MODELLING LONGITUDIAL DISPERSION – AN UPSTREAM TEMPORAL CONCENTRATION PROFILE-INDEPENDENT APPROACH

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Abstract. Primary solute transport processes may be represented using two parameter models such as the ADE or ADZ models. Previously, laboratory measurements have been used to determine best-fit parameter values corresponding to specific drainage structures. This paper shows that the derived ADZ parameter values are not independent of the upstream temporal concentration profile, and that this model fails to provide a robust description of the main solute transport processes operating within a surcharged manhole. Instead it is proposed that the solute transport characteristics of a system may be better represented by the cumulative residence time distribution (CRTD) corresponding to an instantaneous upstream injection.

1. Introduction

Solute transport processes affect the performance of a wide range of water engineering structures. Urban drainage network models that predict the transport of dissolved substances are increasingly used. Some of the models transport the pollutants by advection only, while others also account for the effects of dispersion. The MOUSETRAP model, for example, implements a two-parameter ADE model. An alternative two-parameter model – the ADZ model – has been put forward as being better suited to describe the types of temporal concentration profile (TCP) often observed in practice (Wallis *et al.* [1]; Guymer *et al*. [2]). At present there is limited knowledge regarding appropriate values for the model parameters. Laboratory measurements have been conducted, leading to the identification of best-fit (optimised) parameter values linked to the geometric characteristics of specific drainage structures.

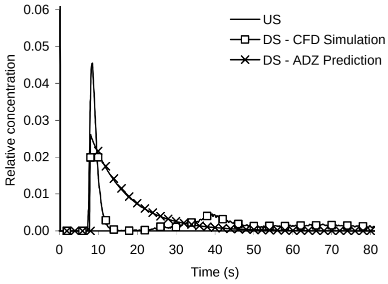
It may previously have been assumed that parameter values determined for a specific structure could be utilised directly within urban drainage network models to predict downstream TCPs, irrespective of the specific upstream TCP. This paper will question this, and propose an alternative modelling approach based on the use of the cumulative residence time distribution corresponding to an instantaneous upstream injection.

2. An assessment of ADZ model robustness

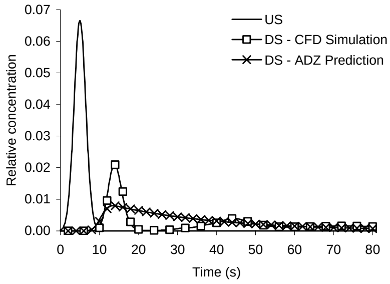
In order to evaluate the potential effects of upstream TCP on derived parameter values, a computational fluid dynamics (CFD) model was used to generate the downstream TCP corresponding to an instantaneous tracer injection, for an 800 mm diameter manhole. The sample trace, shown in Figure 1a, is taken from the high pre-threshold zone, with 150 mm surcharge. At this surcharge depth there is a clear secondary peak in the downstream TCP, shown at around 40 s, which has been attributed to tracer circulating in the surcharged part of the manhole (see Lau *et al.* [3]). Two non-instantaneous upstream profiles were then assumed, both of which are closer to typical field/laboratory profiles: a 10 s Gaussian profile (μ = 5 s, σ = 1.5 s); and a 20 s square pulse (constant concentration). Downstream profiles were synthesised from the simulated instantaneous downstream TCP using superposition (Figures 1b and 1c). An optimization process was used to determine the best-fit ADZ parameter values for each of the three pairs of upstream and downstream TCPs. These values are also presented in Figure 1. It should be noted that the two parameters fitted were α and δ [1, 2]. These are often interpreted to suggest values for the mean travel time ($\bar{t}$) and the residence time (T). T is described as the difference between $\bar{t}$ and the time delay (difference in first arrival times).

All three profiles correspond to the same system, so consistent parameter values should be expected if the model provides a robust description of the system. However, the results show that the inferred parameter values are **very sensitive** to the shape and duration of the inlet profile. For example, the travel time ($\bar{t}$) for the square pulse injection data is nearly <u>three</u> times higher than that derived from the analysis of the instantaneous response. The R_t^2 values are not particularly good for any of the data sets.

Figure 2 presents some sample predictions to show that when the parameter values derived from one pair of profiles are utilized to predict the downstream TCP for a different upstream profile, the prediction is always very poor indeed.



(a)



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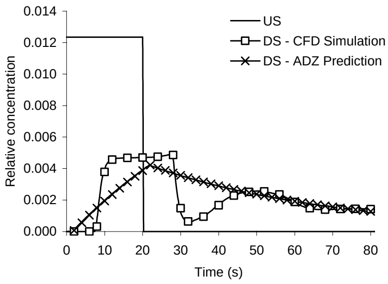


(c)

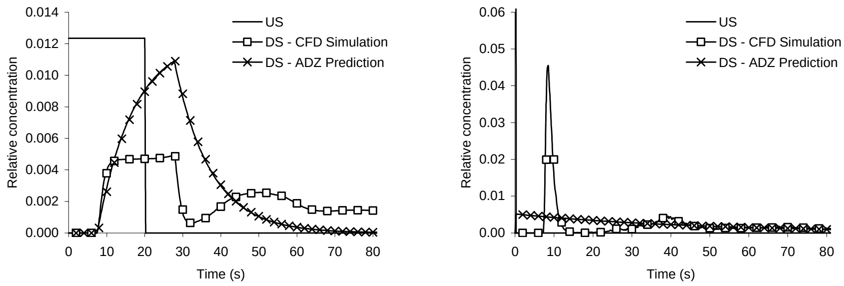
(a) Instantaneous injection
$\bar{t}$ = 17.22 s, T = 9.47 s
δ = 31, α = -0.974, R_t^2 = 0.348

(b) Gaussian superposition
$\bar{t}$ = 32.75 s, T = 27.50 s
δ = 21, α = -0.991, R_t^2 = 0.484

(c) Square pulse superposition
$\bar{t}$ = 48.69 s, T = 48.44 s
δ = 1, α = -0.995, R_t^2 = 0.855

Figure 1. Synthesised responses to three different upstream temporal concentration profiles, compared with best-fit ADZ model predictions; ADZ parameter values.

If the parameter values derived for the instantaneous upstream profile are used to route the square pulse (Figure 2a), the peak is significantly overestimated and occurs later than expected. Similarly, if the parameter values derived for the square pulse upstream profile are then used to route the instantaneous pulse (Figure 2b), the peak is too small and occurs too early. In modelling terms, ADZ parameters derived from non-instantaneous upstream profiles may strictly only be applicable in situations where the upstream temporal concentration matches that for which they were originally derived. **These observations suggest that the first order ADZ model provides a poor representation of the mixing characteristics of the manhole, and that the derived parameter values are not independent of the upstream TCP.**



(a) Square input routed using best-fit ADZ model parameters derived from the instantaneous injection

(b) Instantaneous input routed using best-fit ADZ model parameters derived from the square pulse injection

Figure 2. Effect of routing upstream TCPs with different sets of ADZ parameters.

3. Alternative approach – the cumulative residence time distribution (CRTD) curve corresponding to an instantaneous injection

3.1. *Justification for the approach*

The alternative approach is based upon the use of the *cumulative residence time distribution* (CRTD) corresponding to an *instantaneous* upstream input. The CRTD represents a robust model of a system's fundamental solute transport characteristics [4]. Once the instantaneous response is known (for example, as shown in Figure 1a), the response corresponding to any upstream input may be derived via superposition (as illustrated in Figure 1b and 1c). This holds true for any steady-state non-reacting system. Lau *et al.* [5] have demonstrated the usefulness of the CRTD in describing and understanding the solute transport characteristics of surcharged manholes.

It is not necessarily straightforward to identify the instantaneous cumulative RTD curve from observed data; deconvolution relies on signal-processing techniques to identify appropriate transfer functions. However, CFD-generated data may readily be produced in this form, so it is useful to consider how well the curve needs to be defined to provide a practical predictive tool.

3.2. *Simplified Implementation of the Instantaneous CRTD model*

Figure 3 presents the simulated RTD curve shown in Figure 1a as a CRTD. Data points were reported at 0.25 s intervals, providing a high-resolution description of the system's mixing characteristics. For modelling purposes, it is convenient to simplify this curve into a smaller number of data points. t_{10}, t_{50} and t_{90} are routinely used to characterize the performance of 'mixing vessels'

[4]. However, the steep rising limb and initial peak in the RTD is marked by a sudden change in gradient in the CRTD at t_{30}. Therefore t_{30} and t_{70} were also included, forming a five-point simplification of the CRTD. In experimental work, issues relating to instrument noise and calibration mean that it is difficult to identify t_0 and t_{100} accurately. t_{10} and t_{90} are less sensitive to analysis procedures (e.g. subtraction of background) because they fall on steeper parts of the CRTD. Therefore travel times less than t_{10} or greater than t_{90} were ignored, and the CRTD was redistributed between these two extremes (Figure 3). The simplified RTD shown in Figure 3 provides a better fit ($R_t^2 = 0.73$) to the original high-resolution RTD than the ADZ model (Figure 1a). There is clearly scope for sensitivity analysis to refine the exact choice of points adopted to define a simplified CRTD.

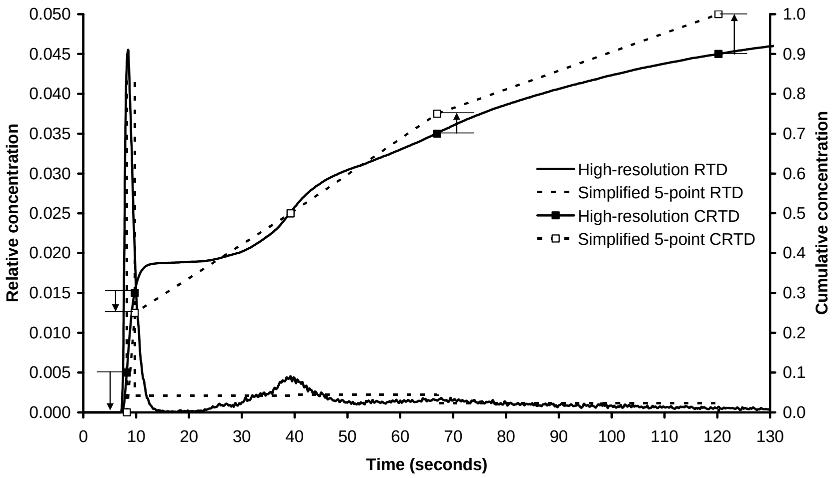


Figure 3. Instantaneous CRTD and RTD for 150 mm surcharge, 800 mm diameter manhole.

Once the RTD corresponding to an instantaneous injection is known for a specific system, the downstream response to *any* upstream input can be determined using superposition. Figure 4 compares three different models of the manhole’s response to a Gaussian upstream input. The ‘full’ RTD profile has been generated using superposition of the original high-resolution CFD-generated RTD. This profile is then compared with two simplified ‘models’ of the RTD: the 5-point RTD and the RTD corresponding to the two-parameter ADZ model (parameter values corresponding to the instantaneous injection, Figure 1a). In both cases superposition of the instantaneous RTDs has been used to generate the downstream response to the Gaussian input.

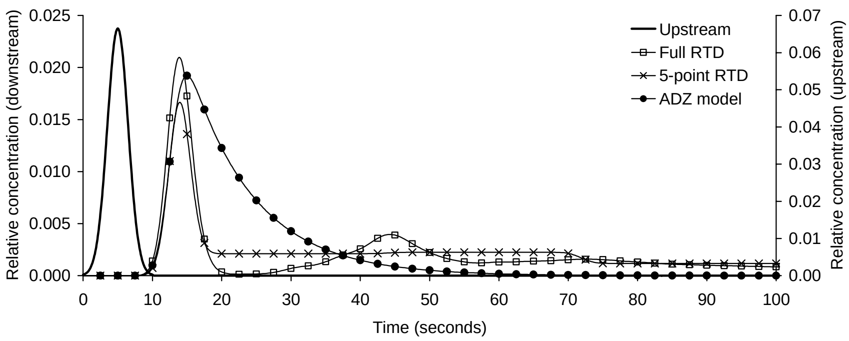


Figure 4. Comparison between 'full', five-point CRTD and ADZ model responses for a Gaussian upstream profile (150 mm surcharge manhole).

It may be seen that the 5-point CRTD model provides a far better fit to the original data set (R_t^2 = 0.9) than the superposed ADZ model of the system's response to an instantaneous injection (R_t^2 = 0.13).

4. Conclusions

- The ADZ model does not provide a good representation of the mixing characteristics of a surcharged manhole.
- Optimised ADZ parameter values are not independent of the upstream TCP.
- CRTD 'curves' characterised by as few as five points (t_{10}, t_{30}, t_{50}, t_{70} and t_{90}) may provide a more robust model of solute transport through urban drainage structures than approaches (such as ADZ) that are reliant upon assumptions about the shape of the upstream TCP.
- Using the instantaneous RTD, downstream TCPs may be obtained for any upstream TCP via superposition. This provides a practical modelling approach that might readily be implemented in urban drainage network modelling packages.

References

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