THEORETICAL AND EXPERIMENTAL INVESTIGATION OF EXPLANATIONS FOR THE ELLSBERG PARADOX

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"Why, look you now how unworthily think you make of me. You would play upon me, you would seem to know my stops. You would pluck out the heart of my mystery, you would sound me from my lowest note to the top of my compass, and there is much music excellent voice in this little organ, yet you cannot make it speak.
'Sblood, do you think I am easier to be played on than a pipe? Call what instrument you will though, you cannot fret me, yet you cannot play upon me"

Hamlet Act 3 Scene 2 Shakespeare

SUMMARY

In this work we investigate theoretically and experimentally the explanations for the Ellsberg Paradox. We review the normative and descriptive models of the explanations of the paradox and the experimental evidences. In chapter III, we present the result of an experiment which explores possible explanations of the Ellsberg paradox and test some of the existing explanations of it. Subjects were asked to evaluate 21 different lotteries. The lotteries were designed in such a way that they would be evaluated the same by an expected utility subject. Different theories however predict different representations of the ambiguous lotteries by non-EU subjects. The experiment showed a consistent replication of the Ellsberg paradox. However, no theory seems able to explain entirely their behaviour. Moreover, subjects seem to have evaluated lotteries according to different explanations in different contexts, which suggests that ambiguity can be perceived in different ways in different environments.In chapter IV, we present the results of an experiment in which we test the theories which explain the Ellsberg paradox but in an insurance context. We build two experimental markets to examine individual evaluations of risk reductions with two different risk-management tools: self-insurance and self-protection. First, we do not find any evidence that the risk-reduction mechanism matters. Second, we find that the presence of ambiguity matters to the valuation of self-insurance and self-protection, although changes in the representation of ambiguity do not alter valuation. Finally, our findings do not provide strong support for the Einhorn-Hogarth ambiguity model.

In the last two chapters, we investigate alternative explanations to our experimental results. In particular, in chapter V we investigate the possible criteria to be used to define different degree of ambiguity when ambiguity is expressed by a second order probability distribution. While in chapter VI we analyze how choices can be context dependent and consequently how the preferences elicited in the experiments may reflect the mental process used in the particular elicitation context.

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PREFACE AND PREVIEW

Since the middle of the 1950's decision theory has become a very active research topic in the areas of economics, psychology, mathematics, statistics and philosophy. Of course different discipline deal with different problems and use different instruments, hence we have a quite heterogeneous field of research. *Within* decision theory, since the beginning of the 80's decision theory under *uncertainty* has become a major topic of research. The distinction between risk and uncertainty goes back to Knight (1921) who referred to risk when a situation can be represented through numerical probabilities and to uncertainty when the situation cannot be represented by numerical probabilities. This distinction makes no sense in the standard theories of decision making under risk and uncertainty: Expected Utility Theory and Subjective Utility Theory. Individuals are assumed either to know the probabilities over the occurrence of events or to be able to assess them subjectively. However, the Ellsberg paradox shows that beliefs about uncertainty cannot be reduces to the single dimension of probability and that what people know about a state's probability does influence their willingness to bet on that state.

Since the seminal work of Ellsberg (1961) economists, psychologists, philosophers, mathematicians, that is to say decision theorists, have investigated the individual decision processes under of uncertainty. In addition they have developed theories which try to accommodate the paradox within the general framework of the theory of rational choice. As in the general field of decision making the contribution of the different disciplines are quite heterogeneous and not always the achievements reached in one discipline spread over to the others.

What we want to do in this work is to contribute to this stream of research. On one hand, we want to verify which is the state of art on the subject, and in doing so we want to analyze contribution from all the disciplines mentioned above. On the other hand, we want to investigate ourselves how people behave and reason when facing uncertainty. We choose to carry our investigation mainly through experimental methods, in the conviction that this methodology can help us not only to test theories of decision making, but to give insights into people's mental processes and strategies. In addition, we try to develop our contribution and we interpret our results in the light of the economic as well as the psychology literature. Uncertainty is a pervasive phenomenon in life and consequently, we believe that the increased knowledge in decision making under uncertainty can give us a better understanding of how people act and how they solve economic problems.

Our research consequently takes three main directions; first, we investigate and test some theories of individual decision making under uncertainty; second, we carry over this investigation in a insurance context, dealing with the same decision problems in a economic context; third, we interpret the results obtained in the light of the new developments of behavioural decision making. As any research, our research too, in the attempt to give answers, ends up raising new questions. This fact should not be taken as the sign of the impossibility of giving an answer but simply as a sign of how long is the journey of which our work is just a stop.

The thesis is organized as follows; in the first chapter we review the theoretical models on the Ellsberg paradox. In particular in our review we try to focus on the intuition of the theories which are often very technical, trying to convey the message and the novelty of each theory. Moreover, we underlie all the possible links between the various models, that as we have already said, belong to different disciplines. Being aware that, very often, how decision making under uncertainty is tackled depends on what we intend for uncertainty, we try to show, in each theory, the connection between the explanation adopted and the particular source of uncertainty identified. We try to show consequently how the different solutions, and the various decision criteria adopted in the different theories depend heavely upon their assumptions about the characteristics of uncertainty. It is however important to note that while some of the models analyzed simply try to describe individual behaviour in face of uncertainty others try to insert this behaviour in the general framework of the rational theory of choice.

In the second chapter, we review some of the experimental work on the Ellsberg paradox. Our aim is that of underlying some aspects of this work that constitute the background for our experimental investigation. Many of the works discussed are by psychologists and are related more to the investigations of the possible sources of uncertainty than to tests of new or existing theories of the Ellsberg paradox. This shows how contribution developed in other discipline can enrich our own research.

In the third chapter, we presents the result of an experiment which explores possible explanations of the Ellsberg paradox and tests some of the existing explanations of it, in a lottery context. Subjects were asked to evaluate 21 different lotteries. The lotteries were designed in such a way that they would be evaluated the same (except two or four depending on the personal beliefs of the subjects) by an Expected Utility subject. Different theories, however, predict different evaluations by non-EU subjects. There are lotteries designed to verify the models of Raiffa (1961), Segal (1987), Kadane (1991), Schmeidler (1989), and Gardenfors and Sahlin (1982,83).

The experiment showed a consistent replication of the Ellsberg paradox. Most subjects value the different representations of the ambiguous lotteries less than the unambiguous one. However, no theory seem able to explain entirely their behaviour. Some theories, however, Schmeidler (1989) and Gardenfors and Sahlin (1983) for example, receive more support then others. Moreover, subjects seem to have evaluated lotteries according to diffeerent explanations in different contexts, which suggests that ambiguity can be perceived in different ways in different environments. The evaluation of the lotteries seems to proceed in stages, and the focus of attention was on different elements at different stages of the process.

In the fourth chapter, we test some theories of decision making under uncertainty in an insurance context. The outcome of the decision problem are consequently losses and not gains as in the previous chapter. We examine individual evaluations of risk reductions with two different risk-management tools: self-insurance and self-protection and under two different conditions: risk and uncertainty. To do so we construct two experimental markets one for self insurance and one for self protection. First, we do not find any evidence that the risk-reduction mechanism matters. Second, we find that the presence of ambiguity matters to the valuation of self-insurance and self-protection, although changes in the representation of ambiguity do not alter valuation. Our findings do not provide strong support either for the Expected utility theory nor for Gardenfors and Sahlin nor for the Einhorn-Hogarth ambiguity models. In the case of losses the individual behaviour seems to respond to less clear rules both in case of risk as well as in case of uncertainty. As far as the single models are concerned only from the analysis based on individual data we can say that a sizable portion of our sample switched from ambiguity aversion to ambiguity preference as the probability of loss changed from low to high values yielding some support to the Einhorn and Hogarth model. However, the mean ratio as well as the mean of the differences between ambiguous and risky prices do not show the monotonically decreasing pattern predicted by the model.

In the fifth chapter we suggest two possible criteria according to which a second order distribution can be considered more ambiguous than another second order distributions and we design an experimental test between the two decision rules. We do this in the attempt to explain some of our experimental results, which cannot be explained within the existing framework. In doing this we also suggest some possible paths for further theoretical research when ambiguity is expressed as a second order probability distribution.

In our sixth and concluding chapter, we link the interpretation of our experimental work with the psychological models of constructive preferences. These psychological models may not only explain some experimental results that cannot be explained otherwise, but they constitute a real challenge not only to the standard theories of decision making but also to most of the alternative and more recent ones.

The fact that preferences can be constructed in the elicitation process or depend on the choice set may be disturbing not only to decision theorists but also to economists.

"And why do you pity us, Patominos" the Sovereign asked.
"For many reasons " the eunuch answered,"but above all because men are subject to the law of change. It is a deceitful law, because no change does exist."
"Do you mean that for the sake of seeking this particular change, I should go to some place?"
"Yes, my Lord," answered Patominos, "So as to be persuaded that there is not any change"
"And would be this enough to cure me?"
"Not the persuasion, my Lord, but the experiences that are necessary to reach this persuasion". Die Geschichte 1002. Nacht, Joseph Roth

ACKNOWLEDGMENTS

Writing a thesis is like taking a long journey, (the image is even more appropriate if the degree is taken in a foreign country); you go through different stages, you learn different things and you meet different people. Hence, even if your thesis is the product of an individual effort it retains traces of your travel mates.

From a scientific standpoint I wish to thank some people from whose suggestions I benefited at different stages of this work. A particular thank is due to my supervisor, John Hey, who has been an acute and indefatigable devil's advocate. I am also indebted to Michele Bernasconi, Michele Cohen, Rosalies Eisenberger, Bill Gerrard, John Kagel, Marco Li Calzi, Graham Loomes, Martin Weber, and an anonymous referee.

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I dedicate this work to those who stayed at home:
Adriana and Erminio, my parents.

Declaration

The results of the experiment described in chapter III have been presented at the II Experimental Economics Italian Conference held in Alessandria in February 1994, and at the International Conference of Foundation of Utility and Risk (FUR VII) held in Oslo in June July 1995. These results has also been published as a Discussion paper of the University of York with the title "Evaluating lotteries with unreliable probabilities: an experimental test of theories of explanations for the Ellsberg paradox". The work has been also submitted to a Journal.

Chapter IV is a the product of a joint work with Carmela Di Mauro.
The results of the experiment reported there have been presented at the International Conference on Foundation of Utility and Risk (FUR VII) held in Oslo in June-July 1994, at the Third Italian Conference of Experimental Economics held in Rome January 1995 and at the Annual Conference of the Royal Economic Society held in Canterbury March 1995. The results of the same experiment have been published with the title "The impact of ambiguity on the evaluation of self-protection and self-insurance " as a Discussion Paper of the University of York. The same work has been submitted to the Journal of Risk and Uncertainty and it is currently under revision.

CHAPTER I

CRITICAL REVIEW OF THE THEORETICAL MODELS ON THE ELLSBERG PARADOX.

I.1 Introduction

The concept of ambiguity was articulated for the first time in the seminal article of Ellsberg. In order to define the concept of ambiguity, Ellsberg started by analysing the definition of uncertainty given by Knight (1921): measurable uncertainty or risk is that which can be represented by numerical probability, whereas "unmeasurable uncertainty" that which cannot be represented by numerical probabilities. This latter situation can occur, according to Knight, when the decision maker is ignorant about statistical frequencies of the event relevant to his decision, when relevant calculations are impossible, when the event is unique, or when an important once-and-for-all decision is concerned.

These kind of distinctions make no sense within the framework of the two leading theories of choices in economics and psychology, namely the Expected Utility Theory (EU) of von Neumann and Morgenstern (1944) and the Subjective Utility Theory (SEU) of Savage (1954).

EU assumes that the probabilities of the outcomes are known. If the individual preferences specified over prospects follow a set of axioms, they can be represented by a real-valued function (in which the preferred choices have higher utility numbers) and the utility of a choice is the expected utility of its possible outcomes. Recently the debate on EU had focused, on the one hand, on alternative underlying axioms and their implications for the utility function, and, on the other hand, on the empirical investigation on the violation of the various axioms. However, it is not an object of this review to describe and analyse any of these theories (see for example Machina (1987), Fishburn (1988) or Weber and Camerer (1987) for the empirical evidence).

The standard implication of SEU is instead that people behave "as though" they assign numerical probabilities, or degrees of belief, to each event and state of the world (probabilities are not known but people are assumed to have subjective probabilities over the states).

Moreover, the basic idea is that it is possible to infer these probabilities or degrees or belief on the likelihood of an event through the willingness of the subject to bet. That is, it is people's willingness to bet on an event which reveals their degree of belief in that event. From the subject's preferences over bets, it is possible to infer the subject's subjective probabilities over the events (our actual choices reveal our beliefs) and, if this ordering satisfies certain axioms, all uncertainty can be reduced to risk. On the other hand, since also utilities are derived from preferences, if these preferences satisfy some axioms then preferences can be represented by expected utility. Consequently the Savage approach combines the idea of subjective probability of De Finetti with the theory of EU of von Neumann and Morgenstern.

The distinction between uncertainty (or ambiguity) and risk challenges this kind of reasoning. In fact, the Ellsberg paradox suggests that there are situations in which people do not behave as if they are able to assign numerical probabilities to events. When this happens and why, how pervasive is this phenomenon, and what people do instead of "assigning numerical probabilities", has been one of the main topics of research and investigation in decision theory in the last ten years.

In this review we try to critically analyse and investigate the possible explanations of the Ellsberg paradox. Our main object is to investigate that part of the literature which has been developing alternative formulations of axioms, decision rules and descriptive behavior to SEU. We will try, however, to analyse also the possible links with other streams of thoughts which tackle the problem of decision making under uncertainty. Since our main goal is review the recent theoretical explanations of the Ellsberg paradox we will not review in this chapter any empirical or experimental evidence of the Ellsberg paradox; nor are we going to review generalizations of EU. Moreover, we will limit our analysis to individual decision making, and consequently we will not consider organizational choice under ambiguity (March and Olsen (1976)). Furthermore, we are not going to review some literature which can be considered related to the problem shown by the Ellsberg paradox: ambiguity tolerance as a personality trait, linguistic ambiguity, and the literature on probability elicitation. Neither are we going to review theories of those authors who tackle the problem of uncertainty considering alternative uncertainty variables like possibility (fuzzy set theory, Zadeh (1978)) and potential surprise (Shackle (1954) or the belief functions of Shafer (1976). This choice has been determined by our main objective (try to confine our analysis to theoretical explanations of the Ellsberg paradox) and not by judgment of value. We think, for example, that Shackle's theory of potential surprise, as well as the fuzzy set theory, can be considered alternative ways of approaching decision making under uncertainty. We also choose not to give any any specific account of the thought of Keynes (1921). However since the literature on ambiguity makes explicit and continuous references to this author, we will refer to his ideas when necessary.

The chapter proceeds as follows. In the first section, we will briefly review SEU and the Anscombe-Aumann' (1963) version of it (A-A). In the second section we will describe the Ellsberg paradox and explain why it violates Savage axioms. In the third section, there will be a guide to the literature. The review will follow and theories will be grouped according to the guidelines. In the final section conclusions will be drawn.

I.2 Decision making under uncertainty: Subjective Expected Utility.

Subjective Expected Utility was developed first by Savage (1954) and then derived in a different way by Anscombe and Aumann (1963). Both developments are relevant in describing generalizations of SEU.

I.2.1 *Savage's approach.*

A decision problem under uncertainty is usually formalized through the notions of states, acts and consequences. When a decision is to be taken this means that one or more acts have to be chosen. In deciding on an act, account must be taken of the possible states of the world, and also of the consequences implicit in each act for each possible state of the world, since the consequence, c, of an act depends on which state of the world, s, will occur (with the term consequence Savage intended anything that can happen to a person). According to Savage, if two different acts have the same consequences in every state of the world, there is no point considering them two different acts at all. An act may therefore be identified with its possible consequences.

More formally:

S is the set of all possible states of the world. It is the universal event which is the event having every state of the world as its element.

s is the generic symbol for a state of the world.

A, B, C, are generic symbols for events. An event is a set of states. A,B,C are subsets of S.

F is the set of all acts and f and g are single acts.

C is the set of all consequences and c is a single consequence.

X is the set of all prizes or outcomes and x is a single prize or outcome.

Formally an act f is a function attaching a consequence to each state of the world, that is to say $c_i = f(s_i)$. In Savage, however, the set of consequences coincides with the set of outcomes, $C=X$ and consequently we can consider $x_i = f(s_i)$. Let us consider for example a football match: X is the set of monetary losses or gains. Each team corresponds to a state of the world and an act $f: S \to X$ is a bet resulting in the amount f(s) if the team corresponding to the state s wins the championship.

Furthermore, if we indicate with $p_i(s\ )$ the subjective probability of the occurence of an event then an act, f, can be also described as a vector $(x(s_1), p(s_1); \ldots\ldots; x(s_n), p(s_n))$ (this notation will be used later).

In Savage's framework individuals are assumed to have preferences over acts. So if we indicate with f and g two acts, with $f \sim g$ we will indicate that the act f is indifferent to the act g (for a particular individual), while with $g \succeq f$ we will indicate that f is at least as preferred as g. The object of SEU is to make possible the description of these preferences by a numerical representation, and a way to obtain such representation is to impose on the preferences plausible conditions or axioms and to show that these axioms imply a real valued functional $V: F \to \mathbb{R}$ such that $V(f) \geq$ (is at least as big as) $V(g)$, if and only if $f \succeq g$ (f is at least as preferred as g). In particular, SEU represents preferences over acts by a numerical utility index u and a probability measure on states p, such that an act f is preferred to an act g if and only if the Subjective Expected Utility of f is bigger than the Subjective Expected Utility of g.

The Subjective Expected Utility of f is defined as

$$SEU(f)=\sum_{s\in S} u(x(s))\ p(s) \qquad (I.1)$$

(I.1) is subjective expected utility representation of the weak order $\succeq$ on F $(\succeq, F)$, p is a unique, finitely additive probability measure on the set of the subsets of S and $u{:}X\to\mathbb{R}$ is a bounded utility function unique up to a positive and affine transformation. This uniqueness gives a cardinal utility on the outcome space derived from ordinal preferences among acts.

To obtain this representation the preferences over acts should satisfy the following axioms:

P1 The relation $\succeq$ on F is a simple ordering (or a **weak order**).

Hence a relation among a set of acts f, g, and h is defined as a simple ordering if and only if for every f, g and h $\in F$

a) Either $f \succeq g$ or $g \succeq f$ (preferences are complete: **completeness**)

b) If $f \succeq g$ and $g \succeq h \Rightarrow f \succeq h$ (preferences are transitive: **transitivity**).

Consider now the event $A \in S$: with ~A we will indicate the event having every state of the world as its element except the states belonging to A (~ A can also be denoted by with S/A or A^c or $\bar{A}$).

P2 If f and g are such that

a) In ~A f agree with[1] g and f' with g',

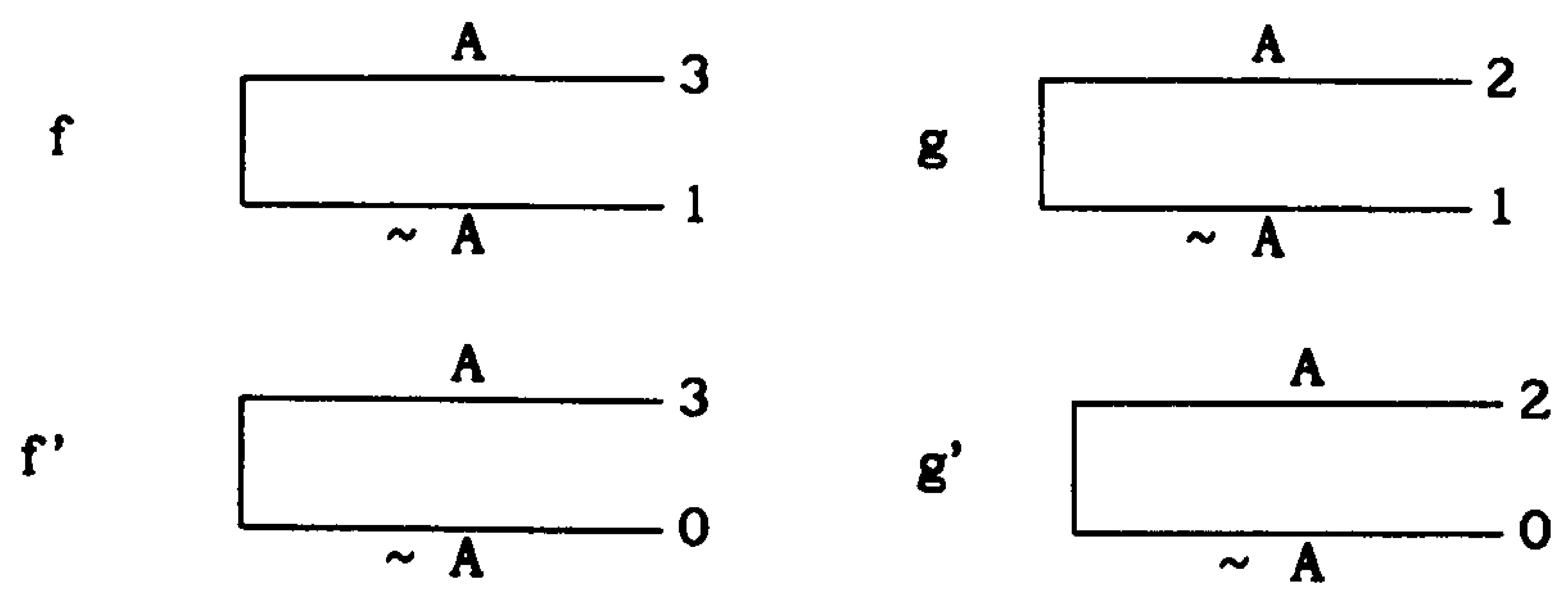
b) In A f agree with f' and g with g'

c) $f \succeq g$ then $f' \succeq g'$.

Consider the following acts where the event A is defined as a red ball is drawn from a bag.

[1] Saying that f "agree with" means that when ~A occurs, under the acts f and g the same consequence c is obtained.

Figure I.1 Illustration of the sure thing principle.



The same can be also express using a matrix form as follows:

	A	~A
f	3	1
g	2	1
f'	3	0
g'	2	0

P2 is also called the **sure thing principle** and it states that if f $\succeq$ g also f' $\succeq$ g'. Since f and g and f' and g' differ only when the event A takes place, the decision maker should not take into any account what happen when A does not occur in defining his or her preferences over acts.

In order to define P3 we need some other concepts and definitions.

A null event:

A $\subseteq$ *S* is null if f is indifferent to g, given A[2], for every f and g $\subseteq$ *F*. According to Savage this is like to say that a person consider A virtually impossible, or in Kreps' words, the event A has probability 0.

"If A is null in this sense then the values acts take on elements of A are irrelevant to all decisions". Savage (1954), p 24.

[2] Given A means once A has occurred.

A constant act: Savage defines as constant acts the acts whose consequence are independent of the state of the world. They are important since they lead to

"a natural definition of preference among consequences in terms of preference among acts". Savage (1954), p 25.

Let us define following Savage $f \equiv c_1$, f is identically c_1, that is to say for every state of the world s, $f(s) = c_1$. Now a preference among consequence can be defined in the following way: For any consequences c_1 and c_2, $c_1 \succeq c_2$; if and only if when $f \equiv c_2$ and $f' \equiv c_2$, $f \succeq f'$

P3 If $f \equiv c_1$, $f' \equiv c_2$ and A is not null; then $f \succeq f'$ given A if and only if $c_1 \succeq c_2$.

Consider the following case :

Figure I.2 Illustration of P3

	s_1	s_2	s_3
f	1	1	1
f'	0	0	0

Where A is equal to (s1,s2,s3). Hence, the axiom states that given A, I should prefer f to f' if and only if I prefer 1 to 0. In particular, the axiom states that

"the knowledge of an event cannot establish a new preference among consequences or reverse the old one", but also assert that, "if the event is not null, no preference among consequences can be reduced to indifference by knowledge of an event" Savage (1954), p 26.

Let us indicate, following Savage, that to *offer a prize in case A obtains* means to make available to a person an act f_A, such that:

$f_A(s) = x_1$ for $s \in A$

$f_A(s) = x_2$ for $s \in \sim A$

Where $x_1 \succ x_2$.

P4 If x_1, x_2, y_1, y_2; A, B; f_A, f_B, g_A, g_B are such that :

1: $x_1 \succ x_2$, $y_1 \succ y_2$

2a $f_A(s) = x_1$ $g_A(s) = y_1$ for $s \in A$

$f_A(s) = x_2$ $g_A(s) = y_2$ for $s \in \sim A$

2b $f_B(s) = x_1$ $g_B(s) = y_1$ for $s \in B$

$f_B(s) = x_2$ $g_B(s) = y_2$ for $s \in \sim B$

3 $f_A \succeq f_B$

then $g_A \succeq g_B$.

In Savage words this postulate assumes that on which of two events a person will choose to stake a given prize does not depend on the prize itself.

In the light of this P4 Savage says that A can be defined as being as least as probable as B, $A \geq B$ if and only if when $x_1 \succ x_2$ and f_A, f_B are such that

$f_A(s) = x_1$ for $s \in A$ $f_A(s) = x_2$ for $s \in \sim A$

$f_B(s) = x_1$ for $s \in B$ $f_B(s) = x_2$ for $s \in \sim B$

Then $f_A \succeq f_B$

Let us consider the axiom in the matrix form:

Figure I.3 Illustation of P4

	A	~A
f_A	x_1	x_2
g_A	y_1	y_2

	B	~B
f_B	x_1	x_2
g_B	y_1	y_2

If we recall now that an act f can be described as a vector $(x(s_1), p(s_1);.....;x(s_n), p(s_n))$, in our case f_A can described as $(x_1(A), p(A); x_2(A), p(\sim A))$ and f_B by $(x_1(B), p(\sim B); x_2(B), p(\sim B))$. The preferences between f_A and f_B can depend only on the fact that the event A is considered more probable that the event B (given that $x_1 \succ x_2$). But if A is considered more probable than B than the axiom states that we also have to prefer g_A to g_B even if we got different prizes, provided that $y_1 \succ y_2$ (in both case we got the best prize under the same event).

P5 There is at least one pair of consequences x_1 and x_2 such that $x_1 \succ x_2$. (which means that there must at least a worth-while prize; this axiom is also called not-triviality)

The axioms P1-P5 have the following important implication which is the derivation of probability from preferences.
Savage defines a relation $\geq.$ as qualitative probability if and only if for all events B, C, D,

1. $\geq.$ is a simple ordering
2 $B \geq. C$ if and only if $B \cup D \geq. C \cup D$. provided that $B \cap D$ and $C \cap D = 0$
3 $B \geq. 0$ and $S >. 0$

Theorem 1 The relation $\geq.$ as applied to events is a qualitative probability.

Postulate P6' and P6 are used by Savage to assign a *numerical* probability to each event and consequently not having to use the concept of *qualitative* probability.

Now we define the concept of probability measure:

a probability measure on a set S is a function p(B) attaching to each $B \subset S$ a real number such that :

1 $p(B) \geq 0$ for every B

2 If $B \cap C = 0$, $p(B \cup C) = p(B) + p(C)$

3 $p(S) = 1$

P6' if $C \succ B$ there is a partition of S the union of each element of which with C is more probable than B.

This last axiom implies that the agreement between qualitative and numerical probability is strict and it is necessary to define as we already said probability measures over events. A stronger version of the axiom is given by P6 and this axiom extend the same kind of requirement, not only to those special acts by which probability is defined, but to acts in general.

P6 If $h \succ g$ and x is any consequence; then there exist a partition of S such that if g and h are so modified on any one element of the partition so as to take the value x at every s there, other values being unchanged: then the modified g remains less then h or g remain less then the modified h, as the case may require. (These modifications cannot change our pattern of preferences, the axioms is stated following Savage).

This axiom has mainly a technical meaning and can be compared with the Archimedean axiom or the continuity axiom. It is to some extent a restriction on how good or how bad an outcome can be. The axiom, for example, fails if there exist an outcome so good that any positive probability of getting it will make the act that contains it better than some other act. This axiom moreover implies that the partition in which S is divided is finite or that S can be divided into finitely many pieces, and implies in connection with the other axioms that the state space must be infinite. The approach of Anscombe and Aumann does, as we shall see later, not require this property of the state space.

P7 For all $A \subseteq S$

$f \succ g(s)$ given A for all $s \in A$ implies $f \succsim g$ given A

$g(s) \succ f$ given A for all $s \in A$ implies $g \succsim f$ given A.

P7 is a dominance condition (or sure thing or independence condition) and it is not required to derive probability from preferences. Fishburn gives a weaker version of the Savage axiom substituting $\succsim$ for $\succ$. Fishburn (1970) p 192.

I.2.2 *Anscombe-Aumann's (1963) approach.*

The other main approach that has been widely used in the models which allow for the behaviour revealed by the Ellsberg paradox is that of Anscombe-Aumann (1963) (A-A). In the Savage set up there is no distinction between objective and subjective probabilities: all the probabilities are subjective. To obtain the representation (see (I.1) of section I.2.1), however, is quite complicated. What A-A do is to infer subjective probabilities through the use of objective random devices. In their own words

"The purpose of this note is to define the person's probabilities in terms of chances by an extension of the von Neumann and Morgenstern theory." Anscombe-Aumann (1963) p 200.

What they do in practice is to enrich the choice set with imaginary objects which are compound lotteries and to construct preferences over them. They moreover introduce the distinction between two kind of lotteries: roulette lotteries in which an uncertain event is associated with a known chance; and horse lotteries in which either chance cannot be associated with the uncertain events in questions or they are unknown (as if we were observing a horse race). Compound lotteries are just lotteries the outcome of which are simple lotteries; they are constructed by iteration from simple lotteries.

In Anscombe-Aumann's model, we consequently have:

S a finite set of states of the worlds (while in Savage *S* must be infinite)

X a set of prizes

$P(X)$ is the set of all probability measures with the finite support on the outcome space $X(s)$ as the set of consequences. These consequences are pure risky lotteries. Hence an act assigns to each state s a lottery $f(s) \in P(X)$. (While in Savage an act is a function that assigns to each state a consequence or a prize , f(s) = x, in A-A, f(s) is a risky lottery, so what Savage call acts are in fact consequences in the A-A framework). A-A assume that the decision maker has preferences over these acts which are called lottery acts.

H is the choice space and it is the set of all functions from outcome (lottery acts) of the horse lottery to probability distributions over prizes (acts in Savage).

In practice a $h \in H$ is

"a betting ticket which specifies, for each possible outcome of the horse race, a roulette wheel lottery that is won by the holder of the betting ticket" Kreps (1988) p 4.

The representation theorem (using Kreps's notation) is given by:

there exist functions $p:S \to [0,1]$ with $\sum_{s \in S} p(s)=1$ and $u:X \to R$ such that

$$f \succ f' \text{ iff}$$

$$\sum_{s \in S} p(s)[\sum_{x \in X} f(s)(x)u(x)] > \sum_{s \in S} p(s)\ [\sum_{x \in X} f'(s)(x)u(x)] \qquad (I.2)$$

where f(s) is a probability distribution on X so f(s)(x) is the probability that f(s) give the prize x. Where each f(s) give some prizes with certainty then we get exactly the Savage representation with f(s) = x.

To reach the above representation theorem the following axioms must hold:

A1 $\succ$ over *H* is a preference relation (this axiom is comparable with Savage's P1)

Individuals have preferences over the horse lotteries. These preferences are complete and transitive (as the preferences over acts of Savage).

A2 $h \succ h'$ and $\alpha \in (0,1]$ imply that $\alpha h + (1-\alpha)g \succ \alpha h' + (1-\alpha)g$ (which is comparable with P2 or the sure thing principle) **mixture independence.**

Given the preference relation between the two horse lotteries h and h' and a probability measure α, the mixed lottery which gives h with probability α and g with probability $(1-\alpha)$ will be preferred to the mixed lottery which gives h' with probability α and g with probability $(1-\alpha)$. In practice, our preferences between these mixed lotteries should not be conditioned by what is the constant factor g.

A3 $h \succ h' \succ h''$ imply there exist $\alpha, \beta \in (0,1)$ such that $\alpha h+(1-\alpha)h'' \succ h' \succ \beta h+(1-\beta)h''$.(This is comparable with P6 or archimedean axiom)

The axiom states that there will be no horse lottery h so good that for $h' \succ h''$, a small probability β of h and a large probability of $(1-\beta)$ of h'' is always better that h'. And that there is no horse lottery h'' so bad that for $h \succ h''$, a large probability α of h and a small probability $(1-\alpha)$ of h'' is always worse than h'.

Stated in this way, with horse lotteries the axiom lacks intuitive appeal. Let us consider the case in which h, h' and h'' instead of being horse lotteries are goods (or bads). Consider the case for example that h is going to Paris for the weekend, h' is staying at home and h'' dying. The axiom states that if h is preferred to h' (I prefer to go to Paris than staying at home) a mixture of h and h''(dying by a flight accident) will not change the preference relation between h and h'(provided that the probability is small enough). In practice I prefer to go Paris even if in order to go to Paris I will take the plane.

Until now we have confined our discussion to the presentation of Savage's and Anscombe-Aumann's models and axioms. There are, however, other two axioms or requirements that preferences should satisfied which are implicit in both frameworks, but that are not separately stated, but which are important because they can be specifically relaxed by some generalizations of or alternative theories to SEU: the invariance principle and the reduction principle.

The Invariance Principle:

The evaluation of prospects does not depend on how the decision problem is presented or described to the decision maker.
This means that different representations of the same problem should yield the same preferences.

"Invariance encompasses two requirements: *description invariance* and *procedure invariance.* Description invariance demand that preferences among options should not depend on the manner in which they are represented or displayed. Two representations that the decision makers, on reflection, would view as equivalent descriptions of the problem should lead to the same choice-even without the benefit of reflection.
Procedure invariance demands that strategically equivalents methods of elicitation will give rise to the same preference order. For example, the standard theory assumes that an individual's preference order can be established either by offering that individual a direct choice between the two options under study, or by comparing their reservation prices. Furthermore, the theory assumes that the two procedures yield the same order." Amos Tversky (1993) p 3.

Reduction of the compound lotteries axiom:

We will indicate with L_i a compound lottery, with f_i a simple lottery with p_i the probabilities of the outcomes x_i of the simple lotteries, and with P_i the probabilities of the simple lotteries f_i which are the outcome of the compound lotteries L_i.
Let us consider lottery $L_1 = (f_1 P_1; \ldots . f_m P_m)$ and let $f_i = (x_1^i p_1^i; \ldots ; x_{ni}^i p_{ni}^i)$ with $i = 1, .., m$ The reduction of compound lottery axiom states that a two stage lottery is reduced to a single stage lottery using the usual rule:
$(f_1 P_1; \ldots f_m P_m) \sim (x_1^1 p_1^1 P_1 ; \ldots ; x_{n1}^1 \; p_{n1}^1 P_1 ; ., ; x_1^m \; p_1^m P_m ; \ldots ; x_{nm}^m \; p_{nm}^m P_m)$

The axiom states that the decision maker is indifferent between a two-stage lottery and its equivalent one-stage lottery, where all uncertainty is resolved in the first stage. That is to say that the decision maker is indifferent about the way in which uncertainty is resolved and he or she cares only about the probabilities of the final outcome.

As we will see in the next section the Ellsberg paradox suggests the presence of individual behaviour that systematically violates two of the main axioms or requirements stated above: the sure thing principle and the additivity of probabilities[3]. Hence most of the models which accommodate the paradox weaken one or more of the axioms described above. We will consequently use the Savage and the Anscombe-Aumann models as benchmarks for our description of the models on ambiguity.

I.3 The Ellsberg paradox.

Ellsberg started from the observation that there are circumstances under which "people do not always assign, or act as though" they assigned, probabilities to uncertain events. The factor which explains the non capability of assigning probabilities to events is the presence of uncertainty. Hence, according to Ellsberg, if we observe that, with respect to certain events, people

"did not obey, nor did wish to obey - even on reflection - Savage's postulates or equivalent rules " Ellsberg (1961) p 646,

we can conclude that we are in presence of uncertainty. The presence of uncertainty is, thus, revealed to us by people's inability to assign probabilities to events.

If this is true, Ellsberg concludes, there is simply no way to infer meaningful probabilities for events from people's choices. Hence, theories that describe uncertainty in terms of probabilities cannot be applied, unless it is possible to devise different operations to measure probability. Moreover, in this case, people cannot be described as expected utility maximizers on the basis of numerical probabilities that they assign to events. In addition, it would be impossible to derive a von Neumann and Morgenstern utility function from their choices among gambles involving those events.

Let us now describe the kind of choices which according to Ellsberg violate the Savage axioms and why. We will follow Ellsberg (1961) in

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3 Ellsberg interprets his paradox also as a possible violation of the ordering axiom P1 or A1. However the following literature on ambiguity mostly explains the paradox as a violation of P2. Probably this is due to the fact that the normative consequences of a violation of P1 are more serious than the one due to a weakening of P2.

the account below.

I.3.1 *The two-colour example.*

We have two urns, urn I and urn II, both of them containing black and red balls; from one of the two urns a ball is drawn at random and we can bet on red or on black.
Betting on red, we will receive a prize of $100 if we draw a red ball (if red occurs), while we will receive no prize if we draw a black ball (non red occurs).
Consider the two choice problem:

Table I.1 Two-colour Ellsberg Paradox

		100	
		R	B
Urn I	RI	$100	$0
	BI	$0	$100
		R(50)	B(50)
Urn II	RII	$100	$0
	BII	$0	$100

We have the following information:
Urn I contains 100 red and black balls *but we do not know* their ratio;
Urn II contains *50 red balls and 50 black* balls.
Suppose that we are asked our preferences over the above pairs of gambles with the aim of inferring from our preferences our subjective probabilities over the various events.
We can be asked

1. whether we prefer RI or BI [4]
2. whether we prefer RII or BII
3. whether we prefer RI or RII
4. whether we prefer BI or BII

Ellsberg considers the case when we are indifferent between RI and BI

[4] From here ahead the bet "betting on red in the first urn" will be called RI while betting on red in the second urn will be indicated by RII, BI and BII are defined analogously.

and between RII and BII (we do not have preference over colour).
As far as questions 3 and 4 are concerned

a. we can be indifferent within each pair of options, that is to say we are indifferent between urn 1 and urn 2 or

b. we prefer to bet on Urn 2 then RII is preferred to RI and BII is preferred to BI or

c. we prefer to bet on Urn 1 then RI is preferred to RII and BI is preferred to BII

If our preferences are like the ones in group b or c we are violating the Savage axioms.

Violation of the complete ordering axiom and of the sure thing principle. The complete ordering axiom, **P1**, states that preference should be complete and transitive. If the complete ordering axiom holds, it is possible to perform certain transformations on the considered choices without affecting the preference ordering. In particular it is possible to substitute for one choice another one which is indifferent to the first one: that is to say if f~g and g~c then f~c.[5]

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[5] In this way we are applying transitivity to an indifference relation.

The sure thing principle, **P2**, states that

"the choice between two actions must be unaffected by the value of the payoffs corresponding to events for which both actions have the same payoff. (i.e. by the value of the payoffs in a constant column) Ellsberg (1961) page 649.

If the axiom holds, we can replace the constant column with another constant column and this operation would not change our preference ordering.

To show this Ellsberg considers the case of an individual who prefers RII to RI.

To show why the paradox implies the violation of the sure thing principle and of the complete ordering axiom, Ellsberg slightly modifies the two choice problems described in Table I.1.

Let us assume that the balls in urn 1 are marked with number 1 and that the balls in urn 2 are marked with the number 2. Let us now consider the case in which the content of the two urns is dropped in a single urn. Now this urn contains 100 black and red balls marked with the number 1 but in an unknown proportion and 50 red balls marked with the number 2 and 50 black balls marked with the number 2.

Table II.2 The modified Ellsberg example

	100		50	50
	RI	BI	RII	BII
Act 1	$100	$0	$0	$0
Act 2	$0	$100	$0	$0
Act 3	$0	$0	$100	$0
Act 4	$0	$0	$0	$100

Let us define other two choices: 5, which corresponds to choose RI or BI (RI or BI, I choose the balls marked with one) and 6, which corresponds to choose RII or BII.

5	$100	$100	$0	$0
6	$0	$0	$100	$100

Ellsberg considers the case of an individual who is indifferent between 1 and 2, between 3 and 4and between 5 and 6.
The same individual however is supposed to prefer 1 to 3[6].

1	$100	$0	$0	$0
3	$0	$0	$100	$0

We apply the sure thing principle between 1 and 3 and substitute the last constant column and we obtain (the superscript will always indicate the new choice obtained from applying either **P1** or **P2.**)

1'	$100	$0	$0	$100
3'	$0	$0	$100	$100

and 1' should be preferred to 3'.
Applying the complete ordering axiom, since 3' is 6, and 6 is indifferent to 5, we can substitute 5 for 6.

1''	$100	$0	$0	$100
3''	$100	$100	$0	$0

Applying the sure thing principle to the first column we obtain

1'''	$0	$0	$0	$100
3'''	$0	$100	$0	$0

Applying the complete ordering axiom, since 1''' is equal to 4, and 3 is indifferent to 4 I substitute 1''' with 3. Since 3''' is equal to 2

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[6] Our individual is indifferent between betting on RI or BI, on RII or BII, is indifferent in betting on RI and BI and RII and BII. However he prefers to bet on RI than on RII.

and 2 is indifferent to 1, I substitute 3''' with 1.

3	\$0	\$0	\$100	\$0
1	\$100	\$0	\$0	\$0

Hence, if our preference order should be the same in spite of the applied transformation our individual should prefer 3 to 1 which is in contradiction with the starting assumption that he prefers 1 to 3.

Violation of Additivity in Probabilities. Let us now consider the previous case of the two urns and consider an individual who prefers RII to RI. From this choice an external observer can infer that the individual regards the probability of red in urn 2 greater than the probability of red in urn 1, that to say p(RII)>p(RI). Let us, moreover, consider the case in which the external observer also sees that our individual prefers BII to BI. From the choices of the individual, it is possible to conclude that our individual not only regards red in the second urn as more probable that red in the first urn; he also regards not-red in the second urn as more probable than not-red in the first urn that is to say p(BII)>p(BI). We know that BI and RI being disjoint events the $p(RI \cup BI) = p(RI) + p(BI) = 1$, the same holds for RII and BII, hence $p(RII \cup BII) = p(RII) + p(BII) = 1$ according to the probability laws. However the Ellsberg paradox shows that people behave as if $p(RII)+p(BII) > p(RI)+ p(BI)$ which is incompatible with additivity in probabilities.
Ellsberg's conclusion is that, in this case, the choices are not revealing judgments of probability at all and so as far as the events above described are concerned, it is not possible to infer probabilities from the choices since some of the Savage axioms are violated.

I.3.2 *Three colour example*

This first example of the Ellsberg paradox can be criticized on the grounds that people rarely face situations in which they are completely ignorant. As far as Urn I is concerned, we do not know anything about the proportion of red and black and hence any proportion can be considered equally likely. To avoid the above criticism Ellsberg considered another example. Consider an urn, urn 3, containing 90 balls; 30 are red and 60 are black and yellow, but in an unknown proportion. One ball is to be drawn from the urn. In this case we are not completely ignorant about the composition of the balls in the urn. We know that there are 30 red balls; what we do not know is the proportion of yellow and black balls.

Consider now the following choices:

Table I.3 Three colour example

First choice

	R(30)	B (60)	Y (60)
R	$100	$0	$0
B	$0	$100	$0

Second choice

	R(30)	B (60)	Y (60)
RY	$100	$0	$100
BY	$0	$100	$100

RY is bet on red or yellow and BY is bet on black or yellow.

We can ask the usual question about which choices we prefer. If we prefer R to B (betting on red is preferred to betting on black) in the and BY to RY (betting on black or yellow is preferred to betting on red or yellow) we are committing the Ellsberg paradox. This kind of pattern violates **P1** (if you prefer R to B you should prefer RY to BY); the two pairs of choices in fact differ just for the constant column. Preferring R to B and BY to RY implies again violation of the additivity property of probabilities. The above pattern of preferences shows an individual who prefers to bet on red than to bet on black but, at the same time, he or she prefers to bet against red than against black. This means that the individual thinks red more likely than black and, at the same time, he or she thinks also that not red is more likely than not-black.

I.4 Ellsberg's definition of ambiguity.

As we have previously said, Ellsberg identified the presence of ambiguity with the fact that people are not acting as though they assign numerical or even qualitative probabilities to the events in question.

If we define ambiguity in this way, we must also define the following two concepts:

a. what is the quality that makes this kind of uncertainty different from others ?

b. which decision rule are people following when displaying Ellsberg paradox behaviour ?.

Ellsberg answered question (a) by saying that

"what is at issue might be called the ambiguity of [the relevant] information, a quality depending on the amount, type, reliability and 'unanimity' of information, and giving rise to one's degree of 'confidence' in estimate of relative likelihoods". Ellsberg (1961) p 657.

Consequently, in the presence of ambiguity, according to Ellsberg, how people actually behave or act can

"depend on another sort of judgment, about the reliability, credibility, or adequacy of his information (including his relevant experiences, advice and intuition) as a whole: not about the relative support he may give to any hypothesis as opposed to another, but about his ability to lend support to any hypothesis at all" Ellsberg (1961) p 659

The lack of reliability in the information makes choices depend on other factors, according to Ellsberg. This can happen, even if we assume, as Ellsberg does, that people can always assign a relative likelihood to a state of nature, reflecting the support that experience, intuition and information can give to different hypotheses.

As we will see later, it is in the relation between the ability to assign relative likelihoods to an event and people's action that we can make a distinction between different positions in the literature. Even in the eventuality that people might be able to assign relative likelihoods to events, in the presence of ambiguity, their actions may depend on some other part of their judgment. In this case, it is important to define what is this other part of their judgment. This other part of their judgment can be weights to attach to probabilities, or be represented in other ways (for example the weight of evidence in Keynes or the competence effect in Heath and Tversky (1991)).

The consequence is that people can assign the same likelihood to an event, or have a given degree of belief, but can act differently, accordingly to other criteria, because what is different is the perception that he or she has of this degree of belief.

In Ellsberg's paper, either the concept of weight or the concept of confidence in one's own judgment characterizes quite explicitly the presence of ambiguity. This, it seems to us, is a consequence of the fact that Ellsberg considers ambiguous a situation where available information is scanty or unreliable, or highly conflicting, or when expectations of different individuals differ widely, or where expressed confidence tends to be low. Moreover if we are in presence of conflicting opinions or evidences, even if the amount of information is high, ambiguity can be high (and the confidence in any particular estimated probability low).

We will illustrate the characterization of ambiguity through the lack of reliability in one's information with the following example taken from Gärdenfors and Sahlin (1982).

Miss Julie is invited to bet on the outcome of three different tennis matches. As far as match A is concerned, she is very well informed about the two players, their physical condition, previous matches and so on. Given all this information, Julie thinks that the match will be even, and consequently that it will be decided by mere chance. As far as the second match B is concerned, she does not know anything of the relative strength of the two competitors, and so, having no information at all, she cannot predict the winner of the match. For match C, the situation is quite similar to that of match B, except for the fact that Julie knows that one of the player is excellent and that the other one is an amateur, so that everybody considers the outcome of the match a forgone conclusion, but she does not know which is which.

It is clear that the willingness of Miss Julie to bet on each of the three matches is probably not the same. The first match is not ambiguous, in the second one Miss Julie does not have any information at all, whereas as far as the third match is concerned, the situation is ambiguous (Miss Julie has some information but this information is unreliable). In this case, what is different is not just the quantity of information but also the quality. This last aspect, the quality of information, is not really present in the urns examples considered by Ellsberg, however it seems to me to show more precisely what constitutes Ellsberg's definition of ambiguity.

To sum up it would appear to be the case that ambiguity can be defined either as:

a) missing information

b) unreliable information (not sure or conflicting)[7].

Both the concepts are present in the literature, and as we will see, each author adopts a particular concept as well as a particular source of ambiguity. For this reason, we will adopt a more operational and general definition of ambiguity.

We will define a risky choice a choice based on known probability and we will define an ambiguous choice a choice based on unknown probability.

With known probability we will indicate probabilities which are specified or which are the outcome of a well known random process.

With "unknown probability" we will refer to probabilities which are not exactly specified, or are more than one, or are referred to the occurence of an event (which are called judgmental probabilities). We decided to use this broad definition in order to be consistent with most of the models analysed in the next section. However is is importance to notice that for the competence model of Heath and Tversky (1991) people may prefer to bet on a judgmental probability than on an equivalent chance, even if the judgmental probability is more ambiguous. According to this theory the source of uncertainty is not the judgment over the occurrence of an event but the relative knowledge of the subject of the elements necessary to form such a judgment.

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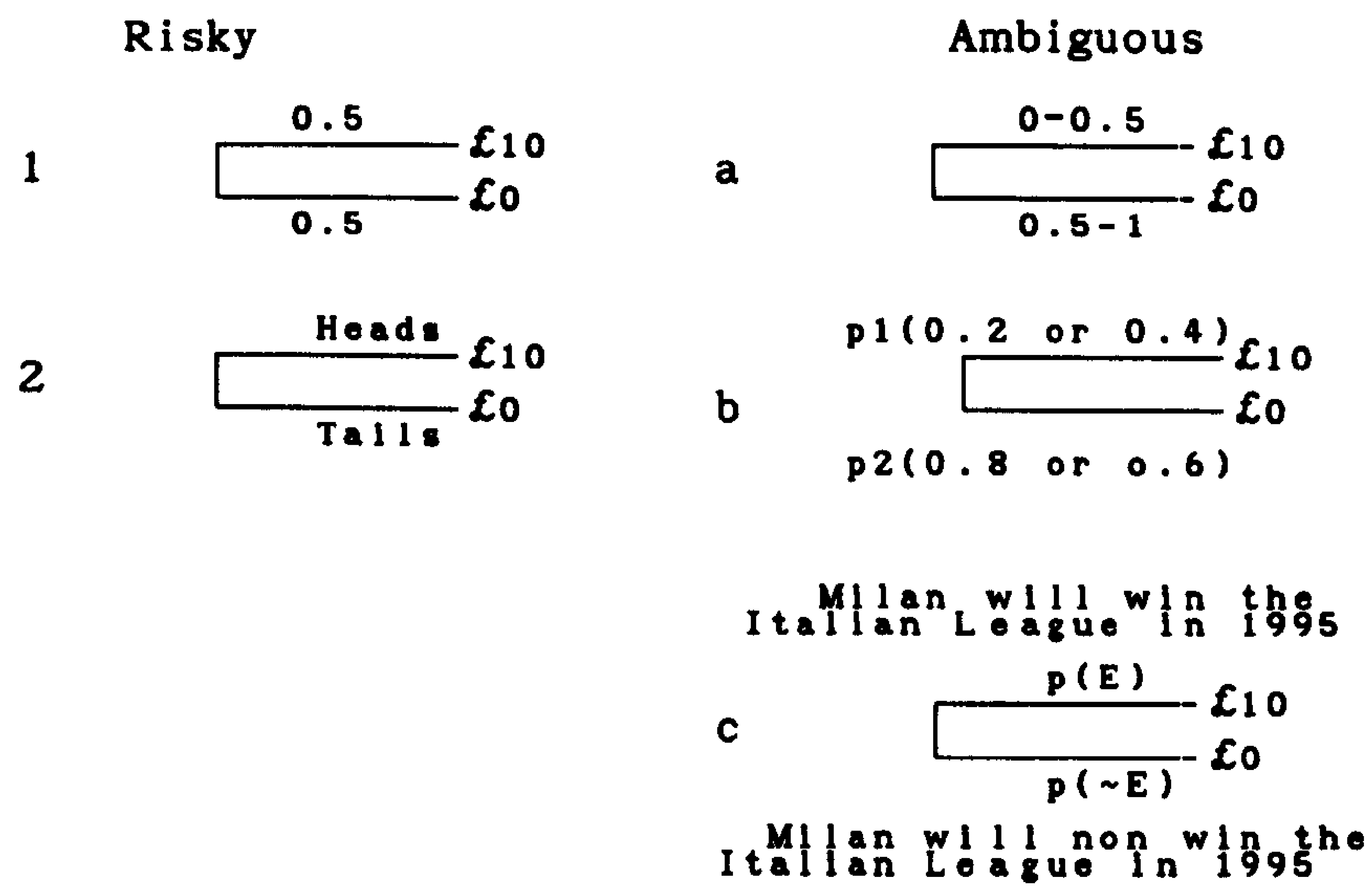
7 In this case what is important is not just the quantity of information but the quality.

Figure I.5 Risky and ambiguous choices



I.5 Theoretical models of explanations for the Ellsberg paradox: guidelines.

In the theoretical literature which tries to devise alternative models which allow for Ellsberg kind of behaviour or which try to respond to the experimental evidence of violation of SEU, we may distinguish four main groups.

a) *Models which simply deny the importance of the violation.* People do commit mistakes and consequently the violation of the axioms of SEU is simply a mistake.

b) *Models which recognize the existence of behaviour not consistent with SEU (I).* These models try to explain the Ellsberg Paradox through a change in the utility of an outcome. For example, the utility of an outcome is modified to reflect a sort of regret, or is discounted when probabilities are unknown.

c) *Models which recognize the existence of behaviour non consistent with SEU (II).* In these models the presence of ambiguity is identified with the existence of more than one probability measure, an unknown probability, an interval of probability, the existence of a decision weight associated with a probability, or a different form of probability with different characteristics.

d) *Behavioural models which are based on the fact that "individual do not posses preferences but mental processes"* (Tversky (1993) talk at the meeting of the International Economic Association held in Turin Italy in October (1993)). The mental processes explain behaviours consistent with the Ellsberg Paradox. It is not possible to have normative theory of decision theory. We can just have a descriptive theory of such a kind of behaviour.

In the following review we will organize the discussion according to the four groups mentioned above. We will give a short account of groups a) and d) mainly to introduce concepts developed in chapters III and V. Most of the formal models of ambiguity can be grouped in b) and c). We will consequently give a more detailed account of these models. Table I.4 is a summary of the analysed models according to the guidelines.

Table I.4 Summary of the review of the leterature according to the guidelines

Models that deny the importance of the violation

Raiffa (1961)

Models that explain the Ellsberg paradox through the modification of the utility

Smith (1969) Winkler (1991) Sarin and Winkler (1992)
Fishburn (1994)

Models which modify the probability

-Models with set of probability measures

Ellsberg (1961) Gardenfors and Sahlin (1982,83)
Levi (1974,86) Gilboa and Schmeidler (1989)

-Models with a second order distribution

Segal (1987)

-Models with adjusted probabilities

Fellner (1961) Einhorn and Hogarth (1985,86,90)

-Models with non-additive probabilities

Schmeidler (1982,89) Gilboa (1987) Nakamura (1990)
Sarin and Wakker (1992,94) Oginuma (1994)

-Models with decision weights

Hazen (1987), Kahn and Sarin (1988) Becker and Sarin (1990)
Hazen and Lee (1991) Tversky and Kahaneman (1992)
Wakker and Tversky (1993)

Psychological Models

Heath and Tversky (1991)

I.5.1 *The mistake interpretation of the Ellsberg Paradox.*

"Some researchers are uncomfortable because the normative theory is not a good descriptive theory of decision making. They would like to consider what is currently observed in some decision-making situations as 'O.K' rather as mistake in term of the normative theory. I believe that just as for me in the case of hydrostatics, the answer lies in education rather than in changing the rules of the game." Howard (1992) p 40.

I.5.1.1 Raiffa's model (1961)

In his note in response to Ellsberg's article Raiffa (1961) suggested the following interpretation of the paradox. People, when analyzing the unknown urn (urn I), do not like it; they are suspicious about it. Hence they evaluate it less than urn II, the one with a known proportion of black and red. This happens because they do not understand the process which generates the data. The subjects do not know in which way the probabilities of the two colours will be defined. The kind of reasoning that can be adopted by an ambiguity averse subject facing urn I can be simplified as follow: "let me consider the unknown urn, I do not understand how the probabilities can be decided, I do not have any insight into the process, so I simply dislike it".
In fact Raiffa writes:

"Immediately I observed what I shall call the two-shift effect. I found that, when relative frequencies or so called objective probabilities were given in numerical form as a data of a decision problem, then these were often used in computing various indices (e.g. expected and actuarial values) which served as a guide to action. But, if certain uncertainties in the problem were in cloudy or fuzzy form, then very often there was a shifting of gears and no effort at all was made to think deliberately and reflectively about the problem ". Raiffa (1961) p 691.

Raiffa's idea is that if we make people reflect on the problem and we explain how the probabilities can be generated, then the violation will disappear. For example, in urn I, the probabilities can be generated by a uniform distribution with a mean value of 1/2 which is exactly the value of the probability of the two colours in urn II. Another way can be to teach people that if they randomize the choice of the two colours than the probabilities of the two colours are again reduced to 1/2. Let us consider the case of drawing a ball from urn I but we do not look at the colour. Now we throw a coin, if tails we will bet on black, if heads we will bet on red. In this way, we randomize our choice using an "objective probability". The probability of red and the probability of black is again 1/2. According to Raiffa, once told, people would realize that calling the colour before or afterwards the drawing is not going to change the problem.

This reasoning is quite simple, however it implicitly assumes that a person can know (or learn) what is a random process and that "all uncertainties" can be reduce to a random process. Is that always possible? In our opinion, if it is possible to teach an individual that given the law of large numbers the probability of head or tails is always 1/2, this is not possible for more complicated random process[8]. Moreover we doubt that all uncertainties can be reduced to random processes. Moreover, experimental evidences show that even if there would be the "need for education to guide choices" (Howard (1990) page 49), people do not always conform to what they are taught. For example, in an experiment done with the three-colour example of the Ellsberg Paradox, Slovic and Tversky (1974) show that people can be immune to persuasion. Immunity to persuasion has been found also by Curley, Yates and Abrams (1986). Immunity to persuasion can maybe be disregarded as evidence of the non-adequacy of the SEU as a descriptive an normative theory of rational choice by people supporting the mistake model. But the fact the people persistently violate the axioms show at least that our arguments in favour are not so compelling.

I.5.2 *Models which explain the Ellsberg paradox through a modification of the utilities.*

" Although ambiguity about probabilities is the ambiguity of concern in this article, I would argue that the influence of this ambiguity on decision-making behaviour generally operate through preferences. Thus attention should be focused on the preference side of modeling rather than on probabilities. The preferences side involve the consequences in the decision model and the value function or the utility function over those consequences" Winkler (1991) page 189.

Let us consider a bet A which gives a payoff of x if an event E happens and a payoff y if the event E does not happen and assume that $x \succ y$.

The value of A can be represented by

$$V(A) = u(x) \, . \, p(E) + u(y) \, . \, p(\sim E) \qquad (I.3)$$

In the approaches described below the ambiguity of the bet is

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[8] In the experiment described in chapter III, for example, when the random process was expressed with a toss of a coin it was understood by more subjects than when it was expressed by a draw of a number from a bag containing numbers from 1 to 13.

expressed through a change in u(x) and u(y) which imply a change in V(A). If we allow the utility of winning a bet be different in the case of ambiguous and unambiguous events, then ambiguity aversion can be in some way consistent with utility maximization. Smith (1969), Franke (1978), Sarin and Winkler (1992), Fishburn (1994) use an utility based approach to the Ellsberg paradox. According to Camerer and Weber (1992) modeling ambiguity aversion through a modification in utilities, or through a modification in the decision weight, or in the probabilities can be seen as a matter of taste. In general a modification on the preference side will suit more the authors who are reluctant to modify the properties of a probability measure[9].

An analysis of the Smith (1969) Sarin and Winkler (1992), Fishburn (1994) papers follows.

I.5.2.1 Smith's (1969) model

Smith's interpretation of the Ellsberg paradox. According to Smith (1969) when people have to express their probability judgment about 'nonstandard' process, the stock price for example, they suffer a sort of utility loss relative to what they experience when they have to express their judgment about more standard processes such as dice games. What makes a probability judgment more demanding in the first case is that there may be real or imagined elements of skill which increase or reduce the subjective value of the outcomes "lose or win". In practice, according to Smith, if the individual loses in a game of

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9 "In all such cases, we are simply saying that the utility of money or other rewards is not independent of the circumstances under which is obtained. The utilities in the payoff matrix may have arguments rather than what appear to be in the 'objective' reward. "Smith (1969) p 325.
In a recent work, Heath and Tversky (1991) also suggest an interpretation of the phenomenon of ambiguity in this direction. The experiment run by them shows that people prefer betting on their own judgment over an equiprobable chance event when they consider themselves knowledgeable but not otherwise. Since judgmental probabilities are more ambiguous than chance events the behaviour described may not be explained by ambiguity aversion. The authors suggest an explanation in terms of the attribution of credit and blame which is very similar to the one of Smith. However, while the Smith explanation is related to the influence of ambiguity on the utility function of the subjects, the one of Heath and Tversky is related to the perception of probability of the subjects and its link with their system of beliefs.

chance he can consider himself a victim of bad luck, but if he makes an incorrect prediction of the rise of a stock price, he can think that he can be blamed by his colleagues for that.

"he may perceive that his colleagues feel that he should have known better, that he is not so smart after all, that they are glad to see his "ignorance" revealed, and so on. Or if he knows nothing about the stock market, then the mysteries and ambiguities in the Dow-Jones may generate special discomfort anxieties when he gamble on such contingencies." (Smith,1961) p 325.

Of course, this is as to say that the utility of money does not depend just on the money itself but on something else; it can depend on the blame or credit of other people, or it can depend on the existence of particular feelings associated with the presence of ambiguity (anxiety or discomfort).

Smith's analyses of the Ellsberg paradox. Let us now recall the two colour Ellsberg example (Table I.1):

In the Urn I we have:

RI :\$100 if red, 0 otherwise and p_1, probability of red, is unknown

BI: 0 if red, \$100 otherwise and q_1, probability of black is unknown

In urn II we have:

RII: 100 if red, 0 otherwise, $p_2=1/2$

BII: 0 if red, 100 otherwise, $q_2=1/2$

Moreover in Urn I urn p_1 is unknown but we can assume it to be equal to n /100. In the say way q_1 can be assumed to be equal to (100-n)/100. Moreover, we know that $p_1+ q_1= n/100 + (100-n)/100 = 1$
Let us now assume that the utility of a choice depends on the monetary outcome x, and on the circumstances under which this monetary outcome x is won. Let us call u_1 and u_2 the utility of the outcome x (\$100) in urn I and urn II respectively.

The fact that we are indifferent between RI and BI implies that

$E(U(RI))= p_1\, u_1(100)+q_1 u_1(0)=$

$E[U(BI)]= p_1\, u_1(0) + q_1\, u_1(100),$

The indifference between RI and BI implies that p_1 and q_1 are[10] equally probable but since $p_1 + q_1 = 1$ then q_1 and p_1 are equal to 1/2.

RII and BII are also indifferent hence implying

$E[U(RII)] = p_2 u_2(100) + q_2 u_2(0) =$

$E[U(BII)] = p_2 u_2(0) + q_2 u_2(100),$

and moreover $p_2 = q_2 = 1/2$.

However the Ellsberg kind of preferences RII>RI imply

$E[U(RII)] = p_2 u_2(100) + q_2 u_2(0) > E[U(RI)] = p_1 u_1(100) + q_1 u_1(0)$

However, from the above reasoning we know that $p_1 = q_1 = q_1 = p_1 = 1/2$.

In this is true then the Ellsberg kind of preferences implies the following inequality $u_2(100) + u_2(0) > u_1(100) + u_1(0)$.

These preferences are consistent only if $u_2(x) > u_1(x)$.

In practice, the utility of a outcome depends on the circumstances under which the outcome is obtained. Smith expresses this monetary loss in the following additive way: $u_2(x) = u_1(x) + \lambda(x)$ where $\lambda(x)$ is the utility loss due to ambiguity. Given a N-M utility function $u_2(x)$ for the standard process, it is possible to derive $u_1(x)$ and $\lambda(x)$ finding the ambiguity premium $\pi(x) > 0$ for each $x > 0$ in a way to make the subjects indifferent between RII and RI[11].

At this point is important to know that for Smith this utility loss is not confined to the unknown probabilities or judgmental probabilities. Smith suggests that even in presence of a standard process (with known probabilities) there will be a sort of utility loss. To show this, Smith suggests that we should represent the lottery in urn I as a 50-50 compound gamble by guaranteeing to the subjects that the number of the red balls in urn I will be determined by a random draw from the integers 0-100. Smith's hypothesis is that probably the preference for urn II gambles over urn I gambles would be changed just a little for many subjects. According to Smith, knowing the data generating process will transform the ambiguity into risk but

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[10] As long as $u_1(100) \neq u_1(0)$.

[11] This has been done in various experimental settings. See the review in chapter II.

will probably not change the preferences of the subjects over the two pairs of gambles. What Smith suggests is that even if we confine ambiguity to a second order distribution and in particular to a uniform second order distribution we will observe ambiguity averse behaviour. In fact this phenomenon has been observed experimentally many times (see Schoemaker (1991)) since this is one of the most common way of expressing ambiguity in a lottery set up experiment. (See also the discussion in chapters III, IV, and V).

I.5.2.2 Sarin and Winkler's model (1992)

Sarin and Winkler develop a more general model of the Smith one in which separation between probabilities and utilities is preserved. In their model the utility loss due to the presence of ambiguity is clearer than in the Smith model since the utilities are made directly dependent on the consequences and on the regret (or rejoicing) due to the consequence that could have been received.

Let us consider a bet A which gives a a payoff of x if an event E happens and a payoff y if the event E (denoted by xEy) does not happen and assume that x>y and in this way we define E as the event associated with x.

The value of A can be represented using a form of SEU by

$$V(A) = u(x) \cdot p(E) + u(y) \cdot p(\sim E) \qquad (I.4)$$

The subject is assumed to express his subjective probability p(E)=p but his or her reaction to the presence of ambiguity is incorporated into the functional form through a modification of the utilities. The modification of the utility will depend on the level of ambiguity and on the difference in the levels of the two payoffs.

If we indicate with v(x/y) the modified u(x) and with v(y/x) the modified u(y) we will have the following functional form:

$$V(A) = v(x/y) \cdot p + v(y/x) \cdot (1-p) \qquad (I.5)$$

As we see the utility of a payoff will depend not only on the payoff that will be received but also on the payoff that will not be received (disappointment). When x-y approaches 0 then v(x/y) → u(x) and the same is true for v(y/x). That is to say that when the payoffs are very close the feeling of regret will be very weak. The modification in the utilities depend also on the level of ambiguity; for the same reason we will have that a individual will experience less regret if put in front to a less ambiguous situation[12].

Sarin and Winkler specified different sets of assumptions about preferences which lead to different modifications of utilities. We will here illustrate the additive representation (they also give a bilinear and a ratio form).

To compare ambiguous and unambiguous lotteries they assume the existence of external devices such as roulette lotteries.

Denoting as before xEy an event lottery which yields x if E and y if ~E and with (x, p, y) a risky lottery which yields x with probability p and y with probability 1-p

The first assumption of the model is the following one.

Assumption 1

If (xEy)~(x,p,y) for some x>y, then (x'Ey')~(x',p,y') for any such x' and y' such that u(x)-u(y) = u (x')-u(y')

This assumption state that the probability premium that is associated with an ambiguous choice stays the same if the amount of the outcome is changed in a way to preserve the difference in utility.

Let us assume that u(x) = x

If $10 if rains and $2 if it does not rain is indifferent to $10 with probability 0.4 and $2 with probability 0.6. Assuming that p(E) = 0.5 my probability premium is 0.1 If the payoffs become $30 and $22 my ambiguity premium should remain unchanged. In practice our attitude to ambiguity, that we elicit through the ambiguity premium we express, is not affected by a change in the outcomes provided the difference between the outcomes does not increase. This is to say that an increase in the difference in the outcomes will affect our feelings of regret or rejoicing. Being the Sarin and Winkler's model based on these feelings

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12 This kind of reasoning however does not say if the level of ambiguity is objective or subjective. For example an equal difference x-y between the two payoffs and an equal interval in the probability seem to imply an equal feeling of regret.

we find quite intuitive that these feelings will be stronger when the occurence or the non occurence of an events leads to two very different outcomes.

If assumption one is true then

$v(x/y) = u(x) + f\ [u(x)-u(y)]$

$v(y/x) = u(y) + f\ [u(y)-u(x)]$

with $f(0) = 0$

if $f(.)$ is linear

$f\ [u(x)-u(y)] = c\ [u(x)-u(y)]$

$f\ [u(y)-u(x)] = d\ [u(y)-u(x)]$

and then linearity implies the following stronger **assumption 2**

If $(xEy) \sim (x,p,y)$ for some $x>y$, then $(xEy) \sim (x,p,y)$ for all $x>y$

which also implies that the probability premium does not depend on the payoffs.

Moreover assumption 2 implies a value function of the following form:

$$V(A) = u(x) \cdot p + u(y) \cdot (1-p) + [pc-(1-p)d]\ [u(x)-u(y)] \quad (I.6)$$

With different sets of assumptions (bilinear for example) they allow for the probability premium to depend on the payoffs; moreover different assumptions leads to different specifications of the $v(./.)$.

This model seems to have some intuitive appeal, especially in its general form, where the modification in the utility depends on the level of ambiguity and on the difference of the payoffs. Its validity is however confined to a single event situation. Moreover, the model does not say if aversion of ambiguity depends on some psychological factor or on objective characteristics. If the latter case some specification or extension about how to incorporate different degree of ambiguity would be appropriate.

I.5.2.3 Variabilities of utilities across states.

In this work Fishburn (1994) introduces an interpretation of the Ellsberg paradox which retains the concept of additive subjective probability as in the case of Smith (1969), Sarin and Winkler (1992), but which departs from SEU because it introduces a factor in the value function which allows for variability of utility across states. In this sense ambiguity aversion can be interpreted as an aversion to variability.

To do so he constructs a model in the framework of A-A, in which preferences are defined over lottery acts H. In order to justify his concept of variability of utility across states, he introduces a distinction between epistemic uncertainty and aleatory uncertainty: epistemic uncertainty (lack of knowledge) is associated with the occurence of the states, while aleatory uncertainty (chance) is associated with lottery probabilities (prospects). The definition mimics definition already present in the literature (A-A for example) but he gives a different interpretation[13]. To insert his definition in the Ellsberg context, for Fishburn the composition of the urn is related to epistemic uncertainty, while the draw of a coloured ball is related to aleatory uncertainty.
We will now depict briefly the model and its properties.

The cav model. As we have already said Fishburn uses an A-A set up. Preferences $\succsim$ are applied to the set H of all functions **h** from S into the set P of all finite support probability distributions p, q on X; where members of H are lottery acts and members of P are lotteries. If we denote with $\pi(s)$ the subjective probability (relative to the occurence) of s (as in Savage) then the value function of a lottery act h will be given by:

$$V(h) = u(h,\pi) - \tau\, \sigma_u(h,\pi) \qquad (I.7)$$

where

$$u(h,\pi) = \sum_{i=1}^{n} \pi(s_i)\, u(p_i) \qquad (I.8)$$

$$\sigma_u(h,\pi) = \left\{ \sum_{i=1}^{n} \pi(s_i) \left[u(p_i) - u(h,\pi) \right]^2 \right\}^{1/2} \qquad (I.9)$$

consequently

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13 In particular he considers epistemic uncertainty something that could be known but it is not. He in practice gives a time interpretation to the distinction.

$$h \succsim h' \text{ iff } V(h) \geq V(h')$$

In (I.7) τ represents the coefficient of aversion to variability; if $\tau = 0$ then the cav model reduces to A-A representation form of SEU.

Letting u($100) = 1 and u($0) = 0 the following table represents the Fishburn's model with a revised version of the two colour Ellsberg example in which the urn contains just two balls.

Table I.5 Fishburn's application of cav to the Ellsberg paradox

	number of black in urn I			
	0 $\pi(0)$	1 $\pi(1)$	2 $\pi(2)$	
BI	0	1/2	1	0 is equal to u (p_0) which is the utility of the lottery which (B,0; R,1)
RI	1	1/2	0	
BII	1/2	1/2	1/2	
RII	1/2	1/2	1/2	

If $\pi(0) = \pi(2) = \alpha$ and $\pi(1) = 1-2\alpha$

If we apply I.7 to BI and RI in the above matrix we obtain:

$$U(BI) = U(RI)$$

$$= 1/2 - \tau \left[\alpha \left(0-1/2\right)^2 + (1-2\alpha)\,(1/2-1/2)^2 + \alpha \left(1-1/2\right)^2\right]^{1/2} \qquad \text{(I.10)}$$

$$= 1/2 - \tau \sqrt{\alpha/2}$$

While

$U(BII) = U(RII) = 1/2$

If we indicate with

$\lambda = 1/2 - \tau \sqrt{\alpha/2}$

according to Fishburn we can say that an individual will be indifferent between BI and the constant lottery act which has the probability λ for $100 and nothing otherwise for each state of the world. In a matrix for:

Table I.6 Fishburn's definition of indifference

BI~CI

	number of black in urn I		
	0	1	2
	$\pi(0)$	$\pi(1)$	$\pi(2)$
BI	0	1/2	1
CI	(\$100,λ;\$0,1-λ)	(\$100,λ;\$0,1-λ)	(\$100,λ;\$0,1-λ)

More generally Fishburn shows the existence of a unique u on P which is a von N-M utility function. The preferences $\succeq$ on P satisfy all the N-M axioms. Fishburn moreover show the uniqueness of π and τ.
As far as λ is concerned Fishburn shows that for each event A such that $\emptyset \subset A \subset S$ there is a unique probability $\lambda_A \in (0,1)$ such that the lottery act (x if A and y if A^c) is indifferent to the constant lottery act that has (x with probability λ_A and y with probability $1-\lambda_A$) in every state (like in Table I.6).
Given two disjoint events A and A^c, λ_A as well as λ_{A^c} are non necessarily additive subjective probabilities (as the Schmeidler (1989) capacities). While $\pi(A)$ and $\pi(A^c)$ are shown to be additive.
Fishburn also provides a system of axioms which preferences upon H must satisfy in order to obtain the representation (I.7). His axiomatisation is however not complete since it is valid just for binary choices. In particular the preferences over binary lottery acts satisfies weak order, continuity and mixture independence as in the A-A model. The only axiom which differs from the A-A model is the comparative independence condition which is equivalent to Savage's P4,

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A4 $h(s)=p$ and $h'(s) = q$ for all $s \in A$ and $h(s) \sim h'(s)$ for all $s \in A^c$
then $$h \succeq h' \Leftrightarrow p \succeq q$$
Two other axioms are needed on the probability λ to assure unicity and additivity of π.
Fishburn's idea of linking the attitude to ambiguity to variability of utilities across state is quite interesting and has intuitive appeal. A form of reaction to variability in utilities even if in a complete different way is also expressed by the models that operationalize

ambiguity through a set of probability measures. In these models, the variability in the utility depends from the fact that we are in front of more utilities according to the different distributions. However the models become quite complicated in order to retain additivity in the probabilities. Moreover, to maintain additivity in the state probabilities Fishburn is forced to introduce λ which is the probability of a constant lottery act $(x\lambda\ 0\ 1-\lambda)$ indifferent to the corresponding lottery act $(x\ A;0\ A^c)$. It is consequently necessary to introduce a sort of "subjective elicited probability" to allow for non additivity.

I.5.3. *Ambiguity expressed as a set of probability measures*

As we mentioned briefly in the introduction, some of the formal models, starting with that of Ellsberg, represent ambiguity as a set of probability measures (SPM). The idea is quite simple: the individual does not have enough information to reach a precise judgment on the occurrence of some event. In fact, he or she possesses an entire class of judgments. The problem become consequently that of establishing a decision rule in order to decide which action to take if this is the case. In this situation, in fact, the simple use of EU does not answer the question since we will have different expected utilities corresponding to the different probability estimates.
Here we will present four models which share the representation of ambiguity as a SPM. However, these four models, Ellsberg (1961), Gardenfors and Sahlin (1982,83), Levi, (1974, 87), and Gilboa and Schmeidler (1989) differ in their structure as well as in the proposed decision rule. However, they have in common the use of a maximin criterion at one stage of the evaluation process even if with different emphasis and motivation.

I.5.3.1 Ellsberg's decision criterion under ambiguity.

According to Ellsberg, there are two elements which characterize the presence of ambiguity:
1) either an individual has many probability judgments (urns example)

or

2) these probability judgments are vague or unsure (Miss Julie example).

As a consequence, in this situation an individual's confidence in his or her assignment of probabilities is very low.

In such a case:

"the judgment of the ambiguity of one's information, of the overall credibility of one's composite estimates, of one's confidence in them, cannot be expressed in terms of relative likelihood or events. (If it could, it would simply affect the final compound probability)" Ellsberg (1961) p 659

Let assume that an individual has to choose between different actions, each of these actions lead to different consequences characterized by different utilities. In this case, ambiguity can be expressed by the fact that the individual possesses an entire set of probability judgments Π on the occurrence of an event. Each of these judgments is characterized by a different degree of reliability.

To compound an overall probability judgment, an individual has to perform the following operations:

1) He or she possibly restricts the entire set Π to some more limited set Π^* in order to consider just the more reliable probability measures.

2) He or she is able to express his or her most reliable (best guess) probability measure π^* (which can be expressed with a different level of confidence).

3) He or she has to apply a decision rule which allows for weighting the existence of more than a probability measure with the best guess probability estimate.

In this way, for example, on the one hand the individual evaluates the situation according to his or her best guess, but at the same time he or she also considers what can possibly happen in the worst case.

Ellsberg's decision rule. Let us assume that an individual has a set of probability measures Π (all the possible reliable π) over the occurrences of some events. With the information he has, he can eliminate some of the probability distributions, hence he reduces Π to Π^* (the most reliable π_1). Moreover between the π_1 he or she is able to form a best guess on the occurrence of the considered events.

Let us consider as an example the first Ellsberg example.

Π is constituted by all the $\pi(RI)$ and $\pi(BI)$ on the range 0 to 100. Let us now assume that for some reason our individual can reduce the set Π to Π^*, which contains just three distributions, (0,1), (1/2,1/2), (1,0). The best guess of our individual is given by π^*= (1/2,1/2)[14]

Ellsberg proposes the following decision rule:

If we indicate with:

ρ = degree of confidence in π^*

x = consequence

π^* = my best guess

A is an act

Π^* = the set which contains all the probability distribution which are still reasonable.

Min EU (A) = minimum expected utility of an act calculated over the entire set Π^*

E* U(A)= expected utility to an act A calculated considering just the best guess distribution π^*

According to Ellsberg's theory the value of an action A is given by:

$$V(A) = \rho \,.\, E^*U(A) + (1-\rho) \,.\, \text{Min } E\, U(A) \qquad (I.11)$$

Give two actions A and B an individual will choose the action with the highest V(.)

$$A \succ B \qquad \text{iff} \qquad V(A) > V(B)$$

As we can see from the model, Ellsberg proposes a linear combination of expected utility and maximin, weighted by the "confidence factor" ρ.

The following example will indicate the application of the Ellsberg model to the two colour Ellsberg problem.

If we indicate with:

ρ = degree of confidence in π^*, assuming 1/4

π^* = my best guess =(1/2, 1/2)

The act that we consider is betting on black in the first urn = BI

Π^* = (0,1), (1/2,1/2), (1,0).

14 For best guess Ellsberg seems to intend the better subjective probability estimate of our individual.

Min EU (BI) = 0

E*U (BI)= 50

$$V(BI) = \rho \cdot E^*U(BI) + (1-\rho) \cdot \text{Min EU (BI)} = = 1/4\ u(50) + 3/4\ u(0) \quad (I.12)$$

If we consider as an action BII (betting on black in the second urn) we will have

$$V(BII) = \rho \cdot E^*(BII) * (1-\rho) \cdot \text{Min E (BII)} = = u\ (50) \quad (I.13)$$

Hence $V(BI) < V(BII)$ so $BII \succ BI$

The same is true also for RI and RII.

In the Ellsberg model, ρ, the level of confidence, depends on the level of information but also on the psychological attitude of the individual. From his account, it seems that, with the same level of information, different individuals can have different level of confidence. If ρ equals 1, the individual will have just a probability judgment and so he will follow EU. The passage from the bigger Π to the smaller Π^* is also determined by the individual's set of information, but Ellsberg does not explain the rules of this reduction.

In the original version with Min EU, the Ellsberg decision rule is a sort of conservative rule[15] (Ellsberg prefers this word to the word pessimism). According to Ellsberg, in a situation of ambiguity such kind of criterion may appeal to a conservative person since it can guarantee a sort of "security" level. This criterion, is a sort of second order criterion, since, if used alone, it implies that people

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[15] The idea underlying his model however is not the one of assuming that the individual is pessimistic, as the one of assuming that the individual are conservative: This means that "our subject does not expect the worst, but he choose to act *'as though' the worst were somewhat more likely than his best estimates of likelihood would indicate.*". Ellsberg 1961 page 667.

The subject distorts his estimate of probability in the direction of putting more emphasis on the less favourable outcomes and in a degree which is in relation with his level of confidence in his best estimate, that is ρ, the level of ambiguity.

would not consider at all those probability judgments for which there is evidence.

According to Ellsberg, this kind of rule will, other things being equal, favour the choice whose expected value is less sensitive to variation of probability distribution within the range of ambiguity. Hence this criterion will favour status quo or present behavior strategies. For these ρ may be high and the range of Π^* small.[16] If the confidence of our subject in his estimate is high, he will conform to the Savage axioms and it would be possible to infer the estimated probabilities from the observed choices.[17]

However, the model can also be suitable to represent cases of optimism if we substitute Min EU for Max EU (we consider the probability vector which corresponds to the maximum expected utility instead of the minimum one)

I.5.3.2 Gärdenfors and Sahlin's model.

Like Ellsberg, Gärdenfors and Sahlin (1982,83) represent ambiguity as a SPM. However, in their model, Gärdenfors and Sahlin emphasized what determines this kind of representation: that is to say the amount and the quality of information in the possession to the agent.

In the "Miss Julie example" reported in section I.4 they write:

"If pressed to evaluate the probabilities of the various possible outcomes of the matches , Miss Julie would say that in all three matches, given the information she has, each of the players has a 50 percent chance of winning. In this situation a strict Bayesian would say that Miss Julie should be willing to bet at equal odds on one of the players winning in one of the matches if and only if she is willing to place similar bets in the two other matches. It seems however perfectly rational if Miss Julie decides to bet on match A, but not on B or C, for the reason that a bet on match A is more *reliable* then a bet on the other. Furthermore she would be very suspicious of anyone offering her a bet at equal odds on match C, even if she could decide for herself which player to back." Gärdenfors and Sahlin (1982) p 314.

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[16] The idea is that for the status quo the degree of confidence ρ in my bet estimated probability is high. At the same time, the set of other still reasonable probability distributions Π^* is narrow, since not only I am very confident in my estimate but also my estimate is more precise. If we consider the set Π^* as an interval of probability the interval will be narrower.

[17] This happens when ρ is equal to 1 and there is just π^* in the set Π^*.

What make the difference in the three probability estimate of Miss Julie is their *epistemic reliability*. This epistemic reliability plays the same role as the level on confidence in Ellsberg's model.

Their model is characterized by two elements:

1) the assumption that people possess a class of probability measures and not just a single probability measure.

2) a new index ρ which is designed to take account of the different reliabilities of the information in the agent's possession.

The process of decision is very similar to that of Ellsberg, but is simpler:

a) first the decision maker has to select a class of probability measures with an acceptable degree of reliability on which to base their decision.

b) relative to this class, our decision maker computes for each act the expected utility

c) he or she chooses finally the act which has the largest minimal expected utility.

The model. As in a Savage's framework, S represent the set of all the states of the world, F the set of all prospects or acts, X is the set of all the consequences or prizes. Gardenfors and Sahlin assume, moreover, that there exists a utility function u(.) mirroring the agent's evaluation of possible outcomes.

What they relax is the assumption that the agent's state of belief, concerning which state is true, can be represented by a unique probability measure. On the contrary, the agent's knowledge and beliefs about the relevant states are represent by a class Π of probability measures defined as the class of all epistemically[18] possible probability measures. The implication of this is that, in the extreme case of complete ignorance, (there is no information at all), all the probability distributions are epistemically possible[19]. On the contrary, in case of perfect information, just one distribution is epistemically

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18 By epistemically possible they intend all those measure which do not contradict the knowledge possessed by the agent.

19 Epistemically means based on our knowledge; hence is like to say all the probability distribution which are reliable according to our knowledge.

reliable and consequently the situation becomes that depicted by the standard Bayesian story.

To each state of the nature there is associated a set of values $\pi(s_i)$ where $\pi \in \Pi$ (It is also assumed that the probability of the outcomes x_{ij} are independent of which alternative is chosen, so that $\pi(x_{ij})=\pi(s_j)$ for all $\pi \in \Pi$ and all acts f_i.

At this point a second order probability measure $\rho(.)$ is introduced. This probability measure is called a measure of epistemically reliability and it is defined over the set Π of epistemic possible probability measures[20].

But what are the properties of ρ?

According to Gardenfors and Sahlin (1982) the only property that ρ need have is that the probability distributions in Π can be ordered with respect to their degree of epistemic reliability. Moreover, it is possible, according to the authors to postulate that ρ can have an upper bound representing the case when the agent has complete information about a probability distribution and a lower bound when the agent has no information about all the probability distributions.

One possible way of considering ρ is as a second order probability measure, that is to say a probability measure defined over the set Π of epistemically possible probability measures. And this is the way in which ρ is considered in Sahlin (1983). However, in Gardenfors and Sahlin (1982) the authors point out that it can also be considered a non-standard probability measure. What is important is that, to rule out some of the probability distributions included in Π, we need a second measure which is related with the degree of our knowledge.

This distinction between the various degrees of epistemic reliability cannot be made just with the use of a probability measure. We need to

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20 "Even though several probability distributions are considered to be epistemically possible some of them are more reliable than others. Some distribution are, for example, backed up by more information then others. This measure of epistemic reliability thus reflects how complete or adequate the knowledge is assessed to be upon which one's first order probability is based". Nils-Eric Sahlin (1983) p 98

"We believe that not all of an agent's beliefs about the states of nature relevant to a decision situation can be captured by a set Π of probability measures. As a second element describing the beliefs relevant to a decision situation, we introduce a (real valued) measure ρ of the epistemic reliability of the probability measures in Π". Gärdenofors and Sahlin (1982) p 318.

use a different kind of measure, that is to say ρ, which measures the level of epistemic reliability of a probability measure.[21]

The decision process. Gardenfors and Sahlin assume that the individual:

a) first reduces the set of all possible epistemic probability distributions of the two alternatives Π to the set Π/ρ_0, where ρ_0 is a level of epistemic reliability that he or she finds satisfactory, that is to say for all $\pi\in\Pi$, $\rho(\pi)\geq\rho_0$

b) secondly, he or she computes the expected utility for each alternative f_1 and f_2 and for each distribution π in Π/ρ_0.

c) Hence he or she determines the minimal expected utility for each alternative.

d) Then he or she selects the alternative which has the largest minimal expected utility.

More formally if $x_{if} = f(s_i)$ is the outcome associated to the occurence of the state si if the act f is chosen, and $x_{if'} = f'(s_i)$ is the outcome associated to the occurence of the state si if the act f' is chosen, and if the utility of an outcome is indicated with u(.), then an individual evaluating the various acts according to Gardenfors and Sahlin's model will prefer f to f' if and only if

$$\min \sum u(f(s_i))\, \pi_{ji} > \min \sum u(f'(s_i))\, \pi_{ji} \qquad \text{(I.14)}$$

If we apply the above decision rule to the two-colour Ellsberg example

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[21] The idea of adding, in order to measure the likelihood of an event or of a statement, to a probability measure another measure or weight can be found, as we can see later, in Keynes work. Also Keynes concept of "weight of evidence" is related to the information we have about an event or a statement. However while in Keynes the concept of weight seems to be more related to the amount of information, in Gärdenfors and Sahlin (1982) the concept of epistemic reliability seems to be more linked to the concept of the quality of the information. The link between Gärdenfors and Sahlin's concept of epistemic reliability and Keynes's concept of weight is clear to them. In their work they quote the following sentence from Keynes. " As the relevant evidence at our disposal increases, the magnitude of the probability of an argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or favourable evidence; but *something* seems to have increased in either case - we have a more substantial basis upon which to rest our conclusion. I express this to say that an accession of new evidence increases the *weight* of an argument. New evidence will sometimes decrease the probability of an argument but it will always increase the "weight". Keynes 1921 p 7.

we will have:

Π/ρ_0 = Π (we do not know anything about the proportion of B and R, that is to say in Π there are all the distributions which vary from 0 to 100 (ρ=0).

If this is true than our individual will prefer BII to BI and RII to RI.[22]

For the agent it is consequently important to choose the desired level of epistemic reliability. According to Gärdenfors and Sahlin (1982), this level of epistemic reliability depends on how large is the risk that he or she is willing to take. If the agent is risk averse he will take a low level of epistemic reliability (he will rule out just a few distributions) if the agent is not risk averse he will take a high level of epistemic reliability.[23]

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[22] In this case, the minimum the minimum expected utility of BI is u(0) (corresponding to a π=0, while the minimum expected utility of BII is u($50), since π=1/2.

If people choose an other level of reliability then the recommended ranking preference would be different. If however we consider (1/2,1/2) the only epistemic reliable distribution then the agent should be indifferent between Bl and BII, and RI and RII as predicted by Expected Utility Theory.

[23] Sahlin defines an agent who takes all possible measures into consideration as a completely epistemic risk averse agent. According to Sahlin we can distinguish between perceived epistemic risk and preferred epistemic risk. The perceived epistemic risk concern the restriction of the set of all epistemic possible probability measures Π to the set of reasonable reliable measures Π/ρ_0. If I take all epistemically possible measures before take any decision I do not take any risk at all. If some of the distribution with $\rho<\rho_0$ are discharged this means that I take some risk. A proposed measure of this risk can be $R(\rho_0)= (1-\rho)\ (\Pi/\rho_0)/\rho(\Pi)$(i).

Hence two agents can can be in a identical epistemic situations, identified by Π and $\rho(.)$ thus according to (i) they can have the same view of perceived epistemic risk and yet have entirely different risk preferences. (I can for example choose a higher level of ρ_0) Let us consider the first example of the Ellsberg paradox. Assume that there are two agents A and B which are identical in their epistemic state. Both of them consider Π to be the class of distribution (1/3,1/3-x,1/3+x) where $0 \leq x \leq 1/3$. Both the agent assign the same epistemic reliability to each of the distributions in Π so that $\rho A(.)=\rho B(.)$. If A prefer a lower degree of risk he can choose $\Pi/\rho_0=\Pi$. B may prefer a higher level of risk and choose Π/ρ_0= (1/3,1/3,1/3) (ρ_0= equiprobable). In this case A will behave as shown by the Ellsberg paradox while B will behave as an utility maximizer.

The two limiting cases of no information or complete information can also be explained by the model of Gärdenfors and Sahlin (1982). In the former case, all the probability distributions over the states are epistemically possible and they have equal epistemic reliability. In this case the minimal expected utility of an alternative is obtained from the distribution which attaches probability one to the worst alternative (in this case the decision criterion is the same as the classical maximin rule). In the latter case, having full information about all the possible states of the world, we will rule out all probability distributions but one, the only one which is epistemically reliable (in this case the decision criterion collapses to that of maximizing expected utility.)

I.5.3.3 Levi's decision theory

In an article in 1987, Levi applies his theory of rational choice to the Ellsberg paradox. For Levi (1974) too, there are situations in which the agent, even when using EU, is not able to reach a final decision since he cannot have a determinate and unique probability judgment. In this case, the individual will possess a set of probability distributions which, according to Levi, contains all the information possessed by the individual. We can call this set Π. Such a set, according to Levi, is determined by the information possessed at a given time, but this set is not characterized by any risk attitude or confidence parameters as the preceding models. Moreover, Levi specifies some attributes that must be possessed by Π. In particular, Π must be non-empty and convex and all the $\pi \in \Pi$ must be finitely additive probability measures. In Levi's words any individual at a given time t possesses a set of probability distributions which represent his "credal state" (beliefs) over the state of nature; and these beliefs are linked with his level of knowledge.
Moreover, Levi assumes the existence of a class G of "permissible" utility measures such that not all these measure need to be linear transformations of another. We will denote these measures by u(.).

In order to be chosen, an act f should posses some characteristics (in fact there are three criteria to be followed in the choice; the application of them follows a lexicographic order):
A) First f should be E-admissible

An action is considered *E-admissible if and only if there is some probability distribution π in Π and some utility function in G such that the expected utility of f relative to π and u is maximal along all the available alternatives.*

B) To be chosen between all the E-admissible acts f must be P-admissible.

An act is considered *P admissible if is E-admissible and 'best' with respect to E-admissible option preservation among all E-admissible options.*[24]

C) Between all the P admissible acts to be chosen an act should be S-admissible.

Definition: A P-admissible alternative is security optimal to an utility function u if and only if the minimum u-value assigned to some possible outcomes of f is at least great as the minimal u-value assigned to any other P-admissible alternative.

An act is S-admissible if is P-admissible and security optimal relative to some utility function in G[25].

If there is no opportunity to defer the choice, according to Levi, the principle of maximizing expected utility theory can be invoked. If all the acts are E-admissible, and all P-admissible, then we have to find another criterion to choose among them.

According to Levi,

"the proposed decision theory identifies situations where the well-known maximin criterion is applied legitimately. Customarily maximin is used to select that option from among all the feasible options which maximizing the minimum gain. This recommendations is legitimate, according to my theory, provided (1) G contains all and only u-functions that are positive affine transformations of one another, and (2) all feasible option are P-admissible. But even if condition (1) is satisfied, it can be the case that the maximin solution from among all the feasible options is not itself E-admissible

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[24] Levi does not provide an adequate explanation of what he intends for "best" with respect to E-admissible option preservation among all E-admissible options. What it seems is that, since in the E-admissible option is included the option of deferring the decision, then the P-admissible option are the E-admissible options superior to deferring the decision. It seem just a criterion to eliminate the possible option of keeping the status quo.

[25] According to Levi in case that there are more then one S-admissible alternatives for the final choice it is possible to assume that the final choice is determined by some random device.

and so cannot be considered to be S-admissible". Levi (1974) p 307.

Let us consider the two-colour Ellsberg example: we know that for Urn I we can have any number of black and red balls between 0 and 100. So we have 101 possible distributions. Hence we compute the expected value relative to each option BI and RI for each probability distribution. (In case of BII we have just one distribution (1/2, 1/2)). Moreover BI and RI are E-admissible. In fact the expected value of both of them reach a maximum relatively to some distributions. For choosing between them we have consequently to adopt a second order criterion and the one suggested by Levi is the maximin criterion. If we compute for each distribution the expected value of an option conditional on the truth of that case we can then assess the corresponding security level. In the case of BII the expected value is 100/2; thus this value can be regarded as the security level since it is the worst possible payoff. As far as BI is concerned the expected value declines from 100 to 0 according to which distribution we consider. Hence if we use this method of computing the expected value (proposed by Wald) the security level associated with BII is higher and consequently the option BII becomes uniquely admissible[26] (The same reasoning applies if we consider the act RII). According to Levi, it is not possible to order the different options according to expected utility considerations, because they are simply not comparable. What is true is that the use of Wald criterion leads to choosing an option which is the best according to one sub class of permissible distributions.

An illustrative example[27]. Let us consider the following example with two states of the world and two acts to show how the models described above can give a different ranking of acts even if they all use a maximin criterion.

Let us consider the following example with two states of the world and three acts[28].

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[26]According to Levi it is not necessary to use the Wald method of computing the maximin, any method could be considered rational because the various subjects can differ in term of their concern for security.

[27]The example is partially taken by Gardenfors and Sahlin(1982).

[28]Outcome are already denominated in utilities.

Figure I.5 Illustration of Gärdenfors and Levi's models

	s1	s2
a1	-10	12
a2	11	-9
a3	0	0

Let us assume that Π of Levi Π/ρ of Gardenfors and Π of Ellsberg are the same Π = {(0.4,0.6); (0.6,0.4)

If we apply Gardenfors and Sahlin we will have that the minimum expected utility min EU (a_1) = -1.2, min EU (a_2)=-1.0, min EU (a_3)= 0. Hence, according to MEUT, $a_3 \succ a_2 \succ a_1$.

According to Levi a1 and a2 are the only E-admissible choice since their EU is maximum respect to some distributions (a3 as always an expected utility of 0, so is never the maximum expected utility and consequently is eliminated as admissible option; a1 has the maximum expected utility (3.2) relative to the distribution (0.4 0.6), while a2 has the maximum expected utility (relative to the distribution (0.6 0.4)); and between the two E-admissible choices, the S-admissible choice is a2, since the minimum expected utility is -1.

Let us now consider Ellsberg. To apply the model of Ellsberg we have to assume, for example, that for our individual ρ = 1/2 (he has the same confidence in each distribution) but his best guess is for some reason (0.4, 0.6).

Figure I.6 Illustration of Ellsberg's maximin

	MinEU	E*U
a1	-1.2	4.2
a2	-1.0	-1.4
a3	0	0

The Ellsberg index will be a1 = 1.5 a2 = -1.2 a3 = 0 and consequently a1≻ a3 ≻ a2.[29]

[29] The rank can change if we choose a different best guess as well as different ρ. However these are individual parameters. The actual value

I.5.3.4 Maximin Expected Utility with a non-unique prior.

Gilboa and Schmeidler's model is an axiomatic model in which the Ellsberg paradox is the result of the fact that, not having enough information on the composition of the urn, the subjects are not able to form an unique prior. Their model is consequently the application of EU in the case of multiple priors; that to say, the subjects possess an an entire set of priors. Moreover, being uncertainty averse, also in Gilboa and Schmeidler the subject adopts a maximin strategy when evaluating the bet. In practice their model can be seen as an axiomatic foundation of the maximin rule.

They use an Anscombe-Aumann framework. In order to allow for uncertainty aversion, their model differ from the A-A in respect to two key axioms: **Certainty independence** (which plays the role of Savage's P2) and **Uncertainty aversion** which has no correspondence in A-A.

To illustrate the two key axioms we need to remember the reader of some definitions:

S is the set of the states of the world, F is the set of all the random lotteries, H is the set of all horse lottery or lottery acts. X is the set of all deterministic outcomes (prizes).

X and S are assumed to be non-empty. H_0 is the set of all $\sum$ measurable finite step function from S to X and H_c are all the constant functions in H_0. Moreover L is assumed to a convex subset on F^S which includes H_c. The preferences are defined over H and they satisfy weak order, continuity monotonicity and non degeneracy.

Certainty independence:

For all f, g in F and h in H_c and for all α in]0,1[:

$f>g$ iff $\alpha f+(1-\alpha)h > \alpha g + (1-\alpha)h$.

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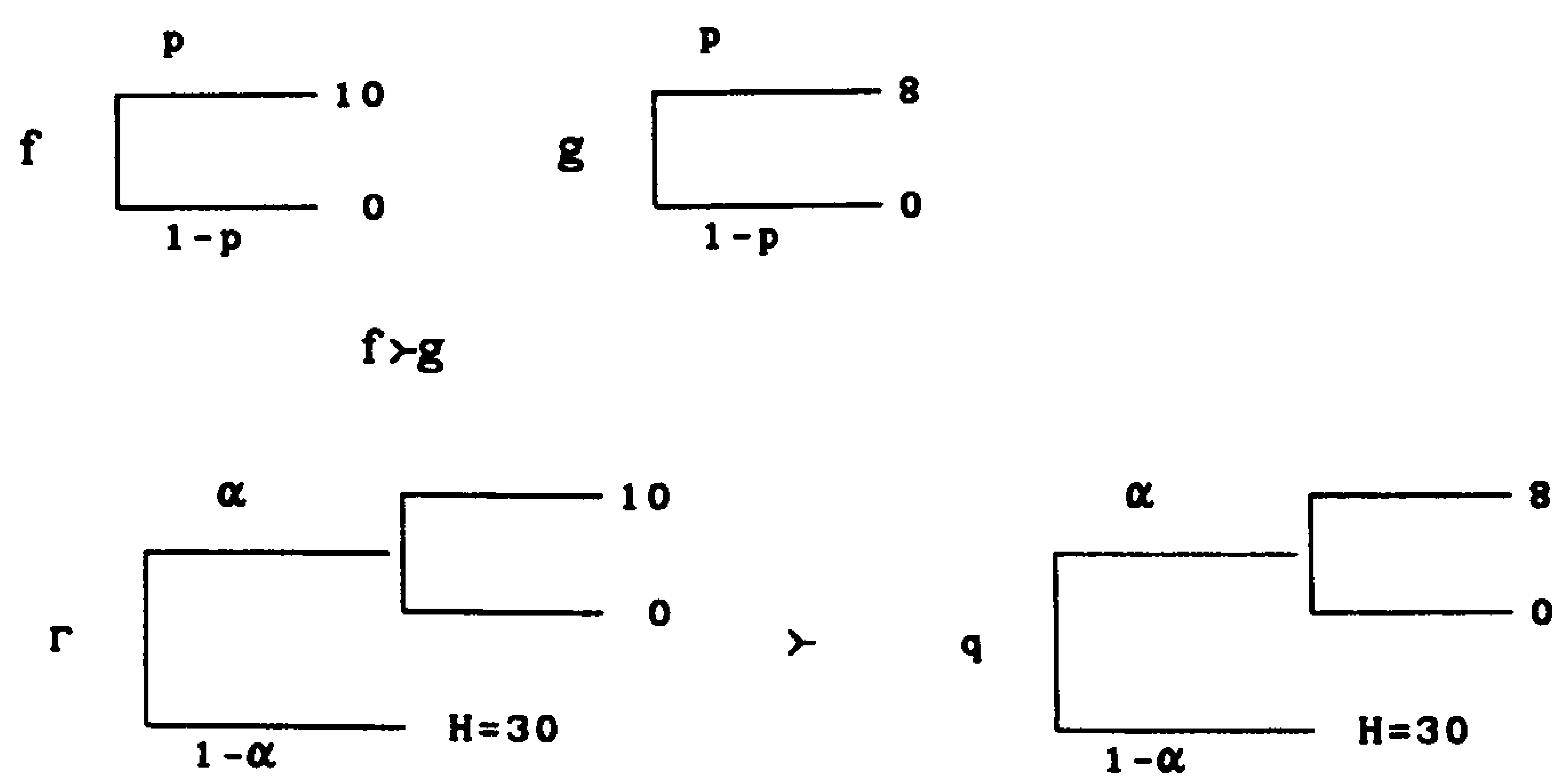
was chosen just for illustrative purpose.

Figure I.7 Illustration of the certainty independence axiom.



This axiom is a weak version of the independence axiom. The mixture of f and g are in fact given in respect of a constant lottery (not a roulette lottery). The idea is that it is much easier for an individual make comparisons with a constant h than with lottery. Hence he will be less likely to reverse his preferences (we have found a very similar axiom in Fishburn (1994))

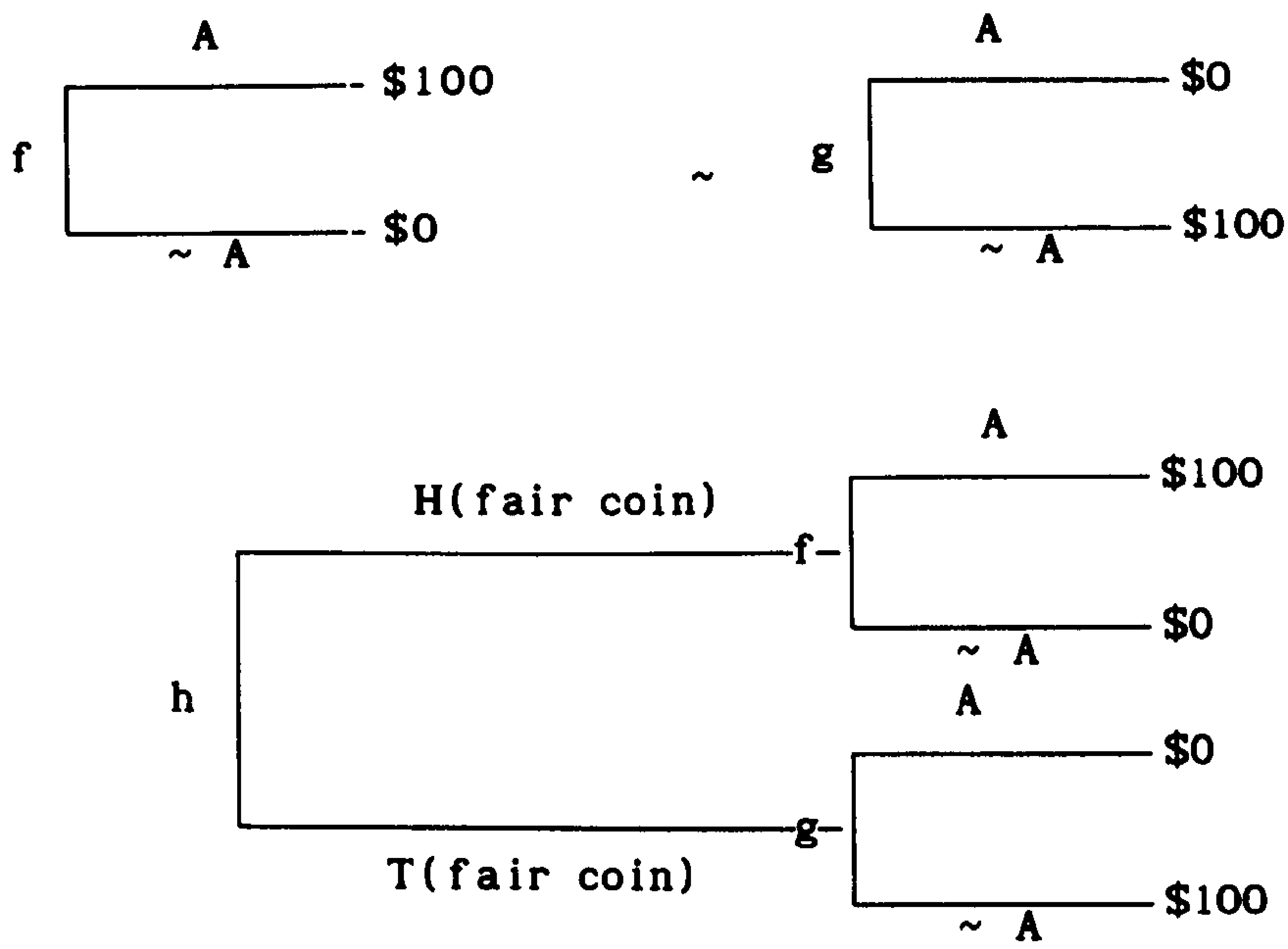
Uncertainty aversion

For all f, g $\in$ H and $\alpha \in$]0,1[:$f \sim g$ $\alpha f+(1-\alpha)g \geq g$

The axiom states that, assuming that two horse lotteries are indifferent, for an uncertainty averse person a random mixture of the two will be at least as preferred as each of the original lotteries.

Let us, for example, consider the following bet:

Figure I.8 Illustration of the uncertainty aversion axiom



$\alpha f+(1-\alpha)g \geq g$ where α can be any value in the interval (0.1)

What the axiom seems to convey is the idea that, if uncertainty averse, the decision maker does not want to commit himself to a particular answer so he prefers a random device to decide. (To some extent this axiom capture the same idea as that of a non-additive probability measure. Non-additivity allows the decision maker not to commit himself in his probability judgment to the occurrence of an event and to the occurrence of the complement event with the same intensity). It is important to note that in Gilboa and Schmeidler's model $\pi(.)$ is an additive probability measure.

If the usual axioms hold, plus these last two, Gilboa and Schmeidler obtain the following representation theorem:

There exists an affine function u:F--> and a non empty, closed and convex set C of finitely additive probability measures on $\sum$ such that:

$$f \geq g \text{ iff } \min_{\Pi \in C} \int u\ f\ d\Pi \geq \min_{\Pi \in C} \int u\ g\ d\Pi \quad \text{(for all } f \text{ and } g \in L_0) \qquad (I.15)$$

Furthermore the function u is unique up to a positive linear transformation. Moreover the set C is unique if non degeneracy is assumed.

According to Gilboa and Schmeidler, one interpretation of their result is an extension of the neo Bayesian paradigm which leads to a set of priors instead of a unique one. However, if the set C is interpreted as the set of all possible distributions then the model can be seen as a personalistic interpretation of the minimax loss criterion of Wald.

"A minimax solution seems, in general, to be a reasonable solution of the decision problem when a a priori distribution in Ω does not exist or is unknown to the experimenter." Gilboa and Schmeidler (1989) p 143.

As we can see the model of Gilboa and Schmeidler seems very similar to that of Gardenfors and Sahlin and Levi. Also in Gardenfors and Sahlin's model in fact we have a set of probability distributions and consequently the criterion of choice is a mixture of expected utility and maximin.

I.5.4 *Ambiguity aversion as aversion to second order distribution (SOP)*

One way to explain aversion to ambiguity is assuming that ambiguity can be represented by the existence of a unique second order distribution. An ambiguity averse person is consequently a person who exhibits "risk aversion" towards this second order distribution. Segal's (1987,91) model adopts this view to explain the Ellsberg paradox. To do so he relaxes the compound lottery axioms and adopts a rank dependent approach in which the probabilities of an event are weighted nonlinearly. Let us now analyze the model in detail.

I.5.4.1 The model

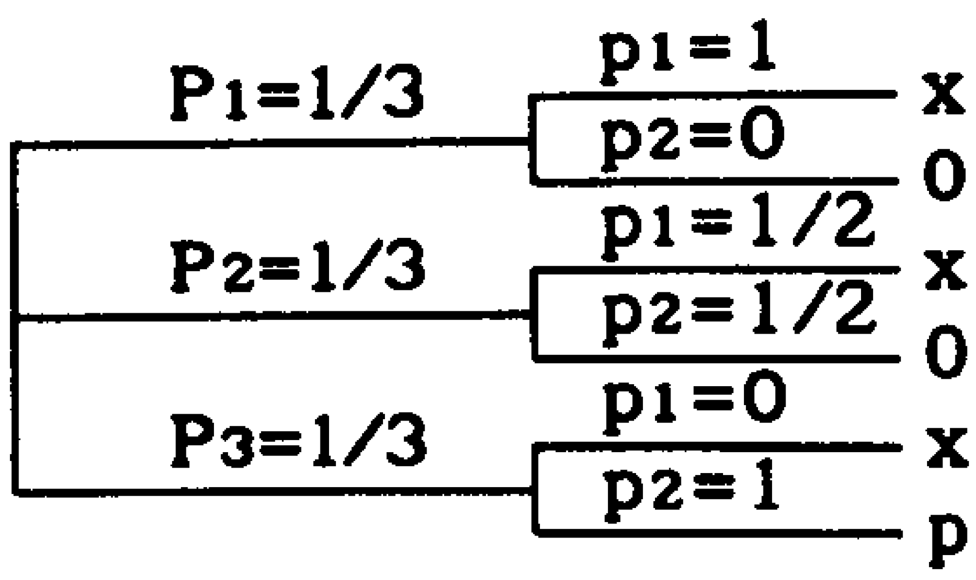
In Segal (1987), Segal suggested that the Ellsberg paradox depends on how people perceive the unknown urn. In particular

" the paper suggests that the ambiguous lottery (x,S;0,~S) (ambiguous in the sense that the decision maker does does not know the probability of S) should be considered a two stage lottery, where the first, imaginary, stage, is over the possible values of the probability of S". Segal (1987) p 177.

Let us consider a lottery defined by the outcomes x and 0 and the corresponding two states of the world S and ~S. Let us consider the case in which we do not know the probability of S. In this case the lottery is defined as an ambiguous lottery, in Segal's interpretation. According to Segal, when confronted with an ambiguous lottery of this kind, a decision maker will consider the lottery as a two stage lottery; the first stage is over over the possible values of the probability of S (over the random variable p, whose outcome is defined by $\tilde{p}$ and its mean value by $\bar{p}$). In the second stage the decision maker participates in the lottery (x, $\tilde{p}$,0, 1-$\tilde{p}$) The decision maker does not know the value of $\tilde{p}$ but Segal assumes that he or she has a subjective probability distribution over its possible values.
The decision maker can for example think that $\tilde{p}$=1 $\tilde{p}$=1/2, $\tilde{p}$=0, where the probability of P($\tilde{p}$=1) = P($\tilde{p}$=1/2) = P($\tilde{p}$=0) = 1/3. The expected value of the distribution of $\tilde{p}$ is 1/2 and we can call it $\bar{p}$. The distribution is assumed to be symmetric around the mean.

See for example figure I.9

Figure I.9 Illustration of Segal's model



where $p_1=\tilde{p}$
and $p_2=1-\tilde{p}$

It is important to note that if the decision maker uses the reduction principle, if he or she calculates the simple probability of winning x or 0 using the usual probabilities rules, then the lottery of the previous example reduces to the simple lottery (x,1/2;0,1/2). In fact Segal argues,

"It is therefore an essential assumption of the approach developed in this paper that the decision maker does not use this reduction assumption." p 177.

The main consequence of not using the reduction of compound lottery axiom is that is not possible to use Expected Utility Theory, since it assumes this axiom. Instead of using Expected Utility Theory, Segal uses anticipated utility theory[30].

The explanation of the Ellsberg Paradox suggested by Segal is composed of three different elements:

a - The way in which people perceive the unknown lottery (as a two stage lottery).

b - The fact that the decision maker does not apply the reduction of compound lottery axiom.

c - The use of anticipated utility theory to evaluate the lotteries.

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30 "It should be empathized that modeling the Ellsberg paradox as a two stage lottery does not depend on the anticipated utility theory, but on the existence of a theory that does not necessarily satisfy the reduction of compound lottery axiom." Segal (1987) p 178.
The point is that with EU we obtain the same result even using the certainty equivalent method.

And each of these three elements is crucial to Segal's explanation of the Ellsberg Paradox.

Having analyzed the first two elements it is now important to see how Segal applies anticipated utility theory to a two-stage lottery.

Let us consider the following simple stage lottery A

$A = (x_1,p_1;...;x_n,p_n)$, where $\sum p_i = 1$, $p_i \geq 0$, which yields the outcome x_i with probability p_i, with $x_i \in X$ and $i=1,..,n$. Moreover it is assumed that $x_1 \leq x_2 \leq ... \leq x_n$.

According to anticipated utility theory the evaluation of the lottery takes one of the following forms[31]:

$$V(x_1,p_1;...;x_n,p_n) =$$

$$u(x_n)\, f(p_n) + \sum_{i=1}^{n-1} u(x_i) \left[f\left(\sum_{j=i}^{n} p_j \right) - f\left(\sum_{j=i+1}^{n} p_j \right) \right] =$$

$$u(x_1)f(p_1) + \sum_{i=2}^{n} \left[u(x_i) - u(x_{i-1}) \right] f\left(\sum_{j=i}^{n} p_j \right) \qquad (I.16)$$

where the decision weight function $F:[0,1] \to [0,1]$ satisfies $f(0)=0$, $f(1)=1$.

Now let L be a compound lottery, $L=(A_1,P_1;...;A_nP_n)$, where $\sum P_i = 1$, $P_i \geq 0$.

With $CE(A_i)$ we indicate the certainty equivalent of the lottery A_i, where the $(CE(A_i),1) \sim A_i$. If the preference relation satisfy the independence axiom then

$(A_1,P_1;...;A_nP_n) \sim (CE(A_1),\ P_1;...;CE(A_n),\ P_n)$. (I.17)

If the preference relation is represented by a anticipated utility function then the $CE(A_i) = u^{-1}(V(A_i))$. Assuming that $CE(A_1) \leq CE(A_2) \leq ... \leq CE(A_n)$ then (I.17) implies that

$$(A_1,P_1;...;A_nP_n) \sim (u^{-1}(V(A_1)),P_1;.. ...;u^{-1}(V(A_n)),P_n) \qquad (I.18)$$

31 They are the same.

If the preference relation satisfies the independence axiom[32] and is represented by (I.16) then by (I.18)) we will have that the value of the two stage lottery can be represented by

$$V(A_1,P_1;\ldots;A_nP_n)=V(A_1)+\sum_{i=2}^{n}\left[V(A_i)-V(A_{i-1})\right]f\left(\sum_{j=i}^{n}P_j\right) \qquad (I.19)$$

Let F* be the distribution function over the possible values of the probability of S in the lottery (x,S;0,~S) and if the range of F* is finite such that the probability of S are $\alpha^*=p_1\leq\ldots\leq p_n=\beta^*$, then by (I.15) the value of the ambiguous lottery (x,S;0,~S) can be expressed equivalently by[33]

$$u(x)\ f(p_1) + u(x) \sum_{i=2}^{n}[\ f(p_i)-f(p_{i-1})]\ f(\sum_{j=i}^{n}P(p_j) \qquad (I.20)$$

where f(.) is a decision weight function f; [0,1] ⇒ [0,1] where f(0)=0 and f(1)=1.

$P(p_i)$ are the probabilities of the first branch of the two-stage lotteries, while p_i are the probabilities in the second branch of the two stage lottery.

According to Segal, to account for ambiguity aversion f(.) should be convex in the whole range and it has to satisfy other requirements. Theorem 4.2 is the one relevant to this point since it states the conditions to be satisfied in order to have ambiguity aversion. The theorem states:

Theorem 4.2. **Let F* be a distribution function over the possible values**

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[32] According to Segal $V(a)=\int_{-\infty}^{\infty} u(x)\ dF_A(x)$ and $V(x_1p_1,..x_np_n) = \sum p_iu(x_i)$

" are the only continuous functions satisfying the Reduction of Compound Lotteries Axiom and the Independence Axiom. Anticipated Utility is compatible with RCLA or IA. Some empirical evidence concerning two-stage lotteries suggests that decision maker accept IA, but not necessarily RCLA." Segal (1987) p 182.

[33] See appendix A for an example of the fact that (I.16) applied to a two stage lottery is equivalent to the direct application of (I.20).

of the probability of S in the ambiguous lottery (x,S;0,~S). Assume that F* is symmetric around $\bar{p}$. If f is convex, if its elasticity is non decreasing, and if the elasticity of $\bar{f}$ is non increasing, then (x,$\bar{p}$, 0,1-$\bar{p}$) is preferred to (x,S;0,~S).[34]

Moreover Segal indicates two functions which satisfy all the conditions of Theorem 4.2; these functions are:

$$f(p)=\frac{e^{p}-1}{e-1} \qquad \text{and} \qquad f(p)=p^{t} \text{ with } t>1.$$

Having defined the condition for ambiguity aversion Segal also defines the conditions according to which one distribution can be consider more ambiguous than another.

Theorem 6.1 defines how an ambiguity averse agent (defined by Theorem 4.2) can rank second order distributions according to their degrees of ambiguity. In order to define theorem 6.1 we need two definitions.

Definition 1

Let F* and G* be two distribution functions on [0,1]. G* is more ambiguous than F* ($G^* \geq_A F^*$) iff G* is a star-shaped spreading of F*, that is to say

a) F* and G* have the same mean value $\bar{p}$.

b) $G^*(p) \geq F^*(p)$ for $p \leq \bar{p}$ and $G^* \leq F^*$ for $p \geq \bar{p}$.

Definition 2

Let 0<p<1. H^*_p is the uniform distribution on [0,2p] when $p \leq 1/2$ and the uniform distribution on [2p-1,1]when $p \geq 1/2$

Theorem 6.1

Let G* and F* be symmetric around $\bar{p}$ such that $H^*_p \geq_A G^* \geq_A F^*$. If f is convex, if its elasticity is non decreasing, and if the elasticity of $\bar{f}$ is non increasing, then the value of the ambiguous lottery (x,S;0,~S) under F* is greater than its value under G*.

---

34 $\bar{f}(p) = 1-f(1-p)$.

The same two functions satisfies all three condition of theorem 6.1

As we will see these two theorems will play an important in the interpretation of the results of the experiment described in chapter III. In particular Theorem 6.1 states conditions of different degrees of ambiguity aversion which try to mimic the conditions for risk aversion. This is consistent with the general idea of Segal, who sees ambiguity as another face of risk. However, in chapter V we will discuss these conditions, which are very difficult to observe. As it is possible to see from the analysis in chapter II too, the links between ambiguity attitude and risk attitude have been explored also by the empirical work. To draw a comparison between risk and uncertainty has for sure some intuitive appeal, especially when ambiguity is operationalized with a second order probability distribution.

I.5.5 *Models with adjusted probabilities.*

Some of the formal models of ambiguity do not allow either for the presence of second order distributions or for a set of probabilities. In these models the decision maker is supposed to be able *to form a unique probability measure which is adjusted* in some way to take into account the fact that she or he is not sure about her or his estimate. Different models depict in different ways how this process of adjustment is made. We will present here three of these models, the one of Fellner (1961) and two of Einhorn and Hogarth (1985,86,90).
Some of the models of adjusted probabilities allow also for the existence of non additive probability measures, even if this is not the main feature of these models. In this section, we underline the existence of the adjustment process. In fact these models can be characterized also by the fact that the decision maker, in order to evaluate an act, does not use probabilities but decision weights. As we will see in the following sections, other models use decision weights in the process of evaluation but the subject does not perform any process of adjustment.

I.5.5.1 Fellner's model

Fellner recognizes the existence of two different situations (reproducing Knight's distinction) the evaluation of risky prospects and the evaluation of uncertain prospects). The risky prospect (a "standard process" in Fellner's words) is characterized by the existence of "objective" probabilities or frequencies,[35] while the uncertain prospect is defined by the absence of this objective frequency. As a consequence the probability judgments relative to the two different situations are not comparable.

Let us consider the following example; a person states that he or she assigns equal subjective probabilities to the occurrence of an event (E) and to the non-occurrence of the same event (~E). Assuming that this probability judgment has been known asking to the subject to choose whether he or she prefers to receive a specific prize if the event does happen, or if he or she prefer receive a prize if the event does not happen, and then asking to the same subjects whether, for a small additional, amount he or she would change his or her answer. In case of a negative response, it is possible to conclude that he or she regards the probability of the event E to be equal to the probability of its complement ~E , $\pi(E) = \pi(\sim E)$.

For Fellner there are important elements which characterize these two probability judgments.

a) These probability judgment may not be additive: for example $\pi(E) = \pi(\sim E) = 0.3$.

b) The probability judgment cannot be compared with probability judgments of equal value given to a risky prospects.(Assuming that $\pi(E) = \pi(\sim E)= 0.5$, this is not like to say that the probability of a head or tail in tossing a fair coin is equal to 0.5. Take, for example the two-colour Ellsberg example. According to Fellner an individual can estimate $\pi(BI) = \pi\ (BII) = 0.5$ and yet still avoid betting on BI. This non commitment to bet reveals according to Fellner the presence of uncertainty. In this case, the subject feels not sure about his judgment and in order to bet on BI he will probably need a $\pi(BI) = 0.6$

c) These probability judgments are determined by the fact he knows that the individual is ignorant of much potentially available information (as we can see also for Fellner it is the lack of information which characterizes the presence of ambiguity).

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[35] The toss of a fair coin is for Fellner a standard process. The distinction between standard process and non standard process is very similar to the distinction between ambiguous event and non ambiguous event in Sarin and Wakker (1993) for example.

Since in a non-standard process our probabilities are biased, to compare these probabilities with those of a standard process, we have to correct or adjust the probabilities. However the "degree" of adjustment will be different for different non standard processes and it will depend on the magnitude of the prizes used to elicit subjective probabilities.

In case of these subjective probability the kind of reasoning process of the individual can be the following:

" the probability of E may be anywhere between 0.3 and 0.7, as is the probability as ~E. If a person reasons in this way it is possible that he acts in a way according to which it is possible to be induced to infer that he assigns a probability of 0.5 to E and its complement; that is he will show indifference between the two events in a probability test. But this indifference which reflects just the fact that the estimated probability is an interval, need not imply that the individual *is equally willing to say "the range 0.7 0.3 can be represented by the figure 0.5," as he to say "red or black from an unbiased deck is 0.5"* Fellner (1961) p 662.

To restore comparability of these subjective probabilities with the objective ones, Fellner suggests an index of correction; this index of correction solve the main purpose of reintroducing additivity for the subjective probability and in the meanwhile it maintains the same proportion between the commitment given to E and the one given to ~E.

Consider the case that $\pi(E) = \pi(\sim E) = 0.3$. If we multiply the two probabilities for 3/5 we obtain the following "corrected" (in Fellner words) probabilities $\pi(E) = \pi(\sim E) = 0.5$; these probabilities have the characteristics of being additive and of preserving the same weight. The ratio between the two probabilities is maintained (ratio 0.3/0.3 = 0.5/0.5)

What happen with "incorrect" probabilities like $\pi(E) = \pi(\sim E) = 0.3$ is that the individual adjusts (bias downward) his probability judgment when a prize is staked on E and when is staked on ~E. Hence to find the unbiased probabilities we have to adjust them upwards.

Fellner's suggestion is to look at these "incorrect" probabilities more as psychological weights than "true" probabilities. Moreover, since individuals are not using corrected probabilities we cannot, according to Fellner, postulate that they are maximizing mathematical expectations of the utility of wealth. They are maximizing something that is similar to the mathematical expectation of utility of wealth but which is based on psychological weights which are represented by distorted probabilities. What Fellner seems to suggest is that the uncorrected probabilities measure a psychological reaction to uncertainty. This means that for an individual an objective probability is psychologically equivalent to a higher subjective probability (uncorrected one) .

Fellner says that, in fact, individuals can underestimate their subjective probabilities even in standard process; but, if this is true, it makes no real difference, since, in any case, uncorrected probabilities are defined relative to the corrected ones. The problem is that the individuals seem to distort more their probabilities when they have to face processes in which they have just a tentative probability judgment. According to Fellner this depends on the fact that while some processes are simple enough and consequently understandable to allow the derivation of significant clues from what is happening in individual instances, in other cases, we face events which, to some extent, are unique. What makes people distort more their probability is the presence of uncertainty and it is for this reason that the slanting inclination is likely to be more for uncertain judgment that for standard process.

I.5.5.2 The anchoring and adjustment model of Einhorn and Hogarth.

In the Einhorn and Hogarth (1985),(1986), the authors develop a formal model of ambiguity in which is used the concept of adjusted probability. The innovations of this model are threefold:

a) The model tries to give a psychological foundation to the adjustment process;

b) It allows for dependence between probabilities and payoffs.

c) It allows for ambiguity preference behaviour. To illustrate a possible case of ambiguity preference behaviour let us consider the following example taken from Einhorn and Hogarth (1986)

Let us consider two urns each containing 1000 balls. In the first urn every ball is numbered from 1 to 1000 and the probability to draw any number is .001, whereas in the second urn there are still 1000 numbered balls but we do not know with which proportion any number is represented. If there is a prize for drawing the number 687 on which urn would you bet? In urn 1 the probability to draw 687 is .001; in urn 2 the probability of drawing a ball with the same number can vary from

0 to 1000;. thus urn 2 involves ignorance and any probability would be equally likely (assuming that there is no preference over number). Einhorn and Hogarth's suggestion is that, in this case, many people would prefer to bet on urn 2 thinking that maybe the probability to have 687 could be higher then .001. That they suggest that people can show aversion to ambiguity for higher probability values and that they can be ambiguity seeking for lower probability values.

Einhorn and Hogarth's model. Einhorn and Hogarth develop a model which tries to explain the behaviour implied in the Ellsberg paradox allowing for sub and super-additive probability, specifying, in the meanwhile, under which conditions the subjects are supposed to avoid or to seek ambiguity.

The model is based on the psychological intuition that people use anchoring and adjustment strategy in which the initial probability is used as a starting point (anchor); then, an adjustment is made to take the idea of ambiguity into account The anchor probability can come from different kinds of sources (a probability salient in memory, the best guess of expert or any other available probabilities).

If the anchor is p the probability which comes out of the adjustment process is s(p)

$$s(p)=p+k \qquad (I.21)$$

where k is the net effect of the adjustment process.

This adjustment process is a sort of mental simulation by which the individual imagines higher and lower values of p[36]; since p can come from any distribution that this kind of simulation can allow one to evaluate which, among the possible distributions, are the more plausible.

The net effect of the adjustment, k, is assumed to be function of three factors:

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[36] Einhorn and Hogarth suggest that people engage mental simulation in which" other values of p are considered by imagining how well they express one's uncertainty".The simulated values are thus" incorporated into the adjustment term, thereby allowing people to maintain sensitivity to both uncertainty and ambiguity" Einhorn and Hogarth (1985) p 436. Speaking about decision under ambiguity the authors distinguish between two kind of uncertainty: uncertainty towards the outcome (we do not know which of the outcome will come out) and uncertainty about the probabilities (ambiguity).

a. the level of the anchor p: Since s(p) lies between 0 and 1, k must lie in the interval $-p \leq k \leq 1-p$

b. the level of ambiguity:. The greater is the level of ambiguity the greater is the level of simulation needed by the process. The level of ambiguity is indicated by the parameter θ where $0 \leq \theta \leq 1$.

c. The person's attitude to ambiguity. For attitude to ambiguity it is meant the relative weight of the imagined probabilities which can be higher or lower then the anchor. $\beta \geq 0$ is the parameter reflecting the relative weight. If one gives more weight to higher probabilities then this results in an upward adjustments to the anchor or vice versa. Hence the sign of k is determined by p and β.

Let $k = k_g - k_s$ (I.22)

where k_s is denotes the effects of imagining smaller value while k_g denotes the effect of imagining greater values.

The size of the simulation however depends on the amount of ambiguity θ.

They assume that k_g and k_s can be represented as a proportion of the maximum adjustment where θ is the constant of proportionality that is

$k_g = \theta(1-p)$

$k_s = \theta p$ (I.23)

if $\theta = 0$ then k_s and k_g are equal to 0.

Since β represents the relative weighting of higher versus lower probabilities, only k_s or k_g has to be weighted to affect k. They suppose that k_s is weighted by β in this way

$k_s = \theta p^\beta$ (I.24)

by substituting (I.23) and (I.24) into (i.22) then we obtain

$k = p + \theta(1-p-p^\beta)$ (I.25)

and substituting into (I.21) we obtain

$s(p) = p + \theta(1-p-p^\beta)$ (I.26)

$s(p) = (1-\theta)p + \theta(1-p^\beta)$ (I.27)

which implies that the judged ambiguous probability is a weighted average of p and $1-p^\beta$ where the weights reflects the amount of ambiguity[37] θ.

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37 θ affects the absolute size of the adjustment factor. When $\theta=0$,that is to say there is no ambiguity then $s(p)=p$. "Thus θ can be thought of as having a magnifying or dampening effect on one's attitude towards ambiguity in the circumstances, (β). For example if perceived ambiguity is small, the tendency to weight different values of probability above

What are the implications of the model?

s(p) is regressive with respect to p which means then keeping θ constant, how much of the range of s(p) is weighted more or less of the anchor depends on the value of β.

If $0<\beta<1$, probabilities lower then the anchor are weighted more heavily then those above the anchor. If $\beta>1$ the probabilities higher than the anchor are weighted more heavily and if $\beta=1$ the imagined probability are equally weighted. Moreover complementary probabilities are additive if p=1 or 0, $\theta=0$ or $\beta=1$ the probabilities add to one, if $\beta>1$ means superadditivity and $\beta<1$ subadditivity.[38]

According to the value of β when $\theta>0$ it is possible to explain ambiguity seeking and ambiguity avoidance behavior.

Hence the model has two parameters which are both functions of individual and situational factors. The parameter θ reflects the perceived ambiguity and the degree to which one simulates values of p that might be possible. On the other hand, θ can reflect also situational factors such as the absolute amount of evidence, the reliability of the source of information and the lack of knowledge regarding the process generating the outcomes. On the other hand, the parameter β reflects the extent to which one differentially weights in imagination possible values of probabilities which are higher or lower then the anchor p. In this way according to Hogarth and Einhorn β can also reflect the pessimistic or optimistic view of the individual. However β can also depend on variables like the size and the sign of the payoffs.[39]

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and below p is of little consequences". Einhorn and Hogarth (1985) p439

Let us consider s(p) and the complementary probability s(1-p) we will have $s(p)+s(1-p) = p+\theta(1-p-p^{\beta})+(1-p)+\theta[1-(1-p)-(1-p^{\beta})]= 1-\theta[1-p^{\beta}-(1-p)]$

[39] "For example if the general effect of ambiguity is to induce caution rather then riskiness, the prospect of an undesirable outcome (et: monetary losses) would induce people to pay more attention in imagination of values of p (loss) than that larger than p; similarly the prospect of a gain would focus attention on smaller values of p (gain). Hogarth and Einhorn (1986), p 449-50. Thus one of the advantages of Einhorn and Hogarth's model is that it tries to explain the behavior of people in making decision in situation of ambiguity by a general psychological model, which, in addition, allows for differences in individual behavior via particular parameters.

Presence of ambiguity and the role of information. One of the characteristics of the Einhorn and Hogarth model is the fact that the adjustment that people make about probabilities is based on a mental simulation in which "what might be" or "what might have been" is combined with "what is" (the anchor). According to Einhorn and Hogarth, the size of the adjustment to the anchor depends crucially on the quality and reliability of the information.

The role of the quality and reliability of information in the Einhorn and Hogarth model is better analyzed if we consider the structure of the following experiment on inference (Einhorn and Hogarth (1985)).

In this experiment about inference people were asked to make probability judgments on the basis of number of reports from a source of limited reliability.

The structure of the problem was the following one:

1 An event occurs

2 The event is observed by witnesses.

3 The witnesses make a report.

Let us consider now two events A and B and A' and B' are the reports of the events A and B. Since there are n witnesses we will have n reports reports can also be seen as the outcomes of n observers reporting on a single trial). The person who has to judge receive f reports in favor of A and c report in favor of the alternative hypothesis B, so that f+c=n and p is equal to f/n.

There are three factors which might have an influence on the evidences and consequently on the level of ambiguity.

a) the dissimilarities between the events A and B.

b) the credibilities of the sources.

c) the numbers of reports or the sample size, n[40].

To take account of the relevance of the information in the adjustment

[40]If n is small ambiguity will be high because few distribution will be ruled out."It is important to note that the judge's assessment of the likelihood of A depends both on the reports observed (p=f/n) and his or her knowledge of the situation. The latter it should be recalled, can be represented by the possible distributions over A that the judge has not been able to eliminate from consideration and that affect the mental simulation process. Thus, in a highly ambiguous situation, the information about the credibility of the source, the dissimilarity of the signal, and the size of the sample does not rule out many distribution". Einhorn and Hogarth (1985) p 440.

model they modified the model[41] in the following way:

$$S(f:c) = p + \vartheta'/n\ (1-p-p^{\beta}). \qquad (I.28)$$

where S(f:c) is the judged probability based on f reports for and c reports against; p is equal to f/n; n is the sample size; ϑ'/n is equal to ϑ the level of ambiguity. According to this version of the model people are supposed to anchor on f/n and then adjust for the unreliability of the source and the amount of information or date. It must be noted that as $n \to \infty$ $S(f:c) \to p$.

Conditional on a given value of ϑ', the model implies that there is a trade off between p and f in determining judged likelihood. One may find the evidence in favor of some hypothesis to be more convincing on the basis of 9:1 than 2:0, however since s(p) asymptotes at p trade off between p and n will occur only at small values of n.

Because $\vartheta = \vartheta'/n$, n also affects the conditions underlying the additively of complementary probabilities.

Specifically

$$s(c:f)+s(f:c) = 1 + (\vartheta'/n)[1-p^{\beta}-(1-p^{\beta})] \qquad (I.29)$$

where s(c:f) is the judged likelihood of the alternative hypothesis based on the same data. As $n \to \infty$ additively will hold regardless of ϑ', β or p. Of course when the evidences are meager we allow for non additive probabilities. The model to some extent provides a psychological link to concerns expressed by others regarding the appropriateness of additivity when evidence is meager. In fact the model predicts that the largest adjustment to the anchor occurs when evidence is meager. In addition as n increases, s(f:c) approaches to p and consequently the weights for what has happened dominates the weight of what might have been; as the absolute amount of evidence increases, the effect of the increasing n is to reduce the amount of non additivity of the complementary probabilities. Moreover the fact that non additivity results from a shift in the direction of the adjustment process is consistent with other effects due to the use of the

[41] The modification of the model is made to take into account two factors the role of evidence/information and the fact that S(f:c) can be a judged decision weight or probability. They actually used this version of the model in the experiment in which they tested ambiguity in presence of judgmental probabilities in non gamble situations.

anchoring and adjustment process, that is to say that the anchor is weighted more than the adjustment.

In Einhorn and Hogarth (1986) the model is enlarged to take into account the acquisition of new information, when it is supposed that new information change the level of ambiguity without changing the anchor probability (the balance between positive and negative evidence remain stable).

Let us call v the amount of new information acquired at time t; let the judged ambiguous probability after time period t be $s(p)_t$ be

$$s(p)_t = p + \theta/v\ (1-p-p^{\beta}). \qquad (I.30)$$

If v increases the effects on the adjustment process due to ambiguity decreases; if v is very large $s(p)_t \to p$ and the complementary probability will approach to additivity as v increases.

However it is not very clear from the model what happens when the balance between positive and negative information does not remain stable.

Let us consider the equation (I.30) and put p=f/n with f numbers of observations (evidences) in favour the considered event. As in equation (I.29) c is the number of observations (evidences) which are against the considered event and f+c=n, n is the total amount of information. Equation (I.30) becomes

$$s(p) = f/n + \theta/n\ [1 - f/n - (f/n)^{\beta}] \qquad (I.31).$$

Let us now consider three cases:

1 We acquire two new pieces of information and one of these information is in favour of our event while the other is against.

2 We acquire two new pieces of information and the two of them are in favor.

3 We acquire two pieces of information and the two are against.

In the first case the anchor will remain the same (this in reality is true only if we start with the same amount of evidence in favour or against), while θ/n will decrease hence the adjustment due to the presence of ambiguity will decrease.

In the second case, the anchor will increase while both the two terms of the second part of the equation will decrease and consequently the overall adjustment due to the level of ambiguity will diminish.

The third case is more ambiguous. The anchor will decrease but while the term ϑ/n will decrease the $[1-f/n-(f/n)]$ will decrease. Consequently the level of the adjustment will be more or less depending on the relative increase or decrease of the two terms. While it is quite sensible that the anchor will decrease if the evidence acquired are all against the event considered is not clear that the level of adjustment due to ambiguity should increase if we possess more information.

Venture theory. Using the model developed before, Hogarth and Einhorn (1990) develop a theory of decision weights in which, while people evaluate outcomes according to prospect theory (Khaneman and Tversky, 1979), they replace probability with decision weights.

As in the model presented above, they first anchor on a stated probability and then adjust their probability by mentally simulating other values. The amount of the simulation depends on the absolute size of the payoffs, on the extent to which the anchor deviates from the extreme 0 and 1, and on the level of perceived ambiguity concerning the relevant probability. Of course the net adjustment will depend on the weights that are given in imagination to values above as opposed to below the anchor in imagination. As in the previous version of the model, the net effect of the adjustment depends on individual and situational variables, from the sign and the size of the payoffs.

The process of formation of the decision weights is represented by the equation (I.21). However k_s and k_g are differently specified with respect to the previous model. In particular

$$k_g = f(\sigma,\vartheta,p,v(x)) \text{ and} \quad (I.32)$$

$$k_s = f(\sigma,\vartheta,p,v(x)) \quad (I.33)$$

where k_s and k_g are increasing functions of outcome uncertainty σ and of perceived ambiguity ϑ, while k_g is a decreasing function of p and k_s is an increasing function of p. The absolute value of v(x) increase both k_s and k_g together with the sign, and determines how much weight is given in imagination to values above and below the anchor.

In general, according to Hogarth and Einhorn, the venture function starts with overweighting, then there is a cross point and then an under weighting. The position of the crossover point depends on the relative weight given to value above or below the anchor; this depends, in turn, on the sign and the size of the payoff. For positive payoff the crossover point is smaller than 0.5 and is smaller for large payoffs. The contrary happens in the domain of losses. How much the decision weight deviates from the probability depends on the outcome uncertainty and on the level of ambiguity and the size of the payoffs. In a multiple gamble, in which there is no ambiguity, also the outcome uncertainty will be reduced and the decision weight will tend to the probability, This, of course, does not happen in the presence of ambiguity. Since the decision weights are non-linear in the probabilities, this means that the complementary probabilities will not necessarily sum to one. The model is then used to make some predictions about the behaviour of the subjects towards risk and ambiguity since it is assumed that the individual will have the same attitude of "caution" in face of risk and ambiguity. In particular as far as ambiguity is concerned they predict that the proportion of ambiguity averse choices will increase as the probability increases, and as payoff increases; the effect of payoffs on ambiguity will be larger at medium as opposed to larger probabilities in the domain of gains. In the domain of losses the proportion of ambiguity averse choices will decreases as probability increases; it will increase as the size of payoffs increases and the effects of payoffs on ambiguity aversion will be larger for medium as opposed to low probabilities.

Einhorn and Hogarth run 4 experiments to test their theory and in fact all the prediction about the behavior in face of ambiguity were valid but the one relative to the probability and payoff interaction were not confirmed, this in the domain of losses as well as in the domain of gain.

The experimental evidence seem consequently to give some support to venture theory as a descriptive theory of decision making under uncertainty. The main advantage of the theory is that can combine cognitive and motivational factors to the explain people behavior. However the model presents some problems. In particular the theory does not clarify which is the process through which people assess the anchor probability. Moreover it is not clear how in the decision weight function the evaluation and the updating of information is included. New information as we have seen can in fact not only modify the level of adjustment due to ambiguity but can also has an influence in the process of formation of the anchor.

I.5.6 *Models with non additive probabilities.*

As we have said, the Ellsberg Paradox constitutes a violation of the sure thing principle or of the additivity of the probabilities. The formal models which are presented in this section allow for non-additivity in the probabilities and generally weaken the sure thing principle to a more limited sets of acts.

Let us remind the reader of the two colour Ellsberg example. In the first urn, (Urn I) we do not know the proportion of red and black balls while in the second urn (Urn II) there are 50 black and 50 red balls.

	s1 R	s2 B
RI	$100	0
BI	0	$100
RII	$100	0
BII	0	$100

The preferences shown by people who commit the Ellsberg Paradox are given by BI~RI, BII~RII but BI < BII and RI<RII ; (BI∪RI) gives the certain event and so p(BI∪RI)=1, on the other hand (BII∪RII) also give the certain event and consequently p(BII∪RII)=1; but from the above preference relation we can infer that p(BI) + p(RI)< p(BII) + p(RII). This means that either p(BI∪RI) is different from p(BI) + p(RI) or that p(BII∪RII) is different from p(BII) + p(RII). Since we know that p(BII) = p(RII) = 1/2 then this mean that p(BI) + p(RI)<1. Models with non additive probabilities, allow for p(BI) + p(RI) be less than 1.

In Schmeidler words

" In the example above, if each of the symmetric and complementary uncertain events is assigned an index 3/7, the number 1/7, 1/7 = 1-(3/7 + 3/7), would indicate the decision maker's confidence in the probability assessment. Thus, allowing non additive (not necessarily additive) probabilities enables transmission or recording of information that additive probability cannot represent". Schmeidler (1989) p 572.

Consequently, Expected Utility with non additive probabilities generalizes SEU allowing the kind of behavior described above. Besides

Cumulative Prospect Theory which will be described in section I.5.7.2, in this group of models we have the models of Gilboa (1987), Schmeidler (1982,89), Wakker (1989), Nakamura (1990), Sarin and Wakker (1992,94) and Oginuma (1994). We are not going to give a complete account of all the models here. Instead, we will try to explain briefly the similarities and the differences between them. We will give a more detailed account of Schmeidler (1982,89) and Sarin and Wakker (1994)[42].

In the table I.7 below (update from Gilboa 1987) the main differences in the various approaches are summarized as far as the general framework is concerned. Wakker (1989) and Nakamura (1990) differ from the approaches described above because they assume some richer structure on the consequences instead of on the states.

Nakamura (1990) extends Gilboa (1987) assuming a fine S and an infinite set X of consequences. Wakker (1989) is an extension of Schmeidler (1984) in which the same representation is reached but the assumption of the availability of lotteries (the set of consequences contains also all the simple lotteries) is replaced by the assumption of continuity of utility.

Sarin and Wakker (1992) can be considered intermediate between Gilboa and Schmeidler, because they do not need to assume extraneous probabilities as in Schmeidler, however they need to assume ambiguous and unambiguous events. Moreover, they use probabilities of the unambiguous events to calibrate probabilities of the ambiguous events.

Oginuma (1994) gives another formulation of a model of expected utility with non additive probability building on Savage and Schmeidler's work. The key characteristic of the paper is the formulation of a new probability concept call I-non additivity probability. Such a kind of probability is partially additive and partially non-additive. Using the concept of I-non additive probability and I-monotonic acts Oginuma (1994) reformulates Schmeidler's and Savage's framework to arrive to a very similar formulation of a non-additive expected utility representation in which I-probability can be defined in a way to be additive or not additive just for I-monotonicy acts.

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42 In general these models allow for sub additive probability measure explaining ambiguity aversion.

Table I.7 Scheme of the models with non additive probabilities

	Objective and subjective probabilities	Only Subjective
Additive probabilities	Anscombe-Aumann (1963)	Savage (1954)
	Ogìnuma (1994)	
Non necesserily additive	Schmeidler (1982)	Gilboa (1987)

We will now describe the models of Schmeidler (1989) and Sarin and Wakker (1992). Both models allow for non-additive probability measures and adopt a preference functional which is called Choquet Expected Utility and is considered a generalization of expected utility to the case of uncertainty.

We now define the concept of capacity or non-necessarily additive probability, and then the Choquet Expected Utility functional form which is common to both the models. Then we well discuss how ambiguous probabilities are derived and the axioms needed to reach the CEU representation. The two models in fact adopt different derivations and a different set of axioms.

Let A denote a subset of S. A contains S and ø. The elements of A are called events. Non-necessarily-additive subjective probabilities, called capacities, are assigned to A. Moreover an event A is said to occur if A contains the true state.

Definition: A function $v{:}A \rightarrow [0,1]$ is a *capacity* if $v(\emptyset) = 0$, $v(S) = 1$ and v is *monotonic* with respect to set inclusion i.e. $A \supset B \Rightarrow v(A) \geq v(B)$. The capacity v is (finitely additive) probability measure if, in addition, v is additive ie.

$$v(A \cup B) = v(A) + v(B)$$

for all disjoint A and B, capacity is convex ranged if for every $A \supset C$ and every μ between v(A) and v (C) there exist $A \supset B \supset C$ such that $v(B) = \mu$

The preference relation $\succeq$ is said to maximize Choquet Expected Utility (CEU) if there exists a capacity v over A and a measurable utility function $u : X \to \mathbb{R}$ such that the preference relation $\succeq$ is represented by $f \to \int u(f(s))dv$; the latter is called the Choquet Expected Utility of f.

If we assume that there are n states of the world $s_1, \ldots, s_n$. Moreover $u(f(s_1)) \leq \ldots\ldots \leq u(f(s_n))$. Then[43]

$$CEU(f)=u(f(s_1)) + \sum_{i=2}^{n-1} \left[u(f(s_i))-u(f(s_{i-1})) \right] v \left(\bigcup_{j=i}^{n} s_j \right) \qquad (I.34)$$

Schmeidler (1989) arrives at the above representation in a Anscombe-Aumann framework in which "physical" (extraneous) probabilities are introduced. Within this model, to each state, an objective lottery over deterministic outcomes is assigned, and the uncertainty is referred to which of the state will prevail. The main difference in the axioms between Anscombe- Aumann model and the

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[43]The representation (I.34) is expressed in term of a measure rather than a transformation of probabilities, but is the same as the (I.16) rank dependent representation is section I.5.4. If we rewrite I.34 as

$$CEU\ (f) = u(f(s_1)).v(s_1)+\sum_{i=2}^{n} \left(u(f(s_i)-u(f(s_{i-1})) \right) v\ (\bigcup_{j=1}^{i} s_i) \qquad (I.35)$$

Hence

$$CEU\ (f) = u(f(s_1)).v(s_1)+\sum_{i=2}^{n} u(f(s_i) \left(v\ (\bigcup_{j=1}^{i})- v\ (\bigcup_{j=1}^{i-1} s_i) \right) \qquad (I.36)$$

Then the equation I.34 can be interpreted regarding the non additive measure v as a weighting function; and the weight of the state i is the difference between the total weight of all the states up to i and the total weight of all the states more than i.

Consider now the case of the events black and red of the two colour Ellsberg example. Applying I.34 rewritten as in I.36 we will have
$u(0) .v(R)+ u(10)\ (v(R \cup B)-v(R))$.

Considering now the case of non ambiguous events (urn II for example). In this case according to Schmeidler v is additive then $v\ (R \cup B) = v(R)+v(B)$. Then if v is additive applying again the I.36 we will have $u(0)\ .v(R)+ u(10)\ (v(R) + v(B) -v(R))= u(0)\ .\ v(R) + u(10)\ v\ (B)$, which shows why CEU reduces to SEU when capacities are additive.

Schmeidler one is the weakening of the mixture independence. The new version of the independence axiom is called **comonotonic independence.**

Defining comonotonicity. Two acts f and g are said to be comonotonic if for no s and t ∈ S, f(s) > f(t) and g(t) > g(s).
Intuitively this means that the outcomes of the two acts under each state of the worlds s and t must move in the same direction. (The best outcome for both acts happen under the same state). The fact that the outcomes move in the same directions makes the two acts "more easily comparable" and consequence the preferences over them can satisfy independence. If the two acts are not comonotonic then they are more difficult to compare and consequently independence can be violated.
Let us consider the following example:

Table I.8 Two comonotonic acts

	s	t	
f	4	2	
g	1	0	

In particular if X is a set of numbers and preferences respect the usual order on numbers, then any two X valued functions g and f are comonotonic if and only if

$$(f(s)-f(t))\ (g(s)-g(t)) \geq 0.$$

Let us consider the two colour Ellsberg example

	s R	t B
f=RI	f(s)=$100	f(t)=0
g=BI	g(s)=0	g(t)=$100

Hence

$(f(s)-f(t))\ (g(s)-g(t)) = (100-0)\ (0-100) \leq 0.$

Then BI and RI are not comonotonic: the two acts are difficult to compare and consequently Schmeidler's model allow for violation of independence.

At this point Schmeidler substitutes the axiom of comonotonic independence for the axiom of mixture independence.

Comonotonic independence.

For all pair wise comonotonic acts f, g, h in H[44], and for all α in)0,1(: f>g implies

$\alpha f + (1-\alpha)\, h > \alpha g + (1-\alpha)\, h$.

Comonotonic independence is of course a weaker axiom than independence since it confines the validity of the independence axiom to comonotonic acts. All the other axioms assumed by Schmeidler are the usual ones, weak order, continuity non degeneracy plus two versions of state independence.

Gilboa's (1987) estension. As we can see from the table at the beginning of this section, Gilboa (1987)'s model extends the result of Schmeidler's to a Savage framework, that is without the necessity of extraneous probabilities. Gilboa's axioms are less intuitive and less clear than Schmeidler's axioms.

Using the Savage framework, Gilboa's definitions arrive at the same representation of preference by a Choquet Expected Utility Functional form. Gilboa's system of axioms is very similar to the Savage ones; as in Schmeidler the main change is a weakening of the sure thing principle to a form which is valid just for comonotonic acts..

Before stating Gilboa **P2** we need some definitions.

Let S be the set of the states of the word, X the set of consequences and F={f:S->X} the sets of acts. Subsets of S are called events. For f, g∈F and A⊂S he defines $f|_A^g$ to be an element of F satisfying

$f|_A^g(s) = f(s)\ \forall\ s \in A^c$ and $f|_A^g(s) = g(s)\ \forall\ s \in A$.

For x∈X Gilboa defines x ∈F to be the constant act x(t)=x ∀x∈S.

≥ will denote a binary relation over F; ≥ ⊂ FxF is a preference relation.

P2 (Gilboa).

For all f_1, f_2, g_1 g_2 ∈ F and A, B ⊂ S and x_1, x_2, y_1, y_2 ∈ X such that $x_1 < y_1, y_2 > x_2$, if

---

[44] L is a convex subset of Y^S which includes L_c, the constant function in L_0, which is set of all ∑measurable finite valued function from S to Y.

a) $f_1|_A^{x1}$, $f_1|_A^{y1}$, $g_1|_A^{x2}$ $g_1|_A^{y2}$ are pair wise comonotonic and so are

$f_2|_B^{x1}$, $f_2|_B^{y1}$, $g_2|_B^{x2}$ $g_2|_B^{y2}$ and

b) $f_1|_A^{x1} \sim f_2|_B^{x1}$, $g_1|_A^{x2} \sim g_2|_B^{x2}$ and $f_1|_A^{y1} \geq f_2|_B^{y1}$

then $g_1|_A^{y2} \geq g_2|_B^{y2}$.

The axiom state that there is a preference order over events.

Let us consider the following case:

$f_1|_A^{x1} \sim f_2|_B^{x1}$ and $f_1|_A^{y1} \geq f_2|_B^{y1}$ where $y1 > x1$

This means that an improvement of the outcome under A is weighted more than an improvement under B since it can transform indifference in weak preference. This, to some extent, is like considering A more probable than B.

The statement may be reversed if there were $g_1|_A^{x2} \sim g_2|_B^{x2}$ such that $g_1|_A^{y2} \leq g_2|_B^{y2}$.

The axiom does not allow for this reversal of preferences. However the compliance with this axiom is limited to monotonic acts. In practice the meaning of comonotonicity is that that each event (A, B) is conceived in the same way in each of the above acts in which it appears as for Schmeidler.

In matrix form the axiom says:

Table I.9 Illustration of Gilboa's axiom

	A	~A		B	~B
	x_1	f_1	~	x_1	f_2
	x_2	g_1	~	x_2	g_2
	y_1	f_1	≥	y_1	f_2
⇒					
	y_2	g_1	≥	y_2	g_2

Sarin and Wakker's model. Wakker and Sarin (1992) also provide an extension of Savage's Expected Utility for decision under uncertainty as in Gilboa (1987). Their representation of Choquet expected utility differs from Schmeidler's in that it does not use the two-stage Anscombe-Aumann representation; moreover they use a system of axioms which has a more intuitive appeal than Gilboa's (1987).

The key axiom of their model is **cumulative dominance.** Cumulative dominance plays the same role for ambiguous acts as the sure thing principle in the Savage system of axioms, but is a weaker axiom (like comonotonic independence for Schmeidler or Gilboa) and it allows for probabilities of some events not to be additive.

The satisfaction of cumulative dominance requires that, if receiving a consequence α or a superior consequence is considered more likely for act f than for act g, for every α, then, the act f is preferred to g. This condition is satisfied by the acts of the Ellsberg paradox, while the sure thing principle is not. For example, in the case of the two-colour problem, we can say that if receiving some sum of money (x>0) or a superior one is considered more likely with the act "betting on white in the second urn", f, than with the act "betting on white in the first urn", g, for any value of x, then the act f is preferred to g.

As in the previous models the representation of preferences is given by

$$CEU(f)=u(f(s_1)) + \sum_{i=2}^{n-1} \left[u(f(s_i))-u(f(s_{i-1})) \right] v\left(\bigcup_{j=i}^{n} s_j \right) \qquad (I.34)$$

where v(.) are the non necessarily additive probabilities or capacities.

In order to reach this representation they enlarge the set of the states of the world in order two include two kinds of events: the ambiguous and unambiguous ones.

They assume that

"the Savage's axioms hold for a sufficiently rich set of 'unambiguous acts', ie. acts measurable with respect to the unambiguous events." Wakker and Sarin p 1255 (1992).

This distinction allow them to require that just the acts which are measurable with respect to unambiguous events satisfy Savage's axioms. General acts are required to satisfy a weaker condition which as we have already said is cumulative dominance. In order to do this, they do not assume an a priori definition of ambiguous or non ambiguous events, but they still assume the existence of a sub-class of events, such those as generated by a roulette wheel that they call unambiguous. This subclass of unambiguous events is rich enough to ensure that all ambiguous events can be calibrated by appropriate bets contingent on unambiguous events. In practice, as in the Savage framework, likelihood comparisons of events can be inferred from preferences over acts. A is considered more likely than B if I prefer bet on A than on B. In case on unambiguous events the likelihood is represented by probabilities. These probabilities in the Sarin and Wakker are used to calibrate the probabilities on the ambiguous events. The assumption is that for any ambiguous event A it is possible to find a matching unambiguous event B of equal likelihood such that a person is indifferent between betting on A and betting on B. Sarin and Wakker call this assigned to A a capacity v(A). This is stated formally by the following condition:

R2 For each ambiguous event A, there exists an unambiguous event B such that $A \sim B$.

Now we state the **cumulative dominance** axiom **(P2)**:
If the event of receiving an outcome consequence α or a superior outcome is considered at least as likely under act f than under act g, for every α, then, $f \succeq g$

Cumulative dominance implies that $(A,\alpha;A^c,\beta) \succeq (B,\alpha;B^c,\beta)$ for some outcome $\alpha \succ \beta$ if and only if this is true for all outcomes $\alpha \succ \beta$. A consequence is that the likelihood order over events is independent from the outcome chosen. Moreover it assures that v(A) is independent of the particular choice of the unambiguous event B which is indifferent to A. Cumulative dominance will ensure that the preferences $\geq$ on A satisfy the usual condition of transitivity and completeness and are in proper agreement with preferences over F.
Moreover, cumulative dominance can be seen as an adaptation of the stochastic dominance condition to the ambiguous acts and in particular

"in this setting, an act (or its probability distribution as generated over consequences) stochastically dominates another if it assign to each cumulative consequence set at least as high a probability. In the present set up, without probability attach to each event, it is natural to say that an act f stochastically ("cumulative") dominates an act g if the decision maker regards each cumulative consequence set at least as likely under f as under g." Wakker and Sarin (1992) p 1262.

An example: different results in the one stage and two stage set up. Let us consider the following example taken from Wakker and Sarin (1992) which also shows how the application of CEU in a two stage and in the one stage set up can give to different evaluation of the same lotteries.

Consider the following lottery. Suppose that a biased and a fair coin are tossed, with H_b and T_b we indicate head and tail in the biased coin and with H_f and T_f we indicate head and tail in the fair coin. Suppose that first the biased coin is tossed and then the fair one. We will have the following states of the world:

s_1	s_2	s_3	s_4
H_bH_f	H_bT_f	T_bH_f	T_bH_f
1	0	0	-1

The same situation can be illustrated with a one stage and a two stage process:

Figure I.10 Wakker and Sarin's one stage formulation

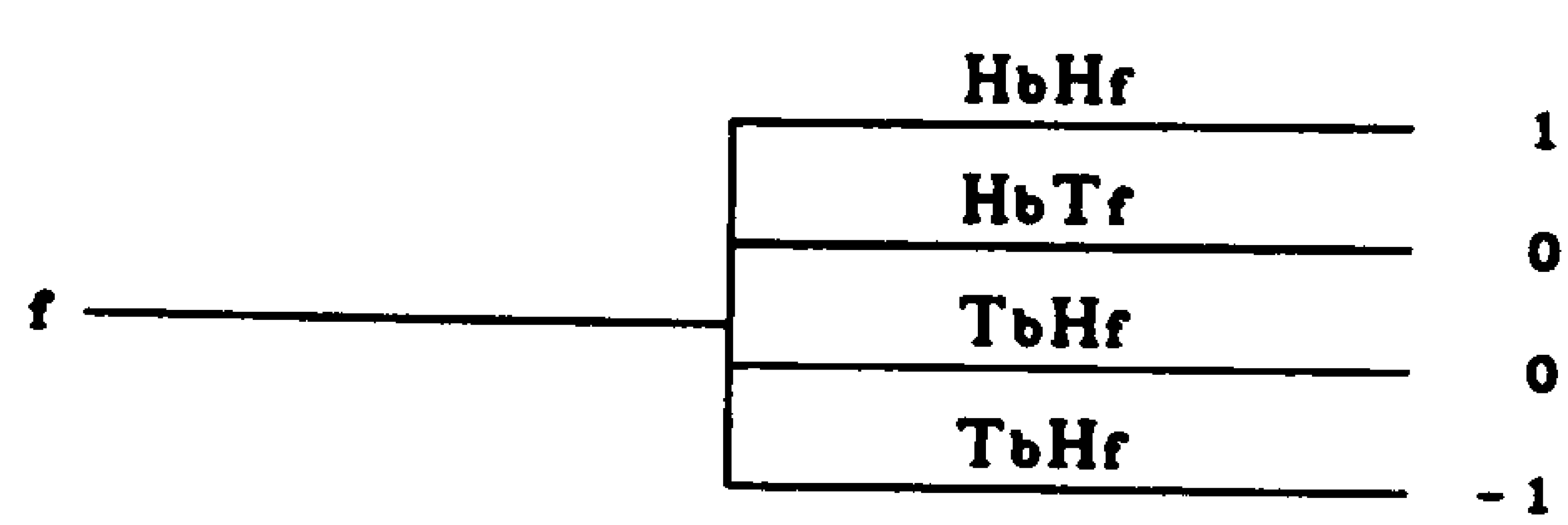
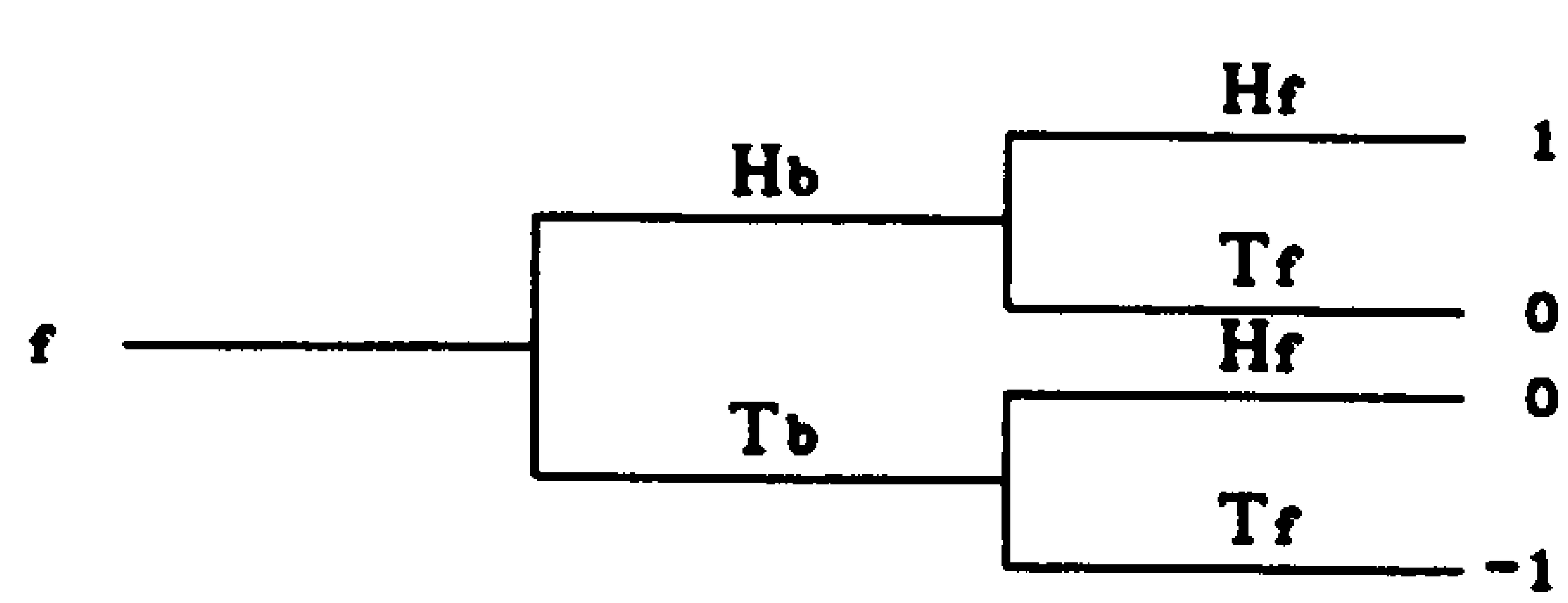


Figure I. 11 Schmeidler's two stage formulation



Suppose then that you are asked to evaluate the above lotteries according to CEU which have the same states of the world and the same utilities (1,0,0,-1) under the same state of the world.

By Choquet expected utility the utility of f is given by:

$$u(f) = \sum_{i=1}^{n-1}(u(f(s_i)) - u(f(s_{i+1}))\ v(s_1,....s_i)+ u(f(s_n)) \qquad (I.37)$$

where $u(f(s_1) \geq ...\geq u(f(s_n)$, s_1 = H_bH_f, s_2 =H_bT_f, s_3 = T_bH_f, s_4 = T_bH_f.

If we apply the above functional form to the Wakker and Sarin one stage set up after having ranked the utilities of the states of the world $(u(f(s_1)) = 1$, $u(f(s_2) = 0$, $u(f(s_3))=0$, $u(f(s_4) = -1$ we will have:

$u(f) = ((1)-(0)).\ v(s_1) + ((0)-(0))\ .\ v(s_1,s_2) + ((0\ )-(-1))\ v(s_1,s_2,s_3) + (-1).$

Sarin and Wakker assume that betting 1 on H_f as well as betting 1 on T_f is equivalent to have 1/2 for sure. On the other hand, betting one on H_b as well as betting 1 on T_b is regarded less preferable. In addition they assume that these two bets may be regarded as equivalent to α < 1/2 (that is to say something less then 1/2)[45].

Now $v(s_1)$ is equal to $\alpha/2$, and $v(s_1,s_2,s_3)$ is equal to $1/2 + \alpha/2$. Hence $u(f) = \alpha/2 + 1/2 + \alpha/2-1= \alpha-1/2$ which is less then zero.

On the other hand if we evaluate the lottery according to Schmeidler, one stage formulation we will have, since in the first stage we are faced with a roulette lottery and consequently CEU reduces to SEU, $C(E_1) = 1/2\ (1) + 1/2\ (-1) = 0$ and $C(E_2) = 1/2\ (0) + 1/2\ (0) = 0$ and, consequently, $u(f) = 0$.

The different result in the evaluation of the lotteries points out a problem with the use of the capacity. Each of the analyzed models depends very strongly, given the same functional form, on the rule used (one stage and two stage set up) in representing the problem and in the assumption relative to the capacity. Provided that the capacity of T_b and H_b is less then the probability of H_f and T_f, the "actual" value will depend on our assumptions on the attitude of the individual

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45 In the Wakker and Sarin's model what is important is that the capacity of the two ambiguous events is less then the probabilities of the corresponding two unambiguous events. They do not assume as Gilboa (1987) that the capacity is equal to the minimum probability; in this case the capacity in fact can be equal to 0 leading again to a different result.

towards ambiguity (Cf. previous footnote) or on the assumption relative two the events[46]. If the individual consider the worst possibility then the capacity can be zero.

As we see all these models allow for the existence of a non necessarily additive probability measure and for a Choquet representation of preferences. To do so they weaken the independence axiom in various ways. In practice the models are very similar, even if the mathematics through which they reach their representation is different. A consequence of this is that the axiom which substitute the sure thing principle does not predict the same in all the models. Here below we can see the prediction of Wakker (1989) and Oginuma (1994) for Gilboa (1987) see above[47].

Table I.10 Wakker(1989)'s representation

	A	~A		A	~A
	α	f	$\leq$	β	g
	γ	f	$\geq$	δ	g
	B	~B		B	~B
	α	s	$\geq$	β	τ
$\Rightarrow$					
	γ	s	$\geq$	δ	τ

[46] For example the capacity in Gilboa is assumed to be symmetric. That is to say consider to disjoint event A and B we will have that v(A) + v(~A) = 1 and v(B) + v(~B) =1 but v(A) + v(B) $\neq$ 1. The same condition is called by Wakker and Sarin additivity respect to A and additivity respect to B.

[47] Gilboa and Wakker are taken from Camerer and Weber (1992)

Table I.11 Oginuma (1994)'s representation

A	~A		B	~B
x_1	f_1	~	x_1	f_2
x_2	g_1	~	x_2	g_2
y_1	f_1	>	y_1	f_2
does not implies				
y_2	g_1	>	y_2	g_2

In Wakker (1989) the acts in the top row and in bottom row are pair wise comonotonic; In Oginuma (1994) the acts are in the first two colons are pair wise comonotonic as well as the acts in the second two colons.

I.5.7 *Models with decision weights*

There is a group of models which allow for the presence of ambiguity using decision weights instead of probabilities; these decision weights associated with the occurrence of an event depend on the ambiguity in the probability of that event. Hence, for example, two events with the same probabilities can have two different decision weights if the associated ambiguity is different. Generally we can describe these models in the following way:

Let us consider the example of an uncertain prospect $X(x_1,E;\ x_2\ \sim E)$

The value function of X can be expressed in the general form:

$$V\ (X) = v(x_1)\ .\ w(E) + v(x_2)\ .\ w(\sim E) \qquad \text{(I.38)}$$

here $w(E) \neq \pi(E)$ which is the SEU subjective probability; w(E) may in some models be a function of $\pi(E)$. The way in which decision weights are related to probabilities and to ambiguity in probabilities is specified in different ways by different models. For example, the decision weight can depend on subjective probability, on the outcome and on the level of ambiguity.

There are a few models of ambiguity which use decision weights[48]. Most of then have been reviewed in other sections (amongst the others, Segal (1987), Fellner (1961) and Einhorn Hogarth (1985,86,90)).

We will here briefly describe two models which uses decision weights: that of Kahn and Sarin (1988) and Becker and Sarin (1990). In the next section we will analyze in more detail two other models: Hazen (1987) and Hazen and Lee (1991), and Tversky and Kahneman (1992) and Wakker and Tversky (1993).

Kahn and Sarin (1988) use non linear weighting functions. This weighting function is used to weight a second order probability distribution where the adjustment reflects aversion to the probability risk implicit in the second order distribution. They allow for ambiguity aversion as well as for ambiguity preference depending on the value of the parameter λ. If $\lambda = 0$ then their model reduces to SEU. According to Camerer and Weber their model resembles theories of disappointment and elation (Bell 1982, Loomes and Sudgen (1982,86), however the disappointment is due to a bad probability outcome and not to a bad outcome.

In Sarin and Becker (1990) the decision weight is interpreted as the probability equivalent of a random variable $\tilde{p}_e$ (Sarin (1992). A function $\psi(w(e))$ (with w(e) we indicate the decision weight) is introduce to compute the decision weight and is given by

$$\psi(w(e)) = E[\psi(\tilde{p}_e)]$$

Hence the decision weight is given by $\psi^{-1}E[\psi(\tilde{p}_e)]$.

Using different shapes for ψ, Sarin and Becker can explain ambiguity-aversion ambiguity-seeking or ambiguity-neutrality (Concave,

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[48] In the models of ambiguity that use decision weights differ from the models under risk with respect to the fact that the decision weight is a function of the subjective probability instead of a function of the probability. It may be important to recall that models which use decision weight and consequently do not assume linearity in probabilities lead to violation of stochastic dominance. See Fishburn (1978), Quiggin (1982). The violation of dominance can be quite troublesome for the normative value of a theory. To avoid this drawback, for example in prospect theory dominated prospect are eliminated in the editing phase.

convex and linear ψ). When ψ is concave then the decision weights are not additive.

In the next section we will describe the Hazen (1987), Hazen and Lee (1991) and the Tversky and Kahneman (1991) models. In the first case decision weights are related to subjective probabilities (in practice subjective probability are calibrated through a comparison with the objective ones), while in the second case they are related to capacities (See the section on non-additive probabilities).

I.5.7.1 Subjectively weighted linear utility and the problem of ambiguity.

Hazen (1987), Hazen and Lee (1991) develop a theory of non-linear utility and subjective probability in which the assessed probabilities are allowed to depend on the outcome associated with an event. The theory is called subjectively weighted linear utility because when objective probabilities are substituted for "subjective" probabilities then the theory is equivalent to the weighted linear utility originated by Chew and MacCrimmon (1979). In particular, the theory

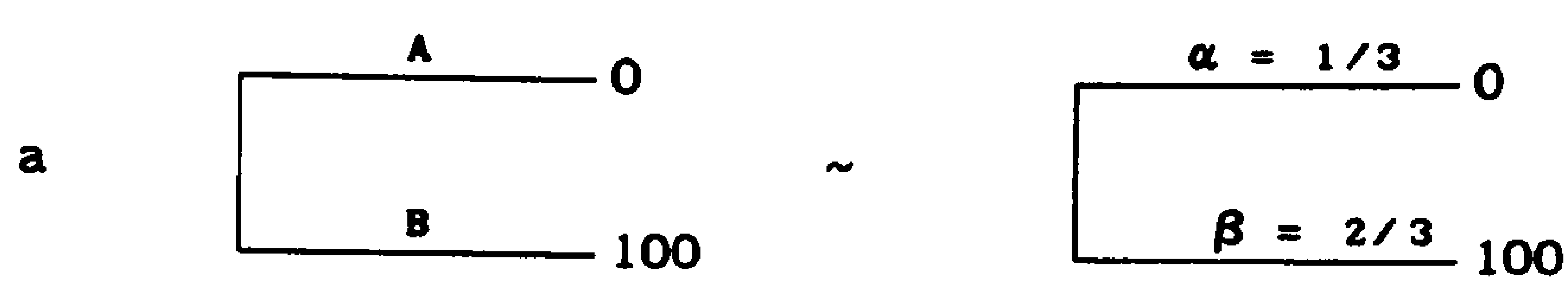
" allows the probability of an event A to depend continuously on the *degree* of desirability of the consequences under A, compared to the consequences under the complement of A. It accommodates ambiguity aversion, but retains transitivity and the independence under pure risk. Hazen (1987) p 263.

The subjective probabilities of an individual relative to the occurrence of an event are elicited in the Hazen and Lee model through the comparison between event lotteries and risky one. Given an uncertain lottery it is always possible to find a risky lottery, with the same outcome, which is indifferent to the uncertain one; through this process of comparison is possible to elicit the "subjective" probability attached to an event. However in the model of Hazen and Lee this process of calibration is done in a particular strange way. Let us assume that we have a first event lottery (if A then .. if B then) with two outcomes. We can be able to find a risky lottery which is indifferent to this event lottery (eliciting the two probabilities π(A), and π(B)). Now we consider another event lottery in which the outcomes are conditional on the occurrence of B (as in the first lottery) and on the occurrence of C, a third event. Also in this case we can find a risky lottery which is indifferent to the ambiguous one and consequently we are able to elicit the two event probabilities π (B), and π(C). Of course π(B) has been elicited in two different contexts and consequently can have two different value. The process goes on in the same way comparing other two lotteries with event A and C. Now let us assume that our individual is confronted with a forth event lottery in which the three outcomes are conditional to the occurrence of A or B or C . At this point Hazen and Lee introduce two axioms or rules that probabilities have to satisfy in order to maintain a certain consistency (which in practice is due to the maintenance of additivity). The two axioms are the Multiplicative Odds axiom and the Proportional Odds axiom. Hence in the third case the subjective probability of the event A B C are in some way adjusted to sum to one but according to these rules which are defined to maintain a sort of proportion between the values obtain in the first separate elicitation.

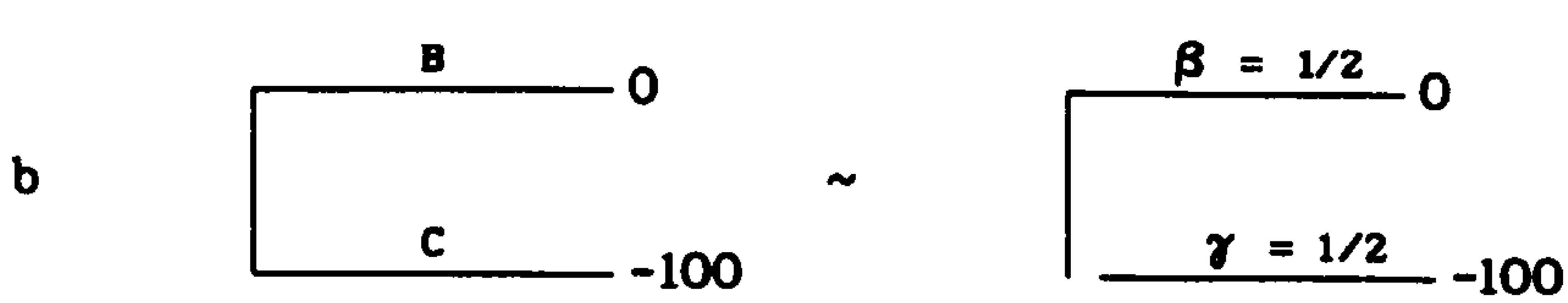
Let us see with two examples how the process works from a mathematical point of view.

Consider the case in which our decision maker shows to be indifferent between the following two lotteries:

Figure I.12 Illustration of the Hazen and Lee's axioms (I)



Moreover the same individual is indifferent between these other two lotteries:



Now if the multiplicative odd axiom holds the following indifferent relation should hold:

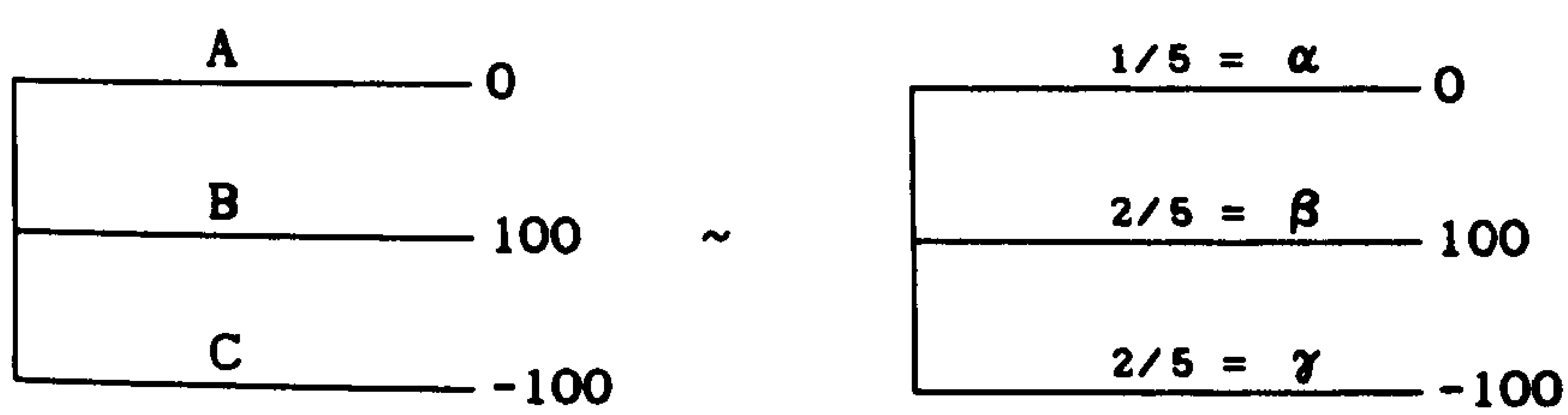
c

In fact if we call π(A) the subjective probability of A; we know that is indifferent to 1/3. π(B) ~ 2/3; moreover π(B)=2π(A). But π(B)= π(C), hence π(C)= 2 π(A). In this way the proportion between the different subjective probabilities is maintained.

Moreover if the proportional odds axiom holds then a lottery with three outcomes 0. 100. -100 will be indifferent to the following risky lottery:

Figure I. 13 Illustration of the Hazen and Lee's axioms (continue)

d

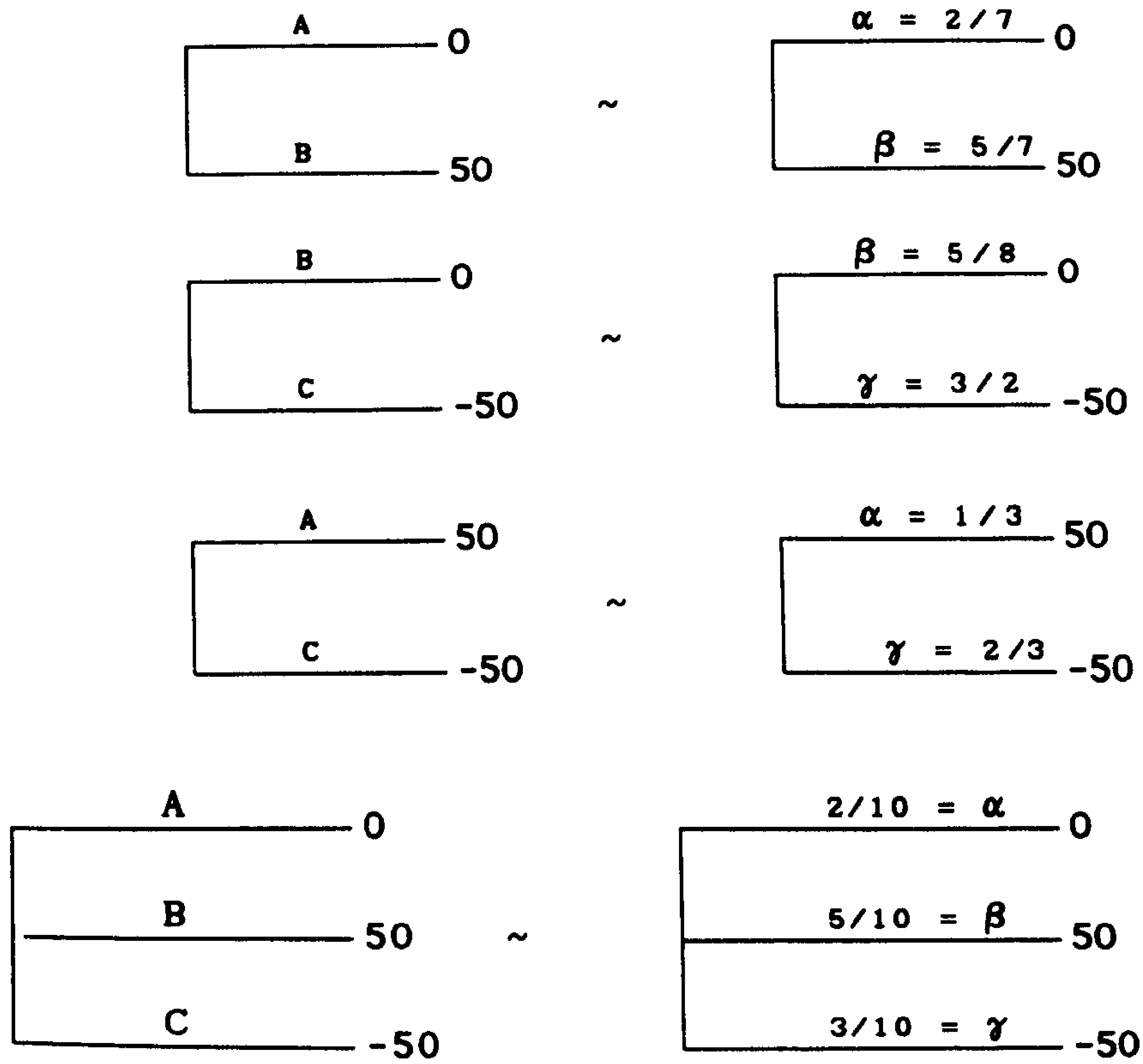


Since we know that π(B) = π(C) = 2 π(A) and in the meantime we want to maintain additivity in the probability then 1 = 2 π(A)+ 2 π(A) + π(A), hence 1 = 2/5 + 2/5 + 1/5

Hazen (1987) asks himself what happens if the absolute size of the payoffs decreases. The hypothesis is that the importance of the ambiguity associated to the contingent lottery will decrease and consequently the event lottery will be more desirable than the risky one.

In this case, for example, we will have followed the same kind of mathematical rule.

Figure I.14 Illustration of the Hazen and Lee axioms (II)



As we said before Hazen and Lee claims that these axioms are need if we want the subjective assessed probabilities to maintain the same coherence properties obeyed by standard probabilities.[49]

From a mathematical point of view the process followed by Hazen and Lee is correct, however in our opinion the elicitation process of this subjective probability seem quite odd. It is like to say that we can elicit subjective probabilities of events calibrating these probabilities comparing lotteries with pairs of events but then we adjust these elicited probabilities in order to maintain coherence when we consider a situation where the set of the events is composed by more than two events. So in the case of the two lotteries with three outcomes (d for example) the probabilities seem to be inferred by the

49 In the original version of the axioms instead of prize Hazen and Lee consider pure risky lotteries.

applications of the two rules and not by a process of comparison with a risky lottery following hence a different process of calibration.

The axiomatic version of their model is developed in a A-A framework. The two above explained axioms on probability plus other seven axioms give the following representation:

$$u(f) = \frac{\sum_{s\in A} \pi(s|\alpha_s)\ \psi(\alpha_s)\ \alpha_s}{\sum_{s\in A} \pi(s|\alpha_s)\ \psi(\alpha_s)} \qquad (I.39)$$

where α_s = u (f(s)) and for all α $\pi(s|\alpha)>0$, $\psi(\alpha)>0$ and $\sum_{s\in A}\pi(s|\alpha)=1$.

The function ψ weights the subjective probabilities in a way that depends on the outcomes f(s). When ψ (.) is constant, since we give the same weight to all subjective probability the (I.39) reduces to SEU. If S is the set X of all consequences x, f(x) = x for all x∈S and w(x) = ψ(u(x) then (.) reduces to the weighted linear utility (Chew (1983))
The main characteristic of the model is the dependence of subjective probability from the size of the outcome (See Camerer and Weber on this point (1991).

"By distinguishing between events and probabilities, the SWLU model permits subjectively assessed probabilities (here termed risk-equivalent probabilities) to depend on payoffs, and allow the distinction between subjective and objective probabilities, something that no pure risk model can do " Hazen and Lee (1990) page 205.

The model however does not seem to have an intuitive appealing. The two axioms on probability in particular seem to have the technical meaning of guarantee additivity but they do not seem have any intuitive base.

I.5.7.2 Cumulative Prospect Theory.

Tversky and Kahneman (1992) develop a model of ambiguity aversion in which they use decision weights which depend on non additive probabilities. Their model is the development of a previous model of choice, (1979) (1986) called prospect theory, which explains the main violations of expected utility theory in risky choices with small numbers of outcomes.
We will describe briefly the original model (1979), to concentrate

then on the main feature of the recent one (1992), which is that which relates directly with uncertainty.

In the original prospect theory, the choice process is divided into two parts: **editing** and **evaluation**. First, the decision maker constructs a representation of the acts, contingencies, outcomes etc. Second, the decision maker assesses the value of each prospect accordingly.

Editing: the decision maker does a preliminary analysis of the offered prospects in order to reach a *simpler representation*[50] of the prospects.

Evaluation: The edited prospects are evaluated and the decision maker chooses the prospect with the highest value.

The editing phase is the same for the 1979 and 1992 models (value function and weighted function are applied to already framed choices), while it is in the evaluation phase that the two models differ.

The main elements of the (1979) model are:

a) a value function which is concave for gain and convex for losses and steeper for losses than for gains;

b) the carriers of value are change in wealth or welfare, rather than the final states, hence the value is calculated with respect to a reference point and there may be a shift in this reference point.

c) a non linear transformation of probabilities which overweight small probabilities and underweight moderate and high probabilities.

In the original model, the value function is applied to "regular prospects": that is to say prospects that have one zero outcome and two non zero outcomes or two non zero outcomes on different size of the reference point.

Let us consider a regular prospect of the kind $X_1 = (x_1, p_1; x_2 p_2,)$

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50 According to Kahneman and Tversky (1979) the editing phase consists in the application of several operations on outcomes and probabilities which can help to simplify the problem. Example of these operations are: coding, combination, segregation, cancellation, simplification, and detection of dominance.
Coding is of the outcomes as gains or losses rather then final states of wealth. Combination is the aggregation of probabilities associated with the same outcome. Segregation is the isolation of the risky part of a prospect from the risk less part. Cancellation is the discharging of components that are shared by the offered prospects. Simplification is given by the rounding of probabilities or outcomes and detection of dominance is the elimination of clearly dominated prospects.

and either $p_1+p_2 < 1$ or $x_1 \geq 0 \geq x_2$

Then in the original version the value function is

$$V(X) = V(x_1)\ w(p_1) + V(x_2)\ w(p_2) \qquad (I.40)$$

Denoting with V (.) the value function and the value of an outcome (V operates on both prospects and on outcomes) and with w the decision weight.

Now let us consider another prospect Y(y_1, q_1; y_2 q_2,) where $q_1+ q_2 =1$ and $y_1>y_2>0$ or $y_1<y_2<0$, that is to say we have strictly negative or strictly positive outcomes. In this case according to prospect theory in the editing phase such prospects are grouped into two parts: a) a risk less component (minimum gain or minimum loss) and b) the risky component (additional gain or additional loss at stake). In this case the evaluation of the prospect is the following one:

Let us consider when $y_1>y_2>0$

$$V(X) = V(y_2) + w(q_1)[\ V(y_2)- V(y_1)] \qquad (I.41)$$

The interpretation of Kahneman and Tversky of (I.41) is that the decision weight is applied to the value-difference which represents the risky part of the prospect and not to $V(y_2)$ which represents the riskless part.

If we look at (I.41) is easy to see that is exactly the same as

$$V(X)=u(x_1) + \sum_{i=2}^{n-1} \left[u\ (x_i)-u(x_{i-1}) \right] f(\sum_{j=i}^{n} p_j) \qquad (I.16)$$

which is the Rank dependent functional form applied to the case where $x_1=y_2$ and $x_2 = y_1$. In general it can be said that the difference between PT and RDEU is that the weighting function depends explicitly on the x_i and not merely on the ordering of the outcomes. See Quiggin (1993).

The more recent version of the model introduces two major novelties:

a) it extends the model to uncertain and not only risky choices and

b) to prospects with any number of outcomes.

In expending the model to prospects with any number of outcomes the new model adopts a rank dependent approach. However, as we will see, the rank is establish respect with the capacity (that to say non additive probability measures). In this sense Cumulative Prospect

Theory is related to Choquet Expected Utility which is defined by:

$$CEU(X)=u(f(s_1)) + \sum_{i=2}^{n-1} \left[u(f(s_i))-u(f(s_{i-1})) \right] v\left(\bigcup_{j=1}^{n} s_j \right) \qquad (I.34)$$

However in Cumulative Prospect Theory, since the risk attitude towards gains may differs from the risk attitude towards losses, also the capacities for gains may differ from the capacities for losses. In fact, as we will see, we will have two decisions weights $w^+$and w^-, where w^+will be defined with respect to v^+(capacity for gain) and w^-will be defined with respect to v^-(capacity for losses).

The new model can, hence, be considered as a generalization of the old one. The main difference between the two is the non-linear transformation of the probability scale is not given by transforming individuals probabilities but cumulative ones in the line of Quiggin (1982), Yaari (1987), Schmeidler (1989) and Wakker (1989).

The new development explains, loss aversion, risk seeking and non linear preferences in terms of a value function and a weighting function as the old version of prospect theory but it also accommodates the Ellsberg paradox kind of behavior.

To sum up, cumulative prospect theory differs from prospect theory mainly because the probability transformation is on the entire distribution function and not on individual probability. In addition, this transformation applies to the cumulative functional separately for gain and for losses and, moreover, in case of uncertain events, capacities are substituted for probabilities.[51]

The model. Let as usual S be a finite set of states of nature: subset of S are called events. It is assumed that just one state obtains and that it is not known in advance to the decision maker. X is defined as the set of consequences or outcomes. Kahneman and Tversky assume that X includes a neutral outcome which is denoted zero (the status quo), while all the other outcomes are defined with respect to the neutral outcome as losses or gains (hence defined by positive or

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[51] The original model of prospect theory did not satisfy stochastic dominance; instead it was assumed that transparently dominated prospects were eliminated in the editing phase.

negative numbers).

An uncertain prospect f is a function from S to X which assigns to each state $s \in S$ a consequence $f(s) = x \in X$. In order to define a cumulative functional the outcomes of each prospects are arranged in increasing order. A prospect is consequently defined with a pair (x_i, A_i), where x_i is given if A_i obtains and $x_i > x_j$, iff $i > j$ and A_i is a partition of S. A prospect is defined as strictly positive or positive if all the outcomes are strictly positive or non negative; negative prospects are defined in the same way as well as strictly negative and strictly positive prospects. The other prospects are called mixed prospects. The positive part of F is defined as f^+ and it is obtained letting $f^+(s) = f(s)$ if $f(s) > 0$ and $f^+(s) = 0$ if $f(s) \leq 0$.

To any prospect f a number V(f) is assigned such that f is preferred to or indifferent to g, iff $V(f) \geq V(g)$.

The new representation of prospect theory is defined in terms of capacities; where a capacity is a function that assigns to each $A \subset S$ a number v(A) satisfying $v(\emptyset) = 0$ and $v(S) = 1$ and $v(A) \geq v(B)$ whenever $A \supset B$.

Cumulative prospect theory asserts that there exists a strictly increasing value function $V: X \to R$ satisfying $V(x_0) = V(0) = 0$ and capacities v^+ and v^- such that for $f = (x_i, A_i)$, $-m \leq i \leq n$,

$$V(f) = V(f^+) + V(f^-)$$

$$V(f^+) = \sum_{i=0}^{n} w_i^+ V(x_i) \quad \text{and} \quad V(f^-) \sum_{i=0}^{n} w_i^- V(x_i) \qquad (I.42)$$

The decision weight denoted with w+, that is associated with a positive outcome, is the difference between the capacities of the events "the outcome is at least as good as x_i" and " the outcome is strictly better than x_i; the decision weight denoted with w-, that is associated with a negative outcome , is the difference between the capacities of the event " the outcome is at least as bad as x_i" and " the outcome is strictly worse than x_i". According to Tversky and Kahneman,

"the decision weight associated with an outcome can be interpreted as the marginal contribution of the respective event, defined in terms of the capacities v^+ and v^-." Tversky and Kahneman (1992) p 301.

If each capacity is additive and consequently is a probability measure, then w is simply the probability of A.

"It follows readily from the definition of w and v that for both positive and negative prospects, the decision weights add to 1. For mixed prospects, however, the sum can be either smaller or greater then 1, because the decision weights for gain and for losses are defined by separate capacities". Tversky and Kahneman (1992) p 301.

Let us illustrate the model with an example taken by the experiment done by the authors.

The subjects had to choose between prospects whose outcomes were contingent on the difference d between the closing values of the Dow-Jones today and tomorrow.

Table I.12 Application of Cumulative Prospect Theory

		A	B	C
		$d<30$	$30 \leq d \leq 35$	$d>35$
Problem 1	f	25	25	25
	g	25	0	75
Problem 2	f'	0	25	25
	g'	0	0	75

If we apply the present model to f (Table above) we will have:

$V(f) = V(25)$ (I.43)

$V(g) = V(75)\, v^{+}(C) + V(25)[\, v^{+}(A \cup C) - v^{+}(C)]$ (I.44)

$V(g') = V(75)\, v^{+}(C)$ (I.45)

$V(f') = V(25)\, v^{+}(B \cup C)$[52] (I.46)

We can use the same example to show the subbadditivity of the v^{+}s.

Since f is preferred to g in problem 1, we have the following preference relation:

$V(25) > V(75)\, v^{+}(C) + V(25)[\, v^{+}(A \cup C) - v^{+}(C)]$ or (I.47)

$V(25) - V(25)[\, v^{+}(A \cup C) - v^{+}(C)] > V(75)\, v^{+}(C)$ or (I.48)

$V(25)[1 - v^{+}(A \cup C) + v^{+}(C)] > V(75)\, v^{+}(C)$. (I.49)

On the other hand, if the subjects prefer g' to f' this implies the

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52 The Ellsberg kind of preference would be f>g and g'>f' showing the usual violation of the sure thing principle.

following relation:

$V(75)\ v^+(C) > V(25)\ v^+(B\cup C)$. (I.50)

Then $[1 - v^+(A\cup C) + v^+(C)] > v^+(B\cup C)$ or (I.51)

$1 - v^+(A\cup C) > v^+(B\cup C) - v^+(C)$ (I.52)

which is to say that

$v^+(S) - v^+(S-B) > v^+(B\cup C) - v^+(C)$. (I.53)

Hence subtracting B from certainty has more impact than subtracting B from C∪B.

Let $v_+(D)$ be equal to $1 - v^+(S-D)$ then (I.53) is equivalent to the subadditivity of v^+ and we will have $v^+(B) + v^+(C) > v^+(B\cup C)$. Subadditivity of the weighting function can also be seen as the expression through the weighting function of the principle of diminishing sensitivity. When we are evaluating outcomes the reference point serves as a boundary that distinguishes the gains from the losses. In this version of prospect theory the concept of reference point is applied also to probabilities and capacities. In the evaluation of uncertainty there are two natural reference points (boundaries) the impossibility and the certainty. Hence the application of the concept of the reference point in case of uncertainty implies that the impact of a given change in the probabilities is different according to its distance from the boundary.

"Diminishing sensitivity therefore gives rise to a weighting function that is concave near 0 and convex near 1. For uncertain prospects, this principle yields subadditivity for very unlikely events and superadditivity near certainty. However the function is not well-behaved near the endpoints and very small probabilities can be either greatly over weighted or neglected altogether". Tversky and Kahneman (1992) p 303.

The axioms. In the axiomatization of cumulative prospect theory these are axioms which play an important role: **Comonotonic independence** and **double matching.**

Let us define $F = \{f:S\to X\}$ as the set of all prospects; with $\succeq$ we indicate a binary preference relation upon F. Moreover, for any f and $g \in F$ and $A\subset S$, $h = fAg$ is defined where $h(s)=f(s)$ if $s\in A$ and $h(s) = g(s)$ if $s \in S-A$. That is to say fAg coincides with f on A and g on S-A.

Kahneman and Tversky assume that:

- $\succeq$ is complete, transitive, and strictly monotonic. If $f\neq g$ and

$f(s) \geq g(s)$ for all $s \in S$, then $f > g$.

- it is satisfied comonotonic independence; that to say whenever the prospects defined as fAg, fAg' f'Ag f'Ag' . are pair wise comonotonic independence is satisfied.[53]

- it satisfies double matching.

Let us now explain double matching. For any prospect f we can denote by f^+ the prospect that results if all losses in f are replaced by the neutral outcome; we can denote by f^- the prospect that results if all gains in f are replaced by the neutral outcome. Let now consider two prospects f and g and suppose that $f^+ \sim g^+$ and $f^- \sim g^-$; double matching consists in the fact that these two indifference relation imply that $f \sim g$

The main result of the theory is stated in the following theorem:

Theorem 1. **Suppose $(F^+, \geq)$ and $(F^-, \geq)$ can each be represented by a cumulative functional. Then $(F, \geq)$ satisfies cumulative prospect theory iff it satisfies double matching and comonotonic independence.**

To sum up, cumulative prospect theory is the "natural" development of prospect theory taking into account ambiguous prospects. All the editing part of the theory remains the same. Cumulative prospect theory can be seen as the application of a Choquet integral to the evaluation of gains and losses. Hence the new model maintains different evaluations for gains and losses with a two part cumulative functional which reflects the principle of diminishing sensitivity (the impact of a change is perceived with a different intensity according to the distance from the reference point). This diminishing sensitivity is also applied to probabilities or capacities. Diminishing sensitivity in the domain of outcomes and diminishing sensitivity in the domain of probabilities were a peculiarity of prospect theory. On the other hand, the main features of the rank dependent utility and of the Choquet expected utility models are the use of transformation of the cumulative distribution (and not of single probabilities) and the use of capacities respectively. Cumulative prospect theory seems to be the attempt to encompass both characteristics in one model , which in the

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53 See previous section for the definition of comonotonicity.

intention of the authors is meant to maintain the descriptive power of prospect theory and generalize it to uncertain prospects[54].

I.6 Psychological literature on uncertainty and probability.

I.6.1 *How people perceive and deal with uncertainty.*

In the last twenty years a consistent part of the psychological literature on cognitive psychology and on decision making has addressed the question of what determines people's beliefs concerning the likelihood of uncertain events and how people assess the probability of an uncertain event or the value of an uncertain quantity.
Most of the researchers effort has focused on the studies of how people judge under uncertainty. The main point of this literature is that people in order to reduce the complex task of assigning probabilities and predicting values, rely on a limited number of heuristic principles. These heuristic principles are quite useful and allow people to perform simpler judgmental operations. However, they may lead to severe and systematic 'errors'.
According to Tversky and Kahneman (1982) p 3

"The subjective assessment of probability resembles the subjective assessment of physical quantities such as distance or size. These judgments are all based on data of limited validity , which are processes according to heuristic rules."

The most common of these rules are representativeness, availability, and anchoring and adjustment.

Representativeness. According to Tversky and Kahneman (1982), most of the probabilistic questions which people have to tackle are of the following type: what is the probability that process B will generate

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[54] It is important to note that this new version of prospect should be applied to already edited prospects. Hence the editing phase remains the same. Moreover the authors seem to suggest that also the decision weights may be sensitive to the formulation of the problems, the number the spacing and the level of outcomes. They think however that to accommodate these effects the theory runs the risk of loosing its predictive power.

the event A or what is the probability that the object A belongs to the class B. In such cases "people typically rely on the representativeness heuristic, in which probabilities are evaluated by the degree to which A is *representative* of B, that is, by the degree to which A *resembles* B. So if A is highly representative of B then the probability of A is considered high and vice versa.[55]

Availability. Another of the heuristic rules is when people assess the probability of an event "*by the ease with which instances or occurrences can be brought to mind*" Tversky and Kahneman (1982) page 11. This happen for example when people, if asked to assess the probability of having a heart attack by a middle age man they think of how many middle age men among acquaintances have got an heart attack. The main consequences in terms of the assessment of probability, through the use of availability, is the fact that a class whose instances appear easier to retrieve will appear more numerous than a class of equal frequency whose instances are less retrievable.[56] Another factor that can influence availability is how much a situation can be easily imagined. Since very often in real life imaginability plays an important role in the evaluation of probabilities then if frequency is assessed by imaginability then situations that are easier to imagine will be also considered more probable.

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[55] One example of how this heuristic yield to erroneous judgment is what is called insensitivity to sample size. People assess the likelihood of a sample result (for example the number of males and females in a group of eight brothers and sisters), by the similarity of this result to the corresponding parameters (that is the average of baby girls or baby boys in population of new born). As a consequence if people assess probabilities according to representativeness, then the judged probability of a sample statistic will be independent from the sample size. Another example is what Tversky and Kahneman call the misconception of chance. People expect that a sequence of events generated by a random process will have the essential characteristics of this random process even when the sequence is short. If asked to define a sequence of 10 heads and tails which according to them seems more random they are very likely to design a sequence in which there are almost the same number of heads and tail and in which heads and tails appears alternatively. In this way they try to represent the fairness of the coin. From a statistical point of view their sequence will have too many alternations and too little runs. They expect to see the law of large numbers reproduced also in a small sample.

[56] Other factors in addition to familiarity can increase the retrievability as salience for example.

Adjustment and Anchoring.

"In many situations, people make estimate by starting from an initial point value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient. Slovich and Lichtenstein,1971) That is, different starting points yields different estimates, which are biased towards the initial values. We call this phenomenon anchoring." Tversky and Kahneman (1982) page 11.

One of the typical consequences of the adjustment and anchoring process is the fact that it yields to biases in the evaluation of conjunctive and disjunctive events. In particular studies of choices among gambles and of probability judgments shows that people tend to overestimate the probability of conjunctive events and to underestimate the probability of disjunctive events. This happen because the stated probability of the elementary event (success at any one stage) provides the natural starting point for the estimation of the probabilities of the conjunctive and disjunctive event[57]. However since the adjustment from the starting point is not sufficient, the final probability remains close to the probabilities of the elementary events. One typical consequence of this bias in evaluating probabilities is the unwarranted optimism in the evaluation that a project will be completed on time.

According to Tversky and Kahneman (1982), the important consequence of this analysis is that these cognitive biases have a major influence for the theoretical and applied role of judged probabilities.

In the standard theory of probability the subjective probability of an event is defined by the set of bets about this event that the subject is willing to accept. However, what Tversky and Kahneman argue is that, even if subjective probabilities sometimes can be inferred from preferences among bet, however

"in reality, subjective probabilities are the ones to determine preferences among bets and are not derived from them, as in the

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57 "A conjunctive event is for example drawing a red marble seven time in succession, with replacement, from a bag containing 90 red marbles and 10 blue. A disjunctive event is drawing a red marble at least once in seven successive tries with replacement from a bag containing 10 red marbles and 90 marbles." Kahnemann and Tversky (1982) p 15
Usually in case of conjunctive events the overall probability of conjunctive event s lower than the probability of each elementary event while in case of disjunctive event the overall probability is higher than the probability of each elementary event.

axiomatic theory of rational choice." Tversky and Kahneman, (1982) p 19.

In particular Tversky and Kahneman object to the fact that as it happens with the standard theory of subjective probability, coherence and internal consistency is the only valid criterion by which judged probabilities should be evaluated.

"For judged probabilities to be considered adequate, or rational, internal consistency is not enough. The judgments must be compatible with the entire web of beliefs held by the individual. Unfortunately, there can be no simple formal procedure for assessing the compatibility of a set of probability judgments with judge's total system of beliefs. The rational judge will never the less strive for compatibility, even thought internal consistency is more easily achieved and assessed. In particular he will attempt to make his probability judgments compatible with his knowledge about the subject matter, the laws of probability, and his own judgmental heuristics and biases." Tversky and Kahneman, (1982) p 20.

To sum up what the psychological literature shows (and it is relevant to our analysis on ambiguity) is that from a psychological point of view the betting heuristic implied by the Bayesian school of probability is unrealistic (preferences are the basis of beliefs and probabilities are derived from preferences between bets). Moreover as Tversky and Kahneman, (1982), say, if the psychological heuristics, described above, are used in well defined sampling process (where objective probabilities are well defined), one can expect these heuristics to play an important role where the evolution of uncertainty is in unique situations and where there is not a "correct" answer. This is particularly important where uncertainty is assessed in term of propensities or confidence. If uncertainty is expressed in terms of confidence, that is to say if the subjective probability represents our degree of belief or our confidence in what we think would happen then there are some of the characteristics of the Bayesian conception of probabilities which seem not at all compelling. Complementary is one of them for example. When uncertainty is expressed in terms of confidence then it is less obvious why probability should add up to one for example. The question of confidence is quite important since the problem of ambiguity is very linked to it. If an ambiguous situation is one in which we do not have all the relevant information then as in the Gardenfors and Sahlin example we can assign a subjective probability to the occurrence of an event but we may have a very weak confidence in our esteem and this can affect and control our decision as in the Ellsberg case.

I.6.2 *The Competence explanation of ambiguity.*

As we have already seen the pattern of preferences shown by the Ellsberg paradox violates the additivity property of probabilities. To account for this, several theories have developed non-additive measures of beliefs or second order probability. However, in the Ellsberg example, as well as in most of the subsequent experimental work, the research on the response to ambiguity is mainly confined to chance processes.

However as Heath and Tversky say, the

"potential significance of ambiguity stems from its relevance to the evaluation of evidence in the real world. Is ambiguity aversion limited to game of chance and stated probabilities , or does it also hold for judgmental probabilities?" Heath and Tversky (1991) p 6.

Starting from the conviction that the aversion to ambiguity observed in a chance set up does not extend to judgmental probability, (which involves epistemic uncertainty), the two authors explore through an experimental investigation an alternative account of uncertainty preferences called competence hypothesis.

The competence hypothesis explanation constitutes, according to us, a link to all the theories which make a distinction between a probability estimate and the confidence in one's own probability estimate.

The core of the competence hypothesis is in the recognition that

"the willingness to bet on an uncertain event depends not only on the estimated likelihood of that event and on the precision of that estimate, it also depend on one's general knowledge or understanding of the relevant context. More specifically, we propose that - *holding judged probability constant*[58] -people prefer to bet in a contest where they consider themselves more knowledgeable or competent that in a contest where they feel ignorant or uninformed." Heath and Tversky (1991), page 7.

According to Heath and Tversky (1991) the main reason for the competence hypothesis is motivational rather then cognitive. In

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[58] The italics is mine.

particular they suggest that the consequences of each bet are not only the monetary payoffs associated to the bets. To the outcome of each bet people associate also credit or blame. The credit and the blame associated to the outcome of the bet depend on the attribution of success or failure. The competence hypothesis is connected with assumption that while in a game of chance success and failure are mainly attributed to luck this does not hold for bets on judgment. In particular Heath and Tversky (1991) suggest that in a situation in which the decision maker has a limited knowledge of the problem, failure may be attributed to ignorance and success to luck. On the other hand in a contest in which the decision makers is an expert, failure may be attributed to chance while success is attributed to knowledge. This asymmetry in the balance of credit to blame will lead to the individuals avoiding betting in situations in which they do not know a lot or in which the do not understand the process.

According to Heath and Tversky (1991) this analysis accounts for the availability of uncertainty preferences whether or not they involve ambiguity. Uncertainty preference can be expressed also if

a) I prefer to bet on the future rather than on the past (see Rothbart and Snyder 1970) or

b) I prefer to bet on a skill rather than on a chance.

In all the three cases (betting on the known probabilities, on future and on skill) my willingness to bet is determined not simply by my probability judgment but my knowledge of the problem relatively to what can be known. For example I prefer to bet on the future than on the past not because the future is less ambiguous than the past. What makes the difference is that "the past, unlike the future is knowable in principle", but not to me (what changes is my relative knowledge).

In this way the competence theory introduces the idea of the presence of a discrepancy between choice and judgment.[59] That is to say that the

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[59] This kind of discrepancy has been underlined by many authors as one of the main consequence of the presence of uncertainty. In the literature of ambiguity as we have seen Ellsberg's original model as well as that of Gardenfors and Sahlin's and Levi's are the models that most underline this distinction. The same kind of distinction, which by the way reproduces the distinction between probability and confidence, is typical of other theories of decision under uncertainty. In particular Shackle's theory of potential surprise and Keynes's idea of probability and its relation with the concept of weight of evidence

subject can prefer to bet on A rather then on B even if he judges B as probable as A. (I assign to the two events the same probabilities but I know more about A so I will choose to bet on A).

For example in one of their experiments (experiment 1) subjects were asked to answer 30 knowledge questions and then to assign their confidence on their answers. After that the subjects were given the opportunity to choose between betting on their answer or on a chance lottery in which the probability of winning was equal to their confidence. The idea was that if people were ambiguity adverse they should prefer to bet on the chance device which is clearly "objective" than to bet on their judgmental probability which are more ambiguous. The results of the experiment seem to confirm the competence hypothesis. The subjects were not ambiguity averse and the ones who knew a lot prefer to bet on their judgment while the subject who knew little prefer to bet on the chance lottery and this *holding the belief constant.*

For Bayesian decision theory the subjective probability obtained from judgment and the subjective probability obtained from choice are consistent. Instead competence theory implies is that the decision weights derived from choices may not reflect the decision maker's beliefs. According to Heath and Tversky (1991), the Ellsberg paradox simply show that people prefer to bet on clear events, not that they consider the red or black in the second urn as less probable. If the degree of belief and the decision weight are two different concepts then

" a person may believe that the probability of drawing the ace of spades from a well-shuffled deck is 1/52, yet in betting on this event he or she may give to it a higher weight." Heath and Tversky (1991) p 23.

Hence the theory of competence suggests that ambiguity can be considered just one of the factors that reduces competence and make people avoid betting in certain situations. In this respect the competence hypothesis can be linked to the other psychological literature which analyses for example (as we have already mentioned above) the preference of betting on future events more than on past events (Rothbart and Snyder 1970) and more recently Levi and Pryor

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seem to stem from the same kind of considerations.

(1987) or the protecting the self from negative consequences in the work of Josephs, Larrick, Steele and Nisbett (1992).

I.6.3 *Linguistic uncertainty.*

In this section we want to review some psychological studies on uncertainty which mainly regards a comparison between verbally and numerically expressed probability measures.

The studies of Budescu, Weinberg and Wallsten (1988) compared the cash equivalents given by subjects for gambles where the probabilities were expressed numerically graphically or verbally.

In the first stage of the experiment the subject had to equate verbally and numerically descriptions to 11 different graphical displays. That is to say the have to choose for example a number or a verbally expression like quite likely, very likely, probable etc for 11 different graphical displays (spinners). In the second stage the subjects had to bid (using the Becker, DeGroot and Marschak device) for simple two outcome gamble in which one outcome was always 0. The uncertain event was draw from the set of questions of the first stage. The gamble varies according to the domain (gains or losses) probabilities, and mode of presentation (numerical graphical and verbal). According to the authors if verbal probabilities are vague and people avoid uncertainty then the prediction was that in the positive domain the subjects should bid less for the verbally expressed probability while in the negative domain the subjects should bid more. These predictions were not sustained. The subjects bid more to obtain verbal than numerical probabilities in the domain of gain and offer more to avoid them in the domain of losses.

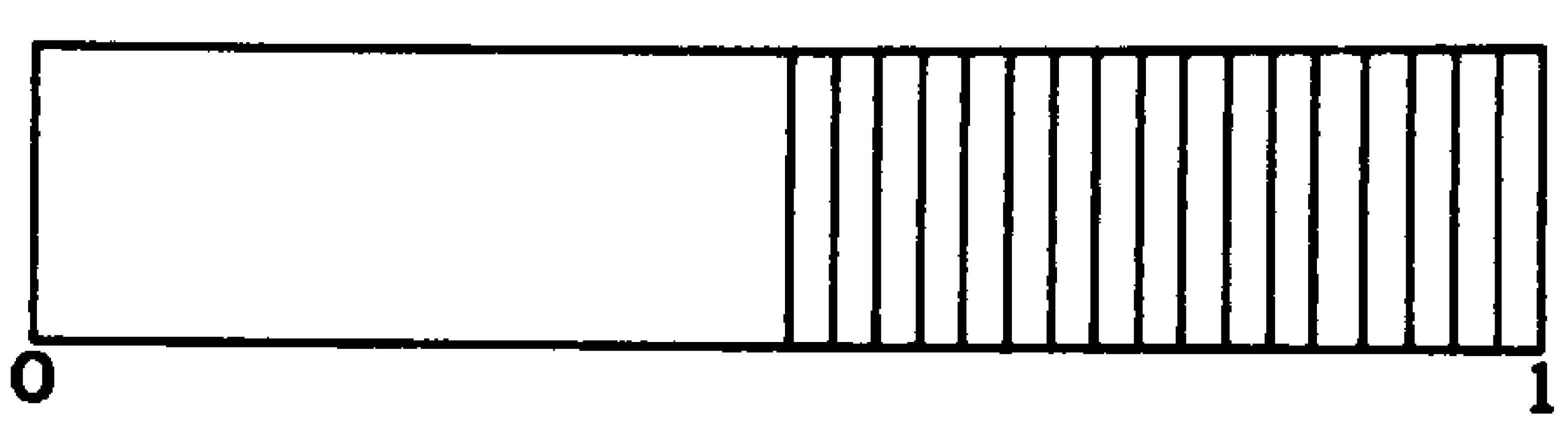
The same experiment was replicated by Wallsten, Budescu and Erev (1988) and the data did not sustained the above prediction either. In this last experiment the subject offered exactly the same amount to obtain verbal as numerical probabilities in the domain of gain and to avoid them in the domain of losses. The only difference is that if the variability of the bids is investigated then the bids in the case of verbally expressed uncertainty are more variable. Wallsten, Budescu and Erev find the result of their experiment quite surprising. To explain their results they develop a model according to which the vague meaning of a linguistic probability expression to an individual is represented by his or her membership function for that phrase.[60] According to their model and individual when required to act on the basic of linguistic probability , he or she behaves in accordance with a specific probability value whose membership is sufficient high (sufficiently high means above a threshold that can vary according to the contest, task or importance of decision). That is to say the model assumes that the probabilities are only considered if their membership value μ is greater or equal to the threshold value ν.

Wallsten, Budescu and Erev (1988) tested their model in two experiments in which subjects made binary choices between linguistic and precise gambles for identical outcomes. For example the subject is presented with the following two gambles:

Figure I.15 Illustration of the Wallsten, Budescu and Erev (1988)'s experiment

- 100 points if white
0 points if red

LIKELY
to lend
on white



The subjects were then asked if they preferred to play the spinner or the phrase gamble.

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60 A membership function is a function over the [0,1] interval which assumes its minimum value usually 0 for probabilities not at all denoted by the phrase it represents; it assumes the maximum value , generally one , for probabilities definitely denoted by the phrase.

In order to test the effect of the threshold outcomes for landing changed from trial to trial (for gains as losses) by two stakes 100 and 1,000. The linguistic gamble was represented by 6 adjectives: doubtful, improbable, possible, good chance and likely. Each phrase was paired with 6 different spinner probabilities of landing on white depending on he phrase. For example for possible the probabilities were 0.23, 0.33,...., 0.73. Each specific choice was presented over a series of three section 9 times.

10 subjects participated in the experiment and the model fitted significantly for 9 subjects.

According to Wallsten, Budescu and Erev (1988), the threshold model is important since

"it allowed widely varying choice patterns to be related to equally diverse membership functions by means of a single model. Optimal threshold values varied across subjects over the entire allowable range, although they did not vary with outcome domain or value, as we had thought they might." Wallsten, Budescu and Erev (1988)p 50.

The surprising result that subjects gave the same response to verbally graphically and numerically expressed probability, while the hypothesis of ambiguity aversion predicted a preference for numerical probability motivated a second group of experiment to test the following idea. In discussing the results of the previous experiment Wallsten pointed out that there may be situations in which one form of communication can be superior to another. In particular Rapoport, Wallsten, Erev and Cohen (1990) organized an experiment to study this problem in a task of sequentially revising opinion or beliefs in the face of uncertain situation. In particular what they wanted to test was the hypothesis of conservatism according to which subjects'estimate of probabilities are conservative that is to say less extreme that the optimal values calculated from the Bayes rule. In particular Rapoport, Wallsten, Erev and Cohen (1990) wanted to test if conservatism was influenced by the presence of verbally or numerically expressed probability, that is to say if was influenced by the mode of expression of uncertainty. The results of their experiment confirm the results of the experiment above described that is to say that there are not differences between the two communication modes.

I.7 Conclusion

As we have see from the present review of the literature the problem posed by the Ellsberg paradox has been tackled by many very different approaches[61]. These different approaches are the results of different fields of inquiry of different disciplines. Very often these approaches share some common feature but their empathize different aspects. The review has been conducted as we have already said grouping the contributions according to the criteria illustrated in Table I.5. As we will see in the following chapter most of the empirical work on the Ellsberg paradox has been in the direction of investigating sources of and attitudes towards ambiguity. This is probably due to the fact that most of the experimental work has been conducted by psychologists. On the other hand, there are few contributions on testing theories and on application to this theories to economic contest. Chapter III and IV want in fact to be our contribution in this respect.

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61 In this review we have analyzed mainly the contributions which tackle the Ellsberg paradox through a modification of the utility or of the probability. However it should be noted that in Fishburn (1991) (1993) a new approach is introduced in which event ambiguity is dealt with as a primitive concept.

CHAPTER II

A REVIEW OF EXPERIMENTAL RESULTS ON THE ELLSBERG PARADOX

II.1 Introduction

In this chapter we review some of the experimental evidence on ambiguity. Ellsberg never did an experiment by himself, however, since the appearance of his article in 1961 others did. This review will not be exhaustive, since we want simply to concentrate our discussion on (for a complete review see Camerer and Weber (1991)) some experimental work which can be considered as the background for the experiment described in the third chapter. Moreover, we are going to review mainly published papers. (Unpublished work is not easily available).

Most of the experimental work on the Ellsberg paradox can be divided in the following groups:

a) replicating the paradox (in a chance or in an event contest, that is to say using lottery or real events);

b) investigating the possible sources of ambiguity (for example manipulating some factors as the range of the second order distribution or testing psychological attitude of subjects with respect to the time resolution of uncertainty or the environment surrounding the resolution of uncertainty;

c) exploring new theories;

d) testing some of the existing theories.

In this respect, the experiment described in chapter III can be classified mainly in the first and in the fourth groups. The experiment is in fact a replication of the paradox in a lottery context and tests some of the theories which attempt to explain the Ellsberg paradox. The novelty of the experiment is mainly in the particular design involved and in the theories tested (even if it is important to notice that at the moment there are very few works which directly test theories on ambiguity which of course we will refer to in the review). The review, consequently, will focus on the elements which are considered important for understanding subsequent work.

The chapter is organized as follows: in the first part we will analyze in general the experimental work on the Ellsberg paradox. The works will be analyzed in chronological order. In the second part we will concentrate on the few works which directly test theories. In the last part we will try to compare the existing work with the experiment presented in chapter III. (See Tables II.1 and II.2 for a summary)

Table II.1 Empirical works on ambiguity Summary Table

Replication of the paradox	Possibele new sources of ambiguity	Investigating new theories	Testing theories
Becker Brownsn (1961)	Becker Brownson (1961)	Einhorn Hogarth (1985)	Curley Yates Abrams (1986)
MacCrimon (1968)	Yates Zkowsky (1976)	Khan Sarin (1988)	Curley Yates (1989)
MacCrimon Larsson (1976)	McCrimmon Larsson (1976)	Heath Tversky (1991)	Keppe Weber (1991)
Einhorn Hogarth (1985) (1986)	Curley Yates (1985)		Mangelsdorff Weber (1992)
Cohen Jaffray Said (1986) (1987)	Einhorn Hogarth (1985)		Bernasconi Loomes (1992)
	Curley Yates Abrams (1986)		
	Schoemaker (1991)		

Table II.2 Cassification of the empirical works according to Various factors

	Incentive	Task	Lottery	Payoffs
Becker Brownson (1961)	Yes/No	C	L	+
MacCrimmon (1968)	No	C/R	L/E	+
Yates Zukowsky (1976)	No	C/P	L	+
MacCrimmon Larsson (1979)	No	R	L/E	+
Curley Yates (1985)	Yes	C	L	+
Einhorn Hogarth (1985) (1986)	No	C	L	+/-
Cohen Jaffray Said (1986) (1987)	Yes	C	L	+/-
Curley Yates Abrams (1986)	Yes	C/P/R	L	+
Kahn Sarin (1988)	Yes	C	L/E	+/-
Curley Yates (1989)	Yes	R	L	+
Schoemaker (1991)	Yes	C/P	L	+/-
Heath Tversky (1991)	Yes/No	C/P	E	+
Keppe Weber (1991)	Yes	CE/PRO	L/E	+
Mangelsdorff Weber (1992)	Yes	PRO	L/E	+/-
Bernasconi Loomes (1992)	No	C	L	*

Legenda C=choice task, P=price task, R= Rank task;
CE=elicitation of the certainty equivalents
PRO=Elicitation of the probabilities, decision weight etc.
L=Lottery enviroment, E= Event enviroment;
+ = choices with gains, - = choices with losses

II.2 Empirical works on the Ellsberg Paradox.

II.2.1 *Replications of the Ellsberg imaginary experiment and extensions.*

Becker and Brownson (1961). Becker and Brownson (1961) ran the first experiment which replicated the Ellsberg paradox. They assumed

"that the ambiguity associated with a given alternative is determined by the nature of the distribution on the probabilities of future events relevant to that action. "Becker and Brownson (1961) p 64.

Since they believed that ambiguity was determined by the distribution on the probabilities of the occurrence of an event, Becker and Brownson (1961) analyzed the difference in behaviour due to different levels of ambiguity, where the various levels of ambiguity were operationalized through different ranges of the possible probabilities of a state. In their experiment, subjects were asked to make 10 decisions; each decision consisted in looking at a pair of urns and state on which urn they preferred to bet and how much they were willing to pay to bet on the preferred urn. The experiment consisted of two parts: in the first part the subjects were screened with a Ellsberg kind of problem with no money involved. Only those subjects who committed the Ellsberg paradox (15 out of 37) participated in the second part of the experiment where real money was involved. Of these subjects all but one were willing to pay a substantial amount of money to draw from their preferred urn. Moreover the preferred urn, for which the subject would pay a premium, was the urn which had the smaller range around the mean, confirming the Becker and Brownson assumption.

MacCrimmon (1968). The first extension in a event context of the Ellsberg paradox is due to MacCrimmon (1968). He replicated the Ellsberg paradox in a chance context and in an event one. In addition, subjects were exposed to written arguments in favour of or against the sure thing principle. Then, they were given the opportunity to reflect on their answers. Thirty eight executives participated in these three experiments; there was no incentive mechanism. In the first experiment they had to express their preference between two situations, one risky (betting on red and drawing a card from a well-shuffled deck) and one uncertain (betting on the fact that the value of a particular stock was higher than a determined amount the day after the experiment[62]). They were exposed to written arguments in favor of and against the sure thing principle and had to state with which arguments they agreed. In this particular experiment, just three subjects violated the sure thing principle, but 11 subjects choose the arguments in favor of the Ellsberg paradox (choosing consequently an answer which conflicted with

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62 They were also offered to bet on the complementary situation, that is to say betting on black and betting on the fact that the stock price would not be higher than a certain amount.

their choices)[63]. The third experiment was very similar to the first one and the choice was between betting on tails or heads and the USA GNP being less than (no less than) $620 billion in 1964. The second experiment was different and involved a choice between investing in two countries with historical frequencies given for one country but not for the other. In the third experiment, the subjects who violated the sure thing principle were 5, while in the second the number of subjects who violated the sure thing principle increased to 19. As we can see, the subjects considered the second situation more ambiguous than the third one[64]. This result was confirmed by the interviews. Between the subjects committing the Ellsberg paradox in the first and in the third experiment just one subject persisted in his choices, all the others said that the distinction between risk and uncertainty was not reasonable. However, in the case of the second experiment more than half of the subjects (10 out of 19) did not want to change their choices.

Yates and Zukowsky (1976). Yates and Zukowsky (1976) reconsidered the Becker and Brownson idea that ambiguity can be

"completely reducible to the range of the induced subjective second order probability distribution". page 21.

The authors constructed three games in order to induce three different second order probability distributions. The first game, G, was defined in a way to induce a second order distribution equivalent to a point estimate corresponding to a situation of risk. The second game, G', was defined in a way to induce a uniform second order distribution (subjects had to draw from a bag containing the 11 numbers from 0 to 10, the chosen number would have determined the number of balls of the winning colour to be put in the bag from which the subject had to draw the final ball). The third game, G'', was a replication of the normal Ellsberg example, a bag containing 10 chips that could be black or red

[63] 18 subjects choose the conforming answer and the remains said that no one of the arguments was logical.

[64] And in fact it is reasonable to consider that it was more ambiguous. Being executives the subjects should have a better knowledge and consequently reach a more "sure" estimate of the probability in an increase in the GNP or in an increase in a stock that making investment decision in an unknown country.

but the proportion of the two colours was unknown. The authors' purpose was not to induce the subjects to think of any particular second order distribution. According to Yates and Zukowsky (1976), if the aversion to ambiguity can be entirely capture by the range of the second order distribution, then the following prediction should hold G $\succ$ G', G $\succeq$ G'' (indifference is possible only if the subject's second order probability distribution for G'' is equal to a point estimate) and G'' $\succeq$ G' (since the range of the induced by the uniform distribution is maximal). In the experiments, the subjects were given six binary choices (G-G', G-G'', G'-G''). For each pair of games, they had to state which was the preferred one. (they were given the three choices above described[65]). In addition, they were asked to state their minimum selling price for each of the games. The incentive mechanism used by the experimenters was the Becker-DeGroot-Marshack device. The subjects sold their lottery tickets or they played the game, during individual sessions at the end of the experiment.

The results of the experiment showed that G was preferred to G'', according to the predictions, but G was not preferred to G' and G' was preferred to G'' (indifference was not permitted in the binary choices)[66]. These last two preferences are not consistent with the predictions described above. The authors concluded their article by saying that the aversion to ambiguity cannot be completely reducible to the range of subjective second order probability distributions. In fact, apart from the range, other factors can be important in determining aversion to ambiguity. In G'' a subject can imagine a second order distribution with the same range as in G', but may attach different probabilities to the first branch (G' and G'' can be imagined as two stage lotteries), thinking that for example some probabilities are more reliable. In such a case, G' and G'' can be represented, for example, by two different second order distributions, with the same range but with a different variance. The results of the experiment in chapter III and the interpretation of these results in chapter IV give a certain support to this hypotheses and this will be one of the topic we will pursue in our future research.

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[65] The games were six because the order was changed.

[66] G$\succ$G' 53%; G$\succ$G" 78% -G'$\succ$ G 46%; G'$\succ$G''68% - G''$\succ$ G 22%; G''$\succ$ G'32%.

MacCrimmon and Larsson (1979). MacCrimmon and Larsson (1979) replicated the Ellsberg paradox and extended the MacCrimmon (1968) experiment. Nineteen students participated in this experiment; they were decision making graduates and there was no incentive mechanism involved. In the first part of the experiment (on the three colour example), they were put in front of a simple replication of the paradox with 90 balls. They were presented with arguments against and for the axioms as in MacCrimmon (1968). The novelty of the experiment was that, besides the original Ellsberg problem, they were confronted with other pairs of choices in which the total number of balls was 100 and in which the probability of red was not fixed to 0.33 but was made to vary (0.20, 0.30, 0.33. 0.34, 0.40, 0.5). The predictions of the authors were for a maximum of violation around 0.33, with a decrease for less than 0.33 (there is a trade off between knowing the probabilities and having a small probability for red) and an increase moving towards p= 0.50 (in this case the probability of red is not just known but also higher). The aim was to test whether the distinction between risk and uncertainty persisted in the case of a wide change in probability. Of the 19 subjects who took part in the experiment, 10 out of 19 committed the Ellsberg paradox. In the second choice (the one with 100 balls instead of 90), 70% of the subjects committed the Ellsberg paradox when the probability of red was 0.33 or 0.34. The percentage of the violation decreased to 15% with a probability of 0.20 and to 0 with a probability of 0.50 as predicted. MacCrimmon and Larsson (1979) concluded that the individual would not maintain the distinction between uncertainty and risk when it is going to cost a significant chance of winning. It would be interesting to repeat the experiment with real money involved.

In the second part of the experiment they tested the two colours Ellsberg paradox with the original description of Ellsberg and with event lotteries like the rise in the price of a particular stock. The subjects were presented with two sets of alternative wagers and they where asked to rank the wagers in each set according to their preferences. The two sets differed in terms of payoffs ($1000 and $1001). Let us consider simply the set with a payoff of $1000 and the wager with the original Ellsberg example. In this case the subjects were asked to rank the 4 possible options. The rate of violation in this case was very high, 84%. Consider now the case of another wager given to the subjects represented by the known urn and by the two options"the price of the Pierce Industries goes down or does not go down". In this latter case, 10 subjects ranked the two urns options before the two stock options, with a percentage of subjects who committed the Ellsberg paradox of 53%. In both situations, the rate of violation of the sure thing principle is quite high and the authors attributed this fact to the kind of task asked, ranking versus choices. This may be true but one more obvious and simpler hypothesis can be the lack of any incentive mechanism. Comparisons between the various results of the experiments are quite often hard to do because of these methodological problems. Different incentive mechanisms as well as different performed tasks (choice task, rank task or pricing task) can indeed have an influence on the behaviour of the subjects.

Curley and Yates (1985). Curley and Yates (1985) investigated if the center and the range of the probability interval can affect ambiguity perception and ambiguity preferences. They represent indeterminacy in probabilities through an interval, so they did not characterized ambiguity as a specified second order distribution. The idea is that of finding another factor, in addition to the range, that can influence the perception of ambiguity, which, they suggest, can be the center of the interval. Their subjects in the experiment were presented with lotteries of the following kind: L (p, \$5; 1-p, \$0). but p was represented by:

P_{max} |————|————| P_{min}

C

Eighty undergraduates participated in the experiment, they were given 30 pairs of lotteries, which differed in the range and the center of the probability interval. They were asked to choose the more preferred lottery of each pair and to express also the strength of their preference through a graphic scale of the following kind:

Rating:

Lottery L |------|------| **Lottery R**

The lotteries consisted in bags containing 100 chips, each red or white, in different proportions. Before drawing the subjects were asked to compose the bag in the following way: "You have a bag containing

0-20 winning chips and 0-20 loosing chips. Now decide which will be the winning chip and add 45 winning chips in the bag and 35 losing chips in the bag. Then draw a chip. If the chip is of the winning colour you will get $5 nothing otherwise."

At the end of the experiment each subject was asked to toss a coin. If the call was successful for the subject, a pair of lotteries was chosen randomly and the preferred lottery was played out for real. Contrary to what was expected by the authors, the perception of ambiguity did not seem to depend on the length of the range (that is to say the differences in the ranges could not account for changes in ambiguity preferences). Moreover there was no predominant switch between ambiguity aversion to ambiguity preference according to the level of probability. However the perception of ambiguity seemed to depend strongly on the level of the center. Ambiguity avoidance increased with the increase of C for range comparison, but quite surprising for the authors, there was almost ambiguity indifference for C = 0.40 and ranges R_1 0.40 and R_2 0.20. Curley and Yates (1985) suggested as an interpretation of this result that the centre might have a strong effect on the perception of ambiguity just when one end of the range is equal to zero. This can be something that is possible to investigate in future, even if around 0 or 1 we have a natural limit in manipulating the intervals. Surprisingly, the result of a diminishing sensitivity to ambiguity preference around 0.40 is replicated in our experiment in chapter IV also in the case of losses. When we operationalized the ambiguity in the probabilities through a probability interval we find that for a probability center of 0.50 and a R = 0.40 aversion to ambiguity is strongly reduced. This may simply suggest that the perception of the probability near 0.5 is less distorted (As for example in Quiggin (1982)).

Einhorn and Hogarth (1985, 86). Einhorn and Hogarth (1985, 86) also ran a series of experiments which replicated the Ellsberg paradox. In particular, they replicated the Ellsberg paradox in the two colours form with two urns with 100 balls. In addition they extended and replicated the paradox in case of a very low probability, 0.001. The subjects were 274 MBA students, they had to answer a questionnaire and they were not paid. The imaginary payoffs of the lotteries were either $100 and 0 or -$100 and 0. In the first experiment, the mere replication one, with a positive outcome, 47% of the subjects showed ambiguity aversion, 19% ambiguity preference and 34% ambiguity neutrality. In the situation with imaginary losses, 30% of the subjects were ambiguity averters, 14% ambiguity seeking, and 56% ambiguity neutral.

In the second kind of situation, subjects were asked to imagine two urns each containing 1000 balls. In the not-ambiguous urn, the balls were numbered consecutively from 1 to 1000. The subjects were told that their payoff was contingent on drawing the number 687. Instead, in the ambiguous case, the subjects were told that any proportion of the 1000 balls could have the number 687 on them. In this latter case, for positive payoffs 30% of the subjects showed ambiguity aversion, 15% ambiguity preferences and 56% ambiguity neutrality. In the case of imaginary losses the results were: 75% of the subjects showed ambiguity aversion, 5% of the subjects showed ambiguity preference and 20% of the subjects showed ambiguity neutrality. Einhorn and Hogarth (1985, 86) results showed an increase in the ambiguity preference behaviour between positive and negative payoffs for a 0.5 probability level, while showing a diminishing ambiguity seeking behaviour from positive to negative payoffs for the low probability level. This seems to suggest that the response to ambiguity is sign dependent. In addition this seems to suggest a over-estimation of low probability in case of losses and an under-estimation of low probability in case of gains.

Cohen, Jaffray and Said (1986) (1987). Cohen, Jaffray and Said (1986) (1987) replicated the Ellsberg paradox and extended it to the domain of losses and investigated possible links between risk attitude and ambiguity attitude. Their experiment was run as follows. Each week, for 10 successive weeks, 134 students participated in the experiment, filling in a questionnaire. The experiment required subjects to make binary choices of the type used by certainty equivalent methods for utility assessment. The subjects were asked to make a series of binary choices between a risky prospect yielding gains (or losses) of 1000 francs with probability p and nothing with probability 1-p, and a sure prospect yielding gain g (or losses l) with different level of g (or l) varying from 0 to 1000 with increments of 50. Binary choices with different probabilities for the non zero outcome (1/2,1/3,1/4,1/6) were given to the subjects. The binary choices also included a lottery with an unknown probability of the Ellsberg kind. In this case the description given to the subjects corresponded to the usual one (100 balls of two colours with unknown proportion), while in the case of risky prospects the probability was not clearly stated, but the subjects had to infer it from a description of a simple event-generating device like the tossing a coin. The subjects did not receive a participation fee for their participation in the experiment. However, they were told that 25% of them were to be randomly chosen and that in that case one randomly chosen lottery was going to be played out for real. In the case that a lottery involving a loss was chosen, their were told and given a 1000 franc bonus. This experiment is particularly interesting because it explores the different attitudes towards risk and towards ambiguity in case of gains and losses and possible relation between them. Let us consider for example the case of the risky lotteries with p=1/2 . In the domain of gains 40% of subjects were risk avers, 28% risk neutral 32% risk seeking. In the domain of losses, 11% risk averters, 19% risk neutral and 70% risk seeking. Moreover, while 41% of the subjects switched from risk aversion to risk loving from the domain of gains to the domain of losses, a good percentage (39%) kept a constant attitude casting doubts on the pervasiveness of the reflection effects. (Kahneman Tversky (1979) subjects who are risk averters in the domain of gains become risk lover in the domain of losses.).

As far as ambiguity is concerned, in the domain of gains 59% of the subjects were found to be ambiguity averse 35% neutral and 6% ambiguity prone. In the domain of losses, 25% of the subjects were ambiguity averse, 42% ambiguity neutral and 33% ambiguity lovers, with a prevalence for ambiguity neutrality in the case of losses. In addition, Cohen Jaffray and Said (1986) did not find any relation between ambiguity attitude and risk attitude. (This last result is confirmed also by Schoemaker(1991) and by the experiment in chapter IV). Hence as far as ambiguity is concerned, the main result of the experiment seems to be the difference in attitude towards ambiguity in case of losses as compared to gains. This seems not to be very surprising, since this difference in attitude holds also in the case of risk. However, this different in attitude seems very sensitive to the incentive mechanism used and to the context used (lottery experiment market experiment etc.; See for example Sarin and Weber (1992), Camerer and Kunreuther (1989), Schomaker (1991). Risk attitudes and ambiguity attitudes comparison is surely something which deserve further research.

Schoemaker (1991). Ambiguity aversion and the links between risk attitude and aversion to ambiguity have been tested also by Schoemaker (1991) in two very interesting and carefully conducted experiments. In the first experiment subjects were asked to choose between a urn with known probabilities but uncertain outcomes[67] and an urn with unknown probabilities. However, in this second case, the urn with unknown probabilities was operationalized with a uniform second order distribution[68]. The problem was posed in the positive as well as in the negative domain. Seventy three MBA students took part in this experiment, they were not paid but 10% of them (randomly chosen) had the opportunity to play the preferred lottery for real[69]. On the whole, the results of the experiment showed that for gains as well as for losses subjects generally disliked the urn with unknown probabilities. This aversion to uncertainty was stronger in the gain domain with respect to the loss domain. Moreover, also in this case, risk attitude seems not to be correlated with ambiguity attitude.

The second experiment was very similar to the first one. In this case, instead of using continuous probabilities, Schoemaker gave to the subjects lotteries involving binary uncertainties. In the case of unknown outcome, the positive outcome could be either $50 or $150 with probability 1/2. In the case of the unknown probabilities, the probability could be either 1/4 or 3/4 with probability 1/2. In this experiment subjects were not asked which lottery they preferred. Instead they were asked

"Suppose that you could rather know the true value of p or x (for free) prior to choosing among the lotteries. Which type of information would

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[67] "Urn A: Contains exactly 50 red and 50 white balls (mixed at random) If you draw a white ball, you get $0. If you draw red, your payoff is going to be some unknown prize between $0 and $100, to be determined by spinning a wheel of fortune. Assume that each amount in this range is equally likely to be the actual prize." Schoemaker(1991) p 298.

[68] "The urn was randomly filled in such a way that each mixture is equally likely" Schoemaker(1991) p 298.

[69] The had the opportunity to play both the gain and the loss lottery. However the loss was enforced only when the subject gained in the gain lottery, that is to say no net losses were inforced. The Becker-De Groot- Marschak mechanism was used.

you or should you prefer? The probability or the prize information?." Schoemaker (1991) p 308.

Schoemaker found that the majority of subjects (62%) preferred to know the number of red balls in the urn (versus 17%) preferring the prize information and 21% being indifferent. When the same problem was posed in the loss domain, the number of subjects interested in the negative prize information rose from 17% to 36%, with 16% being indifferent and 48% preferring the probability information. Overall also the Schoemaker experiment confirmed that people show consistent aversion to uncertainty even when this is expressed with a second order distribution. Aversion to uncertainty seems to be unrelated to risk attitude and loss preferences seem to be more complex and less clear then the gains ones, confirming the results of the Cohen, Jaffray and Said experiments.

II.2.2 *Experiments which test different theories.*

There is a group of empirical works which either test new hypothesis or theories or test existing theories. In this section we will review these works in more detail since some of them have been instrumental in determining the design of the experiment described in chapter III.

Curley, Yates and Abrams (1986). One of the first works which tested some theories (they call them sources of ambiguity) is that of Curley, Yates and Abrams (1986). In this work, Curley, Yates and Abrams (1986) did not simply replicate the Ellsberg paradox but investigated various psychological sources of ambiguity aversion. In particular, they investigated 6 hypotheses of ambiguity aversion: uncertainty aversion; if ambiguity aversion is just an expression of a general aversion to uncertainty (in this case for example ambiguity aversion should be correlated to risk aversion); the hostile hypothesis (subjects perceive that the process with which he ambiguous urn is determined is not random); other evaluation (in making the decision subject anticipates what other can think); self-evaluation (the decision maker anticipates the future evaluation, her or his future evaluation not that of the others); forced choice (the unambiguous lottery is selected just because all other conditions are equal; people apply a lexicographic rule in which ambiguity is a second dimension);

the mistake hypothesis (in choosing the non-ambiguous bag the subjects commit a unconscious mistake, if situations were correctly understood the subjects would avoid the bias). To test all the hypotheses Curley, Yates and Abrams (1986) ran 5 experiments. In all experiments subjects were asked to evaluate and indicate their preferences between two lotteries. The lotteries represented the two colour Ellsberg example with two bags containing 100 red and white chips. If the right colour was chosen subjects could get $5, nothing otherwise. The incentive mechanisms used were generally that of picking one or more subjects at random, and making them to play the preferred lotteries for real. (where it was used another mechanism it will be expressly stated). In the first experiment, 26 psychology undergraduates were involved. They were given a written explanation of the certain Ellsberg lottery and their cash equivalent was elicited through the Becker-De Groot-Marschak device. Then, the second lottery was presented; they were asked which of the two lotteries they preferred and, then, the cash equivalent of the second lottery was elicited with the same procedure. Through the evaluations of the subjects, Curley, Yates and Abrams (1986) defined a risk premium as the difference between the evaluation of the subjects and the expected value of the lottery and an ambiguity premium as the difference between the evaluation of the subject of the risky lottery and the evaluation of the subject of the ambiguous lottery. The authors did not find any relation between risk attitude and ambiguity attitude, consequently they concluded as follows:

"The results clearly fail to support the hypothesis of a general attitude towards uncertainty in that subjects' responses to risk and ambiguity were independent. This suggest that ambiguity avoidance is distinct from risk avoidance. One difficulty with this conclusion is the lower power of the test, which results from the relatively small sample size." Curley, Yates and Abrams (1986), p 239.

The second experiment was designed to test the hostile hypothesis. The 20 subjects were given the same lotteries as in experiment 1. However, in the case of the second lottery they were instructed to "imagine" themselves as the "managers" of lottery two. They were asked to consider how they would have composed the bag of lottery two and, in particular, if it would have been possible, to set the bag in a way that, no matter what colour the players would have chosen, the bag would have been biased against them. Of the 20 subjects of the manipulation group (the one who were given the above description, only 6 (30%) believed that it would be possible to manipulate the bag. In addition, the hostile hypothesis predicts that the subjects who did not consider the eventuality of the bag being biased against them should not behave according to the Ellsberg paradox. In fact this was not the case because also all the other subjects (14) selected the non ambiguous bag showing ambiguity aversion.

To test the forced-choice hypothesis in experiment 3, Curley, Yates and Abrams (1986) constructed four choices and used a rank task. Subjects were asked to rank four lotteries and they received either a flat payment for their participation in the experiment (group one) or had the opportunity to play one of the lotteries for real (group two). 16 + 31 students participated in the experiment. One group of students received lotteries 1 and 2 (the usual ones) and 3 and 4 which were exactly like 1 and 2 but in in which the non zero payoff was 4.99 dollars. The other group was given the same kind of lotteries, but the non zero payoffs were equal to $10 and $9.99. If ambiguity is a second dimension in a lexicographic order subjects should always prefer a lottery with higher expected value. The results of the experiments showed that the number of non lexicographic patterns significantly exceeded the number of lexicographic ones.

In the fourth experiment, Curley Yates and Abrams (1986) tested the other-evaluation and the-self evaluation hypotheses. In the case of the other evaluation, subjects had to announce publicly their choice and the lottery was played in front of the others or they had to stay after the experiment and play the lottery in an individual session. In the high self-evaluation condition, the content of the ambiguous bag was immediately exposed after the subject had played the lottery, while in the low self-evaluation condition the content of the bag was not going to be revealed in any case. For the experiment Lottery 1 and 2 were used followed by the description of the procedure. In their experiment all the subjects had to state which lottery they preferred and, then, one subject was selected and had to play the lottery. 136 subjects, all students, participated in this experiment. According to Curley, Yates and Abrams (1986), the results of the experiment did not give any support to the self-evaluation hypothesis. However, the other-evaluation hypothesis was strongly supported by the data.

"Publicizing subjects' decisions, which increase the likelihood of the evaluation of the decisions by other in the groups, significantly raised the level of ambiguity avoidance."
Curley Yates and Abrams (1986) p 248.

In the fifth experiment, Curley Yates and Abrams (1986) tested the mistake hypothesis; seventy undergraduates participated in this experiment, divided into 3 groups. They were given four pairs of lotteries similar to lotteries 1 and 2 (in fact 2 pairs was given by lotteries 1 and 2). They had to indicate their preferences amongst these four bets which differ from the other cases by the fact that the positive payoff was $10, the chips used were blue and red and the winning colour was specified in the bet (if red comes out you will win, or if blue comes out you will win). First, a pair of bets was offered starting from RII and BII and the subjects were asked to indicate which was the preferred bet or whether they were indifferent between the two bets. In this respect, this was not a forced choice task since indifference was allowed. Moreover, the subjects were asked to indicate the strength of their preferences on a scale of 1 to 5. After the four pair of bets were examined, subjects were given another booklet containing statements about why subjects should react to ambiguity or why subjects should not react to ambiguity. After the presentation of the arguments, the subjects were asked to reconsider the last two choices. The results of this experiment showed that the subjects did not show any systematic colour bias.

Curley, Yates and Abrams (1986) reported the data of the first and of the second choices (the reconsideration after the exposure to the above arguments). In the case of the first choice, 19 subjects were ambiguity seeking, 46 ambiguity averse and 5 neutral. In the second presentation, 22 were ambiguity seeking, 42 ambiguity averters, and 6 ambiguity neutral. Only four times was there a switch from ambiguity preference to ambiguity neutrality, but an equal number of times (4) there was a switch in the opposite direction.

"The reluctance of subjects to change their behaviour after the presentation of a prescriptive counter argument in the present choice situation replicates the results which has been obtained in other choice situations involving ambiguity. The finding is in contrast with the prediction of the mistake hypothesis that subjects would not avoid ambiguity after the "mistake" was pointed out to them. It is not the case the subjects, upon refection, will "correct" themselves after their "error" is highlighted and explained."Curley, Yates and Abrams

(1986) page 252.

To conclude: in the Curley Yates and Abrams (1986)'s experiments all the tested hypothesis except for that on the other-evaluation, are rejected. However, the fact that in the other-evaluations set up, ambiguity avoidance increased give us simply the information that the evaluations by other people might be an important source of ambiguity. We cannot infer that this is the only source of ambiguity. To test this, another experiment is needed. It can be important to notice that in this case, on the overall experiments, almost 70% of the subjects showed ambiguity aversion. However we have to recognized that, even if paid, subjects were paid very little. In fact not only was the expected value of the lotteries low,($5), but also it had to be weighted with the probability of being randomly picked that was 1/16 in the best case and 1/131 in the worst case. In spite of these methodological problems, the Curley Yates and Abrams (1986) work seems to give a quite interesting contribution on the possible sources of ambiguity avoidance behaviour.

Kahn and Sarin (1988). Kahn and Sarin (1988) ran various experiments replicating the Ellsberg paradox under various conditions with the main objective of developing and testing their own model of decision making under ambiguity.[70] In a first informal experiment, Kahn and Sarin (1988) replicated the Ellsberg paradox through a choice between a two hypothetical games: tossing a coin or a thumbtack. If they won the flip they would win $500, nothing otherwise. 54 MBA students participated in this experiment; 18 were ambiguity averse, 21 ambiguity prone and 15 ambiguity neutral. The ambiguity averse people were willing to pay an ambiguity premium on average of $172.37, while the ambiguity lovers a premium of $60.28. In another experiment, 63 students had to choose 15 times between two urns, one with a known proportion of two-colour balls and the other with unknown proportion of two-colour balls. In the second case, even if the proportion was not specified the range was specified. Of course, both urns had the same expected number of red balls (the winning colour was red and was stated by the experimenter). The bets involved a gain of $10 if red and 0 otherwise or winning $10 if red and a loss of $5 otherwise. Gains and

[70] As far as the theoretical model is concerned see chapter I.

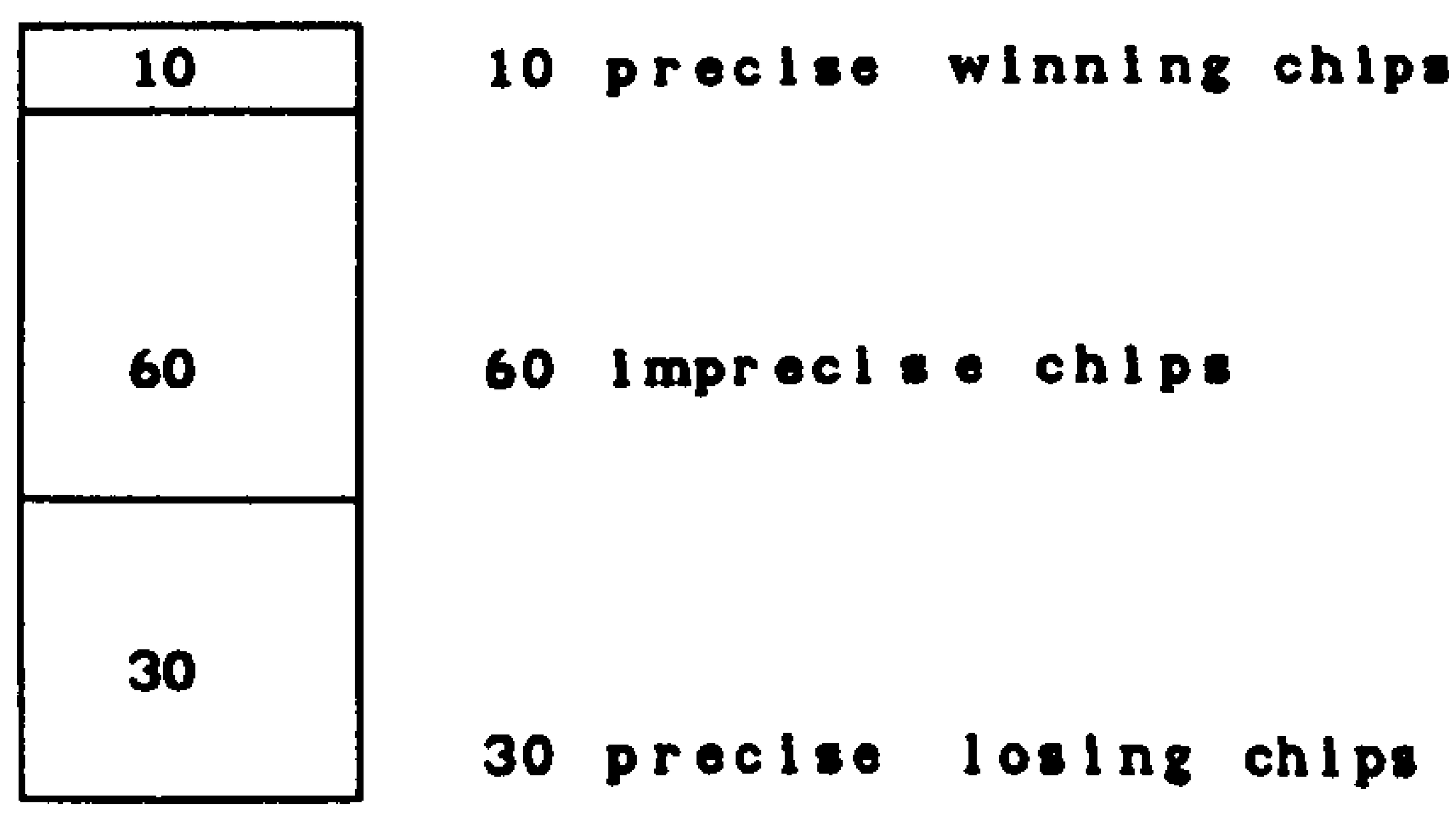
losses were hypothetical but the subjects received a flat payment of $10 dollars for their participation. The experiment showed a significant effect of the range confirming the results of the previous reported experiments. Moreover reaction to ambiguity was shown to depend on the context: if the bet involved just a gain or a gain and a loss. To explore ambiguity reaction in different contexts Kahn and Sarin (1988) ran a further experiment with 60 MBA students. They were told that 25% of them would be chosen at the end to play out a lottery for real. The ones who were not chosen were given $5 for their participation. The choices in the lotteries would come from their responses to a questionnaire. The scenarios given to the students regarded consumer choices (radio warranty decisions, pharmaceutical decisions and quality decisions). The decisions involved different win/loss payoffs and the stated probabilities involved different means and ranges. In addition to the context effect (gain versus losses) found in the previous experiment Kahn and Sarin (1988) found a strong interaction between the mean and the win/loss payoff.

"In the gains domain, there is ambiguity seeking at low mean probabilities and ambiguity aversion at high mean probabilities. In the loss domain, a reflection effect occurs with ambiguity aversion at low mean probabilities and ambiguity seeking at high mean probabilities. These results parallel those observed for risk aversion. A possible explanation of the close resemblance between the finding on risk aversion may be that the same psychological factors are responsible for both effects." p 270 Kahn and Sarin (1988).

These results, however, are in conflict with the Cohen, Jaffray and Said (1986,87) results, showing the necessity of a further investigation of the relation between attitude towards ambiguity and attitude towards risk.

Curley and Yates (1989). Curley and Yates (1989) extended the work of Curley, Yates and Abrams (1986) investigating two main points. On the one hand they studied reactions to ambiguity when ambiguity was operationalized by a probability interval. As in previous studies they varied various factors such as the range of the interval, the centre of the interval and the payoffs of the lotteries. On the other hand, they tested some models: a lexicographic model (ambiguity as a second dimension as in Curley, Yates and Abrams (1986)) and four polynomial models. Thirty one subjects participated in this experiment; at the end they had to play three lotteries for real getting as a maximum amount $30. The task was to order a set of cards (they were asked to rank four lotteries and were give 8 sets of four lotteries) on which there was described a lottery. All the lotteries can be defined by the following graph:

Table II.3 Example of an ambiguous lottery



An ambiguous lottery was defined as a lottery in which the range of the probability interval was positive (R=0 gives a point estimate not an interval). The task was always drawing a chip from a bag of 100 chips. Each bag contained a precise number of chips of the two colours of unknown proportion. The subjects then were asked to add a precise number of the winning colour chips and then a precise number of the losing colour chips. As we already said, the first objective of the experiment was to test the lexicographic rule and the strength of the reaction to ambiguity varying C, R and X (centre, range and outcome). The results of the experiment showed that people were willing to trade off expected value for ambiguity, rejecting consequently the lexicographic model. Ambiguity avoidance predominated at .50 and .75 probability levels. At the .75 probability level they were willing to give up twice the amount of money as at 0.50 probability level (.65 dollars versus .31 dollars). Near 0.25 the subjects were willing to trade off an expectation of $.14 to obtain the more ambiguous option, showing ambiguity proneness.

In the second part of their study, Curley and Yates (1989), examined four polynomial models by means of the theory of polynomial conjoint measurement. They choose this method because they said that it has the advantage of making little scale assumption on the data.

"In particular, only the ordering of options with respect to the undefined ambiguity factor is required for differentiation among the models. " Curley and Yates (1989) page 413.

However this technique seem to have a major drawback - the fact that it does not accommodate any error. This means that any failure of a property is to be interpreted as a failure of the property and any model for which the property is necessary. If we indicate with P the probability factor, A the ambiguity factor and U the utility factor then the four polynomial model can be indicate as follows: (P+A) * U ("the subject arrives to a composite measure of uncertainty combining outcome and process uncertainty, before incorporating the value information" and the modification is on the probability), the distributive model 1;.P*(U+A) (here the modification is on the utility), Distributive model 2; P* U+A (the value of ambiguity is not applied to each outcome separately as in distributive model but "the ambiguity of the option is used as a whole in modifying the value of the option"). Dual distributive model; P*U*A, Multiplicative model (subjects may employ composite multiplicative uncertainty (P*A) or multiplicative composite utilities (U*A) or modify the expectation globally because of the presence of ambiguity (P*U). Also in this study the subjects were asked to rank lotteries similar in the design and procedure to the above described one. All the three factors, R,C,X, were varied. The utility was set to be either $10 or -$10, P was 0.25, 0.50 or 0.75, the range was 0, 20 or 50. Curley and Yates (1989)'s results showed various interactions between the different factors which, however, seem very difficult to interpret. The only clear result was the dependence of reaction to ambiguity according to the probability levels. The switch from ambiguity aversion to ambiguity preference allowed them to reject all those tested models, which did not allow for such kind of behaviour[71]. It is not clear from the article if these results might be a consequence of the particular method used.

Tversky and Heath (1991). Tversky and Heath (1991) proposed and tested a new model of ambiguity reaction called the Competence Hypothesis. The main idea is that aversion to ambiguity observed in a chance set up (involving aleatory uncertainty) may not extend to

[71] In these models are included Fellner (1961) Smith (1969) Gardenfors ((1982), Ellsberg (1961), Toda and Shuford (1965).

judgmental problems (involving epistemic uncertainty). The main point is that the willingness to bet may not depend only on the probability and on the precision of the estimate: it may also be influenced by one's general knowledge or understanding of the relevant context.

"More specifically, we propose that - holding judged probability constant- people prefer to bet in a context where they consider themselves knowledgeable or competent than in a context were they feel ignorant or uninformed".Tversky and Heath (1991) p 7

According to Tversky and Heath (1991), people generally do better in situations which they understand because the consequences of each bet include, beside the monetary payoffs, the credit and the blame associated with the outcome. According to the two authors, these credit and blame depend on the attributions for success and failure. In particular, if the decision maker has a limited understanding of a situation, failure will be attributed to ignorance, while success will probably attributed to luck. If instead the person is an expert on the subject, success will be attributed to competence and failure to bad luck. The most important implication of the competence hypothesis is the existence of a choice-judgment discrepancy, that is to say a preference to bet on A rather than on B even though B is judged to be at least as probable as A.

Tversky and Heath (1991) tested this hypothesis under various conditions and with different incentive mechanisms by running a series of experiments. In the first experiment, subjects answered 30 questions on general knowledge (history, geography, sports etc.). In addition, they were asked to rate their confidence on a scale from 25% (pure guessing, so defined by Tversky and Heath (1991)) to 100% (certainty). After performing these two tasks, they were given the opportunity to choose between betting on their answers and betting on a lottery in which the probability of wining was equal to their stated confidence[72]. According to expected utility theory, subjects should be indifferent between betting on the stated probability or on their judgment. If people are ambiguity averse, in the sense of Ellsberg, they should prefer to bet on the stated probability rather than on a judgment probability which is more ambiguous. The contrary hypothesis called

72 They were also given choices with the complementary probabilities.

chance aversion predicts that people would prefer to bet on their judgment rather than on a matched chance lottery. The regression hypothesis predicts that people will prefer to bet on their judgments when they are knowledgeable, while preferring to bet on the chance lottery when they feel ignorant. In this latter case, Tversky and Heath (1991) expect that the percentage of choices that favor judgment bets over chance bet will increase with the level of probability (since the level of probability is equal to the stated level of confidence). The results of this first experiment confirmed the competence hypothesis. The percentage of people choosing to bet over their judgment increased with the increase in the level of probabilities. This was true for the paid as well for the unpaid group of subjects. In the second experiment, Tversky and Heath (1991) studied the same phenomenon by extending the choice problem to real world events and by eliciting an independent assessment of knowledge. Subjects were sorted out according to their area of expertise (football or politics) In the case of the football group, the subjects were asked to predict the outcome of 14 football games for 5 consecutive weeks. The subjects were moreover asked to assess their knowledge about each game on a scale from 1 to 5. Then according to their rating, the subjects were asked whether they would prefer to bet on the team they choose or a matched chance lottery. The results of the experiment showed that for low levels of knowledge as well as for high level of knowledge the percentage of choices that favour judgment over chance bets increased with the level of probability, but for high levels of knowledge this percentage was always higher than for low level of knowledge. Similar behaviour was also observed in the group of experts in politics. In a third experiment, Tversky and Heath (1991) investigated if people simply prefer to bet on judgment probabilities when the probability of winning is higher than 0.5 and on the chance lotteries otherwise. However the experimental results obtained showed that people preferred to bet on judgment probabilities regardless of the level of p when their knowledge was high. In a fourth experiment, Tversky and Heath (1991) also investigated what they call "expert" prediction. They divided the subjects into groups, according to their knowledge in football and politics and asked each subject to make 40 predictions on future events (20 football and 20 politics), then 20 triples of bets were constructed for each participant. One bet was a chance lottery, a second bet was the subject's prediction in his strong category (high knowledge) and the third bet was subject's prediction in his weak category. They then were asked to rank the bets in each of the triples. As expected by the competence hypothesis, subjects raked higher the chance lottery in their weak category and higher the judgment lottery in their strong category. In the fifth experiment, Tversky and Heath (1991) checked for possible bias in the judgment process giving to the subjects a pricing task that did not involve probability judgments. Sixty eight students were asked to state their cash equivalent for each of 12 bets. The bets were defined for high knowledge and for low knowledge topics. The results of this experiment showed that

" people were paying , in effect, a competence premium of nearly 20% in order to bet on the more familiar propositions. Father more, the average price for the (complementary) high-knowledge bets was greater than that for the low-knowledge bets in 11 out of 12 problems. In accord with our previous foundings, the chance lottery is evaluated above the low-knowledge bets but not above the high-knowledge bets. " Tversky and Heath (1991) p 21

The experiments conducted by Tversky and Heath (1991) seem to give quite strong support to the competence hypothesis. The hypothesis in fact gets support even when tested under different conditions and the subjects performed different tasks: choices, ranking and cash equivalents. What is not completely convincing to me is the difference between what the authors call the discrepancy between choice and judgment. According to this discrepancy people prefer to bet on a judgment than on an equivalent chance bet when they are in a field they know a lot about although the judgment is more vague and consequently more ambiguous by definition. This discrepancy is what makes the authors talk about competence hypothesis in contrast with an uncertainty reaction. However, it can be possible that, when asked to express a judgment, people act in a conservative way. Hence if they say 'the probability that I assign is 0.8, because I know a lot about the subject, I also know that I am being conservative (in fact that I understated my probability estimate). Instead when I do not know nothing about the topic I can think that I have not been enough conservative and that probably I have overstated my probability estimate. In this latter case I would prefer to bet on a chance lottery while in the former case I would prefer to bet on a judgment lottery'.

My knowledge will consequently change my attitude on my probability estimate being my estimate in anyway an estimate and not a precise probability.

Keppe and Weber (1991). The relation between competence or knowledge and ambiguity aversion is also tested in an unpublished experiment by Keppe and Weber (1991). The two authors did not test directly the credit and blame hypothesis, focusing instead on the direct influence of knowledge on ambiguity attitude. In their experiment participated 36 students who received a flat payment of 15 marks. They elicited certainty equivalents and decision weights for simple and event lotteries, for the occurrence of an event and its complement. Indicating with CE(A) and CE(~A) the certainty equivalent of the event A and of its complement, they tested the hypothesis that in case of ambiguity the sum of the two certainty equivalents was directly proportional to the level of knowledge. Moreover they also tested the hypothesis that for ambiguous events the probability event depends on the judged knowledge of the event where a high knowledge implies a higher probability[73]. The results of the experiment confirmed a significant relation between aversion to ambiguity and level of knowledge. The average sum of the certainty equivalents did vary in accordance with the level of knowledge and in the predicted direction. Also the sum of the decision weights was lower in the case of low knowledge events showing that the level of knowledge has a direct influence on the perceived level of ambiguity. The Keppe and Weber 1991 results, however, show more the relation between knowledge and ambiguity perception than in the validity of the Tversky and Heath (1991) theory which is linked to the idea of credit and blame. On the other hand, the connection between ambiguity and knowledge is at the very heart of the Ellsberg definition of ambiguity.

"What is at issue might be called the *ambiguity* of this information, a quality depending on the amount , type, reliability and "unanimity" of information, and giving rise to one's degree of "confidence" in an estimate of the relative likelihoods". Ellsberg (1961) p 657.

Mangelsdorff and Weber (1992). Mangelsdorff and Weber (1992) tested Choquet Expected Utility and compared different ways of

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[73] All the lotteries were designed to have for ambiguity neutral people a probability of 1/2.

eliciting non-additive probabilities. To do so they used the three colours Ellsberg example. They used lotteries with positive outcome of 50 DM or 0. In addition to the classical urn example, they also asked subjects to bet on the share price of the Japanese Dai Ichi Kangyo Bank. In this way they tested ambiguity aversion and Choquet Expected Utility also in a event environment. We will now analyse briefly their results. In the Dai Ichi Kangyo Bank example, subjects were asked to choose between four pairs of lotteries. One lottery (the ambiguous one) was defined on the share price of the Dai Ichi Kangyo Bank (the share price should be greater than a certain amount); the certain lottery was defined with an urn containing 50 black and 50 white balls. Capacities were elicited into two ways: asking people how many balls they wanted to be added in the urn to make the two lotteries indifferent or varying the winning amount. In the standard lotteries representing the three colours example, capacities were elicited changing the amount to be won. In addition to eliciting capacities, in order to test Choquet expected utilities, Mangelsdorff and Weber (1992) tested some of the predictions of the theory. In particular, they made the assumption that for Choquet Expected Utility the two ways of eliciting capacities should give the same results and that capacities are identical when derived from gains and from losses. Seventy four students participated in this experiment, they received a flat payment for their participation. The experiment was organized in two parts. In the first part capacities were elicited as described above(capacities were elicited in the domain of gain as well as in the domain of losses). For each subject also the utility function was elicited. In the second part of the experiment subjects had to evaluate lotteries and they were asked the certainty equivalent for each lottery.

The results of the experiment are reported in table II.4

Table II.4 Manelsdorff and Weber (1992) experimental results

Ellsberg lotteries

Payoffs / Subjects	Averters	Neutral	Lovers	Others
Gains	49%	35%	4.3%	11.7%
Losses	23%	40.5%	26%	10.5%

Dai Ichi Kangyo Bank share price

Payoffs / Subjects	Averters	Neutral	Lovers	Others
Gains	59%	25%	10%	6%
Losses	33%	39%	17%	11%

In addition Mangelsdorff and Weber (1992) found that all the means of the capacities were significantly lower (statistically) than the means of the probabilities in case of gains, while for lotteries involving losses, they were significantly lower only in one case. Moreover the capacities based on changing the numbers of balls were larger than the ones calculated changing the winning amount, and this difference was statistically significant. This fact, that capacities elicited for gains and for losses were found different, and the fact that the capacities elicited with the certainty equivalent method and the ones calculating using the direct elicitation plus the utility function gave different values, are all results that in conflict with the prediction of Choquet Expected Utility. However it may be worth noting that a partial explanation of this results may come from the experimental designed adopted. If for example there exist problems of scale compatibility as in Kahneman and Tversky (1990) this can have caused for example the different results given by the two methods of eliciting capacities. In varying the number of balls subjects may focus on the probability side, while in varying the amount of the winning money, subjects may focus their attention on the price. Moreover a choice task, as well as an evaluation task, as well as a ranking task, even if applied to the same problem, can give different results (See also Payne and Bettman (1992)).

Bernasconi and Loomes (1992) ran also an experiment which replicated the three-colour Ellsberg paradox and tested the Segal theory of ambiguity (for the explanation of Segal theory see chapter I). To do this they designed an experiment which dealt explicitly with multistage lotteries in which the probabilities were known. Hence the Segal "imaginary lottery" was represented by a two stage lottery with a known (fixed by the experimenter) second order distribution. The idea was that, if Segal's model was correct, the violations of the reduction of compound lottery axiom observed in the experiment would have been in the direction consistent with the behaviour predicted by Ellsberg and of the scale reported in earlier experiment. On the other hand, if ambiguity aversion cannot be explained through the violation of the reduction principle, according to Bernasconi and Loomes (1992), there would have been no behaviour consistent with the Ellsberg paradox at all. The experiment was ran in four sessions and, on the whole, 566 subjects were involved. There was no financial incentive. The participants were asked to answer to a questionnaire in which three questions were reported. The first question regarded the first choice of the three colour Ellsberg paradox in the Segal version (two stages). In the second question the subjects were asked whether they wanted to change their previous choice or to stick to it in exchange for a certain amount of money (all imaginary as well as the payoffs). This question was designed to see if the preference reported in question 1 is a strong preference. In the third question, subjects had to answer to a problem which was a modified version of the second choice of the three colour Ellsberg example. To test against colour bias and other factors the question and the order of the colours on the answer sheet was different for the various groups of subjects. The results reported in Bernasconi and Loomes (1992) showed that:

a: The subjects regarded red (not as a colour but as a choice) as different from the other colours. This was true for the subjects who indeed choose red in the first question and for the subjects who choose yellow or bleu. In the second question the only switches were from blue to yellow and vice versa.

b: This preference toward red can be interpreted as aversion to ambiguity only if the subjects in the second choice choose BY. This happened only for the 23.5 % of the people.

According to Bernasconi and Loomes, the data of the experiment showed that the percentage of violation of the reduction principle is relevant but smaller than in other experiments where a standard Ellsberg problem was proposed. The percentage of this violation can be considered an indicator of the fact that Segal's model only partially explains ambiguity aversion. Moreover, the fact that quite a big percentage of subjects first choose red (and who did not want to switch to an other colour in the second question) and than choose YR or BR in the third question showed a kind of violation of the reduction principle which cannot be consistent with Segal's explanation. The proposed explanation (by the two authors) is that for people who violated the reduction principle

"it is not sufficient to characterize their preferences in term of non expected utility functional forms. The way in which an individual mentally represents a multistage problem is also important, since the same non expected utility functions may results in different decisions depending in which representation is used" Bernasconi and Loomes (1992) p 97.

2.3 Some concluding remarks.

As we can see from the above review the empirical investigations on the Ellsberg paradox show the pervasiveness and the persistence of the phenomenon. Most of the experiments described above were replications or extensions of the Ellsberg paradox. Even if is it impossible to make comparisons between the various results, given the different conditions under which the various experiments were run, it is however possible to conclude that the phenomenon is robust. To accommodate the paradox various models of decision making under uncertainty have been developed (see chapter I). We review some of the experiments which tested some of these theories. Also the experiments however show a limited application of the other theories. Subjects seem to be conditioned by the presence of ambiguity in their decision but further research is needed to verify sources and patterns of behaviour. This is also partially the conclusion that we draw in chapter III. In this chapter, in fact, we try to test various theories of decision making under ambiguity trying to replicate some and extend some of the results of previous experiments. (in particular Curley and Yates (1989), Bernasconi and Loomes (1991) and Mangelsdorff and Weber (1992)). As we will see however the design of the experiment is quite particular since the evaluation task is done mainly with two stage lotteries.

It is our opinion that further research is needed in the direction of investigating particular sources of ambiguity and relations (ambiguity versus risk and different attitude in the domain of losses and gains for example) as well as in the direction of testing existing theories. In this respect it can be important to try to replicate the same experiments with different task and different incentive mechanism. The tasks as well as incentives might strongly influence the results of the experiments especially when as in the case of decision making under uncertainty we can test theories through consistency in behaviour (example violation of particular axioms) more than through the adoption of a particular functional form of the utility function.

CHAPTER III

EVALUATING LOTTERIES WITH UNRELIABLE PROBABILITIES: AN EXPERIMENTAL TEST OF EXPLANATIONS OF THE ELLSBERG PARADOX

III. 1 Introduction

According to the traditional analysis of decision making under uncertainty, the decision maker's preference over outcomes, which are represented by a utility function, and his or her belief relation over events, which is represented by a subjective probability function, uniquely define his or her preference over lotteries. In particular, according to the traditional theory, the probability of an event, as judged by any given individual, is a statement of that persons's degree of belief in the occurrence of that event. This implies that an individual can attach a subjective probability to any event, and that every belief about an event can be captured by the single dimension of probability.

The Ellsberg paradox challenges this approach and shows that beliefs about uncertainty cannot be reduced to a single dimension, and suggests that what people know about a state's probability *does* influence their willingness to bet on a state. The Ellsberg paradox, hence, identifies situations in which individual choices between lotteries seem not to allow us to infer a "more likely than" relation over events.

Let us consider the following example which is a replication of the Ellsberg paradox (given to the subjects in our experiment):

1 In front of you there is a bag which contains 12 balls, 6 black and 6 white. The bag is opaque, so you cannot see inside. You are asked to bet on a colour and then you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing. (Lottery E in this experiment)

2 In front of you there is a bag which contains 12 balls. Each ball is either black or white but you do not know how many there are of each. The bag is opaque, so you cannot see inside. You are asked to bet on a colour and then you will draw a ball from the

bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing. (Lottery O in this experiment)

(In the original example the subject is asked to bet simultaneously on a colour and on a bag and then a ball is drawn from the chosen bag; if the ball is of the chosen colour the subject will get $100, otherwise he or she will get $0).

Hence the subject is presented with the following four choices[74]:

B1: bet on black in the first bag;

W1: bet on white in the first bag;

B2: bet on black in the second bag;

W2: bet on white in the second bag;

According to the standard theory, the subject should show the following preference relations B1 ~ W1 ~ B2 ~ W2, or B2 ≻ B1 ~ W1 ≻ W2 or W2 ≻ W1 ~ B1 ≻ B2, depending on his or her prior beliefs about the second bag.

What Ellsberg thought, and subsequent experiments on the paradox showed, is that a substantial group of people, if asked to bet on one of the two colours and on one of the two bags, shows the following pattern of preferences: B1 ~ W1, B2 ~ W2, but B1 ≻ B2 and W1 ≻ W2. This pattern of preferences contradicts the standard axioms of the theory of choice under uncertainty, and in particular the sure thing principle and the additivity of probability (cf. Camerer and Weber (1992) and Gilboa (1987)).

Ellsberg (1961) and others defined this pattern of preferences as *ambiguity aversion* (Cf. Karni and Schmeidler (1991) amongst others). As we have seen from the analysis in chapter I, there is no generally accepted definition of ambiguity in the literature (cf. Camerer and Weber (1992)); neither there is a generally adopted operationalization of ambiguity (see chapter II). Normative and descriptive theories on the Ellsberg Paradox seem to agree that we can talk about ambiguity in case of probability uncertainty, while we can talk about risk in case of outcome uncertainty (cf. Schoemaker (1991)). However, they differ markedly in their explanations as well as in their modellings. Ambiguity aversion is in fact modelled mainly either

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74 We use black and white in this chapter because they are the two colours used in the experiment.

through a change of the probability but also through a change of the utilities (cf. Sarin 1992 amongst others).

The purpose of this chapter is that of exploring, through an experimental analysis, some of these theories with the twofold task of trying to discriminate between different theories and of trying to differentiate between different sources of ambiguity.
As will be described below, this has been done in a particular set up; all the lotteries presented to the subjects are two outcome lotteries, the outcomes in all lotteries are the same (£0 and £25), and all the lotteries (except for the two representing a version of the original Ellsberg paradox) are two stage-lotteries. The choice of a two stage set up was due to the idea of representing ambiguity mainly as a second order distribution. See on this point Yates and Zukoswky (1976) and Segal (1987).

The presentation of the experiment and the discussion of the results will be organized as follows: in the next section we will discuss the design of the experiment and we will present the theories tested in relation to the experimental design. In the third section we will present and discuss the results of the experiment in relation to the various theories. Conclusions and suggestions for further research follow.

III.2 Design and organization of the experiment

Since the original work of Ellsberg (as we have seen in chapter II) a number of experiments have been run in order to verify the robustness of the phenomenon shown by the Ellsberg paradox. These experiments have mainly been replications of the original Ellsberg-thought experiment and subsequent experiments. In addition to replicating the original Ellsberg paradox, in the two colour or in the three colour versions, these experiments investigate ambiguity aversion or partial ambiguity aversion mainly through chance devices. As in some of the experiments reviewed in chapter II, we replicate the original Ellsberg paradox in the two colour form, and we investigate ambiguity aversion through a chance device. In addition, we perform a direct test of particular theories (some of these experiments also do; see Curley and Yates (1989), Schoemaker (1991), Bernasconi and Loomes

(1992) for example). As will be explained in more detail, some of the theories we tested are well-defined ones, while others are more vague or intuitive explanations. What these theories have in common, from the point of view of the design of the experiment, is the fact that they can be represented with a two-stage lottery (except for the original versions of the Ellsberg paradox). Nineteen out of twenty one lotteries given to the subjects to evaluate in this experiment are two-stage lotteries; they all involved two outcomes; a positive outcome of £25 and one of £0. The two-stage set up was made explicit in the following way. Each lottery was constructed in such a way that the subject was presented with some bags from which he or she had to draw a ball. The subject was asked first to bet on a colour, then to choose a bag, and then to draw a ball. The presence of ambiguity was operationalized in different ways according to the various theories; ambiguity was generally operationalized either as a set of probability distributions, or as a second order probability distribution, or through changes of the mechanism by which the bag was chosen before the draw (for example a random device, the experimenter or the subject), according to the models of the various theories[75]. There were also two lotteries in which different levels of ambiguity were introduced through the change of the range of the unknown probability (as in Curley and Yates (1989)). In practice, the subjects were asked to choose a bag and draw balls from it; what was different was either the proportion of white and black balls contained in each bag, or the fact that that proportion was unknown, or the mechanism in order to choose the bag from which to draw.

III 2.1. *The experimental design.*

As has been said above, the theories tested in this experiment have in common, from the point of view of the design of the experiment, the fact that they can be represented with a two-stage lottery. Hence all the lotteries but two presented to the subjects are two stage

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75 As we will see in more detail each model is linked to a particular way of representing ambiguity. In fact, each model describes how people would imagine the ambiguous bag and consequently how they would evaluate it.

lotteries. (For a brief description of the lotteries and the theories see Table III.1) The other characteristic of the lotteries is that all the lotteries involve two outcomes; a positive outcome of £25 and one of £0. It is our opinion that the fact of using the two-colour version of the Ellsberg paradox and lotteries with two outcomes can have an influence in the perception of ambiguity. What we would argue, and this can be partially seen later on, in the discussion of the results, is that the perception of the symmetry of the two events, black and white, is emphasized by the fact that the events are just two. What can be the consequence of this factor in the perception of ambiguity can be a topic of further research; in this experiment, as will be discussed in section III.5, it seems to have reduced the perception[76] of

[76]Using a two outcome set up might represent a constraint in the choice of what theory to test; not all the theories involve two or three outcomes. A natural extension of this experiment would be to use on the one hand a three-colour version of the Ellsberg paradox and to introduce, on the other hand, lotteries with three outcomes. These modifications could help to solve some problems of interpretation due to symmetry and introduce a test on other theories.

Table III.1

Summary of the lotteries given in the experiment

Lottery	Explanation	Theory
	One stage	
E ‡	Certainlottery	Ell. original
O █	Unknown lottery	Ell. original
	Two stage two bags of the *certain* lottery	
	The bag is chosen by	
U "	the subject	Control
B @	a random device	Raiffa
V %	the experimenter	Kadane
	Two stage two bags of the *unknown* lottery	
	The bag is chosen by	
P ▌	the subject	Control
S ←	a random device	Raiffa
Q ┣	the experimenter	Kadane
	Two stage two bags **with different probability** in the second branch	
D -	1/2, 1/2	Maximin
U "	1, 0	
	Information lottery two stage two bags First bag 6 black, 6 white Second bag unknown proportion	
T ┗	1 out of 12	Information
N ├	3 out of 12	
M »	unknown probability in the second branch	
Z >	First bag unknown probability left and right bag 1/2	Schmeidler

Table III.1 cont i nue

Lottery	Explanation	Theory
	Two stage of the certain lottery with more then two bags	
F §	3 bags	Segal and Complexity
A *	5 bags	
G ¢	7 bags	
H ▶	11 bags	
C =	13 bags	
	Two stage of the certain lottery 13 bags	
	The bag is chosen by	
C =	the subject	Control
I ■	random device	Raiffa
	Two stage 3 bags with different probability in the second branch	
F §	1 , 1/2 , 0	Maximin
L !	4/12, 1/2, 8/12	
R <	1/2, 1/2, 1/2	

ambiguity.

III. 2.2 The organization of the experiment.

The experiment was held at the University of York in March and April 93. One group of 21 students were involved in it. There was no selection of subjects by their study topic[77]. As we can see the number of subjects who participated in this experiment was small. This was mainly due to two factors: financial problems (we had to make a trade off between the number of students and the money to stake on each lottery); objective of the experiment (we wanted to know the kind of reasoning adopted by the subjects, hence we choose to interview each subject).

The subjects were asked, first, to examine and evaluate 21 lotteries, and, second, they were asked to come for an interview in which they were asked to describe and explain their evaluations.

At registration, the subjects were given the lotteries to be evaluated at home, the instruction sheet and the answer sheet.

The invitation of the experiment was the same as the instruction sheet and included the following[78]:

You will each be asked to examine and evaluate 21 lotteries. Each lottery will give you a chance to win £25. You are asked to give a price to each lottery. This price can be seen as a minimum selling price for the lottery: that is, the lowest sum of money that you would be prepared to accept in exchange for the lottery. The lotteries will be given to you when you register for the experiment.

The lotteries were given to each subject in a random order which was different for each subject. In addition, to avoid any implicit order, the lotteries were named neither with a number nor with a letter but with a symbol (see Table III.1)[79].

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77 However, after the experiment, we asked just for curiosity, if the subject had attended one or more courses in statistics and maths. 8 subjects out of 11 non EP attended one or two years courses in statistics, while between the EP subjects just two subjects had attended courses in statistics.

78 The complete instruction plus the text of the 21 lotteries is included in appendix B.

79 A letter was then used in the interviews to make communication and

The lotteries were described to the subjects in words; the selected words were the same and followed the same scheme and order, in order to avoid or limit the influence of the description in the evaluation of the lotteries. As example lotteries U and P are reported below.

Lottery "

In front of you there are two bags. Each of the two bags contains 12 balls. One bag contains 12 black balls, while the other contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and, then, first, you will choose a bag and, second, you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery [redacted]

In front of you there are two bags. Each of the two bags contains 12 balls. Each ball is either black or white but you do not know how many there are of each. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Each subject was also given an answer sheet in which he or she was asked to write his or her evaluation of the lotteries and a few notes of explanation. The answer sheet contained a list of symbols each corresponding to each lottery, and the order of the symbols was the same for each subject, but it was different from the order of the lotteries in the envelope but, still random.

As we have already said, the subjects were asked in a interview some questions about their evaluations. The interview was tape recorded; it lasted around 20 minutes and followed a fixed scheme. They were always asked the same questions. First they were asked for a general explanation of the kind of reasoning behind the evaluations. Second we asked " Why did you evaluate this lottery this and that

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the recognition of the lottery easier when listening to the tape.

lottery that" for each group of lotteries.[80] If the lotteries were evaluated the same they were asked "Do you really evaluate these lotteries the same, or at the margin do you find any difference?". Before starting the interviews, the subjects were told to give in their answer sheets with their answers. They were not allowed to change their answers during the interviews even if in some cases during the interviews they realized that they wanted to change some of their evaluations (see interviews in the appendix B).

III.2.3 *The incentive mechanism.*

At the end of the interviews the subjects were paid according to the following incentive mechanism, as explained in the invitation and instruction sheet:

At the end of the interviews the following procedure will be carried out. One of the 21 lotteries will be picked at random: we will look at your evaluation of the randomly picked lottery. Then a number between 0 and 25 will be picked at random; if the number is greater than your price for the lottery you will get that numbers of pounds: otherwise the lottery will be played out and you will be rewarded with either £0 or £25 depending on the outcome of the lottery.

At the moment of the registration the same procedure was explained in other words: moreover at the end of the instruction there was written (this is the Becker, DeGroot and Marschack device, for a discussion of its properties see below):

It is important to know that it is worth your evaluating the various lotteries accurately: if you give to a lottery a price which is less than the value that you place on that lottery you may end up with an amount of money when you would prefer to play out the lottery: while if you give to a lottery a price which is more than the value that you place on that lottery you may end up playing out that lottery when you would prefer to receive that amount of money.

No subjects seem to have had any problem in understanding the mechanism. Moreover some of them came to the interview with a calculation that showed that it was optimal for them to reveal their

80 The groups reflect the grouping in Table III.1

true evaluations (for example subject 21).

In addition to whatever they gained from the procedure described above they also received a £5 participation fee (the subjects were aware of the participation fee since it was written in the instruction).

Some comments are needed in order to justify the choice of the selected incentive mechanism. Usually experimentalists face the problem of choosing a system of incentives which can induce the subjects to take the gamble seriously; that means, on the one hand, to limit the number of random choices or " mistakes", and on the other hand, to make the subject to reveal their true preferences. The standard options present to the experimentalists who deal with experiments involving individual choice with fairly abstract lotteries are

a) make all choices hypothetical

b) use real gambles with small prizes

c) use gambles with large prizes but play just a randomly selected group for real money.

d) Use gamble with large prizes but play out for real the gambles of a randomly selected subset of subjects.

Each of these approaches has its drawbacks. In the case of the first approach, making all choices hypothetical, the usual criticism is that it lacks realism and is weak in terms of incentive[81]. The second approach suffers from the same criticism, though perhaps to a slightly lesser extent.

On the other hand the third approach, which is the one used in this experiment, may introduce some distortions. It may introduce a common ratio effect in that a fractional probability of playing is applied to all the gambles (Schoemaker 1991).

Moreover, the standard method of eliciting certainty equivalents (the Becker, DeGroot and Marschack device) may not be incentive compatible if subjects violate the independence axiom, (Holt (1986) and Karni and Safra (1987)). Moreover, according to Holt there may be particular bias affecting random lottery experiments, and, in particular, if subjects treat a random lottery experiment as a single choice problem: if the compound lotteries are reduced to simple ones by the calculus of

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[81] Thinking costs in terms of effort and time. If the subject is not going to earn anything he or she will tend to minimize time and effort or have some alternative motives for answering.

probabilities, and if the independence axiom of expected utility theory does not hold, then the B-D-M device fails in eliciting the true preferences. An experimental investigation of this hypothesis has been held by Starmer and Sugden (1991), and the two authors reached the following conclusion:

"Holt has shown that random lottery experiments can fail to elicit true preferences if the reduction principle holds and if the independence axiom is violated. In showing that the reduction principle does *not* hold, our results suggest that experimental researchers need not to be too concerned about this particular problem. Of course, this does not eliminate the possibility, mentioned by Holt, that the random-lottery design might be subject to some other source of bias. All we can say is that for choice problems used in our experiment, subjects' responses did not differ much between the random lottery and the real-choice designs. If there are any 'contaminations' effects at work in the experiment, they seems to be fairly weak".p 978.

The use of a incentive compatible system for these kinds of experiment is still a problem that has to be solved. If the incentive mechanism is meant to be used to make the subjects take the experiment seriously, that is to say minimize the possibility of error and increasing the effort of the subjects, then we share Schoemaker's opinion that

"subjects' cognitive strategies are not greatly affected by incentives, when using tasks of medium complexity and stakes". (Schoemaker (1991) p 298).

Experimental psychologists generally support these views.

In this case the aim of the incentive is just the one of giving a monetary reward for the attention and the effort put in the experiment by the subjects. If this is the case, it is our opinion that different incentive mechanism like a flat payment or play out a lottery randomly chosen may give the same results. Subjects can behave "seriously" in this respect even if the choices are hypothetical if they are motivated with non monetary rewards. (cf. Camerer 1989), Kahneman and Tversky (1991); see also Smith and Walker (1993) with a first price auction as incentive mechanism.

We would suggest that the problem of decreasing the probability of errors in the choices of the subjects or the one of increasing the "care" they use in performing the requested task should be considered separately from the one of verifying if the random-lottery system elicit true preferences. If we define the true preferences as Starmer and Sugden (1991)

" We shall say that an individual has a true preference for x over y if he would choose x in an experiment in which he had to choose between x and y, in which this problem was for real, and in which no other problem was for real" p 971.

then the problem is if the random-lottery system elicit true preferences that is to say if it *does not distort* them. And to this problem Starmer and Sugden's conclusions apply.

As far as this experiment was concerned, it was our original opinion that giving high prizes could have an influence on the choices of the subjects. The original idea was that a high reward would give an higher incentive to them. To economize on the cost of recruiting subjects there were two the feasible alternatives:

a) choose to give a very high prize in each lottery and to make just few subjects (picked at random) play one of the lotteries.

or b) I could choose to give a smaller prize but still high and make all the subjects to play a randomly selected lottery.

If alternative a) was adopted then the prize could have been higher, but of course the probability to each subject to be chosen would have been very small, decreasing the potential power of the incentive[82].

In this experiment we adopted alternative b): each subject had to play for real a randomly selected lottery. The interviews were introduced mainly for gaining some insight into the reason behind the evaluations of the subjects. Even if there was no sign that the incentive mechanism was misinterpreted, we would argue that a side effect of the interviews was that of providing the subjects with a strong incentive. It seems to us that to sustain an interview and explain their choices made the subjects more careful about their evaluation[83]. We do not argue that the chosen mechanism was irrelevant but we have the suspicion (given to me by the interviews) that a flat payment could have produced the same results. On the other hand, for the structure of the present experiment, it was more important to know the rationale behind the evaluations and the relative evaluations of

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[82] This was the incentive mechanism used by Schoemaker (1991) in the experiment in which choices were played out for real.

[83] It is also possible to argue that an interview can force a subject to find a "rationale" to his or her choices with the intent, for example, of showing his or her intelligence. We think that this risk is reduce if the interview, as in our case, is just in front of the experimenter. Cf Curley, Yates and Abrams (1986)

the lotteries than their exact absolute value.

These considerations are not intended to dismiss the problem of an incentive compatible system for this kind of experiment; however to verify the consequences of the various incentive mechanisms on the subjects's evaluations or choices, it can be interesting to run the same experiment with different incentive mechanisms. The few existing works that compare choice for real and hypothetical ones seems to suggest a minor influence[84]. But this deserves further investigation.

III. 3 **Results and Discussion.**

III.3.1 *Organization of the discussion*

We will describe the results in the following way: we will divide the subjects into two groups: one group is represented by subjects who did not commit the Ellsberg paradox; the second group is composed by all the subjects who did commit the Ellsberg paradox (we will always define as Ellsberg paradox people (EP, hereafter) the subjects who did commit the EP and non-EP people the subjects who did not commit the EP. This latter group can include Expected Utility people or also people who may be not be Expected Utility but who did non commit the EP).

As far as the first group is concerned the analysis will focus first on the evaluation of the lotteries and, second, on the explicit rationales given by the subjects to their evaluations. As far as the second group is concerned, the analysis will follow the same scheme, but the explanations of the subjects of their evaluations have a particular importance in so far as understanding the consistency or inconsistency of their behaviour with the various theories[85] which try

84 See Schoemaker (1991) and Camerer (1989), Weber, Loomes, Keppe and Meyer-Delius,(1994)

85 We will use the word theory in a very broad sense. According to Oxford dictionary a theory can be " a set of reasoned ideas intended to explain facts or events " but also "opinions or suppositions". Actually some of these theories can be defined as a set of reasoned ideas while others can more easily be included in the category of suppositions. Moreover some of the theories are expressed in models while others can

to explain the Ellsberg paradox. In particular, the behaviour of each subject will be analyzed in the light of each of the theories examined; for each theory we will describe the expected evaluation for each lottery (belonging to the subset of lotteries used to test that particular theory) and consequently the order in which the lotteries should have been evaluated if the explanation of the EP was of the type stated by that particular theory. We will, at the end, analyze for each subject if his or her choices can be considered consistent or inconsistent with one or more of the theories. It is important to note that the evaluations of all the lotteries are not expected to be different for all the theories. In fact some of the lotteries were built just to verify some explanations and can be completely irrelevant for other explanations. As a consequence of this design it is possible to find that the evaluations of a particular subject are consistent (or at least not inconsistent) with more than one theory. To understand better the reasons underlying the Ellsberg kind of behaviour of each subject, we will make use of the explanation or reasoning used by the subjects in the interviews. Indeed we would argue that it is difficult to interpret correctly the evaluations of the subjects without this tool. The analysis of the interviews, moreover, can give some insight into possible explanations of the behaviour of the subjects other than those explicitly considered during the experimental design.

III.3.2 *Results*

Of the 21 subjects who took part in the experiment 10 out of 21 committed the Ellsberg paradox: they evaluated the unknown lotteries differently from the certain ones[86]. Moreover for 9 out of these 10

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be just defined as hypotheses. Where a hypothesis is " idea or suggestion that is based on known fact and it is used as a basis for reasoning or further investigation"; while a model is " a simplified description of a system used in explanation".

86 We will always use the term "the unknown lotteries" to refer to those lotteries in which the proportion of white or black balls in the bag is unknown; those in which the proportion of the two colours is clearly stated will be referred to as " the certain lotteries". We would classified as EP subjects all the subjects who evaluated the certain lotteries equally to the uncertain one, whatever representation of uncertainty was adopted (E evaluated the same as D, B, F, G, H, C, I, R, L, P, S, O, Z, M).(the evaluations of T, N , Q and V may differ

subjects the premium they were willing to pay in order to avoid the unknown lottery was more then 20% of the price given to the certain lottery. Of these 10, 5 were willing to pay more than 50%. Of the 10 subjects who committed EP, 9 were ambiguity averse and 1 ambiguity prone. (See Table III.2 and III.3).

III. 3.3 *Subjects who did not commit the Ellsberg paradox*[87].

III.3.3.1 General results.

Expected Utility Theory. Nineteen[88] lotteries (depending on the personal beliefs of the subjects) out of 21 were designed in such a way that they should be evaluated the same by an Expected Utility subject. All (19) these lotteries in fact collapse to the single lottery (1/2, B; 1/2,W). Consequently the evaluation of these lotteries should depend only on the personal different degrees of risk aversion or preference of the subjects. (The expected value of these lotteries was £12.5).

Lotteries N and T. Two lotteries, N and T were different in this respect:

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across EP subjects. See relevant section).We classified as non EP subjects the subjects who evaluated E differently from O. At this point is also important to notice that in our experiment we do not consider any theory of error (which is the theory that fits better considering the possibility of an error in the evaluations). However we check for the presence of mistakes because we used more than one lottery to test each theory.

[87] Within these subjects, there are some who did not commit the Ellsberg paradox but who cannot be defined for sure as Expected Utility subjects. An EU subject is a subject who behaves as if she or he evaluated all the lotteries according to EU; however some of the subjects evaluated lottery T and lottery N the same as the others when they "should", according to EU, have evaluated these lotteries more.

[88] T and N should be evaluated more than the others. See the evaluation of lotteries V and Q in section III.4.3

Table III.2

Evaluations in pounds of the lotteries by non EP subjects.

	Subjects									
	2	4	6	9	10	12	13	15	17	18	21
Lottery											
E	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
D	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
U	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
B	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
V	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	0
F	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
A	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
G	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
H	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
C	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
I	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
N	14	10	12.5	14	13	15.9	19	14.01	12.5	12.5	14
T	13	11	12.5	13	13	13.9	15	14.01	12.5	12.5	13
F	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
R	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
L	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
P	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
S	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
Q	12	10	12.5	12	13	12.9	0	10.01	12.5	12.5	0
O	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
Z	12	10	12.5	12	13	12.9	12	12.01	12.5	12.5	12
M	15	11	12.5	15	13	12.9	12	12.01	12.5	12.5	12

Table III.3

Evaluations in pounds of the lotteries by EP subjects.

Lottery	Subjects 1	3	5	7	8	11	14	16	19	20
E	13	12	10	12	12	12	13	15	15	13
D	12	12	8	12	12	12	13	16	12	13
U	10	12	8	12	12	12	13	13	13	5
B	10	12	7	12	12	12	13	18	20	5
V	10	12	10	12	12	12	13	17	18	5
F	9	12	8	12	12	11	13	16	15	6
A	10	12	7	12	12	10	13	14	10	7
G	10	12	7	12	12	9	13	16	20	8
H	12	12	8	12	12	8	13	18	10	9
C	12	12	8	12	12	7	13	20	14	2
I	12	10	7	5	12	8	10	10	15	2
N	10	10	8	4	6	10	15	17	15	3
T	6	10	7	4	6	9	7	9	14	3
F	9	12	8	12	12	11	13	16	15	6
R	13	12	10	12	12	12	13	19	10	13
L	12	12	8	12	12	9	13	16	12	10
P	8	10	7	4	6	9	5	10	15	2
S	10	10	8	4	6	9	5	11	12	2
Q	5	12	8	3	6	9	5	5	10	2
O	6	10	7	4	6	9	5	8	20	2
Z	13	12	8	3	12	10	15	12	15	1
M	8	10	7	12	12	10	13	13	17	1

in these two lotteries[89] the conditional probability of drawing a white or a black ball was different from 1/2 since the subject knew the colour of some of the balls put in the bag from which he or she was asked to draw (3 in N and 1 in T). As we have already explained the purpose of using these lotteries was twofold. On one hand, it was a check for the Expected utility subjects; we expected them in fact to evaluate these two lotteries more then the others; specifically lottery N should be evaluated more than T and T more then the rest. On the other hand, in the case of the "Ellsberg subjects" these lotteries were an attempt to measure in some way the influence of adding some information in the case were you do not know the probabilities of the two outcomes of the lotteries; N and T are in fact the only two lotteries in which the level of ambiguity (specifying ambiguity as the range in the probability interval) of the unknown bag varies[90]: we know at least the colour of one out of 12 or three out of 12 balls.

Results. If we look at table II.2, we can see that of non-EP subjects 7 out of 11 evaluated the lotteries N and T more then the others, while 4 gave to the 21 lotteries all the same value. During the interview, however, two of the these four subjects said that the probabilities of getting the desired colour in lotteries N and T was

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[89] In N subject were asked first to draw 3 balls with replacement from a bag containing 6 black and 6 white, and, then, to put them in a bag containing 9 balls which can be either black or white. The final draw is from this latter bag. In T the procedure is exactly the same but in the second bag, which contains 11 balls you have just to put 1 ball.
Take first lottery T : now in case of the first draw we expect an EU subject to belief that the probability of drawing a black or a white is 1/2. In the second stage if we have put a black ball then I know that the other balls in the bag cannot be all whites but 1 black and 11 white or 2 blacks and 10 white etc. so the probability of drawing a black is 1/12+2/12 etc and consequently the expected winning of the lotteries is 13.5.
In case of lottery N the reasoning is the same; however the combinations of balls can be the following ones BBB, WWW, BWW, WBW, WWB, WBB, BWB, BBW. The EV of the lottery when we know that there are 2 balls of the same colour is 14.5 while the EV of the lottery with three balls of the same colour is 15.6 so the EV of N can be calculated as 2/8.15.6+ 6/8.14.5≅ 14.8

[90] As has been already said in section 1 in lotteries T and N the level of ambiguity was operationalized through a change in the range of the unknown probability following Curley and Yates (1987). Moreover since in T the range is bigger then in N, we consider lottery N less ambiguous than lottery T according to the above stated criterion.

higher than in the other lotteries but not sufficiently higher to make them evaluate the lotteries more[91].

Lotteries V and Q. There are two other lotteries whose interpretation in terms of Expected utility theory is not so straightforward: lotteries V and Q. Lotteries V and Q were designed to represent a two stage version of the original Ellsberg paradox. However the choice of the bag is made not by the subject but by the experimenter after the subject had already declared the colour on which he or she will bet. The original idea was that of verifying the theory that states that people avoid betting on the unknown urn just because they distrust the experimenter, and, consequently, this distrust diminishes the utility of the lottery in which the percentage of white and black balls is not known. Making the choice of the bags depend explicitly on the experimenter was a way of focusing directly on this explanation of the Ellsberg paradox. In terms of an expected utility subject we can expect that, if the focus is on the way in which the bag is chosen, then these two lotteries may be evaluated zero, since the experimenter can determine the final outcome of the lottery choosing the bag in which there are none of the balls of the chosen colour[92]. However, the subject can also interpret the two lotteries on the assumption that, since the bags are opaque, neither the subject nor the experimenter will know the contents of the bags.

The evaluation of the two lotteries by the non-EP subjects (the same hold also for EP subjects) will consequently depend on what they assume. However since the distrust in the experimenter has been introduced as an explanation of the aversion to ambiguity, what non-EP people do in this respect is not relevant to test this explanation.

Results. Table III.2 shows that just two subjects evaluated V and

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91 Subject 10 for example said in the interview "These are the only one I thought you have just a slightly more chance really, because you could choose; you knew three of the balls of the bag you draw from: So, if you draw out three black balls or white you get an advantage betting on those but, at the same time, you knew any of the balls in the second bag. So I decided that it was not really worth evaluating them more.. but, I think, you can evaluate them a little more.

92 However to evaluate this lottery 0 it must be assumed that the subject assigns a p=1 to the possibility of being cheated. If the subject assigns a positive probability, but less then 1, then the evaluation of the lottery can be more than 0.

Q less than the other lotteries. In particular subject 21 evealuated both lotteries 0, while subject 13 evaluated just Q 0.
In looking at the evaluations of the non-EP subjects in table II.2 it must be taken into account that lottery M has been misinterpreted by some subjects of the first group[93]. Interpreting the lottery as lotteries N and T the subjects evaluated that lottery more than the others; on the other hand, the subjects who interpreted the lottery correctly gave to it the same value as to all the others[94].

III.3.3.2 The interviews.

The questions asked followed the same scheme. However, while for the EP subjects the entire set of questions was asked, in the case of the non-EP subjects the questions focused especially on the differences between the two lotteries which represented the original Ellsberg paradox and the two groups of the two stage version, that to

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[93] As has already been said the experiment was organized in two different weeks with two different groups of students. After interviewing the first group of students I realized that lottery M has been misinterpreted by some of the subjects. To avoid any further misunderstanding I changed the lottery in the following way:
First version: In front of you there is a bag which contains 12 balls, 6 are black and 6 are white. Then there is a second bag which is empty. From the first bag a ball is to be draw by somebody who is neither you or the experimenter. If **the drawn ball is black** he or she will put a black ball into the second bag. Then after replacing the drawn ball in the first bag a new ball is drawn and a ball of the same colour is put in the second bag.
Second version: ... **If the drawn ball is black he or she will put a black ball into the second bag; if the drawn ball is white she or he will put a white ball into the second bag**. Then, after replacing the drawn ball in the first bag, a new ball is drawn. If the drawn ball is black he or she will put a black ball into the second bag; if the drawn ball is white she or she will put a white ball into the second bag.
In the first case since in the lottery was mentioned only the case of a black ball some of the subjects thought that in the case that a white ball was drawn first no ball was put in the second bag. In this case the probability of drawing a ball of the two colours is different since in the second bag you can just have a black and a white , two blacks or a black or a white. With the correct interpretation you may have in the second bag 2 black 2 whites or a black and a white and consequently the probability of drawing a ball of each colour is 1/2.

[94] When we use the word "correctly" we mean that the subjects did not misinterpret the lottery we do not mean that they behaved according to some theory.

say lotteries E and O, B, U and V and S, P and Q.

Lotteries E and O provide a version of the original Ellsberg lotteries; lottery E is the certain one, while lottery O is the unknown one. Lotteries B, U and V are two-stage versions of lottery E. The subjects are asked to bet on a colour, to choose a bag and then to draw a ball from the chosen bag. The two bags contain 12 balls each; in one bag there are 12 black balls and in the other bag there are 12 white balls. The design of the three lotteries is the same; they differ from each other by one element: how the bag was chosen: by the subject, by the experimenter or by a random device.

The purpose was to discover which kind of reasoning the subjects adopted. In focusing on lotteries E and O most of the subjects recognized the fact that, while in E the probability of drawing black ball or a white ball was known and equal to 1/2, in O these probabilities were not known. However the reasoning by which they justified their evaluations were different. Four subjects declared that since the probability could vary from 0 to 1 they used the average value, another four explicitly used the Principle of Insufficient Reason ("since I do not know I assume that it is half and half"). Three people motivated their choice in a different way. Their reasoning can be summarized as follows: There is a bag and I do not know the proportion of black and white so I do not know the probabilities. However this is not so important, in fact there are two kind of probabilities; the probabilities when you put your hand in the bag and the probability of me saying black or white. Since there are just two colours my probability of saying black or white is still 1/2 and it does not matter which is the real probability present in the bag[95].

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[95] In an experiment reported in Raiffa (1961) used a very similar argument was used to convince the subjects who committed the Ellsberg paradox that they behaviour was non rational. "But then someone - all too often that someone is I - comes up with the following argument: Suppose you withdraw a ball from the urn with unknown composition but do not look at its colour. Now toss a fair (unbiased) coin and call 'red' if heads, 'black' if tails. The 'objective probability of getting a match is now 0.5 and therefore it is just as desirable to participate in the second game as in the first. I have found out that after the student convinces himself it does not matter whether the ball is drawn first or whether the coin is tossed first, that he is most willing to increase his price for the second game up to the price he was willing to pay in the first game."Raiffa (1961) p 693.

In fact the reasoning of some subjects in the experiment was quite

Their evaluation is consequently due to the symmetry of the problem[96]. Symmetry can depend on the presence of either two colours or two outcomes, both equally unknown. This kind of distinction has been present also between subjects who did commit the Ellsberg paradox.

III.3.4 *Subjects who committed the Ellsberg paradox.*

General discussion. In this experiment 10 subjects committed the Ellsberg paradox. Of these 9 were ambiguity averse and 1 ambiguity prone. The evaluation given by ambiguity averse subjects to the ambiguous lottery was less than 80 % of the evaluation given to the certain lottery (O versus E); and for 1/2 of the subjects it was less than 50% of the evaluation given to the certain lottery; it is hence possible to conclude that 1/2 of the Ellsberg subjects showed a strong ambiguity aversion. It may be worth noting that the same subjects did not show a particularly strong risk aversion in evaluating the certain lottery. In fact the expected value of all the lotteries was 12.5 and for most of the subjects the evaluations given to the certain lottery was between 13 and 10. (See table 3). It is also important to note that 10 out of 21 (47.6%) of the subjects participating in this experiment committed the Ellsberg paradox which is a higher percentage than observed in experiments of a similar kind[97].

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similar; they divide the problem into two stages: first the drawing of a ball (black or white) and then the calling of a colour (black or white). Whatever was the result of the draw they still had to call a colour and in this case the probability was anyway one half. Reasoning in this way some of the non-EP people evaluated the two lotteries E and O equally. However, it is worth noting that the same kind of reasoning, plus the fact that people were not temporally indifferent about calling the colour or drawing the ball, motivated a different evaluation of E and O. For example, subject 1 said that he evaluated E more because "once you have put the hand into the bag there is still a fifty-fifty chance of getting the colour that you want".

[96] Raiffa's idea and the reasoning of these subjects seem to suggest that the results could be different if the colour to be called was defined ex ante (the subject is told to bet on white for example). Let us suppose that the subjects were offered two lotteries exactly equal to the certain and the unknown one but in which instead of you are asked to bet on a colour' you put 'you are asked to bet on white', we suspect that even assuming that no subject has any aversion or preference to a colour, the percentage of EP people would increase.

[97] The fact that it is not possible to find a correlation between

III.3.4.1 Segal's explanation of the Ellsberg paradox.

The model. As we have seen in chapter I, Segal (1987) suggested that the Ellsberg paradox depends on how people perceive the unknown urn. In particular, Segal suggests that the ambiguous lottery is perceived as a two stage imaginary lottery in which the first imaginary stage is over the possible values of the probability in the second stage.

In this experiment, the event to be considered is drawing a white ball or a black one, so the first stage is over the possible values of the probability of drawing a white ball or a black ball.

In the second stage the decision maker participates in the lottery in which the probability of black (as well as of white) has been determined by the resolution of uncertainty in the first stage.

Segal assumes that, while evaluating this imaginary two stage lottery, people do not reduce the compound lottery to the equivalent single lottery.

Instead they calculate the certainty equivalents at the second stage. These certainty equivalents become the outcomes of the first stage. At each stage the lotteries are evaluated according to anticipated expected utility.

Then, the explanation of the EP suggested by Segal is composed of three different elements:

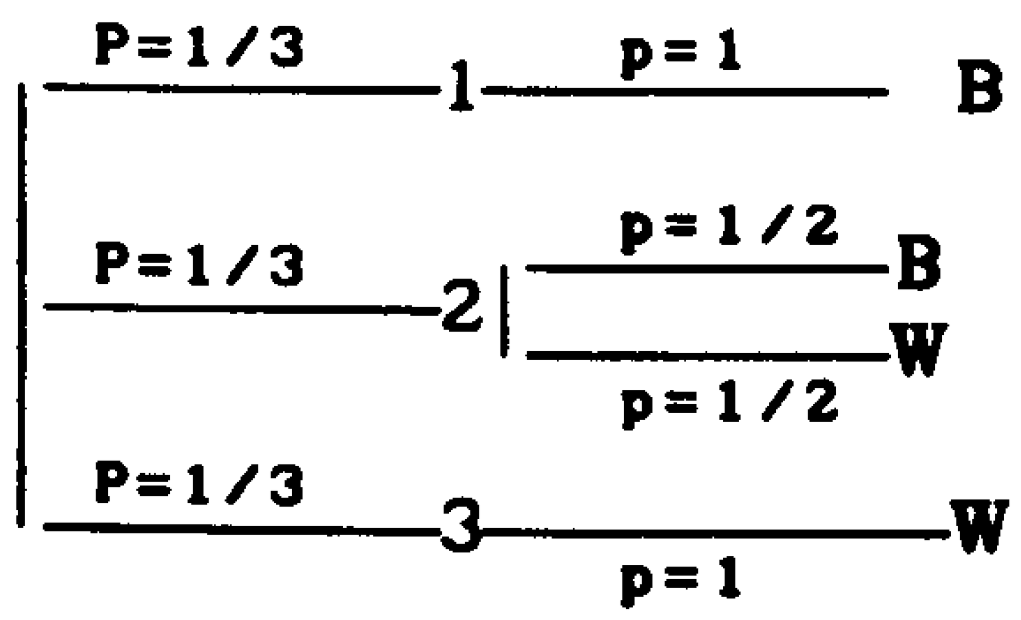
a - The way in which people perceive the unknown lottery (as a two stage lottery).

b - The fact that the decision maker does not apply the reduction of compound lottery axiom.

c - The use of anticipated utility theory to evaluate the lotteries.

Let us now consider the following imaginary two stage lottery and let us assume that we are betting on black. Hence, if black comes out we will get £25, otherwise we will get £0 .

Table III.4   Segal's model



CE(1)= f(1).u(B) = u(25)

CE(2)= u(W) + [u(B)-u(W)].f(1/2)
 = u(0) + [u(25)-u(0)].f(1/2)

CE(3)= f(1). u(W)= u(0)

CE(1), CE(2), CE(3) are the certainty equivalents evaluated applying the anticipated utility functional form (see relevant section in chapter I), where CE(3)<CE(2)<CE(1). Following Segal (1987), we now evaluate the imaginary lottery according to anticipated expected utility. This lottery is now reduced to a single stage lottery in which the outcomes at the first stage are given by the certainty equivalents at the second stage. Applying anticipated utility theory, we have:

CE(3) + [CE(2)-CE(3)]. f(2/3) + [CE(1)-CE(2)]. f(1/3)=

= U(0).f(1) + [u(25). f(1/2)].f(2/3) + [(u(25)-u(25).f(1/2))]
.f(1/3)=

= u(25) [f(1/2). f(2/3) +(1- f(1/2)).f(1/3)]

The experimental design. This experiment partially follows the design of the experiment of Bernasconi and Loomes (1992)[98], that is to say the two-stage representation of the unknown urn has been made explicit as in the lottery of Table III.4 (this is in fact lottery F of our experiment).

There are two groups of lotteries which are relevant to the analysis of Segal's model:

Lotteries U and D; they are two different two-stage representations of lottery E. In each of the two lotteries the subject is asked to bet on a colour and to choose a bag from which to draw a ball. However, in lottery U one bag contains 12 white balls and the other 12 black balls; while in lottery D both bags contain 6 black and 6 white. If

[98] In Bernasconi and Loomes (1992) we have a two-stage representation of the three-colour problem.

evaluated with the anticipated utility functional, the two lotteries should be evaluated the same, since they have the same certainty equivalent. Their evaluation moreover (the evaluation of U and D) should be[99] equal to lottery E. The two-stage set up should not change the evaluation in any case. Consequently we expect the following pattern of preference from a subject using an anticipated utility functional E~U~D.

In the second group there are the lotteries F, A, G, H, C. In lottery F there are three bags , one contains 12 black balls, another contains 12 white balls and the third contains 6 white and 6 black balls. Lotteries A,G,H,C follow the same scheme (See also Table III.I.). In lottery A the number of bags is 5; in lottery G is 7; in lottery H is 11 and in lottery C is 13. As far as the proportion of the two colours in each bags is concerned, in lottery A the proportion of each colour varies over 0, to 3,6,9,12; that to say the probability of having a white ball or a black ball in one bag can be 0 or 3/12 or 1/2 or 9/12 or 1. In lottery G, the proportion each colour in each bag varies over 0, 2,3,5,6,7,9,10,12; in lottery H, it varies over 0, 2,3,4,5,6,7,8,9,11 and in lottery C, it varies over 0, 1,2,3,4,5,6,7,8,9,10,11,12.

F is a two-stage version of lottery E in which the subject has first to choose one bag out of three and then to draw a ball from the chosen bag. If we compare lottery F with E, U and D, a subject following Segal should evaluate them according to the following pattern of preference E~D~U $\succ$ F.

As has been said in the previous paragraph, Segal's explanation of the Ellsberg paradox is that if people perceive the uncertain lottery as a two-stage lottery and do not apply the reduction principle in evaluating that lottery then the one stage and the two-stage lottery should have different evaluations. Hence E and F should be evaluated differently and in particular E should be evaluated more than F (lottery D and U are also two-stage lotteries; however if we evaluate these lotteries with the anticipated utility functional, their value should be equivalent to the value of E)[100]. The fact that these two lotteries are not evaluated equally to E means that they are not

[99] See appendix for calculations.

[100] See the appendix for the evaluation of these lotteries according to AEU.

evaluated according to an Anticipated Utility functional.

Results. Let us consider first the pattern of preference E≻F. If this pattern of preference occurs with the same percentage as the Ellsberg paradox it is possible to conclude that the behaviour of the subjects is consistent with Segal's theory. In fact one can expect to see violation of the reduction principle in the same percentage as the Ellsberg paradox. On the other hand, if the number of subjects who show this pattern of behaviour is consistently less than the number of subjects committing the Ellsberg paradox, then ambiguity aversion cannot be solely explained by Segal's theory.

In the experiment, 5 subjects evaluated lottery E differently from lottery F: in four cases lottery E was evaluated more then lottery F (consistent with Segal's theory) in one case E was evaluated less then F (and this behaviour is not consistent with Segal's explanation since the same subject is ambiguity averse). From this result it seems possible to conclude that Segal's theory can explain at most 40% of the Ellsberg paradox. It is important to note at this point that some of these subjects evaluated E differently from D and from U. U and D are expected to be evaluated equally according to Segal's model; this was true only for one of the subjects; for the other three subjects D was preferred to U. This is worth noting since a pattern of preference of this kind is consistent with the application of MMEU (maximin) (See section III.4.5). And, in fact, if we look at the subjects who exhibit this pattern of preference, we can see that in two out of 4 cases their behaviour was consistent with a maximin rule and in another case was not inconsistent with it.

Let us consider, now, the group of lotteries F,A,G,H,C. Consider first each of these lotteries relative to E. For the same reasoning as above, we expect a subject following Segal to evaluate each of the lotteries less than the certain lottery E. All the 5 lotteries are in fact two-stage versions of the lottery E. The difference between the five lotteries is due to the number of bags from which the subjects have to choose. And in fact the five subjects that evaluated E differently from F also evaluated each of the five lotteries differently from E and in particular they evaluated them less then E.

Let us now consider the relative evaluations of the lotteries F, A, G, H, C. If we take one of the two functional form of the weighting function f indicated by Segal which satisfy theorem 4.2 (see chapter I relative section) and choosing t = 2 we obtain $f(p)= p^2$. If we apply the functional form of the anticipated utility function for a multistage lottery defined by the anticipated utility functional form to all the above mentioned lotteries and and if we use the weighted function $f(p)= p^2$ then we obtain an evaluation of the lotteries which is decreasing with the number of bags. The calculations of the value of the 5 lotteries applying the anticipated utility functional form with the specific functional for $f(p)=p^2$are contained in the appendix.The same calculation has been done with different values of t in the interval [1.10 3] and the result is the same (See Table AB.I in the appendix) in the appendix).

That is to say the pattern of preference which is consistent with Segal model *and* with the conditions stated by theorem 4.2 *and* the specific selected functional form should be the following one: F≻A≻G≻H≻C.

However, the results of the experiment (See table III.3) shows that the subjects who committed the Ellsberg paradox and were ambiguity averse ordered the lotteries exactly in the contrary way: they evaluated higher the lotteries with more bags.

This can suggest that, either Segal*'s* theory may explain a smaller percentage of the Ellsberg paradox, or that *there might* exist another functional form for f(p) which allows for ambiguity aversion and which satisfies the other requirement of theorem 4.2 but which also allows for an evaluation of the lottery which increases with the number of bags.

To summarize, if we look at the subjects' evaluation of the multistage lotteries F,A,G,H,C, only one subject exhibits a pattern of preference consistent with Segal's explanation (in the sense above specified) and only one not inconsistent. For the other subjects the preference relation showed by the subject can cautiously be defined as not consistent with Segal's theory (though only to the extent above specified).

To sum up: as has already been noted, with an explicit two-stage set up, we observe a reduction of 50% of the Ellsberg kind of behaviour which shows that apparently Segal's explanation of the Ellsberg paradox can account for almost 50% of the subjects[101]; if, however, we analyze the

101 This result is very similar to Bernasconi and Loomes's experiment,(23.8), but they used the three-colour Ellsberg example.

behaviour of these subjects, in one only case it can be concluded that the behavior shown by these subject is consistent with the model, in the sense specified above. The order given to the lotteries (inferred by the evaluations) seems in fact to cast doubt on the consistency of the behaviour of these subjects with Segal's theory. What the result of the experiment may suggest is that, as in Segal (1987), ambiguity aversion can be partially explained with the non-application of the reduction principle. However if one subject uses a anticipated utility function and does not apply the reduction principle then his or her order of the lotteries should be different. How can an evaluation increasing with the number of bags of the of 4 subjects be explained? One possible road is verifying if such a kind of preference can be compatible with the non application of the reduction principle but with the use of a utility functional form different from the anticipated utility one (For other two possible explanations see chapter VI).

III.3.4.2 Raiffa (1961)

The model. As we have seen in chapter I Raiffa suggested that one explanation of the Ellsberg paradox may be the fact that people, when analyzing the unknown lottery, dislike it, or they evaluate that lottery in a different way from the certain one, just because they do not understand the process which generates the data. The subjects do not know in which way the probabilities of the two colours will be defined. If we look at the original lottery of the Ellsberg paradox, lottery O in our experiment, the kind of reasoning that can be adopted by an ambiguity averse subject can be simplified as follow: let me consider the unknown lottery, I do not understand how the probabilities can be decided, I do not have any insight into the process, so I simply dislike it.

Raiffa's suggestion is that if we teach people, that in the unknown bag, the probabilities can be thought of as generated by a random process, then they would evaluate the unknown lottery the same as the certain one.

The experimental design. To verify this hypothesis we put into the set of 21 lotteries 3 lotteries (B, S and I) in which the "unknown" process which generates the probabilities of the final outcomes is substituted by a "known, objective" random process, like the tossing of a coin or a random draw. Let us consider the three lotteries B, S and I, in lotteries B and S the bags are chosen by tossing a coin; in I the probability of the second stage is chosen drawing a ball from a bag containing 13 balls, each with a different number on it; the chosen number will correspond to the probability of one of the two colours in the second branch of the decision tree. Lottery B can be compared with the equivalent two-stage-with-known-probability lottery U and lottery S can be compared with the equivalent two-stage-with-unknown-probability lottery P, and lottery I can be compared with the 13-bags lottery C. The relevant lotteries to be considered are hence:

U versus B, U versus E (in which the probabilities are known);

I versus E, I versus C;

S versus O, S versus P (in which the probabilities are not known).

In lotteries B, I and S the probability of the second branch is generated by an objective device; the toss of a coin for B and S, a draw of a number between 0 to 12 for I; if Raiffa's suggestion is correct, then at least some subjects are expected to show the following pattern of preference B~E, S~O, I~E, P~S.

Results. If we look at table III. 3 we can see that in 4 cases out of 10 B~E , in 2 cases I~E, in 6 cases S~O and S~P, but in only two cases B~I, and in 7 cases B~U, in 5 cases U~E, but just in 3 cases I~C.

What can one conclude from this pattern of preferences? Since for 5 subjects out of the 10 committing the Ellsberg paradox the lotteries U and E have the same evaluation, and this is also true for the lotteries P and O, while 6 out of 10 subjects gave the same value to the lotteries P and S and 7 subjects to B and U, it seems possible to conclude that at least 40% of the subjects behave in a way which is consistent with the Raiffa explanation. If, however, we examine the evaluation of the lotteries C and I, the behaviour which can be considered consistent with Raiffa hypothesis seems less frequent. Let us consider lottery C; this is a two-stage version of the lottery E with 13 bags. The subject is asked, first, to choose a bag and, then, to draw a ball; each bag has a different proportion of black and white balls in it; choosing a bag is equivalent to choosing the probability distribution of the second branch of the decision three. Lottery C can be considered equivalent to lottery I where the probability of the second branch is determined by the draw of a numbered ball. From the point of view of Raiffa's hypothesis the two lotteries should be evaluated exactly the same, since they are just two ways of choosing randomly the bag from which to draw. This is true just in two out of the 4 cases which exhibit the preference pattern I~E. In the other two cases lottery C is evaluated more than lottery I; this latter lottery seems to have been perceived as more uncertain. There is one difference between the two lotteries: the choice of the bag is either by the subject or by a random device. In this case it seems as if the random device had increased the element of chance in the lottery[102]. Moreover, lottery I has been evaluated equal to lottery E in just 1 case (lottery C in 4 cases), showing that it has been perceived by 90% of the subjects as more uncertain. It can be worth noting, however, that Raiffa's intuition assumes that people clearly understand what is a random process, that for them equivalent random processes can be considered the same and that they do not show any aversion to randomness, that is they do not dislike the fact that something is decided by chance. This can be true if the random objective process is quite simple like the toss of a coin. The fact that, in the two-bags lotteries, the Raiffa kind of preference is more common, can be explained by the fact that the chosen random process is quite simple and that in any case the choice (all white in one bag, all black in the other bag) was quite straightforward. In the case of lottery I, the choice is not so straightforward, and the random process can be perceived more "random" from a psychological point of view. This interpretation seems to be supported by the evidence that the randomness built into the lottery has been perceived much more in the case of lottery I than in the other two cases (as can be seen from the interviews)[103].

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[102] This behaviour can be linked with the idea of "illusion of control" demonstrated by Langer (1975). See also section III.4.7.2.

[103] One of the subjects declared that what she disliked most was anything determined by chance and a random device was, according to her, a sort of "chance over chance". She said "in general the higher is the degree of casuality (randomness) built in the lottery the less I like to bet on it. I evaluated the lotteries three, two and one according to this criterion..
...but I mean there is too much chance built into it, chance on chance, chance on chance that you know... I just do not want to play."

To sum up: it seems possible to conclude that at least for some subjects, the suggestion that the aversion to uncertainty can disappear if the subject learns to see the "uncertainty" as a random process, can be true. But can this always be true? On the one hand, some random processes can be too complex to understand, and, on the other hand, we doubt that all "uncertainties" can be reduced to a random process.

III.3.4.3 The healthy skepticism of Kadane (1991).

The model. In his work, Kadane (1991) still assumes that people evaluate lotteries according to Expected Utility theory. However his model differs from the traditional one in the following way: if the subject assigns a positive probability of a strategic behaviour of the person making the offer (the experimenter), then he or she will compute the expected utility according to the positive probability assigned to this behaviour. The expected utility of the lottery will be decreased accordingly. The Ellsberg paradox is consequently due to the fact that the subjects evaluate less the unknown lottery just because they think that the experimenter can cheat.

According to Kadane, in the Ellsberg paradox, the method of possible cheating is made very obvious; the experimenter in fact will presumably know the number of white and black balls contained in the bag and consequently the experimenter can decide which bets are on. The kind of reasoning adopted by the subject can be the following one: I am told that in the bag the proportion of black and white is unknown but if the experimenter knows what there is in the bag or can manipulate in some what the content in the bag then if I choose black he can arrange things in a way that in the bag there are more white balls than black. Only by choosing white and or black from the certain bag the subject can protect themselves against such manipulation.[104]

On the other hand another subject, the only one who was ambiguity prone, declared exactly the contrary; he liked everything that was more random.

[104] For Kadane reasoning to be true, however, one have to assume that either the experimenter has more than one bag of "unknown composition", and he is able to choose the bag accordingly to the colour on which the subject has decided to bet, or it is the experimenter who chooses the colour. If the experimenter has just one bag of "unknown composition", whichever is the true composition, if it is the subject who chooses the

"Again the experimenter is supplying the prizes, the urns and chooses which offered alternative will actually obtain" Kadane (1991) page 61

To sum up: the idea is that the Ellsberg paradox makes a reasonably skeptical person suspect the possibility of being cheated.

The experiment design. Let us consider lotteries V and Q; V and Q are two-stage representations of the certain and uncertain lottery. The possibility of being cheated by the experimenter has been introduced explicitly in these two lotteries; the subject is asked to bet on a colour and then the experimenter will choose the bag from which the subject has to draw the ball. In one case, lottery V, the composition of the two bags is known, one bag contains all white balls and the other contains all black balls, while in the other case, lottery Q, the composition of the bag is unknown. If people think that they can be cheated then they can be expected to evaluate these two lotteries less then the certain one or zero.

The results. As as we can see from table III.3, however, not one of the subjects evaluated lottery V (zero or) less then the equivalent two- stage lottery B. In case of lottery Q, 4 out of 10 subjects evaluated it less than the two-stage counterpart P. One possible interpretation of this pattern of preferences may be the following one: in the case of known probabilities, the mere fact that the composition of the bag was known even if the probability of the two colours is either one or zero reduces the level of suspiciousness in the choice; after all, as one subject said, the bags are opaque so no one can see inside. On the other hand in the case of the unknown composition "there are too many unknown values" as one subject said[105]. The choice of the subject not to give zero as the evaluation of lotteries Q and V was generally motivated by the fact that they were not sure of the fact that the experimenter could cheat, it was just a hypothetical possibility. They did not assign a probability equal to 1 to the

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colour on which to bet, then, the subject still has 1/2 probability to bet on the "right" colour.

105 Subject 16 said " Q you do not have any decision in it; it is all up to the experimenter and it was not pure chance; so I might give a slightly less value just in case I do not know, in case the experimenter knew or ..*there are too many values*, so I gave 5 to it.

possibility of being cheated but just a probability greater than 0. In only two cases the lottery Q was evaluated less then 0 and in the other 7 cases was evaluated the same (in one case lottery Q was evaluated more then the equivalent one stage unknown or the two-stage P). The percentage of people showing the fear of being cheated by the experimenter is small in the expected utility group as well as in the group exhibiting the Ellsberg kind of behaviour, however 4 were the subjects who evaluated Q less than P.between the EP subjects. Two subjects out of the three exhibiting an aversion for the choice of the bag made by the experimenter evaluated these lotteries less then the unknown one, showing that more then the cause of the uncertainty the feeling of being cheated can increase the aversion of uncertainty[106].

To sum up, the fear of being cheated does not seem to capture the problem of aversion to ambiguity; even in situations were it is made explicit the fear of being cheated does not seem the main reason for people to avoid the unknown bag; it may at most constitute a further element of ambiguity. However, as far as the design of experiments is concerned it can be worth putting a special care in trying to avoid this source of uncertainty. A careful frame of the experimental environment might help to avoid this risk.

III.3.4.4 Choquet Expected Utility.

The model. In this experiment we tested the two-stage representation of Choquet Expected Utility (Schmeidler (1989). Following Anscombe and Aumann (1963) (see chapter I), Schmeidler adopts a two stage representation. In this framework a lottery can be seen as a two stage process where the state S, in the set up of this experiment white or black, occurs in the first place. Then, in the second stage, conditional on which state of the world occurs, a lottery is played out to determine the outcomes, the final consequences. The framework is

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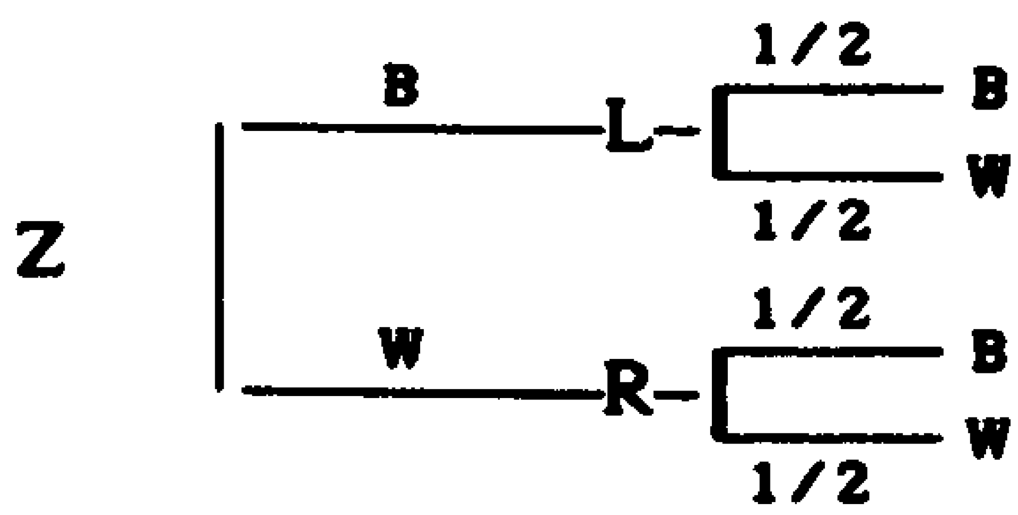
[106] From the interview of subject 1 "With Q the external force is the experimenter, you do not feel as lucky when the experimenter is doing ..you know what I mean. It is an external force that, even if it does not have more control than you do on the bags, it seems to have more control than you do over the experiment in itself.

quite similar to that of Segal[107]. The subjects see the ambiguous lottery as a two stage lottery. However, in the first stage the subjects are presented with an event lottery. The outcomes of this first stage are pure risky lotteries, and these risky lotteries will determine the final consequence. To the occurrence of an event subjects assign capacities, which are not necessarily additive measures of the likelihood of the occurrence of an event. In Schmeidler's (1989) framework, the subjects evaluate the second stage risky lottery according to expected utility. The so obtained certainty equivalents of the second stage become the outcomes of the first stage event lottery, which is evaluated according to Choquet Expected Utility.

The experimental design. Let us consider, as an example, lottery Z. In the first stage the subject has to draw a ball from a bag which contains 12 black or white balls but in an unknown composition. Conditioning on the occurrence of the event Black or White, he or she is asked to draw a second ball from one of the other two bags: if the first draw is black he or she has to draw from the left bag, if the first draw is white she or she has to draw from the right bag. Both bags, the left one and the right one contain 6 white balls and 6 black balls.

Table III.5 Lottery Z



Where in the second branch we have p(B) and p(W) which are additive. While in the first branch we have v(B) and v(W) which are not additive.

[107] From this similarity the criticism of Wakker to Schmeidler model. See Wakker (1989) and Sarin and Wakker (1991). In this experiment we considered Schmeidler version of capacities (See also Camerer and Weber 1992). It is worth noting however that if the unknown lottery is not mentally represented as a two stage lottery but as a single stage as in Sarin and Wakker, the evaluation of the lottery is different from its evaluation in a two stage set up.See chapter I.

The value of this lottery can be computed according to Choquet Expected Utility. It is important to note, however, that another crucial feature of Schmeidler's theory is that, while subjective probabilities (capacities) are dealt in a non additive way, the objective, known probabilities, are dealt with in the traditional way. This means that CEU in case of objective probabilities reduces to EU.

Since CEU reduces to EU if the probabilities are additive (see Table III.5) we will have u(CE(L)) = 1/2 .u(25) + 1/2 .u(0) and u(CE(R)) = 1/2 .u(25) + 1/2 . u(0)

In the second branch, since the probabilities or capacities are not additive, we will have:.

V(Z)=

$$u(CE(L)).v(B)+u(CE(R))\left[v(B\cup W)-v(B)\right]= \qquad \text{(III.1)}$$

$$= u(CE(L) \,.\, v(B) + u\,(CE\,(R))\,v(B\cup W) - u(\,CE(R))\,.\,v(B)$$

since u(CE(R)) is equal to u(CE(L)) then the above expression reduces to u(CE(R)) . v (B∪W).

Since S, the set of all possible outcomes is given by B and W, and we know that v(S) = 1, then we can assume that v(B∪W) (which is the probability of the certain event) is equal to 1. Hence, the evaluation of lottery Z, V(Z), will reduce to u(CE(R)) But Lottery R is equal to lottery E consequently, if subjects behave in a way consistent with Schmeidler's theory, we will expect them to evaluate lottery Z the same as lottery E. Hence, if the subjects evaluate lottery Z the same as E, and this preference relation appears in the same proportion as the Ellsberg paradox, then the Ellsberg paradox can be totally captured by the existence of capacities. If, on the contrary, the percentage of the subjects showing this preference relation is less than that committing the Ellsberg paradox, then, Schmeidler's theory can only partially explain the Ellsberg paradox.

The results. If we look at table III.3 we can se that in this experiment 4 out of the 10 EP subjects evaluated Z the same as E; one evaluated E more than Z, and 5 evaluated Z less than E.

The more frequent motivation to this latter pattern of preference by the subjects was the following one: "I know that what happen in the first stage is almost irrelevant to my final choice but the point is

that any way in the first stage I do not know which is the composition of the bag from which I draw so, since I do not like betting on things that I do not know I evaluate anyway this lottery less".[108]
Moreover, if we analyse in more detail the evaluation of the other lotteries by the subjects who evaluated E the same as Z, it is important to notice that just two out of these 3 subjects behave in a consistent way with Schmeidler's model. In fact, as CEU reduces to EU when the probabilities are known, I would expect these subjects consistently to evaluate lotteries D, U, F, G, H, C the same as lotteries E and Z This pattern of preferences is shown only by two out of 10 subjects in this experiment.
To sum up, it is possible to conclude that in this experiment[109] only few subjects seem to behave in a consistent way with Schmeidler's theory.

III 3.4.5 Maximin criterion for expected utility.

The model. According to Gardenfors and Sahlin (1982), (see chapter I) an ambiguous situation can be depicted as a situation in which beliefs about the states of nature can be represented by a set of probability measures Π (see relevant section in chapter I). However not all of an agent's beliefs about the states of the nature relevant to a decision situation can be captured by a set of probability measures. There is a second element which is relevant to a decision situation: ρ, which is a real value measure of the epistemic reliability of the probability measures in P. To define which distributions are reliable ρ is used, and the level of ρ depends first on the level of information

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[108] Subject 11 said " You got three bags. In one of the bag you do not know which combination there is, and in the second and in the third, you know that there are 12 balls of which 6 are black and 6 are white. So you have to draw a colour from the first bag; so you do not know the choice of taking a white or a black because you do not know the combination that is in there. Then, depending on if you draw a black or a white, you draw a ball from one of the other bags. And I gave then pounds, because, I thought, well, you pick out a white from the first bag, *that is irrelevant*. It does not matter which one you pick, you end up picking up a ball from the second or the third bag, and, so, you got 50% chance of picking up the colour you want. So, even if you got 50% chance I gave 10, *because first you have to pick up from the first bag.*"

[109] See Mangelsdorff and Weber (1992) for a more detailed test of Choquet expected utility in a quite different set up.

of the individual -their epistemic reliability-[110], and, second, on a psychological factor -the fact of being more or less sure of one's own evaluations (if I am sure of my evaluation I will just have one probability measure). Hence, in making a decision an agent considers Π/ρ defined as the set of all the distributions which satisfies his level of ρ, that is to say the distribution which are epistemically reliable according to him.

From the set Π/ρ the individual chooses as follows:

The maximin criterion for EU (MEUT); the alternative with the largest minimum expected utility ought to be chosen.

The experimental design. Gardenfors argues that the pattern of preference displayed by people committing the Ellsberg paradox is consistent with the adoption of a MEUT decision criterion. To investigate this lotteries R, L and F were designed. These three lotteries are two-stage lotteries in which, in the first instance, the subject is asked to choose a bag (out of three) and, in the second instance, the subject is asked to draw a ball from the chosen bag. Each bag consequently represent a probability measure, so the problem can be seen as a case in which the subject has a set of probability measures for the event "drawing a black ball" and each of these probability measures has the same probability. In Gardenfors' terminology we can say that the subject's set of probability measures is composed of equally epistemically reliable probability measures. In the present set-up of this experiment the epistemic reliability is not due to any individual weight of evidence or psychological factor since the various probability measures are known. The problem becomes simply the one of choosing in the situation in which the subject has not just a probability measure but a set of them.[111]

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[110] In this respect the reliability of a probability measure is given by the quality of the information we have. Hence the Gardenfors and Sahlin's idea is very similar to Keynes's concept of 'weight of evidence'. It is in fact the weight of evidence that determine the epistemic reliability of our probability measures. The concept of "epistemic reliability" simply mean reliable according to our level of knowledge.

[111] In the 3 lotteries of the present experiment there is no ambiguity, if by ambiguity it is meant that we do not have any idea of the reliability of the probabilities contained in our set. In this

Lotteries R, L and F differ because the set of probability distributions that are assigned to the draw of a black or a white in each lottery is different. Let us take for example the drawing of a black (the same is valid for white since the distributions are symmetric): in all three lotteries the probability of choosing one of the three bags is 1/3; what is different in the three lotteries is the probability of having a black in the second branch. In particular the minimum probability varies from 0 to 1/2. If I bet on black, for instance, in lottery F the probability of having black can be 0,1/2,1; in lottery L, 4/12,1/2,8/12, while in lottery R the set of probability measures for black will be 1/2, 1/2, 1/2. The decision criterion of Gardenfors is a maximin (in particular Wald maximin) but the worst outcome is not given with respect to the payoffs but with respect to the probabilities. If we compute the expected utility for each probability measure in each set we can see that the minimum expected utility is bigger in lottery R then in lottery L, which is, in turn, bigger than in lottery F. If people use MEUT as a decision criterion it is predicted that they will value R higher than L and L higher than F.

The results. In this experiment 2 out of 10 subjects who committed the Ellsberg paradox show the pattern of preferences implied by MEUT for all the three lotteries and 3 others preferred lottery R to the other two. Are these choices consistent with the MEUT criterion? Three of the subjects explicitly gave in the interview an explanation which resembles the adoption of a maximin criterion. For example subject 1 said " With R, any bag you choose you got 50 % chance of getting a black or a white ball, so this is pretty fair; there is no chance of being persecuted, of having all the other colour balls; with lottery L the bag is still mixed up, but one is fifty percent chance, another is 40, so if you choose the one in which you have just 40, then, you are worse off then in lottery R".

Subject 5 " I put 10 here, because the chance is fifty percent in R;

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experiment the set of the possible probability is defined by the experimenter as well as their reliability. All the probabilities are in fact equally reliable;however there is more than one probability measure and we do not know which state of the world will prevail. The extension of the Gardenfors and Sahlin theory to encompass cases like this seem to be justified. See Gärdenfors and Shalin (1983).

then I thought that it is more equally distributed. F, I do not like it, because there were 12 white balls in one of them; so I thought I might get nothing, and L was not so evenly distributed".

The same kind of reasoning is shown by subject 11. " Because with R which bag you pick out it does not matter, you got fifty percent chance to get whatever colour you want, but with F, depending on which bag you got, you pick out one hundred percent chance of picking out 12 white balls or one hundred percent chance of picking out 12 black balls, actually, I would have given even less value."

Subject 22 was even clearer. " I order lotteries according to the probability of the bag with the wrong colour. Now, because in F there is the bag with the wrong colour, which I do not like. R, R is the one I like better because you can never pick the wrong bag; in every bag you have half and half percentage of winning, and in L you still have ..L is the same as R in the sense that you always... again.. you necessarily will pick a bag in which there is the right colour, but the probability of piking that colour is lower."

Also three of the same subjects preferred lottery D to lottery B which reproduces exactly the same situation than lottery R and F but in a two-stage set up. Hence around 50% of the subjects were ambiguity averse and seemed to behave in a way which is consistent with the Gardenfors and Sahlin decision criterion. The idea of avoiding lotteries in which the probability of the chosen event could be zero was explicitly stated by the subjects and in some cases the rationale of this aversion was explained saying that on one hand the situation was worst and on the other hand the lottery was not "fair".

III.3.4.6 Information lotteries.

The experimental design. As we have already said (describing the results of this experiment for the " non EP " group), lotteries N and Lottery T were introduced with the twofold tasks of checking EU people, on the one hand, and on the other hand, of trying to verify which is the influence of information in an uncertain situation. In this case the lottery presented to the subjects involve betting on a colour and drawing a ball from a bag in which the proportion of black and white balls is partially known (there is a reduction in ambiguity consequently since some of the distributions are ruled out after the draws from the first bag). In fact, in lottery T the subject is asked first to draw a ball from a bag which contains 6 white and 6 blacks ball; then he is asked to put the chosen ball in a second bag containing 11 balls in an unknown proportion. At this point the subject is asked to bet on a colour and to draw a ball from the second bag. In the second lottery, N, the quantity of balls that the subject puts in the second bag becomes 3. (After any draw there is replacement. Hence at the end in the second bag he or she knows the colour of 3 balls out of 12. It is important to note that the subject is asked to bet on one colour after he has put one or three balls in the second bag and consequently just before the final drawing).

Let us consider lottery T; if I put a white ball the probability of drawing a white is higher then the probability of drawing a black[112]; the same holds for lottery N once I have already put two or three white balls in the second bag.

In evaluating, these lotteries an Expected Utility subject can be expected to assign a higher value to lottery N and T than to all the other lotteries. In fact, if the subjects follow some form of updating procedure in the light of new evidence, they will evaluate T and N more and this may happen even if they do not follow Bayesian updating rules. (cf Jaffray (1992) and Gilboa and Schmeidler (1993)). Moreover, our initial idea was that, in the case of an ambiguity averse subject, the new acquired information might need to overcame a sort of threshold level before the subject could feel the necessity of changing his or her prior. This can happen either because changing prior implies effort, hence is costly, or because ambiguity averse people can be pessimistic people who need a substantial increase in information in order to modify their judgment[113].

The results In this experiment 4 out of the 10 EP subjects group evaluated the two lotteries N and T the same, 3 of these 4 evaluated these two lotteries the same as the unknown lotteries, while the fourth evaluated these two lotteries slightly more then the unknown one. In the other 6 cases, lottery N was evaluated more than lottery T, showing

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112 See section III.3.3.1

113 This later phenomenon has been noted in an experiment presented by Cohen, Gilboa Jaffray and Schmeidler in the FUR 1994 Conference on updating ambiguous beliefs. Private conversation with Michele Cohen.

that the reduced ambiguity was "correctly" perceived. Moreover, 3 of these subjects evaluated T equally to O, the unknown one, but evaluated lottery N substantially more (Showing the existence of threshold level?).

To sum up, it is possible to say that while in lottery T the weight of the uncertainty overcomes the weight of information, in lottery N the amount of information was considered sufficient to increase substantially the evaluation of the lottery.

In the present experimental set up the role of information was quite limited: the information received was clearly in the direction of reducing the uncertainty. However it is possible to think of situations in which the increase in the information set can reduce (or not increase) the support of the evidence; that is to say, an increase in the information can as well increase the level of ambiguity. How the information can influence the perception of uncertainty and the aversion to it is an interesting field for further research.

III.3.4.7 Some psychological and intuitive theories.

Does it make any difference in which branch of a compound lottery the uncertainty is present?

The experimental design. In this experiment lottery M has been introduced specifically to verify if in a compound lottery the introduction of uncertainty is perceived in a different way whether it is introduced in the second stage or in the first stage of the lottery.

In lottery M there are two bags, one contains six white and six black balls; the other bag is empty. People are asked to bet on one of the two colours, then a person, who is neither the subject or the experimenter draws two balls with replacement from the first bag and put them in the second bag, without showing the balls neither to the experimenter nor to the subject. At this point, the subject has to draw a ball and he or she will get the prize if the ball he or she has drawn is of the chosen colour. Of course, since the first two draws are with replacement, the probability of each colour is 1/2. However the aim of the lottery is to know how the subject perceives the fact that he or she does not really know what is in the second bag; actually in the second bag there can be 2 black balls, two white balls or one white and one bag ball. The second stage can be consequently represented exactly as lottery F.

Results. In this experiment just 6 out of the 10 subjects who committed the Ellsberg paradox evaluated lottery M less then lottery E, showing that the presence of uncertainty has been perceived also in the second stage.

The fact that the number of subjects who showed this preference relation were less than the number committing the Ellsberg paradox in our opinion, may depend on two factors: either on the fact that in the second bag the balls are two [114]; or that the focus of the subject in a compound lottery is more on the first branch, which in this case has certain probabilities. The first of these two interpretations has some support from the interviews; more than one subject declared i that, while evaluating the lottery, they were considering in how many cases they could choose from bags in which there was at least one ball of the chosen colour (in this case 2 out of 3 and consequently more then in lottery B); the second branch of the lottery can, actually, be represented in a decision tree as lottery F.

Lottery M can so be evaluated more then lottery P or the same as lottery P depending whether individuals consider more important the existence of the uncertainty or the way that this uncertainty is represented. But also in the evaluation M towards P it is important to consider that in the P case the number of unknown balls is 12, while in M they are just 2.

In 3 cases we have M evaluated the same as P and in 6 cases M evaluated more then P, while just in one case P was evaluated more than M, showing that the limited number of balls in lottery M might had had had a positive influence in the evaluation of the subjects.

The hypothesis of control

The Model. One hypothesis that we wanted to test in this experiment was whether the feeling of having control over the experiment has an

[114] The number of balls contained in a bag can have an influence especially if one bag can be considered as a sample of the other. A sample of two can be perceived in a different way than a sample of 6 etc. See Giliotti and Sopher (1990).

influence on the valuations of the decision maker. To verify this we used lotteries U and P. Lottery U is a two-stage representation of the certain lottery E, while lottery P is the two-stage representation of the lottery O.

In lottery U the subject is asked to bet on a colour, to choose a bag (there are two bags one containing 12 black balls and the other containing 12 white balls) and to draw a ball. The same procedure is followed in lottery P, but the two bags contain an unknown proportion of black and white.

In order to verify this hypothesis, lottery B should be compared with lottery U and lottery P with lottery S. What distinguishes these two pairs of lotteries, and is crucial to the verification of our hypothesis, is the fact that, while in lottery U and in lottery P, *the choice of the bag in the first stage is the responsibility of the subject,* while in lotteries B and S the responsibility is given to an objective random mechanism. The rationale of repeating the choice is that the psychological weight of the feeling of having control over the experiment or of taking the responsibility of the choice of the bag can be different in a framework in which the probability of the outcome is known from a situation in which the probability of the outcome is unknown.

This kind of reasoning can be linked to the hypothesis of Heath and Tversky (1991). In their experiment the two authors try to verify the following hypothesis: subjects prefer to play out a lottery, of which they have a subjective probability over the outcomes, when the lottery is related to some field of their knowledge than to play out a roulette lottery, in which the probabilities and the two outcomes are exactly the same as the horse lottery. However, if they have little knowledge about the field related to that bet, then, they will prefer to play out the random objective lottery instead of a lottery with the same but subjective probability over the same outcomes. This kind of preference relation is explained by the Heath and Tversky in the following way: when you know the field on which you are betting, whatever the final outcome will be, you will be considered responsible for your choice, hence you will take the blame or the credit for it. But when your knowledge in the field on which you are betting is weak then, from a psychological point of view, it is better if the blame or the credit is given to luck more than to you.

Lotteries B and P and U and S can be reread in this context; let us consider lotteries B and U, first: in this case, the subject knows the composition of the two bags, he or she has not to use any subjective probability; the only difference is in the first stage of the lottery. In B the subject is responsible for the choice; he or she can be blamed or get credit for the choice; while in U the choice is made by a random device.

The results. Let us now consider lotteries B and U; in this case, the composition of the two bags is known and, consequently, the subject is betting in a situation in which knowledge is strong.

In the first set of lotteries 80% of the subjects did evaluate the two lotteries equally; the usual explanation was that the probabilities were in any case equal to 1/2 and so the mechanism according to which the bag was chosen in the first stage was irrelevant to their choices. In the second set of lotteries, (the composition of the bag was unknown), 30% of the subjects preferred lottery S over lottery P, giving the responsibility for the choice of the bag to the random device. This kind of pattern seem at least not inconsistent with the credit and blame model. What seems to matter to the subjects was not the feeling of having a better control of the experiment if they were to be asked of choosing the bag; the stronger feeling seem to be more that avoiding the responsibility of the choice in an unknown environment; when the environment was clearly defined, the mechanism of choice in the first stage is perceived as irrelevant. This is clearly stated[115] in the interviews of the subjects. The behavior of these subjects appear to be in some ways at least not inconsistent with Heath and Tversky's theory; the fact that the subjects behaved in a more

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[115] Subject 1, for example, "With B you have got to choose the colour of the ball and, then, it is like down to faith, again, when you toss the coin, whether you go for the bag , which has got your colour of the ball that you have chosen: it seem rather down to faith if you are lucky. U it seems of the same sort. It is down to faith which bag you choose, because the coin is just external to the experiment really; the only thing which is different is whether the coin designs where you put your hand in , or, you decide where to put your hand in. But the bags are equal, anyway, they can be mixed up. V is the same as B. The only difference is because i is the experimenter to choose the bag; it is still down to luck again, because V and B are exactly the same; because it is an external force. U is the same because it is still down to luck where you choose."

consistent way with the credit and blame hypothesis in the case of the unknown bags can be explained with the particular contest in which they have to decide in this experiment; the lottery with the unknown probabilities can be quite easily be assimilated to a bet on a field in which your knowledge is very weak, on the other hand in lottery U you do not have much to worry about since anyway the composition of the bag is fixed and known[116].

To sum up, it seems possible to conclude that the feeling of control does not seems to matter a great deal in the evaluation of the lotteries especially when the environment is known or considered clear by the subject. However some of the choices can be more easily, even if cautiously, interpreted in a way quite close to the credit and blame hypothesis of Heath and Tversky (1991).

Ambiguity aversion as aversion to complexity.

The model. Another psychological factor which can have an influence on the explanation of the Ellsberg paradox can be the aversion to complexity. If ambiguity can be represented by a set of probability measures, then to evaluate lotteries becomes more complex; since thinking is costly then the more complicated is to evaluate a lottery, the more is the requested effort and the less will be my evaluation.

The experimental design and the results. To verify this hypothesis I introduce the set of lotteries B, F, A, G, H, C, where complexity is represented by an increase in the number of probability measures that the subject has to consider in evaluating a lottery. If people react negatively to complexity it can be expected that their evaluations of the lotteries would be in inverse proportion to the number of bag. Since this is true just for one subject out of 10 this psychological explanation can be dismissed as an explanation of the Ellsberg paradox. In analyzing those lotteries the focus of the subjects was not on the complexity of the choice, but more on the value

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116 It is important to remember, however, that in the case of lottery I and C, two subjects preferred C to I showing that they prefer to choose the bag directly than to give the choice to a random device when probabilities are known.

Table III.5 Summary of the results.

Theory	Patterns of evaluations	number of subjects
Non EP	E~D~U~B~F~A~G~H~C~I~R~L~P~S~O~Z~M	11
EU	T and N evaluated more than E	7
Kadane	T and Q evaluated 0	2
EP	E≻O	10
Segal	E≻F	5
	F≻A≻G≻H≻C	1
Raiffa	C~I	3
	B~U	7
	E~U	5
	S~P	6
	S~O	6
Kadane	B≻V	0
	P≻Q	4
Schmeidler	E~Z	4
Maximin	R≻L≻F	2
	R≻L~F	3
Information	N≻T	6
Control	B≻U	2
	S≻P	3
Complexity	F≻A≻G≻H≻C	1

of probability of choosing one bag containing at least one ball of the chosen colour.

III.4 Discussion and conclusion

III.4.1 *Rereading the results by subjects*

III.4.1.1 The non EP subjects.

As far as non EP subjects are concerned, they can be divided in two groups. In one group there are the subjects who recognized the different expected value of lotteries T and N (6 out of 10). In the other group all the subject gave the same evaluation to all the lotteries. In the first group however just three of the six subjects seem to have actually calculated the exact value of the lotteries. On the other hand, as has been described in detail in section III.3.1, three of the subjects who evaluated all the lotteries the same recognized a different value of the lotteries T and N in the interviews but this difference was not considered big enough to change their evaluations (see Table III.5 for a summary of the various theories).

III.4.1.2 The EP people.

Now we will look at the results of the experiment referring to the specific evaluations of each subject, to verify if it is possible from their evaluations infer whether they behaved consistently with one or other of the various suggested explanations.
As has been already observed the lotteries were designed to verify various explanations and not all the lotteries are relevant to explain all the hypotheses. This factor can have an influence in the way the data should be interpreted, since some models do not tell us anything about how the subject should evaluate a lottery which was originally designed to verify some other hypothesis or model.
At a first look at the data, it is possible to divide the subjects into two main groups. In one group, we put all the subjects who made a different evaluation of each or almost each of the lotteries, while in the other group we put the subjects who grouped the lotteries mainly in two groups: the certain ones and the uncertain ones; and evaluated them

accordingly.

Let us first consider the first group. In this group there are subjects 1, 11, 16, 19 and 20. we will leave for the moment subject 19 since this deserves special analysis.

Subjects 1 and 20 are the two subjects who used consistently and explicitly the maximin criterion. Moreover for both of them the fact that a lottery could be considered fair or not was the more important factor in their evaluations. And they identify as fair a lottery in which there are no bags with just one colour in it; or, if there is a bag of this type, they evaluate the lottery according to the weight of this bag in the total of the bags. Both the subjects were quite consistent in their evaluations.

Subject number 11 was the only subject who evaluated the lotteries consistently according to Segal's explanation.

It is almost impossible to classify the behaviour of subject 16 according to a specific theory apart from the fact that she always evaluates less the lotteries in which the experimenter chooses the bag.

Subject 19, as has been already said, is the only subject who shows ambiguity loving behaviour. It is almost impossible to find any consistency of the evaluation of this subject with any of the theories. The subject however behaved in a quite consistent way to his own criterion which explicitly stated in the interview. He said that he liked very much to gamble and that he evaluated the lotteries not accordingly to likelihood of the prize but accordingly to the pleasure that a lottery gave to him. "More difficult or complicated I find a lottery more pleasure it will give to me if at the end I succeed hence I will evaluate it more".

The second group of subjects is composed of subjects 5,7,8.14. These subjects mainly divided the lotteries in two groups; in one group they put all the lotteries they considered unknown, and, in the other group, all the lotteries they considered certain, and evaluated them accordingly. They did not distinguish each lottery in their evaluation even if for them there are some of the lotteries which are evaluated in a different way with respect to the known and the unknown probability group. Subject 8 said that she considered certain just lottery E, and that she regarded as uncertain all the other lotteries. Her evaluation

of these latter lotteries was alternatively 8 or 7 pounds and she explained the choice between 7 or pounds as a random way to maximize the possible gain due to the B-D-M device. This can just be interpreted in the sense that since she was not sure between 8 and 7 she just evaluated half of the lotteries 7 and the other half 8.
In the interview, the subject explained her choice as follows: " Just because I played out without any risk, so, I thought, let's evaluate them all about between seven to ten pounds, because the chance that you get that number, when a number is picked, where you just got the money, and you do not play the lottery at all, is over 50%; so I thought, it is better to get some money than nothing, because playing the lottery is 25 or ought so I thought .. I just play without any risk, because there is more chance...
....From 7 to 10 I think I have just randomly chosen, actually...I mean, it depends on how much I thought the lottery will come towards a good end, or a bad end; I did not really calculate the lotteries, but I thought, if there were fifty percent chance I play ten, but if it just sounds suspicious to me, I just put 8 or 7 pounds, just randomly."

To sum up, this latter group seem to focus in their evaluation on just single dimension the presence of uncertainty and the absence of it. In particular these subjects seem to identify the presence of uncertainty with the fact that the probability where not known and absence of it with the known probabilities. They did not seems have given importance to any of the other differences, not even the presence of more then one probability distribution for the events to be considered in the choice.

III.5 Final consideration.

Before concluding, some general comments on the results of this experiment are needed. If we look at the subjects that committed the Ellsberg Paradox and to their evaluations, we can observe (see table III.3) that can be divided into two main groups. One group of subjects made a different evaluation of each or almost each of the lotteries, while an other group just gave roughly two different evaluations: a bigger one to the certain lotteries (usually lottery E and the lotteries with known second order distribution) and a smaller one to the uncertain lotteries. In the first group of subjects most of them seem to follow consistently one model or another [117] at least where possible. In this group, for example, we can find subjects who used consistently and explicitly (recognized from the interviews) the maximin criterion. Subjects belonging to the second group seem to focus in their evaluation on just single dimension: the presence of uncertainty or the absence of it. In particular these subjects seem to identify the presence of uncertainty with the fact that the probabilities were not known, and presence of it with known probabilities. They did not seems have given importance to any of the other differences, not even the presence of more then one probability distribution for the events to be considered in the choice.

It is important to note that most of the subjects evaluated the lotteries by dividing them into groups (they came at the interview with little packs of lotteries). They put the lotteries together according to some criterion, and then, evaluated them separately in each group. It seems that the focus of their attention was, first, for example, on the presence or absence of uncertainty and, then, within each group on other factors, like the number of bags or the mechanism of selection of the bags.

Moreover, the subjects who have evaluated the majority of the lotteries in a different way seem to have adopted different criteria in different contexts, which may suggest that uncertainty is perceived in a different way in different environments (See chapter VI for a more detailed interpretation).[118]

The fact that each subject seems to apply different theories can be due either to the fact that the application of a particular theory by a subject is context based, or that each theory recognized just one source of ambiguity. As we have seen, very often a particular theory operationalizes ambiguity in a particular way. These different

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117 Cf Table III.3 Subject 16 for example follows Segal (1987), while 1 and 2 follow always a maximin rule. Since however each lottery was designed to test a particular theory, a lottery must be considered just in respect to the theory according to which has been designed (a lottery may not reveal anything about other theories.)

118 Each individual seems to have reasoned according to a different theory in the various contexts. The point is that since every theory focus on a different source of ambiguity (different operationalization of ambiguity), most of the subjects recognized all these different sources and evaluate accordingly the corresponding lotteries.

operationalizations may or may not all be perceived by the subjects. This latter interpretation can, it seems, explain why some subjects made a single evaluation for all the unknown lotteries without differentiating between them. For these subjects their ambiguity aversion may just depend on their aversion to the lack of information (unknown proportion). But this aversion is not linked to any particular way of representing ambiguity.

The various theories we have tested may explain how people represent ambiguity in different contexts or how different people generally perceive ambiguity. However they do not seem to give an exhaustive explanation of aversion to uncertainty. They surely identify some of the main sources of ambiguity but they are not able to explain all the ambiguity aversion behaviours.

Are these result partially determined by the present set up of the experiment? Apart from the fact that in any case the design of an experiment is always crucial for the result, it is hard to say how far in this experiment the design may have had a major influence. For sure some of the evaluations of the subjects were due to the fact that the lotteries were all two outcome lotteries and that the problem was always symmetric; symmetry however is one of the characteristics of the original Ellsberg problem. Changing the set up to a multi-outcome one can probably partially change the results.

On the other hand one there is one thing which the results may suggest and that may be crucial in designing a further experiment. Uncertainty in the present set up was actually a limited concept of uncertainty. It was represented through the evaluation of gambles, all with the same two outcomes, £0 and £25.

It was a concept of uncertainty that was mainly confined to the idea of the existence of a set of probability measures or to a second order distribution. But nevertheless even in this particular set up uncertainty was clearly perceived and people showed a strong aversion to it.

Moreover, if a conclusion is to be draw from this experiment is that probably, even in this set up, too many elements were present and were interacting one with the other. The explanations, theoretical or psychological were all to some extent true, but no one could totally capture the behaviour of any one of the subjects. If this is true in such a limited set up then it is hard to figure that any theory could explain the behaviour of people when faced with "real" uncertainty.

What kind of suggestions can be draw from this experiment for further research? On the one, hand we would suggest that to have a deep insight into the phenomenon of decision making under uncertainty, it would be more important to test different elements of one theory than various theories. As we have seen in the analysis of section III.3.4.1, Segal's theory of ambiguity is formed by more than one element and all the elements are crucial to Segal analysis. It could be useful according to me to try to verify the differing importance of the various elements in the explanation of the subjects ' reaction to uncertainty. The importance of the different elements can vary according to the level of ambiguity or kind of outcomes (gain or losses) or their levels. The same theory can moreover be tested in a game of chance set up or in a event set up. The same kind of inquiry can be applied to other theories like Gardenfors' (maximin) or Schmeidler's ones, for example. One the other hand, as far as the explanation of the EP is concerned in general, we think that more experimental investigations on decision making under ambiguity are needed with bets on natural events. In particular we are thinking of investigating the role of information in decision making under ambiguity in a natural event set up.

CHAPTER IV

THE IMPACT OF AMBIGUITY ON THE VALUATION OF SELF-INSURANCE AND SELF-PROTECTION

IV.1. Introduction

In chapter III we tested some of the theories which explain the Ellsberg paradox in a chance context. People had to evaluate lotteries and all the lotteries had a zero or a positive outcome. The aim of this chapter is instead that of testing if ambiguity matters in real life situations. To do so we choose an insurance decision. People had consequently to evaluate how much to spend to self protect or self insure themselves when faced with ambiguous and risky choices. Being an insurance problem the choice of our subjects involve a zero or a negative outcome. This allowed us to test in a completely different context one theory which seemed to have performed quite well in the previous experiment, that is to say the Gardenfors and Sahlin (1982) maximin model, but with choices involving losses. As we have seen from the analysis of the literature in Chapter I and II, some of the theories which explain ambiguity aversion in the case of positive outcomes allow for ambiguity preference in the case of negative outcomes. In this respect the experiment described in this chapter can be seen as a test of this more general hypothesis.

On the other hand we concluded the last chapter by saying that ambiguity theories need to be tested in a event context and not just in a chance context. The following experiment is somehow in-between. People in fact had to evaluate scenarios about the occurrence or non-occurrence of some event and consequently the choice of investing in self-protection or in self-insurance. The probability of the occurrence of the event is described by imprecise statements to convoy the idea of ambiguity. (we choose different imprecise statements according to different theories). Vagueness as imprecision in probability estimates is one of the way in which in real life ambiguity is expressed. In evaluating the various scenarios subjects do not have to refer to a lottery context. However, there still remains the problem that to resolve uncertainty (play out one of the scenarios for real at the end of the experiment in order to pay the subjects) we choose a lottery, that is to say a chance device. This was due mainly (as will be explained later on in much detail) to the fact that we expressed vagueness in the probabilities through (1) an imprecise esteem, (2) a set of probability measure, and (3) an interval probabilities. Hence, the more straightforward way to operationalize it at the end was through a second order distribution. However at the time of their evaluations the subjects did not know how the scenario was going to be played out. We choose not to show it on purpose in order to avoid a possible decrease in the ambiguity perception (Camerer and Kunreuther (1989)). As a practice question deliberately we chose a risky one. As will be seen by the account of the following experiment, testing theories in a more real life context creates more problems for the experimental design, but allows us to know more about theories which are proposed to explain a phenomenon (ambiguity aversion or aversion to uncertainty) which has more to do with every day life situations than to a chance context and whose implications (and policy implications) in real life might be rather important.

IV.2 Self insurance and self protection.

The seminal paper by Ehrlich and Becker (1972) discussed the alternatives to market insurance available to an expected utility maximising consumer or firm who wants to cover against the risk of a loss. An individual can either purchase preventive measures that reduce the probability of a loss (self-protection) or invest in a reduction in the size of the loss (self-insurance).

Several papers have extended the original model on self-protection and self-insurance, showing, for instance, how self-protection is not monotonically related to the degree of risk-aversion (see for instance Dionne and Eeckhout (1985) and Briys, Schlesinger and Schulemburg (1991)). Sweeney and Beard (1992) show that, given two individuals with a von Morgestern utility wealth function and with the same initial wealth , it is impossible that one of the two individuals buys systematically more self-protection than the other, even if no restriction is placed on the individual risk attitude. Concerning the relative preferability of self-protection and self-insurance, Boyer and Dionne (1983), for instance, show that if self-protection and self-insurance expenditures reduce the average loss by the same amount and have the same cost, risk adverse individual will prefer self-insurance to self-protection because the former involves a lower risk.

However if we consider complete self protection and complete self insurance, it can be shown in very simple terms that if the lottery being evaluated involves a loss in the bad state of the world and a payoff of zero (no loss) in the good state of the world, then self-protection that reduces the probability of loss to zero should be equivalent to self-insurance that reduces the loss to zero, because they both eliminate risk completely.

Consider the following simple lottery [p, -L; (1 - p), 0]. Both an investment in complete self-protection or in complete self-insurance give an expected value of 0.

However, since decision making is influenced by the frame under which the decision problem is presented, individuals may evaluate self-protection and self-insurance differently simply because they are perceived as two different ways of reducing risk, even though an expected utility maximiser should be indifferent between the two risk-reduction tools[119].

The equivalence between self-protection and self-insurance should not hold (even for the expected utility maximiser) if the good state of the world involves a gain of G. The value of the lottery [p, -L;(1 - p),+ G] is G with certainty if the individual reduces the probability to zero and (1 - p)G if the individual purchases self-insurance and reduces the loss to zero. In the latter case, individuals should value self-protection more than self-insurance.

In this paper we intend to explore the issue of the existence of a particular framing effect due to the risk reduction mechanism used. In a laboratory experiment we try to elicit the prices that individuals are willing to pay for self-insurance and for self-protection.

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119 The reduction of the potential loss to zero can be perceived differently from the reduction of the probability to zero. Even if the result in term of expected value is the same , in one case the focus is on the outcome while in the other case the focus can be on the probability side.

The issue of the existence of a frame due to the risk-management tool has already been addressed in an experimental study by Shogren (1990), which uses a lottery involving a loss, L, in the bad state of the world and a gain, G, in the good state of the world. In Shogren's experiment, the valuation of self-protection was consistently higher than that for self-insurance, which is interpreted by the author as evidence that risk reduction mechanisms matter to evaluation.[120] However, the use of a lottery involving gains does not allow the isolation of framing effects due to the risk reduction mechanism used. As mentioned above, a utility maximiser would always prefer self-protection to self insurance as this would give a sure gain of G.

In this work it is our intention to modify Shogren's work in order to detect up "pure" framing effects (if any) and to extend it to cover the case of lotteries characterized by ambiguous probabilities. Our first aim is to test whether the different frame provided by the two risk reduction mechanisms really determines a different evaluation of self-insurance and self-protection. We then test whether ambiguity affects the evaluation and the ranking of risk management tools. One might suspect that if individuals are averse to ambiguous probabilities, they might value self-protection less than self-insurance when asked to evaluate a lottery characterized by a known loss and an ambiguous probability.

We construct an experimental design that incorporates two markets: the market for self-insurance and the the market for self-protection (see Table IV.I for a summary).

The design of the experiment is meant to capture three phenomena which are of relevance to theories of decision making under uncertainty.

(1) The first issue concerns whether the presence of ambiguity alters the valuation of self-protection and self-insurance, and whether self-insurance and self-protection are ranked in the same order under risk and under ambiguous probabilities. For instance, an individual may prefer to install a burglar alarm in a house (self-protection) rather than put his valuables in a safe (self-insurance). But if there is no agreement concerning the probability of a burglary, would he still

[120] In addition, Shogren finds that if private versus collective financing mechanisms are used, private mechanisms are valued significantly more.

value the purchase of the alarm more than the safe?

The issue of the effect of ambiguity on the valuation of self-insurance and self-protection will be explored considering three different ways of representing ambiguous probabilities.[121]

Lotteries with different descriptions of ambiguity are administered to different subjects. Together with the ambiguous lotteries, subjects are asked to evaluate risky lotteries where the probability of loss is known with certainty and which are characterized by the same expected probability of loss (relative to the central probability in the ambiguous case. See below).

(2) In addition to exploring the issue of how self-protection and self-insurance are ranked in individuals' preferences, our goal is to provide subjects with both low probability and high probability lotteries.

If ambiguity matters to the evaluation of risk management tools, then by considering a wide range of probability measures, we hope to assess whether attitude to ambiguity varies with changes in the reference probability. Each lottery will be evaluated at four probability levels, 3%, 20%, 50% and 80%.

(3) Finally, the results will be examined in the light of two models of behaviour under uncertainty, namely Einhorn and Hogarth (1985)'s *Model of Anchoring and Adjustment*, Gardenfors and Sahlin (1983)'s *Model of Unreliable Probabilities*. We shall derive predictions for these theories and test whether the results of the experiment accord with either of them.[122]

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[121] As will be explained in detail in section 4.2, the three different ways of representing ambiguity are the following ones: to one group of subjects, we gave a description of a scenario in which the probability of a loss was given as a point estimate but not a precise one (the subjects were told that they had to consider themselves uncertain about such an estimate); for a second group of subjects, the expert's probability estimate was represented by an interval of probabilities (the true probability could fall anywhere in the interval); for a third group of subjects, a set of four probability measures was given, representing the estimates of four experts.

[122] We will interpret the results according to these two models, because the experiment was designed to test these two particular theories. As we will see below the bahaviour of a large group of subjects cannot be explained by any of these two models. We however feel cautious about attribute the behaviour of these subjects to other theories of ambiguity aversion.

Figure IV.1 Summary of the experimental design.

Experimental structure	Hypotheses tested
Risk reduction mechanism	
Self-insurance (SI) Self-protection (SP)	Does the risk reduction mechanism influence valuation?
Valuation of risky versus ambiguous lotteries -Best estimate" -Intervals of probability -Set of probability measures	Does ambiguity matter to the valuation of risk reduction? Does the presence of ambiguity alter the bids for SI and SP? Is there a change in prices when the definition of ambiguity is changed?
Valuation over a range of probabilities of loss	How does valuation change from low-risk to high-risk events?
[p, W-L; (1 - p), W] where p = 3%, 20%, 50%, 80%	Do bids for ambiguous lotteries follow the predictions of the model by Einhorn and Hogarth?

IV.3 Ambiguity in the probability and the valuation of self-insurance and self-protection.

Since Ellsberg (1961) decision theorists have recognized that, when asked to evaluate an uncertain prospect, a large number of individuals make choices from which it is impossible to infer subjective probabilities obeying the classical laws of probability theory. Subadditivity of the "subjective" probabilities of complementary events is often observed in choice. Thus, a descriptive theory of behaviour under uncertainty must allow for sub-additivity (as well as super-additivity) and explain what determines an individual's attitude to uncertainty.

Here we apply two theories of behaviour under probability uncertainty, namely Einhorn-Hogarth (1986), and Gardenfors-Sahlin (1983) to the problem of the valuation of self-protection versus self-insurance. In particular, for each of these models, we derive predictions concerning the attitude that individuals display towards ambiguity at varying probability levels and check whether the data obtained from the experiment support either of these predictions.

Einhorn and Hogarth's model has been found to fit pretty well the price-setting behaviour in an insurance context both by insurers/underwriters and by consumers (see Hogarth and Kunreuther (1985,1989)). Given the strong similarity between the problem we investigate and that explored by Hogarth and Kunreuther (1985,1989), we would have expected more support of our data to the theory. However, we find ambiguous support for the model of anchoring and adjustment.

The model by Gardenfors and Sahlin has been interpreted mainly as a maximin rule and applied to a case in which the set of possible probability distributions and their reliability is exogenously given. Although we realize that this procedure is an over-simplification of the original model, our aim was to check for the percentage of respondents in an insurance/protection experiment who applied a maximin rule to choose their buying price.

With respect to the pricing of self-insurance vs self-protection, as will be seen, both the theories tested predict that if the investment in self-protection is such that it reduces the probability of the loss to zero, and likewise, if expenditure in self-insurance reduces the size of the loss to zero, then, the two risk reduction mechanisms should receive the same valuation, even if the probability of the loss is ambiguous.

Hence, an individual that is averse to ambiguity at a given probability level in the context of self-insurance, should be so also in the context of self-protection. Thus, any differential impact of ambiguity on the valuation of self-insurance and self-protection should be again ascribed to a framing effect linked to the risk reduction mechanism.

IV.3.1. *Expected utility theory predictions.*

Consider an expected utility maximiser consumer with initial wealth who faces the risk of a loss L if the bad state of the world

occurs. Let p be the probability of the loss occurring. If there is no gain in the good state of the world, expected utility predicts that the individual should give the same valuation to a reduction in the loss to zero and to a reduction of the probability of the loss to zero.
Consider the lottery [W - L,p; W, (1 - p)], where W represents the individual's endowment. Then, the maximum willingness to pay to reduce the risk of a loss to zero, P, is identified by:

$$U(W - P) = pU(W - L) + (1 - p)U(W) \qquad (IV.1)$$

It is easy to check that one gets the same expression for a reduction of the size of the loss to zero. If there are no gains in the good state of the world, the individual gets a utility of U(W - P) with certainty both with self-insurance and self-protection.

Since utilities are linear in the probabilities, ambiguity in probabilities should not matter to premium setting by consumers who want to self-protect or self-insure. The "ambiguous" probability can be in fact represented through a second order distribution. As long as the mean of the second order probability distribution coincides with the probability of the correspondent risky lottery ambiguity should not matter.[123] Hence, the prices that subjects are willing to pay to acquire the right to self-protect or to self insure should be the same for both ambiguous and risky lotteries.

<u>Hypothesis 1</u> - An individual asked to bid his maximum willingness to pay to self-insure or self-protect given a lottery of the type (W - L, p; W, (1 - p)), will quote the same premium. In addition to this, he should value ambiguous and risky lotteries equivalently.

IV.3.2. *Einhorn and Hogarth's model of anchoring and adjustment*

In the anchoring and adjustment model it is assumed that

[123] Since we had to choose how to operationalize ambiguity at the end of the experiment, when one scenario was picked out randomly and played out for real in order to pay the subjects, we decided to use always a second order distribution operationalization. This allowed us to use a uniform method while maintaining as much as possible a representation compatible with the chosen theories.

individuals evaluate an ambiguous lottery by forming a subjective assessment of the true probability, S(p), according to the following functional:

$$S(p)= p + \vartheta[(1 - p) - p^{\beta}] \qquad \text{(VI.2)}$$

where p is the anchor, i.e. the starting value which is adjusted upwards or downwards according to people's perception of ambiguity and according to their attitude to ambiguity. The anchor value of the probability is established according to the individual's experience, information set, and can be assumed to coincide with the expected value of the probability distribution. ϑ $(0 \leq \vartheta \leq 1)$ is a parameter that indicates the amount of ambiguity perceived and β is the parameter that indicates the attitude to ambiguity (but not necessarily ambiguity preference or aversion). In particular, $\beta = 1$ means that the individual gives equal weight to adjustments below or above the anchor. $\beta > 1$ implies that individuals attach more weight to adjustments above the anchor and the opposite if $\beta < 1$. $\beta = 0$ implies that adjustment takes place only below the anchor.

If individuals assess probabilities according to the anchoring model, the premium for self-protection or self-insurance is then identified by:

$$U(W - AP)= S(p)U(W - L) + S(1 - p)U(W) \qquad \text{(IV.3)}$$

where AP is the ambiguity premium and S(1 - p) is given by:

$$S(1 - p) = (1 - p) + \vartheta[p - (1 - p)^{\beta}] \qquad \text{(IV.4)}$$

Assuming that the anchor used in forming the value of S(p) coincides with the probability of loss in the risky lottery, we can divide (IV.3) by (IV.1) and obtain:

$$Rc=\frac{U(W - AP)}{U(W - P)}=\frac{S(p)U(W - L) + S(1 - p)U(W)}{pU(W - L) + (1 - p)U(W)} \qquad \text{(IV.5)}$$

If W = L, utility can be conveniently normalized so that U(W - L) = U(0) = 0. In this case:

$$R_c = R_c(p,\vartheta,\beta) = \frac{U(W - AP)}{U(W - P)} = \frac{S(1 - p)}{(1 - p)} \qquad \text{(IV.6)}$$

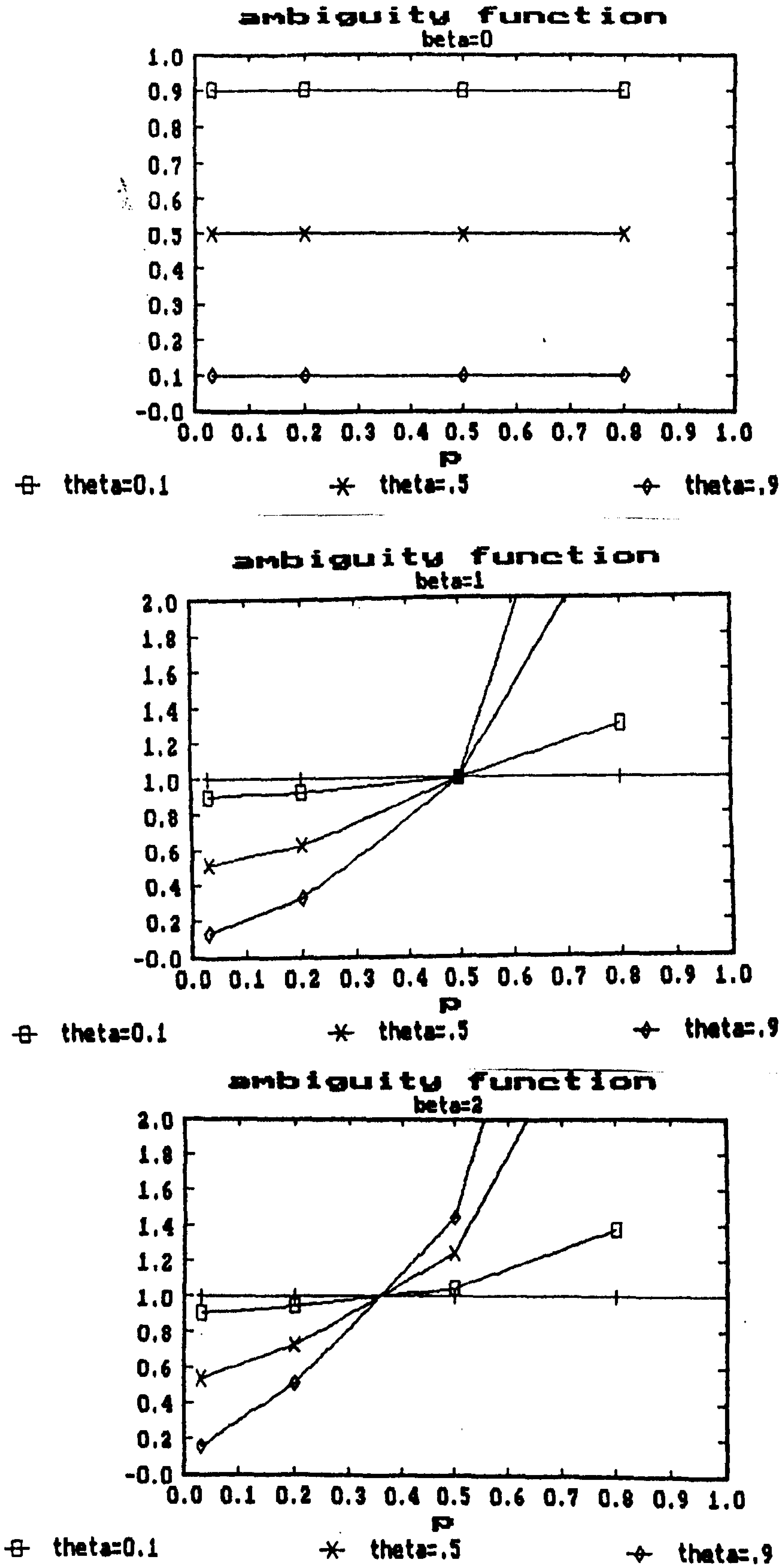
The value of R_c is a function of the individual's β, ϑ, and of the value of the anchor. If the individual is averse to ambiguity, he will be willing to pay a higher premium to self-protect or self-insure against the uncertain lottery, i.e. AP > P. If utility is monotonically related to wealth states, this implies that $R_c < 1$, and in turn that $S(1 - p) < (1 - p)$. It can be easily checked that a necessary condition for the individual to remain persistently ambiguity averse whatever the value of the anchor is that $\beta = 0$. If $\beta \leq 1$, a necessary condition for the individual to be ambiguity averse is that $(1 - p) < .5$. If $\beta \leq 1$, in fact, the individual will give more weight to adjustments below the anchor $(1 - p)$, and hence will deem the state of the world involving a loss more probable. If $\beta > 1$, then the individual can only be ambiguity averse for $(1 - p) > .5$.

If the individual exhibits ambiguity preference, then AP < P, and $R_c > 1$. This implies that $S(1 - p) > (1 - p)$. As before, if $\beta \leq 1$, it must necessarily be $(1 - p) > .5$. For $\beta > 1$ then it must be $(1 - p) < .5$. In this case, the individual will give greater value to adjustments above the anchor $(1 - p)$ and thus he will consider the loss less probable.

In general, for each value of β, as the value of p rises, R_c increases and the individual may eventually switch from ambiguity aversion to ambiguity preference according to its β. Each value of β, in fact, identifies a unique cross-over point from ambiguity aversion to ambiguity proneness.

Figures IV.2 (a,b,c) present the results of some simple simulations carried out using three possible values of ϑ (ϑ = .1,.5,.9) and three possible values of β (β =0,1,2). Figure IV.2 (a) illustrates that for $\beta = 0$, the cross over from ambiguity aversion to ambiguity preference never occurs, whatever the value of ϑ and p : the ambiguity function consistently lie below the unity line. If $\beta = 0$ we should observe the same value of R_c both at low and high probability levels. In Figure IV.2 (b) ($\beta = 1$), the cross-over point occurs when p = .5. Finally, in Figure IV.2 (c) ($\beta = 2$), the individual switches to preference for

Figure IV.2 Simulation of Einhorn and Hogarth's model



ambiguity at a value of p = .38. For each β, the greater the value of θ, the greater the distance of S(1-p) from the anchor at each observation point.

Hypothesis 2 - As the probability of a loss increases from low to high values, individuals will tend to move from ambiguity aversion ($R_c < 1$) to ambiguity preference ($R_c > 1$).

IV.3.3. *Gardenfors and Sahlin' s maximin model*

The model by Gardenfors and Sahlin (1982) argues that the main drawback of Bayesianism is the failure to realize that the information available to a decision maker who has to form subjective probabilities of an event can carry different degrees of *epistemic reliability*.

In the model, it is assumed that people - when confronted with lotteries in which a set of probability measures is substituted for a single probability measure - will evaluate them in the following way: they will compute the expected utility corresponding to each of the probability measures contained in the set and then they will evaluate the lotteries according to their minimum expected utility, i.e. they will place a higher value on the lottery which has the highest minimum expected utility.

Let f and g be two lotteries each characterized by a set of states of the world $S = (s_1, s_2, .., s_j, ..., s_m)$. Beliefs about the states of the world are represented by a set of probability measures for each state of the world s_j, $P(s_j)$, made up of n probability measures $(p_{1j}, p_{2j}, .., p_{ij}, ..., p_{nj})$. The set of all probability measures defined over all states of the world will make up the set $\mathcal{P}$. Under uncertainty, even if several probability distributions are possible, some distributions will be considered by the individual more reliable than others, i.e. some probability measures attached to each state s_j will have a higher degree of *epistemic reliability*. The set $P(s_j)$ which is actually mentally simulated by the individual is one which includes only those probability measures considered to carry a sufficient degree of epistemic reliability (See relevant section in chapter I).

Let $x_{jf} = f(s_j)$ be the outcome associated to the occurrence of state of the world s_j if lottery f is chosen, and $x_{jg} = g(s_j)$ be the outcome associated to state of the world s_j if lottery g is chosen by the decision maker. The utility of each outcome is denoted as normal by $U(\cdot)$. Lottery f will be preferred to lottery g if the minimum expected utility of f is higher than the minimum expected utility of g.

$$f \succ g \text{ if } \min \sum_j U(f(s_j))p_{ij} > \min \sum_j U(g(s_j))p_{ij} \qquad \text{(IV.7)}$$

where $i = 1,\ldots, n$

This is equivalent to saying that individuals will evaluate a lottery according to what the worst situation will be, i.e. using a maximin criterion.

In the problem of choosing the premium for self-insurance or self-protection there are only two states of the world, namely, loss occurs (s1) or does not occur (s2). Subjects are asked to evaluate lotteries of the following kind:

$$L = P(s_1)U(W-L) + P(s_2)U(W) \qquad \text{(IV.8)}$$

where $P(s_1)$ is a vector of n probability measures $(p_1,p_2,..p_i,..p_n)$, with $p_1 < p_2 < p_i < p_n$, and $P(s_2)$ is made up of complementary probability measures $P(s_2) = (1-p_1,1-p_2,..1-p_i,..1-p_n)$. The set of n probability distributions that are included in $P(s_1)$ are those considered to be epistemically reliable by the individual, i.e. those which are assigned at least a minimum level of epistemic reliability.

In our experiment, in order to simplify treatment, we decided to fix the number of epistemically reliable probability distributions exogenously (See an example of the scenario given to subjects in the Appendix). Subjects were told that the probability distributions included in the set "all carried some reliability", although it could not be said which distribution was more reliable than the others. $P(s_1)$ was fixed to be a vector of $n = 4$ probability measures (p_1,p_2,p_3,p_4), with $p_1 < p_2 < p_3 < p_4$, and $P(s_2)$ was made up of complementary probability measures $P(s_2) = (1-p_1,1-p_2,1-p_3,1-p_4)$.

The subject had to choose whether to self-protect (or self-insure) or not by identifying which alternative gave him the highest minimal expected utility. If self-protection (or self-insurance) completely eliminates risk, then to invest a positive amount will always be the best alternative. The maximum willingness to pay will be determined by equating[124]

$$p_4U(W-L) + (1-p_4)U(W) = U(W - AP) \qquad (IV.9)$$

This is equivalent to saying that, when evaluating the expected utility of each lottery in order to choose the maximum premium they are willing to pay to self-protect or self-insure, individuals should give more weight to the situation in which the probability of a loss is higher.[125]
If an individual evaluates the lottery according to the Gardenfors and Sahlin model, therefore, it is easy to see that the individual will always pay a higher price for the ambiguous lottery as compared to the risky lottery, at all probability levels.

Hypothesis 3: Individuals who apply a Gardenfors and Sahlin maximin rule will show ambiguity aversion, at each probability level. Hence, the maximum premium that they are willing to pay in case of ambiguity will always be higher than in case of risk, AP > P.

IV.4 - The experimental design

IV.4.1. *The scenarios.*

In order to test the hypotheses put forward above, we ran 10 experiments, with approximately[126] 8 subjects per session. In five

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[124] This implies that the probability distributions in the set $P(s_1)$ are all considered epistemically reliable. See Gardenfors and Sahlin, op.cit, p.377.

[125] Example: If $P(s_1) = \{.01,.02,.04,.05\}$, and L = W = 10, not investing in protection gives a minimum expected utility of [(.05)U(0) + (.05)U(W)], while purchasing protection gives a utility of W for sure. The maximum willingness to pay for self-protection will then be obtained from (.05)U(10) = U(10 - AP). If utility can be approximated by expected value, this would give a premium AP = 0.5
Compare now this result with the premium that would be quoted by an expected utility maximiser asked to evaluate a risky lottery with probability of loss equal to the mean value of $P(s_1) = \{.01,.02,.04,.05\}$, $\mu = .03$. This would give a premium P = 0.3

[126] Each session was designed to be run with the participation of eight subjects, but in practice we run sessions with 8, or 6 subjects. This

experiments we simulated a market for self-protection and in the remaining five a market for self insurance.
In each experiment, each subject was asked to evaluate 8 scenarios, four referring to a risky prospect and the remaining four to an uncertain prospect. The four risky scenarios are characterized by four different probabilities of loss, namely, 3%, 20%, 50%, 80%. The four uncertain scenarios are characterized by the same four levels of "ambiguous" probabilities. In order to do so, whichever representation of ambiguity we adopted, the means of the second order probability distributions, by which we choose to characterize the ambiguous probability, were 3%, 20%, 50%, 80%. (See section IV.4.2)

IV.4.2 *The incentive mechanism.*

To elicit the preferences of each subjects we adopted a computerised auction mechanism. The auction mechanism was adopted in order to provide subjects with an incentive mechanism capable of inducing truthful revelation of the subjects' willingness to pay. In order to elicit the maximum willingness to pay of all the bidders (winner included) we used a variant of the second price auction.[127] The auction was computerized and an ascending clock was used to help the bidders decide when they wanted to leave the auction, i.e. when the price had hit their maximum willingness to pay. For each market, subjects were asked to place a bid to purchase the right to self insure (or to self-protect); in practice they were asked to press a key when they wanted to leave the auction that is to say when the price shown by the clock reached the maximum price they were willing to pay. Before each bid, each subject was endowed with £10 and was told that he faced the risk of a loss of £10 with probability p.

In the market for self-protection, each subject was asked to quote his or her maximum willingness to pay to reduce the probability of loss to zero, given the magnitude of the loss. In the market for self-insurance, subjects were asked to quote their maximum willingness

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was mainly due to lack of subjects or to the fact that some subjects did not turn up at the very last moment.

[127] This particular auction has been studied by Harstad (1990). On the preference revealing property of the second price auction in case of ambiguity see Karni and Safra (1986)(1987) and Salo and Weber (1994).

to pay to reduce the loss to zero, given the probability of the loss.

When the subjects had expressed their willingness to pay in all the 8 scenarios, one of the 8 scenarios was chosen at random. For that scenario, the first and second highest bid were identified . The player who had made the highest bid acquired the right to self-protection (or self-insurance) and was obliged to pay the second highest bid. The rest of the participants were subject to a random draw to determine whether the negative event (a loss of £10) took place or not and they were paid accordingly. The auction worked as follows: the subjects were shown a screen with the description of the scenario and then they were asked to state the maximum amount of money that they were willing to pay either to reduce the probability of the loss to zero or tho reduce the loss to zero.

They were asked to press a key when they were ready to start. At this point on the screen a clock was displayed, with a price that was steadily increasing. They were asked to indicate their willingness to pay by pressing any key when the price reached the most that they were willing to pay; that is to say when they wanted to leave the auction.

For example the words used in the case of self-insurance were the following ones:

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay, That is, when you want to leave the auction.

Before starting the real experiment subjects were given an hypothetical risky scenario to make them familiar with the problem and with the auction procedure. The risky scenario was chosen instead of the ambiguous one in order to avoid the risk of a reduction in the perception of ambiguity due to a sort of learning effect. On the contrary the literature on the problem of overbidding and underbidding in auctions suggests to introduce the hypothetical question in order to help people to become more familiar with the auction procedure[128]. It is important to notice however that, in this experiment, the auction mechanism was used to elicit subjects' preferences in the case of risky and ambiguous choices. In this case overbidding or under bidding should

[128] Cf..Vickrey, (1961), Kagel, Harstad and Levin (1987), Kagel and Levin (1993), Kagel (1992).

not be a problem since we are interested any way in the ratio of the two premia and there is no reason to think that overbidding should be more/less frequent in a risky than in an ambiguous situation.

The following paragraph explains in detail how the concept of ambiguity is made operative.

The instructions and an example of the lotteries are given in the appendix C.

IV.4.3. *The definitions of ambiguity and their operationalization.*

In the experimental design we use three different ways of representing ambiguous probabilities: (1), a so called "best estimate" scenario [129] (2), a probability interval, and (3), a set of probability measures. The choice of these three ways was determined mainly by three factors.

a. uncertain probability as best estimate probability is the original way of representing ambiguous probabilities by Einhorn and Hogarth (1985,1986,1990),.

b. a set of probability measures is the original way of representing Gardenfors and Sahlin (1982) model.

c. using a definition of ambiguous probability as interval probability (but preserving the same mean and the same extremes of the second order distribution as in Einhorn and Hogarth (1985,1985) and Gardenfors and Sahlin (1982)) gave us the opportunity of testing the two models with a further representation of ambiguous probabilities.

In all the three cases, moreover, it was possible to operationalize ambiguity as a second order probability distribution when we had to resolve the uncertainty and pay the subjects, once an ambiguous scenario was randomly picked up at the end of the experiment and played out for real.

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[129] See Hogarth and Kunreuther (1989). However we do not use the definition "your best estimate" as in Hogarth and Kunreuther (1989) but instead "an expert's estimate". If we represent the "best estimate" as a second order probability distribution, nobody can guarantee that the subject would consider that stated value as the mean and not for example the mode of the distribution. In order to limit this problem of the Einhorn and Hogarth theory we use a symmetric second order distribution in which mode and mean coincide. See also the following footnote.

We faced, in fact, two different problems: to describe an ambiguous situation in a scenario and to make it possible to play out the scenario for real. In both cases we tried to maintain a certain uniformity. In the description of the scenario we always used the same words; and we always referred to the probability estimate as "provided by some expert hired by a Governmental agency"[130]. On the other way, we chose to operationalize ambiguity, once again in a uniform way, once a scenario had to be played out for real,

Hence ambiguity was made operational by specifying a second order distribution for the probability of loss, but in each of the three case this second order distribution had different characteristics. The first two definitions of ambiguity correspond to a different second order probability distribution: the "best estimate" ambiguity corresponds to a second order distribution centered on the "best estimate" value, while the interval of probability corresponds to a uniform second order distribution of the probability measures inside the interval.

The set of probabilities corresponds to a situation where various second order distributions are possible, but the subjects do not know ex ante which particular one will apply.

In all instances, subjects were not told how the lotteries corresponding to the ambiguous scenarios were going to be resolved until they had evaluated the eight scenarios, in order to avoid the possibility that the amount of ambiguity might be reduced during the process of evaluation.[131]

1 - Best estimate probability. Subjects were told that p was the most reliable estimate of the likelihood of a negative event but that the expert who had provided the value was not 100% certain. By doing this

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130 By doing this, we meant to avoid problems tied to source reliability which might affect the valuations of the scenarios used in the aforementioned Hogarth and Kunreuther (1989). See Appendix C.

131 The problem is that if the uncertainty has to be resolved in a lottery context then it had to be specified in some way, but this "specification" may reduce the level of ambiguity. The description of the probabilities in various scenarios was left as vague as possible in order to avoid people to be forced to perceive ambiguity as a second order distribution. See Appendix. This is also one of the reasons we decided not to give any information on the resolution of uncertainty at the beginning of the experiment.May be interesting to note that none of the subjects asked any questions on how uncertainty was going to be resolved.

we tried to induce the subjects to anchor on the value of p (the expert estimate).

As already mentioned, the best estimate ambiguity was characterized in terms of a second order distribution centered on the probability estimate shown in the scenario.

We made this definition of ambiguity operative by asking one of the subjects to draw a ticket out of a bag that contained five tickets, three with the probability estimate given in the scenario and two with values corresponding the the extremes of the intervals. The ticket that was drawn determined the combination of black and white balls that was put in a bag and from which subjects were asked to draw a ball. The anchors provided are 3%, 20%, 50% and 80%.

2 - <u>Probability Interval.</u> Subjects were given a range (p_L, p_H) within which the true probability lay. The intervals were fixed wide enough to allow us to discriminate between results that accord with the various theories tested. The probability intervals provided were:

.01———.05

.05———.35

.35———.65

.65———.95

The average values of these intervals coincide with the probability measure provided in the best estimate scenarios.

To resolve lotteries characterized by this type of ambiguity, we asked subjects to first draw a ticket from a bag containing as many tickets as there were numbers inside the interval. The ticket drawn corresponded to the combination of black and white balls put in the bag from which subjects had to pick a ball.

3 - <u>Set of probability measures.</u> The ambiguity in the probability of the loss is described by the fact that for each scenario four different probability measures are possible: p_1, p_2, p_3, p_4. However, the reliability of each of these estimates is not known, i.e. confidence weights, w_1, w_2, w_3, w_4, attached to each probability are unknown. This is equivalent to saying that the second order distribution of probability $f(p)$ is unknown.

The set of probability measures chosen are the following:

(.01,.02,.04,.05) $E(p) = .03$

(.05,.15,.25,.35) E(p) = .20

(.55,.45,.55,.65) E(p) = .50

(.65,.75,.85,.95) E(p) = .80

Again, the mean value for each set corresponds to the "best estimate" provided in the first type of ambiguous scenario described.

To make ambiguity operational we told subjects that loss could occur with four different probabilities p_1, p_2, p_3, p_4 but that the experimenter did not know which probability measure was the most reliable. We then ask one subject to draw a six-face die from a bag containing 10 biased dice[132]. The die is played and the number drawn corresponds to a bag with a combination of black and white balls corresponding to one of the p_i's. (Therefore there were four bags). One of the subjects was then asked to draw a ball from the selected bag to resolve the lottery once the corresponding scenario was randomly chosen.

IV.5 - Experimental evidence

IV.5.1. *Descriptive analysis.*

The data presented in this section was obtained from running the experiment with 84 undergraduate students at York University in the month of May 1994. Subjects' maximum willingness to pay to purchase the risk-reduction tool (self-insurance or self-protection) was elicited by means of the auction procedure described above.

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[132] Numbers on the dice go from 1 to 4 and stand for the number of probability measures in each set. The number of times a number corresponding to a probability measure figures in a dice gives the weight attached to that probability measure. Given the following 11 die:

A	1,1,1,2,3,4
B	2,2,2,1,3,4
C	3,3,3,1,2,4
D	4,4,4,1,2,3
E	1,1,2,2,3,4
F	1,1,3,3,2,4
G	1,1,4,4,2,3
H	2,2,3,3,1,4
I	2,2,4,4,1,3
J	3,3,4,4,1,2

Dice A gives a weight of 1/2 to p_1, and 1/3 to each of p_2, p_3, and p_4.Each set of weights is equally probable.

Each subject was asked to evaluate 8 scenarios relating to the same risk-management tool (self-insurance or self-protection) and to only one type of ambiguity (either "best estimate" or "interval" or "set of probability measures"). However, each subject provided his or her valuation at each probability level. The scenarios were arranged in random order using the table of random numbers. Subjects participating in different sessions faced therefore the scenarios in different sequences. Also, lotteries appeared one at a time on the screen.

On the whole, four factors were manipulated in the experiment: two of them were *between* subjects factors (risk-reduction mechanism, type of ambiguity), and the other two were *within* subject factors (risky vs ambiguous lottery, probability levels).

Some of our subjects can be considered as task-sophisticated either because they were economics students or because they had previously taken part to other experiments on decision making organized by the EXEC.

Tables IV.1, IV.2, IV.3, and IV.4 below present some qualitative results obtained from our sample of respondents.

Table IV.1 presents the ratio of the mean bid for self-protection to the mean bid for self-insurance. Ratios are calculated at each probability level, for risky lotteries, and for each definition of ambiguity.

Table IV.1 Ratio of mean self-protection to mean self-insurance prices, SP/SI.

	.03	.20	.50	.80
Risky lotteries	.92	.98	1.19*	.98
Amb. lotteries "Best estimate"	2.91*	1.06	1.39*	1.28
Amb. lotteries Interval of probability	.66	.95	1.11	1.55*
Amb. lotteries Set of prob.	1.09	1.14	1.02	1.09

* significant t at the 5% significance level

As the table shows, there is no unambiguous evidence of a "framing effect" due to the risk-reduction mechanism used: the ratios of mean prices for self-protection to mean prices for self-insurance are in the majority of cases not significantly different from one. Self-protection and the self-insurance lotteries were, on average, perceived as indifferent.

In the valuation of the risky lotteries, only at a probability of 50% is the mean valuation of self-insurance significantly lower than the mean valuation for self-protection at the 5% significance level (t = -2, p = 0)[133].

When ambiguous lotteries are compared, there are two definitions of ambiguity in which we find some evidence of a framing effect, namely the "best estimate" and the "interval of probability" type of ambiguity. For the "best estimate" case, the ratio of prices SP/SI is always strictly greater than one and the mean valuations are significantly different at the probability levels of 3% and 50%. However, the sample that evaluated the "best estimate" scenario was our

[133] The t test is performed assuming independence.

smallest (8 subjects for each market) so that a more robust sample should be obtained before accepting the hypothesis that the prices offered for the two risk management tools are significantly different.
For all the other observations, when two "indifferent" lotteries are used, there is no evidence of Shogren (1991)'s hypothesis that the risk reduction mechanism matters.

The risk-reduction mechanism was also manipulated as a within subject factor for some of the participants. Nine participants to one of the self-insurance experiments in the first week were asked to play the self-protection experiment after about a week. Similarly, 5 participants to a self-protection experiment in the first week returned after about a week to play a self-insurance experiment. This procedure provided us with a control group of 14 subjects for whom we could observe matched pairs of prices for the two risk-reduction tools. Table IV.2 gives the mean ratio of the price of self-protection to the price of self-insurance for risky lotteries and at each of the probability levels.

Table IV.2 Mean Ratios of matched prices for self-protection and self-insurance (control group)

	.03	.20	.50	.8
Risky lotteries	1.62 *	1.07	1.32	1.09

As in table IV.1, also for the control group there is no conclusive evidence in favor of a framing effect due to the risk management tool, except at the probability level of 3%. However, the mean ratios are always greater than one. At the individual level, of our 14 subjects in the control group, 4 valued self-protection consistently more than self-insurance, and thus were clearly prey to the "frame" suggested by Shogren.[134]
There remains, of course, also to assess the robustness of this result when specific contexts (inside the insurance frame) are used. In an experiment carried out on 50 undergraduate students in Economics and

[134] Two of them played the self-protection experiment as their second experiment, the other two played self-protection before self-insurance.

Accounting using hypothetical questions, we used the example of house burglary to obtain evaluations of self-insurance and self-protection. Subjects in the self-protection experiment were asked to state the maximum price they were willing to pay to purchase an alarm that would make it impossible for burglars to break in. In the self-insurance experiment, subjects were instead asked to quote a price to purchase a safe that would have made it impossible, in the event of a burglary, to steal the valuables contained in the house. Subjects were also told to assume that in the event of a burglary, the house would not have suffered any damage. In this "house burglary" experiment, the mean price of self-protection was consistently and significantly higher than the mean price for self-insurance, at each probability level.[135]

To analyse more directly the difference between ambiguous prices and risky ones in the two markets we use mainly two indices; the ratio of the two prices as a relative measure and the difference between the two prices as an absolute measure.

Table IV.3 presents the mean ratios of ambiguous to non ambiguous prices for each of the two markets and for each type of ambiguity. This format of the table allows us to test the three hypotheses discussed above with regard to behaviour under uncertainty. The first result that emerges from the data is that ambiguity matters: the mean ratios are always different from one, contrary to the prediction of Expected Utility Theory.

The mean ratios, however, do not provide support for the model of anchoring and adjustment of Einhorn and Hogarth. Nowhere do we find a monotonically decreasing ratio of ambiguous to non ambiguous prices as predicted by that model, whatever the specification of the second order probability distribution.

Hence there doesn't seem to be on average any evidence of anchoring and adjustment, not even in the type of scenarios that should encourage anchoring to the probability measure provided—namely, the "best

135 These results even if not comparable with the ones of the York Experiment (we use different probabilities levels and different incentive mechanism) may suggest that in a non contest free experiment other factors like psychological costs may be at work. These factors may determine preference for self insurance or self protection even when the monetary reward connected with the two reduction mechanism tools are strictly the same. For possible interpretation see Di Mauro Maffioletti (1994).

estimate" definition of ambiguity.

The mean ratios are always greater than one at low probability of loss (3%) indicating ambiguity aversion, but only in two instances do we find ambiguity preference at high probability levels (self-insurance, best estimate scenario, self-insurance set of probability). Rather, the mean ratios (except in the self-insurance, best estimate experiment, self-insurance set of probabilities) are strictly greater than one. On the other hand, mean ratios always greater than one suggest aversion to ambiguity irrespective of probabilities levels which is consistent with the Gardenfors and Sahlin (1983)

Table IV.3 - Means of ratios of ambiguous to non-ambiguous prices

	Probability of loss			
	.03	.20	.50	.80
Self-insurance				
Best Estimate	2.24	.90	.90	.90
Interval of prob.	3.74	1.10	1.10	1.15
Set of prob.	2.5	1.12	1.32	0.87
Self-protection				
Best Estimate	1.62	1.47	1.20	2.25
Interval of prob.	1.18	1.17	1.01	1.35
Set of prob.	1.11	1.93	1.01	1.25

Table IV.4 gives instead the mean of the differences between ambiguity prices and risky prices which we think can give us an indication of the absolute value of the "ambiguity premium"[136].

[136] The values are in pence. Since these are absolute values they depend more directly on the level of probability considered. A positive value indicates ambiguity aversion, a negative one ambiguity proneness.

Table IV.4 - Means of the differences between ambiguous and non ambiguous prices

	Probability of loss			
	.03	.20	.50	.80
Self-insurance				
Best Estimate	-22	-46	-73	- 60
Interval of prob.	15	25	34	- 100
Set of prob.	17	23	73	- 92
Self-protection				
Best Estimate	26	6	-8	152
Interval of prob.	5	43	-3	95
Set of prob.	-11	101	-6	73

Also in this case, it is not possible to notice on average a switching from ambiguity aversion to ambiguity preference from low to high probability levels. As it is possible to notice the ratio and the differences between the two prices not always reveal the same attitude towards ambiguity. (See the following footnote).

To explore further which model best fitted the data, we proceeded to analyze the pattern of the ratios of ambiguous to risky prices at varying probability levels for each of the subjects in the 10 experimental sessions we run. This approach was also justified by the fact that the sample size for some of our experiments was quite small and therefore, the presence of outliers could distort greatly the value of the means.[137]

[137] In fact also the difference is sensitive to the presence of outliers. The situations where ratios and differences give different results toward ambiguity are the ones in which there is more variability between the subjects evaluations. The ratio seem more sensitive to

Table IV.5 gives the proportion of subjects who behaved according to the predictions of expected utility (EU), the anchoring model (AA), and the maximin model, (M), in all the experiments. The last column, (O), gives the proportion of subjects whose behaviour does not accord with any of the three hypotheses tested. The number in small character next to the type of the scenario indicates the sample size. In the case of the best estimate we used the individual ratio between the two prices. The presence of a switching from ambiguity aversion to ambiguity proneness from low to high probability was assumed to be the sign of the use of the anchoring and adjustment model[138]. To verify the use of a maximin we just computed the maximum price considering the worst situation and we compared such a price with the one stated by the subject.

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ambiguity aversion when the ratio is very near to one. We calculate also the ratio of the means and the ratio of the median prices and they generally reveal the same trend; that is to say there is not a general switching from ambiguity aversion to ambiguity proneness from low probability levels to high probability levels.

138 This is what is used in Einhorn and Hogarth (1986) (1990) and in Hogarth and Kunreuther (1989)

Table IV.5 - Proportion of subjects whose behaviour is in accordance with one of the theories tested

	Theories tested			
	EU	AA	M	O
Self-insurance				
Best Estimate (8)	-	.50	-	.50
Interval of prob.(15)	.13	.47	.07	.33
Set of prob. (20)*	.2	.45	.05	.4
Self-protection				
Best Estimate (8)	-	.125	-	.875
Interval of prob.(12)	.25	.25	-	.50
Set of prob. (21)*	.14	.24	-	.62

* includes control group

The behavioural model that receives more support from the analysis of individual data is the anchoring and adjustment one. This model of behavior fits particularly well the valuations for self-insurance, while it receives less support from the self-protection experiments, regardless of the definition of ambiguity. All together, about 36% of our sample behaved according to the prediction of the model. Some of the subjects behaved according to a "weak" version of the anchoring rule. Although the ratio between ambiguity prices and risky prices did not decrease monotonically, still subjects always displayed ambiguity aversion at low probability levels and ambiguity preference at high probability levels. 16 subjects, however, displayed exactly the decreasing monotonic relation between ratios of prices and probability of loss, predicted by the Einhorn and Hogarth model. The number of subjects who behaved according to expected utility was 12: the majority of them were students of economics.

Very few subjects behaved according to the maximin rule at least in its extreme case. It is however not possible to verify if the subjects in evaluating the scenarios considered the probability of the worst situation and then adjusted it up words or considered the best case and the adjusted it down words. In this case their behaviour would resemble the one of a subject who follows an anchoring model.[139] Thus the analysis of the individual data seems not to give any strong support to either of the models, in spite the fact that ambiguity seems to matter for 80 % of the subjects.

As in previous studies, we did not find any correspondence between attitude to risk and attitude to ambiguity: ambiguity aversion and preference were completely independent of risk aversion and preference.[140]

In the following table we report risk and ambiguity attitudes of the various subjects. Along the diagonal we can read the subjects who maintain a constant attitude, that to say they were ambiguity adverse and and risk adverse. The table reported below is referred to the self insurance group and for the 4 probability levels. For the self protection group see appendix C.

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139 A behaviour like this one it is consistent with the subjects which can be called optimists between the pessimists or pessimists between the optimists. This kind of pessimistic optimistic behaviour which mitigates to some extent a pure maximin behaviour has been observed in other contexts. See for example the experiment on updating ambiguous beliefs, Cohen, Gilboa, Jaffrey and Schmeidler (1994).

140 The only regularity we found (but only for 25% of our sample) was a correspondence between risk pronenss and ambiguity proneness at the high probability level of 80%.The same results can be found using the differences instead of the ratio.

Table IV.6 Risk and ambiguity attitudes for the self-insurance group

	Probability 0.03			Probability 0.2		
	AA	AN	AP	AA	AN	AP
RA	10	1	9	12	2	4
RN	2	2	2	3	6	5
RP	9	5	2	4	1	5

	Probability 0.5			Probability 0.8		
	AA	AN	AP	AA	AN	AP
RA	4	1	7	0	3	5
RN	3	4	3	0	4	3
RP	13	3	4	7	4	16

AA Ambiguity averse
AN Ambiguity neutral
AP Ambiguity prone

RA Risk averse
RN Risk neutral
RP Risk prone

Further insight into the data is provided by table IV.7, where we show the median bids and the mean bids for risky and ambiguous lotteries at all probability levels and for both risk-management tools. We provide median bids and mean bids because there is quite a strong skeweness in some of the distributions.

In all experimental conditions, there is ambiguity aversion at low probability of loss. In the self-protection experiments, we also found ambiguity aversion at the high probability level of 80%. For probability levels between 3% and 80% results are mixed.

Table IV.7. Mean and Median bids for all experimental conditions

		Probability of loss							
		.03		.20		.50		.80	
Self-insurance		**M**	**Md**	**M**	**Md**	**M**	**Md**	**M**	**Md**
Best Estimate	R	62	29	262	197	449	495	643	718
	A	40	31	216	181	376	452	583	700
Int. of prob.	R	91	94	266	220	454	500	683	710
	A	106	101	292	264	488	561	538	511
Set of prob.	R	45	30	236	201	407	500	669	707
	A	61	31	272	205	482	500	630	501
Self-protection									
Best Estimate	R	31	30	330	193	496	525	514	650
	A	116	46	228	227	528	500	745	812
Int. of prob.	R	65	48	235	200	546	511	737	800
	A	70	49	279	214	543	521	833	824
Set of prob.	R	78	52	211	216	502	538	619	639
	A	67	65	312	300	495	516	692	775

It has been argued that the fact that ambiguity matters simply depends on lack of familiarity with the problem the subject is asked to evaluate. If the subject is exposed to the comparison between risky and ambiguous version of the stimulus more than once, then ambiguity aversion/preference is bound to disappear. This effect is likely to be stronger if, at the end of each round of the evaluation, the ambiguity is resolved. This point is stressed by Camerer and Kunreuther (1989)[141]:

If we used the same $f(r)$ throughout the experiment, their ambiguity

[141]However a different result is obtain by Sarin and Weber (1991)

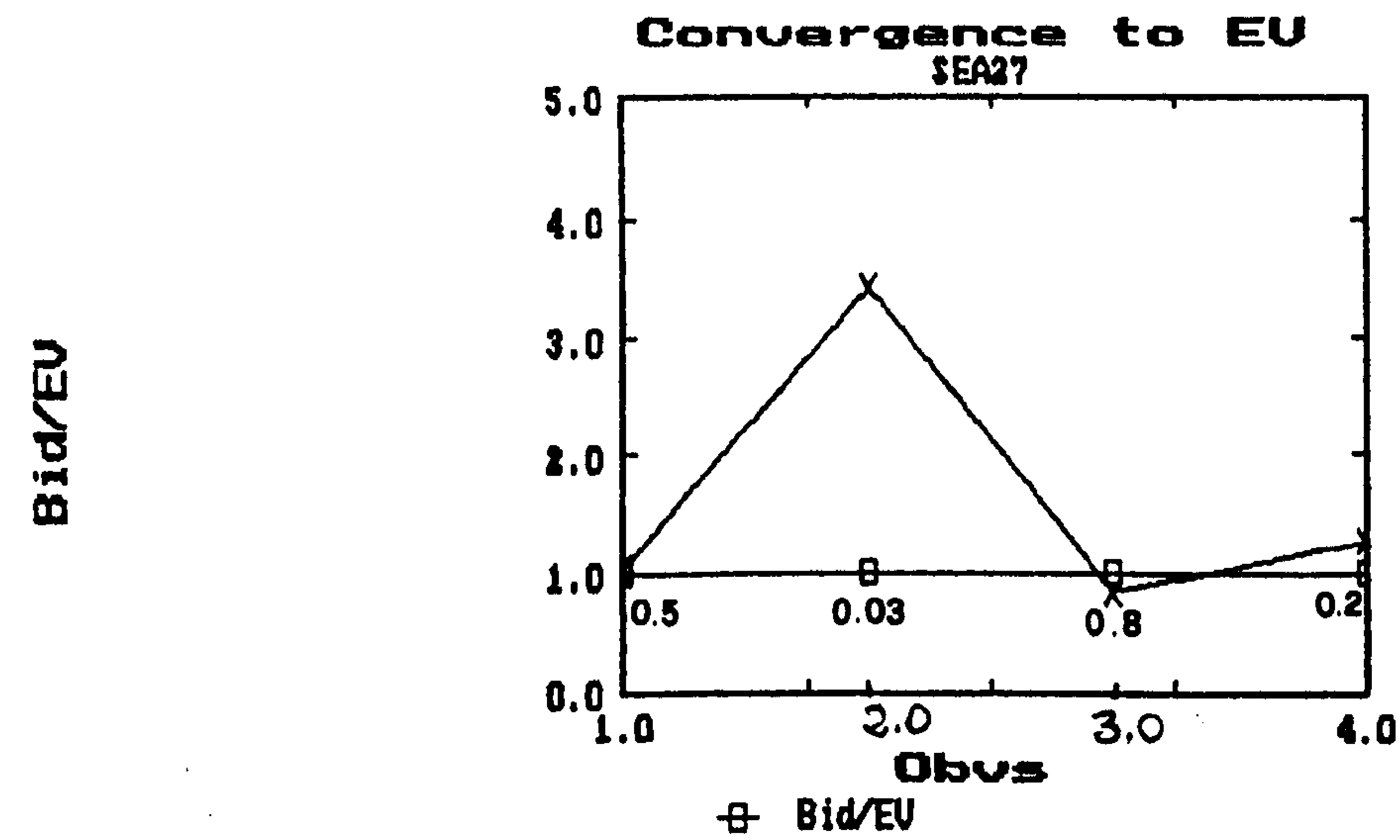
would be reduced as the experiment continued. The effect of this reduction in ambiguity on prices could be confounded with convergence to equilibrium and other kinds of learning in the experiment.(p.285)

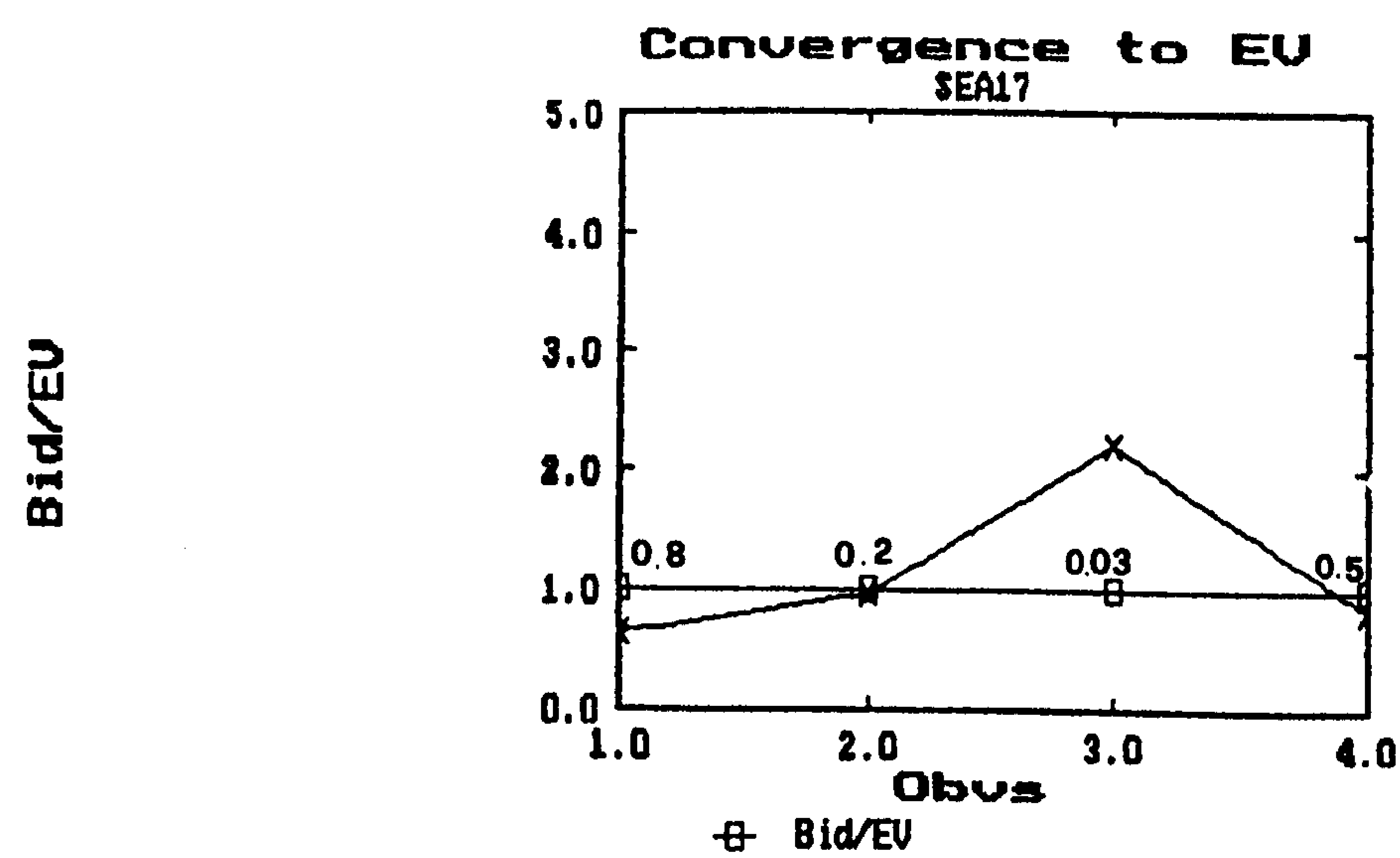
To check whether in the course of the auction, prices for ambiguous lotteries converged to expected value, we made use of diagrammatic analysis. We constructed a diagram for each experimental session we ran (In figures IV.3, we report the diagrams for the control sample. The others diagrams are included in the appendix C).
The horizontal line in the diagram stands for the the case in which the mean bid at each probability value coincides with the expected value of the ambiguous lotteries. The point observations on the broken line are the mean prices for the ambiguous versions of the stimulus. The four observations are numbered from 1 to 4 according to their sequence during the experiment. Since the order in which the lotteries appeared in each experiment had been randomized, each lottery may be assigned a different number in each diagram.

We did not find any evidence of a converging path of mean bids to the expected value. The ratio of the mean bid to the expected value depended not so much on the order in which the lotteries were evaluated, but rather on the probability of loss in each scenario. For instance, when subjects were asked to evaluate an ambiguous lottery with an expected value of the probability of the loss equal to 50%, the ratio BID/EV was always close to one, no matter whether the lottery was the first to be evaluated or the last. Likewise, when the expected probability of loss was 3%, BID/EV > 1, regardless of the position of the lottery in the session (This reduction of ambiguity perception at 0.50 probability level is consistent with the result of Cohen, Said and Jaffray (1985).

This pattern is clearly shown by the two diagrams reported in figure IV.3 which refer to the two control groups which participated to both the experiment on self-protection and on self-insurance. If there was a convergence to EV, this should have been stronger for the participants of the control sessions, given that they had seen the ambiguity resolved already once, and therefore had some information concerning the second order distribution of probabilities. Nevertheless, the oscillations around the EV do not appear to dampen as we move from the first ambiguous lottery to the fourth.

Figure IV.3 Path of mean bids to the expected value for the two control sessions





IV. 5.2. *Analysis of Variance.*

We performed two types of analysis of variance on the data obtained from the experiment. We first conducted a two-way analysis of variance where the dependent variable was the individual ratios of ambiguous to risky bids and using as factors the definition of ambiguity and the risk management tool evaluated. This analysis was undertaken at each probability level. We did not find any evidence of differences in the ratios of ambiguous to risky prices across definitions of ambiguity or across risk reduction mechanism at all probabilities of loss. Hence, the definition of ambiguity does not appear to determine a different behaviour in uncertain situations. In one instance, namely when the probability of loss was 3%, we found a significant interaction effect between the type of ambiguity and the risk reduction tool.

We then carried out a three factor analysis of variance on the price ratios adding the within subject factor of probability to the two between subject factors (the type of ambiguity and the risk reduction tool). We did not find any significant main effect or interaction effect. In particular, the fact that probability is not a significant explanation for price ratios is against the predictions of the model of anchoring and adjustment.

Secondly, we carried out an appropriate analysis of variance on the individual bids using as factors the risk reduction mechanism, the probability level and whether the price referred to a risky or to an ambiguous lottery. As can easily be expected, we find a strong main effect for probability ($F = 12.74$) but no significant effect for the risk reduction tool, and for ambiguity (overall the probability effect seem to overcome the ambiguity one).

However, when ambiguity is used as the only explanatory factor and prices are pooled across the other experimental conditions, we do find a strong effect for ambiguity ($F = 97$) Table IV.8. This last result confirms the analysis of the individual data which show a pervasive ambiguity reaction. Comparing our results to others experimental work we can say that we found a stronger ambiguity reaction than in Camerer and Kunreuther (1979) and this was probably due to the one shot auction we adopted. On the other hand, Eisenberger and Weber (1993) found a stronger ambiguity aversion than in our experiment but they operationalized ambiguity in an event context not in a chance one.

Table IV.8 Analysis of variance

THREE-WAY ANOVA ON RATIOS OF AMBIGUOUS TO RISKY PRICES

SOURCE	SUM OF SQUARES	D.F.	F	PROB(F≤ x)
Type of Amb. (A)	987.19	2	.72	.51
Market (M)	70.88	1	.10	.24
Probability (P)	1301.96	3	.63	.65
Int(A and P)	4277.25	6	1.126	.81
Int(M and P)	3123.12	3	1.6	
Residual	175308	246		

ONE-WAY ANOVA ON INDIVIDUAL BIDS (POOLED SESSIONS)

SOURCE	SUM OF SQUARES	D.F.	F	PROBABILITY
Ambiguity	75446716	1	978	1.00
Residual	41442820.05	559		

IV.6 Conclusions and Policy Implications

This paper has considered experimental markets for two alternative risk-reduction mechanisms in order to obtain individual valuations of risk reductions. Our results show that there is no significant difference in the mean valuation of the self-protection lotteries and of the self-insurance lotteries. Thus, we do not find any unambiguous evidence of a "framing effect" due to the risk reduction tool, as assumed in Shogren (1990).

In our experimental scenarios, the two risk management tools provided the same payoff and thus, the frame was created simply by a difference in wording. It might be that the "framing effect" would be significant if a different context were used, as suggested by the small "house burglary"experiment we have carried out. Alternatively, even with a neutral lottery context, self-protection might be valued more than insurance provided the two scenarios were not as clearly equivalent as in this experiment.

In view of the importance that protection and insurance against environmental risks, product failure and work place safety has assumed in policy making, we think that these issues need to be explored further.

In addition, our subjects have evaluated both risky and ambiguous versions of the scenarios. Three different definitions of ambiguity, characterized in terms of second order probability distributions were provided. Although we found that ratios of ambiguous to risky prices were always different from one, we did not find any significant difference due to the particular definition of ambiguity used.

Hence, the type of ambiguity does not affect the price that individuals are willing to pay to self-protect or to self-insure, although in the majority of cases mean ratios were greater than one, showing that individuals are willing to pay more to protect/insure against ambiguous events.

Finally, given the similarities between the self-insurance and self-protection contexts and market insurance, we have tested whether the model of anchoring and adjustment by Einhorn and Hogarth - which has performed well in the insurance frame - was a good predictor of mean behaviour in our sample. We do not find any conclusive evidence to this end: while the mean ratios do not show the monotonically decreasing pattern predicted by that model, on the other hand the analysis of individual patterns of response shows that some 35% of the sample behaved according to the predictions of the theory. However,the number of subjects which seem to follow the anchoring model diminishes if we use the difference between the prices instead of the ratio. Whether the difference between our results and those obtained by Hogarth and Kunreuther (1989) are due to the fact that there is no unique model of behaviour under uncertainty in the protection/insurance context or to the different incentive mechanism used in this study we are unable to say at the present stage.

CHAPTER V

AMBIGUITY AS SECOND ORDER DISTRIBUTION: SOME COMMENTS

V.1 Introduction

As we have already seen in Chapter I, the models which try to explain attitudes to ambiguity (amongst others see Schmeidler (1989), Gardenfors and Sahlin (1982,83), Ellsberg (1961), Segal (1987)) are often constructed on two main elements :

a) The authors describe *how people perceive* the ambiguous urn (and in doing so they define what for them is ambiguity).

b) They *apply a particular preference functional* to the ambiguous urn.

In doing so, these models implicitly or explicitly *consider a two stage mental process* in the evaluation of the ambiguous urn:

a) *an editing phase* (the subjects try to imagine how it is possible to represent the ambiguous urn);

b) *an evaluation phase* (the subjects apply a particular preference functional to the edited or imagined ambiguous urn.)

The two processes together are used to describe a pattern of preferences which is consistent with the one shown by the Ellsberg Paradox.

One consequence of this is that, since each model adopts a particular description of how subjects imagine the unknown urn, in practice each model proposes a particular definition of ambiguity or identifies a particular source of ambiguity.

One of the most common descriptions of ambiguity (amongst others see Segal (1987), Gardenfors and Sahlin (1982,83) or even Ellsberg (1961)) is the one which considers a situation ambiguous if either the subject is presented or has in mind a set of probability measures or he or she thinks in term of a second order distribution of probabilities. That is to say, the subjects, when analyzing the ambiguous urn, consider themselves as confronted with a distribution over the possible probability values that the two outcomes (black and white) can assume (we will use always the two colour example).

Different models consider different kinds of second order

distributions. More generally, the characteristic of the second order distribution may depend on the level of information, on psychological factors or can be defined objectively by the model.
In Gardenfors and Sahlin (1983), for example, the second order distribution is determined both by the available information and by the psychological attitude of the subjects (Cf. Chapter I). Segal (1987), instead requires this imaginary distribution to be symmetric around the mean (which for him is reasonable in the case of the two-colour and three-colour Ellsberg examples).

The question of how to represent ambiguity is not only a theoretical question linked to what we called above an editing phase, it is also a practical question. Experimentalists when testing models or investigating theories on ambiguity do have to consider how to represent it. This is true especially when they replicate the Ellsberg paradox in a chance context. In fact when they face the problem of playing for real one of the scenarios they have to choose a way to represent ambiguity. Very often ambiguity is represented as a uniform second order distribution (see amongst others Schoemaker (1991) Camerer and Kunreuther (1989), Hogarth and Kunreuther (1989)). In our opinion, this choice, even if reasonable, reduces to some extent the possible psychological impact of ambiguity. In fact we give the same weight to each of the possible outcomes, and this is like assuming that there is no pessimistic or optimistic individual since we give the same weight to the worst and the best outcomes as well as to all outcomes in between. According to Schoemaker (1991), for example, a uniform second order distribution is the weakest way of representing ambiguity.

In this chapter our idea is to explore some elements that, either in the editing phase or in the evaluation process, can cause a different evaluation of different second order distributions. The reason behind this is to try to explain some results of the experiment presented in chapter III, which cannot be explained with the models tested in the same chapter.

In particular we will suggest two possible alternative explanations. Both explanations are consistent with the idea of representing ambiguity as a second order distribution; however they differ in the decision criterion used to evaluate the different distributions.

With the first explanation we want to explore whether it is possible to establish a preference ordering over second order distributions. Moreover we want to explore in which way this preference ordering can be linked to the concept of a more or less ambiguous distribution. In particular the idea is to identify a concept of ambiguity which allows for different evaluations of different second order distributions which can explain the subjects' evaluation in the quoted experiment.

In the second explanation we will propose an interpretation or extension of the Gardenfors and Sahlin model (1982) which can allow for the use of maximin and for the preference ordering shown by the subjects in the experiment.

Since both explanations are consistent with the results of the experiment described in chapter III (relative to the behaviour of some of the subjects), we will finally suggest an experimental test to differentiate between the two models.

The discussion will be organized as follows. First, we want to review the lotteries presented in the experiment, the evaluations given by the subjects and why the theories tested in the experiment cannot entirely explain the preference order given by some of these subjects. Second, we will introduce the concept of different degrees of ambiguity as developed by Segal (1987) and we will put it in relation with the concept of risk aversion. Then we will discuss why it cannot be used to explain the results of the experiment.

Then, we will propose a different definition of degrees of ambiguity in the second order distributions which may be used to explain the results of the experiment.

Third, we will suggest an interpretation of the Gardenfors and Sahlin model which can allow for the same results and seems consistent with the reasoning adopted in the interviews by the subjects. Finally we will discuss possible developments or tests of the two explanations.

V. 2. Ambiguity as a second order distribution

V.2.1 *The lotteries*

In the experiment described in the Chapter III, we presented the subjects with the following lotteries (we will concentrate on three groups of lotteries not on all of them, the reasoning can be applied also to the other lotteries).

Figure V.1 corresponds to the first group, Figure V.2 and V.3 to the second and the third group.

Figure V.1 Decision trees corresponding to lotteries D and B

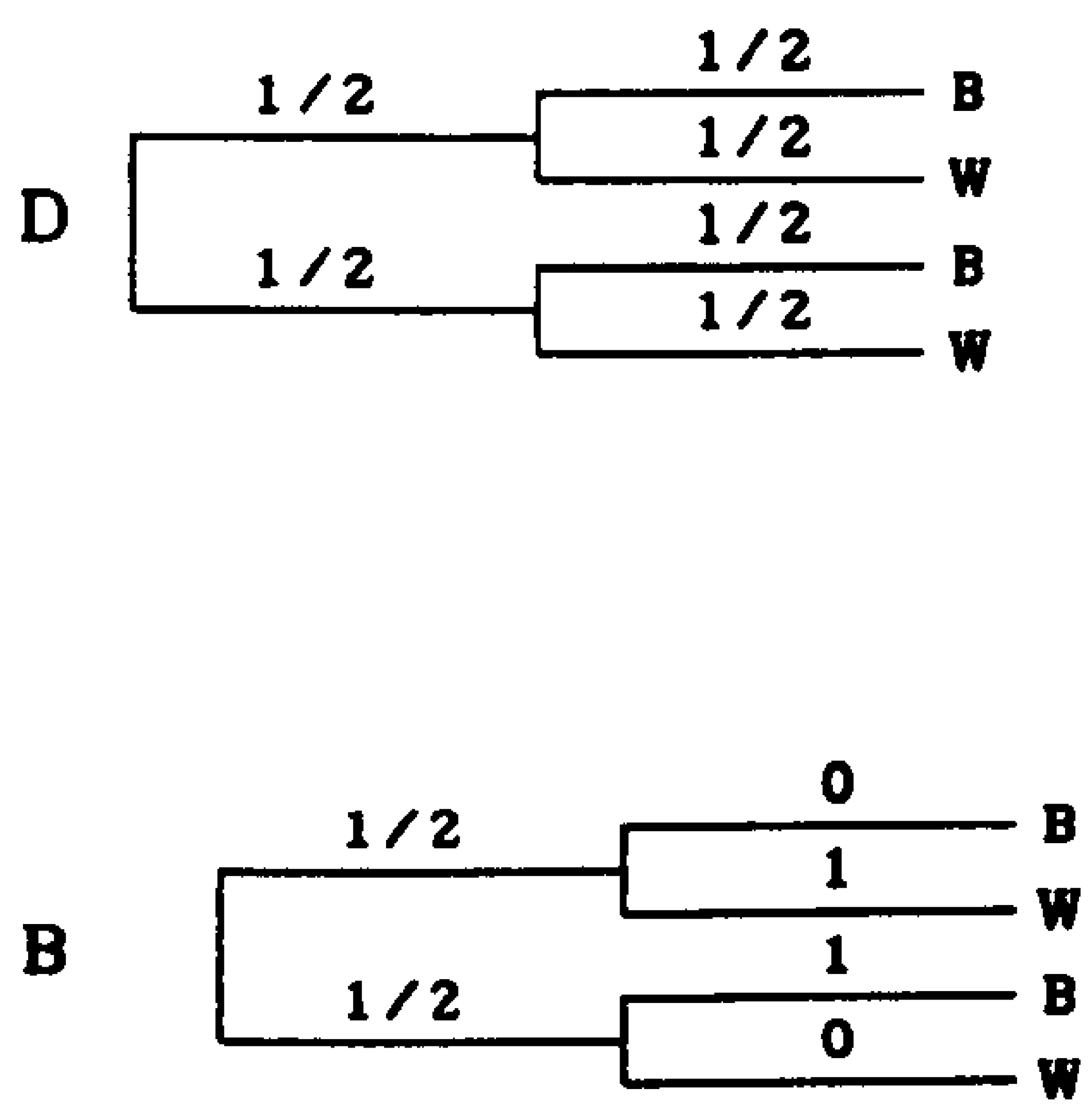


Figure V.2 Decision trees corresponding to lotteries B,F,A,G.

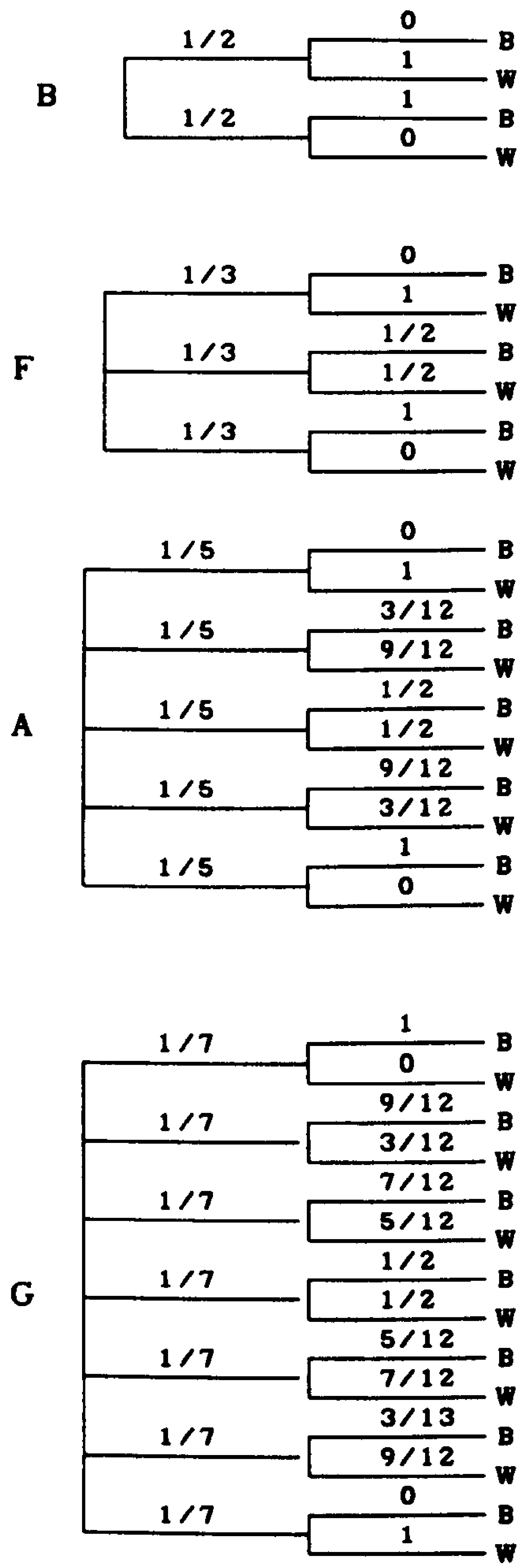
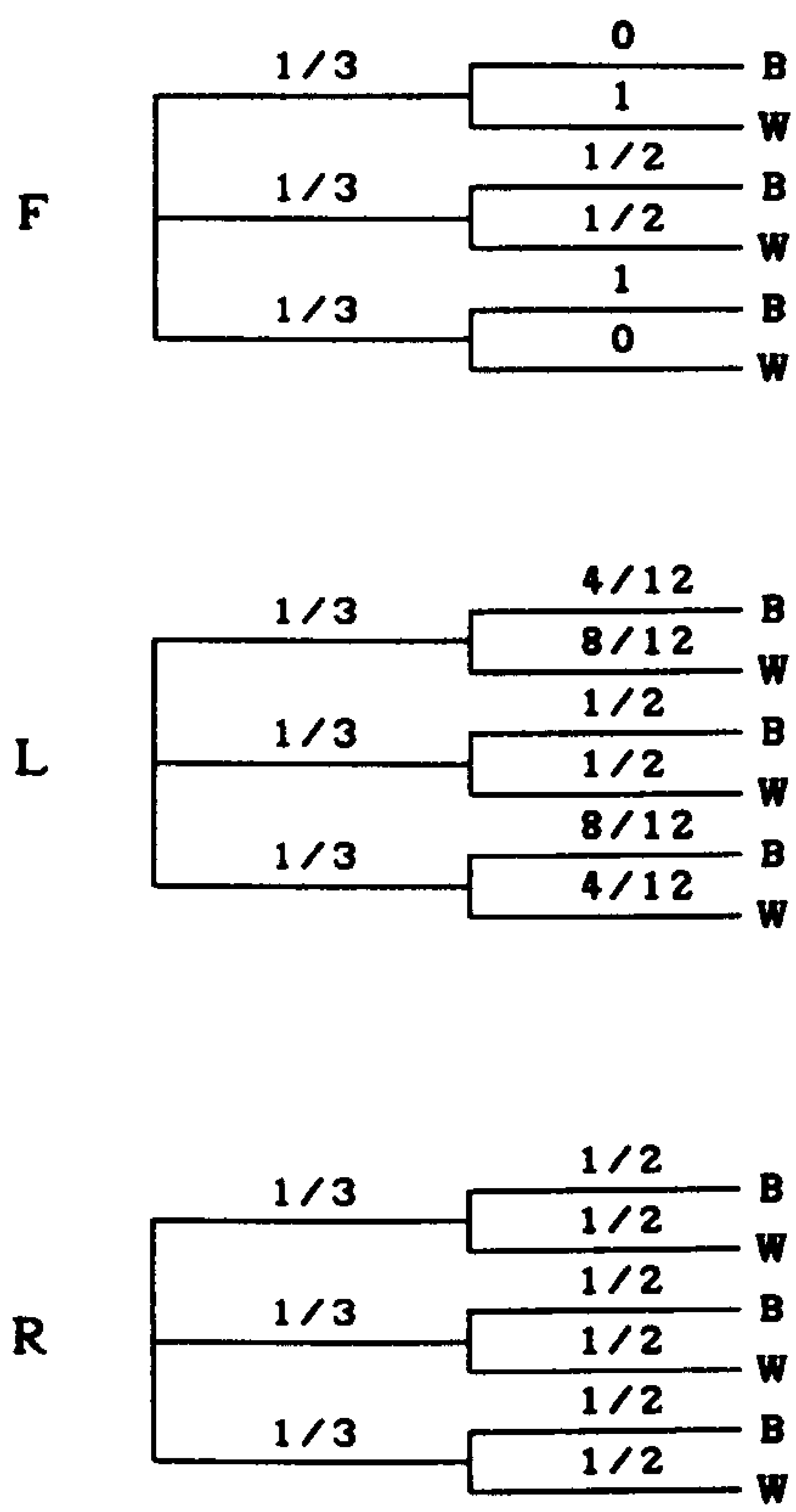


Figure V.3 Decision trees corresponding to lotteries F,R,and L.



All the lotteries , D, B, F, A, G, L, R are two stage lotteries. These lotteries, if evaluated using the reduction principle, collapse to the simple lottery (1/2, B; 1/2, W). Moreover, all of them can be considered different ways of representing the ambiguous urn. In fact they are all different symmetric distributions over the possible values of p (proportion of black and by symmetry of white) with the same mean $\bar{p}=1/2$.

In group 1 and 3 what changes is the probability of obtaining black or white in the second stage, but the probability of obtaining one particular combination of black and white is the same (1/2, for group 1 and 1/3 for group 3).

In group two, instead, what varies is not only the probability assigned to black or white in the second stage, but also the probability of obtaining one bag, since the number of bags in each lotteries increases (in the experiment it goes from two to thirteen). Consequently lotteries D, B, F, A, G, H, L R can all be considered different ways of representing a second order distribution. The outcome of the first stage is in fact the probability of obtaining black in the second stage.

Assuming that these are all ways to imagine the unknown bag, is it possible to define a criterion according to which these different distributions represent different degrees of ambiguity ? Moreover can this concept of ambiguity be incorporated in the evaluation process and consequently determine different evaluations of different distributions?

V.3 **Ambiguity aversion as a form or risk aversion**

As we have seen in Chapter I, one characteristic of Segal's model of ambiguity is that of underlining the intuitive connection between the Ellsberg paradox and the concept of risk aversion.

Risk aversion suggests that a decision maker always prefers a certain value $x = \bar{x}$ to a random variable $\tilde{x}$ with expected value $\bar{x}$. Ambiguity aversion can consequently be intuitively explained with the preference toward a sure probability $p=\bar{p}$ with respect to a random variable $\tilde{p}$ with expected value $\bar{p}$. In Segal's words

"ambiguous lotteries appear to be riskier than 'clear' lotteries, where the probabilities are well defined and are known to the decision maker". p 179.

What makes a decision maker prefer a certain value to a random variable is the presence of variability. It is consequently possible to think of ambiguity aversion as a form of aversion to variability. Moreover if ambiguity is defined as a aversion to variability the concept of variability can be used to define different degrees of ambiguity.

V.3.1 *How to measure ambiguity with respect to risk*

The traditional concept of risk aversion deals with uncertain outcomes rather than uncertain probabilities. Consequently the possible indexes that we can use to measure different degrees of riskiness are the ones that traditionally are used to measure the variability of random prospects.

The variance. One simple way to measure the variability of a distribution is given by the variance, σ^2. If we take the variance of a distribution σ^2 as a measure of variability, we can define an order between distributions according to the variance where the $F \geq_R G$ (F is more riskier then G) if the variance of F is higher then the variance of G ($\sigma_F^2 > \sigma_G^2$). (The variance has been used as a measure of riskiness of a random variable $\tilde{x}$in the sixties and its tractability led to the widespread of the mean variance analysis. However the use of variance to rank risky prospects yields to the violation of first order stochastic dominance Borch (1969). For experimental evidence see Coombs and Lehner (1981). This problem was overcome by the Rothschild and Stiglitz (1970) definition.

Rothschild Stiglitz (1970) definition. One other way of measuring different degree of riskiness is given by the Rothschild Stiglitz (1970) definition of"more variable than".

Let us consider the case of risky prospects (we face uncertain outcomes). Suppose now that we have two distributions; we indicate with F(.) and G(.) their distribution functions on the interval [0,1]. Let us consider distributions which have the same mean: that is to say

$$\int_0^1 [G(x)-F(x)]dx=0 \qquad (V.1)$$

According to Rothschild and Stiglitz (1970) definition a distribution G can be considered more variable than a distribution F if and only if

$$\int_0^1 [G(x)-F(x)]\, dx = 0 \quad \text{and} \quad \int_0^y [G(x)-F(x)]\, dx \geq 0 \quad (V.2)$$

for every y , $0 \leq y \leq 1$

If a distribution G is more variable than another F according to R-S definition, the variance of G is higher than the variance of F. The contrary however does not hold[142].

Segal's definition. Segal (1987) defines directly the concept of "more ambiguous than". The following definition concerns directly the characteristics of the second order distributions.

Definition 1

Let Fand Gbe two distribution functions on [0, 1] G is more ambiguous than F($G \geq_A F$) iff G is star shaped spreading[143] of F, that is :
a) F and G have the same mean value $\bar{p}$;
b) G (p) ≥ F (p) for p ≤$\bar{p}$ and G (p) ≤ F (p)for p ≥$\bar{p}$

In Segal (1987) the distribution function F represent the decision maker ambiguity concerning the probability of the event S in the lottery (x, S; 0,~ S). Hence in order to rank the various degrees of ambiguity Segal defines an order on the set of the distribution functions.

Theorem 6.1 establishes the condition according to which a more ambiguous distribution is evaluated less.

Theorem 6.1

Let G and F be symmetric around $\bar{p}$ such that $H_{\bar{p}} \geq_A G \geq_A F$. If f is convex, if the elasticity of f' is nondecreasing, and if the elasticity of $\bar{f}$' is non increasing, then the value of the ambiguous lottery (x,S: 0, ~S) under F is greater than its value under G.
(where $H_{\bar{p}}$ is the uniform distribution on [0.2p]for p≤1/2 and on [2p-1,1] for p≥1/2)

Now the conditions determined by theorem 6.1 are very strict conditions. Moreover if we apply the Segal Definition 1 of "more ambiguous than" to the various lotteries presented in Figures V.1, V.2,V.3, we will find out that D is less ambiguous than any other

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[142] And all risk averters prefer F to G.

[143] The definition of star-shaped spreading of beliefs is taken from Jones and Ostroy (1984)

lottery, B is the more ambiguous, B is more ambiguous than F (that to say $B \geq_A F \geq_A D$).

However the Segal definition does not allows us to infer any order in ambiguity as far as lotteries F, A, G, H, C are concerned. Moreover the conditions defined by theorem 6.1 are so strict that even in case of lotteries D, B and F, in which Segal definition allows us to say that D is less ambiguous than F and F is less ambiguous than B, we cannot conclude anything applying this theorem about the order in the evaluations of these three lotteries. As we told before Definition 1 allows us to define that $B \geq_A F \geq_A D$. Theorem 6.1 requires however in order to establish an order in the evaluations that the degree of ambiguity of a distribution is related to the distribution $H_{\bar{p}}$; that is to say only if $H_{\bar{p}} \geq_A G \geq_A F$ (and also the condition on f are satisfied) we can conclude that F should be evaluated more than G. In our case we have (applying Definition 1) that $B \geq_A H_{\bar{p}} \geq_A D$. Hence theorem 6.1 cannot be used to establish the order in the evaluation for F and D and B even if the three distribution can be define one more ambiguous than the other according to definition 1...

Moreover as we have seen in Chapters I and III Segal (1987) defined the requirement that the f(.) function has to satisfy in order for a subject to be ambiguity averse (Theorem 4.2) (which are also the requirement of f(.) in the theorem 6.1). (In this Theorem Segal compare some ambiguity with none). As we have already pointed out in Chapter III the evaluations of the lotteries B, D, F, A, G, H, C according to Segal model and adopting a functional form which satisfies the condition of theorem 4.2, that is to say $f(p) = p^t$ with $t>1$ are the following: B has the same value of D. B is evaluated more than F, F more than A, A more than G, G more than H, H more than C..

Hence what we can observe is that if we evaluate lottery B, F and D with the anticipated expected utility and applying the specified functional form of f we obtain that B and D have the same value and F is evaluated less than D and B. However applying definition 1 we can conclude that B is more ambiguous than F, and F is more ambiguous than D. Hence the different degree of ambiguity is not reflected in the evaluations of the lotteries.

For the other lotteries however the ambiguity definition combined with theorem 6.1 do not allow to infer any evaluation order.

On the other hand, some subjects in the experiment gave evaluations which go exactly in the opposite direction of the ones obtained applying anticipated expected utility and the specified f(.) with H is evaluated more than G and G is evaluated more than A and A is evaluated more than F and F evaluated more than B and D is evaluated more than any other lottery.

To sum up:

a) the Segal's definition of more or less ambiguous distribution function does not allow us to infer any ambiguity order about the lotteries described in Figures V.2 and V.3.

b) the order that we find when we apply Segal functional form and evaluate the lotteries accordingly in the case of lottery D, B and F is different from the one given by the application of Segal 's definition of different degrees of ambiguity (for the others lotteries definition 1 is not applicable).

3) Moreover it remains to be explained why some of subjects evaluated the lotteries in a different order with respect to the one given applying Segal functional form with $f(p)= p^t$ with $t> 1$.

Hence we cannot use the Segal definition of more ambiguous than to define an order over the second order distributions represented by the various lotteries in Figure V.1, V.2, V.3 which explain the results of the experiment since Segal definition does not tell us anything at least as far as lotteries F, A, G, H, C are concerned.

V.3.2 *Using the variance as measure of ambiguity*

The variance and the Rothschild and Stiglitz measures of variability are applied to risky prospects. To extend their use to second order distributions and apply them to define different degrees of riskiness of a second order distribution (different degree of ambiguity) we have to assume that people deal with the outcomes of the first stage, which are the probabilities of getting the two colours in the second stage , as real outcomes. (There are no studies to our knowledge of the applications of the variance and of the Rothschild and Stiglitz definition in the contest of second order probability distributions. However from an experimental point of view Curley and Yates (1985) tested the importance of variance as an cause of ambiguity reaction and rejected cautiously this hypothesis. Kahn and Sarin (1988) introduced a parameter related to the variance of the second order distribution in the evaluation functional form of their model. In this chapter however the used of variance is confined to determined different degrees of ambiguity in second order distributions which have the same mean and the same probability ranges, with the exception of lottery D). In this case we can try to measure the different degree of ambiguity using one of the two concepts.

Applying the Rothschild and Stiglitz definition we will find that the evaluation order found in the experiment is consistent with this definition as far as lotteries D B F H are concerned and but not as far A G and H are concerned. For example applying

$$\int_0^y [G(x)-F(x)]dx \geq 0 \qquad (V.1)$$

to the various lotteries while F - A≥ 0, we have A-G <0 as well as G-H < 0, but H-C ≥ 0.

On the other hand if we use the variance as a definition of variability of the lotteries described in Table 1 , 2 and 3 we will find that B is the lottery with the highest variance $\sigma^2_B = 0.25$, while D is the lottery with the smallest variance $\sigma^2_D = 0$ (and also lottery R).

Figure V.4 Variances of the lotteries used in the experiment

$\sigma^2_D = 0$	
$\sigma^2_B = 0.25$	$\sigma^2_F = 0.166$
$\sigma^2_F = 0.166$	$\sigma^2_L = 0.018$
$\sigma^2_A = 0.125$	$\sigma^2_R = 0$
$\sigma^2_G = 0.07$	
$\sigma^2_H = 0.109$	
$\sigma^2_C = 0.09$	

If we consequently apply the variance criterion to the lotteries in Figures V.1, 2, 3 we find that D and R should be evaluated the same and G should be evaluated more than A and A more than F and F more than B,

while C should be evaluated more than H but G should be evaluated more than H and C. (If we look at the actual evaluations of the experiment in chapter III we will see that D>C>H>G>A>F>B which is in accordance with the variance rule except for G).
As well as the evaluations of the lotteries F R and L is concerned, their order should be the same as the one due to the application of a maximin rule that is to say R evaluated more than L, L evaluated more than F.

Except for the case of the lottery G the evaluations of the lotteries based on their variance should increase from B to C.
Hence we suggest that we take the variance of a distribution σ^2 as a measure of variability and we can define an order between distributions according to the variance where the $F \geq_A G$ (F is more ambiguous then G) if the variance of F is higher then the variance of G ($\sigma^2_F > \sigma^2_G$).
In case of ambiguity aversion a person would discount the utility of an ambiguous lottery according to the variability of the second order distribution and consequently he or she would evaluate more the lottery which has the smaller variance.
The variance criterion can consequently explain the evaluation order given to the lotteries in term of "more ambiguous than".

V.3.3 *The modified maximin rule.*

The evaluation order consistent with the application of the variance rule can however be consistent with another explanation.
As we have already say B should be evaluated more than D and R more than L and L more than F according to maximin. The simple application of a maximin to lotteries B F A G H C does not allow for an evaluation order between these lotteries since they all have the same maximin.
The above lotteries have always the same two outcomes 0 and 1: if we simply apply the maximin rule as for example in Gardenfors and Sahlin the minimum expected utility is in each case (Table 1 and 2) always zero. Suppose however that the subject thinks of the probabilities of the second order distribution as pure outcomes and depicts the problem to himself or herself in the following way:

Let us take for example lotteries B and F.

Figure V.5 Editing of lotteries B and F

Lottery B

P=1/2	P=1/2
1	0

Lottery F

P=1/3	P=1/3	P=1/3
1	1/2	0

where with P we indicate the probability in the first branch while with p we indicate the probability in the second branch. The same can be thought for all the other lotteries.

If we consider the three lotteries following a simple maximin rule we will have that lotteries B and F have the same minimum expected utility.

We can however think that people may adopt the following kind of reasoning. For lottery B and Lottery F the worst outcome is the same , that is to say a p value of 0. However, the probability attached to the worst outcome is less in lottery F then in lottery B; hence I prefer this lottery to B.

If we apply this kind of reasoning to all the lotteries described in Table V.1 and V.2 that we will find the following evaluation order over the lotteries C >H >G > A >F >B.

In the original model of Gardenfors and Sahlin the maximin is not applied to a second order distribution. It is applied directly to the various probability measures belonging to the set of all epistemically reliable probability measures. Gardenfors and Sahlin define some criteria to establish which probabilities are epistemically reliable. However, since they do not talk explicitly of second order probabilities, they are vague about the assignment of probabilities to the probability contained in the set. The modified maximin rule can consequently be seen as the natural extension to their model once the concept of a second order distribution is introduced.

What we are suggesting is that people who rank lotteries according to a maximin rule when confronted with lotteries with the same maximin can rank them according to the weight given to the worst outcome. They will choose the lottery which has attached to the worst outcome the smallest probability.

This decision rule may be expressed formally saying that given lottery f and g when

$$\min \sum u\ (f(s))\ p = \sum u\ (g(s))\ p \quad \text{then} \qquad (V.3)$$

$$f \succ g \text{ iff } \quad p(W)_f < p(W)_g$$

where p(W) is the probability assigned at the worst outcome.

When two lotteries have the same maximin one lottery will be preferred to the other one only if the probability of the worst outcome under f is smaller than the probability of the worst outcome under g.

This second explanation is the one that according to us is more similar to the kind of reasoning that the subjects adopted or described in the experiment reported in chapter III. Subjects 1 and 20 for example described the lotteries B and F as "unfair" because the probability of choosing the bag with "wrong" colour in it was to high. On the other hand lottery D and lottery R were described as completely "fair" lotteries since the possibility of choosing the bag with the wrong colour was 0. In all the bags the probability of the two colours was in fact one half. This kind of reasoning give in fact a higher weight to the worst outcome. In lottery B or F since they are symmetric the worst and the best outcome are balanced, however the subjects seemed not to give importance to this. What seems to matter was simply the fact that they had a higher probability of choosing the wrong bag and this is what determined their evaluations.

Although we would suggest that the modified maximin rule is the most plausible interpretation of the results of the experiment, however the lack of data does not allow us to draw such a strong conclusion.

V.4 Conclusion and suggestions.

The lotteries described in Figure V.1, 2 and 3 do not allow us to discriminate between the two decision rules above described . Most of the lotteries in fact would be ranked in the same order according to both the criteria. In fact the variance (but not for lottery G) as well as the weight attached to the worst outcome are decreasing with the number of bags.

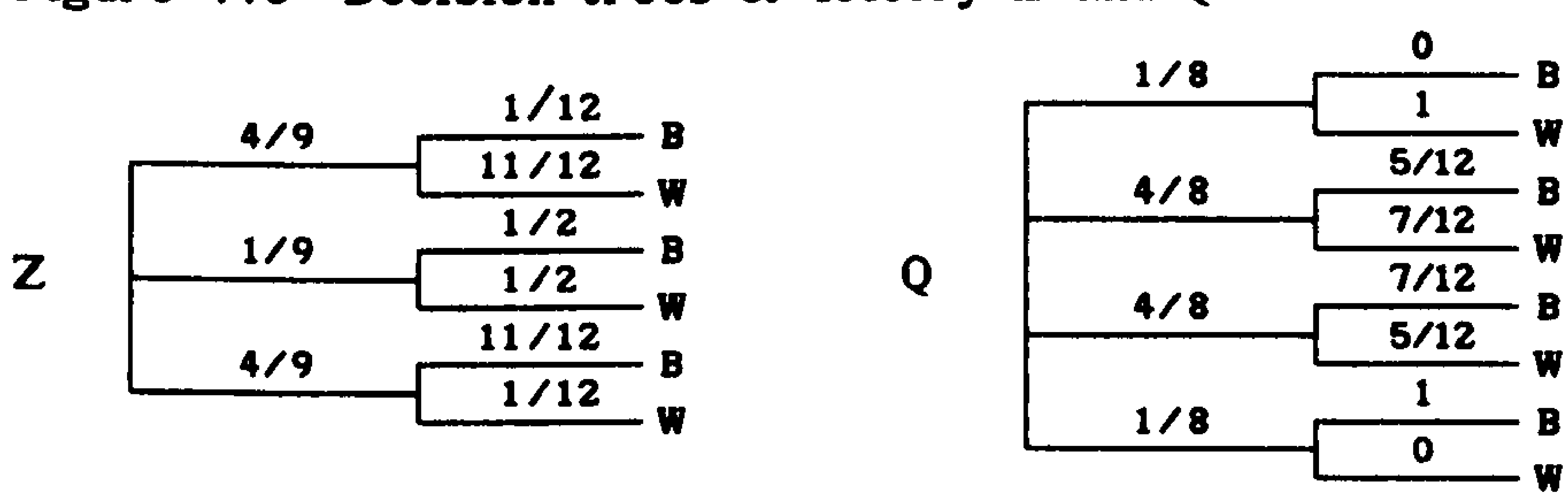
In fact both explanations have their own appeal. An evaluation of the different distributions according to the variance seems to be quite intuitive if we express ambiguity as a second order distribution. In this case in fact the parallel between ambiguity aversion and risk aversion as Segal suggests is quite straightforward. On the other hand, since people use a maximin rules and several models explain ambiguity aversion through the use of a maximin (amongst others Gilboa 1987, Ellsberg, (1961) and Gardenfors and Sahlin (1983)), this modified version of maximin can allow us to establish a preference order in cases where the mere application of a maximin rule gives to the same evaluation.

In order to discriminate between these two possible explanations it is maybe worth to design an experimental test.

One way can be to present the subjects with pairs of lotteries very similar to the one used in the experiment in chapter III but in which the modified maximin criterion and the variance criterion predict different evaluations.

Let us consider for example lotteries Z and Q.

Figure V.6 Decision trees of lottery Z and Q



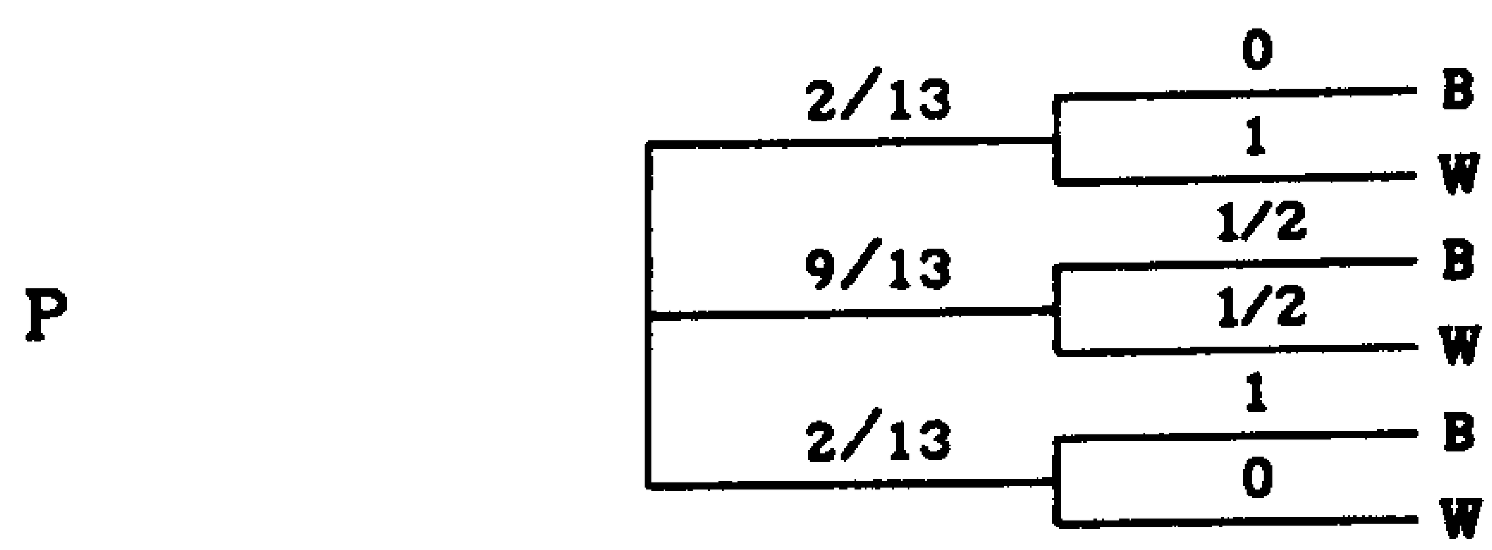
Now if we calculate the variance of lotteries Z and Q we can easily see that the variance is higher in lottery Z than in lottery Q. (0.154, 0.0005)[144]. However if we apply a maximin rule the minimum expected utility is given by lottery Z. If I choose by discounting the value of the lottery according to the variance since lottery Q has a smaller

[144] These two lotteries are not one more variable than the other in term of the R-S definition.

variance I would preferred lottery Q to Z. If, instead, I apply in my evaluation a maximin rule, then, I would prefer lottery Z to lottery Q since lottery Z has the highest minimum expected utility.

Let us consider now these other two lotteries:

Figure V.7 Decision tree for lottery P



$$\sigma^2_P = 0.076$$

and lottery C given in the experiment in which the probability of getting a ball of the right colour varies from 0 to 1 and there are 13 bags with different composition of black and white [0, 1/13; 1/12, 1/13;......1,1/13. As we can see the variance of lottery C is 0.10 while the variance of lottery P. is 0.7.

Consequently if we apply the variance criterion we should evaluate lottery P more then lottery C.

Suppose now instead that we apply the modified maximin criterion.

Lottery P can be thought as:

Figure V.8 Editing of lottery P and C according to the maximin model

2/13	9/13	2/13
0	1/2	1

Lottery C

1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13
0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	1

In this case we can easily see that the probability attached to the worst outcome is smaller in lottery C than in lottery P. Hence if we apply the modified maximin rule we should prefer lottery C to lottery P.

As we have seen it is possible to distinguish between the two criteria and consequently to test their validity in describing people mental processes. Establishing a preference order between second second order distributions can help us to understand better this form of ambiguity. The relevance of this can be high since as we have already said this is the most common operationalization of ambiguity in experimental settings.

CHAPTER VI

SOME COMMENTS ON THE EVALUATION MENTAL PROCESS

VI.1. Introduction

At the end of chapter III, we concluded our analysis of the results of the experiment described there saying that none of the theories tested there seems to explain entirely the phenomenon of ambiguity. In fact we concluded that people seem to have acted in accordance with different theories in different contexts (ambiguity was perceived in different ways according to the different environments).

In this chapter we want to investigate some possible explanations of this behaviour. We do not derive these possible explanations from the adoption of an alternative model of preferences. Instead, the explanation proposed here regards the mental process used by the subjects in order to perform the requested task.

As described in detail in Chapter III, the participants in the experiment were given 21 lotteries to evaluate. If we put aside for a moment what was the aim of that experiment - testing alternative theories of ambiguity[145] - the subjects were simply asked to evaluate 21 objects. Hence, they had to establish through their evaluations a ranking of the lotteries. What we want to investigate is the mental process apparently adopted by the subjects in performing such a task. In particular, by describing this mental process, we will try to investigate if this process has been influenced by the experimental design or whether it is the result of some sort of simplification adopted in the evaluation process by the subjects.

Evaluating 21 lotteries can be considered by some a quite complex task. If such is the case, the subjects might have adopted some strategies in order to simplify this task. The outcome of the evaluation process, the single prices given to the various lotteries, might be seen on the one hand as the result of acting according to different models in different circumstances. On the other hand, however, the same outcome can simply be the result of the simplification strategy adopted by the subjects in

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[145] Each lottery was designed to test a model and was characterized by a particular source of ambiguity.

order to perform a complex evaluation task. The fact that each subject seemed to apply different theories can be, in fact, the result of different stories; it is consequently important to try to distinguish between them.

We will proceed as follows: first we will describe the evaluation process of some of the subjects in the experiments explaining how the evaluation process might be due to the experimental design. Second we will see which conclusion can be drawn from the application of such a evaluation process. Third we will suggest a possible interpretation of the result of the experiment which can link the evaluation process adopted by the subjects to the psychological literature on constructive preferences and beliefs (See Payne, Bettman, Johnson (1992)).

VI.2. **The evaluation process of the subjects.**

VI.2.1 *The description of the experiment and of the lotteries.*

In the experiment described in chapter III the lotteries were given to each subject in a closed envelope. They were put in a random order (different for each subject). In the instructions for the experiment, the subjects were told to give a price to each lottery and that this price could be seen as a minimum selling price for the lottery. They had to evaluate the lotteries at home and they were asked to come for an interview describing their choices afterward. Hence, they were neither asked to perform the evaluation task under any time constraint nor to follow any particular procedure.

In such an evaluation task one might expect people to examine each lottery and compare it to each of the others in order to establish a complete ranking. As described in chapter III this does not seem to be the process followed by the subjects. The subjects seemed instead to have grouped the lotteries in groups and then to have evaluated them within each group.

Before analyzing in detail the process apparently followed, we need to point out some aspects of the design of experiment which may have influenced the strategy apparently adopted.

The lotteries were designed to test different theories. These theories were defined in various ways according to the different sources of ambiguity. Moreover there were groups of lotteries designed to test the same theory. These sets of lotteries share some characteristics (the same source of ambiguity) but they differ one from the other for other respects.

Let us consider for example lotteries F, A, G, H , C; they were all designed to test Segal's (1987) model, hence they were all two stage lotteries. They identified different second order distributions in the probabilities, but these probabilities distributions shared the same mean and the same extreme values.

If we take the whole group of 21 lotteries and we try to analyze it according their similarities we can distinguish various subgroups or subsets according to various characteristics. It is important to notice, at this point, that there were different characteristics objectively present in the lotteries, because they were essential to define or identify different sources of ambiguity. However, if we look at the lotteries simply as objects these factors can be considered simply as different characteristics of these objects.

In the original group we can identify the following characteristics which are shared by different groups of lotteries.

a) One stage lotteries

b) Two stage lotteries

c) Lotteries with known proportion of black and white balls

d) Lotteries with unknown proportion or black and white balls

e) Lotteries with different probabilities in the second stage but always with 3 branches each with probability equal 1/3 at the first stage.

f) Lotteries with probability 1/2 in the first stage but different probabilities in the second stage.

g) Lotteries with probability 1/2 in the first stage and 1 and 0 in the second stage.

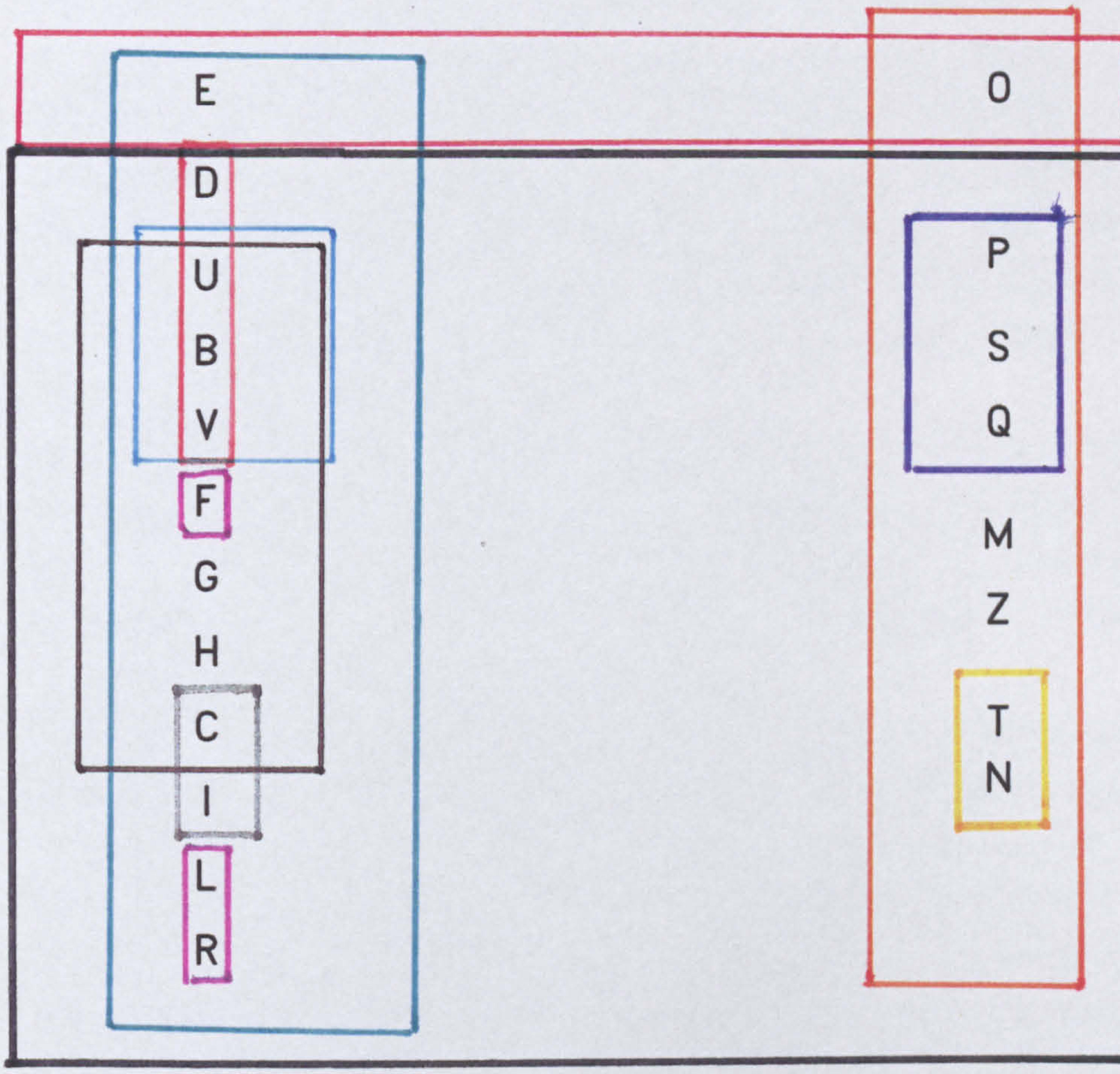
h) Lotteries with different second order distributions but with the same extreme values.

i) Lotteries in which the unknown proportion of black and white was varying.

l) Lotteries with probability 1/2 in the first stage but in which the proportions of black and white in the bags in the second stage was unknown.

m) Lotteries with 13 bags and the same probabilities in the first and in the second branch.

Figure VI.1 Groups of lotteries according to the various characteristics



Legenda

- a red one stage
- b black two stage
- c green known proportion
- d orange unknown proportion
- e violet different probabilities in the second stage but always 3 branches with 1/3 probability
- f pink lotteries with probability 1/2 in the first branch but different probability in the second branch
- g blue lotteries with probability 1/2 in the first branch but 1, 0 in the second branch
- h brown lotteries with differenct second order distributions but always the same extreme value
- i yellow lotteries in which the unknown proportion of black and white was varying
- l navy lottery wit probability 1/2 in the first stage but which the proportion of black and white in the bag in the second stage was unknown
- m grey lotteries with 13 bags

If we look at Figure VI.1, we can see the various subsets of L (the 21 lotteries) defined according to the different characteristics, a, b, c, d , e, f , g, h, i, l, m.

The lotteries were defined for the primary purpose of testing particular theories, however they can be classified or grouped according to different characteristics objectively present. Moreover assuming that these lotteries can be grouped according to these characteristics (from **L**, the set including the 21 lotteries we can form other subets), we can easily see from Figure VI.1 that the groups so defined overlap (some of the subsets derived from **L** according to different characteristics may contain the same object (lottery) contained in other subsets).

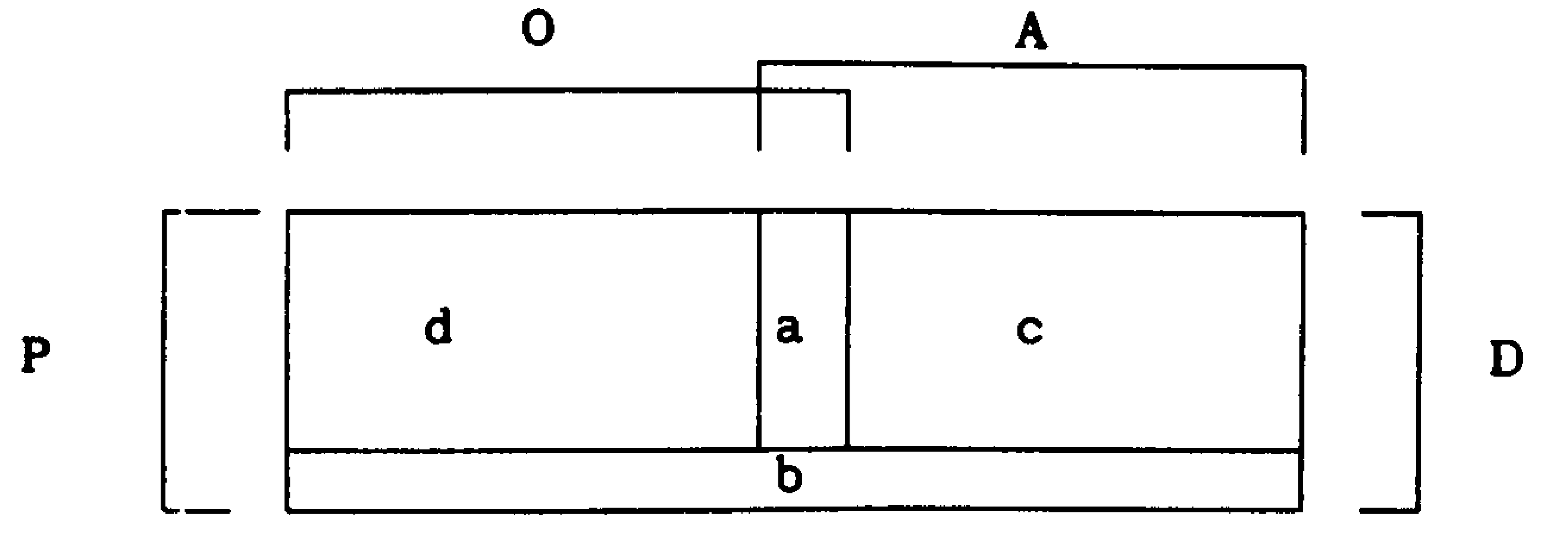
For example suppose we define the subset **A**, of **L**, on the base of the characteristic a, {one stage lotteries}. In **A** we will have lottery E and lottery O: **A** = {E,O}. If we define the subset **B**, on the base of the characteristic b, {two stage lotteries}, then **B**= {D,U,B,V,F,A,G,H,C,I,L,R,P,S,Q,M,Z,T,N}.

Then we can define the subset **D**, on the base of the characteristic d, {unknown proportion of black and white}, and the subset **C**, on the base of the characteristic c, {known proportion of black and white}, we will have **D**= {O,P,S,Q,M,Z,T,N} and **C** = {E,D,U,B,V,F,A,G,H,C,I,L,R}. As we can see O belongs to the set **D** as well as to the set **A** and E belongs to the set **A** as well as to the set **C**. Or in other words assuming that a lottery is characterized on the basis of some characteristics (these characteristic are based on similarities), Lottery A can be characterized by the factor a, one stage, and c, known proportion. O by the factor, a one stage and d, unknown proportion; while D, by b, two stage and c, known proportion, and P, by the factor b, two stage, and d, unknown proportion. As we can see lotteries A and O share the factor a, while they are dissimilar as far as factors c and d are concerned.

This can be expressed graphically in two ways: if we look at Figure VI.1 the various circle represent the possible subsets of **L**. On the other hand if we look at Figure VI.2 we can have a comparison between lotteries E, O, P, D, according to the various factors, d, a, c, b. (This approach can be extended to all the other lotteries but probably using more than one graph[146])

We use the same approach of Tversky (1972)

Figure VI.2 Comparing E D P O according to a b c d.



VI.2.2 *The description of the groups identified by the subjects*

As has already been said briefly in Chapter III, most of the subjects who participated in the experiment evaluated the various lotteries by dividing them into groups. In fact most of them came to the interview with little packets of lotteries[147]. Of course not all the subjects had the same packets since they identified different groups according to different characteristics.

What in general they apparently did was to group the lotteries that they regarded as similar. In fact, one of the subject, Subject 11 stated in the interview " I grouped the lotteries which I thought that were quite similar and in which there good chance of winning and then I made my minimum to 7 pounds because I think I am a little of a gambler. If you don't get it it does not matter. So I thought seven pounds is a fair enough sum."

Of course the factor that makes similar a group of lotteries is also the factor that makes this same group of lotteries dissimilar from the other groups. Moreover, as we have already seen, not all the subjects recognized the importance of all the factors.

However most of the subjects identified the various groups. (It is

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[147] However, some of the subjects (11 out of 21) did not group the lotteries at all. These were those subjects (exept for 1 and 19) who were applying the expected value (they declared so in the interview).
In addition, there were two subjects 1 and 19 which evaluated each lottery singularly, even if in their evaluations (especially in the evaluation of subject 1), all the characteristics above described were recognized and were taken into account in the evaluation.

important to notice here that the various subsets intersect with each other and there is not just one partition of the entire set **L**). Between the subjects who group the lotteries in order to evaluate them, all the subjects identified the groups of the known and unknown lotteries. These two groups were even identified by two subjects that applied the expected value in their evaluations. They recognized the existence of these groups of "riskier lotteries". Three subjects recognized the existence of the one stage group of lotteries and of the two stage group of lotteries. And four subjects recognized almost every group described in Figure VI.1

VI.2.3 *Description of the various stages of the evaluation process*

As we have seen most of the subjects recognized some of the possible groups into which **L** can be divided. We will try now to describe the various stages which this "grouping process" follows and how this "grouping element" entered into the evaluation process and influenced it.

If we try to describe the evaluation process of the subjects, we can recognized a first stage:

a) In order to analyze the problem in a easier way the subjects divided the lotteries into groups according to certain characteristics.

b) This grouping seem to have followed a sort of lexicographic order:

The most important factors (the one which the subjects assign more weight) were the unknown or known proportion of balls (presence of uncertainty). From these factors the subjects derived two groups of lotteries, set **D** and **C**. These two factors can be considered the most important for two reasons: they were the only factors recognized by everybody. Moreover the presence or the absence of the known proportion of balls determined very often the two extremes in the evaluations of the whole set of lotteries. In addition, within these two sets, lottery E and Lottery O played a particular role. They were not recognized always as one stage lottery but as the simplest lotteries. And consequently they were used in the evaluation as a benchmark.

c) Some of the subjects took these two main groups and divided them according to other characteristics that they considered important. This process went on till they could identify groups based on some particularity. In this respect lottery E and Lottery O can also be each considered as one lottery group.

d) They took each group and evaluated the lotteries contained there. These evaluations followed a double consideration. The characteristics used to distinguish one group from the other one determined a higher or a smaller evaluation of the lotteries contained in the first group with respect to the lotteries contained in the other group. However, inside each group, another factor could acquire importance and determine the different evaluations between the various lotteries.

Let us take, for example, lotteries B and D. These two lotteries can be identified as {known proportion, two stage, two bags}. Assuming that we arrive at the identification of this set or group containing the two lotteries, and that in order to identify this set we have used the factor {two bag}. However at this point another factor can acquire importance for the evaluation; in this example, this factor can be the different value of the second order distribution and hence the evaluation between B and D is determined by this last characteristic.

e) In this "grouping by aspects" the subjects sometime insert the same lottery in more than one group. Take for example lottery B. Lottery B can be included in the set {B,D}, {two bags but two different extreme in the probability distribution}; or it can be included in the set {B,U,V}, {two bags, same extremes, different selection mechanism}. This element, the existence of an overlapping) has a twofold influence.

On the one hand, if the subject compares B and D the focus is on the factor that makes the two lotteries dissimilar. The same of course happen when B, V, and U are compared amongst them. In this case the focus will be on the selection mechanism.

Let us now assume that we first grouped the lotteries according to the characteristics {two bags, different extreme values} of the distribution. And then we evaluate the two lotteries comparing these lotteries. Let us suppose that we evaluate lottery D 12 pounds and lottery B 10 pounds. Now assume that we identified the second group in which lottery B can be included, {two bags, same extreme value, different selection mechanism}, B, V, U. At this point we evaluate the three lotteries within this group. B is 10 pounds so the value of U and V is determined in the straight comparison with B. Assuming now that we identified another group, {two stage increasing number of bags same extreme values} in which B can be included {B, F, G, A, H, C}. Then we proceed in evaluating them in the same way.

This kind of procedure may have a major influence in determine the final evaluation of a lottery. Grouping the lotteries to simplify the evaluation task in this way give a particular role to lottery B which is contained in all the three mentioned sets. Lottery B actually is used as a tool for comparison (benchmark) within each set. However since we focus on different elements in ranking the lotteries within each group, this procedure may cause inconsistencies in the evaluations or odd results. Let us consider lottery F for example: its value has been determined directly in comparison with lottery B, within a particular set. The same occurs in case of lottery D and U. It is possible that the evaluation, so established would differ from the one possibly obtained comparing directly F with D and U.
It does not seem from the given evaluations nor from the interview, that the subjects tried at the end to make an overall evaluation.
f) We already said that the subjects when forming a group used a particular characteristic. This characteristic enters into the evaluation of the lotteries within the group with respect to the lotteries outside the group. The possession of a characteristic increases or decreases the attractiveness of a group of lotteries. However, when the lotteries are evaluated within the group the previous characteristic (the one used to form the group) is now a common characteristic of the new group and consequently does not enter in the evaluation which is instead determined by the differences within the group.

It is also important to notice that, even if recognized through the composition of a group, not all the characteristic are evaluated negatively or positively by the subjects. That is to say different subjects give different importance to different characteristics. If we look at Figure VI.2 the area given to a characteristic ,a, can be regarded as the weight given to this characteristic.
The entire evaluation process depends crucially on the characteristics identified by the subjects and on the weight given to each characteristic. The number of characteristics as well as the weight in fact seem to determine in this procedure the number and the elements of the grouping.

VI.3 Final considerations

The process described above can strongly influence the final evaluation of a lottery. The final value assigned to it in fact does not depend only on the utility of the considered lottery with respect to the alternative lotteries. The final evaluation depends crucially on how the process of comparison is constructed. And how this process is construed depends on the characteristic identified by the subjects and the weight assigned to them. Or, even more, the utility of a lottery depends crucially on which group this lottery has been included.

In our discussion we were constrained by the fact that our "objects" to be evaluated were lotteries designed to test alternative theories of explanations of the Ellsberg paradox. However this kind of procedure can be used to evaluate any kind of objects. It is possible to imagine that in order to simplify a complex evaluation task individual will tend to group objects according to some characteristics.

This kind of analysis however raises two main problems:

1) how people identify the various characteristics?

2) is it the characteristics which determine the preferences, or they are just used to determine the various group and then it is the composition of the group that determine the preferences between objects? That is, is the whole process the result of preferences or does the mental process determine the preferences?

The result of the experiment as well as the material of the interview cannot be used to infer any answer to the above two problems. However both aspects have been approached by the psychological literature on decision making.

For example, the choice of the various aspects to be considered in order to form the different choice sets can be determined by characteristic objectively present in the original choice set[148]. The characteristics identified may be determined by other factors as salience representativeness, availability, familiarity, etc, all factors which has been analyzed by the psychological literature (See,

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[148] Characteristic determined by the experimenter like for example in case of the described experiment the different sources of ambiguity.

for example, Kahaneman Slovic and Tversky (1982).

On the other hand, as we noted at the beginning there is a growing part of the psychological literature on behavioural decision making which sees personal preferences as a result of the mental process adopted in the evaluation or of any other task performed by the individuals. According to this literature (See Payne, Bettman, Johnson 1992)

"information and strategies used to construct preferences or beliefs appear to be highly contingent upon and predictable from a variety of task, context, and individual-different factors. Task factors are general characteristics of a decision problem, such as response mode (judgment or choice for example), which do not depend upon the particular values of the alternatives. Context factors such as similarity of the alternatives, on the other hand, are associated with the particular values of the alternative. Task and context factors cause different aspect of the problem to be salient and evoke different process for combining information. Thus, characteristic of the decision problem, such as response mode or similarity, can evoke different strategies that at least partially determine the preferences and beliefs that we observe." page 90.

In this framework , Simonson and Tversky (1992), Tversky and Simonson (1993) develop a model (which also they test experimentally) which allow for context-dependent preferences. In short according to them preferences can be determined by the composition of the choice set. Let us assume that we have two choice sets {x,y,z} and {x, y}, my preference over x and y can be different if expressed in relation to the first set or the second set. They identified two main factors which can determine this different preferences which the call trade off contrast and extremeness aversion. Their entire analysis is related to choices between goods. However it is possible to extrapolate their reasoning and apply it to any kind of objects. If their model can be apply ed to other contexts, then it is possible to go a step ahead. Let us assume, as in the interpretation of the result of the experiment, that people in order to simplify their evaluation process construct, from a unique choice set, different choice sets according to various characteristics. If preferences are context based, we can infer that the preferences and the evaluations expressed towards the objects contained in each of these sets may be different from those that the subject would have expressed when evaluating the objects in the bigger set. This is also what could have happen in the evaluation process in the experiment described in chapter III. Of course we do not have at this stage any evidence that this was the case. However, since the context base model can be quite challenging for the standard decision theory this can be for sure a interesting field of research.

"Much of the recent theoretical work on decision under uncertaintyattempts to reconcile rational choice with observed violations of expected utility theory. Although such attempts have enriched the theory of choice, the psychological analysis of preference and belief indicated that it is not possible in general to reconcile normative and descriptive account of individual choice. The reason for this conclusion - which may be regarded by some as pessimistic or even negative - is that decision making is a constructive process. In contrast to the classical theory that assumes consistent preferences, it appears that people often do not have well-defined values, and that their preferences are commonly constructed, not merely revealed, in the elicitation process. Furthermore, different constructions can give rise to systematically different choices, contrary to the basic principles that underline classical decision theory."

Amos Tversky "Constructive Preferences and Rational Choice" prepared for the special session on "Rationality in Economics" held at the meeting of the International Economic Association in Turin October 1993.

APPENDIX A

(Chapter I)

Let us consider the following compound lottery

L $(A_1,1/3;A_2,1/3;A_3,1/3)$, where $A_1(25,0;0,1)$, $A_2(25,1/2;0,1/2)$, $A_3(25,1;0,0)$, which is exactly lottery F in the experiment reported in chapter III.

Using V {x ,p ;...;x ,p} = (I.16a)

$$u\ (x_n)\ f(p_n)+\sum_{i=1}^{n-1}u(x_i)\left[f\left(\sum_{j=i}^{n}p_j\right)-f\left(\sum_{j=i+1}^{n}p_j\right)\right]=$$

$$u(x_1)f(p_1)+\sum_{i=2}^{n}\left[u(x_i)-u(x_{i-1})\right]f\left(\sum_{j=i}^{n}p_j\right) \quad \text{(I.16b)}$$

to compute evaluate each simple lottery of the compound lottery we obtain

$V(A_1)= u(0)f(1)$

$V(A_2)= u(25)f(1/2)$

$V(A_3)= u(25)\ f(1)$

where the decision weight function $F:[0,1] \to [0,1]$ satisfy $f(0)=0$, $f(1)=1$.

If with $CE(A_i)$ we indicate the certainty equivalent of the lottery A_i, where the $(CE(A_i),1) \sim A_i$; if the preference relation satisfy the indipendent axiom then

$$(A_1,P_1;...;A_nP_n) \sim (CE(A_1),\ P_1;...;CE(A_n),\ P_n). \quad \text{(I.17)}$$

If the preference relation is represented (I.16 a) then the $CE(A_i) = u^{-1}(V(A_i))$.

Assuming that $CE(A_1) \leq CE(A_2) \leq ... \leq CE(A_n)$ then (I.17) implies that

$$(A_1,P_1;...;A_nP_n) \sim (u^{-1}(V(A_1)),P_1;.. ...;u^{-1}(V(A_n)),P_n) \quad \text{(I.18)}$$

In our example

$(A_1,1/3;A_2,1/3;A_3,1/3) \sim (CE(A_1),\ 1/3;...;CE(A_3),\ P_3)$

and $CE(A_1)= u^{-1}\ (V(A_1) = u^{-1}(\ u(0)f(1))$

$CE(A_2)= u^{-1}\ (V(A_2) = u^{-1}(\ u(25)f(1/2))$

$CE(A_1)= u^{-1}\ (V(A_3) = u^{-1}(\ u(25)f(1))$

by (I.16b) then by (I.18) the value of the two stage lottery

$V\ (A_1P_1;A_2P_2;A_3P_3)$ can be represented by

$$V(A_1) + \sum_{i=2}^{n} \left[V(A_i)-V(A_{i-1})\right] \quad f\left(\sum_{j=i}^{n} P_j\right) \quad (I.19)$$

that in our case is

$$u(0)\ f(1) + [u(25)\ f(1/2) - u(0)\ f(1)]\ f(2/3) +$$
$$+ [u(25)\ f(1)-u(25)f(1/2)]\ f(1/3)=$$
$$= u(25)\ f(1/2)\ f(2/3) + u(25)\ f(1/3) - u(25)f(1/2)\ f(1/3)=$$
$$= u(25)\ f(1/2)\ f(2/3) + u(25)\ f(1/3)\ [1-f(1/2)]=$$
$$= u(25)\ [\ f(1/2)\ f(2/3)+ f(1/3)\ [1-f(1/2)]$$

Now if we apply

$$u(x)\ f(p_1) + u(x) \sum_{i=2}^{n}[\ f(p_i)-f(p_{i-1})]\ f(\sum_{j=i}^{n}P(p_j) \qquad (I.20)$$

to the same lottery we obtain:

$P_1 \le P_2 \le P_3$ and $P_1=0,\ P_2= 1/2,\ P_3=1$

$$u(25)f(0) + u(25)\ [f(1/2)-f(0)]f(2/3)+ u(25)[f(1)-f(1/2)]f(1/3)=$$
$$= u(25)f(1/2)f(2/3) + u(25)[f(1)-f(1/2)]f(1/3)=$$
$$= u(25)\ [\ f(1/2)\ f(2/3)+ f(1/3)\ [1-f(1/2)]$$ QED.

Let us consider the following lottery, which is equal to the first branch of lottery F: $L_1(0,1/3;\ 1/2,1/3;\ 1,1/3)$; we want to show that taking one of the two forms (I.16a) and (I.16b) above in the text is exactly the same.

Let us consider $L_i(x_1,p_1;....;x_n,p_n)$ where $i=1,...,n$ and $x_1 \le ... \le x_n$.

The value of the lottery L_1 using (I.16a)

$$V(L_i)= u(x_n)\ \ f(p_n) + \sum_{i=1}^{n-1} u(x_i) \quad \left[f\left(\sum_{j=i}^{n}\ p_j\right) - f\left(\sum_{j=i+1}^{n}\ p_j\right)\right]$$

So $V(L_1)=u(1)\ f(1/3) + u(0)\ [f(1)-f(2/3)] + u\ (1/2)\ [f(2/3)-f(1/3)]$

Since $f(1) = 1\ f(0)=0$ we will have

$u(1)\ f(1/3) + \quad u(0)\ [\ 1 - f(2/3)\ ] + u(1/2)\ [f(2/3) - f(1/3)]$

putting $u(0) = 0\ u(1/2) = 1/2\ u(1) = 1$ the above expression becomes

$f(1/3) + 1/2\ [f(2/3) - f(1/3)]$.

Now we evaluate L_i following the second form indicate by (I.16b).

$$V(L_i) = u(x_1)\, f(p_1) + \sum_{i=2}^{n} \left[u(x_i) - u(x_{i-1}) \right] f\left(\sum_{j=i}^{n} p_j \right)$$

$V(L_1) = u(0)\, f(1/3) + [u(1/2) - u(0)] f(2/3) + [u(1) - u(1/2)]\, f(1/3)$

putting $u(0)=0$ $u(1/2)=1/2$ $u(1)=1$ the above expression becomes

$1/2\, f(2/3) + f(1/3) - 1/2\, f(1/3) = f(1/3) + 1/2\, [f(2/3) - f(1/3)]$ Q.E.D.

APPENDIX B

(Chapter III)

INVITATION

We are looking for 10 volunteers to take part in an experiment. The experiment is to be held in the early part of week 11 (from the 29th to the 31st of March). All 10 volunteers must register for the experiment in week 10: on Wednesday and Thursday from 9 a.m. to 5.30 p.m.. For participation in this experiment, you will receive a participation fee of £5, in addition to whatever you gain from the procedure described below.

You will each be asked to examine and evaluate 21 lotteries. Each lottery will give you a chance to win £25. You are asked to give a price to each lottery. This price can be seen as your minimum selling price for the lottery; that is, the lowest sum of money that you would be prepared to accept in exchange for the lottery. Hence the price you give will represent your evaluation of the lottery. The lotteries will be given to you when you go to register for the experiment; registration will take place in the EXEC office in Derwent, D block, second floor, room 203 (D/D203).

When you register you will also be given an answer sheet in which you will be asked to write your evaluations of the lotteries and a few notes of explanation. A few days later we will meet in the EXEC office and you will be asked some questions about your evaluations of the lotteries. The interview will be tape recorded and will last around 15 minutes. At the end of the interview the following procedure will be carried out. One of the 21 lotteries will be picked at random; we will look at your evaluation of the randomly picked lottery. Then a number between 0 and 25 will be picked at random; if that number is greater than your price for that lottery, you will get that number of pounds; otherwise the lottery will be played out and you will be rewarded with either £0 or £25 depending on the outcome of the lottery. It is important to note that it is worth your evaluating the various lotteries accurately: if you give to a lottery a price which is less than the value that you place on that lottery you may end up with an amount of money when you would prefer to play out the lottery; while if you give to a lottery a price which is more than the value that you place on that lottery you may end up playing out that lottery when you would prefer to receive that amount of money.

ANSWER SHEET

Name: **College:**

Address:

Could you please explain how you arrived at your evaluation?

We will talk about this in more detail at the interview.

Please put your evaluation of the lotteries on the other side of the paper.

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LOTTERIES

Lottery E

In front of you there is a bag which contains 12 balls, 6 black and 6 white. The bag is opaque, so you cannot see inside. You are asked to bet on one of the two colours and then you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery D

In front of you there are two bags. Each of the two bags contains 12 balls, 6 black and 6 white. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery U

In front of you there are two bags. Each of the two bags contains 12 balls. One bag contains 12 black balls, while the other contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery V

In front of you there are two bags. Each of the two bags contains 12 balls. One bag contains 12 black balls, while the other contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, the experimenter will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery B

In front of you there are two bags. Each of the two bags contains 12 balls. One bag contains 12 black balls while the other contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will toss a coin: if it lands heads you will choose the bag on your left; if it lands tails you will choose the bag on your right. At this point you will draw a ball from the chosen bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery F

In front of you there are three bags. Each of the three bags contains 12 balls. One bag contains 12 black balls, a second one contains 6 black balls and 6 white balls and a third contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery A

In front of you there are five bags. Each of the five bags contains 12 balls. One bag contains 12 black balls, a second one contains 9 black balls and 3 white balls, a third one contains 6 black balls and 6 white balls, a fourth one contains 3 black balls and 9 white balls and a fifth one contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery G

In front of you there are seven bags. Each of the seven bags contains 12 balls. One bag contains 12 black balls, a second one contains 9 black balls and 3 white balls, a third one contains 7 black balls and 5 white balls, a fourth one contains 6 black balls and 6 white balls, a fifth one contains 5 black balls and 7 white balls, a sixth one contains 3 black balls and 9 white balls, and a seventh one contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery H

In front of you there are eleven bags. Each of the eleven bags contains 12 balls. One bag contains 12 black balls, a second one contains 11 black balls and 1 white ball, a third one contains 10 black balls and 2 white balls, a fourth one contains 9 black balls and 3 white balls, a fifth one contains 7 black balls and 5 white balls, a sixth one contains 6 black balls and 6 white balls, a seventh one contains 5 black balls and 7 white balls, an eighth one contains 3 black balls and 9 white balls, a ninth one contains 2 black balls and 10 white balls, a tenth one contains 1 black ball and 11 white balls and a eleventh one contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery I

In front of you there are two bags. One bag contains 13 balls. Each ball has a number from 0 to 12. The other bag is empty but it will contain 12 balls which can be either black or white. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then you will draw a ball from the second bag. The proportion of black and white will be determined in the following way. You will draw a ball from the first bag, if the ball you drew has the number three on it, 3 black balls and 9 white balls will be put in the second bag. At this point you will draw a ball from the second bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing. Note that you have to choose the colour on which to bet before you will know the proportion of the black and white balls in the second bag.

Lottery M

In front of you there is a bag which contains 12 balls, 6 are black and 6 are white. Then there is a second bag which is empty. The bags are opaque, so you cannot see inside. From the first bag a ball is to be drawn by somebody who is neither you nor the experimenter. If the drawn ball is black he or she will put a black ball into the second bag; if the drawn ball is white he or she will put a white ball into the second bag. Then, after replacing the drawn ball in the first bag, a new ball is drawn. If the drawn ball is black he or she will put a black ball into the second bag; if the drawn ball is white he or she will put a white ball into the second bag. At this point, without knowing which colours are the two balls put into the second bag, you have to bet on a colour and draw a ball from the second bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery T

In front of you there are two bags. The first bag contains 6 black balls and 6 white balls. The second bag contains 11 balls. Each ball is either black or white but you do not know how many there are of each. The bags are opaque, so you cannot see inside. You are asked to draw a ball from the first bag: if a black ball is drawn then a black ball is put in the second bag; if a white ball is drawn then a white ball is put into the second bag. At this point you are asked to bet on a colour and to draw a ball from the second bag, which now contains 12 balls. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery O

In front of you there is one bag which contains 12 balls. Each ball is either black or white but you do not know how many there are of each. The bag is opaque, so you cannot see inside. You are asked to bet on one of the two colours and then to draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery P

In front of you there are two bags. Each of the two bags contains 12 balls. Each ball is either black or white but you do not know how many there are of each. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery Q

In front of you there are two bags. Each of the two bags contains 12 balls, each ball is either black or white but you do not know how many there are of each. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, the experimenter will choose a bag and, second, you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery C

In front of you there are thirteen bags. Each of the thirteen bags contains 12 balls. One bag contains 12 black balls, a second one contains 11 black balls and 1 white ball, a third one contains 10 black balls and 2 white balls, a fourth one contains 9 black balls and 3 white balls, a fifth one contains 8 black balls and 4 white balls, a sixth one contains 7 black balls and 5 white balls, a seventh one contains 6 black balls and 6 white balls, an eighth one contains 5 black balls and 7 white balls, a ninth one contains 4 black balls and 8 white balls, a tenth one contains 3 black balls and 9 white balls, a eleventh one contains 2 black balls and 10 white balls, a twelfth one contains 1 black ball and 11 white balls and a thirteenth one contains 12 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from that bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery S

In front of you there are two bags. Each of the two bags contains 12 balls. Each ball is either black or white but you do not know how many there are of each. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will toss a coin: if it lands heads you will choose the bag on your left; if it lands tails you will choose the bag on your right. At this point you will draw a ball from the chosen bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery Z

In front of you there are three bags. The first bag contains 12 balls. Each of these 12 balls can be either black or white, but you do not know how many there are of each. The second and the third bags each contain 12 balls, of which 6 are black and 6 are white. The three bags, are opaque so you cannot see inside. First you are asked to bet on one of the two colours and, then, to draw a first ball from the first bag; if you draw a black ball you will have to draw the second ball from the bag on your left, if you draw a white ball you will have to draw the second ball from the bag on your right. At this point you will draw a second ball from the chosen bag. If this second ball is a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery R

In front of you there are three bags. Each of the three bags contains 12 balls. One bag contains 6 black balls and 6 white balls, a second bag contains 6 black balls and 6 white balls and a third one contains 6 black balls and 6 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery L

In front of you there are three bags. Each of the three bags contains 12 balls. One bag contains 4 black balls and 8 white balls, a second bag contains 6 black balls and 6 white balls and a third one contains 8 black balls and 4 white balls. The bags are opaque, so you cannot see inside. You are asked to bet on one of the two colours and then, first, you will choose a bag and, second, you will draw a ball from the bag. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

Lottery N

In front of you there are two bags. The first bag contains 6 black balls and 6 white balls. The second bag contains 9 balls. Each ball is either black or white but you do not know many there are of each. The bags are opaque, so you cannot see inside. You are asked to draw a ball from the first bag, if a black ball is drawn then a black ball is put in the second bag, otherwise a white ball is put into the bag. Then you are asked to draw a second ball from the first bag, after the first one has been replaced in the bag: if a black ball is drawn a black ball is put into the second bag; otherwise a white ball is put into the bag. Then you are asked to draw a third ball from the first bag, after the second one has been replaced in the bag: if a black ball is drawn a black ball is put into the second bag; otherwise a white ball is put into the bag. At this point you are asked to bet on a colour and to draw a ball from the second bag, which now contains 12 balls. If you draw a ball of the colour you have chosen, you will get £25, otherwise you will get nothing.

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Interviews

Subject 1

Generally I ask you to explain to me which is the kind of reasoning that is was behind your evaluations.

For A there were only two bags with all the same colour, so there was a good chance to get a mixture. B, the two bags have just one colour in each, so there is all or nothing; lottery C, there are only two bags with one colour, again, there is a good chance to have a mixture, D seems very fair as there is the same proportion of white and black; for E again there is an equal proportion so it is fair, for F there is not , I think, a 50 per cent chance.. all the same colour in two bags, so that does not seem pretty fair in doing the lottery; G again there are only two bags with all the same colour, so there is a good chance of having a bag which has got a mixture of the two balls in each; for H, again, there are two bags with all one colour in each, so there is a good chance to get a mixture of the two and for I the same again, there is a good chance to have a mixture. For L, all the bags are mixed up, so I feel like to have a better chance of getting the one you choose. T you know some of the proportion of the black and white balls in there, so I assigned a higher value for that N; you do know at least some of the percentage. Each is a fifty percent chance two balls the same colour; for O, I do not know the percentage at all, so you can't assign a very high value to the lottery; P same again you do not know the percentage at all again, so you do not know on what you are betting really; for Q, same again, you still do not know the percentage when you do the lottery so I cannot assign a higher value to it, R is an equal chance so you can assign a higher value to it; the same percentage chance of getting either balls; S again you do not know the percentage at all so I can't give a very high value; T you do know the colour of a lot of the balls but you do not know the colour of the rest of them so I cannot assign a value very high to that lottery; U, you have all one colour in the same bag so it does not seem very fair really; you feel like an all or nothing, B again is a all or nothing feeling, with Z there is an equal chance, so I assign a higher probability to it.

You have put the higher evaluation to E ..

Right

Can you explain me why you have put this evaluation to E.

May I look at the lottery?

Yes of course..

Because it is just one bag which has got six of each in it so it is definite; you just feel you have a 50-50 chance to getting your ball. There is the whole mixture in that so, when you put your hand in, you still got a 50-50 chance.

And pick the lottery D, for instance. Compare it to the one that you have just evaluated you gave an evaluation just lower

It just seems more definite E; just the way I think about the problem.

And then take B, U, and V. You have evaluated them all the same. Can you just explain me why ?.

With B you have got to choose the colour of the ball and, then, it is like down to faith, again, when you toss the coin whether you go to the bag which has got the colour of the ball that you have chosen; it seems rather down to faith if you are lucky. U is the same sort, but it is like the experiment setting up the coin again, it is down to faith which bag you choose, because the coin is just external to the experiment, really the only thing which is different is whether the coin decides, whether you put your hand in , or you decide where to put your hand in but the bags are equal, anyway, they can be mixed up; V is the same as B, the only difference is because it is just the experimenter to choose the bag; it is still down to luck, again, because B and V are exactly the same, because it is an external force; U is the same because it is still down to luck where you choose.

Take F, A, G and H and C; you gave a slightly different evaluation 9 on F, 8 on G, on H, A and C 12. What made you change your evaluation?

With F two out of three bag are just one colour, so once you have chosen your colour then there are just two bags that have some proportion of your colour in, so the chance once you picked a bag, when you put your hand in, there may be no chance of getting your ball, anyway, because there can be all balls of the other colour. With A, again, you got two bags with all the same colour, but in between the other three bags are a mixture, so once you have chosen the bag, there are four out of five bags you can choose in which you got an actual chance of getting the colour you have chosen.. With C again there are 13 bags and just two which have all the same colour, but this time you got 12 out of 13 in which your ball will be in, so you got a greater chance of getting your ball anyway when you have chosen one of the bag;there is just one out of 13 in which you do not have any hope of having your ball out of it. With G even though there are 2 with all the same colours out of 7 bag, the proportion in between seems fair; if you have chosen a black ball, even if there are 6 out of seven bags which contain your balls there is a higher number of the bags that are between the two extremes; so you got greater chance of picking your ball and with H you got eleven bags again you have got 10 bags which would contain your colour but the proportion within it seems reasonably fair to you to pick a ball which you have chosen once you do not chose one of the bag that is at the extremity

And then L, R and F, you gave to L 12, to R 13, to F 9, can you explain me why you have evaluated the three of them in a different way?

With R any bag you choose you got a 50 percent chance of getting a black ball or a white ball, so this is pretty fair; there is no chance of being persecuted of having all the other colour balls; with L the bag is still mixed up but one is fifty percent chance another one is 45 percent chance and the other one is 25 percent chance so if you choose the one in which you have just 25 percent chance then it is worst off that being in lottery R; in F again you got a bag which is fifty percent mix but you got one bag which is all the same colour you have chosen and one bag in which there is none of your balls in it, if you choose that bag you have just no chance of picking the ball that you have chosen it does not seem as fair as the other two.

And then, S, P, Q ,and O; look before to S P and Q and then to O;
you have given the higher evaluation to S then to P then to O then to Q.

For S you got two bags; it is very annoying that both the bags have 12 balls which you do not know, you got an external force that decides which bags you go for, but since you got the two bags, it appears that you got a better chance to pick the bag that has got the colour you want; P is the same but with no external force, it is down to you to choose which bag; as the lottery seems to be down to luck it does not seem to be as fortuitous as the lottery in which it was the coin to decide which bag you draw from; with Q the external force is the experimenter; you don't feel so lucky when the experimenter is doing it, you know what I mean. It is an external force that even if it does not have more control than you do on the bags, it seems to have more control than you do over the experiment in itself; O has just one bag which contains 12 balls so one bag could be just all of the colour that you have not chosen, that is why I did not assign a higher value to it, because there is a higher probability of having all of the colour opposite the one you have chosen because there is just one bag.

Just look at Z, you put a higher evaluation; just explain me why.

It is the last two bags with which you are concerned rather than the first one of the lottery. The first bag just determines from which of the second and the third bag you will take the lottery from and since in the second and in the third bag there is equal chance of getting the black ball or the white ball, you just feel that both bag have a fifty fifty chance; so once you have put the hand into the bag there still a fifty fifty chance of getting the colour that you want.

That's all.

Subject 2

Just explain me in a general way which was the kind of reasoning that you used in evaluating the lotteries.

The main one.. for most of them it was the fact that you pick up a bag at random and you have no idea of the one you picked and the chance that you draw out a white ball or a black one is the same. You can win 25 pound or naught so it means that the expected gain from the lottery is 12.50, so I put 12..
Go on this... this one you got eleven balls and they can be black or white, therefore I assume that there are 5 and a half of each; if you

draw a ball (that can be black and white) and you put it into the bag; so it seems much as you have 6.5 of one colour and 5.5 of the other colour; so, if you choose the colour that you have just put into the bag...(ruined)

And then N

It seems as you got 9 balls so you have 4 and half of each if...(ruined)

Look at V and U

It makes no difference if you choose or I choose the bag so, therefore, B and U are just the same; V is the same because you toss a coin randomly, so randomly you choose a bag and you do not know anyway which balls are in the bag.

And then the other group P, S, Q, and O.

S is the one in which you choose; it is equal to P, because the coin chooses randomly, you choose randomly; in Q the experimenter chooses randomly, therefore, Q is just the same as the other; O is the same as the other because there is just one bag, and it does not really matter which bag you choose since they are chosen randomly.

And just explain me why you put a higher evaluation on T and N

This is because there are eleven balls and, then, you put one ball you know into the bag and, then, you chose a colour; if you choose the same colour you put into the bag you have a better chance. In T you put 3 balls into the bag; this means that you got at least two of one colour and one of the other colour and there might be the case in which you put three of the same colour that means that you have much more chance.

And, then, M which is the other one you gave a higher evaluation.

M is one in which in the first bit if a black ball is picked it is put into the bag and for the second one you have got a fifty fifty chance of putting a black one or a white one; this means that 7.5 percent of the time you can get a black ball and 25 percent of the time you get a white one ..which means that the black one is more probable.

O.K. Thank you

Subject 3

Just explain me, which kind of reasoning did you use in evaluating the lotteries?

O.K. I have not done much, I have not GSG in Maths and so my statistics is not very good. I looked through them all and, as far as I could see, they were ..most of them represented a fifty per cent chance of picking up your colour ball, because the simplest case,when you have one bag, and 6 black balls and 6 white ball in the bag, you have basically a 50-50 chance of picking up the colour but, when I went through it and I worked out the more complicated ones, I am talking about the ones I priced 12 now, I decided that, no matter how complicated you made the lotteries, they all still represent a 50 percent chance, because they where balanced. In the one with seven bags there is one bags with twelve blacks, but there is also one with 12 whites, one with 8 blacks and 4 whites, but there is also one with 4 blacks and 8 whites and then 6 and 6, so, therefore, if you don't know, you can't see in the bags, then, actually, as far as I can work out with my statistics, you still have 50 percent chance of betting on the ball that you have chosen...The only complication I had is that there are some lotteries which are the one I priced 10, which were to some extend blind; in some way you did not know how many balls... I am trying to find them. There is one here which says: in front of you there are two bags... each ball ..... This is a blind one; what you do is that you pull out a number, and you choose the proportion of black and white from that number, but you have to bet on the colour that you choose before you decide the proportion; so there is really no way you could influence that, you can pick up a naught and bet on the wrong colour and there is no way you can win the lottery. Then, I decide just to make them slightly less because I would prefer not to play out the lotteries; as I said before I didn't work out the maths behind; all I could do is just decide to price them slightly less, because I thought I had less chance to win the lottery on those. The reason I priced all the others as twelve is because, as far as I can work out, that gives me the best chance to win the most of money, because it is half way between 0 and 25 which are the extremes you have and it is a 50 - 50 chance.

Can you pick U, V and B? You have evaluated them all the same; can you just explain me why?

O.K. I have to read through them again. O.K. They are basically all the same lottery, there are two bags and there are..one with 12 black and one with 12 white, now, of course it is not an ideal situation but still 50-50 because the bags are opaque, so when you bet on one colour you still don't know you still got a 50-50 chance. The choice is not if you pick out a black ball from the bag the choice is which bag you chose but the odds are still 50 percent, because there are two bags and the only difference between this three lotteries is the way the bag is chosen; in B here you have .. the bag is chosen by tossing a coin, that's 50-50, that is a random game; in this I chose the bag but I mean the bags are opaque so it is like tossing a coin and, then, the experimenter, and in this I was slightly confused because I did not know whether the experimenter would have known what was in the bag and so therefore, he deliberately could choose the bag which would affect what I might decide. I decided at the end that I did not really care, so I just evaluated the same assuming that he did not know so it makes no odds how the bags is chosen, because it is random; no one knows what there is in the bag and, therefore, they are all the same.

And then look at this and this group you also have evaluated the same (E and U,V,B).

Yes there is only one bag this time but there are 6 blacks and 6 whites, there I have to chose a bag all the differences that there were in these lotteries is the different number of stages that you have to go through before you actually chose a ball out, and in this one what you got is one bag with the balls and you just choose a ball and there are 6 black and 6 white and there are 50-50; you can say you just choose a colour and see if luckily you get it; in effect in this lottery, here, it is exactly the same that this one expect that you just separated out the black and the white balls into different bags; you could have another bag, you could have a big bag and inside you could have 6 black and 6 white balls and a little bag also, it could not make any difference because you can't say which is which.

Just take the T and N which you gave 10, slightly less then the others...

Ye these are more difficult one and I am sure that I priced them inaccurately but really I could not decide how to price these at all, because.. I could actually give them even less because ..O.K. These are quite interesting because N would be valued differently depending on the colour of the three balls that you draw out because if you draw out three black balls then you are going to bet on black , but if you draw two black and a white then ..; but even if you do that you know that you have a slightly better chance in theory to pick up black ball that white one, but you still don't know, there could be 9 white balls in the bag; you have no clue of what is in the bag so, therefore, there will be still 9 whites, even if you draw 3 black balls, so there is a great difficulty to decide; then I decide that the only way you could work it out at all would be to assume that the blind bag you didn't see in was equally split or roughly split so it would be like four and five and then depending on what you drawn out. So I just assume that and I priced it slightly lower than twelve because I think I would be worried to play this out, because I do not know which chances I have. What is the next one ...this is the same thing, it is just slightly less complicated, you just have 11 balls; the only sensible thing to do is to draw a colour and bet on that colour.

Now take the S, P, and Q and O .. O. You gave 10 to S....

I gave 10 to Q as well because also this is a blind one, you do not know how they are split at all so ..you gave 12... P is the same thing you got two bags it does not matter really you do not know what colours the balls are so what you can do is the same as blind so I priced it slightly lower for me it is just blind, there are blind chances, there is no way you can guess; as for O, I suspect this is also the same, there is one bag with 12 also and you don't know so the only thing that you can do is bet on one colour ..this is different again is the experimenter who chooses ..the bag in P you choose I eventually decided that it does not really make much difference ...again in this one is just tossing a coin, which is just a random choosing what you are really doing; put a conscious choice into it, it does not make really any difference; you don't know what you choose anyway you may want to toss a coin.

And them just M that you evaluated 10; just explain why

O.K. ..then because, again, this is a quite complicate one, you get ...you don't see what colour are the two balls put into the bag and again the fact I do not know if this is true, but it is how I read it, it does not matter if there are two balls in the bag or 12 or 24, the odds are still the same but you do not know if if is a black or a whiteone or if there are two blacks or two whites; it could be quite easily two whites, it is just 50-50 and 6 of each and you are just asked to draw two balls out so quite easily there can be two whites so you do not lie; there is no way to control the balls so it is why I have evaluated slightly less again, because it is blind and you do not know anyway. And Z which you evaluated 12 .. It is just complicated, in the first stage again you have to choose the bag instead of deciding which bag you want and draw a ball out of that bag; if it is black you choose a right bag ..actually the second and the third bags from which, you are choosing, tell you that six are black and six are white so you got 50-50 chance and both bags have got 6 of each.So, therefore, it is really immaterial what you do at the beginning; which bag you end up with in the first part is the same with the fact that the first bit is blind we do not know how many balls there are;really it is not important at all; because you do not know what are you betting on, when you draw from that; it is a sort of random way of choosing the bag, it would be more complicated if the second and the third have got different numbers, since they have got the same number, it is really immaterial which bag you draw the ball from.

O.K. Thank you.

Subject 4

I ask you which is the general reasoning behind your evaluation.

When I was looking at all the lotteries it seemed to be about 50-50 chance of the ball you have chosen being collected; the fact that you chosen, before drawing the ball out or after it has not really matter. And for some of them, it was really obvious, you had one lotteries with 12 white balls in one bag and 12 black balls in the other bag and you choose a ball and the bag are just 50-50. In others, there was a lot more information given to you; for instance, the one in which it starts off with twelve bags and, then, you have 11 black balls and 1 white; but even then, eventually, you still have the same number of balls of each type and consequently, you still have a fifty-fifty percent chance; there is just confusing information and you have just to spend more time to look through all them; there seems to be a couple where the chance is slightly higher, but only slightly, so it does not really change that much so I change the price accordingly slightly to 11 pounds but ..
I originally started off with 8 pounds and it was a toss between 8 and

9 and 10, then, I thought well with 10, it is a lot of money and you probably get a quite higher chance to pick up; so 10 to 25 is hat to hat so half to a third chance, I said why not? otherwise, I would be quite happy to get a gamble and take 25 pounds or nothing and then you can have just the 5 pounds which you start off. I just thought that I had 50 percent chance to get 25 pounds, which was equivalent to getting no much less of 10 pounds, so I just put 10 pounds. That is how I made my evaluations.

I will just ask you about some groups of lotteries. Can you take B, U, and V. You have evaluated the lotteries all the same; you gave 10 to all of them; are they really equal to you or, at the margin, there are some differences?

May I read ..

Or just explain me why you have evaluated them in this way..

...There is only one bag and in the end you got 6 black balls and 6 white balls and you draw a ball from the bag and that is an even chance to draw out the colour that you have chosen; so it is a 50-50 chance even thought there are less balls to draw out; in this the experimenter has to choose the bag; well it does not really matter who chooses the bag either, because the bags are opaque anyway ...the 6 and 6 has just been translated in 12 whites and 12 blacks, it is still the same ratio so they were absolutely equal and also probably the other one .. The difference is just that you choose the bag or the experimenter chooses the bag but, as I just said before, as for the fact that it is the experimenter; he hasn't any hint, any more knowledge than you do, it is still down to chance so...they are all the same and.. you toss a coin ye that tossing a coin is just a different method of choosing a bag, but it is like you choose the bag , the experimenter chooses the bag, you toss a coin. Still you have no more insight in what is going to come up, and it is just someonemaking a decision for you instead of making it by yourself; but you make a decision without really knowing anything at all, it is still chance, totally down to chance anyway when you toss a coin or you just point out a bag.

May you just take the S, P, Q, and O ?
You have evaluated ten all of them.

That is a little more riskier, but I just thought that because there are just two different colours of balls so it is like having a 50-50 chance to picking out the one you want; I was tempted to put a slightly lower price on that, but I set down and then I thought, if you choose black there can be 12 black balls or no one and so you do bad but ..If you think that you can cancel out like tossing a coin, I think it does not matter who is making the decision for you and it cancels out and become the some as the experimenter, so what I think is that this is the group of the slightly riskier ..I do not know what to say on that because it is the same as the first one (0)..so in this one, you choose the bag but you do not know which is the good bag, so how it will be the chance on that bag.

Take F,A,G,H, you also evaluated them the same so just explain me why, which kind of reasoning you adopted in evaluating them.

I just tried to sort of mentally calculating if there were the same number of black and white balls so ..choosing a bag you do not know what it is inside, but you still have got a 50-50 chance of getting the colour that you want and it was the same with all of these as well, I read through just in case they slightly changed and to see.. there might be one bag missing in the sequence and I thought that would change the experiment but they are all with the same number of black and white balls; it just takes a lot longer to read them, that's all.

And then T and N, you also gave the same evaluation.

This is going back to the experiment where you do not know how many balls there are so this is just a slightly more risky one, but again, because there are just two coloursof balls there is 50 per cent chance to pick out a white one; if you knew how many balls there where in the bags then there could be slightly more blacks than whites, but you do not know that so..
I just put the same price because I thought you can end up with a bag full of the balls you have chosen, but you could end up with a bag with none of those balls so it is even though.....

Take the two that you have evaluated differently which are T and M

..It it does not say if a white ball is drawn it is put in the second bag ..no it does look at them separately.

You still end with 6 blacks and 6 whites then a second ball is drawn and a ball of the same colour is put into the second bag, ye so it seems to having more chance of getting black then the first time that you pick out a ball from the first bag, it is only black that is put into the second bag to be chosen from and then, the second time, it is whatever colour you get; so there are just marginally more chances to picking out black so you chose black anyway ..so I put a higher price on it because there are more chances to play out the lottery .. I do not know if I slightly misread this one ..because it is eleven balls and you actually do know the colour of the balls that you put into the second bag, then you can choose that colour so you do not know the ratio of the balls in the second bag but you know that a white ball or a black ball will be in and you bet on that colour so I thought it is risky but there is still more chance of picking the right colour.

And then Z you evaluated 10.

Well with this one you bet on the colour after drawing a white ball this means that you have less chance to draw another white ball out, then I probably would have put a higher price on that but you have to chose the colour before you have to draw any of the balls at all so it is like taking it out of your hand isn't it?

I suppose that it is slightly riskier, basically I found that ought to be about the same; if I had thought longer I might have changed the price marginally but there was not really that much point and you know changing a price about one pound or two pounds; well I did for those two because I found that definitely they were different so I changed the price therefore, but I thought there was no much need to mess about all the other prices

O.K. Thanks.

Subject 5

Can you just explain to me which kind of reasoning did you use in evaluating your lotteries ?

Yes. Just because I play out without any risk so I thought let's evaluate them all about between seven to ten pounds because the chance that you get that a number is picked when you just get the money and you do not play the lottery at all is over 50 percent; so I thought it is better to get some money than nothing, because playing the lotteries is 25 to naught so I thought ...I just play without any risk because there is more chance.

And which are the reasonings behind the different evaluations of the lotteries ?

From 7 to 10, I think I have just randomly chosen, actually ...I mean, it depends on how much I thought the lottery will come towards a good end or a bad end; so I did not really calculated all the lotteries, but I thought, if there were fifty per cent chance I play 10, but if it just sounds suspicious to me I just put 8 or seven pounds.. just randomly o.k..

Can you take lottery E, B and then V and U so ..You have put 10 on V and on E, are they .. you evaluated them equally..

They are ...V and D are the same.

Then you put B, 7, and U, 8, may you explain me why you put this evaluation, why a different evaluation between U,B, and V.

I think just because I thought try one this way and try this other in that other way to give all of them a chance; I just did not want to play the same on all of them, because I thought that could be more risky some time but not if they were the same.. the same value if there is the same chance I thought let's try both ...

And pick up the P , Q and S and O, look at S, P and Q and then you have evaluated S and Q in the same way which is 8 and P is 7, may you explain me why.

I think that it was for the same reason as before, therefore I think that these are a bit risky, because you don't know how many balls there are of each colour in the bags, so I thought, it might just be one, so I gave seven pounds, since I thought I can try; but I thought that it was not worth ten because there could be just one ball in each of the colour I have chosen. I did not like this.

And then just F, A, G, H, C, you gave to to F, G, C, the same evaluations 8 and then to A and G, 7.

This was the same as before, because I thought, because here you are not sure of how many in each bag, because you do not see in it; so I did not fancy to put 10 on them, and it was the same reason as before, I just wanted to get both, to try.

Take L, R and F; you gave to L, 8, to F, 8 and to R 10.

I put 10 here because the chance are fifty fifty in R. Then I thought that is more equally .. With respect to F I do not like it, because there were 12 white balls in one of them, so I thought I might get nothing, and L that was not evenly distributed.

And then T and N, which you evaluated 8 and 7.

I do not like them, because you do not know how many there are of each; I was not risky at all and .. I think that there are more chances, because you know at least for one bag that it was evenly distributed but for the other one .... this is a little better I think.

And then Z you evaluated this one 8 tell me why

It is the same thing isn't it because in one bag there might be just black balls or just white balls, and ,then, I just thought not be risky at the end.

And then M you have evaluated 7.

Because one is empty so I might get naught I think I play out from the same pattern like when I was not sure if it is fifty percent, I just went a bit lower down so it is a kind of the same thing.

O.K. Thank you.

Subject 6

Just explain me which is the kind of reasoning behind your evaluation in general.

I went through them all, as some of them obviously look the same, so I look to them first. As I looked at all of them, they just came down to the same thing; you could not really deduce anything, they were all the same probability. I might be wrong but this is how I think as far as I reckoned there was no main difference between them, so I put them all the same; I put to all of them 12.50, because it was like medium and I had good chance. Some of them look different because the instructions were different; like whether I pick the ball whether the experimenter picks it, but it did not make any difference.

I ask you about some of them. Take, for example, B, U and, V you have evaluated them equally; are they equal to you or there is a marginal difference or explain just why you have evaluated them equally.

They were all the same I decided .. the choice is taking out a bag at random and I do not know more than taking a ball from the bag, if you choose the bag it does not make any difference.

Just take T and N you also have evaluated these two equally just explain me why they are equal to the rest.

They seem to be the same, because you can not get if it is going to have more whites or blacks in these situations; it is just a little more complicated because there are two bags.

Pick Q, S , P and O; just look at S, P, and Q first, you also evaluated them 12.5. Just explain me why ?

O.K, Because they were exactly the same there is no difference between the various methods of selection and I do really think that I could not make any difference in how I evaluated them.

And between this one and the others?

It is just the same, but there is just one bag; so whether I chose one of the two bags or I take from this bag, it does not make any difference.

And between E and, O you have also evaluated in the same way.

On that one you know that you got a fifty percent chance of getting that right; on the other one, you do not know what chance you are getting but since you do not know you may well assume that there is fifty percent chance, nothing else.

And then just between B and E

Whichever of the two bags you choose, it does not make any difference, because they are the same so..

And between B, I and O that you have evaluated equally?

In I, the proportion of blacks and whites is random; you do not know the exact proportion but you may assume fifty percent.

And then F, G, H, C, you also have evaluated them the same; which kind of reasoning have you adopted?

These are the same; the only difference is the number of bags, and even if there is a different number of whites or blacks in each bag, when you pick a bag you do not know if there is white or black in it, so you again may assume fifty percent chance because this is what is the average.

And then Z which also you evaluated the same.

In the first one you pick, you do not know if there are more blacks or whites; there is no way of guessing, so it is just random if you choose black or white.

M

You do not know of which colour are the two balls in the second bag, so you just guess randomly, there is no way you can guess if there are black or white.

O.K. That's all.

Subject 7

Can you in general explain to me which is the reasoning that you adopted in evaluating the lotteries and then I will ask you about some lotteries in particular.

I hope to get them ready now this or that not ..O.k. then.. So you don't go from the start, do you?

Well, first give me your evaluations and, then, you can go from the beginning to the end and follow your reasoning.

All right. In some cases it was obvious to me that the chances were fifty fifty and, in these cases, I thought, you bet half the money you have in order to make sure that you approximately match the probability that you get the money, when you get fifty percent chance of getting 25 pounds and fifty percent chance of getting nothing, so I bet 12 pounds, because that is the average. Then there were few cases, in the end especially, were it was really hard to figure out what your chances are, because everything seems to be very uncertain and I just gave 4 or 5 for no particular reason but intuition. I looked at the lotteries and I thought, maybe this is the one, and I thought that 25 pounds were o.k., given the gamble, some time you choose the ball and I have to tell the colour in advance, so you could.. you could perfectly easily cheat and put the other colour in and I was a little worried and I do not know.
A few were pretty similar, A, C, F, G, H. They were pretty similar, they were all fifty fifty; you just had a different amount of bags were the number of white or black balls was reduced or increased proportionally, so equaled down to fifty fifty in these case. They were pretty obvious fifty fifty cases, there were B for example and D and E were fifty fifty as well. I think if I did not make big mistakes. This one where there is an arrow going down, took me quite a lot to find out that this was a fifty fifty case as well, but I had to figured out, if there is one ball then it must be ..well ...I do not know, well, shall I have a look?

Yes

Ye there was just one ball, then you might take a second ball. If the first ball drawn is black she or he will put it in the second bag, if the second ball is white then it does not go into the second bag and I do a second draw and then I got a fifty fifty chance because there are six blacks and six whites in the first bag. Then you fill up again, so I got fifty percent that it is black or white and if there were two balls inside then, also, I think that there is a fifty fifty chance, I do not go into much detail, I do not really know.

May you take the B, U, and V: you have evaluated the same way.

I think they were all fifty-fifty, if I did not get anything wrong here the only different.. I think... were the last two, the percentage, the experimenter would choose the bag and, in the other case, I would choose the bag. I think that is was the only major difference, otherwise they were equal. Is that wrong ? I can't remember. I think this was the only difference. I think I have evaluated them similarly, but I thought, if you choose a bag I bet on one of the colour and, then, you choose a bag, you could easily choose a bag in which there is not the colour that I have chosen. I could have valued them differently, but I trust you and I thought that you were not playing unfair, so I said, it does not matter whether I choose the bag or you choose the bag.

Can you take I?

It says Uncertain.

You evaluated 5.

I do not really know why I could not figured out any probability;I just gave 3 or 5 just intuitively, in order to make sure that you could get at least something and may be you are lucky in the lottery and you get .. but I really don't know, I couldn't figure out any probability; there may be some, but I am not very good in probability as that.

Can you just take T and N; you have evaluated them less then the others 4 just can you explain me why?

They are both very uncertain and uncertain when I look through the lotteries and they seem to be very complicated; this one in which you draw three times and you only know three out of twelve and, do not ask me, I couldn't figure out any probability at all.

And then just take L and R and F, which are this one and then that one and the exclamation point; you evaluated anyway all the same.

All fifty ye.

Just explain me why

The first one was one of these which only differs in the quantity of bags which have the same probability, so that I thought it was pretty easy, there was fifty percent chance, and this one, that is one of those, as well isn't it, ye, and it is with three bags, and the last one three bags 6 blacks and 6 whites, it is fifty fifty the same as far as I know as far as my probability math goes.

Just take the Z one which is this other.

I do not think that I am very helpful for you am I ? I really don't know
o.k. I was a bit confused, it said a first ball from the first bag and then it talks about left and right, I was a little confused by that but I do not know why it sounds pretty complicated to me either black or white and you do not know how many.. it sound very uncertain ..six

blacks and six whites which would be fifty fifty ..you can't bet on one of the two colour, well you could, in the case of the second and the third bag, because it is fifty fifty, but since you do not know the number of the first bag, which contains 12 balls, but you do not know which ones; it is hard to figure out which colour you choose, it is pure chance ..what is the bag on my left if I got three bags standing there, I do not know which one are the ones which are fifty fifty, do I?

It is just the second and the third bag which are on your left or on your right

All right so the one that is uncertain is in the middle; all right I got this now then, it is not too uncertain actually ..it does not seem to bad, then, how much I have evaluated 3. I could have evaluated 5 or 6. I really did not know what to do with left and right, but now as you said, it makes sense; if you choose the one you have got on your right side and then one on you left side ,but it is still not very certain I play safe.

And then may you just take S, P and Q which are..this one .. this one this one and then that one you have evaluate S, the arrow and the little rectangle 4 ,and the other symbol three; just explain me why?
And the black square four ..

Yes

I evaluated this one..we had these already do we? is not the one in which the experimenter chooses?

Another one.

Ye it was in another contest, all right.

Forget this one for a while, you have evaluated these all the same.

Yes you can't figure out; it says in all three cases that you do not know; in this case there is one bag, in the other two cases there are two bags, each containing twelve balls and you do not know what the combination is, and from that basis it is really hard to figure out what the lottery is worth; I do not know it is just chance. You have to tell me if I got all these wrong and they were all pretty easy to evaluate and I am too stupid to find out.

Thanks you.

O.K.

Subject 8

Just explain me how you evaluated your lotteries, which kind of criterion is behind your reasoning.

I looked at the probabilities and when they were fifty per cent I chose twelve pounds and the other lotteries where it was uncertain I chose

10, a smaller price that would be 10 pounds. I did not want to take any risk.
Because you say 25 pounds, so you have 50 so I chose 12 instead of 13.

Take the lotteries E and B, U, and V.

O.k.

You gave the same value to all of them.

Ye

May you just explain me why?

You have the probability. You have the same choice to pick one colour, you can't see any difference between them, because you have the same choice, even if the procedure is longer it does not make any difference how and who picks the balls.

Just take F, A, G, H, E, and F also these you evaluated equally you gave the same value.

O.k

Explain me why.

At the end, it comes down to the same total again, even if the distributions inside the bags are different...you got the same chances.

And then, these two: N and T.

In lottery N, in both lotteries there is a bag in which you do not know how many balls there are; you do not know if the distribution is equal or whether there is one ball of one colour and all the rest of the other; so the chance of betting on one colour varies of a small number for that I did not want to set too mach on a number, on a colour, since I did not know how many there were.

And then S, Q, P, and O you gave 6 to all of them you gave the same evaluation, just explain me why and if for you their value is really the same or at the margin there is a difference.

There is no difference; just because you toss a coin or because the experimenter chooses them, it does not make any difference. You still do not know how the colours are distributed and how many balls are there of many colours so..

And then A you evaluated 12, just explain me why.

13 There are 13 balls and between them one randomly will be chosen and it will determine how the distribution will be; if ball number 6 is chosen then the distribution will be even, and before is just like one of the other lotteries, where you got the same relation if you add them together they are equal, there are no more black balls and no more white balls.
So it depends on which bag you get but the chance are equal because there is an equal number of balls.

And then this one also this you evaluated 12: M.

Just because the balls are moved, but since they are replaced again in the first bag the chances of getting a white ball is fifty percent and the chance of getting another white ball, for example, is still fifty percent, again after the ball has been replaced there are two colours in one bag, the chances are fifty fifty; it does not mean anything, it does not matter that there are just two balls in it.

And then this one Z you have evaluated 12.

Because the bag in which you do not know the number of balls does not play any role in the game if a ball is chosen from the left bag or from the right bag the proportion is the same fifty fifty balls so one could toss the coin alteratively or one could left it out and just take a ball

O.K. Thank you.

Subject 9

I just ask you in general which kind of reasoning you used in evaluating the lotteries.

Of most of them what, I actually I find the probability of winning and, then, I looked at the expected winning which was 12 pounds, because in half of the situations there was half of the probability that I will win and half that I will loose, because in some of them like M, I could see for example with M, if I chose black I would have a slightly chance of winning, so that is why the price I put on it was higher, then, the other and the same with T.

May you just take the B, U and V lotteries; may you look at these and explain your evaluation to me; you have evaluated all the same, explain me why you have evaluated the same or if at the margin they are different for you.

With all three of these lotteries one bag has all blacks and the other all whites, but I do not know which is which, so whether I choose or I toss a coin or you choose does not really make much difference, so I do not see that they are really too different.

Just take E and compare with all the three of them.

This one is only different in the sense that in B or V the bags is chosen which determines the outcome, but in E, there is only one bag, but the probability of getting black or white is the same. I do not see .. there is really no difference between them.

And then S, P, and Q.

There were 12 balls and I did not know the colour of any of them, I thought I do not know the proportion of the colour of the balls in the bag, so I thought that the distribution of the colour would be symmetrical: I can assume that there are six of each and assuming that, then, it is the same that the others.

And then F, E, G, H.

Again, these are all symmetrically black and white and since it is symmetric there is still only a fifty percent chance of choosing one ball so if you chose a ball from any of the bag and them ask me to guess the colour it would not make any difference if I guess the colour and then you pick the ball afterwards because it is symmetric so betting white or black is equal.

Just this one M.

With this one if a black ball is drawn out then a black ball is put in, but if a white it is drawn out then nothing happens, and then if the second ball is black or white then, a white or black ball is put in so if black is drawn out first and then there are four possible combinations of colour in the second bag: two blacks, black and white or either black or white and there is more chance of having a black picked out because, if a black ball is taken out from the first bag first, there are more probabilities that a black ball will be put out at the and but, if a white ball is taken out, first, it does not mean that it will be a white ball at the end, and so there is a higher chance that a black ball will be pulled out at the end, so I would bet on the black ball.

Subject 10

Just explain to me, in general, how you have evaluated them.

What I think is that all of them were virtually fifty-fifty, even those in which you do not know how many white balls or black balls were in the bag. You still have to make just a simple choice between black and white and you do not know if you will draw a black, even in the others, so I evaluated all the same but still a little lower of what you expect to win which is 12.50 to have a good chance of getting a reasonable amount if you do not play them through which is a fifty percent chance of getting 25 or naught.

O.K.

Can you take the lotteries B, U, V. I put letters on them, so it is easier to look at them. You have evaluated all of them exactly the same; are they really equal or at the margin the differ in some ways?

In V, you have 50 percent chance of you choosing a colour and then you have to choose a bag, so it is about choosing the right bag. For the colour you still have the same chance to having one bag with the colour you choose, I think. I am afraid I am not good at statistics. For B, I have to toss a coin and you ...this is the same sort of thing anyway ..

E and the others you also evaluated the same.

It is the one with 6 black and 6 white; you just pick one ball and you do not know which colour it is; but you do not know which bag contains black balls or white balls either so it seems to be the same.

And then just take T and N.

These are the only one I thought you have just a slightly betterchance really, because you could choose, you knew the three of the balls of the bags you draw from, so if you drawn out three blacks or whites you get an advantage betting on those, but, at the same time, I do not think you knew any of the balls in the second bag ....so I decided that it was not really worth evaluated them more .. but I think you can evaluate them a little more.

Then P,S and Q.

Now, you do not know what colours are in the bag, which bag is the right one and on which colour you want to choose, but the choice is still a fifty fifty choice, even thought you choose the colour and then you toss a coin, there were 10 white balls and two black balls there are still fifty fifty percent chance, if you choose black or white you still have to make the choice without knowing, you have to make a choice between black and white at it is like tossing a coin: it is just luck: just Q because I do not know whether the experimenter knows what balls are in the bag so you can choose a colour and then he chooses the other bag and I have to choose the colour first and the experimenter have to choose the bag, then if the experimenter does not know then is back to the others.

And then take O please.

Choosing a bag make no difference actually from the choice of taking a ball, because you don't know either the bags nor the balls; if there would be just one bag you still have the same chance.

Can you take O and E.

In this you know that definitely you have 50 percent chance, but O as well, if you consider what a fifty fifty chance is, there isstill a 50-50 chance, even if I do not know what the balls are, or if I do not know what ball to choose, even if all the balls were black there are still the chance between choosing between black and white; when you have made the choice, then, I would have evaluated differently, but I evaluated them before choosing a colour, and I have evaluated all the same.

And now I.

E was the one that you have to choose the bag without knowing if the bag from which you have to draw have 8 or 5 etc but if you add all the balls there is an equal number of white and black balls in the bag and also you chose without knowing there is still an equal chance of getting black or white despite the fact that it look quite the same for all of them you have 12 6 and 6 12 the other have 12 9 3 so they are all basically the same.

Then Z which also evaluated the same.

This is the one you chose a ball from the bag in which you do not know how many blacks or whites there are and depending on what colour you chose, you chose the bag from which you have to make the other choice, but in fact both the bags from which you make the choice between black and white are the same with 6 blacks and 6 whites, so in fact if you miss the whole of taking the first ball out, and then decide, it is the same as E or B: you just have six black and six white.

O.K. Thank you.

Subject 11

Can you just explain to me which kind of reasoning you adopted in evaluating these lotteries?

There are few of them where there are two bags, and each of the two bags has 12 balls and of each bags you do not know if they are black or white; so you had a 50 percent chance of getting the ball that you wanted, if there were 12 balls of your choice in there or none of the balls of your choice and there were quite few like that and then there was also Q and S .. there were two bags one bag containing 12 black balls and the other containing 12 white balls so I thought you have like fifty percent chance and that was like in U and so I put 12 pounds on that because it is half of 25 pounds, leaving to luck if I had to play the lottery or not. The one in which there were a large number of bags, the larger the number of bags, the less likely you were to get the colour of your choice because you had to choose a bag and then to choose a colour when you had the bag, so the chance that you get the bag where there might not be the colour that you have chosen was very probable. There are some where I just put down rough number, for F there was 3 bags one contains 12 black balls one 12 white balls one 6 and 6 of each so I thought, well, depending on which bags you pick out you got more or less chance of having your colour, so I gave 11 so there were few like that A, G, H ,and I are quite similar; there were five bags, seven bags, eleven bags, in which you do not really know which colour there are in each; I grouped the lotteries which I thought were quite similar and in which there were good chances of winning and then I made my minimum at 7 pounds, because I think I am a bit of a gambler, if you do not get it, it does not really matter; so I thought seven pounds is a fair enough sum.

Just take the lottery E.

I got this.

E, U and V and B.

You have evaluated all of them equally, may you explain me why? Is there a marginal difference or are they equal?.

I thought they were all the same as for the probability so I thought perhaps those two U and V I thought they were exactly the same, it does not matter whether you choose the bag or the experimenter chooses the bag, you got 50-50 chance of having the bag where there are the balls you want, and the other E, I thought you got a bag with 50-50 chance of picking up the colour that you want, because there are six black and six white and the other one was B, you toss the coin to see which bag is chosen, I do not think that there is much difference if you choose the bag or the experimenter chooses the bag or you toss a coin. It is just luck what bag you pick up.

May you take F, A, G, H, C you gave 12, 10, 8. and 7; just explain me why.

I thought that if you have 13 bags I put the price of 7 pounds because the chances of you getting the ball that you want are quite remote. Because you first you got to choose a bag and then you got to choose a ball out of the bag, so the chance that you pick the bag that you wanted is one out of 13 and, then, I put seven pounds because the chance that you are going to have a good chance to pick out a number that is above seven, so you can have that money without having to play the lotteries and, then, as I went down to eleven, I put eight, because there is more choice that you get the colour that you want and then I went down to seven you have more chance of getting the colour that you want so I put down ten and then I gave 11 when there were three bags I gave 11, because with two bags I gave 12 with three you have less chance.

Just take N and R and F.

You have evaluated F 11 R 12 and I 9 may you just explain me why.

The F, I gave 11 pounds because you have got one out of three chances to have the bag that have totally your colour in it and again I gave 12 pounds to R. Probably, I have to give the prices on the other way round I should give more money to R then F but there is just a difference of a pound because there are less chance on you with F so ..because with R which bag you pick out it does not matter, because you got fifty percent chance of getting whatever colour you want, but with F depending on which bag you got you pick out one hundred percent chance of picking out 12 white balls and one hundred percent chance of piking out 12 black balls so ...I actually should have given less value but it does not matter; and L, I have given to this less money because there are three bags and there is a combination of 4 and 8 and 6 and 6 and 8 and 4 I gave less money because you only got one out of three chances of picking up one that contains 6 and 6, but there is one out of three bags and you got quite a remote chance of piking the other one in which you have only four balls in the bags instead of eight so I gave to this less money than I gave to the other two because there were less chances of picking the ball that you wanted.

Now T and N.

You gave 9 to T and 10 to N; just explain me why and how you have evaluated in this way.

First with N where you have got a fifty-fifty chance of picking out a black or a white ball from your first bag and then you put it in the second bag but you do not know any-way, so you it is really irrelevant what you pick up and it does not make much difference if you know one ball which you know the colour of and the other which you do not know, so since you draw from the second bag I thought there was not much difference, if you draw first a white or a black ball and then you draw another ball from the first bag, and then depending on what you have drawn in the first case, you can have more chance of having the colour you want and then you put the ball in the second bag and then you draw from that; I put down 10 pounds because...I have to read it ..the second bag you do not know more so I thought that it is luck that you come out with what you wanted; it does not really matter what you have done with the first bag; it is just pure luck; it is like the other one in which you do non know which is the combination, so that was for N and then for T, I put less money in the first bag you have 50-50 chance of picking up a black or a white ball and then you put that into the second bag and there are 11 balls and you do not know which they are anyway, so you can't really say what the combinations are; so it is just pure luck if you get your colour out, I gave less value that the others because.. because in N you got more control on what there is in the second bag than you have in T, because in T you have to put just one ball; in N you have two balls, so you add to 11 you actually are picking three balls there, while in the other you are just picking one ball.

And now please take S, P, Q and O. You have all the same.

I put 9 to all of them O.K. This are like the other one you toss a coin or you choose the bag or the experimenter chooses the bag and in this other you do not have to choose the bag since there is just one bag so it does not really matter if you toss a coin or if you chose or the experimenter chooses you are going to get a bag which you do not know which is the combination of balls in it. So I put the same number because there was no difference who chooses the bag or if you toss a coin, because you got 50 percent chance because you have to choose the colour so if you do not choose the colour .

Take O and E.

I gave 12 to E because you definitely know that you have 6 blacks and 6 whites so you know that once you have picked out the colour you have got fifty percent chance to picking out of the bag; with O you do not know what there is in the bag you have not definitely got fifty percent chance so I gave 9 to it because I had more chance of getting that money or more without having to draw a ball out, it is risky. That is why I gave E a higher value.

Z you gave 10; may you explain me why?.

You've got three bags and in one of the bags you do not know which combination there is and in the second and in the third bag you know that there are 12 balls of which 6 are black and 6 are white, so you have to draw a colour from the first bag, so you do not know the choice of taking a white or a black because you do not know the combination that is in there, then depending on if you draw a black or a white ball you draw a ball from one of the other bags. I gave 10 pounds, because I thought ..well.. you pick out a white from the first bag that is irrelevant.It does not matter which one you pick, you end up piking up a ball from the second or the first bag and so you get 50 percent chance of picking up the colour that you want, so even if you got 50 percent chance, I gave 10, because you first have to pick out from the first bag and then..

M

I thought that you have 50 percent chance because you pick a ball out from the bag that contains 6 blacks and 6 whites; so you do not know which one are you going to get, and the chance of getting two blacks balls from the bag you draw from are 0.5, the chance that there is a combination of black and white is 50 percent, you do not have exactly 50 percent chance of getting the ball you want because you do not know the balls in the bag from which you draw, so I gave 10 because I thought that it is depending on which ball you choose.

Subject 12

Just explain to me in general how you have evaluated these lotteries.

I evaluated them from my knowledge of statistics, I applied the rules; I assumed in the one in which the experimenter has to choose that he was impartial, apart from that it was just probability and the value of the probability has been half in each one, so I just priced them half of 25 pounds.

Just take the one you have evaluated more, T and N and explain me why, look at them.

First of all for T you know exactly what the ball is, opposite to the others where either you do not know or at least you do not know how many of each, so the fact that you know it means that you have a better chance of picking that colour.
The same for N where you have the knowledge of three of them, three out of 12, you know exactly what they are and then I priced them accordingly.

Please just take B, U, and V.

You gave the same evaluation to all of them, just explain if you really evaluated them equally or at the margin you find any difference just explain me your reasoning.

First of all for B it comes down to tossing a coin, the probability of each is really a half so I gave half value. U is the same, instead of tossing a coin you pick a bag since you do not know how the bag is, it is just like picking at random, so the probability of getting your colour is still half as well; for V the experimenter chooses the bag, so as far as I assumed the experimenter was impartial, so I gave them the same value if I was the one to pick the bag.

Look at E and B, you have evaluated them the same; are they really the same as the others according to you ?.

This is just the same, since you have the same number of black and white balls overall, if they were in just one bag you still have to pick them at random, you have a fifty-fifty chance of getting your colour.

Just take S, P, Q and O; you also have evaluated the same explain me why.

First of all for O you do not reckon which colour there are and assume that they are an even selection, on average they will be half, so picking up randomly. There is a fifty-fifty chance of getting the one you want. For P, it is actually the same, you do not know how many of each there are, the fact that you pick one bag or the other does not really matter, on average you will get the same; so you get half. For Q, it is just the same as P but it is the experimenter picking the bags, so assuming that the experimenter knows what you knew; so P would be the same and half and for S it is just the same again, somebody choosing one bag or the other is just as tossing a coin; first of all it does not matter which one you pick if there is the same selection; in both there is a fifty fifty chance whichever one you pick anyway and so I put half as well.

Just Z

Although, yes, first of all, picking a ball at random from the bag..you do not know how many.. I say. We have fifty fifty chance therefore we have a fifty-fifty chance of picking a ball in the second or third bag, since the second and the third bag contains the same and it is a fifty-fifty, so there is no difference which one of 12 so there is a fifty-fifty chance of getting that ball.

O.K. Thank you

Subject 13

Explain to me in general, which kind of reasoning have you adopted in evaluating your lotteries?

It is written down here, do I have to repeat ? I studied a bit of mathematics and a bit of statistics, just I based my guess on something like this; I tried of course; I did not have time to check.

I ask you now to explain to me your evaluation of some particular lotteries. You have given 12 to almost all the lotteries, so I ask you to explain me your evaluation of the lotteries which you have not given 12. You have given 19 and 15 to lotteries T and N just explain me why?

First this one. T first of all; O.K. This one I gave fifty, because I think that in this case here my chance are better than here, where I gave 12. In those I have almost seventy percent of chance to win more than twelve that means 13 pounds, up to 25; I put 12 because of this. In this case my chance are better, so I have more space to go, so I put fifteen, since I have more chance, and it is the same case in N; I have to put three balls I think and then I have to choose and then I choose of course the colour of these three balls, o.k.

Just take please B, U, V, which are this one and this one and.. You have evaluated all of them the same; you have given 12 to the three of them, just explain me why.

I just have fifty percent chance to win and I put 12 because of this criterion, because I have .. percent chance to win more then 12, and this one here then; yes, yes just half percent of win something; ye, half percent.

So for you they are equal?

Yes.

And then take please the S, P, and Q. Just look at them, you gave to Q 10 and to the other two you gave 12 just explain me why?

Yes, the others; I said that because I have half percent, in my opinion of course; and this one, Q I do not know the criteria that the experimenter uses to choose the bag because.. not just to choose the bag of course but what the black balls inside the bags because he has two bags and has put all the black balls here and all the white balls here and he knows, before I go to choose, my opinion, my guess, so if I choose black he can choose the whites; he has just the gain in his hand, so I do not think that is fair.

Now take the V that you have taken before and compare to Q. Not this one .. the percentage sorry; you gave to one 0 to the other 12; just explain me why.

As here it depends on the experimenter he has to choose the bag but he does not know my opinion while here he knows it when he is going to choose the bag but here he does not know my opinion so he has half percent and me too.

That's all, thank you.

Subject 14

I just ask you to explain to me in general which kind of reasoning did you adopt in evaluating the lotteries and then I would ask you some explanations about some group of lotteries.

The price I put on the lotteries is mainly based on probability and I found that most of the lotteries the probability of choosing one colour of the ball is fifty percent, so, since the winner price is 25 pounds, and I think there are fifty percent of the probabilities. The price I like to say is 13; it seems to me suitable I think, for example, if the probability is fifty percent, if you choose one you try twice, maybe you can win 25 pounds, it is 26 so I organized after lotteries I can win one pound so I chosen 13 pounds. Then in some of the lotteries, the probabilities of one colour ball is not clear, since I do not know in each how many colour there are. I do not know how many white colours balls in the box and how many black colour balls in the box, so it is not clear. I think maybe I take some risk further, so just the price would be lower considering the risk, maybe the purchaser thinks the risk will be higher, I put some low prices on lotteries. For others the probability is a little higher or a little lower, the prices are based on the lotteries on the probabilities or uncertainty. This is in general.

So I just ask you to take some of the lotteries; can you take ..if you want to use these ones I put the letters on them so it is easier ..
Just take the B, U, V; you gave to all of them 13 pounds I just ask you

why and if they are equal to you or if at the margin there is a little difference. .

B, this one has two boxes, one contains 12 black balls and the others contains 12 white balls, so if I bet on one of the two colours I think that the probabilities are fifty percent, and this is also.. it is the same one, it is all black and the other is all white so I think that for the two, V and B, the probability is the same, fifty percent and U is ..U, V, B is the same one box contains 12 black balls the other contains 12 white balls, so the probability to choose one colour ball is fifty percent; so V, U and B are the same, so I choose 13 pounds.

Take this one U and this one which is E; you also gave to E 13 pounds; may you just explain me why, look at the two.

The E the box contains 12 balls six are black balls and six are white balls, so counting from mathematics if you choose one ball from the box the probability of choosing one colour ball is fifty percent, so it is equal, the probability is the same fifty percent, so I put the same price.

Then, take I, to this one you gave 10, a little less; just explain me why.

I think the probability to choose one ball of the colour which is my choosing colour, the probability is larger than fifty percent; once contains 13 balls so the number is 12 and I think the proportion of colour put in this box depends on what number you draw from here I think if you ..I mean from six to 12 the total is 7 and the total of the balls is 13 ,this is probably why I choose the colour larger then fifty percent; do you understand what I mean if you choose the minimum number for the probability then larger then fifty percent each is six. That means that if the minimum number I choose is six, I put six black balls into here, in this case, I choose a ball from the box, the minimum probability is fifty percent and if I choose, for example, a number larger than six, for example 7, I put seven black balls inside here; and other five balls in white colour, so I prefer to choose black colour. The probability is larger then fifty percent and the total .. in this moment I choose the number that is larger or equal to six, the probability of this one is 7 over 13, I think, this is larger then fifty percent I choose 10 pounds.

Then just take N and T; to N you gave 15 and to this one, you gave 7, just explain me why please

There are two boxes and one contains 12 balls, six balls are white colour and six ball are black and in the other box eleven balls but I do not know how many white colour balls there are inside here, and how many black balls there are inside here. First, I choose one from this box; if I choose for example a black ball, I put a black ball inside here and now the numbers of the balls is 12 and I think, maybe the probability is ..the number of black balls, the probability of the number of black balls is larger than the white colour ball, it is higher, I compare with this one in which there are 6 white balls and 6 black balls, so maybe if I choose the black balls the probability is larger of fifty percent, so I choose seven I am not sure but I think maybe the probability of the colour I have chosen will be more than for the white ball so I priced 7 pounds here.

And then the other one which is N you put 15.

In this case there are two boxes, all contain 12 balls; there are six black colour balls and six white colour balls inside the box and in the second box there are nine balls, but I do not know what colour each of them, what colour the balls are inside; I know that they can be white or black, but I do not know how many white balls there are inside here and how many black balls, so either the probability I do not know but I had to put inside here 12 balls so I choose, so I should add another three balls inside here and I choose for example three white one two three and put them here, there are three probabilities of the white colour and in this case of course I bet on white colour,ifthree balls are black colour so I will bet on the black colour in these cases the probability to choose what I choose as colour are higher and the last case is that one is white colour and the other two is the black colour so I bet on the black colour, the other is the same the two are white and the one is black and I will bet on the white, so I think that the probability will be higher so I gave fifteen pounds.

And then these five G, F, H, C, A to which you gave 12. Just explain me why in general.

I just used the mathematic in general. I just calculated I think all of them; in these lotteries the probability to choose one colour ball is fifty percent. The probability of each box containing... each box I know how many black balls and how many white balls inside the box, so I can easily calculate the probability to choose one colour of the ball and all of these lotteries the probability of choosing one colour ball is fifty percent. I priced the same price on the lotteries.

And then this one Q, P, and S, you put to all of them five pounds.

In these three lotteries, in each lottery each of the box contains 12 balls but I do not know how many black balls and how many white balls there are in the each, it is just the same I know there are two boxes and each of the box contains 12 balls, but I do not know how many white balls and how many white ball inside the box, so I think I do not know the probability to choose one colour of the ball. I can't make the decision so I think for the lottery buyer it will be a higher risk to bit on one colour so in this case for people psychology, I think, if I do not know anything, I think the risk will be high, so I priced the low price on the lotteries; for example, if I circulate even if I know that there is some short way but I never have been before passed before I prefer the longer way, since it is clear at this is not clear, I cannot take the risk it is the same case.

And all of them are equal to you.

Yes they are equal for me.

And then take this one and this one. You gave to both of them five just explain me why.

In the lottery P there are two boxes and each of the box contains 12 balls but I do not know how many black balls and how many white balls inside the box. I am asked to choose out of the box. I think it is the same, because I do not know how many black balls and white balls inside the two boxes, so for me is the same to choose any of the box. In O, there is just one box and the box contains 12 balls inside but I do not know how many white balls and how many black balls there are inside the box, so in this case the lottery O is the same that lottery P, so I price the same price on the lotteries.

And then this one M you gave as the others you gave 13, explain me why.

In lotteries M there are two bags and one is empty. One contains 12 balls and six are white balls and six are black balls and in this box we have to put two balls and these balls are put by another person a third party; For me, I think for any if I choose to draw a ball from the box, the probability of choosing each of the colour of the balls is fifty percent ,for any person, for the third party, for me the probability is the same, so after I have chosen twice maybe I have chosen one colour, the probability is the same for the examiner for the third party or me it is the same, so I think.

And then this one Z; you gave 13 to this one, just explain me why.

Right this is the case of the three boxes, each of the box contains 12 balls and in the first box I do not know how many black balls and how many white balls there are; here in the second and in the third box I know there are six white balls and six black balls in a box and first I draw one ball from this box, if it is a black one, I put it here, I put the black one here, if it is the white one I put here and before I draw a ball from the box I have to bet on one colour, I can bet on any one I bet on the black one and if I choose black one I put in that here so I choose this box to draw the box to choose the ball. If I choose a black one I know there are 7 black balls inside here, so here it is thirteen, the probability is 7 over 13, so it is larger then fifty percent, so I priced fifteen pound over here.

And then just this one and this one you gave to this 13 and to this 5; just explain me why.

Lottery E is the one that contains 12 balls 6 are black and 6 are white but you choose any one colour of the balls so the probability is fifty percent so I priced 13 pounds; and lottery O is one box and the box contain 12 balls, but I do not know how many black balls, how may white balls inside box so I think the probability I do not know so I think the risk will be higher so I price them low I priced 5 pounds.

Thank you.

Subject 15

I ask you in general, which kind of reasoning you adopted in evaluating the lotteries.

Right, most of them, almost all of them seem to be a straight fifty fifty chance, because you do not know what the balls are going to be, you can't make any decision in which to choose in the first place, so the chance to be correct is fifty fifty, anyway, if I know that there is no way of deciding the only exception to that is Q in which your choice ..the person choosing the bag is you, and it is also after you have plaid your bet at it is also that choice could be made so that you lose, but you do not know that, so I just priced slightly less. And N and T what you actually come to play with the bag of which you know what colour some of the balls are. So you have some slightly chance to decide where to bet, so I priced more for that, and I worked out that the best price for the various lotteries, it actually come out as the probability of winning in the lottery multiply for the 25 pounds and I worked out that for most of the lotteries the best price is 12.50. I actually put down 12.01, but it does not matter as long as it is something between 12 and 13, but not 13. And the others I really did not work out the probabilities but just had a slightly different feelings for the other ones.

I just ask you these two you gave the same evaluations O and E, just explain me why.

Basically, because you do not know what the content of the bag is, as I told you before, you got an equal chance of choosing the right colour; in E there is no bias at all so automatically you got a half percent chance, but in O there is a bias and you do not know, you still do not know which way is the bias so it depends on which you decide to bet in the first place as how likely you are to win and the two chances I think as I worked out will probably cancel out.

And then this three which are B, U and V and you gave to V a slightly lower price, just explain me why you have priced it in this way.

Right, B and U are basically so , again, you do not know which bag is which, so choosing one yourself or tossing the coin is exactly the same, since you do not have any idea you got an equal chance of being right. And then there is the thing with the experimenter is choosing the bag, if the experimenter does not know then it is going to be exactly the same as the other three but before you actually play the bet you do not know, that is why I priced rather less, otherwise it would be the same.

And then this one Z you gave also 12.

This is very similar to the three we have just talked about. Although it does not really matter which bag you choose, both the two bags where you draw the second ball from are identical so it does not matter which you choose in the first place and when you come to choose the ball there are equal numbers so equal probabilities again of the right colour.

And then U and E.

Right. In E you got straight fifty percent chance to get the right colour again because there are even chances again for each colour, in U, it can be fifty percent chance of being correct because of a slightly different reason; basically you do not know which bag contains your chosen colour, although you got again half and half chance of choosing the correct bag They are two completely different mechanisms but they lead to the same chance again.

And then just this one M, you also gave 12.01.

O.K. It is again fifty percent because you have no idea of what is in the bag in the first place, and so you 've got again an equal chance of choosing the right colour in the first place the content of the bag can be all black but you got fifty percent of chance of choosing black, also in that case, if it is a black and a white in there then you got a fifty percent chances of picking black afterwards and the same with white.

Subject 16

I just ask you in general to explain me the kind of reasoning that you adopted in evaluating your lotteries.

I tended to bet more money when I thought that there was a higher chance of winning, and I based on a sort of rough percentage of chance in my head of getting one of the same colour ball, twice. Just I guess.

I would ask you now about some lotteries in particular, please take the E lottery. You have evaluate 15.

This is the one in which you have guaranteed fifty chance of that number. Oh This is the one in which you have guaranteed fifty percent chance of what you bet, the colour you choose; there is only one bag with 12 balls half black and half white, so you are guaranteed certainty, so I just priced a little higher of what the chance was.

And then take D. You gave to this one a slightly higher value than the previous one. Just explain me why.

I just priced higher because since there are two bags, I know that there is a fifty fifty guaranteed chance, but there are two bags and I thought it might be a little easier.

Just take the B, U, and V. You priced B 18 U 15 V 14, just explain me how you have evaluated this lotteries.
B and V I gave almost the same price because in B you have to flip the coin, so you do not have any control, it iscompletely down to chance what is going to happen so even thought you got 50 percent chance relying on the coin, I figured to bet a little higher since there is no decision at all on my part, it is pure chance.
V is quite similar because I did not feel like pure chance as tossing a coin And U this one I priced slightly lower because you have to choose the colour before as well as a bag and I decided that this decreased my chances.

And then just take F, H, C, G, A, F and G; you put 16 to A, 14 to G, 16 to18, and to H to C 20. Just look at all of them and explain me why you gave these evaluations.

F and G I gave the same price to because there are at least fifty percent chances of getting right and I put higher because for G, I decided it was slightly higher than 50 percent chance, because there are so many bags, even though more than half of them have half black or higher;, so there are quite a chance I thought, because there was more then one bag. I put the same price, because I thought again that the chance was at least fifty percent or higher because two out of the three bags have guaranteed fifty percent chance or higher of getting the colour you choose. For A the chances were certainly greater then fifty percent, just because half the bag or more have one ball,almost fifty percent chance or greater of getting the colour you choose and I did not quite evaluated as higher than G because there was no so many bag, so I thought my chances maybe were slightly lower. And for H, you have at least fifty percent chance or more in half the bags of getting the colour you bet on; I priced a little higher than the others, I gave a higher value to C because there are more bags so you get more chance than with H.

And then please take I you gave 10 just explain me why.

I gave less then half value, because I thought that my chance of getting the right colour were slightly less than fifty percent. I think that was the numbers in such to ..me up because I think that there were more things to take into consideration.

And then T and N. To T you gave 9 and to N you gave 17 just explain me why.

T I gave 9 to because I just thought that by drawing a ball and putting one in the other bag you possibly have less then 50 percent when you draw from the second bag, the second ball would be of the same colour, I gave a slightly higher value to N because the chances are slightly more than fifty percent, because you have seen some of the balls so you can make an educated guess of what colour might be.

And then R, F, and L; you gave 19 to ..and 12 two the others two.

L and F, there are at least two bags in which there is a fifty percent chance of getting the one you choose and I put the same because there are at least fifty percent chance you choose the one you have chosen at least in two out of three bags. I gave a slightly higher value out even if fifty percent chances are all across in each bag, in each bag there is fifty percent chance, so I gave it a slightly higher value because whatever bag you choose is going to be fifty percent chance, while in the other it depends on what bag you choose.

And then these, S, Q and P; you gave to S 12, to P 10, and to Q you gave 5; just explain me why.

To S I gave slightly higher value because it is pure chance, since you flip a coin, it is a coin toss, so I do not have any decision in it; for P, I thought the chances were less than fifty percent because you did not know how many balls there are in the bags, so I gave a slightly lower value to that. Q you do not have any decision in it it is all up to the experimenter and it was not pure chance so I might give a slightly less value just in case I do not know; in case the experimenter knew or there are too many unknown values so I gave 5 to it.

And then take P and O, you gave to O 8, and to P 10.

I gave P 10 just because I thought that the chances were less than fifty, but because there are more choices it might be slightly higher.

To E you gave 15 to the other one 8 just explain me why

To E, I gave 15 because I have guaranteed fifty fifty which one you choose, with O you do not know how many balls of each colour, while with E you do know, in the other one you do not have any idea so I gave that one less as a value since you do not know.

Now this one you gave 13; just explain me why.

I gave 13, because no matter what the first draw, you still get fifty percent chance, no matter what colour she draws out because there are still six blacks and six whites.

And this one Z you gave 12; just explain me why.

I gave that just slightly lower than Z just because there is one bag in which you do not know how many balls, how many of each colour there are in it, while with the other two bags you do so, still the probability of the second bit is still roughly fifty fifty, so I priced a little less because there is one bag in which you do not know how many balls, how many of each.

O.K. Thank you.

O.K.

Subject 17

I just ask you in general which kind of reasoning did you adopted in evaluating the lotteries.

I analysed them and I found they look different, but that from the point of view of probability they are the same, and I think that in lotteries there is like a fifty percent chance to win. And this is why I evaluated all of them half of the price, since the price is 25, I evaluated them 12.50. That is all.

I will show you some lotteries and then I will ask you why you have evaluated the same.

Take this one and B U and V. You have evaluated the same so just please explain me why.

Take this there are two bags and any bag contains the same number of balls, which one is black and one is white. And you have to bet on one of the colours and then you choose a bag. And if you said black, yes, you have to choose the bag and you can choose the bag with white balls or black, so you have a fifty percent of each one. And the same is with the colour because if you choose black then if you chose black then the bet is on you but it is again a fifty percent chance.
And this one is exactly the same . Here you choose the bag and here the experimenter chooses the bag, but you do not know which bag is which

one, so it is again fifty percent chance, I do not know if the experimenter can know, but I do not know if it was like this, this one. But on your point of you it is again fifty percent even if the experimenter knows which bag it is and you do not know and the experimenter can help if she knows.

And B..

B is with the coin, the probability of which side of the coin falls down again to half, which is fifty percent. it is again the same whichever bag you will choose white or.. or on your left side or in your right side, I think it is fifty percent in all of them. The difference is that there are more bags or less bags but I think they are going to the same point.

Just take N and T you also have evaluated 12

Again when you draw this ball I think the probability is still fifty percent. In this you have still six white balls, I think that it was again fifty percent. In the second bag, in the one you have 11, you do not know anything about those and you know that they are either black or white, you do not know the colours of these balls and there can be five blacks and six whites and I mean.. I think you can't judge any I think that is still fifty percent because ..

And this one

Here you have 12 balls that can be white or black but because you do not know the number, the best is I think to put fifty percent. You can loose of course as well.

And this one which is Z.

First you ask to bet on one colour and then you draw those balls if you are like.. you got three bags the second and the third are like six and six which is saying fifty percent and the first one you do not know and again I think that is best saying fifty percent.

And then O and P you have evaluated them the same just explain me why.

Because I do not think that if you consider two bags it changes. You got only the information that there is a twelve box balls and they are blacks or whites and you are asked to bet on one of the two colours and you choose a bag, it does not mean anything, it is just a fifty percent that you can choose a bag in which for instance says six plus six and here it is just one bag I don't think it changes anything that there is one bag or two bags.

O.K. Thank you.

Subject 18

I will ask you in general which kind of reasoning did you use and how you have evaluated them.

I was looking at all the bags and whatever lottery it reduces to white or black and you got basically in every lottery a fifty percent chance of having the right colour or having the wrong colour. These were exactly the same I think.

Just take this one and..

They are just saying the same with other words.

O.K. I just put letters because it is easier for me to recognize them through the tape. I will ask you about some lotteries in particular. Then you explain me how you have evaluated them.
Just take this and this one .. this and that one, which are E and U; you gave of course the same value, may you explain me why?

I mean, there are twelve balls and they are equal of each colour so you have fifty percent of having the wrong colour or right colour. And it is the same but you have ..it is like having one bag with 24 balls and taking a ball from that it is the same has having a bag with 12 balls six black and six white and it is just the amount of balls which changes, but for the probability it does not chance anything.

And then these three.

They are just the same there is two bags one with 12 blacks and the other with 12 whites and this one ...it is again the same, I mean if you choose a colour or if you toss a coin or you take it is fifty percent chance.

And then just look at this one which is O.

In one experiment you do not know how many balls from each colour but if you do n experiments, a large number, then, you will have six blacks and six whites and you are choosing randomly.

And then this one which is Z.

In the first bag it is like over here having an experiment in which you do not know which colour and the other two bags you know that they are fifty percent black and fifty percent white.

And then this one which is M.

In each case you have.. because you put the bag back you have fifty percent of chance having a white ball and a black ball in the next bag so you have fifty percent chance of choosing the right colour from the second bag.

And then these two, T and N.

I was thinking in the second bag, you will have on average four and half balls of each colour, and in the first bag you have you draw three balls and it means if you take, for example, hundred of experiments, you will have the chance of .... you will have one and half ball of each colour, which adds up again to six balls of each colour in total. This is the same you have another five and half balls of each colour and when you draw one ball, which is half colour of each it adds again up to six.

O.K.

Subject 19

I just ask you in general which kind of reasoning did you apply in evaluating your lotteries.

It was not my problem; my only concern was that it was a lottery, a game of chance, anything can happen, so I was concerned whether I get it or not. I enjoyed the process of trying to see if my guess was correct or not, and that gave me a pleasure; on the other hand, if I get some money I would love it; I am not too concern on how much I get, though sometimes if I look at the process and I say if I finally succeed I should get a very good amount So I tend to put a higher amount on the processes which I think last long so, if I succeed and I get a very good amount, eventually, for me, then, I love it; but for some of those, sometimes, I think, well, I may loose or gain, so there is no point in me putting a higher amount, provided that I get something. Although, if you think, this reasoning is very inconsistent I have tried to tell you. For me I am not putting my money in, so I do not bother with loosing or gaining but I would be excited, as I said, if I make a choice and it come to me and it will be fine for me; so the prices that I put are not necessarily determined by the reason, by the extent of effort put in it, not the effort, the pleasure.

So I will ask you about some lotteries and and I will ask you why you have evaluated in this way. Take this one and this one, E and D, you evaluated E 15, and D 12. Just explain me why.

Yes, by accident, E was the first one I did and I thought .. o.k. .. it is 25, so I thought to go midway, well if I get 15 out of 25 is good. So it is a good minimum. In D, I compare with what I put with E and I said well if it is too high I could loose, let me come down a bit and give some chance for increase the room of my chance of getting something and not loosing completely, so I went down a bit.

Do you find them different in some way ?

Not exactly these two in particular, not exactly, because I think though it may last shorter, it did not work for me like that I took as a pure matter of chance.

And then I will give you these three B, U and V; you gave 20, 18 and 13; just explains me why.

I think this one in particular, I thought in the process, I tend sometime to get in love with some of the description because even thought they sound similar I tent to feel more emotionally attracted and I increase the amount because I said, this is a precious think, so probably that was one of the reasons why I put 20 for B. As I said, it is pure a matter of chance, sometimes the interest in money also influences; so not to give too high and loose I tend to come down for some of them, so it is purely a question of chance. V 18. Sometimes I am influenced by what I put down previously, especially when I am going to put a price near one close to something I have already written; I am not putting everything 20 20 20 or 15 15 15 and I ask my self shall I go up a bit or shall I go down a bit so these are external influences that come from the amount I have listed before. So sometime is not that I want to give this price or not but just that I want a variety of my choices. So ..

And now take two these and just tell me if you find them equal or different. Anyway you have given different value to them but just tell me if you gave these different prices because you wanted variety or just you find that some of them were different.

As I said I found some of them different, those which I found more elaborated I tend to give higher prices, so if there are any differences that could be, but as I said I was not consistent with my prices as I consider purely as a game of chance; it is not a thing that I am actually bargaining for me; a lottery is a game of chance, so it is not something that I ask to consider the amount of effort, similarity, as so one I think that it depends on luck and no matter how much effort you put in it, so the differences are only a question of variety as I said, I think so, and not in term of similness I do not know if it is a good word.

And then look at these two and just tell me your feelings about them, you gave the same value to them but just give me the feelings you had in reading them.

I think that in these case I consider them equal or very close so I was conscious of similarity and so I gave to them the same price ..but generally given the chance, I do not think I would stick to that but as I said sometimes I consciously compared them and sometimes I did by random.

And now just other two, look at them, you gave very close value, 14 and 15.

In this case, it was not because they are similar one, it is very elaborate and it took quite a long time to read it but at the end I found that the effect following the analysis is the same for both of them, so as I said before I classified them consciously in a similar range. Elaborateness could give me more money but then again you can loose and less elaborateness it costs less but then again if you win is a win so it is good so it can be as good as the other one or as bad as the other one.

O.K. And then these three you gave 12, 10, 15.

I remember that in the process I found that I gave 15, 17 and so on so I have to give something less then half and if someone find himself in a very critical situation one might accept anything that goes on, so even though fifty percent is reasonable, I might go down a bit to forty percent if you give yourself the benefit of the doubt.

And this one Z

You gave 15, I do not remember which one, but occasionally I picked one and I remember I picked something which is similar so I tend to go back to see how much I gave to the previous one and I compare to see if I could award the same price to that one, so I presume that must have been the reason, probably I was trying to make it close with something else.

Subject 20

I ask you in general which kind of reasoning did you adopt in evaluating your lotteries.

I like better lotteries where there is always a chance of picking the right colour from the bag, the chosen colour from the bag. Among these I prefer those which offer a fifty percent chance of winning. When there is a chance of picking a bag where the chosen colour does not exist, I ordered lotteries according to the probability of the bag with the wrong colour. The most preferred is the one in which probability is 1 over 11 and the least preferred is the one in which the bag is 1/2. I would not like to play lotteries where the proportion of balls in some or all the bags is unknown or, and randomly determined. In general the higher is the degree of casuality built in the lottery the lesser I like to bet on it and I evaluated the lottery three, three two and one according to this criterion.

So I ask you some lotteries in particular; take lottery E and D you have evaluated 13 just explain me why.

The same, because they are exactly the same to me, because if I choose D, there are two bags which have six white and six black if I choose one bag, it is like the one with one bag six whites and six blacks.

Then take B, U and V, you gave all of them 5.

Because it does not make any difference to me if I choose the bag, the experimenter chooses the bag or it is a coin or the bag is chosen by a random device. For me it does not make any difference, so it is why I evaluated the same.

And then take F, A, G and H and F, you have evaluated 5, A you have evaluated A 7, G 8 and H 9.

Because here there is the chance of getting... a chance of picking one bag in which the colour that you have chosen it does not exist. And in this case for me, the most preferred one is the one that has one over eleven as chance of getting that bag, and I did prefer the one who has the chance of one third, so it is one eleventh, one seventh, one fifth and one third, basically even if all these are the same I might end with picking up the right or the wrong bag, the bag with the wrong colour and I am interested in the probability of picking that bag, I do not care about the others.

And then just take C and I.

The crazy one.

You gave to both of them just two; just explain me why.

Because here these are ...just a second they are the same ...but there is another one to which I gave two.

Yes it is here.

Yes it is because is just that percentage of whites and blacks is not known and also it looks also out of me to take the wrong or the right bag. The difference between these two is just that the percentage is determined randomly in both with different procedure. But they are the same to me.

And then take T and N.

You gave to both of them 3; just explain me why.

Again it is just here at least you have one bag in which you have ..at least which has a known percentage of balls of both colours and you know that it is half and half, and this is why it is slightly preferred to the one in which the colours are completely unknown; here you have a chance of picking from a bag in which there is a known percentage of the colour, so that is why I is just slightly better to me than the other, but not so much still in my evaluation what counts more is that there is one bag in which the percentage is unknown.

Just take L, R and F again. And then you gave to F 5, to L 10 and to R 15.

Now because to F 5, because in F there is the bag with balls of one colour which I do not like; in R: R is the one I like better because you can never pick the wrong bag, in every bag you have a half and half percentage of winning and in L you still have ..L is the same as R in the sense that you always.. again you necessarily will pick a bag in which there is the right colour but the probability of picking that colour is lower.

And just take S, P and Q, and then O.

First look and S, P and Q and you have evaluated two all of them anyway. Just explain me why.

The two is because I evaluated two when I do not know the percentage of the thing, the percentage of the colour and the thing is that it does not matter to me if just you do not know or well if you do not know the colour then again, the things is that it does not make any difference to me whether I choose the bag the coin chooses it or the experiment chooses it; for me it is the same, because in any case I do not know the percentage in the bag
The O it is exactly the same for the same reason, it does not matter to me .. the other case here ... the outcome when there is one bag is the same answer, I cannot improve if there is one bag or two bags to be chosen if the two bags are equivalent to these one in this lotteries.

And then just take Z.

So this complication does not effect my choice because it is a clear complication when the complication is like in Z and in M it affects my choice because it is just so doggy that I would never bet on these two so that kind of complication does not affect my choices but this. So I evaluated them just because I do not want to play them never; actually I could have evaluated them O.

But because they were complicated or because they were both complicated and unknown.

Because there is so much, there is so much so much it seems I mean obviously the expected value is the same for both but I mean there is to much chance built into it chance on chance on chance chance on chance that you know I just do not want to play.

O.K
Thank you.

Subject 21

I ask you in general to explain to me which kind of reasoning did you adopt in evaluating your lotteries.

I reasoned basically from probabilistic terms and what I find to do is to give to each game first the first game is a randomly selected number between 1 and 25 and you win if it is greater than the value that you assigned to the game and then you are going on to the second game which is the actual game in itself and then working on the expected value ..expected wins of the whole process depending on the value that you gave to this game and then optimizing the winning just optimizing the expected winnings the expected q; then there are some of the games in which you have to take into account well non exact things; so when the experimenter was involved you were unsure how they are going actually to set up, because it would be possible actually break the game, so I was a bit cautious. Some of the highest.. I evaluated one more then 12. This is due to the fact that you know some information about how the bags were sorted, you knew that there were one more black or two blacks and white, three blacks, so I actually increased the expected winnings that you have so..

I just ask you to look at few lotteries and then I will ask you again why you have evaluated them in some way. I just ask you the E and then the U one; you have given the same value, just explain me why.

Well, it is that irrespective of what bag you choose, both have the same proportion of blacks and whites six of each, so choosing any of the bag you have fifty percent chance of actually getting a white or a black, so it is just the same which colour you want bet on, you got half half chance of getting it.

And the U one and E.

This is the two bags. This one is one E you have six blacks and six whites so you have a fifty percent chance of getting the colour that you have decided, so I gave 12 pounds, so it seems a fifty fifty chance. Well in this one the difference is that you have a fifty fifty chance of actually choosing the bag and they are of one colour, once you choose the bags it is pretty down which colour you get, what ever comes in the bag, it is a different position choosing a colour because choosing a bag in that situation it does not make any difference whatsoever.

And then just take that one and the E one again you have evaluated the same just explain me why.

It is fifty fifty, there are six whites and six blacks in one bag, this is because choosing any ball of the bag you do not have the knowledge if it is either black or white, so you assign equal probability to either of those outcomes, you faithfully choose from a bag but if actually the are a fifty fifty... you do not know any information about which colour is determined, actually there is an all spectrum that you can have, there can be 12 black and going through till one with six blacks and six whites in a bag .....come up to still fifty fifty due to symmetry, I mean because you can substitute if you have a sample of 13 possibilities then you can substitutes all blacks for whites etc. and if you choose black or white is going to be a fifty fifty, it is symmetric.

And B and E you have evaluated the same.

In this one you actually choose the bag; it is the bag which determins the colour that you get. It is fifty fifty, this is similar but the decision of what bag is determined by tossing a coin it is fifty fifty, it is random you have a fifty fifty choice it is the sense of perception..

and them M.

Basically there are two possibilities two blacks and two whites, well three possibilities one black and one white; so the chance is fifty, each ball going into the bag has a fifty fifty chance or being black or white black or white so is effectively a bag of... it is similar to the 12 balls in which you do not actually know which colour they are; it is identical only there are two, so you still have a fifty fifty; chance it would not be the same if the bag contained the bag from which you have the drowns off contains 9 black and three whites, so you will have more chance in on black so you will bet on black if you knew it and you will have a higher expected winnings, but they were six blacks and six whites if they were all blacks then you had hundred percent so.

O.K. thank you.

Evaluation of the lotteries relevent for the section III.3.4.1.

Evaluation of the lotteries U, D, F, A, G, H, C, according to (I.20) where $p_1 \leq \ldots \leq p_n$.

Lottery U

$$U = \left(L_1(25,0;\ 0,1),\ 1/2;\ L_2(25,1;\ 0,0),\ 1/2\right)$$

$$V(U) = u(25)\ f(0) + u(25)\ [f(1) - f(0)]\ f(1/2) = u(25)\ f(1/2).$$

Lottery D

$$V=\left(L_1(25,1/2;0,1/2),1/2;\ L_2(25,1/2;0,1/20),\ 1/2\right)$$

$$V\ (D) = f(1/2)\ u(25) + [\ f(1/2) - f(1/2)\ ]\ f(1/2)\ u(25) = f(1/2)\ u(25).$$

Lottery F

$$F= \left(\ L_1(25,0;0,1),1/3;\ L_2(25,1/2;\ 0,1/2),\ 1/3;\ L_3(25,1;0,0),\ 1/3\ \right)$$

$$V(F)= u(25)\ f(0) + u(25)\ [[f(1/2)-f(0)]\ f(2/3) + [f(1)-f(1/2)]\ f(1/3)]=$$

$$= u(25)\ f(1/2)\ f(2/3)\ [\ 1-f(1/2)]\ f(1/3).$$

Lottery A

$$A=\left(L_1(25,0;0,1),1/5;\ L_2(25,3/12,\ 0,9/12),1/5\ ;\ L_3(25,1/2;\ 0,1/2),\ 1/5;\right.$$

$$\left.L_4(25,9/12;0,3/12),1/5;\ L_5(25,1;0,0),\ 1/5\right)$$

$$V(A) = u(25)\ f(0) + u(25)\ [[f(3/12)- f(0)]\ f(4/5) +$$
$$[f(1/2)-f(3/12)]\ f(3/5)+[\ f(9/12) - f(1/2)]\ f(2/5) +$$
$$[f(1)-f(9/12)]f(1/5) =$$
$$=u(25)[[f(3/12)f(4/5)+[f(1/2)-f(3/12)]f(3/5)+$$
$$+[f(9/12)-f(1/2)]f(2/5)+[f(1)-f(9/12)]f(1/5)]$$

Lottery G

$$G=\Big(L_1(25,0;0,1),1/7;\ L_2\ (25,3/12,\ 0,9/12),1/7\ ;\quad L_3(25,5/12;0,7/12),1/7;$$

$$L_4(25,1/2;\ 0,1/2),\ 1/7;L_5(25,7/12;0.5/12),1/7;\quad L_6(25,9/12;0,3/12),1/7;$$

$$L_7(25,1;0,0),\ 1/7\Big)$$

$V(G) = u(25)\ f(0) + u(25)\ [[f(3/12) - f(0)]\ f(6/7) + [\ f(5/12) - f(3/12]$
$f(5/7) + [\ f(6/12) - f(5/12)]\ f(4/7) + [\ f(7/12) - f(6/12)\ ]\ f(3/7) +$
$+ [\ f(9/12) - f\ (7/12)\ ]\ f(2/7) + [\ f(1) - f(9/12)]\ f(1/7) =$
$= u(25)\ \{f(3/12)\ f(6/7) + [\ f(5/12) - f(3/12]$
$f(5/7) + [\ f(6/12) - f(5/12)]\ f(4/7) + [\ f(7/12) - f(6/12)\ ]\ f(3/7) +$
$+ [\ f(9/12) - f\ (7/12)\ ]\ f(2/7) + [\ f(1) - f(9/12)]\ f(1/7)\}$

Lottery H

$H=\Big(\ L_1(25,0;0,1),1/11;\qquad L_2(25,1/12;0,11/12),1/11;$

$L_3(25,\ 2/12;0,10/12),1/11;\qquad L_4(\ 25,3/12;0,9/12),1/11;$

$L_5(25,5/12;0,7/12),1/11;\qquad L_6(25,1/2;\ 0,1/2),1/11;$

$L_7(25,7/12;0,5/12),1/11;\qquad L_8(25,9/12;0,3/12),1/11;$

$L_9(25,10/12;0,2/12),1/11;\qquad L_{10}\ (25,11/12;0,1/12),\ 1/11;$

$L_{11}\ (25,1;0,0),\ 1/11\Big)$

$V(H) = u(25)\ f(0) + u\ (25)\ \{\ [\ f(1/12) - f(0)]\ f(10/11) +$
$+ [\ f(2/12) - f(1/12)]\ f(9/11) + [\ f(3/12) - f\ (2/12)]\ f(8/11) +$
$+ [\ f(5/12) - f(3/12)]\ f(7/11) + [f(1/2)-f(5/12)]\ f(6/11) +$
$+ [\ f(7/12) - f(6/12)]\ f(5/11) + [\ f(9/12) - f(7/12)]\ f(4/11)+$
$+ [\ f(10/12) - f(9/12)]\ f(3/11) + [f(11/12) - f(10/12)]\ f(2/11) +$
$+ [\ f(1) - f(11/12)]\ f(1/11)\} =$
$= u\ (25)\ \{\ [\ f(1/12)\ f(10/11) + [\ f(2/12) - f(1/12)]\ f(9/11) +$
$+ [\ f(3/12) - f\ (2/12)]\ f(8/11) + [\ f(5/12) - f(3/12)]\ f(7/11) +$

$+ [f(1/2)-f(5/12)]\ f(6/11) + [\ f(7/12) - f(6/12)]\ f(5/11) +$

$+ [\ f(9/12) - f(7/12)]\ f(4/11) + [\ f(10/12) - f(9/12)]\ f(3/11) +$

$+ [f(11/12) - f(10/12)]\ f(2/11) + [\ f(1) - f(11/12)]\ f(1/11)\}$

Lottery C

$C= \{ L_1(25,0;0,1), 1/13;$ $L_2(25,1/12;0,11/12),1/13;$

$L_3(25,\ 2/12;\ 0,10/12),1/13$ $L_4(25,3/12;0,9/12),1/13;$

$L_5(25,4/12;0,8/12),1/13;$ $L_6(25,5/12;0,7/12),1/13$

$L_7(25,1/2;\ 0,1/2),1/13;$ $L_8(25,7/12;0,5/12),1/13;$

$L_9(25,8/12;0,4/12),1/13;$ $L_{10}(25,9/12;0,3/12),1/13;$

$L_{11}\ (25,10/12;0,2/12),1/13;$ $L_{12}(25,11/12;0,1/12),1/13;$

$L_{13}\ (25,1;0,0),\ 1/13\ \}$

$V(C) = u(25)\ f(0) + u(25)\ \{[f(1/12) - f(0)]\ f(12/13) +$
$[f(2/12) - f(1/12)]\ f(11/13) + [\ f(3/12) - f(2/12)\ ]\ f(10/13) +$

$+ [\ f(4/12)- f(3/12)]\ f(9/13) + [\ f(5/12) - f(4/12)]\ f(8/13) +$

$+ [\ f(6/12) - f(5/12)]\ f\ (7/12) + [\ f(7/12) - f(6/12)]\ f\ (6/13) +$

$+ [\ f\ (8/12) - f(7/12)\ ]\ f(5/13) + [f(9/12) - f(8/12)]\ f(4/13) +$

$+ [\ f(10/12) - f(9/12)]\ f\ (3/13) + [f(11/12) - f(10/12)]\ f(2/13) +$

$+ [f(12/12) - f(11/12)]\ f\ (1/13) =$

$= u(25)\ \{f(1/12)\ f(12/13) + [f(2/12) - f(1/12)]\ f(11/13) +$

$+ [\ f(3/12) - f(2/12)\ ]\ f(10/13) + [\ f(4/12)- f(3/12)]\ f(9/13) +$

$[\ f(5/12) - f(4/12)]\ f(8/13) + [\ f(6/12) - f(5/12)]\ f\ (7/12) +$

$+ [\ f(7/12) - f(6/12)]\ f\ (6/13) + [\ f\ (8/12) - f(7/12)\ ]\ f(5/13) +$

$[f(9/12) - f(8/12)]\ f(4/13) + [\ f(10/12) - f(9/12)]\ f\ (3/13) +$

$+ [f(11/12) - f(10/12)]\ f(2/13) + [f(12/12) - f(11/12)]\ f\ (1/13).$

Taking $f(p) = p^2$ and evaluatind the above lotteries accodrdingly we obtain the following values:

Lottery B and D

$u(25)\ (1/2)^2$ $(1/2)^2 = 0.25$ $u(25) = 25$ (Assuming linearity)

$25\ .\ 0.25 = 6.25.$

Lottery F

V(F) = u(25) . 0.194 4.84

Lottery A

V(A) = u(25) . 0.175 4.37

Lottery G

V(G) = u(25) . 0.171 4.27

Lottery H

V(H) = u(25) 0.169 4.22

Lottery C

V(C) = u(25) . 0.160 4

Table AB.1 Values of the lotteries used in the experiment for different values of t using $f(p) = p^t$ and u(1) = 1 and u(0) = 0

t	B	F	G	A	H	I
1	0.500	0.500	0.500	0.500	0.500	0.500
1.5	0.354	0.317	0.302	0.299	0.297	0.296
2	0.250	0.194	0.175	0.171	0.170	0.168
2.5	0.177	0.117	0.099	0.096	0.095	0.093
3	0.125	0.069	0.055	0.053	0.052	0.051
3.5	0.088	0.041	0.031	0.029	0.028	0.027
4	0.063	0.024	0.017	0.016	0.015	0.015
4.5	0.044	0.014	0.009	0.009	0.008	0.008
5	0.031	0.008	0.005	0.005	0.004	0.004
5.5	0.022	0.005	0.003	0.003	0.002	0.002
6	0.016	0.003	0.001	0.001	0.001	0.001
6.5	0.011	0.002	0.001	0.001	0.001	0.001
7	0.008	0.001	0	0	0	0

APPENDIX C

(Chapter IV)

INSTRUCTIONS
SELF-INSURANCE EXPERIMENT

You are about to participate in an experiment concerning decision making under risk and uncertainty. The purpose of the experiment is to gain insight into certain features of economic behaviour. If you follow the instructions carefully you can earn money, but you may end up not earning anything, other than the participation fee. You will be paid in cash at the end of the experiment. The mechanism according to which you will be paid will be explained at the end of these instructions.
During the experiment you are not allowed to communicate with the other participants. Communication between participants will lead to the automatic termination of the session.

You will be presented with 8 different scenarios regarding the same kind of problem. Imagine that you are concerned about the occurrence of some event. If this event does occur, you would suffer a loss of money. However, you have the opportunity to take some action at some monetary cost. If you take this action, should the event occur, the loss will be reduced to zero. Each scenario differs according to the probability of the potential loss of £10.
Try to think of each scenario as a real situation.

For each scenario you will be asked to state the maximum amount you are willing to pay to reduce a potential loss to zero. For each scenario, you will indicate your maximum willingness to pay through the following auction mechanism. On the screen you will see a description of the scenario. Below the description, at the bottom of the screen, will be displayed a price which will steadily increase. You will indicate your willingness to pay by pressing any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction. The last person to drop out will acquire the right to reduce the loss to zero and he or she will pay the price at which the second-to-last person dropped out.

At the beginning of the experiment you will be given an endowment of £10. At the end of the experiment, after you have revealed your price for all the eight scenarios, one of the scenarios will be selected with a random device and that scenario will be played out for real. The player who dropped out last in that scenario pays the price of the second-to-last person to drop out and hence he or she will be paid £10 less that amount. The other participants will play the selected scenario out and they will be paid according to the outcome.

The experiment is organised as follows:

Step 1
At the beginning of the experiment, you will be given a hypothetical example in order to help you become familiar with the problem and the auction procedure.

Step 2
You will be given the first scenario. You will be allowed a few minutes to think about it.

Step 3
The auction will take place. You will be asked to press a key when the price reaches the most that you are willing to pay; that is when you want to leave the auction.

Step 4
You will be presented with the other seven situations.

Step 5
At the end of the eight sessions a scenario will be selected at random and played out for real. A person will be asked to pick a number from a bag containing eight tickets numbered from 1 to 8. Each number corresponds to one of the scenarios. If number 5 is picked then the experimenter will enter that number into the control computer. At this point, will be displayed on the screen all the prices at which the subjects dropped out from the auction. If you are the last person to have dropped out for the selected scenario you will have to pay the price at which the second-to-last person dropped out and in this way you will acquire the right to reduce your loss to zero. Hence the last person to have dropped out from the auction for the selected scenario will receive £10 minus the price paid to reduce the loss to zero irrespective of the outcome of the played scenario.
Then the selected scenario will be played out for real.

Step 6
The scenario selected will be played according to the following lottery mechanism: there will be an opaque bag containing 100 balls. The number of black balls corresponds to the chances of loss, while the number of white balls corresponds to the chances of no loss. The proportion of black and white balls will correspond to the various probabilities of the potential loss. The selected scenario will be played out for each subject separately. Each one of the participants will be asked to draw a ball from the bag. After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. in a payoff of £10 for the participant who drew the ball. A black ball results in a loss of £10, i.e. a payoff of £0 for the participant who drew the ball.

The mechanism whereby the lotteries will be played in the different scenarios will be explained in greater detail at the end of the practice question. Please notice that after a scenario has been selected and the lottery corresponding to that scenario has been played out, you will be free to check whether the stated probability corresponds to the combination of white and black balls inside the opaque bag.

SELF-PROTECTION EXPERIMENT

You are about to participate in an experiment concerning decision making under risk and uncertainty. The purpose of the experiment is to gain insight into certain features of economic behaviour. If you follow the instructions carefully you can earn money, but you may end up not earning anything, other than the participation fee. You will be paid in cash at the end of the experiment. The mechanism according to which you will be paid will be explained at the end of these instructions.

During the experiment you are not allowed to communicate with the other participants. Communication between participants will lead to the automatic end of the session.

You will be presented with 8 different scenarios regarding the same kind of problem.

Imagine that you are concerned about the occurrence of some event. If this event does occur you would suffer a loss of money. However, you have the opportunity to take some action at some monetary cost. If you take this action you will be able to reduce the probability of the occurrence of such an event to zero. Each scenario differs according to the probability of the occurrence of such an event.

Try to think of each scenario as a real situation.

For each scenario you will be asked to state the maximum amount you are willing to pay to reduce the probability of the occurrence of such an event to zero.
For each scenario, you will indicate your maximum willingness to pay through the following auction mechanism. On the screen you will see a description of the scenario. Below the description, at the bottom of the screen, will be displayed a price which will steadily increase. You will indicate your willingness to pay by pressing any key when the price reaches the most that you are willing to pay; that is, when you want to leave the auction. The last person to drop out will acquire the right to reduce the probability of the loss to zero and she or he will pay the price at which the second-to-last person dropped out.

At the beginning of the experiment you will be given an endowment of £10. At the end of the experiment, after you have revealed your price for all the eight scenarios, one of the scenarios will be selected with a random device and that scenario will be played out for real. The player who dropped out last in that scenario pays the price of the second-to-last person to drop out and hence she or he will be paid £10 pounds less that amount. The other participants will play the selected scenario out and will be paid according to the outcome.

The experiment is organised as follows:

Step 1
At the beginning of the experiment, you will be given a hypothetical example in order to help you become familiar with the

problem and the auction procedure.

Step 2 You will be given the first scenario. You will be allowed a few minutes to think about it.

Step 3
The auction will take place. You be asked to press a key when the price reaches the most that you are willing to pay; that is to say when you want to leave the auction.

Step 4
You will be presented with the other seven situations.

Step 5
At the end of the eight sessions a scenario will be selected at random and played out for real. A person will be asked to pick a number from a bag containing eight tickets numbered from 1 to 8. Each number corresponds to one of the scenarios. If number 5 is picked then the experimenter will enter that number into the control computer. At this point, will be displayed on the screen all the prices at which each subject dropped out from the auction. If you are the last person to have dropped out for the selected scenario you will have to pay the price at which the second-to-last person dropped out and in this way you will acquire the right to reduce the probability of the occurrence of the event to zero. Hence the last person to have dropped out from the auction for the selected scenario will receive £10 minus the price paid to reduce the probability of the occurrence of the event to zero irrespective of the outcome of the played scenario.
Then the selected scenario will be played out for real.
Step 6
The scenario selected will be played in the following way: there will be an opaque bag containing 100 balls. The number of black balls corresponds to the chances of loss, while the number of white balls corresponds to the chances of no loss. The proportion of white and black balls will correspond to the various probabilities of the occurrence of the event. The selected scenario will be played out for each subject separately. Each one of the participants will be asked to draw a ball from the bag. After every draw the ball will be replaced before the next subject draws another ball. A white ball results in no loss, i.e. in a payoff of £10 for the participant who drew the ball. A black ball results in a loss of £10, i.e. a payoff of £0 for the participant who drew the ball.

The mechanism whereby the lotteries will be played in the different scenarios will be explained in greater detail at the end of the practice question. Please notice that after a lottery has been played, you will be free to check whether the stated probability corresponds to the combination of white and black balls inside the opaque bag.

Example of self-protection scenario:

Risky scenario

Assume that there is a risk of 50% that some event occurs. If this event occurs you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce the probability of the occurrence of such an event to zero.

You will be asked to press any key when the price reaches the most that you are willing to pay; that is when you want to leave the auction.

Ambiguous scenarios:

Best estimate

Assume that there is a potential risk of of the occurrence of some event. There is an estimate of the occurrence of this event; an expert, hired by a Governmental Agency, estimates that the probability of the occurrence of such an event is 50%.However this is the first investigation ever carried out and consequently you experience considerable uncertainty about the precision of this estimate. If this event occurs. you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce the probability of the occurrence of such an event to zero.

Interval probability

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the possible occurrence of this event; an expert hired by a Governmental Agency, estimates that the probability of the occurrence of such an event can be anywhere between 35% and 65%. If this events occur , you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce the probability of the occurrence of such an event to zero.

Set of probability

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the possible occurrence of this event; an expert, hired by a Governmental Agency, estimates that the probability of the occurrence of such an event can be anywhere between 35% and 65%. If this event occurs, you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce the probability of the occurrence of such an event to zero.

Example of self-insurance scenario

Risky scenario

Assume that there is a risk of 50% that some event occurs. If this event occurs you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

Ambiguous scenarios:

Best estimate

Assume that there is a potential risk of of the occurrence of some event. There is an estimate of the occurrence of this event; an expert, hired by a Governmental Agency, estimates that the probability of the occurrence of such an event is 50%.However this is the first investigation ever carried out and consequently you experience considerable uncertainty about the precision of this estimate. If this event occurs, you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

Interval probability

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the possible occurrence of this event; an expert hired by a Governmental Agency, estimates that the probability of the occurrence of such an event can be anywhere between 35% and 65%. If this events occur , you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

Set of probability

Assume that there is a potential risk of the occurrence of some event. There is an estimate of the possible occurrence of this event; an expert, hired by a Governmental Agency, estimates that the probability of the occurrence of such an event can be anywhere between 35% and 65%. If this event occurs, you will suffer a loss of £10.

You are now asked to state which is the maximum amount of money that you would be willing to pay to reduce this potential loss to zero.

Table AC.1. Risk and ambiguity attitudes for the self-protection group

Probability 0.03

	AA	AN	AP
RA	13	2	9
RN	1	5	2
RP	2	3	1

Probability 0.2

	AA	AN	AP
RA	13	2	5
RN	4	3	1
RP	4	2	4

Probability 0.5

	AA	AN	AP
RA	11	2	9
RN	1	4	2
RP	5	0	4

Probability 0.8

	AA	AN	AP
RA	6	2	3
RN	2	5	1
RP	14	2	3

AA Ambiguity aversion
AN Ambiguity neutrality
AP Ambiguity proneness

RA Risk aversion
RN Risk neutrality
RP Risk proneness

Table AC.2 Self-insurance best estimate

R3	A3	R20	A20	R50	A50	R80	A80
1	2	182	159	652	500	902	720
25	20	150	175	506	454	804	761
60	16	175	125	290	453	753	700
33	75	200	230	490	450	200	180
315	105	799	500	499	499	683	799
4	41	199	150	502	300	799	699
11	11	195	187	451	101	500	501
48	50	199	200	201	250	501	301

Table AC.3 Self insurance interval probability

R3	A3	R20	A20	R50	A50	R80	A80
94	100	202	264	500	521	800	362
100	151	210	250	351	560	710	290
199	175	195	190	199	199	500	291
114	157	262	283	413	503	603	511
30	50	205	212	510	500	750	296
250	202	300	301	351	401	401	400
224	206	402	405	385	399	501	278
100	101	200	209	400	500	450	251
2	40	411	457	356	512	786	814
100	149	370	450	530	500	700	750
60	70	220	210	500	501	815	700
20	36	184	303	615	624	815	705
35	21	300	205	600	550	825	825
30	25	200	200	500	500	800	800
5	105	334	443	600	553	120	803

Table AC.4 Self-insurance set of probability

R3	A3	R20	A20	R50	A50	R80	A80
30	30	200	200	501	500	803	801
35	3	200	200	500	500	800	801
31	31	201	201	501	500	801	801
101	70	209	203	150	401	224	221
154	150	402	377	520	555	652	626
25	10	233	240	250	575	705	450
19	76	160	175	330	660	825	810
140	161	299	332	501	530	701	701
91	95	275	425	560	575	765	820
6	100	180	301	606	349	707	750
2	2	150	150	300	800	950	150
1	1	4	5	100	49	500	200
1	1	201	205	475	300	697	701
1	1	320	531	1	80	2	700
36	184	502	531	816	850	907	911

Table AC.5 Self-insurance control group

R3	A3	R20	A20	R50	A50	R80	A80
1	1	50	50	150	250	400	399
24	25	303	233	653	555	853	779
58	100	280	413	613	705	699	718
199	175	195	190	199	199	500	291
6	30	150	77	455	403	601	392

Table AC.6 Self-protection best estimate

R3	A3	R20	A20	R50	A50	R80	A80
30	10	170	20	160	500	60	200
1	1	100	50	350	320	600	650
30	30	200	203	500	500	800	800
2	503	950	95	686	999	199	980
10	35	250	250	549	550	100	699
75	230	726	575	750	575	700	910
47	65	185	325	565	375	825	900
52	56	60	309	410	408	828	824

Table AC.7 Self-protection interval probability

R3	A3	R20	A20	R50	A50	R80	A80
91	100	200	300	615	591	809	840
180	180	350	500	700	600	816	850
35	49	202	232	511	541	801	821
106	60	235	355	503	500	202	950
112	215	509	607	614	706	806	826
9	25	149	202	510	499	820	800
63	51	225	199	501	606	800	840
61	49	200	225	550	625	795	850
30	25	199	175	751	499	699	811
30	30	150	150	300	350	700	800
30	30	200	200	501	500	800	802
30	30	200	200	501	500	800	803

Table AC.8 Self-protection set of probability

R3	A3	R20	A20	R50	A50	R80	A80
1	1	110	100	200	199	300	505
30	50	208	275	505	550	774	800
50	70	210	300	600	500	650	815
52	60	75	205	531	525	199	599
51	100	321	429	601	652	903	901
52	84	325	461	619	588	628	913
111	100	221	300	430	480	500	451
177	25	50	375	316	506	512	263
189	125	275	420	545	457	608	799
35	52	211	360	500	410	803	751
100	56	286	300	609	550	849	805
93	76	236	218	563	528	699	699

Table AC.9 Self-protection control group

R3	A3	R20	A20	R50	A50	R80	A80
11	10	150	149	631	640	852	871
320	290	252	318	655	725	850	885
151	159	450	325	473	500	842	692
31	31	201	201	501	501	801	801
106	105	406	405	606	509	607	605
1	1	49	1	102	302	602	800
104	120	312	370	513	507	508	2

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