OPTIMISATION AND MODELLING TECHNIQUES IN DYNAMIC CONTROL OF WATER DISTRIBUTION SYSTEMS.

by

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SUMMARY

The thesis develops optimisation and modelling techniques with the ultimate aim of control of water distribution systems to produce overall optimised operation. Typical system operating conditions are analysed to determine cost factors and control requirements and hence enable development of system performance criteria. The most significant costs are those for distribution pumping and a range of original optimisation techniques are investigated which will lead to operational improvements for a restricted class of systems. Application of these techniques to more complex systems is shown to be dependent on development of simplified dynamic models. Suitable models are formulated and computer programs are developed to evaluate matching coefficients for very general systems. Combining the optimisation techniques and simplified models enables a computer algorithm to be devised which can be applied to give optimal control of complex systems taking account of all cost factors and operational constraints. The scheme incorporates a simulation of the overall dynamics of a water system, by means of a tailored computer program, which is initially used with historical operating data for validation purposes. The results confirm the theoretical predictions and show that benefits can be obtained from on-line computer controlled operations.

ACKNOWLEDGEMENTS

This project has been a collaborative venture, involving University of Sheffield, Sheffield City Polytechnic and the South Eastern Division of the Yorkshire Water Authority, and could not have been completed without active co-operation and assistance from all parties. I would, therefore, like to express my sincere appreciation to those concerned which includes, but is not limited to:

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FIGURES

TABLES

CHAPTER 1

INTRODUCTION

1. Water distribution systems play an important part in modern life by providing the vast quantities of purified water required for both domestic and industrial purposes. Over the years small localised water undertakings have been amalgamated to form larger networks; this effect has been accelerated by the recent local authority reorganisation scheme resulting in water distribution systems catering for each major city or town and surrounding area. Whilst the smaller networks could be controlled manually, effective operation of these much larger inter-related systems relies on some degree of automatic monitoring and control. The rising standard of living also implies that labour costs will continue increasing and this will give constant encouragement to release of manpower from routine operating tasks to make available for more productive activities. Additionally there has recently been national concern for the reduction of energy consumption, and the electricity cost for distribution pumping has risen dramatically.

It is considered inevitable that management of water supply systems will eventually yield to fully automated control in order to achieve efficient operation of these systems of ever increasing complexities and costs. Existing technology has long been capable of providing computerised equipment hardware for measurement and control, however computer software (in the form of program algorithms) is not in such an advanced state that effective on-line control can be immediately achieved and additional research is required in this latter area.

Of major importance in control is the concept of optimisation of operation which attempts to achieve lowest operating costs consistent with providing a satisfactory service to customers. Such integrated optimisation schemes for these large systems must rely on simplified mathematical models which adequately represent the system dynamics.

Application of sophisticated control techniques ideally requires an existing system with modern control equipment and operating experience. The Doncaster and District Joint Water Board (now the South Eastern Division of the Yorkshire Water Authority), have recently implemented an on-line system of computerised monitoring with limited control features. The current work forms part of a collaborative project to devise new algorithms suitable for overall operational control. In particular, the major part of this thesis is concerned with development of optimisation models and simplified network models which are finally combined to allow presentation of a computer control algorithm suitable for on-line optimal control of complete water distribution systems. For each stage of development the results are validated by use of actual data in conjunction with simulated system operation.

It will be shown that the project is, indeed, a complex one involving optimised control of large scale non-linear dynamic systems subject to unknown disturbances. The optimisation methods must cater for high state and control dimensionality, with further complications of highly non-linear performance indices, and must incorporate both continuous and discrete controls. The following sections outline the detailed complexities and show how they have been approached and resolved in this study.

1.1 Control of Water Distribution Systems

In the region studied boreholes are a typical source of water supply with pumping to the network using parallel combinations of fixed speed pumps. Fixed or variable speed booster pumps together with control valves are normally used for transfer of water between reservoirs of differing pressure zones. In both cases the pumping flows are dependent upon the reservoir levels, and the costs upon electricity unit and demand charges.

Distribution networks consist of large numbers of interconnecting pipes with occasional control valves, both of which have a non-linear relationship between flow and head loss. Approximately constant head reservoirs are connected at various points of the network to provide storage capability and maintain required pressure levels. Individual consumer demands occur at distributed points throughout the network and, since there is usually minimal monitoring, the total demand must be calculated from pump flows and reservoir levels.

Optimisation over a future time period (dynamic optimisation) can only be performed for known consumer demands and a prediction scheme is required which will estimate demand throughout the optimisation period. The data available on which to base such an estimate is essentially past consumption records together with an allowance for known future industrial and residential demands. An automated demand prediction scheme has already been developed as part of this project[91] and consequently has not been covered in this thesis.

Since water networks contain storage the optimisation problem reduces to minimisation of electricity charges and associated costs for the complete network over the entire optimisation period. This can be achieved by control of pumping and storage whilst catering for consumer demands and maintaining desired reservoir levels. The successful application of optimisation methods depends significantly upon the formulation of a dynamic network model for rapid and repeated evaluation of the effect of control strategies upon the network reservoir levels.

It is essential to ensure that theoretical developments are applicable to actual systems and meet all operational constraints. However, there are no truly typical water distribution systems, all are somewhat unique with differing characteristics. In addition it is difficult, if not impossible, to confirm theoretical proposals by manipulation of actual operational systems, this is particularly true for water systems with their extremely limited monitoring capability. As a consequence of these aspects validation of results must rely upon methods for accurate systems simulation using actual operational data where possible and generating additional data as required.

1.2 Presentation of Thesis.

This section shows the layout adopted in the thesis to produce a coherent and unifying theme result in overall optimised control of water distribution systems.

Chapter 2 describes typical features of water distribution systems and gives specific details for the DDJWB system with coverage of system elements, normal system operation and operating costs.

Chapter 3 analyses pumping elements in order to derive models suitable for incorporation in network and optimisation schemes. This includes derivation of an independent control variable to give simulated pump operation on a continuous or discrete basis (allowing for head dependent flows) and derivation of the necessary conditions for operation at maximum pumping efficiency. Also typical pumping operations are analysed to develop and justify cost models, for various optimisation techniques, in terms of electricity charges.

Chapter 4 investigates the application of various optimisation techniques in order to optimise pumping costs in realistic networks. Detailed consideration is given to development of costs in a suitable form for treatment by forward dynamic programming; this technique is applied to a single reservoir system with multiple borehole pumping using combinations of constant flow pumps. It is shown that useful results are achieved but dimensionality can be a severe problem. The method is extended to cover a restricted multi-reservoir system, by means of successive approximations, but the results reveal difficulties in implementation and extensive use of computing time. The conclusions are that the method is feasible but further extensions, to cater for more sophisticated systems, must rely on development of simplified network models.

The general theory of decentralised hierarchical techniques is reviewed and an application is made to a single reservoir zone with pumping from assumed continuously variable capability. The method is developed to include realistic cost factors and covers development of suitable operational performance indices. A comparison of the results, with other similar methods, show that a superior formulation has been achieved giving desirable operating characteristics. Whilst the technique theoretically permits optimisation of very general high dimensional systems, in practice the formulation requires linear system equations and it is shown that modification to cover discrete control variables is difficult to achieve. It is concluded that extension, to allow optimisation of complex multi-reservoir systems, now requires development of simplified linear dynamic models in terms of continuous control variables.

Computer programs are developed by the author for both the above optimisation techniques and written in FORTRAN IV for general purpose applications.

Linear programming techniques are also investigated and the optimisation problem is developed into a mixed variable linear-integer programming formulation. Further analysis shows that solutions could only be obtained with some restrictions on typically sized multi-reservoir systems, but that additional research, using the suggested solution methods, is warranted.

Chapter 5 reviews non-linear water distribution system analysis for both static and dynamic solutions, which form the basis of an existing simulation program,[81] and provides theoretical supplements and program modifications to create a more useful simulation program. An improved version results from incorporation of independent pump and valve control parameters to yield a network simulation capable of responding to either continuous or discrete optimised controls. The program is further enhanced by inclusion of routines, developed by the author, for calculating sensitivity coefficients by direct evaluation of the static solution Jacobian matrix. These coefficients are those required for the linear dynamic models derived in Chapter 6. A description is given of the modified program to show the full implication of the changes and to allow explanation of the particular program features employed in this study.

Chapter 6 examines the possibility of obtaining simplified dynamic model representations of entire, non-linear, distribution networks to cater for overall optimisation of complex systems.

An existing method[16] is reviewed which gives a simplified non-linear model using measurements of commonly available network variables and derived operating constraints. This is proposed as one means for extending the dynamic programming technique of Chapter 4 to cover optimisation of multi-reservoir networks with variable head pumping.

An alternative method, developed by the author[13,94] gives the derivation of linearised models for networks containing any number, or configuration, of reservoirs, pumps, valves, and consumer demands. The theory gives a simplified model in state-space form which defines dynamic reservoir levels in terms of control and disturbance parameters; refinements are made to cater for effects of continuous variation of reservoir levels and allow variable head pumping by inclusion of the pump model derived in Chapter 3. The model is then extended to enable calculation of head variation for selected pressure nodes with further refinements permitting evaluation of average pump flows over each time interval. Two methods of evaluating the model coefficients are examined, one by perturbation of static solutions for each variable in turn, and one by use of the programming modifications developed in Chapter 5. The validity of the theoretical results are confirmed by application to actual networks and comparisons made between different versions of the model. These linear models are concluded to be suitable for extending the decentralised hierarchical techniques of Chapter 4 to cover on-line optimisation of overall system operation.

Chapter 7 combines the pumping station models (of Chapter 3), the decentralised optimisation technique (of Chapter 4) and the linear dynamic model (of Chapter 6) to provide a method leading to on-line optimisation of complex water distribution systems with assessment using the modified simulation program (of Chapter 5).

An existing multi-reservoir network containing head dependent borehole and booster pumping stations and control valves is implemented to allow controllable simulated operation. For this network an equivalent linear dynamic model is developed with coefficients automatically generated by the modified simulation program. Performance indices are derived to suit the optimisation method which, in addition to electricity charges, now take into account system requirements of pump operation at maximum efficiency by optimising potential energy imparted to the water. The optimisation program of Chapter 4 is modified and used to compute optimal pump and valve controls which are applied to the simulated network. The results are shown to be consistent with normal operational experience giving minimised overall costs. Based upon these results a preliminary proposal[13] is made for application of the control algorithm to actual networks by means of an on-line computer.

Chapter 8 discusses and draws conclusions on overall results achieved, shortcomings of the treatment, and requirements for additional complementary research. The conclusions are that an original method has been presented for overall system control which includes all relevant cost factors and uses all available control features. The method results in optimised control values which can be accurately implemented for valves, and any pumps with continuously variable controls, but less accurately implemented for discrete pumps. Additional research is required in this latter area and the Chapter concludes with a review of research possibilities for extensions and improvements.

1.3 Summary of Main Achievements

These contributions all represent significant and original advances over existing work and have resulted in formulation of the computer programs and publication of the referenced papers.

(i) Development of versatile pumping station model for use in optimisation and overall system operation (Chapter 3). This extends existing models to allow both costs and flows to be dependent on the pumping station variable head and an independent control parameter representing proportion of pumps in use.

(ii) Development of theory and computer program (WATDP) for optimisation of pumping costs by dynamic programming[93] (Chapter 4). This is an original application and has resulted in optimised pumping policies for discrete pumps; the method is suitable for immediate application to similar types of systems to those analysed.

(iii) Development of theory and computer program (MULTI 1) for optimisation of pumping costs by hierarchical methods[92] (Chapter 4). This extends existing work by derivation of superior performance indices, directly related to electricity charges, and includes original modifications to optimise maximum demand charges. A comparison of results,with other similar methods, shows that an improved formulation has been obtained giving desirable operational characteristics.

(iv) Modification of theory and computer program (WATSIM) to give more versatile dynamic network simulation (Chapter 5). This extends existing work resulting in a program suitable for interactive use with optimisation and modelling schemes.

(v) Development of theory and computer program (COEF) for obtaining linear dynamic network models[13,94], (Chapters 5 and 6). This is a new approach which, for the first time, gives a linear dynamic model for a wide range of multi-reservoir systems, incorporating all the features required for use in powerful optimisation techniques.

(vi) Development of theory and computer program (MULTI 2) for optimal control of overall system operation[13] (Chapter 7). This extends the previous contributions and then combines them to demonstrate, also for the first time, integrated optimisation, with head dependent pumping and valve controls, using realistic cost factors.

CHAPTER 2

DESCRIPTION OF WATER DISTRIBUTION SYSTEMS

2.1 Introduction

For a study of control techniques in water distribution systems it is necessary to have a detailed knowledge of typical systems and their operation[33]. In addition realistic operating data are required for validation of research results, use in simulation of network operation, and evaluation of cost and network models.

This chapter presents an outline of typical systems and gives specific information on the system controlled by the Doncaster and District Joint Water Board (DDJWB). The information has been obtained by discussion with operating and administrative personnel of the DDJWB and by reference to documents and drawings provided by them[21,26,27]. The values quoted below are as exact as could be determined using standard monitoring facilities. In some instances the data are incomplete by virtue of operating constraints and lack of on-line measuring equipment. The constantly changing nature of the system makes it impossible to keep up to date on a long term research project of this kind, and the system is described as it existed for 1973 with the commencement of centralised supervisory monitoring and control.

The area covered by DDJWB is approximately 300 square miles with a population of 300,000 and a consumption in the region of 60 gallons per head per day for all purposes.

Since the Imperial system of units is still very much in use throughout the water industry this system has been adopted in the thesis. However the following list of factors will suffice for any desired conversion into SI units:

Imperial	SI
1 gal	$0.0045m^3$
1 ft	0.3048m

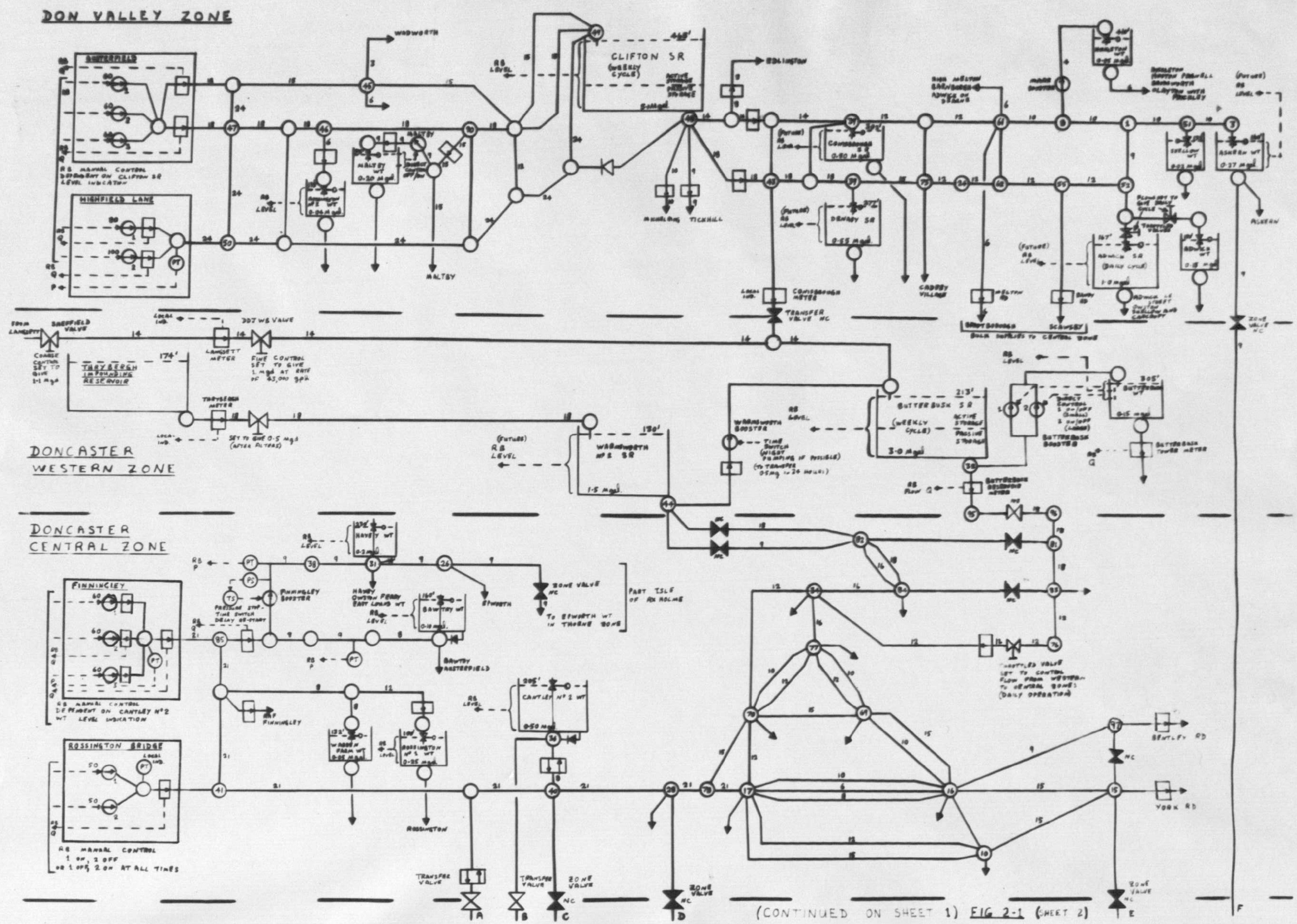
2.2 System Description

Figure 2-1 is a simplified drawing of the DDJWB distribution system showing all major features. An attempt has been made to present the information in a meaningful fashion by adopting a layout in descending order of pressure zones with source flow from left to right. Further details are given in the tables of appendix 2 and the sections below.

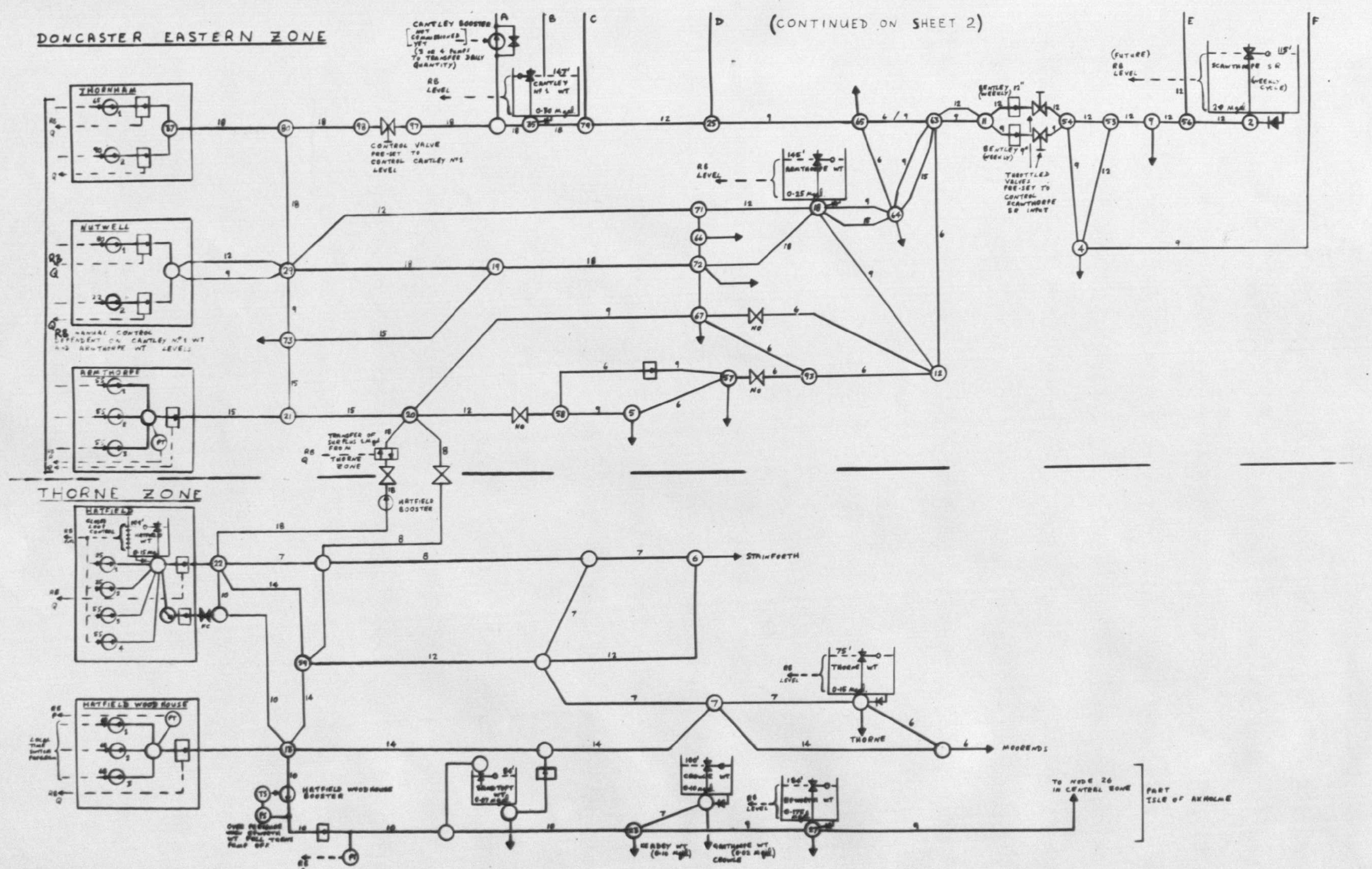
2.2.1 Pumping Stations and Boreholes

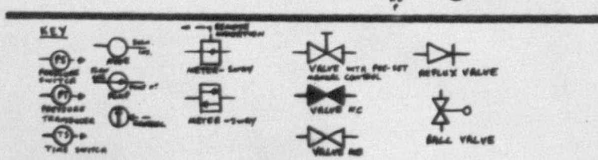
The pumping stations contain borehole pumps for delivery of water from underground boreholes into the network and/or booster pumps for increasing the head of water. Appendix 2, table A2-1, gives a breakdown of the DDJWB pumping stations, on a zone by zone basis, together with pumping parameters.

For boreholes the pumps are fixed speed, submersible or vertical spindle type, driven by electrically operated motors. In operation the pumps deliver to a common main and hence are effectively in parallel. A typical arrangement is to have three equal size pumps of which two are used for normal operation with one as standby. Alternatively two equal size pumps may be provided, with one in use and one as standby. However, this arrangement requires greater standby capacity and is generally less desirable. Pump control equipment includes sequencing for starting and stopping against shut-off control valves with facilities for local or remote activation.



(CONTINUED ON SHEET 1) FIG 2-1 (SHEET 2)





NOTES: 1. NODE NUMBERS REFER TO DDJWB DWG. D6323 SHEET 17
2. RB INDICATES ROSSINGTON BRIDGE, P - PRESSURE, Q - FLOW, MONITORING.

B. Coulbeck 8/10/73

FIG 2-1 (SHEET 1)
DONCASTER AND DISTRICT
JOINT WATER BOARD
DISTRIBUTION SYSTEM

The boreholes consist of steel lined wells, sunk into the Bunter Sandstone which lies to the East of Doncaster, and these provide over 90% of the area water requirements. Whilst the base water level remains reasonably constant the pumping level of any borehole depends heavily upon operation of its own pump and, to a lesser extent, upon operation of pumps in adjacent boreholes. Where possible the boreholes of individual pumping stations have been widely dispersed to reduce pumping interaction. Following changes of pumping combinations the levels usually stabilise within a few minutes and, for normal network simulation purposes, such transient effects can be neglected.

2.2.2 Distribution Network

The network is sub-divided into zones, which cater for each major pressure area, dictated by topography of the region. Each zone contains one or more elevated storage reservoirs to maintain required consumer pressures and there are usually facilities for two-way inter-zonal transfer, via booster pumps and valves, for use during normal operations or in emergencies.

Large sources of supply are fed in by trunk mains of up to 24 inches diameter but the distribution network pipes are usually smaller than this and gradually decrease in size, depending on flow requirements, down to 2 inches diameter at individual consumer levels. Manually operated variable closure valves are incorporated for flow restriction or to valve off areas of the network or trunk mains in the case of leaks. Pressure reducing valves are employed for small localised consumer areas requiring lower pressure levels. For simulation and analysis purposes, pipes having diameters of less than 6 inch are usually ignored and pipe flow delays are neglected.

2.2.3 Reservoir and Service Storage

Appendix 2, table A2-2, gives the locations and parameters of the impounding and service storage reservoirs in the DDJWB area. The purpose of the impounding reservoirs is to provide a source of supply whilst the other reservoirs and water tanks provide service storage capability for the network. The main functions of service storage are to cater for fluctuations in normal demand and to provide reserves of water in case of abnormal demands (e.g. fire fighting purposes) or temporary failure of water sources (i.e. breakage of mains or pumping failures).

The service storage is located adjacent to the trunk mains from the water sources and as near as possible to the point of water usage. At times of peak demand, during the day, water flows from service storage and augments the supply; at times of low demand, during the night, the service storage is replenished. If there is suitable high ground near the point of water usage service storage is provided in the form of a concrete tank at ground level and is known as a service reservoir. In flat areas service storage is built above ground in the form of water towers. It is obviously more expensive to construct storage tanks above the ground so that water towers usually have smaller capacities than service reservoirs. A large water tower may have a capacity of 0.5 million gallons, but a reservoir with a capacity of 1.0 million gallons would be considered to be small.

Most types of reservoirs are fitted with ball-valves to cut off the supply and prevent overflowing when full, also control valves may be used to restrict the flow and prevent the reservoirs emptying or filling too fast.

2.2.4 Centralised Supervisory System

An important feature of modern water distribution systems is the application of centralised instrumentation and control equipment. This can lead to greater operator efficiency and allow more accurate balancing of the network by providing measurement and control facilities for remote stations.

The DDJWB area includes a centralised supervisory system [21] which has just been commissioned. This consists of a main station with telemetry links between eleven out-stations and twenty-five field stations.

The main station, located at the Rossington Bridge borehole pumping station, includes both a master (computer controlled) and a standby (hard-wired program) data logging and control system. The out-stations are located at the other borehole pumping stations with the field stations at reservoirs, water towers, and booster pumping stations. Typical control and measurement facilities for the system are shown in appendix 2, table A2-3. Whilst this system provides an enhanced quick access measurement capability it is still not feasible to monitor individual consumptions. Zonal consumptions can, however, be easily calculated from inflows and reservoir level changes.

2.3 System Operation

Overall system operation essentially consists of management of service storage under both normal and abnormal conditions. To cater for these two differing requirements the total capacity of each reservoir is divided and classified as active storage (which is available for use during normal operation) and passive storage (which is required as standby reserve capacity for emergency purposes). The ratio of active to passive storage is dependent upon several conflicting requirements and empirically determined values can be from 1:1 to 2:1.

Efficient operation is also an important aspect which requires balancing of supplies and relevant storage against consumption under least cost conditions. Operation is simplified if each zone is controlled independently, however, this may not lead to the most efficient or effective method and a compromise is usually desirable, in which some interzonal transfer takes place.

Appendix 2, table A2-4 describes typical operations for the DDJWB system achieved by control of pumps and valves. Traditional methods of manual control are used, based largely on operator experience of consumption patterns, with use of base load pumps to maintain average reservoir levels and additional pumps to cater for peak demand periods and prevent reservoir levels falling below pre-determined values. Whenever possible additional topping up of reservoirs is scheduled to take advantage of night rebate tariffs. During all pumping operations consideration is given to preferential use of most efficient pumps and the desirability of not increasing the electricity maximum demands over those already achieved. With base load pumping, the service reservoirs must have sufficient capacity to operate within limits on a weekly cycle and pumping is scheduled to give full reservoirs each Monday morning at 0800 hours, in preparation for the typically heavy wash-day demand. The smaller capacity water towers are operated on a daily cycle starting off full every morning at 0800 hours.

2.4 Operating Costs

This section defines the controllable costs for direct operation of the DDJWB system which, in this case, reduce to the costs for operation of pumps and valves.

The major operating costs will be those of electricity charges for pumping and appendix 2, table A2-5, gives a breakdown of typical costs for the DDJWB system where the tariffs[28,104] are as follows:

S1ADHV - special tariff 1 based on the Yorkshire Electricity Board (YEB) industrial two part tariff for annual maximum demand charges using high voltage supplies. The difference between this and the standard I2ADHV tariff is that excess annual maximum demand outside peak hours and peak months is charged at a special low rate of £2.625 per kVA. Where peak hours are defined as being from 0730 hours to 1300 hours and 1600 hours to 1900 hours each week-day and peak months as November through March.

S2ADHV - special tariff 2 based on the YEB industrial two part tariff for annual maximum demand charges using high voltage supplies. The difference between this and the standard I2ADHV tariff is that excess annual maximum demand outside peak hours is charged at a special low rate of £1.25 per kVA. Where peak hours are defined as being from 0800 hours to 2000 hours each week-day.

I1QD - YEB industrial block tariff for quarterly maximum demand charges

I2ADLV - YEB industrial two part tariff for annual maximum demand charges using low voltage supplies

I2MDLV - YEB industrial two part tariff for monthly maximum demand charges using low voltage supplies

AMDHV - East Midlands Electricity Board (EMEB) industrial tariff A for monthly maximum demand charges using high voltage supplies

Analysis of these results shows that the annual cost for 1973 was of the order of £100,000 of which £20,000 was incurred as maximum demand charges and £2,500 was a rebate for use of overnight units. The demand charges in this case are not necessarily typical of other systems and would be significantly higher were it not for the special tariffs which have been negotiated. Since the stations are remotely situated the electricity supplies are metered separately for all borehole pumping stations but include any adjacent booster pumps.

An additional cost factor, which is influenced by the operating control strategies will be the maintenance costs for pumps because of wear and tear during pumping and switching.

The final factor is the cost for operating valves which must be considered in relationship to the potential pumping benefits obtained by judicious use of valve controls. The valve operating costs in this instance will be the labour costs for manual adjustment of valves situated several miles apart.

Based on the above cost factors, desirable features of operating strategies are:

(a) Control of system operation which should -

(i) reduce electricity costs by selection of cheapest tariff

(b) Control of pumps, to ensure minimal electricity charges, which should -

(i) reduce unit charges by pumping under most efficient conditions,

(ii) reduce demand charges by limitation on maximum number of pumps used,

(iii) increase night rebate by pumping overnight whenever possible

(c) Control of pumps, to ensure minimal maintenance costs, which should -

(i) reduce starting and stopping stresses by avoiding excessive pump cycling

(ii) reduce temperature stresses by use at maximum efficiency

(d) Control of valves, to ensure minimal overall costs, which should -

(i) reduce labour costs by limiting changes of valve settings

(ii) reduce pumping costs by allowing frequent changes of valve settings

2.5 Conclusions

This chapter has provided the groundwork for a study of the application of control techniques to water distribution systems with specific reference to the DDJWB requirements. In anticipation of follow up work this coverage of the DDJWB area will also form a useful starting point for other researchers. The lack of fully detailed information currently prevents any attempt to match simulation results with actual network results. For this latter purpose a comprehensive on-line pressure and flow survey would be required which could be considerably aided by use of the newly commissioned data logging system.

The main aim has been to give an appreciation of:

(i) Distribution system major components and features so that mathematical models can be developed which fit the practical characteristics. (These can then be used to form complete network models and allow accurate system analysis and simulation).

(ii) System operation, present operating methods and instrumentation capabilities so that requirements of practical automatic controls can be determined. (These will be ultimately required for application of efficient computer control methods to distribution systems)

(iii) Operating costs so that useful cost models can be evaluated which give agreement with actual costs. (These can then be used in development of optimisation methods for subsequent optimised system control)

Detailed consideration of all these aspects is covered in later chapters.

CHAPTER 3.

ANALYSIS OF PUMPS AND PUMPING COSTS

3.1 Introduction.

From an operational viewpoint the most important items on the system are the pumps which are directly controllable and have operating costs dependent on electricity charges and pump usage. For supply purposes these fall into two categories: borehole pumps, which deliver the water from underground boreholes to give a direct supply to the network, and booster pumps, which serve to deliver water from low to high pressure zones within the network. Both sets may be either fixed or variable speed and the control action takes place by means of selection of fixed speed pump combinations or speed control (as appropriate).

Use of the pumps to achieve efficient system operation is a prime consideration and this task can be considerably aided by employment of simulation and optimisation techniques. These methods rely on development of models which which can be used to predict pump flow and allow evaluation of corresponding operating costs. Pump head-flow characteristics are typically non-linear and the flow and costs are both dependent upon pump operation and instantaneous operating conditions within the network. Whilst pump characteristics have been successfully fitted by polynomial functions[7,66,70,81,87] in their present form these are not generally suitable for simulated control purposes and also no effective method has been devised for evaluating fully representative operating costs. Most previous optimised control schemes[2,34,38,44,56,92,93] have used very simple models which assume controllable inflows (independent of network heads) to represent pumping flows and have also assumed that simple direct relationships exist between flows and costs.

This Chapter reviews existing work and seeks to establish controllable head-flow models which can be used to allow a rapid evaluation of pump flow and accurate operating costs in response to dynamic control strategies.

3.2 Pumping Station Head-Flow Characteristics.

Individual pump characteristics are of three main types as shown in figure 3-1, with most centrifugal pumps having characteristics of (i) or (ii). It is assumed that all pumps are fitted with non-return valves and thus the curves only exist in the first quadrant. An actual characteristic for a single fixed speed pump is given in figure 3-2 with superimposed analytical head-flow curves corresponding to the derived models.

For practical applications, individual fixed speed pumps may be combined in parallel combinations, or pump speed may be variable, and it is necessary to consider these features which contribute to the overall pumping station characteristics. It will be noted that these all represent non-linear head-flow relationships and that actual pump flow will be determined by intersection of pump characteristics with the network head-flow response (at the point of inflow).

For mathematical modelling purposes it is necessary to evaluate suitable analytical expressions relating heads and flows for specified operating conditions. A range of models which satisfy these requirements is given below:

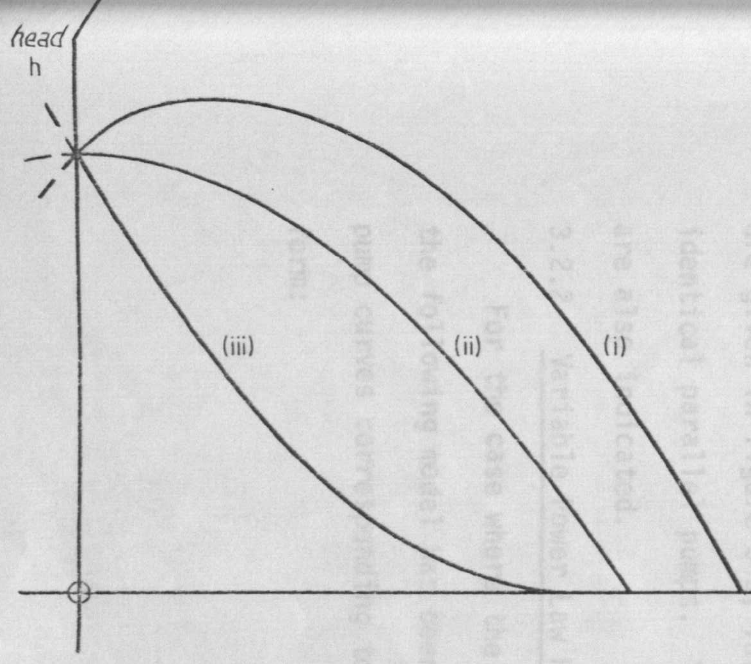


Fig. 3-1 Typyical pump head-flow characteristics

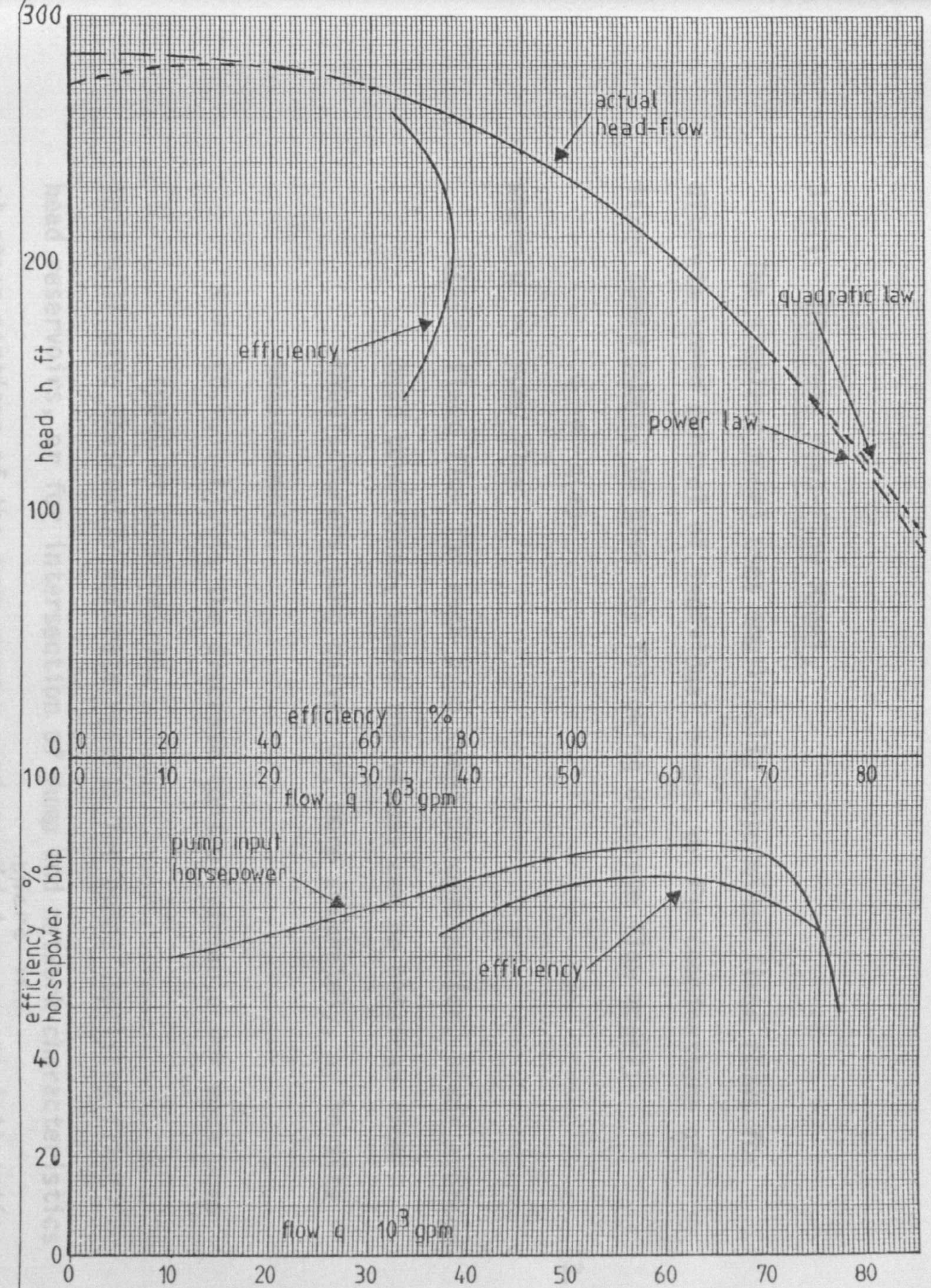


Fig. 3-2 Fixed speed pump characteristics (Hatfield Woodhouse borehole)

3.2.1 Discrete Head-flow Model.

The simplest model uses empirically determined fixed flows for discrete speed control of individual pumps, or for combinations of fixed speed pumps, to give the following discrete flow model:

$$q = q(\ell) \tag{3.1}$$

where q = pump flow

$q(\ell)$ = fixed flow for each pump speed or combination, under average head conditions, where the values can be obtained from manufacturers pump curves or, more accurately, by on-site measurements,

ℓ = (0,1,...L), is the discrete control variable for each pump speed or combination.

This will give reasonably accurate results for pumps supplying fixed head reservoirs, or for intersection of pump and network characteristics at steep portions of the pump curves, and provided there is no severe interaction from adjacent pumping stations. This model can thus cater for pumps with non-linear speed effects or for combinations of pumps with completely different characteristics but only under average external conditions. Typical curves, showing combined pump and network interaction, are given in figure 3-3, for speed variation, and in figure 3-4, for identical parallel pumps. The effects of changes in network operation are also indicated.

3.2.2 Variable Power Law Head-flow Model.

For the case where the dependence of flow upon head is important the following model has been proposed[66,87] which caters for symmetrical pump curves corresponding to type (ii) of figure 3-1. This is of the form:

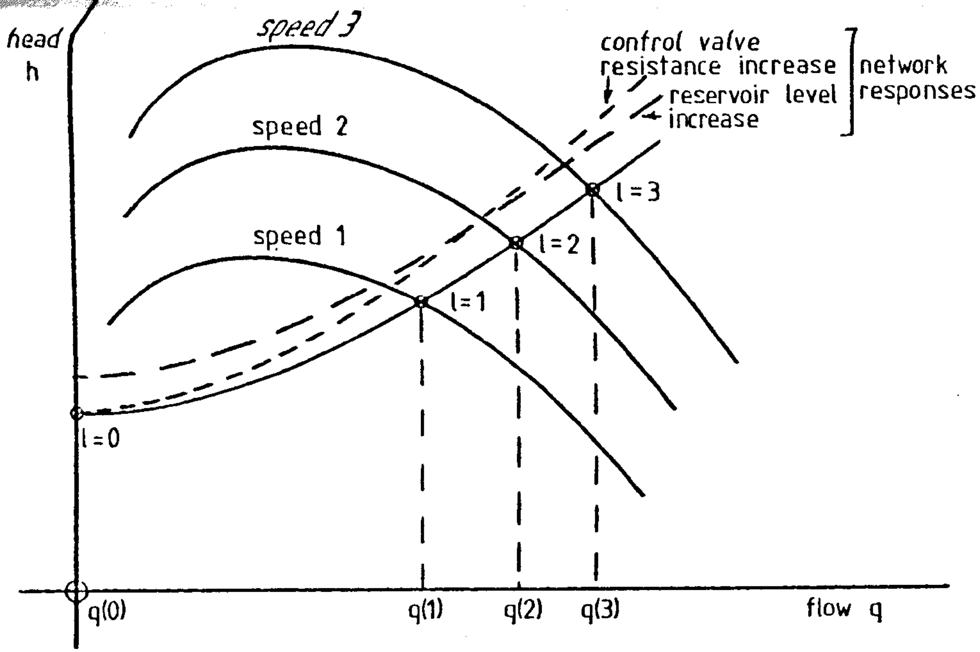


Fig. 3-3 Response curves for discrete speed variation

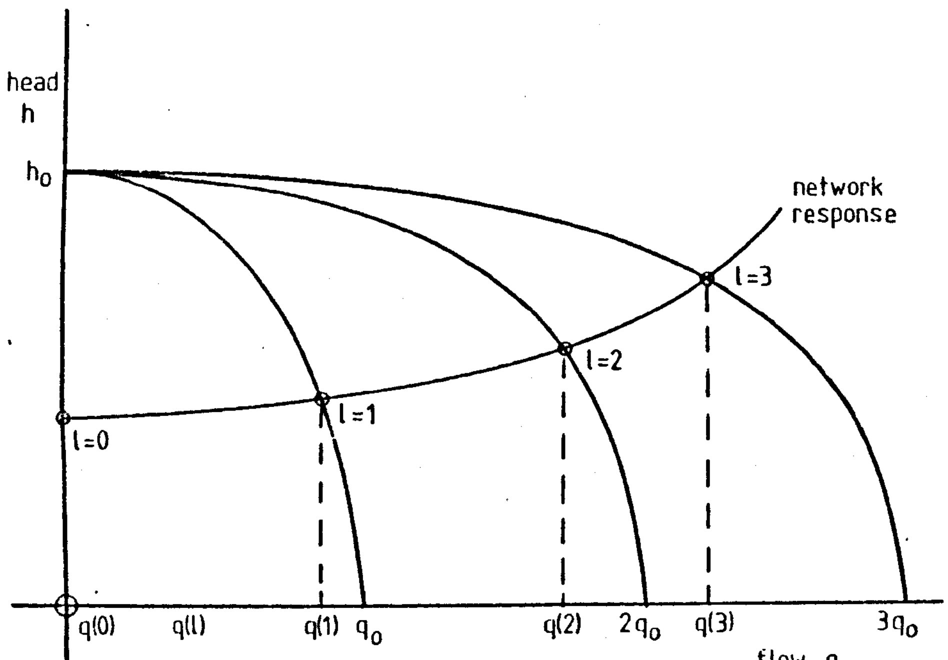


Fig. 3-4 Response curves for discrete parallel pump combinations

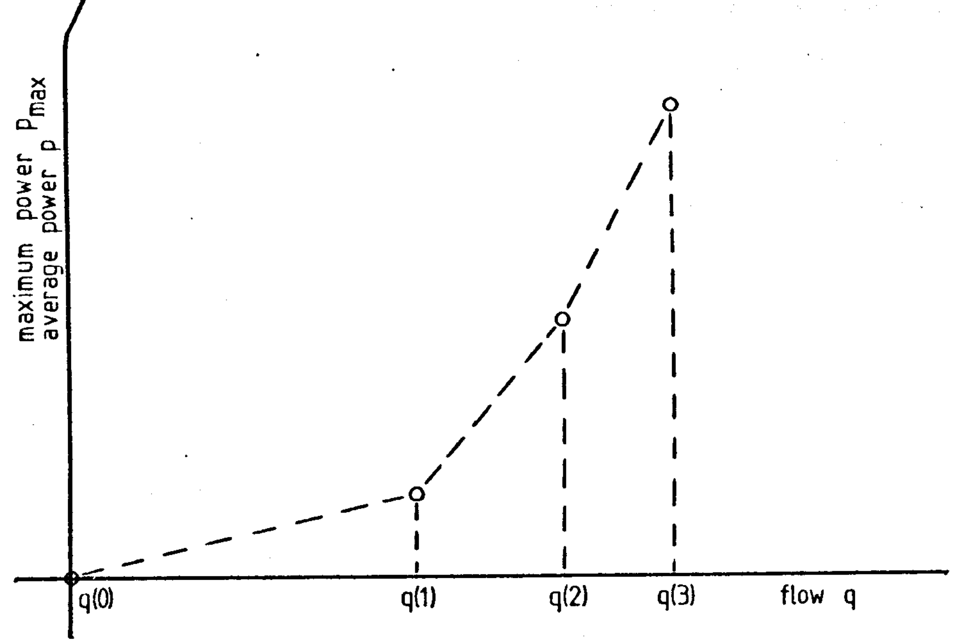


Fig. 3-5 Discrete power function

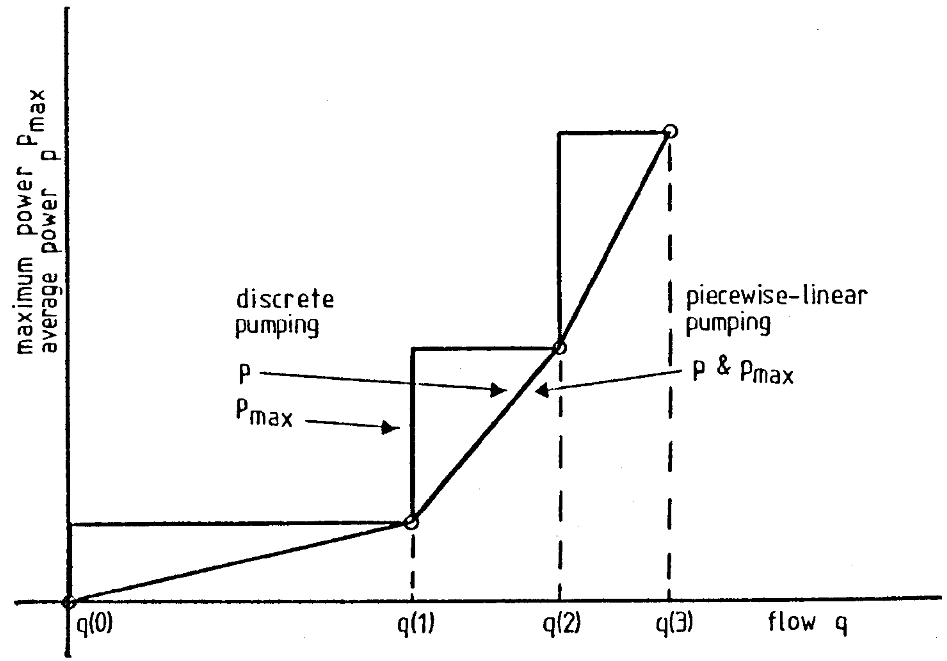


Fig. 3-6 Piecewise-linear power function

$$h = a - b\,q^{c} \quad (3.2)$$

where h = pump head increase

q = pump flow

a,b,c = coefficients derived from the manufacturers supplied characteristics or from on-site measurements for highest accuracy.

Flow at any head ($0 \leq h \leq a$) will then be given by:

$$q = \left(\frac{a-h}{b}\right)^{\frac{1}{c}} \quad (3.3)$$

For the present study a more meaningful relationship can be obtained by rationalising the variables to give:

$$\frac{h}{h_o} = 1 - \left(\frac{q}{q_o}\right)^{c} \quad (3.4)$$

where h_o = head increase for zero flow (cut-off head)

q_o = flow for zero head (cut-off flow)

and the flow for ($0 \leq h \leq h_o$) can be expressed as:

$$\frac{q}{q_o} = \left(1 - \frac{h}{h_o}\right)^{\frac{1}{c}} \quad (3.5)$$

The coefficients of equations (3.4) and (3.5) have been evaluated from the head-flow characteristics of figure 3-2 using the least squares (E04FAF[76]) computer sub-routine. This gave the values h_o = 284.33 ft, q_o = 97.112 x 10^3 gal/h and c = 2.558 and the resulting function is shown superimposed on the actual characteristics.

This latter model can be easily extended to cover combinations of identical pumps by defining a parameter r, as the number of pumps in use.

For series pump combinations, where q_o remains constant but the combined cut-off head varies proportional to number of pumps, this will give:

$$h = r.h_o \left\{ 1 - \left(\frac{q}{q_o}\right)^c \right\} \tag{3.6}$$

and for parallel pump combinations, where h_o remains constant but the combined cut-off flow varies proportional to number of pumps, will give:

$$q = r \; q_o \left(1 - \frac{h}{h_o}\right)^{\frac{1}{c}} \tag{3.7}$$

Typical curves for parallel pump combinations will be as shown in figure 3-4 with the general control parameter, r, replacing the discrete parameter, ℓ.

3.2.3 Variable Quadratic Law Head-flow Model.

For more general types of pump curves (i), (ii) or (iii) of figure 3-1, the pump characteristics can usually be approximated by quadratic expressions[7,70,81] relating head increase to flow as:

$$h = a.q^2 + b.q + c \tag{3.8}$$

where a,b,c = empirically determined coefficients with c equal to cut-off head.

Solving for q for $0 \leq h \leq c$ gives:

$$q = \frac{-b \pm \sqrt{b^2 - 4a(c-h)}}{2a} \tag{3.9}$$

The coefficients of equations (3.8) and (3.9) have been evaluated for the head-flow characteristics of figure 3-2 using the least squares (ICL F4CFΘRPL) computer sub-routine. This gave the values $a = -38.315 \times 10^{-9}$ ft/(gal/h)^2, $b = 1.1131 \times 10^{-3}$ ft/(gal/h) and c = 271.97 ft and the resulting function is shown superimposed on the actual characteristics.

Extending equation (3.9) for the case of r identical pumps in parallel will now result in:

$$q = r\left\{\frac{-b \pm \sqrt{b^2 - 4a(c-h)}}{2a}\right\} \qquad (3.10)$$

while r has been defined as the number of pumps in parallel it does not have to be restricted to integer values but can take on fractional values, to cater for parallel combinations of large and small pumps (of similar characteristics), and can also take on real values to represent continuously variable pumping capability. It will also be noted that the flow is linear in r and that r is independent of head and flow values. Hence r is an independent control parameter which will allow evaluation of head dependent flow, using equation (3.10), for either:

(i) discrete parallel pumping,

(ii) continuously variable parallel pumping.

Similar expressions could be derived for series pumping capability by comparison with equation (3.6), and also for variable speed pumps where the independent control parameter, r, would now represent speed. The present derivation is based on a quadratic representation, which is usually sufficiently accurate, but the same principle applies to higher order expressions and can thus be used for any type of pump curve.
In later use equation (3.10) is inserted directly in the full network computer simulation program (see Chapter 5). The corresponding *linear* network model pump coefficients then automatically reflect the current value of pump head to generate correct head dependent flows (see Chapter 6).

3.3 Pumping Station Cost Characteristics.

Typical cost factor characteristics, as supplied by the manufacturer, are shown on figure 3-2 to which has been added the curve for efficiency versus head which is required to support the following work.

(a) Direct Costs

These correspond to the total electricity charges for energy consumed and the maximum electrical power demanded over the tariff period. The instantaneous electrical power demanded will be:

$$\frac{de}{dt} = \frac{Ch.q}{\eta(h)} \tag{3.11}$$

where $\frac{de}{dt}$ = instantaneous power corresponding to energy rate (eg.kW)

C = conversion constant (eg. 3.777×10^{-6} kW/(gal/h)/ft).

q = pump flow as a function of head and pump control parameter and, from equations (3.7) or (3.10) can be expressed as $k_q(h).r$ (eg.(gal/h)).

h = pump head increase (eg. ft).

$\eta(h)$ = efficiency of pump as a function of head. Since the curve is asymmetrical and $\eta = 0$ for $h = 0$ and $h = h_0$ an appropriate expression will be of the form $h(ah^2 + b\,h + c)$, where a, b and c are empirically determined constants.

For tariff purposes the unit rate is the average value of electrical power over each time interval, Δt, and the maximum electrical demand is defined as the maximum value of electrical power for any half hour average over the complete tariff period (assuming unity power factor electrical equipment).

For average heads and flows over any time interval, Δt, use of equation (3.11) will give these values as:

$$p = k_p(h) . q \qquad (3.12)$$

and

$$p_{max} = \max\left\{k_p(h).q\right\} \qquad (3.13)$$

where p = average power demanded over interval, Δt, corresponding to average energy rate, $\frac{\Delta e}{\Delta t}$ (eg. kW).

p_{max} = maximum power demanded over tariff period (eg. kW)

$k_p(h)$ = power conversion constant which can be obtained from manufacturers pump curves or, more accurately, from on-site measurements and corresponds to $\frac{Ch}{\eta(h)}$ (eg. kW/(gal/h)).

The actual direct costs can then be calculated by application of the appropriate electrical unit and demand tariffs,[28,104] and this aspect is discussed fully in section 4.2.12.

For optimisation with variable heads and flows, where the emphasis is on minimisation of direct pumping costs, a desirable cost function should allow for operation at minimum power input regardless of efficiency. Also, for high accuracy, it is necessary to incorporate direct power input values which then have to be related to the output variables by use of the efficiency curve. From the manufacturers pump characteristics (eg. figure 3-2), it will be noted that minimisation of power input does not necessarily coincide with maximum efficiency and that there is no very convenient correlation between the two curves. However in the next section it is shown that operation at maximum efficiency is important and should lead to an overall optimal solution taking into account additional cost factors.

(b) Indirect Costs.

Whilst the major costs are those of electricity charges,other contributing cost factors are those for pump maintenance because of wear and tear during pumping and switching. These can be minimised by reduction of:

(i) starting and stopping stresses,

and (ii) temperature stresses.

The former requirement implies avoidance of excessive pump cycling which is particularly relevant for on-off control of fixed speed pumps.

The latter requirement can be catered for by operating pumps under maximum efficiency conditions since there will then be the smallest energy loss within the pump which results in heating of water and pump (with possible degradation in performance and increased maintenance costs). This requirement also agrees with efficient system operation since it implies that, for a given quantity of electrical energy, most energy will be imparted to the water; hence the maximum quantity of water will be raised to the highest level within the system under the most favourable conditions. For parallel pump operation it will be shown that the maximum efficiency is a constant and occurs at a constant optimum pump head increase.

3.3.1 Discrete Cost Model.

The simplest model will cater for the direct cost factors only and will use empirically determined fixed heads and flows, corresponding to those of the discrete head-flow model of section 3.2.1, over any time interval, Δt, for the purpose of calculating electricity costs. Under these conditions equations (3.12) and (3.13) will take on the following discrete values and have the form shown in figure 3-5.

$$p = k_p(\ell) . q(\ell) \tag{3.14}$$

$$p_{max} = \max\left\{k_p(\ell) . q(\ell)\right\} \tag{3.15}$$

where $k_p(\ell)$ = conversion constant for each pump speed or combination, under average head conditions, where the values can be obtained from manufacturers pump curves or, more accurately, by on-site measurements.

An example of a portion of a network meeting these requirements is the Don Valley zone of DDJWB and the above model has been used to give optimised pumping strategies using the dynamic programming techniques of section 4.2.

3.3.2 Piecewise-linear Cost Model.

This model is basically the same as the discrete cost model but now the flow is assumed to be linearly variable between each discrete value. The exact form of the piecewise-linear function, shown in figure 3-6, is dependent on whether the pump flow is from continuously variable speed pumps or from discrete parallel pumps. Continuous flow variation from discrete pumps can be justified by assuming that pumping takes place at the next highest feasible pump combination for a proportion of the time interval, Δt; energy costs are then accurately represented for proportional pumping over reduced time intervals. The maximum demands, being discrete, will take on the next highest value corresponding to the maximum flow used over the complete time period. The model is used in the linear and integer programming optimisation formulation of section 4.4.

3.3.3 Linear-quadratic Cost Model.

This is an extension of the discrete and piecewise-linear models which now assumes that the flow is continuously variable and passes through all the discrete values. Experimental measurements show that the resultant curve is of a linear-quadratic form, as shown in figure 3-7, which gives the following relationships for continuous controls:

$$p = aq^2 + bq \tag{3.16}$$

$$p_{max} = aq^2_{max} + bq_{max} \tag{3.17}$$

where a,b = quadratic and linear coefficients evaluated from manufacturers pump curves or on-site measurements for average head conditions.

q_{max} = maximum pump flow achieved over complete time period.

Continuous flow variation from discrete pumps can be justified as for the piecewise-linear model, and a stepped demand function included in figure 3-7. The model is used in the decentralised hierarchical optimisation application of section 4.3.

3.3.4 Linear Plus Quadratic Cost Model.

The previous models have treated the costs as direct functions of the pump flows, which are assumed to be independently controllable, instead of allowing for head dependent functions of variable network operation and independent pump controls. In addition the models have only included direct cost factors and are unable to cope with pump operation under maximum efficiency conditions as required for indirect costs.

A versatile and accurate model, meeting the above requirements, can be developed if the average pump head can always be maintained at an optimal design value corresponding to maximum efficiency for any specified pumping

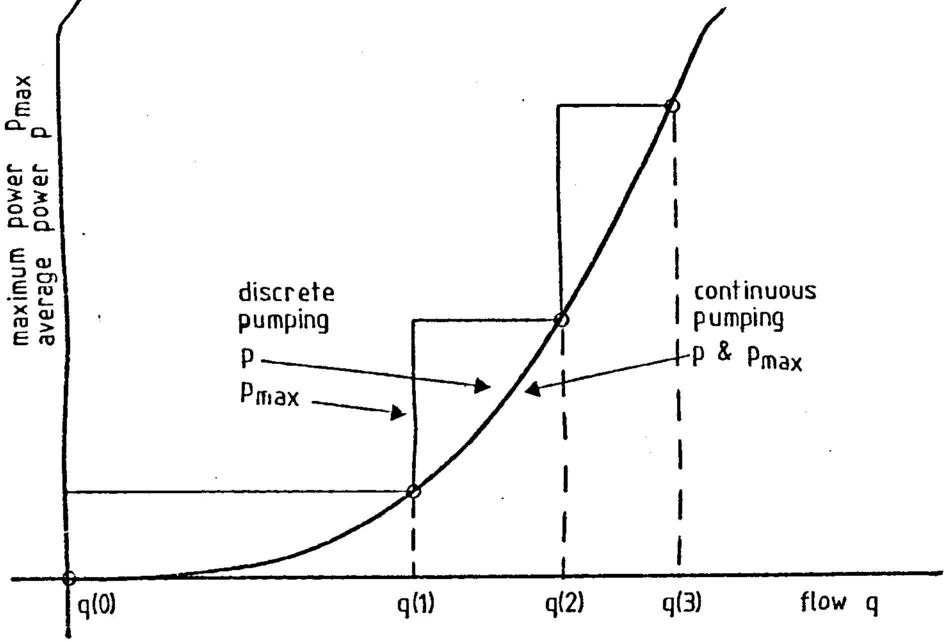


Fig. 3-7 Linear-quadratic power function

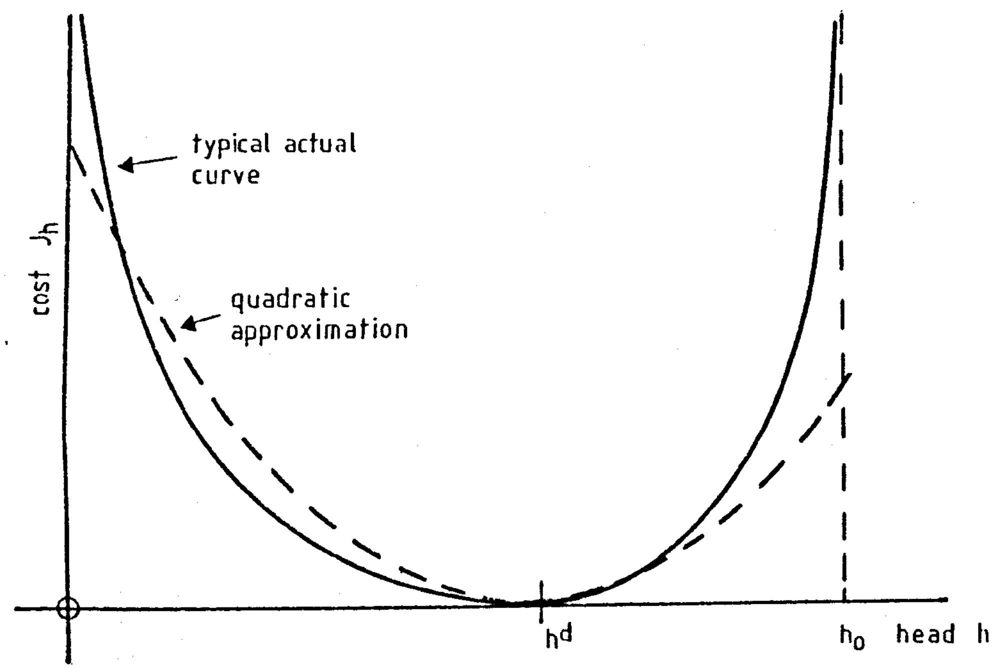


Fig. 3-8 Quadratic pump efficiency cost function

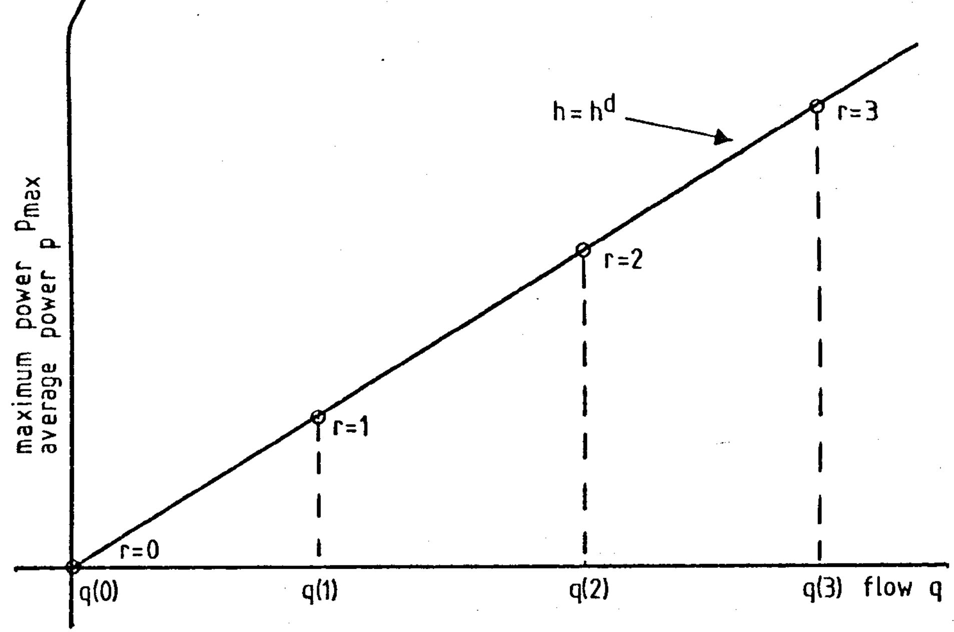


Fig. 3-9 Linear power function

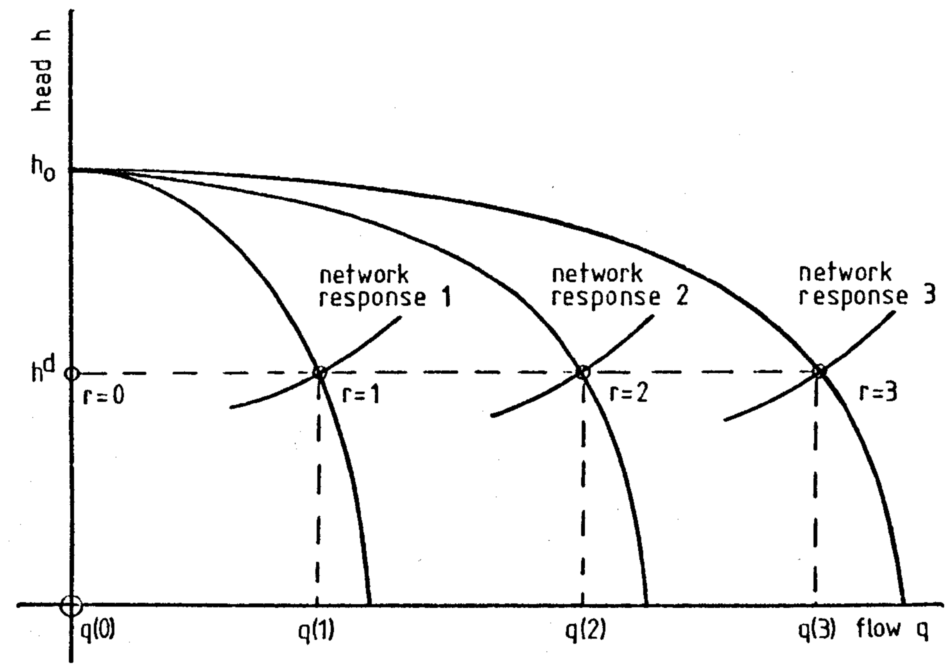


Fig. 3-10 Optimised pump and network response curves

operation. For the case of parallel pumps with similar characteristics the values of both the optimal head and the pumping efficiency, will be constants for all pump combinations. This occurs because operation of parallel pumps at any fixed head will ensure that each pump operates independently as far as electricity input and water output energies are concerned, resulting in a common efficiency versus head curve for all pump combinations. For convenience the effective reciprocal of this type of curve is shown in figure 3-8 to indicate the relative cost of departing from the maximum efficiency condition. Figure 3-8 also shows an approximating cost function of the form:

$$J_h = Q_h(h - h^d)^2 \tag{3.18}$$

where J_h = penalty cost for departing from maximum efficiency condition.

Q_h = empirical cost coefficient determined by comparison with efficiency variation costs or upon desirability of maintaining h closely equal to h^d.

h = pump head increase.

h^d = design pump head increase for maximum efficiency (η_{max}) evaluated from manufacturers pump curves or on-site measurements.

For the condition of average pump head equal to h^d, equations (3.7) or (3.10) can be written as:

$$q = k_q(h^d) . r \tag{3.19}$$

where $k_q(h^d)$ = flow per unit pump at optimal head value and, using equations (3.12) and (3.13), the direct cost factors can then be expressed as:

$$p = k_p' . r \tag{3.20}$$

$$p_{max} = k_p' . r_{max} \tag{3.21}$$

where k_p' = linear power coefficient given by $k_p(h).k_q(h)$ evaluated at $h = h^d$ and $r = 1$ to give $\frac{C\,h^d.\,k_q(h^d)}{\eta_{max}}$.

This corresponds to power per unit pump at maximum efficiency and can be evaluated from manufacturers pump curves or on-site measurements for highest accuracy.

The corresponding function will now be linear in r and q and is shown in figure 3-9.

This model provides an approximate theoretical justification for the previous empirical linear-quadratic cost relationship (of section 3.3.3) by resolving the effects into direct costs, linearly dependent upon flow control, and indirect costs, quadratically dependent upon pumping efficiency. The direct costs are expressed directly and simply in terms of the independent pump control parameter (which can take on continuous or discrete values dependent on pumps and optimisation requirements) and the indirect costs in terms of the pump head. The requirements for operating the pumps at optimal head values can be effected by incorporating equation (3.18) as part of the overall performance index in suitable optimisation techniques.

The application of this model only requires values for power per unit pump and pump head, at maximum efficiency, together with an empirical head deviation penalty weighting factor. The cost model, in conjunction with the variable head-flow model of equation (3.10), is used to allow optimisation of overall system operation in Chapter 7, where the indirect cost factor modifies the system response to maintain average pump heads at their optimal values. The combined pump and network response curves should be as shown in figure 3-10 where it is assumed that the network response can be controlled by means of reservoir levels, control valves and adjacent pumps.

3.4 Conclusions.

This chapter has covered the detailed analysis of pump operation to cater for the theoretical requirements of the simulation, modelling and optimisation methods which follow. The options of combinations of fixed speed pumps or continuous flow variation (by means of speed control, etc.) have been covered by employment of discrete or continuous control parameters and a formulation has been achieved which effectively embeds both types in the same model. Several model versions have been derived which employ these parameters to give controllable head dependent pump flows and operating costs directly related to electricity unit and maximum demand charges.

The head-flow characteristics are suitable for representation of most general types of pumps and are compatible with both full and reduced network models. The pump and network models are combined in Chapters 5 and 6 to give correct inflows for all network operating conditions.

Each of the models has been developed with a particular optimisation technique in mind and the intermediate models are suitable for use with the variety of methods for optimisation of pumping costs in Chapter 4. For the final, most sophisticated version, it has been shown that if the pump head can be controlled at an optimal design value this will result in the most efficient pump operation; under these conditions the performance index then reduces to a particularly simple form which can take into account not only electricity charges but also the effects of varying pumping efficiency. This model is used in Chapter 7, to permit the development of an optimised control algorithm for overall system operation.

The importance of this latter model can be attributed to the fact that:

(i) the direct cost coefficients can be determined accurately from on-site measurements,

(ii) the model includes a factor, suited to efficient optimisation schemes, to enable maximum efficiency operation,

(iii) accurate electricity costs are represented under maximum efficiency conditions.

CHAPTER 4.

OPTIMISATION OF PUMPING COSTS.

4.1 INTRODUCTION.

Optimisation of water distribution systems presents a very complex problem, when all operating factors have to be taken into account, and no entirely satisfactory solution methods are currently available. In order to determine possible solution techniques which will cater for some of the requirements it is necessary to simplify the problem by adopting a compromise between accuracy and feasibility.

Major costs of operation are due to electricity charges for pumping and, as a first step, this Chapter considers optimisation based on these costs only. This provides a feasible method for the further assumptions of directly controllable pump flows which are independent of other network variables. Whilst catering for multiple pumping stations the treatment is limited to systems having independently controllable reservoirs. This latter case is not too restrictive since many networks can be treated as consisting of interconnected pressure zones (each having one reservoir) where the interzonal transfer is known or controllable.

Additional considerations relate to the length of the optimisation period and division into time increments. Ideally these would be infinity and zero respectively but for computational reasons the optimisation period should be as short as possible and the time increment as long as possible; this is particularly relevant when faced with the problem of prediction of fluctuating demands.

Based on operating constraints the minimum optimisation period is determined by future requirements for an optimal quantity of stored water and the maximum time increment is limited by short-term level fluctuations of small reservoirs.

Cost factors also influence these decisions since electricity maximum demand charges are levied at monthly or yearly intervals and night rebates are given for specified overnight units. To satisfy the above conflicting requirements further strategies must be employed, to allow feasible near-optimal cost solutions, which can include: use of non-optimal existing system maximum electricity demands, infrequent long-term and frequent short-term solutions, and short-term solutions under worst-case and normal conditions.

The optimisation problem now reduces to both continuous and discrete control of pumps so as to minimise pumping costs whilst providing predicted demands and operating within system constraints. Classical optimisation techniques[67,100,105] involve definition of system equations, performance indices, constraints, and initial and final states, with a solution yielding an optimal control sequence. The foregoing restrictions permit a simplified formulation of the problem as follows:-

(i) Systems equations.

The slowly varying nature of reservoir levels in relation to operating times allow the system to be described by discrete time dynamic equations. These can be written in terms of storage quantity and pump and demand flows only and will be linear and independent for each zone by virtue of mass balance for single reservoir zones.

(ii) Performance Index.

The costs now reduce to electricity unit and maximum demand charges with night rebates. The assumption of controllable pump flows allows these costs to be formulated and evaluated directly as incremental values, directly related to pump flows, for each time interval over the optimisation period.

(iii) Constraints.

These reduce to preset restrictions on reservoir quantities and pumping flows.

(iv) Initial and final states.

These are the initial and final reservoir quantities. Allowing the terminal state to be free would cause the reservoir to be emptied at the end of the optimisation period, any previously stored water would already have been paid for and the optimisation policy would dictate use of this water rather than incur additional pumping costs. Because of the necessarily limited optimisation period it is essential to provide a sufficient reserve of stored water in order to meet future anticipated demands without incurring heavy electricity maximum demand charges. For short-term optimisation periods a realisable, but non-optimal, solution is to specify a desirable terminal quantity and impose a cost penalty for any deviation from this value.

The control sequence will now be a pump flow profile which should intuitively meet certain requirements for demonstration of an optimal solution. In essence these are: use of most efficient pumping stations to reduce unit charges, maximum pumping levels to be maintained as low as possible to reduce maximum demand charges (allowing reservoirs to take up short-term water demand fluctuations), and topping up of reservoir to take advantage of night rebates.

Various techniques for providing solutions to the optimisation problem have been investigated by the author[92,93] and are detailed below. Wherever possible these have been applied to realistic networks in order to assess their effectiveness and possibilities of extension to more complex systems.

4.2 DYNAMIC PROGRAMMING TECHNIQUES.

The pumping costs optimisation problem has been shown to be of a discrete-time multivariable dynamic type with a non-linear performance index and constraints on states and controls, where the controls can also take discrete values. This type of problem is ideally suited for solution by dynamic programming which, in principle, can handle all of the above requirements and obtain a global optimal solution by evaluation and comparison of the cost of all feasible controls. In practice the conventional dynamic programming procedure is only suitable for low dimensional problems and modifications to the basic procedure must be sought to allow solution of more complex systems.

There are two basic types of dynamic programming, backward, in which optimal trajectories are calculated from all initial and intermediate states leading to a single terminal state, and forward, in which the trajectories are calculated from a single initial state leading to all intermediate and final states. For a practical application the optimisation method must cater for disturbance effects which cause the system to deviate from its optimal trajectory. The use of backward dynamic programming is most suitable in this application but forward dynamic programming can still be applied by re-calculation using the disturbed state as a new initial value.

Whilst there are several examples of the use of dynamic programming in water systems[2,19,20,38,44,56,57], as far as is known there are none which deal with optimisation of water distribution systems including all relevant pumping costs.

4.2.1 Forward Dynamic Programming Solution.

The objective of this section is to show how the conventional forward dynamic programming method can be applied to a simple water supply system to give operational policies which will supply varying water demands at minimum electricity pumping costs.

The Don Valley zone of the Doncaster and District Joint Water Board (DDJWB) was chosen as a suitable water system for analysis. It consists of two independent borehole pumping stations feeding a single service reservoir. The reservoir is operated on a weekly cycle, an attempt being made to start off each week with a full reservoir. Each of the pumping stations use different combinations of electrically powered constant speed pumps. The supply authority is the Yorkshire Electricity Board and the charge tariffs are based on the industrial two part tariff.

A mathematical model of the system has been developed in order to specify the system operation and determine the actual incremental pumping costs in the correct formulation for a dynamic programming solution.

The extensive calculations required for the dynamic programming solution have been implemented on an ICL 1907 digital computer for off-line solution. The results produced indicate the optimum pumping policy for a given terminal level in the reservoir and demand profile. Consequently the technique relies on prediction of the consumer demand over the period of optimisation or the recomputation of the pumping policy following departure of the consumption from the expected pattern.

4.2.1.1 The Dynamic Programming Method.

Several different dynamic programming techniques have been developed[54,56,64], the most appropriate for a particular application being dependent on many factors including the number of state variables and initial and final state constraints and cost functions. A detailed development of the theory of dynamic programming can be found elsewhere.[8,9,12,74]

In particular, forward dynamic programming is most suitable for problems in which all the initial states are known and the majority of final states are unknown. This section considers the suitability of forward dynamic programming for the optimisation of pumping in a water network.

The technique can be formulated as follows:

Given:

(i) System difference equation

$$\underline{X}(k) = \underline{\Phi}[\underline{X}(k-1), \underline{U}(k-1)] \tag{4.1}$$

with $\underline{X}$ - state vector

$\underline{U}$ - control vector

k - index for stage variable having values of k=0,1,....K.

$\underline{\Phi}$ - vector functional

The state of the system at stage k is thus a function of the previous state and the previous control that was applied.

(ii) Performance criterion

$$J = \sum_{k=0}^{K-1} H[\underline{X}(k), \underline{U}(k)] \tag{4.2}$$

with J - total cost as a result of applying a series of controls over all values of k.

H - cost for a single stage.

(iii) Constraints

$$\underline{X} \in \underline{X}^{'}(k) \tag{4.3}$$

$$\underline{U} \in \underline{U}^{\ell}(k) \tag{4.4}$$

with $\underline{X}'(k)$ - set of admissible states at stage k

$\underline{U}^{\ell}(k)$ - set of admissible controls at stage k where ℓ is control variable with $\ell = 0,1,\ldots,L$.

(iv) Initial state

$$\underline{X}(0) = \underline{C}_i \qquad (4.5)$$

with $\underline{C}_i$ - set of initial constants

(v) Final state

May be free or defined by

$$\underline{X}(K) = \underline{C}_f \qquad (4.6)$$

with $\underline{C}_f$ - set of final constants

Find:

The control sequence $\underline{U}(o),\ldots\underline{U}(K-1)$ such that J in equation (4.2) is minimised subject to the system equations (4.1), the constraint equations (4.3),(4.4), and the initial and final state equations (4.5),(4.6).

Solution:

(i) Define $I[\underline{X}(k),k] = \min_{\underline{U}(o),\ldots\underline{U}(k-1)} \{ \sum_{j=o}^{k-1} H[\underline{X}(j),\underline{U}(j)]\} \ldots. \qquad (4.7)$

where $I[\underline{X}(k),k]$ is the minimum cost to reach state $\underline{X}(k)$ at stage k by selection of an optimal control sequence.

(ii) Use principle of optimality[8] to derive an iterative relationship for $I[\underline{X}(k),k]$ as:

$$I[\underline{X}(k),k] = \min_{\underline{U}(k-1)} \{H[\underline{X}(k-1),\underline{U}(k-1)]+I[\underline{X}(k-1),k-1]\} \ldots. \qquad (4.8)$$

The minimum cost in arriving at the present state can consequently be obtained by selection of the previous control so as to minimise the sum of the minimum cost in arriving at the previous state plus the cost of moving from the previous state to the present state.

The cost before any control action is applied is zero which will give the boundary condition:

$$I[\underline{X}(o), o] = 0 \tag{4.9}$$

(iii) Compute the set of optimal controls $\hat{\underline{U}}(k-1)$ for all $\underline{X}$ and all k by iterative solution of equation (4.8) using equation (4.9) as the boundary condition.

(iv) Derive an iterative relationship for the optimal state $\hat{\underline{X}}(k-1)$ by inversion of equation (4.1) to give:

$$\hat{\underline{X}}(k-1) = \underline{\theta}[\hat{\underline{X}}(k), \hat{\underline{U}}(k-1)] \tag{4.10}$$

with $\underline{\theta}$ - vector functional.

(v) Compute the optimal control sequence $\hat{\underline{U}}(K-1), \ldots\ldots, \hat{\underline{U}}(o)$ by iterative solution of equation (4.10) using equation (4.6) as the boundary condition.

Computation Procedure:

If $\underline{X}$ is a continuous variable it must be quantised into discrete states of $\underline{X}^j(k)$ for j = 0, 1...J. At each previous quantised state $\underline{X}^j(k-1)$ where $I[\underline{X}^j(k-1), k-1]$ has just been computed, each admissible control $\underline{U}^{\ell}(k-1)$ is applied; for each corresponding present state of $\underline{X}(k)$ (from equation (4.1)) a check is made to see if it has been the present state for any control applied at previous values of $\underline{X}(k-1)$. If it has not previously been a present state then the value of $I[\underline{X}(k), k]$ (from equation (4.8)) and control $\underline{U}(k-1)$ are stored as tentative minimum cost and optimal control at that point. If it has, then the new value of $I[\underline{X}(k), k]$ is compared with the tentative minimum cost already stored at that point, and, if it is less, the new values of minimum cost and optimal control replace the values stored there. The above procedure is repeated until all the controls have been applied at every quantised state of $\underline{X}(k-1)$. The tentative minimum costs $I[\underline{X}(k),k]$ and optimal controls $\underline{U}(k-1)$ at each $\underline{X}^j(k)$ are then the true minimum costs and optimal controls stored at these points as $I[\underline{X}^j(k),k]$ and $\hat{\underline{U}}[\underline{X}^j(k),k]$.

Repeated application for all values of k will give the set of optimal controls for all optimal state trajectories. The particular optimal control sequence can be obtained by repeated application of equation (4.10) using the boundary condition $\hat{\underline{X}}(K)$ (from equation (4.6)) and associated optimal control $\hat{\underline{U}}[\hat{\underline{X}}(K),K]$ to yield the optimal control sequence $\hat{\underline{U}}(K-1),\ldots\hat{\underline{U}}(0)$. Figure 4.1 shows the application of the above procedures to a simple case.

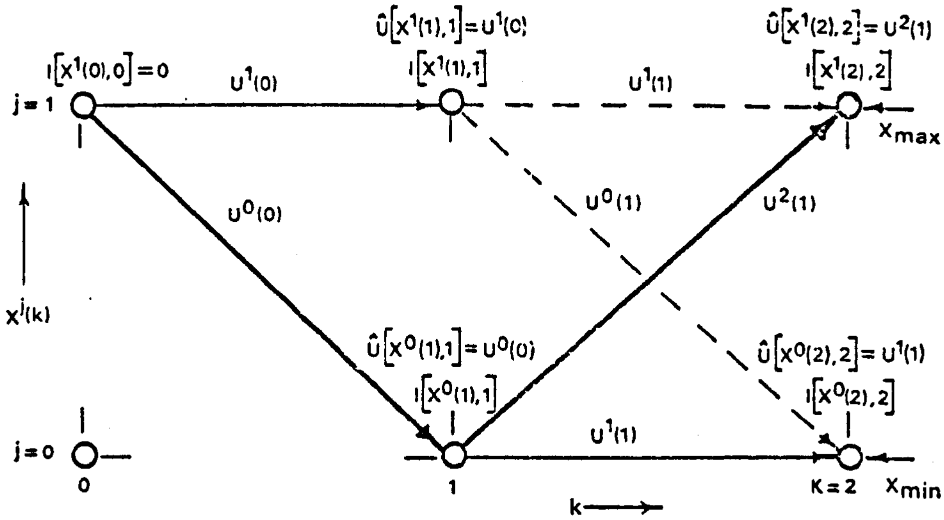
4.2.1.2 Representation of a Pumping Station.

A fundamental pre-requisite to optimisation is the development of a cost model for a pumping station. Figure 4-2 shows a cost model of a pumping station with the parameters to be used in the analysis defined on the diagram. The following simplifying assumptions will be made:

(i) Water flow q is constant, for any particular combination of pumps, over a period of time corresponding to each increment of k.

(ii) Electrical power factor is unity for all pump combinations.

(iii) Electrical power p is proportional to water flow for any pump combination, where k_p is the constant of proportionality.



Key: - - - Possible state trajectories.

—— All optimal state trajectories for $x(0)=x_{max}$

—— Optimal state trajectories for $x(K)=x_{max}$

FIGURE 4-1 FORWARD DYNAMIC PROGRAMMING COMPUTATIONAL PROCEDURE FOR A ONE DIMENSIONAL CASE.

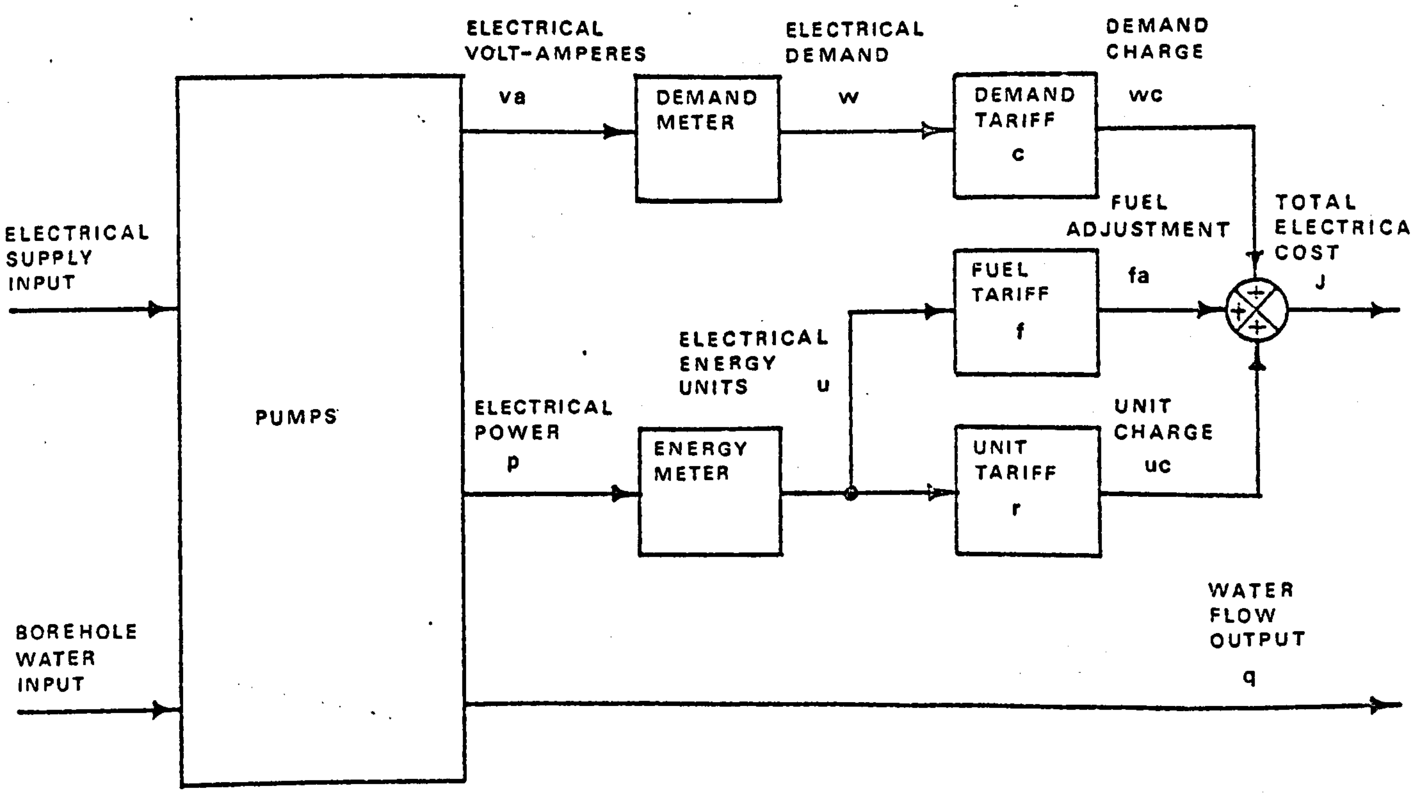


FIGURE 4-2 COST MODEL OF A PUMPING STATION.

Based on these assumptions we can replace time t by the stage variable k (which corresponds to a time increment of Δt) and replace integration with respect to t by summation with respect to k. Using the pump models of sections 3.2.1 and 3.3.1 will give the following expressions where it is assumed that consistent units are used:

Electrical power:

$$p(k) = k_p.q\,(k-1) \tag{4.11}$$

Electrical units rate:

$$u(k) = \sum_{k=o}^{k} p(k) = \sum_{k=o}^{k} k_p.q(k-1) \tag{4.12}$$

Electrical demand:

$$w(k)=\max\{va(k)\} =\max\{p(k)\} =\max\{ k_p\ .q(k-1)\} \tag{4.13}$$

The electricity charges for pumping can now be derived. These have been based on the Yorkshire Electricity Board Industrial two part tariff.[104]

(a) Demand Charge

This is a charge based upon a demand tariff c and the maximum demand over the tariff period (see figure 4-3).

Total demand charge:

$$wc(K) = \int_{0}^{w(K)} c(w)dw \tag{4.14}$$

However for use in dynamic programming this must be written in a form suitable for evaluation over any increment of k. By reference to figs.4-3 and 4-4 a suitable form, which allows for increases in w, is as follows:

Demand charge:

$$wc(k) = \sum_{k=0}^{k} \frac{1}{K} \int_{0}^{w(k-1)} c(w)dw + \sum_{k=0}^{k} \frac{k}{K} \int_{w(k-1)}^{w(k)} c(w)dw \tag{4.15}$$

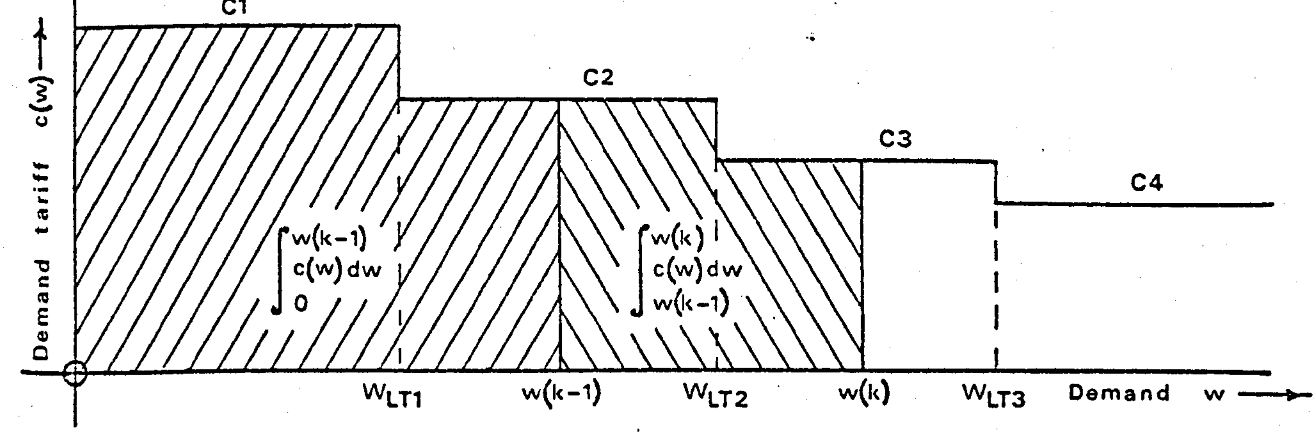


FIGURE 4-3 DEMAND TARIFF.

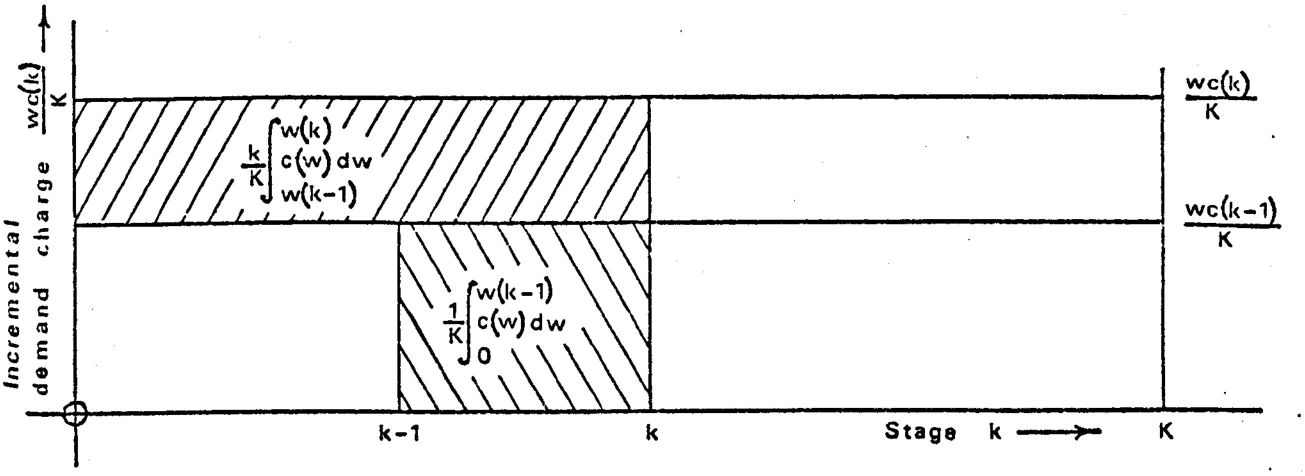


FIGURE 4-4 INCREMENTAL DEMAND CHARGE.

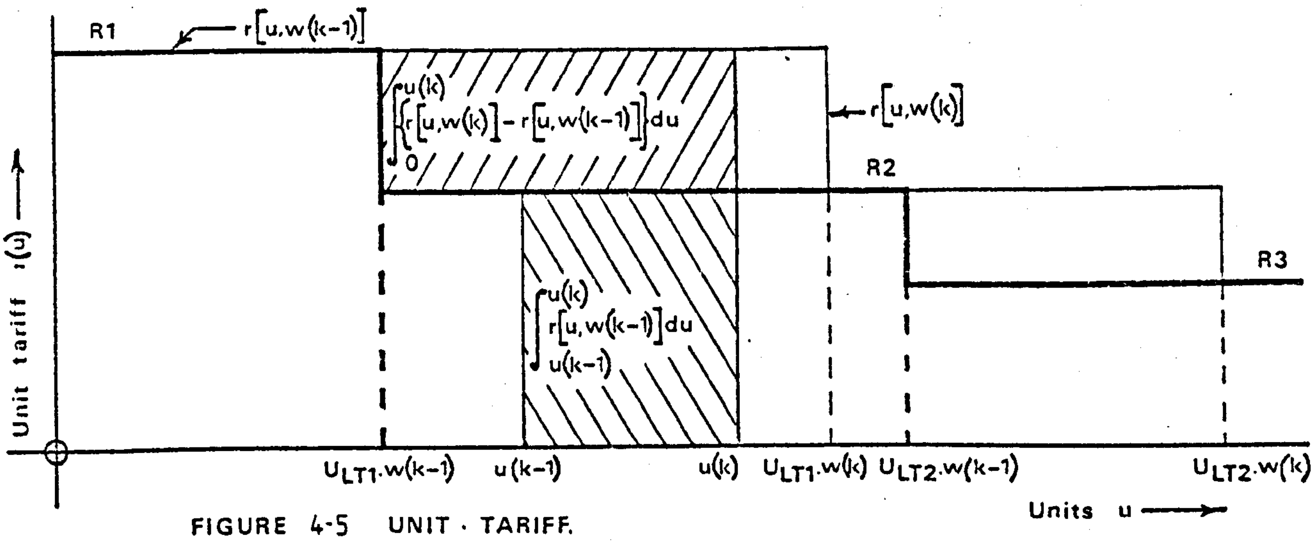


FIGURE 4-5 UNIT · TARIFF.

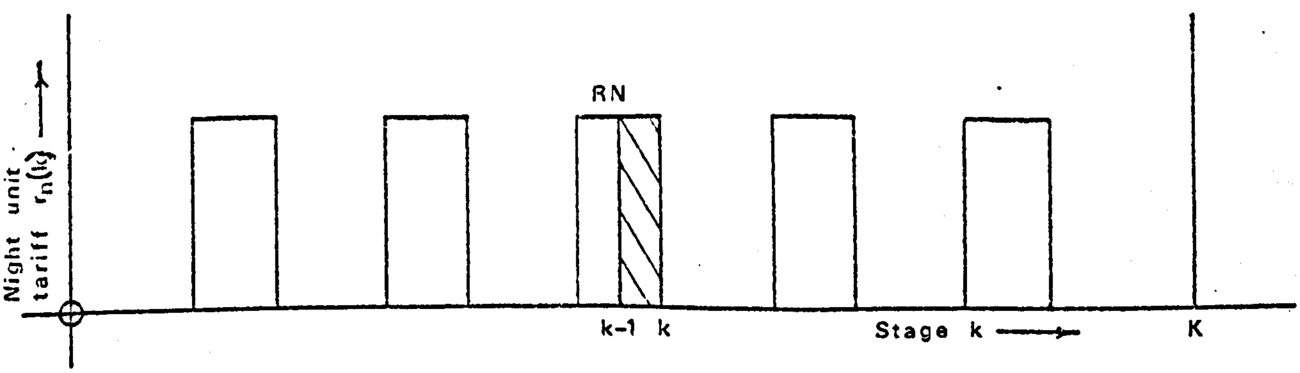


FIGURE 4-6 NIGHT UNIT TARIFF.

Where the first term can be seen to be a standing charge for each increment of k and the second term a penalty charge for any increase in w. Since the demand tariff consists of steps of constant levels the integrals can be evaluated for any particular value of w as follows:-

$$\int_0^{w(k-1)} c(w)dw = C1.w(k-1) \qquad \text{for } 0 < w(k-1) \leq W_{LT1}$$

$$= C1.W_{LT1} + C2\{w(k-1) - W_{LT1}\} \qquad \text{for } W_{LT1} < w(k-1) \leq W_{LT2}$$

$$= C1.W_{LT1} + C2\,(W_{LT2} - W_{LT1}) + C3\{w(k-1) - W_{LT2}\} \qquad \text{for } W_{LT2} < w(k-1) \leq W_{LT3}$$

$$= C1.W_{LT1} + C2(W_{LT2} - W_{LT1}) + C3(W_{LT3} - W_{LT2}) + C4\{w(k-1) - W_{LT3}\} \qquad \text{for } W_{LT3} < w(k-1) \leq \infty$$

(b) <u>Unit Charge</u>

This is a charge based upon a unit tariff r, which is also a function of the maximum demand, and the units used over the tariff period (see figure 4-5).

In addition units used over specified night hours are allowed a rebate in accordance with a night unit tariff r_n (see figure 4-6).

Total unit charge:

$$uc(K) = \int_0^{u(K)} r[u,w(K)]du - \sum_{k=0}^{k} r_n(k).k_p.q(k-1) \qquad (4.16)$$

Again this must be written in a form suitable for evaluation over any increment of k. By reference to figures 4-5 and 4-6 a suitable form, which allows for increases in u and w, is as follows:

Unit charge:

$$uc(k) = \sum_{k=0}^{k} \int_{u(k-1)}^{u(k)} r[u,w(k-1)]du - \sum_{k=0}^{k} r_n(k).k_p.q(k-1)$$

$$+ \sum_{k=0}^{k} \int_{0}^{u(k)} \{ r[u,w(k)] - r[u,w(k-1)]\}du \qquad (4.17)$$

Where the first term is the charge for each increase in u, the second term allows for the night unit rebate and the third term is a penalty charge for any increase in w.

Again the integrals can be easily evaluated by consideration of the unit tariff.

(c) Fuel Adjustment.

This is a cost adjustment based upon the fuel cost to the electricity authorities. It can be allowed for by a fuel tariff f, which is constant for the tariff period, applied to each unit consumed.

Fuel adjustment:

$$fa(k) = \sum_{k=0}^{k} f.k_p.q(k-1) \qquad (4.18)$$

4.2.1.3 Representation of a Water Supply System.

Figure 4-7 shows a schematic diagram of the section of the DDJWB to be analysed.

The following simplifying assumptions will be made:

(i) Water demand d is independent of reservoir quantity g.

(ii) Water demand is constant for each increment of k and can be

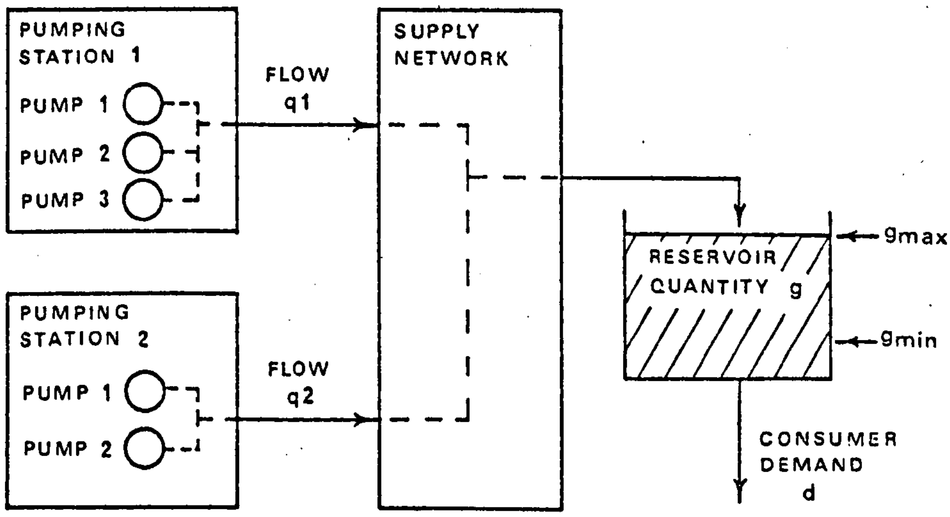


FIGURE 4-7 SCHEMATIC DIAGRAM OF SUPPLY SYSTEM.

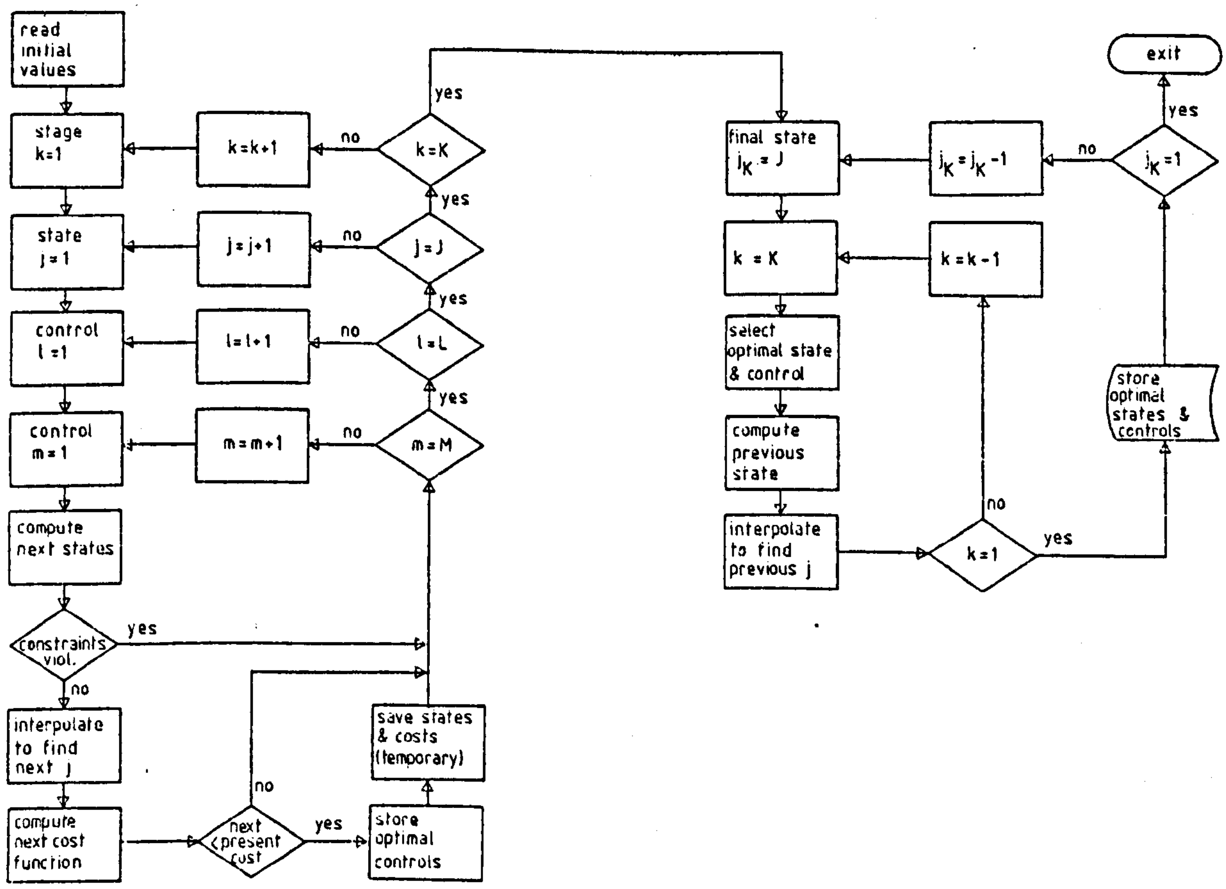


Fig. 4-8 Flow chart for conventional forward dynamic programming solution

predicted for k = 0,1,2,...K-1.

(iii) Water flows q_1, q_2 are constant for each increment of k and there is no interaction between q_1 and q_2.

This leads to the following relationship:

Reservoir quantity:

$$g(k) = \sum_{k=0}^{k} \{q_1(k-1)+q_2(k-1)+d(k-1)\} \cdot \Delta t \qquad (4.19)$$

4.2.1.4 Problem Formulation.

The equations developed in the previous sections can be incorporated into a forward dynamic programming formulation as follows:

(i) System difference equations

These are derived from equations (4.19), (4.12) and (4.13) respectively where the subscripts 1, 2 refer to pumping stations 1 and 2.

$$g(k) = g(k-1)+\{q_1(k-1) + q_2(k-1) - d(k-1)\} \Delta t. \qquad (4.20)$$

$$u_1(k) = u_1(k-1)+k_{p1} \cdot q_1(k-1) \qquad (4.21)$$

$$u_2(k) = u_2(k-1)+k_{p2} \cdot q_2(k-1) \qquad (4.22)$$

$$\left.\begin{aligned} w_1(k) &= w_1(k-1) \text{ for } k_{p1} \cdot q_1(k-1) \leq w_1(k-1) \\ &= k_{p1} \cdot q_1(k-1) \text{ for } k_{p1} \cdot q_1(k-1) > w_1(k-1) \end{aligned}\right\} \qquad (4.23)$$

$$\left.\begin{aligned} w_2(k) &= w_2(k-1) \text{ for } k_{p2} \cdot q_2(k-1) \leq w_2(k-1) \\ &= k_{p2} \cdot q_2(k-1) \text{ for } k_{p2} \cdot q_2(k-1) > w_2(k-1) \end{aligned}\right\} \qquad (4.24)$$

(ii) Performance criterion

This is the sum of the single stage values from equations (4.15), (4.17) and (4.18) respectively.

$$H = wc_1+wc_2+uc_1+uc_2+fa_1+fa_2 \qquad (4.25)$$

where, with appropriate subscripts :

$$wc = \frac{1}{K} \int_{0}^{w(k-1)} c(w)dw + \frac{k}{K} \int_{w(k-1)}^{w(k)} c(w)dw \tag{4.26}$$

$$uc = \int_{u(k-1)}^{u(k)} r[u,w(k-1)]du - r_n(k).k_p.q(k-1)$$

$$+ \int_{0}^{u(k)} \{r[u,w(k)]-r[u,w(k-1)]\} du \tag{4.27}$$

$$fa = f.k_p.q(k-1) \tag{4.28}$$

(iii) Constraints

$$g_{min} < g(k) \leq g_{max}. \tag{4.29}$$

$$0 \leq u_1(k), u_2(k) \tag{4.30}$$

$$q_1(k) = Q_1(\ell) \tag{4.31}$$

$$q_2(k) = Q_2(m) \tag{4.32}$$

$$w_1(k) = K_{p1}(\ell).Q_1(\ell) \tag{4.33}$$

$$w_2(k) = K_{p2}(m).Q_2(m) \tag{4.34}$$

Where $\ell = 0,1,\ldots\ldots.L$ and $m = 0,1,\ldots\ldots.M$ are the control variables which define the constant flows Q_1,Q_2 and conversion constants K_{p1},K_{p2} for each pump combination.

(iv) Initial states

$$g(0) = g_i \tag{4.35}$$

$$u_1(o) = 0 \tag{4.36}$$

$$u_2(0) = 0 \tag{4.37}$$

$$w_1(0) = 0 \tag{4.38}$$

$$w_2(0) = 0 \tag{4.39}$$

(v) Final states

$$g(K) = g_f \tag{4.40}$$

$$u_1(K),\ u_2(K),\ w_1(K), w_2(K) = \text{free} \tag{4.41}$$

4.2.1.5 Computer Solution.

The program requires input data as follows:

Initial, maximum and minimum values of reservoir quantity, g_i, g_{max}, g_{min}.

Final values of stage, state and control variables, K,J,L,M.

Pumping station flows and conversion constants, $Q_1(\ell)$, $Q_2(m)$, $K_{p1}(\ell)$, $K_{p2}(m)$

Water demands for each stage interval, D(k).

Initial values of electrical unit rates and demands, u_1, u_2, w_1, w_2.

Tariff values for unit, demand and fuel charges, r,c,f.

The forward dynamic programming method will give the optimal trajectories leading to all possible end states and a useful feature of this particular method is that terminal cost functions can be easily added to give an optimal end state.

The optimal solutions cover weekly blocks of predicted water demands which can be repeated over several weeks to cover the monthly tariff periods.

A program (WATDP) to implement the above techniques has been developed in Fortran IV (see flow diagram of figure 4-8) and run on an ICL 1907 computer. A processing time of several minutes together with about 20k words of core storage are required to obtain a solution for one week operation. These requirements are not compatible with on-line implementation in a small process computer especially when more complex networks are considered.

4.2.1.6 Analysis of Results.

Operational test data for the DDJWB system shown in figure 4-7 has been analysed in order to define the various system parameters and to give a typical water demand profile for a period of one week. The actual pumping costs and pumping profile for supplying this demand have been used for comparison purposes with the various dynamic programming predictions of costs and pumping profiles.

Pumping station 1 consists of three identical pumps each with a flow of 62,000 gallon/hour. The parameters for each pump combination can be summarised as shown below:

Pump combination	No pumps on	Any one pump on	Any two pumps on	All three pumps on
Control variable, ℓ	0	1	2	3
Flow, $Q_1(\ell)$,gallon/hour	0	62,000	124,000	186,000
Conversion constant, $K_{p1}(\ell)$,kW/(gallon/hour)	0	0.00391	0.00391	0.00391
Demand, w_1, kVA	0	242.42	484.84	727.26

The tariff for this station is a standard industrial two part tariff, as previously described where the unit charge is partially based on the monthly maximum demand and the demand charge is based on the annual maximum demand. The night rebate hours are taken to be 0000 hours to 0800 hours.

Pumping station 2 consists of one small pump with a flow of 75,000 gallon/hour and one large pump with a flow of 93,000 gallon/hour. Practical problems limit the combined output to 133,000 gallon/hour. The parameters for each pump combination can be summarised as follows:

Pump combination	No pumps on	Small pump on	Large pump on	Both pumps on
Control variable, m.	0	1	2	3
Flow, $Q_2(m)$, gallon/hour	0	75,000	93,000	133,000
Conversion constant, $K_{p2}(m)$, kW/(gallon/hour)	0	0.00384	0.00387	0.00421
Demand, w_2, kVA	0	288.00	359.91	559.93

The tariff for this station is the same as station 1 during peak hours, where peak hours are defined as being from 0800 hours to 2000 hours each weekday. The unit charge being partly based on the monthly maximum demand in peak hours and the normal demand charge based on the annual maximum demand in peak hours, however, excess annual maximum demand outside peak hours is charged at a special low rate.

The reservoir can hold a maximum of 5 million gallons and the quantity is not normally allowed to fall below 2.5 million gallons. Current operating policy consists of attempting to start on Monday, 0800 hours of each week with a full reservoir. The allowable reservoir operating quantity is quantised into 100 levels corresponding to increments of 25,000 gallons. Thus $j = 0$ corresponds to 2.5 millions gallons and $j = 100$ corresponds to 5 million gallons. For comparison purposes the initial and final reservoir quantities used are those of the test data at 4.649 million gallons.

Each increment of the stage variable is set to be 4 hours and thus values of k from 0 to 42 correspond to a period of one week starting from Monday, 0800 hours. Night rebate hours correspond to $k = 4,5$; 10, 11; etc. and peak hours correspond to $k = 0,1,2$; 6,7,8; etc to 24,25,26. Since these special tariff hours require 4 hourly increments the water demand must also be specified for every 4 hour period.

Figure 4-9 shows the logged network data and the actual pumping profile. The annual maximum demands which had previously been set up are 484.84kVA for station 1 and 359.91kVA (peak hours), 559.93kVA (outside peak hours) for station 2. The total electricity cost was £1144 for the first week and £4000 for four weeks of the monthly tariff period.

The DDJWB operating policy for the system can be seen to be constant use of two pumps at station 1, with constant use of the large pump at station 2 supplemented by use of the additional small pump outside peak hours, preference being given to night rebate hours. Extensive operating experience has shown that this gives the most economical results. It will also be seen that if the average water demand increases by approximately 1% the maximum output from the present policy will be reached and a new optimal operational policy will need to be determined.

Figure 4-10 shows the dynamic programming solution for the same conditions as for figure 4-9. This indicates an identical pumping policy with a slightly modified pumping profile and the cost for the first week was again £1144. The solution was then run for four weeks with identical results at a cost of £4000.

This result shows that, under the test conditions, representative cost functions have been derived giving correct overall costs and that the dynamic programming method is capable of predicting optimal pumping policies taking into account all the relevant factors.

Figure 4-11 shows the dynamic programming solution assuming that no previous annual demands had been set up. The modified pumping policy now consists of use of all three pumps at station 1 together

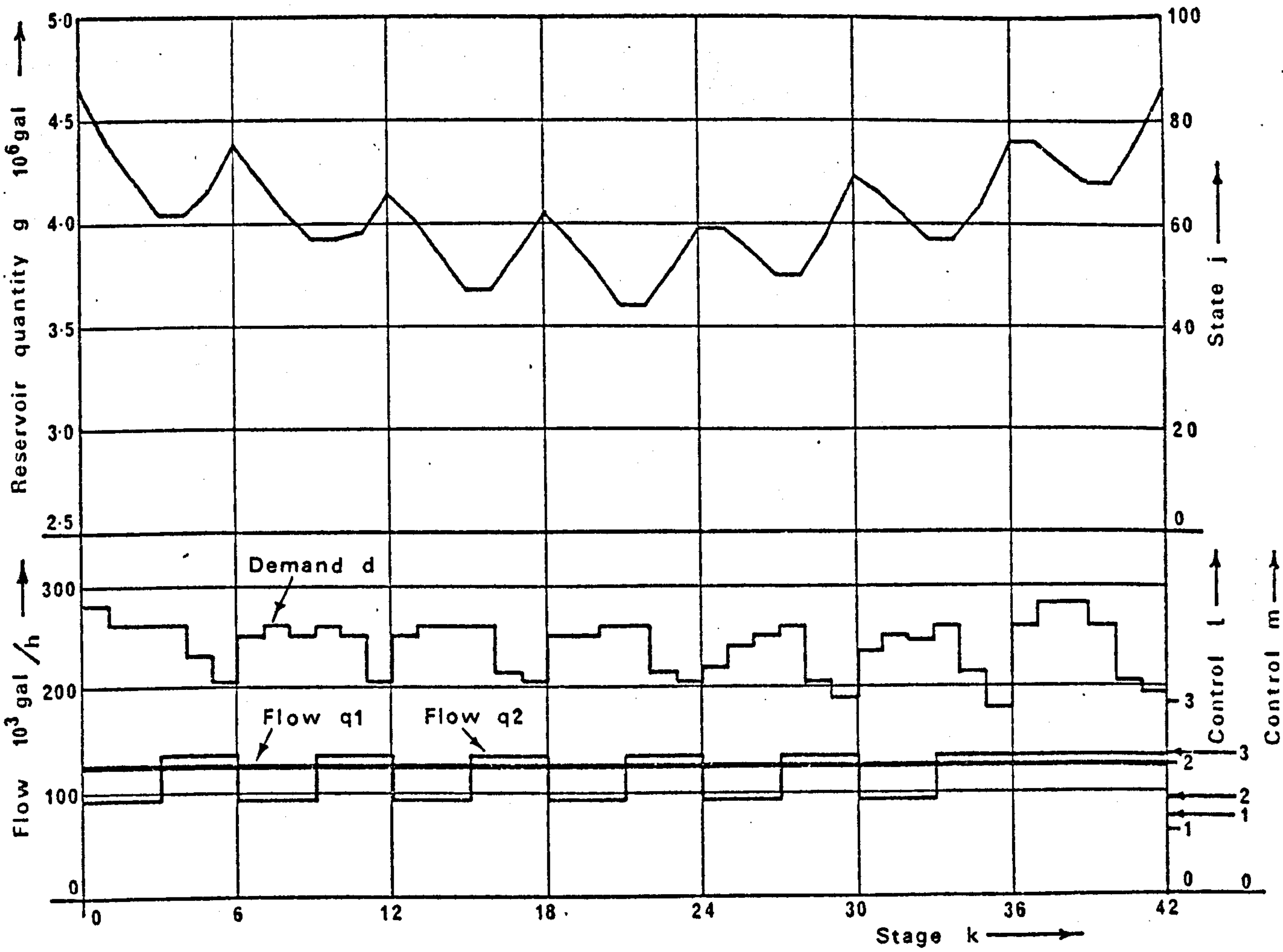


FIGURE 4-9 ACTUAL PUMPING PROFILE.

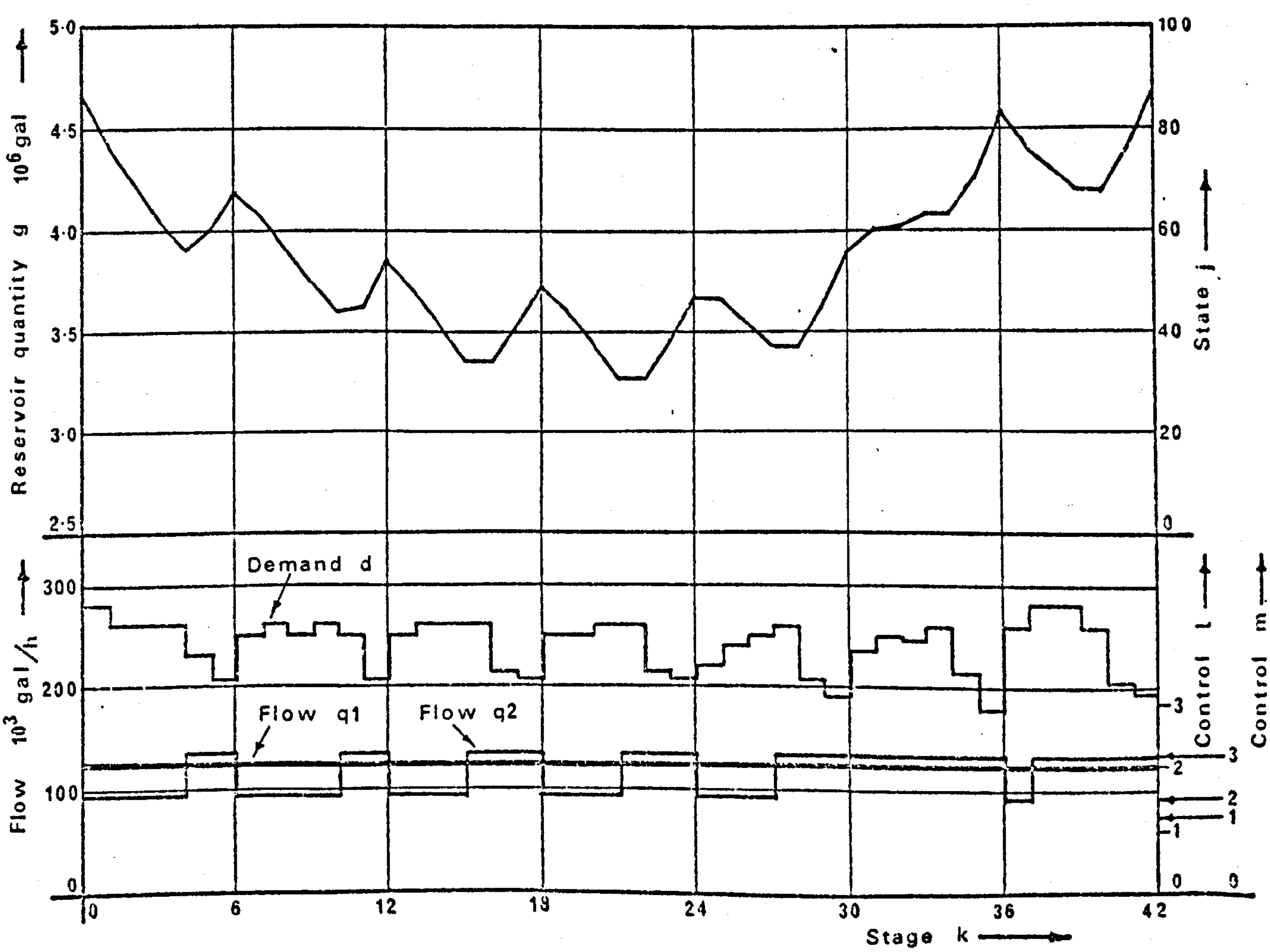


FIGURE 4-10 FIRST PREDICTED PUMPING PROFILE.

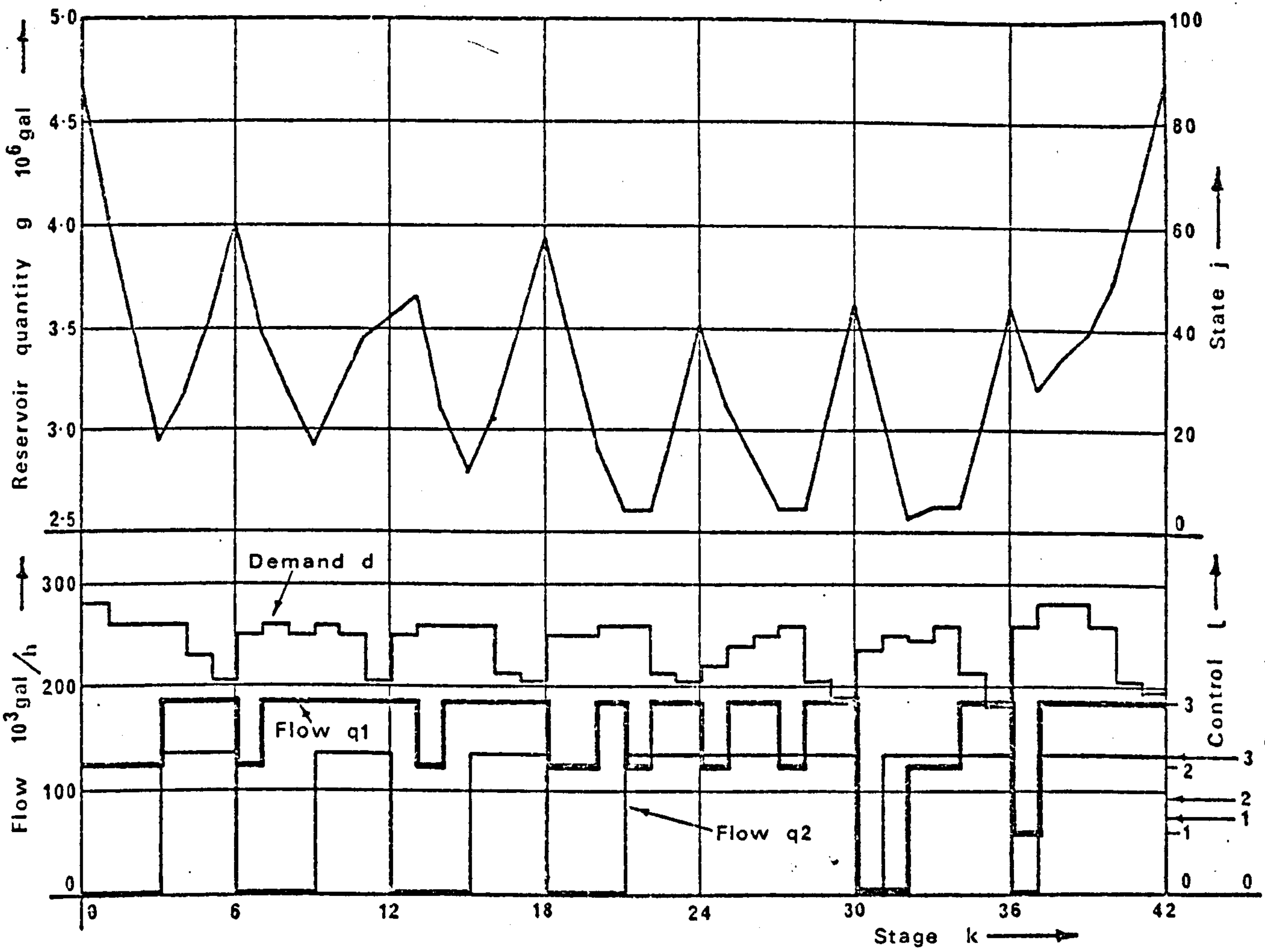


FIGURE 4·11 SECOND PREDICTED PUMPING PROFILE.

with use of both pumps at station 2 outside peak hours and no pumps during peak hours. The cost for the first week was then £1034 and for four weeks £3868.

This result shows that significant savings could be achieved under ideal conditions. However the solution makes no allowance for any lengthy pump maintenance and, in addition, a re-negotiation of the excess demand tariff would probably be required for operation with zero peak demand.

4.2.2 Dynamic Programming Extensions.

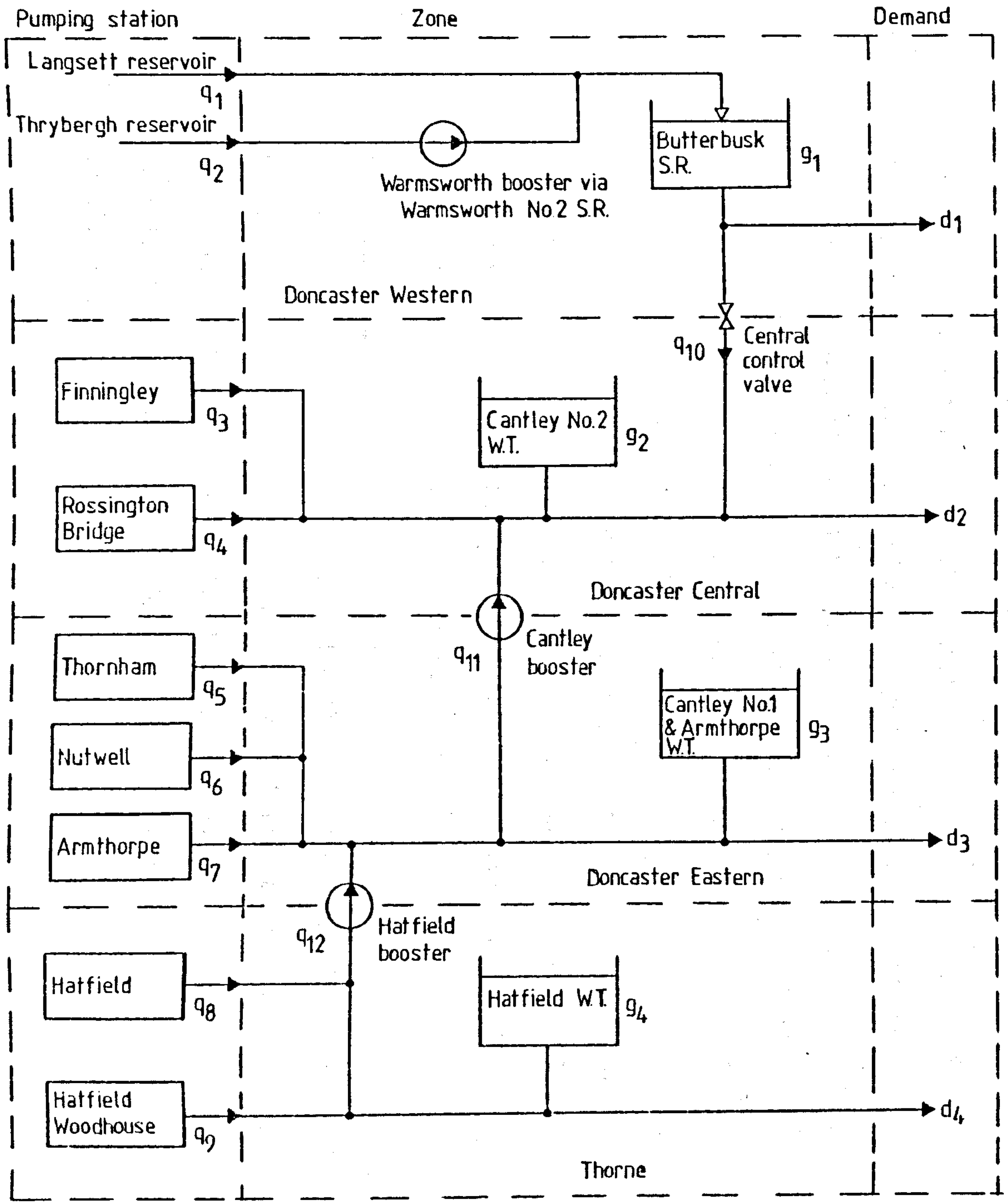
One disadvantage of dynamic programming is that it requires a high-speed memory that is beyond the capacity of present computers when the dimensionality is higher than four or five. Another difficulty usually encountered in the application of dynamic programming is the large amount of computer time required. The methods of successive approximations[10,52,53,54,55,58] and incremental dynamic programming[44,45] have been suggested as means for overcoming the above problems. In the present study both of these techniques have been analysed to determine their suitability for extending the basic dynamic programming methods to cover the higher dimensional cases encountered for multi-zone networks.

(a) Successive Approximations

The principles of this method are based upon the optimisation of one state at a time (the others remaining fixed) so that a sequence of simpler one-dimensional problems are solved with resultant savings in memory and time.

Previously quoted applications[38,44,56,57,99] have relied upon the fact that the system is invertible, this implies that the control dimensionality equals the state dimensionality and the controls can be evaluated uniquely in terms of the state values to allow only one control to be active in conjunction with one free state. For the present case this is not so since the number of controls greatly exceed the number of states. This is shown on the simplified diagram of figure 4-12 where the system has been reduced, to allow application of the successive approximation technique, by forming a series of single reservoir zones with controllable inter-zonal flow. Starting with an initial feasible policy for states and controls the modified method now consists of selecting each reservoir in turn and allowing its level to vary whilst all other reservoir levels remain fixed. Optimisation of the free reservoir being obtained by applying all possible pumping combinations from all zones and selecting the optimal profile (as in the conventional dynamic programming case). The sequence continues until all reservoirs have been optimized at least once and no further improvement in the performance index can be obtained.

In essence this method chooses the cheapest source of water for a reservoir by examining the cost of supply from its own zone and from other zones via inter-zonal transfer. This is precisely what is required but the achievement of this via successive approximations causes some additional problems. Since all other zone reservoirs are held fixed, this implies that their pumping combinations are fixed unless means are available for transfer of any surplus water. This necessary interchange of surplus water can only be achieved if the inter-zonal controls are continuously variable within normal limits.



g = reservoir quantity
d = demand quantity (each k)
q = control quantity (each k)

$$\begin{bmatrix} g_1(k+1) \\ g_2(k+1) \\ g_3(k+1) \\ g_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} g_1(k) \\ g_2(k) \\ g_3(k) \\ g_4(k) \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} q_1(k) \\ q_2(k) \\ q_3(k) \\ q_4(k) \\ q_5(k) \\ q_6(k) \\ q_7(k) \\ q_8(k) \\ q_9(k) \\ q_{10}(k) \\ q_{11}(k) \\ q_{12}(k) \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} d_1(k) \\ d_2(k) \\ d_3(k) \\ d_4(k) \end{bmatrix}$$

Fig. 4-12 Simplified system diagram

In addition, whilst the successive approximations method generates an optimum solution this may only be a local optimum. To guarantee a global optimum it may be necessary to perform repeat solutions starting from different initial feasible solutions.

A computer program, based on the previously described forward dynamic programming algorithm, has been written for implementation of the successive approximations method using the equations given on figure 4-12. This program whilst giving promising results was found to require extensive computer time, since it must cover all pumping combinations, and consequently this technique was not pursued further.

(b) Incremental Dynamic Programming

The operation of this method is based upon the establishment of a corridor around an initial feasible state trajectory[38,44,99]. In its simplest application the corridor is formed by defining fixed state increments, at plus and minus the present state values, and optimisation is performed by choosing control values to give trajectories leading to the defined states in the corridor. This optimisation is repeated with the corridor around the improved state trajectory until convergence is obtained to yield either a local or an absolute optimal solution.

A direct application of the method depends upon having an invertible system with continuous controls and clearly is not feasible for the non-invertible water system with discrete controls.

4.2.3 Discussion.

It has been shown that a conventional dynamic programming method can be applied to simple water systems and will take account of factors such as reservoir constraints, pumping efficiency, maximum demand tariffs, etc. to give optimal pumping policies and evaluate accurate overall pumping costs.

The computer solution is sufficiently general for it to be applied to similar systems, with minor tariff modifications which could prove of immediate benefit for confirmation of present or proposed pumping policies. The present results have only confirmed that an optimum pumping policy is already being used for the system under consideration. However the results are significant since the development of a rigorous computational method of optimisation is a pre-requisite to economic computer control of water systems.

The conventional dynamic programming method has been extended by means of successive approximations to give a computer program formulation (DPSA) for a network consisting of four inter-connected zones. In this case the solution loses some of its attractiveness because of the requirement for continuous inter-zonal controls and the extensive computation times.

Research is continuing on adaptation of these and other dynamic programming techniques more suitable for application to complex water systems consisting of multiple borehole and booster pumping stations interconnected with reservoirs and water towers. These techniques usually require approximate optimal solutions which could be provided by the approach used in section 4.2.1. Implementation of on-line optimal pump control could also necessitate an accurate prediction of the water demand up to one month ahead. The performance of several on-line prediction techniques is currently under investigation[91] together with the development of updating algorithms to account for forecasting errors.

4.3 DE-CENTRALISED HIERARCHICAL TECHNIQUES.

Methods using gradient techniques can be applied to give efficienct optimal solutions but necessitate sufficiently differentiable system equations and performance indices, cannot handle bounded state and control variables without difficulty and require continuous controls. In addition differentiation between local and global optima can present problems and computation time can become excessive for fully integrated optimisation of large scale dynamic systems.

De-centralised methods of solution[60,61,62,95,96,102] can combine the computational advantages of the gradient techniques whilst at the same time removing the associated difficulties. The method essentially consists of creating dual variables which interact with the primal variables to allow de-composition of large scale problems into smaller sub-problems. The independent optimising solutions obtained for each sub-problem can then be co-ordinated by means of the dual variables to give an overall optimal solution.

At the optimum (under specified conditions) the primal and dual functions are equal and form a saddle point. One method of solution consists of searching for this saddle point which, if it exists, solves the problem. The search takes place on a two level hierarchy (using an iterative process) which involves a minimisation of a modified primal function on the first level and a maximisation of the dual function on the second level. The dual function will be continuous and concave and can be maximised using gradient techniques to give a global optimum.

For the present application the most important feature of the de-centralised method lies in the potential for de-composition of high dimensional problems which occur in practice. In particular, use of

Lagrange multipliers as dual variables allow decomposition in time for discrete time problems, and hence converts dynamic optimisation problems into static problems.[61,102] This aspect is reviewed briefly for a general dynamic system described by the state variable vector difference equation:

$$\underline{x}(k+1) = \underline{g}(\underline{x}(k), \underline{u}(k)) \tag{4.42}$$

where $\underline{g}$ is a vector functional and $\underline{x}(k)$ and $\underline{u}(k)$ are state and control variables respectively at stage k.

The constraints on the states and controls are denoted by:

$$(\underline{x}(k), \underline{u}(k)) \in S_1 \tag{4.43}$$

where S_1 represents the set, of any form, of allowed values.

The optimisation problem consists of choosing a sequence of control variables, $\underline{u}(k)$, for k = 0,1...K-1 such that the performance index corresponding to the primal function:

$$J = \sum_{k=0}^{K} f_k (\underline{x}(k), \underline{u}(k)) \tag{4.44}$$

is minimised, and the given initial and final states $\underline{x}(o)$ and $\underline{x}(K)$ are satisfied. f_k is the individual stage performance index.

To convert equation (4.44) to a static optimisation problem the Lagrangian can be expressed as:

$$L(\underline{x},\underline{u}, \underline{p}) = \sum_{k=0}^{K} f_k(\underline{x}(k),\underline{u}(k)) + \sum_{k=0}^{K-1} \underline{p}(k)^T \left[-\underline{x}(k+1) + \underline{g}(\underline{x}(k),\underline{u}(k))\right] \tag{4.45}$$

where $\underline{p}(k)$ is a set of time varying Lagrange multipliers.

Fixing the Lagrange multipliers yields the first level problem

$$\min L (\underline{x},\underline{u}, \underline{p}) \tag{4.46}$$

subject to $(\underline{x}(k),\underline{u}(k)) \in S_1$ for k=0,1...K-1, and $\underline{x}(o),\underline{x}(K)$ fixed.

This decomposes into K independent sub-problems each of the form:

$$\min \left\{ f_k(\underline{x}(k), \underline{u}(k)) - \underline{p}(k-1)^T . \underline{x}(k) + \underline{p}(k)^T . \underline{g}(\underline{x}(k), \underline{u}(k)) \right\} \qquad (4.47)$$

for k = 0,1...K-1, with minimising values of $\underline{x}^*(k)$ and $\underline{u}^*(k)$.

Defining the dual function as:

$$\emptyset(\underline{p}) = \min \left\{ L(\underline{x}, \underline{u}, \underline{p}) \right\} \qquad (4.48)$$

gives the second level problem:

$$\max \emptyset(\underline{p}) \qquad (4.49)$$

subject to $\underline{p} \in S_2$, where S_2 is the set of Lagrange multipliers such that $\emptyset(\underline{p})$ exists.

For the case where a saddle point exists the dual function is concave and continuous in the region and solution to the second level problem can be obtained using a gradient method with gradients obtained from:

$$\nabla_{\underline{p}} \emptyset(\underline{p}) = \frac{\partial \emptyset(\underline{p})}{\partial \underline{p}(k)} = - \underline{x}(k+1) + \underline{g}(\underline{x}(k), \underline{u}(k)) \qquad (4.50)$$

evaluated at the minimising values of $\underline{x}^*(k)$ and $\underline{u}^*(k)$.

The formulation of the problem is seen to be very general and offers the following advantages and features provided a saddle point exists:

(i) High order systems can be decomposed in both time and space to yield simpler sub-problems These are mostly identical in form and can be treated using standardised computer programming techniques.

(ii) Saddle points are defined in terms of maximisation and minimisation rather than stationary points and thus the functions can be non-linear and do not have to be differentiable (however, for differentiable functions, minimisation of the sub-problems can often be efficiently achieved, in closed form, by evaluating the stationary points).

In addition this allows treatment (in principle) of discrete control variables.

(iii) Varying constraints on both states and controls are permitted. Other possible optimisation methods can handle simple constraints on controls but not on states, in this method the treatment of both states and controls becomes identical under decomposition.

(iv) The dual function is always continuous and concave with the result that it can be efficiently maximised using gradient methods to determine a global optimum. Most large computer software packages[76] include gradient optimisation sub-routines which are suitable for this purpose.

Apart from the requirements for a saddle point other disadvantages of the method are:

(i) Primal feasibility (state continuity within constraints) is only achieved when convergence has occurred to give an optimal solution This means that calculations cannot be terminated prematurely to give a near-optimal solution with savings in computing time.

(ii) First level problems have to be solved many times during the iterative calculations and require efficient methods of minimisation. This generally requires quadratic type performance indices for all primal variables.

Whilst the formulation permits use of non-linear functions and discrete variables a saddle point can only be guaranteed to exist for wholly convex programs.[102] The case of linear system equations, quadratic performance index, constraints of upper and lower bounds, together with continuous controls meet the convexity requirements and allow a direct and efficient solution by decentralised techniques. This type of problem is considered in the next section in order to assess the computational aspects of the method and form a basis for investigating its application to practical systems.

4.3.1 Optimisation of Pumping Costs by Hierarchical Methods.

A hierarchical method of optimisation has previously been developed and shown to be useful for application to certain classes of water supply systems in order to optimise pumping costs[34]. However this formulation is not entirely suitable and may fail to give true optimum policies.

This section describes modifications to the basic method which make it more applicable for evaluating optimal control decisions taking into account all relevant factors. In particular, methods have been devised for optimisation of electricity maximum demands and to enable accurate comparison of electricity unit and demand charges.

The method developed in this section has been applied to a simple water supply system and the results show desirable operational features which would result in a reduction of operating cost for continuously variable pumping capability.

4.3.1.1 Review of Hierarchical Optimisation of Linear Quadratic Problems.

The special case of linear equations and quadratic performance indices has been shown to be particularly applicable to water systems when the equations describing the system operation can be linearised, the consumer water demands can be predicted, and the pumps can be controlled to give continuously variable flow outputs[34]. The problem formulation can be developed in terms of the following given conditions:

(i) A system described by the linear difference equation:

$$\underline{x}(k+1) = \underline{A}.\underline{x}(k) + \underline{B}.\underline{u}(k) + \underline{C}(k) \qquad (4.51)$$

where $\underline{x}(k)$ = N dimensional state vector, corresponding to reservoir quantity

$\underline{u}(k)$ = M dimensional control vector, corresponding to pump flow,

$\underline{A}$ = NxN dimensional state identity matrix,

$\underline{B}$ = NxM dimensional control identity matrix,

$\underline{C}(k)$ = N dimensional disturbance vector, corresponding to consumer water demand,

k = 0,1...K, stage variable, corresponding to time increments,

m = 1,2,...M, control vector dimension, corresponding to number of pumping stations,

n = 1,2,...N, state vector dimension, corresponding to number of reservoirs.

(ii) A variational quadratic performance index

$$J(\underline{x},\underline{u}) = \frac{1}{2}\,\underline{x}(K)^T.\,\underline{Q}(K).\,\underline{x}(K) + \frac{1}{2}\sum_{k=0}^{K-1}[\,\underline{x}(k)^T.\underline{Q}(k).\underline{x}(k) + \underline{u}(k)^T.\underline{R}(k).\underline{u}(k)] \tag{4.52}$$

where $J(\underline{x},\underline{u})$ = scalar performance index,

$\underline{Q}(k)$ = NxN dimensional positive definite diagonal matrix of state weighting factors,

$\underline{R}(k)$ = MxM dimensional positive definite diagonal matrix of control weighting factors.

(iii) A set of constraints on the states and controls of upper and lower bounds form

$$\underline{x}_{min} \leq \underline{x}(k) \leq \underline{x}_{max} \tag{4.53}$$

$$\underline{u}_{min} \leq \underline{u}(k) \leq \underline{u}_{max} \tag{4.54}$$

where $\underline{x}_{max}, \underline{x}_{min}$ = N dimensional vectors of state constraints, corresponding to maximum and minimum values of reservoir quantities.

$\underline{u}_{max}, \underline{u}_{min}$ = M dimensional vectors of control constraints, corresponding to maximum and minimum values of pump flows.

(iv) An initial state:

$$\underline{x}(o) = \underline{x}_o \tag{4.55}$$

where $\underline{x}_o$ = N dimensional initial state vector, corresponding to reservoir initial quantity.

The primal problem may then be defined as:

$$J(\underline{x},\underline{u}) \rightarrow \min_{\underline{x},\underline{u}} \tag{4.56}$$

subject to equations (4.51), (4.53), (4.54) and (4.55).

Solution of the primal problem would yield the optimal state and control sequences $\underline{x}^* = (\underline{x}(1),\underline{x}(2),\ldots\underline{x}(K))$, $\underline{u}^* = (\underline{u}(o),\underline{u}(1),\ldots\underline{u}(K-1))$.

In this case the solution can be obtained more efficiently by formulation and solution of the dual problem as follows:

Introducing the Lagrangian function:

$$L(\underline{x},\underline{u},\underline{p}) = J(\underline{x},\underline{u}) + \sum_{k=o}^{K-1} \underline{p}(k)^T \; [-\underline{x}(k+1)+\underline{A}.\underline{x}(k)+ \underline{B}.\underline{u}(k)+\underline{C}(k)] \tag{4.57}$$

where $L(\underline{x},\underline{u},\underline{p})$ = scalar Lagrangian

$\underline{p}(k)$ = N dimensional vector of Lagrange multipliers

enables the dual function to be defined as:

$$\phi(\underline{p}) \rightarrow \max_{\underline{p}} \tag{4.58}$$

where $$\phi(\underline{p}) = \min_{\underline{x},\underline{u}} \; L(\underline{x},\underline{u},\underline{p}) \tag{4.59}$$

subject to equations (4.53), (4.54) and (4.55).

The dual function gradient vector will then be

$$\nabla_{\underline{p}}\phi(\underline{p}) = - \underline{x}(k+1) + \underline{A}.\underline{x}(k) + \underline{B}.\underline{u}(k) + \underline{C}(k) \tag{4.60}$$

and the dual problem can be solved at two levels as follows:

(a) <u>Subordinate (first level) problem</u>.

For fixed $\underline{p}$ solve the following independent minimisation problems to determine initial estimates of $\underline{x}^*, \underline{u}^*$. For the given case where the quadratic weighting matrices are positive definite the minimisation can be efficiently performed by differentiating to find the stationary point. Further, for the special case of diagonal weighting matrices, there is decomposition of the sub-problem state and control dimensions and the minimising elements are all defined independently. If the stationary point occurs outside the bounds the minimising value will be that of the nearest bound.

(i) $\underline{u}^*(k)$ for $k = 0,1,\ldots K-1$

$$\frac{1}{2}\ \underline{u}(k)^T.\ \underline{R}(k).\underline{u}(k) + \underline{p}(k)^T.\ \underline{B}.\underline{u}(k) \underset{\underline{u}(k)}{\rightarrow} \min \qquad (4.61)$$

subject to equation (4.54), to give:

$$\underline{u}^*(k) = -\ \underline{R}(k)^{-1}.\ \underline{B}^T.\underline{p}(k) \qquad (4.62)$$

(ii) $\underline{x}^*(k)$ for $k = 1,2,\ldots K-1$

$$\frac{1}{2}\ \underline{x}(k)^T.\ \underline{Q}(k).\underline{x}(k) - \underline{p}(k-1)^T.\ \underline{x}(k) + \underline{p}(k)^T.\ \underline{A}.\underline{x}(k) \underset{\underline{x}(k)}{\rightarrow} \min \qquad (4.63)$$

subject to equation (4.53), to give

$$\underline{x}^*(k) = \underline{Q}(k)^{-1}.\ (\underline{p}(k-1) - \underline{A}^T.\underline{p}(k)) \qquad (4.64)$$

(iii) $\underline{x}^*(K)$

$$\frac{1}{2}\ \underline{x}(K)^T.\ \underline{Q}(K).\underline{x}(K) - \underline{p}(K-1)^T.\ \underline{x}(K) \underset{\underline{x}(K)}{\rightarrow} \min \qquad (4.65)$$

subject to equation (4.53) to give

$$\underline{x}^*(K) = \underline{Q}(K)^{-1}.\ \underline{p}(K-1) \qquad (4.66)$$

(b) Co-ordination (second level) problem.

(i) For fixed $\underline{p}$, $\underline{x}^*$ and $\underline{u}^*$ compute $\emptyset(\underline{p})$ from equation (4.59) and

$\nabla_{\underline{p}}\emptyset(\underline{p})$ from equation (4.60).

(ii) For fixed $\underline{x}^*$ and $\underline{u}^*$ use a gradient method[37] to find new value of $\underline{p} = \underline{p}^*$ which maximises $\emptyset(\underline{p})$ and makes $\nabla_{\underline{p}}\emptyset(\underline{p}) = 0$.

The complete solution to the dual problem consists of iteration between first and second level problems until convergence is obtained with $\emptyset(\underline{p}^*) = J(\underline{x}^*,\underline{u}^*)$; $\underline{x}^*$ and $\underline{u}^*$ are then the required solutions to the primal problem.

4.3.1.2 Adaptation of Hierarchical Optimisation Technique.

(a) Pumping Costs in a Water Network.

Various pumping station models have been derived in Chapter 3 and the relationship between the various unit and demand charges have been analysed in section 4.2.1.

The unit charge is based upon a unit tariff which is a function of the electricity maximum demand and the units used over the tariff period. A reasonably accurate representation of these effects can be obtained by allowing variation of the unit tariff for each optimisation period. A composite time varying unit tariff ($\underline{T}u(k)$) can then be formed, for each optimisation period, which will include fuel adjustment and night rebates. The most appropriate pumping station cost model for this topic is the linear-quadratic model of section 3.3.3 which shows the relationship between the unit rate and the water flow output to be of the form:

$$\text{Unit rate} = s_m.u_m(k) + r_{mm}.u_m(k)^2 \qquad (4.67)$$

where s_m = linear relationship between pumping station m output and input.

r_{mm} = quadratic relationship between pumping station m output and input.

$u_m(k)$ = pump flow for pumping station m.

The total unit charge for all stations will then be:

$$\text{Unit Charge} = \sum_{k=0}^{K-1} \underline{Tu}(k) \left[\underline{S}^T . \underline{u}(k) + \underline{u}(k)^T . \underline{R} . \underline{u}(k) \right] \qquad (4.68)$$

where $\underline{Tu}(k)$ = MxM dimensional diagonal matrix whose elements correspond to pumping station electricity unit tariff values and include effects of time interval, Δt,(corresponding to each increment of k).

$\underline{S}$ = M dimensional vector with elements of s_m.

$\underline{R}$ = MxM dimensional diagonal matrix with elements of r_{mm}.

The demand charge is based upon a stepped demand tariff and the electricity maximum demand attained over the whole of the tariff period. The present implementation assumes a demand tariff ($\underline{Tw}$) constant for all demands. The relationship between the demands and any station maximum flow output are of the form:

$$\text{Demand} = s_m . w_m + r_{mm} . w_m^2 \qquad (4.69)$$

where w_m = maximum pump flow achieved over optimisation period for pumping station m.

The total demand charge for all stations will then be:

$$\text{Demand charge} = \underline{Tw} \; (\underline{S}^T . \underline{w} + \underline{w}^T . \underline{R} . \underline{w}) \qquad (4.70)$$

where $\underline{Tw}$ = MxM dimensional diagonal matrix whose elements correspond to pumping station electricity demand tariff values.

$\underline{w}$ = M dimensional maximum control vector with elements of w_m.

(b) Performance Index Requirements.

A suitable performance index should allow for accurate time varying comparison of all combinations of unit and demand charges over the complete tariff period and should include a penalty charge dependent on the terminal state only. To avoid the requirement for optimisation over the whole tariff period two complementary indices can be derived and implemented.

A Long Term Performance Index can be used to determine maximum flow requirements for each station based upon optimisation of unit and demand charges. The optimisation would be performed using predicted maximum water demand over any reservoir cycle period and would only be required at the beginning of every tariff period or for any significant change in maximum water demand.

A Short Term Performance Index can be used to determine flow requirements (with previously calculated maximum flow requirements as upper bounds) based upon optimisation of unit charges only. The optimisation would be performed using predicted normal water demand and would be required for each reservoir cycle period.

In practice the short term is a special case of the long term and only the latter performance index need be derived as given below:

(c) Derivation of Performance Index.

On the basis of the above requirements the standard quadratic performance index is not entirely suitable as it takes no account of demand charges and will, in fact, encourage high flows to prevent state deviations during the optimisation period.

Optimisation of demand charges can be incorporated by introduction of an additional linear constraint equation:

$$\underline{w} = \underline{u}(k) + \underline{t}(k) \tag{4.71}$$

subject to

$$0 \leq \underline{u}(k) \tag{4.72}$$

$$0 \leq \underline{t}(k) \tag{4.73}$$

$$0 \leq \underline{w} \leq \underline{w}_{max} \tag{4.74}$$

where $\underline{t}(k)$ = M dimensional control deviation vector.

$\underline{w}_{max}$ = M dimensional upper bound on maximum control vector

Comparison of unit and demand charges can be incorporated by use of equations (4.68) and (4.70).

The complete performance index will now be as given below where $\underline{Q}(k)$ and $\underline{Rt}$ can be given low values in order to minimise unwanted penalties:

$$J(\underline{x},\underline{u},\underline{t},\underline{w}) = \frac{1}{2}\,\underline{x}(K)^T_{\cdot}\,\underline{Q}(K).\underline{x}(K) + \frac{1}{2}\,\underline{Tw}\,(\underline{S}^T_{\cdot}\underline{w} + \underline{w}^T_{\cdot}\,\underline{R}.\underline{w})$$

$$+ \frac{1}{2}\sum_{k=0}^{K-1}\left[\underline{x}(k)^T_{\cdot}\underline{Q}(k)\underline{x}(k) + \underline{Tu}(k).(\underline{S}^T_{\cdot}\underline{u}(k)+\underline{u}(k)^T_{\cdot}\underline{R}.\underline{u}(k)) + \underline{t}(k)^T_{\cdot}\underline{Rt}.\underline{t}(k)\right] \tag{4.75}$$

where $\underline{Rt}$ = MxM dimensional diagonal matrix of control deviation weighting factors.

(d) <u>Formulation of Dual Problem.</u>

(i) Lagrangian

$$L(\underline{x},\underline{u},\underline{t},\underline{w},\underline{p}_x,\underline{p}_w) = J(\underline{x},\underline{u},\underline{t},\underline{w}) + \sum_{k=0}^{K-1}\underline{p}_x(k)^T\,[-\underline{x}(k+1) + \underline{A}.\underline{x}(k) + \underline{B}.\underline{u}(k) + \underline{C}(k)$$

$$+ \sum_{k=0}^{K-1}\underline{p}_w(k)^T\,[-\underline{w} + \underline{u}(k) + \underline{t}(k)] \tag{4.76}$$

where $\underline{p}_w(k)$ = M dimensional vector of Lagrange multipliers.

(ii) Dual function

$$\phi\ (\underline{p}_x,\underline{p}_w) \rightarrow \max_{\underline{p}_x,\underline{p}_w} \qquad (4.77)$$

where $$\phi\ (\underline{p}_x,\underline{p}_w) = \min_{\underline{x},\underline{u},\underline{t},\underline{w}} \{L\ (\underline{x},\underline{u},\underline{t},\underline{w},\underline{p}_x,\underline{p}_w)\} \qquad (4.78)$$

subject to equations (4.53), (4.72). (4.73) and (4.74).

and where $$\nabla\underline{p}_x\ \phi(\underline{p}_x,\underline{p}_w) = -\ \underline{x}(k+1) + \underline{A}.\underline{x}(k) + \underline{B}.\underline{u}(k) + \underline{C}(k) \qquad (4.79)$$

$$\nabla\underline{p}_w\ \phi(\underline{p}_x,\underline{p}_w) = -\ \underline{w} + \underline{u}(k) + \underline{t}(k) \qquad (4.80)$$

(e) Solution of Dual Problem.

The solution follows directly from the procedure given previously. For fixed $\underline{p}_x$ and $\underline{p}_w$ the analytical solutions to the independent minimisation problem for $\underline{x}$*, $\underline{u}$*, $\underline{t}$* and $\underline{w}$* will be:

$$\underline{x}^*(k) = \underline{Q}(k)^{-1}.\ (\underline{p}_x(k-1) - \underline{A}^T.\underline{p}_x(k)) \text{ for } k = 1,2,\ldots K-1 \qquad (4.81)$$

$$\underline{x}^*(K) = \underline{Q}(K)^{-1}.\ \underline{p}_x(K-1) \qquad (4.82)$$

and $$\underline{u}^*(k) = \underline{R}^{-1}.\ \underline{Tu}(k)^{-1}.(\underline{B}^T.\underline{p}_x(k) + \underline{p}_w(k)) - \frac{1}{2}\underline{R}^{-1}.\underline{S} \text{ for } k = 0,1,\ldots K-1, \qquad (4.83)$$

together with $$\underline{t}^*(k) = -\ \underline{Rt}^{-1}.\ \underline{p}_w(k) \text{ for } k = 0,1,\ldots K-1 \qquad (4.84)$$

$$\underline{w}^* = \underline{R}^{-1}.\ \underline{Tw}^{-1}.\ \sum_{k=o}^{K-1}\underline{p}_w(k) - \frac{1}{2}\ \underline{R}^{-1}.\ \underline{S} \qquad (4.85)$$

for values between upper and lower bounds. Outside these bounds the solutions will take on the nearest boundary value.

4.3.1.3 Application to a Water Supply System.

The hierarchical optimisation technique is suitable for application to any complex water supply system in which the system equations can be linearised and the pump flows can be regarded as being continuously variable.

For this application a simple system is required, for which operational data is available, thereby allowing easy evaluation of the results and a check on optimality.

The method described in section 4.3.1.2 has been implemented as a general purpose optimisation program written in Fortran IV, for use on an ICL 1907 computer, and using the Fletcher and Reeves[37,76] method of conjugate gradients for maximising the dual function. This program was used to predict optimal pumping profiles based on the following data:

(a) System Description and Data.

The Hatfield section of the Doncaster and District Joint Water Board has been chosen as a suitable test case. This system consists of a water tower supplied directly from a borehole by use of four constant speed pumps. A closed loop control system is employed which gives on-off pump controls dependent on tower level. The operation is based on a 24 hour cycle period and operating data are logged every 2 hours.

The operational data and electricity tariffs (for 1973) have been evaluated to arrive at the set of nominal values in Table 4-1 where the stage increment is 2 hours starting from 0800 hours, and reservoir quantities are given as deviations from zero (corresponding to 10^5 gallons).

(b) Analysis of Results.

Figure 4-13 shows the resulting variations of reservoir quantity together with each of the actual and predicted pumping profiles for the cases considered below. The data has been taken from Table 4-1 and a cost comparison is made for each case on the

Table 4-1

Data for Hatfield water supply system

$A = 1$	$R = 143$ kVA/$(10^5$gal/h$)^2$ or kW/$(10^5$gal/h$)^2$
$B = 2$	$S = 98$ kVA/$(10^5$gal/h) or kW/$(10^5$gal/h)
$C(k)$ see Figure 1	$Tu(k) = 0.0128$ $(k = 0,1,\ldots 7)$; 0.0117 $(k = 8,9,10,11)$ (£/kWh) for 2h.
$Tw = 0.0338$ (£/kVA for 24h)	$x_o = 0.0$
$k = 0,1,\ldots 11.$	$x_{max} = 0.5$ $(10^5$gal)
$K = 12$	$x_{min} = -0.25$ $(10^5$gal)
$M = 1$	$u_{max} = 0.8$ $(10^5$gal/h)
$N = 1$	$u_{min} = 0.0$
$Q(k) = 0.1$ (£/$(10^5$gal$)^2$)	$Rt = 0.1$ (£/$(10^5$gal/h$)^2$)
$Q(K) = 10.0$ (£/$(10^5$gal$)^2$)	$w_{max} = 0.8$ $(10^5$gal/h)
	$w_{min} = 0.0$

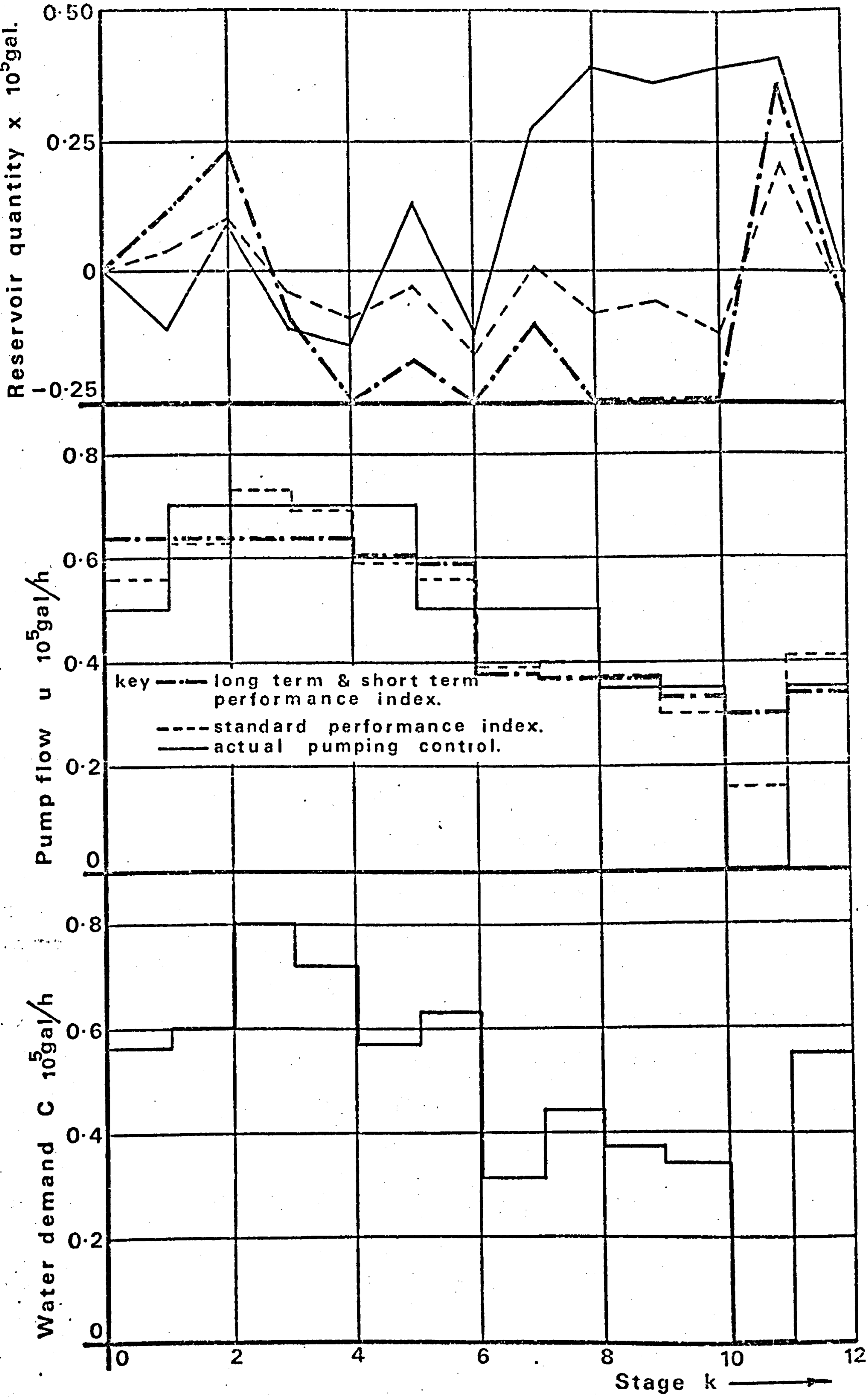


Fig. 4·13 Actual and Predicted Pumping Profiles.

same basis of electricity unit and demand charges only.

(i) Long Term Performance Index (minimising demand and unit charges)

This has been applied to determine the optimum value of maximum flow to cater for maximum water demand over any cycle period. The results show the expected pumping profile of constant flow at the optimised value where permitted by reservoir constraints. The cost was £16.77 and the program required 50 units of computer time.

(ii) Short Term Performance Index (minimising unit charges).

These results, obtained by using the previous optimised value of maximum flow as an upper bound on the pump flow are identical to the long term solution but now require only half the computer time.

(iii) Standard Performance Index (minimising tower quantity deviations and unit charges).

The results, obtained with Q(k) set to 1.0, show excessive maximum flow requirements. The computer time requirement is smaller at 15 units but the cost is now non-optimal at £17.89 which is about the same as required by the simple level control below.

(iv) Actual Pumping Control (minimising tower level deviations).

The results, obtained using the logged data, are close to optimal for the allowed discrete pump flows at a cost of £17.97, thus showing that this method of control is effective for simple systems.

4.3.2 Extensions to Hierarchical Methods.

The results of the previous section have shown that optimisation of pumping costs is amenable to treatment by de-centralised hierarchical techniques under the restrictions of linear system equations, quadratic performance indices and continuously variable pumping. Of these restrictions the requirement of continuous control variables is likely to present the biggest problem for realistic applications involving discrete pumping. In order to extend the usefulness of the hierarchical approach a theoretical and practical investigation has been made of the implications of use of discrete controls.

(a) Discrete Control Formulation.

The modifications necessary to extend the basic method to allow only discrete control variables can be catered for by defining a control constraint of the form:

$$\underline{u}(k) \in S_3 \tag{4.86}$$

where S_3 is now a set of discrete values and hence conforms to a non-convex set of finite points.

For this type of constraint there is no guarantee that a saddle point exists, but if it does exist, and can be found, it will solve the problem. Assuming a saddle point does exist, to find it, it is necessary to solve the independent sub-problem minimisations for the allowed discrete control values. For the typically small number of discrete values a direct search could be made to determine which results in a minimum but a more elegant and efficient method, due to Everett[32], involves defining a continuous differentiable performance function passing through each of the discrete values. This function can then be incorporated into the formulation of the sub-problems and differentiated to determine a continuous value stationary point. Trials of the two adjacent discrete points will then yield the minimising value. For the present study the performance index characteristic has already been arranged to pass through each of the discrete points, thus, modification to the existing computer program merely involves selection of the adjacent discrete value giving the lowest value of the sub-problem.

(b) Results of Investigation.

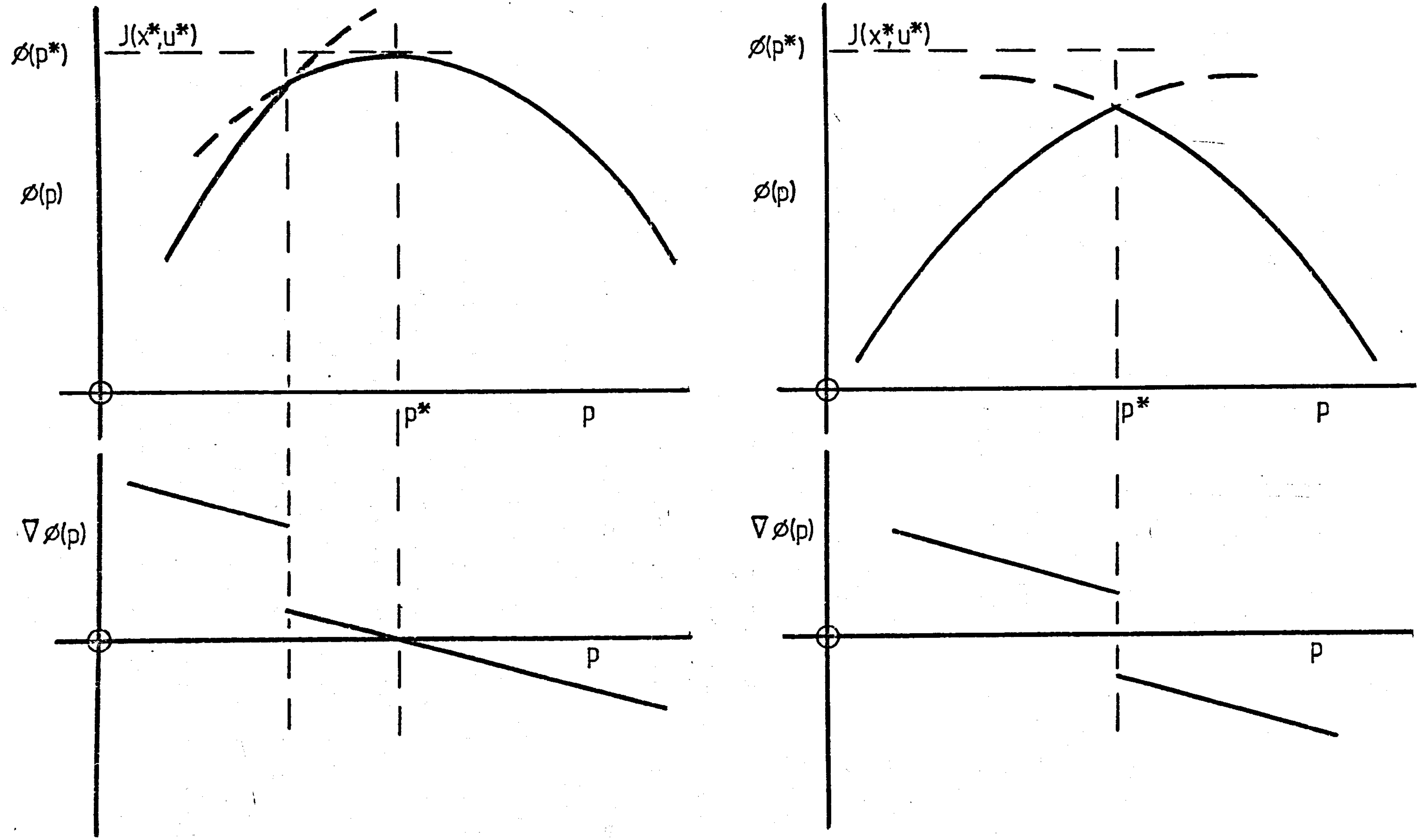
For the above modifications the dual function will still be continuous and concave but will exhibit discontinuous gradients for changes between the discrete values. During initial testing the use of a gradient method for maximisation of the dual function was noted to cause excessive cycling around gradient discontinuities in the region of the maximum. This necessitated a change to an optimisation algorithm based on improvements in the function value (i.e. E04CAF[76]). Whilst this change enabled the maximisation of the dual function the method failed to provide overall optimal feasible solutions.

The reason for the failure is that the method results in an overall primal feasible solution (system equations in balance for all stages) only when all the elements of the dual function gradient are zero (i.e. a saddle point exists since $J(\underline{x}^*,\underline{u}^*) = \emptyset(\underline{p}^*)$ only for $\nabla_{\underline{p}}\emptyset(\underline{p}) = -\underline{x}(k+1) + \underline{g}(\underline{x}(k),\underline{u}(k)) = 0$). Because the dual function gradient can exhibit discontinuities (corresponding to changes between discrete $\underline{u}(k)$ values) it is quite likely, in a multi-stage system, that some of these will occur in the neighbourhood of the optimum. If any gradient element changes discontinously from a positive value to a negative value it can never attain the required zero value to satisfy the system equations. Failure at any stage will give rise to an infeasible solution because of the dynamic nature of the system equations. Figure 4-14 shows a simple geometrical interpretation of these effects in terms of the dual variables.

It could be argued that an overall near optimal solution would result if the solution was to be accepted up to the first stage at which infeasibility occurred, a choice between discrete control values at this stage could then be made to maintain feasibility rather than optimality. Continued computations for optimality could then take place using the forced feasible state values as new initial conditions and could be repeated as often as necessary to the end of the optimisation period. Tests showed that such a scheme was unlikely to be a very promising modification since the frequency of infeasibility was quite high, necessitating extensive re-calculations. Consequently there is currently no easy method for making the choice between discrete values for each infeasible stage, and furthermore there is no indication of the likely departure from the optimum.

4.3.3 Discussion.

Previous work has shown that hierarchical methods are very efficient for on-line control of multi-reservoir water supply systems with continuously variable pumping capability.



(a) saddle point exists

(b) no saddle point exists

Fig. 4-14 Typical dual function and gradients for discrete controls

Section 4.3.1 has given the development of an improved formulation, together with performance indices directly related to electricity charges, which make the method more applicable to optimisation of costs in water supply systems.

A method has also been suggested for handling the optimisation problems of the long terms effects of the electricity maximum demand charges and the short term effects of the electricity unit charges. This is effected by means of two complementary performance indices which can both be applied over short term periods.

The methods have been applied to a practical system and the results show that an optimal pumping policy has been predicted. However, there is little possibility of improvement in the simple system used. More significant savings should be possible for complex systems where the operation depends on many inter-dependent charges and operational constraints.

In general water systems have combinations of fixed and variable pump flow and optimisation methods should cater for these. To this end an investigation into the possibilities for extending the basic method by direct inclusion of discrete controls has been presented. However the results show that, for multi-stage dynamic systems, the likelihood of achieving an optimal feasible solution under these conditions is extremely remote. This is also confirmed by related research[32,61,75].

If use of this powerful optimisation technique is dictated by virtue of its other advantages then other modifications must be found to allow incorporation of discrete controls. One possibility is to use the method to calculate continuous solution values and use an additional algorithm to select close discrete values on an optimal basis whilst maintaining a feasible trajectory. This latter could well prove to be a formidable or even impossible task for fully optimal solutions but it is anticipated that algorithms will be found for close optimal solutions.

4.4 LINEAR AND INTEGER PROGRAMMING TECHNIQUES.

Linear programming theory is well documented and standard computer programs are readily available[48,76]. Thus it is well worthwhile attempting to express the water system optimisation problem in a linear programming format. To achieve this the requirements are for linear system equations, constraints of upper and lower bounds, together with a linear performance index. For the simplified type of system currently studied the system equations are linear but the control variables can be discrete valued and the performance index is non-linear and discontinuous. These difficulties can be resolved (in principle) by use of integer programming techniques which can cater for both discrete values and discontinuities by transformations using 0-1 integer variables. The integrated optimisation problem now becomes a mixed variable type for which solution techniques are available under limited conditions[4,6,11,51].

This section shows how the control of water systems can be cast as a mixed linear-integer problem and discusses possible solutions and difficulties. The formulation is not necessarily the most compact but is designed to illustrate the principles involved in a straightforward manner and give an insight into solution feasibility.

4.4.1 Problem Formulation.

(a) Linear Program.

The standard linear programming problem may be defined in abbreviated form as:

Find $\underline{X} > 0$ to minimise $J = \underline{C}_1^T.\underline{X}$ subject to $\underline{A}_1.\underline{X} \geq \underline{B}_1$ where $\underline{A}_1$ is a coefficient matrix of linear equations in continuous unknowns $\underline{X}$, $\underline{B}_1$ is vector of equation constants, $\underline{C}_1$ is vector of cost factors and J is the total cost.

As a first approximation an entirely linear programming format can be devised using cost functions, for electricity unit and maximum demand charges, which are piece-wise linear over the range of pump combination flows for each pumping station. These cost functions have been justified in the piece-wise linear cost model of section 3.3.1 and are shown in figures 4-15 and 4-16.

The water system equations and constraints, equations (4.51), (4.71) and (4.53) can be re-written as:

$$\underline{x}(k+1) - \underline{A}.\underline{x}(k) - \underline{B}.\underline{u}(k) = \underline{C}(k) \tag{4.87}$$

$$\underline{w} - \underline{u}(k) \geq 0 \tag{4.88}$$

$$\underline{x}(k) \geq \underline{x}_{min} \tag{4.89}$$

$$\underline{x}(k) \leq \underline{x}_{max} \tag{4.90}$$

for $k = 0,1,\ldots K$ and retaining the previous notation.

Noting that figures 4-15 and 4-16 represent convex-separable functions[14] allows specification in terms of the break point co-ordinates as follows:

$$u_m(k) = \underline{e}^T_m(k).\underline{p}_m(k) = \sum_{\ell=1}^{L} e_{\ell m}(k).p_{\ell m}(k) \tag{4.91}$$

where $$\sum_{\ell=1}^{L} p_{\ell m}(k) = 1 \tag{4.92}$$

and $$w_m = \underline{f}^T_m \cdot \underline{q}_m = \sum_{\ell=1}^{L} f_{\ell m} \cdot q_{\ell m} \tag{4.93}$$

where $$\sum_{\ell=1}^{L} q_{\ell m} = 1 \tag{4.94}$$

for $m = 1,2,\ldots M$ and $k = 0,1,\ldots K$.

Where $e_{\ell m}(k)$, $g_{\ell m}(k)$, $f_{\ell m}$, $h_{\ell m}$ = break points defined on Figures 4-15 and 4-16 for each pumping station.

$p_{\ell m}(k)$, $q_{\ell m}$ = sets of continuous variables for each pumping combination of each pumping station.

ℓ = $(1,2,\ldots L)$ is set of pumping combination indices.

m = $(1,2,\ldots M)$ is set of pumping stations.

The corresponding performance index will be given by:

$$J = \sum_{m=1}^{M} \left[\sum_{k=o}^{K-1} \underline{g}_m^T(k)\underline{p}_m(k) + \underline{h}_m^T \cdot \underline{q}_m \right] = \sum_{m=1}^{M} \left[\sum_{k=o}^{K-1} \sum_{\ell=1}^{L} g_{\ell m}(k) \cdot p_{\ell m}(k) + \sum_{\ell=1}^{L} h_{\ell m} \cdot q_{\ell m} \right] \tag{4.95}$$

The solution efficiency depends on the number of constraint equations and the number of variables. Taking typical realistic values of $L = 4$, $M = 8$, $N = 4$ and $K = 12$ will allow calculation of these from:

Equations = $NK + MK + M + LMK + LM = 568$

Variables = $3NK + 3MK + 2M = 448$

This will represent an extensive computer programming problem in terms of data and format but a solution is feasible and will determine optimum values of $\underline{u}(k)$ and $\underline{w}$. These represent pumping flows at each stage and maximum pumping flow achieved over all stages assuming continuous variation of flow is achievable. In practice only discrete values may be possible and the next section attempts to modify the solution and remove the continuity requirement.

(b) Mixed Linear-Integer Program.

One formulation of the mixed linear-integer problem[4] is as follows: Find $\underline{X}, \underline{Y} \geq 0$ to minimise $J = \underline{C}_1^T \cdot \underline{X} + \underline{C}_2^T \cdot \underline{Y}$ subject to $\underline{A}_1 \cdot \underline{X} + \underline{A}_2 \cdot \underline{Y} \geq \underline{B}_1$ where $\underline{A}_1$ is coefficient matrix for continuous unknowns $\underline{X}$, $\underline{A}_2$ is coefficient matrix for 0-1 integer unknowns $\underline{Y}$, $\underline{B}_1$ is vector of equation constants, $\underline{C}_1$ and $\underline{C}_2$ are vectors of cost factors for $\underline{X}$ and $\underline{Y}$ respectively and J is total cost.

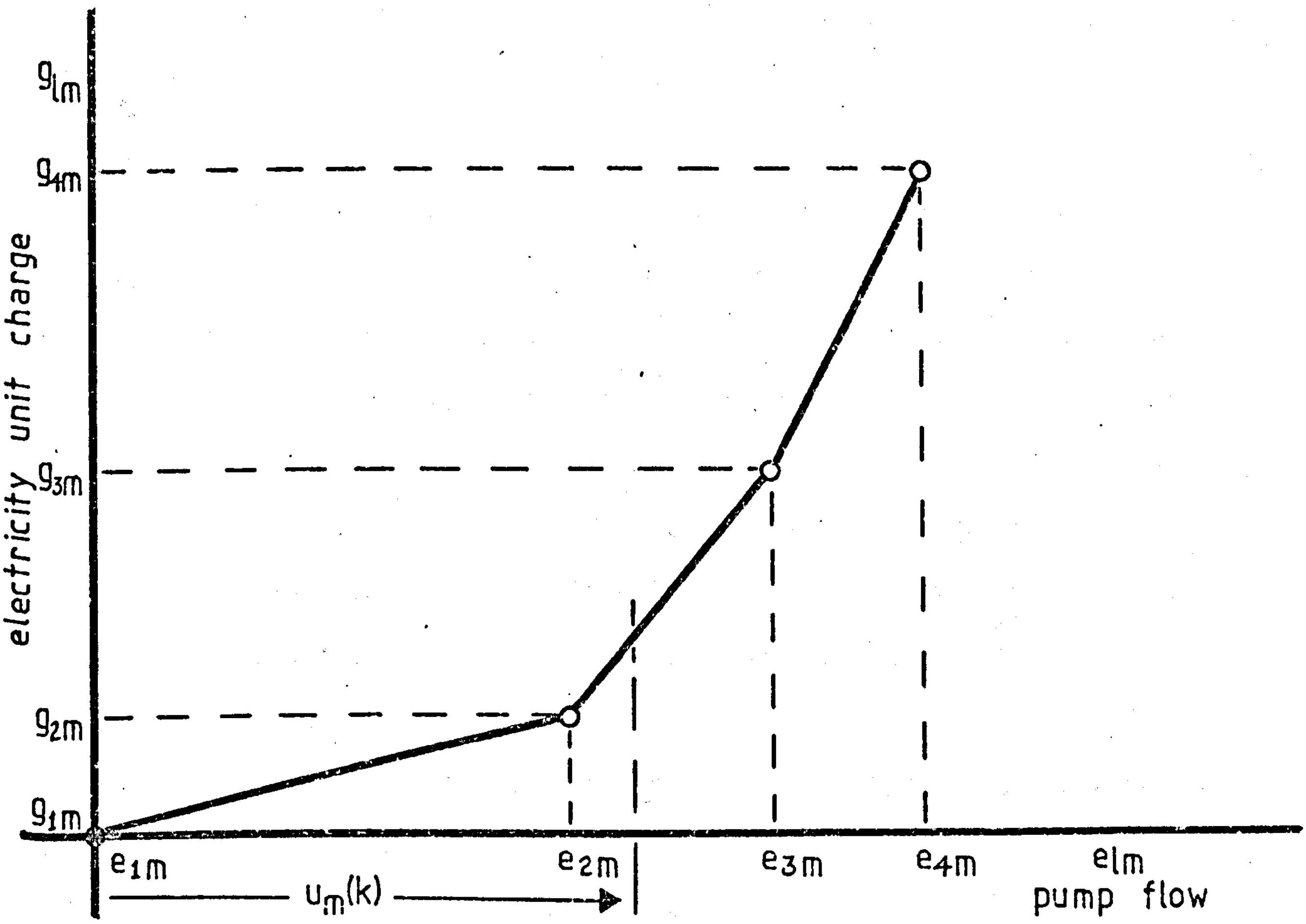


Fig. 4-15 Cost function for electricity unit charges

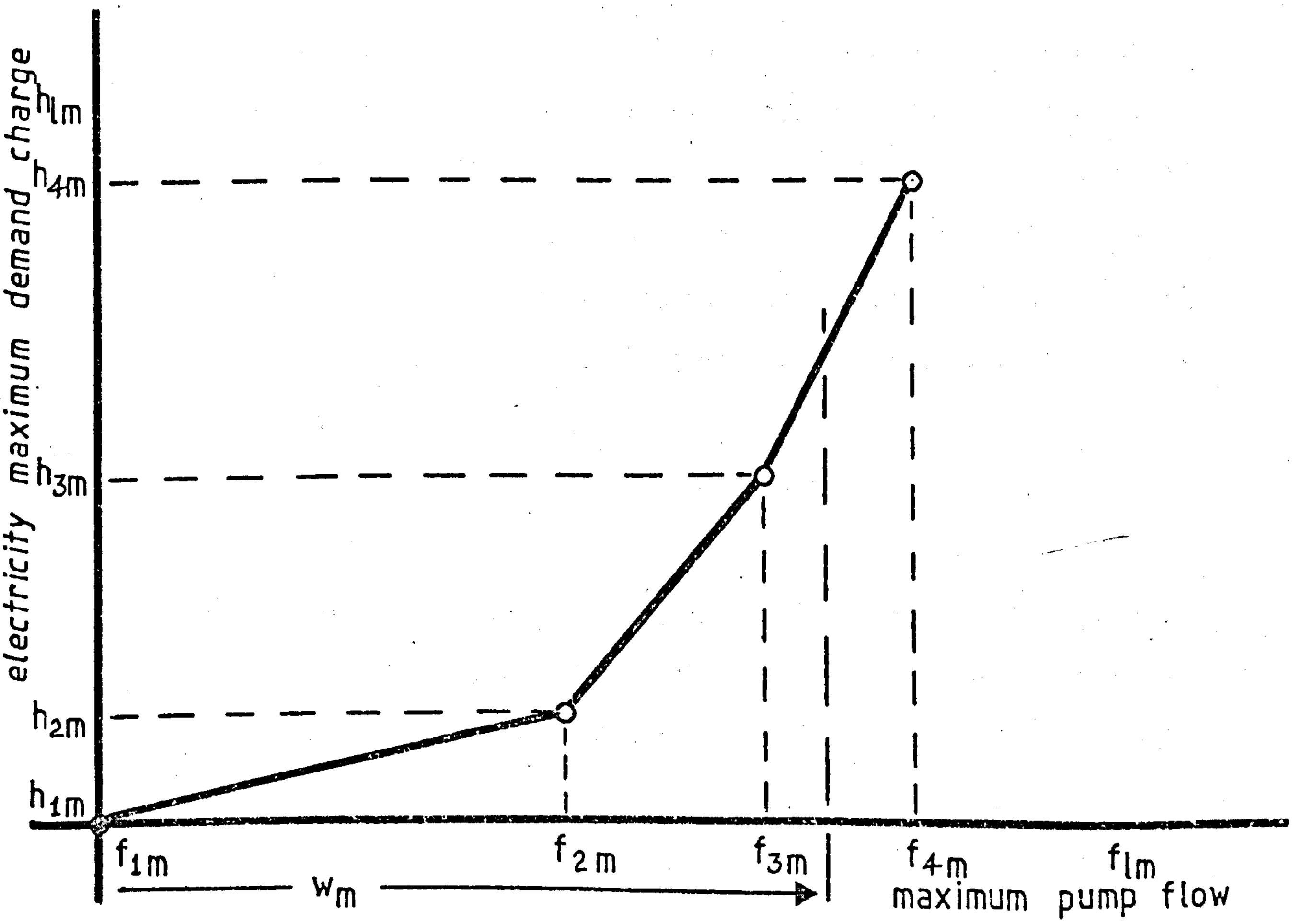


Fig. 4-16 Cost function for electricity maximum demand charges

The linear program can be formulated as a mixed variables program simply by re-defining the continuous variables $p_{\ell m}(k)$ and $q_{\ell m}$ to be 0-1 integer valued. The constraints of equations (4.92) and (4.94) ensure that, for a given value of m over the range of values of ℓ, only one of each of the variables can have a value of 1. Equations (4.91) and (4.93) now constrain $u_m(k)$ and w_m to take on discrete values corresponding to allowed pumping combinations and similarly equation (4.95) ensures that costs are only evaluated for these conditions.

This formulation has the same number of equations but the variables are split as follows:

Continuous variables = NK + MK + M = 152

Integer variables = LMK + LM = 416

Conversion to allow discrete values for both $\underline{u}(k)$ and $\underline{w}$ leads to a large number of integer variables. Benders[11] has noted that solution of mixed integer problems is only feasible for up to 30 - 40 integer variables and thus the formulation must be restricted to this number if a solution is to be possible. One way to achieve this is to allow $\underline{u}(k)$ to be continuous but to restrict $\underline{w}$ to take on discrete values by making only $q_{\ell m}$ a 0-1 integer variable. This will change the variables to the following feasible values:

Continuous variables = NK + MK + M + LMK = 536

Integer variables = LM = 32.

This modification can be justified in terms of pumping operations, with continuous pump flows, by assuming that pumping takes place at the next highest feasible pump combination for a proportion of the time interval (k); costs are thus accurately represented for proportional pumping over reduced time intervals. The maximum demand flow, being discrete, will take on the next highest value corresponding to the maximum flow used over the optimisation period.

Possible solution procedures are due to Beale[6], Benders[11,39] and Land and Doig[51] and, of these, Benders partitioning algorithm would appear to be particularly suitable since it is a primal-feasible method in which the solution can be stopped at any time before convergence to yield an improved control trajectory.

4.4.2 Discussion.

Section 4.4.1 has demonstrated that it is possible to formulate the optimisation problem reasonably accurately by a mixed integer format which corresponds to a piecewise linear cost function for electricity unit charges and a demand tariff stepped at each pump combination.

The analysis shows that for a realistic network, with the specified restrictions, current solution techniques are just feasible; extensions to cover additional operating costs, more complex networks or more accurate network models could well prove impossible. The major problem in this respect is in the application to dynamic systems where the multiplication effect of k for each stage varying parameter results in several hundred constraint equations and variables. Linear programming can handle large numbers of constraints and variables by decomposition to form smaller sub-problems but, in this case, the format of the system equations prevents easy partitioning of the $\underline{A}_1$ matrix. Additionally Balinski[4] has noted that solution procedures involving integer variables produce erratic computational performance with a possibility of excessive solution times. On these counts the method has not been pursued to give a full solution but could well prove a topic for further research.

4.5 OVERALL CONCLUSIONS.

This chapter has covered investigation of various techniques for optimisation of pumping costs and has shown that the optimisation problem, involving both continuous and discrete controls, is extremely difficult to solve for complex systems. Numerical results have been given to show that both the dynamic programming and the decentralised hierarchical methods can give optimised solutions for the restricted systems considered. An assessment has been made of the possibilities of extension to more realistic cases, however, the complexities of the problem have indicated that there is no overall ideal method, each has its own individual areas of usefulness as summarised below:

(a) Dynamic Programming.

System equations can be linear or non-linear.

Controls can be discrete for low dimensional problems but may need to be continuous for higher state dimensionality.

State dimensionality only handled with great difficulty.

Performance indices can be non-linear and discontinuous and thus can accurately represent costs.

Constraints can be handled on both states and controls.

Decomposition of large scale problems difficult to achieve.

Computing requirements of extensive high speed memory with lengthy calculations and non-standard programming format.

Closed loop control can be obtained directly with backward dynamic programming or with re-calculation for forward dynamic programming.

(b) <u>Decentralised Hierarchical</u>

System equations must be linear.

Controls theoretically can be discrete but in practice must be continuous.

State dimensionality causes only slight problems.

Performance indices must be quadratic which may give less accurate representation of costs.

Constraints of upper and lower bounds can be handled on both states and controls.

Decomposition of large-scale problems in both time and space.

Computing requirements of moderate memory and solution time with standardised programming format.

Closed loop control can be obtained by re-calculation.

The presently studied optimisation techniques have, so far, only been applied to simple networks. Practical systems require methods capable of handling the following additional features:

(i) distributed multi-reservoir zones with head dependent inter-zonal flows,

(ii) borehole and booster pumping stations with head dependent flows and pump controls which can be discrete for parallel pumps or continuous for variable speed pumps,

(iii) valve flow dependent on heads and continuous valve controls,

(iv) performance indices allowing for costs of variable pumping efficiency and valve controls.

Further studies make it necessary to evaluate models of these more complex networks which are compatible with the above optimisation methods. This topic is continued in Chapters 5 and 6. The final decision on the best optimisation technique to adopt will depend on the successful evaluation of suitable system models and the extension of the optimisation formulation to meet the system operational requirements.

CHAPTER 5

NETWORK ANALYSIS AND SIMULATION

5.1 Introduction

In an advanced study of water distribution systems an essential pre-requisite in the availability of computer programs for evaluation of network responses under both static and dynamic conditions. Many programs have been developed which can give solutions for static values of network parameters[5,7,22,24,25,30,40,47,49,50,65,66,69,70,72,81,82,84,86,87,101,103,106]. Of these WATSIM[81] is also capable of performing extended period dynamic simulation for a variety of operating conditions. This particular program has been chosen for use because it goes some way towards meeting the requirements of the present study and also because of the possibility of conversion to cover additional requirements in simulation and coefficient evaluation. The formulation of the program is based upon the following concepts[81].

Water distribution networks consist of nodes connected in pairs by network elements such as pipes, pumps, valves, etc. Although the network of any city generally consists of several thousand nodes, the schematic representation can usually be reduced to only a few hundred of the most important or dominant nodes. Each element in the network can be specified by a relationship (usually non-linear) which expresses the flow through the element as a function of the head drop between the nodes at each end of the element. Most networks contain storage elements in the form of elevated reservoirs; these integrate the net inflow to give reservoir levels varying over time periods.

The network is considered to be solved (static solution) when all heads and flows are known at one instant in time, and a typical initial case is when either the head or the flow is known at each node in the network. A set of non-linear equations can be written in terms of known and unknown heads and flows which then have to be solved for the unknown values. The Newton-Raphson method is commonly used for solving the non-linear equations; this involves the iterative calculation of correction values to the unknowns, where the correction values are obtained by solution of a set of linearised simultaneous equations with coefficients contained in a Jacobian matrix of partial differentials.

It is possible to write the defining equations for water networks in terms of either mass balance at nodes or head balance around loops. The nodal equations are closely related to the network diagram and the incidence matrix is sparse since any given node is connected to only a few other nodes. This sparsity characteristic carries over to the Jacobian coefficient matrix of the linearised set of equations which contains as many equations as there are nodes. The nodal approach is also simpler to implement because the initial unknown flows may be specified arbitrarily and loops do not have to be considered. Consequently the nodal approach appears to be easier to use provided that the computer memory and solution times are not excessive as a result of the larger number of nodal equations than loop equations. Fortunately the linearised nodal equations of water distribution systems are amenable to efficient solution schemes employing the sparse matrix method of ordered triangular factorisation[97,98].

The network dynamic solution involves calculation of all heads and flows for each time increment over an extended period and can thus be adapted to allow dynamic simulation of a network under known operating conditions. For this purpose a solution to the reservoir dynamics is obtained using a predictor-corrector integration scheme and used to update the inputs to the static solution in each successive time interval. In the extended period simulation it is necessary to input forecasts of the node demands over the interval of interest. System demand forecasts may be made using time series analysis of historical data [16,36,91] (these are assumed to be available in the present study) and allocated to individual nodes using the concept of proportional loading.

Based on the topography of a region, the service area corresponding to a distribution system is divided into several pressure zones and, consequently, each node is further characterised by its pressure zone identification. Over every interval in the extended period simulation the total outflow at the demand nodes must be equal to the net supply for all the source nodes. This mass balance, when extended to each pressure zone, permits the simulation to be performed in each pressure zone independently of the others.

A useful additional feature of a network program is the ability to analyse responses to changes in operating conditions without necessarily re-solving the whole network[84]. The inclusion of a coefficient solution permits calculation of derivative coefficients related to all important network variables. In chapter 6 these coefficients are shown to be the ones required to evaluate simplified dynamic models of networks.

5.2 Network Models

This section describes the mathematical models usually employed for relevant network elements[81,84] and emphasises any modified formulations required for later developments. Alternative models and models for additional types of elements, may be found in Rao et al[81].

(a) Pipes

The head loss characteristics of a pipe between nodes i and j depends upon the resistance between the nodes and can be modelled using the Hazen-Williams coefficient as follows:

$$q_{ij} = \frac{6.27401}{10^4} C_{HW_{ij}} \cdot D_{ij}^{2.63} \left(\frac{h_j - h_i}{L_{ij}}\right)^{0.54} \tag{5.1}$$

where q_{ij} = flow from node j to node i (cfs)

$C_{HW_{ij}}$ = Hazen-Williams coefficient for pipe,

D_{ij} = diameter of pipe (ins)

L_{ij} = length of pipe (ft)

h_j = head at node j (ft)

h_i = head at node i (ft)

This is usually used in the following form to give a consistent sign for flow as:

$$q_{ij} = r_{ij}^{-0.54} \cdot (h_j - h_i) \cdot |h_j - h_i|^{-0.46} \tag{5.2}$$

where r_{ij} is the resistance between nodes i and j given by:

$$r_{ij} = 852063 \cdot L_{ij} \cdot C_{HW_{ij}}^{-1.85} \cdot D_{ij}^{-4.87} \tag{5.3}$$

This expression can be generalised to:

$$q_{ij} = f_{ij}(h_i, h_j, r_{ij}) \tag{5.4}$$

(b) Parabolic pumps

$$h_i - h_j = a\left(\frac{q_{ij}}{r_{ij}}\right)^2 + b\left(\frac{q_{ij}}{r_{ij}}\right) + c \qquad (5.5)$$

where a, b and c are empirically determined constants and r_{ij} is an independent pump control parameter defining the proportion of total output in use.

Thus:

$$q_{ij} = r_{ij}\left[\frac{-b \pm \{b^2 - 4a(c - |h_i - h_j|)\}^{0.5}}{2a}\right] \qquad (5.6)$$

taking positive root for constant, a, positive, and vice versa, and setting q_{ij} to zero for $|h_i - h_j| > c$ or for $\{b^2 - 4a(c - |h_i - h_j|)\} \leq 0$.

This expression also has the general form of equation (5.4).

(c) Pressure Reducing Valves

These may be modelled by assuming that between nodes i and j there is a valve with a setting equal to H_{PRV}.

If $h_j \geq H_{PRV} \geq h_i$ the valve reduces the head to H_{PRV} to give a head drop of $(H_{PRV} - h_i)$ and flow takes place from j to i given by:

$$q_{ij} = r_{ij}^{-0.54} |H_{PRV} - h_i|^{0.54} \qquad (5.7)$$

If $h_i > H_{PRV}$ the valve shuts off, no reverse flow takes place and hence $q_{ij} = 0$.

If $h_j < H_{PRV}$ and $h_i < H_{PRV}$ the valve acts as a pipe with a head drop of $(h_j - h_i)$.

(d) Non-return Valves

In a pipe fitted with a non-return valve, the loss in head due to the valve itself is usually small and may be either neglected or included in the pipe resistance, thus:

$$q_{ij} = f_{ij}\,(h_i, h_j, r_{ij}) \qquad (5.8)$$

for $h_j > h_i$

Reverse flow is not possible and q_{ij} is set to zero for $h_i \geqslant h_j$.

(e) Control Valves

These may be manually or automatically controlled and are currently modelled for both non-return and two-way valves by assuming that control varies the resistance of the equivalent pipe to give:

$$q_{ij} = r_{ij}^{-0.54}\,(h_j - h_i)\left|h_j - h_i\right|^{-0.46} \qquad (5.9)$$

and the generalised expression of equation (5.4).

Where r_{ij} is now the independent valve control parameter which can vary from $r_{ij_{MIN}}$ for valve fully open to ∞ for valve fully closed.

(f) Fixed Head Reservoirs

These may be boundary reservoirs or boreholes (feeding head dependent pumps, valves,etc.) with fixed or with known head variation, which are modelled as fixed head nodes at the specified levels.

(g) Variable Head Reservoirs

These are treated as fixed head nodes in each static solution (section 5.3) but for dynamic solutions (section 5.4) the heads are updated using the following head-flow relationship for each reservoir:

$$q_i \Delta t = a h_i^3 + b\, h_i^2 + c\, h_i + d \qquad (5.10)$$

where a,b,c & d are empirical constants, determined from the geometry of the reservoir, h_i is reservoir head, q_i is the flow out of the reservoir and Δt is the time period.

(h) Demand Models

The consumption of water at each node in the schematic network representation is dependent upon the types of individual load serviced by the nodes (e.g. industrial, residential,etc). In addition to the spatial variation of the load patterns, each of the load types has its own temporal variation over a 24 hour period. This gives the following model:

$$c_i\,(t) = \sum_{j=1}^{J} \ell_{ij}\; s_j(t) \qquad 5.11$$

where

$c_i(t)$ = total consumption at node i for time t,

ℓ_{ij} = average demand at node i for load type j,

$s_j(t)$ = characteristic curve for load type j at time t, and

J = total number of load types.

5.3 Static Solution

For this the pump and valve controls will be fixed and either the heads or the flows at all the nodes will be known. The static solution then consists of solving for the unknown heads or flows at all the nodes.

The nodal equations may be written:

$$\sum_{\substack{j=1 \\ j \neq i}}^{H+Q} q_{ij} + q_i + c_i = g_i \tag{5.12}$$

for $(i = 1, \ldots. H+Q)$

where

q_{ij} = flow from node j to node i given by $f_{ij}(h_i, h_j, r_{ij})$ in general. The summation is taken to mean only if nodes i and j are connected by a network element.

q_i = reservoir flow entering node i,

c_i = consumption flow entering node i,

g_i = residual flow leaving node i,

H = total number of fixed head nodes,

Q = total number of fixed flow nodes.

A solution is obtained when all the equations are in balance with $g_i=0$.

For unknown variables of $\underline{q}$ $(q_1, \ldots. q_H)$ and $\underline{h}$ $(h_{H+1}, \ldots. h_{H+Q})$ the Newton-Raphson method can be used to give successively improving correction values $\Delta\underline{q}(\Delta q_1, \ldots. \Delta q_H)$ and $\Delta\underline{h}$ $(\Delta h_{H+1}, \ldots. \Delta h_{H+Q})$ to the unknowns at each iteration s as:

$$\begin{bmatrix} q_1^{s+1} \\ \vdots \\ q_H^{s+1} \\ \hline h_{H+1}^{s+1} \\ \vdots \\ h_{H+Q}^{s+1} \end{bmatrix} = \begin{bmatrix} q_1^{s} \\ \vdots \\ q_H^{s} \\ \hline h_{H+1}^{s} \\ \vdots \\ h_{H+Q}^{s} \end{bmatrix} + \begin{bmatrix} \Delta q_1^{s} \\ \vdots \\ \Delta q_H^{s} \\ \hline \Delta h_{H+1}^{s} \\ \vdots \\ \Delta h_{H+Q}^{s} \end{bmatrix} \tag{5.13}$$

Where the correction values can be calculated from:

$$\left[\begin{array}{ccc|ccc} \frac{\partial g_1}{\partial q_1} & \cdots & \frac{\partial g_1}{\partial q_H} & \frac{\partial g_1}{\partial h_{H+1}} & \cdots & \frac{\partial g_1}{\partial h_{H+Q}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial g_{H+Q}}{\partial q_1} & \cdots & \frac{\partial g_{H+Q}}{\partial q_H} & \frac{\partial g_{H+Q}}{\partial h_{H+1}} & \cdots & \frac{\partial g_{H+Q}}{\partial h_{H+Q}} \end{array}\right] \cdot \left[\begin{array}{c} \Delta q_1 \\ \vdots \\ \Delta q_H \\ \hline \Delta h_{H+1} \\ \vdots \\ \Delta h_{H+Q} \end{array}\right] = - \left[\begin{array}{c} g_1 \\ \vdots \\ g_H \\ \hline g_{H+1} \\ \vdots \\ g_{H+Q} \end{array}\right] \qquad (5.14)$$

with the left hand side matrix, $\left[\begin{array}{c|c} \frac{\partial \underline{g}}{\partial \underline{q}} & \frac{\partial \underline{g}}{\partial \underline{h}} \end{array}\right]$, defined as the Jacobian, $\underline{J}$.

By substituting for $\frac{\partial \underline{g}}{\partial \underline{q}}$ from appendix 1 it will be noted that the Jacobian now has the form:

$$\left[\begin{array}{cccc|ccc} 1 & 0 & \cdots & 0 & \frac{\partial g_1}{\partial h_{H+1}} & \cdots & \frac{\partial g_1}{\partial h_{H+Q}} \\ 0 & \ddots & & \vdots & \vdots & & \vdots \\ \vdots & & \ddots & 0 & \frac{\partial g_H}{\partial h_{H+1}} & \cdots & \frac{\partial g_H}{\partial h_{H+Q}} \\ 0 & \cdots & 0 & 1 & & & \\ \hline 0 & \cdots & \cdots & 0 & \frac{\partial g_{H+1}}{\partial h_{H+1}} & \cdots & \frac{\partial g_{H+1}}{\partial h_{H+Q}} \\ \vdots & & & \vdots & \vdots & & \vdots \\ 0 & \cdots & \cdots & 0 & \frac{\partial g_{H+Q}}{\partial h_{H+1}} & \cdots & \frac{\partial g_{H+Q}}{\partial h_{H+Q}} \end{array}\right]$$

which allows a solution for the head correction using only the last Q equations and a reduced Jacobian. The unknown flows can be obtained from equation (5.12) once all heads are calculated to the required accuracy.

Since the set of simultaneous equations are sparse and have to be solved many times ordered triangular factorisation and sparse matrix techniques have been found to be very efficient for this purpose[81,97,98].

In the numerical procedure a check is performed for convergence after each iteration. The criterion for convergence is the amount by which any of the g_i may be different from zero which represents the maximum permissible unbalanced flow at any node. The magnitude of the error criterion is given as:

$$\max_i \; | \, g_i^s \, | \; \leqslant \; \varepsilon \qquad (5.15)$$

where ε is a pre-specified tolerance factor.

5.4 Dynamic Solution

The extended period simulation[81] consists of a sequence of static solutions which are performed at pre-specified intervals. The dynamics of reservoir operation are now included and the schedule of pump and valve settings and load values are used to update the inputs to the static solutions in every time interval. Each variable head reservoir is modelled by a differential equation for the reservoir head as a function of time. These differential equations are integrated in time using a predictor-corrector scheme in the form of a modified Euler procedure.

The dynamic solution consists of cycling through steps (a) to (f) for each of the time stages, k, until the period of simulation is complete starting with the following initial conditions:

(i) reservoir heads and volumes at time stage k; $h_r(k)$&$v_r(k)$, respectively, where $r \varepsilon R$ and R is the set of reservoirs

(ii) boundary heads $h_s(k)$ where $s \varepsilon S$ and S is the set of boundary inflows

(iii) reservoir differential equations in the form:

$$f_r(h,v) = 0 \tag{5.16}$$

(iv) pump and valve control settings at k,

(v) demands at the network nodes $c_j(k)$ where $j \varepsilon J$ and J is the set of load nodes.

(a) A static solution is performed at stage k to determine heads and flows with given pump and valve settings and load demands. This will give reservoir flows, $q_r(k)$, and boundary flows, $q_s(k)$.

(b) Initial calculations are made with flows $q_r(k)$, $q_s(k)$ and $c_j(k)$ assumed to be constant in the interval (k, k+1). The individual reservoir depletion is computed as $q_r(k).\Delta t$ and the total reservoir depletion as $\sum_{r \varepsilon R} q_r(k).\Delta t$, where Δt is the time interval between k and (k+1). The total boundary inflow is computed as $\sum_{s \varepsilon S} q_s(k)\Delta t$ and the total demand outflow as $\sum_{j \varepsilon J} c_j(k)\Delta t$.

(c) Prediction calculations are made using results from (b) where subscript p denotes predictions. The predicted volumetric balance error, for the network, over the interval (k,k+1) is:

$$E_p = \sum_{r \varepsilon R} q_r(k)\Delta t + \sum_{s \varepsilon S} q_s(k)\Delta t + \sum_{j \varepsilon J} c_j(k)\Delta t \tag{5.17}$$

This error is allocated to the rth reservoir in proportion to the flow as:

$$e_{rp} = \frac{q_r(k)}{\sum_{r \varepsilon R} q_r(k)} \cdot E_p \tag{5.18}$$

With this information a predicted volume is calculated for each of the reservoirs at stage (k+1) from:

$$v_{rp}(k+1) = v_r(k) + q_r(k)\Delta t + e_{rp} \tag{5.19}$$

and used to compute the predicted reservoir head h_{rp} (k+1) by solving the reservoir equation $f_r(h,v) = 0$.

(d) A static solution is again performed to determine $q_r(k+1)$ and $q_s(k+1)$ for given values of $h_{rp}(k+1)$, $h_s(k+1)$, $v_{rp}(k+1)$ and $c_j(k+1)$.

(e) Correction calculations are made using results from (d) where subscript c denotes correction.

The corrected volume balance error over the interval (k, k+1) is:

$$E_c = \sum_{r\varepsilon R} \{q_r(k)+q_r(k+1)\}\frac{\Delta t}{2} + \sum_{s\varepsilon S} \{q_s(k)+q_s(k+1)\}\frac{\Delta t}{2} + \sum_{j\varepsilon J} c_j(k)\Delta t. \qquad (5.20)$$

This error is re-allocated to the rth reservoir in proportion to the flow as:

$$e_{rc} = \frac{q_r(k) + q_r(k+1)}{\sum_{r\varepsilon R} q_r(k) + \sum_{r\varepsilon R} q_r(k+1)} \cdot E_c \qquad (5.21)$$

With this information a corrected volume is calculated for each of the reservoirs at stage (k+1) from:

$$v_{rc}(k+1) = v_r(k) + \{q_r(k)+q_r(k+1)\}\frac{\Delta t}{2} + e_{rc} \qquad (5.22)$$

and used to compute the corrected reservoir head $h_{rc}(k+1)$ by solving the reservoir equation $f_r(h,v) = 0$.

(f) A check is made for convergence dependent on the reservoir error criterion, λ_r, and the difference between the predicted and corrected heads.

If $|h_{rp}(k+1)-h_{rc}(k+1)|>\lambda_r$, $h_{rp}(k+1)$, $v_{rp}(k+1)$ are set to $h_{rc}(k+1)$, $v_{rc}(k+1)$ respectively and another iteration performed from step (d).

If $|h_{rp}(k+1)-h_{rc}(k+1)|\leq\lambda_r$, $h_r(k+1)$, $v_r(k+1)$ are set to $h_{rc}(k+1)$, $v_{rc}(k+1)$ respectively, k is incremented by 1 and a solution repeated from step (a).

The simulation [81] allows for control switching during stage intervals but this is not relevant here where the pump and valve controls are assumed to be changed only at each stage k. In addition, other load functions are available where the simulation does not depend upon using the average value of load flow throughout each interval.

5.5 Coefficient Solution

The study of water distribution systems includes investigation of the effects of changes in heads and flows due to changes in operating conditions. In general this necessitates repeat static solutions of the network equations under the new conditions. However it is possible to calculate sensitivity coefficients, defining the changes in heads and flows for small changes in operating conditions, using results from an existing network solution[84,86].

This section describes the calculation of sensitivity coefficients using the simulation program (WATSIM) and incorporating the subroutine (COEF) developed by the author. The coefficients are those for selected dependent variables of heads and flows, due to changes in operating conditions, within the network, caused by variations in selected independent variables of reservoir heads, pump and valve controls, and load values.

Since the coefficients, for all possible independent variables, may not be of interest it is convenient to designate selected independent variables in consecutive order, for each type, by defining:

(i) A vector of reservoir heads $\underline{x}$ $(x_1, \ldots x_N)$ and designating these to N fixed head nodes contained in $\underline{h}$ $(h_1, \ldots h_H)$.

(ii) A vector of pump controls $\underline{u}$ $(u_1, \ldots u_M)$ and designating these to M pump controls contained in network elements r_{ij}.

(iii) A vector of demand disturbances $\underline{y}$ (y_1, ... y_L) and designating these to L consumption nodes contained in $\underline{c}$ (c_1, ... c_{Q+H}).

(iv) A vector of valve controls $\underline{v}$ (v_1, ... v_R) and designating these to R valve controls contained in network elements r_{ij}.

The generalised network equations can now be written as:

$$\underline{f}\ (\underline{x}, \underline{h}, \underline{u}, \underline{v}) + \underline{q} + \underline{y} = \underline{g} \tag{5.23}$$

where $\underline{f}$ = vector (f_1, ... f_{H+Q}) and $f_i = \sum_{\substack{j=1 \\ j \neq i}}^{Q+H} f_{ij}(h_i, h_j, r_{ij})$

$\underline{h}$ = vector of dependent heads (h_{H+1}, ... h_{H+Q})

$\underline{q}$ = vector of dependent flows (q_1, ... q_H)

$\underline{g}$ = vector (g_1, ... g_{H+Q})

Expressions for the coefficients can be obtained by differentiating equation (5.23) under balanced conditions, using each of the independent variable vectors in turn. Typically, for $\underline{x}$, this will give:

$$\frac{d\underline{g}}{d\underline{x}} = \frac{\partial \underline{g}}{\partial \underline{x}} + \left[\frac{\partial \underline{g}}{\partial \underline{q}} \;\middle|\; \frac{\partial \underline{g}}{\partial \underline{h}} \right] \cdot \begin{bmatrix} \dfrac{\partial \underline{q}}{\partial \underline{x}} \\ \hline \dfrac{\partial \underline{h}}{\partial \underline{x}} \end{bmatrix} = \underline{0} \tag{5.24}$$

which can be re-arranged as:

$$\left[\frac{\partial \underline{g}}{\partial \underline{q}} \;\middle|\; \frac{\partial \underline{g}}{\partial \underline{h}} \right] \cdot \begin{bmatrix} \dfrac{\partial \underline{q}}{\partial \underline{x}} \\ \hline \dfrac{\partial \underline{h}}{\partial \underline{x}} \end{bmatrix} = - \frac{\partial \underline{g}}{\partial \underline{x}} \tag{5.25}$$

Expanding in order to investigate the implications of solving this for the unknown coefficients:

$$\left[\begin{array}{ccc|ccc} \frac{\partial g_1}{\partial q_1} & \cdots & \frac{\partial g_1}{\partial q_H} & \frac{\partial g_1}{\partial h_{H+1}} & \cdots & \frac{\partial g_1}{\partial h_{H+Q}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial g_{H+Q}}{\partial q_1} & \cdots & \frac{\partial g_{H+Q}}{\partial q_H} & \frac{\partial g_{H+Q}}{\partial h_{H+1}} & \cdots & \frac{\partial g_{H+Q}}{\partial h_{H+Q}} \end{array}\right] \cdot \left[\begin{array}{ccc} \frac{\partial q_1}{\partial x_1} & \cdots & \frac{\partial q_1}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial q_H}{\partial x_1} & & \frac{\partial q_H}{\partial x_N} \\ \hline \frac{\partial h_{H+1}}{\partial x_1} & & \frac{\partial h_{H+1}}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial h_{H+Q}}{\partial x_1} & \cdots & \frac{\partial h_{H+Q}}{\partial x_N} \end{array}\right] = -\left[\begin{array}{ccc} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial g_{H+Q}}{\partial x_1} & \cdots & \frac{\partial g_{H+Q}}{\partial x_N} \end{array}\right] \quad (5.26)$$

It will be noted that:

(i) The unknown coefficients in the second matrix can be obtained by solving the above sets of simultaneous equations

(ii) The RHS matrix can be formed directly and evaluated for the static solution values of $\underline{q}$ and $\underline{h}$ (see appendix 1 for derivatives)

(iii) The LHS matrix is the full Jacobian which can be formed and evaluated during the static solution

(iv) The static solution is already mechanised to solve such equations on a column by column basis of unknowns

As a consequence of this the static solution can be extended to evaluate the coefficient column vectors for each element of $\underline{x}$ in turn and then form the matrix of coefficients:

$$\left[\begin{array}{c} \underline{A}q \\ \hline \underline{A}h \end{array}\right] = \left[\begin{array}{c} \frac{\partial \underline{q}}{\partial \underline{x}} \\ \hline \frac{\partial \underline{h}}{\partial \underline{x}} \end{array}\right] \quad (5.27)$$

By differentiating equation (5.23) with respect to $\underline{u}$, $\underline{y}$ and $\underline{v}$ and using the same procedure the matrices:

$$\begin{bmatrix} \underline{B}q \\ --- \\ \underline{B}h \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \underline{q}}{\partial \underline{u}} \\ --- \\ \dfrac{\partial \underline{h}}{\partial \underline{u}} \end{bmatrix} \tag{5.28}$$

$$\begin{bmatrix} \underline{C}q \\ --- \\ \underline{C}h \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \underline{q}}{\partial \underline{y}} \\ --- \\ \dfrac{\partial \underline{h}}{\partial \underline{y}} \end{bmatrix} \tag{5.29}$$

$$\begin{bmatrix} \underline{D}q \\ --- \\ \underline{D}h \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \underline{q}}{\partial \underline{v}} \\ --- \\ \dfrac{\partial \underline{h}}{\partial \underline{v}} \end{bmatrix} \tag{5.30}$$

can be evaluated.

Whilst the above procedure evaluates elements of the coefficient matrices for all the dependent variable elements of $\underline{q}$ and $\underline{h}$, not all of these may be required. To facilitate selection of coefficients corresponding to desired variables it is convenient to designate these in consecutive order, for each type, by defining:

(i) a vector of reservoir flows $\underline{q}'$ (q_1' .... q_N') and designate these to N variable flow nodes contained in $\underline{q}$ (q_1 .... q_H).

(ii) a vector of variable heads $\underline{h}'$ (h_1' .... h_P') and designate these to P variable head nodes contained in $\underline{h}$ (h_{H+1} .... h_{H+Q}).

Then finally form reduced matrices of coefficients by selection of:

(a) NxN matrix $\underline{Aq}'\left(\frac{\partial \underline{q}'}{\partial \underline{x}}\right)$ from HxN matrix $\underline{Aq}\left(\frac{\partial \underline{q}}{\partial \underline{x}}\right)$

(b) NxM matrix $\underline{Bq}'\left(\frac{\partial \underline{q}'}{\partial \underline{u}}\right)$ from HxM matrix $\underline{Bq}\left(\frac{\partial \underline{q}}{\underline{xu}}\right)$

(c) NxL matrix $\underline{Cq}'\left(\frac{\partial \underline{q}'}{\partial \underline{y}}\right)$ from HxL matrix $\underline{Cq}\left(\frac{\partial \underline{q}}{\partial \underline{y}}\right)$

(d) NxR matrix $\underline{Dq}'\left(\frac{\partial \underline{q}'}{\partial \underline{v}}\right)$ from HxR matrix $\underline{Dq}\left(\frac{\partial \underline{q}}{\partial \underline{v}}\right)$

(e) PxN matrix $\underline{Ah}'\left(\frac{\partial \underline{h}'}{\partial \underline{x}}\right)$ from QxN matrix $\underline{Ah}\left(\frac{\partial \underline{h}}{\partial \underline{x}}\right)$

(f) PxM matrix $\underline{Bh}'\left(\frac{\partial \underline{h}'}{\partial \underline{u}}\right)$ from QxM matrix $\underline{Bh}\left(\frac{\partial \underline{h}}{\partial \underline{u}}\right)$

(g) PxL matrix $\underline{Ch}'\left(\frac{\partial \underline{h}'}{\partial \underline{y}}\right)$ from QxL matrix $\underline{Ch}\left(\frac{\partial \underline{h}}{\partial \underline{y}}\right)$

(h) PxR matrix $\underline{Dh}'\left(\frac{\partial \underline{h}'}{\partial \underline{v}}\right)$ from QxR matrix $\underline{Dh}\left(\frac{\partial \underline{h}}{\partial \underline{v}}\right)$

5.6 Computer Program for Network Analysis and Simulation

A computer program (WATSIM)[81] has already been developed for the steady state and extended period simulation of water distribution systems. This is based upon the theory for static and dynamic solutions reviewed in sections 5.3 and 5.4.

Extensive modifications have been made to the program to make it more suitable for the present study in optimisation and modelling of water systems. The simulation changes consist of making reservoir nodes eligible as demand nodes, and additions of pump and valve control parameters together with facilities for input of pre-specified values for these over the simulation period. In addition sensitivity analysis has been added according to theory developed in section 5.5. This enables coefficients to be calculated for specified parameters for each time increment of the extended period simulation.

5.6.1 Program Input Parameters

The following is a brief list of possible input data but the actual input is dependent on the type of solution required.

Program control	- units, static/dynamic/coefficient solutions, output requirements, load and solution timing, system limits
Nodes	- node identification, fixed flow values, fixed head values, coefficient identification for reservoir nodes, demand nodes, and variable head nodes
Lines	- element type, node to node identification, element characteristics, coefficient identification for pump and valve elements
Switches	- node or time control parameters, node to node identification of switched elements
Reservoirs	- node identification, reservoir characteristics
Load and Control curves	- curve type identification for load curves, head shape curves and pump/valve control curves, curve number identification, curve levels for each simulation time period
Load and Control allocation	- curve number, node identification for load and head nodes, node to node identification for pump and valve elements
Detail Output	- output format, node identification, element node to node identification, snapshot time listing

5.6.2 Program Description

Fig 5.1 shows a simplified flow chart for the network analysis and simulation program (WATSIM)[81] incorporating the modifications.

The branching logic is driven by T, DT and TNEXT which are simulated time values in hours. T represents the interval currently in use and TNEXT the time at the end of the current standard integration interval, DT. T is initialised at the input value TIN, and the simulation is terminated when it reaches TOUT.

Pump and valve control curves modify the appropriate element values and, for a discontinuous change at T, a static solution is requested, to generate new heads and flows, by setting DT = 0.

The reservoir flow calculations for prediction use the head at time T, except for those boundary reservoirs modelled by head shape curves, which extrapolate on a straight line segment for the current load curve interval.

Load curves are integrated over the load curve interval and, for each load curve, the individual loads are allocated to their respective node by the load allocation vectors. For discontinuous load flow changes at T a static solution is again required and DT will be set to 0 unless previously set.

At this point a check is made for instantaneous solutions (DT = 0) which do not require integration of reservoir volume. If, however, integration is required the head for each reservoir is predicted by calculation of current reservoir volume from current head and modifying it by current outflow projected over time DT suitably adjusted by a factor allocating predicted volumetric balance error proportionally to outflow. This predicted volume is then used in a single variable Newton's method to calculate reservoir head at T + DT from the reservoir head-volume relationship.

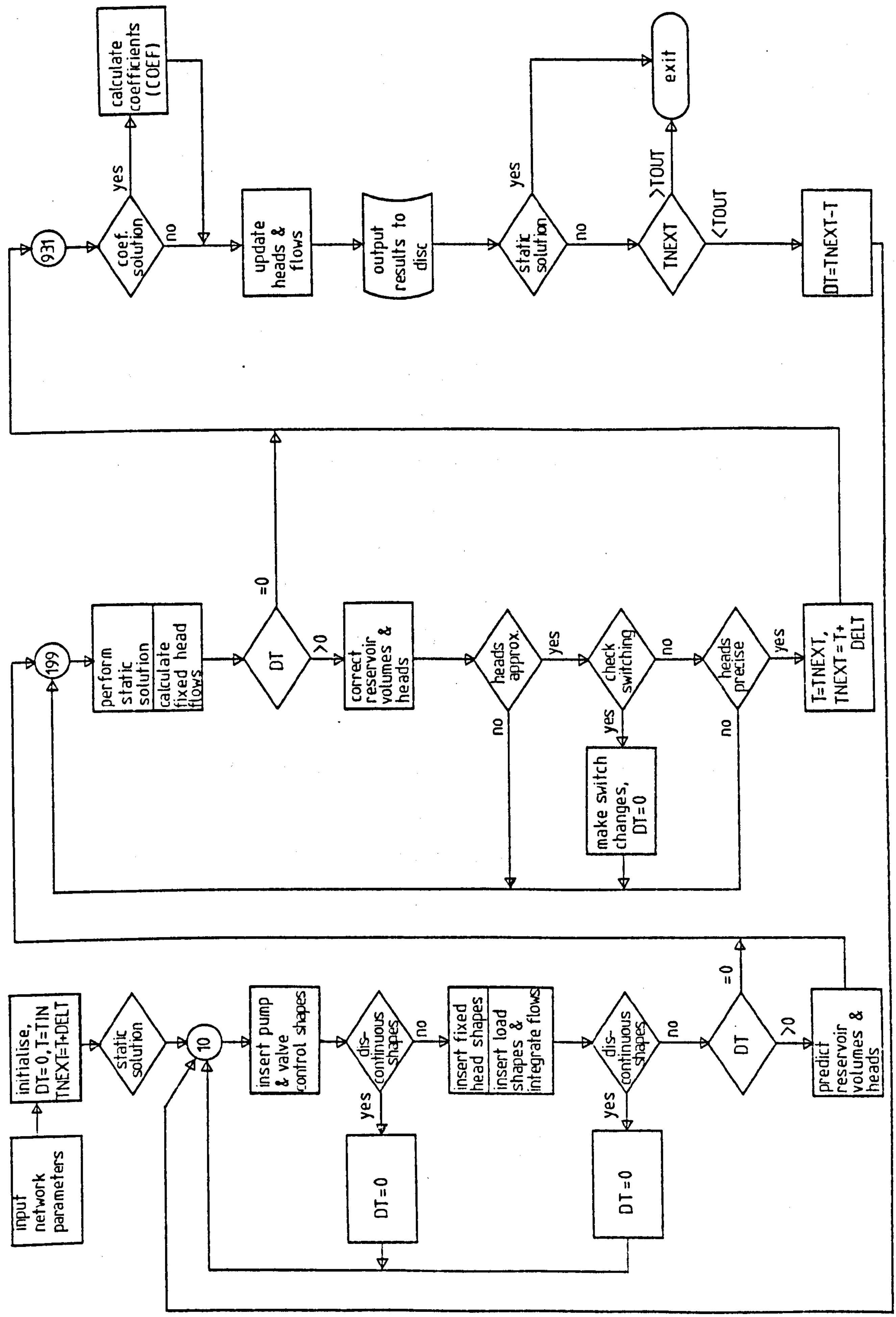


Fig 5·1 Simplified flow chart for network analysis and simulation program

With the reservoir heads established, either from a previous solution or from prediction, the heads at consumption nodes at T + DT are then solved for by a modified Newton-Raphson method which uses the optimally ordered triangular factorisation of the Jacobian in the solution of the linear equations.

After the heads at consumption nodes have been obtained, the flow out of reservoirs at T + DT is re-calculated. A corrected volume based upon this, the outflow at T, and system volumetric balance is calculated. This again produces a head for each reservoir at T + DT by Newton's method. Dependent on the greatest head correction the network may be re-solved for consumption node heads incorporating correction of the reservoir volumes. Switching checks are performed in order to enable the simulation to reflect the effects of the switching actions. If any switch changes occur DT is again set to 0 to generate a static solution. If the reservoir heads are sufficiently accurate the simulation times are updated. Otherwise the reservoir heads are corrected by performing yet another network solution.

Upon completion of the calculations at each time T a coefficient solution can be generated. This uses the subroutine (COEF) and other routines in common with the static solution to calculate derivatives of reservoir flows and load node heads.

At the end of each step the reservoir flows are updated, T is reset and the heads and flows are saved on peripheral storage until required for output at the end of the simulation.

5.6.3 Program Applications

Whilst the possible applications of the program are very wide the following uses are of direct interest in the present study.

(a) Distribution system simulation

In general, distribution systems are not available for experimental type research work and in addition the level of monitoring would probably be inadequate. A convenient substitute is the use of a computer simulation program, matched to the actual network, which will provide realistic responses to any stimuli.

In this case the program input will consist of the network node, line and reservoir parameters with load consumption based on predicted or measured values. Application of pump and valve controls will result in dynamic variation of element and reservoir heads and flows over the simulation period. The pump and valve controls can be from a sequence of known manual operations or in accordance with predicted optimised values.

(b) Evaluation of missing or inadequate data

In order to accurately simulate a water distribution network it is essential to have a detailed knowledge of the individual load values and nodes of application. This is usually impossible to achieve and the next best requirement is for information on the lumped consumption values at representative nodes. Typical monitoring in distribution networks provides hourly records of reservoir levels and control variables; with this data the program can be used to evaluate equivalent lumped consumption flows at all reservoir nodes.

In this case the program input will consist of the network node and line parameters but reservoirs will be treated as fixed head nodes with allowed head variations. Application of known pump and valve controls and reservoir head variations will result in dynamic variation of reservoir flows over the simulation period. The actual reservoir flows can be calculated from the reservoir geometry and the head change, and then subtracted from the simulated reservoir flows to give the equivalent consumption flows.

(c) Evaluation of linear dynamic models

Whilst the program is capable of giving an accurate simulation of a network the computations are quite lengthy and the method is not suitable for calculating optimised controls. For this purpose simplified models must be derived which still adequately represent the system dynamic operation. It will be shown, in chapter 6, that the derivatives obtained from the coefficient solution can be used to define a dynamic model linearised around average operating conditions.

If both dynamic and coefficient solutions are selected, with given input data, time varying coefficients will be evaluated for each time period in the simulation.

Alternatively if both static and coefficient solutions are selected and the input data are average values, over the whole of the simulation period, then average coefficients will be evaluated.

(d) Evaluation of coefficients by perturbation

Evaluation of coefficients, for a linear model, by use of the coefficient solution gives values as point derivatives. Average values for larger changes in variables can be obtained by use of the simulation program, without permitting any updating of the reservoir heads, so that the dynamic solution reverts to a sequence of static solutions.

The input data consists of average values for the first static solution and for each successive solution one of each of the independent control parameters is perturbed and then returned to its original value. The required coefficients can be evaluated by calculating the change in reservoir flow and variable node heads for the given change in pump or valve control, reservoir head, or consumption flow.

5.7 Conclusions

An efficient, general purpose, computer program is required for investigation into methods for advanced control of water distribution systems. The features essential for the present study are an ability to:

(i) model all common types of network elements, control variables and head and flow patterns

(ii) analyse head and flow distribution as a result of either steady state or dynamic inputs

(iii) perform extended period simulations which replicate water system operation

(iv) determine the sensitivity of the heads and flows to changes in operating conditions

whilst other desirable features include a means to:

(v) automatically correct network static parameters to force agreement between network and model for steady state conditions

(vi) automatically correct network dynamic parameters to force agreement between network and extended period simulations

(vii) calculate operating costs based on continuously variable network conditions.

Many programs are available which meet some of the requirements, but, as far as is known, there are none which meet all of them. A program which covers items (i), (ii) and (iii) above and hence goes some way towards meeting the essential requirements has been developed by Rao et al[81] for the US Department of Interior, Office of Water Resources Research. This was written in Fortran IV and has been adapted, by the author, to suit the two presently available computing systems (i.e. IBM 370/135 and ICL 1906S) by means of normal programming modifications.

Further adaptations have also been made to make the program more suitable for the current investigation. In particular the simulation facilities have been enhanced to allow full control of pumps and valves over the complete simulation period. The modifications give both continuous and discrete control by means of parameters independent of network conditions and the program is now capable of simulating water systems for evaluation of either manual or trial optimisation strategies. This feature will allow the simulation program to be used in conjunction with interactive optimised control algorithms which will form a basis for ultimate on-line computer control of water systems. In chapter 7 the program is used to validate proposed schemes for overall system operation.

A further requirement for on-line control is the evaluation of linear dynamic models which are required for efficient calculation of optimal controls. To this end extensive modifications have been made to the program to cover item (iv) and provide facilities for evaluating sensitivity coefficients which can be used to define simplified simulation models. This feature is used in chapter 6 to yield numerical values for the derived model coefficients.

The program can be further extended to cover item (v) by using the same principle of coefficient evaluation in an automatic scheme for alteration of network element resistance values to ensure a match between an actual network and static solution results[24,25,84]. It is also suggested that a similar method could be developed giving dynamic matching as required for item (vi). Pump operating costs for item (vii) can be evaluated from known pump head increase and flow together with pump efficiency characteristics[15,16]. These latter items are suggested as topics for further research in chapter 8.

CHAPTER 6

SIMPLIFIED DYNAMIC MODELS

6.1 INTRODUCTION

Most previous work on optimised control of water distribution systems has been concerned with single reservoir systems, or systems which are related to these by virtue of known flow relationships. This covers multi-zone networks having individual reservoirs or by selection of dominant reservoirs[2,31,38,44,56,57,63,92,93,99].

A typical instance is the single reservoir system used under the further restrictive assumption that input pumping flows are independent of heads and can be controlled at given values. Under these conditions the concept of total volumetric balance applies and the linear one dimensional equations describing the system dynamic operation become:

$$x(k+1) = x(k) + \sum_{m=1}^{M} u_m(k) + \sum_{\ell=1}^{L} y_\ell(k) \qquad (6.1)$$

where x is the stored quantity of water, $u_m (m \in M)$ is the inflow from each of the pumping stations (1,...M), $y_\ell (\ell \in L)$ is the consumption from each demand node (1,...L) and k is the stage of the operation.

This equation applies even for non-linear head-flow relationships within the network but excludes consideration of head dependent pumps and valves. The same principle can be extended to cover the case of multiple zones consisting of one reservoir for each zone, under the same assumptions but allowing for controllable inter-zonal flow, to give the following linear multi-variable formulation:

$$\underline{x}(k+1) = \underline{A}.\underline{x}(k) + \underline{B}.\underline{u}(k) + \underline{C}.\underline{y}(k) \qquad (6.2)$$

where $\underline{x}$ is the N vector of reservoir stored quantities, $\underline{u}$ is the M vector of pumping station and inter-zonal flows, and $\underline{y}$ is the L vector of consumption flows. In this case $\underline{A}$ will be a unit diagonal matrix and $\underline{B}$ and $\underline{C}$ will have unity or zero elements dependent upon system configuration.

More general systems consist of several reservoirs interconnected by non-linear head dependent elements such as pipes, pumps and control valves. Network inflows are typically from multiple pumping stations where the number, or speed, of the pumps on-line can be controlled and the actual pump flows are also dependent on the network head values. Consumption flows are distributed throughout the network and are assumed to be independent of heads and available as known functions of time. Whilst total volumetric balance still applies this will now determine the total volume of water in all reservoirs and will not indicate the relative volumes in individual reservoirs. A simple linear formulation as for equation (6.2) is therefore not feasible. Individual reservoir levels, defining dynamic operation, can only be determined accurately and under any conditions by evaluating all heads and flows. This requires iterative solution of the complete set of non-linear network equations to determine the inter-related values. Such a solution procedure is lengthy, time consuming, and cumbersome, and is totally unsuitable for practical control applications. This conclusion is strengthened when considering optimal control with the requirements of repeated dynamic solutions for each trial change of control variable.

The inefficiency of the solution procedure derives from the accurate calculation of many intermediate values when only the influence of control and disturbance parameters upon the reservoir levels is of interest. By sacrificing extreme accuracy it is possible to derive equivalent reduced sets of equations which relate only the important variables. Solution of such reduced sets can be very fast and this approach paves the way for on-line optimal control of water distribution systems.

This Chapter describes two independent methods for derivation of equations defining simplified dynamic models together with subsequent evaluation of equation coefficients. Whilst the two methods have been developed independently they are somewhat complementary and in certain instances the coefficients are closely related. Taken together the methods present a balanced view of current modelling techniques, each with a specialised area of application relating to previously described optimisation methods. The first of these methods has been developed by De Moyer et al[16], and results in non-linear equations with discrete control variables which can be used for optimisation by means of dynamic programming techniques. The second method has been developed by the author[13,94] and results in linear equations which can be used with any of the optimisation techniques described in Chapter 4.

6.2 NON-LINEAR DYNAMIC MODEL.

It has been noted that alternative models, allowing fast network simulations, are required for optimal control of water distribution systems. One result of this search has been the development of a macroscopic model[15,16,17,18,19,20,71] which calculates only major heads and flows, as opposed to a microscopic model which calculates all heads and flows. The evolution of this macroscopic model, with a vastly reduced number of variables, allows a rapid system balance to be performed. Given pumps on-line at each station, reservoir levels, and total demand, the model enables calculation of pumping station heads and flows, reservoir flows, and selected internal network heads. The static model now consists of a reduced set of non-linear equations derived from conventional network equations and additional empirical relationships. Extension to a dynamic model is catered for by inclusion of the time varying integral relationship between reservoir flow and level. In order to provide optimised operation a measure of the operating cost is required, with the electrical energy used in pumping being calculated using values of heads and flows already determined for the simulation.

6.2.1 Macroscopic Model.

(a) Pumping Station Relationships.

Each pumping station typically consists of sets of pumps which can be put on-line in various preferred combinations. The head increase across the station can be related to the individual pump equation (3.2) by defining general coefficients dependent upon pump combination in use. Taking the power index to be 1.85, for consistency with other equations, gives:

$$\Delta h_m = a_{mj1} + a_{mj2}\, u_m^{1.85} \qquad (6.3)$$
$$j \in J(m)$$

where Δh_m = head increase across station m for j^{th} operating condition.

m = station index number chosen from (1,2,...M).

M = total number of stations.

u_m = station flow for j^{th} operating condition.

a_{mji} = array of station constants.

$J(m)$ = set of operating combinations.

The station suction head, s_m, for all pump combinations can be expressed as:

$$s_m = b_{m1} + b_{m2}\, y^{1.85} + b_{m3}\, u_m^{1.85} \qquad (6.4)$$

where y = total demand

b_{mi} = station supply constants.

Adding these values will give the station discharge head, h_m, as:

$$h_m = s_m + \Delta h_m \qquad (6.5)$$

The electrical energy used in pumping water can be obtained by integrating equation (3.11) to give:

$$e_m = C \int_{t_o}^{t_f} \frac{u_m \cdot \Delta h_m}{\eta_{mj}} \, dt \qquad (6.6)$$

where e_m = electrical energy for station m over simulation period t_o to t_f.

C = constant relating electrical energy to water energy.

η_{mj} = station efficiency for pump combination j.

By use of results from section 3.3 the pumping station efficiency can be expressed as:

$$\eta_{mj} = c_{mj1} + c_{mj2} \cdot u_m + c_{mj3} \cdot u_m^2 \qquad (6.7)$$

where c_{mji} = station efficiency constants for each pump combination.

(b) Network Relationships.

Detailed equations relating individual pipes and nodes are not required provided that the overall head drops from pumping stations to reservoirs can be found. An empirical relationship giving head drop, Δh_{mn}, from the m^{th} station to the n^{th} reservoir has been shown to be:

$$\Delta h_{mn} = d_{m1} + d_{m2}\, y^{1.85} + \sum_{i=1}^{M} d_{m(i+2)}\, u_i^{1.85} \qquad (6.8)$$

where n = reservoir index number chosen from (1,2,...N)

N = total number of reservoirs.

d_{mi} = network head drop constants.

It is possible to evaluate coefficients which will form equations relating each pumping station to each reservoir. However, in practice, one reservoir will be designated as a reference for each station, the particular reservoir being determined by the equation exhibiting the greatest correlation with the data.

Satisfactory service to customers is partially based on a maintenance of water pressure within specified limits. Pressures at internal points in the network can be found from:

$$h_p = e_{p1} + e_{p2}\, y^{1.85} + \sum_{m=1}^{M} e_{p(m+2)}.h_m + \sum_{n=1}^{N} e_{p(n+M+2)}.\, x_n \qquad (6.9)$$

where h_p = pressure at head node p

p = head node index chosen from (1,2,...P)

P = total number of head nodes.

e_{pi} = network head node constants.

x_n = level of reservoir n.

The empirical relationships are based on the assumption that the actual distributed demands throughout the network are proportional to the total demand. This means that only the total demand need be monitored instead of all the individual consumptions. As an aid in assessing the total demand the following relationship must hold:

$$y = \sum_{m=1}^{M} u_m - \sum_{n=1}^{N} q_n \qquad (6.10)$$

where q_n = reservoir inflow.

(c) Reservoir Relationships.

The flow into the n^{th} reservoir, q_n, can be expressed as:

$$q_n = f_{n1} + f_{n2}.y + \sum_{i=1}^{N} f_n(i+2).x_n^{0.54} + \sum_{m=1}^{M} f_n(m+N+2).u_m \qquad (6.11)$$

where f_{ni} = reservoir flow constants.

Because of the storage capabilities the reservoir level will vary with time according to:

$$\dot{x}_n = \alpha_n q_n \qquad (6.12)$$

where $\ddot{x}_n$ = time derivative of n^{th} reservoir level.

α_n = relationship between flow and level based on reservoir geometry.

If α_n is assumed to be independent of level then the time varying level of the reservoir will be given by:

$$x_n(t+\Delta t) = \alpha_n \int_t^{t+\Delta t} q_n(t)dt + x'_n(t) \qquad (6.13)$$

where Δt = time interval over which the reservoir level is to be calculated.

6.2.2 Dynamic Simulation by Non-Linear Model.

For this model a closed form solution cannot be found and the simulation consists of a sequence of static balances interconnected by updated reservoir levels. Use of the simplified empirical equations now allows a rapid static solution and, overall, a rapid simulation can be obtained.

The simulation period will be from times t_o to t_f and all variables will now be denoted in terms of time t.

Equating head drops from each pumping station to their respective reference reservoirs will give the following set of equations:

$$s_m(t) + \Delta h_m(t) = x_n(t) + \Delta h_{mn}(t) \qquad (6.14)$$

$$(m = 1,2,\ldots M)$$

which can be expanded using the previously derived equations (6.3), (6.4), and (6.8).

For any given pump combinations all pumping station flows, $u_m(t)$, can be calculated in terms of known values of total demand, $y(t)$, and reservoir levels, $x_n(t)$. Substitution of calculated values of $u_m(t)$ in equation (6.11) will then allow evaluation of reservoir flows $q_n(t)$ for all reservoirs. Using these values of $q_n(t)$, updated reservoir levels, $x_n(t+\Delta t)$, are determined by numerical integration of equation (6.13). The above sequence is repeated, for appropriate pump combinations, starting from t_o until the simulation is complete at t_f.

During the course of the simulation the pressures, $h_p(t)$, at selected head nodes can be determined by use of equations (6.5) and (6.9) for known values of $y(t)$, $u_m(t)$ and $x_n(t)$.

The total electrical energy, E, used in pumping over the simulation period can be obtained from:

$$E = \sum_{m=1}^{M} e_m \qquad (6.15)$$

with energy for each pumping station, e_m, evaluated using equations (6.3), (6.6) and (6.7).

6.2.3 Evaluation of Coefficients.

The accuracy of the model depends largely upon the accuracy of the equation coefficients (assuming correct formulation of the equations) and one method of evaluation is by a statistical analysis of actual operating data[15,16]. For this task the following data are required on a periodic basis:

Pumping stations - suction head, s_m; discharge head, h_m; flow, u_m; electrical energy, e_m.

Networks - total water demand, y; heads of selected nodes, h_p.

Reservoirs - level, x_n; flow, q_n.

An alternative to use of actual data is to generate the required values by means of an accurate extended period simulation matched to the network.

For the given data a regression analysis is performed to determine the best fit coefficients for the derived equations. An examination of the correlation coefficients facilitates removal of any statistically insignificant terms.

6.2.4 Discussion.

De Moyer et al[15,16], have developed a useful simplified network model based on a set of non-linear relationships between pump and reservoir flows and total system demand. In this case the coefficients are evaluated by regression analysis of operational data. The model caters for control by on-off fixed speed pumps and is not suitable for use with continuous type controls such as valves, variable speed pumps, or in-line booster pumps.

Extensive validation tests have been performed which show that the models are very accurate under strictly proportional demand loading.

For disproportionate loading the accuracy is reduced but may still be acceptable. As part of these tests the model has been applied to give on-line control by use of a simplified single state dynamic programming algorithm[15,19].

6.3 LINEAR DYNAMIC MODEL.

Linear theory is now well established and most suitable for analysis of large scale systems, consequently it is worthwhile searching for a linear model to represent the system under study. This section gives the development, by the author, of a linear dynamic model which describes the basic system operation in terms of control and demand action on reservoir levels. The final version meets the requirements of a wide range of realistic distribution systems by catering for multiple reservoirs which may be interconnected, or supplied, by means of head dependent pipes, pumps and control valves.

The model linear static equations are obtained by linearisation about instantaneous operating points. Conversion to dynamic equations is then obtained by classical solution of reservoir time varying integral relationships for constant coefficients. Several proposals are made for evaluation of the equation coefficients, of which the most efficient uses the coefficient solution feature of the computer program described in Chapter 5.

6.3.1 Review of Network Analysis.

The development of a linear model is based upon the principles of network analysis which are treated in detail in Chapter 5. This section gives a brief overall review of the analysis with emphasis now placed on a format suitable for evaluation of model equations in standard control systems terminology.

A complete network analysis involves calculation of future reservoir levels, over an extended time period, for known pump and valve controls and consumer demands. This can be achieved by formulation and repeated forward solution of static and dynamic network equations as outlined below:

(a) Network Equations.

These can be formulated by considering the flow balance requirements at each node which will give:

$$f_i + q_i + y_i = 0 \qquad (i=1,\ldots N+P) \tag{6.16}$$

where f_i = total element flow at node i.

q_i = reservoir outflow at node i.

y_i = consumer demand at node i.

N = total number of reservoir nodes.

P = total number of variable head nodes.

Summation of element flows at a node can be expressed as:

$$f_i = \sum_{\substack{j=1 \\ j\neq i}}^{N+P} f_{ij}\,(h_i, h_j, r_{ij}) \tag{6.17}$$

where f_{ij} = generalised expression for individual element flow which only has a value for elements connecting nodes i and j.

h_i = pressure head at local node i.

h_j = pressure head at adjacent node j.

r_{ij} = independent element control parameter corresponding to fixed resistance for pipes, variable resistance for valves, and variable flow control for pumps.

It is convenient to introduce control system terminology here to standardise further developments and this is achieved by expressing equation (6.16) as a set of generalised network equations in the following form:

$$\underline{f}\,(\underline{x},\underline{h},\underline{u},\underline{v}) + \underline{q} + \underline{y} = \underline{0} \tag{6.18}$$

where $\underline{f}$ = $(f_1,\ldots f_{N+P})$ vector functional, corresponding to network link flow relationship.

$\underline{x}$ = $(x_1,...x_N)$ storage node state vector, corresponding to reservoir level.

$\underline{h}$ = $(h_1,...h_P)$ non-storage node state vector, corresponding to node pressure head.

$\underline{u}$ = $(u_1,...u_M)$ pump control vector, with u_m corresponding to r_{ij} for pump elements.

$\underline{v}$ = $(v_1,...v_R)$ valve control vector, with v_r corresponding to r_{ij} for valve elements.

$\underline{q}$ = $(q_1,...q_N)$ storage flow vector, corresponding to reservoir outflow.

$\underline{y}$ = $(y_1,...y_L)$ disturbance vector, corresponding to distributed consumer demands defined as network inflows.

L = disturbance vector dimension, corresponding to total number of distributed consumer demands.

M = pump control vector dimension, corresponding to total number of pumping stations.

N = storage node state vector dimension, corresponding to total number of reservoirs.

P = non-storage node state vector dimension, corresponding to total number of variable head nodes in network.

R = valve control vector dimension, corresponding to total number of valves in network.

(b) Static Solution.

The unknown variables will be taken to be $\underline{q}$ and $\underline{h}$ and the static solutions will determine instantaneous value for these with $\underline{x}$, $\underline{u}$, $\underline{y}$ and $\underline{v}$ held constant.

Defining the residual unbalance, $\underline{g}$, in equation (6.18) to be:

$$\underline{g} = \underline{f}\ (\underline{x},\underline{h},\underline{u},\underline{v}) + \underline{q} + \underline{y} \tag{6.19}$$

requires that $\underline{g} = \underline{0}$ for an exact solution.

The Newton-Raphson method is commonly used for solving equations (6.19) which involves iterative calculation of correction values, $\Delta\underline{q}$ and $\Delta\underline{h}$, to the unknown variable from:

$$\underline{J} \begin{bmatrix} \Delta\underline{q} \\ \hline \Delta\underline{h} \end{bmatrix} = -\ \underline{g} \tag{6.20}$$

where $\underline{J}$ is the Jacobian of partial derivatives given by:

$$\underline{J} = \left[\begin{array}{c|c} \dfrac{\partial \underline{g}}{\partial \underline{q}} & \dfrac{\partial \underline{g}}{\partial \underline{h}} \end{array}\right] \tag{6.21}$$

The set of simultaneous equations (6.20) can be solved efficiently for $\Delta\underline{q}$ and $\Delta\underline{h}$ by use of the Gauss-Jordan elimination procedure where all evaluations use the current iteration values. The unknown variables are then updated at each iteration, s, according to:

$$\begin{bmatrix} \underline{q}_{s+1} \\ \hline \underline{h}_{s+1} \end{bmatrix} = \begin{bmatrix} \underline{q}_s \\ \hline \underline{h}_s \end{bmatrix} + \begin{bmatrix} \Delta\underline{q}_s \\ \hline \Delta\underline{h}_s \end{bmatrix} \tag{6.22}$$

iterations ceasing when $\underline{g}$ is sufficiently close to $\underline{0}$.

(c) Dynamic Solution.

Introducing the time variable, t, in order to obtain a solution to the continuous reservoir dynamics, the unknown variable will now be $\underline{x}(t+\Delta t)$ and the dynamic solution will determine values for these using known values of $\underline{x}(t)$ and $\underline{q}(t)$.

The reservoir dynamic relationship can be expressed as:

$$\dot{\underline{x}}(t) = \underline{F}.\underline{q}(t) \qquad (6.23)$$

where $\dot{\underline{x}}(t)$ = time derivative of reservoir level.

$\underline{F}$ = N x N dimensional diagonal reservoir coefficient matrix of elements $(\alpha_1,\ldots.\alpha_N)$.

α_n = n^{th} reservoir coefficient relating water level to quantity. Assumed to be independent of level, which is reasonable for typical reservoirs.

The solution of this yields:

$$\underline{x}(t+\Delta t) = \underline{F} \int_t^{t+\Delta t} \underline{q}(t)dt + \underline{x}(t) \qquad (6.24)$$

where Δt = increment of time corresponding to required dynamic solution values.

Since $\underline{q}(t)$ depends upon $\underline{x}(t)$ equation (6.24) cannot be solved in closed form and numerical integration methods must be used. The procedure given in Chapter 5 solves equations (6.19) and (6.24) iteratively and uses a predictor-corrector integration scheme to update reservoir levels and flows over each time interval.

For typical average operating values of $\underline{u}^a(t)$, $\underline{v}^a(t)$ and $\underline{y}^a(t)$, which are constant over each solution interval, the time varying trajectories of $\underline{x}^a(t)$, $\underline{q}^a(t)$ and $\underline{h}^a(t)$ will be evaluated at each interval, Δt, over the complete simulation period.

6.3.2 Development of Linear Dynamic Model.

This model will be expressed as a set of linear dynamic equations which can be solved explicitly for the dependent variables in terms of known operating conditions.

Now for differential changes in $\underline{x}^a(t)$, $\underline{u}^a(t)$, $\underline{y}^a(t)$ and $\underline{v}^a(t)$ the resultant changes in $\underline{q}^a(t)$ and $\underline{h}^a(t)$ will be given by expanding equation (6.18) about the expected operating values to give:

$$\begin{bmatrix} d\underline{q}(t) \\ \hline d\underline{h}(t) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \underline{q}(t)}{\partial \underline{x}(t)} \\ \hline \dfrac{\partial \underline{h}(t)}{\partial \underline{x}(t)} \end{bmatrix} \cdot d\underline{x}(t) + \begin{bmatrix} \dfrac{\partial \underline{q}(t)}{\partial \underline{u}(t)} \\ \hline \dfrac{\partial \underline{h}(t)}{\partial \underline{u}(t)} \end{bmatrix} \cdot d\underline{u}(t) + \begin{bmatrix} \dfrac{\partial \underline{q}(t)}{\partial \underline{y}(t)} \\ \hline \dfrac{\partial \underline{h}(t)}{\partial \underline{y}(t)} \end{bmatrix} \cdot d\underline{y}(t) + \begin{bmatrix} \dfrac{\partial \underline{q}(t)}{\partial \underline{v}(t)} \\ \hline \dfrac{\partial \underline{h}(t)}{\partial \underline{v}(t)} \end{bmatrix} \cdot d\underline{v}(t) \qquad (6.25)$$

where $d\underline{x}(t) = (\underline{x}(t) - \underline{x}^a(t))$ is differential change from expected value to actual value, with similar definitions for $d\underline{u}(t)$, $d\underline{y}(t)$, $d\underline{v}(t)$, $d\underline{q}(t)$ and $d\underline{h}(t)$.

In terms of matrix coefficients, which remain to be determined, equation (6.25) can be written as:

$$d\underline{q}(t) = \underline{A}q(t).\, d\underline{x}(t) + \underline{B}q(t).d\underline{u}(t) + \underline{C}q(t).d\underline{y}(t) + \underline{D}q(t).d\underline{v}(t) \qquad (6.26)$$

$$d\underline{h}(t) = \underline{A}h(t).d\underline{x}(t) + \underline{B}h(t).d\underline{u}(t) + \underline{C}h(t).d\underline{y}(t) + \underline{D}h(t).d\underline{v}(t) \qquad (6.27)$$

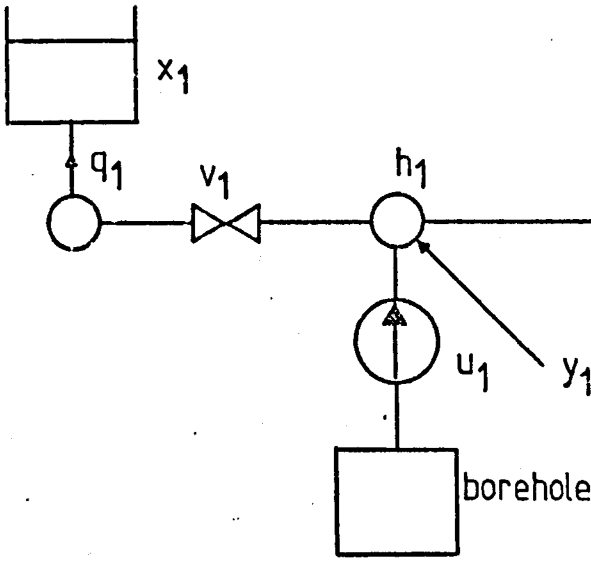
A simplified diagram to illustrate these results is shown in Figure 6-1.

(a) Storage node equations.

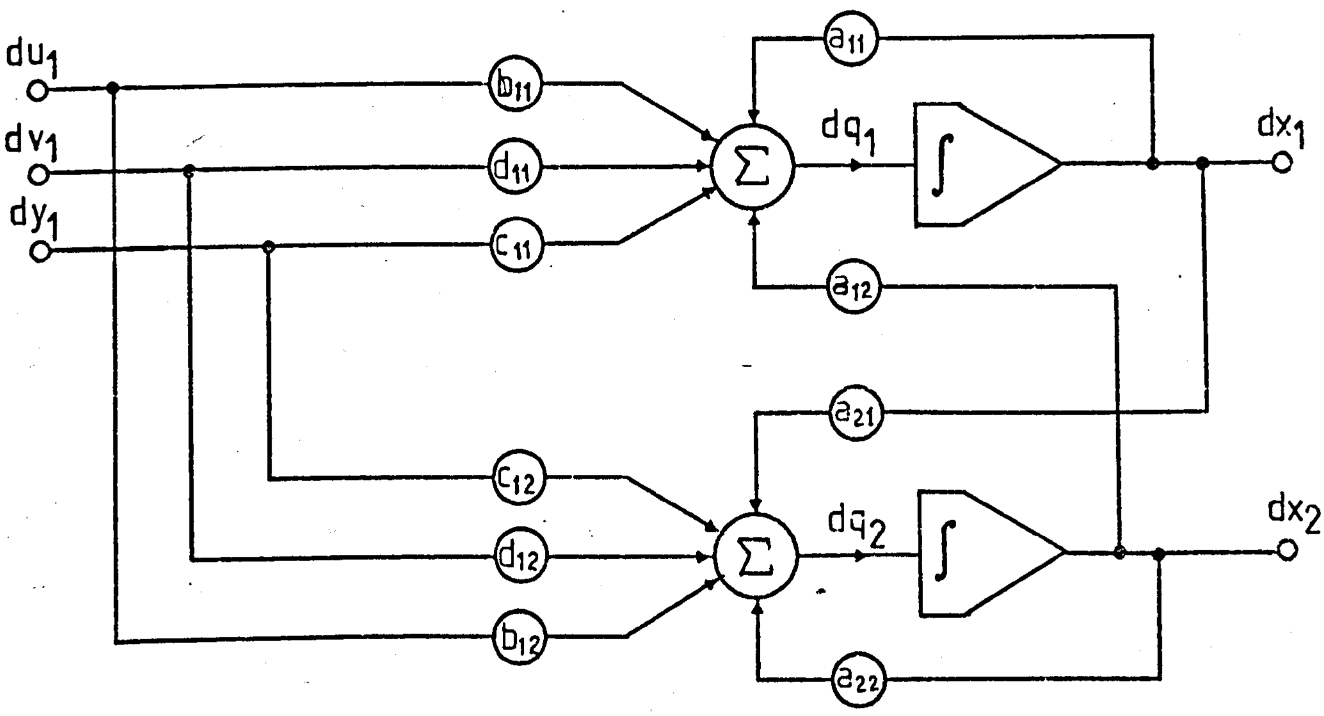
Combining equations (6.23) and (6.26) will give:

$$d\dot{\underline{x}}(t) = \underline{F}\left\{\underline{A}q(t).\, d\underline{x}(t) + \underline{B}q(t).d\underline{u}(t) + \underline{C}q(t).d\underline{y}(t) + \underline{D}q(t).d\underline{v}(t)\right\} \qquad (6.28)$$

which has a solution over each increment, Δt, of:



(a) system



$$\begin{bmatrix} dq_1 \\ dq_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \circ \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \bullet du_1 + \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} \bullet dy_1 + \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix} \bullet dv_1$$

(b) model

Fig. 6-1 Linear model representation for a simple system

$$\left[\varepsilon^{-\underline{F}.\underline{A}q(t).t} \quad d\underline{x}(t) \right]_t^{t+\Delta t} = \int_t^{t+\Delta t} \varepsilon^{-\underline{F}.\underline{A}q(t).t} \, \underline{F} \left\{ \underline{B}q(t).d\underline{u}(t) + \underline{C}q(t).d\underline{y}(t) + \underline{D}q(t).d\underline{v}(t) \right\} d \tag{6.29}$$

Now for $\underline{A}q(t)$, $\underline{B}q(t)$, $\underline{C}q(t)$, $\underline{D}q(t)$, $d\underline{u}(t)$, $d\underline{y}(t)$ and $d\underline{v}(t)$ constant over each interval Δt, but allowed to vary from interval to interval, the solution of equation (6.29) can be expressed as follows (see Elgerd[29] for classical solution details):

$$d\underline{x}(t+\Delta t) - d\underline{x}(t) = \underline{A}'_{exp} \left\{ \underline{A}x(t).d\underline{x}(t) + \underline{B}x(t).d\underline{u}(t) + \underline{C}x(t).d\underline{y}(t) + \underline{D}x(t).d\underline{v}(t) \right\} \tag{6.30}$$

where $\underline{A}'_{exp} = (\varepsilon^{\underline{A}x(t)} - \underline{I})\underline{A}x^{-1}(t)$

$\underline{A}x(t) = \underline{F}.\underline{A}q(t).\ \Delta t$

$\underline{B}x(t) = \underline{F}.\underline{B}q(t).\ \Delta t$

$\underline{C}x(t) = \underline{F}.\underline{C}q(t).\ \Delta t$

$\underline{D}x(t) = \underline{F}.\underline{D}q(t).\ \Delta t$

$\underline{I}$ = unit diagonal matrix.

The average value of reservoir level during each interval is required, for evaluating pressure node average values, and can be calculated from:

$$d\underline{x}^a(t) = d\underline{x}(t) + \frac{1}{\Delta t} \int_0^{\Delta t} \underline{A}'_{exp}.\underline{F}.\tau \left\{ \underline{A}q(t).d\underline{x}(t) + \underline{B}q(t).d\underline{u}(t) + \underline{C}q(t).d\underline{y}(t) + \underline{D}q(t).\ d\underline{v}(t) \right\} d\tau \tag{6.31}$$

which has a solution of:

$$d\underline{x}^a(t) = d\underline{x}(t) + \underline{A}''_{exp} \left\{ \underline{A}x(t).d\underline{x}(t) + \underline{B}x(t).d\underline{u}(t) + \underline{C}x(t).d\underline{y}(t) + \underline{D}x(t).d\underline{v}(t) \right\} \tag{6.32}$$

where $\underline{dx}^{a}(t)$ = average value of reservoir level during interval Δt.

$$\underline{A}''_{exp} = \left\{(\epsilon^{\underline{A}x(t)} - \underline{I})\underline{A}\bar{x}^{1}(t) - \underline{I}\right\} \underline{A}\bar{x}^{1}(t)$$

Equation (6.30) can be re-arranged in standard form as a set of discrete time linear dynamic equations thus:

$$d\underline{x}(t+\Delta t) = \underline{A}'x(t).d\underline{x}(t) + \underline{B}'x(t).d\underline{u}(t) + \underline{C}'x(t).d\underline{y}(t) + \underline{D}'x(t).d\underline{v}(t) \qquad (6.33)$$

where $\underline{A}'x(t) = \underline{I} + \underline{A}'_{exp}.\ \underline{A}x(t)$

$\underline{B}'x(t) = \underline{A}'_{exp}.\ \underline{B}x(t)$

$\underline{C}'x(t) = \underline{A}'_{exp}.\underline{C}x(t)$

$\underline{D}'x(t) = \underline{A}'_{exp}.\ \underline{D}x(t)$

Equation (6.30) can also be expressed in terms of reservoir flow, by use of equation (6.26), to give:

$$d\underline{x}(t+\Delta t) - d\underline{x}(t) = \underline{A}'_{exp}.\ \underline{F}.\ d\underline{q}(t).\ \Delta t \qquad (6.34)$$

As a special case, for reservoirs whose levels change by only a small amount over each time interval, then $\underline{A}x(t) << \underline{I}$ and $\underline{A}'_{exp} \approx \underline{I}$. Equation (6.34) now reduces to $d\underline{x}(t+\Delta t) = d\underline{x}(t) + \underline{F}.d\underline{q}(t).\Delta t$, which is the solution of equation (6.24) for constant reservoir flows, at the initial values, over the interval Δt.

(b) Pressure node equations.

Whilst the time varying heads of pressure nodes are given at each interval by equation (6.27), there is also a requirement (for subsequent optimisation purposes) for calculating average head increase across pumps during each interval. The linear model facilitates this and, as an illustration, consider the case of pump elements connected between two variable head nodes each of which will have head changes given by an expansion of equation (6.27) as follows:

$$dh_i(t) = \sum_{n=1}^{N} a_{in}(t).dx_n(t) + \sum_{m=1}^{M} b_{im}(t).du_m(t) + \sum_{\ell=1}^{L} c_{i\ell}(t).dy_\ell(t) + \sum_{r=1}^{R} d_{ir}(t).\ dv_r(t) \qquad (6.35)$$

where $a_{in}(t)$ = element of matrix $\underline{A}h(t)$ corresponding to variable head node i and reservoir node n, with similar definitions for $b_{im}(t)$, $c_{i\ell}(t)$ and $d_{ir}(t)$.

The generalised expression for head changes across all pumping stations can then be written as:

$$d\underline{z}(t) = \underline{A}z(t).d\underline{x}(t) + \underline{B}z(t).d\underline{u}(t) + \underline{C}z(t).d\underline{y}(t) + \underline{D}z(t).d\underline{v}(t) \qquad (6.36)$$

where $d\underline{z}(t)$ = $(dz_1(t),\ldots dz_m(t),\ldots dz_M(t))$, which is the vector of pumping station head changes.

$dz_m(t) = (dh_i(t) - dh_j(t))$, which is the head change across station m connected between nodes i and j.

$\underline{A}z(t)$ = matrix with elements formed from $(a_{in}(t) - a_{jn}(t))$, with similar definitions for $\underline{B}z(t)$, $\underline{C}z(t)$ and $\underline{D}z(t)$.

The average values of pumping station head changes during each interval can now be obtained by substituting for average reservoir levels from equation (6.32) to give:

$$d\underline{z}^a(t) = \underline{A}'z(t).\ d\underline{x}(t) + \underline{B}'z(t).d\underline{u}(t) + \underline{C}'z(t).d\underline{y}(t) + \underline{D}'z(t).d\underline{v}(t) \qquad (6.37)$$

where $d\underline{z}^a(t)$ = average value of head change across pumping station during interval Δt,

$\underline{A}'z(t) = \underline{A}z(t) + \underline{A}z(t).\ \underline{A}''_{exp}.\ \underline{A}x(t)$

$$\underline{B}'z(t) = \underline{B}z(t) + \underline{A}z(t).\ \underline{A}''_{exp}.\ \underline{B}x(t)$$

$$\underline{C}'z(t) = \underline{C}z(t) + \underline{A}z(t).\ \underline{A}''_{exp}.\ \underline{C}x(t)$$

$$\underline{D}'z(t) = \underline{D}z(t) + \underline{A}z(t).\ \underline{A}''_{exp}.\ \underline{D}x(t).$$

(c) Discrete Time Formulation.

Introducing the stage variable, k, will allow formulation of discrete time equations which are suitable for efficient optimisation methods. In addition, dynamic simulations are usually evaluated in steps of time intervals, Δt, from $t = t_0$ to $t = t_f$. The transformation can be obtained by putting $t = k.\Delta t$ in selected equations to give the following important results:

$$d\underline{x}(k+1)=\underline{A}'x(k).d\underline{x}(k)+\underline{B}'x(k).d\underline{u}(k)+\underline{C}'x(k).d\underline{y}(k)+\underline{D}'x(k).d\underline{v}(k) \qquad (6.38)$$

$$d\underline{z}^a(k) = \underline{A}'z(k).d\underline{x}(k)+\underline{B}'z(k).d\underline{u}(k)+\underline{C}'z(k).d\underline{y}(k)+\underline{D}'z(k).d\underline{v}(k) \qquad (6.39)$$

This set of generalised linear dynamic equations with stage varying coefficients give the deviations from the expected state trajectories for differential changes in the operating values. These equations should also be applicable for small, non-differential, changes in consumer demands under optimal control conditions.

The next section discusses methods of evaluating the coefficients and also shows that a simpler model can be constructed by use of overall average operating values to form stage invariant coefficients. The deviations from these average operating values will be much larger and the results less accurate, but some overall cancellation of errors should occur and it will be shown that, for a typical water network, acceptable results have been achieved.

(d) Model extensions.

Whilst the above modelling procedure can cater for any number of distributed demands it may be desirable to aggregate the effects of these at the N reservoir nodes. This requirement arises because of the difficulty in monitoring and predicting numerous minor demands. It will usually be easier to monitor the reservoir inflows and outflows together with any major metered demands and transform these into non-interacting lumped disturbances with predicted variations based on knowledge of the type of distributed demand (e.g. industrial, domestic, etc.). Since the elements of $\underline{C}x(k)$ determine the proportion of distributed demand taken from each reservoir this can be accomplished by use of the transformation:

$$d\underline{y}'(k) = \underline{C}x(k).d\underline{y}(k) \qquad (6.40)$$

where $d\underline{y}'(k) = (dy'_1(k),.......dy'_N(k))$ is the vector of lumped reservoir disturbances.

A reduction in dimensionality of the model may be possible under limited circumstances (e.g. adjacent reservoirs of similar heights and dimensions). If analysis shows that $dx_i(k) \simeq dx_j(k)$ contracted coefficient matrices can be formed by adding rows i and j for $\underline{A}x(k)$, $\underline{B}x(k)$, $\underline{C}x(k)$ and $\underline{D}x(k)$ and columns i and j for $\underline{A}x(k)$ and $\underline{A}z(k)$.

An additional factor which can affect system operation is change of level of boundary reservoirs. This effect can be easily incorporated in the model by defining an additional independent variable, corresponding to these level changes, with coefficients related to the resultant changes in flow of the internal reservoirs.

6.3.3 Evaluation of Model Coefficients.

The model coefficients cannot be calculated directly since no equations may exist for $\underline{q}(k)$ or $\underline{h}(k)$ in terms of $\underline{x}(k)$, $\underline{u}(k)$, $\underline{y}(k)$ and $\underline{v}(k)$. Such relationships will usually be in terms of states, $\underline{h}(k)$, of intermediate nodes which are themselves dependent on $\underline{x}(k)$, $\underline{u}(k)$, $\underline{y}(k)$ and $\underline{v}(k)$. In spite of these problems methods have been devised, by the author, for obtaining coefficient values. The methods described below initially generate instantaneous network coefficients which are subsequently manipulated to provide the required coefficients for the standardised equations (6.38) and (6.39).

For numerical convenience the required exponential factors can be expressed as:

$$\underline{A}_{exp} = \underline{I} + \underline{A}x + \frac{1}{2!}\underline{A}x^2 + \cdots \tag{6.41}$$

$$\underline{A}'_{exp} = \underline{I} + \frac{1}{2!}\underline{A}x + \frac{1}{3!}\underline{A}x^2 + \cdots \tag{6.42}$$

$$\underline{A}''_{exp} = \frac{1}{2!}\underline{I} + \frac{1}{3!}\underline{A}x + \frac{1}{4!}\underline{A}x^2 + \cdots \tag{6.43}$$

and evaluated by including sufficient terms to give the required accuracy.

(a) Evaluation by Coefficient Solution.

One method of evaluating these stage varying coefficients, based on sensitivity analysis[84], is given by differentiation of equation (6.19) under balanced conditions. For determination of

$$\begin{bmatrix} \underline{A}q(k) \\ \hline \underline{A}h(k) \end{bmatrix}, \quad \begin{bmatrix} \underline{B}q(k) \\ \hline \underline{B}h(k) \end{bmatrix}, \quad \begin{bmatrix} \underline{C}q(k) \\ \hline \underline{C}h(k) \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \underline{D}q(k) \\ \hline \underline{D}h(k) \end{bmatrix}$$

differentiation will be with $\underline{x}(k)$, $\underline{u}(k)$, $\underline{y}(k)$ or $\underline{v}(k)$, respectively and, typically, for $\begin{bmatrix} \underline{A}q(k) \\ \hline \underline{A}h(k) \end{bmatrix}$ this will give:

$$\frac{d\underline{g}(k)}{d\underline{x}(k)} = \frac{\partial\underline{g}(k)}{\partial\underline{x}(k)} + \left[\frac{\partial\underline{g}(k)}{\partial\underline{q}(k)} \;\vdots\; \frac{\partial\underline{g}(k)}{\partial\underline{h}(k)}\right] . \begin{bmatrix} \dfrac{\partial\underline{q}(k)}{\partial\underline{x}(k)} \\ \text{-----} \\ \dfrac{\partial\underline{h}(k)}{\partial\underline{x}(k)} \end{bmatrix} = \underline{0} \tag{6.44}$$

and, by use of results from equations (6.21), (6.25), (6.26) and (6.27),

$$\underline{J}(k) . \begin{bmatrix} \underline{A}q(k) \\ \text{-----} \\ \underline{A}h(k) \end{bmatrix} = - \frac{\partial\underline{g}(k)}{\partial\underline{x}(k)} \tag{6.45}$$

where $\underline{J}(k)$ is a by-product of the static solution obtained for each increment of the dynamic solution and expressions for the right hand side can be obtained from equations (6.16) and (6.18).

Solution of the sets of simultaneous equations (6.45), etc, at $\underline{x}^a(k)$, $\underline{u}^a(k)$, $\underline{y}^a(k)$ and $\underline{v}^a(k)$ will then yield:

$$\begin{bmatrix} \underline{A}q(k) \\ \text{----} \\ \underline{A}h(k) \end{bmatrix} , \begin{bmatrix} \underline{B}q(k) \\ \text{----} \\ \underline{B}h(k) \end{bmatrix} , \begin{bmatrix} \underline{C}q(k) \\ \text{----} \\ \underline{C}h(k) \end{bmatrix} \text{ and } \begin{bmatrix} \underline{D}q(k) \\ \text{----} \\ \underline{D}h(k) \end{bmatrix} .$$

For stage varying values this method requires that the coefficients be evaluated for each value of k during a dynamic simulation. This will necessitate extensive computer time and storage of resulting values.

Under the conditions of zero reservoir flow all reservoir levels will be constant and independent of k, hence simplified results follow if close average operating values are defined, over all k, as $\underline{x}^a$, $\underline{u}^a$, $\underline{y}^a$ and $\underline{v}^a$ which make $\underline{q}^a(k)$ zero and $\underline{h}^a(k)$ constant. Use of these constant average values in a single static solution will allow evaluation of stage invariant coefficients $\begin{bmatrix} \underline{A}q \\ \text{---} \\ \underline{A}h \end{bmatrix}$, $\begin{bmatrix} \underline{B}q \\ \text{---} \\ \underline{B}h \end{bmatrix}$, $\begin{bmatrix} \underline{C}q \\ \text{---} \\ \underline{C}h \end{bmatrix}$ and $\begin{bmatrix} \underline{D}q \\ \text{---} \\ \underline{D}h \end{bmatrix}$.

A computer program is described in Chapter 5 which will automatically generate the above coefficients upon selection of the coefficient solution option. During a dynamic simulation stage varying coefficients will be calculated and for a static solution stage invariant values will be produced.

(b) Evaluation by Off-line Perturbation

Evaluation of the coefficients by sensitivity analysis gives point derivatives which may be inaccurate in non-linear regions or for large deviations. Additionally a computer analysis program is required which uses the nodal equations and in which $\underline{J}$ can be accessed. A further method involves perturbing individual elements of $\underline{x}^a$, $\underline{u}^a$, $\underline{y}^a$ or $\underline{v}^a$, in order to obtain static solutions with all $\underline{q}$ and $\underline{h}$ allowed to take on new values, and evaluation of the coefficient elements individually as average gradients.

Typically for $\begin{bmatrix} \underline{Aq} \\ --- \\ \underline{Ah} \end{bmatrix}$ (with similar results for $\begin{bmatrix} \underline{Bq} \\ --- \\ \underline{Bh} \end{bmatrix}$, $\begin{bmatrix} \underline{Cq} \\ --- \\ \underline{Ch} \end{bmatrix}$ and $\begin{bmatrix} \underline{Dq} \\ --- \\ \underline{Dh} \end{bmatrix}$)

an element a_{ij}, normally given by $\frac{\partial q_i}{\partial x_j}$ or $\frac{\partial h_i}{\partial x_j}$, can be approximated by

$\frac{\Delta q_i}{\Delta x_j}$ or $\frac{\Delta h_i}{\Delta x_j}$ where Δq_i $(i = 1,\ldots.N)$ and Δh_i $(i = 1,\ldots.P)$ can be obtained from a single static solution for a perturbation of Δx_j.

The coefficients will now be more representative if the perturbations are chosen to represent values encountered in practice.

Evaluation of all coefficients in this manner will require more computer time since the static solution must be obtained N + M + L + R times, however each solution will be from a close starting point and the method will be feasible for simple systems. An integrated method for evaluating coefficients in this manner, using a dynamic solution as a sequence of static solutions, is described in section 5.6.3.

(c) Evaluation by On-line Perturbation.

The previous two methods give coefficients which are related to the simulated network and hence they will only give accurate results if the simulated network can be matched to the actual system under all operating conditions. A desirable way of evaluating the coefficients would be by means of an on-line method akin to that described in section 6.2 for evaluation of the non-linear model coefficients. The technique would parallel that of the immediately above off-line method of part (b) in which the independent variables $\underline{x}$, $\underline{u}$, $\underline{y}$ and $\underline{v}$ are perturbed one at a time, and the resultant changes in reservoir flows, $\underline{q}$, and pressure node heads, $\underline{h}$, are monitored. Determination of pump control perturbation effects would present no problem since individual pumps could be switched on and off to yield $\frac{\Delta q_n}{\Delta u_m}$ $(n \in N,\ m \in M)$ and $\frac{\Delta h_p}{\Delta u_m}$ $(p \in P,\ m \in M)$.

Assuming all demand flows lumped at relevant reservoirs then all coefficients $\frac{\Delta q_n}{\Delta y_\ell}$ $(n \in N,\ \ell \in L)$ will have values of unity and $\frac{\Delta h_p}{\Delta y_\ell}$ $(p \in P, \ell \in L)$ will have values of zero.

Valve controls could be varied by specified amounts to give

$$\frac{\Delta q_n}{\Delta v_r}\ (n \in N,\ r \in R) \text{ and } \frac{\Delta h_p}{\Delta v_r}\ (p \in P,\ r \in R).$$

The coefficients most difficult to evaluate would be those for state perturbations involving the effect of reservoir level variations upon adjacent reservoir flows, $\frac{\Delta q_i}{\Delta x_n}$ $(i \in N,\ n \in N)$, and upon adjacent pressure nodes, $\frac{\Delta h_p}{\Delta x_n}$ ($p \in P$, $n \in N$). Because of the storage capability it would be impossible to achieve a rapid change in level. Only slow changes would be possible and then it is likely that other uncontrollable disturbances would have occurred.

This technique has not been investigated in practice but it would appear that, in spite of the associated problems, it is worthy of further investigation.

6.3.4 Application to Practical Water Systems

The proposed method for deriving linear dynamic equations has been applied to typical water networks and validated by comparison of measured and predicted results. The first application uses a two reservoir single zone network and, for a simple demonstration of the principles, the pump flows were assumed to be independent of the network pressure levels and also the reservoir levels were evaluated under the initial assumption that the reservoir flows remained constant over each time increment. The second example treats the case of head dependent pumping for two interconnected zones allowing for exponentially variable reservoir flows over each time increment.

(a) Doncaster Eastern Zone

The Doncaster Eastern Zone of the DDJWB has been chosen as a suitable system for analysis for which network details and historical operating data are available.

The network, shown in figure 6-2, consists of 36 pipe and valve links interconnecting 24 pressure nodes and 2 nodes with storage

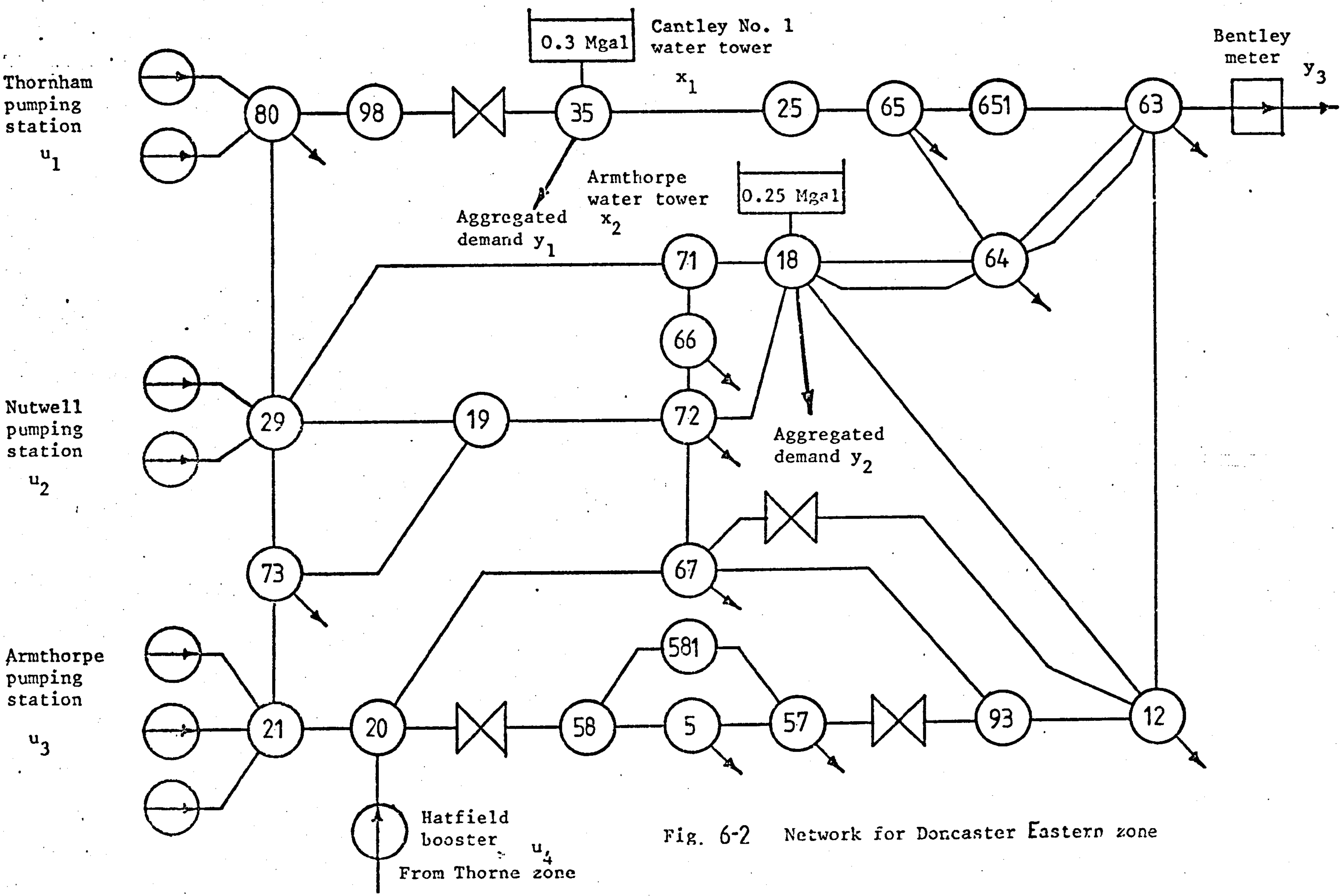


Fig. 6-2 Network for Doncaster Eastern zone

capability provided by 2 water towers with capacities of 0.3 and 0.25 million gallons. Water is supplied to the network by means of 3 independent borehole pumping stations together with inter-zone transfer via one booster pumping station, each station using parallel combinations of fixed speed pumps.

Apart from one large metered demand (read at weekly intervals), individual values for the distributed consumer demands were not available and suitable data for analysis were obtained by performing static network solutions in conjunction with measured reservoir levels. This procedure was carried out for 12x2 hour increments to determine equivalent aggregated reservoir demands for known pump and reservoir flows together with the mean flow value of the metered demand. The pump and derived demand flows $\underline{u}(k)$ and $\underline{y}(k)$, corresponding to the actual operating conditions, are shown in figure 6-3. No information was available on effects of valve operations, consequently all valves were treated as fixed resistance pipe links and catered for in the model by setting the valve control matrix $\underline{D}q$ equal to zero.

For this network the simplified model, corresponding to equation (6.38), can be written in terms of 2 reservoir states, 4 pump controls and 3 demands. To demonstrate the dependence on the flow coefficients, and noting that $\underline{A}'_{exp} = \underline{I}$ (for reservoir flow constant over each interval), equation (6.38) can be re-written as:

$$d\underline{x}(k+1) = \left\{\underline{I} + \underline{F}.\underline{A}q.\Delta t\right\} d\underline{x}(k) + \underline{F}.\underline{B}q.\Delta t.d\underline{u}(k) + \underline{F}.\underline{C}q.\Delta t.d\underline{y}(k) \qquad (6.46)$$

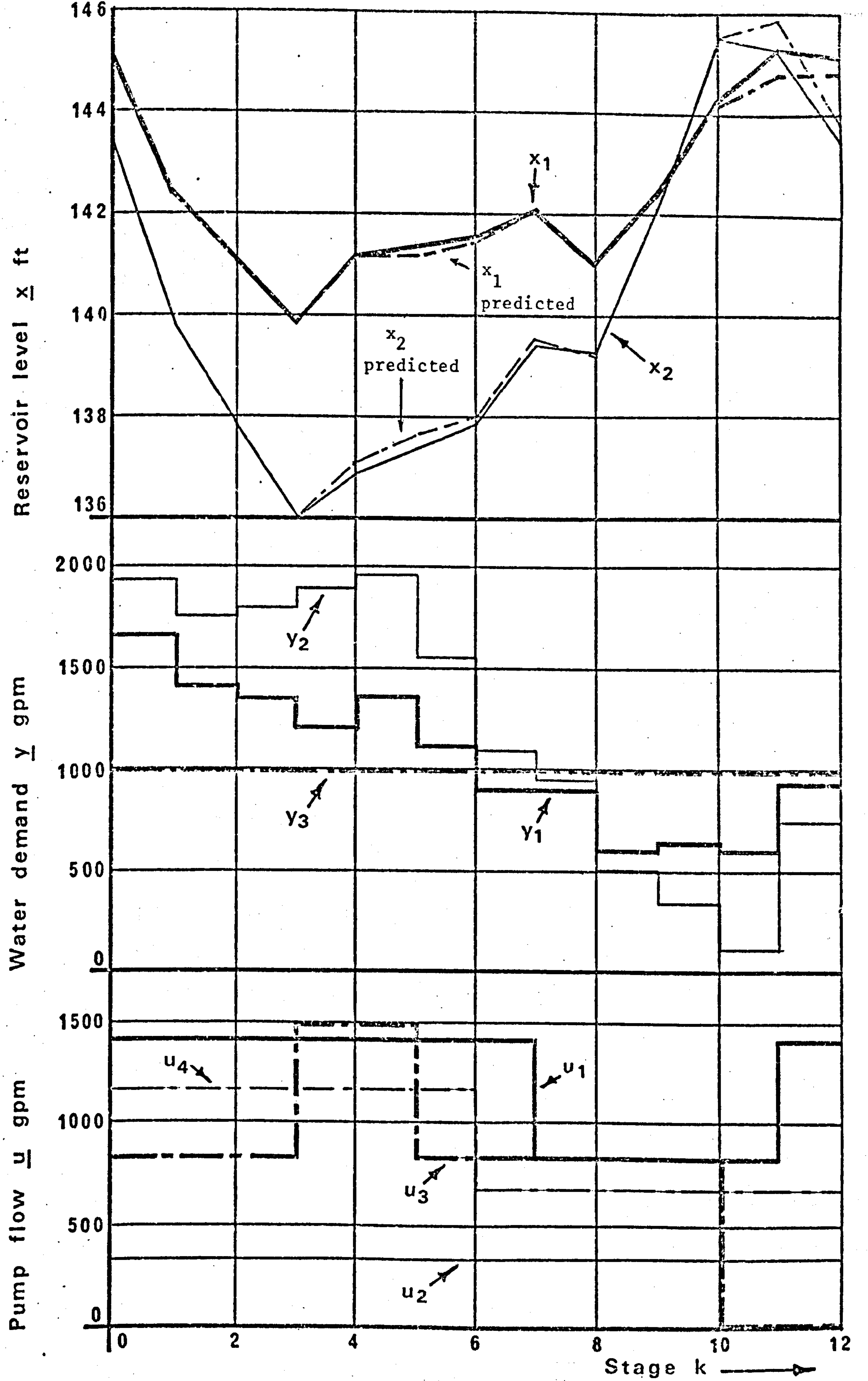


Fig. 6-3 Actual and Predicted Reservoir Levels

NOTE: All values are actual unless specified otherwise.

or:

$$\begin{bmatrix} dx_1(k+1) \\ dx_2(k+1) \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \alpha_1 . \Delta t & 0 \\ 0 & \alpha_2 . \Delta t \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \right\} \begin{bmatrix} dx_1(k) \\ dx_2(k) \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_1 . \Delta t & 0 \\ 0 & \alpha_2 . \Delta t \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} \cdot \begin{bmatrix} du_1(k) \\ du_2(k) \\ du_3(k) \\ du_4(k) \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_1 . \Delta t & 0 \\ 0 & \alpha_2 . \Delta t \end{bmatrix} \cdot \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \cdot \begin{bmatrix} dy_1(k) \\ dy_2(k) \\ dy_3(k) \end{bmatrix} \quad (6.47)$$

where the deviations are given by:

$$d\underline{x}(k) = \underline{x}(k) - \underline{x}^a \quad (6.48)$$

$$d\underline{u}(k) = \underline{u}(k) - \underline{u}^a \quad (6.49)$$

$$d\underline{y}(k) = \underline{y}(k) - \underline{y}^a \quad (6.50)$$

The linearised flow coefficients a_{ij}, b_{ij} and c_{ij} were obtained using the perturbation method described in section 6.3.3(b), whilst the reservoir coefficients, $f_i\Delta t$, are given by known values of level change per unit flow for each reservoir. Static network solutions were used to derive the average operating values $\underline{x}^a$, $\underline{u}^a$ and $\underline{y}^a$ and to ensure that these gave a dynamically balanced network (zero reservoir flows). The calculated values are summarised in tables 6-1 and 6-2.

Equations (6.47), (6.48), (6.49) and (6.50) can now be used to calculate the time varying reservoir levels, $\underline{x}(k)$, for known operating values of pump and demand flows together with initial levels for reservoirs.

Table 6-1. Average Operating Values for Doncaster Eastern Zone.

Thornham borehole pumping station	$u_1^a = 1222$ gpm
Nutwell borehole " "	$u_2^a = 333.3$ gpm
Armthorpe borehole " "	$u_3^a = 805.6$ gpm
Hatfield booster " "	$u_4^a = 916.7$ gpm
Cantley No.1 Water Tower	$x_1^a = 142.3$ ft
Armthorpe " "	$x_2^a = 140.3$ ft
Cantley No.1 demand	$y_1^a = 1060$ gpm
Armthorpe demand	$y_2^a = 1218$ gpm
Bentley meter demand	$y_3^a = 1000$ gpm

Table 6-2. Linearised Coefficients for Doncaster Eastern Zone.

$$\underline{A}q = \begin{bmatrix} -57.15 & 59.47 \\ 57.15 & -59.47 \end{bmatrix} \text{ gpm/ft}$$

$$\underline{B}q = \begin{bmatrix} 0.4595 & 0.4552 & 0.3832 & 0.3715 \\ 0.5405 & 0.5448 & 0.6168 & 0.6285 \end{bmatrix}$$

$$\underline{C}q = \begin{bmatrix} -1.0 & 0.0 & -0.0887 \\ 0.0 & -1.0 & -0.9113 \end{bmatrix}$$

$$\underline{F}.\Delta t = \begin{bmatrix} 0.0068 & 0.0 \\ 0.0 & 0.00816 \end{bmatrix} \text{ ft/gpm}$$

The actual and predicted reservoir levels are compared on figure 6-3 and show that the overall errors are approximately 5% of allowed level variations. Consequently representation of the network by the simplified model gives results sufficiently accurate for normal operational requirements.

The validity of these results is supported by noting that, if x is re-defined as reservoir quantity and the reduction technique of section 6.3.2(d) applied to the coefficient matrices, to simulate a single reservoir zone, the resulting expression is

$$x(k+1) = x(k) + \sum_{m=1}^{4} u_m(k) - \sum_{\ell=1}^{3} y_\ell(k) \qquad (6.51)$$

which corresponds to the classical one dimensional linear equation of water networks.

In addition to the above results the network has also been treated by allowing for exponential reservoir flow variation together with derivation of pumping station pressure levels to give a model corresponding to equations (6.38) and (6.39) with coefficients obtained using the coefficient solution of section 6.3.3(a). In all cases close agreement was observed between actual and predicted results. These are not detailed here since the next section gives comprehensive results for a more complex system.

(b) Doncaster Eastern and Thorne Zones

This two zone system is also used in Chapter 7 for validation of the system optimisation technique and consequently all related network details are given separately in appendix 3 to avoid duplication. In this system the Doncaster Eastern Zone has been extended by relevant parts of the Thorne Zone and now includes variable head pumps supplied, as appropriate, from boreholes (assumed of constant levels). The system diagram is shown in appendix 3, figure A3-1 and the parameters are given in appendix 3, table A3-1.

In order to suit the monitored data the stage increment was chosen as 2 hours starting from 0800 hours, to give a 24 hour dynamic simulation period. The borehole pump head increases were taken to be equal to the pump pressure heads minus the respective borehole levels. For Hatfield booster pump the suction head was taken to be the average level of Hatfield water tower; this latter level also serves as the pressure head for Hatfield borehole pumping station, reducing the number of pressure node equations to 5.

The modified analysis program (WATSIM)was used extensively for data analysis as described in section 5.6.3. The dynamic solution option being used in conjunction with the measured reservoir levels (see figure 6-7) and the known pump and valve controls (see figure 6-5 and figure 6-6) to evaluate the average values of distributed demands (see figure 6-4) for each time interval. The static solution option was then used to determine the overall average operating values (see table 6-3) and finally the coefficient solution option (COEF) was used to calculate the linear model average coefficients (see table 6-4). A simplified diagram of the above sequence of computing operations is shown in figure 7-6.

For this system the simplified model, corresponding to equations (6.38) and (6.39), can now be written in terms of 3 reservoir states, 5 pressure node states, 6 pump controls, 1 valve control, and 5 distributed demands. This allows prediction of the reservoir level and pumping station head trajectories. These predicted values, shown on figures 6-7 and 6-8 respectively, again compare favourably with the actual measured values but with larger errors than the previous two reservoir zone with fixed head pumping.

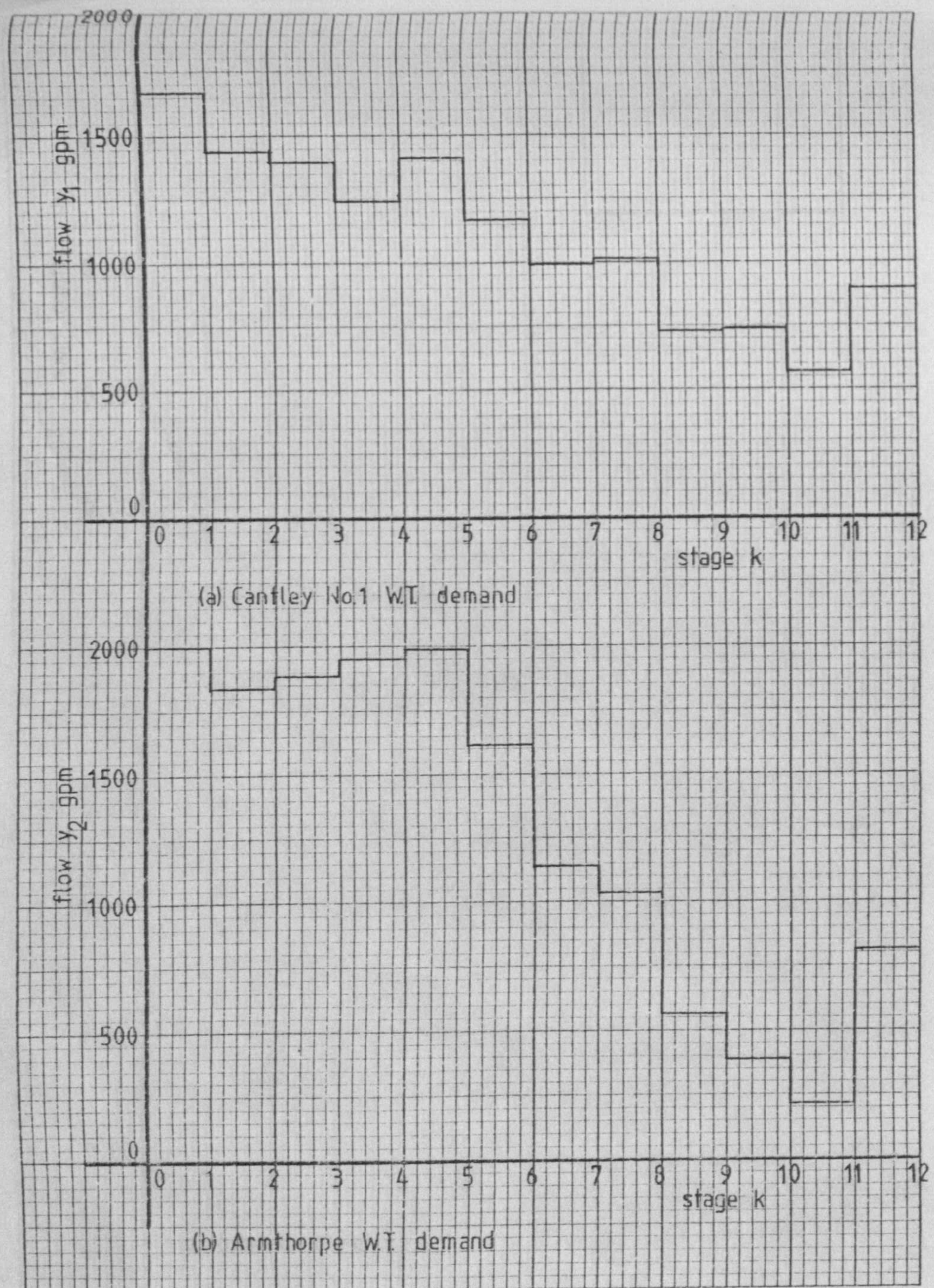


Fig. 6-4 System demands

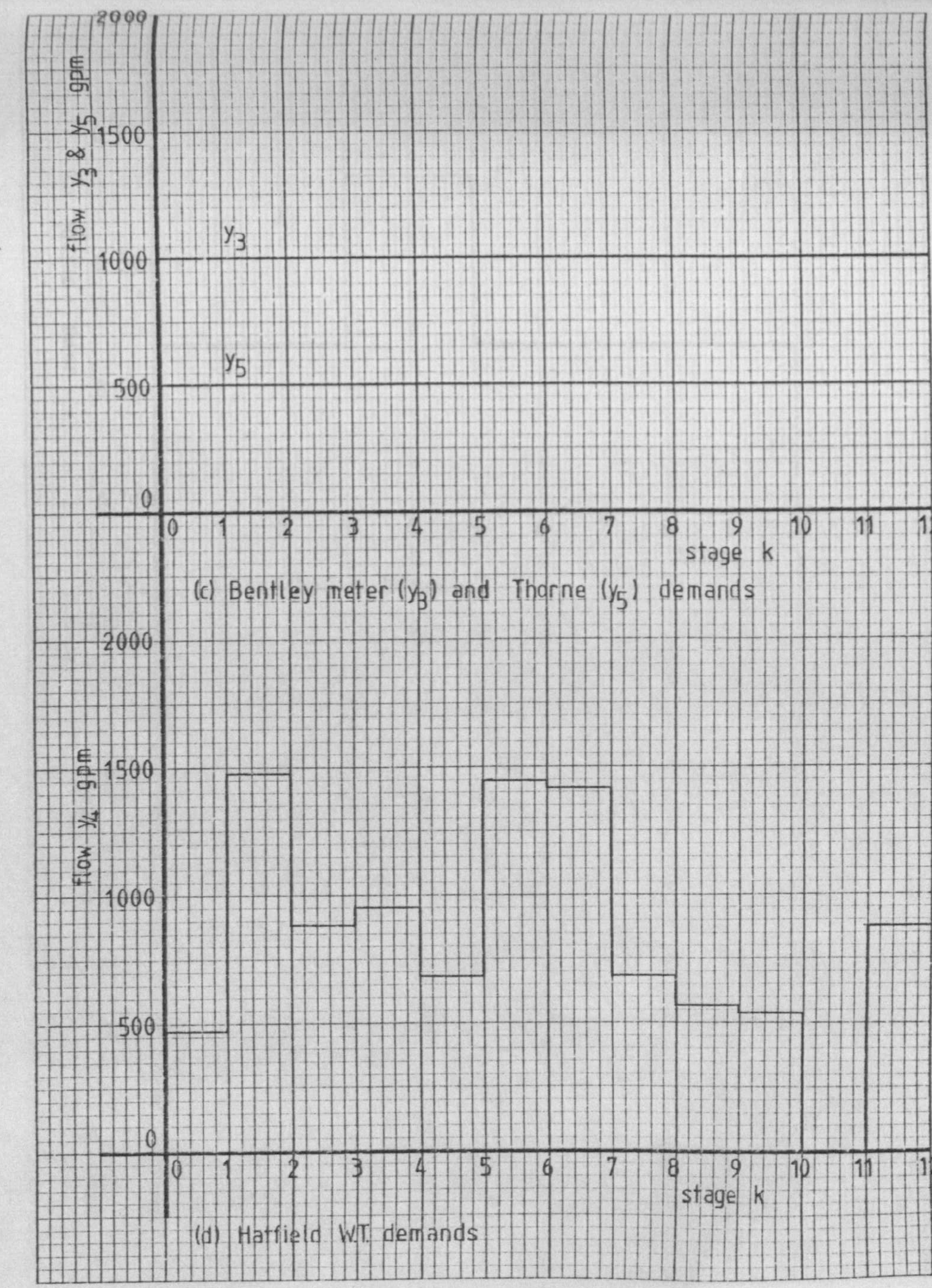


Fig. 6-4 System demands (continued)

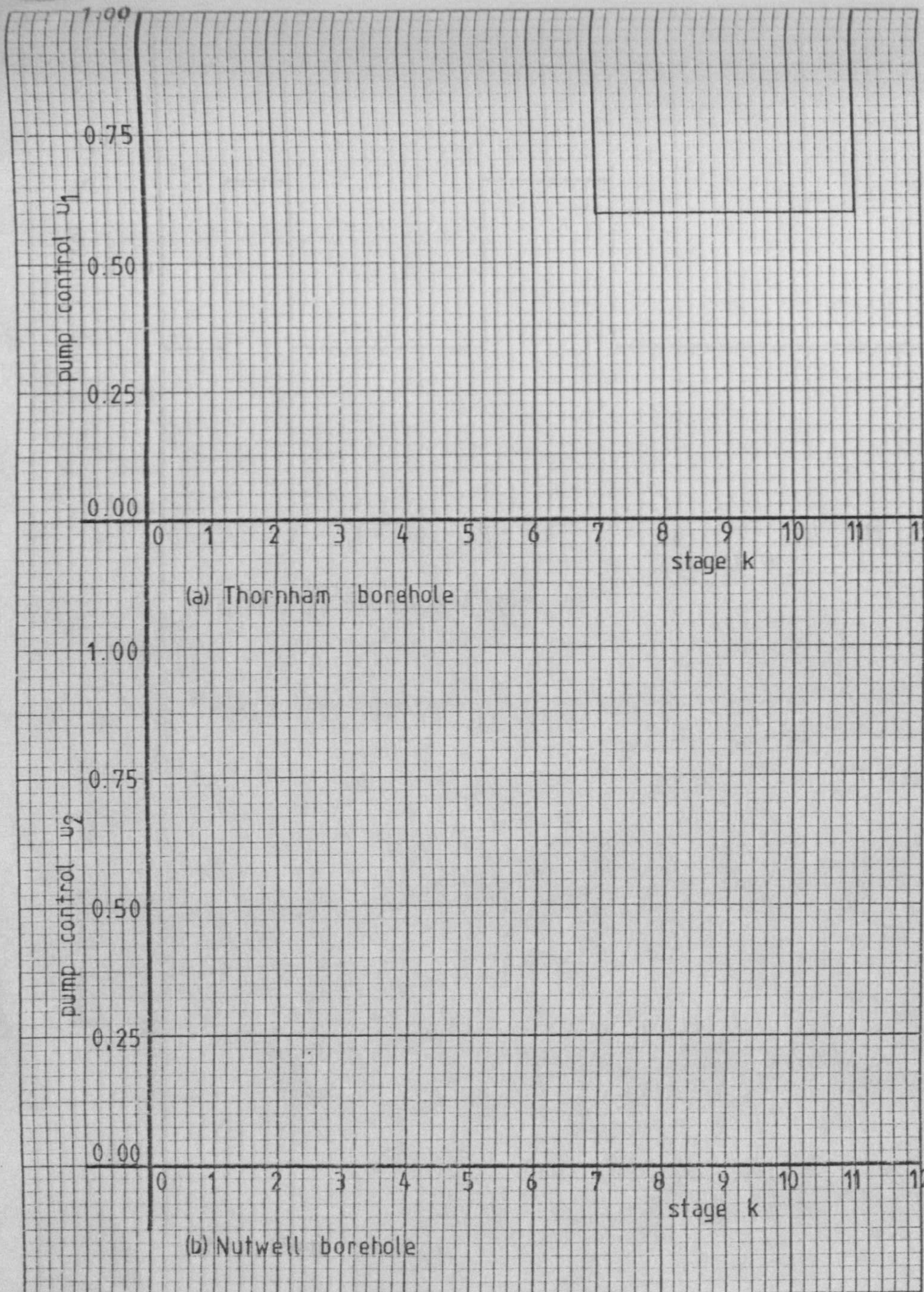


Fig. 6-5 Pump controls

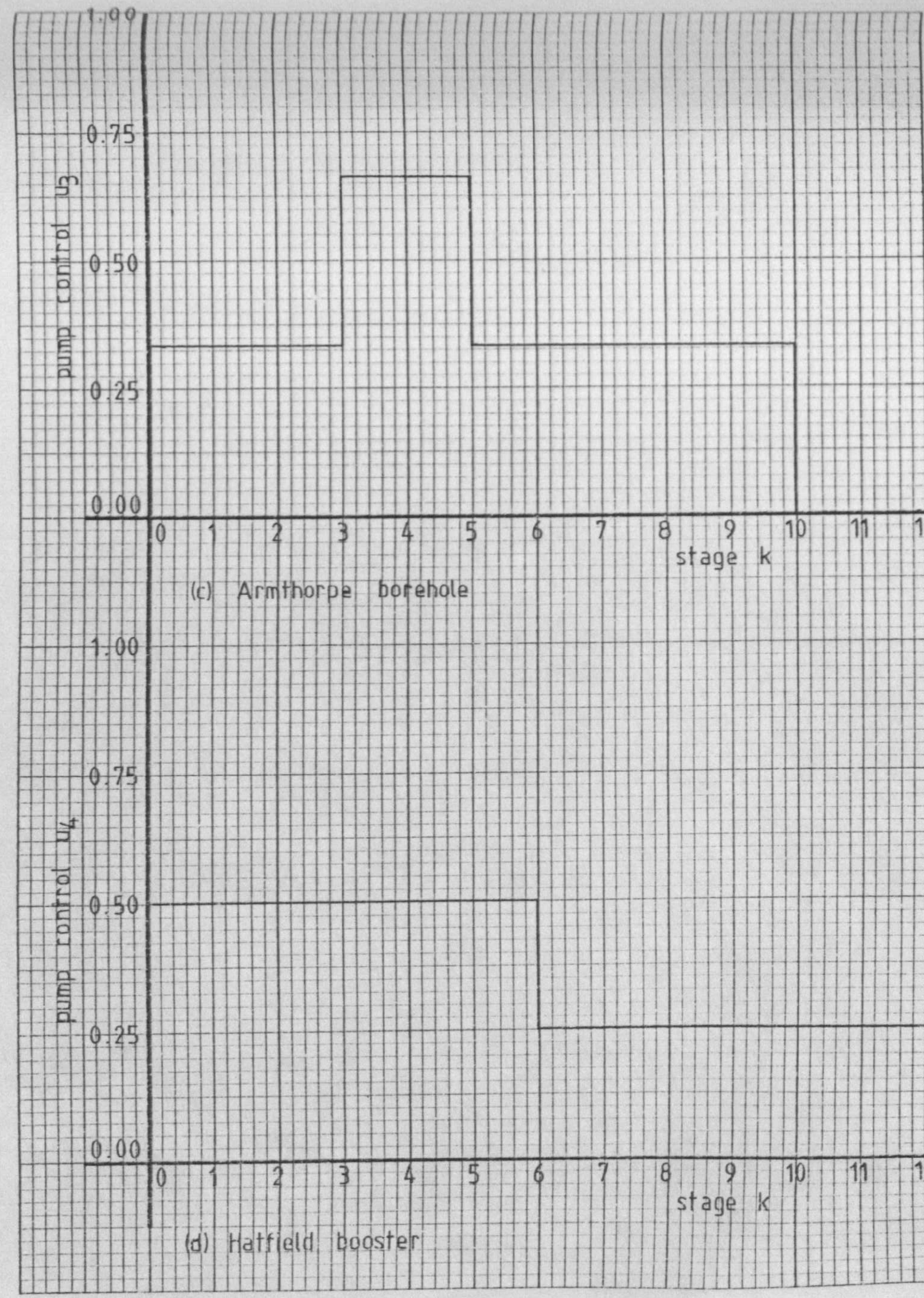


Fig. 6-5 Pump controls (continued)

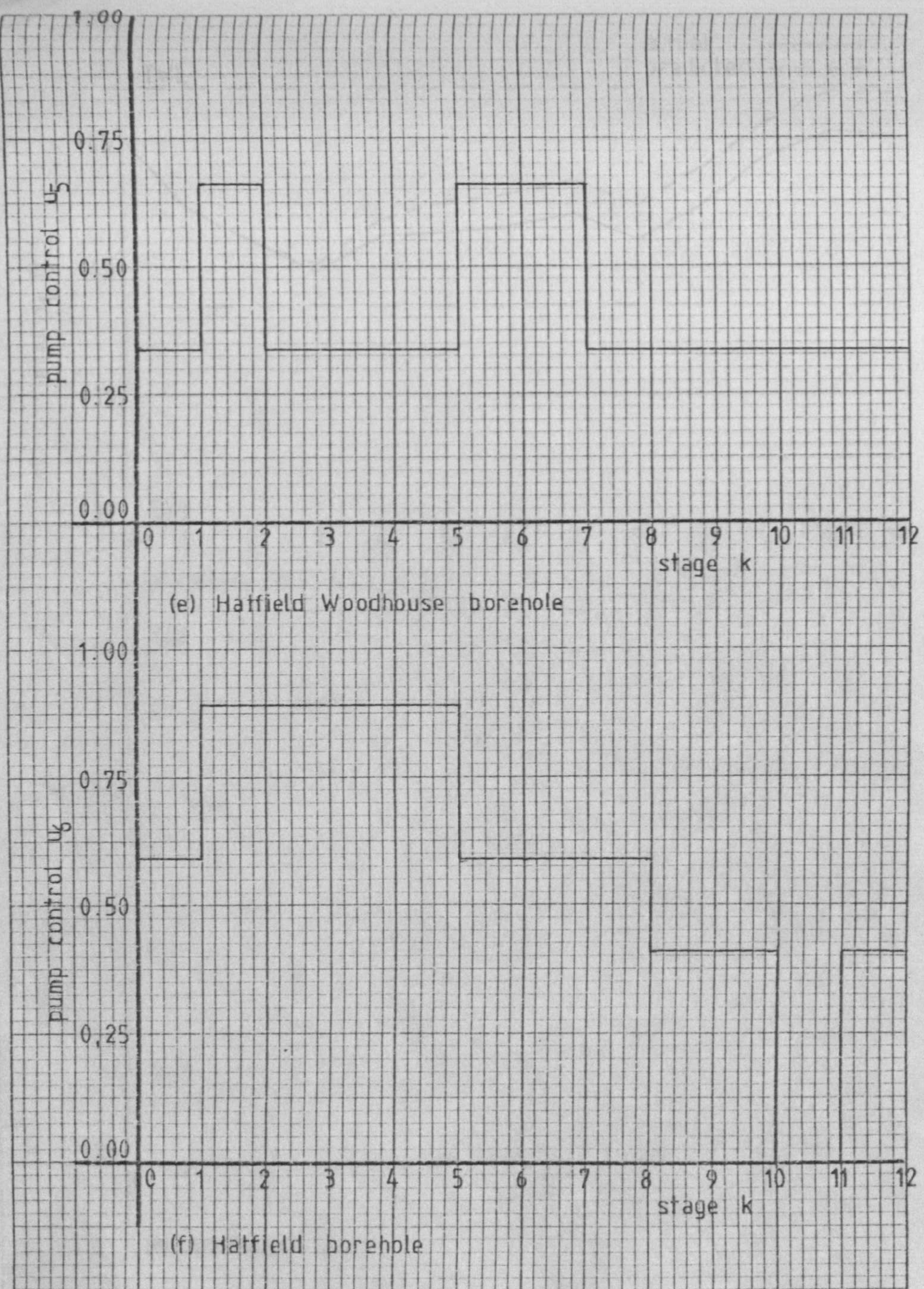


Fig. 6-5 Pump controls (continued)

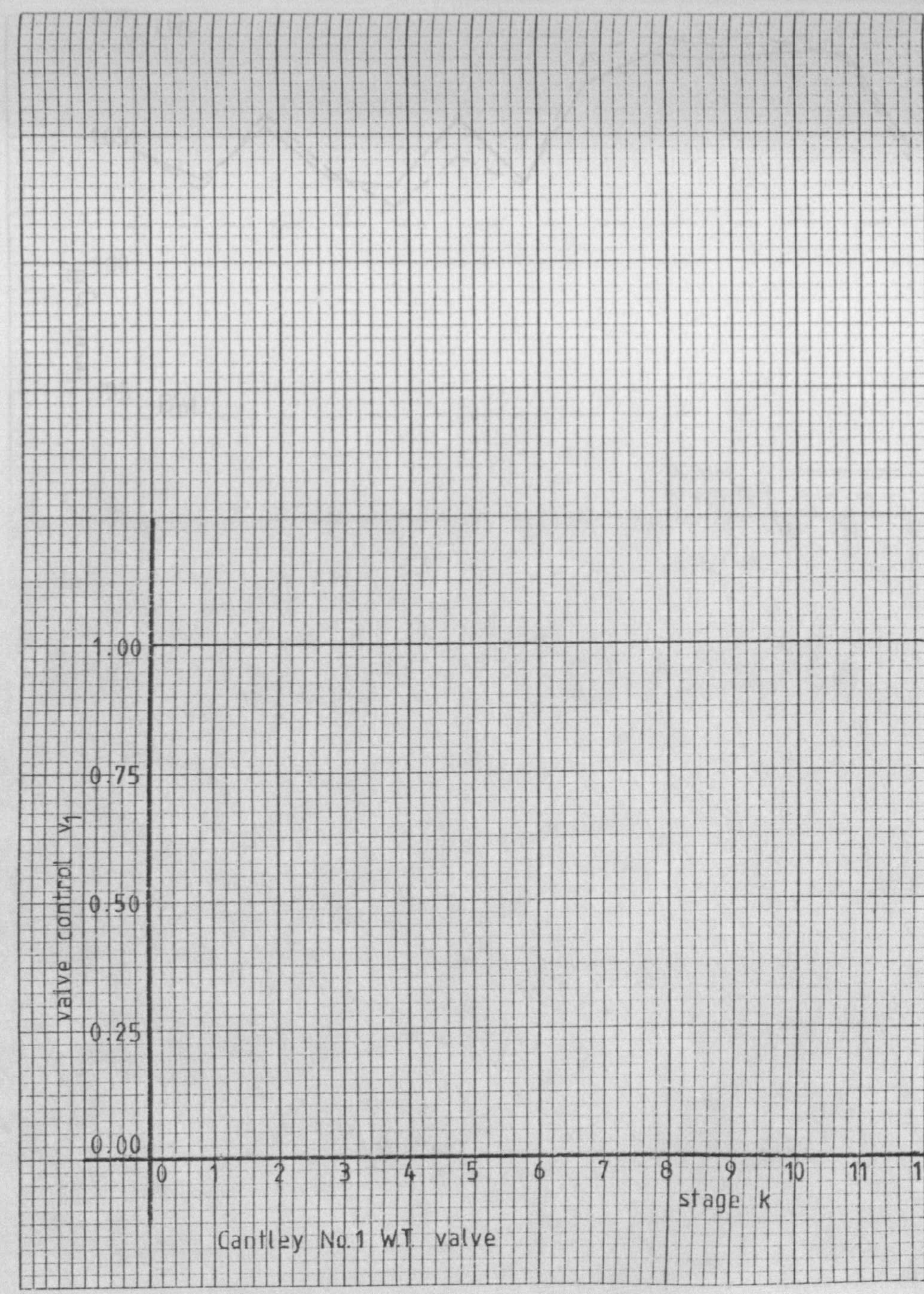


Fig. 6-6 Valve controls

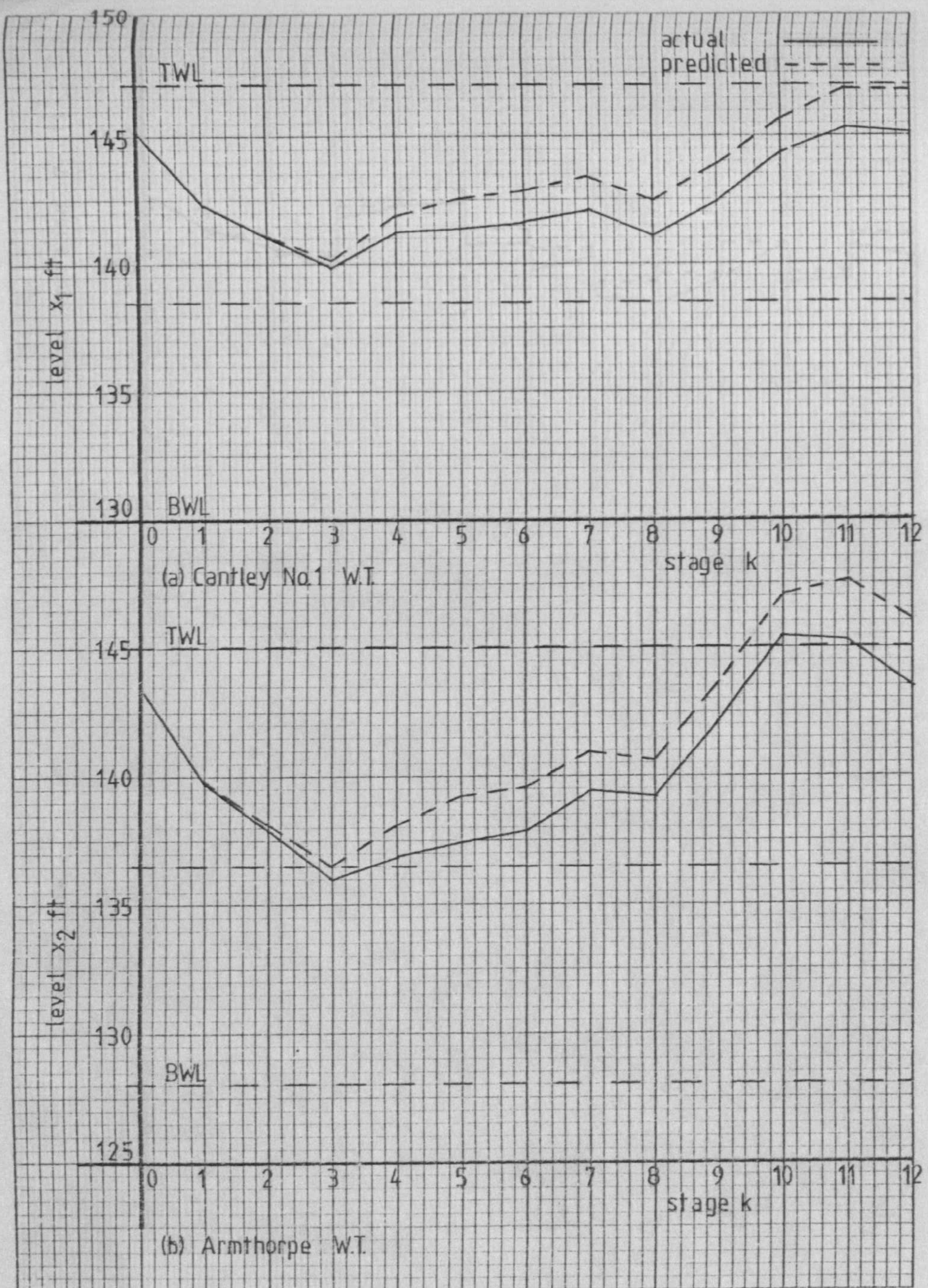


Fig. 6-7 Reservoir levels

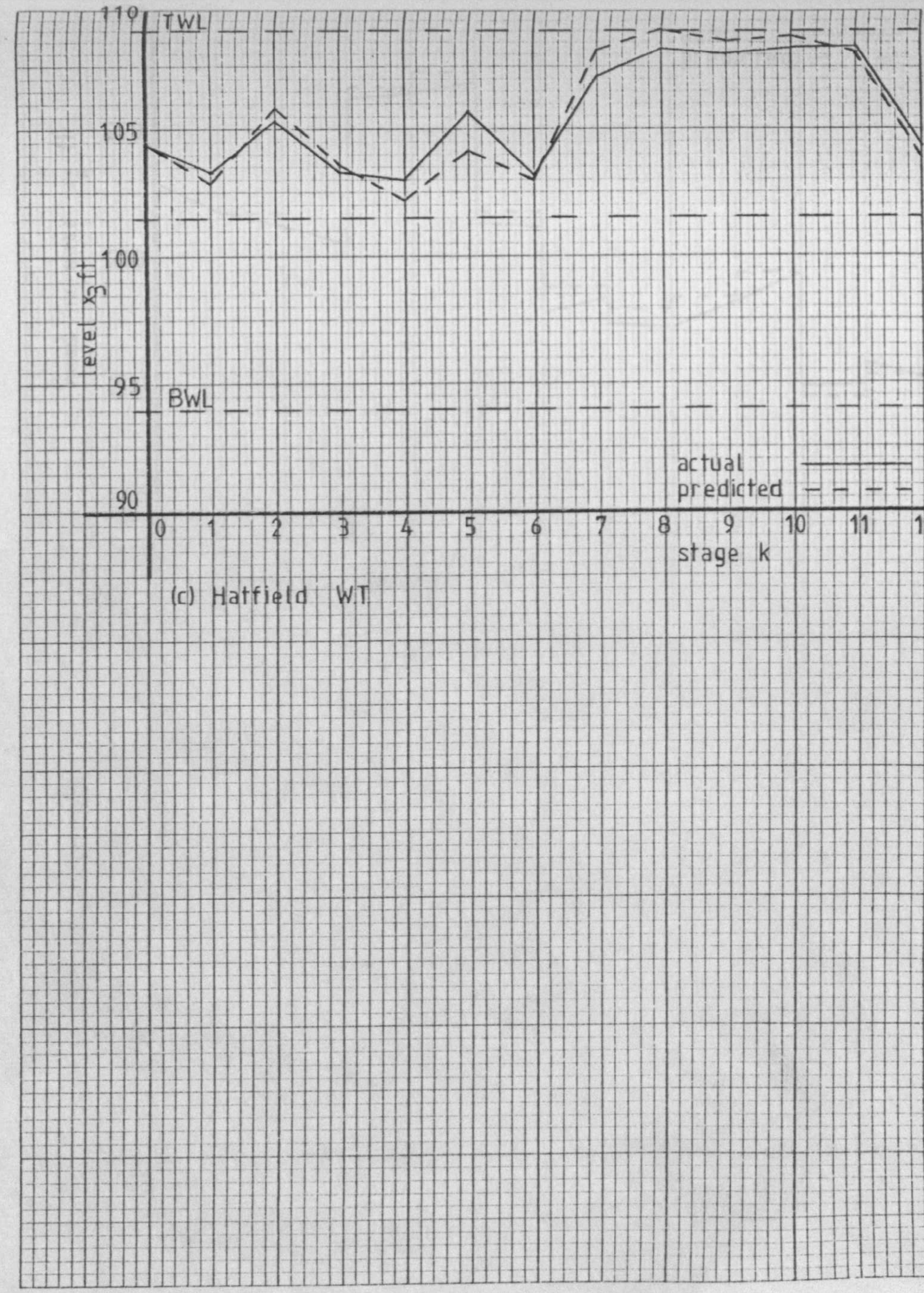


Fig. 6-7 Reservoir levels (continued)

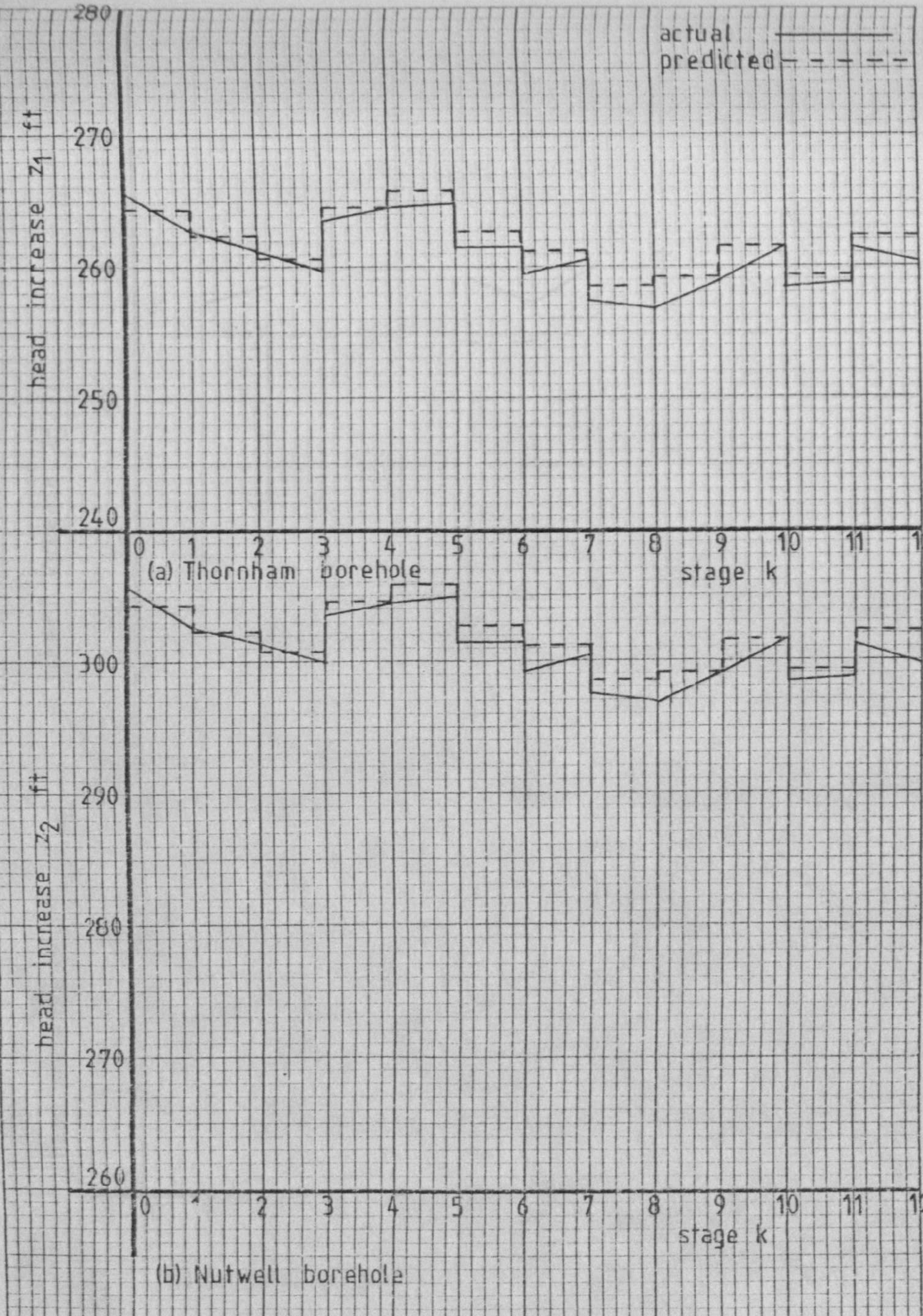


Fig. 6-8 Pumping station heads

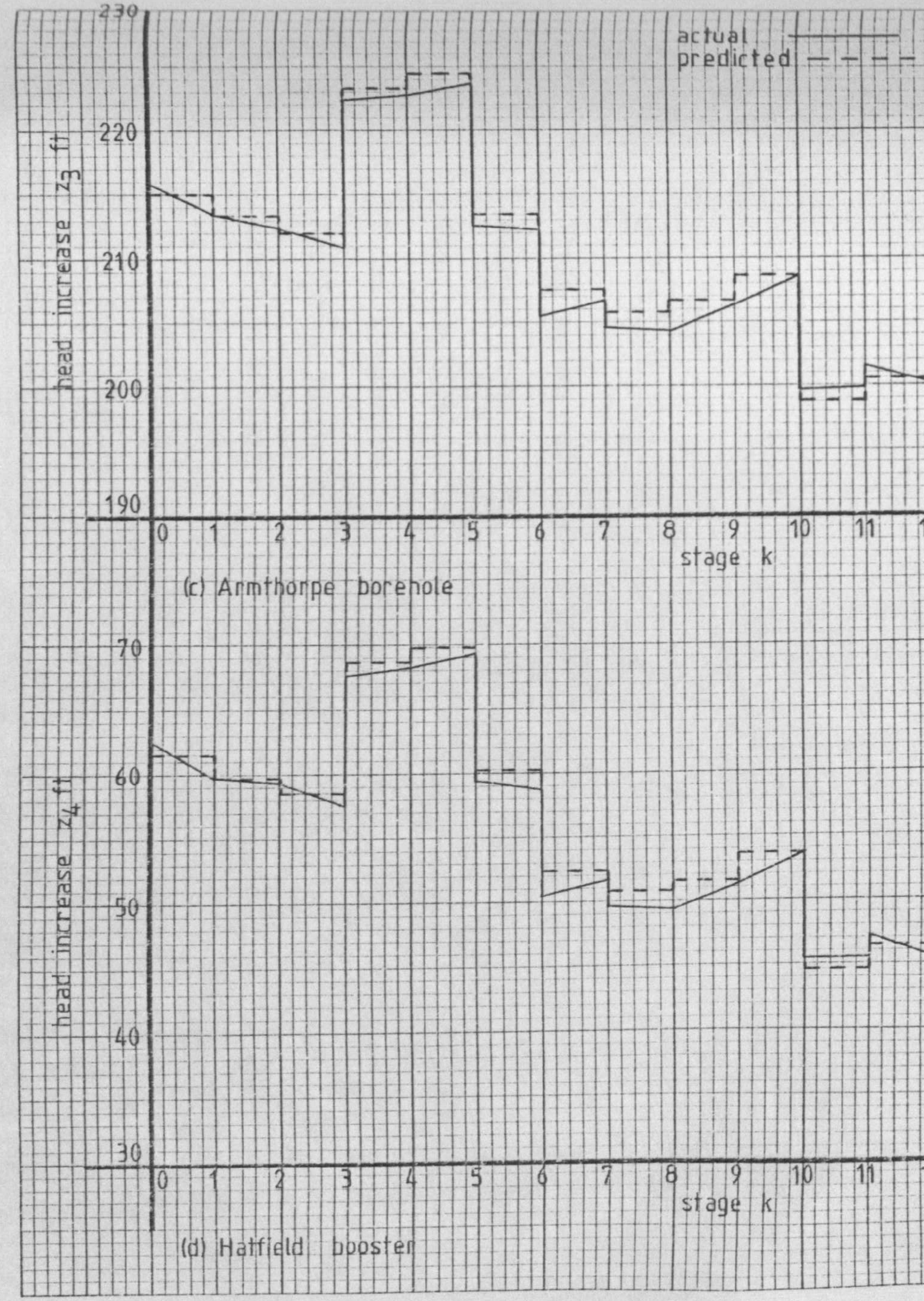


Fig. 6-8 Pumping station heads (continued)

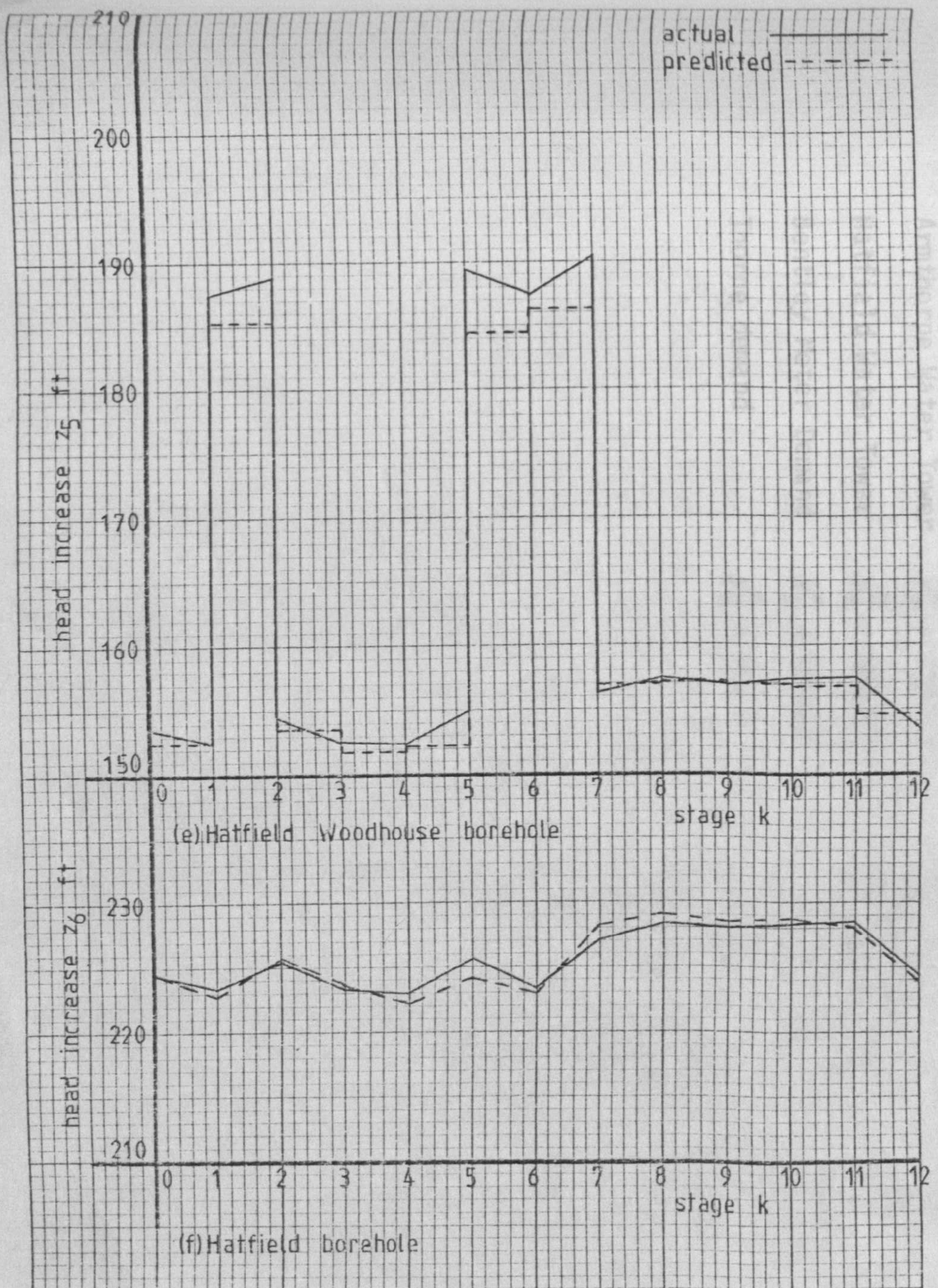


Fig. 6-8 Pumping station heads (continued)

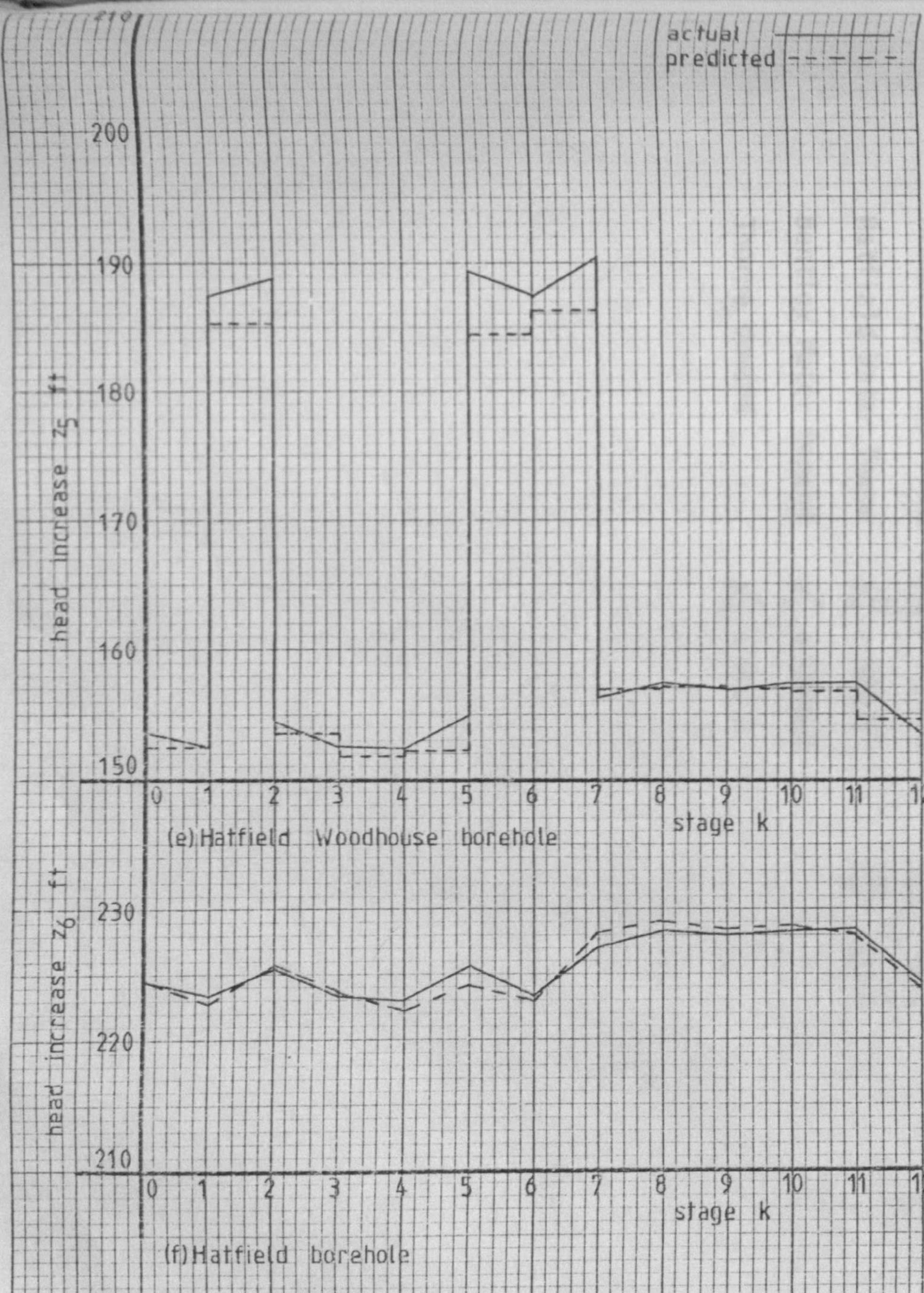


Fig. 6-8 Pumping station heads (continued)

Table 6-3. Average Operating Values for Doncaster Eastern and Thorne Zones.

These values relate to actual operating conditions.

Pumping Station	Pump Control	Pump Head Increase(ft)
Thornham borehole	$u_1^a = 0.8627$	$z_1^a = 260.6$
Nutwell borehole	$u_2^a = 0.2500$	$z_2^a = 300.6$
Armthorpe borehole	$u_3^a = 0.3233$	$z_3^a = 209.0$
Hatfield booster	$u_4^a = 0.3536$	$z_4^a = 54.7$
Hatfield Woodhouse borehole	$u_5^a = 0.4065$	$z_5^a = 161.3$
Hatfield borehole	$u_6^a = 0.5932$	$z_6^a = 225.6$

Reservoirs & Demands	Demand Flow(gpm)	Level(ft)	Valve Control
Cantley No.1.Water Tower	$y_1^a = 1103.0$	$x_1^a = 142.3$	$v_1^a = 1.0$
Armthorpe Water Tower	$y_2^a = 1282.0$	$x_2^a = 140.3$	
Hatfield Water Tower	$y_4^a = 829.1$	$x_3^a = 105.6$	
Bentley Meter Demand	$y_3^a = 1000.0$		
Thorne demand	$y_5^a = 500.0$		

Table 6-4. Linear Model Parameters for Doncaster Eastern and Thorne Zones

These values relate to actual operating conditions.

$$\underline{A}_x' = \begin{bmatrix} 0.7175 & 0.2317 & 0.0228 \\ 0.2780 & 0.6467 & 0.0361 \\ 0.0402 & 0.0532 & 0.8138 \end{bmatrix} \quad \text{ft/ft}$$

$$\underline{B}_x' = \begin{bmatrix} 4.1734 & 3.8530 & 6.6588 & 5.1728 & 0.4197 & 0.1836 \\ 5.2940 & 4.9092 & 10.267 & 8.2450 & 0.7164 & 0.3134 \\ 0.8941 & 0.8292 & 4.3596 & -22.630 & 33.169 & 14.509 \end{bmatrix} \quad \text{ft/unit pump control}$$

$$\underline{C}_x' = \begin{bmatrix} 5.7181 & 1.0915 & 1.4988 & 0.1373 & 0.1193 \\ 1.0915 & 6.5257 & 6.0413 & 0.2342 & 0.2036 \\ 0.1373 & 0.2342 & 0.2309 & 10.840 & 9.4342 \end{bmatrix} \quad \text{ft/}10^3\text{gpm}$$

$$\underline{D}_x' = \begin{bmatrix} 1.4219 \\ 1.3271 \\ 0.2481 \end{bmatrix} \quad \text{ft/unit valve control}$$

$$\underline{A}_z' = \begin{bmatrix} 0.4295 & 0.4541 & 0.0521 \\ 0.4284 & 0.4550 & 0.0522 \\ 0.3395 & 0.4363 & 0.1260 \\ 0.3212 & 0.4286 & 0.1572 \\ 0.0176 & 0.0250 & 0.7862 \end{bmatrix} \quad \text{ft/ft}$$

$$\underline{B}_z' = \begin{bmatrix} 9.6535 & 8.9099 & 15.170 & 11.790 & 0.8966 & 0.3922 \\ 9.6274 & 8.9294 & 15.203 & 11.816 & 0.8985 & 0.3930 \\ 7.5161 & 6.9712 & 37.775 & 28.548 & 2.3087 & 1.0099 \\ 7.0855 & 6.5718 & 34.759 & 35.655 & 2.9043 & 1.2704 \\ 0.3647 & 0.3383 & 1.8936 & -10.244 & 110.60 & 6.5302 \end{bmatrix} \quad \text{ft/unit pump control}$$

Table 6-4 continued

$$\underline{C_z'} = \begin{bmatrix} 1.4474 & 1.9040 & 1.8756 & 0.2930 & 0.2550 \\ 1.4434 & 1.9083 & 1.8791 & 0.2936 & 0.2556 \\ 1.1239 & 1.8435 & 1.8133 & 0.7546 & 0.6567 \\ 1.0586 & 1.8125 & 1.7881 & 0.9492 & 0.8261 \\ 0.0399 & 0.0706 & 0.0704 & 4.8789 & 30.857 \end{bmatrix} \quad \text{ft}/10^3\,\text{gpm}$$

$$\underline{D_z'} = \begin{bmatrix} 2.6958 \\ 2.6886 \\ 2.1005 \\ 1.9806 \\ 0.1094 \end{bmatrix} \quad \text{ft/unit valve control}$$

The largest source of error was found to be due to the use of overall average operating values which resulted in large control and demand deviations. The model translates these deviations into corresponding reservoir inflows and outflows based on coefficients obtained as point derivatives of non-linear functions. Differencing errors can thus occur as a result of combining approximately equal and opposite reservoir flows.

Considering that additional non-linearities had been introduced by incorporating head dependent pumps, and that the extreme control and demand deviations approximate to $\pm$ 100% (of the average operating values) it was concluded that creditable results had been achieved. These results are considered to be sufficiently accurate for the present purpose and to justify use of the model for optimisation purposes.

Where improved accuracy is required the errors should be considerably reduced by evaluating stage varying model coefficients for known typical control and demand profiles. This would, however, require additional computing time and storage.

6.3.5 Discussion

This section has described the development and evaluation of simplified linear dynamic models of complex water distribution systems. In this case the model consists of a set of linear equations, in terms of matrix coefficients and deviations from average operating values, which relate reservoir levels and pumping station heads to control and disturbance parameters. The model treats the case of head dependent pump and valve flow by means of independent parameters which can be used as continuously variable or discrete controls. Refinements to the basic model allow for true exponentially varying reservoir levels and for evaluation of average heads across pumping stations. A computer program has been developed for evaluation of the model coefficients as either constant average values or stage varying values constant over each time increment.

Because of the originality of the work, and hence lack of prior results, the model has proceeded through a number of evolutionary stages with refinements to give greater accuracy. The validation has been by means of realistic networks with results at each stage of the development to verify the model relevance and accuracy.

It has been shown that, for the particular practical systems analysed, the derived linear dynamic equations give results consistent with measured values in spite of the large deviations in state, control and disturbance variables and the relatively small storage capability.

The model has wide application since it does not impose restrictions on the numbers of reservoirs, controls or distributed demands. Furthermore the formulation is suited to the limited monitored system data usually available. The method has the additional advantage of being suitable for on-line use where the coefficients can be rapidly evaluated to meet any change in network configuration as a result of emergency or maintenance requirements. This essentially implies a model hierarchy using a full network simulation at the upper level, catering for network or major operational changes, and a simplified model at the lower level, catering for normal operation.

6.4 OVERALL CONCLUSIONS

This chapter has demonstrated that it is possible to determine simplified mathematical models for complex non-linear water distribution systems containing any number of reservoirs, controls and distributed demands. The models are suitable for use with on-line computer control and require a minimum of monitored data. These features are particularly important in furthering the present study of computer controlled optimisation methods.

Two independent methods of modelling have been presented, one of which considers a non-linear discrete model and the other a linear continuous model. A comparison of their main features is given below:

(a) Non-linear model

- (i) Reduced set of non-linear equations
- (ii) Controls must be discrete
- (iii) Coefficients determined from actual operating data or from network simulation.
- (iv) Requires prediction of total demand.
- (v) Accuracy reduced if individual demands not proportional to total demand.

(b) Linear Model

- (i) Reduced set of linear equations.
- (ii) Controls can be continuous or discrete.
- (iii) Coefficients determined from network simulation with possible use of actual data.
- (iv) Requires prediction of major individual reservoir demands.
- (v) Accuracy reduced for large changes of variables.

Whilst both models achieve the same end result - prediction of system operation, the differing features mean that they will each be suitable for incorporation into different optimisation techniques.

The non-linear model is ideally suited for optimisation by dynamic programming which works best with discrete controls and can be used for non-linear system equations. Dynamic programming also requires evaluation of operating costs over each time increment that can be included in the non-linear model. In relation to the present study the forward dynamic programming method developed in Chapter 4, which currently deals with single reservoir networks, could be usefully extended to handle more complex systems. This is suggested as a topic for further research.

The linear model operates well with most optimisation methods. In particular the hierarchical technique, described in Chapter 4, which requires linear system equations and continuous controls, is capable of giving an overall optimised solution for a complete system. In this area the linear model provides an essential link between optimisation of single reservoir systems with fixed flow pumps and more realistic multi-reservoir systems with head dependent pump and valve flows. Further consideration is given to these aspects in Chapter 7, in order to obtain optimised system operation.

CHAPTER 7

OPTIMISATION OF SYSTEM OPERATION

7.1 Introduction

The main components of theory have now been developed to make possible an integrated overall dynamic optimisation and control algorithm for water distribution systems. The systems considered will be those consisting of interconnected multiple reservoir zones, catering for variable head pumping with continuous and discrete pump controls, and having continuous valve controls. This chapter extends selected optimisation techniques of chapter 4 by use of the linear dynamic models of chapter 6, which incorporate the pump models of chapter 3, to give optimal control strategies in accordance with the requirements of chapter 2. The algorithm is tested on an actual network using the simulation procedures of chapter 5.

The control problem essentially reduces to dynamic optimisation of high dimensional, constrained, non-linear systems containing interactive discrete and continuous control variables. This type of problem is notoriously difficult to solve but, in this instance, two main possibilities have evolved:

(i) use of dynamic programming, which can handle constrained non-linear systems with discrete and continuous control variables, with extensions to cover high dimensional systems

(ii) use of decentralised hierarchical techniques, which can handle high dimensional constrained systems, with extensions to cover non-linear systems with discrete controls

Both of these methods were investigated in chapter 4 where the systems were reduced to form single reservoir zones using volumetric balance relationships and other gross simplifications. However the development of the simplified network models of chapter 6 has removed these restrictions and allows for the possibility of treatment of realistic systems by either method.

The selection of the optimisation method is of the utmost importance and, based upon a comparison of the features and conclusions of chapter 4, the most appropriate is considered to be the decentralised hierarchical method. This technique dictates use of the linear dynamic network model which can be used in conjunction with the variable head-flow pump model to produce an algorithm giving a continuous control solution (but which contains an inherent discrete solution).

A major computational advantage of the chosen method is a means by which all operating costs and system constraints can be incorporated in an integrated fashion.

7.2 Formulation of System Control Model

The optimised control problem requires calculation of pump and valve control trajectories so as to minimise overall costs whilst providing predicted demands and operating within system constraints. This involves definition of appropriate system equations and performance indices which can be used in conjunction with the initial conditions to yield an optimal control sequence solution.

7.2.1 System Equations

The network relationships correspond to the generalised storage node and pressure node equations of the linear dynamic model in terms of stage invariant coefficients. From equations (6.38) and (6.39) these are:

$$d\underline{x}(k+1) = \underline{A}'x . d\underline{x}(k) + \underline{B}'x . d\underline{u}(k) + \underline{C}'x . d\underline{y}(k) + \underline{D}'x . d\underline{v}(k) \quad (7.1)$$

$$d\underline{z}(k) = \underline{A}'z . d\underline{x}(k) + \underline{B}'z . d\underline{u}(k) + \underline{C}'z . d\underline{y}(k) + \underline{D}'z . d\underline{v}(k) \quad (7.2)$$

The pumping station maximum pump control relationship corresponds to equation (4.71) to give:

$$d\underline{w} = d\underline{u}(k) + d\underline{t}(k) \quad (7.3)$$

The equations giving the variables in terms of the deviations from the average operating values are:

$$\underline{t}(k) = \underline{t}^a + d\underline{t}(k) \quad (7.4)$$

$$\underline{u}(k) = \underline{u}^a + d\underline{u}(k) \quad (7.5)$$

$$\underline{v}(k) = \underline{v}^a + d\underline{v}(k) \quad (7.6)$$

$$\underline{w} = \underline{w}^a + d\underline{w} \quad (7.7)$$

$$\underline{x}(k) = \underline{x}^a + d\underline{x}(k) \quad (7.8)$$

$$\underline{y}(k) = \underline{y}^a + d\underline{y}(k) \quad (7.9)$$

$$\underline{z}(k) = \underline{z}^a + d\underline{z}(k) \quad (7.10)$$

With the variables subject to the following set of generalised constraints:

$$\underline{t}_{min} \leq \underline{t}(k) \leq \underline{t}_{max} \quad (7.11)$$

$$\underline{u}_{min} \leq \underline{u}(k) \leq \underline{u}_{max} \quad (7.12)$$

$$\underline{v}_{min} \leq \underline{v}(k) \leq \underline{v}_{max} \quad (7.13)$$

$$\underline{w}_{min} \leq \underline{w} \leq \underline{w}_{max} \quad (7.14)$$

$$\underline{x}_{min} \leq \underline{x}(k) \leq \underline{x}_{max} \quad (7.15)$$

$$\underline{z}_{min} \leq \underline{z}(k) \leq \underline{z}_{max} \quad (7.16)$$

The fixed initial and desirable final values are defined as:

$$\underline{x}(o) = \underline{x}_o \quad (7.17)$$

$$\underline{x}(K) = \underline{x}_K \quad (7.18)$$

with $\underline{y}(k)$ known for k = 0,1, ... K-1.

Where

$\underline{t}(k)$ = M dimensional pump control deviation vector

$\underline{u}(k)$ = M dimensional pump control vector corresponding to proportion of total pumps in use

$\underline{v}(k)$ = R dimensional valve control vector corresponding to valve resistance

$\underline{w}$ = M dimensional maximum pump control vector corresponding to total pumps in use

$\underline{x}(k)$ = N dimensional storage node state vector corresponding to reservoir level

$\underline{y}(k)$ = L dimensional disturbance vector corresponding to distributed consumer water demand

$\underline{z}(k)$ = M dimensional pressure node state vector corresponding to pumping station head increase

subscript min = lower bound on variable

max = upper bound on variable

superscript a = average operating value

d = desired or design value

* = optimal value

$\underline{A}'x$ = NxN dimensional storage node state coefficient matrix

$\underline{B}'x$ = NxM dimensional storage node pump control coefficient matrix

$\underline{C}'x$ = NxL dimensional storage node distrubance coefficient matrix

$\underline{D}'x$ = NxR dimensional storage node valve control coefficient matrix

$\underline{A}'z$ = MxN dimensional pressure node state coefficient matrix

$\underline{B}'z$ = MxM dimensional pressure node pump control coefficient matrix

$\underline{C}'z$ = MxL dimensional pressure node disturbance coefficient matrix

$\underline{D}'z$ = MxR dimensional pressure node valve control coefficient matrix

7.2.2 Performance Index

A requirement for efficient application of the optimisation method is that quadratic cost factors be incorporated for all variables. Also it is essential to allocate realistic costs (which should include any constant uncontrollable costs) to all factors affecting the performance, since only then can system operation be compared on the same basis and allow optimal decisions to be made. To meet the above requirements the individual performance indices have been derived in the correct format and, wherever possible, the weighting factors are representative of true financial operating costs. In cases where the actual costs are negligible the weighting factors have been assigned low relative values to minimise unwanted penalties. In this respect it is important to ensure that variation of reservoir levels is freely permitted (within the constraints) to cater for short period water demands and prevent high electricity demand charges.

The individual performance indices, given below, are mainly based on those previously derived in section 4.3.1 modified by use of the versatile pumping station cost model of section 3.3.4. These are combined to form the performance index for overall system operation as:

$$J = Jt + Ju + Jv + Jw + Jx + Jz \qquad (7.19)$$

with individual indices defined below:

$$Jt = \frac{1}{2} \sum_{k=0}^{K-1} (\underline{t}(k) - \underline{t}^d)^T \underline{R}t\, (\underline{t}(k) - \underline{t}^d) \qquad (7.20)$$

where Jt = cost for deviation of pump control from maximum attained value (see equation (4.75))

$\underline{Rt}$ = MxM dimensional diagonal matrix of pump control deviation weighting factors assigned low relative values to minimise unwanted penalties.

$$Ju = \tfrac{1}{2} \sum_{k=0}^{K-1} \underline{Tu}(k) \left\{ \underline{Su}^T(\underline{u}(k) - \underline{u}^d) + (\underline{u}(k) - \underline{u}^d)^T \underline{Ru}(\underline{u}(k) - \underline{u}^d) \right\} \qquad (7.21)$$

where Ju = pumping cost for electricity unit charges (see equations (3.20) and (4.68)),

$\underline{Tu}(k)$ = MxM dimensional diagonal matrix whose elements correspond to pumping station electricity unit tariff values and include effects of time increment, Δt, for each value of k,

$\underline{Su}$ = M dimensional vector whose elements now correspond to linear relationship of power per unit pump, under maximum efficiency conditions, for each pumping station ,

$\underline{Ru}$ = MxM dimensional diagonal matrix whose elements correspond to quadratic relationship for power per unit pump for each pumping station. Under maximum efficiency conditions this term should be zero but it will be assigned a low relative value to permit a solution by the optimisation method.

$$Jv = \tfrac{1}{2} \sum_{k=0}^{K-1} (\underline{v}(k) - \underline{v}^d)^T \underline{Rv}(\underline{v}(k) - \underline{v}^d) \qquad (7.22)$$

where Jv = cost for valve control deviation from the design value

$\underline{Rv}$ = RxR dimensional diagonal matrix of valve control deviation weighting factors. In this instance it is assumed that valve operating costs are negligible and the weighting factors are assigned low relative values.

$$Jw = \tfrac{1}{2}\, \underline{Tw} \left\{ \underline{Sw}^T (\underline{w} - \underline{w}^d) + (\underline{w} - \underline{w}^d)^T \underline{Rw} (\underline{w} - \underline{w}^d) \right\} \tag{7.23}$$

where Jw = pumping cost for electricity maximum demand charges (see equations (3.21) and (4.70))

$\underline{Tw}$ = MxM dimensional diagonal matrix whose elements correspond to pumping station electricity demand tariff values for optimisation period

$\underline{Sw}$ = M dimensional vector as for $\underline{Su}$. In this instance $\underline{Sw} = \underline{Su}$ but this can be used to take account of differences between the two, because of electrical power factor, etc.

$\underline{Rw}$ = MxM dimensional diagonal matrix as for $\underline{Ru}$. In this instance also $\underline{Rw} = \underline{Ru}$.

$$Jx = \tfrac{1}{2}(\underline{x}(K)-\underline{x}_K^d)^T \underline{Q}x(K)(\underline{x}(K)-\underline{x}_K^d) + \tfrac{1}{2} \sum_{k=o}^{K-1} (\underline{x}(k)-\underline{x}^d)^T \underline{Q}x(k)(\underline{x}(k)-\underline{x}^d) \tag{7.24}$$

where Jx = cost for deviating from the desired storage node state (see equation(4.75))

$\underline{Q}x(k)$ = NxN dimensional diagonal matrix whose elements correspond to state weighting factors. For k = 0,1, ... K-1 the factors will be assigned low relative values and for k = K the factor will be empirically determined dependent upon importance of terminal state value.

$$Jz = \tfrac{1}{2} \sum_{k=o}^{K-1} (\underline{z}(k) - \underline{z}^d)^T \underline{Q}z(\underline{z}(k) - \underline{z}^d) \tag{7.25}$$

where Jz = cost for deviating from the desired pressure node state. Which, in this case, is used to maintain pumping station head increase at the optimal design values and give maximum efficiency operation (see equation (3.18))

$\underline{Q}z$ = MxM dimensional diagonal matrix whose elements correspond to pumping station costs for deviation from maximum efficiency.

7.2.3 Optimisation Equations

These are based on those derived in section 4.3.1 but now include pressure node equations requiring additional Lagrange multipliers. The overall Lagrangian can be expressed as the sum of the overall performance index together with the constraint factors for the maximum pump control, Lw, the storage node state, Lx, and the pressure node state, Lz, to give:

$$L = J + Lw + Lx + Lz \tag{7.26}$$

In terms of the system equations these are:

$$Lw = \sum_{k=0}^{K-1} \underline{p}_w(k)^T \left\{ -d\underline{w} + d\underline{u}(k) + d\underline{t}(k) \right\} \tag{7.27}$$

$$Lx = \sum_{k=0}^{K-1} \underline{p}_x(k)^T \left\{ -d\underline{x}(k+1) + \underline{A}'x.d\underline{x}(k) + \underline{B}'x.d\underline{u}(k) + \underline{C}'x.d\underline{y}(k) + \underline{D}'x.d\underline{v}(k) \right\} \tag{7.28}$$

$$Lz = \sum_{k=0}^{K-1} \underline{p}_z(k)^T \left\{ -d\underline{z}(k) + \underline{A}'z.d\underline{x}(k) + \underline{B}'z.d\underline{u}(k) + \underline{C}'z.d\underline{y}(k) + \underline{D}'z.d\underline{v}(k) \right\} \tag{7.29}$$

where $\underline{p}_w(k)$, $\underline{p}_x(k)$ and $\underline{p}_z(k)$ = time varying Lagrange multipliers of dimensions M, N and M respectively.

The overall Lagrangian can be minimised using the following equations for values between upper and lower bounds, outside these bounds the solutions will take on the nearest boundary value. All values are for $k = 0, 1, \ldots K-1$ unless otherwise stated.

$$\underline{t}(k)^* = -\underline{R}t^{-1}.\,\underline{p}_w(k) + \underline{t}^d \tag{7.30}$$

$$\underline{u}(k)^* = -\underline{R}u^{-1}.\,\underline{T}u(k)^{-1}\left\{ \underline{B}'x^T.\,\underline{p}_x(k) + \underline{B}'x^T.\,\underline{p}_z(k) + \underline{p}_w(k) \right\} - \tfrac{1}{2}\,\underline{R}u^{-1}.\,\underline{S}u + \underline{u}^d \tag{7.31}$$

$$\underline{v}(k)^* = -\underline{Rv}^{-1}\left\{\underline{D}'^T_x.\underline{p}_x(k) + \underline{D}'^T_z.\underline{p}_z(k)\right\} + \underline{v}^d \quad (7.32)$$

$$\underline{w}^* = \underline{Rw}^{-1}.\underline{Tw}^{-1}\sum_{k=o}^{K-1}\underline{p}_w(k) - \tfrac{1}{2}\underline{Rw}^{-1}.\underline{Sw} + \underline{w}^d \quad (7.33)$$

$$\underline{x}(k)^* = \underline{Q}x(k)^{-1}\left\{\underline{p}_x(k-1) - \underline{A}'^T_x.\underline{p}_x(k) - \underline{A}'^T_z.\underline{p}_z(k)\right\} + \underline{x}^d \quad (7.34)$$

$$\underline{x}(K)^* = \underline{Q}x(K)^{-1}.\underline{p}_x(K-1) + \underline{x}^d_K \quad (7.35)$$

$$\underline{z}(k)^* = \underline{Q}z^{-1}.\underline{p}_z(k) + \underline{z}^d \quad (7.36)$$

The minimised Lagrangian is defined to be the dual function, $\phi(\underline{p}_w, \underline{p}_x, \underline{p}_z)$, which can be maximised using the gradients:

$$\nabla\underline{p}_w = -d\underline{w} + d\underline{u}(k) + d\underline{t}(k) \quad (7.37)$$

$$\nabla\underline{p}_x = -d\underline{x}(k+1) + \underline{A}'x.d\underline{x}(k) + \underline{B}'x.d\underline{u}(k) + \underline{C}'x.d\underline{y}(k) + \underline{D}'x.d\underline{v}(k) \quad (7.38)$$

$$\nabla\underline{p}_z = -d\underline{z}(k) + \underline{A}'z.d\underline{x}(k) + \underline{B}'z.d\underline{u}(k) + \underline{C}'z.d\underline{y}(k) + \underline{D}'z.d\underline{v}(k) \quad (7.39)$$

The full decentralised hierarchical solution procedure follows that described in section 4.3.1.

7.3 Application to a Water Distribution System

7.3.1 System Description and Data

The combined Doncaster Eastern and Thorne Zones contain a selection of components typical of water distribution systems and is considered to constitute a suitable validation system posing a complex optimisation and modelling problem. The system diagram is shown in appendix 3, figure A3-1 and the network parameters are given in appendix 3, table A3-1. This system has been previously analysed in section 6.3.4(b) to determine the actual system demands (repeated in figure 7-1) and to yield a linear dynamic model for the actual operating conditions. For optimisation purposes, however, the operating conditions showed significant changes and a modified model has been evaluated (using the computing sequence described in section 6.3.4(b) and shown on figure 7-6) to suit the typical optimisation conditions. The modified average operating values and linear model coefficients are shown in tables 7-1 and 7-2 respectively.

The operational data from appendix 3 and the electricity tariffs have been evaluated to arrive at the set of nominal values for operational constraints and optimisation parameters in table 7-3, where the stage increment is 2 hours, starting from 0800 hours, to give a 24 hours optimisation period.

The borehole pump head increases were taken to be equal to the pump pressure heads minus the respective borehole levels (assumed constant). For Hatfield booster pump the suction head was taken to be the average level of Hatfield water tower; this latter level also serves as the pressure head for Hatfield borehole pumping station, reducing the number of pressure node equations to 5.

7.3.2 Analysis of Results

A computer program (MULTI 2) has been written in Fortran IV for water distribution system optimisation. This is based on the decentralised hierarchical techniques described in section 4.3 and incorporates the generalised equations of section 7.3. This program was used to perform the optimisation calculations but initial results gave slow convergence with excessive computation times. This was assumed to be because of the

Table 7-1 Average Operating Values for Doncaster Eastern and Thorne Zone

These values relate to optimised operating conditions.

Pumping Station	Pump Control	Pump Head Increase (ft)
Thornham borehole	$u_1^a = 0.9211$	$z_1^a = 260.5$
Nutwell borehole	$u_2^a = 0.1000$	$z_2^a = 300.5$
Armthorpe borehole	$u_3^a = 0.3500$	$z_3^a = 210.0$
Hatfield booster	$u_4^a = 0.3750$	$z_4^a = 55.8$
Hatfield Woodhouse borehold	$u_5^a = 0.7444$	$z_5^a = 197.5$
Hatfield borehole	$u_6^a = 0.0000$	$z_6^a = 225.6$

Reservoirs and Demands	Demand Flow (gpm)	Level (ft)	Valve Control
Cantley No. 1 water tower	$y_1^a = 1103.0$	$x_1^a = 142.3$	$v_1^a = 1.0000$
Armthorpe water tower	$y_2^a = 1282.0$	$x_2^a = 140.3$	
Hatfield water tower	$y_4^a = 829.1$	$x_3^a = 105.6$	
Bentley meter demand	$y_3^a = 1000.0$		
Thorne demand	$y_5^a = 500.0$		

Table 7-2 Linear Model Parameters for Doncaster Eastern and Thorne Zone

These values relate to optimised operating conditions.

$$\underline{dx}(k+1) = \underline{Ax}'.\underline{dx}(k) + \underline{Bx}'.\underline{du}(k) + \underline{Cx}'.\underline{dy}(k) + \underline{Dx}'.\underline{dv}(k)$$

$$\underline{dz}(k) = \underline{Az}'.\underline{dx}(k) + \underline{Bz}'.\underline{du}(k) + \underline{Cz}'\ \underline{dy}(k) + \underline{Dz}'.\underline{dv}(k)$$

$$\underline{Ax}' = \begin{bmatrix} 0.7152 & 0.2344 & 0.0244 \\ 0.2812 & 0.6444 & 0.0378 \\ 0.0431 & 0.0556 & 0.8169 \end{bmatrix} \quad \text{ft/ft}$$

$$\underline{Bx}' = \begin{bmatrix} 4.1341 & 3.7607 & 6.6461 & 5.0662 & 0.2871 & 0.1972 \\ 5.3613 & 5.0340 & 9.9470 & 7.8452 & 0.4758 & 0.3269 \\ 0.9589 & 0.9017 & 4.6664 & -21.959 & 21.157 & 14.535 \end{bmatrix} \quad \text{ft/unit pump control}$$

$$\underline{Cx}' \quad \begin{bmatrix} 5.7074 & 1.1060 & 1.5109 & 0.1474 & 0.0939 \\ 1.1049 & 6.5123 & 6.0298 & 0.2443 & 0.1557 \\ 0.1474 & 0.2443 & 0.2408 & 10.860 & 6.9284 \end{bmatrix} \quad \text{ft}/10^3\ \text{gpm}$$

$$\underline{Dx}' \quad \begin{bmatrix} -1.4210 \\ 1.3345 \\ 0.2642 \end{bmatrix} \quad \text{ft/unit valve control}$$

Table 7-2 continued. ...

$$\underline{A}_z' = \begin{bmatrix} 0.4253 & 0.4597 & 0.0558 \\ 0.4179 & 0.4662 & 0.0567 \\ 0.3407 & 0.4250 & 0.1355 \\ 0.3209 & 0.4161 & 0.1700 \\ 0.0138 & 0.0191 & 0.5774 \end{bmatrix} \quad \text{ft/ft}$$

$$\underline{B}_z' = \begin{bmatrix} 9.5065 & 8.6290 & 15.109 & 11.525 & 0.6121 & 0.4205 \\ 9.3197 & 8.7635 & 15.345 & 11.704 & 0.6225 & 0.4277 \\ 7.5283 & 7.0791 & 37.789 & 27.980 & 1.5819 & 1.0868 \\ 7.0662 & 6.2446 & 34.571 & 35.118 & 2.0015 & 1.3750 \\ 0.2870 & 0.2702 & 1.4872 & -7.2841 & 112.56 & 4.7929 \end{bmatrix} \quad \text{ft/unit pump control}$$

$$\underline{C}_z' = \begin{bmatrix} 1.4303 & 1.9299 & 1.8973 & 0.3142 & 0.2004 \\ 1.4017 & 1.9602 & 1.9227 & 0.3195 & 0.2039 \\ 1.1298 & 1.7927 & 1.7672 & 0.8119 & 0.5181 \\ 1.0592 & 1.7568 & 1.7375 & 1.0273 & 0.6554 \\ 0.0313 & 0.0541 & 0.0538 & 3.5810 & 36.866 \end{bmatrix} \quad \text{ft}/10^3\ \text{gpm}$$

$$\underline{D}_z' = \begin{bmatrix} 2.6342 \\ 2.5826 \\ 2.0875 \\ 1.9599 \\ 0.0855 \end{bmatrix} \quad \text{ft/unit valve control}$$

Table 7-3 Operational Constraints and Optimisation Parameters

Vector Element	1	2	3	4	5	6
$\underline{t}$ max	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0
$\underline{t}$ min	0.0	0.0	0.0	0.0	0.0	0.0
$\underline{u}$ max	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0
$\underline{u}$ min	0.0	0.0	0.0	0.0	0.0	0.0
$\underline{v}$ max	10.0					
$\underline{v}$ min	0.75					
$\underline{w}$ max	1.0	1.0	1.0	1.0	1.0	1.0
$\underline{w}$ min	0.0	0.0	0.0	0.0	0.0	0.0
$\underline{x}$ max ft	147.0	145.0	109.0			
$\underline{x}$ min ft	138.5	136.5	101.5			
$\underline{z}$ max ft	450.0	442.0	272.0	93.0	272.0	
$\underline{z}$ min ft	0.0	0.0	0.0	0.0	0.0	
$\underline{t}^d$	0.0					
$\underline{u}^d$	0.0					
$\underline{v}^d$	1.0					
$\underline{w}^d$	0.0					
$\underline{x}^d$ ft	142.3	140.3	105.6			
$\underline{x}_K^d$ ft	145.1	143.5	104.4			
$\underline{z}^d$ ft	264.0	267.0	201.0	50.0	201.0	
$\underline{x}_o$ ft	145.1	143.5	104.4			

Table 7-3 continued. ...

Matrix Diagonal Element			1	2	3	4	5	6
Qx(k)	k = 0,1, ... 11	£/(10ft)^2	1.0	1.0	1.0			
Q(K)	K = 12	£/(10ft)^2	10.0	10.0	10.0			
Qz(k)	k = 0,1, ... 11	£/(100ft)^2	1.0/10.0	1.0/10.0	1.0/10.0	1.0/10.0	1.0/10.0	
Rt		£/(unit pump control)^2	1.0	1.0	1.0	1.0	1.0	1.0
Ru		kW/(unit pump control)^2	13.8	16.0	22.5	6.0	22.5	14.62
Rv		£/(unit valve control)^2	1.0					
Rw		kVA/(unit pump control)^2	13.8	16.0	22.5	6.0	22.5	14.62
Su		kW/unit pump control	138.0	160.0	225.0	60.0	225.0	146.2
Sw		kVA/unit pump control	138.0	160.0	225.0	60.0	225.0	146.2
Tu(k)	k = 0,1, ... 7	£/kW for 2h	0.0128	0.0128	0.0128	0.0128	0.0128	0.0128
Tu(k)	k = 8,9,10,11	£/kW for 2h	0.0117	0.0117	0.0117	0.0117	0.0117	0.0117
Tw		£/kVA for 24 h	0.0338	0.0338	0.0338	0.0338	0.0338	0.0338

Vector and Matrix Dimensions

K = 12
L = 5
M = 6
N = 3
R = 1

large number of variables, and ill-conditioning caused by use of the generalised system equations (now in terms of practical units having different orders of magnitude). Satisfactory solution times were obtained by ensuring that the gradients (corresponding to imbalance of the sets of system equations (7.1), (7.2) and (7.3)) were of approximately equal magnitudes. This was accomplished by multiplying the sets of equations throughout by appropriate factors and, for a convergence criterion of 0.1%, the resulting accuracies were: u ± 0.1%, v ± 0.1%, x ± 0.01ft and z ± 0.10ft. The weighting factor values can also considerably influence the solution efficiency and typical times for the program on an ICL 1906S computer are as given in table 7-4. Whilst these times are not insignificant it should be noted that the full solution will be infrequently required. For instance, the maximum pump controls ($\underline{w}$), having once been established for worst case conditions, will hold for the complete electricity tariff period (e.g. 1 month). Also the times can all be drastically reduced by using close starting values, for the dual variables ($\underline{p}$), obtained from a previous typical solution. As an overall comparison a single 24 hour dynamic simulation, using WATSIM, takes 85 units of time whereas a complete optimisation calculation involving hundreds of trial dynamic trajectories may only take 96 units.

Use of the above program with the linear model and derived data of tables 7-1, 7-2 and 7-3 gave optimised state and control trajectories together with the operating costs (see table 7-4). A direct cost comparison is difficult to make in view of the empirical nature of the pump efficiency weighting factor. To allow for this $\underline{Q}z$ has been set to 1.0 and 10.0 and a comparison is made for each control case on the same basis of electricity unit and demand and pump efficiency costs.

Table 7-4 System Optimisation Results

Control	Dual Variables	Qz £/(100ft)^2 for 2 h	Computing time Units of time	Electricity unit cost £ for 24h	Electricity demand cost £ for 24h	Pump efficiency cost £ for 24h	Total cost £ for 24h
Actual pump control, $\underline{u}(k)$, with pre-set valve control $\underline{v}(k)$.	N/A	1.0	N/A	69.79	23.14	3.79	96.72
Discrete pumping using standard manual operating procedures.		10.0	N/A	69.79	23.14	37.88	130.81
Optimised pump control, $\underline{u}(k)^*$, and valve control, $\underline{v}(k)^*$. Free maximum pump control, $\underline{w}$, and pump head increase, $\underline{z}(k)$.	NK = 36	1.0	26	54.87	17.97	9.01	81.85
Continuous pumping to minimise electricity unit charges		10.0	26	54.87	17.97	90.08	162.92
Optimised pump control, $\underline{u}(k)^*$, within limits of optimised maximum pump control, $\underline{w}^*$, together with optimised valve control, $\underline{v}(k)^*$. Free pump head increase, $\underline{z}(k)$.	NK + MK = 108	1.0	54	55.50	15.70	8.09	79.29
Continuous pumping to minimise electricity unit and demand charges.		10.0	54	55.50	15.70	80.94	152.14
Optimised pump control, $\underline{u}(k)^*$, within limits of optimised maximum pump control, $\underline{w}^*$. Optimised valve control, $\underline{v}(k)^*$, and pump head increase, $\underline{z}(k)^*$.	NK + MK + (M-1)K = 168	1.0	96	56.33	15.80	4.77	76.90
Continuous pumping to minimise electricity unit and demand charges and operate pumps at optimum efficiency.		10.0	700	63.43	17.36	16.75	97.54

Considering the results for optimised continuous pumping under least favourable conditions ($\underline{Q}z$ set to 1.0), table 7-4 shows that a cost improvement of approximately 3% is obtained for each additional control feature included in the optimisation. This result justifies the additional complexity of the model and the optimisation method.

Table 7-4 also shows the costs incurred under actual operating conditions. However, since this is for discrete pumps, the only conclusion that can be drawn is that use of continuously variable pumps could result in a substantial reduction of the indicated operating costs.

The optimised pump and valve controls are shown as figures 7-2 and 7-3, respectively, together with a comparison of the actual operating values. Similarly the resultant reservoir level and pumping station head increase trajectories are shown on figures 7-4 and 7-5. The above figures have all been obtained for the latter case of table 7-4 with the weighting factor, $\underline{Q}z$, set to 10.0 to emphasize the effect of increased pump efficiency.

The significant features of the optimal controls which ensure practical economic operation are:

(i) the electricity maximum demands have been minimised by limiting the maximum proportion of pumps in use at each pumping station,

(ii) the stage variable pump controls operate at the maximum permitted value whenever possible and required,

(iii) the pumping station head increases have been moved as close as possible to the design values to give pump operation at maximum efficiency,

(iv) with one exception the optimised values agree, in general, with the actual pumping policy, thus giving some confidence in the results. The exception, of Hatfield borehole pumping station, is explained by the practical consideration of the current method of closed loop control for Hatfield water tower level,

(v) the control valve must be continuously adjusted to achieve an optimal solution,

(vi) the reservoirs are topped up overnight to take advantage of the electricy night rebate tariff.

In order to confirm that valid and feasible results had been achieved the calculated optimised values were applied to control a dynamic simulation of the actual network (using the modified simulation program, WATSIM, in accordance with section 5.6.3(a)). The resulting simulated trajectories for the reservoir levels and the pumping station head increases are shown on figures 7-4 and 7-5. It will be noted that the simulated results show close agreement with those predicted by use of the linear dynamic model under optimised operation. The improvement in model accuracy,compared to section 6.3.4(b), is explained by the use of the smoothed optimal controls. This final method of validation thus gives full confidence in both the modelling and optimisation techniques.

A simplified diagram for the complete sequence of computing operations used in the analysis is shown in figure 7-6.

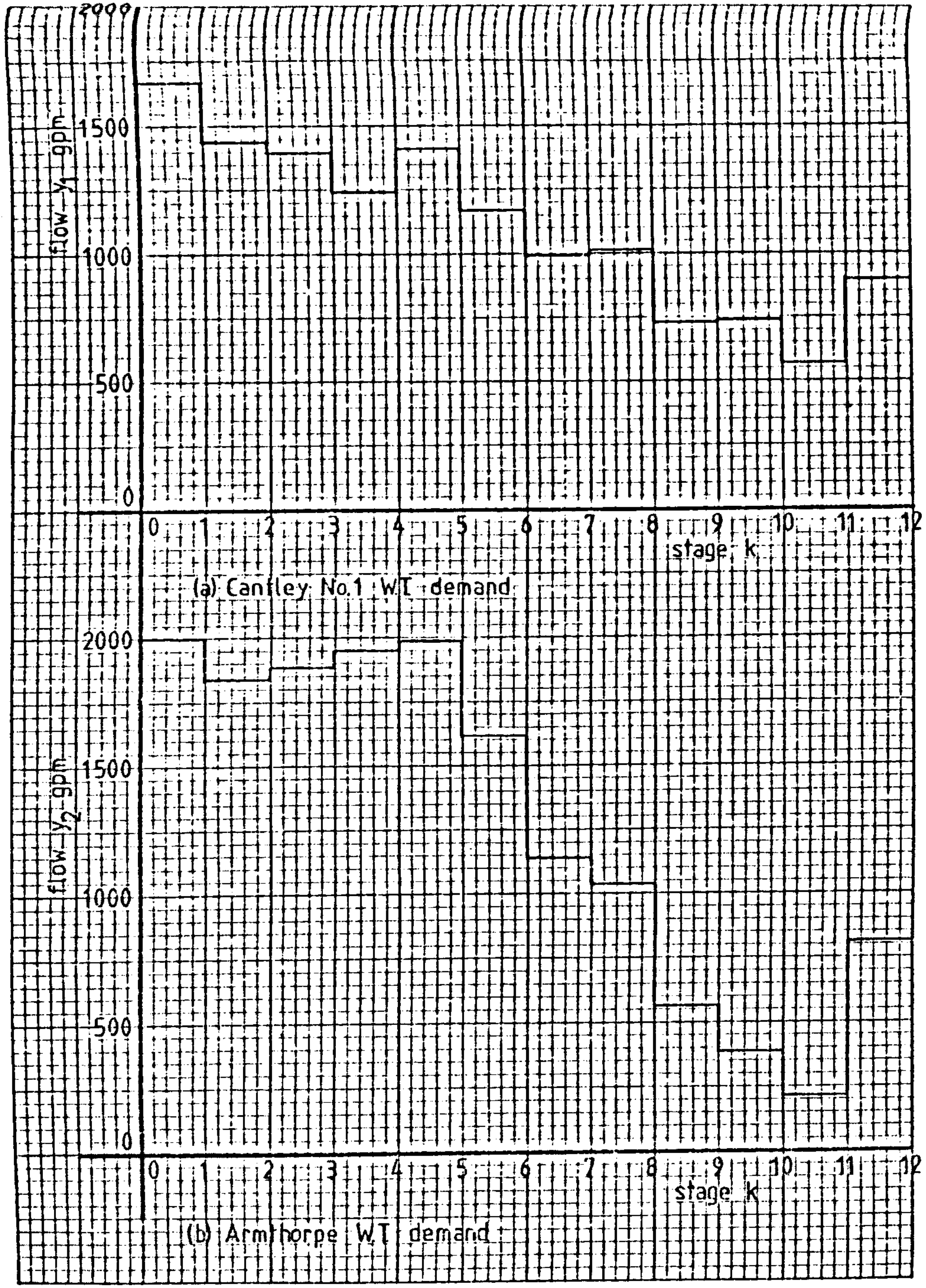


Fig. 7-1 System demands

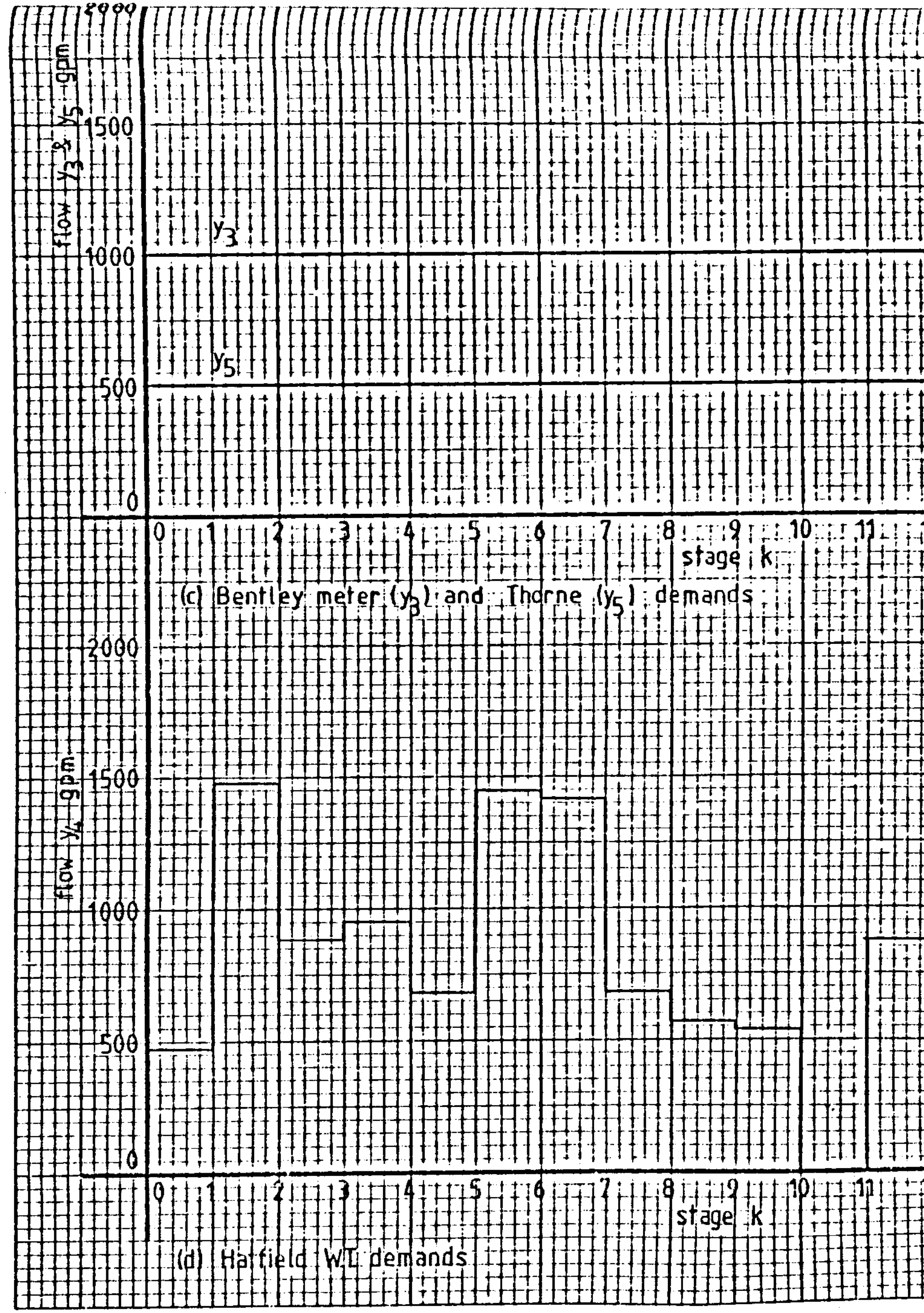


Fig. 7-1 System demands (continued)

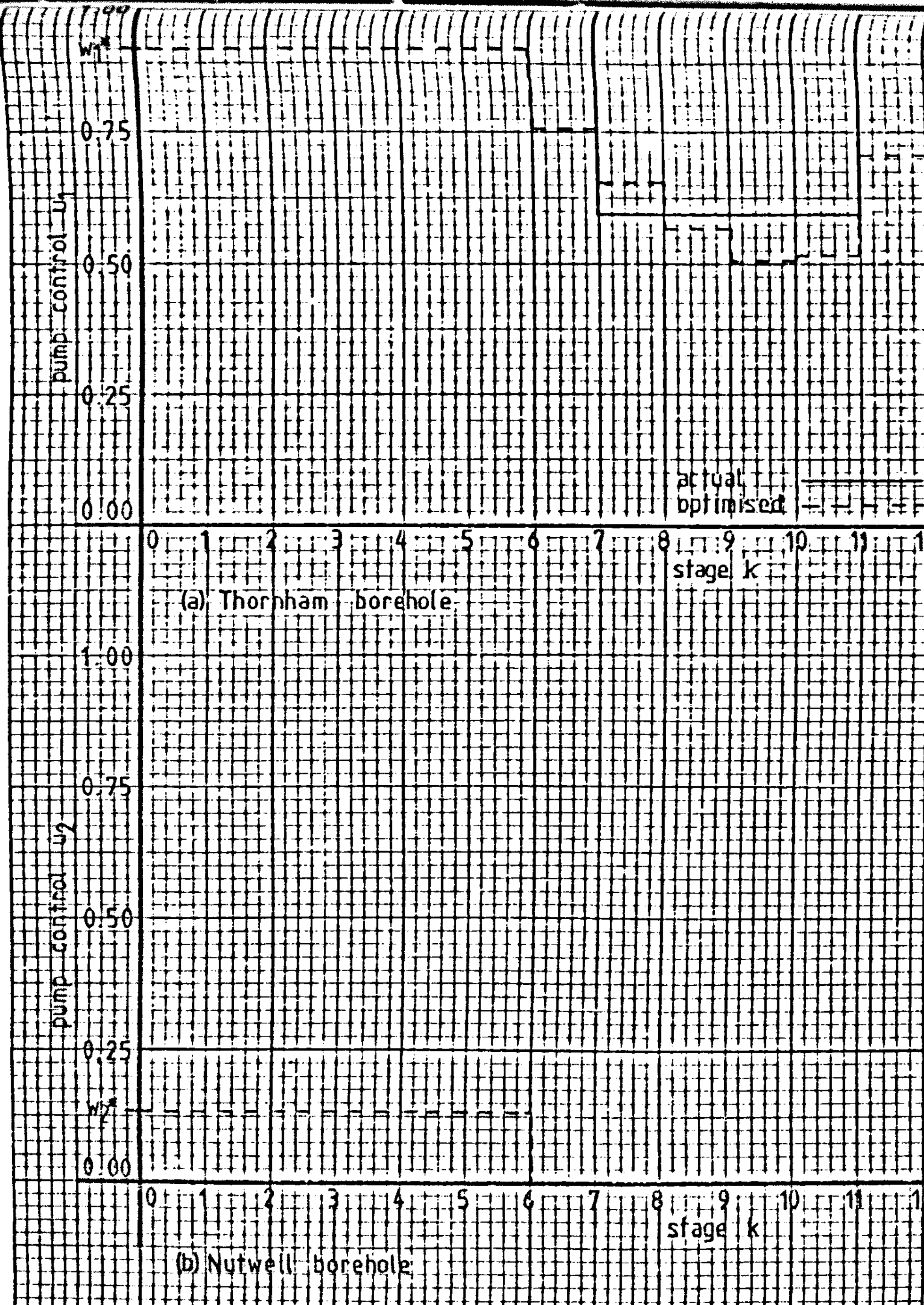


Fig. 7-2 Pump controls

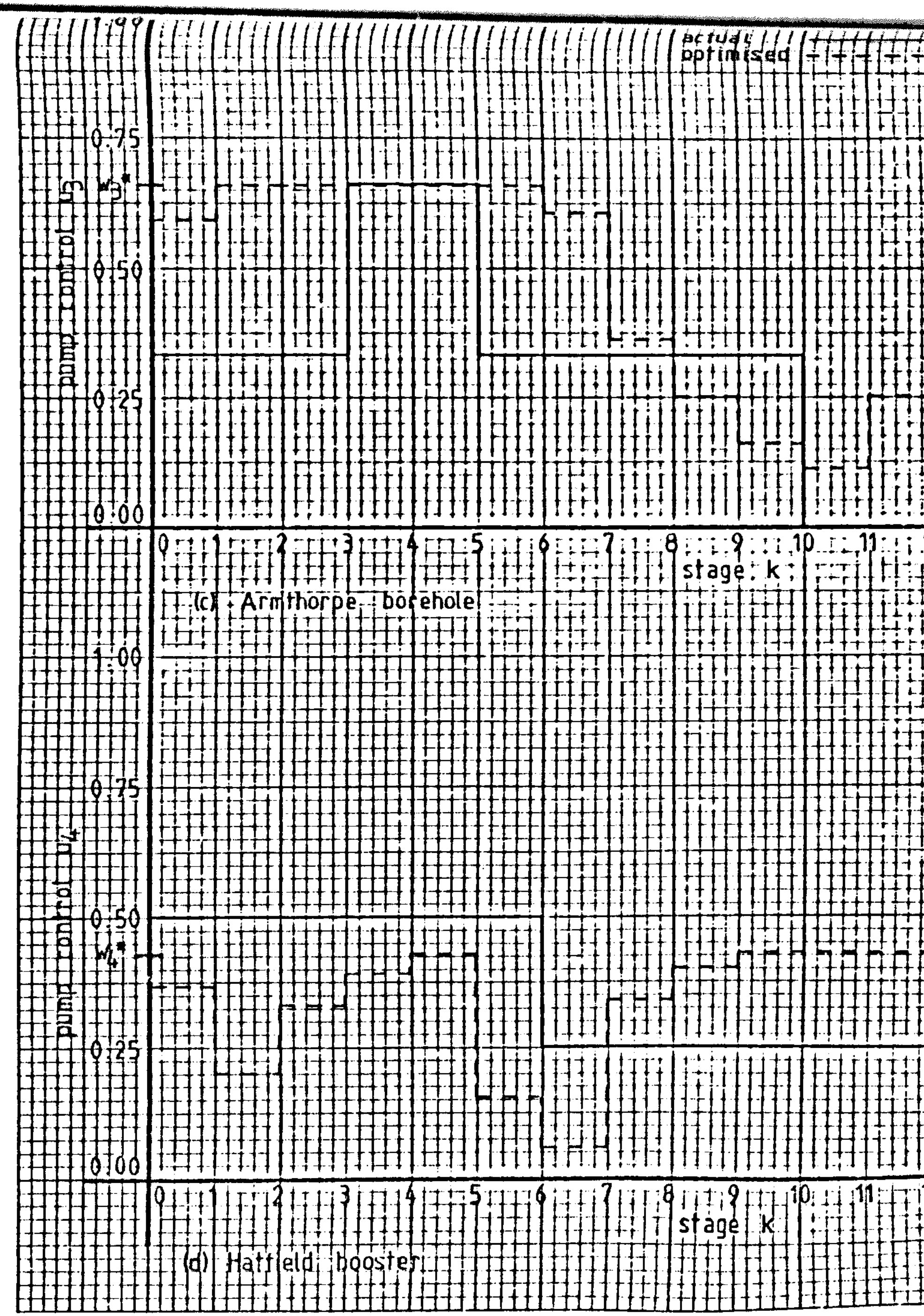


Fig. 7-2 Pump controls (continued)

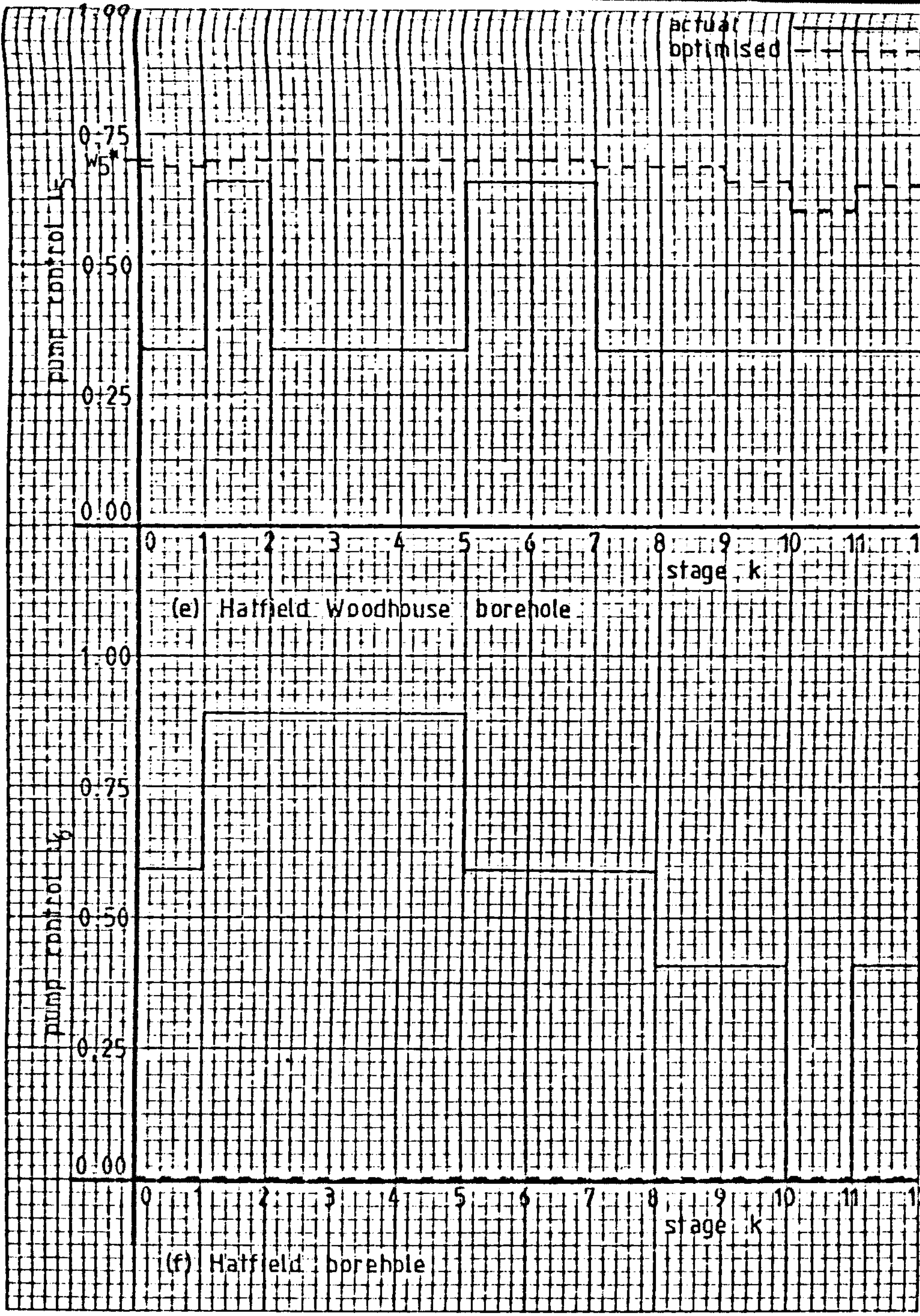


Fig. 7-2 Pump controls (continued)

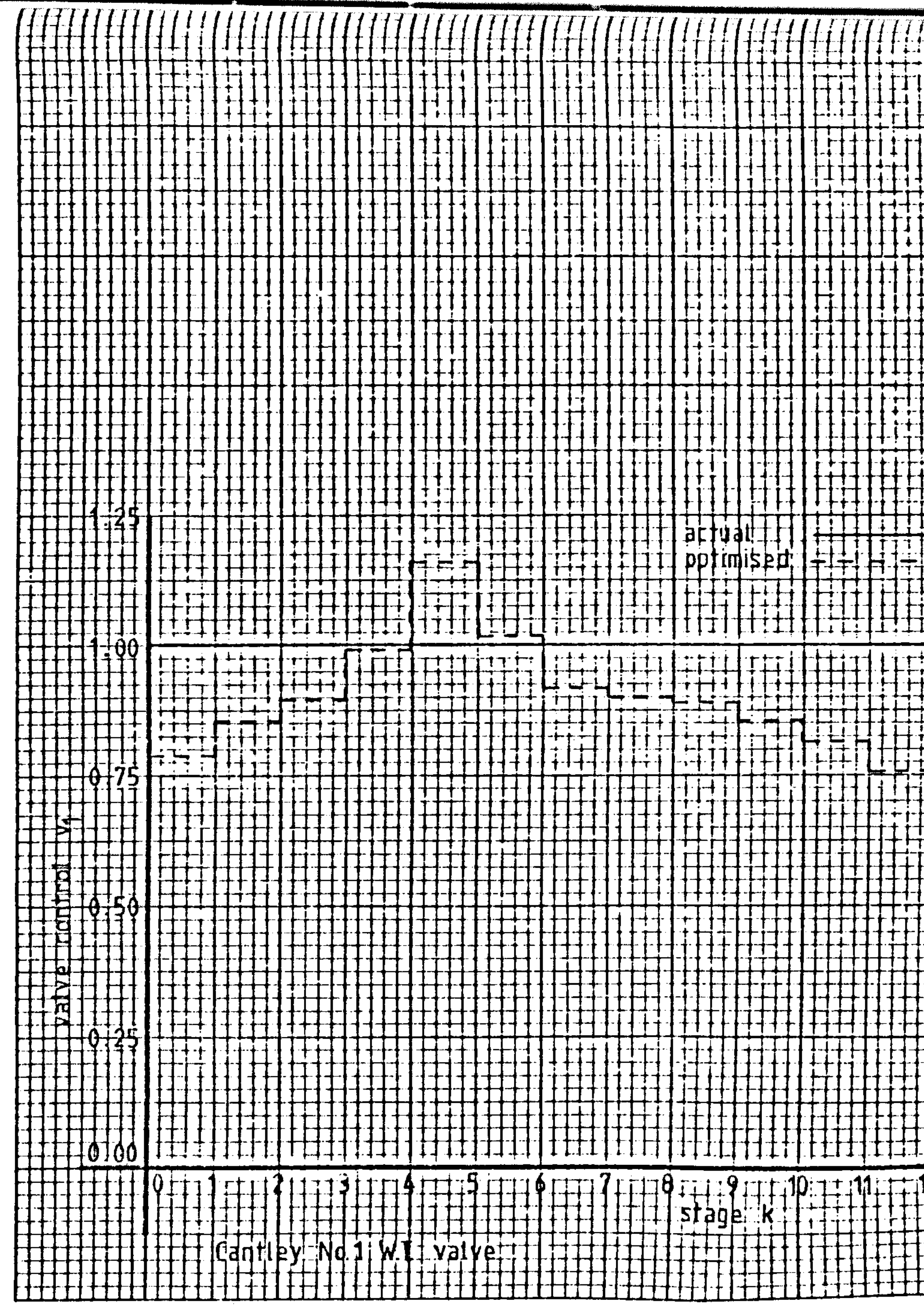


Fig. 7-3 Valve controls

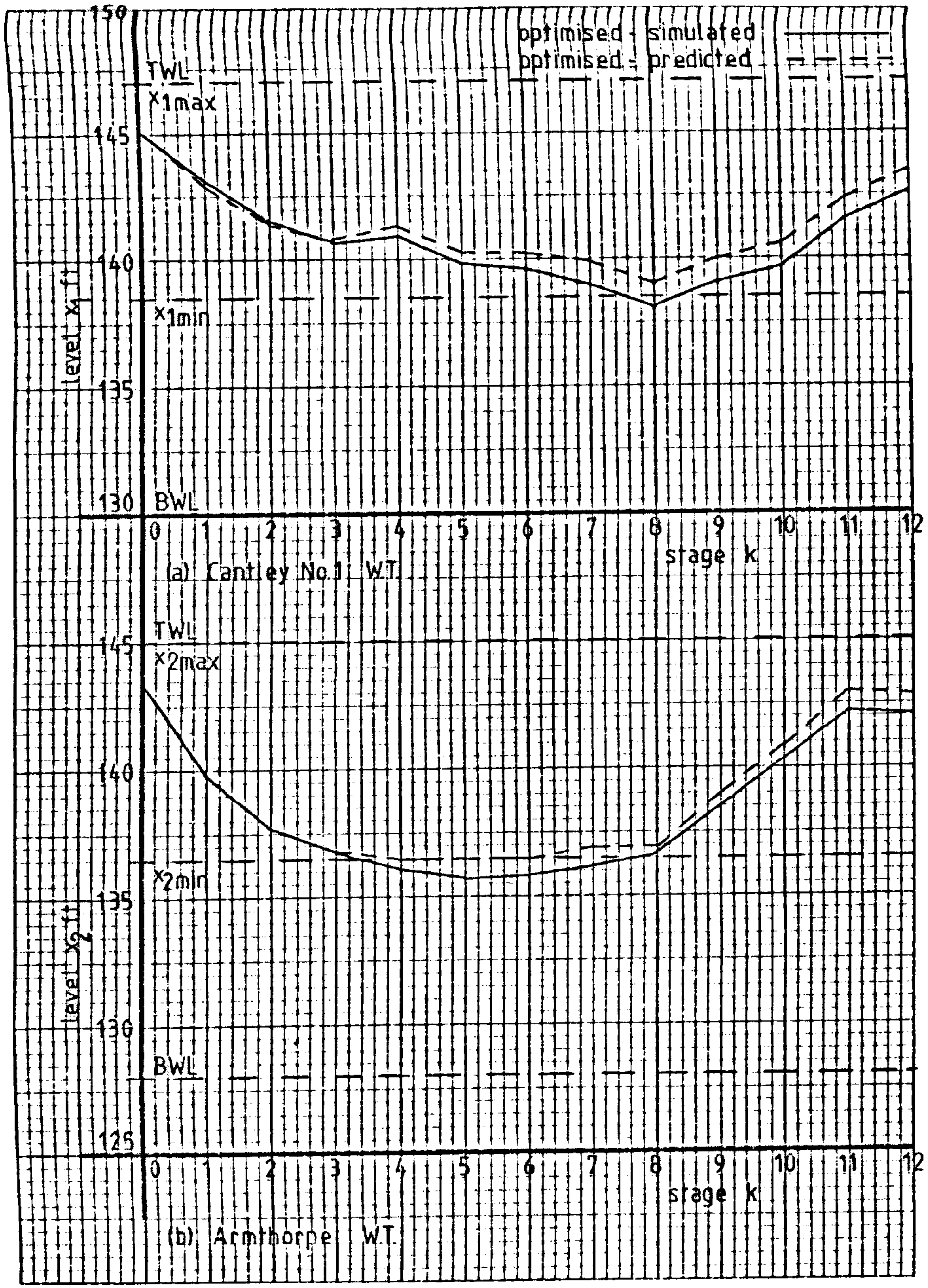


Fig. 7-4 Reservoir levels

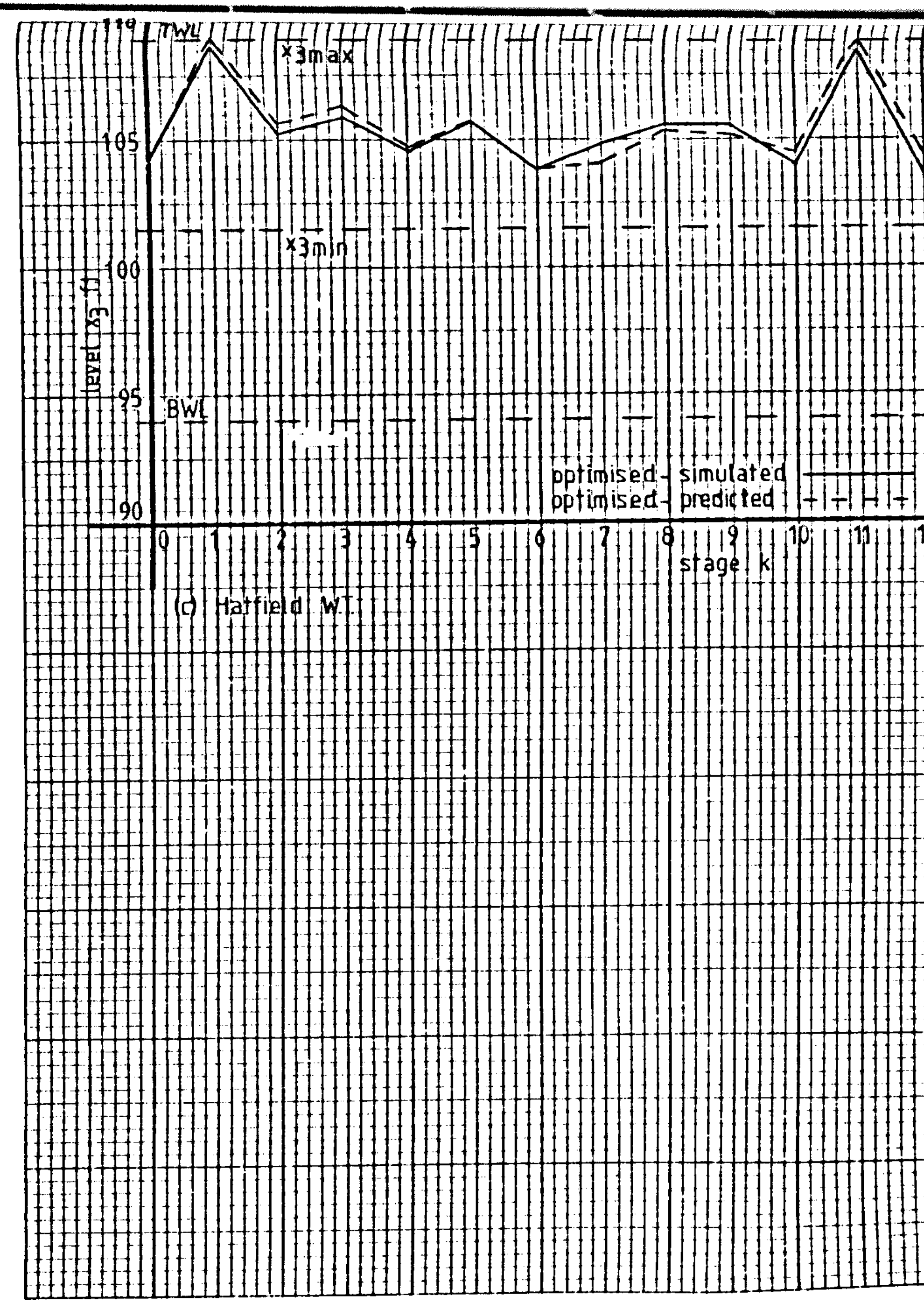


Fig. 7-4 Reservoir levels (continued)

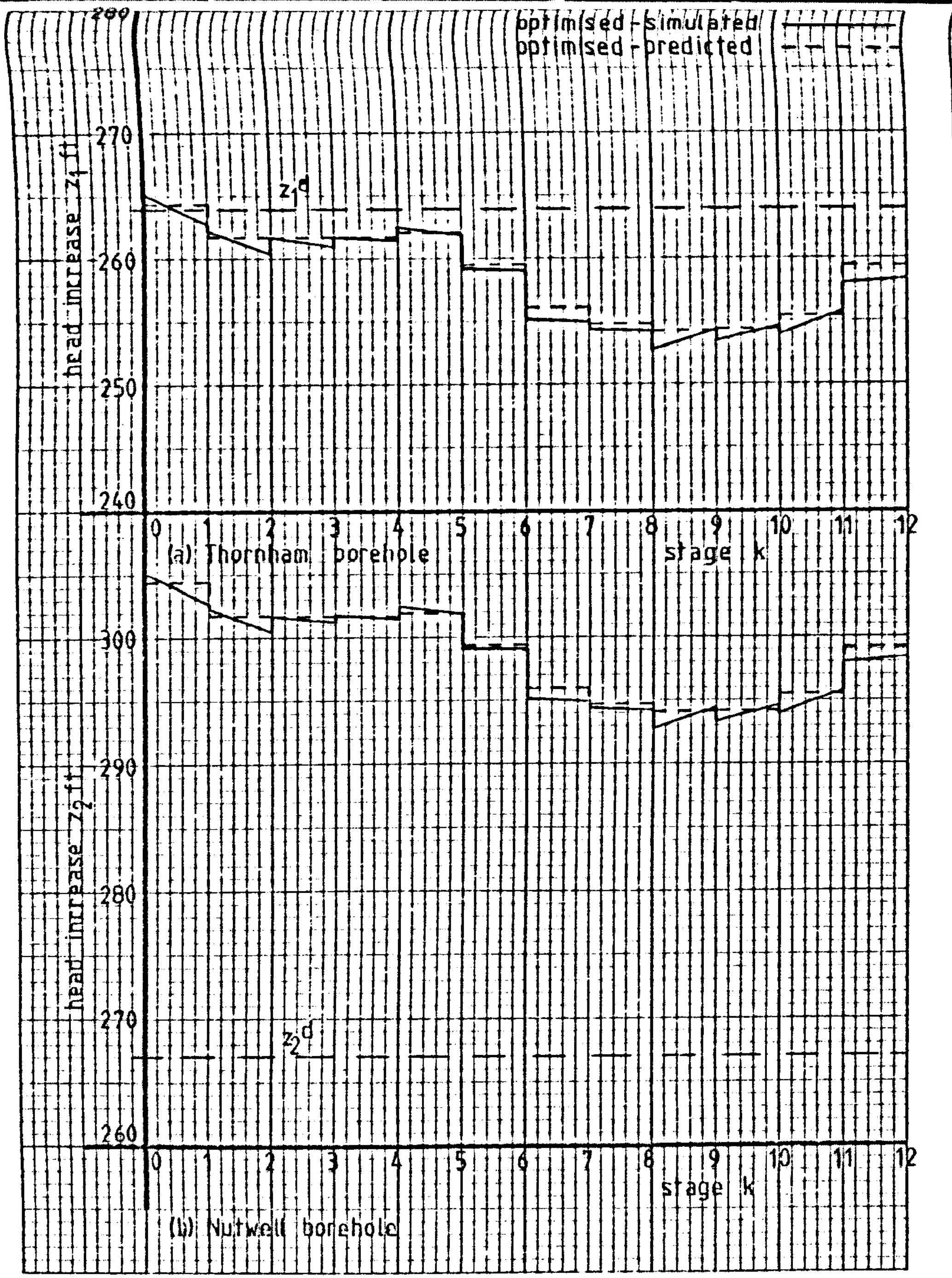


Fig 7-5 Pumping station heads

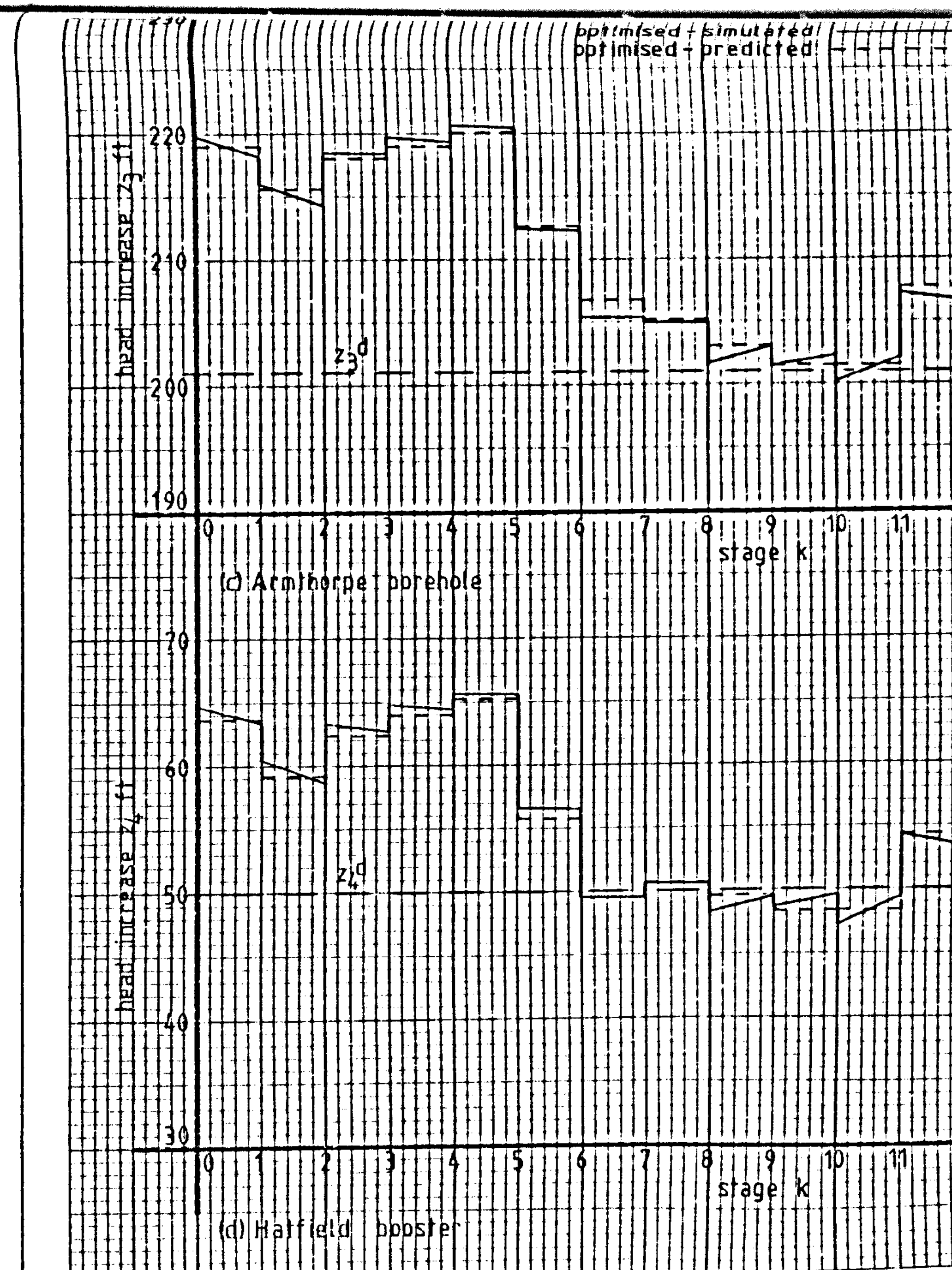


Fig. 7-5 Pumping station heads (continued)

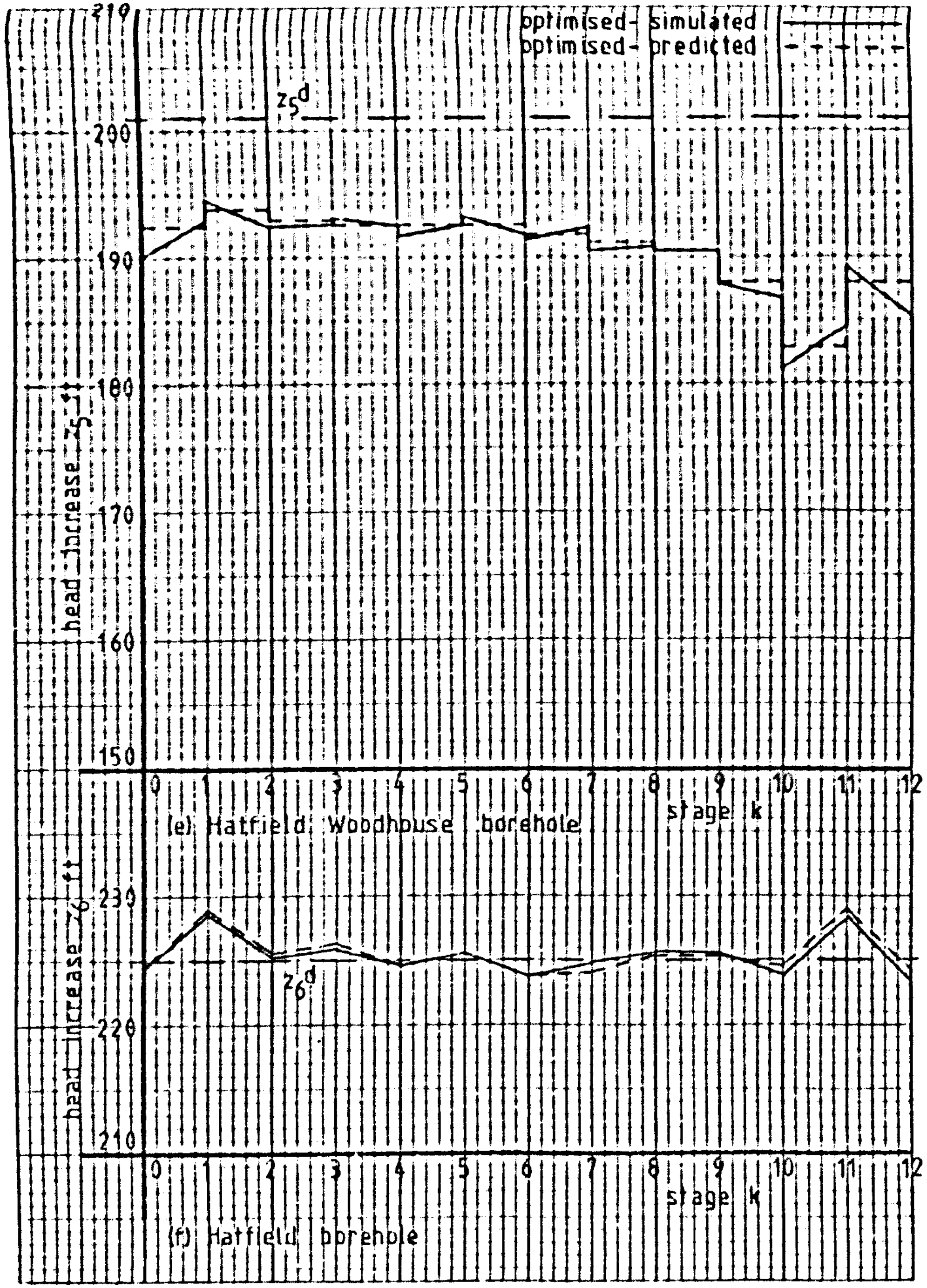


Fig. 7-5 Pumping station heads (continued)

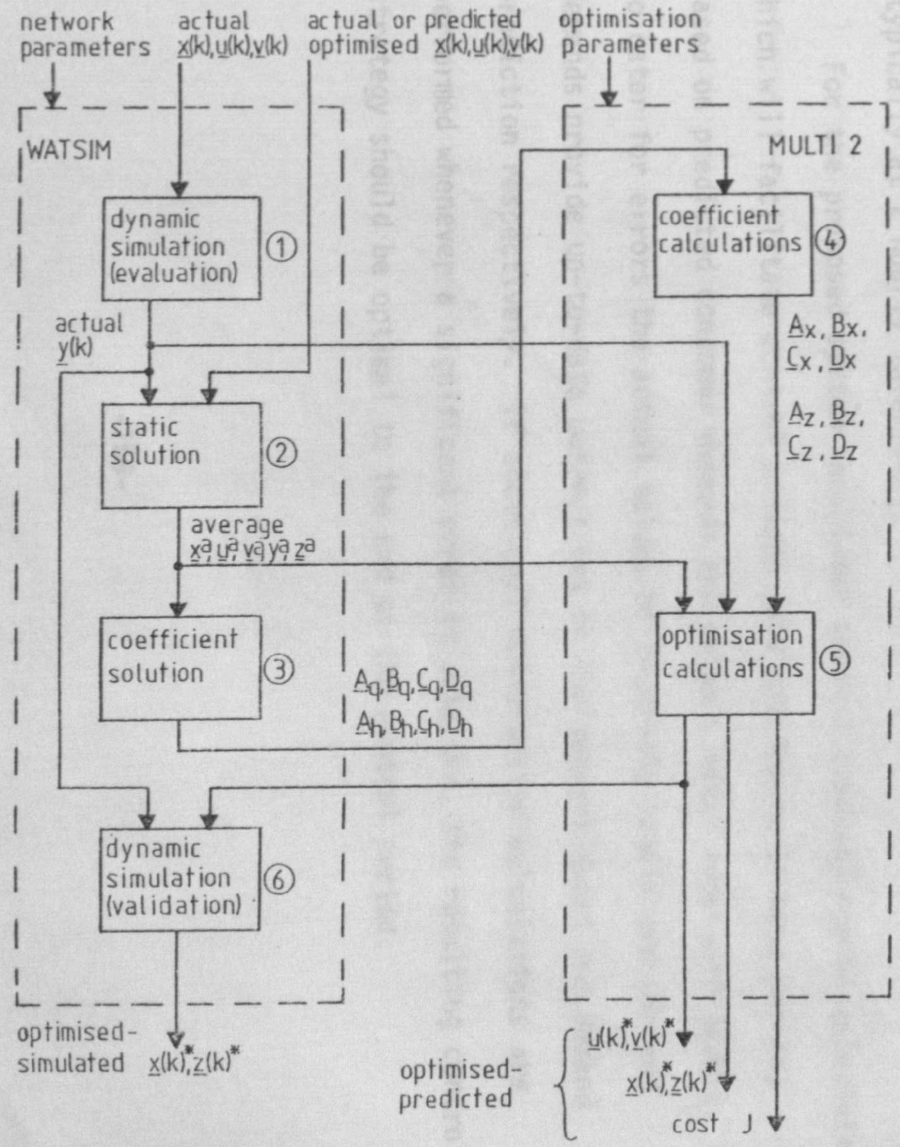


Fig. 7-6 Sequence of computing operations

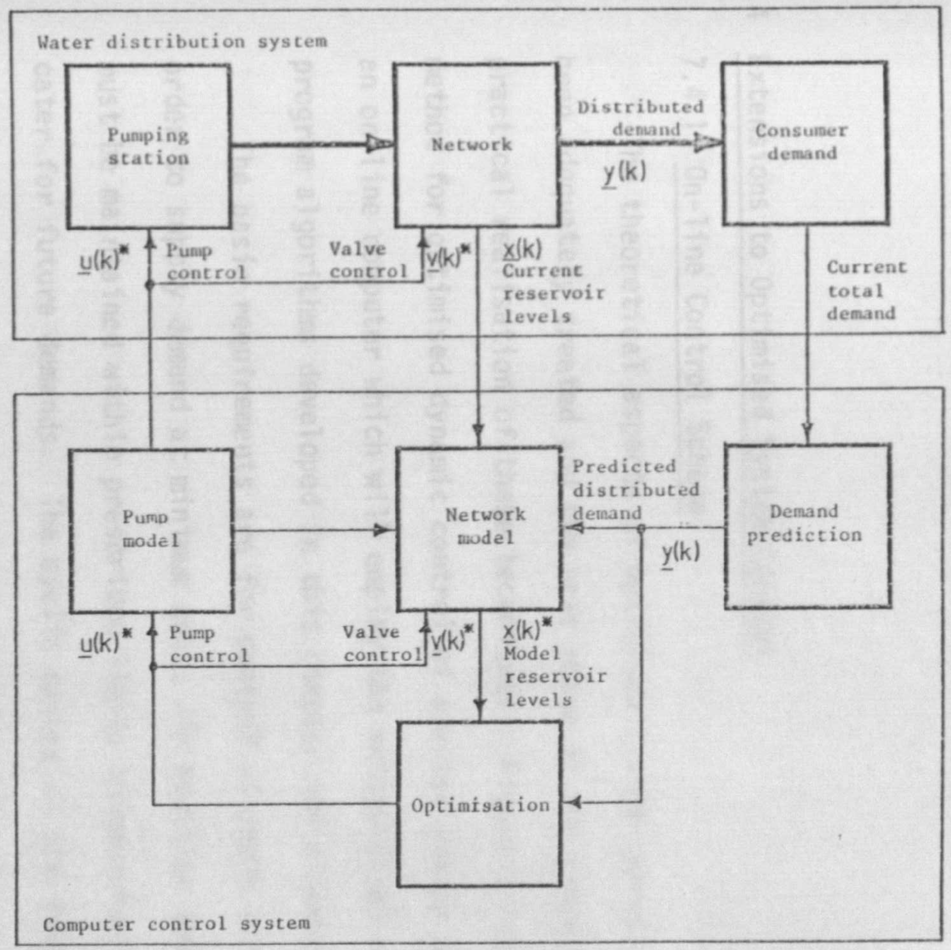


Fig. 7-7 Proposed dynamic control system

7.4 Extensions to Optimised System Control

7.4.1 On-line Control Scheme

The theoretical aspects of optimised system operation have now been adequately treated and the next phase of the project will require practical realisation of these techniques. Figure 7-7 shows a proposed method for optimised dynamic control of a water supply system using an on-line computer which will employ the mathematical models and program algorithms developed in this chapter and elsewhere[59].

The basic requirements are for control of pumps and valves in order to supply demand at minimum cost. In addition reservoir levels must be maintained within prescribed limits to meet emergencies and cater for future demands. The cyclic nature of the demand sets a minimum control period of 24 hours and, for typical reservoirs, control actions need only be taken every 2 hours. Suitable controllers must rely on the limited monitoring of reservoir levels and pump flows (typically at 2 hourly intervals).

For the proposed system an optimal control strategy can be calculated which will facilitate control actions over the future 24 hour period, based on predicted consumer demands and network model reservoir levels. To cater for errors the actual values of reservoir levels and derived demands provide up-to-date corrections to the network model and demand prediction respectively. If additional optimisation calculations are performed whenever a significant error is detected, the resulting control strategy should be optimal to the end of the control period.

7.4.2 Additional Constraint Features

The results have shown that the optimisation method is extremely versatile and can be used to cope with the varying operational constraints met in typical systems. Whilst many of these have been demonstrated there are other useful variations depending on the particular requirements.

The treatment has catered for valves controlled for each stage, k, (e.g.every 2 hours) implying that the valves are all locally situated or capable of being remotely controlled at little cost. For the case of remote valves requiring manual adjustment at significant operating costs it is possible to modify the treatment to allow less frequent valve operation. As an example, for valve adjustment each optimisation period (e.g. every 24 hours) the modifications to model equations would simply involve replacement of $\underline{v}(k)$ by $\underline{v}$ and $d\underline{v}(k)$ by $d\underline{v}$. $\underline{v}$ is now a stage invariant valve control which will have an optimal average value for the complete optimisation period.

Satisfactory service to customers is sometimes determined by adequate pressure levels at specified pressure nodes. The capability for controlling pressure levels has been built into the model but at present is used to maintain pump heads at desired values. However the facility can be easily expanded by defining an additional $\underline{z}$ element for each specified pressure node with corresponding upper and lower bounds and penalty weighting factors for deviation from the desired value.

7.4.3 Discrete Solution

Whilst the optimal control algorithm caters for all variables, to give continuous solutions for both continuous controls (e.g. control valves, variable speed pumps, throttled pumps,etc.) and discrete controls (e.g. fixed speed pump combinations), there is usually a requirement for adjustment of variables where only discrete values are permitted. Since the continuous values have been arranged to pass through the discrete points it is feasible that a method could be devised to seek the optimal discrete values and result in an overall discrete-continuous solution. For this purpose the continuous solution will be a good guide and the simplest method could involve selection of closest discrete values with re-calculation of optimal values for remaining continuous variables. The resulting solution will no longer be fully optimal and it is anticipated that a more rigorous approach could be used to achieve fully optimised performance.

A three level hierarchical decision process is envisaged, for the solution of the above problem, with two way interchange of information occurring via cost sensitivities, passed to next upper level, and resulting upper level decisions passed to next lower level as fixed operating constraints. An optimal search among selected discrete variables will be performed at the upper levels (based on cost sensitivities) and for each set of trial discrete values a complete continuous solution for all remaining variables will then be required at the lower levels (since all variables are interactive). Figure 7-8 shows the decision hierarchy which could be used at each level as described below:

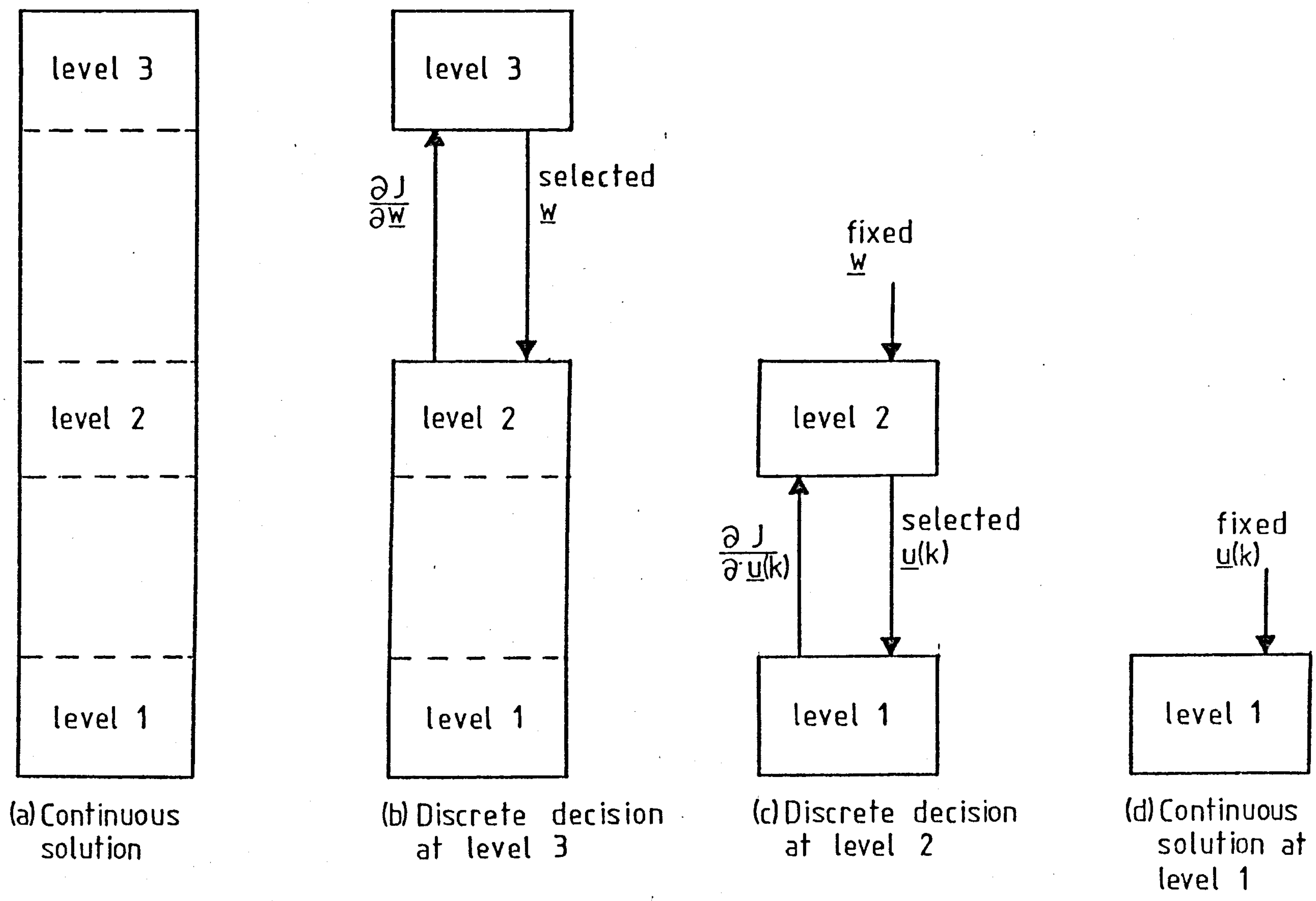


Fig. 7-8 Discrete-continuous solution

Level 3 (upper level)- selection of optimal discrete maximum value of pump control, $\underline{w}$, based on electrical maximum demand cost variation, $\frac{\partial J}{\partial \underline{w}}$. This will decide maximum permitted pumps on-line for base load plant and should only be required at the beginning of every electricy tariff period (e.g. monthly) under worst case consumer water demand.

Level 2 (intermediate level) - selection of optimal discrete value of pump control, $\underline{u}(k)$, based on electrical unit cost variation $\frac{\partial J}{\partial \underline{u}(k)}$. This will decide pump combinations on-line within limits of maximum demand plant fixed by level 3.

Level 1 (lower level) - selection of optimal values of true continuous variables for values of discrete pumps fixed by level 2.

The same control model can be used at all levels since the basic formulation embodies all the essential ingredients to allow both continuous and discrete interactive control variables. In operation this method should cater for minor disturbances by fine control of continuous variables at the lower level and for major disturbances by coarse control of discrete variables at the upper levels.

7.5 Conclusions

A control scheme has been developed meeting the requirements of a computer algorithm for on-line control of practical water distribution systems where the operation depends on many inter-dependent charges and operational constraints.

The systems treated are those consisting of:

(i) multi-reservoir zones with head dependent inter-zonal flows and distributed demands

(ii) borehole and booster pumping stations with head dependent flows and pump controls which can be discrete for parallel pumps or continuous for variable speed pumps

(iii) valves with flows dependent on heads and continuous valve controls,

with costs due to:

(i) direct pumping costs dependent on electricity charges

(ii) indirect pumping costs dependent on variable pumping efficiency

(iii) overall costs dependent on valve controls

As far as is known this scheme is the only one capable of evaluating optimal strategies for such a wide range of system features and has been used to obtain a solution for a realistic distribution network in terms of all continuous control variables. Practical implementation still leaves some problems with selection of any discrete pump combinations to suit the continuous solution values. Nevertheless useful and practical results have been achieved since the continuous solution gives a good indication of likely efficient discrete control strategies and suggestions have been made to cater for discrete selection on an optimal basis.

The present application shows potential cost savings and indicates that use of control valves and other continuous controls is important in overall system optimisation. These results may influence future design of systems for use with computer control where more efficient operation would be possible and would benefit from additional remotely activated continuous controls such as valves and variable speed pumps.

CHAPTER 8

CONCLUSIONS

It has been shown that operation of water distribution systems is extremely complex and that the successful application of automatic control methods is dependent on development of suitable mathematical modelling schemes.

So far all the work that has been done in this area has been limited, by reasons of accuracy, relevancy, and problem complexity, to restricted classes of simplified systems.

This thesis has attempted to redress the balance by developing additional methods for optimisation and modelling, with extended solution capabilities, and in other areas has contributed by a more detailed examination resulting in original refinements to existing methods. Combining these methods has provided an algorithm suitable for overall optimal control of a wider range of water distribution systems than previously possible. The results confirm the benefits of on-line control and may well influence future design to lead to water systems which are more efficient and more amenable to application of advanced control techniques.

The next section gives a critical appraisal of the current research results and show these in context with preceding similar work in dynamic control of water distribution systems. The dissertation concludes with an outline of future research extensions.

8.1 Advances in Dynamic Control of Water Distribution Systems

The advent of sophisticated instrumentation and control equipment has provided the means for more efficient management of water distribution systems. Several such systems[17,21,78] now employ on-line computers for automated monitoring (data logging) which also cater for some data reduction and provide facilities for remote activation of distributed controls. Present operating methods still rely mainly on operator decisions based on the more up to date and higher quality information provided.

The next phase will aim towards fully automatic on-line control with larger process control computers. This application requires advanced concepts in network reduction, optimisation, demand forecasting,etc. Undoubtedly an intermediate step will involve human intervention where the resident improved analysis and simulation facilities will be used to guide the operator towards more efficient overall system operation. The potential benefits of computerised control are well documented[78,82,84] and include economy of operation, system security, reduction of manning levels, etc. The general consensus of opinion amongst other research workers is that direct cost savings of approximately 5% are feasible and that these represent sufficient incentives for implementation, particularly when taken in conjunction with the other benefits.

Several schemes for on-line control have been proposed but the most advanced are considered to be by the following authors:

(i) De Moyer, et al,[15,16,17,18,19,20,41] who derive a non-linear dynamic network model. This is applied, together with a demand forecasting model, to a system consisting of two pumping stations and two reservoirs. System operation is based on control of discrete pumps only, using switching lines which are functions of reservoir levels and demands. A simplified single state dynamic programming algorithm is used to derive near - optimal operating policies with costs based on long-term averaged values of overall electricity charges for average pump heads.

The author recommends additional research aimed at extending the capability of the network model, to include: in-line booster pumps, control valves, and pressure reducing valves, and note that the model accuracy is reduced for large disproportionate changes of industrial demands. Additional desirable extensions would also cover derivation of improved performance indices (to more accurately represent pumping and other costs) and techniques to give fully optimised operation including use of continuous control variables.

(ii) Fallside and Perry[34,35,78], who propose a generalised hierarchy of models for applications in control of a multi-reservoir distribution system. The system is represented by a simplified volumetric balance relationship between six dominant reservoirs and ten pump inflows with non-linearities due to equivalent lumped pipe links (incorporated by reference to De Moyer's non-linear model). The pipe flow is then linearised to permit optimisation by means of alternative decentralised hierarchical methods. Operating costs are long term averaged values of overall electricity charges and system operation is based on continuous flow control of, assumed fixed head, variable speed pumps.

As before for (i) the same restrictions apply for use of the non-linear dynamic model and the authors suggest further research aimed at producing a more systematic method of model reduction, together with incorporation of true pumping costs in the optimisation algorithm.

Additional desirable extensions would also cover: optimisation and modelling with head dependent flow from both fixed and variable speed pumps, optimised control of pumping efficiency and selected pressure levels, and optimisation using valve controls.

Without wishing to detract from the importance of these schemes it can be seen that both of them are deficient in certain aspects of optimisation and modelling which prevent their application to general types of water distribution systems. The proposed scheme of Chapter 7 overcomes these deficiencies by providing an alternative method for on-line control, of very general systems, which leads to overall system optimisation taking into account all relevant cost factors and operating constraints. This proposal is based upon original work in optimisation and modelling techniques which now permit treatment of fixed speed pump combinations, variable speed pumps, and control valves, all with head dependent characteristics. The control strategy is determined by consideration of performance indices involving time varying electricity unit and demand charges under conditions of maximum pumping efficiency. The method has been applied to a realistic multi-reservoir distribution network to achieve a solution in terms of all continuous control variables. Practical implementation requires selection of any fixed speed pump combinations to suit the continuous solution values. This can be allowed for by a suggested control hierarchy which progressively fixes the discrete variables at optimal values and then caters for major demand disturbances by means of discrete variables in the upper control levels and minor disturbances by means of continuous variables at the lowest control level.

Whilst the overall scheme is complete in itself the constituent parts, and other developments, can also stand on their own and are all suitable for on or off-line computerised control applications. The techniques of: pumping station modelling, dynamic programming optimisation, decentralised hierarchical optimisation and linear dynamic models complement existing methods and are considered to represent significant advances in their own right. The computer programs developed in these areas can be used independently, or in conjunction with established methods, to provide an enhanced analysis and simulation capability.

8.2 Summary of Research Extensions

The research contained in this thesis is complete in itself and has provided a scheme for theoretical optimisation of overall system operation. However, additional research is always required which, in this case, can cover: improvements or refinements to the studied topics, alternative methods of achieving the same objectives, and practical implementation of research results. This section outlines the research possibilities, resulting from this study, which are given in main chapter order for correlation with previous topics and discussions.

Chapter 3. Analysis of Pumps and Pumping Costs

An independent control parameter has been derived defining the proportion of total output from parallel constant speed pumps to give flows and costs in terms of varying pump heads. An extension of the method to cover individual variable speed pumps should be fairly easy to incorporate using speed as the independent control parameter.

An indirect pumping cost is due to wear and tear for excessive on-off operation of individual pumps. Some results are available[3,19] but additional research is required to derive suitable performance indices to account for this cost and limit pump cycling in an optimal fashion.

Chapter 4. Optimisation of Pumping Costs

A computer program (DPSA) has been developed to give solutions to optimisation of simple multi-reservoir systems by means of dynamic programming modified by the successive approximations technique (to cater for higher dimensionality). Further research is required in this area which could include use of either of the models of Chapter 6 to provide a possible alternative method for overall system operation allowing for discrete controls.

The optimisation problem has also been formulated as a mixed variables integer-linear program. This format could be applied to typical simple systems and may be suitable for extension to more complex systems using the linear models of section 6.3.

An additional optimisation method, which would appear to be worthy of further investigation, depends on a technique for constraint separation[83]. This is a primal-feasible method, capable of handling state and control constraints and thus meets some of the requirements for optimisation in water distribution systems.

Chapter 5. Network Analysis and Simulation

It has been noted in Chapter 2 that borehole levels can vary significantly for pumping from the respective and adjacent boreholes. When sufficient data can be made available it should be possible to devise a suitable model which would probably be a linear or power law head-drop versus flow relationship. Inclusion of the model in the analysis program would automatically generate corrections in both the simulation and the linear dynamic model results.

It has been noted in Chapter 5 that desirable extensions include automatic correction of network parameters to force agreement between network and simulation for both steady state and dynamic conditions. Techniques exist[24,25,84] for steady state corrections but additional research is required to develop methods for dynamic matching.

Another desirable feature would include calculation of operating costs for simulated operation. Programs exist[15,16] for these calculations based on a loop equation formulation and additional research is required to convert these to the current nodal equation formulation.

Chapter 6. Simplified Dynamic Models

Linear and non-linear models have been formulated, in this study, and results have been obtained for linear dynamic models of networks under typical operating conditions. It would be useful to determine equivalent non-linear dynamic models of the same network and compare the results for accuracy and compatibility.

Evaluation of the linear dynamic model is presently based on a simulation of the network. For a practical implementation it would be desirable to investigate the suggested method, of section 6.3.3, for direct on-line evaluation of model coefficients.

The present linear model uses average values of consumer demand over each time increment and also assumes reservoir linear head-flow relationships. Further research may give a relaxation of these requirements to cater for any continuous time varying demand functions and allow reservoir non-linear relationships.

Chapter 7. Overall System Operation

The implementation of the proposed method would give open loop control and would require re-calculation to cater for unexpected disturbances. A more direct method of closed-loop control is obviously desirable, current work by Singh, et al,[73,88,89,90] indicates the possibility of using the same general solution techniques.

The present scheme results in continuous values for all control variables and selection of adjacent discrete values will be required for some of these. Further research is required on optimal selection methods. Other disadvantages of the scheme are that the solution is only feasible upon convergence and linear system equations are required. A suggested alternative scheme which may overcome these problems is that of generalised reduced gradients (GRG). This has been extensively used in the electricity industry for optimisation of power systems[23,79]. Whilst this latter application is a static one, Abadie[1] has shown the relevance of GRG to simple dynamic problems. Other techniques involving direct non-linear optimisation are given in references[42,63].

Apart from optimal control of existing distribution systems, other related optimisation areas include network design for least cost components and/or least cost operation. It is reasonable to suppose that future extensions will cater for these additional features; typical work and computer programs on these topics is covered in the following references[31,43,46,68,77,84,85].

8.3 Final Observations

It is not the intention to claim that this dissertation has provided a final solution enabling fully automated control of all water systems. However, the work has extended the class of systems for which solutions can be obtained and it is hoped that the development of the alternative technique will stimulate thought in these and other useful directions. It is confidently anticipated that further research, which may, however, depend on mathematical techniques yet to be developed, will eventually yield a solution satisfying the requirements of very general systems.

APPENDIX 1

SUMMARY OF NETWORK EQUATIONS AND PARTIAL DERIVATIVES

See Chapter 5 for notation and usage.

(a) Pipes and control valves.

$$q_{ij} = r_{ij}^{-0.54} (h_j-h_i) \left| h_j-h_i \right|^{-0.46} \tag{A1.1}$$

$$\frac{\partial q_{ij}}{\partial h_j} = 0.54 \, r_{ij}^{-0.54} \left| h_j-h_i \right|^{-0.46} \tag{A1.2}$$

$$\frac{\partial q_{ji}}{\partial h_i} = - \frac{\partial q_{ji}}{\partial h_j} = - \frac{\partial q_{ij}}{\partial h_i} = \frac{\partial q_{ij}}{\partial h_j} \tag{A1.3}$$

$$\frac{\partial q_{ij}}{\partial r_{ij}} = - 0.54 \, r_{ij}^{-1.54} (h_j-h_i) \left| h_j-h_i \right|^{-0.46} \tag{A1.4}$$

$$\frac{\partial q_{ji}}{\partial r_{ij}} = - \frac{\partial q_{ij}}{\partial r_{ij}} \tag{A1.5}$$

(b) Parabolic pumps.

$$h_i-h_j = a\left(\frac{q_{ij}}{r_{ij}}\right)^2 + b\left(\frac{q_{ij}}{r_{ij}}\right) + c \tag{A1.6}$$

$$q_{ij} = r_{ij} \cdot \left[\frac{-b \pm \{b^2 - 4a(c - |h_i-h_j|)\}^{0.5}}{2a} \right] \tag{A1.7}$$

$$\frac{\partial q_{ij}}{\partial h_j} = \pm \; r_{ij} \left\{ b^2 - 4a \, (c - |h_i-h_j| \,) \right\}^{-0.5} \tag{A1.8}$$

$$\frac{\partial q_{ij}}{\partial h_i} = - \frac{\partial q_{ij}}{\partial h_j} \tag{A1.9}$$

$$\frac{\partial q_{ij}}{\partial r_{ij}} = \frac{-b \pm \{ b^2 - 4a \, (c - |h_i-h_j| \,)\}^{0.5}}{2a} \tag{A1.10}$$

(c) Pressure reducing valves.

$$q_{ij} = r_{ij}^{-0.54} \mid h_{PRV} - h_i \mid^{0.54} \tag{A1.11}$$

$$\frac{\partial q_{ij}}{\partial h_i} = -0.54 \; r_{ij}^{-0.54} \mid h_{PRV} - h_i \mid^{-0.46} \tag{A1.12}$$

$$\frac{\partial q_{ij}}{\partial h_j} = 0 \tag{A1.13}$$

(d) Fixed head and fixed flow nodes.

$$\frac{\partial g_i}{\partial c_j} = \frac{\partial g_j}{\partial c_i} = 0 \tag{A1.14}$$

$$\frac{\partial g_i}{\partial c_i} = 1 \tag{A1.15}$$

$$\frac{\partial g_i}{\partial r_{ij}} = \frac{\partial q_{ij}}{\partial r_{ij}} \tag{A1.16}$$

$$\frac{\partial g_j}{\partial r_{ij}} = \frac{\partial q_{ji}}{\partial r_{ij}} \tag{A1.17}$$

$$\frac{\partial g_i}{\partial u_m} = \frac{\partial g_i}{\partial r_{ij}} \tag{A1.18}$$

for m designated as element ij

$$\frac{\partial g_j}{\partial u_m} = \frac{\partial g_i}{\partial r_{ij}} \tag{A1.19}$$

$$\left.\begin{aligned} \frac{\partial g_i}{\partial v_r} &= \frac{\partial g_i}{\partial r_{ij}} \qquad (A1.20) \\ \frac{\partial g_j}{\partial v_r} &= \frac{\partial g_j}{\partial r_{ij}} \qquad (A1.21) \end{aligned}\right] \text{ for } r \text{ designated as element } ij$$

$$\left.\begin{aligned} \frac{\partial g_i}{\partial y_\ell} &= \frac{\partial g_i}{\partial c_i} \qquad (A1.22) \\ \frac{\partial g_j}{\partial y_\ell} &= \frac{\partial g_j}{\partial c_i} \qquad (A1.23) \end{aligned}\right] \text{ for } \ell \text{ designated as node } i$$

$$\left.\frac{\partial g_i}{\partial y_\ell} = \frac{\partial g_i}{\partial c_j}\right] \text{ for } \ell \text{ designated as node } j \qquad (A1.24)$$

(e) Fixed flow nodes.

$$\frac{\partial g_i}{\partial h_j} = \frac{\partial g_j}{\partial h_i} = \frac{\partial q_{ij}}{\partial h_j} \qquad (A1.25)$$

$$\frac{\partial g_i}{\partial h_i} = - \sum_{\substack{j=1 \\ j \neq i}}^{H+Q} \frac{\partial g_i}{\partial h_j} \qquad (A1.26)$$

(f) Fixed head nodes.

$$\frac{\partial g_i}{\partial q_j} = \frac{\partial g_j}{\partial q_i} = 0 \qquad (A1.27)$$

$$\frac{\partial g_i}{\partial q_i} = 1 \qquad (A1.28)$$

$$\frac{\partial g_i}{\partial x_n} = \frac{\partial g_i}{\partial h_i} \tag{A1.29}$$

for n designated as node i

$$\frac{\partial g_j}{\partial x_n} = \frac{\partial g_j}{\partial h_i} \tag{A1.30}$$

$$\frac{\partial g_i}{\partial x_n} = \frac{\partial g_i}{\partial h_j} \quad \text{for n designated as node j} \tag{A1.31}$$

APPENDIX 2

DDJWB DISTRIBUTION SYSTEM PARAMETERS

The tables in this appendix are all referred to in Chapter 2.

Table A2-1 DDJWB Pumping Stations.

Zone	Pumping Station	Pump No.	Design flow (gal/h)	Design flow (gpm)	Nominal Maximum Demand (kVA)	Design Head (ft)
Don Valley	Austerfield borehole	1	69,000	1150.0	242.4	640
		2	69,000	1150.0	242.4	640
		3	69,000	1150.0	242.4	640
	Highfield Lane borehole	1	90,000	1500.0	360.0	762
		2	72,000	1200.0	288.0	762
	Marr booster	1	N/A	N/A	12.4	N/A
	Maltby booster	1	1200	200.0	N/A	100
Doncaster Western	Butterbusk booster	1	N/A	N/A	10.0	N/A
		2	N/A	N/A	17.0	N/A
	Warmsworth booster	1	N/A	N/A	26.0	N/A
Doncaster Central	Finningley borehole	1	63,000	1050.0	136.0	375
		2	63,000	1050.0	136.0	344
		3	63,000	1050.0	136.0	344
	Finningley booster	1	N/A	N/A	25	N/A
	Rossington Bridge borehole	1	65,000	1083.3	117.5	440
		2	65,000	1083.3	117.5	440
	Cantley booster	1	N/A	N/A	N/A	N/A
		2	N/A	N/A	N/A	N/A
		3	N/A	N/A	N/A	N/A
		4	N/A	N/A	N/A	N/A
Doncaster Eastern	Thornham borehole	1	50,000	833.3	82.0	264
		2	85,000	1416.7	138.0	264
	Nutwell borehole	1	72,000	1200.0	120.0	267
		2	24,000	400.0	40.0	267
	Armthorpe borehole	1	60,000	1000.0	75.0	201
		2	60,000	1000.0	75.0	201
		3	60,000	1000.0	75.0	201

Table A2-1 continued

Thorne	Hatfield borehole	1	22,500	375.0	43.0	225
		2	31,500	525.0	60.2	225
		3	45,000	750.0	86.0	225
		4	45,000	750.0	86.0	225
	Hatfield booster	1	40,000	666.7	15.0	50
		2	40,000	666.7	15.0	50
		3	40,000	666.7	15.0	50
		4	40,000	666.7	15.0	50
	Hatfield Woodhouse borehole	1	60,000	1000.0	75.0	201
		2	60,000	1000.0	75.0	201
		3	60,000	1000.0	75.0	201
	Hatfield Woodhouse booster	1	N/A	N/A	N/A	N/A

Table A2-2. DDJWB Reservoirs.

Location and Type	Top Water Level.AOD (ft)	Depth (ft)	Capacity (Mgal)
Thrybergh impounding reservoir	180	-	254
Langsett impounding reservoir	810	-	1400
Adwick-le-Street service reservoir	167	10	1.00
Butterbusk service reservoir	213	18	3.00
Clifton (new) service reservoir	465	17	3.00
Clifton (old) service reservoir	465	11	2.00
Conisbrough service reservoir	302	18	0.50
Denaby service reservoir	277	9	0.55
Scawthorpe service reservoir	115	16	2.00
Warmsworth No.2. service reservoir	130	10	1.50
Armthorpe water tower	145	17	0.25
Askern water tower	160	16	0.27
Bawtry water tower	160	17	0.10
Butterbusk water tower	305	21	0.15
Cantley No.1.water tower	147	17	0.30
Cantley No.2.water tower	205	25	0.50
Crowle water tower	100	15	0.10
East Lound water tower	115	12	0.05
Epworth water tower	184	20	0.175
Garthorpe water tower	70	20	0.02
Hatfield water tower	109	15	0.15
Haxey water tower	230	15	0.20
Hickleton water tower	410	12	0.05
Keadby water tower	91	16	0.10
Maltby water tower	515	16	0.20
Rossington No.1. water tower	100	18	0.25
Rossington No.2. water tower	135	12	0.04
Sandtoft water tower	84	12	0.07
Scawthorpe water tower	180	20	0.02
Skellow water tower	132	12	0.02
Sykehouse water tower	55	12	0.05
Thorne water tower	75	15	0.15
Warren Farm water tower	132	12	0.05

Table A2-3. DDJWB Centralised Supervisory System Facilities.

Alarms	Indications	Measurements	Controls
Intruder	Outstation local control program.	Electricity supply voltage. Electricity kWh meter	Pumps-start/stop
Burst main		Battery voltages	-local/remote
Operating limits exceeded	Pump indications-fault	Water pressures	
Power supplies	-control instruction	Water flow rate	Resets
DC supplies		Water integrator	
Pump auto-change over	-pump state	Rain gauge integrators	
Valvefaults	-valve position	Tower & Reservoir level	
Plant failure - -mechanical -electrical -hydraulic -supply voltage -flood		Valve positions	

Table A2-4 DDJWB System Control Operations

Zone	Control Parameter	Control Type	Normal Operation	Usage
Don Valley	Austerfield borehole pumping station	Manual control dependent on Clifton service reservoir level	Two pumps running One pump standby	Supply to Clifton service reservoir
	Highfield Lane borehole pumping station	Manual control dependent on Clifton service reservoir level	One pump running One pump standby	Supply to Clifton service reservoir
	Marr booster pumping station	Closed loop control dependent on Hickleton water tower level	One pump intermittent	Booster pump for Hickleton water tower
	Maltby booster pumping station	Time switch for off-peak use in conjunction with closed loop control from Maltby water tower	One pump intermittent	Booster pump for Maltby water tower
	Adwick service reservoir control valve	Manual control	Experimental pre-set valve to give daily cycle of Adwick service reservoir level	Controls supply to Adwick service reservoir
	Adwick water tower control valve	Manual Control	Experimental pre-set valve to give daily cycle of Adwick water tower level	Controls supply to Adwick water tower
	Don Valley to Doncaster Western interzone transfer valve	Manual Control	Valve closed	Controls emergency transfer from Clifton service reservoir

Table A2-4 contd. ...

Doncaster Western	Butterbusk booster pumping station	Closed loop control from Butterbusk water tower level	One pump running One pump standby Never two pumps running	Booster pump for Butterbusk water tower
	Warmsworth booster pumping station	Time switch for closed loop control	One pump intermittent	Transfer 0.5 million gallons in 24 hours from Warmsworth No. 2 service reservoir to Butterbusk service reservoir
	Langsett impounding reservoir control valve	Manual control	Experimental pre-set valve to give 1 Mgd at a rate of 43,000 gal/h	Supply to Butterbusk service reservoir
	Thrybergh impounding reservoir control valve	Manual control	Experimental pre-set valve to give 0.5 Mgd	Supply to Warmsworth No. 2 service reservoir
	Doncaster Western to Doncaster Central inter-zone transfer valve	Manual Control	Experimental pre-set valve by daily adjustment	Controls normal transfer from Butterbusk service reservoir
Doncaster Central	Finningley borehole pumping station	Manual Control dependent on Cantley No. 2 water tower level	Two pumps running One pump standby Can have three pumps running	Supply to Doncaster Central zone
	Finningley booster pumping station	Pressure switch for closed loop control from Haxey water tower ball-valve.	One pump intermittent	Booster pump for Haxey water tower
	Rossington Bridge borehole pumping station	Manual control of pump changeover	One pump running One pump standby Always one pump running because of sediment problems	Supply to Doncaster Central zone

Table A2-4 contd. ...

	Cantley booster pumping station	Not commissioned yet	Not commissioned yet	Interzonal transfer from Cantley No. 1 water tower in Doncaster Eastern zone to Cantley No. 2 water tower in Doncaster Central zone
	Doncaster Central to Doncaster Eastern inter-zone transfer valve	Manual Control	Valve closed	Controls emergency transfer from Cantley No. 2 water tower to Cantley No. 1 water tower
Doncaster Eastern	Thornham borehole pumping station	Manual control dependent on Cantley No. 1 and Armthorpe water tower levels	One pump running One pump standby Never two pumps running because of power transformer limitations	Supply to Doncaster Eastern zone
	Nutwell borehole pumping station	Manual control dependent on Cantley No. 1 and Armthorpe water tower levels	One pump running One pump standby	Supply to Doncaster Eastern zone
	Armthorpe borehole pumping station	Manual control dependent on Cantley No. 1 and Armthorpe water tower levels	Two pumps running One pump standby	Supply to Doncaster Eastern zone
	Cantley No. 1 water tower control valve	Manual control	Experimental pre-set valve to control Cantley No. 1 water tower level	Controls supply to Cantley No. 1 water tower
	Scawthorpe service reservoir control valve	Manual control	Experimental pre-set valve to give weekly cycle of Scawthorpe service reservoir level	Controls supply to Scawthorpe service reservoir
	Doncaster Eastern to Thorne interzone transfer valve	Manual Control	Valve closed	Controls emergency transfer from Doncaster Eastern zone to Thorne zone

Table A2-4 contd. ...

Thorne	Hatfield borehole pumping station	Closed loop control from Hatfield water tower level	Up to three pumps running intermittently	Direct supply to Hatfield water tower
	Hatfield booster pumping station	Manual control	Up to two pumps running Two pumps standby	Interzone transfer from Thorne zone to Doncaster Eastern zone
	Hatfield Woodhouse borehold pumping station	Time switch or manual control dependent on Thorne water tower level	Too pumps running One pump standby	Supply to Thorne zone
	Hatfield Woodhouse booster pumping station	Time switch and pressure switch dependent on Epworth water tower level	One pump intermittent	Booster for several water towers in Isle of Axholme area

Table A2-5 *ODJWB Electricity Pumping Costs*

Zone	Pumping Station	Electricity Authority	Tariff	Unit Charge (£/month)	Demand Charge (£/month)	Fuel Adjustment (£/month)	Night Rebate (£/month)	Total Cost (£/month)
Don Valley	Austerfield borehole	YEB	S1ADHV	1346	243	501	59	2031
	Highfield Lane borehole	YEB	S2ADHV	1200	285	463	64	1884
	Marr booster	YEB	I1QD	N/A	N/A	N/A	N/A	N/A
	Maltby booster	YEB	N/A	N/A	N/A	N/A	N/A	N/A
Doncaster Western	Butterbusk booster	YEB	I2MDLV	20	9	5	0	34
	Warmsworth booster	YEB	I2MDLV	44	21	12	0	77
Doncaster Central	Finningley borehole	EMEB }	AMDHV	776	379	235	23	1367
	Finningley booster	EMEB }						
	Rossington Bridge borehole	YEB	I2ADLV	394	106	124	17	607
	Cantley booster	YEB	N/A	N/A	N/A	N/A	N/A	N/A
Doncaster Eastern	Thornham borehole	YEB	I2ADLV	356	108	109	13	560
	Nutwell borehole	YEB	I2MDLV	127	43	39	6	203
	Armthorpe borehole	YEB	I2MDLV	367	144	110	11	610
Thorne	Hatfield borehole	YEB }	I2MDLV	305	136	88	8	521
	Hatfield booster	YEB }						
	Hatfield Woodhouse borehole	YEB }	I2MDLV	288	95	90	11	462
	Hatfield Woodhouse booster	YEB }						
	TOTALS			5223	1569	1776	212	8356

APPENDIX 3

VALIDATION SYSTEM PARAMETERS

All the tables and figures contained in this appendix are for the combined Doncaster Eastern and Thorne Zones, used under variable head pumping, and are referred to in sections 6.3.4(b) and 7.3.

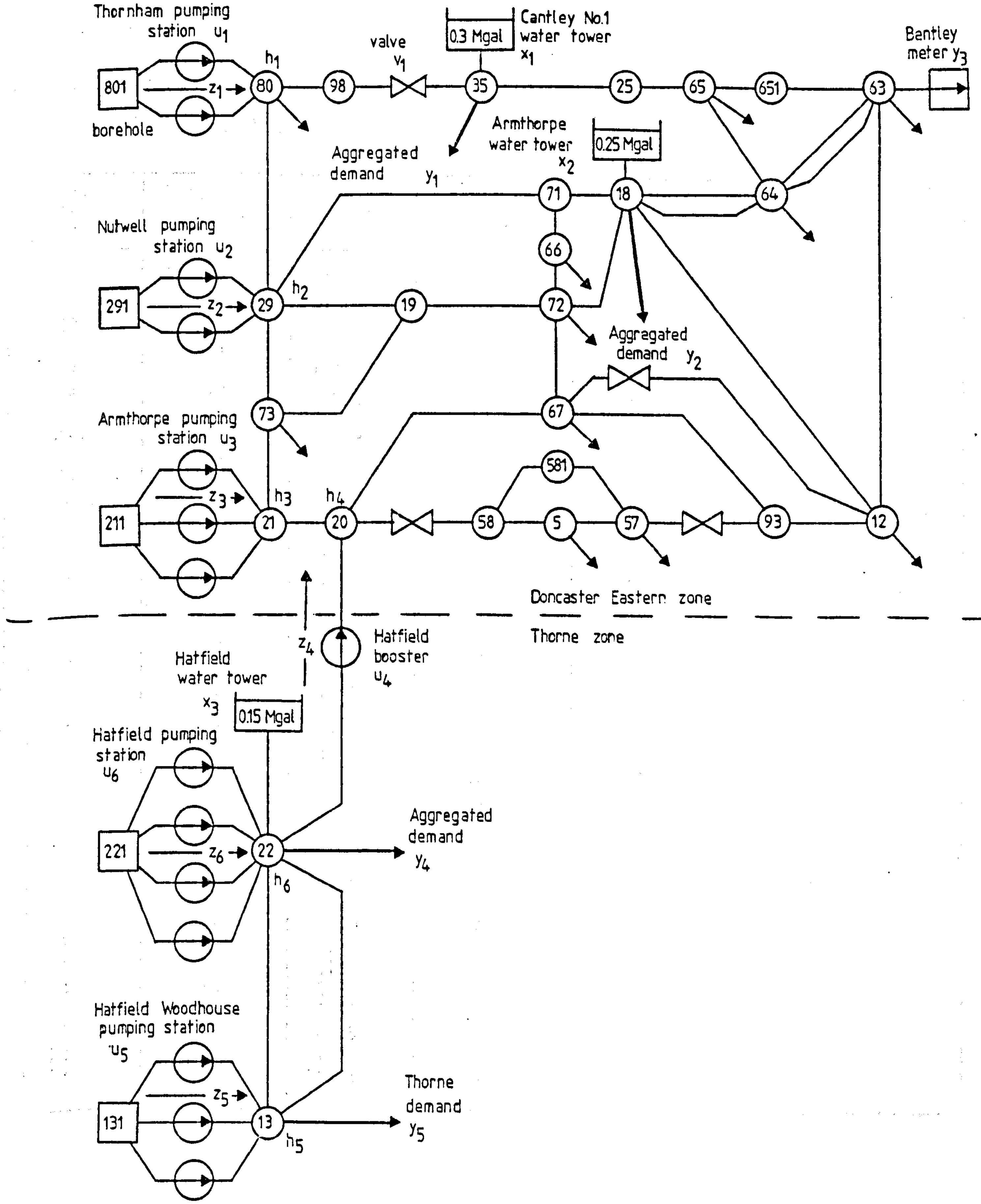


Fig.A3-1 Network for Doncaster Eastern and Thorne zones.

Table A3-1. Network Parameters

(a) Pipe and Valve Links

Origin node	Destination Node	Length, L (ft)	Diameter, D (ins)	Hazen-Williams Coefficient, C_{HW}	Control Coefficient
80	98	1990	18	110	
80	29	1490	18	110	
29	73	9130	9	105	
73	21	1000	15	135	
29	71	11100	12	105	
29	19	5800	18	110	
73	19	3330	15	135	
21	20	3600	15	135	
98	35	6335	18	110	v_1
35	25	10335	12	100	
19	72	2500	18	110	
20	67	3000	9	125	
20	58	2700	12	140	
58	581	1350	6	100	
581	57	2300	9	110	
58	5	3520	9	135	
5	57	2835	6	90	
57	93	2335	6	90	
67	93	2730	6	90	
67	12	7660	6	100	
67	72	6550	9	95	
72	66	2550	9	90	
66	71	1780	9	90	
72	18	5250	18	110	
71	18	820	12	105	
25	65	3375	9	110	
18	64	2440	9	90	
18	64	2440	15	110	
18	12	3055	9	125	
93	12	8825	6	100	
65	651	1070	6	100	
651	63	2770	9	110	
65	64	4180	6	100	
64	63	3660	9	90	
64	63	3660	15	110	
63	12	4615	6	100	
13	22	1400	10	100	
13	22	11805	14	100	

Table A3-1 (continued)

(b) Pumps

Origin Node	Destination Node	Quadratic coefficient, a, (ft/(gpm)^2)	Linear coefficient, b, (ft/gpm)	Constant coefficient, c, (ft)	Control parameter, r,	Control coefficient
801	80	-0.927×10^{-4}	0.0000	450.0	1.0	u_1
291	29	-0.837×10^{-5}	-0.0958	442.0	1.0	u_2
211	21	-0.153×10^{-4}	0.0223	272.0	1.0	u_3
22	20	-0.600×10^{-5}	0.0000	93.0	1.0	u_4
131	13	-0.153×10^{-4}	0.0223	272.0	1.0	u_5
221	22	-0.784×10^{-4}	0.0344	320.0	1.0	u_6

Table A3-1 (continued)

(c) Reservoirs and Boreholes

Node	Linear coefficient, α, (ft/gal)	Water Level AOD (ft)	State coefficient
35	56.66×10^{-6}		x_1
18	67.98×10^{-6}		x_2
22	100.0×10^{-6}		x_3
801		-110	
291		-150	
211		-50	
131		-40	
221		-120	

Table A3-2 Pumping Station Parameters

Combined pump operations at design heads.

Pumping Station	Pump combination (Pump No.)	Design Flow (gpm)	Nominal Maximum Demand (kVA)	Design Head (ft)	Pump Control Parameter
Thornham Borehole u_1	1 2	833.33 1416.67	82.0 138.0	264 264	0.5882 1.0000
Nutwell Borehole u_2	2 1 1 & 2	400.0 1200.0 1600.0	40.0 120.0 160.0	267 267 267	0.2500 0.7500 1.0000
Armthorpe Borehole u_3	1 or 2 or 3 Any two 1,2 & 3	1000.0 2000.0 3000.0	75.0 150.0 225.0	201 201 201	0.3333 0.6667 1.0000
Hatfield Booster u_4	1 or 2 or 3 or 4 Any two Any three 1,2,3 & 4	666.67 1333.33 2000.0 2666.67	15.0 30.0 45.0 60.0	50 50 50 50	0.2500 0.5000 0.7500 1.000
Hatfield Woodhouse Borehole u_5	1 or 2 or 3 Any two 1,2 & 3	1000.0 2000.0 3000.0	75.0 150.0 225.0	201 201 201	0.3333 0.6667 1.0000
Hatfield Borehole u_6	1 2 3 or 4 1 & 4 2 & 4	375.0 525.0 750.0 1125.0 1275.0	43.0 60.2 86.0 129.0 146.2	225 225 225 225 225	0.2941 0.4118 0.5882 0.8824 1.0000

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