THEORY AND MODELS OF FAMILY BEHAVIOUR: LABOUR PARTICIPATION, HOUSEHOLD PRODUCTION, AND FERTILITY CHOICES

MARTINA MENON

PhD THESIS

UNIVERSITY OF YORK

DEPARTMENT OF ECONOMICS AND RELATED STUDIES

MAY 2007

Table of Contents

List of Figures

Abstract

The first chapter studies the labour supply decision of couples in the context of Pareto efficient models. The analysis is carried out by means of the reservation wage theory. In the unitary model, we find that for each spouse there exists a unique combination of reservation wages that allows to derive the participation frontier. Within the collective model, on the other hand, price dependent utility functions means that a unique reservation wage need no longer exist and, in turn, the participation frontier cannot be drawn. However, we show that if the bargaining power index is independent of current market wages the completeness of the reservation wage theory is re-established.

The second chapter examines the production-consumption household collective model under the assumptions of complete and absent markets for the domestic good. We find the conditions of market's structures or household technology that ensure separability between production and consumption-leisure decisions. We perform a qualitative analysis to the household models developed under different market structures.

The last chapter presents a finite-horizon dynamic model of fertility and consumption within a certain environment. Fertility is modelled as a discrete choice and it is the outcome of comparisons between parents' welfare level with and without an extra child. The fertility model emphasizes the effects exerted on the family size decision by the costs of rearing children. We show that in each period of the reproductive life the couple does not have a child if the pure utility gain from not having a child is greater than the utility saving in cost from having the child.

Acknowledgements

Now that this "enterprise" is almost finished I feel bound to thank a number of persons that, in different ways, have been close to me in these years.

I wish to thank Peter Simmons. This study has benefitted from his guidance and discussions.

I express my deepest gratitude to Federico Perali for his invaluable teachings and his constant and unconditional support. This study has greatly benefitted from his encouragement and positive guidance and many dialectic discussions. Probably, without him I would not have undertaken this adventure.

In York I have met a number of good friends. I wish to thank Adriana, Ana, Chiara, Christian, Francesco, Matteo, Marisa, Silvia and Simona. During these years I have shared with them very hard time, but I cannot forget the leisure time spent together.

Last, but not least, I owe heartfelt thanks to my parents, Annamaria and Giuseppe, that with a perfect combination of sweetness and silence they have been close to me in all the important moments of my life.

Declaration

I declare that the chapters contain in the thesis are my own work.

Chapter 1

Introduction

1.1 Motivation for Individual Approaches to Family Behaviour

Since Becker's seminal work, family behaviour has been described by means of a welfare index representing preferences of the family as a whole. Within this approach, referred to as unitary, the family is considered as the elementary decision unit and it is characterized by a unique utility function that is maximized subject to a family budget constraint.

The popularity of the unitary approach is grounded on a number of theoretical and empirical respects. For instance, the integrability properties of the unitary approach allow economists to recover household preferences from the observation of market consumption and in turn it is a necessary condition for normative analysis such as welfare comparison across households or optimal taxation. Besides, the economic properties satisfied by optimal demands derived by the unitary set-up, in particular the symmetry of Slutsky substitution matrix, enable simplifications in empirical estimation. Moreover, household micro data are in general collected at the household level, hence the representation of the family as though it were a unique maximizing agent seems to accustom better the household expenditure surveys.

Despite these attractive properties, it has been widely recognized that the unitary model is not appropriate both to represent theoretical and empirical individual behaviour and to answer important questions of normative analysis. In particular, from a theoretical point of view, the unitary approach conflicts with the neoclassical economic theory in which every agent should be represented by her own preferences. On the other hand, from a welfare viewpoint, it implies that household resources are equally divided between family members. However, it is debatable if household resources are equally shared between individuals and it may be that neglecting intra-household inequality leads to misleading measurement of inequality and poverty (Haddad and Kanbur 1990).

Empirical works have rejected a number of the properties underlying the unitary setting. In the traditional household model it is assumed that income is pooled and then allocated to maximize the household welfare. The income pooling property implies that household outcomes are not affected by who in the family has control on the resources. Besides, the pooling hypothesis has important policy implications. In particular, it has consequences on policy programs in which the target is a specific class of individuals within the household, such as children and women. Results of empirical works suggest that expenditure household data are not consistent with the pooled income hypothesis. Using data from Thailand, Schultz (1990) examines the impact of nonlabour income on fertility rates discovering that more nonlabour income in the women hands tends to raise fertility. Using a Brazilian survey on family health and nutrition, Thomas (1990) attempts to infer how resources are allocated within the family members by focusing on particular household outcomes, such as nutrient intake, child health, survival, and fertility. He finds that nonlabour income under the wives' control has a bigger effect on family health than income controlled by husbands and the assumption of common preferences of unitary model is rejected. Finally, he shows evidence for gender preferences, mothers prefer to devote resources to improving the welfare status of daughters, fathers to sons. Moreover, there exists evidence that neglecting the rule in which families share resources across members can produce misleading welfare analyses (Haddad and Kanbur 1990, Bargain et al. 2006).

A further implication of utility theory is the symmetry of the matrix of substitution terms. However, Slutsky restrictions are often rejected by empirical studies (Blundell, Pashardes, and Weber 1993, Browning and Chiappori 1998). Three main explanations can justify the rejection of Slutsky symmetry. First, the rejection may be a consequence of a mispecification of functional forms applied in estimating demand systems. Second, the rejection may depend on the fact that there does not exist a utility function which is compatible with the data. The last plausible interpretation is that families do not behave as though they were a unique agent and therefore should be accepted that the neoclassical economic theory is not the conventional approach to represent the behaviour of a group of agents.

Given the discussed theoretical and empirical weaknesses of the unitary approach, in the 1980s viable strands, alternative to the traditional setting, have thus been developed. However, before reviewing the alternative models of family behaviour, with the following brilliant insights by Seccombe (1995:146-150) we desire to emphasize the relevance, under many respects, of looking inside the family black box.

"In the period from 1873 to 1914, working-class families came to depend on the primary breadwinner's income to a greater degree than ever before. Studies of family budgets compiled around the turn of the century consistently place the househead's contribution (where he is present and regularly employed) at 70 to 80 per cent of the total family income. This dependency varied over the family cycle and between strata of the proletariat; the higher a man's pay, the greater was his family's reliance upon him. Yet it seems that among all layers of the working class, the male breadwinner's income assumed greater importance in this period. The reasons for this trend are not difficult to discern: a very considerable rise in men's real wages; the curtailment of child labour; and reduced opportunities for women to make money at home.

A deepening reliance on the working man's income meant that the family's fortunes hinged critically upon the division of his wage between his own personal spending money and the housekeeping budget, handled by his wife. [...]

The connection between Saturday-night drunkenness and the mode of wage payment became an issue in the temperance battles of the period. In Scotland, the temperance movement won a major victory in 1853 when pubs were forced to close at eleven o'clock. Per capita alcohol consumption declined from then on, and a great many women must have *blessed the Saturday closing hour as they set off to market with a greater share of their husbands' pay. [...]*

Because the distribution of wages within the family is an informal matter that has not been adequately studied, it is impossible to gauge with any precision the prevalence of different patterns of wage allocation. By my reckoning, three broad variants may be distinguished. A significant minority of men came straight home with their pay, handed it over in its entirety to their wives and took back a modest amount for personal needs at their spouses' discretion or by mutual agreement. From women's standpoint, the whole-wage system (as this variant has been called) was exemplary. Men who adhered to it were universally praised as considerate and kind husbands 'who treat marriage as a real partnership, who regard "my wages" as "our wages," and who plan out the expenditure of joint income with their wives'.

Far more common, however, and probably the dominant pattern, was for working men to hand over a housekeeping allowance, generally a fixed sum keeping whatever was left over for themselves. [...] Men were tempted to keep spouses in the dark as to how much they made in bonuses and overtime so that they could spend these extras' as they wished. In her 1970 study of Middlesbrough, Lady Bell found that a third of ironworkers' wives did not know how much their husbands made.

The fixed allowance had the advantage of being a stable arrangement which provides wives with a predictable income to make ends meet. The essence of the provisioning exercise was to adapt the family's collective needs to the size of the allowance. But when the allowance barely covered the regular weekly expenses of food and rent, it was almost impossible to set aside funds for children's boots, new clothes or unexpected medical bills. When extraordinary expenses arose, women had to ask their husbands for additional funds; special purchases thus took the form of gifts' from Papa. The alternative was to stint on other items; typically, housewives spent less on food, going short themselves to make up the difference. [...]

In a third variant, extremely pernicious but not uncommon, the cash wives obtained was an unpredictable and variable residual—the amount left over after their husbands had *visited the pub or betting shop. [...] Working men who drank' their pay' caused their wives no end of grief. It was impossible to maintain an orderly household under a random income schedule. They needed no prompting from middle-class feminists and temperance crusaders to denounce such men as callous husbands. [...]*

The allocation of the breadwinner's wage was of such importance to women that the criterion of whether a man is a "good husband" [is] the proportion of his wages which he gives to his wife.' Watching their mothers cope, girls developed a strong sense of a fair division. [...]

In Sismondi's view, the modern working man has become accustomed to the fact that he never knows a future beyond next Saturday when he is paid. [...] He has too often been led to think about present comforts so as not to be too afraid of the future suffering his wife and children may bear. Arthur Young preferred the truck system for the same reason: 'An Irishman loves his whiskey as an Englishman does strong beer; but he cannot go to the whiskey house on Saturday night and drink out the support of himself, his wife and his children, not uncommon in the alehouses of England,' As the wage increasingly took the form of a payment to individuals, its subsequent redistribution became a private affair between spouses, widely considered to be no one else's business but their own'. This ethic made it more difficult for women to combat its abuse."

Recently, economists have recognized the importance of considering the family as a collection of individuals rather than as if it were a unique decision unit and, in addition, they have recognized the relevance of inferring how family members share household resources among them. These family models are referred to as collective models. The collective approach to the household can be divided into two broad categories: family approaches that rely on cooperative solutions to bargaining among individuals and family approaches that rely on noncooperative models.

There are two types of cooperative approaches. Models in which it is supposed only that family outcomes are always Pareto efficient (Apps and Rees 1988, 1996, 1997, Browning and Chiappori 1998, Chiappori 1988, 1992, 1997) and nothing is assumed a priori about the nature of the decision process, or equivalently, about the location of the final outcome on the household Pareto frontier. And family cooperative models in which the decision process is explicitly determined using bargaining theory.

The collective model proposed by Chiappori adopts a theoretical framework in which each household member is characterized by her own preferences and, assuming that the decision process results in Pareto efficient outcomes, he shows that when agents are "egoistic" and consumption is purely private, the collective model generates testable restrictions and, from observed market behaviour, one can recover certain structural elements of the decision process, such as individual preferences and the rule that determines the allocation of resources within the family. The decision-making process underlying the collective model can be modelled as if decisions occur in two stages. First, household members agree on the allocation of nonlabour income between them and then each agent independently decides the allocation of her own money resources among different goods.

In bargaining cooperative models (Manser and Brown 1980, McElroy and Horney 1981) households are supposed to be composed of two agents. Preferences of each household member are represented by a well-behaved utility function and each member is endowed with a given level of utility, that represents the pay-off if agreement within the family is not reached. The individual utility function, therefore, depends on a threat point and, in turn, the Nash bargaining solutions are functions not only of prices and income but also of the threat point. This dependence is the critical empirical implication of the cooperative bargaining models.

In contrast to cooperative models, the noncooperative approach does not assume that members necessarily enter into binding and enforceable agreements with each other. They assume that individuals within the household have differing preferences and they act as autonomous individuals. The noncooperative models consider a two-person household in which each individual controls her own income and purchases commodities, subject to an individual nonpooled income constraint. A net transfer of income between individuals establishes the only link between family members. Each individual has a utility function of goods she exclusively consumes and a good consumed in common but not public, conditional on the level of net transfers. When making decisions, each person takes net transfers as given and chooses the goods she will exclusively consume in order to maximize her own utility subject to her individual budget constraint. This family framework yields individual demand functions which depend on prices and net transfers. The Nash equilibrium is the level of goods consumed by both individuals that satisfies both demand function simultaneously. A feature of the noncooperative models is that family outcomes need not be Pareto efficient. However, efficiency may be obtained as a result of independent decision making.

In the remainder of the introduction we describe the general assumptions underlying the collective model and then summarize the organization and major findings of the thesis. In the thesis family behaviour is represented using the unitary and the collective models.

1.2 General Assumptions of the Collective Model

In general, the collective model of household behaviour considers a family with two decision makers, the wife f and the husband m, who consume for their private use the vector of goods $x \in \Re^N$ that is composed of ordinary, assignable and exclusive goods.[1] A good is ordinary when it is a private good is consumed in unobserved proportions by all or some non identifiable household members. This is the common case given the information traditionally available in household expenditure survey. A good is assignable when a strictly private good and is consumed in observed proportions by each member of the household. This may be the case when it is possible to assign the consumption of clothing either to the adult or children component of the household. Finally, a good is exclusive when a strictly private good is consumed by one identifiable member of the household only. An assignable or exclusive good that is frequently considered in literature is leisure.

In the thesis, the assignable good is the individual demand for leisure L^i and the individual consumption of a composite market good c^i represents the ordinary good. Here, we assume that all time not spent at paid work is leisure time. Individual time is thus equal to $T = L^i + l^i$ where l^i is i's labour supply. The family then faces a budget constraint

[1] Throughout the thesis superscripts denote endogenours variables, while subscripts index either exogenous variables or, in the case of functions, the derivatives of the endogenous variables.

that limits the private consumption of market goods and leisure of the two spouses. The budget constraint takes the usual linear market form $\sum_{i=f}^{m} p_i c^i = \sum_{i=f}^{m} w_i l^i + y$, where p_i are market prices of the composite goods, w_i are market wages and y is family nonlabour income. Finally, throughout the thesis we ignore both the consumption of public goods and externalities within the family.

The collective household model relies on three assumptions.

Assumption 1.1 *Individual preferences are assumed to be represented by an egoistic strictly quasi-concave utility function, continuously differentiable and strictly increasing in its elements*

$$u^i = U^i \left(c^i, L^i\right).$$

It is important to underline that "egoistic" preferences are not necessary to recover the individual behaviour and the collective set-up can be extended to a caring utility function $\widetilde{u}^i = \widetilde{U}^i \left[U^f(c^f, L^f), U^m(c^m, L^m)\right]$ without altering the conclusions of the model (Chiappori 1992).

Assumption 1.2 *Household decisions are assumed to result in Pareto-efficient outcomes.*

This assumption does not necessarily imply "harmony" between spouses, thus a collective model may also describe families experiencing marriage dissolution. The rationale motivation of the Pareto efficient assumption is that efficient allocations are likely to emerge from repeated interactions in stationary environments and households are an example of such an environment. However, there can be situations in which the efficient assumption may fail to apply. An example can be when existing social norms impose some patterns of behaviour that deviate from efficient outcomes (Udry 1996). Inefficient outcomes are also plausible for decisions that are not taken frequently, for instance fertility and education choices (Lundberg and Pollak 2003), which imply that the repeated game argument cannot be applied.

Assumption 1.3 *The decision process μ_i is a function continuously differentiable in its arguments of variables that enter the budget constraint, such as market prices, wages and* *nonlabour income.*

The Pareto weight μ_i captures the balance of power in the family and determines the final allocation on the Pareto frontier. If $\mu_i = 1$ then the household behaves as though member i is the effective household dictator, whereas $\mu_i = 0$ it is the opposite scenario. For intermediate values of μ_i, the household behaves as though each spouse has some decision power. In general, in labour supply models the distribution index μ_i is made to depend on wages and nonlabour income. The value of μ_i may also depend on variables that influence the balance of power without affecting both the value of individual utility functions and the household opportunity set. In literature these variables are referred to as distribution factors.

The household behaviour can be described by the maximization of the following weighted household utility function W^h

$$W^h = \widetilde{\mu}_f U^f(c^f, L^f) + \widetilde{\mu}_m U^m(c^m, L^m)$$

subject to the household budget constraint

$$p_f c^f + p_m c^m + w_f L^f + w_m L^m = \sum_{i=f}^{m} w_i T + y,$$

from which the individual demands for leisure and composite market goods are derived

$$\begin{aligned}
\widetilde{c}^i(p_f, p_m, w_f, w_m, y) &= c^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu}_i), \\
\widetilde{L}^i(p_f, p_m, w_f, w_m, y) &= L^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu}_i),
\end{aligned}$$

with $\widetilde{\mu}_i = \mu_i(p_f, p_m, w_f, w_m, y)$ for $i = f, m$. It is worth remarking that the unitary model is a special case of the collective model. This is the case when the weight $\widetilde{\mu}_i$ does not depend on prices and nonlabour income.

The behavioural implications of the collective model require extending the integrability property of standard consumer theory to the following generalization of the Slutsky condition. The generalized Slutsky equation of the collective model has the following form

(Browning and Chiappori 1998)

$$s_{ij} = \left[\frac{\partial L^i}{\partial w_j} + \left(T - L^j\right)\frac{\partial L^i}{\partial y}\right] + \frac{\partial L^i}{\partial \widetilde{\mu}_i}\left[\frac{\partial \mu_i}{\partial w_j} + \left(T - L^j\right)\frac{\partial \mu_i}{\partial y}\right]$$

where the first term between brackets is the conventional Slutsky equation of the unitary model. The corresponding Slutsky matrix is

$$S = \Sigma + R = \Sigma + D_\mu \cdot v'$$

where the matrix Σ summarizes $\left[\frac{\partial L^i}{\partial w_j} + L^j \frac{\partial L^i}{\partial y}\right]$, the vector D_μ is of general term $\frac{\partial L^i}{\partial \widetilde{\mu}_i}$ and the vector v is $\left[\frac{\partial \mu_i}{\partial w_j} + L^j \frac{\partial \mu_i}{\partial y}\right]$.

Equation s_{ij} represents the Slutsky equation performed for a market demand function derived from a collective framework. As shown by Browning and Chiappori (1998), in the collective model the substitution matrix fails to satisfy the symmetry property. In particular, the Slutsky matrix must be equal to the sum of a symmetric semi-definite matrix and a rank-one matrix $S = \Sigma + R$. Note that is the conventional symmetric matrix derived holding the Pareto weight constant. The interpretation of this result is the following. For any given pair of utility functions, a) the household budget constraint determines the Pareto frontier as a function of prices and income, and b) the value of the Pareto weight $\widetilde{\mu}$ determines the location of the household equilibrium on the frontier. Consequently, a change in prices implies a shift of the Pareto frontier. The shift of the Pareto frontier entails the modification of household demands described by the symmetric matrix Σ. However, the value of $\widetilde{\mu}$ varies as well, since it is a function of prices and income. Hence the location of the equilibrium moves along the Pareto frontier. This movement is represented by R.

1.3 Organization and Major Findings of the Thesis

The thesis analyses the family behaviour in the context of Pareto efficient models. In particular, the thesis deals with family labour participation decisions, household production undertaken by the family and fertility choices. The first two chapters are developed within a static framework, on the other hand, the analysis of family fertility choices is carried out within a dynamic framework.

Family Labour Participation Decisions within Pareto Efficient Household Models

The purpose of the first chapter is to describe the labour participation decisions of a two-person family in a given period of time. A family member decides to participate in the labour market if her reservation wage, the value she places on her marginal units of time in leisure and consumption, is equal to the market wage she can earn in the labour market (Heckman 1974). The analysis of individual labour participation decisions is pursued using both the unitary and collective approach to represent the household behaviour. We assume that household preferences have the functional form of a Bergson welfare index defined by the weighted aggregation of individual utility levels. In the unitary model weights are constant and, on the other hand, in the collective model they are functions of variables entering the budget constraint.

In particular, the chapter attempts to give a theoretical characterization to the individual reservation wage function and the participation (nonparticipation) frontier derived within the unitary and collective models. The analysis abstracts from rigidity of male labour supply, involuntary unemployment and household production.

The structure and results of the chapter are as follows.

- Within the unitary model the reservation wage function of each spouse is uniquely identified. In consequence, the participation frontier, defined by the set of prices, wages and nonlabour income bundles for which a household member is indifferent between working or not working, can be identified. The participation frontier divides the market wage plane in four regions. Each region is characterized by an opportunity set such that the spouses are jointly better off either of them working, neither of them working, or one working and the other not.

- In addition, the chapter presents testable restrictions that the individual demands of leisure (labour) belonging to the four labour participation regimes must satisfy in order to be compatible with the structure of household preferences described by the unitary model. In particular, the income pooling hypothesis and conventional Slutsky conditions, negativity and symmetry, must be satisfied by the individual demands for leisure.

- Furthermore, the chapter preforms a qualitative analysis on the individual reservation wage function. In each labour participation regime, we study the effect on the reservation wage function of marginal increases in the market wage of the spouse that works. In order to provide this qualitative analysis we apply Blundell *et al.* (2007) to our framework in which both spouses decide whether to participate or not. We find that the effect on the reservation wage function of marginal increases in the market wage of the spouse that works can be decomposed into two effects: an income effect and the difference in labour supply curves of the working spouse derived in the labour participation regimes in which both spouses work and in which only a spouse works. The latter effect is divided by the labour supply of the other spouse. This result is similar to that obtained by Blundell *et al.* (2007) with the only difference that in Blundell *et al.* (2007) the second term is not divided by the labour supply of the other spouse. This difference originates from the fact that Blundell *et al.* (2007) study the labour participation decisions of one spouse only. Here, our contribution is that we apply this result to characterize the participation frontier. In particular, assuming that the income effect is always positive we define the conditions under which the participation frontier is positively/negatively affected by marginal increases of the wage of the working spouse.

- On the other hand, within the collective model the presence of current market wages in household preferences means that a unique reservation wage need no longer exist. As a consequence, the participation frontier may not be identified unless further assumptions are made. This negative result is consistent with previous findings (Blundell *et al.* 2007, Donni 2003). To overcome this drawback, Blundell *et al.* (2007) and Donni (2003) postulate that the reservation wage is a contraction mapping with respect to market wages ensuring in this way the existence of a unique reservation wage in each participation regime and therefore the existence of a fixed point in the plan of wages from which the participation frontier starts.

- Unlike Blundell *et al.* (2007) and Donni (2003), in order to uniquely derive the individual reservation wage function in each labour participation regime and to identify the participation frontier, in the chapter we assume that the agent's bargaining power, among other variables, is a function of individual expected wages, defined as a function of past working experience, demographic characteristics of the agent and macro-economics indicators. In this way, the Pareto weight does not depend on current market wages. Given this assumption, the reservation wage theory leads to a complete characterization of the labour participation decision also within the collective model of household behavior.

Separability and Joint Production in the Collective Household Model

The second chapter presents the collective approach to the household behaviour with production and consumption-leisure decisions. The household is seen as jointly engaged in production and consumption-leisure decisions. Within this setting the household is involved in producing domestic goods by transforming input factors. At the same time, the household chooses the optimal consumption of market-purchased goods, leisure, and domestic goods produced by the family. Thus the production and consumption-leisure decision processes must be integrated into one single problem.

Economists have devoted substantial attention to the separation property of production and consumption-leisure decisions. In the context of the unitary model, a number of works have studies the separation property both from a theoretical and an empirical point of view (Benjamin 1992, Bardhan and Udry 1999, Chayanov 1986, Lopez 1984, Sen 1966, Singh, Squire, and Strauss 1986). In the context of the collective model, Chiappori (1997) points to the separation property between production and consumption-leisure decisions but then his analysis is confined to show that the decision process is identifiable also taking into account production activities of the family. In this research area, however, there are still some questions that need to be addressed and clarified especially within the collective model.

In the context of a general equilibrium model, Löfgren and Robinson (1999:663) state that "Household production and consumption decisions are nonseparable whenever the household shadow price of at least one production-consumption good is not given exogenously by the market but instead is determined endogenously by the interaction between household demand and supply." However, as we clarify later on, in the case of missing markets for goods produced by families separability of production and consumption decisions may hold true. In particular, production functions with constant returns to scale are a sufficient condition for the separability property to hold. This sufficient condition is general in the sense that it applies to cooperative household models and noncooperative household models as well. Moreover, in the case of missing markets a behavioural aspect that needs to be clarified is that when making production decisions families have a cost minimizing behaviour. We emphasize this point since a number of works (Apps and Rees 1997, Chiappori 1997, Rapoport, Sofer, and Solaz 2003) model the family as maximizing profit on the basis of implicit prices of the domestic good.

In general, most of the collective models with household production assume that families produce a generic output privately consumed in unobservable proportions by their members (Apps and Rees 1997, Chiappori 1997, Donni 2005, Rapoport, Sofer, and Solaz 2003). Differently, we examine the case in which with the same input variables and technology the family produces two different domestic goods privately consumed by its members. In this way, we can study the effects of joint production technologies on the structure of the collective household model.

Furthermore, we perform a comparative statics analysis on individual demands for leisure and domestic good derived from the collective model developed under different market structures. Thus, the generalized Slutsky equation derived by Browning and Chiappori (1998) is extended to collective models with marketable and nonmarketable domestic goods.

The following points describe in detail the content of the chapter.

- For reasons of completeness, the chapter starts by describing the case of marketable domestic goods. When markets are complete, the family produces an aggregated output that can be sold at a given market price or privately consumed by each household member. The price of the domestic good is determined on the marketplace and the separation property between production and consumption-leisure choices holds. As a consequence, the household decides the optimal production plan independently of its optimal consumption-leisure bundle. Conversely, the consumption-leisure choices are affected by the production activities of the family through profit effects.

- The chapter then turns to study the case in which markets for the domestic good are incomplete or absent. Moreover, household production is modelled by means of joint technologies: with the same variable inputs and technology the family produces two different outputs privately consumed by each member. When markets for the domestic goods are missing, the implicit price of the domestic good is endogenous to each household and it is jointly determined by the household's choices. Under the circumstance of absent markets, we find that constant returns to scale are sufficient to ensure that the implicit domestic price does not depend on household tastes and the decision process. This result ensures the separability between production and consumption-leisure choices. In the first stage the family decides the optimal time devoted to the production activities, then it decides the individual consumption of market goods, leisure and domestic goods. It is worth remarking that with missing markets in taking production decisions the family has a cost minimizing behaviour.

- Then, the chapter introduces the case of missing markets for the domestic good with nonconstant returns to scale maintaining the assumption of joint production functions. It is shown that the implicit price of the domestic good depends on the production-consumption variables and, therefore, the household model must be solved jointly with consumption-leisure decisions. In general, within collective models nonseparability of production and consumption-leisure choices has negative consequences also for the household model structure. In particular, the consumption-leisure choices cannot be modelled as if it were a two-stage budgeting process meaning that the sharing rule approach cannot be employed. This finding has crucial consequence for empirical applications. As pointed out by Browning, Chiappori, and Lewbel (2006) identification of the Pareto weight requires knowledge of the unobservable cardinalization of member utilities. Therefore to provide identification of the structural model one needs to know those cardinalizations and the results may strongly depend on the cardinalization chosen by researchers. In addition, in accord with previous findings (Pollak and Watcher 1975), due to joint technologies a closed-form solution to the production-consumption household model cannot be found.

- In order to provide closed-form solutions and to implement the sharing rule approach, in chapter two alternative specifications of the production function are proposed, though it is recognized that they may model very specific household productions. In both alternatives jointness in production is omitted. However, in both alternatives production decisions must be solved jointly with consumption-leisure decisions. A first alternative assumes that domestic goods are produced using two distinct production functions in which both household members allocate a part of their time. Without joint technologies the production-consumption household model has a closed-form solution. However, it cannot be extended to the sharing rule approach. In the second alternative each household member produces by herself the domestic good that she consumes. The advantage of the latter is that, given an appropriate exogenous sharing rule, the household program can be decentralized into two individual programs in which each member chooses production and consumption variables jointly, but independently of production-consumption decisions of the other member.

A Dynamic Model of Consumption and Discrete Fertility Choice

The last chapter presents a finite-horizon dynamic model of fertility and consumption within a certain environment. Fertility is modelled as a discrete choice and it is the outcome of comparisons between parents' welfare level with and without an extra child.

The dynamic model of discrete fertility choice emphasizes the effects exerted on the family size decision by the costs of rearing children. The theoretical model is developed under a set of simplifying assumptions designed to obtain an analytical solution to the household dynamic program. In particular, we assume that household utility is of the CARA form and, neglecting that spouses can have different fertility preferences, we abstract from bargaining processes that can occur within the couple. Furthermore, we assume that parents face perfectly foreseen costs of rearing a child and these costs are equal across children.

The dynamic household model of fertility choice developed in the chapter is similar to that of Wolpin (1984). Studying the connection between child mortality and fertility within a dynamic stochastic model, Wolpin (1984) treats the fertility choice as a discrete endogenous variable. However, Wolpin (1984) aims at estimating the dynamic fertility model and he does not develop an analytical representation of the optimal fertility choice rule.

Within this framework, we show that:

- Given CARA preferences, optimal consumption depends positively and linearly on current disposable resources net of the costs of rearing children. Another feature of optimal consumption derived by CARA preferences is that it may be negative. In our model the probability of negative consumption increases since household wealth available for consumption is reduced by the costs of rearing children. During the reproductive life, in each period of time the couple's decision of having a child leads to a discrete shift in the intercept of optimal consumption and it increases the total costs of rearing children faced by the family. Therefore, it is likely that optimal consumption will fall due to the extra costs related to the presence of a new child. As the number of dependent children increases, *ceteris paribus*, there are less resources available for family consumption other than expenditures for children and consumption decreases.

- In each period of the reproductive life the couple does not have a child if the pure utility gain from not having a child is greater than the utility saving in cost from having the child. In each period of the reproductive life the choice "whether to have an additional child or not" is based upon the costs of rearing children faced by the couple. Fertility choices are also influenced by the coefficient of absolute risk aversion. In particular, as the costs of rearing children increase it is likely that the new birth does not occur. Similarly, if the coefficient of absolute risk aversion increases. The interest rate indirectly affects the fertility choice: an increase in the interest rate positively affects the costs of having an additional child reducing, in this way, the probability that a new birth occurs.

- The chapter presents a simulation exercise of the dynamic model. The fertility outcome is analysed for traditional and non-traditional couples. The former is a family in which there is only one main wage earner. On the other hand, the non-traditional family is characterized by the presence of two wage earners. In this way, we control for different fertility outcomes due to different costs of rearing children faced by the two families. As expected, the traditional couple decides to have more children relative to the non-traditional couple. In particular, the traditional family has two children with a two year gap between the first and second birth. On the other hand, the non-traditional couple has just one child. The consumption pattern and assets accumulation of the two families are influenced by different fertility outcomes. Given a bigger family size, throughout the reproductive life consumption pattern of the traditional family is in general higher relative to consumption of the non-traditional family. To sustain family consumption, through the reproductive life both families have negative assets accumulation. To study how the interest rate may influence parents' fertility choices we also run the simulation assuming that the interest rate is equal to zero. As the interest rate decreases, all the other things being equal, the costs of having a child decreases as well increasing, in this way, the probability of a new birth. With the interest rate equal to zero both traditional and non-traditional families have one more child with respect to the previous scenario. Moreover, setting the interest rate equal to zero has the effect of reducing the spacing of births. Patterns of the simulated consumption and assets accumulation for the two families are similar to those obtained with a positive interest rate.

Chapter 2

Family Labour Participation within Pareto Efficient Models

2.1 Introduction

The purpose of this chapter is to investigate and characterize the labour participation decisions of a two-person family in a given period of time. This purpose is pursued using both a traditional and a collective approach to represent the household behaviour. A family member decides to participate in the labour force if her reservation wage, the value she places on her marginal units of time in leisure and consumption, is equal to the market wage she can earn in the labour market (Heckman 1974). We assume that household preferences have the functional form of a Bergson welfare index defined by the weighted aggregation of individual utility levels where in the unitary model weights are constant and, on the other hand, in the collective model they are functions of variables entering the budget constraint.

Traditionally, the household has been considered as if it were an elementary decision unit maximizing a unique welfare index subject to a family budget constraint. Underlying the unitary model there is the assumption that the family combines all sources of income into a unique income measure. As a consequence of the income pooling hypothesis, family outcomes are not affected by who has control over money and, in turn, resources are equally allocated among family members. Implicit in the unitary model there is the concept either that the topic of intra-household resource allocation is irrelevant or that it can be addressed within the fiction of a dictatorial decision-making process.

However, even though most of its testable properties have been rejected by micro data (Schultz 1990, Thomas 1990, Lundberg, Pollak, and Wales 1997, Fortin and Lacroix 1997, Browning and Chiappori 1998) and the evidence that neglecting the rule in which families share resources across members can produce misleading welfare analyses (Haddad and Kanbur 1990, Bargain *et al.* 2006), the unitary model has been extensively applied both to represent many aspects of the household behaviour and to carry out welfare analyses. In a labour economics perspective, there exists a vast literature that employs the unitary model. This literature covers a wide range of economic and social aspects, such as the analysis of causal relationships between female labour supply and fertility choices.

Recently, other economic models have been developed to study the household behaviour. In contrast to the unitary model, these models are focused on the behaviour of individuals. Manser and Brown (1980) and McElroy and Horney (1981) represent family decisions within a Nash bargaining framework. However, the collective model proposed by Chiappori gets more considerable attention from economists. Chiappori (1988, 1992) adopts an alternative theoretical framework in which each household member is characterized by her own preferences and, assuming that the decision process results in Pareto efficient outcomes, he shows that when agents are "egoistic" and consumption is purely private, the collective model generates testable restrictions and, from observed market behaviour, one can recover certain structural elements of the decision process, such as individual preferences and the rule that determines the allocation of resources within the family. The decision-making process underlying the collective model can be modelled as if decisions occur in two stages. First, household members agree on the allocation of non-labour income between them and then each agent independently decides the allocation of her own money resources among different goods.

In the context of a collective framework, Blundell *et al.* (2007) build a household model of labour participation based on two empirical features of the U.K. labour market.

In particular, Blundell *et al.* (2007) notice that a large proportion of married women do not work at all and the husband's working hours do not present much variation. In general, men work full-time or do not work. This labour market feature produces empirical difficulties when researchers attempt to infer the individual decision process by observing male's labour supply. Therefore, Blundell *et al.* (2007) model husband's working hours as a discrete choice and wife's working hours as a continuous variable censored from below. In doing so, they show that the sharing rule is recovered up to an additive constant and individual preferences are identified. Donni (2003) completes the analysis of labour participation undertaken in Blundell *et al.* (2007) treating the husband's labour choice as a continuous variable.

Using French data, Donni (2006) generalizes this line of research assuming that the individual labour participation is not the result of a free choice and extends the theory of household behaviour under rationing (Neary and Roberts 1980) to collective models. However, the aim of these articles is confined to show that the sharing rule and the underlying individual preferences are identifiable, and to derive testable restrictions in collective frameworks that account for corner solutions.

The present chapter focuses on a different aspect of the analysis of family labour participation. In particular, we attempt to give a theoretical characterization to reservation wages and the participation (nonparticipation) frontier derived within the unitary and collective models. The analysis abstracts from rigidity of male labour supply, involuntary unemployment and household production.

Within the unitary model we find that the reservation wage of each spouse is uniquely identified. In consequence, the participation frontier is represented in the locus of spouses' market wages and it divides the market wage plane in four regions. Each region is characterized by an opportunity set such that the spouses are jointly better off either of them working, neither of them working, or one working and the other not.

On the other hand, within the collective model the presence of current market wages in household preferences, or equivalently in the sharing rule, means that a unique reservation wage need no longer exist. As a consequence, the participation frontier is not well-behaved unless further assumptions are made. To overcome this drawback, Blundell *et al.* (2007) and Donni (2003) postulate that the reservation wage is a contraction mapping with respect to market wages ensuring in this way the existence of a unique reservation wage in each participation regime and therefore the existence of a fixed point in the plan of wages from which starts the participation frontier.

Differently, in labour collective models with corner solutions we assume that agent's bargaining power, among other variables, is a function of individual expected wages, defined as function of past working experience, demographic characteristics of the agent and macro-economics indicators. In this way, we neglect that the Pareto weight depends on current market wages of spouses. Given this assumption we find that the reservation wage theory leads to a complete characterization of the labour participation decision also within the collective model of household behavior.

The chapter is laid out as follows. In Section 2.2 we address the labour participation decisions in the context of the unitary model. In Section 2.3 we characterize the labour supply equations and reservation wages derived by the traditional model of household behaviour. In Sections 2.4 and subsequent, we extend the collective labour supply model (Chiappori 1988, 1992) to corner solutions. The conclusions end the chapter.

2.2 Family Labour Participation within the Unitary Model

We start by investigating the labour participation in the context of the unitary model. The family, composed by two persons f and m, possesses a Bergson welfare function W^h with preferences of each spouse represented by a continuously differentiable, and strictly quasi-concave utility function U^i defined over the consumption of a private composite market good c^i and leisure L^i. We assume that all time not spent at paid work is leisure time. Individual time endowment is normalized to one and leisure is thus equal to $L^i = 1 - l^i$.

The family solves the following constrained program

$$\max_{c^f, l^f, c^m, l^m} \quad W^h = \mu U^f(c^f, 1-l^f) + (1-\mu)\, U^m(c^m, 1-l^m), \tag{2.1}$$

$$\text{subject to} \quad p_f c^f + p_m c^m = w_f l^f + w_m l^m + y,$$

$$\text{and} \quad c^f \geq 0,\ c^m \geq 0,\ 0 \leq l^f \leq 1,\ 0 \leq l^m \leq 1,$$

where μ is a scalar, p_i is the market price of the private composite market good c^i, w_i is the individual market wage[1] and y is the household nonlabour income.

Maximization of program (2.1) with respect to the variables of choice yields the following Kuhn-Tucker first-order necessary conditions

$$\mu_i U^i_{c^i} - \delta p_i \ \leq \ 0, \quad \text{with equality if } c^i > 0, \quad i = f, m, \tag{2.2}$$

$$-\mu_i U^i_{l^i} + \delta w_i \ \leq \ 0, \quad \text{with equality if } l^i > 0, \quad i = f, m, \tag{2.3}$$

$$p_f c^f + p_m c^m \ = \ w_f l^f + w_m l^m + y, \tag{2.4}$$

where δ is the Lagrangian multiplier associated with the budget constraint and hereafter subscripted functions indicate derivatives. In equations (2.2)–(2.3) the constant weight for $i = f$ is equal to $\mu_f = \mu$ and for $i = m$ becomes $\mu_m = 1 - \mu$. Note that in the collective model the weight μ is a function of exogenous variables that determines the location of the family on the Pareto efficient frontier.

In our analysis, individual consumption of the composite market good is assumed to be positive everywhere, therefore, equation (2.2) holds as a strict equality for any combination of market prices, wages and nonlabour income. On the other hand, the labour behaviour of spouses is represented by the following necessary equilibrium conditions.

Both Spouses Participate, $P: \left\{l^f > 0, l^m > 0, p_f c^f + p_m c^m = w_f l^f + w_m l^m + y\right\}$

[1] We assume that wages are constant. Therefore, we ignore the fact that the marginal wage of each agent may depend on the amount of her working hours. From an empirical point of view, this assumption rules out the endogeneity's issue of wage rates.

We begin with the analysis of the interior solution. P denotes the wage plane in which both household members participate in the labour market. Spouses find it optimal to jointly participate in the labour force if and only if at interior values of leisure the individual marginal rate of substitution between working hours and consumption of the composite market good is equal to the ratio of market wage to market price

$$\frac{U^f_{l^f}\left(c^f,1-l^f\right)}{U^f_{c^f}\left(c^f,1-l^f\right)}=\frac{w_f}{p_f},\quad\text{and}\quad\frac{U^m_{l^m}\left(c^m,1-l^m\right)}{U^m_{c^m}\left(c^m,1-l^m\right)}=\frac{w_m}{p_m},$$

from which we can derive the optimal labour supply equations

$$\widetilde{l}^f_P = l^f_P\left(p_f,p_m,w_f,w_m,y\right), \tag{2.5}$$

$$\widetilde{l}^m_P = l^m_P\left(p_f,p_m,w_f,w_m,y\right), \tag{2.6}$$

and the individual demands for leisure are $\widetilde{L}^i_P=1-\widetilde{l}^i_P$ for $i=f,m$. When both spouses work the optimal level of household welfare is $\widetilde{W}^h_P(p_f,p_m,w_f,w_m,y)$.

Given the assumption of additive separability between individual consumption-labour choices of the household utility function W^h, i's marginal rate of substitution is independent of consumption and leisure decisions of j meaning that the labour participation and private consumption of the husband and wife are not jointly determined. In principle, the assumption of additive separability imposes significant restrictions on the structure of the household model. However, empirical evidence rejects the hypothesis that the labour supply of the spouses is jointly determined (Lundberg 1988). Thus, the assumption of additive separability between individual consumption-labour choices does not impose strong restrictions on the structure of model (2.1).

Setting the individual labour equation $\widetilde{l}^i_P$ equal to zero we find the reservation wage of member i by solving for w_i[2]

$$\varpi^{*i}_{P_j}=\varpi^i_{P_j}\left(p_f,p_m,w_j,y\right),\ \text{for } i\neq j=f,m.$$

[2]Equivalently, the reservation wage can be derived equalizing the marginal rate of substitution between labour and consumption to the ratio of the corresponding prices and solving for w_i.

This equation represents the marginal value of time at which i is indifferent between working or not working when the other spouse finds it optimal to supply a positive amount of working hours.

In the following example we clarify how the reservation wage theory works within the unitary model described in (2.1).

Example 2.1 *Let us suppose that each individual has preferences of Cobb-Douglas form*

$$U^i\left(c^i, 1-l^i\right) = \alpha_i \log c^i + \beta_i \log\left(1-l^i\right), \quad i = f, m,$$

where α_i and β_i are parameters. When both spouses work, the household opportunity set is equal to

$$p_f c^f + p_m c^m = w_f l^f + w_m l^m + y.$$

The first-order conditions for an interior solution are

$$\begin{aligned} p_i c^i &= \frac{\mu_i \alpha_i}{\delta}, \\ w_i l^i &= w_i - \frac{\mu_i \beta_i}{\delta}, \end{aligned}$$

where δ is the Lagrange multiplier. Substituting the set of first-order equations into the budget constraint we obtain the expression for the reciprocal of δ

$$\frac{1}{\delta} = \frac{w_f + w_m + y}{\sum_{i=f,m} \mu_i \left(\alpha_i + \beta_i\right)}.$$

Replacing the expression of $1/\delta$ in the first order conditions we get the optimal solution for the individual demands of the composite market good and labour supply

$$\begin{aligned} \widetilde{c}^i &= \frac{\mu_i \alpha_i}{p_i} \frac{w_f + w_m + y}{\sum_{i=f,m} \mu_i \left(\alpha_i + \beta_i\right)}, \\ \widetilde{l}^i &= 1 - \frac{\mu_i \beta_i}{w_i} \frac{w_f + w_m + y}{\sum_{i=f,m} \mu_i \left(\alpha_i + \beta_i\right)}. \end{aligned}$$

Now, to find i's reservation wage we set i's optimal labour supply equal to zero

$$\widetilde{l^i} = 1 - \frac{\mu_i \beta_i}{w_i} \frac{w_f + w_m + y}{\sum_{i=f,m} \mu_i (\alpha_i + \beta_i)} = 0$$

and, then, we solve this equation for w_i

$$\varpi_{P_j}^{*i} = \frac{\mu_i \beta_i}{(1 - \mu_i \beta_i)} (w_j + y), \quad \text{for } i \neq j = f, m,$$

getting i's reservation wage when j is working. Note that if $\mu_i = 1/\beta_i$ *then the denominator of the reservation wage function goes to zero and the reservation wage is not defined.*

Wife Does Not Participate and Husband Participates, $P_M : \{l^f = 0, l^m > 0, p_f c^f + p_m c^m = w_m l^m + y\}$

The labour regime in which the wife chooses not to work and the husband to work, denoted by P_M, will be optimal if and only if

$$\frac{U_{l^f}^f (c^f, 1)}{U_{c^f}^f (c^f, 1)} > \frac{w_f}{p_f}, \quad \text{and} \quad \frac{U_{l^m}^m (c^m, 1 - l^m)}{U_{c^m}^m (c^m, 1 - l^m)} = \frac{w_m}{p_m},$$

where f's marginal rate of substitution is valued at interior solutions for private consumption $c^f > 0$ and at zero working hours $l^f = 0$. On the other hand, m's marginal rate of substitution is valuated at interior solutions both for private consumption $c^m > 0$ and labour supply $l^m > 0$.

In particular, the husband finds it optimal to supply the following positive number of working hours

$$\widetilde{l}_{P_M}^m = l_{P_M}^m (p_f, p_m, w_m, y),$$

and the individual demands for leisure are $\widetilde{L}_{P_M}^m = 1 - \widetilde{l}_{P_M}^m$ and $\widetilde{L}_{P_M}^f = 1$. In this participation regime, at the optimum the family gets a welfare level equal to $\widetilde{W}_{P_M}^h(p_f, p_m, w_m, y)$.

Solving the labour equation $\widetilde{l}_{P_M}^m$ with respect to w_m we have the expression for the husband reservation wage

$$\varpi_N^{*m} = \varpi_N^m (p_f, p_m, y), \tag{2.7}$$

and, since the wife does not work, the reservation wage function ϖ_N^{*m} depends only on market prices and household nonlabour income.

Example 2.2 *Continuing with the previous example, when the wife does not work the household budget constraint becomes*

$$p_f c^f + p_m c^m = w_m l^m + y,$$

and the first-order conditions are

$$\begin{aligned} p_i c^i &= \frac{\mu_i \alpha_i}{\delta}, \quad \text{for } i = f, m, \\ w_m l^m &= w_m - \frac{\mu_m \beta_m}{\delta}. \end{aligned}$$

Similarly, the expression for $1/\delta$ *is*

$$\frac{1}{\delta} = \frac{w_m + y}{\sum_{i=f,m} \mu_i (\alpha_i + \beta_i)},$$

and substituting $1/\delta$ *into the first-order conditions we get the optimal solution of the individual demands for the composite market good and husband's labour supply*

$$\begin{aligned} \widetilde{c}^i &= \frac{\mu_i \alpha_i}{p_i} \frac{w_m + y}{\mu_f \alpha_f + \mu_m (\alpha_m + \beta_m)}, \quad \text{for } i = f, m, \\ \widetilde{l}^m &= 1 - \frac{\mu_m \beta_m}{w_m} \frac{w_m + y}{\mu_f \alpha_f + \mu_m (\alpha_m + \beta_m)}. \end{aligned}$$

Now, to find the male reservation wage we set the optimal labour supply equal to zero that is $\widetilde{l}^m = 0$ *and solve for* w_m

$$\varpi_N^{*m} = \frac{\mu_m \beta_m}{\mu_f \alpha_f + \mu_m \alpha_m} y$$

that gives the husband reservation wage when his wife finds it optimal to supply zero working hours.

Wife Participates and Husband Does Not Participate, $P_F : \{l^f > 0, l^m = 0,$

$p_f c^f + p_m c^m = w_f l^f + y\}$

Similarly, the labour regime in which the wife works and the husband does not work, denoted by P_F, will be optimal if and only if

$$\frac{U^f_{l^f}\left(c^f, 1 - l^f\right)}{U^f_{c^f}\left(c^f, 1 - l^f\right)} = \frac{w_f}{p_f}, \quad \text{and} \quad \frac{U^m_{l^m}\left(c^m, 1\right)}{U^m_{c^m}\left(c^m, 1\right)} > \frac{w_m}{p_m},$$

where f's marginal rate of substitution is valued at interior solutions for individual consumption of the composite good $c^f > 0$ and labour supply $l^f > 0$, and m's marginal rate of substitution is valued at interior solutions for individual consumption of the composite good $c^m > 0$ and at corner solutions for labour supply $l^m = 0$.

The wife finds it optimal to supply the following positive number of working hours

$$\widetilde{l}^f_{P_F} = l^f_{P_F}\left(p_f, p_m, w_f, y\right),$$

with individual leisure demands equal to $\widetilde{L}^f_{P_F} = 1 - \widetilde{l}^f_{P_F}$ and $\widetilde{L}^m_{P_F} = 1$. In this participation regime, the optimal level of household welfare is $\widetilde{W}^h_{P_F}(p_f, p_m, w_f, y)$.

Solving the labour equation $\widetilde{l}^f_{P_F}$ with respect to w_f we have the expression for the wife reservation wage when the husband does not work

$$\varpi^{*f}_N = \varpi^f_N\left(p_f, p_m, y\right). \tag{2.8}$$

As in equation (2.7), the reservation wage ϖ^{*f}_N is function of market prices and household nonlabour income only.

Example 2.3 *To complete the analysis, when the husband does not work the household budget constraint becomes*

$$p_f c^f + p_m c^m = w_f l^f + y,$$

and the first-order conditions are

$$\begin{aligned} p_i c^i &= \frac{\mu_i \alpha_i}{\delta}, \quad for\ i = f, m, \\ w_f l^f &= w_f - \frac{\mu_f \beta_f}{\delta}. \end{aligned}$$

The expression for $1/\delta$ *is*

$$\frac{1}{\delta} = \frac{w_f + y}{\sum_{i=f,m} \mu_i \left(\alpha_i + \beta_i\right)},$$

from which we get the optimal solution of the demand for market goods and wife's labour supply

$$\begin{aligned} \widetilde{c}^i &= \frac{\mu_i \alpha_i}{p_i} \frac{w_f + y}{\mu_m \alpha_m + \mu_f \left(\alpha_f + \beta_f\right)}, \quad for\ i = f, m, \\ \widetilde{l}^f &= 1 - \frac{\mu_f \beta_f}{w_f} \frac{w_f + y}{\mu_m \alpha_m + \mu_f \left(\alpha_f + \beta_f\right)}, \end{aligned}$$

with

$$\varpi_N^{*f} = \frac{\mu_f \beta_f}{\mu_f \alpha_f + \mu_m \alpha_m} y$$

that gives the wife reservation wage when the husband does not work.

Both Spouses Do Not Participate, $N : \left\{l^f = 0, l^m = 0, p_f c^f + p_m c^m = y\right\}$

Finally, the couple decides to not take part in the labour market if and only if

$$\frac{U_{l^f}^f \left(c^f, 1\right)}{U_{c^f}^f \left(c^f, 1\right)} > \frac{w_f}{p_f}, \quad \text{and} \quad \frac{U_{l^m}^m \left(c^m, 1\right)}{U_{c^m}^m \left(c^m, 1\right)} > \frac{w_m}{p_m},$$

spouses' marginal rates of substitution, valued at interior solutions of private consumption and at zero working hour, are greater than the ratio of market wage over the price of market goods. In this case, spouses allocate all time to leisure, thus $\widetilde{L}_N^f = 1$, $\widetilde{L}_N^m = 1$. The optimal welfare level of the family is $\widetilde{W}_N^h(p_f, p_m, y)$.

In this section we have shown that within the unitary framework with linear budget constraint the reservation wage theory is a valid technique to study a two-person family labour decision. We now turn to characterize the unitary model of household labour participation outlined in this section.

2.3 Characterization of the Unitary Model

In this section we derive testable restrictions that the optimal solutions belonging to the four labour participation regimes N, P_F, P_M, P must satisfy in order to be compatible with the structure of household preferences described in (2.1). The characterization of model (2.1) is focused on the individual demand for leisure.

A property that must be satisfied by any economic set-up modelling rational behaviour is that the leisure demand $L^i(\cdot)$ must be homogeneous of degree zero in prices, wages, and nonlabour income. Then, in each labour participation regime, the individual demand for leisure must satisfy the following conditions.

2.3.1 Income Pooling Hypothesis and the Slutsky Equation for Unitary Leisure Demands

Both Spouses Work

Income Pooling Hypothesis

Standard implications of the unitary model are that a) individual members pool their incomes and, further, b) the identity of the income recipient does not matter on household outcomes. As a consequence, when both spouses work, the income pooling hypothesis imposes the following restriction

$$\widetilde{L}^i_{w_j} = \widetilde{L}^i_y, \quad \text{both spouses work,} \tag{2.9}$$

for $i \neq j = f, m$. The income pooling hypothesis underlying the unitary model thus implies that family income will affect household behaviour and moreover the identity of the income recipient does not matter on household outcomes.

The Slutsky Equation

Let $\widetilde{L}^i_P = L^i_P(p_f, p_m, w_f, w_m, y)$ be i's leisure demand curve when both spouses work.

The effect of a marginal variation of w_j on $\widetilde{L}^i_P$ compensated by a change in y such that the family utility is kept constant, that is

$$\frac{\partial \widetilde{L}^i_P}{\partial w_j} dw_j = \frac{\partial L^i_P}{\partial w_j} dw_j + \frac{\partial L^i_P}{\partial y} dy, \quad i,j = f,m,$$

is equal to

$$\frac{\partial \widetilde{L}^i_P}{\partial w_j} = \frac{\partial L^i_P}{\partial w_j} - \left(1 - L^j_P\right) \frac{\partial L^i_P}{\partial y}, \tag{2.10}$$

which is the conventional Slutsky equation.[3] If leisure is a normal good, then the income effect $\frac{\partial L^i_P}{\partial y}$ is positive, and as y increases consumption of leisure increases. The term in brackets $\left(1 - L^j_P\right)$, representing the labour supply of member j, is positive. Now, for $i = j$ the own-wage effect $\frac{\partial L^i_P}{\partial w_i}$ is negative and hence the overall effect (2.10) is negative. On the other hand, for $i \neq j$ the sign of $\frac{\partial L^i_P}{\partial w_j}$ is ambiguous and depends on the fact that i's labour supply is a complement or a substitute for j's labour supply and, without more assumptions, we cannot sign cross-wage effects (2.10).

When both spouses work, compensated wage effects can be summarized in a symmetric and negative semi-definite substitution matrix

$$S_P = \begin{bmatrix} s_{ii} & s_{ij} \\ s_{ji} & s_{jj} \end{bmatrix}, \quad \text{both spouses work,}$$

where $s_{ij} = L^i_{w_j} - l^j L^i_y$ for $i,j = f,m$ is the compensated wage effect, negativity of own-wage effects means that $s_{ii}, s_{jj} < 0$, symmetry of compensated cross-wage effect

[3]The Slutsky equation can be derived as follows (see for instance Silberberg 1990). Let us define the expenditure function $y = y^*(p_f, p_m, w_f, w_m, W^0)$ as the minimum income level that keeps the household welfare W constant to a given level W^0 when w_j changes. In our setting the expenditure function is $y = \sum_{i=f,m} p_i c^i - \sum_{i=f,m} w_i \left(1 - L^i\right)$. By definition

$$L^{*i}\left(p_f, p_m, w_f, w_m, W^0\right) \equiv L^i\left(p_f, p_m, w_f, w_m, y^*(p_f, p_m, w_f, w_m, W^0)\right),$$

where the left-hand side is the compensated demand of i's leisure. Differentiate then both sides of the identity with respect to w_j

$$\frac{\partial L^{*i}}{\partial w_j} = \frac{\partial L^i}{\partial w_j} + \frac{\partial L^i}{\partial y}\frac{\partial y^*}{\partial w_j},$$

where by the envelope theorem $\frac{\partial y^*}{\partial w_j} = -\left(1 - L^j\right)$ and we derive the Slutsky equation

$$\frac{\partial L^{*i}}{\partial w_j} = \frac{\partial L^i}{\partial w_j} - \left(1 - L^j\right)\frac{\partial L^i}{\partial y}.$$

means $s_{ij} = s_{ji}$ and negativity of the determinant of the substitution matrix S_P implies $s_{ii}s_{jj} - s_{ij}^2 < 0$.

Only Spouse i Works

When only member i participates in the labour force, the unitary model implies that

$$\widetilde{L}^i_{w_j} = 0, \qquad j \text{ does not work}, \tag{2.11}$$

when member j does not take part in the labour force, changes of j market wage do not affect the leisure demand of the other spouse. In general, in the collective model restriction (2.11) may not hold true. If the Pareto weight μ is assumed to depend, among other variables, on j's market wage, even if j does not work, then marginal changes of j's market wage affect the labour supply of i through changes in the Pareto weight μ.

The Slutsky Equation

On the other hand, when only member i works i's demand for leisure is $\widetilde{L}^i_{P_I} = L^i_{P_I}(p_f, p_m, w_i, y)$ and the compensated own-wage effect is equal to

$$\frac{\partial \widetilde{L}^i_{P_I}}{\partial w_i} = \frac{\partial L^i_{P_I}}{\partial w_i} - \left(1 - L^i_{P_i}\right) \frac{\partial L^i_{P_i}}{\partial y}, \qquad \text{only } i \text{ works.}$$

Under the assumption of normality, the compensated own-wage effect is negative: as long as leisure becomes more expensive a rational individual decreases her own consumption of leisure.

2.3.2 Characterization of the Participation Frontier

We study the effects of marginal changes of spouses' market wage on the participation frontier. The participation frontier is defined by the set of prices, wages and nonlabour income bundles for which a household member is indifferent between working or not working. The analysis is a generalization of Blundell *et al.* (2007) to the case in which both spouses decide whether to participate or not.[4]

[4] In the context of a unitary model in which the labour participation of spouses is jointly determined, Simmons (2006) provides a similar characterization to the participation frontier.

Only Spouse j Works

The participation decision of member i depends on the difference between the household welfare when i does not work and the other spouse does, $\widetilde{W}^h_{P_J}(p_i, p_j, w_j, y)$, and the household welfare when both spouses work, $\widetilde{W}^h_P(p_i, p_j, w_i, w_j, y)$. Therefore, if

$$\widetilde{W}^h_{P_J}(p_i, p_j, w_j, y) \geq \widetilde{W}^h_P(p_i, p_j, w_i, w_j, y),$$

then member i does not work and $l^i = 0$, otherwise i works and $l^i > 0$.

The participation frontier is thus characterized by the following equality

$$\widetilde{W}^h_P\left(p_i, p_j, \varpi^{*i}_{P_J}, w_j, y\right) = \widetilde{W}^h_{P_J}\left(p_i, p_j, w_j, y\right), \tag{2.12}$$

where $\varpi^{*i}_{P_J} = \varpi^i_{P_J}\left(p_i, p_j, w_j, y\right)$ is i's reservation wage when the other spouse supplies a positive number of working hours. According to equation (2.12), the reservation wage function $\varpi^{*i}_{P_J}$ is such that the household optimal utility functions in the two participation regimes, $\widetilde{W}^h_P$ and $\widetilde{W}^h_{P_J}$, are equal meaning that the family enjoys the same level of welfare regardless of spouses' working status.

To derive the effect of j's wage variations on i's reservation wage function, we totally differentiate relation (2.12), keeping market prices p_i and p_j constant,

$$\frac{\partial \widetilde{W}^h_P}{\partial \varpi^{*i}_{P_J}} \frac{\partial \varpi^i_{P_J}}{\partial w_j} dw_j + \frac{\partial \widetilde{W}^h_P}{\partial \varpi^{*i}_{P_J}} \frac{\partial \varpi^i_{P_J}}{\partial y} dy + \frac{\partial \widetilde{W}^h_P}{\partial w_j} dw_j + \frac{\partial \widetilde{W}^h_P}{\partial y} dy = \frac{\partial \widetilde{W}^h_{P_J}}{\partial w_j} dw_j + \frac{\partial \widetilde{W}^h_{P_J}}{\partial y} dy. \tag{2.13}$$

Using Roy's identity we have

$$\frac{\frac{\partial \widetilde{W}^h_P}{\partial \varpi^{*i}_{P_J}}}{\frac{\partial \widetilde{W}^h_P}{\partial y}} \frac{\partial \varpi^i_{P_J}}{\partial w_j} dw_j + \frac{\frac{\partial \widetilde{W}^h_P}{\partial \varpi^{*i}_{P_J}}}{\frac{\partial \widetilde{W}^h_P}{\partial y}} \frac{\partial \varpi^i_{P_J}}{\partial y} dy + \frac{\frac{\partial \widetilde{W}^h_P}{\partial w_j}}{\frac{\partial \widetilde{W}^h_P}{\partial y}} dw_j + \frac{\frac{\partial \widetilde{W}^h_P}{\partial y}}{\frac{\partial \widetilde{W}^h_P}{\partial y}} dy = \frac{\frac{\partial \widetilde{W}^h_{P_J}}{\partial w_j}}{\frac{\partial \widetilde{W}^h_{P_J}}{\partial y}} dw_j + \frac{\frac{\partial \widetilde{W}^h_{P_J}}{\partial y}}{\frac{\partial \widetilde{W}^h_{P_J}}{\partial y}} dy, \tag{2.14}$$

where

$$\frac{\partial \widetilde{W}_P^h}{\partial \varpi_{P_J}^{*i}} / \frac{\partial \widetilde{W}_P^h}{\partial y} = \widetilde{l}_P^i, \quad \text{both spouses work,} \tag{2.15}$$

$$\frac{\partial \widetilde{W}_{P_J}^h}{\partial w_j} / \frac{\partial \widetilde{W}_{P_J}^h}{\partial y} = \widetilde{l}_{P_J}^j, \quad \text{only } j \text{ works,} \tag{2.16}$$

are Roy's identities derived in two labour participation regimes from which we obtain the labour equation of member i when both spouses work (2.15), and member j when only j works (2.16).

Substituting equations (2.15) and (2.16) into (2.14), we obtain

$$\widetilde{l}_P^i \frac{\partial \varpi_{P_J}^i}{\partial w_j} dw_j + \widetilde{l}_P^i \frac{\partial \varpi_{P_J}^i}{\partial y} dy + \widetilde{l}_P^j dw_j + dy = \widetilde{l}_{P_J}^j dw_j + dy. \tag{2.17}$$

Differentiating the household opportunity set in P_J we find the value of dy

$$dy = -\widetilde{l}_{P_J}^j dw_j.$$

Replacing it into equation (2.17) and re-arranging terms we have

$$\widetilde{l}_P^i \frac{\partial \varpi_{P_J}^i}{\partial w_j} = \widetilde{l}_{P_j}^j - \widetilde{l}_P^j + \widetilde{l}_P^i \frac{\partial \varpi_{P_j}^i}{\partial y} \widetilde{l}_{P_j}^j, \tag{2.18}$$

dividing by $\widetilde{l}_P^i$ both sides of equation (2.18), we obtain the effect of a change of j's market wage on i's reservation wage $\varpi_{P_j}^j$

$$\frac{\partial \varpi_{P_j}^i}{\partial w_j} = \frac{\widetilde{l}_{P_J}^j - \widetilde{l}_P^j}{\widetilde{l}_P^i} + \widetilde{l}_{P_J}^j \frac{\partial \varpi_{P_J}^i}{\partial y}. \tag{2.19}$$

Equation (2.19) is similar to the analysis performed by Blundell *et al.* (2007). The effect of a marginal variation of j's market wage on i's reservation wage can be decomposed into two effects: an income effect $\widetilde{l}_{P_J}^j \frac{\partial \varpi_{P_J}^i}{\partial y}$ and the difference between j's labour supply curves in the two labour participation regimes $\widetilde{l}_{P_J}^j - \widetilde{l}_P^j$. Unlike the analysis of Blundell *et al.* (2007), here the latter effect is divided by i's labour supply curve. This difference originates from the fact that Blundell *et al.* (2007) study the labour participation decision of one spouse only.

To provide interpretations to equation (2.19) that allow to characterize the participation frontier the following assumption is in order.

Assumption 2.1 *We assume that the income effect $\partial \varpi^i_{P_J}/\partial y$ is positive.*

Assumption 2.1 states that as the nonlabour income increases, *ceteris paribus*, i's reservation wage increases as well. In turn, as i's reservation wage increases member i will participate in the labour market for relative higher market wages reducing, in this way, the probability that i will work.

The following situations then can occur.

Proposition 2.1 *Given Assumption 2.1, a positive variation of j's market wage increases i's reservation wage function $\varpi^i_{P_J}$ if and only if one of the following relationships holds*
i) either the difference between j's labour supply curves in the two participation regimes $\widetilde{l}^j_{P_J} - \widetilde{l}^j_P$ is positive,
ii) or the difference between j's labour supply curves in the two participation regimes $\widetilde{l}^j_{P_J} - \widetilde{l}^j_P$ is negative but the income effect is stronger than j's difference in labour supplies, thus $\frac{\widetilde{l}^j_{P_J} - \widetilde{l}^j_P}{\widetilde{l}^i_P} < \widetilde{l}^j_{P_J} \frac{\partial \varpi^i_{P_J}}{\partial y}$.

The implication of Proposition 2.1 on the participation frontier is as follows.

Proposition 2.2 *If i's reservation wage function $\varpi^i_{P_J}$ increases as w_j increases, then the participation frontier, dividing the wage loci in which both spouses work P and only j works P_J, is upwards sloping.*

In order to ease interpretation of Proposition 2.2 suppose that member j is the husband and member i the wife. Then Proposition 2.2 states that as husband's market wage raises the wife participates in the labour market for relative high market wages. From an empirical perspective, the result of Proposition 2.2 is plausible especially within the unitary model in which the identity of the income recipient does not matter on household outcomes and, therefore, an increase of the husband market wage directly translates into an increase of the opportunity set of the whole family.

The following propositions describe the opposite situation.

Proposition 2.3 *Given Assumption 2.1, a positive variation of j's market wage decreases i's reservation wage function $\varpi^i_{P_J}$ if and only if the difference between j's labour supply curves in the two participation regimes $\widetilde{l}^j_{P_J} - \widetilde{l}^j_P$ is negative and the income effect is weaker than j's difference in labour supplies, that is if $\frac{\widetilde{l}^j_{P_J} - \widetilde{l}^j_P}{\widetilde{l}^i_P} > \widetilde{l}^j_{P_J} \frac{\partial \varpi^i_{P_J}}{\partial y}$.*

With the following effect on the participation frontier.

Proposition 2.4 *If i's reservation wage function $\varpi^i_{P_J}$ decreases as w_j increases, then the participation frontier, dividing the wage loci in which both spouses work P and only j works P_J, is downwards sloping.*

Again, let us think at j as if it were the husband and i the wife. Then, Proposition 2.4 argues that as husband's market wage raises the wife reservation wage decreases making in this way her labour participation more likely to occur even for low level of market wages.

In interpreting condition (2.19), we have omitted the result that j's difference in labour supplies $\widetilde{l}^j_{P_J} - \widetilde{l}^j_P$ divided by i labour supply $\widetilde{l}^i_P$ can be equal to the income effect $\widetilde{l}^j_{P_J} \left(\partial \varpi^i_{P_J} / \partial y \right)$. If it is the case, i's reservation wage would be not affected by changes of j's market wage and, in turn, the participation frontier would be not affected by wage variations.

Neither Spouse Works

According to the reservation wage functions (2.7) and (2.8), when both spouses do not work the reservation wage is function of market prices and nonlabour income only. Hence in the wage plane the participation frontier, dividing the labour participation regimes in which neither spouse works and only one spouse works, is not affected by changes of market wages. Generally, one can measure an income effect only and, given Assumption 2.1, reservation wages have a positive relationship with nonlabour income, and reservation wages increase (decrease) as nonlabour income increases (decreases).[5]

In Figures 2.1 to 2.4 we give a graphical intuition of the analysis described in this section. The participation frontier is represented in the locus of spouses' market wages and it divides the market wage plane in four regions. Each participation region is characterized by an opportunity set such that the spouses are jointly better off either of them working P, neither of them working N, or one working and the other not P_i.

Figures 2.1 and 2.2 represent the two polar results discussed in Propositions 2.2 and 2.4. In Figure 2.1 the reservation wage function of both spouses is positively affected by marginal positive variations of the market wage of the working spouse. In Figure 2.2 the reservation wage function of both spouses decreases as the market wage of the working spouse increases.

On the other hand, Figure 2.3 and 2.4 depict the situations in which marginal variations of the market wage operate in opposite direction on the spouses' reservation wage. In particular, looking at Figure 2.3 we have that a positive variation of the husband wage has a negative effect on the wife reservation wage function and, conversely, a positive variation of the wife wage raises the reservation wage of the husband. In Figure 2.4 the opposite situation is illustrated.

2.4 Family Labour Participation within the Collective Model

We now turn to the analysis of labour participation within the collective framework. We first introduce the standard rational collective model of household behaviour and then extend it to corner solutions. As stated in Section 2.2, households are made up of married couples only. The couple faces a budget constraint that limits the private consumption of market goods and leisure of the two spouses. The budget constraint takes the usual linear market form $p_f c^f + p_m c^m = \sum_{i \in f,m} w_i l^i + y$, where p_i are market prices of the composite good privately consumed by each individual and the right-hand side is the full household

[5] Generally, reservation wages are function of demographic characteristics, as education level, age, sex and so forth, and one may empirically observe and measure changing in the level of reservation wage due to at variations on demographic attributes. However, from a theoretical point of view these changing may have ambiguous sings.

income composed of earned incomes $\sum_i w_i l^i$, when spouses work, and household nonlabour income y.

The collective household model relies on two assumptions.

Assumption 2.2 (Individual Preferences) *Individual preferences are assumed to be represented by an egoistic strictly quasi-concave utility function, continuously differentiable and strictly increasing in its elements*

$$u^i = U^i\left(c^i, 1 - l^i\right).$$

Providing for consumption of private goods only, we do not take into account either the consumption of public goods or externalities within families. Although we recognize that this assumption is a severe restriction in a household model, it allows to recover individual preferences and to characterize the decision process observing household consumption rather than individual (Chiappori 1992). Moreover, we assume that leisure $1-l^i$ is privately consumed by both spouses. Fong and Zhang (2001) consider a more general model where leisure can be consumed both privately and publicly. Although the two alternative uses are not empirically distinguishable, they can be identified in general, provided that the consumption of another exclusive good is observed. It is important to underline that "egoistic" preferences are not necessary to recover the individual behaviour and the collective set-up can be extended to a caring utility function $\widetilde{u}^i = \widetilde{U}^i\left[U^f\left(c^f, 1 - l^f\right), U^m\left(c^m, 1 - l^m\right)\right]$ without altering the conclusions of the model (Chiappori 1992).

As we assume in Section 2.2, the household utility function W^h is represented by a Bergson function

$$W^h = \widetilde{\mu} U^f(c^f, 1 - l^f) + (1 - \widetilde{\mu})\, U^m\left(c^m, 1 - l^m\right). \tag{2.20}$$

Different from the unitary model, the Pareto weight $\widetilde{\mu}$ is in general assumed to depend on the variables[6] that enter the budget constraint and is assumed continuously differentiable

[6] Recently, Basu (2006) challenges the common assumption of exogeneity of the sharing rule. He argues that what determines the bargaining power of member i is not just her wage rate but rather what she actually earns, that is $w_i l^i$. Since l^i is a variable of choice, μ gets influenced by the family's decision and

in its arguments. Basically, $\widetilde{\mu}$ captures the balance of power in the family and determines the final allocation on the Pareto frontier. If $\widetilde{\mu} = 1$ then the household behaves as though member f is the effective household dictator, whereas $\widetilde{\mu} = 0$ it is the opposite scenario. For intermediate values of $\widetilde{\mu}$, the household behaves as though each spouse has some decision power. In general, in labour supply models the distribution index $\widetilde{\mu}$ is made to depend on wages and nonlabour income. The value of $\widetilde{\mu}$ may also depend on variables that influence the balance of power without affecting both the value of individual utility functions and the household opportunity set. In literature these variables are referred to as distribution factors.

Assumption 2.3 (Pareto Efficiency) *Household decisions are assumed to result in Pareto-efficient outcomes.*

This assumption does not necessarily imply "harmony" between spouses, thus a collective model may also describe families experiencing marriage dissolution. The rationale motivation of Assumption 2.3 is that efficient allocations are likely to emerge from repeated interactions in stationary environments and households are an example of such an environment. However, there can be situations in which the efficient assumption may fail to apply. An example can be when existing social norms impose some patterns of behaviour that deviate from efficient outcomes (Udry 1996). Inefficient outcomes are also plausible for decisions that are not taken frequently, for instance fertility and education choices (Lundberg and Pollak 2003), which imply that the repeated game argument cannot be applied.

Formally, without public goods and externalities, the assumption of Pareto efficient household decisions implies that for any given price-income bundle (p_f, p_m, w_f, w_m, y) there exists an exogenous weighting function $\widetilde{\mu}$ belonging to $[0, 1]$ such that $(c^i, 1 - l^i)$ for

hence the balance of power is endogenous to the household choice.

$i = f, m$ solves the following constrained program

$$\max_{c^f, l^f, c^m, l^m} \widetilde{\mu} U^f (c^f, 1 - l^f) + (1 - \widetilde{\mu}) U^m (c^m, 1 - l^m), \tag{2.21}$$

$$\text{subject to} \quad p_f c^f + p_m c^m = w_f l^f + w_m l^m + y,$$

$$\text{and} \quad l^f \in [0, 1], \quad l^m \in [0, 1].$$

For interior solutions the first-order conditions, in terms of marginal rate of substitutions, are

$$\begin{aligned} \frac{U^f_{l^f}}{U^f_{c^f}} &= \frac{w_f}{p_f}, \\ \frac{U^m_{l^m}}{U^m_{c^m}} &= \frac{w_m}{p_m}, \end{aligned}$$

that represent the conventional efficient conditions. The household utility function W^h is additively separable in the individual consumption-leisure choices implying that i's marginal rate of substitutions are independent of j's consumption-leisure choices and *vice versa.* Hence the household model can be solved using a two-stage budgeting procedure that in the collective model translates into the "sharing rule approach." The system of the first-order conditions together with the budget constraint

$$w_f l^f + w_m l^m + y - p_f c^f - p_m c^m = 0,$$

generate the following labour supplies

$$\widehat{l}^f_P = l^f_P (p_f, p_m, w_f, w_m, y, \mu (p_f, p_m, w_f, w_m, y)), \tag{2.22}$$

$$\widehat{l}^m_P = l^m_P (p_f, p_m, w_f, w_m, y, \mu (p_f, p_m, w_f, w_m, y)). \tag{2.23}$$

Prices and nonlabour income in the household utility function lead to a specific characterization of the collective model. Interior maxima derived by price dependent preferences

(2.20) do not exhibit all the properties of the traditional demand theory. In particular, they do not satisfy the symmetry of the Slutsky matrix (Pollak 1977, Browning and Chiappori 1998). Browning and Chiappori (1998) generalize the Slutsky conditions to the collective model. They find that the substitution matrix must satisfy the SNR1 condition. This condition states that the collective Slutsky matrix is made of a symmetric, negative semi-definite matrix and a rank one matrix. The rank one matrix originates from the shift of the Pareto weight due to changes in prices, wages and nonlabour income.

Furthermore, in programs where binding maxima are present, such as the analysis of this chapter, price dependent preferences lead to further economic irregularities. Precisely, unless specific assumptions are added to the bargaining power structure or to the reservation wage function, uniqueness of the reservation wage functions fails to exist and, consequently, the participation (nonparticipation) frontier cannot be uniquely drawn. Therefore, the reservation wage theory could not be a suitable approach to model collective behaviour with corner solutions.

In order to show this negative result, we use equation (2.22) to find the wife's reservation wage when the couple is better off when she does not work and her husband does work. Employing the reservation wage approach, as done in Section 2.2, we would set her labour supply equation equal to zero

$$l_P^f\left(p_f, p_m, w_f, w_m, y, \mu\left(p_f, p_m, w_f, w_m, y\right)\right) = 0$$

and then would solve it with respect to w_f. However, since the Pareto weight $\widetilde{\mu}$ depends also on w_f there can be many wage rates for which she is indifferent between working or not working and a unique characterization of the collective model with corner solutions may not be feasible. Comparable findings can be obtained for the other labour participation regimes. To be clear we illustrate this result by means of the following example.

Example 2.4 *Recall the interior solution of spouses' labour supply in Example 1*

$$\begin{aligned}\widetilde{l}^f &= 1 - \frac{\widetilde{\mu}\beta_f}{w_f}\left(w_f + w_m + y\right), \\ \widetilde{l}^m &= 1 - \frac{(1-\mu)\,\beta_m}{w_m}\left(w_f + w_m + y\right),\end{aligned}$$

here, without loss of generality, we assume that $\alpha_i + \beta_i = 1$ *for* $i = f, m$ *and set* $\widetilde{\mu}_f = \widetilde{\mu}$ *and* $\widetilde{\mu}_m = 1 - \widetilde{\mu}$. *Now, within the collective framework the Pareto weight* $\widetilde{\mu}$ *is in general specified as a function of variables that enter the budget constraint. For the sake of the example, let us suppose that* $\widetilde{\mu} = \frac{w_f}{w_m}$ *with* $w_m > w_f$, f*'s labour supply becomes*

$$\widetilde{l}^f = 1 - \frac{\beta_f}{w_m}\left(w_f + w_m + y\right),$$

and m*'s labour supply is*

$$\widetilde{l}^m = 1 - \frac{\beta_m - \frac{w_f}{w_m}\beta_m}{w_m}\left(w_f + w_m + y\right).$$

Now, to derive the reservation wage function for the spouses we apply the standard reservation wage theory as we do within the unitary model. Given the specified structure of the household model, when the husband finds it optimal to work, the wife has a unique reservation wage function equal to

$$\varpi^{*f}_{P_M} = \frac{w_m\left(1 - \beta_f\right)}{\beta_f} - y.$$

On the other hand, when the wife works, the husband reservation wage function is the solution of the following quadratic equation

$$\left(1 - \beta_m\right) w_m^2 - \beta_m y w_m + \left(y + w_f\right) \beta_m w_f = 0$$

that in general the solution of this equation does not lead to a unique reservation wage function. The asymmetry between spouses's reservation wages is a direct consequence *of the particular functional form chosen for the Pareto weight and it does not derive by the rational behaviour of individuals. In household models aiming at estimating occupation decisions then we could find it conceivable that for different occupation decisions individuals have different reservation wages. However, for the discrete choice "whether to participate or not participate in the labour market" rationality implies that individuals have just one "subjective" wage.*

To overcome the latter shortcoming, crucial in empirical analysis with corner solutions, Blundell *et al.* (2007) and Donni (2003) postulate that the reservation wage is a contraction with respect to market wages.[7] The reservation wage function thus has a unique fixed point with respect to w_f and w_m and when both spouses do not work, there will exist one and only one pair of wages such that both spouses are indifferent between working and not working. On the other hand, when member i does not work, there will exist a reservation wage functions of w_j and y such that member i participates in the labour market if and only if her market wage is greater than her reservation wage. Consequently, the participation set is partitioned into four connected sets with a unique intersection.

Conversely, we take a different approach. Precisely,[8]

Assumption 2.4 *We assume that the Pareto weight $\widetilde{\mu}$ is a continuous function of individual expected wages, $E(w_i)$ for i=f,m, market prices, p_f and p_m, and nonlabour income y.*

We think of the expected wage as a prediction of the wage that an agent would get if she were to enter the labour force. It can depend on past working experience and demographic characteristics of the agent as well on macro-economic indicators. As a consequence of Assumption 2.4, the reservation wage theory can be employed to described

[7] Let X be a metric space. A mapping $T : X \to X$ is called contraction map if there exists a constant c with $0 \leq c < 1$ such that $d(T(x), T(y)) \leq cd(x, y)$, $x, y \in X$. An economic application of the contraction mapping theorem is to prove the fixed-point theorems in metric spaces. In other words, a contraction mapping maps a set into a proper subset of itself.

[8] Assumption 2.4 is a special application of Pollak (1977). In a model with price dependent utility functions, Pollak (1977) distinguishes between current market wages, the wages that enter the budget constraint, and normal wages, those that influence preferences alone. Generally, the normal wage function defines normal wages as a function of current and past wages. However, two polar specification are feasible. One specification defines normal wages depending exclusively on current wages. The easiest case is the one in which normal wages depend on past wages but not on current wages.

the labour participation of spouses also in the context of the collective model. As described later, in terms of characterization of the participation decisions, this specification of the Pareto weight $\widetilde{\mu}$ leads to similar results of the unitary model: in each participation regime Assumption 2.4 ensures that spouses have a unique reservation wage function.[9] Potentially, expected wages $E(w_i)$ can depend on current market wages. However, within the collective model it would imply that individual reservation wages are not identified. Therefore, in the analysis of labour participation of family members within a collective model we do not assume that individual expected wages are function of current market wages.

It is worth remarking that both Blundell *et al.* (2007) and Donni (2003) specify the Pareto weight $\widetilde{\mu}$ as a function of current market wages even when spouses do not work. Behind this assumption is the idea that when spouses do not work they might exert their bargaining power within the family on the grounds of their potential wage defined as the current market wage that they may earn entering the labour force. This amounts to assume that agents have perfect information on labour market performances and, in turn, on the wage that they would earn entering the labour force. Differently, we state that when a spouse does not work it is likely that her voice within the family depends, among other variables, on her expected wage.

Similar to the corner solution analysis of Section 2.2, under Assumption 2.4 the labour decision of the couple can be represented by the following necessary conditions and, in addition, the efficient conditions derived by the collective household model are equal to those obtained within the unitary household model.

Both Spouses Participate, $P:\left\{l^f>0, l^m>0, p_f c^f+p_m c^m=w_f l^f+w_m l^m+y\right\}$

Spouses jointly participate in the labour force if and only if

$$\frac{U_{l^f}^f\left(c^f, 1-l^f\right)}{U_{c^f}^f\left(c^f, 1-l^f\right)}=\frac{w_f}{p_f}, \quad \text{and} \quad \frac{U_{l^m}^m\left(c^m, 1-l^m\right)}{U_{c^m}^m\left(c^m, 1-l^m\right)}=\frac{w_m}{p_m},$$

[9] Assumption 2.4 implies that the sharing rule function depends, among other variables, on expected wages rather than current market wages.

and their optimal labour supply equations are

$$\widehat{l}_P^f = l_P^f(p_f, p_m, w_f, w_m, y, \widetilde{\mu}), \quad (2.24)$$

$$\widehat{l}_P^m = l_P^m(p_f, p_m, w_f, w_m, y, \widetilde{\mu}), \quad (2.25)$$

where the Pareto weight is $\widetilde{\mu} = \mu(p_f, p_m, E(w_f), E(w_m), y)$. The individual demands for leisure are $\widehat{L}_P^i = 1 - \widehat{l}_P^i$ for $i = f, m$. When both spouses work the optimal welfare reached by the family is $\widehat{W}_P^h(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$.

Setting i's labour equation $\widehat{l}_P^i$ equal to zero we find the reservation wage of member i by solving for w_i

$$\varpi_{P_j}^{*i} = \varpi_{P_j}^i(w_j, y, \widetilde{\mu}), \text{ for } i = f, m,$$

that gives the marginal value of time at which i is indifferent between working or not working when the other spouse finds it optimal to supply a positive amount of working hours.

Wife Does Not Participate and Husband Participates, $P_M : \{l^f = 0, l^m > 0, p_f c^f + p_m c^m = w_m l^m + y\}$

The regime in which the wife chooses not to work and the husband to work will be optimal if and only if

$$\frac{U_{l^f}^f(c^f, 1)}{U_{c^f}^f(c^f, 1)} > \frac{w_f}{p_f}, \text{ and } \frac{U_{l^m}^m(c^m, 1 - l^m)}{U_{c^m}^m(c^m, 1 - l^m)} = \frac{w_m}{p_m}.$$

The husband supplies the following positive number of working hours

$$\widehat{l}_{P_M}^m = l_{P_M}^m(p_f, p_m, w_m, y, \mu(p_f, p_m, E(w_f), E(w_m), y)),$$

and the individuals demands for leisure are $\widehat{L}_{P_M}^m = 1 - \widehat{l}_{P_M}^m$ and $\widehat{L}_{P_M}^f = 1$. In this labour regime at the optimum the family gets a welfare level equal to $\widehat{W}_{P_M}^h(p_f, p_m, w_m, y, \widetilde{\mu})$.

Solving the labour equation $\widehat{l}_{P_M}^m$ with respect to w_m we find the expression for the husband reservation wage when the wife is not working

$$\varpi_N^{*m} = \varpi_N^m \left(y, \mu \left(p_f, p_m, E\left(w_f\right), E\left(w_m\right), y\right)\right).$$

Wife Participates and Husband Does Not Participate, $P_F : \left\{l^f > 0, l^m = 0, p_f c^f + p_m c^m = w_f l^f + y\right\}$

Similarly, the regime where the wife works and the husband does not work will be optimal if and only if

$$\frac{U_{l^f}^f\left(c^f, 1 - l^f\right)}{U_{c^f}^f\left(c^f, 1 - l^f\right)} = \frac{w_f}{p_f}, \quad \text{and} \quad \frac{U_{l^m}^m\left(c^m, 1\right)}{U_{c^m}^m\left(c^m, 1\right)} > \frac{w_m}{p_m}.$$

The wife find it optimal to supply the following positive number of working hours

$$\widetilde{l}_{P_F}^f = l_{P_F}^f \left(p_f, p_m, w_f, y, \mu \left(p_f, p_m, E\left(w_f\right), E\left(w_m\right), y\right)\right),$$

with individual leisure demands equal to $\widehat{L}_{P_F}^f = 1 - \widetilde{l}_{P_F}^f$ and $\widehat{L}_{P_F}^m = 1$. In this labour regime, the optimal level of household welfare is $\widehat{W}_{P_F}^h(p_f, p_m, w_f, y, \widetilde{\mu})$.

Solving the labour equation $\widetilde{l}_{P_F}^f$ with respect to w_f we have the expression for the wife reservation wage when the husband is not working

$$\varpi_N^{*f} = \varpi_N^f \left(y, \mu \left(p_f, p_m, E\left(w_f\right), E\left(w_m\right), y\right)\right).$$

Both Spouses Do Not Participate, $N : \left\{l^f = 0, l^m = 0, p_f c^f + p_m c^m = y\right\}$

Finally, the optimal choice of the couple is not to work if and only if

$$\frac{U_{l^f}^f\left(c^f, 1\right)}{U_{c^f}^f\left(c^f, 1\right)} > \frac{w_f}{p_f}, \quad \text{and} \quad \frac{U_{l^m}^m\left(c^m, 1\right)}{U_{c^m}^m\left(c^m, 1\right)} > \frac{w_m}{p_m},$$

spouses allocate all their time to leisure, thus $\widehat{L}_N^f = 1$, $\widehat{L}_N^m = 1$. The optimal welfare level of the family is $\widehat{W}_N^h(p_f, p_m, y, , \widetilde{\mu})$.[10]

[10] Given a specific functional form to the individual utility functions of wife and husband, reservation wages can be calculated for both spouses in each of the four different participation regimes. The algebraic derivation of the reservation wages is illustrated in examples 2.1-2.4.

2.5 Characterization of the Collective Model

In this section, we show the testable implications for leisure demands generated by the collective household model characterized by Assumptions 2.2-2.4. In particular, we perform the Slutsky equation for the four participation regimes and, in addition, characterize the participation frontier. The analysis is thus similar to that of Section 2.3.

2.5.1 The Slutsky Equation for Collective Leisure Demands

Both Spouses Work

When both spouses work the Slutsky equation derived by the individual labour equations (2.24) and (2.25) is equal to[11]

$$s_{ij}^{c} = L_{w_j}^{i} - \left(1 - L_P^j\right) L_y^i - L_{\widetilde{\mu}}^i \left(\left(1 - L_P^j\right) \mu_y\right), \quad i, j = f, m. \qquad (2.26)$$

In matrix terms, the generalized substitution matrix can be written as

$$S_P^C = \begin{bmatrix} s_{ii}^c & s_{ij}^c \\ s_{ji}^c & s_{jj}^c \end{bmatrix} = S_P + R = \begin{bmatrix} s_{ii} & s_{ij} \\ s_{ji} & s_{jj} \end{bmatrix} + D_\mu \cdot v', \quad \text{both spouses work,}$$

where the vector D_μ is of general term $L_{\widetilde{\mu}}^i$ and the vector v' is of general term $\left(1 - L^j\right) \mu_y$. Thus the collective Slutsky matrix is the sum of a conventional Slutsky matrix S_P, with the usual properties of negativity and symmetry, and an additional matrix R that is the product of a column vector D_μ and a row vector v. Browning and Chiappori (1998) show that the matrix R has at most rank one. It is worth remarking that in Browning and

[11] When both spouses work, i's demand for leisure is equal to

$$\widehat{L}_P^i \left(p_i, p_j, w_i, w_j, y\right) = L_P^i \left(p_i, p_j, w_i, w_j, y, \widetilde{\mu}\right),$$

where the Pareto weight is $\widetilde{\mu} = \mu\left(p_f, p_m, E\left(w_f\right), E\left(w_m\right), y\right)$. We can define the Slutsky equation of the individual reduced demand for leisure $\widehat{L}_P^i \left(p_i, p_j, w_i, w_j, y\right)$ by its general term

$$s_{ij} = \widehat{L}_{w_j}^i - \left(1 - \widehat{L}_P^j\right) \widehat{L}_y^i.$$

Noticing that $\widehat{L}_{w_j}^i = L_{w_j}^i$ and $\widehat{L}_y^i = L_y^i + L_{\widetilde{\mu}}^i \mu_y$, the Slutsky equation can be written as

$$s_{ij}^c = L_{w_j}^i - \left(1 - L_P^j\right) L_y^i - L_{\widetilde{\mu}}^i \left(1 - L_P^j\right) \mu_y,$$

that represents the Slutsky equation of the individual structural demand for leisure $L_P^i \left(p_i, p_j, w_i, w_j, y, \widetilde{\mu}\right)$.

Chiappori (1998) the vector v comprises price and income effects that are generated by the Pareto weight. Differently, given Assumption 2.4 the vector v is reduced to income effects only. This feature, however, may not affect the rank condition of the matrix R.

Only One Spouse Works

When only i works the Slutsky equation derived by i's leisure demand is

$$s_{ii}^{c} = L_{w_i}^{i} - \left(1 - L_P^i\right) L_y^i - L_{\widetilde{\mu}}^i \left(\left(1 - L_P^i\right) \mu_y\right), \quad i = f, m,$$

that satisfies the same property of equation (2.26). Since j does not work, the compensated cross-wage effect cannot be obviously performed .

2.5.2 Characterization of the Participation Frontier

In this section, the effects on the reservation wage function of changing market wages and nonlabour income are considered. Since the analysis is carried out as done in Section 2.3.2 we briefly present the algebraic passages and then highlight possible differences.

When only member j is working the participation frontier is defined by the following equality

$$\widehat{W}_P^h \left(p_i, p_j, \varpi_{P_J}^{*i}, w_j, y, \widetilde{\mu}\right) = \widehat{W}_{P_J}^h \left(p_i, p_j, w_j, y, \widetilde{\mu}\right), \tag{2.27}$$

where $\varpi_{P_J}^{*i} = \varpi_{P_J}^{i} \left(p_i, p_j, w_j, y, \widetilde{\mu}\right)$ is the wife reservation wage and $\widetilde{\mu} = \mu \left(p_f, p_m, E\left(w_f\right),\right.$ $\left. E\left(w_m\right), y\right)$ is the Pareto weight.

Keeping market prices constant, the total differential of equation (2.27) is equal to

$$\frac{\partial \widehat{W}_P^h}{\partial \varpi_{P_J}^{*i}} \frac{\partial \varpi_{P_J}^{i}}{\partial w_j} dw_j + \frac{\partial \widehat{W}_P^h}{\partial \varpi_{P_J}^{*i}} \frac{\partial \varpi_{P_J}^{i}}{\partial y} dy + \frac{\partial \widehat{W}_P^h}{\partial \varpi_{P_J}^{*i}} \frac{\partial \varpi_{P_J}^{i}}{\partial \widetilde{\mu}} \frac{\partial \mu}{\partial y} dy + \frac{\partial \widehat{W}_P^h}{\partial w_j} dw_j +$$

$$+ \frac{\partial \widehat{W}_P^h}{\partial y} dy + \frac{\partial \widehat{W}_P^h}{\partial \widetilde{\mu}} \frac{\partial \mu}{\partial y} dy = \frac{\partial \widehat{W}_{P_J}^h}{\partial w_j} dw_j + \frac{\partial \widehat{W}_{P_J}^h}{\partial y} dy + \frac{\partial \widehat{W}_{P_J}^h}{\partial \widetilde{\mu}} \frac{\partial \mu}{\partial y} dy.$$

Cancelling out common terms and using Roy's identity

$$\frac{\frac{\partial \widehat{W}_P^h}{\partial \varpi^{*f}_{P_J}}}{\frac{\partial \widehat{W}_P^h}{\partial y_f}}\frac{\partial \varpi^i_{P_J}}{\partial w_j}dw_j + \frac{\frac{\partial \widehat{W}_P^h}{\partial \varpi^{*f}_{P_J}}}{\frac{\partial \widehat{W}_P^h}{\partial y}}\frac{\partial \varpi^i_{P_J}}{\partial y}dy + \frac{\frac{\partial \widehat{W}_P^h}{\partial \varpi^{*f}_{P_J}}}{\frac{\partial \widehat{W}_P^h}{\partial y}}\frac{\partial \varpi^i_{P_J}}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}dy +$$

$$+\frac{\frac{\partial \widehat{W}_P^h}{\partial w_j}}{\frac{\partial \widehat{W}_P^h}{\partial y}}dw_j + \frac{\frac{\partial \widehat{W}_P^h}{\partial y}}{\frac{\partial \widehat{W}_P^h}{\partial y}}dy = \frac{\frac{\partial \widehat{W}_{P_J}^h}{\partial w_j}}{\frac{\partial \widehat{W}_{P_J}^h}{\partial y}}dw_j + \frac{\frac{\partial \widehat{W}_{P_J}^h}{\partial y}}{\frac{\partial \widehat{W}_{P_J}^h}{\partial y}}dy,$$

this expression becomes

$$\widehat{l}^i_P\frac{\partial \varpi^i_{P_J}}{\partial w_j}dw_j + \widehat{l}^i_P\left(\frac{\partial \varpi^i_{P_J}}{\partial y} + \frac{\partial \varpi^i_{P_J}}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right)dy + \widehat{l}^j_P dw_j = \widehat{l}^j_{P_J}dw_j, \tag{2.28}$$

where we have substituted the individual labour supplies derived by Roy's identity, $\frac{\frac{\partial \widehat{W}_P^h}{\partial \varpi^{*f}_{P_J}}}{\frac{\partial \widehat{W}_P^h}{\partial y_f}} = \widehat{l}^i_P$ and $\frac{\frac{\partial \widehat{W}_{P_J}^h}{\partial w_j}}{\frac{\partial \widehat{W}_{P_J}^h}{\partial y}} = \widehat{l}^j_{P_J}$.

Differentiating the budget constraint in P_J we have the expression of dy

$$dy = -\widehat{l}^j_{P_J}dw_j,$$

substituting it into equation (2.28) and re-arranging terms we obtain

$$\frac{\partial \varpi^i_{P_J}}{\partial w_j} = \frac{\widehat{l}^j_{P_J} - \widehat{l}^j_P}{\widehat{l}^i_P} + \widehat{l}^j_{P_J}\left(\frac{\partial \varpi^i_{P_J}}{\partial y} + \frac{\partial \varpi^i_{P_J}}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right). \tag{2.29}$$

Comparing equation (2.29) with equation (2.19) we observe that they differ for the additional income effect $\frac{\partial \varpi^i_{P_J}}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}$ generated by the Pareto weight $\widetilde{\mu}$. It is plausible that positive variations of the nonlabour income has the effect of increasing the Pareto weight. Therefore, within the collective approach to family labour participation presented in the chapter, the Pareto weight has the effect to strengthens the income effect $\frac{\partial \varpi^i_{P_J}}{\partial y}$ and Proposition 3.2 is more likely to occur.

2.6 Conclusions

Although economists have widely recognized the important role played by individualism in modeling family behaviour, there is still no agreement on which approach better represents household behaviour. It is likely that household decisions, such as labour participation or fertility choices, involve a bargaining process between the spouses. On the other hand, it can be the case that decisions taken on a daily basis, such as food consumption, do not require a bargaining approach to model household behaviour. The labour participation analysis presented here is thus studied employing both the unitary and the collective models. For each household model we derive properties that can be tested using appropriate household data helping researchers in the choice of the "ideal" household model to represent family labour participation.

The chapter starts with the analysis of labour participation decisions of married couples within the unitary model. Household preferences are assumed to be additively separable in the individual utility function of the spouses and the budget constraint is assumed to be linear. Within this framework, we find that the reservation wage theory leads to a complete characterization of the labour participation decision. For each spouse we derive a set of reservation wages in different labour participation regimes that allows to characterize the participation frontier. As a result, the participation frontier divides the market wage plane in four regions. Each region is characterized by an opportunity set such that the spouses are jointly better off either of them working, neither of them working, or one working and the other not.

We then analyse the labour participation decision within the collective model. We show that the standard assumptions underlying the collective model, that is Pareto-efficiency, egoistic individual preferences and private consumption, are not sufficient to provide a unique derivation of the reservation wage function and, in turn, the existence of a well-behaved participation set. This result is in line with previous works (Blundell *et al.* 2007, Donni 2003).

To overcome the latter shortcoming, Blundell *et al.* (2007) and Donni (2003) postulate that the reservation wage is a contraction with respect to market wages. The reservation wage function thus has a unique fixed point with respect to individual market wages and when both spouses do not work, there will exist one and only one pair of wages such that both spouses are indifferent between working and not working. On the other hand, when member i does not work, there will exist a reservation wage functions of w_j and y such that member i participates in the labour market if and only if her market wage is greater than her reservation wage. Consequently, the participation set is partitioned into four connected sets with a unique intersection.

Conversely, we take a different approach. We assume that the Pareto weight $\widetilde{\mu}$ is a continuous function of individual expected wages rather than current market wages. The expected wages are defined as a prediction of the wage that an agent would get if she were to enter the labour force. It can depend on past working experience and demographic characteristics of the agent as well on macro-economic indicators. As a result, the reservation wage theory can be applied to described the labour participation of spouses also in the context of the collective model. As described in the chapter, under the assumption that the Pareto weight is function of expected wages rather than current market wages, completeness of the reservation wage theory is re-established. This entails that the characterization of the participation frontier is similar to that obtain within the unitary model of household labour participation.

FIGURE 2.1: Participation Frontier When Spouses' Reservation Wages Are Positively Affected by Wage Variations

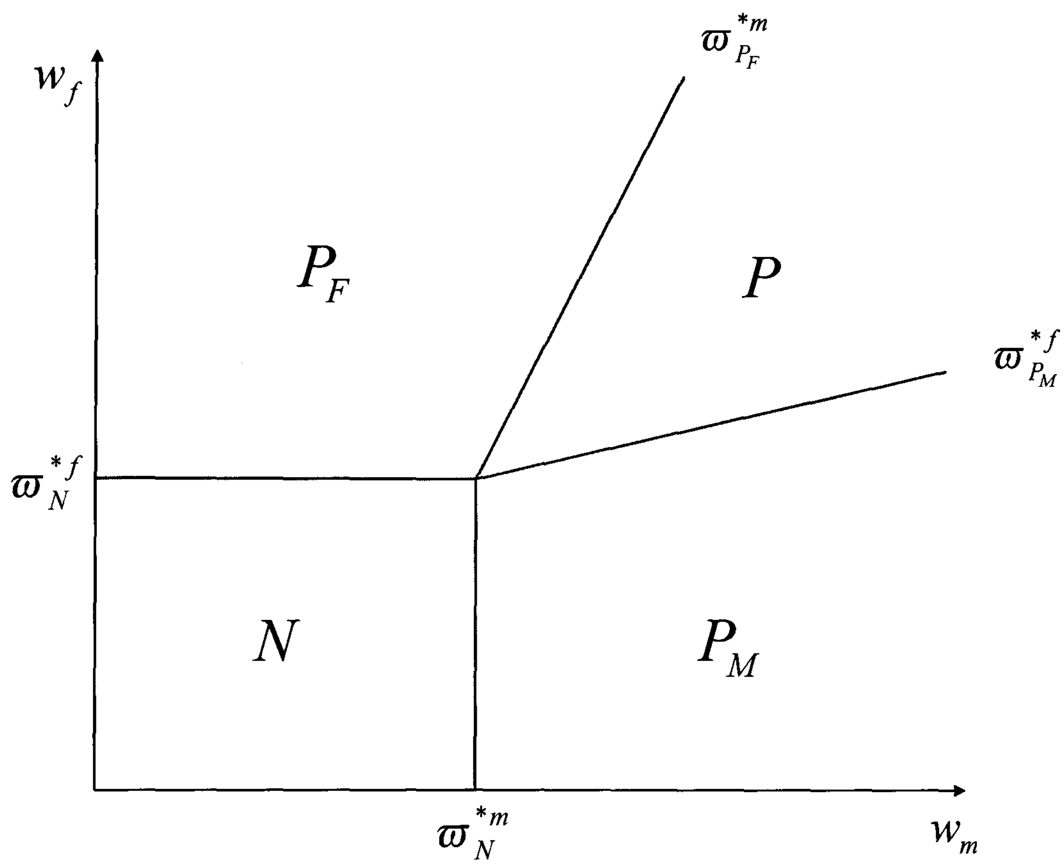


FIGURE 2.2: Participation Frontier When Spouses' Reservation Wages Are Negatively Affected by Wage Variations

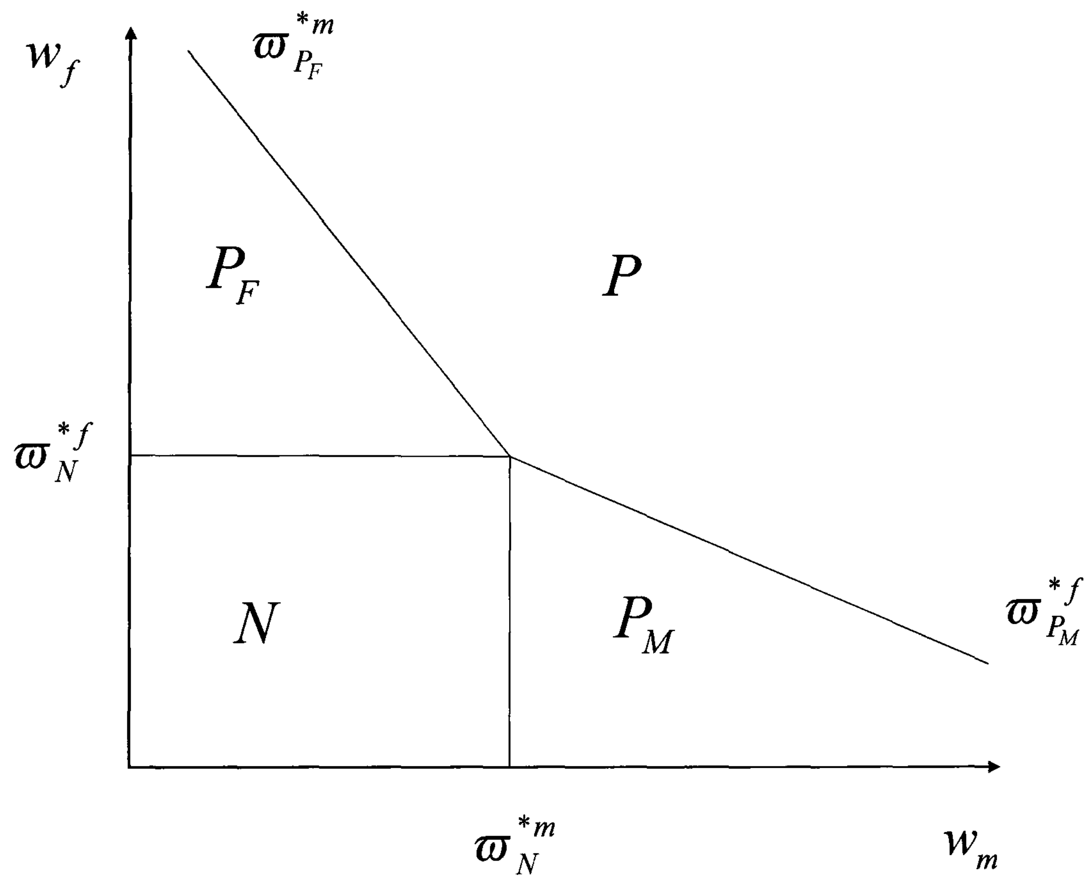


FIGURE 2.3: Participation Frontier When Female's Reservation Wage Is Negatively Affected and Male's Reservation Wage Is Positively Affected by Wage Variations

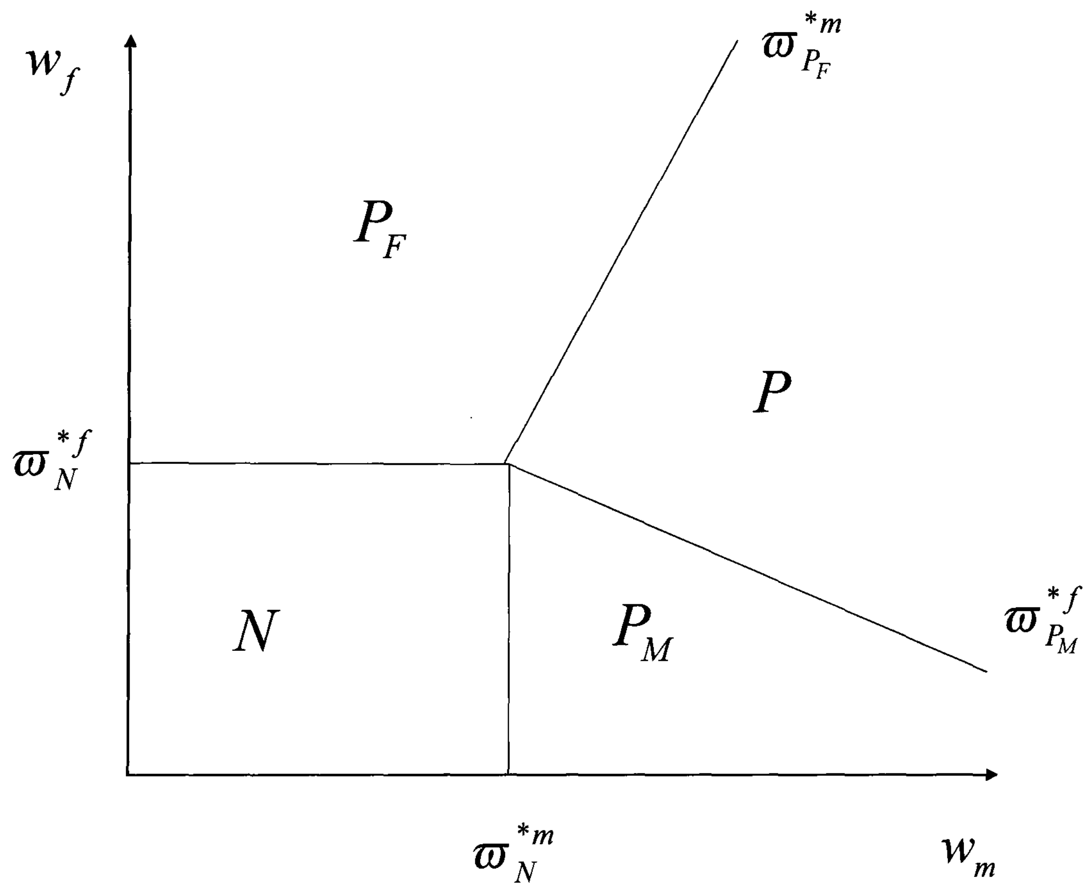
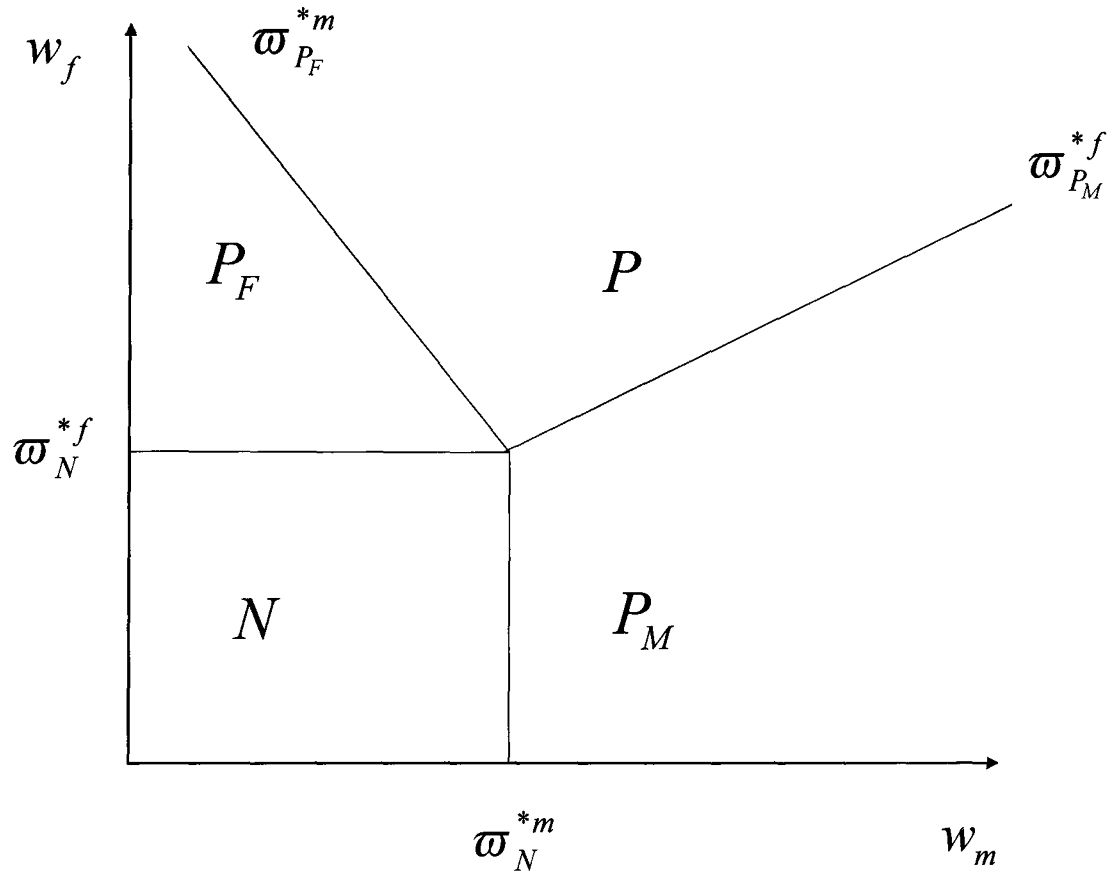


FIGURE 2.4: Participation Frontier When Female's Reservation Wage Is Positively Affected and Male's Reservation Wage Is Negatively Affectd by Wage Variations



Chapter 3

Separability and Joint Production in the Collective Household Model

3.1 Introduction

This chapter presents a collective approach to household behaviour with production and consumption-leisure decisions. In particular, we follow the approach initiated by Apps and Rees (1997) and Chiappori (1997). The household is seen as engaged jointly in production and consumption-leisure decisions. Within this setting the household is involved in producing domestic goods by transforming input factors. We omit purchased inputs and in the model factor demands are working hours supplied by family members. At the same time, the household chooses the optimal consumption of market-purchased goods, leisure, and domestic goods produced by the family. Thus the two decision processes must be integrated into one single problem.

In order to analyze the household as a collection of individuals rather than as an undifferentiated unitary decision unit, the production-consumption household model is then extended to embrace the recent results introduced by Chiappori (1988, 1992, 1997). The collective household model relies on quite general assumptions. In particular, each agent has her own utility function and equilibrium outcomes are Pareto-efficient. The efficiency assumption simply implies that the household's outcome is located on the utility

Pareto frontier and the particular location depends on the decision process of the family. However, the mechanism used by the family members to reach an agreement between them remains unspecified.

Within the collective set-up the household decision process takes place as follows. First, according to a predetermined exogenous rule household members divide the total nonlabour income between them. Then, each agent independently chooses her own optimal consumption bundle subject to her individual total amount of resources.

In the original work of Chiappori it is assumed that individual time is allocated only between market labour and leisure. This assumption neglects that a part of the individual time is spent in housework activities and that the decision of nonparticipation in the outside marketplace can be explained as a rational choice of working at home. Moreover, including household production might lead to inconsistent results concerning the conditions of the recoverability of the collective structure or concerning welfare analyses.

The contribution of Apps and Rees (1997) shares this spirit. Apps and Rees (1997) argue that failure to utilize Becker's (1965) insight can give rise to misleading welfare conclusions. For instance, there can be an equal distribution of market resources but an unequal distribution of household full income. Moreover, a low level of market labour supply will automatically be interpreted as a large consumption of leisure, whereas it may reflect the specialization of one of the members in domestic production. Therefore, they introduce the domestic sector in the general framework of Chiappori. They conclude that the extension of Chiappori's collective models to household production may lead to problems of identification of the sharing rule, unless further assumptions are introduced in the household framework. Notice that Apps and Rees (1997) consider the case of nonmarketable domestic goods only. Under the assumption of marketable domestic goods, Donni (2005) formally shows that omitting the domestic production in collective models of labour supply produces a bias in welfare analyses.

On the other hand, in a model with household production, Chiappori (1997) clearly distinguishes between nonmarketable and marketable domestic goods and then he studies the implications on the identification of the household model structure of accounting for domestic production. In the marketable production case, he shows that testable restrictions on domestic and market labour supply functions can be derived and the sharing rule can be recovered up to *an additive constant.* In the nonmarket case, the sharing rule can be derived up to *an additive function of wages* and the underlying structure of the model is identifiable only if the domestic production exhibits constant returns to scale. However, imposing additional assumptions on the decision process, identification of the sharing rule up to *an additive constant* is still possible.

From an empirical viewpoint, using household data that provide individual information on demographic attributes, and a detailed information on individual time use, earnings and nonlabour income, Apps and Rees (1996) estimate and compare the results of two collective models, a model with household production and a model without. In the model accounting for household production the authors implicitly assume nontradeable domestic goods and a production function with constant returns to scale. In the model without domestic activities they estimate individual demand systems for market goods and leisure consumption. Comparing the estimations, Apps and Rees (1996) show that the two systems yield conflicting results concerning behavioural response, such as wage and income elasticities, and the allocation of nonlabour income within the household members. Using Swedish data, Aronsson, Sven-Olov, and Magnus (2001) estimate two collective models with household production. In particular, in one model the domestic good has a market and in the other it is assumed to be not marketable. They show that leisure demands and household production are significantly influenced by the presence of children and the education of household members.

Recently, Rapoport, Sofer, and Solaz (2003) extend the econometric identification strategy of the sharing rule presented in Chiappori, Fortin, and Lacroix (2002) to the case of household production. They distinguish between market and nonmarket domestic goods. In the former case, given a production function with non-increasing returns to scale, they show that the sharing rule can be identified up to an additive constant. On the other hand, if the domestic good is nonmarketable the sharing rule can be derived up to an additive constant assuming a production function with constant returns to scale.

They estimate the sharing rule with and without household production. Comparing the results of the two estimated sharing rules, they find that the estimated parameters differ among the models and, in addition, taking into account household production increases robustness of the estimations.

The existing literature on collective household models is thus concerned to show that the sharing rule can be identified even when household production is taken into account. However, a central question, both in development economics (Bardhan and Udry 1999) and in the theory of the farm-household (Singh, Squire, and Strauss 1986), is whether production choices are separable from consumption-leisure decisions of the family. When the separation property holds, the household can make its production decisions independently of its consumption and labour-supply decisions. The production side affects consumption and labour supply just through income effects and production decisions are entirely independent of consumption and labour supply decisions. A crucial consequence is that to model production-consumption household choices we can use recursive models. Thus, whether the separation property can be applied has important implications on the structure of the household model and on the estimation of the model.

In the context of the unitary framework, a number of works have studied the separation property both from a theoretical and an empirical point of view (see among others Benjamin 1992, Chayanov 1986, Lopez 1984, Sen 1966, Singh, Squire, and Strauss 1986). Conversely, in the collective literature the separation property is not properly accounted for. Chiappori (1997) points to the separation property between production and consumption-leisure decisions but then his analysis is confined to show that the decision process is identifiable also taking into account production activities of the family.

In general, collective models with household production assume that in the first stage of the decision process household members agree on a rule to share nonlabour income and resources produced by the production activities. As a consequence, it is implicitly assumed that the production decision is always separable from the consumption-leisure choices. However, using recursive models inappropriately can produce misleading representations of the household behaviour in many respects. From an econometric point of view, it results in excluding relevant variables in the functional form specification of the production variables leading to biased estimations. As will be illustrated in the chapter, substitution and income effects differ significantly between recursive and nonrecursive models leading to different Slutsky conditions.

Nonseparability between production and consumption may arise under a wide range of circumstances. It is potentially present whenever the market of at least one production-consumption good is missing. If markets are missing, the implicit price of the domestic good is endogenous to household behaviour and the separation property between production and consumption decisions might not hold. On the other hand, if markets are complete, the household production and individual labour can be sold on the market, or, the same goods and services can be bought on the market at a given price. Hence, households are price takers for every good and production decisions are taken independently from consumption and leisure decisions.

We study the production-consumption household model under the assumptions of complete and absent markets for the domestic good. We find the conditions of market's structures and/or household technology that ensure separability between production and consumption-leisure decisions. These conditions are general in the sense that they can be applied to cooperative household models as well to noncooperative household models. In addition, we perform a qualitative analysis for the household models developed under different market structures. In doing so, we extend the generalized Slutsky equation derived by Browning and Chiappori (1998) to collective models with marketable and nonmarketable domestic goods.

The chapter opens with the analysis of complete markets. Existence of markets for consumption goods and labour supply assure that prices do not vary by agents. Under this circumstance, the family produces an aggregated output that can be sold at a given market price or privately consumed by each household member. The price of the domestic good is determined on the marketplace and the separation property between production and consumption-leisure choices holds. As a consequence, the household decides the optimal production plan independently of its optimal consumption-leisure bundle. Conversely, the consumption-leisure choices are affected by the production activities of the family through profit effects.

In the subsequent sections the marketable assumption of the domestic good is relaxed and, therefore, the implicit price of the domestic good is jointly determined by the household's choices. Moreover, we model the household production by means of joint technologies: with the same variable inputs and technology the family produces two outputs privately consumed by each member. Within this framework, first we analyze the production-consumption household model assuming a production technology with constant returns to scale. Under the circumstance of absent markets, we find that constant returns to scale are sufficient to ensure that the implicit domestic price does not depend on household tastes and, in turn, it ensures the separability between production and consumption-leisure choices.

Then, we assume nonconstant returns to scale. We show that the implicit price of the domestic good depends on the production-consumption variables and, therefore, the household model must be solved jointly with consumption-leisure decisions. In general, within collective models nonseparability of production and consumption-leisure choices has negative consequences on the household model structure. In particular, the consumption-leisure choices cannot be modelled as it were a two-stage budgeting process meaning that the sharing rule approach cannot be employed. This finding has crucial consequence for empirical applications. As pointed out by Browning, Chiappori, and Lewbel (2006) identification of the Pareto weight requires to know the unobservable cardinalization of member utilities. Therefore, to provide identification of the structural model one need to know those cardinalizations and results may strongly depend on the cardinalization chosen by researchers. In addition, in accord with previous findings (Pollak and Watcher 1975), due to joint technologies a closed-form solution to the production-consumption household model cannot be found.

In order to provide a solution to these two drawbacks, that is to find models with closed-form solutions and to implement the sharing rule approach, we propose two alternative specifications of the production function, though we recognize that they may model very specific household productions. In both alternatives we abstract from jointness in production. A first alternative assumes that domestic goods are produced using two distinct production functions in which both household members allocate a part of their time. Without joint technologies the production-consumption household model has a closed-form solution. However, it cannot be extended to the sharing rule approach. In the second alternative each household member produces by herself the domestic good that she consumes. The advantage of the latter is that, given an appropriate exogenous sharing rule, the household program can be decentralized into two individual programs in which each member chooses production and consumption variables jointly, but independently of production-consumption decisions of the other member.

3.2 Marketable Domestic Goods

Formally the model is set-up as follows. Households are composed of two members, f and m, and each of them privately consumes a composite market good in quantity c^i, a home-produced good consumed in quantity z^i and leisure in quantity L^i. We assume that both members are employed in production processes both inside and outside the family. We assume that the household technology to produce z^p can be represented by a generalized production function

$$z^p = h(t^f, t^m),$$

where $h(\cdot)$ represents the household technology used to produce the domestic good z^p and it is assumed strictly increasing, twice differentiable and concave. t^i are hours that each member devotes to the household activities. The household technology can exhibit constant or nonconstant returns to scale. However, in this section we do not restrict the technology to any particular scale. It is worth noticing that z^p is the quantity of output produced by the family that can be sold in the market or consumed by the family members.

The two-member household faces a budget constraint that limits its total consumption given by

$$p_f c^f + p_m c^m + p_z \sum_{i=f}^{m} z^i = \sum_{i=f}^{m} w_i l^i + y + p_z z^p, \tag{3.1}$$

where p_i and p_z[1] are the prices of the market-purchased good and the household good respectively, w_i is the market wage, and y is the household nonlabour income.

Each household member faces a time constraint—she cannot allocate more time to market employment l^i, leisure L^i, and household production t^i, than the total time available to each of them

$$T_i = L^i + l^i + t^i,$$

for $i = f, m$ where T_i is the total stock of i's time.

Before describing the production-consumption household decision process we introduce the following set of assumptions. The first two assumptions are standard in collective models and the last is related to the production side of the model.

Assumption 3.1 (Individual Preferences) *In a collective household model individual preferences are assumed to be represented by an egoistic strictly quasi-concave utility function, continuously differentiable and strictly increasing in its elements*

$$u^i = U^i\left(c^i, z^i, L^i\right).$$

Throughout the chapter, we only provide for consumption of private goods and ignore both the consumption of public goods and externalities within the family. Although we recognize that this assumption is a severe restriction in a household model, it allows to recover individual preferences and to characterize the decision process observing only household consumption rather than individual consumption (Chiappori 1992). Therefore, we assume that also leisure L^i and the domestic good z^i are privately consumed by household members. Fong and Zhang (2001) consider a more general model where leisure can be consumed both privately and publicly. Although the two alternative uses are not empirically distinguishable, they can be identified in general, provided that the consumption of another exclusive good is observed. On the other hand, using a sample of British couples drawn from the British Household Panel Survey, Couprie (2007) provides an econometric identification of the sharing rule based on a model in which the domestic good is treated

[1] Without loss of generality, we assume that z^f and z^m have the same market price p_z.

as a public good.

It is also important to underline that for interior maxima "egoistic" preferences are not necessary to recover the individual behaviour, and the collective set-up may be extended to a caring utility function $u^i = \widetilde{U}^i \left[U^i\left(c^i, z^i, L^i\right), U^j\left(c^j, z^j, L^j\right)\right]$ without altering the conclusions of the model (Chiappori 1992).[2]

Assumption 3.2 (Pareto-Efficiency) *Household decisions are assumed to result in Pareto-efficient outcomes, that is the resource allocation $(c^f, c^m, z^f, z^m, L^f, L^m, t^f, t^m)$ chosen by the household is such that no other feasible allocation $(c'^f, c'^m, z'^f, z'^m, L'^f, L'^m, t'^f, t'^m)$ could make both members better off.*

Spouses are engaged in a long-term relationship with good symmetric information about each others choices and the assumption of Pareto-efficiency in the context of household behaviour is sensible.[3] The assumption of Pareto-efficiency implies that given an allocation of household resources the maximization problem can be decentralized by applying the second theorem of welfare economics.

Assumption 3.3 (Complete Markets) *All markets are assumed to be complete and competitive.*

---

[2] In order to show that caring utility funcitions allow to solve household collective models using a two stage budgeting procedure, consider the following simplified household model with caring individual utility function and without domestic production

$$\max_{c^f, L^f, c^m, L^m} \quad W = \sum\nolimits_{i \in f,m} \mu_i \widetilde{U}^i \left[U^i\left(c^i, L^i\right), U^j\left(c^j, L^j\right)\right]$$

with $\mu_f = \mu$ and $\mu_m = 1 - \mu$, subjet to the household budget constraint $\sum_{i=f}^m p_i c^i + \sum_{i=f}^m w_i L^i = \sum_{i=f}^m w_i T_i + y$.

The marginal utilities of individual i with respect to the variables of choice are

$$\begin{aligned} \mu_i \widetilde{U}^i_{U^i} U^i_{c^i} + \mu_j \widetilde{U}^j_{U^i} U^i_{c^i} &= \delta p_i, \\ \mu_i \widetilde{U}^i_{U^i} U^f_{L^f} + \mu_j \widetilde{U}^j_{U^i} U^f_{L^f} &= \delta w_i, \end{aligned}$$

for $i \neq j = f, m$, δ is the lagrangean multiplier. i's marginal rate of substitution is then equal to

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{pi},$$

that does not depend either on j's preferences or on the decision process μ. This implies that the household model can be solved using a two stage budgeting procedure.

[3] See Udry (1996) for an empirical rejection of the Pareto efficiency assumption using a sample of farm households from Burkina Faso.

Assumption 3.3 establishes that both individual labour supply and household production can be sold on the outside markets, wages and prices are determined on the marketplace and households are price-takers. Therefore, with well-organized markets the household product z^p can be entirely consumed by the family or partly/fully sold on the outside market.[4]

An established result is that perfect markets are a sufficient condition to ensure separability between production and consumption decisions without imposing any other restrictions on household technology. However, we show that complete markets are sufficient but not necessary for separability. In Section 3.3 we will relax Assumption 3.3 and show that under the restriction of a production technology with constant returns to scale the household program is still separable. Precisely, we show that the separation property holds whenever households are price-takers or implicit prices are not affected by household preferences.

The production and consumption optimal decisions of the household then must be the solution of the following constrained maximisation program

$$\max_{\substack{c^f, z^f, L^f, c^m, z^m, L^m, \\ z^p, t^f, t^m}} W^h = \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m(c^m, z^m, L^m), \tag{3.2}$$

$$\text{subject to} \quad z^p = h(t^f, t^m), \quad \text{and}$$

$$\sum_{i=f}^{m} p_i c^i + p_z \sum_{i=f}^{m} z^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y + p_z z^p - \sum_{i=f}^{m} w_i t^i,$$

with nonnegativity constraints on each component of T_i, that is $L^i > 0, t^i > 0, l^i > 0$ for $i = f, m$.[5] Notice that the individual time constraint has been substituted in the opportunity set. The right-hand side of the opportunity set represents the household full income and

[4]In much of the developing world, many farmers grow crops for personal consumption instead of selling their agricultural output for a price that is thought to be a more profitable option. A conjecture to explain this choice is that one or several markets are missing. For instance, in developing countries there can be missing information about the profitability of crops, lack of access to capital, lack of infrastructure necessary to bring the crops to the market, and lack of human capital necessary to adopt successfully a new agricultural technology. On the other hand, in developed countries markets are well organized and complete.

[5]Throughout the chapter we ignore corner solutions.

it is made up of money-value of total time endowment, $\sum_{i=f}^{m} w_i T_i$, nonlabour income and nonmaximized profits, $p_z z^p - w_f t^f - w_m t^m$, generated by the household production. It is worth noticing that given the assumption of complete markets the household product can be sold on the marketplace or entirely consumed by the household and in general we have that $z^p \neq z^f + z^m$.

The objective function W^h is modeled as the weighted sum of the utility function of the spouses with $\widetilde{\mu}_f = \mu \in [0,1]$ and $\widetilde{\mu}_m = 1 - \mu$. The weight μ represents the bargaining power that each member can exert on family resources. If $\mu = 1$ the household welfare is entirely determined by the preferences of member f and it implies that she has total control over the household resources. On the other hand, if $\mu = 0$ the reverse is true, that is member m has full command on the household resources. For intermediate values of μ, W^h embodies preferences of both members. Model (3.2) can represent both the unitary and the collective approach to household behaviour. When μ is independent of budget constraint variables, program (3.2) represents a special case of the unitary model. On the other hand, in the collective model the weight μ is in general a function of exogenous variables including variables entering the budget constraint, such as market prices, wages, nonlabour income and distribution factors.[6] When μ is function of exogenous variables that enter the budget constraint the Slutsky matrix in general is not symmetric and it is made up of a symmetric matrix and a matrix with at most rank one (Browning and Chiappori 1998).

The Lagrangian function corresponding to program (3.2) is

$$\begin{aligned} \mathcal{L} &= \sum_{i=f}^{m} \widetilde{\mu}_i U^i(c^i, z^i, L^i) + \lambda \left[h(t^f, t^m; \theta) - z^p \right] + \\ &\quad + \delta \left[\sum_{i=f}^{m} w_i T_i + y + p_z z^p - \sum_{i=f}^{m} w_i t^i - \sum_{i=f}^{m} p_i c^i - p_z \sum_{i=f}^{m} z^i - \sum_{i=f}^{m} w_i L^i \right]. \end{aligned} \tag{3.3}$$

[6] Distribution factors are exogenous variables that can affect the household behaviour only through their impact on the decision process. In other words, the distribution factors do not affect either the individual preferences or the budget constraint. Examples of distribution factors used in empirical works are divorce ratio and sex ratio (Chiappori, Fortin, and Lacroix 2002), wealth at marriage (Thomas, Contreras, and Frankenberg 1997), and benefits (Rubacolva and Thomas 2000). In general, distribution factors simplify the theoretical identification of the partial derivatives of the sharing rule. In absence of distribution factors there needs to second order differential equations to recover the partial derivatives of the sharing rule.

Given the usual curvature of preferences and a concave production function the following first-order conditions are both necessary and sufficient for an interior solution[7]

$$\begin{aligned}
\mathcal{L}_{c^i} &= \widetilde{\mu}_i U^i_{c^i} - \delta p_i = 0, \\
\mathcal{L}_{z^i} &= \widetilde{\mu}_i U^i_{z^i} - \delta p_z = 0, \\
\mathcal{L}_{L^i} &= \widetilde{\mu}_i U^i_{L^i} - \delta w_i = 0,
\end{aligned}$$

and the budget constraint

$$\mathcal{L}_\delta = 0 \rightarrow \sum_{i=f}^{m} p_i c^i + p_z \sum_{i=f}^{m} z^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y + p_z z^p - \sum_{i=f}^{m} w_i t^i,$$

which imply that the efficient conditions hold

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{p_z}{p_i}, \qquad i = f, m, \tag{3.4}$$

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \qquad i = f, m, \tag{3.5}$$

$$\frac{U^j_{c^j}}{U^i_{c^i}} = \frac{\widetilde{\mu}_j p_j}{\widetilde{\mu}_i p_i}, \qquad i \neq j = f, m. \tag{3.6}$$

At the equilibrium point, the individual's marginal rate of substitutions (3.4) and (3.5) are equal to the ratio between market prices. The marginal rate of substitution across the consumption of the two household members (3.6) depends on μ and it determines the final location on the Pareto efficient frontier.

The first-order necessary conditions of the production decisions are

$$\mathcal{L}_{t^i} = 0 \Rightarrow h_{t^i}(\cdot) = \frac{\delta}{\lambda} w_i \quad i = f, m, \tag{3.7}$$

$$\mathcal{L}_{z^p} = 0 \Rightarrow p_z = \frac{\lambda}{\delta}, \tag{3.8}$$

$$\mathcal{L}_\lambda = 0 \Rightarrow z^p = h(t^f, t^m). \tag{3.9}$$

[7]Throughout the chapter the notation F_x stands for the partial differential of function F with respect to the variable x.

In the case of tradeable domestic goods, equation (3.8) plays a crucial role: it states that at the optimum the ratio between the Lagrange multipliers associated with the opportunity set and technology constraints, respectively, is equal to the market price of the domestic good. Substituting this expression into equation (3.7) we obtain the efficient conditions for the production choice

$$p_z h_{t^i}(\cdot) = w_i \quad i = f, m, \tag{3.10}$$

and the family will produce z^p up to the point where the value of t^i's marginal product is equal to its price.

By equations (3.10) and (3.7), the factor inputs and optimal production of z^p are obtained

$$\widetilde{t^i} = t^i(p_z, w_f, w_m), \quad i = f, m, \tag{3.11}$$

$$\widetilde{z^p} = z^p(p_z, w_f, w_m). \tag{3.12}$$

Note that the reduced demands of inputs and household production are functions only of w_f, w_m and p_z. This condition is sufficient to allow for separation between production and consumption decisions. Thus, optimal program (3.2) can be solved recursively in two stages. In the first stage, the family chooses the optimal level of input and output independently of individual consumption. Once these choices are made, the household decides its optimal consumption bundle that will be influenced by the production decisions through the budget constraint. The separation between production and consumption is a crucial property especially in empirical work. Exploiting the separation property, empirical estimation can be computed in two independent stages. First econometricians can estimate the production parameters and then those related to the consumption variables, avoiding in this way computational problems of jointness of household decision processes. Note that the consumption-leisure variables are affected by the production side of the family through profit effects.

Substituting the optimal production solution into $\mathcal{L}_\delta$, the budget constraint becomes

$$\sum_{i=f}^{m} w_i T_i + y + \pi\left(p_z, w_f, w_m\right) - \sum_{i=f}^{m} p_i c^i - p_z \sum_{i=f}^{m} z^i - \sum_{i=f}^{m} w_i L^i = 0 \tag{3.13}$$

and along with the marginal rates of substitution yield the Marshallian demand functions for the household members

$$\widetilde{c}^i = c^i\left(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}, \widetilde{\pi}\right),$$

$$\widetilde{z}^i = z^i\left(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}, \widetilde{\pi}\right),$$

$$\widetilde{L}^i = L^i\left(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}, \widetilde{\pi}\right),$$

for $i = f, m$ with $\widetilde{\mu} = \mu\left(p_f, p_m, p_z, w_f, w_m, y\right)$. It is worth noticing that in equation (3.13) $\pi\left(p_z, w_f, w_m\right) = p_z \widetilde{z}^p - \sum_{i=f}^{m} w_i \widetilde{t}^i$ represents the optimal level of profits.[8]

Up to this point, we have analyzed the production-consumption household behaviour within the collective model when markets are complete. The household makes its production decisions independently of its consumption and leisure decisions. On the other hand, consumption-leisure decisions are not independent of production decisions. Consumption and leisure depend on market prices, income and the intra-household allocation of resources μ. Total household resources in turn are determined, to some extent, by profits from household's production activity. As a consequence, when making comparative static analyses the inclusion of the profit effect can change the direction and the magnitude of results predicted by models that account only for consumption and leisure. This evidence will be illustrated in Section 3.2.2.

[8] Potentially, profits can be positive, negative or equal to zero. However, in the long run, one can expect that if profits are negative a rational agent should shut down its production activity. This can be extended to economic activities run by households. A relationship between the sign of profits and returns to scale of production functions can be established when dealing with optimal profits derived by the optimization of a constrained program. In this case, economic theory states that with increasing returns to scale there is no solution to the profit-maximizing program. When the production function exhibits constant returns to scale one can distinguish three cases: 1) if marginal costs are greater than market price of the output then the optimal production level is zero, 2) if marginal costs are smaller then the market price of the output then no solution exists to the profit-maximizing program, and 3) when marginal costs are equal to the market price any non negative output level is a solution to the profit-maximizing program and generates zero profits.

3.2.1 The Separation Property: Profit-Maximizing Model and Individual Consumption-Leisure Choices

Assumption 3.3 establishes a sufficient condition for the separability between production and consumption-leisure choices to hold and leads to the decentralization of program (3.2) in two distinct optimal programs.

In the first stage, the household decides the optimal production plan by maximizing the following constrained program

$$\max_{z^p, t^f, t^m} \quad \pi = p_z z^p - w_f t^f - w_m t^m, \tag{3.14}$$

$$\text{subject to} \quad z^p = h(t^f, t^m).$$

With strict concavity of $h(\cdot)$ the first-order conditions are equal to

$$h_{t^i}(\cdot) = \frac{w_i}{p_z}, \qquad i = f, m,$$

where this expression is equivalent to the first-order conditions produced by program (3.2), and the optimal solution is

$$\begin{aligned} \widetilde{t}^i &= t^i(p_z, w_f, w_m), \qquad i = f, m, \\ \widetilde{z}^p &= z^p(p_z, w_f, w_m). \end{aligned}$$

Substituting the last three equations into the objective function the optimal level of profits are derived $\widetilde{\pi} = \pi\left(p_z, w_f, w_m\right)$. The optimal profits can be interpreted as follows. If the domestic good is partially or totally sold on the outside market, $\pi\left(p_z, w_f, w_m\right)$ measures the "objective" profits of the household. On the other hand, if z^p is entirely consumed by the family, $\pi\left(p_z, w_f, w_m\right)$ can be interpreted as the profit imputed to the domestic activities of the family. Note that the production variables do not depend on household preferences, and any exogenous change of the bargaining power μ or the nonlabour income

y does not affect the household production decisions.

In the second stage, according to an exogenous rule, the family decides how to share total economic resources between the household members

$$\varphi_f + \varphi_m = y + \pi\left(p_z, w_f, w_m\right),$$

where $\varphi_f = \varphi\left(p, w, Y\right)$ is the amount of household resources received by member f and $\varphi_m = Y - \varphi\left(p, w, Y\right)$ by member m with $Y = y + \pi\left(p_z, w_f, w_m\right)$. For notational convenience we have set $p = \left(p_f, p_m, p_z\right)$ and $w = \left(w_f, w_m\right)$.

Given the sharing rule interpretation and exploiting the functional separability property of W^h, the household consumption program is equivalent to the following individual model. Each spouse independently chooses her optimal consumption-leisure bundle subject to her opportunity set. The individual model is given by

$$\max_{c^i, z^i, L^i} \quad U^i(c^i, z^i, L^i), \tag{3.15}$$

$$\text{subject to} \quad p_i c^i + p_z z^i + w_i L^i = w_i T_i + \varphi_i\left(p, w, Y\right).$$

From the first-order condition we derive the reduced solution

$$\begin{aligned}
\widetilde{c}^i &= c^i(p_i, p_z, w_i, \varphi_i\left(p, w, y + \widetilde{\pi}\right)), \\
\widetilde{z}^i &= z^i(p_i, p_z, w_i, \varphi_i\left(p, w, y + \widetilde{\pi}\right)), \\
\widetilde{L}^i &= L^i(p_i, p_z, w_i, \varphi_i\left(p, w, y + \widetilde{\pi}\right)),
\end{aligned}$$

where the sharing rule function $\varphi_i\left(\cdot\right)$ is reported in reduced form and, as such, it is a function of exogenous variables.

Under Assumption 3.3 and the additive functional form of W^h, program (3.2) can be split into two distinct optimal programs. One optimal program that accounts for household production decisions and the other for household consumption choices. Second, as a direct consequence of the Second Theorem of Welfare Economics in absence of public goods[9] and externalities, the individual consumption program along with the sharing rule (3.15) lead to the same demands of the household program with the power index μ defined over a set of exogenous variables. Note that the solutions produced by programs (3.14) and (3.15) must be equal to those borne by program (3.2).

3.2.2 Comparative Static Analysis

In this section we analyse the effects of wage, income on the optimal solution. By means of the separation property, production variables are not affected both by household tastes and the decision process μ. It means that output and factor demands do not vary as market prices p_i, nonlabour income y and the decision process μ change. Conversely, through the profit function that enters the budget constraint consumption-leisure choices are affected by changes that occur in the household production activities. In the appendix a formal derivation of the comparative static analysis presented here is drawn.

Gross Substitution Effects Let us recall *i's* leisure demand yielded by program (3.2)

$$\widetilde{L}^i = L^i\left(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}, \widetilde{\pi}\right), \quad \text{for} \quad i = f, m,$$

where $\widetilde{\pi} = \pi\left(p_z, w_f, w_m\right)$ is the optimal profit level and $\widetilde{\mu} = \mu\left(p_f, p_m, p_z, w_f, w_m, y\right)$ is the Pareto weight.

An uncompensated change of w_j on the optimal choice of *i's* leisure has the following effect

$$\frac{\partial \widetilde{L}^i}{\partial w_j} = \frac{\partial L^i}{\partial w_j} + \frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j} + \frac{\partial L^i}{\partial \widetilde{\pi}}\frac{\partial \pi}{\partial w_j}, \qquad \text{with } i, j = f, m,$$

[9] Blundell, Chiappori, and Meghir (2005) extend the collective household model to public consumption. They show that the structure of the model can be identified by observing labour supplies and household demand for public good. Identification of the individual welfare and the decision process requires either a separability assumption of the utility function or the presence of a distribution factor.

where Shephard's lemma $\frac{\partial \pi}{\partial w_j} = -t^j$[10] it follows that

$$\frac{\partial \widetilde{L}^i}{\partial w_j} = \frac{\partial L^i}{\partial w_j} + \frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j} - t^j \frac{\partial L^i}{\partial \widetilde{\pi}}, \quad \text{with } i,j = f,m. \tag{3.16}$$

For $i = j$ we can suppose, *ceteris paribus*, that a positive change in her market wage makes her consumption of leisure more expensive. Thereby, if leisure is a normal good, it is likely that she will substitute away from leisure and towards paid jobs, either l^i or t^i, but the model cannot predict which of the two paid activities she will choose. Thus the first term on the right-hand side of equation (3.16), $\frac{\partial L^i}{\partial w_i}$, is negative. Accounting for household production another effect arises. A change in w_i will change the optimal value of the profit function. An increase in w_i will make the input t^i more expensive, profits will decrease and the household opportunity set will decrease as well by the amount of $t^j \frac{\partial L^i}{\partial \widetilde{\pi}}$.

Moreover, as her market wage changes, it is plausible to expect a variation in the Pareto weight. However, how μ should be affected by a change in w_i depends on the interpretation that one may give to the intra-household allocation process. A first interpretation would point up the "redistribution" purpose of the Pareto weight. Following this view, transfers of household resources turn out to compensate inequalities in paid incomes within the household (Chiappori 1992). In that case, given an increase in w_i, member i will be better off and thus reduces the need for a transfer of household income in her favour. Therefore, relying on this interpretation, it is plausible that $\frac{\partial \mu}{\partial w_i}$ is negative and, if it is the case, the overall sign of equation (3.16) is negative.

A second interpretation, on the other hand, would emphasize the bargaining nature

[10] The maximum value of the profit function is

$$\pi(p_z, w_f, w_m) = p_z h\left(\widetilde{t}^f, \widetilde{t}^m\right) - w_f \widetilde{t}^f - w_m \widetilde{t}^m,$$

differentiating let say with respect to w_f

$$\frac{\partial \widetilde{\pi}}{\partial w_f} = p_z\left(\frac{\partial h}{\partial \widetilde{t}^f}\frac{\partial t^f}{w_f} + \frac{\partial h}{\partial \widetilde{t}^m}\frac{\partial t^m}{w_f}\right) - w_f\frac{\partial t^f}{w_f} - \widetilde{t}^f - w_m\frac{\partial t^m}{w_f},$$

and combining terms yields

$$\frac{\partial \widetilde{\pi}}{\partial w_f} = \left(p_z\frac{\partial h}{\partial \widetilde{t}^f} - w_f\right)\frac{\partial t^f}{w_f} + \left(p_z\frac{\partial h}{\partial \widetilde{t}^m} - w_m\right)\frac{\partial t^m}{w_f} - \widetilde{t}^f.$$

In equilibrium $p_z\frac{\partial h}{\partial \widetilde{t}^i} - w_i = 0$ for $i = f,m$, thus $\frac{\partial \widetilde{\pi}}{\partial w_f} = -\widetilde{t}^f$. Note that the profit change is the same whether or not inputs are held fixed or whether they in principle can vary as the market wage changes.

of μ. In that case, an increase in w_i would lead to an increase of i command on the household resources and $\frac{\partial \mu}{\partial w_i}$ would be positive. In this case, the overall sign of equation (3.16) depends on which of the three effects, the substitution effect, the profit or the bargaining effect, overwhelms the other. Similar gross substitution effects can be drawn for the purchased good and the home-produced good.

Net Substitution Effects Using the optimal demand of leisure, we analyse a change of the market wage w_j compensated by a change of the nonlabour income y such that the household welfare does not vary. The Slutsky equation is also performed for the domestic good.

The compensated effect on i's leisure equation is

$$\frac{\partial L^i}{\partial w_j} = \frac{\partial \widetilde{L}^i}{\partial w_j}|_{dW^h=0} + \left(T_j - L^j - t^j\right)\frac{\partial L^i}{\partial y} - \frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j} - \left(T_j - L^j - t^j\right)\frac{\partial \mu}{\partial y}\right], \quad (3.17)$$

with $i, j = f, m$ where $\widetilde{L}^i|_{dW^h=0}$ is the compensated demand function. Equation (3.17) is the collective generalized Slutsky equation extended to a production-consumption household model. The first two terms compose the conventional substitution matrix, which is symmetric and negative semidefinite. In general $\frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j} - \left(T_j - L^j \;\; -t^j\right)\frac{\partial \mu}{\partial y}\right]$, the term yielded by the Pareto weight, is not symmetric and has at most rank one (Browning and Chiappori 1998).

The introduction of household production does not make much difference in terms of comparative static analysis. In particular, since household members allocate their time to market and domestic jobs the income effect is as conventionally weighted by *i's* labour supply, but accounting for domestic time the number of working hours is net of leisure time. As a consequence, for $i = j$ assuming that leisure is a normal good makes a backward-bending labour supply curve less likely than if agents were solely supplier to the labour market. When her wage increases, she begins to substitute away from the good becoming relatively more expensive, thus it is likely that $\frac{\partial \widetilde{L}^i}{\partial w_i}$ is negative. However, the wage change also changes her opportunity set and her bargaining power within her family. Note that in the unitary setting the latter effect does not appear. Assuming that leisure is a normal good then the income effect is positive, hence $\frac{\partial L^i}{\partial y}$ and $\frac{\partial \mu}{\partial y}$ are both positive. The size of the income effect is determined by j's labour supply $T_j - L^j - t^j$, and considering internal solutions, it is strictly positive. Hence, second and fourth term are both positive. As mentioned above, the sign of $\frac{\partial \mu}{\partial w_i}$ depends on the interpretation that one might give to μ.

In the case of complete markets, the own-price effect of the domestic good is equal to

$$\frac{\partial z^i}{\partial p_z} = \frac{\partial \widetilde{z}^i}{\partial p_z}|_{dW^h=0} + \left(z^p - \sum_{i=f}^m z^i\right)\frac{\partial z^i}{\partial y} - \frac{\partial z^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial p_z} - \left(z^p - \sum_{i=f}^m z^i\right)\frac{\partial \mu}{\partial y}\right].$$

The first term on the right-hand side of the equation is the substitution effect and is negative. Since the domestic good is assumed to be a normal good, $\frac{\partial z^i}{\partial y}$ and $\frac{\partial \mu}{\partial y}$ are both positive. Consequently, the sign and size of the income effect depends on whether the family is a net seller or a net buyer. When the family is a net seller, $z^p - z^i > 0$, the second and last term are positive. On the other hand, when the family is a net buyer, $z^p - z^i < 0$, they change the sign.

In the following sections we study the case of absent markets of domestic goods. We study the conditions assuring separability between production and consumption decisions with missing markets. In addition, we introduce joint production function and discuss the implications of jointness on the structure of the collective household model.

3.3 Nonmarketable Domestic Goods, Constant Returns to Scale and Joint Production

Let us relax Assumption 3.3 and suppose that z^i is nonmarketable with $i = f, m$. Two implications arise. Firstly, there is no exogenous market price for z^i and thus each family has its own implicit price. Secondly, the home-made good is entirely consumed by the family, whereas in the case of complete markets the domestic good can be sold on the market.

The household maximizes the following problem

$$\max_{\substack{c^f,z^f,L^f,c^m,z^m,L^m,\\ t^f,t^m}} W^h = \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m(c^m, z^m, L^m), \tag{3.18}$$

$$\text{subject to} \quad z^f + z^m = h(t^f, t^m),$$

$$\text{and} \quad \sum_{i=f}^{m} p_i c^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i t^i,$$

with $L^i > 0, l^i > 0, t^i > 0$ for $i = f, m$. Program (3.18) is basically similar to (3.2). The substantial differences lie in the composition of the budget constraint and in the production technology. In equation (3.1) the value of the household product appears both on the left-hand side, as expenditure, and on the right-hand side, as revenue. Moreover, on the right-hand side of equation (3.1) the last three terms added together formed the household profits derived by selling z^p. Differently, in the budget constraint of program (3.18) the cost of producing the domestic good appears, $\sum_{i=f}^{m} w_i t^i$, implying that when markets are missing the household behaves as it were a cost minimizer. Finally, the production function exhibits joint technology: by means of the same factors the household produces two different domestic goods privately consumed by each members. Joint production is an assumption that we will maintain also in the following section. Interpretation of the objective function is similar to the verbal interpretation given to problem (3.2).

The Lagrangian function associated with (3.18) is

$$\mathcal{L} = \sum_{i=f}^{m} \widetilde{\mu}_i U^i(c^i, z^i, L^i) + \lambda \left[h(t^f, t^m) - z^f - z^m\right] \\ + \delta \left[\sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i t^i - \sum_{i=f}^{m} p_i c^i - \sum_{i=f}^{m} w_i L^i\right],$$

and it produces the following first order conditions

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \quad i = f, m, \tag{3.19}$$

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{\lambda}{\delta}\frac{1}{p_i}, \qquad i = f, m, \tag{3.20}$$

$$\frac{U^j_{c^j}}{U^i_{c^i}} = \frac{\widetilde{\mu}_j p_j}{\widetilde{\mu}_i p_i}, \qquad i \neq j = f, m, \tag{3.21}$$

$$h_{t^i}(\cdot) = \frac{\delta}{\lambda} w_i \qquad i = f, m, \tag{3.22}$$

plus the technology and budget constraints.

The marginal rates of substitution within and across the consumption of the two members, (3.19) and (3.21) respectively, and the marginal product (3.22) are similar to the case of marketable domestic good. Equation (3.20) says that in equilibrium the individual marginal utility between the domestic and market good is equal to the ratio of $\frac{\lambda}{\delta}$—the relation between the Lagrange multipliers associated with the consumption and production constraint—to the market price of c^i. In Section 3.2 we find that in equilibrium the first-order conditions yield that $\frac{\lambda}{\delta}$ equals the exogenous price of z^p.

Taking the ratio between the marginal products of t^f and t^m, we derive the usual efficient condition for the production activities

$$\frac{h_{t^f}(t^f, t^m)}{h_{t^m}(t^f, t^m)} = \frac{w_f}{w_m},$$

where this expression says that in equilibrium the ratio of wages equals the ratio of marginal products of the two factors. Solving this expression for t^i and substituting it back into the production function the optimal quantity for the domestic time is derived

$$\widetilde{t^i} = t^i (w_f, w_m, z_f, z_m) \qquad i = f, m.$$

It represents the factor demand curve when outputs are held constant. Isolating the last term on the right-hand side of the budget constraint we obtain

$$C (w_f, w_m, z_f, z_m) = \textstyle\sum_{i=f}^{m} w_i t^i (w_f, w_m, z_f, z_m), \tag{3.23}$$

that specifies the minimum total cost of producing any given level of output. It is assumed to be monotonically increasing in z_f and z_m and the prices of the inputs, w_f and w_m.

To solve the set of first-order conditions related to the household consumption, we introduce the following assumption on the household technology.

Assumption 3.4 (Constant Returns to Scale) *Let us assume that the household production function is homogenous of degree one.*[11]

Then it follows that

Proposition 3.1 *When markets of the domestic goods are missing given Assumption 3.4 the implicit price p_z^* of the domestic good is not affected by tastes and the decision process μ of the household.*

Proof. A crucial implication of technologies with constant returns to scale is that the corresponding minimum cost is a function linear in the output, that is

$$C(w_f, w_m, z_f, z_m) = P_z(w_f, w_m) \textstyle\sum_{i=f}^{m} z_i. \tag{3.24}$$

In equilibrium we define the implicit price p_z^* of the domestic good as the marginal cost of producing the domestic good

$$p_z^* \triangleq \frac{\partial C(w_f, w_m, z_f, z_m)}{\partial z_i},$$

that given equation (3.24) it becomes

$$p_z^* = P_z(w_f, w_m).$$

■

This result is a crucial sufficient condition for the separation property to hold when markets of the domestic goods are absent. Another feature of this result is that the

[11] A function $f(x)$ is said to be homogeneous of degree k if $f(tx) = t^k f(x)$ for all values of x and for any positive t. In words, homogeneity means that if all the arguments of the function are multiplied by the same positive constant t then the value of the function ends up to be t^k times its old value. If a function is homogeneous of degree one, then doubling all its arguments doubles the value of the function. If a function is homogeneous of degree zero, then as long as all arguments change by the same percentage the value of the function will remain constant.

implicit price p_z^* is the same for both household members. This follows from assuming production technologies of the form $z^f + z^m = h(t^f, t^m)$. By standard economic theory, in equilibrium marginal costs are equal to the Lagrange multiplier—this result is illustrated in the following remark—[12]and $\frac{\lambda}{\delta}$ in equation (3.20) can be opportunely replaced with $P_z(w_f, w_m)$.

Remark 1 (Linear Homogeneity) *Linear homogeneity of $h(\cdot)$ implies that the minimum cost $C(w_f, w_m, z_f, z_m)$ can be written as a combination of wages and output linear in $z = z_f + z_m$, that is $P_z(w_f, w_m)\, z$. From standard economic theory we know that the marginal cost function equals the Lagrangian multiplier. In our setting the ratio of the Lagrangian multipliers associated with the production and the budget constraints,*

$$\frac{\partial C(w_f, w_m, z_f, z_m)}{\partial z^i} = \frac{\lambda}{\delta}, \tag{3.25}$$

thus exploiting Assumption 3.4 it follows that

$$\frac{\partial C(w_f, w_m, z_f, z_m)}{\partial z^i} = \frac{\lambda}{\delta} = P_z(w_f, w_m).$$

A formal derivation of (3.25) can be obtained as a simple application of the envelope theorem. First, from the optimal program (3.18) the cost minimization problem can be written as

$$\sum_{i=f}^{m} w_i t^i + \frac{\lambda}{\delta}\left[h(t^f, t^m) - z^f - z^m\right]$$

where the Lagrangian multiplier $\frac{\lambda}{\delta}$ is obtained dividing the two terms by δ. The minimum cost is

$$C(w_f, w_m, z_f, z_m) = w_f t^f(w_f, w_m, z) + w_m t^m(w_f, w_m, z),$$

differentiating C partially with respect to z^i

$$\frac{\partial C}{\partial z^i} = w_f \frac{\partial t^f(w_f, w_m, z)}{\partial z^i} + w_m \frac{\partial t^m(w_f, w_m, z)}{\partial z^i}, \tag{3.26}$$

[12]The remark is a simple application of standard economic theory (see for example Silberberg 1990).

from the first-order relations

$$w_i = \frac{\lambda}{\delta}\frac{\partial h(t^f, t^m)}{\partial t^i} \qquad i = f, m,$$

substituting these values into equation (3.26) yields

$$\frac{\partial C}{\partial z^i} = \frac{\lambda}{\delta}\left[\frac{\partial h(t^f, t^m)}{\partial t^f}\frac{\partial t^f\left(w_f, w_m, z\right)}{\partial z^i} + \frac{\partial h(t^f, t^m)}{\partial t^m}\frac{\partial t^m\left(w_f, w_m, z\right)}{\partial z^i}\right]. \tag{3.27}$$

By the constraint

$$z^f + z^m - h(t^f, t^m) = 0$$

and with optimal factor demands it must hold as an identity

$$z^f + z^m - h(\tilde{t^f}, \tilde{t^m}) \equiv 0.$$

Hence, we can differentiate this identity with respect to z^i

$$1 - \frac{\partial h(t^f, t^m)}{\partial \tilde{t^f}}\frac{\partial t^m\left(w_f, w_m, z\right)}{\partial z^i} - \frac{\partial h(t^f, t^m)}{\partial \tilde{t^m}}\frac{\partial t^m\left(w_f, w_m, z\right)}{\partial z^i} \equiv 0,$$

and

$$\frac{\partial h(t^f, t^m)}{\partial \tilde{t^f}}\frac{\partial t^m\left(w_f, w_m, z\right)}{\partial z^i} + \frac{\partial h(t^f, t^m)}{\partial \tilde{t^m}}\frac{\partial t^m\left(w_f, w_m, z\right)}{\partial z^i} \equiv 1$$

thus in equation (3.27) the terms in parentheses equal one and, therefore, at the equilibrium the marginal cost equals the Lagrangian multiplier

$$\frac{\partial C\left(w_f, w_m, z_f, z_m\right)}{\partial z^i} = \frac{\lambda}{\delta}.$$

Now, substituting $P_z\left(w_f, w_m\right)$ into (3.20) yields

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{P_z\left(w_f, w_m\right)}{p_i}, \qquad i = f, m, \tag{3.28}$$

that is the marginal rate of substitution between the domestic good and the market good equals the ratio of the implicit domestic price $P_z(w_f, w_m)$ over the market price. Finally, by means of equations (3.19), (3.21), (3.28) and along with the budget constraint

$$\sum_{i=f}^{m} w_i T_i + y - p_z^* \sum_{i=f}^{m} z_i - \sum_{i=f}^{m} p_i c^i - \sum_{i=f}^{m} w_i L^i = 0, \tag{3.29}$$

where the minimum cost function $C(w_f, w_m, z_f, z_m) = p_z^* \sum_{i=f}^{m} z_i$ has been substituted into the linear budget constraint, we obtain the optimal solution of the consumption choices

$$\widetilde{c}^i = c^i(p_f, p_m, p_z^*, w_f, w_m, y, \widetilde{\mu}), \quad i = f, m$$

$$\widetilde{z}^i = z^i(p_f, p_m, p_z^*, w_f, w_m, y, \widetilde{\mu}), \quad i = f, m$$

$$\widetilde{L}^i = L^i(p_f, p_m, p_z^*, w_f, w_m, y, \widetilde{\mu}), \quad i = f, m.$$

To summarize, in this section we show that production and consumption choices are separable even when markets for the domestic good are missing. In the case of missing markets for the separation property to hold a sufficient condition is that household technology exhibits constant returns to scale. At the equilibrium point the implicit price of the domestic good p_z^* equals the marginal cost. Given constant returns to scale the total cost function is linear in the output quantity and thus the implicit price p_z^* of the domestic good does not depend on household preferences and the decision process μ. The implication is that input factor demands do not depend on household preferences and on μ and the household program can be solved recursively in two stages as in the case of well-organized markets. In the first stage, the household will decide the optimal time devoted to the production activities, then it will decide the optimal consumption of the market goods, leisure and domestic goods.

3.3.1 The Separation Property: Cost-Minimizing Model and Individual Consumption-Leisure Choices

Under Assumption 3.4, the implicit price of the domestic good depends on inputs' prices, w_f and w_m, and on the production technology but not on household preferences. As a consequence, even though markets are missing, household production choices are still separable from the consumption decisions. Differently from the case of marketable domestic goods, here the family has a cost minimizing behaviour. This evidence appears from the budget constraint of program (3.18). Thereby, in the first stage, the household decides the input factor demands by solving the following constrained program

$$\min_{t^f, t^m} \quad C = w_f t^f + w_m t^m,$$

$$\text{subject to} \quad z_f + z_m = h(t^f, t^m),$$

and from the first-order conditions yield the optimal factor inputs

$$\begin{aligned} \widetilde{t^f} &= t^f (w_f, w_m, z_f, z_m), \\ \widetilde{t^m} &= t^m (w_f, w_m, z_f, z_m), \end{aligned}$$

substituting these two equations into the objective function we derive the minimum cost function $C = \sum_{i=f}^{m} w_i t^i (w_f, w_m, z_f, z_m) = p_z^* \sum_{i=f}^{m} z_i$.

In the second stage, similar to the case of marketable domestic good, the household consumption variables are the solution of the following maximizing program

$$\max_{c^f, z^f, L^f, c^m, z^m, L^m} \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m(c^m, z^m, L^m) \tag{3.30}$$

$$\text{subject to} \quad \sum_{i=f}^{m} p_i c^i + p_z^* \sum_{i=f}^{m} z^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y,$$

with first-order conditions equal to

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \qquad i = f, m,$$

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{p^*_z}{p_i}, \qquad i = f, m,$$

$$\frac{U^j_{c^j}}{U^i_{c^i}} = \frac{\widetilde{\mu}_j p_j}{\widetilde{\mu}_i p_i}, \qquad i \neq j = f, m,$$

that together with the budget constraint produce the optimal solution of the consumption-leisure bundle

$$\widetilde{c}^i = c^i(p_f, p_m, p^*_z, w_f, w_m, y, \widetilde{\mu}),$$

$$\widetilde{z}^i = z^i(p_f, p_m, p^*_z, w_f, w_m, y, \widetilde{\mu}),$$

$$\widetilde{L}^i = L^i(p_f, p_m, p^*_z, w_f, w_m, y, \widetilde{\mu}),$$

for $i = f, m$.

Then, according to the second theorem of welfare economics, given an appropriate allocation of household resources, the Pareto-efficient household model (3.30) can be decentralized into two individual consumption programs. Spouses agree on an unspecified rule to allocate nonlabour income and then each spouse solves the following problem

$$\max_{c^i, z^i, L^i} \quad U^i(c^i, z^i, L^i),$$

$$\text{subject to} \quad p_i c^i + p^*_z z^i + w_i L^i = w_i T_i + \varphi_i\left(p_f, p_m, w_f, w_m, y\right),$$

where $p^*_z = P_z\left(w_f, w_m\right)$ and φ_i is the sharing rule function with $\varphi_f + \varphi_m = y$. Notice that in the case of marketable domestic goods household members decide the allocation among them of nonlabour income and optimal profit levels. Thus the sharing rule satisfy $\varphi_f + \varphi_m = y + \pi\left(p_z, w_f, w_m\right)$. Conversely, in a model with minimization of costs the sharing rule represents the intra-household allocation of nonlabour income only.

For the consumption-leisure choice the efficient necessary conditions are

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i},$$

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{p^*_z}{p_i},$$

and substituting back into the budget constraint we get the set of reduced optimal equations

$$\widetilde{c^i} = c^i(p_i, p^*_z, w_i, \varphi_i\,(p_f, p_m, w_f, w_m, y)),$$

$$\widetilde{z^i} = z^i(p_i, p^*_z, w_i, \varphi_i\,(p_f, p_m, w_f, w_m, y)),$$

$$\widetilde{L^i} = L^i(p_i, p^*_z, w_i, \varphi_i\,(p_f, p_m, w_f, w_m, y)),$$

that are equal to the optimal solution obtained solving program (3.30).

3.3.2 Comparative Static Analysis

Similar to Section 3.2.2, we perform gross and net substitution effects for the individual demands of leisure and domestic good derived from program (3.18). In Section 3.2.2 we show that changes of market wages or of the output price p_z affect household consumption directly through conventional substitution effects and indirectly through profit effects. When markets are missing, on the other hand, variations of market wages lead to variation of household consumption directly through substitution effects and indirectly through changes of the implicit price p^*_z.

Gross Substitution Effects Recall *i's* leisure demand produced by program (3.18)

$$\widetilde{L^i} = L^i\left(p_f, p_m, p^*_z, w_f, w_m, y, \widetilde{\mu}\right),$$

where $p_z^* = P_z(w_f, w_m)$, an uncompensated change of w_j gives rise to the following effects

$$\frac{\partial \widetilde{L}^i}{\partial w_j} = \frac{\partial L^i}{\partial w_j} + \frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j} + \frac{\partial L^i}{\partial p_z^*}\frac{\partial P_z}{\partial w_j}. \tag{3.31}$$

A change of w_j affect i's leisure demand through three channels: 1) a direct effect $\frac{\partial L^i}{\partial w_j}$, 2) a change of the Pareto weight $\frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}$, and 3) a change of the implicit price of the domestic good $\frac{\partial L^i}{\partial p_z^*}\frac{\partial P_z}{\partial w_j}$. The first two effects are equal to those derived under the assumption of marketable domestic goods and the interpretation of them is similar to that given to equation (3.16). However, the variation of i's market wage affects also the implicit price of the output. Precisely,

$$\frac{\partial P_z}{\partial w_j} \equiv \frac{\partial^2 C}{\partial z_j \partial w_j} \equiv \frac{\partial t^j}{\partial z_j}$$

assuming that the factor t^j is normal, an increase in output increases the demand for t^j and the overall gross substitution effect depends on which of the three effects overwhelm the others. It is worth remarking that in equation (3.16) the profit effect is negative while in equation (3.31) changes of the implicit price due to changes of j's wage has a positive impact on i's leisure demand. This difference may lead to not comparable gross substitution effects between the household collective models.

Net Substitution Effects The collective generalized Slutsky equation for the individual demand of leisure is

$$\frac{\partial \widetilde{L}^i}{\partial w_j}\mid_{dW^h=0} = \frac{\partial L^i}{\partial w_j} - \left(T_j - L^j - t^j\right)\frac{\partial L^i}{\partial y} + \frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j} - \left(T_j - L^j - t^j\right)\frac{\partial \mu}{\partial y}\right],$$

and for the domestic good is

$$\frac{\partial \widetilde{z}^i}{\partial p_z}\mid_{dW^h=0} = \frac{\partial z^i}{\partial p_z} - \left(z^p - \sum_{i=f}^m z^i\right)\frac{\partial z^i}{\partial y} + \frac{\partial z^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial p_z} - \left(z^p - \sum_{i=f}^m z^i\right)\frac{\partial \mu}{\partial y}\right].$$

In the appendix we give a formal derivation of the Slutsky conditions. The Slutsky conditions derived under the assumptions of complete markets and of missing markets with constant returns to scale are equal. This result is a direct consequence of separability, that is preserved even with missing markets, and duality that establishes equality between the absolute value of the profit effects $\frac{\partial \pi}{\partial w_j}$ and the effect produced by market wage variations on the implicit price $\frac{\partial P_z}{\partial w_j}$. We thus refer readers to the comments reported in Section 3.2.2.

3.4 Nonmarketable Domestic Goods, Nonconstant Returns to Scale and Joint Production

In this section, we study the case of nonmarketable domestic goods with nonconstant returns to scale household technologies. The structure of the household model is as the one described in (3.18) with the following assumption.

Assumption 3.5 (Nonconstant Returns to Scale) *Let us assume that the household production function exhibits nonconstant returns to scale.*

It follows that

Proposition 3.2 *When markets of the domestic goods are missing given Assumption 3.5 the implicit price p_z^* of the domestic good is affected by tastes and by the decision process of the household.*

Proof. The minimum cost function to produce a given quantity of z is equal to

$$C\left(w_f, w_m, z_f, z_m\right) = P_z\left(w_f, w_m, z_f, z_m\right),$$

where the minimum cost function is nonlinear in z_i for $i = f, m$. The implicit price of the domestic good is then equal to

$$P_z^{i*} = \frac{\partial P_z\left(w_f, w_m, z_f, z_m\right)}{\partial z_i},$$

where in general $\frac{\partial P_z}{\partial f \partial z_i}$ depends both on z_f and z_m. As a consequence, P_z^{i*} depends on household preferences and the decision process μ as well as on technology and prices of market goods.[13] ■

Note that differently from Proposition 3.1, the implicit price P_z^{i*} is not the same for the two household members.

When the implicit price of the domestic good depends on household preferences the following problems occur. Firstly, since production decisions depend also on the consumption patterns of the household, recursive models cannot be employed to represent the household behaviour. This is a severe limitation especially for econometric applications. Recursivity is very important for applied works since it makes the problem far more tractable. Secondly, given nonconstant returns to scale the minimum cost function is nonlinear in the output and along with joint production the budget constraint is nonlinear in z as well. The nonlinearity degree depends on the household technology and the functional form chosen for the production function. This means that the production-consumption model may not have an analytical-closed form solution.[14] Lastly, nonseparability of production and consumption decisions together with the assumption that both household members devote time to the domestic production produce that the sharing rule model cannot be employed and the household model cannot be decentralized into two single optimal programs.

Alternative Production-Consumption Household Models

In order to overcome the last two mentioned problems, in the remainder of this section, we propose two alternative specifications of the production function. In both approaches jointness of the production function is excluded but the assumption of nonconstant returns to scale is maintained. In particular, in the first alternative we assume that the household produces z^f and z^m by means of two distinct technologies. In each production function inputs are time allocated by both the household members. Abstracting from jointness allows to find a unique closed-form solution to the household program. However, the production and consumption decisions must be solved jointly and due to the specification of

[13] Generalizing to the case of homothetic production functions the minimum cost function can be written as

$$C\left(w_f, w_m, z_f, z_m\right) = P_z\left(w_f, w_m\right) F\left(z_f, z_m\right)$$

and the implicit price of the domestic good is independent of the output only if the function $F\left(z_f, z_m\right)$ is homogenous of degree one implying a constant returns to scale production function.

[14] Within the context of a consumption technology model, Pollak and Wachter (1975) get similar findings.

the production functions the sharing rule model cannot be employed. Since both household members devote a part of their time to the production of the domestic good privately consumed only by one member, we will refer to this technology specification as a model with "caring" production functions.

In the second alternative we assume that each household member produces by herself the domestic product that she privately consumes. This specification of the production function provides a framework for applying the sharing rule model. However, the production-consumption decisions are taken jointly by each household member. We will refer to this household framework as a model with "separate" production functions.

A. No joint production and "caring" production functions

Without joint production and nonconstant returns to scale, it is possible to derive the household demands for consumption and leisure by specifying particular production technologies. The implicit prices, however, depend on household tastes and as well as on the decision process μ implying that the production-consumption decision must be solved jointly.

Formally, household's production technologies are represented by

$$z^f = h(t^f, t^m) \quad \text{and} \quad z^m = g(\tau^f, \tau^m)$$

where the individual time constraint is now equal to

$$T_i = L^i + l^i + t^i + \tau^i,$$

and according to this identity, individual i allocates her time to the following four activities: leisure, L^i, labour market, l^i, production of z^f, t^i, and of z^m, τ^i.

The household thus solves the following production-consumption program

$$\max_{\substack{c^f, z^f, L^f, c^m, z^m, L^m, \\ t^f, t^m, \tau^f, \tau^m}} W^h = \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m(c^m, z^m, L^m), \tag{3.32}$$

$$\text{subject to} \quad z^f = h(t^f, t^m), \quad z^m = g(\tau^f, \tau^m),$$

$$\text{and} \quad \sum_{i=f}^{m} p_i c^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i t^i - \sum_{i=f}^{m} w_i \tau^i,$$

where the individual time constraint has been opportunely substituted into the household opportunity set. Interpretation of program (3.32) is similar to comments given to the previous two household models.

The Lagrangian function corresponding to program (3.32) is

$$\mathcal{L} = \widetilde{\mu}_f U^f(c^f, h(t^f, t^m), L^f) + \widetilde{\mu}_m U^m(c^m, g(\tau^f, \tau^m), L^m) + $$
$$+ \delta \left[\sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i t^i - \sum_{i=f}^{m} w_i \tau^i - \sum_{i=f}^{m} p_i c^i - \sum_{i=f}^{m} w_i L^i \right],$$

where the production functions have been substituted into the corresponding utility functions.

For an interior solution, the conventional standard efficient conditions for consumption and production variables are

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \qquad i = f, m,$$

$$\frac{h_{t^f}}{h_{t^m}} = \frac{w_f}{w_m},$$

$$\frac{g_{\tau^f}}{g_{\tau^m}} = \frac{w_f}{w_m},$$

that along with the budget constraint yield the household demand functions and inputs which we denote by $c^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, $z^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, $L^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, $t^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$ and $\tau^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$ for $i = f, m$.

The household production-consumption model (3.32) represents a solution to the impossibility of finding a closed-form solution when production functions exhibit simultaneously jointness and nonconstant returns to scale. Given the particular form of the production functions, however, the production-consumption model cannot be decentralized into two individual model and the sharing rule approach cannot be employed.

B. No joint production and "separate" production functions

In the previous section, we suggest that in the presence of nonconstant returns to scale to find a closed-form solution, joint production should be omitted. In this section, we propose household technologies without joint production that are suitable for applying the sharing rule approach.

Formally, household's production technologies are represented by

$$z^f = h_f(t^f) \quad \text{and} \quad z^m = h_m(t^m),$$

where, accordingly, each household member provides by herself the production of the domestic good that she privately consumes.

The household then solves the following production-consumption model

$$\max_{\substack{c^f,z^f,L^f,c^m,z^m,L^m,\\ t^f,t^m}} W^h = \widetilde{\mu}_f U^f(c^f,z^f,L^f) + \widetilde{\mu}_m U^m(c^m,z^m,L^m), \tag{3.33}$$

$$\text{subject to} \quad z^f = h_f(t^f), \quad z^m = h_m(t^m),$$

$$\text{and} \quad \sum_{i=f}^{m} p_i c^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i t^i,$$

where the individual time constraint has been opportunely substituted into the household opportunity set.

The Lagrangian function corresponding to (3.33) is

$$\mathcal{L} = W^h(c^f,z^f,L^f,c^m,z^m,L^m) + \delta\left[\sum_{i=f}^{m} w_i T_i + y - \sum_{i=f}^{m} w_i g_i(z^i) - \sum_{i=f}^{m} p_i c^i - \sum_{i=f}^{m} w_i L^i\right],$$

where $g_i(z^i)$ is the inverse function of the production function, that is $t^i = g_i\left(z^i\right)$. The equilibrium conditions for an interior solution are

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \qquad i = f, m, \tag{3.34}$$

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{w_i g_{iz^i}}{p_i}, \qquad i = f, m, \tag{3.35}$$

together with the budget constraint. Equation (3.34) represents the efficiency condition in the consumption allocation. Equation (3.35) shows that the marginal rate of substitution between the domestic and the market good is equal to the marginal value product divided by the market price. The solution for the household demand functions and inputs are $c^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, $z^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, $L^i(p_f, p_m, w_f, w_m, y, \widetilde{\mu})$, where $t^i\left(p_f, p_m, w_f, \ w_m, y, \widetilde{\mu}\right)$ is obtained by substituting $z^i\left(\cdot\right)$ into $g_i(z^i)$.

The advantage is that, given an appropriate exogenous sharing rule, model (3.33) can be decentralized into the following individual model

$$\max_{c^i, z^i, L^i, t^i} U^i(c^i, z^i, L^i)$$

$$\text{subject to} \quad z^i = h_i(t^i),$$

$$\text{and} \quad p_i c^i + w_i L^i = w_i T_i - w_i t^i + \varphi_i\left(p_f, p_m, w_f, w_m, y\right),$$

for $i = f, m$. The usual efficient conditions for an interior solution are

$$\frac{U^i_{L^i}}{U^i_{c^i}} = \frac{w_i}{p_i}, \qquad i = f, m,$$

$$\frac{U^i_{z^i}}{U^i_{c^i}} = \frac{w_i g_{iz^i}}{p_i}, \qquad i = f, m,$$

with optimal solution given by $c^i(p_i, w_i, \widetilde{\varphi}_i)$, $z^i(p_i, w_i, \widetilde{\varphi}_i)$, $L^i(p_i, w_i, \widetilde{\varphi}_i)$, $t^i(p_i, w_i, \widetilde{\varphi}_i)$ with $\widetilde{\varphi}_i = \varphi_i\left(p_f, p_m, w_f, w_m, y\right)$. As mentioned in Sections 3.2 and 3.3, the solution of program (3.33) must be equal to that produced by the sharing rule model. It is worth highlighting that given Assumption 3.5 separability between production and consumption-leisure decisions fails to apply.

3.4.1 Comparative Static Analysis

In this section we perform gross and net substitution effects for the individual demands of leisure derived by program (3.32) and discuss the differences that nonseparability makes to comparative static analysis. The qualitative analysis is carried out by means of the notion of implicit prices (Strauss 1986).

Gross Substitution Effects i's demand of leisure can be written as

$$\widetilde{L}^i = L^i\left(p_f, p_m, w_f, w_m, y, \widetilde{\mu}, P_z^{i*}\right),$$

where $P_z^{i*} = P_z^i\left(w_f, w_m, z^i\left(p_f, p_m, w_f, w_m, y, \widetilde{\mu},\right)\right)$ is the implicit price of the domestic good privately consumed by member i. Different from model (3.18), given nonconstant returns to scale, P_z^{i*} depends on the optimal output produced and consumed by member i. Therefore, the implicit price of the domestic good P_z^{i*} is, in turn, function of market prices, wages, nonlabour income and the household decision process $\widetilde{\mu}$.

The gross substitution effect of i's leisure demand is equal to

$$\frac{\partial \widetilde{L}^i}{\partial w_j} = \frac{\partial L^i}{\partial w_j} + \frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j} + \frac{\partial L^i}{\partial P_z^{i*}}\left[\frac{\partial P_z^i}{\partial w_j} + \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial w_j} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}\right)\right].$$

The first part of this equation is similar to equations (3.16) and (3.31): in the context of a collective model the wage effect can be decomposed into a Marshallian response, $\frac{\partial L^i}{\partial w_j}$, and a collective effect, $\frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}$, which derived from variations of the Pareto weight. Terms in parenthesis derive from the endogeneity of the implicit price P_z^{i*}. As market wages vary the implicit price presents a direct change $\frac{\partial P_z^i}{\partial w_j}$ and an indirect change produced by a variation of z^i. As shown in Section (3.3.2), $\frac{\partial P_z^i}{\partial w_j} = \frac{\partial t^j}{\partial z^j}$ and assuming that t^j is normal this effect is positive. With nonconstant returns to scale, gross substitution effects become ambiguous and interpretation of these effects is strongly based on a number of assumptions that make it difficult to predict the sign of changes of w_j.

Net Substitution Effects Under the assumption of nonconstant returns to scale and nonseparability of production and consumption-leisure decisions, the collective Slutsky equation is

$$\begin{aligned}\frac{\partial \widetilde{L}^i}{\partial w_j} &= \frac{\partial L^i}{\partial w_j} - l^j \frac{\partial L^i}{\partial y} + \frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j} - l^j \frac{\partial \mu}{\partial y}\right] + \\ &+ \frac{\partial L^i}{\partial P_z^{i*}}\left[\frac{\partial P_z^i}{\partial w_j} + \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial w_j} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}\right) - l^j \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial y} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right)\right],\end{aligned}$$

where $l^j = T_j - L^j - t^j - \tau^j$ is j's labour supply. The compensated effect is composed by the conventional collective Slutsky equation and by changes of the implicit price P_z^{i*}. According to Browning and Chiappori (1998), the collective Slutsky matrix is the sum of a symmetric and negative semidefinite matrix and a rank one matrix. The implicit price P_z^{i*} has three effects on the collective demand for leisure. First, it has a direct wage effect $\frac{\partial P_z^i}{\partial w_j}$ that, as previously discussed, is positive. Second, variations of the implicit price P_z^{i*} due to variations of market wages produce an indirect wage effect $\frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial w_j} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}\right)$. $\frac{\partial z^i}{\partial w_j}$ is negative thus the sign of the wage effect depends crucially on the interpretation that we give to $\frac{\partial \mu}{\partial w_j}$. Third, variations of the implicit price P_z^{i*} due to variations of market wages produce an income effect $l^j \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial y} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right)$. We conclude that with missing markets and nonconstant returns to scale the Slutsky symmetry and sing conditions are not unambiguously interpretable.

3.5 Conclusions

The production-consumption household model presented in the chapter is general under two points of view. The household is seen as a "family-enterprise" producing goods by transforming factor inputs. The factor inputs are time devoted by each family member to the household production. Moreover, consumption decisions of household members are taken in consideration. Thus, the household model describes the family as involved both in production and in consumption decisions. The household model can embrace both urban and rural families in relation to the location of both the household and the entrepreneurial activity. When business activity owned by the family is not undertaken, then the household sells labour either to the job market or to the household. In this case, the general model of a "family-firm" reduces to a "family" engaged in domestic production. The "family-firm" model is a general equilibrium model in miniature where the household enterprise fully reproduces at the micro level the characteristics of a macro society.

The production-consumption household model is also general in the sense that the household is represented as a collection of individuals (Apps and Rees 1988, 1997, and Chiappori 1988, 1992, 1997). Different from the unitary approach, that considers the household as the basic decision unit with a joint preference structure, collective models describe the household as a group of individuals each of whom is characterized by specific preferences. Agents, interacting within a collective decision process, agree on a rule that governs the intra-household allocation of total nonlabour income. The sharing rule function is not directly observable but can be deduced from the available information on private consumption of exclusive or assignable goods (Chiappori 1988, 1992). The collective approach makes no assumption about the rule that governs the allocation of resources within household members. It only requires that household outcomes are Pareto efficient. As a consequence, decision process takes place as if it were a two-stage budgeting process. In the first stage, the household decides how to share combined household resources among each individual. It follows that each member, while choosing the most preferred utility maximizing bundle of goods and leisure, faces an individual budget constraint. Assuming consumption of private goods only, this approach permits to recover up to a constant both private consumption and individual welfare functions (Chiappori 1992).

From an empirical perspective, findings of this chapter highlight that: 1) in the case of complete markets the separation property of production and consumption-leisure holds, hence econometricians can first estimate production equations and then separately consumption-leisure demands. This substantially eases the estimation process; 2) in the case of incomplete markets, the separation property may fail to hold. To preserve the latter, the econometrician has to assume constant returns to scale and then can estimate in two stages the household decisions. When constant returns to scale are not assumed the econometrician has to estimate jointly the household production and consumption decisions.

We study the production-consumption household model under the assumptions of complete and absent markets of the domestic good. This analysis, however, can be extended to absence of other markets, for instance absence of labour markets. We focus on the separation property between production and consumption choices and emphasize the relevance of production and consumption household behaviour both for developing countries, where most households are engaged in agricultural activities, and for developed countries, where families can run business or agricultural activities and/or household members devote a part of their own time to domestic activities. In addition, it is important to account for production-consumption household behaviour when analysing policy intervention in the rural economy. Agricultural policies will affect not only production but also consumption and labour supply of the family members.

Under the assumption of marketable domestic goods, the separation property holds and the production-consumption household model within a collective context can be solved recursively in two stages. In the first stage, the family is engaged in maximizing a profit function independently of its consumption and leisure decisions. Once this choice is made, the family decides the optimal consumption-leisure bundle. The consumption-leisure variables are affected by the production decision through the maximized profits that enter the budget constraint.

We then relax the assumption of complete markets. As a consequence, domestic prices are endogenous to each family and we study which restrictions must be assumed to preserve the separability between production and consumption decisions. Specifically, we show that with missing markets a sufficient condition for the separability to hold is the assumption of constant returns to scale of the household production technology. With constant returns to scale, the minimum cost function to produce a given quantity of z depends linearly on output and consequently the implicit price of the domestic good does not depend on household tastes and the decision process. This assures that the production-consumption household model can be solved recursively. Different from the complete market case, when making production choices the family behaves as it were a cost minimizer.

Lastly, we deal with the case of missing markets of the domestic good under the assumption of nonconstant returns to scale. Within this setting and joint production, the minimum cost function to produce a given quantity of z depends nonlinearly on output and in addition the implicit price of the domestic good is a function of household tastes and the decision process. As a consequence, the production-consumption household model must be solved jointly. Moreover, due to joint technologies a closed-form solution to the production-consumption household model cannot be found. In order to overcome this drawback, we propose two alternative specifications of the production function.

An alternative assumes that the household has two distinct technologies to produce the domestic good consumed privately by the two household members. Without joint technologies the household program has a unique closed-form solution. However, the production and consumption decisions must be solved jointly and due to the specification of the production functions the sharing rule model cannot be employed. A second specification of the production function that we propose is a viable way to apply the sharing rule model. We assume that each household member produces by herself the domestic product that she privately consumes. However, the production-consumption decisions are taken jointly by each household member.

The collective models of production-consumption household behaviour developed under different market structures are enlarged with a qualitative analysis. In particular, we extend the generalized collective Slutsky equation to incorporate the production choices of the family both with marketable and nonmarketable domestic goods. Either with complete markets for the domestic good or missing markets and constant returns to scale production functions, the introduction of household production does not make much difference in terms of comparative static analysis. The collective Slutsky conditions satisfy the SNR1 property (Browning and Chiappori 1998). This property states that the collective Slutsky matrix is made of a symmetric, negative semidefinite matrix and a rank one matrix. The rank one matrix originates from the shift of the Pareto weight due to changes of prices and nonlabour income. On the other hand, with absence of markets for the domestic good and production functions with nonconstant returns to scale the comparative static analysis fails to predict unambiguously the sign and symmetry of the Slutsky matrix.

Appendix: Comparative Static Properties

In this appendix we formally derive the Slutsky equation of the collective demands for leisure and domestic goods of the three household models presented in the chapter. As usual, the following results hold at the equilibrium point only.

The Case of Marketable Domestic Goods

Let us recall the second stage of the household consumption program when markets are assumed to be complete

$$\max_{c^f, z^f, L^f, c^m, z^m, L^m} W^h = \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m(c^m, z^m, L^m), \tag{A.1}$$

$$\text{subject to} \quad \sum_{i=f}^{m} p_i c^i + p_z \sum_{i=f}^{m} z^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y + \pi(p_z, w_f, w_m),$$

with optimal solution equal to

$$\widetilde{c}^i(p_f, p_m, p_z, w_f, w_m, y) = c^i(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}), \tag{A.2}$$

$$\widetilde{L}^i(p_f, p_m, p_z, w_f, w_m, y) = L^i(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}), \tag{A.3}$$

$$\widetilde{z}^i(p_f, p_m, p_z, w_f, w_m, y) = z^i(p_f, p_m, p_z, w_f, w_m, y, \widetilde{\mu}), \tag{A.4}$$

where $\widetilde{\pi} = \pi(p_z, w_f, w_m)$ are the optimal profits and $\widetilde{\mu} = \mu(p_f, p_m, p_z, w_f, w_m, y)$ is the bargaining index.

Let dw_j indicate an infinitesimal change in the j's market wage and let it be compensated by a variation in the nonlabour income dy. Hence, using equation (A.3) the following expression is derived

$$\frac{\partial \widetilde{L}^i}{\partial w_j} |_{dW^h=0, d\widetilde{\mu}=0}\, dw_j = \frac{\partial L^i}{\partial w_j} dw_j + \frac{\partial L^i}{\partial y} dy + \frac{\partial L^i}{\partial \widetilde{\mu}} \left(\frac{\partial \mu}{\partial w_j} dw_j + \frac{\partial \mu}{\partial y} dy \right). \tag{A.5}$$

Totally differentiating the budget constraint yields

$$\sum_{i=f}^{m} T_i dw_i + dy + \frac{\partial \pi}{\partial p_z} dp_z + \frac{\partial \pi}{\partial w_i} dw_i + \frac{\partial \pi}{\partial w_j} dw_j = \sum_{i=f}^{m} p_i dc^i + \sum_{i=f}^{m} c^i dp_i + p_z \sum_{i=f}^{m} dz^i + \sum_{i=f}^{m} z^i dp_z + \sum_{i=f}^{m} w_i dL^i + \sum_{i=f}^{m} L^i dw_i.$$

and solving for dy we obtain

$$dy = -\sum_{i=f}^{m} \left(T_i - L^i\right) dw_i - \frac{\partial \pi}{\partial p_z} dp_z - \frac{\partial \pi}{\partial w_i} dw_i - \frac{\partial \pi}{\partial w_j} dw_j + \sum_{i=f}^{m} p_i dc^i + \sum_{i=f}^{m} c^i dp_i + p_z \sum_{i=f}^{m} dz^i + \sum_{i=f}^{m} z^i dp_z + \sum_{i=f}^{m} w_i dL^i,$$

where by Shephard's lemma we have that $\frac{\partial \pi}{\partial p_z} = z^p$ and $\frac{\partial \pi}{\partial w_i} = -t^i$.

As in Browning and Chiappori (1998), the comparative statics is performed keeping both the *household utility function* and the *Pareto weight* constant; in this way, while the budget constraint is rotating due to simultaneously changes in wage and income, the household, as a whole, remains on its original indifference curve. See Browning, Chiappori, and Weiss (2006) for a geometric representation of the substitution and income effects of the collective model. The total differential of the household welfare is

$$\sum_{i=f}^{m} \widetilde{\mu}^i \left(U^i_{c^i} dc^i + U^i_{z^i} dz^i + U^i_{L^i} dL^i\right) + \sum_{i=f}^{m} U^i d\widetilde{\mu}^i = 0, \tag{A.6}$$

and using the first-order conditions, $\widetilde{\mu}^i U^i_{c^i} = \delta p_i, \widetilde{\mu}^i U^i_{z^i} = \delta p_z, \widetilde{\mu}^i U^i_{L^i} = \delta w_i$ together with the assumption that $d\widetilde{\mu}^i = 0$ for $i = f, m$, equation (A.6) becomes

$$p_f dc^f + p_z dz^f + w_f dL^f + p_m dc^m + p_z dz^m + w_m dL^m = 0. \tag{A.7}$$

Given equation (A.7), the total differential of the budget constraint turns out to be

$$dy = -\sum_{i=f}^{m} \left(T_i - L^i - t^i\right) dw_i - z^p dp_z + \sum_{i=f}^{m} c^i dp_i + \sum_{i=f}^{m} z^i dp_z. \tag{A.8}$$

We are studying the effect of a compensated change of w_j only, thus expression (A.8) is equal to

$$dy = -\left(T_j - L^j - t^j\right) dw_j.$$

Finally, substituting the expression of dy into equation (A.5) the compensated wage effect equals

$$\frac{\partial \widetilde{L}^i}{\partial w_j} = \frac{\partial L^i}{\partial w_j} - \left(T_j - L^j - t^j\right) \frac{\partial L^i}{\partial y} + \frac{\partial L^i}{\partial \widetilde{\mu}} \left[\frac{\partial \mu}{\partial w_j} - \left(T_j - L^j - t^j\right) \frac{\partial \mu}{\partial y}\right].$$

Let us now turn on the own-price effect of the domestic good. From (A.4) we have

$$\frac{\partial \widetilde{z}^i}{\partial p_z} |_{dW^h=0, d\widetilde{\mu}=0}\, dp_z = \frac{\partial z^i}{\partial p_z} dp_z + \frac{\partial z^i}{\partial y} dy + \frac{\partial z^i}{\partial \widetilde{\mu}} \left(\frac{\partial \mu}{\partial p_z} dp_z + \frac{\partial \mu}{\partial y} dy\right), \qquad \text{(A.9)}$$

and (A.8) is

$$dy = -\left(z^p - \textstyle\sum_{i=f}^{m} z^i\right) dp_z,$$

substituting then this equation into (A.9) the own-price effect results in

$$\frac{\partial \widetilde{z}^i}{\partial p_z} = \frac{\partial z^i}{\partial p_z} - \left(z^p - \textstyle\sum_{i=f}^{m} z^i\right) \frac{\partial z^i}{\partial y} + \frac{\partial z^i}{\partial \widetilde{\mu}} \left[\frac{\partial \mu}{\partial p_z} - \left(z^p - \textstyle\sum_{i=f}^{m} z^i\right) \frac{\partial \mu}{\partial y}\right],$$

where $\frac{\partial \pi}{\partial p_z} = z^p$.

The Case of Nonmarketable Domestic Goods: Constant Returns to Scale

Let us recall the second stage of the household consumption program when markets are absent and the production function exhibits constant returns to scale

$$\max_{c^f, z^f, L^f, c^m, z^m, L^m} W^h = \widetilde{\mu}_f U^f(c^f, z^f, L^f) + \widetilde{\mu}_m U^m\left(c^m, z^m, L^m\right),$$

$$\text{subject to} \quad \sum_{i=f}^{m} p_i c^i + \sum_{i=f}^{m} w_i L^i = \sum_{i=f}^{m} w_i T_i + y - C\left(w_f, w_m; z_f, z_m\right),$$

where $\widetilde{C} = C\left(w_f, w_m; z_f, z_m\right)$ is the minimum cost function faced by the family to produce the optimal amount of z_f and z_m totally consumed by the family. The optimal solution bears by the program are

$$\begin{aligned}
\widetilde{c}^i\left(p_f, p_m, w_f, w_m, y\right) &= c^i\left(p_f, p_m, w_f, w_m, y, \widetilde{\mu}\right), \\
\widetilde{L}^i\left(p_f, p_m, w_f, w_m, y\right) &= L^i\left(p_f, p_m, w_f, w_m, y, \widetilde{\mu}\right), \\
\widetilde{z}^i\left(p_f, p_m, w_f, w_m, y\right) &= z^i\left(p_f, p_m, w_f, w_m, y, \widetilde{\mu}\right),
\end{aligned}$$

Now, the comparative statics analysis in this section proceeds in like manner the case of marketable domestic goods. Therefore, the compensated wage effect on the demand of leisure is

$$\frac{\partial \widetilde{L}^i}{\partial w_j}\,|_{dW^h=0,d\widetilde{\mu}=0}\, dw_j = \frac{\partial L^i}{\partial w_j}dw_j + \frac{\partial L^i}{\partial y}dy + \frac{\partial L^i}{\partial \widetilde{\mu}}\left(\frac{\partial \mu}{\partial w_j}dw_j + \frac{\partial \mu}{\partial y}dy\right),$$

To find the expression for dy, then we totally differentiate the budget constraint

$$\sum_{i=f}^{m} T_i dw_i + dy - \sum_{i=f}^{m}\frac{\partial C}{\partial w_i}dw_i - \sum_{i=f}^{m}\frac{\partial C}{\partial z^i}dz^i = \sum_{i=f}^{m} p_i dc^i + \sum_{i=f}^{m} c^i dp_i + \sum_{i=f}^{m} w_i dL^i + \sum_{i=f}^{m} L^i dw_i.$$

Since we are holding both the *household utility function* and the *bargaining index* constant the total differential of the household welfare function,

$$\sum_{i=f}^{m}\widetilde{\mu}^i\left(U^i_{c^i}dc^i + U^i_{z^i}dz^i + U^i_{L^i}dL^i\right) + \sum_{i=f}^{m} U^i d\widetilde{\mu}^i = 0,$$

becomes

$$p_f dc^f + C_{z^i}dz^f + w_f dL^f + p_m dc^m + C_{z^i}dz^m + w_m dL^m = 0,$$

where we have opportunely substituted the first-order conditions, $\widetilde{\mu}_i U^i_{c^i} = \delta p_i$, $\widetilde{\mu}_i U^i_{z^i} = \delta C_{z^i}$, $\widetilde{\mu}_i U^i_{L^i} = \delta w_i$, and that $d\widetilde{\mu}^i = 0$ for $i = f, m$. Substituting the last expression in the total differential of the budget constraint we have

$$dy = -\left(T_j - L^j - t^j\right)dw_j,$$

where the identity $\frac{\partial C}{\partial w_i} = t^i$ has been opportunely substituted. Eventually, the compensated wage effect is

$$\frac{\partial \widetilde{L}^i}{\partial w_j}\mid_{dW^h=0,d\widetilde{\mu}=0}=\frac{\partial L^i}{\partial w_j}-\left(T_j-L^j-t^j\right)\frac{\partial L^i}{\partial y}+\frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j}-\left(T_j-L^j-t^j\right)\frac{\partial \mu}{\partial y}\right].$$

Let us turn on the own-price effect on the domestic good and use the same arguments as above we have

$$\frac{\partial \widetilde{z}^i}{\partial p_z}\mid_{dW^h=0,d\widetilde{\mu}=0} dp_z=\frac{\partial z^i}{\partial p_z}dp_z+\frac{\partial z^i}{\partial y}dy+\frac{\partial z^i}{\partial \widetilde{\mu}}\left(\frac{\partial \mu}{\partial p_z}dp_z+\frac{\partial \mu}{\partial y}dy\right), \tag{A.12}$$

the total differential of the budget constraint becomes

$$dy=-\left(z^p-\sum_{i=f}^m z^i\right)dp_z,$$

then replacing the value of dy in equation (A.12) we have

$$\frac{\partial \widetilde{z}^i}{\partial p_z}\mid_{dW^h=0,d\widetilde{\mu}=0}=\frac{\partial z^i}{\partial p_z}-\left(z^p-\sum_{i=f}^m z^i\right)\frac{\partial z^i}{\partial y}+\frac{\partial z^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial p_z}-\left(z^p-\sum_{i=f}^m z^i\right)\frac{\partial \mu}{\partial y}\right].$$

The Case of Nonmarketable Domestic Goods: Nonconstant Returns to Scale

Let us recall i's collective demand for leisure of program (3.32)

$$\widetilde{L}^i\left(p_f,p_m,w_f,w_m,y\right)=L^i\left(p_f,p_m,w_f,w_m,y,\widetilde{\mu},P_z^{i*}\right)$$

where $P_z^{i*}=P_z^i\left(w_f,w_m,z^f\left(p_f,p_m,w_f,w_m,y,\widetilde{\mu},\right)\right)$ is the implicit price of the domestic good privately consumed by i.

On $\widetilde{L}^i\left(p_f,p_m,p_z,w_f,w_m,y\right)$ a change of j's market wage yields the following effects

$$\frac{\partial \widetilde{L}^i}{\partial w_j}=\frac{\partial L^i}{\partial w_j}+\frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}+\frac{\partial L^i}{\partial P_z^{i*}}\left[\frac{\partial P_z^i}{\partial w_j}+\frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial w_j}+\frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}\right)\right], \tag{A.13}$$

and a change of the nonlabour income y has the following effects

$$\frac{\partial \widetilde{L}^i}{\partial y}=\frac{\partial L^i}{\partial y}+\frac{\partial L^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}+\frac{\partial L^i}{\partial P_z^{i*}}\left[\frac{\partial P_z^i}{\partial y}+\frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial y}+\frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right)\right]. \tag{A.14}$$

Substituting equations (A.13) and (A.14) into the conventional Slutsky equation, $\frac{\partial \widetilde{L}^i}{\partial w_j} - l^j \frac{\partial \widetilde{L}^i}{\partial y}$, we obtain the collective Slutsky equation

$$\begin{aligned}\frac{\partial \widetilde{L}^i}{\partial w_j} &= \frac{\partial L^i}{\partial w_j} - l^j \frac{\partial L^i}{\partial y} + \frac{\partial L^i}{\partial \widetilde{\mu}}\left[\frac{\partial \mu}{\partial w_j} - l^j \frac{\partial \mu}{\partial y}\right] + \\ &\quad + \frac{\partial L^i}{\partial P_z^{i*}}\left[\frac{\partial P_z^i}{\partial w_j} + \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial w_j} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial w_j}\right) - l^j \frac{\partial P_z^i}{\partial \widetilde{z}^i}\left(\frac{\partial z^i}{\partial y} + \frac{\partial z^i}{\partial \widetilde{\mu}}\frac{\partial \mu}{\partial y}\right)\right]\end{aligned}$$

where $l^j = T_j - L^j - t^j - \tau^j$ is j's labour supply.

Chapter 4

A Dynamic Model of Consumption and Discrete Fertility Choices

4.1 Introduction

This chapter presents a finite-horizon dynamic model of fertility and consumption within a certain environment. Fertility is modelled as a discrete choice and it is the outcome of comparisons between parents' welfare level with and without an extra child.

Especially in advanced societies, where contraceptive methods are easily available at a relatively low price and nowadays are generally accepted as common practices of birth control (Goldin and Katz 2002), spouses can schedule the number, timing and spacing of births over their reproductive life and the fertility behaviour can be modelled as an endogenous variable. However, the fertility choice is also affected by socioeconomic circumstances specific to each household, such as parents' income, level of education, altruism and the costs of children, and by environmental circumstances faced by families living in the same country in a given period of time, such as child care services available at a relative low price, availability of part-time jobs, flexibility of the labour market, tax system and in general family policies that provide financial support to households with dependent children.

Since Becker's pioneering application of economic analysis to various aspects of human behaviour, a number of works have studied the interrelation between socioeconomic variables and the fertility decision. Becker himself (1960) classifies children with durable goods, such as cars or houses, and shows that the theory of the demand for durable goods is a useful set-up in analysing the demand for children and in understanding the effect of family income on fertility.

In the context of the theory of consumer behaviour, Willis (1973) presents a static model of lifetime fertility. Couples obtain their utility by the maximization of a household welfare function whose arguments are home-produced commodities, quantity and quality of children. The level of utility that the household can achieve depends on its budget constraint and technologies to produce the consumed commodities and quantity and quality of children.

Michael (1973) analyses the mechanisms through which couples' education affects their fertility behaviour. He assumes that households derive their utility from consumption of home-produced commodities, and the number and quality of children. The household produces both the commodities and children according to a production function in which inputs are purchased market goods and parents' time. In this framework, parents' level of education might influence their fertility choice by affecting tastes, wealth, the production function and/or the value of time of each parent. Investments in education raise the potential earnings of the spouses and in turn might affect the relative prices of bearing and rearing children, influencing the demand for children. It is especially the increase in the market wage of the wife that negatively influences the demand for children. Moreover, Michael (1973) recognizes that the parents' education can determine the quality and quantity of children. He argues that more-educated couples can be particularly productive in enhancing human capital in their children and education can lower the relative cost of fertility control and more-educated couples would choose contraceptive methods which on the average are more effective in preventing pregnancy.

Becker and Barro (1988) study the implications of parents' altruism toward children on their fertility choice. Altruism is modelled using a dynastic utility function that depends on the consumption, fertility and the number of descendants in all generations. Within this context, fertility is an endogenous variable to each household and the household budget constraint includes the interest rate, among other variables such as wage rate, costs of raising children and bequest. The condition of utility maximization related to the parents' fertility choice requires the marginal benefit of an additional child to be equal to the marginal cost of producing that child. As a consequence, the first order conditions to maximize utility imply that fertility in any generation depends positively on the real interest rate. They also find that fertility responds to variations in the degree of altruism: if the cost of rearing children is constant over time, fertility depends positively on interest rates and on the degree of altruism.

A number of works study how policy intervention affects the fertility decision. In particular, it is studied the effects that tax systems and child subsidies produce on the demand for children. It is widely recognized that welfare policies that provide financial support to families with children positively affect the fertility choice of the families. Studying the inverse relationship between female labour participation and fertility rates, Apps and Rees (2004) show that countries which have individual rather than joint taxation, and which support families through child care facilities rather than child payments, are likely to have both higher female labour supply and higher fertility. Comparing the evolution of completed fertility patterns between Swedish women and women in neighbouring countries, Björklund (2006) finds that the extension of Sweden's family policy aiming at reducing the costs of children has raised the level of fertility and has shortened the spacing of births. However, Björklund (2006) finds that a generous family policy is not able to change the negative relationship between wive's education level and fertility.

In developing countries, that are characterized by missing markets and absence of pension schemes, the reproductive behaviour of families is also affected by insurance motives. Parents consider their offspring as a form of investment against risks of income insufficiency in old age or in other economic adverse circumstances, such as unemployment or illness (Cain 1983).

The purpose of the chapter is to explore fertility outcomes as a discrete choice in a life-cycle context. The dynamic analysis of fertility choices emphasizes the effects exerted on the family size decision by the costs of rearing children. The dynamic model is developed under a set of simplifying assumptions designed to obtain an analytical solution to the dynamic program. In particular, we assume that the instantaneous utility function of parents is of the CARA form and, neglecting that spouses can have different fertility preferences, we abstract from bargaining processes that can occur within the couple. Furthermore, we assume that parents face perfectly foreseen costs of rearing children and these costs are equal across children. Within this framework, we show that in each period of the reproductive life the couple does not have a child if the pure utility gain from not having a child is greater than the utility saving in cost from having the child. However, the model has neither implications for quality of children nor for wife's participation to the labour force.

We then carry out a simulation exercise of the dynamic model. The fertility outcome is analysed for traditional and non-traditional couples. The former is a family in which there is only one main wage earner. On the other hand, the non-traditional family is characterized by the presence of two wage earners. In this way, we are able to control for different fertility outcomes due to different costs of rearing children faced by the two families. As expected, the traditional couple decides to have more children relative to the non-traditional couple. In particular, the traditional family has two children with a two year gap between the first and second birth. On the other hand, the non-traditional couple has just one child. The consumption pattern and assets accumulation of the two families are influenced by different fertility outcomes. Given a bigger family size, throughout the reproductive life consumption pattern of the traditional family is in general higher relative to consumption of the non-traditional family. To provide sustainable consumption through the reproductive life, both the families get heavily into debt.

The dynamic household model of fertility choice developed in this chapter is similar to that of Wolpin (1984). Studying the connection between child mortality and fertility within a dynamic stochastic model, Wolpin (1984) treats the fertility choice as a discrete endogenous variable. However, Wolpin (1984) aims at estimating the dynamic fertility model and he does not give an analytical representation of the optimal fertility choice rule. The estimation proposed by Wolpin (1984) is based on integrating the numerical solution of the model with a maximum likelihood procedure.

The contribution of the chapter is twofold. First, using a dynamic model we explore fertility outcomes as a discrete choice in a life-cycle context. Hence, fertility is modelled as a discrete choice and it is the outcome of comparisons between parents' welfare level with and without an extra child. To our knowledge, from a theoretical perspective this approach has been applied to various family decisions but not to fertility choices.

Second, the chapter develops a simulation exercise of the dynamic fertility model. In particular, the fertility outcome is analysed for traditional and non-traditional couples. The former is a family in which there is only one main wage earner. On the other hand, the non-traditional family is characterized by the presence of two wage earners. In doing so, we control for different fertility outcomes due to different costs of rearing children faced by the two families. Results of the simulation are as expected: the traditional couple decides to have more children relative to the non-traditional couple. In particular, the traditional family has two children with a two year gap between the first and second birth. On the other hand, the non-traditional couple has just one child. Moreover, different family sizes impact on the consumption pattern and assets accumulation differently. Given a bigger family size, throughout the reproductive life consumption pattern of the traditional family is in general higher relative to consumption of the non-traditional family and to provide sustainable consumption through the reproductive life the traditional family gets heavily into debt.

The remainder of the chapter is organized as follows. In Section 4.2 we lay out the dynamic model of fertility choice and consumption. In Section 4.3 we present the simulation analysis of the dynamic model differentiating fertility outcomes by family types. Section 4.4 draws conclusions.

4.2 A Life-Cycle Model of Fertility and Consumption

We consider the parents' decision of consumption and fertility over their life-cycle. They marry at time f, have a reproductive life that lasts up to F and die at T with $T > F$.[1]

[1] Without loss of generality we assume that births occur within married couples.

The fertility choice is made sequentially. At the beginning of period $t \in [f, F]$ the couple decides whether to have a child, and in $t+1$ decides whether to have an additional child, and so on up to F. We abstract from the likelihood of either multiple births or child mortality and the variation in the number of children $\Delta_t n$ can assume only two discrete values: $\Delta_t n = 0$ if the couple does not have an additional child, $\Delta_t n = 1$ if the couple has the additional child. A period t is defined to be that length of time within which a single birth may occur. Contraception is assumed to be perfect during reproductive life and once spouses decide to have a child the birth occurs with certainty.

We assume that the couple has intertemporally separable preferences defined over a finite horizon and preferences depend on a single composite consumption good c_t and the number of children n_t

$$W = \sum_{t=f}^{T} \rho^t u(c_t, n_t),$$

where $\rho > 0$ is the rate of time preference. The parents' instantaneous felicity function is represented by a CARA utility function[2]

$$u(c_t, n_t) = 1 - \eta_t \exp(-\beta c_t), \tag{4.1}$$

with $\eta_t = f(n_t)$ equals 1 whenever $n_t = 0$ and $\eta_t' < 0$, and $\beta > 0$ is the coefficient of absolute risk aversion. We assume that the number of children n_t interacts multiplicatively with consumption and hence the consumption and the fertility choices are not separable implying that the marginal utility of c_t is affected by the number of children of the family.

The stock of household assets evolves from one period to the next according to

$$A_{t+1} + c_t + Pn_t = (1+r) A_t + y_t, \tag{4.2}$$

where we assume that there are no liquidity constraints, and the couple should die with

[2] We think of the family as if it were a unique decision maker and therefore we model family behaviour using a unitary framework. However, we recognize the relevance of using alternative approaches to model family fertility behaviour. For instance, Rasul (2007) develops and tests a model of household bargaining over fertility where parents have different preferences toward children. He finds that if couples bargain with commitment, fertility outcome representes both spouses' fertility choice. On the other hand, if couples bargain without commitment, the effect of each spouse's preference on fertility outcomes depends on the threat point in marital bargaining and the distribution of bargaining power.

no wealth and no debt, therefore $A_{T+1} = 0$. Household nonlabour income A_t is measured at the beginning of time t. For the sake of simplicity, the interest rate r is assumed to be constant through time. In each period of time parents face perfectly foreseen costs of rearing a child[3] P and they are assumed to be equal for all the children.[4]

The law of motion for the stock of children n_t is

$$n_t = n_{t-1} + \triangle_t n, \quad t = f+1, ..., F, \tag{4.3}$$

where $\triangle_t n$ can take two values 0 or 1,

$$n_t = n_{F,} \quad for\ t = F+1, ..., T,$$

and with initial condition

$$n_f = 0, \tag{4.4}$$

that is spouses start marriage with no children.

As described in the remainder of this section, the household maximization problem given by (4.1)-(4.4) is solved using the principle of optimality (Bellman 1957). The solution is obtained by backward recursion. At the start of each period of time t the discrete fertility choice is the result of comparisons between parents' utility levels with and without a birth.

4.2.1 The Value Function

Let introduce the definition of the variables of choice and parameters that enter the utility function and the opportunity set:

$P =$ denotes the per period costs of rearing a child faced by the couple; these costs are perfectly foreseen by spouses;

$r =$ denotes the interest rate and is assumed to be constant through time;

$\rho =$ denotes the rate of time preference or discount factor with $\rho > 0$;

$\beta =$ denotes the coefficient of absolute risk aversion with $\beta > 0$;

[3] Note that real costs of rearing children fall due to the interest rate r.

[4] In Section 4.3 we give a definition of the costs of rearing children and, in addition, we give information on how in principle these costs can be estimated.

$c_t =$ denotes household consumption measured at the end of each period of time t;

$n_t =$ denotes the number of children measured at the end of each period of time t;

$y_t =$ denotes household labour income measured at the end of each period of time t;

$A_t =$ denotes household nonlabour income measured at the beginning of each period of time t.

We conjecture that at the beginning of period t the value function $V(A_t)$ is of the same functional form as the utility function (Merton 1971, Berloffa and Simmons 2003)

$$V(A_t) = \lambda_t - N_t \exp(-\beta \delta_t A_t). \tag{4.5}$$

where λ_t, N_t and δ_t are functions to be determined.

Optimal Consumption and the Value Function in the Last Period of the Couple's Lifetime

In the last period of life T, the reproductive life has ended and the couple does not decide whether to have a child. All the financial assets are consumed by the family thus $A_{T+1} = 0$ and the optimal consumption rule is simply equal to

$$c_T = (1+r) A_T + y_T - P n_F,$$

where the stock of children n_F is constant and equal to the number of children born between f and F. The instantaneous utility function is obtained by replacing the optimal consumption c_T in the instantaneous utility function

$$1 - \eta_F \exp[-\beta((1+r) A_T + y_T - P n_F)],$$

and the value function, measured at the start of the period, is

$$V(A_T) \equiv 1 - \eta_F \exp(-\beta c_T) + \rho V(A_{T+1}), \tag{4.6}$$

with $\eta_F = f(n_F)$. Using the fact that $A_{T+1} = 0$ and substituting the optimal consumption into (4.6) we can establish that

$$\lambda_T - N_T \exp\left(-\beta\delta_T A_T\right) \equiv 1 - \eta_F \exp\left[-\beta\left(\left(1+r\right)A_T + y_T - Pn_F\right)\right]$$

from which we get the solution for the unknowns λ_t, δ_t and N_t

$$\begin{aligned}
\lambda_T &= 1, \\
\delta_T &= (1+r), \\
N_T &= \eta_F \exp\left(-\beta\left(y_T - Pn_F\right)\right).
\end{aligned}$$

Optimal Consumption and the Value Function in Periods $t \in (F,T)$

At the beginning of period $t \in (F,T)$, when the reproductive life of the couple has ended, the couple solves the following optimal program

$$\max_{c_t, A_{t+1}} \; V\left(A_t\right) = 1 - \eta_F \exp\left(-\beta c_t\right) + \rho V\left(A_{t+1}\right) \tag{4.7}$$

$$\text{subject to} \quad A_{t+1} + c_t + Pn_F = \left(1+r\right)A_t + y_t.$$

To solve the optimal problem we substitute $V\left(A_{t+1}\right)$ along with the budget constraint into (4.7) thus

$$\max_{c_t} \; V\left(A_t\right) = 1 - \eta_F \exp\left(-\beta c_t\right) + \rho\left\{\lambda_{t+1} - N_{t+1}\exp\left[-\beta\delta_{t+1}\left(\left(1+r\right)A_t + y_t - c_t - Pn_F\right)\right]\right\},$$

taking then the first-order condition with respect to c_t and opportunely re-arranging we find the optimal policy for the household consumption for $t \in (F,T)$

$$\exp\left(-\beta c_t\right) = \left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_F}\right)^{\frac{1}{1+\delta_{t+1}}} \exp\left[-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t + y_t - Pn_F\right)\right], \tag{4.8}$$

and the instantaneous utility function $u\left(c_t, n_t\right)$ is

$$1 - \eta_F\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_F}\right)^{\frac{1}{1+\delta_{t+1}}} \exp\left[-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t + y_t - Pn_F\right)\right].$$

Now we check the conjecture that $V(A_t) \equiv \arg\max_{c_t} 1 - \eta_F \exp(-\beta c_t) + \rho V(A_{t+1})$

$$\begin{aligned} \lambda_t - N_t \exp(-\beta\delta_t A_t) &\equiv 1 - \eta_F \exp(-\beta c_t) + \rho\{\lambda_{t+1} - N_{t+1} \\ &\exp[-\beta\delta_{t+1}((1+r)A_t + y_t - c_t - Pn_F)]\}. \end{aligned} \tag{4.9}$$

From equation (4.8) we can derive the expression for $\exp(\beta\delta_{t+1}c_t)$

$$\exp(\beta\delta_{t+1}c_t) = \left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_F}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}} \exp\left[\beta\frac{\delta_{t+1}^2}{1+\delta_{t+1}}((1+r)A_t + y_t - Pn_F)\right]. \tag{4.10}$$

Substituting (4.8) and (4.10) into (4.9) yields

$$\begin{aligned} \lambda_t - N_t \exp(-\beta\delta_t A_t) &= 1 - \eta_F \left(\frac{\rho\delta_{t+1}N_{t+1}}{\eta_F}\right)^{\frac{1}{1+\delta_{t+1}}} \exp\left[-\frac{\beta\delta_{t+1}}{1+\delta_{t+1}}((1+r)A_t + y_t\right. \\ &\left.-Pn_F)\right] + \rho\lambda_{t+1} - \rho N_{t+1}\exp[-\beta\delta_{t+1}((1+r)A_t + y_t - Pn_F)] \\ &\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_F}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}} \exp\left[\beta\frac{\delta_{t+1}^2}{1+\delta_{t+1}}((1+r)A_t + y_t - Pn_F)\right], \end{aligned}$$

and re-arranging terms we have established our conjecture (4.5)

$$\begin{aligned} \lambda_t - N_t \exp(-\beta\delta_t A_t) &\equiv 1 + \rho\lambda_{t+1} - \frac{\eta_F(1+\delta_{t+1})}{\delta_{t+1}} \left(\frac{\rho\delta_{t+1}N_{t+1}}{\eta_F}\right)^{\frac{1}{1+\delta_{t+1}}} \\ &\exp\left[-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}(y_t - Pn_F)\right]\exp\left[-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}(1+r)A_t\right], \end{aligned} \tag{4.11}$$

with the following recurrence relations

$$\lambda_t = 1 + \rho\lambda_{t+1},$$

$$\delta_t = \frac{\delta_{t+1}}{1+\delta_{t+1}}(1+r),$$

and

$$N_t = \frac{\eta_F(1+\delta_{t+1})}{\delta_{t+1}} \left(\frac{\rho\delta_{t+1}N_{t+1}}{\eta_F}\right)^{\frac{1}{1+\delta_{t+1}}} \exp\left[-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}(y_t - Pn_F)\right].$$

and so forth up to the period $T - F$. In order to explain how the backward recursion method works, in the Appendix we solve this system of equations for the periods $T-1$ and $T-2$.

4.2.2 The Fertility Choice Rule

At the beginning of period $T-F$, that is the last period of time in which the couple can decide whether to have a child or not, the couple solves the following program

$$\begin{aligned}\max_{c_{T-F},n_{T-F},A_{T-F+1}} V\left(A_{T-F}\right) &= 1-\eta_{T-F}\exp\left(-\beta c_{T-F}\right)+ \\ &\quad \rho V\left((1+r)A_{T-F}+y_{T-F}-c_{T-F}-Pn_{T-F}\right),\end{aligned}$$

subject to the household asset constraint. Substituting the expression of $V\left(A_{T-F+1}\right)$, we obtain

$$\begin{aligned}\max_{c_{T-F},n_{T-F}} V\left(A_{T-F}\right) &= 1-\eta_{T-F}\exp\left(-\beta c_{T-F}\right)+\rho\left\{\lambda_{T-F+1}- N_{T-F+1}\right. \\ &\quad \left.\exp\left[-\beta\delta_{T-F+1}\left((1+r)A_{T-F}+y_{T-F}-c_{T-F}-Pn_{T-F}\right)\right]\right\}.\end{aligned}$$

Taking the first-order condition with respect to c_{T-F}, we find the optimal path for the consumption

$$\begin{aligned}\exp\left(-\beta c_{T-F}\right) &= \left(\rho\delta_{T-F+1}\frac{N_{T-F+1}}{\eta_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}} \\ &\quad \exp\left[-\beta\frac{\delta_{T-F+1}}{1+\delta_{T-F+1}}\left(\left(1+r\right)A_{T-F}+y_{T-F}-Pn_{T-F}\right)\right],\end{aligned} \tag{4.12}$$

and the value of the optimal utility function is

$$\begin{aligned}&1-\eta_{T-F}\left(\rho\delta_{T-F+1}\frac{N_{T-F+1}}{\eta_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}} \\ &\exp\left[-\beta\frac{\delta_{T-F+1}}{1+\delta_{T-F+1}}\left(\left(1+r\right)A_{T-F}+y_{T-F}-Pn_{T-F}\right)\right],\end{aligned}$$

from which we can expand derivation of the fertility rule as

$$\min\left[\eta^0_{T-F}\left(\frac{N^0_{T-F+1}}{\eta^0_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^0_{T-F}\right)}{1+\delta_{T-F+1}}\right),\right. \quad (4.13)$$

$$\left.\eta^1_{T-F}\left(\frac{N^1_{T-F+1}}{\eta^1_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^1_{T-F}\right)}{1+\delta_{T-F+1}}\right)\right],$$

where the superscript 0 stands for the couple's decision "not to have a child" and 1 "to have a child." Putting equations (4.12) and (4.13) back into the value function we have that

$$\begin{aligned}
&\lambda_{T-F}-N_{T-F}\exp\left(-\beta\delta_{T-F}A_{T-F}\right)\equiv\\
&\equiv 1+\rho\lambda_{T-F+1}-\exp\left(-\beta\frac{\delta_{T-F+1}\left(1+r\right)}{1+\delta_{T-F+1}}A_{T-F}\right)\frac{1+\delta_{T-F+1}}{\delta_{T-F+1}}\left(\rho\delta_{T-F+1}\right)^{\frac{1}{1+\delta_{T-F+1}}}\\
&\qquad\min\left[\eta^0_{T-F}\left(\frac{N^0_{T-F+1}}{\eta^0_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^0_{T-F}\right)}{1+\delta_{T-F+1}}\right),\right.\\
&\qquad\qquad\left.\eta^1_{T-F}\left(\frac{N^1_{T-F+1}}{\eta^1_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^1_{T-F}\right)}{1+\delta_{T-F+1}}\right)\right],
\end{aligned}$$

and equating coefficients we solve the unknowns $\lambda_{T-F}, \delta_{T-F}$ and N_{T-F}

$$\begin{aligned}
\lambda_{T-F} &= 1+\rho\lambda_{T-F+1},\\
\delta_{T-F} &= \beta\frac{\delta_{T-F+1}\left(1+r\right)}{1+\delta_{T-F+1}},\\
N_{T-F} &= \frac{1+\delta_{T-F+1}}{\delta_{T-F+1}}\left(\rho\delta_{T-F+1}\right)^{\frac{1}{1+\delta_{T-F+1}}}\\
&\quad\min\left[\eta^0_{T-F}\left(\frac{N^0_{T-F+1}}{\eta^0_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^0_{T-F}\right)}{1+\delta_{T-F+1}}\right),\right.\\
&\qquad\left.\eta^1_{T-F}\left(\frac{N^1_{T-F+1}}{\eta^1_{T-F}}\right)^{\frac{1}{1+\delta_{T-F+1}}}\exp\left(-\beta\frac{\delta_{T-F+1}\left(y_{T-F}-Pn^1_{T-F}\right)}{1+\delta_{T-F+1}}\right)\right].
\end{aligned}$$

In order to explicitly write the optimal fertility choice, let us write

$$\eta_{T-F}^{0} = f\left(n_{F-1}\right), \quad \eta_{T-F}^{1} = f\left(n_{F-1}+1\right), \tag{4.14}$$

and

$$\begin{aligned}
N_{T-F+1}^{0} &= f\left(n_{F-1}\right)^{D_{T-F}(\delta)} \exp\left(-\beta G_F\left(\delta\right) P n_{F-1}\right) K_F(y_{T-F+1},...,y_T), \\
N_{T-F+1}^{1} &= f\left(n_{F-1}+1\right)^{D_{T-F}(\delta)} \exp\left(-\beta G_F\left(\delta\right) P(n_{F-1}+1)\right) K_F(y_{T-F+1},...,y_T),
\end{aligned}$$

where $K_F\left(\cdot\right)$ captures the evolution of the labour income y_t from $T-F$ to T, $G_F\left(\delta\right)$ and $D_{T-F}\left(\delta\right)$.summarise the evolution of δ through time Substituting the expression N_{T-F+1}^{0} into N_{T-F+1}^{1} we get

$$N_{T-F+1}^{1} = N_{T-F+1}^{0}\left[\frac{f\left(n_{F-1}+1\right)}{f\left(n_{F-1}\right)}\right]^{D_{T-F}(\delta)} \exp\left(-\beta G_F\left(\delta\right) P\right). \tag{4.15}$$

Re-arranging equation (4.13) the optimal fertility choice corresponds to

$$\begin{aligned}
&\min\Bigg[\left(\eta_{T-F}^{0}\right)^{\Delta}\left(N_{T-F+1}^{0}\right)^{\frac{1}{1+\delta_{T-F+1}}} \exp\left(\beta\Delta P n_{T-F}^{0}\right)\exp\left(-\beta\Delta y_{T-F}\right), \\
&\left(\eta_{T-F}^{1}\right)^{\Delta}\left(N_{T-F+1}^{1}\right)^{\frac{1}{1+\delta_{T-F+1}}} \exp\left(\beta\Delta P n_{T-F}^{1}\right)\exp\left(-\beta\Delta y_{T-F}\right)\Bigg],
\end{aligned}$$

where $\Delta = \frac{\delta_{T-F+1}}{1+\delta_{T-F+1}}$, replacing equations (4.14) and (4.15) and $n_{T-F}^{0} = n_{F-1}$ and $n_{T-F}^{1} = n_{F-1}+1$

$$\begin{aligned}
&\min\Bigg[\left(f\left(n_{F-1}\right)\right)^{\Delta}\left(N_{T-F+1}^{0}\right)^{\frac{1}{1+\delta_{T-F+1}}} \exp\left(\beta\Delta P n_{F-1}\right)\exp\left(-\beta\Delta y_{T-F}\right), \\
&\left(\left(f\left(n_{F-1}+1\right)\right)^{\Delta}\left(N_{T-F+1}^{0}\left(\frac{f\left(n_{T-F-1}+1\right)}{f\left(n_{T-F-1}\right)}\right)^{D_{T-F}(\delta)} \exp\left(-\beta G_F\left(\delta\right) P\right)\right)^{\frac{1}{1+\delta_{T-F+1}}} \\
&\exp\left(\beta\Delta P(n_{F-1}+1)\right)\exp\left(-\beta\Delta y_{T-F}\right)\Bigg],
\end{aligned}$$

cancelling out common terms, we obtain

$$\begin{aligned}(f(n_{F-1}))^{\Delta} &> (f(n_{F-1}+1))^{\Delta} \\ &\left[(f(n_{F-1}+1)/f(n_{F-1}))^{D_{T-F}(\delta)} \exp(-\beta G_F(\delta) P)\right]^{\frac{1}{1+\delta_{T-F+1}}}.\end{aligned}$$

Re-arranging terms

$$(f(n_{F-1}))^{\Delta+D_{T-F}(\delta)} > (f(n_{F-1}+1))^{\Delta+D_{T-F}(\delta)} \exp(-\beta G_F(\delta) P)^{\frac{1}{1+\delta_{T-F+1}}},$$

and solving for the child function $f(n_{F-1})$

$$f(n_{F-1}) > f(n_{F-1}+1)\left[\exp(-\beta G_F(\delta) P)\right]^{\frac{1}{\delta_{T-F+1}+(1+\delta_{T-F+1})D_{T-F}(\delta)}}. \tag{4.16}$$

In each period of time $t < F$ equation (4.16) can be generalized as

$$f(n_{t-1}) > f(n_{t-1}+1)\left[\exp(-\beta G_t(\delta) P)\right]^{\frac{1}{\delta_{t+1}+(1+\delta_{t+1})D_{T-t}(\delta)}}. \tag{4.17}$$

Equation (4.17) describes the optimal fertility rule for each $t \in [f, F]$: the couple does not have another child in t if the pure utility gain from not having a child $f(n_{t-1})$ is greater than the utility saving in cost from having the child, $f(\cdot)\left[\exp(-\beta G_t(\delta) P)\right]^{\frac{1}{\delta_{t+1}+(1+\delta_{t+1})D_t(\delta)}}$; otherwise the couple finds it optimal to have an additional child.

In each period of the reproductive life the choice "whether to have an additional child or not" is based upon the costs of rearing children P faced by couples. Fertility choices are also influenced by the coefficient of absolute risk aversion β. In particular, we observe that as the costs of rearing children P increase it is likely that the birth does not occur. Similarly, if β increases.

The interest rate r indirectly affects the fertility choice through δ. According to the recurrence equations, δ is increasing in the interest rate. Therefore, as the interest rate raises, *ceteris paribus*, the last term on the right-hand side of equation (4.17) increases raising in this way the costs of having an additional child and hence reducing the probability of having a new birth.

4.2.3 The Optimal Consumption Rule

Taking the logarithm of equation (4.8) and solving for c_t we obtain the optimal consumption policy in each t

$$c_t = -\frac{\ln(\rho\delta_{t+1})}{\beta(1+\delta_{t+1})} - \frac{1}{\beta(1+\delta_{t+1})}\ln\left(\frac{N_{t+1}}{\eta_t}\right) + \frac{\delta_{t+1}}{1+\delta_{t+1}}\left((1+r)A_t + y_t - Pn_t\right), \quad (4.18)$$

with $\eta_t = \eta_F$ and $n_t = n_F$ for $t \in (F,T]$. In the last period of the couple lifetime T, the optimal consumption function is

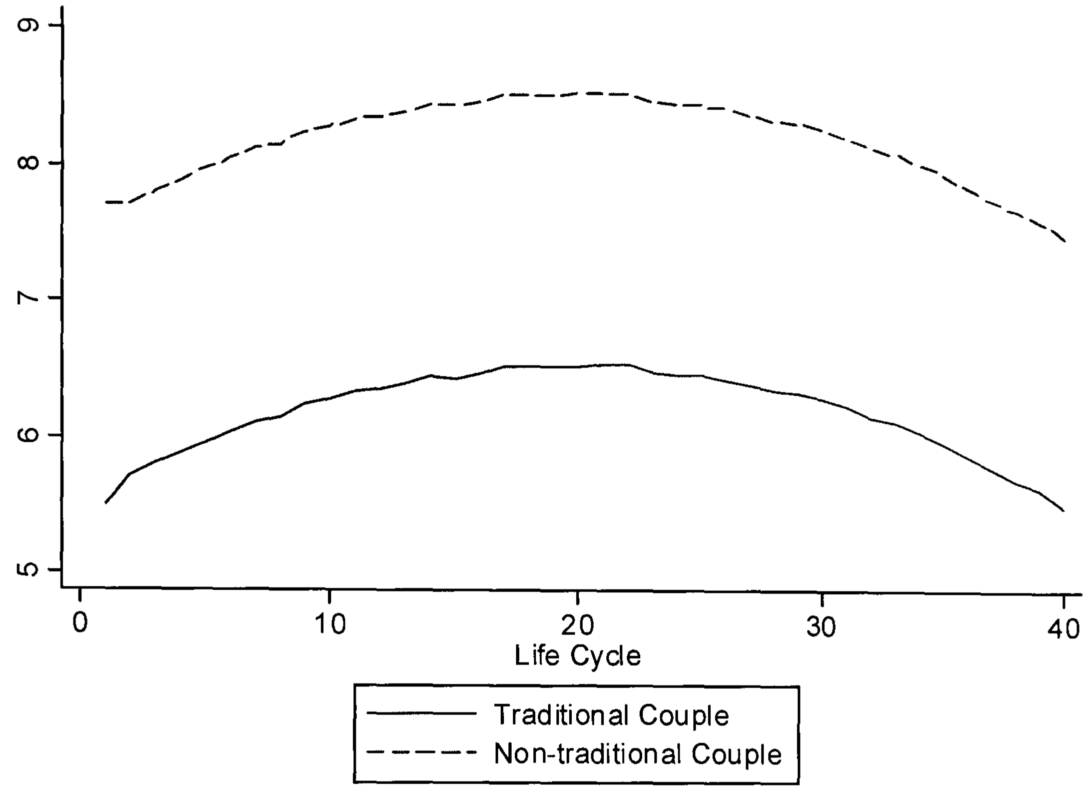
$$c_T = (1+r)A_T + y_T - Pn_F.$$

As usual with CARA preferences, optimal consumption (4.18) depends positively and linearly on current disposable resources net of the costs of rearing children, $(1+r)A_t + y_t - Pn_t$. Another feature of optimal consumption derived by CARA preferences is that c_t may be negative.[5] In our model the probability of negative consumption increases since household wealth available for consumption is reduced by the costs of rearing children. During the reproductive life, the couple's decision of having a child in each $t \in [f,F]$ leads to a discrete shift in $\frac{N_{t+1}}{\eta_t}$ and it increases the total costs of rearing children Pn_t faced by the couple. Therefore, it is likely that optimal consumption will fall due to the extra costs related to the presence of children. As the number of dependent children increases, *ceteris paribus*, there are less resources available for family consumption other than expenditures for children and c_t decreases. A similar effect would occur if the costs of rearing children P raise.

The intercept of the optimal consumption c_t is composed of time preference effect $\ln(\rho\delta_{t+1})$ and the effect of future fertility choice $\ln\left(\frac{N_{t+1}}{\eta_t}\right)$. The time preference effect

[5] A feature of optimal consumption derived by CARA preferences is that consumption may be negative. One can find conditions on exogenous variables under which optimal consumption is positive in all t (Berloffa and Simmons 2003). On the other hand, one may impose the restriction on the optimization that consumption must be positive in any date. However, in doing so the tractability of the model could be lost and one must beware of the possibility that the model can produce absurd predictions.

FIGURE 4.1: Labour Income Profiles Over the Life Cycle by Traditional and Non-Traditional Families



$\ln(\rho\delta_{t+1})$ is negative thus the first term in equation (4.18) increases consumption. It is more difficult to predict the sign of the effect of future fertility choices $\frac{N_{t+1}}{\eta_t}$ on optimal consumption. If $\frac{N_{t+1}}{\eta_t} > 1$ future fertility choices hold down current consumption. In this way, couples depress current consumption to face the future costs of new births. The interest rate r affects consumption through different ways. It positively affects nonlabour income A_t. Moreover, as previously mentioned, δ is a function increasing in r. Thus, the interest rate r raises the slope $\frac{\delta_{t+1}}{1+\delta_{t+1}}$ of the optimal consumption and increases the coefficient of future fertility choice.

4.3 A Simulation Exercise: Fertility Choices of Traditional and Non-Traditional Couples

This section describes the model's simulation. The simulation is carried out for two stylised scenarios. In particular, we consider the fertility choice of traditional and non-traditional couples. According to Apps and Rees (2001), the traditional couple is a family in which there is one main wage earner, in general the husband. On the other hand, in the non-traditional couple both spouses works. In this way, we attempt to model different family size patterns that occur since families with different labour market participations face different opportunity costs of children. Thus we expect families with higher opportunity costs of children, in the simulation the non-traditional couple, will have fewer children. We suppose that spouses marry at twenty-eight years old, reproductive life lasts up to forty and, thus, fertility choices are simulated for a period of twelve years.

In order to mimic earnings over couples' life cycle, labour incomes y_t are designed to have a bell-shaped profile according to the following equation (Berloffa and Simmons 2003)

$$y_t = \gamma + 0.05\exp((-(t-20)/20)^2) + 0.05\epsilon_t \quad \text{with} \quad \epsilon_t \sim U(0,1).$$

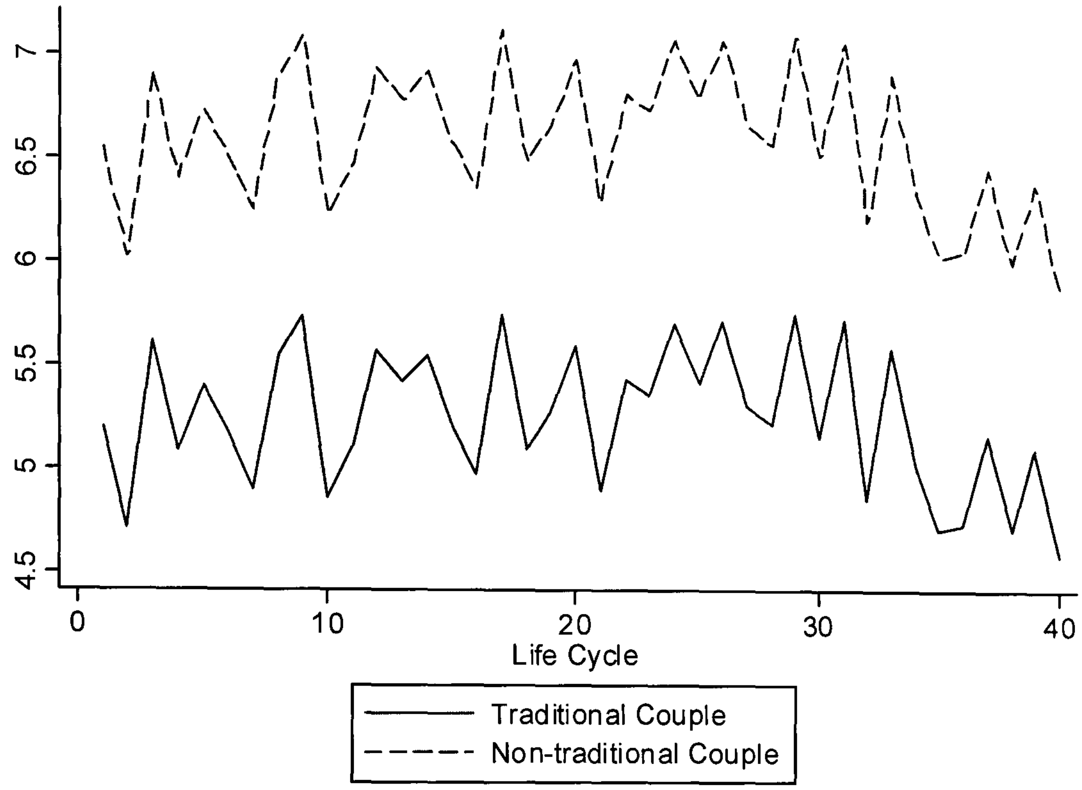
To differentiate labour incomes by family types we assume that the intercept γ takes a value of 6.5 for the traditional couple and of 8.5 for the non-traditional couple. In addition, in order to capture wage fluctuations due to economic trends of labour markets, we allow the labour income profile to slightly vary through time according to a noise term ϵ_t uniformly distributed. In simulating the labour income profile y_t we attempt to reduce heterogeneity between the two families. We thus assume that adult members of the two families are employees, with similar levels of education, and the non-traditional family gets higher wage incomes only because there are two wage income recipients.

In Figure 4.1 the labour income profile y_t is depicted by traditional and non-traditional family. Over the life cycle, labour incomes are designed to be always higher for the non-traditional couple, since there are two wage earners, they get the maximum at $t = 20$ then labour incomes start to decrease.

The interest rate is held constant at $r = 0.025$, the discount factor is $\rho = 1/1.15$ and the coefficient of absolute risk aversion is $\beta = 1.63$.[6] Initial assets A_f are equal to 3. These figures are held constant across the two families. As illustrated in the remainder of the section, the costs of rearing a child together with the level of labour incomes vary across traditional and non-traditional families. We also present the fertility choice simulation

[6] The value of these terms is based on Berloffa and Simmons (2003).

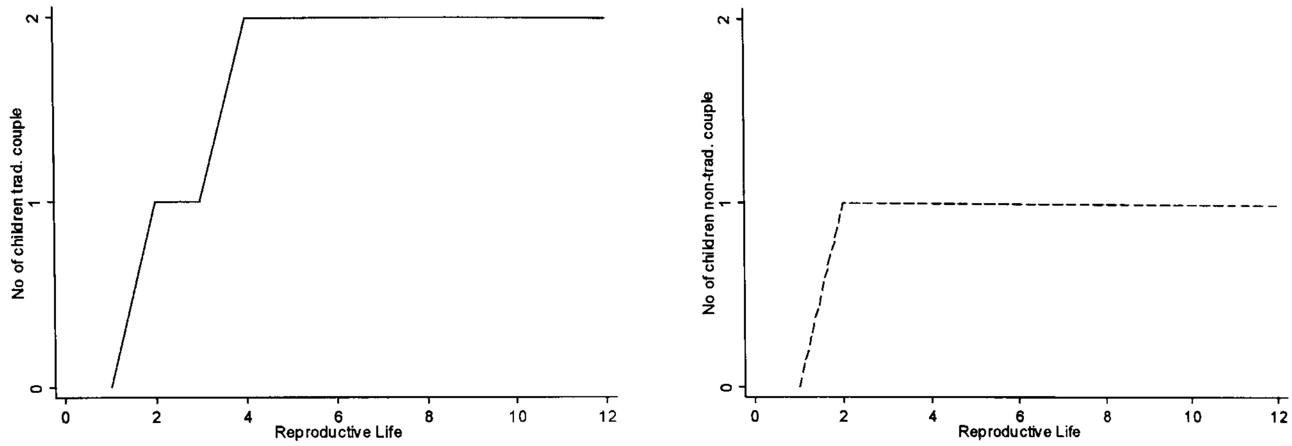
FIGURE 4.2: Costs of Rearing a Child Over the Life Cycle by Traditional and Non-Traditional Families



assuming $r = 0$ but maintaining the value of the other parameters unchanged.

The presence of children radically changes the organization of the family. Dependent children affect the expenditure patterns of a family. Parents substitute away adult good expenditures for children good expenditures (Deaton, Ruiz-Castillo, and Thomas 1989). Empirical evidence shows that there exists a positive correlation between family size and the budget share of food expenditure. As the family size increases, *ceteris paribus*, the budget share for food expenditures increases. Parents' allocation of time is also affected by children. Especially in the first years of children's life when they need supervision, women devote a good deal of their time to look after children. In order to do that, women often decide either to work part-time or to cease from work. Children influence also the demand for housing, portfolio and migration decisions. However, for these household decisions there is not systematic evidence of how children affect them. Since household outcomes are significantly affected by the presence of children, economists devote a good deal of their research agenda to study the effect of children on household behaviour and, from an empirical perspective, how to model the effect of children on household demands (Barten

FIGURE 4.3: Number of Children and Birth Spacing Over the Reproductive Life by Traditional and Non-Traditional Families



1964, Gorman 1976, Lewbel 1985, Pollak and Wales 1981).

According to Browning (1992), the presence of children raises some questions.[7] An issue examined by Browning (1992), related to our purpose of defining the costs of rearing a child, is the expenditure question, that is how much parents spend on their children. How much a family spends for children defines the costs of rearing a child and, to some extent, it explains fertility outcomes.

We define the costs of rearing a child as follows.

Definition 4.1 (Costs of rearing children) *The costs of rearing a child P is a function, increasing in all its terms,*

$$P = h(C_M, C_L^c, C_T^c)$$

of the costs of maintenance a child C_M, the costs of non-necessary goods consumed only by children C_L^c and the cost of adult time devoted to look after children C_T^c.

The costs of maintenance a child C_M refer to another issue raised by Browning (1992). Precisely, children arise "the needs question: how much incomes does a family with children need compared to a childless family?" (Browning 1992:1440). Equivalence scales answer the needs question and, therefore, we propose to estimate the costs of maintenance by means of them. Equivalence scales are used to make welfare comparison between

[7]Browning (1992) arises four questions associated with the presence of children in the family. However, here we report only the issues related to our analysis.

FIGURE 4.4: Consumption Patterns Over the Reproductive Life by Traditional and Non-Traditional Families

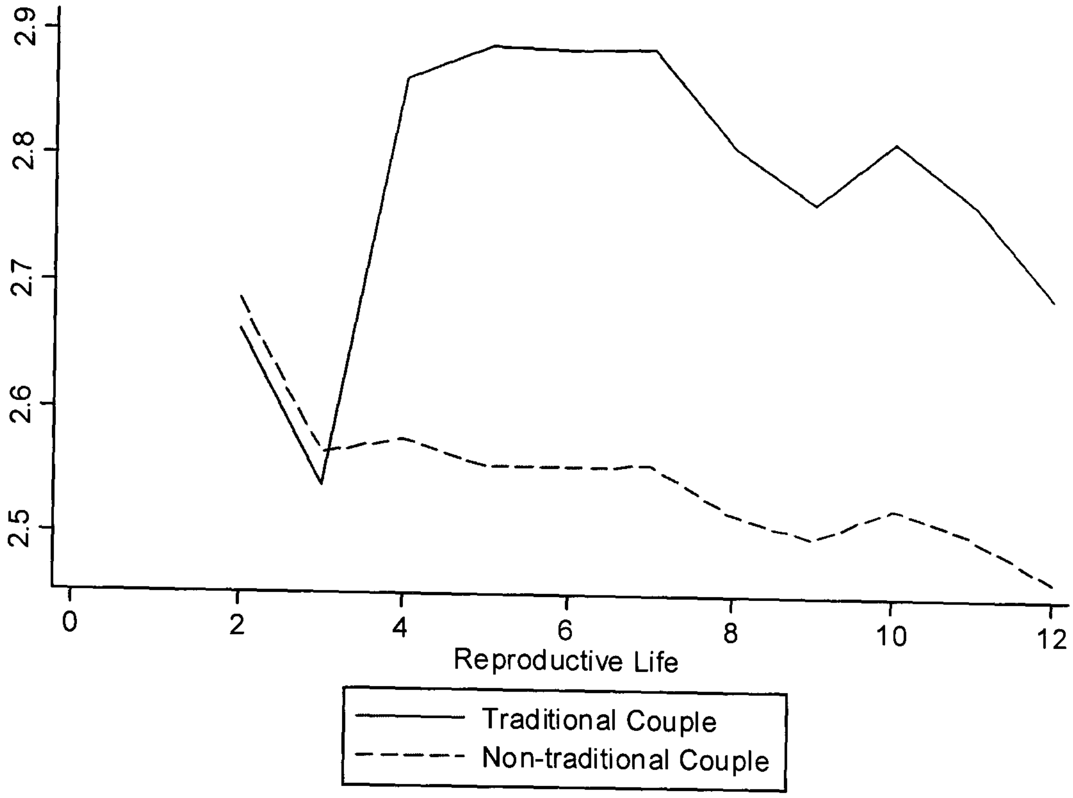
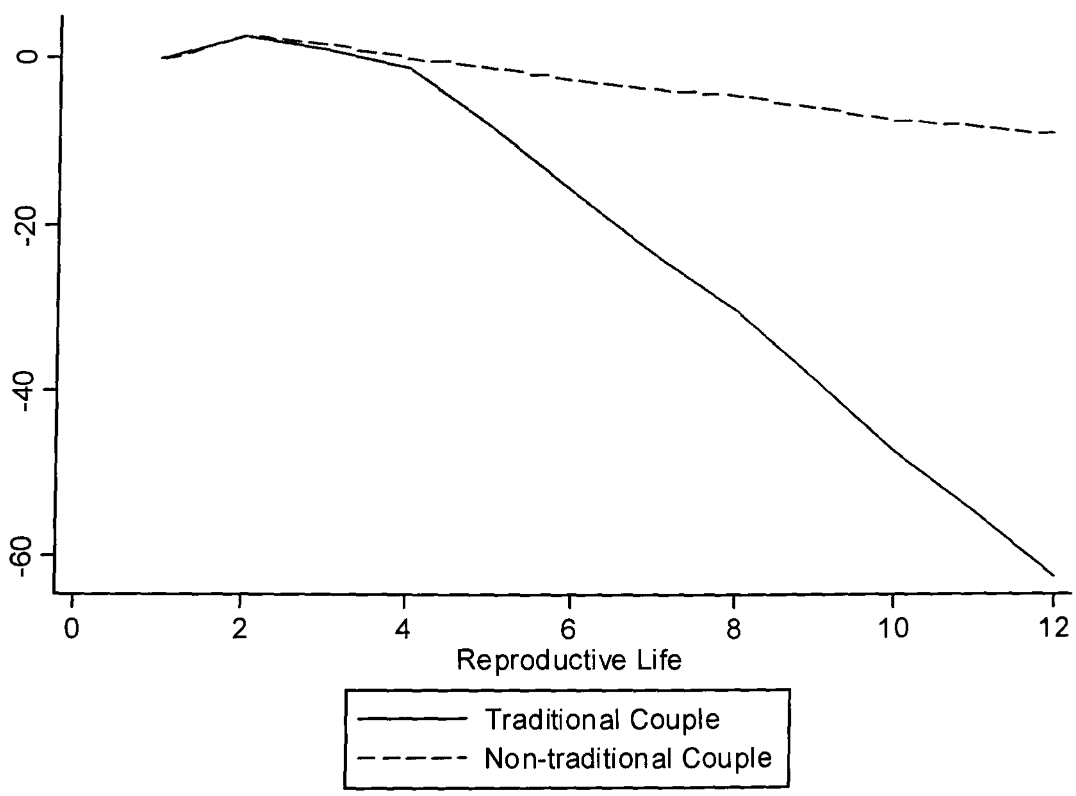


FIGURE 4.5: Asset Accumulations Over the Reproductive Life by Traditional and Non-Traditional Families



families with different demographic characteristics and to serve this objective they must satisfy the property of income independency (Blackorby and Donaldson 1991, Blundell and Lewbel 1991). As a consequence, the costs of maintenance account only for necessity good expenditures and as such C_M is independent of family current disposable income. For the purpose of the simulation, we set $C_M = 2.8$[8] and it does not vary across the two family types. As for labour income profiles, to reflect price variations through time we modify the costs of maintenance adding a random effect uniformly distributed.

There exists a negative correlation between the presence of children and female labour supply. This evidence shows that in general the opportunity costs of children C_T^c are borne by the wife that decides to work part-time or to cease from work when there are young children in the family. If this is the case, family current disposable income decreases and the presence of children may lead to negative effects on family consumption of adults. To estimate the value of time devoted by parents to child care one can choose between the opportunity costs and the market cost approach (Fitzgerald, Swenson, and Wicks 1996, Harvey 1996). The former approach values the time allocated to domestic activities using the wage that individuals earn in the labour place. An empirical issue arises when individuals do not work since we cannot observe their market wages. On the other hand, the market cost approach measures the value of domestic goods and services using the market price that one would pay to buy the good in the outside market. Underlying the market cost approach is the idea that domestic and market goods are each other substitutes.

Finally, the costs of rearing a child P account for the costs of non-necessary goods consumed only by children C_L^c. As the opportunity costs of children, these costs depend on family current disposable income. In principle, it is straightforward to measure C_L^c: it suffices to survey "who gets what" in the household. However, there are very few household surveys that record item expenditures at the individual level. In absence of such data to infer the costs of non-necessary goods consumed only by children we need to use indirect methods. An indirect method is to regress household expenditures on

[8] This choice is based on a recent estimation of equivalence scales for Italy (Menon and Perali 2007).

children (Douthitt and Fedyk 1990). Another set of methods for identifying expenditures on children when we observe only household expenditure is to use information on adult goods (Deaton, Ruiz-Castillo, and Thomas 1989). Lazear and Michael (1988) propose an interesting approach. Their idea consists in identifying total expenditure on adults and from this we can find the expenditure on children by subtraction.

In the simulation, in each t the costs of non-necessary goods consumed only by children C^c_L and the opportunity costs C^c_L are simulated by imputing the 30 per cent of current labour income y_t for the traditional couple and the 40 per cent of current labour income y_t for the non-traditional couple. The costs of rearing a child[9] are illustrated in Figure 4.2. Over the life cycle, by construction, the non-traditional couple faces higher costs of rearing a child than the traditional couple.

As expected, the non-traditional couple ends its reproductive life with fewer children than the traditional couple (Figure 4.3). In particular, fertility patterns of the two families show that the traditional couple has two children with two year gap between the first and the second child. On the other hand, the non-traditional couple has only one child. Both families has the first child in the second year of marriage.

In Figure 4.4 consumption patterns are reported by family types. In the periods of time in which the two families has equal family size, consumption of the non-traditional family is higher relative to consumption of the traditional family. Consumption of the traditional family suddenly jumps when the second child is born, this effect is produced by the discrete shift of the intercept, and then throughout the reproductive life it remains higher than consumption of the non-traditional family. However, consumption patterns of the two families are depressed by the costs of rearing children. As a result, consumption of both the families decreases during the reproductive life. The non-traditional couple has smoother consumption patterns than the traditional couple. After five years of marriage the two families get into debts. However, the non-traditional family registers less negative

[9] The costs of rearing a child P has been calculated as $P = C_M + C^c_L + C^c_T$. In particular, in each t for the traditional family the costs of rearing a child is given by $P = (2.8 + \epsilon_t) + 0.30 \cdot y_t$, where ϵ_t is a noise uniformly distributed and $0.30 \cdot y_t$ summarises the costs of non-necessary goods consumed only by children, C^c_L, and the cost of adult time devoted to look after children, C^c_T. For the non-traditional family the costs of rearing a child becomes $P = (2.8 + \epsilon_t) + 0.40 \cdot y_t$.

FIGURE 4.6: Number of Children Over the Reproductive Life by Traditional and Non-Traditional Families with r=0

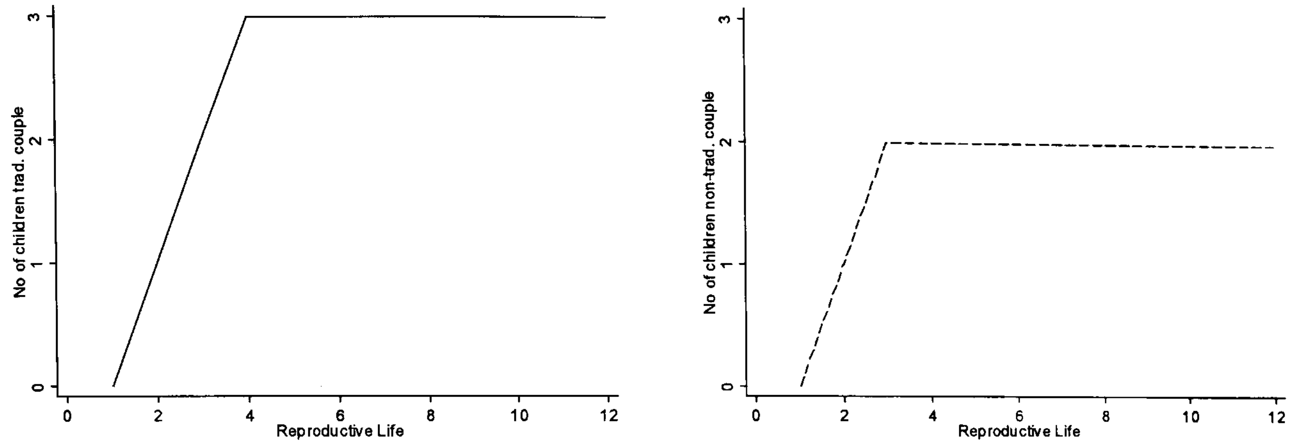
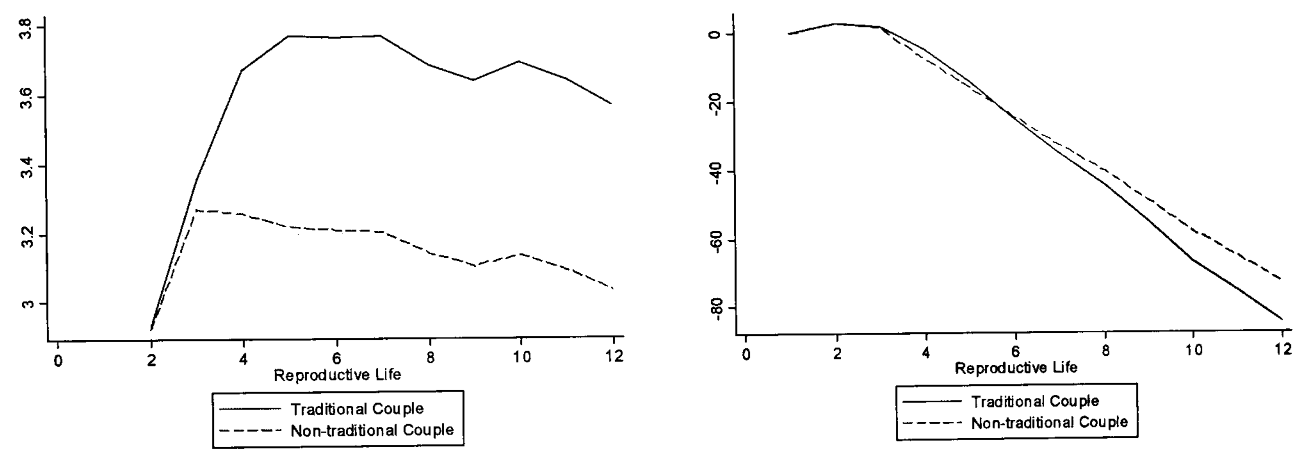


FIGURE 4.7: Consumption Patterns and Assets Accumulations Over the Reproductive Life by Traditional and Non-Traditional Families with r=0



assets accumulation, hence it has more wealth, due to higher labour income profile and a smaller family size relative to the traditional couple.

To illustrate how the interest rate may influence parents' fertility choices we run the simulation assuming that the interest rate is equal to zero $r = 0$. As stated in Section 4.2.2, the interest rate r affects fertility choices through δ. In particular, as the interest rate decreases, all the other things being equal, the costs of having a child decreases as well increasing, in this way, the probability of a new birth. This effect is depicted in Figure 4.6. Both families have one more child with respect to the previous scenario. Moreover, setting the interest rate equal to zero has the effect of reducing the spacing of births.

Figure 4.7 provides a graphical presentation of the consumption pattern and assets accumulation of the two families assuming $r = 0$. The left-hand panel shows the simulated consumption pattern for the two family types: when the size of the two families increases consumption increases as well and then starts to decrease. The right-hand panel shows the assets accumulation of the two families. Similar to Figure 4.5, the two families have negative assets accumulations but with $r = 0$ the difference in assets accumulation of the two families is reduced.

We have also simulated fertility and consumption choices assuming $\beta < r$. Since on fertility and consumption choices the coefficient of absolute risk aversion has the same effect of the interest rate, we have obtained similar results to the scenario with the interest rate equal to zero.

4.4 Conclusions

Given the relevance of the human reproductive behaviour, a number of researchers from various scientific disciplines have devoted attention in understanding it. The fertility behaviour has raised the attention of economists because of its unexpected trends. Malthus argued that with increasing income people would marry earlier and therefore had more children. Moreover, an increase in income would reduce child mortality and, as a consequence, countries getting richer would experience a population growth.

Contrary to Malthus' view, since the late 1960s the fertility rate has registered a drastic decline in almost all the industrialized countries. The fertility decline has been accompanied by a sharp change in the structure of the reproductive behaviour of households. In the last decades couples have decided to postpone births affecting also the spacing of births. In general, with the increase of average age at first marriage, couples tend to reduce the spacing between subsequent births.

Family size outcomes are affected by a number of variables. The number of children is the result of sharing a family project and stability of the relationship between spouses. The number of children is also affected by economic aspects, such as easy availability of child care services, availability of part-time jobs, flexibility of the labour market, and family policies that provide financial support to households with dependent children. Therefore, to understand fertility outcomes of families one should potentially control for a number of socioeconomic aspects.

The purpose of this chapter is to describe family fertility outcomes as a discrete choice within a dynamic model. At the start of each period of the reproductive life, the fertility choice is the outcome of comparisons between parents' welfare level with and without an extra child. Our dynamic model of discrete fertility choice emphasizes the effects exert on the family size decision by the costs of rearing children. The dynamic model is developed under a set of simplifying assumptions designed to obtain an analytical solution to the dynamic program. We assume that the instantaneous utility function of couples is of the CARA form and, neglecting that spouses can have different fertility preferences, we abstract from bargaining processes that can occur within the couple. Furthermore, we assume that parents face perfectly foreseen costs of rearing children and these costs are equal across children. Within this framework, we show that in each period of the reproductive life the couple does not have a child if the pure utility gain from not having a child is greater than the utility obtained from saving in costs from having the child. The model has neither direct implications for quality of children nor for the wife's participation to the labour force.

We perform a simulation exercise of the fertility choice model. In particular, we describe the fertility outcome of traditional and non-traditional couples. The former is a family in which there is one main wage earner. On the other hand, the non-traditional family is characterized by the presence of two wage earners. We find that the traditional couple decides to have two children with a two year gap between the first and second birth. On the other hand, since both spouses work, the non-traditional couple faces higher opportunity costs and, in turn, higher costs of rearing children relative to the traditional couple. Thus the non-traditional couple has just one child. The consumption pattern and assets accumulation of the two families are influenced by their fertility outcomes. Given a bigger family size, during the reproductive life consumption of the traditional family is in general higher relative to consumption of the non-traditional family. In addition, both families registers a negative assets accumulation. The traditional family, however, gets heavier into debts than the non-traditional family.

Appendix: Derivation of the Optimal Consumption

Derivation of the optimal rule for consumption in t

The optimal household program is

$$\max_{c_t,\Delta_t n} 1 - \eta_t \exp(-\beta c_t) + \rho \{\lambda_{t+1} - N_{t+1} \exp[-\beta\delta_{t+1}((1+r)A_t + y_t - c_t - Pn_t)]\}.$$

taking the first-order condition

$$\eta_t \exp(-\beta c_t)\beta - \rho\{N_{t+1}\exp[-\beta\delta_{t+1}((1+r)A_t + y_t - c_t - Pn_t)]\beta\delta_{t+1}\} = 0,$$

solving for $\exp(-\beta c_t)$

$$\begin{aligned}
\exp(-\beta c_t)\exp(-\beta\delta_{t+1}c_t) &= \rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\exp\{-\beta\delta_{t+1}[(1+r)A_t + y_t - Pn_t]\},\\
\exp[-\beta c_t(1+\delta_{t+1})] &= \rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\exp\{-\beta\delta_{t+1}[(1+r)A_t + y_t - Pn_t]\},\\
\exp(-\beta c_t)\exp(1+\delta_{t+1}) &= \rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\exp\{-\beta\delta_{t+1}[(1+r)A_t + y_t - Pn_t]\},
\end{aligned}$$

and the optimal consumption policy results equal to

$$\exp(-\beta c_t) = \left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{\frac{1}{1+\delta_{t+1}}}\exp\left\{-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}[(1+r)A_t + y_t - Pn_t]\right\}. \tag{A.1}$$

Derivation of the value function in t

$$\begin{aligned}
\lambda_t - N_t\exp(-\beta\delta_t A_t) &= 1 - \eta_t\exp(-\beta c_t) + \rho\{\lambda_{t+1} - N_{t+1}\\
&\quad \exp[-\beta\delta_{t+1}((1+r)A_t + y_t - c_t - Pn_t)]\},
\end{aligned}$$

$$\begin{aligned}
\lambda_t - N_t\exp(-\beta\delta_t A_t) &= 1 - \eta_t\exp(-\beta c_F) + \rho\{\lambda_{t+1} - N_{t+1} \qquad \text{(A.2)}\\
&\quad \exp[-\beta\delta_{t+1}((1+r)A_t + y_t - c_t - Pn_t)]\exp(\beta\delta_{t+1}c_t)\},
\end{aligned}$$

where using equation (A.1) opportunely rearranged, we can derive the expression for $\exp\left(\beta\delta_{t+1}c_t\right)$

$$\exp\left(\beta\delta_{t+1}c_t\right)=\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}}\exp\left\{\beta\frac{\delta_{t+1}^2}{1+\delta_{t+1}}\left[\left(1+r\right)A_t+y_t-Pn_t\right]\right\}.$$

Substituting the last equation into (4.19) we have

$$\begin{aligned}\lambda_t-N_t\exp\left(-\beta\delta_tA_t\right) &= 1-\eta_t\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{\frac{1}{1+\delta_{t+1}}} \qquad (A.3)\\ &\exp\left(-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t+y_t-Pn_t\right)\right)+\rho\lambda_{t+1}-\\ &\rho N_{t+1}\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}}\\ &\exp\left(-\beta\delta_{t+1}\left(\left(1+r\right)A_t+y_t-Pn_t\right)\right)\\ &\exp\left(\beta\frac{\delta_{t+1}^2}{1+\delta_{t+1}}\left(\left(1+r\right)A_t+y_t-Pn_t\right)\right),\end{aligned}$$

$$\begin{aligned}\lambda_t-N_t\exp\left(-\beta\delta_tA_t\right) &= 1+\rho\lambda_{t+1}-\eta_t\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{\frac{1}{1+\delta_{t+1}}}\\ &\exp\left(-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t+y_t-Pn_t\right)\right)\\ &-\rho N_{t+1}\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}}\\ &\exp\left(-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t+y_t-Pn_t\right)\right),\end{aligned}$$

$\rho N_{t+1}\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}}$ can be written as

$$\frac{\eta_t}{\delta_{t+1}}\rho N_{t+1}\frac{\delta_{t+1}}{\eta_t}\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{-\frac{\delta_{t+1}}{1+\delta_{t+1}}}=\frac{\eta_t}{\delta_{t+1}}\left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{\frac{1}{1+\delta_{t+1}}}$$

and

$$\lambda_t - N_t \exp\left(-\beta\delta_t A_t\right) = 1 + \rho\lambda_{t+1} - \left(1 + \frac{1}{\delta_{t+1}}\right)\eta_t \left(\rho\delta_{t+1}\frac{N_{t+1}}{\eta_t}\right)^{\frac{1}{1+\delta_{t+1}}}$$
$$\exp\left(-\beta\frac{\delta_{t+1}}{1+\delta_{t+1}}\left(\left(1+r\right)A_t + y_t - Pn_t\right)\right).$$

Derivation of the optimal rule for consumption in $T-1$

In $T-1$, the optimal household consumption is equal to

$$\exp\left(-\beta c_{T-1}\right) = \left(\frac{\rho\delta_T}{\eta_F}\right)^{\frac{1}{1+\delta_T}} \exp\left\{-\beta\frac{\delta_T}{1+\delta_T}\left[\left(1+r\right)A_{T-1} + y_{T-1} - Pn_F\right]\right\} N_T^{\frac{1}{1+\delta_T}},$$

the value function is

$$\lambda_{T-1} - N_{T-1}\exp\left(-\beta\delta_{T-1}A_{T-1}\right) = 1 + \rho\lambda_T - \frac{\eta_F\left(1+\delta_T\right)}{\delta_T}\left(\frac{\rho\delta_T N_T}{\eta_F}\right)^{\frac{1}{1+\delta_T}}$$
$$\exp\left[-\beta\frac{\delta_T}{1+\delta_T}\left(y_{T-1} - Pn_F\right)\right]$$
$$\exp\left[-\beta\frac{\delta_T}{1+\delta_T}\left(1+r\right)A_{T-1}\right],$$

with

$$\begin{aligned}
\lambda_{T-1} &= 1 + \rho\lambda_T = 1 + \rho, \\
\delta_{T-1} &= \frac{\delta_T\left(1+r\right)}{1+\delta_T} = \frac{\left(1+r\right)^2}{2+r}, \\
N_{T-1} &= \frac{\eta_F\left(1+\delta_T\right)}{\delta_T}\left(\frac{\rho\delta_T}{\eta_F}\right)^{\frac{1}{1+\delta_T}} \exp\left[-\beta\frac{\delta_T}{1+\delta_T}\left(y_{T-1} - Pn_F\right)\right] N_T^{\frac{1}{1+\delta_T}} = \\
&= \frac{\eta_F\left(1+\delta_T\right)}{\delta_T}\left(\frac{\rho\delta_T}{\eta_F}\right)^{\frac{1}{1+\delta_T}} \exp\left[-\beta\frac{\delta_T}{1+\delta_T}\left(y_{T-1} - Pn_F\right)\right] \\
&\quad \left(\eta_F\exp\left(-\beta\left(y_{T-1} - Pn_F\right)\right)\right)^{\frac{1}{1+\delta_T}},
\end{aligned}$$

where $\delta_T = 1 + r$.

Derivation of the optimal rule for consumption in $T-2$

In $T-2$, the optimal consumption of spouses evolves as

$$\begin{aligned}\exp\left(-\beta c_{T-2}\right) &= \left(\frac{\rho\delta_{T-1}}{\eta_F}\right)^{\frac{1}{1+\delta_{T-1}}} \\ &\quad \exp\left\{-\beta\frac{\delta_{T-1}}{1+\delta_{T-1}}\left[\left(1+r\right)A_{T-2}+y_{T-2}-Pn_F\right]\right\}N_{T-1}^{\frac{1}{1+\delta_{T-1}}},\end{aligned}$$

the value function is

$$\begin{aligned}\lambda_{T-2}-N_{T-2}\exp\left(-\beta\delta_{T-2}A_{T-2}\right) &= 1+\rho\lambda_{T-1}-\frac{\eta_F\left(1+\delta_{T-1}\right)}{\delta_{T-1}}\left(\frac{\rho\delta_{T-1}N_{T-1}}{\eta_F}\right)^{\frac{1}{1+\delta_{T-1}}} \\ &\quad \exp\left[-\beta\frac{\delta_{T-1}}{1+\delta_{T-1}}\left(y_{T-2}-Pn_F\right)\right] \\ &\quad \exp\left[-\beta\frac{\delta_{T-1}}{1+\delta_{T-1}}\left(1+r\right)A_{T-2}\right],\end{aligned}$$

with

$$\begin{aligned}\lambda_{T-2} &= 1+\rho\lambda_{T-1}=1+\rho\left(1+\rho\right), \\ \delta_{T-2} &= \frac{\delta_{T-1}\left(1+r\right)}{1+\delta_{T-1}}=\frac{\left(1+r\right)^3}{3+r}, \\ N_{T-2} &= \frac{\eta_F\left(1+\delta_{T-1}\right)}{\delta_{T-1}}\left(\frac{\rho\delta_{T-1}}{\eta_F}\right)^{\frac{1}{1+\delta_{T-1}}}\exp\left[-\beta\frac{\delta_{T-1}}{1+\delta_{T-1}}\left(y_{T-2}-Pn_F\right)\right]N_{T-1}^{\frac{1}{1+\delta_{T-1}}}= \\ &= \frac{\eta_F\left(1+\delta_{T-1}\right)}{\delta_{T-1}}\left(\frac{\eta_F\left(1+\delta_T\right)}{\delta_T}\right)^{\frac{1}{1+\delta_T}}\left(\frac{\rho\delta_{T-1}}{\eta_F}\right)^{\frac{1}{1+\delta_{T-1}}}\left(\frac{\rho\delta_T}{\eta_F}\right)^{\left(\frac{1}{1+\delta_T}\right)^2} \\ &\quad \exp\left[-\beta\frac{\delta_{T-1}}{1+\delta_{T-1}}\left(y_{T-2}-Pn_F\right)\right]\exp\left[-\beta\frac{\delta_T}{1+\delta_T}\left(y_{T-1}-Pn_F\right)\right]^{\frac{1}{1+\delta_T}} \\ &\quad \left(\eta_F\exp\left(-\beta\left(y_{T-1}-Pn_F\right)\right)\right)^{\left(\frac{1}{1+\delta_T}\right)^2},\end{aligned}$$

with $\delta_{T-1}=\frac{(1+r)^2}{2+r}$.

Chapter 5

Conclusions

The thesis examines three central aspects of family behaviour with potentially relevant policy implications. In particular, the thesis deals with the labour participation decisions of family members, household production undertaken by family members and fertility choices of parents. The first two topics are modelled within a static framework using the unitary and collective approaches to family behaviour. On the other hand, the model of family fertility choices is developed within a dynamic framework in the context of the unitary model. In doing so, we assume that parents have equal preferences toward children.

Although economists have widely recognized the important role played by individualism in modeling family behaviour, there is still no agreement on which approach better represents family behaviour. It is likely that family decisions, such as labour participation, involve a bargaining process between the spouses. On the other hand, it may be the case that decisions taken on a daily basis, such as food consumption, do not require a bargaining approach to model household behaviour.

The labour participation analysis presented in the thesis is thus studied employing both the unitary and the collective models. In doing so, the properties that characterize the two family models can be tested and, in principle, it can be suggested which household model fairly represents the labour participation decisions of family members.

The chapter opens with the analysis of labour participation decisions of married couples within the unitary model. Household preferences are assumed to be additively separable in the individual utility function of the spouses and the budget constraint is assumed to be linear. Within this framework, the reservation wage theory leads to a complete characterization of the labour participation decision. For each spouse a set of reservation wages in different labour participation regimes is derived. The set of reservation wages allows characterisation of the participation frontier. As a result, the participation frontier divides the market wage plane in four regions. Each region is characterized by an opportunity set such that the spouses are jointly better off either of them working, neither of them working, or one working and the other not.

We then analyse the labour participation decisions within the collective model. In line with previous works (Blundell *et al.* 2007, Donni 2003), the standard assumptions underlying the collective model, that is Pareto-efficiency, egoistic individual preferences and private consumption, are not sufficient to provide a unique derivation of the reservation wage function and, in turn, the existence of the participation frontier.

To overcome the latter shortcoming, Blundell *et al.* (2007) and Donni (2003) postulate that the reservation wage is a contraction with respect to market wages. The reservation wage function thus has a unique fixed point with respect to individual market wages and when both spouses do not work, there will exist one and only one pair of wages such that both spouses are indifferent between working and not working. On the other hand, when member i does not work, there will exist a reservation wage function of w_j and y such that member i participates in the labour market if and only if her market wage is greater than her reservation wage. Consequently, the participation set is partitioned into four connected sets with a unique intersection .

Conversely, we take a different approach. We assume that the Pareto weight $\widetilde{\mu}$ is a continuous function of individual expected wages rather than current market wages. The individual expected wage is defined as the prediction of the wage that an agent would get if she were to enter the labour force. It can depend on past working experience and demographic characteristics of the agent as well on macro-economic indicators. As a result, the reservation wage theory can be applied to described the labour participation of spouses also in the context of the collective model. As described in the first chapter, under the assumption that the Pareto weight is function of expected wages rather than current market wages, completeness of the reservation wage theory is re-established. This entails that the characterization of the participation frontier is similar to that obtained within the unitary model of family labour participation.

The labour participation analysis presented in the thesis can be extended to study child labour participation in developing countries. In particular, it is relevant to study the relationships, if any exists, between the rule governing resources allocation among adults and children and child labour participation. Moreover, it is interesting to compare how resources are shared between adults and children across households where children do not work and where children work. The comparison could be extended to the case where working children maintain ownership of their resources and where they do not. Using a collective approach we can study who gains in the family from child labour. Altruistic working children may also improve family equality, such as the state of being well-nourished and in good health and opportunities to access education, across household members. Finally, estimation of the sharing rule within this context can be used to determine the best policies in terms of the highest improvement in well-being of the household social welfare function and in terms of measures of polarization of the incomes of the household and equality of opportunities across members.

The thesis then considers the family as an "enterprise" producing goods by transforming factor inputs. The factor inputs are time devoted by each spouse to the household production. Moreover, consumption decisions of household members are taken in consideration. Thus, the family model describes the family as jointly involved in production and in consumption decisions. The family model can embrace both urban and rural families in relation to the location of both the household and the entrepreneurial activity. When business activity owned by the family is not undertaken, then the household sells labour either to the job market or to the household. In this case, the general model of a "family-firm" reduces to a "family" engaged in domestic production. The "family-firm" model is a general equilibrium model in miniature where the household enterprise fully reproduces at the micro level the characteristics of a macro society. The production-consumption household model is also general in the sense that the household is represented as a collection of individuals (Apps and Rees 1988, 1997, and Chiappori 1988, 1992, 1997).

We study the production-consumption household model under the assumptions of complete and absent markets of the domestic good. This analysis, however, can be extended to absence of other markets, for instance absence of labour markets. We focus on the separation property between production and consumption choices and emphasize the relevance of production and consumption household behaviour both for developing countries, where most households are engaged in agricultural activities, and for developed countries, where families can run business or agricultural activities and/or household members devote a part of their own time to domestic activities. In addition, it is important to account for production-consumption household behaviour when analysing policy intervention in the rural economy. Agricultural policies will affect not only production but also consumption and labour supply of the family members.

Under the assumption of marketable domestic goods, the separation property holds and the production-consumption household model within a collective context can be solved recursively in two stages. In the first stage, the family is engaged in maximizing a profit function independently of its consumption and leisure decisions. Once this choice is made, the family decides the optimal consumption-leisure bundle. The consumption-leisure variables are affected by the production decision through the maximized profits that enter the budget constraint.

We then relax the assumption of complete markets. As a consequence, the implicit price of the domestic good is endogenous to each family. We thus study which restrictions should be assumed to preserve the separability between production and consumption decisions. Specifically, we show that with missing markets a sufficient condition for the separability to hold is the assumption of constant returns to scale of the household production technology. With constant returns to scale, the minimum cost function depends linearly on output and, consequently, the implicit price of the domestic good, defined by the marginal cost functions of the domestic good, does not depend on household tastes and the decision process. This assures that the production-consumption household model can be solved recursively. Different from the complete market case, when making production choices the family behaves as it were a cost minimizer.

Lastly, we examine the case of missing markets of the domestic good under the assumption of nonconstant returns to scale. Within this setting and joint production, the minimum cost function depends nonlinearly on output and, as a result, the implicit price of the domestic good is a function of household tastes and the decision process. As a consequence, the production-consumption household model must be solved jointly. Moreover, due to joint technologies a closed-form solution to the production-consumption household model cannot be found. In order to overcome this drawback, we propose two alternative specifications of the production function.

An alternative assumes that the household has two distinct technologies to produce the domestic good consumed privately by the two household members. Without joint technologies the household program has a unique closed-form solution. However, the production and consumption decisions must be solved jointly and due to the specification of the production functions the sharing rule model cannot be employed. A second specification of the production function that we propose is a viable way to apply the sharing rule model. We assume that each household member produces by herself the domestic product that she privately consumes. However, the production-consumption decisions are taken jointly by each household member.

In the thesis we also extend the generalized collective Slutsky equation to incorporate the production choices of the family both with marketable and nonmarketable domestic goods. Either with complete markets of the domestic good or missing markets and constant returns to scale production functions, the introduction of household production does not make much difference in terms of comparative static analysis. The collective Slutsky conditions satisfy the SNR1 property (Browning and Chiappori 1998). This property states that the collective Slutsky matrix is made of a symmetric, negative semidefinite matrix and a rank one matrix. The rank one matrix originates from the shift of the Pareto weight due to changes of prices and nonlabour income. On the other hand, with absence of markets for the domestic good and production functions with nonconstant returns to scale the comparative static analysis fails to predict unambiguously the sign and symmetry of the Slutsky matrix.

The last purpose of the thesis is to describe the family fertility decisions. Fertility choices of the parents are treated as discrete choices. In particular, at the start of each period of the reproductive life, the fertility choice is the outcome of comparisons between parents' welfare level with and without an extra child. The dynamic model is developed under a set of simplifying assumptions designed to obtain an analytical solution to the dynamic program. We assume that the instantaneous utility function of couples is of the CARA form and, neglecting that spouses can have different fertility preferences, we abstract from bargaining processes that can occur within the couple. Furthermore, we assume that parents face perfectly foreseen costs of rearing children and these costs are equal across children. Within this framework, we show that in each period of the reproductive life the couple does not have a child if the pure utility gain from not having a child is greater than the utility obtained from saving in costs from having the child.

The chapter performs a simulation exercise of the fertility choice model. Predictions of the simulation are consistent with demographic trends of countries, such as Italy, in which family policies do not provide sufficient and targeted financial support to families with dependent children. As a consequence of inadequate family policies, the total fertility rate is lower relatively to countries with generous family policies (Apps and Rees 2004).

In particular, we describe the fertility outcome of traditional and non-traditional couples. The former is a families in which there is one main wage earner. On the other hand, the non-traditional family is characterized by the presence of two wage earners. We find that the traditional couple decides to have two children with a two year gap between the first and second birth. On the other hand, since both spouses work, the non-traditional couple faces higher opportunity costs and, in turn, higher costs of rearing children relative to the traditional couple. Thus the non-traditional couple has just one child.

The consumption pattern and assets accumulation of the two families are influenced by their fertility decisions. Given a bigger family size, during the reproductive life consumption of the traditional family is in general higher relative to consumption of the non-traditional family. In addition, both families dissave. The traditional family, however, gets heavier into debt than the non-traditional family.

The fertility dynamic model emphasizes the effects exerted on the family size decision by the costs of rearing children. Hence, the chapter briefly addresses the issue of how to measure the costs of rearing children. It is argued that the costs of rearing children corresponds to the amount of income spent or foregone to have and raise children. These costs, as with those associated with other goods, depend both on the quantity, that is the number of children, and the quality, in the sense of quality of life that parents can guarantee to their children by investing time and other material resources in acquiring both consumption goods and investment goods such as education, medical care, and others. The costs of rearing a child are a fundamental part of the information affecting the endogenous choice of having a child. Besides, these costs are not maintained constant for different families because there are differences in quality. For instance, the child of a rich family can complement the education received in public school with the private supply of education. The costs of rearing children reproduces the information known to families at the moment they decide to have a new child. For this reason, to design adequate family policies policy-makers should have in mind these costs. Indeed, one of the major questions regarding social policies is concerned with determining the amount of transfers from the state to families necessary to satisfy the society's objectives given the government budget constraint. Moreover, the efficacy of family policies is critically affected by intra-household resource allocation rules.

References

Apps, P. F. and R. Rees (1996): "Labour Supply, Household Production and Intra-Family Welfare Distribution," *Journal of Public Economics*, 60(2): 199-219.

Apps, P. F. and R. Rees (1997): "Collective Labor Supply and Household Production," *Journal of Political Economy*, 105(1): 178-90.

Apps, P. F. and R. Rees (2001): "Household Production, Full Consumption and the Costs of Children," *Labour Economics*, 8(6): 621-48.

Apps, P. F. and R. Rees (2004): "Fertility, Taxation and Family Policy," *Scandinavian Journal of Economics*, 106(4): 745-63.

Aronsson, T., D. Sven-Olov, and W. Magnus (2001): "Estimating Intrahousehold Allocation in a Collective Model with Household Production," *Journal of Population Economics*, 14(4): 569-84.

Bardhan, P. and C. Udry (1999): *Development Microeconomics*, Oxford: Oxford University Press.

Bargain, O., M. Beblo, D. Beninger, R. Blundell, R. Carrasco, M.-C. Chiuri, F. Laisney, V. Lechene, N. Moreau, M. Myck, J. Ruiz-Castillo, and F. Vermeulen (2006): "Does the Representation of Household Behavior Matter for Welfare Analysis of Tax-benefit Policies? An Introduction," *Review of Economics of the Household*, 4(2): 99-111.

Barten, A. P. (1964): "Family Composition, Prices and Expenditure Patterns," in *Econometric Analysis for National Economic Planning: 16th Symposium of the Colston Society*. eds. P. Hart, G. Mills, and J. K. Whitaker, London: Butterworth.

Basu, K. (2006): "Gender and Say: a Model of Household Behaviour with Endogenously Determined Balance of Power," *Economic Journal*, 116(511): 558-80.

Becker, G. S. (1960): "An Economic Analysis of Fertility," in *Demographic and Economic Change in Developed Countries*, 209-31, Universities-National Bureau Conference Series 11, Princeton, N. J.: Princeton University Press.

Becker, G. S. (1965): "A Theory of Allocation of Time," *Economic Journal*, 75(299): 493-517.

Becker, G. S. and R. J. Barro (1988): "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 103(1): 1-25.

Bellman, R. E. (1957): *Dynamic Programming*, Princeton, N. J.: Princeton University Press.

Benjamin, D. (1992): "Household Composition, Labor Markets, and Labor Demand: Testing for Separation in Agricultural Household Models," *Econometrica*, 60(2): 287-322.

Berloffa, G. and P. Simmons (2003): "Unemployment Risk, Labour Force, Participation and Savings," *Review of Economic Studies*, 70(3): 521-39.

Björklund, A. (2006): "Does Family Policy Affect Fertility? Lessons form Sweden," *Journal of Population Economics*, 19(1): 3-24.

Blackorby, C. and D. Donaldson (1991): "Adult-Equivalence Scales, Interpersonal Comparisons of Well-Being, and Applied Welfare Economics," in *Interpersonal Comparisons of Well-Being*, eds. J. Elster and J. E. Roemer, Cambridge: Cambridge University Press .

Blundell, R. and A. Lewbel (1991): "The Information Content of Equivalence Scales," *Journal of Econometrics*, 50(1-2): 49-68.

Blundell, R., P.A. Chiappori, and C. Meghir (2005): "Collective Labor Supply with Children," *Journal of Political Economy*, 115(6): 1277-306.

Blundell, R., P. A. Chiappori, T. Magnac, and C. Meghir (2007): "Collective Labor Supply: Heterogeneity and Non-Participation," *Review of Economic Studies*, 74(2): 417-45.

Blundell, R., P. Pashardes, and G. Weber (1993): "What Do We Learn about Consumer Demand Patterns from Micro Data?," *American Economic Review*, 83(3): 570-97.

Browning, M. (1992): "Children and Household Economic Behavior," *Journal of Economic Literature*, 30(3): 1434-75.

Browning, M. and P. A. Chiappori (1998): "Efficient Intra-household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 66(6): 1241-78.

Browning, M., P. A. Chiappori, and A. Lewbel (2006): "Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power," Working Paper, Department of Economics, Boston College.

Browning, M., P. A. Chiappori, and V. Lechene (2006): "Collective and Unitary Models: A Clarification," *Review of Economics of the Household*, 4(1): 5-14.

Browning, M., P.A. Chiappori, and Y. Weiss (2006): *Family Economics*, Cambridge: Cambridge University Press (forthcoming).

Cain, M. (1983): "Fertility as an Adjustment to Risk," Population and Development Review, 9(4): 688-702.

Chayanov, A. V. (1986): *The Theory of Peasant Economy*, Madison: The University of Wisconsin Press (original edition 1926).

Chiappori, P. A. (1988): "Rational Household Labor Supply," *Econometrica*, 56(1): 63-90.

Chiappori, P. A. (1992): "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100(3): 437-67.

Chiappori, P. A. (1997): "Introducing Household Production in Collective Models of Labor Supply," *Journal of Political Economy*, 105(1): 191-209.

Chiappori, P. A., B. Fortin, and G. Lacroix (2002): "Marriage Market, Divorce Legislation, and Household Labor Supply," *Journal of Political Economy*, 110(1): 37-72.

Chiappori, P. A. and I. Ekeland (2006): "The Micro Economics of Group Behavior: General Characterization," *Journal of Economic Theory*, 130(1): 1-26.

Couprie, H. (2007): "Time Allocation within the Family: Welfare Implications of Life in a Couple," *Economic Journal*, 117(1): 287-305.

Deaton, A., J. Ruiz-Castillo, and D. Thomas (1989): "The Influence of Household Expenditure Patterns: Theory and Spanish Evidence," *Journal of Political Economy*, 97(1): 179-200.

Donni, O. (2003): "Collective Household Labour Supply: Nonparticipation and Income Taxation," *Journal of Public Economics*, 87(5-6): 1179-98.

Donni, O. (2005): "Labor Supply, Home Production and Welfare Comparisons," Discussion Paper No 1511, IZA.

Donni, O. (2006): "Collective Female Labor Supply: Theory and Application," *Economic Journal*, forthcoming.

Douthitt, R and J. Fedyk (1990): *The Cost of Raising Children in Canada*, Toronto and Vancouver: Butterworths.

Fitzgerald, J. M., M. S. Swenson, and J. H. Wicks (1996): "Valuation of Household Production at Market Prices and Estimation of Production Functions," *Review of Income and Wealth*, 42(2): 165-80.

Fong, Y-K. and J. Zhang (2001): "The Identification of Unobservable Independent and Spousal Leisure," *Journal of Political Economy*, 109(1): 191-201.

Fortin, B. and G. Lacroix (1997): "A Test of the Unitary and Collective Models of Household Labour Supply," *Economic Journal*, 107(442): 933-55.

Goldin, C. and L. F. Katz (2002): "The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions," *Journal of Political Economy*, 110(4): 730-70.

Gorman, W. M. (1976): "Tricks with Utility Functions," in *Proceedings of the 1975 AUTE Conference, Essays in Economic Analysis*, Sheffield, eds. M. J. Artis and A. R. Nobay, Cambridge: Cambridge University Press.

Haddad, L. and R. Kanbur (1990): "How Serious Is Neglect of Intra-household Inequality?," *Economic Journal*, 100(402): 866-81.

Harvey, S. J. (1996): "The Valuation of Household Production: How Different Are the Opportunity Cost and Market Price Valuation Methods?," Working Paper, Department of Economics, University of Hartford.

Heckman, J. (1974): "Shadow Prices, Market Wages, and Labor Supply," *Econometrica*, 42(4): 679-94.

Lazear, E. P. and R. T. Michael (1988): *Allocation of Income Within the Household*, Chicago: Chicago University Press.

Lewbel, A. (1985): "A Unified Approach to Incorporating Demographic or Other Effects into Demand Systems," *Review of Economic Studies*, 52(1): 1-18.

Löfgren H. and S. Robinson (1999): "Nonseparable Farm Household Decisions in a Computable General Equilibrium Model," *American Journal of Agricultural Economics*, 81(3): 663-70.

Lopez, R. E. (1984): "Estimating Labor Supply and Production Decisions of Self-Employed Farm Producers," *European Economic Review*, 24(1): 61-82.

Lundberg, S. (1988): "Labor Supply of Husbands and Wives: A Simultaneous Equations Approach," *Review of Economics and Statistics*, 70(2): 224-35.

Lundberg, S. and R. A. Pollak (2003): "Efficiency in Marriage," *Review of Economics of the Household*, 1(3): 153-67.

Lundberg, S., R. A. Pollak, and T. J. Wales (1997): "Do Husbands and Wives Pool their Resources?," *Journal of Human Resources*, 32(3): 463-80.

Manser, M. and M. Brown (1980): "Marriage and Household Decision Making," *International Economic Review*, 21(1): 31-44.

McElroy, M. and M. J. Horney (1981): "Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand," *International Economic Review*, 22(2): 333-49.

Menon, M. and F. Perali (2007): "The Costs of Maintenance a Child," Working Paper, Department of Economics, University of Verona, Italy.

Merton, R. C. (1971): "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3(4): 373-413.

Michael, R. T. (1973): "Education and the Derived Demand for Children," *Journal of Political Economy*, 81(2, Part 2): S128-S164.

Neary, J. P. and K. W. S. Robert (1980): "The Theory of Household Behaviour under Rationing," *European Economic Review*, 13(1): 25-42.

Pollak, R. A. (1977): "Price Dependent Preferences," American Economic Review, 67(2): 64-75.

Pollak, R. A. and M. L. Watcher (1975): "The Relevance of the Household Production Function and Its Implications for the Allocation of Time," *Journal of Political Economy*, 83(2): 255-77.

Pollak, R. A. and T. J. Wales (1981): "Demographic Variables in Demand Analysis," *Econometrica*, 49(6): 1533-51.

Rapoport, B., C. Sofer, and A. Solaz (2003): "Household Production in a Collective Model: Some New Results," Working Paper, Team.

Rasul, I. (2007): "Household Bargaining Over Fertility: Theory and Evidence from Malaysia," *Journal of Development Economics*, forthcoming.

Rubalcava, L. and D. Thomas (2000): "Family Bargaining and Welfare," Working Paper No 00-10, RAND.

Schultz, T. P. (1990): "Testing the Neoclassical Model of Family Labor Supply and Fertility," *Journal of Human Resources*, 25(4): 599-634.

Seccombe, W. (1995): *Weathering the Storm*, London, New York: Verso.

Sen, A. K. (1966): "Peasants and Dualism with and without Surplus Labor," *Journal of Political Economy*, 74(5): 425-50.

Silberberg, E. (1990): *The Structure of Economics: A Mathematical Analysis*, New York: McGraw-Hill, Inc.

Simmons, P. (2006): "Joint Labour Participation," Working Paper, Department of Economics, University of York, U.K..

Singh, I. J., L. Squire, and J. Strauss (1986): *Agricultural Household Models: Extensions, Applications, and Policy*, Baltimore: John Hopkins Press.

Strauss, J. (1986): "The Theory and Comparative Statics of Agricultural Household Models: A General Approach," in *Agricultural Household Models. Extensions, Applications, and Policy*, eds. Singh, I. J., L. Squire, and J. Strauss, Baltimore: John Hopkins Press.

Thomas, D. (1990): "Intra-Household Resource Allocation: An Inferential Approach," *Journal of Human Resources*, 25(4): 635-64.

Thomas, D., D. Contreras, and E. Frankenberg (1997): "Child Health and the Distribution of Household Resources at Marriage," Working Paper, RAND, University of California at Los Angeles.

Udry, C. (1996): "Gender, Agricultural Production, and the Theory of the Household," *Journal of Political Economy*, 104(5): 1010-46.

Willis, R. J. (1973): "A New Approach to the Economic Theory of Fertility Behavior." *Journal of Political Economy*, 81(2, Part 2): S14-S64.

Wolpin, K. I. (1984): "An Estimable Dynamic Stochastic Model of Fertility and Child Mortality," *Journal of Political Economy*, 92(5): 852-74.