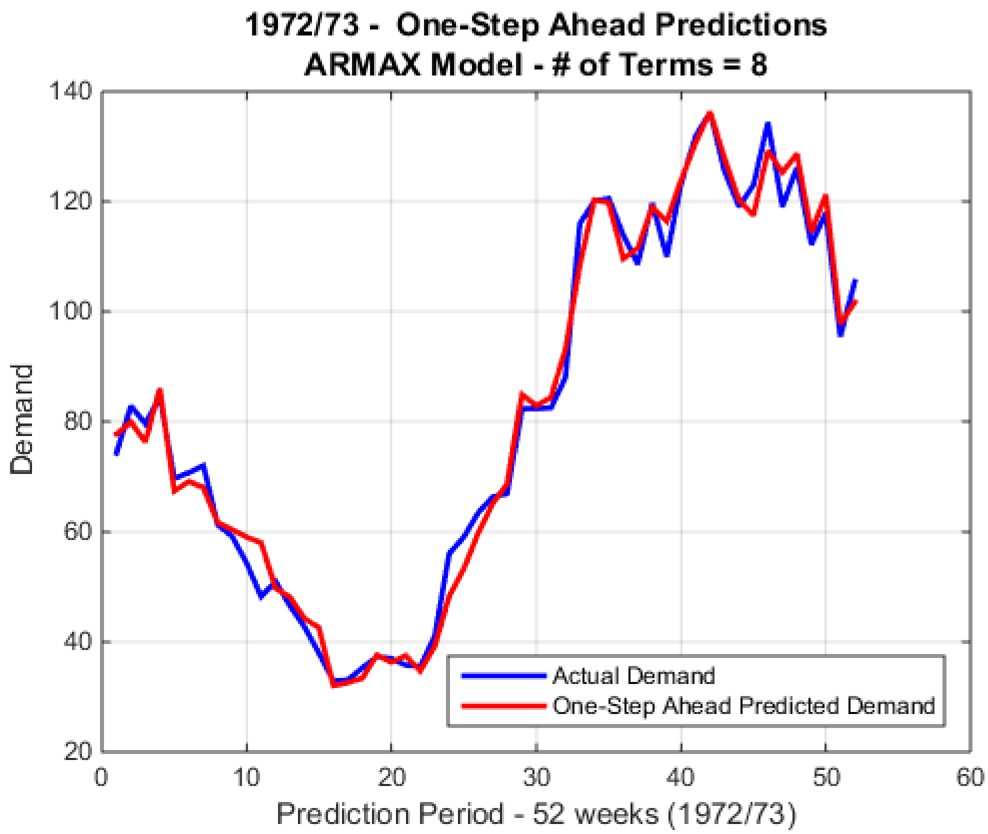

Figure H.3: Predicted vs Actual Demand for 1972/73 (ARMAX model)

H.3 Model Analysis- NARX/NARMAX

The next step was to test different model orders, thus generating a possible NARX or NARMAX model. A 2nd Order NARX model was tested, initially, using the same variables as the linear ARX model (i.e. $y_{k-1}, y_{k-2}, y_{k-52}$ and x_k, x_{k-1}, x_{k-2}). This generated 27 terms. Eight terms were selected on reaching the thresholds, with an ERR total of 87.48%. However, the ACF of the residuals showed that there was additional information especially around lags 52 and 104. Hence Moving Average terms (linear) were added $e(k-52)$, and $e(k-104)$. The details are described in Section 5.4.3.2

H.4 Forecasting Future Multi-Step ahead Demand

This is described in detail in Section 5.4.3.4 and not duplicated here.