Analysis of the Project Supply Chains: Coordination and Fair Allocation

by

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

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March, 2017

Abstract

This research investigates how project contracts can coordinate the supply chain between a project manager and contractor and if the solutions can be ensured as equitable. The main features of this type of supply chain are the trade-offs between the selection of a higher rate of resource consumption with a consequent higher cost to the contractor and a lower rate of resource consumption leading to later delivery and a reduction of the project-reward to the project manager. This broader problem could lead to a coordination problem for the overall supply chain. This research proposed a solution to this broader problem in two different scenarios: Take it or leave it scenario and negotiation scenario. Finally, the fair allocation of the risks and benefits and the related decision-making issues are addressed as one of the behavioural barriers to the supply chain coordination.

The coordination issues in a take it or leave it scenario are addressed using time-based and fixed price project contracts using Stackelberg games. Models of coordination were proposed with time-based contracts, but the fixed price contracts failed to coordinate. The coordination problems in negotiation scenario are addressed with the Nash's bargaining, the Kalai Smorodinsky bargaining, and the utilitarian approach. A cost plus contract has been found to dominate the solutions over any cost sharing contract and fixed price contract for Nash's bargaining and Kalai Smorodinsky bargaining cases. Finally, the issues of fairness of allocation of risks and benefits as one of the challenges of supply chain coordination, have been investigated. Among the various definitions, this research used the definition of fairness as inequity aversion as proposed by Fehr & Schmidt (1999). The fixed price contracts were found to coordinate the supply chain under consideration alongside the time-based contracts if the members had fairness concern.

Some of the key features of this research include the incorporation of various probability distributions for the project completion time and cost, the inclusion of various forms of risk preference, and addressing the challenges of fair allocation in project supply chains.

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Acknowledgements

My Ph.D. journey has been an eventful one right from the word go to reach to this stage. It would have been incomplete without the support and help I received from various people at different capacity. Thus, my sincere gratitude and thanks to all these people who have been a key part of this journey.

My Ph.D. journey began with Dr. Andrew Brint and Dr. Alok Choudhary since the time when I applied and successfully secured the scholarship to pursue my dream. Their comments, guidance, and support have been the key cornerstones of my journey of this Ph.D. Meeting with Dr. Brint every week to discuss issues around my Ph.D. has always been more than just helpful. I would also like to extend my thanks and gratitude to my two other supervisors. After Dr. Choudhary left the Management School, I have got the chance to work with Dr. John Fry and then Dr. Chantal Cantarelli. Their help and support have not only been just helpful, but also very insightful to improvise my work.

My PhD. journey would have been incomplete without the administrative help and support from the wonderful people from the research office at the management school, The University of Sheffield. I would like to take this opportunity to express my sincere thanks and gratitude to them.

Further thanks extend to the examiners of my Ph.D. viva: Professor. Jiyin Liu from Loughborough University and Dr. Michael Simpson from The University of Sheffield. Their suggestion to make the necessary amendments to this thesis and the possible options to extend my research work in future, are invaluable.

I would also like to acknowledge the support I received from my colleagues and my friends who have always been full of ideas, debates, and fun. Without their company, help and support my Ph.D. life would have been a trivial journey.

Most importantly my heart goes to my family. My mum has been the key instrumental person for me to get the inspiration to excel in life starting from a very small town. She encouraged me to believe that it is possible to achieve despite being raised in a small town. My mum, dad, and my sister have inspired me in every possible aspect of my Ph.D. My dad's explanation of mathematical models from a Physicist's perspective has always been a delight

for me. Of course, the fun-filled moments I have shared with my sister during this Ph.D. life are the key stress busters. I thank you all for the sacrifices you have made to support my journey.

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Chapter 1

Introduction

> "No man is an island"
>
> *John Donne*

The quote by the English poet John Donne means everyone needs to rely on others to get some of the jobs done for their day to day life. To put it in other words, people are dependent on each other in society. This is also true for modern organizations as they depend on the actions of the other organizations in the supply chain. However, these organizations have their own goals and objectives, and so there are problems of misalignment and lack of coordination of decisions. This research argues that this lack of coordination is not beneficial for either individual organisations or for their overall sum. This research has the broad objective to find the solutions for coordination problems for the organizations in a project supply chain.

A popular saying is that the nature of competition will increasingly not be between companies, but rather between supply chains. Intense market competition, short product life cycles, and increased customer service expectations prompt the need for ever more efficient supply chain management through better coordination and cooperation among all members. Coordination is very critical to the success of the performance of the entire supply chain as poor coordination leads to increased inefficiency and costs. Due to this importance, it has received considerable attention in the literature. However, most of the attention has been paid to product supply chains with order quantities and price as the decision variables. Limited attention has been paid to supply chains in the project settings where completion time and cost are the key elements. The importance of coordination in project supply chains is no less than in product supply chains. Various problems arise in absence of coordination in project supply chains such as time and cost overrun. Therefore, this research focuses on the coordination issues for project supply chains. A dyadic supply chain consisting of a project manager and a contractor organization, is the basis for this research. Different forms of project contracts: fixed-price, time-based, and cost-based, have been considered as the tools to achieve the coordination. Concepts of game-theory have been used as the theoretical underpinning. The first two objectives are to investigate how the project supply chain can be coordinated using the contracts in a take it or leave it situation and a bargaining situation. The third objective is to investigate the issues of fairness in allocation as one of the barriers to supply chain coordination.

Before these objectives of the research are addressed, this chapter introduces the background theme of this research. The main discipline area for this research is supply chain management. The area of supply chain coordination sits under the bigger umbrella of supply chains and supply chain management. Hence, this chapter introduces supply chain coordination as an element of the broader paradigm of supply chain management, followed by its importance and motivation, and the scope of this research.

1.1 Supply chain management and its elements

A supply chain comprises of all the players and all of their activities which are directly or indirectly meeting the requirements of the end customer (Chopra & Meindl 2007), whereas supply chain management is defined as a system approach to manage the entire flow of information material, services and finance (Chase et al. 1998). The members of a supply chain can operate in a centralized setting with a single decision maker or in a decentralized setting with multiple decision makers. In the case of centralized settings, the goals, and objectives of the members are aligned due to the presence of a single decision maker and they aim to optimize the system-wide performance. In the case of decentralized settings, due to the presence of multiple decision makers with multiple individual goals and objectives, the overall goals and objectives may not be aligned and may be conflicting in nature (Li & Wang 2007). This leads to sub-optimal overall benefit in comparison to the centralized settings. from this consideration, centralization may be ideal situation to have. However, decentralized systems are very common in the business environment due to the presence of various other benefits such as proximity to the market and a better understanding of the market (Lee & Whang 1999).

Each stage in the supply chain has a buyer-supplier relation between its members. Every member except the end consumer adds value to the input procured from their supplier and puts it forward to their respective customer. Thus, each individual pair is interlinked with others in the form of a chain relationship with numerous activities (Cooper et al. 1997). These activities can be classified into the following clusters (Wisener et al. 2005)

- Purchase Elements - Raw material procurement and supply management issues
- Production Elements Scheduling, capacity, and production techniques
- Distribution Elements Locations, transportation, logistics, warehousing and store
- Integration Elements- Collaboration in decision making, financial and non-financial flow in order to improve overall profit and customer value delivery

Several authors have used the terms supply chain integration, collaboration, cooperation, and coordination synonymously. However, authors including Camarinha-Matos et al. (2009) and Cohen & Roussel (2005) have argued that these terms differ depending upon the degree of integration. There seems to be a lack of unique definition of these terms. However, summarizing these debates, it can be said that the integration continuum ranges from arm's length transaction based short term relationship to fully integrated long term relationships. This research starts its investigation with the short to medium term integration approach (coordination) in a decentralized supply chain in order to identify mechanisms for the achievement of long-term collaboration. It is important to note that it may be counter-productive to integrate with all the members of the supply chain due to increased complexity (Lambert & Cooper 2000). These authors suggested the consideration of integration with the members who are critical to the success of the organization of concern. Thus, the present research considers the supply chain members who are critical to the success of the focal firm as well as the overall supply chain.

In order to review the problem areas of supply chain coordination, the coordination of supply chain is defined in the next subsection section followed by the justification of the selection of this topic.

1.2 Supply chain coordination and its importance

Malone & Crowston (1990, 1994) defined coordination in their coordination theory as managing dependencies between activities. On a similar note, supply chain coordination has been defined by several authors such as

- Supply Chain (SC) members taking actions jointly to improve the supply chain surplus (Chopra & Meindl 2007).
- Two or more companies implementing a long-term collaborative approach in order to create value, which could not have been possible working individually (Cohen & Roussel 2005).
- Simatupang & Sridharan (2002) stated, “Two or more independent organizations working jointly to execute supply chain operations with a greater degree of success in comparison to operating SC in isolation” (p.19).

Summarizing all these definitions, supply chain coordination can be considered as two or more independent organizations working in collaboration to achieve improvement in the following areas in comparison to non-collaborative and isolated approach of working

- Value proposition to customers
- Reduction in overall supply chain cost
- Overall supply chain profit

Early researchers highlighted that the supply chain can be coordinated when it is centralized. This is due to the presence of a single decision maker who optimizes the risks and benefits for the supply chain whereas, in a decentralized setting, every member would optimize their individual decisions. However, Cachon (2003) defined that if the coordination mechanisms can encourage supply chain members to take decisions in a decentralized supply chain such that the total benefit confirms to the benefits of a centralized supply chain without compromising the optimal individual benefits, then the decentralized supply chain can be considered as coordinated. This definition has been used as a working definition by many authors in the literature for proposing mathematical models for supply chain coordination. This research follows this as the working definition of supply chain coordination.

As stated in the last sub-section, the concept of supply chain coordination is a continuum from arms length or transaction-based relationships to close relationships (Cooper et al. 1997). Traditionally supply chain members have worked in an arm's length relationship. However, the modern day business environment has several challenges such as: an increased cost of resources; shorter product life cycle; increased competition among business entities; increased customer value expectations at a lower price; and changing external regulations. These have created a need for the organizations to come up with a coordinated supply chain system, which would not only exceed customer expectations, but also improve the profit figure. In addition, Simchi-Levi et al. (2003) have highlighted the need to effectively integrate suppliers, manufacturers, warehouses, and stores, so that goods are produced and distributed at the right time, in the right quantities, to the right locations and at the same time also to minimize system-wide costs while satisfying service requirements and meeting environmental regulations. However, in the presence of various challenges, the collaborative efforts are not always exercised in the decentralized settings. Many authors highlighted various barriers of supply chain management. These barriers can be classified into five broad categories as below

1. **Incentive barriers:** The existence of misaligned goals and objectives among members leads to sub-optimum results. Quite often, the optimum solution requires one or member of the supply chain to move away from their individual optimal solution. As a result, members tend to make decisions as per the local optimum instead of the global optimum. This leads to inefficiency in overall supply chain performance (Chopra & Meindl 2007, Harland et al. 2007, Fawcett et al. 2008)

2. **Financial barriers:**. Lack of access to financial resources limits the acquisition of other resources and investment opportunities to support coordination and ultimately achieve long-term integration (Briscoe et al. 2004, Chopra & Meindl 2007, Harland et al. 2007).

3. **Behavioural and other decision-making barriers:** Lack of trust among members, relational barriers, fear of loss of control, resistance to change, lack of understanding of benefits, and lack of top management commitment lead to further decision-making problems. e.g.- lack of rationality while making decisions on sharing risk and benefits

among members of supply chain (Cetindamar et al. 2005, Chopra & Meindl 2007, Harland et al. 2007, Fawcett et al. 2008, Forslund & Jonsson 2009).

4. **Information related barriers:** The poor flow of information constrains decision making and ultimately leads to sub-optimum results. This is contributed by many factors such as: poor information system which leads to lack of end to end visibility; unwillingness to information sharing; poor quality of shared information and so on (Barratt 2004, Chopra & Meindl 2007, Fawcett et al. 2008, Forslund & Jonsson 2009).

5. **Operational barriers:** Inconsistent processes and performance metric selections lead to increased variability of the overall supply chain performance. In addition, operational problems in the form of: problems of longer lead time due to geographic distance, inconsistent forecasting techniques, and poor inventory management further leads to increased variability of the overall performance (Briscoe et al. 2004, Forslund & Jonsson 2009).

Owing to these barriers, often the supply chains fail to coordinate properly and thereby fail to reap the benefits of the coordination. Some of the impacts of a non-coordinated supply chain are realized in the form of certain phenomena such as the bullwhip effect, double marginalization, overstocking and under-stocking (Simchi-Levi et al. 2003). The bullwhip effect is the order amplification when it moves away from the demand generation point at the end consumer end (Lee et al. 1997). Double-marginalization is the overall supply chain's profit reduction and final products price increase, due to the reservation of individual profit by each member of supply chain (Simchi-Levi et al. 2003). As a result, organizations end up with too much or too little stock, an inappropriate customer service level, and high cost.

The internal problems mentioned in the last paragraph makes the organizations vulnerable in the uncertain business environment with a twofold problem of lesser profit and dissatisfied customers, with few further repercussions. Some of the recent examples of such business environmental threats are: Closure of production sites of Toyota after a recent earthquake in south Japan (Kanaracus 2016), impact on the forecast growth figure of Nestle due to its infamous product recall of Maggi noodles from Indian market (Gibbons 2016), raw material shortage and lead time increase in BMWs plant (Automotive News, 2013); and loss of supply chain control and horse meat scandal of British retailer Tesco (Johnson 2013). These external threats are not caused by the lack of coordination; but their existence makes supply chains more vulnerable to these external threats if not coordinated (Chen et al. 2013). The authors further argued that supply chain coordination can reduce the extent of vulnerability of the supply chains to these external risks. The benefits of supply chain coordination can be realized in the form of reduced overall supply chain cost, reduced overstocking and understocking situations, improved customer service and improved quality (Wisener et al. 2005).

The supply chain coordination can be achieved using various means e.g. tools such as joint decision-making, supply chain contracting (Arshinder & Deshmukh 2008), and information sharing (Sahin & Robinson 2002, Arshinder & Deshmukh 2008). Several researchers including (Yao et al, 2008; Savaskan et al, 2004; Baiman, 2000; Chen et al, 2000; Cachon and Fisher, 2000; Chen, 1999) proposed coordination models with centralized and decentralized supply chain settings with the above-mentioned coordination mechanisms. Vendor Managed Inventory (VMI), Collaborative Planning, Forecasting, and Replenishment Techniques (CPFR), Joint replenishment and Joint New Product development are few of the tools which are used in modern day manufacturing and other related industries to facilitate supply chain coordination (Arshinder et al, 2008). This research has selected contracting mechanisms for the coordination of supply chain with a project manager and a contractor.

1.3 Motivation

Supply chain coordination has received considerable attention in the literature due to its enormous importance as mentioned in earlier subsections. The literature survey (documented in the next chapter) on supply chain coordination suggests the existence of two main streams of research direction: the importance of the role of coordination in supply chain coordination (Fisher et al. 1994, Lee et al. 1997, Ramdas & Spekman 2000, Horvath 2001); and the other one is the proposal of coordination mechanisms (Corbett et al. 2004, Giannoccaro & Pontrandolfo 2004, Bernstein & Federgruen 2005*a*, Kim & Oh 2005, Chen et al. 2006, Ding & Chen 2008). One of the most notable observations from these two research streams is the lack of focus on one of the challenges of coordination of how to allocate the benefits after coordination. Evidence from literature and practice suggests if mechanisms of coordination can not mitigate this challenge, then the benefits derived are not sustainable and are of a temporary nature e.g. Termination of the contractual relation between Walmart Canada and Lego group upon the rejection from Lego group to reduce the price in the Canadian market. Lego group kept the price same as in the American market and reaped additional benefit due to the appreciation of Canadian dollar (Georgiades 2008). Moreover, the failure of coordination and the early termination of contractual relationship have been documented in the absence of a proper fair allocation mechanism after coordination in the literature of Katok & Pavlov (2013). In addition, Griffith et al. (2006) found evidence of a positive relation between fair allocation mechanism and supply chain performance.

Another notable observation from the literature review is the emphasis on product supply chains. Relatively little has been explored on supply chains in a project setting. The difference between these two settings are well differentiated in the work by Slack et al. (2010) in the figure 1.3.

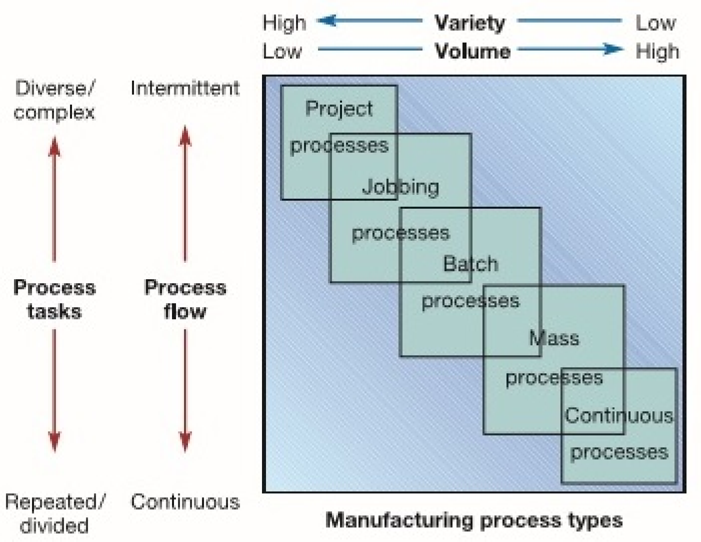


Figure 1.1: Different Process Types (Source: Slack et al. (2010))

It can be seen from this figure that the project process has got an unique and lowest volume of output. In addition, the projects have a definite start and definite end period. Once the project ends, the product operation begins.

The importance of projects is not a new phenomenon. Projects were present even in ancient time e.g. construction of pyramids during ancient Egyptian time (Kwon et al. 2010). The notable characteristics of projects are the specific start time and finish time. Unlike its counterpart product based manufacturing supply chains, it does not have the perpetual operation as it will cease at some point of time. The majority of the concepts of project management come from the construction projects and construction management. In fact, most of the tools and techniques for effective project management reside in the literature of civil engineering such as Critical Path Method (CPM) and Project Evaluation Review Techniques (PERT) (Kwon et al. 2010). However, while these tools can model uncertainty in project completion time (Klastorin 2004), they cannot model how changing contract incentives can encourage or discourage increasing the amount of resources allocated to the project contractor and hence, the likely completion time. Moreover, these tools are deployed at organizational level. Each organization involved in the project would have their own estimation of completion times using the tools CPM or PERT. However, there is a paucity of application of these tools in literature and practice which could effectively manage supply chain issues in a project setting. This is due to certain differences between the general product supply chains with the project supply chains (Kwon et al. 2010). These are as follows

- The projects have a definite start and a definite end point. The operations which transform the raw materials to a finished product, begin after that.
- The main source of uncertainty in product supply chain is the demand, whereas in the case of project supply chains the completion time and costs are the sources of uncertainty.
- The main decision variable in a product supply chain is the order quantity, whereas the resource consumption rate is the main decision variable in the case of project supply chains.
- The decision variable order quantity becomes visible in the case of product supply chains. However, the resource consumption rate cannot be easily monitored in the case of project supply chains.

Due to these basic differences, supply chain coordination in project settings is different from the supply chain coordination in product setting. Relatively little attention has been paid to the coordination issues in the project supply chains. However, the importance of coordination is no less for project supply chains in comparison to the product supply chains as various problems are associated with the project supply chains in the absence of proper coordination. Time and cost overrun are the two most prominent of them. Myriad examples can be found where the project went over time with a significant cost overrun for e.g. significant time and cost overrun in the rebuilding project of the Wembley stadium in UK (Moore 2009).

The above mentioned observations from literature and practice have been the main motivation for this research. Hence, the next subsection entails the scope of this research.

1.4 Scope of this research

The research gaps mentioned in the last subsection are the motivation of this research. This research seeks to propose a solution to the problems of supply chain coordination in a project setting with the following aim: **To propose the mathematical models of coordination in project supply chain and finally to overcome the challenges of fair allocation of risks and benefits.**

This research aim will be achieved with three different objectives in this research. Project contracts have been used as coordination mechanism and concepts from game theory have been used to explain the underlying decision-making process.

- **Obj.1:** To investigate if supply chain coordination models in a take it or leave it scenario using project contracts can be proposed.

- **Obj.2:** To investigate and propose optimal coordinating solutions in a bargaining scenario.

- **Obj.3:** To investigate what is a fair allocation of risks and benefits when coordinating the supply chain.

In all of these three objectives, a dyadic project supply chain consisting of a project manager organization and a contractor organization have been considered. The applicability of the derived models is explored with numerical examples. Construction projects have been used as a case example. The numerical values that are used, are based on the prevailing practice in the UK construction sector.

1.5 Expected Contribution

By achieving the objectives mentioned above, this research tries to contribute in several areas. The first objective will extend the existing models of supply chain coordination in a take it or leave it type of project environment.This will bridge the gap of lack of use of project contracts as a tool for coordination mechanisms for the supply chain.

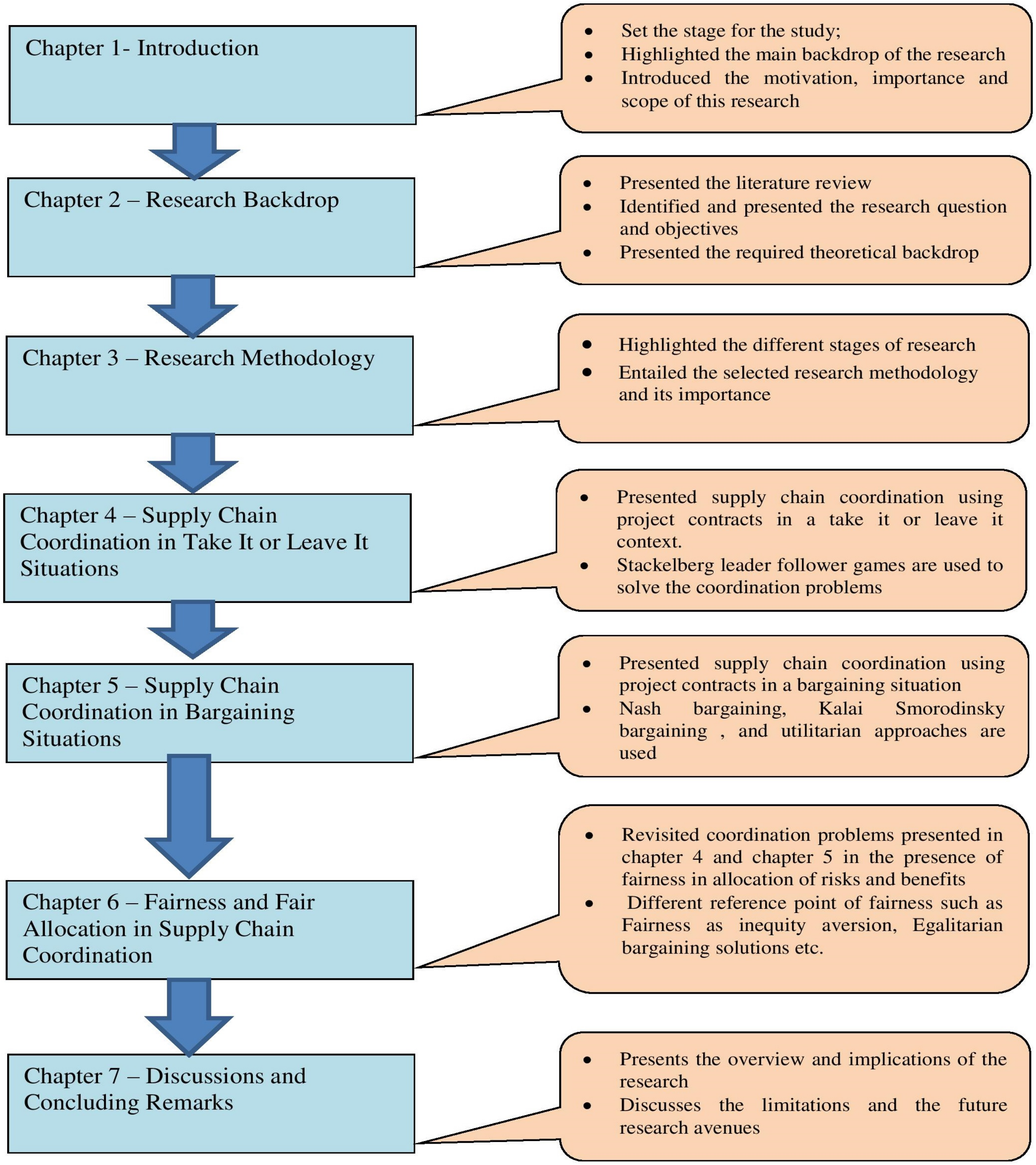
The second objective will contribute to the coordination mechanism for the environment with negotiation opportunities. Additionally, this is also expected to contribute to the application of bargaining theories and models to the supply chain coordination which is a relatively less addressed area in the literature.

The third objective is an attempt to extend the concepts of fair allocation from behavioural economics to the supply chain coordination. The literature review presented in next chapter reveals that behavioural & decision-making issues are one of the less addressed barriers of supply chain coordination. This research attempts to bridge the gap of this less addressed challenge by addressing the fair allocation issues.

Apart from the academic contributions, these objectives are also expected to contribute to the practice of supply chain coordination principles in real life projects. The literature review in later chapter reveals a reliance on a specific type of simple statistical distributions when it comes to consider the deciding variables of projects such as the completion time and the completion cost. This research is expected to bridge this gap by proposing solutions which are more close to reality by selecting more applicable distributions.

1.6 Summary of the thesis

This report is organized in the following manner



Chapter 1- Introduction

- Set the stage for the study;
- Highlighted the main backdrop of the research
- Introduced the motivation, importance and scope of this research

Chapter 2 – Research Backdrop

- Presented the literature review
- Identified and presented the research question and objectives
- Presented the required theoretical backdrop

Chapter 3 – Research Methodology

- Highlighted the different stages of research
- Entailed the selected research methodology and its importance

Chapter 4 – Supply Chain Coordination in Take It or Leave It Situations

- Presented supply chain coordination using project contracts in a take it or leave it context.
- Stackelberg leader follower games are used to solve the coordination problems

Chapter 5 – Supply Chain Coordination in Bargaining Situations

- Presented supply chain coordination using project contracts in a bargaining situation
- Nash bargaining, Kalai Smorodinsky bargaining , and utilitarian approaches are used

Chapter 6 – Fairness and Fair Allocation in Supply Chain Coordination

- Revisited coordination problems presented in chapter 4 and chapter 5 in the presence of fairness in allocation of risks and benefits
- Different reference point of fairness such as Fairness as inequity aversion, Egalitarian bargaining solutions etc.

Chapter 7 – Discussions and Concluding Remarks

- Presents the overview and implications of the research
- Discusses the limitations and the future research avenues

Chapter 2

Research Backdrop

This chapter sets the background of this research. At first, the literature review is presented, followed by the research gaps in the literature. Based on those research gaps, the focus of this research is presented in the form of research questions, objectives, and relevant theoretical underpinning.

2.1 Literature Review

This section reviews the existing research in the area of supply chain coordination in order to identify the existing research gaps. This helps the development of the problem framework for this research. This section will be subdivided into the following subsections:

- Arguments on different types of literature review
- Planning of the review
- Conducting the review
- Reporting the findings from the review
- A summary of literature to highlight the research gaps.
- Identified research gaps

2.1.1 Arguments on different types of literature review

An effective literature review is one of the most important basic building blocks for any academic research (Webster & Watson 2002). It creates the platform for knowledge advancement and theory creation by linking the developments from past research to the present. Any literature review process is of high scientific value in terms of knowledge creation when it is thorough, fair and evidence-based (Adolphus, 2013; NUI Galaway, 2014). Literature could be conducted in narrative format and systematic literature review format (Garg et al. 2008). A systematic literature review provides more objective and evidence-based approach to identify the literature. On the contrary, a narrative review is a more traditional approach which offers a more theoretical and detailed overview of the research topic. Both of these methods have shortcomings and benefits. The narrative method offers a more comprehensive review (Garg et al. 2008). However, it lacks in the details of the underpinning method selected for the review and thereby, it is less replicable. This shortcoming is overcome by systematic literature review (Tranfield et al. 2003, Colicchia & Strozzi 2012) . However, this method is also not a fool proof solution. One of the notable shortcomings of a systematic literature review is its bias due to selective publication selection (Garg et al. 2008). Thus, the authors argued the need for a more careful approach to the literature review to overcome these shortcomings. In order to maintain a balance between unbiased nature of the review and replicable nature of the review, this research would not be tied up with any particular label of doing the literature review. This research has borrowed some ideas of a systematic literature review as well. At the same time, in order to avoid any bias of rejecting the relevant source of literature, this research applied some ideas of narrative review. This research did not undertake any meta-analysis.

As a starting point, the three step evidenced based approach of (Tranfield et al. 2003) is followed. The authors suggested to conduct the literature review in three stages: Plan the review, conduct the review, and report the findings.

2.1.2 Plan of the review

Review Protocol

The review protocol was decided at this stage. It's a detailed plan of steps to be taken for conducting the review. The C (Context), I (Intervention), M (Mechanism), and O (Outcome) logic suggested by Denyer & Tranfield (2009), has been considered for setting up the review protocol. According to the authors, the CIMO logic should consist of the following

Context (C)

Individual, relationship, institutional settings or wider system being studied

Intervention (I)

Action or activity being studied

Mechanisms (M)

- Identification of conditions under which the interventions work.
- Identification of mechanisms that explain the relationships between interventions and outcomes.

Outcome (O)

- Effects of interventions
- The measurement of the outcomes
- Any intended and unintended effects of the interventions

This research studied the context of supply chains coordination in the project setting. However, to begin with, this research studied the context of supply chain coordination for product supply chains and project supply chains. The intervening activities are the barriers to the coordination and sources of uncertainty. Coordination mechanisms proposed by Arshinder & Deshmukh (2008) are the mechanisms. As a result of the intervention, finally, it was analysed and found how the models of coordination fared with respect to the challenges and under what type of uncertainty.

A review framework is derived based on the CIMO explained above.

Barriers Mechanisms	Incentive Based Barriers	Financial Barriers	Behavioural/ Decision Making Barriers	Information Related Barriers	Operational Barriers
Formal Contractual Mechanisms					
Informal Relational Mechanisms					

Figure 2.1: Literature Review Framework

The horizontal axis represents the barriers to supply chain coordination as described in the introduction chapter of this thesis. The vertical axis represents the contractual mechanisms. According to (Arshinder & Deshmukh 2008), the coordination can be achieved with contracting, information system, information sharing, and joint planning/joint decision making. This can be summarised as a continuum of formal contracting to informal relational mechanisms such as information sharing and joint decision making. Thus, this research has classified the mechanisms from two different dimensions: formal contractual mechanisms and informal relational mechanisms.

Search Criteria

One of the first steps was to identify the search criteria. In order to maintain the high standard of findings, only published works from peer-reviewed journal articles were considered in the search (Newbert, 2007; David and Han, 2004). For this purpose, the journals from the latest ABS (Association of Business School) and AACSB (The Association to Advanced Collegiate Schools of Business) Journal Ranking are considered. Any unpublished works and conference proceedings are excluded from the search list. In order to maintain the high standard quality of the review, this research has put a higher emphasis on the ABS 3* or 4*, and AACSB A^+ journals. However, in certain cases, depending on the requirement, importance, and relevance of the findings, published papers in the other peer-reviewed journals are also selected for this review.

Another important criterion used for journal article selection was the date of publications.

The objective of this study is based on the supply chain coordination in the vertical direction of the supply chain and assessing how the challenges of coordination have been addressed. Spengler (1950) is one of the first authors to analyse the issues of double marginalization in a non-coordinated supply chain. This terminology identifies the reduction of overall profit of the supply chain due to individual profit reservations by the members of the supply chain. As a result, this causes incentive barriers. As per the knowledge of the author of this research, this is the first published work that analysed the existence of barriers to supply chain coordination. Thus, this research has selected published works post 1950 only. Although it was identified in 1950, the realisation of the importance of coordination started in mid-1980s and early 1990s. In fact, from the late 1990s authors started to propose coordination mechanisms. Thus, most of the selected papers and published works for this review are starting from that era to till date with few exceptions as necessary.

Keyword selection

Parallel to the determination of the search criteria, a list of relevant keywords was identified. In the first phase, 40 keywords were generated with the help of existing narrative literature review articles, keywords from journal articles searched based on random keywords e.g. - supply chain coordination. After careful consideration and relevancy, 10 keywords were selected as below

1. Supply Chain Integration Or Supply Chain Coordination Or Supply Chain Cooperation Or Supply Chain Collaboration Or Supply Chain Partnership Or Supply Chain Alliance
2. Collaborative Planning, Forecasting, and Replenishment Techniques or CPFR
3. Vendor Managed Inventory or VMI
4. Multistage or Multi-echelon Inventory
5. Supply Chain Contracting or Supply Chain Contracts
6. Information Sharing And Supply Chain
7. Joint Decision Making or Collaborative Decision Making and Supply Chain
8. Joint Cost Reduction in Supply Chain or Collaborative Cost Reduction in Supply Chain

9. Joint Inventory Management or Collaborative Inventory Management and Supply Chain
10. Vertical Integration in Supply Chain

2.1.3 Conduct the review

Based on the research protocol identified in subsection 2.1.2, the literature review was conducted through searching for journal articles using the selected 10 keywords. Web of Science was the main search engine used for this purpose. In order not to miss any important existing body of literature, the keywords/key-phrases were also searched from search engines including Science Direct, Scopus, and Google scholar. At first, these searches generated over 5000 search results. This included conference proceedings and other unpublished works. This was filtered by including only journal articles from the list of journals selected earlier. This generated 1017 articles. This was further filtered by carefully reading the abstracts. This resulted in 388 screened articles. After reading the main body of the identified literature, several articles that are not directly relevant to the identified research question, are further screened out. This resulted in a final selection of 206 papers. Out of these 206 papers identified, 23 of them have been found to be describing a certain phenomenon of supply chain coordination or some antecedent of the coordination mechanisms. These papers did not propose any coordination mechanism as such. Hence, these are not included in the framework in fig 2.1. However, the findings from these 23 papers are used to support the arguments proposed for this review.

The distribution of the papers by their source journal and years are presented in figures 2.2 and 2.3.

2.1.4 Report the findings

The findings from the set of selected literature are reported according to the classification framework shown in fig. 2.1. The set of literature is classified according to the nature of coordination mechanisms used in the paper. Then for each case, it was assessed how these mechanisms have addressed the barriers.

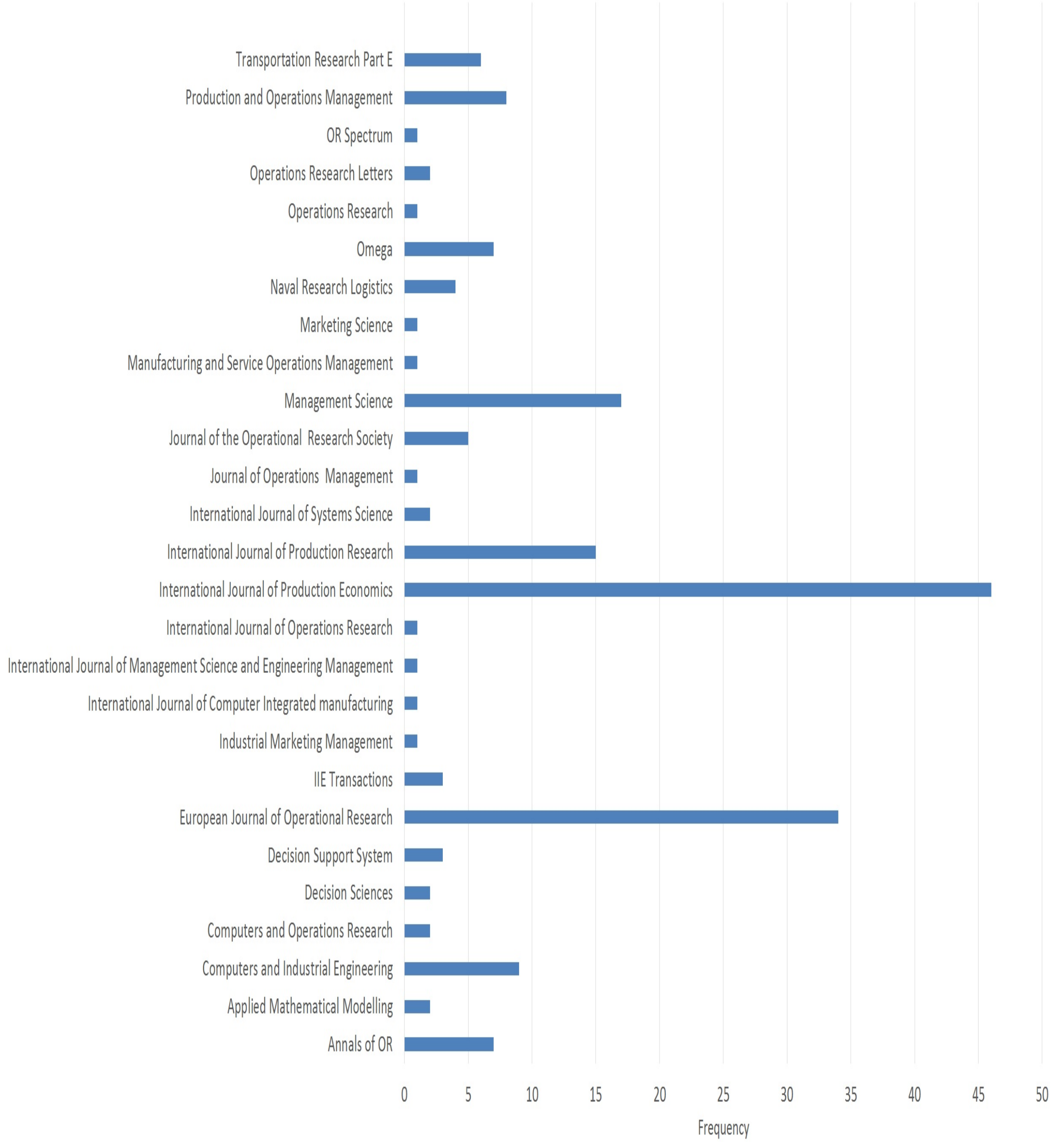


Figure 2.2: Distribution of Journals

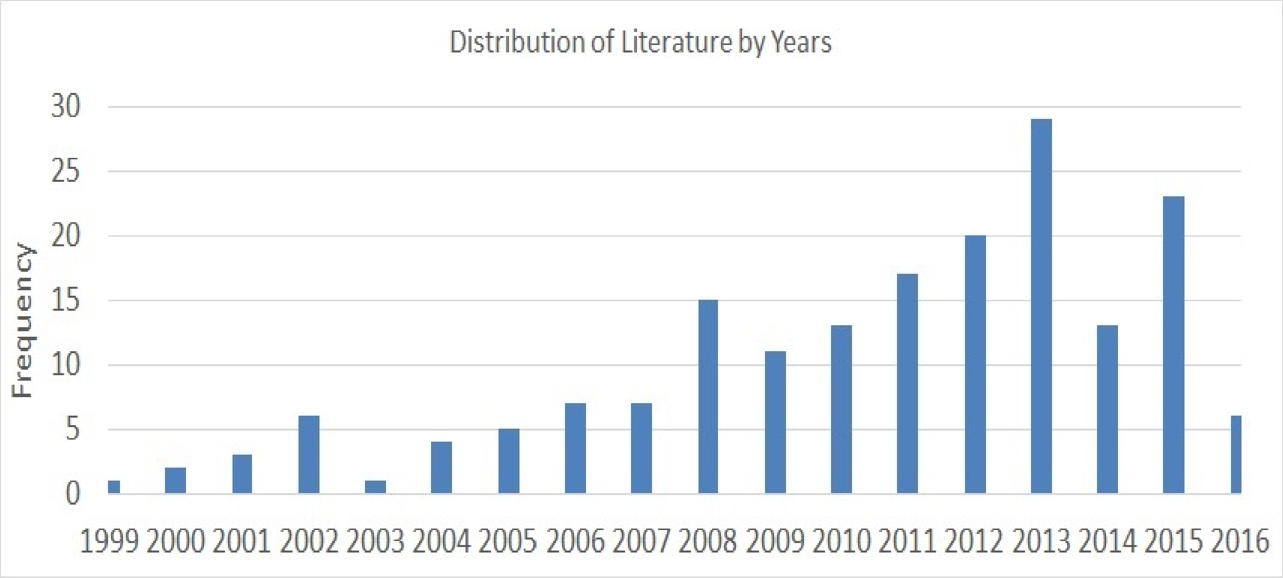


Figure 2.3: Distribution of Years

Formal Contractual Mechanism

Contracting between supply chain members has been used as a form of coordination mechanism in order to motivate the involved members to take decision for the best interest of the overall supply chain. Cachon (2003) presented a review with various contracts to coordinate supply chain with incentive conflicts. There are two major groups of contract mechanisms used in literature: Price only (Wholesale price contracts) and Price with risk sharing (Side payment, Buyback, and Rebate) contracts. Authors including Chen et al. (2012) argued the incapability of price only contracts to coordinate the supply chain and proposed a need for using risk sharing contracts.

Side payment contract is one of the risks sharing contractual mechanisms used in literature. This contractual type has taken the form of revenue sharing, two part-tariff, and other penalty based forms to coordinate the supply chain.

Revenue Sharing Contracts:

With a revenue sharing contract, the seller charges the buyer with a wholesale price and the buyer also pays the seller a percentage of his(her) revenue (Cachon 2003). Revenue sharing has been one of the popular contractual mechanisms used as a tool to coordinate a supply chain for different structure for example: profit in a dyadic supply chain (Hou et al. 2009, El Ouardighi & Kogan 2013, Saha 2013), three stage or a multi-stage supply chain (Giannoccaro & Pontrandolfo 2004, Van Der Rhee et al. 2010, Zeng 2013, Moon et al. 2015), in a competing supply chain with a common member (Geng & Mallik 2007, Zhang et al. 2012, Cao et al. 2013, Cao 2014, Jiang et al. 2014, Xu et al. 2014) and without a common member (Ai et al. 2012). Henry & Wernz (2015) used revenue sharing contract for a three level supply chain using multi-scale decision theory. This allowed the model to address the uncertainty at every stage of the supply chain rather than only at the final stage. Authors including Linh & Hong (2009) and Chen & Wei (2012) used revenue sharing contract to propose a coordination mechanism for a multi-period supply chain. Coordination with the issues like demand disruption (Zhang et al. 2012), and demand and cost disruption simultaneously (Cao et al. 2013) have been considered using the revenue sharing contracts as well. Revenue sharing has also been considered to coordinate the supply chains where operating issues other than profit have been considered as a decision variable such as optimal capacity utilization (Gupta & Weerawat 2006, Geng & Mallik 2007, Ha & Tong 2008*b*), service quality with retail price (Xiao et al. 2011), effort level (Ha & Tong 2008*b*), and corporate social responsibility issues (Panda 2014, Panda et al. 2015). Gupta & Weerawat (2006) and Xiao & Jin (2011) used this type of contracts in a "Make to Order" type of supply chains. Xiao & Jin (2011), Xiao et al. (2011) considered quality assurance as another operating decision variable for coordination.

This type of contract has also been used to address reverse supply chain issues such as optimal return Zeng (2013), Govindan & Popiuc (2014). Revenue sharing contract was found to be the coordinating contract for a supply chain with sustainability investments under cap and trade regulation (Dong et al. 2014). The revenue sharing contracts have also been used in conjunction with the bargaining games (Hou et al. 2009, Li et al. 2009). This type of contracts have also been used to in order to introduce the existence of informal relational coordination mechanisms such as information sharing (Kong et al. 2013, Zhang & Chen 2013, Henry & Wernz 2015) or in conjunction with other coordinating mechanisms such as demand information sharing (Zhou & Wang 2012), and vendor managed inventory (VMI) Chen & Wei (2012), Chen (2013). The revenue sharing contract has also been used along with other form of contracts such as payback mechanism (Tang & Kouvelis 2014) , rebate based contracts (Hu et al. 2013, Saha 2013), cost sharing contracts (Kunter 2012) and cooperative investment contract (Zhang et al. 2015). Khouja et al. (2010) used revenue sharing contracts to address the coordination issues for price and profit in a rental information goods supply chain. Xiao & Jin (2011) used revenue sharing contract to coordinate a fashion apparel supply chain under lead-time depended demand. Palsule-Desai (2013) proposed revenue depended on revenue sharing contracts to coordinate the Bollywood film distribution supply chain.

Profit Sharing Contracts:

Profit sharing is another side payment form of contractual mechanism which has been used in supply chain coordination literature. Wei & Choi (2010) used wholesale pricing with profit sharing contract to coordinate a supply chain under mean -variance decision-making framework. Yue et al. (2013) used the profit-sharing mechanism which can coordinate a manufacturer-retailer supply chain with optimal price discount and advertising investment decisions.

Two-part Tariff Contracts:

Two-part tariff is another form of side payment contract that has been used to coordinate the supply chain. It is used for coordination of supply chains with different structures such as: in a dual-channel supply chain (Chen et al. 2012), in a one retailer-two suppliers supply chain (Lee & Yang 2013). This has even been used in a more complicated supply chain for example in a complicated electricity distribution supply chain (Oliveira et al. 2013) and in a serial supply chain (Majumder & Srinivasan 2006). It has also been used to coordinate with multiple decision variables such as manufacturer induced quality and retailer induced effort (Gurnani & Erkoc 2008, Ma et al. 2013). Two-part tariff contracts have also been used to coordinate supply chain with environmental sustainability issues such as the greening effort in the paper of Swami & Shah (2013*a*) and collection decisions in a closed loop supply chain in the paper by Hong et al. (2015).

Other Side Payment Contracts:

There are some other forms of side payment contracts that have been used in literature to coordinate a supply chain for example side payment with lead-time quotation (Xu et al. 2010), modified revenue sharing to a profit sharing contract (Wang et al. 2012), and cost sharing contract to outsource a financially constrained call centre unit of a focal company (Xu, Cheng & Sun 2015). Penalty based contracts were also used as a form of side payment contracts in literature for example late fee mechanism based contract in the literature of Cachon & Zhang (2006), and penalty based service level contracts in the paper of Sieke et al. (2012).

All the models with side payment contracts mentioned in the papers in the last couple of paragraphs have either completely or near completely addressed the barriers of misaligned incentives of the members of the supply chain. However, the major limitation with these proposed side- payment contracts is the dependence of profit share/revenue share or any lump sum payment on bargaining power. A further implication of this on the share and future decision making of the players was mostly unexplored in these papers.

Price Discounts:

Discounts and rebates are another commonly used forms of contracting mechanisms in the supply chain coordination literature. Price reduction can be depended on several forms of price discounts such as price mark down (Cai et al. 2009, Wang & Webster 2009, Chung et al. 2011), advanced purchase discount (Liu et al. 2014), price subsidy rate contract (Xiao et al. 2005), discounting factor (Chaharsooghi et al. 2011), transportation cost (Cai et al. 2013), and returns (Chen 2011*a*). Zhou (2009) used price discounting to entice retailers in a group of retailers to order as per a unified annual order quantity. These type of contracts have been used for more complex structures of the supply chain such as supply chain with single supplier-multi retailers supply chain (Bernstein & Federgruen 2005*b*). The authors proposed the linear and the nonlinear price discount contracts to coordinate the supply chain.

Quantity Discount Contracts:

On the contrary, the quantity discounting contracts are designed in a form to encourage the buyer to purchase higher quantity. The wholesale price is a function of quantity sold. It can take all unit quantity discount where the wholesale price is a decreasing function of quantity purchased (Cachon 2003). This contractual form has been used for a coordination of different supply chain structures for example: a dyadic two-echelon supply chains (Corbett & De Groote 2000, Hsieh et al. 2008, Xiao & Qi 2012, Zhang, Dong, Luo & Segerstedt 2014, Saha & Goyal 2015, Giri & Bardhan 2016), single buyer-multiple suppliers (Lee & Yang 2013, Huang et al. 2015), one manufacturer-multiple retailers (Xiao et al. 2007, Chen & Xiao 2009, Zhou 2009, Su & Mukhopadhyay 2012, Xing & Liu 2012, Ogier et al. 2013), and multi-stage supply chain (Panda et al. 2015). The quantity discounting has addressed several issues for coordination such as: influencing ordering behaviour and thereby reducing the overall supply chain cost (Corbett & De Groote 2000, Zissis et al. 2015); pricing and environmentally- friendliness level of product (Giri & Bardhan 2016); avoidance of false failure-return (Huang et al. 2011); return policy of a retailer in a closed loop supply chain

(Yoo et al. 2015); ordering and pricing decision (Hsieh et al. 2008); allocation of purchase orders among multiple suppliers (Huang et al. 2015); corporate social responsibility (Panda et al. 2015) and enlarging lot size (Zhang, Dong, Luo & Segerstedt 2014). This type of discounting has been used under different operating and business environmental conditions for example asymmetric information (Corbett & De Groote 2000, Hsieh et al. 2008, Lee & Yang 2013, Zissis et al. 2015), deterministic demand (Giri & Bardhan 2016), inventory and retail price depended demand (Saha & Goyal 2015), price, delivery time and reliability of delivery depended demand (Xiao & Qi 2012), and post demand disruptions (Xiao et al. 2007, Chen & Xiao 2009). Quantity discounting contracts have been used in conjunction with other forms of contractual parameters such as trade credit (Zhang, Dong, Luo & Segerstedt 2014). Authors including Lu & Wu (2015) identified the different nature of contractual preferences among the members of the supply chain. The authors identified a form quantity discount contract that eliminated the conflict of interests among the members of the supply chain. Some authors including Lau et al. (2012) used volume discounting to coordinate the supply chain with price and sales effort sensitive demand.

Rebate Contracts:

Rebate is another form of discounting contract that has received attention in the supply chain literature. It's very popular when the retailer as a buyer further sells the items of the supplier to another buyer. The retailer pays the supplier a wholesale price. However, the supplier compensates the retailer by a rebate per unit once the quantity sold is above a certain threshold limit (Cachon 2003). Taylor (2002) found a manufacturer-retailer supply chain could be coordinated using a target sales rebate along with appropriately designed return contract. However, Chiu et al. (2015) found that target rebate should be combined with fixed order quantity to coordinate the supply chain. The rebate could take a form of a weather depended rebate as well (Chen & Yano 2010). The authors used this to design Pareto improving contracts to coordinate the supply chain. Target rebates have been used to address the issue of false failure return by enticing the retailer to increase the effort to reduce false failure return and thereby coordinating the supply chain Ferguson et al. (2006). This type of contract has also been used in a multi-channel supply chain such as Xing & Liu (2012) designed a target rebate contract in a single manufacturer with online and brick and mortar retailer. The author used this contract to coordinate the supply chain by avoiding the decreasing effort level of the brick and mortar retailer due to the existence of online channel. Few authors used target rebate in conjunction with other mechanisms to coordinate a supply chain such as: Chen & Wei (2012) and Wong et al. (2009) used it along with vendor managed inventory (VMI); Chiu et al. (2011) used it along with wholesale price and returns to coordinate a supply chain facing price depended demands; Giri et al. (2016) used it alongside buy-back and penalty to design a contract that can coordinate a three layer supply chain facing uncertain demand.

Coordination involving using buybacks, returns, and other reverse supply chain issues have also received considerable attention in the literature.

Buy-back Contracts:

Pasternack (1985) is one of the pioneer authors who investigated the problems of sub-optimal results in a supply chain with returns of unsold items. The authors used buy-back contracts to coordinate the supply chain. Buy-back contracts need the retailer to purchase the items at a fixed wholesale price. However, the supplier buys the unsold items at a less price than the wholesale price at the end of the selling season (Cachon 2003). Buy-back is one of the commonly devised contracts to coordinate the supply chains (Hou et al. 2010, He & Khouja 2011, Özen et al. 2012, Chung & Erhun 2013, Feng et al. 2015, Yan & Zaric 2016). This contracting mechanism has been used for different supply chain structures such as dyadic supply chain (Deng et al. 2013, Ruiz-Benitez & Muriel 2014), two supplier(one back up and one main)-one buyer supply chain (Hou et al. 2010), and for two competing supply chains (Wu 2013*a*). This contracting mechanism has been used under various different business environmental conditions such as supply disruption (Hou et al. 2010), and consumer return (Ruiz-Benitez & Muriel 2014, Xu, Li, Govindan & Xu 2015). The buy-back contracts have been used to coordinate the supply chain under several operating conditions as constraints such as budget constraint (Feng et al. 2015), products with limited shelf life (Chung & Erhun 2013), and information asymmetry (Deng et al. 2013). Wang & Webster (2007) proposed a buyback model with risk neutral manufacturer-loss averse retailer with an assumption of information symmetry of retailer's loss aversion. This was extended by Chung & Erhun (2013). Various issues have been considered for coordination with buy-back contracts for example retailer's effort (Krishnan et al. 2004, Yan & Zaric 2016), contract with a deadline for consumer return (Xu, Li, Govindan & Xu 2015), to coordinate a dyadic supply chain with consumer returns (Xiao et al. 2010) and others.

The buy-back contracts have been used in conjunction with other coordination mechanisms such as joint forecasting (Özen et al. 2012). The authors showed that a three parameter buy -back contract with joint forecasting can coordinate the supply chain with a manufacturer and n no of retailers. Shreds of evidence were also found where the buy-back mechanism has been combined with other contractual mechanisms such as revenue sharing (Feng et al. 2015), with penalty and sales rebate (Giri et al. 2016), with promotional; cost-sharing agreement (Krishnan et al. 2004), with quantity flexibility (Xiong et al. 2011), with quantity discount (Yang et al. 2015). The notable limitations of these papers is the limited emphasis on how to share the benefits of risk and benefit post coordination. This issue was addressed by (Devangan et al. 2013). The authors designed an individually rational buy-back contract for a supply chain with a retailer who faces inventory depended demand. Using Shapley value from the cooperative game theory, the authors ensured the fairness of the shared benefits.

Return Contracts:

Returns, recycling, and re-manufacturing are very common phenomena these days in any product supply chain. In fact, literature in supply chain coordination has included a diverse range of issues covered by return contract to coordinate the supply chain such as the return of unsold goods (Chen 2011*a*), consumer returns (Xiao et al. 2010) and product re-manufacturing (Savaskan et al. 2004). Again this type of contract has been used for different supply chain structures such as simple dyadic (Chen 2011*a*, He & Zhao 2012, Jeong 2012), and three-level supply chain (Ding & Chen 2008). This has been considered for coordination under various operational issues such as including partial information asymmetry (Jeong 2012), and limited stochastic salvage capacity (Lee & Rhee 2007). Various business environmental issues have been also used while coordinating the supply chain for example single period demand uncertainty (Xiao et al. 2010), two-period demand uncertainty (Chen & Xiao 2011*a*), supply and demand uncertainty (He & Zhao 2012), and declining wholesale price (Taylor 2001). return for recycling and re-manufacturing (Savaskan et al. 2004). Return contracts have been used in conjunction with other contracts including rebates (Chiu et al. 2011), and with buy-back contract contingent upon consumer return deadline (Xu, Li, Govindan & Xu 2015). Authors including Ding et al. (2011) have even combined return policies with information sharing to propose a coordination solution.

Trade Credit Contracts:

Trade credit is another form of supply contract that has been used in supply chain literature for coordination. It is a short-term loan to the buyer from the seller which allows the seller to delay or defer the payment to the seller (Lee & Rhee 2011). This type of coordination mechanism has been used to coordinate two-echelon dyadic supply chains (Jaber & Osman 2006, Arkan & Hejazi 2012, Chen & Wang 2012, Du et al. 2013, Giri & Maiti 2013) and three level supply chain (Moussawi-Haidar et al. 2014). It has been used in conjunction with other mechanisms to coordinate the supply chains such as profit sharing (Jaber & Osman 2006, Giri & Maiti 2013), discounted interest rate (Moussawi-Haidar et al. 2014), quantity discount (Zhang, Dong, Luo & Segerstedt 2014), and wholesale price discount (Du et al. 2013). Lee & Rhee (2010, 2011) found that some of the contractual mechanism are not sufficient to coordinate the supply chain sufficient unless they re combined with trade credit provision. The authors found if the retailers finance direct from financial institutes, then popular mechanisms such as quantity discounts, buy-back, two-part tariff, revenue sharing, and mark down are not sufficient to coordinate the supply chain. Chen & Wang (2012) showed that using trade credit with limited liability can coordinate a two-level supply chain with budget constraints. However, few of the cases trade credit was found to be incapable of coordinating supply chain (Luo & Zhang 2012). The authors found the trade credit option to coordinate the supply chain in the case of symmetric information. However, the trade credit failed to coordinate the supply chain under asymmetric information case.

One of the major limitation of these studies is the limited exploration of the risks associated with the trade credit. One of the major challenges is the establishment of credibility. Besides this, these papers did not explicitly discuss how the benefits of the coordination are shared among the members of the supply chain.

Quantity Flexibility Contracts:

Apart from flexibility regarding the time to pay in the form of trade credit, the members of the supply chain can use flexible quantities arrangements to coordinate the supply chain. Tsay (1999) is one of the pioneer authors who used this concept of quantity flexibility to coordinate the supply chains. The author defined this as buyer's commitment to buy no less than certain percentage below forecast value and seller's commitment to deliver not above forecast value. This has been used in conjunction with other contractual mechanisms such as price discountsChung et al. (2014), and buy-back contracts (Xiong et al. 2011). Quantity flexibility contract has also been used to coordinate a supply chain with multi-objectives along with the profit maximization (Shi & Chen 2008). The authors combined this quantity flexibility along with whole sale price contracts to find out a Pareto optimal solution for the supply chain.

Other Contractual Forms:

Other notable form includes the inclusion of penalty in contract terms such as Cachon & Zhang (2006) designed a menu of contracts consisting of simple contracts with a penalty in the form of late fee and specified lead time requirements. The author found that this contract can nearly coordinate a single buyer multi-supplier supply chain to minimize the total procurement cost. Gan et al. (2005) proposed a risk-sharing contract that can coordinate the supply chain with a risk neutral supplier and a down-side risk-averse retailer. Lee, Rhee & Cheng (2013) designed a quality compensation contract that compensates the retailer for defective items and this can achieve full coordination. Again, these papers did not highlight how the derived risks and benefits of coordination are shared among the members of the supply chain. Authors including Ryu & Yücesan (2010) used a fuzzy approach in conjunction with multiple contracting mechanisms: quantity discounts, profit sharing, and buy-back contracts to propose a coordination mechanism for a newsvendor supply chain. An option contract is another form of coordination contracts used for supply chain coordination (Zhao et al. 2010).

Informal Relational Mechanism

In the introduction chapter of this thesis, the continuous development of supply chain relationship was discussed. It starts with arm's length relationship and progresses over time towards a more mutual collaborative relationship. Thus, it is more likely to start with a formal contractual relationship. Over time, a trust-based relationship develops between the members of the supply chain. This helps the supply chain members to achieve coordination using less formal mechanisms for example information sharing (Cachon & Lariviere 2001, Fiala 2005, Kong et al. 2013). The last subsection has reviewed the literature development in the area of supply chain coordination using the formal mechanisms. This section explores the literature with more informal relational mechanisms of coordination including the deployment of information technology, information sharing, and collaborative developments such as collaborative planning, collaborative forecasting, collaborative inventory management, and others.

Information technology as a Facilitator:

As stated in the paper of Fiala (2005), deployment of information technology tools is one of the facilitators of information sharing. This was supported by Arshinder et al. (2007), Fawcett et al. (2007). These authors identified information technology as one of the drivers of information sharing in the supply chain. Information technology as a facilitator tool has been widely used to promote information sharing and thereby to coordinate the supply chain such as the deployment of enterprise resource planning (ERP) (Kelle & Akbulut 2005), Radio Frequency Identifier (RFID) (Szmerekovsky & Zhang 2008, Lei et al. 2015), and use of Electronic Data Interchange (EDI) (Hill & Scudder 2002). An important observation by Lei et al. (2015) is the justification of RFID technology. In their study, the authors used RFID technology in conjunction with revenue sharing contracts in order avoid inventory error and further improve the benefits of coordination. If the importance of inventory error is not high or the if the investment for RFID is very high, then the benefits derived is very marginal.

Contractual Mechanisms as a Facilitator:

As mentioned earlier, the presence of formal contracting has been identified as one of the antecedents of information sharing in supply chain (Cachon & Lariviere 2001, Kong et al. 2013, Pezeshki et al. 2013, Zhang & Chen 2013). Few authors have identified the existence of a formal contractual mechanism alongside information sharing as a necessary requirement or as a complementary mechanism to coordinate the supply chain for example with risk sharing contract (Chen et al. 2006, Wakolbinger & Cruz 2011), with subsidy contract (Gao 2015), with two-part tariff contract (He et al. 2008), and with buy-back contract (Kurata & Yue 2008). These authors also addressed the issues of the impact of information sharing on the double marginalization and the quality of shared information. However, these studies have not revealed how derived risks and benefits are shared and the impact of that on supply chain coordination. Other issues that are not explored, are behavioral and socio-cultural challenges for information sharing in the literature of Kong et al. (2013) and Zhang & Chen (2013)

Other Antecedents of Information Sharing:

There are other antecedents and drivers of information sharing addressed in the literature for example risk sharing rule (Xiao & Yang 2009), cost sharing rule with embedded trust (Han & Dong 2015), and nature of strategic interaction (Xiao & Yang 2009, Du et al. 2012). Again these authors did not explore how these benefits of information sharing are going to be shared among the members of the supply chain. Other limitations are: cases with the service investment as private information (Xiao et al. 2010); and lack of focus on the willingness to share information over time (Xiao et al. 2010, Du et al. 2012).

The challenge of willingness to share information has been addressed in the papers of Li & Zhang (2008), Özer et al. (2011), and Voigt & Inderfurth (2012). Another driver of the information sharing has been the trust based relationship among the members of the supply chain as identified in the literature of Özer et al. (2011), Voigt & Inderfurth (2012), Inderfurth et al. (2013), Hung et al. (2014), Han & Dong (2015). In fact, Özer et al. (2011) argued with the help of controlled laboratory experiment that existence of trust and trustworthiness can ensure cooperation and information sharing even in absence of reputation building complex contracts. Li & Zhang (2008) explored the case in a one manufacturer-multi retailers environment. The author concluded the existence of guaranteed confidentiality from manufacturer's side would promote information sharing from the retailer's end. Cai et al. (2010)

identified how reciprocation of good and reliable behaviour can positively influence the information sharing in the Chinese supply chain context with the help of an empirically tested framework. The major limitations of these studies (Cai et al. 2010, Özer et al. 2011) are: the lack of generalization; and the selected research method. In addition, the challenges of achievement of these antecedents of information sharing (trust, confidentiality) are not explored. Moreover, these studies also have not highlighted how the benefits of information sharing can be shared among the members of the supply chain.

Information Sharing as a Mechanism:

Various methods have been used to measure the benefits of information sharing to coordinate the supply chain such as linear programming (Albrecht & Stadtler 2015), Analytical Hierarchy Process (AHP) along with fuzzy logic (Arshinder et al. 2007), simulation with agent-based modelling (Datta & Christopher 2011), Markov decision process (Davis et al. 2011), and agent-based forecasting using Genetic Algorithm (GA) (Liang & Huang 2006)

Different types of information sharing method and different types of shared information have been used for supply chain coordination such as primal information which deals with input or withdrawal information of central resources (Albrecht & Stadtler 2015), negotiation based information in a make to order supply chain (Chan & Chan 2009), set up and holding cost information from supplier (Karabatı & Sayın 2008), demand forecast information (Liang & Huang 2006), upstream product rollover information (Li & Gao 2008), production capacity and resource constraints (Thomas et al. 2015), and inventory level information (Kulp et al. 2004).

The information sharing has been used for coordination of supply chain addressing various issues such as reducing inventory variance (Costantino et al. 2014), the impact of information sharing on different ordering policies (Costantino et al. 2015), to solve the huge mismatch between actual and forecast demand (Datta & Christopher 2011), and the reduction of cost and lost sales (Davis et al. 2011). In relation to these, Lee et al. (1997) have argued information sharing as one of the potential solutions for reducing the impact of order distortion and order amplification across the supply chain known as the ”Bullwhip Effect”. The benefits have been documented in greater detail in the papers by the authors including Wu & Katok (2006), Croson & Donohue (2006), Ouyang (2007), and Bailey & Francis (2008). Croson & Donohue (2006) found from the experimental evidence that the complete eradication of bullwhip effect is not possible. However, the presence of information sharing could reduce the extent of bullwhip effect and there by improving the overall supply chain performance. However, these studies have not highlighted: the challenges of information sharing and truthful information; and quality of shared information. In an extension, Li (2013) identified endogenous mechanisms of bullwhip effects and established the importance of information sharing on the reduction of this. Authors also highlighted uncertainty of having full information sharing between supply chain members. However, this study did not reveal the optimum balance of information to share and not to share. Moreover, one of the major limitations of the work by Croson & Donohue (2006) and Wu & Katok (2006) is the selection of the research method as laboratory experiment only.

There is a school of thought questioned the need for any information sharing (Cachon & Lariviere 1999, Ha & Tong 2008*a*). In an earlier study, Cachon & Lariviere (1999), found the inability of the supply chain to reach dominant equilibrium under the Pareto optimal or individually responsive allocation mechanisms with truthfully shared information. Although the results support order inflating behavior, the reasons behind such order inflating behavior are mostly unexplored in these studies. Ha & Tong (2008*a*) argued whether information sharing in the supply chain will be beneficial or not, depends on the investment required for information sharing and the contract type. Despite all these, it has been found as a tool to coordinate the supply chain in practice. In fact, experimental results show this method to be the prerequisite for application of other informal coordination mechanisms (Spiliotopoulou et al. 2015).

Information sharing has been one of the cornerstone for further development of relational coordination mechanisms between the members of the supply chain in the form of several collaborative developments and collaborative decision making such as vendor managed inventory (VMI), collaborative planning, forecasting, and replenishment techniques (CPFR), and collaborative planning.

Vendor Managed Inventory:

Vendor managed inventory is a supply chain initiative where the supplier is in charge of maintaining a certain level of stock at buyer (retailer) location (Çetinkaya & Lee 2000). This allows the supplier to have a view of the actual demand or reduce the inconsistency of forecasting techniques across the supply chain. There are multiple benefits of implementation of

VMI. These include: reduction of overall supply chain cost (Lee et al. 2016), clear visibility of actual demand in downstream of the supply chain (Wong et al. 2009) and thus avoidance of having distorted demand information, reduction of stock-outs (Gao 2015), reduction of high inventory carrying cost (Çetinkaya & Lee 2000), and increase in batch size and reduction in overall cycle time (Bazan et al. 2014, Chakraborty et al. 2015). In fact, these benefits have been proposed as a solution to the bullwhip effect Lee et al. (1997). Çetinkaya & Lee (2000) proposed an analytical model for coordinating inventory and transportation decisions in a VMI environment. Dong & Xu (2002) further argued that VMI could be more beneficial to the individual members (supplier) in the longer run rather than in shorter run. The authors including Lee & Ren (2011) further added that implementation of VMI helps the supply chains to avoid the negative impacts of external uncertainties. The authors showed that a supplier-retailer supply chain can reduce a greater amount of cost with VMI in the presence of exchange rate uncertainty than in the absence of the exchange rate uncertainty.

VMI has often been used to coordinate the supply chain in conjunction with the formal contractual mechanisms such as: sales rebate contracts (Wong et al. 2009), revenue sharing and side payment (Chen et al. 2010, Chen 2013, Xiao & Xu 2013), revenue sharing with linear rebate (Chen & Wei 2012), holding cost subsidy contract (Nagarajan & Rajagopalan 2008), and all unit quantity discount and two-part tariff (Toptal & Çetinkaya 2006). Other mechanisms used alongside VMI include : lost sales cost sharing (Lee et al. 2016), and consignment stock arrangement (Bazan et al. 2014). Various operating conditions have been used while developing the coordinating solution with VMI such as limited storage capacity (Lee et al. 2016), deteriorating conditions for goods (Chen & Wei 2012, Xiao & Xu 2013), and for the product with shorter product life (Toptal & Çetinkaya 2006). Various business environmental conditions have also been considered in literature while deploying VMI as a coordination tool for the supply chains such as deterministic demand (Mateen & Chatterjee 2015) and exchange rate uncertainty (Lee & Ren 2011).

The majority of the papers have considered various mathematical models to propose the coordination model including dynamic programming with retailer led Stackelberg games (Chen 2013), and Markov model along with dynamic programming (Gao 2015). Application of the concepts of game theory has been another important cornerstone methodology of these sets of literature. Concepts of Nash equilibrium, Stackelberg games and Pareto optimality are few of the frequently concepts as found in the literature of Wong et al. (2009), Chen et al. (2010), Chen & Wei (2012), Chen (2013), and Xiao & Xu (2013).

Furthermore, authors including Yao et al. (2007) and Duan & Liao (2013) have extended how the benefits of vendor managed inventory (VMI) are shared among the members of the supply chain. The authors have proposed some side payment distribution to less benefited members for uneven distribution cases. However, any impact of bargaining power on this and issues related to an effective distribution of the benefits have not been highlighted.

Collaborative Forecasting Techniques:

Collaborative approaches in planning and forecasting techniques have been another popular area of research in the supply chain. Aviv (2001) showed that the importance of the collaborative forecast tends to increase with the increase in diversity of the forecast techniques in the supply chain. The author further argued that the benefits of collaborative forecasting are more when it is combined with some other collaborative approaches such as efficient consumer response (ECR). However, the author did not discuss the incentive parameters and financial implications of the forecasts. Gao (2015) used collaborative planning forecasting and implementation (CPFR) techniques to constantly monitor advance supply signals such as operational viability and financial health and then manage the disruption risk. The author then designed a contract in conjunction with CPFR to coordinate the supply chain. Özen et al. (2012) studied the impacts of collaborative forecasting and forecast information sharing in a single manufacturer-multi retailer supply chain. The authors found that if the retailers posses some asymmetric information then, sharing forecast information may not be beneficial. However, in the presence of joint forecasting combined with a three parameter contract can coordinate the overall supply chain. Information and communication technologies (ICT) have also been considered for adopting to facilitate CPFR in the literature of (Danese 2006). The authors used case study methods to study these phenomena. There are other methods used to study the benefits of application of CPFR for such as Yang & Fan (2016) used simulation to find out the benefits of using CPFR alongside information sharing to reduce the bullwhip effect in a supply chain. Ramanathan & Gunasekaran (2014) used empirical methods by surveying respondents from textile sector to find out the impact of CPFR to achieve long-term collaborative relations between the supply chain members. Authors including Shin & Tunca (2010) argued the possibility of over investment for forecasting techniques if the downstream retailers in the supply chain are at an arms race approach for forecasting in Cournot competition. The authors found the inability of normal market-based supply contracts to coordinate the supply chain. The authors further proposed a uniform-price divisible good auction-based contract to coordinate the supply chain.

Collaborative Planning Approaches:

Collaborative planning has also been used as a coordination mechanism. Zimmer (2002) proposed a single period ordering and delivery plan to coordinate a supply chain in just in time setting. Kaya et al. (2013) proposed a coordination policy between transportation and production in a single supplier-single retailer deterministic inventory system. Bajgiran et al. (2016) used mixed integer programming along with heuristic techniques to propose an integrated annual planning that coordinates the harvesting, procurement, production, distribution, and sale activities of the lumber supply chain. Dudek & Stadtler (2005, 2007) proposed a negotiation based non-hierarchical plan to coordinate a supply chain using mathematical techniques. Jung et al. (2008) proposed a framework for planning in a decentralized setting. The solutions in this setting are close to the first best solutions from centralized setting and thus, these can be considered as near optimal solutions. Kim & Ha (2003) proposed a coordinating lot-splitting strategy to allow frequent small lot size deliveries of supply materials in a buyer-seller supply chain in just in time (JIT) environment for the finite horizon. Puettmann & Stadtler (2010) proposed a collaborative plan for intermodal transportation providers. Steinrücke (2011) used a mixed integer decision-making model to achieve the coordination across the production quantities and time for all the members of the supply chain in an aluminum supply chain network where the members are dispersed across the world. The authors proposed heuristics techniques to propose the coordination model which represent the continuous planning period and minimizes the total transportation and production cost. Taghipour & Frayret (2012) used mathematical programming techniques to propose a negotiation based planning mechanism that can achieve near optimal solution for coordination. Zhao & Wang (2002) used a simple forward algorithm to coordinate joint pricing-production/ordering decision in a manufacturer-distributor supply chain for as finite horizon. Zhao et al. (2016) proposed an optimal inventory coordination solution for inventory replenishment and achieving the global optimal total cost across the multi-stage supply chain for an infinite horizon. Egri & Váncza (2012) proposed a lot-sizing decision-making approach along with appropriate payment scheme to coordinate a supply chain under asymmetric information.

Other Collaborative Approaches:

There are other collaborative approaches used in literature where authors addressed solution in the form of optimal inventory level or by having an optimal order cycle time. Cheung & Lee (2002) proposed a model with shipment coordination and stock rebalancing together in a single supplier multiple retailer supply chain. Glock (2011) proposed a mathematical model with integrated inventory to coordinate single buyer-multi suppliers supply chain. The model was shown to minimize the total procurement cost. Chen & Chen (2005) proposed four decision-making models for optimal inventory replenishment and production policies in two echelons of a supply chain. Jaber & Goyal (2008) proposed a model for coordinating order quantities in a centralized three level supply chain with common order cycle time. Saharidis et al. (2009) proposed a Markovian queueing model for coordinating a two-stage supply chain with joint decision making on stocks. Jorinaldi & Zhang (2013) proposed an integrated production and inventory control model for the whole manufacturing supply chain system including reverse logistics, using mixed integer non-linear programming techniques. However, the major limitation of these studies is not having any insight about issues and relevant impacts of the risk and benefit sharing from these mechanisms.

2.1.5 Summary of the findings

In sections 2.1.4 the findings from the literature are presented. The main purpose of this review was to assess how the selected papers have addressed the challenges in 2.1. This is reported in figures 2.4 and 2.5. Several areas can be highlighted from these set of literature.

- **Structure of the supply chain:** Various forms including two level dyadic, two level divergent, two level convergent, three-level linear, multi-level multiple members and two competing supply chains, have been used in supply chain coordination literature.
- **Variables for Coordination :** Several supply chain related issues have been considered for coordination. Profit and cost are the two most common of these variables. Other variables found in the literature include: lead time, sales effort of retailers, greening effort in an environmentally conscious supply chain, quality assurance, inventory level, and corporate social responsibility effort.

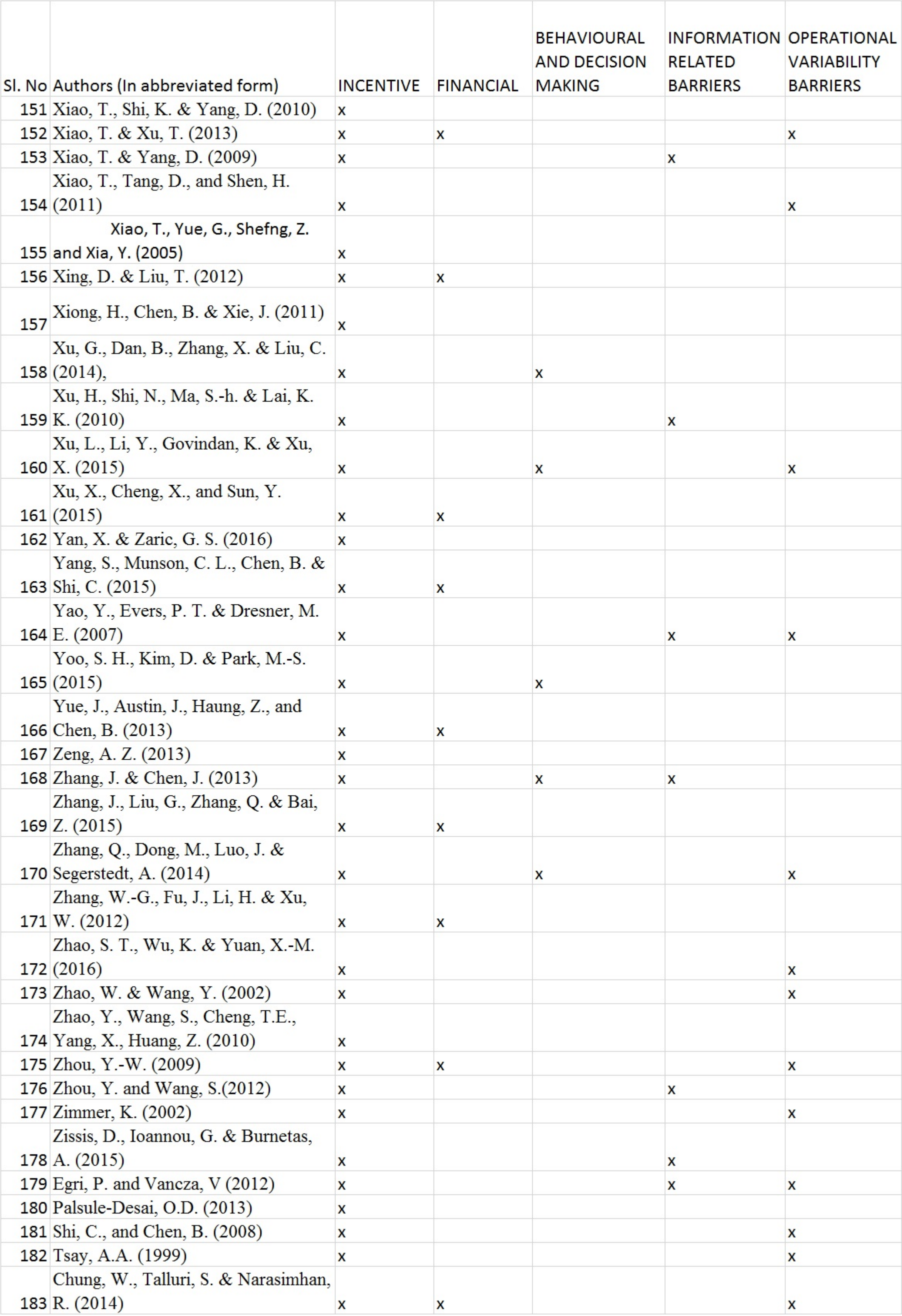
Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
1	Ai, X., Chen, J. & Ma, J. (2012),	x			x	
2	Albrecht, M. & Stadtler, H. (2015)	x			x	
3	Arkan, A., and Hejazi, S.R. (2012)	x	x			x
4	Aviv, Y. (2001)	x			x	
5	Bailey, K. & Francis, M. (2008)					x
6	Bajgiran, O. S., Zanjani, M. K. & Nourelfath, M. (2016)		x		x	
7	Bazan, E., Jaber, M. Y., Zanoni, S. & Zavanella, L. E. (2014)				x	x
8	Bernstein, F. & Federgruen, F. (2005)	x	x			x
9	Cachon, G. P. & Lariviere, M. A. (2001)	x		x	x	x
10	Cachon, G. P. & Zhang, F. (2006)	x	x			x
11	Cai, G. G., Zhang, Z. G. & Zhang, M. (2009)	x	x			x
12	Cai, X., Chen, J., Xiao, Y., Xu, X. & Yu, G. (2013)	x	x			x
13	Cao, E. (2014)	x	x		x	x
14	Cao, E., Wan, C. & Lai, M. (2013)	x	x			
15	C¸ etinkaya, S. & Lee, C.-Y. (2000)	x			x	x
16	Chaharsooghi, S. K., Heydari, J. & Kamalabadi, I. N. (2011)	x	x			x
17	Chakraborty, A., Chatterjee, A. & Mateen, A. (2015)	x				x
18	Chan, H. K. & Chan, F. T. (2009)	x			x	x
19	Chen, F. Y. & Yano, C. A. (2010)	x	x			x
20	Chen, H., Chen, J. & Chen, Y. F. (2006)	x			x	x
21	Chen, J. (2011)	x	x			
22	Chen, J.-M., Lin, I.-C. & Cheng, H.-L. (2010)	x	x			x
23	Chen, J., Zhang, H. & Sun, Y. (2012)	x				
24	Chen, K. & Xiao, T. (2009)	x				
25	Chen, K. & Xiao, T. (2011)	x				
26	Chen, L.-T. (2013)	x				
27	Chen, L.-T. & Wei, C.-C. (2012)	x				
28	Chen, T.-H. & Chen, J.-M. (2005)	x	x	x		x
29	Chen, X., and Wang, A. (2012)	x	x			
30	Cheung, K. L. & Lee, H. L. (2002)					x

Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
31	Chiu, C.-H., Choi, T.-M., Hao, G. & Li, X. (2015)	x		x		
32	Chiu, C.-H., Choi, T.-M. & Tang, C. S. (2011)	x	x			
33	Chung, W., Talluri, S. & Narasimhan, R. (2011)	x	x			
34	Chung, Y. T. & Erhun, F. (2013)	x				
35	Corbett, C. J. & De Groote, X. (2000)	x				x
36	Costantino, F., Di Gravio, G., Shaban, A. & Tronci, M. (2015)	x			x	x
37	Davis, L. B., King, R. E., Hodgson, T. J. & Wei, W. (2011)	x				x
38	Deng, X., Xie, J. & Xiong, H. (2013)	x		x	x	
39	Devangan, L, Amit, K, Mehta, P, Swami, S, and Kripa, S (2013)	x		x		
40	Ding, D. & Chen, J. (2008)	x				
41	Ding, H., Guo, B., & Liu, Z. (2011)	x			x	x
42	Dong, C., Shen, B., Chow, P., Yang, L., & Ng, C. (2014)	x	x			
43	Dong, Y. & Xu, K. (2002)	x				x
44	Du, R., Banerjee, A., and Kim, S.L. (2013)	x	x			x
45	Duan, Q. & Liao, T. W. (2013)	x			x	x
46	Dudek, G. & Stadtler, H. (2005)	x			x	
47	Dudek, G. & Stadtler, H. (2007)	x			x	
48	El Ouardighi, F. & Kogan, K. (2013)	x				
49	Feng, X., Moon, I. & Ryu, K. (2015)	x	x		x	
50	Ferguson, M., Guide Jr, V. D. R. & Souza, G. C. (2006)	x				
51	Gan, X., Sethi, S., & Yan, H. (2005)	x		x		
52	Gao, L. (2015)	x			x	x
53	Geng, Q. & Mallik, S. (2007)	x				x
54	Giannoccaro, I. & Pontrandolfo, P. (2004)	x				
55	Giri, B. & Bardhan, S. (2016)	x				
56	Giri, B., Bardhan, S. & Maiti, T. (2016)	x				
57	Giri, B., and Maiti, T. (2013)	x	x			
58	Glock, C.H. (2011)					x
59	Govindan, K., and Popiuc, M.N. (2014)	x				
60	Gupta, D. & Weerawat, W. (2006)	x				x

Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
61	Gurnani, H. & Erkoc, M. (2008)	x			x	
62	Ha, A. Y. & Tong, S. (2008*b*)	x				
63	Han, G. & Dong, M. (2015)	x		x	x	
64	He, C., Marklund, J. & Vossen, T. (2008)	x			x	
65	He, X. & Khouja, M. (2011)	x				
66	He, Y. & Zhao, X. (2012)	x	x	x		x
67	Henry, A. and Wernz, C. (2015)	x				
68	Hill, C. A. & Scudder, G. D. (2002)				x	
69	Hong, X., Xu, L., Du, P. & Wang, W. (2015)	x	x			
70	Hou, J., Zeng, A. Z. & Zhao, L. (2009)	x			x	
71	Hou, J., Zeng, A. Z. & Zhao, L. (2010)	x				
72	Hsieh, C.-C., Wu, C.-H. & Huang, Y.-J. (2008)	x	x			x
73	Hu, F., Lim, C.-C. & Lu, Z. (2013)	x				x
74	Huang, X., Choi, S.-M., Ching, W.-K., Siu, T.-K. & Huang, M. (2011)	x				
75	Huang, Y.-S., Ho, R.-S. & Fang, C.-C. (2015)	x				
76	Jaber, M. & Goyal, S. (2008)	x				x
77	Jaber, M. Y.& Osman, J.H. (2006)	x	x			x
78	Jeong, I.-J. (2012)	x			x	
79	Jiang, L., Wang, Y. & Yan, X. (2014)	x				
80	Jorinaldi & Zhang, D. (2013)					x
81	Jung, H., Chen, F. F. & Jeong, B. (2008)				x	
82	Karabatı, S. & Sayın, S. (2008)	x				x
83	Kaya, O., Kubali, D., and Oemeci, L. (2013)	x				x
84	Khouja, M., Rajagopalan, H. K. & Sharer, E. (2010)	x				
85	Kim, S.-L. & Ha, D. (2003)	x				x
86	Kong, G., Rajagopalan, S. & Zhang, H. (2013)	x		x	x	
87	Krishnan, H., Kapuscinski, R. & Butz, D. A. (2004)	x				
88	Kulp, S. C., Lee, H. L. & Ofek, E. (2004)				x	
89	Kunter, M. (2012)	x				
90	Kurata, H. & Yue, X. (2008)	x			x	

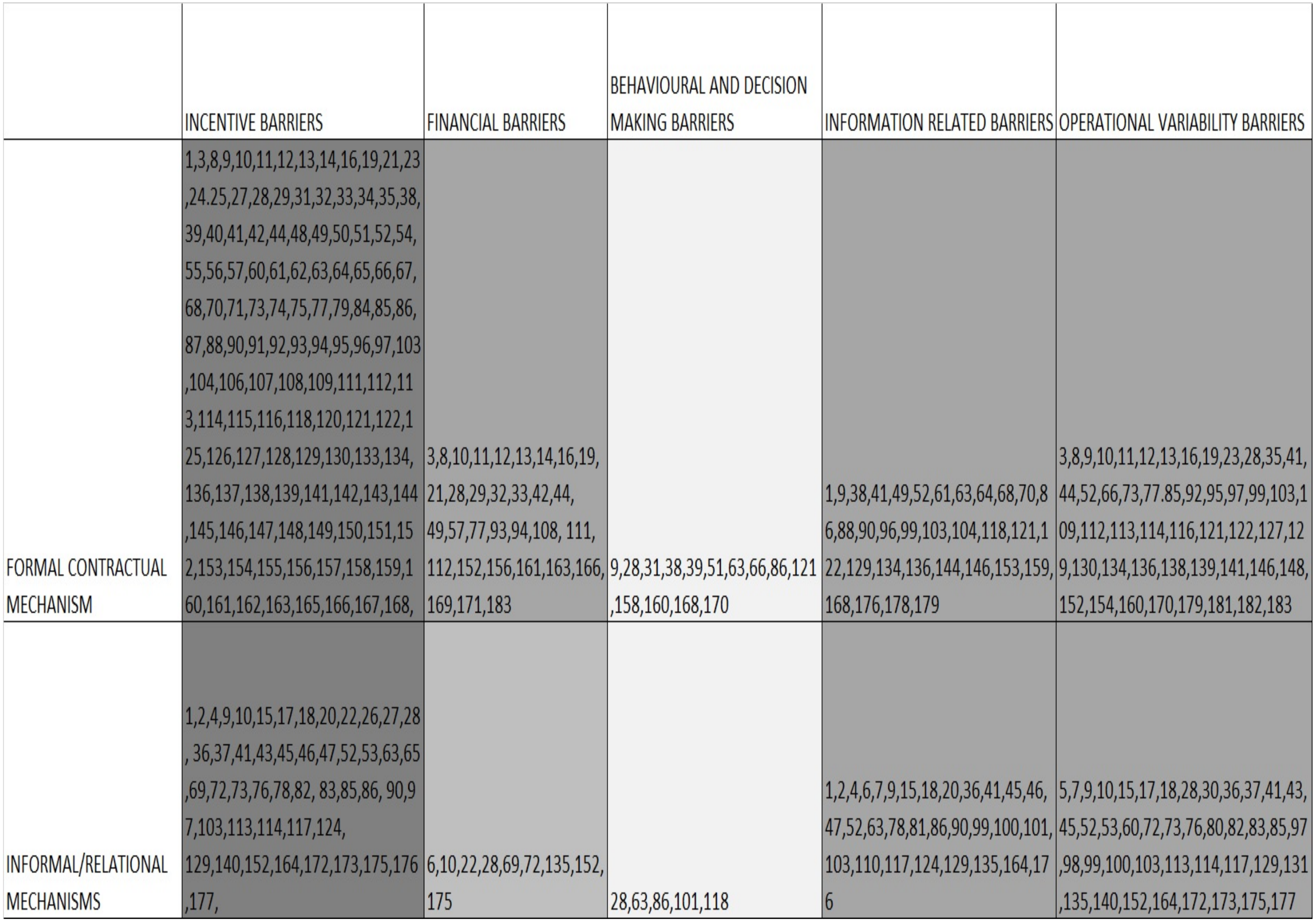
Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
91	Lau, H.S.,Su, C., Wang, Y., and Hua, Z. (2012)	x				
92	Lee, C. H. & Rhee, B.-D. (2007)	x				x
93	Lee, C. H. & Rhee, B.-D. (2010)	x	x			
94	Lee, C. H. & Rhee, B.-D. (2011)	x	x			
95	Lee, C. H., Rhee, B.-D. , & Cheng, T.(2013)	x				x
96	Lee, C.-Y. & Yang, R. (2013)	x			x	
97	Lee, J.-Y., Cho, R. K. & Paik, S.-K. (2016)	x				x
98	Lee, J.-Y. & Ren, L. (2011)					x
99	Lei, Q., Chen, J., Wei, X. & Lu, S. (2015)	x			x	x
100	Li, C. (2013)				x	x
101	Li, L. & Zhang, H. (2008)			x	x	
102	Li, S., Zhu, Z. & Huang, L. (2009)	x				
103	Li, Z. & Gao, L. (2008)	x			x	x
104	Liang, W.-Y. & Huang, C.-C. (2006)				x	
105	Linh, C. T. & Hong, Y. (2009)	x				
106	Liu, Z., Chen, L., Li, L. & Zhai, X. (2014)	x				
107	Lu, L. & Wu, Y. (2015)	x				
108	Luo, J. and Zhang, Q. (2012)	x	x			
109	Ma, P., Wang, H. & Shang, J. (2013)	x				x
110	Mateen, A., and Chaterjee, A.K. (2015)				x	
111	Moon, I., Feng, X.-H. & Ryu, K.-Y. (2015)	x	x			
112	Moussawi-Haidar, L., Dbouk, W., Jaber, M.Y., and Osman, I.H. (2014)	x	x			x
113	Nagarajan, M. & Rajagopalan, S. (2008)	x				x
114	Ogier, M., Cung, V.-D., Boissiere, J. & Chung, S. H. (2013)	x				x
115	Oliveira, F. S., Ruiz, C. & Conejo, A. J. (2013)	x				
116	Ouyang, Y. (2007)					x
117	Ozen, U., Sosic, G. & Slikker, M. (2012)	x			x	x
118	Ozer, O., Zheng, Y. & Chen, K.-Y. (2011)				x	
119	Panda, S. (2014)	x				
120	Panda, S., Modak, N., Basu, M. & Goyal, S. (2015)	x				

Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
121	Pezeshki, Y., Baboli, A., Cheikhrouhou, N., Modarres, M. & Jokar, M. R. A. (2013)	x		x	x	x
122	Puettmann, C. & Stadtler, H. (2010)				x	x
123	Ruiz-Benitez and Muriel (2014)	x				
124	Ryu, K. and Yucesan, E. (2010)	x			x	
125	Saha, S. (2013)	x				
126	Saha, S. & Goyal, S. (2015)	x				
127	Saharidis, G. K., Kouikoglou, V. S. & Dallery, Y. (2009)	x				x
128	Savaskan, R. C., Bhattacharya, S. & Van Wassenhove, L. N. (2004)	x				
129	Shin, H. and, Tunca, T.I. (2010)	x			x	x
130	Sieke, M. A., Seifert, R. W. & Thonemann, U. W. (2012)	x				x
131	Steinrucke, M. (2011)					x
132	Su, X. & Mukhopadhyay, S. K. (2012)	x				
133	Swami, S. & Shah, J. (2013)	x				
134	Szmerekovsky, J. G. & Zhang, J. (2008)				x	x
135	Taghipour, A. & Frayret, J.-M. (2012)		x		x	x
136	Tang, S.Y., and Kouvelis, P. (2014)	x			x	x
137	Taylor, T. A. (2001)	x				
138	Taylor, T. A. (2002)	x				x
139	Thomas, A., Krishnamoorthy, M., Singh, G. & Venkateswaran, J. (2015)					x
140	Toptal, A. & C¸ etinkaya, S. (2006)	x				x
141	Van Der Rhee, B., Van Der Veen, J. A., Venugopal, V. & Nalla, V. R. (2010)	x				
142	Wang, C. X. & Webster, S. (2007)	x				
143	Wang, C. X. & Webster, S. (2009)	x				
144	Wang, Y., Lau, H. & Hua, Z. (2012)	x			x	
145	Wei, Y. and Choi, T.M. (2010)	x				
146	Wong, W.-K., Qi, J. & Leung, S. (2009)	x			x	x
147	Wu, D. (2013)	x				
148	Xiao, T. & Jin, J. (2011)	x				x
149	Xiao, T. & Qi, X. (2012)	x				
150	Xiao, T., Qi, X. & Yu, G. (2007)	x				



Sl. No	Authors (In abbreviated form)	INCENTIVE	FINANCIAL	BEHAVIOURAL AND DECISION MAKING	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
151	Xiao, T., Shi, K. & Yang, D. (2010)	x				
152	Xiao, T. & Xu, T. (2013)	x	x			x
153	Xiao, T. & Yang, D. (2009)	x			x	
154	Xiao, T., Tang, D., and Shen, H. (2011)	x				x
155	Xiao, T., Yue, G., Shefng, Z. and Xia, Y. (2005)	x				
156	Xing, D. & Liu, T. (2012)	x	x			
157	Xiong, H., Chen, B. & Xie, J. (2011)	x				
158	Xu, G., Dan, B., Zhang, X. & Liu, C. (2014),	x		x		
159	Xu, H., Shi, N., Ma, S.-h. & Lai, K. K. (2010)	x			x	
160	Xu, L., Li, Y., Govindan, K. & Xu, X. (2015)	x		x		x
161	Xu, X., Cheng, X., and Sun, Y. (2015)	x	x			
162	Yan, X. & Zaric, G. S. (2016)	x				
163	Yang, S., Munson, C. L., Chen, B. & Shi, C. (2015)	x	x			
164	Yao, Y., Evers, P. T. & Dresner, M. E. (2007)	x			x	x
165	Yoo, S. H., Kim, D. & Park, M.-S. (2015)	x		x		
166	Yue, J., Austin, J., Haung, Z., and Chen, B. (2013)	x	x			
167	Zeng, A. Z. (2013)	x				
168	Zhang, J. & Chen, J. (2013)	x		x	x	
169	Zhang, J., Liu, G., Zhang, Q. & Bai, Z. (2015)	x	x			
170	Zhang, Q., Dong, M., Luo, J. & Segerstedt, A. (2014)	x		x		x
171	Zhang, W.-G., Fu, J., Li, H. & Xu, W. (2012)	x	x			
172	Zhao, S. T., Wu, K. & Yuan, X.-M. (2016)	x				x
173	Zhao, W. & Wang, Y. (2002)	x				x
174	Zhao, Y., Wang, S., Cheng, T.E., Yang, X., Huang, Z. (2010)	x				
175	Zhou, Y.-W. (2009)	x	x			x
176	Zhou, Y. and Wang, S.(2012)	x			x	
177	Zimmer, K. (2002)	x				x
178	Zissis, D., Ioannou, G. & Burnetas, A. (2015)	x			x	
179	Egri, P. and Vancza, V (2012)	x			x	x
180	Palsule-Desai, O.D. (2013)	x				
181	Shi, C., and Chen, B. (2008)	x				x
182	Tsay, A.A. (1999)	x				x
183	Chung, W., Talluri, S. & Narasimhan, R. (2014)	x	x			x

Figure 2.4: How barriers of coordination have been addressed in literature



	INCENTIVE BARRIERS	FINANCIAL BARRIERS	BEHAVIOURAL AND DECISION MAKING BARRIERS	INFORMATION RELATED BARRIERS	OPERATIONAL VARIABILITY BARRIERS
FORMAL CONTRACTUAL MECHANISM	1,3,8,9,10,11,12,13,14,16,19,21,23,24.25,27,28,29,31,32,33,34,35,38,39,40,41,42,44,48,49,50,51,52,54,55,56,57,60,61,62,63,64,65,66,67,68,70,71,73,74,75,77,79,84,85,86,87,88,90,91,92,93,94,95,96,97,103,104,106,107,108,109,111,112,113,114,115,116,118,120,121,122,125,126,127,128,129,130,133,134,136,137,138,139,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,165,166,167,168,	3,8,10,11,12,13,14,16,19,21,28,29,32,33,42,44,49,57,77,93,94,108, 111,112,152,156,161,163,166,169,171,183	9,28,31,38,39,51,63,66,86,121,158,160,168,170	1,9,38,41,49,52,61,63,64,68,70,86,88,90,96,99,103,104,118,121,122,129,134,136,144,146,153,159,168,176,178,179	3,8,9,10,11,12,13,16,19,23,28,35,41,44,52,66,73,77.85,92,95,97,99,103,109,112,113,114,116,121,122,127,129,130,134,136,138,139,141,146,148,152,154,160,170,179,181,182,183
INFORMAL/RELATIONAL MECHANISMS	1,2,4,9,10,15,17,18,20,22,26,27,28, 36,37,41,43,45,46,47,52,53,63,65,69,72,73,76,78,82, 83,85,86, 90,97,103,113,114,117,124,129,140,152,164,172,173,175,176,177,	6,10,22,28,69,72,135,152,175	28,63,86,101,118	1,2,4,6,7,9,15,18,20,36,41,45,46,47,52,63,78,81,86,90,99,100,101,103,110,117,124,129,135,164,176	5,7,9,10,15,17,18,28,30,36,37,41,43,45,52,53,60,72,73,76,80,82,83,85,97,98,99,100,103,113,114,117,129,131,135,140,152,164,172,173,175,177

Figure 2.5: Literature Review Frame Work B

- **Method used:** Diverse range of methods were found in the selected literature for devising the appropriate coordination mechanisms for the supply chains. The most commonly used are mathematical modelling. In few cases, empirical studies with conceptual framework were used. However, these frameworks were mostly used to identify any causal and mediating relation between variables that are associated with supply chain coordination. Several concepts from game theory have been found to be used to solve the coordination problems in these papers. Stackelberg games, Nash equilibrium, Nash's bargaining and Pareto optimality are among the most frequently used concepts. It has been noticed that these game theory concepts were used for more simplified supply chain structures such as dyad, or triads (convergent, divergent, and linear). With the increase in a number of supply chain echelons and number of members, the complexity increases. More complicated optimization techniques such as mixed integer programmming, other heuristics and meta-heuristics techniques have been used to find near-optimal solutions in these complex supply chains.

- **Sources of Uncertainty:** The sources of uncertainty have been mostly observed to be price, demand, and supply.

From the framework in figure 2.5, it is evident that majority of the coordination mechanisms have addressed the incentive problems and thereby solved the problems of misalignment of goals and objectives of the supply chain members. In figure 2.5, the boxes have been highlighted in gray colour. Darker the shed is, higher the number of papers addressed that particular barrier of the supply chain. The behavioural and decision-making challenges are found to be one of least addressed barriers of supply chain coordination. these observations led to the following research gaps in the literature

2.1.6 Research Gaps

Summary of the findings from the literature suggest a considerable amount of research in the area of supply chain coordination. Despite of this, many a times members of the supply chains struggle to coordinate. One of the possible reasons could be the barriers to coordination (As identified in chapter 1). Analysis with the help of the frameworks presented in figures 2.1 and 2.5, the research gaps are identified. The observations from the summary of literature have helped to identify the following research gaps

- Majority of the literature have addressed the incentive barriers of supply chain coordination. However, limited shreds of evidence were found how to address the behavioural and decision-making barriers. One such notable barrier is the allocation of risks and benefits post-coordination of the supply chain. Authors have left this to be distributed either arbitrarily or on the bargaining power of the members.

- There are other behavioural variables which have a direct or indirect effect on controlling the supply chain coordination mechanisms and thereby the supply chain coordination itself. As mentioned in the papers by Özer et al. (2011), Voigt & Inderfurth (2012), Inderfurth et al. (2013), Hung et al. (2014), and Han & Dong (2015), existence of trust between supply chain members is one of the essential variable for implementation of one of the supply chain coordination mechanisms, information sharing. However, the barriers of trust and how these are overcome for implementation of supply chain coordination have been studied on very limited occasions.

- There are other soft issues such as lack of top management support (Briscoe et al. 2004, Fawcett et al. 2008) , and resistance to change (Briscoe et al. 2004) have been identified as barriers to supply chain collaborative relationship. Although their effects are realizable in the longer term, but their implication in the short term could be significant. However, these have received limited attention while proposing the models of supply chain coordination.

- Risk preference is another form of behavioural challenge which has received less attention in the literature. Most of the cases, authors have assumed the members to be risk neutral. Only in limited occasions, authors have addressed the variable risk preferences among the members of the supply chain such as Gan et al. (2005), Wang & Webster (2007), Chen & Xiao (2009), Chen & Wang (2012), Deng et al. (2013), Zhang, Dong, Luo & Segerstedt (2014), and Chiu et al. (2015).

- Financial constraints in the form lack of availability of financial resources to the member of the supply chains have received relatively less attention in the literature. Few papers including Xu, Cheng & Sun (2015) have addressed this issue. However, the authors suggested a need for more future research involving more complexity in the situation. One such complexity is the combined challenges of risk aversion and lack of access to the financial resources.

- The selected literature have a good mix of coordination mechanisms for deterministic and stochastic environments. However, the main source of uncertainty has been considered as demand for the item(s) to be sold and sometimes its (their) price(s) i.e. the majority of the papers have focussed on product supply chains. In a very limited occasion, coordination issues have been explored for supply chains with time and cost as sources of uncertainty such as in the case of project supply chains.
- In continuation to the last research gap identified, service supply chains have also received relatively less attention in comparison to its counterpart physical good supply chains.

These research gaps mentioned in this subsection have prepared the backdrop for this present research. This research aims to address few of the relevant research gaps. Thus, the next section discusses where the research is positioned, why it is important to pay attention to those research gaps, followed by research question and objectives and finally a framework to explain the input areas for this research.

2.2 Research Positioning

In last subsection, this research summarized the research gaps and limitations based on the literature review conducted. This research aims to address the following research gaps

- Supply chain coordination with time and cost as sources of uncertainties
- The allocation problem post-coordination for the risk and benefits
- Variable risk preference of the members of the supply chain.

2.2.1 Importance of the research gaps

Proper allocation of risk and benefits of supply chain coordination is very critical to the success of the supply chains for survival. Katok & Pavlov (2013) and Wu (2013*b*) found evidence of negative impact in absence of proper fairness considerations in risk and benefit sharing on the supply chain coordination with behavioural experiment with human participants. In fact, Katok & Pavlov (2013) found termination of the contractual relation between the members of the supply chain. Even in practice, lack of fairness in the allocation of risk and benefits led to the termination of the contractual relationship, for example, the early termination of the contractual relation between Walmart Canada and Lego group (Georgiades 2008). Liu et al. (2012) cited the termination of the contractual relationship between Chinese home appliance Gome and Air Conditioner manufacturer Gree due to the unfair promotional price being charged by the retailer during summer time. In addition to the problems, Griffith et al. (2006) found a positive relation between supply chain performance and fairness. Due to these importances, this research would address this research gap.

Time and cost over run are two most frequently cited reasons for project failures. In fact the projects with members in a non-coordinated supply chain run into time and cost overrun very often, for example, Denver airport's new baggage system installation project ran into over time and over budget (Moore 2009). The authors cited poor information flow and misaligned incentives as two most important reasons behind this failure. This research has already identified less emphasis of supply chain coordination literature considering and time and cost as sources of uncertainty. Thus, this present research aims to address this research gap. The research will address the research gap with the help of the construction supply chain. The reason for selection of the construction industry is justified because of its enormous importance and very limited research on addressing challenges and proposing a proper mechanism to achieve supply chain coordination for the long term.

On this backdrop with the research gap areas presented, the next subsection highlights the research aim, research question, and the objectives.

2.2.2 Research Question, and Objectives

This section identifies the research question and defined research aims and objectives. Research gaps presented at the beginning of section 2.2 suggest the existence of opportunities for knowledge creation. The research gaps selected for this research presents certain research input areas.

Identification of Research Inputs

The first is Supply chain coordination with time and cost as sources of uncertainties. Supply chains in project environment fit this description best. Hence, this research has selected a dyadic supply chain in a project set up with one main project manager organization and a contractor organization. In section 2.1.4, two broad categories of coordination mechanisms (Formal contracts and Informal relational mechanism) have been identified for supply chains. It was also argued that implementation of informal relational mechanisms are possible after formal contractual mechanisms. Although Özer et al. (2011) argued that existence of trust even in absence of contractual mechanisms can promote information sharing, but majority of the authors supported the need for formal contractual relation in first place. In fact Arshinder et al. (2007) presented a framework that highlights how the coordination mechanisms are devised in different stages of the coordination over time as shown in figure 2.6.

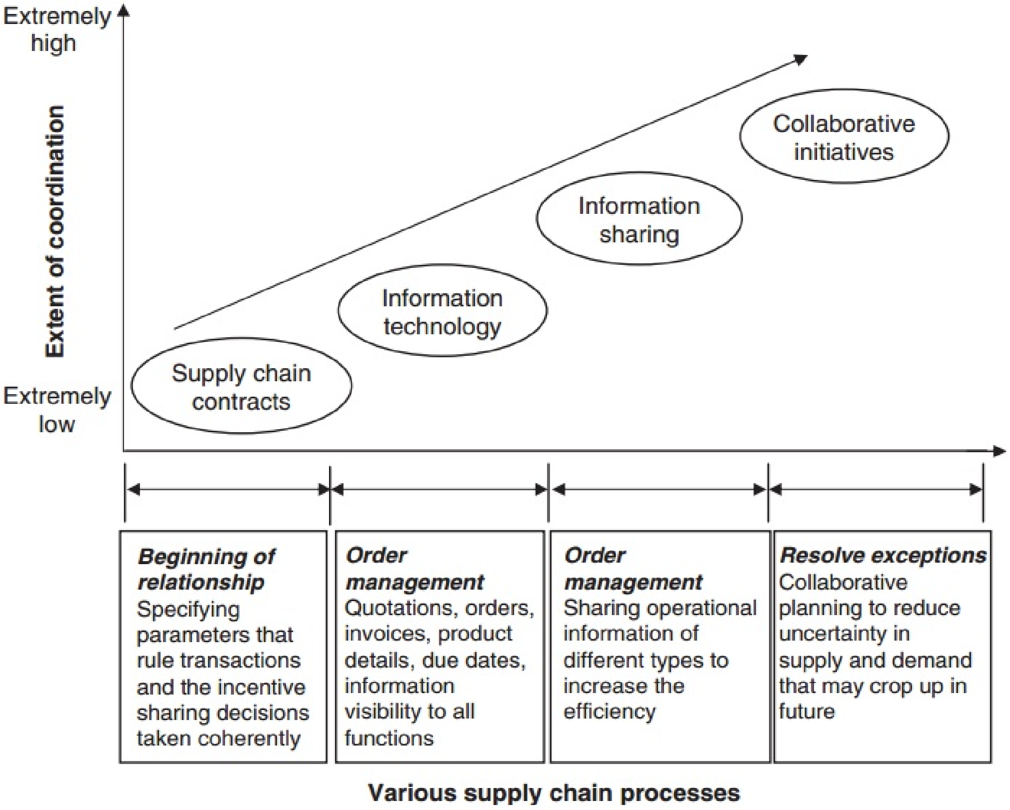


Figure 2.6: Coordination Mechanisms in the Supply Chain Processes (Source: Arshinder et al. (2007)

Based on this observation, this research has selected contractual mechanisms to test if the supply chain under consideration can be coordinated or not. The need for use of contractual mechanisms to coordinate the supply chains was highlighted in the literature of Kwon et al. (2010) and Lippman et al. (2013). The authors have highlighted the limited shreds of evidence of use of project contracts to coordinate the project supply chains. Kwon et al. (2010) used a time-based project contract to coordinate a dyadic supply chain in a take it or leave it situation with completion time as a source of uncertainty. On the contrary, Lippman et al. (2013) investigated the problems in a bargaining situation. Moreover, Lippman et al. (2013) used differential preferences for risk perception between the members. The relevant literature review on project supply chains is presented in chapter 4 and chapter 5. These models are restricted by the choice of the selected nature of probability distributions such as the exponential distribution of time (Kwon et al. 2010) and normally distributed completion cost (Lippman et al. 2013). It is discussed in chapter 4 and 5 that the completion time and completion cost can take various other forms of probability distribution in practice. This renders a question mark on the applicability of the existing models. The present research is positioned to derive coordination models for different possible probability distributions of completion time and completion cost.

The concept of fairness is a popular phenomenon in Economics more precisely behavioural economics. However, the application of the concepts of the fairness is relatively new in supply chain coordination. As per the best knowledge of the author, Cui et al. (2007) were one of the pioneer authors who used the concepts of fairness for channel coordination. Again, the relevant literature review is presented in chapter 6.

Research Question

Combining the research gaps presented, this research aims to answer the research question presented in fig. 2.7. The research question is highlighted in the black box in figure 2.7. This question is designed with inputs from various areas mentioned in the gray boxes above it with arrows pointing towards it. The output of this research contributes to several areas highlighted in the white boxes underneath the research question.

This research used mathematical models as input and generate mathematical models as output. Mathematical models tend to break down due to complexity with network kind of

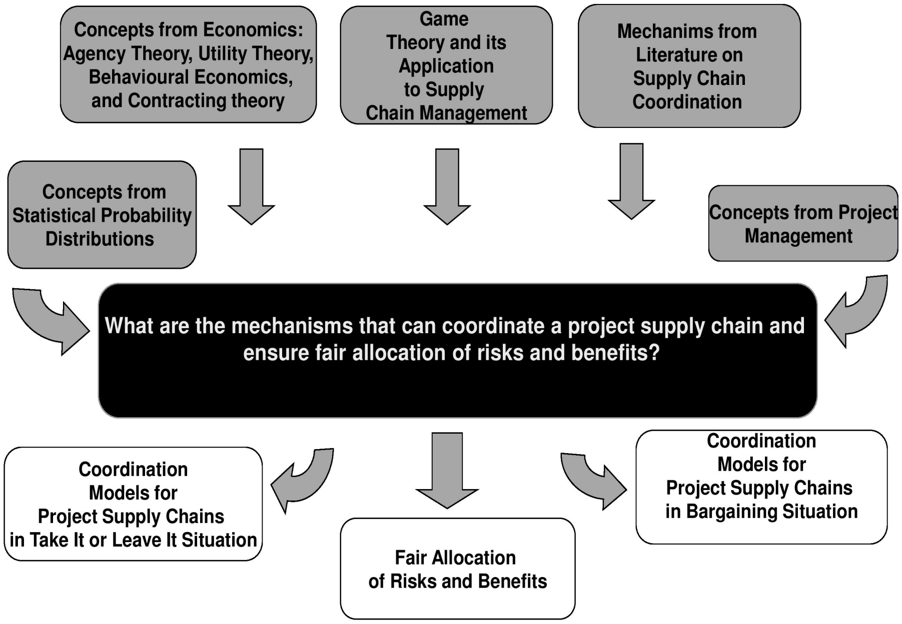


Figure 2.7: Research Question

structures with multiple supply chain members (Huang et al. 2003). That is why a two-stage supply chain is selected. The research question highlighted in fig. 2.7 is answered with the following research aim

To investigate the coordination in a project supply chain using mathematical models and finally to overcome the challenges of fair allocation of risks and benefits.

This aim is broken down into three objectives

- **Obj.1:** To investigate and propose supply chain coordination models in a take it or leave it scenario using project contracts. This research extended the models proposed by Bayiz & Corbett (2005) and Kwon et al. (2010).
- **Obj.2:** To investigate and propose optimal coordinating solutions in a bargaining scenario. This research extended the models proposed by Lippman et al. (2013).
- **Obj.3:** To investigate if the supply chain can be coordinated with fairly allocated risks and benefits in the scenarios mentioned in objective 1 and 2.

Chapter 3

Research Methodology

In chapter 2, this research has identified the research question, research aim, and research objectives. In order to answer the research question and fulfill the objectives, this chapter explains which research method(s) can achieve this. For this purpose, this research has referred to several journal articles which have discussed the issues around the selection of research method. Based on the findings from these journal articles and considering the research question, this research has designed the research method in this section.

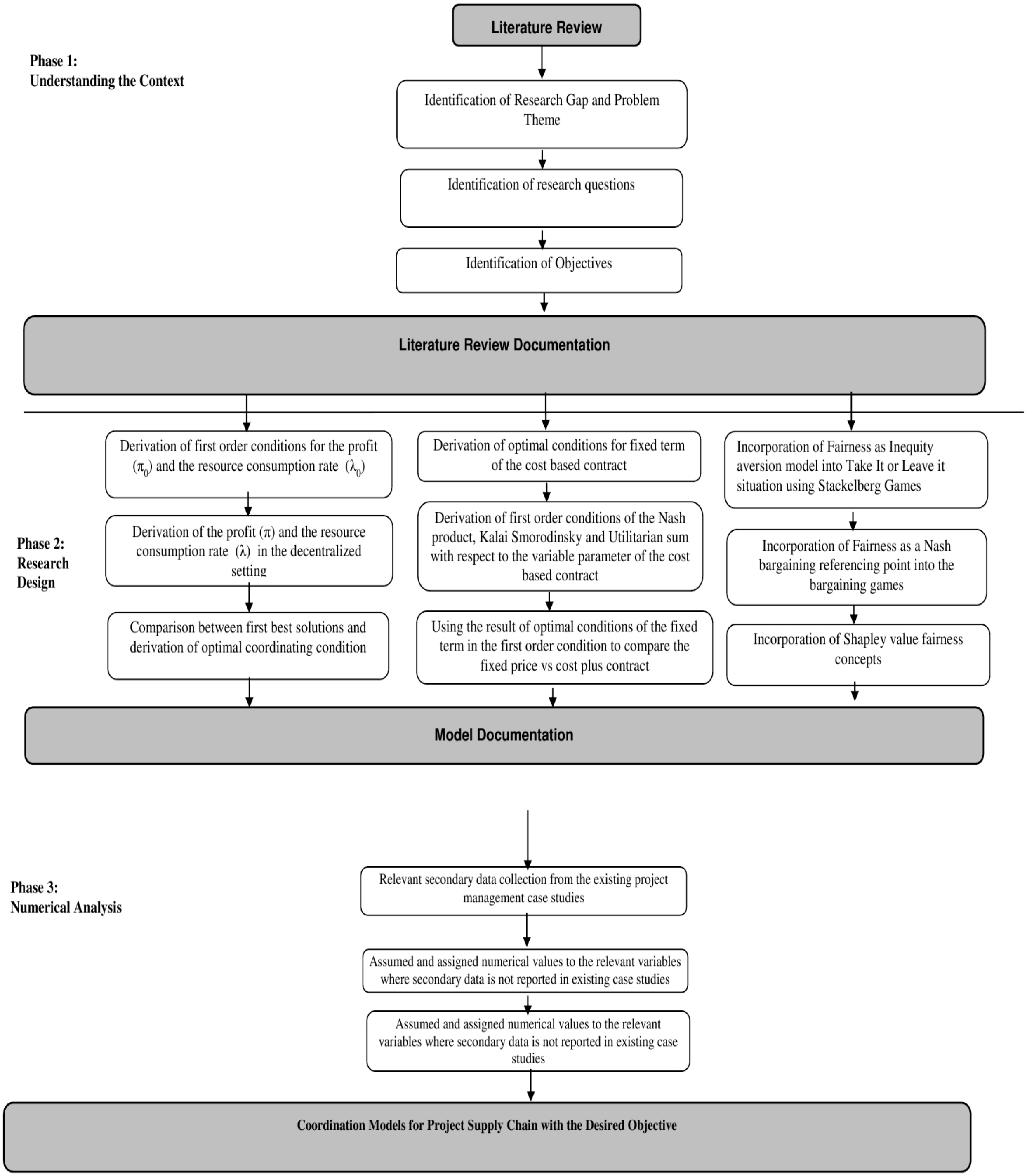
This section is divided into three main subsections explaining the phases of this research as highlighted in fig. 3.1.

The research design is explained in the second phase.

3.1 Phase 1: Understanding the context

In the first phase of this research, research gaps and areas, where the proposed research can contribute to the knowledge base, are identified. Funded projects with existing problem area from the research institute in concern, industry-sponsored projects with existing problems and extensive literature reviews are generally used for identification of research gaps (Webster et al, 2002). This research has conducted an extensive literature review for this purpose. At the beginning, broad ideas were generated from the previous background knowledge on this subject area. This was followed by an initial literature review which narrowed the focus to particular research gaps in the area of supply chain management. After careful consideration, a very broad objective was decided. Keeping this broad objective in mind, based on further

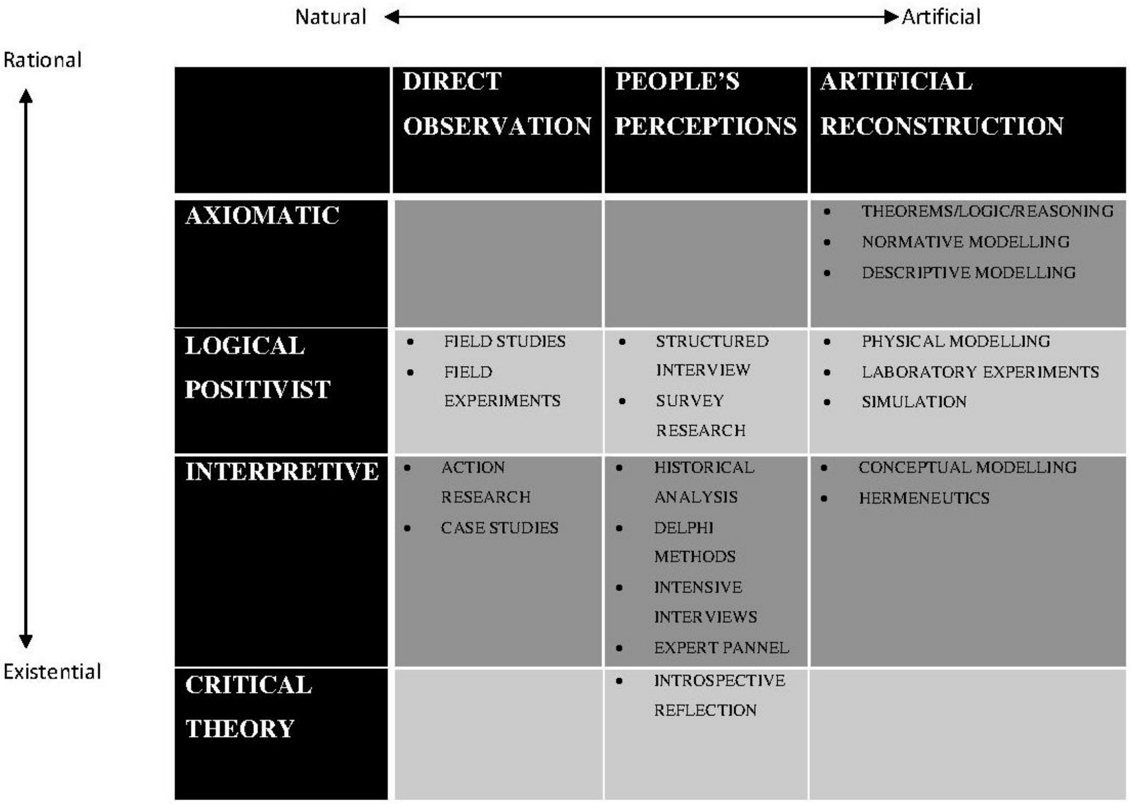
Phases of Present Research



literature review, generated ideas and with the help of pre-existing literature review articles (Cooper et al. 1997, Kanda et al. 2008), taxonomical framework for literature classification were identified (mentioned in fig. 2.1). This framework helped organize the literature. From an extensive literature review, several ideas were generated to identify the research gaps. In order to maintain a consistency and coherency in the gaps identified, idea screening was carried out. The screened, coherent and consistent ideas helped the formulation of the research question presented in figure 2.7 in chapter 2. In order to answer this research question, a set of objectives were identified, which will serve the objectives of this research project. All these findings were documented in the literature review chapter of this report.

3.2 Phase 2: Research Design

In order to effectively design any particular research, an understanding of the research paradigm is the first step (Frankel et al. 2005, Meredith et al. 1989). Thus, this research investigates the issues around the research paradigm and then proceeds to the research design. The research paradigm has been defined from various perspectives (Meredith et al. 1989, Frankel et al. 2005, Matthews & Ross 2010). Considering these views, this research has considered it as a set of common beliefs, practices and shared norms among researchers from a particular area to look into the problems of the world pertinent to that area. The paradigm should contain three main elements: Ontology, Epistemology, and Methodology (Denzin & Lincoln 1994, Frankel et al. 2005). Ontology refers to how the reality exists; epistemology refers to how researchers perceive knowledge (Matthews & Ross 2010). These two elements of paradigm influence the third element methodology, which explains how researchers gain knowledge (Frankel et al. 2005). Several possible classifications of paradigms have been identified: based on techniques used to collect data; methods used to analyze the data; the immediate purpose of the research; nature of units being studied; time/duration of data collection and others (Meredith et al. 1989). The authors proposed a framework based on some previous work on a two-dimensional framework: rational to existential (R/E continuum) in one dimension which explains the epistemological elements; and the other one is from natural to artificial (N/A continuum) in another dimension to explain how the reality exists(fig. 3.2). The framework is chosen for its thoroughness. The R/ E dimension explains more of philo-



	DIRECT OBSERVATION	PEOPLE'S PERCEPTIONS	ARTIFICIAL RECONSTRUCTION
AXIOMATIC			• THEOREMS/LOGIC/REASONING • NORMATIVE MODELLING • DESCRIPTIVE MODELLING
LOGICAL POSITIVIST	• FIELD STUDIES • FIELD EXPERIMENTS	• STRUCTURED INTERVIEW • SURVEY RESEARCH	• PHYSICAL MODELLING • LABORATORY EXPERIMENTS • SIMULATION
INTERPRETIVE	• ACTION RESEARCH • CASE STUDIES	• HISTORICAL ANALYSIS • DELPHI METHODS • INTENSIVE INTERVIEWS • EXPERT PANNEL	• CONCEPTUAL MODELLING • HERMENEUTICS
CRITICAL THEORY		• INTROSPECTIVE REFLECTION	

Figure 3.2: Framework for research method (Source: Meredith et al. (1989))

sophical aspects of the research paradigm and the degree of dependence of the research on its context. It has four stages: Axiomatic, logical positivist, interpretive and critical theory. On the other hand, the N/A dimension explains how the information is used in the research. This dimension is divided into three stages: Direct observation, people's perception, and artificial construction (Craighead et al. 2007, Dunn et al. 1994, Meredith et al. 1989). These stages are briefly explained as follows based on the definitions of Craighead et al. (2007), Dunn et al. (1994), and Meredith et al. (1989).

R/E Dimension

Axiomatic: The dimension is most rational and scientific with the theorem proved research.

Logical Positivist: This presents a perspective in which the research can be isolated from the research context. The research is objective in nature.

Interpretive: The researchers consider the human being studied and the context as inseparable part of the study. This is more subjective in nature.

Critical theory: This perspective transcends the research beyond the positivist and interpretive dichotomy and tries to establish an interrelationship between these two perspectives.

N/A Dimension

Direct observation: Researchers observe reality directly. This is considered as the most natural and objective in the framework.

People's perception: This allows the use of information from another person's perspective and is presented as mid-range perspective in this dimension between the most natural and most artificial.

Artificial Construction: This has been considered to be the most artificial perspective on this dimension of this framework. This perspective allows the researchers to collect and use the information from the artificial model building and by simulating the reality.

The findings of research conducted in logistics and supply chain management as reported by Craighead et al. (2007) and Dunn et al. (1994) are presented in fig 3.2. The most notable finding from their study is the predominant use of logical positivist paradigm in supply chain research. This supports the findings of Meredith et al. (1989); the authors presented the historic developments of the research paradigm in operations management and showed how this area has historically been influenced by operations research and management science streams. Using the framework of Meredith et al. (1989) for research paradigm, the next subsection highlights the research paradigm and method selection for this research.

Research Paradigm

(Frankel et al. 2005) suggested four factors on which the selection of research methodology depends based on previous researchers' findings. These are: Format of the research question (What, Why, Who, How etc.); Nature of the phenomenon under study (Contemporary or Historical); Extent of control required over behavioral events; and researcher's philosophical stance. The philosophical view of this research is more rational than existential. The reason for that is the nature of the research problem:

- Can contracting mechanisms coordinate the decentralized project supply chain in a take it or leave it situation?
- Can contracting mechanisms coordinate the decentralized project supply chain in a bargaining situation?
- Can fairness in risk and benefit sharing be incorporated in these mechanisms?

The nature of these research questions, highlight the need for a more objective and context independent inquiry into these areas. Moreover, Dunn et al. (1994) argued the need for a scientific and objective inquiry into logistics and supply chain studies with latent variables (variables measured indirectly with the measure of some other direct variables) such as the measure of supply chain integration. That is why this research will follow the rationalist paradigm. As stated earlier, this paradigm will allow this research to isolate the research from its context.

Method Selection

This research used axiomatic quantitative methods. According to Bertrand & Fransoo (2002), the quantitative models are derived based on a set of variables over a specific domain. At the same time, these models establish the causal quantitative relation between the variables. There are several benefits of using this approach for this research.

- It allows the research to study the underlying phenomenon independent of the context. (Meredith et al. 1989)
- It allowed this research to control the phenomenon, the relationship between variables and the observations.
- It helped this research in explaining the outcome and how that outcome is derived. This makes, the results more verifiable and reliable with greater precision.

However, there are some disadvantages of the quantitative modelling method e.g.

- The interpretation of results would be restricted to the model or in other words the abstract nature of the model limits the applicability of the method (Meredith et al. 1989, Bertrand & Fransoo 2002)
- The effect of the human factor on the results gets ignored (Bertrand & Fransoo 2002)
- The lack of applicability as the counterpart empirical theory may be tested with real data. (Kaipia 2007, Swamidass 1991)

According to Bertrand & Fransoo (2002), the axiomatic research works tend to have got less documented research methodology section unlike its counterpart empirical research works.

However, the authors identified an early research by Mitroff et al. (1974) to clearly documenting the research method used in axiomatic research. The framework by Mitroff et al. (1974) is presented below. According to Bertrand & Fransoo (2002), the Mitroff's model has got four phases

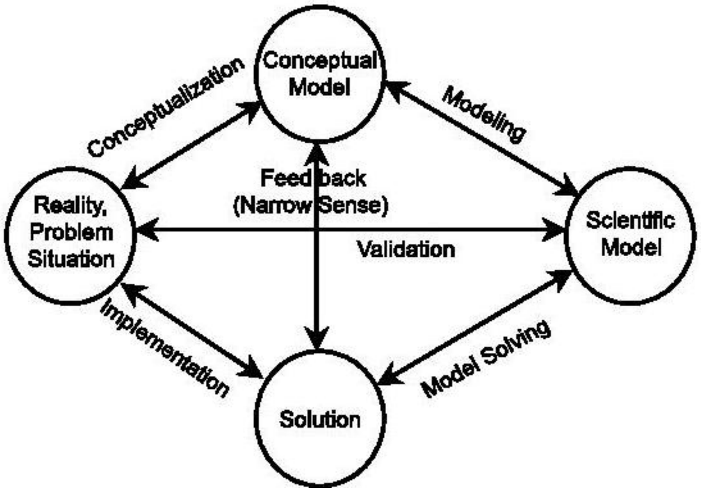


Figure 3.3: Research model by (Mitroff et al. 1974) (Source: Mitroff et al. (1974))

- Conceptualization
- Modeling
- Model solving
- Implementation

Mitroff et al. (1974) mentioned that any research can start at any of these stages and ends at any of these stages. However, the authors highlighted some of the short cut methods such as conceptual "modeling-model solving- narrow feedback". Authors highlighted that sometimes researchers make the mistake of considering model solving for the process of implementation. In addition, the authors also highlighted "conceptualization-narrow feedback-implementation" as another short-cut cycle. The researchers tend to make the mistake of considering conceptualization for modeling.

Mitroff et al. (1974) suggested four methods : Axiomatic Descriptive, Axiomatic Normative, Empirical Descriptive, and Empirical Normative. With axiomatic descriptive, researchers use existing conceptual models to generate scientific models. Modeling becomes central to this methodology. On the contrary, with axiomatic normative methods, model solving is central to the research method.

Despite various criticisms, axiomatic modelling has been the predominant research methodology in the logistics and supply chain research. This is due to the strong influence on the operations management research paradigm from operations research and management (Golicic & Davis 2012, Spens & Kovács 2006, Meredith et al. 1989). In fact, some of the shortcomings of the axiomatic modelling approach have already been addressed in a model proposed by Mitroff et al. (1974) in figure 3.3.

This research adopts the axiomatic normative modeling approach. One of the main constructs of this research is the supply chain coordination. The introduction chapter of this research has highlighted the debates over the definition of supply chain coordination. It was also mentioned that this research is following the definition of supply chain coordination proposed by Cachon (2003). The literature review section has explained how this conceptualized definition has been widely used to propose and generate solutions in the form of models for supply chain coordination in the literature of Cachon & Zhang (2006), Lee & Rhee (2007), Ding & Chen (2008), Cai et al. (2009), Chen et al. (2012), Chen & Wei (2012), Ma et al. (2013), Cao et al. (2013),Saha (2013), Cao (2014) and many others. Moreover, these models were shown to be capable of allowing full coordination for the supply chain. Thus, this research also follows the axiomatic approach for addressing the objectives addressed in chapter 2.

Game theoretic models were identified as one of the input blocks for the research question. The concepts of game theory are at the cross road of mathematics and economics. It has got a wide application in economics. John von Neumann and Oskar Morgenstern are credited as the father figures of modern game theory (Cachon & Netessine 2004). Any decision-making process involving multiple decision makers with each of them dependent on each other can be effectively analyzed by the tools of game theory (Osborne 2004, Cachon & Netessine 2004). Game theory is especially beneficial if a decision-making process involves multiple decision makers with each of them having a possible set of actions. The game theoretic models capture the interaction between the decision makers Osborne (2004). Apart from the above-mentioned requirements to have multiple decision makers with each having their own set of decisions, if the decision makers have a preference for a certain set of actions over others, then the concepts of strategic games can analyse the situations (Osborne 2004, Cachon & Netessine 2004).

The concepts of game theory have been very popular in the supply chain literature. The supply chain decision making often involves multiple decision makers. They have a set of decisions and a preferred set of decisions among the entire set for implementation. Due this basic fit the with requirements, the concepts of game theory have been widely applied in supply chain decision-making problems. Any interested readers may find a detailed survey about the use of game theory in supply chain management in the book chapter by Cachon & Netessine (2004). There are various possible classifications for the applications of game theory. However, this research followed the classification by Cachon & Netessine (2004).

This research has two decision makers: a project manager organization and a contractor organization. Both of them have a set of actions/decisions. However, they would be inclined to take the decision that would optimize their profit or utility depending on the case. In other words, they have a preferred set of decisions to make. This research investigated three different decision-making problems in a supply chain which are explained in there different objectives in the next few chapters. It is assumed that both the members of the supply chain (the project manager and the contractor) have got the full knowledge about the variables which could affect the decisions making such as cost, profit, and others except for the resource consumption rate.

Methods for Objective 1:

The first objective was to propose coordination mechanisms in take it or leave it situations. This research has considered a two-level dyadic project supply chain. The sequence of the solution procedure is explained in figure 3.4

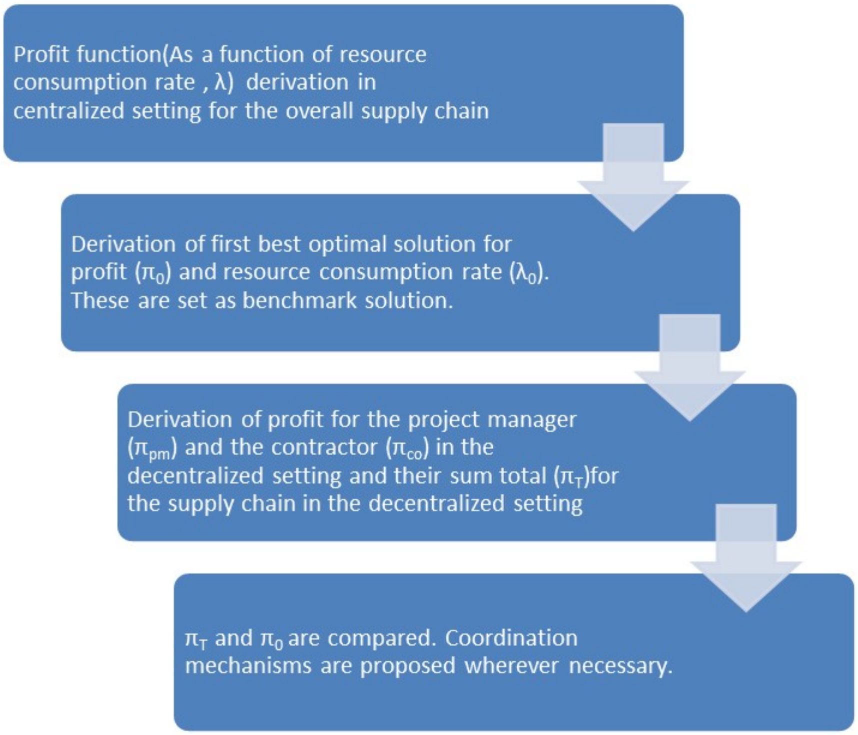


Figure 3.4: Sequence of solution

As explained in the figure 3.4, the benchmark solutions are derived from the centralized supply chain. In the centralized setting, the project manager and the contractor both belong to the same organization with the same goals and objectives. Thus, a single decision maker is assumed to optimize the profit and the resource consumption rate for the overall supply chain.

In the decentralized setting, the project manager and the contractor belong to different organizations. For the first objective, they are assumed to have the objectives of maximizing their individual profit given certain constraints. Hence, their objectives may not be aligned and coordination mechanisms might be required to coordinate these. Stackelberg games from game theory are one of the most popularly used concepts for similar situations in the literature selected in the chapter 2 of this thesis (Lee & Rhee 2007, Karabatı & Sayın 2008, Chen et al. 2012, Chen & Wei 2012, Lau et al. 2012, Hong et al. 2015, Giri & Bardhan 2016). Strategies are announced as a sequence in Stackelberg game move game (Simaan & Cruz Jr 1973). There is a first mover who moves first as a leader with a certain strategic offer that maximizes (minimizes) his or her profit (cost). If the follower agrees, then the leader and the follower needs to perform their respective tasks as agreed during the game. On the contrary, if the follower disagrees to the offer from the leader, then the game terminates and they end up having the profit (cost) if they don't participate in the agreement (Osborne 2004). The first objective only explores the situations with take it or leave it conditions. The project manager is considered to be the more powerful member (in terms of bargaining power and power position in the supply chain) than the contractor. The project manager has a project. Some parts of the project need to be outsourced. Thus, the project manager (she) moves first with an offer to the contractor (he). If the contractor, agrees, then they reach an agreement; otherwise, they go their separate ways. Due to the similarity of the sequence of actions, this research has adopted the Stackelberg games for solving the coordination problem for this take it or leave it situation. It is also assumed that the project manager is the leader and the contractor is the follower.

As a first mover, the project manager could offer a zero profit, grab everything on offer and maximize her benefits. However, this may not be accepted as the contractor would have some minimum expectations of profit to be earned from outside this project. This imposes a constraint to the project manager to offer at least what the contractor expects to earn from outside options. The project manager envisages this minimum value of the profit which she needs to offer to the contractor. In fact, any offer from the project manager equal to or above the minimum which the contractor can get outside this contract, would entice the contractor to accept the contract. However, for any value above this minimum value, the contractor would accept the contract, but it would reduce the profit of the project manager. Hence, those values are not the best response of the project manager. Given the constraints, the best response for the project manager is to offer the minimum the contractor can expect to earn to outside this contract. Given this offer, the contractor would select a resource consumption rate that allows them to earn equal to what they can earn outside this contractual offer. This becomes the best response for the contractor. This also satisfies the definition of Pareto optimal solution. Tadelis (2013), suggests a solution would be Pareto optimal if none of the players of a game can be better off without worsening at least one of the players.

The act of envisaging the minimum expected profit required to entice the contractor to agree to the offered contract is called the backward induction method. Cachon & Netessine

(2004) supported this point of view that dynamic games including Stackelberg games use these backward induction methods to solve these games. Thus, this research also followed the backward induction method to solve the games with the concepts of Stackelberg's leader-follower games.

Methods for Objective 2:

The second objective is to investigate if coordination mechanisms can be proposed for the supply chain under consideration where negotiation and renegotiation of contractual agreement are possible. Unlike the take it or leave it situations, if the contractual offer is rejected by the contractor, then the project manager has the options for re-offering the contracts. That means the game is not terminated after one round of rejection, rather renegotiated.

Negotiation is represented by a bargaining process. Bargaining process can be modelled by cooperative games (Cachon & Netessine 2004, Nagarajan & Sošić 2008). The authors argued that cooperative game theory has received relatively less attention in supply chain literature than in comparison to non-cooperative game theory. Several bargaining models are proposed in the literature. The cooperative games, unlike non-cooperative games, allow the processes to be studied that leads to the outcomes (Nagarajan & Sošić 2008).

Two or more players have a got a set of feasible outcomes and any one of these will be implemented if it is agreed unanimously by all the players. On the contrary, the players will reach a disagreement point outcome if they fail to reach an agreement. Nagarajan & Sošić (2008) suggested if there are feasible outcomes better than disagreement point payoff, then the members have the incentive to reach an agreement to achieve that outcome. The author further added, if at least two players differ regarding the outcome, then the bargaining may take place.

The present research has got only two players as two members of the supply chain. In objective 1, if the players differ in their preferred outcome, then the game was considered to be terminated. However, the second objective relaxed this restriction. Hence, the members of the supply chain under consideration i.e. the project manager and the contractor can engage in a bargaining process to reach a unanimously agreed outcome which gives them a better outcome than terminating the game and reaching the disagreement point outcome. There are several bargaining models which can capture this situation. However, Nagarajan & Sošić (2008) argued that Nash bargaining is one of the most popularly used bargaining processes. In fact, the authors further argued that the experimental bargaining theory suggests the stronger empirical evidence of the existence of Nash bargaining theory. Moreover, few of the available studies from the set of literature selected in chapter 2 have used Nash bargaining as a bargaining model (He & Zhao 2012). Thus, this research used Nash bargaining as the staring point of the bargaining games. The models proposed in the literature of Lippman et al. (2013) are used for further extension. In addition, the bargaining models proposed by Kalai-Smorodinsky and Utilitarian models are also used to see if the results have any similarity to what is achieved using the Nash bargaining games.

Nagarajan & Sošić (2008) highlighted the main results of Nash bargaining in a two player bargaining problem, with F as a closed convex feasible set of $\mathbb{R}^2$ and $d = (d_1, d_2)$ as a disagreement vector. According to the author, the feasible outcome selected as solution satisfies the following axioms

1. Symmetry : Identical players should receive an identical allocation.
2. Feasible: Sum total of the allocation of the players should not be higher than what is on offer.
3. Pareto optimal: As explained earlier, no player can be better off without making other players worse off
4. Linear Transformation : The solutions can also be presented with a linear change of scale by a factor
5. Independence of Irrelevant Alternatives: If the alternative solutions that are not selected as optimal, are removed, then the bargaining solution won't change

With the above axioms, the Nash bargaining solution will satisfy the following

$$arg \max_{x=(x_1,x_2)\in F, x\geq d} : (x_1 - d_1)(x_2 - d_2)$$

Methods for Objective 3:

The objective 3 is to investigate the solutions proposed in objective 1 and objective 2 in the presence of fairness consideration. This research has followed a definition of fairness as inequity aversion proposed by Fehr & Schmidt (1999). This is one of the most popular models of fairness used in the literature. The citation count as per Web of Science is around 2000.

It is further assumed that the member of the supply chain (the project manager or the contractor) would be a utility maximizer which might include some non-profit component depending on the case.

For the take it or leave it situations, the sequence of solutions is slightly modified as shown in figure 3.5

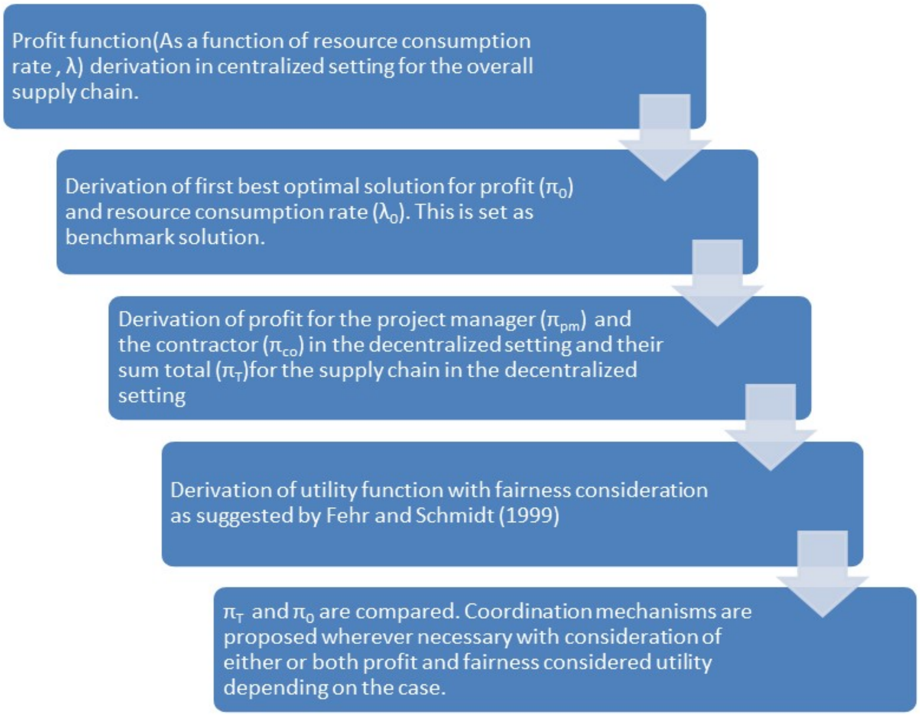


Figure 3.5: Sequence of Solutions for Take It or Leave it Situation with fairness consideration

Again like in objective 1, the situation can be best explained by Stackelberg games.

3.3 Numerical Analysis

After the model development and model solving phase is over, the numerical analysis conducted. The proposed models can be applied in any project context with the selection of suitable variable and parameter values. However, this research has prepared the numerical example section based on construction supply chains from the UK.

Now the question is why there is a need for these kinds of collaborative supply chain practices in the construction sector? According to UKCG (2012), the construction sector value chain worth around 14% of the total UK's GDP. According to this report, it had contributed around £1203 mn. surplus to the UK's economy between the years 2009-11. The report also says that every £1 invested in the construction sector generates £2.84 mn economic activity. Due to its huge importance, it has even featured in the top three sectors for government support just after railway and health.

Despite the importance of the construction sector in UK economy, the report published by ECLLP (2013), states that construction supply chain integration is still an area which needs careful attention and improvement. This conclusion was based on a survey conducted among the UK construction practitioners by the Business Innovation and Skills department UK. The main purpose of the survey was to identify how the UK construction companies are performing with respect to the suggested supply chain integration approaches in the Latham report published in 1994. However, the responses suggest the supply chain integration as one of the lesser highlighted areas of construction projects. In chapter 1, this research has explained certain barriers of supply chain coordination. In chapter 4, this research further elaborated the different nature of project supply chains with respect to product supply chains with a special reference to the construction supply chain.

The relevant information such as project value duration etc. was collected from the existing case studies as secondary data sources from the Association of Project Management, UK (https://www.apm.org.uk/). However, there are certain variables/parameters whose values are confidential in nature and are not shared as publicly available data. These values are assigned/assumed as relevant.

There are certain distribution specific parameters used in the models proposed in this research. Again, some realistic and relevant values were assumed while preparing the numerical examples.

The next three chapters highlighted how the identified objectives have been addressed in this research. Each of these chapters consists of the details of the backdrop of the objectives, the proposed models, and their numerical analysis results.

Chapter 4

Supply Chain Coordination using Project Contracts in Take it or Leave it situations

The importance of supply chain coordination was highlighted in chapter 1, and the related development in the literature was presented in chapter 2. These previous chapters also highlighted that there are various problems associated with the projects in absence of the proper supply chain coordination. Time and cost overruns are the two main problems which arise in the absence of proper supply chain coordination. A few real-life projects such as Wembley stadium renovation project and Denver airport baggage handling system installation project were cited as cases of projects with subsequent time and cost overruns (Moore 2009). In fact, the Denver airport project was quoted by the author to have a significant coordination problem. On the contrary, there are few examples from practice that can be referred with coordination among the members of the supply chain as the key to the project success such as Turner and Townsend set an example of proper coordination of supply chain activities to complete the construction project of the University of Exeter's newly developed Business School. This helped the project management team of Turner and Townsend to complete the project within the budget of £14 million, and approximately three years estimated time.

The focus of this chapter is to address the first objective of this research. That is how project contracts can coordinate the supply chains for a more general set of distributions. The steps of solving the coordination problem were explained in chapter 3. As decided, earlier in chapter 1, the supply chain in the decentralized setting would be considered coordinated when total benefits conforms to the maximum benefits which can be derived from the centralized setting as defined by Cachon (2003)

In a supply chain context, contracts can specify parameters such as order quantity, price, time and delivery (Kanda et al. 2008). Due to this specificity, supply chain contracts can help the total supply chain achieve coordination (Giannoccaro et al. 2004). Based on the principle proposed by Cachon (2003), several authors proposed coordination models for supply chain as discussed in the literature review in chapter 2. In all of these papers, the authors used certain contractual terms as incentives to motivate the members of the supply chain to take decisions that are aligned with the overall goals and objectives. These contractual terms can take the form of flexible payments such as trade credit (Chen & Wang 2012), side payments such as a two-part tariff (Corbett et al. 2004), revenue sharing (Giannoccaro et al. 2004), discounting contractual terms such as price discount (Bernstein & Federgruen 2005*a*, Chen 2011*b*), quantity discount (Li & Liu 2006), price plus subsidy rate (Xiao et al. 2005), discounting using reverse supply chain conditions such as buy back (Chen 2011*b*), and returns (Chen & Xiao 2011*b*). However, these models are limited to the demand being the source of the uncertainty and the order quantity/price being the decision variable to be optimized.

Very little is known about coordination in project supply chains. In a project supply chain (with a project manager and a contractor), the project manager can verify the project completion time and the cost upon completion. However, it is difficult for the project manager to verify the resource consumption rate of the contractor when the members belong to different organizations. This could lead to a misalignment of contractor's selected resource consumption rate and the optimal resource consumption rate. As a result, it could lead to time and cost overruns.

Tools and techniques used for effective project management mainly reside in the literature of civil engineering as its origin is from there (Kwon et al. 2010). As mentioned earlier, some commonly used tools from project management such as CPM and PERT work under deterministic to low uncertainty, but not in more uncertain environments (Klastorin 2004). Moreover, only a limited amount of research evidence has been found in the supply chain coordination literature with cost and time as the sources of uncertainty such as project supply chain. However, the importance is not negligible in this case. In a survey study, Akintoye et al. (2000) identified supply chain coordination as one of the key requirements for the success of the construction sector in the UK, but project supply chains often fail to coordinate.

A few authors have proposed conceptual models for coordinating construction supply chains such as an inter-organizational learning model (Love et al. 2002), a framework to influence co-development of decisions (Crespin-Mazet & Ghauri 2007), and a system-wide information system (Hadaya & Pellerin 2010). However, none of these models are quantitative in nature. Recently, Bayiz & Corbett (2005), Kwon et al. (2010) and Lippman et al. (2013) proposed coordination models using project contracts for the project supply chains. However, these models assumed specific functional forms for the project completion time and the completion cost: an exponential function for completion time (Bayiz & Corbett 2005, Kwon et al. 2010) and a normal distribution for completion cost (Lippman et al. 2013). However, in practice, project completion times are often modelled as the uniform distribution in simulation (Lee, Arditi & Son 2013), the beta distribution (Golenko-Ginzburg 1988), the gamma distribution (Roy & Roy 2013), and the Weibull distribution (Abdelkader 2004). It has been not investigated in detail if the existing models will work with these distributions or not. Therefore, there is a need to explore if coordination models can be proposed for project supply chains. To address this issue, this research aims to fulfill the first objective as mentioned before.

Objective 1. *To extend the coordination model proposed by Kwon et al. (2010) with a general set of continuous distributions for project completion time in a Stackelberg model with ultimatum games.*

This research considered the basic model proposed by Kwon et al. (2010) and Bayiz and Corbett (2005) for the extension. The following are the main features of the extensions to the existing model

- Model proposed by Kwon et al. (2010) used exponentially discounted cash-flows. Bayiz & Corbett (2005) used a linearly decreasing cash-flow. In this research, both exponential and non-exponential discounted cash-flows (linear, quadratic and so on) are considered for modelling.
- Unlike the existing models, this research considers an additional cost (C_o). This is independent of the resource consumption rate
- The models proposed in this research are not restricted by the nature of the distribution

function for the completion time and the completion cost.

The existing models are extended for various forms of continuous distributions for project completion time. It is analysed with take it or leave it (TIOLI) contracts with the help of concepts from Stackelberg games and ultimatum games.

4.1 Problem description

A dyadic supply chain is considered with a project manager and a contractor. The project manager is referred as she and the contractor is referred to as he. The project manager can verify the completion time and cost, but not the resource consumption rate selected by the contractor. Thus, in the absence of proper incentives, it may be in the contractor's interest to select a resource consumption rate that leads to a non-optimal overall project completion time and cost. To avoid this, the project manager offers a contract P(T,C), where T is the project completion time and C is the cost to the contractor. As mentioned in chapter 3, Stackelberg games from game theory are used to analyse the situation.

The project manager has a project value of q ($q = q_0$ in the beginning) that decreases with time. The reduction in project value comes from two factors. Firstly if a project is delivered late, its effective lifetime may be shortened, for example, if a software project is delivered late, then the benefits that would have accrued from an earlier delivery may be lost. This is modelled as a polynomial, usually taking it to be a linear loss of value. On the contrary, there are some projects where the product life does not change and the end of life is a fixed time from the date of completion of the project e.g. power plant and bridge projects. The second loss of value stems from discounting the project value. For short and medium-term projects, the discounting will be very small and does not have a significant effect. However, for a long-term project, the time value of the money can have a considerable impact on the cash flow. Thus, any cash-flow in the long term project is exponentially discounted to take into account the time value of money. A continuous exponential discounting is considered to take into account the time value of money. The discounting factor can be calculated with the help of the prevailing discounting rate suggested by the country under consideration. For presentational simplicity, it is assumed that both the project manager and the contractor use the prevailing discounting rate.

The project manager needs to outsource some part of the project to an external contractor. Thus, she offers a contract P(T,C) to the contractor. The contractor can decide either to accept or reject the contract. If he rejects the contract, then the game terminates. Hence, the game is considered as an ultimatum game with a take it or leave it contract. In practice, there is a possibility that the contractual terms would be subjected to further bargaining. However, this case is not part of this chapter and is discussed in next chapter. If the contractor accepts the contract, then he needs to select the resource consumption rate (λ) to complete the project. This rate is assumed as constant throughout the project once selected. Upon completion, the project manager verifies the completion cost and time and makes the payment according to the contractual agreement.

In a centralized setting, the project manager and the contractor, belong to the same organization with the same goals and objectives. The profit is calculated as the difference between the expected value of the project and the expected cost of the project. The project manager would select the resource consumption rate (λ_0) that maximizes this profit. As suggested in Cachon (2003), these optimal values of resource consumption rate (λ_0) and the corresponding supply chain profit (π_0) have been considered as the **first best solutions** of the supply chain under consideration.

However, in a decentralized setting, the contractor, and the project manager belong to different organizations, and so they have different goals and objectives. The contractor would select the value of the resource consumption rate (λ) that would maximize his profit. This value of the resource consumption rate may not be aligned with the optimal value (λ_0). This would lead to a non-optimal overall profit as an outcome for the entire supply chain. However, Cachon (2003) defined that the supply chains could be coordinated if this is avoided through properly designed contracts. Thus, to avoid this moral hazard of selecting a non-optimal λ, the contract P(T,C) needs to be designed properly. Central to the contract design is the incentive compatibility and individual rationality constraints. The incentive compatibility constraint ensures the maximization of the profit for the member given the selected resource consumption rate (λ). The individual rationality constraint ensures the participation in the contractual agreement. This means members are better off in terms of profit by selecting the contract under consideration than selecting any outside contract.

4.2 Model terminologies

The proposed model requires three main variables: the project value, the cost incurred by the contractor, and the resource consumption rate.

The contractor's resource consumption rate is λ. The project completion time depends on λ and its probability density function is denoted as $f_\lambda(T)$. Mean completion time (μ) depends on the resource consumption rate (λ). This dependency is modelled as

$$\mu_\lambda = \frac{\mu_1}{\lambda^A} \qquad \text{[Where, } 0 < A \leq 1\text{]} \tag{4.1}$$

The restriction on A is because it is assumed that increasing the resource consumption rate does not improve the efficiency at the same rate.

As mentioned earlier, the loss of project value from the late delivery is modelled as having two components: A polynomial element from a shortened operating life and a discounting element. For short-term projects, the discounting element is not significant. This situation is followed in the next sub-section. Then, the following section looks at long term projects with exponential discounting with and without polynomial reduction as well.

4.2.1 Polynomial reduction of project rewards

The project value is reduced as a polynomial function of the completion time to model the lost income from the late delivery as the project manager is not able to benefit from the project during [0,T].

$$q(T) = q_0(1 - \psi T^m) \qquad \text{[where m is an integer and } m \geq 1\text{]} \tag{4.2}$$

Thus, the expected value of the project is

$$E(q) = \int_0^\infty q_0(1 - \psi T^m) f_\lambda(T) dT = q_0[1 - \psi . E\{T^m\}] \tag{4.3}$$

The project cost (C) depends on the resource consumption (λ). In prior research in the areas of reverse supply chain returns (Savaskan & Van Wassenhove 2006), and green supply chain management (Swami & Shah 2013*b*), the authors used $C = k\lambda^2$. In order to maintain a generalizable form, the cost per unit time is assumed as $k\lambda^n$. The value of n satisfies $n > 1$, as increasing the resource consumption rate leads to an extra cost.

For short-term projects, the effects of discounting monetary values are very low and so these are disregarded and the exponential discounting is not applied. Thus, the expected total cost of the project is

$$E(C) = \int_0^\infty \left(\int_0^T k\lambda^n dt \right) f_\lambda(T)dT \ = k\lambda^n E(T) \tag{4.4}$$

As mentioned in the section 4.1, the project manager outsources a part of the project to the contractor. This means the project manager incurs some additional costs that do not depend on λ such as raw material costs, and overhead costs. These costs all together are denoted by C_o.

Hence, the expected overall profit for the project is

$$\pi = E(q) - E(C) - C_0 = q_0[1 - \psi E\{T^m\}] - k\lambda^n E(T) - C_o \tag{4.5}$$

In the centralized setting, λ is chosen to maximise the project's profit and the resulting profit is denoted by π_0. In the decentralized setting, the project manager offers a time-based contract P(T,C) = g-hT. The first term of the contract g is the base term. The second term h is a penalty for the completion time. Although the time penalty can take other forms, Kwon et al. (2010) stated that a linear form is most popular in practice. Using equations (4.3) and (4.4), the project manager's and the contractor's profits are derived as respectively

$$\begin{aligned}\pi_{pm} &= E(q) - E\{(P(T,C)\} - C_o \\ &= \int_0^\infty q(T)f_\lambda(T)dT - \int_0^\infty P(T,C)f_\lambda(T)dT - C_o \\ &= q_0\{1 - \psi E(T^m)\} - \{g - hE(T)\} - C_o \end{aligned} \tag{4.6}$$

$$\pi_{co} = E\{P(T,C)\} - E(C) = \{g - hE(T)\} - \{k\lambda^n E(T)\} \tag{4.7}$$

4.2.2 Exponential Discounting of cash-flows

For long-term projects, the cash-flows decrease continuously with a discounting parameter (α), where $\alpha > 0$. Thus, the expected project value upon completion for projects with recoverable product life upon completion is

$$E(q) = \int_0^\infty (q_0 e^{-\alpha T} f_\lambda(T)dT = q_0 E\{e^{-\alpha T}\} \tag{4.8}$$

The expected project value upon completion for projects with irrecoverable product life upon completion (i.e. polynomial reduced) is

$$E(q) = q_0 \left[1 - \psi \int_0^\infty \left\{ \int_0^T t^{m-1} e^{-\alpha t} dt \right\} f_\lambda(T)dT \right] \tag{4.9}$$

In practice, the loss of revenue due to project delay is more likely to be linear i.e. m = 1 (the best case of no loss of revenue is equation 4.8) and so this case is analysed further. Thus, the expected value of the project in equation (4.9) becomes

$$E(q) = q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] \tag{4.10}$$

The exponential discounting is used for the expected cost as used by (Kwon et al. 2010). The expected cost is calculated as below

$$E(C) = \int_0^\infty k\lambda^n f_\lambda(T) \left(\int_0^T e^{-\alpha t} dt \right) dT = \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\} \right] \tag{4.11}$$

As before, the project manager will incur some additional costs which are independent of the resource consumption rate. It is assumed that these costs are incurred straight away by the project manager and so C_o is not discounted. Hence, the overall profit is

$$\begin{aligned} \pi &= E(q) - E(C) - C_o \\ &= \begin{cases} q_0 E\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\} \right] - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] - \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\} \right] - C_o & \text{for irrecoverable product life with m=1} \end{cases} \end{aligned} \tag{4.12}$$

Again in a centralized setting. λ is chosen to maximize π and the resulting π will be denoted by π_0. As mentioned earlier, the project manager would offer a contract P(T,C) =g-hT to the contractor in the decentralized setting. The profit function for the project manager and the contractor are as follows

$$\pi_{pm} = E(q) - E\{(P(T,C)e^{-\alpha T}\} - C_o$$
$$= \begin{cases} q_0 E\{e^{-\alpha T}\} - \int\limits_0^\infty P(T,C)\, e^{-\alpha T} f_\lambda(T) dT - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] - \int\limits_0^\infty P(T,C)\, e^{-\alpha T} f_\lambda(T) dT - C_o & \text{for irrecoverable product life} \end{cases} \tag{4.13}$$

$$\pi_{co} = E\{P(T,C)e^{-\alpha T}\} - E(C) = \left[\int\limits_0^\infty P(T,C)\, e^{-\alpha T} f_\lambda(T) dT \right] - \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\}\right] \tag{4.14}$$

4.3 Models without cash discounting

For short term projects, the polynomial reduction of project reward is considered as mentioned in the section 4.2.1. Four different forms of probability distributions (uniform, gamma, beta, and Weibull) for the completion time are investigated.

4.3.1 Centralized Setting

In the centralized setting, λ is chosen to maximise the profit function in equation (4.5). This gives the **first best** solutions for the resource consumption rate (λ_0) and the profit (π_0). These first best solutions form the benchmark for assessing the decentralized setting. Equation (4.5) involves the mth moment of the probability density function (PDF) of the completion time. The PDF value $f_\lambda(T)$ of the uniform, gamma, beta, and Weibull distributions are as follows

(Evans et al. 1993)

$$f_\lambda(T) = \begin{cases} \frac{1}{\theta} & \text{for uniform distributed time with } 0 \leq T \leq \theta \\ \frac{T^{w-1}e^{-\frac{T}{\theta}}}{\Gamma(w)\theta^w} & \text{for gamma distributed time with scale parameter } \theta \text{ and shape parameter w} \\ \frac{T^{u-1}(\theta-T)^{v-1}}{\theta^{u+v-1}B(u,v)} & \text{for beta distributed time with } 0 < T \leq \theta \text{ and u \& v as shape parameters} \\ \frac{sT^{s-1}}{\theta^s}e^{-(T/\theta)^s} & \text{for Weibull distributed time with scale parameter } \theta \text{ and shape parameter s} \end{cases} \tag{4.15}$$

For these PDF s, the expected value of the mth moment E(T^m) can be calculated in terms of the scale parameter θ of the distributions as $E(T^m) = \int\limits_0^\infty T^m f_\lambda(T)dT$ (Evans et al. 1993). Using the values of $f_\lambda(T)$

$$E(T^m) = \begin{cases} \left\{\frac{\theta^m}{(m+1)}\right\} & \text{For uniform distributed time} \\ \theta^m \prod\limits_{i=1}^{m}(w+i-1) & \text{For gamma distributed time} \\ \theta^m \prod\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right) & \text{For beta distributed time} \\ \theta^m\left[\Gamma\left(1+\frac{m}{s}\right)\right] & \text{For Weibull distributed time} \end{cases} \tag{4.16}$$

Using the values of the first moment of each distribution i.e μ (Evans et al. 1993) and the observation from equation (4.1), the expected value of the mth moment can be expressed in terms of λ as below

$$E(T^m) = \begin{cases} \left\{\frac{(2\mu_1)^m}{(m+1)\lambda^{mA}}\right\} & \text{For uniform distributed time} \\ \frac{\mu_1^m}{w^m\lambda^{mA}} \prod\limits_{i=1}^{m}(w+i-1) & \text{For gamma distributed time} \\ \frac{\mu_1^m}{\left(\frac{u}{u+v}\right)^m\lambda^{mA}} \prod\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right) & \text{For beta distributed time} \\ \frac{\mu_1^m}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA}}\left[\Gamma\left(1+\frac{m}{s}\right)\right] & \text{For Weibull distributed time} \end{cases} \tag{4.17}$$

Proof. Using the value of PDF for gamma distributed function from the equation 4.15, the expected value of m^{th} moment for gamma distributed time becomes

$$\begin{aligned}
E(T^m) &= \left(\frac{\theta^m \Gamma(m+w)}{\Gamma(w)}\right) \int\limits_0^\infty \frac{T^{m+w-1} e^{-\frac{T}{\theta}}}{\Gamma(m+w)\theta^{m+w}} dT \\
&= \frac{\theta^m \Gamma(m+w)}{\Gamma(w)} \\
&= \frac{\theta^m (m+w-1)!}{(w-1)!} \\
&= \theta^m \prod_{i=1}^{m} (w+i-1) \\
&= \frac{\mu_1^m}{w^m \lambda^{mA}} \prod_{i=1}^{m} (w+i-1) \\
&\text{[From the equation 4.1, } w\theta = \frac{\mu_1}{\lambda^A}]
\end{aligned}$$

For the beta distributed time,

$$\begin{aligned}
E(T^m) &= \left(\frac{\theta^m B(u+m,v)}{B(u+v)}\right) \int\limits_0^\infty \frac{T^{m+u-1}(\theta-T)^{v-1}}{\theta^{u+m+v-1} B(u+m,v)} dT \\
&= \frac{\theta^m B(u+m,v)}{B(u,v)} \\
&= \theta^m \frac{\Gamma(u+m)\Gamma(v)}{\Gamma(u+m+v)} \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} \\
&= \theta^m \prod_{i=1}^{m} \left(\frac{u+i-1}{u+v+i-1}\right) \\
&= \frac{\mu_1^m}{\left(\frac{u}{u+v}\right)^m \lambda^{mA}} \prod_{i=1}^{m} \left(\frac{u+i-1}{u+v+i-1}\right) \\
&\text{[From the equation 4.1, } \frac{\theta u}{u+v} = \frac{\mu_1}{\lambda^A}]
\end{aligned}$$

For the Weibull distributed time, it is assumed that $\left(\frac{T}{\theta}\right)^s = x$. By taking derivative both side, it can also be shown that $sT^{s-1}dT = \theta^s dx$. It can also be shown that $T^m = \theta^m x^{\frac{m}{s}}$. Thus, using these values in the expected value of the m$^{\text{th}}$ moment

$$E(T^m) = \int_0^\infty T^m \left\{ \frac{sT^{s-1}}{\theta^s} e^{-(T/\theta)^s} dT \right\}$$
$$= \int_0^\infty \theta^m x^{\frac{m}{s}} e^{-x} dx$$

By definition $\int_0^\infty x^{(1+\frac{m}{s})-1} e^{-x} dx = \Gamma(1+\frac{m}{s})$. Hence, $E(T^m) = \theta^m \Gamma(1+\frac{m}{s})$. $\square$

Substituting the values from equation (4.17) into the equation (4.5), the following expression for the centralized profit are derived

$$\pi = \begin{cases} q_0 - (q_0\psi)\left[\frac{(2\mu_1)^m}{(m+1)\lambda^{mA}}\right] - k\mu_1\lambda^N - C_o & \text{for uniform distributed time} \\ q_0 - q_0\psi\left[\frac{\mu_1^m}{w^m\lambda^{mA}}\prod_{i=1}^{m}(w+i-1)\right] - k\lambda^N\mu_1 - C_o & \text{for gamma distributed time} \\ q_0 - (q_0\psi)\left[\frac{\mu_1^m}{\left(\frac{u}{u+v}\right)^m\lambda^{mA}}\right]\left[\prod_{i=1}^{m}\frac{(u+i-1)}{(u+v+i-1)}\right] - k\lambda^N\mu_1 - C_o & \text{for beta distributed time} \\ q_0 - (q_0\psi)\left(\frac{\mu_1^m}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA}}\right)\left[\Gamma(1+\frac{m}{s})\right] - k\lambda^N\mu_1 - C_o & \text{for Weibull distributed time} \end{cases} \tag{4.18}$$

[where $N = n - A$]

The optimal value λ_0 of the resource consumption rate can be found by differentiating the above equation (4.18) and setting the results equal to zero:

$$\lambda_0 = \begin{cases} \left[\frac{2^m mAq_0\psi\mu_1^{m-1}}{kN(m+1)}\right]^{\frac{1}{mA+N}} & \text{for uniform distributed time} \\ \left[\frac{mAq_0\psi\mu_1^{m-1}\prod_{i=1}^{m}(w+i-1)}{kNw^m}\right]^{\frac{1}{mA+N}} & \text{for gamma distributed time} \\ \left[\frac{mAq_0\psi\mu_1^{m-1}}{kN\left(\frac{u}{u+v}\right)^m}\left\{\prod_{i=1}^{m}\frac{(u+i-1)}{(u+v+i-1)}\right\}\right]^{\frac{1}{mA+N}} & \text{for beta distributed time} \\ \left[\frac{mAq_0\psi\mu_1^{m-1}\left\{\Gamma(1+\frac{m}{s})\right\}}{kN\left\{\Gamma(1+\frac{1}{s}\right\}^m}\right]^{\frac{1}{mA+N}} & \text{for Weibull distributed time} \end{cases} \tag{4.19}$$

Substituting the values of λ_0 from the above equation (4.19) into equation (4.18), gives the first best solution of the profit π_0. In this section, the project duration is considered as short to medium. Thus, the impact of completion time is not very high on cash-flows. However, to account for the impact of time on cash-flows, the project value has been considered as a decreasing function of time as mentioned in equation (4.3). In the next few subsection, models are prepared with these considerations for : uniform, gamma, beta, and Weibull distributed time function T in a decentralized setting.

4.3.2 Decentralized Setting with Time Based Contracts

In the decentralized settings, the contractor would select a resource consumption rate (λ) that maximizes his profit in equation (4.7). The project manager cannot verify the selected resource consumption rate of the contractor (λ). As mentioned in section 4.1, it may be the case that $\lambda \neq \lambda_0$. To avoid this misalignment, the project manager needs to offer a contract P(T,C) to the contractor that satisfies the incentive compatibility and individual rationality constraints explained in section 4.1.

For a time-based contract, P(T,C) = g-hT, the optimal conditions for the contract parameters are derived in the following proposition

Proposition 1. *The parameters h* and g* of an optimally coordinated time-based contract P(T,C)=g-hT satisfy the following*

$$h^* = \begin{cases} \frac{kN}{A}\left[\frac{2^m mAq_0\psi\mu_1^{m-1}}{kN(m+1)}\right]^{\frac{N+A}{mA+N}} & \textit{[T is uniform between 0 and } \theta\textit{]} \\ \frac{kN}{A}\left[\frac{mAq_0\psi\mu_1^{m-1}\prod_{i=1}^{m}(w+i-1)}{kNw^m}\right]^{\frac{N+A}{mA+N}} & \textit{[T is gamma distributed with parameters (w, } \theta\textit{)]} \\ \frac{kN}{A}\left[\frac{mAq_0\psi\mu_1^{m-1}}{kN\left(\frac{u}{u+v}\right)^m}\left\{\prod_{i=1}^{m}\frac{(u+i-1)}{(u+v+i-1)}\right\}\right]^{\frac{N+A}{mA+N}} & \textit{[T is beta with parameters (u,v)]} \\ \frac{kN}{A}\left[\frac{mAq_0\psi\mu_1^{m-1}\left\{\Gamma(1+\frac{m}{s})\right\}}{kN\left\{\Gamma(1+\frac{1}{s})\right\}^m}\right]^{\frac{N+A}{mA+N}} & \textit{[T is Weibull distributed with (s, } \theta\textit{)]} \end{cases} \tag{4.20}$$

$$g* = \frac{m(N+A)}{mA+N}q_0 - \left[\frac{m(N+A)}{mA+N}\pi_0 - \pi_{out}\right] - \frac{m(N+A)}{mA+N}C_o \tag{4.21}$$

[For any continuous distributed time function]

Proof. The contract offered by the project manager to the contractor takes a linear form of P(T,C) = g- hT. The profit functions of the members follow the equations (4.6) and (4.7) .

The values of E(T) are same as mean value (μ) in equation (4.1). This gives us the following

$$\pi_{co} = g - h\left(\frac{\mu_1}{\lambda^A}\right) - k\lambda^N \mu_1 \qquad \text{[Where N = n-A]} \tag{4.22}$$

Now differentiating this π_{co} with respect to λ, setting that equal to zero and finally rearranging the terms, the value of λ is derived in terms of h, namely $\lambda^{N+A} = \frac{hA}{kN}$. This value of resource consumption rate (λ) would maximize the contractor's profit in the decentralized setting. The project manager envisages this by backward induction method. Thus, she would require to offer a contract that makes this resource consumption rate equal to the first best solution (λ_0). Hence, equating this λ with the λ_0 (equation 4.19), the optimal conditions for h for uniform, gamma, beta, and Weibull distributed project completion time are derived in conditions in equation (4.20).

The individual rationality constraint requires the contract to ensure a minimum profit (π_{out}) for the contractor. This is the profit that can be earned by the contractor in the event this contract not materialized. The optimal values of h from equation (4.20) are used in the profit equations of the contractor (equation 4.22). Now using the value of λ_0 for each distribution, the contractor's profit function can easily be shown as $\pi_{co} = g - k\lambda_0^N \mu_1 \left[\frac{N}{A} + 1\right]$. This equation is bounded by a value at least equal to the π_{out}. That is

$$g - k\lambda_0^N \mu_1 \left[\frac{N}{A} + 1\right] \geq \pi_{out} \tag{4.23}$$

For each distribution, from the first best value of resource consumption rate (λ_0) from equations (4.19), the ψq_0 values are rearranged as a function of λ_0. Then, these rearranged values are used to replace of ψq_0 in the equations of the first best profit conditions in equation (4.18) for respective distributions. From these, it can be easily shown that $k\lambda_0^N \mu_1 = (q_0 - \pi_0 - C_o)\left(\frac{mA}{mA+N}\right)$. Using this observation in equation (4.23), the coordinating condition for g are derived as

$$g \geq \frac{m(N+A)}{mA+N} q_0 - \left[\frac{m(N+A)}{mA+N}\pi_0 - \pi_{out}\right] - \frac{m(N+A)}{mA+N} C_o \tag{4.24}$$

[For any continuous distributed time function]

This coordinating condition above should also optimize the profit of the project manager mentioned in equation (4.6). The h value is bound by the equal sign in the equation. However, for any value of g at least equal to the right-hand side of equation (4.24), the coordinating conditions are satisfied. Thus, multiple values of g can coordinate the supply chain. In equation (4.6), any increase in value of g would reduce the profit of the project manager. Thus, to optimize the profit of the project manager, the g value has to be restricted to a minimum value that coordinates the supply chain. From this requirement, the optimal condition g* in equation (4.21) is derived. □

4.3.3 Decentralized Setting with Fixed Price Contracts

For a fixed price contract, the offered contract becomes P(T,C) = f. This value of P(T,C) is used in the profit functions for the project manager (4.6) and the contractor (4.7). From the above equation of the contractor's profit, it can be easily shown that λ = 0 for the contractor to maximize his profit i.e. the completion time would be very large. Hence, the project manager's profit would be negative. Using this observation, the following is proposed

Proposition 2. *A fixed price contract P(T,C) = f, fails to coordinate the supply chain when* $q = q_0(1 - \psi T^m)$ *and the project completion time T follows any form of continuous distribution.*

4.4 Model with exponential discounting

In the last section, the models of supply chain coordination are derived for the short term projects with a polynomial reduction of the project-reward. This section extends the models of coordination to long term projects with and without this polynomial reduction i.e. in both the cases, exponential discounting is used to take into account the time value of money over the project term. This section used the basic model proposed by (Bayiz & Corbett 2005) and Kwon et al. (2010) for the extension. As before, the models are extended to other continuous distributions of completion times that are commonly used for project completion times: uniform, gamma, beta, and Weibull distributions. As mentioned in section 4.2.2, two cases are considered: when delay causes irrecoverable revenue loss and when it does not.

4.4.1 Centralized Setting

The first best solutions of the centralized supply chain are taken as the benchmark solutions. The profit function follows from equation (4.12). From this equation, the first best solutions for profit (π_0) and resource consumption rate (λ_0) can be derived. Equation (4.12) contains the term $E[e^{-\alpha T}]$. For the uniform, gamma, beta, and Weibull distributed time, these are (Evans et al. 1993)

$$
\begin{aligned}
E\{e^{-\alpha T}\} &= \int\limits_0^\infty e^{-\alpha T} f_\lambda(T)dT \\
&= \begin{cases}
\left(\frac{1-e^{-\alpha\theta}}{\alpha\theta}\right) & \text{for uniform distributed time} \\
\left(\frac{1}{1+\alpha\theta}\right)^w & \text{for gamma distributed time} \\
1+\sum\limits_{m=1}^{\infty}\theta^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left[\frac{(-\alpha)^m}{m!}\right] & \text{for beta distributed time} \\
1+\sum\limits_{m=1}^{\infty}\theta^m\left[\Gamma(1+\frac{m}{s})\right]\left[\frac{(-\alpha)^m}{m!}\right] & \text{for Weibull distributed time}
\end{cases}
\end{aligned}
\tag{4.25}
$$

Projects with recoverable product life upon completion with delay

Using this value of discounted time in equation (4.12), the total supply chain profits for recoverable product life are (The values of θ are replaced in terms of λ from the first moment of each distribution. Please see the proof below.)

$$
\pi = \begin{cases}
q_0\lambda^A\left[\frac{1-e^{-\alpha\theta}}{2\mu_1\alpha}\right]-\left[\frac{k\lambda^n}{\alpha}\right]+\left[\left(\frac{k\lambda^{n+A}}{2\mu_1\alpha^2}\right)(1-e^{-\alpha\theta})\right]-C_o \\
\text{for uniform distributed time} \\
q_0\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w-\left(\frac{K\lambda^n}{\alpha}\right)\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w\right]-C_o \\
\text{for gamma distributed time} \\
\left[q_0+\frac{k\lambda^n}{\alpha}\right]\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]-\frac{k\lambda^n}{\alpha}-C_o \\
\text{for beta distributed time} \\
\left[q_0+\frac{k\lambda^n}{\alpha}\right]\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma\left(1+\frac{1}{s}\right)}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]-\frac{k\lambda^n}{\alpha}-C_o \\
\text{for Weibull distributed time}
\end{cases}
\tag{4.26}
$$

Proof. The mean values for the distributions are as below

$$\mu = \begin{cases} \frac{\theta}{2} & \text{for uniform distributed time with } 0 < T \leq \theta \\ w\theta & \text{for gamma distributed time with shape parameter w and scale parameter } \theta \\ \frac{\theta u}{u+v} & \text{for beta distributed time with shape parameters u,v and } 0 < T \leq \theta \\ \theta\,\Gamma\left(1+\frac{1}{s}\right) & \text{for beta distributed time with shape parameter s and scale parameter } \theta \end{cases}$$

The mean value (μ) is replaced as a function of resource consumption rate (λ) in equation (4.1). From there, the θ is expressed as a function of λ and replaced in the values of E$\{$e$^{-\alpha T}\}$ in the equation (4.25). and finally in the equation (4.26) ☐

Differentiating equation (4.26), the first order condition for λ , **with uniform distributed time** can be derived as

$$\left(\frac{q_0 A}{\lambda}\right)\left[\frac{\lambda^A(1-e^{-\alpha\theta})}{2\mu_1\alpha} - e^{-\alpha\theta}\right] - \left[\frac{kn\lambda^{n-1}}{\alpha} - \frac{(n+A)(1-e^{-\alpha\theta})(k\lambda^{n+A-1})}{2\mu_1\alpha^2} + \frac{kAe^{-\alpha\theta}\lambda^{n-1}}{\alpha}\right] = 0 \tag{4.27}$$

Again differentiating the above equation (4.26), the first order condition for λ with **a gamma distributed time** can be derived as below

$$\left[\frac{q_0 A w^{w+1}\alpha\mu_1\lambda^{Aw-1}}{(w\lambda^A+\alpha\mu_1)^{w+1}}\right] + \left[\frac{K\lambda^{n-1}\{(nw\lambda^A+\alpha n\mu_1+A\alpha\mu_1 w)(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1})^w - nw\lambda^A - \alpha n\mu_1\}}{\alpha(w\lambda^A+\alpha\mu_1)}\right] = 0 \tag{4.28}$$

Differentiating (4.26) with respect to λ for **a beta distributed time**. the first order condition can be derived as

$$\left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left[\sum_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right] + \left[q_0+\frac{k\lambda^n}{\alpha}\right]\left[\sum_{m=1}^{\infty}\left\{\frac{(-mA)\mu_1(u+v)^m}{u^m\lambda^{mA+1}}\right\}\left(\prod_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right] = 0 \tag{4.29}$$

Differentiating (4.26) with respect to λ for a **Weibull distributed time**. the first order condition can be derived as

$$\left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left[\sum_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^{A}\Gamma\left(1+\frac{1}{s}\right)}\right\}^{m}\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\left\{\frac{(-\alpha)^{m}}{m!}\right\}\right]$$
$$+\left[q_0+\frac{k\lambda^{n}}{\alpha}\right]\left[\sum_{m=1}^{\infty}\left\{\frac{(-mA)\mu_1^{m}}{\left(\Gamma(1+\frac{1}{s})\right)^{m}\lambda^{mA+1}}\right\}\left\{\Gamma\left(1+\frac{m}{s}\right)\right\}\left\{\frac{(-\alpha)^{m}}{m!}\right\}\right]=0 \qquad (4.30)$$

Solving, the value of λ from the equations (4.27), (4.28), (4.29) and (4.30) gives the value of the first-best optimal value of resource consumption rate λ_0 for uniform, gamma, beta, and Weibull distributed time in this case. Using these value of λ_0 in the equation (4.26) gives the first best profit function (π_0) for the supply chain under consideration

It can be seen that the first order conditions for beta and the Weibull distributed time do not give a closed form solution for λ_0. Hence, the first best profit function for the beta and Weibull distributed time are not a closed form solution

Projects with irrecoverable product life upon completion with delay

Using the value of discounted time from equation (4.25) in the equation (4.12), the profit function can be derived for the projects with outputs whose product life can not be extended upon completion of the project. (The values of θ are replaced in terms of λ from the first moment of each distribution as shown earlier)

$$\pi = \begin{cases} q_0\left[1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}\lambda^A\{\frac{1-e^{-\alpha\theta}}{2\mu_1\alpha}\}\right]-(\frac{k\lambda^n}{\alpha})+\left[(\frac{k\lambda^{n+A}}{2\mu_1\alpha^2})(1-e^{-\alpha\theta})\right]-C_o \\ \text{for uniform distributed time} \\ q_0\left[1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w\right]-\left(\frac{K\lambda^n}{\alpha}\right)\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w\right]-C_o \\ \text{for gamma distributed time} \\ \left[\frac{q_0\psi}{\alpha}+\frac{k\lambda^n}{\alpha}\right]\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right] \\ +q_0\left[1-\frac{\psi}{\alpha}\right]-\frac{k\lambda^n}{\alpha}-C_o \\ \text{for beta distributed time} \\ \left[\frac{q_0\psi}{\alpha}+\frac{k\lambda^n}{\alpha}\right]\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\gamma\left(1+\frac{1}{s}\right)}\right\}^m\{\Gamma(1+\frac{m}{s})\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right] \\ +q_0\left[1-\frac{\psi}{\alpha}\right]-\frac{k\lambda^n}{\alpha}-C_o \\ \text{for Weibull distributed time} \end{cases} \tag{4.31}$$

Differentiating equation (4.31), the first order condition for λ , **with uniform distributed time**, can be derived as

$$\begin{aligned} &\left(\frac{q_0\psi A}{\lambda\alpha}\right)\left[\frac{(1-e^{-\alpha\theta})\lambda^A}{2\mu_1\alpha}-e^{-\alpha\theta}\right] \\ &-\left[\frac{kn\lambda^{n-1}}{\alpha}-\frac{(n+A)(1-e^{-\alpha\theta})(k\lambda^{n+A-1})}{2\mu_1\alpha^2}+\frac{kAe^{-\alpha\theta}\lambda^{n-1}}{\alpha}\right]=0 \end{aligned} \tag{4.32}$$

Similarly differentiating the profit equation for gamma distributed time in equation (4.31), we can derive the first order condition for λ , **with gamma distributed time**, as

$$\begin{aligned} &\left[\frac{q_0\psi Aw^{w+1}\mu_1\lambda^{Aw-1}}{(w\lambda^A+\alpha\mu_1)^{w+1}}\right] \\ &+\left[\frac{K\lambda^{n-1}\{(nw\lambda^A+\alpha n\mu_1+A\alpha\mu_1 w)(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1})^w-nw\lambda^A-\alpha n\mu_1\}}{\alpha(w\lambda^A+\alpha\mu_1)}\right]=0 \end{aligned} \tag{4.33}$$

Again solving the equations (4.32) and (4.33) gives the first best optimal conditions for the resource consumption rate (λ_0) for the uniform and the gamma distributed time. Using the value of λ_0 in the equation (4.31), the first best profit functions can be derived. Similar to the case of projects with recoverable operational product life, beta and gamma distributed project completion times do not give a closed form solution in this case of projects with irrecoverable operational product life.

4.4.2 Decentralized Setting with Time Based Contracts

In the decentralized setting, the project manager offers a contract P(T,C)= g-hT to the contractor. The equations (4.13) and (4.14) give the profit functions for the project manager and the contractor. The contractor would select a λ that maximizes his profit in equation (4.14). In a similar way to section 4.3.2, differentiating equation (4.14) and setting the results equal to zero, gives the conditions for λ. Equating this value with the first best solution derived from the centralized setting, the optimal conditions for g and h can be derived. Using these conditions along with the individual rationality conditions, the optimal conditions can be derived. For the uniform and gamma distributed time with probability density function (PDF) in equation (4.15), the following lemmas 1 and 2, are derived

Lemma 1. *With the monetary reward discounted with (α), a time-based contract with P(T,C)=g-hT, (Where T is **uniformly** distributed between time 0 and θ) can coordinate the decentralized project supply chain when the contract parameters g and h satisfy*

$$g = \begin{cases} \left(\frac{h}{\alpha}\right)\left[\frac{\lambda^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-2\mu_1\alpha^2\theta e^{-\alpha\theta}}{(1-e^{-\alpha\theta})\lambda^A-2\mu_1\alpha e^{-\alpha\theta}}\right] + q_0 & \textit{for recoverable product life} \\ \left(\frac{h}{\alpha}\right)\left[\frac{\lambda^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-2\mu_1\alpha^2\theta e^{-\alpha\theta}}{(1-e^{-\alpha\theta})\lambda^A-2\mu_1\alpha e^{-\alpha\theta}}\right] + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for irrecoverable product life} \end{cases} \tag{4.34}$$

and

$$h \leq \begin{cases} \left(\frac{\alpha}{\lambda_0^A}\right)(\pi_0 - \pi_{out} + C_o)\left[\frac{\{(1-e^{-\alpha\theta})\lambda_0^A\}-(2\mu_1\alpha e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1-e^{-\alpha\theta})}\right] & \textit{For recoverable products} \\ \left(\frac{\alpha}{\lambda_0^A}\right)\left[\pi_0 - \pi_{out} + C_o + q_0(\frac{\psi}{\alpha}-1)\right]\left[\frac{\{(1-e^{-\alpha\theta})\lambda_0^A\}-(2\mu_1\alpha e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1+e^{-\alpha\theta})}\right] & \textit{For irrecoverable products} \end{cases} \tag{4.35}$$

Proof. The profit function for of the contractor for a time based contract P(T,C) = g - hT, where T is uniform distributed (PDF mentioned in equation 4.15) as mentioned earlier is

$$\pi_{co} = \int_0^{\infty} (g - hT)e^{-\alpha T} f_\lambda(T)dT - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

(From equation 4.14)

$$= \left(\frac{\lambda^A g}{2\mu_1 \alpha}\right)(1 - e^{-\alpha\theta}) - \left(\frac{\lambda^A h}{2\mu_1 \alpha^2}\right)[1 - e^{-\alpha\theta}(\alpha\theta + 1)] - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}] \quad (4.36)$$

Replacing the value of $E\{e^{-\alpha T}\}$ from the equation (4.25), then differentiating the equation (4.36) with respect to λ and then setting that expression equal to zero, the following condition is derived

$$\left(\frac{gA}{2\mu_1 \alpha \lambda}\right)\left[\{(1 - e^{-\alpha\theta}\lambda^A\} - (2\mu_1 \alpha e^{-\alpha\theta})\right] - \left(\frac{Ah}{2\mu_1 \alpha^2 \lambda}\right)\left[\lambda^A\{1 - e^{-\alpha\theta}(\alpha\theta + 1)\} - 2\mu_1 \alpha^2 \theta e^{-\alpha\theta}\right]$$
$$- \left[\frac{kn\lambda^{n-1}}{\alpha} - \frac{(n + A)(1 - e^{-\alpha\theta})(k\lambda^{n+A-1})}{2\mu_1 \alpha^2} + \frac{kAe^{-\alpha\theta}\lambda^{n-1}}{\alpha}\right] = 0 \quad (4.37)$$

The above equation gives the value of λ that would maximize the profit of the contractor. In this expression, the value of $\frac{dE(C)}{d\lambda}$ is replaced from the first order condition of λ_0 from equations (4.27) and (4.32) for the cases of projects with recoverable product life and irrecoverable product life respectively. Rearranging the terms, the condition for g in equation (4.34) in lemma 1 can be derived.

By the individual rationality constraint, the present contract should ensure a minimum profit of π_{out} for the contractor. Thus,

$$\left(\frac{\lambda^A g}{2\mu_1 \alpha}\right)(1 - e^{-\alpha\theta}) - \left(\frac{\lambda^A h}{2\mu_1 \alpha^2}\right)[1 - e^{-\alpha\theta}(\alpha\theta + 1)] - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}] \geq \pi_{out}$$

The values of g from the equation (4.34) is used in the above condition.

$$\begin{cases} \left[\left(\frac{h}{\alpha}\right)\left[\frac{\lambda^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-2\mu_1\alpha^2\theta e^{-\alpha\theta}}{(1-e^{-\alpha\theta})\lambda^A-2\mu_1\alpha e^{-\alpha\theta}}\right]+q_0\right]\left(\frac{\lambda^A}{2\mu_1\alpha}\right)(1-e^{-\alpha\theta}) & \\ -\left(\frac{\lambda^A h}{2\mu_1\alpha^2}\right)[1-e^{-\alpha\theta}(\alpha\theta+1)]-\frac{k\lambda^n}{\alpha}[1-E\{e^{-\alpha T}\}]\geq\pi_{out} & \text{For recoverable products} \\ & \\ \left[\left(\frac{h}{\alpha}\right)\left[\frac{\lambda^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-2\mu_1\alpha^2\theta e^{-\alpha\theta}}{(1-e^{-\alpha\theta})\lambda^A-2\mu_1\alpha e^{-\alpha\theta}}\right]+q_0\left(\frac{\psi}{\alpha}\right)\right]\left(\frac{\lambda^A}{2\mu_1\alpha}\right)(1-e^{-\alpha\theta}) & \\ -\left(\frac{\lambda^A h}{2\mu_1\alpha^2}\right)[1-e^{-\alpha\theta}(\alpha\theta+1)]-\frac{k\lambda^n}{\alpha}[1-E\{e^{-\alpha T}\}]\geq\pi_{out} & \text{For irrecoverable products} \end{cases}$$

The value of E(C) i.e $\frac{k\lambda^n}{\alpha}[1-E\{e^{-\alpha T}\}]$ is replaced from the profit equations (4.26) and (4.31). Now rearranging the terms, the coordinating conditions for h in equation (4.35) in lemma 1. □

Lemma 2. *A time-based contract with P(T,C) = g - hT, (where T is* ***gamma*** *distributed with(w,θ)) can coordinate the decentralized project supply chain when the monetary reward is discounted exponentially with (α), and the contract parameters satisfy*

$$g=\begin{cases} q_0+h\left[\frac{w(\mu_1\alpha-\lambda^A)}{\alpha(w\lambda^A+\alpha\mu_1)}\right] & \textit{For recoverable product life} \\ & \\ q_0(\frac{\psi}{\alpha})+h\left[\frac{w(\mu_1\alpha-\lambda^A)}{\alpha(w\lambda^A+\alpha\mu_1)}\right] & \textit{For irrecoverable product life} \end{cases} \tag{4.38}$$

$$h\leq\begin{cases} (\pi_0-\pi_{out}+C_o)\left\{\frac{\alpha(w\lambda^A+\alpha\mu_1)^{w+1}}{(w^{w+1})(\lambda_0^{Aw+A})}\right\} & \textit{For recoverable product life} \\ & \\ [\pi_0-\pi_{out}+C_o+q_0(\frac{\psi}{\alpha}-1)]\left\{\frac{\alpha(w\lambda^A+\alpha\mu_1)^{w+1}}{(w^{w+1})(\lambda_0^{Aw+A})}\right\} & \textit{For irrecoverable product life} \end{cases} \tag{4.39}$$

Proof. The profit function for of the contractor for a time based contract P(T,C) = g - hT, where T is uniform distributed (PDF mentioned in equation 4.15) as mentioned earlier is

$$\pi_{co} = \int_0^\infty (g - hT)e^{-\alpha T} f_\lambda(T)dT - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}] \qquad \text{(From equation 4.14)}$$

$$= gE\{e^{-\alpha T}\} - h\int_0^\infty Te^{-\alpha T} f_\lambda(T) - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

$$= gE\{e^{-\alpha T}\} - h\int_0^\infty Te^{-\alpha T} f_\lambda(T) - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

It is defined that $\frac{1}{\theta} = \phi$. Replacing this in $f_\lambda(T)$ in the equation (4.15) and using this in the equation above gives

$$\pi_{co} = gE\{e^{-\alpha T}\} - h\int_0^\infty Te^{-\alpha T}\left[\frac{\phi^w e^{-\phi T} T^{w-1}}{\Gamma(w)}\right]dT - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

$$= gE\{e^{-\alpha T}\} - h\left[\frac{\Gamma(w+1)}{\Gamma(w)}\right]\left[\frac{\phi^w}{(\alpha+\phi)^{w+1}}\right]\int_0^\infty \frac{(\alpha+\phi)^{w+1}e^{-(\alpha+\phi)T}T^{w+1-1}}{\Gamma(w+1)}dT$$

$$- \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

$$= gE\{e^{-\alpha T}\} - h\left[\frac{\Gamma(w+1)}{\Gamma(w)}\right]\left[\frac{\phi^w}{(\alpha+\phi)^{w+1}}\right]\int_0^\infty \frac{(\alpha+\phi)^{w+1}e^{-(\alpha+\phi)T}T^{w+1-1}}{\Gamma(w+1)}dT$$

$$- \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

By definition of the gamma distribution, $\int_0^\infty \frac{(\alpha+\phi)^{w+1}e^{-(\alpha+\phi)T}T^{w+1-1}}{\Gamma(w+1)}dT = 1$

$$\pi_{co} = gE\{e^{-\alpha T}\} - h\left[\frac{\Gamma(w+1)}{\Gamma(w)}\right]\left[\frac{\phi^w}{(\alpha+\phi)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

$$= gE\{e^{-\alpha T}\} - hw\left[\frac{\phi^w}{(\alpha+\phi)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}[1 - E\{e^{-\alpha T}\}]$$

Replacing the value of ϕ in terms of θ, then finally in terms of λ and using the value of

$E\{e^{-\alpha T}\}$ from the equation (4.25),

$$\pi_{co} = g\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w - h\left[\frac{\mu_1 w^{w+1}\lambda^{Aw}}{(\mu_1\alpha+w\lambda^A)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w\right] \tag{4.40}$$

Now differentiating the equation (4.40) with respect to λ and then setting that expression equal to zero, the following condition is derived

$$\begin{aligned}&\left[\frac{gA\mu_1\alpha w^{w+1}\lambda^{Aw-1}}{(w\lambda^A+\mu_1\alpha)^{w+1}}\right] - \left[\frac{Ahw^{w+2}\mu_1\lambda^{Aw-1}(\mu_1\alpha-\lambda^A)}{(w\lambda^A+\mu_1\alpha)^{w+2}}\right]\\&+\left[\frac{K\lambda^{n-1}\{(nw\lambda^A+\alpha\mu_1 n+A\alpha\mu_1 w)(\frac{\lambda^A}{\lambda^A+\alpha R})^w - nw\lambda^A-\mu_1\alpha n\}}{\alpha(\lambda^A+\alpha R)}\right]=0\end{aligned} \tag{4.41}$$

The above equation gives the value of λ that would maximize the profit of the contractor. In this expression, the value of $\frac{dE(C)}{d\lambda}$ is replaced from the first order condition of λ_0 from equations (4.28) and (4.33) for the cases of projects with recoverable product life and irrecoverable product life respectively. Rearranging the terms, the condition for g in equation (4.38) in lemma 2 can be derived.

Again, by the individual rationality constraint, the present contract should ensure a minimum profit of π_{out} for the contractor. Thus,

$$g\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w - h\left[\frac{\mu_1 w^{w+1}\lambda^{Aw}}{(\mu_1\alpha+w\lambda^A)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w\right] \geq \pi_{out}$$

The values of g from the equation (4.38) is used in the above condition.

$$\begin{cases}\left[q_0+h\left\{\frac{w(\mu_1\alpha-\lambda^A)}{\alpha(w\lambda^A+\alpha\mu_1)}\right\}\right]\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w - h\left[\frac{\mu_1 w^{w+1}\lambda^{Aw}}{(\mu_1\alpha+w\lambda^A)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w\right] \geq \pi_{out}\\ \text{For recoverable products}\\ \\ \left[q_0(\frac{\psi}{\alpha})+h\left\{\frac{w(\mu_1\alpha-\lambda^A)}{\alpha(w\lambda^A+\alpha\mu_1)}\right\}\right]\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w - h\left[\frac{\mu_1 w^{w+1}\lambda^{Aw}}{(\mu_1\alpha+w\lambda^A)^{w+1}}\right] - \frac{k\lambda^n}{\alpha}\left[1-\left(\frac{w\lambda^A}{w\lambda^A+\mu_1\alpha}\right)^w\right] \geq \pi_{out}\\ \text{For irrecoverable products}\end{cases}$$

The value of E(C) i.e $\frac{k\lambda^n}{\alpha}[1-E\{e^{-\alpha T}\}]$ is replaced from the profit equations (4.26) and (4.31). Now rearranging the terms, the coordinating conditions for h in equation (4.39) in lemma 2. □

The coordinating conditions in lemmas 1 and 2 should again optimize the profit of the project manager mentioned in equation (4.13). The g value has a specific bound with the equal sign and is a function of h in the equations (4.34) and (4.38) in lemma 1 and 2 respectively. However, multiple values of h can coordinate the supply chain. For any h value at most equal to the right-hand side of the equations (4.35) and (4.39) would coordinate the supply chain. In the profit equation of the project manager (equation 4.13), any reduction in the value of "h" would reduce the profit of the project manager and vice versa. Thus, the value of h is restricted to a maximum value that optimizes the profit of both the members, given the constraints in the equations (4.35) and (4.39). Thus, considering the optimizing conditions of all the members, the following are proposed:

Proposition 3. *The contract parameters g* and h*, of an optimal time-based contract P(T,C) = g - hT, (T is* ***uniformly*** *distributed between time 0 and θ) satisfy the following when cash-flows are exponentially discounted with (α)*

$$g* = \begin{cases} \left(\frac{\pi_0 - \pi_{out} + C_o}{\lambda_0^A}\right) \left[\frac{\lambda_0^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-(2\alpha^2\mu_1\theta e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1+e^{-\alpha\theta})}\right] + q_0 \\ \textit{For recoverable product life} \\ \left(\frac{\pi_0 - \pi_{out} + C_o + q_0(\frac{\psi}{\alpha}-1)}{\lambda_0^A}\right) \left[\frac{\lambda_0^A\{1-e^{-\alpha\theta}(\alpha\theta+1)\}-(2\alpha^2\mu_1\theta e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1+e^{-\alpha\theta})}\right] + q_0(\frac{\psi}{\alpha}) \\ \textit{For irrecoverable product life} \end{cases} \tag{4.42}$$

and

$$h^* = \begin{cases} \left(\frac{\alpha}{\lambda_0^A}\right) (\pi_0 - \pi_{out} + C_o) \left[\frac{\{(1-e^{-\alpha\theta})\lambda_0^A\}-(2\mu_1\alpha e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1-e^{-\alpha\theta})}\right] \\ \textit{For recoverable products} \\ \left(\frac{\alpha}{\lambda_0^A}\right) \left[\pi_0 - \pi_{out} + C_o + q_0(\frac{\psi}{\alpha} - 1)\right] \left[\frac{\{(1-e^{-\alpha\theta})\lambda_0^A\}-(2\mu_1\alpha e^{-\alpha\theta})}{e^{-\alpha\theta}(\alpha\theta-1-e^{-\alpha\theta})}\right] \\ \textit{For irrecoverable products} \end{cases} \tag{4.43}$$

Proof. As mentioned in section 4.4.2, any h value less than or equal to the right hand side of the equation (4.35) would entice the contractor to participate in the game. However, any reduction in h value means a reduction in the profit for the project manager and thus the results will not be Pareto optimal and not in equilibrium. Thus, consideration of the equilibrium solution would lead to the h* values in equation (4.43) in proposition 2. Using these values of h* in equation (4.34), the coordinating conditions for g in equation (4.42) in proposition 2 can be derived. □

Similar to the case of uniform distributed time, the following is proposed for the gamma distributed time

Proposition 4. *The contract parameters g* and h*, of an optimally coordinated time-based contract P(T,C)=g-hT, (T is* ***gamma*** *distributed with (w,θ)) satisfy the following when the cash-flows are exponentially discounted with* (α)

$$g* = \begin{cases} q_0 + (\pi_0 - \pi_{out} + C_o)\left[\frac{(w\lambda_0^A + \alpha\mu_1)^w(\alpha w - \lambda_0^A)}{(w^w)(\lambda_0^{Aw+A})}\right] \\ \textit{For recoverable product life} \\ q_0(\frac{\psi}{\alpha}) + [\pi_0 - \pi_{out} + C_o + q_0(\frac{\psi}{\alpha} - 1)]\left[\frac{(w\lambda_0^A + \alpha\mu_1)^w(\alpha w - \lambda_0^A)}{(w^w)(\lambda_0^{Aw+A})}\right] \\ \textit{For irrecoverable product life} \end{cases} \tag{4.44}$$

and

$$h^* = \begin{cases} (\pi_0 - \pi_{out} + C_o)\left\{\frac{\alpha(w\lambda^A + \alpha\mu_1)^{w+1}}{(w^{w+1})(\lambda_0^{Aw+A})}\right\} & \textit{For recoverable product life} \\ [\pi_0 - \pi_{out} + C_o + q_0(\frac{\psi}{\alpha} - 1)]\left\{\frac{\alpha(w\lambda^A + \alpha\mu_1)^{w+1}}{(w^{w+1})(\lambda_0^{Aw+A})}\right\} & \textit{For irrecoverable product life} \end{cases} \tag{4.45}$$

Proof. Following the same set of steps as shown in the case of uniform distribution, the coordinating conditions for g and h can be derived for gamma distributed time with PDF in equation (4.15) in a similar way to the uniform distributed time. □

The values of $E\{e^{-\alpha T}\}$ were shown in the section 4.4.1 to take a non-closed form for a beta and a Weibull distributed times. Based on this, the following is proposed

Proposition 5. *For a beta or Weibull distributed project completion time T with exponentially discounted cash-flows, the closed form of coordinating conditions are not available.*

4.4.3 Decentralized Setting with Fixed Price Contracts

A fixed price contract P(T,C) = f is used to derive the models when cash-flows are exponentially discounted. This value of P(T,C) is used in the equations (4.13) and (4.14). Differentiating the profit function of the contractor with respect to λ, and setting that equal to zero, the optimal value of λ is solved. Equating this value with the first best solution, the required coordinating condition is derived as $f \geq q$. If $f = q$, then the profit to be earned by the project manager would be zero. Thus, the following is proposed:

Proposition 6. If cash-flows are exponentially discounted and the project completion time is a decreasing function of resource consumption rate (λ), then a fixed price contract cannot coordinate the project supply chain under consideration. This also supports the findings of Kwon et al. (2010). This is applicable for project completion time with any form of continuous distributions.

4.5 Numerical Analysis and Results

In this section, the models proposed in the last few sections are illustrated with numerical values. The proposed models can be illustrated with the data from any project supply chain. However, in this research, the models are mostly illustrated to be fit in with the data from the construction sector. This is because of the enormous importance of the construction sector to the economy. As per the UK Contractor Group (2012), every £1 spent in the construction sector, generates £2.84 in output. Moreover, the construction sector value chain is worth 14% of UK's overall GDP (UKCG, 2012).

4.5.1 For polynomial discounted cash-flows

It is assumed that the project value follows the following equation $q(T) = 30 - 1.5T$ where ψ = 0.05 and q_0 = £ 30,000. In practice, the value of A in equation (4.1) would usually be less than 1 with a maximum value of 1, but for simplicity, it is assumed that A = 1 in this numerical analysis. The resource cost per unit time has been considered as $k\lambda^n$. Following Savaskan & Van Wassenhove (2006) and Swami & Shah (2013*b*), we assume that $n = 2$. k can be considered as the resource price per unit per unit time. Assuming an average wage of £10 per hour, with 20 hours. per week, the approximate value of k is assigned as £ 200 per week. In the existing literature by Kwon et al. (2010), the authors did not consider the costs incurred by the project manager which are independent of λ. According to Potts & Ankrah (2014), these costs can vary between 50-70 percent of the overall cost. The C_o value is considered as £ 15,000. The minimum profit to be earned by the contractor if the current contract fails, is assumed π_{out} = £1,800. The value of λ was derived as resource unit per week.

For polynomial discounting, if m=1, then the project reward is discounted linearly. From equations (4.18) and (4.19), we get the unique set of solutions for the first best profit and first best resource consumption rate for the uniform, gamma, beta and Weibull distributed completion time as $\pi_0 = q_0 - q_0.\psi\left[\frac{\mu_1}{\lambda_0^A}\right] - k\mu_1\lambda_0^N - C_o$ and $\lambda_0 = \left[\frac{Aq_0\psi}{kN}\right]^{\frac{1}{A+N}}$ respectively. The optimal values of h* and g* from equations (4.20) and (4.21) are: $h^* = q_0\psi$ and $g^* = q_0 - (\pi_0 - \pi_{out}) - C_o$. The profits are presented in figure 4.1 and in table D.1 (in appendix D). An increase in the value of h while keeping g constant reduces(increases) the profit of the contractor (project manager) and vice-versa. The total profit increases at first. Then, it reaches the maximum first best solution for profit value of 6.24 at g = 10.56 and h = 1.5. After that, it starts to decline. This can be explained by setting the value of h=0. When h attains a value of zero, the contract becomes a fixed price contract equivalent. It yields a value of selected resource consumption rate (λ) as zero and the profit functions become undefined (-∞). This fails to coordinate the supply chain. This supports the findings from the proposition 2. Thus, any positive entry of h above zero would increase the profit of both the members of the supply chain and eventually total profit at first. However, further increasing h reduced the contractor's profit and increased the project manager's profit. In the beginning, the increase in project manager's profit overcomes the decrease in contractor's profit. As a result, the total profit increased slowly until it reached the first best solution. After that, the reduction of contractor's profit was found to be substantial. As a result, the total profit starts to drop from

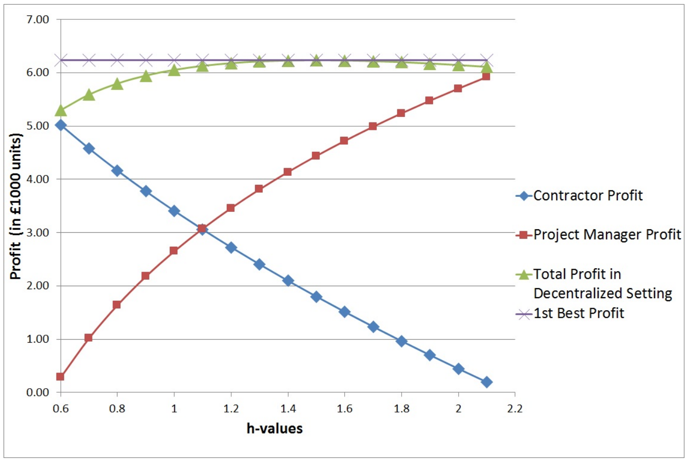


Figure 4.1: Profit values and Efficiency for polynomial discounted cash-flows with m=1

the maximum value with every positive increment of h.

On the contrary, keeping h constant at the optimal value of 1.5, changing the g values was found to have no impact on the total profit of the decentralized setting. However, this has an impact on the individual profit share of the members in the decentralized setting. Increasing g while keeping h constant increases (reduces) the profit of the contractor (project manager) and vice-versa. Thus, setting h constant at the optimal solution, multiple values of g can yield the total decentralized profit equal to the first best solution. However, if the members of the supply chain can earn a minimum profit outside the present contract, then there exists an individual rationality constraint. This makes some of the solutions to be invalid in the given context.

Furthermore, the values of the exponent m are changed. λ_0 was found to be increasing with the increase in increase in m. This caused a decrease in first best profit π_0. In fact, after m started attaining some higher values of 3 or 4 or above, the optimal π_0 values, started to become negative. On those, cases the proposed model would become invalid for the supply chain under consideration. The results are presented in table D.2 in Appendix D.

4.5.2 For exponential discounted cash-flows

For the numerical example, the effective value of project reward is assumed as q_0 = £4 million. α is the continuous discounting rate. According to the Govt. of UK., Cabinet Office (2015), the present discount rate in practice is 3.5%. This research assumed the value as 0.04. C_o is assumed as £2 million. It is also assumed that μ_1 is 10 years and π_{out} = £0.25 million.

Furthermore, the per hour wage for the work is assumed as £15. The worker will be employed for 150 hours each month of the year. This approximates to a value of k = 0.03 per year. n is assumed to be 2 as before (to be consistent with the previous literature) and again A is assumed as 1

For recoverable product life

In this case, it was assumed that the product life is recoverable upon completion of the projects. Thus, any project reward reduction due to anticipated revenue loss is not considered. The results for the profits are presented in figures 4.2 and 4.3.

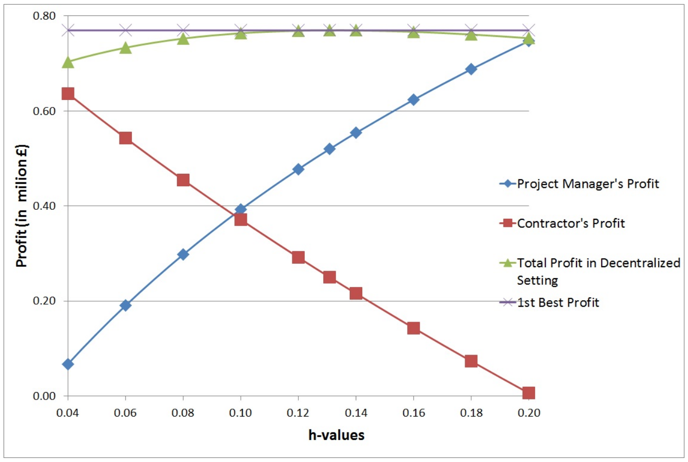


Figure 4.2: Profit values for Uniform Distribution with exponential cash-flows and time-based contracts

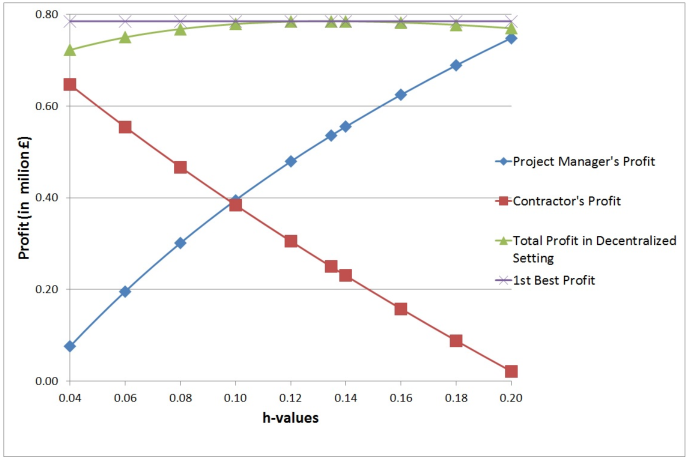


Figure 4.3: Profit values for Gamma Distribution with exponential cash-flows and time-based contracts

Once again in the decentralized setting, keeping g as constant at the optimal value (1.55 for uniform, and 1.54 for gamma), increasing h was found to be reducing (increasing) the profit of the contractor (project manager) and vice-versa. Again the total profit increased at first. Then, it reached the maximum first best solution of the profit value (0.77 for uniform and 0.78 for gamma) for the optimal values of h (0.13 for bothe the distributions). After that, these values started to decline.

Similar results are found as found in section 4.5.1 while keeping h constant at the optimal values and changing g. Changing g was not found to have any considerable impact on the overall profit.

For irrecoverable product life

Similar results were derived for this case as for the case with recoverable product life. The product life of the outcome of the project cannot be recovered if there is any delay in completion. Thus, a polynomial reduction of project reward was considered with the discounting factor $\psi = 0.05$ times the project value per unit time. The results are presented in the figures 4.4 and 4.5.

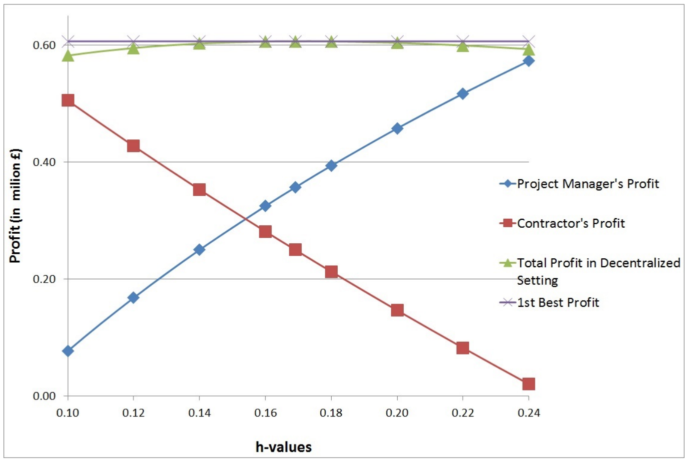


Figure 4.4: Profit values for uniform distributed time with exponential cash-flows and time-based contracts for irrecoverable product life

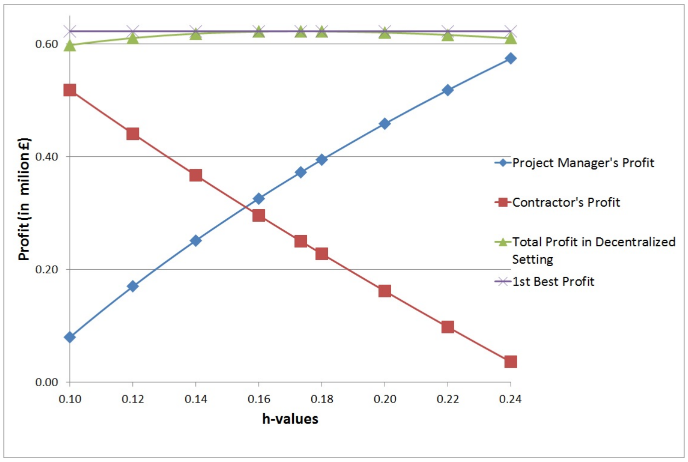


Figure 4.5: Profit values for gamma distributed time with exponential cash-flows and time-based contracts for irrecoverable product life

Once again in the decentralized setting, keeping g as constant at the optimal value (1.72 for uniform, and 1.71 for gamma), increasing h was found to be reducing (increasing) the profit of the contractor (project manager) and vice-versa. Again the total profit increased at first. Then, it reached the maximum first best solution of profit value (0.61 for uniform and 0.62 for gamma) for optimal values of h (0.17 for both the distributions). After that, these values started to decline.

Comparing the results of the two scenarios of the long-term projects in sections 4.5.2 and 4.5.2, there are similar results for both the distributions. Both the contract parameters have increased in the case of the polynomial reduction in comparison to the nonreduction. However, the first best profit has decreased in the case of the polynomial reduction of project reward in comparison to the case with no reduction of reward.

4.6 Chapter Summary

This chapter presented the contractual solution to the problems of non-coordination in a project supply chain with the help of Stackelberg leader-follower games in a take it or leave it situation. The project manager was considered as a leader and the contractor was considered as a follower. Starting from the models proposed by Bayiz & Corbett (2005) and Kwon et al. (2010) as a reference, this chapter proposed the models to cover distributions more commonly used for project completion times (Uniform, Gamma, Beta, and Weibull).

For short-term projects, the irrecoverable loss of revenue from delayed completion was considered as the motivating factor for the early completion of the projects. The proposed models with polynomial reduction of project reward with respect to time is an extension of the linear reduction proposed by Bayiz & Corbett (2005). The contractual conditions for g and h were found to be similar irrespective of the nature of the distribution. On the one hand, the condition for g depends on the π_0, π_{out}, C_o, m,n, and A. The π_0 value would change depending upon the nature of the distribution. On the other hand, the optimal conditions for h depend on k, m, N, A, and λ_0. The value of λ_0 would change depending on the type of distribution. Accordingly, the expressions for g and h would change depending on the nature of the distribution.

Similarities among the solutions were also found in some other forms. The uniform distributed T was found as a special case of beta distributed T with u=1 and v=1. This was also supported by the same optimal conditions for g* and h* by putting u=1 and v=1 and using other numerical values. Similarly if w=1, then the gamma distributed T would become exponential. Similarly, when s=1, the Weibull distribution becomes an exponential distribution.

Interestingly, for m = 1, i.e. for the case of linear reduction of project value with respect to time, the optimal coordinating conditions for g and h were found to be the same irrespective of any distribution. This was derived from the numerical analysis of the models. It was also found that with the increase in the value of m, the first best profit (π_0) decreases for any distribution of project completion time. This was due to the rapid increase in the resource consumption rate to avoid high penalty of delay. As a result, the coordination opportunities became restrictive. In fact, after the m value started attaining values such as 3 or 4, the first best profit became negative. The proposed models may mathematically ensure the contracts to achieve the same profit as the first best profit. However, in practice, this is never a solution due to the profits being negative.

The models for long-term projects were derived with polynomial reduction of project reward alongside the exponential discounting. The exponential discounting of the cash-flows were considered to take into account the time value of money. As an extension to the paper of (Kwon et al. 2010), this research considered two cases: one with the recoverable operational life upon project completion and the other with irrecoverable operational life upon project completion. In both these cases, a wider range of distribution of completion times (Uniform, Gamma, Beta, and Weibull) was considered. For the case of irrecoverable operational product life upon project completion, a reduction of project reward as a polynomial function of completion time was considered. In addition, all the cash-flows were exponentially discounted. However, for the case of recoverable operational product life, any reduction of project reward was not considered, but only exponential discounting of cash-flows to take into account the time value of money. The tractable optimal solutions for a uniform and gamma distributed time were found. However, the closed form of solutions was not found for beta and Weibull distributed.

The optimal coordinating conditions of the parameters g and h of a time-based contract were found to be different for the different distributions. However, using the values of w=1, μ_1, A=1 and $C_o = 0$, the optimal conditions for gamma distribution converts to the optimal conditions of an exponential distribution proposed in the original research of Kwon et al. (2010).

The fixed price contracts failed to coordinate (in any of the cases this research explored) the project supply chain under consideration in the Stackelberg game settings. This supports the findings of Kwon et al. (2010). However, the fixed price contracts are still very popular in practice due to their simplicity in application.

Chapter 5

Supply Chain Coordination using Project Contracts: With Bargaining Games

In chapter 4 of this thesis, the supply chain coordination models were presented with Stackelberg games. The two main features of these type of models were: the nature of the game as a take it or leave it situation and the first mover's advantage. The project manager was the first mover in those games. She offered a take it or leave it type of contract to the contractor. If the contractor disagreed with the offered contract, the game terminated. Moreover, the project manager had the first mover's advantage due to the bargaining power she had. This type of situation is very common in projects in practice such as construction projects. However, pieces of evidence of bargaining in project setting from practice were found in the study of Bajari et al. (2009). The authors mentioned about the negotiated contracts in the North California building construction sector region during 1995-2000. Most of the building contracts in Dubai were re-negotiated post economic downturn in 2008 (Bertenshaw 2012).

The application of bargaining models in supply chain management (with special reference to supply chain coordination) has received relatively less attention in comparison with other approaches of game theoretic models. As mentioned earlier, most empirical evidence indicates that the solutions of the bargaining process approaches the Nash's bargaining solution. This bargaining concepts have been used in the literature of Gan et al. (2011), Gjerdrum et al. (2002), He & Zhao (2012), He & Zhao (2012), Hezarkhani & Kubiak (2010), Huang & Li (2001), Li et al. (2009), Nagarajan & Bassok (2008), Yan (2011), and Ye & Xu (2010). Nash bargaining has been used for several supply chain issues such as optimization of inventory (Gjerdrum et al. 2002, Ye & Xu 2010), cooperative advertising (Huang & Li 2001), cost sharing (Ye & Xu 2010), and profit sharing (Li et al. 2009, Yan 2011). The extended version of Nash bargaining model has also been used in different supply chain structures with more than two members such as multi-echelon supply chain (He & Zhao 2012), and single assembler-n supplier assembly supply chain (Nagarajan & Bassok 2008). One of the limitations of supply chain coordination literature identified in chapter 2 is the less focus on the risk preference of the members of the supply chain. Members have been assumed as risk neutral. However, authors including Gan et al. (2011), He & Zhao (2012), and Huang & Li (2001) used a Nash bargaining approach to propose optimal solutions with differential risk preference of the members of the supply chain. As an extension to Nash bargaining, authors including Hezarkhani & Kubiak (2010) and Lin et al. (2010) used generalized Nash bargaining which takes into account the differential bargaining power of the supply chain members. The notable feature of these supply chains is that they are supply chains with product demand as a decision variable. Other notable use of bargaining models in supply chain includes Rubinstein bargaining model (Zhang, Wang & Ren 2014) and a few other non-zero sum bargaining algorithm Sucky (2005, 2006) .

In contrast to the supply chains mentioned in last paragraph, the application of bargaining concepts in a project setting is very limited. Chapter 4 highlighted the limited research on the supply chain coordination in a project setting (Kwon et al. 2010, Lippman et al. 2013). The few models which exist are based on take it or leave it situations (Bayiz & Corbett 2005, Kwon et al. 2010, Lippman et al. 2013). It is highlighted in chapter 4 that very few models of supply chain coordination exist in the literature. (Bayiz and Corbett, 2005; Kwon et al, 2010; Lippman et al., 2013). Most importantly models with project contracts in bargaining situations are very rare. To the best knowledge of the author of the present research, only the models proposed by Lippman et al. (2013) have considered bargaining games between the members of the supply chain. However, these models are also restricted by the nature of the statistical distributions for the cost variable. The authors used a normally distributed cost function for their model. However, in practice, cost data usually have high Skewness and Kurtosis values. This has been supported in the literature of Back et al. (2000) where the authors used a triangular distribution. Thus, the existing model proposed by (Lippman et al. 2013) might not work correctly with a cost function distributed as a non-normal continuous distributions. This has motivated the present research to investigate and extend the model proposed by (Lippman et al. 2013) with the cost data following other continuous distributions. These are investigated with the help of Nash bargaining models primarily. This is because of its reputation of usage in supply chain literature as a tool. Moreover, chapter 3 highlighted that mostly of the empirical bargaining solutions approach Nash bargaining. In addition, this research also investigated the models with Kalai Smorodinsky bargaining approach and Utilitarian bargaining approach.

5.1 Problem Description

A dyadic supply chain with one project manager and one contractor is considered. As mentioned in chapter 4, the project manager is referred as she and the contractor is considered as he. Similar to chapter 4, this research assumes the project manager and the contractor both as risk neutral in the first case. However, there are cases in practice (particularly in construction projects) where the situation is different. The project manager belong to a large scale organization with financial ability to take risks from projects whereas, their counterpart contractor belong to a small scale organization with limited ability to take risk. Thus, in this context, the project manager is considered to be risk neutral and the contractor is considered as risk averse. Thus, this research assumes the project manager as risk neutral and the contractor is assumed as risk neutral at first. Then, the models are also derived with risk averse contractors as well. In order to maintain consistency with the existing literature, the bargaining models are analysed using the utility maximizing supply chain members.

By definition from the investment management theory, the utility function for a risk neutral member should have a constant marginal return (Haugen 2001, Levy & Levy 2002) i.e. it should satisfy the following $\frac{d^2u_i}{dz^2} = 0$ [where $u_i(z)$ is the utility function of member i and z is the wealth value]. This requirement is satisfied by a linear form of utility function with respect to wealth. Thus for, the risk neutral project manager and the risk neutral contractor, the utility functions are as follows

$$u_{pm}(z) = z \tag{5.1}$$

$$u_{co}(z) = z \tag{5.2}$$

From the expected utility theory proposed by Neumann and Morgenstern (1947) as mentioned in (Von Neumann & Morgenstern 2007), a risk averse contractor's utility function should satisfy the diminishing marginal return (Haugen 2001, Levy & Levy 2002, Davies & Satchell 2007), i.e. the utility function should satisfy $\frac{d^2 u_{co}}{dz^2} < 0$. This condition can be satisfied as long as the utility function is concave in nature (Levy & Levy 2002). This can be achieved by using: logerthemic form, exponential form, and quadratic form (Haugen 2001). However, the decreasing exponential form of utility function among the other concave forms is the most common one (Corner & Corner 1995). Use of exponential form of risk averse utility function yields the constant Arrow-Pratt absolute risk aversion coefficient. This would ensure no change in the risk premium with respect to the absolute risk aversion. Thus, the utility function for the contractor is assumed as below

$$u_{co}(z) = 1 - e^{-\eta z} \tag{5.3}$$

The sequence of events are as follows

- The project manager has a project of value q
- She needs to outsource the project to an external contractor. So she offered a cost based contract P.
- The contractor can either accept or reject the contract.
- If he rejects the contract, then unlike the ultimatum game in chapter 4, the game doesn't terminate. The game would be subjected to bargaining.
- When he accepts the contract, then he needs to select a resource consumption rate as mentioned earlier in chapter 4. It can be easily shown that for optimal conditions, the resource consumption rate is not affected by the the contract parameters of a cost based contract or it doesn't affect the optimal contract parameters of the cost based contract.
- Upon completion, the project manager verifies the cost of completion and makes the payment to the contractor.

5.1.1 For cost based contracts

For a cost based contract, P takes the form of P = a+ bX; where X is the cost function to the contractor. The cost based contract has two parameters a and b. It is assumed that $a > 0$ and $b \in [0, 1]$. a is the fixed component of the contract. b is the variable component of the contract. When b=0, the cost based(sharing) contract is equivalent to a fixed price contract. When b=1, the contract is equivalent to a cost plus contract. For ease of exposition, the time value of money is ignored from this model. Thus, the expected utility functions can be derived as follows from the equations (5.1, (5.2), and (5.3)

For the project manager

$$U_{pm} = E[q - (a + bX)] = q - a - bE(X) \tag{5.4}$$

For a risk neutral contractor

$$U_{co} = E[(a + bX) - X] = a - (1 - b)E(X) \tag{5.5}$$

For a risk averse contractor

$$\begin{aligned} U_{co} &= 1 - e^{-\eta\{(a+bX)-X\}} \\ &= 1 - E[e^{-\eta\{a-(1-b)X\}}] \\ &= 1 - e^{-\eta a}E[e^{\eta(1-b)X}] \end{aligned} \tag{5.6}$$

[Where the X is the cost function (a random variable)].

5.1.2 For time based contracts

It is assumed that the time based contract takes the form of P=g-hT. g is the fixed element of the contract and h is the variable element of the contract that entices the contractor to finish early. It was also shown in chapter 4 that the cost incurred by the contractor depends on the resource consumption rate λ and takes the form $k\lambda^n$ per unit time and a total of $k\lambda^n T$ over time T. The expected time taken to complete the project as μ_1 if one unit of resource is deployed (as assumed before in chapter 4). Thus, the average time for completion of the project becomes $\mu_T = \frac{\mu_1}{\lambda^A}$. Using this, the cost of the contractor becomes $\left[k\left(\frac{\mu_1}{\mu_T}\right)^{\frac{N}{A}} T\right]$. The utility functions of the project manager and the contractor are derived as below

For the project manager

$$U_{pm} = E[q - (g - hT)] = q - g + hE(T) \tag{5.7}$$

For a risk neutral contractor

$$\begin{aligned} U_{co} = E[(g - hT) - k\lambda^n T] &= g - hE(T) - E\left\{k\left(\frac{\mu_1}{\mu_T}\right)^{\frac{n}{A}} T\right\} \\ &= g - hE(T) - \Omega\mu_T \\ &\left[\text{where } \Omega = k\left(\frac{\mu_1}{\mu_T}\right)^{\frac{n}{A}}\right] \end{aligned} \tag{5.8}$$

For a risk averse contractor

$$\begin{aligned} U_{co} = 1 - E[e^{-\eta\{(g-hT)-k\lambda^n T\}}] &= 1 - e^{-\eta g} E[e^{\eta hT}] E\left[e^{\eta k\left(\frac{\mu_1}{\mu_T}\right)^{\frac{n}{A}} T}\right] \\ &= 1 - e^{-\eta g} E[e^{\eta hT}] E\left[e^{\eta \Omega T}\right] \end{aligned} \tag{5.9}$$

T is assumed as a random variable in all the above cases. Now μ_T is the expected value of T i.e. $\mu_T = E(T)$. Thus, taking the expectation of the expectation becomes constant number which is no longer a random variable. Thus, taking expected value for these will be the value itself. Hence, these are considered as constant. Thus, Ω becomes constant.

5.2 Bargaining Models of Supply Chain Coordination with Cost Based Contracts: Nash's Bargaining

Given $b \in [0, 1]$, the project manager and the contractor would maximize the Nash product N(a,b) as below

$$N^* = max : N(a, b) \tag{5.10}$$

where

$$N(a, b) = U_{PM}(a, b) * U_{co}(a, b) \tag{5.11}$$

5.2.1 For risk neutral project manager and risk neutral contractor

For risk neutral project manager and contractor, the optimal Nash's product becomes

$$\max_{a,b\in[0,1]} N(a,b) = \{q - a - b\mu\}\{a - (1-b)\mu\} \qquad [\text{where } \mu = \mathrm{E(x)}\,] \tag{5.12}$$

Differentiating the equation (5.12) with respect to a and setting equal to zero, the first order condition for a can be derived as below

$$a = \frac{q + (1-2b)\mu}{2} \tag{5.13}$$

Thus, for a fixed price contract with b = 0, the optimal a_0 satisfies $a_0 = \frac{q+\mu}{2}$. Using these values, the utility for the project manager and the contractor become

For the project manager

$$U_{pm} = q - \frac{q+\mu}{2} = \frac{q-\mu}{2} \tag{5.14}$$

For the contractor

$$U_{co} = \frac{q+\mu}{2} - \mu = \frac{q-\mu}{2} \tag{5.15}$$

On the contrary, for a cost plus contract with b =1, the optimal a_1 satisfies $a_1 = \frac{q-\mu}{2}$. Using these observation in the utility equations of the project manager and the contractor, it can be shown that their utilities are the same as those found in equations (5.14) and (5.15).

Thus, the optimal utilities derived from Nash's bargaining for a risk neutral project manager and risk neutral a contractor are equal and half of the value of the maximum possible value of the utility. More importantly, the the solution is same for fixed price and cost plus contracts in this case. Due to simplicity of implementation, fixed price contracts may be preferred by the members in practice in these cases.

5.2.2 For risk neutral project manager and risk averse contractor

As mentioned in the problem description, there are situations in practice where the contractors belong to a small organization in comparison to the project manager. Due to this financial constraint, the contractor would be more risk averse in comparison to the project manager.

Thus, this subsection considers the project manager as risk neutral and the contractor as risk averse. The utility functions for the project manager and contractor are as defined in the equations (5.4) and (5.6). Lippman et al. (2013) used a normally distributed cost function X. However, this is very unlikely in practice. Thus, this research extends the model with different forms of probability distribution for cost function X including: gamma, exponential, beta; and Weibull.

As defined earlier, the expected value $E(X) = \mu$. Furthermore, it is defined that $E[e^{-\eta(1-b)X}] = W$. As mentioned by Lippman et al. (2013), this research also assumes the term "W" represents the expected risk exposure of the contractor. Thus, using these values in the equation (5.11) gives

$$N(a, b) = (q - a - b\mu)(1 - e^{-\eta a}W) \tag{5.16}$$

In order to get the optimal first best value, differentiating equation (5.16) with respect to a and setting that equal to zero gives the first order condition as

$$\eta W(-a - b\mu + q)e^{-\eta a} + We^{-\eta a} - 1 = 0 \tag{5.17}$$

On the one hand, with b =0, the contract becomes a fixed price contract. On the other hand, with b=1, the contract becomes pure cost plus contract. Thus,

$$\begin{cases} a = a_0 \text{ and } W = W_0 & \text{for fixed price contracts} \\ a = a_1 \text{ and } W = W_1 & \text{for cost plus contracts} \end{cases}$$

It can be easily shown when b = 1, $W_1 = 1$. Both these contract's (fixed price and cost plus) parameters should satisfy the first best condition in equation (5.17). Thus, using the values of a and W from above, the following conditions can be derived

For fixed price

$$W_0 \left[\eta(-a_0 + q) + 1\right] e^{-\eta a_0} - 1 = 0 \tag{5.18}$$

For cost plus

$$[\eta(-a_1 - \mu + q) + 1]e^{-\eta a_1} - 1 = 0 \tag{5.19}$$

The W value can be calculated based on the concepts of moment generating functions as below

$$W = E[e^{tX}] = \begin{cases} \frac{1}{(1-\phi t)^{\omega}} \\ \text{gamma distributed cost with shape parameter}\,\omega\text{ and scale parameter}\,\phi \\ \frac{1}{(1-\phi t)} \\ \text{exponential distributed cost with scale parameter}\,\phi \\ 1+\sum\limits_{i=1}^{\infty}\left(\phi^i \prod\limits_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{t^i}{i!} \\ \text{beta distributed cost with shape parameters c \& d and scale}\,\phi \\ 1+\sum\limits_{i=1}^{\infty}\left[\frac{t^i\phi^i}{i!}\Gamma\left(1+\frac{i}{S}\right)\right] \\ \text{Weibull distributed cost with shape parameter s and scale parameter}\,\phi \end{cases} \tag{5.20}$$

where $t = \eta(1-b)$

Thus, for a fixed price contract with b = 0 and a cost plus contract with b=1, the following can be derived

$$W = W_0 = \begin{cases} \frac{1}{(1-\eta\phi)^{\omega}} \\ \text{gamma distributed cost following equation (5.20)} \\ \frac{1}{(1-\eta\phi)} \\ \text{exponential distributed cost following equation (5.20)} \\ 1+\sum\limits_{i=1}^{\infty}\left(\phi^i \prod\limits_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{\eta^i}{i!} \\ \text{beta distributed cost following equation (5.20)} \\ 1+\sum\limits_{i=1}^{\infty}\left[\frac{\eta^i\phi^i}{i!}\Gamma\left(1+\frac{i}{S}\right)\right] \\ \text{Weibull distributed cost following equation (5.20)} \end{cases}$$

and

$W = 1$ for cost plus contracts with b=1 for all the selected distributions (5.21)

Thus, the mean value μ for the cost functions are as follows

$$\mu = \begin{cases} \omega\phi & \text{gamma distributed cost with shape parameter } \omega \text{ and scale parameter } \phi \\ \phi & \text{exponential distributed cost with scale parameter } \phi \\ \phi\left(\frac{c}{c+d}\right) & \text{beta distributed cost following equation (5.20)} \\ \phi\Gamma\left(1+\frac{1}{s}\right) & \text{Weibull distributed cost with shape parameter s and scale parameter } \phi \end{cases} \tag{5.22}$$

Using these values of W (Including W$_0$) and μ in equation (5.17), the optimal condition for the contract parameter a can be determined for the selected distributions of cost function. This is summarised in the following lemmas.

Lemma 3. *The optimal value of contract parameter* a_0 *of a cost based contract P= a+bX that maximizes the Nash product, satisfies the following conditions if b = 0*

$$\begin{cases} \left[\frac{1}{(1-\eta\phi)^{\omega}}\right]\left[\eta(-a_0+q)+1\right]e^{-\eta a_0}-1=0 & \textit{for a gamma distributed cost} \\ \left[\frac{1}{(1-\eta\phi)}\right]\left[\eta(-a_0+q)+1\right]e^{-\eta a_0}-1=0 & \textit{for an exponentially distributed cost} \\ \left[1+\sum\limits_{i=1}^{\infty}\left(\phi^i\prod\limits_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{\eta^i}{i!}\right]\left[\eta(-a_0+q)+1\right]e^{-\eta a_0}-1=0 & \textit{for a beta distributed cost} \\ \left[1+\sum\limits_{i=1}^{\infty}\left\{\frac{\eta^i\phi^i}{i!}\Gamma\left(1+\frac{i}{S}\right)\right\}\right]\left[\eta(-a_0+q)+1\right]e^{-\eta a_0}-1=0 & \textit{for an Weibull distributed cost} \end{cases} \tag{5.23}$$

Lemma 4. *The optimal value of contract parameter* a_1 *of a cost based contract P= a+bX* *that maximizes the Nash product, satisfies the following conditions if b = 1*

$$\begin{cases} \left[\eta(-a_1 - \omega\phi + q) + 1\right] e^{-\eta a_1} - 1 = 0 & \textit{for a gamma distributed cost} \\ \left[\eta(-a_1 - \phi + q) + 1\right] e^{-\eta a_1} - 1 = 0 & \textit{for an exponentially distributed cost} \\ \left[\eta(-a_1 - \frac{\phi c}{c+d} + q) + 1\right] e^{-\eta a_1} - 1 = 0 & \textit{for a beta distributed cost} \\ \left[\eta\left\{-a_1 - \phi\Gamma\left(1 + \frac{1}{s}\right) + q\right\} + 1\right] e^{-\eta a_1} - 1 = 0 & \textit{for an exponential distributed cost} \end{cases} \tag{5.24}$$

Lippman et al. (2013) proposed a method to identify the optimal value of b for $b \in [0, 1]$. However, this method is difficult to be applied when X follows a non-normal continuous distribution. Thus, an alternate method proposed in their literature is used in this research. The alternate method is based on sign tests of the derivatives. Using this alternate method, Nash product or the individual utility functions of the members of the supply chain can be shown as either increasing or decreasing in b. Accordingly, it can be concluded whether fixed price, cost based or cost plus contract dominates the solution. Thus, differentiating equation (5.11) with respect to b

$$\frac{dN(a,b)}{db} = U_{co}\left(\frac{dU_{PM}}{db}\right) + U_{PM}\left(\frac{dU_{co}}{db}\right) \tag{5.25}$$

Differentiating equation (5.4) with respect to b, and using the value of $E(x) = \mu$

$$\frac{dU_{PM}}{db} = -\frac{da}{db} - \mu \tag{5.26}$$

Rearranging the terms from equation (5.17),

$$(q - a - b\mu) = \frac{1 - e^{-\eta a}W}{\eta e^{-\eta a}W} = \frac{1 - A}{\eta A} \qquad \text{[where } A = e^{-\eta a}W\text{]} \tag{5.27}$$

Differentiating equation(5.27)

$$-\frac{da}{db} - \mu = -\frac{1}{\eta A^2}\frac{dA}{db} \tag{5.28}$$

Now

$$\frac{dA}{db} = -\eta e^{-\eta a} W \frac{da}{db} + e^{-\eta A} \frac{dW}{db} \tag{5.29}$$

Using this values of W and $\frac{dA}{db}$ from (5.29) in equation (5.28)

$$-\frac{da}{db} - \mu = -\frac{1}{\eta A^2}\left[-\eta A \frac{da}{db} + \frac{A}{W}\frac{dW}{db}\right]$$

or

$$\left[-\frac{da}{db} - \mu\right]\left[1 + \frac{1}{A}\right] = \left[-\left(\frac{1}{\eta A W}\right)\frac{dW}{db} - \left(\frac{1}{A}\mu\right)\right]$$

or

$$\left[-\frac{da}{db} - \mu\right]\left[1 + \frac{1}{A}\right] = -\frac{1}{A}\left[\left(\frac{1}{\eta W}\right)\frac{dW}{db} + \mu\right] \tag{5.30}$$

As mentioned earlier, $U_{co} = 1 - e^{-\eta a}W$. Based on the assumption made in equation (5.27), $U_{co} = 1 - A$. Thus, differentiating both side with respect to b

$$\frac{dU_{co}}{db} = -\frac{dA}{db} \tag{5.31}$$

Thus, the signs of $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$ depend on the signs of $\frac{dW}{db}$ and $\frac{dA}{db}$ respectively. The sign tests of these derivatives depend on the nature of distribution of the cost function X. Thus, the next few subsections analyse the case for gamma, exponential, beta, and Weibull distributed cost functions.

Gamma Distributed Cost

$$\frac{dW}{db} = -\frac{\eta\phi\omega}{\{1-\eta\phi(1-b)\}^{\omega+1}} = -\frac{W\eta\phi\omega}{\{1-\eta\phi(1-b)\}} \tag{5.32}$$

As assumed before, $\eta > 0$. It can also be shown that $q - a_0 > 0$; otherwise the project manager's utility would be negative and she would never participate in the bargaining. From the equation (5.23), it can be shown that $\frac{1}{(1-\eta\phi)^{\omega}} > 0$ when η and ϕ both are positive. Thus, $\eta\phi < 1$.

It is also assumed before that $0 \leq b \leq 1$. This leads to $0 \leq \eta\phi(1-b) < 1$ and $1 - \eta\phi(1-b) > 0$. Thus, the value of $\frac{dW}{db}$ is negative. This means W is a decreasing function of b.

Now using the value of μ from the equation (5.22) and the value of $\frac{dW}{db}$ from equation (5.32) in equation (5.30),

$$\begin{aligned}\left[-\frac{da}{db}-\omega\phi\right]\left(1+\frac{1}{A}\right) &= -\frac{1}{A}\left[\left(\frac{1}{\eta W}\right)\left\{\frac{-\eta W\phi\omega}{1-\eta\phi(1-b)}\right\}+\omega\phi\right] \\ &= -\frac{\omega\phi}{A}\left[\frac{-1+1-\eta\phi(1-b)}{\{1-\eta\phi(1-b)\}}\right] \\ &= \left[\frac{\omega\eta\phi^2(1-b)}{A\{1-\eta\phi(1-b)\}}\right]\end{aligned}$$

or

$$\left[-\frac{da}{db}-\omega\phi\right] = \left(\frac{1}{1+A}\right)\left[\frac{\omega\eta\phi^2(1-b)}{\{1-\eta\phi(1-b)\}}\right] \tag{5.33}$$

In the equation (5.33), the right hand side of the equation is positive with $0 \leq b < 1$ and zero with b =1. Thus, the value of the term $(-\frac{da}{db} - \omega\phi)$ is: positive with $0 \leq b < 1$; and zero with b =1 for a gamma distributed cost function X. Using this observation in equation (5.26), U_{PM} is found to be increasing in b in the range $0 \leq b < 1$ and attains the maximum with b = 1. This means $q - a_1 - b\omega\phi > q - a_0$ i.e. $a_1 + \omega b\phi < a_0$.

Rearranging the terms from equation(5.28) and using the value of μ,

$$\left[-\frac{da}{db}-\omega\phi\right]\left(\eta A^2\right) = -\frac{dA}{db}$$

From equation (5.33), $-\frac{da}{db} - \omega\phi > 0$ with $0 \leq b < 1$ and zero with b =1. It is also assumed earlier that $\eta > 0$. Thus, $-\frac{dA}{db} > 0$. This leads to the conclusion that

$$\frac{dU_{co}}{db} \geq 0$$

This means U_{co} is increasing in b with $0 \leq b < 1$ and attains the maximum with b=1.

Using these above observations in equation (5.25), it can be shown that $\frac{dN}{db} > 0$ [for $0 \leq b < 1$] and $\frac{dN}{db} = 0$ [for b=1]. These observations are summarized in the following lemma

Lemma 5. *With a cost based contract P=a+bX (where cost function X follows a gamma dis-*

tribution), The Nash product and the utility functions of the project manager & the contractor are higher under cost plus contract than under the fixed price contract or any cost sharing contract ($0 < b < 1$*).*

Exponential Distributed Cost

Similar to the case in gamma distributed cost, following the same argument, it can be shown that $(1 - \eta\phi) > 0$ or $\eta\phi < 1$, otherwise the project manager would never participate in the bargaining. It is also assumed before that $0 \leq b \leq 1$. Thus, $0 \leq 1 - \eta(1-b)\phi \leq 1$

Now

$$\frac{dW}{db} = -\frac{\eta\phi}{\{1 - \eta\phi(1-b)\}^2} = -\eta\phi W^2 \tag{5.34}$$

As assumed earlier, $\eta > 0$ and $\phi > 0$. Thus, the value of $\frac{dW}{db}$ is negative. This means W is a decreasing function of b and from the observation in the beginning of the section, $W \geq 1$.

Similar to the calculations with gamma distributed cost, using this value of $\frac{dA}{db}$, W and μ in equation (5.30) and rearranging the terms, it can be shown that

$$\left[-\frac{da}{db} - \phi\right] = \left(\frac{1}{1+A}\right)\left[\frac{\eta\phi^2(1-b)}{\{1 - \eta\phi(1-b)\}}\right] \tag{5.35}$$

In equation (5.35), the right hand side of the equation is positive with $0 \leq b < 1$ and zero with b =1. Thus the term $-\frac{da}{db} - \phi > 0$ with $0 \leq b < 1$; and zero with b =1 for an exponentially distributed cost function. Using the observation from equation (5.26), U_{pm} is found to be increasing in the range $0 \leq b < 1$; and maximum with b =1. This means $q - a_1 - b\phi > q - a_0$ or $a_1 + b\phi < a_0$

Similar to the gamma distributed cost function, using the observation from equation (5.35) in equation (5.28), it can be shown that $-\frac{dA}{db} \geq 0$. This leads to the conclusion that for an exponentially distributed cost,

$$\frac{dU_{co}}{db} \geq 0$$

This means U_{co} is increasing in b with $0 \leq b < 1$ and attains the maximum with b = 1.

Using these above observations in equation (5.25), it can be shown that $\frac{dN}{db} > 0$ [for $0 \leq b < 1$] and $\frac{dN}{db} = 0$ [for b=1]. These observations are summarized in the following lemma

Lemma 6. *With a cost based contract P=a+bX (where cost function X follows an exponential distribution), The Nash product and the utility functions of the project manager & the contractor are higher under a cost plus contract than under a fixed price contract or any cost sharing contract* ($0 < b < 1$)*.*

Beta Distributed Cost

Differentiating the equation (5.20) with respect to b for the beta distributed cost gives

$$\frac{dW}{db} = -\sum_{i=1}^{\infty} \left\{ \prod_{r=0}^{i-1} \phi^i \left(\frac{c+r}{c+d+r} \right) \right\} \frac{i\eta^i (1-b)^{i-1}}{i!} \tag{5.36}$$

As mentioned earlier, it is assumed that $0 \leq b \leq 1$ and $\eta > 0$. Thus, the value of $\frac{dW}{db}$ is negative with $0 \leq b < 1$, and becomes zero with b = 1. By taking the second derivative of W with respect to b, $\frac{d^2W}{db^2}$ would become positive. This means W is a convex and decreasing function of b with $0 \leq b < 1$ and attains the minimum value of one with b = 1.

Using the value of μ of beta distributed cost from equation (5.22) in equation (5.30)

$$\left[-\frac{da}{db} - \frac{\phi c}{c+d} \right] \left(1 + \frac{1}{A} \right) = -\frac{1}{A} \left[\frac{1}{\eta W} \left(\frac{dW}{db} \right) + \left(\frac{\phi c}{c+d} \right) \right] \tag{5.37}$$

Expanding the values from equation (5.36)

$$\begin{aligned} \frac{dW}{db} &= -\left(\frac{\eta\phi c}{c+d} \right) - \left(\frac{2\eta^2(1-b)\phi^2 c(c+1)}{2!(c+d)(c+d+1)} \right) - \left(\frac{3\eta^3(1-b)^2\phi^3 c(c+1)(c+2)}{3!(c+d)(c+d+1)(c+d+2)} \right) - \cdots \\ &= -\left(\frac{\eta\phi c}{c+d} \right) \left[1 + \left(\frac{\eta(1-b)\phi(c+1)}{(c+d+1)} \right) + \left(\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} \right) + \cdots \right] \end{aligned} \tag{5.38}$$

Using this value of $\frac{dW}{db}$ in the right hand side of the equation(5.37)

$$\begin{aligned} &-\frac{1}{A} \left[\frac{1}{\eta W} \left(\frac{dW}{db} \right) + \left(\frac{\phi c}{c+d} \right) \right] \\ &= \frac{\left(\frac{\eta\phi c}{c+d} \right) \left[1 + \left(\frac{\eta(1-b)\phi(c+1)}{(c+d+1)} \right) + \left(\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} \right) + \cdots \right]}{\eta AW} - \frac{1}{A} \left(\frac{\phi c}{c+d} \right) \end{aligned}$$

or

$$-\frac{1}{A}\left[\frac{1}{\eta W}\left(\frac{dW}{db}\right)+\left(\frac{\phi c}{c+d}\right)\right]$$
$$=\frac{1}{A}\left(\frac{\phi c}{c+d}\right)\left[\frac{1+\left(\frac{\eta(1-b)\phi(c+1)}{(c+d+1)}\right)+\left(\frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)}\right)+\cdots}{1+\eta(1-b)\left[\frac{\phi c}{c+d}\right]+\frac{\eta^2(1-b)^2}{2!}\left[\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right]+\cdots}-1\right]$$

[expanding W]

or

$$-\frac{1}{A}\left[\frac{1}{\eta W}\left(\frac{dW}{db}\right)+\left(\frac{\phi c}{c+d}\right)\right]$$
$$=\frac{1}{A}\left(\frac{\phi c}{c+d}\right)\left[\frac{\eta(1-b)\phi\left(\frac{(c+1)}{(c+d+1)}-\frac{c}{c+d}\right)+\frac{\eta^2(1-b)^2\phi^2}{2!}\left(\frac{(c+1)(c+2)}{(c+d+1)(c+d+2)}-\frac{c(c+1)}{(c+d)(c+d+1)}\right)+\cdots}{1+\eta(1-b)\left[\frac{\phi c}{c+d}\right]+\frac{\eta^2(1-b)^2}{2!}\left[\frac{\phi^2 c(c+1)}{(c+d)(c+d+1)}\right]+\cdots}\right]$$

or

$$-\frac{1}{A}\left[\frac{1}{\eta W}\left(\frac{dW}{db}\right)+\left(\frac{\phi c}{c+d}\right)\right]$$
$$=\frac{\eta(1-b)\phi^2}{A}\left(\frac{c}{c+d}\right)\left[\frac{\left(\frac{d}{(c+d)(c+d+1)}\right)+\frac{\eta^2(1-b)^2\theta^2}{2!}\left(\frac{2d(c+1)}{(c+d)(c+d+1)(c+d+2)}\right)+\cdots}{W}\right]$$

[converting the denominator to W]

As shown before, the denominator in the above equation (W) is positive. The parameters η, c, and d are all positive. It is also shown that A is positive. Thus, when $0 \leq b < 1$, the left hand side of the equation becomes positive. When b=1, the left hand side is zero. Using this observation in equation (5.37), it can be shown that $-\frac{da}{db} - \frac{\phi c}{c+d} \geq 0$ with $0 \leq b \leq 1$. This again concludes that U_{PM} is increasing in b with $0 \leq b < 1$ and attains the maximum value with b = 1. This means $q - a_1 - \frac{\theta u}{u+v} > q - a_0$ or $a_1 + \frac{\theta u}{u+v} < a_0$.

Using the observations above in the last paragraph in equation (5.28), it can be shown $-\frac{dA}{db} \geq 0$ for $b \in [0, 1]$. Thus, similar to the calculations of the gamma and exponential distributions, it can be shown $\frac{dU_{co}}{db} > 0$ for $0 \leq b < 1$ and attains the maximum at b = 1 i.e. U_{co} is increasing for any $b \in [0, 1]$.

Using these observations in the equation (5.25), it can be shown that $\frac{dN}{db} > 0$ for $0 \leq b < 1$

and is maximum at b = 1. These observations are summarized in the following lemma

Lemma 7. *With a cost based contract P=a+bX (where cost function X follows a beta distribution), The Nash product and the utility functions of the project manager & the contractor are higher under cost plus contract than under the fixed price contract or any cost sharing contract (*$0 < b < 1$*).*

Weibull Distributed Cost

Differentiating the equation (5.20) with respect to b

$$\frac{dW}{db} = -\sum_{i=1}^{\infty} \phi^i \left\{ \frac{i\eta^i (1-b)^{i-1}}{i!} \right\} \Gamma\left(1+\frac{i}{S}\right) \tag{5.39}$$

As mentioned earlier, it is assumed that $0 \leq b \leq 1$. Thus, the value of $\frac{dW}{db}$ is negative with $0 \leq b < 1$, and becomes zero with b=1. By taking the second derivative of W with respect to b, $\frac{d^2W}{db^2}$ would become positive. This means W is a convex and decreasing function of b with $0 \leq b < 1$ and attains the minimum value of one with b=1.

Using the value of μ for Weibull distributed cost in equation (5.30)

$$-\left[\frac{da}{db} + \phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\right]\left(1+\frac{1}{A}\right) = -\frac{1}{A}\left[\frac{1}{\eta W}\left(\frac{dW}{db}\right) + \phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\right] \tag{5.40}$$

Expanding the right hand side of the equation (5.39)

$$\frac{dW}{db} = -\eta\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\} - \frac{2\eta^2\phi^2(1-b)}{2!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\} - \frac{3\eta^3\phi^3(1-b)}{3!}\left\{\Gamma\left(1+\frac{3}{S}\right)\right\} - \cdots \tag{5.41}$$

Using this value of $\frac{dW}{db}$ and the value of W from equation (5.20) in the right hand side of the equation(5.40)

$$\begin{aligned} &-\left[\frac{da}{db} + \phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\right]\left(1+\frac{1}{A}\right) \\ &= -\frac{1}{A}\left[\frac{-\eta\phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\} - \frac{2\eta^2\theta^2(1-b)}{2!}\left\{\Gamma\left(1+\frac{2}{s}\right)\right\} - \cdots}{\eta W} + \phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\right] \end{aligned}$$

or

$$-\left[\frac{da}{db}+\phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\right]\left(1+\frac{1}{A}\right)$$
$$=\frac{1}{A}\left[\frac{\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta\phi^2(1-b)}{1!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}{1+\eta\phi(1-b)\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta^2\phi^2(1-b)^2}{2!}\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\cdots}-\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}\right]$$

or

$$-\left[\frac{da}{db}+\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}\right](1+A)$$
$$=\left[\frac{\eta(1-b)\phi^2\left\{\Gamma\left(1+\frac{2}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta^2(1-b)^2\phi^3}{2!}\left\{\Gamma\left(1+\frac{3}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}{1+\eta\phi(1-b)\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta^2\phi^2(1-b)^2}{2!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}\right]$$

or

$$-\left[\frac{da}{db}+\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}\right]\left(1+\frac{1}{A}\right)$$
$$=\frac{\eta(1-b)\phi^2}{A}\left[\frac{\left\{\Gamma\left(1+\frac{2}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta(1-b)\theta}{2!}\left\{\Gamma\left(1+\frac{3}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}{W}\right]$$

As shown before, the denominator in the above equation (W) is positive. The parameters η, ϕ, and S are all positive. It is also shown that A is positive. Furthermore, numerically, it can be shown that all the terms on the numerator consisting of difference between two Gamma functions are also positive. Thus, when $0 \leq b < 1$, the left hand side of the equation becomes positive. When b = 1, the left hand side is zero. Using this observation in equation (5.40), it can be shown that

$$-\frac{da}{db}-\phi\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}\geq 0 \qquad \text{with } 0 \leq b \leq 0$$

Using this observation in equation(5.26), it can further be shown U_{PM} is increasing in b with $0 \leq b < 1$ and attains the maximum value with b=1.

Using the above observations from the last paragraph in equation (5.28) for the Weibull distributed case, it can be shown that $-\frac{dA}{db} > 0$ for $b \in [0, 1]$. Thus, similar to the other distributions, it can be shown $\frac{dU_{co}}{db} > 0$ for $0 \leq b < 1$ and and attains a maximum value at b = 1 i.e. U_{co} is increasing in b, for $b \in [0, 1]$. Using these observations in the equation (5.25), it can be shown that $\frac{dN}{db} > 0$ for $0 \leq b < 1$ and maximum at b = 1. These observations are summarized in the following lemma

Lemma 8. *With a cost based contract P=a+bX (where cost function X follows a Weibull distribution), The Nash product and the utility functions of the project manager & the contractor are higher under cost plus contract than under the fixed price contract or any cost sharing contract (*$0 < b < 1$*).*

Similarly, the calculations can be extended for cost functions following other continuous distributions and it can be shown that cost sharing contracts are capable of offering a dominating solution to ensure a win-win solution. Hence, combining the findings from lemmas 5, 6, 7, and 8, the following is proposed

Proposition 6. *With a cost based contract P=a+bX (where cost function X follows any non-normal skewed continuous distribution), the Nash product and the utility functions of the project manager & the contractor are higher under cost plus contract than under the fixed price contract or any cost sharing contract (*$0 < b < 1$*). The optimal condition for the fixed parameter satisfies the condition in the equation (5.24) in lemma (4)*

Generalized Nash's Bargaining Set

In the last sub-section, the bargaining power of the members of the supply chain were considered to be equal. However, in practice this is very unlikely. Members are likely to have differential bargaining power in the supply chain. It is assumed that the bargaining power of the project manager is τ and the contractor is $(1-\tau)$. Thus, the generalized Nash's product would be

$$GN = U_{pm}^{\tau} \cdot U_{co}^{1-\tau} \tag{5.42}$$

U_{pm}, and U_{co} are calculated from the equations (5.4), and (5.6) respectively.

Thus to identify the optimal condition for a, the equation (5.42) is differentiated with respect to a as below

$$\begin{aligned}
\frac{dGN}{da} &= \frac{d}{da}\left[U_{pm}^{\tau} \cdot U_{co}^{1-\tau}\right] \\
&= \tau\left(\frac{U_{co}}{U_{pm}}\right)^{1-\tau}\frac{dU_{pm}}{da} + (1-\tau)\left(\frac{U_{pm}}{U_{co}}\right)^{\tau}\frac{dU_{co}}{da} \\
&= \tau\left(\frac{1-e^{-\eta a}W}{q-a-b\mu}\right)^{1-\tau}(-1) + (1-\tau)\left(\frac{q-a-b\mu}{1-e^{-\eta a}W}\right)^{\tau}\left(\eta e^{-\eta a}W\right)
\end{aligned}$$

Thus, for the first order condition

$$\tau\left(\frac{1-e^{-\eta a}W}{q-a-b\mu}\right)^{1-\tau}(-1)+(1-\tau)\left(\frac{q-a-b\mu}{1-e^{-\eta a}W}\right)^{\tau}\left(\eta e^{-\eta a}W\right)=0$$

or

$$\begin{aligned} q-a-b\mu &= \frac{\tau}{1-\tau}\left[\frac{1-e^{-\eta a}W}{\eta e^{-\eta a}W}\right] \\ &= \frac{\tau}{1-\tau}\left[\frac{1-A}{\eta A}\right] \qquad (5.43) \\ &\text{[As assumed earlier } A=e^{-\eta a}W] \end{aligned}$$

The contract parameter b follows the condition $0 \leq b \leq 1$. In order to identify the optimal value within this range, the equation (5.42) is differentiated with respect to b as below

$$\begin{aligned} \frac{dGN}{db} &= \frac{d}{db}\left[U_{pm}^{\tau}\cdot U_{co}^{1-\tau}\right] \\ &= \tau\left(\frac{U_{co}}{U_{pm}}\right)^{1-\tau}\frac{dU_{pm}}{db}+(1-\tau)\left(\frac{U_{pm}}{U_{co}}\right)^{\tau}\frac{dU_{co}}{db} \qquad (5.44) \end{aligned}$$

To find out the optimal value, the sign test of the first order derivative is required. Thus, the right hand side of the equation (5.44) is analysed for each and individual element.

As assumed earlier, $0 \leq \tau \leq 1$. Thus, neither τ nor $(1-\tau)$ can be negative. U_{co} and U_{pm} both have to be positive, otherwise the members of the supply chain would not participate in the bargaining. Thus, the sign of the derivative $\frac{dGN}{db}$ depends on the signs of $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$.

Differentiating both side of the equation (5.43) with respect to b

$$\left[-\frac{da}{db}-\mu\right]=\frac{1}{\eta}\left(\frac{\tau}{1-\tau}\right)\left[\left(-\frac{1}{A^2}\right)\frac{dA}{db}\right] \qquad (5.45)$$

Using the value of $\frac{dU_{pm}}{db}$ from equation (5.26),

$$\frac{dU_{pm}}{db}=\frac{1}{\eta}\left(\frac{\tau}{1-\tau}\right)\left[\left(-\frac{1}{A^2}\right)\frac{dA}{db}\right]$$

As mentioned earlier, both τ and $(1-\tau)$ are positive. The value of $\left[\left(-\frac{1}{A^2}\right)\frac{dA}{db}\right]$ depends on the nature of distribution. Earlier in this section (5.2.2), the value of $\left[\left(-\frac{1}{A^2}\right)\frac{dA}{db}\right]$ is shown as positive with $0 \leq b < 1$ and as zero with b=1 for gamma, exponential, beta and Weibull distributed cost respectively. Thus, it can be shown that $\frac{dU_{pm}}{db} \geq 0$. This leads to the conclusion that the utility of the project manager is increasing for $b \in [0, 1)$ and is maximum at b=1.

As explained in section 5.2.2, $U_{co} = 1 - A$ and $\frac{dU_{co}}{db} = -\frac{dA}{db}$. Rearranging the terms from equation (5.45)

$$\left(\frac{1}{\eta A^2}\right)\left(-\frac{da}{db} - \mu\right)\left(\frac{1-\tau}{\tau}\right) = -\frac{dA}{db}$$

As assumed earlier, $\eta > 0$. Thus, the right hand side of the above equation is positive for the same reasons mentioned for the project manager. Thus $\frac{dU_{co}}{db}$ is increasing in $0 \leq b < 1$ and maximum at b = 1. Thus, similar to the project manger, the utility of the contractor is positive for $b \in [0, 1)$ and maximum at b = 1 for the cost functions with gamma, exponential, beta, and Weibull distributed.

Using the above observations in equation (5.44), it can be shown that $\frac{dGN}{db} \geq 0$ for $b \in [0, 1]$. this means the generalized Nash product is increasing in b for $b \in [0, 1]$ and maximum at b=1. Thus, similar to the case of Nash's bargaining, the cost plus contract dominates the solutions of any cost sharing contracts (for $0 < b < 1$) and the solution of the fixed price contract.

Proposition 7. *With a cost based contract P=a+bX (where cost function X follows any non-normal skewed continuous distribution), the generalized Nash product and the utility functions of the project manager & the contractor are higher under cost plus contract than under the fixed price contract or any cost sharing contract (*$0 < b < 1$*). The optimal condition for the fixed parameter satisfies the optimal condition below*

$$a_1 = \begin{cases} \eta e^{-\eta a_1}(1-\tau)(q - a_1 - \omega\phi) - \tau(1 - e^{-\eta a_1}) = 0 & \textit{gamma distributed cost} \\ \eta e^{-\eta a_1}(1-\tau)(q - a_1 - \phi) - \tau(1 - e^{-\eta a_1}) = 0 & \textit{exponential distributed cost} \\ \eta e^{-\eta a_1}(1-\tau)(q - a_1 - \frac{\phi c}{c+d}) - \tau(1 - e^{-\eta a_1}) = 0 & \textit{beta distributed cost} \\ \eta e^{-\eta a_1}(1-\tau)\{q - a_1 - \phi\Gamma\left(1 + \frac{1}{S}\right)\} - \tau(1 - e^{-\eta a_1}) = 0 & \textit{Weibull distributed cost} \end{cases} \tag{5.46}$$

Proof. The optimal value of a for the generalized Nash baragianing satisfies the equation (5.43). Using the values of W, a, and μ for b = 1 i.e. $W = 1$, $a = a_1$, and μ from the equation (5.22) into the equation (5.43) and rearranging the terms leads to the first order condition for

a_1 in the proposition 7. □

5.3 Bargaining Models of Supply Chain Coordination with Cost Based Contracts: Kalai and Smorodinsky Bargaining

The utility functions for the project manager and the contractor remain the same as described in equations (5.4) and (5.6) respectively. According to the Kalai Smorodinsky rule (Kalai and Smorodinsky, 1975), the optimal solution is

$$K(Z,d) = arg \max_{z_i} \left\{ \min_{(i \in 1,2)} \frac{z_i - d_i}{a_i(Z,d) - d_i} \right\} \tag{5.47}$$

Where i denotes either the project manager or the contractor; z_i is the pay off to the member i; d_i is the disagreement pay-off; and $a_i(Z,d)$ is the aspiration pay-off to the member i. $a_i(Z,d)$ is defined as below

$$a_i(Z,d) = arg\, max(z_i) \tag{5.48}$$

Thus, the Kalai Smorodinsky solution K(Z,d) maximizes the individually rational pay-off normalized with respected to the aspiration point pay off.

It is assumed before that d_i is zero. In other words the disagreement pay-off for both the members are assumed as zero. The aspiration point for the project manager and the contractor are respectively as follow

For a project manager

$$a_{pm}(Z,d) = q - E(X) = q - \mu \tag{5.49}$$

For a risk neutral contractor

$$a_{co}(Z,d) = q - \mu \tag{5.50}$$

For a risk averse contractor

$$a_{co}(Z,d) = 1 - e^{-\eta q} E\{e^{\eta X}\} = 1 - e^{-\eta q} V \tag{5.51}$$

[where $\mu = E(X)$ and $V = E\{e^{\eta X}\}$]

5.3.1 For a risk neutral project manager and a risk neutral contractor

In order to satisfy the condition for optimal K, the minimum of the normalized utilities of the project manager and the contractor should be maximized. When the minimum of values of these two fractions are maximized, they become equal in value. If they are unequal at their maximum values, then that violates the conditions for Kalai Smorodinsky's basic condition. Thus, at an optimal solution

$$\frac{q-a-b\mu}{q-\mu} = \frac{a-(1-b)\mu}{q-\mu}$$

or

$$a + b\mu = \frac{q+\mu}{2} \tag{5.52}$$

For a fixed price contract with b=0, the optimal a_0 becomes $a_0 = \frac{q+\mu}{2}$ and for a cost plus contract with b = 1, the optimal a_1 becomes $a_1 = \frac{q-\mu}{2}$

Using these values of a_0, a_1, and b in the utility equations of the project manager and the risk neutral contractor, it can easily be shown that $U_{pm} = U_{co} = \frac{q-\mu}{2}$.

This is same for fixed price and cost plus contract. Thus, similar to the case of Nash's bargaining with risk neutral members, for Kalai Smorodinsky bargaining, the maximum utility is equally split amongst the members. Like Nash's bargaining, this is same for fixed price and cost plus contract. Thus, due to simplicity, the fixed price contract may be preferred over the cost plus contract in practice with this similar situation.

5.3.2 For a risk neutral project manager and a risk averse contractor

Using these values of aspiration point, the normalized individual rationalities of the members of the supply chain are as follows

For the project manager

$$U_{pmn} = \frac{q-a-b\mu}{q-\mu} \tag{5.53}$$

For the contractor

$$U_{con} = \frac{1-e^{-\eta a}W}{1-e^{-\eta q}V} \qquad \text{[W follows equation (5.20)]} \tag{5.54}$$

In order to satisfy the condition for optimal K, the minimum values of the two fractions on the right hand side of the equations (5.53) and (5.54) should be maximized. As mentioned in the last sub-section, when the minimum of these two fractions are maximized, they become equal in values. Thus, at the optimal solution

$$\frac{q-a-b\mu}{q-\mu}=\frac{1-e^{-\eta a}W}{1-e^{-\eta q}V} \tag{5.55}$$

As defined earlier in section (5.2), $b=1$, $a=a_1$ for a cost plus contract; and $W=1$. On the contrary, $b=0$, $a=a_0$ for a fixed price contract; and $W=W_0$. Thus using these values in the optimal condition for Kalai Smorodinsky Solution in equation (5.55) goves

For fixed price contracts

$$\frac{q-a_0}{q-\mu}=\frac{1-e^{-\eta a_0}W_0}{1-e^{-\eta q}V} \tag{5.56}$$

For cost plus contracts

$$\frac{q-a_1-\mu}{q-\mu}=\frac{1-e^{-\eta a_1}}{1-e^{-\eta q}V} \tag{5.57}$$

In order to identify if the solution with the fixed price or the cost plus contract dominates, the sign tests for the first order derivative $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$ are required. To determine the sign on the right hand side of the above equation, both sides of the equation (5.55) are differentiated with respect to b

$$\left(-\frac{da}{db}-\mu\right)\left(\frac{1}{q-\mu}\right)=\left[\eta e^{-\eta a}W\frac{da}{db}-e^{-\eta a}\frac{dW}{db}\right]\left(\frac{1}{1-e^{-\eta q}V}\right)$$

or

$$\left(-\frac{da}{db}-\mu\right)=\left[\eta e^{-\eta a}W\frac{da}{db}-e^{-\eta a}\frac{dW}{db}\right]\left(\frac{q-\mu}{1-e^{-\eta q}V}\right)$$

or

$$(-\frac{da}{db}-\mu)(1+\eta e^{-\eta a}WB)=\left[-e^{-\eta a}\frac{dW}{db}-\eta e^{-\eta a}W\mu\right]B \tag{5.58}$$

[where $B=\frac{q-\mu}{1-e^{-\eta q}V}$]

Now the term B is positive as the maximum possible utilities of the members have to be positive for participation of the members. The values of W and $\frac{dW}{db}$ would change depending on the nature of distribution of X. Thus, the next few subsections discuss how the Kalai Smorodinsky solutions work for different types of probability distributions of cost X.

Gamma Distributed Cost

As mentioned earlier, for a gamma distributed cost X, W satisfies the condition mentioned in the equation (5.20). Using the values of $\frac{dW}{db}$ from equation (5.32) in equation (5.58) and rearranging the value in terms of W gives

$$\begin{aligned}(-\frac{da}{db} - \mu)(1 + \eta e^{-\eta a} W B) &= B \left[e^{-\eta a} \left\{ \frac{\eta \omega \phi W}{(1 - \eta(1 - b)\phi} \right\} - \{\eta e^{-\eta a} W \mu\} \right] \\ &= B\eta e^{-\eta a} W \left[\frac{\omega \phi - \mu + \eta \mu (1 - b)\phi}{(1 - \eta(1 - b)\phi} \right] \\ &= B\eta e^{-\eta a} W \left[\frac{\eta \mu (1 - b)\phi}{(1 - \eta(1 - b)\phi} \right]\end{aligned}$$

or

$$(-\frac{da}{db} - \mu) = \frac{B\eta e^{-\eta a} W \left[\frac{\eta \mu (1-b)\phi}{\{1 - \eta(1-b)\phi\}} \right]}{(1 + B\eta e^{-\eta a} W)} \tag{5.59}$$

For a gamma distributed cost with parameters (scale : ϕ and shape: ω), it is shown in section 5.2.2 that $1 - \eta(1 - b)\phi > 0$ for $0 \leq b < 1$ and becomes zero for b = 1. W was also shown as positive in the case of models with Nash's bargaining. Thus, the right hand side of the above equation (5.59) is positive. Using this observation with the equation (5.26), it can be concluded that U_{pm} is increasing in b, for $0 \leq b < 1$ and attains a maximum value for b = 1.

In the equation (5.58), the right hand side represents $B\left(\frac{dU_{co}}{db}\right)$. It was already shown in the last paragraph that this is positive for $0 \leq b < 1$ and zero at b = 1. Thus, U_{co} is increasing in b , for $0 \leq b < 1$ and attains the maximum value at b = 1. These observations lead to the following lemma

Lemma 9. *With a cost based contract P = a+bX (where X follows a gamma distribution with shape parameter ω and scale parameter ϕ), the Kalai Smorodinsky value K, and the utility functions of the project manager and the contractor are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts ($0 < b < 1$).*

Exponential Distributed Cost

Using the values of $\frac{dW}{db}$ from the equation (5.34) and W from the equation (5.20) in equation (5.58)

$$\begin{aligned}(-\frac{da}{db}-\mu)(1+B\eta e^{-\eta a}W) &= B\left[e^{-\eta a}\left\{\eta\phi W^2\right\}-\left\{\eta e^{-\eta a}W\mu\right\}\right]\\ &= B\eta e^{-\eta a}W\left[\frac{\mu-\mu+\eta\mu(1-b)\phi}{(1-\eta(1-b)\phi}\right]\\ &= B\eta e^{-\eta a}W\left[\frac{\eta\mu(1-b)\phi}{(1-\eta(1-b)\phi}\right]\end{aligned}$$

or

$$(-\frac{da}{db}-\mu)=\frac{B\eta e^{-\eta a}W\left[\frac{\eta\mu(1-b)\phi}{(1-\eta(1-b)\phi}\right]}{(1+B\eta e^{-\eta a}W)} \tag{5.60}$$

For an exponentially distributed cost (with scale parameter: ϕ), it is shown in section 5.2.2 that $1-\eta(1-b)\phi>0$ for $0\leq b<1$ and becomes zero for b = 1. W was also shown as positive in the case of models with Nash's bargaining. Thus, the right hand side of the above equation (5.60) is positive. Using this observation with the equation (5.26), it can be concluded that U_{pm} is increasing in b, for $0\leq b<1$ and attains a maximum value for b = 1.

Similar to the case of gamma distributed cost, the right hand side in the equation (5.58) represents $B\left(\frac{dU_{co}}{db}\right)$. Thus, U_{co} is increasing in b , or $0\leq b<1$ and attains the maximum value at b = 1.

These observations lead to the following lemma

Lemma 10. *With a cost based contract P = a+bX (where X follows an exponential distribution with scale parameter ϕ), the Kalai Smorodinsky value K, and the utility functions of the project manager and the contractor are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts (*$0<b<1$*).*

Beta Distributed Cost

Using the value of $\frac{dW}{db}$ from equation (5.38) and the expanded form of W from equation (5.20) in equation (5.58) [For a beta distributed cost X ($0<X<\phi$; where ϕ is the scale parameter) with shape parameters c and d.]

$$
\begin{aligned}
&(-\frac{da}{db} - \mu)(1 + B\eta e^{-\eta a}W) \\
&= -Be^{-\eta a}\left[\frac{\eta\phi c}{c+d}\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\}\right] \\
&- B\eta e^{-\eta a}\mu\left[1 + \frac{\eta\phi(1-b)c}{(c+d)} + \frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)} + \cdots\right] \\
&= Be^{-\eta a}\eta\mu\left[\eta(1-b)\phi\left\{\frac{(c+1)(c+d) - c(c+d+1)}{(c+d)(c+d+1)}\right\}\right] \\
&+ Be^{-\eta a}\eta\mu\left[\frac{\eta^2(1-b)^2\phi^2}{2!}\left\{\frac{(c+1)(c+2)(c+d) - c(c+1)(c+d+2)}{(c+d)(c+d+1)(c+d+2)}\right\}\right] + \cdots \\
&= e^{-\eta a}\eta\mu\left[\eta(1-b)\phi\left\{\frac{d}{(c+d)(c+d+1)}\right\} + \frac{\eta^2(1-b)^2\phi^2}{2!}\left\{\frac{(c+1)2d}{(c+d)(c+d+1)(c+d+2)}\right\}\right]
\end{aligned}
$$

From the above equation, it can be shown that the right hand side of the equation is positive. This is because c and d are both assumed as positive. Following the steps shown in gamma and exponential distributed cost, it can be again shown that $\frac{dU_{pm}}{db} > 0$ and $\frac{dU_{co}}{db} > 0$ for $0 \leq b < 1$ and zero at b = 1. This leads to the following lemma

Lemma 11. *With a cost based contract P = a+bX (where X follows a beta distribution with scale ϕ, and shape parameters c & d), the Kalai Smorodinsky value K, and the utility functions of the project manager and the contractor are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts (*$0 < b < 1$*).*

Weibull Distributed Cost

Using the value of $\frac{dW}{db}$ from the equation (5.41) and the expanded form of W from the equation (5.20) in equation (5.58) [For a Weibull distributed cost with shape parameter S and scale parameter ϕ]

$$
\begin{aligned}
&(-\frac{da}{db} - \mu)(1 + B\eta e^{-\eta a} W) \\
&= -e^{-\eta a} B\left[-\eta\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\} - \frac{2\eta^2\theta^2(1-b)}{2!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\} - \cdots\right] \\
&- B\eta e^{-\eta a}\mu\left[1 + \left(\frac{\eta(1-b)\phi}{1!}\right)\Gamma\left(1+\frac{1}{S}\right) + \frac{\eta^2(1-b)^2\phi^2}{2!}\Gamma\left(1+\frac{2}{S}\right) + \cdots\right] \\
&= B\eta e^{-\eta a}\left[\eta(1-b)\phi^2\left\{\Gamma\left(1+\frac{2}{S}\right) - \Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{1}{S}\right)\right\}\right] \\
&+ B\eta e^{-\eta a}\left[\frac{\eta^2(1-b)^2\phi^3}{2!}\left\{\Gamma\left(1+\frac{3}{S}\right) - \Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{2}{S}\right)\right\}\right] + \cdots
\end{aligned}
$$

It was assumed earlier that $\eta > 0$; and W and B were shown as positive for the requirement of participation in the bargaining game. Based on the derivation mentioned in the sub-section (5.2.2), the right hand side of the above equation can be numerically shown as positive for $0 \leq b < 1$ and zero with b=1. This, leads to the conclusion that $-\frac{da}{db} - \mu > 0$ for $0 \leq b < 1$ and $-\frac{da}{db} - \mu = 0$ for b=1. Following the steps shown in gamma and exponential distributed cost, it can be again shown that $\frac{dU_{pm}}{db} > 0$ and $\frac{dU_{co}}{db} > 0$ for $0 \leq b < 1$ and zero at b = 1. This leads to the following lemma

Lemma 12. *With a cost based contract P = a+bX (where X follows a Weibull distribution with scale ϕ, and shape parameter S), the Kalai Smorodinsky value K, and the utility functions of the project manager and the contractor are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts (*$0 < b < 1$*).*

Similar to the calculations for gamma, exponential, Beta, and Weibull distributed cost functions, it can shown that the solutions from the cost plus contracts dominate the solutions and are able to offer win-win solutions for the project manager and the contractor. Hence, combining the findings from lemmas 9, 10, 11, and 12, the following is proposed

Proposition 8. *Using the Kalai Smorodinsky bargaining with a cost based contract P = a+bX (where X can follow any non-normal continuous distribution, and a & b are contract parameters), the solutions for the Kalai Smorodinsky value K, and the utilities of the project manager & the contractor are the dominant solution for a cost plus contract (b = 1). This dominates the solutions from any cost sharing contract (*$0 < b < 1$*) and fixed price contract* *(b = 0). The optimal value of contract parameter a_1 satisfies the following*

$$a_1 = \begin{cases} \frac{q-a_1-\omega\phi}{q-\omega\phi} - \frac{1-e^{-\eta a_1}}{1-e^{-\eta q}\frac{1}{(1-\eta\phi)^{\omega}}} = 0 & \textit{for a gamma distributed cost} \\ \frac{q-a_1-\phi}{q-\phi} - \frac{1-e^{-\eta a_1}}{1-e^{-\eta q}\frac{1}{(1-\eta\phi)^{\omega}}} = 0 & \textit{for an exponential distributed cost} \\ \frac{q-a_1-\frac{\phi c}{c+d}}{q-\frac{\phi c}{c+d}} - \frac{1-e^{-\eta a_1}}{1-e^{-\eta q}\left[1+\sum_{i=1}^{\infty}\left(\phi^i\prod_{r=0}^{i-1}\frac{c+r}{c+d+r}\right)\frac{\eta^i}{i!}\right]} = 0 & \textit{for and beta distributed cost} \\ \frac{q-a_1-\phi\Gamma\left(1+\frac{1}{S}\right)}{q-\phi\Gamma\left(1+\frac{1}{S}\right)} - \frac{1-e^{-\eta a_1}}{1-e^{-\eta q}\left[1+\sum_{i=1}^{\infty}\left\{\frac{\eta^i\phi^i}{i!}\Gamma\left(1+\frac{i}{S}\right)\right\}\right]} = 0 & \textit{for a Weibull distributed cost} \end{cases} \tag{5.61}$$

Proof. Since the cost plus contract dominates the other solutions, thus the optimal value is b = 1. Hence, the optimal value for a is $a = a_1$. The optimal condition for a_1 should satisfy the condition in equation (5.55). The value of μ was replaced using equation (5.22). As defined, $W = E\left(e^{\eta(1-b)X}\right)$. Thus, (for b = 0), $V = W_0$ as V is assumed as $E(e^{\eta X})$. Replacing the values of V for the selected distributions from the equations (5.21), the optimal conditions for a_1 in the proposition 8 can be derived. ☐

5.4 Bargaining Models of Supply Chain Coordination with Cost based Contracts: Utilitarian Approach to Bargaining

According to the Utilitarian rule, the sum of the utilities during the bargaining is maximized. Thus,

$$U(Z,d) = arg\ \max_{u\in Z}\sum_{i=1}^{2}u_i \tag{5.62}$$

This research derives the model for two cases: in the first case with both risk averse members; and in the second case with a risk neutral project manager and a risk averse contractor. The analysis is shown in the next two sub-sections.

5.4.1 For a Risk Neutral Project Manager and a Risk Neutral Contractor

As mentioned earlier, the utility functions for the project manager and the risk neutral contractor follow equations (5.4) and (5.5). Thus, the equation (5.62) for the case of both risk neutral members is as follows

$$U(\psi, d) = arg \max_{u \in \psi}[(q - \mu)] \tag{5.63}$$

Thus, differentiating the above equations with respect to either contract parameters a or b, would yield the first order condition as zero. This means, that for the utilitarian approach with both risk neutral members, the solution is indifferent for fixed price or cost plus contracts. Due to the simplicity of the application, members of the supply chain might be inclined to use fixed price contracts in practice.

5.4.2 For a Risk Neutral Project Manager a Risk Averse Contractor

Using the utility functions from the equations 5.4 and 5.6 in the equation (5.62)

$$U(S, d) = arg \max_{u \in S} \left[(q - a - b\mu) + (1 - e^{-\eta a} W)\right] \tag{5.64}$$

In order to get the optimal solutions for contract parameters a, the equation (5.64) is differentiated with respect to a and set it equal to zero as below.

$$\frac{dU(S,d)}{da} = -1 + \eta e^{-\eta a} W = 0$$

Rearranging the terms of the above equation, the first order condition for a is as follows

$$a = \frac{1}{\eta} \log_e(\eta) + \frac{1}{\eta} \log_e(W) \tag{5.65}$$

In order to find the optimal conditions for b ($0 \leq b \leq 1$), the equation (5.64) is differentiated with respect to b and rearranging the terms

$$\frac{dU(S,d)}{db} = \left(\frac{da}{db}\right) \left(\eta e^{-\eta a} W - 1\right) - \mu - e^{-\eta a} \left(\frac{dW}{db}\right) \tag{5.66}$$

Now

$$\frac{da}{db} = \frac{1}{\eta W}\left(\frac{dW}{db}\right) \tag{5.67}$$

Thus, using this value of $\frac{da}{db}$ the equation (5.66) becomes

$$\frac{dU(S,d)}{db} = -\frac{1}{\eta W}\left(\frac{dW}{db}\right) - \mu \tag{5.68}$$

The values of W and $\frac{dW}{db}$ would vary depending on the nature of distribution. Thus, the next few subsections discuss how the model would work for different distributions of cost function X.

Gamma Distributed Cost

Using the values of W for gamma distribution from the equation (5.20) and $\frac{dW}{db}$ from equation (5.32) in equations (5.66) and (5.67)

$$\frac{da}{db} = -\frac{\phi\omega}{1-\eta(1-b)\phi} \tag{5.69}$$

and

$$\begin{aligned}
\frac{dU(S,d)}{db} &= -\frac{\phi\omega}{\{1-\eta(1-b)\phi\}}\left[\frac{\eta e^{-\eta a}}{\{1-\eta(1-b)\phi\}^{\omega}} - 1\right] - \mu + \left[\frac{e^{-\eta a}\eta\omega\phi W}{\{1-\eta(1-b)\phi\}}\right] \\
&= \frac{-\phi\omega\eta e^{-\eta a} + \{1-\eta(1-b)\phi\}^{\omega}(\omega\phi) - \mu\{1-\eta(1-b)\phi\}^{\omega+1} + e^{-\eta a}\eta\omega\phi}{\{1-\eta(1-b)\phi\}^{\omega+1}} \\
&= \frac{\{1-\eta(1-b)\phi\}^{\omega}\mu\left[1-\{1-\eta(1-b)\phi\}\right]}{\{1-\eta(1-b)\phi\}^{\omega+1}} \\
&= \frac{\mu\eta(1-b)\phi}{\{1-\eta(1-b)\phi\}}
\end{aligned} \tag{5.70}$$

As mentioned earlier, $\{1-\eta\phi(1-b)\} > 0$. Thus, the the U(S,d) is increasing in b in this model for $0 \leq b < 1$ and zero at b = 1.

Now differentiating U_{pm} with respect to b

$$\begin{aligned}\frac{dU_{pm}}{db} &= -\frac{da}{db} - \mu \\ &= \frac{\phi\omega}{1-\eta(1-b)\phi} - \mu \\ &= \frac{\mu\eta(1-b)\phi}{1-\eta(1-b)\phi} \qquad \text{[using the value of } \mu \text{ for the gamma distributed cost]} \end{aligned} \tag{5.71}$$

Using the observation from equation (5.70), this can be shown that right hand side of equation (5.71) is positive for $0 \leq b < 1$ and zero at b = 1. Thus, U_{pm} is increasing in b for $0 \leq b < 1$ and maximum at b =1.

Differentiating U_{co} with respect to b $\frac{dU_{co}}{db} = \eta e^{-\eta a} W \frac{da}{db} - e^{-\eta a} \frac{dW}{db}$. Using the value of $\frac{da}{db}$ from equation (5.67), it can be shown that $\frac{dU_{co}}{db} = 0$. This also means the second order derivative is also zero. Thus, this leads to inconclusive findings regarding the movement of U_{co} with respect to b. Thus, for an utilitarian bargaining approach, the utility of the contractor is neither increasing nor decreasing as b moves from 0 to 1.

Thus, summarising the above observations, the following lemma is proposed

Lemma 13. *With a cost based contract P = a+bX (where X follows a gamma distribution with scale ϕ, and shape parameter ω), the Utilitarian sum U, and the utility functions of the project manager are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts ($0 < b < 1$). However, the contractor's utility doesn't change for any values of b for $b \in [0, 1]$.*

Exponential Distributed Cost

Using the values of W from equation (5.20) and $\frac{dW}{db}$ from equation (5.34) in equations (5.66) and (5.67) gives

$$\frac{da}{db} = -\frac{\phi}{1-\eta(1-b)\phi} = -\phi W \tag{5.72}$$

and

$$\begin{aligned}\frac{dU(S,d)}{db} &= -\phi W\left[\eta e^{-\eta a}W - 1\right] - \mu + \left[e^{-\eta a}\eta\phi W^2\right] \\ &= \phi W - \mu \\ &= \mu\left[\frac{1}{\{1-\eta(1-b)\phi\}} - 1\right] \qquad \text{[Replacing the mean value, } \mu = \phi] \\ &= \frac{\mu\eta(1-b)\phi}{\{1-\eta(1-b)\phi\}} \end{aligned} \tag{5.73}$$

Thus, similar to the gamma distributed cost, the U(S,d) is increasing in b for $0 \leq b < 1$ and zero for b = 1.

Now, differentiating U_{pm} with respect to b

$$\begin{aligned}\frac{dU_{pm}}{db} &= -\frac{da}{db} - \mu \\ &= \frac{\phi}{1-\eta(1-b)\phi} - \mu \\ &= \frac{\mu\eta(1-b)\phi}{1-\eta(1-b)\phi}\end{aligned} \tag{5.74}$$

Using the observation from equation (5.73), it can be seen that the right hand side of equation (5.74) is positive for $0 \leq b < 1$ and zero at b = 1. Thus, U_{pm} is increasing in b for $0 \leq b < 1$ and zero at $b = 1$.

Similar to the gamma distributed case, it can be easily shown $\frac{dU_{co}}{db} = 0$. That means U_{co} doesn't change for any change in $b \in [0, 1]$.

Thus, summarising the above observations, the following lemma is proposed

Lemma 14. *With a cost based contract P = a+bX (where X follows an exponential distribution with scale ϕ), the Utilitarian sum U, and the utility functions of the project manager are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts ($0 < b < 1$). However, the contractor's utility doesn't change for any values of b in $b \in [0, 1]$.*

Beta Distributed Cost

Using these values of W for the beta distribution from equation (5.20) and $\frac{dW}{db}$ from equation (5.36) or (5.36) in equations (5.66) and (5.67)

$$\frac{da}{db} = -\frac{\mu}{W}\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\} \tag{5.75}$$

[The mean value of beta distributed cost $\mu = \frac{\phi c}{c+d}$]

and

$$\begin{aligned}
&\frac{dU(S,d)}{db} \\
&= -\frac{\mu}{W}\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\}\left[\eta e^{-\eta a}W - 1\right] \\
&- \mu + e^{-\eta a}\left[\eta\mu\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\}\right] \\
&= \frac{\mu\left[\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\} - W\right]}{W} \\
&= \frac{\mu\left[\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\} - \left\{1 + \frac{\eta\phi(1-b)c}{c+d} + \frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)} + \cdots\right\}\right]}{1 + \frac{\eta\phi(1-b)c}{c+d} + \frac{\eta^2\phi^2(1-b)^2c(c+1)}{2!(c+d)(c+d+1)} + \cdots}
\end{aligned}$$

From subsection 5.2.2, it can be shown that the right hand side of the above equation is positive for $0 \leq b < 1$ and zero for b = 1. This leads to the conclusion that $U(S,d)$ is increasing in b for $0 \leq b < 1$ and reaches a maximum at b = 1.

Now, differentiating U_{pm} with respect to b

$$\begin{aligned}
\frac{dU_{pm}}{db} &= -\frac{da}{db} - \mu \\
&= \frac{\mu}{W}\left\{1 + \frac{\eta(1-b)\phi(c+1)}{(c+d+1)} + \frac{\eta^2(1-b)^2\phi^2(c+1)(c+2)}{2!(c+d+1)(c+d+2)} + \cdots\right\} - \mu \\
&= \frac{dU(S,d)}{db}
\end{aligned}$$

Thus, $\frac{dU_{pm}}{db} > 0$ for $0 \leq b < 1$ and zero at b = 1. Thus, the utility of the project manager increases for $b \in [0,1)$ and maximum at b = 1. It is also interesting to see that $\frac{dU(S,d)}{db} = \frac{dU_{pm}}{db}$. That means $\frac{dU_{co}}{db} = 0$ and so the utility of the contractor does not change for any change in $b \in [0,1]$.

Thus, summarising the above observations, the following lemma is proposed

Lemma 15. *With a cost based contract P = a+bX (where X follows a beta distribution with scale ϕ and shape parameters c & d), the Utilitarian sum U, and the utility functions of the project manager are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts ($0 < b < 1$). However, the contractor's utility doesn't change for any values of b for $b \in [0, 1]$.*

Weibull distributed cost

Using the values of W and $\frac{dW}{db}$ from equations (5.20) and (5.41) in equations (5.66) and (5.67)

$$\frac{dU(S,d)}{db}$$

$$= \frac{1}{\eta}\left[\frac{\eta\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{2\eta^2\phi^2(1-b)}{2!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\}+\frac{3\eta^3\phi^3(1-b)}{3!}\left\{\Gamma\left(1+\frac{3}{S}\right)\right\}+\cdots}{1+\left(\frac{\eta(1-b)\phi}{1!}\right)\Gamma\left(1+\frac{1}{S}\right)+\frac{\eta^2(1-b)^2\phi^2}{2!}\Gamma\left(1+\frac{2}{S}\right)+\frac{\eta^3(1-b)^3\phi^3}{3!}\Gamma\left(1+\frac{3}{S}\right)+\cdots}\right]-\left[\phi\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}\right]$$

$$= \left[\frac{\eta(1-b)\phi^2\left\{\Gamma\left(1+\frac{2}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta^2(1-b)^2\phi^3}{2!}\left\{\Gamma\left(1+\frac{3}{S}\right)-\Gamma\left(1+\frac{1}{S}\right)\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}{1+\eta\phi(1-b)\left\{\Gamma\left(1+\frac{1}{S}\right)\right\}+\frac{\eta^2\phi^2(1-b)^2}{2!}\left\{\Gamma\left(1+\frac{2}{S}\right)\right\}+\cdots}\right]$$

As mentioned in the section (5.2.2), the right hand side of the above equation can be shown to be positive numerically in the interval $0 \leq b < 1$. and becomes zero with b=1. Thus, U(S,d) is increasing in b for $0 \leq b \leq 1$ and attains the maximum value at b = 1. Thus, similar to the cases with gamma, exponential and beta distributed costs in sub-section (5.4.2), it can be shown that $\frac{dU_{pm}}{db} = \frac{dU(S,d)}{db}$ and $\frac{dU_{co}}{db} = 0$. Thus, similar to the cases with other distributions, the project manager's utility is increasing for $0 \leq b < 1$ and becomes maximum at b = 1. However, for the contractor, the utility remains remains unchanged for $b \in [0, 1]$.

Thus, summarising the above observations, the following lemma is proposed

Lemma 16. *With a cost based contract P = a+bX (where X follows a Weibull distribution with scale ϕ and shape parameters S), the Utilitarian sum U, and the utility functions of the project manager are higher under the cost plus contract (b = 1) than under the fixed price contract (b = 0) or any other cost sharing contracts ($0 < b < 1$). However, the contractor's utility doesn't change for any values of b for $b \in [0, 1]$.*

Combining the findings from lemmas 13, 14, 15, and 16, the following is proposed

Proposition 9. *Using the utilitarian bargaining approach with a cost based contract P=a+bX, the solutions derived from a cost plus contract (b = 1) dominates the utilitarian sum U(S,d) and the utility value of the project manager over the solutions derived from any other cost based contract (*$0 < b < 1$*) or fixed price contract (b = 0). However, the utility value for the contractor remains same for* $b \in [0, 1]$*. The optimal value of a satisfies the condition in equation (5.65). The W value in the equation (5.65) satisfies the conditions in the equation (5.20).*

It can be easily shown that the contractor would earn a higher profit with the fixed price contract in this case. As a result, it can not be clearly said if the contractor would be better off with a cost plus contract in this case. Moreover, another important shortcomings of this bargaining approach is that it does not conform to the individual rationality constraint.

5.5 Bargaining Models of Supply Chain Coordination with Time Based Contracts: Nash's Bargaining

With a given contract P= g- hT, (where T is a random time variable), if the project manager and the contractor would negotiate based on the Nash bargaining, then they would maximize the Nash product as below

$$N(g, h) = U_{PM}(g, h) * U_{co}(g, h) \tag{5.76}$$

5.5.1 For a Risk Neutral Project Manager and a Risk Neutral Contractor

The utility functions of the project manager and the contractor follow equations (5.7) and (5.8) respectively. These are substituted to the equation (5.76) Differentiating this equation with respect to g gives

$$q + (2h + \Omega)\mu_T - 2g = 0 \tag{5.77}$$

From this, $g* = \frac{q+(2h+\Omega)\mu_T}{2}$. Hence, $\frac{dg*}{dh} = \mu_T$. Using this in $\frac{dU_{pm}}{dh} = -\frac{dg}{dh} + \mu_T$, it can be shown that $\frac{dU_{pm}}{dh} = 0$. Similarly, it can be shown $\frac{dU_{co}}{dh} = \frac{dg}{dh} - \mu_T = 0$. Hence, the utilities of the project manager and the contractor are not changing with respect to any change in h. Using these observations in $\frac{dN(g,h)}{dh}$, it could be shown the Nash product also remains unchanged with respect to any change in h. Thus, selection of either fixed price or time based contracts would not make any difference to either the project manager or the contractor. However, the fixed price contracts are easy to implement and hence, the members of the supply chain would be inclined to use that.

5.5.2 For a Risk Neutral Project Manager and a Risk Averse Contractor

The utility values from the equations (5.7) and (5.9) are substituted to equation (5.16). Then, differentiating with respect to g and setting that equal to zero is the first order condition

$$\frac{dN(g,h)}{dg} = e^{-\eta g} W_T \left\{ \eta(-g + h\mu_T + q) + 1 \right\} - 1 = 0 \tag{5.78}$$

[where $W_T = E\{e^{\eta(h+\Omega)T}\}$].

The W_T value can be considered as the risk exposure of the contractor and can be calculated based on the concepts of moment generating functions as below

$$W_T = E[e^{tT}] = \begin{cases} \frac{1}{(1-\theta t)^w} \\ \text{gamma distributed cost with shape parameter } \theta \text{ and scale parameter w} \\ \frac{1}{(1-\theta t)} \\ \text{exponential distributed cost with scale parameter } \theta \\ 1 + \sum_{m=1}^{\infty} \left(\theta^m \prod_{j=0}^{m-1} \frac{u+j}{u+v+j} \right) \frac{t^m}{m!} \\ \text{beta distributed cost with shape parameters u \& v and scale } \theta \\ 1 + \sum_{m=1}^{\infty} \left[\frac{t^m \theta^m}{m!} \Gamma\left(1 + \frac{m}{s}\right) \right] \\ \text{Weibull distributed cost with shape parameter s and scale parameter } \theta \end{cases} \tag{5.79}$$

where $t = \eta(h + \Omega)$

Differentiating the equation (5.76) with respect to h

$$\frac{dN(g,h)}{dh} = U_{co}\left(\frac{dU_{PM}}{dh}\right) + U_{PM}\left(\frac{dU_{co}}{dh}\right) \tag{5.80}$$

Differentiating equation (5.7) with respect to h, and using the value of $E(x) = \mu_T$

$$\frac{dU_{PM}}{dh} = -\frac{dg}{dh} + \mu_T \tag{5.81}$$

Rearranging the terms from equation (5.78),

$$(q - g + h\mu_T) = \frac{1 - e^{-\eta g}W_T}{\eta e^{-\eta g}W_T} = \frac{1 - A_T}{\eta A_T} \qquad \text{[where } A_T = e^{-\eta g}W_T\text{]} \tag{5.82}$$

Differentiating equation (5.82)

$$-\frac{dg}{dh} + \mu_T = -\frac{1}{\eta A_T^2}\frac{dA_T}{dh} \tag{5.83}$$

Now

$$\frac{dA_T}{dh} = -\eta e^{-\eta g}W_T\frac{dg}{dh} + e^{-\eta A_T}\frac{dW_T}{dh} = -\eta A_T\frac{dg}{dh} + e^{-\eta A_T}\frac{dW_T}{dh} \tag{5.84}$$

Using the values of W_T from the equation (5.79) and the value of $\frac{dA_T}{dh}$ from (5.84) in equation (5.83)

$$-\frac{dg}{dh} + \mu_T = -\frac{1}{\eta A_T^2}\left[-\eta A_T\frac{dg}{dh} + \frac{A_T}{W_T}\frac{dW_T}{dh}\right]$$

or

$$\left[-\frac{dg}{dh} + \mu_T\right]\left[1 + \frac{1}{A_T}\right] = \left[-\left(\frac{1}{\eta A_T W_T}\right)\frac{dW_T}{dg} + \left(\frac{\mu_T}{A_T}\right)\right]$$

or

$$\left[-\frac{dg}{dh} - \mu_T\right]\left[1 + \frac{1}{A_T}\right] = -\frac{1}{A_T}\left[\left(\frac{1}{\eta W_T}\right)\frac{dW_T}{dh} - \mu_T\right] \tag{5.85}$$

As mentioned earlier, $U_{co} = 1 - e^{-\eta g}W_T$. Based on the assumption made in equation (5.82), $U_{co} = 1 - A_T$. Thus, differentiating both side with respect to h

$$\frac{dU_{co}}{dh} = -\frac{dA_T}{dh} \tag{5.86}$$

Thus, the signs of $\frac{dU_{pm}}{dh}$ and $\frac{dU_{co}}{dh}$ depend on the signs of $\frac{dW_T}{dh}$, and $\frac{dA_T}{dh}$ respectively. The sign tests of these derivatives depend on the nature of distribution of the time function T.

Differentiating the value of W_T for gamma distributed time with respect to h gives

$$\frac{dW_T}{dh} = \frac{\eta w \theta}{\{1 - \eta\theta h\}^{w+1}} = \frac{\eta w \theta W_T}{\{1 - \eta\theta(h + \Omega)\}} \tag{5.87}$$

As assumed before, $\eta > 0$. It can also be shown that $q - g + h\mu_T > 0$; otherwise the project manager's utility would be negative and she would never participate in the bargaining. Hence, from the equation (5.78), it can be shown that $W_T = \frac{1}{\{1-\eta\theta(h+\Omega)\}^w} > 0$. This true for any positive w. Hence, $\frac{dW_T}{dh}$ is positive. Thus, the risk exposure of the contractor increases with increase in h.

Based on the calculations from chapter 4, $\mu_T = w\theta$ and the value of $\frac{dW_T}{dh}$ from equation (5.87) in equation (5.85),

$$\begin{aligned}\left[-\frac{dg}{dh} + w\theta\right]\left(1 + \frac{1}{A_T}\right) &= -\frac{1}{A_T}\left[\left(\frac{1}{\eta W_T}\right)\left\{\frac{\eta w\theta W_T}{1 - \eta\theta(h + \Omega)}\right\} - w\theta\right] \\ &= -\frac{w\theta}{A_T}\left[\frac{1 - 1 + \eta\theta h}{\{1 - \eta\theta h\}}\right] \\ &= -\left[\frac{w\eta\theta^2(h + \Omega)}{A_T\{1 - \eta\theta(h + \Omega)\}}\right]\end{aligned}$$

or

$$\left[-\frac{dg}{dh} + w\theta\right] = -\left(\frac{1}{1 + A_T}\right)\left[\frac{w\eta\theta^2(h + \Omega)}{\{1 - \eta\theta(h + \Omega)\}}\right] \tag{5.88}$$

In the equation (5.88), the right hand side of the equation is negative. Thus, the value of the term $(-\frac{dg}{dh} + w\theta)$ is negative for a gamma distributed time T when h is positive and zero at h =0.

Rearranging the terms from equation (5.83) and using the value of μ,

$$\left[-\frac{dg}{dh} - w\theta\right]\left(\eta A_T^2\right) = -\frac{dA_T}{dh}$$

Differentiating both side of the equation (5.88) with respect to h, it can be shown that $-\frac{d^2g}{dh^2} < 0$. Now $\frac{d^2U_{pm}}{dh^2} = -\frac{d^2g}{dh^2}$. Using this observation in equation (5.81), U_{PM} is found to be concave and decreasing in h. This leads to the conclusion that $\frac{dU_{co}}{dh} < 0$ for any positive h and zero at h = 0.

It is shown $-\frac{dg}{dh} - w\theta < 0$ for any positive h value and $\eta > 0$. Thus, $-\frac{dA_T}{dh} < 0$. It can also be shown that $\frac{d^2U_{co}}{dh^2} < 0$. This means U_{co} is concave and decreasing in any positive h.

Using these above observations in equation (5.80), it can be shown that $\frac{dN(g,h)}{dh} < 0$ for any positive value of h and zero at h = 0.

Similar to this case of gamma distributed time, it can be shown that the utilities and the Nash product are maximum at h=0 and decreasing at any positive value of h for other continuous time distribution. This leads to the following proposition

Proposition 10. *With a time based contract P=g-hT (where time function T follows any continuous distribution), the Nash product and the utility functions for project manager & the contractor are maximum under a fixed price contract (with h = 0). These values decrease with the increase in positive h values. In other words the solution for the fixed price contracts dominates the solutions of the time based contracts*

5.6 Bargaining Models of Supply Chain Coordination with Time Based Contracts: Kalai and Smorodinsky Bargaining

The utility functions for the project manager and the contractor remain the same as described in equations (5.7) and (5.8) for the risk neutral case and (5.9) for the risk averse case respectively.

The Kalai Smorodinsky model mentioned in the equation (5.47) is applied to the time based contract (with P= g- hT) case. Thus, the aspiration point for the project manager and the contractor are respectively as follow

For the project manager

$$a_{pm}(Z, d) = q - \Omega\mu_T \tag{5.89}$$

For the risk neutral contractor

$$a_{co}(Z, d) = q - \Omega\mu_T \tag{5.90}$$

For the risk averse contractor

$$\begin{aligned} a_{co}(S,d) = 1 - E[e^{-\eta\{q-k\left(\frac{\mu_1}{\mu_T}\right)^{\frac{n}{A}}T\}}] &= 1 - e^{-\eta q}E\{e^{\eta\Omega T}\} \quad \text{as assumed earlier} \\ &= 1 - e^{-\eta q}\rho \quad \text{where } \rho = E\{e^{\eta\Omega T}\} \end{aligned} \tag{5.91}$$

5.6.1 For a risk neutral project manager and a risk neutral contractor

The utility functions of the project manager and the contractor follow equations (5.7) and (5.8). As shown in the case of the cost based contracts, the normalized utilities of the project manager and the contractor are equal at the optimal value. Hence,

$$\frac{q - g + h\mu_T}{q - \Omega\mu_T} = \frac{g - h\mu_T - \Omega\mu_T}{q - \Omega\mu_T} \tag{5.92}$$

From the above equation, the optimal condition for g becomes, $g* = \frac{q+(2h+\Omega)\mu_T}{2}$. This is the same solution as found in the case of Nash bargaining with time based contracts (Shown in section 5.5.1). Hence, it follows from the section 5.5.1 that $\frac{dU_{pm}}{dh} = 0$, and $\frac{dU_{co}}{dh} = 0$. It can be easily shown that $\frac{dK(Z,d)}{dh} = 0$. Thus, the selection of either the fixed price or the time based contract would not make any difference to either the project manager or the contractor. As stated earlier, due to simplicity, the members of the supply chain (the project manager and the contractor) would be inclined to use the fixed price contract.

5.6.2 For a Risk Neutral Project Manager and a Risk Averse Contractor

Similar to the analysis for cost based contracts, the Kalai Smorodinsky bargaining solutions should satisfy the condition mentioned in the equation (5.47). Using these values of aspiration point, the normalized individual rationality of the members of the supply chain are as follows

For the project manager

$$U_{pmn} = \frac{q - g + h\mu_T}{q - \Omega\mu_T} \tag{5.93}$$

For the contractor

$$U_{con} = \frac{1 - e^{-\eta g}\rho W_T}{1 - e^{-\eta q}\rho} \tag{5.94}$$

[The utility before normalization follows equation (5.9)and $W_T = E\{e^{\eta(h+\Omega)T}\}$]

In order to satisfy the condition for optimal K, the minimum values of the two fractions on the right hand side of equations (5.53) and (5.54) should be maximized. As mentioned earlier in the case of cost based contracts, when the minimum of these two fractions are maximized, they become equal in value. Thus, at the optimal solution

$$\frac{q - g + h\mu_T}{q - \Omega\mu_T} = \frac{1 - e^{-\eta g}\rho W_T}{1 - e^{-\eta q}\rho} \tag{5.95}$$

In order to identify if the solution with fixed price or time based contract dominates, it is required to identify how the utility functions of the project manager and the contractor and the K value performs with respect to the movement of h .

To determine this, both sides of equation (5.95) are differentiated with respect to h.

$$\left(-\frac{dg}{dh} + \mu_T\right)\left(\frac{1}{q - \Omega\mu_T}\right) = \left[\eta e^{-\eta g}\rho W_T\frac{dg}{dh} - e^{-\eta g}\rho\frac{dW_T}{dh}\right]\left(\frac{1}{1 - e^{-\eta q}\rho}\right)$$

or

$$\left(-\frac{dg}{dh} + \mu_T\right) = \left[\eta e^{-\eta g}\rho W_T\frac{dg}{dh} - e^{-\eta g}\rho\frac{dW_T}{dh}\right]B_T \quad \text{where} \quad \left[B_T = \left(\frac{q - \Omega\mu_T}{1 - e^{-\eta q}\rho}\right)\right]$$

or

$$(-\frac{dg}{dh} + \mu_T)(1 + \eta e^{-\eta a}\rho W_T B_T) = \left[-e^{-\eta g}\rho\frac{dW_T}{dh} + \eta e^{-\eta g}\rho W_T\mu_T\right]B_T \tag{5.96}$$

Now the term B_T is positive as the maximum possible utilities of the members have to be positive for participation of the members. The values of W_T and $\frac{dW_T}{dh}$ change depending on the nature of the distribution of T.

For a gamma distributed time with scale parameter θ and shape parameter w, the W_T satisfies the condition mentioned in the equation (5.79) Using the values of $\frac{dW_T}{dh}$ from equation

(5.87) in equation (5.96) and rearranging the value in terms of W_T

$$\begin{aligned}(-\frac{dg}{dh}+\mu_T)(1+\eta e^{-\eta a}\rho W_T B_T) &= B_T\left[-e^{-\eta a}\rho\left\{\frac{\eta w\theta W_T}{(1-\eta(h+\Omega)\theta)}\right\}+\{\eta e^{-\eta g}\rho W_T\mu_T\}\right]\\ &= B_T\eta e^{-\eta g}\rho W_T\left[\frac{-w\theta+\mu_T-\eta\mu_T(h+\Omega)\theta}{(1-\eta(h+\Omega)\theta)}\right]\\ &= -B_T\eta e^{-\eta g}\rho W_T\left[\frac{\eta\mu_T(h+\Omega)\theta}{(1-\eta h\theta)}\right]\end{aligned}$$

or

$$(-\frac{dg}{dh}+\mu_T) = -\frac{B_T\eta e^{-\eta g}\rho W_T\left[\frac{\eta\mu_T(h+\Omega)\theta}{(1-\eta(h+\Omega)\theta)}\right]}{(1+\eta e^{-\eta a}\rho W_T B_T)} \tag{5.97}$$

For a gamma distributed time with parameters (scale : θ and shape: w), it is shown in section 5.5 that $(1-\eta(h+\Omega)\theta > 0)$. W_T was also shown as positive. Thus, the right hand side of the above equation (5.97) is negative for any positive value of h and becomes zero when h = 0. Thus, it can be concluded that U_{pm} is decreasing in h for any positive value of h.

In the equation (5.96), the right hand side represents $B_T\left(\frac{dU_{co}}{dh}\right)$. It has already been shown in the last paragraph that this is negative for any positive value of h and zero when h = 0. Hence, it can be shown that U_{co} is decreasing in h.

Similar to the case of gamma distributed time, it can be shown the utilities of the project manager and the contractor and the Kalai Smorodinsky value K decreases with increase in any positive h. Hence, the following is proposed

Proposition 11. *With a time based based contract P = g-hT (where T follows any continuous distribution), the Kalai Smorodinsky value K, and the utility functions of the project manager & the contractor are higher under the fixed price contract (h = 0) than under any of the time based contract.*

5.7 Bargaining Models of Supply Chain Coordination with Time based Contracts: Utilitarian Approach to Bargaining

In this case, the total utility is maximized according to the equation (5.62). As described earlier, this research derives the model for two cases: in the first case with both risk averse members; and in the second case with a risk neutral project manager and a risk averse contractor. The analysis is shown in the next two sub-sections.

5.7.1 For a Risk Neutral Project Manager and a Risk Neutral Contractor

As mentioned earlier, the utility functions for the project manager and the risk neutral contractor follow the equations (5.7) and (5.8). Thus, with a time based contract P =g - hT, equation (5.62) for the case of both risk neutral members is as follows

$$U(Z, d) = arg \max_{u \in Z} [(q - \Omega \mu_T)] \tag{5.98}$$

Thus, differentiating the above equations with respect to the either contract parameters g or h, yields the first order condition as zero. This means, that for the utilitarian approach with both risk neutral members, the solution is indifferent for fixed price or time based contracts. Due to the simplicity of the application, the members of the supply chain would be inclined to use fixed price contracts in practice.

5.7.2 For a Risk Neutral Project Manager and a Risk Averse Contractor

With the utility functions of the project manager and the contractor following equations (5.7) and (5.9), the optimization problem becomes

$$U(Z, d) = arg \max_{u \in Z} \left[(q - g + h\mu_T) + (1 - e^{-\eta g} \rho W_T)\right] \tag{5.99}$$

In order to get the optimal solutions for contract parameters g, equation (5.99) is differentiated with respect to g and the equation is set to zero.

$$\frac{dU(Z, d)}{dg} = -1 + \eta e^{-\eta g} \rho W_T = 0$$

Rearranging the terms of the above equation, the first order condition for g is as follows

$$g* = \frac{1}{\eta} \log_e(\eta) + \frac{1}{\eta} \log_e(W_T) + \frac{1}{\eta} \log_e(\rho) \tag{5.100}$$

In order to find the optimal conditions for b ($0 \leq b \leq 1$), equation (5.64) is differentiated with respect to b and rearranging the terms gives

$$\frac{dU(Z,d)}{dh} = \left(\frac{dg}{dh}\right)\left(\eta e^{-\eta g}\rho W_T - 1\right) + \mu_T - e^{-\eta g}\rho\left(\frac{dW_T}{dh}\right) \tag{5.101}$$

Now from equation (5.100)

$$\frac{dg}{dh} = \frac{1}{\eta W_T}\left(\frac{dW_T}{dh}\right) \tag{5.102}$$

Thus, using this value of $\frac{dg}{dh}$ equation (5.102) becomes

$$\frac{dU(Z,d)}{dh} = -\frac{1}{\eta W_T}\left(\frac{dW_T}{dh}\right) + \mu_T \tag{5.103}$$

The values of W_T and $\frac{dW_T}{db}$ would vary depending on the nature of distribution. Using the values of W_T for gamma distributed time from equation (5.79) and $\frac{dW_T}{dh}$ from equation (5.87) in equations (5.102) and (5.103)

$$\frac{dg}{dh} = \frac{w\theta}{1 - \eta(h+\Omega)\theta} \tag{5.104}$$

and

$$\frac{dU(S,d)}{db} = -\frac{w\theta}{1 - \eta(h+\Omega)\theta} + w\theta = -\frac{\mu_T \eta(h+\Omega)\theta}{\{1 - \eta(h+\Omega)\theta\}} \quad \text{[Using } \mu_T = w\theta\text{]} \tag{5.105}$$

Now differentiating U_{pm} with respect to h

$$\frac{dU_{pm}}{dh} = -\frac{dg}{dh} + \mu_T = -\frac{w\theta}{1 - \eta(h+\Omega)\theta} + \mu_T = -\frac{\mu_T \eta(h+\Omega)\theta}{\{1 - \eta(h+\Omega)\theta\}} \quad \text{[Using } \mu_T = w\theta\text{]} \tag{5.106}$$

Thus, it is observed that $\frac{dU(Z,d)}{dh} = \frac{dU_{pm}}{dh}$. Thus, $\frac{dU_{co}}{dh} = 0$. Moreover, it can be argued that the utility of the project manager and the utilitarian sum U(Z,d) is negative for any positive value of h and zero with h = 0. Thus, the utility of the project manager and the utilitarian sum are decreasing functions of h. However, the utility of the contractor would not change in h as $\frac{dU_{co}}{dh} = 0$.

Similar to the gamma distributed time, similar results can de derived for time functions with other form of continuous probability distributions. This leads to the following proposition

Proposition 12. *With a time based contract P = g-hT (where T follows a continuous time distribution), the Utilitarian sum U, and the utility functions of the project manager are higher under the fixed price contract (h = 0) than under any time based contract ($h > 0$). However, the contractor's utility does remain the same for any h value. Thus, the fixed price contracts was found to dominate any time based contracts using the utilitarian bargaining approach.*

5.8 Numerical Example

The last few sections have presented the models of supply chain coordination using bargaining games. This section tests the models numerically.

5.8.1 Nash Bargaining

It is assumed the value of the project upon completion is, q = £10. The models are derived for gamma, exponential, beta and Weibull distributed cost. The parameter η is assumed as 0.2 in the beginning. The other distribution specific values are assumed as below

- For a gamma distributed cost, the following are assumed
    - shape parameter ω = 2
    - scale parameter ϕ = 2

  Thus, the mean value of the cost, μ = £4.
  Firstly, the value of W is calculated for the gamma distribution using the numeric values. Using the conditions from equations (5.18) and (5.19), the optimal value of a is calculated for b=0, 0.5 and 1. Using these values, the values of U_{pm}, U_{co} and Nash product N are calculated. The results are presented in figure 5.1 below and in table E.1 in appendix E.1.

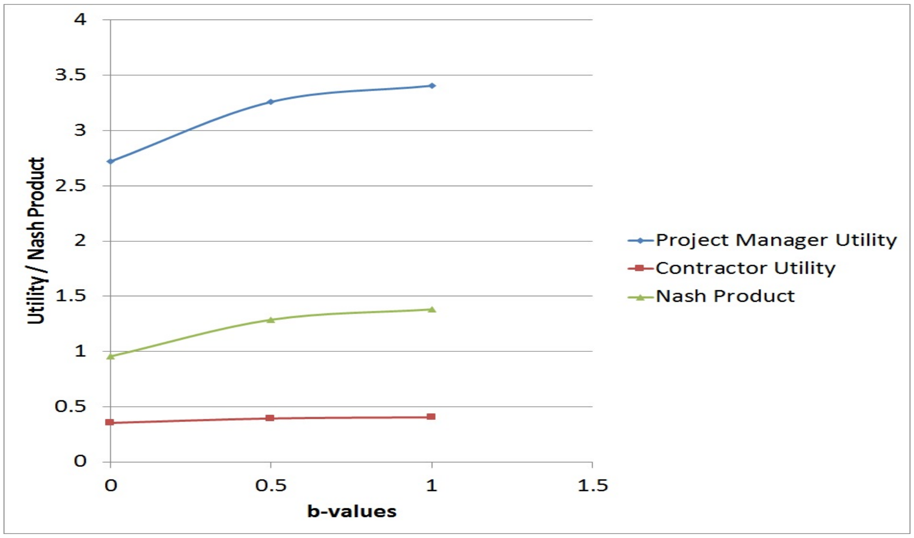


Figure 5.1: Individual Utilities/Nash Product vs. ”b” values: gamma distributed cost

- For an exponential distributed cost, the shape parameter $\omega = 1$. Assuming the scale parameter $\phi = 2$, the values of W, a, U_{pm}, U_{co} and Nash product N are calculated for b = 0, 0.5 and 1 . The results are presented in figure 5.2 below and in table E.2 in appendix E.1.

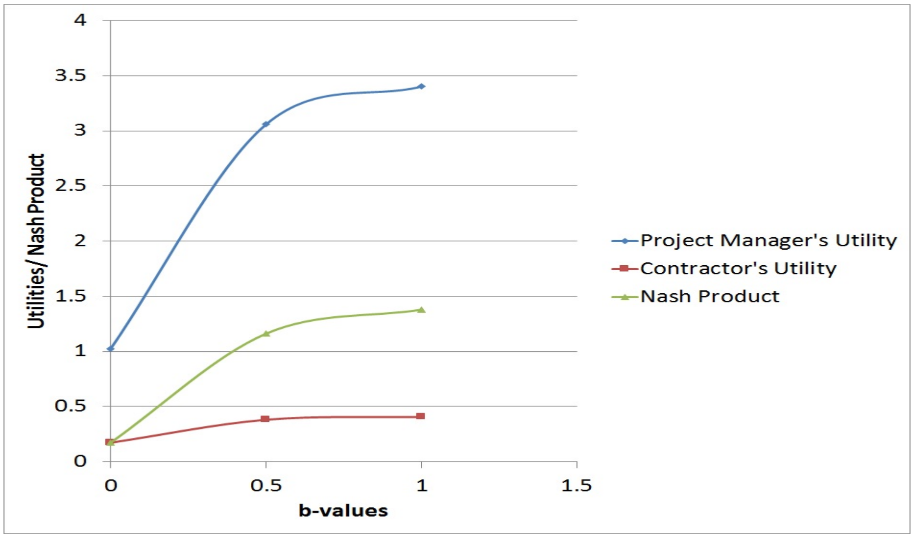


Figure 5.2: Individual Utilities/Nash Product vs. ”b” values for exponential distributed cost

- For a beta distributed cost, shape parameters are assumed as c = 2 and d = 3. The scale is assumed as $\phi = 7$. Using these values, the values of W, a, U_{pm}, U_{co} and Nash product N are calculated for $b = 0, 0.5$ and 1. The results are presented in figure 5.3 below and in table E.3 in appendix E.1.

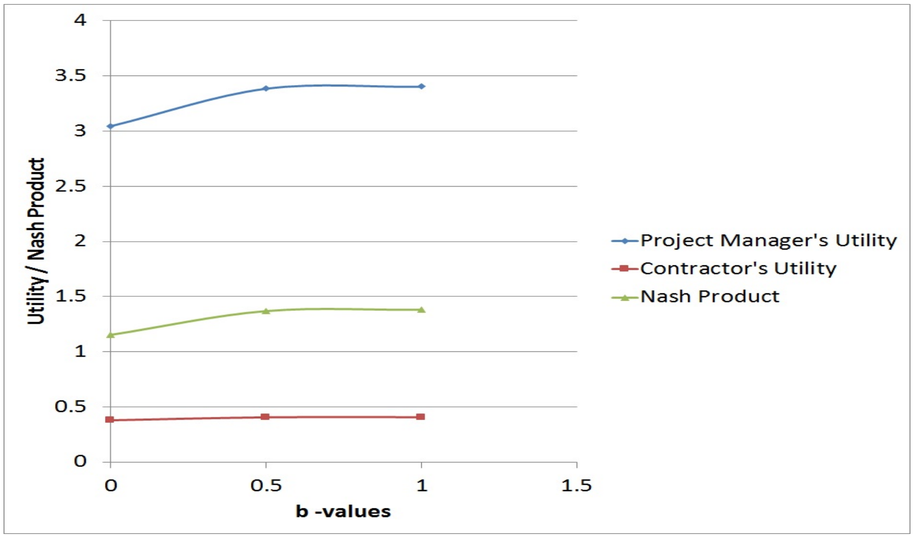


Figure 5.3: Individual Utilities/Nash Product vs. ”b” values: beta distributed cost

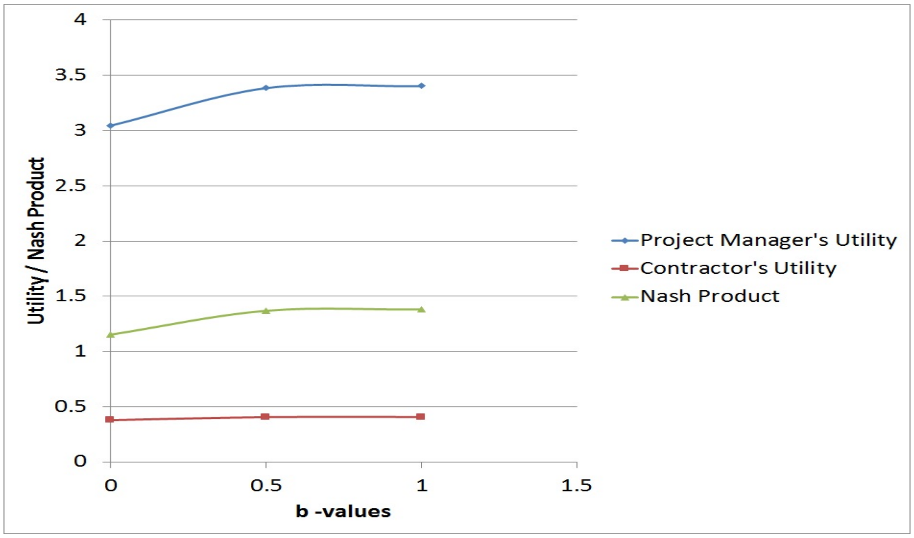


Figure 5.4: Individual Utilities/Nash Product vs. ”b” values: Weibull distributed cost

- For a Weibull distributed cost, it is assumed the shape parameter $S = 2$ and scale $\phi = 6$. Again using these observations, the values of W, a, U_{pm}, U_{co} and Nash product N are calculated for $b = 0, 0.5$ and 1. The results are presented in figure 5.4 below and in table E.4 in appendix E.1.

It can be seen from tables E.1, E.2, E.3, and E.4 that the values of U_{pm}, U_{co}, and Nash product are highest for in $b = 1$, followed by $b = 0.5$, and then $b = 0$. Similar is the observation from figures 5.1, 5.2, 5.3, and 5.4. The values of U_{pm}, U_{co}, and Nash product are increasing in b for $b \in [0, 1]$. Thus, the results of a cost plus contract dominates the solutions for fixed price contract and any cost sharing contract with $0 < b < 1$. This supports the original findings from the models.

5.8.2 Kalai Smorodinsky Bargaining

The values of q and η are assumed as before similar to the case of Nash's bargaining. Numeric examples were prepared for gamma distributed cost, exponential cost, beta distributed cost, and Weibull distributed cost. The shape parameters (ω, c, d, and S as applicable to the case) and the scale parameter ϕ are assumed as the same value as in the case of Nash's bargaining.

Using these, the values of W, a, U_{pm}, U_{co}, and the Kalai Smorodinsky Function K are determined for b=0, 0.5 and 1. The results are presented in distributed cost figures 5.5, 5.6, 5.7, and 5.8 below and tables E.5, E.6, E.7, and E.8 in appendix E.2.

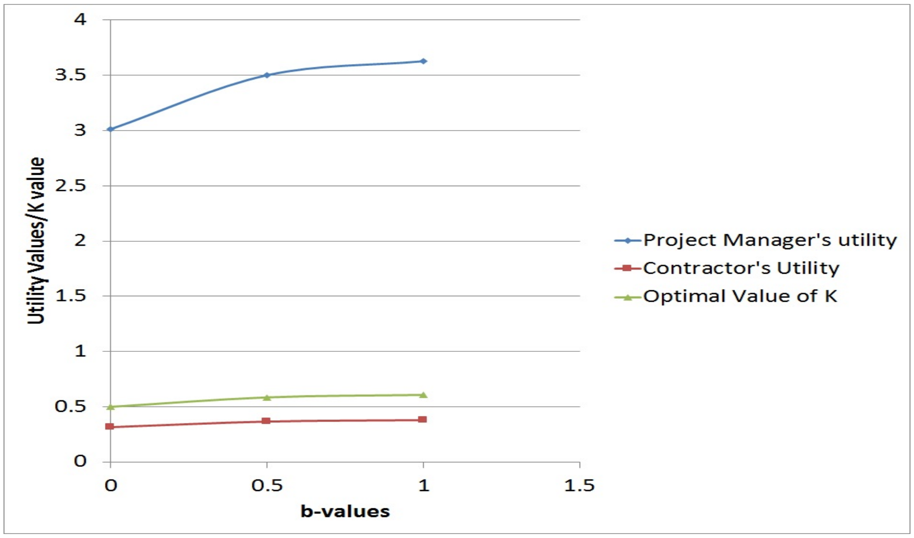


Figure 5.5: Individual Utilities/Kalai Smorodinsky value K vs. b values for gamma distributed cost

Similar to the observation in the case of Nash's bargaining, the values of U_{pm}, U_{co}, and K are found to be increasing in the value of b, with $b \in [0, 1]$. Thus, once again the results of a cost plus contract dominates the solutions from a fixed price contract and from any cost sharing contract with $0 < b < 1$. This once again supports the original findings from the model.

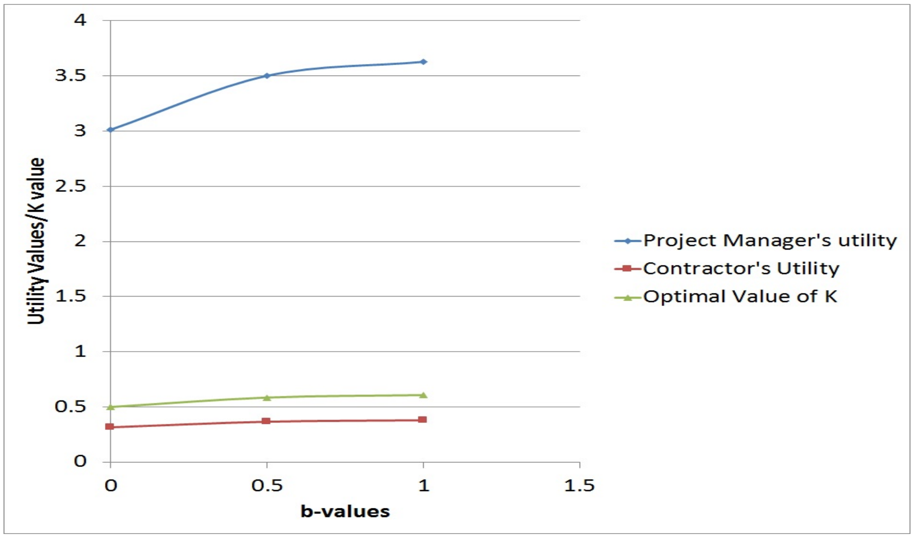


Figure 5.6: Individual Utilities/Kalai Smorodinsky value K vs. b values for exponential distributed cost

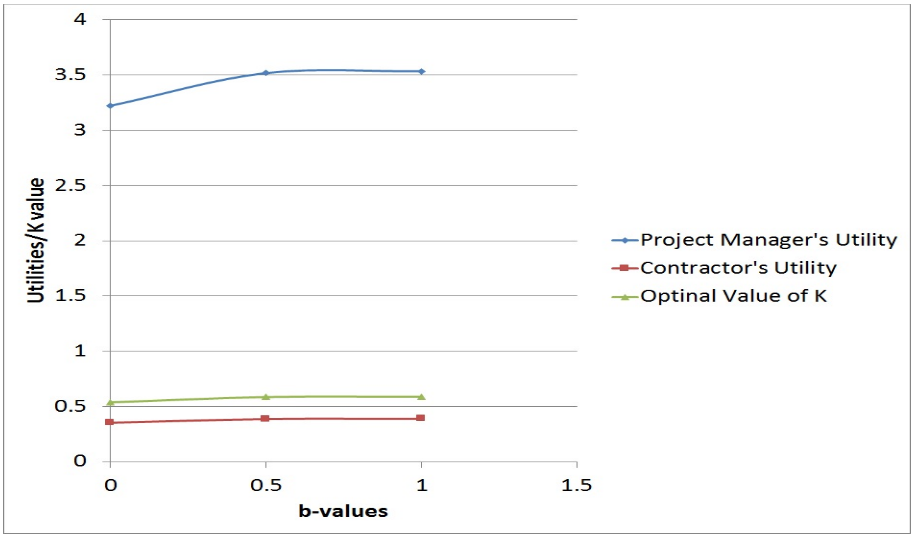


Figure 5.7: Individual Utilities/Kalai Smorodinsky value K vs. b values for beta distributed cost

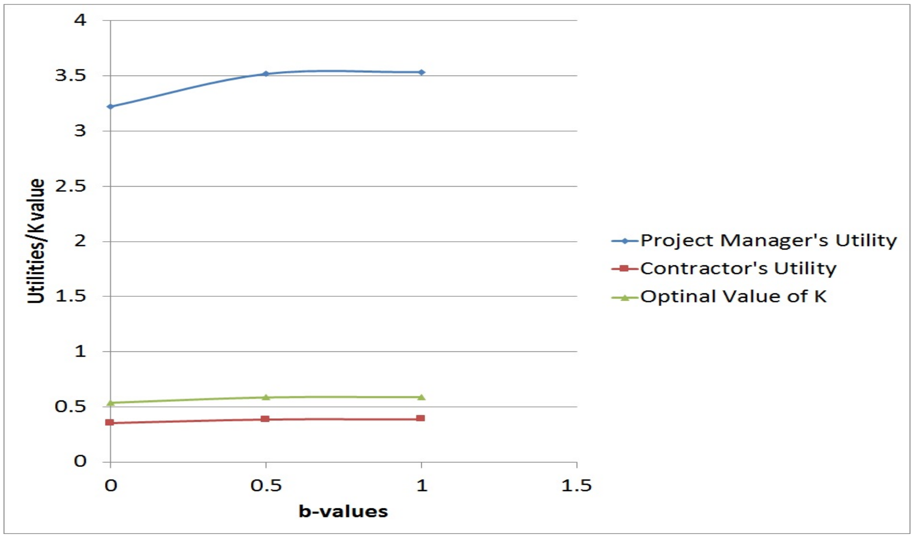


Figure 5.8: Individual Utilities/Kalai Smorodinsky value K vs. b values for Weibull distributed cost

5.8.3 Utilitarian Bargaining

Using the optimal value of a from equation (5.65) in the equation of utility function of the contractor and rearranging the terms

$$U_{co} = 1 - \frac{1}{\eta} \tag{5.107}$$

As mentioned earlier, $U_{co} > 0$, otherwise contractor won't participate in the bargaining. Thus, for the utilitarian bargaining approach $\eta > 1$. Thus, unlike the case of Nash's bargaining and Kalai Smorodinsky bargaining, the risk aversion parameter η can not take lower values. To put it in other words, the utilitarian approach can be applicable for more risk averse members.

q is assumed as 5 units. η is assumed as 1.2. Again, the analysis was conducted for gamma, exponential, beta, and Weibull distributed cost.

- For a gamma distributed cost, the scale parameter is assumed as ϕ = 0.4 and the shape parameter is assumed as ω = 2. This leads to the mean value of cost as 0.8 units.
- For an exponential distribution, the scale parameter is assumed ϕ = 0.8. Thus, the mean value of the cost is 0.8 units.
- For a beta distributed cost, the shape parameters c and d are assumed as 2 and 3 as before. The scale (ϕ) is assumed as 2. The mean value becomes 0.8 units

- For a Weibull distributed cost, the shape parameter (s) is assumed as 2 and scale (ϕ) as 0.9

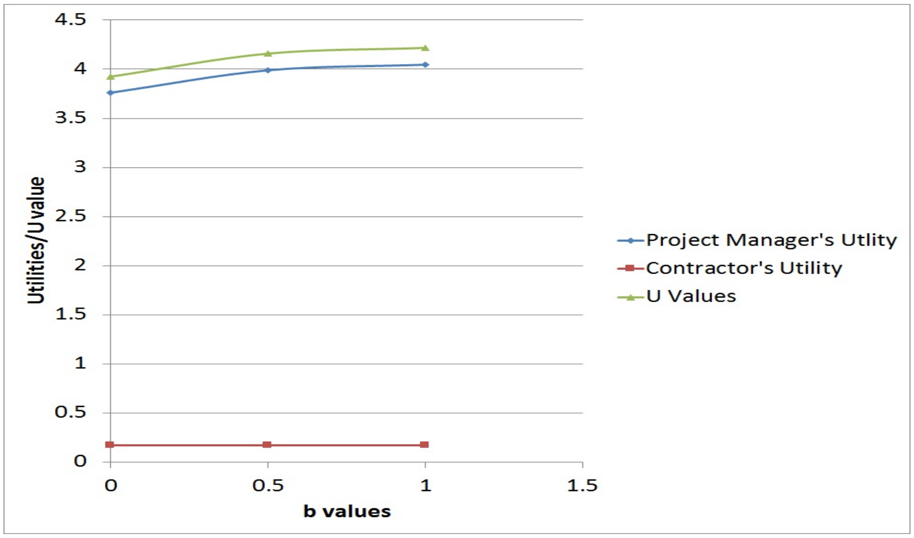


Figure 5.9: Individual Utilities/Utilitarian sum value U Product vs. b values for gamma distributed cost

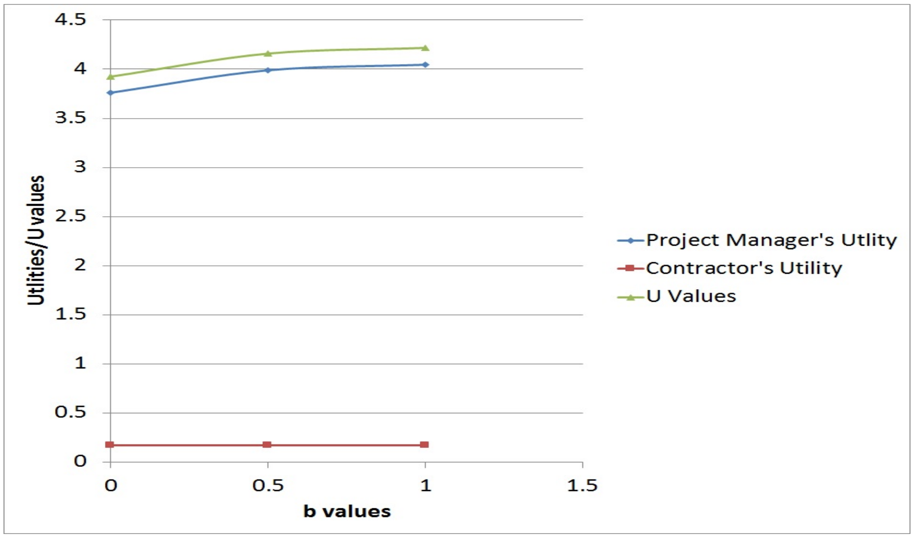


Figure 5.10: Individual Utilities/Utilitarian sum value U Product vs. ”b” values for exponential distributed cost

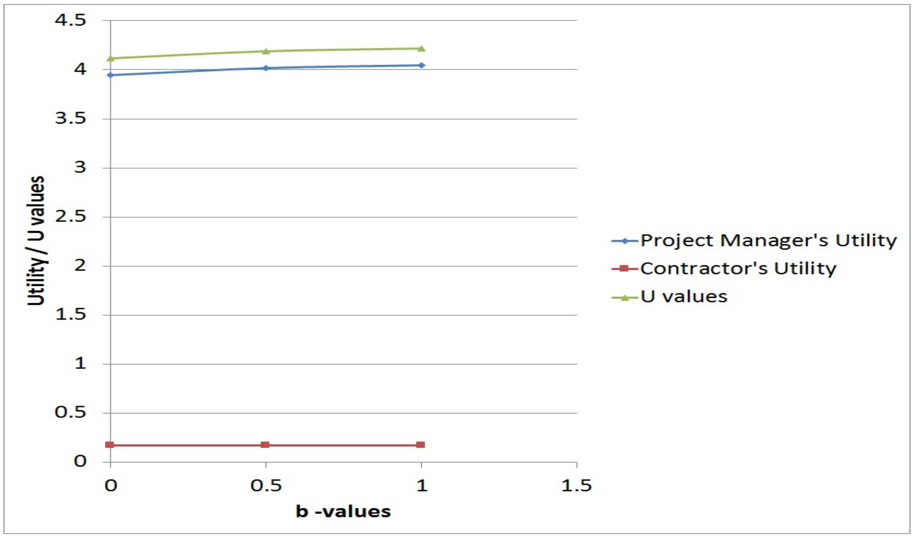


Figure 5.11: Individual Utilities/Utilitarian sum value U Product vs. b values for beta distributed cost

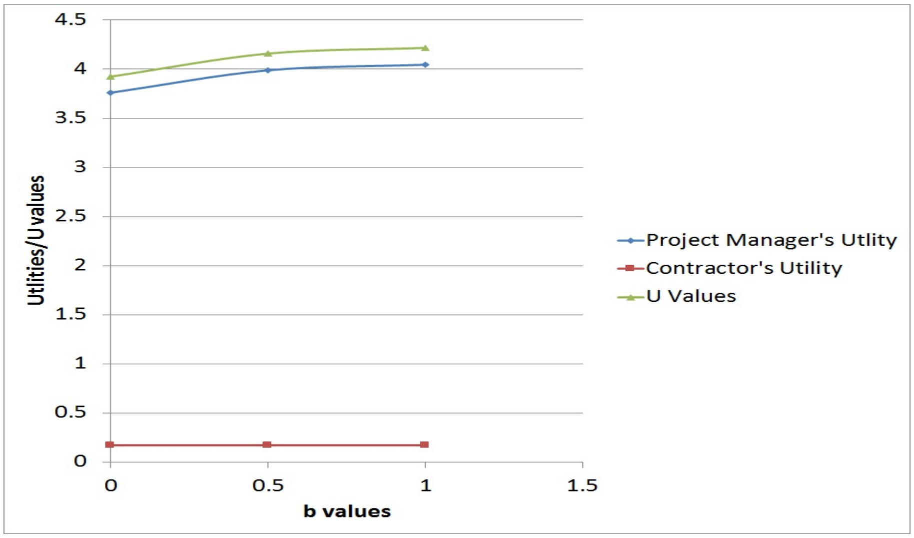


Figure 5.12: Individual Utilities/Utilitarian sum value U Product vs. b values for Weibull distributed cost

5.8.4 Comparison among the results

The last few subsections have presented the results of the numerical applications using the proposed bargaining models. There are notable similarities in the results from different bargaining models. Both the Nash and Kalai Smorodinsky bargaining applications to the supply chain under consideration yields similar results. In both cases, the fixed price contract solutions are dominated for both the members of the supply chain. The project manager and the contractor both were found to have higher utility with the increase in the deciding contract parameter $b \in [0, 1]$. Hence, the cost plus contract with b=1 was found to dominate any other solutions in both Nash and Kalai Smorodinsky bargaining cases. Apart from individual utilities, Nash product in the case of Nash bargaining or the Kalai Smorodinsky value K in the case of Kalai Smorodinsky bargaining case were also found to dominate the solutions in the case of cost plus contracts with b=1.

In comparison to the Nash bargaining and Kalai Smorodinsky bargaining, the Utilitarian approach was found to have some distinctly different features. The sum utility function U was found to have a dominating solution for the cost plus contract with b=1 for all the distribution selected. This was mainly due to the existence of the dominating solutions of the utility function of the project manger at b = 1 (Cost plus contract) than in comparison to the solutions with b = 0.5 and b = 0.0. However, the utility of the contractor was found to be same for any values in the range $b \in [0, 1]$. In fact the utility of the contract was found to be independent of the nature of the probability distribution of the project cost. This can be explained from the equation (5.107). As evident from the equation, the U_{co} was found to be independent of any distribution specific parameters of the cost function. The utility of the contractor was found to be dependent only on the parameter η.

5.9 Chapter Summary

This chapter has investigated how to reach the optimal solution to coordinate supply chain with negotiation between the project manager and the contractor. This chapter used

- A cost based contract P= a+bX (Where X is a continuous cost function; a is the fixed component of the contract; and b is the variable part of the contract with $b \in [0, 1]$).
- A time based contract with P=g-hT (where g is the fixed part and h is the penalty per unit to entice the contractor for early completion)
- In either of the above contracts, if the variable part becomes zero (b or h), it becomes a fixed price contract

The bargaining approach used includes Nash bargaining, Kalai Smorodinsky bargaining, and Utilitarian approach. The models were prepared for two different situations: both the members are risk neutral, and the project manager is risk neutral and the contractor is risk averse.

For the case when both the supply chain members (the project manager and the contractor) are risk neutral, no clear dominance of solutions were observed by comparing the fixed price with either time based or cost based contracts. This, may entice the members of the supply chain under consideration to implement fixed price contract instead of implementing any complicated contracts.

On the contrary, if the project manager is risk neutral and the contractor is the risk averse, then the results are different and a dominance of solutions were observed. In comparison between the time based and the fixed price contracts, the solutions from the fixed price contract were found to dominate the solutions from any time based contracts. In comparison between the cost based contract, and the fixed price contract, the results from the cost based contracts were found to dominate the results from the fixed price contracts. In fact, the cost plus contract with b = 1 was found to dominate any other cost sharing contracts with $0 < b < 1$. This dominance was found to be strictly dominating for the case of Nash and Kalai Smorodinsky bargaining for all the members of the supply chain under consideration and the respective bargaining parameters (the Nash product N and the Kalai Smorodinsky bargaining value K). However, the contractor's utility was found to be independent of the variable part of the offered contract (either b or h depending on the case) in the case of Utilitarian approach of bargaining. Hence, the utility of the contractor remained unchanged with respect to the fixed price contract in the utilitarian bargaining approach.

Chapter 6

Fairness and Fair Allocation in Project Supply Chain

This research has addressed the issue of supply chain coordination in take it or leave it situation in chapter 4 and extended it to bargaining situations in chapter 5. However, issues on the allocation of risk and benefits of supply chain coordination were not discussed. Chapter 2 highlighted the problems arise in absence of fair allocation of risk and benefits.

Justice or fairness has been conceptualized since the time of Aristotle and Plato (Liu et al. 2012). However, the authors concluded based on past research evidence how justice or fairness has been perceived differently in different contexts. The concept of fairness or justice has been studied for a long time in various economic and social exchange (Adams 1965, Lind & Tyler 1988, Greenberg & Cropanzano 1993).Various studies in economics and marketing have addressed the importance of fairness in the social exchange (Frazier 1983, Heide & John 1988, Corsten & Kumar 2005).

Liu et al. (2012) highlighted how four dimensions of justice have been developed in the literature over the last few decades. These are distributive, procedural, interpersonal and informational. The authors considered the first two dimensions as part of structural fairness and the last two as part of social fairness.

Despite its importance and long rooted existence in the economics and other social science related literature, the applications in supply chain related exchanges is relatively new. As per the knowledge of the author of the present research, Cui et al. (2007) is one of the pioneer authors who investigated the issues of distributive fairness in supply chain coordination. Following their study, authors including Loch & Wu (2008), Caliskan-Demirag et al. (2010), and Ho et al. (2014) proposed various models of supply chain coordination with fairness considerations. However, these studies were conducted with product supply chains with supply contracts and the quantity demanded as the decision variable. This chapter presents the models as an extension to the early studies in the project supply chain setting with the project contracts.

The question is what is the importance of fairness in the context of supply chain coordination. The distributive and procedural fairness have been found to have a positive impact on long term relation between a firm and its distributor in a supply chain and ultimately on the overall performance (Griffith et al. 2006). Absence of fairness has been found as one of the factors leading to the failure of supply chain coordination relationship in some recent studies. Katok & Pavlov (2013) explored the reasons leading to the termination of coordinated contractual relationships in a supply chain using behavioural laboratory experiments. The authors found lack of inequality aversion, incomplete information and bounded rationality as three reasons for this failure. This finding also supports the findings of Wu (2013*b*) where the authors found rejecting behaviours from retailers in a supply chain when experiencing unfair offers from the suppliers. Some of the examples from practice corroborate the importance of the need for the fair allocation of the risks and benefits in the coordinated relationship such as the termination of the contractual relationship between Walmart Canada and Lego group upon rejection by Lego group to reduce the price in the Canadian market. Lego group kept the price same as in the American market and reaped additional benefits due to the appreciation of Canadian dollar (Georgiades 2008). Similar was the case with the breakdown of contractual relationships between Chinese home appliance retailer Gome and air condition manufacturer Gree (Liu et al. 2012).

Traditionally, it used to be believed that the participants only care about the rational profit maximization as their objective in contractual agreements. However, some experimental studies have shown the existence of fairness considerations from the participants (Loch & Wu 2008, De Bruyn & Bolton 2008). More interestingly, this kind of caring behaviour has been observed not only in the take it or leave it environments, but also in the bargaining environments as well (Camerer 2003). The classification of justice or fairness in buyer-seller relationship by Liu et al. (2012) provides the basic starting point. The interesting fact is the consideration of distributive fairness as one of the forms of fairness consideration in supply chain literature. Authors including Fehr & Schmidt (1999), Bolton & Ockenfels (2000), and Charness & Rabin (2002) have defined fairness from somewhat different perspectives. Fehr & Schmidt (1999) defined fairness from an inequity aversion perspective, whereas the authors in two other studies defined fairness from reciprocity perspective. This has been supported in the literature of Falk & Fischbacher (2006). Some other notable extensions have been documented in literature specific to the supply chain coordination such as peer-induced fairness (Ho et al. 2014). In a recent study by Du et al. (2014), the authors were critical of the models proposed by Fehr & Schmidt (1999) from applicability point of view. The authors used Nash bargaining solution as the fairness reference point solution. This research summarises this debate by the existence of context-specific nature of fairness consideration. This has been supported in the literature by Liu et al. (2012).

This research has used the definition of fairness proposed by Fehr & Schmidt (1999) as the reference for the take it or leave it situations and in some case for bargaining situations as well. One of the main reasons is its ability to best describe the context of the coordination problem considered for this research. Based on the definition by Fehr & Schmidt (1999), Cui et al. (2007) identified that simple wholesale price contracts can coordinate a manufacturer-retailer supply chain when members are fairness concerned. This model of Cui et al. (2007) has been extended in various different directions such as: models with non-linear demand (Caliskan-Demirag et al. 2010); and when the supplier's fairness concerns are private information in a supplier-retailer supply chain (Katok et al. 2014). The authors found that under this situation the wholesale price contract can coordinate the supply chain as it did in the case of information symmetry in the models of Cui et al. (2007). Voigt & Inderfurth (2012) used the concepts of inequity aversion in a similar context with asymmetric holding cost information. There are other supply chain contexts where fairness in allocation has been considered such as cooperative advertising (Yang et al. 2013).

This research did not find any evidence of any supply chain coordination model including fairness consideration alongside profit maximizing objective in the project settings. Thus, this chapter addresses the third objective of this research

Objective 3. *To investigate if the supply chain can be coordinated with fairly allocated risks and benefits in the scenarios mentioned in objectives 1 and 2.*

The first part presents the analysis in take it or leave it situations with Stackelberg games. The approach proposed by Cui et al. (2007) has been used as the reference for this. The second part follows this up with some analysis of fairness considerations in bargaining situations.

6.1 Problem Description

As described earlier in chapter 4, the coordination problem is analysed with Stackelberg leader-follower games in the take it or leave it situation. The project manager is considered as the leader and the contractor is considered as the follower. In the supply chain literature, by Cui et al. (2007) and Caliskan-Demirag et al. (2010), the authors used a fixed wholesale price contract to see if it can coordinate the supply chain under consideration with the existence of the fairness concern. This research uses a fixed price contract to investigate if it can achieve the coordination requirements in the event of the presence of fairness concern.

Following the definition of the fairness in Fehr & Schmidt (1999) and the approach by Cui et al. (2007), the utility equation of the member i in a two member supply chain (with member i and member j) is

$$U_i(\lambda, P(T,C)) = \pi_i + D_i(\lambda, P(T,C)); i \in \{pm, co\} \tag{6.1}$$

The first part of the equation (6.1) corresponds to the monetary profit of the member i of the supply chain. The second part of the equation i.e. $D_i(\lambda, f)$ is the member i's disutility due to inequity or unfairness. As per this model, the member would incur some disutility if (s)he earns more than or less than the profit (s)he believes is fair or equitable. This fair equitable profit perceived by the member is compared against the profit of the other member. Let $\gamma\pi_{pm}$ and $\delta\pi_{co}$ are the equitable profit as perceived by the contractor and the project manager respectively in the supply chain under consideration (where $\gamma > 0$ and $\delta > 0$). Based on the suggestion of Caliskan-Demirag et al. (2010), these factors γ and δ are exogenous to the members and are calculated based on the outside options available to the members of the supply chain under consideration. α_i and β_i ($i \in \{pm, co\}$) are the disutility to the member per unit due to earning less (disadvantageous inequity) and more (advantageous inequity) in comparison to the other member. Authors including Fehr & Schmidt (1999), Cui et al. (2007) and Caliskan-Demirag et al. (2010), suggested based on previous research that the member is more sensitive to disadvantageous inequity (earning less) than advantageous utility (earning more). Thus, it is assumed that $\alpha_i \geq \beta_i$. It is also assumed that $0 < \beta_i < 1$ in accordance with the existing literature. Thus, based on the definition of Fehr & Schmidt (1999), the disutility due to inequity or unfairness is defined as below

$$D_{pm}(\lambda, P(T,C)) \quad = -\alpha_{pm}[max\{(\delta\pi_{co} - \pi_{pm}), 0\}] - \beta_{pm}[max\{(\pi_{pm} - \delta\pi_{co}), 0\}] \tag{6.2}$$

and

$$D_{co}(\lambda, P(T,C)) \quad = -\alpha_{co}[max\{(\gamma\pi_{pm} - \pi_{co}), 0\}] - \beta_{co}[max\{(\pi_{co} - \gamma\pi_{pm}), 0\}] \tag{6.3}$$

where $\alpha_{pm} \geq \beta_{pm}$; $0 < \beta_{pm} < 1$; $\alpha_{co} \geq \beta_{co}$; and $0 < \beta_{co} < 1$.

As mentioned in the chapter 4, the coordination problem is solved using backward induction method from game theory. Given an offer of a contract price of P(T,C), the contractor would select a resource consumption rate λ that maximizes his profit if the contractor does not have any fairness concern. However, in the event of the presence of a fairness concern, the contractor will select a λ that maximizes his utility as below

$$U_{co} = \pi_{co} - \alpha_{co}[max\{(\gamma\pi_{pm} - \pi_{co}), 0\}] - \beta_{co}[max\{(\pi_{co} - \gamma\pi_{pm}), 0\}] \tag{6.4}$$

The project manager would incorporate this requirement of λ in her take it or leave it offer and selects the value of P(T,C) that maximizes her profit (if she is not fairness concerned) given the constraint of λ. In the event, the project manager is fairness concerned, she selects a value of P(T,C) that maximizes her utility as below

$$U_{pm} = \pi_{pm} - \alpha_{pm}[max\{(\delta\pi_{co} - \pi_{pm}), 0\}] - \beta_{pm}[max\{(\pi_{pm} - \delta\pi_{co}), 0\}] \tag{6.5}$$

The next few subsections explore the coordination problems with fairness concerned members for different types of contracts used in chapter 4.

6.2 Supply Chain Coordination with Fixed Price Contracts in a Take it or Leave it Situation under Fairness Concern

In the literature of Fehr & Schmidt (1999) and Caliskan-Demirag et al. (2010), authors used wholesale price contract in a retail supply chain with fairness concerned members to coordinate the supply chain. The authors argued if a simple wholesale price contract can coordinate a supply chain with fairness concerned members, then there is a limited need to go for complicated contracts.

As mentioned in chapter 4, the contractor then selects the resource consumption rate λ after the project manager offered him a contract P(T,C). However, a fixed price contract with P(T,C)= f was found to fail to coordinate the supply chain as shown in the 4. Here in this chapter, the same set of approaches would be repeated, but with the presence of fairness concerns of the members of the supply chain (the project manager or the contractor). The utility of the contractor following equation (6.1) is as below

6.2.1 For Short Term Projects

As mentioned in the chapter 4, the first best supply chain profit in the centralized setting follows the following equation

$$\pi_0 = q_0\{1 - \psi E(T^m)\} - k\lambda_0^N \mu_1 - C_o \tag{6.6}$$

As mentioned in chapter 4, the project manager and the contractor's profit in decentralized setting are

$$\pi_{pm} = q_0\left[1 - \psi E(T^m)\right] - f - C_o \tag{6.7}$$

$$\begin{aligned} \pi_{co} &= f - k\lambda^n E(T) \\ &= f - k\lambda^N \mu_1 \end{aligned} \tag{6.8}$$

The E(T^m) values satisfy the equation (4.17) and the λ_0 values satisfy the conditions in the equation (4.19) from chapter 4.

Fairness Concern Contractor and Profit Maximizing Project Manager

The fairness concern contractor will chose a resource consumption rate λ that maximizes his utility. As mentioned earlier in chapter 4, this resource consumption rate λ can not be monitored by the project manager. The game is solved using backward induction.

Using these values of profits from above, the utility function of the contractor becomes

$$U_{co} = \begin{cases} (f - k\lambda^N\mu_1) - \alpha_{co}[\gamma\{q_0 - q_0\psi E(T^m) - f - C_o\} - (f - k\lambda^N\mu_1)] \\ \text{for } \{f - k\lambda^N\mu_1\} < \gamma[q_0\{1 - \psi E(T^m)\} - f - C_o] \\ \\ (f - k\lambda^N\mu_1) - \beta_{co}[(f - k\lambda^N\mu_1) - \gamma\{q_0 - q_0\psi E(T^m) - f - C_o\}] \\ \text{for } \{f - k\lambda^N\mu_1\} \geq \gamma[q_0\{1 - \psi E(T^m)\} - f - C_o] \end{cases} \tag{6.9}$$

The first case corresponds to disadvantageous disutility case, equitable case, and the second one for the advantageous inequity case. As mentioned earlier, the contractor would select a resource consumption rate λ that maximizes his utility. Thus, when $\{f - k\lambda^N\mu_1\} < \gamma[q_0\{1 - \psi E(T^m)\} - f - C_o]$, the contractor would select a λ that satisfies

$$\frac{dU_{co}}{d\lambda} = -kN\lambda^{N-1}\mu_1 - \alpha_{co}\left[-\gamma q_0\psi\frac{dE(T^m)}{d\lambda} + kN\lambda^{N-1}\mu_1\right] = 0 \tag{6.10}$$

The values of E(T) and E(T^m) depend on the nature of selected distribution.

From the equation (4.17), the value of $\frac{dE(T^m)}{d\lambda}$ is derived as

$$\frac{dE(T^m)}{d\lambda} = \begin{cases} -\frac{mA(2\mu_1)^m}{(m+1)\lambda^{mA+1}} & \text{For uniform distributed time} \\ -\frac{mA\mu_1^m}{w^m\lambda^{mA+1}}\prod_{i=1}^{m}(w+i-1) & \text{For gamma distributed time} \\ -\frac{mA\mu_1^m}{\left(\frac{u}{u+v}\right)^m\lambda^{mA+1}}\prod_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right) & \text{For beta distributed time} \\ -\frac{mA\mu_1^m}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA+1}}\left[\Gamma\left(1+\frac{m}{s}\right)\right] & \text{For Weibull distributed time} \end{cases} \tag{6.11}$$

Using the values from (6.11) in the equation (6.10)

$$\frac{dU_{co}}{d\lambda} =$$
$$\begin{cases} -kN\lambda^{N-1}\mu_1 - \alpha_{co}\left[\gamma q_0\psi\frac{mA(2\mu_1)^m}{(m+1)\lambda^{mA}} + kN\mu_1\lambda^{N-1}\right] = 0 & \text{For uniform distributed time} \\ -kN\lambda^{N-1}\mu_1 - \alpha_{co}\left\{\gamma q_0\psi\frac{mA\mu_1^m\prod_{i=1}^{m}(w+i-1)}{w^m\lambda^{mA+1}} + kN\mu_1\lambda^{N-1}\right\} = 0 & \text{For gamma distributed time} \\ -kN\lambda^{N-1}\mu_1 - \alpha_{co}\left[\gamma q_0\psi\frac{mA\mu_1^m\prod_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)}{\left(\frac{u}{u+v}\right)^m\lambda^{mA+1}} + kN\mu_1\lambda^{N-1}\right] = 0 & \text{For beta distributed time} \\ -kN\lambda^{N-1}\mu_1 - \alpha_{co}\left\{\gamma q_0\psi\frac{mA\mu_1^m\left[\Gamma\left(1+\frac{m}{s}\right)\right]}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA+1}} + kN\mu_1\lambda^{N-1}\right\} = 0 & \text{For Weibull distributed time} \end{cases} \tag{6.12}$$

Rearranging the terms from the above equation, it can be shown (for the case of uniform distribution) that $\lambda^{m+A} = -\frac{\alpha_{co}\gamma q_0 mA}{(1+\alpha_{co})(m+1)kN\mu_1}$. Hence, solving the equation (6.12), the root of the equation either becomes negative or the real roots of the equation can not be found. Thus, if the contractor experiences a disadvantageous inequity, then the contractor's practical best option is to select $\lambda = 0$. This could never be able to coordinate the supply chain under consideration. Based on this observation, the following is proposed

Proposition 13. *A fixed price contract fails to coordinate a project supply chain with a fairness concerned contractor and a profit maximizing project manager, if the contractor experiences disadvantageous inequity.*

In the second case from equation (6.9) when, $\{f - k\lambda^N\mu_1\} \geq \gamma[q_0\{1 - \psi E(T^m)\} - f - C_o]$, the selected resource consumption rate should satisfy the following

$$\frac{dU_{co}}{d\lambda} = (-kN\lambda^{N-1}\mu_1) - \beta\left[(-kN\lambda^{N-1}\mu_1) + \gamma q_0\psi\frac{dE(T^m)}{d\lambda}\right] = 0 \tag{6.13}$$

Using the values of $\frac{dE(T^m)}{d\lambda}$ from the equation (6.11) in the equation (6.13),

$$\frac{dU_{co}}{d\lambda} = \begin{cases} -kN\lambda^{N-1}\mu_1 - \beta_{co}\left[-\gamma q_0\psi\frac{mA(2\mu_1)^m}{(m+1)\lambda^{mA+1}} - kN\mu_1\lambda^{N-1}\right] = 0 \\ \text{For uniform distributed time} \\ -kN\lambda^{N-1}\mu_1 - \beta_{co}\left\{-\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}(w+i-1)}{w^m\lambda^{mA+1}} - kN\mu_1\lambda^{N-1}\right\} = 0 \\ \text{For gamma distributed time} \\ -kN\lambda^{N-1}\mu_1 - \beta_{co}\left[-\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)}{\left(\frac{u}{u+v}\right)^m\lambda^{mA+1}} - kN\mu_1\lambda^{N-1}\right] = 0 \\ \text{For beta distributed time} \\ -kN\lambda^{N-1}\mu_1 - \beta_{co}\left\{-\gamma q_0\psi\frac{mA\mu_1^m\left[\Gamma\left(1+\frac{m}{s}\right)\right]}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA+1}} - kN\mu_1\lambda^{N-1}\right\} = 0 \\ \text{For Weibull distributed time} \end{cases} \tag{6.14}$$

Rearranging the terms of the above equation (6.14) in terms of λ

$$\lambda = \begin{cases} \left[\frac{\gamma\beta_{co}mAq_0\psi(2\mu_1)^m}{kN\mu_1(m+1)(1-\beta_{co})}\right]^{\frac{1}{mA+N}} & \text{For uniform distributed time} \\ \left\{\frac{\gamma\beta_{co}mAq_0\psi\mu_1^m\sum\limits_{i=0}^{m}(w+i-1)}{w^m kN\mu_1(1-\beta_{co})}\right\}^{\frac{1}{mA+N}} & \text{For gamma distributed time} \\ \left[\frac{\gamma\beta_{co}mAq_0\psi\mu_1^m\sum\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)}{\left(\frac{u}{u+v}\right)^m kN\mu_1(1-\beta_{co})}\right]^{\frac{1}{mA+N}} & \text{For beta distributed time} \\ \left\{\frac{\gamma\beta_{co}mAq_0\psi\mu_1^m\Gamma\left(1+\frac{m}{s}\right)}{\{\Gamma\big(1+\frac{1}{s}\}\big)^m kN\mu_1(1-\beta_{co})}\right\}^{\frac{1}{mA+N}} & \text{For Weibull distributed time} \end{cases} \tag{6.15}$$

Now taking the second order derivative of U_{co} with respect to λ,

$$\begin{aligned} \frac{d^2U_{co}}{d\lambda^2} &= \{-kN(N-1)\lambda^{N-2}\mu_1\} - \beta_{co}\left[(-kN(N-1)\lambda^{N-2}\mu_1) + \gamma q_0\psi\frac{d^2E(T^m)}{d\lambda^2}\right] \\ &= \{-kN(N-1)\lambda^{N-2}\mu_1\}(1-\beta) - \beta_{co}\gamma q_0\psi\frac{d^2E(T^m)}{d\lambda^2} \end{aligned} \tag{6.16}$$

As assumed before $0 \leq \beta_{co} < 1$. It can be easily shown that the second order derivative of the E(T^m) are positive for the uniform, gamma, beta and Weibull distributed completion times. Thus, $\frac{d^2U_{co}}{d\lambda^2} < 0$. This means the values of λ found in the equation (6.15) would maximize the utility function of the fairness concerned contractor in the decentralized setting.

As mentioned earlier, in order to coordinate the supply chain, $\lambda = \lambda_0$. Hence, the following must be satisfied

$$\left[\frac{\gamma\beta m A q_0 \psi (2\mu_1)^m}{kN\mu_1(m+1)(1-\beta)}\right]^{\frac{1}{mA+N}} = \left[\frac{mAq_0\psi(2\mu_1)^m}{(m+1)KN\mu_1}\right]^{\frac{1}{mA+N}}$$

[For uniform distributed time]

$$\left[\frac{\gamma\beta_{co} m A q_0 \psi \mu_1^m \sum\limits_{i=0}^{m}(w+i-1)}{w^m kN\mu_1(1-\beta_{co})}\right]^{\frac{1}{mA+N}} = \left[\frac{mAq_0\psi\mu_1^{m-1}\sum\limits_{i=0}^{m}(w+i-1)}{w^m kN}\right]^{\frac{1}{mA+N}}$$

[For gamma distributed time]

$$\left[\frac{\gamma\beta_{co} m A q_0 \psi \mu_1^m \sum\limits_{i=1}^{m}(\frac{u+i-1}{u+v+i-1})}{\left(\frac{u}{u+v}\right)^m kN\mu_1(1-\beta_{co})}\right]^{\frac{1}{mA+N}} = \left[\frac{mAq_0\psi\mu_1^{m-1}\sum\limits_{i=1}^{m}(\frac{u+i-1}{u+v+i-1})}{\left(\frac{u}{u+v}\right)^m kN}\right]^{\frac{1}{mA+N}}$$

[For beta distributed time]

$$\left[\frac{\gamma\beta_{co} m A q_0 \psi \mu_1^m \Gamma\left(1+\frac{m}{s}\right)}{\left\{\Gamma\left(1+\frac{1}{s}\right\}\right)^m kN\mu_1(1-\beta_{co})}\right]^{\frac{1}{mA+N}} = \left[\frac{mAq_0\psi\mu_1^{m-1}\Gamma\left(1+\frac{m}{s}\right)}{\left\{\Gamma\left(1+\frac{1}{s}\right\}\right)^m kN}\right]^{\frac{1}{mA+N}}$$

[For Weibull distributed time]

From these above equations, it can be shown

$$\beta = \frac{1}{1+\gamma} \tag{6.17}$$

The project manager takes into account the possible selected value of λ as calculated in (6.15) in her profit function. Thus, her optimization problem becomes

$$\max_{f} : U_{pm} = \pi_{pm} = q_0 \left[1 - \psi \frac{(2\mu_1)^m}{(m+1)\lambda^{mA}}\right] - f - C_o$$

St.

$$f \geq \frac{\gamma[q_0\{1 - \psi E(T^m)\} - C_o] + k\lambda^N \mu_1}{1+\gamma} \tag{6.18}$$

$$f \geq \frac{-\gamma\beta_{co}[q_0\{1 - \psi E(T^m)\} - C_o] + k\lambda^N \mu_1(1-\beta_{co})}{1-\beta_{co}-\beta_{co}\gamma} \tag{6.19}$$

It can be easily shown that $\frac{\gamma[q_0\{1-\psi E(T^m)\}-C_o]+k\lambda^N\mu_1}{1+\gamma} - \frac{-\gamma\beta_{co}[q_0\{1-\psi E(T^m)\}-C_o]+k\lambda^N\mu_1(1-\beta_{co})}{1-\beta_{co}-\beta_{co}\gamma} = \frac{\gamma\pi_0}{(1+\gamma)(1-\beta_{co}-\beta_{co}\gamma)}$. Since this is positive, the constraint condition in (6.19) is redundant. Thus, the project manager selects a value of f that maximizes her profit given the constraints in equation (6.18) i.e.

$$\frac{dU_{pm}}{df} = -1 < 0 \tag{6.20}$$

As the first order condition is negative, the project manager's utility decreases with f. Now applying the constraint from equation (6.18), any value f more than or equal to the right hand side maximizes the utility of the contractor. However, the project manager would select the value of f that maximizes her utility (profit in this case). Considering all these, and replacing the values of E(T^m) the following is proposed

Proposition 14. *A fixed price contract can coordinate the project supply chain under consideration with a fairness concerned contractor and a profit maximizing project manager, if the contractor's monetary profit is more than the equitable profit expectation of the contractor and the following conditions are satisfied*

$$\beta = \frac{1}{1+\gamma} \tag{6.21}$$

$$f = \begin{cases} \frac{\gamma[q_0\{1-\frac{\psi(2\mu_1)^m}{(m+1)\lambda_0^{mA}}\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for uniform distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{w^m\lambda_0^{mA}}\prod_{i=1}^{m}(w+i-1)\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for gamma distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{\left(\frac{u}{u+v}\right)^m\lambda_0^{mA}}\prod_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for beta distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda_0^{mA}}\left[\Gamma\left(1+\frac{m}{s}\right)\right]\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for Weibull distributed time} \end{cases} \tag{6.22}$$

Fairness Concern Contractor and Project Manager

This subsection explores how the optimal condition changes if the project manager also becomes fairness concerned. Again, the coordination problem is solved using the backward induction method. The contractor would select the resource consumption rate λ that maximizes his utility in the equation (6.9). Thus, as mentioned in the section (6.2.1), the contractor selects $\lambda = 0$ in the case of advantageous inequity mentioned in proposition 13. In the second case with advantageous inequity, the λ value again should satisfy the values in the equation (6.15). The optimal condition to achieve the coordination should satisfy the equation (6.21) in proposition 14.

Now, unlike the case of profit maximizing project manager, the fairness concerned project manager would maximize her utility mentioned in (6.5). Replacing the values of π_{pm} and π_{co} from the equations (6.7) and (6.8), the utility function becomes

$$U_{pm} =$$

$$\begin{cases} U_{pm1} = [q_0\{1-\psi E(T^m)\} - f - C_o] - \alpha_{pm}[\delta(f - k\lambda^N\mu_1) - \{q_0(1-\psi E(T^m)) - f - C_o\}] \\ \text{when } \{q_0(1-\psi E(T^m)) - f - C_o\} - \delta(f - k\lambda^N\mu_1) < 0 \\ U_{pm2} = [q_0\{1-\psi E(T^m)\} - f - C_o] - \beta_{pm}[\{q_0(1-\psi E(T^m)) - f - C_o\} - \delta(f - k\lambda^N\mu_1)] \\ \text{when } \{q_0(1-\psi E(T^m)) - f - C_o\} \geq \delta(f - k\lambda^N\mu_1) \end{cases} \tag{6.23}$$

where $\beta_{pm} \leq \alpha_{pm}$, $0 \leq \beta_{pm} < 1$. Similar to Cui et al. (2007), the profit maximizing project manager is a special case of the above with $\alpha_{pm} = 0$ and $\beta_{pm} = 0$.

The contractor considers $\gamma\pi_{pm}$ as the equitable profit and the project manager considers

$\delta\pi_{co}$ as the equitable profit. This means the contractor and the project manager consider $\frac{\gamma}{1+\gamma}\pi_0$ and $\frac{\delta}{1+\delta}\pi_0$ as their equitable share of the supply chain profits respectively. As defined in Cui et al. (2007), the sum of these two pay-offs is considered as the equity capable channel payoff (ECCP) i.e. $ECCP = \frac{\gamma}{1+\gamma}\pi_0 + \frac{\delta}{1+\delta}\pi_0 = \left(\frac{\gamma\delta+\gamma+\delta+\gamma\delta}{\gamma\delta+\gamma+\delta}\right)\pi_0$.

Again as defined in Cui et al. (2007), when $\delta\gamma > 1$ i.e. $ECCP > \pi_0$, the supply chain is considered as acrimonious channel with the members together expect to generate more monetary payoff the supply chain is capable of producing. Thus, there will be some inequity in existence. On the contrary, with $\delta\gamma \leq 1$ i.e. $ECCP \leq \pi_0$, the channel is considered as harmonious channel (Cui et al. 2007).

Thus, the fairness concerned project manager's maximization problem for the case of disadvantageous inequity becomes

$$\max_{f} : U_{pm1} = [q_0\{1 - \psi E(T^m)\} - f - C_o] - \alpha_{pm}[\delta(f - k\lambda_0^N\mu_1) - \{q_0(1 - \psi E(T^m)) - f - C_o\}]$$

St.

$$f \geq \frac{\gamma[q_0\{1 - \psi E(T^m)\} - C_o] + k\lambda^N\mu_1}{1+\gamma} \tag{6.24}$$

Again, the project manager will select a value of f that maximizes her utility function U_{pm1} i.e.

$$\frac{dU_{pm1}}{df} = -1 - \alpha_{pm}(\delta + 1) < 0 \tag{6.25}$$

Thus, again the utility function of the project manager is a decreasing function of f. Thus applying the constraint from the equation (6.24), the optimal value of f would satisfy

$$f_a^* = \frac{\gamma[q_0\{1 - \psi E(T^m)\} - C_o] + k\lambda^N\mu_1}{1+\gamma} \tag{6.26}$$

Now in order for the project manager to incur disadvantageous inequity, the following should be satisfied

$$\{q_0(1 - \psi E(T^m)) - f - C_o\} - \delta(f - k\lambda^N\mu_1) < 0 \tag{6.27}$$

Now replacing these values of f from equation(6.26) and λ_0 in the expression (6.27), it can be shown

$$\{q_0(1-\psi E(T^m)) - f - C_o\} - \delta(f - k\lambda^N\mu_1) = \pi_0\left[\frac{1-\delta\gamma}{1+\gamma}\right] < 0$$

Since π_0 is positive in order for the participation of the members of the supply chain, the project manager to incur disadvantageous inequity when $\delta\gamma > 1$. In other words, the project manager would incur disadvantageous inequity in acrimonious supply chain. The project manager would select the $f = f_a^*$ value as long as her utility is non-negative. Thus,

$$U_{pm1}$$
$$= [q_0\{1-\psi E(T^m)\} - f - C_o] - \alpha_{pm}[\delta(f - k\lambda_0^N\mu_1) - \{q_0(1-\psi E(T^m)) - f - C_o\}] \geq 0$$

$$= \left[q_0\{1-\psi E(T^m)\} - \frac{\gamma[q_0\{1-\psi E(T^m)\} - C_o] + k\lambda^N\mu_1}{1+\gamma} - C_o\right] + \alpha_{pm}\pi_0\left[\frac{1-\delta\gamma}{1+\gamma}\right] \geq 0$$
$$= \frac{\pi_0}{1+\gamma}\{1 + \alpha_{pm}(1-\delta\gamma)\} \geq 0$$

or

$$\alpha_{pm} \leq \frac{1}{\delta\gamma - 1} \tag{6.28}$$

From the above observations, the optimal condition for the project manager is summarised in the following lemma

Lemma 17. *If both the project manager and the contractor are fairness concerned, the supply chain is coordinated with the fixed price contract satisfying the condition in the equation (6.26) when the following are satisfied*

$$\beta_{co} \geq \frac{1}{1+\gamma},$$
$$\delta\gamma > 1,\ and\ \alpha_{pm} \leq \frac{1}{\delta\gamma - 1} \tag{6.29}$$

On the contrary to the disadvantageous inequity, the project manager incurs advantageous inequity when when the following is satisfied,

$$\{q_0(1-\psi E(T^m))-f-C_o\}-\delta(f-k\lambda^N\mu_1)\geq 0 \tag{6.30}$$

Thus, the optimization problem of the project manager becomes

$$\max_f : U_{pm2} \tag{6.31}$$

$$= [q_0\{1-\psi E(T^m)\}-f-C_o]-\beta_{pm}[\{q_0(1-\psi E(T^m))-f-C_o\}-\delta(f-k\lambda_0^N\mu_1)]$$

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$$f \geq \frac{\gamma[q_0\{1-\psi E(T^m)\}-C_o]+k\lambda^N\mu_1}{1+\gamma} \tag{6.32}$$

The value of f that maximizes project manager's utility in the above equation should satisfy

$$\frac{dU_{pm2}}{df} = -1+\beta_{pm}(1+\delta) \tag{6.33}$$

If $-1+\beta_{pm}(1+\delta)\leq 0$, then $\frac{dU_{pm}}{df}\leq 0$. This means the project manager's utility would be a decreasing function of f. Thus, the project manager selects the minimum value that satisfies the constraint in the equation (6.32) i.e. $f_h^* = \frac{\gamma[q_0\{1-\psi E(T^m)\}-C_o]+k\lambda^N\mu_1}{1+\gamma}$. Moreover, the conditions mentioned in equation (6.21) in proposition 14 is applied to the λ for the purpose of coordination. Using these values of f and λ in the utility function of the project manager, it can easily be shown that the project manager would incur this advantageous inequity in the event of $\pi_0\left[\frac{1-\delta\gamma}{1+\gamma}\right]\geq 0$ i.e. $\delta\gamma\leq 1$. This means, the project manager would incur advantageous inequity in the harmonious supply chain. Summarising these, the optimization conditions are presented in the following lemma

Lemma 18. *If both the project manager and the contractor are fairness concerned, the supply chain is coordinated with the fixed price contract*

$$f_h^* = \frac{\gamma[q_0\{1-\psi E(T^m)\}-C_o]+k\lambda^N\mu_1}{1+\gamma} \tag{6.34}$$

when the following are satisfied

$$\beta_{co} \geq \frac{1}{1+\gamma}, \delta\gamma \leq 1 \textit{ and } \beta_{pm} \leq \frac{1}{1+\delta} \tag{6.35}$$

If $[-1 + \beta_{pm}(1+\delta)] > 0$ i.e $\beta_{pm} > \frac{1}{1+\delta}$, then $\frac{dU_{pm}}{df} > 0$. This means, the utility of the project manager would increase with f. Thus, higher the value of f, higher the utility of the fairness concerned project manager and there is no upper bound to the optimal solution for f. However, the profit of the project manager decreases with any increase in f. As a result after a certain higher values of f, the profit of the project manager would become exactly the same as her expected fair profit and further increase in f would make the profit negative.It can be easily shown from the lemma 18 that $\pi_{co} = \frac{\gamma\pi_0}{1+\gamma}$ at the offered value of contract f_h^*. This is the fair allocation of the profit for the contractor. Hence, increasing the value of f after this would never be able to allocate the profit fairly. Thus, this research won't consider this case. Combining the findings from lemmas 17 and 18, and using the expected values of the m$^{\text{th}}$ moments of the completion time for different distributions from the equation (4.17) in the optimal contract price, the following is proposed

Proposition 15. *In a fairness concerned supply chain (with both the project manager and the contractor are fairness concerned), the project manager can coordinate the supply chain with the following fixed price*

$$f^* = \begin{cases} \frac{\gamma[q_0\{1-\frac{\psi(2\mu_1)^m}{(m+1)\lambda_0^{mA}}\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for uniform distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{w^m\lambda_0^{mA}}\prod_{i=1}^{m}(w+i-1)\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for gamma distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{\left(\frac{u}{u+v}\right)^m\lambda_0^{mA}}\prod_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for beta distributed time} \\ \frac{\gamma[q_0\{1-\frac{\psi\mu_1^m}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda_0^{mA}}\left[\Gamma\left(1+\frac{m}{s}\right)\right]\}-C_o]+k\lambda_0^N\mu_1}{1+\gamma} & \textit{for Weibull distributed time} \end{cases} \tag{6.36}$$

if and only if the contractor has a non-zero positive disutility parameter β_{co}, that $\beta_{co} \geq \frac{1}{1+\gamma}$ and when any of the following is satisfied

1. $\delta\gamma > 1$, *and* $\alpha_{pm} \leq \frac{1}{\delta\gamma-1}$
2. $\delta\gamma \leq 1$, *and* $\beta_{pm} \leq \frac{1}{1+\delta}$.

Fairness concerned project manager and profit maximizing contractor

The coordination problem is again solved using backward induction method. For a given value of fixed price contract f, the contractor selects a resource consumption rate λ that maximizes his profit. The project manager would anticipate this value of λ by backward induction and would offer a fixed price f that maximizes her own utility. This should also satisfy the constraint to achieve the λ that maximizes the contractor's profit.

Similar to the calculation shown in chapter 4, the λ that maximizes the contractor's profit in equation (6.8), should satisfy $-kN\mu_1\lambda^{N-1} = 0$. In other words, the selected value of resource consumption rate would be zero. This leads to the following proposition

Proposition 16. *A supply chain with a fairness concerned project and a profit maximizing contractor, can not be coordinated using a fixed price contract with the resource consumption rate as the decision making variable.*

6.2.2 For Long Term Projects

As described in chapter 4, there could be two different types of scenario for the long term projects: the projects with recoverable operational life of the product, and the projects with irrecoverable operational life of the product, in the event the project completion is delayed. Chapter 4 analysed the scenario for profit maximizing project manager and profit maximizing contractor. This section extends that analysis to fairness concerned members. As described in chapter 4, the profit functions in the centralized setting satisfy the condition in the equation 4.12. The first best resource consumption rate $\lambda = \lambda_0$ satisfies the following

$$\frac{d\pi}{d\lambda} = \begin{cases} q_0 \frac{dE\{e^{-\alpha T}\}}{d\lambda} - \frac{kn\lambda^{n-1}}{\alpha}\left[1 - E\{e^{-\alpha T}\}\right] + \frac{k\lambda^n}{\alpha}\left[\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right] = 0 & \text{for recoverable product life} \\ \left(\frac{q_0\psi}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda} - \frac{kn\lambda^{n-1}}{\alpha}\left[1 - E\{e^{-\alpha T}\}\right] + \frac{k\lambda^n}{\alpha}\left[\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right] = 0 & \text{for irrecoverable product life} \end{cases} \tag{6.37}$$

In the decentralized setting, the project manager offers a contract P(T, C) = f to the contractor. Thus, their individual profits become (Using the observations from the equations 4.13 and

4.14)

$$\pi_{pm} = \begin{cases} q_0 E\{e^{-\alpha T}\} - \int\limits_0^{\infty} P(T,C)\, e^{-\alpha T} f_{\lambda}(T) dT - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] - \int\limits_0^{\infty} P(T,C)\, e^{-\alpha T} f_{\lambda}(T) dT - C_o & \text{for irrecoverable product life} \end{cases} \tag{6.38}$$

$$\pi_{co} = \left[\int\limits_0^{\infty} P(T,C)\, e^{-\alpha T} f_{\lambda}(T) dT \right] - \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\}\right] \tag{6.39}$$

Since the fixed price contract is used, P(T,C) = f. Thus, the expected value becomes

$$E\{P(T,C)e^{-\alpha T}\} = f \int\limits_0^{\infty} e^{-\alpha T} f_{\lambda}(T) = f E\{e^{-\alpha T}\} \tag{6.40}$$

Using this value of $E\{P(T,C)e^{-\alpha T}\}$ in equations (6.38) and (6.39), the following modified profit functions of the project manager and the contractor in the decentralized supply chain are derived

$$\pi_{pm} = \begin{cases} q_0 E\{e^{-\alpha T}\} - f E\{e^{-\alpha T}\} - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] - f E\{e^{-\alpha T}\} - C_o & \text{for irrecoverable product life} \end{cases} \tag{6.41}$$

$$\pi_{co} = E\{P(T,C)e^{-\alpha T}\} - E(C) = f\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha} \left[1 - E\{e^{-\alpha T}\}\right] \tag{6.42}$$

$$\text{where } E\{(P(T,C)e^{-\alpha T}\} = \left[\int\limits_0^{\infty} P(T,C)\, e^{-\alpha T} f_{\lambda}(T) dT \right]$$

As shown in chapter 4, the $E\{e^{-\alpha T}\}$ values can be derived from the equation (4.25) (Evans et al. 1993).

Fairness Concern Contractor and Profit Maximizing Project Manager

If the contract is fairness concerned, the he selects a resource consumption rate λ in the decentralized setting which maximizes his utility function as below

$$U_{co} =$$

$$\begin{cases} \left[fE\{e^{-\alpha T}\} - \tfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\alpha_{co}\left[\gamma\left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\} - \left\{fE\{e^{-\alpha T}\} - \tfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right]^+ \\ -\beta_{co}\left[\left\{fE\{e^{-\alpha T}\} - \tfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} - \gamma\left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\}\right]^+ \\ \text{for recoverable product life} \\ \\ \left[fE\{e^{-\alpha T}\} - \dfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\alpha_{co}[\gamma\{q_0\{1 - \dfrac{\psi}{\alpha} + \dfrac{\psi}{\alpha}E(e^{-\alpha T})\} - fE\{e^{-\alpha T}\} - C_o\} - \{fE(e^{-\alpha T}) - \dfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\}]^+ \\ -\beta_{co}[\{fE\{e^{-\alpha T}\} - \dfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\} - \gamma\{q_0\{1 - \dfrac{\psi}{\alpha} + \dfrac{\psi}{\alpha}E(e^{-\alpha T})\} - fE\{e^{-\alpha T}\} - C_o\}]^+ \\ \text{for irrecoverable product life} \end{cases} \tag{6.43}$$

Similar to the case of short term project, this utility function can be broken into two cases.

The disadvantageous inequity occurs when

$$\begin{cases} \left[\left\{fe^{-\alpha T}\right\} - \tfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] - \left[\gamma\left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\}\right] < 0 \\ \text{for recoverable product life} \\ \left[\left\{fE\{e^{-\alpha T}\} - \tfrac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] - \left[\gamma\left\{q_0[1 - \tfrac{\psi}{\alpha} + \tfrac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\right\}\right] < 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.44}$$

The utility function becomes $U_{co} = U_{co1}$, where

$$U_{co1} = \begin{cases} \left[fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\alpha_{co}\left[\gamma\left\{q_0E(e^{-\alpha T}) - fE(e^{-\alpha T}) - C_o\right\} - \left\{fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] \\ \text{for recoverable product life} \\ \left[fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\alpha_{co}\left[\gamma\left\{q_0(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E(e^{-\alpha T})) - fE(e^{-\alpha T}) - C_o\right\} - \left\{fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.45}$$

For an offered value of f, the fairness concerned contractor will select a resource consumption rate λ that maximizes the utility mentioned in the equation (6.45). Thus, the selected λ satisfies the following

$$\frac{dU_{co1}}{d\lambda} = \begin{cases} (1+\alpha_{co})\left[f\frac{dE\{e^{-\alpha T}\}}{d\lambda} - \left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right] \\ -\alpha_{co}\left[\gamma\left\{q_0\frac{dE\{e^{-\alpha T}\}}{d\lambda} - f\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right\}\right] = 0 \\ \text{for recoverable product life} \\ (1+\alpha_{co})\left[f\frac{dE\{e^{-\alpha T}\}}{d\lambda} - \left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right] \\ -\alpha_{co}\left[\gamma\left\{\left(\frac{q_0\psi}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda} - f\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right\}\right] = 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.46}$$

In order to coordinate the supply chain, this selected λ should be at least equal to the first best solution λ_0 i.e. $\lambda \geq \lambda_0$. Thus replacing the value of $\frac{dE(C)}{d\lambda}$ i.e. the term $\left[-\left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right]$ from the equation (6.37) in the equation

(6.46) and rearranging the terms, the following conditions are derived

$$\frac{dU_{co1}}{d\lambda} = \begin{cases} (1+\alpha_{co}+\gamma\alpha_{co})\frac{dE(e^{-\alpha T})}{d\lambda}(f-q_0)=0 & \text{for recoverable product life} \\ (1+\alpha_{co}+\gamma\alpha_{co})\frac{dE(e^{-\alpha T})}{d\lambda}(f-\frac{q_0\psi}{\alpha})=0 & \text{for irrecoverable product life} \end{cases} \tag{6.47}$$

Differentiating the $E\{e^{-\alpha T}\}$ with respect to λ, it can be shown that $\frac{dE(e^{-\alpha T})}{d\lambda} > 0$ for the statistical distributions selected for this research. Thus, in order to achieve coordination, the offered contract has to be at least equal to the project value at the start of the project i.e. $f \geq q_0$ for a project with recoverable product life upon completion, and $f \geq \frac{q_0\psi}{\alpha}$ for a project with irrecoverable product life upon completion. It is assumed earlier that $\psi > \alpha$ in order to make sure that the impact of any project delay is taken into consideration.Thus, summarising the above observation, the following is proposed

Proposition 17. *A fixed price contract fails to coordinate a supply chain with a profit maximizing project manager and a fairness concerned contractor when the cash-flows are exponentially discounted and the contractor experiences any disadvantageous inequity.*

On the contrary, the advantageous inequity occurs when the following are satisfied

$$\begin{cases} \left[\{fE(e^{-\alpha T})\} - \frac{k\lambda^n}{\alpha}\left\{1-E\left(e^{-\alpha T}\right)\right\}\right\} - \gamma\left\{q_0E(e^{-\alpha T}) - fE(e^{-\alpha T}) - C_o\right\}\right] \geq 0 \\ \text{for recoverable product life} \\ \left[\left\{fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1-E\left(e^{-\alpha T}\right)\right\}\right\} - \gamma\left\{q_0\left(1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}(e^{-\alpha T})\right) - fE(e^{-\alpha T}) - C_o\right\}\right] \geq 0 \\ \text{for irrecoverable product life} \end{cases}$$

or

$$f \geq \begin{cases} \frac{\gamma q_0E(e^{-\alpha T})+\frac{k\lambda^n}{\alpha}\left\{1-E\left(e^{-\alpha T}\right)\right\}-\gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \text{for recoverable product life} \\ \frac{\gamma q_0[1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E\{e^{-\alpha T}\}]+\frac{k\lambda^n}{\alpha}\left\{1-E\left(e^{-\alpha T}\right)\right\}-\gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \text{for irrecoverable product life} \end{cases} \tag{6.48}$$

The utility function becomes $U_{co} = U_{co2}$, where

$$U_{co2} =$$

$$\begin{cases} \left[fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\beta_{co}\left[\left\{fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} - \gamma\left\{q_0E(e^{-\alpha T}) - fE\{e^{-\alpha T}\} - C_o\right\}\right] \\ \text{for recoverable product life} \\ \left[fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\beta_{co}\left[\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} - \gamma\left\{q_0(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E(e^{-\alpha T})) - fE(e^{-\alpha T}) - C_o\right\}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.49}$$

Again, the contractor selects a λ that maximizes his utility function in the equation (6.49). Thus, it should satisfy

$$\frac{dU_{co2}}{d\lambda} = \begin{cases} (1-\beta_{co})\left[f\frac{dE\{e^{-\alpha T}\}}{d\lambda} - \left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE(e^{-\alpha T})}{d\lambda}\right] \\ +\beta_{co}\left[\gamma\left\{q_0\frac{dE\{e^{-\alpha T}\}}{d\lambda} - f\frac{dE(e^{-\alpha T})}{d\lambda}\right\}\right] = 0 \\ \text{for recoverable product life} \\ (1-\beta_{co})\left[f\frac{dE(e^{-\alpha T})}{d\lambda} - \left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda}\right] \\ +\beta_{co}\left[\gamma\left\{\left(\frac{q_0\psi}{\alpha}\right)\frac{dE(e^{-\alpha T})}{d\lambda} - f\frac{dE(e^{-\alpha T})}{d\lambda}\right\}\right] = 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.50}$$

Again for the purpose of the coordination the selected λ should be at least equal to the first best solution λ_0 i.e. $\lambda \geq \lambda_0$. Thus replacing $\frac{dE(C)}{d\lambda}$ i.e. the term $-\left(\frac{kn\lambda^{n-1}}{\alpha}\right)\left\{1 - E(e^{-\alpha T})\right\} + \left(\frac{k\lambda^n}{\alpha}\right)\frac{dE\{e^{-\alpha T}\}}{d\lambda}$ from the equation (6.37) in the equation (6.50)

$$\frac{dU_{co2}}{d\lambda} = \begin{cases} (1-\beta_{co}-\gamma\beta_{co})\frac{dE(e^{-\alpha T})}{d\lambda}(f - q_0) = 0 & \text{for recoverable product life} \\ (1-\beta_{co}-\gamma\beta_{co})\frac{dE(e^{-\alpha T})}{d\lambda}(f - \frac{q_0\psi}{\alpha}) = 0 & \text{for irrecoverable product life} \end{cases} \tag{6.51}$$

It can be easily shown that $\frac{dE(e^{-\alpha T})}{d\lambda} > 0$. For the coordination to be achieved, $(f - q_0) < 0$. If $\beta_{co} = \frac{1}{1+\gamma}$, then $\frac{dU_{co}}{d\lambda} = 0$. For $\lambda \geq \lambda_0$, $\frac{dU_{co}}{d\lambda} \geq 0$. Thus, $(1 - \beta_{co} - \beta_{co}\gamma) \leq 0$. i.e. $\beta_{co} \geq \frac{1}{1+\gamma}$.

The project manager anticipates these requirements of f in order to entice the contractor for coordination. Thus, she takes it into consideration before offering the contract f and her optimization problem becomes

$$\max_{f} : \pi_{pm} = \begin{cases} q_0 E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o & \text{for irrecoverable product life} \end{cases} \tag{6.52}$$

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$$f \geq \begin{cases} \frac{\gamma q_0 E(e^{-\alpha T}) + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \text{for recoverable product life} \\ \\ \frac{\gamma q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \text{for irrecoverable product life} \end{cases} \tag{6.53}$$

The project manager selects a λ that maximizes her profit, i.e. $\frac{dU_{pm}}{d\lambda} = \frac{d\pi_{pm}}{d\lambda} = -E\{e^{-\alpha T}\} < 0$. This is true for for both recoverable and irrecoverable product life.

Thus, again similar to the case of of short term projects with no discounting of cash-flows, the project manager's utility is a decreasing function of f. Thus, in order to maximize the profit, given the constraints in the equation (6.53), the optimal value of f should satisfy condition in the following lemma

Lemma 19. *A fixed price contract can coordinate the project supply chain when the cash-flows are exponentially discounted, with a fairness concerned contractor and a profit maximizing project manager, if the contractor experiences advantageous inequity, the cash-flows are exponentially discounted and the following conditions are satisfied*

$$\beta_{co} \geq \frac{1}{1+\gamma} \tag{6.54}$$

$$f^* = \begin{cases} \frac{\gamma q_0 E(e^{-\alpha T}) + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \textit{for recoverable product life} \\ \frac{\gamma q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \textit{for irrecoverable product life} \end{cases} \tag{6.55}$$

For the values $\beta_{co} > \frac{1}{1+\gamma}$, the utility of contractor might increase, but at the same time his monetary profit would keep on decreasing and after a certain increase in λ, it would become negative, but the overall utility will still increase due aversion of to high per unit price of advantageous inequity. This research avoids this type of scenario due to practicality considerations. Using the value of the moment generating functions of the distributions, the following are proposed

Proposition 18. *A fixed price contract can coordinate the project supply chain when the cash-flows are exponentially discounted, with a fairness concerned contractor and a profit maximizing project manager, if the contractor's monetary profit is more than the equitable profit expectation of the project manager and the following conditions are satisfied*

$$\beta_{co} = \frac{1}{1+\gamma} \tag{6.56}$$

1. *for an uniform distributed time*

$$f = \begin{cases} \dfrac{\gamma q_0 \lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right) + \frac{k\lambda^n}{\alpha} \left\{ 1-\lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right) \right\} - \gamma C_o}{(1+\gamma)\lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right)} & \textit{for recoverable product life} \\ \dfrac{\gamma q_0 [1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha} \lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right) + \frac{k\lambda^n}{\alpha} \left\{ 1-\lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right) \right\} - \gamma C_o}{(1+\gamma)\lambda^A \left(\frac{1-e^{-\frac{2\mu_1 \alpha}{\lambda}}}{2\alpha\mu_1} \right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.57}$$

2. *for a gamma distributed time*

$$f = \begin{cases} \dfrac{\gamma q_0 \left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w + \frac{k\lambda^n}{\alpha} \left\{ 1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w \right\} - \gamma C_o}{(1+\gamma)\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w} & \textit{for recoverable product life} \\ \dfrac{\gamma q_0 [1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha} \left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w + \frac{k\lambda^n}{\alpha} \left\{ 1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w \right\} - \gamma C_o}{(1+\gamma)\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1} \right)^w} & \textit{for irrecoverable product life} \end{cases} \tag{6.58}$$

3. *for beta distributed time*

$$f = \tag{6.59}$$

$$\begin{cases} \dfrac{(\gamma q_0 - \frac{k\lambda^n}{\alpha})\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} & \textit{for recoverable prod life} \\ \dfrac{\gamma q_0\left(1-\frac{\psi}{\alpha}\right)+\left(\frac{q_0\psi}{\alpha}-\frac{k\lambda^n}{\alpha}\right)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m\left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} & \textit{for irrecoverable prod life} \end{cases} \tag{6.60}$$

4. *for Weibull distributed time*

$$f = \tag{6.61}$$

$$\begin{cases} \dfrac{\left(\gamma q_0 - \frac{k\lambda^n}{\alpha}\right)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} & \textit{for recoverable prod life} \\ \dfrac{\gamma q_0\left(1-\frac{\psi}{\alpha}\right)+\left(\frac{\psi}{\alpha}-\frac{k\lambda^n}{\alpha}\right)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} & \textit{for irrecoverable prod life} \end{cases} \tag{6.62}$$

Fairness Concern Contractor and Project Manager

This subsection explores how the optimal condition changes if the project manager also becomes fairness concerned. Again, the coordination problem is solved using the backward induction method. The contractor would select the resource consumption rate λ that maximizes his utility in the equation (6.43). Thus, as mentioned earlier the contractor would still select $\lambda = 0$ when he experiences disadvantageous inequity (As mentioned in proposition 17). In the second case with advantageous inequity, the λ value again should satisfy the values in the equations (6.50) and (6.51) and the optimal condition $\beta_{co} \geq \frac{1}{1+\gamma}$ in order to achieve the coordination. The optimnal condition for f mentioned in the equation (6.53) satisfies this requirement.

Now, again unlike the case of profit maximizing project manager, the fairness concerned project manager would maximize her utility mentioned in (6.5). Now replacing the values of

π_{pm} and π_{co} from the equations (6.7) and (6.8), the utility function becomes

$$U_{pm} = \begin{cases} \left[fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] \\ -\alpha_{pm}\left[\delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} - \left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\}\right]^+ \\ -\beta_{pm}\left[\left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right]^+ \\ \text{for recoverable product life} \\ \left[fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right] - \alpha_{pm}[\delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} \\ -\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\right\}]^+ - \beta_{pm}[\{q_0(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E(e^{-\alpha T})) \\ -fE(e^{-\alpha T}) - C_o\} - \delta\left\{fE(e^{-\alpha T}) - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}]^+ \\ \text{for irrecoverable product life} \end{cases} \tag{6.63}$$

[where $\beta_{pm} \leq \alpha_{pm}$, $0 \leq \beta_{pm} < 1$ as before].

As mentioned in the section 6.2.1, the contractor and the project manager consider $\frac{\gamma}{1+\gamma}\pi_0$ and $\frac{\delta}{1+\delta}\pi_0$ as their equitable share of the supply chain profits respectively. As defined in Cui et al. (2007), the equity capable channel payoff is ECCP $= \left(\frac{\gamma\delta+\gamma+\delta+\gamma\delta}{\gamma\delta+\gamma+\delta}\right)\pi_0$.

When the following are satisfied, the project manager incurs disadvantageous inequity

$$\begin{cases} \left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} < 0 \\ \text{for recoverable product life} \\ \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} < 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.64}$$

Then, the the project manager's utility becomes

$$U_{pm1} = \begin{cases} \big[\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\big] \\ -\alpha_{pm}\big[\delta\big\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\big\{1 - E\big(e^{-\alpha T}\big)\big\}\big\} - \big\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\big\}\big] \\ \text{for recoverable product life} \\ \big[q_0\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E(e^{-\alpha T})\} - fE\{e^{-\alpha T}\} - C_o\big] \\ -\alpha_{pm}[\delta\big\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\big\{1 - E\big(e^{-\alpha T}\big)\big\}\big\} \\ -\big\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\big\}] \\ \text{for irrecoverable product life} \end{cases} \tag{6.65}$$

Thus, the fairness concerned project manager's maximization problem becomes

$$\max_{f} : U_{pm} = U_{pm1} \qquad \text{[where } U_{pm1} \text{ follows the equation (6.65)]}$$

Sub. to

$$\text{f} \quad \text{satisfying the equation in (6.53)} \tag{6.66}$$

Again, the project manager will select a value of f that maximizes her utility function U_{pm1} i.e. $\frac{dU_{pm1}}{df} = -1 - \alpha_{pm}(\delta + 1) < 0$;

Thus, again the utility function of the project manager is a decreasing function of f. Now applying the constraint from the equation (6.66), the optimal value of f would satisfy the optimal condition of f mentioned in equation (6.55). Now replacing these values of f and λ_0 in the expression (6.64), it can be shown that $\pi_0 \left[\frac{1-\delta\gamma}{1+\gamma}\right] < 0$.

Proof. Using the optimal value of f from the equation (6.55) and $\lambda = \lambda_0$ in the left hand side of the equation (6.64)

$$
\begin{aligned}
&\left[q_0E(e^{-\alpha T}) - \frac{\gamma q_0 E(e^{-\alpha T}) - \gamma C_o + k\lambda^n\{1-E(e^{-\alpha T})\}}{(1+\gamma)E(e^{-\alpha T})}E(e^{-\alpha T}) - C_o\right] \\
&- \delta\left[\frac{\gamma q_0 E(e^{-\alpha T}) - \gamma C_o + k\lambda^n\{1-E(e^{-\alpha T})\}}{(1+\gamma)E(e^{-\alpha T})}E(e^{-\alpha T}) - k\lambda_0^n\{1-E(e^{-\alpha T}\}\right] \\
&= \left[q_0E(e^{-\alpha T}) - k\lambda_0^n\{1-E(e^{-\alpha T})\} - C_o\right]\left[\frac{1-\delta\gamma}{1+\gamma}\right] \\
&= \pi_0\left[\frac{1-\delta\gamma}{1+\gamma}\right]
\end{aligned}
$$

$\square$

Similar to the case of short term projects with no cash discounting, π_0 is positive for the long term projects with cash discounting in order for the participation of the members of the supply chain. Thus,the project manager incurs disadvantageous inequity when $\delta\gamma > 1$. In other words, the project manager would incur disadvantageous inequity in acrimonious supply chain (based on the definition in Cui et al. (2007)). The project manager would select the optimal f value from the equation (6.55) as long as her utility is non-negative. Thus, using thsi optimal f value in the equation (6.65) and using the observation in the proof above.

$$
U_{pm1} = \begin{cases}
\left[q_0E(e^{-\alpha T}) - \frac{\gamma q_0 E(e^{-\alpha T}) + \frac{k\lambda_0^n}{\alpha}\{1-E(e^{-\alpha T})\} - \gamma C_o}{(1+\gamma)E(e^{-\alpha T})}E(e^{-\alpha T}) - C_o\right] - \alpha_{pm}\pi_0\left[\frac{\delta\gamma-1}{1+\gamma}\right] \geq 0 \\
\text{for recoverable product life} \\
\left[q_0\{1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E(e^{-\alpha T})\} - \frac{\gamma q_0 E(e^{-\alpha T}) + \frac{k\lambda_0^n}{\alpha}\{1-E(e^{-\alpha T})\} - \gamma C_o}{(1+\gamma)E(e^{-\alpha T})}E(e^{-\alpha T}) - C_o\right] \\
-\alpha_{pm}\pi_0\left[\frac{\delta\gamma-1}{1+\gamma}\right] \geq 0 \\
\text{for irrecoverable product life}
\end{cases}
$$

or

$$
U_{pm1} = \begin{cases}
\frac{\left[q_0E(e^{-\alpha T}) - k\lambda_0^n\{1-E(e^{-\alpha T})\} - C_o\right]}{1+\gamma} - \alpha_{pm}\pi_0\left[\frac{\delta\gamma-1}{1+\gamma}\right] \geq 0 & \text{for recoverable product life} \\
\frac{\left[q_0\{1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E(e^{-\alpha T})\} - k\lambda_0^n\{1-E(e^{-\alpha T})\} - C_o\right]}{1+\gamma} - \alpha_{pm}\pi_0\left[\frac{\delta\gamma-1}{1+\gamma}\right] \geq 0 & \text{for irrecoverable product life}
\end{cases}
$$

or

$$
U_{pm1} = \frac{\pi_0}{1+\gamma}\{1-\alpha_{pm}(\delta\gamma-1)\} \geq 0
$$

or

$$\alpha_{pm} \leq \frac{1}{\delta\gamma - 1} \qquad\qquad [\because \pi_0 > 0, \text{ and } (1+\gamma) > 0]$$

From all the above observations, the optimal condition for the project manager is summarised in the following lemma

Lemma 20. *The supply chain with a fairness concerned project manager and a fairness concerned contractor is coordinated with the fixed price contract below when cash-flows are discounted exponentially*

$$f_a^* = \frac{\gamma q_0 E(e^{-\alpha T}) - \gamma C_o + \frac{k\lambda^n}{\alpha}\{1 - E(e^{-\alpha T})\}}{(1+\gamma)E(e^{-\alpha T})} \tag{6.67}$$

In addition, the following conditions must satisfied

$$\beta_{co} \geq \frac{1}{1+\gamma}, \; \delta\gamma > 1, \; \textit{and} \; \alpha_{pm} \leq \frac{1}{\delta\gamma - 1} \tag{6.68}$$

On the contrary to the disadvantageous inequity, when the following is satisfied, the project manager incurs advantageous inequity when

$$\begin{cases} \left\{q_0 E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} \geq 0 \\ \text{for recoverable product life} \\ \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\} \geq 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.69}$$

Thus, the optimization problem of the project manager becomes

$$\max_{f} : U_{pm} = U_{pm2} = \tag{6.70}$$

$$\begin{cases} \left[\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right] \\ -\beta_{pm}\left[\left\{q_0E\{e^{-\alpha T}\} - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] \\ \text{for recoverable product life} \\ \left[q_0\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E(e^{-\alpha T})\} - fE\{e^{-\alpha T}\} - C_o\right] \\ -\beta_{pm}\left[\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - fE\{e^{-\alpha T}\} - C_o\right\} - \delta\left\{fE\{e^{-\alpha T}\} - \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\}\right\}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.71}$$

Sub.to

$$\text{f} \quad \text{satisfying the equation in (6.53)} \tag{6.72}$$

In order to find the optimal value of f, the first order derivative of the project manager's is derived as below

$$\frac{dU_{pm2}}{df} = [-1 + \beta_{pm}(1 + \delta)]E(e^{-\alpha T}) \tag{6.73}$$

Since $E(e^{-\alpha T}) \geq 0$, so if $-1 + \beta_{pm}(1 + \delta) \leq 0$, then $\frac{dU_{pm}}{df} \leq 0$. This means the project manager's utility would be a decreasing unction of f. Thus, the project manager will select the minimum value that satisfies the constraint in the equation (6.53). Moreover, the conditions mentioned in equation (6.56) in lemma 18 is applied to the resource consumption rate to satisfy $\lambda = \lambda_0$ for the purpose of coordination. Using theses value of f and λ, in the utlity function of the project manager, it can easily be shown that the project manager would incur this advantageous inequity in the event of $\pi_0\left[\frac{1-\delta\gamma}{1+\gamma}\right] \geq 0$ i.e. $\delta\gamma \leq 1$. This means, the project manager would incur advantageous inequity in the harmonious channel. Summarising these, the optimization conditions are presented in the following lemma

Lemma 21. *If both the project manager and the contractor are fairness concerned, the supply chain is coordinated with the fixed price contract with cash-flows are discounted exponen-*

tially if

$$f_h^* = \begin{cases} \frac{\gamma q_0 E(e^{-\alpha T}) + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \textit{for recoverable product life} \\ \\ \frac{\gamma q_0 [1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha} E\{e^{-\alpha T}\}] + \frac{k\lambda^n}{\alpha}\left\{1 - E\left(e^{-\alpha T}\right)\right\} - \gamma C_o}{(1+\gamma)E\{e^{-\alpha T}\}} & \textit{for irrecoverable product life} \end{cases} \tag{6.74}$$

when the following are satisfied

$$\beta_{co} \geq \frac{1}{1+\gamma},\ \delta\gamma \leq 1 \textit{ and } \beta_{pm} \leq \frac{1}{1+\delta} \tag{6.75}$$

Similar to the case of short term projectrs with no cash discounting, at the above optimal value of the contractor would earn a fair profit of $\frac{\gamma \pi_0}{1+\gamma}$ and the project manager would earn $\frac{\pi_0}{1+\gamma}$.

If $[-1 + \beta_{pm}(1 + \delta)] > 0$ i.e $\beta_{pm} > \frac{1}{1+\delta}$, then $\frac{dU_{pm}}{df} > 0$. This means, the utility of the project manager would increase with increase in f. Thus, higher the value of f, higher the utility of the fairness concerned project manager and there is no upper bound to the optimal solution for f. After a certain higher values of f, the profit of the project manager would become negative, but due to higher value of disutility aversion per unit due to advantageous inequity would compensate this. It is also not very logical to increase the value of f after the value mentioned in the above lemma as it would never allocate the profit fairly. Thus, this research does not consider this case. Combining the findings from lemmas 20 and 21, and using the expected values of the m$^{\text{th}}$ moments of the completion time for different distributions from the equation (4.17) in the optimal contract price, the following is proposed

Proposition 19. *In a fairness concerned supply chain (with both project manager and the contractor are fairness concerned), the project manager can coordinate the supply chain with the following fixed price when the cash-flows are discounted*

1. *for an uniform distributed time*

$$f^* = \begin{cases} \dfrac{\gamma q_0 \lambda^A \left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right) + \frac{k\lambda^n}{\alpha}\left\{1-\lambda^A\left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right)\right\} - \gamma C_o}{(1+\gamma)\lambda^A\left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right)} & \textit{for recoverable product life} \\ \dfrac{\gamma q_0 [1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}\lambda^A \left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right) + \frac{k\lambda^n}{\alpha}\left\{1-\lambda^A\left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right)\right\} - \gamma C_o}{(1+\gamma)\lambda^A\left(\frac{1-e^{-\frac{2\mu_1\alpha}{\lambda}}}{2\alpha\mu_1}\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.76}$$

2. *for a gamma distributed time*

$$f^* = \begin{cases} \dfrac{\gamma q_0 \left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w + \frac{k\lambda^n}{\alpha}\left\{1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w\right\} - \gamma C_o}{(1+\gamma)\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w} & \textit{for recoverable product life} \\ \dfrac{\gamma q_0 [1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w + \frac{k\lambda^n}{\alpha}\left\{1-\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w\right\} - \gamma C_o}{(1+\gamma)\left(\frac{w\lambda^A}{w\lambda^A+\alpha\mu_1}\right)^w} & \textit{for irrecoverable product life} \end{cases} \tag{6.77}$$

3. *for beta distributed time*

$$f^* = \begin{cases} \dfrac{(\gamma q_0 - k\lambda^n\alpha)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m \left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right] + \frac{k\lambda^n}{\alpha} - \gamma C_o}{(1+\gamma)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m \left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} \\ \textit{for recoverable prod} \\ \dfrac{\gamma q_0\left(1-\frac{\psi}{\alpha}\right) + \left(q_0\frac{\psi}{\alpha} - \frac{k\lambda^n}{\alpha}\right)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m \left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right] + \frac{k\lambda^n}{\alpha} - \gamma C_o}{(1+\gamma)\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^A}\right\}^m \left(\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right)\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} \\ \textit{for irrecoverable prod} \end{cases} \tag{6.78}$$

4. *for Weibull distributed time*

$$f^* = \begin{cases} \frac{\left(\gamma q_0 - \frac{k\lambda^n}{\alpha}\right)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} \\ \textit{for recoverable prod} \\ \frac{\gamma q_0\left(1-\frac{\psi}{\alpha}\right)+\left(\frac{q_0\psi}{\alpha}-\frac{k\lambda^n}{\alpha}\right)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]+\frac{k\lambda^n}{\alpha}-\gamma C_o}{(1+\gamma)\left[1+\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^A\Gamma(1+\frac{1}{s})}\right\}^m\left\{\Gamma(1+\frac{m}{s})\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right]} \\ \textit{for irrecoverable prod} \end{cases} \tag{6.79}$$

if and only if the contractor has a non-zero positive disutility parameter β_{co}, *such that* $\beta_{co} \geq \frac{1}{1+\gamma}$ *and when any of the following is satisfied*

1. $\delta\gamma > 1$, *and* $\alpha_{pm} \leq \frac{1}{\delta\gamma - 1}$
2. $\delta\gamma \leq 1$, *and* $\beta_{pm} \leq \frac{1}{1+\delta}$.

Fairness concerned project manager and profit maximizing contractor

The coordination perotblem is again solved using backward induction method. For a given value of fixed price contract f, the contractor selects a resource consumption rate λ that maximizes his profit. The project manager anticipates this value of λ by backward induction and offers a fixed price f that maximizes her own utility as well as satisfies the constraint to achieve the λ that maximizes the contractor's profit.

Similar to the calculation shown in chapter 4, the λ that maximizes the contractor's profit in equation (6.39), should satisfy $f = q_0$. This means the project manager need to offer the contractor a contract value that leaves her with zero profit. Thus, it is proposed

Proposition 20. *A supply chain with a fairness concerned project manager and a profit maximizing contractor, can not be coordinated using a fixed price contract with the resource consumption rate as the decision making and the cash-flows are expoinentially discounted.*

6.3 Supply Chain Coordination with Time Based Contracts in a Take it or Leave it Situation under Fairness Concern

In last section, the models were derived for fixed price contracts. The supply chain under consideration can be coordinated with fairness concerned members under certain considerations. However, there are limitations to it as shown in the last section. Thus, it would be worthy to explore how incorporation of fairness changes the profit allocation when the fixed price contract fails to coordinate despite the existence of fairness consideration.

This section analyses the impact of fairness concern on supply chain coordination when the offered contract takes a linearly decreasing function of time (as described in chapter 4) i.e. $P(T,C) = g - hT$.

6.3.1 For Short Term Projects

Fairness Concern Contractor and Profit Maximizing Project Manager

The fairness concern contractor selects a resource consumption rate λ that maximizes his utility. As mentioned earlier in chapter 4, this resource consumption rate λ is not visible and can not be monitored by the project manager. The game is solved using backward induction. As mentioned in chapter 4, the project manager and the contractor's profit in decentralized setting follow the equations (4.6) and (4.7) respectively. Using these values of profits, the utility function of the contractor converts to the following

$$U_{co} = \begin{cases} (g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \alpha_{co}[\gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - (g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)] \\ \text{for } \{g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1\} < \gamma[q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ (g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \beta_{co}[(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\}] \\ \text{for } \{g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1\} \geq \gamma[q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \end{cases} \tag{6.80}$$

The first case corresponds to the disadvantageous inequity case, and the second case corresponds to the advantageous inequity case. As mentioned earlier, the contractor would select a resource consumption rate λ that maximizes his utility. Thus, when $\{g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1\} \leq \gamma[q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o]$ i.e. when the contractor experiences disadvantageous inequity, the contractor would select a λ that satisfies

$$\frac{dU_{co}}{d\lambda} = \left[\frac{hA\mu_1}{\lambda^{A+1}} - kN\lambda^{N-1}\mu_1\right] - \alpha_{co}\left[-\gamma q_0\psi\frac{dE(T^m)}{d\lambda} - \gamma\frac{hA\mu_1}{\lambda^{A+1}} - \frac{hA\mu_1}{\lambda^{A+1}} + kN\lambda^{N-1}\mu_1\right] = 0 \tag{6.81}$$

Using the values of $\frac{dE(T^m)}{d\lambda}$ from (6.11) in the equation (6.81)

$$\frac{dU_{co}}{d\lambda} = \begin{cases} \frac{hA\mu_1}{\lambda^{A+1}}\{1 + \alpha_{co}(1+\gamma)\} - kN\lambda^{N-1}\mu_1(1+\alpha_{co}) - \alpha_{co}\left[\gamma q_0\psi\frac{mA(2\mu_1)^m}{(m+1)\lambda^{mA+1}}\right] = 0 \\ \text{For uniform distributed time} \\ \frac{hA\mu_1}{\lambda^{A+1}}\{1 + \alpha_{co}(1+\gamma)\} - kN\lambda^{N-1}\mu_1(1+\alpha_{co}) - \alpha_{co}\left[\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}(w+i-1)}{w^m\lambda^{mA+1}}\right] = 0 \\ \text{For gamma distributed time} \\ \frac{hA\mu_1}{\lambda^{A+1}}\{1 + \alpha_{co}(1+\gamma)\} - kN\lambda^{N-1}\mu_1(1+\alpha_{co}) - \alpha_{co}\left[\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)}{\left(\frac{u}{u+v}\right)^m\lambda^{mA+1}}\right] = 0 \\ \text{For beta distributed time} \\ \frac{hA\mu_1}{\lambda^{A+1}}\{1 + \alpha_{co}(1+\gamma)\} - kN\lambda^{N-1}\mu_1(1+\alpha_{co}) - \alpha_{co}\left[\gamma q_0\psi\frac{mA\mu_1^m\left[\Gamma\left(1+\frac{m}{s}\right)\right]}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA+1}}\right] = 0 \\ \text{For Weibull distributed time} \end{cases} \tag{6.82}$$

For the purpose of coordination, $\lambda = \lambda_0$. Using the values of λ_0 from the equation (4.19)

$$\frac{dU_{co}}{d\lambda} = \frac{hA\mu_1}{\lambda_0^{A+1}}\{1 + \alpha_{co}(1+\gamma)\} - kN\lambda_0^{N-1}\mu_1(1+\alpha_{co}) - \alpha_{co}\left[\gamma kN\lambda_0^{N-1}\mu_1\right] = 0 \tag{6.83}$$

Solving the equation (6.83), the optimal condition for h is derived as below

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.84}$$

Apart from maximizing the contractor's utility, the offered contract P(T,C) = g- hT should also satisfy the positive utility requirement of the contractor i.e. $U_{co} \geq U_{out}$, where U_{out}

is the minimum utility contractor is expected to achive in the event he moves out of this contractual agreement. For the purpose of simplicity, this research assumes this $U_{out} = 0$.

Thus,

$$(g - h\frac{\mu_1}{\lambda_0^A} - k\lambda_0^N\mu_1) - \alpha_{co}[\gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda_0^A} - C_o\} - (g - h\frac{\mu_1}{\lambda_0^A} - k\lambda_0^N\mu_1)] \geq 0$$

or

$$g \geq \frac{\alpha_{co}\gamma[q_0\{1 - \psi E(T^m)\} - k\lambda_0^N\mu_1 - C_o] + h\frac{\mu_1}{\lambda_0^A}\{1 + \alpha_{co} + \alpha_{co}\gamma\} + k\lambda_0^N\mu_1(1 + \alpha_{co} + \alpha_{co}\gamma)}{(1 + \alpha_{co} + \alpha_{co}\gamma)}$$

using the values of π_0 and h

$$g \geq \frac{\alpha_{co}\gamma\pi_0}{(1 + \alpha_{co} + \alpha_{co}\gamma)} + k\lambda_0^N\mu_1\left(\frac{N + A}{A}\right) \tag{6.85}$$

Rearranging the terms from the equation (6.6)

$$k\lambda_0^N\mu_1 = q_0 - q_0\psi E(T^m) - C_o - \pi_0 \tag{6.86}$$

Now replacing the value of $\mathrm{E(T^m)}$ from the equation (4.19) in the equation of (4.17)

$$E(T^m) = \frac{kN\lambda_0^N\mu_1}{q_0\psi mA} \tag{6.87}$$

Using this observation from (6.87) in the equation (6.86) and rearranging the terms, it can be shown

$$k\lambda_0^N\mu_1 = (q_0 - C_o - \pi_0)\left(\frac{mA}{mA + N}\right) \tag{6.88}$$

Replacing the value of $k\lambda_0^N\mu_1$ from the equation (6.88) in the equation (6.85)

$$g \geq \frac{\alpha_{co}\gamma\pi_0}{(1 + \alpha_{co} + \alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N + A)}{mA + N}\right\} \tag{6.89}$$

From the requirement of the disadvantageous inequity,

$$\{g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1\} \leq \gamma[q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \tag{6.90}$$

or

$$g(1+\gamma) \leq \gamma\left[q_0\{1-\psi E(T^m)\} - k\lambda_0^N\mu_1 - C_o\right] + h\frac{\mu_1}{\lambda^A}(1+\gamma) + k\lambda_0^N\mu_1(1+\gamma)$$

or

$$g \leq \frac{\gamma}{1+\gamma}\pi_0 + h\frac{\mu_1}{\lambda^A} + k\lambda_0^N\mu_1 \tag{6.91}$$

Using the optimal condition of h and the value of $k\lambda^N\mu_1$ from the equation (6.88)

$$g \leq \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.92}$$

Thus, combining the observations, of the optimal values of h from equation (6.84), and the values of g from the equations (6.89) and (6.92), the following lemma is derived

Lemma 22. *A time based contract P(T,C) =g-hT, can maximize the utility function of the fairness concerned contractor in the equation (6.80) when the contractor encounters disadvantageous inequity if the following are satisfied*

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.93}$$

where the values of λ_0 *follows the equation (4.19)*

and

$$g \in \left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}, \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right] \tag{6.94}$$

The project manager anticipates this by the backward induction process and would offer a contract P(T,C) =g-hT that maximizes her profit. Thus, her optimization problem becomes

$$\max_{g,h} U_{pm} = \pi_{pm} = q_0 \left[1 - \psi E(T^m)\right] - \left[g - hE(T)\right] - C_o \tag{6.95}$$

Sub. to.

$$\text{where g is satisfying the conditions in equation (6.94)} \tag{6.96}$$

$$\text{where h is satisfying the conditions in equation (6.93)} \tag{6.97}$$

The constraint for the parameter h has equal sign in the equation (6.93). However, the g value could vary in the given range mentioned in the equation (6.94). The value of $\frac{\partial U_{pm}}{\partial g} = -1 < 0$. Thus, the project manager's profit is a decreasing function of g. Hence, for the optimization purpose, the project manager would select the minimum g value in the given constraint in the equation (6.94). Based on this observation, the following is proposed

Proposition 21. *The optimal conditions of a time based contract that maximizes the profit of the project manager and the utility of the fairness concerned contractor, and achieves the optimal coordinating conditions for the over all supply chain, satisfy the following*

$$h^* = \frac{kN\lambda_0^{N+A}}{A} \tag{6.98}$$

$$g^* = \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.99}$$

Proof. In the equation (6.94), the difference between the values $\left[\frac{\gamma}{1+\gamma}\pi_0\right]$ and $\left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)}\right]$ is $\frac{\gamma\pi_0}{(1+\gamma)(1+\alpha_{co}+\alpha_{co}\gamma)}$. This is a positive number. Hence, it can be shown that

$$\left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right] < \left[\frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right]$$

□

Using these optimal values g^* and h^*, the profits of the contractor becomes

$$\begin{aligned}\pi_{co} &= g^* - h^* \frac{\mu_1}{\lambda^A} - k\lambda_0^N \mu_1 \\ &= \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - k\lambda_0^N \mu_1 \left(\frac{N+A}{A}\right)\end{aligned}$$

Using the observation $k\lambda_0^N \mu_1$ from the equation (6.88) in the above equation

$$\pi_{co} = \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} \tag{6.100}$$

Thus, the profit of the project manager becomes

$$\pi_{pm} = \pi_0 - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} = \frac{(1+\alpha_{co})\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \tag{6.101}$$

Thus, none of the profits are the one's with equitable distribution.

On the contrary to the disadvantageous inequity case, the contractor experiences an advantageous inequity when $\{g - h\frac{\mu_1}{\lambda^A} - k\lambda^N \mu_1\} \geq \gamma[q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o]$. Thus again, the selected resource consumption rate, λ should satisfy the following (By differentiating U_{co} from the equation 6.80 with respect to λ)

$$\frac{dU_{co}}{d\lambda} = \left(h\frac{A\mu_1}{\lambda^{A+1}} - kN\lambda^{N-1}\mu_1\right)(1-\beta_{co}) + \beta_{co}\gamma\left\{q_0\psi\frac{dE(T^m)}{d\lambda} - \left(h\frac{A\mu_1}{\lambda^{A+1}}\right)\right\} = 0 \tag{6.102}$$

Using the values of $\frac{dE(T^m)}{d\lambda}$ from equation (6.11) in equation (6.102),

$$\frac{dU_{co}}{d\lambda} = \begin{cases} \left(h\frac{A\mu_1}{\lambda^{A+1}}\right)(1-\beta_{co}-\gamma\beta_{co}) - kN\lambda^{N-1}\mu_1(1-\beta_{co}) + \beta_{co}\gamma q_0\psi\frac{mA(2\mu_1)^m}{(m+1)\lambda^{mA+1}} = 0 \\ \text{For uniform distributed time} \\ \left(h\frac{A\mu_1}{\lambda^{A+1}}\right)(1-\beta_{co}-\gamma\beta_{co}) - kN\lambda^{N-1}\mu_1(1-\beta_{co}) + \beta_{co}\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}(w+i-1)}{w^m\lambda^{mA+1}} = 0 \\ \text{For gamma distributed time} \\ \left(h\frac{A\mu_1}{\lambda^{A+1}}\right)(1-\beta_{co}-\gamma\beta_{co}) - kN\lambda^{N-1}\mu_1(1-\beta_{co}) + \beta_{co}\gamma q_0\psi\frac{mA\mu_1^m\prod\limits_{i=1}^{m}\left(\frac{u+i-1}{u+v+i-1}\right)}{\left(\frac{u}{u+v}\right)^m\lambda^{mA+1}} = 0 \\ \text{For beta distributed time} \\ \left(h\frac{A\mu_1}{\lambda^{A+1}}\right)(1-\beta_{co}-\gamma\beta_{co}) - kN\lambda^{N-1}\mu_1(1-\beta_{co}) + \beta_{co}\gamma q_0\psi\frac{mA\mu_1^m\left[\Gamma\left(1+\frac{m}{s}\right)\right]}{\left\{\Gamma\left(1+\frac{1}{s}\right)\right\}^m\lambda^{mA+1}} = 0 \\ \text{For Weibull distributed time} \end{cases} \tag{6.103}$$

Again for the purpose of coordination, $\lambda = \lambda_0$. Thus, using the values of λ_0 from the equation (4.19) in the above equation (6.103)

$$\left(h\frac{A\mu_1}{\lambda_0^{A+1}}\right)(1-\beta_{co}-\gamma\beta_{co}) - kN\lambda_0^{N-1}\mu_1(1-\beta_{co}-\beta_{co}) = 0$$

or

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.104}$$

Now taking the second order derivative of U_{co} with respect to λ,

$$\begin{aligned} \frac{d^2U_{co}}{d\lambda^2} &= \{-kN(N-1)\lambda^{N-2}\mu_1\} - \beta_{co}\left[(-kN(N-1)\lambda^{N-2}\mu_1) + \gamma q_0\psi\frac{d^2E(T^m)}{d\lambda^2}\right] \\ &= \{-kN(N-1)\lambda^{N-2}\mu_1\}(1-\beta) - \beta_{co}\gamma q_0\psi\frac{d^2E(T^m)}{d\lambda^2} \end{aligned} \tag{6.105}$$

As assumed before $0 \leq \beta_{co} < 1$. The second order derivative of the E(T^m) are positive for the uniform, gamma, beta and Weibull distributed completion times. Thus, it can be shown that $\frac{d^2U_{co}}{d\lambda^2} < 0$. This means for the values of λ found in the equation (6.103) would maximize the utility function of the fairness concerned contractor.

As mentioned the equation in the case of disadvantageous inequity, the offered contract should ensure the contractor to earn a non-negative utility i.e. $U_{co} \geq 0$. Thus,

$$(g - h\frac{\mu_1}{\lambda_0^A} - k\lambda_0^N\mu_1) - \beta_{co}[(g - h\frac{\mu_1}{\lambda_0^A} - k\lambda_0^N\mu_1) - \gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\}] \geq 0$$

or

$$g \geq \frac{h\frac{\mu_1}{\lambda^A}(1 - \beta_{co} - \beta_{co}\gamma) + k\lambda_0^N\mu_1(1 - \beta_{co} - \beta_{co}\gamma) - \beta_{co}\gamma\{q_0 - q_0\psi E(T^m) - k\lambda_0^N\mu_1 - C_o\}}{(1 - \beta_{co} - \beta_{co}\gamma)}$$

Using the value of h from (6.104) and rearranging the terms

$$g \geq k\lambda_0^N\mu_1\left(1 + \frac{N}{A}\right) - \frac{\beta_{co}\gamma}{1 - \beta_{co} - \beta_{co}\gamma}\pi_0$$

Replacing the value of $k\lambda_0^N\mu_1$ from the equation (6.88) in the above inequity

$$g \geq (q_0 - \pi_0 - C_o)\frac{m(N + A)}{mA + N} - \frac{\beta_{co}\gamma}{1 - \beta_{co} - \beta_{co}\gamma}\pi_0 \tag{6.106}$$

From the requirement of the advantageous inequity to take place,

$$\{g - h\frac{\mu_1}{\lambda_0^A} - k\lambda_0^N\mu_1\} \geq \gamma[q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o]$$

Based on the calculations in the case of disadvantageous inequity to derive the condition shown in inequity (in equation 6.92), it can be shown

$$g \geq \frac{\gamma}{1 + \gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N + A)}{mA + N}\right\} \tag{6.107}$$

Thus, combining the requirements from (6.106) and (6.107)

$$g \geq max : \left\{(q_0 - \pi_0 - C_o)\frac{m(N + A)}{mA + N} + \frac{\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0, \frac{\gamma}{1 + \gamma}\pi_0 + (q_0 - C_o - \pi_0)\frac{m(N + A)}{mA + N}\right\} \tag{6.108}$$

Now the difference between the above two values

$$(q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0 - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$$
$$= \left\{\frac{\gamma\pi_0}{(1+\gamma)(\beta_{co} + \beta_{co}\gamma - 1)}\right\}$$

If $(1+\gamma)(\beta_{co}+\beta_{co}\gamma-1) > 0$ i.e. $\beta_{co} > \frac{1}{1+\gamma}$, then $g \geq (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0$. On the contrary, if $(1+\gamma)(\beta_{co} + \beta_{co}\gamma - 1) < 0$ i.e. $\beta_{co} < \frac{1}{1+\gamma}$, then $g \geq \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$

The optimization problem for the project manager becomes

$$\max_{g,h} U_{pm} = \pi_{pm} = q_0\left[1 - \psi E(T^m)\right] - \left[g - hE(T)\right] - C_o$$

Sub. to

h satsifies the condition in the equation 6.104 (6.109)

g satsifies the condition in the equation 6.108 (6.110)

Again, the h constraint for the project project manager is bound by the equal sign. However, the g constraint is bound by inequal sign. $\frac{dU_{pm}}{dg} = -1 < 0$ i.e. the project manager's utility is decreasing in g. Thus, when $(1+\gamma)(\beta_{co} + \beta_{co}\gamma - 1) > 0$ i.e. $\beta_{co} > \frac{1}{1+\gamma}$, then the constraint for g in the equation (6.108) becomes

$$g \geq (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0$$

Since the project manager's utility decreases in g, the minimum offer from the project manager is

$$g = (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0$$

Using this value of g, the value of h from the equation (6.104), and using the observation from the equation (6.88) the contractor's profit becomes

$$\begin{aligned}
\pi_{co} &= g - h\frac{\mu_1}{\lambda^A} - k\lambda_0^N\mu_1 \\
&= (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0 - \left\{\frac{kN\lambda_0^{N+A}}{A}\right\}\left(\frac{\mu_1}{\lambda^A}\right) - k\lambda_0^N\mu_1 \\
&= \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0
\end{aligned}$$

Thus, the project manager's profit becomes

$$\begin{aligned}
\pi_{pm} &= \pi_0 - \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0 \\
&= \frac{\beta_{co}-1}{\beta_{co}+\beta_{co}\gamma-1}\pi_0
\end{aligned}$$

Since $(\beta_{co}+\beta_{co}\gamma-1) > 0$, and $0 \leq \beta_{co} < 1$, the project manager can not earn a non-negative profit in this case Thus, the possibility of coordination is non existent if $\beta_{co} > \frac{1}{1+\gamma}$.

On the contrary, if $(1+\gamma)(\beta_{co}+\beta_{co}\gamma-1) < 0$ i.e. $\beta_{co} < \frac{1}{1+\gamma}$ then constraint for g in the equation (6.108) becomes $g \geq \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$.

Since the project manager's utility decreases in g, the minimum offer from project manager is

$$g = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$$

The offer for h from the project manager is still be the same as shown in the equation (6.104). Thus, the contractor's profit becomes

$$\begin{aligned}
\pi_{co} &= g - h\frac{\mu_1}{\lambda^A} - k\lambda_)^N\mu_1 \\
&= \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - \frac{kN\lambda_0^{N+A}}{A} - k\lambda_0^N\mu_1 \\
&= \frac{\gamma}{1+\gamma}\pi_0
\end{aligned}$$

Thus, the project manager's profit becomes

$$\pi_{pm} = \pi_0 - \frac{\gamma}{1+\gamma}\pi_0 = \frac{1}{1+\gamma}\pi_0$$

Thus, summarising these above observations, the following is proposed

Proposition 22. *The optimal conditions for a time based contract that maximizes the profit of the project manager, utility of the fairness concerned contractor and thereby achieves the optimal coordinating conditions for the over all supply chain, satisfy the following*

$$\begin{aligned} h^* &= \frac{kN\lambda_0^{N+A}}{A} \\ g^* &= \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \end{aligned} \tag{6.111}$$

with $\beta_{co} < \frac{1}{1+\gamma}$ *and* λ_0 *follows the equation (4.19)*

Unlike, the case of disadvantageous inequity for the contractor, the contractual conditions here not only coordinates the supply chain, but also ensure the equitable share of the profit.

Now summarising the findings from the propositions 21 and 22, the following corollary is derived

Corollary 1. *Comparing the equations (6.99) and (6.111), the project manager earns a lower profit in the later case. Hence, chances of offering a contract with the term g higher than the value mentioned in the equation (6.99) is very unlikely from a profit maximizing project manager. Hence, the existence of the optimal solution presented in the equation (6.111) in proposition22 is unlikely.*

Proof. It can be shown

$$\begin{aligned} &\frac{\alpha_{co}\gamma\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} - \frac{\gamma\pi_0}{1+\gamma} \\ &= \frac{-\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)(1(1+\gamma)} < 0 \end{aligned}$$

Using this observation in the comparison between the values of g from the equations From the equations (6.99) and (6.111), the above mentioned corollary can be proved. □

Fairness Concern Contractor and Project Manager

This subsection explores how the optimal condition changes if the project manager also becomes fairness concerned. Again, the coordination problem is solved using the backward induction method. The contractor selects the resource consumption rate λ that maximizes his utility in the equation (6.80).

Now, unlike the case of profit maximizing project manager, the fairness concerned project manager maximizes her utility mentioned in (6.5). Now replacing the values of π_{pm} and π_{co} from the equations (4.6) and (4.7), the utility function becomes

$$U_{pm} = \begin{cases} U_{pm1} = [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ -\alpha_{pm}[\delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\}] \\ \text{when } \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \leq \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \\ U_{pm2} = [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ -\beta_{pm}[\{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)] \\ \text{when } \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \geq \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \end{cases} \tag{6.112}$$

where $\beta_{pm} \leq \alpha_{pm}$, $0 \leq \beta_{pm} < 1$.

Similar to the explanation in the case of the analysis with the fixed price contracts, the contractor considers $\gamma\pi_{pm}$ as the equitable profit and the project manager considers $\delta\pi_{co}$ as the equitable profit. As defined ealier in Cui et al. (2007), the sum of these two pay-offs is considered as the equity capable channel payoff (ECCP) i.e. $ECCP = \left(\frac{\gamma\delta+\gamma+\delta+\gamma\delta}{\gamma\delta+\gamma+\delta}\right)\pi_0$. Again as defined in Cui et al. (2007), when $\delta\gamma > 1$ i.e. $ECCP > \pi_0$, the supply chain is considered as acrimonious channel and when $\delta\gamma \leq 1$ i.e. $ECCP \leq \pi_0$, the channel is considered as harmonious channel (Cui et al. 2007).

Thus, if the contractor experiences a disadvantageous inequity, the fairness concerned project manager's maximization problem in the case of her disadvantageous inequity becomes

$$\begin{aligned} &\max_{g,h} : U_{pm1} = [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ &- \alpha_{pm}[\delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\}] \end{aligned} \tag{6.113}$$

Sub. to

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.114}$$

$$\begin{cases} g \in \left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}, \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right] \\ \text{if the contractor experiences a disadvantageous inequity} \\ g \geq max : \left((q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0, \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right) \\ \text{if the contractor experiences an advantageous inequity} \end{cases} \tag{6.115}$$

Again, the project manager will select a value of f that maximizes her utility function U_{pm1} i.e. $\frac{dU_{pm1}}{dg} = -1 - \alpha_{pm}(\delta + 1) < 0$. Thus, again the utility function of the project manager is a decreasing function of g. Thus, the project manager selects the minimum g value in the constraints in the equation (6.115). Based on the calculation shown earlier, it can be shown that when the contractor encounters a advantageous inequity and $\beta_{co} + \beta_{co}\gamma - 1 > 0$ i.e. $\beta_{co} > \frac{1}{1+\gamma}$, then profit of the contractor and the project manager become

$$\begin{aligned} \pi_{co} &= \frac{\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0 \\ \pi_{pm} &= -\frac{1 - \beta_{co}}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0 \end{aligned} \tag{6.116}$$

Thus, for the utility maximizing project manager, her utility becomes

$$\begin{aligned} U_{pm1} &= \pi_{pm} - \alpha_{pm}[\delta\pi_{co} - \pi_{pm}] \\ &= -\frac{1 - \beta_{co}}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0 - \alpha_{pm}\left[\frac{\delta\beta_{co}\gamma}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0 + \frac{1 - \beta_{co}}{\beta_{co} + \beta_{co}\gamma - 1}\pi_0\right] < 0 \end{aligned}$$

Thus, the cases with $\beta_{co} > \frac{1}{1+\gamma}$ cannot coordinate the supply chain and are not considered. Based on the calculation shown earlier , the values of the contract parameter the project manager would select given the constraints

$$\text{h value satisfies the equation (6.114)} \tag{6.117}$$

and

$$g^* = \begin{cases} \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \\ \text{when the contractor encounters a disadvantageous inequity} \\ \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \\ \text{when the contractor encounters an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases} \tag{6.118}$$

Now in order for the project manager to incur disadvantageous inequity, the following should be satisfied

$$\{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \leq 0 \tag{6.119}$$

Now replacing these values of g from the equation (6.118) & h from equation (6.114) and λ_0 from the equation (4.19) in the expression (6.119) **when the contractor experiences disadvantageous inequity**

$$\left[q_0\{1-\psi E(T^m)\} - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^{N+A}\mu_1}{A} - C_o\right]$$
$$-\delta\left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - \frac{kN\lambda_0^N\mu_1}{A} - k\lambda_0^N\mu_1\right] \leq 0$$

or

$$\left[\pi_0 - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^{N+A}\mu_1}{A} + k\lambda_0^N\mu_1\right]$$
$$-\delta\left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - k\lambda_0^N\mu_1\frac{N+A}{A}\right] \leq 0$$

or

$$\frac{\{1+\alpha_{co}(1-\delta\gamma)\}\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \leq 0 \tag{6.120}$$

when the contractor experiences advantageous inequity

$$\{q_0(1-\psi E(T^m)) - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o\}$$
$$-\delta(\frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - \frac{kN\lambda_0^N\mu_1}{A} - k\lambda^N\mu_1) \leq 0$$

or

$$\left[\pi_0 - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} + k\lambda_0^N\mu_1\right]$$
$$-\delta\left[\frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - k\lambda_0^N\mu_1\left(\frac{N+A}{A}\right)\right] \leq 0$$

or

$$\frac{\pi_0(1-\delta\gamma)}{1+\gamma} \leq 0 \tag{6.121}$$

Since π_0 is positive in order for the participation of the members of the supply chain, and α_{co} is also assumed as positive, the project manager incurs disadvantageous inequity when $\delta\gamma > 1$. In other words, the project manager would incur disadvantageous inequity in acrimonious supply chain.

The selected contract parameters should ensure a non-negative utility for the project manager i.e. $U_{pm1} \geq 0$. Using the values of the contract parameters from the equations (6.114) , & (6.118), in the utility function of the project manager in the equation (6.112) and based on the calculations above in the conditions in (6.120) and (6.121)

$$U_{pm1} = \begin{cases} [q_0\{1-\psi E(T^m)\} - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o] \\ -\alpha_{pm}\frac{\{-1+\alpha_{co}(\delta\gamma-1)\}\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \geq 0 \text{ when the contractor experiences a disadvantageous inequity} \\ \\ [q_0\{1-\psi E(T^m)\} - \frac{\gamma\pi_0}{(1+\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o] \\ -\alpha_{pm}\frac{\pi_0(\delta\gamma-1)}{1+\gamma} \geq 0 \text{ when the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases}$$

or

$$U_{pm1} = \begin{cases} [\pi_0 - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} + k\lambda_0^N\mu_1] \\ -\alpha_{pm}\frac{\{-1+\alpha_{co}(\delta\gamma-1)\}\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \geq 0 \text{ when the contractor experiences a disadvantageous inequity} \\ \\ [\pi_0 - \frac{\gamma\pi_0}{(1+\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} + k\lambda_0^N\mu_1] - \alpha_{pm}\frac{\pi_0(\delta\gamma-1)}{1+\gamma} \geq 0 \\ \text{when the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases}$$

Replacing the value of $k\lambda_)^N\mu_1$ from the equation (6.88) in the above conditions and rearranging the terms

$$U_{pm1} = \begin{cases} \frac{(1+\alpha_{co})-\alpha_{pm}\{-1+\alpha_{co}(\delta\gamma-1)\}}{1+\alpha_{co}+\alpha_{co}\gamma}\pi_0 \geq 0 \\ \text{when the contractor encounters a disadvantageous inequity} \\ \\ \frac{1-\alpha_{pm}(\delta\gamma-1)}{1+\gamma}\pi_0 \geq 0 \\ \text{when the contractor encounters an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases} \tag{6.122}$$

From the conditions mentioned in (6.122), and using the optimality conditions mentioned in equations ((6.118)) and (6.114), the following lemma is derived

Lemma 23. *If both the project manager and the contractor both are fairness concerned, the optimal conditions of the time based contracts contract should satisfy the following in an acrimonious supply chain i.e.* $\delta\gamma > 1$

1. *h value satisfies the condition in the equation (6.114).*

   and

2. *(a)*

$$g^{*} = \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.123}$$

$$\textit{with}\ \ \delta\gamma > 1, \alpha_{pm} \le \frac{1+\alpha_{co}}{\{\alpha_{co}(\delta\gamma-1)-1\}}\ \&\ \alpha_{co} > \frac{1}{\delta\gamma-1}$$

(b)

$$g^{*} = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.124}$$

$$\textit{with}\ \ \delta\gamma > 1, \beta_{co} < \frac{1}{1+\gamma}\ \&\ \alpha_{pm} \le \frac{1}{\delta\gamma-1}$$

On the contrary to the disadvantageous inequity, the project manager experiences advantageous inequity in a harmonious supply chain when $\delta\gamma < 1$. The sum of the overall expected fair profit of the members of the supply chain for this case is less than overall maximum profit the supply chain can generate. If the following is satisfied, the project manager incurs advantageous inequity

$$\{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \ge 0 \tag{6.125}$$

Thus, the optimization problem of the project manager becomes

$$\begin{aligned} \max_{g,h} : U_{pm2} &= [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ &- \beta_{pm}[\{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)] \end{aligned} \tag{6.126}$$

Sub.to

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.127}$$

$$\begin{cases} g \in \left[\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}, \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right] \\ \text{if the contractor experiences a disadvantageous inequity} \\ g \geq max : \left\{(q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0, \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}\right\} \\ \text{if the contractor experiences an advantageous inequity} \end{cases} \tag{6.128}$$

The value of first derivative of the project manager's utility is

$$\frac{dU_{pm2}}{dg} = -1 + \beta_{pm}(1+\delta) \tag{6.129}$$

If $-1 + \beta_{pm}(1+\delta) \leq 0$, i.e. $\beta_{pm} \leq \frac{1}{1+\delta}$, then $\frac{dU_{pm}}{dg} \leq 0$. This means the project manager's utility would be a decreasing function of g. Thus, the project manager would select the minimum value that satisfies the constraint in the equation (6.128). Thus, when the contractor experiences disadvantageous inequity, then, the project manger selects $g = \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$. On the contrary, if the contractor experiences an advantageous inequity, then it depends on the sign of the expression $\beta_{co} + \beta_{co}\gamma - 1$ (As shown before). If $\beta_{co} + \beta_{co}\gamma - 1 > 0$ i.e $\beta_{co} > \frac{1}{1+\gamma}$, then $(q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} + \frac{\beta_{co}\gamma}{\beta_{co}+\beta_{co}\gamma-1}\pi_0 \geq \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$. It was shown earlier if $\beta_{pm} \geq \frac{1-\beta_{co}}{(1-\beta_{co})+\delta\beta_{co}\gamma}$, then the project manager's utility is non-negative when $\beta_{co} > \frac{1}{1+\gamma}$. However, as shown in the equation (6.116), the profit of the project manager is negative. The utility is positive due to higher inequity aversion. Hence, again, like previous cases, the situation $\beta_{co} > \frac{1}{1+\gamma}$ is not considered. Thus, in a fairness concerned supply chain, with both the members experience advantageous inequity, the optimal g value is $g = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}$. This satisfies the constraint in the equation (6.128) and maximizes the project managers' utility . Thus, summarising these above observations, when $-1 + \beta_{pm}(1+\delta) \leq 0$, i.e. $\beta_{pm} \leq \frac{1}{1+\delta}$, the project manager selects the optimal h* that satisfies the equation (6.114)and g values as below

$$g* = \begin{cases} \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \\ \text{if the contractor experiences a disadvantageous inequity} \\ \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \\ \text{if the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases} \tag{6.130}$$

These values of g and h should ensure a non-negative utility for the project manager i.e. $U_{pm2} \geq 0$. Thus,

$$U_{pm2} = \begin{cases} [q_0\{1-\psi E(T^m)\} - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o] \\ -\beta_{pm}[\{q_0\{1-\psi E(T^m)\} - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o\} \\ -\delta(\frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - \frac{kN\lambda_0^N\mu_1}{A} - k\lambda_0^N\mu_1)] \geq 0 \\ \text{if the contractor experiences a disadvantageous inequity} \\ [q_0\{1-\psi E(T^m)\} - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o] \\ -\beta_{pm}[\{q_0\{1-\psi E(T^m)\} - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + \frac{kN\lambda_0^N\mu_1}{A} - C_o\} - \\ \delta(\frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} - \frac{kN\lambda_0^N\mu_1}{A} - k\lambda_0^N\mu_1)] \geq 0 \\ \text{if the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases}$$

Using the observations from (6.120) and (6.121) in the above inequities

$$U_{pm2} = \begin{cases} [\pi_0 - \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}] \\ -\beta_{pm}\frac{\{1+\alpha_{co}(1-\delta\gamma)\}\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \geq 0 \\ \text{if the contractor experiences a disadvantageous inequity} \\ [\pi_0 - \frac{\gamma}{1+\gamma}\pi_0 - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\}] \\ -\beta_{pm}\frac{\pi_0(1-\delta\gamma)}{1+\gamma} \geq 0 \\ \text{if the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases}$$

or

$$U_{pm2} = \begin{cases} [\frac{(1+\alpha_{co})\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)}] - \beta_{pm}\frac{\{1+\alpha_{co}(1-\delta\gamma)\}\pi_0}{1+\alpha_{co}+\alpha_{co}\gamma} \geq 0 \\ \text{if the contractor experiences a disadvantageous inequity} \\ \frac{\pi_0}{1+\gamma} - \beta_{pm}\frac{\pi_0(1-\delta\gamma)}{1+\gamma} \geq 0 \\ \text{if the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma} \end{cases}$$

It can be shown from the above inequities that the project manager would experience non-negative inequity when

$$\beta_{pm} \leq \frac{1+\alpha_{co}}{1+\alpha_{co}(1-\delta\gamma)} \qquad \text{if the contractor experiences a disadvantageous inequity}$$
$$\beta_{pm} \leq \frac{1}{1-\delta\gamma} \qquad \text{if the contractor experiences an advantageous inequity and } \beta_{co} < \frac{1}{1+\gamma}$$

The prerequisite for applying the optimal g and h values from the equation (6.130) is $\beta_{pm} \leq \frac{1}{1+\delta}$. Now it can be easliy shown that for positive values of α_{co}, β_{co}, β_{pm}, δ, and γ, $\frac{1}{1+\delta} < \frac{1+\alpha_{co}}{1+\alpha_{co}(1-\delta\gamma)}$ and $\frac{1}{1+\delta} < \frac{1}{1-\delta\gamma}$. This means , for $\beta_{pm} \leq \frac{1}{1+\delta}$, the optimal value of g and h in equation (6.130) would ensure a positive utility for the project manager. Summarising these, the optimization conditions are presented in the following lemma

Lemma 24. *If the project manager and the contractor both are fairness concerned, the optimal conditions of the time based contracts contract should satisfy the following in a harmonious supply chain when the cash flows are not discounted*

1. *The h value satisfies the conditions in the equation(6.114) and*

2. *(a)*

$$g^* = \frac{\alpha_{co}\gamma\pi_0}{(1+\alpha_{co}+\alpha_{co}\gamma)} + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.131}$$

$$\textit{with } \delta\gamma < 1, \beta_{pm} < \frac{1}{1+\delta}$$

(b)

$$g^* = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.132}$$
$$\textit{with } \delta\gamma < 1, \beta_{co} < \frac{1}{1+\gamma} \;\&\; \beta_{pm} < \frac{1}{1+\delta}$$

If $[-1+\beta_{pm}(1+\delta)] \geq 0$ i.e $\beta_{pm} \geq \frac{1}{1+\delta}$, then $\frac{dU_{pm2}}{dg} > 0$. This means, the utility of the project manager would increase with g. Thus, higher the value of g, higher the utility of the fairness concerned project manager. Hence, with the contractor experiencing the disadvantageous inequity, the project manager would select the maximum possible value of g in constraint in equation (6.128). Thus, with $\beta_{pm} > \frac{1}{1+\delta}$ and the contractor experiencing disadvantageous inequity, the project manager would select $g = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - \pi_0 - C_o)\left\{\frac{m(N+A)}{mA+N}\right\}$. As shown before, $\beta_{pm} \leq \frac{1}{1-\delta\gamma}$ in order to get a positive utility using this optimal value. If the contractor experiences advantageous inequity, then the optimal value of h should be satisfying the condition in point 1 and g should be satisfying the condition for g in point 2b in lemma 24. At these values, the contractor achieves his perceived fair profit. There is no upper limit for g (from equation 6.128). Thus after this values from lemma 24 are attained, any further increase in g values would fail to allocate any fair profit for the members. After certain the profit of the project manager becomes negative.Thus, these cases are excluded. These observations are summarized in the following lemma

Lemma 25. *If both the project manager and the contractor both are fairness concerned, the optimal conditions of the time based contracts contract should satisfy the following in a harmonious supply chain*

$$h^* = \frac{kN\lambda_0^{N+A}\mu_1}{A}$$

and

$$g^* = \frac{\gamma}{1+\gamma}\pi_0 + (q_0 - C_o - \pi_0)\left\{\frac{m(N+A)}{mA+N}\right\} \tag{6.133}$$

$$\textit{with } \delta\gamma < 1, \frac{1}{1+\delta} \leq \beta_{pm} \leq \frac{1}{1-\delta\gamma}$$

Summarisng the findings from lemmas 23, 24, and 25, the following is proposed

Proposition 23. *In a fairness concerned supply chain (with both project manager and the contractor are fairness concerned), the project manager can coordinate the supply chain with the following time based contracts in the following cases*

$$h^* = \frac{kN\lambda_0^{N+A}}{A}$$

and any of the following

1. *g satisfying the condition in the equation (6.123) with* $\delta\gamma > 1, \alpha_{pm} \leq \frac{1+\alpha_{co}}{\{\alpha_{co}(\delta\gamma-1)-1\}}$ & $\alpha_{co} > \frac{1}{\delta\gamma-1}$
2. *g satisfying the condition in the equation (6.124) and* $\delta\gamma > 1, \beta_{co} < \frac{1}{1+\gamma}$ & $\alpha_{pm} \leq \frac{1}{\delta\gamma-1}$
3. *g satisfying the condition in the equation (6.131) with* $\delta\gamma < 1, \beta_{pm} < \frac{1}{1+\delta}$
4. *g satisfying the condition in the equation (6.132) with* $\delta\gamma < 1, \beta_{co} < \frac{1}{1+\gamma}$ & $\beta_{pm} < \frac{1}{1+\delta}$
5. *g satisfying the condition in the equation (6.133) with* $\delta\gamma < 1, \frac{1}{1+\delta} \leq \beta_{pm} \leq \frac{1}{1-\delta\gamma}$

From the proposition 23, the following corollary is deduced

Corollary 2. *Comparing the findings from 23, it can shown that some of the solutions may not exist. These are summarised in the following corollary*

1. *In the acrimonious channel, the project manager experiences the disadvantageous inequity. The* U_{pm} *is a decreasing function of g. Thus, the project manager would most like to offer the contractor a minimum possible g value in option 1 in proposition 23 and thus, the occurrence of advantageous inequity to the contractor is less likely.*

2. *In the harmonious channel, the project manager experiences the advantageous inequity. Again, the project manager's utility is a decreasing function of g when* $\beta_{pm} < \frac{1}{1+\delta}$ *and would select the minimum possible value which ensures a positive utility and* $\lambda = \lambda_0$*. Thus, the project manager is likely to select the g value in option 3 again and the contractor is unlike to experience any advantageous inequity.*

For coordination of the supply chain, again the contractor should also select a $\lambda = \lambda_0$ which is possible when the condition mentioned in equation (6.54) in proposition 14 is satisfied.

Fairness Concerned Project Manager and Profit Maximizing contractor

The game is again solved using backward induction method. For a given value of time based contract contract P(T,C) =g-hT, the contractor selects a resource consumption rate λ that maximizes his profit. The project manager would anticipate this value of λ by backward induction and would offer a contract P(T,C) =g-hT that maximizes her own utility as well as satisfies the constraint to achieve the λ that maximizes the contractor's profit.

Similar to the calculation shown in chapter 4, the λ that maximizes the contractor's profit in equation (4.7), should satisfy

$$\frac{d\pi_{co}}{d\lambda} = \frac{hA\mu_1}{\lambda^{A+1}} - kN\lambda^{N-1}\mu_1 = 0$$

From this, it can be shown $h = \frac{kN\lambda^{N+A}}{A}$. In order to coordinate the supply chain, h should ensure that $\lambda = \lambda_0$. Thus $h^* = \frac{kN\lambda_0^{N+A}}{A}$

The offered contract should also ensure the contractor to earn a minimum profit of π_{out}. Thus,

$$g - \frac{kN\lambda_0^{N+A}}{A} - k\lambda_0^N\mu_1 \geq \pi_{out}$$

From the calculations shown earlier, the value of $k\lambda_0^N\mu_1$ can be replaced from the equation (6.88) in the above inequity as below

$$g - (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} \geq \pi_{out}$$

or

$$g \geq (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out}$$

These above values of g and h would become constraint for the fairness concerned project manager who maximizes her utility. The utility of the project manager follows the equation

(6.112). Thus, her optimization problem becomes

$$\max_{g,h} U_{pm} = \begin{cases} U_{pm1} = [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ -\alpha_{pm}[\delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) - \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\}] \\ \text{when } \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} < \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \\ \\ U_{pm2} = [q_0\{1-\psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \\ -\beta_{pm}[\{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)] \\ \text{when } \{q_0(1-\psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \geq \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \end{cases}$$

St

$$g \geq (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out}$$

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.134}$$

In the case of advantageous inequity, the project manager selects a g that maximizes U_{pm1}. Thus,

$$\frac{dU_{pm1}}{dg} = -1 - \alpha_{pm}(\delta + 1) < 0$$

Again, the project manager's utility is a decreasing function of g. Thus, the project manager selects the minimum g value in the above constraint in the equation (6.134). Thus the optimal solutions are presented in the following lemma

Lemma 26. *The optimal contractual parameters of a time based contract satisfies the following in a supply chain with fairness concerned project manager and a profit maximizing contractor when the project manager experiences a disadvantageous inequity.*

$$g = (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out} \tag{6.135}$$

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.136}$$

In the event, the project manager incurs advantageous inequity, the project manager selects a g that maximizes his utility U_{pm2}. Thus,

$$\frac{dU_{pm2}}{dg} = -1 + \beta_{pm}(1+\delta)$$

Thus, if $-1 + \beta_{pm}(1+\delta) \leq 0$ i.e. $\beta_{pm} \leq \frac{1}{1+\delta}$, then project manager's utility would be a decreasing function of g and the results follow the lemma 26.

On the contrary, if $-1+\beta_{pm}(1+\delta) > 0$ i.e. $\beta_{pm} > \frac{1}{1+\delta}$, then the project manager's utility becomes and increasing function of g. Thus, any increase in g would keep on increasing the value of the utility of the project manager. At the same time, the profit of the project manager keep on decreasing with any increase in the value of g and after a certain value is reached, the profit becomes negative. As mentioned earlier, these cases are not considered for this research. However, if an upper bound can be set for this value g, then a realistic solution can be achieved. Since the constraint for g doesn't have any upper bound in the equation (6.134), additional constraint is required to set a bound. It is further assumed that g can be maximized up to a value that ensures the fair share of the profit to both the project manager and the contractor as per the project manager. At this point, the profits are $\pi_{co} = \frac{\pi_0}{1+\delta}$ and $\pi_{pm} = \frac{\delta\pi_0}{1+\delta}$. Thus replacing the value of π_{co} from the equation (6.100),

$$g - h\frac{\mu_1}{\lambda^A} - k\lambda_0^N\mu_1 = \frac{1}{1+\delta}\pi_0$$

Even at this stage, the optimal h value remains unchanged as it has got an equal sign at the constraints in (6.134). Hence, the optimal value of h is used in the above equation and it becomes

$$\begin{aligned}
g - \frac{kN\lambda_)^N\mu_1}{A} - k\lambda_0^N\mu_1 &= \frac{1}{1+\delta}\pi_0 \\
g - kN\lambda_0^N\mu_1\left(\frac{N+A}{A}\right) &= \frac{1}{1+\delta}\pi_0 \\
g &= \frac{1}{1+\delta}\pi_0 + kN\lambda_0^N\mu_1\left(\frac{N+A}{A}\right)
\end{aligned}$$

As shown earlier, the value of $kN\lambda_0^N\mu_1$ is replaced from the equation (6.88) in the above and the optimal of g that ensures the equitable profit for the contractor and the project manager

$$g = \frac{1}{1+\delta}\pi_0 + (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} \tag{6.137}$$

Thus, the optimization problem of the project manager becomes

$$\max_{g,h} U_{pm2} = [q_0\{1 - \psi E(T^m)\} - g + h\frac{\mu_1}{\lambda^A} - C_o] \tag{6.138}$$

$$- \beta_{pm}[\{q_0(1 - \psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} - \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N \mu_1)] \tag{6.139}$$

$$\text{when } \{q_0(1 - \psi E(T^m)) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \geq \delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N \mu_1)$$

St

$$g \in \left[(q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out} \text{ , } \frac{1}{1+\delta}\pi_0 + (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N}\right]$$
$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.140}$$

Now the constraint for g in the above equation (6.140, has got two limits within which the g would be increasing leading to increase in U_{pm2}. In order to have the non-empty set, it is assumed

$$(q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out} < \frac{1}{1+\delta}\pi_0 + (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N}$$
$$\pi_{out} < \frac{1}{1+\delta}\pi_0$$

If $\pi_{out} > \frac{1}{1+\delta}\pi_0$, then there won't be any change in the solution mentioned in lemma 26. Since $\beta_{pm} > \frac{1}{1+\delta}$ and the project manager's utility is increasing in g, the project manager would be selecting the maximum value of g in the constraint above. These are summarised in the following lemma

Lemma 27. *The optimal contractual parameters of a time based contract satisfies the following with fairness concerned the project manager and with the profit maximizing contractor* *when the project manager experiences advantageous inequity*

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.141}$$

$$g = \begin{cases} g = (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out} & \textit{if } \beta_{pm} \le \frac{1}{1+\delta} \\ g = \frac{1}{1+\delta}\pi_0 + (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} & \textit{if } \beta_{pm} > \frac{1}{1+\delta} \end{cases} \tag{6.142}$$

Summarisng the findings of lemmas 26 and 27, the following is proposed

Proposition 24. *A supply chain consisting of a fairness concerned project manager and a profit maximizing contractor could be coordinated with a time based contract P(T,C) = g-hT, if*

$$h = \frac{kN\lambda_0^{N+A}}{A} \tag{6.143}$$

and

1. $$g = (q_0 - C_o - \pi_0)\left\{\frac{m(N+A}{mA+N}\right\} + \pi_{out} \tag{6.144}$$

   when the project manager experiences a disadvantageous inequity or advantageous inequity with very lower utility loss per unit due to earning more than the contractor, with $\beta_{pm} \le \frac{1}{1+\delta}$.

2. $$g = \frac{1}{1+\delta}\pi_0 + (q_0 - \pi_0 - C_o)\frac{m(N+A)}{mA+N} \tag{6.145}$$

   if $\frac{1}{1+\delta}\pi_0 > \pi_{out}$ *when the project manager experiences a considerable utility loss per unit due to earning more than the contractor so that* $\beta_{pm} > \frac{1}{1+\delta}$.

In the above proposition 24, the first optimal condition can coordinate the supply chain and ensure the contractor to earn a minimum profit of π_{out} as it was found in the case of a supply chain without any fairness consideration. In fact, the optimal conditions are same as found in chapter 4. This certainly can not guarantee the fair solution unless $\pi_{out} = \frac{\pi_0}{1+\delta}$. In fact in the second case, it was assumed that $\pi_{out} < \frac{\pi_0}{1+\delta}$ to allow the project manager to increase her offer of g in order to improve her utility and pushing the solution to the fair one.

6.3.2 For long term projects

As described in chapter 4, there could be two different type of scenarios for the long term projects: projects with recoverable operational life of the product, and projects with irrecoverable operational life of the product, in the event the project completion is delayed. Chapter 4 analysed the scenario for a profit maximizing project manager and a profit maximizing contractor. This section extends that analysis to fairness concerned members. As described in chapter 4, the profit functions in the centralized setting follows the equations (4.12 and 4.26) and the first best resource consumption rate $\lambda = \lambda_0$ satisfies the requirements in equation (6.37). In the decentralized setting, the project manager offers a contract P(T,C) to the contractor. Thus, their individual profits follows the equation (6.38) and (6.39). Since the time based contract, P(T,C) = g-hT is used, the expected value becomes

$$\begin{aligned} E\{P(T,C)e^{-\alpha T}\} &= E\{(g-hT)e^{-\alpha T}\} \\ &= g\int_0^\infty e^{-\alpha T} f_\lambda(T) - h\int_0^\infty T e^{-\alpha T} f_\lambda(T) \\ &= gE\{e^{-\alpha T}\} - h\int_0^\infty T e^{-\alpha T} f_\lambda(T) \end{aligned} \tag{6.146}$$

Using this value of E[P(T,C) $e^{-\alpha T}$] in the equations (6.38) and (6.39), the following modified profit functions of the project manager and the contractor in the decentralized supply chain are derived

$$\pi_{pm} = E(q) - E\{(P(T,C)e^{-\alpha T}\} - C_o$$
$$= \begin{cases} q_0 E\{e^{-\alpha T}\} - gE\{e^{-\alpha T}\} + h\int\limits_0^\infty Te^{-\alpha T} f_\lambda(T) - C_o \\ \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\{e^{-\alpha T}\}] - gE\{e^{-\alpha T}\} + h\int\limits_0^\infty Te^{-\alpha T} f_\lambda(T) - C_o \\ \text{for irrecoverable product life} \end{cases} \tag{6.147}$$

$$\pi_{co} = E\{P(T,C)e^{-\alpha T}\} - E(C) = g\{e^{-\alpha T}\} - h\int\limits_0^\infty Te^{-\alpha T} f_\lambda(T) - \frac{k\lambda^n}{\alpha}\left[1 - E\{e^{-\alpha T}\}\right] \tag{6.148}$$

The $E\{e^{-\alpha T}\}$ values follow the values calculated in the $E\{e^{-\alpha T}\}$ in the equation (4.25). For calculation simplicity, it is assumed that $E(e^{-\alpha T}) = E$, and $\int\limits_0^\infty Te^{-\alpha T} f_\lambda(T)dT = I$. The expected cost becomes $E(C) = \frac{k\lambda^N}{\alpha}(1 - E)$. It is further assumed $E(C) = C_\mu$. Using these notations, the centralized profit, the project manager's decentralized profit, and the contractors' decentralized profit become

$$\pi_0 = \begin{cases} q_0 E - C_\mu - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - C_\mu - C_o & \text{for recoverable product life} \end{cases} \tag{6.149}$$

$$\pi_{pm} = \begin{cases} q_0 E - gE + hI - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o & \text{for recoverable product life} \end{cases} \tag{6.150}$$

$$\pi_{co} = gE - hI - C_\mu \tag{6.151}$$

Fairness Concern Contractor and Profit Maximizing Project Manager

If the contract is fairness concerned, then he selects a resource consumption rate λ in the decentralized setting which maximizes his utility function as below

$$U_{co} = \begin{cases} [gE - hI - C_\mu] - \alpha_{co}[\gamma\{(q_0 E - gE + hI - C_o\} - \{gE - hI - C_\mu\}]^+ \\ -\beta_{co}[\{gE - hI - C_\mu\} - \gamma\{(q_0 E - gE + hI - C_o\}]^+ \\ \text{for recoverable product life} \\ [gE - hI - C_\mu] - \alpha_{co}[\gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} - \{gE - hI - C_\mu\}]^+ \\ -\beta_{co}[\{gE - hI - C_\mu\} - \gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\}]^+ \\ \text{for irrecoverable product life} \end{cases} \tag{6.152}$$

Similar to the case of short term project, this utility function can be broken into two cases.

The disadvantageous inequity occurs when

$$\begin{cases} \{gE - hI - C_\mu\} - \gamma\{q_0 E - gE + hI - C_o\} \leq 0 \\ \text{for recoverable product life} \\ \{gE - hI - C_\mu\} - \gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} \leq 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.153}$$

The utility function becomes $U_{co} = U_{co1}$, where

$$U_{co1} = \begin{cases} [gE - hI - -C_\mu] - \alpha_{co}[\gamma\{q_0 E - gE + hI - C_o\} - \{gE - hI - C_\mu\}] \\ \text{for recoverable product life} \\ [gE - hI - C_\mu] - \alpha_{co}[\gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} - \{gE - hI - C_\mu\}] \\ \text{for irrecoverable product life} \end{cases} \tag{6.154}$$

For an offered value of P(T,C) = g-hT, the fairness concerned contractor selects a resource consumption rate λ that maximizes the utility mentioned in the equation (6.154). Thus, the selected λ satisfies the following

$$\frac{dU_{co1}}{d\lambda} = \begin{cases} (g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - \frac{dC_\mu}{d\lambda}) - \alpha_{co}[\gamma(q_0\frac{dE}{d\lambda} - g\frac{dE}{d\lambda} + h\frac{dI}{d\lambda}) - (g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - \frac{dC_\mu}{d\lambda})] = 0 \\ \text{for recoverable product life} \\ (g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - \frac{dC_\mu}{d\lambda}) - \alpha_{co}[\gamma(q_0\left(\frac{\psi}{\alpha}\right)\frac{dE}{d\lambda} - g\frac{dE}{d\lambda} + h\frac{dI}{d\lambda}) - (g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - \frac{dC_\mu}{d\lambda})] = 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.155}$$

In order to coordinate the supply chain, this selected λ should be at least equal to the first best solution λ_0 i.e. $\lambda = \lambda_0$. Now λ_0 should satisfy the equation (6.37). Replacing the abbreviated version in the equation (6.37)

$$\frac{d\pi_0}{d\lambda} = \begin{cases} q_0\frac{dE}{d\lambda} - \frac{dC_\mu}{d\lambda} = 0 & \text{for recoverable product life} \\ q_0(\frac{\psi}{\alpha})\frac{dE}{d\lambda} - \frac{dC_\mu}{d\lambda} = 0 & \text{for recoverable product life} \end{cases} \tag{6.156}$$

Thus replacing the values of $\frac{dC_\mu}{d\lambda}$ from the equation (6.156) in the equation (6.155) and rearranging the terms, the following conditions are derived

$$\frac{dU_{co1}}{d\lambda} = \begin{cases} (1 + \alpha_{co} + \gamma\alpha_{co})(g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - q_0\frac{dE}{d\lambda}) = 0 & \text{for recoverable product life} \\ \\ (1 + \alpha_{co} + \gamma\alpha_{co})\frac{dI}{d\lambda}(g\frac{dE}{d\lambda} - h\frac{dI}{d\lambda} - \frac{q_0\psi}{\alpha}\frac{dE}{d\lambda}) = 0 & \text{for irrecoverable product life} \end{cases} \tag{6.157}$$

Again, for simplicity of calculation, it is assumed $\frac{dE}{d\lambda} = E'$ and $\frac{dI}{d\lambda} = I'$. As mentioned before, it can be shown that $\frac{dE(e^{-\alpha T})}{d\lambda} > 0$ for the uniform, gamma, beta and Weibull distributions. From the equation (6.157), the g values are derived as below

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \text{for irrecoverable product life} \\ h\frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) \end{cases} \tag{6.158}$$

The offered contract should also ensure a non-negative utility i.e. $U_{co} \geq 0$.

Thus,

$$U_{co1} = \begin{cases} [gE - hI - C_\mu] - \alpha_{co}[\gamma\left\{q_0 E - gE + hI - C_o\right\} - \{gE - hI - C_\mu\}] \geq 0 \\ \text{for recoverable product life} \\ [gE - hI - C_\mu] - \alpha_{co}[\gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} - \{gE - hI - C_\mu\}] \geq 0 \\ \text{for irrecoverable product life} \end{cases}$$

Replacing the value of g from the equation (6.158) in the above equation and rearranging the terms,

$$U_{co} = \begin{cases} (h\frac{I'}{E'}E - hI)(1 + \alpha_{co} + \alpha_{co}\gamma) + q_0 E(1 + \alpha_{co} + \alpha_{co}\gamma) - C_\mu(1 + \alpha_{co}) \\ -\alpha_{co}\gamma q_0 E + \alpha_{co}\gamma C_o \geq 0 \text{ for recoverable product life} \\ (h\frac{I'}{E'}E - hI)(1 + \alpha_{co} + \alpha_{co}\gamma) + q_0(\frac{\psi}{\alpha})E(1 + \alpha_{co} + \alpha_{co}\gamma) - C_\mu(1 + \alpha_{co}) \\ -\alpha_{co}\gamma q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] + \alpha_{co}\gamma C_o \geq 0 \text{ for irrecoverable product life} \end{cases}$$

or

$$U_{co} = \begin{cases} (h\frac{I'}{E'}E - hI)(1 + \alpha_{co} + \alpha_{co}\gamma) + \pi_0(1 + \alpha_{co}) + C_o(1 + \alpha_{co} + \alpha_{co}\gamma) \geq 0 \\ \text{for recoverable product life} \\ (h\frac{I'}{E'}E - hI)(1 + \alpha_{co} + \alpha_{co}\gamma) + \pi_0(1 + \alpha_{co}) \\ +q_0(\frac{\psi}{\alpha} - 1)(1 + \alpha_{co} + \alpha_{co}\gamma) + C_o(1 + \alpha_{co} + \alpha_{co}\gamma) \geq 0 \\ \text{for irrecoverable product life} \end{cases}$$

Rearranging the values, h values are derived as below

$$h \leq \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(I-\frac{I'}{E'}E\right)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \text{for recoverable product life} \\ \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E-I\right)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)}\frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \text{for irrecoverable product life} \end{cases} \tag{6.159}$$

The contractor experiences the disadvantageous inequity when the following is satisfied

$$\begin{cases} (gE - hI - C_\mu) - \gamma(q_0E - gE + hI - C_o) \leq 0 & \text{for recoverable product life} \\ (gE - hI - C_\mu) - \gamma(q_0(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E) - gE + hI - C_o) \leq 0 & \text{for irrecoverable product life} \end{cases}$$

Replacing the value of g from the equation (6.158) and using the observation from the equation (6.149)

$$\begin{cases} -h(I - \frac{I'}{E'}E)(1+\gamma) + \pi_0 + (1+\gamma)C_o \leq 0 & \text{for recoverable product life} \\ -h(I - \frac{I'}{E'}E)(1+\gamma) + \pi_0 + q_0(\frac{\psi}{\alpha} - 1)(1+\gamma) + (1+\gamma)C_o \leq 0 & \text{for irrecoverable product life} \end{cases}$$

or

$$h \geq \begin{cases} \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I - \frac{I'}{E'}E)(1+\gamma)} & \text{for recoverable product life} \\ \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{q_0(\frac{\psi}{\alpha} - 1)}{(I - \frac{I'}{E'}E)} + \frac{C_o}{(I - \frac{I'}{E'}E)} & \text{for irrecoverable product life} \end{cases} \tag{6.160}$$

Given the requirements from the equations (6.158), (6.159), and(6.160), the optimization problem of the contractor becomes

$$\max_{g,h} : U_{pm} = \pi_{pm} = \begin{cases} q_0E - gE + hI - C_o & \text{for recoverable product life} \\ q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o & \text{for recoverable product life} \end{cases} \tag{6.161}$$

Sub. to

$$g \begin{cases} h\frac{I'}{E'} + q_0 & \text{for recoverable product life} \\ h\frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) & \text{for irrecoverable product life} \end{cases} \tag{6.162}$$

$$h \in \begin{cases} \left[\frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)(1+\gamma)} \text{ , } \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)}\right] \\ \text{for recoverable product life} \\ \left[\frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} \text{ , } \frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.163}$$

It can easily be shown for any values of $\gamma > 0$, that

$$\begin{cases} \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} < \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\big(I-\frac{I'}{E'}E\big)} + \frac{C_o}{\big(I-\frac{I'}{E'}E\big)} \\ \text{for recoverable product life} \\ \left\{\frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)}\right\} < \left\{\frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\big(I-\frac{I'}{E'}E\big)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{\big(I-\frac{I'}{E'}E\big)}\right\} \\ \text{for irrecoverable product life} \end{cases}$$

Since $\frac{dU_{pm}}{dh} = I$, it is positive, otherwise the expected value of the offered contract would be negative. Hence, the project manager would select the maximum value of h, given the constraint in the equation (6.163). These above observations are summarised in the following lemma

Lemma 28. *The time based contract P(T,C)=g-hT ensures the contractor to earn the optimal utility and $\lambda = \lambda_0$ in a supply chain with profit maximizing project manager and a fairness concerned contractor experiencing disadvantageous inequity if the following are satisfied*

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) & \textit{for irrecoverable product life} \end{cases} \tag{6.164}$$

$$h = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \textit{for recoverable product life} \\ \\ \frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \textit{for irrecoverable product life} \end{cases} \tag{6.165}$$

On the contrary to the disadvantageous inequity, if the contractor experiences the advantageous inequity, then her utility function becomes

$$U_{co2} = \begin{cases} [gE - hI - C_\mu] - \beta_{co}[\{gE - hI - C_\mu\} - \gamma \{q_0 E - gE + hI - C_o\}] \\ \text{for recoverable product life} \\ [gE - hI - C_\mu] - \beta_{co}[\{gE - hI - C_\mu\} - \gamma \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\}] \\ \text{for irrecoverable product life} \end{cases} \tag{6.166}$$

Again for a given contract of P(T,C)=g-hT, the contractor selects a λ that maximizes his utility in the equation (6.166). Thus, it should satisfy the following

$$\frac{dU_{co2}}{d\lambda} = \begin{cases} \{gE' - hI' - C'_\mu\} - \beta_{co}\left[\{gE' - hI' - C'_\mu\} - \gamma\{q_0 E' - gE' + hI'\}\right] \\ \text{for recoverable product life} = 0 \\ \{gE' - hI' - C'_\mu\} - \beta_{co}\left[\{gE' - hI' - C'_\mu\} - \gamma\{q_0\left(\frac{\psi}{\alpha}\right)E' - gE' + hI'\}\right] = 0 \\ \text{for irrecoverable product life} \end{cases} \tag{6.167}$$

For the purpose of coordination, $\lambda = \lambda_0$. Thus, it should also satisfy first order condition in the equation (6.156). Thus replacing the value of C'_μ from this first order condition to the equation (6.167).

$$\frac{dU_{co2}}{d\lambda} =$$

$$\begin{cases} \{gE' - hI' - q_0 E'\} - \beta_{co}\left[\{gE' - hI' - q_0 E'\} - \gamma\{q_0 E' - gE' + hI'\}\right] = 0 \\ \text{for recoverable product life} \\ \\ \{gE' - hI' - q_0\left(\frac{\psi}{\alpha}\right)E'\} - \beta_{co}\left[\{gE' - hI' - q_0\left(\frac{\psi}{\alpha}\right)E'\} - \gamma\{q_0\left(\frac{\psi}{\alpha}\right)E' - gE' + hI'\}\right] = 0 \\ \text{for irrecoverable product life} \end{cases}$$

or

$$\frac{dU_{co2}}{d\lambda} = \begin{cases} \{gE' - hI' - q_0E'\}(1 - \beta_{co} - \beta_{co}\gamma) = 0 \\ \text{for recoverable product life} \\ \{gE' - hI' - q_0\left(\frac{\psi}{\alpha}\right)E'\}(1 - \beta_{co} - \beta_{co}\gamma) = 0 \\ \text{for irrecoverable product life} \end{cases}$$

It is assumed that $1 - \beta_{co} - \beta_{co}\gamma \neq 0$. Thus, from the above equation, the can be shown

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \text{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \text{for irrecoverable product life} \end{cases} \tag{6.168}$$

The offered contract should ensure the contractor a non-negative utility i.e. $U_{co} \geq 0$. Thus,

$$U_{co2} = \begin{cases} [gE - hI - C_\mu] - \beta_{co}[\{gE - hI - C_\mu\} - \gamma\{q_0E - gE + hI - C_o\}] \geq 0 \\ \text{for recoverable product life} \\ \\ [gE - hI - C_\mu] - \beta_{co}[\{gE - hI - C_\mu\} - \gamma\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\}] \geq 0 \\ \text{for irrecoverable product life} \end{cases}$$

Replacing the values of g from the equation(6.168)

$$\begin{cases} [h(\frac{I'}{E'}E - I)(1 - \beta_{co} - \beta_{co}\gamma) + q_0E(1 - \beta_{co} - \beta_{co}\gamma) & \\ -C_\mu(1 - \beta_{co}) + \gamma\beta_{co}q_0E - \gamma\beta_{co}C_o] \geq 0 & \text{for recoverable product life} \\ \\ [h(\frac{I'}{E'}E - I)(1 - \beta_{co} - \beta_{co}\gamma) + q_0\left(\frac{\psi}{\alpha}\right)E(1 - \beta_{co} - \beta_{co}\gamma) & \\ -C_\mu(1 - \beta_{co}) + \gamma\beta_{co}q_0\left\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right\} - \beta_{co}\gamma C_o] \geq 0 & \text{for irrecoverable product life} \end{cases}$$

Using the value of π_0 from the equation 6.149 and rearranging the variables in the above equation

$$h \leq \begin{cases} \frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for recoverable product life} \\ \frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{q_0\{\frac{\psi}{\alpha}-1\}}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for irrecoverable product life} \end{cases}$$

The contractor experiences advantageous inequity if the following is satisfied

$$\begin{cases} (gE - hI - C_\mu) - \gamma(q_0E - gE + hI - C_o) \geq 0 & \text{for recoverable product life} \\ (gE - hI - C_\mu) - \gamma\left\{q_0\left(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right) - gE + hI - C_o\right\} \geq 0 & \text{for irrecoverable product life} \end{cases}$$

Using the value of g from the equation (6.168)

$$\begin{cases} h(\frac{I'}{E'}E - I)(1+\gamma) + q_0E(1+\gamma) - C_\mu - \gamma q_0E + \gamma C_o \geq 0 \\ \text{for recoverable product life} \\ h(\frac{I'}{E'}E - I)(1+\gamma) + q_0\left(\frac{\psi}{\alpha}\right)(1+\gamma)E - C_\mu - \gamma q_0\left(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right) + \gamma C_o \geq 0 \\ \text{for irrecoverable product life} \end{cases}$$

Replacing the value of π_0 from the equation 6.149 in the above condition and rearranging the variables

$$h \leq \begin{cases} \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for recoverable product life} \\ \\ \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for irrecoverable product life} \end{cases}$$

Summarising these above observations, the optimization problem for the project manager becomes

$$\max_{g,h} : U_{pm} = \pi_{pm} = \begin{cases} [q_0 - gE + hI - C_o] & \text{for recoverable product life} \\ \left[q_0(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E) - gE + hI - C_o\right] & \text{for irrecoverable product life} \end{cases} \tag{6.169}$$

Sub. to

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \text{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \text{for irrecoverable product life} \end{cases}$$

$$h \leq \min \begin{cases} \left\{ \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} \text{ , } \frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} \right\} \\ \text{for recoverable product life} \\ \left\{ \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} \text{ , } \frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{q_0\{\frac{\psi}{\alpha}-1\}}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} \right\} \\ \text{for irrecoverable product life} \end{cases} \tag{6.170}$$

Now for the case of the projects with products whose operational life can be recovered upon completion in the event of any delay.

$$\left[\frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)}\right] - \left[\frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)}\right]$$
$$= \left[\frac{\pi_0\gamma}{(I-\frac{I'}{E'}E)(1+\gamma)(1-\beta_{co}-\beta_{co}\gamma)}\right]$$

Thus, if $(1-\beta_{co}-\beta_{co}\gamma) < 0$ i.e. $\beta_{co} > \frac{1}{1+\gamma}$, then project manager selects

$$h = \frac{\pi_0(1-\beta_{co})}{(I-\frac{I'}{E'}E)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)}$$

Thus, the profit of the project manager becomes

$$\begin{aligned} \pi_{pm} &= q_0E - gE + hI - C_o \\ &= q_0E - h\frac{I'}{E'}E - q_0E + hI - C_o \\ &= \frac{\pi_0(1-\beta_{co})}{(1-\beta_{co}-\beta_{co}\gamma)} \end{aligned}$$

Since $\beta_{co} < 1$ and as assumed that $(1-\beta_{co}-\beta_{co}\gamma) < 0$, the profit for the project manager becomes negative. Similarly, for the projects with products whose operational life could not be recovered upon completion in the event of the delay, the project manager earns a negative profit if $\beta_{co} > \frac{1}{1+\gamma}$. Hence, the cases with $\beta_{co} > \frac{1}{1+\gamma}$ are not considered any more.

On the contrary, if $\beta_{co} < \frac{1}{1+\gamma}$, then $h \leq \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)}$ (for recoverable product life) or $h \leq \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)}$ (for irrecoverable product life). Since $\frac{dU_{pm}}{dh} = I > 0$, the project manager would select the maximum value of h given the constraint in equation (6.170). Summarising all the above arguments, the following lemma is derived.

Lemma 29. *The time based contract P(T,C)=g-hT ensures the contractor to earn the optimal utility and $\lambda = \lambda_0$ in a supply chain with a profit maximizing project manager and a fairness concerned contractor (the contractor experiencing advantageous inequity with $\beta_{co} < \frac{1}{1+\gamma}$) if the following are satisfied*

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) & \textit{for irrecoverable product life} \end{cases} \tag{6.171}$$

$$h = \begin{cases} \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \textit{for recoverable product life} \\ \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \textit{for irrecoverable product life} \end{cases} \tag{6.172}$$

Now for the value of E follows the equation (4.25). Replacing the value of θ in terms of λ (As shown in chapter 4), the E' values are as follows

$$E' = \begin{cases} \frac{(1-e^{-\alpha\theta})A\lambda^{A-1}}{2\mu_1\alpha} - (Ae^{-\alpha\theta}\lambda^{-1}) & \text{for uniformly distributed time} \\ \frac{Aw^{w+1}\alpha\mu_1\lambda^{Aw-1}}{(w\lambda^A+\alpha\mu_1)^{w+1}} & \text{for gamma distributed time} \\ \sum_{m=1}^{\infty}\left[-(mA)\left\{\mu_1\left(\frac{u+v}{u}\right)\right\}^m \frac{1}{\lambda^{mA+1}}\left\{\prod_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right] & \text{for beta distributed time} \\ \sum_{m=1}^{\infty}\left[-(mA)\left\{\frac{\mu_1}{\Gamma\left(1+\frac{1}{s}\right)}\right\}^m \frac{1}{\lambda^{mA+1}}\left\{\Gamma\left(1+\frac{m}{s}\right)\right\}\left\{\frac{(-\alpha)^m}{m!}\right\}\right] & \text{for Weibull distributed time} \end{cases} \tag{6.173}$$

The value of the I is for different distributions are as follows

$$
\begin{aligned}
I &= \int\limits_{0}^{\infty} T e^{-\alpha T} f_{\lambda}(T) dT \\
&= \begin{cases}
\frac{\lambda^{A}}{2\mu_1\alpha^2}\left\{1 - e^{-\alpha\theta}(\alpha\theta + 1)\right\} & \text{for uniform distributed time} \\
\mu_1 \left\{\frac{\lambda^{Aw} w^{w+1}}{(\alpha\mu_1 + w\lambda^{A})^{w+1}}\right\} & \text{for gamma distributed time} \\
\left[\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1(u+v)}{u\lambda^{A}}\right\}^{m}\left\{\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right\}\left\{\frac{(-\alpha)^{m-1}}{(m)!}\right\}\right] & \text{for beta distributed time} \\
\sum\limits_{m=1}^{\infty}\left\{\frac{\mu_1}{\lambda^{A}\Gamma\left(1+\frac{1}{s}\right)}\right\}^{m}\Gamma\left(1 + \frac{m}{s}\right)\left\{\frac{(-\alpha)^{m-1}}{(m-1)!}\right\} & \text{for Weibull distributed time}
\end{cases}
\end{aligned}
\tag{6.174}
$$

The values of I' for different distributions are as follows

$$
I' = \frac{dI}{d\lambda} \tag{6.175}
$$

$$
= \begin{cases}
\frac{A}{2\mu_1\alpha^2\lambda}\left[\lambda^{A}\left\{1 - e^{-\alpha\theta}(\alpha\theta + 1)\right\} - 2\mu_1\alpha e^{-\alpha\theta}(\alpha\theta + 1) + 2\mu_1\alpha e^{-\alpha\theta}\right] & \text{uniform distributed time} \\
\frac{A\mu_1 w^{w+2}\lambda^{Aw-1}(\alpha\mu_1 - \lambda^{A})}{(w\lambda^{A} + \mu_1\alpha)^{w+2}} & \text{gamma distributed time} \\
\sum\limits_{m=1}^{\infty}\left[-(mA)\left\{\mu_1\left(\frac{u+v}{u}\right)\right\}^{m}\frac{1}{\lambda^{mA+1}}\left\{\prod\limits_{i=1}^{m}\frac{u+i-1}{u+v+i-1}\right\}\left\{\frac{(-\alpha)^{m-1}}{(m-1)!}\right\}\right] & \text{beta distributed time} \\
\sum\limits_{m=1}^{\infty}\left[-(mA)\left\{\frac{\mu_1}{\Gamma\left(1+\frac{1}{s}\right)}\right\}^{m}\frac{1}{\lambda^{mA+1}}\left\{\Gamma\left(1 + \frac{m}{s}\right)\right\}\left\{\frac{(-\alpha)^{m}}{m!}\right\}\right] & \text{Weibull distributed time}
\end{cases}
\tag{6.176}
$$

From the proof of lemmas 1 and 2 from chapter 4, it can be shown that

$$
\frac{I'}{E'} = \begin{cases}
\frac{\lambda_0^{A}\{1 - e^{-\alpha\theta}(\alpha\theta + 1)\} - 2\mu_1\alpha^2\theta e^{-\alpha\theta}}{\alpha\{(1 - e^{-\alpha\theta)\lambda^{A} - 2\mu_1\alpha e^{-\alpha\theta}\}}} & \text{for uniform distributed time} \\
\frac{w(\mu_1\alpha - \lambda^{A})}{\alpha(w\lambda^{A} + \mu_1\alpha)} & \text{for gamma distributed time}
\end{cases}
\tag{6.177}
$$

Similarly, using the value of E from the equation (4.25) and using the observations from the proof of the lemmas 1 and 2, it can be shown

$$
\left(I - \frac{I'}{E'}E\right) = \begin{cases}
\frac{e^{-\alpha\theta}(\alpha\theta - 1 - e^{-\alpha\theta})}{(1 - e^{-\alpha\theta})\lambda_0^{A} - 2\mu_1\alpha e^{-\alpha\theta}} & \text{for uniform distributed time} \\
\frac{w^{w+1}\lambda_0^{Aw+A}}{\alpha(w\lambda^{A} + \alpha\mu_1)^{w+1}} & \text{for gamma distributed time}
\end{cases}
\tag{6.178}
$$

As shown in chapter 4, the values of E take no -closed form for beta and Weibull distributed time. Thus, it can be easily shown that the values of I, I', E', $\frac{I'}{E'}$, and $I - \frac{I'}{E'}E$ are also non-closed form.

Earlier it was assumed that$(1 - \beta_{co} - \beta_{co}\gamma) \neq 0$. If this assumption is relaxed, then the equation 6.170 becomes

$$h \leq \min \begin{cases} \left\{ \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I - \frac{I'}{E'}E)} \text{ , } \infty \right\} \\ \text{for recoverable product life} \\ \left\{ \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha} - 1\right)}{(I - \frac{I'}{E'}E)} + \frac{C_o}{(I - \frac{I'}{E'}E)} \text{ , } \infty \right\} \\ \text{for irrecoverable product life} \end{cases} \tag{6.179}$$

However, for $(1 - \beta_{co} - \beta_{co}\gamma) = 0$ i.e $\beta_{co} = \frac{1}{1+\gamma}$, $\frac{dU_{co}}{d\lambda} = 0$ even if $\{gE' - hI' - q_0E'\} \neq 0$ (for recoverable product life) or $\left\{gE' - hI' - q_0\left(\frac{\psi}{\alpha}E'\right)\right\} \neq 0$ (for irrecoverable product life. Hence, summarising the findings from lemmas 28 and 29, and from the above observations the following is proposed

Proposition 25. *The supply chain with a fairness concerned contractor and a profit maximizing project manager can be coordinated using a time based contract P(T,C) = g- hT if the following are satisfied*

1. *if the contractor experiences an disadvantageous inequity*

$$h = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E - I\right)} + \frac{C_o}{\left(\frac{I'}{E'}E - I\right)} & \textit{for recoverable product life} \\ \frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E - I\right)} + \frac{q_0\left(\frac{\psi}{\alpha} - 1\right)}{\left(\frac{I'}{E'}E - I\right)} + \frac{C_o}{\left(\frac{I'}{E'}E - I\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.180}$$

or

2. *if the contractor experiences an advantageous inequity with disutility per unit is not high i.e. $\beta_{co} \leq \frac{1}{1+\gamma}$ and the following are satisfied*

$$h = \begin{cases} \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I - \frac{I'}{E'}E)} & \textit{for recoverable product life} \\ \frac{\pi_0}{(I - \frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha} - 1\right)}{(I - \frac{I'}{E'}E)} + \frac{C_o}{(I - \frac{I'}{E'}E)} & \textit{for irrecoverable product life} \end{cases} \tag{6.181}$$

and

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) & \textit{for irrecoverable product life} \end{cases} \tag{6.182}$$

It was shown earlier that $\frac{\pi_0}{(\frac{I'}{E'}E-I)(1+\gamma)} < \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E-I\right)}$. Hence, the the following corollary is deduced based on this finding

Corollary 3. *Comparing the two options from the findings of the proposition 25, it can be shown that the option 2 for the optimal of h is unlikely to be offered by the project manager. This is because the optimal value of h would be higher in option 1 (because* $\frac{\pi_0}{(\frac{I'}{E'}E-I)(1+\gamma)} < \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E-I\right)}$*), and the project manager's profit is increasing in h.*

Fairness Concern Contractor and Project Manager

This subsection explores how the optimal condition changes when both the project manager and the contractor are fairness concerned. Again, the coordination problem is solved using the backward induction method. The contractor would select the resource consumption rate λ that maximizes his utility in the equation (6.152). Thus, as mentioned earlier for the case of profit maximizing project manager and the fairness concerned contractor , the contractor selects $\lambda = \lambda_0$ in the case of disadvantageous inequity if the conditions in the equations (6.162) and (6.163) are satisfied.

The contractor selects $\lambda = \lambda_0$ for the case of advantageous inequity if the values of the contract parameters satisfy the equations (6.170) . The project manager earns a negative profit for the values of $\beta_{co} > \frac{1}{1+\gamma}$ (as shown for the case with a fairness concerned contractor and a profit maximizing project manager). Hence, these cases are not considered again. Thus, the contractor would select $\lambda = \lambda_0$ if $\beta_{co} < \frac{1}{1+\gamma}$ and if the contractual parameters satisfy the equations (6.171) and (6.172).

Unlike the profit maximizing project manager, the fairness concerned project manager maximizes her utility mentioned in (6.5). Now replacing the values of π_{pm} and π_{co} from the equations (6.150) and (6.151), the utility function becomes

$$U_{pm} = \begin{cases} [q_0E - gE + hI - C_o] - \alpha_{pm}\left[\delta\left\{gE - hI - C_\mu\right\} - \left\{q_0E - gE + hI - C_o\right\}\right]^+ \\ -\beta_{pm}\left[\left\{q_0E - gE + hI - C_o\right\} - \delta\left\{gE - hI - C_\mu\right\}\right]^+ \\ \text{for recoverable product life} \\ \left[q_0\left\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right\} - gE + hI - C_o\right] \\ -\alpha_{pm}\left[\delta\left\{gE - hI - C_\mu\right\} - \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\}\right]^+ \\ -\beta_{pm}\left[\left\{q_0\left\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right\} - gE + hI - C_o\right\} - \delta\left\{gE - hI - C_\mu\right\}\right]^+ \\ \text{for irrecoverable product life} \end{cases} \tag{6.183}$$

As discussed before, if $\delta\gamma > 1$ i.e. $ECCP > \pi_0$, the supply chain is considered as acrimonious channel. On the contrary, if $\delta\gamma \leq 1$ i.e. $ECCP \leq \pi_0$, the supply chain is considered as harmonious channel (Cui et al. 2007).

Project manager disadvantageous inequity, contractor disadvantageous inequity

When the following are satisfied, the project manager experiences a disadvantageous inequity

$$\begin{cases} \left\{q_0E - gE + hI - C_o\right\} - \delta\left\{gE - hI - C_\mu\right\} < 0 & \text{for recoverable product life} \\ \left\{q_0\left\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right\} - gE + hI - C_o\right\} - \delta\left\{gE - hI - C_\mu\right\} < 0 & \text{for irrecoverable product life} \end{cases} \tag{6.184}$$

Then, the the project manager's utility becomes

$$U_{pm1} = \begin{cases} [q_0E - gE + hI - C_o] - \alpha_{pm}\left[\delta\left\{gE - hI - C_\mu\right\} - \left\{q_0E - gE + hI - C_o\right\}\right] \\ \text{for recoverable product life} \\ \left[q_0\left\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right\} - gE + hI - C_o\right] \\ -\alpha_{pm}\left[\delta\left\{gE - hI - C_\mu\right\} - \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.185}$$

If the contractor experiences a disadvantageous inequity, then the optimization problem for the fairness concerned project manager becomes

$$\max_{g,h} : U_{pm} = U_{pm1} \text{ in equation } (6.185) \tag{6.186}$$

Sub. to

- g satisfying the condition in the equation (6.162)
- h satisfying the equation (6.163)

Again, the project manager will select a value of h that maximizes her utility function U_{pm1}. Thus, the first order condition of λ should satisfy

$$\frac{dU_{pm1}}{dh} = I + \alpha_{pm}(\delta + 1)I > 0 \tag{6.187}$$

The project manager's utility is an increasing function of h. Thus, applying the constraint of the constraint in the equation (6.163), the project manager would select the maximum value of h given the constraint; so, the optimal value of h would satisfy

$$h_a^* = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for recoverable product life} \\ \frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)(I-\frac{I'}{E'}E)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for irrecoverable product life} \end{cases} \tag{6.188}$$

The optimal value of g_a^* should satisfy the condition in the equation (6.162). Replacing these values of g_a^* in the expression (6.184)

$$\begin{cases} \left[q_0 E - q_0 E - h_a^* \left(\frac{I'}{E'}E\right) + hI - C_o\right] - \left[\delta(q_0 E + h_a^* \left(\frac{I'}{E'}E\right) - hI - C_\mu\})\right] < 0 \\ \text{for recoverable product life} \\ \left[q_0 \left(1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\right) - q_0 \left(\frac{\psi}{\alpha}\right) E - h_a^* \left(\frac{I'}{E'}E\right) + hI - C_o\right] \\ - \left[\delta(q_0 \left(\frac{\psi}{\alpha}\right) E + h_a^* \left(\frac{I'}{E'}E\right) - hI - C_\mu\})\right] < 0 \\ \text{for irrecoverable product life} \end{cases}$$

or

$$\begin{cases} h_a^*\left(I-\frac{I'}{E'}E\right)(1+\delta)-C_o(1+\delta)-\delta\pi_0<0 & \text{for recoverable product life}\\ q_0\left(1-\frac{\psi}{\alpha}\right)(1+\delta)+h_a^*\left(I-\frac{I'}{E'}E\right)(1+\delta)-C_o(1+\delta)-\delta\pi_0<0 & \text{for irrecoverable product life}\end{cases}$$

Now replacing the values of h_a^* from (6.188) in the above condition

$$\begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)}(1+\delta)-\delta\pi_0<0 & \text{for recoverable product life}\\ -q_0\left(\frac{\psi}{\alpha}-1\right)(1+\delta)+\frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)}+q_0(\frac{\psi}{\alpha}-1)-\delta\pi_0<0 & \text{for irrecoverable product life}\end{cases}$$

or

$$\frac{\pi_0\left\{1+\alpha_{co}(1-\delta\gamma)\right\}}{1+\alpha_{co}+\alpha_{co}\gamma}<0 \tag{6.189}$$

The above expression is negative only if $\delta\gamma>1$. Hence, like the previous cases, the project manager experiences disadvantageous inequity in the acrimonious supply chain and advantageous inequity in the harmonious supply chain. The contractual parameters should also ensure the project manger to earn a non-negative utility.

$$U_{pm1}=\begin{cases} [q_0E-gE+hI-C_o]-\alpha_{pm}\left[\delta\left\{gE-hI-C_\mu\right\}-\left\{q_0E-gE+hI-C_o\right\}\right]\geq 0\\ \text{for recoverable product life}\\ \left[q_0\left\{1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E\right\}-gE+hI-C_o\right]\\ -\alpha_{pm}\left[\delta\left\{gE-hI-C_\mu\right\}-\left\{q_0[1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E]-gE+hI-C_o\right\}\right]\geq 0\\ \text{for irrecoverable product life}\end{cases} \tag{6.190}$$

Using the optimal value of g from the equation (6.162) and the observation from the condition in (6.189)

$$\begin{cases} \left[h\left\{I-\frac{I'}{E'}E\right\}-C_o\right]-\alpha_{pm}\frac{\pi_0\{-1+\alpha_{co}(-1+\delta\gamma)\}}{1+\alpha_{co}+\alpha_{co}\gamma}\geq 0 & \text{for recoverable product life}\\ \left[q_0\{1-\frac{\psi}{\alpha}\}+h\left(I-\frac{I'}{E'}E\right)-C_o\right]-\alpha_{pm}\frac{\pi_0\{-1+\alpha_{co}(-1+\delta\gamma)\}}{1+\alpha_{co}+\alpha_{co}\gamma}\geq 0 & \text{for irrecoverable product life}\end{cases}$$

Using the value of $h = h_a^*$ from the equation (6.188), the following condition can be derived

$$\left[\frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)}\right] - \alpha_{pm}\frac{\pi_0\left\{-1+\alpha_{co}(-1+\delta\gamma)\right\}}{1+\alpha_{co}+\alpha_{co}\gamma} \geq 0$$

for both recoverable product life and irrecoverable product life

Since $\alpha_{co} > 0$ and $\delta\gamma > 1$, so if $\pi_0(1+\alpha_{co}) - \alpha_{pm}\left\{-1+\alpha_{co}(\delta\gamma-1)\right\} \geq 0$ then $\alpha_{pm} \leq \frac{(1+\alpha_{co})}{-1+\alpha_{co}(\delta\gamma-1)}$. It was assumed earlier that $\alpha_{pm} > 0$. Thus, it can be shown $\alpha_{co} > \frac{1}{\delta\gamma-1}$

Project manager disadvantageous inequity, contractor advantageous inequity

As mentioned earlier, the contractor selects $\lambda = \lambda_0$ if the optimal contractual parameters satisfy the conditions in the equations (6.170). Moreover, to avoid any negative profit for the project manager, $\beta_{co} \leq \frac{1}{1+\gamma}$. Thus, the optimization problem for the project manager becomes

$$\max_{g,h} : U_{pm} = U_{pm1} \text{ mentioned in the equation } (6.185) \tag{6.191}$$

Sub. to

- $\beta_{co} \leq \frac{1}{1+\gamma}$
- g and h satisfying the conditions in (6.170)

As shown before $\frac{dU_{pm1}}{dh} > 0$. Thus, the project manager selects the maximum value of h given the constraint in the equation (6.170). Hence, the optimal condition for the g and h should become

$$g_a^* = \begin{cases} h\frac{I'}{E'} + q_0 & \text{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \text{for irrecoverable product life} \end{cases} \tag{6.192}$$

$$h_a^* = \begin{cases} \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for recoverable product life} \\ \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)} + \frac{C_o}{(I-\frac{I'}{E'}E)} & \text{for irrecoverable product life} \end{cases} \tag{6.193}$$

Using these optimal values in the condition (6.184),

$$\begin{cases} \frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)}(I-\frac{I'}{E'}E)(1+\delta)+\frac{C_o}{(I-\frac{I'}{E'}E)}\left(\frac{I'}{E'}E-I\right)(1+\delta)-C_o-\delta(q_0-C_\mu)<0 \\ \text{for recoverable product life} \\ -q_0\left(\frac{\psi}{\alpha}-1\right)(1+\delta)+\frac{\pi_0}{(I-\frac{I'}{E'}E)(1+\gamma)}(I-\frac{I'}{E'}E)(1+\delta)+\frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{(I-\frac{I'}{E'}E)}(I-\frac{I'}{E'}E)(1+\delta) \\ +\frac{C_o}{(I-\frac{I'}{E'}E)}(I-\frac{I'}{E'}E)(1+\delta)-C_0-\delta\left\{q_0\left(1-\frac{\psi}{\alpha}+\frac{\psi}{\alpha}E\right)-C_\mu\right\}<0 \\ \text{for irrecoverable product life} \end{cases}$$

or

$$\frac{\pi_0}{(1+\gamma)}(1+\delta)-\delta\pi_0<0 \quad \text{for both recoverable product life and irrecoverable product life}$$

or

$$\frac{\pi_0(1-\delta\gamma)}{1+\gamma}<0 \quad \text{for both recoverable product life and irrecoverable product life} \quad (6.194)$$

Thus, again the project manager experiences disadvantageous inequity if $\delta\gamma>1$ i.e. in an acrimonious supply chain.

The offered contract must ensure a non-negative utility for the project manager. Thus,using the values of g_a^* from the equations (6.192) and the observations from the condition (6.194), in the conditions mentioned in (6.190), the following are derived

$$\begin{cases} \left[h\left(I-\frac{I'}{E'}E\right)-C_o\right]-\alpha_{pm}\left[\frac{\pi_0(\delta\gamma-1)}{1+\gamma}\right]\geq 0 & \text{for recoverable product life} \\ \left[-q_0\left(\frac{\psi}{\alpha}-1\right)+h\left(I-\frac{I'}{E'}E\right)-C_o\right]-\alpha_{pm}\left[\frac{\pi_0(\delta\gamma-1)}{1+\gamma}\right]\geq 0 & \text{for irrecoverable product life} \end{cases}$$

Replacing the value of h_a^* from the equation (6.193) in the above condition.

$$\frac{\pi_0-\alpha_{pm}\pi_0(\delta\gamma-1)}{1+\gamma}\geq 0 \qquad \text{for both recoverable and irrecoverable product life}$$

Thus, $\alpha_{pm}\leq\frac{1}{\delta\gamma-1}$. From the above observations, the optimal condition for the project manager is summarised in the following lemma

Lemma 30. *If both the project manager and the contractor both are fairness concerned,* *the supply chain is coordinated with the time based contract below with cash-flows are discounted exponentially (the project manager experiences disadvantageous inequity in an acrimonious supply chain) if*

1. *when the contractor experiences disadvantageous inequity and the following are satisfied*

$$h_a^* = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E-I\right)} + \frac{C_o}{\left(\frac{I'}{E'}E-I\right)} & \textit{for recoverable product life} \\ \frac{\{\pi_0\}(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(\frac{I'}{E'}E-I\right)} + \frac{q_0(\frac{\psi}{\alpha}-1)}{(\frac{I'}{E'}E-I)} + \frac{C_o}{\left(\frac{I'}{E'}E-I\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.195}$$

   $\delta\gamma > 1$, $\alpha_{pm} \leq \frac{1+\alpha_{co}}{\alpha_{co}(\delta\gamma-1)-1}$, *and* $\alpha_{co} > \frac{1}{\delta\gamma-1}$

2. *when the contractor experiences advantageous inequity and the following are satisfied*

$$h_a^* = \begin{cases} \frac{\pi_0}{\left(\frac{I'}{E'}E-I\right)(1+\gamma)} + \frac{C_o}{\left(\frac{I'}{E'}E-I\right)} & \textit{for recoverable product life} \\ \frac{\pi_0}{\left(\frac{I'}{E'}E-I\right)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(\frac{I'}{E'}E-I\right)} + \frac{C_o}{\left(\frac{I'}{E'}E-I\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.196}$$

   with $\beta_{co} \leq \frac{1}{1+\gamma}$, $\delta\gamma > 1$, *and* $\alpha_{pm} \leq \frac{1}{(\delta\gamma-1)}$

and

$$g_a^* = \begin{cases} h_a^* \frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h_a^* \frac{I'}{E'} + q_0(\frac{\psi}{\alpha}) & \textit{for irrecoverable product life} \end{cases} \tag{6.197}$$

Project manager advantageous inequity, contractor disadvantageous inequity

On the contrary to the disadvantageous inequity, when the following is satisfied, the project manager experiences advantageous inequity when

$$\begin{cases} \{q_0E - gE + hI - C_o\} - \delta\{gE - hI - C_\mu\} \geq 0 & \text{for recoverable product life} \\ \left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} - \delta\{gE - hI - C_\mu\} \geq 0 & \text{for irrecoverable product life} \end{cases} \tag{6.198}$$

The contractor selects $\lambda = \lambda_0$ if he experiences disadvantageous inequity and if the conditions in the equations (6.162) and (6.163) are satisfied. Thus, the optimization problem of the project manager becomes

$$\max_{g,h} : U_{pm2} = \begin{cases} \{q_0E - gE + hI - C_o\} - \beta_{pm}\left[\{q_0E - gE + hI - C_o\} - \delta\{gE - hI - C_\mu\}\right] \\ \text{for recoverable product life} \\ \left[q_0\{1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E\} - gE + hI - C_o\right] \\ -\beta_{pm}\left[\left\{q_0[1 - \frac{\psi}{\alpha} + \frac{\psi}{\alpha}E] - gE + hI - C_o\right\} - \delta\{gE - hI - C_\mu\}\right] \\ \text{for irrecoverable product life} \end{cases} \tag{6.199}$$

Sub. to

1. g satisfies the conditions in the equation (6.162)
2. h satisfies the conditions in the equation (6.163)

The project manager would select the contractor parameters that maximizes her utility in the equation (6.199) given the constraints of g and h in the equations (6.162) and (6.163). The value of g has a equal sign in the constraint and it depends on h. The value of h has inequity in the constraint equation. Now, the values of $\frac{dU_{pm2}}{dh} = (I - \beta_{pm}I - \beta_{pm}\delta I)$. Thus, if $(I - \beta_{pm}I - \beta_{pm}\delta I) > 0$, i.e. $\beta_{pm} < \frac{1}{1+\delta}$ then the utility of the project manager would be an increasing function of h. Hence, the project manager would select a maximum value of h given the constraint in the condition in (6.163). Thus, the optimal conditions are mentioned in the following lemma

Lemma 31. *If both (the project manager and the contractor) are fairness concerned, with the contractor experiences disadvantageous inequity, the optimal contractual conditions in a harmonious supply chain are as follows if* $\beta_{pm} < \frac{1}{1+\delta}$.

$$g^*_{ad} = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for recoverable product life} \end{cases} \tag{6.200}$$

$$h = h^*_{ad} = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(I-\frac{I'}{E'}E\right)} + \frac{C_0}{\left(I-\frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\pi_0(1+\alpha_{co})}{(1+\alpha_{co}+\alpha_{co}\gamma)\left(I-\frac{I'}{E'}E\right)} + \frac{C_0}{\left(I-\frac{I'}{E'}E\right)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.201}$$

On the contrary, if $(I - \beta_{pm}I - \beta_{pm}\delta I) < 0$, i.e. $\beta_{pm} > \frac{1}{1+\delta}$, then the project manager's utility would be a decreasing function of h. However, the profit of the project manager is still increasing in h. Thus, the project manager would select the lowest possible value for h in the constraint condition in the equation (6.163) if that ensures a positive profit for the project manager. The lowest possible value in that constraint should be

$$h = \begin{cases} \frac{\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\gamma)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \text{for recoverable product life} \\ \frac{\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \text{for irrecoverable product life} \end{cases} \tag{6.202}$$

For this value of h, and g satisfying the equation (6.200), the profit value of the project manager is

$$\begin{aligned} \pi_{pm} &= \begin{cases} h\left(I - \frac{I'}{E'}E\right) - C_0 & \text{for recoverable product life} \\ -q_0\left(\frac{\psi}{\alpha} - 1\right) + h\left(I - \frac{I'}{E'}E\right) - C_0 & \text{for irrecoverable product life} \end{cases} \\ &= \frac{\pi_0}{1+\gamma} > 0 \end{aligned}$$

It can be easily shown that $\pi_{co} = \frac{\gamma\pi_0}{1+\gamma}$. Thus, any further increase in h would reduce the value of the profit of the project manager and it would attain a negative value after certain increase in h value. Moreover, these values of h would never be able to allocate the fair profit for the contractor. Thus, these values are not considered in this case.

Hence, for $\beta_{pm} > \frac{1}{1+\delta}$, the project manager would select the above mentioned h value. The optimal conditions are summarised in the following lemma

Lemma 32. *If both (the project manager and the contractor) are fairness concerned, with the contractor experiences disadvantageous inequity, the optimal contractual conditions in a* *harmonious supply chain are as follows if* $\beta_{pm} > \frac{1}{1+\delta}$.

$$g = g^*_{ad} = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for recoverable product life} \end{cases} \tag{6.203}$$

$$h = h^*_{ad} = \begin{cases} \frac{\pi_0(1+\alpha_{co})}{(1+\gamma)\left(I-\frac{I'}{E'}E\right)} + \frac{C_0}{\left(I-\frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\pi_0(1+\alpha_{co})}{(1+\gamma)\left(I-\frac{I'}{E'}E\right)} + \frac{C_0}{\left(I-\frac{I'}{E'}E\right)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.204}$$

Project manager advantageous inequity, contractor advantageous inequity

If the contractor experiences an advantageous inequity, then he would select $\lambda = \lambda_0$ as long as the contractual parameters g and h satisfy the conditions in the equation (6.170). As shown earlier (with the fairness concerned contractor and the profit maximizing project manager), if $\beta_{co} > \frac{1}{1+\delta}$, then

$$\begin{cases} \frac{\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\gamma)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} > \frac{\pi_0(1-\beta_{co})}{\left(I-\frac{I'}{E'}E\right)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} \\ \text{for projects with recoverable prdocut life} \\ \frac{\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)}\frac{C_o}{\left(I-\frac{I'}{E'}E\right)} > \frac{\pi_0(1-\beta_{co})}{\left(I-\frac{I'}{E'}E\right)(1-\beta_{co}-\beta_{co}\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} \\ \text{for projects with irrecoverable prdocut life} \end{cases}$$

For $\beta_{co} > \frac{1}{1+\gamma}$, then the project manager earns a negative profit and hence, these cases are rejected. Thus, the optimization problem of the project manager becomes

$$\max_{g,h} : U_{pm} = U_{pm2} \text{ as mentioned in the equation } (6.199) \tag{6.205}$$

Sub.to

- g and h conditions satisfy the requirements in the conditions in (6.170) and
- $\beta_{co} < \frac{1}{1+\gamma}$

The project manager again selects a value of h that maximizes her utility which follows the equation (6.199). Thus, if $\frac{dU_{pm2}}{dh} = (I - \beta_{pm}I - \beta_{pm}\delta I) > 0$ i.e. $\beta_{pm} < \frac{1}{1+\delta}$, then the utility of the project manager increases with h in this case. Thus, the contractor would select the contractual parameters g and h in the equations in (6.171) and (6.172) when he experiences an advantageous inequity. It has been shown earlier that the contractor's utility is non-negative for the values of the contract parameters in (6.171) and (6.172). The project manager earns a positive profit of $\pi_{pm} = \frac{\pi_0}{1+\gamma}$ as it is shown before the lemma 32. The profit for the contractor becomes $\frac{\gamma\pi_0}{1+\gamma}$. Hence, the utility of the project manager becomes

$$\begin{aligned} U_{pm2} &= \pi_{pm} - \beta_{pm}(\pi_{pm} - \delta\pi_{co}) \\ &= \frac{\pi_0}{1+\gamma} - \beta_{pm}\left[\frac{\pi_0(1-\delta\gamma)}{1+\gamma}\right] \\ &= \frac{\{1 - \beta_{pm}(1-\delta\gamma)\}}{1+\gamma} \end{aligned}$$

It was shown earlier that the project manager experiences an advantageous inequity in the harmonious supply chain; so $\delta\gamma < 1$. Hence, contractual parameter g and h from the conditions in (6.171) and (6.172), would ensure a non-negative utility for the project manager if $1 - \beta_{pm}(1-\delta\gamma) \geq 0$ i.e. $\beta_{pm} \leq \frac{1}{1-\delta\gamma}$. It can be easily shown that $\frac{1}{1+\delta} < \frac{1}{1-\delta\gamma}$

Hence, the optimal conditions are summarized in the following lemma

Lemma 33. *If both the members of the supply chain (the project manager and the contractor) are fairness concerned, with the contractor experiences disadvantageous inequity, the optimal contractual conditions in a harmonious supply chain are as follows if* $\beta_{pm} < \frac{1}{1+\delta}$ *and* $\beta_{co} < \frac{1}{1+\gamma}$.

$$g^*_{aa} = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for recoverable product life} \end{cases} \tag{6.206}$$

$$h = h^*_{aa} = \begin{cases} \frac{\pi_0}{\left(I - \frac{I'}{E'}E\right)(1+\gamma)} + \frac{C_o}{\left(I - \frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\pi_0}{\left(I - \frac{I'}{E'}E\right)(1+\gamma)} + \frac{q_0\left(\frac{\psi}{\alpha} - 1\right)}{\left(I - \frac{I'}{E'}E\right)} + \frac{C_o}{\left(I - \frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.207}$$

On the contrary, if $(I - \beta_{pm} I - \beta_{pm} \delta I) < 0$ i.e. $\beta_{pm} > \frac{1}{1+\delta}$, then the utility of the project manager decreases in h. Now in the constraint condition for h mentioned in (6.170), has no lower limit for h. Thus, lowering the h value would keep on increasing the utility. However, the profit function for the project manager is an increasing function of h. Thus, after a certain lower value of h is attained, the profit of the project manager would become negative. It is also not very practical to set a lower limit below the value mentioned for h in the lemma 33. This is because, the contractor would be ensured a fair profit when g and h satisfy the conditions in lemma 33 and it also satisfies the coordination requirements for the resource consumption rate i.e.$\lambda = \lambda_0$. Thus, even if $\beta_{pm} > \frac{1}{1+\delta}$, the h value need not to be reduced below the optimal conditions mentioned in lemma in 33. The coordination could be achieved with this solution as long as $\beta_{pm} \leq \frac{1}{1-\delta\gamma}$.

Summarising all the findings from the lemmas 30 ,31, 32, and 33, the following is proposed

Proposition 26. *In a fairness concerned supply chain (with both project manager and the contractor are fairness concerned), the project manager can coordinate the supply chain with the following time based contracts when the cash-flows are exponentially discounted in the following cases*

$$h^* = \frac{kN\lambda_0^{N+A}}{A}$$

and any of the following

1. *If both the project manager and the contractor experience disadvantageous inequity in the acrimonious supply chain, the h satisfying the condition in the equation (6.195) with* $\delta\gamma > 1, \alpha_{pm} \leq \frac{1+\alpha_{co}}{\{\alpha_{co}(\delta\gamma-1)-1\}}$ & $\alpha_{co} > \frac{1}{\delta\gamma-1}$
2. *If the project manager experiences disadvantageous inequity and the contractor experiences advantageous inequity in the acrimonious supply chain, the h satisfies the equation (6.196) with* $\delta\gamma > 1$, $\beta_{co} < \frac{1}{1+\gamma}$ *, &* $\alpha_{pm} \leq \frac{1}{\delta\gamma-1}$
3. *If the project manager experiences advantageous inequity and the contractor experiences disadvantageous inequity, the h satisfies the equation (6.201) with* $\delta\gamma < 1$, *&* $\beta_{pm} < \frac{1}{1+\delta}$ *and h follows (6.204) with* $\delta\gamma > 1$, *&* $\beta_{pm} > \frac{1}{1+\delta}$

4. *If both the project manager and the contractor experience advantageous inequity, then h satisfies (6.207) with* $\beta_{co} < \frac{1}{1+\gamma}$ *&* $\beta_{pm} \leq \frac{1}{1-\delta\gamma}$

Fairness Concerned Project Manager and Profit Maximizing contractor

The game is again solved using backward induction method. For a given value of time based contract contract P(T,C) =g-hT, the contractor selects a resource consumption rate λ that maximizes his profit. The project manager would anticipate this value of λ by backward induction and would offer a contract P(T,C) =g-hT that maximizes her own utility as well as satisfies the constraint to achieve the λ that maximizes the contractor's profit.

Similar to the calculation shown in chapter 4, the contractor would the λ that maximizes the contractor's profit in equations (6.148) or (6.151). Thus, the following must be satisfied

$$gE' - hI' - \frac{dC_\mu}{d\lambda} = 0 \tag{6.208}$$

Replacing the value of $\frac{dC_\mu}{d\lambda}$ from the equation (6.156) in the above equation, the optimal condition for g could be derived. This optimal condition is same as derived in the equation (6.158) or (6.168). Unlike the case of the fairness concern contractor, the optimal condition for h value would be different. This has to ensure the minimum profit of π_{out} for the contractor (by individual rationality constraint). Thus, from the equation (6.151), $gE - hI - C_\mu \geq \pi_{out}$. Using the optimal condition for g (As identified same as in the equation (6.158) or (6.168)), the optimal conditions for h can be shown as

$$h \leq \begin{cases} \frac{(\pi_0 - \pi_{out} + C_o)}{\left(I - \frac{I'}{E'}E\right)} & \text{for recoverable product life} \\ \frac{\left\{\pi_0 - \pi_{out} + q_0\left(\frac{\psi}{\alpha} - 1\right) + C_o\right\}}{\left(I - \frac{I'}{E'}E\right)} & \text{for irrecoverable product life} \end{cases} \tag{6.209}$$

The project manager anticipates these optimal conditions required to ensure the participation of the contractor. Hence, her optimization problem becomes

$$\max_{g,h} : U_{pm} =$$

$$\begin{cases} U_{pm1} & \text{as mentioned in the equation (6.185) when the condition in (6.184) is satisfied} \\ U_{pm2} & \text{as mentioned in the equation (6.199) when the condition in (6.198) is satisfied} \end{cases}$$

St.

$$\begin{cases} \text{g following the conditions in either equations (6.158) or (6.168)} \\ \text{h following the conditions in equation (6.209)} \end{cases}$$

The project manager would select the contract parameters that maximize her utility in the equation above, given the constraints of g and h. The value of g has a equal sign in the constraint and it depends on h.

It was shown in equation (6.187) that U_{pm1} is increasing in h with the project manager experiencing disadvantageous inequity. Thus, the project manager would select the maximum h values as given the constraint conditions above. These h values could be used in the optimal condition for g to find out the optimal values of g. This is summarized in the following lemma

Lemma 34. *The optimal contractual parameters of a time based contract satisfies the following where the cash flows are exponentially discounted. The supply chain consists of a fairness concerned project manager and a profit maximizing contractor; the project manager experiences a disadvantageous inequity.*

$$h = \begin{cases} \frac{(\pi_0 - \pi_{out} + C_o)}{\left(I - \frac{I'}{E'} E\right)} & \textit{for recoverable product life} \\ \frac{\left\{\pi_0 - \pi_{out} + q_0\left(\frac{\psi}{\alpha} - 1\right) + C_o\right\}}{\left(I - \frac{I'}{E'} E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.210}$$

$$g = \begin{cases} q_0 + \frac{(\pi_0 - \pi_{out} + C_o)\left(\frac{I'}{E'}\right)}{\left(I - \frac{I'}{E'} E\right)} & \textit{for recoverable product life} \\ q_0\left(\frac{\psi}{\alpha}\right) + \frac{\left\{\pi_0 - \pi_{out} + q_0\left(\frac{\psi}{\alpha} - 1\right) + C_o\right\}\left(\frac{I'}{E'}\right)}{\left(I - \frac{I'}{E'} E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.211}$$

On the contrary, if the project manager experiences advantageous inequity, then U_{pm} would decrease or increase in h depending on the value of the β_{pm}. It was shown earlier that the values of $\frac{dU_{pm}}{dh} > 0$ if $\beta_{pm} < \frac{1}{1+\delta}$. Then the utility of the project manager would be an increasing function of h. Hence, the project manager would select a maximum value of h given the constraint above. As a result, the situation becomes similar to the explained in the case of project manager experiencing the disadvantageous inequity. Thus, the optimal solutions would follow the conditions mentioned in the equations (6.210) and (6.211) in lemma (34)

Lemma 35. *The optimal contractual parameters of a time based contract (P(T,C)=g-hT) satisfies the conditions mentioned in the equations (6.210) and (6.211) in lemma 34 in a supply chain with fairness concerned project manager and a profit maximizing contractor. The project manager experiences an advantageous inequity in this case which is not too high i.e.* $\beta_{pm} < \frac{1}{1+\delta}$.

If $\beta_{pm} > \frac{1}{1+\delta}$, then $\frac{dU_{pm}}{dh} < 0$ (As shown before). From the constraint condition from (6.209), the h value has an upper limit in the constraint. However, it does not have any lower bound in the constraint. As shown earlier in similar cases, the utility function of the project manager would increase with the decrease in the value of h. However, the profit function of the project manager would still be decreasing with the decrease in h. Thus, after a certain reduction of h, the profit of the project manager would become negative, but the utility may be still increasing. This is due to the high inequity aversion compensating the loss of profit figure in the utility function. Again, this scenario is not favourable as the negative profit is not considered for this research. Thus, the h value is only allowed to decrease until it assigns a fair allocation of the profit. With the maximum of current optimal values from the given constraints in (6.210) and (6.211), the profit of the contractor becomes π_{out} (As shown in chapter 4). According to the perception of the project manager, she should earn $\delta\pi_{co}$ as a fair share of the profit. Hence, the fair share should be for the project manger $\frac{\delta}{1+\delta}\pi_0$ and $\frac{1}{1+\delta}\pi_0$ for the contractor. Thus, if $\pi_{out} < \frac{\pi_0}{1+\delta}$, then the project manager could reduce her offer for the h value in (6.209) when $\beta_{pm} > \frac{1}{1+\delta}$ until the profit of the contractor becomes $\frac{\pi_0}{1+\delta}$. Otherwise, it is not pragmatic to reduce the value below what is mentioned in the condition (6.210).

Lemma 36. *The optimal contractual parameters of a time based contract (P(T,C)=g-hT) satisfies the either of the following conditions in a supply chain with fairness concerned project manager and a profit maximizing contractor when the project manager experiences a disadvantageous inequity which is not too low i.e.* $\beta_{pm} > \frac{1}{1+\delta}$.

1. *If the value of* $\pi_{out} \geq \frac{\pi_0}{1+\delta}$*, then the optimal values should satisfy the conditions as mentioned in the equations (6.210) and (6.211) in lemma 34*

2. *If $\pi_{out} < \frac{\pi_0}{1+\delta}$, then, the optimnal conditions become*

$$g_a^* = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for recoverable product life} \end{cases} \tag{6.212}$$

$$h_a^* = \begin{cases} \frac{\delta\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\delta)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\delta\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\delta)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.213}$$

Proof. To ensure a fair profit for the contractor , the project manager should select a h that ensures $\pi_{co} = \frac{\pi_0}{1+\delta}$. Thus, using the value of π_{co}, the following can be derived

$$\frac{\pi_0}{1+\delta} = gE - hI - C_\mu$$

Using the value of g from the above lemma in the above equation, the optimal value mentioned in above lemma can be easily be found. □

Summarising the findings of lemmas 26 and 27, the following is proposed

Proposition 27. *A supply chain consisting of a fairness concerned project manager and a profit maximizing contractor could be coordinated with a time based contract P(T,C) = g-hT, if*

$$g = \begin{cases} h\frac{I'}{E'} + q_0 & \textit{for recoverable product life} \\ h\frac{I'}{E'} + q_0\left(\frac{\psi}{\alpha}\right) & \textit{for recoverable product life} \end{cases} \tag{6.214}$$

and

1.

$$h = \begin{cases} \frac{(\pi_0-\pi_{out}+C_o)}{\left(I-\frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\left\{\pi_0-\pi_{out}+q_0\left(\frac{\psi}{\alpha}-1\right)+C_o\right\}}{\left(I-\frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.215}$$

when the project manager experiences a disadvantageous inequity or advantageous inequity with very low utility loss per unit due to earning more than the contractor, with $\beta_{pm} \leq \frac{1}{1+\delta}$.

2.

$$h = \begin{cases} \frac{\delta\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\delta)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \textit{for recoverable product life} \\ \frac{\delta\pi_0}{\left(I-\frac{I'}{E'}E\right)(1+\delta)} + \frac{q_0\left(\frac{\psi}{\alpha}-1\right)}{\left(I-\frac{I'}{E'}E\right)} + \frac{C_o}{\left(I-\frac{I'}{E'}E\right)} & \textit{for irrecoverable product life} \end{cases} \tag{6.216}$$

if $\frac{1}{1+\delta}\pi_0 > \pi_{out}$ *and when the project manager experiences a considerable utility loss per unit due to earning more than the contractor so that* $\beta_{pm} > \frac{1}{1+\delta}$.

In the above proposition (24), the first optimal condition can coordinate the supply chain and ensure the contractor to earn a minimum profit of π_{out} as it was found in the case of a supply chain without any fairness consideration. In fact, the optimal conditions are same as found in chapter 4. This certainly can not guarantee the fair solution unless $\pi_{out} = \frac{\pi_0}{1+\delta}$. In fact in the second case, it was assumed that $\pi_{out} < \frac{\pi_0}{1+\delta}$ to allow the project manager to increase her offer of g in order to improve her utility and pushing the solution to the fair one. However, if $\pi_{out} > \frac{\pi_0}{1+\delta}$, then the project manager has to offer π_{out} for the contractor's participation. This may not guarantee her the fair profit to be earned.

6.4 Fairness and Bargaining

Chapter 5 presented the models of bargaining to show how the win-win situations can be achieved if the project manager and the contractor engage in negotiation. Two different scenarios were presented: both are risk neutral in one scenario;and the project manager is risk neutral, but the contractor is risk averse in the other scenario.

However, the chapter 5 did not discuss the issues around fairness in bargaining scenario. Rachmilevitch et al. (2011) mentioned that bargainers have got two main issues to solve while participating in the bargaining process. These are fairness and efficiency. The authors also highlighted that these two are in a trade-off. The authors argued that the bargaining problem is harmonic if the egalitarian solution and utilitarian solution agree with each other.

The authors further argued that the solution lies between the maximum and the minimum values of the payoff derived from the egalitarian and utilitarian solution and Nash bargaining solution balance the trade-off between the fair allocation and the efficiency.

In fact, an Egalitarian approach has been considered as the solution that fairness concerned members prefers in the bargaining if the interpersonal utility comparison is allowed (Binmore 2014). The author argued this based on experimental evidence. In fact, fairness motivation has been observed in non-cooperative bargaining approaches such as Rubinstein bargaining (Camerer 2003). The author argued that the share of the benefits tend to cluster around equal split even with differential bargaining power due to fairness concern.

Chapter 5 presented how the optimal solutions encourage the risk neutral project manager and the risk neutral contractor to equally split the benefits derived from the project completion. Hence, the theories of equal split and achieving fairness have been supported.

On the contrary, the second situation is somewhat different and challenging. The contractor is risk averse and his utility function takes it into account by incorporating non-linearity.

This research follows the argument proposed by Rachmilevitch et al. (2011) that the feasible fair allocation could possibly lie between the maximum and the minimum solutions from the Egalitarian solution and the Utilitarian solution. Chapter 5 presented the utilitarian approach. The egalitarian approach proposed by Kalai (1977) as below

$$E(Z,d) = \max_{z \in (Z,d)} : \left\{ \min_{(i=pm,co)} (z_i - d_i) \right\} \tag{6.217}$$

Applying this above equation, it can be easily be shown that the utility functions of the project manager and the contractor would become equal at the optimal solution. This can easily be achieved for the case when both the members are risk neutral.

However, chapter 5 did not analyse if this Egalitarian approach can be applied for the case of risk neutral project manager and risk averse contractor. Applying the Egalitarian optimization condition with the utility functions for cost based contracts from the equations (5.4) and (5.6) for the cost based contracts

$$(q - a - b\mu) = 1 - e^{-\eta a} W \qquad \text{for W = } [e^{\eta(1-b)X}] \tag{6.218}$$

Differentiating both side with respect to b and rearranging the terms

$$\left(-\frac{da}{db} - \mu\right)\left(\eta e^{-\eta a} W + 1\right) = \left[-e^{-\eta a}\frac{dW}{db} - \mu\eta e^{-\eta a} W\right]$$

The above equation is very similar to the observation from the equation (5.58) in section 5.3.2 in chapter 5. The only exception in the equation (5.58) was the normalization term B which was positive as well. Thus, following the same set of steps (from equation 5.58), it can be shown that $\frac{dU_{pm}}{db} > 0$ and $\frac{dU_{co}}{db} > 0$ for $0 \leq b < 1$ and 0 at b =1 .Thus, U_{pm} and U_{co} are increasing in $b \in [0, 1)$ and attains the maximum at b = 1. Hence, like other bargaining models, the cost plus contract with b = 1 dominates the solutions for other cost sharing contracts ($0 < b < 1$) and for fixed price contracts (b = 1). Similarly, for the time based contracts, it can be shown that the the solutions for any time based contract is dominated by the solutions from the fixed price contracts.

It can be very easily shown that this Egalitarian approach is applicable for certain limited cases with $U_{pm} < 1$. Moreover, several authors including Birkeland & Tungodden (2014) raised questions on the applicability of the equal share. The authors argued that the bargainers may not find equal share to be fair if they have got some initial endowment. This was evident in some recent experimental studies including the paper of Cappelen et al. (2010). This encourages the need for a more practical approach of sharing the risk and benefits.

6.4.1 Ex Ante vs Ex Post Fairness Consideration along With Risk Preference

With risk neutral members(both the project manager and the contractor in this case), the optimal solution approaches an equal split of the profit (as shown earlier). Existing literature considered this as a fair solution in some cases in absence of any initial endowment. However, things become more complicated in the presence of differential risk preference of the members of the supply chain.

One school of thought considers the fairness consideration and the risk preference to be independent (Bolton & Ockenfels 2000, Brennan et al. 2008). However, recent experimental evidence suggests the existence of a correlation between the fairness consideration and the risk preference (Krawczyk & Le Lec 2010, Fudenberg & Levine 2012, Brock et al. 2013).

The fairness consideration discussed so far for these cases (either in this chapter or in chapter 5) considered the fair allocation of risk once it is realized. In the literature, it has been defined as **Ex post fairness** (Fudenberg & Levine 2012). On the contrary, the bargainers were found to have some fairness consideration before the bargaining took place in the experimental evidence of the literature of Krawczyk & Le Lec (2010), and Brock et al. (2013). This fairness concern was present in the presence of uncertainty about the risk. López-Vargas (2014) defined this as the **Ex ante Fairness**. The reasons behind the existence of these type of fairness concerns are still at its early stage in literature and subjected to debate (López-Vargas 2014). The scope of the present research is restricted outside this debate. This research incorporates the **Expected Inequality Aversion model** proposed by Fudenberg & Levine (2012) to incorporate any ex-ante fairness consideration along with ex-post fairness consideration.

$$U = \Delta u(E(x, y)) + (1 - \Delta)E(u(x, y)) \tag{6.219}$$

However, authors suggested to use this model for the cases where u(x,y) follows the inequity aversion model for fairness suggested by Fehr & Schmidt (1999). This assumption was extended by López-Vargas (2014) with the generalized expected inequality aversion model. The author suggested the model can be applied with u(.) as a concave function.

The utility function proposed for risk averse contractor in the present research is a concave function (As shown in the equations 5.6 and 5.9). It was discussed earlier that the project manager is likely to be risk neutral in practice due to her financial position than in comparison to the contractor. Thus, the utility function for the project manager would be considered as a linear function as shown before.

Applying the generalized expected inequality aversion model, the updated utility function of the risk averse contract becomes

$$\begin{aligned} U_{co}(\Delta) &= \Delta U_{co}(E(z)) + (1 - \Delta)E(U_{co}(z)) && \text{[where } z = a - (1 - b)X\text{]} \\ &= \Delta E\left[1 - e^{-\eta a} e^{\eta(1-b)\mu}\right] + (1 - \Delta)\left[1 - e^{-\eta a} E\{e^{\eta(1-b)X}\}\right] && \text{[where } U_{co}(z) = 1 - e^{-\eta z}\text{]} \\ &= \Delta\left[1 - e^{-\eta a} Y\right] + (1 - \Delta)\left[1 - e^{-\eta a} W\right] && \text{[where } Y = E\{e^{\eta(1-b)\mu}\}\text{]} \end{aligned} \tag{6.220}$$

If the project manager is fairness concerned, her utility function is assumed as

$$\begin{aligned} U_{pm} = (q-a-b\mu) - \alpha_{pm}\left[\delta\{a-(1-b)\mu\} - \{q-a-b\mu\}\right]^{+} \\ - \beta_{pm}\left[\{q-a-b\mu\} - \delta\{a-(1-b)\mu\}\right]^{+} \end{aligned} \tag{6.221}$$

Utilitarian Approach

The utilitarian approach satisfies the equation (5.62). The contractor's modified utility function satisfies the equation (6.220). If the project manager experiencing the advantage inequity, then the utility of the project manager from the equation (6.221) becomes $U_{pm} = (q-a-b\mu) - \beta_{pm}\left[\{q-a-b\mu\} - \delta\{a-(1-b)\mu\}\right]$. Thus, the optimization problem for the utilitarian approach becomes

$$\max : U(Z,d) =$$
$$\begin{cases} (q-a-b\mu) - \alpha_{pm}\left[\delta\{a-(1-b)\mu\} - \{q-a-b\mu\}\right] & \\ +\Delta\left[1-e^{-\eta a}Y\right] + (1-\Delta)\left[1-e^{-\eta a}W\right] & \text{for disadvantageous inequity} \\ (q-a-b\mu) - \beta_{pm}\left[\{q-a-b\mu\} - \delta\{a-(1-b)\mu\}\right] & \\ +\Delta\left[1-e^{-\eta a}Y\right] + (1-\Delta)\left[1-e^{-\eta a}W\right] & \text{for advantageous inequity} \end{cases} \tag{6.222}$$

Thus, the first order condition for a should satisfy

$$\frac{dU(Z,d)}{da} = \begin{cases} -(1+\alpha_{pm}+\alpha_{pm}\delta) + \Delta(\eta e^{-\eta a}Y) + (1-\Delta)(\eta e^{-\eta a}W) = 0 \\ \text{for disadvantageous inequity case} \\ -(1-\beta_{pm}+\beta_{pm}\delta) + \Delta(\eta e^{-\eta a}Y) + (1-\Delta)(\eta e^{-\eta a}W) = 0 \\ \text{for advantageous inequity case} \end{cases} \tag{6.223}$$

Hence, from the above equation, the optimal value for a becomes

$$a = \begin{cases} \frac{1}{\eta}\log_e \frac{\eta\{\Delta Y+(1-\Delta)W\}}{(1+\alpha_{pm}+\alpha_{pm}\delta)} & \text{for disadvantageous inequity case} \\ \frac{1}{\eta}\log_e \frac{\eta\{\Delta Y+(1-\Delta)W\}}{(1-\beta_{pm}-\delta\beta_{pm})} & \text{for advantageous inequity case} \end{cases} \tag{6.224}$$

To find out the optimal value of b in $b \in [0, 1]$, the equation (6.223) is differentiated with respect to b.

$$\frac{dU(Z,d)}{db} = \frac{dU_{pm}}{db} + \frac{dU_{co}}{db} \tag{6.225}$$

Now $\frac{dU_{pm}}{db} = (-\frac{da}{db} - \mu)\{(1 - \beta_{pm}) - \beta_{pm}\delta)\}$ or $\frac{dU_{pm}}{db} = (-\frac{da}{db} - \mu)\{(1 + \alpha_{pm} + \alpha_{pm}\delta)\}$ and $\frac{dU_{co}}{db} = \eta e^{-\eta a}\frac{da}{db}\{\Delta W + (1-\Delta)Y\} - e^{-\eta a}\{\Delta\frac{dW}{db} + (1-\Delta)\frac{dY}{db}\}$. Thus, both the derivatives of the utility functions of the project manager and the contractor depend on the value of $\frac{da}{db}$. The $\frac{da}{db}$ can be derived as follows

$$\frac{da}{db} = \frac{\Delta\frac{dY}{db} + (1-\Delta)\frac{dW}{db}}{\eta\{\Delta Y + (1-\Delta)W\}} \tag{6.226}$$

Thus, both of the derivatives $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$ are depended on $\frac{dY}{db}$ and $\frac{dW}{db}$.

The value of $\frac{dY}{db} = -\eta\mu Y$ (Taking derivative of Y both side from the assumption in the equation 6.220). As discussed earlier in chapter 5, $\eta > 0$ and $\mu > 0$. Hence, $\frac{dY}{db} < 0$.

Using the value of $\frac{dW}{db}$ for a gamma distributed cost from the equation (5.32) and the mean value of gamma distributed cost,

$$\frac{dU_{pm}}{db} = \begin{cases} (-\frac{da}{db} - \mu)\{(1 + \alpha_{pm}) + \alpha_{pm}\delta)\} & \text{for advantageous inequity cases} \\ (-\frac{da}{db} - \mu)\{(1 - \beta_{pm} - \beta_{pm}\delta)\} & \text{for diadvantageous inequity cases} \end{cases}$$

$$= \begin{cases} \left[-\frac{-\eta\mu\Delta Y - (1-\Delta)\frac{W\eta\phi\omega}{1-\eta\phi(1-b)}}{\eta\{\Delta Y + (1-\Delta)W\}} - \mu\right]\{(1 + \alpha_{pm} + \alpha_{pm}\delta)\} & \text{for disadvantageous inequity cases} \\ \left[-\frac{-\eta\mu\Delta Y - (1-\Delta)\frac{W\eta\phi\omega}{1-\eta\phi(1-b)}}{\eta\{\Delta Y + (1-\Delta)W\}} - \mu\right]\{(1 - \beta_{pm} - \beta_{pm}\delta)\} & \text{for advantageous inequity cases} \end{cases}$$

$$= \begin{cases} \left[\frac{(1-\Delta)\frac{W\eta\mu}{1-\eta\phi(1-b)} - \eta(1-\Delta)W\mu}{\eta\{\Delta Y + (1-\Delta)W\}}\right]\{(1 + \alpha_{pm} + \alpha_{pm}\delta)\} & \text{for disadvantageous inequity cases} \\ \left[\frac{(1-\Delta)\frac{W\eta\mu}{1-\eta\phi(1-b)} - \eta(1-\Delta)W\mu}{\eta\{\Delta Y + (1-\Delta)W\}}\right]\{(1 - \beta_{pm} - \beta_{pm}\delta) & \text{for advantageous inequity cases} \end{cases}$$

$$= \begin{cases} \left[\frac{(1-\Delta)\eta^2 W\mu\phi(1-b)}{\eta\{\Delta Y + (1-\Delta)W\}\{1-\eta(1-b)\phi\}}\right]\{(1 + \alpha_{pm} + \alpha_{pm}\delta)\} & \text{for disadvantageous inequity cases} \\ \left[\frac{(1-\Delta)\eta^2 W\mu\phi(1-b)}{\eta\{\Delta Y + (1-\Delta)W\}\{1-\eta(1-b)\phi\}}\right]\{(1 - \beta_{pm} - \beta_{pm}\delta)\} & \text{for advantageous inequity cases} \end{cases}$$

It was shown earlier in chapter 5 that $\{1 - \eta(1-b)\phi\} > 0$. W, (1-b), and ϕ are all positive. It can be very easily shown from the equation (6.224) $(1 - \beta_{pm}) - \beta_{pm}\delta > 0$, otherwise the utilitarian approach can not be applied to this problem scenario. It is assumed that $0 < \Delta < 1$, otherwise, the Expected Inequity Aversion Model proposed in literature would reduce the utility of the contractor with the increase ex post fairness concern for sharing the risk. This is not very practical. Hence, it becomes evident from the above mentioned derivative that $\frac{dU_{pm}}{db} > 0$. Now

$$\begin{aligned}\frac{dU_{co}}{db} &= \eta e^{-\eta a}\frac{da}{db}\{\Delta W + (1-\Delta)Y\} - e^{-\eta a}\{\Delta\frac{dW}{db} + (1-\Delta)\frac{dY}{db}\}\\ &= \eta e^{-\eta a}\left[\frac{\Delta\frac{dY}{db} + (1-\Delta)\frac{dW}{db}}{\eta\{\Delta Y + (1-\Delta)W\}}\right]\{\Delta W + (1-\Delta)Y\} - e^{-\eta a}\{\Delta\frac{dY}{db} + (1-\Delta)\frac{dW}{db}\}\\ &= 0\end{aligned}$$

Hence, the utility of the contractor is unchanged with respect to any change in b.

Egalitarian Approach

As shown earlier, the egalitarian solution satisfies the condition in (6.217). Hence, the following must be satisfied if the contractor has got fairness consideration along with the risk averse preference and the project manager has got fairness consideration as well

$$\Delta(1 - e^{-\eta a}Y) + (1-\Delta)(1 - e^{\eta a}W) = \begin{cases} (q - a - b\mu)(1 + \alpha_{pm}) - \alpha_{pm}\delta\{a - (1-b)\mu\} \\ \text{for advantageous inequity cases} \\ (q - a - b\mu)(1 - \beta_{pm}) + \beta_{pm}\delta\{1 - (1-b)\mu\} \\ \text{for disadvantageous inequity cases} \end{cases}$$

The optimal value of a should satisfy the above condition. In order to find the optimal value of b in $b \in [0, 1]$, both the sides of the above equation is differentiated with respect to b

$$\begin{aligned}&\eta e^{-\eta a}\frac{da}{db}\{\Delta W + (1-\Delta)Y\} - e^{-\eta a}\left\{\Delta\frac{dY}{db} + (1-\Delta)\frac{dW}{db}\right\}\\ &= \begin{cases} \left(-\frac{da}{db} - \mu\right)\{1 + \alpha_{pm} + \delta\alpha_{pm}\} & \text{for disadvantageous inequity cases} \\ \left(-\frac{da}{db} - \mu\right)\{(1 - \beta_{pm}) - \delta\beta_{pm}\} & \text{for advantageous inequity cases} \end{cases}\end{aligned}$$

or

$$-e^{-\eta a}\left\{\Delta\frac{dY}{db}+(1-\Delta)\frac{dW}{db}\right\}-\eta e^{-\eta a}\mu\{\Delta Y+(1-\Delta)W\}$$

$$=\begin{cases}\left(-\frac{da}{db}-\mu\right)\left[\{1+\alpha_{pm}+\delta\alpha_{pm}\}+\eta e^{-\eta a}\{\Delta W+(1-\Delta)Y\}\right]\\ \text{for disadvantageous inequity cases}\\ \left(-\frac{da}{db}-\mu\right)\left[\{(1-\beta_{pm})-\delta\beta_{pm}\}+\eta e^{-\eta a}\{\Delta W+(1-\Delta)Y\}\right]\\ \text{for advantageous inequity cases}\end{cases}$$

Again the nature of the derivatives are depended on the values of $\frac{dY}{db}$ and $\frac{dW}{db}$. The $\frac{dW}{db}$ value depends on the nature of probability distribution of the cost function.

Using the value of $\frac{dW}{db}$ from the equation (5.32) for the gamma distributed cost in the last equation

$$\begin{cases}\left(-\frac{da}{db}-\mu\right)\left[\{(1+\alpha_{pm})+\delta\alpha_{pm}\}+\eta e^{-\eta a}\{\Delta Y+(1-\Delta)W\}\right] & \text{for disadvantageous inequity cases}\\ \left(-\frac{da}{db}-\mu\right)\left[\{(1-\beta_{pm})-\delta\beta_{pm}\}+\eta e^{-\eta a}\{\Delta Y+(1-\Delta)W\}\right] & \text{for advantageous inequity cases}\end{cases}$$

$$=-e^{-\eta a}\left\{\Delta\frac{dY}{db}+(1-\Delta)\frac{dW}{db}\right\}-\eta e^{-\eta a}\mu\{\Delta Y+(1-\Delta)W\}$$

$$=e^{-\eta a}\left\{\Delta\eta\mu Y+(1-\Delta)\frac{W\eta\phi\omega}{1-\eta\phi(1-b)}\right\}-\eta\mu e^{-\eta a}\{\Delta Y+(1-\Delta)W\}$$

$$=\eta e^{-\eta a}\mu\left[\left\{\Delta Y+\frac{(1-\Delta)W}{1-\eta\phi(1-b)}\right\}-\{\Delta Y+(1-\Delta)W\}\right]$$

$$=\frac{e^{-\eta a}\eta^2\phi(1-b)\mu(1-\Delta)W}{1-\eta\phi(1-b)}$$

Thus, the sign of $-\frac{da}{db}-\mu$ depends on the sign of $(1-\Delta)$ and $\{(1-\beta_{pm})-\delta\beta_{pm}\}$ or $(1+\alpha_{pm}+\alpha_{pm}\delta)$. It is easy to show that $(1+\alpha_{pm}+\alpha_{pm}\delta)>0$.

If this is positive, then $-\frac{da}{db}-\mu$ is also positive. Thus, it can be shown from the optimal condition of the Egalitarian approach that the $\frac{dU_{pm}}{db}$ and $\frac{dU_{co}}{db}$ both become positive for this case for $0\leq b<1$ and zero at b = 1. Thus, again U_{pm} and U_{co} are increasing in $b\in[0,1)$ and maximum at b =1 if $(1-\Delta)>0$. This is summarized in the following proposition

Proposition 28. *The solution of a cost plus contract (b=1) dominates the solution of any cost sharing contracts (*$0<b<1$*) and the solutions of a fixed price contract (b = 0) in a bargaining situation with a fairness concerned project manager and a contractor with the* *following conditions are satisfied.*

1. *The project manager is inequity averse with utility following the condition mentioned in the equation (6.221). If the project manager experiencing advantageous inequity, then* $\beta_{pm} < \frac{1}{1+\delta}$.
2. *The contractor has got both ex ante and ex post fairness concern. The weight assigned by the contractor on ex ante fairness* Δ *should satisfy* $0 < \Delta < 1$

6.5 Numerical Analysis

6.5.1 For fixed price contracts

As stated earlier in chapter 4, the following values are assumed

- $q_0 = 30.$
- $\psi = 0.05$
- $\mu_1 = 8$
- m = 1
- k=0.2 per unit resource per month
- $C_o = 15$

Using these assumed values, the first best value for resource consumption rate becomes $\lambda_0 = 2.74$ and the first best profit becomes $\pi_0 = 6.24$ (As shown in chapter 4).

In addition, the values δ, γ, α_{co}, β_{co}, α_{pm}, and β_{pm} are assigned as per the case concerned (Stated accordingly).

The contractor is fairness concerned only

The optimal value of λ is achieved (i.e. $\lambda = \lambda_0$) if the relation shown in the equation (6.17) is satisfied. Assuming $\gamma = 0.8$, the optimal β_{co} can be calculated from the equation (6.17) as

$\beta_{co} = 0.56$. On the contrary, it is assumed $\alpha_{co} = 0.8$ if the contractor experiences disadvantageous inequity. Using these values in the equation (6.55) along with the distribution specific parameters mentioned in chapter 4, the optimal value of f is derived as $f^* = 7.15$

Using these assumed and calculated values in the equations (6.4), (6.7), and (6.8), the fairness concerned contractor's utility and the profit, and the profit of the project manager can be determined numerically. The results are presented in figure 6.1 and table F.1 in appendix F.1.

It can be seen from the table F.1, that the contractor selects a zero value of resource consumption rate if the offered contract is below the optimal value of 7.15. At the optimal value of 7.15, the resource consumption rate (λ) attains a value of 2.74. Even after the offered contract value is higher than 7.15, it did not entice the contractor select a different resource consumption rate (λ) other than 2.74. This was because λ was found to be independent of f as defined in equation (6.15).

Now, rearranging the right-hand side of the equation (6.9), the utility function of the contractor becomes

$$U_{co} = (f - k\lambda^N \mu_1)(1 - \beta_{co}) + \beta_{co}\gamma\{q_0 - q_0\psi E(T^m) - f - C_o\}]$$

As λ values are not changing, the cost to the contractor would not change. The value of the $E(T^m)$ which is depended on λ, is also constant. Thus, $\frac{d\pi_{co}}{df} = 1$ and $\frac{dU_{pm}}{df} = -1$. Hence, any increase in f would increase the profit of the contractor linearly and the project manager's profit would decrease linearly by the same amount. It can also be shown that $\frac{dU_{co}}{dh} = (1 - \beta_{co} - \beta_{co}\gamma) = 0$ for $\beta_{co} = 0.56$. This would lead to unchanged U_{co} which can be found in the fig. 6.1.

Any offer from the project manager less than 7.15 leads the $\lambda = 0$ (Mathematically the solution is negative, but practically that is not possible). Hence, the project manager's profit and the contractor's utility become a large negative number -M. The figure 6.1 does not reveal the motivation to select $\lambda = \lambda_0$. As shown earlier in equation (6.1), the value of λ depends on the behavioural parameter β_{co}. Hence, the value of the β_{co} is changed and the corresponding λ values were noted. The results are presented in figure 6.2. From figure 6.2, it was observed that any $\beta_{co} < 0.56$ yields an optimal value of $\lambda < 2.74$ (where $\lambda_0 = 2.74$ is the first best solution). Any values $\beta_{co} > 0.56$, would induce a selection of a high valued λ. This would

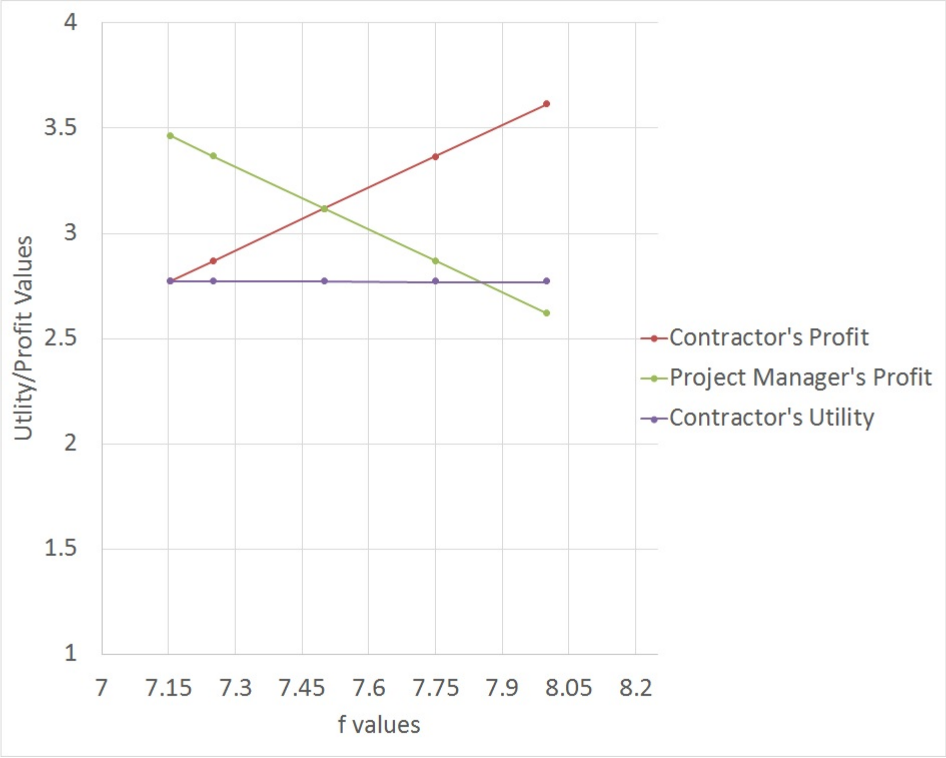

Figure 6.1: Utility or Profit vs f values: Fairness concerned contractor and profit maximizing project manager in the short term project

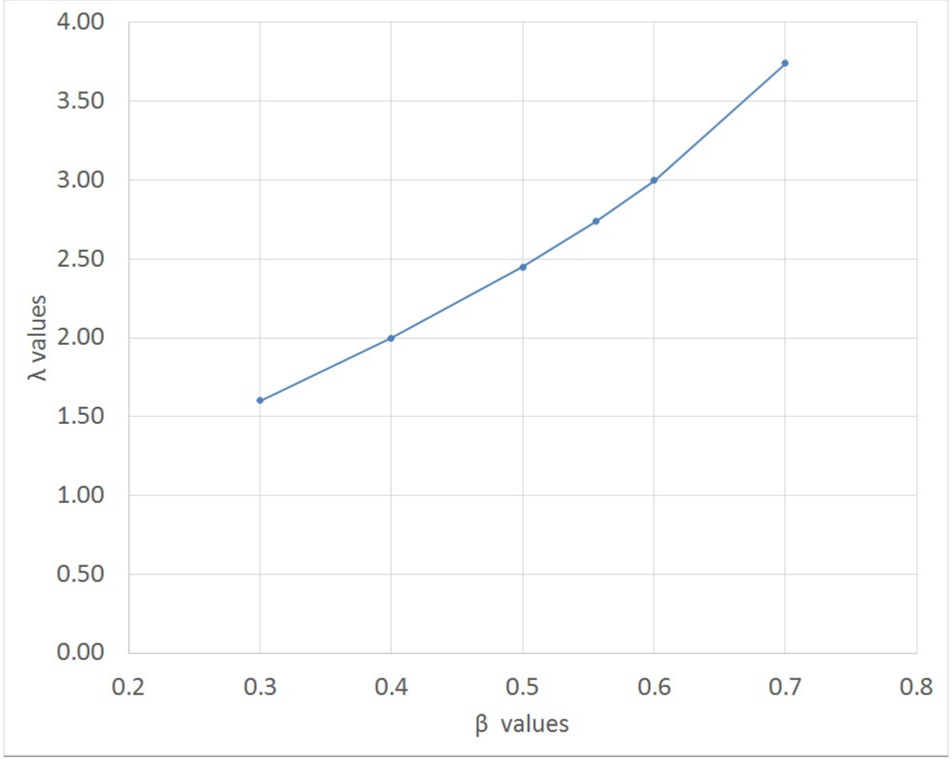

Figure 6.2: λ values vs β values for fixed price contracts

reduce the contractor's profit below his expected profit. Thus, the supply chain coordination for this case depends not only on offered fixed price, but also on the disutility of the contractor per unit. Thus, achievement of coordination and fair allocation is left with the chance of the behavioural perception of the contractor about the utility.

Keeping the $\beta_{co} = 0.56$, changing the λ values, the movement of π_{pm}, π_{co}, and U_{co} are noted. The results are presented in figure 6.3.

Figure 6.3: Utility or Profit vs λ values for Fixed price contracts short term projects: Contractor only fairness concerned

It is observed that the project manager's profit and the contractor's utility increased in λ in the beginning, until it reached the value of 2.74 (The first best solution). The contractor's profit was found to be monotonously decreasing in λ.

The contractor experienced an advantageous inequity with $\lambda < 2.74$. This was because his profit was higher than her expected fair profit when $\lambda < 2.74$. The utility function of the contractor becomes

$$U_{co} = (f - k\lambda^N \mu_1)(1 - \beta_{co}) + \beta_{co}\gamma\{q_0 - q_0\psi E(T^m) - f - C_o\} \tag{6.227}$$

Thus, $\frac{dU_{co}}{d\lambda} = (kN\mu_1\lambda^{N-1})(1 - \beta_{co}) - \beta_{co}\gamma q_0\psi\frac{dE(T^m)}{d\lambda}$. Since $\frac{dE(T^m)}{d\lambda} < 0$ and $\beta_{co} < 1$,

$\frac{dU_{co}}{d\lambda} > 0$. On the contrary, when $\lambda > 2.74$, the contractor's profit became less than her expected fair profit. As a result, the contractor experienced a disadvantageous inequity. His utility function this time is $U_{co} = (f - k\lambda^N\mu_1)(1 + \alpha_{co}) - \alpha_{co}\gamma\{q_0 - q_0\psi E(T^m)\}$. From this, it can be shown $\frac{dU_{co}}{d\lambda} = -kN\mu_1\lambda^{N-1}(1 + \alpha_{co}) + \alpha_{co}\gamma q_0\psi\frac{dE(T^m)}{d\lambda}$. As shown earlier that $\frac{dE(T^m)}{d\lambda} < 0$, the $\frac{dU_{co}}{d\lambda}$ is negative. Thus, the contractor's utility starts decreasing in λ when $\lambda > 2.74$. Since $\alpha_{co} > \beta_{co}$, the rate of reduction of U_{co} (when $\lambda > 2.74$) is higher than the rate of increase of U_{co} (when $\lambda < 2.74$).

Based on the findings in the last paragraph, it can be said that the optimal solution of the resource consumption rate (λ) is 2.74 when $\beta_{co} = 0.56$. This is same as the first best solution of resource consumption rate as derived earlier ($\lambda_0 = 2.74$). Hence, the contractor would select this value of resource consumption rate in the decentralized setting when his disutility per unit due to earning more than his expected fair profit ($\beta_{co} = 0.56$).

The Project Manager and the Contractor both Fairness Concerned

As described earlier, the project manager experiences disadvantageous inequity in the acrimonious supply chain. The contractor's advantageous disutility factor β_{co} is set equal to optimal required 0.56. δ is assumed as 1.4 and γ is assumed as 0.8. The optimal value of f is derived as 7.15 from the equation mentioned earlier. The disutility due to disadvantages inequity incurred by the project manager (α_{pm}) is assumed as 1.20.

Again similar to the case with a fairness concerned contractor and a profit maximizing project manager, the values of the profits and the utilities are noted for different values of f with $\beta_{co} = 0.56$ and $\alpha_{pm} = 1.20$. The results are presented in figure 6.4. Again from figure 6.4, any offer from the project manager below 7.15 would not encourage the contractor to select a non-zero positive resource consumption rate. This is due to the contractor earning less than in comparison to her expected fair pay-off profit. Anything above 7.15, the contractor's profit will increase, but the utility remained the same. This was due to the fact that $\frac{dU_{co}}{df} = (1 - \beta_{co} - \beta_{co}\gamma) = 0$ for $\beta_{co} = 0.56$. However, the project manager's utility and profit both started to decrease with any increase in f value after 7.15. From the equation (6.23), the project manager's utility can be derived as

$$U_{pm} = [q_0\{1 - \psi E(T^m)\} - f - C_o](1 + \alpha_{pm}) - \alpha_{pm}\delta(f - k\lambda^N\mu_1) \qquad (6.228)$$

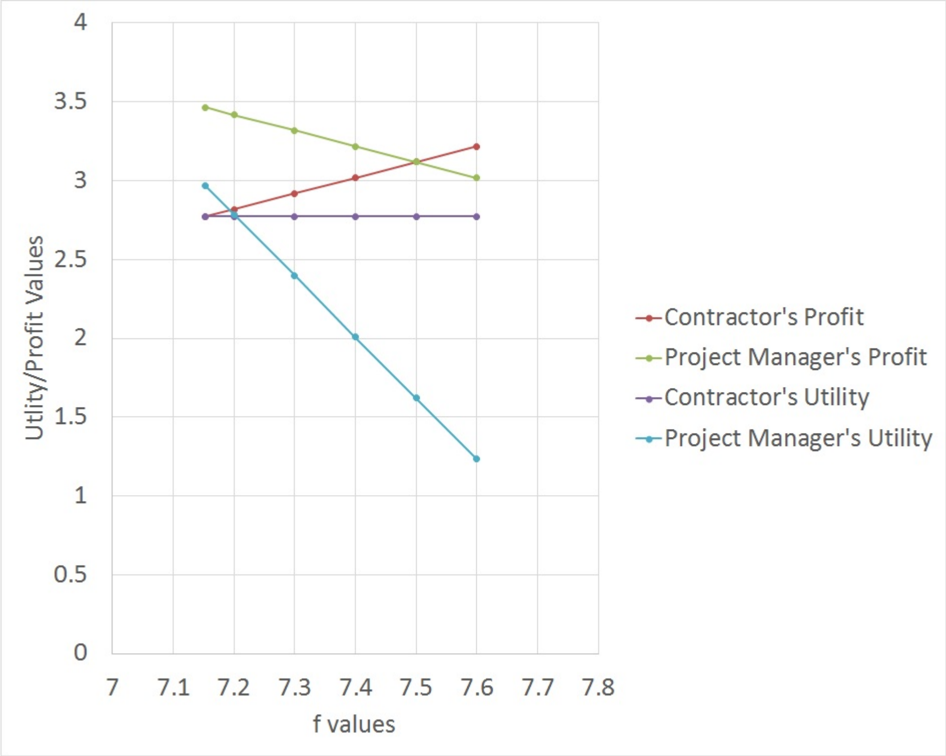

Figure 6.4: f values vs utilities or profits: Fairness concerned contractor and project manager in the acrimonious supply chain

The first part of the above equation i.e. $[q_0\{1 - \psi E(T^m)\} - f - C_o](1 + \alpha_{pm})]$ represents the project manager's profit. From the above equation, it can be seen that the project manager's utility is depended on f, and λ (As rest of the parameters in the above equation can be reasonably considered as constant). Similar to the case mentioned with a fairness concerned contractor and a profit maximizing project manager, the λ value was found to be independent of f and it was found to be controlled by contractor's disutility per unit β_{co}. Hence, λ and $E(T^m)$ are constant. As a result, the project manager's profit becomes a linearly decreasing function of f. It was also found that the project manager's utility is decreasing as a function $(1 + \alpha_{pm} + \alpha_{pm}\delta)f$. This means the project manager's utility is decreasing linearly with f at higher rate that the project manager's profit. It was also observed from the figure 6.4 that the project manager's utility has always been less than her profit. Even at f= 7.15, her utility which is maximum is less than her profit. However, the contractor's profit and utility are same at f = 7.15. Since both the contractor and the project manager are fairness concerned and utility maximizer, the contractual offer f = 7.15 is the optimal solution in this case. The solution can ensure a fair profit for the contractor. However, the project manager's profit can

Figure 6.5: Utility or Profit vs λ values for Fixed price contracts short term projects: Fairness Concerned Contractor and Project Manager in Acrimonious Supply Chain

never be ensured as the fair in this case.

For a constant β_{co} value, changing λ value is found to have the similar effect as shown in the case of the contractor is fairness concerned only. The results are presented in figure 6.5. The project manager's and the contractor's profits were found to be monotonously increasing and decreasing respectively with the increase in λ. The utility function of the project manager was found to be rapidly increasing in λ until it reached $\lambda = 2.85$. The project manager's profit was less than her expected fair profit and thus, she experienced a disadvantageous inequity with $\lambda < 2.85$. From the equation (6.228) above, $\frac{dU_{pm}}{d\lambda} = -q_0\psi(1+\alpha_{pm})\frac{dE(T^m)}{d\lambda} + \alpha_{pm}\delta k N \mu_1 \lambda^{N-1}$. Since $\frac{dE(T^m)}{d\lambda} < 0$, the term $\frac{dU_{pm}}{d\lambda}$. is positive. Thus, with any increase in λ, the project manager's utility increased as well. After $\lambda > 2.85$, the project manager's profit started to become more than her expected fair profit ($\delta\pi_{co}$). As a result, she started to experience an advantageous inequity and her utility becomes $U_{pm} = [q_0\{1-\psi E(T^m)\} - f - C_o](1-\beta_{pm})] + \beta_{pm}\delta(f - k\lambda^N\mu_1)$. From this, it can be shown that $\frac{dU_{pm}}{d\lambda} = -q_0\psi\frac{dE(T^m)}{d\lambda}(1-\beta_{pm}) - \beta_{pm}\delta k N \lambda^{N-1}\mu_1$. Using the value of $\frac{dE(T^m)}{d\lambda}$ can be derived from equation (6.11) with m=1, $\frac{dU_{pm}}{d\lambda} = q_0\psi\left(\frac{\mu_1 A}{\lambda^{A+1}}\right)(1-\beta_{pm}) - \beta_{pm}\delta k N \mu_1 \lambda^{N-1}$. Numerically, it can be shown that the absolute value of the first part of this equation is less than the second part. As a result, λ i.e. $\frac{dU_{pm}}{d\lambda} < 0$ with $\lambda > 2.85$ i.e. U_{pm} is decreasing in this case with the increase in λ as shown in the figure (6.5). However, due to existence of the first positive term, the rate of decrease in U_{pm} is not as great as the rate of increase of U_{pm} when $\lambda < 2.85$.

The contractor's utility was somewhat similar to what has been mentioned in the case with the contractor fairness concerned only, not the project manager. The contractor experienced an advantageous inequity with $\lambda < 2.74$. This was because his profit was higher than her expected fair profit when $\lambda < 2.74$. The utility function of the contractor follows the equation (6.227). Thus, $\frac{dU_{co}}{d\lambda} = (kN\mu_1\lambda^{N-1})(1-\beta_{co}) - \beta_{co}\gamma q_0\psi\frac{dE(T^m)}{d\lambda}$. Since $\frac{dE(T^m)}{d\lambda} < 0$ and $\beta_{co} < 1$, $\frac{dU_{co}}{d\lambda} > 0$. On the contrary, when $\lambda > 2.74$, the contractor's profit became less than her expected fair profit and he experienced a disadvantageous inequity. His utility function this time was $U_{co} = (f - k\lambda^N\mu_1)(1+\alpha_{co}) - \alpha_{co}\gamma\{q_0 - q_0\psi E(T^m)\}$. It can be shown $\frac{dU_{co}}{d\lambda} = -kN\mu_1\lambda^{N-1}(1+\alpha_{co}) + \alpha_{co}\gamma q_0\psi\frac{dE(T^m)}{d\lambda} < 0$ (As shown earlier that $\frac{dE(T^m)}{d\lambda} < 0$). Thus, the contractor's utility starts decreasing in λ when $\lambda > 2.74$. Since $\alpha_{co} > \beta_{co}$, the rate of reduction of U_{co} (when $\lambda > 2.74$) is higher than the rate of increase of U_{co} (when $\lambda < 2.74$).

At $\lambda = 2.74$, the contractor was ensured to earn a fair profit. Hence, he would be tempted to select this value of λ as the resource consumption rate. However, from figure (6.5), it is evident that the project manager is not ensured a fair profit at this value of resource consumption rate (λ). For her, the fair profit can be achieved at a higher resource consumption rate $\lambda = 2.85$. This difference is due to the higher expectation from the project manager of the fair profit (As $\delta = 1.4$). As a result, the total expectation of the fair profit of the project manager and the contractor is more than the supply chain could produce. Hence, a fair profit could not be guaranteed to both of them.

On the contrary to the acrimonious supply chain, somewhat different results were observed in the case of harmonious supply chain. Previously, the coefficient (δ) of project manager's expectation of fair profit expectation was 1.4. As a result, the total of the expected fair profits of the contractor and the project manager was higher than the maximum profit the supply chain could have produced. Now in the harmonious supply chain, the $\delta = 1.25$ i.e. the project manager's expectation of the fair profit has now reduced from a previous value. Similar to the case in the acrimonious supply chain, β_{co} is set as 0.56, γ is assumed as 0.8, and α_{pm} is assumed as 1.20. The value of β_{pm} is less than or equal to $\frac{1}{1+\delta}$ (As per proposition 15). Thus, it is assumed that $\beta_{pm} = 0.44$. With these assumed values, the values of the profits and utilities are derived for different values of f. These are presented in figure 6.6.

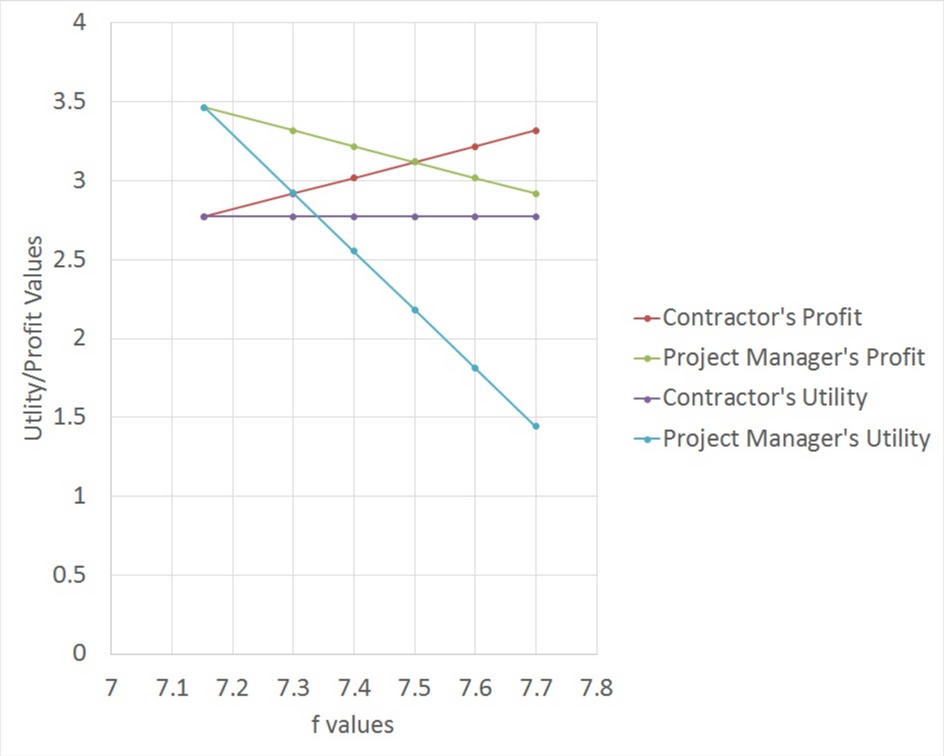

Figure 6.6: f values vs utilities or profits: Fairness concerned contractor and project manager in the harmonious supply chain with $\delta = 1.25$

The contractor's utility and the profit values followed the same pattern as it was found in the case of the acrimonious supply chain. The project manager's profit and the utility function also followed the similar trend as it was the case in the acrimonious channel. However, unlike the acrimonious supply chain (making a comparison between figures 6.4 and 6.6), the project manager's profit and the utility function coincided at f = 7.15. After that, the project manager started to earn less than her expected fair profit in a similar way she did in the case of the acrimonious supply chain. Thus, in the harmonious supply chain, f = 7.15 not only ensured the a fair profit for the contractor, but it also ensured a fair profit for the project manager as well. As explained in the case of the acrimonious supply chain in figure 6.5, the optimal resource consumption rate (λ) selected by the contractor was 2.74 at f =7.15. Thus, this value of resource consumption rate ($\lambda = 2.74$) also ensured the fair profit for the project manager unlike the case in the acrimonious supply chain where the required resource consumption rate, $\lambda = 2.85$ ensured a fair profit for the project manager. Hence, the supply chain can be coordinated as well as it ensured a fair profit for both the members of the supply chain.

If the project manager's expectation of the fair profit further decreases $\delta = 1.1$, then, the value of f at which she earns her fair profit further increases. This is presented in figure 6.7. f=7.35 was found to be the value when the project manager earned a fair profit this time.

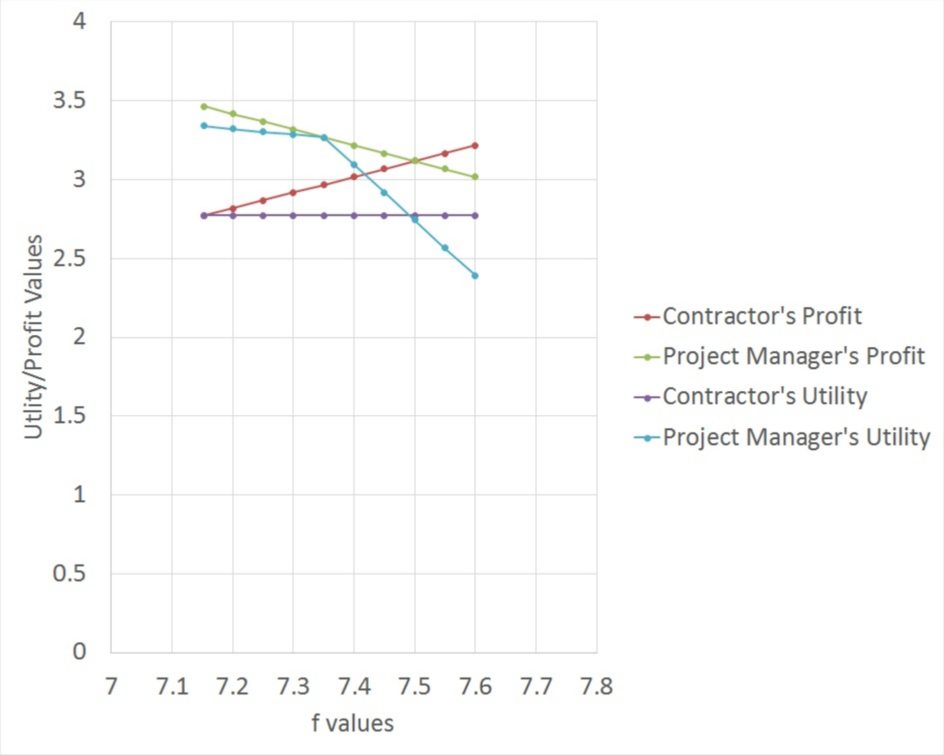

Figure 6.7: f values vs utilities or profits: Fairness concerned contractor and project manager in the harmonious supply chain with $\delta = 1.1$ and $\beta_{pm} < \frac{1}{1+\delta}$

From the figure 6.7, it was found that the project manager earned a profit more than her expected fair profit for any values $7.15 < f < 7.35$. The project manager experienced an advantageous inequity in this range. However, it was found that the project manager's utility was decreasing in f. This was because his $\frac{dU_{pm}}{df} = -1 + \beta_{pm} + \beta_{pm}\delta = -0.076 < 0$ for $\beta_{pm} = 0.44 < \frac{1}{1+\delta}$. As a result, the project manager would prefer to offer f=7.15 as it is maximizing his utility. This would ensure a fair profit for the contractor as per his perception, but the project would experience some advantageous inequity.

On the contrary, if the project manager has a higher disutility per unit for earning more than her expected fair profit ($\beta_{pm} = 0.53 > \frac{1}{1+\delta}$), then $\frac{dU_{pm}}{df} = 0.113$. As a result for $7.15 < f < 7.35$, the project manager's utility would be increasing in f and she would be

Figure 6.8: f values profits: Fairness concerned contractor and project manager in the harmonious supply chain with $\delta = 1.1$ and $\beta_{pm} > \frac{1}{1+\delta}$

tempted to select f=7.35 where her profit becomes exactly equal to her expected fair profit and her utility is also maximum. The contractor's utility is maximized at $\lambda = 2.74$. Hence, he would still select this as his resource consumption rate. The results are presented in figure 6.8.

Summarising, the numerical results, some of the important findings from model section for the fixed price contracts is re- established. It is evident that the fixed price contract is capable of offering coordination if the contractor is fairness concerned (with profit maximizing and fairness concerned project manager) and earning at least equal to his expected fair profit i.e. he is experiencing advantageous inequity. However, this was found to be heavily depended on the disutility per unit (β_{co}) of the contractor for earning more than his expected fair profit. This parameter is a behavioural decision making parameter. Hence, the achievement of coordination is somewhat left on chance.

6.5.2 For time based contracts

Unlike the fixed price contracts, the time based contract used in this research has two components: the fixed component (g) and the variable penalty component per unit time (h) (As discussed earlier in 4). The assumptions made in the beginning of section 6.5.1 remain the same.

For the contractor is fairness concerned only

It is further assumed that $\alpha_{co} = 0.8$ and $\beta_{co} = 0.40$. When the contractor experiences disadvantageous inequity, then optimal h and g values are derived using the observations from proposition 21. The first best profit (π_0) and first best resource consumption rate (λ_0) have been derived as 6.24 and 2.74 units respectively (This is same as derived in chapter 4).

The optimal g values were found to be 10.40 (if the contractor experienced a disadvantageous inequity) and 11.54 (if the contractor experienced an advantageous inequity) and the optimal value of h was found to be 1.5 units. Keeping the g value constant at 10.40, any change in h changed the profit values of the contractor (π_{co}) and the project manager (π_{pm}), the utility of the contractor (U_{co}) and the resource consumption rate of the contractor (λ) values in the decentralized setting. This is presented in figure 6.9 and in table F.3 in appendix.

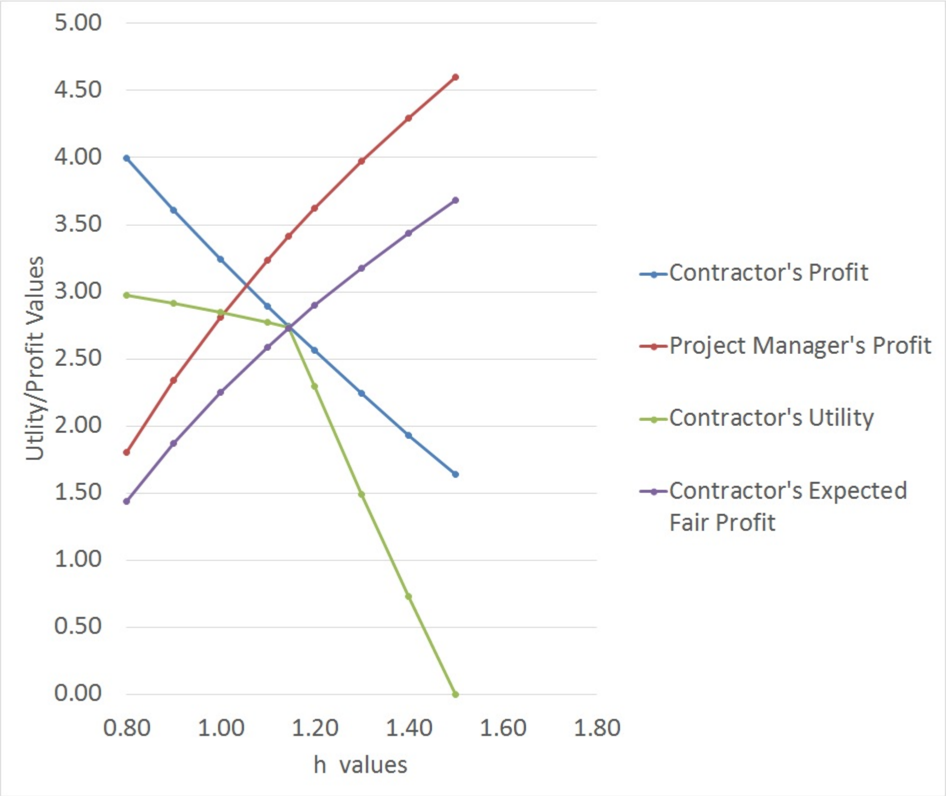

Figure 6.9: Profit/Utility values vs. h values: The contractor fairness concern only

The movement of π_{pm} and π_{co} are somewhat similar to what has been observed in chapter 4. This was due to the fact that λ is controlled by h as it can be found earlier $h = \frac{kN\lambda^{N+A}}{A}$. The interesting fact is the movement of U_{co} due to change in h. The project manager experienced an advantageous inequity for h¡1.20. It was found to have some marginal increase in U_{co} for any value below h =1.20. This can be explained from the utility function of the contractor which is,

$$U_{co} = (g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)(1-\beta_{co}) + \beta_{co}\gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \quad (6.229)$$

as derived from the equation (6.80)

Replacing the value of $E(T^m)$ from the equation (4.17) for m=1, it can be shown $U_{co} = (g - h\frac{\mu_1}{\lambda^A} - kN\lambda^N\mu_1)(1-\beta_{co}) + \beta_{co}\gamma\{q_0 - q_0\psi\frac{\mu_1}{\lambda^A} - g + h\frac{\mu_1}{\lambda^A} - C_o\}$. From this, the first order derivative of U_{co} is derived as $\frac{dU_{co}}{dh} = \{-\frac{\mu_1}{\lambda^A} + (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) - kN\mu_1\lambda^{N-1}\frac{d\lambda}{dh}\}(1-\beta_{co}) + \beta_{co}\gamma\{(\frac{q_0\psi A\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) + \frac{\mu_1}{\lambda^A} - (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh})\}$. Using the values of $\frac{d\lambda}{dh}$ and h, numerically it can be shown that the first part of the equation is negative with, but the second part is positive with marginally lower absolute value due to lower β_{co}. As a result, $\frac{dU_{co}}{dh} < 0$. Hence, the contractor's utility was found to be decreasing marginally. Intuitively, it can be said that any increase in h led to the increase in resource consumption rate, λ and the cost to the contractor. However, this led to the project manager's profit increase due to lesser time to complete the project. Thus, the difference between contractor's profit and his expected fair profit is decreasing. This marginal decrease in disutlity was added to the decrease in the contractor's profit decrease. As a result, his utility declined marginally. At h=1.20, the contractor's profit was exactly same as his expected fair profit.

However, the value of the utility of the contractor dropped sharply for any increase in h beyond 1.20. This can be accounted due the fact that the contractor was experiencing the disadvantageous inequity for any increase in h beyond this point. The contractor's utility can be shown as

$$U_{co} = (g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1)(1+\alpha_{co}) - \alpha_{co}\gamma\{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\} \quad (6.230)$$

as derived from the equation (6.80)

From the above equation $\frac{dU_{co}}{dh} = \{-\frac{\mu_1}{\lambda^A} + (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) - kN\mu_1\lambda^{N-1}\frac{d\lambda}{dh}\}(1+\alpha_{co}) - \alpha_{co}\gamma\{(\frac{q_0\psi A\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) + \frac{\mu_1}{\lambda^A} - (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}))\}$. Using the values of h and $\frac{d\lambda}{dh}$, it can be shown numerically that the first part of the equation is negative. However, this time the second part is also negative. As a result, it can be shown that $\frac{dU_{co}}{dh} < 0$ for $h > 1.20$. The larger disutility per unit due to earning less than his fair profit is more than the disutlity for earning more than his fair profit i.e ($\alpha_{co} > \beta_{co}$). This is causing a sharp decline. Intuitively, the contractor's profit was further decreasing with any increase in h even after $h > 1.20$. In addition, the difference between his expected fair profit and his own profit kept on increasing. As a result, his utility started to decline more sharply. At h=1.50, the contractor selects a resource consumption rate $\lambda = 2.74$ and his utility becomes zero. It was assumed earlier that the contractor would accept the contract as long as his utility ensures the utility value he could have got by accepting the contractual offer outside this contract i.e. the utility (U_{out}) for the profit π_{out}. For the simplification of calculation, it is assumed $U_{out} = 0$. Thus, the maximum value of h that the contractor would accept is h = 1.50.

Keeping h constant at h=1.5, the g values were changed to observe the movement of U_{co}, π_{co}, and π_{pm}. The results are presented in table F.4 and figure 6.10.

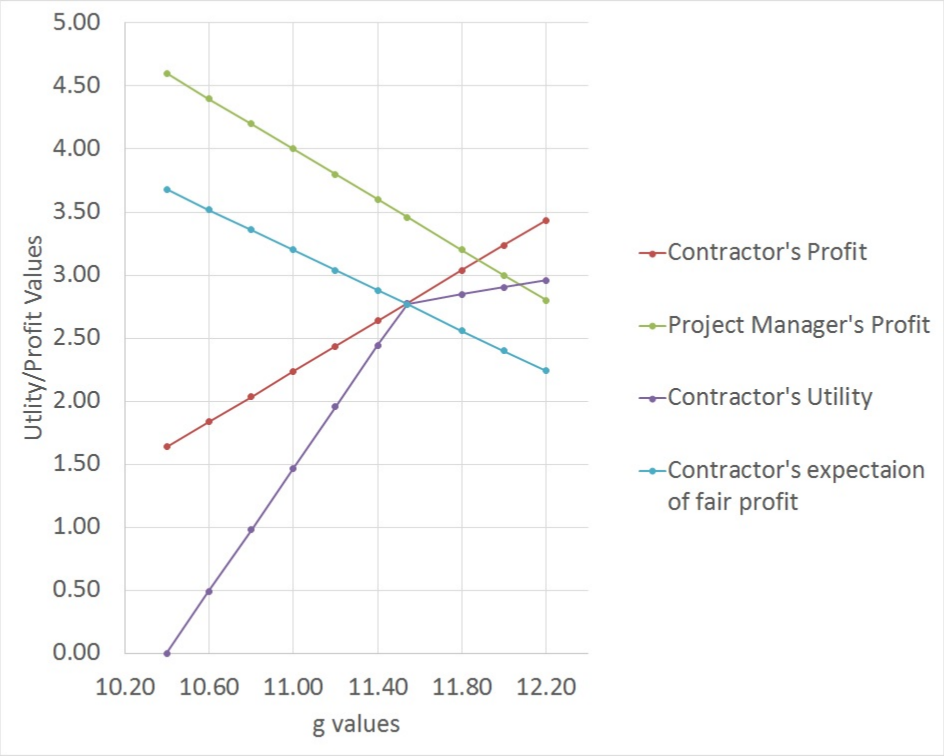

Figure 6.10: Profit/Utility values vs. g values: The contractor fairness concern only

It is interesting to see that the contractor's profit was found to be increasing in g. On the contrary, the project manager's profit was found to be decreasing in g. The optimal value for g from the equation (6.111) was derived as 11.54. At g=11.54 and h=1.5, the solutions are coordinating (as $\lambda = \lambda_0 = 2.74$)) the supply chain as well as allocating the profit fairly (As the contractor profit is equal to her expected fair profit). For any value of g less than 11.54, the contractor's profit is less than her expected fair profit. As a result, he experienced a disadvantageous inequity. Thus, $\frac{dU_{co}}{dg} = (1 + \alpha_{co} + \alpha_{co}\gamma) > 0$. This was because g did not have any impact on the selection λ and h was constant. Thus, any increase in g led to increase in the contractor's utility linearly. Similarly, it can be easily shown that the project manager's utility function is a linearly decreasing function of g. At g=11.54, the contractor's profit and her expected fair profit were the same. With $g > 11.54$, the contractor started to earn more than her expected fair profit. As a result, the contractor earned an advantageous inequity for $g > 11.54$. Thus, it can be shown that $\frac{dU_{co}}{dg} = (1 - \beta_{co} - \beta_{co}\gamma)$. If the value of $(1 - \beta_{co} - \beta_{co}\gamma) < 0$ i.e. $\beta_{co} > \frac{1}{1+\gamma}$, then the contractor's utility would be an increasing function of g. As argued in proposition 22, the value of β_{co} is assigned as $\beta_{co} < \frac{1}{1+\gamma}$. Thus, the contractor's utility would be increasing in g, with $g \geq 11.40$. As $(1 + \alpha_{co} + \alpha_{co}\gamma) > (1 - \beta_{co} - \beta_{co}\gamma)$, the increase in contractor's utility with $g > 11.54$ was quite lower than in comparison to $g < 11.54$.

Based in the observations from the figures 6.9 and 6.10, it can said that for g=11.54 and h=1.5 ensures the fair profit to the contractor as well as the project manager is capable of coordinating the supply chain. However, the question is whether the project manager would offer that higher value of g=11.54? It depends on the U_{out} values. For ease of calculation, this research assumed that $U_{out} = 0$. Hence, as long as the offered contract ensures the contractor a non-negative utility, he accepts it. This was achieved at g=10.40. Hence, the project manager won't have the incentive to offer $g > 10.40$ as it would reduce his profit. This further supports the findings from the corollary 1 that existence of solutions in proposition 22 is unlikely. However, if the contractor expects a positive and higher utility outside this contract, then he might refuse to accept any g which would fail to ensure this minimum utility. In those cases, the project manager would require to offer $g > 10.40$. Hence, with the increase in U_{out}, the offer g would also required to be increased and it would continue up to a maximum of g=11.54. If the contractor's minimum utility expectation outside this contract becomes 2.77 (based on the results from the table F.4), then the project manage would have no option but to offer g=11.54.

Figure 6.11: Profit/Utility values vs. h values: The contractor and the project manager both fairness concern

For the both the contractor and the project manager are fairness concerned only

Acrimonious supply chain

It is assumed that $\alpha_{co} = 9$, $\alpha_{pm} = 1.5$, $\gamma = 0.8$ and $\delta = 1.4$. Since $\gamma\delta > 1$, this is called as an acrimonious supply chain. α_{co} is assigned more than $\frac{1}{1-\delta\gamma} = 8.33$ and α_{pm} is assigned as $\alpha_{pm} < \frac{1}{\delta\gamma-1}$.

When the contractor experiences a disadvantageous inequity, then optimal h and g values are derived using the observations from proposition 23. The first best profit (π_0) and first best resource consumption rate have been derived as before 6.24 and 2.74 units respectively. The optimal values of g were found to be 11.37 (if the contractor experiences disadvantageous inequity) and 11.54 (if the contractor experiences advantageous inequity) and the optimal h was found to be 1.5 same as before.

Keeping g constant at the optimal value of 11.37, changing h was found to provide similar results as shown in the case with the fairness concerned contractor and the profit maximizing project manager. The results are presented in figure 6.11 The profits of the contractor and the project manager were found to be decreasing and increasing respectively with any increase in h. The contractor's profit was more than his expected fair profit for $h < 1.45$. As a result, he experienced an advantageous inequity. His utility was declining at a slower rate with the increase in h due to the same reason mentioned in the case with a fairness concerned contractor and a profit maximizing project manager. At h=1.45, the contractor's profit (π_{co}) is exactly the same as his expected fair profit ($\gamma\pi_{pm}$). For $h > 1.45$, the contractor's utility declined very sharply. The contractor's profit is less than his expected fair profit. As a result, he started to experience disadvantageous inequity. His disutility per unit for earning less than his expected profit is much higher ($\alpha_{co} = 9$) in comparison to the case with fairness concerned contractor and profit maximizing project manager ($\alpha_{co} = 0.8$). As a result, the contractor's utility declined at a much faster rate and at h=1.50, it became zero. The project manager's profit has monotonously increased with any increase in h. However, it has been less than his expected fair profit for $h < 1.50$. Thus, the project manager's utility function could be rearranged from the equation (6.112) as below

$$U_{pm} = \{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\}(1 + \alpha_{pm}) - \alpha_{pm}\delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \tag{6.231}$$

Replacing the value of $E(T^m)$ from the equation (4.17) for m=1, it can be shown $U_{pm} = \{q_0 - q_0\psi\frac{\mu_1}{\lambda^A} - g + h\frac{\mu_1}{\lambda^A} - C_o\}(1 + \alpha_{pm}) - \alpha_{pm}\delta(g - h\frac{\mu_1}{\lambda^A} - kN\lambda^N\mu_1)$. From this, the first order derivative of U_{pm} is derived as $\frac{dU_{pm}}{dh} = \{(\frac{q_0\psi A\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) + \frac{\mu_1}{\lambda^A} - (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh})\}(1 + \alpha_{pm}) - \alpha_{pm}\delta\{-\frac{\mu_1}{\lambda^A} + (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) - kN\mu_1\lambda^{N-1}\frac{d\lambda}{dh}\}$. Using the values of $\frac{d\lambda}{dh}$ and h, numerically it can be shown that the first part of the equation is positive and the second part becomes positive due to the existence of the negative sign before $\alpha_{pm}\delta$. As a result, $\frac{dU_{pm}}{dh} > 0$. It is interesting to note that $\frac{d\pi_{pm}}{dh} = \{(\frac{q_0\psi A\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) + \frac{\mu_1}{\lambda^A} - (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh})\}$. Since the second part of the term $\frac{dU_{pm}}{dh}$ becomes positive, it can be easily shown that $\frac{dU_{pm}}{dh} > \frac{d\pi_{pm}}{dh}$. This is why the project manager's utility was found to be more rapidly increasing than in comparison to her profit with any increase in h with $h < 1.50$ (Please see figure 6.11).

At h=1.50, the project manager's earned profit (π_{pm}) is equal to his expected fair profit ($\delta\pi_{co}$). Hence, the project manager needs to offer a higher h value (h=1.50) to achieve a fair profit than the contractor can achieve one (with h=1.45). This was because her expectation of the the fair profit was much higher than the contractor's expectation ($\delta > \gamma$).

Keeping h constant at the optimal value of h=1.5, changing the g values offered some insightful findings. The results are presented in table F.5 and figure 6.12.

Figure 6.12: Profit/Utility values vs. g values: The contractor and the project manager both fairness concern a

At g=11.37, the contractor's utility was zero. Any value g offered above 11.37 increased the contractor's utility and profit. The contractor's profit increase was similar to what had been found in chapter 4. The contractor's profit was less than his expected fair profit for $11.37 < g < 11.54$. Hence, he experienced a disadvantageous inequity in this case. The contractor's utility follows the equation 6.230. From, this it can be said $\frac{dU_{co}}{dg} = (1 + \alpha_{co} + \alpha_{co}\gamma)$, where as rate of change of his profit is $\frac{d\pi_{co}}{dg} = 1$. It is assumed earlier that $\alpha_{co} = 9$. As a result, the contractor's utility increased at a rapid rate with any increase in g. At g=11.54, the contractor's expected profit was same as his fair profit. As a result, g=11.54 and h =1.5, the solutions are coordinating the supply chain as well as allocating the profits fairly as per contractor's perception. For any offer after $g > 11.54$, the contractor's profit became more than his expected fair profit. As a result, he started to experience the advantageous inequity. His utility can be calculated from the equation (6.229); so $\frac{dU_{co}}{dg} = (1 - \beta_{co} - \beta_{co}\gamma)$. As assumed earlier, $\beta_{co} < \frac{1}{1+\gamma}$, the value of $\frac{dUco}{dg}$ is positive. It can be easily shown that $(1 + \alpha + \alpha_{co}\gamma) \gg (1 - \beta_{co} - \beta_{co}\gamma)$. Thus, the rate of increase in the contractor's utility was very small for $g > 11.54$.

On the contrary to the contractor's profit and utility, the project manager's utility and profit both the decreased with any increase in g for $g > 11.37$. In fact the project manager's profit was found to be less than her expected fair profit for $g > 11.37$. Hence, the project manager's utility would follow the equation (6.231). From this, it can be found that $\frac{dU_{pm}}{dg} = -(1 + \alpha + \alpha_{pm}\delta) < 0$ where as $\frac{d\pi_{pm}}{dg} = -(1 + \alpha_{pm})$. This is why the project manager's utility is decreasing at a higher rate than her profit with any increase in g. This has been true for a higher value of $\alpha_{pm} < \frac{1+\alpha_{co}}{\alpha_{co}(\delta\gamma-1)-1} = 125$ and $\alpha_{pm} > \frac{1}{\delta\gamma-1} = 8.33$. For $\alpha_{pm} = 10$, the results are more or less similar. However, the reduction of the project manager's utility with increase in g for $g > 11.37$ and increase in project manager's utility with any increase in h $h < 1.50$, both were quite rapid due to higher disutility (α_{pm}).

Based on the discussion above from the figures 6.11 and 6.12, it can be concluded that the project manager would be tempted to offer the contractor a contract g=11.37 and h=1.50. This is because it was assumed earlier that the contractor would accept any offer that ensures $U_{out} = 0$. At g=11.37 and h=1.5, the contractor's utility is zero, the project manager's utility was maximum, and the project manager's profit was ensured a fair profit. Hence, this supports the findings from corollary 2.

Now if the contractor's opportunity to earn a better utility from any other contracts improves, then the project manager needs to offer a better contractual offer (either by reducing h or increasing g). The project manager would not be interested to change h as it would demotivate the contractor to select the first best resource consumption rate ($\lambda = \lambda_0$). Thus, she would be increasing the g to meet the contractor's minimum requirement of $U_{out} > 0$. Once g=11.54, the contractor is ensured a fair profit. However, any of these g values would fail to ensure the project manager a fair profit. The offered contract could either ensure a fair profit for the contractor or the project manager depending on what is the value of U_{out}.

Harmonious supply chain

If the project manager's or the contractor's expectation about the fairness profit decreases such that $\delta\gamma < 1$, then the supply chain becomes harmonious supply chain with a chance to allocate fair profit. It is assumed $\gamma = 0.8$, but $\delta = 1.25$ unlike the acrimonious supply chain with $\delta = 1.4$. It is also assumed that $\alpha_{pm} = 9$. However, the project manager experiences an advantageous inequity this time. In the first case, the β_{pm} is assumed to be not too high and

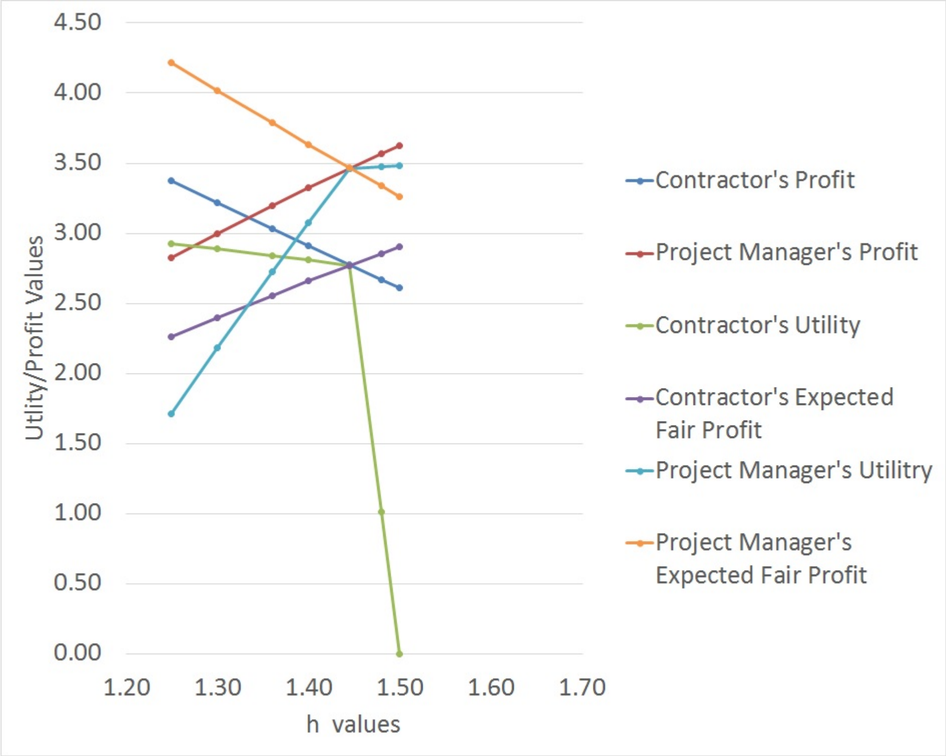

Figure 6.13: Profit/Utility values vs. h values: The contractor and the project manager both fairness concern in harmonious supply chain

$\beta_{pm} < \frac{1}{1+\delta} = 0.45$. Hence, it is assigned as 0.4.

Keeping g constant at 11.37, and changing h was found to have similar results as found previously. It was again found to be controlling λ, π_{pm} and π_{co}. The results are presented in fig 6.13. Similar to the case of acrimonious supply chain, the project manager and the contractor's profit increased and decreased respectively with any increase in h. The project manager's profit was less than her expected fair profit for $h < 1.445$ and she experienced a disadvantageous inequity. Thus, her utility function would again follow the equation (6.231) mentioned in the case of acrimonious supply chain. From this equation it can be again shown $\frac{dU_{pm}}{dh} > \frac{d\pi_{pm}}{dh} > 0$. Due to this, similar results were found as it was found in the case of acrimonious supply chain.

The values of h=1.445 and g= 11.37 ensured the fair profit to both the contractor and the project manager. However, this would fail to ensure the first best resource consumption rate to be selected by the contractor as $\lambda = \lambda_0$ at h=1.50 (As shown in chapter 4). For any $h < 1.50$, $\lambda < \lambda_0$. Hence, g=11.37 and h=1.445 ensured fair profit for both the members, but it failed to coordinate the supply chain.

For $h > 1.445$, the project manager started to experience an advantageous inequity. Hence, her utility function becomes

$$U_{pm} = \{q_0 - q_0\psi E(T^m) - g + h\frac{\mu_1}{\lambda^A} - C_o\}(1 - \beta_{pm}) + \beta_{pm}\delta(g - h\frac{\mu_1}{\lambda^A} - k\lambda^N\mu_1) \tag{6.232}$$

the first order derivative of U_{pm} is, $\frac{dU_{pm}}{dh} = \{(\frac{q_0\psi A\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) + \frac{\mu_1}{\lambda^A} - (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh})\}(1 - \beta_{pm}) + \beta_{pm}\delta\{-\frac{\mu_1}{\lambda^A} + (\frac{hA\mu_1}{\lambda^{A+1}})(\frac{d\lambda}{dh}) - kN\mu_1\lambda^{N-1}\frac{d\lambda}{dh}\}$. Using the values of $\frac{d\lambda}{dh}$ and h, numerically it can be shown that the first part of the equation is positive and the second part is negative and overall the right hand side becomes marginally positive As a result, $\frac{dU_{pm}}{dh} > 0$. This is why the project manager's utility was increasing very marginally for $h > 1.445$.

The utility of the project manager is higher with g=11.37 and h =1.5 than at g=11.37 and h=1.445. This motivates the utility maximizer project manager to change the offered the contract. The project manager won't offer any h more than 1.50 as this would fail to ensure the contractor a non-negative utility. Now keeping h constant at h=1.5, g values are changed and the corresponding utility and profit values are observed. The results are presented in table F.6 and figure 6.14.

For $11.37 < g < 11.54$, the contractor's profit was less than his expected fair profit. As a result, he experienced a disadvantageous inequity. Hence, his utility could be derived from the equation (6.230). Hence, $\frac{dU_{co}}{dg} = (1 + \alpha_{co} + \alpha_{co}\gamma)$ which is much higher than $\frac{d\pi_{co}}{dg}$ due to high α_{co}. As a result, the contractor's utility increased at a a higher rate than his profit with increase in g ($11.37 < g < 11.54$). At g=11.54, the contractor's profit became equal with his expected fair profit. From figure 6.14, it can also be shown that at g=11.54, the project manager's profit equals with his expected fair profit. Similar to the case in a acrimonious supply chain, the project manager's profit and utility both were found to be decreasing in any increase in g. However, unlike the acrimonious supply chain, the project manager was experiencing an advantageous inequity for $11.37 < g < 11.54$ this time. From the equation (6.232), it can be found $\frac{dU_{pm}}{dg} = -1 + \beta_{pm} + \beta_{pm}\delta$. Since the value of β_{pm} is assigned a low value of 0.4 ($\beta_{pm} < \frac{1}{1+\delta}$), the value of $\frac{dU_{pm}}{dg}$ is negative. Hence, her profit would be decreasing in any increase g when h is fixed at 1.50. Since, the project manager would try to maximize her utility, she would have limited motivation to increase g to 11.54 as it would reduce her utility.

Figure 6.14: Profit/Utility values vs. g values: The contractor and the project manager both fairness concern harmonious supply chain a

Now if the contractor's disutility factor for the advantageous inequity is not too small, then results were found as somewhat different for changing g while keeping h=1.5. It is assumed that $\beta_{pm} = 0.7$ i.e. $\frac{1}{1+\delta} < \beta_{pm} < \frac{1}{1-\delta\gamma}$, then the solution achieved not only can coordinate the supply chain, but also achieves the fair allocation. The contractor's profit is exactly same as the his expected fair profit. This was because $\frac{dU_{pm}}{dg} = -1 + \beta_{pm} + \beta_{pm}\delta > 0$. As a result, the project manager's utility was increasing in g with $11.37 < g < 11.54$. This motivated the project manager to increase g from 11.37 to 11.54. At g=11.54 and h=1.5, the project manger and the contractor both had their profit equal to their expected fair profit. At the same time the supply chain was coordinated as well. The results are presented in table F.7 and figure 6.15.

For the project manager is fairness concerned only

In lemma 26 and in proposition 24, it was shown if the project manager experiences disadvantageous inequity, then her utility and profit both decreases in g. Hence she would offer the minimum possible value to ensure the contractor's participation. The results are the same

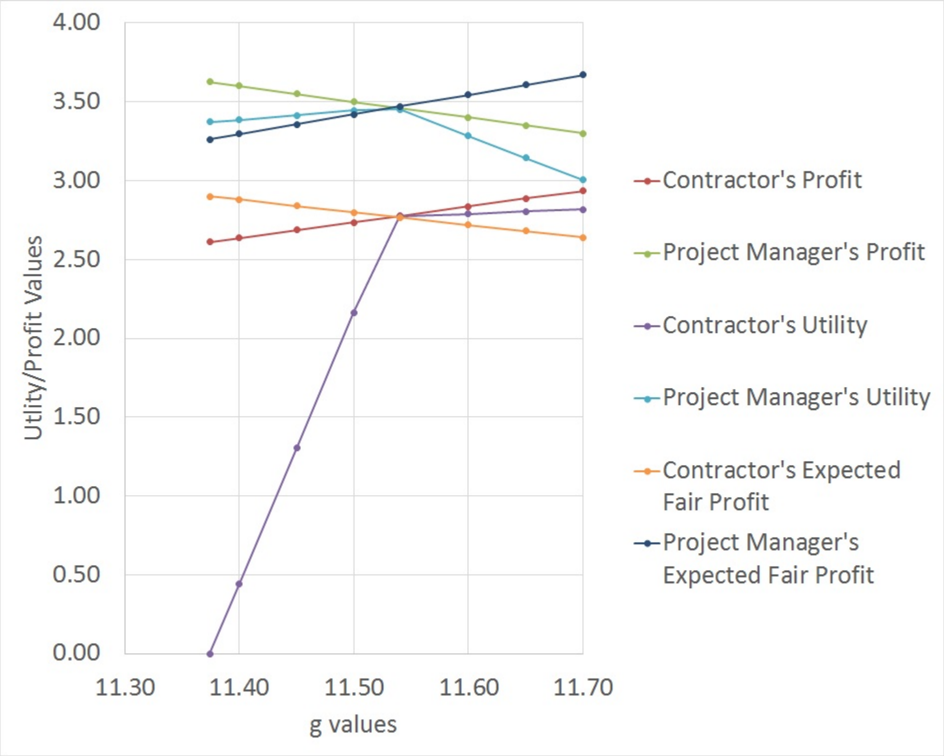

Figure 6.15: Profit/Utility values vs. g values: The contractor and the project manager both fairness concern harmonious supply chain b

as what have been presented in chapter 4.

Now if the project manager experiences an advantageous inequity, then two situations are considered. In the first case, the project manager's disutility is not too large and $\beta_{pm} < \frac{1}{1+\delta} = 0.44$ with $\delta = 1.25$. In the second case, the beta is set higher than this value of 0.44.

Using the models from the proposition 24, the optimal value of h is found to be 1.5. The optimal g value was found to be 10.56 when the $\beta_{pm} = 0.4$. The optimal g value becomes 11.26 with $\beta_{pm} = 0.5$.

Keeping g constant at 10.56 and changing h values, the changes in the contractor's and the project manager's profits, and the project manager's utility function were observed. The results for the case with $\beta_{pm} = 0.4$ are presented in figure 6.16 and in table F.8. The contractor's profit was found to be decreasing in any increase in h. The project manager's profit was found to be less than her expected fair profit for $h < 1.195$ and she experienced a disadvantageous inequity. Her utility function would follow the equation (6.231). At h=1.195, the project manger profit becomes exactly equal to her expected fair profit. For $h > 1.195$, the project manager's profit was found to be more than her expected fair profit. For, $h > 1.195$,

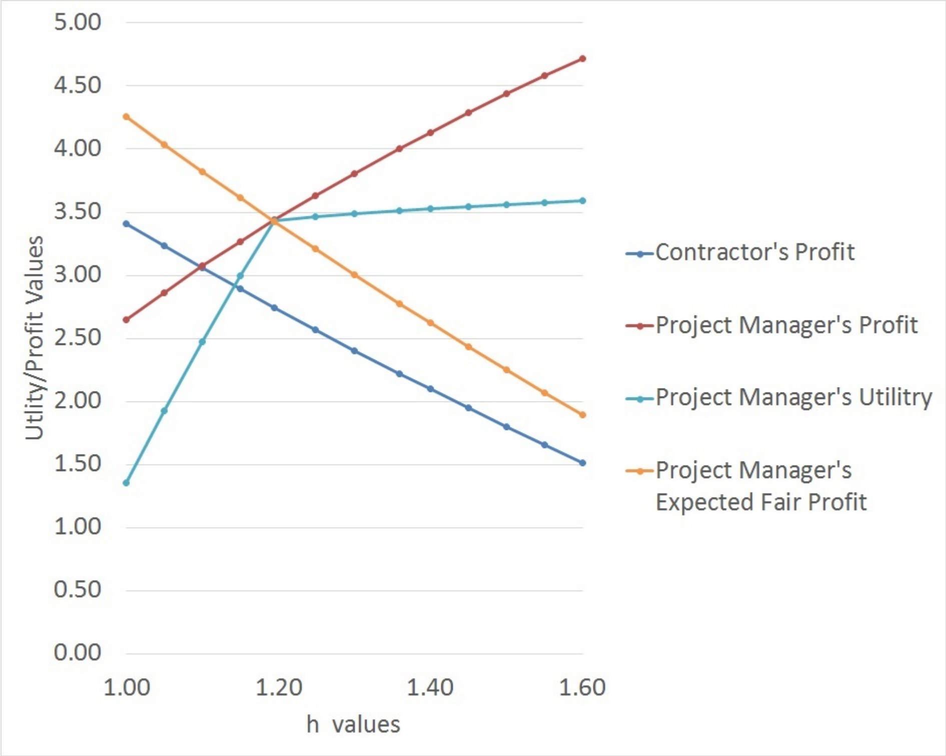

Figure 6.16: Profit/Utility values vs. h values: The project manager fairness concern only a

the project manager's utility follows the equation (6.232) . From the equations (6.231) and (6.232), it was shown earlier that $\frac{dU_{pm}}{dh} > 0$ for both the cases of disadvantageous inequity and advantageous inequity when $\beta_{pm} = 0.4$. It was also shown earlier that $\frac{dU_{pm}}{dh}$ was higher when the project manager experienced a disadvantageous inequity than when she experienced an advantageous inequity. That's why her utility was increasing at a faster rate for $h < 1.,195$ than for $h > 1.195$. Since, $\frac{dU_{pm}}{dh} > 0$, the utility maximizing project manager would be interested to increase the value from h=1.195 even though h=1.195 ensures a fair profit. Now the contractor's profit decreases with increase in h. He would accept the offer as long as it ensures a minimum profit of $\pi_{out} = 1.80$. At h=1.50, his profit becomes 1.80. Thus, the project manager would increase the profit upto h=1.50.

Keeping h constant at the optimal value of h=1.5, changing g values have offered some interesting insights. The results are presented in table F.9 and in figure 6.17. The project manager's profit was higher than her expected fair profit at g=10.56 and h=1.50 and she experienced an advantageous inequity. The project manager's utility and profit both were found to be decreasing in g for the $\beta_{pm} = 0.4 < \frac{1}{1+\delta}$. This was because $\frac{dU_{pm}}{dg} = -1 + \beta_{pm} + \beta_{pm}\delta = -0.1 < 0$ (As it can be derived from the equation 6.232). At g=11.54, the project

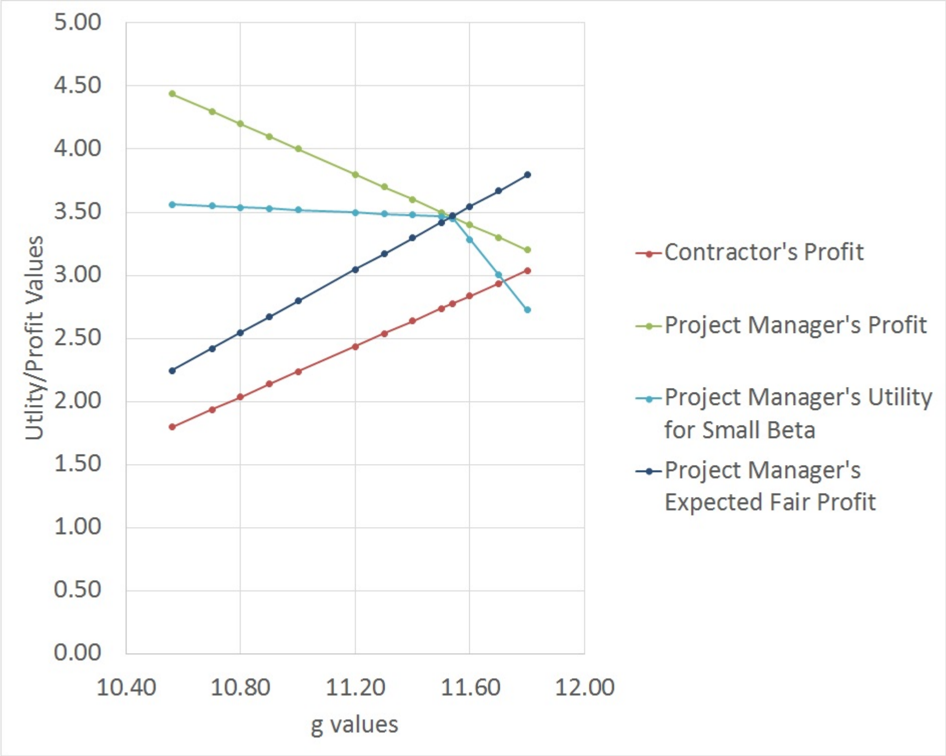

Figure 6.17: Profit/Utility values vs. g values: The project manager fairness concern only for small advantageous disutility

manger's profit was found to be same as her expected fair profit. After that, her profit started to decline sharply. This was because she started to experience a disadvantageous inequity for $g > 11.54$ and $\frac{dU_{pm}}{dg} = -1 - \alpha_{pm} - \alpha_{pm}\delta = -2.8 < 0$. Summarising the above observations, the project manager's utility would be decreasing with any increase in g. Thus, she wouldn't be interested to offer $g > 10.56$. As a result, the optimal solution becomes g=10.56 and h=1.5 when $\beta_{pm} < \frac{1}{1+\delta}$. This would ensure a coordinated supply chain, but fails to ensure any fair profit.

On the contrary, with h = 1.50, the project manager's utility function was increasing with any increase in g if $\beta_{pm} = 0.7 > \frac{1}{1+\delta}$. This was because $\frac{dU_{pm}}{dg} = -1 + \beta_{pm} + \beta_{pm}\delta = 0.575 > 0$. This increase in her utility continued until g=11.54. At g=11.54, the project manager's profit was found to be exactly same as his expected fair profit i.e. $\pi_{pm} = \frac{\pi_0}{1+\delta}$. For $g > 11.54$ where, the project manager's utility started to decline. This was because the project manager's profit had started to decrease below her expected fair profit $\delta\pi_{co}$ and her rate of change of utility was quite faster ($\frac{dU_{pm}}{dg} = -2.8 < 0$). The results are presented in figure 6.18 and table F.10. Summarising these observations, it can be said that the project manager now has got the incentive to increase g from 10.56 to 11.54. The contract with g=11.54 and h=1.50 can ensure the project manger a fair profit and her utility is also maximum.

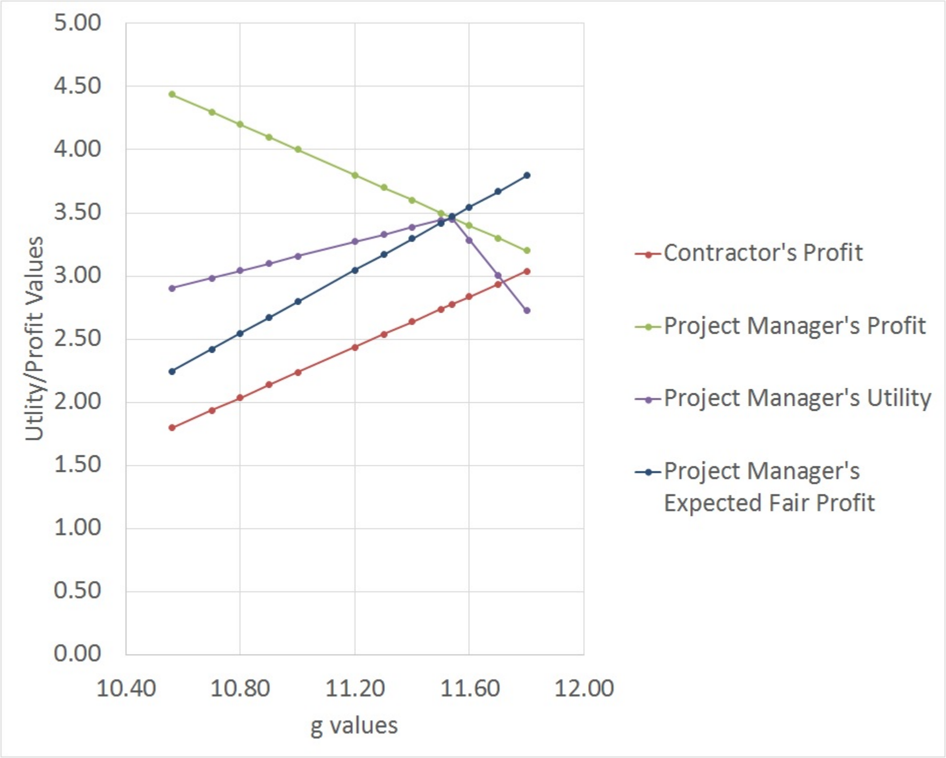

Figure 6.18: Profit/Utility values vs. g values: The project manager fairness concern only for considerable advantageous disutility

6.6 Chapter Summary

This chapter addressed the third objective of this research i.e. to investigate the issues of fair allocation of risk and benefits of supply chain coordination. The definition of fairness has been found to be multifaceted. However, considering the context of this research, the definition by Fehr & Schmidt (1999) has been selected. This is based on the premise of inequity aversion which has been used in the literature of Cui et al. (2007), and Caliskan-Demirag et al. (2010). This chapter is an extension to their work in the project supply chain setting.

The benefits of the presence of fairness concern, is the ability of the fixed price contract to achieve coordination (Cui et al. 2007). However, this has not been completely true for the supply chain under consideration in this research. The fixed price contract could have coordinated the supply chain only when the contractor was fairness concerned (either with a profit maximizing project manager or with a fairness concerned project manager). Moreover, he required to be earning more than his expected fair profit i.e. the coordination was possible only with the advantageous inequity case. If the contractor was offered less than his expectation of the fair profit, then the fixed price contract failed to encourage the contractor to select any non-zero positive resource consumption rate. More interestingly, the selection of global optimal resource consumption rate in the decentralized setting was also found to be depended on the disutility factor of the contractor per unit ($\beta_{co} = \frac{1}{1+\gamma}$). The numerical analysis on this further highlighted some interesting facts about it. It was found that anything below or above of this condition for β_{co} failed to achieve the coordination for the resource consumption rate (λ_0).

In the case with both of the members of the supply chain (the project manager and the contractor) become fairness concerned, the coordination and achievement of fair allocation of profit was found to be depended on the interaction of the parameters of their expectations about the fair profit and on the disutility values per unit. If the project manager was the only fairness concerned member of the supply chain, then the fixed price contract failed to coordinate the supply chain.

In short, it can be said that the fixed price contract performed better in the case of the supply chain members having some fairness concern than in the supply chain with only profit maximizing members. However, it is still limited by the nature of the individual decision making or behavioural variables. These limitations were somewhat found to be overcome by the time based contracts P(T,C)= g-hT.

The time based contract was found to offer more cases where the supply chain could have been coordinated than the fixed price offered, if either or both the members of the supply chain were fairness concerned.

When the contractor was the fairness concerned member, then the project manager achieved coordination with a contract parameter slightly higher than what she needed to offer for the profit maximizing contractor (comparing the g value, it was required to be 10.56 in the case of profit maximizing contractor, whereas in the case of fairness concern contractor, it became 11.37). However, the project manager was not found to offer any value of g which assures the contractor to earn a fair profit.

When the project manager became fairness concerned, then the nature of her offer was depended on the interaction of her expected fair profit proportion (δ) and the contractor's expected fair profit proportion (γ). It is defined in literature of Cui et al. (2007), the supply chain fails to allocate everyone their expected fair proportion when $\delta\gamma > 1$. The project manager was found to experience a disadvantageous inequity. Again, the results were found to be somewhat similar to what has been observed in the case with contractor being the only fair member. The project manager could coordinate the supply chain with a certain optimal solution, but that could not ensure the fair profit for the contractor.

On the contrary, when $\delta\gamma < 1$, the supply chain was capable of allocating fair share of the profit to everyone. The project manager experienced advantageous inequity. However, the contractor receiving the fair profit or not was found to be depended on the disutility of the project manager. If the disutility factor β_{pm} was not too high ($\beta_{pm} < \frac{1}{1+\delta}$), then the results were found to be more or less similar to what was just explained in the last few cases. Only when the project manager had an considerably higher β_{co}, the utility increased in the contract parameters and she offered a contract that not only ensured the coordination, but also assigned a fair profit for the contractor. Thus, summarising the findings of the time based contracts, it can be said that it performed better than the fixed price contracts. However, the existence of the optimal solutions were still found to be depended on parameters which are more of behavioural in nature.

Finally this chapter also used the bargaining approaches generally used in literature for allocating the risks and benefits in a fair way. This research also used the expected inequality aversion model proposed by Fudenberg & Levine (2012). This has allowed this research to incorporate risk aversion and fairness in the same bargaining model following the improvisation on the original model by López-Vargas (2014). This chapter used the utilitarian approach and the egalitarian approach for this. The results are similar to what has been found in chapter 5. Again, the contractor was found to be indifferent between the fixed price and any cost based contract for the utilitarian approach. However, the project manager was found to be better off with cost plus contract.

Chapter 7

Discussion and Concluding Remarks

This research has addressed the supply chains coordination issues in a project setting. This chapter summarizes the findings, discusses the implications (both academic and practical) of the findings, and delineates the future research directions.

7.1 Overview and Implication of the findings

The introduction chapter (chapter 1) introduced the readers to the importance of supply chain and highlighted how supply chains can suffer from sub-optimal performance (especially in terms of cost and profit). It also highlighted how organizations in supply chains are exposed to external risks such as natural and man-made disasters to a greater degree in the absence of supply chain coordination. One of the main motivations was the failure to coordinate the supply chains despite its importance. This is mainly due to the presence of several barriers to the supply chain coordination as argued in several works of literature highlighted in chapter 1. This was further elaborated in chapter 2 with the detail literature review. A literature search framework was created in fig. 2.1. This framework helped this research to classify how the existing studies on supply chain coordination have addressed the barriers of supply chain coordination. The literature review suggested certain key research gaps which have been less addressed in the literature, but that are of considerable importance in practice. These research gaps helped to formulate the research question addressed in fig. 2.7. This research question is answered with three objectives. These three objectives have been addressed the chapters 4, 5, and 6.

7.1.1 Implications for Objective 1

Chapter 4 of this research presented the contractual solution to the problems of non-coordination in a project supply chain in the take it or leave it type of situation. Decision making problems in these type of situations are often solved to as an ultimatum games (Osborne 2004) . This research solved this with the help of Stackelberg leader-follower ultimatum games with the project manager as the leader and the contractor as the follower. The proposed models were extensions of the models proposed by Bayiz & Corbett (2005) and Kwon et al. (2010). One of the main contributions was the enhancement of the models to cover distributions more commonly used for project completion times (Uniform, Gamma, Beta, and Weibull). Bayiz & Corbett (2005) used normal distributed time, whereas Kwon et al. (2010) used an exponential distributed time. Chapter 4 highlighted how project completion time can take different forms of probability distributions (Golenko-Ginzburg 1988, Abdelkader 2004, Roy & Roy 2013, Lee, Arditi & Son 2013). Another notable highlight was the derivation of the models for short-term projects with polynomial reduction of project reward with respect to time. This was an extension to the linear reduction of the model by Bayiz & Corbett (2005).

For the long term projects, the basic models of (Kwon et al. 2010) was extended. Apart from the use of different distributions, the polynomial reduction of project reward was also considered alongside the exponential discounting. The discounting of any cash flow becomes significant in the longer run of the projects. The prevailing discount rate is around 4 % in the UK (Cabinet Office 2015). Thus, in the short run, the impact of this on the cash flow could be so small that it is effectively negligible. Hence, the discounting is considered for the long term projects only. As explained in chapter 4, some of the projects have the product outputs whose operational life is not much affected by any delay in the project completion. Any revenue loss can be recovered over the course of its operational life such as Power plant Projects. Thus, there is no need for assigning any penalty in either linear or polynomial form to take into account of the revenue loss. There would be still the loss value of the project reward due to the time value of money. Thus, this was addressed with the exponential discounting. On the contrary, some of the projects have the product outputs whose operational life can not be recovered in the event of a delay such as software projects. Due to stiff market competition and the short nature of the operational life, any delay in the project completion means there would be considerable revenue loss which could not be recovered. These cases have both the element of project value reductions: revenue loss due to delay in project and loss of project value due to the time value of money. These type of cases were not explored in the literature of Kwon et al. (2010). Hence, this research contributes to this. One of the notable features of this case is the dependency on the ratio of $\frac{\psi}{\alpha}$, where ψ is the reduction factor of the project value per unit time to take into account any revenue loss and α is the prevailing discounting factor for the cash flows. If $\psi \leq \alpha$, the impact of revenue loss would become very small and it can be neglected.

The fixed price contracts failed to coordinate (in any of the cases explored in this research) the supply chain under consideration in Stackelberg game settings. This finding supports the findings of Kwon et al. (2010). However, the fixed price contracts are still very popular in practice due to their simplicity in application. These may work reasonably well when the risk associated with the project is very small. Although not very common, linear time-based contracts have been used in practice such as after the Northridge earthquake in 1994, the City of Los Angeles offered a time-based contract to Clint Meyers (Kwon et al. 2010). The linear time-based contract in the form P(T,C)=g-hT was found to coordinate the supply chain under consideration in the majority of the cases.

Changing the shape parameters of some distributions (gamma and beta) was found to shift the expected completion time towards the left or right tail. A decrease in the first best profit was observed with the movement of the expected value towards the right. Thus, a proximity of the expected completion time and worst case time is detrimental for the first best profit.

Project management literature has heavily emphasized on the use of beta distribution for completion time (Roy & Roy 2013). This is due to its nature as a family of distributions. From this distribution, the other form of distributions such as triangular and uniform can be developed as special cases. Another notable characteristic of the proposed models is the use of parameters specific to the distribution type. The biggest question is how to estimate the values of these distribution specific parameters? For uniform distribution, usually, two parameters are required: a most optimistic estimation and the other one worst case scenario. This research assumed the most optimistic value to be zero for the simplification of calculation. The other parameter θ determines the scale or stretch of the distribution. At the time of contracting, the project manager and the contractor both make some estimates about these two completion times. In literature and practice, project evaluation review techniques

(PERT) have been a frequently used tool to have an estimate for these two completion times. The same is the case for the beta distribution as well. Summarizing the project management literature, it can be said that the beta distribution is combined with PERT techniques to derive the most optimistic, most likely and worst-case times (Davis 2008). The author derived conditions to show that the sum of the parameters of the beta distribution (u and v) should satisfy the condition $4 \leq (u + v) \leq 8$. Using this condition, the proposed model with the beta distribution can be used in practice. In order to use the proposed models of this research in practice, the user needs to assign the values of the model parameters. The value of α, $q(T)$ and π_{out} can be estimated from usual practice.

Despite this wide range of applicability of the beta distribution as a form of project completion time, it has got certain limitations for the proposed model in this research. The present research failed to derive a closed form solution for coordination models with beta and Weibull distributed time if the cash flows are discounted. This was due to the non-existence of a closed form moment generating function for the beta and Weibull distributions.

Some interesting results were observed from the numerical analysis section of chapter 4. The applicability of the models is dependent on the power of the polynomial reduction (m) of the project value with respect to time. This was observed in the case of short-term projects without any discounting of the cash flow. The first best solution was found to be negative in most of the cases for higher values of m. It was also found that the proposed models can achieve 100 % efficient solution mathematically. However, from an applicability point of view, these model would be difficult to be implemented in those cases. One probable explanation for this type of limitations is the higher requirement of the resource consumption in order avoid rapid reduction of the project value with respect to time.

7.1.2 Implications for Objective 2

This objective investigated how to reach the optimal solution with negotiation between the project manager and the contractor. This optimal solution should be a win-win for both members. This is an extension of the game setting considered in the objective 1. In objective 1, the members of the supply chain (the project manager and the contractor) were not allowed to go for a negotiation if the contractual agreement was not successful at the first attempt. This was the reason why these types of games were called ultimatum games. These types of ultimatum games are considered as a special case of the bargaining games. Objective 2 addressed the issues of achieving win-win solutions using bargaining games. Hence, objective 1 can be considered as a special case of objective 2 in some way. There is a distinguishable difference between these two objectives. This research used a sequential mover leader-follower game i.e. Stackelberg Games for addressing the coordination problem in objective 1. However, this research used simultaneous mover bargaining games for objective 2.

This research extended the models proposed by Lippman et al. (2013). the authors used a normal distributed cost and Nash Bargaining approach to find out the win-win solution. However, a normally distributed cost is very unlikely in practice (Back et al. 2000). One of the main contributions of this research is the use of various skewed distributions (gamma, exponential, beta, and Weibull) for project completion cost. In addition, this research also investigated the bargaining solutions using the approach proposed by Kalai Smorodinsky bargaining and Utilitarian approach alongside the Nash bargaining.

The models were prepared for two different situations: with both the members are risk neutral, and with one risk neutral and one risk averse member. In the second situation, the contractor was considered risk averse and the project manager was considered risk neutral. It is more likely that the contractor would be a part of a small scale organization (in terms of financial size) in comparison to the project manager's organization. As discussed in the chapter 2, the existence of supply chain coordination models which consider the differential risk preference of the members of the supply chain is limited. The proposed models in chapter 5 were the attempts to bridge this research gap.

Optimal results indicated an equal share of the total benefit between the project manager and the contractor if both of them are risk neutral. This has been found as true for fixed price, any cost-based and any time-based contracts. Due to the simplicity of application, fixed price contracts are likely to be the preferred form of contract in this case.

One of the notable features of the risk averse member is the nature of utility function considered. A linear form of the utility function is a very common form for risk neutral members, whereas a concave form is used to represent the risk averse member's utility. This is to make sure the risk averse member has a diminishing marginal return (Haugen 2001, Levy & Levy 2002, Davies & Satchell 2007). Various forms of utility functions for risk-averse members have been considered in the literature such as hyperbolic functional form, an exponential form,and a quadratic form. However, a majority of the models in the literature used exponential form (Corner & Corner 1995). This may be due to the fact of having a constant absolute risk aversion ratio. This avoids the need for an increase in any risk premium with the increase in the wealth value. Thus, this research also used this form for the utility function to represent the risk averse contractor's utility. Results were found to be different when the contractor became risk averse than in comparison to results for a risk neutral contractor. The solutions from fixed price contracts were found to dominate the solutions from a time-based contracts. However, the solutions of the cost plus contract were found to dominate the solutions of any other contract. The project manager's profit from the cost plus contracts dominated the profits derived from the other contracts. However, the contractor's profit was better for the case of fixed price contracts than the cost plus or any cost-sharing contracts. It was the effect of risk aversion which changed the preference of the contractor in situation. Due to the uncertainty of the completion cost, the contractor has got the least amount of risk bearing from the cost plus contract. The contractor's utility function changed accordingly to reflect the risk aversion. The win-win solution for the project manager is in terms of the expected profit and for the contractor, it is in terms of the risk exposure.

There is a minor exception to the results explained in the last paragraph. For the case of utilitarian bargaining, the contractor's utility was found to be independent of the type of contract. Since contractor's risk exposure is same for the cost based and fixed price contract, he may prefer a fixed price contract. However, the project manager was found to be better off with the cost plus contract.

Bargaining and negotiation are in existence in real life project setting. Bajari et al. (2009) found half of the private sector building contracts in north Carolina were negotiated. Post economic downturn, the building contracts in Dubai were also negotiated (Bertenshaw 2012). There could be much more examples of contract negotiation in a project setting.

7.1.3 Implications for Objective 3

This objective investigated fairness in the allocation of risk and benefits of the supply chain coordination as one of the behavioural issues. Chapter 6 presented the detailed analysis of this. Several laboratory experiments have confirmed undesirable results such as early termination of contracts when there is a lack of fairness in allocation (Katok & Pavlov 2013).

This has been true in practice as well such as the contractual termination between Walmart Canada and Lego group (Georgiades 2008), and the contractual termination between Chinese home appliance retailer Gome and air conditioner manufacturer Gree (Liu et al. 2012). The literature review also revealed a limited attention on the issues of fair allocation of risk and benefits in supply chain literature despite its importance. The objective 3 was an attempt to bridge this gap.

This research extends the approach proposed by Cui et al. (2007). The authors used the inequity aversion model proposed by Fehr & Schmidt (1999) for a product supply chain along with the supply contract. This research extended this to the project setting.

The first part of chapter 6 presented the analysis for the case of the take it or leave it situations. Similar to chapter 4, the coordination problem in the take it or leave it the situation was addressed using Stackelberg games with the help of backward induction. Again the project manager was considered as the Stackelberg leader moving first and the contractor was considered as the Stackelberg follower moving second. Similar to the findings of Cui et al. (2007) and Caliskan-Demirag et al. (2010), the fair allocation of profit in the coordinated supply chain depends on the disutility factors of the members of the supply chain (the project manager and the contractor in this case).

Similar to the case of product supply chains (Cui et al. 2007), the project supply chain can be coordinated with fixed price contracts in the presence of fairness consideration of the members of the supply chain. However, this research found several limitations to this. The fixed price contract failed to coordinate if the contractor earned a profit less than his expected fair profit i.e in the case of the contractor experiencing disadvantageous inequity. Even when the contractor is experiencing advantageous inequity i.e. his profit is more than his perception of fair profit, it depends on the disutility factor he assigns. If it is too small, then neither the coordination nor the fair allocation is possible. Moreover, the fixed price contracts failed to coordinate the supply chains if only the project manager is fairness concerned, but not the contractor.

In order to overcome these above shortcomings of the models using fixed price contracts, this research also investigated if the presence of fairness concerns can change the results for the models with time-based contracts. Unlike, the case with the fixed price contracts, the time-based contracts are able to coordinate the supply chain when the contractor earns a profit less than his expected fair profit (i.e. when he experiences disadvantageous inequity). This was true when only the contractor was fairness concerned or both of them were fairness concerned. However, the time-based contract failed to ensure the allocation of profit which is fair according to the contractor except in one case. The exception was the case when the contractor experienced disadvantageous inequity, and the project manager experienced an advantageous inequity which is not too small ($\beta_{Pm} \leq \frac{1}{1+\delta}$).

Unlike the case with fixed price contracts, the time-based contracts were also found to be capable of coordinating the supply chain when the contractor was not fairness concerned, but the project manager was. If the project manager experiences disadvantageous inequity, then the offered contract was found to ensure the minimum profit expectation of the contractor (π_{out}) and thereby coordinate the supply chain. However, it was not sure if the profit earned by the contractor was meeting the fairness requirement or not. The same was the case with the project manager experiencing advantageous inequity and her disutility per unit due to inequity was not too large ($\beta_{co} < \frac{1}{1+\delta}$). The things were different when the project manager's disutility per unit is not too low ($\beta_{pm} \geq \frac{1}{1+\delta}$) and the project manager experiencing advantageous inequity. If the minimum profit that the contractor can earn outside, is more than the project manager's fair profit expectation about the contractor, then the offered contract was found to ensure a profit of π_{out} to the contractor. This is more than the project manager's expectation of the fair profit of the contractor. Hence, the project manager would not be able to earn the fair share in this case. On the contrary, if the minimum profit the contractor can earn outside is less than the project manager's fair profit expectation about the contractor, then the higher disutility parameter of the project manager enticed her to offer a contract which ensures a fair allocation for both the project manager and the contractor.

The models were also derived for the case of bargaining games. Authors including Binmore (2014) highlighted that egalitarian approach of bargaining tends to offer a solution which is often accepted as the fair solution by the members of the supply chain. This research found that the fair solutions using this egalitarian and other bargaining approach tend to approach an equal share of the overall risk and benefits. This was found to be true with both the project manager and the contractor are risk neutral. It was also found that allocations changed if the contractor had some impacts from another behavioural decision-making variable namely the risk perception. This research found very limited research in this area where the authors considered both the fairness concern and the risk preference of the member in the supply chain transactions. Thus, this research used the expected inequity aversion model proposed by Fudenberg & Levine (2012) and improvised by López-Vargas (2014). This allowed the present research to combine the definition of fairness proposed by Fehr & Schmidt (1999) along with the risk aversion of the contractor. The utilitarian approach and the egalitarian approach have been used to see how these models work. The solutions were found to be dependent on the values of the disutility parameters of the project manager α_{pm} or β_{pm} and the contractor's relative weightage between the ex-ante fairness perception and ex-post risk sharing mechanisms.

7.2 Limitations and Future research avenues

The main aim of this research has been achieved with certain restrictive assumptions. Some of these assumptions may offer certain limitations to the scope of this research.

One of the main limitations of the models proposed in this research is the structure of the supply chain. This research only considered a dyadic supply chain with a project manager and a contractor. In addition, the information symmetry of certain variables and parameters can pose certain limitations to the applicability of the findings of the research. This information symmetry means that the decision variables and parameter values are known to all the participating members of the supply chain. This includes the assumptions about time, cost, risk preference of the participating supply chain members, and the parameters pertinent to the fairness perception of the member of the supply chain (γ, δ, α_{co}, α_{pm}, β_{co}, and β_{pm}).

The proposed models failed to offer any coordinating solution in certain cases such as fixed contract failed to coordinate the profit maximizing supply chain. Other instances include the unavailability of closed form solutions with beta and Weibull distributed project completion times. However, the fixed price contracts are still very popular in practice due to the simplicity of applications.

The other notable limitations include simplification of any context-specific decision-making characteristics. Few of the issues have been assumed in this research as given or the decision maker makes a rational decision. It is assumed that the members of the supply chain would use the bargaining power rationally and chances of a dictator game are very less likely. This was based on the premise of thought that all the participants in any economic transaction have got some degree of fairness. However, there is a contradictory school of thought in practice specially in construction projects. It is often believed based on the common practice that less powerful members (in terms of bargaining power) often receive the least possible pay-off in construction projects. The two situations of having to sign contracts with other project managers or not having any other options outside the one on hand may have different motivations for the contractors. This research has assumed the contractor would participate in the game if the current contractual offer is better than the rest. It would be interesting explore how a contractor makes the decision on whether to participate or not, and to prioritize his fairness concern along with profit maximizing concerns or not considering these.

Use of various fairness reference point (definition of fairness) for different context may generate different results. Du et al. (2014) used Nash bargaining solution as the fairness reference point to propose a fair allocation mechanism for the supply chain members in a newsvendor setting. The insightful findings was the inability of the wholesale price (an equivalent of fixed price) contracts to coordinate the supply chain. Thus, it is intriguing to test the proposed models using various other reference point. Moreover, in the bargaining situation, this research assumed the idea proposed by Binmore (2014) that egalitarian approach tends to be the fair solution. However, in the presence of initial endowment, these may not be the feasible solutions. Any loss of efficiency due to the fairness concern could be the other area which requires further attention.

Another limitation stems from the applicability point of view. The proposed models are generalizable due to a greater extent. However, it would be interesting to explore if any additional challenges arise in its application phase and if these can be overcome using more rigorous empirical foundations.

These limitations are not the end of the world. Rather, these have highlighted the opportunities for future research to generate new ideas and theories. The future research avenues may include the following but not limited to

- Derivation of models with information asymmetry
- models with variable resource consumption rate (λ)
- Incorporation of the learning effect of the contractor on the resource consumption rate

(λ)

- Incorporation of the bargaining power of the members of the supply chain and their impact.
- Derivation of the models with more complex structures with more than two members
- Application of heuristics and meta heuristics techniques to propose solutions where normal mathematical techniques failed offer any closed form solution
- Use of other fairness reference points
- Use of Shapley values, α fairness and other methods to analyse the trade-off between the fairness and the efficiency

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Appendix A

Dissemination arising from this thesis

Conferences Attended/Papers Presented

- Presented a paper on: Challenges and Mechanisms for Decentralized Supply Chain Coordination at White Rose Doctoral Conferences, University of York, 2014.
- Presented a paper on: Coordination of project supply chains at WRSSDTC Business and Management Pathway Sustainability Conference, University of Sheffield, 2015.
- Presented paper on: "**Modelling Supply Chain Coordination using Project Based Contracts**" at European Operations Management Association Conference, 2015 (EUROMA, 2015) at University of Neuchatel, Switzerland.
- Presented paper on "**Fairness in Profit Allocation in a Coordinated Supply Chain**" at 27th European Conference on Operational Research, 2015 (EURO, 2015) at University of Strathclyde, United Kingdom.
- Presented paper on: "**Fairness and Project Supply Chain Coordination**" at 58th Annual OR Society Conference, 2016 at the University of Portsmouth, United Kingdom

Submitted Items for Publication

Some of the findings from chapter 4 of this thesis has been submitted to **Annals of Operations Research** for Publication (Under review).

Appendix B

List of Acronyms

B.1 For chapter 4

- T = Effective Project Completion Time
- λ= Resource Consumption Rate
- λ_0= First best Resource Consumption Rate
- π_0 = First best Profit
- π_{pm} = Project manager's profit
- π_{co} = Contractor's profit
- π_{out} = Contractor's minimum profit outside this contract
- k = Resource price per unit of resource per unit time
- C = Cost
- C_o = Cost that is independent of λ
- q_0 = The project value at the beginning of the project
- ψ = The decreasing parameter of project value per unit time (For the polynomial discounting case)
- $q(T)$ = Effective project value upon completion

- g and h : Contract parameters of the time based contract
- f: fixed price contract parameter
- θ = Scale parameter
- (u and v): Shape parameters of beta distributed time "T"
- w: Shape parameter of gamma distributed T
- s: Shape parameter of Weibull distributed T
- $f_{\lambda}(T)$ = Probability Density Function for T
- μ_1 = Mean completion time when one unit of resource is deployed
- α = Exponential discounting factor

B.2 For chapter 5

- All the items are applicable what has been assumed for chapter 4 as applicable
- η = Degree of risk aversion with $\eta > 0$
- U_{pm} =The project manager;s utility
- U_{co}= The contractor's utility
- a and b: The fixed and variable parameters of the cost based contract with $b \in [0, 1]$
- μ = Expected value of random variable cost X
- μ_T = Expected value of completion time T
- ϕ = Scale parameter of a cost distribution
- ω = Shape parameter of a gamma distributed cost
- c and d = The shape parameters of the beta distributed cost
- S= Shape parameter of the Weibull distributed cost

- W= The contractor's expected risk exposure due to cost uncertainty
- W_T= The contractor's expected risk exposure due to time uncertainty
- N(a,b) or N(g,h) = Nash product for the cost based contract or time based contract cases
- K(Z,d) = Kalai Smorodinsky maximum individually rational pay-off
- U(Z,d)= Utilitarian sum
- Z= feasible set of the solutions for the bargaining approach
- d = disagreement point pay-off

B.3 For chapter 6

- All the items are applicable what has been assumed for chapter 4 as applicable
- δ = The project manager's expectation of fair profit with respect to the contractor's profit
- γ = The contractor's expectation about the fair profit with respect to the project manager's profit
- α_{co} = The contractor's disutility per unit due to earning less than his expected fair profit
- β_{co} = The contractor's disutility per unit due to earning more than his expected fair profit
- α_{pm} = The project manager's disutility per unit due to earning less than her expected fair profit
- β_{pm} = The project manager's disutility per unit due to earning more than her expected fair profit

Appendix C

Glossary of Terms

- **Ultimatum Games:** Mostly suited for take it or leave it type of offering scenario. If the offer is rejected by the respondent, then the game terminates.

- **Stackelberg game principles:** A type of leader follower game often used in ultimatum. The leader enters the game with an offer. The follower then decides whether to accept or reject. If accepted, the game continues, otherwise it terminates in the ultimatum game scenario. Thus, its a sequential mover game (Cachon and Netessine, 2004, Osborne, 2004)

- **Backward Induction Methods:** A type of techniques often used in leader follower games. The leader analyses all the possible ex post decisions and associated pay offs from an ex ante point of view. (S)He anticipates the best ex post alternative the follower is likely to implement in response to his /her offer and accordingly finds his or her best response. This best response is the offer that the leader is likely to make.

- **Pareto Optimal Results:** An allocation set where it is impossible for any individual player to be better off without making others worse off (Cachon and Netessine, 2004; Osborne, 2004)

- **First best solution:** For the centralized supply chain, the profit function has been differentiated with respect to the resource consumption rate and set equal to zero. From this the resource consumption rate can be solved which is called the first best solution. Using this first solution in the profit equation gives the first best profit.

- **Supply Chain Coordination Working Definitions:**
  - If the sum of individual profits in the decentralized setting is same as the first best profit and every individual earns at least his or her minimum possible earning outside this offer (based on Cachon, 2003). In other words, the solution would reach the Pareto optimal solution.
  - A win-win approach: This ensures that the players/members of the supply chain both are better off by participating in the game (contractual agreement) than not participating
  - Chapter 4 used the definition of Cachon (2003). In chapter 5 and 6, this research used certain other behavioural variables (Risk preference in chapter 5 and fairness in chapter 6). A win-win approach best explains the optimal equilibrium solutions.

Appendix D

Results For Chapter 4

Table D.1: Results for Coordinating conditions for Models without Cash Discounting [with $\lambda_0 = 2.74$ and $\pi_0 = 6.24$]

λ	g	h	π_{co}	π_{pm}	π_{total}	Efficiency = $\left(\frac{\pi_0 - \pi_{total})}{\pi_0}\right)$
0.00	10.56	0	-	-	-	-
1.73	10.56	0.6	5.02	0.28	5.30	84.99
1.87	10.56	0.7	4.58	1.02	5.59	89.67
2.00	10.56	0.8	4.16	1.64	5.80	93.00
2.12	10.56	0.9	3.78	2.17	5.95	95.39
2.24	10.56	1	3.41	2.65	6.06	97.10
2.35	10.56	1.1	3.06	3.07	6.13	98.31
2.45	10.56	1.2	2.73	3.46	6.18	99.12
2.55	10.56	1.3	2.41	3.81	6.21	99.64
2.65	10.56	1.4	2.10	4.13	6.23	99.92
2.74	10.56	1.5	1.80	4.44	6.24	100.00
2.83	10.56	1.6	1.51	4.72	6.23	99.93
2.92	10.56	1.7	1.23	4.99	6.22	99.72
3.00	10.56	1.8	0.96	5.24	6.20	99.42
3.08	10.56	1.9	0.70	5.47	6.18	99.02
3.16	10.56	2	0.44	5.70	6.15	98.54
3.24	10.56	2.1	0.19	5.92	6.11	98.01

Table D.2: The values of λ_0, π_0, g and h for different values of m

Distribution	m	λ_0	π_0	g	h
Uniform	1	2.74	6.24	10.56	1.50
	2	5.43	1.97	19.17	5.89
	3	7.33	-0.63	25.24	10.73
Gamma	1	2.74	6.24	10.56	1.50
	2	5.65	1.45	19.87	6.38
	3	8.11	-2.30	27.74	13.15
Beta	1	2.74	6.24	10.56	1.5
	2	5.31	2.25	18.80	5.65
	3	7.12	-0.19	24.59	10.14
Weibull	1	2.74	6.24	10.56	1.5
	2	5.34	2.17	18.91	5.72
	3	7.24	-0.45	24.97	10.49

Table D.3: Results for coordinating conditions for models with exponential discounting for products with recoverable product life: Uniform distributed time

g	h	λ	π_{co}	π_{pm}	π_{total}	Efficiency $= \frac{\pi_0-\pi_{total}}{\pi_0}$
1.55	0.04	1.43	0.64	0.07	0.704	91.41
1.55	0.06	1.57	0.54	0.19	0.733	95.24
1.55	0.08	1.70	0.45	0.30	0.752	97.74
1.55	0.10	1.83	0.37	0.39	0.764	99.21
1.55	0.12	1.96	0.29	0.48	0.769	99.89
1.55	0.13	2.05	0.25	0.52	0.770	100.00
1.55	0.14	2.09	0.22	0.55	0.769	99.97
1.55	0.16	2.20	0.14	0.62	0.766	99.59
1.55	0.18	2.32	0.07	0.69	0.761	98.85
1.55	0.20	2.43	0.01	0.75	0.753	97.84

Table D.4: Results for coordinating conditions for models with exponential discounting for products with recoverable product life: Gamma distributed time

g	h	λ	π_{pm}	π_{co}	π_{total}	Efficiency $= \frac{\pi_0-\pi_{total}}{\pi_0}$
1.54	0.04	1.42	0.08	0.65	0.722	92.00
1.54	0.06	1.55	0.20	0.55	0.750	95.51
1.54	0.08	1.68	0.30	0.47	0.768	97.83
1.54	0.10	1.81	0.39	0.38	0.779	99.22
1.54	0.12	1.94	0.48	0.31	0.784	99.88
1.54	0.13	2.02	0.54	0.25	0.785	100.00
1.54	0.14	2.06	0.56	0.23	0.785	99.98
1.54	0.16	2.17	0.62	0.16	0.782	99.65
1.54	0.18	2.28	0.69	0.09	0.777	98.97
1.54	0.20	2.39	0.75	0.02	0.770	98.03

Table D.5: Results for coordinating conditions for models with exponential discounting for products with irrecoverable product life: Uniform distributed time

g	h	λ	π_{co}	π_{pm}	π_{total}	Efficiency $= \frac{\pi_0 - \pi_{total}}{\pi_0}$
1.72	0.10	1.89	0.51	0.08	0.582	95.96
1.72	0.12	2.02	0.43	0.17	0.595	98.10
1.72	0.14	2.14	0.35	0.25	0.603	99.36
1.72	0.16	2.25	0.28	0.32	0.606	99.92
1.72	0.17	2.32	0.25	0.36	0.607	100.00
1.72	0.18	2.36	0.21	0.39	0.606	99.96
1.72	0.20	2.47	0.15	0.46	0.604	99.57
1.72	0.22	2.58	0.08	0.52	0.599	98.82
1.72	0.24	2.68	0.02	0.57	0.593	97.80

Table D.6: Results for coordinating conditions for models with exponential discounting for products with irrecoverable product life: Gamma distributed time

g	h	λ	π_{pm}	π_{co}	π_{total}	Efficiency $= \frac{\pi_0 - \pi_{total}}{\pi_0}$
1.71	0.10	1.87	0.08	0.52	0.598	96.06
1.71	0.12	1.99	0.17	0.44	0.611	98.10
1.71	0.14	2.11	0.25	0.37	0.618	99.33
1.71	0.16	2.22	0.33	0.30	0.622	99.91
1.71	0.17	2.29	0.37	0.25	0.623	100.00
1.71	0.18	2.33	0.39	0.23	0.622	99.98
1.71	0.20	2.43	0.46	0.16	0.620	99.64
1.71	0.22	2.54	0.52	0.10	0.616	98.97
1.71	0.24	2.64	0.57	0.04	0.610	98.03

Appendix E

Results for Chapter 5

E.1 Results for Nash Bargaining

Table E.1: Results from Nash's bargaining for gamma distributed cost

b	a	W	U_{pm}	U_{co}	N
0.00	7.28	2.78	2.72	0.36	0.95
0.50	4.74	1.56	3.26	0.39	1.29
1.00	2.60	1.00	3.40	0.41	1.38

Table E.2: Results from Nash's bargaining for exponential distributed cost

b	a	W	U_{pm}	U_{co}	N
0.00	8.98	5.00	1.02	0.17	0.17
0.50	4.94	1.67	3.06	0.38	1.16
1.00	2.59	1.00	3.40	0.41	1.38

Table E.3: Results from Nash's bargaining for beta distributed cost

b	a	W	U_{pm}	U_{co}	N
0.00	6.95	2.50	3.04	0.38	1.15
0.50	4.61	1.50	3.38	0.40	1.36
1.00	2.60	1.00	3.40	0.41	1.38

Table E.4: Results from Nash's bargaining for Weibull distributed cost

b	a	W	Upm	Uco	N
0.00	7.22	2.73	2.77	0.36	0.98
0.50	4.73	1.60	3.05	0.38	1.15
1.00	2.43	1.00	3.14	0.39	1.21

E.2 Results from Kalai Smorodinsky Bargaining

Table E.5: Results from Kalai Smorodinsky bargaining for gamma distributed cost

b	a	W	V	U_{pm}	U_{co}	K
0.00	6.99	2.78	2.78	3.02	0.31	0.50
0.50	4.50	1.56	2.78	3.50	0.36	0.58
1.00	2.37	1.00	2.78	3.63	0.38	0.61

Table E.6: Results from Kalai Smorodinsky bargaining for exponential distributed cost

b	a	W	V	U_{pm}	U_{co}	K
0.00	8.48	5.00	5.00	1.52	0.08	0.25
0.50	3.82	1.67	5.00	4.17	0.22	0.70
1.00	1.42	1.00	5.00	4.58	0.25	0.76

Table E.7: Results from Kalai Smorodinsky bargaining for beta distributed cost

b	a	W	V	U_{pm}	U_{co}	K
0.00	6.78	2.50	2.50	3.22	0.36	0.54
0.50	4.48	1.50	2.50	3.51	0.38	0.59
1.00	2.47	1.00	2.50	3.53	0.40	0.60

Table E.8: Results from Kalai Smorodinsky bargaining for Weibull distributed cost

b	a	W	V	U_{pm}	U_{co}	K
0.00	6.44	2.73	2.73	3.56	0.25	0.76
0.50	4.50	1.60	2.73	5.06	0.34	1.08
1.00	2.84	1.00	2.73	6.27	0.44	1.34

E.3 Utilitarian Bargaining

Table E.9: Results from Utilitarian bargaining for gamma distributed cost

b	a	W	U_{pm}	U_{co}	U
0.00	1.24	3.69	3.75	0.17	8.92
0.50	0.61	1.73	3.99	0.17	9.15
1.00	0.15	1.00	4.04	0.17	9.21

Table E.10: Results from Utilitarian bargaining for exponential distributed cost

b	a	W	U_{pm}	U_{co}	U
0.00	2.83	25	2.17	0.17	2.33
0.50	0.69	1.92	3.90	0.17	4.07
1.00	0.15	1.00	4.05	0.17	4.21

Table E.11: Results from Utilitarian bargaining for beta distributed cost

b	a	W	U_{pm}	U_{co}	U
0.00	1.05	2.94	3.94	0.17	4.12
0.50	0.58	1.67	4.02	0.17	4.18
1.00	0.15	1.00	4.05	0.17	4.24

Table E.12: Results from Utilitarian bargaining for Weibull distributed cost

b	a	W	U_{pm}	U_{co}	U
0.00	1.06	2.99	3.94	0.17	4.10
0.50	0.54	1.60	4.10	0.17	4.26
1.00	0.15	1.00	4.13	0.17	4.30

Appendix F

Results for Chapter 6

F.1 With fixed price contracts

Table F.1: Results for f values vs. π_{pm}, π_{co}, and U_{co}

f	λ	π_{co}	U_{co}	π_{pm}
6.50	0.00	6.50	-M	-M
7.00	0.00	7.00	-M	-M
7.15	2.74	2.77	2.77	3.46
7.25	2.74	2.87	2.77	3.37
7.50	2.74	3.12	2.77	3.11
7.75	2.74	3.37	2.77	2.87
8.00	2.74	3.62	2.77	2.62

Table F.2: Results for f vs. π_{pm}, π_{co}, U_{co}, and U_{pm}

f	λ	π_{co}	U_{co}	π_{pm}	U_{pm}
6.50	0.00	6.50	-M	-M	-M
7.00	0.00	7.00	-M	-M	-M
7.15	2.73	2.77	2.77	3.46	1.80
7.25	2.73	2.86	2.77	3.36	0.78
7.50	2.73	3.11	2.77	3.11	-1.87

F.2 For time based contracts

Table F.3: Results for Profit/Utilities vs h with time based contracts: The contractor fairness concerned only

h	π_{co}	π_{pm}	U_{co}
1.00	3.24	2.81	2.85
1.10	2.89	3.24	2.77
1.15	2.74	3.41	2.74
1.20	2.56	3.62	2.29
1.30	2.24	3.97	1.49
1.40	1.93	4.30	0.73
1.50	1.64	4.60	0.00

Table F.4: Results for Profit/Utilities vs g with time based contracts: The contractor fairness concerned only

g	π_{co}	π_{pm}	U_{co}
10.40	1.64	4.60	0.00
10.60	1.84	4.40	0.49
10.80	2.04	4.20	0.98
11.00	2.24	4.00	1.47
11.20	2.44	3.80	1.95
11.40	2.64	3.60	2.47
11.54	2.77	3.47	2.77
11.80	3.04	3.20	2.85
12.00	3.24	3.00	2.90
12.20	3.44	2.80	2.96

Table F.5: Results for Profit/Utilities vs g with time based contracts: The contractor and the project manager both fairness concerned in an acrimonious supply chain

g	π_{co}	π_{pm}	U_{co}	U_{pm}
11.37	2.61	3.63	0.00	3.62
11.40	2.64	3.60	0.44	3.51
11.45	2.69	3.55	1.30	3.28
11.50	2.74	3.50	2.16	3.06
11.54	2.77	3.46	2.77	2.88
11.60	2.84	3.40	2.79	2.61
11.65	2.89	3.35	2.80	2.39
11.70	2.94	3.30	2.82	2.17

Table F.6: Results for Profit/Utilities vs g with time based contracts: The contractor and the project manager both fairness concerned in harmonious supply chain a

g	π_{co}	π_{pm}	U_{co}	U_{pm}
11.37	2.61	3.63	0.00	3.48
11.40	2.64	3.60	0.44	3.48
11.45	2.69	3.55	1.30	3.47
11.50	2.74	3.50	2.16	3.47
11.54	2.77	3.46	2.77	3.46
11.60	2.84	3.40	2.78	3.28
11.65	2.88	3.35	2.80	3.15
11.70	2.94	3.30	2.82	3.00

Table F.7: Results for Profit/Utilities vs g with time based contracts: The contractor and the project manager both fairness concerned in harmonious supply chain b

g	π_{co}	π_{pm}	U_{co}	U_{pm}
11.37	2.61	3.63	0.00	3.37
11.40	2.64	3.60	0.44	3.39
11.45	2.69	3.55	1.30	3.42
11.50	2.74	3.50	2.16	3.44
11.54	2.77	3.46	2.77	3.46
11.60	2.84	3.40	2.78	3.28
11.65	2.89	3.35	2.80	3.14
11.70	2.94	3.30	2.82	3.00

Table F.8: Results for Profit/Utilities vs h with time based contracts: The project manager fairness concerned

h	π_{co}	π_{pm}	U_{pm}
1.00	3.41	2.65	1.36
1.05	3.23	2.87	1.93
1.10	3.06	3.07	2.47
1.15	2.89	3.27	2.99
1.20	2.74	3.44	3.43
1.25	2.57	3.63	3.46
1.30	2.41	3.81	3.49
1.36	2.22	4.01	3.51
1.40	2.10	4.13	3.53
1.45	1.95	4.29	3.55
1.50	1.80	4.44	3.56
1.55	1.66	4.58	3.58
1.60	1.51	4.72	3.59

Table F.9: Results for Profit/Utilities vs g with time based contracts: The project manager fairness concerned a

g	π_{co}	π_{pm}	U_{pm}
10.56	1.80	4.44	3.56
10.70	1.94	4.30	3.55
10.80	2.04	4.20	3.54
10.90	2.14	4.10	3.53
11.00	2.24	4.00	3.52
11.20	2.44	3.80	3.50
11.30	2.54	3.70	3.49
11.40	2.64	3.60	3.48
11.50	2.74	3.50	3.47
11.54	2.78	3.46	3.45
11.60	2.84	3.40	3.28
11.70	2.94	3.30	3.00
11.80	3.04	3.20	2.72

Table F.10: Results for Profit/Utilities vs g with time based contracts: The project manager fairness concerned b

g	π_{co}	π_{pm}	U_{pm}
10.56	1.80	4.44	2.90
10.70	1.94	4.30	2.98
10.80	2.04	4.20	3.04
10.90	2.14	4.10	3.10
11.00	2.24	4.00	3.16
11.20	2.44	3.80	3.27
11.30	2.54	3.70	3.33
11.40	2.64	3.60	3.39
11.50	2.74	3.50	3.44
11.54	2.78	3.46	3.45
11.60	2.84	3.40	3.28
11.70	2.94	3.30	3.00
11.80	3.04	3.20	2.72