AN INTERPRETATION OF THE STRUCTURE OF THE WORLD BASED ON WHITEHEAD'S NOTION OF DIPOLARITY

A New Ontological and Physical Framework for Theories of Quantum Gravity

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Fundamentally, there is love.

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Abstract

The main aim of the thesis is to explore the structure of the world from a Whiteheadian process theoretic perspective and suggest it as a framework for an algebraic approach to Quantum Gravity. In particular, throughout the thesis, the analysis focuses on the notion of dipolarity that appears in Whitehead's philosophy. The thesis begins with an exploration of Whitehead's process of becoming which is followed by a presentation of sheaf theory as the mathematical basis. Based on the analysis of Whitehead's philosophy in combination with physical theories, the features of the structure of the world are explored. The process of becoming is translated as a transformation from determinable dispositions to determinate manifestations. In virtue of dispositions, the structure of the world acquires a causal status. The introduction of causality in terms of becoming is accompanied by the introduction of relata in the structure of the world. There is then a comparison between our Neo-Whiteheadian thesis and both structuralism and constructivism. In particular, the comparison with constructivism is justified by the fact that we adopt a view about the generation of abstract notions out of concrete instances or manifestations of determinable dispositions, where manifested quantities are related mereologically. On the other hand, the comparison with structuralism is motivated by the holistic character of the process of becoming. The thesis ends with exploring laws and the process notion of dipolarity.

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List of Abbreviations

The following abbreviations are used in this thesis.

ADG – Abstract Differential Geometry

EFE – Einstein's Field Equations

ESR – Epistemic Structural Realism

GTR – General Theory of Relativity

LQG – Loop Quantum Gravity

OSR – Ontic Structural Realism

QFT – Quantum Field Theory

QM – Quantum Mechanics

STR – Special Theory of Relativity

SR – Structuralism

WDW-E – Wheeler DeWitt Equation

1 INTRODUCTION

The truth is simple but not reducible

The parts are in the whole and the whole is manifested in the parts

1.1 WHITEHEAD AND PHYSICAL THEORIES

Whitehead's philosophy has started gaining the attention of quantum physicists because he develops his ontology being aware of relativity theory and secondly, he introduces the notion of occurrence which seems similar to the reduction of the wave function in quantum physics (Hättich, 2004; Stapp, 2004). Regarding the former (section 2.5.2), Whitehead refers to the notion of temporal duration which is analogous to the relativistic surfaces of simultaneity, namely, the occasions found in the same temporal duration are in 'unison of becoming'. As for the latter, just as in those quantum interpretation of measurement that include the subject, he claims, that there is a mental side that plays a selective role, where the selection is understood as a transition from potential to actual. On the other hand, in order to combine elements from both relativity theory and quantum theory, Whitehead had to deal with the inconsistencies between the two theories and in particular, he chooses the indeterminism of quantum physics over the determinism of relativity theory. Not all tenses are treated on a par, rather the future remains open. In order then to deal with the tenseless view - the non-distinction between past, present and future - and the determinism of relativity, Whitehead takes that the temporal durations do not actually describe simultaneous occasions, rather the subject perceives a continuum of actual occasions in ‘unison of becoming’. The continuum of events is not something characterizing an objective world, but the continuum is generated out of parts in the subject just as the quantum state can be perceived by the subject which makes the decision and selection among the possibilities. In this way, he tries to accommodate the idea of different observers perceiving different occasions in ‘unison of becoming’.

Hättich (see explanation in Stapp (ibid.)) rejects this last point, namely, the ‘unison of becoming’ for different observers. But Hättich appears to assign the unison of becoming to an objective world whereas the unison of becoming is something perceived and not actually characterizing the external world. He then notes that Quantum Field Theory incorporated relativistic features in a more sophisticated way, compared to Whitehead’s framework developed earlier. The argument is that relativistic Quantum Field Theory (QFT) can be formulated with respect to a unique hypersurface of simultaneity, and in that case the idea of occasions in ‘unison of becoming’ can make sense considering that along these surfaces potentialities convert to actualities. Retaining the idea of unison of becoming albeit not dependent on observers, Hättich claims that the world can be conceived as a succession of temporal durations where each duration is associated with an occurrence that consists of all the occurrence of the previous durations and at the same time, it divides into actual occasions found in disjoint spacetime regions, where this mereological relation between occurrences is inspired by Whitehead's view.

But the idea of singling out a surface of simultaneity is not devoid of problems both in physics and in Whitehead's ontology. In physics, the Hamiltonian formulation of General Theory of Relativity (GTR) singles out a hypersurface of simultaneity, but since the Hamiltonian being equal to zero entails no evolution in time, the hypersurfaces represent the same state of affairs in time. This outcome gives rise to the problem of time (see section 1.2).

In addition, singling out a hypersurface of simultaneity is inconsistent with the four-dimensional diffeomorphism invariance of GTR, where there is no specific foliation of the four-dimensional spacetime and in addition, in order to choose a foliation, one should stand as a super-observer outside the universe and be able to answer whether a spacelike event in distance from us has occurred. But one has limited access to her causal past and not instantaneous access to the entire universe.[1]

Regarding Whitehead's ontology, there is no continuum of events in an objective world in Whitehead's ontology. The extensive continuum that unites occasions in the same temporal duration is a potentiality and not a characteristic of the concrete world. The continuum one experiences is generated by the subject and does not actually link simultaneous occasions in an objective world which is made of atoms of spacetime.

[1] See reactions to the existence of a super-observer in (Sorkin 2007, Markopoulou 1997, 1998, 1999). In Sorkin's causal set approach and probabilistic dynamics, there does not exist a super-observer that would offer a distinguished path of the causet because this would contradict general covariance. Therefore, there is a 'multiplicity of "nows"' defined by the different observers.

Hence Whitehead's view pushes us towards an algebraic approach in physics, while singling out a hypersurface implies the presupposition of a spacetime continuum.

If then one would wish to give a more precise account of Whitehead's ontology, it would be more suitable to focus on his mereotopological treatment along with the notion of dipolarity which is an expression of an inseparable treatment of mind and body. Whitehead postulates that such mutually exclusive categories should not be treated separately, but in an inseparable way by a unifying structure. Whitehead provides a holistic picture which gives emphasis on relations, so that the potential candidates of relata (occurrences) should not be seen as distinct separate monads but as monads in relation to everything. Each monad is in relation to both the physical and the mental and although some philosophers interpreted Whitehead's philosophy dualistically, we will understand it in a dipolar way.[2] Therefore, the interplay between Quantum Mechanics (QM) and GTR should capture the bidirectional relation between quantum and classical as an extension of the dipolarity between mind and body. In particular, the whole is generated out of the parts and vice versa, the parts become out of a selecting process that the whole is subjected to. In this context, a) the quantum (the whole) is generated in the subject which makes the selection for the manifestation found in an objective world and b) the continuum is generated by parts found in the objective world.

So, in this context, the zero of the Hamiltonian in relativistic spacetime would be something that characterizes the potential side. In fact, Whitehead's framework offers a suggestion for the recovery of time, namely, the transition from the potential to the

[2] Despite the use of the 'authorial we', common in academia, this thesis is the sole work of its author.

actual means that the actualization of the continuum is the becoming of atoms of space and time. Hence, time is not presupposed but from a potential state, eventually comes into existence. Then, in virtue of the transition from being possible to becoming concrete actuality, there is a transition from being timeless to becoming temporal, so that the zero Hamiltonian expresses the timeless character of becoming actualized. In such an interpretation, it is not that any object including the universe that exists timelessly. But it is the becoming that occurs timelessly. Hence, the timeless and the becoming temporal are entangled in one and though being limited, the observer experiences the timeless from inside, since she has access to some part of the universe.

Of course, one should still need to explain the multiplicity of presents. There is a plurality of presents and a tenseless view of time, since the present is defined with respect to an occasion. Still, just as the dipolar relation applies to electrons and chairs, it applies to the occurring universe. A unique present would be defined for the universe and ultimately for all occurrences, but one should show how occurrences each defining its own present are interrelated to each other and they are all related to the present occurrence of the entire universe.

But what reasons does one have to support that spacetime is a potentiality? The singular behaviour of spacetime can function as an indication for a pointless ontology and as a reason to shift to an ontology of regions and to an algebraic formulation of GTR that eliminates spacetime points (section 4.2.1). A relation between hypersurfaces could then be established mereologically and spacetime will be regarded as a construction of the mind generated mereologically.

The elimination of singularities from relativistic spacetime shows that the parts-regions should be treated with importance just as the whole-spacetime continuum (detailed analysis in section 4.2.1). If the parts are associated with notions such as localization, holistic notions such as nonseparability in QM and equivalent distributions of the metric in GTR should be correlated with localized notions so that a more holistic structure that brings together the whole and the parts is developed. In physics, the local and the global are treated in Yang-Mills theories where a gauge field interacts with matter so that local symmetry transformations are reconciled or gauge invariance is established. For Whitehead, any kind of field along with the spacetime continuum is taken as something non-concrete while it becomes concrete when it is actualized. The actualization of a field is understood as its confinement in some chunk of spacetime, where this chunk represents the actualized atom of spacetime. The dipolarity means that there is a bidirectional relation between parts and whole or local and global, where the localization is with respect to some context (actualized atom of spacetime). Whitehead's treatment of fields can motivate a common handling of the quantum world and the GTR domain, which is focused on the interactions rather than the kinematical structure. Yang-Mills theories show how the interactions offer an interplay between the local and the global so in an algebraic formulation where the interactions are localized with respect to some region, the relation between global and local can find correspondence to Whitehead's bidirectional relation between local and global (section 4.2.2). Focusing on QFT and not on QM anymore, one needs to release the tension between QFT and GTR, which is owed to the rejection of the fixed kinematical structure of spacetime by GTR, something that QFT incorporates. Removing a continuous spacetime manifold from the background, there should be a focusing on the interactions as QFT instructs. So, just as the other interactions, i.e. the electromagnetic and weak/strong nuclear interactions, the gravitational field can be considered another interacting field that couples with quantum fields albeit without the presupposition of a continuous spacetime.[3] Since the spacetime continuum is rejected, the metric cannot be used in order to describe a universal interaction between fields. Rather one can focus on the potential or gauge field in order to describe the interactions. Just as in QFT, the gauge field pairs with the matter field in the relevant interactions, instead of the metric it is the potential one can employ in order to describe the gravitational interactions (see a common framework for all interactions in chapter 3 and discussion in section 4.2).

1.2 WHITEHEAD'S NO-SCALES PERSPECTIVE AND QUANTUM GRAVITY

This section continues the discussion on how Whitehead's view should be interesting for physicists and in particular for the development of Quantum Gravity (QG). Whitehead's process of becoming which is the fundamental element of existence applies to everything, hence at any scale. In Whitehead's ontology, the structure captured by the process of becoming, provides interrelation of everything and applies to the individual (from human to particles) as well as to a society of individuals (where a series of becomings defines an individual and a society refers to many series of becomings related to each other). So, the relationship between individual and itself and the relationship between individual and other individuals are treated in a unifying way by

[3] Contrast with Cao (2001) who proposed an ontological approach which is used as a conceptual framework for the development of a theory of QG. His main assumption is that the conceptual structure of the theory represents a causal-hierarchical structure which is fundamental and from which all entities, properties, relations emanate. At the fundamental level, the gravitational field interacts with the other fields without the involvement of a continuous manifold and, therefore, the continuous general relativistic spacetime is an emergent entity.

the process of becoming. In the next paragraphs, we will try to see how a no-scales perspective can be important for the formulation of a QG theory.

QG is expected to give a description of the gravitational field and the spacetime structure at the Planck scale where quantum mechanical effects dominate. In their attempt to formulate a successful theory of QG, physicists have encountered a series of problems. One problem has to do with the unattainability of experiment at such scales. Moreover, canonical QG theories such as Loop Quantum Gravity (LQG) have to deal with the emergence of classical spacetime out of a fundamental quantum reality. Furthermore, they need to tackle the problem of time which has to do with locating time in quantum gravity theories (and not about finding time at the classical limit). Note that the problem of time is distinct from the issue of the emergence of time. Although the notion of time in particular will not be a part of the investigation, still there will be exploration of relevant notions such as change at the Planck level and as for the problem of emergence, this will be used as an argument for shifting to a new ontological framework which treats levels in the same way so that emergence is avoided (see next section). So, let me briefly outline some issues concerning the problem of time and the problem of emergence.

The problem of time is associated with the Hamiltonian constraint which describes the dynamics both in classical case and in canonical QG. In the classical case, the Hamiltonian constraint relates points from one spacelike hypersurface to another, so that each hypersurface corresponds to a state. But since the Hamiltonian being equal to zero entails no evolution in time, the hypersurfaces represent the same state of affairs in time. In terms of observables, this is expressed by the vanishing of the Poisson bracket of the Hamiltonian with the observables. In canonical QG, the Poisson bracket is replaced by commutation relations and by analogy, the observables commute with the constraints. That is, observables remain invariant and therefore, there is no change.[4]

Concerning the emergence of spacetime, the main idea is that since according to LQG, physical spacetime becomes granular at the Planck scale, then continuous spacetime is an emergent phenomenon (Butterfield and Isham, 1999; Isham, 1992; Lam and Esfeld, 2012; Wüthrich, 2012). Butterfield and Isham (op. cit.), discussing in what sense time and more generally spacetime emerge in large scales, claim that in virtue of a limiting procedure, the spin network states are related to the spacetime of GTR, viz. the spin network states give spacetime in the classical limit. More generally, one can distinguish two options of emergence; either the continuous spacetime structure emerges out of fundamental non-spatiotemporal quantum entities or alternatively, it is a characteristic of the quantum gravitational level and the emergence of the classical spacetime is interpreted as re-description that corresponds to the classical level.

However, the first option of emergence is not devoid of problems as Lam and Esfeld (op. cit.) point out. Firstly, it is not clear how concrete physical entities can be conceived if the relevant structures do not involve spacetime which differentiates them from abstract mathematical structures. Moreover, as they argue, it is not clear how differentiation should be defined without the involvement of spatiotemporal notions. For example, in LQG, the differentiation is given with respect to an underlying spatial manifold in which spin networks are found. So, such objections would support the appearance of spatiotemporality at the fundamental level.

[4] There has been different reactions to the claim that there is no evolution primarily because such a conclusion contrasts with what one experiences. See the problem of time in (Belot and Earman, 2001; Isham, 1992, sec. 3.3.1-3.3.3; Kuchar, 1993; Rickles, 2006; Rovelli, 2001, 2002).

In the above cases, QG theories assume that the macrocosm is treated differently from the microcosm. What we want to defend is a common treatment of the two levels. Firstly, the theory needs to provide a unifying structure of all levels in order to avoid problematic metaphysics of emergence. This means that there is no fundamental level where upper level is reducible or emerges from it. Therefore, in the spirit of Occam's razor, we do not regard a fusion of structures. Rather, a structure should provide a unifying framework for 'all scales' without making distinctions into levels, so that neither emergence nor reduction is espoused. The reason is that what is fundamental is this unifying structure and nothing else privileges fundamentality. We claim that this framework is simpler than others' scenarios, in the sense that one can avoid any complicating procedure that would bridge theories at the Planck level with theories at great scales. What one needs to do is simply describe the behaviour of the system in the same way in all scales without having to worry in what respects the quantum level differs from the classical and whether new features appear. (Actually, the non-fundamentality of a scale of a minimum size (in this case the Planck scale) would be motivated by what history of science has taught us, namely, that there is an even deeper level; there has been a series of discoveries starting from atoms and going to subatomic particles-electrons, protons, neutrons, and to elementary particles and nothing tells us that there should be a stop somewhere.)

Secondly, the measurement would become attainable at Planck scale, if that scale was treated as the scales where experiments are not inhibited by experimental limitations. The entanglement commonly gains the attentions in the holistic interpretations while the appearance of a determinate measurement outcome is not taken into consideration. However, we claim that a holistic picture of the world should involve an unstructured element in the form of a measurement outcome; the measurement outcome should be treated with importance if the structure treats holistically mutually exclusive categories, namely, the mental and the physical.

Finally, Lam's and Esfeld's arguments that spacetime at the fundamental level is necessary for ensuring concreteness of the system and providing differentiation are contentious. a) Algebraic differentiation can be defined with respect to the covariant derivative (which is composed of the gauge field) for all scales (section 3.3); in this way, a spatiotemporal notion viz. the covariant derivative is retained while at the same time the idea of an underlying manifold is rejected, and b) if spacetime is taken as an indicator of concreteness, this does not have to be related to the whole continuum but each concrete part can be defined with respect to some region (chapters 4 and 5).

1.3 STRUCTURE OF THE THESIS

Before outlining the structure of the thesis, the reader will need to keep in mind that there are points where we introduce in our interpretation notions that Whitehead did not include. For example, we argue that Whitehead's dipolarity can accommodate tropes. The main idea is that employing the notion of dipolarity, certain outcomes can be derived and these conclusions may not have appeared in Whitehead's thesis.

The main aim of the chapters is to analyze the notion of dipolarity both in a mathematical/physical and philosophical way. The chapters will be structured as follows. Chapter 2 makes an analysis of Whitehead's process philosophy. There is analysis of the mereological interrelation of occurrences which comes to replace the interrelation via a spacetime continuum. Furthermore, there is an investigation of notions related to time and in particular, what is present and how present occurrences are related mereologically with past occurrences.

Chapter 3 investigates sheaf theory which is suggested to implement the dipolarity between parts and whole. Sheaf theory that provides a dialogue between the local and the global and contrary to fiber bundles, does not employ a background spacetime. The main idea is the chunking of spacetime to give parts and subsequently, the gluing of the parts to give a whole. This will establish a bidirectional treatment between parts and whole. Sheaf theory provides a mereological ordering between regions which is transferred to the mereological relations between matter fields coupled with some gauge field and defined with respect to some region. On the basis of these mereological relations, change will be developed. Contrary to common claims of no change at the fundamental level (although there have been attempts to put evolution and change at the Planck level e.g. see (Markopoulou, 1997, 1998, 1999)), since a sheaf theoretic framework applies to any region of some size, change will transcend all scales. Sheaf theory is finally combined with cohomologies which capture change algebraically.

Chapter 4 initially discusses the removal of spacetime singularity as a motivation for treating the parts with significance and interpreting the relation between matter and potential in a dipolar way. The analysis is accompanied by some examples of Yang-Mills interacting systems. Then, the chapter studies the holistic dipolar relation between mind and external world. Currently, in literature, a holistic attitude in philosophy and physics is associated with notions such as entanglement, indeterminacy, indistinguishability etc. In philosophy, Ontic Structural Realism (OSR) gives a structural holistic picture of the world with other philosophers eliminating the relata and others espousing the existence of relata, although the latter have structural properties or are put on the same ontological footing with relations. If holism is commonly linked to the whole and the indeterminate, it will be here claimed that holism concerns mutually exclusive categories that should be seen as an inseparable whole. The relata can be associated with the notion of substance or object (although these notions will be removed from the ontology as we will see in the thesis) found in the external world while the inseparable treatment of relations and relata will be translated as an inseparable treatment of mind and body. Contrary to Epistemic Structural Realism (ESR) which assumes that epistemologically, what the subject knows objectively about the external world is the structure, here due to the inseparable treatment between the external world and the mental, the subject has epistemological access to the individual. In addition, since in constructivist approaches physical quantities become measurable at any scale and are generated out of series of overlapping parts (which can be taken in measurements), our ontology that employs the notion of dipolarity between whole and parts is a support for a hybrid of constructivism and structuralism (SR) and hence, it bridges the alleged gap between the two views.[5] Our constructivist attitude expresses that science should not be alien to one's experience of the world. Instead, what we are called to do is to explore nature and develop an empirical relation with it so that science is not a system of rules and structures imposed but developed via interaction with the world. Experience then permits us to penetrate into the rational order with which the universe became.

[5] E.g. see Brouwer's (1975, pp. 17–19) intuitionism and no restriction of measurement at some scale.

So the structure of the world adopts elements from both constructivism and SR, but it rejects basic elements of both, namely the non-existence of structures external to the mind and the rejection of a constructivist attitude by SR. One also needs to note that Whitehead did not discuss constructivism, but his view bears similarities with it, since as in intuitionism the continuum unfolds and space and time is in state of becoming. In any case, as mentioned, our interpretation of the structure may involve extra elements that do not appear in Whitehead's discussion. Hence, a conglomeration of different ontologies - SR and constructivism - is suggested while at the same time the primary source of influence is Whitehead's thesis.

Chapter 5 refers to the elimination of the notion of substance and its replacement by the notion of structure of becoming. Chapter 6 explores the case of dispositions having causal powers (since there is no time on the basis of which causality could be understood), where causality involves the entire process of transforming from a disposition to a manifestation. Since the continuum is eliminated from the background, change and causation should be disentangled from temporality. Although the usual understanding of causation is the one according to which the cause is temporally prior to its effect, there have been arguments that refuse that the basis of the causal direction is the temporal direction (Schaffer, 2008).[6] Thus, in literature, there are metaphysical accounts where the distinction between cause and effect does not depend on temporal ordering. The alternative proposal that we will discuss provides a dispositional analysis

[6]Firstly, the time travel denies that the causal order is reduced to the temporal order because it is logically consistent to assume that we can travel back in time and consequently the effect to be prior to the cause. Secondly, according to the simultaneous argument, both effect and cause can occur simultaneously. One can refer to the causal theories of time where it has been argued that it is the time order that should be reducible to the causal order and not vice versa.

of causality.[7] Applying dipolarity, a different dispositional view will be formulated from the ones usually appearing in discussions e.g. dispositions will not privilege fundamentality.

Finally, chapter 7 discusses dipolarity in combination to laws. Some structuralists provide a structural picture of the world by starting with the laws which express a structure of relations and this approach contrasts dispositionalism where instances play dispositional role and function as the basis from which laws are supposed to be derived. In our framework where neither laws nor determinate dispositions found in instances will be fundamental. Our scheme acknowledges the reality of laws independently of dispositions but these laws are treated inseparably from the manifestations. Dispositions then come to play an internalization role for the laws so that laws or universals do not stand anymore separately in a Platonic heaven but they are internalized. Hence, laws should not be taken as imposed, rather laws should develop out of one's interaction with the objective world. Finally, it is claimed that in virtue of the pair matter field/gauge field as well as the pair operator/matter field taken to embody certain possibilities analyzed in dispositional terms, symmetries, laws, invariant and conserved quantities can be accommodated in a scheme of nested dispositions.

[7]See also other proposals that provide a distinction between cause and effect in terms of counterfactuals (Horwich, 1987; Lewis, 1986; Paul, 2009; Reichenbach, 1956), probabilities (Reichenbach op. cit.), agency and intervention (Woodward, 2009). Still, it has been argued that some of these proposals do involve a temporal identification of the cause and the effect (Williamson, 2009, pp. 190–193).

2 WHITEHEAD'S PROCESS PHILOSOPHY

2.1 INTRODUCTION

In his book *Process and Reality* (1978),[8] Whitehead develops his metaphysical doctrine, the Philosophy of Organism, in which he applies the notion of *concrescence* which stands for the process of becoming concrete (or actual), so that the being is found on becoming. The following sections will explore the structure (of the world) captured by the process and the relations built in it. Initially, the notion of dipolarity is discussed and it is clarified in what sense Whitehead's holistic approach differs from other philosophers. His view on the generation of the continuum by sequences of parts that overlap exemplifies the notion of dipolarity. Space and time is investigated in this context according to which the actual world is a world of discrete things. The chapter ends with exploring how the notion of dipolarity combines with other notions such as laws. This will assist us in our exploration of the structure; structuralists invoke symmetries and laws in order to give an objective structural account of the world. In

[8] Originally published in 1929.

chapter 7, we will give our own perspective on how laws can be understood if dipolarity is employed.

2.2 THE PROCESS OF BECOMING

Whitehead distinguishes between two major categories: the potentialities and the actualities (or actual entities or final realities). The potentialities are subdivided into *general* and *real potentialities* (ibid., p. 65). So the characterization real applies to both actualities and potentialities with the exception of general potentialities.

The general potentiality is absolute, involving all the possibilities arising from the multiplicity of eternal objects. Eternal objects, which are qualities, such as colors, sounds, bodily feelings, tastes, and smells, are purely non-real potentialities.

The real potentiality represents the determinable 'conditioned by the correlate universe' or conditioned by data in the related world (ibid., p. 23). It is this conditioning that renders the potentiality real. The determinable character of the real potentiality contrasts with the determinateness of actual entities.

> [I]n each concrescence whatever is determinable is determined, but that there is always a remainder for the decision of the subject-superject of that concrescence. This subject-superject is the universe in that synthesis, and beyond it there is nonentity. (ibid., pp. 27-28).

Potentialities cover a range of cases and actualities concern the realization of one of the potentialities. The subject - which gives the perspective - makes a selection among potentialities so that the determinable transforms to determinate. Hence, the real potentiality gives the perspective whereby some actual entity is defined. The characterization superject is justified by the fact that an actual entity takes input from the objective world (past actualities) (ibid., pp. 29, 83, 87). An actual entity is always subject-superject, meaning an experiencing subject and a superject defined by experiences, so that none of them is taken in isolation from the other. The superject expresses the objective character of the subject, where this objectivity means that the subject cannot be treated in isolation from the objective world because it takes input from it.

Potentialities and actualities are not treated separately:

> [T]he notion of 'complete abstraction' is self-contradictory. For you cannot abstract the universe from any entity, actual or non-actual, so as to consider that entity in complete isolation. (ibid., p. 28).

This means that determinables expressed in real potentialities and determinates expressed in the final actual entities are put on the same ontological footing. Likewise, actual entities and eternal objects cannot be taken in abstraction from each other. In general, categories, which are traditionally treated as mutually exclusive such as abstract/concrete, universals/particulars, actual/potential, physical/mental should not be treated individually, but in an inseparable way as a part of a unifying structure. This is expressed as dipolarity in comparison to duality (ibid., p. 45). Contrary to the Cartesian dualism, where mind and body constitutes two separate substances, the Philosophy of

Organism can be read in such a way so that the mental side and the physical side are in unity.

This interrelation between non-actual and actual entities is grounded on their being united in the process of becoming of an actual entity. The actual entity develops relations with past actual occurrences as well as with eternal objects. That past actual entities become immanent in the present actuality is called objectification and is related to the functionality of *feeling* or equivalently, *positive prehension* (see also (Garland, 1982)). The feeling is constituted by the following five components: '(i) the "subject which feels and evaluates the prehensions; (ii) the "initial data" which are to be felt; (iii) the "elimination" in virtue of negative prehensions; (iv) the "objective datum" which is felt; (v) the "subjective form," which is how that subject feels that objective datum' (op. cit., p. 221). Let us consider these components in turn.

Two kinds of prehensions and respectively, two kinds of poles of concrescence are distinguished: *conceptual prehensions* of the mental pole, which refer to the relations between actual entities and eternal objects and *physical prehensions* of the physical pole which are the relations between actual entities. Both kinds of prehensions can be either positive (Whitehead uses the term feeling for positive prehension) or negative, and the actual entity prehends positively or negatively the eternal object or other actual entities respectively.

Past actual occurrences function as the initial data found in the physical pole and act as real potentialities for the concrescence of a new actual entity. A physical prehension is positive for those actual entities that the respective actual entity selects and in that way 'the potentiality of one actual entity is actualized in another actual entity' (ibid., p. 23).

An actual occasion prehends positively (negatively) another past actual occasion, if the latter does (does not) have an impact on the becoming of the former. Moreover, in the mental pole, the general, perspective independent, potentiality or abstract boundless possibility will transform to and be limited to the real, perspective dependent, potentiality (ibid., p. 220). The subject conceptually prehends eternal objects, so that a conceptual prehension is positive when the actual entity selects some eternal objects while rejects others and in that case eternal objects are said to ingress into actual occasion so that the latter will acquire certain characteristics. So, the mental pole provides both eternal objects and the initial data, which are real potentialities of past occurrences, with subjectivity. The perspectives induced are taken to be the different potentials that an actual occasion can be. Note that the mental pole applies to all entities, from particles to human beings (ibid., p. 92). One should not then confuse subjectivity with human presence. The existence of the mental pole does not imply that actual occasions have consciousness, rather in the concrescence there is a side responsible for the evaluation of potentialities. Although panpsychism is likely to cause inconvenience, we agree with Simons (2013) that one should not be discouraged from investigating and taking into consideration the merits of the philosophy of organism. This panpsychism could be justified by the fact that even things such as fossils, black holes etc. and not only humans can act as memories so that functions that the human mind performs can also be assigned to things. Moreover, as we will see in later chapters, the evaluation can be explained in terms of a system of nested dispositions, symmetries and generated laws that applies to all concrescences. In addition, we sympathize with Whitehead's panpsychism as a reasonable framework in the sense that God unites the two poles so that pure potentialities are held in waiting to ingress, when they have not actually done so, by God. So Whitehead's panpsychism is not pantheistic rather panentheistic because

God is not exhausted by the universe, but He contains and interacts with it (Viney, 2014, sec. 4). This is consistent with God's mediation for the unity of the two poles so that everything is interrelated through God.

The realization of one of the many potential modes occurs in the last phase of satisfaction. In this phase, an occasion becomes actual by selecting one of the many potential modes. Because of the evaluation of the various possibilities by the subject, there are elements omitted and are not present in the actual entity, since in the satisfaction, indeterminacy cannot be the case, for example contrary elements cannot be both present (Whitehead, op. cit., p. 45).

Upon satisfaction, the actual entity perishes and transforms to object and to a real potentiality which functions as initial datum to subsequent concrescences (the effects of an occasion can be directed to many subsequent occasions and not to a particular one).[9] That is why it is said that the actual entities privilege immortality; upon satisfaction, although they lose their subjectivity manifested in the mental pole, they obtain objective immortality because, as real potentialities, they remain forever in subsequent occasions. In other words, an actual entity can always be recovered on those subsequent occasions which positively prehend it.

The objective datum captures the prehension of past occurrences by the current one. The relatedness between the actual occasion and the occurrences in its causal past is termed *nexus* (*nexús* for the plural form), so that the objective datum is a nexus. For a simple physical feeling which is the feeling of a subject which has a single actual entity as its

[9] Here the characterization 'object' emphasizes the transformation from subject to object. The characterization appears also in eternal objects and enduring objects introduced later, but in a different way.

initial datum, the objective datum is the feeling of this single actual entity (ibid., p. 236). There can be many such feelings so that the subject initially experiences 'a multiplicity of simple physical feelings', where each feeling corresponds to an initial datum (ibid., p. 236). These data together constitute the initial data for the new occurrence which are subjected to evaluation so that aspects of past actual entities that are not consistent for the integration are eliminated by negative prehensions before the new objective datum is finally derived (ibid., p. 340).

The nexus of actual entities related to a concrescence is called 'the actual world' of that concrescence (one should not be confused by the terminology and assume the existence of many actual worlds) (ibid., p. 23). An actual entity feels only entities which have already come into being and belong to its causal past. If B belongs to the causal past of A, A feels B, whereas B does not feel A (see example in (ibid., p. 226)). The prehension of B by A means that the nexus B-A comes into existence; initially, B comes into existence, then A prehends B along with other prehensions and finally, A comes into existence as a prehension which incorporates the feeling of B. If there is a third actual entity C belonging to the actual world of A and is felt by A directly, A negatively prehends the nexus between C and B. But if C belongs to the actual world of B, then A feels C via the mediation of B or equivalently, A feels C in its nexus with B. An occurrence has an internal structure which incorporates information inherited by previous occurrences, information that captures the relation between an occasion and the nexus of past occurrences. Present actualities are then internally related to occurrences of the past. On the other hand, they do not bear information of future occurrences. Still, they become data for them, so there is still an interrelation directed towards future occurrences.

In fact, every concrescence prehends all the universe. Via prehensions including the negative ones, each actuality is related to the entire world, eternal objects and actualities no matter how remote they are (ibid., pp. 41, 44). Negative prehensions still express interrelation in the sense that the subject rejects something as unsuitable (both acceptance and rejection contribute to one's experience of the world). So, via prehensions, an occurrence mirrors the world. As it has been argued, in Whitehead, 'the world is in the individual' (or the individual mirrors the world) and vice versa, 'an individual is in the world' (Viney, 2014, sec. 4).

2.3 MIRRORING OF THE WORLD IN THE INDIVIDUAL

To stress out what the mirroring is, one can compare Whitehead to Leibniz who also assumes that each monad mirrors the entire world. Although Whitehead's organic theory and Leibniz' monadology (in (Russell, 2008)) present similarities, they differ in how they treat the interrelation between the basic elements (monads for Leibniz, occurrences for Whitehead) of their ontology (Johnson, 1959).

As in Whitehead (see later sections), for Leibniz, uniformity, contrary to diversity, is a characteristic of abstraction, for example, time, space and other mathematical entities are abstractions while actual things (as well as things in possible worlds) are made up of discrete quantities (Barbour, 1982). These discrete quantities correspond to monads, so that actual things, made up of infinite number of monads, have infinite variety (apart from the actual world, all possible worlds are comprised of monads). Monads are the fundamental constituent of the world, which means that variety and not uniformity is a characteristic of the world. Each monad is unique, satisfying Leibniz's principle of the 'Identity of Indiscernibles', according to which two things cannot be exactly the same. Likewise, in Whitehead, an actual occurrence is distinct from any other entity it unifies.

Although both philosophers claim that the actual world constitutes of discrete things (excluding the fact that for Whitehead, actual infinity is a characteristic of God and not of actual occurrences, that is, God is the only actuality being infinite), monads are windowless, namely, there are no inputs coming in and outputs going out whereas Whitehead's occurrences are 'windows' because they are related via prehensions to eternal objects and past occurrences (Simons, 2013). Leibniz' monads remain independent from each other so that each is not subjected to external influences. This is the reason why monads change internally. However, the mutual exclusiveness of monads becomes problematic considering Leibniz' assumption that each monad mirrors the entire world. Each monad being unique satisfying the Identity of Indiscernibles, represents a different universe, though they are 'different perspective representations' of the same universe.[10] This means that a monad builds relations with the others, where relations express relative distances and succession between monads. Indeed, monads have external relations but these are not genuine, because extrinsic relations of monads supervene on their intrinsic properties.[11] In order to establish a binding between monads, Leibniz evokes God. However, a resolution that does not invoke God would be

[10] See Barbour's (1999) timeless view according to which the entire structure which involves the heaps of possibilities and the heaps of actualities is characterized as timeless. In Barbour's interpretation correlated to Leibniz' monadology, each configuration represents a different universe, while at the same time it is a perspective of the same universe. Barbour's dynamics are actually described by the Hamiltonian formulation of GTR. In the epilogue of the thesis, we present in brief a topic of future research on time and how an approach motivated by our study on Whitehead would deal with it. In such a research the differences with a Leibnizian perspective would become apparent.

[11] For more on the reducibility of the extrinsic properties on intrinsic ones see (Jauernig, 2010).

preferential to the one that does. Moreover, if God provides the linkage between monads, the description of monads as independent entities is now denied. In addition, Whitehead reacts to God's goodness as bonding between monads because this cannot ground confused perception (op. cit., p. 27). In order God to unite things, He should be benevolent. Since there is confused perception (which is like a passion whereas distinct perception is positive), God allows bad things to happen, but this undermine His benevolence or His omnipotence. That is, God is benevolent, hence he would prevent confusion and evil, but since bad things happen, He is not omnipotent else God is not benevolent.

On the other hand, for Whitehead, the origin of confusion can be very well explained in terms of interaction among actual entities.[12] According to Whitehead, there is perception via our senses, but our inferences are faulty (ibid., p. 64, 65). The perception can be delusive. For example, one can hallucinate the existence of a chair. In that case, the sense data involved are not found in the real constitution of a chair but in the internal constitution of the organs of the human body, e.g. if one takes drugs and has hallucinations, then sense data are the product of taking drugs. Since there is

[12] One basic difference between Leibniz' and Whitehead's descriptions of God is that the former assumes God existing independently of His creation whereas the latter assumes God being with the world (Johnson, 1959, p. 294). For Leibniz, God has sovereignty over monads while for Whitehead, God is not the sole controller but participates and assists the creative process. A limitation of God that is found in Whitehead's philosophy, in philosophy of religion, it is also found in the kenotic model according to which God empties Himself from some of His properties to make Himself present in the world, so it would be interesting to see differences and similarities between the two theories. Moreover, regarding the role of God in the context of physics, Torrance (1972) comparing Newtonian physics with modern physics, explains that Newtonian physics give a mechanistic view of the world, where the mechanism of the universe is dependent on a sovereign God who rules everything. But quantum theory questioned the adequacy of a mechanistic deterministic universe. Likewise, GTR had an impact on the traditional mechanistic view of an absolute spacetime and emphasized the role of the observer in how reality is understood. Hence, Torrance claims that a mechanistic universe should be replaced by an organic universe.

interconnection of everything, our perception of the actual external world obstructed by the internal constitution of the body is explicable.

In organic philosophy, actual entities are not isolated but involved in interactions with each other. Each actuality prehends all the universe, so that via prehensions, an occurrence mirrors the entire world. Leibniz also argues that each monad mirrors the entire universe, but there is no such assumption that the internal constitution of a monad appears in others (Mander, 2013). Since occurrences are related, there is need to impose internal change on them. Change applies on the nexus and not on actual entities, which are only subjected to a process of becoming but not to change (Whitehead, op. cit., 80) (see change in later sections).

The next section shows how the process of becoming and this mirroring are implemented mereologically. In particular, there is a mereological relation between potentialities and actualities. It will be shown that due to dipolarity mentioned earlier, there is a transition from the whole to the parts and vice versa, from the parts to the whole. The section initially gives as an example the continuum which is a real potentiality and represents the whole, while atoms of space and time are actual and represent the parts. The analysis proceeds with mereotopology which aims to capture the relationship between the parts and the whole and ends with the effect this relationship has on the characterization of universals and particulars. Because of the dipolarity, it is shown that this characterization is not unique so that universals are not abstract and particulars not concrete. Rather there can be correlations such as abstract particulars (though, this term is not applied by Whitehead).

2.4 PART-WHOLE RELATIONSHIP

2.4.1 THE CONTINUUM AS REAL POTENTIALITY

When contrasting his ontology to others, Whitehead claims that:

> Both Descartes and Locke, in order to close the gap between idea representing and tactual entity represented,' require this doctrine of the sun itself existing in the mind.' But though, as in this passage, they at times casually state it in order to push aside the epistemological difficulty, they neither of them live up to these admissions. They relapse into the tacit presupposition of the mind with its private ideas which are in fact qualities without intelligible connection with the entities represented. (ibid., p. 76).

In comparison to Descartes' and Lock's view, Whitehead's ontology aims at closing the gap between the representation and the represented. In order to represent the actual world, the mind perceives the things continuously via the extensive (spacetime) continuum which is categorized as real potentiality. The continuum acquires reality because it does not stand independently of the external world. Its reality is expressed in the paragraph:

> This extensive continuum is 'real', because it expresses a fact derived from the actual world and concerning the contemporary actual world. All actual entities are related according to the determinations of this continuum; and all possible actual entities in the future must exemplify these determinations

> in their relations with the already actual world. The reality of the future is bound up with the reality of this continuum. It is the reality of what is potential, in its character of a real component of what is actual. Such a real component must be interpreted in terms of the relatedness of prehensions. (ibid., pp. 66-67).

The continuum is real because it is generated from data received from the external actual world and in turn, the entities in the actual world are associated with some atoms of space and time which are actualizations of the real continuum. This means that the shape of a body is perceived to have a particular geometrical perspective with respect to us, but what seems to be a continuum is in fact a potentiality, so that the continuous expanse of earth and sky that one perceives is real but non-concrete. Apart from a real character, the continuum has a potential status, because it is the possibilities of division which comprise a continuum. That is, a continuum is divisible or has a potentiality for division (ibid., pp. 61-62). The process of actualization of the extensive continuum, termed coordinate analysis or division, involves the atomization of space and time (ibid., pp. 72, 123-124).

So, space and time are treated as other entities that come into and go out of existence. When space and time occur, they have some determinate extension, so that each occurrence has its personal quantum space and time. As Simons (2009) puts it, the occurrence is accompanied by the becoming of a 'bubble of spacetime'. Therefore, '[t]here is a becoming of continuity, but no continuity of becoming' (ibid., p. 35), in the sense that each actual occasion associated with some atom of space and time are themselves continuously extensive, but the becoming does not occur in an extensive spacetime.

Referring in particular to space (the atomization of time is in section 2.5.1), the quantum of space is an actualization of the continuum along with the physical field. The extensive continuum and the physical field, which can be any kind of field (gravitational, electromagnetic etc.), are potentialities (in fact, the physical field is a real potentiality), which are treated as one, so that empty space refers to the potentiality of the spacetime continuum in unity with the field. Along with the continuum, a physical field becomes concrete actuality when it is atomized. The field is a global entity and the finite extension that the continuum and the field acquires upon actualization could be understood as the localization of the field. The distinction between local and global does not have to do with the restriction of one's observation at a local region since there is no concrete spacetime for that, rather it is a distinction between the satisfaction of concrescence (reflected in the value of the measured property), which represents the local element and the potential side of concrescence, which represents the global element.

So, upon actualization, the field and the continuum becomes a particular actual entity so that matter does not occupy space. Here lies the difference with Newton's view which assumes the actuality of the extensive continuum (Whitehead, op. cit., p. 71). In Newtonian ontology (Newtonian substantivalism), space and time are actual and exist independently of matter which comes to fill in the space. On the contrary, for Whitehead, there is no such actually concrete background spacetime. Rather, space participates actively in the generation of new things-matter by being itself actualized. '[L]ife is a characteristic of "empty space" and not of space "occupied" by any corpuscular society' (ibid., p. 105). Life or creative advance is the outcome of the actualizations of the field or empty space. Whitehead (ibid., pp. 91-92, 105, 177) makes the following distinction between actual occasions: those being 'moments in the life-histories of enduring [..] objects' (i.e. non-living objects such as electrons, living objects and objects with conscious knowledge) and those (i.e. the physical field) in empty space. The difference between these two categories is that in the latter case, there is no chain of past occasions and it is due to the empty space that there is creative advance and not 'reiteration from the past'. So, the difference between a common concrescence, for example a moment in the life of an electron, and creative advance is that the latter is not understood as a serial advance (ibid., p. 35) but as something inherently novel.

One could interpret Whitehead's view as a version of endurantism. Perdurantists think that objects have both spatial and temporal parts while endurantists think that objects have only spatial parts. For an endurantist, an object has different spatial parts at different times, because it is subjected to change (for example my body exchanges molecules with the environment, I grow nails etc.). So, parts of an object belong to the past and others to the future. Perdurantists deny that an object can have different parts at different times, because this undermines the identity of the object. Rather, they think of the whole object atemporally; it is the same 'me' yesterday, today and tomorrow among my parts. So, objects have a four-dimensional extension, but their temporal extension cannot be divided into parts, although they acknowledge the existence of spatial parts. The Whiteheadian view is then a case of endurantism; an object has different occurrences mereologically ordered, that is, an 'enduring object' e.g. a molecule, the universe, humans, is a society of actual entities serially ordered (ibid., p. 35); taking objects as a series of occurrences shows Whitehead's favouritism towards occurrences over substances (Simons, 2013). Then contrary to the common view of endurantism, time does not provide an ordering any more, rather in combination with space, it constitutes part of the object.

2.4.2 MEREOTOPOLOGY

The outcome of the previous section is that continuity belongs to the category of potentials because extension which is undivided has the possibility of being divided into atomic finite entities. Since the spacetime continuum is a mere potentiality, processes are not located in space and time. Therefore, their ordering is not defined spatiotemporally, rather it is determined mereologically in consideration that the relation between the represented and the representation is part-whole.

The represented is discrete, while the representation involves the real continuum, which becomes actualized and discrete e.g. the mountain is represented and perceived continuously, while actually, it is made of atoms. Since the space incorporates all the range of possibilities and its actualization results in the generation of a particular actuality, '[creativity] is that ultimate principle by which the many, which are the universe disjunctively, become the one actual occasion, which is the universe conjunctively. It lies in the nature of things that the many enter into complex unity.' (Whitehead, op. cit., p. 21). There is then a transition from a disjunctive unity of many to conjunctive manifestation of the many in one entity, so that a novel entity is disentangled from the 'disjunctive "many" which it leaves' (ibid.), becoming part of other actual occurrences that it finds upon realization. In this way, '[t]he many become one, and are increased by one.' (ibid.). Therefore, past occurrences are projected in the current occurrence via prehensions, so that a past occurrence is a part of the actual world of the current one and the current occurrence is part of the actual world of its future one while relationship between an actual occurrence and the extensive continuum (which combines all possibilities) is a part-whole one.

There should then be a bidirectional part-whole relationship, because the whole (representation) is derived from the parts (represented) and vice versa, the whole is atomized to parts. The gap between conceptual and physical closes, in the sense that they do not stand independently and there is a transformation from actual to potential and vice versa. This means that the representation should not be seen as purely conceptual because the subject represents the external world as an extension of itself. Hence the representation should reflect the physical mereological relation between the current occurrence and previous ones. An inseparable treatment of the representation and the represented is then understood as follows. The subject represents the 'external' world, but the very same representer should be seen as a physical mereological extension of the represented. In that sense, the occurrence mirrors the external world not only mentally since it represents the external world but also physically.

Although the mereological relation between the represented and the representation is a relation between concrete parts and non-concrete whole, still mereology does not abide by any assumption about concreta or abstracta; the whole as the parts can be concrete (e.g. the legs of the chair) and the parts as the whole can be abstract (e.g. the sides of a rectangle). The characteristic of this case is that the parts and the whole are of different type, so that both concreta and non-concreta are related mereologically.

The part-whole relationships developed in the theory of prehensions was aimed by Whitehead to be in accordance with the theory of extensiveness. This means that the part-whole relationship should be combined with topological relations between regions (since the parts are associated with atoms of space), hence the ordering of occurrences is defined mereotopologically.[13] This integration of mereological notions with topological ones is now known as mereotopology.[14]

Mereology is the theory of parthood relations, that is, relations between parts of a greater whole and between parts and the whole. The parthood is captured by the order relation. If P denotes the parthood relation, so that Pxy is understood as x is part of y, then the parthood is governed minimally by the following axioms: (1) Pxx (reflexivity), (2) $(Pxy \wedge Pyz) \rightarrow Pxz$ (transitivity), (3) $(Pxy \wedge Pyx) \rightarrow x=y$ (antisymmetry). Given these three axioms, one can introduce other predicates such as (4) $PPxy =_{df} Pxy \wedge \neg Pyx$ (proper parthood; the difference between parthood and proper parthood is that the latter is a partial order whereas the former is a strict (or irreflexive) partial order) and (5) $Oxy =_{df} \exists z(Pzx \wedge Pzy)$ (overlap).

One can then introduce topological notions and apply partial ordering to topological regions, considering that regions can be connected with each other and share common parts. A region can be included in another (in fact, every region includes other regions since the topological analysis is atomless), two regions may overlap by including the same third region or be externally connected, that is, connected but not overlapping. In particular, if C is the binary relation connectedness standing between x and y and

[13] Mathematical notions has already been introduced in Whitehead's book *An Enquiry Concerning the Principles of Natural Knowledge* (1919, pt. III).

[14] For more on mereotopology see Simons (1987, sec. 2.10.2) and Lucas (2000, chap. 10).

denoted as Cxy, then (1') Cxx (reflexivity) and (2') Cxy $\rightarrow$ Cyx. The enclosure relation is defined as Exy $\leftrightarrow$ (Czx $\rightarrow$Czy) (transitivity) and if E is anti-symmetric i.e. (Exa $\leftrightarrow$ Exb) $\leftrightarrow$ (a=b), it becomes a partial ordering, so that Pxy $\rightarrow$ Exy and Oxy$\rightarrow$Cxy.

Based on mereotopology, Whitehead aims to derive the abstract geometrical entities from concrete relations of 'whole and part'.

> The explanatory purpose of philosophy is often misunderstood. Its business is to explain the emergence of the more abstract things from the more concrete things. It is a complete mistake to ask how concrete particular fact can be built up out of universals. The answer is, 'in no way.' The true philosophic question is, [h]ow can concrete fact exhibit entities abstract from itself and yet participated in by its own nature? In other words, philosophy is explanatory of abstraction, and not of concreteness. It is by reason of their instinctive grasp of this ultimate truth that, in spite of much association with arbitrary fancifulness and atavistic mysticism, types of Platonic philosophy retain their abiding appeal; they seek the forms in the facts. Each fact is more than its forms, and each form 'participates' throughout the world of facts. The definiteness of fact is due to its forms; but the individual fact is a creature, and creativity is the ultimate behind all forms, inexplicable by forms, and conditioned by its creatures. (Whitehead, 1978, p. 20).

So, geometrical elements do not play a basic role in his ontology. Rather, it is the (actual) occurrences that are the fundamental entities. That is why, the properties of the extensive continuum (e.g. the extensive order) are not derived from 'relationships of metrical geometry' (ibid., p. 66), but are the outcome of part/whole relations.

In order then to define geometrical notions such as point, line and surface, Whitehead (ibid., p. 298) applies the mathematical concept of the abstractive set.[15] The abstractive set is a set of regions so that if A and B are two members of the set, then the one region is non-tangentially included in the other, and no region is included in every member of the set. In other words, the abstractive set is a set of embedded regions. Abstractive sets can converge to points, lines and surfaces and in that way, one can develop a standard point-set topology. Let us take for example the case of a point and an abstractive set of embedded regions which become smaller and smaller. Intuitively, this series converges to a spatial point. Since a point can be got by many series of regions, Whitehead defines the point as the co-convergence of all these abstractive sets. Co-convergence is defined as the covering of an abstractive set a from another abstractive set b where, covering is understood as each member of a including some members of b. Such abstractive sets are called 'equivalent'.

Theoretically, there is no upper or lower bound on the size of region so that the mechanism of concrescence can apply to any scale. Hence, an actual entity can range from any entity of the microcosm (particles etc.) to any entity of the macrocosm (human, planets etc.) and, therefore, the respective actualized time and space intervals can be of any order. Actual entities have some size but it can be any size (Ross, 1983, pp. 110, 183, 197–8). Thus, Whitehead is not anti-realist concerning non-directly observable things.

[15] An abstractive set is 'a generalization of Cauchy sequences' (Lucas, op. cit., pp. 274-276).

To close this section, there have been reactions to whether set theory is appropriate for the implementation of Whitehead's theory of extension. The mathematics (which involved set theory) that Whitehead employed and has been criticised as flawed (Lucas, 2000, pp. 277-279) or inadequate to capture his philosophy on mereotopological connections in correlation to his theory of prehensions (Epperson and Zafiris, 2013, p. 120).[16] Whitehead's approach is holistic and there is an emphasis on relations. It is commonly taken that set theory presupposes the existence of distinct relata whereas category theory provides a framework where relata are eliminated, so that it is relations that play major role and if relata are not removed, category theory can give an inseparable treatment of relata viz. relata are not taken in isolation. These are shown in section 4.3.4; it is explained that not only does category theory and in particular sheaf theory capture the internal relations that an actual occasion bears to its past occurrences, but it also implements the dipolarity that characterizes the prehensions (mereologically, the dipolarity will be understood as a transformation of the whole to parts and vice versa of the parts to give a new whole).[17]

2.4.3 UNIVERSALS AND PARTICULARS

In his interpretation of Whitehead's ontology, Simons (2009, 2013) gives emphasis on Whitehead's dualism of actual entities versus eternal objects, and the resultant dualism of prehensions that arises from it. Being pure non-actual potentialities, eternal objects

[16] As Lucas explains, there are pathological cases such as when two abstractive sets which are disconnected may still converge to the same point which results in this point not uniquely being characterized.

[17] For the bidirectionality between parts and whole in mathematics (including in sheaves), see (Bell, 2004).

are out of time, and are only real vicariously insofar as they do ingress and participate in actuality. As Simons claims, it is basically platonic participation where participation is taken as ingression of universals into actualities; except for the terminological difference between the Whiteheadian ingression of eternal objects into particulars and the Platonic instantiation of universals by particulars, eternal objects for Whitehead are only real as far as they ingress into actualities, otherwise they are nothing, merely pure potentialities. On the other hand, an actual entity is constituted of actual conceptual prehensions and physical prehensions, and it is never literally 'out of time'. It is the eternal objects that it conceptually prehends that are out of time.

Therefore, Simons disagrees with our view that potentialities reside in the mental pole: they are apprehended in the mental pole of an actual entity, but as what they are themselves they remain separate from the physical universe. This means that conceptual prehensions and physical prehensions (and likewise the mental and the physical) should be seen separately.

So, our difference with him lies on our emphasis of Whitehead's dipolarity and not on dualism. We do not focus on this separation between eternal objects and actualities and likewise, between universals and particulars, rather we take literally that '[the perfect realization] implants timelessness on what in its essence is passing', so that it is not that through its conceptual feelings an actual entity gives an eternal object an indirect toehold on actuality in time, but an actualization involves both the timeless and the temporal. An actual entity will then be taken to be both out of time and in time, so that not only the actual entity gives the eternal object a toehold in time but vice versa, an eternal object gives an actual entity a toehold in the timeless.

Our approach can be motivated by Whitehead characterizing as a misconception the assumption that actual entities are particulars and eternal objects are universals.

> The philosophy of organism denies the premises on which this distinction is founded. It admits two ultimate classes of entities, mutually exclusive. One class consists of 'actual entities,' which in the philosophical tradition are misdescribed as 'particulars'; and the other class consists of forms of definiteness, here named 'eternal objects,' which in comparison with actual entities are misdescribed as 'universals.' (op. cit., p. 158).

According to Whitehead's doctrine of universal relativity, it is misleading to say that a universal can participate in the description of many particulars while a particular does not participate in the description of other particulars (ibid., pp. 48, 149). The reason is that it is not only universals that are involved in the description of an actuality, but also that actualities participate in the description of other actualities. In that sense, every particular is a universal because it has a public role to play. A particular (or the actual entity upon satisfaction) enjoys universality for it contributes to the realization of other actual entities. Likewise, a universal manifests particularity when the subject prehends eternal objects in the mental pole. Due to the ingression of eternal objects and finally, their participation in every actuality, the universal acquires private character. In other words, an eternal object has a dual role to play (ibid., pp. 184, 290). It is a universal therefore it has a public character. But due to its ingression in every actuality, it gains a private character and is associated with some atom of space. That is, the eternal object loses its universality, due to the integration between conceptual and physical prehensions.

Hence, Whitehead concludes, the distinction between universals and particulars is blurred. Simons (2009b, 2013) would disagree with this idea. For Simons, the particular does not behave as a universal when an entity serves as a datum for others. It is prehended as a particular, but being the particular it is, it exemplifies numerous universals, and a later actual entity 'sees through' earlier actual entities to the universals they exemplify. According to Simons' interpretation, the universal characterizations of actual entities in the above passage is figurative, not literal, therefore actual entities can only appear as particulars.

According to our interpretation, in order to introduce universality to concrete particulars and particularity to abstract objects, one can entertain the option of abstract particulars (the abstract characterization of tropes can be justified by the involvement of the mental side. For more on abstract vs. concrete tropes, see section 5.4), even if Whitehead does not use this characterization. Although the notion of tropes appeared in Stout's (1921) work and hence, before the publication of *Process and Reality*, it is entertained primarily in later philosophy (Maurin, 2014, sec. 1).

One could object that an abstract universal acquires particularity when it is instantiated, so why should we consider the case of abstract particulars? This is justified by our decision to treat the mind and the body inseparably, where this inseparability is expressed as a bidirectionality between the whole and the parts. One characteristic of tropes is that they capture the interplay between the parts and the whole and the unrepeatability that characterizes each occurrence, that is, the direction whole-to-parts and parts-to-whole in the compresence and resemblance relations respectively. The compresence relation shows that many tropes (whole) are coexistent in some concrete particular (part) and the resemblance relation shows that many tropes (parts) coalesce to give a trope that has a more universal character (whole) since it captures the resemblance between tropes.

Since abstract universals are negatively or positively prehended, so that there is a transformation from pure to real potentiality, the relevant real potentiality can be taken as abstract particular. A pure potentiality or an abstract universal becomes particularized in the respective subject of the concrescence which imposes the perspective; a real potentiality can be then associated with an abstract particular. How an abstract universal and an abstract particular can be related is a matter of investigation considering that the former is repeatable while the latter is not; one can easily notice that employing the resemblance relation between tropes, an abstract universal is derived if there is exact similarity between the tropes which is a trivial case considering that one would expect degrees of resemblance between tropes.

In addition, since an actuality upon perishing transforms to a real potentiality, particulars cease to be concrete. A local potentiality can be treated as an abstract particular; it is related to a certain atom of space and each thing has its own history of occurrences and mereological relations. The resemblance relation between abstract particulars can then justify the universality of concrete particulars prehended by other occurrences.

To conclude this section, it is claimed that there are abstract universals, abstract particulars and concrete particulars. Does any of them have fundamentality over the other? The answer is negative.

Whitehead puts on the same ontological level actualities and eternal objects. 'The fundamental types of entities are actual entities, and eternal objects; and [..] the other types of entities only express how all entities of the two fundamental types are in community with each other, in the actual world.' (op. cit., p. 25). Other types of entities apart from actual entities and eternal objects are listed in *The Categories of Existence* (ibid., p. 22). For example, a multiplicity consisting of actual entities is less fundamental than the actualities. Likewise, a nexus which is different from a multiplicity, for the former is not a set of actualities rather it describes the relations between them, is less fundamental than the interrelated actualities. Indeed, in a creative advance, there is no prior actuality, but there are always the eternal objects or equivalently, both poles are present in concrescence.

However, it should not be the case that abstract universals and concrete particulars are more fundamental than entities such as abstract particulars, which involve the participation of prehensions for the establishment of the relevant transformations. This would contradict the main principle that nothing can be taken in abstraction. The abstract cannot be taken separately from the concrete, which implies that the involved prehensions are not given less priority than eternal objects and actual entities. In other words, it is the entire structure that describes the concrescence that is fundamental.

2.5 TIME

In a previous section, it was argued that the continuum constitutes potentiality, while its actualization results in atoms of space and time. This section focuses on time. An actuality does not occur gradually. The process of becoming is not something that extends in time as the time unfolds, that is, as there is no underlying space, there is also no underlying time where a process takes place. The elimination of points of a continuum is in accordance with Whitehead's realist attitude that the actual should be in accordance with what one experiences. 'It needs very little reflection to convince us that a point in time is no direct deliverance of experience. We live in durations, and not in points.' (ibid., p. 142).

The analysis proceeds to show how atoms of time are involved in the general process scheme. In particular, the mental pole is the timeless domain of potentialities while the physical pole is the temporal domain of actualities, where temporality is understood as occurrence of atoms of time. The section ends with the notions of present and of contemporary occurrences.

2.5.1 THE BECOMING OF TIME

As mentioned, the actualization of time concerns the becoming of an atom or quantum of time.[18] In Whitehead's metaphysics, each actual entity enjoys its own quantum of physical time:

[18] Since the life of an object is a manifestation of a creative advance, the life-span of an enduring object such as the universe originated due to actualization of the field. This original actual occasion counts as the original atom of time which constitutes the original discrete moment in the life-history of the enduring object. Time then is not prior to the universe. In a way, this reminds Augustine's (2002, bk. 11) view that time is not prior to the universe, but time comes into being along with the universe. The reason is that time, like objects, is created by God who exists timelessly and is the cause of everything and hence, of time as well (ibid., bk. 12).

> The actual entity is the enjoyment of a certain quantum of physical time. But the genetic process is not the temporal succession: such a view is exactly what is denied by the epochal theory of time. Each phase in the genetic process presupposes the entire quantum, and so does each feeling in each phase. The subjective unity dominating the process forbids the division of that extensive quantum which originates with the primary phase of the subjective aim. The problem dominating the concrescence is the actualization of the quantum in solido. (op. cit., p. 283)

In this quote, it is stated that the quantum of time involves both the potentiality, manifested in the mental pole, and the actuality, manifested in the physical pol, of an occasion, that is, all the stages of the concrescence, from the initial phase to the satisfaction. The quantum of time involves the initial phase where subjectivity is manifested as well as the completion and the transformation of the subject to object. It is then wrong to consider that the quantum of time involves only the satisfaction of an occasion which is related to the physical pole, but it embraces the entire process of becoming.[19] In addition, since the time becomes with the actual entity e.g. universe, it is not that potentiality is prior in time to actuality (or in terms of mind-body relations, the mind does not precede temporally the body, but both body and mind come as a package). So the time interval occurs timelessly (and not in time).

The involvement of the mental pole implies that a quantum of time differs in accordance with the subjective aim or perspective. A day for a human may not be a long period but this is not the case for a particle. Since there is no perspective that enjoys privilege, the

[19] See more on time in (Hurley, 1986, pp. 99–104).

duration of a quantum of time is not uniquely determined, but the time span is relative to the respective perspective. For example, in relativistic spacetime there are various potential decompositions of the invariant spacetime interval into potential timelike and potential spacelike intervals. The actualization of one of these potential timelike intervals may infer time dilation, while the actualization of one of these potential spacelike intervals may infer length contraction (Epperson, 2004, p. 171).

Considering now that, according to Whitehead, only the actual is temporal and that potentialities are timeless, the fundamental reality is neither timeless nor temporal. Rather, it is a combination of temporal actuality and timeless potentiality.

> The perfect realization is not merely the exemplification of what in abstraction is timeless. It does more: it implants timelessness on what in its essence is passing. The perfect moment is fadeless in the lapse of time. Time has then lost its character of 'perpetual perishing'; it becomes the 'moving image of eternity.' (op. cit., p. 338)

In the above quote, Whitehead argues that a concrescence involves both potentiality and actuality, so that the eternal things participate in the temporal ones.[20] Note that timeless is a characteristic of potentiality, no matter if it is real or pure (ibid., p. 40). It is not then only the eternal objects that are timeless. This is because:

> Every actual entity is 'in time' so far as its physical pole is concerned, and is 'out of time' so far as its mental pole is concerned. It is the union of two

[20] The eternal and the temporal are united due to the mediation of the divine element of the world that incorporates both the actuality of the things which are temporal and the timelessness of the things which are potential (Whitehead, op. cit., p. 40).

> worlds, namely, the temporal world, and the world of autonomous valuation. (ibid., p. 248).

Since a concrescence involves both poles, the eternal things participate in the temporal ones. The interaction between the two poles as well as the notions of temporal and timeless are related to the notion of change.

In order to define change, Whitehead (ibid., p. 24) defines the notion of event which is a nexus of interrelated actual occasions (an actual occasion constitutes an event with only one member (ibid., p. 73)). That is, change concerns actual entities which prehend each other. The requirement for a nexus is because a single actual occasion is not subjected to any change. An actual entity taken alone remains immovable because any difference in motion cannot be captured by space and time, since an actual occasion does not move in space and time. Likewise, in a nexus of actual occasions, actual occasions cannot be attributed with motion. Rather, any change is the outcome of differences between actual occasions in the nexus. Therefore, Whitehead's account deviates not only from Newtonian physics, since there is no actual space and time existing in the background, but also from Descartes' philosophy on motion. Contrary to Descartes' plenum where bodies move, here there is a plenum of actual entities but these actual entities do not move (in space and time).

So, though change seems to be continuous, it is in fact a succession of occurrences characterized by their own atom of time and related logically and mereotopologically. If then an occurrence is a part of a greater one, then their relationship is described mereotopologically and there is no need for defining a location of the occurrence simply because, there is no space in the background.

Finally, since change is defined on the nexus of occurrences and time is an occurrence, in order to give an account of time e.g. its passage, one needs firstly to give an account of change between occurrences. That is, in order to measure time, we need to consider the relevant changes between nexus of actual occasions; a partial order of (or a nexus of) occurrences (or measurement outcomes) is defined while time intervals are taken by the relevant differences between subsequent measurement outcomes.

2.5.2 PRESENTNESS AND CAUSALITY

In Whitehead's thesis, temporal duration is the analogue of the relativistic surfaces of simultaneity, with the difference that the temporal duration has a temporal thickness and does not refer to an instant of time.[21] A temporal duration involves an actual entity M and its contemporaries. The contemporary actual entities are in 'unison of becoming' (ibid., p. 320) which means that they do not feel each other, because an actual entity does not have its contemporary actualities in its causal past, that is, contemporary entities are causally unrelated.

[21] Simultaneity has already been discussed in his (1919) book, where Whitehead disagrees with Einstein's operationalist description of simultaneity (according to operationalism, only measurable quantities are meaningful). Einstein's definition of simultaneity was based on the assumption that the speed of light remains constant (for more on Einstein's operationalism see (Brown and Pooley, 1999, 2004)). Simultaneity can then be determined by actual measurements, namely, that by sending light signals, an observer can determine which events are simultaneous. Einstein then argues that since moving clocks genuinely slow down, they do not measure real time. Likewise, real length is also undetectable, therefore, real time and real length are non-existent. Whitehead (op. cit., 53) rejects the idea that a definition of simultaneity should be based on presupposing certain features of light. As Bain (1998) explains, Whitehead objects to the meaning that simultaneity is assigned in terms of the behaviour of light signals. However, in Process and Reality, in accordance with Einstein, Whitehead (ibid., p. 61) claims that contemporary events are causally disconnected, so causality functions as a limitation for simultaneity (White, 1983).

Consider for example the diagram below. Here, C does not feel A and A does not feel C, because A and C are contemporaries not bearing unity of relatedness given by a feeling. An actual entity may belong to an infinite number of durations and, for each duration, an actual entity has different contemporaries, for example C belongs in durations D1 and D2 and is contemporary with A and B respectively. This is in analogy to Special Theory of Relativity (STR) where different simultaneity surfaces define different simultaneous events. The characteristic of the relation of contemporaneity (R) is that it is reflexive (A is contemporary with A), symmetric (if A is contemporary with C, then C is contemporary with A) but not transitive (if A is contemporary with C and C is contemporary with B, then it is not that A is contemporary with B). Although A and C, as well as C and B are contemporary, A causally feels B.

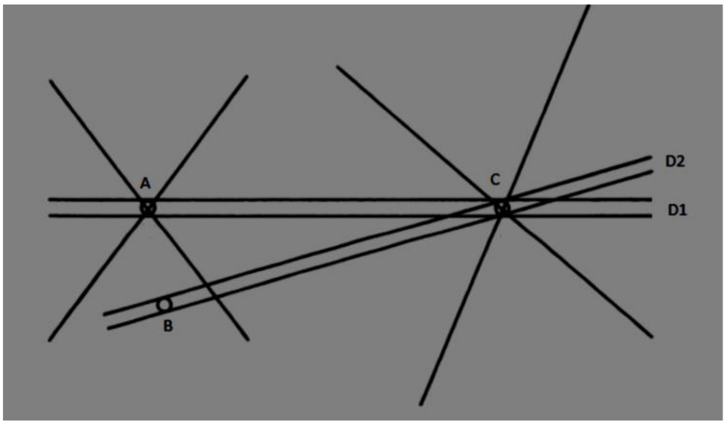

Figure 2.1: Durations D1 and D2 defining the contemporaries of occurrence C.[22]

Although the contemporary occurrences are unrelated, a kind of interrelation can still be recovered for contemporary occurrences that are causally connected with the same past actual occurrence. Whitehead introduces the notion of presentational immediacy in order to establish a relationship between the relevant contemporary occasions.

[22] Source: (Hurley, 1986, p. 107).

Presentational immediacy has to do with what entities one perceives as contemporary through the senses. What counts as contemporary events differs in accordance with the perspective. For example, the front parts of my desk and my laptop are in my presentational immediacy, but from another perspective it is the rear parts of them that they are in my presentational immediacy. Whitehead calls the respective duration, the 'presented duration' which is operative in the mental pole. So, contemporary entities should be seen in two ways, firstly, as actualities belonging to the same duration and each defining its own actual world and secondly, as contemporaries as perceived in the mode of presentational immediacy. If there was only causality, there would be no unity between contemporary events. This interconnection of contemporary entities via presentational immediacy is achieved due to the extensiveness of space, which provides the spatial interrelations. The actual moon and I contemporary with it are related potentially, so that there are only actual concrete connections between causally related entities.

The mode of presentational immediacy does not involve simple feelings, rather both conceptual and physical feelings (ibid., p. 63). Whitehead describes the relevant nexus as 'the contemporary nexus perceived in the mode of presentational immediacy, with its regions defined by sensa.' (ibid., p. 126) In the mode of presentational immediacy, the subject prehends the nexus of contemporary occasions via the mediation of eternal objects (i.e. sense data) which are unspatialized and transmitted from one occurrence to another (ibid., pp. 113-114). Sense data are taken from the subject's past e.g. the occasions of the moon one perceives belong to the backward light cone of the actual moon which is contemporary with her (ibid., pp. 64, 311, 318). So, the moon image is a representation referring to the antecedent occasions of the moon. When one stares at the moon, she prehends sense data concerning past occurrences that she prehends causally. For example, if B and C are contemporary and bear a simple physical feeling with A, B and C perceive the same sense data and prehend the same conceptual feeling. Now, if B and C are contemporary but do not have access to the same sense data (i.e. their separation is great), then B and C have different conceptual and causal relations. The point is that the location of contemporary nexus in the presentational immediacy involves contemporary entities that are in such distance so that the one sees past occurrences of the other. By introducing the mode of presentational immediacy, Whitehead confines the number of contemporaries of an occasion M. As Hurley (1986, pp. 99-104) points out, Whitehead exploits the fact that it is not only that for every inertial frame of reference different contemporaries for an occasion M are considered, but also that for every occasion M, certain actual occasions are in its presentational immediacy. As we will argue in section 3.1, in order to take spacetime events which are not separated far apart, a finite number of causally related events will be taken. This is consistent with the becoming of an atom of spacetime, where one needs to consider a finite number of near neighbours.

Since contemporaries are not physically connected, there is no unique seriality in the becoming of actual entities (Whitehead, op. cit., 35, 65-66). Each contemporary actuality belongs to a different actual world defined by the respective actual entity and the actual occurrences in its causal past. So, although in the diagram above, a serial order due to inclusion relationship between A and B (B is in the light cone of A) can be defined, the same cannot be said for contemporary entities. As a consequence, in the above example, A and the relevant nexus BA comes into existence after B, but one cannot determine whether C comes into existence along with B or along with A. If C becomes along with B, A becomes after C and if C becomes along with A, B becomes before C.

Nevertheless, one can define a non-serial order (Lango, 2000). The reason is that the relation feels is irreflexive, asymmetric and transitive, that is, it is a strict partial order. In fact, this is in agreement with the relativity of simultaneity of STR, according to which there is a partial causal ordering of events. Let us assume for example the following causal set:

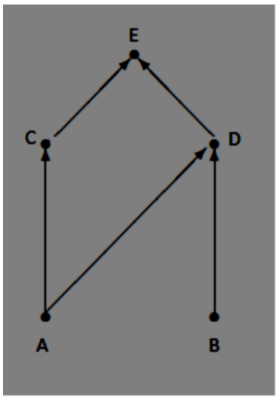

Figure 2.2: Actual occurrences related to their predecessors via the relevant nexús.

What one knows from the above diagram is that C and D are two actual entities in E's actual world, A is in the actual world of C and B is in the actual world of D. Equivalently, C feels A, D feels A, D feels B and E feels C and D. Note that contemporaneity is dependent on the duration, so that with respect to one duration, A and B are contemporary while with respect to another duration, C and B are contemporary. So, A occurs and becomes datum for the occurrence of C. Since C positively prehends A, A-C nexus comes into existence.

Note also that in correlation with the discussion on the becoming of time atoms, the occurrence of C does not come temporally after the occurrence of A, rather the occurrence of the time bubble C stands in a whole part relation with the occurrence of the time bubble A. Change is then defined mereologically and represented by the link/nexus A-C.

Furthermore, each new occurrence reflects its past occurrences due to the becoming of the respective nexus. For example, when C comes into existence, the prehension of A by C comes into existence as well, so that the present occurrence C reflects the nexus A-C. Therefore, the present is characterized by the current occasion plus all its causal past. Likewise, when D feels both A and B, the present occasion D reflects all its causal paths. As a consequence, although A and B are causally unrelated, they are both reflected at the present occasion. As for E, this causally prehends directly the past occurrences (C, D) and via them, it prehends indirectly the nexús (A-C, A-D, B-D). Hence, not only do causally related occurrences overlap at occasion E, but also the previous causally unrelated ones. One issue here is whether A-C-E should be taken as equivalent to the path A-D-E and hence one of them could be discarded, in the sense that they have the same starting and ending nodes.[23] On the one hand, E takes input from both C and D; C, D upon satisfaction, become potentialities which combine to give E. So, the present for E should be defined by the nexus (A - (C x D) - E) x (B - D - E). Different paths notify the life span of different enduring objects. The nexus of all actualities that constitute the span of life of an enduring object forms the historic route of the object (Whitehead, op.

[23] Discussion on alternative paths, their equivalence and which should be regarded the actualized one(s) appears with regard to causal sets (Callender and Wüthrich, 2014, pp. 7–8; Earman, 2008, p. 158; Sorkin, 2007, pp. 6–7). Callender and Wüthrich (2014, pp. 7–8) interpret equivalent paths in Sorkin's (2007) and Rideout's and Sorkin's (1999) Classical Sequential Growth (CSG) which employs causal set theory, as a defence for a tenseless view of time consistent with general covariance.

cit., pp. 187, 287). If paths interact, this signifies the interaction between enduring objects in the current occasion. Hence, paths are not equivalent. On the other hand, there is no evidence about which path, A-C-E or A-D-E, is taken, therefore the paths should be considered as equivalent. Such an outcome would be consistent with general covariance and eventually a tenseless view of time in GTR. Still although Whitehead's ontology suggests the existence of many presents, one for each occurrence still a tenseless view can be objected on the grounds that the universe as a whole is an occurrence that defines a global present. It would then be more appropriate to study the notion of general covariance in correlation to mereological understanding of dipolarity.

So, since past and future occurrences are potentialities while the present occurrences are the only actual, the latter are the ones that can define a present. Although past occurrences perished, in a way they are still actual for they are prehended by present occurrences, so that the relevant nexus is actual. So, '[t]he systematic scheme, in its completeness embracing the actual past and the potential future, is prehended in the positive experience of each actual entity.' (ibid., p. 72), so that '[t]he future is merely real, without being actual; whereas the past is a nexus of actualities.' (ibid., p. 214).

The above definition of presentness contrasts with a definition that involves surfaces of simultaneity. If presentness was defined in terms of temporal durations or the contemporary occasions in the presentational immediacy, it would be characterized as potential and not actual. The extensive continuum that unites contemporary occasions is a potentiality. It is then wrong to define presentness in terms of unconnected contemporary occasions. Also remember that the extensive continuum incorporates all the tenses. 'It underlies the whole world, past, present, and future', so that all tenses are real in their potentiality (ibid., p. 66). There can be many occurrences, each defining a present, but there is no such hypersurface that unites them.

Moreover, the above interpretation of present involved the subject who perceives sense data, but at the same time the physical causal connections are objective. Although generally speaking there is no base for the intersubjectivity and objectivity, since empirical knowledge is based on (private) sense data, still objectivity is recoverable because the causal relation between physical objects is objective and public. The objectivity of such a definition of presentness is in accordance with GTR since causal-connectedness in relativistic spacetime is an absolute relation independent of frame of references viz. all inertial systems will find the same sign of the difference between the time occurrences of two timelike separated events, as long as relativistic spacetime is temporally oriented (Earman, 1969, p. 279).[24]

Finally, since time is generated, there is no time with respect to which the present should be defined. As a consequence, the notion of tenses is disentangled from time. If presentness was defined as happenings at the same time and the happening concerns time itself, it follows that time-occurrences take place at the same time which occurs. This is avoided since the present is defined by a net of causal relations. The definition of present then involves the entire process of emission and reception of the signal in a relativistic spacetime. Two events are present although they occur at different times; if t

[24] If temporal orientability is not a presupposition, a non-temporally orientable spacetime is locally temporally orientable, for there is an open neighbourhood which is temporally orientable. As will be discussed in section 4.2.2.1, gauge invariance is expected to resolve inconsistencies between the orientations of local sections. Presentness should then be discussed with regard to the relation between the local and the global (see (Horwich, 1987)).

is the time the signal is transmitted from a remote object S to earth and t' is the time the signal is received, then both events are present.[25]

2.6 HOW TO UNDERSTAND DIPOLARITY IN PHYSICAL THEORIES

2.6.1 WHITEHEAD'S LOGICAL PROCESS

In the literature, there are different approaches of Whitehead's philosophy. One of them is Epperson's and Zafiris' (op. cit.) who emphasize the logical aspect of Whitehead's view in their physical framework. Whitehead's concrescence has been characterized as a logical process and not a process in time (also see (Simons, 2009)). Firstly, Whitehead argues that '[a] physical pole is in its own nature exclusive, bounded by contradiction: a conceptual pole is in its own nature all-embracing, unbounded by contradiction.' (op. cit., p. 348) It seems then that at the conceptual phase where the subject prehends eternal objects, the principle of non-contradiction (PNC) does not hold, and it is upon satisfaction, that PNC is satisfied (see paraconsistent logic and intuitionistic logic where contradiction can hold). Secondly, he (ibid., p. 11) argues that propositions having

[25] Although what is commonly argued is that the signal one receives now is regarded as present, in his presentist 'causal theory' standing opposite to tensed theories of instantaneous present, Godfrey-Smith's (1979) claimed that it is the signal that was transmitted by S light years ago that should be involved in the definition of presentness. Thus the state of a remote object S 'n years ago' is co-present with the state one observes 'now'. The reason is that it is wrong to say that n light years is the duration that the signal took in order to travel from S to here, because purely spatial separations are rejected by STR.

contextual character are partially true. The proposition is embedded in a context, so that it does not capture the information of the entire universe.

Epperson and Zafiris (op. cit.) apply intuitionistic logic to their interpretation of Whitehead's concrescence. Intuitionistic logic provides the framework for a) the truth-value of a proposition being multi-valued or equivalently, belonging to a domain greater than the classical {0, 1}, and b) the truth-values being contextual.[26] Epperson and Zafiris employ dipolarity in combination with sheaf theory to implement a) and b).

Topos theory and especially a topos of presheaves (which is the category of sheaves of sets on a topological space), where the internal logic is intuitionistic, can tackle the contextual and multi-valued character of propositions which are related to quantum mechanical phenomena.[27] One can derive global entities that may exist in a larger presheaf than the spectral presheaf (which is associated with a spectral (non-Boolean) algebra) is embedded in (Isham and Butterfield, 1998). This approach can provide a global valuation of all propositions, though a proposition is assigned a 'partial' truth value. This means that, although a proposition 'a' cannot be declared to be true or false, still, there can be a (Borel) function f such that f(a) is true; the f(a) is a coarse-grained but one can define more levels of coarse-graining.

Epperson and Zafiris (op. cit.) (see also (Zafiris 2006)) show that the contextuality of propositions and the projection to a set of Boolean events are translated and captured sheaf theoretically in the pair of functors L: Sets→W and R: W→Sets, where W is the category of quantum event algebras (intuitionistic logic) and Sets is the category of

[26] Apart from the violation of the principle of non-contradiction (PNC) ([¬(a ∧ ¬a)]), intuitionistic logic accepts the violation of the principle of excluded middle (PEM) (a ∨ ¬a=1).

[27] However, see other kinds of logic e.g. paraconsistent logic and superposition.

Boolean event algebras (Boolean logic). Hence there is a bidirectional mapping which is consistent with the dipolarity between categories in Whitehead's philosophy.[28]

However, one can provide a more general framework that is still consistent with Whitehead's basic idea though disentangled from any particular kind of logic and there are reasons to do so.

Shimony (1986) demonstrated that the consistence between measurements made with respect to the same frame of reference or different ones shows that actualization should not be taken that a physical quantity in entangled state acquires definite value (or the proposition becomes true or false). For Shimony, the notion of potentiality and the notion of entanglement are strongly connected, in the sense that both concepts are characterized by indefiniteness. He refers to potentiality as the 'combination of indefiniteness of value with definite probabilities of possible outcomes'. Still, as he claims that actualization on measurement should be disentangled from definiteness and the Boolean set {1, 0} so that not only the potentialities but also the actualities are characterized by indefiniteness.

We agree with Shimony in the sense that although a measurement results in some determinate outcome, we need to consider that outcomes are combined to give some greater result. Hence, the presupposition of a transition from a non-Boolean logic to a Boolean one raises concerns. However, as mentioned sheaf theory permits the mapping

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[28] A related idea appears in Markopoulou's and Smolin's (1997) and Markopoulou's (1997, 1998, 1999) attempt to combine spin networks with causal sets. They take the functor between the category of causal sets and the category of sets. Spin networks, the candidate of the gravitational field at the Planck scale, evolve according to a causal structure underlied by intuitionistic logic. What Markopoulou does is to create an evolving set of causal pasts as well as an evolving set of truth-values. Also see Isham's (2003a) sheaf theoretic analysis which considers propositions about histories which are statements about causal sets.

of the opposite direction, that is, from Boolean to non-Boolean, hence one can still recover an indefinite valuation. That is, if the map R: W→Sets from the category of self-adjoint operators with discrete spectra to the category of set results in definite truth values {0, 1} taken upon measurement, then a subsequent combination of measurement outcomes could be accommodated from the map L: Sets→W (see later sections e.g. 3.3 on combination of measurement results by sheaf theory).

However, the framework should be more general and logic independent. Consistency between measurements made with respect to the same frame of reference or different ones should be of primary concern, but this can be treated in terms of physical concepts and not logical ones such as gauge invariance as will become clear throughout the thesis, therefore a mapping to Boolean logic should not be a precondition.

Shimony's argument that both potentialities and actualities can be characterized by indefiniteness shows that the determinate or determinable character of a physical quantity is contextual, namely a measurement outcome is characterized by definiteness if it is seen in isolation and by indefiniteness if it is combined. This indeterminacy would be treated dispositionally in our analysis (chapters 6 and 7). The disposition is a determinable which satisfies the requirement of a range of values which may be contradictory and the manifestation results in a determinate quantity.

Hence, a more general resolution that applies sheaf theory will be given without the presupposition of a certain logic. As we will see, a sheaf theoretic treatment treats a mereological contextual character of generating series of physical quantities and is not a defence of a particular logic. Although it is regarded that the logic in the category of sheaves is intuitionistic, a presheaf associates categories of any type (e.g. the categories of causal sets, of groups, of Hilbert spaces, etc.) and not only the categories of non-Boolean and Boolean algebras. So, sheaf theory provides a general framework which is independent of the underlying logic.

Since we will go dispositionally, the next subsection analyzes Whitehead's view of capacities and laws.

2.6.2 LAWS AND CAPACITIES

According to Whitehead,

> [..] a system of 'laws' determining reproduction in some portion of the universe gradually rises into dominance; it has its stage of endurance, and passes out of existence with the decay of the society from which it emanates. (op. cit., p. 91)

> We here arrive at the notion of physical quantities which vary from individual to individual; this is the notion of the systematization of individual differences, the notion of 'Law.' (ibid., p. 98)

One discerns a kind of local governance, in the sense that a system of laws controls the reproduction of some occurrences. A law does not stand independently of the occurrences, but like every other thing in Whitehead's ontology it becomes, comes into existence and decays.

A law has a certain endurance which is determined by the life of the society of occurrences from which it is obtained. The law is context relative where this context is the nexus of the actual objective world, therefore it is objective (ibid., pp. 67, 110). The elements of a law are arbitrary and defined by the widest society of actual occurrences, or by a 'cosmic epoch' as Whitehead expresses it (ibid., pp. 91, 98). Whitehead discerns a succession of cosmic epochs where the laws of nature differ. One can then discern epochs e.g. epochs characterized by electronic and protonic occurrences (remember that an electron and a proton are enduring objects described by a society of electronic and protonic occasions respectively).

Whitehead makes use of capacities to give an account of laws. The occasions have the capacity to reproduce, that is, because of their capacity, there are more occurrences. These occasions are the cause of the laws. The characteristic of Whitehead's view is that not only is there a direction from capacities to laws but the picture is reversed. The capacity for reproduction is in virtue of these same laws while failure in the reproduction scheme is translated as the law not being totally obeyed (ibid., p. 91).[29] In general, the individual occurrences exist in virtue of the law of the society, but at the same time, the laws 'come into being' through its individual members (ibid., pp. 90-91).

For example, in electromagnetism:

> These occasions are the reasons for the electromagnetic laws; but their capacity for reproduction, whereby each electron and each proton has a long

[29] If an epoch is dominated by electromagnetic occasions, and this dominance is close to completeness, then the law is dominant, else the dominance is not complete and the law is statistical (ibid., p. 98).

> life, and whereby new electrons and new protons come into being, is itself due to these same laws. (ibid., p. 91)

For example the quanta of energy and the associated rhythmic periods are described by a law, but at the same time they are derived from protons and electrons, so that the law is in virtue of protonic and electronic occurrences (ibid., p. 79).

How can we make sense of the above? These can be explained by the dipolarity between the mental and the physical in combination with the notion of capacity.

The notion of capacity is related to the notion of potentiality. Both pure and real potentialities have capacities. Eternal objects have abstract capacities (ibid., p. 214) and a proposition has a capacity to be realized (ibid., p. 224). A conceptual feeling has the capacity to be an actualized determinate of an occurrence (ibid., pp. 239-240). That is to say, it has the capacity to determine the character of an occurrence. Hence, in general a potentiality has the capacity to determine. Likewise, potentialities that are past occurrences have capacities:

> The character of an actual entity is finally governed by its datum; whatever be the freedom of feeling arising in the concrescence, there can be no transgression of the limitations of capacity inherent in the datum. The datum both limits and supplies. (ibid., p. 110)

The datum is the nexus of past occurrences which is prehended by the subject. It is a potentiality (past occurrences transform to potentialities upon perishing) that acts as a datum and governs a future actuality. Hence, capacities which govern or determine actualities are potentialities. But a law can govern if it is actual not potential. This actuality is provided to the law by past occurrences (or the initial datum which was an actuality) and if there is no past occurrences, then '"God" is that actuality in the world, in virtue of which there is physical "law."' (ibid., p. 283), so even if there are no past occurrences playing the role of the initial actual state, still all the system is restricted by the ultimate actuality which is God who is actual but timeless.

To recapitulate, a law being a potentiality becomes actualized and governs occurrences which in turn give rise to a law. A system of laws is generated, endures for some occurrences and perishes. An occurrence isolated by external influences internally prehends (negatively or positively) the entire world. Each occurrence inherits information from the past and adds new. A system of laws captures these differences between occurrences.

One last remark is that although the law as presented above concerns only occurrences that are in the nexus, this does not seem entirely true. Whitehead argues that one needs to derive an interaction between societies and the laws of nature, where these societies are characterized by the perceptive mode in presentational immediacy (ibid., p. 327). Remember that presentational immediacy confines the number of contemporary actual occurrences. It seems then that the law should take into consideration the role of presentational immediacy.

The above discussion will be applied as follows in chapters 6 and 7. Capacity will be understood as determinable disposition captured by the pair field/matter so the actualization will appear as the (determinate) manifestation of the field in a matter. Since due to dipolarity, determinates play an equally important role as determinables, laws (which give relations between determinables) will not be fundamental (although dispositions will not be taken fundamental either) and this will justify our approach to derive the law from determinable dispositions. In addition, although the subject constitutes a part of occurrence, the context of a concrescence is the objective world that acts as a datum. This will have an impact on the recovery of objectivity in the derivation of laws.

2.7 CONCLUDING REMARKS

The interrelation of everything does not sound strange once the findings of modern physics are considered. Currently, holism is a common understanding motivated by the nonseparability appearing in QM.[30] However, in comparison to common interpretation of QM measurement, a Whiteheadian interpretation should be based on a 'non-temporal mereological' transition from potentialities (determinables associated with the entangled state) to actualities (the determinate value of the measured physical quantities), which is consistent with Whitehead's theory of prehensions. The non-temporal mereological transition from potentiality to actuality reflects the non-existence of a background time, so that the potentiality is not prior in time to actuality.

Real potentialities will be treated in terms of actual non-concrete dispositions that become actualized or manifested. Causality will be related to dispositions (which will stand for the notion of potentiality). Although causality is commonly defined at the level of concrete actualities due to the dipolarity between mental and physical, causality is also related to the realm of potentialities. We again contrast Simons according to whom

[30] See discussions on nonseparability and QM in (Howard, 1985; Teller, 1986; French, 1989; Maudlin, 2011).

it is through positive physical prehensions that causality operates, that is, causality is confined at the realm of concrete actualities. For us, since dipolarity expresses a bidirectionality between parts and whole, a full account of causality should be based on a mereological treatment between the potential and the actual.

Whitehead's framework exhibits an actualism in real potentialities and the concrete actualities while it accommodates other entities apart from actual ones; past actualities act as input in the generation of real potentialities of current occurrences and moreover, pure potentialities are confined to real ones to give an actual thing. In Whiteheadian philosophy, a limitation of possibilities is imposed in the transformation of abstract potentialities to real potentialities. In this phase, the infinity of possibilities is confined to real potentialities so that the relevant satisfaction will follow. Hence, contrary to modal realism which assumes that all possibilities (for some, there are also possible abstract worlds) are real with no restrictions, in Whitehead's view, there are restrictions. Moreover, modal realism is denied also in the sense that the range of possibilities does not imply the existence of other possible worlds standing separately from the actual one. Real potentialities are strongly related to what is actual since possibilities are conditionalized by an initial state determined by actual past occurrences. It is important to note at this point that in order to bring the discussion in more recent debates in modality, the terms abstract non-actual possibilities, abstract actualities and concrete actualities will substitute Whitehead's terminology of pure potentialities, real potentialities and concrete actualities.

3 SHEAF THEORY

Abstract

This chapter presents the algebraic framework of sheaf theory. The aim is the elimination of spacetime from the background and the construction of physical quantities out of mereological relations. An equivalent treatment between the macro and micro levels is implemented via a unifying structure of coverings of a topological space which are characterized by different degrees of graining.

3.1 INTRODUCTION

Historically, it was Grothendieck (1960) whose topology employs open coverings, on the basis of which, sheaves as well as other constructions such as cohomology can be defined. The notions of coverings, sheaves and cohomologies will be employed for the construction of a system that unifies the macro-level with the micro-level and eventually, offers quantization.

Although the continuum appears at the beginning of the construction, in sheaf theory, it is eventually removed from the background. Theories can begin with a topological space, but this is used for the construction of the sheaf and cohomology so that at the end, it is rejected. Even if manifolds are employed, these are not external standing as a background but are internalized and treated as objects of a category. An emphasis of space(time) atoms (regions, posets, manifolds) entails that a wave function will not be a function $\psi(x, t)$ of a spacetime point but a function $\psi(A(U))$ of an algebraic object $A(U)$, where U is the space(time) atom.

The procedure for the derivation of sheaves can be summarized in the following steps:

a. Atomization of spacetime
b. Algebrization
c. Quantization

Let's see these steps in turn.

Atomization

To atomize spacetime, one takes a covering $\{U_i: U_i \in X, i \in I\}$ of a topological space X, namely, a topological space X is covered by open subsets U_i such that their union $\cup U_i = X$. Subsequently, the covering can be associated with a poset or a causet, so that there is no more a continuum of events.[31] The mereological ordering between open sets

[31] The difference between posets and causets is that the latter have causal (or chronological) significance while the former have only topological (see discussion by Sorkin (1991, 1995) who, in his causal set approach, claims that the metrical aspects of the continuum can be recovered from a causal set approach at the fundamental level).

expressed by inclusion relations $U_i \subseteq U_k$ and overlapping $U_i \cap U_j$ is likewise captured in causal sets.

A causal set provides a partial causal ordering of events (in this case region-events), where causality is determined by the past and future light cones, so that event p which resides in the past (future) light cone of event q causally precedes (succeeds) the q event. A causal set is a locally finite, partially ordered set (or poset) C with the binary relation $\langle$, which expresses precedence so that: (1) if $x \langle y$ and $y \langle z$ then $x \langle z$, $\forall x, y, z \in C$ (transitivity), (2) if $x \langle y$ and $y \langle x$ then $x = y$ (antisymmetry) (3) for $x, z \in C$, the set $\{y \mid x \langle y \langle z\}$ of elements y lying between x and z is finite (local finitarity). The antisymmetry reflects the assumption that the binary relation is not reflexive, where a reflexive relation is denoted as $x \langle x$ and translated as x precedes itself; however, there are causal set scenarios which take the binary relation $\langle$ to be a reflexive relation.[32]

Finer coverings result in finer causets or posets. For example, Sorkin (1991) provides a coarse-graining algorithm of finitary substitutes where a covering of a topological space X is associated with a poset (or a causet). In a nutshell, finite open coverings $\{U_i\}$ of an open region X define different degrees of graining. The coverings can then be ordered by refinement relations '$\preceq$' so that for an open covering $\{U_i\}$ which is coarser than $\{U_j\}$, $\{U_i\} \preceq \{U_j\}$ or $U_i \preceq U_j$ for simplicity reasons. Subsequently, from the open sets in $\{U_i\}$, the respective poset (or causet) is derived. In particular, the quotient space $X/\sim_{\{U_i\}} = P_i$ is a poset (or causet) derived out of equivalence classes of points in X. The refinement relation '$\preceq$' then applies from coarser to finer posets, viz. $P_i \preceq P_j$.

[32] Irreflexivity, supported by Malament's (1977) theorem, means that there are no closed causal curves although the latter are supported by GTR. Irreflexivity is taken that a new element coming into existence cannot succeed its occurrence. For more see (Earman, 1974, p. 17; Wüthrich, 2006, p. 99).

As we will see throughout the thesis, the main motivation is that coarse graining is employed for the localization of the measurement of quantum quantities; a fine, micro-level description will be treated in a more pragmatic, coarse, macro-level way.

In addition, if the covering concerns some finite number of nearest neighbours and consequently, a finite number of causally related events, the atomization achieves localization to some region or causal set respectively. In particular with respect to causal sets, the recovery of a local notion of causality is important, considering that the problem that causal set theorists face is the non-localized character of the causal relation $x \langle y$, so that the two events can be separated far apart, for example the first event can correspond to the birth of the universe and the latter one to its death. In addition, this localization is with respect to some context of controllable size which is more promising solution than taking the entire universe for measurement.

Algebrization

The second step is the algebrization, which gives a mapping from open sets or causets to a relevant algebra. For example it has been shown that there is a relation between posets and incidence algebras (Rota, 1964), where in the finite case, an incidence algebra has matrices with ones in ij locations capturing the order relation $i \rightarrow j$ of posets and with zeros in the remaining locations (Sorkin, 2003). Or Isham (1996) takes the Heyting algebra that he associates with quantum history propositions related by a refinement relation; the idea is similar namely, classical behaviour emerges at a coarse-grained scale and decoherence is seen in terms of quantum history propositions that associate quantum description at a finer resolution where the logic is intuitionistic to a classical description at a coarser resolution where the logic is Boolean (see section 3.3).

In the following sections, the refinement relation between coverings is translated as coarsening relation between the associated algebras, namely for algebra $\Omega_i^k(U_i)$ of k-th degree, $\Omega_j^k(U_j)\preceq\Omega_i^k(U_i)$ for open coverings $U_i\preceq U_j$ or in terms of algebras being associated with poset/causets, $\Omega_j^k(P_j)\preceq\Omega_i^k(P_i)$ for $P_i\preceq P_j$. The reversal of the ordering relation is translated as the poset category being contravariant to the algebra category.

Quantization

The third step concerns the quantization of the system. This is achieved by the association of the structures in previous steps with structures that offer quantization. E.g. the mereological relation between regions can transfer to mereological ordering between Hilbert spaces. Apart from the mereological relation between regions, other frameworks such as topological spaces, posets, causal sets, algebras that capture the mereological ordering of regions can likewise be applied so that the mereological ordering expressed in such structures is conveyed in the mereological ordering between Hilbert spaces.

The following section outlines the sheaf framework that implements the above.

3.2 THE SHEAF THEORETIC FRAMEWORK

3.2.1 PRESHEAVES AND SHEAVES

Sheaf theory provides an algebraic framework implementing a mereological treatment of overlapping regions, where these regions are not defined with respect to a background spacetime. Starting from the category of regions (where regions are expressed as topological spaces, partially ordered sets, causal sets, manifolds), one can obtain constructions that refer to other categories e.g. the category of algebras, the category of Hilbert spaces etc. This is due to the fact that sheaf theory shows how two categories are interrelated.

A category C consists of a) objects a, b, ..., b) morphisms or maps or arrows $f: a \to b$, ... from a source object (in this case, a) to a target object (respectively, b), and c) a binary relation $\circ$, called composition of morphisms, so that if $f: a \to b$ and $g: b \to c$ then the composition of f and g gives $g \circ f: a \to c$ which satisfies: c1) the identity axiom $id_x: x \to x$ such that $id_b \circ f = f = f \circ id_a$ and c2) the associativity axiom $h \circ (g \circ f) = (h \circ g) \circ f$, for $h: c \to d$.

In sheaf theory, presheaf is a contravariant functor or a mapping between categories, so that a functor associates objects and morphisms of the categories. In particular, a contravariant functor F associates an object a in category D with an object F(a) in category E.[33] Likewise, it associates the respective morphisms, that is, each morphism

[33] Case studies are a) the derivation of the presheaf between the poset category and the category of incidence algebras in (Raptis, 2001; Raptis and Zapatrin, 1999, 2001), b) the derivation of the presheaf of Hilbert spaces over the

$$f: a \rightarrow b \text{ in } D$$

with a morphism

$$F(f): F(b) \rightarrow F(a) \text{ in } E$$

The objects a and b of category D and respectively, the objects F(a) and F(b) of category E are connected with each other by arrows which are order-preserving maps between them. The difference in the direction of the arrows in F(f) is justified by the fact that the contravariant factor reverses the direction of the f morphism. The contravariant functor preserves identity and compositions morphisms, i.e.

$$F(id_a) = id_{F(a)} \text{ and } F(g \circ f) = F(f) \circ F(g)$$

for $f: a \rightarrow b$ and $g: b \rightarrow c$.

As an example, one can consider a presheaf F defined on topological space X. F is a functor such that, for an open set U of X, there is an object F(U) in category E and for $V \subseteq U$ in X, there is a morphism, so-called restriction morphism,

$$res_{V,U} : F(U) \rightarrow F(V)$$

in E. Since E is a category of F(U) objects and restriction morphisms, the identity axiom expressed by the identity morphism

$$res_{U,U}: F(U) \rightarrow F(U)$$

category of causal sets (or posets, topological spaces or groups) and the consideration of morphisms as analogues of operators in (Isham, 2003a, 2003b, 2003c) c) the definition of functors between the category of three-dimensional spaces and the category of Hilbert spaces, and observables, of which the expectation values are topological invariants in (Witten, 1988).

and the associativity axiom

$$res_{W,V} \circ res_{V,U} = res_{W,U}$$

for $W \subseteq V \subseteq U$ are satisfied. F(U) are the sections of F over U while a global section is a section over X.

A presheaf converts to its corresponding sheaf if it satisfies the following two conditions:

a) Locality: if $\{U_i\}$ is an open covering of an open set of U and $s, t \in F(U)$ are two sections of F over U such that $res_{U_i,U}(s)=res_{U_i,U}(t)$ for each set U_i of the covering then s=t.

b) Gluing: if $\{U_i\}$ is an open covering of an open set of U, $U_{ij}=U_i \cap U_j$ is the overlapping of sets U_i, U_j of the covering and $s_i \in F(U_i)$ is a section of F over U_i such that $res_{U_{ij},U_i}(s_i)=res_{U_{ij},U_j}(s_j)$ for all i, j, then there is a unique section s in F(U) such that $res_{U_i,U}(s)=s_i$ for each i.

3.2.2 SIEVES

The above definitions a) and b) concerned some open cover, but there can be a generalization to the category of open covers on some topological space X. The categorical term that can capture open covers is the covering sieve, e.g. for an open set on the topological space X, a series of open covers of the open set is associated with a covering sieve, where the reference contexts are related via morphisms. Hence, instead of a covering of a topological space by open sets, the notion of covering is associated with a family of morphisms that end to the same context.

In particular, if O(X) is the category of partially ordered sets, a U-sieve is a family S of O(X) morphisms with codomain U, such that:

a) f: V→U for a context V which is a coarse-graining of the context U.
b) If (f: V→U)∈S and (g: W→V) is an O(X) morphism, then (S(g)(f): W→V→U)∈S

The b) expresses the pullback operation on a sieve denoted as S(g)(f). It shows that the pulling back of U-sieve by f: V→U results in a new sieve g which is defined on V and is the totality of morphisms targeting to V. In addition, considering that a U-sieve is a set of arrows of which all elements are coarse-grainings of U, any two U-covering sieves S, R have the same refinement, namely S∩R is a U-covering sieve.

Then a presheaf A of algebras on O(X) means that for f: V→U, A(f): A(U) →A(V). Subsequently, the presheaf converts to its corresponding sheaf by a process called sheafification. Using the notion of germ of a sheaf, which is nothing else than an equivalence class of compatible sections over a covering sieve, sheafification captures how overlapping sections are related. The sheaf derivation of the respective germ involves the gluing of sections and expresses the requirement that sections of A over O(X) can be glued together to give a unique section as long as their restrictions agree on the overlappings. A germ is specified by the equivalence relation (or the stalk) defined with respect to a point x in the intersection of the sections, which means that there is a neighbourhood of x at the intersection where sections have the same restriction so that the equivalence classes give the stalk A_x of the presheaf A.

In particular, the notion of germ is related to the notion of colimit which stands for the categorical limit of algebras. For an algebra A(U), the colimit is defined

$$\mathrm{Colim}_K[A(U)]=D(x)/\sim_K$$

where D(x) is the disjoint union of all A(U), i.e.

$$D(x) = \coprod_{U \in K} A(U)$$

and K is the set of open subsets of topological space X containing x. The $\mathrm{Colim}_K[A(U)]$ of sets of A(U) is the equivalence relation in D(x), so that p is equivalent to q (p~q) for p∈A(U) and q∈A(V), if they have the same restriction to the same open set in K i.e. $res_{U\cap V,U}(p)=res_{U\cap V,V}(q)$. The stalk of A at the point x, denoted A_x is the above colimit. The germ is then defined by the formula

$$i_{W,x}(p)=p_x=\mathrm{germ}_x p$$

p∈A(W) and

$$i_{W,x}:A(W)\rightarrow A_x$$

is the inclusion morphism for open set W containing the x in the intersection U∩V. Hence, germs are taken by the maximum localization or refinement of a net of sets and offers maximum localization of the algebras.

Different coverings results in different refinements so that fine graining associated with the quantum regime is brought together with coarse graining associated with the classical level. The aim was to find a structure that brings together the quantum and the classical in an inseparable way; this structure is captured by the notion of covering sieve which is a historical covering structure of contexts so that the quantum is related to the classical. The inseparable treatment is translated as a restriction of the micro to the macro (localization procedure) and as an extension of the macro to the micro (gluing mechanism). There are then two directions, namely a localization of the quantum algebraic structure with respect to reference contexts and a gluing of compatible local sections so that local information becomes a global one.

3.3 EXAMPLES: DIFFERENTIATION AND THE RELATION BETWEEN MACRO AND MICRO[34]

This section provides examples that employ sheaf theory.

1. *de Rham cohomology*

Differentiation can be provided by the de Rham complex which determines higher degree sheaf morphisms denoted as d^n: $\Omega^n \rightarrow \Omega^{n+1}$. In general, the de Rham complex is given by:

$$0 \rightarrow \Omega^0(M) \xrightarrow{d^0} \Omega^1(M) \xrightarrow{d^1} \Omega^2(M) \rightarrow \ldots \rightarrow \Omega^n(M) \xrightarrow{d^n} \Omega^{n+1}(M) \rightarrow \ldots$$

[34] See also (Mallios and Zafiris, 2011; Zafiris, 2007).

where $\Omega^0(M)$ is the space of smooth functions on M and $\Omega^i(M)$ is the space of i-forms. Algebraic differentiation denoted as d^i is then given by morphisms between the space of differential forms of some degree.

The above scheme can extend to a de Rham complex that involves sheaves of algebras of n-forms, namely:

$$0 \to \mathbb{R} \xrightarrow{id} A \xrightarrow{d^0} \Omega^1(A) \xrightarrow{d^1} \Omega^2(A) \to \ldots \to \Omega^n(A) \xrightarrow{d^n} \Omega^{n+1}(A) \to \ldots$$

where $\Omega^0(A)\equiv A$ is the sheaf of algebras of $\mathbb{R}$-valued functions, $\Omega^i(A)$ is the sheaf of algebras of differential i-forms on X, id represents the constant sheaf that gives identity morphisms (Mallios, 2006, 2009). The classical 'd' is now replaced by the algebraic differentiation 'd^i' which acts on the sections of the sheaf algebra A(U) defined with respect to some open covering. As a consequence, a dynamical mechanism is provided with no spacetime in the background.

As an example, Mallios (section 4.2.2.2) takes E to be an A-module sheaf that represents the matter field and the morphism which experesses differentiation is the connection D on E, viz.

$$D: E \to \Omega^1(A) \otimes E = \Omega^1(E)$$

which satisfies

$$D(as) = aD(s) + s \otimes da$$

where a is a complex differentiable function, s is a section of E and d is exterior differentiation.

2. *Čech cohomology*

In subsection 1., the cochain was presented with respect to some local frame U but this can extend to the general case to include many local frames while by successive refinements, a common local frame can be defined.

In particular, the de Rham is isomorphic to the Čech cohomology, which is generated by the Čech complex:

$$C^0(U,A) \overset{\delta^0}{\rightarrow} C^1(U,A) \overset{\delta^1}{\rightarrow} C^2(U,A) \rightarrow \ldots \rightarrow C^n(U,A) \overset{\delta^n}{\rightarrow} C^{n+1}(U,A) \rightarrow \ldots$$

where U ranges over the open coverings. The Čech cohomology is $\check{H}^i(U, A)$, for A the sheaf of algebras, so that the sequence of cohomology A(X) modules is given by $\check{H}^i(U, A)$ of X. This is due to the fact that Čech cohomology is generated by open coverings of different refinement, namely, $\check{H}^n(X,A) = \lim\limits_{\vec{U}} \check{H}^n(U,A)$, where the colimit applies to all coverings of X and expresses the equivalence relation on the disjoint union between refinements provided by different open coverings of X.

A quantum operator would then be associated with a d^i morphism while different degrees of refinement express how the quantum is inseparable from the classical. Taking the pair $(\Omega^i(E), d^i)$, where $\Omega^0(E) \equiv E$, the operator is expected to be associated with a morphism defined as d^i: $\Omega^i(E) \rightarrow \Omega^{i+1}(E)$, for i=0, 1, 2,...

The gluing mechanism expands from sections in $A(U_i)$ to sections $p_i \in \Omega^k(E(U_i))$, where p_i are local observables measured over the contexts U_i. With respect to the point x in the intersection $U_{ij}=U_i \cap U_j$, one can consider the equivalence relation $p_i \sim p_j$ for

$p_i \in \Omega^k(E(U_i))$, $p_j \in \Omega^k(E(U_j))$ for some $k \in \mathbb{N}$. Under the process of sheafification, local observables p_i, p_j taken under different measurement circumstances are related to each other, so that a new observable will be determined at the overlapping.

For example, for matter fields E and F and the covariant derivatives D_E and D_F respectively which are related if the following diagram is commutative:

$$\begin{tikzcd} E \arrow[r, "D_E"] \arrow[d, "\varphi"'] & \Omega(E) \arrow[d, "\varphi\otimes 1"] \\ F \arrow[r, "D_F"'] & \Omega(F) \end{tikzcd}$$

that is, if $D_F \circ \varphi = (\varphi\otimes 1) \circ D_E$, where φ is an isomorphism between E and F. This means that D_E and D_F are gauge equivalent. In the special case where E=F, Isom(E, E)=AutE. Hence, for different matters fields or the same matter field, at the overlapping there is gauge invariance so that measurement outcomes with respect to different contexts are reconciled.

3. *Category of sheaves*

Based on Kochen-Specker theorem, Isham (1996) and Epperson and Zafiris (op. cit.) derive the non-Boolean quantum global algebra of observables out of Boolean measurement outcomes in some coarse-grained resolution of measurement. So, the way to interrelate the macro and the micro is to establish a correspondence

$$\mathrm{M}: A_C \rightarrow A_Q$$

between the category A_Q of quantum observable algebras $\mathrm{A_Q}$ and the category A_C of classical observable algebras $\mathrm{A_C}$.[35] If $\mathrm{Sets}^{(Ac)op}$ is the category of presheaves of classical algebras where $(A_C)op$ is the opposite category of A_C (where the direction of morphisms is reversed), there is functor

$$\mathrm{F}: A_Q \rightarrow \mathrm{Sets}^{(Ac)op}$$

Quantum observable algebras are then treated by classical R-algebras of classical observables. Inversely, information regarding the quantum regime is recoverable by gluing sections of sheaves of local commutative algebras, which means that F has an adjoint functor:

$$\mathrm{L}: \mathrm{Sets}^{(Ac)op} \rightarrow A_Q$$

Taking covers on the category A_C, one can have access to the quantum regime of observable structure. A sequence of sheaves is then generated as follows:

$$F(A_Q) \xrightarrow{d^0} F(\Omega^1(A_Q)) \xrightarrow{d^1} F(\Omega^2(A_Q)) \rightarrow \ldots \rightarrow F(\Omega^n(A_Q)) \xrightarrow{d^n} F(\Omega^{n+1}(A_Q)) \rightarrow \ldots$$

[35] The characterization classical refers to commutative algebras which contrasts quantum algebras which are non-commutative.

3.4 CONCLUSION

The above analysis show that a morphism is associated with some quantum operator which is coupled with some matter. Morphisms then provide the dynamical change when they pair with a matter field. This coupling between matter and the d^i will be stressed out throughout the thesis, and will reappear in section 4.2.2.2, where there is reference to interaction field systems; in these systems, the d^i will be the covariant derivative which will capture changes.

4 A HYBRID OF STRUCTURALISM AND CONSTRUCTIVISM AS REFLECTED IN A DIPOLAR RELATION BETWEEN SUBJECT AND OBJECT

Abstract

The aim of this chapter is to show that the continuum is constituted by true parts which appear in constructions for the removal of the singularity of the relativistic spacetime. This motivates a constructivist approach for the generation of the continuum but due to a holistic understanding of the generating process, constructivist is paralleled to structuralism and it is shown that a hybrid structuralist/constructivist view can be formulated.

4.1 INTRODUCTION

This chapter is divided into two many sections, section 4.2 and section 4.3. The aim of the first section is to show that the appearance of singularities in the relativistic spacetime can be used as a motivation for treating the mental and the physical as an inseparable whole. This idea will extend to interacting field systems. The discussion is motivated by Eddington's interpretation of Einstein's Field Equations (EFE) which shows the effect of the gravitational field on matter and vice versa; the gravitational field-spacetime appears in the mental side and the matter is associated with the external world. We will claim that a mutual effect motivates an inseparable treatment of the mental and the physical.

The second section shows that due to a bidirectional or dipolar non-separate treatment of mind and body, contrary to direct and indirect realism, there is no external world separate from the subject, therefore, there is no scepticism about its existence. Due to dipolarity that appears in Whitehead's ontology, a view which can be considered as a hybrid of SR and constructivism can be formulated. In addition, it is argued that contrary to constructivist views, there are mathematical structures external to the mind (or extra structures than the ones that are constructed) and contrary to SR, structures are generated constructively.

4.2 BIDIRECTIONALITY BETWEEN PARTS AND WHOLE

4.2.1 SINGULARITIES AND BIDIRECTIONALITY BETWEEN PARTS AND WHOLE

In this section, we will present arguments related to the appearance of singularities in relativistic spacetime, where these arguments will motivate our support of a bidirectionality between parts and whole and eventually an inseparable treatment of mind/body.

The parts

It has been shown that the relativistic spacetime includes singularities, viz. points where there is a breakdown in the geometry of relativistic spacetime. In order to deal with this anomaly, there have been suggestions which attempt to eliminate the singularity. For example, Schmidt (1971) provided a rigorous approach, so called b-boundary construction, in order to include the missing (singular) points in the manifold. Singular points can be included in the extension M' of the initial manifold M, where M'=MU∂M and ∂M defines the boundary of the area of all missing points. So spacetime singularities are localized on the boundary of M and the question is whether in the extended spacetime singularities are truly eliminated. It has been shown that not all singularities are removable by extending spacetime rather there can be a division into inessential and essential (or irremovable) singularities (Earman, 1995, pp. 33–40). A point that lies on the boundary is an inessential singularity if the spacetime M can be extended to M’ and a point on the boundary of M is a point of M’; this means that the geodesic can extend beyond the singularity. The essential singularities are those that cannot be removed in the extended manifold. For example a curvature singularity is a point on the boundary where b-incomplete curves terminate and the curvature does not have a limit.

In addition, in some spacetimes, it has been shown that the inclusion of the singularity in the manifold renders the spacetime points non-Hausdorff separated which means that all spacetime points share a common region in their neighbourhood with the singular point, so that there finally exist overlapping regions rather than points with disjoint neighbourhoods. For example the singular point of the FLRW model, which is the Standard Model of modern cosmology, is not Hausdorff separated from spacetime points (and the singularity is an (essential) curvature singularity) (Bosshard, 1976; Johnson, 1977). However, the Hausdorff criterion has been considered as an essential feature of localizing the singularity and the attempt to localize it fails in that case. It has then been questioned whether the b-boundary construction is the appropriate way to define singularities.

Another kind of construction is cobordism that associates different manifolds and creates wormholes free of singularities.[36] A wormhole is created by taking the cobordism M, where a cobordism between two n-dimensional manifolds (e.g. a cobordism between two 3-spheres) Σ and Σ' is an n+1-dimensional manifold M of which the (n-dimensional) boundary is the disjoint union of Σ and Σ'. For example, by applying homology, Gibbons (1993) and Gibbons and Hawking (1992a, 1992b) (see also (Chamblin, 1995)) take the cobordism between two spaces of different topology in order to create a wormhole or a tunnel with ends in the two spaces. Taking the homology group of the boundary ∂M and chain of homologies, they derive topological information such as the number of kinks and the number of wormholes (or handles) where the number of kinks expresses the number of times the light cone of a non-singular time orientable Lorentz metric g_L tips over (changes from timelike to spacelike

[36]A wormhole is understood as a tunnel with ends in different regions of spacetime. It is like a short-cut in spacetime. A wormhole can be constructed either classically or in a quantum way. In the former case, the wormhole is made by a singularity-free warping of spacetime, but as Geroch (1967) showed, this involves time twisting. In the latter case, the wormhole is created due to gravitational vacuum fluctuations. For more on wormholes, see (Thorne, 1994, chap. 14).

and vice versa). The scheme of cobordism can be expand to include other mathematical objects. For example, Topological Quantum Field Theories (TQFTs) provides an algebraic treatment of entire manifolds (and hence independence of any background) (Atiyah, 1988, 1989, 1990; Witten, 1988). The basic elements of TQFT are cobordisms and functors between the category of manifolds and the category of Hilbert spaces. One can then define the cobordism category Bord_M of which objects are n-dimensional manifolds and morphisms are cobordisms between them. Witten (ibid.) takes the Hodge-de Rham homology, where the boundary operator ∂ of the chain complex describes a quantum mechanical tunnelling from an initial space Σ to a final space Σ' for manifold M with boundary $\partial M = \Sigma'^{*} \coprod \Sigma$.

The above paragraphs were introduced in order to show that spacetime has true parts. It is commonly considered that spacetime does not have true parts from which it is generated, where an overlapping indicates the existence of true parts. There are no parts prior to whole from which spacetime is built. Hence, the whole (spacetime continuum) is prior to the parts. However, in constructions for the removal of singularities, it is implied that the parts should be treated with importance. Firstly, in the b-boundary construction, being non-Hausdorff separated shows that the extended manifold has true overlapping parts. Although non-Hausdorff separateness appears in some models and the singularity may not be removable in the extended manifold, the singular behaviour of spacetime can function as an indication for an ontology of regions and as a reason to shift to an algebraic formulation of GTR that eliminates spacetime points.[37] Secondly, algebraic approaches that make use of cobordisms, categories between manifolds and

[37] E.g. see discussion in (Bain, 2009); the continuum can be mapped to Einstein Algebras which are combined with sheaf theory for the extended manifold. It is claimed that algebra alone is not adequate to offer a sufficient resolution; in tensor models in the extension M', the structure is preserved locally whereas sheaf theory offers a preservation of a global differentiable structure.

the relevant homologies show that there are true parts. Taking the category of entire manifolds and define morphisms between them shows that the parts – different spaces – are treated with importance while a mereological relation between them provides a unity between them so that each part – space is not independent of the others (see section 3.1). Disunity of space means that space is split into two or more segments, so that each segment can be treated individually. For example, if there was a background time, space would divide into separate pieces in time. But here, there is no evolution in time and a correlation between topological spaces does not occur in time rather there is a mereological relation between them.[38] Therefore, a topological change from one space to another is not accompanied with division of space in separate pieces which means that the other spaces do not define other possible worlds standing independently of ours.[39]

If then spacetime is removed from the background, it is humans who impose the continuum upon reality. Actually, focusing on the extended manifold and not on the initial manifold is consistent with the continuity one perceives; a world which accommodates infinities should be taken as strange to the subject, on the grounds that one does not perceive things popping in and out of spacetime.

The idea that the continuum is a mental construction can then be an indication for adopting a constructivist approach. That the continuum is not made of distinct points but rather is derivative of overlapping regions is claimed by both Whitehead and

[38] This is also motivated by a relativistic conception of spacetime. As Callender and Weingard (2000) explain, a disunity of space due to a time separation does not apply to relativistic spacetime since space cannot be seen independently of time.

[39] Historically, it was Descartes (1984, pt. II, principle 22) who rejected the plurality of worlds and argued that space identified with matter should be a unity. Opposing Aristotle, he claimed that there is one matter for heaven and earth, hence there cannot be many worlds.

constructivists such as Brouwer. Classically, according to the atomistic view, each point of a continuum has its own position on the line of the real numbers, e.g. there is a mapping of continuous time to the continuum of the real numbers and of durationless time instants to the points on the continuum. So, when an extensive continuum is presupposed, the arithmetic can be taken as a mapping to real numbers. On the other hand, in constructivism, the arithmetization is generated out of differences and likewise, time intervals and spatial differences can be taken by the relevant differences between subsequent measurement outcomes related mereologically.[40]

For example, Brouwer (1975), a defender of intuitionism, provided a construction of the continuum in the form of sequences of which the terms are not defined in advance; the continuum is in a perpetual state of growth and it cannot be separated into isolated points, but it is an assemblage of continually growing parts that overlap. Hence, there is no objectivity of empirical space which ensures that the mind has access to the external world but any experience is stripped of any mathematical notion and hence of space (ibid., p. 68-71). So, in intuitionism, humans impose the continuum upon reality.

However, treating the parts with significance due to the elimination of singularity should not be interpreted as an indication that parts gain fundamentality over the whole. Firstly, if the parts are prior, problems are encountered. In particular, if parts were prior to the whole, the following two cases can be distinguished. The first case is that parts are mental experiences. This can be supported by intuitionism where the mind generates

[40] In fact, in intuitionistic constructions, the domain of definition can exceed the domain of real numbers. Also refer to Witten's (1988) approach where the expectation values of observables belong to the domain of complex numbers. In addition, see sheaf theoretic generation of the arithmetic with respect to some region in (Mallios, 2006, p. 73); the arithmetic is taken by gauge invariant quantities. See also Synthetic Differential Geometry, which assumes the infinitesimal and not geometrical points as fundamental entities of the continuum (Bell, 2005, chap. 10). Due to the employment of the infinitesimal, the domain of definition is greater than the one of the real numbers, namely it includes quantities such that for the increment δx, $(\delta x)^2=0$ without $\delta x=0$.

the whole out of overlapping parts. Intuitionism relates subsequent measurement outcomes and generates series of parts mereologically related. However, we would like to recover an objectivity even if the construction is found in the subject. Actually, the existence of singularity introduces objectivity since in the case of the non-Hausdorff separated singular point, this is shared by all parts independently of observers. The recovery of objectivity brings us to the second option where parts are found in the objective world. In this case, although the continuum is generated in the subject, the input comes from the external world, therefore the construction is not purely subjective. The problem in this scenario is that modalities are not accommodated, that is, the only possibilities are only the actualized ones, the ones found in instances-parts in the external world.

The whole

Moreover, the parts should not be taken as prior because there are reasons to believe that the whole is equally important. The relativistic spacetime as a whole can be defended by the fact that according to GTR, the gravitational field is non-localizable at a point. In vacuum solutions of EFE where the stress-energy tensor in the right hand side is zeroed and therefore matter, viz. particles, radiation, and non-gravitational force fields, is removed, the gravitational field remains non-eliminable. Therefore spacetime is not completely empty, for it is always occupied by the gravitational field, which it has been argued, constitutes another matter field.

In addition, this importance of the whole or global also appears in the case of quantum entities. Particles do not have determinate position in spacetime (with Bohm’s theory being an exception) and even if one would argue that particles do have a determinate position (for some the Uncertainty principle should not be taken that it implies that quantum particles do not have determinate positions, rather they may have a determinate position, or a determinate velocity, but not both), still there is problem of indeterminacy in QFT (Bain, 2011). In interpretations of QFT, there are two camps disagreeing on whether particles or fields should be taken as more basic, but both camps deal with the problem of indeterminacy. In a particle ontology, particles are non-localized in a determinate position. In a field ontology, in contrast to classical fields of which values are scalar quantities defined on spacetime points, a quantum field is actually operator-valued, and since a field operator has a spectrum of eigenvalues, it does not refer to determinate values (Teller, 1997). Therefore, if quantum effects are taken into consideration, not only does the gravitational field display a global character but other matter (particles, fields) as well.[41]

So the whole is also important but just as the parts should not be prior, there are arguments that defend that the whole should also not be taken as prior to parts. Let us assume the priority of the whole. For example, the continuum found in the subject could be supported by a defender of Kantianism, although objectivity would be recoverable in the sense that space and time are a priori mathematical structures (while the things in themselves are non-spatial and non-temporal and are responsible for sending us the sensory data). One does not have knowledge for things in themselves but one can derive

[41] Treating subsequently both particles and fields as parts localized to some spacetime atom can be motivated by common characteristics that they display. Firstly, both particles and fields can be assigned momentum and energy. It has been argued that particles make fields, fields act on particles etc., so that when one speaks about a particle, she can refer to it as a particle-field (e.g. boson-field) (Feynman et al., 2013, pp. 10–9). Moreover, as quantum particles are described by wave functions, in QFT quantum fields are described by wave functionals. The state of a quantum mechanical particle is described by a wave function $\psi(x)$, while for a field, the wave functional $\psi(\varphi(x))$ gives the states of a field $\varphi(x)$, so positions are substituted by field values at points.

objective features of the world of experience from a priori structures (spatial and temporal) located in the mind.

One first obvious objection to Kant's subjective ideal Euclidean space which is necessary for the conception of the external world is that other kinds of geometries apart from the Euclidean geometry capture the physical structure. If one considers the extended manifold and an algebraic formulation of GTR, this would be rephrased as non-existence of an a priori algebraic structure isomorphic to Euclidean space. Such an outcome questions the objectivity of an a priori spacetime.

Actually, this plurality of geometries is justified by the appearance of the singularity. As mentioned, the removal of singularities allows the topology change which is the mereological relation between entire spaces and the appearance of observer is important here since a space is defined with respect to some frame of reference. Space in relativity can possess different properties according to the frame of reference, e.g. in de Sitter spacetime, space can be seen as finite in one frame of reference and as infinite in another. So, the parts appear to have important role to play.

As a consequence, someone influenced by Kant would have to show in what sense there is a plurality of topologies and how objectivity is recoverable e.g. in the form of isomorphism between the structure of perceptions and the structure of the physical world (see structuralist approaches later).

Einstein's Field Equations

For example, to get a grasp of a holistic idea of spacetime found in the subject, one can invoke Eddington (who does not support Kantianism, but acknowledges that Kantian philosophy is closer than other ontologies with regard to modern physics) (see a structuralist interpretation of Eddington's view in (French, 2003)). Eddington (1920) rejects a bidirectional reading of EFE, where according to the left to right reading, spacetime determines the motion of matter, and according to right to left reading, matter determines the curvature of spacetime. What Eddington denies is the duality between the continuum of events and matter that appears in a bidirectional reading. Matter described as substance does not cause spacetime to curve, rather the equation shows how the mind evaluates the quantity of the left-hand side. By eliminating the right to left reading, he does not begin with a material substance, but matter being dependent on the mind becomes structural. For quantum objects, this means that although particles are indistinguishable, namely there is no intrinsic property that distinguishes them, they are still distinct and properties, such as mass, charge etc. that play a distinguishing role are extrinsic or structural (in particular, they can be defined in terms of group theory). Hence, although indistinguishability is subjective in its origin, still objectivity is recovered in the subject through the structure.

However, this indistinguishability that characterizes the quantum objects also denotes spacetime points. The spacetime structure captured by the metric is not adequate for the identification of points, where spacetime is not identified as a bare manifold without the metric and the metric tensor provides the chronogeometrical and gravitational structure of spacetime (Stachel, 1993, p. 142). Shifting points around the manifold gives equivalence class of metrics of points related by a diffeomorphism, as appears in discussions with regard to the hole argument.[42] That is, the same relations hold between points, therefore spacetime points are indistinguishable.

However, a left to right reading means that the whole plays prior role. But attempts to remove the singularity from spacetime are an indication for supporting the importance of the parts as well. If then a left to right reading introduces subjectivity, a right to left reading which offers actualization in an objective world is responsible for the recovery of objectivity.

To close, chapter 6 shows that one can take the pair (matter, field), endow it with conceptual status and still recover the distinction between mind and body which are treated inseparably in this case.

To recapitulate

It has been argued that both parts and whole are treated with equal importance as an inseparable whole so that neither parts nor whole are fundamental. As in a possible constructivist approach, there is a part-to-whole transition but due to the rejection of the fundamentality of the parts over the whole and vice versa, our neo-Whiteheadian interpretation also assumes that the whole splits into parts, so that a growing tree does not picture the process as in constructivism.[43] It is rather a growing network of links

[42] For more on the identification of spacetime and its relation to the gravitational field, see the hole argument and the relevant debate between substantivalists and relationalists (Butterfield, 1989; Earman, 1989; Earman and Norton, 1987; Hoefer, 1996; Maudlin, 1988; Pooley, 2006).

[43] E.g. Dummett (2000) depicts the intuitionist generation of physical quantities in terms of trees where new branches appear that show how the whole tree grows. The measuring magnitude of some quantity such as time corresponds to a node in a tree while the links of the tree give the partial order of segments that differ by a

connecting nodes, where nodes divide or merge into new nodes. That is, there are growing sequences of overlapping parts and dividing wholes.

In addition, one should not consider that both parts and whole are found in the objective world. Our choice to introduce the mental side has to do with the notion of continuity. The continuum (the whole) is something experienced by the subject and not existing in an objective world. Then, in order to avoid reference only to the subject, parts and whole should be of different kind. By taking a dipolar relation between parts and whole, two directions and two categories are distinguished, so that it is not only that the subject is involved in the construction but the objective world functioning as the context of the occasion affects the construction, so that the construction is quasi-subjective. The difference between whole and parts is a difference between the subjective and the objective in the sense that the whole expresses the totality of possibilities that the mind evaluates while the part is found in the objective world and expresses the actualization of the whole. It is not just a mere system of beliefs that the subject has and may be true of false, because this would be a defence for a mind/body dualism (except if one manages the notion of belief in a physicalist way). The dipolarity means that both subject and object coalesce to give an objective physical reality. (That the subject shapes the objective world can be justified dispositionally, if one thinks that the mind has power to cause things, as will be discussed in chapter 6.)

To close, an inseparable treatment of subject and object entails that this dipolarity between subject and object refers to everything: the mind and everything - the body

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margin of error +/-δ with the previous value, so that what eventually is measured are overlapping intervals and not durationless instants. Also, see syntax trees of languages in cognitive and computer science.

from genes to particles - in unity with the mind. There is unity between subject and object that transcends everything. In that sense, panpsychism is justifiable.

4.2.2 YANG-MILLS THEORY

This section and the next section 4.2.3 aim to demonstrate how this dipolarity between whole and parts can be understood in modern physics. In virtue of the inseparable treatment between whole and parts, the non-localized fields (gravitational and quantum fields) could be localized to some region in such a way so that there is a direction from the global and the local and vice versa. This section starts with how the global and the local are correlated in interacting field systems where the analysis involves spacetime, and later focuses on Mallios' algebraic approach of interacting fields which applies the algebraic framework of sheaf theory.

4.2.2.1 INTERACTING FIELD SYSTEM

An interplay between the local and the global appears in Yang-Mills theories which demonstrate the interaction between field and matter. Auyang (1995, pp. 58–59) summarizes the following steps for the derivation of an interacting field system.[44] a) *The*

[44] Excluding the fact that Auyang (ibid., 40–42, 57–59, 215–218) applies fiber bundles, her methodology resembles ours, in the sense that local symmetry is the outcome of a two-step procedure: the world is divided into pieces that are afterwards glued together. In addition, fiber bundles have been suggested as the alternative framework to sheaf theory e.g. see (Haag, 1996, p. 326) for QFT interactions; like fiber bundles, sheaf offers a dialogue

localization of symmetry transformations. Symmetry transformations are localized on each point x in the matter field by taking parameters e.g. the phase factor θ of a matter field, which are subjected to transformations as functions of position e.g. $\theta(x)$. b) *The imposition of global invariance*. Global invariance means that the symmetry transformations are reconciled, if the Lagrangian of the field remains invariant under local transformations.[45] c) *The introduction of an interaction field*. Step b) becomes realized through a coupling between the phase of the matter field and the potential of an interaction field.

The above will be illustrated in the following three examples: by the interaction, firstly, of electron with electromagnetic field, secondly, of the light cone structure with the gravitational field and thirdly, of gauge bosons with the Higgs field.

Consider firstly the Aharonov-Bohm (A-B) effect and $\psi(x)$ which describes the wave function of the electron or the electron field. Under phase transformation represented by the unitary group U(1), the $\psi(x)$ transforms to $\psi'(x)=e^{i\theta(x)}\psi(x)$. The phase $\theta(x)$ is dependent on the position x, so that the symmetry U(1) is localized. What is needed so that the Lagrangian remains invariant under local phase transformations, is the introduction of an electromagnetic potential. By the joint local transformation of the electromagnetic potential $A_\mu'(x)=A_\mu(x)-\nabla\theta(x)$ and the phase $\theta(x)$, the Lagrangian can

between local transformations and global invariance, but sheaf theory focuses on regions rather than points and gets rid of the continuum, while fiber bundles presuppose the existence of spacetime. Haag (ibid., p. 128) suggests sheaf theory, on the grounds that in QFT, the relevant point-like interactions are loaded with incurable infinities. The energy density becomes infinite for distance that approaches zero. The resolution is the energy not to exceed the rest mass energy of the electron and to put a minimum length scale which replaces the continuous space.

[45] The terms global and local are used in the distinction between symmetries depending on constant parameters e.g. θ(=constant) and symmetries parametrized by arbitrary smooth functions defined on spacetime e.g. $\theta(x)$. The terms do not mark the distinction between the universe as a whole and local regions of the universe, and do not have connection with non-locality that appears in quantum theory.

become invariant. This means that a change in θ is accompanied by a change in both the electromagnetic potential and the electron field. The coupling between the local phase shift and the potential is provided by a coupling term which in this case is $\psi(x)\gamma^{\mu}A_{\mu}(x)\psi(x)$. The coupling term gives the interaction between the electron and the electromagnetic field, so that phases and potentials are not taken separately but are coupled to form one system.[46]

The reconciliation of local transformations via the potential also becomes apparent in GTR.[47] Here, one makes use of the tangent space TM_x, which is the collection of all tangent vectors at a point x in M or the directional derivatives at each point of the curve. Orthonormal frames named tetrads defined on the tangent space can rotate according to the Poincaré group which gives the transformations of the tetrad. The tangent space TM_x above a point x in M provides a localization of the light cone structure over each point; this localization is a local symmetry captured by the Poincaré group. However, like the phases in the previous example, the orientations of the tetrad, which define the orientations of the lightcone, change with x, which means that the light cones on different points have different orientations (this contrasts with STR where there exists only one orientation). Since the rotation of tetrad is different for each point, the matter distributions i.e. distributions of measuring devices will give different experimental results due to different experimental deployments. A reconciliation between the

[46] See an interpretation of A-B effect by Wallace (2014) who, contrary to others, takes that the A-B effect is loaded neither of rejection of gauge invariance nor of rejection of locality. He claims that such problems are avoided if the A-field is taken jointly with the matter field.

[47] In GTR, the analogue of gauge invariance is general covariance according to which the coordinate system can change under diffeomorphisms. The problem is that gauge field theories refer to local interactions while GTR does not provide such interactions (Belot and Earman, 2001; Weinstein, 1998). Auyang (op. cit., pp. 40-42, 215-218) takes general covariance as a local symmetry and treats gravity as a fundamental interaction; general covariance is a principle of local symmetries or gauge invariance since the orientations of the frames of reference can vary throughout the manifold and by employing fiber bundles, different tangent spaces are related in virtue of the coupling of different orientations of orthonormal frames with the gravitational potential.

different disjoint tangent spaces is provided by the gravitational potential, so that locally there is no disagreement between the tetrads. The gravitational potential couples to orientations and the relevant distributions of measuring devices. The above are implemented by fiber bundles. The tangent spaces are disjoint so, in order to contrast tangent spaces at two points, one needs a curve uniting both points and a connection. In this way, a vector can be parallel transported along a curve so that orientations between distant vectors (and therefore between distant events) can be compared. That the frames of reference are treated as matter fields is consistent with our main assumption that everything is treated as an occurrence. A measuring device is an enduring object understood as mereologically related occurrences of becoming. This argument will be called upon many times throughout the thesis.

The third example comes from the Higgs mechanism which is part of the Standard model, where all the interactions are treated by Yang-Mills theory. In particular, the Higgs mechanism describes the generation of corpuscular or massive entities by a field without the a priori involvement of masses (Higgs, 1964, 1966). The main idea is based on the assumption that since the symmetries describing the interactions of particles imply that the particles should be massless so that there is gauge invariance, in order to derive massive particles, the Higgs mechanism accepts that symmetry can break while gauge invariance is retained. In the vacuum state, the Higgs field φ, a scalar field of four components, having an everywhere non-zero strength, breaks the symmetry SU(2)xU(1) of the electroweak interaction, of which the gauge bosons are the three SU(2) W bosons (W^+, W^0, W^-) and the U(1) B^0 boson, all of which are massless. The components of the Higgs field then couple with SU(2) and U(1) gauge bosons which become the now-massive W^+, W^- and Z^0 bosons and the U(1) massless photon. Gauge invariance is taken by the coupling between a vector field A_μ (which represent the gauge bosons) and the

Higgs field both before and after the spontaneous symmetry breaking. Note that the Higgs mechanism shows that due to the satisfaction of gauge invariance, new symmetries appear.

The above show that the potential is fundamental feature of global invariance under local transformations and is treated as a pair with the matter field. The next section shows how the potential couples with matter in Mallios' algebraic formulation.

4.2.2.2 MALLIOS' SHEAF THEORETIC DESCRIPTION

In his theory termed Abstract Differential Geometry (ADG), Mallios (2006, 2009) offers an algebraic approach of Yang-Mills fields. He employs cohomology and sheaf theory to give an account of observables and measurement outcomes. Mallios' algebraic work captures sheaf theoretically the gauge principle by the coupling of the covariant derivative with matter field. Mallios starts with the pair (E, D) where E represents the matter and is a vector sheaf on topological space X, $D=(\partial_\mu - ieA_\mu)dx^\mu$ is the covariant derivative on E, ∂_μ is the partial derivative and A_μ is the gauge field, and explores the interaction between them and eventually the interaction between the gauge field A_μ and the matter. The notion of derivative reflected on the covariant derivative D is given categorically as a sheaf morphism D: $E \to \Omega^1(E)$ and the relevant de Rham complex is:

$$E \overset{D^0}{\to} \Omega^1(E) \overset{D^1}{\to} \Omega^2(E) \to \ldots \to \Omega^n(E) \overset{D^n}{\to} \Omega^{n+1}(E) \to \ldots$$

Mallios and Raptis (2001, 2002a, 2002b, 2004) suggest the above sheaf theoretic scheme as the appropriate framework for a quantum theory of gravity which is based on the sheaf theoretic relation between matter and gauge field.[48] Due to its inbuilt quantum features, ADG does not require any particular process of quantization; it provides a general framework working in all scales and there is no need for any particular process of quantization of gravity (Mallios and Raptis, 2004, pp. 374–5).

Gauge invariance, which we take as a necessary condition for observability, is a two-step process, namely, an open covering $\{U_i\}$ of spacetime is defined and sections over the open cover can be glued together to give a unique section as long as the sections agree on the overlapping $U_{ij}=U_i \cap U_j$. A covering is a local frame of matter field E, so that for a local frame $\{U_i\}$ of E, a gauge transformation of E with respect to the local frame is given by

$$g \equiv \prod_{i,j} GL(n, A(U_{ij}))$$

where $U_{ij} \neq \emptyset$, $n \epsilon N$ is the rank of E, Π is the Cartesian product and A is a sheaf of algebras on X (Mallios, 2006, pp. 18-20, 101-109). A local gauge of E is then defined by an open set $U_{ij} \subseteq X$, so that E is restricted to U_{ij} viz. $E|_{U_{ij}}$. Introducing the D of E, the realizations of the D of E are gauge equivalent, if the D is restricted on local gauges U_i of E. More generally, gauge equivalence is a relation between gauge equivalent covariant derivatives D with respect to an automorphism of E ($GL(n, A)_{|U} = AutE_{|U}$). This is an expression of the principle of general covariance, defined by the relevant

[48] As a case study, one can refer to Selesnick's (1984) sheaf cohomological treatment of the Higgs mechanism; by employing the (E, D), he implements the interaction between the gauge bosons (which acquire mass) with the Higgs field.

group action. Hence, a symmetry group viz. automorphism gives the transformations between the various sections viz. of equally admissible systems of reference.[49]

What are the implications of the above understanding of observability? In sheaf theory, there are two kinds of relations: a) a relation directed from the global (the whole) to the local (the parts) which is expressed by morphisms which reverse the direction of the inclusion relation between sets, and b) a relation directed from the local (the parts) to the global (the whole) which is expressed in the gluing mechanism; gluing is an expression of gauge invariance which means that every local observable captured by the germ is internally related to a global structure. Once more, the distinction between local and global does not have to do with the restriction of one's observation at a local region of spacetime since there is no spacetime in the background, rather the local expresses that an actual observed quantity is related to some spatial extension while the global expresses the range of possible outcomes.

4.2.3 COARSE-GRAINING INDUCED DECOHERENCE

This section refers to quantum decoherence as another example of how dipolarity between parts and whole can be understood. Based on different degrees of refinement, a mereological association between the macro and the micro can be understood as follows.

[49] It becomes evident that one can have a gauge theory where points are eliminated and the potential acts on regions and not on points. This is an objection to the argument (e.g. see (Weinstein, 1998)) that in contrast to gauge invariance which takes differences in the field at a point, diffeomorphism considers changes of the field by its mapping from one point to another (and in that sense, an ontology of regions should substitute an ontology of points). Both gauge invariance and diffeomorphism can apply to regions.

As in Many Worlds Interpretation (MWI) of quantum measurement, there is no external observer, rather the macro and the micro, from the Schrödinger's cat and the observer to the particles are in superposition and described by one wave function. From the two kinds of decoherence, environment induced decoherence and coarse-graining induced decoherence, the latter one we deal with. The former kind requires an external factor to the system, namely the environment causes decoherence and the appearance of macroscopic objects. The latter kind of decoherence is an intrinsic process. The correlation between the observed and the quantum is expressed by the mereological correlation between different scales according to the two directions – from the finer (quantum) to the coarser (classical) and vice versa. There is no coupling of the quantum system with the environment rather quantum observable behaviour is expressed in macroscopic algebras of observables, where classical behaviour emerges at a coarse-grained scale. The quantum global algebra of observables can be recovered by a complete covering provided by many devices at different overlapping measurement scales. Hence, device and environment are not seen as external factors but as internal localizing factors of a global algebra of fine-grained microscopic observables. A global quantum description of a system of quantum entities (observer, measuring device, quantum particle) can be done locally in some coarse-grained resolution of measurement where quantities are determinate. One does not have to specify quantum observables globally, but local determinate germs can do the specification. Decoherence is then the process of reduction of a quantum system to a localized system of determinate behaviour. Employing the topological approach of sheaves where the algebras of observables are identified with sections of a sheaf, decoherence via a coarse-graining process provides localization by restriction to coarser contexts.

In particular, taking relations to be the basic constituent of reality, decoherence is a function of history outcomes. For each set of histories described by sieves, coarser-grained histories can be constructed while overlapping between histories/sieves results in finer-grained outcomes. The superposed state is recoverable by taking the direction from the coarser to the finer; in a process of continuous becomings, a new entangled state is taken by overlapping of new becoming histories so that nothing is repeatable. As the system evolves locally contextualized events can subsequently be intergrated to give a global objective outcome.

The introduction of the mental element is justified as follows. Only at the macro level, one can discuss about separate elements and consider them in isolation from others. Deriving quantum states based on local coarse contexts demands an abstraction that the mind can perform. This contrasts with MWI that eliminates the mind on the grounds that in decoherence, there is no such perception of a superposed state so perception is consistent with the single reality we observe. In our scenario, acknowledging the existence of true parts and the derivation of the whole from the parts justifies the perception of a quantum state. The perception of the continuum then goes hand in hand with the perception of the quantum state. For $A(U_n)$ algebra and sheaf $S_n(A(U_n))$ where U_n an open cover of X, as U_n becomes more refined, the sheaves S_n converge to the sheaf of germs S(X), that is, $\lim_{n\to\infty} S_n\big(A(U_n)\big) = S(X)$.

So the main point is that the existence of the mental element is justified by the fact that the treatment of singularities shows that there are true parts. Since a) the continuum is something experienced by the subject at the finest graining e.g. the chair has true parts and the chair as a continuum is generated by the parts at the finest graining by the mind and b) at the finest graining, there is superposition, the continuum we experience is then not approached in a classical way, but it is correlated with the quantum state while the mental side participates in the entire process of decoherence and the recovery of superposed state.[50]

In chapter 6, it will be claimed that probability in quantum decoherence will be interpreted as propensity.

4.3 THE STRUCTURE OF BECOMING

The holism expressed as an inseparable mereological treatment of subject and object motivates us to study dipolarity in correlation to SR. In support of SR, Lam (2006, 2008) makes the following claim with regard to the non-Hausdorff separateness of spacetime points. Since all spacetime points share a common region in their neighbourhood with the singular point, the attempt to localize the singularity fails, hence one cannot associate local entities and properties to the singular behaviour of spacetime. That is, one should reject the existence of local entities such as boundary points or local properties instantiated by boundary points. Rather, he argues, the singular behaviour is a global (non-local) feature of spacetime or equivalently, spacetime bears a singular feature that does not supervene on the existence of any local entities or properties. Therefore, the singular behaviour of spacetime opposes Humean supervenience which gives a highly localized picture. Rather, it acts as a defence for OSR since the singularity acts in a unifying way in the sense that the parts – spacetime points or regions – cannot be treated independently from one another. Hence, there is an

[50] See also (Mallios and Raptis, 2004) for the treatment of the continuum.

emphasis on the structure of relations and not to individuals possessing some intrinsic properties. Although Lam emphasizes the importance of the whole, he misses to note the existence of true parts. Therefore, as we will see a different interpretation from a structuralist one can be given if one thinks in terms of becoming and not of being, namely, spacetime continuously becomes out of overlapping parts.

In particular, the next sections study how the mental and the physical can be treated as an inseparable whole and how the structure that describes this coalition can be understood. SR appears in two forms, the ESR and the OSR. They both focus on the structure of the unobservable world with the difference that ESR makes epistemological claims and OSR ontological ones and in particular, for ESR, what one only knows is the structure of the unobservable world while for OSR, what exists is the structure. Roughly speaking, claims regarding what one knows or what exists beyond the structure – whatever this may be, e.g. individuals, objects, intrinsic properties - are not part of the discussion and if they are involved, they should be accommodated according to the main scheme of Structural Realism that focuses on the structure.

Due to the dipolarity between mind and body, the subject and the object will be components in our interpretation of the structure of the world. Throughout the chapter there will be epistemological and ontological claims in order to unfold the notion of inseparability between subject and object. It will appear that both ESR and OSR are relevant and in fact, the gap between them to dissolve. This dissolution is clarified in section 4.3.1. In addition, the adoption of a constructivist attitude can be justified by the importance that the realization of the mathematical structure is treated, which means that the objective world participates in our epistemological and ontological understanding of the structure in a constructive way. That's why section 4.3.2 discusses about mathematics and in particular, mathematical SR and constructivism. Mathematical SR is also mentioned because it has been argued that the introduction of a non-structural element in ontology is important for the differentiation between mathematical and scientific SR. In our framework, in order to introduce a non-structural element, we will think as follows: individuation independently of the structure can be provided if there are initial relata-individuals out of which structures are generated. We will then adopt a partly constructivist attitude which starts from actual physical systems for the derivation of abstract quantities.

Sections 4.3.1-4.3.2 are basically used for clarification purposes to pave the way to the main discussion that takes place in the following sections. In particular, section 4.3.3 revisits Whitehead's dipolarity according to which there is a unifying treatment of the external world with the subject so that the external world becomes 'internalised'. Section 4.3.4 contrasts our interpretation of the notion of dipolarity with constructivism and SR. It is shown that like in constructivism, there is a mereological generation of constructions but contrary to constructivism, it is claimed that there are extra structures from the ones constructed. It is demonstrated that the dipolarity can be implemented in terms of category theory and mappings between categories. Finally, section 4.3.5 refers to mathematical representation in correlation to dipolarity and section 4.3.6 closes with some conclusive remarks on the idea that the structure we are searching for is not a defense for neither ante rem SR nor in re SR, hence it is supposed to accommodate causality and modality.

4.3.1 ESR VS. OSR – AN EVAPORATION OF THE DISTINCTION

As mentioned ESR is the one form of SR. In particular, in ESR, there are positions affected by Kant's philosophy, which attempt to recover objectivity or inter-subjectivity in the subject. According to ESR's argument from perception, there is an isomorphism between the structure of perceptions and the structure of the physical world (actually, this is an argument of Indirect ESR (IESR), which is a descendant of indirect realism and espouses that one can only have access to sense data, and everything else, chairs, particles, charge etc. should count as external unobservable world of which one only knows the structure).[51] The knowledge of perceptions can be both structural and non-structural, but it is only the structural aspects of perceptions that encode information of the external world, that is, the non-structural properties of relations or of monadic properties cannot be communicated with other minds. The reason is that two minds do not have identical perceptions of the same object, hence the non-structural properties of perceptions do not give information about the common external world. According to the argument from representations, the structural knowledge of the external world can be acquired in terms of representations; isomorphism employed on mathematical objects provides a representation of the external world. For example in particle physics, the group representations allow invariance of the system and therefore, introduce objectivity.

[51] For both Direct and Indirect ESR, one can only know the structure of the unobservable world. The difference is that Direct ESR assumes that one has structural and non-structural knowledge of the observable world, which includes those objects or properties that can be accessed through senses (e.g. chairs, meter readings etc.). See exposition in ESR in (Frigg and Votsis, 2011; Psillos, 2001).

Regarding OSR, although this is different from ESR, OSR philosophers tend to bridge the gap between them. For example, French's and Ladyman's (2003a, 2003b) radical OSR has an epistemological justification for the elimination of the relata. Specifically, the elimination of relata is epistemologically grounded on the observation that in the quantum entangled state, there are no distinguishable individuals. The entangled quantum state shows that particles have the same intrinsic and relational properties which means that quantum particles are indistinguishable; therefore either particles should be eliminated or else they should be individuated by other criteria such as bare particularity, haecceity etc. Instead of introducing the mysterious notion of haecceity, French and Ladyman support eliminitavism.

The same attitude is also followed by Esfeld. In his (2004) paper, he makes it explicit that in order to close the gap between ESR and OSR, he considers that there are relata but these have only relational properties. The methodology is similar to French and Ladyman, the difference lies on how QM is interpreted, namely the entangled state is non-supervenient upon the non-relational properties of individuals (Teller, 1986). Since in that case there is a gap between epistemology and ontology in the sense that epistemologically one only has access to the structure while ontologically there are individuals with intrinsic properties, Esfeld claims that the gap can be closed if the relata have structural properties. Since knowledge is restricted to the structure, one can have a structural knowledge of the relata. So for Esfeld, there can be individuals albeit distinguished only by structural properties and neither relations nor relata are prior or secondary.

Just as in the quantum case, such a methodological shift from epistemology to ontology appears in relativistic spacetime. Both versions of SR consider the equivalence class of metrics of points related by a diffeomorphism.[52] It has then been argued that either diffeomorphism should be taken as a motivation for eliminating the points (radical position of OSR) or the diversity of points is grounded on the relations they stand in where these relations are described by the metric structure.

However, such methodological shifts from epistemology to metaphysics are not devoid of criticism (Morganti, 2004). Firstly, the reality can be richer from what one knows. There is reality which is hidden so that one's knowledge is embedded in a greater ontological framework. Secondly, one can speak of individuals even in the quantum case. There are theories (e.g. Bohmian or hidden variable interpretation of QM) that assume the existence of a determinate position, though one is not in the position to know it; in such frameworks, the limitation in knowledge is expressed in the employment of probabilities.

In addition, OSR does not take into consideration the instantiation, for example for an eliminativist, relata are instantiations of universals but being confined to appearances, one does not have epistemic access to these instances, hence relata should be eliminated. But the structure is instantiated and therefore, particularized.[53] Instantiation, implementation, realization of the structure brings forth notions such as individuality, intrinsic properties, particularity, therefore if epistemology rejects individuality, ontology should be something more than ESR or equivalently, OSR is not sufficient in its current description to offer a full picture of reality. Criticizing OSR, Esfeld (2013)

[52] For more on the identification of spacetime and its relation to the gravitational field, see the hole argument and the relevant debate between substantivalists and relationalists (Butterfield, 1989; Earman, 1989; Earman and Norton, 1987; Hoefer, 1996; Maudlin, 1988; Pooley, 2006).

[53] E.g. see instances of structures in (Mertz, 2003).

claims that in its current formulation OSR is like ESR and OSR would be sustainable if it involves the instantiation or realization of the structure of a given theory.

As a conclusion, either ESR is enough and OSR does not offer something additional or OSR has to involve the notion of instantiation/realization. What will be claimed in this chapter is that the ontology accommodates the analogue of instances (which will be called becomings or actualizations of the structure) found in an objective actual world in such a way so that the whole (structure) is not treated with significance over the parts (analogue of instances) while a correlation between epistemology and ontology will not be exhausted by a shift from epistemology to ontology. In particular, in order to establish a bridge between ontology and epistemology, (unobservable) particulars in the external world participate in one's knowledge. Not only will there be a shift from epistemology to ontology which can ground structural ontological claims but vice versa, a shift from ontology to epistemology will mean that there is knowledge of the particular of the external world. In a way the distinction between ESR and OSR evaporates, because the subject and the object participate both in knowledge and in defining the ontology. Actualized structures in the objective world will be taken to shape one's knowledge hence it will not be ESR which maintains that one does not have access to realized individuals. The suggested approach is consistent with the limitation of knowledge; such an approach will not nullify the idea that there is structure which is greater than the accessible one. Quite the opposite. One has access to some structure and this is justifiable by the fact that the external world is seen in unity with the subject. But there can be structures that are beyond one's knowledge and these structures does not involve such a coalition between subject and object, as will be discussed throughout the chapter.

To close, since we do not start from epistemology to build our metaphysics, but both epistemology and ontology coalesce in building a framework, epistemological and ontological claims will interchange throughout the discussion.

4.3.2 MATHEMATICAL APPROACHES AND (PHYSICAL) STRUCTURAL REALISM

1. *Mathematical Structuralism*

In literature, Structural Realism is examined in correlation to mathematical SR, because the notion of the mathematical structure appears in physical theories and the mathematical structure is instantiated by a physical system.

Just as in scientific SR, the issue of the elimination of objects appears in mathematical SR and in case there is individuation of mathematical objects, this should not be grounded on intrinsic properties rather the diversity of objects if existent is based on the structure. In literature, Shapiro’s ante rem mathematical SR assumes that the mathematical structure is fundamental; structures are abstract, platonic entities that are at least independent of (and possibly prior to) the systems that exemplify them. The problem is that either one has to accept that there are no objects on the grounds that distinct but indiscernible mathematical objects are identical or that there are objects individuated by haecceity. The former means that a place in the structure can be occupied by any number (e.g. in some progression, the third place can be occupied by any number hence number three can be any object), therefore a number would not be an individual, as Benacerraf (1965) claims and he concludes that in a structuralist construal, numbers cannot be objects because abstract objects should be called individuals if there is intrinsic individuation rather than they are identified by the structure. If the latter is espoused, namely mathematical objects are individuated by a haecceity, then they have more than structural properties and therefore, ante rem SR should collapse to traditional Platonism. Ante rem SR rejects the presumption of objects having some intrinsic essence before they enter the structure. Shapiro who is a non-eliminativist, takes that in his ante rem SR that structures and their places are prior to any physical structure that exemplifies them. Shapiro as others mathematical and physical structuralists wants to keep the objects since structures need objects, but they identify these objects as having structural properties and thus being secondary to the structure which they occupy. In general, non-eliminativist mathematical structuralists currently take that numbers are mathematical objects in virtue of being placeholders.

Regarding now the relation between mathematical and physical structure, it has been argued by structuralists that if the physical theory is deciphered in terms of the mathematical structure, then the physical structure is indistinguishable from the mathematical one. From one perspective, a singlet state will be regarded as a concrete structure that exemplifies the abstract structure and the spins are concrete objects of the structure that come to replace the abstract objects which are mere placeholders in the abstract structure. In that sense, one can understand that in ante rem structures, abstract objects being places in the structure are existent, since when the structure is exemplified the places are occupied by the concrete objects. However, even in this case, the alleged concrete objects can be characterized as abstract if they are only identified by structural properties just as abstract objects do. Evoking Dummett who, exploiting Benacerraf's argument, denounces as mystical the idea of abstract objects identified only by structural properties of mathematical structures, Busch (2003) made exactly the same comment with regard to physical structures, namely if objects are identified only by structural properties, this leads to mysticism. We would agree with the idea that if there is indiscernibility objects possess only structural properties, this implies something ideal. Even if one invokes weak discernibility, as Saunders (2003) does in the sense that in a singlet state, there are individuals which are weakly discernible, because there is a reflexive relation viz. having opposite spin between the two particles that distinguishes them, still individuals individuated by the structure cannot be seen separately. If then ontology is built on those grounds, then it lacks the reality of the physical.

It has then been suggested that the distinction between the abstract mathematical structure and the instantiated physical concrete structure cannot be given in purely structural terms (Ladyman, 2007, p. 39). On that subject, Psillos (2006) contrasts ante rem SR to in re SR. In ante rem SR, mathematical structures are like universals, they exist independently of the systems that exemplify them while in in re SR, the systems are prior to the abstract mathematical structure, hence structures are abstractions out of certain systems. Since according to in re SR, there is no abstract structure standing over and above concrete systems that instantiate it, the in re SR takes that objects of the structure are the objects of the systems that have the structure. Hence, objects should not have merely the structural properties, but additional properties since they are prior to the structure and therefore, they acquire their individuality and identity independently of the structure. On the other hand, in ante rem SR, the objects of the structure are places and in no case, they are identified with the objects of the system that exemplify the structure.

The benefit of espousing in re SR is that since the mathematical structures exist in the systems that instantiate them, the structures are causal, contrary to the (abstract) ante rem structures, which are not part of the causal world and therefore do not accommodate causality (Busch, op. cit.). However, although causality is handled, in re SR has been criticised for not accommodating extra structures that may not be instantiated by any system, e.g. paraparticles etc., something that ante rem SR does (Psillos, op. cit.).[54] One then needs a framework which accommodates a) causality and b) a range of possibilities which is greater than the one that can be actualised. On that ground, we claim that the structure should be neither ante rem because it should be causal nor in re because it should involve modalities.

In a similar fashion, French argues for a) and b) but in parallel, he removes the non-structural element from his ontology. In particular, discussing the interplay between the mathematical abstract structure and the physical concrete one, he refers to the group structure and the representation. Concerning the group theoretic representation of elementary particles, French (2014, pp. 266–268) argues that the distinction between the abstract and the concrete is problematic. The group G realized by representations is taken as abstract while the homomorphism group as concrete. However, this picture is distorted if one considers that the group G yields all the possibilities so that there is a mapping to all possible representations – particle and paraparticle representations.[55]

[54] The idea that the structure is neither in re nor ante rem is espoused by French (2014, pp. 205–212); the world is neither abstract, because it is causal, nor a system that is structured, for in that case the system is prior to the structure and therefore separate from the structure. For French, the abstractness of the structure renders the structure inherently modal. However, his OSR is an ontology of being whereas our view is an ontology of becoming.

[55] In particular, Permutation Invariance (according to which observables commute with the permutation operator) encodes all the possible particle statistics and parastatistics and yields the distinction between bosons and fermions (French, 2014, pp. 264-270). The permutation group acts on Hilbert space which is subdivided into subspaces that correspond to symmetric representations of bosons, the antisymmetric representations of fermions

Considering now the distinction between the abstract and the concrete, since only the bosonic and fermionic representations correspond to concrete actualities, the paraparticle representations are abstract possibilities. This entails that not only the group but also the representations cover the abstract and possible. Moreover, since only the representation of Bose-Einstein and Fermi-Dirac statistics are concrete, G yields the concrete because the representations make sense only on the basis of the respective group G. So, from another perspective, the group is not abstract, but as concrete as the representation. Hence, the representations do not fully cover the actual, as the group does not fully yield the abstract. This argument reflects French's general view that the structure (which in this case incorporates both the representations and the group structure) is neither concrete nor abstract and that both the group and the representations are constituents of the structure of the world. This contrasts with the alternative view that the representations are ontologically significant and the group plays just a mathematical descriptive role, since in that case, one needs to make clear the distinction between the group and its automorphism.

The main outcome is that a group which is something abstract, albeit actual (where the group gains its actuality from the representation) can be characterized as concrete and since the structure needs to accommodate modalities, the group is not purely concrete rather it displays dual role. The consequence of such an explanation is that the group structure represents both the structure of theoretical elements and the structure of sensations, hence the group structure communicates information for both the

and the representations of paraparticles (French and Rickles, 2003). This means that a particle belonging to a certain subspace does not have accessibility to other subspaces given that the Hamiltonian is symmetric. So, the Permutation Invariance constrains the observables into certain categories and the Hamiltonian says why there cannot be hopping from one category to another, i.e. why fermions cannot become bosons.

consciousness and the external world.[56] In French's resolution, the mind-body dualism can be rejected because one would need a non-structural element that would differentiate between the two categories (mind and body). So for French, the structures are mental, albeit objective. As a consequence, although French (2014, pp. 205–212) claims that the structure is neither purely in re nor ante rem, he interprets both the in re and the ante rem characterizations structurally.

On the other hand, our framework differs from French's firstly, since there is distinction between mind and body and therefore the non-structural element will be incorporated and secondly, the instantiations or better the becomings of the physical structure in the objective world will be included in the description, therefore there is distinction between physical and mathematical structure. That is, the representation of fermions and bosons is the physical structure found in the subject while the particular becomings (some electron x) are found in the objective world. This last point will be discussed in section 4.3.5.

We will then assume the following:

a. The structure of the world is neither in re nor ante rem.
b. The abstract encompasses both mathematical structures such as groups and paraparticle representations as well as conceptual structures such as particle representations which differ from the former because they are actual.
c. The concrete is the actualized physical structures.
d. There is a range of structures, namely, the mathematical structure, the conceptual structure, the actualized physical structure and the structure of the

[56] As an example, see French's (2003, pp. 246–7) interpretation of Eddington's employment of groups in a structuralist way without invoking the mind.

world or the structure of becoming which is the greatest structure of all and encompasses all the above structures.

With regard to b., there are actual abstract possibilities, and one of them is actualized. But, we think that not all abstract objects are actual. So the structure of the world incorporates both an abstract domain of actual possibilities and of non-actual possibilities and a domain of concrete actualities. The conceptual relations refers to those actual abstract structures that encode information of the external world and the mind is aware of. The distinction between conceptual and concrete relations lies on the distinction between mind and body and the introduction of a non-structural element.

2. *Constructivism*

Generally speaking, constructive mathematics accepts that a mathematical entity exists as long as it is constructed (Bridges and Palmgren, 2013). There are different representatives; others do not involve language in their construction (e.g. Brouwer), while others use logic in order to ground mathematical constructions (e.g. Bishop, Heyting), but there is commonly rejection of the existence of mathematical structures external to the mind or the brain, as a Platonist would argue.

According to Brouwer's (1975) intuitionism, mathematics is a free creation of the human mind; a mathematical object e.g. a number exists as long as the human mind creates it. Brouwer employed the notion of free choice sequences, so that only a segment of the sequence, denoted as $<a_0, a_1, ..., a_k>$ where $a_0, a_1, ..., a_k$ are the elements of the sequence, is known at each stage of the construction. Therefore, the derived arithmetic is in a perpetual state of growth and it cannot be separated into isolated points, but it is an assemblage of continually growing parts that overlap. This construction is not generated linguistically. Brouwer distinguishes between mathematics and mathematical language. The mind generates freely mathematics. Mathematics precede logic or logic is subsequently abstracted from the mathematical constructions of the mind. Mental constructions are developed out of sensory experience, hence the knowledge of the world is subjective, e.g. numbers do not belong to an objective domain but they are constructed out of measuring or counting things. For Brouwer (op. cit., pp. 59–60, 99), quantities such as mass are nothing but a coefficient in the construction and although he defines objectivity as invariance of the coefficient under transformations, this kind of objectivity Brouwer refers to should not be taken literally, since the construction is still subjective. The fact that a quantity is stabilized to some value after a great number of repetitions, this can be taken as a better approximation but not as a recovery of objectivity. In addition, although the construction is repeatable by any human subject, so that it is not the isolated subject but it is a community of subjects that yields mathematical constructions and therefore, the involvement of many minds could imply a kind of objectivity, still the problem of subjectivity is transferred from the person to the community which now functions as a greater subject. Since more than one subject is involved to yield an objective outcome, one should show what this element is that ensures objectivity.

One can also find constructivism in correlation to the functioning of the brain, so that the operation of the mind is not over and above the operations of the brain. The notion of recursion (see recursive constructive mathematics) can appear in these constructions. Recursion is an ingredient of language, for example, in cognitive science where the functionality of the brain can be captured by the functionality of a computer, recursion and machine computability are equivalent, namely, recursive functions are mechanically computable functions.

However, we would not like to reduce the mind's functionality to the brain's one. According to (Platonist) Gödel, the power of human mind infinitely exceeds the power of any finite machine (Sieg, 2006). If the axioms that the human mind conceives can be listed by a Turing machine, then the human mind cannot prove the consistency of mathematics. On the grounds of incompleteness theorem, Gödel concludes that either mathematics are incomplete and there are unsolvable problems if there is a finite rule or the human mind overcomes the power of the finite machine. He then argues that the operations of the human mind are not reducible to the operations of the brain and that the power of the mind infinitely overcomes the power of the brain. Even if one does not espouse the unlimited power of the mind since man is a finite entity with limited capacity and therefore the constructions are also characterized by finitude, still the importance of the mind in the constructions can be supported for other reasons. E.g. in Brouwer's view, the mind generates freely so that functions are not recursive described by some language, rather they are free choice sequences.

The outcome of the above is that in constructivism, contrary to the objectivity that characterizes the mathematical representation in structuralist views, it can imply subjectivity or alternatively, it can display objectivity e.g. in recursive constructions where the mathematical constructions are related to the recursion that characterizes the language or the mental processing.

In our approach, as mentioned, we will provide an interpretation of the structure of the world that will be considered as a hybrid of SR and constructivism and hence, it bridges the alleged gap between the two views. The structure of the world adopts elements from both philosophies, but it rejects basic elements of both, namely constructivism's assumption of the non-existence of external structures and SR's rejection of relata-parts out of which structures are built. Contrary to constructivism, there are mathematical structures external to the mind and existing independently of our knowledge. In a constructivist approach, the mathematical structure is an abstraction out of the physical and in that sense, it resembles the in re SR. But since the structure incorporates elements of the ante rem SR, there will be extra structures external to the mind so that modalities are accommodated.

4.3.3 REVISITING WHITEHEAD'S DIPOLARITY

Whitehead expresses his realist attitude when he considers that an actual entity is always subject-superject, meaning an experiencing subject and a superject characterized by objectivity. His realist approach can be associated with indirect ESR, on the grounds that what is perceived is continuous whereas the actual world is discrete; if we perceived directly the external world, we would expect that there is no such big mismatch between the perceived and the world. In addition, panpsychism viz. that everything is treated in the same way can be another argument for indirect ESR, which does not make any distinction between the macrocosm and the microcosm with respect to what is perceived. On the other hand, contrary to indirect realism, the external world is not separate and independent of the subject (hence one does not end up being sceptic about his existence). Objectivity of the superject means that the subject cannot be treated in isolation from the objective world. The physical prehension of the objective world is actually a mirroring of the external world in the superject, that is, past actual occasions become immanent in the present actuality. As in indirect realism, due to time lag between the transmission of a signal and its receiving, the sense data one receives concern antecedent occasions and therefore, a representation refers to the past events, but in addition since the external world is not separate, one is physically related to the past occurrences in a non-separate way. In the mind-body dualism, one's physical body is a part of the external world, but here the difference is that this body should not be seen separately from the other physical bodies, rather the one body is a mereological extension of others. Hence, if a physical body is seen as an occurrence which has a mental and a physical side, there is a mereological relation that holds between occurrences. Mereological relation between occurrences can be grounded on the mereological relation between the region of an occurrence and the regions of other occurrences. The objective world is then understood as atoms of space ordered mereologically. Or better, the ordering of occasions is not defined spatiotemporally, but in terms of the part-whole relationship combined with topological relations between regions.

The objective physical world or the parts act as input, in the form of sense data, to the subject and hence the parts coalesce to give a whole. The initial data are subjected to evaluation by the subject before the new objective reality is finally derived. Hence, there is a whole to part relation between the totality of possibilities and the final objective datum.

In section 4.3.5, it will be shown that with respect to mathematical representation, the above can be interpreted as the gap between the represented (objective world) and the representer (subject) closes. Section 4.3.4 constrasts our framework with constructivism and SR. Although Whitehead's ontology is holistic, namely, there exists a web of occurrences bearing relations with each other and there is an emphasis on the internal relations that an actual occasion bears to its past occurrences, still this should not be an indication for a structuralist view rather, based on the importance with which Whitehead treats the data from the objective world which are combined mereologically, there is a constructivist attitude. Comparing his view to Kant's, Whitehead (op.cit., p. 88) claims that 'The philosophy of organism is the inversion of Kant's philosophy.' The process theory aims to show how objective data function as an input to the subject which provides the relative perspective (ibid., pp. 72, 152, 155–156), whereas the Critique of Pure Reason aims to describe how the subject gives an 'objective' world of appearances. Hence, we aim to give an alternative approach from the one that a structuralist influenced by Kantian philosophy would give.

4.3.4 COMPARISON WITH CONSTRUCTIVISM AND STRUCTURALISM

In this section, there is a comparison between our interpretation of Whitehead's dipolarity and both constructivism and SR. The first argument that shows difference with constructivism and SR is motivated by the relation between parts and whole. The second argument which is the main one shows that contrary to SR the structure of the world has a non-structural element in the form of individuals-relata for the constructive generation of new structures.

The 'non-structural' is defined as follows:

> Non-structural refers to physical structures which are relata for the generation of the conceptual structure.

The non-structural characterization usually refers to intrinsic properties, where the intrinsic means that these properties are independent of the relations they will stand in. However, there is no reason why the non-structural should not characterize other kinds of entities such as structures themselves out of which new structures are generated. If the former structures are characterized as first order and the latter as second order structures, then due to dipolarity, just as Fa is the concrete instantiation of abstract universal F, the first order structure is the realization of the second order structure or the first order structure is physical and the second order structure is abstract (conceptual).

4.3.4.1 THE ARGUMENT FROM THE NON-FUNDAMNTALITY OF PARTS

One can observe that taking both directions results in a different perspective with regard to the role of individuals. If parts are distinct and separate, then they are prior to or independent from the whole, since generally speaking, the priority of the parts means that there are individuals which have independent existence from each other and are individuated independently of the structure. That parts are individuals, but they are not treated separately can be used as a motivation for OSR which emphasizes holism.

Therefore, parts appear to have dual character, namely seen as both separately and inseparably. This can be justified as follows. In the direction from parts to whole, parts are initially taken as distinct and separate, but, the fact that they generate a relation (e.g. entangled state) means that they cease to be seen separately. Subsequently, in the direction from whole to parts, parts are again seen separately. So taking both directions shows that in a series of continuous becomings, something should be seen separately as well as inseparably from the whole and in that case, as will become apparent later, discernibility of the individuals is justified because each occurrence has its own history of mereological relations.

4.3.4.2 THE ARGUMENT FROM THE EXISTENCE OF THE NON-STRUCTURAL ELEMENT DUE TO RECURSION

Due to the elimination of the spacetime continuum from the background, the approach presented will be algebraic. The objective reality is the totality of spatiotemporal atoms (or parts) mereologically related, while conceptual structures are generated out of mereological relations of overlapping spatiotemporal atoms. Taking coverings of different refinement, the (subjective) continuum (or the whole) is generated out of mereological relations found in the objective world at the finest graining (at the limit). And vice versa due to dipolarity, the continuum becomes actualized (or objective) to some spatiotemporal atom so that a new quantity is derived with respect to some spatiotemporal atom. Hence, quantities will be defined with respect to some spatiotemporal context, which means that the mereological ordering between regions in the objective world is transferred to the associated physical quantities. That is, properties e.g. energy, momentum etc. associated with some region, generate structures on the basis of some ordering relations.

To illustrate the above, let us consider the following formulas and mathematical assumptions. Let open sets of a topological space X, a U-sieve, a V-sieve and a sheaf Ω^k of algebras which can be associated with some matter ψ and observable values and the dynamics are given by morphisms d^k, as discussed in section 3.3. One could consider the following series which shows the mereological relation between objects of the category Ω^k.

(1)	$\Omega^k(\psi(U)) \rightarrow \Omega^k(\psi(U_1))$	$U_1 \rightarrow U$
(2)	$\Omega^k(\psi(V)) \rightarrow \Omega^k(\psi(V_1))$	$V_1 \rightarrow V$
(3)	$\Omega^k(\psi(U_1)) \rightarrow \Omega^k(\psi(U_1 \cap V_1))$	$U_1 \cap V_1 \rightarrow U_1$
(4)	$\Omega^k(\psi(V_1)) \rightarrow \Omega^k(\psi(U_1 \cap V_1))$	$U_1 \cap V_1 \rightarrow V_1$

The above morphisms provide restrictions of sections $p_i \in \Omega^k(\psi(U_i))$. The sections will be associated with some value of a property. Properties are associated with some region so that the mereological relation between regions captured by sieves is transferred to the mereological relation between objects in category of algebras E. An object $\Omega^k(\psi(U))$ is related via morphism with $\Omega^k(\psi(U_1))$ and subsequently related via morphism with $\Omega^k(\psi(U_1 \cap V_1))$ (which will also be related via a morphism to another object in E). So an object e.g. $\Omega^k(\psi(U_1))$ is linked to a series of previous morphisms (formula 1) and subsequent ones (formula 3).

The argument that will appear in this section is the following. The above framework can be a support for physical SR since the basic building element is the morphism which is a relation. However, there should be a stronger evidence that the above structure is not mathematical; we need a non-structural element that distinguishes the mathematical from the physical. It will be argued that some relations-morphisms in this web of structures can be characterized as physical because they constitute the realization of the abstract structure but in parallel, abstract structures are generated out of these physical relations. The latter point introduces a constructivist approach to mathematics which is also supported by the fact that the above framework has been interpreted by some constructively and not structurally. Let's see this issues in turn.

One can firstly notice that these formulas can be used as a defence for constructivism which presupposes unstructured notions such as those of operation and collection which are epistemically and logically prior to structural notions (such as group, ring, category etc.) (e.g., a group consists of a collection of elements endowed with a binary operation). One collects unstructured data that subsequently organizes in order to derive symmetries and structural notions. In particular, the above formulas a) presuppose the notion of collection, viz. locally defined data are attached to the open sets of a topological space, b) employ the notion of category which, for a constructivist, presupposes notions such as those of class, operation and set and c) they employ the concept of functor between categories which, for a constructivist, presuppose the concept of operation.[57] So one needs to presuppose sets U_1 and V_1, take their overlapping and the operators that relate objects, so that a new object $\Omega^k(\psi(U_1 \cap V_1))$ at the overlapping is defined.

That operation and elements are epistemically and logically prior is where a structuralist will disagree. It was earlier mentioned that numbers are not taken independently of the

[57] See sheaf theory and constructivist claims in (Bell, 1981; Feherman, 1977) and sheaf theory and structuralist claims in (Awodey, 2004; Landry, 2013; Lawvere, 1964, 2005).

structure, because this will introduce a non-structural notion; this structuralist attitude can extend namely, anything taken prior and separately from the structure will introduce a non-structural notion. A structuralist would claim that elements outside of the mathematical context are not mathematical; if elements are taken separately and treated independently of the operations e.g. union, intersection etc., then this is meta-mathematics, on the grounds that mathematics presuppose structural notions (e.g. see Shapiro's (2005) view). Hence, sheaf theory can be interpreted both structurally and constructively. However, a hybrid SR/constructivism can be formulated if one employs Awodey's (2004) idea. Although Awodey is a mathematical structuralist and his suggestion to remove the foundational ladder so that only relations will remain aims to support SR, still, a hybrid SR/constructivism can be defended with regards to the recursion that the formulas express as will be analyzed in the rest of this sub-section. The involvement of individual relata out of which structures are built renders the structure physical; as mentioned, it is not sufficient to say that the structure is physical just because it involves operators and physical/mathematical notions such as the Hilbert space.

Let us consider Awodey's structuralist interpretation that structures are built out of morphisms in a category. Awodey claimed that one can remove the foundational ladder once it is used and take the mathematical structure to be algebraic. This means that even if one begins with some foundation e.g. sets, as a constructivist does, ultimately, there is no need to refer to such a foundation. We would agree with Awodey's view, in the sense that starting with a topological space X covered by open subsets, one can then take the respective category (e.g. category of open sets in our example) and via the functor, associate it with another category. So the continuum appears at the beginning of the construction but it is eventually removed and one ends up having only a structure of morphisms and in addition, there is no need to presuppose the notion of collection because contexts (regions in this case) themselves are objects in a category and are related in terms of morphisms.

On that ground, the (1)-(4) formulas provide an algebraic or structural picture in the sense that a) out of morphisms in a category (category of sets and category of algebras), structures are built and b) out of functors, the relation between structures is established. The building concept is the morphism so that there are no relata, rather a structure of relations. For example, $\Omega^k(\psi(U_1))$ should be treated structurally; it is an object in a category described by the morphism $I\rightarrow\Omega^k(\psi(U_1))$, where I is the terminal object and participates in the identity morphism $1: \Omega^k(\psi(U_1))\rightarrow\Omega^k(\psi(U_1))$ and in morphisms that provide restrictions to a subregion.

However, such a radical structuralist interpretation is not devoid of objections by structuralists themselves. The relata which radical OSR would want to eliminate in category theory are not objects of the relevant category, but the elements of the objects of that category (e.g. see (Bain, 2013)). In other words, a physical understanding of relations and relata means that physical objects stand in relations which are represented by relations between elements of the objects and not by morphisms among objects. So, one can take that category structure works in a higher level than the physical structure OSR is interested in. This means that one should consider local sections $p_i\in\Omega^k(\psi(U_1))$ which play the role of relata and should study the relations between these local sections.

As mentioned, the physical significance of sections is found in their association with the value of some property, so that in a measurement, these are related to the measured outcome. In particular, let quantities $p_1\in\Omega^k(\psi(U_1))$ and $p_2\in\Omega^k(\psi(V_1))$ which are taken (e.g. in two different measurements) by morphisms (1) and (2), that capture the direction from the whole to the parts, since the local observed quantities p_1 and p_2 are defined with respect to the contexts U_1 and V_1 respectively which are coarse grainings of the initial respective regions U and V. These determinate quantities are subsequently related due to the overlapping of their regions according to (3) and (4), that is, there is equivalence relation $res_{U1\cap V1,U1}(p_1) = res_{U1\cap V1,V1}(p_2)$ for sections $p_1 \in \Omega^k(\psi(U_1))$ and $p_2 \in \Omega^k(\psi(V_1))$.

So local sections or observational contents p_1 and p_2 are related via morphisms that give restrictions to the overlapping and they themselves are defined in terms of morphisms from the terminal object. This outcome is in accordance to formulas (1) to (4) which show how sections are restricted from one open set to another. Therefore, even if one considers local sections of objects of a category as candidates of relata, the notion of morphism appears again, since the relation between sections is a morphism when the aim is to derive observable quantities in a measurement.[58]

[58] Contrast our resolution to Bain's (2013) and Lam's and Wüthrich's (2013). Bain claims that the category Sh(X) of sheaves of sets on X, where sheaves play the role of objects and morphisms are morphisms between sheaves for each open set of X, does not get rid of relata. His argument is that the objects of the category Sh(X) does not always have global sections which are the candidates of relata; a sheaf always has local sections, but it is not guaranteed that there will be a continuous extension of local sections to global ones defined over X. He concludes that the absence of global sections is an indication that the category of sheaves lacks relata and such an outcome could function as a support for a category theoretic radical OSR where the category is in particular, the category of sheaves. Lam and Wüthrich, objecting to Bain's argument, claim that even if global sections are not present, there are still local sections, that can be interpreted as local elements and hence as relata. One should then show what kinds of relations local sections stand in where these relations are not C-morphisms, since the latter concern the relation between objects of the category. However, the derivation of global sections has physical significance and should constitute a presupposition of a framework. The existence of global sections is significant for observability and is taken as a satisfaction of global gauge invariance which means that local observables can be glued in the intersection of their contexts (refer to Atiyah (1979) who takes a cohomology exact sequence to derive global sections). The relation between local sections is then a morphism relation just as in the case of global sections. Since the gluing acts on observational contents which are morphisms and thus relations, sheaf theory seems to give primary concern on the relations between elements rather than the elements

So according to the above analysis, an object $\Omega^k(\psi(U_1))$ and its sections are analyzed structurally with no involvement of relata. At this point, a radical OSR would probably claim that it is relations all the way down starting from the higher level of mathematics and the objects of a category described in terms of morphisms and going to physical structures and sections which are also described by morphisms. But such an interpretation would not involve the distinction between mathematical and physical structures by a non-structural element. However, due to recurrence a non-structural element is recovered in the form of relata defined as structures themselves. Since a section in $\Omega^k(\psi(U_1))$ participates in the generation of later structures captured by future restrictions, it should also be seen as a relatum albeit defined as a structure of relations. The generation of new structures results in a new section defined in $\Omega^k(\psi(U_1 \cap V_1))$ with respect to a new context $U_1 \cap V_1$. There is then a continuous becoming of new relations, where a structure of relations acts as an input for the generation of subsequent structures of relations with respect to a new context. That is, something is a structure of relations with respect to some context and as a relatum with respect to another context in a series of continuous becomings.

In such a framework, one can then distinguish two kinds of relations, abstract conceptual relations and concrete relations, e.g. taking the limit and the finest graining in the overlapping, the continuum is recovered in the subject while sections associated with some realization in the objective world are concrete relations. To get a grasp why there are two kinds of relations, one can consider the following argument. For the sake of demonstration, let us take the formula {X, R}, where X is the domain of individuals

themselves (see also (Mallios, 2007, p. 268)). On the other hand, if one would like to introduce other kinds of relations between local sections which are not related to observability, she should show what these relations are and how they are included in the greater framework.

and R are the relations holding between these individuals. This formula is employed by pure structuralists who defend the view that it is structures all the way down, viz. the process of analysis does not stop somewhere (atoms). In particular, pure structuralists take a right-to-left reading of the formula {X, R}. For a pure structuralist, the relations in R have ontological priority so that the objects of X are relations and in that sense, infinite regress appears. On the other hand, for us, the reading occurs in both directions. The occurrence prehends all the causal (or mereological) relations or the section in $\Omega^k(\psi(U_1))$ captures the totality of mereological relations and in that sense there is a right-to-left reading of the formula {X, R}, where X here represents the occurrence or the section in $\Omega^k(\psi(U_1))$. So the relations in R become the objects of X. At the same time, an occurrence acts as a datum for subsequent one or the section in $\Omega^k(\psi(U_1))$ acts as input for other relation, therefore, according to a left-to-right reading, the objects of X which are relations subsequently feed the relations R (which actually express the gluing mechanism). Although the infinite regress is avoided, there appears circularity in this dual reading which can be avoided if those relations expressed in R and those expressed in X are distinguished in a way. In our interpretation, it was claimed that the relations in R are conceptual while what becomes actualized are concrete physical relations in the objective reality expressed in X. So conceptual relations expressed in R give rise to physical objective relations expressed in X, which subsequently act as relatum along with other relata for new conceptual relations (R') that the subject will evaluate. Hence, the circularity is avoided due to the difference between subjective and objective relations.

Therefore, if the distinction between mathematical structure and physical structure lies on the introduction of a non-structural element, it has been claimed that the realization of morphisms is consistent with the introduction of a non-structural element if one exploits the recursion that can be found in morphisms in sheaf theory. In that case, due to recursion that characterizes the mathematical structure, the actualized structures function as relata in subsequent morphisms. These relata are individuals out of which new abstract structures are generated. Considering that sections are associated with particular measurement outcomes, we introduce realization in an objective world. Alternatively, one could assume that measurement outcomes are found in the appearances but the structures would be found in the subject.[59] So measurement outcomes found in an objective world means that not only do the values of a physical quantity overlap in subsequent measurements that an observer performs, but also these values must be in accordance with the values that other observers get in their measurements. The synchronization of different observers does not imply subjectivity, because the input to a measurement is a datum from an objective world.

So even if one employs the building notion of morphism as Awodey does, eventually one introduces the notion of relata so that the actualization of the structure is captured. In order to deal with the infinite regress, Awodey eliminates the notion of relata by

[59] A relevant idea appears in Van Fraassen's (1980) constructive empiricism which makes the three-fold distinction between phenomena, appearances and theory. Phenomena are the observable entities and appearances are the content of observation and are perspectival (appearances are of phenomena and not of mental images). From appearances, data models (e.g. data points on a graph) and surface models (that are the refinement of data models e.g. fill in missing points in data models) are constructed and from these models, theoretical models are finally derived. The point with which we would agree with van Fraassen is that he gives an emphasis on measurement outcome which is context dependent, viz. data are gathered under certain circumstances, e.g. certain experimental setting etc. Something observable with naked eye is different from this thing observed by a telescope, namely the context is different. However, there are two drawbacks. Firstly, being dependent on the context (measurement or interpretation), appearances may not be real and likewise, the theoretical models isomorphic to the constructed models do not capture reality. Hence, models are not true representations of reality. Secondly, the structures of appearances which are the only knowable would be in re (Psillos, 2006, p. 566). For van Fraassen, mathematical models are derivative of concrete physical phenomena, so as Psillos (ibid.) comments structural empiricism may accept excess structure over the appearances (there can be many incompatible with each other theoretical models of appearances), but still these structures are linked to the structure of appearances.

analysing everything in terms of the primary notion of morphism so that it is not relations between some relata which are subsequently analyzed in terms of relations. Although Awodey considers only morphisms in a category in order to avoid the infinite regress that appears in pure SR where it is structures all the way down, we have shown that in virtue of recursion, there is distinction between mathematical and physical structure if relata are not eliminated.

The explanation of the constructive notion of recursion implies the introduction of relata which are relations themselves. Of course, it would be compatible with Structural Realism to take relations as relata of other relations.[60] But the circularity was avoided due to the distinction between two kinds of relations. This distinction then introduces constructivism. Initial relations (candidates of relata) generate structures.

Our resolution does not imply the priority of the parts/relata over the whole/structure of relations, because the structure itself provides new relata for subsequent occasions.[61] The structure is not taken separately from the relata since the structure described in $\Omega^k(\psi(U_1))$ gives rise to relata. Hence, neither the abstract structure nor the concrete relata in the objective world are fundamental, but what is fundamental is the structure of becoming where the whole gives new relata and the parts generate new abstract conceptual structures. In addition, this distinction between conceptual and physical

[60] E.g. French is an eliminativist about objects, so where these are relata, they are eliminated as well. Also, although Mertz's (2003) view is a support for fundamentality of the parts, he acknowledges the existence of unrepeatable relata that are relations themselves, namely, there are individuated relation n-adic unrepeatable instances, R^n_i, R^n_j, R^n_k, ... along with n-adic intensions or universals Rn where, these relations instances play the role of relata in the formation of more complex structures so that eventually a hierarchy of levels emerges out of relations.

[61] Observe the similarity with Esfeld's and Lam's (2008) (also see (Esfeld, 2004)) moderate SR, according to which relations and relata are put on the same ontological level. However, for Esfeld and Lam, objects are bare particulars and the properties they have are structural, whereas for us, relata *are defined by and do not have* structural properties.

relations where none is prior to the other conforms to our epistemological aim. Priority of the whole (as a structuralist may claim) or the parts (as a constructivist may claim) would imply that invoking only the mind for epistemology with no involvement of the external world suffices. Due to the equal treatment of parts and whole, both mind and body participate in epistemology; the mind perceives information from individual relata in the external world and structural information is generated.

4.3.5 MATHEMATICAL REPRESENTATION

The representation is a homomorphism from the symmetry group GL(A(U)) to the automorphism group Aut(ψ(F(U))) of the vector sheaves, where A is a sheaf of algebras on X and U is some context. Namely for each element a of a group GL(A(U)), the representation

$$\mathrm{Ad}: GL(A(U)) \rightarrow \mathrm{Aut}(\psi(F(U)))$$

is given by $\mathrm{Ad}_s(a)=asa^{-1}$, where $a \in GL(A(U))$ and $s \in \psi(F(U))$.

At the overlapping $U_{ij}=U_i \cap U_j$, a transformation of ψ with respect to the local contexts is given by

$$g \equiv \prod_{i,j} GL(A(U_{ij}))$$

where Π is the Cartesian product. Automorphism then gives the transformations between the various contexts and since the germ gives the reconciliation between different contexts, Aut stands for the group sheaf of germs of automorphisms.

The mathematical representation is dynamical updated by new data found in the objective world. In a series of continuous becomings, the representation is continuously generated by new parts coming into existence in the objective world. The external world is in constant change and the mathematical representation captures continuously new mereological connections.

In addition, that the abstract is inseparable from the concrete entails that the (abstract) conceptual (particle) representations are related to the (actual) concrete physical structure inseparably. The totality of possibilities are evaluated by the subject. This evaluation entails that the objective world will be shaped by the selection of the subject and subsequently the objective world will contribute to updating the representation. Hence, in comparison to indirect realists, the representation and the represented are interconnected so that the gap between them closes.

4.3.6 ANTE REM VERSUS IN RE STRUCTURALISM REVISITED

This last section will close our analysis by referring to ante rem vs. in re SR. As mentioned, a framework should account for a) causality and b) a range of possibilities which is greater than the realizable one.

With respect to the latter point in combination with the formalism of presheaves that we used in order to capture dipolarity, one can distinguish between extra unrealizable structures, the range of actualizable possibilities and the actualized physical reality, where the unrealizable possibilities are distinguished from realizable ones by some symmetry conditions so that subsequently the sheaf implements the relation between the realizable possibilities and the final concrete actuality. For example, Doplicher et al. (1969) define the field algebra G(O) of a region O and covers both the unobservable parastatistical and observable statistical fields. So G(O) is larger than the collection of local algebras G_1(O) of operators associated with observable fields, but due to superselection rules, there is restriction to the subset G_1(O) of G(O) which remains invariant under gauge transformations. A presheaf which captures the set of realizable states restricted to an algebra G_1(O) can then be defined (Haag, 1996, pp. 108–111). Therefore, the presheaf involves only the physical representations with no intrusion of those parastatistical representations which are unphysical. In such a structure, there is a mereological relation between the totality of possibilities and the actualizable particle possibilities, as well as a dipolar relation between the actual possibilities and the concrete actuality. Hence, one can acknowledge the existence of superfluous abstract structures while some abstract structures will be internalized and treated inseparably with the objective world which means the subject will evaluate the abstract real possibilities so that the concrete actuality will appear.

As for the first point on causality, one needs to discuss how the subject and the object as an inseparable whole treat causality. Although causality is commonly associated with the physical level and not the mental level, still one should not think that causality holds between concrete particulars, because nothing including causality can be stripped off the entire process of becoming which involves both the mental and the physical. Invoking the structure of becoming in the explanation of causality would entail that there can be causality without the mathematical structure being in re, since the structure of becoming does not give priority on the concrete systems of the objective world. Although it is commonly argued that the mathematical structure must be in re, so that causality is accommodate, still the structure of becoming is supposed to capture the causal relations without the mathematical structure being in re.

In addition, with regard to some structuralists' argument that one needs objects to retain causality (Psillos, op. cit., sec. 4), based on the bidirectional reading of the formula {X, R}, we can provide a structuralist answer without the involvement of objects possessing intrinsic non-structural properties. Contrary to French who rejects the notion of relata, we can give an account of causality where relata are defined by structural properties. Moreover, concerning Chakravartty's (2003, pp. 872–3) argument that the existence of relata is necessary for they are 'an active principle' responsible for the transformation of a set of relations into another, this power is not necessarily found at the object. Chakravartty takes objects - observable and unobservable concrete particulars - to have properties that confer on them certain capacities or dispositions to act in a certain way while the manifested behaviour of an object is conditioned by the properties of and the interaction with other particular(s), so that the manifestation is the concrete structure generated by the relations between causal properties-relata (Chakravartty, 2007, chaps. 3–5). In our framework, this power could be located at the process of becoming (for more see chapter 6). Objects are secondary to the process of becoming, namely, they are defined as a history of occurrences so that occurrences should be the primary seat of powers and if one wants to say that objects have such power this is in virtue of the dispositional properties that characterize the occurrences. Although there is a distinction between concrete physical relations and abstract conceptual relations, and disposition could be found in conceptual relations, still disposition do not stand independently of the manifested concrete relations, so that the seat of potency is found in the process of becoming (see analysis in chapter 6).

Furthermore, defining the occurrence as a process of becoming provides resolutions in problems appearing in causality relation between individual events which are concrete particulars (Davidson, 1967). Firstly, an ontology of events renders the relevant structure in re (Psillos, op. cit., p. 569).[62] Secondly, apart from the fact that the notion of event presupposes the existence of spacetime, there is also a problem with the localizability of the event in spacetime. Causality should involve events with determinate attributes and if the location of the event is defined by particle that undergoes that change, the location of particles is far from determinate.

To conclude, we espouse neither ante rem nor in re SR because there are extra unrealizable abstract structures which do not stand independently of the structure of becoming that brings close the abstract and the concrete, and at the same time the physical system is not prior to the abstract structure. For example, the concrete system of particles is not prior since there are parastatistical representations and the abstract system of possibilities does not stand separately from the concrete actualized system due to the inseparable treatment between abstract and concrete. In other words, the structure of becoming aims to close the gap between abstract and concrete, hence, there is a metaphysical linkage between mental possibilities and abstract things in a Platonic

[62] There are also alternatives to event-event type of causality (Campbell, 1981, p. 480): condition-event (e.g. the poor construction of a house caused its collapse during the earthquake), event-condition and condition-condition, where conditions represent abstract particulars, but again the concern is that fundamentality of tropes renders the structure in re (see more on tropes in sections 5.4 and 5.5).

heaven, just as there is a metaphysical linkage between mental possibilities and concrete things in the objective concrete world.

4.4 CONCLUSION

To close, it was shown that one can retain the reality of the structure and at the same time, give a constructivist account of obtaining the structure. In this way, the structure of the world that describes this relation between subject and object embeds the non-structural element within.

The framework presented shows that neither the mind has an authoritative attitude over the body nor the body has an authoritative attitude over the mind. The aim was to oppose the highly intellectualization which is prevalent among philosophers influenced by the Kantian theory.

5 ELIMINATION OF SUBSTANCE AND THE DIPOLAR RELATION

Abstract

The holistic approach presented in the previous chapter will be used as a justification for the elimination of the notion of substance and the employment of tropes which do not presuppose the existence of a background spacetime. The framework applied will be sheaf theory.

5.1 INTRODUCTION

The loss of individuality of particles in entanglement has motivated philosophers to remove the notion of material substance from their ontology. Apart from those structuralists for whom reality is found in the structure while substance possessing priority is eliminated, the same attitude with respect to substance appears in other ontologies. For Simons (1994), substances do not have basic character; his trope ontology rejects the primitive, unanalysable status of substances, rather substances are analyzed in terms of tropes, where tropes are divided into essential and non-essential, where the essential ones constitute a kernel. Also, in Seibt's (2002) Axiomatic Process Theory, particles should not be taken as fundamental, but they should be understood as composed of processes, e.g. in QFT, the scattering process of two particles can be analyzed in an infinity of possible interactions.[63] Seibt's free processes are concrete individuals but not particulars because they are not fully determinate, that is, they do not have a determinate (single) spatiotemporal location rather they refer to many spatiotemporal regions. So, for a measurable property of a process, there are distribution patterns of that property.

In this chapter, the notion of substratum will be removed and the notion of substance will be secondary to the notion of process. However, contrary to Seibt, we will see that the elimination of substance does not entail the elimination of concrete particulars and individuals. The determinacy will be associated with concreteness and indeterminacy with abstractness (Rosen, 2014, sec. 3). This will give us more flexibility in accommodating modality; if what is fundamental is concrete (whatever this may be:

[63] As Kuhlmann (2000a, 2000b) explains, a process theorist would not take the infinity of Feynman diagrams to imply the infinity of particles which participate in a single scattering process. For a process theorist, particles are not thought of as moving from one point to another at time instants, rather they are understood in terms of their interactions, their creation or their annihilation. Hence, each single diagram is not taken separately from the others, rather particles will be defined by the totality-infinity of diagrams/interactions. Consequently, for a process ontologist, the lack of individuality due to superposition does not constitute a problem, since objects are secondary to processes. However, the main objection to this approach, just as to the approaches which build the ontology on the basis of quantum entanglement, has to do with the realization of the structure which brings forth the notion of determinacy. The realization in Feynman diagrams is related to the dynamical possibilities of paths between two states, the initial prepared state and the final measured state; one can then calculate the probability of a measured state given a prepared initial one.

processes (characterized by indeterminacy or not), particulars etc.), then it is not clear how the structure will incorporate the abstract.

5.2 THE ABSTRACT VERSUS THE CONCRETE

In the previous chapter the notion of abstractness referred to mathematical structures such as paraparticle representations which are non-actual as well as those mathematical structures such as particle representations which are actual. The latter were the ones that were characterized as conceptual because they capture the relation between subject and object. This chapter will retain this distinction between actual abstract and actual concrete but it will also introduce the notion of determinacy in the definition which will be consistent with what discussed in the previous chapter.

Generally speaking, in philosophy, concrete and abstract entities are commonly distinguished on the grounds that the former are in space and time and have causal efficacy whereas the latter are considered non-spatiotemporal or causally inefficacious or both (Rosen, 2014).[64] Moreover, it has been argued that even if an object is in spacetime, it should be characterized as abstract, if it fails to occupy a determinate region of space (or spacetime), so that an abstract object either is not located in a determinate region or lacks causal efficacy or both. Not all espouse that view; being spatiotemporal is vital in characterizing something as concrete even if this something is not in determinate location.

[64] According to Frege's Way of Negation (Rosen, 2014, sec. 3), abstract objects are devoid of features that appear in concrete objects.

However, it has been claimed that any distinction on the grounds of causal efficacy and spatiotemporality is not adequate in the context of contemporary fundamental physics (Ladyman and Ross, 2007, pp. 159–161; Lam, 2007, pp. 46–7, 101–2). Firstly, in modern physics, one would have to say in what sense the spacetime structure is causal. The idea that the spacetime structure causes matter to behave in a certain way is not appropriate – one would need to find a way to characterize this relation as causal, for example, to define a cause timely separated by its effect. In 3+1 Hamiltonian formulation of GTR where three-dimensional spatial hypersurfaces can be defined, there is no time with respect to which the causal ordering will be determined.

In addition, the inclusion of singular points in the manifold questions the concreteness of spacetime. Firstly, the existence of points which lack determinate boundaries questions the concreteness of spacetime. If spacetime and its constituents - Hausdorff separated points - are taken as concrete, then the existence of non-Hausdorff separated points which lack determinate boundaries questions the concreteness of spacetime. This can be related to the alleged relationship of abstractness with indeterminacy. Since points do not have determinate boundaries, they can be considered neither concrete nor particulars and likewise the extensive manifold (and of the associated gravitational field) is not concrete as well-albeit actual. Secondly, if one considers the cobordism M between spaces Σ and Σ', then the unity between them so that none is seen separately entails that the extended manifold M as well as its constituents Σ and Σ' can be considered as non-concrete because Σ and Σ' lack determinate boundaries that divides them.

One could object that determinacy and concreteness should not be related. For example, Seibt's free processes are concrete but not determinate and Esfeld (2009) takes the structures of entanglement to be concrete. But such approaches give priority to the whole which we have already rejected.

The non-concreteness of spacetime provides a justification for its elimination from the background. As discussed, in an algebraic account, there is an association of atom of spacetime (regions, manifolds etc.) though these atoms are not found in spacetime. What is concrete are the parts having determinate boundaries and what is abstract is taken by the overlapping of regions. Hence, in our approach, the distinction between the actual concrete and the actual abstract will be related to the notion of (in)determinacy.

(T_0) The actual abstract will be considered as determinable, contextual, conceptual and inseparable from the actual concrete which will be taken as determinate, contextual and physical.

Both the abstract and the concrete are characterized as actual and this is expressed in the contextuality with respect to some spacetime region (or the entire manifold if it is entire manifolds mereologically and inseparably related). On the other hand, abstractness is related to the mind which evaluates a range of possibilities. The distinction between determinables and determinates will be discussed further in the next section.

5.3 DETERMINATES/DETERMINABLES AND MEREOLOGY

It is a common idea among philosophers that fundamental entities are those which are maximally determinate. In particular, Armstrong ((1985, pp. 114–115)) rejected the existence of determinable universals. Instead of determinable universals, he refers to classes of determinates which are classes of universals characterized by some ordering (in particular ordered by relations of partial identity). For example, there is no such property as ‘being a mass’. If there was, the balance-weight would have both the property K - ‘being one kilogram’ and M(K) - mass of one kilogram. Likewise, there is no such property colour. If colour is the determinable property and shade is the fully determinate property, redness lies between colour and shade. Hence, a red blouse would have all these properties between colour and shade. Therefore, Armstrong assumes the existence of classes of fully determinate properties.

In such an approach, the fixing of all entities is grounded on the fixing of the fundamental ones. The fundamentality of determinates should be understood as once the more specific -determinate- is fixed, the less specific -determinable- is fixed as well and it is not the other way around; what God had to do was to define the fully determinate and the determinable would be an abstraction of determinates. Therefore, the determinate properties are more fundamental than the determinable properties.

However, such an approach would imply the existence of a continuum with respect to which the ordering and the fixation takes place. E.g. the class ‘red’ could be defined by the continuous spectrum of hues and therefore, a red blouse which is associated with some surface, would be characterized by all the determinate properties, each one related to some spatial point. Hence, the red blouse is a continuum of determinate values. But considering that the continuum has true overlapping parts and is generated out of these parts by the subject, then God does not fix determinate values rather the involvement of determinable quantities is justified as follows. If determinate property p_1 e.g. $colour_1$ or $mass_1$ is found in region U_1 and determinate property p_2 e.g. $colour_2$ or $mass_2$ is found in region U_2, then although the two particulars have non-resembling sets of properties, they still have the resembling determinable properties, colour and mass. Moreover, determinables can be mapped to multiple determinates, even determinates that are not instances in the actual world. Hence, a determinable is not determined by the instances, that is, a non-modal fact expressed by a determinate cannot give ground for a modal fact expressed by a determinable.

However, as mentioned, the structure is neither ante rem nor in re and that the whole and the parts have equal treatment. Since neither parts nor whole are fundamental, likewise neither are determinates nor determinables endowed with fundamentality but both determinate and determinable are treated with equal status. Although, in general, the relation between determinable and determinate is different from the mereological relation, here the mereological relation between spatiotemporal regions is expressed in the relation between determinates and determinables.

The equal treatment of determinates and determinables is relevant to Wilson's (2012) support for the inclusion of the determinables in the fundamental base, along with the determinates. The first criterion that Wilson invokes is the naturalness of fundamental properties which make the particulars that possess them similar in certain respects. Determinates are more specific than determinables, hence the particulars that share the same determinate property will resemble each other more than particulars having the same determinable property. Therefore, determinates should be regarded fundamental. However, Wilson rejects that the criterion of naturalness implies the fundamentality of determinates, because particulars may not possess resembling determinate properties but they may have resembling determinable properties. In that case, naturalness appears in determinable properties. As for the fixing criterion for fundamentality, Wilson concludes that a complete base should ground both the modal and non-modal facts; both determinables giving the ground for modal facts and determinates playing the role of existential witnesses about the happenings of this world should be included in the base of fundamental entities.

One issue here is how an interconnection between determinates and determinables is understood when both of them are put on the fundamental base. If determinates were fundamental, there would be an asymmetric ontological dependence between a fundamental base and its dependants where this asymmetric relation is translated as supervenience relation of determinables upon determinates. Alternatively if determinables are over and above determinates, one can invoke the instantiation relation which is supposed to link determinable universals and determinate particulars. However, there is an explanatory gap concerning the way universals and particulars combine together because instantiation is burdened with obscurity; the problem is that as a relation, instantiation should also be instantiated (Daly, 1994, pp. 258–260).

Taking the option that neither determinates nor determinables is fundamental over the other, their relation could be expressed as a symmetric dependence relation which contrasts with the asymmetric relation in a fundamentality claim and in addition, in our approach the mereological symmetric relation between them is translated as symmetric relation between parts and whole. Such a resolution means that the range of determinables accommodates more possibilities than the one actualized.

Moreover, with respect to Wilson's criterion of naturalness, there is no need for exact similarity between determinates, since determinables have equal status with determinates. Hence, one can distinguish degrees of resemblance between properties.[65] Consider unequal quantities q_1 and q_2 associated with regions 1 and 2 respectively. An overlapping between these regions is taken as a resemblance relation between q_1 and q_2. The fact that there is overlapping of regions and not well-defined boundaries is an indication that a) the quantities q_1 and q_2 are associated with some determinable property and b) that the overlapping introduces indeterminacy and hence the idea that something determinate is concrete whereas something determinable is abstract, finds correspondence to the view that abstractness is related to indeterminacy.

In particular, due to bidirectionality between whole and parts, it follows that:

(T_1) A determinable actual abstract quantity stands in a bidirectional whole-part relation with a determinate actual concrete quantity.

In a constructivist approach of continuous becomings, concreteness characterize determinate properties with respect to some region of determinate boundaries while abstractness characterize determinable properties with respect to some overlapping region. In a series of becomings, if determinates $q_{1|U1}$ and $q_{2|U2}$ (parts) come to combine on the overlapping, a determinable $q_{3|U1\cap U2}$ (whole) is defined with respect to the intersection area. Alternatively, if region U_4 stands in a whole part relation with regions U_5 and U_6, determinable $q_{4|U4}$ (whole) is in relation with new $q_{5|U5}$ and $q_{6|U6}$ (parts).

As explained, sheaf theory provides the appropriate framework that can treat whole and parts equally and hence such traditionally mutually exclusive categories as

[65] The resemblance relation between determinates can be expressed as resemblance relation between tropes and be treated topologically in terms of sheaves as explained in section 5.4. E.g. the determinable (e.g. (mass, charge)) can be derived mereotopologically from the related concrete particulars (e.g. ($mass_1$, $charge_1$), ($mass_2$, $charge_2$)).

determinables-determinates with the same importance and in an inseparable way, so that there is no miraculous transition from being determinable to becoming determinate. As an example, let us consider the growing process of 3.3; the morphism d^0 is associated with a quantum operator while the actualizations are captured by sections $p_1 \in \Omega^1(\psi(F(U_1)))$ and $p_2 \in \Omega^1(\psi(F(V_1)))$. Since the quantum operator is not associated with a determinate value rather with a range of eigenvalues associated with some region, the derivation of p_1 and p_2 captures the direction from the whole to the parts. However, these determinate quantities are subsequently related due to the overlapping and more generally, the gluing mechanism gives the transition from the classical to the quantum and therefore, it results in the appearance of an abstract determinable with respect to $U_1 \cap V_1$.

Another example is the case of interacting fields in Yang-Mills theory (presented sheaf theoretically by ADG (section 4.2.2.2)). Determinate quantities e.g. (E_U, D_U) and (E_V, D_V) are recovered by the coupling of matter field and potential. These quantities are derived out of inclusion relations between regions and the relevant morphisms (from the whole to the parts). Inversely, a determinable ($E_{U \cap V}$, $D_{U \cap V}$) is recovered at the overlapping and more generally, gauge invariance captures the combination of determinate quantities (parts) which results in something determinable (whole).

More generally, the sheaf theoretic framework allows the distinction of levels of determinateness. A less determinate or more determinable property is the refinement of a more determinate or less determinable property. Different degrees of determinacy is provided by coverings of different degrees of refinement, so that the finest graining characterizes the determinable of the greatest degree. The distinction between determinables and determinates and mereology will be employed in the next section for recovering the resemblance between tropes in a mereotopological way.

5.4 ELIMINATION OF THE SUBSTANCE – A TROPE THEORETIC VIEW

In literature, Campbell (1981, 1990) rejects the idea of concrete particulars defined by the totality of universals along some element of particularization - substratum, of which the existence in the background is denied by modern physics. In his trope theory, Campbell defines tropes as abstract particulars, where local properties are abstracted by the mind from the other properties with which they are in conjunction (section 5.4.1). Likewise, in our framework, due to the involvement of the mental side, tropes will be understood as abstract particulars, but as we will see, the point of introducing the mental side is to give a justification of the experienced continuum which is nevertheless generated by atoms in the physical world due to inseparability of the actual abstract from the actual concrete. The next section refers to the understanding of tropes as either abstract or concrete particulars, before we present our view and why tropes are taken as abstract. The notion of substance is eliminable while the mereological treatment of (in)determinacy motivates a trope analysis.

5.4.1 ABSTRACT OR CONCRETE TROPES

Campbell

In literature, according to Campbell (1990, pp. 2-4), tropes are abstract particulars. Properties such as the colour of the chair and the texture of the blouse are abstract because they can be abstracted by the mind from the other properties with which they are in conjunction. This does not entail that such properties are products of the mind, rather they exist independently of the mind. In addition, abstract is not translated as something indefinite, purely theoretical, non-spatiotemporal. The colour of the blouse is found at some spacetime location. Tropes are particulars because they exist in a unique spacetime location just as the respective concrete particular.

Campbell's argument for taking that tropes are abstract and not concrete has to do with the discrepancy between the apparent existence of parts with some spatiotemporal boundary and the basic trope which has no boundaries and is not constituted of parts. For Campbell, field tropes filling the entire spacetime are basic while tropes such as mass or shape of an object which are associated with some reference frame locally are derivative. The derivative tropes appear out of localization of basic field tropes to some region. These tropes are quasi-tropes because they are pseudo-parts. A quasi-trope is a chunk of field treated separately from the others, though it is not separate because any boundaries are imposed by the human. So quasi-tropes are not fully objective real tropes, because they are appearances (ibid., p. 155).

The lack of actual boundaries is justified by Campbell's following arguments (ibid., pp. 136-141). Let us take the colour of the chair. If the surface of the chair is truly divided into parts so that a trope is associated with each part, then there should be a unique true partition. However, there are indefinitely many ways an area can be partitioned hence, taking a certain partition and putting boundaries are conventional and not natural. Since there is no certain pattern of division of the whole into parts or equivalently, there are no natural boundaries, the chair cannot be identified by many colour tropes where each trope is associated with some region (ibid., p. 137).

Moreover, in the dynamic context, the existence of temporal parts of the chair entails that tropes are continuously replaced by new ones, and this is a problem since the preservation of the identity of the chair is owed to tropes. If then tropes are divided into instantaneous temporal parts, tropes cannot play an identification role, except if this temporal partition is not fundamental but derivative, in the sense that only derivative tropes are dissective.

What Campbell suggests is then that basic tropes should be partless, changeless and with unambiguous boundaries. On the grounds that spacetime has no true parts, for Campbell (ibid., pp. 145-147), basic tropes are spacetime-filling fields (gravitation, electromagnetism, the weak and the strong nuclear fields).

However, Campbell's approach has two drawbacks that will be raised in the next section again. Firstly, as Morganti (2009) objecting to Campbell argues, since field intensities are determinates and fields are determinables, what is basic should be something simpler than the entire field. Secondly, spacetime has true parts.

Simons

A trope ontology is not necessarily related to dualism, rather it can be a support for materialism; tropes are not interpreted as abstract particulars and there is no involvement of experienced sensations in the interpretation. For example, for Simons (op. cit.), abstract objects have neither spatial nor temporal location whereas concrete objects have at least temporal location. So, when he comes to define tropes, he characterizes them as concrete, where concrete tropes can be defined if one distinguishes between dependent and independent particulars, so that tropes are dependent concrete particulars while independent concrete particulars are bundles of tropes or a totality of compresent tropes.

His nuclear theory of tropes postulates that objects are made of a bundle of essential tropes ('nucleus') and non-essential tropes and the individual essence of substances is grounded on this kernel. Basic tropes are searched at the fundamental level of particles, namely, mass, charge, and quantum of spin are basic tropes constituting the nucleus, while position, kinetic energy, momentum and direction of spin are contingent tropes. Employing the notion of nucleus, Simons wants to tackle with a) the infinite regress that appears in compresence of tropes and b) a tropic preservation of identity.

With regard to issue a), one needs to answer how tropes come together to give a unified object and for Simons who denies the existence of a substratum who could play a unifying role, compresence relation has this unifying responsibility.[66] However, compresence relations should also be taken as additional tropes that are compresent themselves. For an infinite number of tropes, one ends up with infinite regress, and

[66] If the substratum plays a unifying role, in the case of two tropes, the compresence relation could be a three-place relation which links a place with two tropes. This place functions as substratum, in virtue of which, the whole bundle is a unity. An alternative scenario that involves the substratum would be the compresence relation identified as a many-place relation where many places unite the tropes into a whole system.

hence unification is unattainable. In order to resolve this problem, Simons takes the compresence relation to be internal and not external to the tropes, so that infinite regress is avoided, that is, the compresence relation is (unanalysable) basic or supervenient on the nature of the related tropes and thus, it does not get ‘troped’. The compresence relation is internal, if two tropes essentially co-occur whereas if two tropes contingently co-occur (e.g. although a colour trope needs a surface trope, some surface trope S contingently co-occur with some colour trope C), such a compresence relation is not included in the nucleus determination and therefore for stopping the infinite regress.

As a consequence, with regard to b), on the basis of essential tropes, a compresence relation being internal to the nature of tropes can justify retention of identity so that individuality is granted by the nucleus. Contrary to a substratum theory, where bare particulars do not come into and go out of existence, and therefore substances being indestructible survive change, here the nucleus bundles can be replaced by a new nucleus. For example, two particles with opposite spin taken separately have different nuclei. When these particles are superposed, then the two trope bundles appear in the new substance. Hence, the initial bundles combine to give a new bundle. Subsequently, upon measurement, the nucleus of the superposed state is destroyed and two new nuclei are generated.

After this presentation of Campbell’s and Simons’ views, the next section presents our framework and shows that, assuming that there are true parts, the identity is preserved due to inseparable mereological relation between parts and whole and not due to the distinction between contingent and fixed properties.

5.4.2 TROPE DESCRIPTION OF INTERACTING FIELD SYSTEM

Treating the material world with importance does not necessarily imply the recovery of substance. A bidirectional reading of Einstein's equations entails that matter found in the material world is not basic and the inclusion of singular point on the boundary justifies the elimination of the substratum idea, hence there are no substances acquiring their individuality by any substratum. If one wants to retain the notion of substance, this should be secondary analyzed in terms of a more basic ontology. In particular, we consider a trope ontology which rejects the primitive, unanalysable status of substances so that substances are analyzed in terms of tropes.

The overlapping of regions can be taken as a support for particulars being identified by tropes in the following sense. The resemblance relation, which relates tropes in different individuals that resemble each other, comes with degrees so that one can distinguish complete similarity and partial similarity between tropes, for example the spin of one particle can be completely similar to the spin of another while the blueness of my pen resembles in some extent the blueness of my jacket. Since topology can be used as the basis for the implementation of notions such as neighbourhood, nearness, degrees of resemblance can be expressed by the mereotopological overlapping and thus correspond to a partial order. In addition, the existence of a background spacetime is not necessary in an ontology of tropes. It is commonly found that tropes are constrained spatiotemporally in a single position that they share with other tropes. Still as Campbell (1981, p. 486) argues, tropes cannot exist without a formed volume but this volume is a feature of things, not of space(time). Hence, tropes are defined with respect to some context U associated with the volume trope of the concrete particular.

For example, with regard to the cases we have studied, since the derivation of an observable quantity presupposes the satisfaction of gauge invariance, a concrete particular is derived out of the coupling of the gauge field with a matter field.[67] In terms of tropes which are compresent in the same individual, this pairing can be translated as the compresence of the matter field and gauge field tropes, so that the physical and the mental coalesce to give a concrete actuality with respect to some context (U, V or U∩V) (see section 4.1 which studies the bidirectional reading of EFE, where the gravitation field is associated with the mental side and the matter fields with the physical side; also see section 4.2.2 which studies the pairing of matter field and gauge field). The covariant derivative expresses differentiation acting on section and can be associated with properties such as energy if gauge transformations are employed.[68] Such properties are relational since the D expresses change which is algebraically captured by morphisms expressing ordering relation. That is, a sheaf morphism D^i: $\Omega^i(E)\to\Omega^{i+1}(E)$ is related with properties such as energy which are coupled with matter E to give a concrete particular expressed in the node $\Omega^{i+1}(E)$ in the cochain (the actualized eigenstate). In addition, matter can also be defined by the totality of tropes (e.g. spin, mass etc.) with respect to some context U. Matter is still described in terms of symmetries as a structuralist would claim, but the automorphism group is contextual and generated from input found in the objective world so that the whole (the automorphism group) is not prior to the parts as these appear in the gluing of sections.

[67] In literature, Mormann (2003, n.d.) treats trope theory in terms of the topological theory of sheaves which captures the relation of compresence and the relation of resemblance. In particular, he defines the domain B of individuals or concrete particulars (such as a particle) represented by a subset of a topological space and the domain E of qualitons or tropes (such as mass). The mapping from E to B is a compresent relation between tropes for the same individual and the mapping from B to E provides the resemblance relation between tropes in a topological way. Our framework is an extension of Mormann's idea where the trope framework is combined with interaction field systems where the state of fields remains invariant under transformations.

[68] E.g. if the wave function transforms to $\psi'(x)=e^{i\theta(x)}\psi(x)$ and the potential to $A_\mu'(x)=A_\mu(x)-\nabla\theta(x)$, then the Schrödinger equation $H\psi=-\hbar^2/(2m)(A_\mu(x)-iq\nabla)^2\psi+V\psi$ remains invariant under gauge transformations.

In addition, overlapping of regions can be understood as resemblance between the tropes in the common region. Gauge invariance involves the patching and gluing of sections $\Omega^{i+1}(E)$ which play the role of abstract particulars, where the gauge invariance can apply to the same matter field or different matter fields (section 3.3). There is a global to local and vice versa bonding (that can be implemented sheaf theoretically) due to gauge invariance; firstly, the global expressed in the non-localizable potential and matter fields are linked to the local or the individual which is constituted by the compresent potential and matter tropes and secondly, different concrete particulars and the relevant tropes resemble each other so that different individuals are linked to the global.

The contextual character of the determinacy of the tropes gives a flexibility in the framework. If tropes are fully determinate, a trope theorist cannot give an account of shared determinable properties in terms of some resemblance relation which is not exact similarity, since determinable properties are not accommodated, e.g. there is no resemblance relation between the fully determinate tropes spin-up and spin-down while resemblance is recovered if one considered the property spin as the shared determinable trope.

Contrasting our view to Campbell, with regard to the apparent existence of spatiotemporal boundaries that identify different parts and the elimination of boundaries from reality, the removal of the singularity from spacetime shows that spacetime has true overlapping parts. There are parts of matter (which can be a microscopic object and a macroscopic one), where these parts are not separated by some empty space. In this context, the colour of the chair is divided into colour-parts defined with respect to some context. Moreover, in a continuous process of becomings, contexts overlap so that although parts have boundaries these are not strictly separated, for the whole is derivative out of them. Consequently, we do not need to take the entire field as basic, as Campbell does. By splitting spacetime into chunks, we are able to implement dipolarity between determinates and determinables, so that neither is fundamental.

In addition, the difference between our abstract understanding of tropes and Simons' concrete understanding of tropes lies on his presupposition of a background time. Although just as us, Simons takes that parts (the nuclei of individual particles) combine to give a whole (a new nucleus) and the whole splits to new parts so that neither parts nor whole seem to have priority, he treats parts and whole as concrete because there is a background time with respect to which the evolution is defined. When identical particles do not interact but they are in isolation, their identity is defined over time, but when they interact (or become superposed), their identity is not defined over time. However, for Simons, the problem of non-individuality of interacting particles can be resolved on the grounds that what exists is the new nucleus that refers to the superposed state and any discussion about individual particles is off the table because these went out of existence (their nuclei perished). Due to the existence of the nucleus, the identity of substances derivative of tropes can be retained over time. On the other hand, for us, there is no background time to define different substances (individual particles or a system of interacting particles where there are no individuals distinguished) and their evolution in time. The mereological relation between individual particle and interacting particles should then be defined with no time. This justifies the distinction between abstract and concrete, where the concrete are individual particles and the abstract are superposed states. (In fact, there are no lonely isolated particles. Contrast that to French's argument on lonely object scenarios. According to French, there cannot be a lonely particle, since properties are structurally defined by the relevant relations described by the law. E.g. the property charge of the particle is a property of which the features are defined by Coulomb's law. In our view, a particle does not exist separately in the sense that an individual particle should be seen as an interacting system.)

In our view, that tropes were not taken as concrete is justified by the non-existence of spacetime. Spacetime is a product of mind generated by the parts and likewise, the continuous perceived ranges of colours are products of mind. This justifies the abstract characterization of the colour trope while its compresence to some part with other tropes justifies its particularity. Moreover, although the initial regions can be partitioned in multiple ways or mathematically, there is an infinity of coverings that a topological space can be partitioned and likewise, an infinity of causets or posets, this does not necessarily mean that the whole is basic. Campbell's argument was based on the assumption that the spacetime continuum does not have true parts, but if this is not the case, the partition is not necessarily purely subjective.

As for the preservation of identity, although the existence of parts of the chair or of the matter field entails that there is continuously the generation of new tropes, there is still preservation of the identity of the chair or the matter, in the sense that the whole and the parts are related mereologically and parts are not separate from each other by some space or time, but parts are united by a mereological relation between them. Any change or transformation does not occur in space or in time but is mereological, therefore the identity is preserved as long as the mereological ordering is retained. Hence, there is no need to distinguish between fixed and contingent properties in order to define a nucleus that preserves in transformations. For example, the chair whose colour C is compresent with surface trope S is mereologically related to its true part and the relevant tropes C_1 and S_1. The retention of identity in this case is translated as the retention of the ordering relation between whole and parts; this relation described by the process of becoming unites whole and parts and parts with each other. With respect to potential and matter tropes, the matter trope (that can be defined by the compresence of some fixed tropes (e.g. spin) and some contingent one (e.g. spin direction)) is compresent with the contingent surface trope. So although there are contingent tropes in the identification of the substance, the identity is still retained since the contingent surface tropes of parts are ordered in an inseparable way. Hence, it is not necessary that basic tropes are found at the fixed properties of particles but also in dynamical properties, such as energy, momentum, potential etc. In addition, the compresence of the field trope, matter trope and surface trope is internal and there is no external entity that functions as a substratum. As explained in chapter 4, it is a web of relations that can give credence of the pairing of matter and connection.

To close, although tropes can explain the interrelation between the actual abstract and the actual concrete, it is the process of becoming which is fundamental. This is emphasized in the next section.

5.5 THE STRUCTURE OF BECOMING IS FUNDAMENTAL

One objection to those who would espouse the fundamentality of tropes is how modalities are accommodated. If tropes are concrete particulars, then abstract possibilities such as paraparticles or talking donkeys which are not found in concrete particulars cannot become part of the description. Or if tropes are abstract particulars where this abstractness is product of the mind and these particulars are in spacetime, then again abstract possibilities are not included in the framework. The main point is that the physical system should not be prior to abstract possibilities. One can even enjoy the idea that uninstantiated universals not found in particulars should not be excluded from the framework. This means that the structure of the world accommodates not only tropes but also abstract universals, and none should privilege fundamentality.

Our initial aim was to incorporate process elements in the ontology. What we suggest is the fundamentality of the structure of becoming. So, the notion of substance which does not have primitive, unanalysable status was analyzed by the structure of becoming or transforming from determinable to determinate and vice versa. Moreover, the bidirectionality achieves the unrepeatability of concrete particulars that appears in trope ontology. Each context-spacetime atom is unique hence the concrete particulars being compresent tropes that express ordering relations are unrepeatable and the resemblance relation being fed by new contextual tropes is also unrepeatable. The (E, D) pair ensures the individual essence of substances; the individuality of each occurrence is grounded on the uniqueness of the causal history expressed in the ordering relation between topological spaces. Hence, our framework renders the existence of a substratum for avoiding the repeatability of particulars unnecessary and it provides a justification of the unrepeatability of particulars.

In addition, the process of becoming captures the perishing and the becoming. Contrary to a substratum theory, where bare particulars do not come into and go out of existence, and therefore substances being indestructible survive change, here a concrete particular is not prior to and independent of its tropes e.g. as a bare particular, but it is completely defined in terms of matter and field tropes, for example, in the Higgs mechanism, the Higgs field couples with different vector field before and after the symmetry breaking hence what counts for a concrete particular depends on what matter and gauge fields couple (section 4.2.2.1). Employing the terminology ‘perishing and becoming’, after the symmetry breaking, the old vector field has perished because it does not couple.

5.6 CONCLUSION

It was shown that mind does not subdue matter and the interaction between matter (which would play the role of substance) and the field (which plays the role of a global entity) entails a particularization which is understood at the level of process and not of presupposed particulars so that the elimination of substratum and the substance idea is justifiable.

6 A DISPOSITIONAL PROCESS ACCOUNT OF BECOMING CAPTURED SHEAF THEORETICALLY

Abstract

Based on the analysis of abstractness and determinacy in combination with the pair (matter, d^i), this chapter provides a process theoretic account of modality where a process of becoming is implemented via determinable nested dispositions. A mereological sheaf theoretic account that implements a bidirectional connection between the actual potentialities and the concrete actualities is applied, so that the two categories are not treated separately. As a consequence, determinable dispositions which play the role of potentialities are not prior to concrete actual manifestations, rather what is fundamental is the process of becoming which bridges the potentialities and the actualities. It is shown that the treatment of quantum operators as well as of interacting field systems in a sheaf cohomological theoretic way results in the distinction of many generative levels, where each level is seen as a

manifestation with respect to a lower degree level and as a disposition with respect to a higher degree level.

6.1 PRELIMINARIES

In this chapter, we will offer an account of modality in terms of process of becoming. In the literature on modality, there have been discussions regarding how the concrete and abstract characterizations apply to actual world and possibilities (or possible worlds). For most authors there is plenty of non-concreteness about the actual, e.g. Kripke (1980) takes possible worlds to be abstract states characterized as either actual (e.g. an abstract state where the actual me does not write this essay) or non-actual (e.g. an abstract state that involves a non-actual individual), for Lewis (2001), there is much more concreteness in the possible than modal non realists think, where these possibilities are non-actual (excluding the actualized possibility) and for ersatz modal realism, there is only one world, the actual one and countless abstract actual representations which are not worlds rather ersatz worlds, one of which is actualized although all are included in the one concrete actual world.[69]

We will agree with most authors that there are actual abstract potentialities, where these potentialities are treated as dispositions and not as possible worlds in our scenario. We will then focus on one world, the actual one and on actual abstract potentialities, one of which is actualized. But unlike ersatz modal realism, we think that not all abstract objects are actual, namely there can be properties that are strange, far from being actual.

[69] See exposition of main positions in (Menzel, 2015).

Hence, we understand the structure of the world as incorporating both an abstract domain of actual potentialities and of non-actual possibilities and a domain of concrete actualities. Still we will focus on the interrelation between the actual potential and the actual concrete, where this interrelation treated sheaf theoretically is the process of actualization of the actual potentiality. Sheaf theory can provide an actualist framework where all potentialities are actual albeit not all actualized. This actualist attitude is justified by the fact that in a sheaf theoretic framework, there are overlapping topological spaces, therefore there is no such complete separation which is presupposed in an ontology of unconnected possible worlds. Since topological spaces will be related, they will be considered as becoming part of the actual world.

The range of actual potentialities will be captured in terms of determinable dispositions. Although some authors would react that there is nothing especially potential about a determinable apart from the obvious point that since a determinable is a family of mutually exclusive properties, a single object having one of its determinates renders impossible that object having a different one of its determinates, still we showed that the characterization of determinable was also used to express mutually implicative properties. It has been argued that determinates associated with the same determinable (e.g. negative and positive charge are associated with the property charge) cannot overlap whereas determinates that are associated with different determinables e.g. mass and charge can overlap (Lewis, 2001, pp. 154–155). However, according to our framework, due to resemblance between properties associated with overlapping spaces, even determinate properties such as energy and momentum associated with the same determinable property can overlap. This scenario presupposes that properties are given as functions of regions and not of points.

Definition (T_1) in 5.3 will then be understood as follows in this chapter:

(T_2) A determinable quantity is an actual abstract potentiality standing in a bidirectional whole-part relation with the determinate concrete actual manifestation.

The introduction of propensity in analysis can be justified as follows. As mentioned, decoherence is a function of history outcomes, where histories are characterized by some degree of refinement. Although many histories are accommodated in the structure of the world (even if histories concern different manifolds (sheaves are employed on manifolds rather than region of a topological space), these histories can overlap so no world is completely isolated and inaccessible), this does not imply that every outcome state in a superposition is actual rather one branch can be taken to be actualized in the actual world. This contrasts MWI, where all the different branches are actualized since every possibility is actual and everything that is probable to happen happens in some world. The fact that in our framework there are two directions – from the finer (quantum) to the coarser (classical) and vice versa – is an indication that not all branches are actualized, in the sense that the classical manifested reality confines the infinite number of histories to the ones that are manifested in the actual world. The range of possible outcomes is restrained by what is actualized which is consistent with what we experience in a macro-level. In such a context, probability will be interpreted as propensity.[70] An entangled state infers potential existence e.g. Schrödinger's cat

[70] See decoherence as the transformation of quantum probabilities to classical ones; due to different levels of refinement, the classical probability is expected to emerge out of coarsening the scale of resolution so that quantum observable behaviour is ultimately described by macroscopic observable behaviour (Epperson and Zafiris, 2013, pp. 335–345). A measurement outcome will be predicted probabilistically with respect to some local context so that the system has classical probabilistic description based on history of relations rather than outcomes at certain times. See a history version of quantum theory in (Isham, 1996), which takes probabilities of

exists potentially and not actually as a superposition of live and dead states and actualization will be interpreted as manifestation of a disposition. Measurement will be interpreted as any other physical processes in terms of dispositions, an interaction between quantum entities and a relation between different scales.

In particular, since a bidirectional relation between determinates and determinables is established, neither (determinable) potentialities nor (determinate) actualities privilege fundamentality (see next section). Hence, the notion of potentiality is here different from disposition, in the way the latter usually appears in discussions, because dispositions are not understood as essential. This outcome contrasts with the main dispositional accounts which take dispositions as basic. There are three points about why the notion of disposition is employed and how its non-fundamentality appears. Firstly, what is fundamental is the structure of becoming which incorporates dispositions (viz. actual potentialities), non-actual possibilities and concrete actualities. Secondly, simply evoking the distinction between determinables and determinates for describing the structure of the world is not sufficient, because a notion (viz. disposition) that embodies potency or power is needed to justify the becoming of a concrete actuality. This opposes philosophers such as Wilson, who although she involves both determinates and determinables in her view; her ontology describes what exists, whereas we describe the structure of the world in terms of becoming and explain how the relation between the determinate and determinable is established. Thirdly, although dispositions are taken to impose restrictions in the sense that they function as the causes of manifestations/concrete actualities and therefore, what is manifested is restricted by

history propositions. The additivity criterion of probabilities applies for consistent sets of histories where there is no quantum interference; in that case, the probabilities for coarser-grained history is the sum of the finer-grained histories.

the relevant potencies, it will be shown in later section that in a scheme of nested dispositions, dispositions themselves are subjected to constraints and in particular they are restrained by the concrete actuality. In that respect, dispositions should not be regarded as fundamental. An advantage of our view is that due to the assumption that dispositions are not treated independently of the manifestation, dispositions are manifested continuously, so that there is no such a thing as an eternally unmanifested disposition. Consequently, although the being of a potency is found in its potentialities, dispositions gain reality for they do not stand separately from the manifestation. If dispositions were fundamental on the grounds that the power which is responsible for an actualization precedes the manifestation or the occurrence of the manifestation is dependent on the presence of powers and not vice versa, then there would appear the risk that a disposition is eternally unmanifested.

The structure of the chapter is the following. Section 6.2 shows that the pair (matter, d^i) is endowed with dispositional status in a sheaf theoretic framework. Section 6.3 expands the outcome to more levels; since a cohomology has many generative levels, likewise one can distinguish degrees of determinable nested dispositions and determinate manifestations, where the term ‘nested’ indicates that a manifestation can bring forth a new disposition.

6.2 (MATTER, d^i) PAIR BEING DISPOSITIONAL

6.2.1 PRELIMINARIES – POTENTIAL CANDIDATES FOR DISPOSITIONS

One first matter in a dispositional account is that one needs to start with some fundamental properties that will be assigned with dispositional status. The problem is which these fundamental properties are.

One way to determine the properties is to start with a statement e.g. law and its components can be interpreted dispositionally. For example, the inertial mass can be taken as the disposition of accelerating at F/m rate (manifestation) under the exertion of force F (stimulus) (Bird, 2007, p. 100). However, a statement can afford more than one interpretations. In the above example, force F can be regarded *as* the disposition that produces acceleration F/m (manifestation) of a mass m (stimulus).

One way to deal with that is to consider only fixed properties such as colour, mass and electric charge as dispositional. But even in that case, the overdetermination phenomenon appears again. In classical physics, mass appears in Newton's second law of motion and Newton's law of universal gravitation, hence, there are two kinds of masses and therefore, two dispositions, one inertial and one gravitational, which happen to be equal (Bird, ibid., p. 215).[71] One mass m is the disposition of the manifestation

[71] The equivalence of gravitational and inertial mass, acting as an important heuristic impetus for Einstein, is reconciled in his principle of equivalence. Due to the equality between the two masses, the gravitational force experienced locally by an object accelerating towards the Earth at a rate g(=9.81m/s^2) is the same as the acceleration of an inertially moving body observed from an accelerated frame of reference in free space at a rate g. From that, Einstein deduced that free fall is actually inertial (non-accelerated) motion. Although the

M_1 (acceleration at F/m rate) under stimulus S_1 (force F) and the other one is the disposition of the manifestation M_2 (acceleration at Gm/r^2 rate) under stimulus S_2 (mass m') (observe that in the latter case, mass appears also in the stimulus, which means that the dispositionalist distinction between stimulus property and dispositional property is undermined and there appears overdetermination once more). But there is nothing that would imply that one of them will be manifested, rather both dispositions corresponding to two physical quantities having the same value will produce manifestations. In order to avoid such overdetermination issues, either mass should not be considered as fundamental dispositional property or it should have another character such as being categorical, that is, either mass is derivative out of some fundamental disposition or it should be regarded as categorical (not derived dispositionally).

One objection to the above argumentation and eventually the appearance of overdetermination is that the law was presupposed in the dispositional analysis, that is, one analyzes a disposition having already the law in hand. However, a dispositionalist's explanation should not be based on laws since the main aim of metaphysically prioritizing dispositions is the discovery of new laws and symmetries. After a series of mathematical and physical manipulations, a dispositional property gives rise to a law. It is these calculations that should be involved in the analysis of what accounts for dispositions. What kind of dispositional structure will lead to the appearance of laws is of course a matter of investigation. The point is that for a dispositionalist who takes dispositions to be fundamental, laws are not posited, hence Newton's second law and his law of universal gravitation will be re-evaluated in the new framework where mass may not be a fundamental disposition.

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equivalence between inertial and gravitational mass is reconciled in GTR, the problem of mass arises in the relativistic context, where mass appears as a conserved quantity or as an invariant quantity.

Another approach where laws are not involved in the explanation appears in Morganti's (2009) trope analysis, which is based on the observed properties of particles as presented in the Standard Model. Colour, mass and electric charge constitute categorical properties of particles while spin, which has contingent direction, constitutes dispositional property, where a dispositional trope produces measurement results with some probability. For Morganti, the basic tropes involve both fixed categorical tropes as well as dispositional tropes, hence not all properties are produced dispositionally.[72]

However, such an analysis can become problematic if one considers other factors such as the frame dependence of a physical quantity. The mass that Morganti considers as categorical is the rest mass or rest energy which remains invariant in all frames of reference, but is not conserved. There is also the relativistic mass or total energy which is not invariant in all frames of reference, but is conserved. There are then two options that one can follow in order to deal with the two kinds of masses. One can claim that the only candidate for mass is the invariant (constant) mass while the energy should only be a conserved quantity (this is a claim made by Weinberg (1972, p. 33)), hence mass and energy are treated separately. This means that one can take the invariant mass to be a categorical property, as Morganti does, while the total energy which is not fixed can be explained dispositionally. Alternatively, one advocates the equivalence between mass and energy hence mass or energy appears in two forms, relativistic mass and rest mass. As earlier, the overdetermination due to the appearance of two kinds of masses as possible candidates for the basic disposition could be resolved if mass is not taken as a basic property but as a derivative of some dispositional framework.

[72] There are also scenarios such as Simons' (1994) who takes for example only the spin magnitude as basic due to its certain character and the spin direction as non-basic due to its contingency.

The idea that not only the spin, but also mass can be taken dispositionally generated by some fundamental disposition can motivate a scenario which start only with dispositional properties (and exclude categorical properties in the fundamental properties). In a constructivist approach, it can be argued that all properties are constructed out of sequences of overlapping parts and not presupposed as constants; constancy is translated as a stabilization of the sequence to a certain outcome.

In our approach that will be discussed in the next section, the pair (ψ, d^i) will be endowed with dispositional status. There we will meet again the phenomenon of overdetermination. For the moment, with respect to mass, the Higgs field, constituted by four massless bosons, along with a vector field (potential) could be regarded as the disposition to generate massive bosons and a massless photon. But the mass is not dispositional, because it is not the mass which has the potency to generate massive particles. In addition, it would also be interesting to research how the two kinds of masses – conserved and invariant - could be treated. Section 7.3.2 refers to symmetries, laws and dispositions and in particular it refers to gauge invariance and conservation which are interrelated via Noether's variational problem. So if the invariant mass was constructed in some way out of mereological series, the gauge invariance condition that characterizes the dispositions/manifestations scheme could be correlated with the conservation of mass/energy and therefore the conserved mass.

6.2.2 (MATTER, d^i) DISPOSITION

In sheaf theoretic framework, one could locate the following three different candidates for the role of dispositions: a) the d^i quantity, b) the matter ψ and c) the pairing between matter and the d^i quantity with no involvement of stimulus. We will give arguments for the c) option, examining the case that d^i expresses a quantum operator and the case that d^i expresses the covariant derivative.

d^i as a quantum operator

The d^i quantity can be a good candidate for disposition since, being associated with a differential, it indeed provides changes as dispositions do. For example, the Hamiltonian would be taken as the disposition that generates evolution; the energy expressed in the Hamiltonian which is first degree differential would produce manifested quantity expressed in $\Omega^1(\psi(F(U))$. However, not only the d^i quantity can play the role of disposition, but matter as well; the wave function is a disposition that produces measurement results with some probability. If both matter and the d^i quantity can then play dispositional role, this brings us to the third option which can be taken as more appropriate considering that the stimulus is characterized by degrees.

In particular, the argument is the following. Firstly, d^i appears with degrees expressed in the different eigenvalues that the respective operator can yield when it combines with matter. In general, there is an infinite set of eigenvalues, for example, the position and momentum operators can yield infinite eigenvalues and the Hamiltonian operator can yield an infinite number of (continuous or discrete) energy eigenvalues. Likewise, ψ is taken either by summation or integration of eigenstates. Therefore, if the properties expressed in d^i play the role of disposition and ψ plays the role of stimulus and considering the general case of infinite set of eigenvalues, the manifestations and stimuli are infinite and the relevant tracks are infinite in number as well. The disposition of infinite tracks is multi-track in comparison to the single-track disposition of a single manifestation and single stimulus.

Multi-track dispositions appear when there exist more than one manifestation or/and stimulus in comparison to single-track dispositions of a single manifestation and single stimulus (Bird, op. cit., pp. 21-24). As Bird argues, impure dispositions (e.g. with a disjunction of manifestations ($M_1 \vee M_2 \vee M_3 \vee ...$) and a disjunction of corresponding stimuli ($S_1 \vee S_2 \vee S_3 \vee ...$)) cannot be considered as fundamental, for there should be an explanation why and if a certain track is preferred. The infinitely many tracks can be handled, if dispositions are treated in terms of restricted possibility instead of counterfactuals, where the restriction of the infinitely many tracks is provided by what is actual or manifested.[73] A state of affairs is then possible if it is the manifestation of an actual disposition, that is, there should be a limitation of dispositions to this world, so that there is no need to assume an infinity of possible worlds.

[73] This resolution appears in Vetter's (2014) actualist view that the stimulus can be removed from a dispositional account if dispositions are treated in terms of graded restricted possibility. Like us, Vetter belongs to those philosophers who argue that dispositions can be determinables and not determinates. Her argument is that apart from dispositions (e.g. fragility) which come with degrees, stimulus e.g. the exertion of a mechanical force is determinable and is related to a continuous scale of determinates which are ordered by a 'greater than' relation. Disposition is then multi-track, where the tracks are infinite in number. Hence, she concludes that the stimulus should not be handled in terms of counterfactual conditionals but in connection to graded restricted possibility while the infinitely many tracks will be confined by what is actual. In other words, a disposition is individuated merely by its manifestation. Similar claims have been made by Martin (2010), Borghini and Williams (2008), who give a modal actualist account of possibility where possibility is determined by dispositions found in the actual world, so that modality is not understood in the context of possible worlds. Hence, in such views, the reality of potency is grounded on the reality of (actual) manifestation conditions, so eternally unactualized possibilities are impossibilities.

This is a different approach from the standard model of dispositions, where modality is understood in terms of counterfactuals and a disposition is individuated by its stimulus and its manifestation. If the stimulus ceases to be external, it is internalized. The pair (ψ, d^i) has dispositional status in the sense that it is endowed with possibilities restricted by the actual manifestation (e.g. it cannot be the wave function of a paraparticle), with no reference to an external stimulus. For example, the wave function and the Hamiltonian are not taken separately, rather their pairing (ψ, d^0) is dispositional giving the manifested quantity expressed by the respective node $\Omega^1(\psi(F(U)))$. The above suggestion has also the advantage of justifying the actuality of abstract possibilities. The limitation of the range of possibilities by the actual manifestations endows potency with reality (and in addition, ensures that a potency cannot remain confined forever, but at some point, there will be actualization; since powers do not exist independently of their manifestations, but the occurrence of the manifestation is dependent on the presence of powers and vice versa).

d^i as the covariant derivative

In the mind-body dualism, disposition is frequently related to the mental. The mental has the power or inclination to something which is manifested in the bodily reaction. As mentioned, with respect to GTR, the prevalence of the mind over the body appears in a left-to-right reading of EFE. Introducing dispositions, this can be interpreted as the properties of spacetime (or of the gravitational field) being dispositional and causing changes in the kinetic properties of matter.[74] More precisely, matter is the stimulus, the structural properties of spacetime described by the metric are to be regarded as dispositional, and the acceleration of the particle is the manifestation. However, if the mental and the physical are treated equally, then both the mind and the body can be endowed with dispositional status. A bidirectional reading of EFE means that apart from spacetime being dispositional, matter causes the modification of the dynamical properties of spacetime. That is, matter is dispositional, the structural properties of spacetime points are the stimulus and the curvature of the gravitational field-spacetime is the manifestation. This bidirectionality can be translated as the mental under the presence of the physical causes some physical manifestation and the physical under the presence of the mental causes some mental manifestation.

That the dispositionalist distinction between properties having some determinate character (viz. either disposition or manifestation) is undermined can be linked to Bird's argument on action-reaction and dispositions. The spacetime of GTR is the example Bird uses to support his claim that not only should a property be taken as dispositional but as a manifestation as well so that it both causes and is subjected to changes: 'According to the action-reaction principle, something is a potential cause only if it is a potential effect also. [..] In dispositional essentialist terms, we can see that by being potential manifestations of dispositional essences, spatial and temporal properties may also have dispositional essences themselves.' (op. cit., p. 166). So, a dispositionalist can account for the notable feature of spacetime that is, it (spacetime) is subjected to change, and is in turn responsible for change. Bird (ibid., pp. 164-166) argues that in physical theories, which assume that spacetime is a dynamic entity acting and being acted upon

[74] Historically, for Aristotle a thing has dunamis (=*δύναμις* in Greek) or the potency to produce movement or kinesis (=*κίνησις* in Greek) (Cohen, 2014, sec. 12).

and not a static background, spatiotemporal properties can be taken as dispositional having causal power. He basically contrasts GTR spacetime which can have causal powers with classical substantival spacetime which being unchanging cannot be seen either as effect or as a cause. Although contrary to Bird, we claim that the metrical properties are taken either as stimulus or disposition, while the curvature is the manifested quantity, the main motivation is the same.

However, the appearance of two alternatives for disposition and the undermining of the dispositionalist distinction between stimulus and disposition entails overdetermination. Such kind of overdetermination is handled if one takes the pair (matter, potential). Instead of matter and gauge fields being regarded as both dispositions and stimuli, it is claimed that one can take the pair matter and potential as dispositional.[75] Matter is not taken separately from the potential which in this case represents the continuum or the gravitational field. In other words, the two dispositional cases are reduced to one, by eliminating the external stimulus and by considering the pair matter/potential in dispositional terms. In this way, not only are the mental and the physical treated equally but also inseparably.

In addition, the pair matter/potential endowed with dispositional status can also be supported by the appearance of multi-track disposition. Both matter $\psi'(x)=e^{i\theta(x)}\psi(x)$ and potential $A_\mu'(x)=A_\mu(x)-\nabla\theta(x)$ are subjected to symmetry transformations which give an infinite range of possible changes of the phase $\theta(x)$. Therefore if the one e.g. the

[75] Note that it is matter and not some certain property e.g. mass that appeared as disposition and as stimulus in our dispositional interpretation which was based on Einstein's equations. This evokes natural kinds in the dispositional claim, where natural kinds concern a web of relations (matter and potential are relations while their interaction can be understood trope theoretically in terms of compresence and resemblance relations). For the attribution of dispositional essences to natural kinds (which can be particles, chemical elements, galaxies, interactions, fields etc.), see (Bigelow et al., 1992).

potential plays the role disposition and the other e.g. the matter field plays the role of stimulus, the disposition is multi-track, since there is an infinite number of dispositions and stimuli. The appearance of multi-track disposition can be avoided, if the pairing matter/potential has dispositional essence, because it incorporates certain possibilities, while the stimulus as something external to the disposition can be eliminated. The range of possibilities is restricted to actual possibilities, since unrealizable possibilities are impossibilities while the range of actual possibilities is captured the pair matter/potential.

Overdetermination

The subject of overdetermination will not be discussed in extent, but relevant issues will be noted. The variety of options for d^i implies a kind of overdetermination. Apart from the problem that d^i may refer to different operators and therefore, morphisms may represent different quantities that express change e.g. energy or momentum which both refer to first degree change (if energy is taken as derivative with respect to time), d^i may be used to refer to the covariant derivative. One then needs to show which is more fundamental, e.g. the coupling between momentum and matter, the coupling between energy and matter or the coupling between potential and matter.

One major assumption is the gauge invariance which was taken as a major precondition of observability. On those grounds, the covariant derivative plays major role. Considering that the gluing mechanism acts on any observational content, then energy or momentum outcomes would be operators expressed with respect to the potential. This is supported by the elimination of the metric from the background, hence the operators would not be defined with respect to some spacetime, but some substitute of it, in this case the gauge field.

We will not work more on the overdetermination, but it would be important to investigate in depth what should be involved in the pair (matter, morphism) and what the relation between operators and potential should be (e.g. such a relation appears in LQG as discussed in brief in the epilogue of the thesis as a suggestion of future research).

6.3 THE PROCESS OF BECOMING AND NESTED DISPOSITIONS

The previous section has shown that dispositions are captured in the pair matter and d^i quantity. This section will show that these dispositions are nested, where a manifestation brings about new dispositions. As mentioned, the relation between potentialities and actualities can be explained in terms of nested dispositions. Generally speaking, in a scheme of nested dispositions, a manifestation of a disposition functions as a disposition for subsequent manifestations. The general idea is that new dispositions can appear along with the manifestation of previous dispositions and a chain of dispositions $\{A \rightarrow B \rightarrow C \rightarrow ...\}$ is formed, where B is the manifestation of A, C the manifestation of B etc. For example, Thompson (2011) shows that 'a structure of multiple generative levels' is produced out of nested dispositions (e.g. potential energy $V(x)$ is the disposition that produces force $F=-dV/dx$ in the presence of a test particle and subsequently, force F is the derivative disposition that produces acceleration F/m of a mass m).

In a chain of sheaves, the nestedness of dispositions appears in the following three ways:

a) Ordering relation between sheaves of different degree.

A chain of sheaves exhibits many generative levels expressed by the degree of the sheaf hence, the cohomological scheme of dispositions resembles the scheme of nested dispositions. The de Rham complex

$$0 \to \mathbb{R} \xrightarrow{id} \psi(\mathrm{U}) \xrightarrow{d^0} \Omega^1(\psi(\mathrm{U})) \xrightarrow{d^1} \Omega^2(\psi(\mathrm{U})) \to \ldots \to \Omega^n(\psi(\mathrm{U})) \xrightarrow{d^n} \Omega^{n+1}(\psi(\mathrm{U})) \to \ldots$$

shows that $\Omega^1(\psi)$ (or better sections in $\Omega^1(\psi)$) captures the manifestations of (ψ, d^0), $\Omega^2(\psi)$ captures the manifestations of $(\Omega^1(\psi), d^1)$ etc. As a result, a chain of multiple dispositional levels is produced, where each node in the chain should be seen both as a manifestation and as a disposition. The k^{th} node expresses the manifestations of the $(k-1)^{th}$ disposition and also expresses the disposition of the manifestations in $(k+1)^{th}$ node, that is, the pair $(\Omega^k(\psi), d^k)$ is the disposition that brings forth manifestation in $\Omega^{k+1}(\psi)$. (As earlier, the $\Omega^k(\psi)$ and d^k were not taken separately and there was no external stimulus; taking either d^k or $\Omega^k(\psi)$ as possible candidates for dispositions can entail overdetermination of causes for the same manifestation in $\Omega^{k+1}(\psi)$ and in addition, the different possibilities captured in $\Omega^k(\psi)$ and d^k would involve multi-track dispositions.)

For example, the curvature R of the connection of matter ψ is the manifestation of the connection represented by $d^1 \circ d^0$ coupled with matter ψ, viz. $d^1 \circ d^0: \psi \rightarrow \Omega^2(\psi)$ and $R|_U \in \Omega^2(\psi(U))$.

b) Mereological relation between objects of category

The morphism d^k captures the change $\Omega^{k+1}(\psi(U_1)) \rightarrow \Omega^{k+1}(\psi(U_2))$, $U_2 \subseteq U_1$ where U_1 and U_2 subsets of some covering $\{U\}$. Hence, a new disposition $(\Omega^k(\psi(U_2)), d^k{}_2)$ appear along with the manifestation in $\Omega^{k+1}(\psi(U_2))$ of a previous disposition $(\Omega^k(\psi(U_1)), d^k{}_1)$. This is depicted in the following commutative diagram:

$$\begin{tikzcd}
\Omega^k(\psi(U_1)) \arrow[r, "d^k{}_1"] \arrow[d, "\varphi"'] & \Omega^{k+1}(\psi(U_1)) \arrow[d, "\varphi\otimes 1"] \\
\Omega^k(\psi(U_2)) \arrow[r, "d^k{}_2"'] & \Omega^{k+1}(\psi(U_2))
\end{tikzcd}$$

where $d^k{}_2 \circ \varphi = (\varphi\otimes 1) \circ d^k{}_1$. Hence, the change that the system is subjected to is taken by a mereological series of nested dispositions of some degree, where dispositions $(\Omega^k(\psi(U_{i+1})), d^k{}_{i+1})$ appear along with manifestations in $\Omega^{k+1}(\psi(U_{i+1}))$ of previous dispositions $(\Omega^k(\psi(U_i)), d^k{}_i)$.

Based on growing sequences of manifested quantities interrelated mereologically, one can then derive conclusions about how a determinable property changes, e.g. if the manifested property is its energy then how the energy changes appears in the form of sequences where the terms are related mereologically.

c) Ordering relation in a sieve

Likewise, the same diagram appears for $U_2 \to U_1$ where U_1 and U_2 subsets of different coverings, due to the association of the relevant commutative diagram of the Čech complex:

$$\begin{tikzcd}
C^k(\psi(U_1)) \arrow[r, "\delta^k_1"] \arrow[d, "\rho"'] & C^{k+1}(\psi(U_1)) \arrow[d, "\rho\otimes 1"] \\
C^k(\psi(U_2)) \arrow[r, "\delta^k_2"'] & C^{k+1}(\psi(U_2))
\end{tikzcd}$$

where $\delta^k_2 \circ \rho = (\rho\otimes 1) \circ \delta^k_1$ with the de Rham complex:

$$\begin{tikzcd}
\Omega^k(\psi(U_i)) \arrow[r, "d^k_i"] \arrow[d, "\tau"'] & \Omega^{k+1}(\psi(U_i)) \arrow[d, "\tau\otimes 1"] \\
C^k(\psi(U_i)) \arrow[r, "\delta^k_i"'] & C^{k+1}(\psi(U_i))
\end{tikzcd}$$

where $\delta^k_i \circ \tau = (\tau \otimes 1) \circ d^k_i$. Therefore, the manifestation in $\Omega^{k+1}(\psi(U_2))$ of disposition $(\Omega^k(\psi(U_1)), d^k)$ is accompanied with the appearance of a new dispositions $(\Omega^k(\psi(U_2)), d^k)$.

To close, the fact that manifestations have a concrete character whereas dispositions have a non-concrete one has the consequence that it is not dispositions all the way down. If in a scheme of nested dispositions, the manifestation is not distinguished by the disposition, then there is the risk that manifestations may not be observable due to this infinite regress.[76] But in our scheme, manifestations differ from dispositions since only manifestations are concrete determinate things, hence the infinite regress that appears in case all properties were dispositional disappears.

6.4 CONCLUSIVE REMARKS

On the basis of the sheaf framework, the nested dispositions offer a mereological dialogue between determinate manifestations and determinable dispositions. The range of possibilities appearing in the pair (matter, d^i) is restricted by the manifested properties just like the range of possible values of fragility is restricted by the manifestation if there is one. And just as subsequently a piece of a broken vase is also characterized by a fragility of some degree and therefore a disposition to break, manifestations in the quantum context can subsequently bring about new dispositions. In other words, it is the structure of becoming that treats inseparably the abstract and the concrete and assures a manifestation takes place.

This framework has the advantage that it can apply to all scales in the same way since a region can be of any size. So it is theoretically possible that there is experimental

[76] See pandispositionalism (McKitrick, 2003, 2010; Molnar, 2003, pp. 194–8), according to which all properties are dispositional.

accessibility to any scale. On the basis of the suggested framework, measurement is incorporated in such a way so that neither measurement is presupposed for taking determinate outcomes nor experimental data play just a verification role (one collects experimental data, arranges them into patterns, imposes relevant constraints (e.g. gauge invariance) and derives conclusions about whether the relevant structure e.g. law is verified). Metaphysically speaking, what actually occurs is expressed by nested dispositions so that measurement outcomes which are recorded manifested quantities capture the nested dispositional framework. That is, measurement outcomes are treated just like all manifestations. Therefore, although a measurement outcome itself is not dispositional and does not have any potency, just as manifestations, it plays a selective role on the disposition that will act.

7 A PROCESS ACCOUNT OF INTERNALIZING LAWS AS A WAY TO RECOVER GOVERNANCE

Abstract

This chapter provides a process theoretic account of laws where a process of becoming is implemented via determinable nested dispositions. By employing a mereological account that implements a bidirectionality between parts and whole, neither dispositions nor laws are taken as fundamental, rather what is fundamental is the structure of becoming. In this way, it is shown how Mumford's elimination of laws, on the grounds that they do not play governmental role, can be addressed. It is claimed that laws' governance can be recovered and that there is a kind of action-reaction relation between dispositions and laws.

7.1 INTRODUCTION

One objection to OSR is that it fails to give an account of causality due to the elimination of individual objects which are considered vital in causation. In order to include causality in ontology, French claimed that the structure is modal, where causal phenomena are explained in terms of necessity, possibility, and probability. He (2014, ch. 9, 10) regarded that modality is found on laws and symmetries and not on dispositional properties while other structuralists such as Esfeld and Chakravartty proceeded with incorporating dispositions in their structural approach, e.g. according to Esfeld's (2009) dispositional claim, since there are structures of entanglement and not individual objects, as well as metrical structures and not spacetime points with intrinsic identity, Esfeld argues that causality should be found at the level of the structure, so that just as one endows properties with causal powers, one needs to allow the same for structures.

However, concerns have been raised about how structures such as symmetries and laws can be treated in correlation to dispositions (e.g. (French op. cit., pp. 231, 264-265)). The aim of this chapter is to show how laws and dispositions can be parts of the same scheme. In particular, we will discuss about Mumford's argument on dispositions and the elimination of laws on the grounds that laws' governmental role is questioned, and we will see how the governmental status of laws can be recovered dispositionally (section 7.2). Then we will proceed with symmetries and dispositions and see how these two can be accommodated by the structure of becoming (section 7.3).

7.2 LAWS AND DISPOSITIONS

7.2.1 PRELIMINARIES

In literature, there are two main ways that one can describe the relation between laws and instances. According to the universal theory of laws, the relation between the law and the instances is summarized by the formula

$$(N(F, G) \Rightarrow \forall x(Fx \rightarrow Gx)) \qquad (1)$$

where the left hand side expresses that two universals F and G are related via relation N and the right hand side is a universal quantification over particular instances Fx and Gx. In a dispositional account of laws, one starts from particular instances for the derivation of law, that is, if some x has dispositional property Dx, then under stimulus Sx, there is manifestation Mx and on the basis of the universal quantification

$$\forall x((Dx \,\&\, Sx) \rightarrow Mx) \qquad (2)$$

the law is obtained, so that the conjuncts are more fundamental than the conjunction (Bird, 2007, pp. 21-24, 46). The law is internal or supervenient on the dispositional property in the sense that the dispositions yields the law. However, concerns were raised by Mumford about whether laws should be retained in both cases, viz. if laws are relations between universals or if laws are derivative from dispositional properties; his main argument is that laws in both cases do not play a governmental role.

This section aims to explore Mumford's argument, raise criticisms and show how governance of laws is recoverable. The subsections are structured as follows. Section 7.2.2 presents the two main views about what a law of nature is – the regularity theory and the universals theory. Section 7.2.3 presents Mumford's argument. Sections 7.2.4 and 7.2.5 introduce the new scheme that relates laws and dispositions and shows how laws can play governmental role. Finally, sections 7.2.6 and 7.2.7 provide extra commentary on the new scheme.

7.2.2 WHAT IS A LAW OF NATURE?

The Regularity Theory

There are different elaborations of regularity theory, but focusing on the naive regularity theory, the main idea is that a statement is a law if and only if the statement is i) universal, ii) contingent, iii) omnitemporally and omnispatially true and iv) is construed of logical connectives, quantifiers and non-local, empirical predicates (Armstrong, 1985, p. 12). Hence, the law that Fs are Gs is interpreted as all Fs are Gs, i.e. $\forall x(Fx \rightarrow Gx)$.

There are some problems that the theory has been criticized with. According to the argument from unrealized physical possibilities, there are physical possibilities which are never actualized, although they are not excluded by laws, e.g. evolutionary theory does not factor out the case that there could be white-feathered race of ravens living in snowy regions; although this is not the case, evolutionary theory can justify such a possibility. However, naive regularity theory would rule out possibilities as white-feathered ravens as unphysical. Something analogous appear in the argument from uninstantiated laws where they tell about cases non-existent. For example, Newton's First Law which refers to bodies which are not subjected to any force, although there are no inertial bodies. Or there can be functional laws of the form Q=f(P) i.e. a quantity Q is a function of quantity P, where there are values of P that are never instantiated, although the law refers to all values, instantiated and uninstantiated. For regularity theory, such cases/laws should be considered as non-existent.

Moreover, regularity theory would reject the idea of scientific laws applying locally. Statements are omnitemporal and omnispatial. If there was such locality, how would regularity theory distinguish between law statements from non-law ones? But there have been suggestions (apart from Whitehead) that want laws to apply in a local level. Of course, the existence of local laws is not a worry only for the regularity theory but also for the universals theory as well since laws are relations between universals. Armstrong (ibid., pp. 100-101) attempts to provide a resolution in the case of universal theory by introducing the notion of quasi-universal which is neither a universal, because it refers to a particular context e.g. a certain cosmic epoch or Smith's garden, nor a particular because it is repeatable. Either the F or G would then be a quasi-universal which would be related via the necessitation relation N with a universal. Of course, this is an intuitive resolution and needs to be explored. The point is that both the universals theory and the regularity theory need to find a generalization for local laws.

In addition, regarding the probabilistic laws, a regularity theorist would have to interpret the probability that F is a G as the frequency of Gs among Fs. The relative frequency of Gs among Fs is the frequency of Gs which tends to be stabilized to some number when a greater number of samples is taken. However, the problem that the regularity theory faces is that the frequency of a result in a distribution is expected to match the probability of the result under an infinite number of trials, which is impossible. But even if one assumes infinite collections, the probability should not be the limiting frequency of an infinite sequence (ibid., p. 31). The reason is that each event in the sequence is independent of all the other events in the sequence and therefore, any distribution is permissible. Even if some distributions are more probable than others, all distributions are equally possible. Therefore, a probabilistic law should not be identified by the limiting frequency. In general, the collection be finite or infinite, probability is something more than frequency of events.

The Universals Theory

Rather than starting with the collection of necessitation relation appearing in individual cases, the universal theory takes the necessitation relation between universals. For Armstrong (op. cit.), the statement 'all Fs are Gs' is a law of nature if there is contingent necessitation relation N between the predicates F and G and there is an entailment relation as described by (1). The contingency of the necessitation relation means that it can be the case that N and Fa hold but not Ga, for example at some point in time, F and G are not related in the same way, so that 'all Fs are Gs' does not hold anymore. In general, if F initially appears in the set of laws such as N(F, G), N(F, H), N(F, I) etc., it could be possible that at some point these laws do not exist, rather laws such as N(F, G*), N(F, H*), N(F, I*) etc. appear. In order to accommodate the case that F and G are related differently at some other time, Armstrong invokes possible worlds so that F and G will be related differently in different worlds but not in the same world. This resolution is justified on the grounds that it could not be the case that in the same world a necessitation relation between F and G can be different because this would contradict the main assumption of the universality-omnispatiotemporality of the relation N between the universals F and G.

The above can also extend to involve probabilistic laws although Armstrong analyzes the case of probabilistic laws in the same way he analyses the deterministic laws, with the difference that the necessitation relation N becomes a probability of necessitation (N:P), namely (N:P)(F, G) $\Rightarrow$ (Fx$\rightarrow$Gx) gives the probability F to be G. Although coarse-graining induced decoherence involves probabilities, we will discuss the general case of deterministic law for convenience reasons.

One main concern for N(F, G) is how a relation that is not necessary can necessitate a link between Fa and Ga in the world. Armstrong (op. cit., pp. 88-93) attempts to give a resolution by taking that the necessitation relation is primitive. In particular, he takes that N(F, G) can be seen both as a relation between universals and as a universal itself, hence the law would be instantiated in the same way as universals are. In particular, for a state of affairs Fa, F property is instantiated as a first order universal by a particular a. For the state of affairs N(F, G), F and G are second order particulars that instantiate a second order universal N. So a second order particular is a first order universal. In general, a universal of order n is also a particular of order n+1. A chain of instantiations can then be justified as follows: first order state of affairs instantiate first order universals which instantiate second order universals, hence first order state of affairs instantiate second order universals. But the problem is not resolved in this way. The law is considered both a second order universal and a state of affairs. So it is still possible that Fa holds but Ga does not and therefore the N universal is not instantiated.

As for uninstantiated laws, these are counterfactuals expressing what the case would be in certain unrealized situations. For Armstrong, uninstantiated universals do not exist, but if such universal existed, then the respective law would hold as well. The main point is that laws are abstractions of things and therefore, cannot exist independently of things. But the main objection to Armstrong is that a theory that accommodates uninstantiated universals is preferable. For example, in Lewis' modality, universals uninstantiated in our world are instantiated in another possible world. Or Kripke takes possible worlds to be abstract states characterized as either actual or non-actual.

As will be shown, in Mumford's argumentation, the main point of the argument has to do with the externality of laws to properties. The contingency of laws justifies why the laws are external to the properties they are supposed to govern. The externality of the nomic relation means that the relation N is not intrinsic to the relata-properties F and G or there is nothing intrinsic to F and G that renders N(F, G) a law, hence the law can vary independently of the properties, e.g. Newton's law is external to properties but because it is external, it could be different and apply to something different. So it may be the case that F and N holds but not G. If F was internally related to G, then there would be no need to invoke an external relation to get G, rather F would necessitate G. But Armstrong wants to retain the relation N in order to introduce other worlds. To do that, he introduces the relation N which is contingent, so that it is not always that F necessitates G. This is consistent with the assumption that for Armstrong all properties are categorical, which means that F is not dispositional and therefore does not have the potency to bring about G.

7.2.3 MUMFORD'S ARGUMENT

In his Central Dilemma (2004, chap. 9), Mumford summarizes his argument as follows: if laws have a governing role then either a) they are external to the related properties or b) they are internal to them. If option a) holds and laws stand independently of properties, then it is not clear how laws can exert governance. Mumford (ibid., pp. 93–94, 102–103) primarily here addresses Armstrong's view; although Armstrong assumes that the relation N expresses a nomic necessitation, still he claims that the instantiations of the F and G properties is contingent, therefore there is the possibility that the properties are not co-instantiated. Since the relation N does not necessarily hold between F and G, one cannot give an account of the law's governance based on the instantiation of the universals. In addition, the externality of laws has the disadvantage that it implies the metaphysically problematic notion of quiddity, in the sense that the identity of a property being independent of the law should be in virtue of quiddity. On the other hand, if option b) holds, then laws are internal to properties because they are reduced or supervenient to them, but then they do not play a governing role; if laws govern powerless properties, attributing properties with powers renders laws unnecessary. Since neither option a) nor option b) are appropriate, lawlessness is the other alternative. There is no need of laws since their existence would imply their possession of a governmental role which is either non-explicable or non-justifiable.

However, the following criticisms can be raised and will pave the discussion to our perspective.

Criticism 1 (to option b of Mumford's argument): The fundamentality of (determinate) dispositions is questionable

The internality of laws that Mumford presents is consistent with the standard idea of dispositionalism which takes dispositions as basic so that dispositions yield laws. On these grounds, Mumford is eliminativist with respect to laws. However, if dispositions were not fundamental then laws would not necessarily base their existence on dispositions. The non-fundamentality of dispositions could entail that they lose some of their power to define things and in that sense, the recovery of laws' governance could be justifiable.

But what reasons do we have to believe that dispositions are not fundamental? If determinate dispositions were prior then the physical concrete particulars (that has the disposition) would privilege priority over abstract possibilities. If then dispositions of concrete objects are prior to the abstract then non-actual abstract possibilities would not be accommodated by the structure of the world. In addition, if dispositions were fundamental, they would be granted independent existence from their manifestations. But then there is nothing to ensure the reality of the potency, for potentiality can remain mere potentiality. Some reality to the potency can be provided by the stimulus and manifestation conditions, but still even in the case the stimulus is real, it is not guaranteed that a disposition will be manifested even if a stimulus was present e.g. the fragility of the vase may not be manifested if the force exerted is less than some threshold. On the other hand, as discussed in chapter 6, the bidirectionality between dispositions and manifestations ensures that potency will be ultimately expressed.

So we will assume that laws' existence does not depend on dispositions, but we recover governance by employing dispositions. Espousing features of dispositionalism does not necessarily entail that one prioritizes dispositions over laws, so that one can retain the reality of laws independently of dispositions and in parallel, give a dispositional account for the recovery of the governmental status of laws. At the same time, as will be shown, neither are laws fundamental, but the general structure of becoming that brings laws and dispositions together.

Criticism 2 (to option b of Mumford's argument): The fundamentality of determinates is questionable

Armstrong's universal theory of laws is based on the assumption that there only exist fully determinate universals. But as mentioned, there are reasons to believe that there are determinables apart from determinates. To summarize the arguments from the previous chapters, the fixing of determinate properties with respect to a continuum is denied since, due to existence of singularities in the spacetime continuum, the continuum is constituted by true parts. In that case, two particulars associated with two overlapping parts may not have the same resembling determinate properties, but they may have the same determinable property. In addition, determinables cover a range of determinates greater than the one that is instantiated. So modal facts can be accommodated if determinables are treated with importance and moreover, with regard to laws, these refer to relations between determinables and therefore to all values, instantiated and uninstantiated. That is, for $Q=f(P)$ functional law, Q and P are determinables and this justifies the non-instantiation of some values of P.

In particular, we will explore a framework where there is a bidirectionality (dipolarity) between determinates and determinables, so none is fundamental. The process of becoming is defined as a mereological bidirectional relation between parts and whole which is transferred to a bidirectional interlink between determinates and determinables, where parts and whole are contextual and defined with respect to an atom of space(time). So, the inseparable treatment between laws and instances will be understood as a bidirectional mereological relation between parts and whole. Many philosophers do not hold that there is a mereological relation between laws and instances (e.g. Lewis) while in Armstrong's view, there is a mereological relation in the sense that the universal is part of its instances and the same can be said for trope theorists (e.g. Campbell), in the sense that the trope is particularized just like the concrete particular is. The difference in our model is that we start with mereological relation between regions which is consequently expressed in the relation between determinables and determinates as well as laws and instances.

Since determinables do not have priority over determinate instances and likewise, determinate instances do not have priority over determinables due to bidirectional relations, neither laws will have priority over determinate instances nor, as dispositionalists would claim, determinate instances will be fundamental. Hence, it will not be a contrast between a top-down approach from laws to instances and a bottom-up approach from dispositional (determinate) instances to laws rather the level of determinables and the level of determinates will be treated as a whole in terms of process. A formula that would describe our framework (although we will provide detailed formulas in later sections) is:

$$\text{laws} \underset{\leftarrow}{\rightarrow} \text{instances}$$

which will be an extension of the formula

$$\text{determinable dispositions} \overset{\rightarrow}{\leftarrow} \text{determinate manifestations} \qquad (3)$$

where determinable dispositions capture the range of abstract actual possibilities, one of which is actualized or becomes concrete. If then supervenience is one direction relation with the instances being fundamental, our framework is implemented by a bidirectionality between instances and laws which is an extension of the bidirectionality between determinates and determinables.

7.2.4 A NEW PERSPECTIVE OF LAWS 1/2: FROM WHOLE TO PARTS

One first issue is that the bidirectional link between whole and parts is associated with an ontology of relations. The parts/relata generate the whole/relations but the whole/relations is the one that becomes and is manifested in the parts. This account will be based on the analysis in the previous chapter which implements the framework of category theory and sheaf theory with no continuum spacetime in the background. The analysis will be divided into two parts; in this section, there is analysis of laws for the direction from whole to parts and in the next section, there is analysis of laws for the direction from parts to whole. Laws will be conceptualized by the topological theory of sheaves and cohomologies as described in the previous chapter.

Let us consider the category of open sets, the category of dispositions D of some degree k and the category of manifestations M of some degree k+1. How these last two categories will be defined mathematically (in terms of groups, Hilbert spaces etc.) was discussed in previous chapter, for the moment we define the categories of dispositions and manifestations without having a certain mathematical category in mind. Morphisms correlate objects that belong to the same or different category. In particular, open sets are related by the following ordering relation

$$U_3 \to U_2 \to U_1$$

where the direction of the link expresses a mereological ordering from the parts to the whole and there are the contravariant functors such that:

$$D\psi(U_1) \to D\psi(U_2) \to D\psi(U_3)$$

and

$$M\psi(U_1) \to M\psi(U_2) \to M\psi(U_3)$$

where ψ describes the state of a physical system defined with respect to some region U_i and M is the determinate property manifested with respect to some region U_i. The first formula expresses the mereological succession of dispositions and the second the mereological succession of manifestations characterized by their own region. The matter ψ refers to a system, e.g. a system of particles or a system of one particle, so the formula concerns different states of a physical system with respect to some region. One can expand the formulas to include interactions between systems which is not addressed in this chapter.

Dispositions and manifestations are related via:

$$
\begin{tikzcd}
D\psi(U_1) \arrow[r] \arrow[d] & M\psi(U_1) \arrow[d] \\
D\psi(U_2) \arrow[r] & M\psi(U_2)
\end{tikzcd}
$$

Diagram 7.1: Mereotopological relation between dispositions and manifestations

which is a simplified version of the diagrams in section 6.3. The pair (D, $\psi(U_1)$) is the determinable disposition that brings about new manifestation.

$M\psi(U_2)$ is defined with respect to some region U_2 which stands in part whole relation with region U_1 while new disposition $D\psi(U_2)$ appears along with the manifestation $M\psi(U_2)$. There appears nested dispositions in the sense that the manifested state of the system is associated with new dispositions for the generation of subsequent manifestations. In this generative sequence of new dispositions, determinate quantities are derived while the range of possibilities in dispositions is restrained by manifested quantities of previous dispositions. Actually we exploited this restrictive role that manifestations play in order to exclude the stimulus from the formula. As claimed, since both determinable dispositions and stimulus appears with degrees, the manifestations and stimuli are infinite and the relevant tracks are infinite as well. The infinitely many tracks can be dealt with, if dispositions are treated in terms of restricted possibility and not in terms of counterfactuals where a stimulus is distinguished. Then the restriction of the infinitely many tracks is provided by what is actual or manifested.

Coming to the laws, the law is expected to be $N(D\psi(U_i) \rightarrow M\psi(U_j))$. In this framework, both dispositions and manifestations are morphisms and the matter is an object in a category described by a morphism. So the law describes the relation between different structures, the dispositional structure and the manifested one. Contrasting this to regularity theory, taking D to be determinable entails that the law covers a great range of possible manifestations. A disposition is understood as abstract potentiality which will become concrete manifested relation upon actualization. Based only on manifestations, the law cannot be recovered, since the law can employ in more cases than the ones manifested.

If the law was internal to D and M, then a different disposition D* would mean that the law would be different. But then there would be no need to invoke the existence of laws. The disposition would suffice to govern its own manifestations. In fact, manifestations can give rise to many laws that conform to the manifestations but there is one law that also covers the unmanifested cases (Armstrong op. cit., p. 38).

What we would then need is a law that is external. Here, one can distinguish two cases. The first one is Armstrong's approach of a contingent law. The law would vary independently of $D\psi(U)$. This means that the law could be different and in that case it would apply to a different world that D* holds. But the problem is how it governs if it is external. As mentioned, Armstrong want to keep the relation N external and contingent so that he introduces possible worlds. However, this can avoided if one thinks in the following way. The law is something necessary and external and covers all possible cases even those that are not instantiated. This makes the structure modal. Due to the localization provided by the contextualization with respect to some region, it is the case that any of the following scenario N(D, M), N(D, M*) or N(D*, M) can hold. But this does not make the law contingent. The law is necessary and does not change. Its unchangeable character can be understood if one considers the parts to whole direction discussed in the next section. In addition, it will also be explained how the law governs, since its externality questions its governmental status.

7.2.5 A NEW PERSPECTIVE OF LAWS 2/2: FROM PARTS TO WHOLE

Both regularity theory and the universals theory are based on the employment of universals. The idea is that a law should express something that is repeatable. This also appears in Armstrong's introduction of quasi-universals for local account of laws; these universals should be repeatable. However, it is contentious whether repeatability should be a necessary condition for lawhood. Consider for example an ontology of tropes; it would be expected that the resemblance relation between tropes offers the universality of the law (see tropes and laws in (Fuhrmann, 1991)). In an ontology of tropes, laws expressing objectivity between instances would be captured by some resemblance class of tropes. The resemblance relation offers the direction many-to-one, that the regularity theory emphasizes. Many tropes resemble each other so that many particulars are linked to a universal (where the characterization 'universal' is used to notify the universality of the resembling property and not an abstract repeatable universal), while a concrete particular is constituted by compresent tropes, so that many universals are linked to an individual. If T is the domain of tropes, B is the domain of individuals or concrete particulars, there is a projection $p:T \rightarrow B$ to the common base for the compresent tropes and the mapping p^{-1} provides all the qualitons that resemble a universal.

Actually, as mentioned in a previous chapter, the localization to some spatial volume and the non-repeatability of properties motivates the use of tropes. Hence, a third case of laws that is based on tropes would be relevant to our approach. The following paragraphs present our approach. We will not work on tropes but one could associate the resemblance between manifestations with the resemblance between tropes.

Let us consider again the mereological ordering

$$M\psi(U_1)\rightarrow M\psi(U_2)\rightarrow M\psi(U_3)$$

and

$$res_{U_i,U_j}\colon M\psi(U_j)\rightarrow M\psi(U_i).$$

The reversed ordering shows that for a manifestation $s\in M\psi(U_1)$, there is a restriction $res_{U_i,U_j}(s)$ of the s from the greater to the smaller region. Since manifestations are ordered with respect to the ordering between regions, they are related via associativity, that is:

$$res_{U3,U2} \circ res_{U2,U1}=res_{U3,U1}$$

but not via commutativity, that is:

$$res_{U3,U2} \neq res_{U2,U3}$$

The resemblance relation between manifestations will be expressed at the overlapping of regions, namely:

$$res_{Uij,Ui}(s_i)=res_{Uij,Uj}(s_j)$$

where $s_i \in M\psi(U_i)$ is a section of $M\psi$ over U_i. This relation shows that manifestations s_i and s_j represented by sections have the same restriction at the overlapping.

In such a framework, a necessitation relation between manifestations is taken for all combinations of i, j, namely:

$$\forall i, j \; res_{U_{ij},U_i}(s_i)=res_{U_{ij},U_j}(s_j) \Rightarrow \exists \text{ unique } s \text{ in } M\psi(U) \text{ s.t. } \forall i \; res_{U_i,U}(s)=s_i \quad (4)$$

Contrasting this account to regularity theory, the above statement is i) not universal, ii) contingent, iii) contextual, hence not omnispatiotemporally true and iv) local, empirical data are combined to give a global outcome.

Diagram 7.1 and equation (4) are combined to give the following simplified formula for law:

$$D\psi(U_i) \quad \underset{\leftarrow i=j;j=l}{\rightarrow} \quad M\psi(U_j),\, U_j \subseteq U_i \rightarrow (M\psi(\cap_{\forall m} U_{jm}),\, U_{jm}= U_j \cap U_m) \qquad (5)$$

where the double arrow corresponds to the diagram and the single arrow to the equation above.

In the direction from parts to whole, the law derived, let us name it law_i, is recovered by the manifestations. The law_i demonstrates an internal character with regard to the determinate manifestations which determines the law_i. The law_i stands internally to the continuously growing sequence of overlapping parts. So the law_i is generated as the sequence develops. In other words, the law becomes. The law_i can be one of the scenario N(D, M), N(D, M*) or N(D*, M). For example, the D can represent any gauge field coupled with different candidates of matter field; the choice of the gauge field, matter field and the relevant spatiotemporal region defines the relevant epoch. A different D or M will entail a change in the law_i which governs D. Hence, there is a synchronization or covariation of D, M and law_i. Although the general law does not change, the becoming law_i changes so that a different D and M are linked to a different law_i. One can note here that although the general law expresses the relation between determinables, the law_i is derived out of determinate quantities, but this is justifiable due to the bidirectionality between determinates and determinables. Therefore, there is realist overtone; one still accepts the reality of laws independently of dispositions so that laws are not be diminished to human driven products, although one can access them via a scheme of nested dispositions so that the derived statements approximate the real laws. The unchangeable character of the law can be justified by the contingency of law_i; law_i should be changeable with respect to something that remains constant. It is via comparison that we come to know whether the derived law_i is a good approximation of the general law; hence the general law should not be contingent.

Consequently, the law encompasses all possible laws denoted law_i; it is law_i which is generated by the instances but the general law is not found in the manifestations as in regularity theory but stands externally to the structure of relations. Since the law's existence is not dependent on the manifestations, the N is not exhausted by the manifestation and therefore governance is retained.

In addition, it has been argued that if laws are supervenient on dispositions that governs and the existence of laws is retained, there appears an overdetermination due to governance of laws in addition to potencies and such a problem can be avoided if laws and dispositions do not govern the same thing (Bird 2006, p. 449). On the other hand, in our framework, the overdetermination is avoided because the law is not supervenient on dispositions, dispositions govern manifestations which determine the law_i and the law in return govern dispositions. Dispositions are the way they are in virtue of laws. So not only do dispositions impose restrictions (as commonly argued in the sense that they function as the causes of manifestations and therefore, what is manifested is restricted by the relevant potencies), but dispositions themselves are also subjected to constraints and in particular they are governed by the law. (Hence, neither laws nor dispositions should be regarded as fundamental.)

As a consequence and with regard to Mumford's (op. cit., p. 155) worry that one cannot evoke the governance of laws in a dispositional account in order to retain laws since something cannot govern something upon which it supervenes, here via an action-reaction relation, laws have an effect upon dispositions. So laws do not become impoverished by the potency of dispositions to determine, for dispositions are disentangled from total governance; both dispositions and laws are endowed with partial governance and none plays a central governing role. As a result, the recovery of a notion of governance justifies the retention of laws in the framework.

7.2.6 CONCRETENESS AND LAWHOOD

The aim of this section is to introduce an extra element in the criteria of governance, namely the law should share some concreteness with its governing concrete things. If universals and laws reside in a Platonic heaven, how can then an abstract law govern something that is concrete? A platonic heaven is too 'distant' to be approachable and laws should be brought closer to the concrete actuality.

Armstrong assumes that universals exist in their instances, that is, there do not exist uninstantiated universals. Likewise, laws are relations between universals which are themselves universals. Laws being universals exist in their instances, which are particular causal sequences. Therefore, he denies Platonism and in this way, although laws are abstractions out of concrete things, one can say that they gain some concreteness from the concrete things where they are found. As a consequence, although according to Mumford's argument, the law governs the properties and not concrete things as planets and chairs, the law's governance does not concern abstract universals residing in a Platonic heaven and therefore, it ultimately applies to something concrete.

In our approach, the significance with which concrete manifestations are treated means that laws can be traced, viz. generated by growing sequence of overlapping parts, and hence, brought down from the platonic heaven of abstraction. Although the law is abstract, law_i expresses the becoming or the internalization of the law so that the law_i does not reside in a Platonic heaven. One can still acknowledge the existence of laws in abstraction, but based on the process of becoming, laws do not stand any more separately from the concrete system of manifestations but laws just as everything else becomes or better becomes concrete. Hence, the law_i capturing the mereological relation between manifestations acquires concreteness while by not being exhausted by manifestations, the law also exhibits abstractness.

To close, extending Mumford's argument on governance, we then claim that laws should display a kind of concreteness in order to play a governmental role. An abstract law separate from concrete particulars, it is non-governing.

7.2.7 A UNIFYING PICTURE

Adopting a generation of abstract notions out of manifestations of dispositional properties, where manifested quantities are related mereologically justifies a constructivist attitude. We do not start with laws and derive properties; rather, we start with a series of nested dispositions, where manifestations feed subsequent dispositions upon which laws supervene. This generating aspect of our framework reflects a constructivist attitude where the construction is objective. One can contrast our approach with Brouwer's (1913, p. 55) one. Brouwer states that in order to organize causal sequences of phenomena (where phenomena are appearances or experiences of things and causal sequences of phenomena are repeatable sequences), counting and measuring play a vital role so that natural laws are treated by the relations between the outcomes of counting and measuring. So, according to Brouwer, nature's causality (regularity) is a construction of mind and not an objective characteristic of the world. On the other hand, our approach is basically not from measurements to laws, but from dispositions to laws and since manifestations, and therefore, measurement outcomes, are objective, the construction is not subjective as in Brouwer. Moreover, the dipolarity between the dispositions and manifestations has another consequence. If methodologically and metaphysically, one starts from the laws, like a structuralist, this implies human's accessibility to the law in itself; the law is a priori and based on intuition while experimental data merely play a verification role of a discovered law. What we showed is that one can internalize the laws via the process of becoming and have accessibility to the laws. In this way, measurement outcomes being themselves manifestations of dispositions are related to the physical laws metaphysically via a scheme of nested dispositions and do not just verify the law.

Furthermore, there is a unifying picture of laws. The reason lies in the common structure applying to all occurrences and expressed as dipolarity between dispositions and manifestations. So one can employ dispositions and still advocate unification. The existence of a unifying structure applying to all occurrences has also the consequence that there is no reason why measurement should not continue even beyond the Planck scale. An experimental inaccessibility of a scale should be translated as a limitation imposed by current scientific development (theories, devices) and not as a limitation imposed by the structure of the world, since all occurrences are treated in the same dispositional way.

However, since the law is treated with respect to some context and some pair matter/potential (or matter/quantum operator), it is not the universal law a fundamentalist would advocate.[77] For a fundamentalist, that the law applies in all locations and at all times presupposes a concept of space and time. If spacetime is removed and the world is a world of occurrences characterized by some context, then the relevant universal claim would be that the law applies to all occurrences. In our suggestion, one can expand from a law to a system of laws based on interactions

[77] One can contrast our view with Cartwright's (1999). Firstly, for Cartwright, a law does not govern the world, rather, there is a plurality of laws applying locally to patches of the world (ibid., pp. 28-33). Laws are *ceteris paribus* statements holding for a particular patch which is not subjected to any external interferences. Secondly, she takes a law to be the result of capacities and it is in virtue of these capacities that the law governs locally, where capacities are understood as determinable dispositions (ibid., p. 64). Thirdly, she accepts a form of scientific realism about theories; she explains that what she opposes is not realism but fundamentalism (ibid., pp. 23-28) (see also Hoefer's (2003) criticism). The reality of laws can be supported by the fact that there are elements in her thesis such as capacities which are ontological, stripped of any pragmatism (op. cit., pp. 23-28). At the same time, there is a constructivist attitude (see (Paul, 2002)). There is the human element since a statement counts as a law if one derives the same outcome under repetition of the same experiment. A series of repeated events in a particular region of spacetime gives what Cartwright calls a nomological machine which is isolated and shielded from external disturbances so that one can test the regularity of a situation, that is, whether the capacities of the objects are expressed repeatedly in the same environment (op. cit., ch. 3).

between matter fields, but such a system of laws would apply a nexus of occurrences mereologically related at some part of the universe.

One could entertain the thought of a law applying to the universe as whole, e.g. if the context is the entire manifold. However, such an implementation would presuppose a super-observer that can see the universe as a whole and in addition, this approach is problematic when we come to the level of measurement where we need a local controllable picture.

7.3 SYMMETRIES AND DISPOSITIONS

In the previous section, we discussed laws. This section continues the discussion referring to conservation principles and symmetries. Generally speaking, arguing for the determination of fundamental physical properties by symmetries is a rejection of dispositionalism, since dispositionalists deny that a dispositional property can be co-determined by non-dispositional essences. As Bird (2007, p. 214) expresses it: ‘Properties are already constrained by their own essences and so there is neither need nor opportunity for higher-order properties to direct which relations they can engage in. The dispositional essentialist ought to regard symmetry principles as pseudo-laws.’ So, constraints and conservation principles should be derivatives of dispositions and not an extra constraint imposed in advance (ibid., 10.3.2).

However, considering the success of modern physics which wants properties such as mass and spin to be specified via symmetry groups, a dispositionalist needs to make sense of the fundamental characterisation of properties in causal terms.

In the following paragraphs, it will be shown that the presented scheme of determinable dispositions can provide an account of symmetries. The tension between the two definitions of properties in terms of dispositions and in terms of symmetries is removed, because neither dispositions nor symmetries gain fundamentality.

7.3.1 SYMMETRIES AND ABSTRACT VERSUS CONCRETE

Any of the two options – priority of determinate dispositions (as a dispositionalist would claim) or priority of symmetries –appears insufficient with the main assumption that the structure is neither in re nor ante rem and the realizations of the mathematical structure are captured (section 4.3). Determinate concrete dispositions found in concrete particulars should give account of symmetries but in such an account the symmetries would be found in the concrete and the structure would not be ante rem. Alternatively, if one starts from the symmetries as French does, then the structure found in the subject lacks actualization in the objective world.

In a framework where neither the abstract nor the concrete are fundamental, one can introduce dispositions without presupposing objects having determinate dispositional properties which are essential. Dispositions as determinable potentialities (which we assume) are not fundamental, therefore they are not prior to symmetries. Dispositions themselves are specified in terms of symmetries, namely, the potential and the matter has some symmetry e.g. the potential has an SU(2) symmetry. The determinable character of disposition can be associated with the abstract character of the group symmetry while the coupling gives a manifestation.

In addition, dispositions, as the causes of manifestations, impose restrictions but in turn, they are subjected to constraints-symmetries. Not only are they specified by some symmetry, but the manifestations play a constraining role for dispositions. That is, the manifestation-representation restrains subsequent dispositions.

7.3.2 DISPOSITION FOR THE DERIVATION OF SYMMETRIES - EXAMPLES

Conservation laws and gauge invariance

Furthermore, other kinds of symmetries will be derivative of dispositions and this is justifiable since neither symmetries are taken as fundamental. (Actually, new symmetries characterized by the i^{th} degree of the respective pair ($\Omega^i(E)$, D^i) can appear.) The satisfaction of gauge invariance, viz. the invariance of the Lagrangian under local group transformations entails that conservation principles are also expected to be derivative considering that according to Noether's (1918) principle, both local and global symmetries of the action (taken by the integral of the Lagrangian) of a physical system are connected with conservation laws. In particular, it was shown by Brading and Brown (2000) that taking Noether's variational problem (the action does not change under a variation of the dependent or independent variables, i.e. $\delta S=0$) for gauge theories, whose action remains invariant under global and local gauge transformations, conserved quantities are derived. In the case of global gauge symmetries, as in Noether's first theorem (where symmetries are related to conserved currents), they derived conserved currents. Likewise, it was shown that under certain assumptions, local gauge symmetry along with the satisfaction of gauge field equations entail a conserved quantity.

So, theoretically speaking, more symmetries and conservation principles emanate from the matter/potential disposition.

Conservation of energy-momentum in GTR

Due to the satisfaction of gauge invariance, there is conservation of a quantity both locally and globally. Our scenario can then be proved valuable for the alleged non-conservation of energy-momentum in GTR. Contrary to what happens in Classical Mechanics where energy and momentum conservations constitute separate laws, in GTR, energy and momentum of the matter fields apart from the gravitational field are both carried by the energy-momentum tensor T^{ab}. It has been argued that the law of energy-momentum conservation in GTR is expressed as $T^{ab}{}_{;b}=0$. However, this has two limitations (Hoefer, 2000). Firstly, it excludes the gravitational field from the calculations. The justification for this was that the other forms of energy can be treated in terms of local interactions but this does not apply to the case of the non-localizable gravity. Secondly, the differential form does not behave in the same way as the integration by parts form. These two formulations should be equivalent if a formula would represent a conservation law. The reason is that the energy-momentum should be conserved both globally and locally, where local conservation is taken by integration over regions.[78] But $T^{ab}{}_{;b}=0$ cannot be written as an integral conservation law. Therefore, it is not a genuine conservation law, for it does not hold locally.

However, the equal treatment of differentiation and integration is contentious. The integration presupposes that there can be a separation into parts. But if there are overlapping parts (this is supported by the inclusion of the singularity in the extended manifold), then the derivation of a quantity that corresponds to the whole can be a more complicating procedure (considering that overlapping can entail that a quantity is calculated twice). The inconsistency between the differentiation and integration may be handled by approaches such as cohomology and sheaf theory which incorporate the element of differentiation and the notion of overlapping regions.

In addition, by taking an algebraic coupling between matter and the potential of the relevant field restricted to chunks of spacetime, one expects to attain localization of the field, including the gravitational one and the satisfaction of gauge invariance will ensure the conservation locally and globally. That gravity is non-localizable in GTR should be taken as a limitation of the theory, so that the lack of energy conservation should not be taken as a loss of energy but as an indication for a theory replaced by a new one. This is an objection to Hoefer (op. cit.) who concludes that the gravitational field does not possess energy and there is a genuine loss (or gain) of energy.

[78] In literature, Bird (2006, 10.3.2) suggests that conservation laws emanate from the dispositional essence of the property 'being a world (like ours?)' which is instantiated by our world (also see (Livanios, 2010)). But such a solution is ad hoc and being a macro property, the being-a-world applies only globally and does not justify local conservation.

Dispositions employed for symmetry recovery-Higgs mechanism

According to Esfeld (2009), structures just as properties are endowed with causal powers. Esfeld (2004) (as well as Dorato (2011)) assumes the case that the stimulus is absent from the characterization of dispositions and he interprets particle decay and spontaneous symmetry breaking dispositionally without the presence of stimulus. The actuality of potency found in concrete structures is used by Esfeld as a reason why dispositions can exist without stimuli; since dispositions are actual, there is no need for an actual stimulus that would grant them with actuality upon manifestation. However, the actuality of potency should not be taken as opposing a scenario of potentialities since potentialities can be actual, albeit non-concrete. In addition, if the claim is that a disposition should be concrete apart from actual, our suggestion is that one can consider the system (actual potentiality-concrete actuality) so that the system as a whole combines concreteness with abstractness in an inseparable way. Therefore, contrary to Esfeld, we assume that the notion of abstract potentiality and the notion of entanglement are strongly connected.

Moreover, excluding the fact that he considers that the structure of entanglement is concrete, one worry here is that the elimination of stimulus contradicts the main definition of dispositions in terms of stimulus and manifestation conditions (see French's seminar paper) and one should provide an exposition that justifies the removal of the stimulus. Esfeld's justification is contentious because the vital issue for someone who espouses dispositions is to give an account of symmetries, namely, in terms of dispositions, symmetries appear or are recovered and not the other way around, that is, there is a symmetry breaking and this signifies the presence of a disposition (section 7.3.2). For example according to Castellani (2003), the breaking of symmetry does not entail that there is no symmetry present, rather the state where the symmetry is broken is of lower symmetry compared to the state where the symmetry is not broken. In other words, there are layers of symmetry. Applying group theory, this is translated as the initial symmetry group being broken into one of its subgroups and the breaking can be described by the relevant relations between the initial symmetry group and its subgroup. For example, due to the variance of the vacuum state under the action of charges, Goldstone (massless) bosons appear due to the corresponding global spontaneous symmetry breaking. The Goldstone bosons can then disappear to give rise to gauge bosons that obtain mass while at the same time the symmetry is promoted to a local one. So, any symmetry is the outcome of a higher order symmetry that is broken. As Castellani explains, if there was an absolute symmetry (viz. invariance under all possible symmetry transformations), then there would be no differentiation and therefore no definite entity. Hence, a lower symmetry than the absolute one guarantees the existence of definite things.

We agree with Castellani that there is the derivation of new symmetries, but what Castellani does is to provide an explanation in terms of being (actual particles appears because new symmetries appear) whereas we provide an explanation in terms of becoming. For example, the Higgs mechanism is not interpreted as a mechanism where there is no external stimulus (although ultimately we eliminated the stimulus but for certain reasons as explained in the previous chapter), rather a mechanism where symmetries are recovered via gauge invariance, which in turn is related to the coupling between matter and potential. Breaking of symmetry means disunity between matter and potential and recovery of symmetry means pairing between matter and potential.

(With regard to our discussion on the relation between mind and body, breaking of symmetry would be associated with a disunity between subject and object, e.g. there is no such unity of subject and object anymore for Socrates, although the matter that remains behind participates in causal objective connections, whereas recovery of symmetries would be translated as unity between subject and object.)

7.4 CONCLUSIVE REMARKS

Based on a series of nested dispositions, a constructivist approach to laws and symmetries was presented. The laws and symmetries are endowed with reality, although one can generate them via a scheme of nested dispositions. With regard to laws, this entailed the recovery of governmental role of laws.

8 EPILOGUE

The end of the thesis has been reached. To complete our discussion, let us recapitulate our main steps and principal conclusions, and in light of them, suggest some future avenues of research.

In our Whiteheadian approach, we showed how alleged mutually exclusive categories can be brought under one common framework. We mostly focused on notions of determinates/determinables, parts/whole and potential/actual and we claimed that there is a symmetric relation expressed as dipolarity between such categories. This dipolarity was captured sheaf theoretically which basically implements the symmetric relation between parts and whole. Based on that relation, other relations such as the one between actual and potential are built.

Since the occurrence involves both the physical pole and the mental one, there are constraints imposed by the physical. There is the locality constraint which is the restriction imposed by the spatiotemporal segment. The actualization (of the field) is restricted by the spatiotemporal boundaries of the piece. In addition, gauge invariance imposes extra constraint; in terms then of extensional mereology, the world is made of spatiotemporal pieces overlapping to give bigger spatiotemporal parts while the set of observables which have the same restriction to some set give an equivalence class. In particular, gauge invariance was understood as a manifestation due to the coupling of the quantity represented by the morphism d^i (covariant derivative, quantum operator) with some matter field. The interaction between the physical and the mental was captured by the structure of the world described in terms of becoming or as a transition from determinable dispositions to determinate manifestations, where the notion of disposition was introduced because it embodies potency and can explain the becoming of an actuality/manifestation. This continuous transformation from abstract potentiality to concrete actuality, and vice versa, was correlated to a constructivist derivation of quantities out of sequences of overlapping parts, so nothing can be seen in isolation but instead as a part of a greater whole.

Furthermore, we explored the relation between determinable dispositions and laws. In agreement with dispositionalists, methodologically we claimed that one can start with nested dispositions in order to trace laws and derive symmetries/conservation principles. This approach contrasts structuralist views that make laws and symmetries fundamental so that causality is found on laws and symmetries and not on dispositional properties.

Although we wish that the suggested scheme - and in particular the dipolarity between potential and actual - will be proved to be beneficial in discussions in metaphysics of physics, we will here draw attention to two possible ways in which the current study might be expanded.

Firstly, the aim of future research is to explore the problem of time. On the basis of algebraic differentiation in cohomologies of sheaves, the recovery of change and time should be re-evaluated (one should here note that the unifying treatment of the Planck level and the macrocosm implies that change transcends all scales). In the sheaf algebraic framework, time in the background is eliminated, so that the zeroing of the Hamiltonian constraint in the Wheeler DeWitt Equation (WDW-E) should be reinterpreted. In the nested dispositional account, the Hamiltonian constraint should not be taken as a lack of change rather in terms of a process captured in dispositional terms, where the process is understood as becoming actualized or transforming from a disposition/potentiality to a manifested actuality. In such an interpretation, the Hamiltonian formulation could suggest a growing view of the universe.

In particular, it would be helpful, if the WDW-E and the constraints are discussed in the framework suggested in LQG (Ashtekar et al., 2003, 2006; Ashtekar and Lewandowski, 2004; Rovelli, 2004, 2010). LQG has elements in the theory that can be adaptable to a sheaf theoretic framework. Firstly, the theory applies Ashtekar variables which is the pair ($E^a{}_i$, $A^i{}_a$), where $A^i{}_a(x)$ is an SU(2) gauge field or the connection and $E^a{}_i$, i=1, 2, 3 is its complementary variable or its conjugate momentum, which means that the action of the operator E_i on the $\psi(A)$ is taken as the functional derivative of the $\psi(A)$:

$$E^a{}_i \, \psi(A) = -i \, \delta\psi(A)/\delta A^i{}_a$$

Secondly, the wave functional representing spin networks is a possible candidate for matter and the covariant derivative D_a would be the analogue of the morphism. The covariant derivative coupled with the conjugate momentum of the matter gives gauge invariance:

$$D_a \, E^a{}_i \, \psi(A) = 0$$

where $D_a E^a{}_i = \partial_a E^a{}_i + \epsilon_{ij}{}^k A^j{}_a E^a{}_k$. This entails that the functional $\psi(A)$ remains invariant under rotational SU(2) transformation. The two constraints,

the diffeomorphism: $F^i{}_{ab} E^b{}_i \psi(A) = 0$

where $F^i{}_{ab} = \partial_a A^i{}_b - \partial_b A^i{}_a + \epsilon^i{}_{jk} A^j{}_a A^k{}_b$ is the field strength tensor or the curvature of the gauge field $A^i{}_a$ and

the Hamiltonian constraint: $H^+ \psi(A)= -\epsilon_{ijk} F^k{}_{ab} E^a{}_i E^b{}_j \psi(A) = 0$

would subsequently be calculated on a sheaf cohomology framework in terms of morphisms of some degree.[79] As a consequence, in such an algebraic context where there is no continuous time in the background, change should be recovered out of ordering relations.

Another way that the current research can expand would involve the metaphysics of black holes. In chapter 5, we referred to singularities and sheaf theory, so a detailed explanation of the actual and the potential in correlation to black holes would provide a clearer picture of the ontology. One idea could involve the entropy of black holes; based on the calculation of the Lorentzian action and the constraints in the WDW-E, the next step could be the calculation of the entropy, and the relation between the action and the Hamiltonian for horizons of infinite area (e.g. acceleration horizons) and black hole horizons of finite area. Since there is no spacetime in the background, the dynamics and the associated energy will be defined with respect to morphisms that associate space regions. Based on the analysis in the thesis, the configuration space of the system can be defined as the set of objects Ob(Q) in category Q (e.g. a category of topological spaces,

[79] See Thiemann (1998a, 1998b) for a construction of the Hamiltonian.

posets, 3-dimensional manifolds etc.), where objects are connected with each other by morphisms or cobordisms e.g. a cobordism between two 3-dimensional regions Σ and Σ'.[80] One can expand the formalism and consider functors from the category of cobordisms to the category *A* of gauge connections on regions (e.g. A(Σ) is an object of *A*) and subsequently, associate the matter field E with the connection, i.e. E(A(Σ)) (one could note that things such as planets, particles, observers treated as path connecting spacetime points in relativistic spacetime, should be treated as connected regions).

According to the above analysis, there is an ordering relation between Σ and Σ' which is transferred to an ordering relation between E(A(Σ)) and E(A(Σ')). E(A(Σ')) is subsequently related via another morphism with E(A(Σ'')) etc., so that a growing series of related regions is developed, although the system should not be taken as if it evolved in time from an initial state at t_0 to a final one at time t_1. The entropy will then be studied with respect to the ordering relation between regions and the ordering relation between sections of sheaves.

To close this thesis, there is more work to be done on metaphysics and the structure of the world in correlation to modern physics. But we believe that some interesting points were extracted, and we hope both that sheaf theory as a framework of investigation will gain more attention from philosophers of physics and that based on the notion of dipolarity, more insights will be induced.

[80] E.g. see (Atiyah, 1988) which offers a reconciliation between a zero Hamiltonian and change via the Feynman path-integral approach, viz. the transition paths from one ground state to another.

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