THE INFLUENCE OF TEXTURE ON THE FATIGUE BEHAVIOUR OF STEEL

by

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A Thesis submitted for the degree of Doctor of Philosophy, in the University of Sheffield.

April, 1984.

To MOM and DAD,

with love. O X

ACKNOWLEDGMENTS

The content of this thesis is original, except where due reference has been made, and refers to work carried out in the Department of Metallurgy, University of Sheffield and at G.K.N. Group Technological Centre, Wolverhampton, between October, 1979 and January, 1983. Thanks are due to Prof. B.B.Argent and Dr.P.Watson for their permission to use laboratory facilities at the aforesaid establishments respectively. The work was funded by the S.E.R.C., and a C.A.S.E. award from G.K.N. Group Technological Centre, Wolverhampton.

I wish to thank my parents for their constant love and support in all things. Without them, I would have been unable to complete this project.

I owe a debt of gratitude to my supervisor, Prof. G.J.Davies, who has proved to be a sound adviser and sympathetic confidant throughout my time in Sheffield. I am grateful to Dr.R.M.Bateman and Dr.M.A.Burke for the expertise they so freely shared. Dr.N.Hurd, and Mr.S.Kemp, both of G.K.N., were excellent sources of aid and information with fatigue analyses, and Mr.V.Kumar (G.K.N.) provided first-class technical assistance.

Thanks to the patience and friendliness of all the technical staff in the Department of Metallurgy, University of Sheffield, most of the specimens for the project were made in the department. However, I must thank Mr.I.Lomas in particular for the manner in which he has shared his knowledge when I have required it! Prof.R.W.K.Honeycombe, of the Department of Metallurgy, University of Cambridge, was kind enough to let me have the use of an automatically timed furnace. I am also grateful to Mrs.R.Greenway who has typed the manuscript.

Finally, I wish to thank my Lord and Saviour Jesus Christ, without whom I can do nothing.

Barry Thomas Sturman-Mole,
Sheffield, 1984.

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GLOSSARY

This glossary contains <u>only</u> those terms <u>not</u> defined in the text.

da/dN	crack extension per cycle
C	constant for a given material
m	constant for a given material
P	constant for a given material
ε_p	plastic strain
$\varepsilon_{p_{cum}}$	cumulative plastic strain
τ_s	saturation shear stress
$\Delta\gamma_p$	plastic shear strain
C_{11}, C_{12}, C_{44}	elastic coefficients
S_{11}, S_{12}, S_{44}	elastic compliances
E	Young's modulus of elasticity
ρ	density
t	thickness of material in plane of bending
L	length
x_i	constant (x_1 = 22.37, x_2 = 61.64, x_3 = 120.8, etc)
f_i	resonant frequency at x_i
n	strain hardening exponent
n'	cyclic hardening exponent
$\frac{\Delta\varepsilon_e}{2}$	elastic strain amplitude at saturation
$\frac{\Delta\varepsilon_p}{2}$	plastic strain amplitude at saturation
$\frac{\Delta\varepsilon}{2}$	total strain amplitude of saturation
S'	cyclic yield stress
K'	cyclic hardening exponent

SUMMARY

This investigation attempts to assess the effect of preferred crystallographic orientation on the fatigue properties of mild steel. Quantitative texture analysis has been performed, using the crystallite orientation distribution function (c.o.d.f.) so that mechanical properties could be quantitatively predicted.

The fatigue properties are evaluated by producing textured plates of different texture types and severities and machining specimens at specific orientations to the rolling direction of the plate. Smooth, flat specimens were tested under fully reversed strain amplitude control to generate strain-life and cyclic stress-strain data. The data was analysed using the parametric approach of Morrow to determine a set of characteristic material parameters. Also, fatigue crack propagation studies were conducted on single edge notch plate specimens machined at specific angles from the rolling direction of textured plate. Data in the form of crack length vs number of cycles (produced under constant load amplitude control) was analysed to produce crack propagation rate vs stress intensity amplitude data which may be parametrically expressed by the Paris constants C and m.

The anisotropy of fatigue behaviour may be predicted from texture measurements, and the cyclic stress-strain data display anisotropy which is related to the type and severity of the texture. The fatigue data display increasing anisotropy as texture severity increases, until a point after which grain size effects become dominant. The anisotropy of fatigue life data is shown to be a function of the product $\Delta\sigma\Delta\epsilon$ per cycle which is directly dependent on the cyclic stress-strain curve.

Volume 1

TEXT

CHAPTER 1

Introduction

Fracture by the progressive growth of fatigue cracks under repeated loading, i.e. fatigue, must now be considered, in conjunction with corrosion effects, as the principal cause of in-service failures of engineering structures and components. Now it is normal to consider engineering materials to be isotropic, and typical values of properties, such as yield stress and elastic modulus, are therefore usually presented in lists of reference data. In practice, however, isotropy is rare, and, after forming, most metallic solids are predominantly anisotropic insomuch as they exhibit differences of properties in different directions. Although severe anisotropy in hexagonal metals has been recognized as an influential factor in fatigue, little evidence is available that similar investigations have been carried out on either b.c.c. or f.c.c. materials, which both normally display greater degrees of isotropy.

Anisotropy arises from either

(a) crystallographic anisotropy where the crystal grains in a polycrystalline aggregate are arranged in a non-random manner to give a preferred crystallographic orientation or,

(b) microstructural anisotropy, where there is a substantial variation in grain size and shape, and the microstructural constituents are arranged in a non-random way.

Although engineers usually require isotropic materials, it is becoming obvious that controlled anisotropy can be advantageous, e.g. for improved formability, for directional strength. It has been recognized that many of the problems encountered during forming may be due to texture, e.g. earing.

This work examines the effect of crystallographic anisotropy (texture) on the fatigue failure of low carbon steel, and the quantitative predictability of any induced anisotropy in the materials employed has been assessed by using a quantitative technique for the determination of texture data. Such a method of texture analysis utilizes the crystallite orientation distribution function (c.o.d.f.) which may be used, in conjunction with suitable deformation models, to predict the mechanical properties. A description of the application is given in Chapter 3.

Chapter 2 opens with a brief resumé of the fatigue failure process, before continuing to present a literature survey of the influence of texture on the fatigue properties of metals. The effects on cubic metals in both monocrystal and polycrystal form are reported.

Fatigue properties were assessed by using the cyclic stress-strain analysis reported in Chapter 5, and standard techniques of crack propagation measurements were analysed as described in Chapter 5. The cyclic stress-strain analysis considers that the cyclic deformation resistance of a material may be defined by a set of interdependent material parameters, related by the relationship between the cyclic stress and cyclic strain amplitudes generated by the loading conditions. Therefore fatigue data is collected in the form of a strain amplitude-life curve, and the coincident stress amplitudes are also recorded. Fatigue crack propagation specimens were cycled under constant load, and the crack lengths were recorded by optical measurements. The experimental details are given in Chapter 4.

The experimental results are given in Chapter 6. Chapter 7 presents the discussion, which compares the findings of the texture analysis from each material, and the effect of texture on the parametric characterisation of the cyclic properties, and on the localised failure process, is discussed. There is a critical examination of the applicability of the cyclic stress-strain approach to the quantitative investigation of factors affecting fatigue properties.

Conclusions and suggestions for further work are given in Chapter 8.

Two appendices give the mathematical derivation of the c.o.d.f. from pole figure data, and the cyclic stress-strain analysis computer program.

CHAPTER 2

Literature Review

2.1 Introduction

When considering fatigue behaviour, it is normal practice to assume that metals and alloys are isotropic, despite the fact that cubic metals possess, to a greater or lesser degree, some form of preferred orientation (texture) which leads to anisotropic elastic and plastic properties.

In this chapter, the various fatigue parameters are presented and discussed briefly. The relevant data concerning body-centred cubic (b.c.c.) and face-centred cubic (f.c.c.) single crystals is examined, before turning to textured cubic polycrystals.

The various methods by which metals and alloys can exhibit anisotropic behaviour when undergoing fatigue are highlighted, and the extent to which assumptions of isotropy must be qualified are indicated.

2.2 Fatigue Parameters

Various review articles[1-4] give detailed accounts of the fatigue process. Fatigue failure in metals and alloys normally involves the same processes. To examine the effects of crystallographic preferred orientation in fatigue, it is therefore in order to examine the results by clarifying them into four groups, i.e.

S-N data.

Cyclic response data.

Phenomenological data referring to fatigue failure.

Quantitative data concerning crack propagation.

2.2.1 S-N Data

The S-N (stress-life) curve provides the simplest method of describing the fatigue response of a material[5]. Parameters other than stress may be used to measure the response of a material undergoing fatigue. For example, strain-life curves or stress intensity factor-life curves may be plotted. In each case, the parameter is maintained at constant amplitude for the duration of the test, annd the results are presented in the form of a plot of the controlling parameter amplitude against the number of reversals to failure.

2.2.2 Cyclic Response Data

Metals are metastable under application of cyclic loads, and their stress-strain response can be drastically affected due to repeated plastic strains. Depending on the initial state, and it's test condition, a metal may cyclically harden, soften, remain stable, or display variable behaviour, before stabilising to produce a 'saturation' response where the cyclic strain produced is proportional to the applied cyclic stress amplitude.

2.2.2.1 Cyclic Hardening and Softening

Determination of constant amplitude fatigue lives of specimens is customarily performed under conditions of controlled stress or controlled strain. In actual engineering structures, stress-strain gradients do exist, and there will usually be a certain degree of structural constraint of the material at critical locations. Such a condition is analagous to strain control[5]. Therefore, it is more advantageous to characterize material response under completely constrained, or strain cycling conditions[6]. By plotting the stress amplitude against reversals from controlled strain test results, cyclic strain hardening and softening can be observed (Figure 1). Some metals are cyclically stable, in which case their monotonic stress-strain behaviour adequately describes their cyclic response. In materials which harden or soften, the steady state condition is usually achieved in about 20%-40% of the total fatigue life. In many materials, this cyclic response is unique, and unaffected by prior mechanical processing, although some alloys do exhibit a cyclic stress-strain response which is history dependent[7]. Many two-phase alloys, however, do not exhibit a saturation stage during fatigue, but show either continual softening or initial hardening immediately followed by gradual softening prior to failure.

2.2.2.2 The Cyclic Stress-Strain Curve

The cyclic behaviour of materials is best described in terms of a stress-strain hysteresis loop[5, 8, 9] (Figure 2). Such a curve shows the corresponding stress and strain amplitudes during the saturation stage. For completely reversed strain controlled conditions with zero mean stress, the total width of the loop, or total strain range, is $\Delta\varepsilon$, and the total height of the loop, or total stress range, is $\Delta\sigma$. In order to construct a cyclic stress-strain curve, the tips of the stabilized hysteresis loops from comparison specimen tests at various controlled strain amplitudes are connected as shown in Figure 3.

The cyclic stress-strain curve can be compared directly to the monotonic or tensile stress-strain curve to assess quantitatively cyclically-induced changes in mechanical behaviour (Figure 4).

Determination of this curve is time consuming, and such a curve requires many specimens. Further, the method is limited to those materials which exhibit a saturation stage. In the absence of a saturation stage, the problem may be approached by defining the cyclic stress-strain curve as the cyclic response after a given fraction of the life. Suggested solutions to the problem are the multiple step test, and the incremental step test[8, 10].

Anisotropy of cyclic properties should, therefore, be apparent in changes in either the cyclic stress-strain curve, or the rate of cyclic work hardening as a function of orientation.

2.2.3 Phenomenological Aspects of Fatigue Behaviour

Metallo graphic observations of fatigue failures indicate that, prior to crack nucleation, considerable modification of surface topography takes place and that fatigue crack propagation occurs in a single step-wise manner with a step associated with each cycle[10].

Gough[11] has shown that a metal deforms under cyclic strain by slip on the same atomic planes and in the same crystallographic directions as in unidirectional strain. In unidirectional deformation, slip is usually widespread throughout all the grains, but, in fatigue, some grains will show slip lines while others will exhibit no evidence of slip. Slip lines are generally formed (in bands) during the first few thousand cycles of stress. Successive cycles produce additional slip bands, but the number of slip bands is not directly proportional to the number of stress cycles. Cracks are usually found to occur in the regions of heavy deformation parallel to what was originally a slip band. Slip bands have been observed at stresses below the fatigue limit of ferrous materials[12]. Therefore, the occurrence of slip during fatigue does not in itself mean that a crack will form.

Crack nucleation occurs at persistent slip bands (p.s.b.'s) or at cell [4], grain[14] or twin[15] boundaries which may become 'persistent'. Local irreversibility of slip leads to inhomogeneities, resulting in topographical development at these sites. The intrusions which develop from such inhomogeneities become preferred sites for crack nucleation. These p.s.b.'s are embryonic fatigue cracks, since they open into wide cracks on the application of small tensile strains. Once formed, fatigue cracks tend to propagate initially along slip planes, although they may later take a direction normal to the maximum applied tensile stress. Ordinarily, fatigue crack propagation is transgranular[12].

It can be expected that crack nucleation from p.s.b.'s should display greater crystallographic anisotropy, in a manner consistent with slip symmetry, rather than nucleation from other sites, and therefore it may be anticipated that surface feature development would be orientation dependent.

Fatigue fracture surfaces are often characterised by striations known as 'beachmarks' or 'tidemarks', indicating the position of the crack front at the end of each cycle[10].

Striations are most prominent at high rates of crack propagation at 90° to the stress axis. The position of the crack front can, therefore, be shown by observation of such surfaces. The local direction of propagation during any cycle can also be recognized, and thus reveal the influence of crystallographic effects.

2.2.4 Quantitative Aspects of Crack Propagation

Material resistance to fatigue crack propagation may be regarded as a specific material property. It is usual to correlate the crack advance rate with the amplitude of the applied stress intensity factor ΔK. Crack propagation data may therefore be conveniently represented by plotting da/dN against ΔK on log co-ordinates.

Several mathematical descriptions of fatigue crack propagation rates have resulted[16,17,18] from the similarity of form of such plots (Figure 5). The description due to Paris[16] is the simplest, i.e.

$$\frac{da}{dN} = C(\Delta K)^m \qquad \ldots\ldots \quad (1)$$

Anisotropic fatigue properties should, therefore, be displayed in terms of variations in C and/or m in the Paris equation.

2.3 Fatigue of b.c.c. Single Crystals

Dislocation slip in b.c.c. crystals is similar to slip in f.c.c. crystals. However, it should be noted that there are two significant differences between slip in b.c.c. and slip in f.c.c. structures

(a) In b.c.c. crystals, slip is not confined solely to the closest packed planes as in f.c.c. crystals, and

(b) The possibility of asymmetric slip occurs in b.c.c. crystals, due to the fact that slip of screw dislocations may occur on one plane in tension, but on a different plane in compression <u>if the same stress axis is used</u>[19].

Fatigue behaviour in b.c.c. crystals displays greater anisotropy[20] than in f.c.c. crystals, even though cross-slip is relatively easy[21]. Therefore, asymmetric slip is the fundamental anisotropic effect[20], and not the confinement of slip to close packed planes, as in the case of f.c.c. crystals.

2.3.1 S-N Data of b.c.c. Single Crystals

In bending fatigue tests, Hempel[22] found little effect of orientation in 0.006%C iron crystals on the S-N curve. After plotting the macroscopic fatigue bending stress against the number of cycles to failure, he postulated that the data could be represented by a simple curve (Figure 6). However, since slip in b.c.c. crystals is not confined to the closest packed planes, the applied stress should be resolved onto the operative {110}, {112} or {123} slip plane with a common ⟨111⟩ slip direction. There are 48 slip systems, but since the planes are not so close-packed as in the f.c.c. structure, higher shearing stresses are usually required to produce slip. Kettunen[23] re-analysed Hempel's[22] data and presented them in terms of the resolved shear stress amplitude of the operating pencil glide or ⟨111⟩ slip system (Figure 7). Displayed in the same figure are the S-N curves for polycrystalline material (after using the appropriate Taylor[85] factor* to calculate the equivalent shear stress from the applied shear stress). The data are very similar to the single crystal data. Kettunen[23] concluded that the fatigue behaviour of b.c.c. iron and f.c.c. copper single crystals is similar.

However, when the applied stress (for the crystals which were used in Hempel's experiments) was resolved with a ⟨111⟩ direction onto the operative {110}, {112} or {123} slip planes, it was found that for the same bending stress the shear stress resolved onto the operative slip plane would be the same to within 10%. Therefore, only a limited range of orientation factor is involved in relating the macroscopic bending stress to the resolved shear stress. There is no data from orientations of the stress axis near ⟨110⟩ where

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* See section 2.6.4.1

the Schmid factor is only 60% of that for the crystals tested. Further, Nine[24] has shown that the impurity content of b.c.c. niobium single crystals should be considered when determining fatigue slip characteristics. In particular, oxygen was found to inhibit the asymmetry of slip, and to produce p.s.b.'s at the surface, similar to those observed on fatigued copper crystals. Hempel's[22] polycrystalline iron prior to strain annealing contained 480 ppm oxygen, plus other interstitial solutes.

Nine[19,26,27], Etemad[28], Hempel[22], Doner[29], Mughrabi and Wuthrich[30] have all demonstrated the effect of asymmetric slip on the fatigue behaviour of iron, molybdenum and niobium. Mughrabi and Wuthrich[30] suggest that the fatigue limit of iron single crystals is due to see-saw motion of edge dislocations at low stresses. Screw dislocations are produced when the stress range is increased, and this leads to asymmetric slip and ultimate failure.

Nine[24] argued that, since the greater ease of cross slip in b.c.c. crystals should render them relatively more isotropic than f.c.c. crystals, the anisotropy of fatigue properties in b.c.c. crystals must be due solely to asymmetric slip. Asymmetric slip occurs when crystals are oriented such that the resolved shear stress on the operative slip plane is significantly greater than on any other slip plane. Asymmetric slip derives from the asymmetry in the critical resolved shear stress necessary for screw dislocation glide on the various systems. Nine[19,26] showed that the degree of asymmetry of slip displayed by a single crystal in fatigue can be correlated with a decrease in fatigue life. He showed that iron crystals fatigued in torsion display asymmetric slip behaviour if $\langle 123 \rangle$ is near the stress axis, but do not display similar properties if the stress axis is near $\langle 112 \rangle$.

Tests on iron, molybdenum and niobium single crystals[27, 30] exhibit the fact that rapid localised damage leads to accelerated failure, and large unreversed strains may be a forerunner to crack nucleation. This accelerated nucleation may be inhibited by suppressing asymmetric slip. In niobium[24], for example, asymmetric slip can be suppressed by the concentration of interstitial solute elements. The resulting surface deformation resembles the p.s.b.'s found on the surface of fatigued copper crystals.

2.3.2 Cyclic Response Data of b.c.c. Single Crystals

During tests on α-iron single crystals in torsion, Nine[31] concluded that the observed slip planes differed for forward and reverse shear formations. Similar effects of slip plane asymmetry have been observed on α-iron[30] and also on niobium[33,34] single crystals in push-pull fatigue experiments at constant $\Delta\varepsilon_p$. In these cases, asymmetric slip manifested itself mainly in shape changes of the originally round cross sections which were found to become increasingly elliptical during cyclic hardening.

Mughrabi et al[32] carried out similar tests on α-iron mono- and polycrystals. Figure 8 displays the development of the shape changes (in terms of the ratio d_{max}/d_{min} of the major to minor axis[32] in decarburised α-iron single crystals. For similar values of $\varepsilon_{p_{cum}}$, the shape changes are small at low ε_p, and increase considerably with increasing $\Delta\varepsilon_p$ up to $\Delta\varepsilon_p$ 5 x 10^{-3}. Beyond this amplitude, the dependence on $\Delta\varepsilon_p$ becomes negligible. Because the asymmetric slip which is responsible for the shape changes is a result of the non-equivalence of forward and reverse glide of the screw dislocations, Figure 8 demonstrates the fact that the screw dislocations perform only small displacements in the range of cyclic microstrains of some 10^{-4}, but glide more extensively as $\Delta \varepsilon_p$ approaches values characteristic of cyclic macrostrains ($\sim 5 \times 10^{-3}$).

Mughrabi et al[32] describes the cyclic σ v ε_p curve of decarburized α-iron single crystals as having three distinct sections (Figure 9). In the region of higher applied stress, but very low $\Delta\varepsilon_p$ ($\leq 2 \times 10^{-4}$),σ_s increases sharply[35,36]. This increase is not associated with the cyclic hardening but is due to the growing effective stress σ_s^* as the screw dislocations begin to move[32,35]. Up to $\Delta \varepsilon_p \leq 10^{-3}$, a small plateau is observed. In comparison, f.c.c. single crystals have a relatively extended plateau[37], which is due to p.s.b.'s, but the shorter plateau which is evident in this case is a result of the impeded cyclic hardening due to the limited glide of the screw dislocations[37]. Above $\Delta \varepsilon_p \sim 5 \times 10^{-4}$, cyclic hardening is enhanced by increasing glide of screw dislocations and enforced secondary slip which lead to a cell structure[38]. At all values of $\Delta \varepsilon_p \geq 2 \times 10^{-3}$, the effective saturation stress component σ_s^* is significantly greater than $\sigma_{s,G}$. This behaviour is consistent with conclusions drawn by Mughrabi et al[32], from asymmetric slip. Mughrabi and Wuthrich[30] describe region A as quasi-reversible microstrain, and region C as irreversible cyclic macrostrain. The transition from A to B should therefore define the fatigue limit.

Doner et al[29] investigated fatigue hardening in niobium single crystals. Figure 10 shows the fatigue hardening curves for single crystals oriented for single (S) and multible (M) slip, The $\{101\}$ $\langle 111 \rangle$ slip system is the most active in the multiple slip orientation. Both S and M oriented specimens yield cyclic data which correlates well with the cyclic stress-strain curve equation[8].

$$\tau_s = k\left(\frac{\Delta \gamma_p}{2}\right)^n \qquad \ldots\ldots \quad (2)$$

n is found to lie within the range of values (0.1 - 0.2) reported for most metals[8]. Agrawal and Stephens[41] supply data which suggest that the cyclic stress-strain curve of polycrystalline niobium differs only slightly from that of single crystals of niobium.

Etemad and Guiu [28] reported on the asymmetry of the flow stress in molybdenum single crystals of different axial orientations. A range of constant total strain limits was used. They demonstrated that whenever any plastic strain occurs, then asymmetry of slip is also present. Two crystal orientations were investigated, $\langle 100 \rangle$ and $\langle 110 \rangle$. At high strains, the shape of the cyclic stress-strain curve resembles more that of the unidirectional stress-strain curves. At stresses greater than 300 MPa, the $\langle 100 \rangle$ crystals exhibited a significantly higher cyclic hardening rate. As the cyclic plastic strain $\Delta\varepsilon_p$ increases, so does $\sigma_{tens} - \sigma_{comp}$, the asymmetry of the saturation stress, with a greater asymmetry in the $\langle 100 \rangle$ crystals than in the $\langle 110 \rangle$ crystals at the same strain. Because these data were accumulated while the specimens were in a saturated and reversible state, any influence of variations in dislocation substructure and hardening rates is eliminated.

The initiation of plastic deformation (micro-yielding) must be independent of crystal orientation, due to the fact that the asymmetry appears at about the same stress level for both of the crystal orientations. Therefore, the plastic anisotropy cannot be readily related to the intrinsic lattice friction nor to the asymmetry of slip.

2.3.3 Phenomenological Aspects of Fatigue Cracks in b.c.c. Single Crystals

Single crystals of b.c.c. metals are seen to exhibit asymmetric deformation when fatigued torsionally or axially, even when undergoing net zero strain. This property results from the asymmetric properties of the b.c.c. structure[44]. For asymmetric deformation to occur, dislocations must move on different index slip planes, depending on the sense of the shear stress and cross-slip between slip planes as the stress reverses. Asymmetric deformation leads to large rapid local deformation, and therefore provides convenient sites for crack nucleation and subsequent propagation more quickly than in f.c.c. metals.

Nine[19] demonstrated that iron single crystals, oriented along {123} and fatigued in torsion, exhibited heavily localised deformation at two positions 180° apart over the crystal cross-section. Mughrabi and Wuthrich[32] have explained such an effect of asymmetric slip. The shape changes observed in a specimen of circular cross-section are presented in Figure 11; i.e., as the absolute value of plastic deformation is increased, so is the degree of ellipticity of the specimen cross-section. (Ellipticity is given as $d(\phi)/d_o(\phi)$, where d_o is the original diameter).

Neumann[33] observed a similar effect in push-pull testing of niobium single crystal specimens. The asymmetric deformation manifested itself by forming an S-shaped gauge length in the specimen, and twisting the central region of the gauge length about the specimen axis. This is due to cyclic deformation by slip with a common slip vector b_p on one glide plane in tension and on another in compression being equivalent to slip along b_p on the plane containing b_p and the specimen axis. Figure 12 illustrates how such a deformation, represented by the shearing of parallel lamellae, changes the shape of an undeformed specimen of square cross-section.

Nine[27] has observed the movement of material during torsional fatigue of b.c.c. single crystals. The movement of material in only one crystallographic direction under net zero torsional strain was observed in single crystals of iron, niobium and molybdenum.

Fiducial lines were lightly scribed at equal azimuthal intervals, parallel to the gauge length, around each sample. The displacement of these lines then clearly demonstrated the degree and direction of surface deformation. The active slip systems were identified by measuring the angle of slip lines to the fiducial marks. The fiducial marks reveal 'peaks' of high asymmetric movement. The relative resolved shear stress at the peaks can be determined after identifying the active slip planes present at these peaks. When the relative resolved shear stress is plotted as a function of azimuthal angle about the crystal (Figure 13) the common positions of asymmetric slip can be determined. Nine[27] carried out this analysis, and found that positions corresponding to the maxima of the relative resolved shear stresses corresponded to the locations of the pronounced asymmetric slip lines. The number and position of such maxima are functions of the orientation of the single crystal under examination. Strong asymmetric deformation occurs at two azimuthal positions for iron single crystals near the $\langle 123 \rangle$ axial orientation, and at four azimuthal positions near the $\langle 011 \rangle$ orientation. The $\langle 112 \rangle$ orientation displays no evidence of asymmetric slip.

In iron, slip systems were generally found to act in pairs during fatigue. Three different slip systems were identified: $\{011\}$ $\langle 111 \rangle$, $\{112\}$ $\langle 111 \rangle$, and $\{123\}$ $\langle 111 \rangle$[31]. The orientation dependence of slip planes was found to be a function not only of the resolved shear stress on each slip system but also the critical shear stress, τ_c, necessary to move dislocations in a given direction on each slip plane.

The large localized strains which are accumulated during asymmetric slip provide ideal sites for fatigue crack nucleation.

Single crystals oriented along $\langle 123 \rangle$ appear to display more severe and more localized slip markings than crystals oriented along $\langle 011 \rangle$[27]. The severe asymmetry provides a ratchet mechanism, which produces higher accumulated deformation for a given number of cycles than is apparent in f.c.c. single crystals[25]. Asymmetric deformation further increases the speed of local deformation because the asymmetric slip is ordered, concentrating most of the deformation into narrow bands which move in only one direction. These compounding effects lead to very rapid local accumulation of fatigue damage.

The effect of these shape changes in single crystals may point to detrimental effects on the fatigue life of polycrystals[32]. Shape changes of individual grains, due to grain boundary conditions, are suppressed in the interior of the polycrystal. Therefore, interior grain boundary cracking may not, after all, be due to incompatibility of shape changes. However, conditions at the surface are entirely different. Restrictions are much less severe, and considerable surface roughening (deformation) due to grain shape changes can occur. During experiments on α-iron polycrystals, Mughrabi and Wuthrich[30] observed that grain boundaries were the origin of fatigue microcracks. The experiments demonstrate that the degree of grain boundary fatigue cracking is a direct consequence of the cyclic strain rate.

There is little evidence which exists concerning fatigue crack growth in b.c.c. single crystals. Neumann[45,46] has shown that fatigue cracks in Fe-3%Si grow by a 'coarse-slip' mechanism. This mechanism is also concluded to occur in the fatigue of copper single crystals. Flat fracture surfaces and straight crack fronts were a common feature in the Fe-3%Si samples, relative to the copper crystals, possibly due to the fact that a number of configuarations are possible. Of more significance, however, is the fact that the fatigue cracks in

Fe-3%Si were found to propagate at larger crack opening displacements by a cyclic cleavage mechanism.

Crack propagation rates of austenitic and ferritic stainless steel single crystals have been examined by Rieux et al[47]. Similarly to Neumann[45], they found that macroscopically flat fracture surfaces were obtained for $\{100\}$ fracture planes in the austenitic specimens. However, contrary to Neumann's observations, flat fractures were obtained by propagating fatigue cracks on $\{100\}$ planes in the $\langle 011\rangle$ direction, and also on $\{110\}$ $\langle 111\rangle$ and $\{112\}$ $\langle 110\rangle$. In both b.c.c. and f.c.c. crystals, the macroscopically flattest surfaces were produced in the orientations with the greatest propagation velocity. Rieux et al[47] postulate that the process of fatigue crack growth is due to the accommodation of strain at the crack tip by shear in two bands which are inclined to the stress axis. Neumann argues similarly, except that the strain is accommodated by multiple slip in narrow bands, and not by slip on single planes.

2.3.4 The Effect of Impurities on the Slip Behaviour of b.c.c. Single Crystals

Nine[24] investigated the effect of impurities on the slip behaviour of niobium single crystals. Bowen et al[42] argue that substitutional impurities promote $\{123\}$ slip in niobium. Although asymmetric slip in niobium single crystals was expected, Nine[24] observed p.s.b. behaviour only when a relatively large amount (380 ppm) of interstitial oxygen was added.

Since asymmetric deformation in fatigue is related to cross-slip of dislocations on stress reversal, the change from expected asymmetric deformation to persistent slip with the addition of interstitial impurities suggests that such impurities inhibit cross-slip. This is consistent with the findings of Smialek and

Mitchell[48], who investigated the inhibitions of cross-slip in tantalum.

Clearly, it is imperative to determine whether fatigue crack initiation in technologically useful b.c.c. materials is due primarily to asymmetric slip behaviour, or p.s.b. behaviour due to interstitial impurities.

2.4 Fatigue of f.c.c. Single Crystals

2.4.1 S-N Data of f.c.c. Single Crystals

Roberts and Honeycombe[49] attempted to assess the dynamic stress-strain relationships in aluminium crystals during fatigue in cyclic push-pull tests. A study of changes in the stress-strain hysteresis loops showed how hardening during fatigue was dependent, amongst other things, on the orientation of the crystal relative to the stress axis. The ultimate fatigue life appears to be determined by the degree of plastic deformation during the early part of the fatigue life. All the specimens were tested under constant stress amplitude control, therefore the hysteresis loop width at the beginning of cyclic treatment was always greater for the specimens in the soft (easy glide) orientation. For harder orientations, the hysteresis loop width remained relatively constant. As demonstrated in Figure 14, crystals oriented for deformation by easy glide had significantly shorter fatigue lives than those with their applied stress axes close to the $\langle 001 \rangle$ - $\langle 110 \rangle$ boundary of the stereographic triangle.

Broom and Ham[50] subjected copper single crystals of two orientations to alternating stresses, so that complete failure occurred within 5×10^5 cycles. The applied stress axis of crystal A was within the easy glide region of the stereographic triangle (Figure 15) and failed at a shear stress of ±339 MPa. Crystal B was oriented so that the applied stress axis lay on the ⟨001⟩ - ⟨111⟩ symmetry line of the stereographic triangle; the crystal failed at a shear stress of ±346 MPa. The fatigue lives of the specimens were 4.9×10^5 and 5.2×10^5 cycles respectively. Clearly, it can be argued that there is a slight anisotropic effect on the fatigue life, but more duplex slip should be expected in the B crystals than was actually observed. As expected, the specimens of both Roberts and Honeycombe[49] and Broom and Ham[50] oriented for easy glide failed at significantly lower applied stresses than did crystals of other orientations.

2.4.2 Cyclic Response Data of f.c.c. Single Crystals

The establishment of a stable stress-strain relationship (similar to work hardening in monotonic tensile tests) in the first 20% of life is the first stage of cyclic response in f.c.c. metals. This commonly observed phenomenon can be a function of orientation. The strain hardening rate of high purity copper single crystals during cycles of reversed plastic strain has been studied by Kemsley[51] and Paterson[52], and it is apparent that it depends markedly on the orientation of the crystal with respect to the applied stress axis. While undergoing constant plastic strain amplitude cycling, crystals with axes oriented near ⟨100⟩ or ⟨111⟩ directions display a high rate of strain hardening, while those in the middle of the stereographic triangle, or near ⟨110⟩, show a very low rate of strain hardening, i.e. easy glide, as demonstrated previously. The rate of strain hardening for crystals oriented near the ⟨110⟩ - ⟨111⟩ boundary was observed to be uniform, unlike the crystals oriented near the ⟨110⟩ - ⟨100⟩ and ⟨100⟩ - ⟨111⟩ boundaries, where the strain hardening rate is low at first, and then increases

(Figure 16). It is noteworthy that a crystal lying near ⟨110⟩ displayed a slow strain hardening rate, very similar to an identical specimen in a monotonic tension test. These results were subsequently confirmed by Wadsworth[53] who tested copper crystals in a similar manner. Wadsworth[53] also showed that crystals oriented for duplex slip displayed a greater and more erratic strain hardening rate than those oriented for single slip.

Similar results were obtained in stress controlled cyclic testing of aluminium single crystals by Roberts and Honeycombe[49]. By monitoring the strain width of the cyclic hysteresis loop during the fatigue test, they observed an anisotropy of hardening (Figure 14) similar to that of copper found by Paterson[52]. The most rapid cyclic hardening occurred in crystals oriented on the ⟨100⟩ - ⟨111⟩ boundary. The orientation dependence of the hardening rate could also be correlated with the fatigue life behaviour. Under stress control, it was found that crystals of hard orientations exhibited longer lives than those crystals of soft orientations under similar test conditions. Sastry et al[54] demonstrated that fatigue hardening in silver is orientation dependent. An investigation was carried out under constant shear strain amplitude on two sets of differently oriented crystals (Figure 17). At the same shear strain amplitude, crystals of orientation B display higher strain hardening rates than crystals of orientation A. This is in accord with previously cited works [51-53]. Orientation B is closer to the ⟨100⟩ - ⟨111⟩ boundary, and electron microscopy indicated that for this orientation, deformation produces a greater dislocation density to which can be attributed the more rapid hardening rate. Also, in crystals with the same shear strain amplitude, the saturation stress differed in crystals of different orientation <u>and</u> the dislocation substructures developed at saturation in these crystals were also different. Curiously, the stress-strain curve of silver was plotted as being independent of orientation.

Feltner and Laird[7] showed that deformation in both monotonic and cyclic tests has many similar characteristics, and Bhat and Laird[56] have suggested that the cyclic stress-strain curve is independent of orientation and should be expressed in terms of shear stress and shear strain on the primary slip plane. The cyclic stress-strain curve comprises three regions:-

(a) a region below which p.s.b.'s cannot form,

(b) a plateau region where the saturation stress is independent of applied strain during which stage the reversible plastic strain is carried by the p.s.b.'s, and

(c) a region where the saturation stress increases with the applied shear strain in which the whole of the specimen has the structure of a p.s.b. and the cell size within the p.s.b. can adjust to accommodate the applied strain.

Fatigue deformation and damage in (b) and (c) is confined to p.s.b.'s which lie parallel to the primary slip plane. Assuming a Taylor factor of 3.06, and assuming a random aggregate, the same idea was also applied to polycrystalline behaviour. The longitudinal stress and strain data were converted to shear stress - shear strain data using the equations,

$$\tau = \frac{\sigma}{3.06} \qquad \ldots\ldots (3)$$

$$\Delta\gamma_p = 3.06\,\Delta\varepsilon_p \qquad \ldots\ldots (4)$$

Data for both mono- and polycrystals were plotted on the same axes (Figure 18) and Bhat and Laird[56] concluded that agreement between mono- and polycrystalline data is good.

Similarly, Kettunen[57] compared S-N data of mono-and polycrystal copper. Although the agreement between the two sets of data was not as good as that for iron (see section 2.2) the correlation was acceptable.

2.4.3 Phenomenological Aspects of Fatigue Cracks in f.c.c. Single Crystals

Metallographic evidence concerning the crystallographic effects in the fatigue of f.c.c. single crystals has been obtained from studies of the topographic development of the sides of smoothly polished crystals and of the appearance of fracture surfaces.

A model based on the concept of cross-slip is given by Mott[58] to explain how a slip band can develop into a crack if dislocations in the band are free to move backwards and forwards. All the tests were carried out at the temperature of liquid helium, which establishes the fact that, although thermally activated processes such as the diffusion of vacancies or chemical processes such as oxygen attack can play a role at higher temperatures, they are not essential to fatigue. Mott[58] emphasises the differences in hardening under cyclic and unidirectional straining. These are:-

(a) There is much less gross bending under cyclic stressing, i.e. there are no regions in which the number of dislocations of one sign differs greatly from the number of those of opposite sign, and

(b) The dislocations responsible for hardening in cyclic stressing seem to have a higher density of jogs. This may be due to the repeated cutting of other dislocations.

The behaviour of a metal under cyclic straining displays three stages (see section 2.3.2). Wood[59] shows that, in the intermediate region, some slip lines broaden and become persistent, and therefore slip must be continuing, even though hardness is steady. Mott[60] verifies this claim by showing that the stress (σ_i) necessary to move a dislocation in a work-hardened lattice should, in general, be less than that required to generate new dislocations (σ_i') so that in this intermediate region of the curve the applied stress lies between σ_i and σ_i'.

Friedel[61,62] postulates that, depending on the substructure from which they start, the formation of numerous slip lines in the first stage of slip (Figure 19a) after the first stage of hardening (easy glide) is completed. Slip bands are due to cross-slip[63], which eliminates piled up groups at the ends of the lines, and therefore allows the sources S, S' and S" (Figure 19b) to generate more and more dislocations. Local softening occurs when slip lines broaden into slip bands. Broadening is probably due to cross-slip, i.e. the continual backwards and forwards motion of a dislocation on a long slip line AB (Figure 20) past a neighbouring short line CD, will eventually lead to cross-slip and the consequent annihilation of the dislocation. This then allows S to generate more dislocations.

Bowden and Tabor[64] observed how clean fatigued metal surfaces adhere when they touch. This, and the fact that fatigue cracks are normally initiated on the surface led Forsyth[65] to look for and observe extrusions. Extrusion formation is suppressed in some alloys by cooling, but both Forsyth[65] and Mott[58] postulated that extrusions are present whenever fatigue cracks initiate. Wood[59] suggests they may not always be apparent because of immediate oxidation in air. Cottrell and Hull[66] have suggested a mechanism by which extrusions and intrusions can occur as a result of slip on two sets of planes (Figure 21). Mott[58] and Fujita[67] suggest a more complex alternative, where two edge dislocations of opposite signs are on planes 0.1 nm or less apart, and attract each other so strongly that a crack is opened up and other dislocations moving along the same planes move into the crack and make it wider. If this is correct, then the condition for the initiation of fatigue is that slip should occur on two parallel planes within a few atomic distances of each other.

Forsyth[65] reports that the extrusion effect in aluminium alloys is usually associated with some persistent condition of the slip bands. It appears that extrusion may occur when the fatigue stress produces any changes in structure resulting in local softening. Agreeing with Cottrell[66], Forsyth[65] proposed that the slip process which results in extrusion appears to be in the nature of reverse glide in which two sets of planes near to each other slip in preferred directions.

Avery et al[68] established the importance of cross-slip in the slip band extrusion process. They fatigued copper single crystals of two orientations, both orientations intended to give single slip. Orientations of crystal A had a Schmid factor of 0.21 on the cross-slip system, while crystals of orientation B had a cross-slip Schmid factor of 0.026. It was shown that the slip band extrusion rate, after the crystals had cyclically hardened, was a function of the shear stress on the cross-slip system (Figure 22). The rate of topographical development was greater in crystal A (which possessed the larger cross-slip Schmid factor). Slip band extrusion and intrusion can occur rapidly, in bursts of dislocation movement. More important, a strong orientation dependence of the rate is indicated. Further, it suggests that the applied stress resolved onto dislocations in the cross-slip system is more significant in determining the extrusion-intrusion rate than stress on the primary system acting to drive dislocations into the cross-slip system.

Neumann[45,70] has attempted to describe Stage II propagation in terms of strain bursts along intersecting slip bands at the crack tip. Clearly this involves the concept that Stage II propagation occurs via. intersecting cross-slip process at an angle to the crack plane. This is consistent with the 'plastic blunting' mechanism of Laird and Smith[71]. Neumann[45] has published an analysis of the effect of crack plane orientation on the fracture surface appearance of fatigued copper single crystals. It is referred to as the 'coarse slip' model. By testing notched samples in 4-point bending, the crack plane and propagation direction should be clearly defined. The only crack planes which gave macroscopically flat fracture surfaces were the $\{100\}$ planes. Propagation on these planes in either the $\langle 100 \rangle$ direction (notch root parallel to the $\langle 010 \rangle$ direction) or the $\langle 110 \rangle$ direction (notch root parallel to the $\langle 110 \rangle$ direction) was found to give flat fracture surfaces with reasonably straight crack front striations (Figure 23). According to the coarse slip model (Figure 24) which Neumann[45] apparently verified with SEM photographs of copper single crystals (Figure 25) fatigue crack propagation occurs by slip in alternating coarse bands at the crack tip to cause separation. The crack front should therefore be determined by the intersection of $\{111\}$ planes, i.e. the microscopic crack front should be aligned along a $\langle 110 \rangle$ direction. For a crack to have a macroscopically straight front, it should consist of segments of two different $\langle 110 \rangle$ directions. Neumann [45] observed that crack propagation in the $\langle 100 \rangle$ direction produces striations that macroscopically parallel to $\langle 010 \rangle$ but consist of small segments parallel to the $\langle 110 \rangle$ directions (Figure 26).

2.5 Summary of Fatigue of Cubic Single Crystals

2.5.1 S-N Data

There is little quantitative information concerning the effect of crystal orientation on the S-N curve. However, it is qualitatively clear that the curve is orientation dependent both for b.c.c. and f.c.c. metals. Asymmetric slip in b.c.c. metals produces more rapid failure than in f.c.c. materials in the same conditions. In b.c.c. and f.c.c. materials where asymmetry of slip is inhibited, the S-N relationships which are exhibited can be reduced as a first approximation to a curve on which the stress parameter is the resolved shear stress amplitude on the primary slip plane. Similarly, the fatigue limit of metal single crystals of both b.c.c. and f.c.c. structures have been shown to be orientation dependent.

2.5.2 The Cyclic Response of Cubic Single Crystals

The rate of cyclic hardening is orientation dependent. The cyclic work hardening rate of b.c.c. single crystals in which asymmetric slip is possible is proportional to the degree of asymmetry of slip. In f.c.c. structures, the cyclic work hardening rate increases with slip activity on secondary slip systems, and crystals oriented with the stress axis near to the $\langle 100 \rangle$ - $\langle 111 \rangle$ boundary of the stereographic triangle harden more rapidly than those crystals oriented initially for easy glide. The amplitude of the resolved shear stress at saturation appears to be a function of the applied shear strain amplitude for f.c.c. metals only. The possibility of asymmetric slip in b.c.c. single crystals produces an orientation dependence of the saturation shear stress amplitude, and of the cyclic shear stress-strain curve. The effect of multiple slip on the stress-strain curves of b.c.c. and f.c.c. mono- and polycrystals is difficult to assess due to lack of data.

2.5.3 Phenomenological Aspects of Nucleation and Growth of Fatigue Cracks in Cubic Single Crystals

Fatigue crack nucleation is exacerbated by irreversibility of plastic strain, which produces intrusion-extrusion pairs, so that a site for crack initiation is provided. In some orientations, b.c.c. single crystals rapidly accumulate highly localised surface damage as a result of asymmetric slip. P.s.b/'s are the source of fatigue crack nucleation in f.c.c. single crystals, and, as cross-slip increases, so does the rate of nucleation. As the shear stress on the cross-slip system increases, so does the rate of intrusion-extrusion topography within a given slip band. Therefore, the rate of development of such a topography must be orientation dependent.

Stage I propagation occurs along slip bands parallel to the $\{111\}$ primary slip planes. The "coarse slip" or "plastic blunting" models require the activation of two intersecting slip systems at the crack tip. The influence of orientation on Stage II crack growth is obvious, due to the strict conditions required to produce macroscopically flat fracture surfaces and straight crack fronts in single crystals of copper.

2.5.4 Quantitative Aspects of Crack Growth in Cubic Single Crystals

This section refers only to f.c.c. single crystals, due to the lack of available data correlating fatigue crack growth with texture in b.c.c. single crystals. When the stress axis lies near to the $\langle 100\rangle - \langle 111\rangle$ boundary of the stereographic triangle, rapid Stage I fatigue crack growth rates can be seen. This is reflected in the "m" parameter in the Paris equation. It has been suggested[79], however, that although fatigue crack propagation rates in f.c.c. single crystals are orientation dependent, this dependence is rather weak.

2.6 Fatigue Properties in Textured Polycrystals

2.6.1 The Interactions Between Texture and Fatigue in Cubic Metals

Clearly, from previous evidence (see section 2.4) the mechanical and physical properties of single crystals are orientation dependent. However, in a bulk material, if the orientations in the aggregate are random,. then the average properties of the material will be isotropic. Only when a degree of preferred orientation (texture) exists in the material will the properties display any anisotropy. It is usual to analyse polycrystalline properties in terms of a random aggregate of crystals. However, the production of specimens for any type of test necessiates, at some stage, some thermomechanical processing, and it has been long-established[72] that such treatments induce some degree of anisotropy. Bhat and Laird[56] applied Taylor's orientation factors for polycrystals, and analysed data in terms of resolved shear stress and shear strain. Their findings are coincident with those of Laird[73]; i.e., the cyclic stress-strain curves of polycrystals are very similar to the curves of the equivalent monocrystals oriented for slip on one system only. Now, the Taylor factor is not isotropic for textured materials, so the macroscopic cyclic stress-strain curve will be determined by the relative orientation of the stress-strain axes to the principal components of the texture.

Considering dislocation behaviour in cyclic deformation, Feltner and Laird[74] showed that there exist many similarities between the mechanisms of Stage I monotonic hardening in f.c.c. single crystals and low strain fatigue hardening in both mono- and polycrystals, and between Stages II and III monotonic hardening in single crystals and hardening behaviour in high strain fatigue. Similarities between mono- and polycrystalline behaviour in fatigue are not confined to dislocation behaviour. Kettunen[57], for example, has shown that similarities also apply to fracture aspects of fatigue. Furthermore, Laird[73] has shown that the fatigue strain limit predicted from the cyclic stress-strain curve for copper single crystals oriented for single slip agrees closely with that for polycrystalline material. No texture analysis was carried out during any of these tests, but clearly, the results display a certain amount of anisotropy. One can only conclude that the polycrystalline specimens used did not consist of random aggregates of crystals, but were preferredly oriented (textured).

Some reports have indicated that the fatigue process itself may induce some degree of preferred orientation[76, 77]. Hayashi and Suzuki[76] cycled polycrystalline copper under fully reversed loading and, after recording texture changes, concluded that the fatigue process tends to randomise any texture which may be present. However, Inakazu and Yamamoto[77] disagreed with this conclusion. They found that under torsional fatigue, the most stable orientations were $\{110\}\langle 111\rangle$ and that other crystals tended to rotate towards these.

They went on to argue that the fatigue limit is controlled by the texture, with the stable orientation giving a higher value for that limit. Inakazu and Yamamoto[77] further postulated that grain rotations require increasing components of cross-slip, which provides the topography required for crack nucleation.

However, Burke[78] found that the texture severity actually decreased after fatigue cycling. It is obvious that more work is required before understanding of the development (or otherwise) of texture during fatigue is achieved.

2.6.2 The Influence of Texture on the Fatigue Life of Cubic Metals

Le May and Nair[80-82] tested, in fully reversed bending, three f.c.c. materials, namely Al-2.5%Mg, tough pitch copper and super pure aluminium. After cutting specimens at 0° or 90° to the rolling direction, fatigue specimens were obtained which were identical in all respects save orientation. Typical results are shown in Figure 27. {110} ⟨112⟩ textures were exhibited by the cold rolled sheet materials, while the annealed copper showed a {100} ⟨001⟩ (cube) texture. The annealed aluminium displayed only a random texture. The S-N curves for the annealed materials were coincident, but the transverse specimens of the cold rolled materials clearly exhibited better fatigue properties than the specimens cut at 0°. However, these experiments did not rule out the possible effects of microstructural anisotropy due to grain shape variation. For example, in the cold rolled fatigue specimens, differences in both texture and grain size (intercept parallel to stress axis) are manifested.

Nevertheless, Le May and Nair(80-82) postulated that the differences in fatigue properties were due to texture, and not microstructural inhomogeneities.

Burke(78) experimented briefly with SAE 4161, and came to the conclusion that, even in heavily textured material, the fatigue behaviour would be dominated by the inclusion content and morphology.

2.6.3 Phenomenological Aspects of Fatigue Failure in Textured Cubic Materials

Le May and Nair(80-82) determined that the modes of crack initiation and propagation are determined by texture. In the cold rolled materials, many surface microcracks were linked, and crack initiation took place, presumably along substructure cell boundaries as described by the 'H' mechanism of Wood(4). Crack nucleation in the annealed cube textured copper occured by the development of p.s.b.'s. In the randomly textured alloy, surface damage consisted of both types, the 'H' mechanism being more prominent at higher stress levels(81). Due to the constraints imposed on the crack tip stress field by the texture, the mode of crack propagation, particularly the transition from Stage I to Stage II, is affected. It should be remembered that even in very severe textures, only a small volume of material is suitably oriented, but at least the mechanism by which texture may control crack initiation is indicated. Arnell and Teer[83] showed that topographic development at surface grains was consistent with the slip systems that would be activated in a single crystal of that orientation under the same applied stress, except in grains in which the stress axis lay near the $\langle 100 \rangle$ - $\langle 110 \rangle$ boundary of the stereographic triangle. Therefore, although it may not be possible to consider crack propagation in terms of localised events, the nucleation and very early growth of fatigue cracks have been correlated with the orientation of individual grains.

Laird[2] has commented that analysis of Stage I failure is speculative due to the fact that the fracture surfaces are relatively featureless. Forsyth et al[121] suggested that crack propagation involves two competing processes, producing a zone of ductile fracture and a zone of brittle cleavage within each fatigue striation. It is recognised that Stage I failure occurs on slip planes, probably by a variation of the sliding-off mechanism. However, the appearance of crystallographically faceted fracture surfaces does not necessarily imply failure in the Stage I mode. Garrett and Knott[122] found facets on the fatigue fracture surface of another aluminium alloy. Although little plastic deformation was evident, crack propagation due to cleavage was discounted on the basis of environmental effects.

Similar crystallographically dependent mechanisms have been shown to operate in b.c.c. materials. Fukui et al[13] observed that fatigue crack propagation in Fe-3%Si tended to occur on $\{100\}$ or $\{110\}$ planes. The striation pattern was complex because the crack propagation, which is initially inclined to the {100} and {110} planes, tends to produce a curved surface, so that the crack plane bent around it until it became parallel to the crystallographically specified plane. The striation direction was identified as being parallel to the line of intersection of the macroscopic fracture plane and the {100} or {110} planes. Richards[21], with similar material, found that the fracture surface topography was a function of the orientation of the stress axis with respect to the texture.

2.6.4 Quantitative Aspects in Cubic Metals

2.6.4.1 Yield Stress

The Sachs' model[84] assumes that all crystals deform independently, but Taylor's[85] concept is that each crystallite undergoes the same total deformation as the polycrystalline specimens overall. Taylor[85] further suggests that, during slip, the operative combination of slip systems is such that boundary conditions with the least total shear are satisfied. Bishop and Hill[86] compared the work of Sachs[84] and Taylor[85] and arrived at the conclusion that Taylor's theory has much the greater validity.

Several attempts[87-89] have been made to use texture data in the prediction of material properties. When Bishop and Hill's[86] analysis was applied, there was definite correlation between the various sets of results. The relative magnitudes of the errors found by Kallend and Davies[89] indicate that truncation of the W_{lmn} series at the twentieth order introduces no greater errors than those inherent in the x-ray measurements. Davies et al[90] demonstrated that satisfactory approximations of predicted properties can be obtained by truncating the analysis at the fourth order term. If the angular variation of the elastic modulus in the plane of the sheet is known, then an analytical prediction of the variation of plastic properties can be made. Advantage is taken of the fact that, for cubic metals, both the elastic and plastic properties are influenced predominantly through the zeroth and fourth order coefficients[91].

2.6.4.2 Elastic Modulus

Hill[92] proposed taking the mean value of the Voigt[93] and Reuss[94] averages as a reasonable approximation for calculating the elastic properties of polycrystals. To calculate the elastic modulus from texture data, only fourth order (see secion 3.4.2) coefficients are required[90,95]. Bunge and Roberts[87] and Davies[72] used quantitative texture data to predict the angular variation of the elastic modulus in the sheet plane for cold rolled and Al-killed steel.

2.7 The Consideration of Texture and Anisotropy in Relation to Metal Forming

2.7.1 Textures in Rolled and Annealed b.c.c. Materials

2.7.1.1 Introduction

After forming, most metallic solids are substantially both crystallographically and microstructurally anisotropic. Crystallographic texture is normally described by linking together a crystallographic specification with a feature of the specimen goemetry. Sheet textures are described as being of the {hkl} ⟨uvw⟩ variety. Then the {hkl} planes are considered to lie in the rolling plane, with their ⟨uvw⟩ directions parallel to the rolling direction. Deviation of the {hkl} planes from the rolling plane can be as great as 50° about the rolling direction[97], and a spread of the ⟨uvw⟩ directions from the rolling direction is usual. If rolling is followed by annealing, then a recrystallisation texture is formed, which is usually in the form of a rotation around common ⟨110⟩ poles of the original rolling texture[96].

The "{hkl} ⟨uvw⟩" notation is described as the "ideal orientation" for sheet metals. However, preferred orientations are best described by the c.o.d.f. (see section 3.2).

2.7.1.2 Textures in Cold Rolled b.c.c. Sheets

The predominant texture in cold-rolled iron and steel is {100} ⟨011⟩. Typical pole figures for iron are reproduced in Figure 28, in which the subsidiary orientations {112} ⟨110⟩ and {111} ⟨112⟩ are indicated as well as the principal one. Barrett and Levenson[98] deformed single crystals of iron to simulate the behaviour of grains when an aggregate is rolled, and found that rotations of crystals and crystal fragments had final orientations within the dense areas of the aforementioned pole figures for polycrystalline material. They also suggested that the {111} ⟨110⟩ orientation described the remainder of the texture. Although the deviation of {hkl} planes about the rolling direction may be as great as 50° (see section 2.7.1.1), this is sensitive to the intensity level at which the spread is measured, and this spread almost certainly decreases with increasing deformation[99]. Goss[100] and Nusbaum and Brenner[101] have studied cold rolling textures of low carbon steel sheet and found that as the temperature of cold rolling is reduced, the degree of preferred orientation increases at a given reduction, although the texture type appears to remain constant. Goss[100] rolled low carbon steels at a number of temperatures, and found the normal b.c.c. texture at low temperatures, a 'random' texture at intermediate temperatures, and a {110} ⟨100⟩ texture at temperatures greater than 370°C. He came to the conclusion that the effect could be due to a temperature-dependence of deformation modes, but Nusbaum and Brenner[101] suggest it could also be due to a surface-texture effect.

The development of rolling texture in iron and steel has been closely followed by Bennewitz[99], Haessner and Weik[102], and Möller and Stablein[124]. All three descriptions of texture development are in close accord. The starting texture in each case rotated to {112} ⟨110⟩, which subsequently rotated to about the rolling direction {001} ⟨110⟩ or {111} ⟨110⟩.

The cross-rolling texture of b.c.c. metals consists largely of the {100} ⟨011⟩ orientation. Since only this component of the straight rolling texture has four-fold symmetry about the sheet normal, this is to be expected.

2.7.1.3 Textures in Hot Rolled b.c.c. Sheets

In contrast to the amount of published data relating to cold-rolled and cold-rolled and annealed low carbon steel sheets, there are no quantitative texture data available in the literature for hot-rolled steels. This is probably due to the fact that textures produced in commercially hot-rolled steels have generally been considered weak[39]. It has been established[69] that process variables such as rolling temperature, holding time after rolling, reduction per pass and the number of passes determine the structure and texture of hot rolled steel. Several workers[39,40,43,44,55] have studied the effect of the finish-rolling temperature and concluded that the texture falls into one of three types according to whether the material is in the wholly austenitic, duplex, or wholly ferritic condition respectively, at end of rolling. Hancock and Roberts[43] have shown that in the case of a 0.4%C rimming steel, there is little change in the rolling texture between ambient temperature and 680°C. Morris[69] demonstrated that the severity of texture in a carbon-maganese-niobium-vanadium steel was temperature dependent.

2.7.1.4 Textures in Annealed b.c.c. Sheets

Annealing textures for steel sheet have been quantitatively described by Morris and Heckler[75] and Bunge and Roberts[87]. Goodwill[79] investigated the effect of prior rolling reduction on the annealing textures developed in a rimmed, an aluminium-killed and a titanium-bearing low carbon steel. At low reductions around 20%, all the steels retained, essentially, the rolling texture, but at intermediate reductions, each of the materials exhibited a unique recrystallisation behaviour. Rimmed steel selectively developed orientations in order of decreasing energy of cold work, giving rise to a prominent $\{111\}$ $\langle uvw \rangle$ fibre texture, and there was no evidence of recovery or growth constraints. The aluminium-killed steel showed a similar texture type, but recovery and early subgrain development were inhibited by aluminium nitride particles, giving rise to preferred development of those orientations with high stored energy. The titanium-bearing steel showed different peak orientations in the $\{111\}$ $\langle uvw \rangle$ zone, and a more severe* texture.

2.7.2 The Influence of Microstructure on the Directionality of Properties

Besides crystallographic anisotropy, directionality of mechanical properties can also be due to microstructural anisotropy[110,111], where there is considerable variation in grain size and shape, and the phases are arranged non-randomly. This is normally due to the directional nature of most thermo-mechanical processes, resulting in

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*TSP - the texture severity parameter - is a quantitative measure of the strength of a texture. It is defined in section 3.2.1.

elongated phases in the product.

Planes of weakness arise along the matrix-inclusion interface (Figure 29) where inclusions are elongated in the direction of working, and differences in the volume and shape of inclusions clearly influence the fractureproperties.

Alternate layers of ferrite and pearlite are due to pearlite bonding in steels, which then behaves similarly to a composite lamellar material. Changing the shape of grains can be expected to influence grain-size dependent properties[112], since the effective grain size is a function of the testing direction.

A serious difficulty surrounding the study of directionality in steels lies in isolating the influence of individual microstructural features, although Davies[112] has shown that standard quantitative observations on the shape and distribution of phases can reveal the degree of anisotropy present.

2.7.2.1 Pearlite Banding

Layered pearlite in ferrite is exhibited to some degree by all forms of carbon steels (Figure 30). Cairns and Charles[113] concluded that it is due to dendritic segregation of slowly diffusing elements such as manganese in the ingot structure. During heat treatment, the carbon distribution is affected so that ferrite-pearlite banding occurs in the rolling direction.

Grange[114] compared longitudinal and transverse properties of four steels which ranged from 'clean' and unbanded to 'dirty' and banded. Banding had no effect on the tensile strength or the yield strength, and further, no effect on tensile properties was displayed, although ductility was reduced. However, the banded 'dirty' steel had poorer properties all round than the banded 'clean' steel, which indicates that interaction effects were significant. However, Heiser and Hertzberg[125] attributed anisotropy of ductility in ferrite-pearlite and ferrite-martensite steels to aligned inclusions alone, after testing banded 'dirty' and homogenized 'dirty' steel.

2.7.2.2 Inclusions

Second phase particles in alloys have important effects on strength, toughness and ductility. Alloys rely heavily on the strengthening effects of finely dispersed particles. Dispersion-strengthened alloys commonly suffer some embrittlement because of the increased strength[103]. However, quantitative data on the mechanical effects of inclusions on the directionality of mechanical properties are meagre.

The fatigue limit of a steel of a given tensile strength decreases with increasing inclusion size. Some workers have shown that the fatigue limits of high tensile steels depend on the size[104] and shape[105] of non-metallic inclusions. When inclusions in original steel billets are elongated during the forming of subsequent bars, they present a larger flow area on a transverse section than on a longitudinal one, and this gives rise to significant differences in the fatigue limits of specimens cut in the transverse and longitudinal directions. Boyd[106] carried out rotating-bending fatigue tests on En 25 steel specimens, cut either longitudinally or transversely from the initial worked blank and heat treated to give tensile strength between 930 MNm^{-2} and 2000 MNm^{-2}. The fatigue limits of the longitudinal specimens increased as the tensile strength increased, but the fatigue limit of the transverse specimens remained constant irrespective of tensile strength.

It would seem that although inclusions are of little importance in determining the fatigue limits of longitudinal specimens, their projected shapes on transverse sections are sufficient to determine the onset of cracking, which occurs at a stress level less than that necessary to cause slip band cracking in the matrix.

Embury et al[115] have demonstrated that weak interfaces in mild steel laminates can increase the work required to produce a fracture, due to delamination at the interface which relaxes the state of triaxial tension ahead of the crack.

It is difficult to negate the effect of inclusions by heat treatment. They are best controlled at source. (See section 4.3).

2.7.2.3 Grain Size and Distribution

Variation in grain size and shape are normally associated with single phase metals. Variation can be estimated using standard quantitative metallographic techniques.

The Hall-Petch relation[116,117] describes the experimentally observed dependence of the lower yield stress, σ_y, on the grain diameter, d, in iron as:

$$\sigma_y = \sigma_o + kd^{-\frac{1}{2}} \qquad \text{......(5)}$$

where σ_o and k are constants. For a material with an equiaxed grain structure, the relationship adequately describes the observed behaviour, but highly elongated grains might be expected to induce yield stress anisotropy in a material due to the anisotropy apparent in the grain diameter d. In support of this, Grange and Mitchell[118] found the longitudinal yield strength in an ausformed steel correlated well with the grain size measured in the thickness direction.

The probability of grain boundary failure is increased in rolled materials, due to the fact that a considerable proportion of the grain boundary area is parallel to the rolling plane. In a series of Charpy tests on longitudinal and transverse specimens of a 0.07%C, 1.4%Mn control rolled steel, Coleman et al[119] observed extensive fissuring along planes parallel to the rolling plane.

The effect of grain size anisotropy is due to inhomogeneous deformation, non-uniform thermo-mechanical treatment through the bulk, cr the effects of segregation through recrystallisation. Therefore, careful heat treatment can reduce the effects of such anisotropy.

2.8 The Correlation of Fatigue Failure and Texture

Despite the fact that tensile anisotropy arising from texture is now reasonably well understood, and may be used to advantage in the forming of sheet metal components[107,108], little has been done to examine the effects of preferred orientation on fatigue properties, particularly in b.c.c metals.

Normally, fatigue specimens of the material to be tested are taken at various orientations with respect to the rolling direction, and their S-N curves recorded. These curves, and the corresponding cyclic stress-strain curves display any differences in fatigue behaviour which may be present. Texture data are also recorded from the material, and comparisons between fatigue properties and texture can be made (see Chapters 3 and 4).

Le May and Nair[80] carried out a simple investigation on three f.c.c. materials. They clearly showed that cold rolling of Al-Mg gives rise to considerable anisotropy of fatigue properties. The same material was much stronger in the transverse direction than the longitudinal direction, which was explained in terms of the operative fatigue damage and crack propagation mechanisms and the existing preferred orientation. Cold rolled and annealed Al-Mg was weaker in the transverse direction: the workers claim this is due to elongated CuO inclusions lying in the longitudinal direction. Annealed superpure aluminium (effectively inclusion free) exhibited a slightly higher tensile strength in the transverse direction. Also, annealed copper with a typical planar isotropic cube texture was produced which displayed identical longitudinal and transverse fatigue properties.

CHAPTER 3

The Analysis of Texture Date and the Description of Crystallite Orientation in Textured Materials

3.1 Analysis

Three independent parameters are required in order to describe completely the orientation of a single crystal within a polycrystalline sample. The orientation of a particular crystal direction with respect to sample axes may be described by two angular co-ordinates, but a third angular co-ordinate is required to define the rotational position about the axis. To specify the orientation of each crystal in a polycrystalline material is impractical, but a useful description of the material texture can be made by specifying the distribution and relative frequencies of crystallites which occur in the various orientations.

3.2 The Crystallite Orientation Distribution Function

The crystallite orientation distribution function (c.o.d.f.) is the function which is capable of describing the distribution of crystallites in a polycrystalline material. Both Roe[120] and Bunge[109] showed that the c.o.d.f. could be derived from ordinary pole figures. The Roe[120] convention (Figure 31) is used throughout this work.

The c.o.d.f. describes the probability of a unit volume crystallite having an orientation with respect to a set of arbitrary reference axes in the sample material. Roe[120] used the three Euler angles ψ, θ and ϕ to relate the crystallite axes to the specimen axes (Figure 32). The angles θ and ψ define the orientation of the $\bar{z}$ axis of the crystallite in the sample, and ϕ specifies the rotation of the crystallite about this axis. The crystal axes $O\bar{X}$, $O\bar{Y}$ and $O\bar{Z}$ are made to coincide with the [100], [010] and [001] directions in cubic crystals, and the reference axes in the sample refer to the rolling and transverse directions and the rolling plane normal, respectively.

The c.o.d.f. $w(\psi, \theta, \phi)$ expresses the probability of a unit volume crystallite having an orientation with respect to a set of reference axes, specified by the Euler angles ψ, θ and ϕ. It is such that

$$\int_0^{2\pi} \int_0^{2\pi} \int_{-1}^{1} w(\psi,\theta,\phi)\, d\psi\, d\xi\, d\phi = 1 \qquad \ldots\ldots (6)$$

where $\xi = \cos\theta$.

The c.o.d.f. is generated as a series expansion with coefficients W_{lmn}, and given as a series of generalised spherical harmonics in the form

$$w(\psi,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} W_{lmn} Z_{lmn}(\xi) \exp(-im\psi) \exp(-in\phi) \cdots\cdots (7)$$

where W_{lmn} are the series coefficients, and $Z_{lmn}(\xi)$ is a generalisation of the associated Legendré polynomial.

The orthotropic texture displayed by materials in the cubic system contains all the distinct orientations in the range

$$\begin{aligned} 0 &\le \psi \le \pi/2 \\ 0 &\le \phi \le \pi/2 \\ 0 &\le \theta \le \tan^{-1}\left(\frac{1}{\cos\phi}\right), \quad \phi \le \pi/4 \\ 0 &\le \theta \le \tan^{-1}\left(\frac{1}{\sin\phi}\right), \quad \phi \ge \pi/4 \end{aligned} \qquad \ldots\ldots (8)$$

Therefore, specific ideal orientations in Euler space are represented by points in the ranges

$$\begin{aligned} 0 &< \psi < \pi/2 \\ 0 &< \theta < \pi/2 \\ 0 &< \phi < \pi/2 \end{aligned} \qquad \ldots\ldots (9)$$

Using contour mapping, the c.o.d.f. plots the probability of a crystallite having a given orientation in Eulerian space, with unity representing the intensity of a sample displaying no texture. Davies et al[123] have used the points described in equation (9) to present indexing charts (Figure 33) taken at constant ϕ sections of Eulerian space, so that the task of interpreting the c.o.d.f.'s is simplified.

In practice, the series shown in equation (7) is truncated at a point where truncation and experimental errors are of similar orders of magnitude[126], i.e. the twentieth order.

$$w(\psi,\theta,\phi) = \sum_{l=0}^{20} \sum_{m=-l}^{+l} \sum_{n=-l}^{+l} W_{lmn} Z_{lmn}(\xi) \exp(-in\phi) \exp(-im\psi) \cdots\cdots \quad (10)$$

A more detailed analysis of the relationship between the crystallite pole distribution and the c.o.d.f. can be found in Appendix 1.

3.2.1 Texture Severity Parameter

The texture severity parameter (T.S.P.), derived by Kallend[126], gives the mean standard deviation of the c.o.d.f. from that of an untextured sample, i.e.

$$(\text{T.S.P.})^2 = \frac{1}{8\pi^2} [w(\Omega) - 1/8\pi^2]^2 \, d\Omega \quad \cdots\cdots (11)$$

where Ω is a general orientation. Therefore, the T.S.P. can be found by utilizing the texture coefficients and using:

$$(\text{T.S.P.}) = 4\sqrt{2}\,\pi^2 \left[\sum_{l=1}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} W^2 \right] \quad \cdots\cdots (12)$$

The T.S.P., and the maximum function height of the c.o.d.f. (see section 3.2), give a ready comparison of the severity of various textures, without recourse to detailed analysis.

3.2.2 The Integration of the c.o.d.f.

Kallend[126] also derived a method, by integration of the c.o.d.f. between stipulated limits, of calculating the quantity of material which lies in a particular region of Euler space; for example, near certain ideal orientations. Since the integral over all of Euler space is unity (see equation (6)), a direct measure of the volume fraction of crystallites in the stipulated orientation is obtained.

3.3 The Accuracy of Texture Data

There are two sources of error in the c.o.d.f., namely, truncation and experimental errors. Examples of experimental errors are misalignment of texture sample slices, inaccurate cutting of the specimen slices, incorrect setup of the specimen in the goniometer circle, and statistical variations of the intensity of the x-ray beam (although this was twice measured over the time period of a typical pole figure run and found to have a standard deviation of $<0.7\%$).

Truncation errors are introduced into both pole figure data and the c.o.d.f., due to the use of the finite series. The W coefficients may be determined to the 22nd order from only two independent pole figures, but a third pole figure allows the errors to be estimated, utilizing the coefficients previously found by a least squares regression technique[126]. The truncation errors in the pole figure data are easily obtained because both the complete and truncated functions are available.

The truncation error in $w(\psi, \theta, \phi)$ is more difficult to obtain due to the fact that the series expansion is the only available estimate of the function. Kallend[126] describes a method which involves extrapolation of the series coefficients from the known values, and is used in the computer analysis of the c.o.d.f. (see section 5.2).

3.4 The Application of the c.o.d.f. to the Prediction of Cyclic Properties

The c.o.d.f. is a completely quantitative description of the texture, therefore it is not restricted merely to describing textures, but is flexible enough to be used for a variety of applications.

3.4.1 Prediction of Mechanical Properties

Kallend and Davies[127], basing their calculations on the orthogonality relation of spherical harmonics as shown by Pursey and Cox[91], described the prediction of material anisotropy. $g(\psi, \theta, \phi)$, the anisotropic property of a single crystal, may be developed as a series of spherical harmonics with G_{lmn} as the coefficients.

The average value is

$$\bar{g} = 4\pi^2 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} G_{lmn} W_{lmn} \qquad \ldots\ldots (13)$$

in the direction $\psi = 0$, $\theta = 0$, $\phi = 0$.

To calculate the value of $\bar{g}$ at angles of α, β and γ with respect to the reference axes $\bar{X}$, $\bar{Y}$, $\bar{Z}$, the axes $\bar{X}$, $\bar{Y}$, $\bar{Z}$ are rotated by α, β, γ. The new W coefficients (W') are defined by

$$W'_{lmn} = \left(\frac{2}{2l+1}\right)^{\frac{1}{2}} \sum_{p=-l}^{l} W_{lpn} Z_{lpm}(\cos\beta)\exp(-ip\alpha)\exp(-im\gamma) \qquad \ldots\ldots (14)$$

which reduces to

$$\bar{g}(\alpha) = 4\pi^2 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} G_{lmn} W_{lmn} \cos n\alpha \qquad \ldots\ldots (15)$$

when anisotropy in the plane of a flat material only is considered[126].

3.4.1.1 Prediction of Elastic Modulus

The orientation dependence of the elastic modulus in cubic crystals is governed by the function

$$\gamma = l_1^2 l_2^2 + l_2^2 l_3^2 + l_1^2 l_3^2 \qquad \text{..... (16)}$$

where l_i is the direction cosine relating the direction considered with the x_i axis. This function can be shown to depend only on zeroth and fourth order spherical harmonics[91]. The mean value of $\gamma(\psi, \xi, \phi)$ (the modulus distribution function) over all the crystals in a textured sheet for a given direction in the sheet is given from equation (15) i.e.

$$\bar{\gamma}(\alpha) = 4\pi^2 \sum_{l=0}^{4} \sum_{m=-l}^{l} \sum_{n=-l}^{l} G_{lmn} W_{lmn} \cos(m\alpha) \qquad \text{..... (17)}$$

The Voigt[93] upper bound solution determines the elastic stiffness by considering that all grains undergo an identical strain, and is given by:

$$E_v = \frac{(C_{11} + 2C_{12})(C_{11} - C_{12} - 3\bar{\gamma}C)}{(C_{11} + C_{12} - \bar{\gamma}C)} \qquad \text{..... (18)}$$

where $C = C_{11} - C_{12} - 2C_{44}$.

The Reuss[94] lower bound solution effectively averages the elastic compliance, assuming that all grains undergo the same stress, and is given by:

$$E_R = 1/[S_{11} - 2(S_{12} - S - 0{\cdot}5S_{44})\bar{\gamma}] \qquad \text{..... (19)}$$

Hill[92] indicates that the arithmetic mean of the two previous postulates demonstrates more clearly the behaviour of weakly textured materials.

Average elastic moduli calculations were carried out using the program written by Kallend[126] and modified by Bateman[128].

3.4.1.2 Prediction of Yield Stress Anisotropy

In order to correlate texture with the cyclic yield stress, the yield stress anisotropy was predicted. Various assumptions are required. The lower bound Sachs'[84] model assumes that all the crystals in the aggregate deform independently, and that the tensile stress is the same in each grain. The upper bound Taylor[85] - Bishop and Hill[86] model assumes that each crystallite undergoes the same total deformation as the polycrystalline specimen overall, and postulates that, during slip, the operative combination of slip systems would be that which satisfied the boundary conditions with the least total shear. Kocks[129] has concluded that the upper bound theory has the greater validity.

The amount of plastic work per unit volume done by a tensile or compressive stress σ_{xx} in the x direction is

$$dW = \sigma_{xx} d\varepsilon_{xx} \qquad \text{..... (20)}$$

This can be equated with the work due to shear on the active slip systems:

$$dW = \tau \sum_i d\gamma_i \qquad \text{..... (21)}$$

Equations (20) and (21) combine to give a generalised Schmid factor (the Taylor factor) relating the applied stress for flow with the critical stress for slip. i.e.

$$M = \frac{\sigma_{xx}}{\tau} = \frac{d\gamma_i}{d\varepsilon_{xx}} = \frac{dW}{\tau d\varepsilon_{xx}} \qquad \text{..... (22)}$$

Equation (21) may be written as

$$\tau = \frac{dW}{\sum_i d\gamma_i}$$

For slip to occur, the shear stress must be raised to, but not above, the critical value for slip. This is achieved by minimizing the total shear $\sum d\gamma_i$ (Taylor's principle) or maximizing the work dw (Bishop and Hill's principle).

To apply the analysis to textured sheets, Hosford and Backofen [130] suggested that values of M should be determined for a given strain increment $d\varepsilon_{xx}$ in the direction of loading, as a function of r, where

$$r = \frac{d\varepsilon_{yy}}{d\varepsilon_{yy}+d\varepsilon_{zz}} = \frac{R}{1+R} \qquad \ldots\ldots (23)$$

The minimum value of $\bar{M}(r)$ corresponds to the expected behaviour, and the values of $\bar{M}(r)$ and r at the minimum identify the relative strength σ_{xx}/τ, and the strain ratio r. Equation (15) becomes

$$\bar{m}(r,\alpha) = 4\pi^2 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} G(r)_{lmn} W_{lmn} \cos m\alpha \qquad \ldots\ldots (24)$$

3.4.1.3 Prediction of the Plastic Strain Ratio

By using the Hosford and Backofen [130] criterion with the calculated angular variation of the flow stress, the angular variation of the plastic strain ratio can be predicted [89,153]. Bishop and Hill's [86] analysis shows that only a limited number of stress states are capable of fulfilling the yield criterion, and by applying a maximum work principle for given incremental strains, the active stress state and the corresponding generalized Taylor factor may be found. This can then be averaged over all orientations in the textured sheet to give an average Taylor factor, $\bar{M}$. A series ($\bar{M}(q)$) of these averages can then be evaluated for different assumed values of the contraction ratio, q, given by

$$q = \frac{d\varepsilon_{22}}{d\varepsilon_{22}+d\varepsilon_{33}} = \frac{r}{1+r} \qquad \ldots\ldots (25)$$

Therefore, the different values of $\bar{M}(q)$ are determined for a given strain increment $d\mathcal{E}_{33}$ in the loading direction as a function of q. This procedure should be valid for small plastic strains, as it assumes there is no change in texture as a result of small plastic strains.

CHAPTER 4

Experimental Methods

4.1 Introduction

All of the experimental methods employed were already well established techniques in the fields of strain life testing[20], texture analysis[72], and subsequent mechanical property predictions[72] from the texture data. The four materials used were selected so that three specific categories of rolled b.c.c. metals were investigated. The preliminary preparation of the materials was based on the well documented[131] behaviour of steels during heat treatment, and the rolling procedures used were similar to those used by other workers[79]. Texture specimens were prepared using the method of Elias and Heckler[132], and polished using a technique developed by Burke[78]. Texture analyses were obtained from x-ray goniometry data, and fatigue properties were characterized by constant amplitude - life and fatigue crack propagation experiments using specimens machined at specific orientations with respect to the rolling direction. All data were supported by standard optical and scanning electron microscopical techniques.

4.2 Rolling Practice

Reductions of 80% (cold rolled) were required in some cases. The starting thickness of all the materials used was 25mm. Rolling was carried out either on a Marshal-Richards two-high 400 tonne reversing mill with 41.9cm diameter polished rolls*, or a pair of Robertson two-high 400 tonne mills**, the first mill having 45cm

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* Courtesy of Inco Limited, Wiggin Street, Birmingham.

** Courtesy of Birmingham University Dept. of Industrial Metallurgy.

diameter rolls, and the finishing mill having 35cm diameter rolls. All three mills operate at a constant torque producing a rolling speed of approximately 0.5 msec.

In all cases, the rolls were lubricated with WD40 lubricating oil to reduce friction and attempt to ensure homogeneous deformation(79). The direction of rolling was reversed at each pass to gain orthotropic symmetry of deformation. Due to the torque capacities of the mills, the reduction per pass was never greater than 1.5mm, and in most cases never exceeded 0.5mm. To minimise heating effects, the sheets were cooled in water after six passes through the mill.

4.3 Materials

Mild, intermediate and severe textures were required for the investigation in order to estimate the degree to which the fatigue properties of steel are governed by the texture of the steel. To achieve this, two batches of steel were used.

(a) SAE 4042

(b) An experimental steel*, slab No. API 5LX X65.

SAE 4042 is standard production hot rolled plate with a low inclusion content, i.e. a relatively 'clean' steel.

The experimental slab was extremely low in sulphur content, and hence effectively free of MnS inclusions. It had an as-received yield strength of 65,000 p.s.i. (456.9 MPa). (Table 1 gives the compositional details of the two materials).

Low inclusion-content steels were used so that any influence on the fatigue properties due to such inclusions was at a minimum. Burke(78) produced evidence which demonstrates that Mn S inclusions

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* Courtesy of B.S.C., Teeside.

have a dominant effect on the fatigue properties of steel, therefore the attempt was made to control the problem at source (see section 2.7.2.2). Burke's[78] data are used for comparison in Chapters 6 and 7.

The processing details and material codes are as follows.

4.3.1 SAE 4042

Two recrystallized textures were produced from this steel. The alignment of fatigue specimens in the grips of the fatigue testing apparatus (see section 4.5.1.2) is critical: therefore the fatigue specimens must be flat.

The final heat treatments were therefore carried out prior to machining, due to the fact that correction shaped fatigue specimens, but with machining-induced residual stresses, were considered more desirable than mis-shapen fatigue specimens.

The composition of SAE 4042 is given in Table 1.

4.3.1.1 S10BM

SAE 4042 was spheroidized at 680°C for approximately 12 hours before being furnace cooled. The material was then cold-rolled 80% before being normalized by heating to 860°C for 4 hours in nitrogen prior to air cooling.

4.3.1.2 S12BM

The preparation was identical to that of S10BM, except for the final heat treatment. After rolling, the material was heated to 710°C for 12 hours, for re-spheroidization. However due to a malfunction in the temperature controls of the furnace, heavy decarburization, and a complex microstructure, occurred. An attempt to simplify the microstructure was made by heating the steel at

900°C for 4 hours in nitrogen before water cooling, and then holding at 700°C for 72 hours prior to furnace cooling.

4.3.2 API 5LX X65

Three texture types were produced from this material; two cold-rolled textures of different severities, and a recrystallized texture. The microstructure of the recrystallized material was equi-axed, therefore any difference in the properties of the 0°, 45° and 90° oriented specimens must be directly attributable to the difference in orientation. The behaviour of the cold-rolled materials should elucidate the relative importance of texture and grain shape effects. In these materials, the mean grain size intercept parallel to the stress axis is far greater in the 0° than in the 90° direction, but the 0° and 90° directions are identical with respect to texture. Therefore any difference in the fatigue behaviour of the specimens can be attributed to grain size effects. Since the 45° orientation differs from the others with respect to both texture and grain size, comparison of the cyclic fatigue properties in this orientation with those at 0° and 90° should separate textural and microstructural effects. The difference in texture severity of the cold-rolled materials will indicate the degree to which microstructural and crystallographic anisotropy affect the performance of the material.

Any difference in mechanical properties which is manifested in the recrystallized material should be due to textural effects only, since an equi-axed microstructure should be present.

The composition of API 5LX X65 is given in Table 1.

4.3.2.1 CR68

A 25mm thick slab of API 5LX X65 was held at 950°C in nitrogen for 2 hours before being water quenched. Then the material was heated to 650°C for 4 hours before being air cooled. The material was then cold rolled to 68% reduction to produce CR68.

4.3.2.2 CR80

The preparation was identical to that of CR68 except the final rolling reduction was to 80%.

4.3.2.3 R80AN

The preparation was identical to that of CR80, but, after rolling, a box annealing program was used. i.e. An increase of 2°C per minute to 700°C, then held for 24 hours before reducing at 2°C per minute down to the ambient temperature. An argon atmosphere was used during this final heat treatment.

4.4 Texture Measurement

4.4.1 Introduction

The reflection technique most commonly used to obtain pole figures is that of Schultz[133]. However, to obtain complete pole figures, which are necessary for quantitative texture measurements, the method was supplemented by using specimens in which the plane normal lies at the centre of a quadrant of the pole figure, as described by Lopota and Kula[136]. Then, the crystallite orientation distribution function could be calculated from sets of the measured single quadrant pole figures.

4.4.2 Specimen Fabrication

The specimen technique employed was based on the work of Elias and Heckler[132], so that the x-ray data, which are based on a spiral which is centred at equal angles to the rolling, transverse and normal directions, could utilize the available computer programmes (see section 5.2.1). The prediction of mechanical properties requires 'through-thickness' averaging, and Elias and Heckler's[132] method provides for this.

Textural orthotropic symmetry is displayed in cubic metals after undergoing thermomechanical treatment. Therefore it is possible to produce an average composite specimen by stacking the components of the specimen in the manner described by Morris[69]. These specimens were used, and orthotropism was confirmed by taking direct pole figures from specimens parallel to the rolling plane.

4.4.3 Specimen Preparation

After rolling, the material was at least 5mm thick. It is very difficult to cut accurately at 45° to the rolling direction as required by Elias and Heckler[132], so composite specimens were produced by cutting strips at 90° to the rolling direction, and, after cleaning and degreasing, stacking them in the manner shown in Figure 34, and glueing them with 'Araldite' epoxy resin. The required slice was then cut from the bonded block by a cutting wheel while the block was gripped in a specially designed jig (Figure 35).

The specimen was mechanically polished to 0.25μm finish, before being chemically polished in a 2:1 solution of orthophosphoric and nitric acid.

4.4.4 Pole Figure Acquisition

A Siemens texture goniometer, similar to that described by Neff[134], was used to measure the composite specimen pole figures. A molybdenum target, operating at 40kv and 18mA was used to produce incident radiation. A zirconium filter removed the K_β component of the radiation, so that the K_α component ($\lambda \sim 0.711$Å) was used as the incident radiation.

The specimen was set up (Figure 36) so that it rotated about an axis normal to it's surface, and that axis was in the plane of the goniometer circle. Simultaneously, the specimen was rotated about an axis in it's surface coplanar with the incident and diffracted beams. By setting, critically, the rolling plane normal at an angle of 60° to the plane containing the incident and diffracted beams, the diffracted x-ray intensities are measured along a special path in the stereographic projection (Figure 37) from which the pole figure is constructed. The goniometer was set up so that the pitch of the spiral was 5°.

Reflections from the $\{110\}$, $\{200\}$ and $\{211\}$ planes were recorded. There is no detectable reduction in the diffracted intensity of the radiation at angles up to 65°[126,135] from the centre of the spiral, and as measurements up to 55° are sufficient to completely cover a quadrant of the pole figure, no defocussing effects were experienced.

The incident beam angle was set approximately by using the horizontal goniometer circle, and the position of the reflected beam was found by moving the scintillation counter detector about the 2θ position until the peak intensity was located. The detector slit was then widened to 6mm to ensure that the whole of the peak was recorded. After each run, background measurements were taken at a point well removed from the Bragg 2θ position. Measurements were recorded in the form of an aggregate count every 5 seconds onto paper tape. Due to the crystallographic orthotropism exhibited by cubic materials, only one composite specimen pole figure at each $\{110\}$, $\{200\}$ and $\{211\}$ reflection needed to be recorded.

4.5 Mechanical Testing

Constant strain amplitude-life tests and crack propagation tests were carried out to assess the fatigue response of the materials.

4.5.1 Strain-Life Testing

4.5.1.1 Specimen Preparation

Specimens were cut at 0°, 45° and 90° to the rolling direction from all the materials. Each blank was machined to F.G.8 specifications (Figure 38). The gauge lengths were longitudinally polished to a 1200 grit finish.

4.5.1.2 Fatigue Testing

An MTS closed loop servo-hydraulic fatigue testing machine (Figure 39) under constant strain amplitude control about zero mean stress was used to conduct the tests. It was fitted with a 2.5 tonne or 5 tonne load cell, depending on the material under test, so that the full scale range was never exceeded. A clip gauge extensometer measured the strain, and control was achieved with a variable amplitude sine wave. The two points on the specimen gauge length at which the knife edges of the extensometer had contact were, in the first experiments, protected with narrow strips of adhesive tape, but this proved to lead to an unsatisfactorily wide range of values for Young's modulus.

The problem was solved by substituting 'Tippex' for the adhesive tape, which provided a tough and rigid bed for the knife edges. To avoid the problems of cross-over error at zero stress and the gripping system applying lateral stresses, Wood's metal grips were used.

Each test was continuously monitored by an oscilloscope connected to the load cell output, and intermittently monitored by load-strain hysteresis loops on an x-y recorder. The loops were taken at such intervals that the number of cycles at the penultimate loop was always greater than or equal to the total number of cycles to failure. The loops were recorded at a frequency of 0.1 Hz, although tests were run in the range 0.5 Hz - 30 Hz. With at least one specimen from each batch, loops were recorded at frequencies of 0.1 Hz, 0.2 Hz, and 0.5 Hz. Specimen failure was defined as complete separation.

4.5.2 Crack Propagation Testing

Crack propagation tests were carried out on CR80 and R80AN. These materials were chosen, because, after the strain-life tests, it was apparent that any textural effects of fatigue were most clearly manifested in these two materials.

4.5.2.1 Specimen Preparation

Single edge notch (SEN) specimens were machined (Figure 40) in three orientations, the long axis of each specimen being of 0°, 45° or 90° to the rolling direction (Figure 41). The central region of each specimen was longitudinally polished to a 0.5μm finish, so that any cracks would appear approximately normal to the direction of polishing.

4.5.2.2 Crack Propagation Testing Method

An Amsler Vibraphone (Figure 42) under constant load amplitude conditons was used to test the specimens in tension-tension. The $\sigma_{min}/\sigma_{max}$ value was maintained at 0.5 in an attempt to keep the crack tip as sharp as possible. Crack monitoring was achieved by observing the progress of the crack with a travelling microscope equipped with a metric vernier, on each side of the specimen. Two microscopes were used to compensate for the fact that, in some cases, crack growth was non-uniform. The crack length was measured at intervals of 50K cycles. In cases where the crack did not propagate on the plane normal to the stress axis, the crack length measured was that which projected the crack front onto the plane normal to the stress axis.

4.5.3 Proof Stress Testing

4.5.3.1 Specimen Preparation

The specimen preparation was identical to that described in Section 4.5.1.1, plus the fact that a 5mm diameter hole was drilled into each end of the specimen, and through each grip, so that a 5mm diameter steel bar could be passed through the grips and the ends of the specimens, ensuring that the specimens did not slip out of the grips. Also, spare material was used to prepare specimens at 22.5° and 67.5° to the RD, so that the proof stress and r ratio.. could be investigated.

4.5.3.2 Proof Stress Testing Method

A similar MTS testing machine to the one which was used for fatigue testing (see Section 4.5.1.2) was employed to carry out monotonic tensile tests on the specimens. A 10 tonne load cell was used. The ram speed was 5mm min^{-1}, and tests were continued until complete separation occurred.

All the tests were started in load control. Where load ranges were exceeded, the test automatically switched to strain control. The automatic mode change ensured that the specimen was protected from spurious loads, and was effected by a computer program*, which also analysed the data.

4.6 Elastic Modulus Measurement

The variation of Young's modulus in the plane of a rolled sheet is conveniently recorded by the transverse resonant vibration method of Nortcliffe and Roberts[137] (Figure 43).

Two piezo-electric gramophone pick-ups support horizontally suspended rectangular specimens. A sine wave oscillator excites one pick-up, which excites vibrations in the specimen. These are transmitted along the bar, and detected by the second pick-up, from which the signal was fed into an oscilloscope. The position of resonance was recorded, and the frequency was measured on a second counter.

Chalmers and Quarrell[138] give the relationship between

$$f_i = x_i \frac{F(t.d)}{L^2} \left(\frac{E}{\rho}\right)^{\frac{1}{2}} \qquad \ldots\ldots (26)$$

$$\text{where } F(t.d) = \frac{t}{4\pi\sqrt{3}}$$

Specimens at 0°, 45° and 90° were cut from the rolled sheet, and ground to uniform size.

4.7 Observations

4.7.1 Grain Size Measurements

The grain size and shape of all materials were ascertained by

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* Courtesy of S.J.Kemp, G.K.N.

standard metallographic techniques. Low power optical microscopy was also employed on fractured specimens, prior to scanning electron microscopy.

4.7.2 Scanning Electron Microscopy

A Phillips PSEM500 was utilized to observe fracture surfaces and slip markings at magnifications up to x 20,000.

4.8 Hardness Testing

A Vickers' diamond pyramid hardness machine was used, utilizing a 1.7cm eyepiece and a 10 Kg load. The effect of texture in the plane of the sheet was minimized by rotating the specimens at approximately 0°, 45° and 90° to the rolling direction and taking the arithmetic mean of the results.

CHAPTER 5

Data Analysis and Management

5.1 Introduction

Analyses of data from fatigue and crack propagation tests, and texture measurements, were carried out using the methods and computer programs described in this chapter.

5.2 Analysis of Texture Data

5.2.1 Data Management

A brief description of the analysis of the pole figure data has been given in Section 3.2. A complete description is given in Appendix 1. The x-ray data, which are output on paper tape, give intensities along a spiral track covering one quadrant of the pole figure (Figure 37). The intensities were normalized over the whole of the pole figure*, and transferred to a polar grid (Figure 44) which has reference points at 5° intervals circumferentially and radially from the centre of the pole figure. The intensity at each of the points was found by linear interpolation between the corresponding four nearest data points on the special grid. Pole figures and c.o.d.f.'s were printed on a computer controlled plotter using contouring facilities.

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* The analyses and predictions were performed on the Cambridge University IBM 370 165 computer, via SERCNET, using the original programs of Kallend[126] after modification and extension by Morris[69] and Bateman[128] into PL/1.

5.2.2 The Utilization of Texture Data

A description of the uses for which the c.o.d.f. can be used has already been given in Section 3.4.

5.3 The Analysis of Fatigue Data

5.3.1 Introduction

The specimens were tested in a conventional servo-hydraulic fatigue testing machine, previously described in Section 4.5.1.2. There were two parts to the testing programme,

(a) Monotonic tension test, and

(b) Constant amplitude, fully reversed fatigue tests on smooth specimens under strain control to obtain data ranging from the endurance limit to the low cycle region of the S-N curve.

The tests were based on the parametric method of fatigue testing.

5.3.2 The Parametric Approach to Fatigue Testing

The parametric approach depends on the material parameters which determine the cyclic behaviour of the material. It is now used extensively[5, 139, 504] to identify design criteria so that fatigue failure may be averted. The proportions of saturation response, crack nucleation and propagation with respect to each other are ignored, and only the fatigue life to a point where a specific amount of fatigue damage has accumulated is considered. In this case, this is where a small laboratory specimen completely separates. The problem of being unable, with this approach, to distinguish the separate stages of deformation is outweighed by the fact that the question of determining, for example, when an intrusion within a slip band ceases to be a notch and becomes a short crack, is unnecessary.

Stress-strain hysteresis loops (Figure 2) applied strain amplitudes and the number of reversals to failure supply the data. The loops were taken at such intervals throughout the test that at least one loop demonstrated a life greater than half of the total life of the specimen, i.e. the saturation stage. The data were taken from these loops. The parameters used were as follows,

$\frac{\Delta\varepsilon_t}{2}$ - the applied strain amplitude

$\frac{\Delta\varepsilon_p}{2}$ - the plastic strain amplitude at saturation

$\frac{\Delta\sigma}{2}$ - the stress amplitude required to give the applied strain amplitude at saturation

$2N_f$ - the number of reversals to failure

E - Young's modulus of elasticity.

(The cyclic stress-strain approach considers that the fatigue resistance of a material is determined by the relationship between the cyclic strain amplitude and the corresponding cyclic stress amplitude, in a similar manner to the monotonic stress-strain curve). Mitchell[5] has shown how the cyclic stress-strain curve may be constructed from several stress-strain hysteresis loops (Figure 3).

The cyclic stress-strain response is therefore an aggregate of the elastic and plastic regions of the cycle, and is characterized by

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \qquad \text{......(27)}$$

where $\frac{\Delta\varepsilon}{2}$ is the elastic strain amplitude at saturation.

This can be written as

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n}} \qquad \text{......(28)}$$

Similarly, the strain life data may be analysed using parameters unique to the material. According to Basquin [140], the fatigue life of materials just above the fatigue limit obeys the relation

$$\frac{\Delta\sigma}{2} = M(N_f)^{-b} \qquad \text{......(29)}$$

At the opposite end of the S-N curve, the low cycle region, the Coffin-Manson [141,142] relationship describes the fatigue life as

$$\frac{\Delta\varepsilon_p}{2} = P(N_f)^{-c} \qquad \text{......(30)}$$

The material response to the applied strain is the determining factor in the fatigue life. The relationship between fatigue life and the imposed strain amplitude has been given by Mitchell [5] and is based on the Basquin and Coffin-Manson laws by apportioning damage between the elastic and plastic strain components.

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \underbrace{\varepsilon_f'(2N)^{-c}}_{\text{Plastic}} + \underbrace{\frac{\sigma_f'}{E}(2N)^{-b}}_{\text{Elastic}} \qquad \text{......(31)}$$

Mitchell [5] who showed that the parameters ε_f', σ_f', c and b are not independent but are related by n' and K'.

$$n' = \frac{b}{c} \qquad \text{......(32)}$$

$$K' = \frac{\sigma_f'}{(\varepsilon_f')^{n'}} \qquad \text{......(33)}$$

To summarize, four fatigue properties, unique to each material, have been given.*

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* Smith et al [143] have suggested a simpler parameter by which fatigue life can be represented, i.e. $2N_f = \text{const}\ (\Delta\sigma\,\Delta\varepsilon\ \ E)^{\frac{1}{2}}$ as have Rebbeck and Watson [44].

σ_f' - fatigue strength coefficient

ε_f' - fatigue ductility coefficient

b - fatigue strength exponent

c - fatigue ductility exponent

5.3.3 Computer Program

The strain amplitude life data were analysed using a computer program written by Burke[78]. The data inputted are from each specimen in a series of tests, the total strain amplitude, the plastic strain amplitude at saturation, the equivalent total stress amplitude, and the number of reversals to failure. The first three parameters are shown in Figure 2. Young's modulus of elasticity, determined by the resonance method, is input separately. A copy of the program is given in Appendix II. The program was run, via SERCNET, on an IBM 370/165 computer at the University of Cambridge.

A least squares regression fit to equations (27) and (30) transferred the data to logarithmic axes. A comparison of the cyclic stress-strain and strain-life analyses is possible due to the fact that the six parameters are interdependent. The output was in the form of printed material parameters and from these regression lines were calculated.

5.3.4 The Accuracy of Fatigue Results

The accuracy of the material parameters calculated by the fatigue program depends entirely on the statistical nature of the data and the precision with which the hysteresis loops are measured. The computer program accommodates discrepancies and uses two methods to analyse the data.

(i) The elastic strain amplitude is calculated by taking the difference between the total strain amplitude and the plastic strain amplitude.

(ii) The elastic strain amplitude is calculated as the quotient of the stress amplitude and the static elastic modulus.

At no stage was a mean elastic modulus used, because, in some cases, data points demonstrate elastic strains greater than the imposed strain. Burke(78) carried out a detailed analysis of the method of acquiring the fatigue results, and produced the computer program (see Section 5.3.3) accordingly. The program employs the method of (ii) above. (See also Chapter 6).

5.4 The Analysis of Crack Propagation Data

5.4.1 Introduction

Crack propagation data were obtained as a set of crack length measurements (a) as a function of the number of cycles (N) at contact load amplitude (ΔP). Fatigue crack growth is often described as a function of the applied stress intensity at the crack tip, and the simplest relation is that suggested by Paris(16) and given in equation (1).

5.4.2 Computer Program

The data were analysed using the computer program written by Davenport(145) and modified by Cadman (156). The program is based on the spline curve fitting procedure described by McCartney and Cooper(146). The a vs N data are fitted to a polynomial of ℓth degree, incorporating n number of splines. Differentiation of the expression at each data point can be achieved. The program employs the compliance functions of Walker and May(147) to calculate the cyclic stress intensity (Δ K) from the applied load amplitude (Δ P) the mean load ($\bar{P}$) and the crack length (a). From these raw data, tabulated data of crack propagation rate (da/dN) are obtained as a function of the magnitude of the applied stress intensity.

The program was run on the University of Sheffield ICL 1906S computer. Four plots were obtained;

(a) normalised crack length v normalised number of cycles

(b) crack propagation rate v normalised crack length

(c) log (da/dN) v log (ΔK)

(d) calculated normalised crack length v number of cycles data from plot (c).

The program also calculates the values of C and m in equation (1) using a least squares regression method.

5.4.3 The Accuracy of Crack Propagation Data

Spline fitting is a very powerful analytical tool, due to the fact that it permits a series of data points to be very closely followed by the curve fitting routine. The graph plotting facility permits the data to be reviewed and the 'fitted' parameters to be compared with the raw data in order that the optimum values of n and L (the number of splines and the degree of polynomial) may be chosen. The precision of the technique is hampered by the accuracy (or otherwise) of the experimental data i.e. at large values of n and L the curves tend to follow the scatter in the raw data[148]. Generally, therefore, the empirical data restricted the usefulness of the analytical program, and the values of n and L were normally limited to 2 and 4, respectively.

Optical measurement of the crack length is the major source of error in the experiments, compounded by a smaller degree of error in the load cell output of the Vibraphone. There are two sources of error inherent in the optical method of measuring crack length;

(a) the uncertainty of where the crack actually ends,

(b) the probability that the crack does not have a flat profile throughout the thickness of the specimen.

With care, the errors due to (a) can be substantially reduced, and reproducibility of results achieved. The errors due to (b) are more complex.

The relationship between the crack trace and the crack front profile is not simple, and, further, it has been shown that crack growth is decelerated by a free surface[149]. Therefore, the crack front in the centre of the specimen is normally advanced relative to the crack trace at the free surface. However, the crack front should maintain a uniform profile, and therefore the crack propagation <u>rate</u> should be unaffected, and the error introduced by this assumption affects only the crack position.

More serious problems were encountered with specimens taken at 45° to the rolling direction. Crack traces on both faces of the SEN specimens were found to be rotated from the notch direction such that on one face the rotation was clockwise, and on the other face, anti-clockwise. The lengths of the traces were also found to be unequal, although this latter problem was by no means unique to the 45° specimens, at least in the early stages of propagation. The crack length was, therefore, calculated by measuring the projected crack length in the notch direction on each face, and calculating the arithmetic mean. Unfortunately, this produces two further sources of error. The compliance factors for the K calibration assume uniform crack fronts, therefore it is difficult to justify the use of the mean projected crack length. Also, the program calculates the stress intensity ΔK_I, which of course assumes mode I opening.

However, such a complex crack front includes some degree of modes II and III opening (Figure 45). It is therefore inaccurate to use ΔK_I to describe this type of crack. However, the primary purpose of the experiments was to compare the rates of crack propagation in textured steel specimens of different orientations, and, since non-plane strain conditions at the crack front are a direct consequence of the crystallographic texture, the method of comparison is considered valid in this case. The calculation of K is based on linear elastic fracture mechanics, and, since gross plasticity was observed in several of the specimens, its relevance was questionable in any case.

CHAPTER 6

Results

6.1 Microstructural Characterization

6.1.1 Texture Data

6.1.1.1 Introduction

The textures of all five materials are shown in Figures 47-51. A typical direct pole figure is shown in Figure 46 to demonstrate the orthotropic texture in the plane of the sheet which was displayed by all of the materials.

6.1.1.2 S10BM

The texture of S10BM is shown in Figure 47. The severity parameter of 0.26 reflects the fact that the degree of crystallographic texture in the material is low. Also the maximum function height (m.f.h.) of 1.8 x random indicates that individual orientations are not excessively pronounced. The material exhibits a weak $\{111\}$ <uvw> texture together with a weak $\{hkl\}$ <011> component, where (001) $[\bar{1}\bar{1}0]$ is the orientation which is most prominent. Integration of the c.o.d.f., however, shows that only 1.5% (12%*) of the material volume is oriented within 10° of the $\{100\}$ <011> position, while approximately 30% of the material is within 10° of the $\{111\}$ <uvw> texture. Sharp textures normally result in large truncation errors. The low truncation errors in this case are therefore another indication of the weak texture displayed.

6.1.1.3 S12BM

The S12BM c.o.d.f. is shown in Figure 48. It consists predominantly

*For comparison, the figure in brackets is the corresponding volume fraction percentage for a non-textured sample.

of a {111} <uvw> texture, which peaks at {111} <112>. A smaller peak at {100} <011> is also evident, although the volume of crystallites in this orientation is less than 2% of the total, at least 40% of the total being within 10° of the {111} <uvw> texture. Low truncation errors again add further evidence that the overall texture is weak.

6.1.1.4 <u>CR68</u>

The CR68 c.o.d.f. is shown in Figure 49. The texture is a relatively severe (m.f.h. = 5.2, t.s.p. = 9.93) branched {111} <uvw> and {hkl} <110> texture, typical of the textures obtained in cold-rolled low carbon steel sheet[69]. The texture component displaying the greatest severity is {100} <011>, but only 4% of the total volume of the material is oriented within 10° of this texture, while almost 50% of the volume is within 10° of the {111} <uvw> fibre texture. The truncation error in the analysis for this material is accordingly higher.

6.1.1.5 <u>CR80</u>

Figure 50 presents the c.o.d.f. for this material. A m.f.h. of 12.8, and a t.s.p. of 1.63 indicate a severe texture. High truncation errors corroborate this evidence. A branched texture is clearly in evidence with 10% of the total volume displaying the most pronounced texture of {100} <011>, and at least 65% of the total material volume is within 10° of the {111} <uvw> texture.

6.1.1.6 <u>R80AN</u>

A significantly different texture has been achieved with this material (Figure 51). The volume fraction of the material given over to the {hkl} <110> texture is equivalent to that conceded to the {111} <uvw> component, i.e. 28%. The severity of the texture is high (m.f.h. 47, t.s.p. 1.02) and,as a result, the truncation errors are also high.

6.1.2 Metallographic Observations

The grain structures of the materials were examined using standard polishing and etching procedures. All the materials except S12BM consisted of a dual phase ferrite-pearlite structure, to a greater or lesser degree, interspersed with occasional oxides. S12BM was of carbides suspended in a ferrite matrix. There was little evidence of MnS inclusions in both S10BM and S12BM, and effectively none in the other three materials. The linear intercept method was used to measure grain sizes. The recrystallized materials were observed to consist of equiaxed structures, while elongated grains were a distinguishing feature of the two cold-rolled materials, CR68 and CR80. R80AN displayed a coarser grain size with respect to any of the other materials, due to the time spent undergoing the anneal. The results of the metallographic observations are given in Table 2. The grain diameters for CR68 and CR80 are those diameters taken normal to the axis of elongation.

6.2 Mechanical Property Predictions

Table 3 shows the predicted elastic and plastic properties of all the materials. The elastic properties were predicted using the methods of Reuss[94], Voigt[93] and Hill[92], and the prediction of the plastic properties utilized the Taylor[85], Bishop and Hill[86] model.

6.3 Elastic Modulus Measurements

Equation (26) was used to calculate the sonic modulus of each batch of specimens. The data used, and the modulii obtained, are given in Table 4. Young's modulus in the long direction of each modulus specimen was found using the appropriate coefficient. The arithmetic mean of the modulii found at clearly distinguishable resonances was taken to be the modulus of the specimens.

This modulus may then be compared with the modulii predicted from texture measurements and monotonic tension tests. The same test is a particularly useful analytical tool when the modulus of a soft material is required, where a monotonic test may prove to be unsatisfactory due to the fact that, at very low strain, the elastic and plastic microstrains cannot be separated.

Figure 52 shows that the predicted and experimental values of E are in agreement

6.4 Plastic Property Measurements

6.4.1 The Plastic Strain Ratio, r.

Table 5 presents the results of plastic strain ratio measurements on specimens from all batches of material. Due to the nature of the computer program which controlled the tensile tests, it was not possible to take measurements at, say, 10% uniform tensile elongation, therefore all readings were taken after complete separation had occurred. Figure 53 shows both the measured and predicted variation of strain ratio with angle from rolling direction for all specimen types, including specimens cut at 22.5° and 67.5° to the RD. The measured results follow the trend of the predicted results, but display considerable scatter, although, in all cases, the experimental values are greater than the predicted values. In general, the higher the experimental value of r, the greater was the error in prediction. This is a result of using the criterion of Hosford and Backofen[130] to predict r where the value of the contraction ratio q_{min}, of which the Taylor factor $\bar{M}$ is a minimum, is derived from crystallographic texture data.

i.e.
$$r = \frac{q_{min}}{1 - q_{min}} \qquad \text{......(35)}$$

For a completely randomly oriented cubic material, $\bar{M}$ has a value of 3.06 (the minimum) at $q = 0.5$. As the degree of anisotropy increases, q_{min} (corresponding to $\bar{M}_{min}$) increases. Therefore, r becomes more sensitive to errors in the derived q_{min}. Furthermore, the $\bar{M}$ v q curve can become very flat, and therefore q_{min} is difficult to locate. In this case, a small texture change could alter the position of q_{min} significantly. The prediction of r values using the Taylor[85], Bishop Hill[86] analysis appears to give reasonable results when the material under investigation possesses a low degree of normal anisotropy (r close to unity). This is borne out by the experimental r values displayed by S10BM and S12BM. The values are much closer to the predicted values than those of the other three materials, and the texture severity in S10BM and S12BM is much less than in the other materials.

6.4.2 The Flow Stress

Both the predicted (relative) and the experimental (relative) flow stresses are presented in Figure 54. Data taken from specimens cut at 22.5° and 67.5° to the RD are also included. The flow stress required to initiate and maintain plastic flow in a given direction α° from the rolling direction in the rolling plane of a polycrytalline sheet material can be predicted from the product of the averaged

Taylor factor (found from equations (24) and (25) at $q = q_{min}$) and the critical resolved shear stress, , for flow in a single crystal.

$$\sigma_{\alpha} = \bar{M} \; (q = q_{min}, \alpha) \qquad \text{......(36)}$$

Comparing values of the Taylor factors derived from texture data for various orientations of the polycrystalline sample thus provides a relative measure of the flow stresses in these directions and avoids determining absolute values of τ. The data in Figure 54 are, therefore, presented in terms of relative flow stresses versus angle to the rolling direction.

Flow stress values were obtained from load elongation curves of computer-controlled monotonic tests. When values were taken at 0.2% strain, large discrepancies between actual and predicted relative yield stresses were apparent, with the experimental values not even following the trend of the predicted values. Kocks[129] has reviewed the theories correlating single crystal behaviour with that of polycrystals, namely those of Sachs[84] and Taylor[85] - Bishop and Hill[86], and concluded that the available evidence supported a theory which attains strain compatability, i.e. the Taylor-Bishop and Hill approach. In both of these procedures the assumption is made that the property under consideration is additive, that crystal interaction is negligible (Sachs) or accounted for by an averaging process (Taylor) and that grain boundary material has no effect. Kocks[129] shows that the Taylor[85]- Bishop and Hill[86] approach appears to validate these assumptions at strains greater than 1%. The experimental data given in Figure 54 were therefore redetermined at 1% strain, and it is clear that, although individual results may differ slightly from the predicted results, the experimental results do follow the trend of the predicted curves.

6.5 The Cyclic Stress- Strain Investigation

Tables 6 - 20 give the results of the stress-strain strain life tests for all the materials. The specimen orientation with respect to the rolling direction is simply added to the end of the material code. e.g. S10BM0 indicates S10BM specimens cut at 0° to the rolling direction, and CR8045 indicates CR80 specimens cut at 45° to the rolling direction. All the data given in the tables were taken from hysteresis loops recorded nearest to the half-lives of the specimens taken at a frequency of 0.1Hz. Data from tests in which premature failures occurred due to knife edge fretting etc., were noted but not used in the strain-life analysis. Several of this type of failure were experienced in the early part of the testing programme, but the application of two or three coats of 'Tippex' (see Section 4.5.1.2) to each specimen gauge length at the points where the extensometer made contact eliminated the problem. Scrutiny of the raw data reveals considerable variation of the elastic modulus as measured from the hysteresis loops. This phenomenon is not uncommon[154], and in this case, can be attributed to two sources.

(a) the method of calculating the elastic modulus, and

(b) the idealized treatment of the hysteresis loop shown in Figure 2.

In the first case, the modulus is found by using the difference between the total strain amplitude and the plastic width of the hysteresis loop as the denominator when $E = \sigma/\varepsilon$ is employed. A large fractional error occurs in E, due to the fact that, when one large quantity is subtracted from another large quantity, each with it's own error, a small quantity remains. When used as the denominator, this produces a large fractional error. In the second case, two discrepencies may be noted between an actual hysteresis loop taken at approximately the half-life of a specimen, and the idealized loop shape, namely

(b1) The 'straight' sides of the loop often start to curve <u>before</u> crossing the strain axis, and

(b2) The turning points at the loop tips never appear as sharp reversals when a sinusoidal mode of testing is used.

Instead, the turning points appear rounded, so that the stress and strain maxima are not coincident.

This second phenomenon (b2) is due to stress relaxation effects together with effects associated with the applied sinusoidal control signal[78]. The applied strain rate continually decreases near the tips of the loops, and an anelastic strain component is, therefore, produced. The loops were recorded at three speeds (Section 4.5.1). The loops with the sharpest points were those recorded at 0.1Hz.

Despite these effects, it is normal practice[155] to employ sinusoidal wave forms in constant strain amplitude testing, even though the idealized loop shape is rarely obtained. Therefore elastic, plastic and anelastic strains should all be considered in the analysis. However, because truly elastic deformation cannot produce damage, the Basquin[140] and Coffin-Manson[142] relationships do account for any anelasticity. The analysis described by Figure 2 is, thus, a reasonable approximation, and by ascribing the plastic strain to the strain width of the loop, and,the elastic strain to the stress amplitude divided by the static elastic modulus, the errors are accommodated.

The static modulus was determined as described in Section 4.6. The difference between the total strain as measured in Figure 2, and the recalculated strain found by adding the elastic and plastic strains found by the methods described above, is small. A typical set of data can be found in Table 21.

This technique has been found to be the most consistent method of analysis[78]. Since the analysis is founded on the (empirical) laws of Coffin[141] and Basquin[140], the data employed in the technique described above are those to which the laws pertain.

The strain-life data yielded material constants which are given in Table 22, together with the constants from the stress-strain investigation. Figures 55-69 show the cyclic stress-strain curves for all the materials, and Figures 70-84 give the strain-life curves including the elastic and plastic lines. Figures 85 and 86 give summaries of the elastic and plastic strain-life ranges, respectively.

6.6 Fatigue Crack Propagation Investigation

The fatigue crack propagation data were analysed as described in Section 5.4. Tables 23 and 24 tabulate the values of C and m as defined by equation (1) and also the value of ΔK_6 (the threshold stress intensity amplitude as suggested by Rieux et al[47]), the stress intensity amplitude required to attain a growth rate of 10^{-10} m/cycle.

The cracks which were propagated in specimens oriented at 45° to the rolling direction generally deviated from the trace of the plane perpendicular to the stress axis by between 5° and 13°. In both CR80 and R80AN, the sense of the deviation of the crack in the 45° specimens was the same on both faces of each specimen. i.e. on each face the crack plane trace was inclined towards the top (or bottom) grip. In the case of CR80, it appears that the crack plane trace tended to rotate so that propagation was aligned along the long dimension of the grains.

The data clearly fall into two subsets characterized by the Paris exponent m. One set, associated with CR80, has values greater than 2.5, and the second set, associated with R80AN, has values less than 2.3. Further, since C and m are interdependent, a more precise correlation may be made using the values of ΔK_6, i.e. the values of ΔK_6 are lower for R80AN than for CR80. The specimens within these two subsets which display inclined growth (i.e., 45° specimens of CR80 and R80AN) require a higher stress intensity amplitude to enforce a crack growth rate of 10^{-10}m/cycle, i.e., under the same stress intensity amplitude, the specimens which display an inclined crack exhibit slower crack propagation than those which show crack growth normal to the stress axis.

6.7 Scanning Electron Microscopy Observations

6.7.1 Strain-Life Specimens

6.7.1.1 Specimen Sides

It was not possible to polish the specimens to such a degree that slip lines could be observed on the surface. Due to the difficulties encountered in machining the flat plate specimens to the required dimensions, significant score marks were left on the edges of the specimens. After considerable effort, using grinding papers, all of these marks were erased, except for some of the more pronounced longitudinal ones. So, although most of the finished surfaces were as smooth as possible, they were not flat enough to highlight any microscopic slip markings which might be present. However, due to the fact that S12BM was very soft, it is possible, in Figure 87, to see a few intrusion-extrusion pairs, together with considerable surface deformation. Also, regardless of the material, secondary cracks were occasionally visible (Figure 88) and slip bands normal to the stress axis were visible to the naked eye on

R80AN specimens. The intrusion-extrusion pairs may result from one of the slip configurations described by Arnell and Teer[83], i.e. 'single slip bands', 'intersection slip bands', 'Herring-bone', and 'severe surface rumpling'. Despite the profusion of slip configurations, Arnell and Teer[83] found that fatigue cracks grew only from the long, parallel slip band configuration. Although the Zener[173] model of crack nucleation (which assumes that the coalescence of a number of edge dislocations lying on the same slip plane should open up a crack on a plane normal to the slip plane) assumes that a twin boundary acts as a barrier to the movement of dislocations, the roles of twin boundaries (and grain boundaries in polycrystalline materials) are not clear. It appears, however, that such boundaries may help to establish an embryonic crack which has formed within a p.s.b.

6.7.1.2 Fracture Surfaces

Fully reversed strain cycling complicates the investigation of specimen fracture surfaces by causing the fracture surfaces to impinge on each other during the compressive half of the cycle, and therefore causing 'smearing' to occur. However, it is rare for fracture surfaces to undergo complete extinction. All the fracture surfaces displayed the usual stages of fracture, i.e.,

(a) a featureless zone near the nucleation site,

(b) a secondary zone where typical fatigue features could be seen (usually normal to the stress axis, but sometimes inclined, and,

(c) an area of ductile shear caused by final, monotonic fracture.

Stage I growth[2] of the embryonic fatigue cracks is exhibited by the fracture surfaces near the initiation sites (Figure 89). The annealed materials displayed two distinct features of Stage II growth,

(i) Fatigue striations (Figure 90) and cleavage steps.

(ii) Ductile fracture surfaces.

Indeed, Figure 91 is an excellent example of the void-sheet mechanism. The cuplets are clearly oriented in a non-random manner.

The Stage II regions in the cold-rolled materials exhibited obvious facet-life textures with cleavage striations in the crack growth region (Figure 92a). Both the 0° and 90° specimens generally display crack initiation and growth which is inclined to the stress axis particularly in the case of the 68%-rolled material. Figure 93 is a low magnification S.E.M. photograph which clearly shows how two separate, inclined cracks have caused failure. In all cases, there was extensive internal delamination of the cold-rolled materials, in planes parallel to the faces of the specimens (Figure 94) due to bending.

6.7.2 Crack Propagation Specimens

6.7.2.1 Specimen Sides

Both materials (CR80 and R80AN) produced cracks in the 45° oriented specimens which were inclined to the stress axis by 15°-30°. The softer specimens produced a large, macroscopic plastic zone (Figure 95) with slip bands visible to the naked eye, resulting in gross cross-section yielding.

6.7.2.2 Fracture Surfaces

Figure 96 shows a surface typical of all the fracture surfaces, the striations on the fracture surfaces of the softer material being particularly prominent. At longer crack lengths, the crack morphology becomes less planar, exhibiting macroscopic irregularities. It is worth noting that, in all cases, fatigue striations were not obvious on fracture surfaces near to the notch root, due to the fact that crack propagation rates were less than those encountered in the strain-life tests, because all the tests were terminated before the crack reached 65% of the specimen width.

6.8 Hardness

Hardnesses of all the materials are given in Table 26.

CHAPTER 7

Discussion

The literature review has indicated that the fatigue properties of cubic single crystals can be expected to be orientation dependent. By expressing the cyclic stress-strain curves, in terms of the resolved shear stress and strain amplitudes, Laird(158) has shown that materials deforming by wavy slip would be expected to display a unique cyclic stress-strain curve. The anisotropy of cyclic behaviour is, therefore, dependent upon the Taylor factor. Kettunen(57) has attempted to rationalise the behaviour of single crystals (and polycrystals) using the Taylor factor. Both of these approaches assumed that the polycrystals used were free of texture, and therefore exhibited the Taylor factor associated with a randomly oriented aggregate of crystals, i.e. 3.06.

Avery et al(68) demonstrated the effect of orientation on the rate of crack nucleation in single crystals, and a corresponding dependence on the orientation dependence of the development of surface topography in polycrystals was noted by Arnell and Teer(83), i.e. grains which developed long straight slip bands were oriented so that the stress axis lay at the centre of the stereographic triangle. It appears that the rate of crack propagation in single crystals is also influenced by the crystal orientation(81). However, these data may not be directly relevant to the fatigue of polycrystals since crack growth data have been correlated with the tendency towards multiple slip(18) and, in polycrystals, it would be expected that the constraints of neighbouring grains would promote polyslip in all grains.

The only data which have inspected the effect of preferred orientation on cyclic properties are those of Nair and LeMay[80], and Burke[78]. Both sets of workers found differences in the fatigue properties of textured metals when those properties were measured in the transverse and longitudinal directions of the material, but, in materials which display either a 'cube' or 'random' texture, the two orthogonal orientations produce much more similar fatigue behaviour. Le May and Nair[80], nevertheless, were only able to distinguish and correlate distinct differences in fatigue behaviour when there were considerable differences in texture e.g. they argue that the S-N data for the annealed copper polycrystals form a single curve. However, it could be agreed that the orientations are not identical, but that the scatter bands of the two sets of data overlap. Also, their use of incomplete pole figures is not a complete description of the texture. For example, the incomplete pole figure method would clearly show a strong cube texture, but this does not necessarily mean that the 0° and 90° orientations are texturally identical, due to minor components which may be present (but not displayed) and therefore any anisotropy of mechanical properties would be affected. Also, LeMay and Nair[80] only found differences in the fatigue performances of cold-rolled material; therefore, it has not been proved (or disproved) that these differences are attributable to texture only, since in one orientation the long grain boundaries are parallel to the stress axis while in the other they are perpendicular.

7.1 Strain-Life

The strain-life/cyclic stress-strain approach used in this investigation considers that, under fatigue conditions, the endurance of a material is determined by the mechanical response under cyclic loading, where such a response can be adequately described by the cyclic stress-strain curve. The implication is, therefore, that for a given applied strain amplitude, the material will develop a unique stress amplitude. Such a description of the stable stress-strain response is simplified if materials exhibit the 'classical' response of hardening or softening followed by a saturation stage terminated by fracture. In this case, the cyclic stress-strain curve is determined by the saturation stress amplitudes as a function of the applied strain amplitudes. In many cases, however, a saturation stress amplitude cannot be determined due to the fact that the material under test simply displays a continuous slow softening. For example, Abel[159] observed that the saturation apparently exhibited by single crystals of copper is actually a very slow rate (10^{-7} per cycle) of softening. Since the lack of saturation is often more marked than in the aforesaid example, it is necessary to define a comparative point where the stress and strain amplitudes can be taken. This point is taken as the half-life[6].

Constructing the cyclic stress-strain curve from saturation values implies that the material structure attains a stability under the cyclic conditions, and it follows that the rate of accumulation of fatigue damage is determined by the dislocation substructure. Feltner and Laird[74] suggested that 'wavy-slip' materials should display unique cyclic stress-strain curves independent of prior history. Laird et al[160] have recently clarified the question of the uniqueness of the cyclic stress-strain behaviour of 'wavy-slip' materials. The same investigations found, conversely, materials which exhibited low stacking fault energy were observed to display cyclic stress-strain responses where were dependent on previous mechanical history, i.e. the 'saturation' stress at a given strain amplitude would be raised by pre-strain. Although the uniqueness of the cyclic stress-strain curve of copper has been widely accepted, Tuler and Morrow[161] produced data which do not concur with the general view, since they found that heavily cold-worked copper displayed a cyclic saturation stress which was not uniquely determined by the applied strain amplitude. The data of Laird et al[160] contained a cyclic stress-strain curve for heavily cold-worked copper which was considerably higher than a similar curve for annealed material. Similarly, Lukas and Klesnil[162] reported that, although the cyclic stress-strain curves of copper in the annealed and 20% tensile pre-strained conditions coincided, the curves taken at the 30% and 40% pre-strained conditions displayed higher stress levels.

Very little similar work has been carried out with steel, although Hagiware and Kawabe[163] have observed that the strength of a cold-rolled and aged maraging steel increases continuously as the amount of cold reduction increases.

The cyclic stress-strain curves shown in Figures 55-69 show demarcations which are compatible with the aforementioned examples. i.e., the cold-worked materials display cyclic stress-strain curves which rise considerably above the curves of the annealed materials. It is noteworthy that the 80% cold-rolled material exhibits a higher curve than the 68% cold-rolled material, and the furnace-cooled materials show lower curves than the air-cooled material. The same trends are highlighted more distincly in the low strain regions of the strain-life curves (Figures 70-84). At high strain, the total strain-life curves lie within the same approximate region, but some cross-over of curves is apparent.

In the high cycle regimes of the strain-life curves, all the cold-worked materials data lie above the cold-rolled and annealed data. The plot of the Basquin relationship (Figure 85) shows that the cold-worked materials lie above the annealed materials, while

(Figure 86) shows the Coffin relationship of all the materials, where the cold-rolled data tend to be below those of the annealed steel data.

Kemsley[164] was the first to indicate that there may be differences in the fatigue lives of cold-rolled materials and annealed materials. He observed both annealed and cold-worked copper specimens under cantilever bending, and the cold-worked specimens produced longer fatigue lives. Nair[80] presented similar data (see Figure 27) to show that annealed materials have inferior fatigue properties to cold-rolled materials. The variety of S-N curves constructed from various modes of controlled tests make it difficult to compare any work with any previously published work. Moreover, the stress levels calculated during cantilever bending usually ignore the strain hardening properties of the specimens, although the method does facilitate the collection of S-N data. Karjalainen[165] has indicated that this is a common source of error which produces discrepancies between fatigue data measured by uniaxial cycling and those from bending.

The textural effects on fatigue are displayed by marking specimens of different orientations from the same plates, which maintains constant all factors except crystallographic orientation.

The textures of the cold-rolled plate materials are given in Figures 49 and 50. They display identical features, but CR80 has a more severe texture. The mechanical property prediction given in Table 3 indicate that, in the case of CR68, the 0° and 90° oriented specimens should be almost identical, while the 45° oriented specimens should have different mechanical properties. These predicted trends are still apparent in the case of CR80, although the demarcation is not so obvious. Reference to Table 2 will show an aspect ratio for the grains contained in CR80 which is much greater than for those in CR68. It is probable, therefore, that the grain size and shape in CR80 is beginning to exert an effect on the mechanical properties of the material different to that exerted by the crystallographic texture. Both predicted and experimental mechanical properties of all materials are given in Figures 52-54, where differences (in CR68 and CR80) between properties of 45°, compared to those at 0° and 90°, are apparent. In both materials, the cyclic stress-strain data (Figures 61-66) indicate that, for a given level of imposed strain, the stress amplitude developed by the 45° orientation is lower than in the 0° and 90° specimens. Further, the strain-life curves (Figures 76-81) show that the 45° orientation in both materials produces a superior life at all strain levels under constant strain amplitude cycling than do the 0° and 90° orientations. This phenomenon, whereby the material that displays the stronger cyclic stress-strain curve also displays the inferior life under strain-life conditions agrees with the premises of the cyclic stress-strain method of fatigue analysis, and with the parametric approach which was employed. (See Section 5.3.2). The work of Morrow[8], which shows that in order for the constants in the life equation to be related to the cyclic stress-strain curve the accumulating of fatigue change must be related to the hysteresis work involved during cycling, is borne out. That is, if σ_f' and ε_f' represent a point on the cyclic stress-strain curve, it is because the loop area defines the degree of damage accumulation. In this investigation, the parameters characterising the cyclic stress-strain curve agree when calculated either from the strain-life or the saturation stress and strain amplitude values, which is commensurate with the fact that the material with the lower stress-strain curve exhibits the superior life under imposed strain controlled cycling.

The agreement between the 0° and 90° orientation apparently indicates that there is little grain orientation effect on the fatigue properties of CR68, and that the difference in the fatigue properties observed in the 45° specimens can be ascribed solely to crystallographic texture. However, there is evidence in the present results to support Kettunen's[57] argument that grain size influences the endurance limit of the material. The grain aspect ratios in CR80 are clearly greater than in CR68, and the endurance limit of the 90° oriented CR80 is clearly lower than that of 90° oriented CR68. Also, the experimental fatigue properties of CR80 at 0° and 90° are clearly not identical, which indicates that, as cold reduction increases, the asymmetry of fatigue properties in these two orientations will probably become more pronounced.

The elastic modulus measurements of CR68 are in the ratio

$$\frac{E_{45}}{E_{90}} = \frac{197}{222} = 0.887,$$

(from Table 4) and the ratio of the predicted moduli is 0.880. The ratio of the cyclic yield stresses for the same material, taken from Table 22, is 0.945, while the ratio from the predicted yield stresses is 0.993 (from Table 3). It would therefore appear that, under conditions of imposed strain cycling, the orientation which exhibits the higher flow stress will display poorer fatigue resistance, and that this can be predicted from the texture measurements.

Of the three cold-rolled and annealed materials, only R80AN developed a significant texture (t.s.p. = 1.02) while S10BM and S12BM only attained t.s.p.'s of 0.26 and 0.37 respectively, the higher annealing temperatures tending to randomise the texture. This is clearly reflected in Table 3, where there is little difference between the predicted mechanical properties at 0°, 45° and 90° for either S10BM or S12BM. There is, however, a clear and significant asymmetry about the predicted properties of R80AN. The fatigue data for both S10BM and S12BM exhibit much greater similarity between the three orientations in each set, than do the same data for the two sets of cold-rolled material. In the case of both S10BM and S12BM, the 0° orientation displays the slightly superior life, and the 45° the inferior life, while the data for the 90° orientations' data are midway in both sets of data. The 45° orientations of both materials also display the lowest cyclic stress-strain curves. The scatter in all of the data, however, is greater than that apparent in the data from the cold-rolled materials. This scatter is more obvious in the cyclic stress-strain data than in the strain-life plots. The correlation coefficientshowever, indicate that the data are more accurately displayed by their individual curves, rather than by one curve which includes all the data from the three orientations of one material. Clearly, though, the three data sets of both materials are easily covered by the 'factor of two' criterion which is usually considered acceptable with fatigue data[166]. Quantitatively, the predicted values of both elastic modulus and yield stress are in agreement with the values calculated from the cyclic stress-strain data, and, for S10BM,

$$\frac{S'_{45}}{S'_{90}} \quad : \quad \frac{M_{45}}{M_{90}} \quad = \quad 1.01:1.00$$

where S' is the 2% offset cyclic stress amplitude and M is the corresponding predicted Taylor factor.

Due to the fact that the texture of R80AN was so much more severe than that of either S10BM or S12BM, a greater indication of the likely effects of texture on the fatigue properties of a cold-rolled and annealed steel is available. The predicted properties given in Table 3 and Figures 52-54 indicate that the properties in the 0°, 45° and 90° directions will all differ. This is borne out in the results shown in Figures 52-54, 67-69, and 82-84. In all the cyclic tests, the material displayed a slow, continuous rate of hardening. The 0°

orientation displays a superior life to the 45° orientation, which is itself superior to the 90° orientation. The marked differences in the stress-strain curves (where the 90° orientation is the highest curve, and the 0° orientation the lowest curve) confirm that the differences in the points of intersection of the elastic and plastic lines on the strain-life curves between the three orientations is significant. The scatter in all of the data is again greater than that apparent in the data from the cold-rolled materials, particularly in the stress-strain plots. However, even allowing for the fact that the correlation coefficient for the 0° data is only 0.882, compared to approximately 0.96 for the other two orientations, it is clear that the 0° orientation displays much the superior life. In all three cases, the data sets are again adequately covered by the 'factor of two' criterion[166]. The predicted values of both elastic modulus and yield stress agree quantitatively with the values generated by the cyclic stress-strain data, and

$$\frac{S'_o}{S'_{45}} \quad : \quad \frac{M_o}{M_{45}} \qquad 0.93 \quad : \quad 0.97$$

Kettunen[101], Bhat and Laird[56] and Mughrabi[35] have suggested that the cyclic stress-strain curves of single cyrstals and polycrystalline metals may be compared by employing the Taylor factor to transform the tensile stress and strain amplitudes to shear-stress and shear-strain amplitudes.

Also, although the Taylor factor actually relates to deformation on more than one slip system, Laird[73] has tried to show that the fatigue limit of polycrystalline copper may be predicted from the value obtained from single crystals oriented for single slip. (Strictly, the Taylor factor relates the microscopic tensile stress to the arithmetic sum of the shear strains on the individual slip systems[167].

If the cyclic stress-strain data are analysed in the manner of Bhat and Laird[56], the data clearly fall into two sets (Figure 97) which are coincident with the cold-rolled and cold-rolled and annealed conditions respectively. The data generated by the individual sets of materials were calculated from the cyclic stress-strain analyses shown in Tables 6-20 and are given in Table 25.

$$\text{Now,} \quad \frac{\Delta\tau}{2} = \left(\frac{\Delta\sigma}{2}\right)\Big/M \qquad \ldots\ldots(37)$$

$$\text{and} \quad \frac{\Delta\gamma_p}{2} = M\left(\frac{\Delta\varepsilon_p}{2}\right) \qquad \ldots\ldots(38)$$

$$\text{Now, if} \quad \frac{\Delta\sigma}{2} = K' \left(\frac{\Delta\varepsilon_p}{2}\right)^{n'} \qquad \ldots\ldots(39)$$

$$\text{and} \quad \frac{\Delta\tau}{2} = K'' \left(\frac{\Delta\gamma_p}{2}\right)^{n''} \qquad \ldots\ldots(40)$$

$$\text{then} \quad n'' = n' \qquad \ldots\ldots(41)$$

$$\text{and} \quad K'' = \frac{K'}{M^{(1+n')}} \qquad \ldots\ldots(42)$$

Figure 97 shows that the cyclic stress-strain data of the annealed materials forms a reasonable single data set, although the more strongly textured material (R80AN) exhibits much more scatter than does the weakly textured material. Also, the R80AN data tend to lie above the data from S10BM. This is probably due to the smaller grain size which is apparent in R80AN, and to compositional differences (which can be observed in Table 1) between the two materials.

The correlation of the shear stress-shear strain data for the cold-rolled materials is also shown in Figure 97. Here, the two materials are identical except for the amount of cold work to which they have been subjected. Clearly, although both data sets form a concise single data set with very little scatter, the more heavily rolled material, i.e. CR80 produces data which lie above that of CR68. This agrees with the observations of Laird[160], who has demonstrated that the cyclic stress-strain curve of wavy slip materials is not independent of prior cold work. The cyclic stress-strain curve is unique because the (imposed) cyclic deformation has the ability to disturb the dislocation structure formed by the original cold work, and to develop a unique sub-structure defined only by the amplitude of the imposed cyclic deformation. Cells[7], walls[7] and ladders[168] are the usual dislocation structures found in fatigue. They are all low energy configurations[169], and so the initial high energy dislocation structure must be rearranged into a sub-structure containing cells, walls and ladders. This occurs during the rapid hardening/softening stage of cyclic deformation. Winter[170] has described the microstructure of single crystals of some wavy slip materials in terms of the p.s.b. structure which carries both the plastic strain and a matrix structure of a higher plastic resistance which consists of dense arrays of dislocation dipoles and multipoles, and is, therefore, a configuration of higher energy. The metastable structure is preserved because, since the plastic strain is concentrated in the p.s.b.'s, the dislocation motion necessary to rearrange the dipoles and multipoles does not occur in the matrix structure.

It is the initial cold work and the applied strain amplitude, therefore, which determine the appearance of the rapid hardening/softening stage, and it is the ability of the strain amplitude to break down the cold-worked dislocation structure which leads to the achievement of the stable 'saturation' stage. The differences in cold-worked reduction (and possibly minor differences in the reduction process, e.g. state of lubrication of the rolls) produce different initial structures, and hence different sub-structures at the onset of cycling. Therefore the cyclic shear stress-shear strain curves are different for the two cold-worked materials. It should be noted that, in all cases, the annealed materials exhibited a greater degree of work hardening than did the cold-rolled materials. Indeed, R80AN showed a significant continuous hardening through to failure. It might be expected, therefore, that the cyclic stress-strain response would be similar for both the recrystallized and cold-worked materials. Laird[158] has proposed that cold-worked materials would, if fracture could be prevented, attain a state of saturation which could be determined by a history independent cyclic stress-strain curve.

The textural dependance of the cyclic stress-strain response of low carbon steel may be rationalized, therefore, by using the Taylor factor to reduce the cyclic stress-strain curve to a resolved shear stress-shear strain curve. However, the differences between the sets of data indicate that it must be concluded that, while the effect is apparent, and can be predicted quantitatively within a given aggregate of crystals, it plays a secondary role to effects such as cold work. In the presence of microstructural anisotropy (for example due to MnS inclusions) the effect of texture would be further reduced[78]. An understanding of the mechanisms involved in fatigue failure is required before attempting to relate single crystal properties to stress or strain-life data of polycrystals. Crack nucleation and growth into fatigue damage occurs by accumulation of locally non-reversed plastic strain[2]. It is unfortunate that the strain amplitude-life curve has to be analysed in terms of both the elastic and plastic components of the strain, due to the fact that microplasticity can occur under nominally elastic conditions. Fatigue failure cannot, therefore be ascribed to true elastic strain, and even anelastic strain should be reversible. Non-reversible microplasticity may therefore be present in any elastic strain-life correlations.

All fatigue failures occurred in accordance with the observations of Arnell and Teer[83], who employed x-ray microdiffraction to show that fatigue failures in aluminium were associated with grains which showed a specific set of slip bands, i.e. those with stress axes at the centre of the stereographic triangle. Therefore the failure process depends on the cyclic deformation experienced by a grain in the 'soft' orientation, and the textural dependence of the fatigue process should be determined by the relation between the macroscopic and microscopic stresses and strains. Relating such macroscopic stress-strain states to microscopic stress-strain states is analagous to the problem of notch root crack nucleation (Figure 98). Neuber's[171] rule was utilized by Topper et al[172] to relate the macroscopically imposed stress state to the stress and strain at a notch root.

Similar to the assumption that notched body fatigue is dependent on the local stress state, it will be assumed that the fatigue of polycrystals is dependent on the stress state in a grain of 'soft' orientation. The function $\frac{\Delta\sigma}{2} \; \frac{\Delta\varepsilon}{2}$ describes the accumulation of fatigue damage in terms of the hysteresis work which is absorbed during a single cycle, although Morrow[8] makes it clear that this factor is directly applicable only to the plastic work, and the function should then be multiplied by a stage factor describing the hysteresis loop. Equations 37 and 38 can be used to show that

$$\Delta\sigma\Delta\varepsilon = \Delta\tau\Delta\gamma \qquad \ldots\ldots(43)$$

Strictly, this pertains only to plastic work, but it will be used to describe the total work, due to the nature of the deformation in the supposedly elastic region of the strain-life curve. Figures 99 and 100 show the data points $\Delta\sigma\Delta\varepsilon/4$ vs $2N_f$, and the predicted curves (calculated from the values of σ_f', ε_f', b, c in Table 22) of the same function, for the cold-rolled material CR80 and the recrystallized material R80AN respectively. Due to equation 43, the figures actually present the local shear work i.e. $\Delta\tau\Delta\gamma$ against the fatigue life. Assuming fatigue damage is consistently accumulated in the same manner, then the fatigue life data will be dependent upon a function of the local shear stress and shear strain amplitudes as long as the premise that the cyclic stress-strain response of a material can be expressed as a unique shear stress-shear strain curve is correct. Now, since fracture always occurred in a similar manner, and a polycrystalline material will always contain a number of favourably oriented grains, then the assumption that the deformation may be characterized by damage accumulation in the region which failure occurs, is reasonable. By plotting the data in this manner, Figure 99 shows that the three sets of data points form, effectively, one data set. A similar effect is shown for the annealed material, R80AN, in Figure 100.

Figure 99 should be compared with Figures 79 - 81, and Figure 100 with Figures 82 - 84. It is clear that materials of identical origin and processing are reduced to a single data set irrespective of textural differences. (There is still an obvious difference between the two single data sets of both materials, i.e., the annealed material continues to display a lower endurance limit at all points of the curve). Figure 101 shows $\frac{\Delta\sigma\Delta\varepsilon}{4}$ against reversals for CR68. The three data sets are again reduced to a single data set. The predicted curves are also shown. It should be compared with Figures 76 - 78, and Figure 99.

Comparing Figures 99 and 101, there are discernable consistent differences between these two cold-worked materials, particularly in the high cycle region. These differences are due to the amount of cold work present.

7.2 Fatigue Fracture

Except in the long life strain-life tests, where fatigue crack growth is reasonably slow, the information available on fracture behaviour is negligible. The smaller cross-sectional area of the specimens decreases rapidly, and hence propagation is fast. Hull(174) points out that there is growing evidence to support the theory that the presence of slip in b.c.c. materials is not necessarily an indication that twinning has occurred, therefore the slip bands visible on the softer materials do not particularly suggest that nucleation has been caused by the intersection of twins. However, Hull(174) did observe, in a range of crystal orientations, crack nucleation at a number of different twin intersections in molybdenum. There is no hard evidence to imply that crack nucleation was related to grain orientation or to the orientation of the specimen reference axes, but the occurrence of intrusion-extrusion pairs on some of the specimens indicates the presence of one of the slip configurations suggested by Arnell and Teer(83), which suggests that grain orientation controls nucleation.

The smearing effects on the fracture surfaces caused by the compressive deformations while undergoing tensile-compression strain cycling does not simplify identification of the salient features. However, ductile striations and facets can be used to describe the main features found in Figures 90 and 92a respectively. The ductile striations were more prevalent in the annealed materials and at higher stresses in the cold-worked materials (i.e. in the low cycle regime, or after considerable crack growth in the high cycle regime). The facet-like features are more prevalent in the lower stress amplitudes, and are found closer to the site of nucleation, or the notch in the case of the crack propagation specimens. Hertzberg[175] interprets the appearance of striations similar to those shown in Figure 90 in a crystallographic manner. When metals plastically deform, only specific slip systems are operative. For example, in the b.c.c. system, slip occurs predominantly in <117> type directions, and on type {011} planes. Assuming that the sides of striations are parallel to the operative slip systems, then the striation sides should be parallel to the {011} slip planes. The position of these planes should then determine the angle that the striations make with the advancing crack front. From this model, and its dependence upon the particular orientation of each grain, it is possible to explain why large angles sometimes occur between adjacent areas of striations, and why unfavourable orientations may give rise to poorly defined or even non-existent striations. Nair and LeMay[80,81] observed ductile striations and facet-like features in annealed materials and low strain amplitude tested materials respectively, and used Wood's[4] models of fatigue failure to compare the differences in fracture surface appearance. Nair and LeMay[80,81] suggested that facets are produced by a fracture made of limited ductility, which involved Wood's[4] 'F'-mechanism of brittle crack propagation along dislocation cell boundaries. Awatani et al[168], however, have used the T.E.M. to show that fatigue cracks propagate preferentially transgranularly, and do not employ the grain boundaries. Due to the severe deformation produced by the fatigue cracks, determination of the crystallographic orientation of the facets was not achieved. Further, because of the magnitude of the deformation at the fracture surface, brittle fracture cannot confidently be ascribed as the source of the facets. It is just as likely they are due to crystallographic constraints on the deformation at the crack tip. Priddle and Walker[176] reported similar facets on the fracture surfaces of 316 stainless steel, which were most prolific when the reverse plastic zone of the crack tip equalled the grain size.

Although the crystallographic orientation of the facets observed in this investigation is not known, their presence can be associated with lower crack propagation rates. e.g., in Figure 92, for example, they are in an optically lighter area of the fracture surface, and also they are found at lower strain amplitudes and closer to the origin of the crack.

7.3 Fatigue Crack Propagation

The ability to re-orient crack growth away from the normal to the stress axis seems to be the major effect of texture. (Due to the difficulties involved in producing thinner specimens with the available material, all specimens used in the investigation were the same thickness.) Gross plastic yielding was clearly apparent in the R80AN specimens, leading to considerable (25%) reduction of thickness at the crack tip.

The CR80 specimens which were oriented at 45° to the rolling direction exhibited a fracture which was inclined at an angle from the plane normal to the stress axis. This was also the case for 45° specimens of R80AN. The 45° oriented CR80 specimens consistently showed lower crack propagation rates when projected onto the plane normal to the stress axis (see Table 23). The same is true of the 45° specimens of R80AN (Table 24). Although the actual crack propagation plane has been projected to coincide with the plane normal to the stress axis, the use of ΔK_I, in this case, is merely a mathematical function of the crack length and applied load, and is not attempting to consider the actual stress field at the tip of the crack which is at an oblique angle to the stress axis.

Many workers have investigated the effect of specimen thickness on the rate of crack propagation. In most of the investigations, as thickness is reduced, a macroscopic slant orientation of the crack at 45° to the through-thickness direction is observed, indicating that plane strain conditions cause such a phenomenon. All of the fracture surfaces in the current investigation developed macroscopically planar fractures. Several workers[e.g., 45-47] have observed that greater crack propagation rates were obtained when planar fracture surfaces were present.

The stress intensity of the tip of a crack inclined to the stress axis is effectively dependent on ΔK_I only, but the stress intensity at the tip of a crack slanted to the through thickness is dependent on components of ΔK_I, ΔK_{II} and ΔK_{III}. However, the mode I stress intensity in the two cases is not the same. For the same loading conditions and the same projected crack length the mode I opening stress intensity at the tip of a slant crack is much less than at the tip of a flat crack[177].

$$\text{i.e.} \qquad \Delta K_1 = KA \sin^2$$

$$\frac{da}{dN_1} = \frac{da}{dN_{abs}} \cos\theta$$

where θ is the angle between the crack plane and the stress axis, and $\frac{da}{dN_{abs}}$ is the rate of crack extension which occurs along the crack. Garrett[178] has shown that the point of inflexion in crack propagation curves as a function of applied ΔK is due to a tensile-to-shear transition and is effectively caused by a reduction in the mode I stress intensity amplitude associated with the reorientation of the crack. Recently, Sik and Barthelemy[179] have used an energy density criterion of damage accumulation to show that under constant loading conditions, deviation of the crack orientation from the direction normal to the stress axis, lying in the plane perpendicular to the stress axis, should result in a decrease in the rate of crack propagation.

The microscopic views of the fracture surfaces appeared similar, particularly in the regions close to the notch root. The softer (R80AN) specimens displayed higher rates of crack propagation, and, therefore, there are more shear features apparent, presumably due to the influence of monotonic deformation[179]. Under conditions of low stress intensity, the microscopic modes of crack propagation would appear to be very similar in both fully reversed strain controlled fatigue (Figure 92) and thick plate S.E.N. specimens (Figure 96).

(The inclined fracture developed in this investigation therefore occurs in the orientation which is predicted to develop the higher r-value.) Due to the thickness of the material only approximate plane strain conditions prevail at the crack tip for all three orientations. For the 0° and 90° orientations, the shear bands at the crack tip may be formed at about 45° to the rolling direction. The r-ratio is large in this region (see Table 3), and therefore shear along the bands ε_s as shown in Figure 102 is possible with the suppression of deformation along the line of intersection of the two bands. The crack is then able to propagate in the manner described by Rieux[47], and so planar fracture is produced which enables rapid propagation to take place. If the bands in the 45° specimens form at approximately 45° to the tensile axis, then the predicted r-value suggests that in-plane shears are suppressed at the expense of through-thickness shears. i.e. the ε_{th} shears in Figure 102, are promoted. Due to the thickness of the specimens, however, the crack tip is constrained to resist the through-thickness strain by the areas of the specimen through which the crack has passed. The deformation of these crack tips may, therefore, be achieved by producing through-thickness strains which are in the opposite sense in each band. The shears at the crack tip then consist of mode I, mode II, antiplane shear, and opening. Under the influence of these shears and constraints, the crack maintains its planar mode. This complex group of shears and constraints continues to operate as the crack grows, and the lower growth rate may be due to the mixed mode of crack opening described by Garrett[178].

The mode of fatigue failure is apparently unaffected by the influence of texture, according to the similarity of the fractures highlighted by microfractography. The constraints imposed by the bulk of the material upon the slip processes which may act at a given site in the crack front must be considered before rationalizing the influence of texture on fatigue crack propagation. The crack front may not accelerate through a grain oriented favourably for crack growth, due to the arresting effect of adjacent grains. Therefore, the requirements of compatability of strain are more important than local crystallographic conditions. Similarly, the microscopic crack plane is probably determined by the orientation of the crack in surrounding grains, and not only the local crystallography, as proposed by Neumann[45].

CHAPTER 8

Conclusions and Suggestions for Further Work

8.1 Conclusions

1. Both the cyclic stress-strain curves and the constant strain amplitude-life curves of low carbon steel are affected in the presence of crystallographic anisotropy.

2. Texture analysis is able to predict the effect of crystallographic anisotropy or the cyclic stress-strain curve of steel. As the severity of anisotropy increases, so does the magnitude of it's influence on the cyclic stress-strain relationship. However, the effects of grain size and shape dominate beyond an unspecified point. Presumably, significant amounts of non-metallographic inclusions would also force crystallographic anisotropy into a secondary role.

3. By using the appropriate Taylor factor, M, the cyclic stress-strain curves of materials with the same starting condition may be rationalized to a single cyclic shear stress-shear strain ($\Delta\tau$ v $\Delta\gamma$) curve. The strain history of the material appears to affect the cyclic stress-strain curve. However, the softer (annealed) materials did undergo a greater degree of work hardening.

4. It is proposed that the cyclic stress-strain curve of a given material is unique because the imposed cyclic deformation has the ability to disturb the dislocation structure formed by the

original cold work, and to develop a unique sub-structure, defined only by the amplitude of the imposed cyclic deformation, which uniquely determines the cyclic stress-strain curve.

5. In accordance with the parametric approach to fatigue investigation and the premises of the cyclic stress-strain of fatigue analysis, the orientations within a material which develop the greater resistance to constant strain cycling display the softer cyclic stress-strain curve. Further, the strength of the cyclic stress-strain curves may be predicted from texture measurements.

6. A function independent of crystallographic orientation was observed when the strain amplitude-life data was re-analysed in terms of the parameter $\Delta\sigma\Delta\varepsilon/4$ vs $2N_f$. It has been proposed that the function $\Delta\sigma\Delta\varepsilon$ (the shear stress amplitude in the weakest grain) is identical to $\Delta\tau\Delta\gamma$ (the shear strain amplitude in the weakest grain). Fatigue damage was consistently accumulated in the same manner within a given material. It is therefore reasonable to assume, due to the uniqueness of the cyclic shear stress-strain curve, that the failure process is related to damage accumulation in the weakest grain.

7. The cyclic stress-strain analysis is subject to significant errors when damage is ascribed to the elastic and plastic regimes incorrectly, even though the errors involved in the assessment of texture effects is reduced when the fatigue properties are expressed in terms of σ_f', ε_f', b, c, n' and K'. Unfortunately, texture significantly affects the elastic modulus of a given material, and the modulus is often used to determine

the strain ranges. The correct choice of modulus therefore has a significant effect on the result of the cyclic stress-strain analysis.

8. Any small differences in the fatigue process due to the texture may be obscured because of the statistical nature of fatigue data. The errors involved in fatigue analysis are significantly greater than those in texture measurements.

9. Crystallographic anisotropy causes crack propagation specimens taken at 45° to the rolling direction to produce cracks inclined to the stress axis. In all of these cases, when the crack was projected onto the plane normal to the stress axis, the crack propagation rate was found to be less than that of cracks in specimens taken at 0° and 90° to the rolling direction.

8.2 Suggestions for Further Work

1. Strain-life investigations on materials which have undergone varying amounts of severe deformation should be carried out to determine the degree to which grain size and shape interferes with the effects of crystallographic anisotropy.

2. It is highly likely that aligned non-metallographic inclusions override the effects of crystallographic anisotropy. An investigation to confirm or deny this should be carried out.

3. Both 'thick' and 'thin' crack propagation specimens may be produced, so that specimens producing 'flat' and 'slant' type cracks may be compared. Difficulty may be experienced in producing a severe texture in a 'thin' specimen, due to the required

starting thickness of the material.

4. A single phase metal should be investigated to determine whether or not a significant difference in the fatigue properties is apparent between it and a multiphase metal, or between it and an f.c.c. material.

5. Super pure samples of textured polycrystalline Fe may be used to determine the importance of asymmetric slip, which has been shown (see Chapter 2) to influence the fatigue of single crystals.

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APPENDIX I

The Mathematical Relationship Between the c.o.d.f. and the Pole Distribution

The data are measured as a set of data points $q(\tau,\eta)$ describing the orientation distribution of the pole i as a function of the polar and azimuthal angles, $\cos^{-1}\tau$ and η, with respect to the sample axes. These may be described by the series expansion

$$q_i\ (\tau,\eta) = \sum_{L=0} \sum_{m=L}^{L} Q^i_{Lm}\ P^L_m\ (\tau)\ e^{-im\eta}$$

where $Q_{Lm}{}^i$ are the series coefficients and P^m_L are the associated Legendré polynomials.

The coefficients Q^i_{Lm} may be determined due to the orthoqonality of Legendré polynomals, i.e.,

$$\int_{-1}^{+1} P^m_L\ (\tau).P^m_L\ .\ (\tau)\ d(\tau) \begin{array}{l} = 1 \text{ if } L = L' \\ = 0 \text{ if } L \neq L' \end{array}$$

therefore

$$Q^i_{Lm} = \frac{1}{2\pi}\int_0^2 \int_{-1}^{+1} q_i\ (\tau\eta)P^m_L\ (\tau)e^{-im\eta}\ d\tau\ \ d\eta\ .$$

The c.o.d.f. may also be expressed as a series of spherical harmonics, i.e.

$$w\ (\psi,\xi,\phi) = \sum_{L=0}^{\infty} \sum_{m=-L}^{L} \sum_{n=-L}^{L} W_{Lmn}\ Z_{Lmn}\ (\xi)e^{-im\psi}\ e^{-in\phi}$$

where Z_{Lmn} is a generalisation of the associated Legendré function and W_{Lmn} are the series coefficients.

The relationship between the c.o.d.f. and the $q_i\ (\tau,\eta)$ and therefore between the coefficients Q^i_{Lm} and W_{Lmn} is obtained by setting a temporary co-ordinate system x'y'z' such that the z' axis coincides with the i-plane normal. Thus,

$$\psi = \eta$$

and $$\xi = \tau ,$$

which leads to

$$q_i (\tau,\eta) = \int_0^{2n} W' (\tau \eta \phi')d\phi'$$

where $W' (\psi', \xi', \phi')$ is the c.o.d.f. with respect to the temporary co-ordinate system. Expanding q_i and W', we find

$$\sum_{L=0}^{\infty} \sum_{m=-L}^{L} Q^i_{Lm} p^m_L(\tau) e^{-im\eta} = \int_0^{2n} \sum_{L=0}^{\infty} \sum_{m0-L}^{L} \sum_{n=-L}^{L} W'_{Lmn} Z_{Lmn}(\tau) e^{-im\eta} e^{-in\phi'} d\phi'.$$

Integrating,

$$\sum_{L=0}^{\infty} \sum_{m=-L}^{L} Q^i_{Lm} p^m_{Lm} (\tau) e^{-im\eta} = 2n \sum_{L=0}^{\infty} \sum_{m=-L}^{L} W_{Lm0} Z_{Lm0}(\tau) e^{-im\eta}$$

now, sine $p^m_i (\tau) = Z_{Lm0}(\tau)$,

by comparison,

$$Q^i_{Lm} = 2m W'_{Lm0}$$

The relationship between the coefficients W_{Lmn} and the coefficients W'_{Lmn} is due to the Legendré addition theorem, i.e.

$$W'_{Lmn} = \left[\frac{2}{2L + 1}\right]^{\frac{1}{2}} \sum_{p=-L}^{+1} W_{Lmp} (\cos \theta_i) e^{-ip\Phi_i}$$

This represents, for a given value of L, a set of linear simultaneous equations with 2L + 1 unknowns. However, crystal and specimen symmetry restrict the values of Q^i_{Lm} and W_{Lmn}. For cubic crystal symmetry and orthotropic specimens symmetry:

(i) Q^i_{Lm} and W_{Lmn} are real,

(ii) When $L = 2$, $W_{Lmn} = Q_{Lm} = 0$

$L \neq 2K$, $W_{Lmn} = Q_{Lm} = 0$

$m \neq 2K$, $W_{Lmn} = Q_{Lm} = 0$

$n \neq 2K$, $W_{Lmn} = Q_{Lm} = 0$

(iii) $W_{Lmn} = W_{Lm\bar{n}} = W_{L\bar{m}n} = W_{L\bar{m}\bar{n}}$

(iv) $Q_{Lm} = Q_{L\bar{m}}$

(v) For fixed values of L and m, the W_{Lmn} values are linearly related.

Thus,
$$Q^{i}_{Lm} = \frac{1}{2\pi}\int_{0}^{2\pi}\int_{-1}^{+1} q_i(\Upsilon\eta)\,P^{m}_{L}(\Upsilon)\cos m\eta\, d\Upsilon\, d\eta$$

$$W(\psi,\xi,\phi) = \sum_{L=0}^{\infty}\sum_{m=-L}^{L}\sum_{n=-L}^{L} W_{Lmn}\, Z_{Lmn}(\xi)\cos(m\psi+n\phi)$$

and the set of simultaneous equations becomes

$$Q^{i}_{Lm} = 2\pi\left[\frac{2}{2L+1}\right]^{\frac{1}{2}}\sum_{p=-L}^{+L} W_{Lmp}P^{p}_{L}(\cos\theta_i)\cos(p\Phi_i)$$

Taken together with the conditions (i) to (v), this set of equations may now be solved. The W_{Lmn} coefficients may be determined up to the 22nd order by measuring two complete pole figures.

APPENDIX 2

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FATIGUE:
  PROCEDURE (PARMLIST) OPTIONS(MAIN);
/*THIS PROGRAM FITS A SET OF STRAIN-LIFE FATIGUE DATA TO THE */
/*MATERIAL PARAMETERS SIGMA',EPSILON',b,c,K',N' AS */
/* IN THE RELATION, */
/*Et/2=SIGMA'/E*(2Nf)**-b + EPSILON'*(2Nf)**-c */
/*USING A LEASTSQUARES FIT ON LOGARITHMIC STRAIGHT LINES,THE */
/* DATA IS INPUTTEDAS TOTAL-STRAIN,PLASTIC-STRAIN,STRESS AND */
/* # OF REVERSALS TOFAILURE. */
/* The first time through the program uses the static elastic */
/* elastic modulus and the stress amplitude to determine the */
/* elastic strain amplitude,the second time  it uses the  */
/* differenc between the total strain and the strain width of */
/* the hysteresis loop as the elastic strain  amplitude  */
  DECLARE
     PARMLIST CHAR(12) VAR,
     BPARM(2) CHAR(6),
     APARM CHAR(12),
     STATMOD FLOAT DECIMAL INITIAL(0),
     ROUTE FIXED(1,0) INITIAL(0),
     MODIFE(50) FLOAT DECIMAL INITIAL((50)0),
     (NN,
     I,
     J,
     N) FIXED(4,0) INITIAL(0),
     (ELPSO,
     ELPSTP) FLOAT DECIMAL INITIAL(0),
     ( EPLSO,
     PLST) FLOAT DECIMAL INITIAL(0),
     BASEDATA(50,6)FLOAT DECIMAL INITIAL((300)0),
     LOGDATA(50,6) FLOAT DECIMAL INITIAL((300)0),
     (RPLSO,
     RPLST,
     EPLST,
     EPLSTP) FLOAT DECIMAL INITIAL(0),
     ( PLSCOUNT,
     PLSREV,
     SQPLST,
     SQPLSREV,
     PLSCROSS,
     PSTST,
     STRESS,
     SQSTRESS)FLOAT DECIMAL INITIAL(0),
     ( ELSCOUNT,
     ELST,
     ELSREV,
     SQELST,
     SQELSREV,
     ELSCROSS) FLOAT DECIMAL INITIAL(0),
     MODL FIXED(2,0) INITIAL(0),
     ( MODULUS,
     SQMOD,
     SIGMASQ,
     SIGMA,
     AVEMOD) FLOAT DECIMAL(12) INITIAL(0),
     (ALPHA,
     LOGC,
     EPSILONF,
```

```
        PLASR) FLOAT DECIMAL INITIAL(0),
        ( BETA,
        LOGB,
        BASQUIN,
        SIGMAEFF) FLOAT DECIMAL INITIAL(0),
        ( ELASR,
        ENPRIME,
        KAYPRIME,
        OFFSET) FLOAT DECIMAL INITIAL(0),
        ( LOGK,
        KPRIME,
        NPRIME,
        CYCLICR,
        SPRIME) FLOAT DECIMAL INITIAL(0),
        TITLE CHARACTER(8);
     DECLARE
        UNLIST ENTRY EXT;
        ON ENDFILE (FDATA)
           GO TO START;
        NN=0;
        APARM=PARMLIST;
        CALL UNLIST(APARM,',',BPARM,N);
        TITLE=BPARM(1);
        STATMOD=BPARM(2);
  LAB:
        NN=NN+1;
        GET FILE(FDATA) LIST((BASEDATA(NN,I) DO I=1 TO 4));
        GOTO LAB;
  START:
        NN=NN-1;
        DO I=1 TO NN;
           BASEDATA(I,6)=STATMOD;
           BASEDATA(I,5)=BASEDATA(I,3)/BASEDATA(I,6);
  /* N.B.USING STRESS AND ELASTIC MODULUS TO CALCULATE */
        END;
  /* ELASTIC STRAIN*/
        GO TO CALC;
  AGAIN:
        DO I=1 TO NN;
           BASEDATA(I,5)=BASEDATA(I,1)-BASEDATA(I,2);
           BASEDATA(I,6)=BASEDATA(I,3)/BASEDATA(I,5);
        END;
  CALC:
        DO I=1 TO NN;
           DO J=1 TO 6;
              LOGDATA(I,J)=LOG(BASEDATA(I,J));
           END;
        END;
        PLSCOUNT=0;
        PLST=0;
        PLSREV=0;
        SQPLST=0;
        SQPLSREV=0;
        PLSCROSS=0;
        PSTST=0;
        STRESS=0;
        SQSTRESS=0;
        ELSCOUNT=0;
```

```
      ELST=0;
      ELSREV=0;
      SQELST=0;
      SQELSREV=0;
      ELSCROSS=0;
PLASTIC:
      DO N=1 TO NN;
         IF BASEDATA(N,2)<E-03 THEN
            GOTO ELASTIC;
         PLSCOUNT=PLSCOUNT+1;
         PLST=PLST+LOGDATA(N,2);
         PLSREV=PLSREV+LOGDATA(N,4);
         SQPLST=SQPLST+LOGDATA(N,2)**2;
         SQPLSREV=SQPLSREV+LOGDATA(N,4)**2;
         PLSCROSS=PLSCROSS+LOGDATA(N,4)*LOGDATA(N,2);
         PSTST=PSTST+LOGDATA(N,2)*LOGDATA(N,3);
         STRESS=STRESS+LOGDATA(N,3);
         SQSTRESS=SQSTRESS+LOGDATA(N,3)**2;
ELASTIC:
         ELSCOUNT=ELSCOUNT+1;
         ELST=ELST+LOGDATA(N,5);
         ELSREV=ELSREV+LOGDATA(N,4);
         SQELST=SQELST+LOGDATA(N,5)**2;
         SQELSREV=SQELSREV+LOGDATA(N,4)**2;
         ELSCROSS=ELSCROSS+LOGDATA(N,4)*LOGDATA(N,5);
      END;
      MODULUS=0;
      SQMOD=0;
YOUNGS:
      DO MODL=1 TO NN;
         MODULUS=MODULUS+BASEDATA(MODL,6);
         SQMOD=SQMOD+BASEDATA(MODL,6)**2;
      END;
      SIGMA=ABS((SQMOD-MODULUS**2/NN)/(NN-1));
    SIGMA=SQRT(SIGMA);
      AVEMOD=MODULUS/NN;
      ALPHA=-(PLSCROSS-PLST*PLSREV/PLSCOUNT)/(
       SQPLSREV-PLSREV**2/PLSCOUNT);
      LOGC=(PLSREV*PLSCROSS-PLST*SQPLSREV)/(
       PLSREV**2-PLSCOUNT*SQPLSREV);
      EPSILONF=EXP(LOGC);
      BETA=-(ELSCROSS-ELST*ELSREV/ELSCOUNT)/(
       SQELSREV-ELSREV**2/ELSCOUNT);
      LOGB=(ELSREV*ELSCROSS-ELST*SQELSREV)/(
       ELSREV**2-ELSCOUNT*SQELSREV);
      BASQUIN=EXP(LOGB);
      BASQUIN=EXP(LOGB);
      SIGMAEFF=BASQUIN*AVEMOD;
      ENPRIME=BETA/ALPHA;
      KAYPRIME=SIGMAEFF/EPSILONF**ENPRIME;
      OFFSET=KAYPRIME*(0.002)**ENPRIME;
      RPLSQ=(PLSCROSS-PLST*PLSREV/PLSCOUNT);
      RPLST=PLSCOUNT* SQRT(SQPLSREV/PLSCOUNT-(PLSREV/PLSCOUNT)**2);
      RPLST=RPLST*SQRT(SQPLST/PLSCOUNT-(PLST/PLSCOUNT)**2);
      PLASR=RPLSQ/RPLST;
      ELPSQ=(ELSCROSS-ELST*ELSREV/ELSCOUNT);
      ELPSTP=ELSCOUNT*
      SQRT(SQELSREV/ELSCOUNT-(ELSREV/ELSCOUNT)**2);
```

```
     ELPSTP=ELPSTP*SQRT(SQELST/ELSCOUNT-(ELST/ELSCOUNT)**2);
     ELASR=ELPSO/ELPSTP;
CYCLIC:
     LOGK=(PSTST*PLST-STRESS*SQPLST)/(PLST**2-SQPLST*PLSCOUNT);
     KPRIME=EXP(LOGK);
     NPRIME=(PSTST-STRESS*PLST/PLSCOUNT)/(
      SQPLST-PLST**2/PLSCOUNT);
     SPRIME=KPRIME*0.002**NPRIME;
     CYCLICR=(PSTST-STRESS*PLST/PLSCOUNT)/PLSCOUNT;
     CYCLICR=CYCLICR/SQRT(SQPLST/PLSCOUNT-(PLST/PLSCOUNT)**2);
     CYCLICR=CYCLICR/
     SQRT(SQSTRESS/PLSCOUNT-(STRESS/PLSCOUNT)**2);
TABLE:
     PUT PAGE;
     PUT SKIP(4) EDIT('FATIGUE DATA ANALYSIS FOR ',TITLE)(
      COLUMN(10),A(28),A(8));
     IF ROUTE=0 THEN
        PUT SKIP
        EDIT(
        'Fatigue analysis employing static modulus and stress amplitude
 to calculate the elastic strain range'
         )(
        COL(16),A(100));
     ELSE
        PUT SKIP
        EDIT(
        'Fatigue analysis employing total strain and plasti strain to c
alculate the elastic strain range'
         )(
        COL(18),A(97));
     PUT SKIP(3) EDIT('STRAIN-LIFE DATA')(COLUMN(53),A(18));
     PUT SKIP(2) EDIT('CYCLIC PLASTIC PARAMETERS')(
      COLUMN(40),A(25));
     PUT SKIP EDIT('COFFIN EXPONENT = ',ALPHA)(
      COLUMN(30),A(18),E(14,6));
     PUT SKIP EDIT('CYCLIC FRACTURE STRAIN = ',EPSILONF)(
      COLUMN(30),A(25),E(14,6));
     PUT SKIP EDIT('CORRELATION COEFFICIENT RP = ',PLASR)(
      COLUMN(30),A(29),E(14,6));
     PUT SKIP(2) EDIT('CYCLIC ELASTIC PARAMETERS')(
      COLUMN(40),A(25));
     PUT SKIP EDIT('BASQUIN EXPONENT = ',BETA)(
      COLUMN(30),A(19),E(14,6));
     PUT SKIP EDIT('CYCLIC FRACTURE STRESS = ',SIGMAEFF)(
      COLUMN(30),A(25),E(14,6));
     PUT SKIP EDIT('CORRELATION COEFFICIENT RE = ',ELASR)(
      COLUMN(30),A(29),E(14,6));
     PUT SKIP
     EDIT('MEAN ELASTIC MODULUS = ',AVEMOD,'STANDARD DEVIATION = ',
     SIGMA)(COLUMN(30),A(23),E(14,6),A(21),E(14,6));
     PUT SKIP
     EDIT('CYCLIC STRESS-STRAIN PARAMETERS FROM STRAIN-LIFE DATA')(
      COLUMN(40),A(53));
     PUT SKIP EDIT('CYCLIC STIFFNESS CONSTANT = ',KAYPRIME)(
      COLUMN(30),A(28),E(14,6));
     PUT SKIP EDIT('CYCLIC HARDENING EXPONENT = ',NPRIME)(
      COLUMN(30),A(28),E(14,6));
     PUT SKIP EDIT('0.2% OFFSET STRESS = ',OFFSET)(
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        COLUMN(30),A(21),E(14,6));
      PUT SKIP(3) EDIT('CYCLIC STRESS-STRAIN ANALYSIS')(
        COLUMN(50),A(29));
      PUT SKIP(2) EDIT('CYCLIC STIFFNESS CONSTANT = ',KPRIME)(
        COLUMN(30),A(26),E(14,6));
      PUT SKIP EDIT('CYCLIC HARDENING EXPONENT = ',ENPRIME)(
        COLUMN(30),A(28),E(14,6));
      PUT SKIP EDIT('0.2% OFFSET STRESS = ',SPRIME)(
        COLUMN(30),A(28),E(14,6));
      PUT SKIP EDIT('CORRELATION COEFFICIENT RC = ',CYCLICR)(
        COLUMN(30),A(29),E(14,6));
/*END*/
      DO I= 1 TO NN;
         MODIFE(I)= BASEDATA(I,2)+BASEDATA(I,5);
      END;
      PUT SKIP(3) EDIT('DATA DEVELOPED BY THIS CALCULATION')(
       COL(31), A(34));
      PUT SKIP
      EDIT('Total strain','Plastic strain','Stress','Reversals to',
      'Elastic strain','Elastic','Corrected strain')(
       COL(1),A(12),COL(17), A(14),COL(35),A(6),COL(52),A(12),COL(68),
      A(14),COL(86),A(7), COL(104),A(16));
      PUT SKIP
      EDIT('Amplitude','Amplitude','Amplitude','Failure', 'Amplitude',
      'Modulus','Amplitude')(
       COL(4),A(9),COL(20),A(9),COL(37),A(9),COL(55), A(7),COL(71),
      A(9), COL(88),A(7),COL(110),A(9));
      DO I= 1 TO NN;
         PUT SKIP
         EDIT(BASEDATA(I,1),BASEDATA(I,2),BASEDATA(I,3),BASEDATA(I,4),
         BASEDATA(I,5),BASEDATA(I,6),MODIFE(I))(
          COL(1),E(14,6),COL(17),E(14,6), COL(35),E(14,6),COL(52),
         E(14,6),COL(68),E(14,6),COL(86),E(14,6),COL(104), E(14,6));
      END;
      ROUTE=ROUTE+1;
      IF ROUTE =1 THEN
         GO TO AGAIN;
FINISH:
  END FATIGUE;
```