THE MEASUREMENT AND DETECTION OF RESIDUAL STRESSES IN COLD DRAWN TUBES.

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CONTAINS PULLOUTS

TEXT BOUND CLOSE TO THE SPINE IN THE ORIGINAL THESIS

PREFACE AND ACKNOWLEDGEMENTS.

The subject of residual stresses which occur in metals after many processing operations, is probably one of the most widely written subjects in the engineering and metallurgical fields. Despite the widespread interest which has now been maintained for almost seventy years, during which time much progress has undoubtedly been made, the accurate determination of residual stresses has not appeared possible. In an attempt to place the detection and measurement of residual stresses in cold drawn tubes on a more factual basis, the present investigation was carried out in the University under the aegis of the Sheet Metal Working Committee of the British Iron and Steel Research Association.

It is not claimed that the present investigation solves the problem of residual stresses in tubes. To be more specific, the work carried out can only be considered as a start to the solution of the problem. However, the techniques developed during the work enable the calculation of what are believed to be reasonably correct values of the residual stresses present in tubes and it is thought that these can be applied quite satisfactorily to a study of the influence of the many variables which occur in the process of tube drawing. The number of possible investigations which now seem open are so numerous that no attempt is made in this work to make any suggestions for future development. It should be stated, however, that the techniques developed demand very careful experimentation and any single test is of necessity of long duration, two factors which might weigh heavily against the techniques and show the necessity for the development of other (possibly more approximate) methods for more widespread application.

Throughout the course of this work the author has been deeply indebted to Professor H. W. Swift for his continuous interest in the problem and for his kindly advice which was forthcoming on many occasions. Mention must also be made of Messrs. Botros, Montgomery, Howard, Roake and Whiteley, who, as research students working under the author's supervision, were responsible for much of the development work described in sections 3.2, 3.4.1, 3.4.3, 3.4.5, 4.2, 5.2.3, 6.2 and 6.3 of this report. The valuable advice given by many of the author's colleagues at the University is also acknowledged, particularly Messrs. Freeman, Fisher, Harris and Thomas for their assistance with the development of the strain gauge techniques, the computation of the results and the machining of specimens as and when this was necessary.

Thanks are also due to Messrs. Howells, Ltd., Accles and Pollock, Ltd., Tube Investments Ltd., (through Dr. J. W. Jenkin) and Chesterfield Tube Co. (through Mr. Marsh) for supplying many of the tubes used in this work, to R. B. for arranging the heat treatment of the tubes used in Part 7 of this work, in the Sheffield works of which he is a director, and to Mr. Evans (Sheffield Testing Works) for calibrating the strain gauge mounted drawing pin on a testing machine of larger capacity than was available in the University.

This list of acknowledgements cannot be concluded without collective mention of the members of the Sheet Metal Working Committee of the British Iron and Steel Research Association. On several occasions the progress of the work has been discussed by this Committee and many suggestions for its improvement and extension have been made.

Finally, attention is drawn to the following extract of a letter from Dr. J. W. Jenkin, which accompanied two of the tubes which have been used in this investigation.

"It will be appreciated that the tubes supplied to you are in a far greater state of residual stress than any tube which would be supplied to a customer. They have been left in this state solely for the purpose of your investigation."

E. M. L.

Sheffield, 1955.

SUMMARY.

THE DETECTION AND MEASUREMENT OF RESIDUAL STRESSES IN COLD DRAWN TUBES.

Residual stresses are defined as those stresses which exist in a body when all external constraints are removed.

Following a review of the most important published works related to residual stresses and particularly to those present in cold drawn tubes, it is concluded that there are available two mechanical methods of measuring residual stresses, each being dependent upon the ability to release and measure internal strains. In both cases strains are released by upsetting the balance of force across a section by the removal of metal, but in one case the resultant strains are measured directly and in the other by a process known as bending deflection.

Of the three published analyses related to the two methods of strain release, two are considered sufficiently exact for general application (although one of these is too cumbersome for widespread use), while the third contains unnecessary approximations. It is shown that this analysis can be amended quite easily without destroying the simplicity of the method.

Tests are reported which show that all the analyses and both methods of strain release yield almost identical results when applied to specimens prepared from the same tube, provided certain requirements are met with regard to specimen length and width. Several tests are necessary with bending deflection; single tests with direct strain release.

The experimental techniques developed for this work demand a chemical pickling process for the removal of layers of metal from a specimen and resistance strain gauges for the measurement of the induced changes (this latter with direct strain release only).

The residual stresses which occur in hollow drawn tubes are investigated together with the factors influencing their production.

CONTENTS

1. INTRODUCTION.

The term residual or internal stress is used to refer to a stress or a stress system which is induced in an article during its manufacture and which does not disappear in the natural relaxation of the article when all external constraints are removed (Orowan, 1947)*. If portions of a solid or a number of solids forming together one rigid mass, are hindered from assuming the natural length which they would have in the absence of any hindrance, this solid or number of solids is said to be under stress. The stressed condition may be brought about (a) by forces acting on the solid (or solids) from outside (external forces) or (b) without any observable action from outside after all external action has ceased. These latter stresses are known as internal stresses and the solid can be referred to as being "self strained" (Heyn, 1914).

Residual stresses may occur in an engineering component on either a microscopic or a macroscopic scale (Soete, 1949). Microscopic stresses vary from grain to grain of the material and are an intrinsic property of the material itself. They are generally caused by inhomogeneities in the texture of a material and are often referred to as textural stresses. A typical example of them is the stress which can exist in a steel as a result of the different coefficients of expansion of the cementite and ferrite lamellae occurring within the pearlite matrix. Macroscopic stresses are of much greater magnitude and are of more importance to engineers. They may be formed as the resultant of microscopic stresses but more generally they are formed by such things as shrinkage

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* A complete list of references is given at the end of this thesis.

after casting or welding, heat treatment (particularly rapid quenching), cold working (radial drawing, strip and sheet rolling, extrusion, etc.), the forcible joining of improperly fitted components (e.g. incorrect bolting or rivetting) and so on. These stresses are sometimes referred to as "body stresses" and it is with these and more particularly with the ones produced by the cold drawing of tubes that this thesis is concerned.

The fact that residual stresses are set up in metals during processing has long been recognized. The importance which is attached to the knowledge of the magnitude and direction of these stresses may well be assessed by the number of symposia concerned with the subject which have been held from time to time by learned societies in different parts of the world (e.g. A.S.T.M., 1918; Faraday Society, 1921; A.S.T.M., A.I.M.E., 1944; Inst. of Metals, 1947; A.S.M., 1951) and by the large number of references to published papers etc., all connected with the subject which can be quickly found in most technical libraries.

Unless an engineering component is used in a completely stress-relieved state, the residual stresses present may have an important bearing on the service life of the part; in some cases residual stresses may prove beneficial (e.g. high residual compressive stresses increase the fatigue strength of railway wagon axles) (Horger and Neifert, 1942), but in many cases their effect may lead to disastrous consequences and result in almost spontaneous fracture of the component containing the stress.

(An interesting example of the failure of deep drawn cups under the influence of residual stresses, happened in the author's presence some years ago. In connection with a deep drawing problem, a number of 2 in. dia. cups were drawn from circular blanks prepared

FIG. 1.1. DEEP-DRAWN ALUMINIUM-MAGNESIUM ALLOY CUPS WHICH SPLIT SPONTANEOUSLY AFTER CLEANING.

from 0.036 in. thick sheet of an aluminium magnesium alloy. For several months these cups were placed in a store during which period no visible change occurred in them. On removing from store, however, and after cleaning in readiness for an exhibition, many of the cups burst open along a number of longitudinal cracks which apparently developed quite suddenly in the walls of the cups. Several of the fractured cups are shown in Fig. 1.1.)

As a general rule, manufacturers attempt to remove the residual stresses induced in a component during the manufacturing process, by suitable heat treatment, but in many cases this step is not advisable as it generally results in a loss of both strength and dimensional accuracy of the components.

Despite the vast amount of published information, the measurement of residual stresses is still only approximate in nature and apart from a few isolated publications, most of the available results refer to the stresses occurring in particular components without attempting to penetrate the fundamentals of the problem.

The investigations made in the course of this work are an attempt to place the problem in a more correct perspective, particularly as it applies to cold drawn tubes and other tubular components. The initial stages of the investigation have been concerned with the development of suitable experimental techniques for the measurement and detection of residual stresses and in a later stage the results of this development have been applied to the measurement of residual stresses in a number of hollow drawn tubes with different wall thicknesses and prepared in a number of single stage diametral reductions.

2. SURVEY OF PREVIOUS INVESTIGATIONS RELATED TO THE MEASUREMENT OF RESIDUAL STRESSES IN TUBES.

2.1. Theoretical methods available for the detection and measurement of residual stress.

Residual stresses in a body are set up as a result of a mismatch of different elements of that body. In order that these elements may be brought together to form a continuous or complete whole body, each individual portion must adjust itself so that it is compatible with the other elements. The adjustment of each element is effected by elastic strain and it is this strain which instigates the residual stress system. Since residual stresses cannot be measured directly using similar methods to those used in the measurement of applied stresses, measurements are made of the strains which exist in a residually stressed body, and from these measurements the stresses are computed. In other words, residual stresses are measured indirectly by measuring the elastic strains within the stressed body.

Many methods have been proposed from time to time to detect and measure the internal strains in a residually stressed body, but generally these can be classified under three main headings:-

(i) direct strain release and measurement,

(ii) strain release and measurement by bending deflection

and (iii) X-ray or other non-destructive detection of strain.

2.1.1. Direct strain release and measurement.

The procedure embodied in the direct release and measurement of strain in a residually stressed body, involves in general terms a reversal of the process by which the strains were produced, i.e. elements are removed from a stressed body and observations are made of the resultant deformations (or mismatch) in the remaining material.

The first significant development in the field of direct strain release was made by Bauer and Heyn in 1911. They

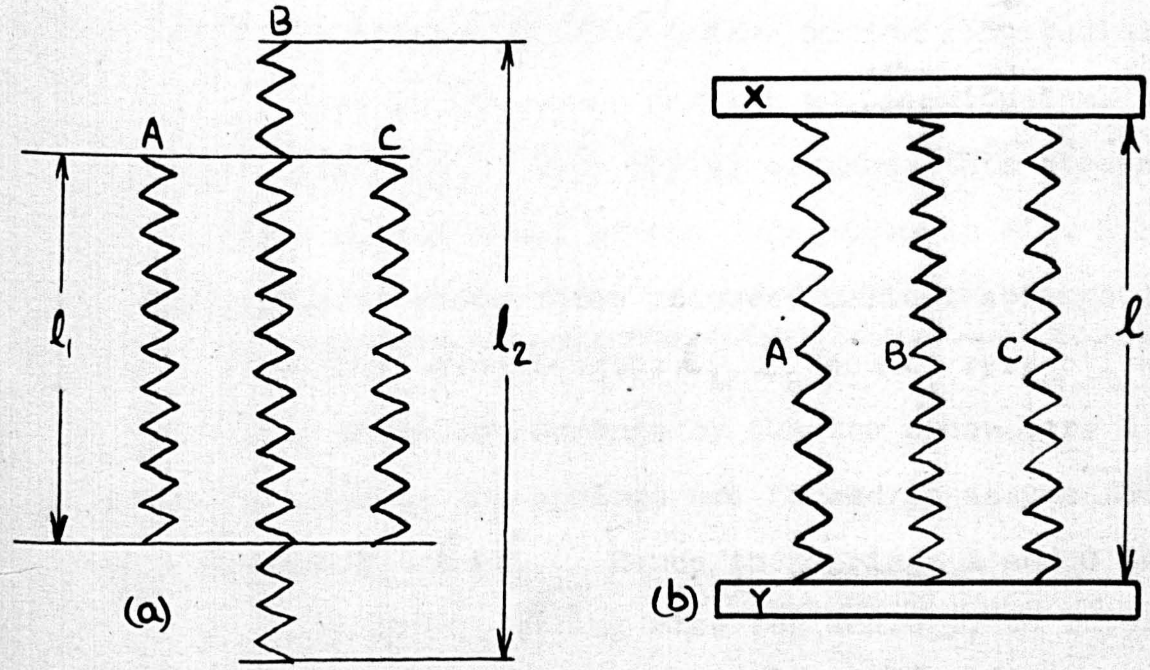


FIG: 2.1. HEYN'S SIMPLE SPRING MODEL.

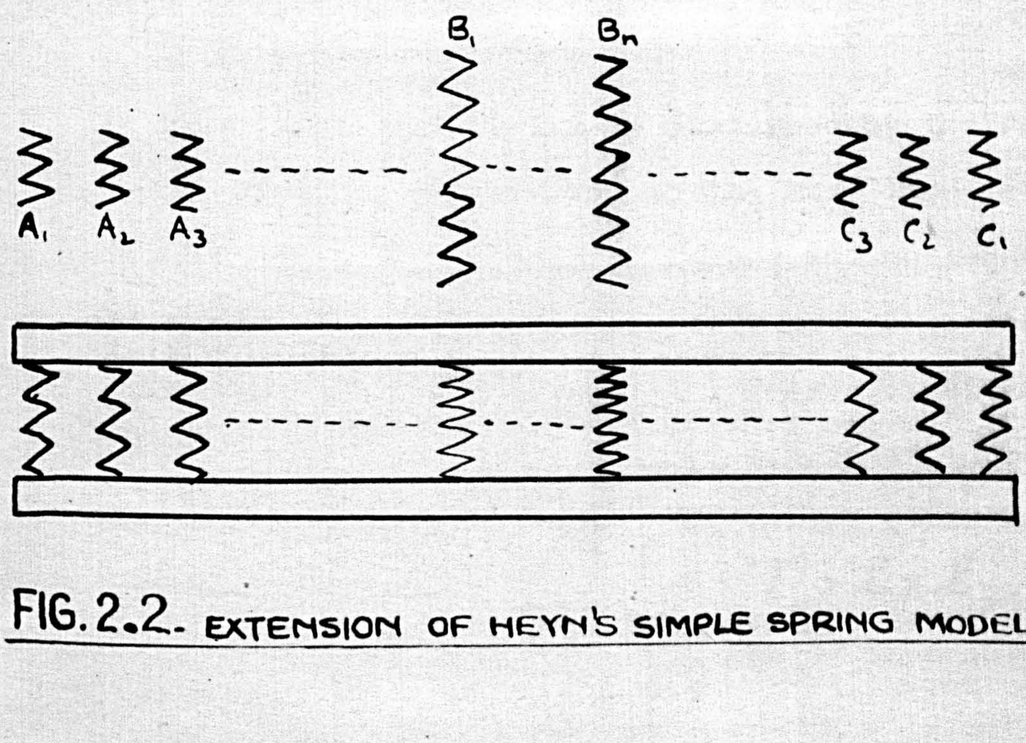


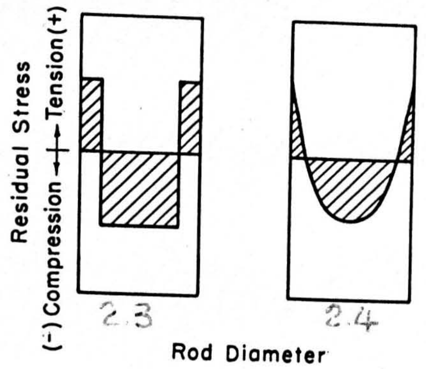
FIG. 2.2. EXTENSION OF HEYN'S SIMPLE SPRING MODEL.

developed a rather specialised method which is more important for the way in which it illustrates the technique rather than for the accuracy of its analysis. The particular stress system on which Bauer and Heyn based their analysis was one contained in a cylindrical rod which carried longitudinal tensile stresses in its outer portion and longitudinal compression in its core. Heyn (1914) compared this stress system with a spring model of the type shown in Fig. 2.1.

Fig. 2.1(a) shows three unloaded helical springs A, B and C which have free lengths l_1, l_2 and l_1 respectively. On connecting these two springs by the two cross bars X and Y as in Fig. 2.1(b) the springs are forced to assume the common length l where $l_1 < l < l_2$. Hence the springs A and C increase in length elastically, giving rise (by analogy) to internal tensile stresses, while spring B contracts giving rise to internal compression. The two springs A and C tend to bring the two bars X and Y nearer to each other with a force P_1, while the spring B tends to separate them with a force P_2. To satisfy the equilibrium of the system the force must be balanced and consequently $2P_1$ must be numerically equal to P_2.

If this force equilibrium is disturbed by the removal of the outer springs, the central spring must immediately shed its load to restore the balance of forces and consequently the distance between the two connecting bars must increase to l_2, the original length of the spring B. Hence, by analogy, the release of the outer tensile stresses causes an increase in the length of the remaining section. If, on the other hand, the conditions are reversed in Fig. 2.1 and $l_1 > l > l_2$, the removal of the outer compression will result in an overall shortening of the remaining spring.

If this simple model is replaced by the rather more complex model shown in Fig. 2.2, which contains many more springs, and if the springs are now taken to indicate stresses, and the two crossheads the boundaries of a residually stressed body, the analogy can be followed through to show the whole



FIGS. 2.3 and 2.4. SIMPLE AND COMPLEX STRESSES IN A ROD. (After Baldwin, 1949).

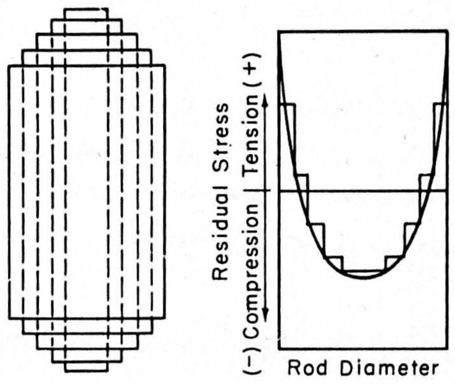


FIG. 2.5. HEYN'S RESIDUAL STRESS DETERMINATION APPLIED IN A SUCCESSION OF SMALL STEPS TO A CYLINDRICAL ROD. (After Baldwin, 1949).

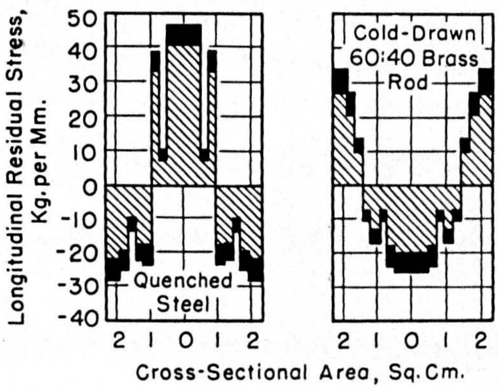


FIG. 2.6. COMPARISON OF RESIDUAL STRESSES DETERMINED BY HEYN'S METHOD (SHADED) AND SACHS' METHOD (BLACK). (After Portevin, 1928).

principle of residual stress measurement by direct strain release. Considering the removal of the outer tensions A, the length of the residually stressed body (original length $= \ell$) will increase by an amount dl. According to Hooke's law, the stress p_{ℓ}, relieved by this expansion is related to the strain by the expression

$$p_{\ell_1} = E\, de_1$$

where $de_1 = d\ell_1/\ell$ and E = Young's modulus for the material.

The force (F_i) imposed over the remaining section of the body must counterbalance the force (F_o) in the removed outer portion. The force in the remaining section is the product of the relieved stress and the area A_1 of the section. If A_o is the initial area of the body and A_1 is the area after removal of the outside ($dA_1 = A_o - A_1$) and considering the mean stress over the removed section as being equal to $\bar{p}_{\ell_1}$, then

$$F_o = \bar{p}_{\ell_1}\, dA_1$$

$$= F_i = p_{\ell}\, A_1 = A_1 E de_1$$

Hence $$\bar{p}_{\ell_1} = A_1 E \frac{de_1}{dA_1} \qquad \text{.......................(2.1)}$$

This expression itself is sufficient to satisfy the stress pattern shown in Fig. 2.3 (Baldwin, 1949), but it does not apply to a more general stress pattern of the type illustrated in Fig. 2.4, where the stress varies continuously throughout the section of the residually stressed body.

In order to determine the residual stress pattern shown in Fig. 2.4 occurring in a symmetrical section (rod or tube), the method of Bauer and Heyn is still applicable as will be realized from a further consideration of Fig. 2.2. If successive thin layers are removed from the cylindrical specimen and the resultant deformations in the remaining section are observed after each stage of layer removal, then the resultant stress pattern can be approximately determined as illustrated in Fig. 2.5.

After the first removal, equation 2.1 can be applied and the residual stress in the removed layer calculated. During the removal of this layer, a stress of magnitude $p_{\ell_1} = Ede_1$ is superimposed on the remaining section so that when the next layer is removed, the stress removed in it calculated according to equation 2.1 is in excess of the actual stress originally present in that section by an amount p_{ℓ_1} $(= Ede_1)$

Hence
$$\bar{p}_{\ell_1} = A_1E\frac{de_1}{dA_1}$$

$$\bar{p}_{\ell_2} = A_2E\frac{de_2}{dA_2} - Ede_1$$

Proceeding further, $\bar{p}_{\ell_3} = A_3E\frac{de_3}{dA_3} - Ede_1 - Ede_2$

and so on. Finally the stress relieved with the nth layer is
$$\bar{p}_{\ell_n} = A_nE\frac{de_n}{dA_n} - \sum_0^n Ede_1$$

or more generally, where the removed layers are of differential thickness,

$$\bar{p}_{\ell} = E(A\frac{de}{dA} - e)\text{............} \quad (2.2)$$

where e is the total strain in the remaining section after each layer removal.

Equation 2.2 may be readily applied to the determination of longitudinal stresses in a cylindrical specimen by a graphical analysis (Lynch, 1951). Finite layers are carefully removed from the surface of the specimen and the total strain (e) is measured after each removal. This strain is then plotted as a function of the area of the remaining section and a smooth curve drawn. The quantity de/dA is then the slope of the experimental curve at any particular point under consideration.

The method of Bauer and Heyn, dealt with in detail in the preceding paragraphs is, however, only approximate since longitudinal stresses only are considered, and as previously mentioned it is more important for illustrating the principles involved in residual stress measurement by direct release of strain rather than for the accuracy of its analysis. The presence of residual stresses in the circumferential and radial directions will definitely affect the longitudinal strains measured using Bauer and Heyn's technique. Portevin (1928) used the Bauer and Heyn technique coupled with (a) the Heyn analysis and (b) the more exact Mesnager - Sachs analysis (discussed later) for the determination of the residual longitudinal stresses in a quenched steel rod and in a cold drawn β (60/40) brass rod. His results, reproduced in Fig. 2.6 show clearly that the actual longitudinal stresses were underestimated by as much as 30% by the use of the Heyn analysis.

The limitations in the Bauer and Heyn analysis were recognized by Mesnager (1919) who developed a method for round bars and other cylindrical components. The experimental techniques proposed by Mesnager were identical with those of Bauer and Heyn but both circumferential and longitudinal strains were measured at each stage of the metal removal process. The analysis was based on the theory of thick cylinders under internal or external pressure and by substitution of the measured quantities in a series of expressions, Mesnager was able to determine the three principal residual stresses in a cylindrical body. Sachs (1927) greatly simplified the calculations necessary in the Mesnager method by the inclusion of graphical analysis similar to that already mentioned in connection with Bauer and Heyn's equations. Today, the method of Mesnager and Sachs is very widely known as the Sach's boring method and it is with this designation that it will be referred to hereafter.

In developing the boring method, Mesnager and Sachs made the following assumptions:-

(i) The elastic theory of thick cylinders under internal or external pressure applies.

(ii) The stress distribution in a residually stressed specimen is symmetrical about the axis of the specimen and is constant along the whole length of the specimen at any particular distance from the axis of symmetry.

(iii) The removal of a layer of material is accompanied by an equal change in stress at all similar points in the cross-section, i.e. the material is wholly homogeneous and has constant and fixed values of Young's modulus and Poisson's ratio.

In the Sach's version of the final equation, a further assumption is inherent in that:-

(iv) It is assumed that the drawing of a smooth curve through the experimentally derived points does not incur any error in the results and that infinitesimal changes can be related to the slope of a curve drawn through the experimental points.

These assumptions, which are relatively few in number, particularly in view of the magnitude of the problem, are all justifiable and while obviously restricting the method to the study of residual stresses in cylindrical objects in which the stresses vary only in one Cartesian direction (the radial direction) they do not detract from the usefulness of the analysis and the experimental procedures.

The Sach's boring method was initially developed to determine the entire residual stress distributions in solid cylindrical components, but it can be applied with greater accuracy to hollow components such as tubes or cylinders. (With solid specimens a hole must first of all be drilled through the centre along the axis of symmetry, before any systematic metal removal process **can** be started. With hollow specimens, however, this initial drilling is not necessary, as the starting surface is already present.) The experimental technique consists of boring out, in steps, the interior of a cylindrical specimen. Before and after each stage of the metal removal process the diameter, length and wall thickness of the specimen are measured at carefully marked reference points on the outer surface. The two measured unit strains (λ, δ in the longitudinal and circumferential directions respectively) based on the initial dimensions of the specimen, and the current area of the bore enable the stresses to be calculated according to the following expressions:-*

$$\left.\begin{aligned} p_{\ell} &= \frac{E}{1-\sigma^2}\left[(A_o - A_d)\frac{d\psi}{dA_d} - \psi\right] \\ p_c &= \frac{E}{1-\sigma^2}\left[(A_o - A_d)\frac{d\theta}{dA_d} - \left(\frac{A_o + A_d}{2A_d}\right)\theta\right] \\ p_r &= \frac{E}{1-\sigma^2}\left[\left(\frac{A_o - A_d}{2A_d}\right)\theta\right] \end{aligned}\right\} \quad \ldots\ldots\ldots(2.3)$$

where p_{ℓ}, p_c, p_r are respectively the residual longitudinal, circumferential and radial stresses at the point in the tube wall defined by the area A_d.

A_o = original area circumscribed by the outside diameter.

A_d = current cross sectional area of bore.

E = Young's modulus for the specimen material.

σ = Poisson's ratio.

θ, ψ = Parameters $\delta + \sigma.\lambda$ and $\lambda + \sigma.\delta$ respectively.

In the determination of the strains, considerable care has to be exercised to get measurements at constant

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* The full derivations of these equations are given in Appendix 5.

temperatures or to correct for temperature changes. The accuracy of the method is considerably increased if the actual measured strain values are plotted against the bored cross sectional area to give smooth curves and the values of the derivatives $\frac{d\theta}{dA_d}$ and $\frac{d\psi}{dA_d}$ taken from these curves.

The same method of calculation can be applied to the removal of metal from the outside diameter, measuring the change of length (λ) and the change in internal diameter (δ). In this case the equations 2.3 have to be amended to:-

$$\left.\begin{aligned} p_\ell &= \frac{E}{(1-\sigma^2)}\left[-(A_D - A_i)\frac{d\psi}{dA_D} - \psi\right] \\ p_c &= \frac{E}{(1-\sigma^2)}\left[-(A_D - A_i)\frac{d\theta}{dA_D} - \frac{(A_D + A_i)}{2A_D}\theta\right] \\ p_r &= \frac{E}{(1-\sigma^2)}\left[\frac{(A_D - A_i)}{2A_D}\theta\right] \end{aligned}\right\} \quad \ldots(2.4)$$

where A_D is the variable cross section of the material removed, and A_i is the original cross section of the bore.

Since the publication of the Mesnager - Sachs analyses for the determination of residual stresses in cylindrical bodies, the principles of residual stress detection and measurement by direct strain release have been applied to components of irregular shapes (Mathar, 1934, Soete, 1949) but at present the techniques are rather unsatisfactory and the analyses very difficult to handle (internal strains are measured in any three or more directions and from these the principal strains have to be derived either by laborious geometrical constructions or tedious mathematical manipulation), and apart from their mention they will not be considered further at this stage. (A full review of these methods, known generally as local strain release methods, has been made by Hiendelhofer, 1948).

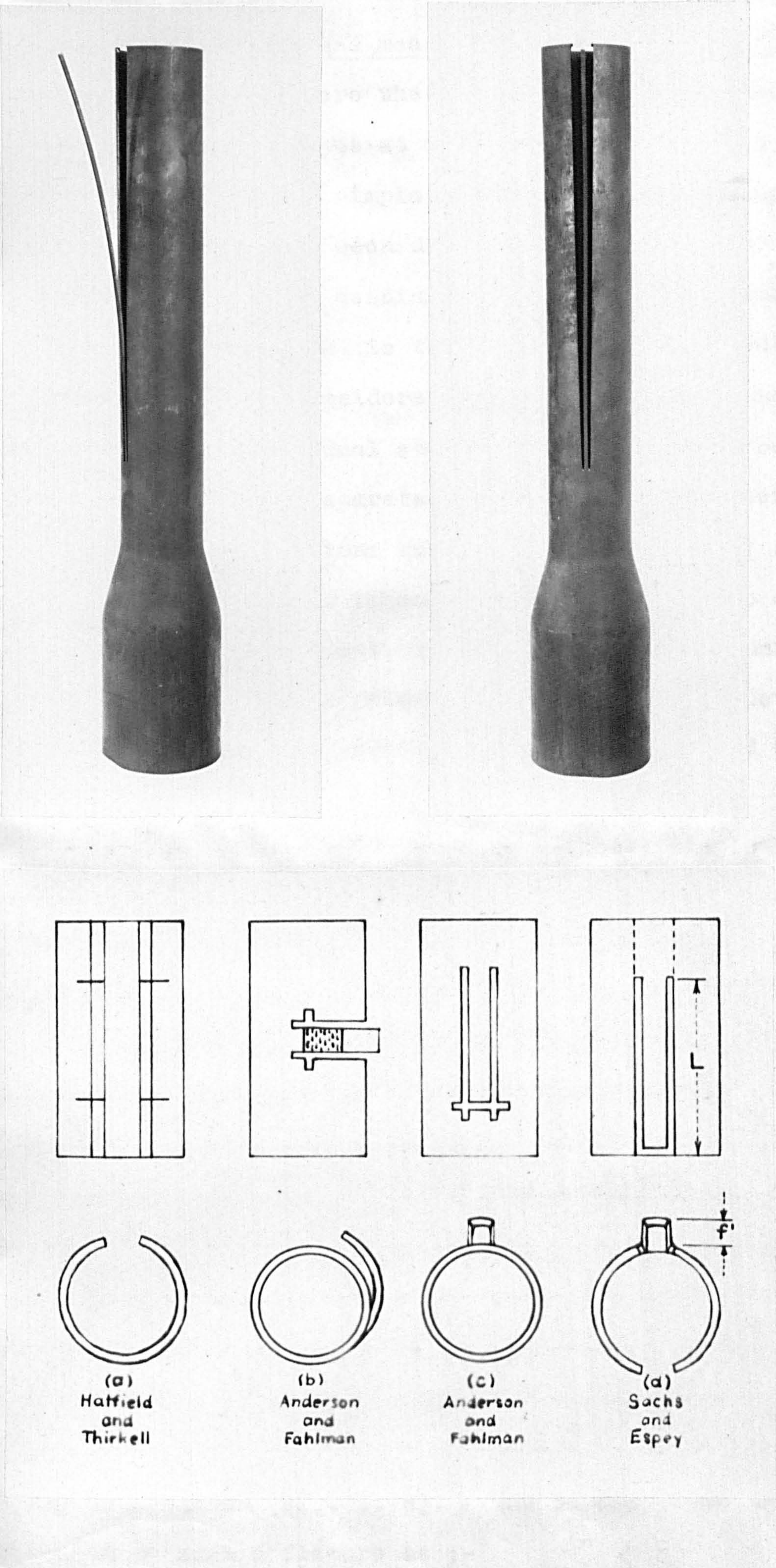

FIG. 2.7. SIMPLE BENDING DEFLECTION METHODS (After Sachs and Espey, 1941).

(Photographs show a specimen prepared by the author to demonstrate the methods).

2.1.2. Strain release and measurement by bending deflection.

In components where the distribution of residual stresses is such that the stresses at one surface are tensile and at the other compressive, simple methods for determining the residual stresses have been developed. These methods are now generally known as bending deflection methods and the basic principle of them is that sections are cut out from the specimens under consideration in such a manner that a major part of the residual stress is relieved by a bending of the part which is separated from the adjacent material.

Stress distributions suitable for analysis by such means are usually found in cold drawn tubes or deep drawn shells and the development of the analyses and techniques for this type of strain release have largely been developed for measuring residual stresses in such cold worked components.

As with direct strain release methods, simple forms of bending deflection were first evolved, giving rise to approximate results and from these more elaborate and complete analyses were developed.

The simple method appears to have been proposed by Hatfield and Thirkell (1919) but variations of it have been proposed on numerous occasions by investigators such as Anderson and Fahlman (1924/5), Pinkerton and Tait (1926), Sachs and others (1932), Sachs and Espey (1941) and Swift (1940)

In all these methods, the circumferential stress is evaluated from the change in diameter occurring when a length of tube is slit in a longitudinal direction (Fig. 2.7a) or when a circumferential tongue is partly cut out from the specimen (Fig. 2.7b). The bending moment M released by such a flexure is :-

$$M = \frac{E\,I}{1-\sigma^2}\left(\frac{1}{R_0} - \frac{1}{R_1}\right) = \frac{E\,I}{1-\sigma^2}\,\frac{R_1 - R_0}{R_0\,R_1} \quad \ldots\ldots\ldots(2.5)$$

where E = Young's modulus.

σ = Poisson's ratio.

I = the second moment of area of the section

R_0, R_1 = mean radius of specimen before, after slitting, respectively.

The release of this bending moment corresponds to the release of stresses, which in a thin section vary linearly from one surface to the other, and which have the value p_c in any element distant x from the outer surface where :-

$$p_c = \frac{E}{1-\sigma^2} \frac{(t-2x)}{2} \frac{R_1 - R_0}{R_0 . R_1} \quad \text{..............(2.6)}$$

t being the thickness of the tube wall.

The maximum stress values are at the surface and are given by

$$p_{c_{max.}} = \pm \frac{E}{1-\sigma^2} \frac{t}{2} \frac{R_1 - R_0}{R_0 \quad R_1} \quad \text{............(2.7)}$$

Fundamentally, the same equation applies to longitudinal stresses which can be calculated from the deflection of longitudinal strips or tongues cut from the tubes (Fig. 2.7 c and d). In this case

$$p_l = E\,(t - 2x)\,\frac{\delta}{L^2} \quad \text{...................(2.8)}$$

where p_l = released longitudinal stress

L = gauge length

and δ = the end deflection from its original position of the gauge length L.

The simple bending deflection method can be extended to enable the stress distribution in tubes to be determined in a more quantitative and exact manner, by extending the simple slitting methods of Hatfield, etc. to samples from which successive surface layers of material have been removed by pickling or machining. The total stress distribution can be derived from sets of experiments in which successive layers are removed from either the outside or inside surface of tubular specimens. The pickling or machining can be done either before or after slitting.

In the latter case a single specimen can be used (Sachs and Espey, 1941)* while in the former case a number of specimens, probably between 15 and 25 in number must be employed.

Such a procedure appears to have been first suggested by Fox (1930) who, however, did not succeed in developing a proper and complete mathematical solution. The fundamental theory of the method, based on the Bach-Winkler theory of curved beams was developed by Davidenkov (1932) whose complete analysis is reproduced as Appendix 2 to this report. The Davidenkov equations which are extremely complicated and laborious in application are reproduced below:-

$$p_c = p_{c1} + p_{c2} + p_{c3}$$

$$p_{c1} = \frac{E}{1-\sigma^2} \frac{\Delta D_o}{D_m} \frac{\left\{\frac{t}{2} - x + \frac{t^2}{6 D_m}\right\}}{\left\{\frac{t}{2} - x + \frac{D_m}{2}\right\}}$$

$$p_{c2} = \frac{E}{1-\sigma^2} \frac{a^2}{(D-a)^2} \frac{dD}{da} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

$$p_{c3} = \frac{2}{3} \frac{E}{1-\sigma^2} \frac{1}{(D_o-2a)D_o} \left\{ -3a\Delta D + \frac{D_o-2a}{D_o} \int_{D_t}^{D_a} y\,dD + \frac{D_o-a}{D_o^2} \int_{D_t}^{D_a} y^2\,dD \right\}$$

where p_c = total residual circumferential stress at a point in the tube wall distant x from the outer surface (or when the current wall thickness = a)

p_{c1}, p_{c2}, p_{c3} = components of this total stress released at different stages (see Appendix 3 and Fig. 11.1)

ΔD_o = change in outside diameter of the tubular specimen on slitting

D_m = mean diameter of tube before and after slitting

t = initial thickness of tube wall.

D = current outside diameter of tube

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* This statement is shown to be incorrect as a result of the author's investigations described later in this report.

D_o = initial outside diameter of tube.

ΔD = change of outside diameter after removal of material of thickness $x = t - a$

D_a = outside diameter of slit tube when wall thickness is "a".

D_t = outside diameter of slit tube when wall thickness is t.

y = thickness of tube wall as it varies progressively from its initial thickness t to the value "a" at the stage under consideration.

Although Davidenkov himself does not mention the fact, these equations are only applicable when material is being removed from the inner (bore) surface of specimens. When material is being removed from the external cylindrical surface the expressions for p_{c2} and p_{c3} must be adjusted (see Appendix 2 for further details).

The Davidenkov analysis for longitudinal stresses is much simpler and the resultant equations from which the stress at any point is calculated can be summarized as follows :-

$$\left.\begin{aligned} p_\ell &= p_{\ell_1} + p_{\ell_2} + p_{\ell_3} \\ \text{where } p_{\ell_1} &= E\,\frac{(t-2x)}{b^2}\,\Delta y \\ p_{\ell_2} &= \frac{-Ex^2}{3b^2}\,\frac{dy}{dx} \\ p_{\ell_3} &= \frac{2}{3}\,\frac{E}{b^2}\left[-3x\,\Delta y_x + \int_{y_x}^{y_t} x\,dy\right] \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(2.10)$$

where p_ℓ = total residual stress at any point in tube wall distant x from the outside surface.

p_{ℓ_1}, p_{ℓ_2}, p_{ℓ_3} = components of the total stress released at different stages in the metal removal process.

Δy = initial deflection at the centre of a strip of length 2b.

Δy_x = deflection at the centre of the same strip when its wall thickness has been reduced to a value x.

Sachs and Espey (1941) considered the Davidenkov equations unnecessarily complicated, particularly for application to thin walled tubes and they succeeded in developing the following more simple expressions for the residual circumferential stresses (their expressions for the longitudinal stresses are no simpler and less exact than the ones given by Davidenkov and although these are derived in Appendix 3 together with the circumferential stress equations, they are not reproduced here).

$$\left.\begin{aligned} p_c &= p_{c1} + p_{c2} + p'_{c3} + p''_{c3} \\ p_{c1} &= \frac{E}{1-\sigma^2} \frac{(t-2x)}{D_m^2} \Delta D_o \\ p_{c2} &= \frac{-E}{3(1-\sigma^2)} \frac{(t-2x)}{D_m^2} \frac{dD}{dx} \\ p'_{c3} &= \frac{E}{1-\sigma^2} \frac{(t-x)\Delta D}{D_m^2} \\ p''_{c3} &= -\frac{1}{t-x}\int_0^x (p_{c2} + p'_{c3})\, dx \end{aligned}\right\} \quad \ldots\ldots\ldots\ (2.13)$$

(The designation of the stresses p_{c2}, p'_{c3} and p''_{c3} has been altered substantially from that in the original published version of Sachs and Espey to bring it into line with the previously mentioned Davidenkov analysis.)

The only other theoretical work of note in connection with bending deflection methods, is that of Knights (1951) who made a detailed comparison of the analyses of Davidenkov and Sachs and Espey and reached the obvious conclusion that the Davidenkov analysis is in all respects more fundamental (and presumably more accurate) than the Sachs analysis. Knights suggested certain simplifications to the Davidenkov analysis which, he claimed, did not impair the accuracy of the method and which at first glance obviously make the analysis much easier and less laborious in application (see Section 3.1).

2.13 <u>2.1.4. X-ray and other non-destructive strain measurement.</u>

The application of X-rays to the measurement of residual stresses has increased considerably in the past few years.

The method used is one in which measurements are made of the purely elastic strains which are present in the crystal lattice within the body of the stressed material under investigation. In this case the gauge length over which the strains are measured, is the interplanar spacing of the unstrained lattice and the strain is determined by the change in this interplanar spacing, aas measured by the diffraction of X-rays.

This method has two important related features: (a) stresses can be determined without having to make measurements in the unstressed material and (b) it is the only really non-destructive way of measuring residual stresses. Since it is fundamentally a means of measuring strains of an interplanar nature, the gauge lengths under consideration are very small and much smaller than can ever be hoped for using any mechanical or electrical method of measuring strain.. Localized stresses and large stress gradients can consequently be detected whereas mechanical and electrical strain gauges can only determine average stresses over a relatively large area.

Unfortunately, the wide application of X-rays to residual stress detection and measurement has several serious disadvantages. The cost of equipment is high and the long exposure periods necessary demand the installation of costly shielding devices or the housing of the equipment in a specially built laboratory. The method at its present stage of development is purely qualitative except for surface stresses which can be determined quantitatively. The most serious disadvantage, however, is that other physical factors as well as internal stresses can affect the interplanar spacings and because of this the method can only be applied to metals which exhibit reasonably sharp diffraction lines. Consequently, any material which has been severely cold worked or subjected to severe thermal stressing during heat treatment, does not lend itself at all readily to X-ray stress analysis, since the diffraction lines in such materials are too wide and ill-defined for accurate measurement.

In view of the serious limitations to the application of X-ray techniques to the measurements of residual stresses in tubes, these methods are not here considered any further and any additional information required about them can be found in papers by Barrett (1934, 1936, 1937 and 1943), Finch (1949), Lynch (1951) and Thomas (1941).

The utilisation of the magnetic properties of a material to determine residual stress has been suggested by Becker (1939), Förster and Stambke (1941), Webb (1938) and others. Any application of magnetic principles must be extremely restrictive in nature and while they may be used for determining the residual stresses in such things as iron or nickel wires it seems unlikely that such methods will ever enjoy widespread application.

2.1.5. Qualitative methods of detecting residual stresses.

While qualitative analysis of residual stresses falls outside the scope of this work, it is perhaps desirable that a brief mention should be made of their existence. Commercially, these methods have a widespread application because of their low cost and the rapidity with which they can supply an answer, even though this when obtained is only qualitative in nature.

Etching or stress corrosion is the most popular way of revealing internal stresses. If exposed to suitable reagents many residually stressed materials crack in the regions of high tensile stress, thus making the method a kind of accelerated test for corrosion or season cracking. The reagents used for these tests are many in number and vary for different materials and alloys (e.g. an acid solution of mercurous nitrate is used to ascertain the

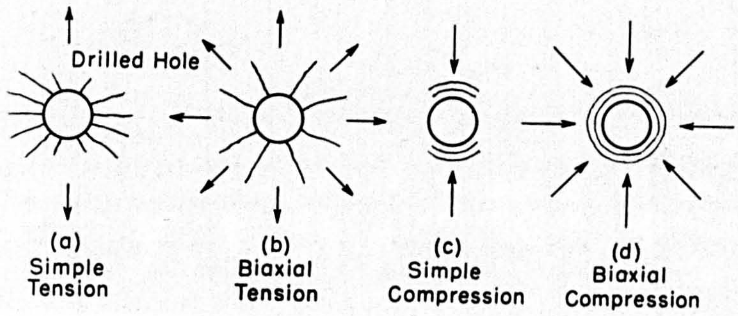


FIG. 2.8. TYPICAL CRACK PATTERNS FOR VARIOUS SURFACE STRESSES USING COMBINATION OF HOLE METHOD (MATHAR, 1934) AND BRITTLE LACQUER. (After Gadd, 1946).

liability to season cracking of brasses, while common salt solutions can be used for the same purpose with most aluminium alloys).

Brittle lacquer coatings are also widely used to reveal the presence of residual stresses in a material. A specimen under test is first coated with a brittle lacquer which is then allowed to dry thoroughly or is chilled to increase its sensitivity. A small hole (generally about 0.125 in. dia.) is then drilled through the coating and into the residually stressed material. The resultant surface strains which develop around the drilled hole produce cracks in the lacquer coating, the pattern of which gives some indication of the type of stress present in the specimen under test (Fig. 2.8; Gadd, 1946).

2.2. Experimental investigations.

There is available an enormous amount of literature dealing with experimental investigations concerned with the measurement of residual stresses in cold drawn tubes and it is proposed here to make mention of only the most important contributions in the field. A number of papers not specifically referred to in the text of this report are given in the second part of the bibliography; these papers are of interest to anyone studying the residual stresses which are present in solid cylindrical components (rods, wires, etc.) and in flat plates or strips, but are not particularly relevant to the subject matter under investigation.

2.2.1. Direct strain release methods.

As previously mentioned, the principle of residual stress measurement by the direct release of strains is to remove small annular layers of material from either the inner (bore) or outer surfaces of cylindrical components and to measure the resultant axial and circumferential strains which develop in the remaining section at each stage of the metal removal process. Sachs (1927) proposed that the metal removal should be performed by machining (boring or turning) and that only light cuts should be taken at each stage to minimise the possibility of superimposing machining stresses on the stresses which were being measured. As far as can be ascertained, this method of metal removal has been carried out by all investigators to date who have used the Mesnager - Sachs development of the Heyn process.

Many investigators have pointed out the practical difficulties involved in the application of the Sachs boring method to the measurement of internal stresses. Extremely small elastic strains have to be measured and only in exceptional circumstances can mechanical extensometers be employed. Furthermore, temperature effects can cause strains of the same order of magnitude as those being measured, and every effort must be made to take measurements either at a constant temperature or to be able to compensate for temperature effects. Despite these difficulties, the Sachs boring method has become probably the most widely used method of stress determination in cylindrical components, and accounts of its application have been given by Bucholtz and Buehler (1933), Sachs (1939), Kempf and Van Horn (1942), Horger and others (1943) and Bucholtz and Buehler (1952). The majority of these investigators used optical or dial comparators to measure the strains, although Bucholtz and Buehler (1952) concluded that micrometers or caliper gauges could be used, provided a gauge length in excess of 10 ins. was employed.

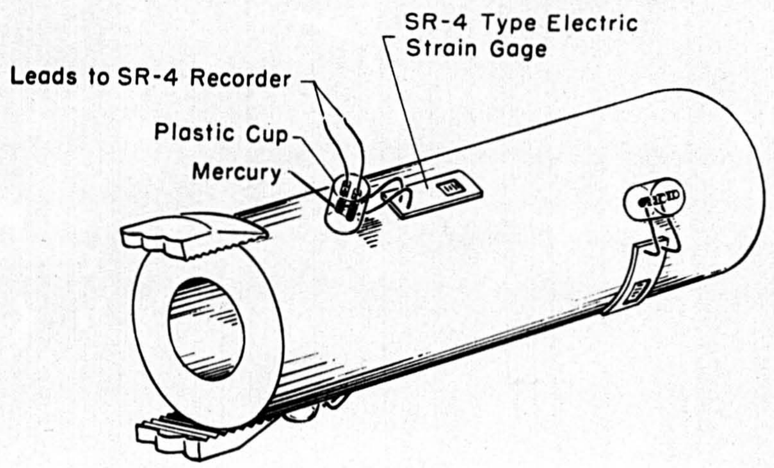


FIG. 2.9. ELECTRICAL RESISTANCE STRAIN GAUGES MOUNTED ON A TUBE FOR SACHS' BORING TESTS. (After Lynch, 1951).

In 1943, Horger and others used electric resistance strain gauges to measure the axial and circumferential strains in an investigation to determine the residual stresses in solid components. The introduction of such gauges to the field of residual stress measurement appears to be a most significant and useful step and further investigations in which they have been employed have been reported by Timoshenko (1947) Greaves and others (1944), Knights (1951), Whiteley (1953) and Loxley and Whiteley (1953). Ford (1948) expresses doubt about the validity of electrical resistance strain gauges taken over relatively long periods of time and suggests that these doubts should be cleared up before the application of such gauges becomes too widespread. (This has been carried out in the present investigation as described later in Section 4.)

Another possible unsatisfactory feature of the application of strain gauges to residual stress measurement, is that it is frequently necessary to "make" and "break" the leads between the gauges and the measuring bridge. If it were impossible to do this without changing the contact resistances the value of the method would be negatived. MacPherson (1944) investigated this point and found that, provided careful and clean soldering techniques were adopted, the effect of joint resistance changes could be neglected. An ingenious device to overcome the disadvantages of continuously breaking and re-making soldered connections to strain gauges has been suggested by Lynch (1951). This device, which demands the fixing of plastic cups to hold mercury to effect a joint between the strain gauge and recording instrument connections, is shown in Fig. 2.9, but as far as can be ascertained it has never been used in any experimental investigation.

As poihted out by Knights (1951) the major drawback to the Sachs' boring method (or indeed to any direct strain release tests) is the fact that layers are removed by machining. The effect of such machining on the residual stress system being measured is not known in detail, but some indication of the magnitude of the induced stresses can be obtained by reference to works of Ruttmann (1936) and Hendriksen (1948). Although Hendriksen concluded that the residual stresses induced by planing and turning could be of the order of the ultimate strength of the material, he also found that they were concentrated in a very narrow band of the material near to the machined face. The depth of this band is relatively independent of the depth of cut and the band is usually removed in the next cutting operation, although of course new stresses are induced in this stage. Although these machining stresses are not cumulative, they introduce a serious difficulty into the application of direct strain release methods to the measurement of residual stresses, and previous to the work described later in this report, no attempt to overcome it has ever been made.

2.2.2. Bending deflection methods.

The bending deflection method of circumferential strain measurement consists of preparing a ring specimen from the parent tube and slitting it longitudinally along any one radial plane. The amount by which the outer diameter of the tube in a plane perpendicular to the one containing the slit, alters from its original unslit value, is a measure of the stress released. (This is the basic principle of the approximate methods of Hatfield and Thirkell, etc.). By reducing the tube wall thickness by pickling or machining, and by measuring the current outer sprung diameter and wall thickness at each stage, the residual stress distribution throughout the tube wall can be found. For longitudinal stresses, a longitudinal tongue is cut out from the tube wall, and the amount of its deflection and curvature from the original straight line is a measure of the released stress. Further stress components are released by reducing the tongue thickness by acid attack, and the longitudinal stress distribution throughout the walls of the tube may be found.

The application of bending deflection methods to the measurement of residual stresses in tubes has one big advantage over the methods involving direct strain release; provided the residual stresses in the component being tested are of sufficient magnitude, the diametral and curvature changes which occur during strain release by bending deflection are much larger than the corresponding changes in direct strain release, and can be measured more accurately and easily by the use of micrometers or simple dial comparators.

Davidenkov (1932) appears to be the first person to suggest the application of layer removal by pickling (acid attack) rather than by machining, although he does not appear to have used such a process himself. The application of the Davidenkov analysis to the measurement of residual stresses in tubes appears to have been confined to works by Jenkin (1937) and Knights (1951). Both these workers have commented on the length of time and the labour involved in the determination of the residual stress distribution in a component from the experimental results using the Davidenkov analysis.

Apart from the two experimental investigations of Jenkin and Knights already referred to, all previous investigations of the effects of drawing variables on the residual stresses in tubes have been concerned with brass or copper tubes or cartridge cases. The work of Knights alone stands out for the studies he made of the influence of the drawing variables on the stress distributions in steel tubes.

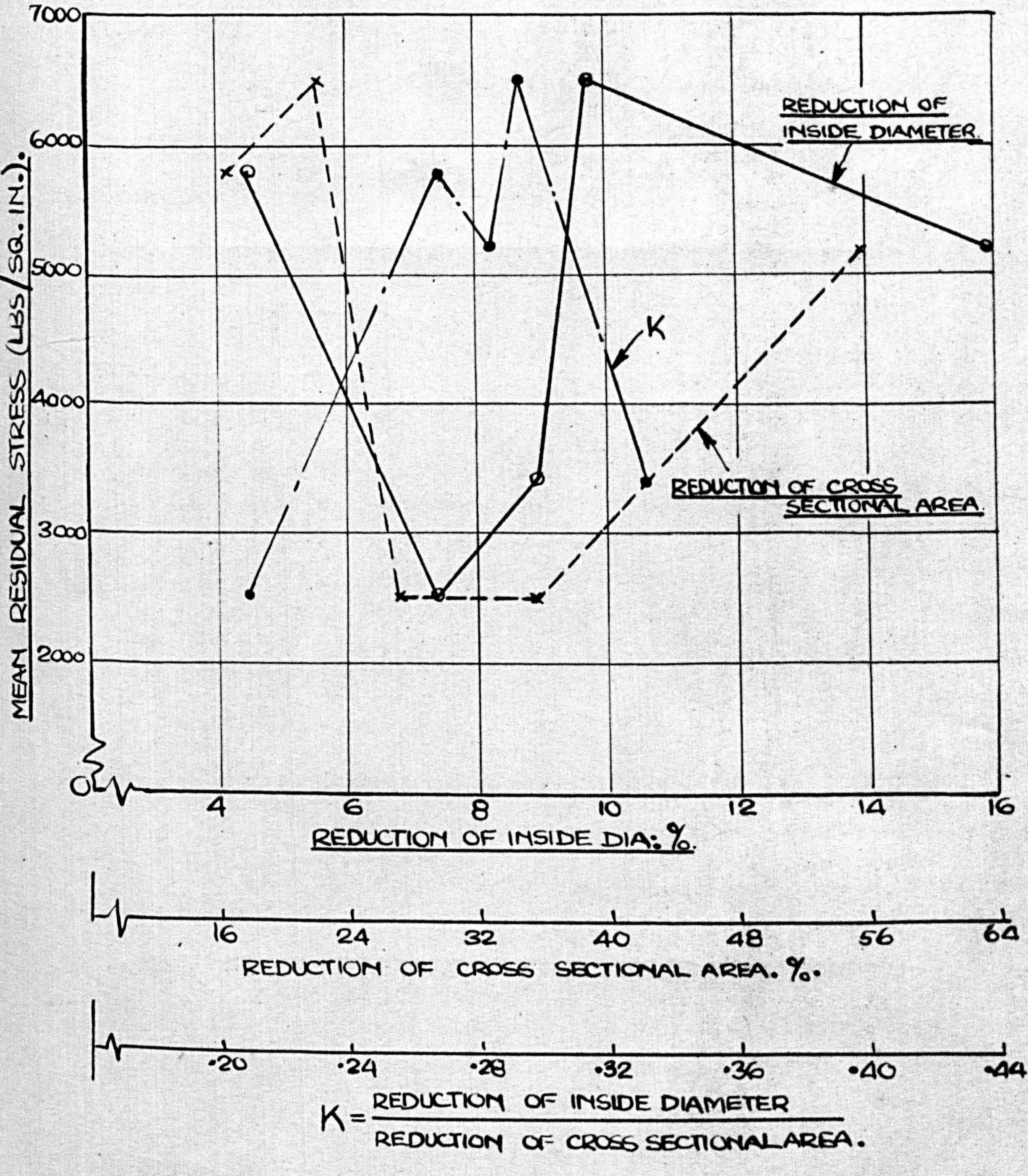


FIG: 2.10. SUMMARY OF RESULTS OF ANDERSON AND FAHLMAN.

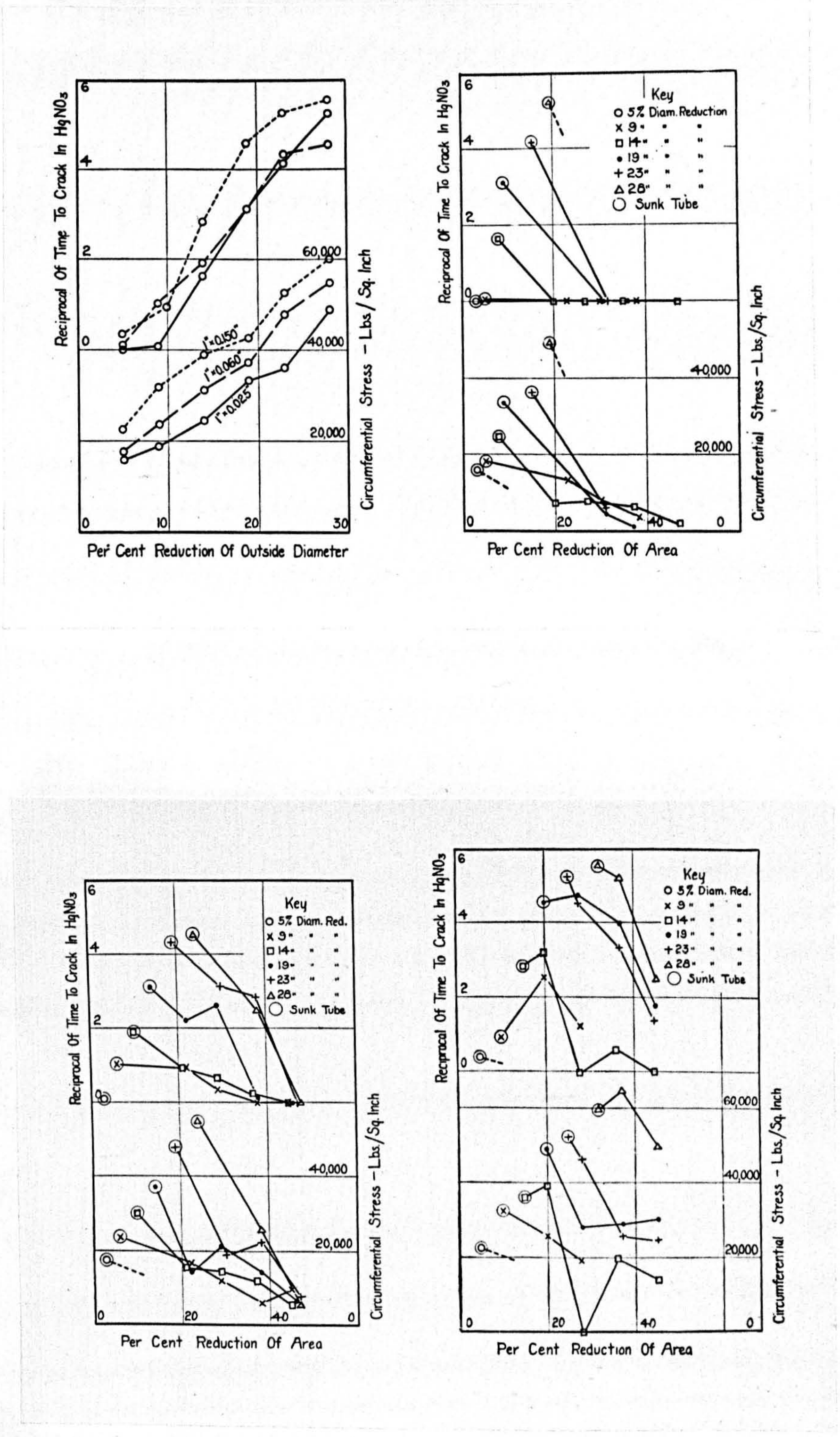


FIG. 2.11. EFFECT OF REDUCTION OF DIAMETER AND AREA ON THE INTERNAL STRESSES IN HOLLOW DRAWN BRASS TUBES, INITIALLY OF 10 IN. DIAMETER. (After Crampton, 1930).

One of the first investigations concerned with the influence of drawing variables on the residual stresses in tubes is that of Anderson and Fahlman (1924) who made approximate measurements of the residual longitudinal stresses in five batches of cartridge brass tubing. Their results are summarized in Fig. 2.10 and they concluded that the cold reduction of area and the hardness of worked brass tubes are not criteria of the magnitude of the residual stresses in such tubes. The stresses produced tend to increase with reduction of internal diameter and decrease with increase in reduction of cross-sectional area.

The first really systematic investigation into the problem was made by Crampton (1930) who again applied approximate bending deflection methods to the measurement of stresses in brass tubes. Tubes, all of 1.0 in. initial diameter and with wall thicknesses of 0.150, 0.060 or 0.025 in. were used, and these were reduced by hollow or mandrel drawing through 20° taper dies to various outside diameters. The stress measurements made were confined to those in the circumferential directions and no comparison with material properties were attempted. The more relevant of Crampton's results are given in Fig. 2.11 and the most important conclusions were that the intensity of internal stresses in high-brass tubes are (i) increased by (a) increase in wall thickness relative to the diameter (b) hollow drawing instead of mandrel or plug drawing and (c) increase of diametral clearance; (ii) unaffected by the hardness of the material and (iii) decreased by increase in the reduction of area for any particular diametral reduction.

An account of the residual stress distribution set up in the walls of hollow drawn cartridge brass tubes of 0.50 in. diameter and 0.032 in. thickness was given in 1943 by Sachs and Espey. Several dies

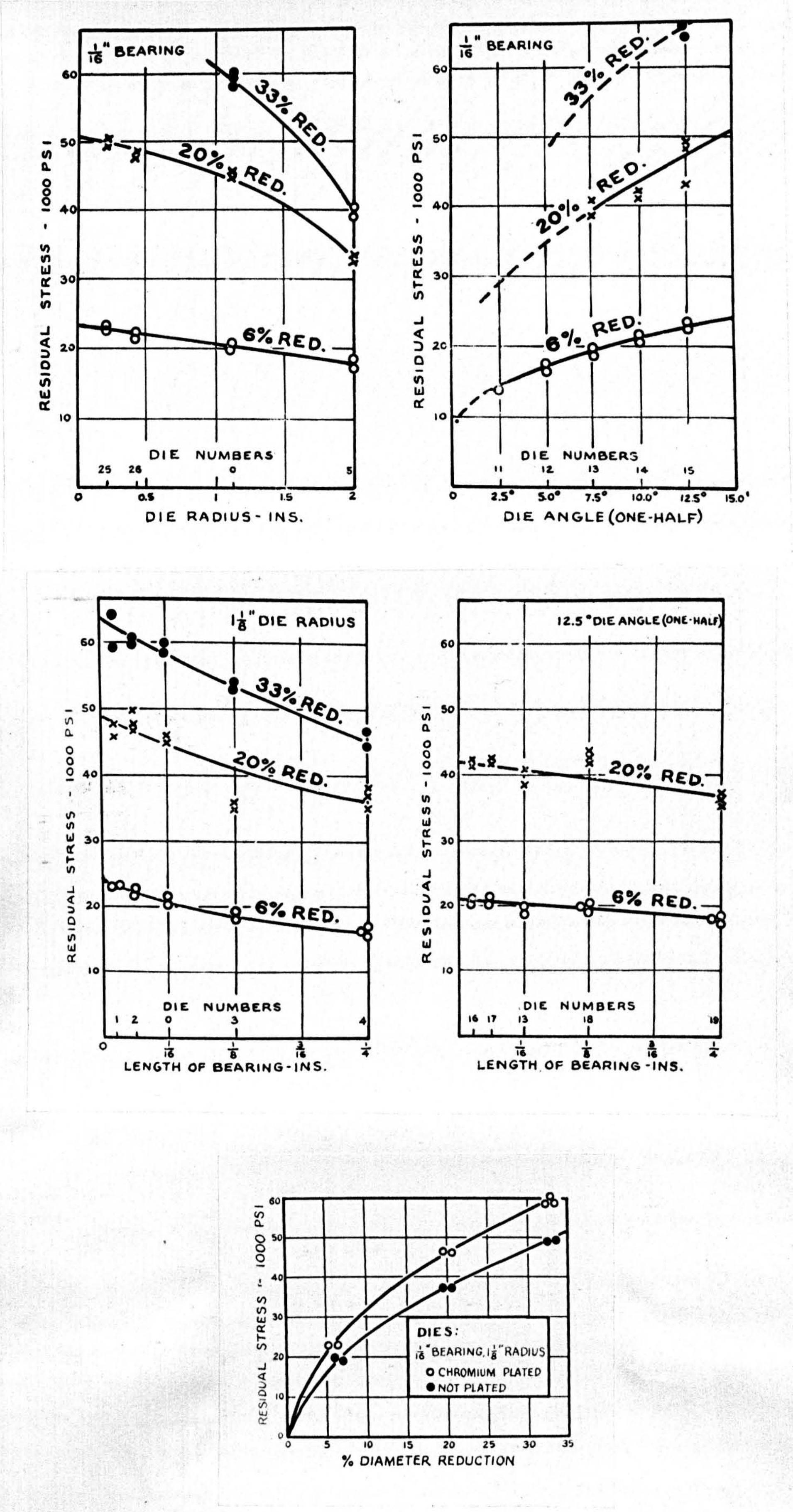


FIG. 2.12. EFFECT OF DIE RADIUS AND DIE ANGLE, DIE LAND AND DIE FINISH ON THE RESIDUAL STRESSES IN CARTRIDGE BRASS TUBES. (After Sachs and Espey, 1943).

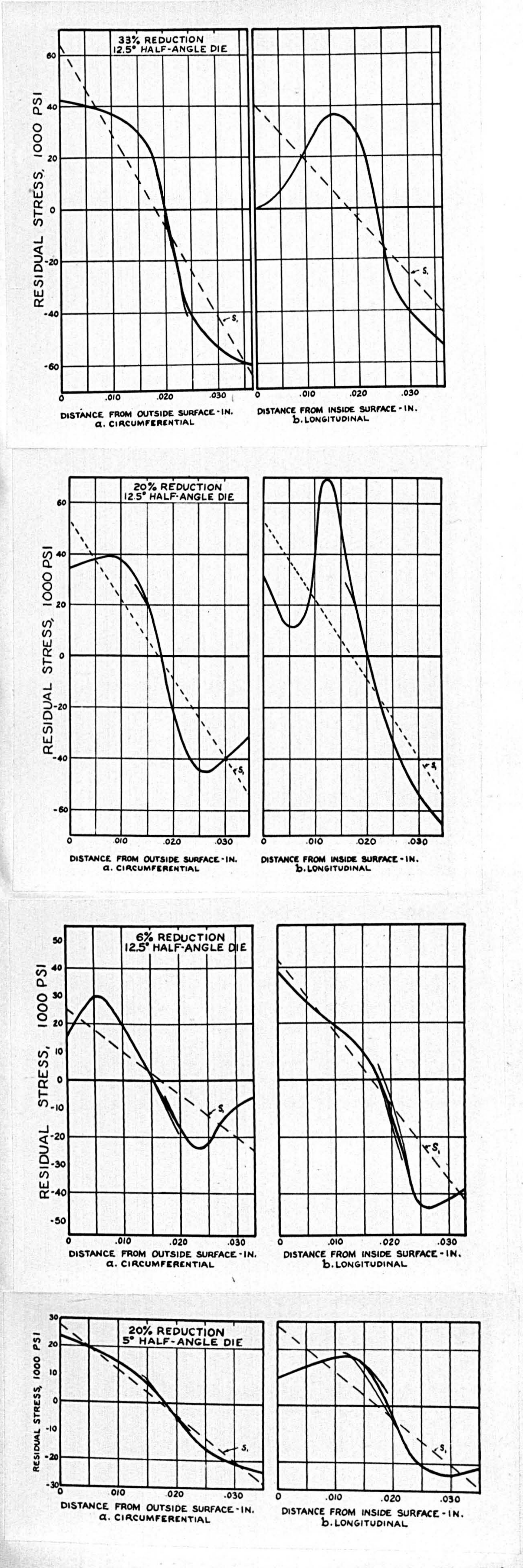


FIG. 2.13.

DISTRIBUTION OF CIRCUMFERENTIAL AND LONGITUDINAL STRESSES IN CARTRIDGE BRASS TUBES.

(Dotted lines indicate stress derived by simple method.)

(After Sachs and Espey, 1943).

of various forms and giving different diametral reduutions were used. For the greater part of the investigation circumferential stresses only were measured and these by a simple bending deflection method. Some of the findings of Sachs and Espey are reproduced in Fig. 2.12 and the main conclusions reached were that residual stresses increase with increasing diametral reduction for (a) increasing die profile radii (Fig. 2.12.a), (b) increasing die angles (Fig. 2.12.b) and (c) increasing bearing length (length of the constant diameter region at the die throat) (Fig. 2.12.c). The surface condition of the drawing die can also have an appreciable effect on the residual stresses in drawn tubes as evidenced by Fig. 2.12.d.

Sachs and Espey also extended their studies on the tubes previously mentioned, to the determination of the total stress distributions in them using their own complete bending deflection analysis. In every case stress distributions obtained were essentially tensile in the outer half of the tube wall and compressive in the remainder. The maximum tensile stress was found to occur some distance in the tube wall from the outer surface, while the maximum compressive stress occurs at or near to the inner (bore) surface. The distribution curves of residual circumferential stresses do not vary as much from a linear distribution as do the residual longitudinal stresses. Some of the distribution curves obtained by Sachs and Espey are reproduced in Fig. 2.13.

Other investigators have reported similar stress distributions to those obtained by Sachs and Espey (Jenkin 1937; Knights, 1951; Montgomery, 1950; Howard, 1951; Loxley 1950/1/3). In 1944 Sachs, Espey and Clark reported the results of an investigation in which it was found that the magnitude of residual circumferential stresses in ironed cups (analogous to mandrel drawn tubes) increased with increasing die angle and decreased with the total reduction

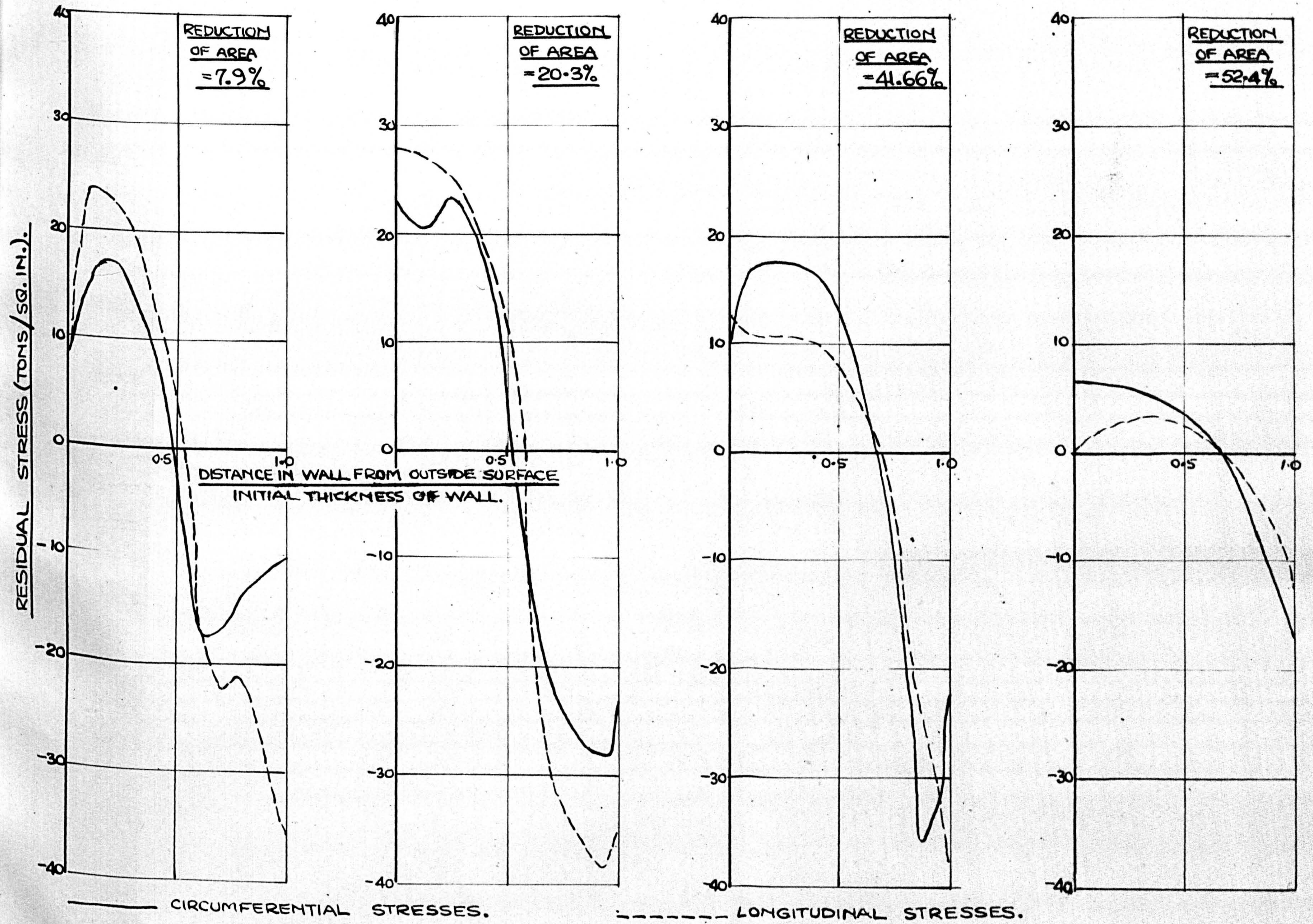


FIG: 2.14. RESIDUAL STRESSES IN HOLLOW DRAWN STEEL TUBES. (After Knights, 1951).

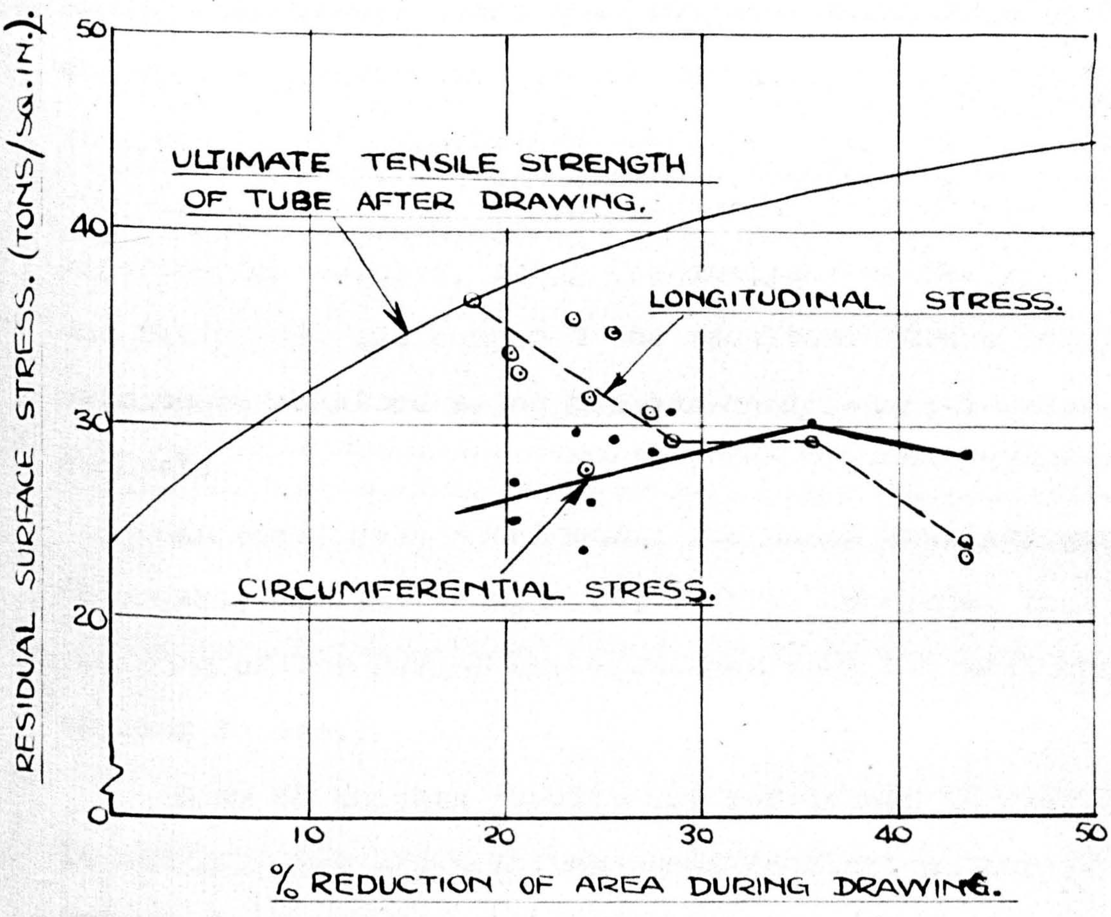


FIG: 2.15. RESIDUAL BENDING STRESSES (RELEASED ON SLITTING) IN LOW CARBON STEEL TUBES SUNK (HOLLOW DRAWING) THROUGH 15° TAPER DIES. (After Knights, 1951).

of area of the cup walls. In some cases with high reductions, the residual stresses were found to be compressive at the outer surface and it was suggested that there is a tendency for such stresses to develop as the total conical angle of the drawing die is decreased.

In recent years an intensive study of theinfluence of drawing variables on the residual stresses which occur in cold drawn steel tubes has been made by Knights (1951) who used a modification of the Davidenkov method in the analysis of his experimental results. (He, did, however, calculate several stress distributions from single sets of experimental results, using the analyses of Sachs and Espey and Davidenkov and compared the resultant stress patterns with those obtained using his own modified Davidenkov analysis.

In every case considered, the three analyses gave remarkably similar results and Knights concluded that his own form of the Davidenkov equations were the most convenient to use.)

Some of Knights results are reproduced in Figs. 2-14 and 2.15 and his most important conclusions were:-

(a) Simple bending tests usually indicate the general residual stress level with sufficient accuracy for general use. With sunk (hollow drawn) tubes and with tubes drawn with little thickness change, the residual bending stresses (surface stresses released by simple slitting operations) have numerical significance in relation to the maximum residual stresses (Fig. 2.14.)

(b) The most important variables in determining the magnitude of residual stresses in steel tubes are the ratio of sink to draft (proportion of reduction of area achieved by diametral reduction to that obtained by reduction in thickness) and the percentage reduction of area in the drawing pass (Fig. 2.15).

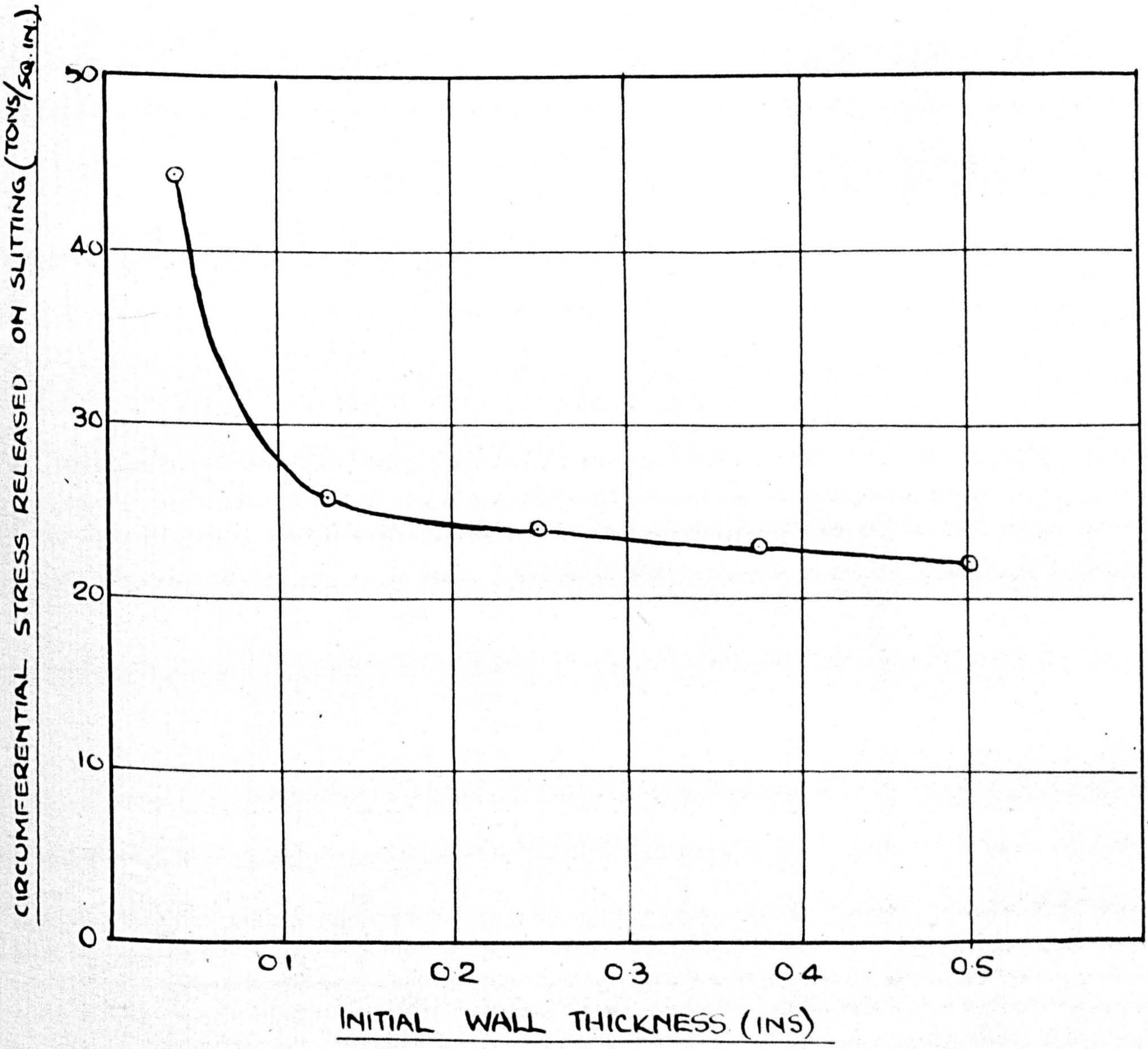


FIG: 2.16. INFLUENCE OF TUBE WALL THICKNESS ON THE STRESSES RELEASED ON SLITTING. (MILD STEEL TUBES. 4¼" INITIAL OUTSIDE DIA. DRAWN THROUGH 3½" DIA STRAIGHT TAPER DIE. EACH SPECIMEN OF 3·2" WIDTH) AFTER BOTROS.

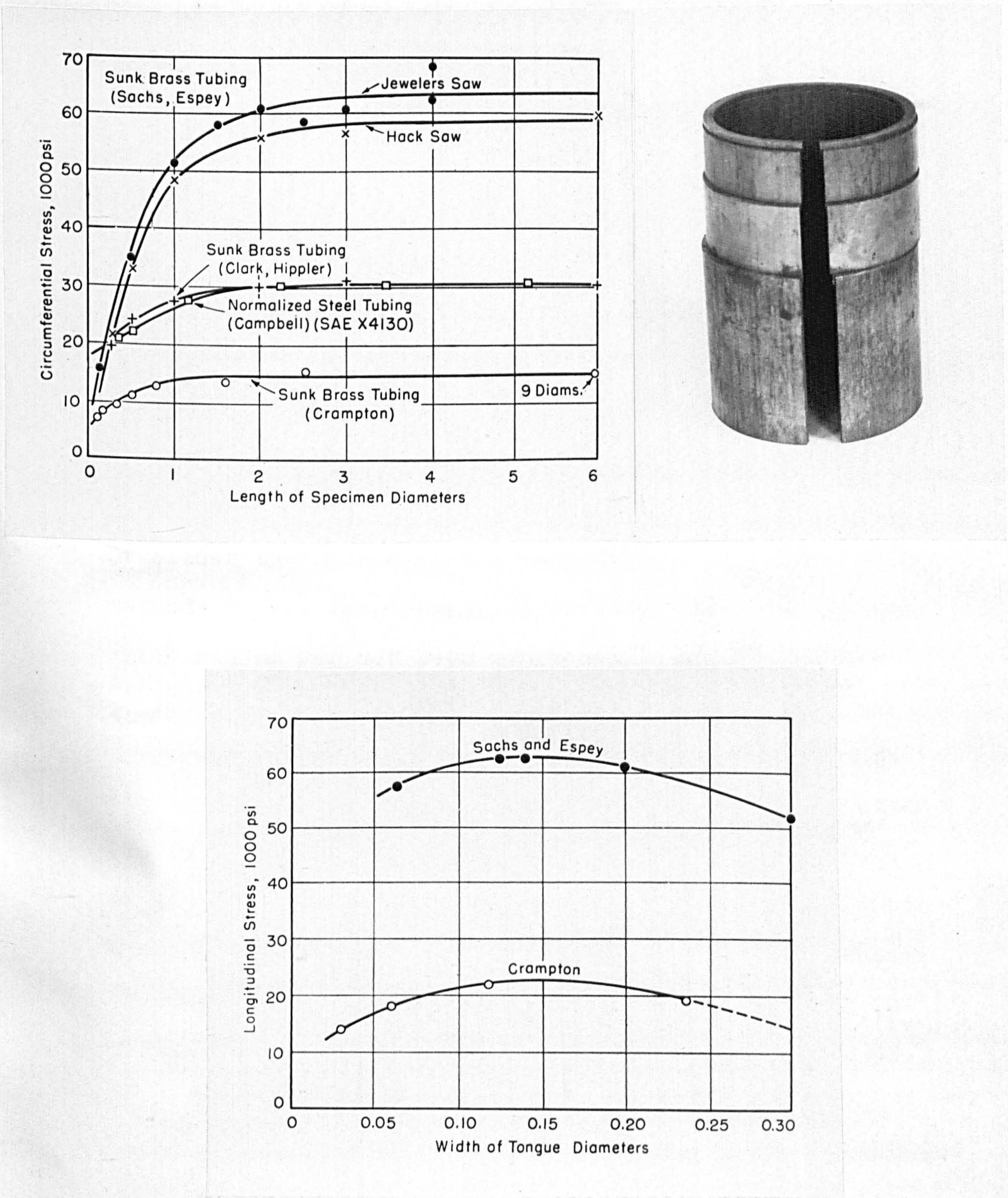


FIG. 2.17. THE "LENGTH" AND "WIDTH" EFFECTS IN RESIDUAL STRESS MEASUREMENT BY BENDING DEFLECTION METHODS.
(After Sachs and Espey, 1941).

Botros (1950) used the approximate simple bending deflection method to study the influence of wall thickness on the residual stresses in hollow drawn tubes and from the results given in Fig. 2.16 he concluded that the magnitude of the residual stresses in tubes all subjected to the same drawing condition (diametral reduction) are influenced by the wall thickness of the tubes, the stress values tending to reduce as the wall thickness increases. For small values of the ratio tube thickness to tube diameter the stress increases rapidly with decrease of thickness.

There is one serious disadvantage to the application of either the simple or the complete bending deflection methods to the measurement of residual stresses in tubes which has so far not been mentioned. All the mathematical relationships derived for the calculation of circumferential and longitudinal stresses at any point in a tube wall are respectively independent of the specimen length or width. Many investigators (Crampton, 1930; Sachs and Campbell, 1941; Botros, 1950; Knights, 1951 and others) have all recorded that the released stresses are not independent of the specimen length or width until either exceeds a particular value of the tube diameter, (Fig. 2.17). This effect is known as the "length effect" (circumferential stresses) or the "width effect" (longitudinal stresses) and it will be discussed further in Section 5 of this report.

No investigator appears to have made any serious attempt to compare the residual stress distributions in drawn tubes, or for that matter in any cylindrical component, obtained by bending deflection ~~or~~ and direct strain release methods. Such a comparison would undoubtedly give a measure of assurance to the validity of either one or both of the methods of approach. The only

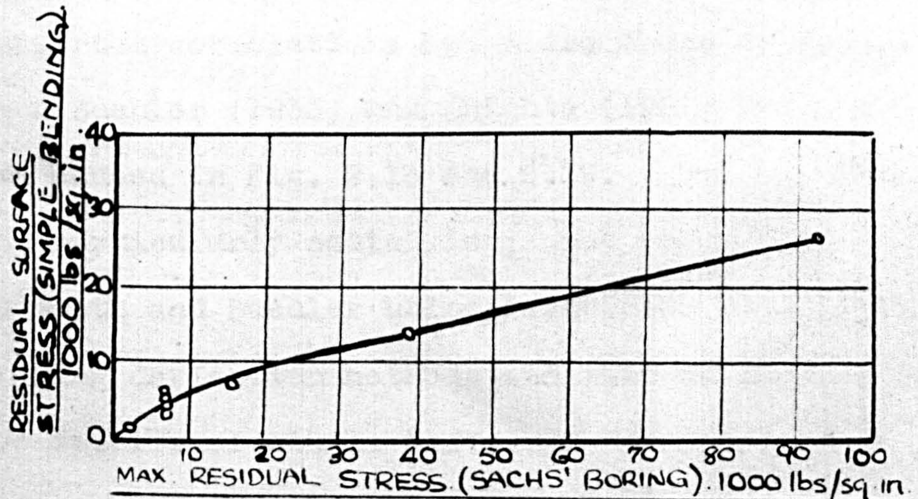


FIG. 2.18. COMPARISON OF RESIDUAL STRESSES IN STEEL CYLINDERS DETERMINED BY THE SACHS BORING METHOD AND SIMPLE BENDING DEFLECTION. (AFTER BUSCHOLTZ AND BUEHLER)

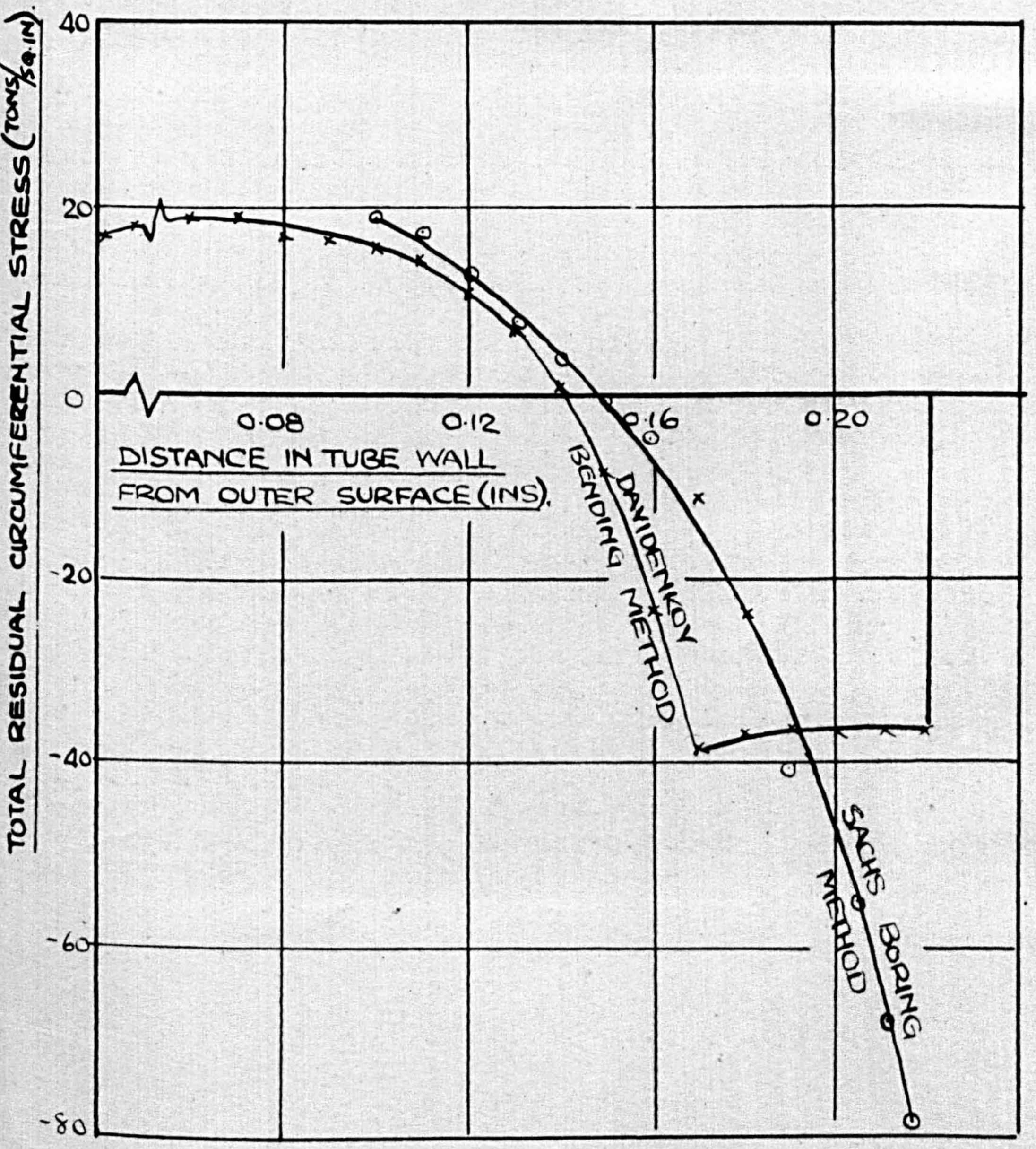


FIG: 2.19. COMPARISON OF THE RESIDUAL STRESSES IN A HOLLOW DRAWN TUBE DETERMINED BY THE SACHS BORING METHOD AND BY THE DAVIDENKOV BENDING METHOD. (AFTER KNIGHTS) reported correlations known are those of Bucholtz and Buehler (1933) and Knights (1951) which are reproduced in Fig. 2.18 and 2.19. Neither of these are particularly satisfying; the comparison made by Bucholtz and Buehler being restricted to approximate bending deflection methods and that of Knights to the results of single tests.

3. COMPARISON OF THE BENDING DEFLECTION ANALYSES AND THE DEVELOPMENT OF EXPERIMENTAL TECHNIQUES FOR THEIR APPLICATION.

3.1. Critical review of the available analyses.

Apart from the approximate analyses of Hatfield and Thirkell (1919) etc., enabling the calculation of the surface stresses released on slitting a tubular specimen or on cutting a longitudinal strip out of such a specimen, the only two analyses based on the bending deflection method of strain release which are applicable to cylindrical components are those of Davidenkov (1932) and Sachs and Espey (1941) (see pages 12 and 13).

In both these analyses the residual stresses are considered as being released in three stages, giving rise to three components p_1, p_2 and p_3 from which the total stress p at any point in the tube wall is calculated. (Sachs and Espey state that their analysis gives rise to four stress components, but in actual fact two of these components are released at the same stage in the metal removal process and a clearer conception of them is obtained if they are considered as a single unit).

The first stress component p_1 is calculated from the diametral change (or the change in curvature) which occurs as a result of the simple slitting operation (or tongue separation). The second stress component p_2 is that contained in the current elemental surface layer of the specimen under test, when that particular layer is about to be removed. The equilibrium of the remaining section is upset by the removal of the stress p_2 in the surface layer, and this is immediately restored by the imposition of a direct force over the section, giving rise to the release of direct stresses and by a bodily change in the "sprung" diameter of the specimen which releases bending stresses from the section. Consequently, when a layer of material is ready to be removed from the specimen, the stress in it is no longer that which was present in that particular layer after the initial bending stress release due to slitting, but one which differs from the original stress by the component p_3.

(In the Sachs and Espey analysis, the component stresses were considered in a different order and were labelled differently from the way they are treated in this report. The present treatment is used in preference to the original form, as it enables direct comparison of the two analyses.)

For the derivation of the longitudinal stresses, both analyses rely on the simple bending theories as related to straight beams, and should yield identical results. However, the Sachs and Espey analysis contains certain inaccuracies which the originators claim were necessary if the analysis was not to become unwieldly in application. These errors are discussed later in connection with residual circumferential stresses and will not be considered further at this stage. Because of the inherent weaknesses in the Sachs and Espey analysis and since this analysis is no more simple in application than the corresponding Davidenkov one (despite Sachs claim to the contrary) it was not considered necessary or advisable to use it in the present investigation.

Davidenkov's analysis for the complete determination of the residual circumferential stresses in tubes (Appendix 2) is in all respects the more fundamental of the two approaches, but it leads to a complicated expression for the stress component p_{c3} which involves a considerable amount of mathematical computation in its solution. The analysis is based on the curved beam theory of bending in contrast to the simple theory of bending used by Sachs and Espey (appendix 3).

(In fairness to Sachs and Espey on this point, it should be mentioned that their method was developed essentially for thin walled tubes where the simple bending theory is sufficiently exact in application.) Apart from the assumptions regarding the symmetry of the residual stress distributions and the homogeneity of the material which are present in all residual stress analyses based on strain release, the only other assumption made by Davidenkov is the constancy of the relevant "sprung" diameter throughout the metal removal process. (That this assumption is justifiable is obvious from the experimental results quoted later in this work. In no single test carried out during the present investigations, has the unpickled "sprung" diameter of a specimen changed by more than 2% of its original value during the metal removal process, and more generally the maximum variation in a particular specimen has been more nearly half this value.)

Sachs and Espey, on the other hand, introduce two assumptions into their analysis which are not so easily justified. The first of these is the adoption of a constant mean diameter to be used in all the several expressions giving the different stress components. This assumption is justifiable only if the method is restricted in application to thin walled tubes where the diameter - thickness ratio is relatively high and if the constant mean diameter is given a different value in each of the expressions defining the component stresses. (For further information on this point see Appendix 4.) The second assumption lies in the derivation of the stress component p''_{c3} i.e. that stress component released by the imposition of a direct force over the section whenever a layer of material is removed. It is implied that this

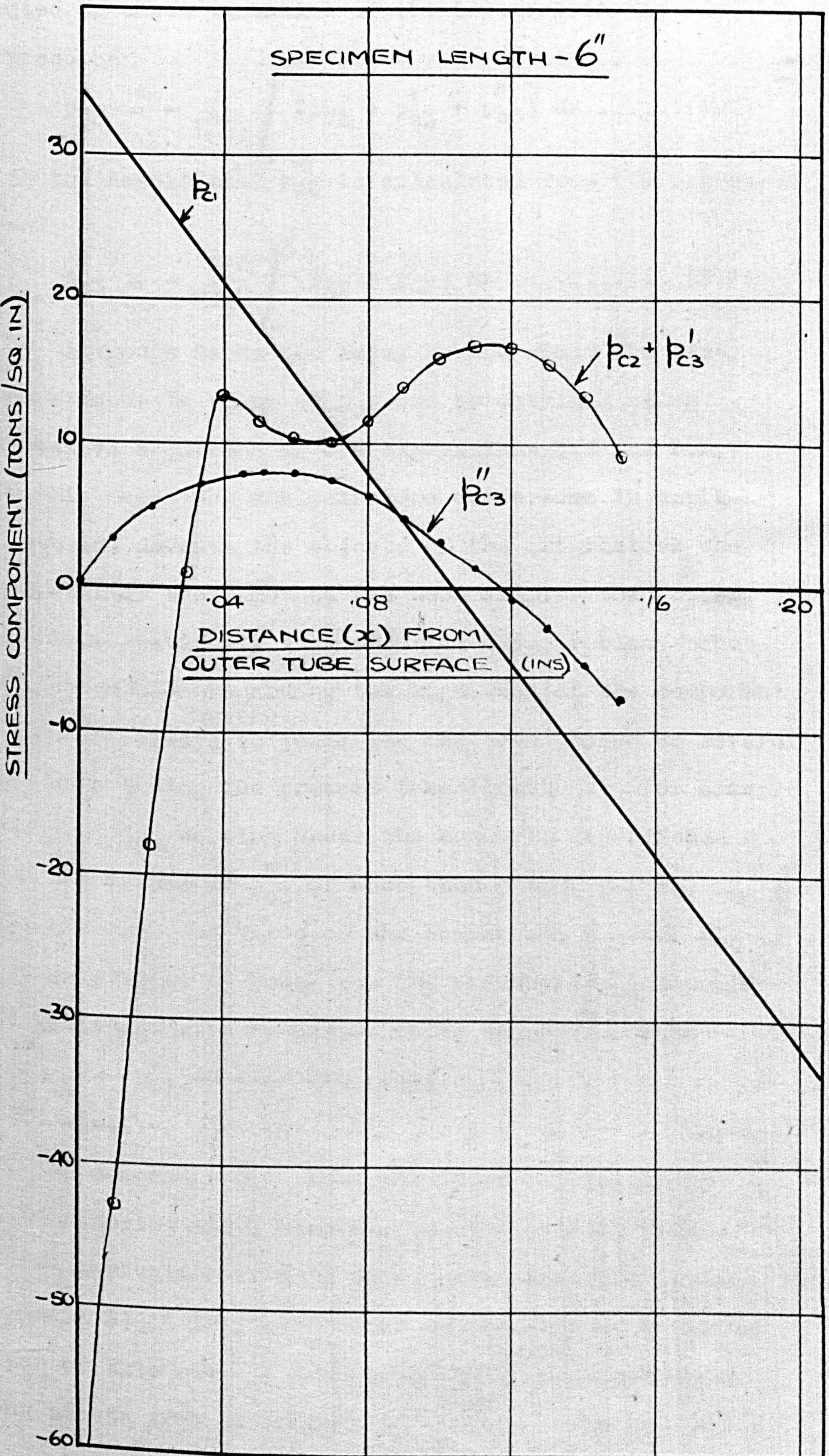


FIG: 3.1. CIRCUMFERENTIAL STRESS COMPONENTS (SACHS AND ESPEY ANALYSIS) (AFTER HOWARD 1951).

stress component is small in comparison with the summation of the components p_{c2} and p'_{c3} and can be neglected in the computation of the integral in the expression

$$p''_{c3} = -\frac{1}{t-x}\int_0^x (p_{c2} + p'_{c3} + p''_{c3})\,dx \quad \ldots\ldots (3.1)$$

with the result that p''_{c3} is calculated from the expression

$$p''_{c3} = -\frac{1}{t-x}\int_0^x (p_{c2} + p'_{c3})\,dx \quad \ldots\ldots\ldots\ldots (3.2)$$

Although Sachs and Espey do not state the fact, a more accurate value of p''_{c3} can be obtained using successive solutions of the expressions 3.2 and 3.1, but this makes the analysis more cumbersome in application and defeats the objects of the originators who claimed that their method was much simpler than other solutions previously suggested for this problem. That this assumption regarding the magnitude of the component p''_{c3} cannot always be justified has been proved on several occasions during the present investigation. For example, Howard (1951) working under the author's supervision reported values of p''_{c3} of more than 8 tons per sq. in. when the combined value of the components p_{c2} and p'_{c3} were only about 10 tons/ sq. in. and where $\int_0^x p''_{c3}dx$ could not satisfactorily be neglected in comparison with $\int_0^x (p_{c2} + p'_{c3})dx$ (see Fig. 3.1).

Finally, the expression given by Sachs and Espey for the component p'_{c3} i.e. that stress released by bending during metal removal, is not strictly valid. In the derivation of this expression (Equation 11.3, Appendix 3) it is implied that the removal of a finite layer of thickness x_1 from a specimen, releases a bending stress from the remaining section given by

$$M = \frac{2EI\Delta D}{(1-\sigma^2)\,D_m^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

where ΔD is the bodily change in diameter on removal of the layer of thickness x_1. This gives rise to the release of a stress defined by the expression

$$p'_{c3} = \frac{E}{1-\sigma^2} \frac{(t-x_1)\Delta D}{D_m^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

In actual fact, the bending moment M is not released at one stage but is successively released by the progressive removal of thin annular surface layers of the material up to the point in the tube wall defined by x_1. Consequently the released moment is more strictly defined by

$$M = \frac{2E}{1-\sigma^2} \int_0^{\Delta D} \frac{I dD}{D_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

and the released bending stress at x_1 by :-

$$p'_{c3} = \frac{E}{1-\sigma^2} \int_0^{\Delta D} \frac{(t + x - 2x_1)}{D_m^2} dD \quad \ldots\ldots\ldots\ldots (3.6)$$

where dD = diametral change on removal of a layer of thickness dx, and x = depth in wall of all layers of material removed before x_1 is reached. The expressions (3.6) and (3.4) may yield values for p'_{c3} which show no significant differences depending upon the relative values of t and x_1 and upon the form of the ΔD - x relationship. However, in most cases which have been considered in the present work, the Sachs and Espey expression (3.4) tends to over-estimate the value of the component p'_{c3} and indirectly the value of p''_{c3} as given by (3.2).

If the method of solution proposed by Davidenkov is applied to the derivation of the component p_{c3} using the simple theory of bending adopted by Sachs and Espey, the following expression results (see Appendix 4).

$$p_{c3} = p'_{c3} + p''_{c3} = \frac{2}{3} \frac{E}{(1-\sigma^2) D_m^2} \left[(2t-3x_1)\Delta D + \int_{D_0}^{D_{x_1}} x dD \right] \quad \ldots\ldots (3.7)$$

The form of this expression is no more difficult to handle than the two expressions (3.2) and (3.4) which on summation define the same stress component. Furthermore equation 3.7 is basically more fundamental than the corresponding Sachs and Espey expressions. (The expression 3.7 should really be attributed to Knights (1951) who reached an identical expression in a successful attempt to simplify the Davidenkov relations. He, however, reached the solution by analogy with the Davidenkov equations for the derivation of residual longitudinal stresses and not by application of first principles as has been done in the present work.)

3.2. Development of experimental techniques for the application of the bending deflection methods of strain release to the detection and measurement of residual stresses in tubular components.

Although the basic principles for determining residual stresses by bending deflection methods of strain release have already been given in this report, it is perhaps pertinent to reiterate them.

For the determination of circumferential stresses, the method consists of preparing a ring specimen from a parent tube and after careful measurement, slitting it longitudinally along any one radial plane. The amount by which the outer diameter of the specimen in a plane perpendicular to the one containing the slit alters from its original unslit value is a measure of the stress released. By successively reducing the tube wall thickness by machining or pickling, and by measuring the current outer "sprung" diameter at each stage, the residual stress distribution throughout the tube wall can be found. For longitudinal stresses, a longitudinal tongue is cut from the tube wall and the amount of its deflection and curvature from the original straight line is a measure of the released stress. Further stress components are released by reducing the tongue thickness by pickling (adid attack) and the longitudinal stress distribution throughout the walls of the tube can be determined.

At an early stage of the present work it was realised that metal removal by machining was not ideal in any way as stresses may be induced by such a process which, over perhaps a relatively short depth near the machined surface, are of the same order of magnitude as the stresses being determined. After many preliminary trials to remove material evenly and carefully from steel plates by such processes as electrolytic (anodic) polishing and pickling in various chemicals, a method of layer removal by pickling in nitric acid was successfully developed. (For a full account of this preliminary work see Montgomery, 1950.) Such a method of layer removal had actually been suggested by Davidenkov (1932) but until Knights (1951) carried out his investigations, the method had not been widely adopted.

The experimental techniques and procedure eventually adopted for all the bending deflection tests made in the course of this work were as follows:

A tubular specimen of predetermined length (decided according to considerations discussed later in Section 6) was carefully parted off from a parent tube using a high speed band saw and taking every possible precaution to avoid overheating of the specimen in the region of the saw cut. After careful measurement of the length, wall thickness and diameter of the specimen using micrometers which had been previously checked against slip gauges, the specimen was slit longitudinally along one radial plane using the previously mentioned band-saw and taking all the precautions previously employed. The "sprung" diameter at several points along the plane perpendicular to that containing the slit was then measured and the specimen was coated on all its surfaces except one cylindrical surface by painting with a solution of special "stopping-off" wax dissolved in trichlorethylene. The treated specimen was then allowed to dry thoroughly in air, after which it was completely immersed in a 15%

solution of nitric acid maintained at a temperature of $110^\circ \pm 10^\circ$F. Continuous agitation of the acid solution and movement of the specimen in the acid bath during the ensuing pickling process was carried out, and pickling was continued until sufficient metal had been removed from the unprotected surface to reduce the initial wall thickness of the specimen by about 0.005 in. or by approximately $\frac{1}{50}$ of the initial wall thickness, whichever was the smaller value. The specimen was then removed from the pickling bath, thoroughly washed with water and then completely measured at several points to obtain accurate mean values of the current wall thickness and "sprung" diameter. The pickling and measuring processes were then successively repeated until the remaining wall thickness was substantially less than half the initial value. The acid pickling process removed surprisingly uniform layers of material from the unprotected surface of the specimen, particularly if due attention was paid to continuous agitation of the acid and to rotational movement of the specimen, and provided the acid solution was frequently renewed. With almost every specimen treated in this manner, the quality of the pickled surface was only slightly inferior to that of the original surface (in a few cases a distinct improvement was noticed after pickling) and any eccentricity originally present between the bore and outer surfaces of the specimen was always maintained.

An acid pickling process was developed for parting off specimens from the parent tube (see Section 6) and for slitting the specimens longitudinally, but this was not generally adopted throughout the investigation because of the prohibitive times involved. (A single specimen prepared completely by acid pickling from a 2.125 in. diameter tube with walls approximately 0.20 in. thick took some 190 hours to prepare during which period the

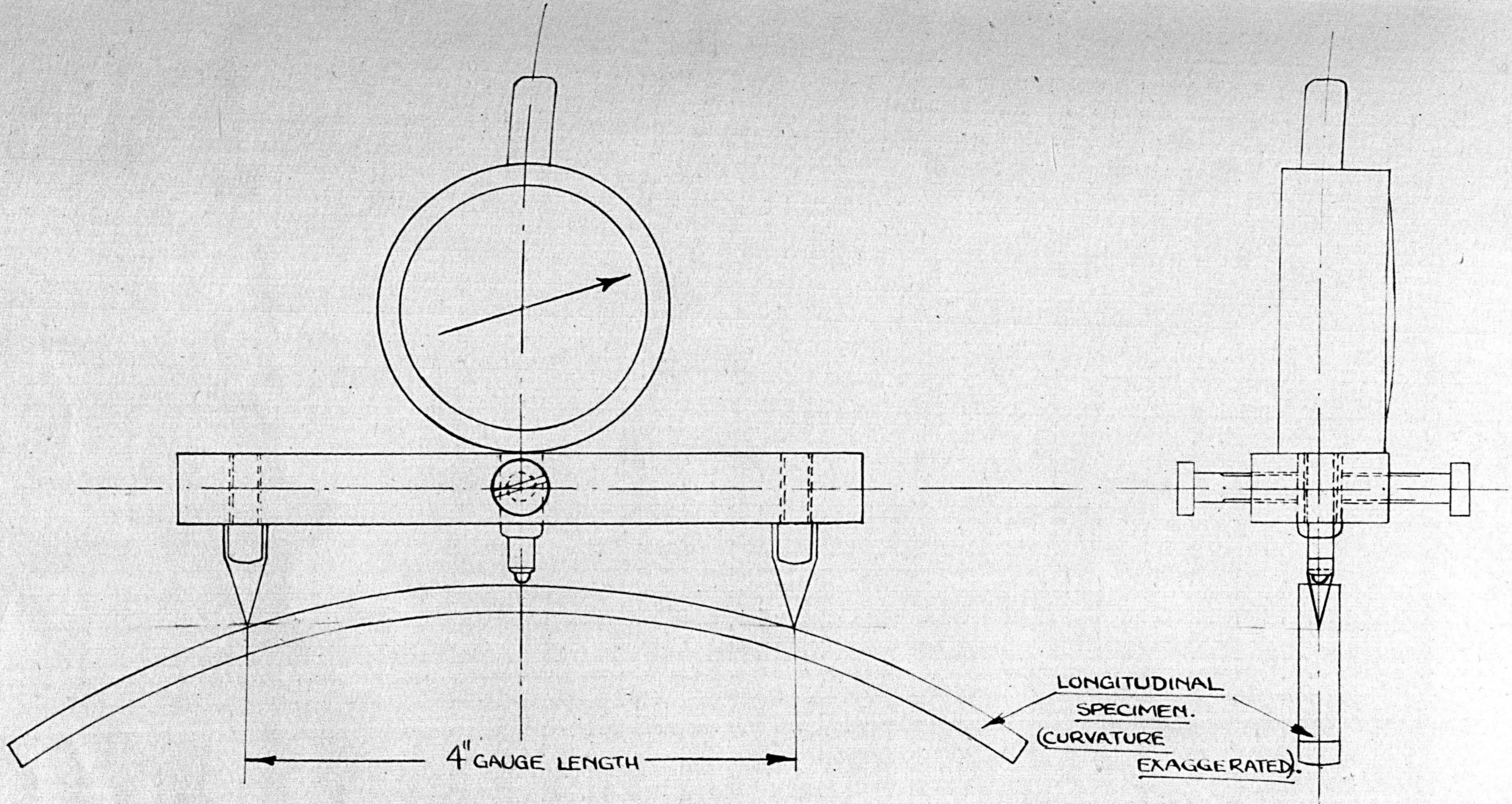


FIG: 3.2. GAUGE FOR MEASURING THE CURVATURE OF LONGITUDINAL BENDING SPECIMENS.

specimen had to be under almost continuous observation. Such times as that quoted were not uncommon in the application of the process.)

For the detection and measurement of residual longitudinal stresses, strip specimens were prepared from the parent tube by cutting with two $\frac{1}{16}$ in. thick milling cutters separated by a bush so that the resultant specimen width was approximately 0.15 x the diameter of the stock tube. (This width was selected as a result of the researches of Knights and Crampton, previously mentioned on Page 27). These specimens were then "stopped off" on the appropriate faces with wax and pickled using the same solution, techniques and precautions as previously described for ring specimens. The resultant curvature changes were measured using the three point curvature gauge shown diagrammatically in Fig. 3.2.

During a meeting in which the progress of this investigation came under discussion (Loxley 1950 and 1951) it was suggested that a gaseous evolution during the removal of layers of material by an acid pickling process, may affect the magnitude of the residual stresses in the part being tested, in a similar way to shot-peening or hydrogen embrittlement.

Since the acid pickling process seemed destined to play an important part in this investigation, it was necessary to establish whether this in fact was the case.

For this purpose a piece of steel measuring 6 in. x $\frac{1}{2}$ in. x $\frac{1}{4}$ in. was carefully annealed and its resulting curvature in the longitudinal direction carefully measured using a three point method over a gauge length of 5 in. The specimen was then completely "stopped-off" with wax except on one $\frac{1}{2}$ in. wide face which was then subjected to acid attack. The resulting curvature was measured after approximately 0.010 in. thickness of material had

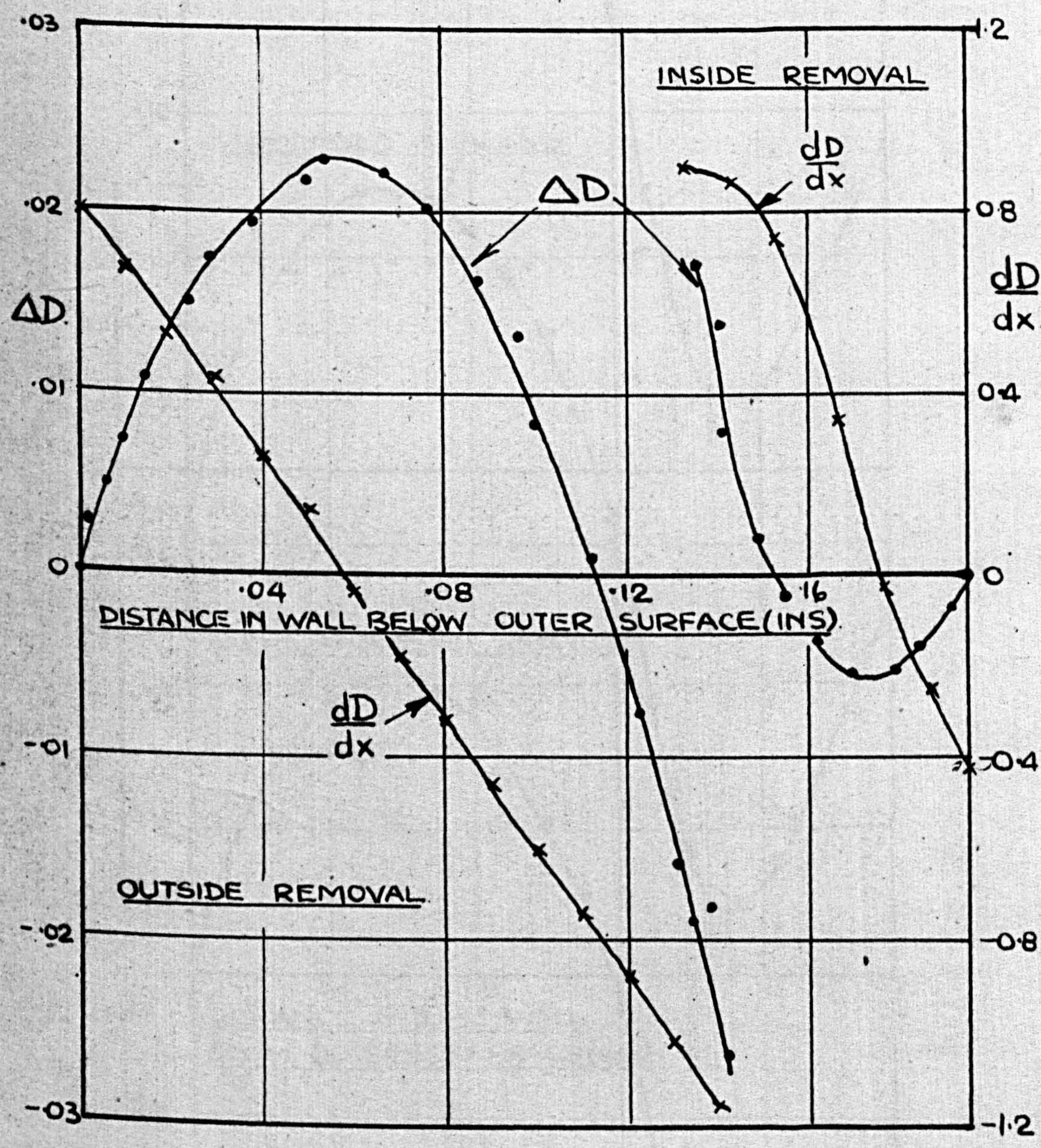


FIG. 3.3a COMPARISON OF ANALYSES. EXPERIMENTAL RESULTS. SPECIAL TUBE.

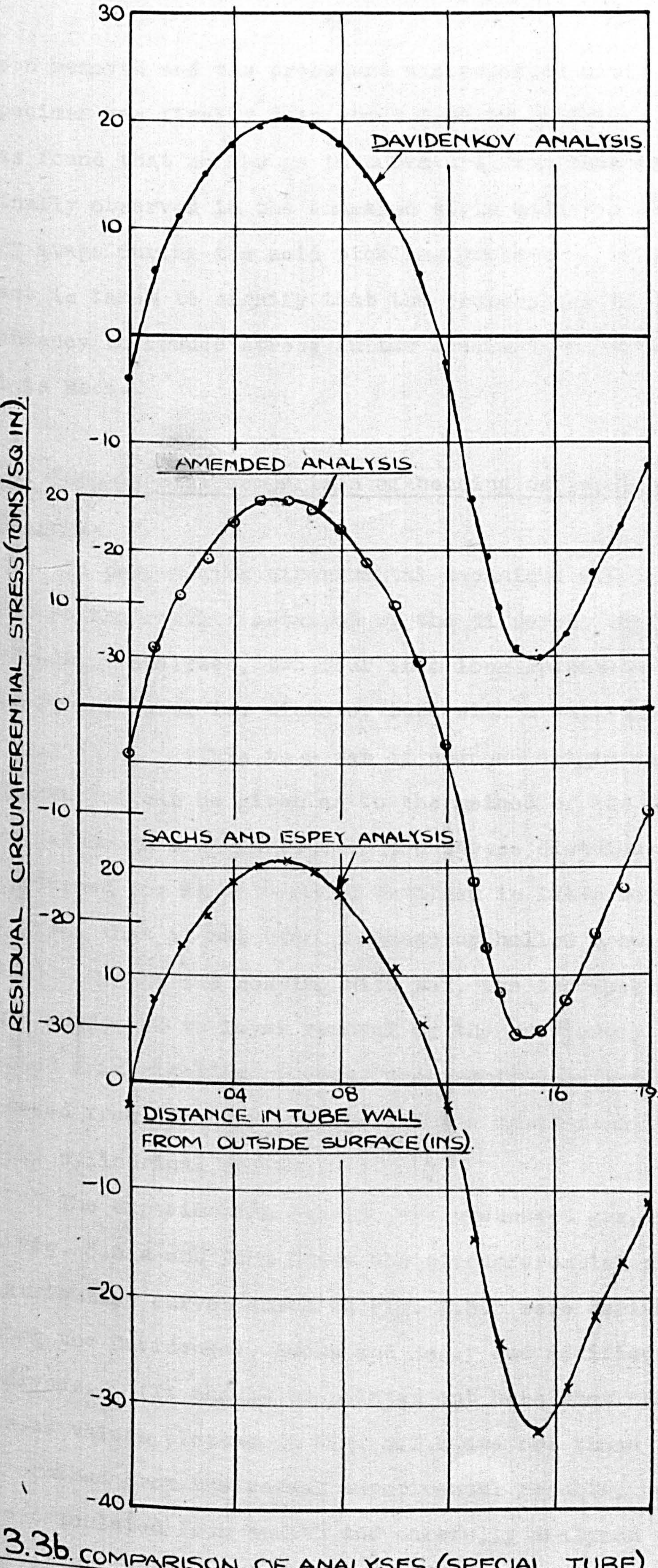


FIG: 3.3b. COMPARISON OF ANALYSES. (SPECIAL TUBE).

been removed and the procedure was repeated until the specimen was finally only about 0.05 in. thick. It was found that no change in curvature from that originally observed in the annealed strip occurred at any stage during the acid pickling process. This fact is taken to signify that the process has no tendency to induce stress in the specimens on which it is used.

3.3. Experimental comparison of bending deflection analyses.

To perfect the experimental technique and to compare the results obtained by the different bending deflection analyses, two four inch long specimens were cut from a 2.501 in. diameter tube with a wall thickness of 0.195 in. (This tube was of unknown origin and no information can be given as to the method of its processing, although a comparison of the stress distribution determined for it with those obtained in later work, suggests that it had been produced by hollow drawing.) After slitting and coating with wax, the two specimens were subjected to layer removal by the previously mentioned acid pickling process, one specimen having layers removed from its bore surface and the other from its outer cylindrical surface.

The experimental results are presented graphically in Fig. 3.3.a and from these the circumferential stress distribution curves shown in Fig. 3.3.b were derived using the Davidenkov, Sachs and Espey and modified analyses. (It should be pointed out here that the stress values plotted in Fig. 3.3.b are not those determined from the actual experimental results, but are calculated from smooth and carefully analysed curves drawn through the experimental results at each stage in the computation of the three stress components

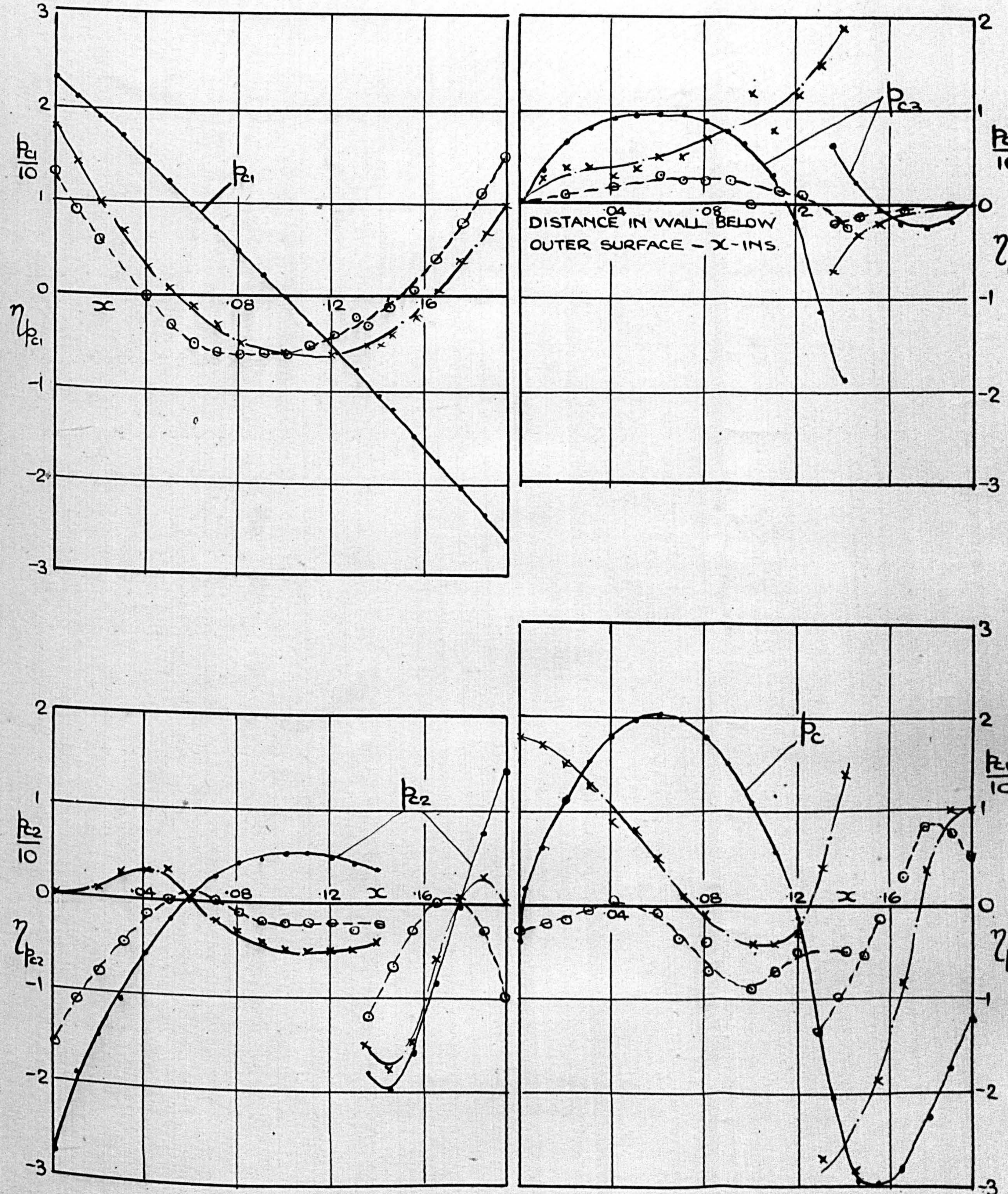


FIG. 3.4. COMPARISON OF ANALYSES. STRESS COMPONENTS AND THEIR VARIATION. (SPECIAL TUBE).

○ ○ ○ AMENDED ANALYSIS.

× × × SACHS AND ESPEY ANALYSIS.

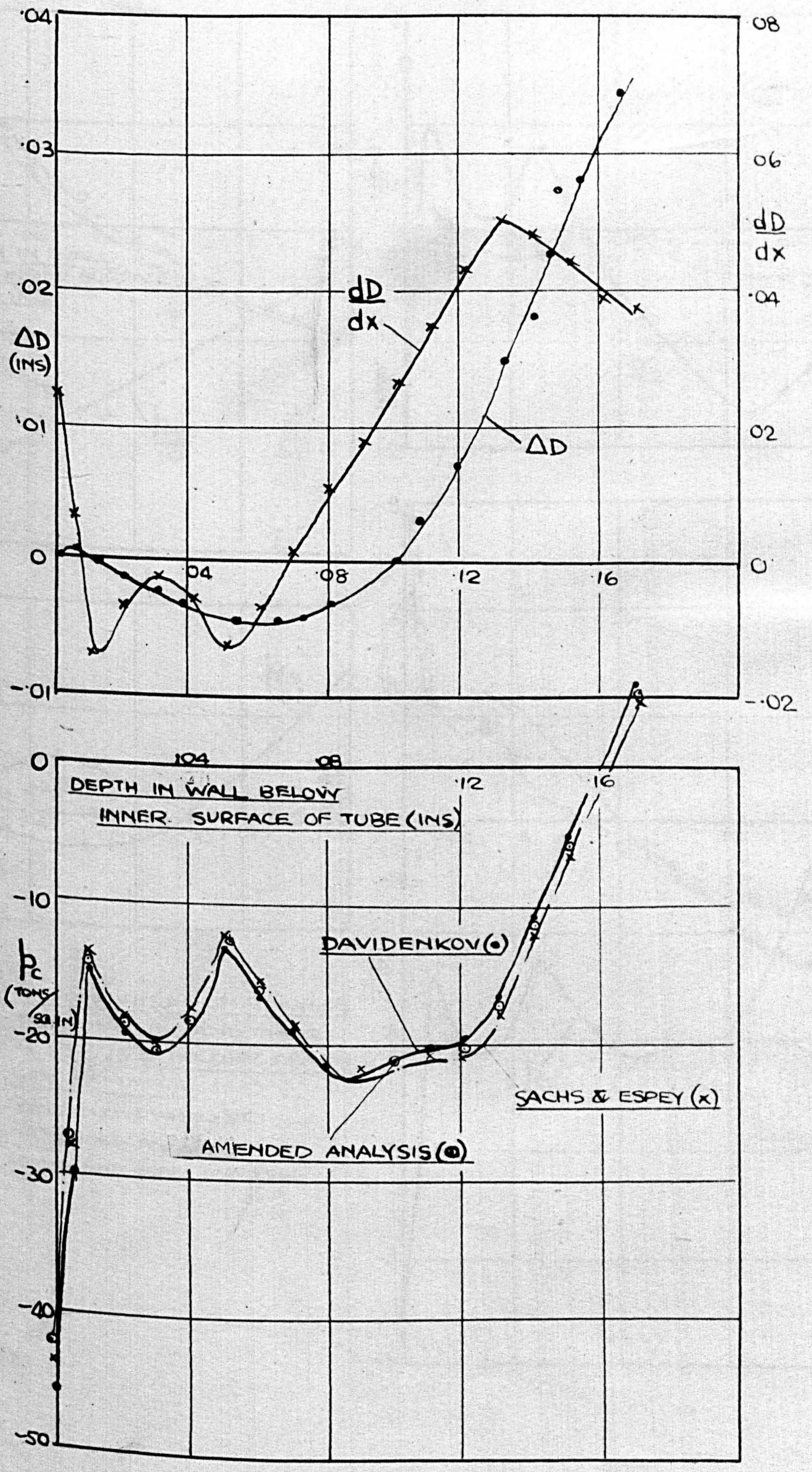


FIG: 3.5. COMPARISON OF ANALYSES. (3·375" ϕ TUBE)
(DETAILS AS ON FIG. 3.6).

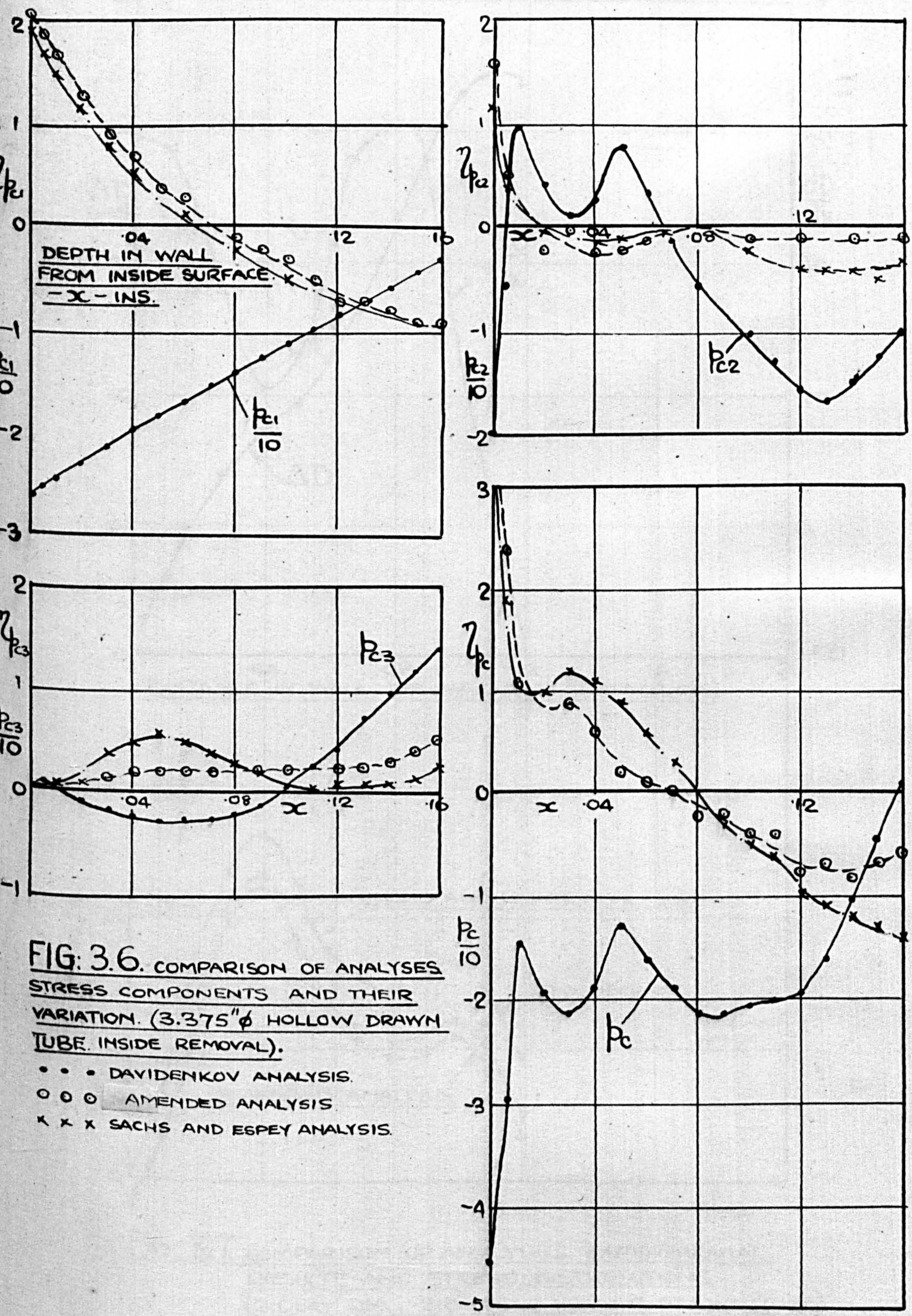


FIG: 3.6. COMPARISON OF ANALYSES STRESS COMPONENTS AND THEIR VARIATION. (3.375"ϕ HOLLOW DRAWN TUBE. INSIDE REMOVAL).

• • • DAVIDENKOV ANALYSIS.

⊙ ⊙ ⊙ AMENDED ANALYSIS

x x x SACHS AND ESPEY ANALYSIS.

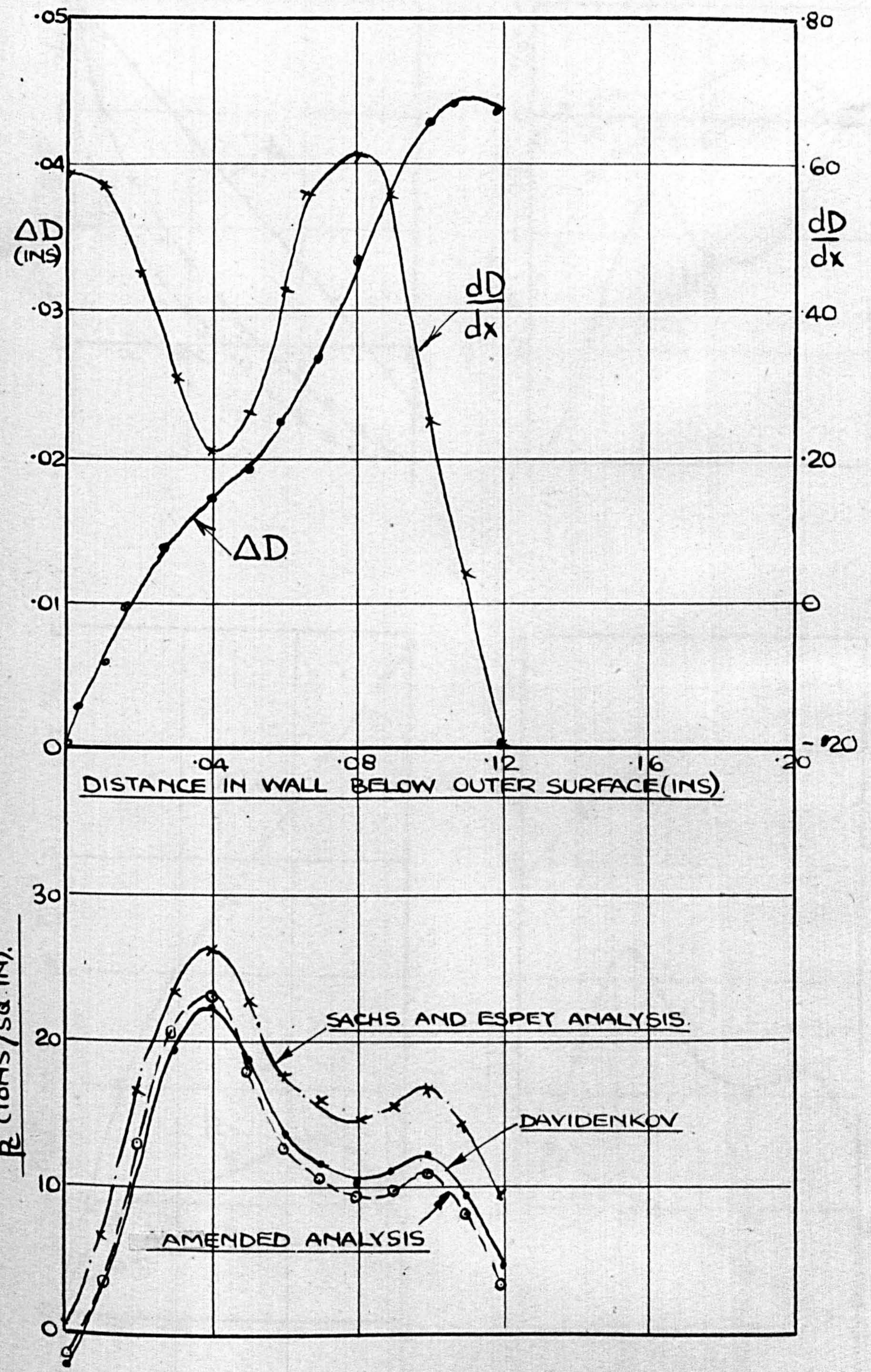


FIG: 3.7. COMPARISON OF ANALYSES. EXPERIMENTAL RESULTS AND STRESS DISTRIBUTIONS. (2.006" DIA, ·218" WALL; HOLLOW DRAWN TUBE).

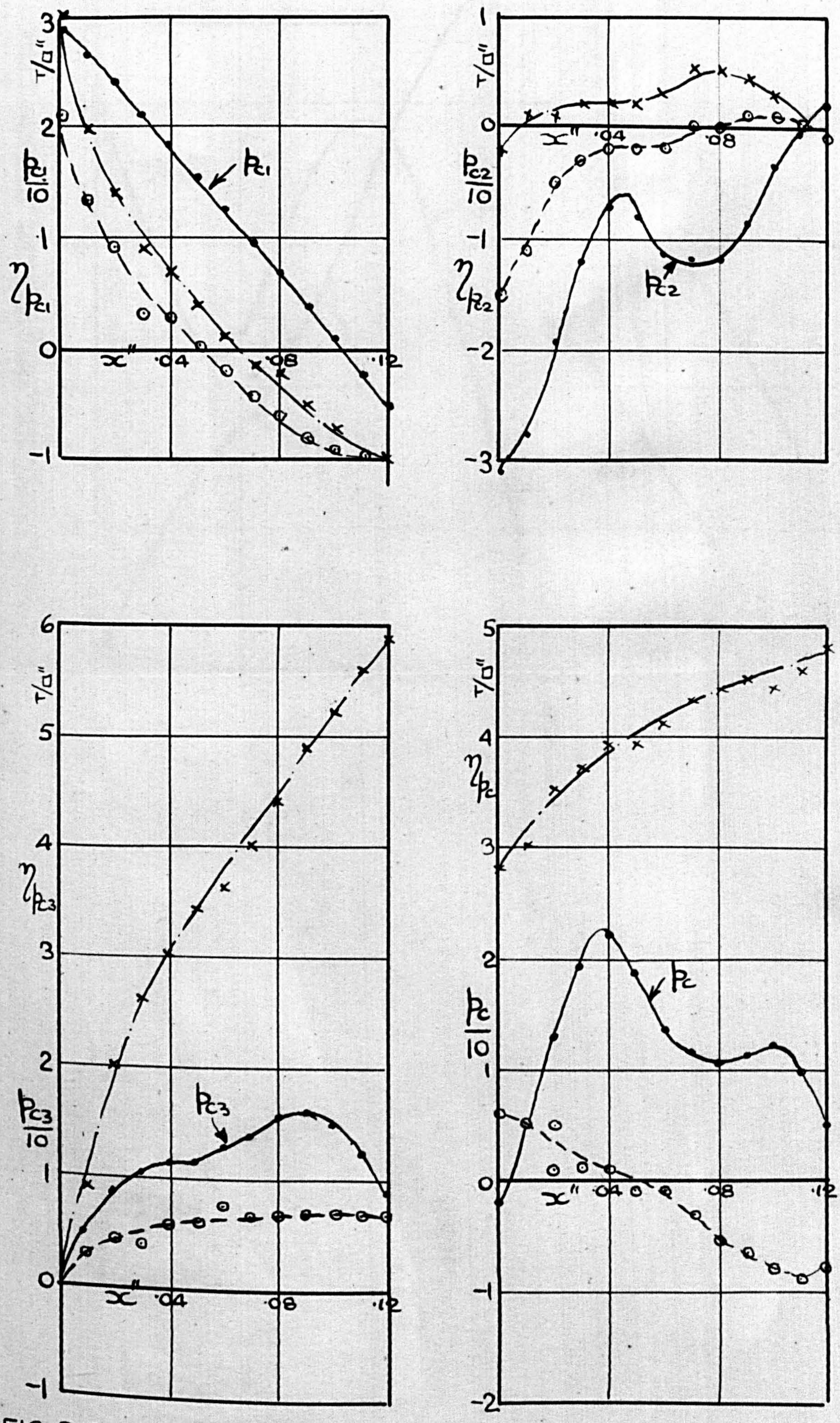


FIG: 3.8. COMPARISON OF ANALYSES. STRESS COMPONENTS AND THEIR VARIATION. (DETAILS AS ON FIG: 3.7)

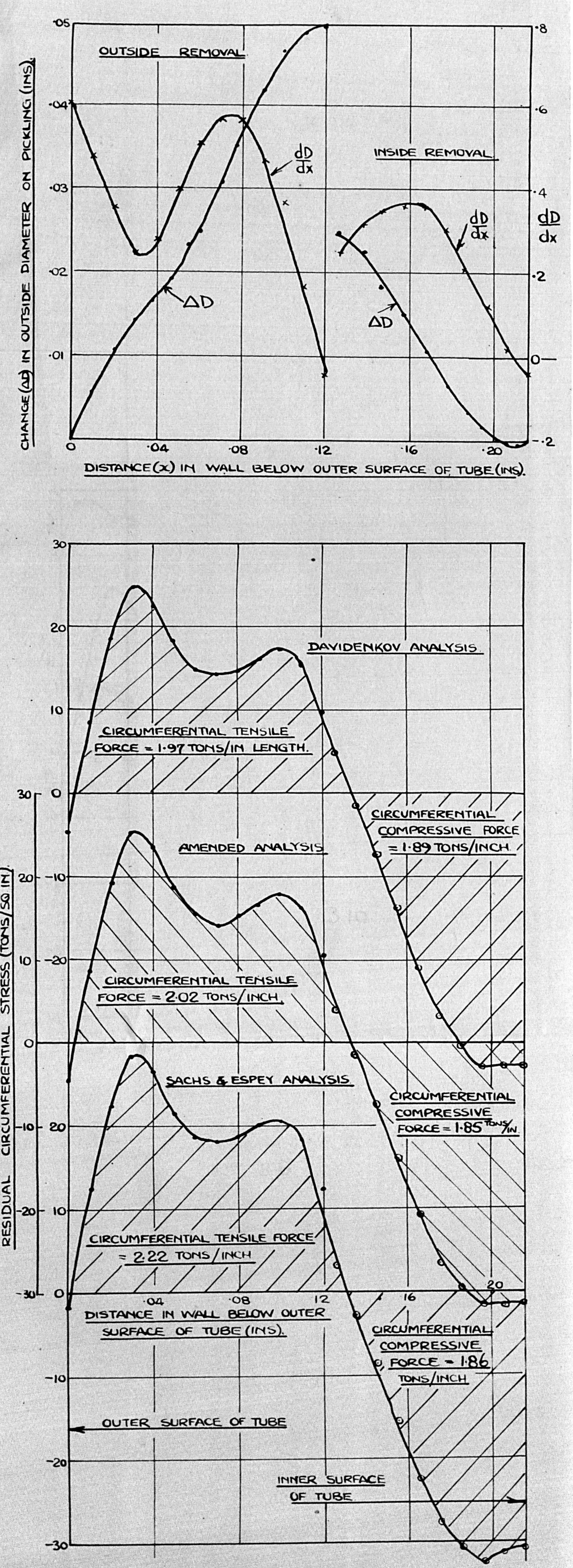


FIG. 3.9 COMPARISON OF ANALYSES. EXPERIMENTAL RESULTS AND STRESS DISTRIBUTION CURVES. (2·25" ø TUBE 0·216" WALL).

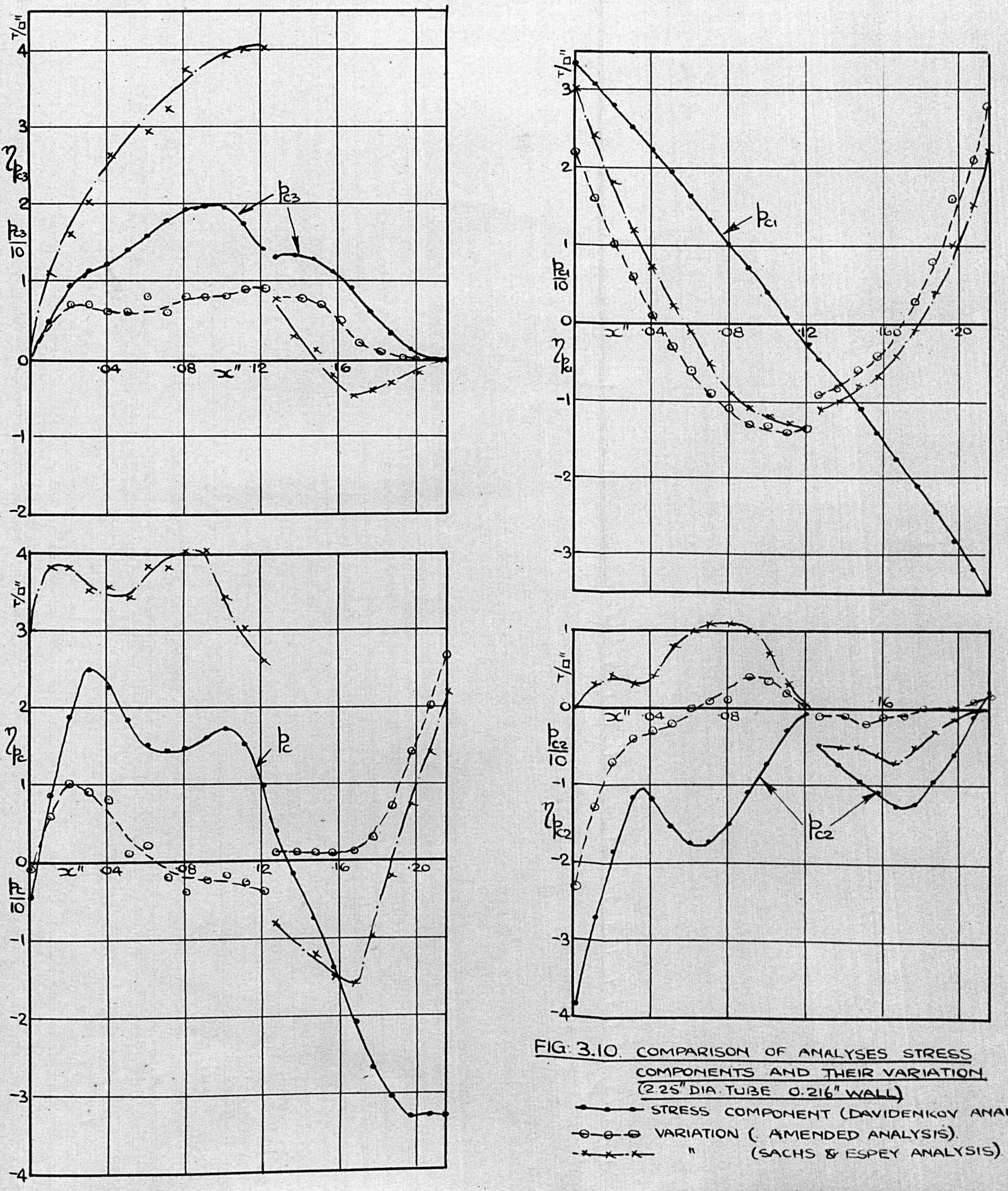


FIG: 3.10. COMPARISON OF ANALYSES STRESS COMPONENTS AND THEIR VARIATION (2.25" DIA. TUBE 0.216" WALL)

—•—•— STRESS COMPONENT (DAVIDENKOV ANALYSIS.

-⊖-⊖-⊖ VARIATION (AMENDED ANALYSIS).

-×-×-× " (SACHS & ESPEY ANALYSIS).

from which the total stress is calculated. This procedure is standard throughout this work.) The three curves shown in Fig. 3.3.b are obviously of a similar form but closer inspection of them reveals slight differences in the magnitude of the stress at any particular point in the tube wall. These differences are more clearly seen in Fig. 3.4, which shows the variation (η) between the stress components p_{c1}, p_{c2} and p_{c3} as derived by the Sachs and Espey and modified analyses and the corresponding values calculated by the Davidenkov equations. Superimposed on each of the graphs in Fig. 3.4 is the distribution curve of the relevant stress component as calculated by the Davidenkov analysis. The following conclusions are suggested from a careful study of Fig. 3.4 and these are generally confirmed in Figs. 3.5 to 3.10 inclusive, which show the experimental results, the derived stress distribution curves and the variation in the stress component curves as obtained from four other specimens from other tubes treated in an identical manner to that described above.

(i) The maximum variation between the values of the stress component p_1 (the stress released on slitting) occurs at the surfaces of the specimen. For this component, there is little, if anything, to choose between the Sachs and Espey and the modified analyses.

(ii) With the stress component p_{c2} (the stress removed with a layer of material) the modified analysis gives lower stress values than either the Davidenkov or the Sachs and Espey analysis at the outer surface of the specimen, although for most of the distance through the tube wall it results in stress values which are very similar to those given by the Davidenkov analysis.

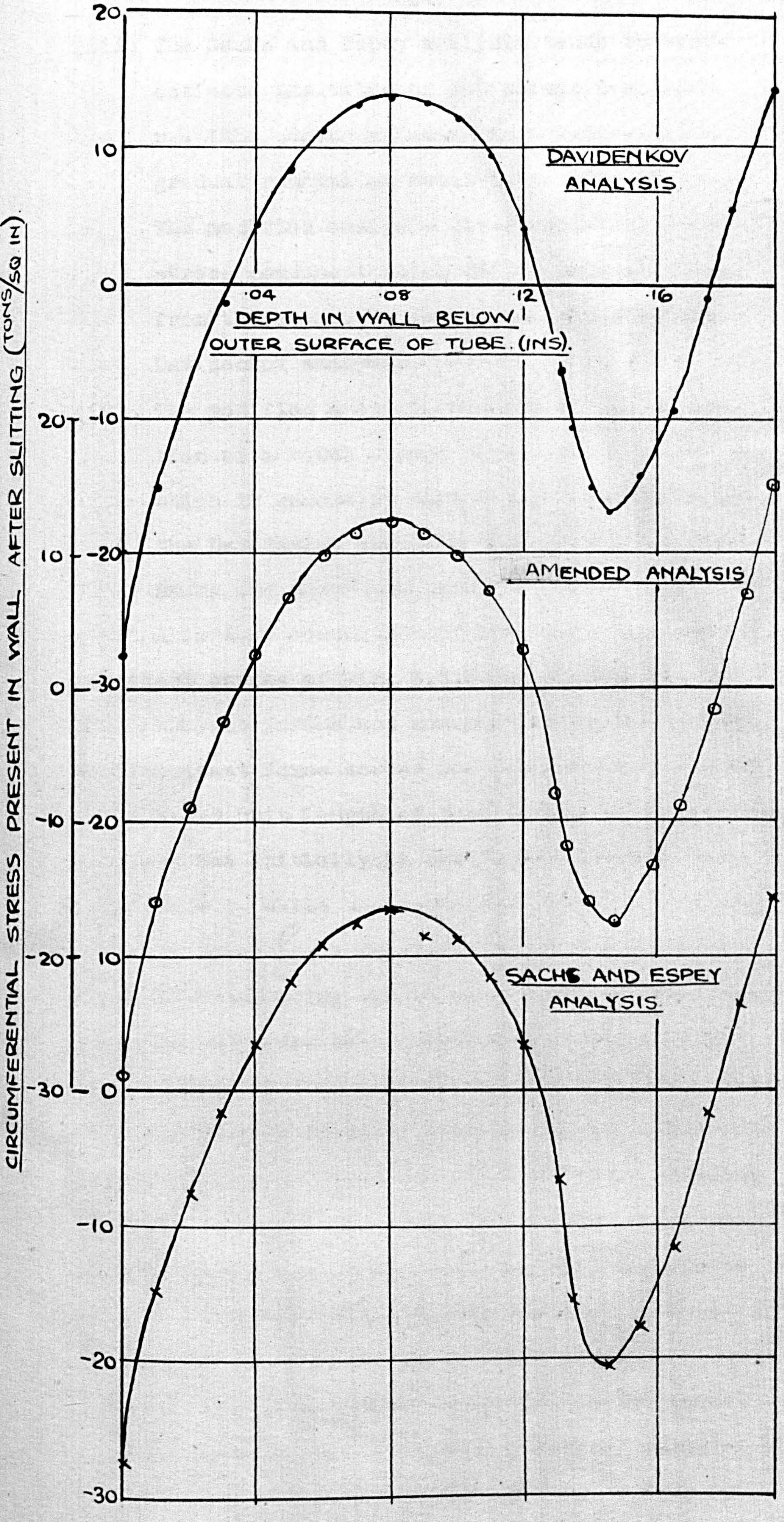


FIG. 3.11. COMPARISON OF ANALYSES. STRESS DISTRIBUTION AFTER SLITTING (SPECIAL TUBE).

(iii) The Sachs and Espey analysis tends to overestimate the value of the stress component p_{c3} (the stress released in a layer by the gradual removal of metal up to that layer). The modified analysis gives values of this stress component which differ only slightly from the corresponding values given by the Davidenkov analysis.

(iv) The modified analysis results in the derivation of a total stress distribution curve which is generally more nearly that given by the Davidenkov analysis than does the original Sachs and Espey analysis.

A further comparison of the analyses from the resultant curves of Fig. 3.3.b can be made by studying the resultant forces and moments across the section. The resultant force across the residually-stressed section per unit length of tube should be zero since the tube was initially in static equilibrium, i.e. $\int_c^x p_c \, dx = 0$, while the resultant moment across the section, i.e. $\int_0^x p_c \, x \, dx$ should equal the moment released on slitting the tube. Both the resultant force and the resultant moment across the section can be considered in two stages, one being associated with the tensile stressed regions and the other with the regions of compressive stress. After slitting the tube, stresses remain in the section which are of magnitude $p_c - p_{c1}$ and both the resultant force and the resultant moment across the section must now be zero as complete equilibrium attains. The distribution of the stress remaining in the specimen after the initial slitting operation, calculated from the results summarized in Fig. 3.3.b, is given in Fig. 3.11. Graphical integration to solve the expressions $\int_0^x p_c \, dx$ and $\int_0^x p_c \, x \, dx$ for

		DAVIDENKOV ANALYSIS	AMENDED ANALYSIS	SACHS-ESPEY ANALYSIS
RESULTANT CIRCUMFERENTIAL STRESS DISTRIBUTIONS (FIG. 3.3)				
FORCE (TONS/IN.)	TENSILE	1.57	1.53	1.65
	COMPRESSIVE	1.59	1.58	1.61
	RESULTANT	-0.02	-0.05	0.04
BENDING MOMENT ABOUT OUTER SURFACE (TONS INS./IN.)	* ANTI-CLOCKWISE	0.095	0.093	0.097
	CLOCKWISE	0.263	0.261	0.268
RESULTANT BENDING COUPLE (TONS INS./IN.)		0.168	0.168	0.171
BENDING COUPLE RELEASED ON SLITTING ǂ (TONS INS./IN.)		0.156	0.156	0.156
STRESS DISTRIBUTIONS AFTER SLITTING (FIG. 3.11)				
FORCE (TONS/IN.)	TENSILE	0.94	0.92	0.97
	COMPRESSIVE	0.91	0.87	0.91
	RESULTANT	0.03	0.04	0.06
BENDING MOMENT ABOUT OUTER SURFACE (TONS INS./IN.)	* ANTI-CLOCKWISE	0.091	0.089	0.094
	CLOCKWISE	0.087	0.084	0.089
RESULTANT BENDING COUPLE (TONS INS./IN.)		0.004	0.005	0.005

* SENSE OF BENDING MOMENTS TAKEN TO CONFORM WITH STRESS DISTRIBUTIONS IN FIGS. 3.3 AND 3.11. (TENSILE FORCES YIELD ANTI-CLOCKWISE MOMENTS).

ǂ DIAMETRAL CHANGE ON SLITTING = 0.048 INS.

TABLE 3.I. COMPARISON OF ANALYSES. FORCES AND MOMENTS.

the tensile and compressive stressed regions given in both Figs. 3.3.b and 3.11 result in the comparison of forces and moments given in Table 3.I.

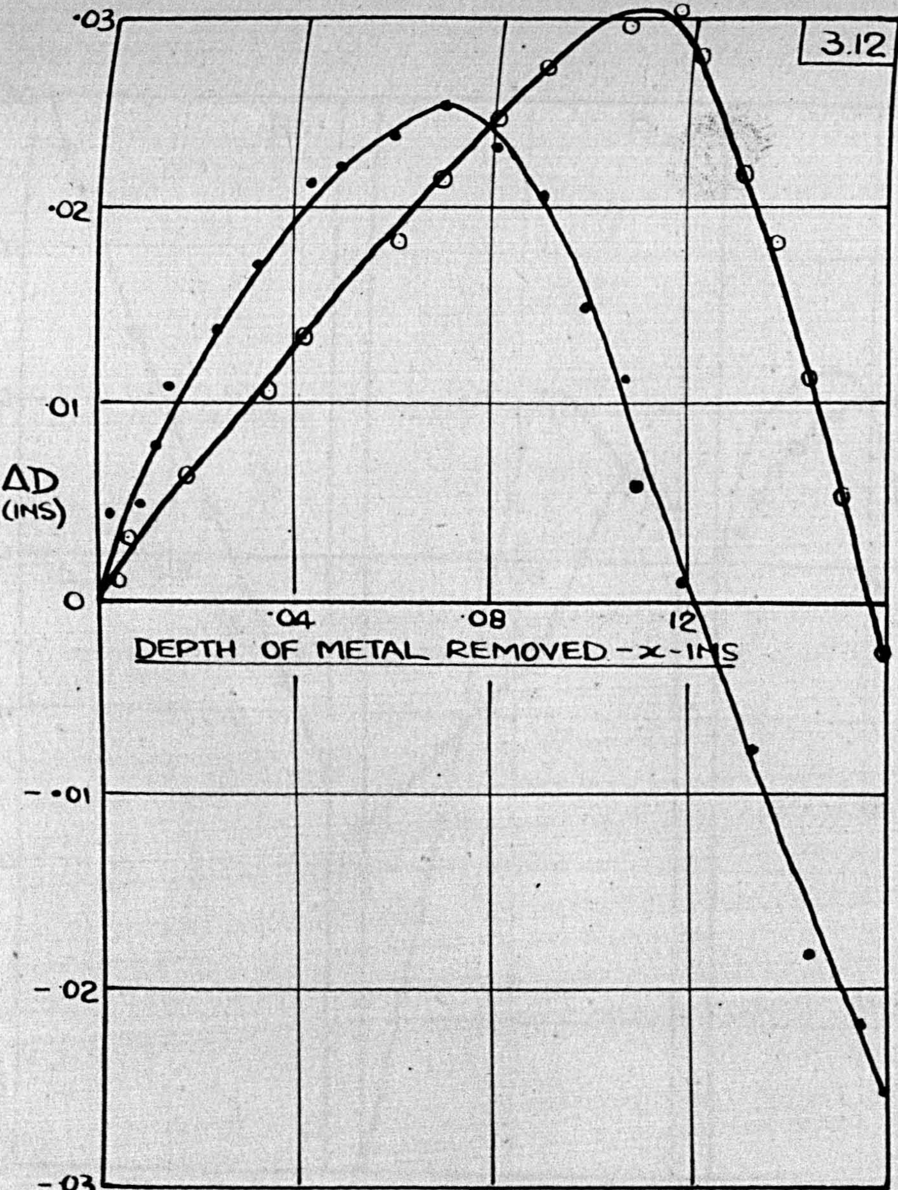
The total stress distribution curves of Fig. 3.3.b and the stress distribution curves after slitting (Fig. 3.11) each yield effectively balanced forces across the section, the variation between the tensile and compressive net forces being never greater than 5% and in most cases substantially less then this. (Agreement of this order is considered remarkable because of the inherent inaccuracies in the computation from the experimental results - see Appendix 6, Section 14.) All the analyses yield a resultant bending couple greater than that actually released on slitting, although this effect cannot be considered significant in view of the experimental results described later in Section 3.4.4 (see page 48). After slitting, however, the calculated resultant moment across the section is approximately zero.

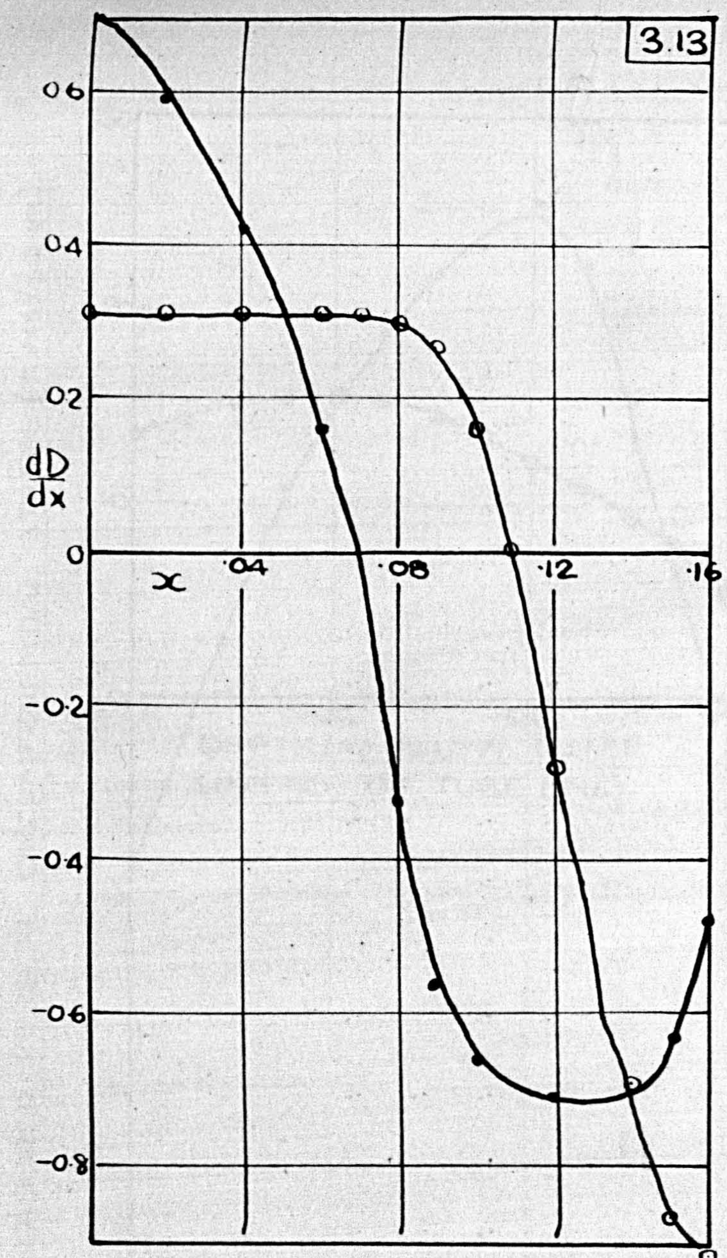
From these experimental considerations of the three analyses considered in this section of the work, it is concluded that although the Davidenkov analysis is the most fundamental and presumably most exact of all the analyses, there is no justification for its application to thin walled tubes. The modified Sachs and Espey analysis which is much simpler in application gives results which differ only slightly from the results given by the Davidenkov analysis, but the original Sachs and Espey analysis can result in errors (associated with the stress component p_{c3} - see Fig. 3.8) which may be of a serious nature.

Consequently, all the circumferential stress distributions given later in this work as being derived by bending deflection methods have been obtained by application of the modified Sachs and Espey analysis, except where otherwise stated.

TUBE REFERENCE	A	B	C
Method of processing	Hollow drawn	Hollow drawn	Hollow drawn
Tube material	Mild steel	Mild steel	Mild steel
Initial outside diameter	3.375 in.	3.375 in.	4.25 in.
Final outside diameter	2.121 in.	2.223 in.	2.926 in.
Initial wall thickness	-	-	0.125 in.
Final wall thickness	0.218 in.	0.221 in.	0.131 in.
Type of die used	Straight taper	Patent "bottle-neck"	Straight taper
Total conical angle of die	30°	-	30°
Tools used in drawing	Commercial draw bench	Commercial draw bench	Experimental apparatus
Tube drawn by	Tube Investments, Ltd.	Tube Investments, Ltd.	Sheffield University

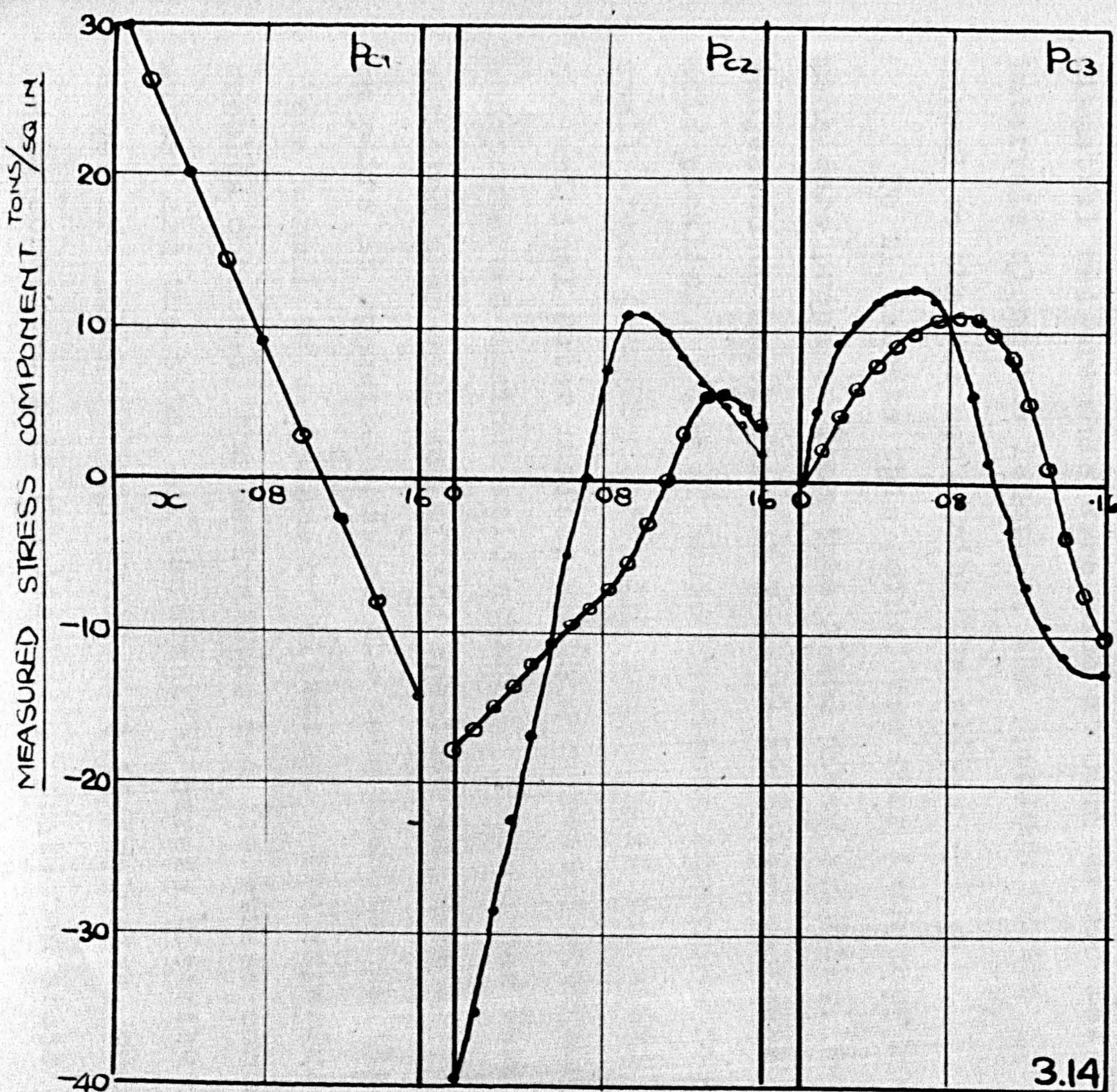
TABLE 3.II. SPECIFICATION OF TUBES USED FOR THE EXPERIMENTAL PROGRAMME.

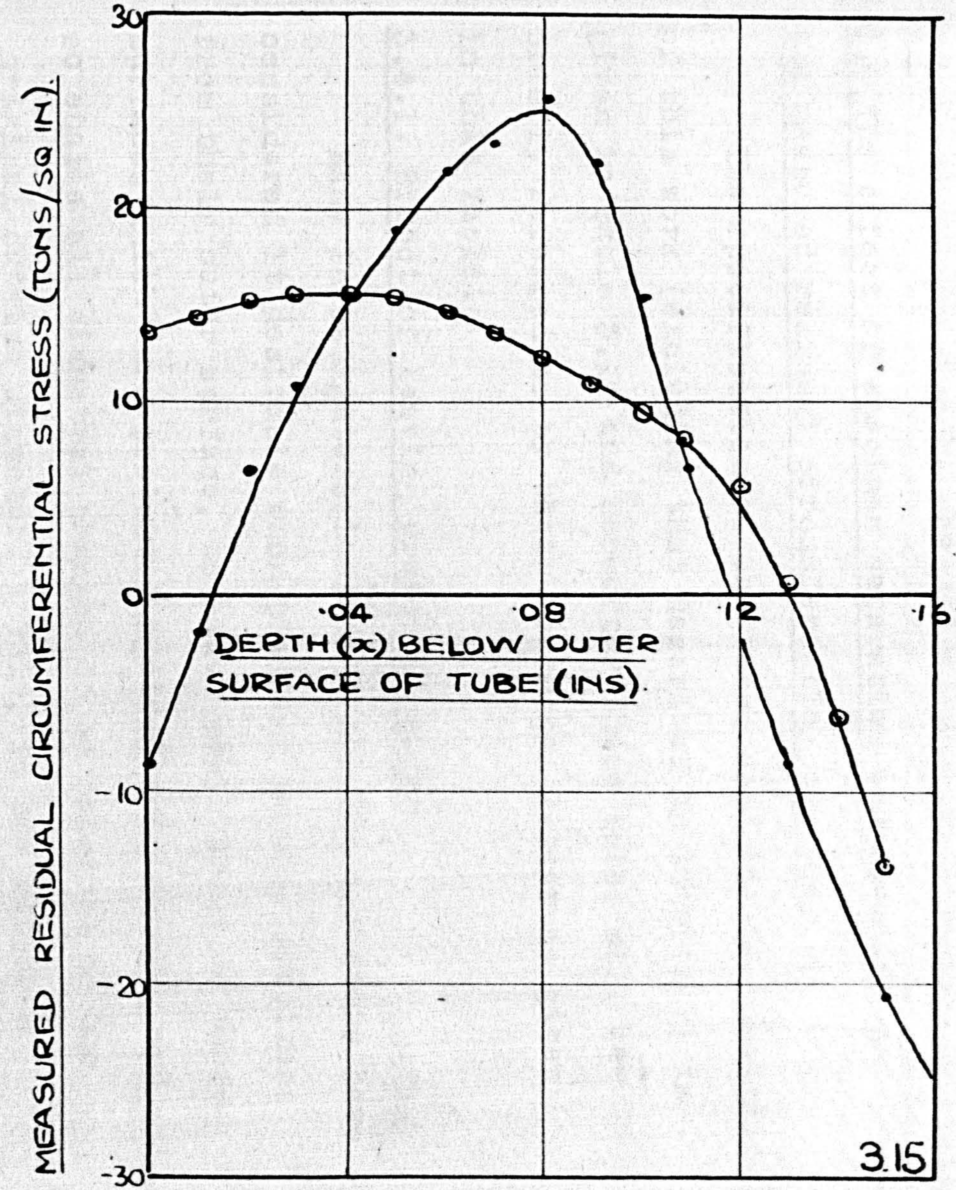




FIGS: 3.12 & 13. INFLUENCE OF LEAVING PROTECTED AREAS. EXPERIMENTAL RESULTS AND DERIVATIVES.

o o o o PROTECTED AREAS. —•—•—•—•— NO PROTECTED AREAS.





FIGS. 3.14 & 15 EFFECT OF LEAVING PROTECTED AREAS. STRESSES AND COMPONENT STRESSES

—•—•— NO PROTECTED AREAS —o—o—o— PROTECTED AREAS.

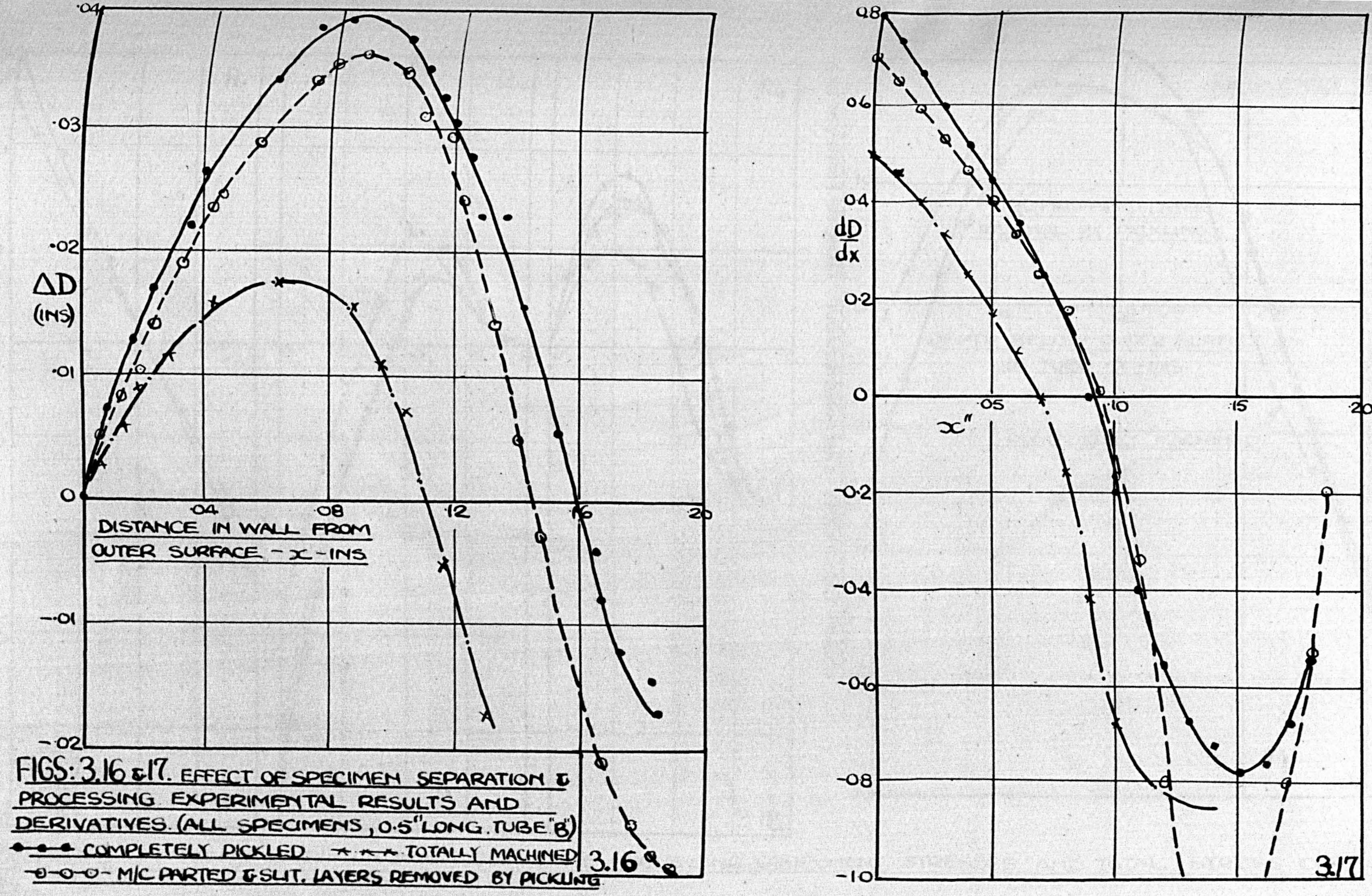
3.4 The effect of specimen separation and handling on the measured residual circumferential stresses in tubes.

The specification of the tubes used in the remainder of this work (excluding that described in Section 7) is given in Table 3.II and hereafter these tubes will be briefly referred to by their reference letter as given in this Table.

3.4.1. Effect of leaving protected areas.

Many investigators including Knights and Sachs have considered it desirable when pickling tubular specimens from their outer surface, to leave small areas on these surfaces which were protected against acid attack, and so enable all diametral changes to be referred to the original outside diameter. This, in the author's opinion, was completely unnecessary and could possibly result in serious errors in the derived results, particularly by restricting the measurement of the "sprung" diameter to two points and so reducing the accuracy of its measured value. (In all the work described herein, diametral measurements were made at 0.25 to 0.5 in. intervals along the length of the specimen in the plane normal to the one containing the slit.)

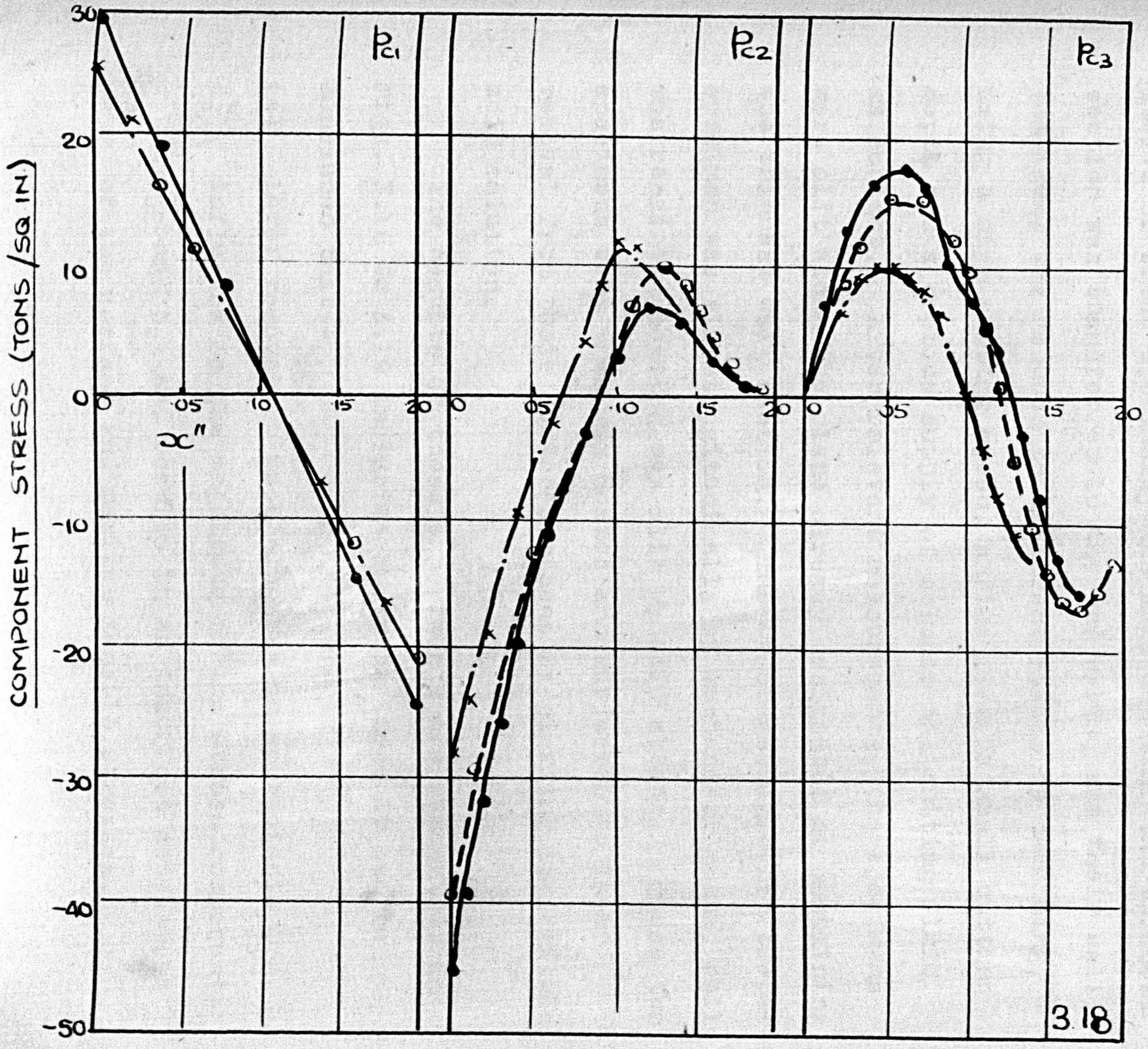
To illustrate this point, two four inch long specimens were prepared from tube A and after slitting, were subjected to layer removal from their outer surfaces by acid pickling. One of these specimens contained four small areas (roughly 0.375 in. long x 0.25 in. wide) of the original outer surface which were coated with wax and protected against acid attack. These areas were situated near each end of the specimen and were arranged in pairs at opposite ends of the tube diameter in the plane normal to the plane of slitting. The experimental results obtained, the derivatives and the stress component curves and the total circumferential stress distribution curves are shown in Figs. 3.12 to 3.15 inclusive, and it can be concluded that the leaving of protected areas

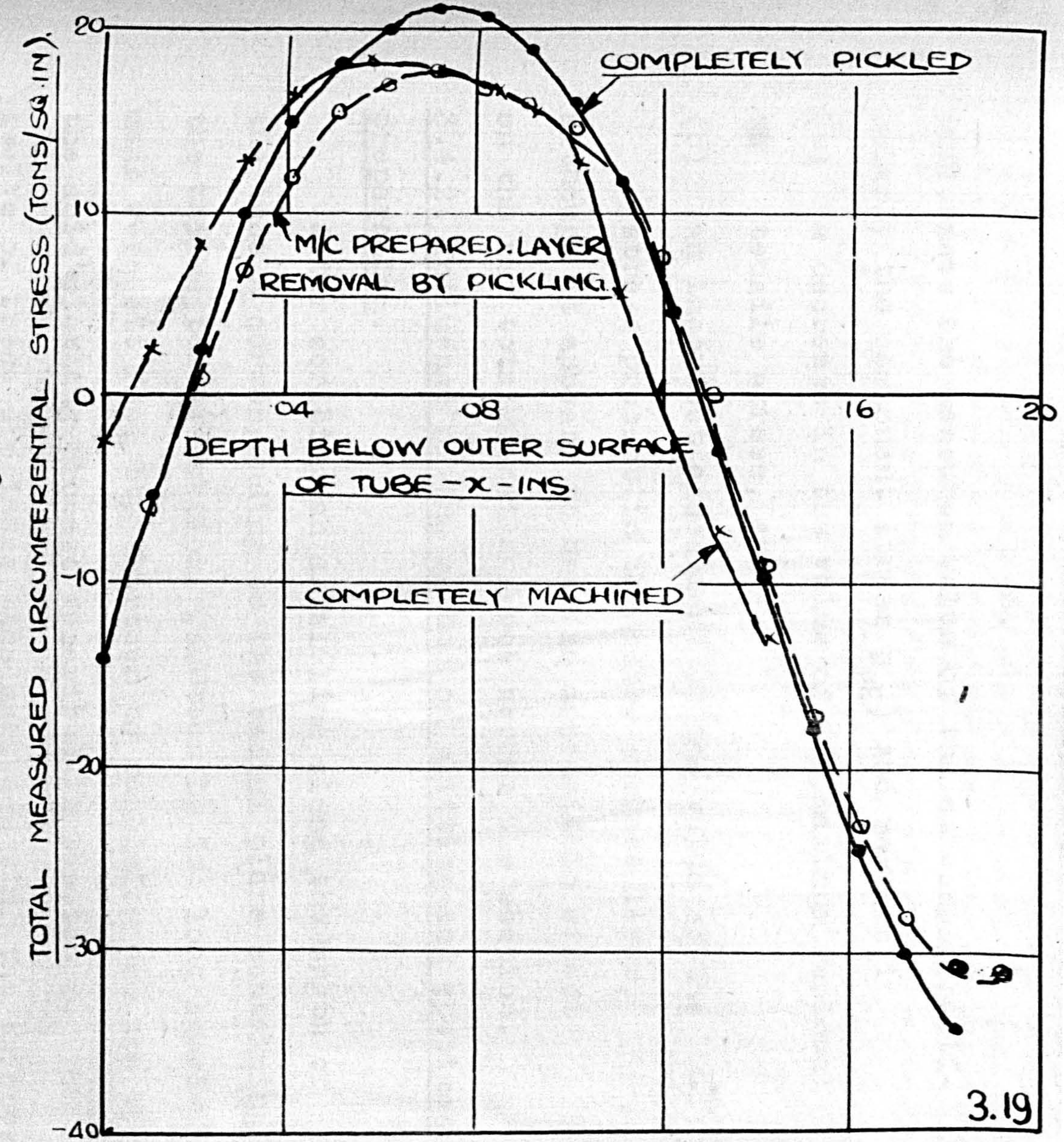


FIGS: 3.16 & 17. EFFECT OF SPECIMEN SEPARATION & PROCESSING EXPERIMENTAL RESULTS AND DERIVATIVES. (ALL SPECIMENS, 0·5" LONG. TUBE "B")

—●—●—● COMPLETELY PICKLED ×—×—× TOTALLY MACHINED

—o—o—o— M/C PARTED & SLIT. LAYERS REMOVED BY PICKLING





FIGS: 3.18 & 3.19. EFFECT OF SPECIMEN PREPARATION AND PROCESSING COMPONENT STRESSES AND TOTAL STRESS CURVES.

influences the measured value of the stresses (particularly the component stress p_{c2}) and results in

(a) a decrease in the value of the maximum residual tensile stress, and

(b) an incorrect assessment of the value of the stress present in the outer surface layer,

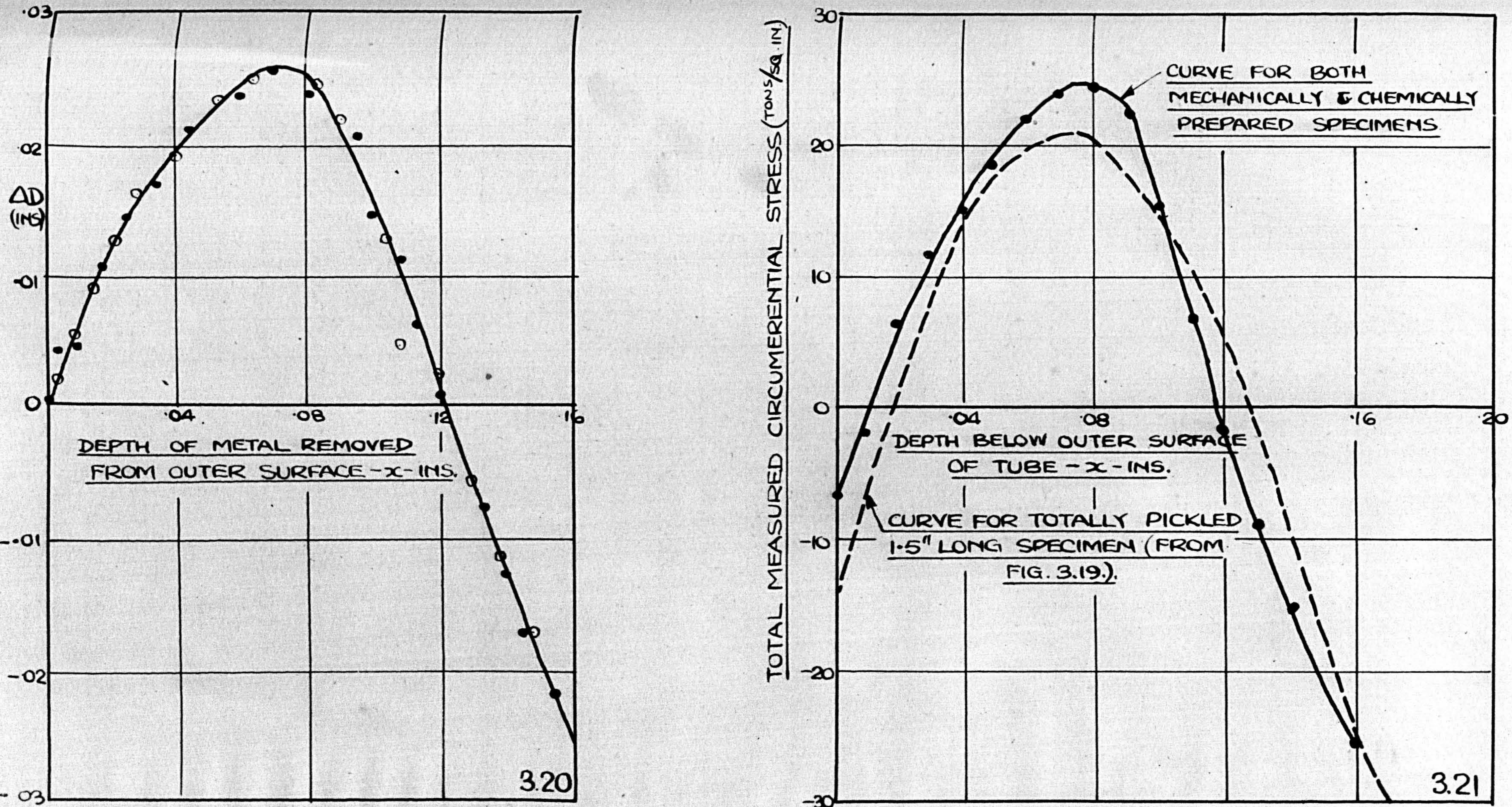
although it appears to have no appreciable effect on the net total tensile force across the section.

3.4.2. Influence of the method of slitting and specimen preparation.

After scribing an axial line along the length of the tube B corresponding to the radial plane containing the thinnest wall section, fourteen 1½ in. long specimens were cut from it, thirteen by machining and one by the acid pickling process described in Section 6. On twelve of these specimens, the mark denoting the thinnest section was transferred to the end walls, and their wall thickness was reduced to different values by machining, using a high cutting speed, a fine feed, a small cutting depth and a copious supply of soluble oil cutting fluid as coolant. After relevant measurements, the specimens were slit along the plane already designated as containing the thinnest wall section. The other mechanically prepared specimen was slit longitudinally by mechanical means, while the remaining section was slit by acid attack. These two specimens were then wax coated and subjected to layer removal from their outer surfaces by pickling in a nitric acid solution.

The results of these experiments are summarized in Figs. 3.16 to 3.19 inclusive and from these it can be concluded that:-

(a) layer removal by machining has an appreciable influence on the magnitude of the stress components p_{c2} and p_{c3} and results in an incorrect assessment of the surface and maximum tensile stresses.



FIGS: 3.20 & 21. EFFECT OF SPECIMEN PREPARATION AND PROCESSING EXP^{L} RESULTS AND STRESS CURVES. 4" LONG SPECIMENS - TUBE "B". LAYERS REMOVED BY PICKLING ●–●–● MECHANICALLY PREPARED AND SLIT. ○–○–○ CHEMICALLY PREPARED AND SLIT.

(b) With the short specimens employed, total mechanical preparation also results in an incorrect assessment of the magnitude of the maximum residual tensile stress.

To study this latter point further, two 4 in. long specimens were prepared and slit from tube B, one by machining and one by pickling. These two specimens had successive layers removed from their outer surfaces by pickling and the resultant variation of the change in diameter with the depth of the removed layer is shown in Fig. 3.20. As will be seen, there was no appreciable difference in the results obtained from the two specimens and in fact a single curve can be drawn through the experimental points obtained during the two tests. The resultant total stress distribution curve obtained from the mean experimental results is shown in Fig. 3.21, on which diagram the previously obtained curve for the $1\frac{1}{2}$ in. long totally pickled specimen is superimposed. The two curves shown in Fig. 3.21 are not strictly comparable as unfortunately the $1\frac{1}{2}$ in. long and the 4 in. long specimens were not slit along the same generator, but the agreement in the form of the curves is quite good, although the stress values show differences in magnitude at different points through the wall. (This point is discussed in further detail in the next section.)

As a result of the experiments summarized in Figs. 3.20 and 3.21 it is concluded that provided the specimens are sufficiently long, there is no need to use the very lengthy pickling process in their preparation - the simpler and quicker machining process followed by layer removal by pickling will yield the same results.

3.4.3. Influence of specimen length.

For studying the effect of the length of the specimen on the derived stress distribution curve, specimens taken from the tube A (Table 3.II) were employed.

Careful inspection and measurement of this tube revealed that, although it was very regular along its length and of remarkably constant outside diameter, its bore was eccentric and the wall thickness varied between 0.208 in. and 0.221 in. However, the thinnest section was always along the same radial plane and this was marked by a line scribed along the length of the tube before any parting or slitting operation was carried out. In subsequent handling, the tubular specimens parted off from this tube, were always slit along this scribed line, i.e. they were always slit along the plane containing the thinnest wall section.

Several specimens varying in axial length from 0.5 in. to 6.0 in. were separated from this tube and after relevant measurements had been made the specimens were slit longitudinally. Both these operations were carried out on a bandsaw and every possible precaution was taken to avoid damaging and overheating the specimens. After coating the requisite surfaces with "stopping-off" wax, removal of surface layers of material from these specimens was carried out using the acid pickling process previously described. Frequent measurements of wall thickness and outside diameter were made on each specimen during the pickling process.

The tests carried out can be briefly summarized as follows:-

Test 1. 0.5 in. long specimen. Outside layer removal.

Tests 2 - 7. 1 in. to 6 in. long specimens respectively (specimen length increasing in increments of 1 in.) Outside layer removal.

Tests 8 and 9. 3 in. and 5 in. long specimens respectively. Inside layer removal.

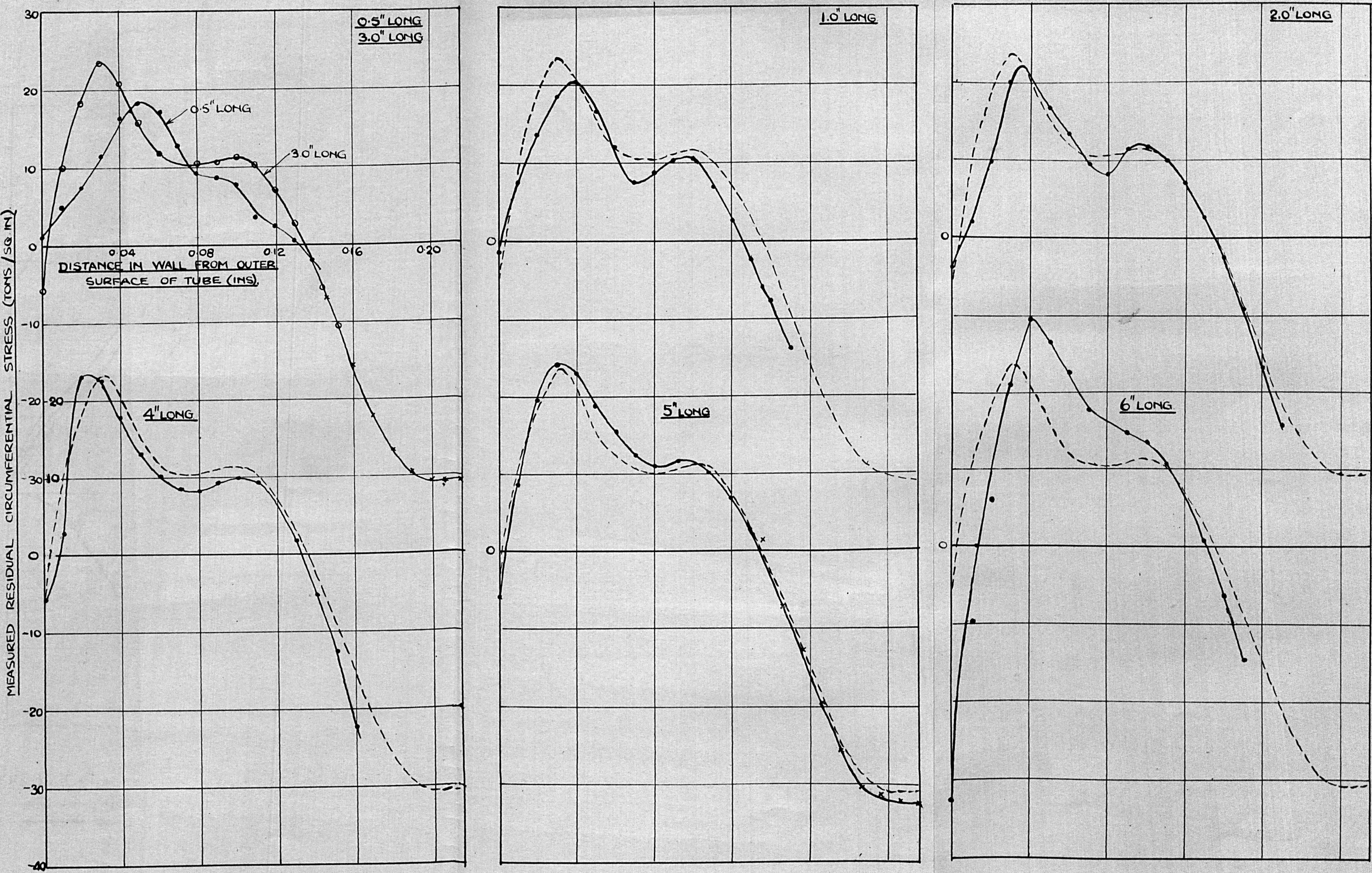


FIG: 3.22 EFFECT OF SPECIMEN LENGTH ON THE MEASURED RESIDUAL CIRCUMFERENTIAL STRESSES IN TUBES. (ALL SPECIMENS FROM ONE TUBE AND SLIT ALONG SAME GENERATOR).

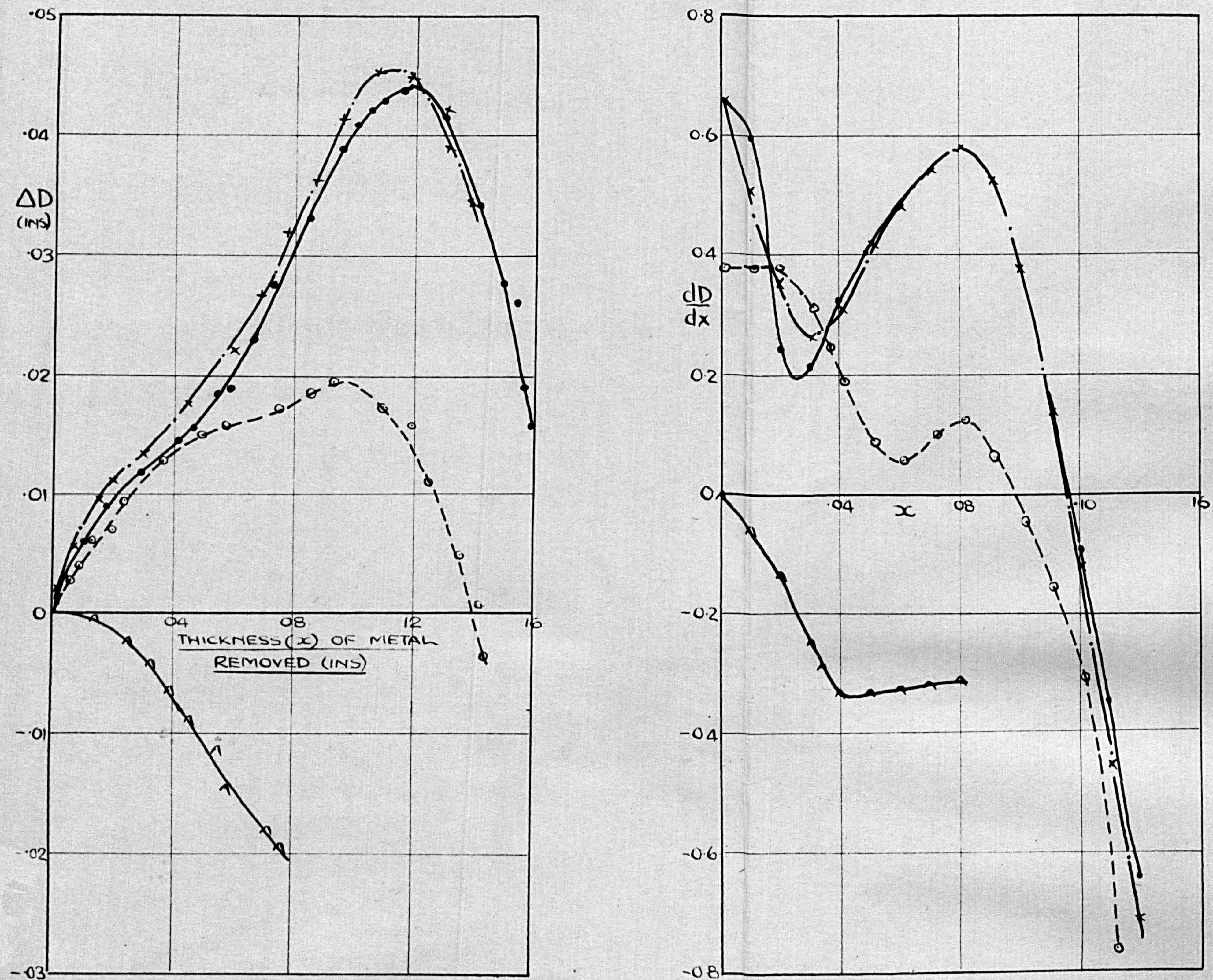


FIG: 3.23. EFFECT OF SPECIMEN LENGTH TYPICAL EXPERIMENTAL RESULTS

- o — o — o — 0·5" LONG
- ● — ● — ● 3·0" "
- x . x . x 5·0" LONG (OUTSIDE REMOVAL)
- Δ — Δ — Δ " " (INSIDE ").

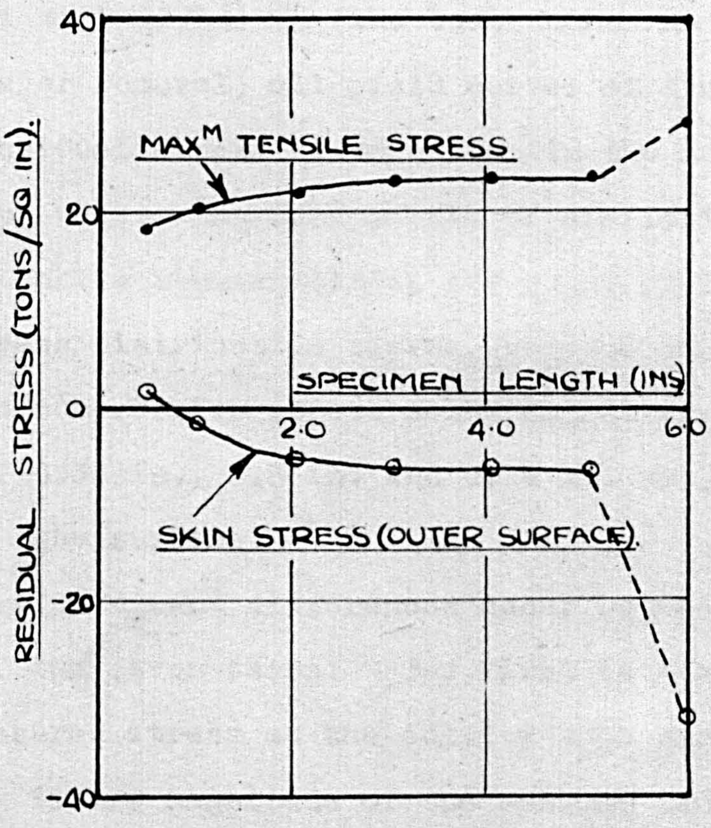


FIG: 3.24. EFFECT OF SPECIMEN LENGTH ON SURFACE AND MAXIMUM STRESSES.

The total measured residual stress distribution in the tube wall as determined from the results of each of these nine tests, is presented graphically in Fig. 3.22. In this diagram the chain dotted curve superimposed on each curve for comparative purposes, is the one derived from the results obtained with the 3 in. long specimen. Typical experimental results are illustrated in Fig. 3.23.

Reference to Fig. 3.22 will show that the results of tests 1 to 7 inclusive (the tests concerned with outside layer removal) all yield curves of similar type for the residual stress distribution in the outer half of the tube wall. Salient points of similarity are the peak tensile stress values, the point of inflection in the stress distribution curves, and the change from tensile to compressive stress occurring at approximate depths of 0.35 in., 0.8 in. and 0.14 in. respectively below the tube surface.

Two significant differences occur between the results of the seven tests. The first is the difference in the measured stress at the outside tube surface and the second is the magnitude of the maximum tensile stress. Both these stress values are plotted against specimen length in Fig. 3.24. It will be noticed that the maximum measured tensile stress value generally increases as the length of the specimen increases and that the measured surface stress increases algebraically as the specimen increases in length up to 5 in. The reverse effect is then observed with a very pronounced increase in compressive stress with further increase in specimen length up to 6 in.

Attention should be drawn to the apparent difference in type of the residual stress distribution curves obtained for the ½ in. and 6 in. long specimens as compared with those obtained for the remaining five specimens. It is believed that the curve obtained for the ½ in. long specimen is somewhat inconsistent and unreliable because of a springing effect observed during the acid pickling process of layer removal, which made the ring specimen take up the form of a split spring washer. A repeat test was made on a further 6 in. long specimen, together with specimens of three other lengths and in no case was the variation in the derived residual stress greater than 6% of the values shown plotted, and the change from tensile to compressive stress occurred at the same position in the tube section as in the previous test.

Further reference to Fig. 3.22 will show that the results of the two tests in which inside layer removal was considered gave similar results for the stress distribution in the inner half of the tube wall. The only apparent difference in these results is in the measured compression near the inside tube surface and even this difference is too small to warrant further consideration at the present stage. Comparison of the areas under the tensile and compressive stress curves obtained by the conjunction of the relevant residual stress distribution curves for inside and outside layer removal for the 3 in. and 5 in. long specimens, show that these areas are of equal value to within 4.4%. In view of the inherent errors in the analysis, agreement of this order can be taken as a general verification of the results in view of the fact that the mean hoop stress across the tube wall must be zero, and also as some check on the form of the distribution obtained.

It appears from the results summarized in Fig. 3.22 that tests made on specimens with different lengths cut from a particular tube will yield different stress distribution curves by the bending deflection methods of stress analysis. It is known, however, from repeat tests carried out in the course of this work (see Fig. 3.25) that specimens of identical length yield results which show no more than 5 or 6% variation in any of the stress components or in the resultant total residual stress at any point in the tube wall.

3.4.4. Influence of plane of slitting.

A tubular specimen, approximately 21 in. long, was sawn off from the 2.125 in. diameter parent tube fully specified in Table 3.II, care being taken initially to mark clearly on the specimen which end was nearest to the leading end of the parent tube during its manufacture. The thinnest wall section of this specimen, which occurred in the same radial plane along its whole length, was then marked by scribing a line along its outer surface. Nine equally spaced 2 in. long sections were then marked out along the tube and each section (except the two ends) was then further scribed with a longitudinal line along its outer surface, each line being displaced 45° around the tube in a clockwise direction from its near leading end neighbour.

These nine two inch long specimens were then separated from the tube and after relevant measurements had been made, they were slit longitudinally along the previously marked sections, the two end specimens of course being slit along the radial plane corresponding to the thinnest wall section. After coating the inner surfaces, slit edges and ends with "stopping-off" wax, layers of material were removed from the outer surface of the specimens using the acid pickling process previously described. Frequent measurement of wall thickness and outside diameter were made on each specimen during the pickling process and all the usual precautions regarding wax coating and drying, pickling temperatures, etc., were taken during the tests.

The total measured residual circumferential stress distribution in the wall of each specimen was then calculated from the experimental results using the modified Sachs and Espey analysis.

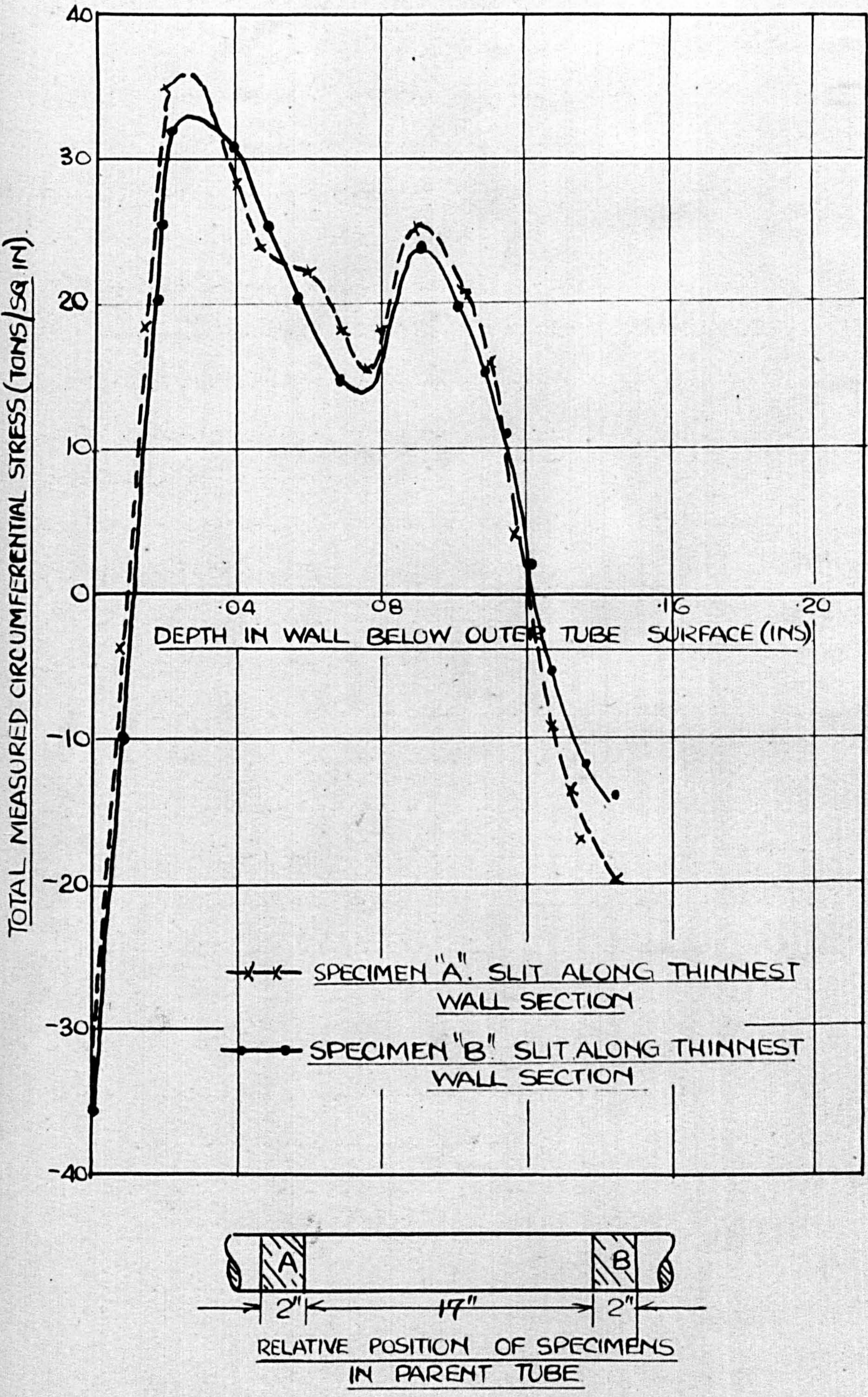
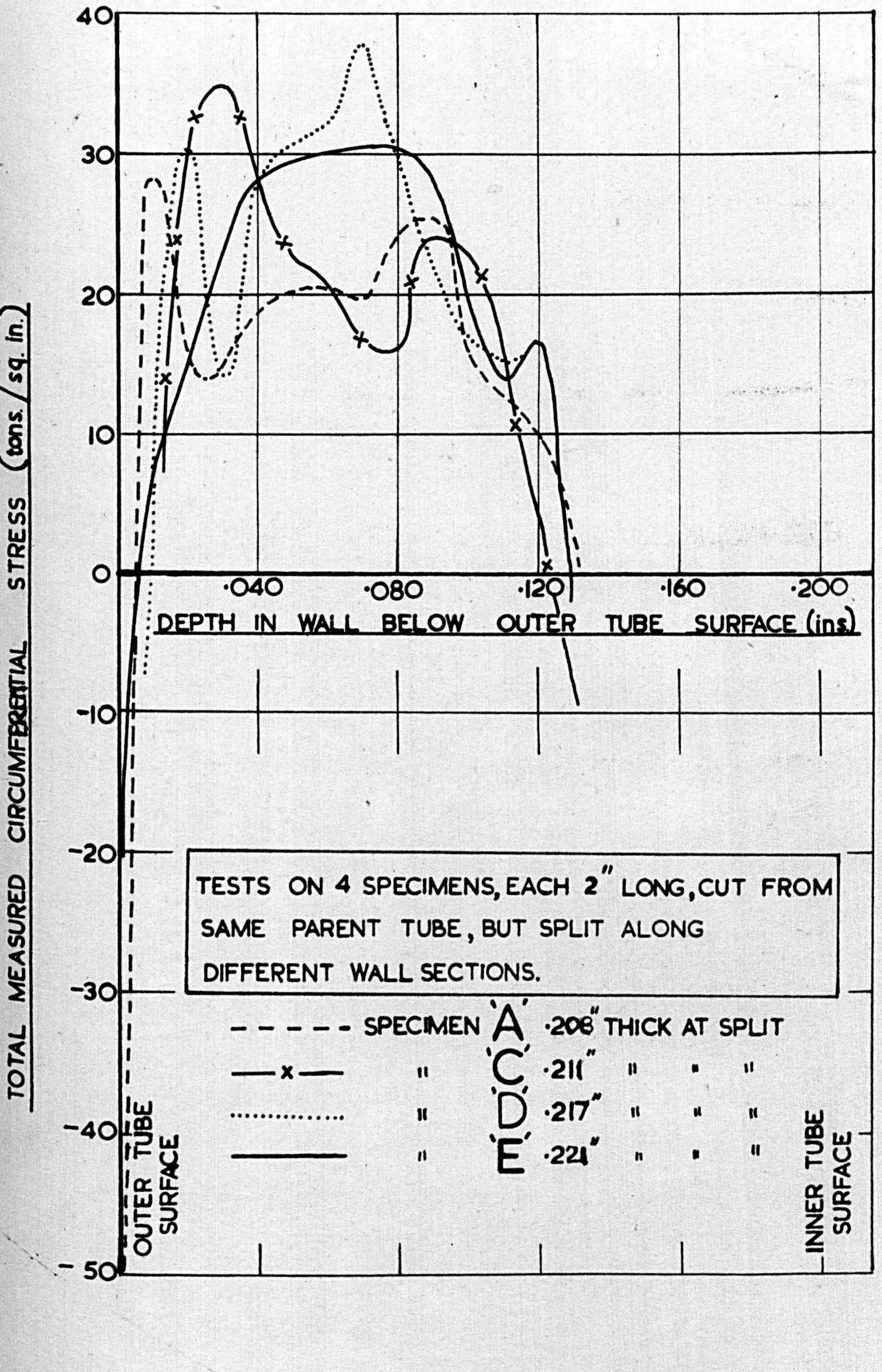


FIG. 3.25

CONSISTANCY OF BENDING DEFLECTION TEST RESULTS



TOTAL MEASURED CIRCUMFERENTIAL STRESS (tons./sq. in.)
40
30
20
10
0
-10
-20
-30
-40
-50
·040
·080
·120
·160
·200
DEPTH IN WALL BELOW OUTER TUBE SURFACE (ins.)
TESTS ON 4 SPECIMENS, EACH 2" LONG, CUT FROM SAME PARENT TUBE, BUT SPLIT ALONG DIFFERENT WALL SECTIONS.
SPECIMEN 'A' ·208" THICK AT SPLIT
" 'C' ·211" " " "
" 'D' ·217" " " "
" 'E' ·221" " " "
OUTER TUBE SURFACE
INNER TUBE SURFACE

FIG. 3.26

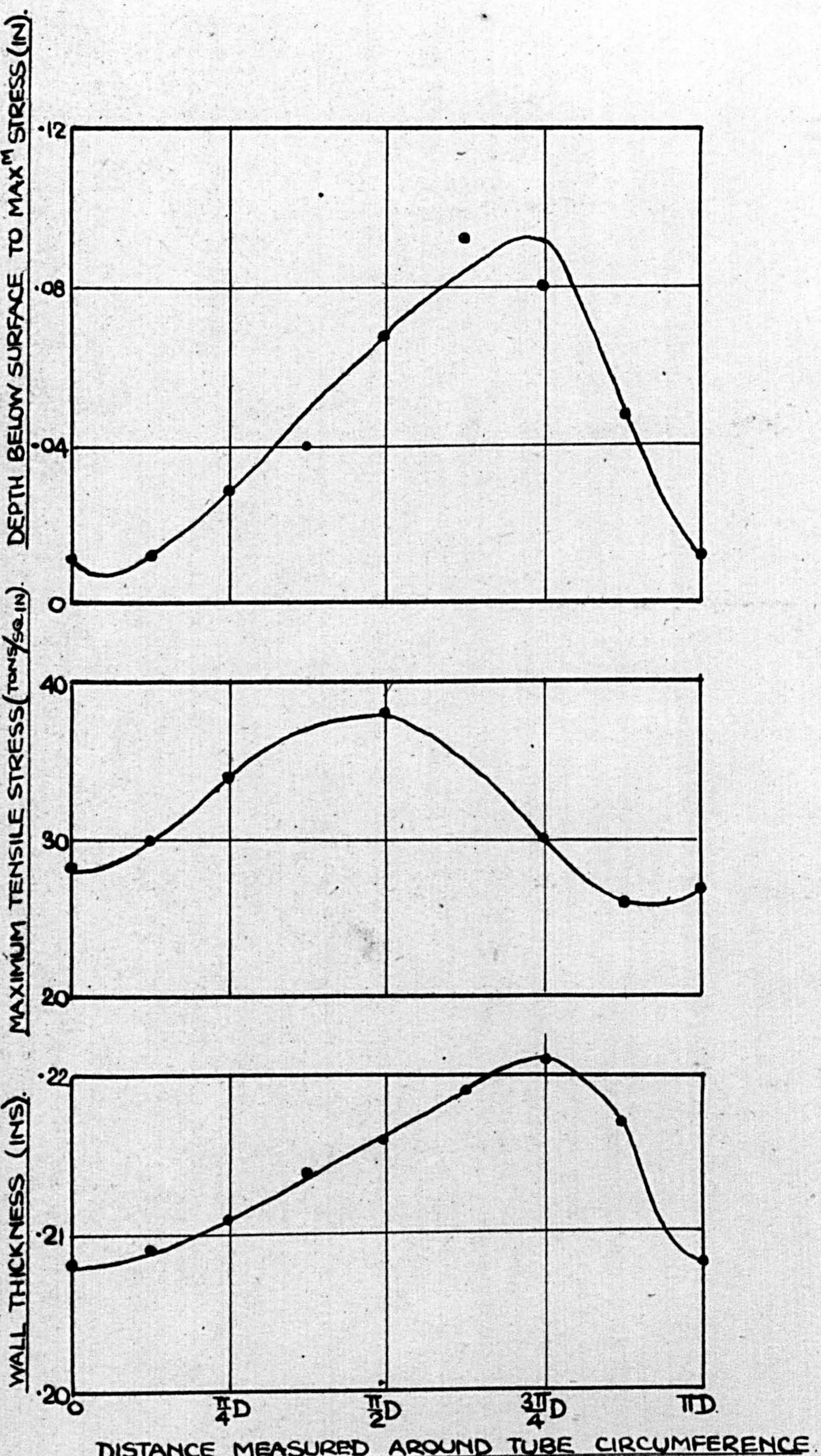


FIG: 3.27. EFFECT OF SLITTING PLANE ON SURFACE & MAXIMUM RESIDUAL STRESSES.

The stress distribution curves obtained from tests on the two end specimens, i.e. those split along the plane of thinnest wall section, are shown in Fig. 3.25. It will be observed that there is general agreement between the results of the two tests, both in regard to magnitude of stresses and form of distribution curve, and it can be concluded that, provided sufficient care is taken, it does seem possible to repeat experimental results on specimens of equal size which have been cut from one parent tube and slit along the same relative position.

Sample results of the tests on the nine specimens are shown as stress distribution curves in Fig. 3.26. The results of four tests only are given here for purposes of clarity and the four selected refer to specimens which were slit along planes at right angles to each other. A summary of the salient features of the nine tests is given in Fig. 3.27, which shows the depth below the outer surface of each tube to the point of maximum stress, the magnitude of the maximum stress and the variation of wall thickness all presented as a function of the plane of slitting as represented by the distance from the plane of thinnest wall section measured around the circumference of the tube.

As a result of the tests summarized in Fig. 3.26 it can be inferred that stress distributions obtained for specimens of equal size cut from the same parent tube but slit along different relative positions in the wall, while being similar in type, differ widely in magnitude. Furthermore, examination of Fig. 3.27 will show that there appears to be no relationship between the magnitude of the released stresses and the position of slitting the specimen relative to the variation in wall thickness of the parent tube, i.e. maximum or minimum stresses do not correspond to any point in the wall corresponding to maximum

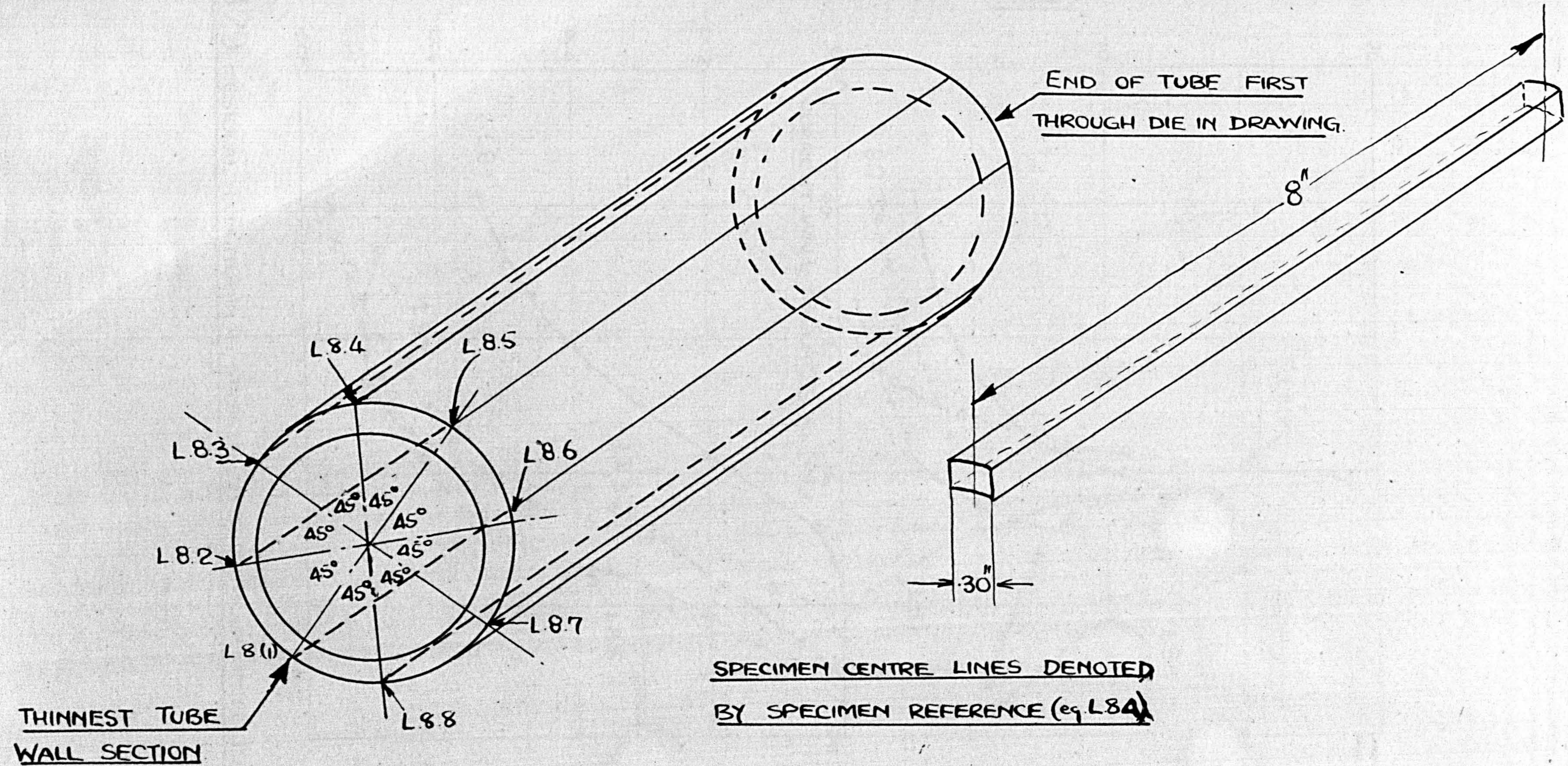


FIG. 3.28 LOCATION AND DIMENSIONS OF LONGITUDINAL SPECIMENS.

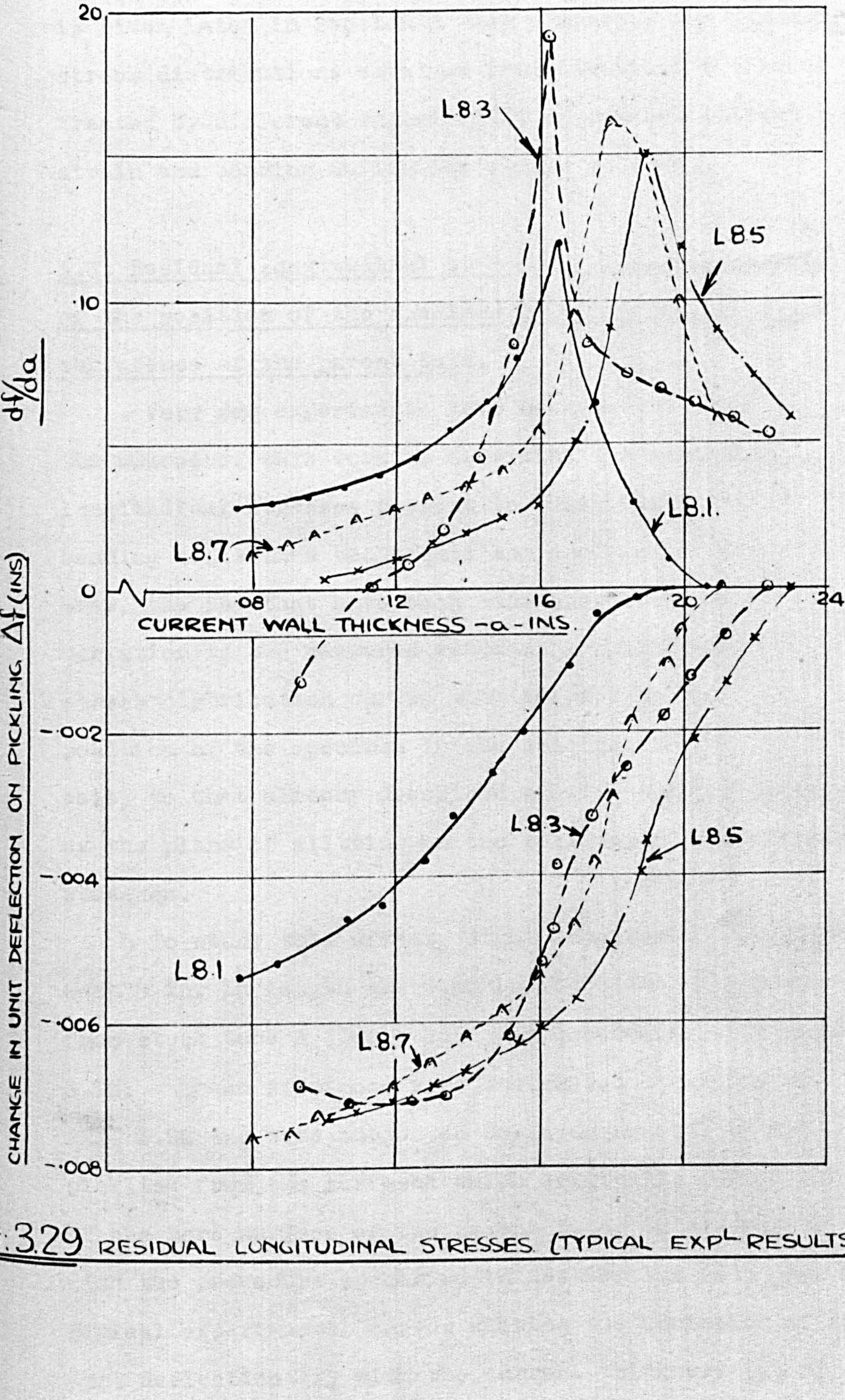


FIG. 3.29 RESIDUAL LONGITUDINAL STRESSES. (TYPICAL EXP^{L} RESULTS).

or minimum wall thicknesses, but to some point randomly oriented around the tube wall.

A further confirmation of the variation of the residual stress distribution with the plane of slitting is given later in Section 5 when comparing the residual stress distributions obtained from identical specimens treated by different experimental approaches (direct strain and bending deflection strain release).

3.5. Residual longitudinal stresses and the influence of the position of the specimen relative to the circumference of the parent tube.

Very few experiments have been carried out in the course of this work to determine the residual longitudinal stresses present in tubes using the bending deflection techniques and analyses. However, the few that have been made show a similar variation in the measured residual longitudinal stress distribution curves with respect to the position of the specimen in the original tube wall, to that already described showing the influence of the plane of slitting on the residual circumferential stresses.

To study this effect, eight longitudinal specimens, each 8 in. in length and 0.30 in. in width were prepared from stock tube A (Table 3.II) in accordance with Fig. 3.28. These specimens were designated according to Fig. 3.28 and were subjected to layer removal by acid pickling from the surfaces which originally formed part of the bore surface of the parent tube, in accordance with the procedure specified in Section 3.2 (see page 34). Typical experimental curves showing the variation of the unit deflection (f) with the current thickness (a) of the specimen are shown in Fig. 3.29 together with the variation of the derivatives df/da. The resultant stress

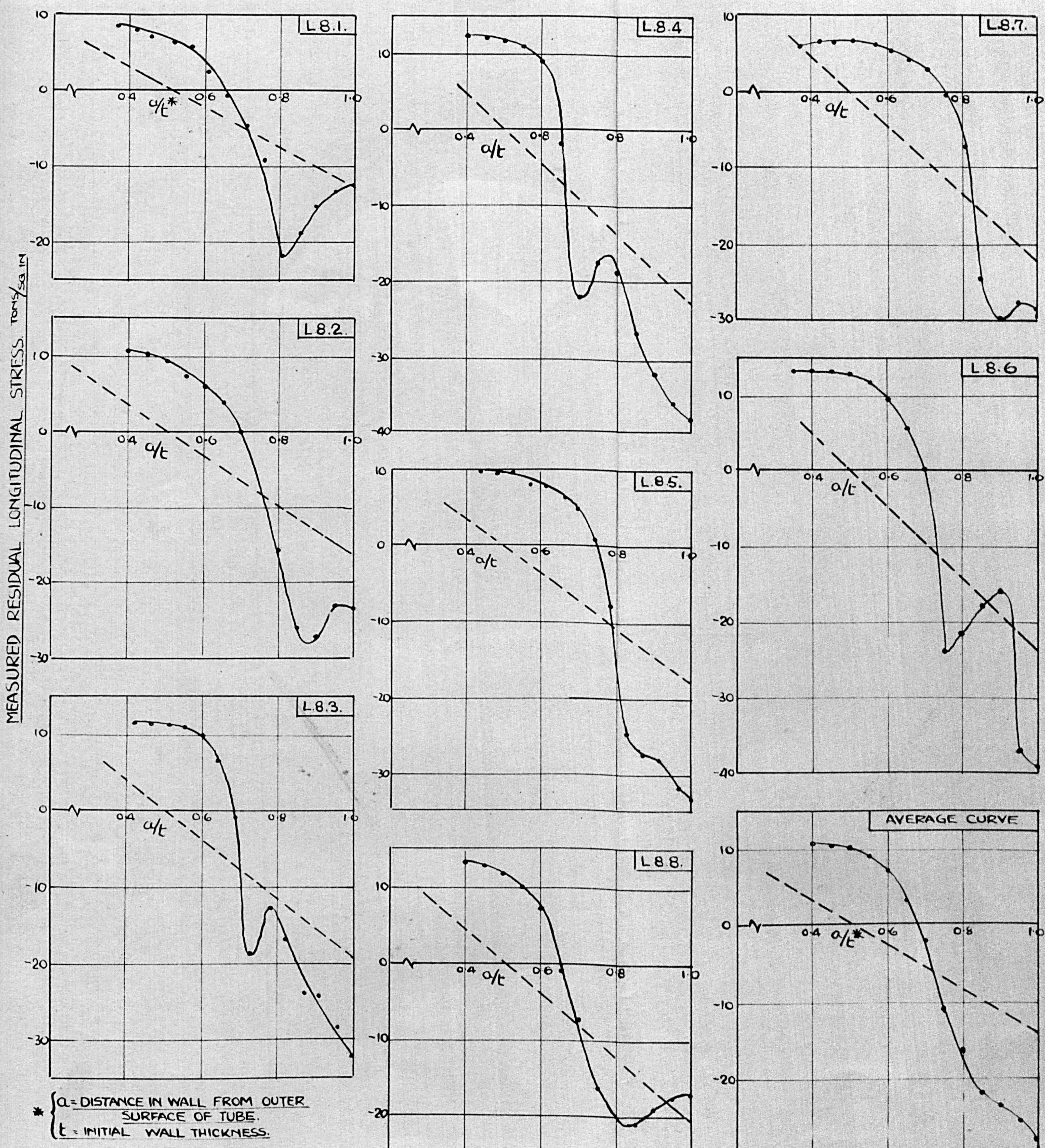


FIG: 3.30 RESIDUAL LONGITUDINAL STRESSES. INFLUENCE OF POSITION OF SPECIMEN IN TUBE WALL.

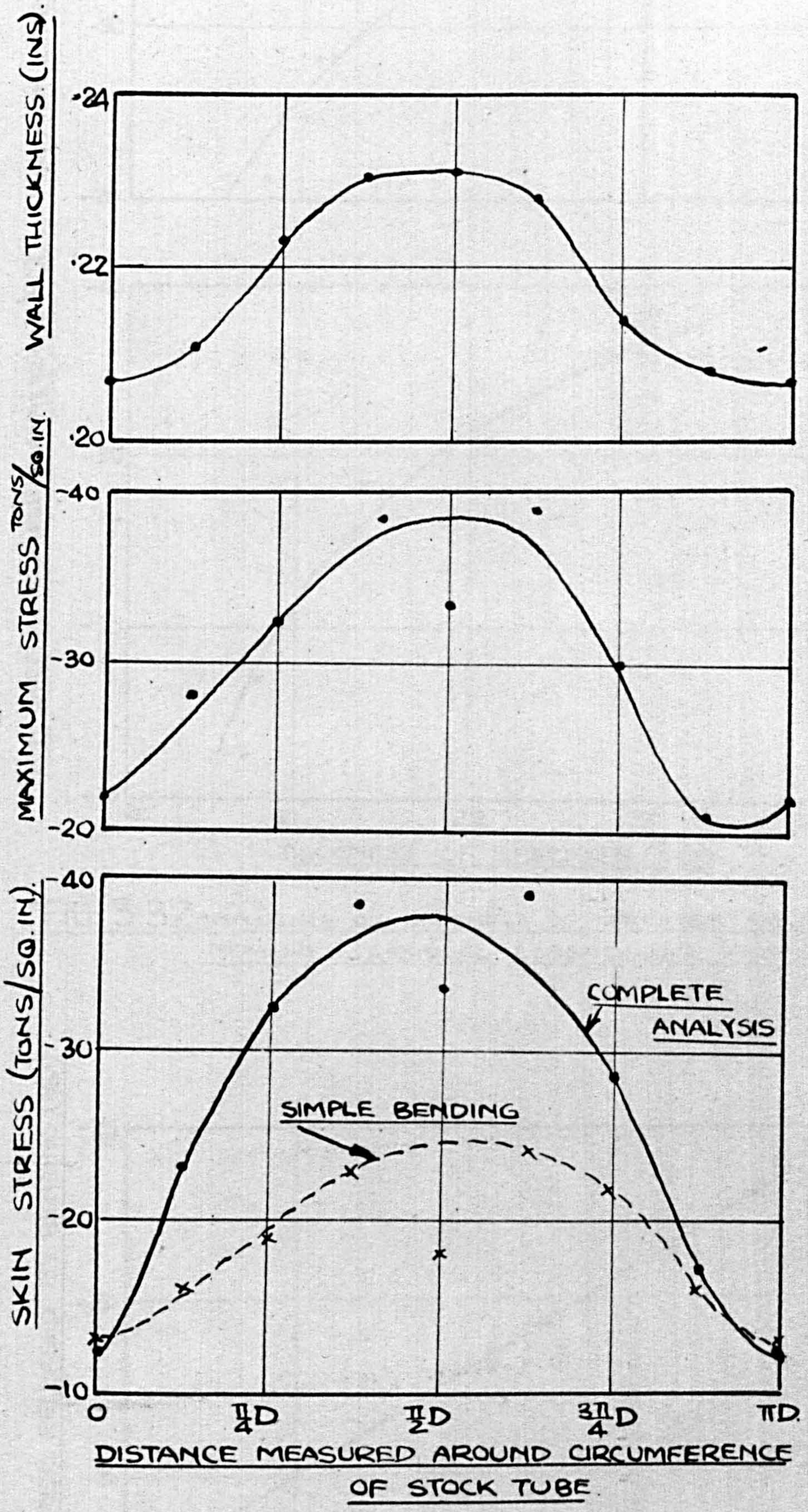


FIG. 3.31. RESIDUAL LONGITUDINAL STRESSES. SUMMARY OF TESTS SHOWING THE INFLUENCE OF THE PLANE OF SECTION.

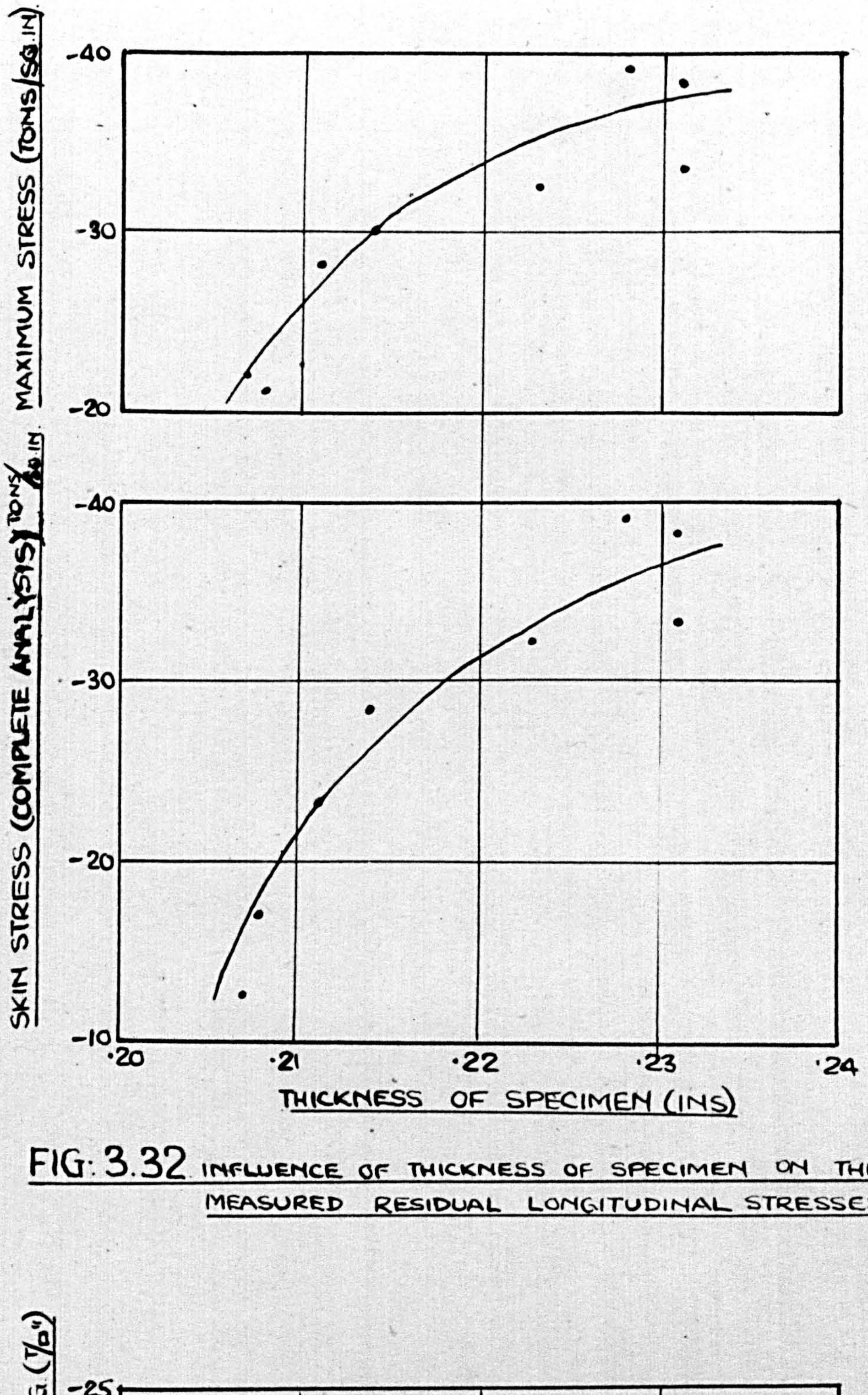


FIG: 3.32 INFLUENCE OF THICKNESS OF SPECIMEN ON THE MEASURED RESIDUAL LONGITUDINAL STRESSES

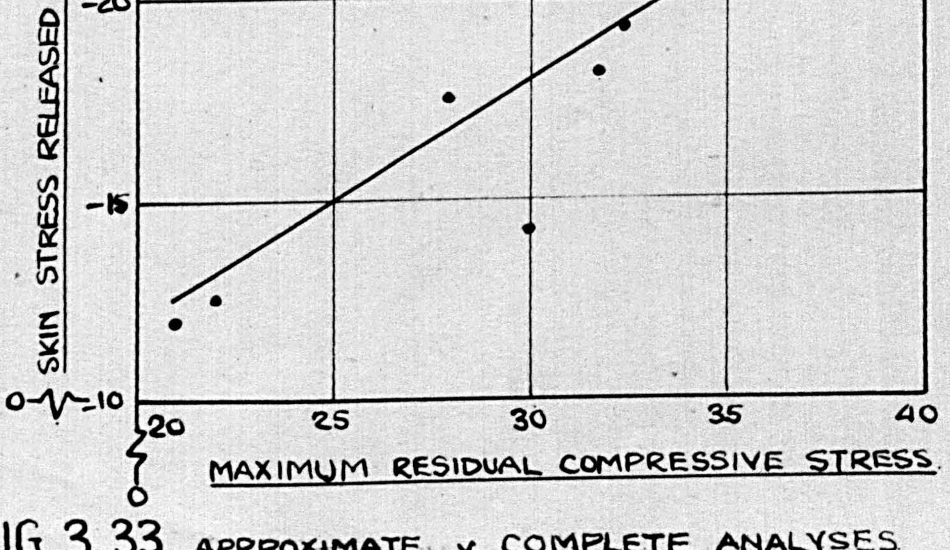


FIG. 3.33. APPROXIMATE v COMPLETE ANALYSES (RESIDUAL LONGITUDINAL STRESSES).

distribution curves for all the specimens as calculated from the experimental results using the Davidenkov analysis, are shown in Fig. 3.30, together with the superimposed curves (shown dotted) showing the bending stress released from each specimen during its initial separation from the stock tube. It will be noticed that while the sense of the measured stresses at all points in the wall was essentially the same for all the specimens, considerable differences in the magnitudes of the measured stresses were obtained. The differences occurring at salient points are shown in Fig. 3.31, from which it appears that both the maximum residual compressive stress and the maximum stress at the inner surface are both related to the thickest wall section and vice versa for the minimum stress values. These two latter observations are seen more clearly in Fig. 3.32 which shows the variation of the surface and maximum stress with the initial thickness of the specimen. From the results shown graphically in Fig. 3.33 it appears that a linear relationship may exist between the surface stress and the maximum compressive stress in different specimens taken from the same tube, but why this should be so is not fully understood.

4. THE APPLICATION OF THE SACHS BORING METHOD TO THE DETECTION AND MEASUREMENT OF RESIDUAL STRESSES IN COLD DRAWN TUBES.

4.1. Introduction.

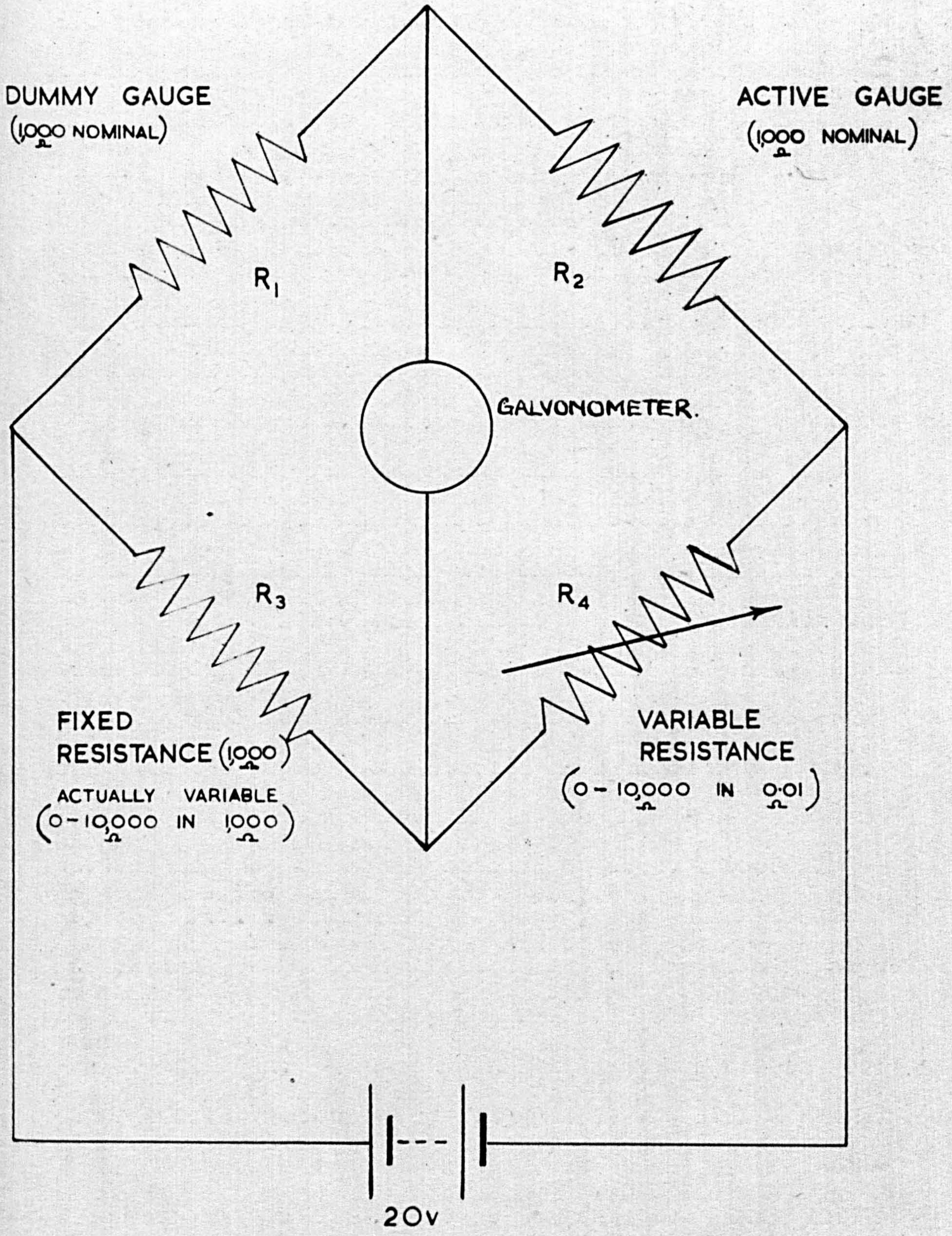
Methods of determining the residual stress distribution in cold drawn tubes by direct strain analyses, i.e. analyses based on the direct release and measurement of internal strains, are well known and have been very widely used by many investigators. The best known and probably most useful of such methods, is the Sachs' boring method (see Appendix 5), in which successive layers of material are removed from either the inner or the outer cylindrical surfaces of a tubular specimen and the consequent length and diametral changes measured at each stage. When applying this method workers have always been faced with the problems of apparatus sufficiantly sensitive to measure the extremely small strains consequent upon such a procedure and with means of layer removal which do not induce additional stresses into the specimen under test.

4.2. Development of a suitable experimental techniques and procedure.

After careful consideration it was decided to develop the Sachs' boring method using electric resistance strain gauges in conjunction with a simple Wheatstone bridge circuit and a sensitive mirror galvanometer for measurement purposes, and an acid pickling process of layer removal similar to that used in the previously described work on bending deflection methods of stress analysis.

Neither the use of strain gauges nor the application of the acid pickling process were new approaches since both had been previously used by other investigators. The application of an acid pickling process to the removal of layers of material from specimens fitted with strain gauges was, however, an innovation, and in itself presented many difficulties. Apart from the need for protection of surfaces which were not to be subjected to acid attack, there was also the necessity of protecting the strain gauges and their leads from being affected by acid. However, after many exploratory tests, many of which gave absolutely negative results, all the difficulties were in time surmounted and a satisfactory test procedure was evolved as outlined below. (A full account of these development tests was given by Whiteley, 1953).

A tubular specimen of the required length was carefully parted off from a parent tube and at least two strain gauges with high length/width ratios were secured to it in both longitudinal and circumferential directions on the outer cylindrical surface. Every precaution was taken during the fitting of these strain gauges to ensure that they were uniformly secured over their entire area and that they were not distorted or damaged in any way. After leaving the specimen in a warm dry atmosphere for several days to allow the strain gauges to thoroughly dry out, P.V.C. coated copper wires, each about twelve inches long, were carefully soldered to the strain gauges which were then completely covered over with a compound rubber solution. After again leaving the specimen for several days in a warm dry atmosphere, the strain gauge leads were bent at points away from the soldered connections and all brought together to leave the tube parallel to its axis. A thin sheet of pure rubber was then lightly stretched and wrapped around the outer surface of the specimen so that the latter projected about $\frac{1}{4}$ in. at each end of the rubber. The strain gauge leads were then inserted in rubber tubing which was slit open at one end to allow it to overlap the sheet rubber protection, and the whole was then secured in position with rubber solution. After drying, the ends of the specimen and



WHEATSTONE BRIDGE STRAIN GAUGE CIRCUIT

FIG. 4.1.

the unprotected outer surface were painted with a solution of "stopping-off" wax in trichlorethylene as used in previous experiments and then left overnight to allow the wax to harden thoroughly.

While securing the strain gauges to the test specimen, a matched strain gauge was carefully secured to the outer surface of an identical tube to fulfil the role of "dummy" gauge and temperature compensator. This gauge was also provided with leads of 12 in. approximate length and protected with compound rubber solution against the vagaries of atmospheric conditions. A wooden box with dimensions somewhat larger than those necessary to contain both tubes, was made and lined with cotton wool.

When the "dummy" and specimen tubes were ready for test, they were placed side by side in the box and their strain gauge leads were connected into the Wheatstone bridge circuit shown in Fig. 4.1. After allowing sufficient time for the two tubes to attain the same temperature, the initial resistance balance of the bridge for each of the gauges on the test specimen was obtained. The latter specimen was then taken out of the box and immersed in a bath of a 15% solution of nitric acid maintained at a temperature of 110 $\pm$ 10° F. The pickling process was allowed to proceed until a layer of approximately 0.005 in. thick had been removed from the inner cylindrical surface (the unprotected surface) of the specimen. The latter was then removed from the acid, washed thoroughly in cold water, had its wall thickness carefully measured and then returned to the box by the side of the temperature compensating tube. After allowing sufficient time for the two tubes to attain the same temperature, the bridge balance was obtained for each of the gauges on the active (test)

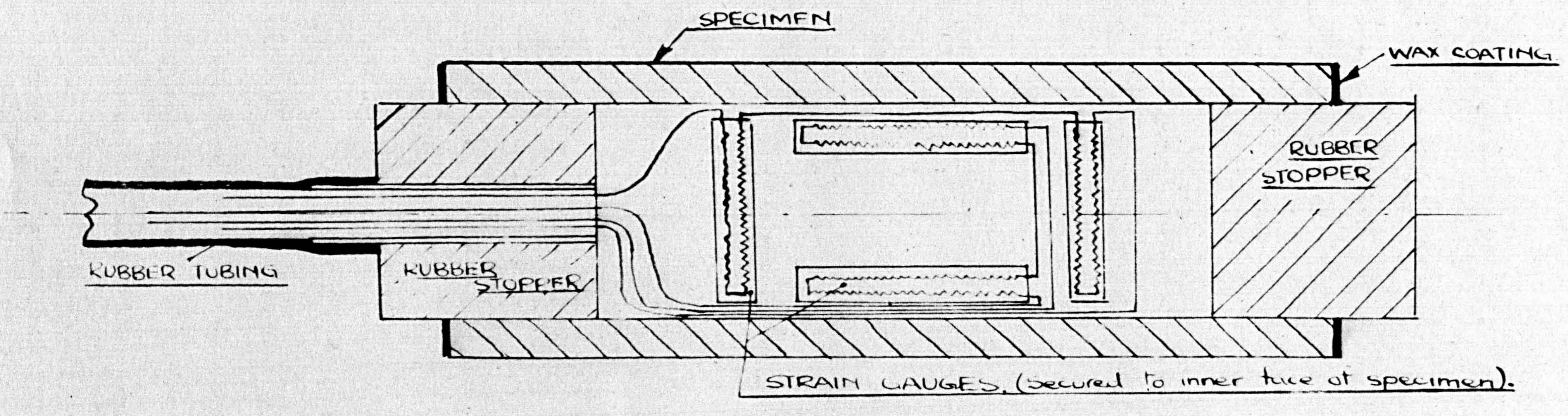


FIG: 4.2. PROTECTION OF SPECIMEN AND STRAIN GAUGES FOR OUTSIDE LAYER REMOVAL BY PICKLING. (DIAGRAM ALSO SHOWS SERIES CONNECTIONS TO STRAIN GAUGES).

specimen. (The period necessary for the two specimens to assume the same temperature was usually in the order of 24 hours.) The pickling and measuring process was then continuously repeated until the wall thickness of the specimen had been reduced to some 30-40% of its original value. The residual circumferential and longitudinal stress distributions were then calculated from the measured quantities using the Sachs' analysis detailed in Appendix 5.

Although the maker's gauge factor was used when determining the magnitude of the strains from the bridge balancing resistance, this was not accepted without a preliminary confirmatory test. (Details of this test, together with the relevant information about the strain gauges used in the investigation, are given in Appendix 6).

At a later stage of the work it became necessary to develop the application of the method to enable external layer removal to be carried out. This simplified the problem of protecting the strain gauges but increased the difficulties of strain gauge mounting, since these had now to be fitted to the inner (bore) surface. This has, however, been accomplished in the course of this work on tubes with minimum bore diameters of approximately 2 in. by adopting normal strain gauge fitting techniques. To secure strain gauges to the inner cylindrical surface of tubes with a bore diameter of less than 2 in. may prove somewhat more difficult and has not been attempted at this stage. With internally fitted strain gauges protection from acid attack was accomplished by the use of rubber tubing and rubber stoppers as shown in Fig. 4.2. The end faces of the tubular specimen were protected in the usual way by painting with a solution of stopping-off wax in trichlorethylene.

SACHS BORING METHOD

4.0" Long Specimen - Tube "A" (Table 3.II).

ψ and θ against bore area

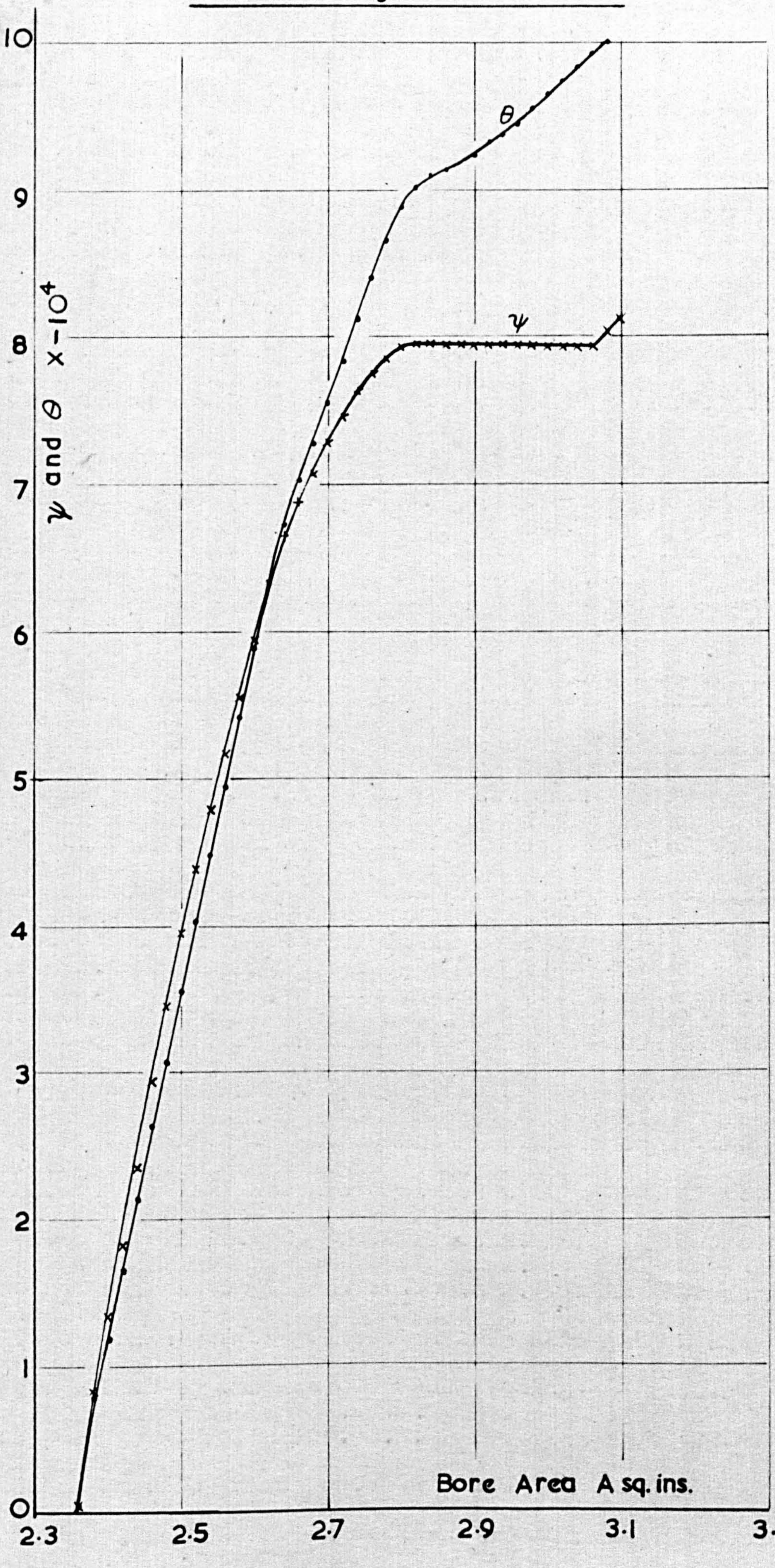


FIG: 4.3.

SACHS BORING METHOD

4" Long Specimen (Tube "A" - Table 3.II)

$\frac{d\psi}{dA}$ and $\frac{d\theta}{dA}$ against bore area

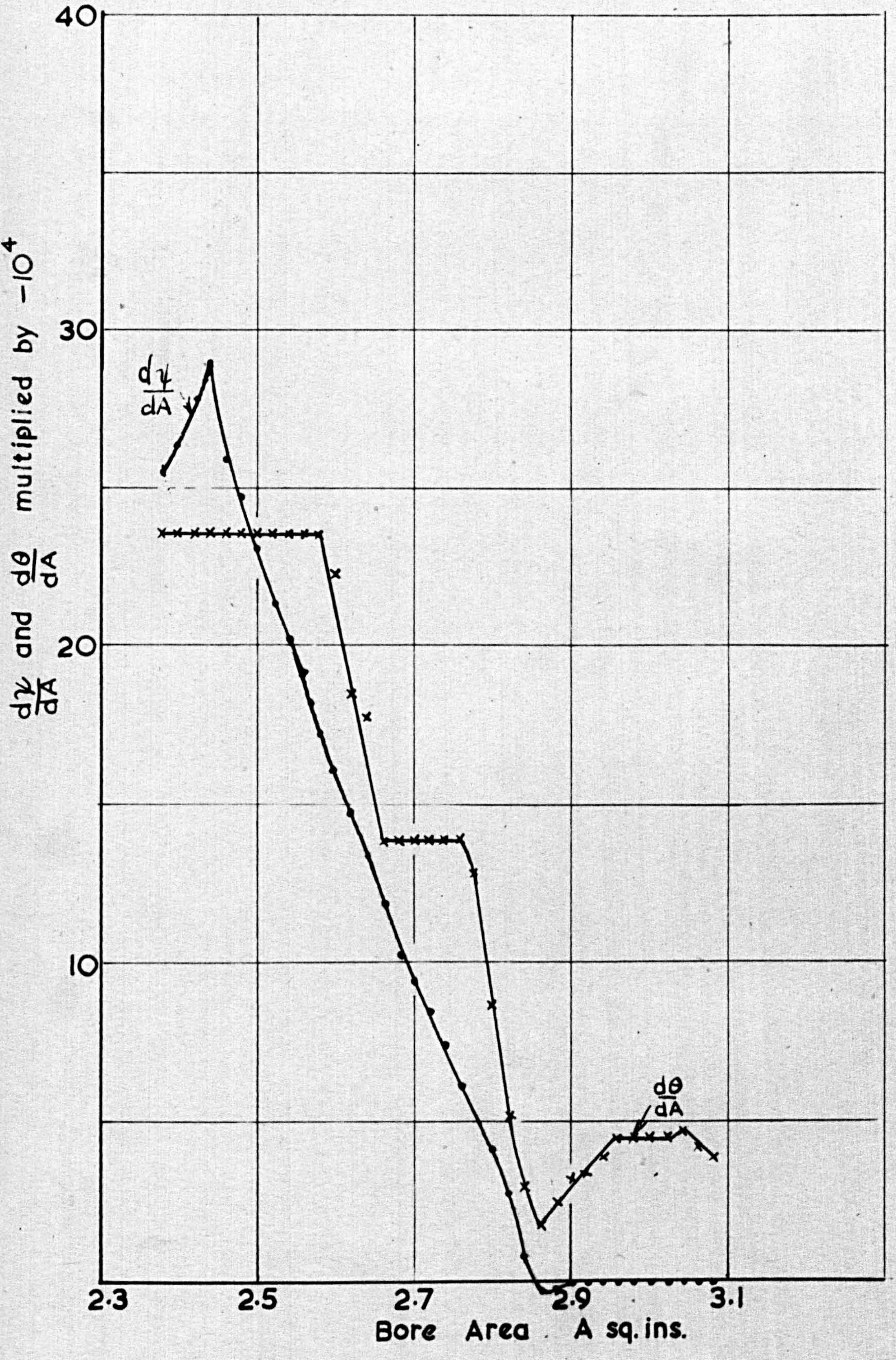


FIG: 4.4.

Sachs Boring Method

4.0" Long Specimen - Tube "A" - Table 3.II.

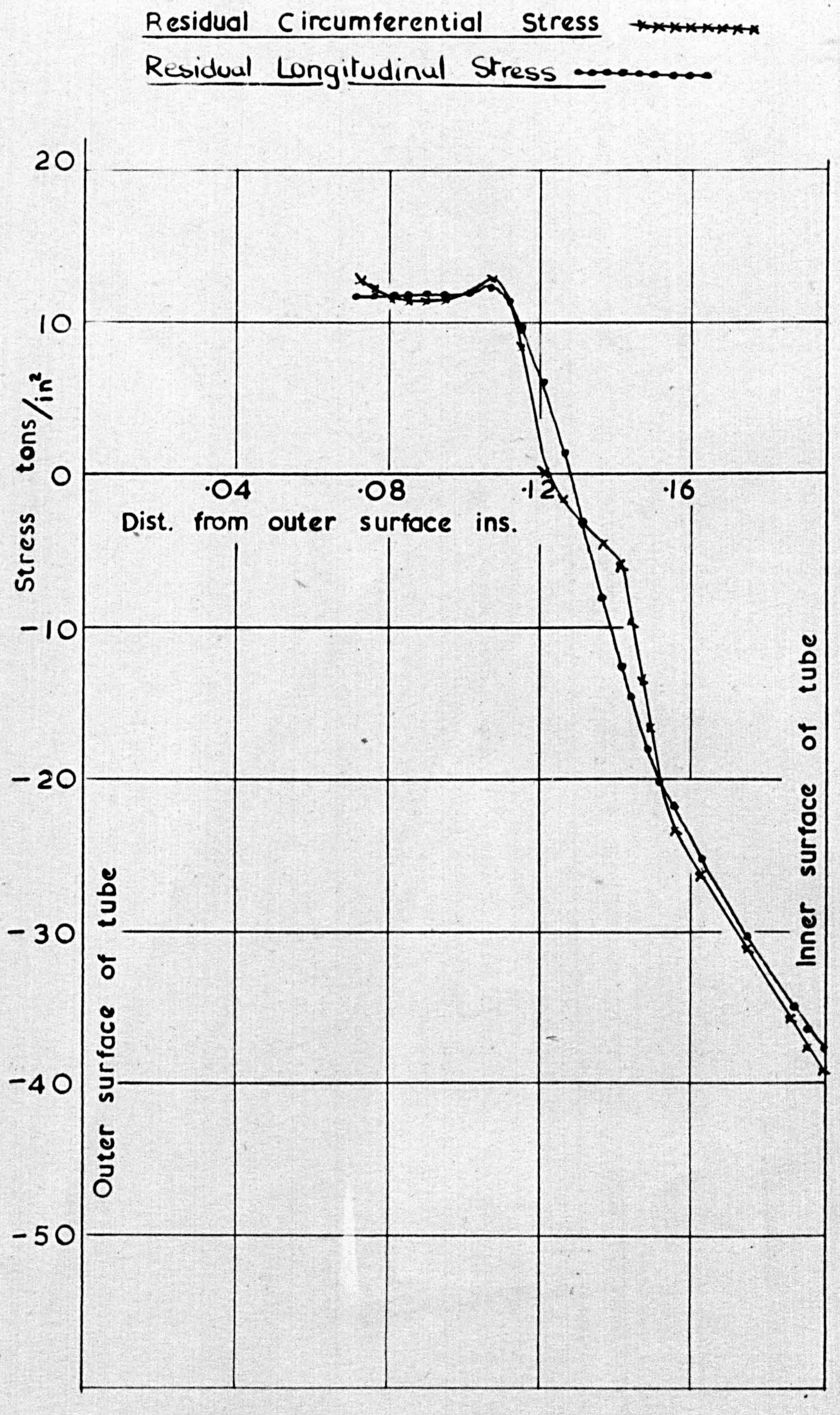


FIG: 4.5.

4.3. Residual stress distributions determined by the use of the Sachs' boring method and the influence of specimen length and method of processing.

4.3.1. Preliminary test.

As a preliminary test to finally establish the experimental techniques, a four inch length was carefully parted off from tube A, (Table 3.II) and fitted with strain gauges and protective coating on its outer cylindrical surface in accordance with the procedure given in Section 4.2. This specimen was then subjected to layer removal from its inner surface by acid pickling and measurements of the resultant longitudinal and diametral changes were made at each stage of the metal removal process. The experimental results obtained are shown in Fig. 4.3 as curves showing the variation of the parameters θ and ψ with the current bore area,

where $\theta = \delta + \sigma.\lambda$

and $\psi = \lambda + \sigma.\delta$

δ being the tangential (hoop) strain and λ the longitudinal strain measured at the outer surface of the tubular specimen at each stage of the metal removal process and σ = Poisson's ratio = 0.3. From the relevant values of θ and ψ and the derivatives $\frac{d\theta}{dA}$ and $\frac{d\psi}{dA}$ (shown in Fig. 4.4), the total residual circumferential and longitudinal stresses present at any point in the tube wall were determined using the Sachs analysis. The resultant stress distribution curves are shown in Fig. 4.5.

4.3.2. Effect of specimen length on the magnitude and distribution of the residual stresses derived by the Sachs' boring method.

To study whether the length of a specimen had any influence on the calculated stresses (as with bending deflection methods) four tubes with different lengths ranging from 2.5 in. to 6 in. were carefully parted

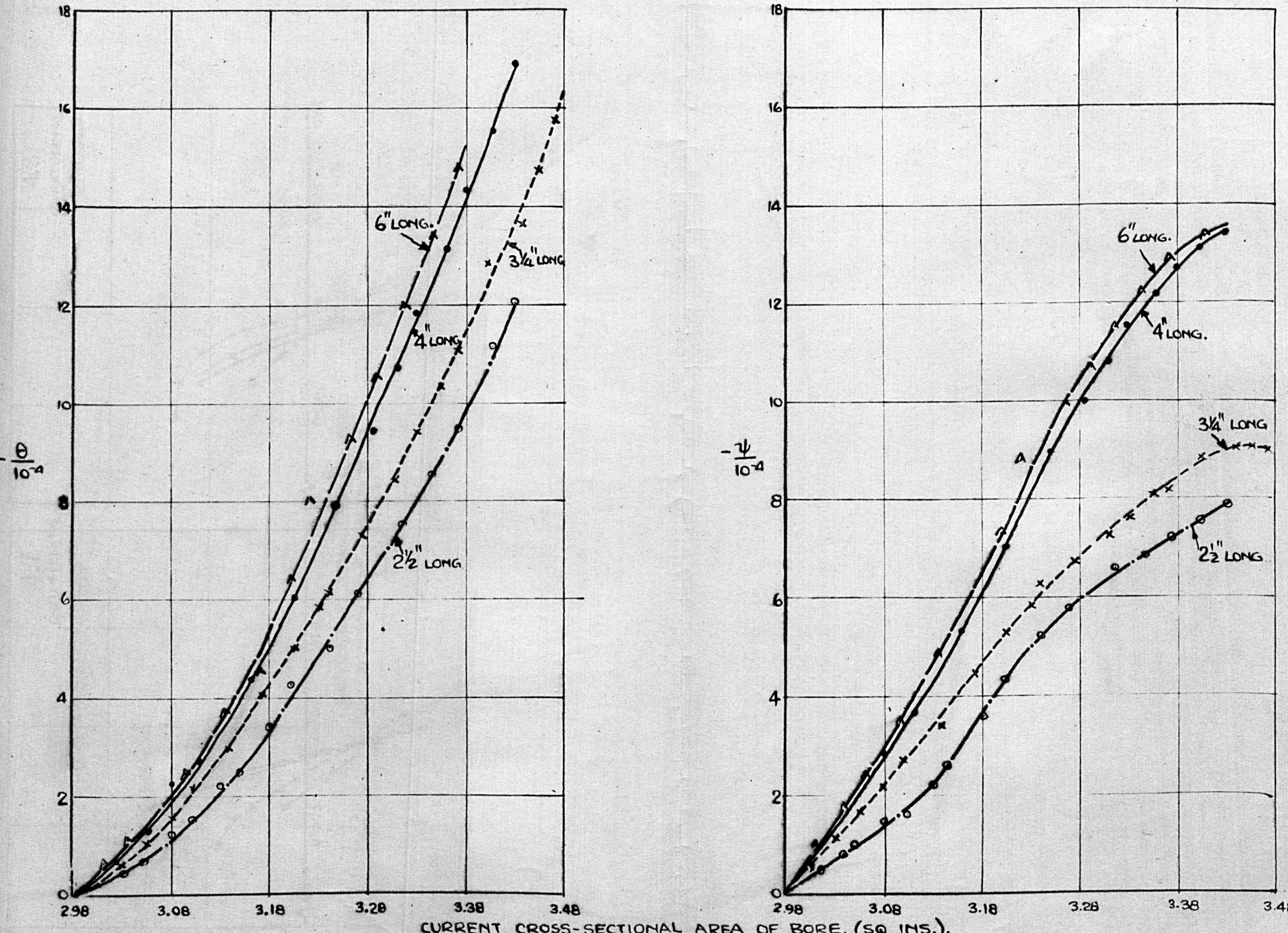
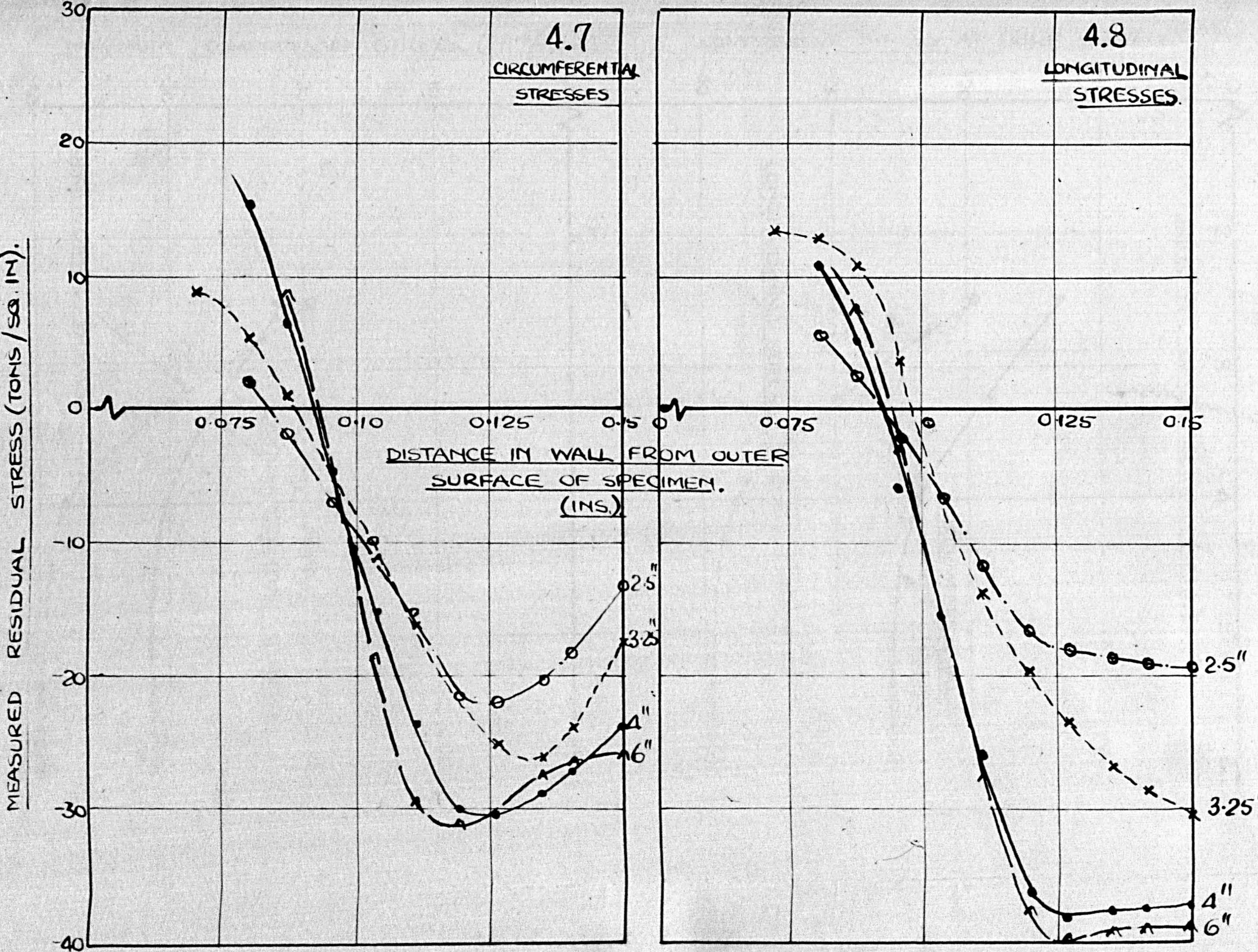


FIG: 4.6. SACHS BORING TESTS. INFLUENCE OF SPECIMEN LENGTH. EXPERIMENTAL RESULTS.



FIGS: 4.7 AND 8. SACHS BORING TESTS. INFLUENCE OF SPECIMEN LENGTH STRESS DISTRIBUTION CURVES.

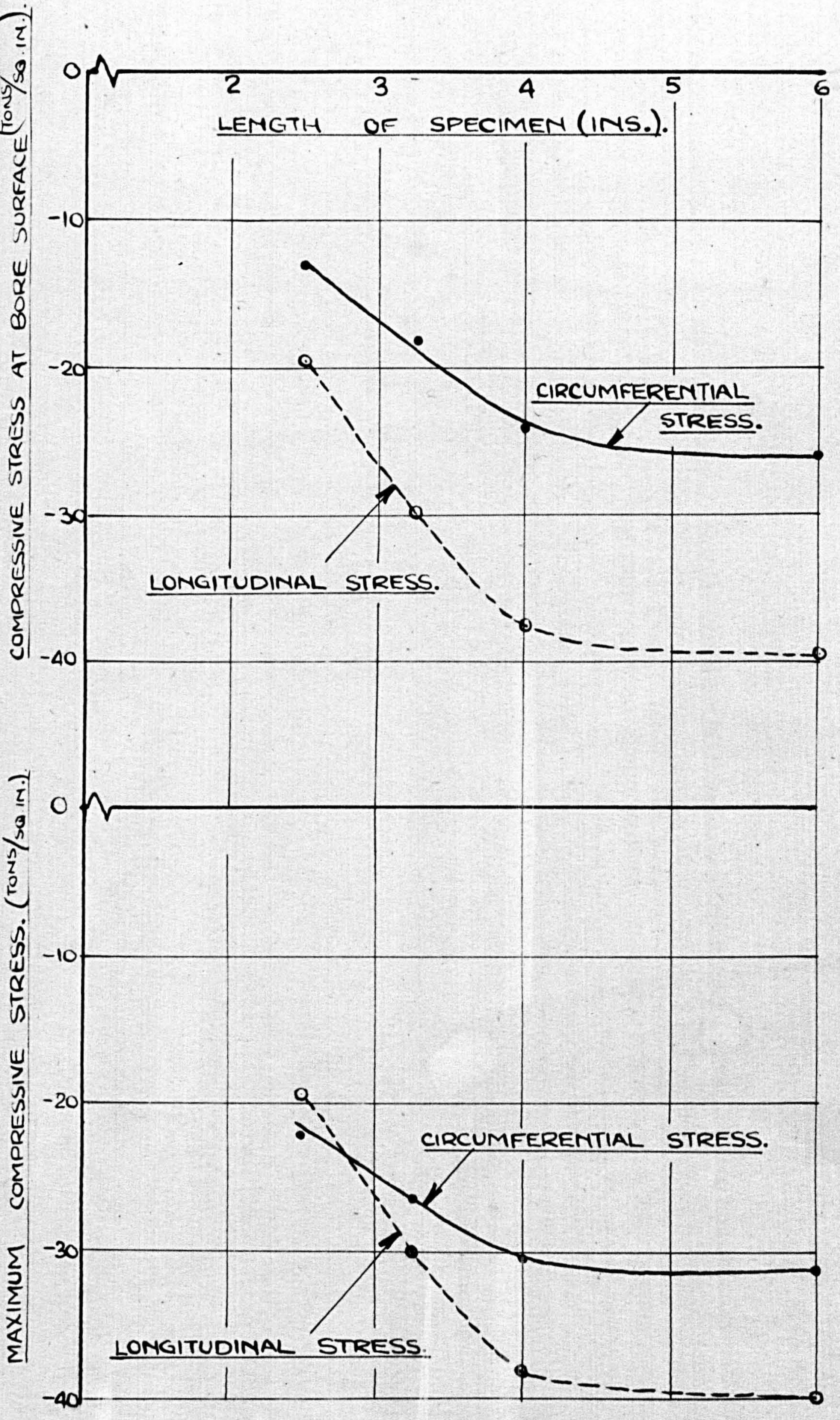


FIG: 4.9. SACHS' BORING TESTS. INFLUENCE OF SPECIMEN LENGTH ON THE MEASURED RESIDUAL STRESSES.

from a 2.25 in. diameter tube with 0.15 in. thick walls which had been hollow drawn through a patent bottle neck die, from 3.0 in. diameter. (A specimen shorter than 2.5 in. was not considered, as all the strain gauges available were 2.0 in. in length). Strain gauges were secured to the outer surfaces of these specimens and these were completely protected from acid attack. Removal of surface layers by pickling from the inside ensued and the resultant curves showing the variation of the parameters ψ and θ with the current cross-sectional area of the bore are shown in Fig. 4.6. The distribution of the residual circumferential and longitudinal stresses as determined by the Sachs' analysis from the experimental results are given in Figs. 4.7 and 4.8 respectively. The similarity in the form but not the magnitude of the stress distribution curves given on each diagram is immediately obvious, and consideration of Fig. 4.9 gives a better appreciation of the most important differences. It is obvious that specimen length is important in this work and that if a specimen is not sufficiently long, serious underestimations of the magnitude of the residual stresses may result.

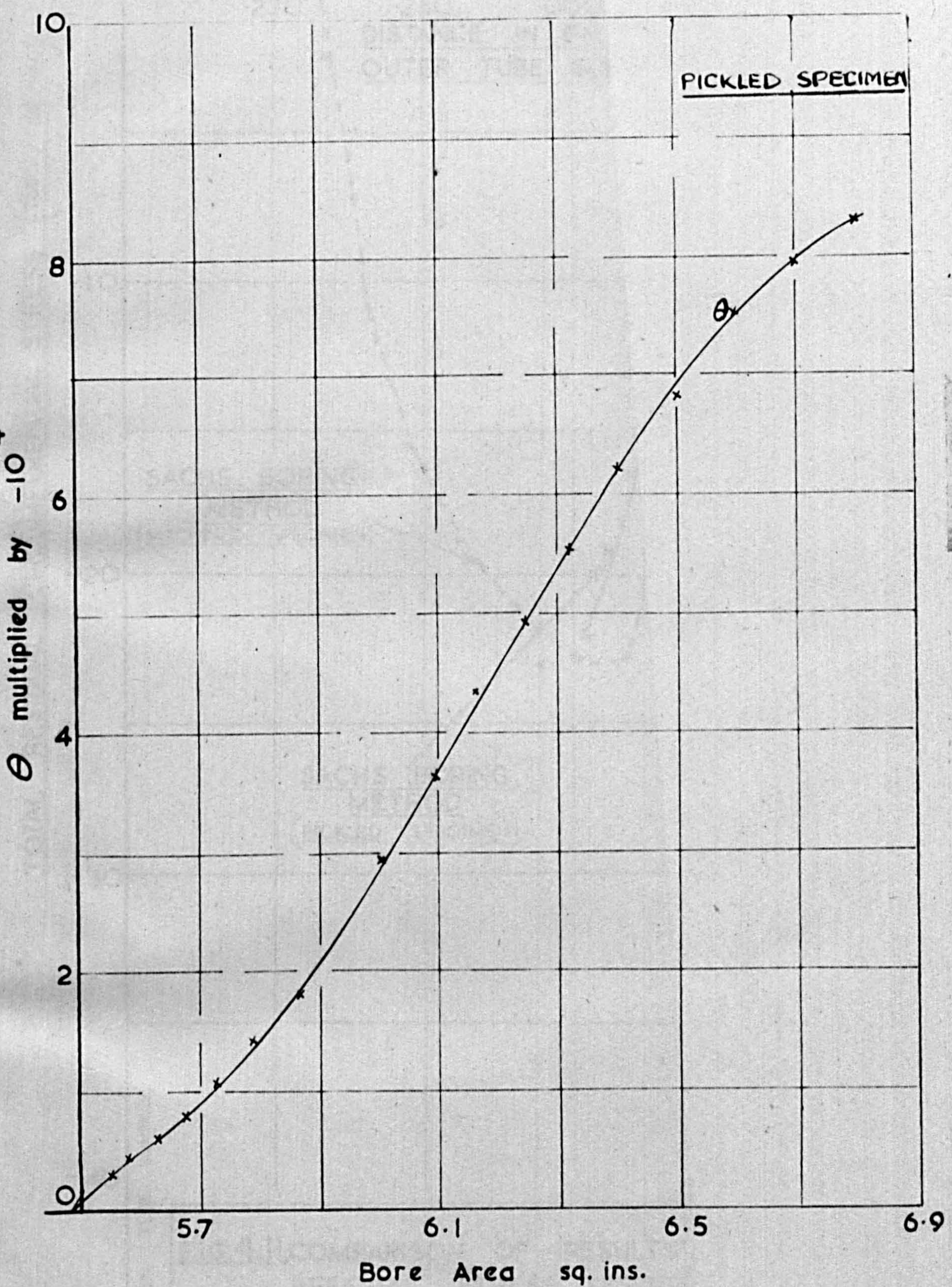
4.3.3. Effect of machining on the residual stress distributions determined by the boring method.

To investigate whether layer removal by machining was really detrimental or not to the results obtained by boring tests, two specimens, each four inches in length were parted off from tube C (Table 3.II). Both these specimens were fitted with strain gauges on their outer cylindrical surface and both were suitably protected, one to prevent acid attack and the other to prevent the gauges being affected by a soluble oil cutting fluid. Successive layers of material were then removed from the bore surfaces of these specimens

SACHS BORING METHOD

4.0" Long Specimens – Tube "C" (Table 3.II).

θ against bore area



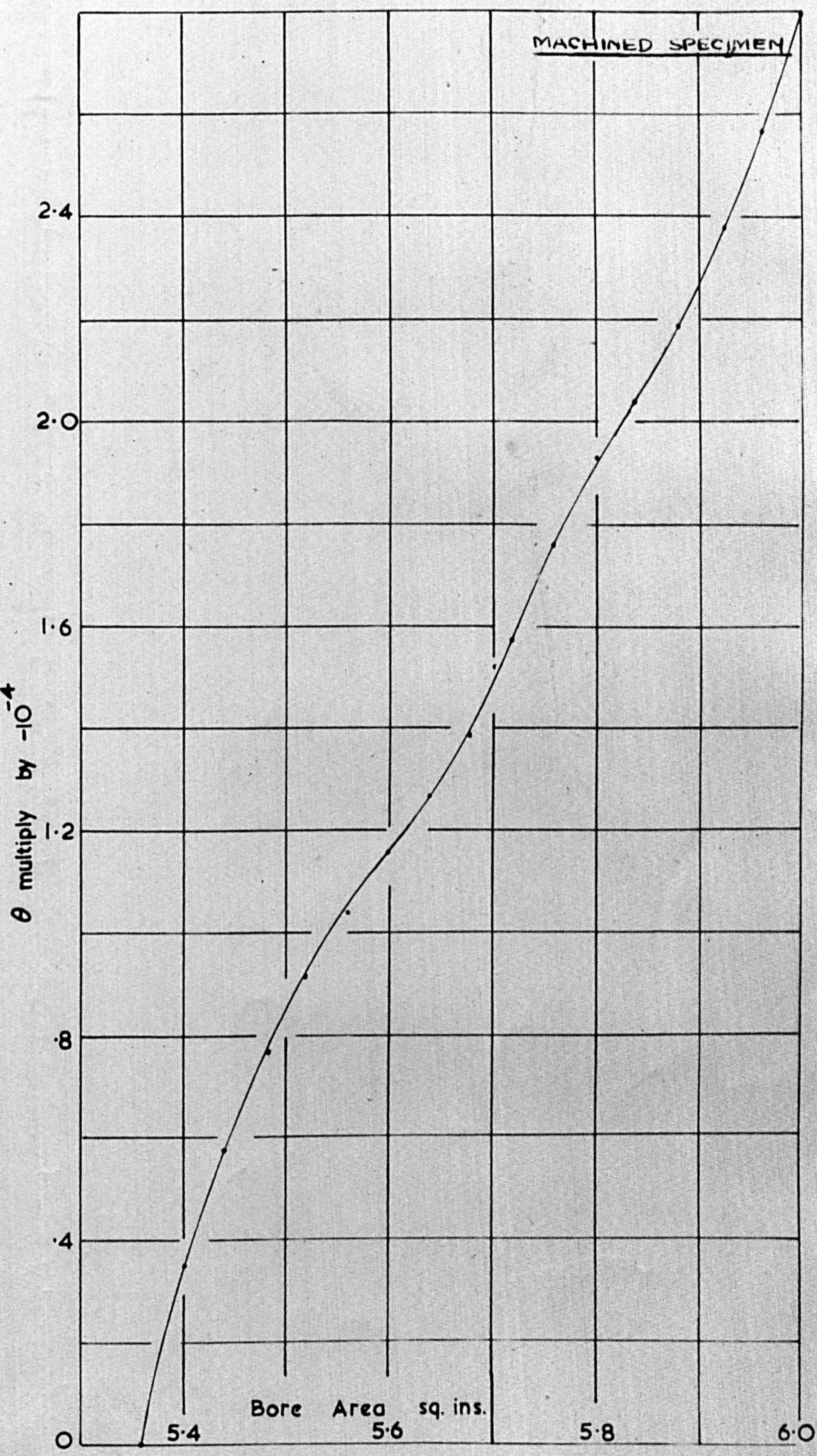


FIG: 4.10.

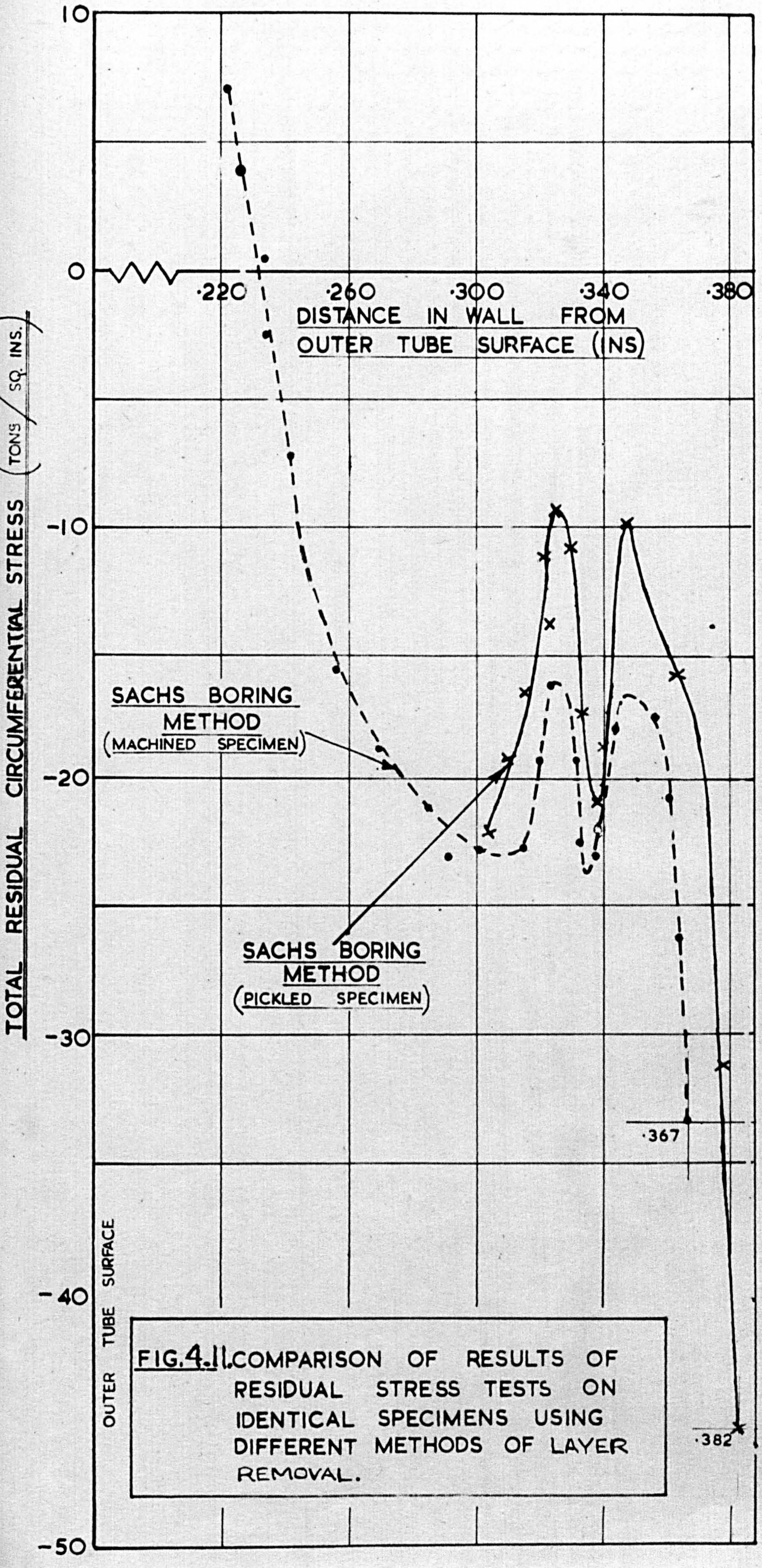


FIG. 4.11. COMPARISON OF RESULTS OF RESIDUAL STRESS TESTS ON IDENTICAL SPECIMENS USING DIFFERENT METHODS OF LAYER REMOVAL.

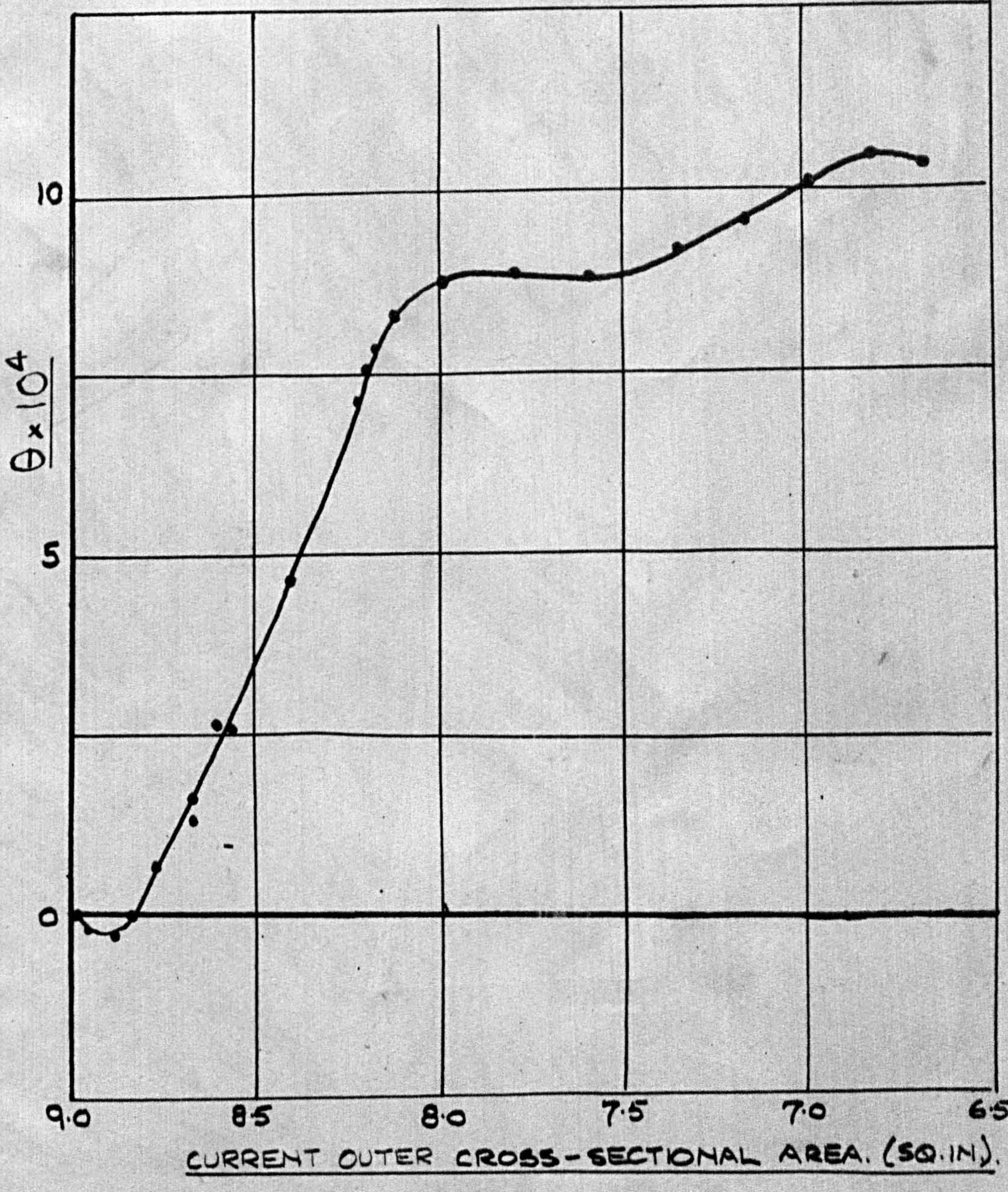


FIG: 4.12. PICKLED SPECIMEN - OUTSIDE LAYER REMOVAL.

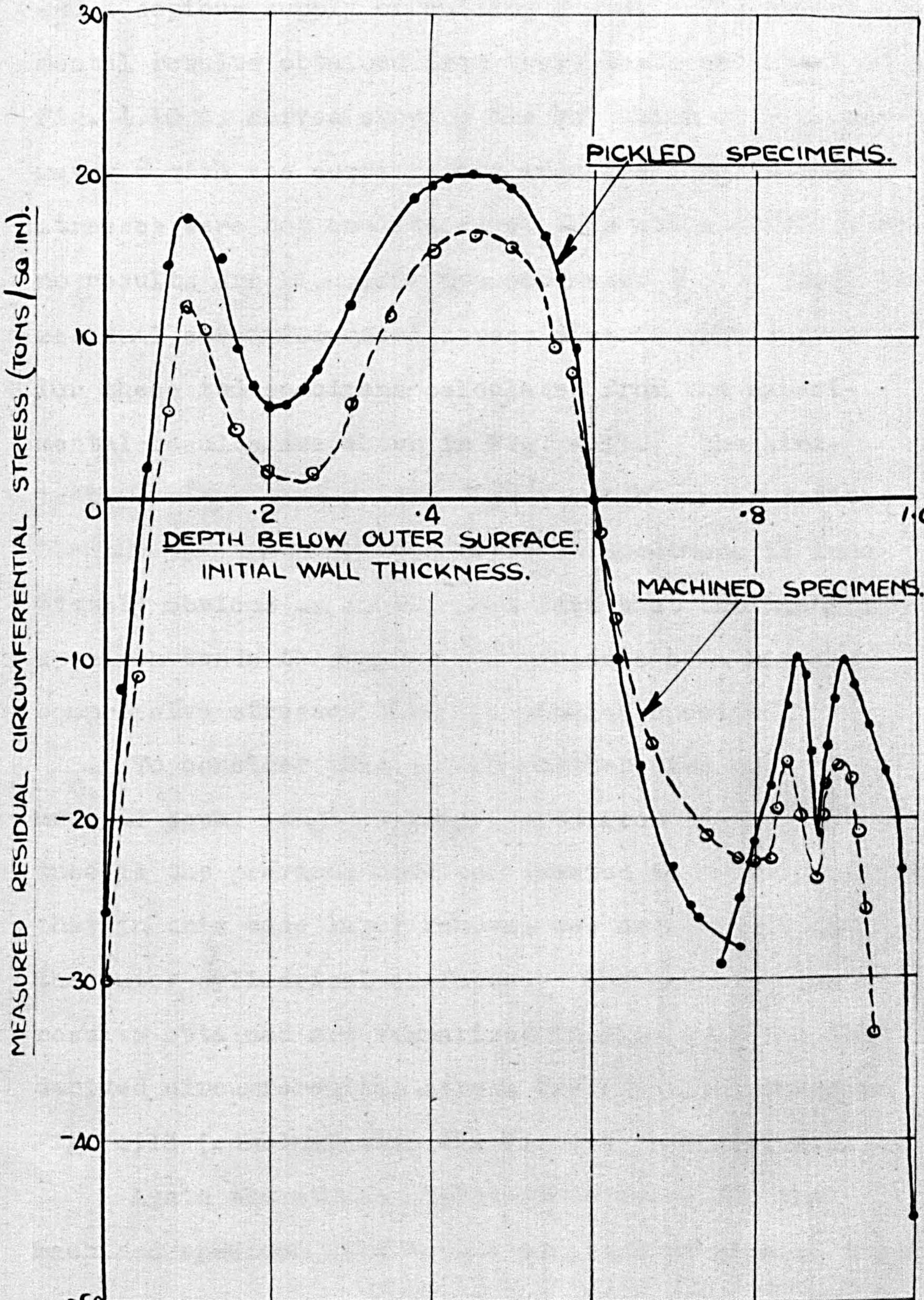


FIG: 4.13. INFLUENCE OF LAYER REMOVAL BY MACHINING ON THE MEASURED RESIDUAL STRESSES IN TUBES. (SACHS' BORING METHOD).

in one case by the acid pickling process generally adopted, and in the other by machining, using a very high cutting speed, a fine feed, a small depth of cut and a copious supply of cutting fluid. The experimental results obtained from these tests are shown in Fig. 4.10 as curves showing the variation of the parameter θ with the current bore area (as longitudinal stresses were not considered at this stage of the work, no results are given for the parameter ψ). The residual circumferential stress distribution curves for these two specimens calculated from the experimental results are shown in Fig. 4.11. The similarity in the form of the distribution curves derived for the "pickled" and the machined specimens is immediately obvious as is also the fact that the machined specimen tends to suggest the presence of a higher compressive stresses than the pickled specimen.

To consider this effect further, two more specimens of equal length were prepared from the same stock tube as the previous ones and treated as before, except that in this case layer removal was carried out from the outer cylindrical surfaces. The experimental results obtained are summarized in Fig. 4.12 and the derived circumferential stress distribution curves in Fig. 4.13 (combined with the results from Fig. 4.11).

Again the stress distribution curve for the machined specimen lies on the compressive side of the corresponding curve obtained for the pickled specimen and as the difference in the magnitude of the stresses at all points in the tube wall is appreciably constant it must be concluded that machining tends to induce compressive stresses. (Consideration of the balance of forces derived from Fig. 4.13 gives further confirmation of this. The net force / inch length of specimen as determined by graphical integration of the resultant stress curve for the pickled specimen is 0.06 tons in the tensile direction, which corresponds to only 3% difference in the tensile and compressive areas of the stress distribution curve. The corresponding result obtained from the machined specimen shows a net compressive force of 1.9 tons, with the total compressive area under the stress curve being approximately twice that of the tensile regions.)

4.4. Check on the validity of strain gauge readings over a relatively long period of time.

As each test on strain gauge fitted specimens was of necessity of long duration (sometimes extending over more than three weeks) it was considered advisable to check the validity of strain gauge readings over such a long period. For this purpose three tubular specimens of identical length were cut from a previously hollow drawn tube and each was fitted with two longitudinally mounted strain gauges which were suitably protected with compound rubber solution against atmospheric effects. One of these specimens was set on one side for use as a "dummy" in the strain gauge circuit and the other two were carefully checked for length using a vertical dial comparator which had been previously set against slip gauges. The initial bridge balance was observed for the strain gauges on the test specimens and periodically throughout the duration of the tests, layers of metal were removed from the bore surfaces of the specimens to bring about changes in the lengths of the specimens. Daily, throughout the tests, the length of each specimen was checked with the dial comparator, the bridge balancing resistance was obtained for each of the gauges on the test specimens, and the room temperature was observed. The results, taken over a period of some seven weeks, during which time the room temperature at the daily reading was never outside the range 67° F ± 2° F,

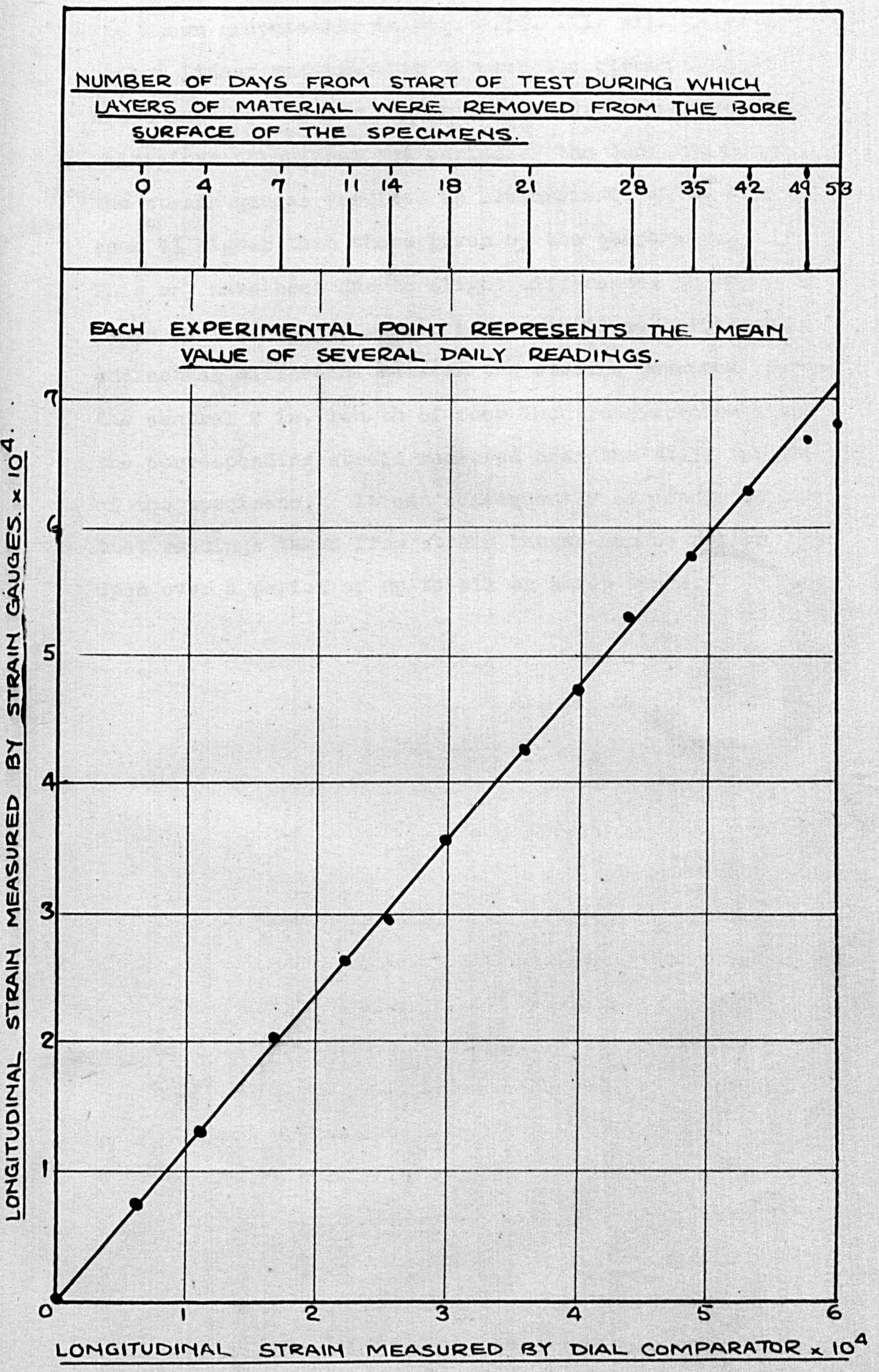


FIG. 4.14. CHECK ON THE VALIDITY OF STRAIN GAUGE READINGS OVER AN EXTENDED PERIOD OF TIME.

are shown graphically in Fig. 4.14. It will be seen that a linear relationship between the strain measurements made by the comparator and the strain gauges was maintained throughout the period of the test, although the strain gauges resulted in measurements which were some 4% higher than those given by the comparator. This may have been due to slight differences in the gauge factors of the strain gauges or it may represent an actual difference between the strains measured over the central 2 in. length of four inch long specimens and the corresponding strain measured over the whole length of the specimens. It can consequently be concluded that readings taken from strain gauges can be relied upon over a period of up to six or seven weeks.

5. COMPARISON OF THE BENDING DEFLECTION AND DIRECT STRAIN RELEASE METHODS.

5.1. Introduction.

Although the different methods of strain detection in tubular specimens and the subsequent stress analyses have been known for some considerable time, no satisfactory correlation has ever been obtained between residual stress distributions obtained for identical specimens subjected to direct or bending deflection strain release. This has probably been due to unsatisfactory mechanical methods of releasing strains and also to the lack of equipment sufficiently sensitive to measure the extremely small strains involved.

Once the procedure outlined in the previous section of this report had been established it seemed an ideal opportunity to compare the results of tests made to determine the residual stress distribution in a tube by the Sachs' boring method (direct strain release) with those of previous tests made on similar specimens using a bending deflection method of strain detection. Such a correlation at this stage seemed very desirable since if it proved successful it would give added confidence in the progress of the research, while if differences were found to exist between the results of the two methods of strain release it might at least indicate what means should be adopted to modify both the experimental and analytical approaches to the present problem.

5.2. Residual circumferential stresses.

5.2.1. Specification of tube used.

Unfortunately, since the stock of tube used in all the previous tests had become exhausted, it was impossible to carry out the test programme as originally planned, and two new tubes had to be prepared for this investigation. The complete specification of these tubes is as follows:-

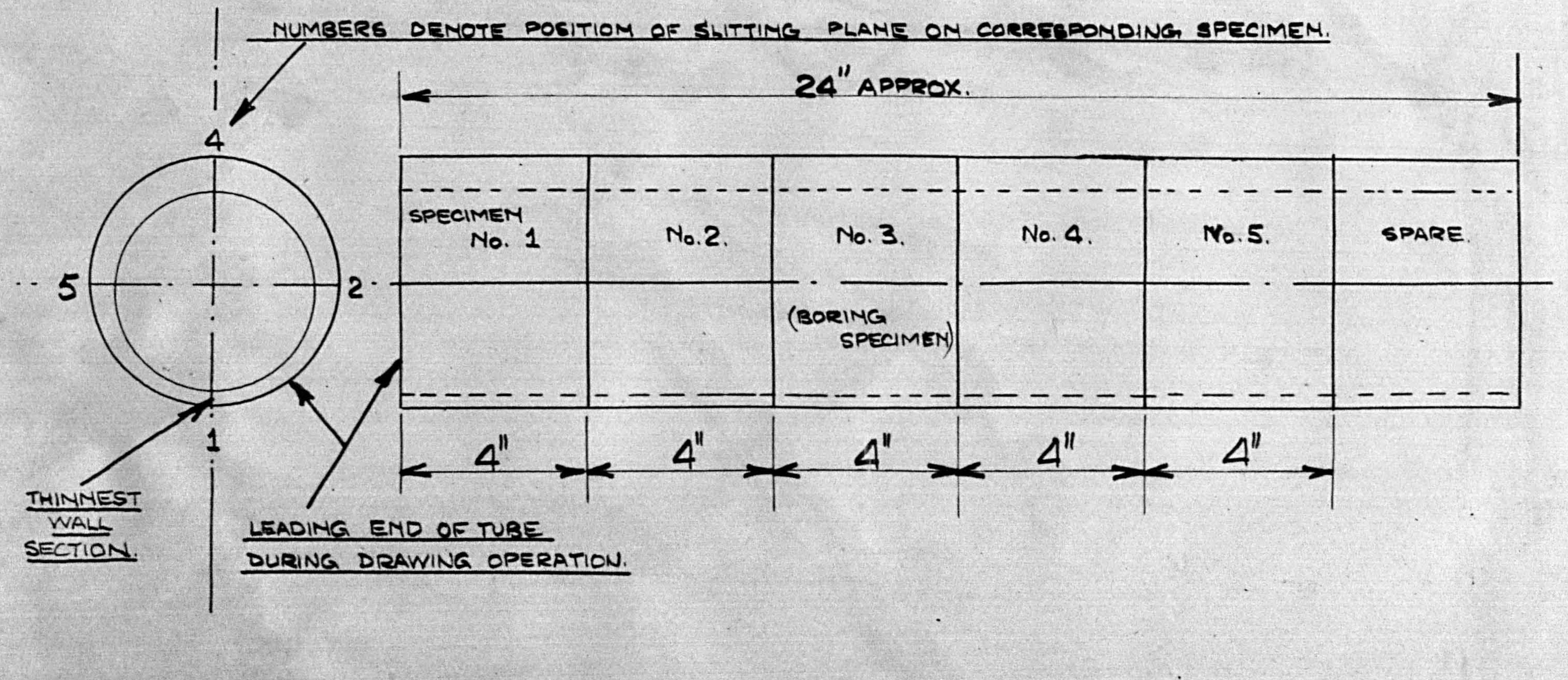


FIG: 5.1. LOCATION OF SPECIMENS AND SLITTING PLANES ON PARENT TUBE.

Material Mild steel.

Initial outside diameter ....4.125 in.

Initial bore diameter .... 3.500 in.

Final outside diameter .... 3.375 in.

Final bore diameter .... 2.605 in.(average).

Final wall thickness .... 0.385 in.(average).

Variation of final thickness around the tube........................... ±0.021 in.

Tubes hollow drawn through a 30° included angle straight taper die at a drawing speed of 16 ft. per minute.

The end five inch lengths of each tube were sawn off and discarded. Two 2.00 in. long specimens were then parted off from each of the resultant tubes (one being taken from each end) and after their diameters and wall thicknesses had been measured, they were slit longitudinally along the radial plane containing the minimum wall section. The outside diameters of these specimens were then measured in the plane at right angles to the one containing the slit. The mean change in diameter resulting from the slitting operation on both specimens was 0.0392 in. ± 0.0004 in. and it was therefore concluded that the residual stresses along the length of the tubes were sufficiently constant and the tube was suitable for the present investigation.

5.2.2. Summary of tests and discussion of results.

The two tubular specimens, which were each about 24 in. in length, were scribed with an axial line along their outer cylindrical surface, denoting the position of minimum wall thickness. Five four inch lengths were then parted off from each tube, and eight of these were slit longitudinally in accordance with the details of Fig. 5.1. Two sets of five specimens were thus prepared, four in each set being slit along different generating planes and the other remaining unslit. After the relevant

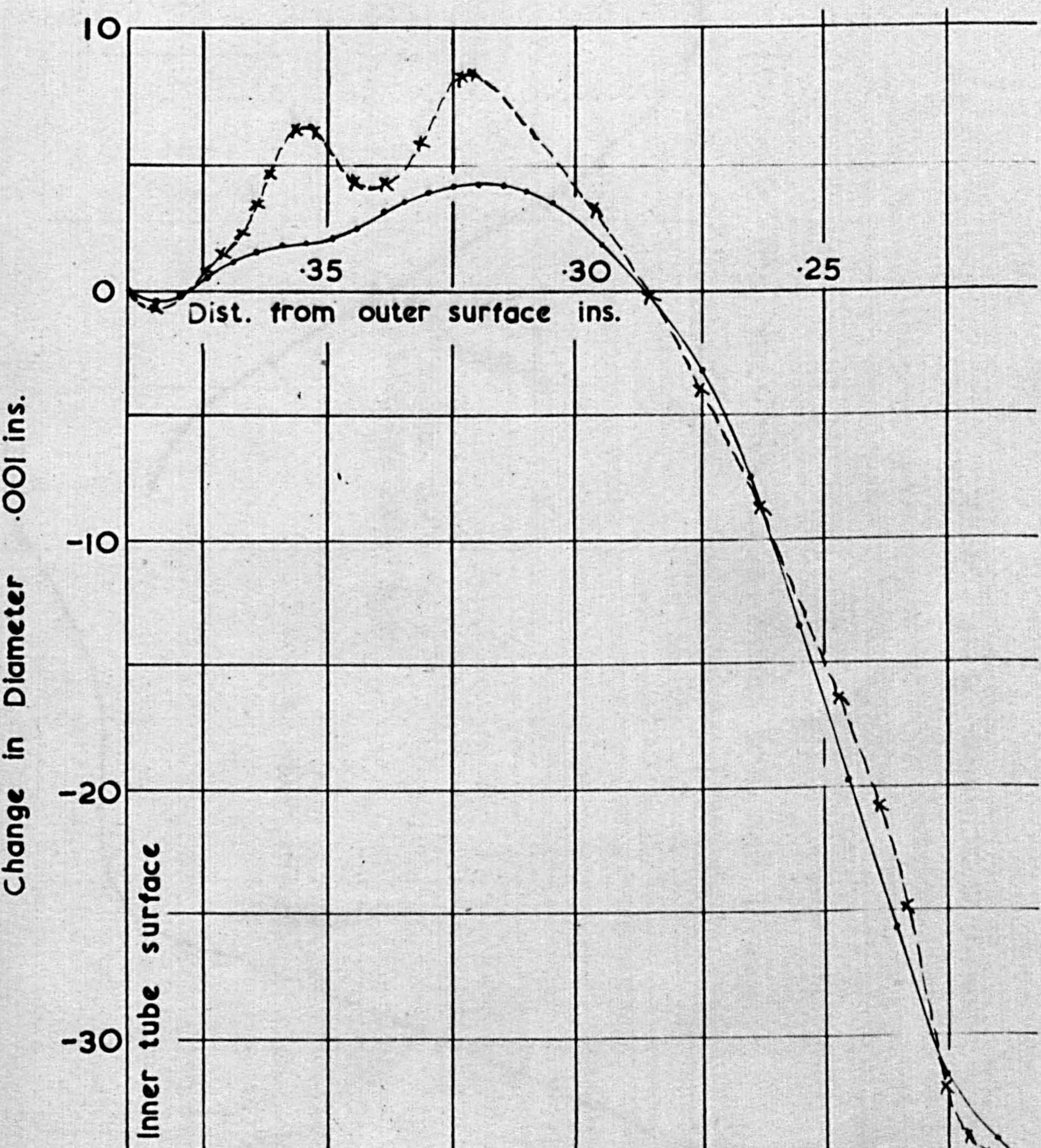


FIG: 5.2. BENDING DEFLECTION TESTS. TYPICAL EXPERIMENTAL RESULTS FOR INSIDE LAYER REMOVAL. (SPECIMENS OF EQUAL LENGTH, FROM ONE TUBE, SLIT ALONG DIFFERENT GENERATORS.)

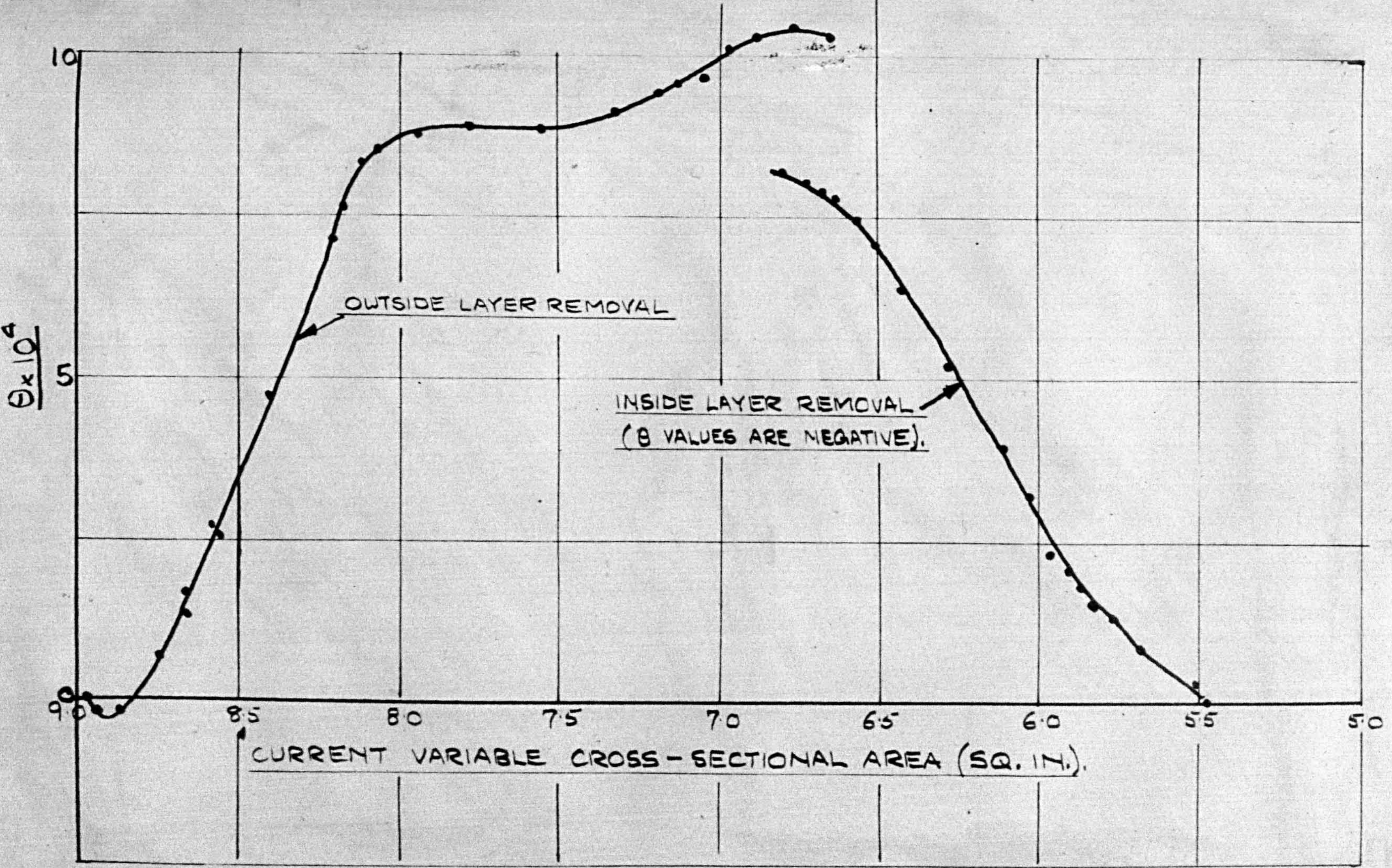


FIG. 5.3. BORING SPECIMENS. θ-A CURVES.

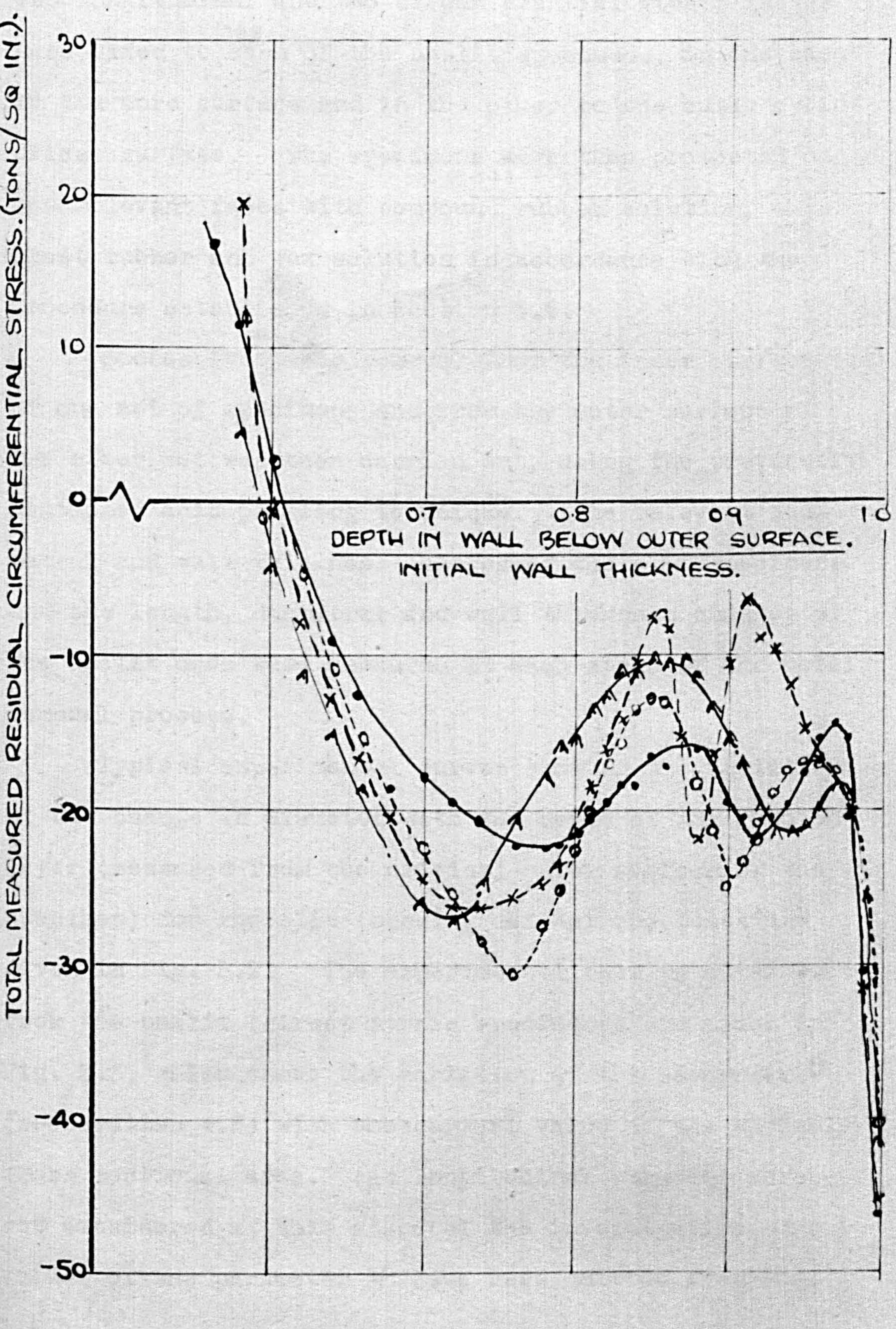


FIG: 5.4. TYPICAL INDIVIDUAL DISTRIBUTION CURVES FROM BENDING DEFLECTION TESTS WITH DIFFERENT SLITTING PLANES.

measurements had been made the slit specimens of each group were prepared for bending deflection tests, one group of specimens being left unprotected on their outer surface (to allow external layer removal) and the other set unprotected on their bore surface. Two longitudinal and two circumferential strain gauges were fixed to each of the unslit specimens, in one case to the bore surface and in the other to the outer cylindrical surface. The specimens were then protected on the relevant faces with compound rubber solution, thin sheet rubber and wax solution in accordance with the procedure established in section 3.2.

Successive layer removal from the inner surface of one set of specimens and from the outer surface of the other set was then carried out, using the previously mentioned acid pickling technique. The relevant diametral and wall thickness changes of the slit specimens and the length, diametral and wall thickness changes of the unslit ones were measured at each stage of the metal removal process.

Typical experimental curves showing the variation of the change in diameter with the depth of the removed layer (measured from the original outer surface of the specimen) for the slit (bending deflection) tubes are given in Fig. 5.2. The experimental results obtained from the unslit (direct strain specimens) are given in Fig. 5.3, which shows the variation of the parameter θ (see Section 4.3) with the current value of the variable cross sectional area. (As longitudinal stresses were not considered at this stage of the investigation, the values of the parameter ψ have been omitted from Fig. 5.3).

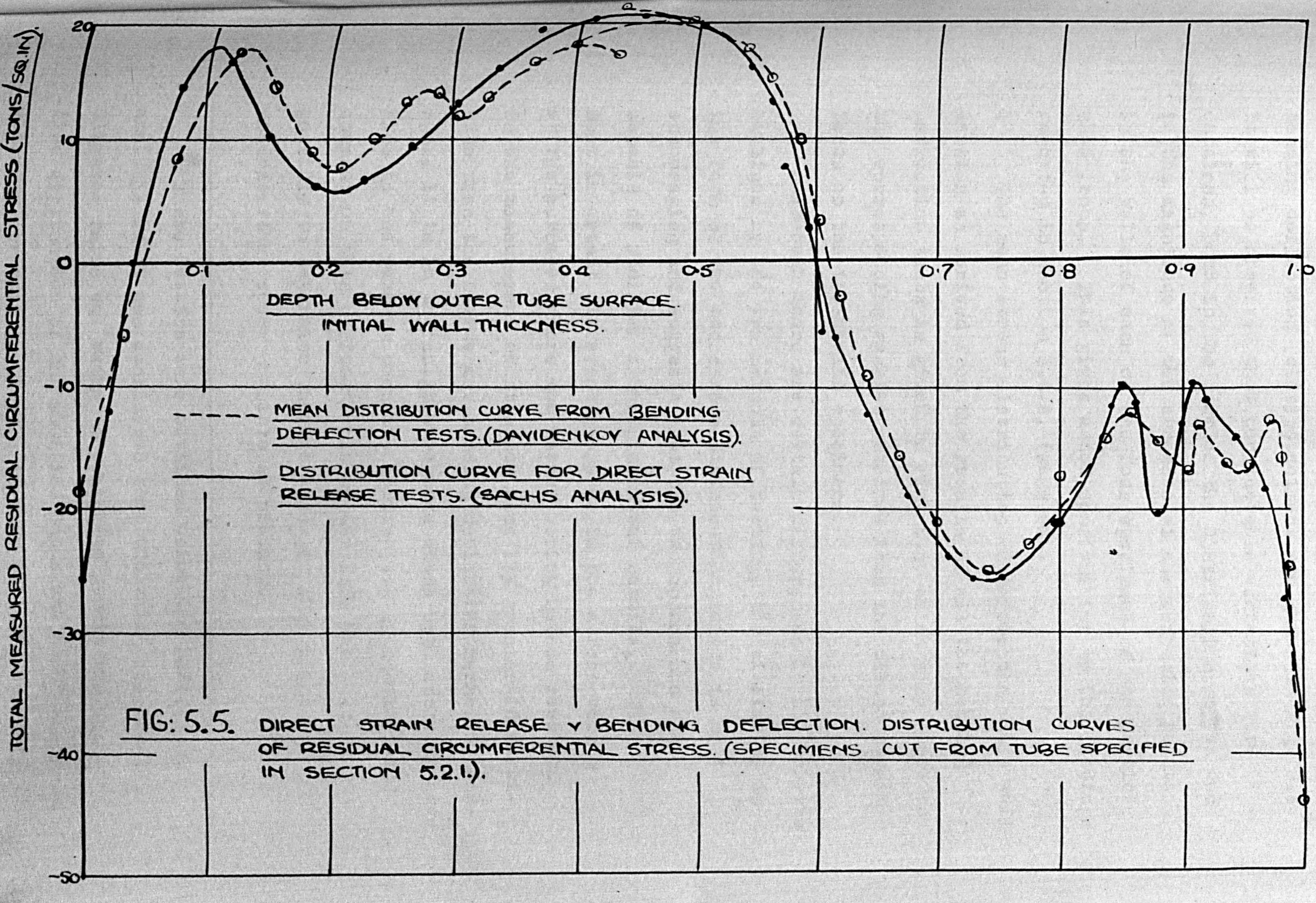


FIG: 5.5. DIRECT STRAIN RELEASE v BENDING DEFLECTION. DISTRIBUTION CURVES OF RESIDUAL CIRCUMFERENTIAL STRESS. (SPECIMENS CUT FROM TUBE SPECIFIED IN SECTION 5.2.1.).

The resultant stress distribution curves obtained from the experimental results given by each of the eight bending deflection (slit) specimens are shown in Fig. 5.4.* It will be observed that considerable variations were obtained, both in the form of the distribution curves and in the magnitude of the stresses at corresponding points in the walls of each of the four specimens of one particular group. This gives added confirmation to the findings reported in Section 3.4.4, (page 48).

The mean stress distribution curve for the whole wall section as derived from the results of all the bending deflection tests is given in Fig. 5.5, which also contains the corresponding results obtained from the direct strain tests on the two unslit specimens.

Although there are certain differences in the results obtained by the two methods of testing, the similarity in the form of the two stress distribution curves is most encouraging and suggests that the two approaches are capable of yielding almost identical results. The most important differences appear in the stress values at the surfaces where the Sachs' boring method resulted in the measurement of stresses some 17% greater in magnitude than the Davidenkov bending deflection method. However, it is in the surface regions where both analyses give rise to the maximum possible errors (see Appendix 7) and comment on this difference is better left until it is possible to determine surface stresses with greater accuracy than seems possible at present.

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* These results were obtained using the Davidenkov analysis as it was considered that the wall thickness / diameter ratio was rather too high to permit the unqualified application of the modified Sachs and Espey analysis, which is strictly applicable to thin walled tubes only.

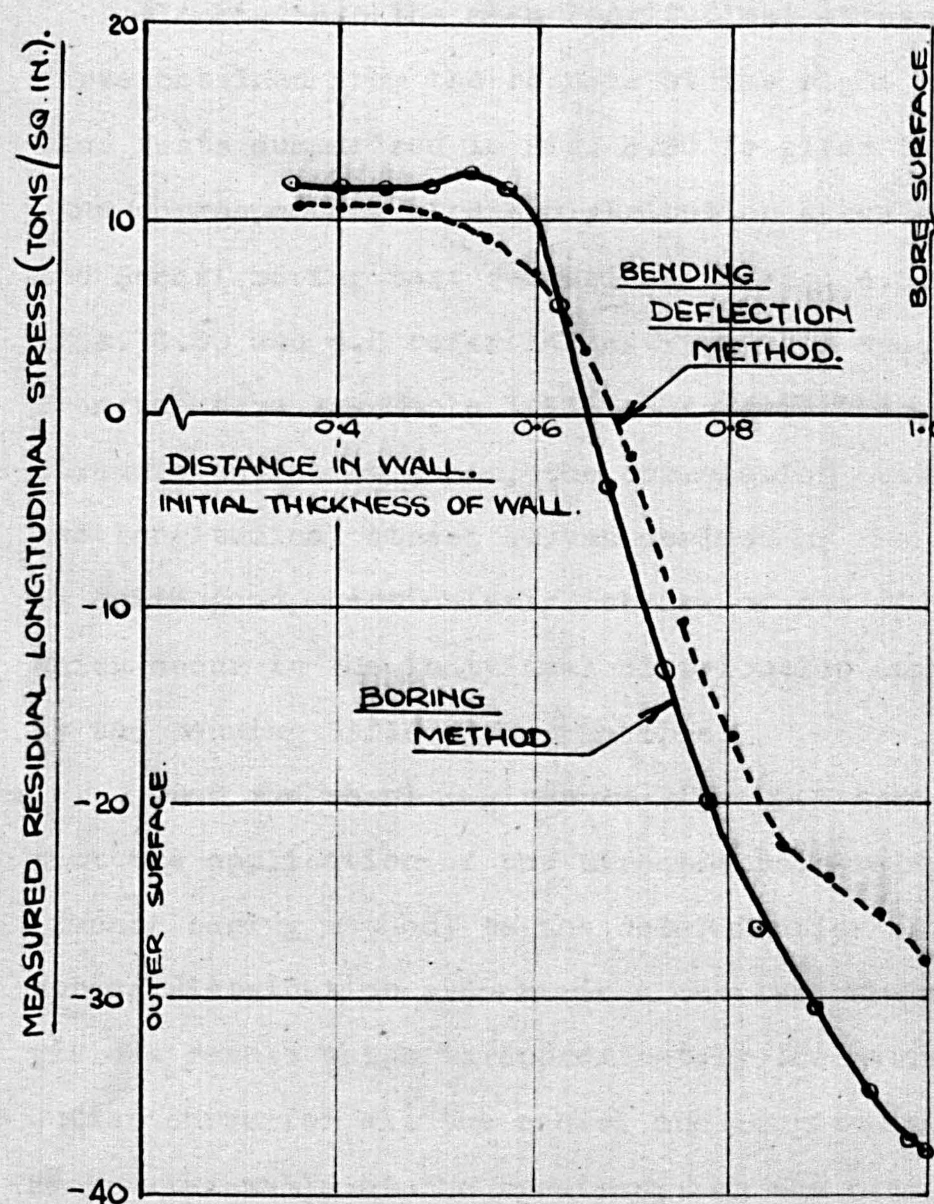

FIG: 5.6. STRESS DISTRIBUTIONS IN IDENTICAL SPECIMENS, DETERMINED BY DIFFERENT METHODS.

5.3. Longitudinal stress distribution curves.

For a comparison of the longitudinal stress distribution curves derived from the results of bending deflection and direct strain release tests on specimens prepared from the same stock tube, it was not necessary to carry out special tests as sufficient information was already available to enable the comparison to be made.

In Fig. 5.6 the mean longitudinal stress distribution curve obtained from the results of the eight bending deflection tests summarized in Fig. 3.30 is given together with the corresponding stress distribution curve obtained from the Sachs' boring test summarized in Fig. 4.5. (Both Figs. 3.30 and 4.5 refer to tests made on specimens cut from the tube A - Table 3.II). As with the residual circumferential stresses, the correlation between the two longitudinal stress curves derived by the two methods is quite good, particularly in view of the wide differences which occur in the individual distribution curves determined by the bending deflection principle.

From the results given in Figs. 5.5 and 5.6 it appears that the application of the direct strain release method (Sachs' boring method) to the determination of the residual stress distribution present in a tubular component, will yield a result which is approximately the average distribution curve for all the radial and longitudinal elements around the walls of the specimen; on the other hand, the bending deflection method of strain release, will result in the derivation of a stress distribution curve which is very dependent upon the position of slitting or sectioning in the original tube wall, and which, in the case of circumferential stresses, is probably influenced more by the stresses present in the section diametrically opposite the slit than by the stresses at any point in the section.

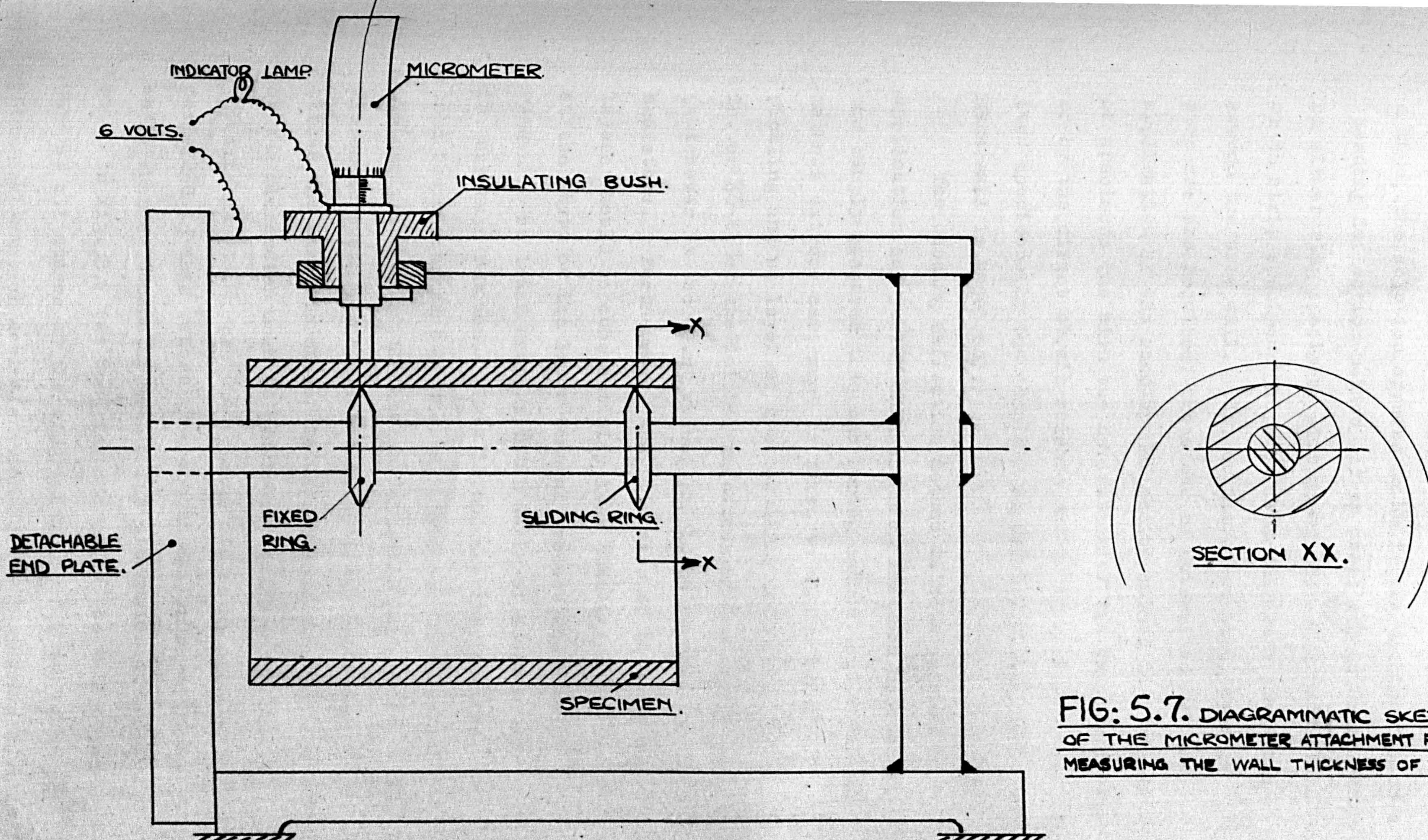

FIG: 5.7. DIAGRAMMATIC SKETCH OF THE MICROMETER ATTACHMENT FOR MEASURING THE WALL THICKNESS OF TUBES.

Consequently, if it is considered necessary to determine the maximum values of the residual stresses present in a tube, the solution appears to lie in the application of a bending deflection method and to carry out several tests on either ring specimens slit along different generators or on strip specimens prepared from different positions around the tube wall. If, on the other hand, average stresses are required, these can be determined by the application of the bending deflection method to a number of specimens (as with the derivation of the maximum stress) or by the direct strain release method to a single specimen (two specimens if the complete distribution throughout the tube wall is required).

The bending deflection method is both quicker in application and yields changes which are much more easily and simply measured than the boring method, but with both methods extreme care must be taken to ensure uniform pickling and no layer of greater depth than about 0.005 in. or $\frac{1}{50}$ th of the initial thickness of the tube wall (whichever is the least value) should be removed at any one stage. Furthermore, the thickness of the removed layers should be substantially less than these values at the start of the layer removal process and should remain so at least until a total depth of about 0.010 in. has been removed. This introduces difficulties into the measurement of the wall thickness changes in the early stages of the stress relieving process. Micrometers can be used for this purpose, provided either the same micrometer is used throughout the test or the micrometer used is previously calibrated against slip gauges. (In the later stages of the work described in this report when the significance of these early changes became apparent, the micrometer attachment shown in Fig. 5.7 was used for measuring the wall thickness at all stages of the experiments. The use of an electrical contact device ensured that the pressure applied to the micrometer was constant whenever readings were taken).

6. THE "LENGTH EFFECT" IN THE MEASUREMENT OF RESIDUAL CIRCUMFERENTIAL STRESSES IN TUBES.

6.1. Introduction.

As previously pointed out in Section 2, all the mathematical relationships derived for the calculation of circumferential and longitudinal stresses at any point in a tube wall, from the strains measured by bending deflection methods, are respectively independent of the specimen length(L) and the tongue width (b). Many investigators have suggested that correct results are only obtained if the value of L or b exceeds some proportion of the tube diameter (D) (Fig. 2.17) but many values for the critical ratios $^L/D$ and $^b/D$ have been quoted and agreement between different workers seems remote. Furthermore, Sachs (1941) has suggested that different results are obtained by different ways of separating specimens of identical length from a parent tube, e.g. by milling, sawing, grinding, etc., but very few experimental results are available in support of this claim.

Thred suggestions have been advanced to explain this length effect. These are:-

(a) that anticlastic bending due to residual circumferential stresses has different effects on the flexural rigidity of long and short specimens.

(b) that a difference in flexural rigidity occurs as above, but is due to the presence of residual longitudinal stresses in the tube walls.

(c) that when specimens are being parted from the parent tube, the local heating due to the cutting medium has the effect of partially annealing, or in some way relieving the stress at the ends of the specimen where the parting action has taken place. This local heating, or other disturbance would probably be approximately the same for specimens of any length and consequently short specimens would have a greater proportion of their residual stress relieved in this manner, than would longer specimens, (Sachs, 1941).

As detailed later, the suggestions "a" and "c" have been systematically examined in the course of this work. The second suggestion "b" has not been studied as fully. Elementary considerations of the longitudinal stresses which can exist in tubular specimens of varying lengths suggest that the rigidity of a specimen should increase as the length of the specimen increases and not vice versa as is necessary to prove the length effect. (Because of the discontinuity of the section, the longitudinal stresses present in a specimen must be zero at the free ends and must increase progressively to their maximum (and presumably constant) values at some distance away from the ends of the specimen. Such a stress release in the end regions of specimens will be relatively more severe in short specimens than in longer ones and consequently the latter should be more rigid.

6.2. The effect of different cutting methods on the release of bending stresses by slitting.

(Length effect suggestion "c").

6.2.1. Mechanical separation.

For studying the effects of different mechanical methods of specimen separation and slitting, a number of ring specimens ranging in axial length from 0.016 in. to 15.0 in. were parted off from a 2.926 in. dia. tube, with a wall thickness of 0.131 in. which had been hollow drawn from 4.25 in. dia. through a 30° total conical angle straight taper die. Rings of various lengths were severed from the stock tube by turning, milling, sawing and grinding, care being taken during the cutting process to avoid excessive heat and to eliminate cutter and work vibrations, as far as possible. (Before parting the specimens from

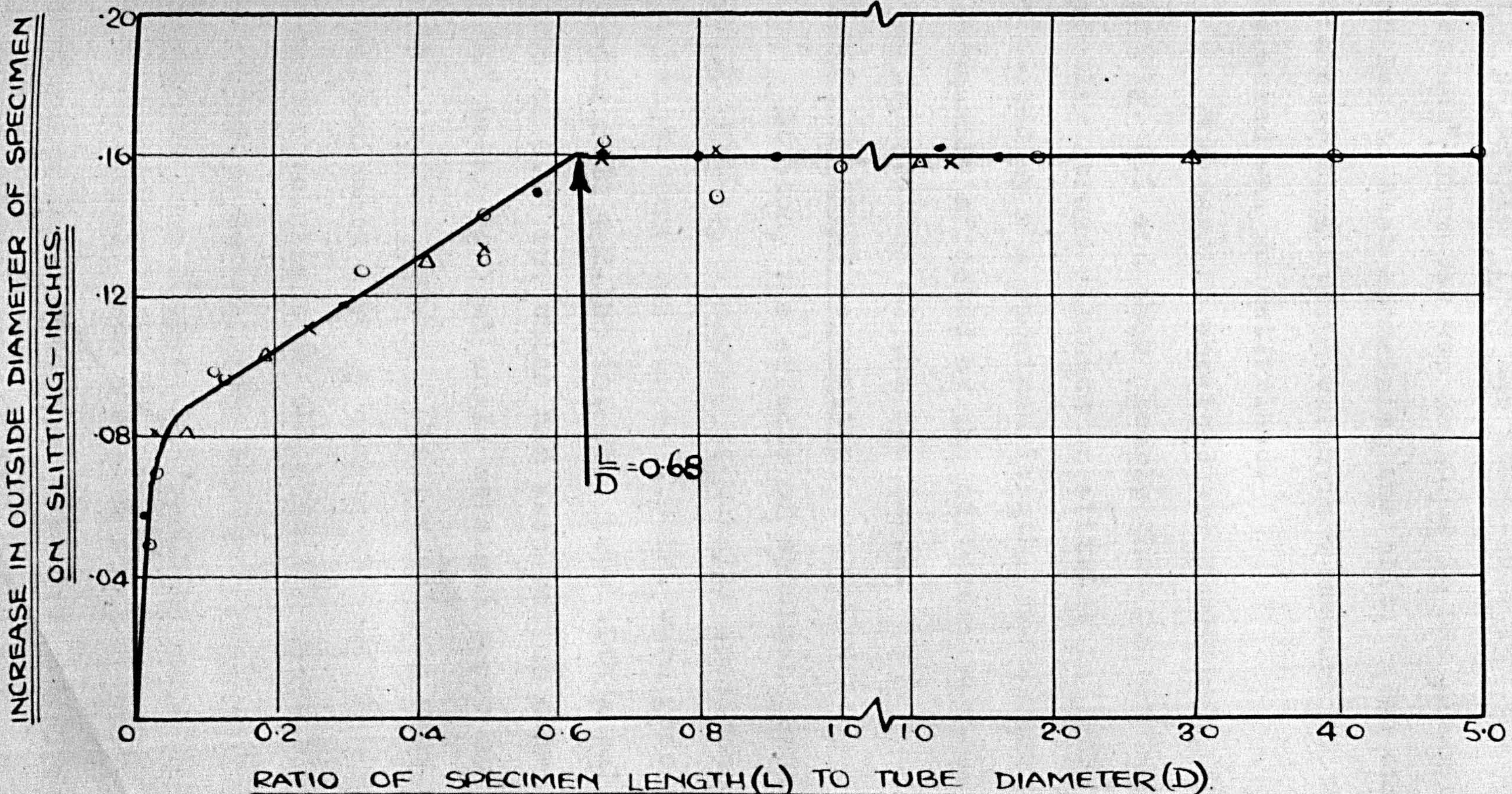

FIG. 6.1. EFFECT OF DIFFERENT MECHANICAL METHODS OF SPECIMEN PREPARATION ON THE RELEASE OF RESIDUAL CIRCUMFERENTIAL STRESSES IN TUBES.

○ SPECIMENS PREPARED BY TURNING, ● BY SAWING, △ BY MILLING AND × BY GRINDING

ALL SPECIMENS CUT FROM THE SAME STOCK TUBE AND SLIT ALONG THE PLANE CONTAINING THE THINNEST WALL SECTION the parent tube a longitudinal line was scribed along it at the point corresponding to the thinnest wall section to ensure that the tubes were later slit along the same radial plane). Each specimen was then slit longitudinally using the same process as was used in its separation from the stock tube and the resultant "sprung" diameter was measured in the plane at right angles to that containing the slit. The change in diameter on slitting which is a measure of the stress released was so determined.

The results obtained are presented graphically in Fig. 6.1 and show clearly that the circumferential bending stress released on slitting is definitely affected by the specimen length, but either unaffected, or equally affected by the different methods used in the preparation of the specimens.

6.2.2. Chemical separation.

To study the effects of specimen preparation still further, a number of tubular specimens ranging in length from 0.5 in. to 4.0 in. were prepared by sawing from a 2.125 in. dia. tube with a wall thickness of 0.22 in. which had been hollow drawn through a 30° total conical angle straight taper die from a 3.375 in. dia. tube. These specimens were slit longitudinally also by sawing and the changes in diameter resultant on the slitting operation were measured. Several more tubular specimens, each at least one inch longer than would be finally required, were parted off from the same stock tube also by sawing and these specimens were completely coated with a special "stopping-off" wax dissolved in trichlorethylene. After allowing sufficient time for the wax to thoroughly dry, two circumferential grooves at the requisite distance apart (the required specimen length plus approximately 0.2 in.) were cut through the wax coating around the central portion of each tube by means of a carpenter's gauge. Corresponding grooves were then made in the wax coating on the inside tube wall surfaces and the specimens were completely immersed in a 15% solution of nitric acid. Chemical action occurred at the points of acid - metal contact, i.e. around the grooves, and proceeded to penetrate into the tube walls until finally the central portion was completely severed from each tube. During this and all subsequent pickling operations the acid temperature was maintained at 100° F ± 10° F by constant agitation and cooling whenever necessary. The short rings so prepared, possessed very jagged edges which were then substantially smoothed off by suspending the specimens in a horizontal plane above the acid bath and just allowing the edge under treatment to penetrate the acid surface. (The reason for the apparent waste of stock tube involved in severing the small rings from the initially longer tubes was to eliminate any effects caused by the initial mechanical separating process.)

The acid separated ring specimens were then carefully cleaned, measured up and re-coated with wax. A longitudinal groove was cut through the wax coating of each specimen and the rings were again immersed in the acid solution. Chemical action proceeded along the grooves and ultimately the rings were split open. They were then removed from the acid, washed, scraped and cleaned and their diameters measured in the plane normal to the one containing the slit, in the same way as all the previously prepared rings.

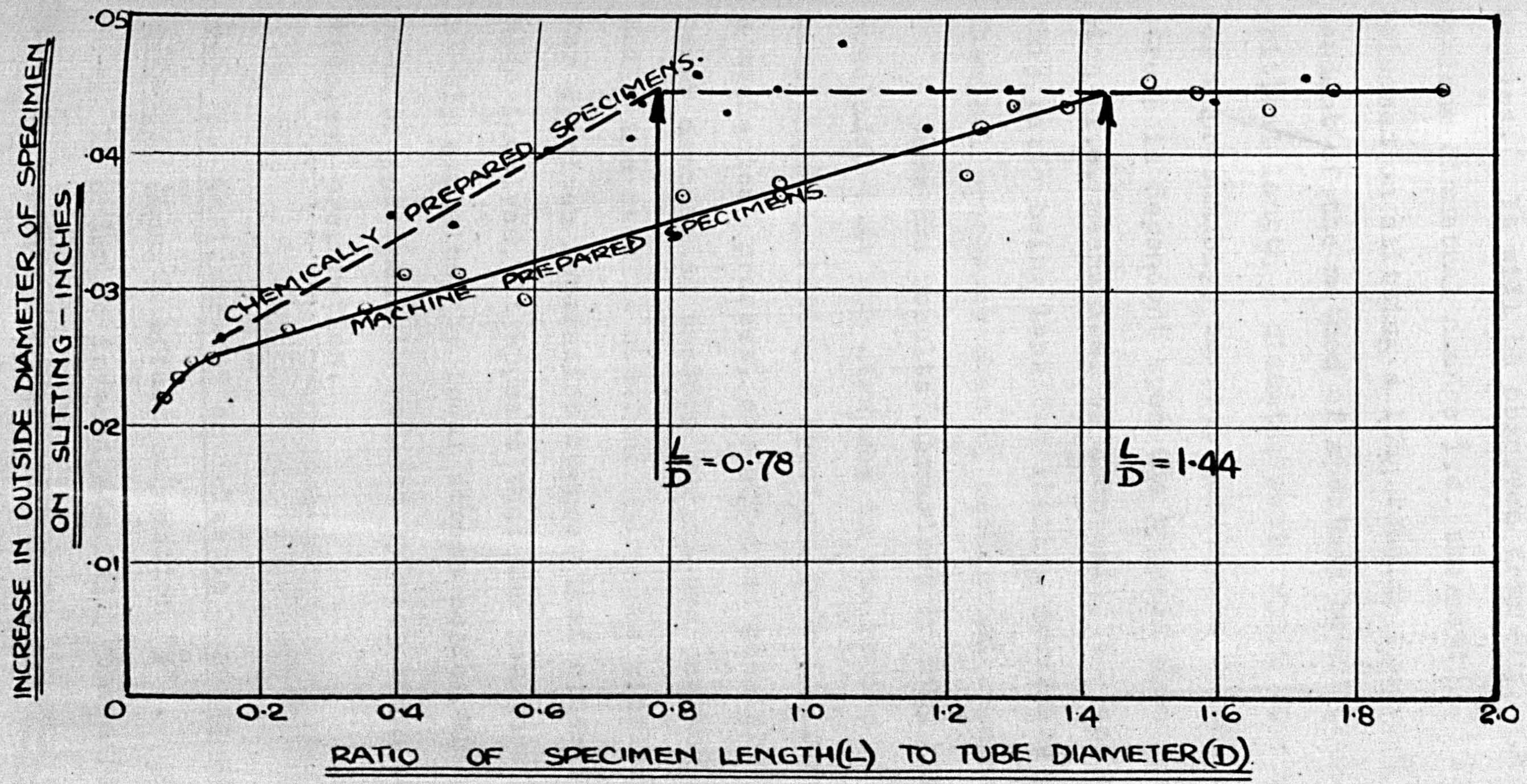

FIG: 6.2. EFFECT OF CHEMICAL AND MECHANICAL METHODS OF SPECIMEN PREPARATION ON THE RELEASE OF RESIDUAL CIRCUMFERENTIAL STRESSES IN TUBES.

ALL SPECIMENS CUT FROM THE SAME TUBE AND SLIT ALONG THE PLANE CONTAINING THE THINNEST WALL SECTION

The results obtained from the two series of tests are presented graphically in Fig. 6.2, the solid curve relating to the completely mechanically prepared specimens and the dotted curve to the chemically prepared samples. It will be observed from Fig. 6.2 that the maximum diametral change i.e. the maximum hoop stress released on slitting a long tubular specimen is independent of the method of specimen separation. The critical length / diameter ratio which must be exceeded for a (presumably) correct assessment of the residual stress is dependent upon the type of specimen preparation (i.e. mechanical or chemical), the actual value of this ratio being less for similar specimens prepared by chemical processes than by mechanical separation.

From the results summarized in Fig. 6.2 it may be concluded that the stresses in the region of the "cuts" made when separating a specimen from its parent tube by a mechanical process, are affected possibly by heat and other associated factors during the parting operation. While the results obtained using the pickling process for specimen separation suggest that other factors are also involved, it appears that the length effect can be partly explained by local stress relief at and near the ends of specimens introduced by mechanical methods of specimen preparation.

6.3. The influence of anticlastic bending on the length effect. (Length effect suggestion "a").

In order to investigate fully the effects of anticlastic bending in this connection, it was decided to reproduce on a scale convenient for measurement, conditions which obtain in a slit tube carrying residual circumferential stress. In principle this was to be achieved by using a thin walled tube of large diameter without internal stress, slitting it and applying opposing couples to its walls in a plane perpendicular

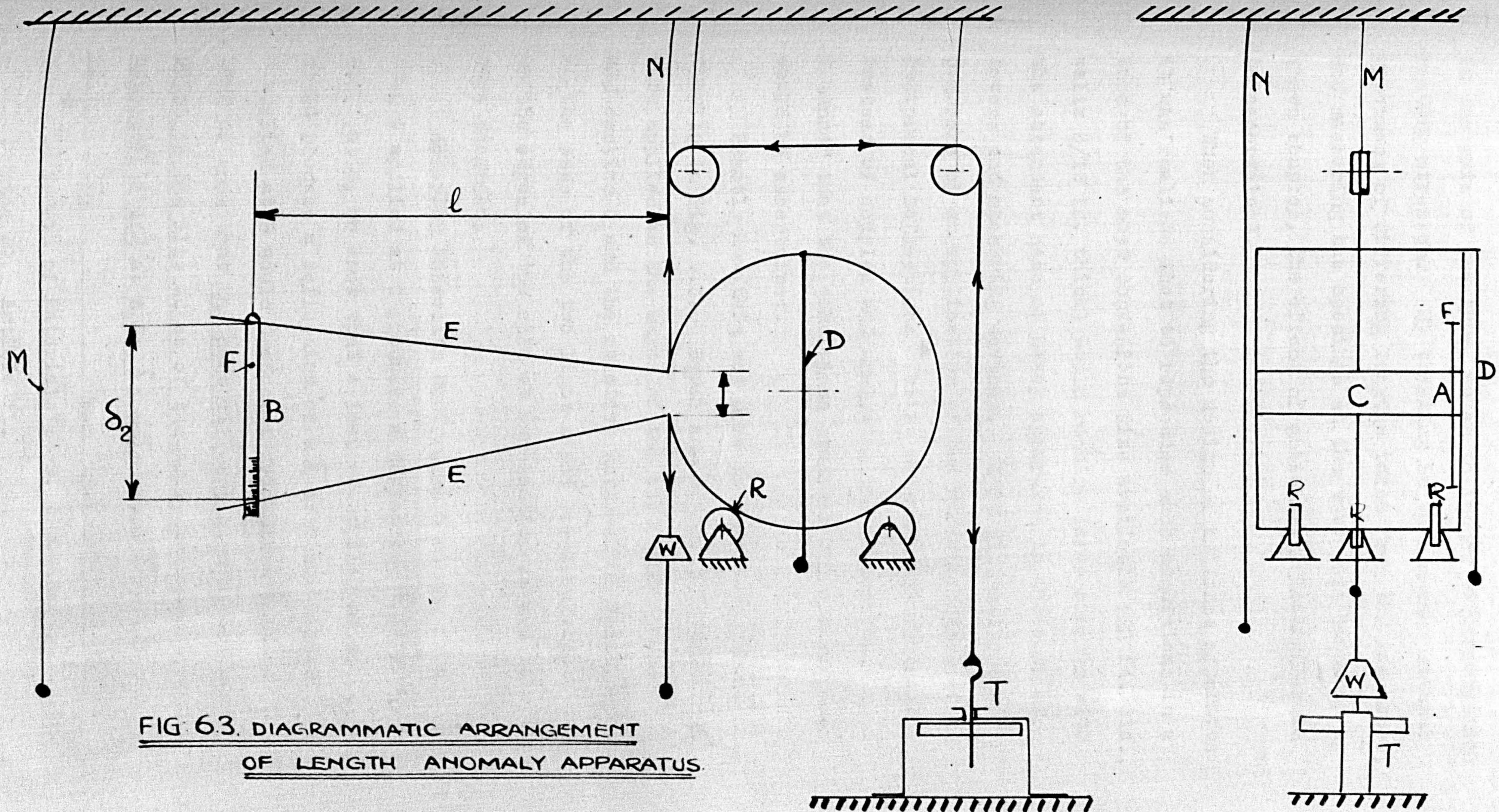

FIG 6.3. DIAGRAMMATIC ARRANGEMENT OF LENGTH ANOMALY APPARATUS.

to the axis of the tube, so as to induce circumferential bending stresses. By treating in this manner several specimens of differing lengths taken from the same tube, and measuring the openings at the tube slit produced by known couples, the effect of anticlastic bending could be ascertained.

When considering the design of suitable apparatus it was realised that to reproduce such conditions in a tube of the most convenient size available (12 in. dia., walls 3/16 in. thick) would require large couples with the attendant need of heavy apparatus in the form of levers and measuring devices. This difficulty was overcome by an application of the Maxwell Theory of Reciprocal Deflections* using moderate direct loads instead of couples and analysing the results in such a manner that the conditions produced by pure couples could be ascertained.

Exactly how this was done is shown diagrammatically in Fig. 6.3. Equal and opposite loads W were applied to the edges of the slit tube at its mid-section C and the relative deflections δ_1 and δ_2 of the ends of the two light rigid levers clamped to the edges of the slit at any section such as A were recorded.

Applying Maxwell's Reciprocal Theorem, if a load W applied at C produces a deflection δ_1 at A and δ_2 at B, we know that a load -W applied at A would produce a deflection of $-\delta_1$ at C and a load W applied at B would produce a deflection δ_2 at C. Thus we know that the nett deflection at C, the mid point of the slit edges of the tube, produced by a couple Wl acting at A is $\delta_2 - \delta_1$ where l is the

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* "Strength of Materials". S. Timoshenko. Chapter 10, Article 71.

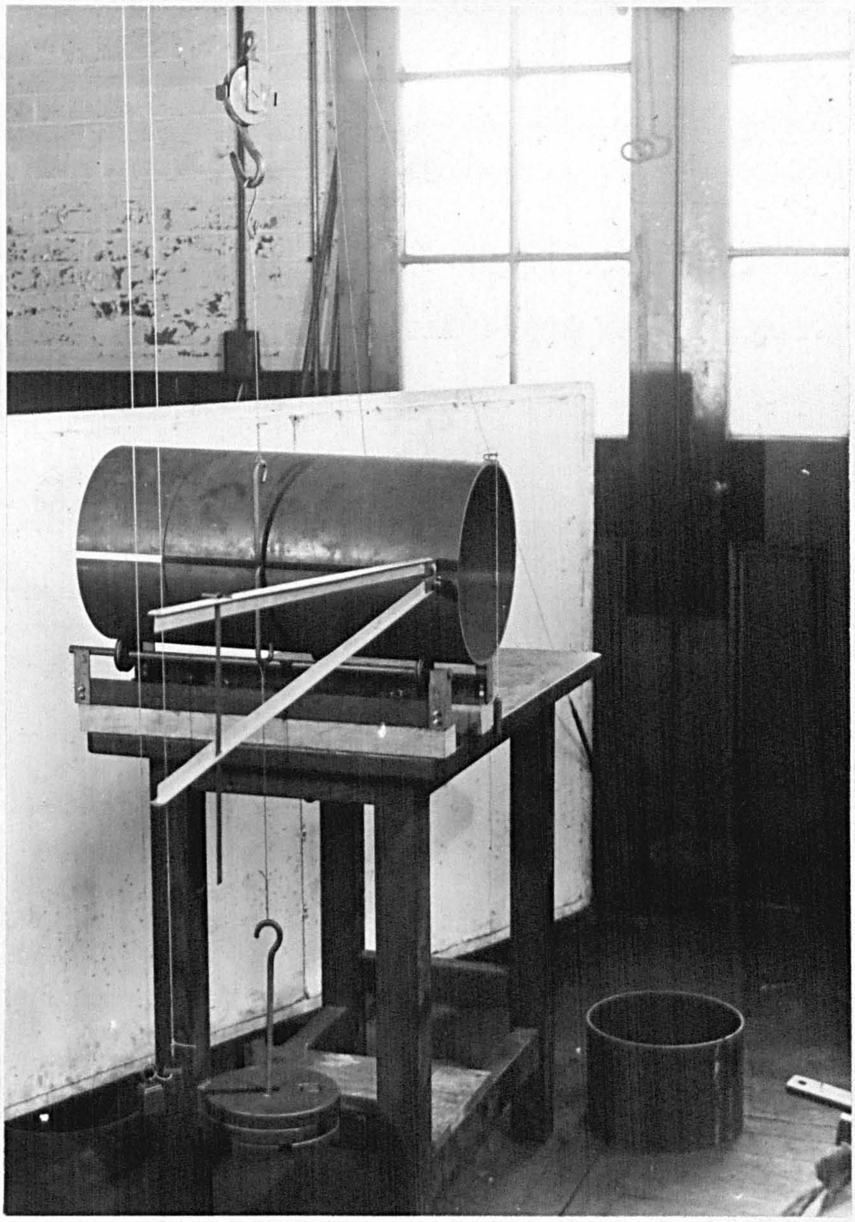


FIG. 6.4. THE LENGTH ANOMALY APPARATUS.

DETAILS OF LENGTH ANOMALY APPARATUS

SCALE:- FULL SIZE

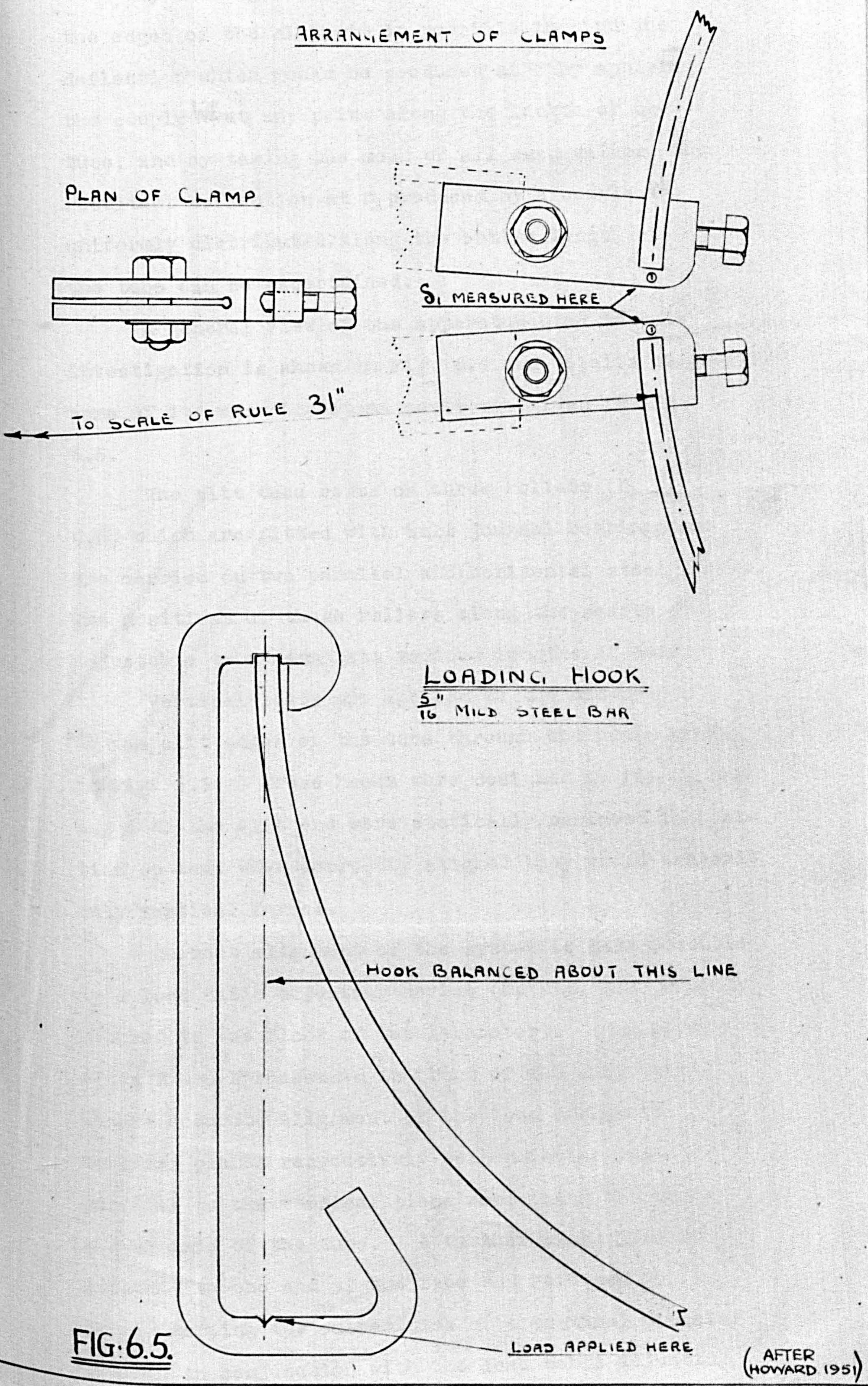


FIG. 6.5.

distance from A to B in the horizontal radial plane containing the centre line of the slit in the tube.

By setting the levers at different points along the edges of the slit, it is possible to find the deflection which would be produced at C by applying the couple Wl at any point along the length of the tube, and by taking the mean of all such values, the resultant deflection at C produced by a couple Wl uniformly distributed along the entire length of the tube can be ascertained.

A general view of the apparatus used in this investigation is shown in Fig. 6.4 and details of some of its more important parts are given in Fig. 6.5.

The slit tube rests on three rollers (R, Fig. 6.3) which are fitted with ball journal bearings and are carried on two parallel and horizontal steel shafts. The positions of these rollers along the shafts are adjustable to accommodate various lengths of tube.

Vertical loads are applied to the mid-points of the slit edges of the tube through the hooks shown in Fig. 6.5. These hooks were designed to fit on the edges of the slit and were statically balanced in position so that when correctly aligned they would transmit only vertical forces.

Correct alignment of the system is made possible by a load cable adjusting device (T, Fig. 6.3) rigidly secured to the floor of the laboratory. Two plumb lines M and N suspended in front of the slit tube ensure accurate alignment of the load cables in vertical planes respectively perpendicular and parallel to the vertical plane containing the horizontal axis of the tube. A further plumb line D attached to one end of the tube and referred to points marking the extremities of a vertical diameter is used in conjunction with the load cable adjusting attachment to ensure that the centre line of the slit is always in the horizontal radial plane.

Two aluminium levers E mounted in the same vertical plane normal to the tube axis are rigidly fixed to the slit edges of the tube by steel clamps (Fig. 6.5) which, when released, permit movement of the levers along the slit edges and allow for adjustment of the angular position of the levers relative to each other. The upper lever carries a steel rule F (visible in Fig. 6.4) which is pinned through at its upper end to enable it to pivot in a notch near the free end of the lever. This rule hangs vertically and passes freely through a slot in the flange of the lower lever and enables the relative deflection δ_2 of the levers to be measured. Adjustable dividers are used to measure the deflection δ_1 at the slit. These are set to points marked on the sides of each clamp which correspond to the mean circumference of the tube.

Six slit tubes ranging in axial length from 4 in. to 24 in., each prepared from the 12 in. diameter (nominal) tube, were used for the investigation.

Before attempting any systematic measurements the elasticity of the system was verified. This was essential because Maxwell's theorem applies only to elastic systems, and also convenient as readings might be taken while the tubes were under zero and maximum loads only, in the knowledge that intermediate readings would follow a linear relationship. This check was accomplished by loading the 4 in. long tube with gradually increasing loads and taking readings of the two relevant deflections under each loading condition. This procedure was continued until a load calculated to produce a bending stress of approximately 10 tons per sq. in. in the outer fibres of the tube wall had been reached. The experiment was then repeated for unloading conditions.

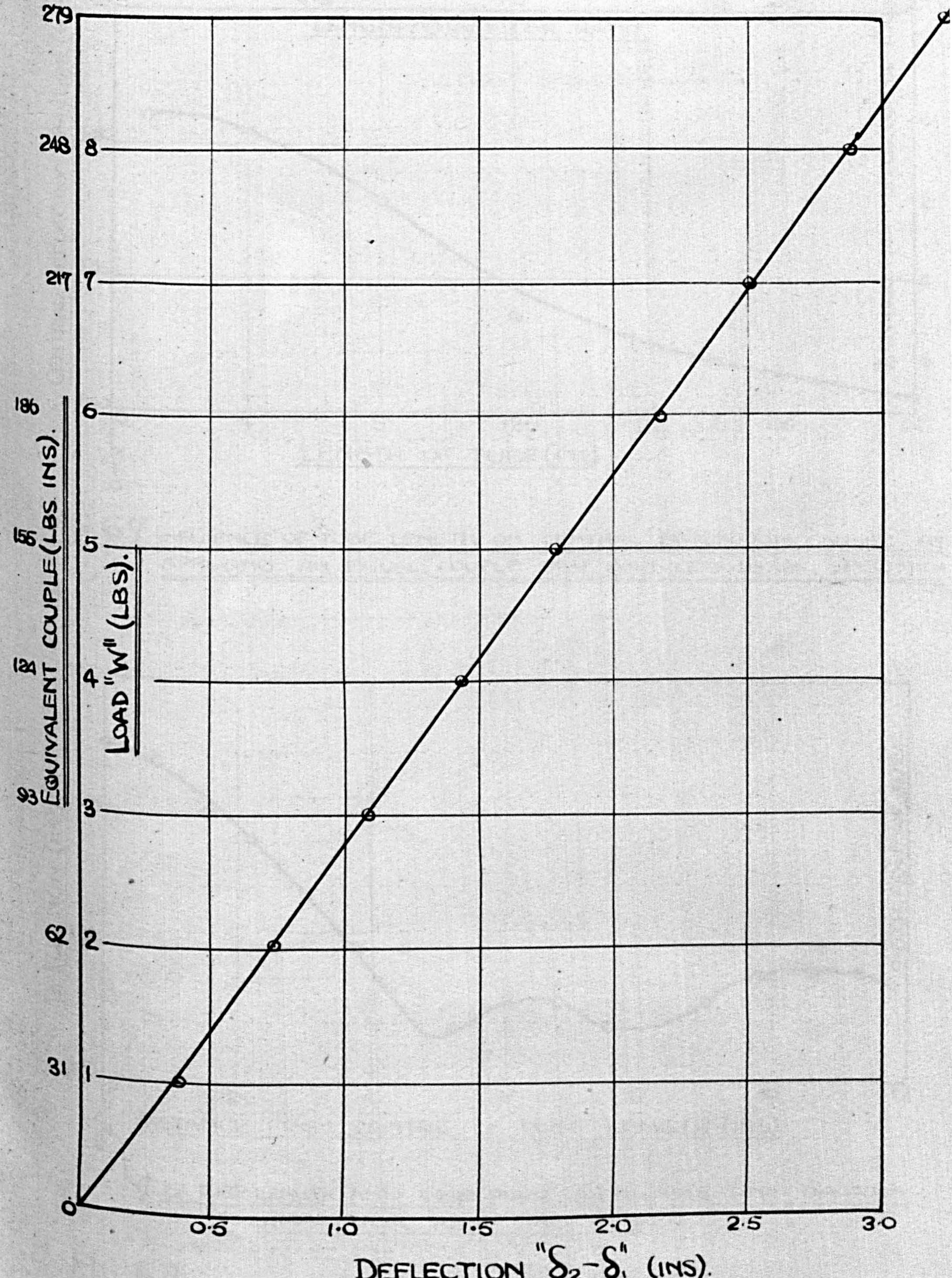


FIG: 6.6. TO VERIFY THE ELASTICITY OF THE LENGTH ANOMALY APPARATUS.

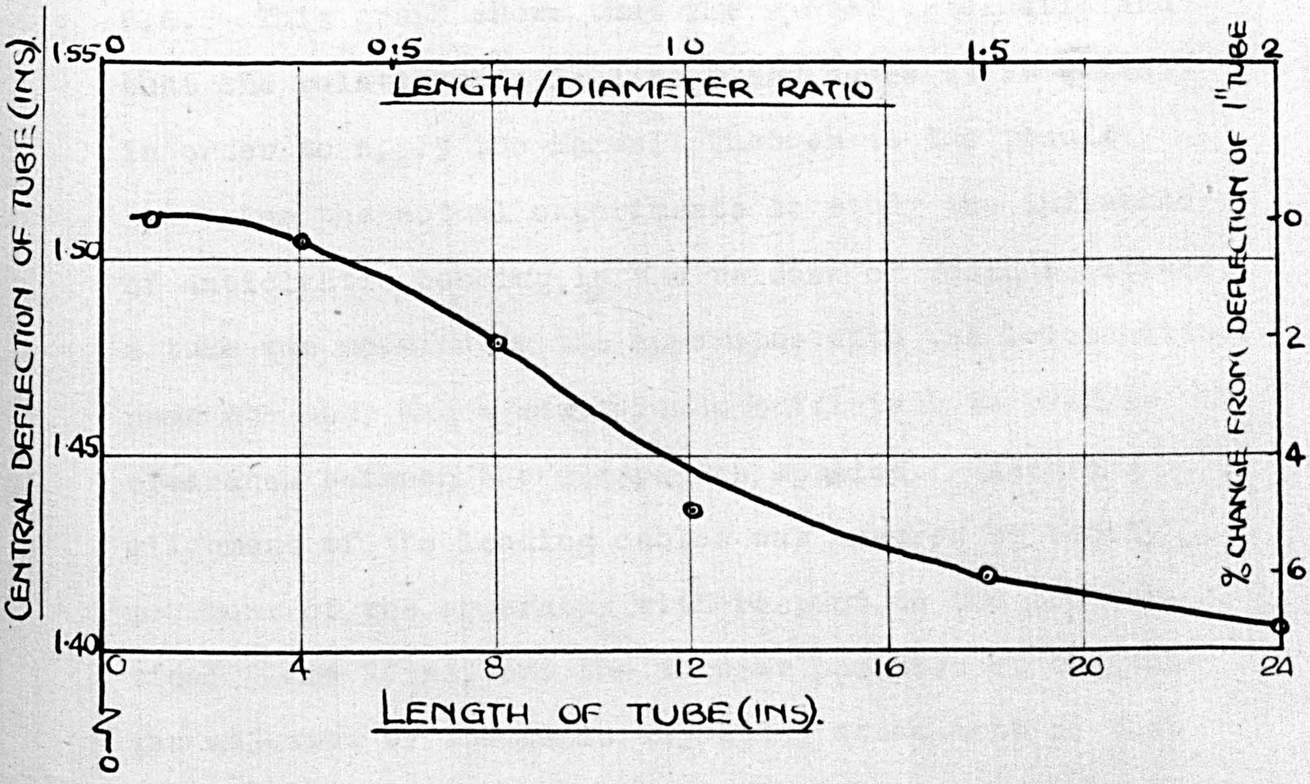


FIG: 6.7 INFLUENCE OF TUBE LENGTH ON CENTRAL DEFLECTION, CAUSED BY APPLYING AN EQUAL COUPLE PER UNIT LENGTH OF SPECIMEN

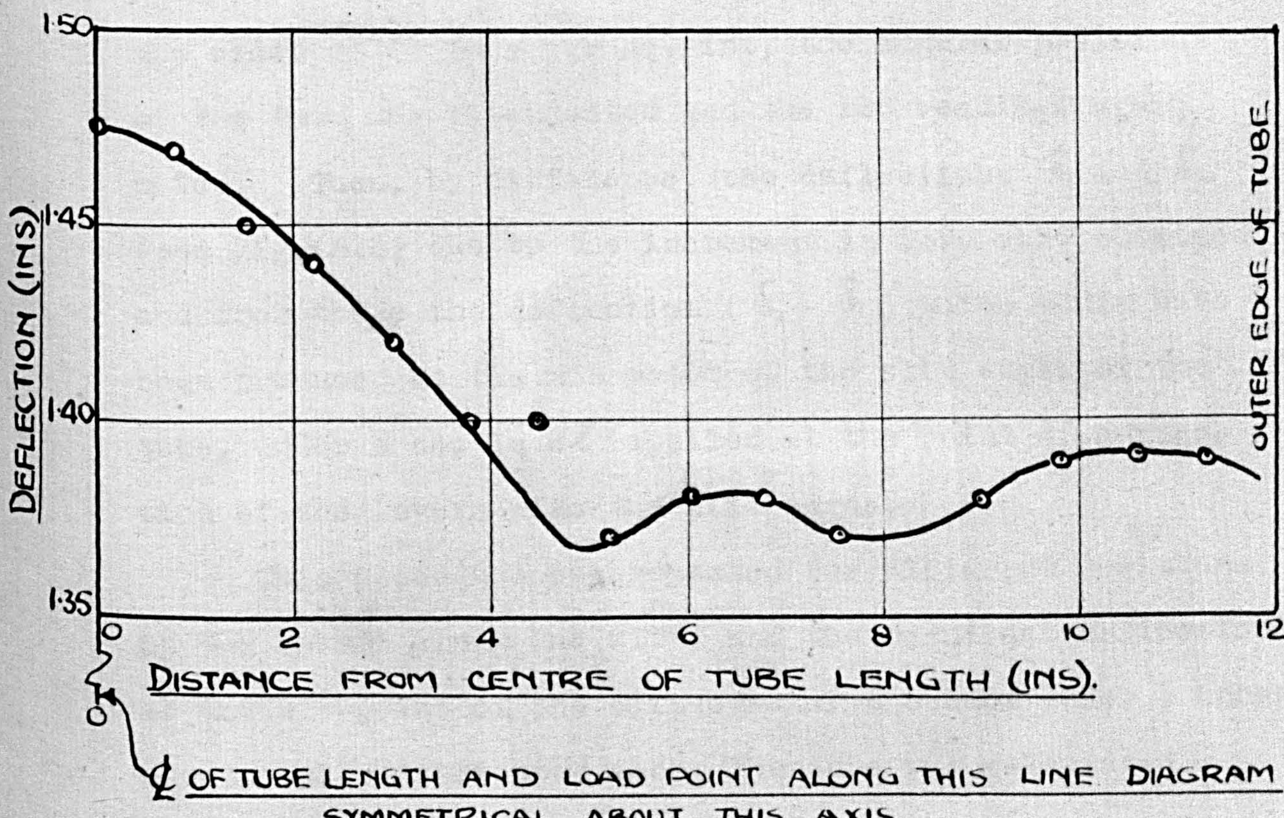


FIG: 6.8. CONTOUR OF SLIT EDGE OF 24" LONG TUBE.

Typical values of the effective couple plotted against the corresponding deflection are shown in Fig. 6.6. This graph shows that the system is elastic and that the relationship is linear and hence it is quite in order to apply the Maxwell Theorem to the result.

For the actual experiments to study the influence of anticlastic bending, in the release of residual stress, a tube was mounted in the apparatus with the levers fitted near one end, and a small load, sufficient to produce clearance between the clamps was applied. Accurate alignment of the loading cables was ensured by bodily movement of the apparatus with respect to the two vertical plumb lines, and the angular position of the tube was adjusted by the cable adjusting attachment so that the centre line of the slit lay in a horizontal plane.

The scale reading of the steel rule and the deflection at the slit were recorded. A further load W was added such as would produce a skin stress in the tube of the order of 10 tons per sq. in., the angular position of the tube was re-adjusted and the two readings again noted. Then, by difference, the deflections δ_1 and δ_2 (see Fig. 6.3) due to the increment in load were obtained and from these the deflection ($\delta_2 - \delta_1$) which would have been produced at the mid point of the slit edges of the tube, under a couple $W\ell$ applied at the point of connection of the levers with the slit edgds.

This procedure was repeated for different positions of the levers along the slit, and the resultant deflection at the mid point of the slit edge of the tube under a known pure couple distributed along the tube was calculated. This procedure was carried out for each of the six tubes under investigation.

The results thus obtained are presented graphically in Fig. 6.7 and the deflected form of the slit edge of the 24 in. long tube under a pure couple acting at its mid-section is shown in Fig. 6.8. The curve in Fig. 6.7 shows clearly that under circumferential stresses anticlastic curvature, which should develop more freely in narrow rings, does not reduce the deflection caused by a given couple per unit length of the walls of a tubular specimen. In fact, the shorter specimens are shown to be less rigid than longer ones and so would produce a greater deflection under a given release of bending stress, an effect which is opposite in sense to the length effect under investigation and which cannot therefore explain it.

As a result of these tests it can be concluded that anticlastic bending under circumferential stresses is not capable of explaining the length effect in the release of residual hoop stresses in tubes.

7. RESIDUAL STRESSES IN HOLLOW DRAWN TUBES.

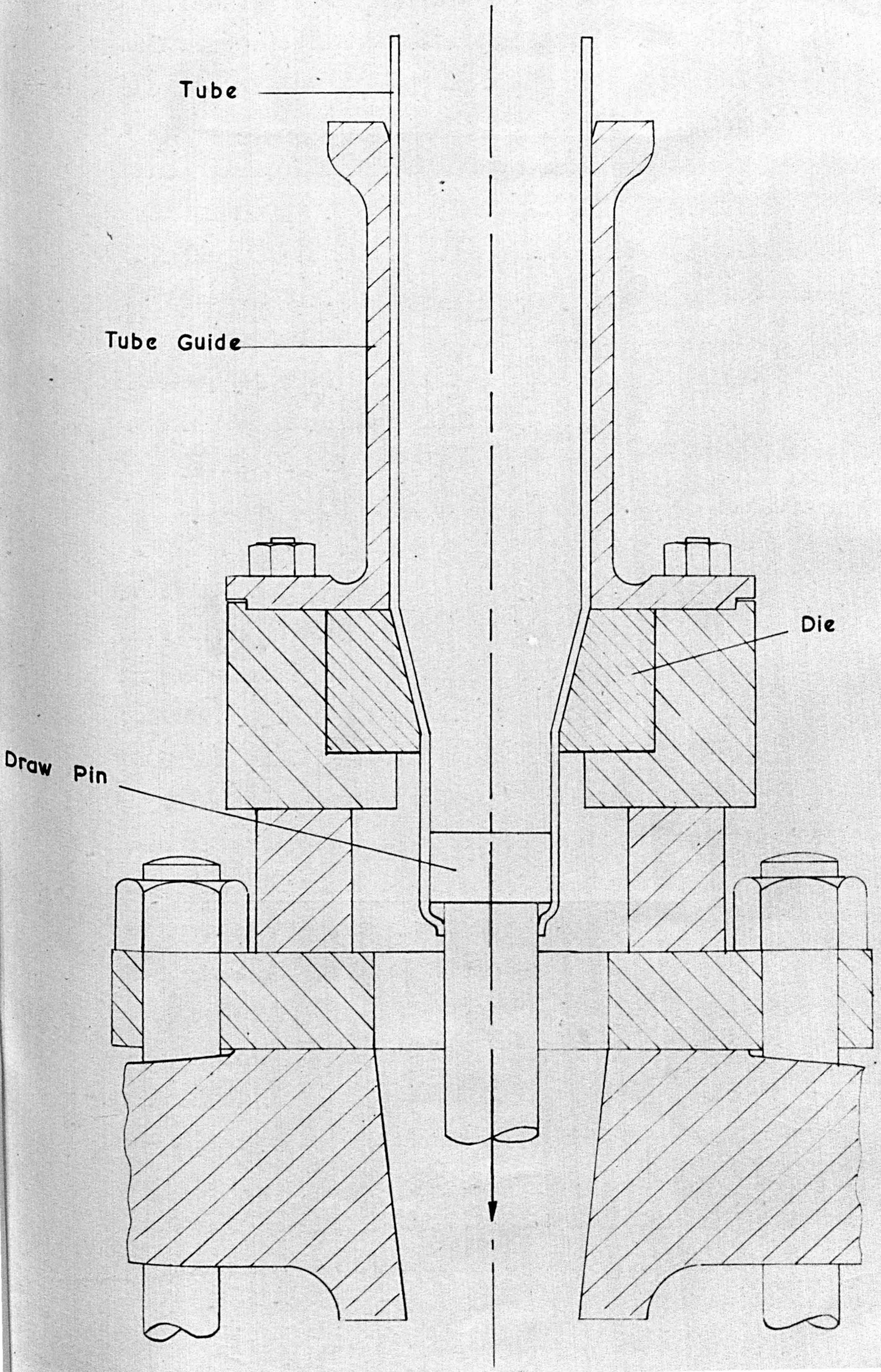
7.1. Introduction.

As mentioned in an earlier section of this work, many investigators have attempted to establish the residual stress distributions in tubes after drawing by various processes. The degrees of success and the accuracy of the results obtained have been largely determined by the nature of the investigations which have generally been limited to the study of residual stresses in specific components e.g. cartridge cases, and have not been concerned with the general aspects of the problem. The work of Knights (1951) is, however, one exception to this statement and he alone appears to have made an attempt to treat the problem in a general way.

The number of variables to be considered in any general study of tube drawing are many in number and must be severely restricted in any one particular investigation carried out by a single investigator, if satisfactory results are to be obtained in a reasonable period of time. Consequently, the work described herein has had to be restricted to a study of the residual stresses which occur in mild steel tubes when they are hollow drawn (commercially "sunk") through conical dies. Although the investigation has concentrated on tubes initially of the same diameter, it is believed that similar stress distribution will obtain in tubes of other diameters which are processed in a similar manner, although the magnitude of the residual stresses may be quite different. The choice of the variables (initial outside diameter and wall thicknesses of tubes, die diameter and taper and drawing lubricant) considered in this investigation were governed by the stock of materials and apparatus available.

Fig. 7.1.a

DIE ARRANGEMENT FOR TUBE DRAWING



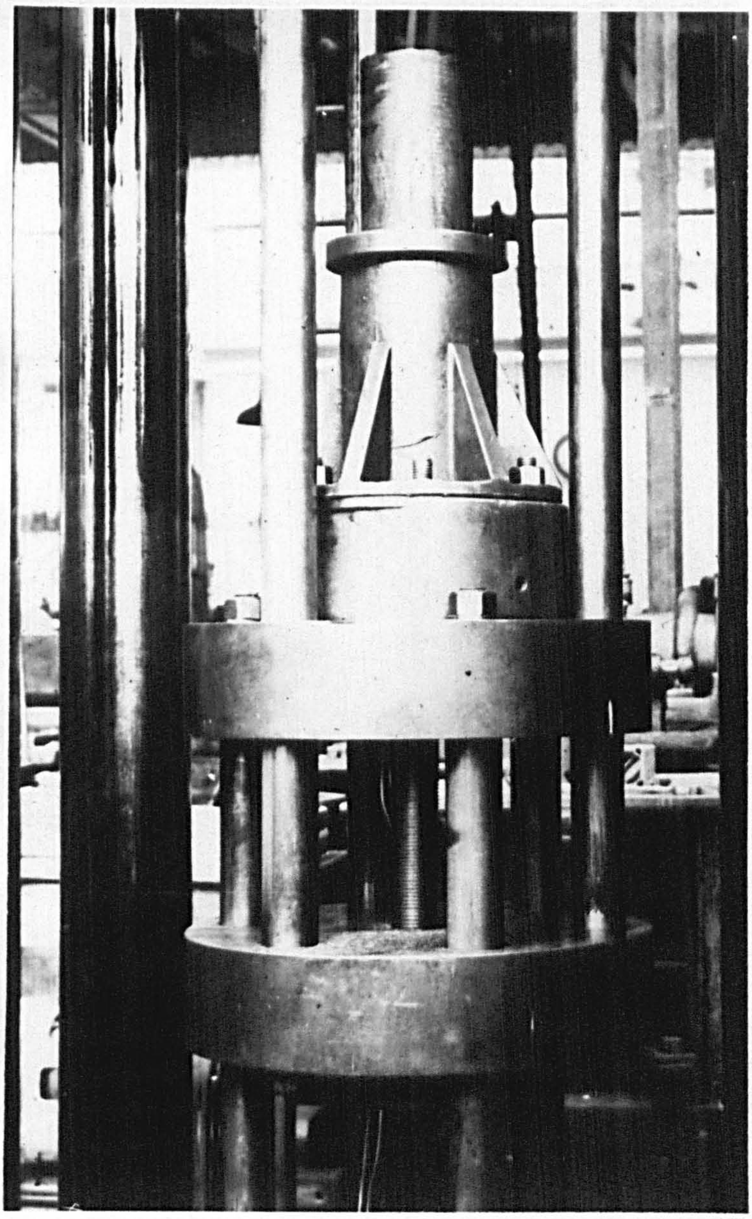


FIG. 7.1b. TUBE DRAWING APPARATUS.

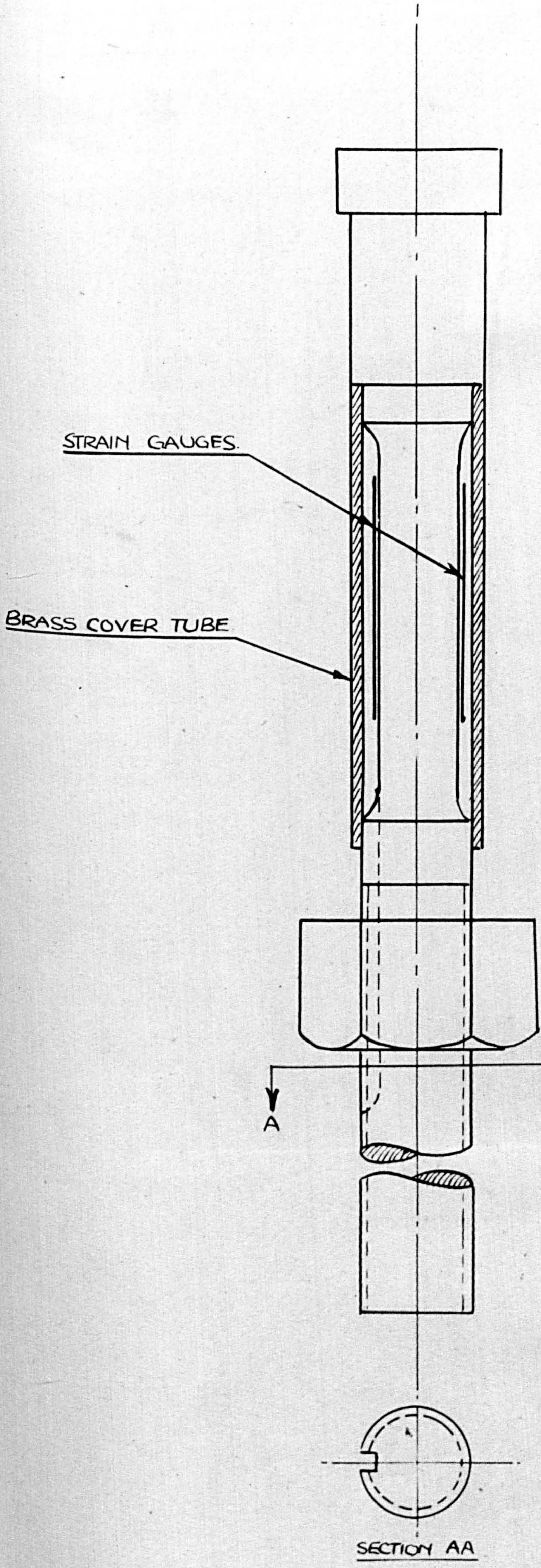


FIG. 7.2.

STRAIN GAUGE MOUNTED DRAWING PIN

7.2. Apparatus, materials and procedures for drawing tests.

All the tubes used in this investigation were hollow drawn through 15° straight taper dies mounted in the apparatus shown in Fig. 7.1.a, which in turn was assembled in a tensile cage attachment fitted in the operating space of a 100 ton long stroke vertically mounted hydraulic press (Fig. 7.1.b). A constant drawing speed of 14 feet per minute was adopted for the tests and except where otherwise stated the tubes were lubricated with a 3 : 1 mixture (by weight) of tallow and graphite. Drawing loads were measured electrically using a micro-ammeter, Wheatstone bridge and the special strain gauge mounted drawing pin shown in Fig. 7.2, the latter being calibrated before use as indicated in Appendix 7. Full details of the drawing procedures and apparatus can be found in reports by Clark and Swift (1945) and by Botros (1949, 1950).

The initial length and outside diameter of the tubes used were 24 in. (approx.) and 4.25 in. respectively and specimens were available in each of the following five wall thicknesses:- 0.50, 0.375, 0.250, 0.125 and 0.08 in. The tubes had all been nozzled before supplying to the University and had been in stock for some considerable time. Before drawing, all the tubes were thoroughly cleaned and then normalised at 920° C for 15 minutes in an electric furnace fitted with a gas curtain to prevent excessive scaling. After heat treatment the tubes were pickled in a commercial phosphate solution to remove all traces of scale and then washed thoroughly in water at 80° C for a period of about 15 minutes. Following a suitable drying period the tubes were coated with the drawing lubricant and processed within 24 hours of heat treatment.

After heat treatment, three tubes with wall thicknesses of 0.50, 0.25 and 0.125 in. were cut open and tensile specimens to B.S. 18 were prepared from them, three specimens being cut from each tube. These specimens

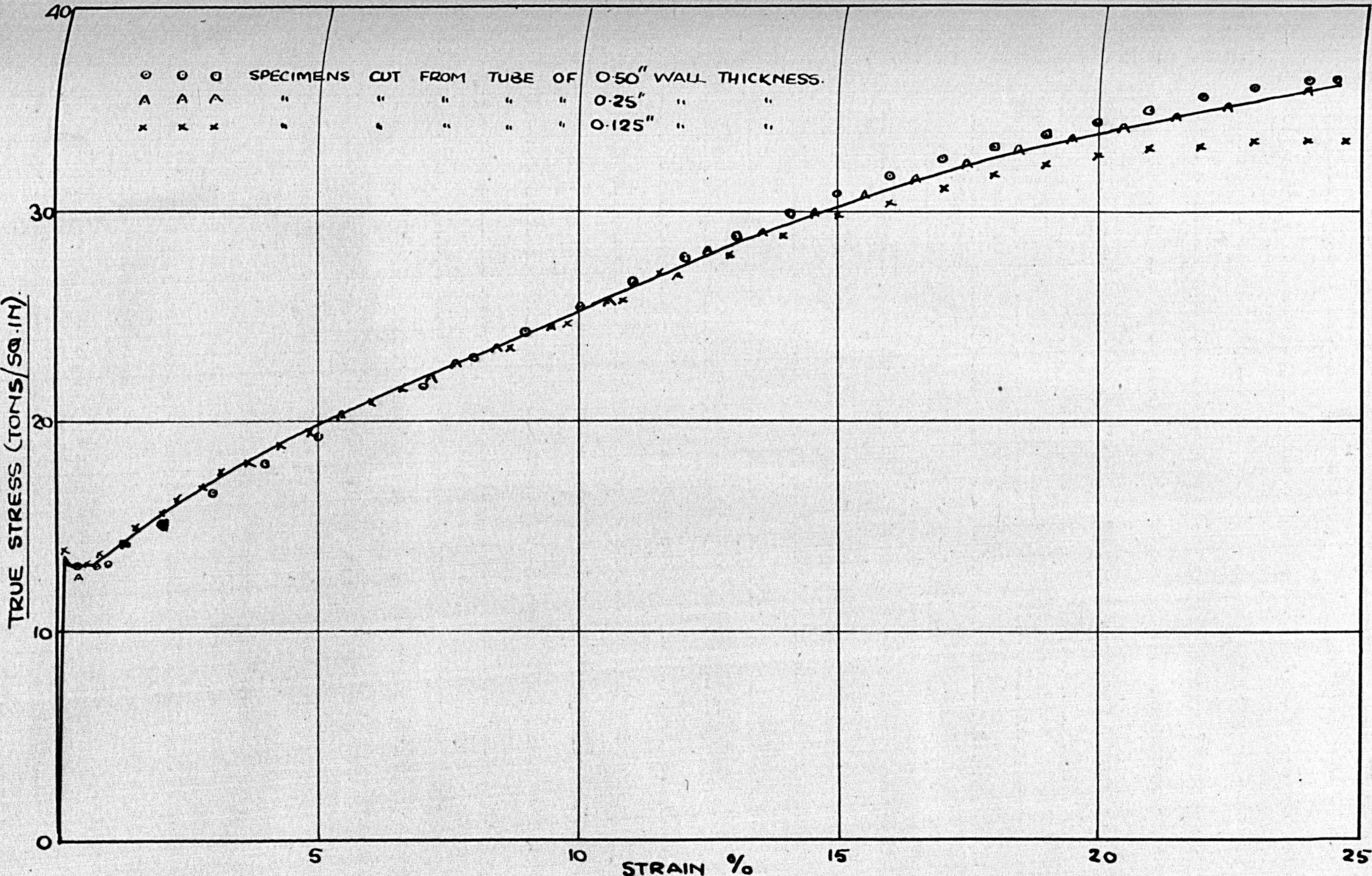


FIG: 7.3. STRESS/STRAIN CHARACTERISTICS OF TUBE MATERIAL AFTER NORMALISING.

WALL THICKNESS (ins)	HARDNESS (VPN).	LOWER YIELD STRESS. (tons/sq in)	AT INCIDENCE OF "NECKING"	
			STRESS (tons/sq in)	STRAIN (%)
0·50	113	13·0	36·3	24·8
0·25	110	12·7	34·8	22·6
0·125	115	13·2	33·5	25·2

TABLE: 7. I. PROPERTIES OF TUBE MATERIAL AFTER NORMALISING.

TUBE REFERENCE	INITIAL OUTSIDE DIAMETER (ins).	INITIAL WALL THICKNESS (ins).	FINAL OUTSIDE DIAMETER (ins)	FINAL WALL THICKNESS (ins)	DRAWING LOAD (tons)	RED^N IN AREA (%)	AVERAGE STRAIN. %		
							THICKNESS ε_t	AXIAL ε_l	HOOP ε_h
A. 1	4·251	·500	3·375	·503	60·2	22·9	0·60	29·7	-23·8
A. 2	4·251	·377	3·374	·384	49·4	21·3	1·87	27·1	-23·0
A. 3	4·246	·250	3·376	·260	33·5	19·0	4·00	23·5	-22·4
A. 4	4.248	·124	3·375	·138	22·6	12·8	11·20	14·7	-21·6
B. 5	4·250	·499	3·502	·489	50·0	21·9	-2·00	27·8	-20·0
B. 6	4·255	·375	3·502	·370	37·8	20·3	-1·33	25·5	-19·4
B. 7	4·249	·250	3·501	·250	26·7	18·7	0	23·0	-18·8
B. 8	4·248	·124	3·502	·129	17·4	15·1	4·00	17·7	-18·3
B. 9	4·251	·081	3·500	·088	9·8	11·1	8·75	12·5	-18·2
C. 10	4·251	·501	3·627	·485	40·3	18·9	-3·20	23·2	-16·5
C. 11	4·255	·372	3·626	·361	30·25	18·4	-2·93	22·6	-16·0
C. 12	4·258	·249	3·623	·243	21·0	18·0	-2·40	21·8	-15·6
C. 13	4·246	·125	3·627	·124	11·2	15·9	-0·80	18·9	-15·2
C. 14	4·250.	·080	3·625	·081	7·85	13·8	1·25	16·1	-15·0

TABLE: 7. II. SUMMARY OF RESULTS OF DRAWING TESTS. (MILD STEEL TUBES. GRAPHITE & TALLOW LUBRICATION).

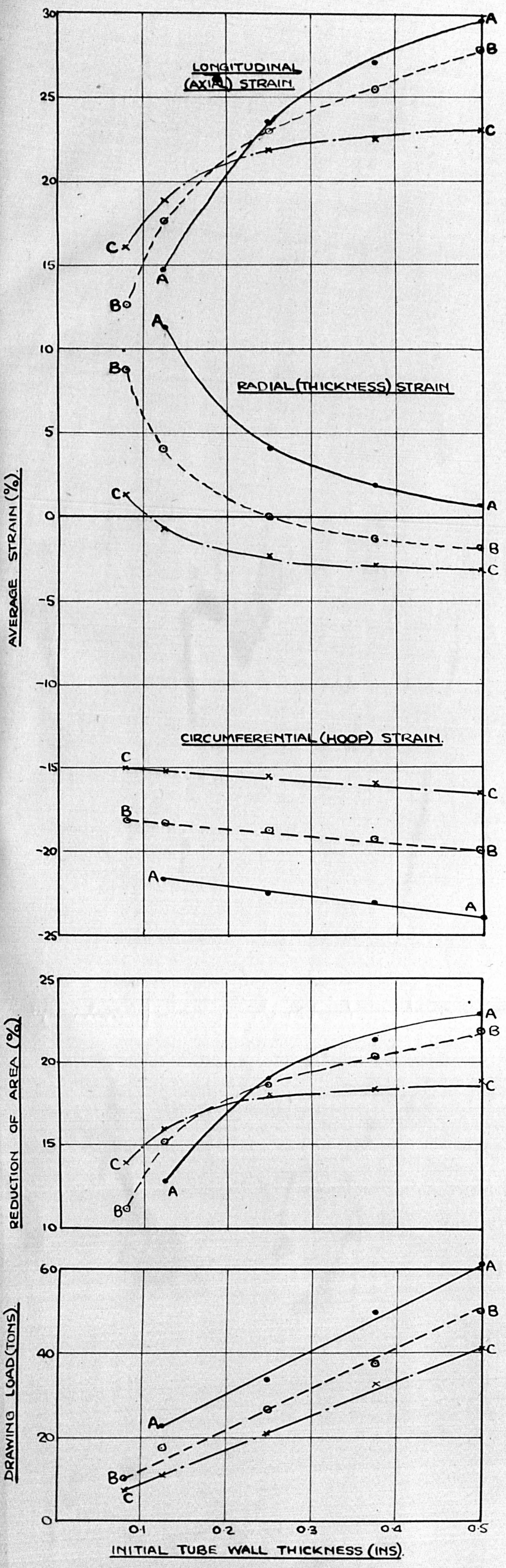


FIG. 7.4. SUMMARY OF THE RESULTS OF THE DRAWING TESTS.

were tested in a motorised Hounsfield Tensometer using a Lindley dial gauge extensometer for recording strains up to 5% and rule and dividers thereafter, and the resultant average true stress - conventional engineering strain curve up to the point at which necking commences (see Nadài, 1950) is shown in Fig. 7.3. Slight differences in the stress - strain curves obtained for the three different thicknesses were observed but these are not considered significant in the present investigation and the general similarity of the curves coupled with the similarity of the hardness of the specimens (see upper part of Table 7.I) has been taken as an indication that the tubes were prepared from a similar quality of steel.

Before drawing, all the relevant dimensions (outside diameter, bore diameter and wall thickness) of each tube were measured and three circumferential rings at 6 in. axial spacing were marked on the tubes. After drawing through one of the three dies with throat diameters of 3.375, 3.500 and 3.625 in. all the tube dimensions were again checked and from these measurements average radial (thickness), circumferential (hoop) and longitudinal (axial) strains and the reduction in area of the tube wall sections were computed. Although this investigation was not primarily concerned with the development of strains in tube drawing these results are summarized in Table 7.II and Fig. 7.4. The results actually represent an extension of the work carried out by Botros (1950) and the chained curve in each group of curves in Fig. 7.4 are very similar to those he obtained. (No work has previously been undertaken on the other two reductions).

(In Table 7.II and in the succeeding three tables the references quoted for the materials all follow the same pattern - a letter A, B or C is prefixed to the serial number of the tube to indicate whether the tube

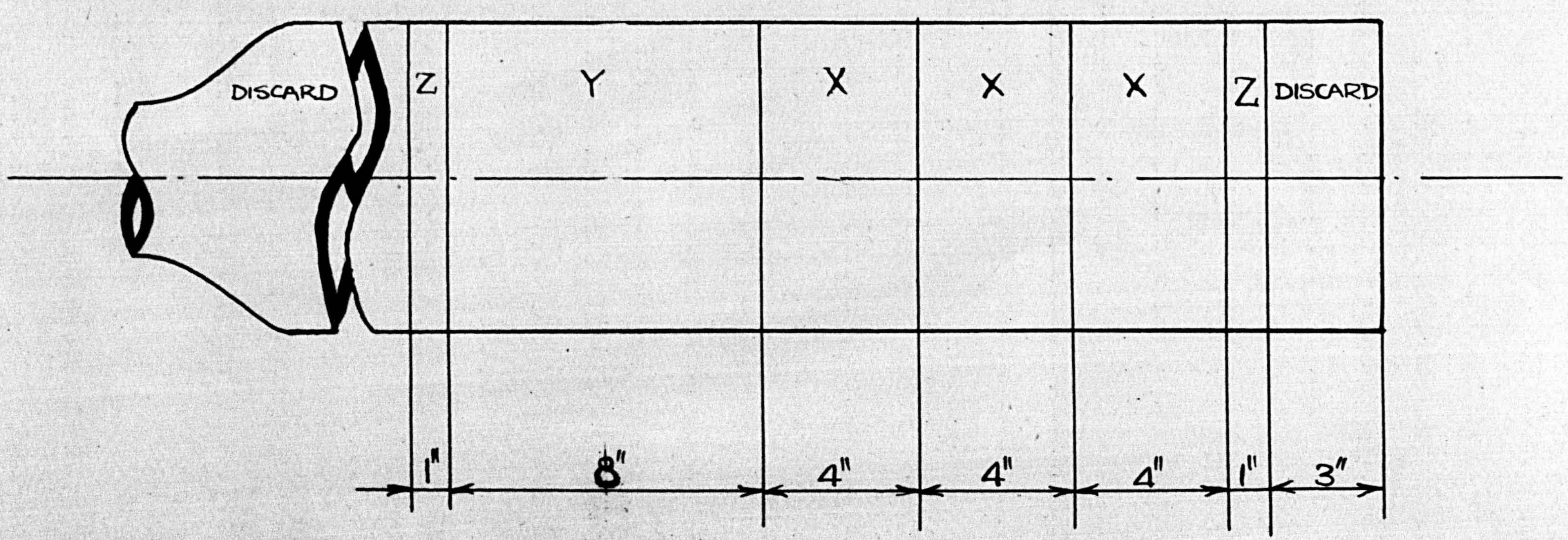
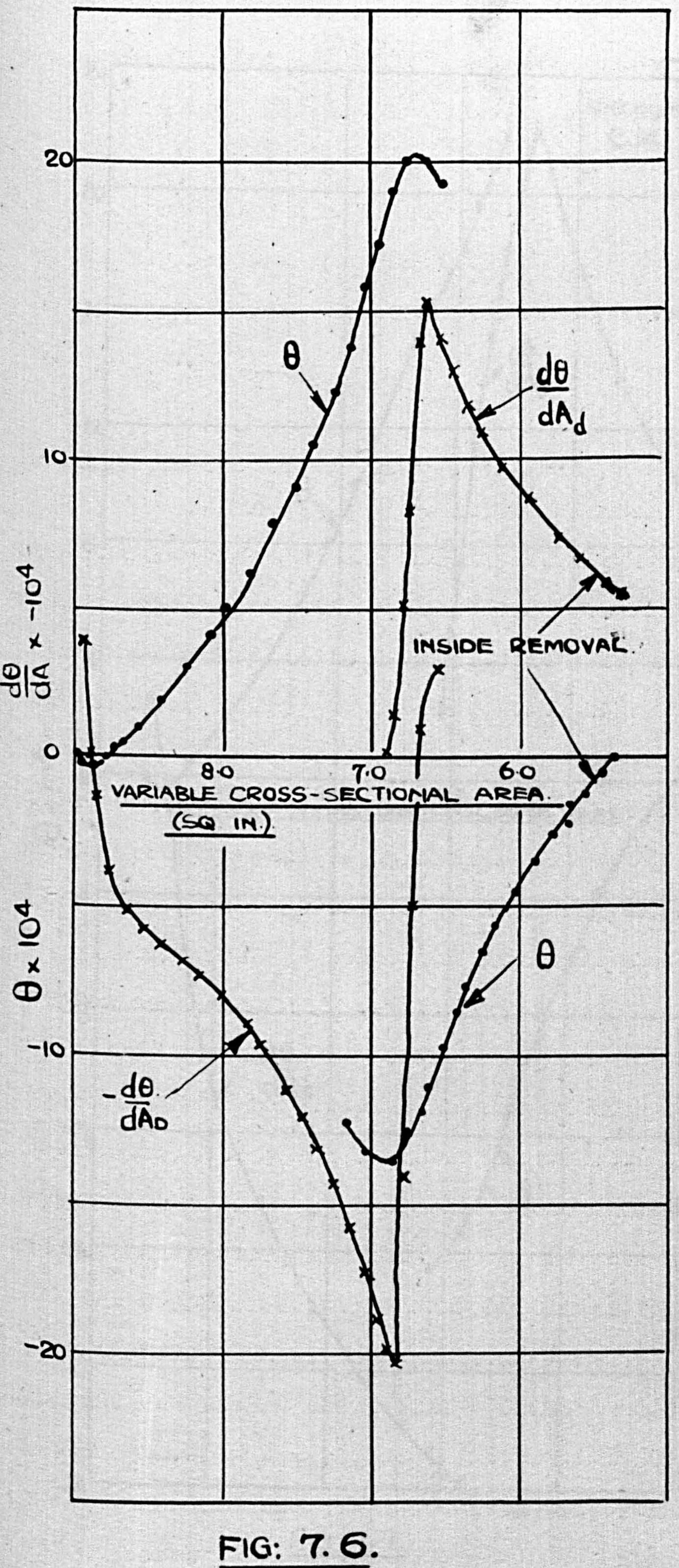


FIG: 7.5. LOCATION OF SPECIMENS ON HOLLOW DRAWN TUBES was drawn through the 3.375, 3.500 or the 3.625 in. diameter die respectively. All tubes with serial numbers up to and including 14 were drawn using tallow and graphite as lubricant and those with higher serial numbers were drawn with other lubricants. In the diagrams the denoting of a curve by the reference AA, BB or CC signify that the results apply to tubes drawn through the 3.375, 3.500 or 3.625 in. diameter die respectively).

7.3. Residual stress investigations.

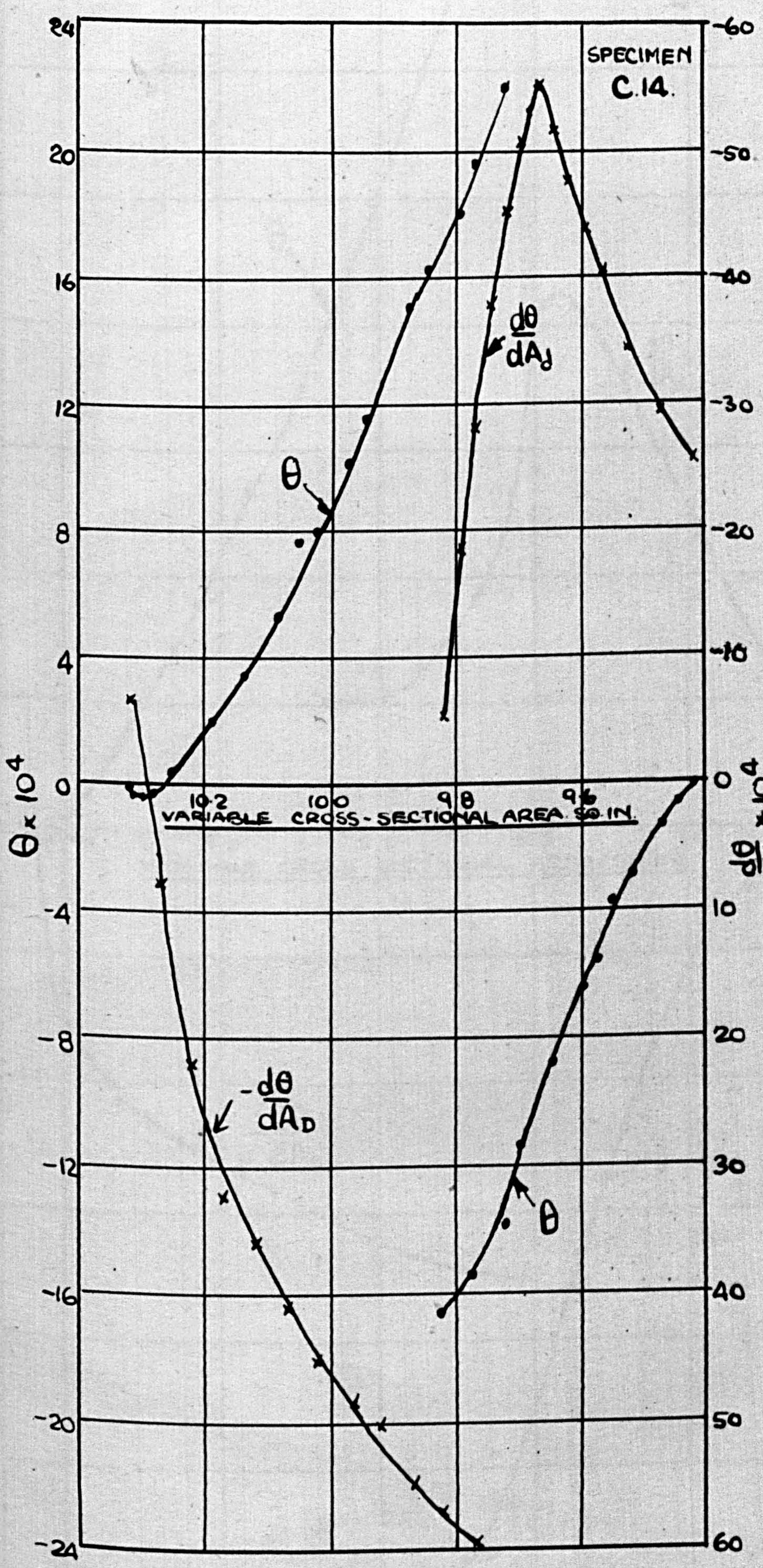
(Since the supply of tubes was rather limited and it was realised that several identical specimens are necessary to obtain the residual stress distributions by bending deflection methods (see Section 3) it was decided to apply the boring test techniques (Section 4) to the study of residual stresses for the present programme. Simple bending deflection tests were, however, carried out on specimens prepared from each tube although these were not subjected to a full analysis).

After drawing each tube was sectioned into eight samples as shown in Fig. 7.5. The two end pieces were discarded, and the two one inch lengths (Z) taken from each end of the tube were slit along their thinnest wall section and the sprung diameter measured. No two such samples taken from any one tube showed variations in change in diameter on slitting of more then 3.3% and this was considered sufficient justification for assuming a constant residual stress pattern along the length of the tube from which the actual specimens were taken. Two of the 4 in. lengths (X) were fitted with strain gauges (one to the bore surface and one to the outside surface) and subjected to direct strain release tests in accordance with the techniques and procedure described in Section 4. The remaining 4 in. specimen was slit longitudinally along



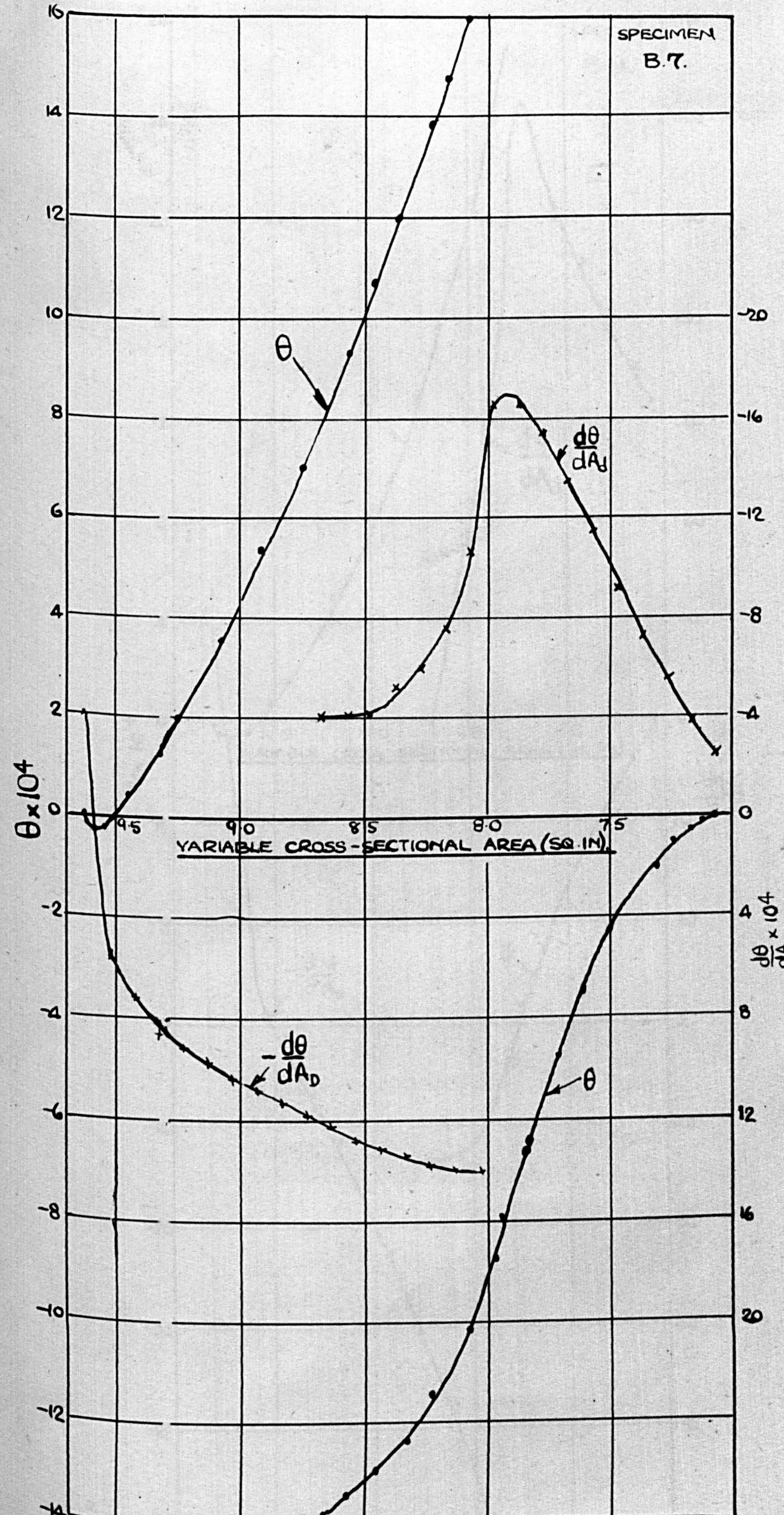
20
10
0
-10
-20
$\frac{d\theta}{dA} \times -10^4$
$\theta \times 10^4$
8.0
7.0
6.0
VARIABLE CROSS-SECTIONAL AREA.
(SQ IN.)
θ
$\frac{d\theta}{dA_d}$
INSIDE REMOVAL.
θ
$-\frac{d\theta}{dA_D}$

FIG: 7. 6.



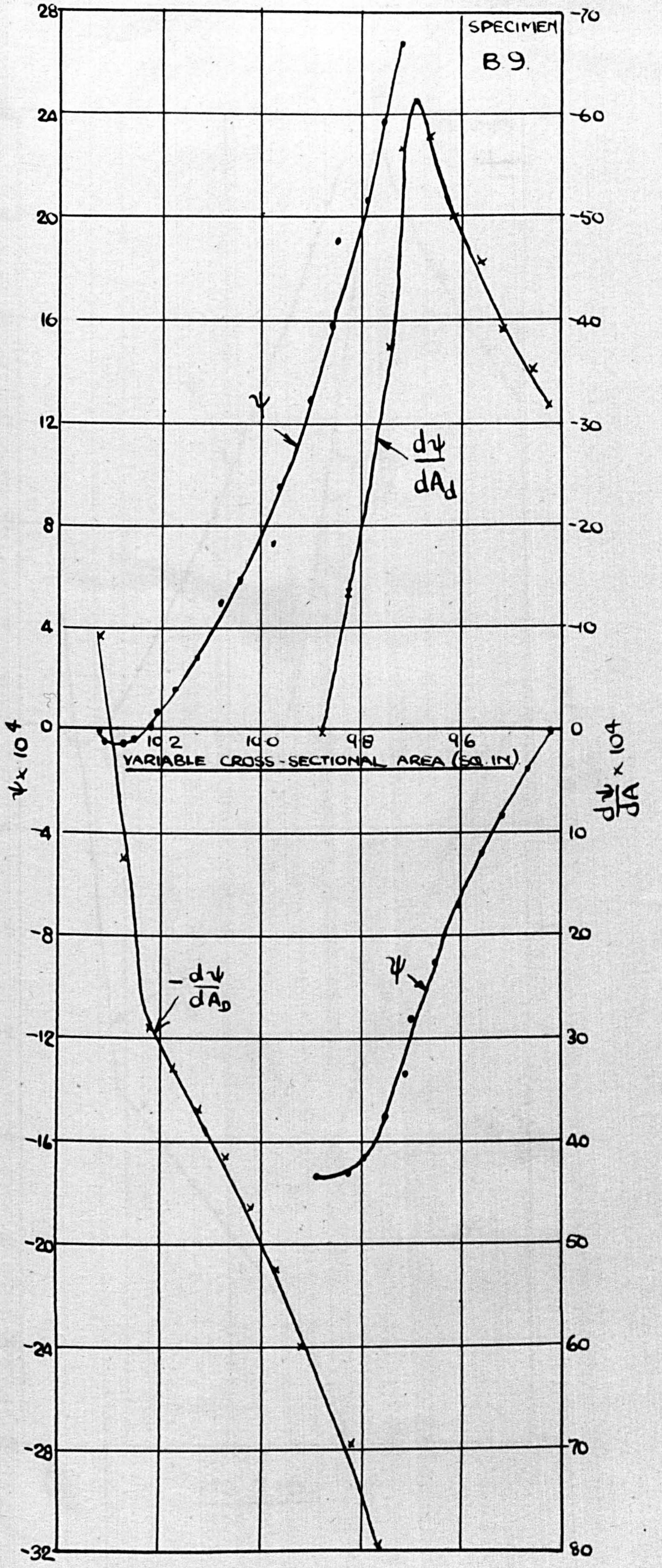
SPECIMEN
C.14.
24
20
16
12
8
4
0
-4
-8
-12
-16
-20
-24
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
θ x 10⁴
dθ/dA x 10⁴
10·2
10·0
9·8
9·6
VARIABLE CROSS-SECTIONAL AREA SQ.IN.
dθ/dA_J
θ
-dθ/dA_D
θ

FIG. 7.7.



SPECIMEN
B.7.
θ × 10^4
dθ/dA × 10^4
VARIABLE CROSS-SECTIONAL AREA (SQ. IN)
9.5
9.0
8.5
8.0
7.5
θ
dθ/dA_I
-dθ/dA_D
θ
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-20
-16
-12
-8
-4
0
4
8
12
16
20

FIG: 7.8.



SPECIMEN
B 9.
ψ
dψ/dA_d
ψ × 10^4
dψ/dA × 10^4
VARIABLE CROSS-SECTIONAL AREA (SQ. IN.)
10.2
10.0
9.8
9.6
ψ
−dψ/dA_D
28
24
20
16
12
8
4
0
-4
-8
-12
-16
-20
-24
-28
-32
-70
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
80

FIG. 7.9.

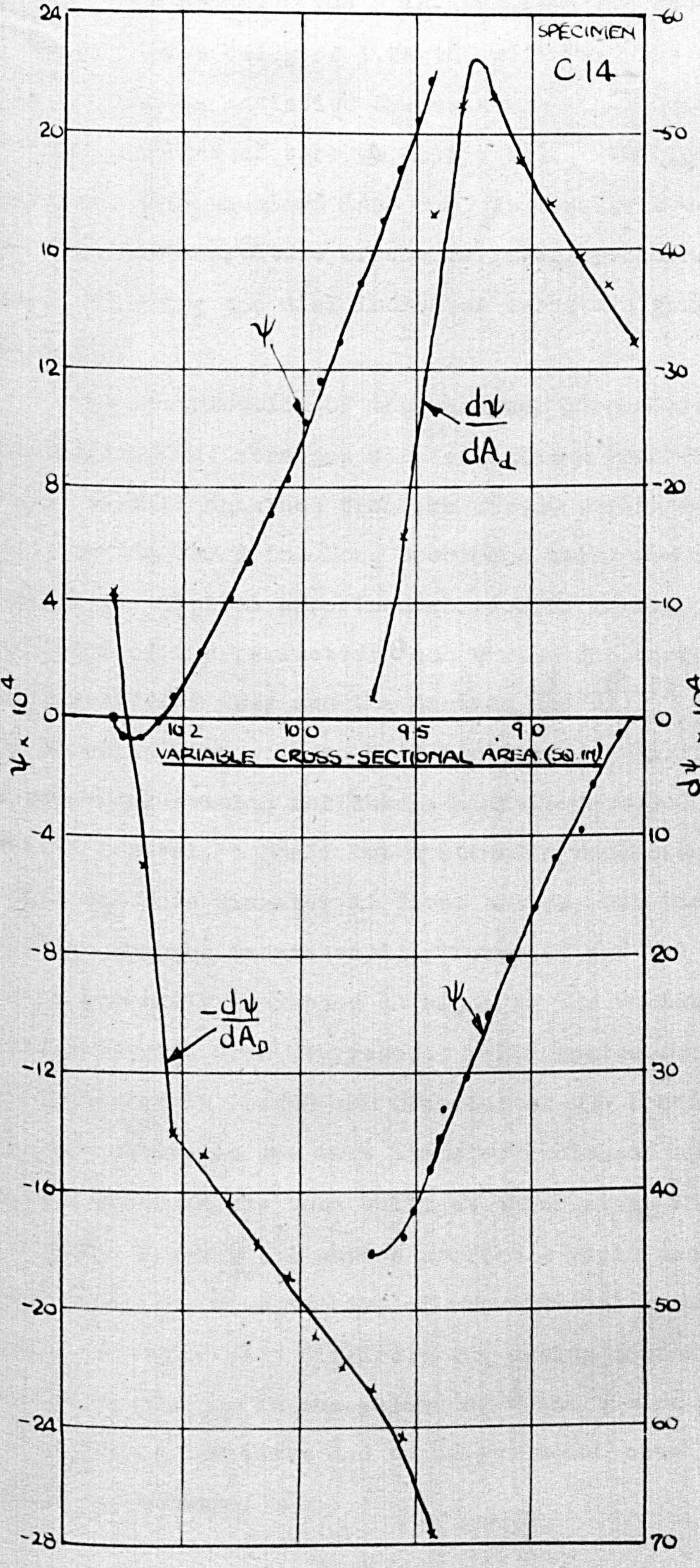
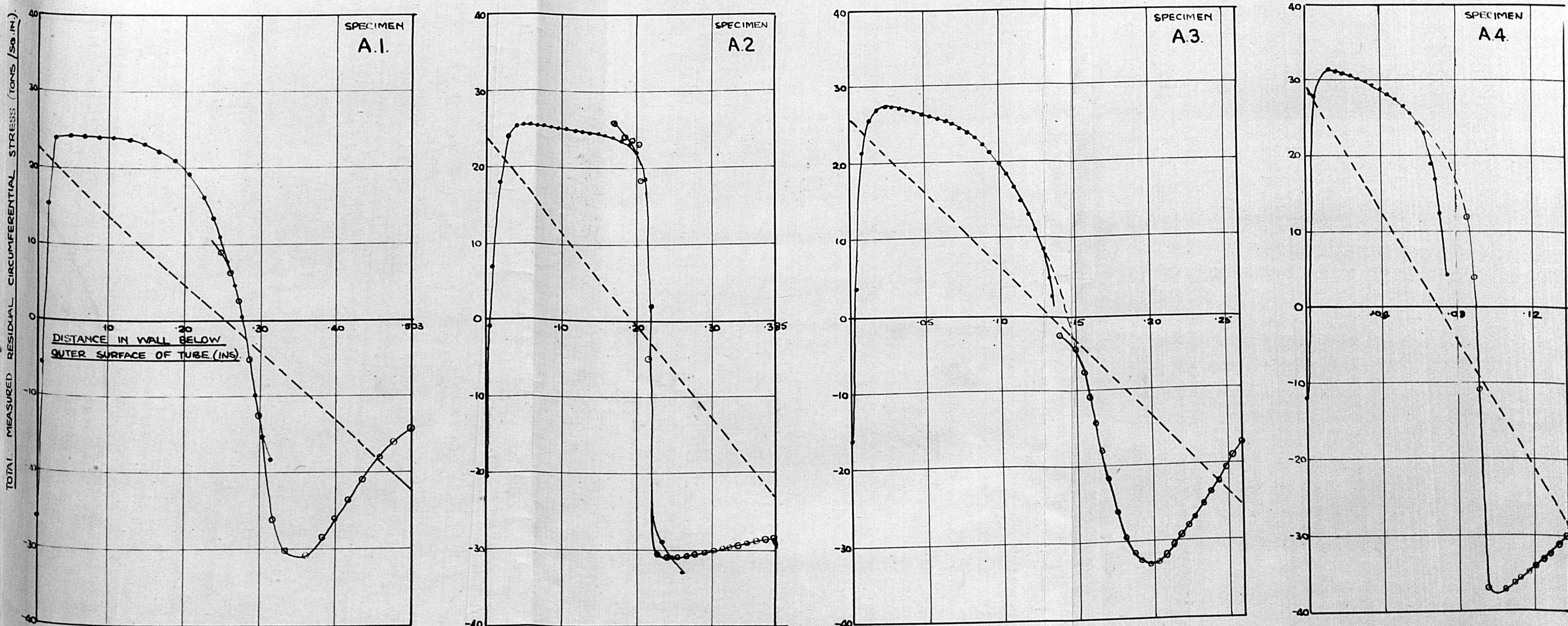

SPECIMEN
C14
ψ
dψ/dA_d
-dψ/dA_D
VARIABLE CROSS-SECTIONAL AREA (SQ. IN)
10.2
10.0
9.5
9.0
ψ × 10^4
dψ/dA × 10^4
24
20
16
12
8
4
0
-4
-8
-12
-16
-20
-24
-28
-60
-50
-40
-30
-20
-10
10
20
30
40
50
60
70

FIG 7.10

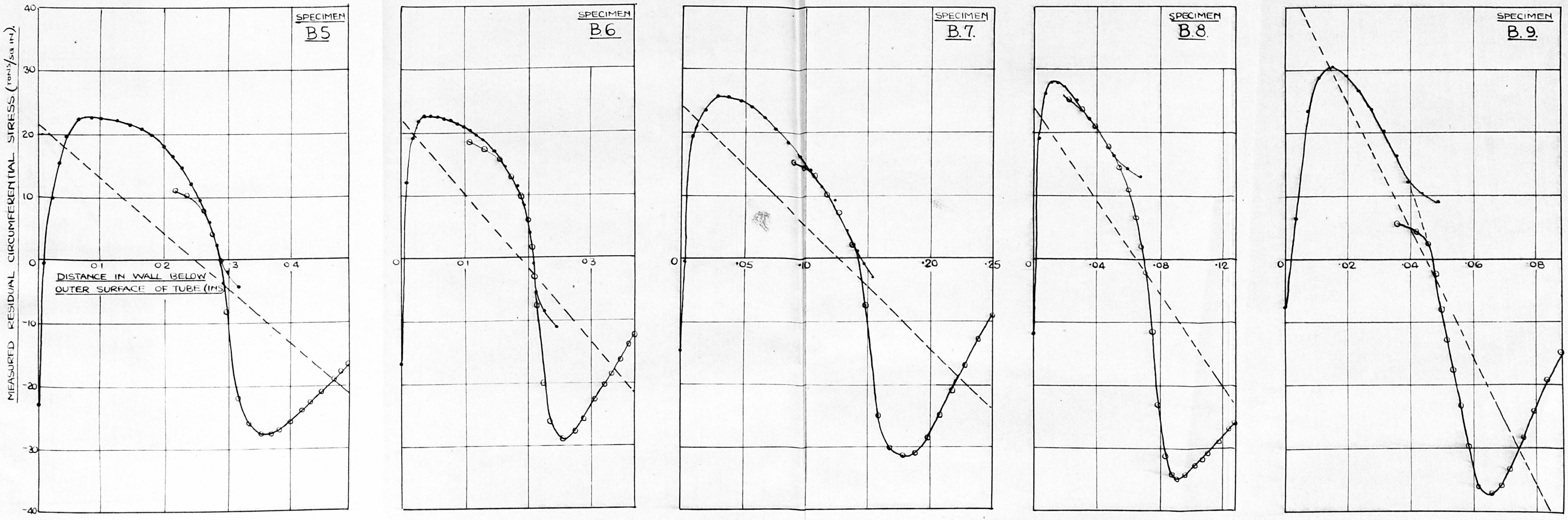
its thinnest wall section and the change in diameter on slitting was noted. The 8 in. specimen (Y) was slit into 6 strips, three being of 1.25 in. width and the others of such a width as satisfied the relationship:- width of strip : diameter of tube $\simeq$ 0.15 : 1.0. The $1\frac{1}{4}$ in. wide specimens were machined into B.S. 18 tensile specimens and the resultant curvature of the narrower specimens was measured using the dial indicator curvature gauge (see Section 2).

The distribution of the residual circumferential and longitudinal stresses were calculated from the experimental results obtained from the direct strain release tests on the two 4 in. long specimens using the Sachs analysis. Typical experimental results showing the variation of the parameters θ and ψ with the variable cross sectional area and the derived $\frac{d\theta}{dA}$, $\frac{d\psi}{dA}$ - A curves are shown in Figs. 7.6 to 7.10 inclusive. All the experimental results followed a similar pattern. With outside removal, a small but systematic reduction in length and bore diameter at first ensued, but the sense of these changes became rapidly reversed and the curves generally increased in slope as the variable cross-sectional area decreased. With inside removal the reduction in length and diameter of the specimens was progressive as the bore diameter increased up to a certain point in the tube wall, at which stage the slope of the θ - A and ψ - A curves started a rapid decline. (At this stage the obtaining of experimental results became generally very difficult suggesting that large and rapid changes in the values of θ and ψ were possible and generally the tests had to be abandoned once this region was reached.



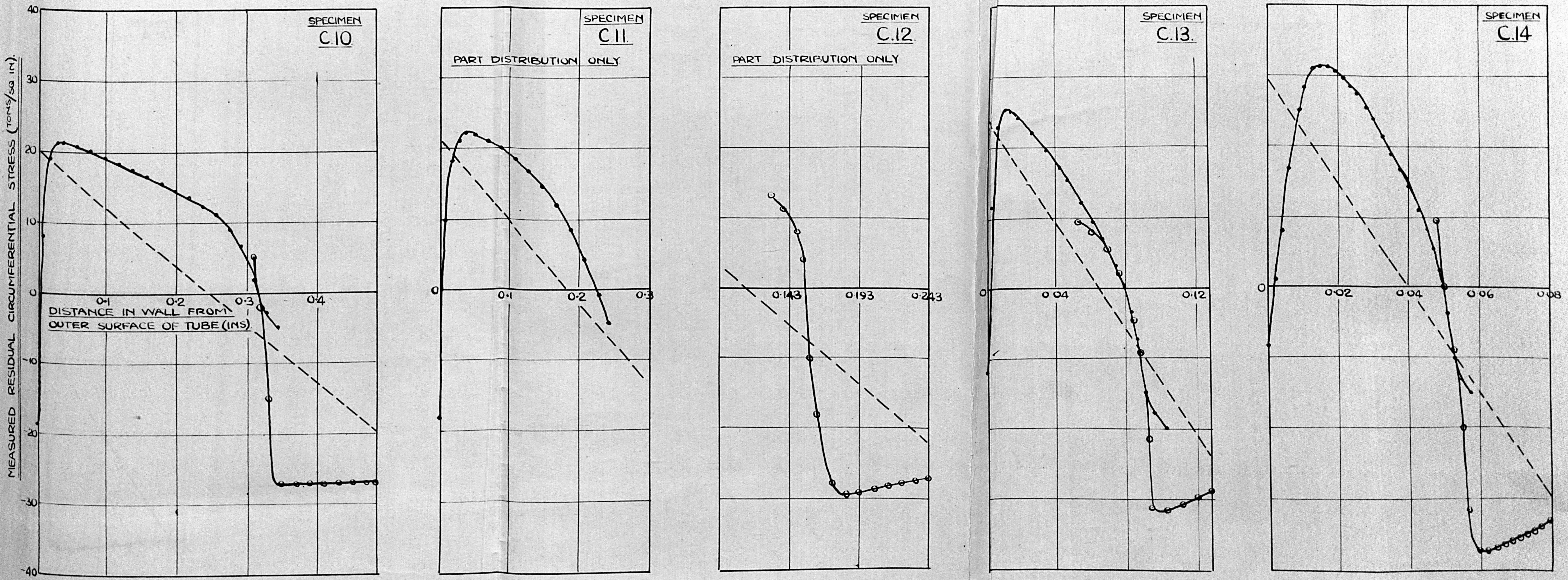
NOTE DIFFERENCES IN THE HORIZONTAL (DEPTH) SCALE.

FIG: 7.11. RESIDUAL CIRCUMFERENTIAL STRESSES. (3.375" DIA: TUBES)



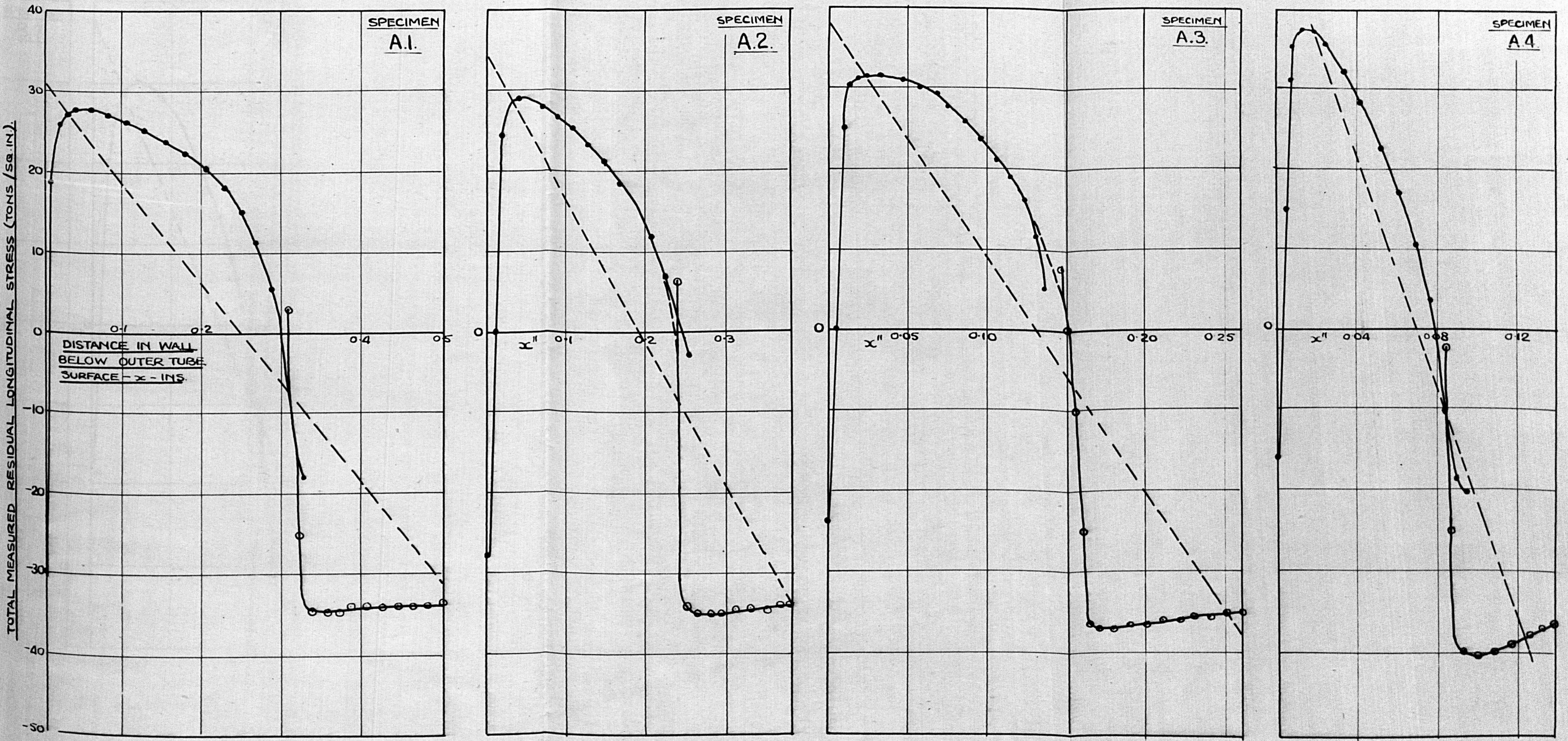
NOTE. THE HORIZONTAL (DISTANCE) SCALES ARE NOT THE SAME ON EACH DIAGRAM.

FIG: 7.12. RESIDUAL CIRCUMFERENTIAL STRESSES (3.50" DIA TUBES).



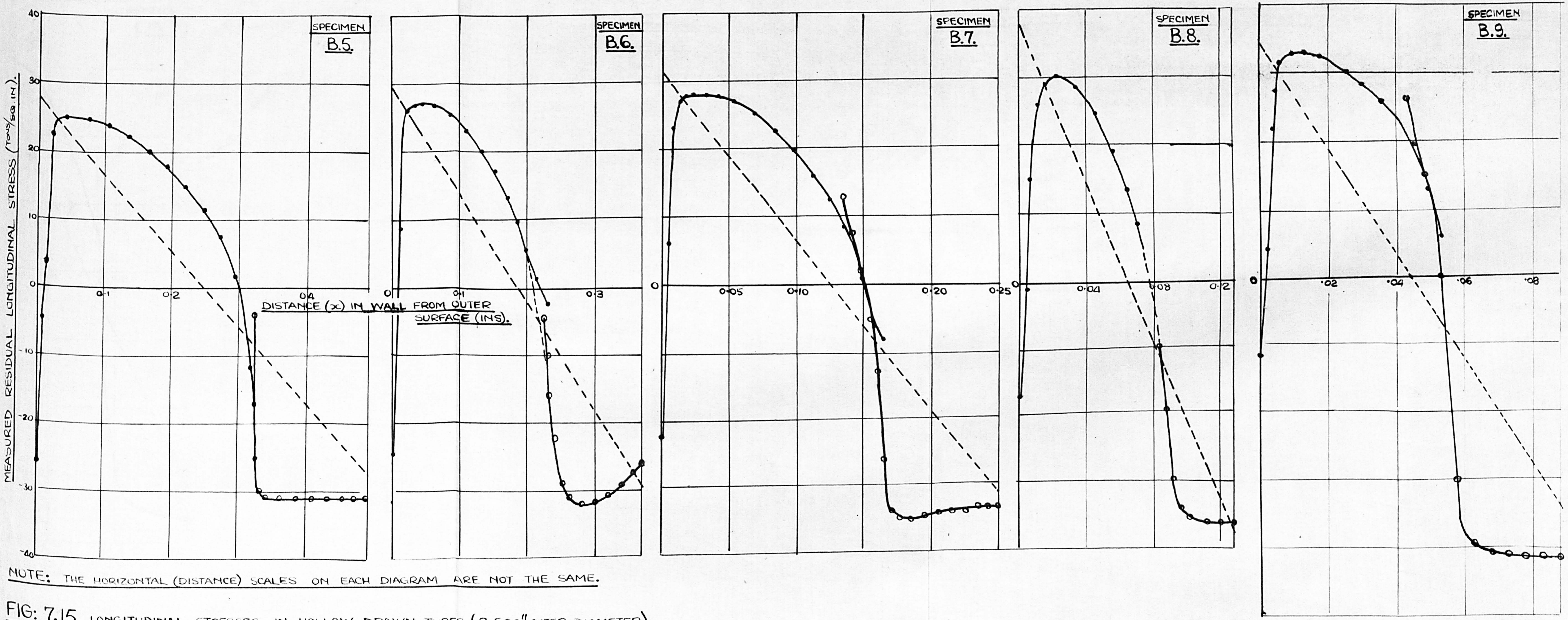
NOTE. THE HORIZONTAL (DISTANCE) SCALES ARE NOT THE SAME ON EACH DIAGRAM

FIG: 7.13. RESIDUAL CIRCUMFERENTIAL STRESSES (3.625" DIA TUBES).



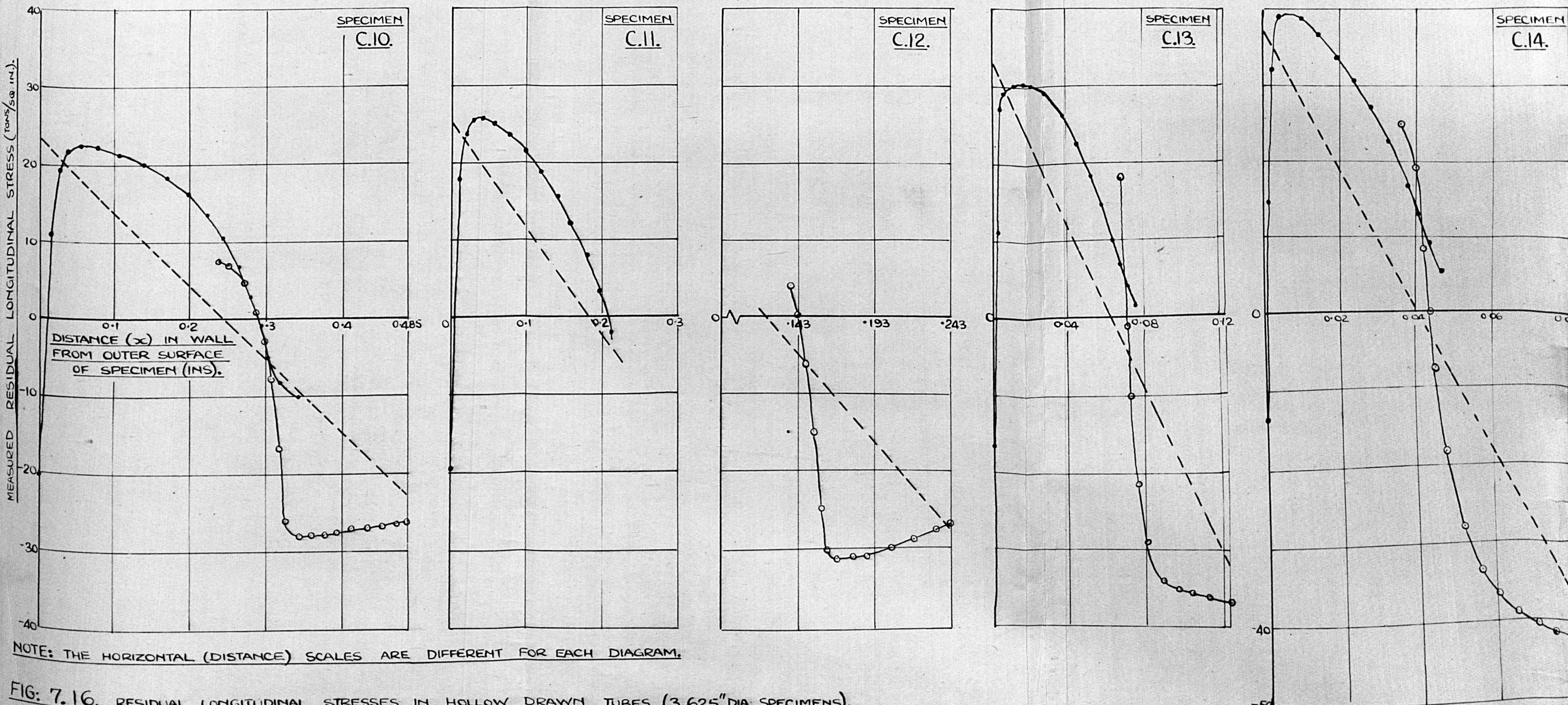
NOTE: HORIZONTAL (DISTANCE) SCALES ARE DIFFERENT ON EACH DIAGRAM.

FIG. 7.14. RESIDUAL LONGITUDINAL STRESSES. (3.375" ϕ HOLLOW DRAWN TUBES)



NOTE: THE HORIZONTAL (DISTANCE) SCALES ON EACH DIAGRAM ARE NOT THE SAME.

FIG: 7.15. LONGITUDINAL STRESSES IN HOLLOW DRAWN TUBES (3.500" OUTER DIAMETER).



NOTE: THE HORIZONTAL (DISTANCE) SCALES ARE DIFFERENT FOR EACH DIAGRAM.

FIG: 7.16. RESIDUAL LONGITUDINAL STRESSES IN HOLLOW DRAWN TUBES. (3.625" DIA SPECIMENS).

	BENDING DEFLECTION TESTS.					BORING (DIRECT STRAIN RELEASE) TESTS.							
TUBE REFERENCE	DIAMETRAL CHANGE ON SLITTING. (ins).	UNIT DEFLECTION ON SLITTING (ins).	WIDTH OF STRIP. (ins).	RELEASED SKIN STRESS. (tons/sq in)	RELEASED MOMENT. (tons in/in)	STRESS AT OUTER SURFACE (tons/sq in)	MAXIMUM TENSILE STRESS (tons/sq in)	MAXIMUM COMPRES-SIVE STRESS (tons/sq in)	STRESS AT INNER SURFACE (tons/sq in).	RESIDUAL BENDING COUPLE (tons in/in)	TENSILE FORCE. (tons/in)	COMPRESSIVE FORCE (tons/in)	NET FORCE (tons/in)
(a) CIRCUMFERENTIAL STRESSES.													
A. 1	·026	–	–	22·6	0·955	–26·0	24·0	–31·0	–14·0	1·032	5·08	5·06	0·02
A. 2	·029	–	–	23·4	0·575	–20·4	25·3	–31·1	–29·7	0·890	4·80	4·92	–0·12
A. 3	·068	–	–	25·6	0·289	–16·1	27·4	–33·1	–17·0	0·310	2·85	2·76	0·09
A. 4	·159	–	–	28·8	0·091	–12·0	31·3	–38·0	–30·0	0·107	1·98	1·74	0·24
B. 5	·028	–	–	21·5	0·858	–23·0	22·7	–27·5	–15·8	0·920	4·78	4·70	0·08
B. 6	·041	–	–	22·0	0·498	–16·8	22·9	–29·0	–12·0	0·530	3·15	3·25	–0·10
B. 7	·072	–	–	24·0	0·250	–14·6	25·7	–31·2	–9·0	0·274	2·55	2·48	0·07
B. 8	·168	–	–	26·0	0·072	–11·8	28·0	–34·9	–26·0	0·079	1·64	1·52	0·12
B. 9	·482	–	–	45·0	0·066	–7·5	30·4	–37·4	–15·0	0·041	0·97	0·94	0·03
C. 10	·029	–	–	20·0	0·785	–18·7	21·4	–26·9	–26·9	0·820	4·40	4·38	0·02
C. 11	·0435	–	–	20·9	0·455	–18·0	22·1	*	*	*	3·10	*	*
C. 12	·074	–	–	22·0	0·217	*	*	–29·7	–27·0	*	*	2·32	*
C. 13	·174	–	–	24·0	0·062	–12·0	25·4	–31·7	–29·0	0·063	1·16	1·11	0·05
C. 14	·390	–	–	29·5	0·032	–8·3	31·6	–37·5	–33·0	0·036	0·93	0·92	0·01
(b) LONGITUDINAL STRESSES.													
A. 1	–	·0190	·50	30·8	1·285	–30·0	28·0	–34·5	–34·0	1·645	6·20	6·29	–0·09
A. 2	–	·0281	·50	34·1	0·800	–28·0	29·7	–35·0	–34·4	0·995	4·85	4·92	–0·07
A. 3	–	·0473	·50	38·4	0·400	–23·7	32·0	–36·8	–35·0	0·501	3·30	3·41	–0·11
A. 4	–	·1215	·50	47·6	0·124	–16·0	37·3	–41·5	–37·1	0·161	1·85	1·96	–0·11
B. 5	–	·0169	·525	27·4	1·142	–26·0	25·0	–31·0	–31·0	1·476	5·30	5·36	–0·06
B. 6	–	·0240	·525	29·2	0·682	–24·4	27·1	–32·0	–26·2	0·867	3·85	4·04	–0·19
B. 7	–	·0392	·525	31·8	0·331	–22·1	28·0	–35·0	–33·4	0·430	3·10	3·09	0·01
B. 8	–	·0920	·525	37·9	0·097	–16·9	30·4	–36·1	–36·1	0·122	1·40	1·43	–0·03
B. 9	–	·172	·525	45·2	0·048	–11·8	33·0	–41·5	–41·5	0·046	1·30	1·29	0·01
C. 10	–	·0141	·545	23·0	0·960	–20·3	22·5	–28·0	–26·5	1·212	4·57	4·64	–0·07
C. 11	–	·0208	·545	25·3	0·592	–19·3	26·0	*	*	*	3·56	*	*
C. 12	–	·0301	·545	28·0	0·292	*	*	–31·0	–27·2	*	*	2·69	*
C. 13	–	·0812	·545	33·0	0·086	–16·7	30·2	–36·7	–36·7	0·108	1·58	1·69	–0·11
C. 14	–	·180	·545	47·0	0·050	–13·4	39·6	–42·0	–42·0	0·050	1·22	1·16	0·06

* THESE VALUES COULD NOT BE OBTAINED BECAUSE OF FAILURE OF STRAIN GAUGES ON THE RELEVANT SPECIMENS.

TABLE: 7. III. SUMMARY OF RESIDUAL STRESS TESTS.

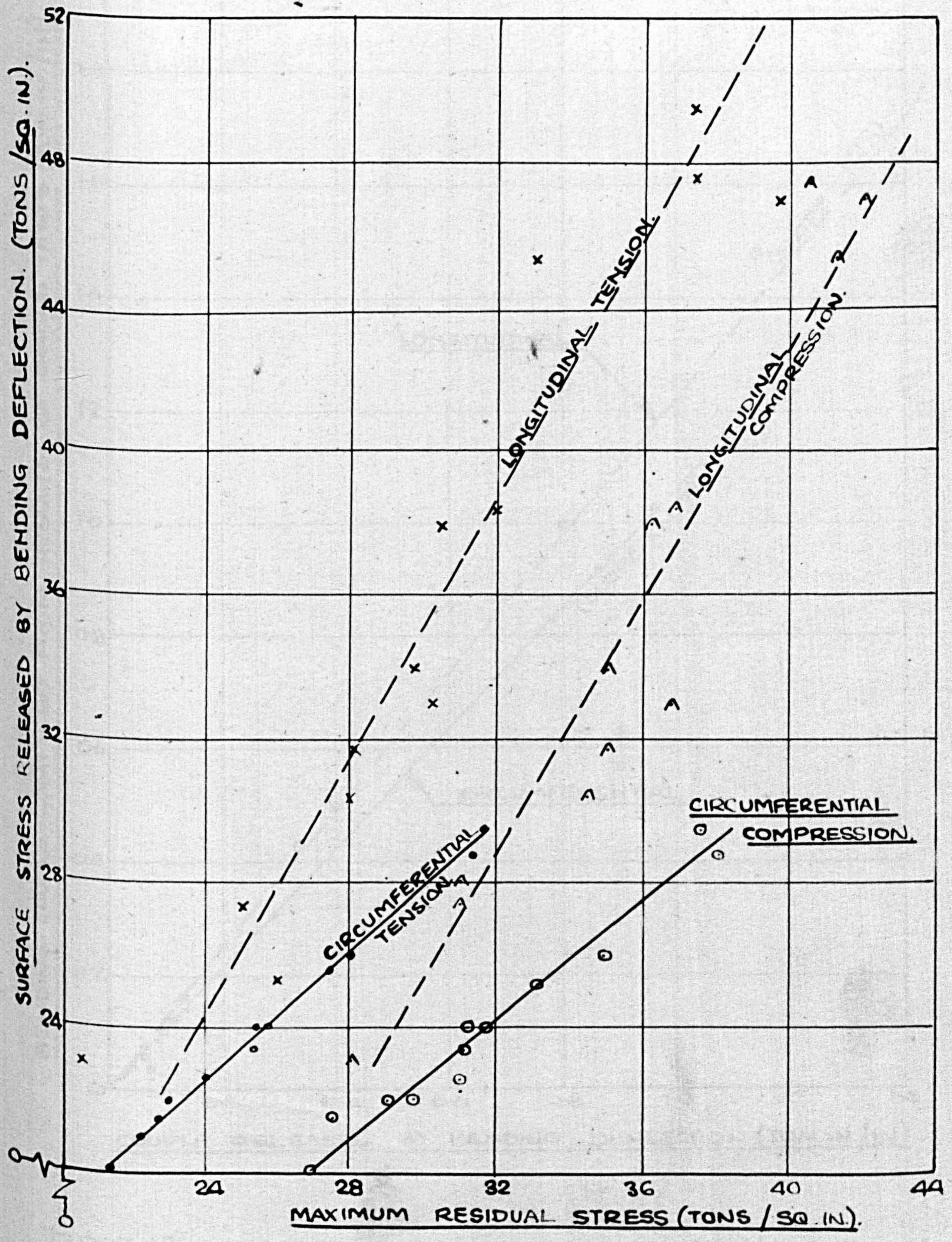


FIG. 7.17. DIRECT STRAIN v BENDING DEFLECTION. COMPARISON OF STRESSES.

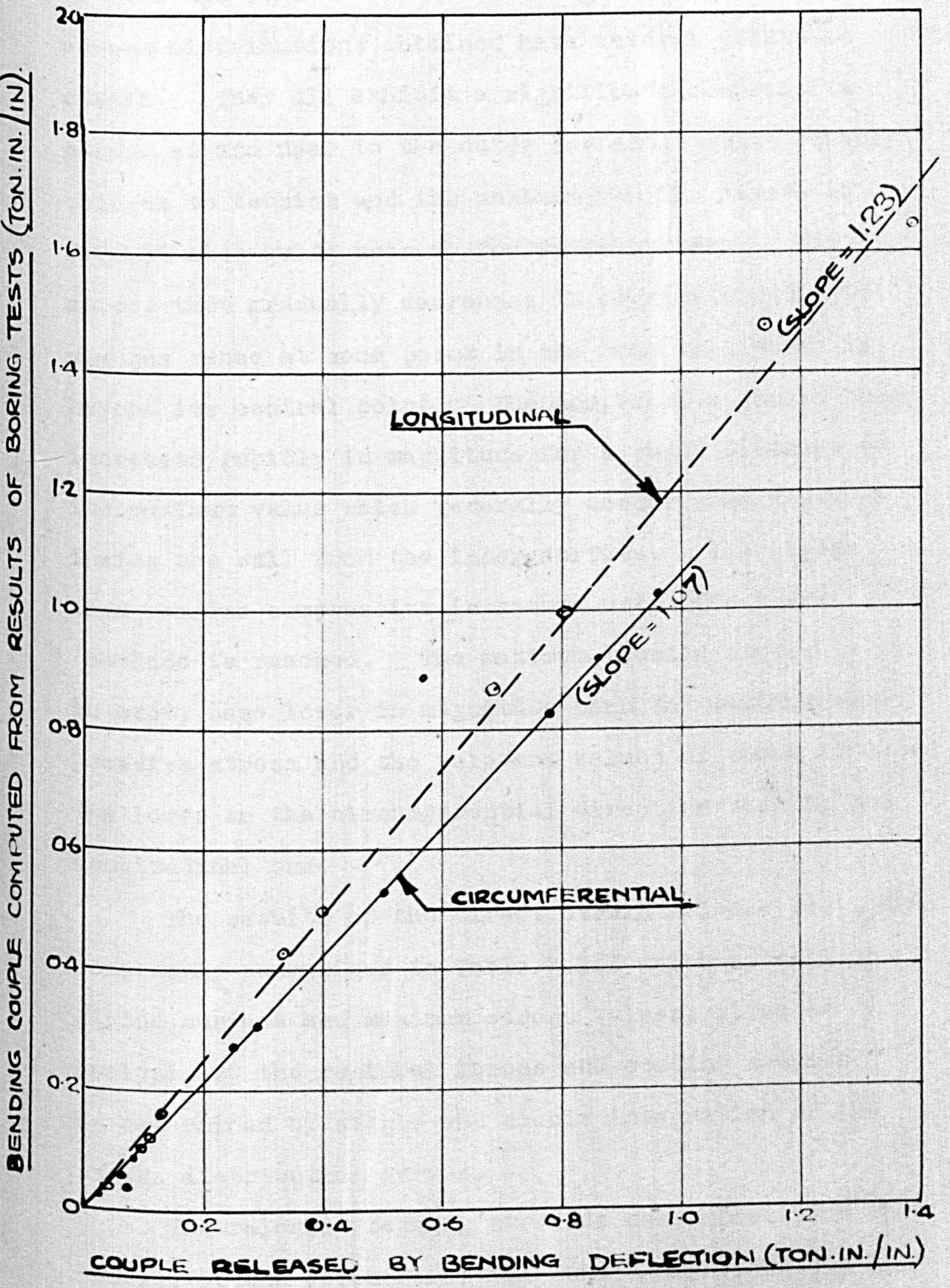


FIG: 7.18. DIRECT STRAIN v BENDING DEFLECTION COMPARISON OF BENDING COUPLES.

The derived circumferential and longitudinal residual stress distributions are given in Figs. 7.11 to 7.13 and 7.14 to 7.16 respectively. All the residual stress distributions obtained have several points in common. They all exhibit a significant compressive stress at and near to the outer surface. This rapidly changes to tension and the maximum tensile stress is reached also quite near to the outer surface. The stress then gradually decreases in magnitude until it changes sense at some point in the tube wall which is beyond its central point. The compressive stress then increases rapidly in magnitude for a short distance to its maximum value which generally occurs some distance inside the wall from the inner surface. The stress then remains compressive in nature until the inner surface is reached. The maximum tensile stress is in every case lower in magnitude than the maximum compressive stress and the relevant values of these stresses are lower in the circumferential direction than in the longitudinal one.

The results of the direct strain release tests are completely summarized in Table 7.III which as well as giving surface and maximum stress values, gives an analysis of the residual forces and bending couples as determined by single and double integration of the stress distribution curves.

The released bending stresses determined from the results of the circumferential bending specimen (X) and the three longitudinal specimens (Y) and calculated according to the simple straight beam theory of bending are superimposed as chained lines on Figs. 7.11 to 7.16 respectively and also included in Table 7.III.

In Figs. 7.17 and 7.18 respectively comparisons are given of the maximum residual stress as determined from the boring tests and the surface stress released

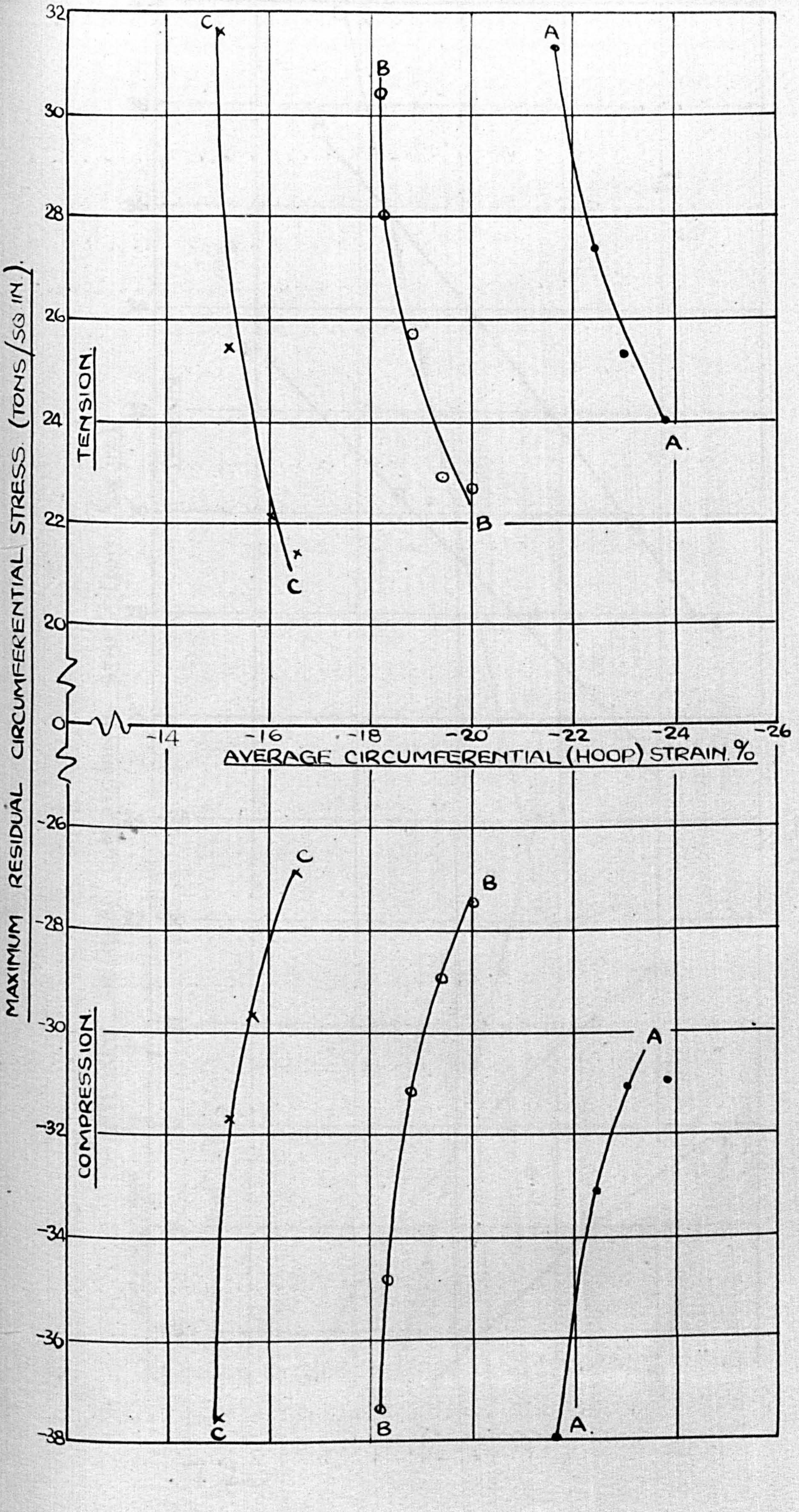


FIG. 7.19. VARIATION OF MAXIMUM CIRCUMFERENTIAL STRESS WITH THE CIRCUMFERENTIAL STRAIN.

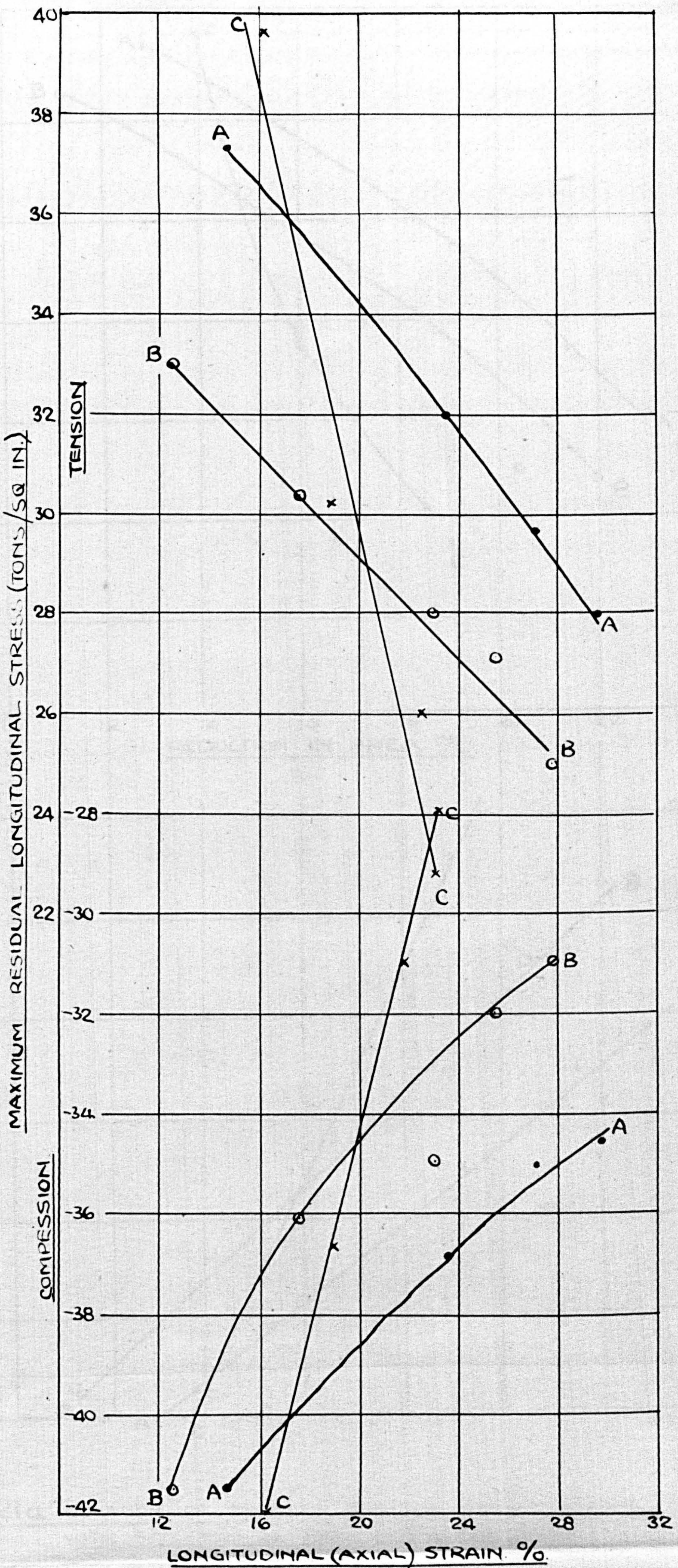


FIG: 7.20 VARIATION OF MAXM LONGITUDINAL STRESS WITH THE AXIAL STRAIN

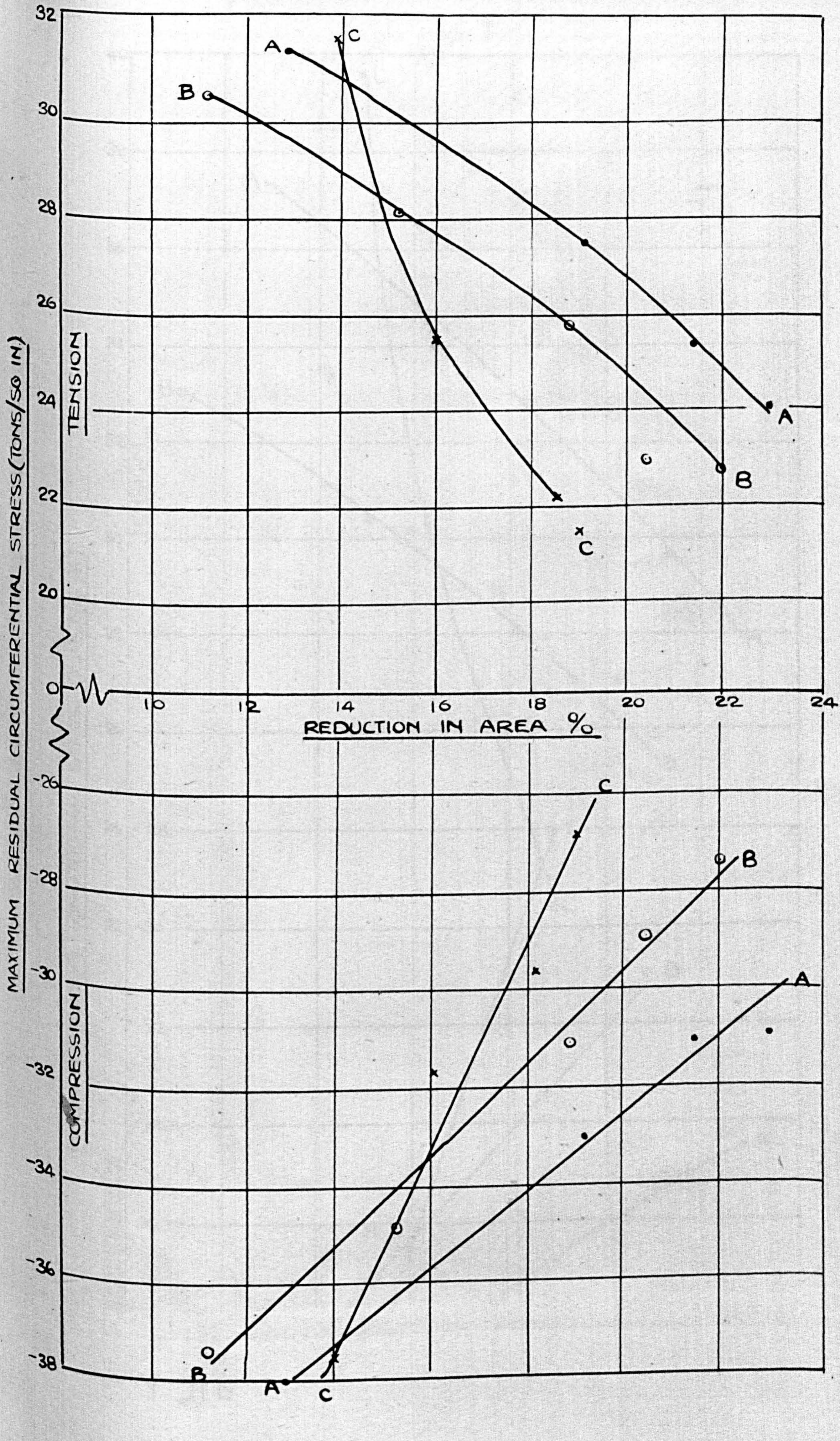


FIG: 7.21a VARIATION OF MAXIMUM RESIDUAL CIRCUMFERENTIAL STRESS WITH REDUCTION IN AREA DURING DRAWING

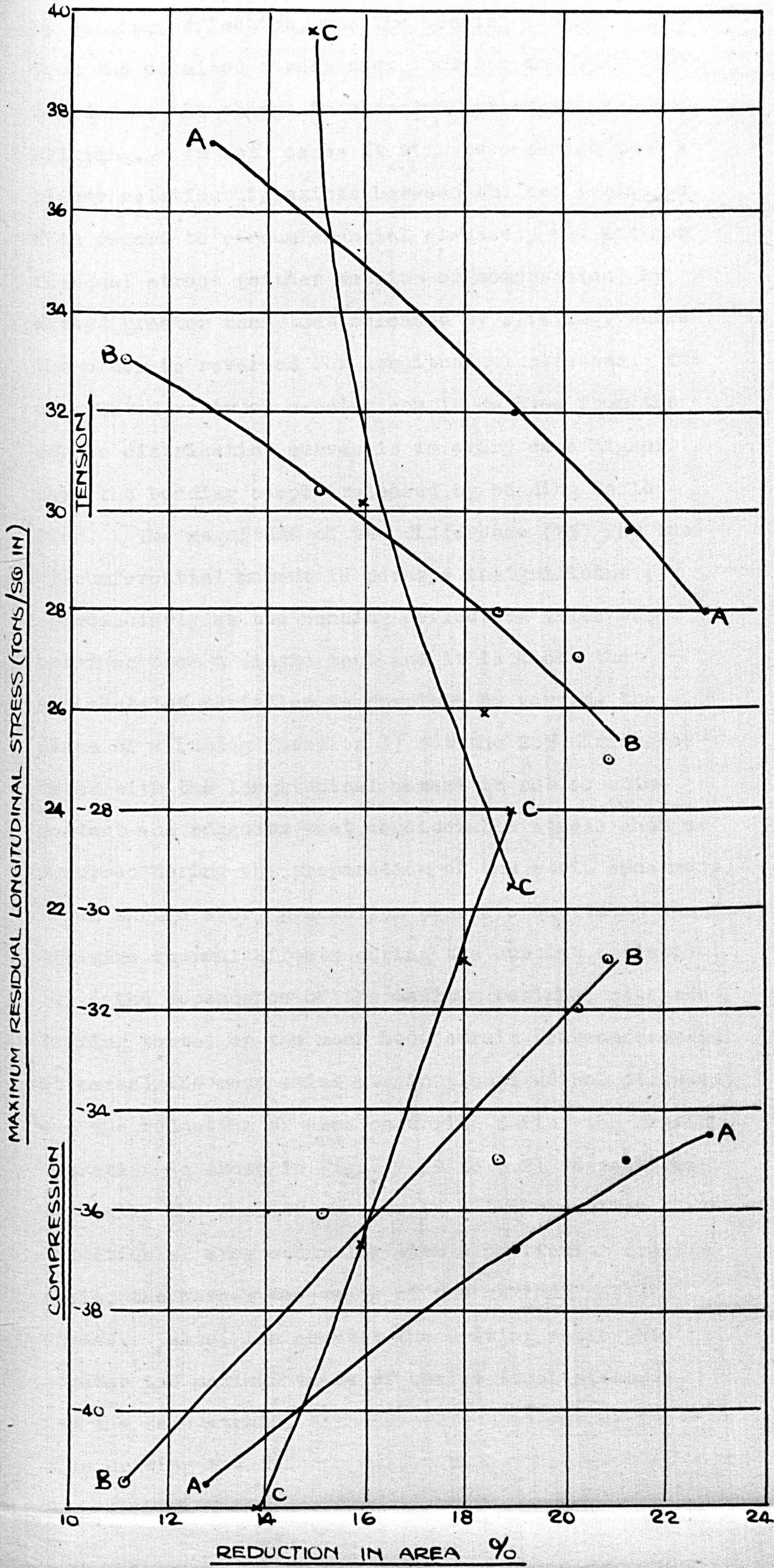


FIG. 7.21b VARIATION OF MAXIMUM RESIDUAL LONGITUDINAL STRESS WITH REDUCTION IN AREA DURING DRAWING.

by bending deflection, and the bending couple computed from the obtained stress distributions and that calculated from the change in diameter (or curvature) on slitting. In both cases it will be observed that a linear relationship exists between the two variables. With regard to circumferential stresses, the maximum residual stress (either tension or compression) is always greater than that released by slitting, while the order is reversed for longitudinal stresses. The calculated residual bending couple derived from the stress distribution curves is in every case higher than the bending couple released by bending deflection. The magnitude of the difference (7%) for the circumferential moment is perhaps insignificant , particularly as the bending deflection value was obtained from a single test and it is known that at least 10% variation is possible by varying the plane of slitting (Section 3) but the 23% difference noted with the longitudinal moment is not so unimportant and suggests that considerable stress changes occurred during the preparation of the strip specimens, even though every precaution possible was taken to minimise thermal effects during the cutting operation.

The dependence of the maximum residual stresses (boring tests) on the mean hoop strain (circumferential stresses), the mean axial strain (longitudinal stresses) and the reduction of area occurring during the drawing operation is shown in Figs. 7.19 to 7.21 respectively. Generally the smaller the strain and the smaller the reduction of area occurring with a particular drawing ratio, the larger the value of the maximum residual stress. Also, the greater the drawing ratio the greater the maximum value of the residual stresses for the same strain, although this does not apply when drawing the thinner walled tubes through the

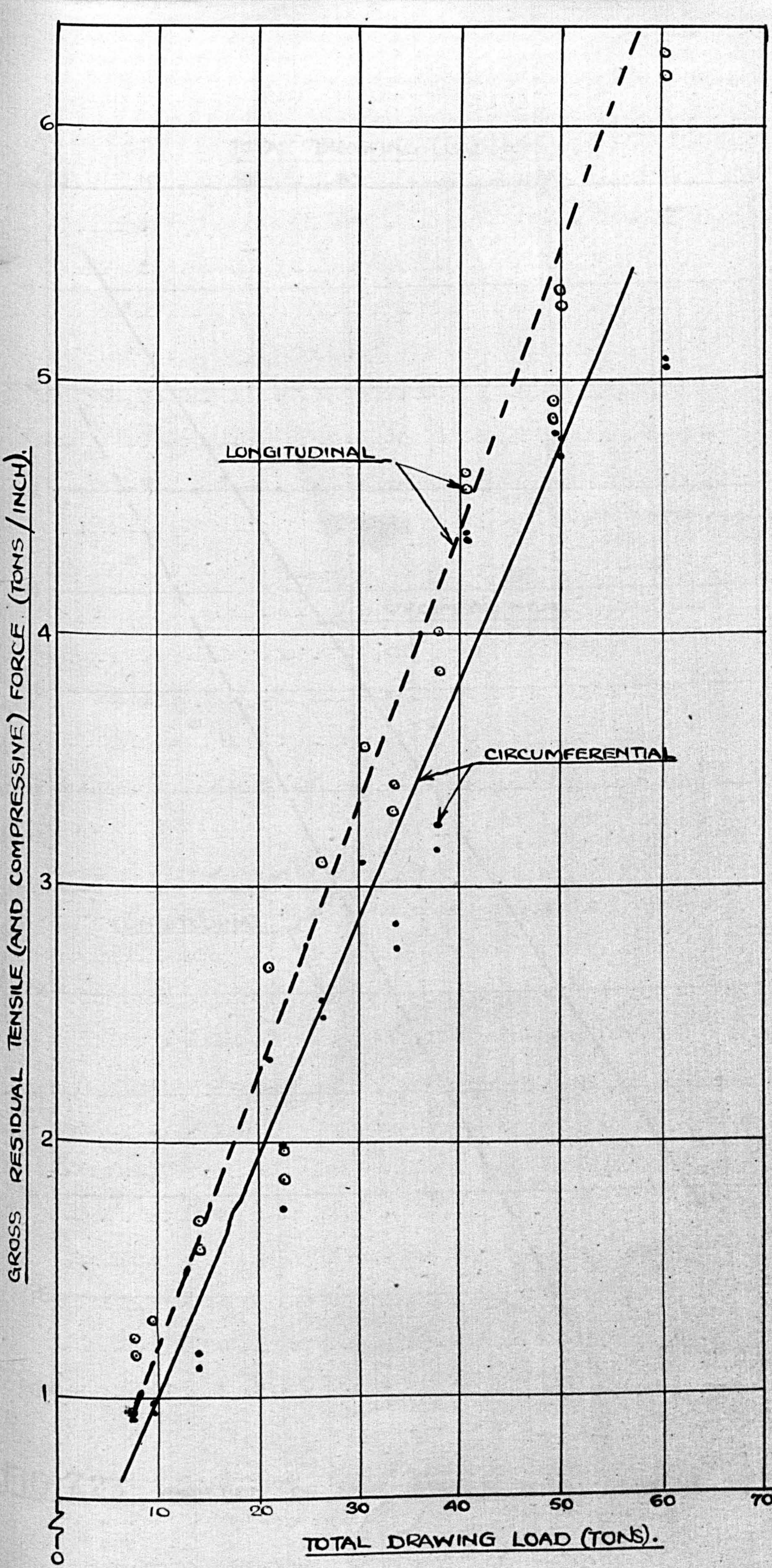


FIG: 7.22. VARIATION OF GROSS FORCE WITH DRAWING LOAD.

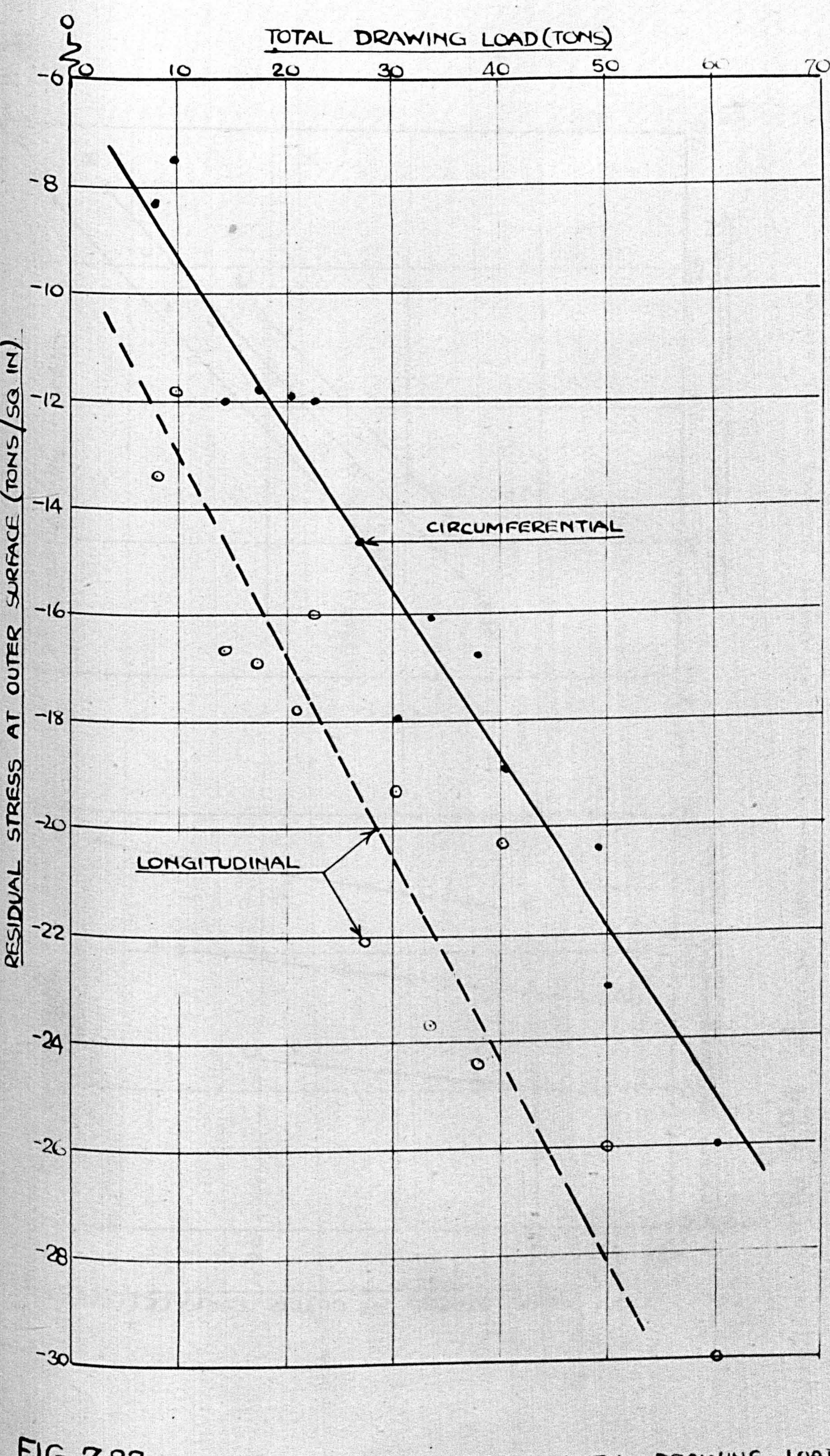


FIG: 7.23. VARIATION OF SKIN STRESS WITH DRAWING LOAD

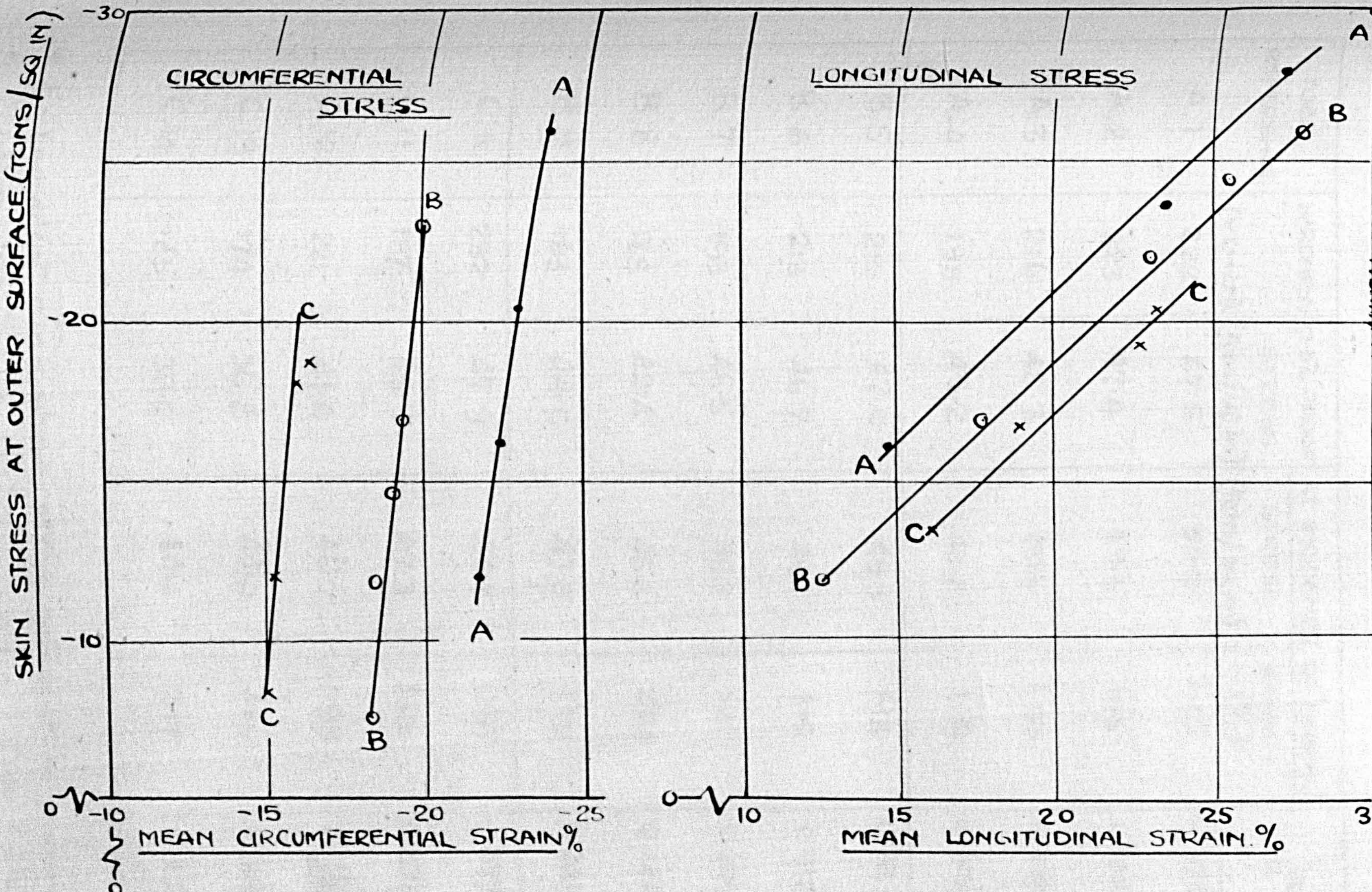


FIG: 7.24 VARIATION OF SKIN STRESS AT OUTER SURFACE WITH STRAIN

TUBE REFERENCE	HARDNESS (V.D.P.N.)	0.5% PROOF STRESS (tons/sq in)	AT INCIDENCE OF "NECKING" STRESS (tons/sq.in)	STRAIN (%)	$\mathcal{E}$*
A.1	222	47.5	50.2	1.15	0.376
A.2	223	47.4	49.3	1.13	0.356
A.3	216	48.3	50.6	1.03	0.335
A.4	198	45.2	48.7	1.32	0.315
B.5	215	47.0	49.3	1.13	0.333
B.6	213	46.7	49.1	1.15	0.312
B.7	193	45.5	48.3	1.21	0.297
B.8	168	42.4	45.2	2.04	0.266
B.9	163	44.0	47.0	1.19	0.249
C.10	232	47.9	50.7	0.97	0.277
C.11	187	45.3	47.3	1.19	0.274
C.12	221	41.4	46.0	1.93	0.263
C.13	174	36.8	44.0	2.60	0.241
C.14	165	38.0	44.4	2.17	0.221

* $\mathcal{E}$ = Equivalent r.m.s. shear strain present in material after drawing $= \sqrt{\left\{\frac{1}{3}\sum(\varepsilon_h - \varepsilon_t)^2\right\}}$ (see table 7.II).

TABLE, 7.IV: MECHANICAL PROPERTIES OF TUBE MATERIALS AFTER DRAWING.

3.625 in. die diameter (curves CC Fig. 7.20). In this case the maximum residual longitudinal stress - axial strain relationship actually intersects the curves for the other two reductions which follow the more expected pattern. (A similar trend is observed in the relationship between residual stress and reduction in area - Figs. 7.21 a and b - the curves for the lighter reduction again intersecting the curves for the other two reductions).

In Fig. 7.22 the relationship between the gross residual force in one particular direction (net force ≏ 0) and the total drawing load is given. This diagram suggests a linear relationship between the variables (see Appendix 8) and suggests that the general residual stress level (not the maximum values or the way in which the stress is distributed through the walls) is determined by the force necessary to draw the tube. This latter value is itself dependent upon many factors such as material strain hardening, die design, reduction, etc. (Chung and Swift, 1952) and consequently it seems reasonable to assume that the general level of residual stress can also be associated with these factors.

A similar inference can be drawn from Fig. 7.23 which again suggests a linear relationship, this time between the compressive stress at the outer surface and the drawing load. That the skin stress at the outer surface is also dependent upon the mean circumferential or longitudinal strain induced in the drawing operation for a particular drawing ratio is evident from Fig. 7.24.

The equivalent representative shear strain present in each tube after the drawing operation as calculated by the root mean square method proposed by Swift (1946 is listed in Tables 7.III and 7.IV, the latter table also summarizing the results of the tensile tests carried out on the specimens prepared from each tube after drawing.

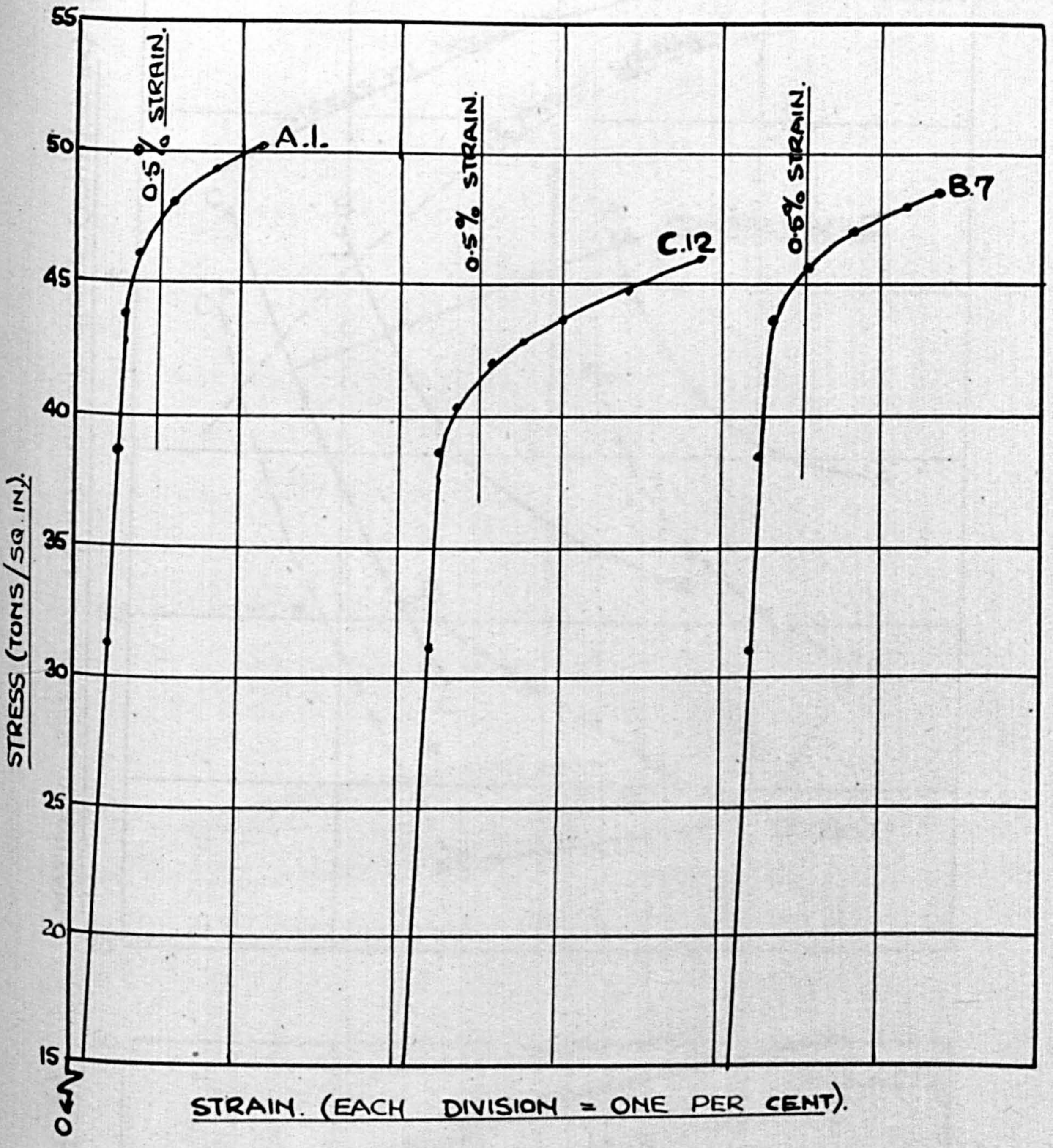


FIG: 7.25. TYPICAL STRESS/STRAIN CHARACTERISTICS OF TUBE MATERIAL AFTER DRAWING.

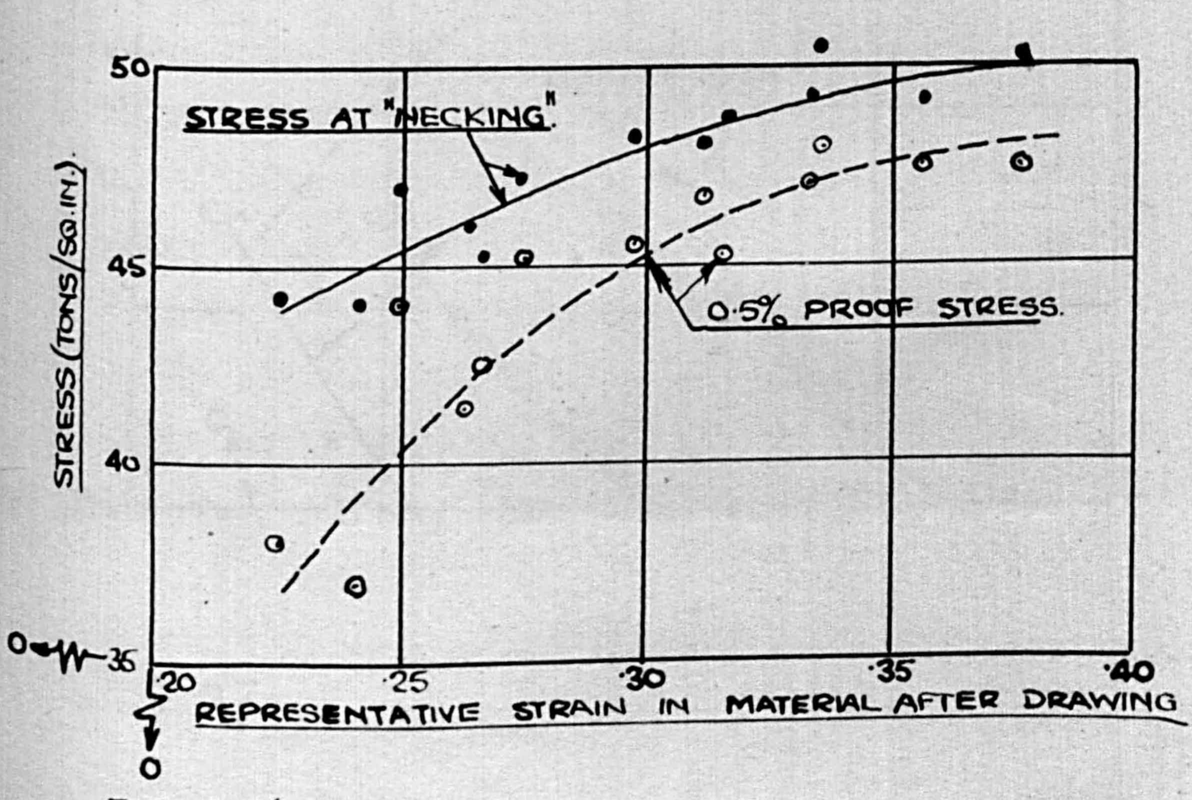


FIG. 7.26. VARIATION OF STRENGTH OF TUBE MATERIAL WITH STRAIN IN MATERIAL AFTER DRAWING.

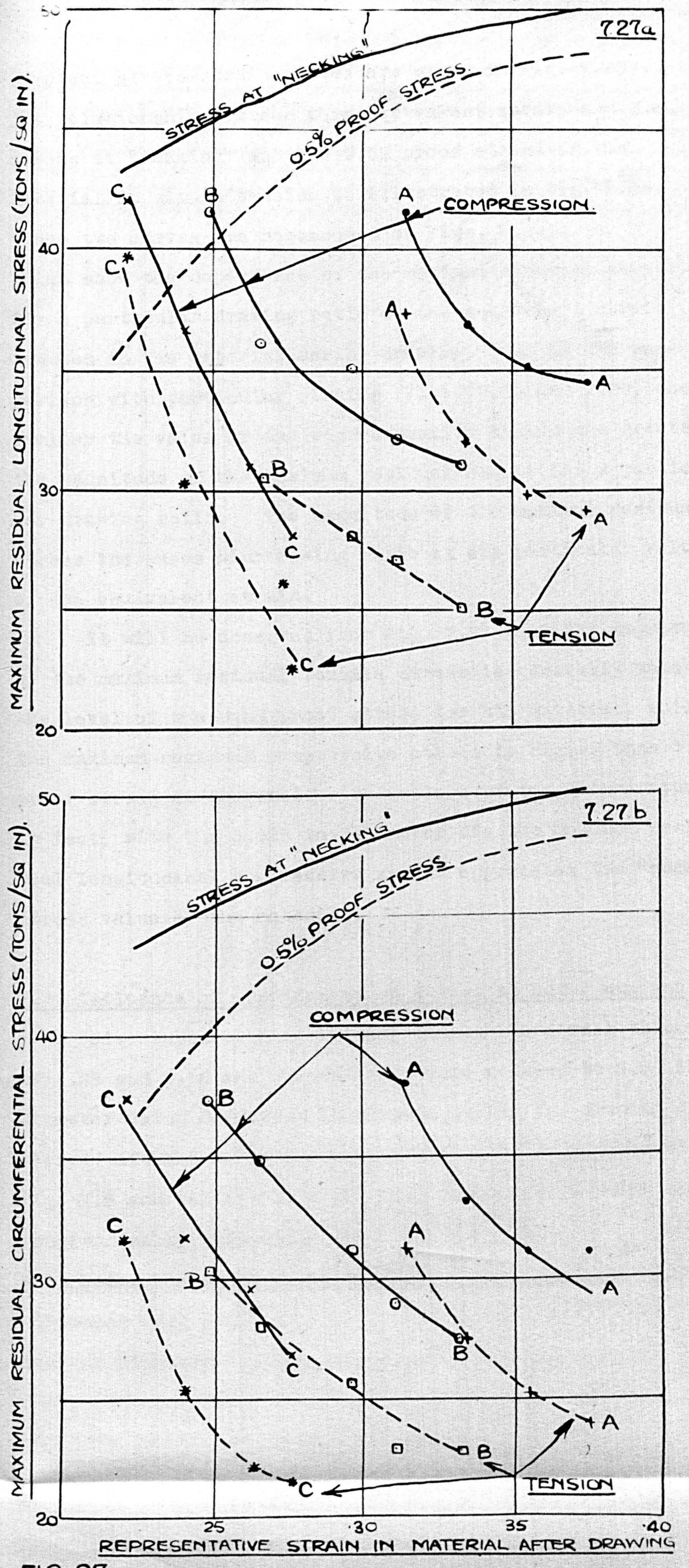


FIG: 27. INFLUENCE OF STRAIN ON MAX^{M} RESIDUAL STRESSES.

(Typical stress-strain curves are given in Fig. 7.25). The relationship between this equivalent strain and the stress at "necking" and the 0.5% proof stress of the material in simple tension is illustrated in Fig. 7.26. These two curves are reproduced on Figs. 7.27 a and b which show the dependence of the maximum residual stress for a particular drawing ratio on the equivalent strain induced in the material during drawing. As in the comparison with particular strains (Figs. 7.19 and 7.20) the smaller the value of the representative strain the greater the magnitude of the maximum residual stress for a particular drawing ratio. The magnitude of the maximum residual stress increases with drawing ratio at any particular value of the equivalent strain.

It will be observed from Fig. 7.27 that the magnitude of the maximum residual tensile stress is generally below the level of the 0.5% proof stress for the material, while the maximum residual compressive stress is higher than the proof stress at the smaller reductions of the thinner tubes. In fact, with the 3.625 in. diameter die the maximum residual longitudinal compressive stress approaches the "necking" stress value of the material.

7.4. Influence of lubrication on stress at outer surface.

Five tubes each of initial diameter and wall thickness of 4.25 and 0.25 in. respectively were reduced to 3.50 in. diameter using different lubricants during the drawing operation. After drawing these were cut up in accordance with Fig. 7.5 and two of the 4 in. long specimens (X) were slit longitudinally, one along the plane containing the thickest wall section and the other along the plane containing the thinnest wall section. As indicated in Table 7V the average diametral change of the two specimens from any one particular tube, after drawing, were well within the expected tolerance of ± 5% of the mean value and over the whole range of tubes the released surface stress was

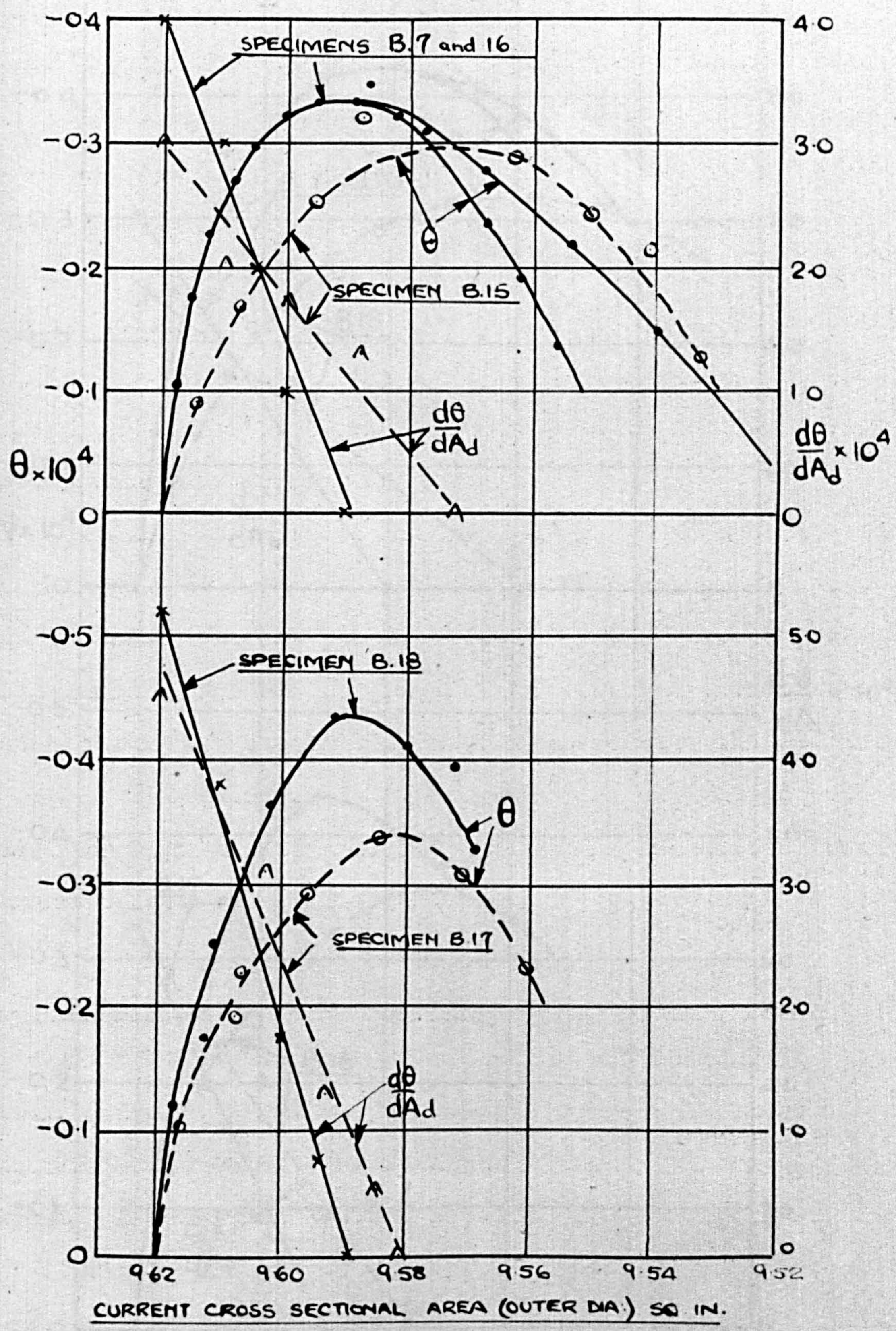


FIG 7 28. LUBRICATION TESTS. EXPERIMENTAL RESULTS.

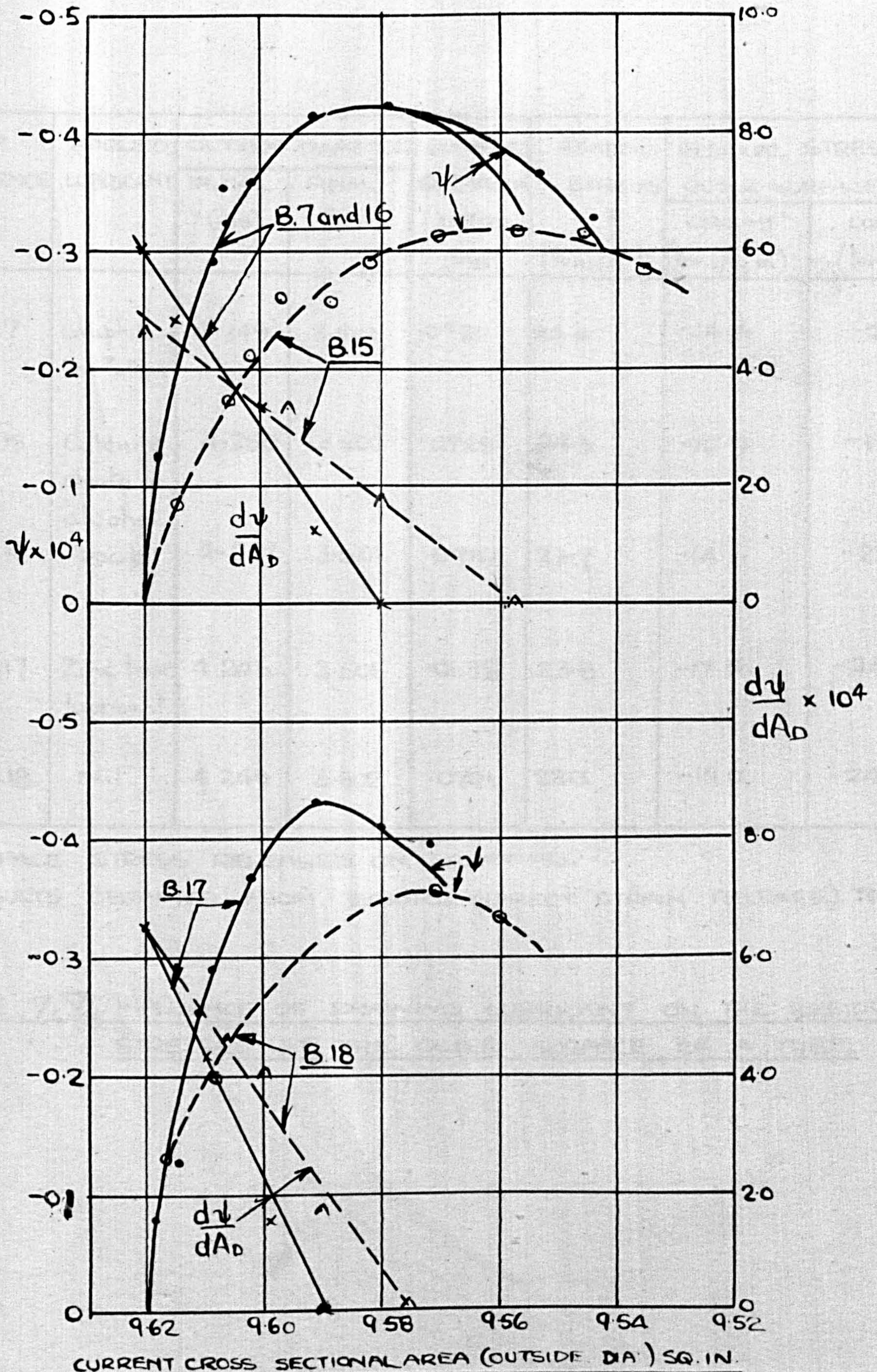


FIG. 7.29. LUBRICATION TESTS. EXPERIMENTAL RESULTS.

TUBE REFERENCE	APPLIED LUBRICANT	OUTSIDE DIAMETER INITIAL (ins)	OUTSIDE DIAMETER FINAL (ins)	CHANGE OF DIA: ON SLITTING (ins).	BENDING STRESS * (tons/□")	RESIDUAL STRESS AT OUTER SURFACE ‡ CIRCUMF^L (tons/sq.in)	RESIDUAL STRESS AT OUTER SURFACE ‡ LONGITUD^L (tons/sq.in)
B.7	Graphite-in-Tallow	4·249	3·501	·0721	24·0	−14·6	−22.1
B.15	Calcium oleate in alcohol.	4·250	3·500	·0729	24·3	−10·9	−18·1
B.16	Soap	4·247	3·501	·0710	23·7	−14·6	−22·1
B.17	Zinc base lubricant	4·246	3·501	·0698	23·8	−17·5	−24·0
B.18	Nil	4·249	3·502	·0701	23·3	−19·0	−24·0

* SURFACE STRESS RELEASED ON SLITTING.

‡ RESULTS OBTAINED FROM BORING (DIRECT STRAIN RELEASE) TESTS.

TABLE 7.V. INFLUENCE OF DRAWING LUBRICANT ON THE RESIDUAL STRESSES AT THE OUTER SURFACE OF A TUBE.

remarkably constant. This was taken as an indication that lubrication has little or no effect on the general distribution of residual stress throughout a tube and consequently no boring tests were carried out to determine the complete stress pattern.

Boring tests were, however, carried out on one specimen from each tube fitted with strain gauges on the inside surface and subjected to layer removal from the outside. Every precaution was taken to obtain as many results as possible within the first 0.010 in. of metal removal and the resultant experimental curves of the parameters θ and ψ plotted against the variable cross-sectional area are given in Figs. 7.28 and 7.29. These figures also show the derived values of the derivatives $\frac{d\theta}{dA}$ and $\frac{d\psi}{dA}$ from which the values of the residual circumferential and longitudinal skin stresses quoted in Table 7.V have been determined.

It will be seen from Table 7.V that although lubrication does not appear to influence the general pattern of residual stress in a tube, it does influence the stress at the outer surface, and generally, the more efficient the lubricant the lower the magnitude of this surface compression. This suggests that surface compression at and near the outer surface of a hollow drawn tube in the region where the tube material has been almost in contact with the die material, is influenced by lubrication and may be caused by frictional heating or frictional shear stress effects.

8. CONCLUSIONS.

Although all the conclusions derived from this investigation have already been stated during the discussion of the many different stages of the work, the most important of them are briefly summarized below.

8.1. Bending deflection strain release.

1. The Davidenkov analysis is considered the most fundamental of all the residual stress analyses based on bending deflection methods of strain release, but its use leads generally to an unnecessary amount of labour in the computation of the residual circumferential stresses from the experimental results. For longitudinal stresses, the Davidenkov analysis is no more difficult in application than any other similar analysis.

2. The Sachs and Espey analysis for thin walled tubes, although simple in application, introduces several unnecessary sources of error. These can be easily corrected while still maintaining the simplicity of the method. These corrections give rise to a modified analysis which, in thin tubes gives results very similar to those derived by the much more complicated Davidenkov analysis.

3. The acid pickling process for removing metal from steel specimens is satisfactory in all respects (provided sufficient care is taken in its application), and does not affect the magnitude of the released stresses as is the case with layer removal by machining.

4. The leaving of wax protected areas on the outer surfaces of specimens pickled from their outer surfaces is to be avoided as it leads to considerable errors in the form of the derived stress distribution curves, though not necessarily in the magnitude of the maximum stresses.

5. The measured residual circumferential stresses in a

tubular specimen are influenced greatly by the length of the specimen and unless the specimen is more than a certain length (and possibly shorter than some other greater length) incorrect results will be obtained. The length range to give valid results cannot be stated without previous investigation.

6. With specimens of the correct length, mechanical preparation does not appear to affect the derived stress distribution curves, although successive layer removal by machining is definitely detrimental.

7. The residual stresses present in a tube are not generally the same at all points around the tube wall. Wide variations in the derived stress curves are obtained from identical specimens from the same tube which have been slit or sectioned along different generators, although two identical specimens slit along the same generator yield similar results.

8.2. Direct strain release.

8. The Sachs' boring method and analysis is considered sufficiently exact for general application.

9. Electric resistance strain gauges are suitable for measuring the direct strains released during the successive removal of surface layers from a specimen.

10. No errors are introduced into the experimental results by the application of strain gauges to the measuring process.

11. Layer removal by machining from a tubular specimen, induces compressive stresses in the specimen which are of sufficient magnitude to have an appreciable influence on the values of the derived residual stresses.

12. A similar length effect to that observed with bending deflection methods exists with the direct strain release method. Unless specimens are greater than some experimentally predetermined length, incorrect stress values will result from any tests carried out on the specimens.

13. Direct strain release tests of the boring type appear to result in the determination of average residual stresses at any particular depth in the specimen wall and consequently to the derivation of mean stress distribution curves.

8.3. Bending deflection v. Direct strain release.

14. The average stress distribution curves obtained from similar specimens by both methods are appreciably the same.

15. The direct strain method is much more conservative of material than the bending deflection method, but its application demands a much more careful experimental approach.

16. For both methods, the depth of the removed layer at any stage should not exceed 0.005 in. (or 0.02 x the initial wall thickness if this is the smaller value) and during the early stages of metal removal should be substantially less than this value.

8.4. The length effect.

17. The length effect cannot be explained by either anticlastic bending effects or by increase in the lateral rigidity of a specimen due to the presence of longitudinal stresses.

18. Local stress relief during the separation of a specimen from the parent tube, by heating or other associated forms, cannot in itself explain the length effect although it does appear to have a contributory effect.

8.4. Residual stresses in hollow drawn tubes.

19. The residual stress distribution present in such tubes is essentially tensile in the outer 60% (approximately) of the tube wall and compressive over the remainder.

20. Rapidly changing compressive stresses exist at and near to the outer surface of hollow drawn tubes, where the material has been almost in contact with the drawing die. This surface stress is affected by friction and can be reduced by the use of an efficient drawing lubricant.

21. The maximum tensile stress occurring in a specimen is generally lower than the corresponding compressive stress.

22. The stress released by bending on slitting a ring specimen or on preparing a tongue specimen bears a linear relationship to the maximum stress values in the specimen, although the latter are always of the higher magnitude.

23. The residual stresses which develop in a hollow drawn tube are a function of:-

(i) The drawing load.

(ii) The principal strains and the r.m.s. shear strain which develop in the tube during drawing.

(iii) The drawing ratio (reduction of outside diameter).

Generally, the smaller the root-mean-square shear strain developing in a tube during drawing for a particular drawing ratio, the higher the magnitude of the maximum residual stresses which develop in the tube although for a particular strain value the stress values increase with increase in the drawing ratio.

24. Local residual stress values which approach the "necking" stress value of the material can apparently exist in the walls of tubes, although generally the maximum stress values lie below the level of the 0.5% proof stress of the "as drawn" material.

9. APPENDIX 1.

NOMENCLATURE ADOPTED AND ASSUMPTIONS PRESENT IN ALL RESIDUAL STRESS ANALYSES.

9.1. Nomenclature.

E = Young's modulus.

σ = Poissons' ratio.

D_o = initial outside diameter of specimen.

D = current outside diameter of specimen.

d_o = initial bore diameter of specimen.

d = current bore diameter of specimen.

A_o = initial area circumscribed by the outer diameter.

A_D = current area circumscribed by the outer diameter.

A_i = initial cross sectional area of bore.

A_d = current cross sectional area of bore

t = initial wall thickness of specimen

a = current wall thickness of specimen

y = variable wall thickness of specimen

x, x_1 = depth below original surface to layer in wall under consideration

D_m = mean diameter of tube (see Appendix 4).

f = deflection of strip on bending, measured over a length 2b.

f_1 = deflection of strip on preparation.

ΔD_1 = change of diameter on slitting

ΔD = change of "sprung" diameter on metal removal

δ = diametral strain occurring on metal removal

λ = longitudinal strain occurring on metal removal

$\psi = \lambda + \sigma.\delta$

$\theta = \delta + \sigma.\lambda$

p_c = residual circumferential stress.

p_{c1}, p_{c2}, p_{c3} = components of p_c

p_ℓ = residual longitudinal stress.

$p_{\ell 1}, p_{\ell 2}, p_{\ell 3}$ = components of p

p_r = residual radial stress.

9.2. Assumptions present in all residual stress analyses.

The following assumptions are present in all the published analyses relating to residual stresses in tubes.

(a) The metal is effectively homogeneous and has constant values of the elastic constants E and σ.

(b) The residual stresses are distributed with rotational symmetry about the axis of the tube under consideration.

(c) The tube is circular in section and its inner and outer wall surfaces are concentric.

The last assumption is very rarely absolutely true for deep drawn tubes and consequently the assumption (b) is also not strictly applicable. However it is shown in the experimental work described in the main body of this work, that reasonably exact solutions are possible using the analyses quoted in Appendices 2, 3, 4 and 5 although the common assumptions (b) and (c) are not strictly valid.

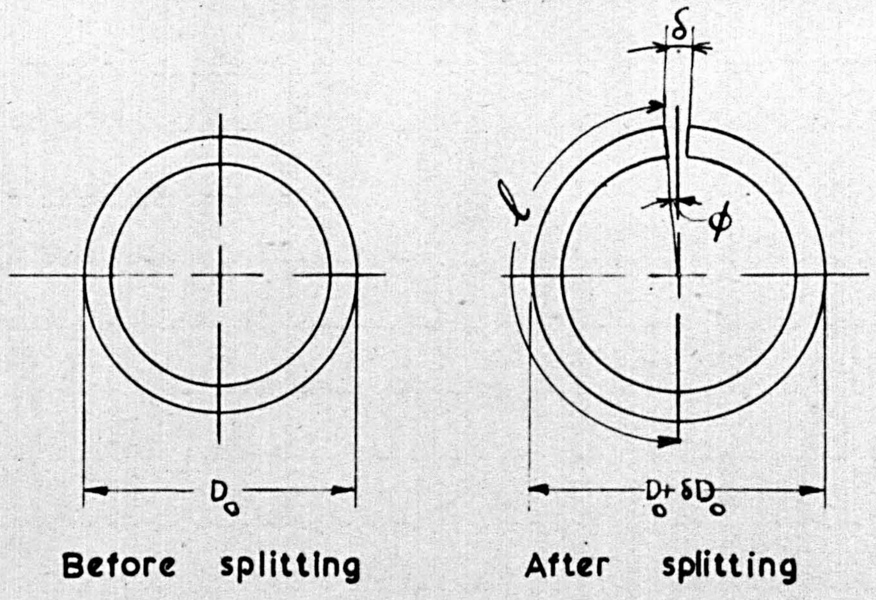


FIG: 10.1.

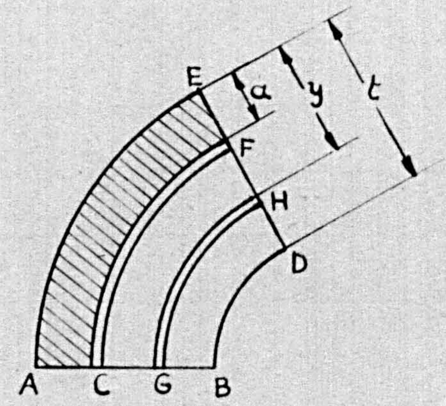


FIG: 10.2.

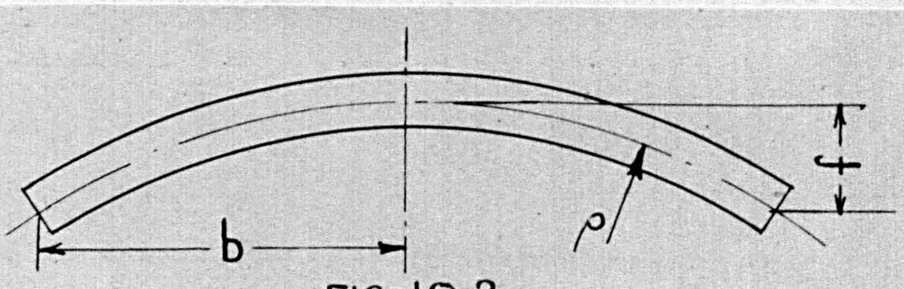


FIG: 10.3.

after each stage of the metal removal process. The distribution of the residual longitudinal stresses in the original specimen is then calculated from the measured deflections and the current thickness of the strip specimen.

In both the determination of circumferential and longitudinal stresses, the total stress is calculated as the sum of three components:-

(a) The stress relieved on slitting the tube.

(b) The stress disappearing when a small layer of material is removed, and

(c) The stress relieved at a particular point in the original cross section by the gradual removal of all material interior (or exterior) to the point considered.

10.2. Circumferential stresses.

The following analysis is given for the particular case of layer removal from the inner cylindrical surface of a specimen. A similar method of solution, leading to slightly different expressions (quoted later) can be followed for outside metal removal.

10.2.1. Stresses relieved by slitting the specimen.

When the specimen is slit longitudinally a bending moment of magnitude M is released and the outside diameter D_o increases by an amount ΔD_o (Fig. 10.1).

Considering one half of the slit ring and applying Castigliano's theorem (neglecting the small strain energy term introduced by shear stresses).

$$U = \text{total strain energy} = \frac{1}{2E'I}\int_0^{\ell} M\, dx$$

where $E' =$ modified elastic constant $= \frac{E}{1-\sigma^2}$

$E =$ Young's modulus, $\sigma =$ Poisson's ratio.

$I =$ Second moment of area of section about plane of bending.

(Other notation as on Fig. 10.1).

Now $\delta x = R.\,\delta\theta$

Hence $U = \frac{1}{2E'I}\int_0^{\pi} M^2 R d\theta = \frac{M^2 R \pi}{2E'I}$

Also $\frac{\partial U}{\partial M} = \phi$ where ϕ = angular deflection $= \frac{\delta}{R}$

$$= \frac{MR\pi}{E'I}$$

Since ΔD_o is the change in diameter on slitting

$$2\delta = \pi.\Delta D_o$$

$$= 2R\phi$$

$\therefore M$ = released moment $= \frac{E'I\Delta D_o}{2R^2}$

i.e. $M = \frac{E' t^3 \Delta D_o}{6D^2_m}$ per unit length of tube (10.1)

where t = initial thickness of tube wall

D_m = mean diameter on slitting the tube.

From Winkler's theory of curved beams, the circumferential stress component p_{c1} released at all points of the tube wall is given by:

$$p_{c1} = -\frac{M}{hA}\frac{(Y - h)}{(Y + R)}$$

where Y = distance of layer considered from axis containing the centroid of the section (positive when measured outwards).

A = area of section

h = distance of centroid from neutral axis

Now $h = \frac{-R(A' - A)}{A'}$

where A' = modified area $\simeq A + \frac{I}{R^2} = A(1 + \frac{t^2}{12R^2})$

Hence $h = -\frac{Rt^2}{12R^2 + t^2} \simeq \frac{t^2}{12R}$

If a = depth of point considered below the outer surface

$$Y = \frac{t}{2} - a$$

and hence:
$$p_{c1} = \frac{E'\Delta D_o}{D_m} \frac{\left\{\frac{t}{2} - a + \frac{t^2}{6D_m}\right\}}{\left\{\frac{t}{2} - a + \frac{D_m}{2}\right\}} \quad \text{..........} \quad (10.2)$$

Note. ΔD_o is considered positive for increasing diameter and this gives the normal sign convention to the relieved stresses across the tube.

10.2.2. Stress suddenly disappearing when a small layer is removed.

From Fig. 10.2 it can be seen that when all the layers from B to C have been removed, there exists in an elemental layer such as CF, the circumferential stress p_{c2}. When this layer is removed the stress will disappear and the loss of force equivalent to $p_{c2}.\delta a$ per unit length of tube will effect the remaining cross section of thickness "a" in two ways:

(a) as equivalent to the action of a direct force $p_{c2}\,\delta a$.

and (b) equivalent to a bending moment $\delta M = -p_{c2}\delta a.\frac{a}{2}$

The first of these causes introduces a uniformly distributed stress of magnitude δp_{c2} over the remaining section, where

$$\delta p_{c2} = p_{c2}\frac{\delta a}{a}$$

If the layer to be removed is equally strong in all directions no diametral change will occur in the remaining section, since this is symmetrical. In any case, the effect will be insignificant and can be neglected at this stage.

The second cause is the bending of the ring under a constant moment M, which involves a diametral change in the specimen:

From equation 10.1 written in differential form

$$\delta M = \frac{E'}{6(D-a)^2}\, y^3.\ \delta D.$$

(D - a = mean diameter of unslit tube at this stage).

If δD is considered positive when the outside diameter increases and "a" is positive when measured inwards

then $\delta M = - p_{c2} . \delta a . \frac{a}{2}$ (negative since δ a is negative)

and $p_{c2} = - \frac{E' a^2}{3(D-a)^2} \frac{dD}{da}$ .... (10.3)

The values of p_{c2} at any point in the tube wall can then be calculated using a curve drawn through the experimental points relating D and a.

10.2.3. Stress relieved by the gradual removal of layers of metal up to the point considered.

The effect of removing a small layer GH (Fig. 10.2) of thickness dr on the stress in the layer CF is determined and the effect of removing all layers up to CF may then be obtained by integration.

The effect of removing the layer GH is regarded as equivalent to superimposing on the remaining section a direct force and a moment.

The stress induced by the superimposed force = dp'_{c3}

where $dp'_{c3} = p_{c2} . \frac{\delta y}{y} = - \frac{E' y dD}{3(D_o - y)^2}$ .... (10.4)

(The outside diameter, D_o in this expression, may be regarded as constant, since the changes it undergoes are small).

From equation 10.2 the superimposed moment produced, in the layer CF, the stresses :-

$$dp''_{c3} = \frac{E'}{D_o - y} \left\{ \frac{\frac{y}{2} - a + \frac{y^2}{6(D_o - y)}}{\frac{y}{2} + a + \frac{D_o - y}{2}} \right\} dD$$

$$= \frac{E'}{D_o - 2a} \left\{ \frac{y - 2a}{D_o - y} + \frac{y^2}{3(D_o - y)^2} \right\} dD \quad \text{.......... (10.5)}$$

Summation of equations 10.4 and 10.5 followed by integration gives the total effect of the release of stresses at all points interior to the layer CF on the stresses present in the layer CF after slitting the specimen.

i.e. $$p_{c3} = -\int_a^t (dp'_{c3} + dp''_{c3})$$

$$= \frac{2E'}{3(D_o-2a)} \int_{D_t}^{D_a} \left\{ \frac{aD_o}{(D_o-y)^2} + \frac{y-4a}{D_o-y} \right\} dD$$

Expanding the expressions inside the integral by the binomial theorem, neglecting terms including powers of y of higher order than y^2, and integrating where possible leads to the expression:

$$p_{c3} = \frac{2}{3} \frac{E'}{(D_o-2a)D_o} \left\{ -3a(D_a-D_t) + \frac{D_o-2a}{D_o} \int_{D_t}^{D_a} y.dD + \frac{D_o-a}{D_o^2} \int_{D_t}^{D_a} y^2 dD \right\} \quad \text{............ (10.6)}$$

The value of p_{c3} at any point in the tube wall can then be evaluated by graphical integration of the curves drawn through the experimental points relating y and y^2 to D.

The total residual circumferential stress at any point in the tube wall is given by the summation of equations 10.2, 10.3 and 10.6.

i.e. $$p_c = p_{c1} + p_{c2} + p_{c3} \quad \text{................ (10.7)}$$

As mentioned previously, consideration of external layer removal leads to expressions for the stress components p_{c2} and p_{c3} which differ slightly from those given in Equations 10.3 and 10.6 for internal metal removal. The analysis is modified by considering the bore diameter (d) as being constant (instead of the external diameter) and the resultant expressions are :-

$$p_{c2} = \frac{E'a^2}{3(d+a)^2} \frac{dD}{da} \quad \text{........................ (10.3a)}$$

and $$p_{c3} = \frac{2E'}{3(d+2a)d} \left\{ 3a(D_a-D_t) - \frac{d+2a}{d} \int_{D_t}^{D_a} ydD - \frac{d+a}{d} \int_{D_t}^{D_a} y^2 dD \right\} \quad \text{.............. (10.6a)}$$

In these expressions, dD again refers to the change in outside diameter and is considered positive when the diameter increases; y and a are measured from the inside surface of the tube.

10.3. Longitudinal stresses.

The longitudinal stresses are again determined by a procedure which relieves them in three stages in a similar way to that used for circumferential stresses. In this case, however, instead of slitting a ring to determine the stresses relieved by bending, an axial strip or tongue is cut from the tube, and the amount by which its mid point deflects from the straight is a measure of its stress state. Successive pickling to remove thin layers of material from one of the original tubular surfaces and measurement of the mid point deflection and the current thickness of the strip, provides the information from which the stress distribution through the thickness of the specimen can be calculated. (Davidenkov himself suggests the use of any form of precision straightness indicator, such as a Zeiss Optimeter, for making the deflection measurements).

The following method of analysis can be used for layer removal from either surface of the tube and provided the same sign convention is adopted in both cases, identical formulae ensue. To maintain a uniform treatment throughout this appendix, the analysis which follows is again based on internal layer removal.

The method of solution is based on the simple theory of bending which is found in all text-books dealing with the elastic behaviour of materials.

10.3.1. Stresses relieved by initial deflection of strip specimen.

When the strip specimen is totally severed from the parent tube, it deflects out of its own plane (see Fig. 10.3) into a curved strip of radius ρ. Applying simple theory, the moment of the released stress is given by:

$$M = \frac{EI}{\rho}$$

and if f is the deflection of the strip measured over a length 2b ;

$\rho = \frac{b^2}{2f}$ (f being assumed very small in comparison with ρ).

and hence $M = \frac{2EIf}{b^2}$

Extending the simple theory of bending to its next stage and applying obvious geometrical relationships, it can easily be shown that the released longitudinal stress at any point in the tube wall distant "a" from the outer surface is given by;

$$p_{\ell 1} = \frac{2E(\frac{t}{2} - a)}{b^2} f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10.8)$$

10.3.2. Stress suddenly disappearing when a small layer is removed.

In what follows the shape of the cross section is assumed to be rectangular and for narrow strips this assumption is obviously justifiable. The bending will be assumed positive if the strip deflects so that its original outer surface is on the inner side of the bend. Positive bending therefore corresponds with the release of tensile stresses in the layer removed.

The moment released by the removal of a layer of thickness da at a distance "a" below the outer surface is

$$dM = -\frac{1}{2}(a.p_{\ell 2}.da) = \frac{2EI}{b^2}\, df \quad \text{per unit width of specimen,}$$

from which

$$p_{\ell 2} = -\frac{4EI}{ab^2}\frac{df}{da} = -\frac{Ea^2}{3b^2}\frac{df}{da} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10.9)$$

(the negative sign arising from the sense in which da is measured).

10.3.3. Stress relieved at a layer by the gradual removal of all layers interior to it.

The stress component $p_{\ell 3}$, relieved by the removal of all layers up to the layer considered, is derived in a manner analogous to that used in the determination of equations 10.4, 10.5 and 10.6.

Removal of a layer of thickness dy, distant y from the outer surface, results in the imposition of a direct stress and a bending stress on the stress in the layer da at "a".

The direct stress $dp'_{\ell 3}$ is given by

$$dp'_{\ell 3} = - \frac{Ey^2}{3b^2} \frac{df}{dy} \frac{dy}{x} = - \frac{Ey}{3b^2} df$$

and the bending stress $dp''_{\ell 3}$ by :

$$dp''_{\ell 3} = \frac{2E(\frac{y}{2} - a)}{b^2} df$$

from which

$$p_{\ell 3} = - \int_{f_a}^{f_t} (dp'_3 + dp''_3)$$

i.e. $$p_{\ell 3} = \frac{2E}{3b^2} \int_{f_a}^{f_t} y df - \frac{2Ea}{b} (f_a - f_t) \quad \text{.......... (10.10)}$$

The integral $\int_{f_a}^{f_t} y.df$ can be evaluated graphically from a graph drawn through the experimental values relating y and f. The total residual longitudinal stress at any point in the original thickness is again given by

$$p_\ell = p_{\ell 1} + p_{\ell 2} + p_{\ell 3} \quad \text{...................... (10.11)}$$

i.e. by the summation of equations 10.8, 10.9 and 10.10.

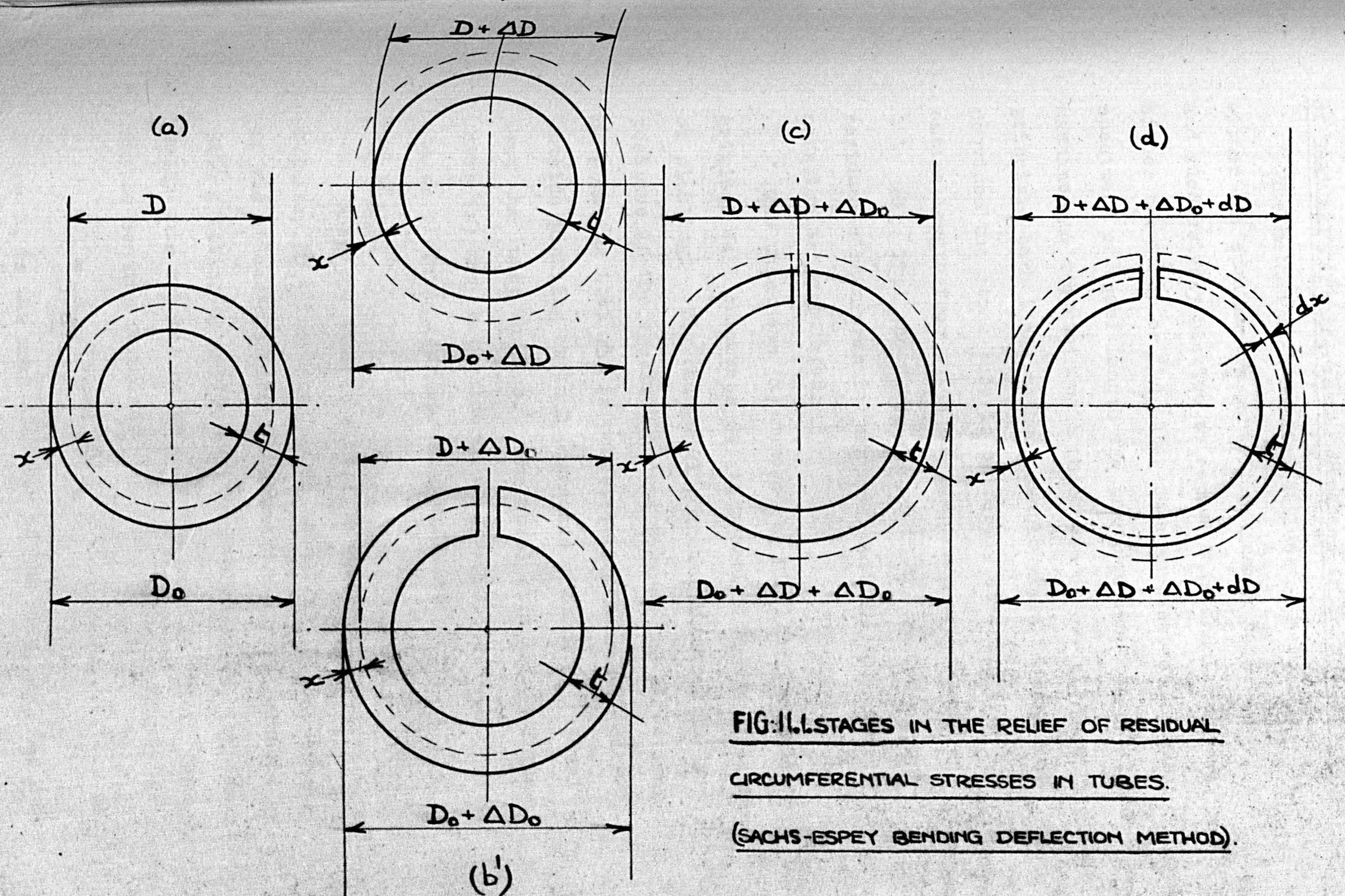


FIG: 11.1. STAGES IN THE RELIEF OF RESIDUAL CIRCUMFERENTIAL STRESSES IN TUBES.

(SACHS-ESPEY BENDING DEFLECTION METHOD).

11. APPENDIX 3.

The Method of Sachs and Espey (1941) for the Measurement of Residual Circumferential Stresses in Thin Walled Tubes.

The method is a bending deflection method in which either one specimen is parted off from a parent tube, slit and successively pickled to remove thin annular layers from either the inner or the outer tube surface, or a number of specimens of equal length are prepared and then pickled or machined to a certain wall thickness before slitting. In principle both these methods should yield identical results provided the stock tube is uniformly stressed over the entire length from which the specimens are taken.

Fig. 11.1 shows the stages in the relief of the residual circumferential stresses and the following notation is postulated.

The thickness (x) removed by pickling from the original wall thickness (t) is positive in the direction of pickling. The change (ΔD) of the diameter (D) is positive if the specimen curves towards the direction of pickling i.e. ΔD is positive if D increases with outside pickling or if D decreases with metal removal from the inner surface. The total stress (p_c) in the fibre (x) may be considered as being relieved in three stages yielding four stress components p_{c1}, p_{c2}, p'_{c3} and p''_{c3}.

When the tube is slit, the original unturned outer diameter (D_o) increases by an amount ΔD_o (Fig. 11.1.B). If the stress distribution through the wall released by this slitting operation is linear with depth, (fundamentally this may not be true if the wall is thick) then by simple bending formulae:

$$p_{c1} = \frac{E}{1-\sigma^2} (t-2x) \frac{\Delta D_o}{D_m^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11.1)$$

where D_m is the mean diameter of the tube, and the other factors have meanings consistent with the notation given in Appendix 1.

After pickling has reduced the wall thickness by an amount x (Fig. 11.1.c) two types of stress have been relieved. A component p''_{c3} is set free in the relief of the direct stresses in the removed layer. If p_c is the sum of the four components present, then for equilibrium:

$$p''_c \; (t-x) + \int_0^x (p_c - p_{c1}) \, dx = 0$$

so that $p''_{c3} = -\frac{1}{t-x} \int_0^x (p_c - p_{c1}) \, dx$ .... (11.2)

but as p_c is not known except as $p_c = p_{c1} + p_{c2} + p'_{c3} + p''_{c3}$, the value of p''_{c3} cannot yet be determined.

The component p'_{c3} is also set free in the removal of the same layer (x). It is that stress which is relieved at the depth x due to a bodily change in diameter of the specimen when all the metal up to x is removed. If the change in "sprung" diameter on removal of the layer is ΔD, then this change is ascribed solely to the removal of the layer. This is considered equivalent to causing a change of diameter ΔD in a tube of wall thickness (t-x) which from elementary bending theory, gives the stress released as

$$p'_{c3} = \frac{E}{1-\sigma^2} \; \frac{(t-x)\Delta D}{D^2_m} \quad \text{.................... (11.3)}$$

Finally a diametral change dD is caused by the removal of a layer of thickness dx at a depth x, and is evidence of a component p_{c2}. As p_{c2} is relieved, a change of bending moment occurs given by:

$$dM = - p_{c2} \, dx \, \frac{(t - x)}{2}$$

which, by bending formulae, produces a diametral change dD, if

$$dM = \frac{E}{1-\sigma^2} \; \frac{dD}{D_m^2} \; 2 \, I$$

For a unit length of tube $I = \frac{(t - x)^3}{12}$

and thus $p_{c2} = - \frac{E}{1 - \sigma^2} \frac{(t - x)^2}{3 \, D_m^2} \frac{dD}{dx}$ .... (11.4)

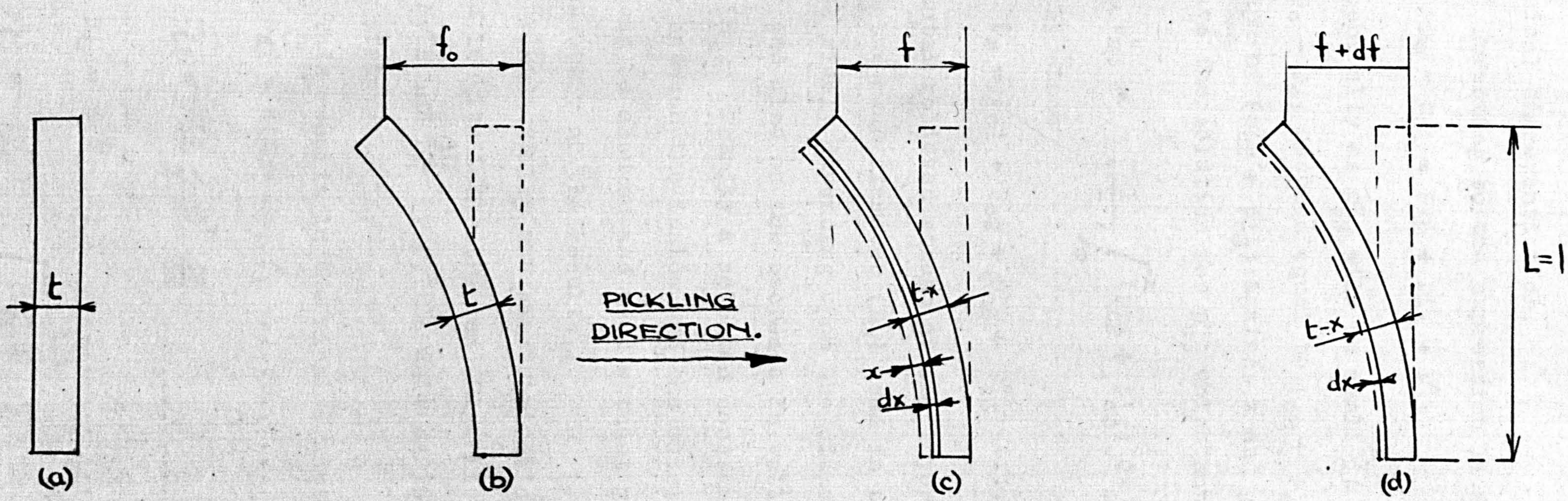


FIG: 11.2. STAGES IN THE RELEASE OF RESIDUAL LONGITUDINAL STRESSES. (SACHS & ESPEY).

This equation is soluble after plotting a graph showing the variation of ΔD with x and obtaining the value of the derivative $\frac{dD}{dx}$ from it by a process of graphical differentiation.

Reverting to the expression 11.2, we have that

$$p_c - p_{c1} = p_{c2} + p'_{c3} + p''_{c3}$$

and generally as p''_{c3} will be small it can be neglected and $p_c - p_{c1}$ assumed equal to $p_{c2} + p'_{c3}$. Hence plotting a graph of $(p_{c2} + p'_{c3})$ against x and graphically integrating we have the solution for p''_{c3} which now reads

$$p''_{c3} = -\frac{1}{t - x}\int_0^x (p_{c2} + p'_{c3})\, dx \quad \ldots\ldots\ldots\ldots (11.5)$$

Finally since

$$p_c = p_{c1} + p_{c2} + p'_{c3} + p''_{c3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11.6)$$

the total relieved stress at any layer "x" is calculated.

The mean diameter (D_m) used in the above expressions, varies widely depending upon the diametral changes which occur and also on the decrease in wall thickness during pickling. Usually a selected constant value of D, somewhere between the inside and the outside diameter of the tube can be assumed and used without greatly impairing the accuracy of the results.

Fundamentally the same derivation of formulas apply to the longitudinal stresses (Fig. 11.2) after replacing $\frac{E}{1-\sigma^2}$ by E and $\frac{\Delta D}{D_m^2}$ by f, the unit deflection of a longitudinal strip.

This results in the final equations :-

$$p_{l1} = E(t - 2x)\, f_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.7)$$

$$p_{l2} = -\frac{E(t - x)^2}{3}\frac{df}{dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.8)$$

$$p'_{l3} = E(t - x)\, f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.9)$$

and $$p''_{l3} = -\frac{1}{(t - x)}\int_0^x (p_{l2} + p'_{l3})\, dx \quad \ldots\ldots\ldots\ldots (11.10)$$

and p_{ℓ} = total longitudinal stress = $p_{\ell 1} + p_{\ell 2} +$

$$p'_{\ell 3} + p''_{\ell 3} \quad \ldots. \quad (11.11)$$

12. APPENDIX 4.

Modifications to the Sachs and Espey Analysis.

Following a critical examination of the Sachs and Espey analysis for the determination of the residual circumferential stresses in tubes (see Section 3.1), it was decided that certain unnecessary inaccuracies were inherent in the method of solution, which could be avoided by a slightly different mathematical treatment or by more precise definition of the constant mean diameter. In this appendix the modifications necessary to give the Sachs - Espey analysis a more fundamental basis are derived and it is shown that the resultant expressions do not increase in any way the labour involved in the computation of the several stress components in which the total stress is calculated.

It is considered that the expressions 11.1 and 11.4 (Appendix 3) are sufficiently exact for the derivation of the stress components p_{c1} and p_{c2}, provided that the constant mean diameter (D_m) is more precisely defined. In equation 11.1 the value for the diameter D_m which should be used is the one which is implied in the derivation of the expression for the stress component p_{c1}:

i.e. $D_m^2 = (D_o - t)(D_o - t + \Delta D_o)$ .... (12.1)

where D_o = initial outside diameter, t = initial wall thickness and ΔD_o = change in outside diameter on slitting. In the derivation of the expression for the component p_{c2} (Equation 11.4) the implied mean diameter is given by the expressions:

$$D_m^2 = (D_o - t + x)(D_o - t + x - \Delta D) \quad .... \quad (12.2)$$

and $$D_m^2 = (d_o + t - x)(d_o + t - x + \Delta D)$$

the first expression relating to layer removal from the inner surface and the second to layer removal from the outer surface. (In 12.2 the sense of ΔD is taken to conform with the postulations of Sachs and Espey regarding positive diametral changes and the factor d_o and x relate to the initial bore diameter and the depth below the original surface respectively). Since ΔD is generally small in comparison with D_o and as the method is only applicable to relatively thin sections where a linear bending stress distribution can be assumed, the expressions 12.2 can be substantially simplified and given constant values without introducing any appreciable errors into the computation. If, for example, layer removal from either surface is only considered to take place until the wall thickness of a specimen has been reduced to approximately half of its original thickness, i.e. $x_{max} = \frac{t}{2}$, the expressions 12.2 can be suitably amended as follows:

$$D_m^2 = (D_o - \frac{3t}{4})^2 \text{ for inside removal}$$

$$\text{and} \quad D_m^2 = (d_o + \frac{3t}{4})^2 \text{ for outside removal} \quad \ldots\ldots (12.3)$$

As pointed out in Section 3.1 the expressions 11.3 and 11.5 for the stress components p'_{c3} and p''_{c3} are somewhat approximate and an analogous treatment to that adopted by Davidenkov for the derivation of these components leads to a more exact solution.

Referring to Figs. 11.1 and 12.1, the removal of a layer of thickness dx at a distance x below the original pickled surface, alters the stresses present in a layer distant x_1 below the original surface ($x_1 > x$) in two ways.

The release of the stress p_{c2} at x with the layer removed causes the imposition of a direct force over the remaining section and also the release of a bending moment from the remaining section. These changes release stresses of magnitude dp'_{c3} and dp''_{c3} in the layer x_1, where

$$dp'_{c3} = - p_{c2} \frac{dx}{t-x} = \frac{E}{1-\sigma^2} \frac{(t-x)\, dD}{3D_m^2}$$

and $dp''_{c3} = \frac{dM}{I}$ (distance of x_1 from neutral axis when thickness $= t-x$)

$$= \frac{2E}{(1-\sigma^2)} \frac{dD}{D_m^2} \left(\frac{t-x}{2} + x - x_1 \right)$$

$$= \frac{E}{1-\sigma^2} \frac{(t+x-2x_1)}{D_m^2} dD$$

Combining these two expressions and integrating the result, the following expression defining the component p_{c3} ensues.

$$p_{c3} = \frac{2}{3}\left(\frac{E}{1-\sigma^2}\right) D_m^2 \left[(2t-3x_1)\Delta D + \int_0^{\Delta D} x \, dD\right] \ldots\ldots \quad (12.4)$$

where ΔD = change in "sprung" diameter when the wall thickness has been reduced by an amount x_1 and D_m has the same definition (12.3) as in the derivation of the component p_{c2}.

An identical expression to 12.4 has previously been quoted by Knights (1951) who, however, obtained it from direct substitution of appropriate factors in the Davidenkov expression giving the longitudinal stress component $p_{\ell 3}$ (see Equation 10.10 and its derivation) and not from first principles as has been done in the present treatment. Knights, however, referred all diametral changes to the original outside diameter - whether this was destroyed or not in the pickling process - and did not consider values of the constant mean diameter based on the bore diameter when removing metal from the outside surface of a specimen.

The computation of the stress component p_{c3} from the expression 12.4 does not involve any more labour than from the corresponding expressions (11.3 and 11.5) given by Sachs and Espey. In fact the expression is slightly easier to handle than the Sachs and Espey ones, since the graph giving the relationship between x and ΔD has to be drawn to enable the computation of p_{c2} and there is no need to prepare another curve for the process of graphical integration as is necessary in the original form of the analysis.

As both the Davidenkov and Sachs and Espey analyses for the derivation of residual longitudinal stresses are based on the simple bending of beams, any modification of the latters' expression for the component $p_{\ell 3}$ would only result in the derivation of the corresponding more fundamental equation given by the former, and consequently this is not considered here, nor has the Sachs - Espey analysis for longitudinal stresses been applied in the present investigation, for the same reason.

13. APPENDIX 5.

Sachs' Boring Method.

13.1. Introduction.

The Sachs' boring method for the detection and measurement of residual stresses, is a method involving the direct release of stresses by a gradual and systematic removal of material containing the stresses, and the measurement of the strains which develop in the remaining material by the natural restoration of static equilibrium during the metal removal process.

Applied to a tubular specimen, the procedure involves the removal of annular layers of metal from either the inner or the outer cylindrical surface and the measurement of the diametral and longitudinal changes which occur in the remaining section at each stage of the removal process. It will be appreciated that these changes in length and diameter can only be of elastic order of magnitude and hence extreme care must be taken when making measurements to ensure that errors are not introduced by temperature changes, etc.

From the measurements made during the above sequence of operations, the distributions and magnitudes of the three principal residual stresses can be calculated using the results of the following analysis.

13.2. The analysis of the Sachs' boring method.

The following method of analysis is based on the well known theory of thick cylinders under radial pressures developed by Lamé and which is to be found in any standard text-book concerned with the strength of materials.

For convenience it is preferable in the present analysis to amend the original Lame formulae to make all the stresses positive when tensile (it is more usual to denote the radial component as positive when

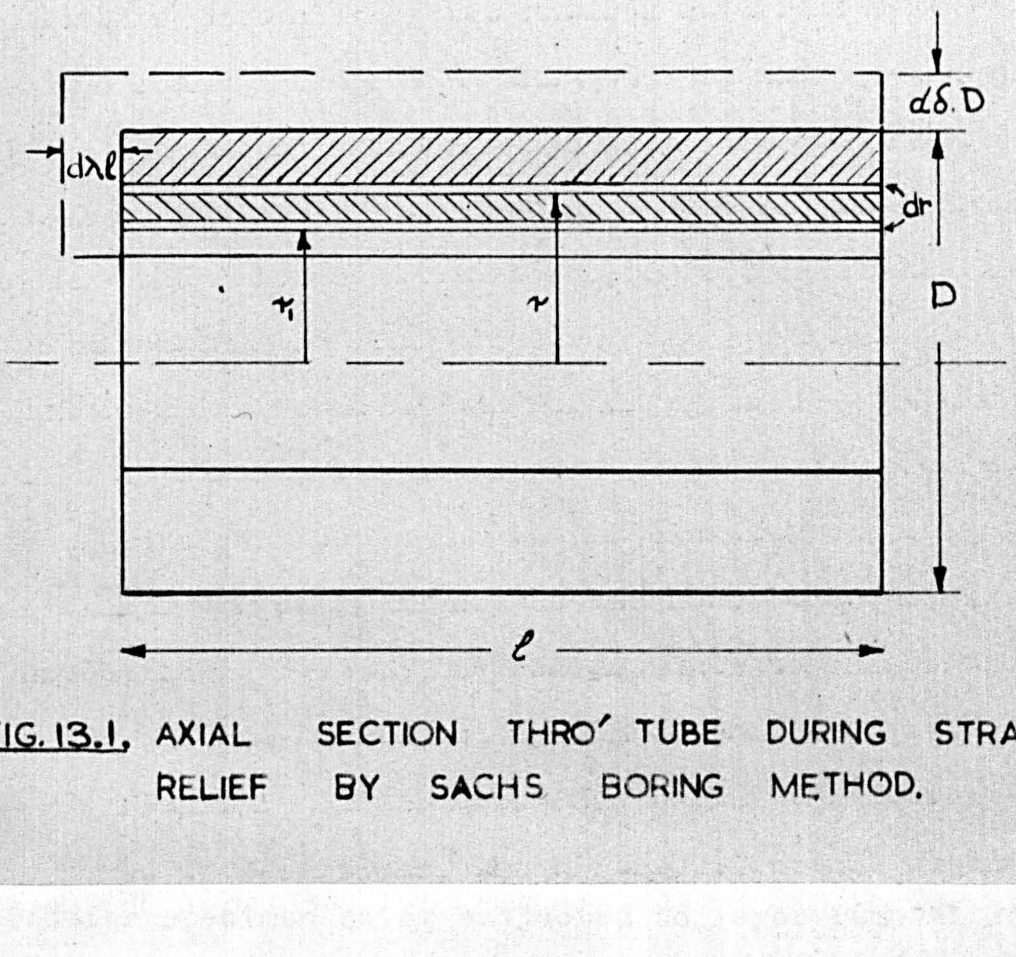


FIG. 13.1, AXIAL SECTION THRO' TUBE DURING STRAIN RELIEF BY SACHS BORING METHOD.

compressive) and because of this, the amended equations are quoted as follows:

(a) Equilibrium equation: $p_c - p_r = R \frac{dp_r}{dR}$ ...... (13.1)

(b) Stresses at any point in cylinder wall when external pressure is zero (internal tension = p_i).

$$p_r = p_i \left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(\frac{R_o^2 - R^2}{R^2}\right) \text{ .... (13.2)}$$

$$p_c = -p_i \left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(\frac{R_o^2 + R^2}{R^2}\right) \text{ .... (13.3)}$$

(c) Stresses at any point in cylinder wall when internal pressure is zero (external tension = p_o).

$$p_r = p_o\left(\frac{R_o^2}{R_o^2 - R_i^2}\right)\left(\frac{R^2 - R_i^2}{R^2}\right) \text{ .... (13.4)}$$

$$p_c = p_o \left(\frac{R_o^2}{R_o^2 - R_i^2}\right)\left(\frac{R^2 + R_i^2}{R^2}\right) \text{ .... (13.5)}$$

where

p_r, p_c = radial and hoop stresses respectively at any point in cylinder wall at radius R.

R_o, R_i = External, internal radii of cylinder.

R = Radius to any element in cylinder wall.

The following detailed analysis is given for a tubular specimen being subjected to layer removal from its inner (bore) surface. A similar method of solution is followed for metal removal from the outer cylindrical surface and where the results lead to expressions of slightly different form these are quoted in square brackets [ ] immediately following the expression derived in the detailed analysis, and given the same identification with the addition of the letter "a"; thus equation (13.11) refers to internal layer removal while equation (13.11a) refers to external layer removal.

When an annular layer of metal of thickness dr is removed from the bore of a specimen at such a radius as r (Fig. 13.1) the length and outer diameter of the remaining tubular section change by amounts $\ell d\lambda$ and $D_o d\delta$

respectively where ℓ and D_o were the length and external diameter respectively before layer removal and $d\lambda$ and $d\delta$ are respectively the longitudinal and diametral strains (positive if the basic dimensions increase) consequent upon the layer removal.

As layers of metal interior to that at radius r are removed, the stresses at r undergo continuous alteration, so that when the layer dr at r is finally removed, the stresses which disappear with it are not those originally present in that particular element.

If p_r, p_c and p_ℓ correspond respectively to the radial, circumferential and longitudinal stresses originally at radius r, then:

$$p_r = p_{r1} + p_{r2}$$
$$p_c = p_{c1} + p_{c2} \quad \ldots\ldots\ldots\ldots (13.6)$$
$$p_\ell = p_{\ell 1} + p_{\ell 2}$$

where the suffix 1 refers to the components of the original stresses which are present in a layer when it is removed and the suffix 2 refers to those components which are relieved at a particular radius by the gradual removal of metal interior to it.

<u>13. 2.1. Directly removed stress components</u> p_{r1}, p_{c1}, $p_{\ell 1}$.

When the internal radius of the tube is r, there exist in the immediate layer of thickness dr (which is about to be removed) uniform circumferential and longitudinal stresses of magnitude p_{c1} and $p_{\ell 1}$ respectively and a radial component p_{r1} which varies from zero at the free surface to dp_r at the radius r + dr.

When the layer of thickness dr is removed, strains $d\lambda$ and $d\delta$ occur at the outer surface of the tube, equivalent to stresses of magnitude $dp_{\ell o}$ and dp_{co} respectively, where:

$$dp_{co} = \frac{E}{1-\sigma^2}(d\delta + \sigma . d\lambda) \quad \ldots\ldots\ldots\ldots (13.7)$$

and $$dp_{\ell o} = \frac{E}{1-\sigma^2}(d\lambda + \sigma . d\delta) \quad \ldots\ldots\ldots\ldots (13.8)$$

E and σ being respectively Young's modulus and Poisson's ratio of the material.

The removal of the longitudinal force $p_{\ell 1} dA_d$ from r where dA_d = change in bore area = $2\pi r.dr$, causes the distribution of an equal force over the remaining cross section. Assuming that this force is uniformly distributed, then

$$p_{\ell 1} dA_d = dp_{\ell o} (A_o - A_d) \quad \text{.....................} \quad (13.9)$$

where A_o and A_d refer to the initial outside and current bore areas respectively of the specimen. (To be more exact, the current external cross-sectional area of the specimen should be used instead of A_o but the changes which occur in this value are so small compared with the magnitude of A_o that they can be neglected without impairing the accuracy of the solution).

Elimination of $dp_{\ell o}$ from equations (13.9) and (13.8) gives:

$$p_{\ell 1} = (A_o - A_d) \frac{E}{1-\sigma^2} \frac{d(\lambda + \sigma.\delta)}{dA_d} \quad \text{.........} \quad (13.10)$$

$$\left[p_{\ell 1} = -(A_D - A_i) \frac{E}{1-\sigma^2} \frac{d(\lambda + \sigma.\delta)}{dA_D} \right] \quad \text{.........} \quad (13.10a)$$

where A_D, A_i are current external, initial internal areas of the tubular specimen.

Equilibrium of the elemental cylinder of radius r and thickness dr, requires that

$$dp_r = p_{c1} \frac{dr}{r} \quad \text{...........................} \quad (13.11)$$

$$\left[dp_r = -p_{c1} \frac{dr}{r} \right] \quad \text{...........................} \quad (13.11a)$$

The removal of the internal tension dp_{r1} at the radius r + dr, is analagous to the imposition of an internal pressure of equal magnitude on the remaining section. Applying equation (13.3) to the conditions obtaining on the remaining section leads to the expression

$$dp_{co} = \frac{2r^2}{\frac{D\delta}{4} - r^2} dp_r \quad \text{....................} \quad (13.12)$$

Combination of equations (13.7), (13.11) and (13.12) results in the expression:

$$p_{c1} = (A_o - A_d)\ \frac{E}{1-\sigma^2}\ \frac{d(\delta + \sigma.\lambda)}{dA_d} \quad \ldots\ldots\ldots \quad (13.13)$$

$$\left[p_{c1} = -(A_D - A_i)\ \frac{E}{1-\sigma^2}\ \frac{d(\delta + \sigma.\lambda)}{dA_D} \right] \quad \ldots\ldots\ldots \quad (13.13a)$$

The radial component (p_{r1}) removed at the radius r lies between the values 0 and dp_r and can be neglected at this stage.

13.2.2. Gradually removed stress components p_{r2}, p_{c2}, $p_{\ell 2}$.

When the bore radius has increased to r_1 and a layer of thickness dr is further removed, stresses dp_r, p_{c1}, $p_{\ell 1}$ are released. Their release gives rise to deformations corresponding to stresses of magnitude dp_{co} and $dp_{\ell o}$ in the outer skin and of magnitude dp_{r2}, dp_{c2} and $dp_{\ell 2}$ at the radius r.

Provided that the distribution of changes of longitudinal stress is uniform over the remaining cross-section, then:

$$dp_{\ell 2} = dp_{\ell o} = \frac{E}{1-\sigma^2}(d\lambda + \sigma\, d\delta) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13.14)$$

The stress relieved at the radius r by the removal of all layers interior to it, is then:

$$p_{\ell 2} = -\int_{\frac{d}{2}}^{r} dp_{\ell 2} = -\frac{E}{1-\sigma^2}(\lambda + \sigma.\delta) \quad \ldots\ldots\ldots\ldots\ldots \quad (13.15)\ (13.15a)$$

(The negative sign in equation 13.15 is consequent upon the fact that the release of a tensile stress at radius r_1 causes the release of a compressive stress at all radii external to it).

The value of the infinitesimal circumferential stress component dp_{c2} is derived from (13.3) by considering the section remaining after the removal of the layer dr at radius r_1, as a thick cylinder under an internal pressure of magnitude dp_r. The resultant deformations at the outer surface are such that:

$$dp_{co} = \frac{2r_1^2}{\frac{D_o^2}{4} - r_1^2}\ dp_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13.16)$$

and at the radius r,

$$dp_{c2} = \frac{\left(\frac{D_o^2}{4} + r^2\right)}{\left(\frac{D_o^2}{4} - r_1^2\right)} \frac{r_1^2}{r^2} \; dp_r \quad \ldots\ldots\ldots\ldots\ldots (13.17)$$

Elimination of r_1, dp_r and dp_{co} from equations (13.7), (13.16) and (13.17) gives:

$$dp_{c2} = \frac{E}{1 - \sigma^2} \frac{(A_o + A_d)}{2A_d} \left(d\delta + \sigma\, d\lambda\right) \quad \ldots\ldots (13.18)$$

and since A_d is constant at a given radius,

$$p_{c2} = -\int_{\frac{d}{2}}^{r} dp_{c2} = -\left(\frac{A_o + A_d}{2A_d}\right) \frac{E}{1 - \sigma^2} \left(\delta + \sigma.\lambda\right) . (13.19)$$

$$\left[p_{c2} = -\left(\frac{A_D + A_i}{2A_D}\right) \frac{E}{1 - \sigma^2} \left(\delta + \sigma.\lambda\right)\right] \ldots\ldots\ldots (13.19a)$$

Finally by a treatment analogous to that used in the derivation of p_{c2} (using equation 13.2 in place of 13.3);

$$dp_{r2} = -\frac{\left(\frac{D_o^2}{4} - r^2\right)}{\left(\frac{D_o^2}{4} - r_1^2\right)} \frac{r_1^2}{r^2} \; dp_r \quad \ldots\ldots\ldots\ldots\ldots\ldots (13.20)$$

which on combination with (13.16) and (13.7) and followed by integration gives:

$$p_{r2} = -\int_{\frac{d}{2}}^{r} dp_{r2} = \frac{E}{1-\sigma^2}\left(\frac{A_o - A_d}{2A_d}\right) \left(\delta + \sigma.\lambda\right) \quad \ldots\ldots (13.21)$$

$$\left[p_{r2} = \frac{E}{1-\sigma^2}\left(\frac{A_D - A_i}{2A_D}\right) \left(\delta + \sigma.\lambda\right)\right] \ldots\ldots\ldots\ldots\ldots (13.21a)$$

13.2.3. Residual stress at any point in tube wall.

Combining the respective equations for the stress components by the expressions 13.6 and by writing the parameters ν and θ respectively for the factors $(\lambda + \sigma.\delta)$ and $(\delta + \sigma.\lambda)$, the following expressions result, giving the total residual stress at any point in the tube wall in terms of the measured variables.

Circumferential stress $= p_c = \frac{E}{1-\sigma^2}\left\{(A_o - A_d)\frac{d\theta}{dA_d} - \frac{(A_o + A_d)}{2A_d}\theta\right\} \ldots\ldots (13.22)$

$$\left[p_c = \frac{E}{1-\sigma^2}\left\{-(A_D - A_i)\frac{d\theta}{dA_D} - \frac{(A_D + A_i)}{2A_D}\theta\right\}\right] \ldots\ldots (13.22a)$$

Longitudinal stress $= p_\ell = \frac{E}{1-\sigma^2}\left\{(A_o - A_d)\frac{d\psi}{dA_d} - \psi\right\}$ ... (13.23)

$$\left[p_\ell = \frac{E}{1-\sigma^2}\left\{-(A_D - A_i)\frac{d\psi}{dA_D} - \psi\right\}\right] \text{ .... (13.23a)}$$

Radial stress $= p_r = \frac{E}{1-\sigma^2}\left\{\frac{A_o - A_d}{2A_d}\right\}\theta$ .... (13.24)

$$\left[p_r = \frac{E}{1-\sigma^2}\left(\frac{A_D - A_i}{2A_D}\right)\theta\right] \text{ .... (13.24a)}$$

In the application of this method, ψ and θ are calculated at each stage of the metal removal process, and the values are plotted against the current cross-sectional area (A_d) of the bore (in the case of internal layer removal). From smooth curves drawn through these experimental results the values of the factors $\frac{d\psi}{dA_d}$ and $\frac{d\theta}{dA_d}$ are derived, it being assumed

(i) that no error is entailed in drawing a smooth curve through the graphical presentation of experimental results, and

(ii) that the slope of a curve represents changes due to the removal of an elemental layer of material at the point considered.

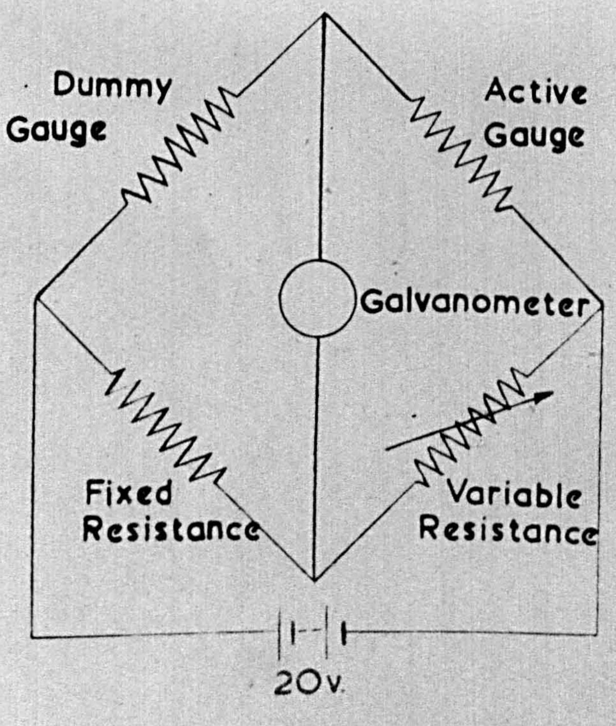
14. APPENDIX 6.

STRAIN GAUGE EQUIPMENT AND CALIBRATION OF GAUGES.

14.1. Equipment used.

The electrical resistance strain gauges used in this work were selected as the most suitable for this particular application. They had a nominal resistance of 1000 ohms and a high length to width ratio. This ensured that the effect of strains perpendicular to the direction of those being measured was as small as possible. The gauges were made of "Nichrome" wire, which has a very low temperature coefficient of expansion, thus reducing the effect of temperature variations to a minimum. Although the gauges were of the self-adhesive type this feature was not used, since the gauges so attached have occasionally been found to be unreliable.

A circuit diagram is shown below; the resistances used were of the normal decade box type. The fixed arm consisted of a box varying from 0 to 10,000 ohms in steps of 1,000 ohms. The variable arm was



Strain Gauge Circuit

made up of five decade boxes in series. They were so arranged that any resistance from 0 to 10,000 ohms could be obtained in steps of .1 ohms. A final slide wire box was incorporated in this arm to give direct readings to .01 ohms, it being possible to estimate to .001 ohms.

Two galvanometers were used, a micro-ammeter with a full scale deflection of 25 micro-amps for coarse readings and a sensitive mirror galvanometer with a half scale deflection of approximately 5 micro amps to obtain the final accurate balance.

The 20 volt supply was from two high tension batteries; as only a small current was taken during the actual measurements the load on them was not excessive.

The circuit was wired up with P.V.C. coated bell wire and arranged in a perspex topped box. The gauge, supply and galvanometer terminals were arranged on the top of the box to enable the circuit to be connected and disconnected easily. A switch was placed in one of the leads to each galvanometer so that they could be isolated when not in use.

14.2. Method of Calculation.

If initially, with no current flowing through the galvanometer, the resistances are as shown in the diagram, then the well known condition for balance is:-

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \text{.............................} \quad (1)$$

When the gauge is strained its resistance will change to some value $R_2 + \Delta R_2$. To maintain the balance, that is, no current flowing through the galvanometer, the resistance R will have to be changed to some value $R_4 + \Delta R_4$. It is assumed at this stage that R_1 and R_3 do not change.

The condition for balance then is:

$$\frac{R_1}{R_2 + \Delta R_2} = \frac{R_3}{R_4 + \Delta R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Eliminating $\frac{R_1}{R_3}$ from equations (1) and (2)

$$\frac{R_2}{R_4} = \frac{R_2 + \Delta R_2}{R_4 + \Delta R_4}$$

which simplifies to

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

For a given gauge factor defined by the relation

$$\text{Strain} = \frac{\Delta R_2}{R_2 \times F}$$

where F is the gauge factor the strain may be calculated, using equation (3) from the relation

$$\text{Strain} = \frac{\Delta R_4}{R_4 \times F} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

It can easily be shown, in a similar manner that if both R_1 and R_3 change by the same amount due to temperature effects the validity of the calculation is not affected.

Note. The balancing value of bridge resistance referred to in Section 2 is the value of R_4 when no current flows through the galvanometer.

Calibration of Gauges.

The purpose of this calibration was to ascertain whether the gauge factor, which the makers supply with each batch of gauges, could be used to calculate the strains in Equation (4).

The test was carried out on the specimen used in the preliminary test (Section 4) and took the form of a compression test in a multi-lever testing machine. Only the longitudinal gauge was calibrated since the accuracy with which the strain in a circumferential direction could be calculated, from the results of a compression test on a short specimen, is not very great.

CALIBRATION OF LONGITUDINAL GAUGES

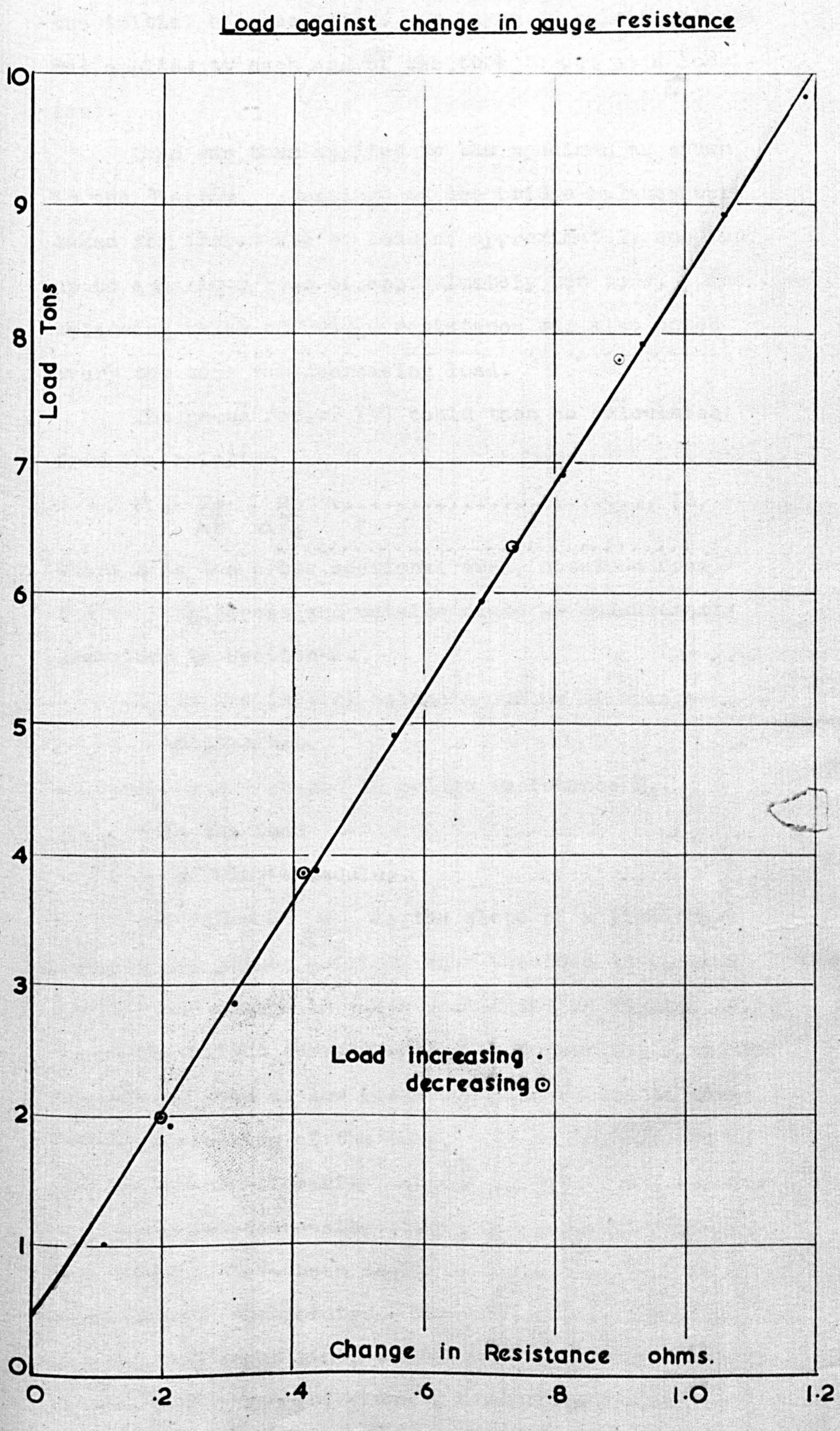


FIG: 14.1.

The active and dummy specimens were placed as close together as possible in the testing machine and the initial balance noted. A layer of calcium oleate was applied to each end of the tube to act as a lubricant.

Load was then applied to the specimen as shown in the diagram. Readings of the bridge balance were taken for increments of load of approximately one ton, up to a maximum load of approximately ten tons. The balancing value of bridge resistance was also noted every two tons for decreasing load.

The gauge factor (F) could then be calculated from the relation

$$F = \frac{R_4}{AE} \cdot \frac{W}{\Delta R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where A is the cross sectional area, obtained from the wall thickness and outside diameter measurements described in Section 2.

R_4 is the initial balancing value of bridge resistance.

ΔR_4 is the change in bridge resistance R_4.

W is the load

E is Young's Modulus.

The value of $\frac{W}{\Delta R_4}$ is the slope of a line drawn through the points obtained when the load is plotted against the change in gauge resistance as in Fig. 14.1.

The slight curvature of the calibration curve for increasing load at low loads is probably due to some slight barrelling of the tube. It is unlikely to be due to the non-linearity of the gauges, since it does not occur for decreasing load. Consequently the first two readings have been neglected when drawing a straight line through the points. The remainder of the readings lie on a straight line, the slope of which when substituted in Equation (6) gives a gauge factor of 2.16, assuming that E is 13,000 tons per in. The value given by the makers is 2.20, the small difference observed (2%) may be ignored and the makers gauge factor confidently accepted for the gauges used in this work.

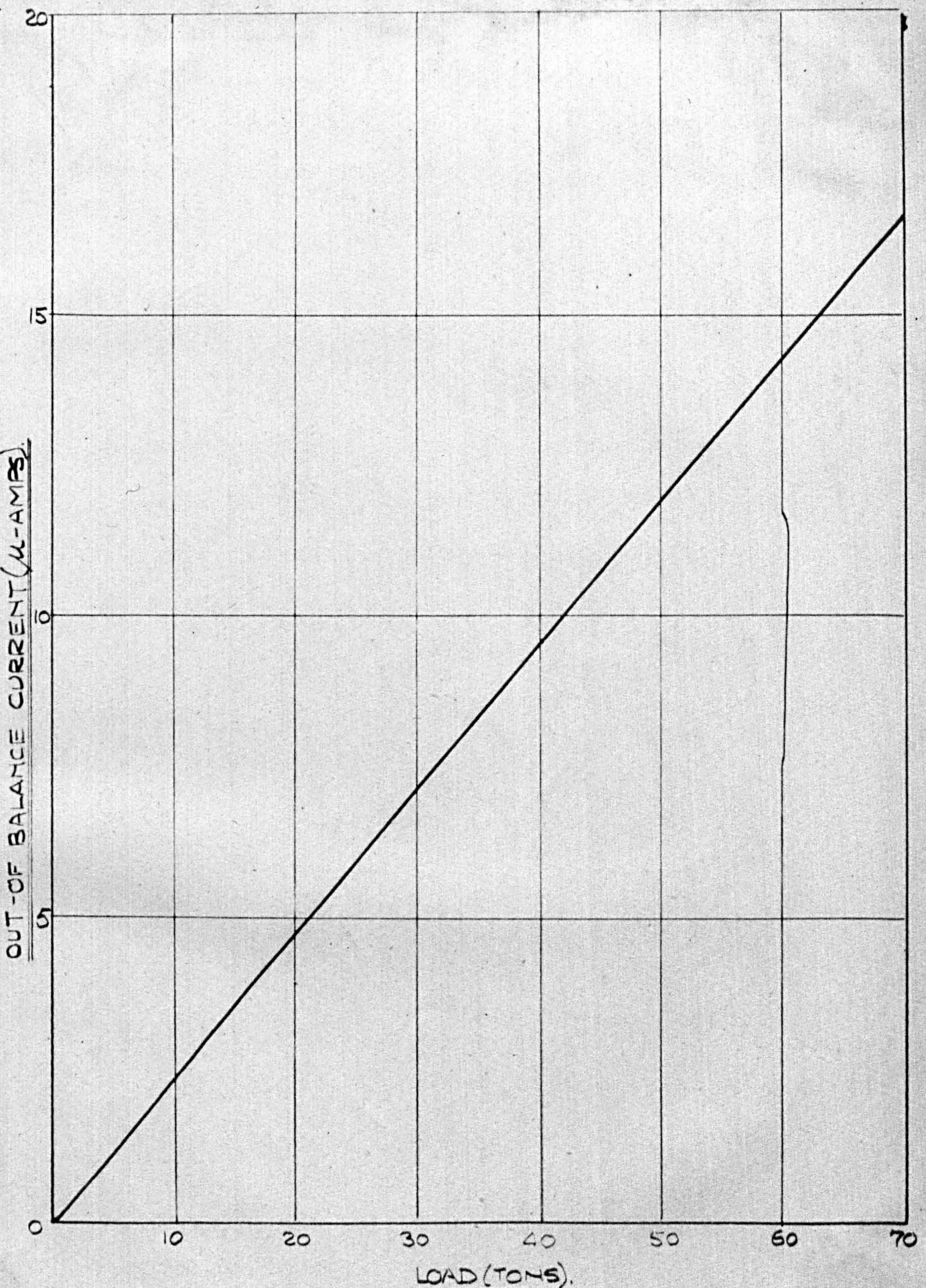


FIG. 15.1. DRAWING PIN CALIBRATION CURVE.

15. APPENDIX 7.

Calibration of drawing pin.

The drawing pin (Fig. 4.2) was fitted with two axially mounted resistance strain gauges, each of 1000 ohms resistance. Two matched gauges were also fixed partially to the drawing pin to act as dummy gauges and temperature compensators.

The strain gauges were connected into a Wheatstone bridge circuit with an operating voltage of 30 volts and incorporating a D.C. micro-ammeter for measuring the out-of-balance current across the bridge.

Calibration of the drawing pin to enable the conversion of the micro-ammeter readings into the drawing load transmitted through the pin, was carried out on a 100 ton hydraulic straining single lever testing machine at Sheffield Testing Works.

The results of this calibration are given in tabular form below and in graphical form in Fig. 15.1.

Load (tons)	Micro-ammeter reading* (μ-amps)	
	Loading	Unloading
5	1.2	1.2
10	2.4	2.3
15	3.6	3.5
20	4.7	4.7
25	5.9	6.0
30	7.1	7.1
35	8.3	8.3
40	9.5	9.5
45	10.7	10.7
50	11.8	11.9
55	13.0	13.1
60	14.2	14.3
65	15.5	15.6
70	16.7	-

* average of four tests.

16. APPENDIX 8.

Analysis of errors in the experimental results and the derived stresses.

The major sources of error in residual stress determination are not introduced in the experimental measurements but are due to the variations in the wall thickness of different specimens and the difficulty in deriving the differential coefficients used in the analyses.

In the present work the probable accuracy of the measured observations is relatively high. The bending deflection method demands the measurement of wall thickness and diametral changes, both of which can be measured using calibrated micrometers to within ± 0.0002 in. This corresponds to a mean accuracy of about ± 2%. In the boring tests, changes in gauge resistance of 0.01 ohms can be readily detected which correspond to an accuracy of 1% in the maximum values of the observed parameters θ and ψ Since careful precautions were taken to avoid errors due to temperature compensation and zero drift in the strain gauge measurements, the accuracy of the strain parameters θ and ψ coupled with the corresponding bore area, will generally be well within the range ± 2%.

The errors in the calculation of the results are largely the results of the graphical differentiation process used to determine the values of the derivatives $\left(\frac{dD}{dx}, \frac{d\psi}{dA} \text{ etc}\right)$ The maximum error entailed in these measurements is probably of the order of ± 10% at points where the slope of the curve under consideration is changing rapidly and at the beginning of the experimental curve where the latter becomes discontinuous. This error in the derivative is more likely to have a greater influence on the accuracy of the stresses determined by the boring tests rather than by bending deflection tests, although in neither case will the maximum error be greater than ± 10% and this at and near the surface of the specimens.

Subsidiary errors will also be introduced into the stresses determined by bending deflection methods by the graphical evaluation of the integrals in the relevant analyses. The application of Simpson's rule to the process of graphical integration is reasonably accurate and the error introduced by this process is not very great, particularly in comparison with the errors introduced by the opposite process of graphical differentiation. Consequently the subsidiary errors can be considered insignificant.

Actually, because of the variation of the several components in which the total stress at any particular point in the wall of a tubular component is calculated, it is extremely difficult to state an overall percentage error for the residual stress determinations obtained in this work. It is believed, however, that the measured stress values quoted are accurate to within ± 10% at the surface of the specimens and to within ± 2 tons per sq. in. at all other points in the tube wall.

The greatest errors in the determination of residual stress distributions in tubes, however, are caused by the variation in the wall thickness which is likely to be present in any particular specimen. The normal tolerance on the wall thickness of commercial tubes is ± 10% and while the tubes used in this work have had a much smaller thickness variation than this (the maximum variation recorded being ± 7% and the average variation for all the tubes tested being only a little over half this amount), the effect is still likely to be significant.

In the analysis of the experimental results, statistical methods have been applied to establish the validity of any simple functional relationship (linear) which appears to exist between any two variables. The method of least squares has been applied to obtain the "best line" through the experimental results and no linear relationship proposed has a correlation coefficient (see "Facts from Figures",

A. J. Moroney - Pelican) numerically less than 0.92 and in most cases the degree of correlation is more than 0.95.

17. APPENDIX 9.

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