A multidimensional dynamic framework for handling simple interruption phenomena, anaphoric pronouns and definite descriptions.

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Submitted in accordance with the requirements for the degree of

Doctor of Philosophy

The University of Leeds

School of Computer Studies

December 1998

The candidate confirms that the work submitted is his own and that appropriate credit has been given where reference has been made to the work of others.

Abstract

The last two decades have witnessed the development of theories dealing with discourse. Contributions from Linguistics, Philosophy, and Logics have helped to expand the knowledge of the field. But, independently of the researchers' viewpoint, all theories make the same basic claim, namely: *discourse is a complex structured abstract entity*. If a theory is to succeed it must take into account such structure. Therefore, questions such as "What structures do discourses have?", "How many structure sorts can be assigned to discourses?", "Can discourse structure sorts be unified?", and the like, must be be asked (and answered). This thesis proposes a formal semantical model as an answer to the first question.

This is not a work in Linguistics; it, however, is based on a series of well established results from Linguistics research. For instance, it presupposes that discourses can be hierarchically organized having *discourse segments*, or discourse blocks, as their basic components. Also, it presupposes that coordination and subordination are the basic relations among discourse blocks. However, differently from the literature where the hierarchical organization corresponds to the well known tree structure, leading in one way or another to stack oriented processing, this work proposes a "list" oriented processing.

The stack oriented processing has been vindicated in the literature as a tool for processing discourses. Among many other things, the stack-block structure allow us to explain how anaphoric relations are possible even across segment borders. This would be fine for *continuous* discourses. But, it clearly fails to account for interruptions.

Sometimes, a discourse can be interrupted and resumed later on. A simple example of this phenomenon occurs when someone introduces a person to a group and discovers that that person is not so well-known as (s)he thought. Trying to recover the situation, a parenthetical background explanation would be given; after that the discourse might be resumed naturally. But, if the resumption is almost always taken in a backward direction (and therefore the stack model will work fine), there might be the case that a "forward" move

into the parenthesis is needed. As a consequence the stack model ought to be relaxed (the stack is not a stack, after all) or hardened (stacks of stacks, and the stack-like have been prescribed for these stack-rascal cases.) Therefore, a stack oriented processing does not seems to be the "natural" model, at least not as natural as a list (or tuple) might be. A nice property is that all "relevant properties" modelled by a stack-theory are preserved under the list model such as, for example, the search for possible referents of definite noun phrases and pronouns.

This is a work in Semantics; this thesis proposes a semantical system for handling nested discourses with interruptions. Since these issues remind us of programming languages concepts; dynamic logic which has been extensively used to model programming languages forms our basic building block. This thesis can be seen as a generalization of Groenendijk and Stokhof's (1991) system, since it takes Groenendijk and Stokhof's (1991) Dynamic Predicate Logic (DPL) as the dynamic theory to be further developed. Groenendijk and Stokhof's (1991) system deals with anaphoric pronouns occurring in a "plain" discourse, i.e., in a linear sequence of sentences. The cornerstone of Groenendijk and Stokhof (1991) is the insight that a sentence is a function between information states. Since the present work presupposes hierarchically structured discourses, a multidimensional function should be used instead. As a consequence of the better structuring "attached" to discourse structure, the present work allow us to keep track not only of anaphoric relationships but also of definite noun phrases which could be rendered under the uniqueness restriction (which I take as a relative concept.)

This thesis not only generalizes previous work, but also opens the way for a new series of semantical systems. Going even further in the programming languages paradigm, we might say that the present system has only dealt with a few parameters. For example, we assume that all "dimensions" share the same domain of individuals. Also, we don't take into account any rhetorical relations (these might be seen as a kind of parameter communicating different things amongst different dimensions). But this is left as future work.

Acknowledgments

Anthony G. Cohn for his supervision, guidance, encouragement, patience, and many helpful comments given within a realistic time-scale, especially during the writing up stage. This research project would not have been possible without his help.

Célia Ghedini Ralha for being a very special postgraduate friend with whom I learned to share everything in life. Thanks for her never ending support, inspiration, continuous encouragement and love.

My friends and colleagues at the AI Lab for their chats and help; indeed thanks to everybody for making the SCS such a friendly and productive place.

The Brazilian community at Leeds for their friendship and help, specially Eugenia Lamounier for looking after Nadine.

My parents, sister and brothers for being responsible for who I am today. With their love, care, sacrifices, and noble examples which have instilled in me the deepest feelings of love, gratitude and respect.

A special thanks to my daughter Nadine G. Ralha for sharing not only her mother but also her father since her first months of life. Thanks for making me to realize how life is much more important and happy after you.

A special thanks to my colleagues at Departamento de Ciências de Computação e Estatística, Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista, campus de São José do Rio Preto, to whom I own my leave of absence to pursue a PhD degree.

Finally, thanks to the Brazilian Government for supporting this research through a scholarship from CNPq, a leave of absence from Departamento de Ciências de Computação e Estatística, Instituto de Biociências, Letras e Ciências Exatas da Universidade Estadual Paulista, campus de São José do Rio Preto, and a small amount of support from EPSRC grant GR/H/78955.

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Chapter 1

Introduction

1.1 Research Context

This research work deals with natural language semantics and is the result of a cross-fertilization between two compelling research fields, which we might roughly call "logic semantic theories for natural languages" and "linguistically based discourse theories."

Characteristic of linguistic theories is the attempt to explain not only the regularities among sentence structure components (and therefore to come up with a compositional semantics of natural language sentences), but also (in a "general perspective") the regularities among groups of sentences. Central to the discourse discussion are questions such as (to cite only a few): Are discourses a hierarchically structured entity? If so, which structures are there? Are these structures related to what makes a discourse coherent? What is the relationship between these structures and anaphoric reference? Also characteristic of these approaches is the cognitive, psycho-linguistic settings into which their development is made.

Characteristic of logic semantic theories is the *computational paradigm* adopted. In it, the analogy drawn makes a sentence a computational process with the potential of modifying the information one might have before the sentence has been conveyed. So, sentences are modelled as "update functions" between information states. This model has achieved impressive results. For example, under the compositionality criteria (basic to almost all logic theories since Frege), anaphoric intra-sentential (as well as inter-sentential) reference has been solved. As a consequence of the sentential barrier breakdown, these logical theories “stepped forward” to a more general setting: the discourse context.

Although some approaches have adopted the word “discourse” into their names or “advertising slogan,” we must be aware that they all, indeed, deal with discourses as plain as possible (discourses are equated to a linear sequence of sentences. No hierarchical structure is even taken into consideration.) This certainly does not invalidate the computational paradigm. On the contrary, it shows that there is a problem still to be solved.

Put in a simple way, the problem is to develop a logical semantic system capable of dealing with a more complex range of structured discourses (and a more complex range of linguistic phenomenon occurring inside and among the discourse structures, such as, for example, pronominal anaphoric reference, long distance reference, interruptions, the use of definite noun phrases, etc.) under the rule of computational paradigm. **And that is what this thesis is all about**.

1.2 Goals of this Research

The main goal of this research is to show that:

1. it is possible to bring “linguistically based discourse theories” into “logic semantic theories,”
2. the previous move improves our comprehension of some linguistic phenomena,
3. the improved comprehension might be used to give feedback to “linguistically based discourse theories,”
4. the model developed here could be used as the starting point for new generation of dynamic logics.

Items 1, 2, and 3, above, characterize the cross-fertilization already mentioned. The move from “linguistics” to “logics” seems to provide an improved model for understanding a logical analysis of some natural language phenomena, since it helped us to expand the ontology taken into account by traditional dynamic semantics. The new ontology ought to include new entities corresponding to discourse and its constituents (namely, discourse blocks). The traditional dynamic settings are, still, rather conservative; for them, the top most entity (namely, sentence) is only a formula and therefore a discourse (which they take as a sequence of formulas) is only a formula, since a conjunction of formulas is still a formula.

It is clear that these new entities ought to introduce a new scope dimension into the information states model. The dimension I am referring to here is related to the physical limits of discourse blocks. This dimension, which is missing from traditional dynamic settings, would be used to model some linguistic phenomenon. For example:

anaphoric pronouns on noun phrases It is a well known fact that anaphoric pronouns might be used inside (but not outside) the discourse block where their antecedent noun phrases are first introduced. As a consequence, anaphoric relationships should be kept local to discourse blocks.

long distance reference Long distance reference occurs when the antecedent of an anaphoric pronoun (or anaphoric noun phrase) is located far away from its anaphor. But this distance depends only on the metric adopted. So, long distance reference might be seen as a short distance one if the lengthy intermediate material behaves like a digression, which in this case, should constitute a sub-block (i.e., a sub-discourse).[1] Once more, the referential relationship is kept local to the discourse block.

To cope with such new ontological entities and their implications, a new information states model ought to be developed. This new model ought to keep track of the

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[1] An analogy to the programming language Lisp would help here. Let (a1 a2 (b1 b2 . . . b100) a3 . . . a10) be a list. How far is a3 from a2? For a Lisp programmer the answer is easy: two units of distance. For a non programmer the answer is easy as well: one hundred and one units of distance. The Lisp programmer sees (b1 b2 ... b100) as a single object for which the internal elements are not relevant for answering the question made. The same metric could be used for explaining the long distance anaphoric puzzle.

local nature of scope. The analogy to Lisp would help us once more. Lists (a mathematician would read tuple instead of list) are recursive structures composed of atoms or lists. And discourses are no different: they are recursive structures composed of sentences or discourses. To keep some phenomenon local to the index position they occur, a tuple is the semantic tool we are after. The information state model conforming to the data presented can be constructed upon the "traditional" information state model in the format of a tuple of traditional information states. In other words, the new information state is multi-dimensional.

The move from "logics" to "linguistics" seems to provide an improved model for understanding some linguistic phenomena, since substituting the list processing for the stack processing model (used by all linguistics theories I have seen) should provide us with better solutions for problems posed by, for example, discourse interruption and discourse resuming. However, I am not a linguist and so will not attempt to a full linguistic evaluation.

The multi-dimensional dynamic logic proposed looks like the top of an iceberg. The present formulation is kept as simple as possible, but open to further generalizations as well. Some generalizations would be of methodological nature (and in some sense conservative, since we should expect the preserving of basic results). Others, not so much.

For example, we assume that for each discourse block the universe of discourse is the same (a not very realistic assumption) and equals the whole universe. If we drop this assumption, new possibilities will appear. For example, I might have defined the universe as the union of the tuple of local discourse block universes. We also used total assignment functions (an inheritance from traditional Tarskian semantics). I might have adopted partial valuation functions instead. But these proposals seem to fit into the methodological category.

A set of possibilities are available and could be attached to every dimensional component. But the one I like most is to imagine that every component of the multidimensional information state is composed by a set of assignment functions and a set of rhetorical functions. These rhetorical functions would allow us to transfer specific phenomena (or entities) from one dimension (discourse block) to another. This would impose an even greater dynamic character on an already dynamic framework. This certainly fits into a new radically distinct category. We have to wait for the right time to come.

1.3 Outline of the Thesis

This thesis is divided into two halves. In the first half we provide the background for the thesis' second half, where a new logical framework is presented and discussed. An incremental construal of the argument that ***discourses are highly structured abstract entities*** *(where anaphora, uniqueness of definite descriptions, among many other phenomenon)* ***might be better analysed*** is adopted for the first part. The length of this part reflects the many roots we searched for support.

The main part proposes a new logical framework, which is suitable for processing some types of complex discourses, and develops a new semantic system in tune with it. The proposed framework, which deviates from classical Tarskian semantics settings, adheres to the new information states semantic paradigm.[2] But, if we accept that discourses are hierarchically structured objects, our ontology should reflect this; as a consequence, sentences can not any longer be taken as **uni-dimensional** update functions on **uni-dimensional** information states. To overcome these problems, this thesis proposes a **multi-dimensional** information state model. More than a simple extension, this model (of multi-dimensional update functions) paves the way for a great number of new extensions, capable of dealing with broader range of linguistic phenomenon, which we can already envisage a quite interesting ones for future work. After all, we are not seals on the top of an iceberg engaged in a discussion about the whole iceberg size. We are aware of the iceberg we have just

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[2] The terms *computational paradigm*, *information states semantic paradigm*, and *dynamic semantics* will be used hitherto interchangeably in this work.

“discovered”. So:

Chapter one introduces the problem we are going to deal with, and establishes the research context.

Chapter two covers anaphora in more detail. It presents a good number of examples of anaphora as well as a syntactic tool, the Government Binding Theory (GB), due to Chomsky, for dealing with it. GB’s strengths and weakness are also discussed. Moreover, the material presented makes clear the need for world knowledge, word meaning, inference and default referents since the referent for an anaphor can be almost anything – be it explicit, implicit or absent from discourse. The next two chapters deal with these topics from different viewpoints.

Chapter three draws a contrast between unstructured and structured approaches for discourse representation from a linguistic point of view. The linguistic data and linguistic theories presented argue for a hierarchical structure for discourse processing. A stack oriented processing is advocated by these theories as an adequate tool for explaining how (and when) anaphoric relations are allowed (or not) across discourse segments’ borders. The structural restrictions inherited from the stack model allow us to explain linguistic phenomenon such as, for example, which referents are actually available, and the use of a full noun phrase instead of a pronoun. But, instead of accepting a stack model we argue for a list model; after all, we can simulate a stack on a list. Above all, lists are general enough to allow one to represent multi-dimensional entities. Most importantly, this chapter motivates the development of a logically based theory taking not sentences but discourse segments as the basic information unit.

Chapter four discusses theories dealing with unstructured discourses. All logically based theories for discourse found in the literature are of this modality; discourse is always presented as a linear sequence of sentences. However, these approaches, in one way or another, are able to cope with problems such as (1) the relationship between indefinites and pronouns occurring inside conditional clauses (the so-called *donkey sentences*), and (2) the relationship between noun phrases and pronouns anaphoric on them occurring in it or in some previous sentence. Also, the natural language dynamic interpretation framework, referred to as *dynamic semantics* is presented and discussed. Differently from traditional logic approaches, where meaning is equated to truth conditions, dynamic approaches are based on a completely different basic notion. It is the information change potential of a sentence that is taken as constituting its meaning.

Chapter five, at last, presents the problem and proposes a methodology to solve it. The problem is related to structured discourses where interruptions might occur with posterior resuming of it. So to speak, this kind of discourse is rooted in chapter three. However, the stack methodology does not fit as a natural model for this discourse class. The solution, on the other hand, is rooted in chapter four. The cross-fertilization of both "views" lead to the proposed solution.

Chapter six is where the development of the formal system is done and its properties are presented.

Chapter seven is a collection of examples showing how the formal system copes with it.

Finally, chapter eight summarizes the overall research and points out its limitations and contributions, as well as recommendations for future work.

Chapter 2

Syntactically Focused Approaches

This chapter deals with the problematic character of anaphora and its associated döppelganger partner, reference, in connection with Natural Language Processing (NLP).

As is well known, *reference* plays a central role in language; theories developed to model natural language should take a close view of *reference-anaphora* pair into account. Philosophers, linguists, and, more recently, AI workers have studied such phenomena from different viewpoints. Roughly speaking, we might classify these viewpoints into three basic categories which might be labelled as "syntactic," "semantic," and "pragmatic." Each category has witnessed the development of theories spread through the time-line.

For the "syntactic" category, two fundamental approaches are represented in the literature by Russell's (1905) Theory of Descriptions[1] and Chomsky's works on syntactic and binding theory (as known as GB) which inspired most of the NLP research up to the mid 70's; here the time gap is around half a century. For the "semantic" category, fundamental approaches are given by Kamp's (1981) Discourse Representation Theory (DRT), Heim's (1982) File Changing Semantics, and Groenendijk and Stokhof's (1991) Dynamic Predicate Logic (DPL); they all have in common the fact that they revive Frege's dichotomy between sense and reference. Finally, for

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[1] I am assuming the reader is acquainted to this work; comments on it will appear in the scope of this thesis; no other mention to Russell's work will appear in this chapter.

the "pragmatic" category, fundamental approaches are exemplified by Strawson's (1950) criticism of Russell's (1905) work and Grosz and Sidner's (1986) proposal.

Although "syntactic" approaches fail to take into account most of relevant aspects of NLP they provide introductory background concepts. According to the trichotomy given above, this chapter should be seen as the syntactic one while Chapter 3 should be seen as the pragmatic one and Chapter 4 the semantic one.

2.1 A Brief Introduction to Anaphora

Being one of the most fundamental natural language phenomenon, anaphora is also one of the most puzzling and pervasive problems; its problematic character shows up in every theory dealing with natural language independently of the viewpoint adopted. It is present in logically, linguistically, philosophically, and pragmatically oriented theories corresponding to the syntactic, semantic, and pragmatic trichotomy already explained.[2] Moreover, due to its immanent dynamics, complexities, and subtleties, no theory has successfully covered the issue of explaining how words are able to denote concepts and "refer back"[3] to objects, concepts and entities in general.

Anaphora will be used here in the general sense to refer to the relationship in natural language wherein a proform is interpreted by reference to another term, usually a name, or noun phrase (NP), in a sentence or discourse. We can say also that anaphora is the device of making in discourse a short reference to some "object(s)", real or abstract,[4] in the expectation that the discourse participants be able to "recover" the full reference and therefore determining the identity of the

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[2] We are not going to take into account psycho-linguistic, and therefore, cognitive theories for anaphora acquisition. However, such theories help to strength the pervasiveness character of anaphora; psycholinguistic research has pointed out that children's mastering of anaphora occurs at a fairly uniform age, **being quite independent of the child's level of general syntactic development**. The last argument seems to strength the idea that anaphora could not be fully accounted by purely (traditional) syntactic approaches. To get better acquainted to such issues, see Chomsky (1969), and Lust (1986).

[3] The term anaphor derives from the Greek meaning *pointing back*. However, what is being pointed back might occur in a forward as well as backward direction which are termed cataphor and anaphor resp. And for *deictic* cases, "outside" of discourse.

[4] Asher (1993) deals with abstract objects, and reference to them, in English.

entity referred to. The reference is called an *anaphor* and the entity referred back to is usually called the *referent* or *antecedent*. A reference and its referent are said to be *coreferential*.[5]Although the terms *referent, antecedent*, and *coreference* are not problem free we will adopt the general usage.

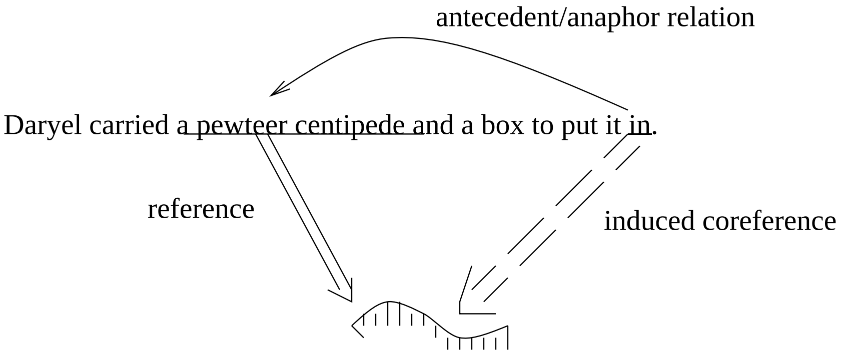

Figure 2.1: Coreference as a induced relation.

Usually the anaphoric reference is lexically, or phonetically, shorter than the referent, as for instance in (1.a), and (1.b).

(1) a – Daryel carried *a pewter centipede* and a box to put *it* in.

b – Ross took Nadia and Sue ϕ Daryel.

Paraphrases, however, provide counter-examples for shorter reference case, as in (2).[6]

(2) Most of the city's federal buildings were dark, but chandeliers shone brightly from the National Portrait Gallery. Inside *the building in which Walt Whitman once read his poetry to wounded Union troops and Abe Lincoln held his second Inaugural Ball*, a black-tie assemblage of guests stood chatting.

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[5] Strictly speaking, the traditional semantic view of reference is one in which the relationship of reference is taken to hold between expressions in a text and entities in the real world, and that of coreference between expressions in different parts of a text. However, Brown and Yule (1983, page 192) states that

> co-referential forms are forms which "instead of being interpreted semantically in their own right ...make reference to something else for their interpretation (Haliday and Hasan (1976, page 31))." These forms direct the hearer/reader to look elsewhere for their interpretation.

Figure 2.1 summarizes these points and explains why the term coreference is employed as I stated.

[6] Example taken from Hirst (1981, p. 26).

A possible alternative "definition" for shorter reference is accepting that the anaphoric reference provides less information, or is less specific, than the referent. This is not true, either, since example (3) refutes such assumption; the reference is clearly more informative than the referent.

(3) Maaike went to *a sunny country* last year. She wanted to go to Spain, but eventually went to *Portugal*.

Anaphor is a complex issue that cannot be approached from a single point of view. Pragmatically and semantically related issues will be discussed in Chapter 3 and Chapter 4 respectively. It is worth noting that the literature on anaphora deals mostly with the *intrasentential* kind. This is specially true for syntactic and semantic approaches, although, more recent semantic theories have been designed to take care of *intersentential* kind which traditionally has been addressed by pragmatically discourse related approaches.

The next few sections will be dedicated to present a basic taxonomy for anaphora, based on Hirst's (1981) work, and discuss anaphora from a syntactic point of view, where General Binding Theory concepts play a fundamental role.

2.2 A Basic Taxonomy

A "grammar" for anaphora ought to take into account the distributional differences among anaphor types and their interpretation in terms of the specifics of their domain, whether syntactic, semantic or pragmatic (in the sense introduced in the beginning of this chapter). Adhering to this viewpoint, Hirst (1981) presents a fine-grained anaphora taxonomy, which is summarized in Table 2.1.

Pronominal anaphora is the most common kind of anaphoric relationship in the literature; for them, Hirst distinguishes three main categories. Although he didn't refine his pronoun pronominal variety, the "literature" goes further splitting that category into three other subcategories corresponding to *deictic*, *e-type* and *bound* types, to be presented in section 2.3. For the remaining types, a short characteriza-

Type of anaphor	Lexical realization
pronominal	
• pronouns	"he", "she", "it", "one", ...
• epithets	"the idiot", "that stinking lump of camel excrement", ...
• surface count	"the former", "the latter", "same", low ordinals, ...
prosentential	"it", "so", ...
pro-verbial	"do"
proactional	"do so", "do it"
proadjectival | prorelative	"such", "so", ...
temporal	"then", temporal relations
locative	"there", locative relations
ellipsis	ϕ

Table 2.1: Hirst's (1981) anaphora classification.

tion, achieved by the use of prototypical examples, will be provided in the next few paragraphs. However, before this, it is worth pointing out that Hirst claims that all types of anaphora in Table 2.1 plus paraphrase are indeed special instances of *definite reference*, for which he presents a proposal dealing with semantical **ISA** hierarchy augmented/amended by pragmatic factors such as *focus of attention*, *consciousness*, and *activatedness*, to cite but a few. This reflects the state of the art at the time. Interestingly, the distinctions made were similar to the Fregean dichotomy between sense and reference, which started to become part of many logical-semantical theories thereafter. Such distinctions try to solve, for instance, problems exemplified in (4) where *it* refers to the **same** entity which the gherkin sandwich refers to. For the paycheck example *it* does not refer to the same entity referred to by the antecedent; instead, they pick up their referents from the very same sortal class.

(4) a – Mr Bean made a gherkin sandwich but didn't eat it.

b – The man who gave his paycheck to his wife was wiser than the man who gave it to his mistress.

In an indirect way, we will present the definite reference anaphora case making use of the partition presented in table 2.1. As tiles of a puzzle fit together to produce a complete picture, so the tools below will integrate to provide an improved model of anaphoric reference. Each "subcase" will indicate the need of tools from syntax, semantics and pragmatics fields.

2.2.1 Definite Reference: case by case

Epithets When used anaphorically, epithets cannot take pronouns as antecedents (cf. Lakoff (1976)). This seems to direct us to "syntactic" theories; traditional ones, such as GB, for intrasentential cases as (5.a) or discourse grammar ones, such as Prüst, Scha and van der Berg (1994) and Polanyi (1988), for cases as (5.b).

(5) a – 'What's that?' asked Terrier, bending down over . . .
'The bastard of that woman from the rue aux Fers who killed her babies!'[7]

b – Mary used Ross' credit card so much, the poor guy had to declare bankruptcy.

Surface Count Reference Noun phrases like *the former* and *the latter* can be used anaphorically as in (6). The reference is directly guided by syntactic surface structure.

(6) If I have to choose between a car or an elephant, I will go for the former.

Although ordinal numbers could be used in this sense, it seems unnatural to pick up an inner element from a list, as in (7):

(7) John went to a car boot sale and bought a penguin pet, nails, a pair of spectacles, a bunch of dried flowers, a broken mainframe console, a pair of ex-NASA thecno-trousers, and a money maker machine. He declared that he preferred the sixth.

Prosentential Reference This category includes pronouns and words such as *such*, and *so* used to refer, not to previous NPs, but to situation(s) evoked by, as exemplified in (8) (from Anderson (1976)).

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[7] The epithet refers to Jean-Baptiste Grenouille, the main character of Patrick Süskind's book *Perfume*.

(8) Your wife was under the impression that you would be away tonight, and as you can see, I thought so too.

Strained Reference This category includes cases where the referent is not "explicitly" present in any previous NP; the referent, however, is risen to attention by a lexically similar term, as exemplified in (9) (cited in Hirst (1981)). The pronoun *it* refers to the guitar which is only indirectly brought to discourse by the NP *a guitarist*.

(9) John became a guitarist because he thought that *it* was a beautiful instrument.

Pro-verbial Reference This is the case for verbal phrase anaphora, as in (10). Notice that *to do* is the unique English pro-verbial verb (cf. Hirst (1981)).

(10) a – Maaike likes belly-dancing.
b – She hates waltzing.
c – Saskia does too.

Proactional Reference This is the case when an anaphor refers to the action(s) taking place in a previous event, as in (11). This kind is built with *do* in conjunction with *so*, *it* and demonstratives.

(11) Nadia removed a herring from her pocket and began to fillet it. Ross *did so* too.

However, as Hirst (1981) observes, there is no clear border separating proactions and pro-verbs; sometimes replacing *does it* or *does so* for *does* do not change the originally intended meaning.

Pro-adjectival Reference Words like *such* might be used to refer to adjectival forms, such as in (12).

(12) I was looking for a purple wombat, but I couldn't find *such* a wombat.

Temporal Reference This is the case when an anaphoric reference is made to a time or an event as in (13).

(13) a – In the mid-sixties, free love was rampant across campus. It was *then* that Sue turned to Scientology.

b – In the mid-sixties, free love was rampant across campus. *At that time*, however, bisexuality had not come into vogue.

Locative Reference This is the case when an anaphoric reference is made to a place as in (14).

(14) The Church of Scientology met in a secret room behind the local Colonel Sanders' chicken stand. Sue had her first dianetic experience *there*.

Ellipsis This is the case when the anaphor is completely null.

(15) Nadia brought the food for the picnic, and Daryel ϕ the wine.

The examples given above do not exhaust each item of the basic classifications. For instance, example (10), the pro-verbial case, shows the intricacies for coordinative verbal phrase anaphora since (10.c) might refer to (10) as a whole or only to (10.b).[8]

2.3 Pronominal Anaphora

As already mentioned, pronouns have been classified into three categories, namely, *deictic*, *bound*, and the *e-type*, accordingly to their characteristic behaviour patterns and intended analyses.

The interpretation of deictic pronouns are determined in relation to specific features of the speech-act; in a two person conversation, the speaker and addressee's identity together with the time and place depends on the speech event. Because of that, deictic pronouns have been traditionally analysed as free variables of a predicate calculus.

The bound approach to pronominal anaphora can be analysed in a purely syntactic tradition, as in the Government Binding Theory, and in a semantic line, as done in the dynamic logic settings. Dynamic settings follow the motto that pro-

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[8] For more detailed coverage on VP anaphora, see Prüst et al. (1994).

nouns should be seen as syntactically free variables that are, somehow, semantically bound.

The e-type analysis are similar to the bound analysis in that both assume that pronouns are, somehow, semantically bound variables. The difference between them is basically that in the bound analyses pronouns are identified to variables while the e-type analyses pronouns are interpreted as “going proxy” to definite descriptions. E-type accounts realize these ideas by taking e-type pronouns as quantifiers. Again, a logical semantic setting is present.

To be in accordance with this chapter’s name and goals, and the previous exposition, it seems that the proper place to give more details of pronominal analysis should be postponed until chapter 4, the one dealing with logical approaches, has been reached.

2.4 Government Binding Theory

Binding Theory, Chomsky (1981), is a theory developed to syntactically mirror the principles governing anaphoric behaviour of noun phrases (NPs). In order to achieve this, the notion of *command* is introduced and NPs are classified in three categories, namely, *ana* for reflexives and reciprocals, *pro* for pronouns, and *np* for non-pronominal full NPs. Each category is governed by a principle restricting anaphoric possibilities. The principles are usually referred to as:

Principle A: If an anaphor is of type *ana*, it must be locally bound to an antecedent.

Principle B: If an anaphor is of type *pro*, it must be locally free.

Principle C: If an anaphor is of type *np*, it must be free.

Auxiliary notions of locally boundness/freedom are defined as:

- An NP is *locally bound* if it is coindexed with a locally commanding NP.
- An NP is *locally free* if it is not coindexed with a locally commanding NP.

- An NP is *free* if it is not coindexed with a commanding NP.

The notion of *command* remains unexplained; in the traditional Binding Theory (GB) this notion, called c-command, is defined configurationally based on the surface syntactic tree representation. In GB c-command is defined as:

- A c-commands B iff the first branching node dominating A dominates B.
- A locally c-commands B iff A c-commands B, and A and B are contained in the same minimal S or NP.

Figure 2.2 shows that the NP *the boys* locally c-commands the anaphor *themselves*.

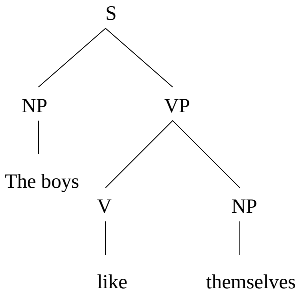

Figure 2.2: A locally c-commanding configuration.

That all full NPs behave the same as far as binding relations are concerned is not problem free. Proper nouns, non-pronominal definite NPs, indefinite and quantified NPs are all subject to Principle C, and therefore equally treated. However, data displayed in (16)–(19) refutes GB criteria since all these sentences respect Principle C — the full NPs are free and coindexed with some NP but not with a commanding NP. Moreover, (*a*) sentences are better than the (*b*) ones.[9]

(16) a – When he_i arrived home, John_i kissed his wife.

b – When he_i arrived home, every man_i kissed his wife.

(17) a – John_i is a fool but John_i doesn't mean any harm.

b – A man_i is a fool but a man_i doesn't mean any harm.

(18) a – John_i's mother loves John_i's father.

b – A boy_i's mother loves a boy_i's father.

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[9] These examples are from Dorrepaal (1994)

(19) a – His$_i$ friends say John$_i$ is very intelligent

b – His$_i$ friends say [every boy in my class]$_i$ is very intelligent

Solutions have been proposed, but they were not adopted by the standard version of the Binding Theory. Postal (1971), for instance, claims that when a definite pronoun is to the left of an NP, this NP must be definite for it to serve as antecedent. Wasow (1972) suggests that the relevant distinction is between referring expressions, which he calls *determinate*, and other NPs. The former class includes specific NPs and generic NPs. For him, non-specific non-generic NPs are indeterminate. These constraints, if adopted, would be added to Principle C.

Another type of solution is shift to the semantic representation differences as exemplified in (19) a and b. Although defensible that properties like determinate or definite belong to semantics rather than syntax, that shift does not solve the problem. Reinhart (1983) and Dorrepaal (1994) argue that the unavailability of anaphora for cross-over cases, like (19.b), should be ascribed to properties of surface constituent order rather than scope.

Dorrepaal (1994) proposes an alternative to GB preserving as much as possible of GB's original goals; the basic difference to standard GB is a mechanism of controlled coindexing affecting Principle C.

In most linguistic theories, the anaphora problem is approached from only one viewpoint. Syntactic theories are mostly concerned with syntactic intrasentential constraints; they also require that antecedents be more specific, i.e. have more descriptive content, than the anaphors. But, the specificity constraint is the Achilles' heel for most of syntactic approaches since it is better accounted for at the discourse level.

2.5 Summary

The aim of this chapter was to present a basic taxonomy for anaphora as general as possible in order to show how difficult the development of an automatic anaphora solver would be. From a methodological point of view such an automatic solver presupposes the existence of a general framework (in some kind of general logical language) capable of representing anaphoric phenomena. However, due to the number of anaphoric sorts, we have shown that the common practice is to take into account a small number of cases. This typically includes noun phrases such as pronouns and definite descriptions, when used anaphorically.

The material presented makes clear the need for world knowledge, word meaning, inference and default referents since the referent for an anaphor can be almost anything – being it explicit, implicit or absent from discourse. The next two chapters deal with these topics.

Chapter 3

Discourse Focused Approaches

3.1 Initial Remark

The aim of this chapter is to draw a contrast between unstructured and structured approaches for discourse representation. A non-structured discourse might be understood as a linear sequence of sentences/utterances. As a consequence, the discourse does not play a role except for delimiting the extension for scope relations. However linguistic data has been prescribing/pointing to a tree structured hierarchy as the best model for representing the relationships among discourse segments. So a stack oriented processing has been prescribed as the tool for processing discourses. Moreover the block structure explains how anaphoric relations are allowed (or not) across discourse segments' borders. This is even more astonishing when we take into account that these theories are older than the unstructured ones which, by the way, have taken programming languages as a paradigm.

By now, we might realize that structured approaches convey more information since, for instance, the stack imposes restrictions over which referents are actually available, the use of a full noun phrase instead of a pronoun, etc. Plain theories lack all this richness. This chapter motivates the development of a logically based theory taking not sentences but discourse segments as the basic information unit.

Since all "linguistic" theories for discourse segmentation are, in some sense, based on three approaches, I will focus on them.

3.2 Semantics and Pragmatics of Natural Language

Pragmatics, as the study of language in context, is often difficult to distinguish from semantics, as the study of the connection between the language sign system and the world it represents. Although the phenomena covered by both viewpoints are almost the same, a difference of perspective is still present. “Semanticists” advocate that meaning is a property of a text, dialogue or discourse. For the less strict semanticists, meaning can not be seen detached of context. To determine the meaning, large amount of contextual knowledge are needed and used. Contextual knowledge is used under different disguises and names such as ontologies (complete with default reasoning),[1] and lexical/semantic preference approaches, which help us to interpret metonymies, metaphor, and other non-literal language and facts about discourse structure and other language regularities to determine the overall plan and purpose of the discourse.[2] However, the parts of context used tend to be compartmentalized and represented as universal static knowledge sources (cf. Farwell and Helmreich (1995, p. 4)). On the other hand, “pragmaticists” advocate meaning as a property of people since only people can engage in intentional thought and action. Pragmaticists generally agree that human language is used not just for reflecting the world, but for the purpose of describing complex mental models of how things are not as well as how things are. The last point of view is literally expressed in Farwell and Helmreich’s (1995) article intitled “Contextualizing Natural Language Processing”.

From the viewpoint expressed above, the approaches we are going to discuss might be labeled as *semantic* theories.

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[1] Lascarides and Asher (1991) might be included in this group. As we will see in subsection 3.3.1 they use defeasible reasoning to infer discourse relations providing a way to deliver different interpretations for similar syntactic structures in a temporal import. Defeasible rules represent causal laws and Gricean-style pragmatic maxims that codify world knowledge as well as linguistic knowledge.

[2] Grosz and Sidner (1986) should be included into this group.

3.3 Three linguistically oriented well-known approaches

A discourse is usually understood as a sequence of utterances/sentences; each utterance/sentence may be assumed to contribute something to the meaning of that discourse as a *whole* (Prüst et al. (1994)). However, the meaning of a discourse cannot be regarded as the simple conjunction of the meanings of the utterances/sentences that constitute it. Sentences, and discourse segments in general, are usually involved in complex semantic dependencies. Parallelism is a good source for simple and clarifying examples of these dependencies. The two sentences

(1) a. John likes visiting relatives

b. (and) Peter likes visiting friends.

can independently have an "active" reading or a "passive" one (i.e., *John likes to visit relatives* for the active reading and *John likes relatives who visit* for the passive reading). The parallel coordination does not allow for a mixed[3] reading; this case shows how discourse compartmentalizes ambiguity. If we replace (1b) for *Peter likes visiting marketplaces* then the active reading for it superimposes onto the ambiguous (1a) sentence. So, a discourse is more than a sequence of connected sentences.

But, discourse seems to be a pervasive concept. Its meaning is usually taken from the common ground; therefore, it is unlikely to be directly approached. Hopefully, there exist indirect ways to determine the nature of discourses, the essence of its building blocks, and the relationships between these building blocks. Questions are the tools for this research endeavour. Questions such as *what makes a coherent discourse*?[4] And this is what the literature in discourse is all about.

Hobbs (1979), Hobbs (1985), Reichman (1978), Grosz and Sidner (1986), Mann and Thompson (1987b), Polanyi (1988) and Scha and Polanyi (1988) proposed the most significant theories dealing with the questions above. At first glance, these

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[3] Not allowed: (1.a) active and (1.b) passive and vice-versa.

[4] Notice that this question is indeed a twofold one, namely: (i) What are the basic building blocks for making a discourse? (ii) What kinds of relationships do hold between the building blocks?

theories do not have any common point but the hierarchical structure claim. Independently of the cited authors' distinctive points of view and background, the analysis of discourse data allowed them to attribute hierarchical structure to discourse, and relations of "*different sorts*" between the discourse segments. The theories vary in the fine grain structure of their analysis of discourse structure. The extremes are best exemplified by Grosz and Sidner's (1986) approach, which makes use of three levels of description for discourse, and Rhetorical Structure Theories, which propose an open-ended set of relations between discourse segments; Mann and Thompson (1987b), for example, introduces a set of twenty three relations such as Evidence, Cause, Contrast and Elaboration. Although there is no direct mapping from one theory to another some rules of thumb hold: for example, Elaboration is a type of subordination (cf. Inder and Oberlander (1994, p. 4)).

Not surprisingly, the hierarchical structure proposed is modeled through tree structures. Tree structures display all relevant characteristics since nodes at same level might be used to represent coordination. Nodes at different levels, displaying the ancestor-descendent relationship, might be used for subordination. Tree structures are suitable for representing not only plain coordination/subordination relations but also information about the segments, such as its extension (i.e. where the segments start and end; every subtree of a given node conveys such information) and the relationship among them, such as, for example, *Explanation* and *Narration.* Figure 3.1 is an abstraction exemplifying narration as the coordinative force and explanation as the subordinative one. Moreover, all theories advocate the *right frontier* as the locus of all possible points for attachment of incoming sentences/utterances.[5] The default attachment point is the bottom right-hand node of the tree; however, various discourse cues can cause attachment to occur further up on the right frontier.

Coordination and subordination play an important role not only for natural languages but also for artificial ones. Let us take the Algol programming language as a preliminary example. As is well known, Algol is a *nested block* oriented lan-

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[5] Gardent (1994) argues that in cases where one discourse segment is semantically related to two or more discourse segments, a graph, instead of a tree, provides a better solution.

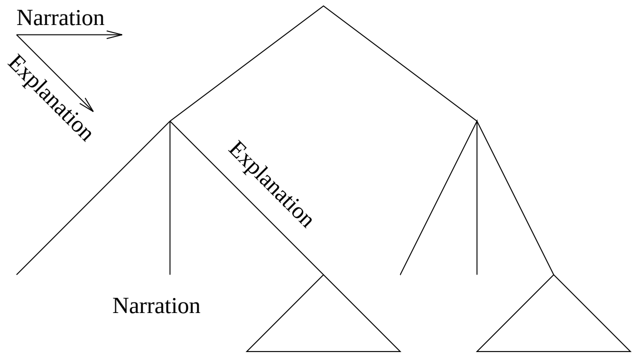

Figure 3.1: An Abstract/Hypothetical Discourse Tree

guage. The concept of nesting represents, essentially, the idea of (co/sub)ordination since blocks at the same level represent process coordination while embedded blocks represent process subordination.

Programming languages as a paradigm for natural language processing is not a new claim. As a paradigm, it has been used some times disguisedly, some times explicitly. We can enroll Grosz and Sidner (1986) and Polanyi (1988) in the first group and Groenendijk and Stokhof (1991) in the second one. Grosz and Sidner (1986) propose a stack as the way to compute the focus of attention, one of the three components of their theory. Polanyi (1988) uses a tree as the natural model for her theory, the Linguistic Discourse Model (LDM). These two theories share the idea that discourses are made of possibly embedded segments standing in a coordination or subordination relationship. Dynamic Predicate Logic (DPL), Groenendijk and Stokhof (1991), was the first logical theory to take it explicitly.[6]

The block structure provide us with a rough answer for the original questions above. In some sense, we have produced some evidence for a kind of grammar for discourse dealing with the possible *syntactic* attachment points for incoming sentences. But to discuss the *semantic* counterpart that makes all cited theories distinctive we should take them individually.

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[6] But only in a shallow form since a discourse is taken as a linear sequence of sentences. As a programming language it reminds me of Basic. However, DPL and other logics are the target for chapter 4.

3.3.1 Rhetorical Structure Theories

Rhetorical Structure Theory (RST) addresses the development of a comprehensive theory of text organization. The theories developed, such as Mann and Thompson (1987a), Mann and Thompson (1988), Dahlgren (1988), Hovy (1991), Moore and Paris (1991), follow a very basic pattern, since they all provide a set of rhetorical schemata rules for a wide variety of purposes trying to capture what it means for a text to be coherent.

To achieve such goals, the following questions should be answered:

- what are the *smallest building blocks*, or *atomic parts*, of a organized text?
- how can these parts be arranged?
- how can the parts be connected together to form a whole text?

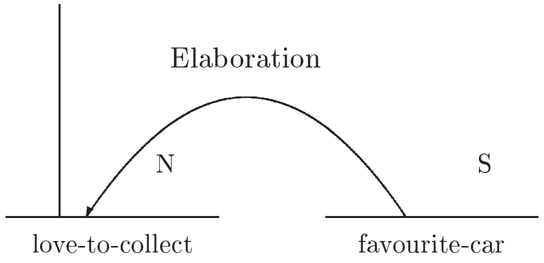
In this way, the theory could be used as part of a text analysis system, applied to generation and/or interpretation process. And, indeed, most of "rival" alternative approaches, in the interpretation set up, take RST as a useful and important part of their own internal structure. Polanyi (1988) and Prüst et al. (1994), for instance, fall into this category.

Rhetorical approaches, such as the ones already named, characterize text structure in terms of functional relations holding between parts of the text. These theories also take texts as hierarchally structured objects making explicit the resources available for use in interpretation or generation. But, instead of using coordination and subordination terminology, we find an equivalence on the use of terms like *nucleus* (N) and *satellite* (S), Dale (1993), as already pointed out on page 23.

In Dale (1993, topic 3, transp 53 and 55) we find the following example and also the definition for the Elaboration relation as displayed in figure 3.2.

(1) I love to collect classic automobiles.

My favourite car is my 1899 Duryea.



relation name:	Elaboration
constraints on **N**:	none
constraints on **S**:	none
constraints on the **N** + **S** *combination*:	**S** presents additional detail about the situation or some element of subject matter which is presented in **N** or inferentially accessible in **N** in one or more of the ways listed below. In the list, if **N** presents the first member of any pair, then **S** concludes the second: 1. set : member 2. abstract : instance 3. whole : part 4. process : step 5. object : attribute 6. generalization : specific
the effect:	Reader recognizes the situation presented in **S** as providing additional details for **N**. Reader identifies the element of subject matter for which detail is provided.
the locus of the effect:	**N** and **S**.

Figure 3.2: Definition for the Elaboration relation (cf. Dale (1993))

Although Rhetorical Theories propose an open-ended set of relations between parts of a text, they also suggest the existence of grammars dealing with discourse. Polanyi (1988), Scha and Polanyi (1988), Lascarides and Asher (1991), Lascarides, Asher and Oberlander (1992), and Prüst et al. (1994) present us with sophisticated grammars; in Lascarides and Asher (1991), Lascarides et al. (1992), Lascarides and partners work on Discourse Relations, Discourse Representation Theory and Defeasible Reasoning. Using defeasible reasoning to infer discourse relations, they provide a way to deliver different interpretations for similar syntactic structures, in a temporal import. In their framework, defeasible rules represent causal laws and Gricean-style pragmatic maxims codify world knowledge as well as linguistic knowledge. Prüst et al. (1994) present us with an extension for Scha and Polanyi (1988) in a context of typed multi-sorted logic.

The main criticism of Rhetorical Theories (RT) is based on the fact that if RTs provide an account to discover/model relationships holding between discourse segments, they all fail to explain, or provide an account to model, three kinds of semantic phenomena. They fail to provide explanation for the way discourse structures change the propositional content of the constituents being related in the discourse. They also fail to explain anaphora resolution where the antecedents refer to abstract entities (Asher (1993)). Finally, they do not account for temporal reasoning in discourse (Lascarides and Asher (1991)).

Let us make again a parallel to computing. Imagine that discourse segments are subprograms taking a discourse from an input state s_i to an output state s_{i+1}, as exemplified below:

$$s_i \boxed{\alpha} s_{i+1} \boxed{\beta} s_{i+2}$$

The dynamics of "first-order" subprograms.

Differently from "first-order" subprograms, where the input state and the subprogram determine the output state, "higher-order" rhetorical subprograms have to take into account an extra parameter, a rhetorical relation, as displayed below:

$$s_i \boxed{\alpha} \overset{Rh}{s_{i+1}} \boxed{\beta} s_{i+2}$$

The dynamics of "higher-order" subprograms.

The output state s_{i+2} is dependent on the input state s_{i+1} as well as on the nature of the rhetorical relation *Rh*. But rhetorical relations modify the computational behaviour of subprograms they are input. They themselves behave like subprograms. So to speak, the computation done inside each discourse block is mediate by another (kind) of subprogram – the rhetorical subprogram – which is given as input to it.

From a functional viewpoint, each discourse segment might be seen as a computable (partial) function taking two different sorted inputs, a state and a rhetorical relation. To model incoherence we have two choices. On one hand, we might adopt partial functions as models.[7] On the other hand, we might adopt a trap strategy assigning a particular trap value as output for those undefined input values. If the rhetorical relation makes the discourse incoherent, relative to the input state, then the output from β must reflect this fact.

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[7] Recall that partial functions are not defined for all input values. When applied to values that do not belong to their domain, partial functions diverge, i.e., do not produce or assign an output value.

Although incoherence affects the whole discourse, it is localized to the open part of discourse, namely, the *right frontier* of the discourse parse tree (for more details on discourse parse, see Lascarides and Asher (1991) and Polanyi (1988)). Recovering from incoherence would be a matter of backtracking, trying to recover a correct rhetorical relation, as in cases of misunderstanding.[8] The worst case occurs when no recovering is possible. If no rhetorical relation could be recovered for it, the block might have been linked to the wrong point. And, if no place could be found to attach the block, then something went wrong. But if the last case occurs, it is likely to happen only on a single participant discourse situation such as the reading of book (or thesis) where the author's intention could not be accessed. For dialogues, however, it is possible to extend recovering via segment deletion, since the other(s) participant(s) could help to overcome the troublesome situation. In any case, this does not change the point made here since deletion should be done on the most right element of the discourse parse tree.

Since rhetorical theories do not invalidate the programming language analogy we proceed to the analysis of Grosz and Sidner's theory, which advocates the use of a small set of intentional relations instead of rhetorical ones, and to the tree based theories, which incorporate rhetorical relations into their own account.

3.3.2 Grosz & Sidner's theory

Grosz and Sidner (1986) present us with a theory based on linguistic and non linguistic notions intending to stress the role of purpose and processing in discourse. They model the discourse structure as a composite of three separate but interrelated components: *linguistic structure*, *intentional structure* and the *attentional state*. They conceive the last two structures as the non linguistic ones.

As they themselves state on page 175, "the linguistic structure deals with the structure of the sequence of sentences. It consists of segments of the discourse into

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[8] In these cases, the discourse segment has not to be changed; what has to be recovered is the correct rhetorical relation attached to the discourse block. See Dahlgren (1988), particularly chapter 8, for more details on this point.

which the sentences naturally aggregate. The intentional structure captures the discourse relevant purposes, expressed in each of the linguistic segments as well as relationships among them. The attentional state is an abstraction of the focus of attention of the participants as the discourse unfolds. The attentional state, being dynamic, records the objects, properties, and relations that are salient at each point of the discourse. The distinction among these components is essential to provide an adequate explanation of such discourse phenomena as cue phrases, referring expressions, and interruptions."

3.3.2.1 Linguistic Structure

To explain the linguistic structure, the authors make an analogy to sentence development. At the sentential level, following particular patterns, words aggregate to make sentences. At the discourse level, a similar process occurs. According to particular roles and intentions, sentences aggregate into blocks, discourse segments, contributing not only for the particular discourse segment but also to the overall discourse.

A close reading of the authors' viewpoints should be enough to realize that they have implicitly accepted the existence of an automata dealing with discourse. Differently to syntacticians,[9] who developed formal grammars for discourse, Grosz and Sidner informally describe some characteristics of discourse segmentation, such as, for example, the "neighbourhood relation" between two consecutive sentences. For any two consecutive sentences, there are cases when they belong to the same segment and cases when they do not. And, there are cases when non-consecutive sentences belong to the same discourse segment. Figure 3.3 exemplifies these cases.

Once more, the resemblance to a programming language becomes evident. Firstly, a typical Algol-like block structure is described in the last sentences as shown in figure 3.3. For this figure, sentences (a), (d) and (h) are not consecutive although belonging to the same segment/block. On the other hand, sentences (a) and (b) are

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[9] See section 3.3.3, page 34.

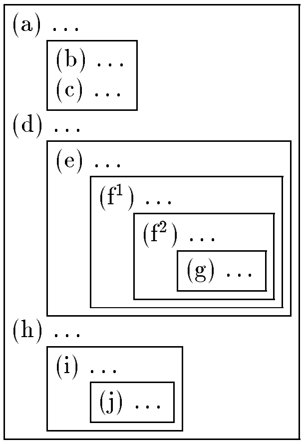

Figure 3.3: A hypothetical Algol-like block structure

in different block/segments. Secondly, the concept of subprogramming underlying the block structure is triggered by the contribution it could make to the overall discourse.

The other two components of the theory, namely, the intentional structure and the attentional state, allow us to go further into the analogy with an Algol-like programming language.

3.3.2.2 Intentional structure

Intentions play a primary role in explaining discourse structure, defining discourse coherence, and providing a coherent conceptualization of the term *discourse* itself. In their paper, Grosz and Sidner have integrated two previous research work lines, dealing with focusing in discourse and intention recognition, providing basic material to generalize these notions to a broader range of discourses.[10] The model they propose for intentional structure is very simple when compared to alternative rhetorical based theories. They make, indeed, a strong criticism of rhetorical theories claiming that a fixed set of rhetorical patterns, such as the ones in Hobbs (1979), Mann and Thompson (1983), Reichman (1981), are unlikely to cover the so diverse intentions underlying discourse.

Every discourse has a foundational purpose, which the authors refer to as Discourse Purpose (DP), reflecting the intention that underlies engaging in a particular

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[10] The previous works were in the "context" of task oriented discourse.

discourse. Since a discourse is built on discourse segments, each segment has associated a proper *Discourse Segment Purpose* (DSP) which contributes to the overall DP. Based on relevant structural relationships among intentions, modeled by DPs and DSPs, the authors avoided the adoption of rhetorical relations into their theory.

The authors have identified two structural relations: *dominance* and *satisfaction–precedence*. A DSP_1 **dominates** DSP_2, or conversely DSP_2 **contributes to** DSP_1, when the first is intended to provide part of the satisfaction for the second. A DSP_1 **satisfaction–precedes** DSP_2 whenever DSP_1 must be satisfied before DSP_2.

Notice, however, that the two structural relations correspond to subordination and coordination, respectively. As a consequence, the authors can transpose DPs and DSPs to the Attentional State part of their theory.

3.3.2.3 Attentional State

The third component of the theory, the attentional state, is the one to which the analogy to programming languages becomes more salient. The authors think of it as a computational device modeling the focus of attention as the discourse unfolds. Being inherently dynamic, the attentional state is responsible for recording the objects, properties, and relations that are salient at each point in the discourse. It is modeled by a set of *focus spaces* and a set of transition rules. These rules specify the conditions for adding or deleting spaces according to changes in the attentional state. The authors call the process of manipulating spaces *focusing*.

Each discourse has associated with it a focus space containing the entities that are salient – either because they have been mentioned explicitly in the segment or because they became salient in the process of producing or comprehending the utterances in the segment. Moreover, the focus space includes the DSP reflecting the fact that the context participants are focused not only on what they are talking about, but also on why they are talking about it. The focus space structure enables certain processing decisions to be made locally. Particularly, it limits the information that must be considered in recognizing the DSP as well as that considered in identifying the referents of certain classes of referring expressions.

The authors claim that a *stack* is the best model for dealing with focusing structure characteristics. However, such stacks should not be taken in a strict sense.[11] The space in the stack's top is the most salient one, but information in lower spaces might be accessed from higher ones (but less so than the information in the higher spaces). The stacking of focus spaces displays the relative salience of the entities in each space during the corresponding segments portion of the discourse.

Via the focusing structure, the attentional state constrains the search for referents of definite noun phrases, pronouns and the like. For example, there are different constraints on the use of pronouns and reduced definite noun phrases within a segment than across segment boundaries, as figure 3.5 shows. This concept is supported by authors' statement, on page 178, where they say: "The segmentation of discourse constrains the use of referring expressions by delineating certain points at which there is a significant change in what entities (objects, properties, or relations) are being discussed. While discourse segmentation is obviously not the only factor governing the use of referring expressions, it is an important one."

It is amazing the similarity here compared to Algol-like languages. Firstly, segments might introduce local entities which should be available for pronominal or definite reference only into the segment. Figures 3.4 and 3.5 exemplify such behaviour; for the tent example, the definite reference on sentence 14 refers back to the indefinite noun phrase on sentence two since both sentences belong to the very same segment. For the movie essay, however, sentences one and four, belonging to distinct segments, make impossible the use of definite or pronominal reference. In some intuitive sense, this compares to local declarations for programming languages. A local variable, for instance, has as scope the block or subprogram where it has been declared (except when not being locally declared it has been declared in one of the block's ancestors).[12] Secondly, the stack model proposed corresponds to the

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[11] In the sense that only the top element is accessible.

[12] Being subordinated, a pronoun/variable might refer back to the entity defined on the main clause.

DS0
1. A I'm going to camping next weekend. Do you have a two-person tent I could borrow?
2. B Sure. I have a two-person backpacking tent.

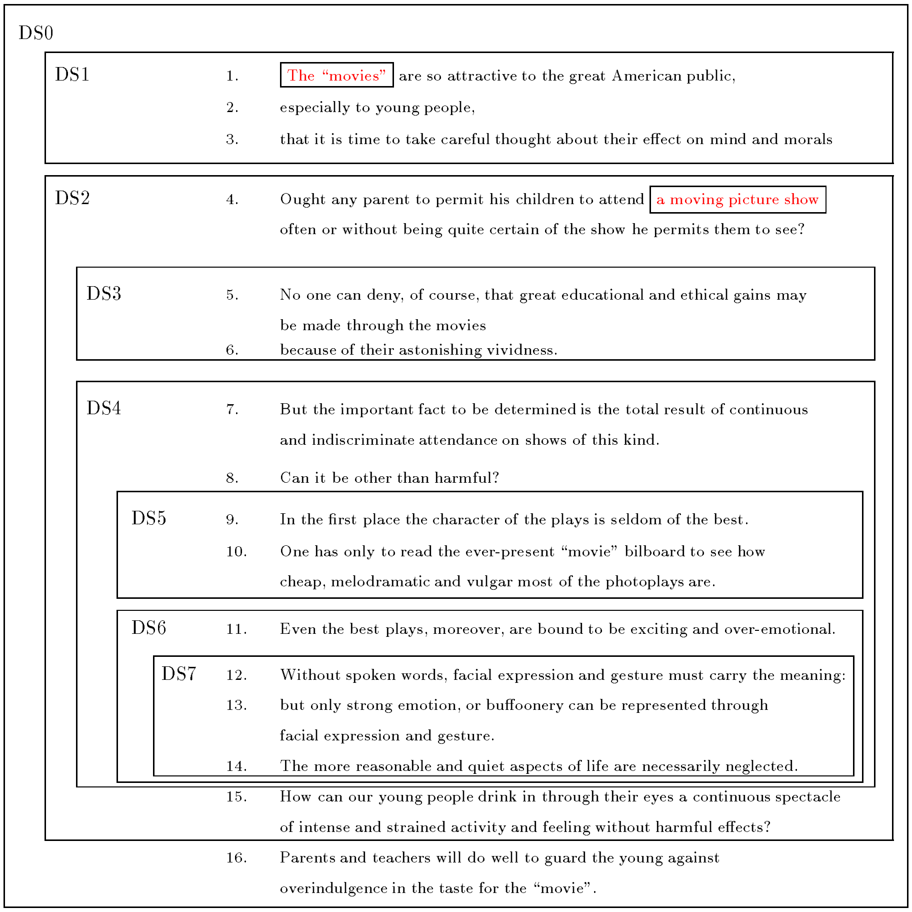
DS1
3. A The last trip I was on there was a huge storm.
4. It poured for two hours.
5. I had a tent, but I got soaked anyway.
6. B What kind of tent was it?
7. A A tube tent.
8. B Tube tents don't stand up well in a real storm.
9. A True.

10. B Where are you going on this trip?
11. A Up in the Minarets.
12. B Do you need any other equipament?
13. A No.
14. B Okay. I'll bring the tent in tomorrow.

Figure 3.4: The tent example

execution time abstract machine related to block-structure programming languages. Suppose now that to accomplish the intended task, embedded blocks are needed for some block. So, during the execution of the dominant block, which is on the top of the stack, embedded blocks will be processed. At that point, a push operation will occur on the stack putting a new embedded block in there. The unfinished block is now less salient than the top one. However, the unfinished block might contain definitions for entities referred, but not defined, into the top one. Lastly, since the lexical scope is the same for both approaches, the search for referents follows the same discipline, i.e., looking for them through the activation records (or focus spaces) until a definition is found.

An important remark should be made here. Since I have been talking about Algol-like language, I would like to make explicit the one to one mapping between the discourse segmentation block and the Algol block structure. As such, the Algol concept is best understood as an anonymous subprogram whose invocation point corresponds to its definition point. This differs radically from named subprograms which can be invoked from different points into the program (obeying the lexical convention.) This allows for differentiating the lexical link from the dynamic link. The dynamic link records the base address of the calling subprogram while the lexical link points to the stack address where the non-local entities should be found. As a consequence, if a program does not use named subprograms the execution stack corresponds to the lexical definition. The converse, however, is not true.



1. The “movies” are so attractive to the great American public,
2. especially to young people,
3. that it is time to take careful thought about their effect on mind and morals
4. Ought any parent to permit his children to attend a moving picture show often or without being quite certain of the show he permits them to see?
5. No one can deny, of course, that great educational and ethical gains may be made through the movies
6. because of their astonishing vividness.
7. But the important fact to be determined is the total result of continuous and indiscriminate attendance on shows of this kind.
8. Can it be other than harmful?
9. In the first place the character of the plays is seldom of the best.
10. One has only to read the ever-present “movie” bilboard to see how cheap, melodramatic and vulgar most of the photoplays are.
11. Even the best plays, moreover, are bound to be exciting and over-emotional.
12. Without spoken words, facial expression and gesture must carry the meaning:
13. but only strong emotion, or buffoonery can be represented through facial expression and gesture.
14. The more reasonable and quiet aspects of life are necessarily neglected.
15. How can our young people drink in through their eyes a continuous spectacle of intense and strained activity and feeling without harmful effects?
16. Parents and teachers will do well to guard the young against overindulgence in the taste for the “movie”.

Figure 3.5: The movies essay

Although events anaphora could be a case for the general full Algol-like case, I am going to stick to the former case, i.e., to the anonymous block structure.

3.3.3 Syntactic Approaches

Scha and Polanyi’s (1988) framework is considered a paradigm of the tree based, syntax semantics isomorphism approach[13] to discourse theories (cf. Gardent (1994, p. 2)).

Proponents of the syntax semantics isomorphism support the idea that discourse can be described by means of a discourse grammar which, essentially, consists of a typed unification based sentence grammar augmented with a set of discourse grammar rules. The introduction of a discourse grammar provides the tools for predicting the tree structure of discourse as well as an isomorphism between discourse syntax

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[13] This explains this section’s name.

and discourse semantics. The tree structure comes up as a consequence of rewrite rules, since the discourse grammar proposed is basically a context-free one. The grammar consists of rules which describe how to build up various kinds of structurally different *Discourse Constituent Units* (DCU).

As we have already pointed out, coordination and subordination play an important role and Scha and Polanyi (1988, page 574) distinguish the following kinds of DCUs:

Subordinations. Subordination is a binary structure in which the first element remains accessible. They are units in which all or most of the structurally relevant features are inherited from the left constituent.[14] In *semantic subordinations*, such as *rhetorical subordinations* and *topic-dominant chains*, there is a semantic relationship between the components. *Interruptions*, however, are semantically very different, although structurally analogous. In this case, there is no semantic connection whatsoever between the two constituents.

Binary Coordinations Binary coordinations are structures in which the second element has equal status to the first. As a consequence, the first element becomes inaccessible to the other one. Under this category, the authors, include *rhetorical coordination* (the counterparts of the rhetorical subordinations), and *adjacency pairs* which are concerned with the *interactional* dimension of the discourse.

N-ary Coordinations These are flat structures containing arbitrarily many elements, of which, at any time, only the most recent one is accessible. *Lists*, *monotonic lists*, and *narratives* fall into this category. This case could be seen as an extension to the binary coordination when one assigns a right recursive structure to the context-free grammar rules.

Scha and Polanyi (1988) give an extensive grammar for discourse parsing while Polanyi (1988) gives an informal description of the parsing process.

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[14] Unlike the sentential subordination, in discourse subordination, the subordinating element is always located to the left of the subordinated one.

Parsing follows a step by step strategy, attaching every incoming sentence to the right edge of an existing discourse parse tree (see figures 3.6 and 3.7). An important assumption of the parsing process is that at any point it only uses information on the right edge of the existing discourse tree. **This means that interlocutors just need to be aware of *the stack of information* which corresponds to the labels on the right edge of the tree, rather than the complete details of the discourse so far** (see Scha and Polanyi (1988, p. 576)).

(a) John is a very good athlete.
> (b) He can run a four-minute mile.
> (c) He throws a mean hardball, too.

(d) And John is very smart.
> (e) Won all the prizes at his graduation.
>> (f^1) Even I was surprised
>>> (f^2) that he won the Spanish prize.
>>>> (g) He didn't even like Spanish.

(h) Anyway, he's a disaster at parties.
> (i) He's too shy.
>> (j) Last week, he went to a party at Bill's house . . .

Figure 3.6: An example from Polanyi (1988, p. 620)

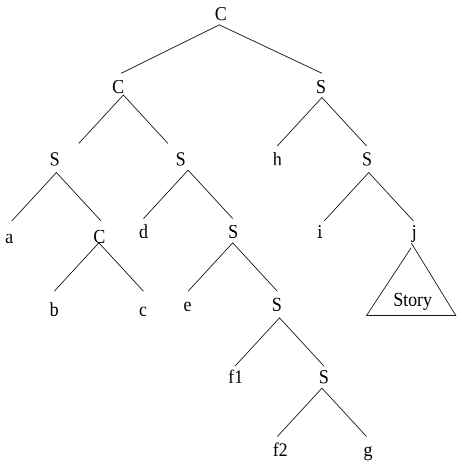

Figure 3.7: Discourse Parse Tree for example in figure 3.6

Prüst et al. (1994) present us with a very sophisticated extension of the ideas sketched above. Using a many-sorted typed logic, the authors define the Most Specific Common Denominator (MSCD), a kind of unification procedure, aiming to solve Verb Phrase Anaphora.[15] The unification procedure, or more correctly, the underlying logic, takes DCUs as logically complex terms.

The general format of DCUs, as well as the rhetorical coordination rule, are depicted below:

$$\text{cat } [\phi_1 : \alpha_1, \ldots, \phi_n : \alpha_n]$$

where "cat" is the start symbol of the context-free grammar, ϕs are the attributes, and αs are expressions standing for the value-sets of the attributes.

$$\begin{aligned} & \text{dcu}[\phi_1 : \mathit{mscd}(\alpha_1, \beta_1), \ldots, \phi_n : \mathit{mscd}(\alpha_n, \beta_n), \text{sem} : a \ \&\ b \ \&\ R(a, b)] \\ \Rightarrow \quad & \text{dcu}_1[\phi_1 : \alpha_1, \ldots, \phi_n : \alpha_n, \text{sem} : a] \\ & \text{dcu}_2[\phi_1 : \beta_1, \ldots, \phi_n : \beta_n, \text{sem} : \lambda x : R(x, b)] \end{aligned}$$

This rule parses semantic coordinations involving an explicitly indicated binary coordinating rhetorical relation R, such as "*therefore*", "*accordingly,*" for instance. Notice that the meaning of the relation is incorporated in the semantics of the clause in which it occurs; it denotes a predicate on propositions. The function "*mscd*" computes the most specific common denominator of its arguments in the hierarchy of value-expressions of relevant attribute.

Back to the programming languages analogy, we find here explicit references to concepts such as stack and parse tree. The stack, corresponding to the leaves on the right edge of the tree, models what Grosz and Sidner (1986) call focusing. Also, the discourse grammar rules presented provide a unique possibility for attachment of new incoming sentences, viz. the right edge. Inheritance, when allowed, "propagates" entities, such as discourse referents, for example, from the left parent (not necessarily the immediate previous sentence) to the right daughter. As a consequence, a block structure comes up naturally.

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[15] The missing verb phrase is recovered by computing the MSCD between parallel sentences.

3.4 Conclusions

This chapter is one of the cornerstones of this research work which claims that *it is possible to deal* ***logically*** *with discourses from a* ***dynamic*** *point of view.* The keyword here is ***discourse***, which I will take as general as possible, i.e., discourses as hierarchically complex structures. To give support for the forthcoming logical development, and provide insightful comments on the subject of discourse structure, a summary of the most important linguistic theories on discourse structure was presented.

The methodology to be approached in chapter 6 (namely, the dynamic semantics) grows upon an analogy to programming languages. The analogy made by logically based discourse theories is related to the concept of state of program execution. This analogy seems to be sufficient when one takes discourses as no more than a simple linear sequence of sentences. However, it is possible to go deeper into the programming analogy borrowing not only the state execution concept but also data structure concepts. And, as presented in this chapter, the data structure model is *the one* present (sometimes explicitly, sometimes implicitly) in all of the leading linguistic theories accounting for discourse modelling.

The typical data structure used by linguistic theories (dealing with discourse) is known as stack. The stack model is used to provide us with explanation (and impose constraints on) linguistic phenomena such as anaphoric reference, interruptions and the use of definite (and indefinite) noun phrases. However, the kind of stack used is not really very stack-like and I have argued that a list would be a better structure.

To sum up, this chapter presents a summary of the leading linguistics approaches to discourse modelling. It also provides the background setting for the (linguistic) claim that discourses are very complex entities and, how some data structures can explain the constraints ruling some linguistic phenomena.

Chapter 4

Logic Focused Approaches

4.1 Initial Remarks

The aim of this chapter is to present, discuss and compare different logical approaches dealing with discourse structure. By its very nature, discourses are *dynamic* and anaphoric relations are the most striking examples. However, we must realize the naïve use for terms such as *dynamic* and *discourse*. Most theories make discourses a "plain" first step generalization for scope binding operations. This entails, for example, that existentially introduced objects are available for reference through the whole discourse. Such an effect is achieved by the careful use of indexes or variables throughout discourse. Questions related to variable renaming and scope for existentials are in the kernel of such theories. DRT, DPL and their offspring are the leading examples for this framework. [1] This chapter paves the way for the need for more structured Discourse Theories.

4.2 Introduction

This chapter provide us with a brief overview on theories addressing the issue of *discourse structure* from a logical point of view. *Discourse* seems to be a rather vague

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[1] As it was said in this thesis' abstract, the present research work takes DPL as its basis, and therefore, this thesis should also be included in the end as one of DPL's offspring.

word used to bridge a gap between natural language semantics and pragmatics. Being a typically pragmatic notion, discourse had been left aside from traditional logical approaches. However, the more recently linguistically based approaches have taken this notion into account (see Kamp (1981), Kamp and Reyle (1993), Groenendijk and Stokhof (1991), to cite but a few) and brought it to the "semantic battle field". For this kind of semanticism, meaning is taken as a property of texts, dialogues or discourses. This does not imply that meaning could be completely determinable apart from context as happens in ordinary Predicate Calculus (PC). Actually, large amounts of all sorts of knowledge are used to determine the meaning of a discourse.

Research on issues such as ontologies, lexical/semantic preferences, metaphor, and discourse structure has been shedding light onto discourse meaning theories. Integrating several of these trends, we can cite Lascarides and Asher (1991), Grosz and Sidner (1986), Polanyi (1988), and Prüst et al. (1994). Lascarides and Asher (1991), for example, use a set of rhetorical relations and defeasible reasoning to modeling temporal relations in discourse; Grosz and Sidner (1986), and Polanyi (1988), present us with a research on discourse structure and discourse regularities to determine the overall plan and purpose of the discourse. And Prüst et al. (1994) present a theory dealing with verb phrase anaphora in which a discourse grammar, taking into account rhetorical and discourse constraints, establish the parallelism between the syntactic and semantic components. Such grammar is latter modelled by a multi-sorted typed logic which is the formal device to recover the antecedent of the anaphorical verb phrase.

Spelling out rhetorical relations and other pragmatic issues, a question one should ask is *which factors might a purely semantic "discourse" theory take into account*. This question has been acknowledged as a very hard one. To begin with, let us assume a discourse as "simple" as possible (in the sense that it could be formalized in PC). The first generalization step toward a "real" discourse should take into account a more general quantification theory in which natural language quantifiers such as *most*, *a few*, and others could be accounted for. Generalized Quantification

Theory (GQT) was developed to shed light onto this topic and will be presented in section 4.3. Bearing in mind the role played by quantification, it would not be surprising to see (some kind of) GQT underlying all further discourse generalizations.

GQT and PC share the same inadequacy to deal with *intersentential* relationships as exemplified by pronominal anaphora as in (1) below.

(1) A man walks in the park. He whistles.

It is easily seen that the pronoun *he*, in the second sentence, is beyond the scope of its referent, the NP *a man*, introduced in the first sentence. Therefore, the next generalization step would be related to scope-binding issues, if we want to deal with intersentential anaphoric relationships.

Most theories tackling the last issue adopt a *dynamic* approach; the static-dynamic contrast is meant to emphasize the static character of scope (which was traditionally tied up to syntactic structures as presented in GB)[2] and the dynamic *semantic* character of binding (therefore, providing us with ways to bring *syntactically* free variables to the scope of quantificational static structures). This has been achieved in different ways, such as, for instance, the use of *unselective quantification*, sometimes associated to reversing binding direction, and the *weakening* of existentials associated to unselective discourse quantifiers. This topic will be presented in section 4.5.

In one way or another, all dynamic approaches assume that an upcoming sentence changes the informational state someone would have built upon previous discourse. Theories addressing this issue will be presented in section 4.5.1.

Although we have pointed out only a very few generalization steps towards a semantically based discourse theory, the interaction among them covers the literature on the topic.

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[2] See section 2.4, page 16

4.3 Logical form and quantification

In English and other natural languages, quantifying expressions like *all*, *no*, *every*, *some* are always accompanied by nominal expressions that seem intuitively to restrict the universe of discourse to individuals to which the nominal applies. Although quantification in PC[3] bears a connection with quantification in English, such a connection is not straightforward. Nominals like *man* in (2) below are usually represented by a predicate in PC.

(2) a – Every man snores.

b – $\forall x[man(x) \rightarrow snore(x)]$

c – Some man snores.

d – $\exists x[man(x) \wedge snore(x)]$

However, such representations do not emphasize the intuition that nominals like *man* do play, indeed, quite a different role from predicates like *snore*. Moreover, PC's formulas change not only the quantifier but also the connective in a complex formula over which the quantifier has scope. In contrast, the English sentences differ only in the quantifying expression used. (3) shows the way to make the dependence of the quantifier on the nominal explicit with the further advantage of making clear the need for no connectives when considering simple sentences as those in (2).

(3) a – $[\forall x : man(x)]\, snore(x)$

b – $[\exists x : man(x)]\, snore(x)$

Logics using this kind of quantification impose that the range of quantifiers be restricted to those individuals satisfying the formula immediately following the quantifying expression. The quantifiers are then interpreted as usual requiring that

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[3] PC is based on Frege's insight of analyzing quantified statements as having two components where one component is a singular sentence with a place-holder element like a pronoun and the other component is such that it states how many of the possible values assigned to the place-holder are such that the singular sentence is true relative to that value of the place-holder. Truth conditions for quantified statements are defined for the singular sentence relative to some value for the place-holder. In the second stage, truth conditions are defined in terms of generalizations about values assigned to the singular sentence.

$$
\begin{aligned}
[\![\text{every } \alpha]\!] &= \{X \subseteq U \mid [\![\alpha]\!] \subseteq X\} \\
[\![\text{some } \alpha]\!] &= \{X \subseteq U \mid [\![\alpha]\!] \cap X \neq \emptyset\} \\
[\![\text{no } \alpha]\!] &= \{X \subseteq U \mid [\![\alpha]\!] \cap X = \emptyset\} \\
[\![\text{most } \alpha]\!] &= \{X \subseteq U \mid X \cap [\![\alpha]\!] \text{ is bigger than } \bar{X} \cap [\![\alpha]\!]\} \\
[\![\text{the } \alpha]\!] &= \{X \subseteq U \mid \text{ for some } u \in U, [\![\alpha]\!] = \{u\} \text{ and } u \in X\} \\
[\![\text{two } \alpha]\!] &= \{X \subseteq U \mid X \cap [\![\alpha]\!] \text{ contains two or more elements } \} \text{ or} \\
& \{X \subseteq U \mid X \cap [\![\alpha]\!] \text{ contains exactly two elements } \}
\end{aligned}
$$

Table 4.1: Generalized Quantifiers Interpretation

all (3.a) or **some** (3.b) of the assignments of values to x satisfying the restricting formula must also satisfy what follows.

Both approaches work equally well for "traditional" quantifiers as those in (2). However, quantifiers like *most*, which can not be represented in PC, are easily accounted for in the *restricted quantification* approach. Example (4.c) is true *iff* more than half the assignments from the restricted domain of men are also assignments for which *snore(x)* is true. (4.b) is a dead-end since there are no combinations between most assignments and connectives →, ∧ capable to express (4.a) in PC.

(4) a – Most men snore.

b – $most\ x[man(x)\ ?\ snore(x)]$

c – $[most\ x : man(x)]\ snore(x)$

(4) shows that *most* is not first-order definable since the semantics for it will have to resort to what essentially amounts to quantification over higher-order entities like sets (cf. Chierchia and McConnell-Ginet (1990, chap3, note 2, page 444)).

Ideas employed in (4) can be applied to other quantificational structures as long as we take (i) full NPs (Det + nominal) as logical quantifiers and (ii) sets of sets as the semantic objects interpreting NPs. Table (4.1) shows the semantic interpretation for some cases. It is not difficult, now, to see that for any NP α and predicate β, $\alpha\beta$ is true iff $[\![\beta]\!] \in [\![\alpha]\!]$.

Assuming ***compositionality*** as a methodological criteria to follow, the next step deals with the question of assigning a semantic interpretation for determiners; however, the previous exposition has already provided us with the answer. Recall

For every $Y \subseteq U$,

$$\begin{array}{lcl}
\llbracket \text{every} \rrbracket(Y) & = & \{X \subseteq U \mid Y \subseteq X\} \\
\llbracket \text{some} \rrbracket(Y) & = & \{X \subseteq U \mid X \cap Y \neq \emptyset\} \\
\llbracket \text{no} \rrbracket(Y) & = & \{X \subseteq U \mid X \cap Y = \emptyset\} \\
\llbracket \text{most} \rrbracket(Y) & = & \{X \subseteq U \mid X \cap Y \text{ is bigger than } \bar{X} \cap Y\} \\
\llbracket \text{the} \rrbracket(Y) & = & \{X \subseteq U \mid \text{ for some } u \in U, Y = \{u\} \text{ and } u \in X\} \\
\llbracket \text{two} \rrbracket(Y) & = & \{X \subseteq U \mid X \cap Y \text{ contains two or more elements } \} \text{ or} \\
 & & \{X \subseteq U \mid X \cap Y \text{ contains exactly two elements } \}
\end{array}$$

Table 4.2: Determiners Interpretation

that determiners combine with nominals, which are taken as properties (i.e., sets), to yield NPs, which are taken as sets of sets. Therefore, a function from sets of individuals to sets of sets is the natural candidate for the semantic interpretation of determiners. Table (4.2) displays the semantic interpretation for the determiners in table (4.1). It is now clear that we can analyse the meaning of *most men*, for instance, as specified in terms of the meanings of *most* and *men* as $\llbracket \text{most} \rrbracket(\llbracket \text{men} \rrbracket)$ which yields the generalized quantifier $\{X \subseteq U \mid X \cap \llbracket men \rrbracket \text{ is bigger than } \bar{X} \cap \llbracket men \rrbracket\}$.

The previous discussion made a point towards moving from determiners as quantifiers, as it occurs in PC, to full NPs as quantifiers; this identity is best known as *generalized quantifiers*.

Complex NPs constructions involving possibly several determiners are smoothly accounted for in the generalized quantifier framework. Complex determiners such as *some man and some woman*, for instance, are interpreted as

$$\{X \subseteq U \mid \llbracket man \rrbracket \cap X \neq \emptyset\} \cap \{X \subseteq U \mid \llbracket woman \rrbracket \cap X \neq \emptyset\}$$

and therefore equivalent to

$$\{X \subseteq U \mid \llbracket man \rrbracket \cap X \neq \emptyset \text{ and } \llbracket woman \rrbracket \cap X \neq \emptyset\}$$

For negation, as in *not every woman*, set-theoretic complementation will do the task since

$$\mathcal{P}(U) - \{X \subseteq U \mid \llbracket woman \rrbracket \subseteq X\} = \{X \subseteq U \mid \llbracket woman \rrbracket \not\subseteq X\}$$

It is not hard to see that embedding generalized quantifiers into an intentional logic would give us a way to represent natural language sentences. With this in mind, we can analyse (5) as showed in (6) and (7) (where we have assumed that *and* and *but* are truth-conditionally identical and also an intentional framework along (the Montagovian) Dowty, Wall and Peters (1981) line.) Also, when possible, we present a PC truth-conditionally equivalent formula.

(5) a – Most but not all men snore.

b – Not all but some men snore.

(6) a – Most but not all men snore.

b – $[\text{most}' \wedge [\neg\text{every}']](\text{man}')(\text{snore}')$

c – $[\text{most}'(\text{man}') \wedge [\neg\text{every}'](\text{man}')](\text{snore}')$

d – $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \mathit{card}(X \cap \llbracket\text{man}'\rrbracket) > \mathit{card}(\bar{X} \cap \llbracket\text{man}'\rrbracket)\} \cap$
$\{X \subseteq U \mid \llbracket\text{man}'\rrbracket \nsubseteq X\}$

e – $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \mathit{card}(X \cap \llbracket\text{man}'\rrbracket) > \mathit{card}(\bar{X} \cap \llbracket\text{man}'\rrbracket)\}$
and $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \llbracket\text{man}'\rrbracket \nsubseteq X\}$

f – PC formula: ????

(7) a – Not all but some men snore.

b – $[[\neg\text{every}'] \wedge \text{some}'](\text{man}')(\text{snore}')$

c – $[[\neg\text{every}'(\text{man}')] \wedge \text{some}'(\text{man}')](\text{snore}')$

d – $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \llbracket\text{man}'\rrbracket \nsubseteq X\} \cap \{X \subseteq U \mid \llbracket\text{man}'\rrbracket \cap X \neq \emptyset\}$

e – $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \llbracket\text{man}'\rrbracket \nsubseteq X\}$
and $\llbracket\text{snore}'\rrbracket \in \{X \subseteq U \mid \llbracket\text{man}'\rrbracket \cap X \neq \emptyset\}$

f – $\llbracket\text{man}'\rrbracket \nsubseteq \llbracket\text{snore}'\rrbracket$ and $\llbracket\text{man}'\rrbracket \cap \llbracket\text{snore}'\rrbracket \nsubseteq \emptyset$

g – PC formula: $\neg\forall x[\text{man}'(x) \rightarrow \text{snore}'(x)] \wedge \exists x[\text{man}'(x) \wedge \text{snore}'(x)]$

Concluding this section, we point out that the generalized quantifiers approach allow us to study a wide variety of empirical properties of natural language. Questions related to *polarity*, *conservativity*, and *monotonicity*, for example, have been analysed with it; for instance, determiners such as *every*, *some*, and *no* are conservative (assuming conservativity defined in such a way that $\delta(\alpha)(\beta) \leftrightarrow \delta(\alpha)(\alpha \wedge \beta)$ equivalence holds) since sentences like ***Det*** *man snores* are truth-equivalent to ***Det*** *man is a man who snores*, where ***Det*** $\in$ **{*every, some, no*}**. Assume now that a definition for *right upward monotonicity* has been given allowing the inference pattern $\delta(\alpha)(\beta \wedge \delta) \rightarrow \delta(\alpha)(\beta)$. As illustrated in (8), determiners like *some*, *many*, and *every* have this property while others such as *no*, and *few* lack it. Since the discussion of this issue would unnecessarily extend the present section we refer the interested reader to Gärdenfors (1987) and the references therein.

(8) a – Some student is Italian and blond → some student is blond.
b – No students are Italian and blond $\not\rightarrow$ no students are blond.

Although a nice framework for the study of quantification, generalized quantifiers approach (GQT), in the format presented here, does not solve all problems related to quantification. The most notorious problems are posed by pronouns. Traditionally, pronouns are identified with variables which ought to be bound to some quantificational structure; *deictic* pronouns, on the other hand, are analysed as free variables. Anaphoric pronouns, however, are "odd" since: (i) they refer to some previously introduced entity and therefore suggesting that they might have been bound into the scope of some quantifier; (ii) they occur outside the scope of any quantifier. In other words, anaphoric pronouns are syntactically free variables that ought to be bound.

The "equation" involving quantifiers and pronouns has been tackled from different viewpoints. Firstly, there are the *dynamic* theories. Secondly, there are the theories originated along the lines of Heim (1982), and Kamp (1981) (such as Discourse Representation Theory (DRT)), and File Change Semantics, Heim (1983). Independently of the viewpoint adopted, pronouns are still approached from one of two perspectives, namely, *E-type* and *bound*. The next sections are devoted to a short but concise explanation of these matters.

4.4 Discourse Representation Theory

DRT is nowadays a theory of semantic content of natural language sentences, discourses and texts, as well as, more recently, of the content and structure of thought. As a theory of the content of sentences and texts, DRT is designed to identify and encode the semantic connections between the successive sentences of a text (cf. Reyle and Gabbay (1994, p. 343)).

DRT was the first of a group of theories to approach a dynamic notion on meaning. However, motivation for the development of a dynamic semantics was already pointed out in Stalnaker (1974) and Stalnaker (1976).

DRT analyzes meaning in two steps. The first one, a semantic representation for a discourse is built up through the Discourse Representation Structure (DRS) construction algorithm. The construction algorithm is a set of rules for constructing the box representation generated and related to noun phrases.[4] *This representation is built up sentence by sentence.* If j sentences have been processed to yield the DRS K_j, then the processing of S_{j+1} will yield a DRS that combines with K_j to form an extended DRS K_{j+1}. The second step is related to DRT interpretation proper. It is accomplished by the *correctness definition*, which provides instructions for homomorphically embedding a DRS in a model so as to yield correct truth conditions for a discourse. In other words, the interpretation of a discourse in DRT takes two steps: firstly, the construction of a DRS, then the proper embedding of the DRS into a model. The dynamic interpretation effect is accomplished by the combination of the correctness definition with the construction algorithm. The dynamic meaning of S_{j+1} is that function which takes us from the truth conditions of K_j to the truth conditions of K_{j+1}.

In the literature, two aspects of DRT are questioned. Firstly, there is the compositional aspect. It is said that the construction algorithm is not fully compositional (Groenendijk and Stokhof (1990), to cite one). Secondly, it is questioned if the rep-

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[4] The determiner heading the NP is, usually, the responsible for the particular box representation used.

resentational level is really necessary to achieve the dynamics of anaphoric relations. For the first problem, fully compositional extensions have been developed by some researchers while others have been arguing for the need of such intermediate level.[5]

The original DRT framework presented in Kamp (1981) contained common nouns, verbs, the determiners *a* and *every*, and the English conditional expression *if . . . then*. Also, the *construction procedure* followed a top down analysis (contrasting to Asher (1993) who adopted a bottom up analysis). To get acquainted with the way DRT works, let us start by presenting the DRS for (9) below (ignoring the semantic contributions of tense):

(9) A boy kicked Fred.

$$(\mathrm{K1})\quad \begin{array}{|l|}\hline \quad \mathrm{x,\ y} \\ \hline \mathrm{boy(x)} \\ \mathrm{Fred(y)} \\ \mathrm{kick(x,y)} \\ \hline \end{array}$$

(K1) describes graphically an abstract, information structure, a DRS, with two parts. One part is called the *universe* of the DRS, the other its *condition set*. So, a DRS can be formally stated as an ordered pair consisting of its universe and condition set, written as $\langle \mathrm{U_K}, \mathrm{Con_K} \rangle$. The DRS (K1) has as its universe two "discourse individuals," x and y, and as its condition set a collection of property ascriptions to x and y. The conditions in $\mathrm{Con_{K1}}$ are formed from unary and binary DRS *predicates* and discourse referents as arguments. For the fragment considered here, DRS predicates are generated from nouns or verbs. The condition set of (K1) says that x is a boy, y is Fred and x kicks y. To give the truth conditions for (9), we need to define a *proper embedding* for (K1). A proper embedding for (K1) in an (extensional) model $\mathrm{M} = \langle \mathrm{D}, [\![ ]\!] \rangle$, consisting of a domain D of individuals and an interpretation function $[\![ ]\!]$. $[\![ ]\!]$ is a function $\mathbf{g}$ that maps x and y onto elements of the domain of M such that $\mathbf{g}$(x) is a boy in M, $\mathbf{g}$(y) is Fred in M, and $\mathbf{g}$(x) kicks $\mathbf{g}$(y) in M. If we define (9) to be true in M just in case (K1) has a proper embedding in M, we get the right truth conditions.

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[5] See Asher (1993) for a defense for indirect interpretation, via an intermediate level, especially for pronoun and anaphora resolution.

Having shown a DRS and the truth conditions it determines for a simple sentence, we are ready to move towards the treatment of a multisentential discourse. Adding the sentence *Fred cried* to (9) we get the discourse:

(10) A boy kicked Fred. Fred cried.

We already know that (K1) is the structure the first sentence yields. To get a DRS for all of (10), the DRS created from the first sentence serves as a context for processing the second sentence. In processing the second sentence, the conditions and discourse referents introduced by the second sentence are entered into the condition list and universe already created in processing the first sentence; such processing produces (K2) below.

(K2)

x, y, z
boy(x)
Fred(y)
kick(x,y)
cry(z)
Fred(z)

The advantage of using the DRS built up from previous discourse as a context for the interpretation of the next sentence arises in the process of anaphora resolution. Anaphoric pronouns introduce a peculiar sort of condition. Many conditions come with a determinate, context free-content, but others do not. In particular those that are introduced by anaphoric pronouns introduce a discourse referent that must be linked with some other discourse referent in order to give the condition a complete meaning. Such conditions are called *incomplete conditions* and are of the form $\mathbf{z} = ?$. All other conditions are complete conditions.

The distinction between incomplete conditions and complete conditions carries over to DRSs. Complete DRSs are those containing only complete conditions; incomplete DRSs contain at least one incomplete condition. The condition contributed by the pronoun *he* in (11), $\mathbf{z} = ?$, is responsible for the DRS (K3) being incomplete.

(11) A boy kicked Fred. He cried.

(K3)

x, y, z
boy(x)
Fred(y)
kick(x,y)
cry(z)
$\mathbf{z} = \mathbf{?}$

In order to complete the DRS, the question mark in this condition needs to be replaced by a discourse referent. The task of anaphora resolution is to find an appropriate discourse referent other than **z** and to turn $\mathbf{z} = \mathbf{?}$ into an identity assertion. For (11), y is an appropriate discourse referent[6] introduced by the processing of the first sentence. After identifying y with **z**,[7] we have the following completed DRS for (11):

(K′3)

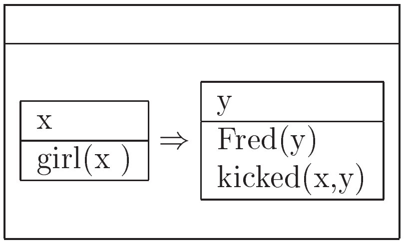
x, y, z
boy(x)
Fred(y)
kick(x,y)
cry(z)
$\mathbf{z} = \mathbf{y}$

(K′3) has a proper embedding in a model M just in case M contains Fred and a boy such that the boy kicked Fred and Fred cried.

There are more complex DRSs that themselves contain DRSs. Any condition containing one or more DRSs as a constituent is called a *complex condition*. (12) gives rise to one:

(12) Every girl kicked Fred.

(K4)



The DRS for (12) has two DRSs that are constituents of a complex condition. Such constituents are called *sub*DRS*s*. Let us call them (K4.1), the one on the left, and (K4.2), the one on the right. Naturally, the notion of a proper embedding for such

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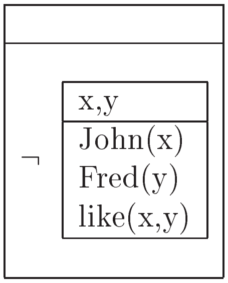
[6] A discourse referent, from DRT perspective, is an element of the DRS that serves as a context for the processing of discourse subsequent sentences.

[7] Tony Cohn pointed to me that the reading $x = z$ is available as well, because *he*, the boy, might have regretted his action.

a DRS must take these subDRSs into account. It does so in the following way: f is a proper embedding for (K4) in a model M iff every extension (superset) of f that properly embeds (K4.1) in M can be extended to a proper embedding of (K4.2) in M. Not only do many determiners like *every* introduce such complex DR-theoretic structures, but also conditionals, ⇒, and other operators, such as negation, ¬, do as well. Thus, a sentence like (13) yields the DRS below it:

(13) John does not like Fred.

(K5)



(K5) has a proper embedding just in case there are no objects x and y such that x is John, y, Fred, and x likes y.

I hope that the reader not acquainted to DRT has now grasped the basics of the theory. We might naturally extend this fragment by adding a stock of operators corresponding to determiners. And, we might also present the basics of the DRT construction algorithm. However, such a course of action is not relevant for the purposes of this chapter. For the definitive and complete exposition of the theory, see Kamp and Reyle (1993).

To sum up, DRT characteristics, as might be inferred from the previous "crash course,"[8] could be stated as:

- it handles indefinite noun phrases as non-referential, non-quantificational, restricted free variables,
- it uses operators to bind indefinite noun phrases which are much richer than those of predicate logic,
- it treats anaphoric pronouns as plain bound variables,
- it uses polyadic connectives and quantifiers which may bind multiple variables

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[8] All examples in this section are from Asher (1993).

simultaneously.

Although DRT has achieved a respectable position, it is not problem free, as already stated. One of the first criticisms of it was related to the compositionality principle. It is frequently said that DRT does not respect it. However, there are versions which respect it, as for example, the system proposed by Pinkal (1991). With respect to (polyadic) quantification, Partee (1984) calls attention to the proportion problem which results from the view of quantifiers as binary relations between relations of indefinite arity. The truth conditions for (14.a) are given by (14.b) and are correct when Q is *all*, *some*, *no*, or *not all*. But they do not hold for *most*. For this quantifier, (14.b) would be incorrect if there are two men, one of which owns two cars that he washes on Sundays while the other owns just one car which he washes on Saturdays.

(14) a – Q men who own a car wash it on Sunday.
b – $[\mathrm{Q}xy : \mathrm{M}(x) \wedge \mathrm{C}(y) \wedge \mathrm{O}(x,y)]\ \mathrm{W}(x,y)$

Within DRT the proportion problem is discussed by Kadmon (1990) where she proposes, for the determiner *most*, an analysis in which material from the restriction is copied to the scope.[9] This strategy leads to the weak reading for *most men who own a car wash it on Sunday*: the cars owners need not wash all of their cars in order to make it true. In the literature, the strong reading, where all cars have to be washed, is also suggested. The problem of weak and strong readings is discussed in Rooth (1987), Chierchia (1992), and Dekker (1993), among many others, and studied in depth by Kanazawa (1993). Notice, however, that Kadmon's suggestion still leaves the problem whether there is a uniform way in which it can be made to work for all quantifiers. For DPL, such a method exists (cf. Does (1993, p. 7)).

4.5 Dynamic theories I – the bound perspective

Dynamic theories depart from traditional ones since *the information change potential* of a sentence is regarded as constituting its meaning. Put in other way, the basic idea

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[9] This analysis closely resemble the E-type one.

is that the meaning of a sentence does not uniquely depend on its truth conditions, but rather "in the way it changes (the representation of) the information of the interpreter" (Groenendijk and Stokhof (1991)).

The key notion in the above characterization is that of information. Depending on the ontology, information may concern the values of variables, or even possible worlds, or even world-time intervals, or whatever parameters we decide to take into account. As Groenendijk and Stokhof (1990) already pointed out, "information is about indices".

To get right to the point, assume that I is a set of indices; assume also any subset of I as characterizing an information state. The power set of I, $(\mathcal{P}(I))$, is the set of all information states; it is naturally the least informative state since all possibilities are present in it. On the other hand, singletons are the most informative states while the empty set corresponds to the absurd state. It is not hard to see that information updating is conveyed through functions from information states to information states.

Theories of dynamic meaning could be grouped together according to the properties displayed in Table (4.3). Each property aims to model some peculiar aspect of the dynamics of discourse. So, for example, Veltman's (1990) Update Semantics, (US), and Veltman's (1996) Defaults in Update Semantics, which are dynamic semantic theories for the language of epistemic propositional logic and default reasoning, resp., involve update of information about the world according to the *"eliminative"* model. This means that updating an information state s with a sentence will take us to an information state which contains at least as specific information about the world as s. On the other hand, Groenendijk and Stokhof's Dynamic Predicate Logic, (DPL), which is a dynamic semantic interpretation of the language of first order predicate logic (keeping stock of the possible values of variables introduced while a discourse unfolds), is a "pointwise" *distributive* system. In other words, interpretation in DPL may involve the introduction of new possibilities as we will soon show. And finally, Dekker (1993), Groenendijk and Stokhof (1990), and Does

- τ is eliminative iff for every state s, $\tau(s) \subseteq s$
- τ is distributive iff for every state s, $\tau(s) = \bigcup_{i \in s} \tau(\{i\})$

Table 4.3: Update varieties

(1993) present us with a unified account of DPL in a update semantics format.

4.5.1 An Update System for Epistemic Propositional Logic

Veltman's (1990) US is a propositional logic with an extra epistemic operator $\diamond$ (might). US deals with information about the world; the meaning of a US formula is characterized through update of information about the world. All US formulas in which $\diamond$ does not occur express factual information about the world. On the other hand, formulas $\diamond\phi$ express that one's information about the world is compatible with ϕ, something along the line "as far as my information is concerned, it might, but need not, be the case that ϕ".

Update of information about the world consists in eliminating possibilities. For instance, the interpretation of an atomic sentence p in a state s involves the update of s brought about by eliminating the worlds from s which are inconsistent with p. The resulting state only contains possible worlds in which p is true.

US interpretation is defined as an update function $[\![\ ]\!]$ on the domain of information states. It is defined with respect to a model $\mathcal{M} = \langle W, F \rangle$ consisting of a set of worlds W and a interpretation function F that assigns sets of worlds to proposition letters. In what follows, $s[\![\phi]\!]_{\mathcal{M}}$ indicates the result of updating an information state s with ϕ with respect to a model $\mathcal{M}$, that is, the result of applying the function $[\![\phi]\!]_{\mathcal{M}}$ to s. As usual, reference to $\mathcal{M}$ is omitted whenever this does not lead to confusion. Interpretation is defined as follows:

Definition 1: Update Semantics

$$\begin{array}{lcl} s[\![p]\!] & = & \{i \in s \mid i \in F(p)\} \\ s[\![\neg\phi]\!] & = & s - s[\![\phi]\!] \\ s[\![\phi \wedge \psi]\!] & = & s[\![\phi]\!][\![\psi]\!] \\ s[\![\diamond\phi]\!] & = & \begin{cases} s & \text{if} \quad s[\![\phi]\!] \neq \emptyset \\ \emptyset & \text{if} \quad s[\![\phi]\!] = \emptyset \end{cases} \end{array} \qquad \square$$

Definition 1, *which is a functional characterization for Update Semantics stated in a postfix notation*, says that proposition letters are assigned an information content that intersects with the input information state. Negation is associated with state subtraction (or set difference) and sentence conjunction with sequencing, i.e., function composition. To interpret a conjunction of two formulas ϕ and ψ in a state s, we first interpret ψ in the state resulting from the update of s with ϕ. Interestingly, the operator $\diamond$ acts as a test: in an information state s, $\diamond\phi$ tests whether s can be *consistently* updated with ϕ. If ϕ is acceptable in s, then $\diamond\phi$ is true in that state and the interpretation of $\diamond\phi$ in s returns s. However, if we already know that ϕ is false, then $\diamond\phi$ is rejected and its interpretation returns the absurd state, i.e., the empty set.

A remarkable point about $\diamond\phi$ is its "instability" exemplified in (15).

(15) a – A dog is barking at the moon. ... It might be Fido. ... It is Rex.

b – A dog is barking at the moon. ... It is Rex. ... It might be Fido.

The instability is due to the fact that at some stage $\diamond\phi$ may be true (if ϕ is not excluded at that stage), whereas at a later stage it is false (if the possibility that ϕ has been excluded in the meantime). As a natural consequence from instability, conjunction can not be commutative, as easily seen from (15.a) and (15.b). Also, *distributivity* does not hold; a formula $\diamond\phi$ tests a global property of a state s, namely, its consistency with ϕ, which does not hold of all subsets of s. Let us assume, for example, that ϕ is a predicate, $s = \{i, j\}$, $\{i\}[\![\phi]\!] = \{i\}$ and $\{j\}[\![\phi]\!] = \emptyset$. Therefore, $s[\![\phi]\!] \neq \emptyset$ and $s[\![\diamond\phi]\!] = s$ since $s[\![\phi]\!] = \{i \in s \mid i \in F(\phi)\} = \{i\} \neq \emptyset$. However, $\{i\}[\![\diamond\phi]\!] = \{i\}$ and $\{j\}[\![\diamond\phi]\!] = \emptyset$ and therefore $\bigcup_{i\in s}\{i\}[\![\diamond\phi]\!] = \{i\}$.

To understand the contrast seen in (15.a) and (15.b) we provide the reader with the following definitions:

Definition 2: ϕ is consistent iff for some state s, $s[\![\phi]\!] \neq \emptyset$ ☐

Bearing in mind that $s[\![\phi \wedge \psi]\!] = s[\![\phi]\!][\![\psi]\!]$ we can see that $\diamond p \wedge \neg p$ is consistent whereas $\neg p \wedge \diamond p$ is not.

Another basic notion of US is that of acceptance:

Definition 3: ϕ is accepted in s, $s \models \phi$, iff $s \subseteq s[\![\phi]\!]$ ☐

And, in terms of acceptance, the following notion of entailment is defined

Definition 4: $\phi_1, \ldots, \phi_n \models \psi$ iff

for all models $\mathcal{M}$ and states s, $s[\![\phi_1]\!]_{\mathcal{M}} \ldots [\![\phi_n]\!]_{\mathcal{M}} \models \psi$ ☐

This definition says that a conclusion follows from a sequence of premises $\phi_1, \ldots, \phi_n$ if whenever an information state s is updated with $\phi_1, \ldots, \phi_n$, in that order, the result is an information state which accepts ψ.

For more details and alternative notions of entailment available in the original US formulation, see van Benthem (1991), and Veltman (1990).

As a further development into the framework of US, Veltman (1996) presents the reader with systems of update semantics covering sentences in which modal qualifications such as *presumably*, *probably*, *must*, *may*, as well as *might*, occur. Next section presents a summary of Veltman's (1996) work.

4.5.2 Veltman's (1996) Update Semantics Framework

Veltman (1996) is a refined and self-contained work on the update semantics subject aiming: (i) to introduce the framework of update semantics as well as to point out the kind of semantic phenomena which may successfully be analysed in it; (ii) to give a detailed account of one such phenomenon, namely *default reasoning*.

To better understand Veltman's 96 (US based) analysis for default reasoning, we should point out the differences between his approach and "traditional theories." Firstly, US analysis differs from traditional theories in virtue of its definition of *logical validity*. The standard "static" definition states that an argument is valid if its premises cannot be true without its conclusion being true as well. As a consequence, the heart of these theories consists in a specification of truth conditions. The definition of US states that *one knows the meaning of a sentence if one knows* ***the change*** *it brings about in the information state of anyone who accepts the news conveyed by it.* In this sense, meaning becomes a dynamic concept since the meaning of a sentence is modelled as an update function on information states. Secondly, for traditional theories, default reasoning is considered a special kind of reasoning handling ordinary sentences. Veltman's framework, on the other hand, equates default reasoning to ordinary reasoning handling special kind of sentences (sentences including special operators such as *might*, *presumably*, *normally*, and *necessarily*).

To define an update semantics for a language $\mathcal{L}$, Veltman (1996) specifies a set Σ of relevant information states, and a function $[\,]$ that assigns to each sentence ϕ an operation $[\phi]$ on Σ. The resulting triple $\langle \mathcal{L}, \Sigma, [\,] \rangle$ is called an update system; if σ is a state and ϕ a sentence, '$\sigma[\phi]$' denotes the result of updating σ with ϕ. Since $[\phi]$ is a function and σ the argument, it would have been more in line with common practice to write '$[\phi](\sigma)$', but the postfix notation is more convenient for dealing with texts. Now we can write '$\sigma[\psi_1]\ldots[\psi_n]$' for the result of updating σ with the sequence of sentences $\psi_1 \ldots \psi_n$ (cf. Veltman (1996, page 221)).

The naïve characterization given above might lead us to problems. This might happen if we identify the process of updating an information state with the addition of informational content of a sentence ϕ to the information we already have. However, this kind of updating is true only for ***additive*** update systems.

Definition An update system $\langle \mathcal{L}, \Sigma, [\,] \rangle$ is *additive* iff there exists a state $\mathbf{0}$, the *minimal* state, in Σ and a binary operation $+$ on Σ such that

(i) the operation $+$ has all the properties of a join operation:

$$\begin{aligned} \mathbf{0} + \sigma &= \sigma \\ \sigma + \sigma &= \sigma \\ \sigma + \tau &= \tau + \sigma \end{aligned}$$

$$(\rho + \sigma) + \tau \quad = \quad \rho + (\sigma + \tau)$$

(ii) for every sentence ϕ and state σ, $\sigma[\phi] = \sigma + \mathbf{0}[\phi]$ $\Box$

Whenever (i) holds Σ is called an information lattice. Cases such that $\rho + \tau = \tau$ are denoted by $\sigma \leq \tau$ (in Veltman's words, τ is *at least as strong as* τ).

Veltman (1996, pages 222, 223) defines some principles, namely, the principles of idempotence, persistence, strengthening and monotony, which bear a close relationship to additive update systems as stated in proposition 1.2, page 223. For convenience, we will repeat these principles, proposition 1.2, as well as the characterization of the notion of *acceptance* below.

Acceptance in US Let σ be any state and ϕ be any sentence. *ϕ is accepted in σ*, $\sigma[\phi] \Vdash \phi$, iff $\sigma[\phi] = \sigma$.

The Principle of Idempotence: For every state σ and sentence ϕ, $\sigma[\phi] \Vdash \phi$.

The Principle of Persistence: If $\sigma \Vdash \phi$ and $\sigma \leq \tau$, then $\tau \Vdash \phi$.

The Principle of Strengthening: $\sigma \leq \sigma[\phi]$.

The Principle of Monotony: If $\sigma \leq \tau$, then $\sigma[\phi] \leq \tau[\phi]$.

Proposition 1.2 *An update system $\langle \mathcal{L}, \Sigma, [\,] \rangle$ is additive iff* (i) *Σ is an update lattice on which $[\,]$ is total, and* (ii) *the principles of Idempotence, Persistence, Monotony and Strengthening hold.*

To say a few words on some of these principles, notice that persistence, for example, naturally explains the processing of the following sequence of sentences

Somebody is knocking at the door. … Maybe it's John. … It's Mary.

This sequence shows that expectations can be overruled by facts. On the other hand, it is not natural to accept that someone still expects something else after knowing the facts, as exemplified by the following sequence of sentences

Somebody is knocking at the door. … Maybe it's John. … It's Mary. … Maybe it's John.

Idempotence, for instance, offers a natural explanation for paradoxical sentences such as *"This sentence is false"* since, it is impossible to change the information state we are in such a way that we come to accept the sentence. As shown in Groeneveld (1994), the paradoxicality of this sentence resides in the fact that every time we try to accommodate the information it conveys, we have to change our mind (cf. Veltman (1996, p. 223)).

Being a dynamic theory, US should present us with different characterization for notions such as *validity.* Indeed, in Veltman (1996), we can see three different definitions which we repeat below for convenience.

Validity 1: An argument is valid$_1$ iff updating the minimal state $\mathbf{0}$ with the premises $\psi_1, \ldots, \psi_n$ in that order, yields an information state in which the conclusion ϕ is accepted. Formally:

$$\psi_1, \ldots, \psi_n \Vdash_1 \phi \text{ iff } \mathbf{0}[\psi_1] \ldots [\psi_n] \Vdash \phi$$

Validity 2: An argument is valid$_2$ iff updating any information state σ with the premises $\psi_1, \ldots, \psi_n$ in that order, yields an information state in which the conclusion ϕ is accepted. Formally:

$$\psi_1, \ldots, \psi_n \Vdash_2 \phi \text{ iff for every } \sigma,\ \sigma[\psi_1] \ldots [\psi_n] \Vdash \phi$$

Validity 3: An argument is valid$_3$ iff one cannot accept all its premises without having to accept the conclusion as well. Formally:

$$\psi_1, \ldots, \psi_n \Vdash_3 \phi \text{ iff } \sigma \Vdash \psi_n \text{ for every } \sigma \text{ such that } \sigma \Vdash \psi_1, \ldots, \sigma \Vdash \psi_n$$

Interestingly, these three notions turn out to be equivalent for any additive update system (cf. Veltman (1996, proposition 1.3, page 224)). However, this fact is not always true. Notice, for example, that validity$_3$ is monotonic, validity$_2$ is at least left monotonic while validity$_1$ is neither right nor left monotonic.[10]

As it has been pointed out, the three validity notions are equivalent for additive update systems; therefore, one can develop an update system based on any one of

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[10] Adding "new" premises, in any order, do not change the validity$_3$ argumentation. On the other hand, order is fundamental for validity$_2$ argumentation. Finally, validity$_1$ conforms to the principle of *Sequential Monotony*, which can be characterized by the following property: if $\psi_1, \ldots, \psi_n \Vdash_1 \phi$ and $\psi_1, \ldots, \psi_n, \theta_1, \ldots, \theta_k \Vdash_1 \chi$, then $\psi_1, \ldots, \psi_n, \phi, \theta_1, \ldots, \theta_k \Vdash_1 \chi$. Also, validity$_1$ complies with the following version of the principle of *Cut Elimination*, which Veltman calls *Sequential Cut*: If $\psi_1, \ldots, \psi_n \Vdash_1 \phi$ and $\psi_1, \ldots, \psi_n, \phi, \theta_1, \ldots, \theta_k \Vdash_1 \chi$, then $\psi_1, \ldots, \psi_n, \theta_1, \ldots, \theta_k \Vdash_1 \chi$.

these notions. Interestingly, this is not the case for nonadditive systems. For the systems developed in Veltman (1996), validity$_1$ is the right choice to adopt since the schematic argumentation below is not valid$_2$ or valid$_3$.

$$\begin{array}{ll}
\text{premiss 1:} & P\text{'s normally are } R \\
\text{premiss 2:} & x \text{ is } P \\
\hline
\text{conclusion:} & \text{Presumably, } x \text{ is } R
\end{array}$$

Notice that this argument remains valid$_1$ even when one learns more about the object x, provided there is no evidence that the new information is relevant to the conclusion, as exemplified by

$$\begin{array}{ll}
\text{premiss 1:} & P\text{'s normally are } R \\
\text{premiss 2:} & x \text{ is } P \\
\text{premiss 3:} & x \text{ is } Q \\
\hline
\text{conclusion:} & \text{Presumably, } x \text{ is } R
\end{array}$$

However, if on top of the premises 1, 2 and 3 the rule ‘Q’s normally are not R’ is adopted, the argument is not valid$_1$ any more. If all one knows is

$$\begin{array}{ll}
\text{premiss 1:} & Q\text{'s normally are not } R \\
\text{premiss 2:} & P\text{'s normally are } R \\
\text{premiss 3:} & x \text{ is } P \\
\text{premiss 4:} & x \text{ is } Q
\end{array}$$

then it remains open whether one can presume that x is R. It seems obvious that the object x must be an exception to one of the rules. However, there is no reason to expect it to be an exception to one rule rather than to the other. Adding further default rules may make the balance tip. If, for instance, we add ‘Q’s normally are P. as a premise, we get the following valid$_1$ argument:

premiss 1: Q's normally are P
premiss 2: Q's normally are not R
premiss 3: P's normally are R
premiss 4: x is P
premiss 5: x is Q
conclusion: Presumably, x is not R

In the presence of the principle 'Q's normally are P', the principle Q's normally are not R takes precedence over the principle P's normally are R. A concrete example given by Veltman (1996) is shown in the following reading: 'x is P becomes 'x is an adult', 'x is Q' becomes 'x is a student' and 'x is R becomes 'x is employed'.

As remarked before, none of these arguments is valid$_2$ or valid$_3$. This is so because in their definitions a quantification over the set of states is at stake. This means that in checking the validity$_2$ or validity$_3$ of an argument, one must reckon with the possibility that more is known than is stated in the premises. Conclusions drawn from default rules, however, are typically drawn 'in the absence of any information to the contrary'; they may have to be withdrawn in the light of new information. Therefore, in evaluating a default argument it is important to know exactly which information is available. That is why Veltman (1996) concentrates on the notion of validity$_1$.

After presenting the basic ideas just summarized, Veltman (1996) proceeds to the presentation of formal systems dealing with the epistemic possibility operator might and default reasoning. Since the epistemic system is basically the same as the one summarized in section 4.5.1 and default reasoning is not relevant to the present work we proceed to the next section which deals with a distributive update system for predicate logic. The distributive system to be presented is the basis of the present research work.

4.5.3 Dynamic Predicate Logic Proper

4.5.3.1 Prolegomena

Groenendijk and Stokhof's DPL is an insightful landmark on the compositional logically based analysis of natural language discourse structure. It was the first theory to take into account the intersentential, as well as intrasentential, anaphoric relation among pronouns and existential noun phrases taken as their referents. As might be correctly inferred, the theory was developed to conform to the compositional criteria sorting out the anaphoric relationship between pronoun and existential noun phrase.

To achieve the goal, Groenendijk and Stokhof made very simple assumptions based on the idea that anaphoric pronouns are somehow bound variables falling outside the syntactic scope of existential noun phrases they are related to. As a consequence of this assumption, the binding process ought to be modelled at the semantic level, and therefore, they could stick to the traditional predicate logic syntax since they didn't take into account generalized quantifiers.

Also, discourse is taken as simple as possible: *discourse is a linear sequence of sentences.* So, the natural language sequence operator "." could be mapped to the traditional conjunction operator "$\wedge$." It is clear that any semantic theory dealing with the *dynamic* binding power emanating from an existential quantifier should also make provision for the conjunction ability of passing on that binding power. In a nutshell, the syntactic part of DPL could be made identical to the standard predicate logic; however, the semantic part of DPL could not be made identical to the standard semantics of predicate logic.

Differently from PC, DPL is not a truth conditional semantic theory. DPL assumes that an upcoming sentence changes the *informational state* someone would have built upon previous discourse. So, it is the information change potential of a sentence that is regarded as constituting its meaning. It is clear that the nature of any characterization of information state would depend on the ontology. For DPL, the ontology is concerned with the values of variables.[11] Having pointed out all this

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[11] Recall that pronouns are taken as free variables dynamically bound to existentials.

aspects, it is not difficult to imagine what Groenendijk and Stokhof have taken for defining an information state: *an information state is nothing but a set of assignment functions from the set of* DPL *variables to the domain of individuals.* Therefore, the interpretation of any DPL formula would take into account the information state someone is at. However, instead of following the functional approach just explained in section 4.5.1, Groenendijk and Stokhof (1991) present the notion of information state in a disguised fashion. In the relational format presented in Groenendijk and Stokhof (1991), information states are merged into pairs of *input-output* assignment functions. The equivalence between the relational format and the functional format developed in Groenendijk and Stokhof's later works[12] is granted by the following definition, taken from Vermeulen (1993).

Definition: Let G be the set of assignment functions. Let $\sigma \in \mathcal{P}(G)$ be an information state. Let $[\![\phi]\!]_{gs} \in G \times G$ be the interpretation of ϕ as a relation on assignments. Then $(\!(\phi)\!)_{gs}$, the interpretation of ϕ as an update function, is defined by the following property:

$$\sigma(\!(\phi)\!)_{gs} = \{g \in G \mid \exists f \in \sigma :\ f[\![\phi]\!]_{gs} g\} \qquad \square$$

Having presented the basics of Groenendijk and Stokhof's DPL, we proceed to present a short summary of their theory in its original relational formulation.[13]

4.5.3.2 DPL

The syntax of DPL is the same as the one of ordinary predicate logic. So, the non-logical vocabulary of DPL consists of: n-place predicates symbols, individual constants, and variables. Logical constants are negation $\neg$, conjunction $\wedge$, disjunction $\vee$, implication $\rightarrow$, the existential and universal quantifiers $\exists$ and $\forall$, and identity $=$.

Definition 1 (Syntax)

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[12] See, for instance, Groenendijk and Stokhof (1990).

[13] For the full article, we refer the reader to Groenendijk and Stokhof (1991).

1. If $t_1, \ldots, t_n$ are individual constants or variables, R is an n-place predicate letter, then $Rt_1 \ldots t_n$ is a formula.
2. If t_1 and t_2 are individual constants or variables, then $t_1 = t_2$ is a formula.
3. If ϕ is a formula, then $\neg\phi$ is a formula.
4. If ϕ and ψ are formulas, then $(\phi \wedge \psi)$ is a formula.
5. If ϕ and ψ are formulas, then $(\phi \vee \psi)$ is a formula.
6. If ϕ and ψ are formulas, then $(\phi \rightarrow \psi)$ is a formula.
7. If ϕ is a formula, and x is a variable, then $\exists x\phi$ is a formula.
8. If ϕ is a formula, and x is a variable, then $\forall x\phi$ is a formula.
9. Nothing is a formula except on the basis of 1–8. □

Definition 1 plays the role of showing us that the set of DPL formulas is the same as the PC one. And since DPL is a semantic theory we proceed to give the formal characterization of interpretation.

A model $\mathcal{M}$ is a pair $\langle D, F\rangle$, where D is a non-empty set of individuals, F an interpretation function having as its domain the individuals constants and predicates. If α is an individual constant, then $F(\alpha) \in D$; if α is an n-place predicate, then $F(\alpha) \subseteq D^n$. An assignment g is a function assigning an individual to each variable: $g(x) \in D$. G is the set of all assignment functions. Next, Groenendijk and Stokhof define the interpretation of a term t: $[\![t]\!]_g = g(t)$ if t is a variable, and $[\![t]\!]_g = F(t)$ if t is an individual constant. Finally, Groenendijk and Stokhof define the interpretation function $[\![]\!]^{DPL}_{\mathcal{M}}$ as follows.[14]

Definition 2 (Semantics)

1. $[\![Rt_1 \ldots t_n]\!] = \{\langle g, h\rangle \mid h = g \;\&\; \langle [\![t_1]\!]_h \ldots [\![t_n]\!]_h\rangle \in F(R)\}$
2. $[\![t_1 = t_2]\!] = \{\langle g, h\rangle \mid h = g \;\&\; [\![t_1]\!]_h = [\![t_2]\!]_h\}$
3. $[\![\neg\phi]\!] = \{\langle g, h\rangle \mid h = g \;\&\; \neg\exists k : \langle h, k\rangle \in [\![\phi]\!]\}$
4. $[\![\phi \wedge \psi]\!] = \{\langle g, h\rangle \mid \exists k : \langle g, k\rangle \in [\![\phi]\!] \;\&\; \langle k, h\rangle \in [\![\psi]\!]\}$

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[14] As usual, Groenendijk and Stokhof suppress subscripts and superscripts whenever this does not lead to confusion.

5. $[\![\phi \vee \psi]\!] = \{\langle g,h\rangle \mid h = g \;\&\; \exists k : \langle h,k\rangle \in [\![\phi]\!] \vee \langle h,k\rangle \in [\![\psi]\!]\}$

6. $[\![\phi \rightarrow \psi]\!] = \{\langle g,h\rangle \mid h = g \;\&\; \forall k : \langle h,k\rangle \in [\![\phi]\!] \Rightarrow \exists j : \langle k,j\rangle \in [\![\psi]\!]\}$

7. $[\![\exists x\phi]\!] = \{\langle g,h\rangle \mid \exists k : k[x]g \;\&\; \langle k,h\rangle \in [\![\phi]\!]\}$

8. $[\![\forall x\phi]\!] = \{\langle g,h\rangle \mid h = g \;\&\; \forall k : k[x]h \Rightarrow \exists j : \langle k,j\rangle \in [\![\phi]\!]\}$ □

In standard semantics of first order predicate logic, the interpretation of a formula is a set of assignment functions – those functions which verify the formula. In the dynamic framework of DPL, the semantic object expressed by a formula is a set of ordered pairs of assignments. A closer look at definition 2 shows that except for conjunction and existential quantification both views, for all practical purposes, conflate due to the clause $g = h$. Such formulas are called *tests* because they function as a kind of test on incoming assignments: if the test succeed, the input assignment is passed on as an output assignment.

What happens when an existentially quantified formula is interpreted dynamically? The answer is that a pair $\langle g,h\rangle$ is in the interpretation of such an existential formula if and only if when such a formula is evaluated with respect to g, h is a possible outcome of the evaluation process. Since g and h are assignments of elements from the domain to variables, the difference between an input assignment g and an output assignment h can only be that a different object is assigned to one or more variables. This is precisely the point where the dynamic binding power of DPL comes from.

If existentials act like dynamic binding generators, conjunctions act like transducers pushing forward to the second conjunct the dynamic binding that might have been generated on the first conjunct. This analogy would be more clearly understood through the inspection of the analysis of an example such as $[\![\exists x Px \wedge Qx]\!]$.[15] Notice that in this example, the second occurrence of x is outside the scope of $\exists x$ in the first conjunct. However, it gets bound by the existential quantifier as showed

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[15] This formula might be seen as formalizing the natural language discourse composed by the following two sentences: *A man walks in the park. He whistles.*

by the calculation below.

$$\begin{aligned}
[\![\exists x Px \wedge Qx]\!] &= \{\langle g,h\rangle \mid \exists k : \langle g,k\rangle \in [\![\exists x Px]\!] \;\&\; \langle k,h\rangle \in [\![Qx]\!]\} \\
&= \{\langle g,h\rangle \mid \exists k : k[x]g \;\&\; k(x) \in F(P) \;\&\; h = k \;\&\; h(x) \in F(Q)\} \\
&= \{\langle g,h\rangle \mid h[x]g \;\&\; h(x) \in F(P) \;\&\; h(x) \in F(Q)\}
\end{aligned}$$

As displayed in the last line, the second occurrence of x gets bound with the same strength as the first occurrence of x in the first conjunct. This entails that for DPL there is no difference *in meaning* between the formula $[\![\exists x Px \wedge Qx]\!]$ and $[\![\exists x(Px \wedge Qx)]\!]$. [16] This result, which is not valid in PC, might be generalized for any formulas ϕ and ψ without further problems.

Another remarkable kind of discourse is exemplified by the so called *donkey sentences* whose prototypical format is given by DPL's formula $(\exists x\phi) \rightarrow \psi$. It happens that, in DPL, the last formula is equivalent in meaning to $\forall x(\phi \rightarrow \psi)$. These DPL equivalences are referred to, in the dynamic literature, as the Scope Theorems. These theorems will be demonstrated in the sequence (see page 70.)

It is time to state the notions of truth, validity and entailment.

Definition 3 (Truth) ϕ is *true with respect to g in $\mathcal{M}$* iff $\exists h : \langle g,h\rangle \in [\![\phi]\!]_{\mathcal{M}}$. □

Definition 4 (Validity) ϕ is *valid* iff $\forall\mathcal{M}\forall g :$ ϕ is true with respect to g in $\mathcal{M}$. □

Definition 5 (Contradictoriness) ϕ is a *contradiction* iff $\forall\mathcal{M}\forall g$: ϕ is false with respect to g in $\mathcal{M}$. □

In standard logic, ϕ entails ψ if and only if whenever ϕ is true, ψ is true as well. In virtue of definition 3, it is possible to define an analogue of this notion for DPL.

Definition 6 (s-entailment) $\phi \models_s \psi$ iff $\forall\mathcal{M}\forall g$: if ϕ is true with respect to g in $\mathcal{M}$, then ψ is true with respect to g in $\mathcal{M}$. □

It is well known that in standard predicate logic, the notion of entailment coincides

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[16] Notice that $\exists x Px$ does not occur as a subformula in $\exists x(Px \wedge Qx)$, and therefore does not conform to the compositional criterium which has dominated logic (and semantics) since the days of Frege.

with the notion of meaning inclusion. As it would be expect, in virtue of DPL's dynamic character, the same coincidence does not hold. As Groenendijk and Stokhof themselves stated in page 66, "in DPL, meaning is a richer notion than in PL, where interpretation and satisfaction coincide. Meaning inclusion implies s-entailment, but not the other way around."

The problem with the notion of s-entailment is that it is not a truly dynamic notion. To see why, let us point out the fact that this notion does not correspond to implication. For example, although it holds that $\models_s \exists x Px \to Px$, it **does not hold** that $\exists x Px \models_s Px$. The notion of s-entailment does not account for binding relations between premiss and conclusion that do happen to hold for implication, where an existential quantifier in the antecedent can bind variables in the consequent. However, in natural language, such relations do occur. From *A man walks in the park wearing a hat*, we may conclude *he wears a hat*, where the pronoun in the conclusion is anaphorically linked to the indefinite noun phrase in the premiss.

To find another notion of entailment in tune with the dynamic philosophy proposed, Groenendijk and Stokhof have taken the programming metaphor once more. If we look at a sentence as a kind of program, a reasonably intuitive notion would be: ϕ entails ψ if every successful execution of ϕ guarantees a successful execution of ψ. In other words, ϕ entails ψ iff every assignment that is a possible output of ϕ is a possible input for ψ.

Definition 7 (Entailment)

$$\phi \models \psi \text{ iff } \forall \mathcal{M}, \forall g, \forall h : \langle g, h \rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists k : \langle h, k \rangle \in [\![\psi]\!]_{\mathcal{M}}. \qquad \square$$

The notion of dynamic entailment just defined corresponds to the interpretation of (dynamic) implication. This relationship can be set out as a Deduction Theorem for DPL.

Deduction Theorem $\phi \models \psi$ iff $\models \phi \to \psi$.

Proof [17]

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[17] The theorems (deduction and the two scope theorems) are from Groenendijk and Stokhof (1991), but the proofs below are all mine.

$Suppose$	1	$\phi \models \psi$
$1, entailment\ def$	2	$\forall \mathcal{M}, \forall g, \forall h : \langle g, h\rangle \in \llbracket\phi\rrbracket_{\mathcal{M}} \Rightarrow \exists k : \langle h, k\rangle \in \llbracket\psi\rrbracket_{\mathcal{M}}$
$Suppose$	3	$g = h\ \&\ \forall k : \langle h, k\rangle \in \llbracket\phi\rrbracket_{\mathcal{M}}$
$3, \wedge\ elim$	4	$\forall k : \langle h, k\rangle \in \llbracket\phi\rrbracket_{\mathcal{M}}$
$4, 2, MP$	5	$\exists j : \langle k, j\rangle \in \llbracket\psi\rrbracket_{\mathcal{M}}$
$3, 5, DMT$	6	$g = h\ \&\ \forall k : \langle h, k\rangle \in \llbracket\phi\rrbracket_{\mathcal{M}} \Rightarrow \exists j : \langle k, j\rangle \in \llbracket\psi\rrbracket_{\mathcal{M}}$
$6, \llbracket\phi \to \psi\rrbracket\ def,\ set\ theory$	7	$\langle g, h\rangle \in \llbracket\phi \to \psi\rrbracket_{\mathcal{M}}$
$7, \exists\ intro$	8	$\exists h : \langle g, h\rangle \in \llbracket\phi \to \psi\rrbracket_{\mathcal{M}}$
$8, truth\ def$	9	$\phi \to \psi\ is\ true\ with\ respect\ to\ g\ in\ \mathcal{M}$
$9, \forall\ intro$	10	$\forall \mathcal{M}, \forall g : \phi \to \psi\ is\ true\ with\ respect\ to\ g\ in\ \mathcal{M}$
$10, validity\ def$	11	$\phi \to \psi\ is\ valid,\ i.e.,\ \models \phi \to \psi$ $\square$

In Groenendijk and Stokhof (1991) the reader will find a delightful exploration on the grounds of dynamic semantics; therefore, we refer the interested reader to the full length article.

Before moving towards DPL's offspring, we would like to present some peculiar results which do hold in DPL but not in standard logic. These results are related to the notions of scope and binding, and provide us with the formal compositional tools to analyse donkey sentences as the ones below.

(1) A man walks in the park. He whistles.

(2) If a farmer owns a donkey, he beats it.

(3) Every farmer who owns a donkey, beats it.

In standard logic, donkey sentences as (1), (2), and (3) get the right interpretation if we let an existential quantifier have wider scope over the sentential connective. Doing so, we arrive at

(1a) $\exists x[man(x) \wedge walk\_in\_the\_park(x) \wedge whistle(x)]$

(2a) $\forall x \forall y[[farmer(x) \wedge donkey(y) \wedge own(x,y)] \rightarrow beat(x,y)]$

(3a) $\forall x \forall y[[farmer(x) \wedge donkey(y) \wedge own(x,y)] \rightarrow beat(x,y)]$

Notice that the translation of the first sentence in (1), which would be $\exists x[man(x) \wedge walk\_in\_the\_park(x)]$, does not occur as a subformula in (1a). At first sight, (1a) can not be produced from (1) in a step-by-step procedure, i.e., in a compositional way. In a compositional approach, we would rather translate (1) as (1b):

(1b) $\exists x[man(x) \wedge walk\_in\_the\_park(x)] \wedge whistle(x)$

From the standard predicate logic point of view, (1b) is not a proper translation of (1), since in (1b) the last occurrence of the variable x is not bound by the existential quantifier, and hence the anaphoric link in (1) is not accounted for. However, suppose we could interpret (1b) in such a way that it is equivalent with (1). Evidently, (1b) would be preferred to (1a) as a translation of (1), since it could be the result of a compositional procedure. In DPL, this analysis is possible due to the theorem:

Scope Theorem 1 $(\exists x\phi) \wedge\ \psi$ iff $\exists x(\phi \wedge \psi)$

Proof

$[\![(\exists x\phi) \wedge \psi]\!]_{\mathcal{M}} = \{\langle g,h\rangle \mid \exists k : \langle g,k\rangle \in [\![\exists x\phi]\!]_{\mathcal{M}}\ \&\ \langle k,h\rangle \in [\![\psi]\!]_{\mathcal{M}}\}(1)$

$\langle g,k\rangle \in [\![\exists x\phi]\!]_{\mathcal{M}}$ iff $\langle g,k\rangle \in \{\langle g,h\rangle \mid \exists k : k[x]g\ \&\ \langle k,h\rangle \in [\![\phi]\!]_{\mathcal{M}}\}$ iff

$\exists j : j[x]g\ \&\ \langle j,k\rangle \in [\![\phi]\!]_{\mathcal{M}}$ (2)

Therefore, from (1) and (2), we get that

$\{\langle g,h\rangle \mid \exists k : \exists j : j[x]g\ \&\ \langle j,k\rangle \in [\![\phi]\!]_{\mathcal{M}}\ \&\ \langle k,h\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$

$\{\langle g,h\rangle \mid \exists j : j[x]g\ \&\ \exists k : \langle j,k\rangle \in [\![\phi]\!]_{\mathcal{M}}\ \&\ \langle k,h\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$

$\{\langle g,h\rangle \mid \exists j : j[x]g\ \&\ \langle j,h\rangle \in [\![\phi \wedge \psi]\!]_{\mathcal{M}}\} = [\![\exists x(\phi \wedge \psi)]\!]_{\mathcal{M}}$ □

Cases (2) and (3) are more dramatic than the previous one. Although (2) and (3) contain indefinite terms, which normally translate as existentially quantified phrases, we need universal quantification to account for their meaning in these examples. Notice, moreover, that the corresponding universal quantifiers $\forall x$ and $\forall y$ have to be given wide scope over the whole formula, whereas the indefinite noun phrases in (2) and (3) to which they correspond appear inside the antecedent of an implication in the case of (2), and way inside the relative clause attached to the subject term *every farmer* in the case of (3). Again, if we use standard predicate logic as our representation formalism, these kinds of examples prevent us from uniformly translating indefinite noun phrases as existentially quantified phrases. In DPL, this analysis is possible due to the theorem:

Scope Theorem 2 $(\exists x\phi) \to \psi$ iff $\forall x(\phi \to \psi)$

Proof

$[\![\phi \to \psi]\!]_{\mathcal{M}} = \{\langle a, b\rangle \mid a = b \;\&\; \forall c : \langle b, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\}$ (by def).
Therefore $\langle k, j\rangle \in [\![\phi \to \psi]\!]_{\mathcal{M}}$ iff $k = j \;\&\; \forall c : \langle j, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}$
iff $\forall c : \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}$

$[\![\forall x(\phi \to \psi)]\!]_{\mathcal{M}} = \{\langle g, h\rangle \mid h = g \;\&\; \forall k : k[x]h \Rightarrow \exists j : \langle k, j\rangle \in [\![\phi \to \psi]\!]_{\mathcal{M}}\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : k[x]h \Rightarrow (\exists j : \forall c : \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}})\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : k[x]h \Rightarrow (\forall c : \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}})\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : k[x]h \;\&\; \forall c : \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : \forall c : k[x]h \;\&\; \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : \forall c : k[x]g \;\&\; \langle k, c\rangle \in [\![\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall k : \forall c : \langle g, c\rangle \in [\![\exists x\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$
$\{\langle g, h\rangle \mid h = g \;\&\; \forall c : \langle g, c\rangle \in [\![\exists x\phi]\!]_{\mathcal{M}} \Rightarrow \exists d : \langle c, d\rangle \in [\![\psi]\!]_{\mathcal{M}}\} =$
$[\![(\exists x\phi) \to \psi]\!]_{\mathcal{M}}$ $\square$

The dynamics of DPL's implication and entailment, shown in the Deduction Theorem, allows us to account for the dynamic relationship that occurs between premises and conclusion in natural language reasoning. From *An old lady came in wearing a blue dress*, one may conclude *So, she wore a dress*, where the pronoun *she* occurring in the conclusion is anaphorically bound to the indefinite noun phrase *an old lady* in the premiss. This line of reasoning is justified not only by the Deduction Theorem but also by the result below:

Theorem $\exists x Px \models Px$

Proof

$Suppose$	1	$\langle g,h\rangle \in \llbracket \exists x Px \rrbracket_{\mathcal{M}}$
$1, def$	2	$\langle g,h\rangle \in \{\langle g,h\rangle \mid \exists k : k[x]g \ \&\ \langle k,h\rangle \in \llbracket Px \rrbracket_{\mathcal{M}}\}$
$2, set\ theory$	3	$\exists k : k[x]g \ \&\ \langle k,h\rangle \in \llbracket Px \rrbracket_{\mathcal{M}}$
$3, def$	4	$\exists k : k[x]g \ \&\ k = h \ \&\ k(x) \in \mathcal{F}(P)$
4	5	$\exists h : h[x]g \ \&\ \langle h,h\rangle \in \llbracket Px \rrbracket_{\mathcal{M}}$
$1, 5, DMT$	6	$\langle g,h\rangle \in \llbracket \exists x Px \rrbracket_{\mathcal{M}} \Rightarrow \exists k : \langle h,k\rangle \in \llbracket Px \rrbracket_{\mathcal{M}}$
$6, \forall intro$	7	$\forall \mathcal{M}, \forall g, \forall h : \langle g,h\rangle \in \llbracket \exists x Px \rrbracket_{\mathcal{M}} \Rightarrow \exists k : \langle h,k\rangle \in \llbracket Px \rrbracket_{\mathcal{M}}$
$7, entailment\ def$	8	$\exists x Px \models Px$ $\square$

Finishing this section, we wish to call the reader's attention to the problem posed to us by "natural language reasonings" in which pronouns are introduced in intermediary steps. The following example, from Groenendijk and Stokhof (1990, page 70), illustrates this point:

1.	It is not the case that nobody walks and talks.	$\neg\neg\exists x[Px \wedge Qx]$
2.	So, somebody walks and talks.	$\exists x[Px \wedge Qx]$
3.	So, he walks.	Px
4.	So, somebody walks.	$\exists x Px$
5.	So, it is not the case that nobody walks.	$\neg\neg\exists x Px$

Notice that the pronoun *he* occurring in 3 is bound by *somebody* in 2. In Groenendijk and Stokhof (1990, page 70) words, "although 1 implies 2, and 2 implies 3, 1 does not imply 3, precisely because 1 cannot, and should not, bind the pronoun in 3. But in the transition from 2 via 3 to 4, 3 can be omitted. And the same holds for all other intermediate steps. So, in the end, 5 is a consequence of 1".

4.5.3.3 Summary

It is the power to push forward variable bindings from the left to the right conjunct that allows for existential quantifiers to bind variables yet to come. This means that variables outside the syntactic scope of existentials and pronouns anaphoric to antecedent noun phrases mirror the same phenomenon. This explains how DPL achieves its goal of developing a compositional logical framework dealing with intersentential, as well as intrasentential, anaphoric relationships.

4.5.4 Dynamic Predicate Logic Varieties

In this section we sketch some systems developed on the grounds of Dynamic Predicate Logic; in order to facilitate comparison, we discuss systems in a functional format developed after the original presentation of DPL,[18] such as the ones in Groenendijk and Stokhof (1990), Dekker (1993) and Dekker (1994).

All dynamic logic theories are based on DPL and therefore they all deal with a compositional analysis of anaphoric intersentential as well as intrasentential relations as shown in (16) below.

(16) a – A man walks in the park. He whistles.

b – Every farmer who owns a donkey, beats it.

For these sentences, ordinary translations would be as in (17).

(17) a – $\exists x(\text{man}(x)\wedge \text{walk\_in\_the\_park}(x))\wedge \text{whistles}(x)$

b – $\forall x((\text{farmer}(x) \wedge \exists y(\text{donkey}(y)\wedge \text{own}(x,y))) \rightarrow \text{beat}(x,y))$

According to the *static* semantics of predicate logic, these formulas do not express what sentences (16) mean. This is so because the *pronoun-variables*[19] are not bound by the existential quantifiers to which they refer to. The semantic relationship between pronouns and their antecedents is established in a compositional way by associating pronouns with variables, and defining the interpretation algorithm as a function "updating" information about possible values of variables. Moreover, this

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[18] For the original relational formulation of DPL, which has been summarized in the previous section, see Groenendijk and Stokhof (1991)

[19] We can distinguish three main approaches dealing with the semantics of anaphoric relationships. Firstly, there is what has been called the **bound variable** approach which can be subdivided under the labels *representational* and *compositional*. DPL fits the last category while Kamp (1981) fall under the representational (and non compositional, by the way) label. The third approach, on the other hand, corresponds to the so-called E-type framework, Evans (1977), Heim (1990), Neale (1990), Does (1993) which, roughly speaking, identifies pronouns with descriptions.

treatment of indefinites binding free occurrences of the variables they quantify over induces a semantical change on the other quantifiers and connectives. All these facts are put together, in an implicit form, in table 4.4 below and, in an explicit form, in the Scope Theorems below.

$$\boxed{\begin{array}{lll} s[\![Pt_1 \dots t_n]\!] & = & s \cap \{g \in G \mid \langle g(t_1), \dots, g(t_n)\rangle \in F(p)\} \\ s[\![t_1 = t_2]\!] & = & s \cap \{g \in G \mid g(t_1) = g(t_2)\} \\ s[\![\neg\phi]\!] & = & s - \downarrow[\![\phi]\!] \\ s[\![\phi \wedge \psi]\!] & = & s[\![\phi]\!][\![\psi]\!] \\ s[\![\exists x\phi]\!] & = & s[x][\![\phi]\!] \end{array}}$$

Table 4.4: The functional characterization of DPL's semantics. G is the set of all assignment functions, $s[x] = \bigcup_{g \in s}\{h \mid g \approx_x h\}$, $g \approx_x h$ are assignments that differ at most with respect to the value they assign to x, and $\downarrow[\![\phi]\!] = \{i \in G \mid \{i\}[\![\phi]\!] \neq \emptyset\}$.

Table 4.4 shows the functional characterization of DPL's semantics. Straightforwardly, the essential dynamic feature of DPL is the dynamics of existential quantifier binding free occurrences beyond its syntactical scope. This fact is reflected in the Scope Theorems, which hold unconditionally for DPL (cf. stated by Groenendijk and Stokhof (1991, p. 63, 65) and proofs on page 69 of the present work.)

Scope Theorems:

(a) $(\exists x\phi) \wedge \psi \Leftrightarrow \exists x(\phi \wedge \psi)$

(b) $(\exists x\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$

Proof

(a) The interpretation of $\exists x\phi \wedge \psi$ is the sequence of updates $[\![\exists x\phi]\!] \circ [\![\psi]\!]$, and therefore $(s[x] \circ [\![\phi]\!]) \circ [\![\psi]\!]$. Since function composition is an associative operation, we get $s[x] \circ ([\![\phi]\!] \circ [\![\psi]\!])$ and therefore the desired result.

(b) Before we can prove this item, we need to define the following equivalences.

1) $\phi \rightarrow \psi \quad = \quad \neg(\phi \wedge \neg\psi)$

2) $\phi \vee \psi \quad = \quad \neg(\neg\phi \wedge \neg\psi)$

3) $\forall x\phi \quad = \quad \neg\exists x\neg\phi$

Notice that

$$\begin{array}{lcll} \exists x\phi \rightarrow \psi & = & \neg(\exists x\phi \wedge \neg\psi) & \text{by equivalence (1) above} \\ \neg(\exists x\phi \wedge \neg\psi) & = & \neg(\exists x(\phi \wedge \neg\psi)) & \text{by item (a)} \\ \neg(\exists x(\phi \wedge \neg\psi)) & = & \forall x(\neg(\phi \wedge \neg\psi)) & \text{by equivalence (3) above} \\ \forall x(\neg(\phi \wedge \neg\psi)) & = & \forall x(\phi \rightarrow \psi) & \text{by equivalence (1) above.} \end{array} \qquad \square$$

The scope theorems allow us to prove the equivalence[20] shown in (18).

(18) $$(\exists x(farmer(x) \wedge \exists y(donkey(y) \wedge owns(x,y)))) \rightarrow beats(x,y)$$
$$\Updownarrow$$
$$\forall x(farmer(x) \rightarrow \forall y((donkey(x) \wedge owns(x,y)) \rightarrow beats(x,y)))$$

Notice that the scope theorem (item b) provides the so-called *strong reading.* For donkey sentences, which have a universal import, the strong reading is a welcome result. However, some sentences have a *weak* (existential) *reading* as shown in (19).

(19) If I have a dime in my pocket, I'll put it in the parking meter.

On its most natural reading (19) says that if I have one or more dimes in my pocket, then I'll throw one in the meter. One is unlikely to interpret it as saying that I'll throw all the dimes I have in my pocket in the meter.

It is possible to define a notion of weak implication, along the lines that Pelletier and Schubert (1988) argue for, assigning to conditional sentences the weak truth

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[20] The proof is as follows.

(1)··· $(\exists x(farmer(x) \wedge \exists y(donkey(y) \wedge owns(x,y)))) \rightarrow beats(x,y)$

from (1), using the donkey equivalence
$(\exists x\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$ we get

(2)··· $\forall x((farmer(x) \wedge \exists y(donkey(y) \wedge owns(x,y))) \rightarrow beats(x,y))$

from (2), using the classical equivalence
$((a \wedge b) \rightarrow c) \Leftrightarrow (a \rightarrow (b \rightarrow c))$ we get

(3)··· $\forall x(farmer(x) \rightarrow (\exists y(donkey(y) \wedge owns(x,y)) \rightarrow beats(x,y)))$

using the donkey equivalence $(\exists x\phi) \rightarrow \psi \Leftrightarrow \forall x(\phi \rightarrow \psi)$
on the subformula $(\exists y(donkey(y) \wedge owns(x,y)) \rightarrow beats(x,y))$
occurring in (3) we get
$\forall y((donkey(y) \wedge owns(x,y)) \rightarrow beats(x,y))$. Therefore

(4)··· $\forall x(farmer(x) \rightarrow \forall y((donkey(y) \wedge owns(x,y)) \rightarrow beats(x,y)))$

conditions preserving however the internal dynamics of the implication.[21]

Weak and strong readings apart, in DPL and EDPL, Dekker's update revision of DPL, the information carried over interpretation is information about the values of variables achieved through the use of sets of assignments of individuals to variables; such sets are called information states. For DPL, information states are sets of *total* assignments whereas in EDPL they are *partial*.[22]

Such a "little" change allow us to account for two different aspects of information growth. As Dekker (1993, page 12) pointed out "update of information consists either in getting more information about the values of variables, by the elimination of partial variable assignments, or in extending the domain of partial variable assignments[23] (or, of course, in a mixture of both)". This change also embraces the existential quantifier. In both systems, the existential quantifier introduces arbitrary valuations of the bound variable. However, instead of DPL's re-instantiation scheme,[24] domain extension is used by EDPL. All in all, that change provides EDPL with an authentic update semantics, in the same sense stated in section 4.5.1.

Closely related to Dekker's EDPL, Dekker's (1994) Predicate Logic with Anaphora, (PLA), is built on the following ideas:

- There is independent motivation to keep pronouns apart from variables. For instance:
    - Assigning pronouns to a new and specialized term category, entails that bound and anaphoric pronouns and variables are kept apart from one another at the syntactic level.
    - Pronouns and variables display a different semantic behaviour in the scope of modal or epistemic operators (cf. Groenendijk and Stokhof (1994)).

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[21] See, for instance, Chierchia (1992) for such an account. Dekker (1993) present us with a fully developed argumentation about weak and strong readings of conditionals which he managed to fit in a general framework of universal adverbial quantification.

[22] A function is called total when it is defined for all elements in its domain. Otherwise, it is partial.

[23] Roughly speaking, this means that one gets more informed when one knows more about some specific thing or when one knows about more things.

[24] Re-instantiation might be paraphrased as *forget about any "old" values assigned to that variable and assign new values to it.*

In the following definitions D is the domain of individuals, V the set of variables used, S the set of all information states, and X any subset of variables		
DPL	$S = \mathcal{P}(D^V)$	
EDPL	$S^X = \mathcal{P}(D^X)$ $S = \bigcup_{X \subseteq V} S^X$	
PLA	$S^n = \mathcal{P}(D^n)$	is the set of information states about n subjects
	$S = \bigcup_{n \in \mathcal{N}} S^n$	is the set of information states

Table 4.5: Information states for DPL, EDPL, and PLA.

  - The ordinary notions of scope and binding can be sustained without any further modification.

- Information growth is achieved in the same way as stated for EDPL.
- Subjects are *partial*, since their identity need not be absolutely determined. Furthermore, subjects are *interdependent*, since the value of one subject may depend on that of another (cf. Dekker (1994, p. 5)).

To achieve all the points listed above, PLA's information states deal with information about values themselves, instead of sets of assignments of individuals to variables, as is the case in DPL and EDPL. These values are modelled by tuples of individuals that are the values of variables. Table 4.5 shows the definitions for the information state notion for the three systems. It also reveals the interrelationship among the systems: EDPL extends DPL in the sense that the former carries over information about not only the values assigned to variables but also the variable sets themselves.[25] On the other hand, PLA pass on the values themselves. Therefore, it might be the case that its semantics provides a "heuristics" for pronominal anaphora resolution. This is indeed the case, since in the language a new set of terms corresponding to anaphoric pronouns is defined as $\{p_i \mid i \in \mathcal{N}\}$. The index i is to be understood as pointing to the $i+1$ last introduced subject of the state s and case $e \in s$ with respect to which it is evaluated.

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[25] As already pointed out, this makes possible to model the two ways of information growth.

All the points together conform to Karttunen's philosophy of indefinites setting up discourse referents which would be available for future (co-)reference.[26] The interpretation of an existentially quantified formula $\exists x\phi$ follows the traditional static way, i.e., its interpretation with respect to some assignment g is stated in terms of the interpretation of ϕ with respect to any assignment $g[x/d]$ which at most differs from g in that it assigns an individual d to x. However, differently from static theories, d gets added to the cases considered possible after interpreting ϕ with respect to $g[x/d]$.

By keeping pronouns apart from variables, PLA differs from other dynamics settings with respect to the scope theorem, which, obviously, does not hold in PLA. Moreover, they differ in some other aspects as, for example, the α-conversion which holds for PLA but not for DPL or EDPL.[27] In fact, Dekker (1994) proved that PLA is a proper extension, and not a modification, of ordinary predicate logic. In this respect, "PLA stands on a par with the so-called E-type pronoun approaches, claimed advantage of which has always been that they keep as much as possible to classical semantics" (Dekker (1994, p. 12)).

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[26] McCawley (1981) explains the Karttunenian approach using an axiomatic formulation of group theory as a metaphor. Notice that for postulates (c) and (d) below, the role played by e is quite different since in (c) e is an existentially bound variable while in (d) it is a constant. However, in both postulates they are conceived as referring to the same entity.

A set G with a binary operation . is a group if and only if
a. ('Closure') $(\forall x : x \in G)(\forall y : y \in G)(x.y \in G)$
b. ('Associativity') $(\forall x : x \in G)(\forall y : y \in G)(\forall z : z \in G)(x.(y.z) = (x.y).z)$
c. ('Identity') $(\exists e : e \in G)(\forall x : x \in G)(x.e = e.x = x)$
d. ('Inverse') $(\forall x : x \in G)(\exists x^{-1} : x^{-1} \in G)(x.x^{-1} = x^{-1}.x = e)$

To sum up, Karttunen notes that existential NP's have, in addition to the function of binding a variable in forming existential propositions, as in the traditional static analysis, the function of bringing into being constants (which Karttunen christens *discourse referents*) that may figure in all or part of the subsequent discourse and which correspond to the entity that the existential proposition asserts to exist.

[27] Such substitution is not admissible in DPL because it changes the binding potential of the quantified formula.

4.6 Pronominal Anaphora revisited

In chapter 2 I have stated that pronominal anaphora could be accounted for from two viewpoints, namely, the syntactic and semantic ones. The syntactic GB framework was summarized but the semantic ones were postponed. It is time now to revisit the topic using the semantic insights already provided by the previous sections.

In the literature, we find a three-fold classification for pronouns, namely, deictic, anaphoric and E-type. And since so much work has been done concerning them, let us take a closer look at them.

4.6.1 Deictic

An expression is used *deictically* when its interpretation is determined in relation to specific features of the speech-act; the identity of those participating as speaker(s) and addressee(s) together with the time and place depends on the speech event.

(20) It is true Dear, *that driver* is looking at us.[28]

It is clear that the referent of *that driver* is whoever is reading the advertisement

(21) I want to know why you are here.

For the classical sentence (21), *I*, *you* and *here* refer to whoever is uttering the sentence, whoever is being addressed and wherever the sentence is being uttered.

(22) – I had a trunkful ...they found out what he is good for.
– I demand ...
– They made him a clown.[29]

In the dialog (22), the reference for *I* changes according to the elephant speaker while *he*, *him* refers to Dumbo and *they* to the circus' people.

Among the most obvious deictic elements are the personal and object pronouns, and their possessive counterparts as well as demonstratives and locatives. Deictic

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[28] Bus advertisement that I have seen running by Leeds metropolitan area.

[29] A summarized elephants' dialog from Disney's movie *Dumbo*.

are also the inflectional category of tense and a variety of temporal expressions such as *then, later, today, ...* including prepositional phrases (PPs) such as *on Sunday*, adverbs like *soon* and phrases ending in *ago.* Finally, definite NPs with **the**, such as *the door*, can also be used deictically as in (23.c) referring to the door where the sentence is uttered.

As usual in natural language issues, there is no categoric division between classes of deictic and non-deictic expression; the same lexical item might be used in both senses depending on the context, as in (23).

(23) a – They'll arrive *soon.* (deictic)

b – They *soon* discovered their mistake. (non-deictic)

c – Please, close *the door.* (deictic)

d – When he finally reached her house, he found that *the door* was open. (non-deictic)

e – Max came to Australia when he was five, and has lived *here* ever since. (both)

f – Sue's coming in today – *we*'re having lunch together. (both)

In (23.e) *here* deictically refers to the place where the sentence has been uttered. It also anaphorically refers to Australia; the mixed reading assigns *somewhere in Australia* as the semantic interpretation for the expression.

Traditionally, deictic pronouns have been associated to free variables so that their denotations depend on the assignment functions. The systems surveyed do not pay attention to this category; instead, the bound and E-type varieties are the centre of attention.

4.6.2 Bound Analysis

Bound analysis has been carried over to a syntactic and semantic fashion as exemplified by GB, DRT, and DPL (to cite but a few, where the first falls under the syntactic label and the others under the semantic label). Syntactic approaches have as a major advantage the fact that the logical interpretation form associated to the surface sentence assigns a bound variable to anaphoric pronouns as in (24.a). As a consequence, the searching for antecedents is minimized. For (24.a), the logical form would be (24.b).

(24) a – The boys like themselves.

b – $\exists x(boy(x) \wedge like(x,x))$

However, syntactic approaches do not cover most of pronominal anaphora, since they are basically concerned with intrasentential anaphora.[30] So, research on the topic shifted from syntax to semantics.

As we already seen, DRT and DPL are semantic theories where quantification and binding depart from usual. For them, anaphoric pronouns are syntactically free but semantically bound variables. Instead of looking for syntactic methods to bind pronouns under the scope of some quantifier, these theories make use of semantic tools such as unselective discourse binding operators (DRT) and dynamic binding (DPL). Figures 4.1 and 4.2 show, respectively, how DRT and DPL[31] handle the micro-discourse in (25).

(25) If a farmer owns a donkey, he beats it.

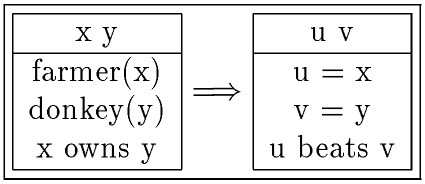

Figure 4.1: DRT representation for (25).

$$(\exists x(farmer(x) \wedge \exists y(donkey(y) \wedge owns(x,y)))) \rightarrow beats(x,y)$$
$$\Updownarrow$$
$$\forall x(farmer(x) \rightarrow \forall y((donkey(y) \wedge owns(x,y)) \rightarrow beats(x,y)))$$

Figure 4.2: DPL representations for (25).

---

[30] Intersentential cases, that seems to be the most frequent anaphoric phenomenon in spontaneous discourse events, do not belong to traditional approaches to syntax.

[31] See footnote 20 on page 74 for the proof of this equivalence.

			PL	DPL
(i)	$((\phi \wedge \psi) \wedge \chi) \Leftrightarrow (\phi \wedge (\psi \wedge \chi))$		x	x
(ii)	$(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$	DPL's scope theorem		x
(iii)	$((\phi \wedge \psi) \rightarrow \chi) = \neg((\phi \wedge \psi) \wedge \neg\chi)$ $\Updownarrow$ $\neg(\phi \wedge (\psi \wedge \neg\chi)) = (\phi \rightarrow (\psi \rightarrow \chi))$		x	x
(iv)	$(\exists x\phi \rightarrow \psi) = \neg(\exists x\phi \wedge \neg\psi)$ $\Updownarrow$ $\neg\exists x(\phi \wedge \neg\psi) = \forall x(\phi \rightarrow \psi)$	DPL's universal reading		x

Table 4.6: PL and DPL equivalences for donkey anaphora resolution

The advantage of semantic methods is the broad modelling coverage allowed. The basic drawback is that they all favour only one reading, the universal reading, for certain kind of sentences known as *donkey sentences.* A contrast between universal and existential readings is given in (26.b) and (27.b) below.

(26) a – Every farmer who owns a donkey beats it.

b – Every farmer who owns a donkey beats every donkey which he owns.

(27) a – Every man who owns a hat will wear it to the concert.

b – Every man who owns a hat will wear (exactly) one hat which he owns to the concert.

Table 4.6 gives us a clear indication of how universal readings, item (iv), are achieved in DPL. It also shows how existential scope gets extended over conjunction, item (ii). Items (i) and (iii), that hold in PC and DPL, are displayed intending to provide all the equivalences needed to, compositionally, transform the existential clause into the universal one (of Fig. 4.2).

Because of the methodological failure to account for both readings in a unique unified framework, new varieties of these theories (DPL and DRT), using the so-called E-type pronoun,[32] have been proposed.

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[32] The term E-type pronoun reflects the type of analysis used.

4.7 Dynamic theories II – the E-type perspective

As already pointed out, the category assigned to pronouns has a strong influence on how the problems with pronouns are analysed. Here, we can list two lines of thought: firstly, there are schools of thought that map pronouns to terms, as seen in sections 4.4 and 4.5. Secondly, there are the ones that map *E-type pronouns*[33] to quantifiers interpreted in context "going proxy" to definite descriptions; this line of thought is known as E-type analysis.

This kind of analysis, started by Evans (1977), Cooper (1979) and Evans (1980) and developed in recent years by Lappin and Francez (1994), Lappin (1989), Heim (1990), and Neale (1990), assigns a definite description selecting the object, or set of objects for plural, satisfying an open sentence obtained from the clause in which the pronoun's antecedent occurs, as the interpretation of anaphoric pronouns.

According to the original E-type analysis of Cooper (1979) and Evans (1980), (28.a) would be analyzed as (28.b).

(28) a – Every man who owns a donkey beats it.

b – Every man who owns a donkey beats the donkey he owns.

c – Every man who owns a donkey beats every donkey which he owns.

Recall that the preferred reading for (28.a) is (28.c); the problem with (28.b) is that (28.a) does not seem to entail the existence of a unique donkey for each donkey owner. The uniqueness problem has already been pointed out by Heim (1982); there, (29) has been used to emphasize the uniqueness problem.

(29) Everyone who bought a sage plant here bought five others along with it.

Due to the success achieved by rival line of thought in analyzing pronominal anaphora, the E-type analysis was put aside for awhile. If DRT and DPL give the strong (universal) reading correctly assigned to donkey sentences, they do fail to give the weak (existential) reading for a variety of donkey type as in (30).

(30) a – Every person who has a hat will wear it to the concert.

---

[33] This term is due to Evans (1980) and is also referred to as pronouns of laziness.

b – Every person who has a credit card pays his bill with it.

c – Every person who had a dime in his pocket put it into the meter.

Recent E-type proposals by Chierchia (1992), Does (1993), and Lappin and Francez (1994), allow us to analyze strong and weak donkey sentences. The difference between them, is that the first presents a non-unified framework relying on extra-grammatical factors.[34] The latter developed a unified framework for both readings; the correct reading is triggered by structural parametric values depending on pragmatic factors. Finally, Does (1993) presents an E-type analysis in a dynamic setting taking into account results from Kanazawa (1993). Also, the last two proposals are general enough to take into account adverbs of quantification.

The dynamic proposal of Does (1993) combines several interesting aspects.

- Firstly, it tracks the footsteps of DPL.
- Secondly, it adheres to GQT philosophy. Using Kanazawa's (1993) insights on the dynamics of GQT, [35] van der Does avoids the *pronoun problem*.[36]
- Thirdly, he makes E-type pronouns quantifiers sensitive to scope. This point is indeed related to the question whether (or not) E-type pronouns refer. Evans argues that they do; he argues that E-type pronouns as terms which have their reference determined by description are scopeless rigid designators. In Phillips (1985), Evans comments on scope with respect to psychological attitudes, negation, modalities, and time, gives support to the scopeless view. On the other hand, Neale (1990, pp. 185-189) and Does (1993, sect. 6) show that the data is more complex.
- Finally, he proposes and uses *choice functions* for solving the problems posed by classical E-type analyzes of singular pronouns.

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[34] The same can be said for Heim (1990). Indeed, on page 169, she herself says "Not all existing versions of the E-type analysis rely as heavily on pragmatics as Cooper's and mine."

[35] Kanazawa's work gives formal support to the claim that in some cases the dynamic treatment of dynamic generalized quantifiers allows for a principled choice between the weak and strong readings.

[36] The pronoun problem is related to the Geachian truth conditional analysis of donkey sentences within an E-type framework, which seems to give to such sentences always a strong (or always a weak) reading.

Such a combination of features seems to be enough to keep us in touch with this E-type approach.

The Dynamic Quantifier Logic, DQL, proposed by Does (1993) incorporates the previous ideas within a formal system. This system is defined in two stages which might be thought of as a standard static logic to which a separate dynamic component to handle the context generated by a text is added.

The language is a fairly standard version of PC with the addition of: (i) quantifier symbols ‘pro_{sg}’ and ‘pro_{pl}’ for singular and plural pronouns, respectively, (ii) two place determiner signs ‘all’, ‘some’, ‘D’, . . . , (iii) three implicational connectives, $\rightarrow_c$, $\rightarrow_a$, $\rightarrow_k$, which allow to discern classical, anaphoric and kataphoric referring expressions, (iv) formation rules dealing with (i), (ii) and (iii). The “dynamic module” is the cornerstone of Does (1993) proposal since to interpret E-type pronouns as quantifiers contextually restricted (through the use of choice functions) it is crucial that the part of the text in which they occur generates a context to supply their restriction. For him, contexts are of a syntactical figure analyzed as *partial functions* from variables to formulas. The author defines the context change potential of a formula as :

Definition 1: Context Change

For each formula ϕ assign a function $\llparenthesis\phi\rrparenthesis$ from contexts to contexts. Using a postfix notation, we have

$$
\begin{array}{rl}
\text{(i)} & \mathbf{c}\llparenthesis Rx_1 \ldots x_n\rrparenthesis \simeq \mathbf{c} \\
\text{(ii)} & \mathbf{c}\llparenthesis x = y\rrparenthesis \simeq \mathbf{c} \\
\text{(iii)} & \mathbf{c}\llparenthesis \neg\phi\rrparenthesis \simeq \mathbf{c}\llparenthesis\phi\rrparenthesis \\
\text{(iv)} & \mathbf{c}\llparenthesis \phi \rightarrow \psi\rrparenthesis \simeq \mathbf{c}\llparenthesis\phi\rrparenthesis\llparenthesis\psi\rrparenthesis \\
\text{(v)} & \mathbf{c}\llparenthesis [c\ x]\phi\rrparenthesis \simeq (\mathbf{c} \cup \{\langle x, x = c\rangle\})\llparenthesis(\phi)^x\rrparenthesis \\
\text{(vi)} & \mathbf{c}\llparenthesis [\mathrm{D}x : \phi]\psi\rrparenthesis \simeq (\mathbf{c} \cup \{\langle x, (\phi)^x \wedge_a (\psi)^x\rangle\})\llparenthesis(\phi)^x\rrparenthesis\llparenthesis(\psi)^x\rrparenthesis \\
\text{(vii)} & \mathbf{c}\llparenthesis [\mathrm{pro}_{\mathrm{sg}}\ x]\phi\rrparenthesis \simeq \begin{cases} (\mathbf{c}^{-x} \cup \{\langle x, x = \eta x\mathbf{c}(x) \wedge_a (\phi)^x\rangle\})\llparenthesis(\phi)^x\rrparenthesis & \text{if } \mathbf{c}(x)\downarrow \\ \mathbf{c}\llparenthesis(\phi)^x\rrparenthesis & \text{if } \mathbf{c}(x)\uparrow \end{cases} \\
\text{(viii)} & \mathbf{c}\llparenthesis [\mathrm{pro}_{\mathrm{pl}}\ x]\phi\rrparenthesis \simeq \begin{cases} (\mathbf{c}^{-x} \cup \{\langle x, \mathbf{c}(x) \wedge_a (\phi)^x\rangle\})\llparenthesis(\phi)^x\rrparenthesis & \text{if } \mathbf{c}(x)\downarrow \\ \mathbf{c}\llparenthesis(\phi)^x\rrparenthesis & \text{if } \mathbf{c}(x)\uparrow \end{cases} \qquad \square
\end{array}
$$

This definition conforms to the idea that processing a text, from left to right, the context should register the information given by possible antecedents (which is relevant to the interpretation of E-type pronouns). Differently from DPL, DQL treats contexts as *structured objects* which are created as we go along, reflecting not only the dynamic increasing of information, but also the partial character of contexts while functions. Notice, also, that this definition makes clear that only quantificational expressions affect context. Atomic sentences, negation and implications, which do **not** affect context, play a role of adding and pushing ahead information provided by their subformulas.

To interpret formulas in context, the author makes use of *choice functions* as the formal device associated to singular pronouns. For the empty set Does (1993) opts for assigning the null object '•', which is disallowed for occurring in the extension of relations. The model theory developed is fairly standard, except for the inclusion of a special element, the null object. Definition 2 presents the interpretation in context. Note that a model $\mathcal{M} = \langle D, * \rangle$ is a notational convention standing for a triple $\mathcal{M} = \langle E, D, * \rangle$ where $E = D \cup \{\bullet\}$, and $\bullet \notin D$ and $D \neq \emptyset$.

Definition 2: Interpretation in context

Let $\mathcal{M} = \langle D, * \rangle$ be a model, $\mathbf{c}$ a context, $\mathbf{h}$ a choice function for D, and $\mathbf{a}$ an assignment for $\mathcal{M}$. The truth of χ in $\mathcal{M}$ with respect to $[\mathbf{a},\mathbf{c},\mathbf{h}]$ – notation: $\mathcal{M} \models \chi$ $[\mathbf{a},\mathbf{c},\mathbf{h}]$ – is defined recursively.

a. $\mathcal{M} \models Rx_1 \ldots x_n\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\langle \mathbf{a}(x_1), \ldots, \mathbf{a}(x_n) \rangle \in R^*$
b. $\mathcal{M} \models x = y\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathbf{a}(x) = \mathbf{a}(y)$
c. $\mathcal{M} \models \neg\phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathcal{M} \not\models \phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$
d. $\mathcal{M} \models \phi \rightarrow_c \psi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathcal{M} \not\models \phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ or $\mathcal{M} \models \psi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$
e. $\mathcal{M} \models \phi \rightarrow_a \psi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathcal{M} \not\models \phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ or $\mathcal{M} \models \psi\ [\mathbf{a},\mathbf{c}(\llparenthesis\phi\rrparenthesis),\mathbf{h}]$
f. $\mathcal{M} \models \phi \rightarrow_k \psi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathcal{M} \not\models \phi\ [\mathbf{a},\mathbf{c}(\llparenthesis\psi\rrparenthesis),\mathbf{h}]$ or $\mathcal{M} \models \psi\ [\mathbf{a},\mathbf{c}(\llparenthesis\phi\rrparenthesis),\mathbf{h}]$
g. $\mathcal{M} \models [\mathrm{c}\ x]\phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathrm{c}^* \in \widehat{x}.\llbracket(\phi)^x\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}}$
h. $\mathcal{M} \models [\mathrm{D}\ x : \phi]\psi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathbf{D}(\widehat{x}\llbracket(\phi)^x\rrbracket_{\mathbf{a},\mathbf{c}(\llparenthesis(\psi)^x\rrparenthesis),\mathbf{h}}, \widehat{x}\llbracket(\psi)^x\rrbracket_{\mathbf{a},\mathbf{c}(\llparenthesis(\phi)^x\rrparenthesis),\mathbf{h}})$
i. $\mathcal{M} \models [\mathrm{pro}_{sg}\ x]\phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathbf{h}(\llbracket\mathbf{c}(x)\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}}) \in \widehat{x}.\llbracket(\phi)^x\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}}$ *and* $\mathbf{c}(x)\downarrow$
$\mathcal{M} \models [\mathrm{pro}_{pl}\ x]\phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathbf{pro}(\widehat{x}.\llbracket\mathbf{c}(x)\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}}, \widehat{x}.\llbracket(\phi)^x\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}})$ *and* $\mathbf{c}(x)\downarrow$
$\mathcal{M} \models [\mathrm{pro}\ x]\phi\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *iff* $\mathcal{M} \models (\phi)^x\ [\mathbf{a},\mathbf{c},\mathbf{h}]$ *and* $\mathbf{c}(x)\uparrow$

Here, $c^* \in \widehat{x}.\llbracket(\phi)^x\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}}$ *is the set* $\{d \in D \mid \mathcal{M} \models \phi\ [\mathbf{a}[d/x],\mathbf{c},\mathbf{h}]\}$ *The assignment* $\mathbf{a}[d/x]$ *is identical to* $\mathbf{a}$ *unless* $\mathbf{a}(x) \neq d$*. When used, terms* $\eta x\phi$ *are interpreted by* $\llbracket\eta x\phi\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}} = \mathbf{h}(\widehat{x}.\llbracket\phi\rrbracket_{\mathbf{a},\mathbf{c},\mathbf{h}})$. □

Variables play a double role for systems like DRT, DPL, and DQL, since they function as indices for anaphoric links as well as place holders for binding "operations." The articulation of both roles is achieved in DQL through the $(-)^x$ operation which erases all occurrences of [pro x] within the scope of any expression.[37]

For deictic and E-type pronouns, which are interpreted as referring expressions – or contextually restricted quantifiers – an inspection of def 1 shows that a context is defined for a variable x if it has processed a sentence[38] with a proper name or a determiner binding x. Therefore, due to the double role of variables, this means that a possible antecedent for [pro x] has been found (cf. Does (1993, p. 22)). This means that if a context **c** is defined[39] an unbound pronoun should be interpreted as an E-type pronoun, which is a **choice** from the set $\widehat{x}.\mathbf{c}(x)$, if [pro x] is singular, and a quantifier 'pro' restricted by this set, otherwise. Notice, moreover, that for undefined contexts $\mathbf{c}(x)$, an unbound [pro x] functions deictically.

Now that the basic ideas have been stated, instead of presenting the formal development leading to the characterization of entailment, and the like, it would be worth presenting the theory in action.

Let us start with one sentence length "discourses" displaying anaphoric and kataphoric situations.

For (31) below, the pronoun *he* is not within the scope of the proper name. Sentence (31.a) is represented by (31.b) which generates the context set shown in (31.c). This context is defined for x; therefore, def. 2.i makes (31.b) equivalent to (31.d), which uses the eta-term $\eta x(x = \mathrm{j})$ to indicate a choice from the singleton set $\widehat{x}.x = \mathrm{j}$ (therefore, $\eta x(x = \mathrm{j}) = \mathrm{j}$). Therefore, (31.d) is equivalent to (31.e).

(31) a – If John loves music he admires Mozart.
b – $([\mathrm{j}\ x]\mathrm{L}\ x) \rightarrow ([\mathrm{pro}_{sg}x][\mathrm{m}\ y]\mathrm{A}xy)$
c – $\{\langle x, x = \mathrm{j}\rangle, \langle y, y = \mathrm{m}\rangle\}$

---

[37] In this way, the author provides an account for making a **pronoun** (occurring within the scope of a quantificational expression binding x) **a bound variable**. Notice his use of this *erasing* operation in the definitions of context change potential, def. 1, and the interpretation in context, def. 2. Therefore, context does not affect such bound variables-pronouns; they are taken care of by total assignments in the usual way.

[38] As usual, processing is done on a left to right basis.

[39] Recall the characterization of contexts as *partial functions*.

d – $\mathrm{Lj} \rightarrow \mathrm{A}\eta x(x = \mathrm{j})\mathrm{m}$
e – $\mathrm{Lj} \rightarrow \mathrm{Ajm}$

Example (32) below, shows how dependencies among choices are accounted for. The context (32.c) results from the logical form (32.b) of (32.a).

(32) a – If a cardinal meets another cardinal, he blesses him.
b – $[\text{an } x : \mathrm{C}x]\ [\text{an } y : \mathrm{C}y \wedge x \neq y]\mathrm{M}xy \rightarrow [\mathrm{pro}_{sg}\ x][\mathrm{pro}_{sg}\ y]\mathrm{B}xy$
c – $\{\langle x, \mathrm{C}x \wedge [\text{an } y : \mathrm{C}y \wedge x \neq y]\mathrm{M}xy\rangle, \langle y, \mathrm{C}y \wedge x \neq y \wedge \mathrm{M}xy\rangle\}$
d – $\mathrm{B}\eta x\mathbf{c}(x)\eta y(\mathrm{C}y \wedge \eta x\mathbf{c}(x) \neq y \wedge \mathrm{M}\eta x\mathbf{c}(x)\mathrm{y})$
e – $[\text{every } x : \mathrm{C}x][\text{every } y : \mathrm{C}y \wedge x \neq y \wedge \mathrm{M}xy]\mathrm{B}xy$

The consequence of the conditional is represented by (32.d), which leaves the value $\mathbf{c}(x)$ implicit. This means that if a cardinal meets another cardinal, the context will pick a P from the cardinals meeting another cardinal and then a P′ from the cardinals different from but met by P. So, cardinal P blesses P′. This way, DQL makes the choice of 'him' dependent upon a choice of 'he.' This dependency complies with the general phenomena that in these cases the scope relations of the pronouns should coincide with that of their antecedents.

Related to this class of examples, Does (1993) observes on page 26 that (32.b) "reports on a disposition of cardinals to bless the colleagues they meet. Therefore, the choices involved should be rather arbitrary. Within an extensional framework the closest one could get to this reading is perhaps the use of a conditional like:

$$\mathcal{M} \models \phi \Rightarrow_{all} \psi[\mathbf{a},\mathbf{c},\mathbf{h}] \text{ iff}$$

$$\text{If } \mathcal{M} \models \phi[\mathbf{a},\mathbf{c},\mathbf{h}] \text{ then for } \textit{all } \mathbf{h}' : \mathcal{M} \models \psi\ [\mathbf{a},\mathbf{c}(\llbracket\phi\rrbracket),\mathbf{h}']$$

This conditional gives the consequent of (32.a) its strong reading, where it means (32.e). Notice that by varying the italicized quantifier in the definition of $\Rightarrow_{all}$, one seems to get a semantics for adverbs of quantification along the lines of Groenendijk and Stokhof (1991, 81-82)."

MiG sentences are accounted for in DQL as in (33) below. Note that the bound pronoun $[\mathrm{pro}_{sg}\ x]$ in (33.a) is not copied into the context.[40] For this reason the semantics of DQL produces (33.c). If (33.c) is true then there is a unique MiG that

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[40] This analysis follows up on Neale (1990, 196-197). Neale observes that $[\mathrm{pro}_{sg}\ x]$ is bound while $[\mathrm{pro}_{sg}\ y]$ is E-type.

chased and was hit by the shooting pilot. As a consequence, (33.c) turns out to be equivalent to the subject wide scope reading.[41]

(33) a – The pilot who shot at it hit the MiG that chased him.
b – $[\text{the } x : \mathrm{P}x \wedge [\mathrm{pro}_{sg}\ y]\mathrm{S}xy][\text{the } y : \mathrm{M}y \wedge [\mathrm{pro}_{sg}\ x]\mathrm{C}xy]\mathrm{H}xy$
c – $\{\langle x, \mathrm{P}x \wedge [\mathrm{pro}_{sg}\ y]\mathrm{S}xy \wedge [\text{the } y : \mathrm{M}y \wedge \mathrm{C}xy]\mathrm{H}xy\rangle, \langle y, \mathrm{M}y \wedge \mathrm{C}xy \wedge \mathrm{H}xy\rangle\}$
d – $[\text{the } x : \mathrm{P}x \wedge \mathrm{S}x\eta y(\mathrm{M}y \wedge \mathrm{C}yx \wedge \mathrm{H}xy)][\text{the } y : \mathrm{M}y \wedge \mathrm{C}yx]\mathrm{H}xy$

Complex discourses displaying intersentential anaphora are handled as in (34) below. The logical form of (34.a) is (34.b), whose antecedent generates the context $\mathbf{c}(x) \mapsto \mathrm{M}x \wedge \mathrm{W}x$. Recall that, for such cases, all intersentential pronouns must be e-type since the notions of scope and binding are the standard ones for this kind of analysis.

(34) a – Just one^1 man walks in the park. He$_1$ whistles.
b – $[\text{just one } x : \mathrm{M}x]\mathrm{W}x.\ [\mathrm{pro}_{sg}\ x]\mathrm{WH}x$
c – $[\text{just one } x : \mathrm{M}x]\mathrm{W}x.\ \mathrm{WH}\eta x(\mathrm{M}x \wedge \mathrm{W}x).$

In (34), the pronoun 'he' is interpreted as a choice from the set $\widehat{x}.\mathbf{c}(x)$, which, by the antecedent sentence, is the singleton set $\widehat{x}.\mathrm{M}x \wedge \mathrm{W}x$.

This approach works for conservative as well as non-conservative determiners such as 'just one' and 'only' respectively.

Closing this section, it would be worth to call attention for the fact that, according to def. 1, pronouns update the formula associated with their variable. If discourse (34) had been extended by sentence 'He airs his dog' we might get the following discourse

Just one^1 man walks in the park. He$_1^2$ whistles. He$_2$ airs his dog.

in which the first pronoun is interpreted as a choice from the men who walk. However, this pronoun changes the value of $\mathbf{c}(x)$ from $\mathrm{M}x \wedge \mathrm{W}x$ to $x = \eta x(\mathrm{M}x \wedge \mathrm{W}x) \wedge \mathrm{WH}x$. As a consequence, the second pronoun is interpreted in the new context, i.e., as the previously chosen walking man, who is now required to whistle.

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[41] According to Karttunen, (33.a) has two non-equivalent readings depending on the relative scope of the descriptions.

4.8 Summary

In the present chapter we have presented a way of dealing with quantified expressions in general and also with intra as well as inter-sentential anaphora.

The introduction of a more complex semantic category, the category of generalized quantifiers, allowed us to do a number of things. First, it provided us with a compositional semantics for NPs, which appears to be impossible on a standard first-order approach. Second, it enabled us to state and hypothesize an explanation for a substantive universal characteristic of natural language determiners. Third, it might enable us to come up with a simple and precise classificatory criteria for NPs allowing us to characterize the distribution of negative polarity items as well as the behaviour of certain items in the presence of others, if we had discussed this issue (for a concise discussion on these matters, see Keenan and Moss (1985), Keenan (1995), Chierchia and McConnell-Ginet (1990), but mainly Kanazawa (1994).)

The truth-conditional and model-theoretic approach to meaning developed into the GQT format has a real empirical concern and a enormous relevance for linguistic theory. Without it, some nontrivial properties of language would be lost. The GQT kind of semantics, although limited in its scope, deeply contributes to the effort of characterizing what a human language is.

The standard approach to model-theoretic semantics for natural language which has been referred to as *static semantics* can be characterized as follows: the meaning of a sentence is identified with its truth-conditional content. As a consequence, the interpretation of a sentence with respect to some model $\mathcal{M}$ is given by a recursive definition of the truth of a sentence with respect to $\mathcal{M}$ and some other parameters specified in $\mathcal{M}$ (such as assignments of values to variables, possible worlds, points in time, speaker, hearer, and so on (Groenendijk and Stokhof (1990)). Using the term *index* to cover whatever parameters are in use, the meaning of a sentence in $\mathcal{M}$ can be identified with the set of indices with respect to which it is true in $\mathcal{M}$. Other semantic notions are defined in terms of this one; entailment, for instance, could be defined as meaning inclusion in all $\mathcal{M}$. And the notion of updating an information state with a sentence is defined as taking the conjunction of the information state with the information content, i.e., the truth-conditional content, of the sentence.

The natural language dynamic interpretation framework, referred to as *dynamic semantics*, is based on a completely different basic notion. It is the information change potential of a sentence that is taken as constituting its meaning. Therefore, the notion of the interpretation of a sentence with respect to a model $\mathcal{M}$ is given by a recursive definition of the result of updating an information state with the sentence. The meaning of a sentence with respect to $\mathcal{M}$ can then be identified with the update function associated with the sentence in $\mathcal{M}$. This already brings out the fundamental difference between a static and a dynamic semantical systems. Whereas in the former the notion of information content is the basic recursive notion, in the latter it is the notion of information change that plays this role. Finally, as Groenendijk and Stokhof (1990) point out, the dynamic notion of meaning brings along new possibilities for defining entailment.

The several issues surveyed in this chapter cover different problems posed to, and extensions on, traditional logic approaches to natural languages. In a progressive development, we show a *compositional* approach to general quantifying (GQT) since compositionality is not only central to logic but also to liguistics and philosophy of language. Also, the development of GQT has come up with new results such as a taxonomy for determiners classification which shed light onto the comprehension of the constraints posed by them to anaphoric relations. Since GQT still sticks to the PC conservative ontology, real discourses can not be addressed seriously in it. A step forward is made by dynamic approaches, such as DRT and DPL, for which, a richer ontology is considered. As consequence, a more "complex" notion of discourse is achieved and anaphoric relations can be solved. Then, several distinctive offspring (of DPL) were presented intending to show how the notion of update can be improved to deal with problematic aspects of the original formulation. But, the more evident aspect of all approaches presented (and, to the best of my knowledge, all literature concerning with this issue) is that no one has ever gone far enough to take discourses as structured entities as chapter 3, and references therein, claims. But this issue is the target of chapter 5.

Chapter 5

The problem being tackled

5.1 Initial Remarks

The development of dynamic semantic approaches, such as US, DPL, PLA, and DRT among many others, showed that sentences could be used as devices for modeling the dynamics of information changing (or information growth). From these **traditional** dynamic semantic viewpoints, a discourse is a linear sequence of sentences whose syntactic form resembles PC. However, the semantic counterpart is much more sophisticated than PC in explaining how to keep track of all the information related to each state. The theory explains the way to compute the next state given an input sentence and a state. As a consequence of a discourse having a very simple structure, it would be sequentially processed in a very similar way as regular languages are recognized by finite automata.

What happens if we replace the input sentence by a set of sentences bearing some built in structure? Literature in linguistics has shown that discourses are a complex phenomena carrying information that is impossible to be attached to single sentences. Figure 5.1 clearly displays a non-linear structure similar to block structures found in Algol-like languages.[1] As such, the figure seems to be suggesting the use of more powerful devices for processing “complex” discourse structures. Imagine, for instance, that this very same interrupted discourse had been resumed

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[1] See also Chap. 3 for more examples and explanation.

after Mr. Lewis managed to keep the pets under control (locking the pets in his house library! (the very last place I'd let a pet (any pet) stroll away.)) Notice that in this case a more powerful computational device is required, since we need to recover the state where the conversation had been disrupted by the pouncing pets. There remains the question of seeing how much computational power we need.

This is so because, in some sense, classical dynamic settings are *one dimensional* and this explains why we can not represent and compute complex discourse relations (or, if you prefer, discourse structures) in classical dynamic settings since complex discourse structures are *multidimensional*. At first glance, it seems to entail changes to syntax as well as semantic. The language would distinguish among several notions of scope such as, for instance, sentential scope (or classical scope, be it dynamic or static) and block-segment scope. It is clear that different kinds of relationships are in order for modeling inter-block or intra-block anaphoric links. (For instance, pronouns can be used inside a discourse block instead of the noun phrase it replaces. However, an anaphoric noun phrase must be used outside the block it was first mentioned.)

Using programming languages as a paradigm, US, DPL and offspring reminds me of BASIC. I'd like to step forward to ALGOL!

5.2 Introduction

The literature in dynamic semantics is **all** focused on "one dimensional" discourses. By one dimensional discourse I mean that the only underlying structure available is a **linear** sequence of sentences. This is fine for dealing with simple anaphoric pronouns. As we have showed in chapter 4, the one dimensional dynamic settings provided us with the ability to extend the variable binding operation across sentence boundaries.

Chapter 3 presented a "defense" of the need for introducing more structure into theories dealing with discourse. Examples 3.5, 3.4, 3.6 and 3.7 display clearly such claim. These examples make it evident that the DPL style dynamic binding is not "infinitely" stretchable;[2] it is at least limited to discourse blocks which display a nested structure a la Algol. This implies that dynamic binding operations and the dynamics of anaphoric relationships, in general, should have a limited scope. And the block structure is such a limit. However, as we will see, even the block structure might be dynamically extended throughout discourse. Therefore, we might think of two kinds of scope relations, namely: (i) the DPL dynamic intersentential one, characterized by DPL scope theorems, and (ii) a new (inter/intra)block one. This seems to suggest that discourses are, in some sense, *multi-dimensional* structures.

5.3 Towards a multidimensional dynamic logic

Cooper (1996) describes the following scenario in connection to the example presented in figure 5.1.[3]

> Imagine that, in their US household, the Lewises have not only a cat but also a dog, both of whom have been dashing around the room, brushing past your teacup and causing you some apprehension. Eventually, the situation quiets down and David Lewis engages you in calming conversation. He starts to speak to you:

The dog is under the piano and the cat is in the carton.
The cat will never meet our other cat,
because our other cat lives in New Zealand.
Our New Zealand cat lives with the Cresswells and their dog.
And there he'll stay,
because the dog would be sad if the cat went away. ‖
The cat's going to pounce on you.
And the dog's coming too.

Figure 5.1: The Lewises scenario I

Notice that ‖ signalizes a change in the focus of attention (cf. Grosz and Sidner (1986) and Lewis (1979)) because it is no longer the conversationally salient New

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[2] Compare to Groenendijk and Stokhof (1991, page 65) where they state "...its binding power extends indefinitely to the right". The binding power they are referring to is generated by indefinite noun phrases which they take as existential quantifiers.

[3] Cooper's example is based on a similar example from Lewis (1979). Cooper himself acknowledges it. Cooper's use of this example is due to the fact that his work is concerned with the role of situations in a situation theoretic treatment of generalized quantifiers.

The dog is under the piano and the cat is in the carton.

> **The cat will never meet our other cat,**
> **because our other cat lives in New Zealand.**
> **Our New Zealand cat lives with the Cresswells and their dog.**
> **And there he'll stay,**
> **because the dog would be sad if the cat went away.** ‖

The cat's going to pounce on you.
And the dog's coming too.

Figure 5.2: An Algol-like block structure for the Lewises scenario I

Zealand cat which is being referred to but rather the approaching American one. Also, for this scenario, it signalizes the end of the inner subordinate discourse block.

In order to make comparisons with other proposals and draw an analogy with nested programming languages it is worth quoting Cooper, who on pages 76-77 says

> On Lewis' account this corresponds to a change in focus from a more salient cat to a less salient cat. For Barwise and Perry,[4] it represents a change from a resource situation supporting infons about a cat and a dog in New Zealand to one supporting infons about a dog and a cat in this room. For Lewis, the reference back to the US dog could require just as much accommodation as the reference back to the US cat, unless the US and New Zealand animals are bundled up in different context sets. For Barwise and Perry, the accommodation gives us back a whole previous resource situation. Thus on the Barwise and Perry view you would not expect a change to be signalled for the dog, provided you had divided up the resource situations in an intuitive way. Similarly, if Lewis were to continue the conversation about the New Zealand dog, for example, replacing the last sentence with *It's amazing how much affection the dog shows for other animals in the house*, one has the feeling that the reference to the cat pouncing would have to be clearly marked off as parenthetical in some way. What is switching here is whole situations, not just individuals or arbitrary sets of individuals determined independently from the situations that are being talked about.

Although Barwise and Perry remark were concerned with situation theory,[5] their remark is akin in spirit to Grosz and Sidner's (1986, p. 175) theory.[6]

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[4] The reference here is to Barwise and Perry (1983)

[5] Barwise and Perry introduced the notion of resource situation to deal with definite descriptions. Through resource situation they were able to preserve the intuition that definite descriptions have a uniqueness requirement. However, they do not equate uniqueness to universal unicity. In other words, a definite description such as *the dog* does not require that there is one and only one dog in the whole universe of discourse.

[6] See section 3.3.2 page 28 in this thesis for the relevant material.

For the sake of simplicity, let us concentrate on discourses displaying a neatly and properly embedded block structure such as the one being discussed thus far (fig. 5.2 shows the block structure for the Lewises' scenario I).[7] And, for the sake of concreteness, let us take the Lewises' scenario as the paradigmatic example. Let us assume yet that there are only two situations, or discourse blocks, on Lewises' scenario I, namely: the US household and the New Zealand one. Once more, an analogy with a programming language would be worth drawing.

The first analogy I am proposing is regarding these discourse blocks as a kind of subprogram; sometimes discourse blocks mimic the prototypical subroutine behaviour as shown in fig. 5.2. But, sometimes their behaviour follows the ***coroutining*** pattern as explicitly indicated by the double occurrence of ‖ in fig. 5.3. The second analogy is related to the programming language idea that identifiers, in a general sense, must be defined before use.[8] In this sense, the occurrences, in a bold emphasized tipeface, of definite descriptions correspond to the declaration statements while the remaining occurrences correspond to executable statements, which could be seen as the anaphoric use of noun phrases. The third analogy is related to Algol visibility laws; based on such laws a variable, i.e., a pronoun, might be anaphoric on values of noun phrases already present in the very same discourse frame (or, maybe, in its immediate ancestor.) As a consequence, a noun phrase introduced into an internal discourse frame would not be referred to by a pronoun occurring in any external discourse frame.[9] Finally, the accommodation signalled by ‖ could be seen as indicating ***coroutine resuming***. This particular analogy is even more evident if we look at the Lewises' scenario II in fig. 5.3. The first occurrence of ‖ signals a parenthetical warning for the guest; therefore a resuming of the US household coroutine is needed. And after the parenthetical warning has been closed, the NZ household is resumed. Notice that none of the values of noun phrases have been lost

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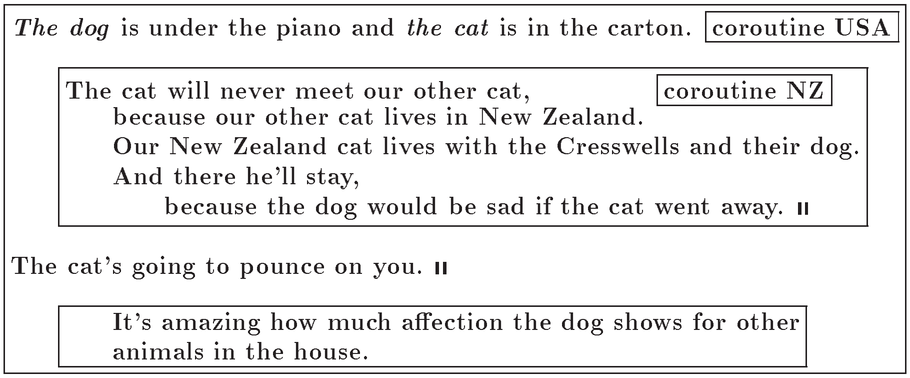
[7] We are not taking into account any other possible topological relation between discourse blocks even though a skillful linguist would possibly create examples where two discourse blocks overlap.

[8] Therefore, constants and variables must be defined before their first occurrence into a executable statement.

[9] See chap. 3, sect. 3.3.3, page 37 where reasons for such impossibility are given.



The dog **is under the piano and** ***the cat*** **is in the carton.** **coroutine USA**

The cat will never meet our other cat, **coroutine NZ**
because our other cat lives in New Zealand.
Our New Zealand cat lives with the Cresswells and their dog.
And there he'll stay,
because the dog would be sad if the cat went away. ‖

The cat's going to pounce on you. ‖

It's amazing how much affection the dog shows for other animals in the house.

Figure 5.3: An Algol-like block structure for the Lewises scenario II

in this coroutining shifting. And for the scenario I, the ‖ warning brings us back to the US household; therefore, the cat and dog referred to in the sentences following ‖ are the American ones. Notice how the coroutine analogy strengths the dynamic scope of discourse blocks: sentences might be added to the ancestor block of a closed sub-block of an unfolding discourse. Sentences might also be added to a block in a "discontinuous sequence" as shown in scenario II (see figure 5.3).

Figure 5.3 suggests that it would be possible to characterize two types of scope theorems in the same line of DPL. The DPL (intersentential) scope theorem might be retained if we take the dynamic binding working inside individual blocks. But the same idea could be followed for blocks; in this sense a block might be "sparsely" distributed across a conversation without loosing its coherence. In this sense each coroutine resuming would correspond to a block stretching.

5.4 The problem

The question one might ask is if a unified dynamic framework for dealing with "complex" discourses displaying a coroutining behaviour and such that the discourse blocks do not hold any interblock anaphoric relationship. This last property will be termed the *impenetrable hypothesis*. The next section sketches the nature of information states needed for modeling such kind of discourses. And a positive answer is given in the next chapter.

5.5 The dynamics of complex discourses

DPL departs from static logical approaches by regarding sentences as moving an agent along information states. Since DPL only covers plain "unstructured" discourses – in the sense discussed in the previous sections – its definition of information state, as a set of assignment functions, suffices. It is sufficient since such discourses are unidimensional and only one dimension is present in the information state definition.

As we are dealing with a kind of structured discourse, exhibiting a coroutining behaviour, a multidimensional information state would be the kind of generalization we are after. The basic idea is to allow a new dimension for every nested discourse block present in the discourse. So, if D is a domain of individuals and V the set of variables of some formal language (L), then any sequence $s = s_1, s_2, s_3, \ldots$ where every component s_i, $i > 0$, is a set of assignment functions from V to D, i.e., $s_i \subseteq D^V$, is a multidimensional information state.

The multidimensional character of information states just proposed should allow us to keep track of information related to each dimension. In this way, when we move from one block to another, we shift from one dimension to another. The update of information conveyed into the new block will be processed in the new dimension without losing of a single bit of information related to the other blocks. Shifting from one block to another corresponds to the coroutining process described early in this chapter. Notice that the information state model proposed prevents the dynamic binding from crossing discourse block boundaries,[10] since a typical information state would look like $\{g \mid g \in D^V\}, \{g \mid g \in D^V\}, \ldots$, where dynamic binding emanates from each dimensionally localized g (which are assignment functions on *one* argument (which is the variable lying around in that very same block)).

For example, imagine a hypothetical logical discourse as depicted by

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[10] Boundary crossing ought to be done through re-introduction of full noun phrases. See chap. 3 for relevant material.

$$\boxed{\begin{array}{l} \exists x(Dx \wedge Cx) \\ \qquad \boxed{\begin{array}{l} Dx \\ \exists x(Cx \wedge Pxy) \\ Qx \end{array}} \\ Hx \end{array}}$$

where formulas $\exists x(Dx \wedge Cx)$ and Hx occur, in this order, in the outermost block, and formulas Dx, $Dx \wedge \exists x(Cx \wedge Pxy)$, and Qx occur, in this order, in the innermost block. Because of the block structure, the dynamic binding emanating from the existential quantifier occurring in the outermost block ought to be confined into the outermost block. The same for the innermost (or any other block if present). Because of the multidimensional character proposed for information states, the variable x in Hx, in the outermost block is dynamically bound to the existential $\exists x(Dx \wedge Cx)$. On the other hand, the variable x in Dx, in the innermost block, **does** occur free while x in Qx is dynamically bound to the existential $\exists x(Cx \wedge Pxy)$.

Support for all the points presented here is present in the work of Grosz and Sidner (1986). Notice the similarity between the analogy proposed and what Grosz and Sidner (1986) call focusing. Related to focusing, Grosz and Sidner (1986, p. 180) claim that "The focusing structure is a stack. Information in lower spaces is usually accessible from higher ones (but less so than the information in the higher spaces)"; a few paragraphs ahead, still in the same page, they state "the stacking of focus spaces reflects the relative salience of the entities in each space during the corresponding segment's portion of the discourse." Then, on page 191, they say "A second role of the focusing structure is to constrain the OCP's search for possible referents of definite noun phrases and pronouns". It is clear that Grosz and Sidner's use of the stack concept departs from the stack concept as defined in abstract data type theory.

It is worth remarking that we have so far only discussed semantic aspects of a new dynamic logic; syntactic issues have not been addressed yet. Problems may be expected related to this aspect, but such problems will be addressed in next chapter where we develop the logic system in full. And, as usual, we expect that the light we are shedding onto the dynamics of discourse structure will bring us a new range of problems; however, these problems will have to await future work.

5.6 Summary

In this chapter we presented the problem target of this thesis and the methodology we expect to solve it with. In order to provide the reader with a better comprehension of the issues involved, an analogy with a programming paradigm was drawn.

The conclusion drawn from the data presented here and the literature on discourse theories, is that a better information state model is needed if we want to push discourse analysis further. It is clear that in order to represent structured discourse a new range of entities are needed. And this, naturally, argues for a improved ontology.

It is also clear that some decisions ought to be made as exemplified by the very nature of information states. We have assumed that information states are sequences of sets of assignment functions in D^V; therefore the same domain of individuals is attached for all discourse blocks. If this assumption would make easier the development of the first "instance" of formal system coping with structured discourses, it also would not seem natural from a linguistic point of view. It would be more natural to assume different domains for every discourse block. But it should be remarked that for every decision to be made, a new system would be proposed as future work.

Chapter 6

The formal system

6.1 Introduction

The system developed in this chapter builds on the seminal work of Groenendijk and Stokhof (1991). However, Groenendijk and Stokhof's DPL, as well as rivaling semantic theories dealing with discourses, suffers from ***uni***-*dimensionality* which is reflected directly from its characterization of information states. For DPL any set of assignment functions is taken as an information state. Although not free of problems, this conceptualization is fine as long as we take discourses as plain sequences of sentences. As an immediate consequence, DPL's ontological top most entity, the discourse, is modelled through formulas.[1]

In chapter 3, we made a point of showing that discourses are *not* unidimensional: discourses are indeed complex hierarchically structured entities. In spite of this fact, it is still possible to undertake a semantic analysis of complex discourses in the same philosophy started by Groenendijk and Stokhof (1991). All that is needed is to pass from unidimensional to ***multi***-*dimensional* conceptualizations. Multidimensional information states, for example, are envisaged as tuples of sets of assignment functions. The multidimensional framework presents us with ontological as well as philosophical questions.

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[1] Recall that DPL's syntax is the same as for PC. Hitherto, the word 'formula' will be used in this sense.

From an ontological point of view, complex discourses cannot be represented by formulas since the structure of the discourse has to be somehow accounted for. Besides, formulas (sentences) have a relative place to "live," the discourse block they occur at. This lead us to the need of characterizing a new class of entities (DBPL's sequences and texts) and connectives (DBPL's $\bullet$), operators and relations between these new first class citizens.

Since the new ontology poses us with new possibilities, some decisions ought to be made. Questions such as "Do discourse blocks have different discourse domains?", or "Should we grant cross-block interference?" As a first step into the multidimensional setting, we decided to keep the system as simple as possible; for example, we decided not to grant multidimensional interference. We term this the *impenetrable hypothesis*. And also, we decided to "distribute" the same domain to all discourse blocks. Although simple, such a system is capable of dealing with many sorts of natural language discourses.

We hope that the previous exposition has shown the similarities and dissimilarities of the present system and Groenendijk and Stokhof (1991).

This chapter is structured in the following way. The present section makes a short explanation of what the reader should expect to find in this chapter. Section 6.2 presents the syntax and semantics of DBPL and therefore it is the section dealing with the characterization of DBPL's ontological entities. The way the definitions of DBPL's formulas and texts are made, makes clear that all phenomena occurring at any dimensional index get confined to the very same index. Section 6.3 introduces the dynamic counterpart for notions such as truth and entailment. For the definition of DBPL's text entailment, we thought of a multidimensional Cartesian product of DPL's entailment notion (which is related to implication.) Since implication is *internally (but not externally) dynamic*, DBPL text entailment is a dynamic notion. This approach allows us to model the idea that anaphoric pronouns occurring at a conclusion text may refer back to indefinites previously introduced by the premiss without losing the dimensional niche the pronouns and indefinites may occur at.

Section 6.4 presents the properties the system has. Related to each individual dimension index, DPL's properties might be expected to hold. Related to the whole text some properties are also expected to hold; for example, text commutativity does not hold in DBPL (in the same way that we cannot commute pages in a book). Section 6.5 presents some additional remarks and finally a summary of the issues presented in the chapter is the issue of the last section.

6.2 Dynamic Blocked Predicate Logic

The vocabulary of DBPL is almost the same as the one for PC (and DPL). The notable difference is related to the introduction of a new unidimensional multisorted sentential conjunctive connective (•) and a new multidimensional interblock dummy functor (▶◀) which, for all practical purposes will be handled as a new kind of parenthesis.

Definition 1: Vocabulary of DBPL

- The non-logical vocabulary of DBPL consists of:
  - an infinite stock of n-place predicative symbols, for every natural number n.
  - an infinite stock of symbols for individual constants.
  - an infinite stock of symbols for variables.[2]
  - a special set of delimiters and connectives, including only the following symbols: (,), ▶, ◀, and the full stop connective • (which is to be interpreted as sentential conjunction.)
- The logical vocabulary consists of negation $\neg$, conjunction $\wedge$, disjunction $\vee$, implication $\rightarrow$, the universal quantifier $\forall$, the existential quantifier $\exists$, and identity $=$. □

We proceed to the definition of DBPL terms and formulas in a classical conservative

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[2] Henceforth, the set of variables will be denoted by V.

first-order fashion, except for the "absence" of functional symbols.[3]

6.2.1 The syntax of DBPL

Definition 2: DBPL terms

The set of all DBPL terms is formed by the union of the set of all individual constant symbols and the set V of variables. □

Definition 3: DBPL formulas

1. If $t_1, \ldots, t_n$ are individual constants or variables and R is a n-place predicate, then $Rt_1 \ldots t_n$ is a formula.
2. If t_1 and t_2 are individual constants or variables, then $t_1 = t_2$ is a formula.
3. If ϕ is a formula, then $\neg\phi$ is a formula.
4. If ϕ and ψ are formulas, then $(\phi \wedge \psi)$ is a formula.
5. If ϕ and ψ are formulas, then $(\phi \vee \psi)$ is a formula.
6. If ϕ and ψ are formulas, then $(\phi \rightarrow \psi)$ is a formula.
7. If ϕ is a formula and x a variable, then $\exists x\phi$ is a formula.
8. If ϕ is a formula and x a variable, then $\forall x\phi$ is a formula.
9. Nothing is a formula except on the basis of 1–8 □

So, the set of DBPL formulas is the same as the set of DPL formulas (which is the same set of standard PC formulas.) For PC and DPL, formulas are first class citizens in the sense that there are no other objects defined upon them. In this sense, DBPL formulas are only second class, since they play a role in the definition of DBPL first class citizens, namely, DBPL texts.

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[3] For first-order theories in which equality is definable, function symbols can always be eliminated in favour of predicate symbols. Instead of $f^n(x_1 \ldots x_n)$ we can write $A^{n+1}(y_1 \ldots y_n y_{n+1})$ for some appropriate predicate letter for which the following is a theorem:

$$\forall y_1 \ldots \forall y_n \forall y_{n+1} \forall y'_{n+1} (A^{n+1}(y_1 \ldots y_n y_{n+1}) \wedge A^{n+1}(y_1 \ldots y_n y'_{n+1})) \rightarrow (y_{n+1} = y'_{n+1})$$

Where the notion of equality is available, we can always use *functional predicate symbols* (with appropriate axioms) in the place of function symbols (cf. Hatcher (1968, p. 64)). More on this subject will be discussed in section 6.5, page 149.

Definition 4: DBPL pre-sequences, sequences and texts

Let ϕ be a DBPL formula. Then

1. ϕ is a DBPL pre-sequence (of length 1).
2. If α is a DBPL pre-sequence (of length n), then $\alpha \bullet \phi$ is a DBPL pre-sequence (of length $n+1$).
3. If α is a DBPL pre-sequence (of length n), then $\blacktriangleright \alpha \blacktriangleleft$ is a DBPL plain text (of depth 1 and length n).
4. All DBPL plain texts are DBPL texts (preserving the same length and depth (depth $= 1$)).
5. If $\blacktriangleright \alpha \blacktriangleleft$ is a DBPL text (of depth $n+1$), then α is a DBPL sequence (of depth n and length $= \mathit{length}(\alpha)$).
6. If $\blacktriangleright \alpha \blacktriangleleft$ is a DBPL text (of depth n and length m), then $\blacktriangleright \alpha \blacktriangleleft$ is a DBPL sequence (of depth n and length 1).
7. If α, and β are DBPL sequences (of length m and 1, resp.), then $\alpha \bullet \beta$ is a DBPL sequence (of $\mathit{depth}(\alpha \bullet \beta) = \mathit{max}(\mathit{depth}(\alpha), \mathit{depth}(\beta))$ and length $m+1$).
8. If α is a DBPL sequence, then $\blacktriangleright \alpha \blacktriangleleft$ is a DBPL text (of depth n, where $n = \mathit{depth}(\alpha) + 1$ and length $= \mathit{length}(\alpha)$).
9. Nothing is a DBPL pre-sequence, sequence, plain text, or text except on the basis of 1–8. □

Texts are not only the topmost entities for DBPL but also, to the best of my knowledge, new entities for semantic theories of discourse literature. The following examples are given in order to make the reader acquainted with them.

Example 6.1 *Some DBPL entities:*

1. *$\exists x(Px \wedge Qx)$ is a DBPL formula such that both occurrences of x are bound to the existential quantifier. $\exists x(Px) \wedge Qx$ is also a DBPL formula for which the first occurrence of x is clearly bound while the second one* seems to be free. *We*

will soon show that the second occurrence is, indeed, under the dynamic scope *of the existential quantifier.*

2. *Since* $\exists x(Px) \wedge Qx$ *and* $\exists x(Px \wedge Qx)$ *are each* DBPL *formulas they are also* DBPL *pre-sequences of length 1. On the other hand,* $\exists x(Px \wedge Qx) \bullet Rx$ *is a* DBPL *pre-sequence of length 2 since* $\exists x(Px \wedge Qx)$ *is a* DBPL *pre-sequence of length 1 and* Rx *is a* DBPL *formula. A distinctive character of* DBPL *pre-sequences is that the parenthetical* $\blacktriangleright\blacktriangleleft$ *is not allowed to occur in them.*

3. *As already shown,* $\exists x(Px \wedge Qx) \bullet Rx$ *is a pre-sequence of length 2. Therefore,* $\blacktriangleright\ \exists x(Px \wedge Qx) \bullet Rx\ \blacktriangleleft$ *is a* DBPL plain text*. This plain text might be seen as the* DBPL *counterpart for the plain micro-discourse (6.1), below,*

   (6.1) A man walks in the park. He whistles.

   where predicative letters P*,* R*, and* Q *denote the predicates* ***man****,* ***whistles****, and* ***walks-in-the-park*** *respectively. Notice that the anaphoric relationship between the pronoun in the second sentence and the noun phrase in the first sentence is captured by the variable* x *in* Rx *which is under the* dynamic scope[4] *of the existential quantifier in* $\exists x(Px \wedge Qx)$*. Notice, moreover, that for this particular* DBPL *plain text, a block structure picture would be given by figure 6.1.*

$$\boxed{\begin{array}{l}\exists x(\mathbf{man}\,x \wedge \mathbf{walks\text{-}in\text{-}the\text{-}park}\,x)\\[1em] \mathbf{whistles}\,x\end{array}}$$

Figure 6.1: Block structure for DBPL plain text (6.1)

4. *By clause 4 of definition 4, which states that all* DBPL *plain texts are* DBPL *texts, we get that* $\blacktriangleright\ \exists x(Px \wedge Qx) \bullet Rx\ \blacktriangleleft$ *is a* DBPL *text. The converse, however, is not true. Therefore, not all* DBPL *texts are* DBPL *plain texts. The emphasis put on the plain texts clause aims to reinforce the case of absence of block structure*

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[4] Any variable other than x might be used; but then, the anaphoric link will be lost.

in all other dynamic theories laying around.

5. *To build up the double recursive notion of* DBPL *text an auxiliary notion has been introduced. The trick is to allow* DBPL *texts be* DBPL *sequences and then build up new and more complex* DBPL *texts upon these sequences. So, as we have seen before,* $\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft$ *is a* DBPL *text (of depth 1). Therefore,*

   $$\begin{cases} \exists x(Px \wedge Qx) \bullet Rx & \text{is a DBPL sequence of depth 0.} \\ \blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft & \text{is a DBPL sequence of depth 1.} \end{cases}$$

   Since $\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft$ *is a* DBPL *sequence of depth 1, by clause 8 of definition 4,* $\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft$ *is a* DBPL *text of depth 2.*

6. *Since* $\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft$ *is a* DBPL *text of depth 2 it is also a* DBPL *sequence of depth 2. On the other hand,* $\exists x(Px \wedge Qx) \bullet Rx$ *is a* DBPL *sequence of depth 0. Therefore, by lemma 6.3 (page 119)* $\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft \bullet \exists x(Px \wedge Qx) \bullet Rx$ *is a* DBPL *sequence of depth 2 since* $max(2, 0) = 2$ *and length 3 (since* $length(\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft) + length(\exists x(Px \wedge Qx) \bullet Rx) = 1 + 2 = 3.$*)*

7. *Since* $\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft \bullet \exists x(Px \wedge Qx) \bullet Rx$ *is a* DBPL *sequence of depth 2,* $\blacktriangleright\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft \bullet \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft$ *is a* DBPL *text of depth 3 for which the following block structure picture, i.e. figure 6.2, would be assigned to.* □

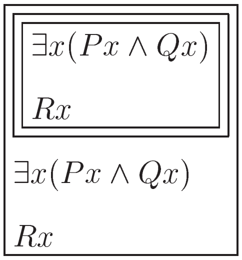

Figure 6.2: Block structure for $\blacktriangleright\blacktriangleright\blacktriangleright \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft\blacktriangleleft \bullet \exists x(Px \wedge Qx) \bullet Rx \blacktriangleleft$

The previous examples should help us to establish the correspondence between embedded blocks and the proper occurrences of the parenthetical delimiters $\blacktriangleright\blacktriangleleft$.

Moreover, they make clear that DBPL *plain texts* are DBPL texts “suffering” from the absence of any further discourse block structuring. So, in some sense, this case (namely, the plain texts case) corresponds to a simple sequence of sentences of plain unstructured discourses.

6.2.2 Semantics of DBPL

The semantics is defined with respect to a model $\mathfrak{M} = \langle \mathcal{D}, \mathcal{F} \rangle$ consisting of a non empty set of individuals $\mathcal{D}$ and an interpretation function $\mathcal{F}$ that assigns individuals from $\mathcal{D}$ to constant symbols and sets of n-tuples of individuals from $\mathcal{D}$ to n-ary relation expressions.

Differently from static approaches, any dynamic semantic theory ought to mirror the intuition that individual sentences operate on the information state one might have before the sentences have been processed. This implies that sentences ought to be seen as a kind of *update function* between information states. This approach to semantics is not new; it is indeed the breakthrough provided by DPL for simple sequences of sentences of any *plain unstructured discourse*. Moreover because of this, DPL’s notion of information states, which are thought of as sets of assignment functions, lacks any kind of structure.

As we are dealing with *structured discourses*, i.e. discourses exhibiting a nested block structure à la Algol,[5] a multidimensional information state would be the kind of generalization we are after. The basic idea is to allow a new dimension for every nested discourse block present in the discourse. Therefore, a DBPL information state would be modelled by tuples of DPL information states, i.e., tuples of assignment functions sets.

There is a clear correspondence amongst nested blocks and tuple components. Figures 6.2, and 6.3 as well as the drawings already given throughout several chapters of the present work, should help us to clarify the last point since they reflect, more

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[5] See pages 33, 34, 107, and 109 for examples.

naturally, the block structure for DBPL texts. Let us take

$$(6.2) \qquad \blacktriangleright\ \exists x(Dx \wedge Cx) \bullet \blacktriangleright\ Dx \bullet \exists x(Cx \wedge Pxy) \bullet Qx \blacktriangleleft \bullet Hx \blacktriangleleft$$

for instance; this text is composed of two nested blocks and therefore an n-tuple, where $n \geq 2$, would be the right structure to deal with information states for this particular example. It is clear that for $n = 1$ any n-tuple of sets of assignment functions couldn't deal with the example given due to the lack of dimensions. To make sure we will not run into this problem we take tuples as infinite sequences of sets of assignment functions. Moreover, the multidimensional approach allows us to keep some phenomena *confined* to the dimension they occur. For example, the formulas $\exists x(Dx \wedge Cx)$ and Hx occur, in this order, in the outermost block. Therefore, the variable x occurring in Hx is dynamically bound by the existential quantifier occurring in $\exists x(Dx \wedge Cx)$. Because of the multidimensional character adopted for information states, the variable x in Dx, in the innermost block, does occur free[6] (see figure 6.3, page 109.)

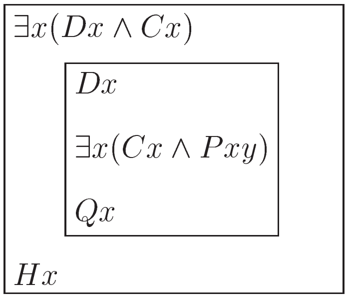

Figure 6.3: Block structure diagram for (6.2), page 109.

Definition 5: Information States

1. a ω-tuple is any sequence with infinitely denumerable components.
2. Let $\mathcal{D}$ be a non-empty domain of individuals and V the set of DBPL variables.

   A *multi-dimensional information state* is a ω-tuple where every component

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[6] This assertion is made under the *impenetrable assumption*. By impenetrable I mean that each dimensional component does not get affected by any phenomena occurring in any other dimensional component. This hypothesis, if relaxed, might give rise to new families of logics.

is a set of assignment functions in $\mathcal{D}^V$. To emphasize the multidimensional character of any information state s, we will use the notation s^n instead. Also, s^n_i and $\Pi(i, s^n)$ will be employed to refer to the i^{th} element of s^n. And s^0 will be used to refer to the ω-tuple s^n such that for each natural number i, if $i > 0$ then $s^n_i = \emptyset$. $\square$

Notice that the definition of multi-dimensional information states provides us with infinitely denumerable sequences of sets of assignment functions. If we had taken finitary sequences of sets of assignment functions instead of the course taken, we would run into trouble for cases such as $s^n[\![\Upsilon]\!]^i$ where $n < \mathit{depth}(\Upsilon)$ due to insufficiency of dimensional components.

Remark 6.1 *Note that the definition of multi-dimensional information states is based on the assumption that all dimensions "share" the same domain of individuals.* The possibility of having different domains for distinct dimensions has to await future work. $\square$

Remark 6.2 *We will use the terms n-dimensional information state and multi-dimensional information state interchangeably whenever this does not lead to confusion. Also, as usual, reference to $\mathfrak{M}$ will be omitted whenever this does not lead to confusion. Therefore, we use $s^n[\![]\!]^i$ instead of $s^n[\![]\!]^i_{\mathfrak{M}}$.* $\square$

Three kinds of information states are of special interest, namely, the absurd states, the ignorance state and the maximal states. The absolute absurd state $s^0 = \emptyset = \langle \emptyset, \ldots, \emptyset, \ldots \rangle$ is reached when incoming material is in a contradiction relation with previously introduced information in all dimensional levels. Relative absurd states are states where for some, but not all, dimensional indices i, $s^n_i = \emptyset$. The interesting point here is related to the finitary character of real life discourses. As finitary entities, discourses have a finite depth and therefore a relative absurd state might be considered "absolute" (relative to some discourse) whenever all components of s^n, from 1 to the depth of the discourse, are empty. The ignorance state, on the other hand, models the absence of knowledge; it should be taken as the initial state for any semantic valuation since all possibilities are open. Therefore the ignorance state corresponds to the minimal information one might have and the set of all assignment functions seems to be the ideal candidate for it. Maximal information, on the other hand, is represented by singleton sets of assignment functions. Therefore, variables have their value determined; this means that no other options are possible for every variable occurring in the discourse.

Recall that the definition for DBPL formulas characterizes the same set of PC formulas and therefore DPL. So, it comes as no surprise that the semantics for DBPL formulas conflate to DPL if we disregard the multidimensional nature of information states employed by DBPL (or, equivalently, if we consider DPL's information states notion as a particular case of the DBPL one where $n = 1$.) Moreover, due to the *multidimensional* nature of information states adopted, DBPL formulas ought to be interpreted according to the dimension index reflecting the embedding level the formulas occur in a DBPL text. The relativity of such interpretation, which might seem odd at first sight, is what allows us to use the "same" definite noun phrase to represent the two different cats talked about in the Lewises' scenario on page 95. For it, the American cat is the one who inhabits the outermost block while the New Zealand one inhabits the innermost block. These noun phrases could be represented as a conjunction of two predicates, ***cat*** and ***lives-in***, standing respectively for *cat* and *lives-in*. So, $\exists! x(\boldsymbol{cat}\, x) \wedge \boldsymbol{lives\text{-}in}\, x, \textit{USA}$ should be present in, and would affect only, the outermost block. Contrastingly, formula $\exists! x(\boldsymbol{cat}\, x) \wedge \boldsymbol{lives\text{-}in}\, x, \textit{NZ}$ should be present in the other block. Notice now that for each block, i.e. dimension, the existential quantifier "sets up" all possible discourse referents for x, which will be "refined" afterwards by formulas ***cat***, ***lives-in***, and whatever will get x as an argument. Figure 6.4 not only neatly synthesizes these points but mainly shows how the Russellian analysis of definite descriptions, based upon the uniqueness condition, could be sustained into the multidimensional approach.

As already pointed out, DBPL formulas might be seen as *update functions on multidimensional information states.* The update produced by DBPL formulas conforms

$\exists! x(\boldsymbol{cat}\, x) \wedge \boldsymbol{lives\text{-}in}\, x, \text{\textsc{usa}}$
$\exists! y(\boldsymbol{dog}\, y)$
$\vdots$
$\boldsymbol{is\text{-}under\text{-}the\text{-}piano}\, y$: the dog is under the piano
$\boldsymbol{is\text{-}in\text{-}the\text{-}carton}\, x$: the cat is in the carton

> $\exists! x(\boldsymbol{cat}\, x) \wedge \boldsymbol{lives\text{-}in}\, x, \text{\textsc{nz}}$
> $\exists! y(\boldsymbol{dog}\, y)$
> $\vdots$
> $\boldsymbol{loves}\, y, x$: the dog loves the cat

$\boldsymbol{is\text{-}going\text{-}to\text{-}pounce\text{-}on\text{-}you}\, x$: the cat is going to pounce on you
$\vdots$

Figure 6.4: A DBPL variant for Lewises scenario on page 95.

to the many different ways logical constants behave when anaphoric relationships are taken into account. Conjunction, for instance, is not only internally dynamic, (which means that an antecedent in its first argument could bind an anaphor in its second argument), but also externally dynamic since it also passes on bindings to sentences to come. The question one might ask at this point about the other logical constants is whether they are externally or internally dynamic (or both). Answers to this question are found elsewhere in the literature, but Groenendijk and Stokhof (1991) provide us with a concise one. But the basics are all present in figure 6.4. Notice that:

- **existentials** and **conjunctions** are internally as well as externally dynamic. Existentials pose us with some choices for the reference system to be developed. Take $Dx \wedge \exists x Px$ for example. The existential might (or not) take care of the variable quantified on. The position we are going to adopt is to take existentials as *downdates*, because whatever the value x has been assigned to will be lost and new references will be established. Moreover, the dynamic binding originating from existentials are passed on by both conjunctions.
- **atomic formulas** do not have dynamic effects of their own although they function as *eliminative updates* since they test whether an input assignment

> satisfies the condition it embodies. If so, the input assignment is passed on as output, if not it is rejected. So, the output is a subset of the input set.

Except for existentials and conjunction, all other logical constants are externally static. This means that bindings are not passed on by negation, implication, disjunction or even universals. It is time now for intuitions to give place to their formal counterparts.

Definition 6: Semantics of DBPL formulas

Let s^n and r^n be any n-dimensional information states. Suppose, also, that $s^n_j = r^n_j$ for every index dimension j other than i. Then

1. $s^n[\![Rx_1 \ldots x_m]\!]^i = r^n$ where $r^n_i = \{k \in s^n_i \mid \langle k(x_1), \ldots, k(x_m)\rangle \in \mathcal{F}(R)\}$
2. $s^n[\![x = y]\!]^i = r^n$ where $r^n_i = \{k \in s^n_i \mid k(x) = k(y)\}$
3. $s^n[\![\neg\phi]\!]^i = r^n$ where $r^n_i = \{g \in s^n_i \mid \{g\}[\![\phi]\!] = \emptyset\}$
4. $s^n[\![\phi \wedge \psi]\!]^i = s^n[\![\phi]\!]^i[\![\psi]\!]^i$
5. $s^n[\![\exists x\phi]\!]^i = s^n[x]^i[\![\phi]\!]^i$

where $s^n[x]^i = r^n$ such that $r^n_i = \{k[x/d] \mid k \in s^n_i \; \& \; d \in \mathcal{D}\}$. As usual, $k[x/d]$ is the assignment g such that g agrees with k on the values of all the variables except, possibly, x and such that $k(x) = d$. □

Definition 7: Classical closure of DBPL formulas

Let α be any DBPL formula and $\mathfrak{M}$ an arbitrary model. Then

$\downarrow[\![\alpha]\!]_{\mathfrak{M}} \;=\; \{g \in D^V \mid \{g\}[\![\alpha]\!]_{\mathfrak{M}} \neq \emptyset\}.$ □

Remark 6.3 *Note that due to the* shared domain assumption *stated in remark 6.1,* $\downarrow [\![\alpha]\!]$ *does not change among dimensional indices. Therefore, for every natural number* $i > 0$, $\downarrow[\![\alpha]\!]^i = \{k \in \mathcal{D}^V \mid \{k\} \; [\![\alpha]\!] \neq \emptyset\}$ □

Remark 6.4 *The notation used in definition 6 is, indeed, an abbreviation for*

1. $s^n[\![Rx_1 \ldots x_m]\!]^i = \langle s^n_1, \ldots, s^n_i[\![Rx_1 \ldots x_m]\!], \ldots, s^n_n, \ldots\rangle$
$$= \langle s^n_1, \ldots, \{k \in s^n_i \mid \langle k(x_1), \ldots, k(x_m)\rangle \in \mathcal{F}(R)\}, \ldots, s^n_n, \ldots\rangle$$
2. $s^n[\![x = y]\!]^i = \langle s^n_1, \ldots, \{k \in s^n_i \mid k(x) = k(y)\}, \ldots, s^n_n, \ldots\rangle$
3. $s^n[\![\neg\phi]\!]^i = \langle s^n_1, \ldots, \{g \in s^n_i \mid \{g\}[\![\phi]\!] = \emptyset\}, \ldots, s^n_n, \ldots\rangle$
4. $s^n[\![\phi \wedge \psi]\!]^i = \langle s^n_1, \ldots, s^n_i[\![\phi]\!][\![\psi]\!], \ldots, s^n_n, \ldots\rangle$
5. $s^n[\![\exists x\phi]\!]^i = \langle s^n_1, \ldots, s^n_i[x][\![\phi]\!], \ldots, s^n_n, \ldots\rangle$

The only advantage of this "verbose" notation is to make it easy to see the evaluation of each dimensional block under the government of the impenetrable assumption. □

Definition 6 is given in a format intending to show not only the set of primitive logical constants for DBPL but also how dynamic binding is accomplished. For instance, note that conjunction, which is interpreted as function composition, allows pushing forward any possible binding that might occur during the semantic evaluation of previous formulas (giving by the assignments already present in the state s^n or "freshly" bindings risen from the first conjunct). In some sense, conjunction works like a passive transducer since the real source of dynamic binding is the existential quantifier. In this case, the existentially quantified variable behaves like an active booking system keeping track of values in the *dynamic* scope of the existential quantifier. Notice, also, how items 4 and 5 manage to keep the dynamics of conjunction and existential quantification confined to only one dimension by using the same superscript index i. These points are better presented in examples 6.2 and 6.3 below, where we disregard all indices other than a certain i.[7]

Example 6.2 *Let* $V = \{x, y, \ldots\}$ *be the set of variables, and suppose also, that* $\mathcal{D} = \{1, 2, 3, 4\}$ *is the domain of discourse and* $\mathcal{F}(\phi) = \{3, 4\}$ *for a predicate* ϕ*. Assume that*

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[7] Therefore, if we take $i = n = 1$, these examples would be valid for DPL too.

$i_1 = \{(x,1),(y,1),\ldots\}$ $i_5 = \{(x,2),(y,1),\ldots\}$ $i_9 = \{(x,3),(y,1),\ldots\}$
$i_2 = \{(x,1),(y,2),\ldots\}$ $i_6 = \{(x,2),(y,2),\ldots\}$ $i_{10} = \{(x,3),(y,2),\ldots\}$
$i_3 = \{(x,1),(y,3),\ldots\}$ $i_7 = \{(x,2),(y,3),\ldots\}$ $i_{11} = \{(x,3),(y,3),\ldots\}$
$i_4 = \{(x,1),(y,4),\ldots\}$ $i_8 = \{(x,2),(y,4),\ldots\}$ $i_{12} = \{(x,3),(y,4),\ldots\}$
$i_{13} = \{(x,4),(y,1),\ldots\}$ $i_{14} = \{(x,4),(y,2),\ldots\}$ $i_{15} = \{(x,4),(y,3),\ldots\}$
$i_{16} = \{(x,4),(y,4),\ldots\}$

and let $s^n_i = \langle \ldots, \{i_2, i_4\}, \ldots \rangle$.

By definition, $s^n\llbracket \exists x\phi x \rrbracket^i = s^n[x]^i\llbracket \phi x \rrbracket^i$*. But* $s^n[x]^i$ *is the information state obtained from* s^n *by "forgetting" all information* s^n *might have about* x*, in the dimension index* i*, i.e.,* downdating s^n_i *with respect to* x*. So,*

$s^n[x]^i = \langle \ldots, \{i_2, i_6, i_{10}, i_{14}, i_4, i_8, i_{12}, i_{16}\}, \ldots \rangle$*. Therefore*

$s^n[x]^i\llbracket \phi x \rrbracket^i = \langle \ldots, \{i \in s^n[x]^i \mid i(x) \in \mathcal{F}(\phi)\}, \ldots \rangle = \langle \ldots, \{i_{10}, i_{12}, i_{14}, i_{16}\}, \ldots \rangle$ □

Example 6.3 *As example 6.2 except that* $\phi = (\exists x\psi x) \wedge \theta x$*,* $\mathcal{F}(\psi) = \{3,4\}$ *and* $\mathcal{F}(\theta) = \{4\}$*. By definition* $s^n\llbracket (\exists x\psi x) \wedge \theta x \rrbracket^i = s^n\llbracket (\exists x\psi x) \rrbracket^i\llbracket \theta x \rrbracket^i = s^n[x]^i\llbracket \psi x \rrbracket^i\llbracket \theta x \rrbracket^i$*. By example 6.2, we already know that* $s^n[x]^i\llbracket \psi x \rrbracket^i = \langle \ldots, \{i_{10}, i_{12}, i_{14}, i_{16}\}, \ldots \rangle$*.* ***Therefore*** $s^n[x]^i\llbracket \psi \rrbracket^i\llbracket \theta x \rrbracket^i = \langle \ldots, \{i_{10}, i_{12}, i_{14}, i_{16}\}, \ldots \rangle\llbracket \theta x \rrbracket^i = \langle \ldots, \{i_{14}, i_{16}\}, \ldots \rangle$*.*

□

Example 6.2 clearly shows how the formula $\exists x\phi x$ **sets up** values – the possible discourse referents – for x. The updated state $\langle \ldots, \{i_{10}, i_{12}, i_{14}, i_{16}\}, \ldots \rangle$ keeps track of such information. Notice, also, that i_{10}, i_{14} and i_{12}, i_{16} agree with i_2 and i_4, resp., on all other values for variables. The values the *input* state could have for variables other than x are preserved by assignments on the *output* state.

Example 6.3, on the other hand, shows how to extend the scope of existentially quantified variables beyond the syntactic border. Note that the occurrence of x in θx is syntactically free. However, the kinematics projected into definition 6 brings it back to the scope of the quantifier which, by the way, occurs in the first conjunct of a conjunction.

A closer look at negation shows that it is also a bit "tricky". Negation is indeed a set difference operation,[8] which ultimately removes from the input state all assignments that classically validate any formula ϕ. Therefore, any existentials which might occur as part of any negated formula ϕ will have their dynamic binding power blocked by negation. Let us take $(\neg\exists x\phi x)\wedge\psi x$ for example. In this case, x in ψx occurs free. The dynamic power originating from $\exists x\phi x$ gets blocked inside negation because of the set difference operation; the set difference operation removes from the input state s any assignments originating from the existential $\exists x\phi x$. In other words, $s[\![(\neg\exists x\phi x)\wedge\psi x]\!] = s[\![\neg\exists x\phi x]\!][\![\psi x]\!] = (s-\downarrow[\![\exists x\phi x]\!])[\![\psi x]\!])$.

Examples 6.2 and 6.3, as well as the comment about negation, provide us with evidence to take negation, conjunction and existential quantification as primitive logical constants for DBPL. The remaining logical constants can be defined in terms of the primitive set[9] by

$$\begin{array}{lcl}\phi\rightarrow\psi & \backsimeq & \neg[\phi\wedge\neg\psi]\\ \phi\vee\psi & \backsimeq & \neg[\neg\phi\wedge\neg\psi]\\ \forall x\phi & \backsimeq & \neg\exists x\neg\phi\end{array}$$

and therefore their semantic interpretation can be given by

$$\begin{array}{lcl}s[\![\phi\rightarrow\psi]\!] & = & s-\downarrow[\![\phi\wedge\neg\psi]\!]\\ s[\![\phi\vee\psi]\!] & = & s-\downarrow[\![\neg\phi\wedge\neg\psi]\!]\\ s[\![\forall x\phi]\!] & = & s-\downarrow[\![\exists x\neg\phi]\!]\end{array}$$

The interesting bit here is related to implication: implication is *internally dynamic*. The internal dynamic effect is achieved by the use of conjunction in its definition. So, free variables occurring in the consequent might be bound to some existential into the antecedent. This is more easily seen if we take $s[\![\phi\rightarrow\psi]\!]$ as $\{g\in s\mid\{g\}[\![\phi]\!]=\emptyset\vee\{g\}[\![\phi]\!][\![\psi]\!]\neq\emptyset\}$ which is equivalent to $s-\downarrow[\![\phi\wedge\neg\psi]\!]$.

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[8] It is a trivial set theory exercise to show that $\{g\in s_i^n\mid\{g\}[\![\phi]\!]=\emptyset\} \ = \ s_i^n-\downarrow[\![\phi]\!]$. The proof goes thus:
$i\in(s_i^n-\downarrow[\![\phi]\!])$ iff $i\in s_i^n$ & $i\not\in\downarrow[\![\phi]\!]$. But $i\not\in\downarrow[\![\phi]\!]$ iff $i\not\in\{g\in D^V\mid\{g\}[\![\phi]\!]\neq\emptyset\}$ iff $\{i\}[\![\phi]\!]=\emptyset$. Therefore, $i\in s_i^n$ & $\{i\}[\![\phi]\!]=\emptyset$, i.e., $i\in\{g\in s_i^n\mid\{g\}[\![\phi]\!]=\emptyset\}$.

[9] There is a reason for using $\backsimeq$ instead of the usual "full equivalence" symbol $\equiv$. For the reasons why we could not take universal quantifier and disjunction, nor universal quantifier and implication, and therefore, having the usual "full equivalence", we refer the reader to Groenendijk and Stokhof (1991, page 61).

Contrasting to PC, which allows a broader range of primitives and therefore a broader interdefinability of logical constants, DBPL and DPL are more restricted in this aspect. But PC is a static theory where binding and scope collapse into the same concept. DBPL on the other hand, is a dynamic theory in which these aspects do not collapse. Recall that existential formulas have the power of binding variables outside their syntactic scope and conjunction allows to propagate dynamic binding through the function composition mechanism. Therefore, the set of primitives presented conforms to the dynamics we are after. Moreover, no other set of primitives does work for DBPL. The reason for this, together with some other logical properties of the system, will be discussed later in this chapter.

If we restrict ourselves to formulas, the DBPL system will collapse to DPL. But DBPL differs from DPL by its multidimensional aspect, which is syntactically characterized in definition 4 as *texts*. And, since definition 4 is built up on the auxiliary notions of pre-sequence and sequence, the semantic interpretation for these concepts conforms to the following intuitive ideas:

- $\star$ For DBPL sequences, the dynamic interpretation is almost self-suggesting: the $\bullet$ connective is mapped to the function composition operator. Therefore, as each sentence updates the previous information state, such updated states will be used as the input state for the next “sentence” in the sequence.
- $\star$ For DBPL *sub-texts*,[10] the dynamic interpretation is also self-suggesting: the $\blacktriangleright\blacktriangleleft$ parenthesis take us from the dimension in which we are evaluating the *sub-text* to a higher one, where we ought to evaluate the sequence defining the sub-text. This process, naturally, reflects the level of embedding the sub-text might be occurring at inside a text. And since the embedding process might be done *ad infinitum*, the corresponding *dimensional shifting* should reflect this inertial point of view.

Notice that the combination of both ideas preserves the *compositionality criteria*.

Definition 8: Semantics of DBPL pre-sequences, sequences and texts

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[10] A sub-text could be seen as a text occurring in the sequence that defines the broader text.

Let ϕ be a DBPL formula and s^n a multidimensional information state. Suppose, also, that i is a natural number such that $i > 0$. Then

1. If α is a DBPL pre-sequence of length 1 and ϕ is the formula that equals α, then $s^n[\![\alpha]\!]^i = s^n[\![\phi]\!]^i$. (* it reduces to the formula's interpretation *)
2. If α is a DBPL pre-sequence of length $m > 1$, then $s^n[\![\alpha \bullet \phi]\!]^i = s^n[\![\alpha]\!]^i[\![\phi]\!]^i$.
3. If α is a DBPL sequence such that α is $\blacktriangleright \beta \blacktriangleleft$ for some DBPL sequence β, then $s^n[\![\alpha]\!]^i = s^n[\![\blacktriangleright \beta \blacktriangleleft]\!]^i = s^n[\![\beta]\!]^{i+1}$. (* shift from the ongoing working dimension to the next one *)
4. If α, β are sequences (of length n and 1 resp.), then $s^n[\![\alpha \bullet \beta]\!]^i = s^n[\![\alpha]\!]^i[\![\beta]\!]^i$. (* index distributivity *)
5. **If $\blacktriangleright \alpha \blacktriangleleft$ is a text, then** $s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!] = s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!]^{i=0}$ $\square$

Definition 8 is stated in such a way that evaluation of pre-sequences, item 2, and sequences, item 4, is made in a right-to-left basis. This evaluation splits the whole (pre-)sequence into two "halves", where the first half is of length $n-1$ and the second one is of length 1. However, we could state this in a stronger form saying that

(6.3) If α and β are sequences of arbitrary length, then $s^n[\![\alpha \bullet \beta]\!]^i = s^n[\![\alpha]\!]^i[\![\beta]\!]^i$

A natural consequence of this theorem is that the right-to-left order does not play a real role; we might have based definition 8 on a left-to-right basis without changing the desired effect.[11]

But, to state (6.3) we need first to develop some auxiliary lemmas, which will be used in the theorem's proof. At some point of its proof it will be necessary to answer the following question: *Which DBPL entities are of length 1?* The answer quite naturally would be: *pre-sequences of length 1 and sequences of length 1.* In

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[11] What is really at stake here is function composition. Recalling that phrases and texts are update functions on information states we should note that $s^n[\![\alpha]\!]^i[\![\beta]\!]^i$ denotes the function composition in a postfix format. So, $s^n[\![\alpha]\!]^i[\![\beta]\!]^i$ might be read, in a more set theoretic fashion, as $[\![\beta]\!]^i([\![\alpha]\!]^i(s^n))$.

fact, pre-sequences are also sequences, even though this fact has not been explicitly granted in definition 4.

Lemma 6.1

All DBPL pre-sequences are DBPL sequences.

Proof

supp	*1.*	α *is a pre-sequence of length* $m, m \geq 1$
1, definition 4.3	*2.*	$\blacktriangleright \alpha \blacktriangleleft$ *is a* ***plain text*** *of depth 1 and length* m
2, definition 4.4	*3.*	$\blacktriangleright \alpha \blacktriangleleft$ *is a* ***text*** *of depth 1 and length* m
3, definition 4.5	*4.*	α *is a sequence of depth 0 and length* m ☐

Lemma 6.2 ***sequence decomposition (weak form)***

Let α *be any DBPL sequence of length* $m, m \geq 1$*. Then, there exist* m *DBPL sequences of length 1 such that* $\alpha = \alpha_1 \bullet \alpha_2 \bullet \ldots \bullet \alpha_m$

Proof

The proof is made by induction on the complexity of α*.*

Base step*: for any sequence* α *of length 1. Trivial, since the only DBPL sequences of length 1 are either:*

(1) DBPL pre-sequences of length 1 (lemma 6.1) or

(2) DBPL sequences of type $\blacktriangleright \beta \blacktriangleleft$*, for some DBPL sequence* β*. By definition 4.6,* $\blacktriangleright \beta \blacktriangleleft$ *is a DBPL sequence of length 1.*

Inductive step*: for any sequence* α *of length* m*,* $\alpha = \alpha_1 \bullet \alpha_2 \bullet \ldots \bullet \alpha_m$

The only way of making bigger sequences is given by definition 4.7. So, if α' *is any DBPL sequence of length* $m+1$ *then there exist two DBPL sequences* α *and* β *of length* m *and 1 resp. such that* $\alpha' = \alpha \bullet \beta$*. Applying the inductive hypothesis on* α *we get the desired result.* ☐

Lemma 6.3 ***sequence addition***

If α *and* β *are DBPL sequences of length* m *and* n *resp., then* $\alpha \bullet \beta$ *is a DBPL sequence* *of length* $m+n$.

Proof

By lemma 6.2, for any DBPL *sequence* β *of length* n *there exist* n DBPL *sequences of length 1 such that* $\beta = \beta_1 \bullet \beta_2 \bullet \ldots \bullet \beta_n$. *By straightforward* n *applications of definition 4.7 we get that* $\alpha \bullet \beta_1 \bullet \ldots \bullet \beta_n$, *i.e.,* $\alpha \bullet \beta$ *is a* DBPL *sequence of length* $m+n$. □

Lemma 6.4 ***sequence decomposition (strong form)***

Let α *be a sequence of length* m, $m > 1$. *Then*

there exist, at least, two sequences α_1 *and* α_2 *such that* $\alpha = \alpha_1 \bullet \alpha_2$

Proof

The proof is made by cases.

(1) suppose $length(\alpha) = 2$. *Then by lemma 6.2, there exists* 2 DBPL *sequences* α_1 *and* α_2 *of length 1 such that* $\alpha = \alpha_1 \bullet \alpha_2$.

(2) suppose $length(\alpha) > 2$. *By lemma 6.2, there exist* m DBPL *sequences* $\alpha_1, \ldots, \alpha_m$ *of length 1 such that* $\alpha = \alpha_1 \bullet \ldots \bullet \alpha_m$. *Let* i *be any number* $1 \leq i < m$. *Two cases are still possible. (i) If* $i = 1$, *then* α_i *is a sequence of length 1. By* $m - i$ *successive applications of definition 4.7 we get the sequence* $\alpha_{i+1} \bullet \ldots \bullet \alpha_m$ *of length* $m - i$. *Call them* α_1 *and* α_2 *resp. By lemma 6.3 we get the desired result. (ii) If* $i > 1$, *then by* i *successive applications of definition 4.7 we get the sequence* $\alpha_1 \bullet \ldots \bullet \alpha_i$ *of length* i. *By similar argumentation, we get another sequence, namely,* $\alpha_{i+1} \bullet \ldots \bullet \alpha_m$ *of length* $m - i$. *Call them* α_1 *and* α_2 *resp. Again, by lemma 6.3 we get the desired result.* □

Lemma 6.5 ***Index distributivity over sequence formation operator*** $\bullet$

Let α *be a* DBPL *sequence of length* m, *i.e.* $\alpha = \alpha_1 \bullet \alpha_2 \ldots \bullet \alpha_m$, *where for all* $i, 1 \leq i \leq m, \alpha_i$ *is a* DBPL *sequence of length 1. Then*

$$s^n[\![\alpha]\!]^i = s^n[\![\alpha_1 \bullet \alpha_2 \ldots \bullet \alpha_m]\!]^i = s^n[\![\alpha_1]\!]^i[\![\alpha_2]\!]^i \ldots [\![\alpha_m]\!]^i$$

Proof

The theorem is a trivial consequence of definition 8, clause 4, and its proof is made by induction on the length of α.

The inductive step: for all $j \leq m, s^n[\![\alpha_1 \bullet \alpha_2 \ldots \bullet \alpha_j]\!]^i = s^n[\![\alpha_1]\!]^i[\![\alpha_2]\!]^i \ldots [\![\alpha_j]\!]^i$

Then for any DBPL *sequences* α' *of length* m *and* β *of length 1*

$s^n[\![\alpha' \bullet \beta]\!]^i = s^n[\![\alpha']\!]^i[\![\beta]\!]^i$ *by definition 8.4. It follows from the inductive step that* $s^n[\![\alpha']\!]^i[\![\beta]\!]^i = s^n[\![\alpha_1]\!]^i[\![\alpha_2]\!]^i \ldots [\![\alpha_m]\!]^i[\![\beta]\!]^i$ □

In some sense, this sequence splitting is weak since it allows for distributing the dimensional index throughout all "atomic" sequence components. A stronger version would say that the dimensional index could be distributed in a prefix-suffix basis, for all pairs of prefixes-suffixes that might compose the sequence.[12]

Lemma 6.6 ***sequence distributivity theorem (strong form)***

If α *and* β *are sequences of arbitrary length, then*

$$s^n[\![\alpha \bullet \beta]\!]^i = s^n[\![\alpha]\!]^i[\![\beta]\!]^i$$

Proof

Suppose β *is a sequence of length* m, $m \geq 1$. *Then, by lemma 6.2 there exist* m *sequences of length 1 such that* $\beta = \beta_1 \bullet \ldots \bullet \beta_m$. *Therefore,* $s^n[\![\alpha \bullet \beta]\!]^i$ *can be rewritten as* $s^n[\![\alpha \bullet \beta_1 \bullet \ldots \bullet \beta_m]\!]^i$. *By* m *applications of definition 8.4 we get* $s^n[\![\alpha]\!]^i[\![\beta_1]\!]^i \ldots [\![\beta_m]\!]^i$. *Now, by function associativity we get* $s^n[\![\alpha]\!]^i([\![\beta_1]\!]^i \ldots [\![\beta_m]\!]^i)$ *i.e.* $s^n[\![\alpha]\!]^i[\![\beta]\!]^i$. *Therefore,* $s^n[\![\alpha \bullet \beta]\!]^i = s^n[\![\alpha]\!]^i[\![\beta]\!]^i$ □

Theorem 6.1 ***sequence splitting theorem (strong form)***

Let α *be any sequence of length > 1. For any two sequences* α_1, α_2 *such that* $\alpha = \alpha_1 \bullet \alpha_2$ *it holds that*

$$s^n[\![\alpha]\!]^i = s^n[\![\alpha_1 \bullet \alpha_2]\!]^i = s^n[\![\alpha_1]\!]^i[\![\alpha_2]\!]^i$$

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[12] For example, if α is a sequence of length 3 then the possible prefix-suffix pairs might be: $(\alpha_1, \alpha_2 \bullet \alpha_3)$, and $(\alpha_1 \bullet \alpha_2, \alpha_3)$.

Proof

Follows from lemmas 6.4 and 6.6. □

From now on, whenever this does not lead to confusion, theorem 6.1 will be used interchangeably with definition 8.4.

Items 2 and 4 of definition 8 still deserve a comment. Both items deal with distribution of the dimensional index through pre-sequences and sequences components. Since item 2 deals with pre-sequences, which necessarily are only made of formulas, this entails that all components of a pre-sequence will be evaluated at the same dimensional index. Item 4, as well as theorem 6.6, deals with general sequences, which might be made of any DBPL entities. Although item 4 distributes the dimensional index through the sequence components, **this does not mean that all sequence components will be evaluated at the same dimensional index**, since, by item 3, the internal sequence structure does matter. Therefore, it is wrong to assume that $s^n[\![\alpha \bullet \beta]\!]^i = \langle s^n_1, \ldots, s^n_i[\![\alpha]\!][\![\beta]\!], \ldots, s^n_n, \ldots\rangle$ holds unconditionally. This is especially clear when α and β are sequences of different depths. It is clear that the facts just explained together with items 4 and 5 of definition 6 conform to the intuitive ideas underlying the impenetrable hypothesis.

As might be seen, definition 8 deals with DBPL sequences in such a way that the shifting among dimensions does not move us from the information state reached (for each individual dimension) as the discourse unfolds. The following example shows how the dimensional shiftings are performed.

Example 6.4 *Suppose that* $V = \{x, y, \ldots\}$ *is the set of variables, and also, that* $\mathcal{D} = \{1, 2, 3, 4\}$ *is the domain of discourse and* $\mathcal{F}(P) = \{(3,1), (4,2), (4,4)\}$, $\mathcal{F}(Q) = \{2, 4\}$ *for predicates* P, *and* Q. *Assume that*

$$\begin{array}{lll}
i_1 = \{(x,1),(y,1),\ldots\} & i_5 = \{(x,2),(y,1),\ldots\} & i_9 = \{(x,3),(y,1),\ldots\} \\
i_2 = \{(x,1),(y,2),\ldots\} & i_6 = \{(x,2),(y,2),\ldots\} & i_{10} = \{(x,3),(y,2),\ldots\} \\
i_3 = \{(x,1),(y,3),\ldots\} & i_7 = \{(x,2),(y,3),\ldots\} & i_{11} = \{(x,3),(y,3),\ldots\} \\
i_4 = \{(x,1),(y,4),\ldots\} & i_8 = \{(x,2),(y,4),\ldots\} & i_{12} = \{(x,3),(y,4),\ldots\} \\
i_{13} = \{(x,4),(y,1),\ldots\} & i_{14} = \{(x,4),(y,2),\ldots\} & i_{15} = \{(x,4),(y,3),\ldots\} \\
i_{16} = \{(x,4),(y,4),\ldots\} & &
\end{array}$$

and let $s^n = \langle \{i_1, i_2\}, \{i_{16}\}, \ldots\rangle$.

Finally, let the DBPL *text be* $\blacktriangleright \exists x Pxy \bullet Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft\blacktriangleleft$.

The update of s^n *with* $\blacktriangleright \exists x Pxy \bullet Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft\blacktriangleleft$ *determines* $\langle \{i_{14}\}, \{i_{14}, i_{16}\}, \ldots\rangle$ *for output, i.e.,*

$$\langle \{i_1, i_2\}, \{i_{16}\}, \ldots\rangle \llbracket \blacktriangleright \exists x Pxy \bullet Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft\blacktriangleleft \rrbracket = \langle \{i_{14}\}, \{i_{14}, i_{16}\}, \ldots\rangle$$

The computation of the output state is as follows.

$$\begin{aligned}
& s^n \llbracket \blacktriangleright \exists x Pxy \bullet Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft\blacktriangleleft \rrbracket & = \\
& s^n \llbracket \exists x Pxy \bullet Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& s^n \llbracket \exists x Pxy \rrbracket^1 \llbracket Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& s[x]^1 \llbracket Pxy \rrbracket^1 \llbracket Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_1, , i_2\}[x], \{i_{16}\}, \ldots\rangle \llbracket Pxy \rrbracket^1 \llbracket Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_1, i_5, i_9, i_{13}, i_2, i_6, i_{10}, i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket Pxy \rrbracket^1 \llbracket Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_9, i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket Qy \bullet \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_9, i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket Qy \rrbracket^1 \llbracket \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket \blacktriangleright \exists y Qy \bullet Pxy \blacktriangleleft \rrbracket^1 & = \\
& \langle \{i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket \exists y Qy \bullet Pxy \rrbracket^2 & = \\
& \langle \{i_{14}\}, \{i_{16}\}, \ldots\rangle \llbracket \exists y Qy \rrbracket^2 \llbracket Pxy \rrbracket^2 & = \\
& \langle \{i_{14}\}, \{i_{16}\}, \ldots\rangle [y]^2 \llbracket Qy \rrbracket^2 \llbracket Pxy \rrbracket^2 & = \\
& \langle \{i_{14}\}, \{i_{13}, i_{14}, i_{15}, i_{16}\}, \ldots\rangle \llbracket Qy \rrbracket^2 \llbracket Pxy \rrbracket^2 & = \\
& \langle \{i_{14}\}, \{i_{14}, i_{16}\}, \ldots\rangle \llbracket Pxy \rrbracket^2 & = \\
& \langle \{i_{14}\}, \{i_{14}, i_{16}\}, \ldots\rangle & \Box
\end{aligned}$$

Notice that the semantics for DBPL texts meets the constraints we are looking for: firstly, each dimension corresponds to discourse blocks, i.e. DBPL *sub-texts*, depending only on the depth the sub-texts occur at. Admitting that

$$\blacktriangleright\ \alpha\bullet\ \blacktriangleright\ \beta\ \blacktriangleleft\ \bullet\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\blacktriangleleft \tag{6.4}$$

is a text such that α, β, γ and δ are DBPL formulas (as shown in figure 6.5), and that s^n is a multidimensional information state, the update of s^n with (6.4) follows the steps shown in figure 6.6, page 124.

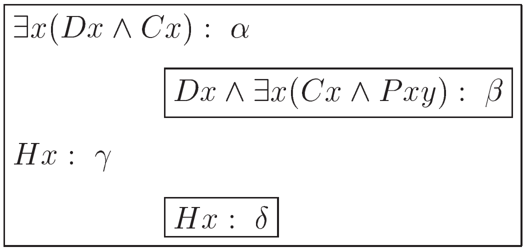

Figure 6.5: Block structure picture for (6.4)

1.	$s^n\llbracket\ \blacktriangleright\ \alpha\bullet\ \blacktriangleright\ \beta\ \blacktriangleleft\ \bullet\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\blacktriangleleft\ \rrbracket =$	
	$= s^n\llbracket\ \blacktriangleright\ \alpha\bullet\ \blacktriangleright\ \beta\ \blacktriangleleft\ \bullet\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\blacktriangleleft\ \rrbracket^0$	by def. 8.5
2.	$= s^n\llbracket\alpha\bullet\ \blacktriangleright\ \beta\ \blacktriangleleft\ \bullet\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\ \rrbracket^1$	from 1 by def. 8.3
3.	$= s^n\llbracket\alpha\rrbracket^1\llbracket\ \blacktriangleright\ \beta\ \blacktriangleleft\ \bullet\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\ \rrbracket^1$	from 2 by def. 8.4
4.	$= s^n\llbracket\alpha\rrbracket^1\llbracket\ \blacktriangleright\ \beta\ \blacktriangleleft\ \rrbracket^1\llbracket\gamma\bullet\ \blacktriangleright\ \delta\ \blacktriangleleft\ \rrbracket^1$	from 3 by def. 8.4
5.	$= s^n\llbracket\alpha\rrbracket^1\llbracket\ \blacktriangleright\ \beta\ \blacktriangleleft\ \rrbracket^1\llbracket\gamma\rrbracket^1\llbracket\ \blacktriangleright\ \delta\ \blacktriangleleft\ \rrbracket^1$	from 4 by def. 8.4
6.	$= s^n\llbracket\alpha\rrbracket^1\llbracket\beta\rrbracket^2\llbracket\gamma\rrbracket^1\llbracket\ \blacktriangleright\ \delta\ \blacktriangleleft\ \rrbracket^1$	from 5 by def. 8.3
7.	$= s^n\llbracket\alpha\rrbracket^1\llbracket\beta\rrbracket^2\llbracket\gamma\rrbracket^1\llbracket\delta\rrbracket^2$	from 6 by def. 8.3
8.	$\langle s^n_1\llbracket\alpha\rrbracket, s^n_2, \ldots\rangle\llbracket\beta\rrbracket^2\llbracket\gamma\rrbracket^1\llbracket\delta\rrbracket^2$	from 7 since α is a formula
9.	$\langle s^n_1\llbracket\alpha\rrbracket, s^n_2\llbracket\beta\rrbracket, \ldots\rangle\llbracket\gamma\rrbracket^1\llbracket\delta\rrbracket^2$	from 8 since β is a formula
10.	$\langle s^n_1\llbracket\alpha\rrbracket\llbracket\gamma\rrbracket, s^n_2\llbracket\beta\rrbracket, \ldots\rangle\llbracket\delta\rrbracket^2$	from 9 since γ is a formula
11.	$\langle s^n_1\llbracket\alpha\rrbracket\llbracket\gamma\rrbracket, s^n_2\llbracket\beta\rrbracket\llbracket\delta\rrbracket, \ldots\rangle$	from 10 since δ is a formula
12.	$\langle s^n_1\llbracket\alpha\bullet\gamma\rrbracket, s^n_2\llbracket\beta\bullet\delta\rrbracket, \ldots\rangle$	

Figure 6.6: Information states computation for (6.4) when α, β, γ and δ are DBPL formulas (for example, the ones shown in figure 6.5.)

Step 7 is general enough to show us how to compute the final multidimensional information state. Since by hypothesis α, β, γ, and δ are all DBPL formulas, the final result is as shown in step 12. Suppose now that α is $\blacktriangleright\blacktriangleright \exists xPx \blacktriangleleft\blacktriangleleft$ and β, γ, and δ are DBPL formulas as depicted in figure 6.7. The information states computation should now follow the steps shown in figure 6.8, page 125.

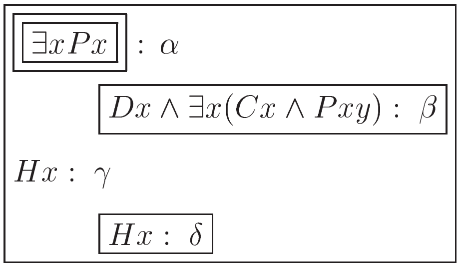

Figure 6.7: Block structure picture for (6.4) when α is $\blacktriangleright\blacktriangleright \exists xPx \blacktriangleleft\blacktriangleleft$

7.	$= s^n\llbracket\alpha\rrbracket^{1}\llbracket\beta\rrbracket^{2}\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 6 by def. 8.3
8.	$= s^n\llbracket \blacktriangleright\blacktriangleright \exists xPx \blacktriangleleft\blacktriangleleft \rrbracket^{1}\llbracket\beta\rrbracket^{2}\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 7 and equality
9.	$= s^n\llbracket \blacktriangleright \exists xPx \blacktriangleleft \rrbracket^{2}\llbracket\beta\rrbracket^{2}\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 8 by def. 8.3
10.	$= s^n\llbracket\exists xPx\rrbracket^{3}\llbracket\beta\rrbracket^{2}\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 9 by def. 8.3
11.	$\langle s_1^n, s_2^n, s_3^n\llbracket\exists xPx\rrbracket, \ldots\rangle\llbracket\beta\rrbracket^{2}\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 10 and definition 6
12.	$\langle s_1^n, s_2^n\llbracket\beta\rrbracket, s_3^n\llbracket\exists xPx\rrbracket, \ldots\rangle\llbracket\gamma\rrbracket^{1}\llbracket\delta\rrbracket^{2}$	from 11 since β is a formula
13.	$\langle s_1^n\llbracket\gamma\rrbracket, s_2^n\llbracket\beta\rrbracket, s_3^n\llbracket\exists xPx\rrbracket, \ldots\rangle\llbracket\delta\rrbracket^{2}$	from 12 since γ is a formula
14.	$\langle s_1^n\llbracket\gamma\rrbracket, s_2^n\llbracket\beta\rrbracket\llbracket\delta\rrbracket, s_3^n\llbracket\exists xPx\rrbracket, \ldots\rangle$	from 13 since δ is a formula
15.	$\langle s_1^n\llbracket\gamma\rrbracket, s_2^n\llbracket\beta \bullet \delta\rrbracket, s_3^n\llbracket\exists xPx\rrbracket, \ldots\rangle$	
16.	$\langle s_1^n\llbracket\gamma\rrbracket, s_2^n\llbracket\beta \bullet \delta\rrbracket, s_3^n[x]\llbracket Px\rrbracket, \ldots\rangle$	from 15, definition 6.5

Figure 6.8: Information state computation for (6.4) when β, γ and δ are DBPL formulas and α is $\blacktriangleright\blacktriangleright \exists xPx \blacktriangleleft\blacktriangleleft$.

For all cases, the affected dimensions are the ones displayed as superscripts (step 7 for the first case and step 10 for the second.) This is necessarily the case in virtue of the recursive construal of definition 8.

Secondly, the characteristic dynamic binding of DPL is still available. Notice that DBPL dynamic binding is restricted to each particular dimension. For

$$\blacktriangleright \exists x(Dx \wedge Cx)\bullet \blacktriangleright Dx \wedge \exists x(Cx \wedge Pxy) \blacktriangleleft \bullet Hx \blacktriangleleft,$$

we can see that in the inner text, i.e. $\blacktriangleright Dx \wedge \exists x(Cx \wedge Pxy) \blacktriangleleft$, the first occurrence of x in Dx is statically and dynamically free; the combined effect of the existential quantifier and sentential connective occurring in the outer text, i.e. the underlined text in $\blacktriangleright \underline{\exists x(Dx \wedge Cx)\bullet} \blacktriangleright Dx \wedge \exists x(Cx \wedge Pxy) \blacktriangleleft \underline{\bullet Hx} \blacktriangleleft$, dynamically binds the "free" occurrence of x in Hx. So, the multidimensional model for information states allows us to confine the kinematics of binding to each particular dimension. This confinement is even more easily seen from the two formal derivations shown in figures 6.6 and 6.8 (pages 124 and 125, resp.) and from the informal pictorial representation of the same DBPL texts as shown in figures 6.5 and 6.7 (pages 124 and 125, resp.).

Thirdly, the proposed model retains the information state, for each dimension, during the interpretation of sub-texts occurring at the same level independently of possibly intervening material occurring at different levels. This is convenient for situations such as the one described in Lewises scenario II, in figure 5.3, page 97. This discourse has two blocks occurring at the same level corresponding to the resuming of the topic being talked about just before the interruption. It is clear that no piece of information has been lost.[13] In fact, the conversation follows based on the assumption that the other participants retained the information already given. This phenomenon is what I have called co-routining in the previous chapter. So, definition 8 models this co-routining behaviour.

Fourthly, the multidimensional character of information state adopted allows us to use definite descriptions in a Russellian way. For PC (and DPL as well), which might be seen as a kind of unidimensional DBPL-like system, unicity and uniqueness collapse into one another. However, for multidimensional cases, where the uniqueness condition would not affect any dimension but the one where the description occurs at, uniqueness does not mean unicity. Unicity is therefore a much stronger concept meaning that for all possible dimensions there exist one and only one object satisfying some condition. For the Lewises' scenario, the definite noun phrase *the cat*, for instance, is used in two different blocks referring to different cats. Since only one cat "inhabits" each block, the uniqueness condition is satisfied.

Finally, by not conflating sequence conjunction and formula conjunction it is possible to keep a better recording of the "internal" structure of DBPL texts (without

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[13] See, for instance, figure 6.6, step 12, and figure 6.8, step 16, for details of how this is accomplished.

loosing any of the above cited characteristics.) This better structuring would help us, for example, to discover better splitting points for text segmentation (or the other way around).[14] It is easily seen that if α and β are DBPL sequences, then $\alpha \bullet \beta$ is a DBPL sequence which might be *broken* in some reasonable place.[15] This is what the theorem below states.

Theorem 6.2 ***DBPL text splitting (strong form)***

Let α and β be any DBPL sequences of any length and s^n any information state. Then

$$s^n[\![\blacktriangleright \alpha \bullet \beta \blacktriangleleft]\!] = s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!][\![\blacktriangleright \beta \blacktriangleleft]\!]$$

Proof

$$\begin{array}{rcll} s^n[\![\blacktriangleright \alpha \bullet \beta \blacktriangleleft]\!] & = & s^n[\![\blacktriangleright \alpha \bullet \beta \blacktriangleleft]\!]^0 & \textit{(def. 7.5)} \\ & = & s^n[\![\alpha \bullet \beta]\!]^1 & \textit{(def. 7.3)} \\ & = & s^n[\![\alpha]\!]^1[\![\beta]\!]^1 & \textit{(lemma 6.6, page 121)} \\ & = & s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!]^0[\![\beta]\!]^1 & \\ & = & s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!][\![\blacktriangleright \beta \blacktriangleleft]\!]^0 & \\ & = & s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!][\![\blacktriangleright \beta \blacktriangleleft]\!] & \square \end{array}$$

Before stating the multidimensional notions related to meaning, truth and equivalence, we ought to develop extra machinery in order to cope with most of DBPL's text results. In fact, most of the results to be presented, will be stated in a two-fold approach. Firstly, we will employ a reduction strategy in order to prove theorems for DBPL entities that can be "reduced" to DPL's "equivalent" ones. This is the case for DBPL's formulas and DBPL's pre-sequences. That DBPL's formulas are DPL formulas is not difficult to see since for both systems the definition for formulas follow the same pattern. Although DBPL sequences do not have a DPL counter-part, pre-sequences might be seen as DPL formulas if we replace the $\bullet$ conjunction by the more tradi-

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[14] These facts could be rephrased as stating that *the text composed by juxtaposing sequences α and β, i.e. $\blacktriangleright \alpha \bullet \beta \blacktriangleleft$, induces the same output as the juxtaposition of the texts made up upon α and β, i.e. $\blacktriangleright \alpha \blacktriangleleft\blacktriangleright \beta \blacktriangleleft$* . Notice that this "*equivalence*" preserves the dynamic character we are after.

[15] A DBPL sequence can be broken in two (or more) DBPL sequences only if each of these sequences are balanced with respect to $\blacktriangleright\blacktriangleleft$. Notice that $\blacktriangleright\blacktriangleleft$ do not occur in DBPL pre-sequences, therefore any occurrence of $\bullet$ in a DBPL pre-sequence would be used as a reasonable splitting point.

tional $\wedge$.[16] Doing so, the result for formulas would be immediately transposed for pre-sequences.

Secondly, we will employ a "synchronization" strategy in order to prove most of the results for DBPL first class citizens, i.e., DBPL texts (which cannot be reduced, in any way, to any DPL entities in virtue of their multidimensionality). The synchronization strategy, which is related to the impenetrable assumption, will take every individual unidimensional information state, for every dimensional index i, of any multidimensional information state s^n, to compute the result of updating s^n with a DBPL text. But, to undertake this computation we need to know which objects inhabit every dimensional niche. Therefore, some extra concepts, notation and auxiliary results ought to be developed before we proceed to the next section.

As we have said before, we need to develop some extra technical tools in order to prove DBPL theorems related to texts. One of these tools is a projection function, $Proj$, which is a binary function taking as input a natural number i and a DBPL text $\blacktriangleright \Psi \blacktriangleleft$ and giving as output a DBPL pre-sequence containing all DBPL formulas occurring at the specified level i in their natural order of occurrence. So, for example, if α_1 and α_2 are DBPL pre-sequences, then $Proj(1, \blacktriangleright \alpha_1 \bullet \blacktriangleright \Phi \blacktriangleleft \bullet \alpha_2 \blacktriangleleft) = \alpha_1 \wedge \alpha_2$,[17] independently of the internal structure of Φ. However, such a function poses us with technical problems exemplified by $Proj(1, \blacktriangleright\blacktriangleright \Phi \blacktriangleleft\blacktriangleleft)$ for which case the output is null. Since DBPL did not make provision for entities such as the *null sequence* and the *null text* we ought to make some provisos for dealing with cases like $Proj(1, \blacktriangleright\blacktriangleright \Phi \blacktriangleleft\blacktriangleleft)$, and, also, the subsequent problems related to the introduction of such *null entities* into the framework. One of these problems is related to the following question: what does it mean to update a multidimensional state with a null text. These points will be handled in sequel.

Definition 9: Null sequence and null text

The null text is a "text" of depth 0 and length 0. The null sequence is a "sequence"

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[16] For pre-sequences, the "multi-sorted" connective "$\bullet$" will be applied to, and only to, DBPL formulas. In this case, the bullet operator "might be seen" as equivalent to the traditional $\wedge$.

[17] Notice that $Proj$ maps $\bullet$ to $\wedge$.

of length 0. The null text and the null sequence will be denoted by ϵ. □

Notice that definition 9 does not grant the null entities a special place among DBPL's entities. Indeed, neither the null text nor the null sequence are DBPL entities. The introduction of such exotic objects is due to the total character assumed for the $Proj$ function, which must always produce a value.[18]

Definition 10: null sequence update

If ϵ is the null sequence then $s^n[\![\epsilon]\!]^i = s^n_i$ for all multidimensional states s^n and dimensional indices i. □

Definition 11: null text update

If ϵ is the null text then $s^n[\![\epsilon]\!] = s^n$ for all multidimensional states s^n. □

It seems reasonable to interpret the null entities in two ways: firstly, as producing an absurd state. After all, it is not natural to be presented with an empty page or any other empty object without running into a contradictory feeling. On the other hand, we might stay in the same information state as before since the null text does

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[18] Trading once more on the programming language paradigm, I would point out the striking similarity between DBPL texts and *lisp* lists. Although I do not intend to develop here the necessary repertoire of functions needed in order to characterize $Proj$, I would like to give a Common Lisp version of $Proj$. This is accomplished by the following functions.

```
(defun proj (n list_of_lists)
   (cond
      ((null list_of_lists) nil)
      ((equal n 1) (remove-if #'null (mapcar #'atom_proj list_of_lists)))
      (t
        (proj (1- n) (ze_append (remove-if #'null (mapcar #'list_proj list_of_lists)))))))

(defun atom_proj (simbolo)
   (cond ((atom simbolo) simbolo)
      (t nil)))

(defun list_proj (arg)
   (cond ((listp arg) arg)
      (t nil)))

(defun ze_append (lista)
   (cond ((null lista) nil)
      (t (append (car lista) (ze_append (cdr lista))))))
```

not provide us with any information. For technical reasons, we choose the latter option since it will make the proof of the next theorems easier.

As a straightforward consequence of definition 10, it holds that

Theorem 6.3 *Let* ϵ *be the null sequence and* $\mathfrak{M}$ *an arbitrary model. Then*

$$\downarrow [\![\epsilon]\!]_{\mathfrak{M}} = D^V$$

Proof

By definition 7, $\downarrow [\![\epsilon]\!]_{\mathfrak{M}} = \{g \in D^V \mid \{g\}[\![\epsilon]\!]_{\mathfrak{M}} \neq \emptyset\}$. By definition 10, $\{g\}[\![\epsilon]\!]_{\mathfrak{M}} = \{g\}$. Therefore $\downarrow [\![\epsilon]\!]_{\mathfrak{M}} = D^V$. □

It is clear that there exist a closer connection between DBPL's text evaluation and $Proj$. This relationship is as follows.

$$s^n[\![\blacktriangleright \Psi \blacktriangleleft]\!] = \langle s^n_1[\![Proj(1, \blacktriangleright \Psi \blacktriangleleft)]\!], \ldots, s^n_i[\![Proj(i, \blacktriangleright \Psi \blacktriangleleft)]\!], s^n_{i+1}, s^n_{i+2}, \ldots\rangle$$

where i is the depth of $\blacktriangleright \Psi \blacktriangleleft$.

Having developed the necessary tools, we can now proceed to the next section.

6.3 Meaning, Truth and Equivalence

Static semantic systems were primarily devised as devices to model the world. As a consequence, their underlying languages should reflect the relationship between language and the world they model. Therefore, truth and falsity were in the very kernel of such a relationship. Contrastingly, in dynamic settings, it is *information about the world*, and not the world itself, that language is related to. If the notions of truth and falsity occupy a central position in standard static semantics, their places are expected to be filled by more appropriate information oriented notions which are usually referred to in the literature by names such as consistency, support, satisfaction and the like (cf. Groenendijk, Stokhof and Veltman (1994, p.123)).

Definition 12: Support (or compatibility) in DBPL

Let α be a DBPL formula, $\blacktriangleright \Upsilon \blacktriangleleft$ a DBPL text, s^n a multidimensional information state, and $\mathfrak{M}$ a model. Then,

Formulas: s^n supports α with respect to $\mathfrak{M}$ and a dimensional index $i > 0$ iff

$\Pi(i, s^n) \subseteq \downarrow [\![\alpha]\!]_{\mathfrak{M}}$

Notation: $s^n \models^i_{\mathfrak{M}} \alpha$

Texts: s^n supports $\blacktriangleright \Upsilon \blacktriangleleft$ with respect to $\mathfrak{M}$ iff for all i, $1 \leq i \leq depth(\blacktriangleright \Upsilon \blacktriangleleft)$ & $Proj(i, \blacktriangleright \Upsilon \blacktriangleleft) \neq \epsilon \Rightarrow \Pi(i, s^n) \subseteq \downarrow [\![Proj(i, \blacktriangleright \Upsilon \blacktriangleleft)]\!]_{\mathfrak{M}}$.

Notation: $s^n \models_{\mathfrak{M}} \blacktriangleright \Upsilon \blacktriangleleft$ □

Remark 6.5 *Recalling definition 7, definition 12 might have been stated in an equivalent form as displayed below.*

Formulas: s^n *supports* α *with respect to* $\mathfrak{M}$ *and a dimensional index* $i > 0$ *iff*

for all g, $(g \in \Pi(i, s^n) \Rightarrow \{g\}[\![\alpha]\!]_{\mathfrak{M}} \neq \emptyset)$

Texts: s^n *supports* $\blacktriangleright \Upsilon \blacktriangleleft$ *with respect to* $\mathfrak{M}$ *iff for all assignments* g *and natural numbers* i, $(1 \leq i \leq depth(\blacktriangleright \Upsilon \blacktriangleleft)$ & $Proj(i, \blacktriangleright \Upsilon \blacktriangleleft) \neq \epsilon$ & $g \in \Pi(i, s^n) \Rightarrow \{g\}[\![Proj(i, \blacktriangleright \Upsilon \blacktriangleleft)]\!]_{\mathfrak{M}} \neq \emptyset)$ □

The equivalence between definition 12 and remark 6.5 is granted by the following theorem.

Theorem 6.4 $s^n_i \subseteq \downarrow [\![\alpha]\!]_{\mathfrak{M}}$ *iff for all* g, $(g \in s^n_i \Rightarrow \{g\}[\![\alpha]\!]_{\mathfrak{M}} \neq \emptyset)$.

The proof is trivial since by definition of $\downarrow [\![\alpha]\!]_{\mathfrak{M}}$, $\downarrow [\![\alpha]\!]_{\mathfrak{M}} = \{g \in D^V \mid \{g\}[\![\alpha]\!]_{\mathfrak{M}} \neq \emptyset\}$. Therefore, $s^n_i \subseteq \downarrow [\![\alpha]\!]_{\mathfrak{M}}$ iff for all g, $g \in s^n_i \Rightarrow \{g\}[\![\alpha]\!]_{\mathfrak{M}} \neq \emptyset$. □

The support definition equivalent form stated in remark 6.5 helps us to more easily understand that an information state s^n supports a DBPL text (or put the other way around, a DBPL text is compatible with s^n, the information state one might have reached thus far) iff the update of s^n with the text takes one to a relative non-absurd information state.[19] Being "local entities", DBPL formulas could

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[19] Recall that for any DBPL text $\blacktriangleright \Psi \blacktriangleleft$, a relative multidimensional absurd state is any multidimensional information state such that for each dimensional index ranging from one to the depth

only affect the dimensional "niche" they occupy in a DBPL text. This means that a DBPL formula should be compatible (or incompatible) with the dimensional index of a multidimensional information state. It does mean also that different instances of a formula, occurring into different dimensional indices, might produce different answers: in one index the update produced by the formula might be compatible to the "local (uni)dimensional information state" while just the opposite for the other index.

Having defined the notion of support we can now characterize the notion of entailment and validity in terms of it. Before doing this, we will characterize the notion of multidimensional agreement which plays an important role in the concepts to be defined.

Definition 13: Multidimensional agreement in DBPL (weak form)

Let $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft$ be DBPL texts. Then $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_{n-1} \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Upsilon_n \blacktriangleleft$ iff $\mathit{depth}(\blacktriangleright \Upsilon_n \blacktriangleleft) = \mathit{depth}(\blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_{n-1} \blacktriangleleft)$ and for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Upsilon_n \blacktriangleleft) \Rightarrow (\mathit{Proj}(i, \blacktriangleright \Upsilon_n \blacktriangleleft) \neq \epsilon \Rightarrow \mathit{Proj}(i, \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_{n-1} \blacktriangleleft) \neq \epsilon)$. □

Notice that the kind of multidimensional agreement stated in definition 13 is in some sense a weak form since it does not state that all texts in the sequence must agree among themselves; if this is the case (let us call it strong multidimensional agreement), then the sequence $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_{n-1} \blacktriangleleft$ multidimensionally agrees with $\blacktriangleright \Upsilon_n \blacktriangleleft$. For strong multidimensional agreement we can, indeed, permute $\blacktriangleright \Upsilon_n \blacktriangleleft$ with any other text in the sequence and the multidimensional agreement still holds. However, for weak agreement, permutation should not preserve, in general, the multidimensional agreement as exemplified by $\blacktriangleright \phi_1 \blacktriangleleft, \blacktriangleright\blacktriangleright \phi_2 \blacktriangleleft\blacktriangleleft$ which multidimensionally agree with $\blacktriangleright\blacktriangleright \phi_3 \blacktriangleleft \bullet \phi_4 \blacktriangleleft$. Note that if we permute $\blacktriangleright\blacktriangleright \phi_3 \blacktriangleleft \bullet \phi_4 \blacktriangleleft$ with $\blacktriangleright \phi_1 \blacktriangleleft$, then the weak agreement is destroyed.

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of the text $\blacktriangleright \Psi \blacktriangleleft$ for which $\mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) \neq \epsilon$, the unidimensional information states for such indices are all the empty set. In other words, independently of the "irrelevant" dimensions – the dimensions not present in the text $\blacktriangleright \Psi \blacktriangleleft$ – the relative absurd multidimensional information state will not support $\blacktriangleright \Psi \blacktriangleleft$.

Notice that multidimensional agreement allows us to define multidimensional connectives induced from the unidimensional ones. So, we can define $\overset{m}{\wedge}$, $\overset{m}{\Rightarrow}$, and $\overset{m}{\neg}$, for instance, as

- ⋆ $\blacktriangleright \Upsilon \blacktriangleleft \overset{m}{\wedge} \blacktriangleright \Psi \blacktriangleleft$ is a DBPL text $\blacktriangleright \Phi \blacktriangleleft$ iff $\blacktriangleright \Upsilon \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Psi \blacktriangleleft$ and $\blacktriangleright \Psi \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Phi \blacktriangleleft$ and for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Psi \blacktriangleleft)$ & $\mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) \neq \epsilon \Rightarrow \mathit{Proj}(i, \blacktriangleright \Upsilon \blacktriangleleft) \wedge \mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) = \mathit{Proj}(i, \blacktriangleright \Phi \blacktriangleleft)$
- ⋆ $\blacktriangleright \Upsilon \blacktriangleleft \overset{m}{\Rightarrow} \blacktriangleright \Psi \blacktriangleleft$ is a DBPL text $\blacktriangleright \Phi \blacktriangleleft$ iff $\blacktriangleright \Upsilon \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Psi \blacktriangleleft$ and $\blacktriangleright \Psi \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Phi \blacktriangleleft$ and for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Psi \blacktriangleleft)$ & $\mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) \neq \epsilon \Rightarrow \mathit{Proj}(i, \blacktriangleright \Upsilon \blacktriangleleft) \rightarrow \mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) = \mathit{Proj}(i, \blacktriangleright \Phi \blacktriangleleft)$
- ⋆ $\overset{m}{\neg} \blacktriangleright \Upsilon \blacktriangleleft$ is a DBPL text $\blacktriangleright \Phi \blacktriangleleft$ iff $\blacktriangleright \Upsilon \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Phi \blacktriangleleft$ and for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Upsilon \blacktriangleleft)$ & $\mathit{Proj}(i, \blacktriangleright \Upsilon \blacktriangleleft) \neq \epsilon \Rightarrow \neg(\mathit{Proj}(i, \blacktriangleright \Upsilon \blacktriangleleft)) = \mathit{Proj}(i, \blacktriangleright \Phi \blacktriangleleft)$

According to the definitions given above, $\blacktriangleright \phi \bullet \blacktriangleright \psi \blacktriangleleft\blacktriangleleft \overset{m}{\Rightarrow} \blacktriangleright \alpha \bullet \blacktriangleright \beta \blacktriangleleft\blacktriangleleft$ is the DBPL text $\blacktriangleright \phi \rightarrow \alpha \bullet \blacktriangleright \psi \rightarrow \beta \blacktriangleleft\blacktriangleleft$.

We can now start discussing entailment in DBPL.

Text entailment might be defined as

$$\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft \models \blacktriangleright \Psi \blacktriangleleft$$

iff for all models $\mathfrak{M}$ and information states s^n,

$$s^n [\![\blacktriangleright \Upsilon_1 \blacktriangleleft]\!]_{\mathfrak{M}} \ldots [\![\blacktriangleright \Upsilon_n \blacktriangleleft]\!]_{\mathfrak{M}} \models_{\mathfrak{M}} \blacktriangleright \Psi \blacktriangleleft.$$

Looking at the premiss and conclusion texts we can think of three relationships holding among them. For the first case, let us assume that there are no restrictions on them, i.e., the premises and conclusion can be any texts. For the second case, let us assume that the premises weakly multidimensionally agree with the conclusion, i.e., $\blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Psi \blacktriangleleft$. As this weak multidimensional agreement does not imply that all texts multidimensionally agree among themselves, this would be the point for the third relationship. Let us denote these entailments by $(1)\models$, $(2)\models$ and $(3)\models$, respectively. As a consequence of these proposed "definitions" (each definition would include the respective restriction), the following results hold.

Theorem 6.5 *for any* DBPL *formula* ϕ, $\blacktriangleright \phi \blacktriangleleft, \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \ (1)\models \blacktriangleright \phi \blacktriangleleft$.

Proof

Let $\mathfrak{M}$ and s^n be any model and multidimensional information state, resp.

Since ϕ is a formula we get that $depth(\blacktriangleright \phi \blacktriangleleft) = 1$. So, $Proj(1, \blacktriangleright \phi \blacktriangleleft) = \phi \neq \epsilon$ and $\Pi(1, s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket\llbracket \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \rrbracket) = s^n_1\llbracket\phi\rrbracket$. We know also that

$\downarrow\llbracket Proj(1, \blacktriangleright \phi \blacktriangleleft)\rrbracket = \downarrow\llbracket\phi\rrbracket = \{g \in D^V \mid \{g\}\llbracket\phi\rrbracket \neq \emptyset\}$ (*)

However, $s^n_1\llbracket\phi\rrbracket = \{g \in s^n_1 \mid \{g\}\llbracket\phi\rrbracket \neq \emptyset\}$ (**)

Therefore, from (*), (**) and set theory (**) $\subseteq$ (*). By definition of support, $s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket\llbracket \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \rrbracket \ (1)\models \blacktriangleright \phi \blacktriangleleft$. And since $\mathfrak{M}$ and s^n are arbitrary, by definition of (1)entailment we get $\blacktriangleright \phi \blacktriangleleft, \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \ (1)\models \blacktriangleright \phi \blacktriangleleft$ □

Theorem 6.6 *for any* DBPL *formula* ϕ, $\blacktriangleright \phi \blacktriangleleft, \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \ (2 \mid 3)\not\models \blacktriangleright \phi \blacktriangleleft$.

Proof

Trivial, since by definition the texts do not multidimensionally agree. □

Theorem 6.7 *for all* DBPL *formula* ϕ, $\blacktriangleright \phi \blacktriangleleft \ (1 \mid 2 \mid 3) \not\models \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft$.

Proof

- $(1)\models$

  Suppose by contradiction that $\blacktriangleright \phi \blacktriangleleft \ (1)\models \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft$. Then, by definition of (1)entailment, for all models $\mathfrak{M}$ and multidimensional information states s^n, $s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket_{\mathfrak{M}} \ (1)\models_{\mathfrak{M}} \ \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft$. Let us take a multidimensional information state r^n such that one and only one component (the first component) does indeed support ϕ and all other dimensional components support $\neg\phi$. It is clear that such r^n when updated by $\blacktriangleright \phi \blacktriangleleft$ will produce an information state that does not support $\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft$ (since $\Pi(2, r^n) \not\subseteq \downarrow\llbracket Proj(2, \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft)\rrbracket_{\mathfrak{M}}$, because $\Pi(2, r^n) \cap \downarrow\llbracket\phi\rrbracket_{\mathfrak{M}} = \emptyset$), which is an absurd.

- $(2 \mid 3)\models$

  Trivial, since by definition the texts do not multidimensionally agree. □

By similar argumentation, the converse of theorem 6.7, theorem 6.8 below, does hold.

Theorem 6.8 *for all* DBPL *formula* ϕ, $\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ (1 \mid 2 \mid 3)\not\models \blacktriangleright\ \phi\ \blacktriangleleft$. □

The problem with $(1)\models$ is that it does not allow us to follow a uniform procedure when deduction is accounted for. The basic problem is exemplified by $\blacktriangleright\ \phi\ \blacktriangleleft, \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi\bullet \blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft$. If we want to preserve the usual deduction style, we should expect that $\blacktriangleright\ \phi\ \blacktriangleleft\ (1)\models\ \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ \overset{m}{\Rightarrow}\ \blacktriangleright\ \phi\bullet \blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft$ holds. But what does $\overset{m}{\Rightarrow}$ mean in such dimensionally mismatched texts? Also, any multidimensional information state supporting the premiss $\blacktriangleright\ \phi\ \blacktriangleleft$ when updated by $\blacktriangleright\ \phi\ \blacktriangleleft$ must support the conclusion $\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\overset{m}{\Rightarrow}\blacktriangleright\ \phi\bullet \blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft$. To solve the $\overset{m}{\Rightarrow}$ mismatching, two possibilities are open:

1. "relaxing" $\overset{m}{\Rightarrow}$ making it to denote a text where the mismatched dimensions are kept and for all other dimensions we get that $\phi \rightarrow \psi$ where ϕ comes from the premiss and ψ from the conclusion. So, $\blacktriangleright\ \phi\ \blacktriangleleft\ (1)\models\ \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ \overset{m}{\Rightarrow}\ \blacktriangleright\ \phi\bullet \blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft$ becomes $\blacktriangleright\ \phi\ \blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi\bullet \blacktriangleright\ \phi \rightarrow \phi\ \blacktriangleleft\blacktriangleleft$ and therefore
   $$(1)\models\ \blacktriangleright\ \phi \rightarrow \phi\bullet \blacktriangleright\ \phi \rightarrow \phi\ \blacktriangleleft\blacktriangleleft$$
   which would be the intended result. But now, let us consider the following case. $\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft, \blacktriangleright\ \phi\ \blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi\ \blacktriangleleft$ iff $\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi\ \blacktriangleleft\overset{m}{\Rightarrow}\blacktriangleright\ \phi\ \blacktriangleleft$ i.e. $\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi \rightarrow \phi\ \blacktriangleleft$ iff $(1)\models\ \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\overset{m}{\Rightarrow}\blacktriangleright\ \phi \rightarrow \phi\ \blacktriangleleft$
   iff $(1)\models\ \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\ \bullet\phi \rightarrow \phi\ \blacktriangleleft$. But this is obviously absurd since there would be multidimensional states where the second component would support $\neg\phi$ instead of ϕ.

2. As we have seen in the previous item, keeping mismatched dimensions in the resulting text leads us to inconsistency. So, we can think of throwing mismatched dimensions away. But this also leads us to problems as exemplified by $\blacktriangleright\ \phi\ \blacktriangleleft, \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\ (1)\models\ \blacktriangleright\ \phi\bullet \blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft$. For this case, we get that

$\blacktriangleright \phi \blacktriangleleft$ $(1)\models$ $\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft \overset{m}{\Rightarrow} \blacktriangleright \phi\bullet \blacktriangleright \phi \blacktriangleleft\blacktriangleleft$. But $\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft$ and $\blacktriangleright \phi\bullet \blacktriangleright \phi \blacktriangleleft\blacktriangleleft$ do not dimensionally agree and therefore we should throw some components of one of them away. Which text should be thrown away? It seems quite obvious to discard the "useless" hypothesis, but which one is (are) the useless one(s)?

The problem is related to the **global/simultaneously** binding work emanating from all relevant dimensions[20] as stated in definition 12 which cannot be split when we walk back through the premises sequence in order to pass on the last premiss to the conclusion. This can only be done when no dimensional splitting occurs. And since $(1)\models$ does not fulfill this criteria it must be rejected as a candidate for characterizing the entailment relation. Notice, however, that $(2)\models$ and $(3)\models$ respect the "simultaneous global binding" criteria. Moreover, the pre-definitions given for $(2)\models$ and $(3)\models$ show that these concepts are empirically equivalent with respect to entailment. They both take care of the dimensional interplay between premises and conclusion; all relevant dimensions are there, no more no less. So, we feel justified to adopt $(2)\models$ for defining entailment.

Definition 14: Entailment in DBPL

Let $\phi_1, \ldots, \phi_n$ and ψ be DBPL formulas. Let, also, $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft$ and $\blacktriangleright \Psi \blacktriangleleft$ be any DBPL texts such that $\blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft$ multidimensionally agree with $\blacktriangleright \Psi \blacktriangleleft$. Then

Formulas: $\phi_1, \ldots, \phi_n \models \psi$ iff for all models $\mathfrak{M}$ and information states s^n, and natural numbers $i > 0$, $\quad s^n[\![\phi_1]\!]^i_{\mathfrak{M}} \ldots [\![\phi_n]\!]^i_{\mathfrak{M}} \models_{\mathfrak{M}} \psi$

Texts: $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft \models \blacktriangleright \Psi \blacktriangleleft$ iff for all models $\mathfrak{M}$ and information states s^n, $\quad s^n[\![\blacktriangleright \Upsilon_1 \blacktriangleleft]\!]_{\mathfrak{M}} \ldots [\![\blacktriangleright \Upsilon_n \blacktriangleleft]\!]_{\mathfrak{M}} \models_{\mathfrak{M}} \blacktriangleright \Psi \blacktriangleleft$. □

Definition 15: Validity in DBPL

Let Υ be a DBPL text and α a DBPL formula. Then

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[20] The relevant dimensions we are referring to here are the ones for which the $Proj(i, \Upsilon) \neq \epsilon$ for any arbitrary DBPL text Υ and natural number i, $1 \leq i \leq depth(\Upsilon)$.

Formulas: $\models \alpha$ iff for all models $\mathfrak{M}$, multidimensional information states s^n, and natural numbers $i > 0$, $s^n_i \models_{\mathfrak{M}} \alpha$

Texts: $\models \Upsilon$ iff for all models $\mathfrak{M}$ and multidimensional information states s^n, $s^n \models_{\mathfrak{M}} \Upsilon$ □

The following theorems show how the impenetrable hypothesis underlies the entailment relation. Also, they emphasize the point that premiss texts and the conclusion text must agree in all relevant dimensional indices. Obviously, the most trivial agreement one could get for any DBPL text is given by the following result.

Theorem 6.9 *for any DBPL text* Υ, $\Upsilon \models \Upsilon$

Proof

The proof is trivial since for all i, $1 \leq i \leq \mathit{depth}(\Upsilon)$ such that $\mathit{Proj}(i, \Upsilon) \neq \epsilon$, $\mathit{Proj}(i, \Upsilon)$ is a DPL formula. Since for any DPL formula ϕ, $\phi \models_{\text{DPL}} \phi$, we get the desired result. □

The next theorem shows how dynamic binding is preserved under entailment.

Theorem 6.10 $\blacktriangleright \exists x Px \blacktriangleleft \models \blacktriangleright Px \blacktriangleleft$

Proof

Let $\mathfrak{M}$ be an arbitrary model and s^n an arbitrary multidimensional information state. Then

$$\begin{aligned} s^n[\![\blacktriangleright \exists x Px \blacktriangleleft]\!]_{\mathfrak{M}} &= s^n[\![\exists x Px]\!]^1 && \textit{by definition } 7.5 \\ &= \langle s^n_1[\![\exists x Px]\!], s^n_2, \ldots \rangle && \textit{by remark } 6.4 \\ &= \langle s^n_1[x][\![Px]\!], s^n_2, \ldots \rangle && \textit{by definition } 6.5 \end{aligned}$$

Two cases are required:

(i) $s^n_1 = \emptyset$.

For this case $s^n_1[x] = \emptyset$ and therefore $\emptyset[\![Px]\!]_{\mathfrak{M}} = \{g \in \emptyset \mid g(x) \in \mathcal{F}(P)\} = \emptyset$ i.e. $s^n[x][\![Px]\!]_{\mathfrak{M}} = \emptyset$ i.e. $s^n[\![\exists x Px]\!]^1_{\mathfrak{M}} = \emptyset$

On the other hand, $\downarrow[\![\mathit{Proj}(1, \blacktriangleright Px \blacktriangleleft)]\!]_{\mathfrak{M}} = \downarrow[\![Px]\!]_{\mathfrak{M}} = \{i \in D^V \mid \{i\}[\![Px]\!]_{\mathfrak{M}} \neq \emptyset\} = \{i \in D^V \mid \{g \in \{i\} \mid i(x) \in \mathcal{F}(P)\} \neq \emptyset\}$ but since $g \in \{i\} \Rightarrow g = i$ then we get that

$\{i \in D^V \mid \{g \in \{i\} \mid i(x) \in \mathcal{F}(P)\} \neq \emptyset\} = \{i \in D^V \mid i(x) \in \mathcal{F}(P)\}$. Since $\emptyset \subseteq X$ for all sets X then we get that $s_1^n\llbracket \exists x Px \rrbracket_{\mathfrak{M}} \subseteq \downarrow \llbracket Proj(1, \blacktriangleright\ Px\ \blacktriangleleft) \rrbracket_{\mathfrak{M}}$ and since $depth(\blacktriangleright\ Px\ \blacktriangleleft) = 1$ we get that for all i, $1 \leq i \leq depth(\blacktriangleright\ Px\ \blacktriangleleft) \Rightarrow s_i^n \subseteq \downarrow \llbracket Proj(1, \blacktriangleright\ Px\ \blacktriangleleft) \rrbracket_{\mathfrak{M}}$. Therefore, by definition 8, $s^n\llbracket \blacktriangleright \exists x Px \blacktriangleleft \rrbracket_{\mathfrak{M}}$ supports $\blacktriangleright\ Px\ \blacktriangleleft$ with respect to $\mathfrak{M}$. Since $\mathfrak{M}$ and s^n are arbitrary, then by definition 9, we get the expected result.

(ii) $s_1^n \neq \emptyset$.

Suppose now that $i \in s_1^n\llbracket \exists x Px \rrbracket_{\mathfrak{M}}$. Then $i \in s_1^n[x]\llbracket Px \rrbracket_{\mathfrak{M}}$. By definition 6, $i \in \{k \in s_1^n[x] \mid k(x) \in \mathcal{F}(P)\} \therefore i(x) \in \mathcal{F}(P)$. On the other hand

$\downarrow\llbracket Proj(1, \blacktriangleright\ Px\ \blacktriangleleft) \rrbracket_{\mathfrak{M}} = \downarrow\llbracket Px \rrbracket_{\mathfrak{M}} = \{i \in D^V \mid \{i\}\llbracket Px \rrbracket_{\mathfrak{M}} \neq \emptyset\} =$

$= \{i \in D^V \mid \{g \in \{i\} \mid g(x) \in \mathcal{F}(P)\} \neq \emptyset\} = \{i \in D^V \mid i(x) \in \mathcal{F}(P)\}$.

Therefore $i \in \downarrow\llbracket Proj(1, \blacktriangleright\ Px\ \blacktriangleleft) \rrbracket_{\mathfrak{M}}$ and by set theory

$$s_1^n\llbracket \exists x Px \rrbracket_{\mathfrak{M}} \subseteq \downarrow\llbracket Proj(1, \blacktriangleright\ Px\ \blacktriangleleft) \rrbracket_{\mathfrak{M}}$$

By definition 8, $s^n\llbracket \blacktriangleright \exists x Px \blacktriangleleft \rrbracket_{\mathfrak{M}}$ supports $\blacktriangleright\ Px\ \blacktriangleleft$ with respect to $\mathfrak{M}$. Since $\mathfrak{M}$ and s^n are general, by definition 9 we get that $\blacktriangleright \exists x Px \blacktriangleleft \models \blacktriangleright\ Px\ \blacktriangleleft$. $\square$

Indeed, theorem 6.10 holds for $\exists x Px$ as premiss and Px as conclusion provided that both formulas occur at the same dimensional index,[21] i.e., $\blacktriangleright^n \exists x Px \blacktriangleleft^n \models \blacktriangleright^n Px \blacktriangleleft^n$ since both texts multidimensionally agree and the same argumentation presented in the proof of theorem 6.10 can be used.

The next theorem deals with a case where premises and conclusion show a "dimensional gap". Note that the first premiss is a text of depth 1 while the second premiss a text of depth 3 where the first and second dimensional projection are null, i.e., $Proj(1, \blacktriangleright\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\blacktriangleleft) = \epsilon = Proj(2, \blacktriangleright\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\blacktriangleleft)$. The conclusion is a text of depth 3 where the second dimensional projection is null, i.e., $Proj(2, \blacktriangleright\ \phi \bullet \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\blacktriangleleft) = \epsilon$.

Theorem 6.11 *for any* DBPL *formula* ϕ, $\blacktriangleright\ \phi\ \blacktriangleleft, \blacktriangleright\blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\blacktriangleleft \models \blacktriangleright\ \phi \bullet \blacktriangleright\blacktriangleright\ \phi\ \blacktriangleleft\blacktriangleleft\blacktriangleleft$

Proof

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[21] The superscript n in $\blacktriangleright^n$ stands for the sequence of $\blacktriangleright$ repeated n times $(n \geq 1)$. Analogously for $\blacktriangleleft^n$.

First of all, the premises and conclusion multidimensionally agree. So, for all models $\mathfrak{m}$ and information states s^n, for

$s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket_{\mathfrak{m}}\llbracket \blacktriangleright\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft \rrbracket_{\mathfrak{m}} \models_{\mathfrak{m}} \blacktriangleright \phi \bullet \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft$

to hold it must be the case that for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \phi \bullet \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft) \Rightarrow$ $\Pi(i, s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket_{\mathfrak{m}}\llbracket \blacktriangleright\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft \rrbracket_{\mathfrak{m}}) \subseteq \downarrow\llbracket \mathit{Proj}(i, \blacktriangleright \phi \bullet \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft)\rrbracket_{\mathfrak{m}}$. Therefore

For $i = 1$, we get that $\Pi(1, s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket_{\mathfrak{m}}\llbracket \blacktriangleright\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft \rrbracket_{\mathfrak{m}}) = s^n_1\llbracket\phi\rrbracket_{\mathfrak{m}}$

$= \{g \in s^n_1 \mid \{g\}\llbracket\phi\rrbracket \neq \emptyset\}$ (1)

$\downarrow\llbracket \mathit{Proj}(1, \blacktriangleright \phi \bullet \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft)\rrbracket_{\mathfrak{m}} = \downarrow\llbracket\phi\rrbracket_{\mathfrak{m}} = \{g \in D^V \mid \{g\}\llbracket\phi\rrbracket \neq \emptyset\}$ (2)

Therefore, (1) $\subseteq$ (2)

For $i = 2$, $\Pi(2, s^n\llbracket \blacktriangleright \phi \blacktriangleleft \rrbracket_{\mathfrak{m}}\llbracket \blacktriangleright\blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft \rrbracket_{\mathfrak{m}}) = s^n_2\llbracket\epsilon\rrbracket = s^n_2$ $(1')$

$\downarrow\llbracket \mathit{Proj}(2, \blacktriangleright \phi \bullet \blacktriangleright\blacktriangleright \phi \blacktriangleleft\blacktriangleleft\blacktriangleleft)\rrbracket_{\mathfrak{m}} = \downarrow\llbracket\epsilon\rrbracket_{\mathfrak{m}} = \{g \in D^V \mid \{g\}\llbracket\epsilon\rrbracket \neq \emptyset\}$

$= \{g \mid g \in D^V\}$ $(2')$

$(1') \subseteq (2')$

For $i = 3$, analogous to case where $i = 1$

Therefore, the result holds. □

Theorem 6.11 shows that if dimensional "holes" are present in the premises then the same dimensional holes ought to be present in the conclusion text (if entailment holds).

What is at stake is the fact that DBPL's text entailment takes care of all dimensions of premises and conclusion (DBPL) texts. Unmatched dimensions among premises and conclusion ruin the unidimensional entailment flow throughout such unmatched dimensions. In other words, entailment definition says that all dimensions present in the conclusion text must be somewhere present in the premises (though not necessarily vice versa) and the premises must give support for all dimensions in the conclusion.

Notice that DBPL's notion of entailment is "doubly" dynamic.[22] Firstly, DBPL's

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[22] Pursuing the programming languages paradigm even further, the double character referred to by the word *double* might be easily explained if we "compile" a DBPL text as a set of $\langle\langle \mathit{level,offset}\rangle, \mathit{formula}\rangle$ instructions. The level–offset pairs indicate in a *bidimensional picture* the position each $\mathit{formula}$ instruction occurs at in the text. Entailment ought to mirror the *local-global* dichotomy of DBPL entities: DBPL formulas induce local effects while DBPL texts induce global effects provided that the *impenetrable assumption* be accepted as is indeed the case.

formula entailment corresponds, in the usual way, to the interpretation of implication. It means, also, that pronouns in the conclusion may refer back to subjects introduced in the *local* premises, as easily seen from $\exists x Fx \models Fx$. As Dekker (1993, p. 10) pointed out, this corresponds to the following reasoning:

If a man comes from Rhodes, he likes pineapple juice. A man I met yesterday comes from Rhodes. So, he likes pineapple juice.

$\exists x(Mx \wedge Rx) \rightarrow Lx,\ \exists x(Mx \wedge Rx) \models Lx$

Secondly, DBPL's text entailment corresponds, as might be expected, to the interpretation of multidimensional implication. Since unidimensional implication is a dynamic notion so is multidimensional implication and therefore entailment. Put in other way, multidimensional implication might be seen as the Cartesian product of unidimensional implication. We can also think of premises and conclusion as pages in a sequence since pronouns in a concluding page may refer back to subjects introduced in previous pages. If we think of previous pages as premises then the entailment relation, as defined above, captures these ideas which are then reflected in its plenitude throughout the Deduction Theorem.

Thirdly, notice that, in a certain sense, dynamic entailment is not monotonic. Recall that the order of binding is relevant for DBPL and therefore the premises' order may interfere with the conclusion. For example, it holds that $\blacktriangleright \exists x Px \blacktriangleleft \models \blacktriangleright Px \blacktriangleleft$ but $\blacktriangleright \exists x Px \blacktriangleleft, \blacktriangleright \exists x Qx \blacktriangleleft \not\models \blacktriangleright Px \blacktriangleleft$ since the premiss $\blacktriangleright \exists x Qx \blacktriangleleft$ interferes with the anaphoric pronoun x in the conclusion "text page"; the pronoun x could not any longer be referring back to the first premiss.[23]

DBPL, also licenses deduction theorems reflecting its multidynamic character as already explained.

Theorem 6.12 (Deduction Theorem)

Let $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft$ *be any DBPL texts which multidimensionally agree with the*

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[23] To see why, imagine for example, that the intersection of $\mathcal{F}(P)$ with $\mathcal{F}(Q)$ is empty; in this case the conclusion does not hold since it depends on the value assigned to x. As already pointed out, existentials behave like downdates discarding previous information one might have about the variable. As the second premiss terminates the dynamic binding emanating from the first one, the conclusion does not hold. Recall that the definition of support "takes care" of all dimensions and therefore any "troublemaker" block will ruin the dynamics of the whole discourse.

DBPL text $\blacktriangleright \Psi \blacktriangleleft$. *Then*

Formulas: $\phi_1, \ldots, \phi_n \models \psi$ *iff* $\phi_1, \ldots, \phi_{n-1} \models \phi_n \to \psi$

Texts: $\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft \models \blacktriangleright \Psi \blacktriangleleft$ *iff* $\models \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \overset{m}{\Rightarrow} \blacktriangleright \Psi \blacktriangleleft$

Proof

Formulas: The proof is made by reduction to DPL's deduction theorem, since the set of DBPL and DPL formulas are exactly the same and each dimensional index of a multidimensional information state is a DPL information state.

Texts: The proof is made by reduction to DPL's deduction theorem, which will be applied to every dimensional index i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Psi \blacktriangleleft)$, in the following way. Suppose $\blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft$ and $\blacktriangleright \Psi \blacktriangleleft$ multidimensionally agree. Suppose also that

$$\blacktriangleright \Upsilon_1 \blacktriangleleft, \ldots, \blacktriangleright \Upsilon_n \blacktriangleleft \models \blacktriangleright \Psi \blacktriangleleft$$

Then by definition of entailment, for all models $\mathfrak{M}$ and multidimensional information state s^n,

$$s^n \llbracket \blacktriangleright \Upsilon_1 \blacktriangleleft \rrbracket_{\mathfrak{M}} \ldots \llbracket \blacktriangleright \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}} \models \blacktriangleright \Psi \blacktriangleleft$$

By theorem 6.2

$$s^n \llbracket \blacktriangleright \Upsilon_1 \blacktriangleleft \rrbracket_{\mathfrak{M}} \ldots \llbracket \blacktriangleright \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}} = s^n \llbracket \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}}$$

and therefore

$$s^n \llbracket \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}} \models \blacktriangleright \Psi \blacktriangleleft$$

By definition of support, this means that for all i, $1 \leq i \leq \mathit{depth}(\blacktriangleright \Psi \blacktriangleleft)$ & $\mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) \neq \epsilon \Rightarrow \Pi(i, s^n \llbracket \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}}) \subseteq \downarrow \llbracket \mathit{Proj}(i, \Psi) \rrbracket_{\mathfrak{M}}$ So, let us take such an i. Then

$$\Pi(i, s^n \llbracket \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \rrbracket_{\mathfrak{M}}) = s^n_i \llbracket \mathit{Proj}(i, \blacktriangleright \Upsilon_1 \blacktriangleleft) \wedge \ldots \wedge \mathit{Proj}(i, \blacktriangleright \Upsilon_n \blacktriangleleft) \rrbracket_{\mathfrak{M}}$$

and $s^n_i \llbracket \mathit{Proj}(i, \blacktriangleright \Upsilon_1 \blacktriangleleft) \wedge \ldots \wedge \mathit{Proj}(i, \blacktriangleright \Upsilon_n \blacktriangleleft) \rrbracket_{\mathfrak{M}} \subseteq \downarrow \llbracket \mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft) \rrbracket_{\mathfrak{M}}$. So,

$$s^n_i \llbracket \mathit{Proj}(i, \blacktriangleright \Upsilon_1 \blacktriangleleft) \wedge \ldots \wedge \mathit{Proj}(i, \blacktriangleright \Upsilon_n \blacktriangleleft) \rrbracket_{\mathfrak{M}} \models_{\mathfrak{M}} \mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft)$$

Therefore, by DPL's deduction theorem we get

$$\models_{\mathfrak{M}} \mathit{Proj}(i, \blacktriangleright \Upsilon_1 \blacktriangleleft) \wedge \ldots \wedge \mathit{Proj}(i, \blacktriangleright \Upsilon_n \blacktriangleleft) \to \mathit{Proj}(i, \blacktriangleright \Psi \blacktriangleleft).$$

Since it holds for any i, by $\overset{m}{\Rightarrow}$ definition we get that

$$\models \blacktriangleright \Upsilon_1 \bullet \ldots \bullet \Upsilon_n \blacktriangleleft \overset{m}{\Rightarrow} \blacktriangleright \Psi \blacktriangleleft$$

$\square$

6.4 What properties does the system have?

All properties of DBPL are related to the dynamics of the binding mechanism used in it. Traditionally, the binding process is stated in a way saying that bound variables are the ones under the syntactic scope of a quantified formula while free variables are the ones outside the syntactic scope of a quantified formula.

To characterize the dynamic version we need to change the traditional characterization of bound variables. The intuitive idea is that any existential formula not only binds the variable quantified over but also makes it somehow active. Moreover, previously activated variables are not free even they are not syntactically bound in the formulas they occur at. This dynamic binding mechanism is formally characterized in definition 16 through the notions of *binding pairs*, *active quantifier occurrence* and *free variable* conforming to the following notational convention.

Remark 6.6 *Let Ψ be a DBPL text of depth m. Then*

$bp\,(\Psi)$ *is a m-tuple of sets of* binding pairs *in* Ψ.

$aq\,(\Psi)$ *is a m-tuple of sets of* active quantifier occurrences *in* Ψ.

$fv\,(\Psi)$ *is a m-tuple of sets of* free occurrences of variables *in* Ψ.

We will also use bp_i*,* aq_i *and* fv_i *to indicate the dimensional index the binding pair, active quantifier occurrence and free variable sets are concerned with.* □

Recall that the depth of a text reflects the dimensional embedding of discourse blocks occurring at it. So, the notions of *bp*, *aq* and fv are multidimensional and the previous remark makes sure that there is a way to keep track of these notions for every discourse block. Alternatively, we might have defined an m-tuple of triples where the first component (of the triple) was the binding pairs set, the second component was the active quantifier occurrence set and the third the free variable set. The only advantage of the formulation presented in the remark 6.6 is to keep notation as simple as possible.

Definition 16: Scope and Binding

For all DBPL texts of depth n and dimensional index i, $1 \leq i \leq n$ and DBPL formulas ϕ, ϕ_1, and ϕ_2, and DBPL text $\blacktriangleright \psi \blacktriangleleft$,

1. $bp_i\,(Rx_1, \ldots, x_m) = \emptyset$

   $aq_i\,(Rx_1, \ldots, x_m) = \emptyset$

   $fv_i\,(Rx_1, \ldots, x_m) = \{x_i \mid x_i \textit{ is a variable occurring at } Rx_1, \ldots, x_m\}$

2. $bp_i\,(\neg\phi) = bp_i\,(\phi)$

   $aq_i\,(\neg\phi) = \emptyset$

   $fv_i\,(\neg\phi) = fv_i\,(\phi)$

3. $bp_i\,(\phi_1 \wedge \phi_2) = bp_i\,(\phi_1) \cup bp_i\,(\phi_2) \cup \{\langle \exists x, x\rangle \mid \exists x \in aq_i\,(\phi_1) \ \& \ x \in fv_i\,(\phi_2)\}$

   $aq_i\,(\phi_1 \wedge \phi_2) = aq_i\,(\phi_2) \cup \{\exists x \in aq_i, (\phi_1) \mid \exists x \notin aq_i\,(\phi_2)\}$

   $fv_i\,(\phi_1 \wedge \phi_2) = fv_i\,(\phi_1) \cup \{x \in fv_i\,(\phi_2) \mid \exists x \notin aq_i\,(\phi_1)\}$

4. $bp_i\,(\phi_1 \bullet \phi_2) = bp_i\,(\phi_1) \cup bp_i\,(\phi_2) \cup \{\langle \exists x, x\rangle \mid \exists x \in aq_i\,(\phi_1) \ \& \ x \in fv_i\,(\phi_2)\}$

   $aq_i\,(\phi_1 \bullet \phi_2) = aq_i\,(\phi_2) \cup \{\exists x \in aq_i, (\phi_1) \mid \exists x \notin aq_i\,(\phi_2)\}$

   $fv_i\,(\phi_1 \bullet \phi_2) = fv_i\,(\phi_1) \cup \{x \in fv_i\,(\phi_2) \mid \exists x \notin aq_i\,(\phi_1)\}$

5. $bp_i\,(\phi\bullet \blacktriangleright \psi \blacktriangleleft) = \langle bp_1, \ldots, bp_i\,(\phi), bp_{i+1}\,(\psi), \ldots\rangle$

   $aq_i\,(\phi\bullet \blacktriangleright \psi \blacktriangleleft) = \langle aq_1, \ldots, aq_i\,(\phi), aq_{i+1}\,(\psi), \ldots\rangle$

   $fv_i\,(\phi\bullet \blacktriangleright \psi \blacktriangleleft) = \langle fv_1, \ldots, fv_i\,(\phi), fv_{i+1}\,(\psi), \ldots\rangle$

6. $bp_i\,(\exists x\phi) = bp_i\,(\phi) \cup \{\langle \exists x, x\rangle \mid x \in fv_i\,(\phi)\}$

   $$aq_i\,(\exists x\phi) = \begin{cases} aq_i\,(\phi) \cup \{\exists x\} & \textit{if } \exists x \notin aq_i\,(\phi) \\ aq_i\,(\phi) & \textit{otherwise} \end{cases}$$

   $fv_i\,(\exists x\phi) = fv_i\,(\phi) \textit{ minus the occurrences of } x \in \ \phi$ $\square$

Notice how clause 6 displays the dynamic binding power emanating from existentials and how clauses 3 and 4 manage to pass on such a binding. For formulas, $\bullet$ is just like its "cousin," $\wedge$, since sequences of DBPL formulas inhabit the same block. However, it is possible for a DBPL sequence to be formed by a DBPL formula and a DBPL text, as for example, in $Px\bullet \blacktriangleright \exists xQx \blacktriangleleft$. For such cases, and due to the impenetrable assumption, the second conjunct ought to be taken as "invisible" (for the first conjunct) since it corresponds to a whole new discourse block. This situation is accounted for by item 5 which reflects a form of synchronization between the consecutive dimensional indices: the first conjunct will pass on the bindings it knows about while the second does the same.

Having stated a formal characterization of dynamic binding, we can undertake now a discussion on the properties of the system.

Trading on the "page" and "subpage" analogy,[24] some properties such as associativity, for example, would be expected to hold even in a dynamic setting where active bindings might get blocked by new occurrences of existentials. Since we follow the left to right convention for writing, new occurrences would, necessarily, appear on the right (or subsequent pages) of previous ones and therefore pronouns[25] anaphoric on these existentials will be under the scope of the right-most one; so, the *binding potential* is preserved by both DBPL conjunctions, i.e. $\wedge$, DBPL's formula conjunction and $\bullet$, DBPL's sentential conjunction, which are interpreted through function composition.[26] Therefore, associativity holds at any DBPL level, be it "global" or "local".

Theorem 6.13 ***DBPL sequence associativity***

Let α, β, and γ be any DBPL sequences. Then

$$s^n [\![\alpha]\!]^i [\![\beta \bullet \gamma]\!]^i = s^n [\![\alpha \bullet \beta]\!]^i [\![\gamma]\!]^i$$

Proof

This result is a immediate consequence of lemma 6.5, page 120, and function associativity. □

---

[24] The page analogy is based on text splitting theorem (theorem 6.2, page 127), and text deduction theorem (theorem 6.12, page 140). The first says that we can always split up a text (at any adequate point) and the second allows us to move the split part(s) from the premiss to the conclusion.

[25] Recall that pronouns are being conflated to variables.

[26] Recalling that DBPL texts are update functions on multidimensional information states, we might have characterized update functions f^n as $\prod$-functions, i.e., n-tuples of functions $f^n = \langle f_1, \ldots, f_n \rangle$ such that $f^n(x^n) = \langle f_1(x_1), \ldots, f_n(x_n) \rangle$. If f^n and g^n are $\prod$-functions, then $f^n \circ g^n(x^n) = f^n(g^n(x^n)) = f^n(\langle g_1(x_1), \ldots, g_n(x_n) \rangle) = \langle f_1(g_1(x_1)), \ldots, f_n(g_n(x_n)) \rangle = \langle f_1 \circ g_1(x_1), \ldots, f_n \circ g_n(x_n) \rangle$. So, the global composition $f^n \circ g^n$ induces the local $f_i \circ g_i$ ones. It is a trivial exercise to show that $\prod$-composition is associative. These facts play a very important role in the forthcoming material, i.e., in the proofs of next results.

Theorem 6.14 ***DBPL text associativity***

$$s^n([\![\blacktriangleright \alpha \blacktriangleleft]\!][\![\blacktriangleright \beta \blacktriangleleft]\!])[\![\blacktriangleright \delta \blacktriangleleft]\!] = s^n[\![\blacktriangleright \alpha \blacktriangleleft]\!]([\![\blacktriangleright \beta \blacktriangleleft]\!][\![\blacktriangleright \delta \blacktriangleleft]\!])$$

Proof

This result is a consequence of splitting theorem 6.2, page 127, and function associativity. □

Since DBPL is a multidimensional generalization of DPL, as far as formulas are concerned, DPL properties are expected to hold in DBPL.[27] Indeed, DPL properties do hold in every DBPL dimensional index by virtue of the construal in definition 6. Therefore, the so-called "donkey equivalences,"[28] as well as conjunction(s) associativity does hold in DBPL while conjunction(s) commutativity, reflexivity, idempotency does not, in general, hold. These results, which are fully explained in the relevant DPL literature (and chapter 4 of this thesis), induce similar versions for DBPL texts. The difference between formula results and text equivalents relies on the multiplicity character introduced by the multidimensional information state adopted. This means that to show that some DPL property holds for a DBPL text we need to assure the property for all dimensions. For the converse, all that is needed is to find out a single "badly behaving block" where the property does not hold. In virtue of these facts, it is easy, now, to understand why

1. DBPL conjunctions are not, in general, commutative. The page analogy would be helpful here: we cannot (in general) commute pages on a book without changing the information conveyed. Imagine a discourse where pronouns occurring at any dimensional index i, under the rule of an active quantifier occurring previously at the same dimensional index, makes perfect sense. If we commute the "pages" we would loose the original anaphoric relationships; new anaphoric

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[27] Trading on an analogy to vectorial spaces, we can think of scalar product as the generalization tool moving us from DPL to DBPL as far as formulas are concerned. So, DPL properties play the scalar role while the multidimensional state plays the vector role. A proviso: this analogy only works by virtue of the impenetrable assumption which keeps things confined to each dimension they happen to occur in.

[28] See table 4.6, page 81.

relationships might be established and the discourse as a whole might lose its coherence.

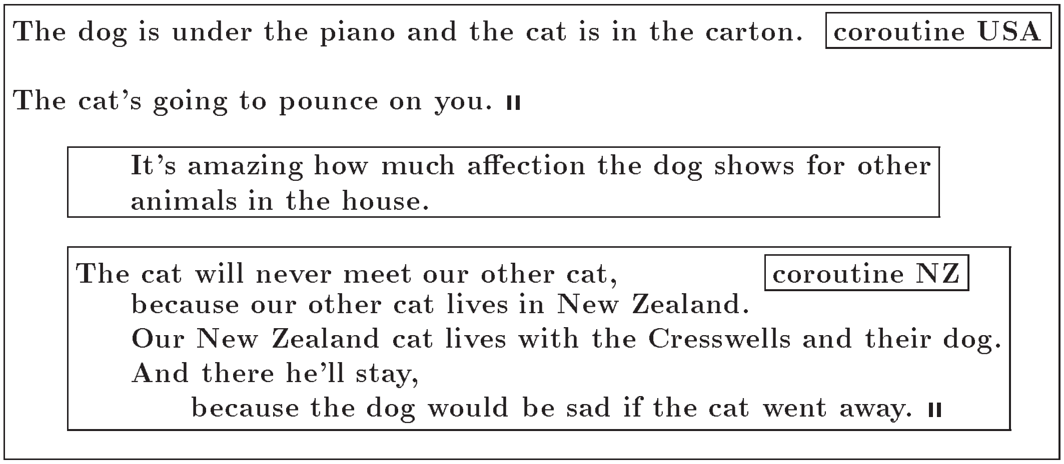
Anaphoric relationships, for example, would be lost or even get distorted by attaching pronouns to the wrong existentials. This is what happens for the DBPL text below

$\blacktriangleright \ldots \blacktriangleright \exists x(Wx \wedge WKx) \bullet WHx \bullet \exists x(Mx \wedge WKx) \bullet ADx \bullet\ \ldots \blacktriangleleft \ldots \blacktriangleleft$ (where predicates W, WK, WH, M, AD stand for *woman, walks_in_the_park, whistles, man,* and *airs_his_dog* resp.), when we commute $WHx \bullet \exists x(Mx \wedge WKx)$ getting $\blacktriangleright \ldots \blacktriangleright \exists x(Wx \wedge WKx) \bullet \exists x(Mx \wedge WKx) \bullet WHx \bullet ADx \bullet\ \ldots \blacktriangleleft \ldots \blacktriangleleft$. The original discourse has an inner block where one was talking about a woman and a man walking in the same park. Moreover, one has stated that the woman was whistling while the man was airing his dog. After commuting the sequence, we get a different discourse where the only information about the woman is that she was walking in the park while the walking man was not only airing his dog but also whistling.

Co-routining discourses would provide us with another source of counter-examples; for these cases the second block is the one where some topic under previous discussion is being resumed. If we reverse the blocks we will get a resuming block occurring at a point before the interrupted one. Compare, for example, the discourse in figure 6.9 with the one in page 97.

A natural conclusion from the previous examples is that for DBPL the way a text is built up does matter. It matters for a 'microscopic level' since formula elements' order mirror the natural order in which anaphoric relationship are established. But, it also matters for 'macroscopic level' since sequence elements' order mirror the natural order in which structured discourses are built up. And since order does matter commutativity can not be expected to hold anymore.

2. DBPL conjunctions are not, in general, idempotent. That DBPL $\bullet$ is not idempotent is exemplified by $\blacktriangleright \ldots \blacktriangleright Px \wedge \exists x Qx \blacktriangleleft \ldots \blacktriangleleft$ and $\blacktriangleright \ldots \blacktriangleright Px \wedge \exists x Qx \bullet Px \wedge \exists x Qx \blacktriangleleft \ldots \blacktriangleleft$. For the second DBPL text, the second occurrence of Px is


The dog is under the piano and the cat is in the carton.

The cat's going to pounce on you. ⏸

It's amazing how much affection the dog shows for other animals in the house.

The cat will never meet our other cat,
because our other cat lives in New Zealand.
Our New Zealand cat lives with the Cresswells and their dog.
And there he'll stay,
because the dog would be sad if the cat went away. ⏸

Figure 6.9: Commuting blocks for Lewises scenario II.

under the active scope of the quantifier $\exists x Qx$. As a consequence, $Px \wedge \exists x Qx$ and $Px \wedge \exists x Qx \bullet Px \wedge \exists x Qx$ are not equivalent since $s^n[\![Px \wedge \exists x Qx]\!]^i$ and $s^n[\![Px \wedge \exists x Qx \bullet Px \wedge \exists x Qx]\!]^i$ should not be equivalent, independently of the dimensional index i, the multidimensional information state s^n and the model $\mathfrak{M}$ considered.

3. DBPL entailment relations are not, in general, reflexive. $\blacktriangleright\ Px \bullet \exists x Qx\ \blacktriangleleft\ \models\ \blacktriangleright\ Px \bullet \exists x Qx\ \blacktriangleleft$ is a counter-example. The reason is that in the occurrence of this text as conclusion, the variable x in the first conjunct gets bound by the quantifier in the occurrence of the text as a premiss, whereas in the occurrence of the text as a premiss it is free. On the other hand, if the active quantifier variables (AQV) of a text $\blacktriangleright\ \Psi\ \blacktriangleleft$ do not intersect the free variables (FV) of $\blacktriangleright\ \Psi\ \blacktriangleleft$, then $\blacktriangleright\ \Psi\ \blacktriangleleft\ \models\ \blacktriangleright\ \Psi\ \blacktriangleleft$

4. DBPL entailment relations are not, in general, transitive. The cases that pose problems to transitivity can be characterized as follows. Suppose $\blacktriangleright\ \Phi\ \blacktriangleleft\ \models\ \blacktriangleright\ \Psi\ \blacktriangleleft$ and $\blacktriangleright\ \Psi\ \blacktriangleleft\ \models\ \blacktriangleright\ ?\ \blacktriangleleft$. If we want to conclude from this that $\blacktriangleright\ \Psi\ \blacktriangleleft\ \models\ \blacktriangleright\ ?\ \blacktriangleleft$, then problems may arise if $x \in FV(?)$ and $x \in AQV(\Psi)$. Consider $\blacktriangleright\ \neg\neg\exists x Px\ \blacktriangleleft\ \models\ \blacktriangleright\ \exists x Px\ \blacktriangleleft\ \blacktriangleright\ \exists x Px\ \blacktriangleleft\ \models\ \blacktriangleright\ Px\ \blacktriangleleft$. It is clear that the first entails the second and the second entails the third, without the first entailing the third. On the other hand, consider $\blacktriangleright\ \exists x Px\ \blacktriangleleft$, $\blacktriangleright\ \exists x Px\ \blacktriangleleft$ and $\blacktriangleright\ Px\ \blacktriangleleft$. This is a case where nothing goes wrong. So, not all cases where $\blacktriangleright\ ?\ \blacktriangleleft$ contains

a free occurrence of x, and $\blacktriangleright \Psi \blacktriangleleft$ contains an active occurrence of $\exists x$ ought to be excluded. Evidently, what also matters is what $\blacktriangleright \Phi \blacktriangleleft$ "says" about x, in the dynamic sense of what constraint it puts on whatever free occurrence of x that are still to come. Roughly speaking, what $\blacktriangleright \Phi \blacktriangleleft$ says about variables which occur freely in $\blacktriangleright\ ?\ \blacktriangleleft$ and which are bound in $\blacktriangleright \Psi \blacktriangleleft$, should be at least as strong as what $\blacktriangleright \Psi \blacktriangleleft$ says about them.

One property, however, deserves a special treatment. This property is the pointwise character of DPL update which is stated, in DPL, as $s[\![\phi]\!] = \bigcup_{i \in s}\{i\}[\![\phi]\!]$.[29] This property is pointwise because in order to compute the output state we only need to know each individual component on the input state. What this really means is that $(\bigcup_{i \in s}\{i\})[\![\phi]\!] = \bigcup_{i \in s}(\{i\}[\![\phi]\!])$.[30] The straightforward transposition of distributivity, from the unidimensional setting of DPL to the multidimensional setting of DBPL, related to any DBPL text Υ, would be accomplished by $s^n[\![Proj(i,\Upsilon)]\!]^i = \bigcup_{k \in s^n_i}\{k\}[\![Proj(i,\Upsilon)]\!]$, for all dimensional index i between 1 and $depth(\Upsilon)$.

Theorem 6.15 ***(DBPL text distributivity)***

Let $\mathfrak{M}$, s^n and Υ be an arbitrary model, multidimensional information state and DBPL text, resp. Then for all i, $1 \leq i \leq depth(\Upsilon)$,

$$\Pi(i, s^n[\![\Upsilon]\!]_{\mathfrak{M}}) = \bigcup_{r \in s^n_i}\{r\}[\![Proj(i,\Upsilon)]\!]^i_{\mathfrak{M}}$$

Proof

The proof is trivialized by resorting to DPL's distributivity theorem. Notice that for all i, $1 \leq i \leq depth(\Upsilon)$, $Proj(i,\Upsilon)$ is a DPL formula. Therefore,
$s^n_i[\![Proj(i,\Upsilon)]\!]_{\mathfrak{M}} = \bigcup_{r \in s^n_i}\{r\}[\![Proj(i,\Upsilon)]\!]_{\mathfrak{M}}$ by DPL distributivity theorem. Since

$$\begin{aligned} s^n[\![\Upsilon]\!]_{\mathfrak{M}} &= \langle s^n_1[\![Proj(1,\Upsilon)]\!], s^n_2[\![Proj(2,\Upsilon)]\!], s^n_3[\![Proj(3,\Upsilon)]\!], \ldots, \\ &\quad s^n_{depth(\Upsilon)}[\![Proj(depth(\Upsilon),\Upsilon)]\!], s^n_{depth(\Upsilon)+1}, s^n_{depth(\Upsilon)+2}, \ldots\rangle \end{aligned}$$

---

[29] Recall that for DPL any information state s is a set of assignment functions.

[30] Recall from set theory that for any set X, $X = \bigcup_{i \in X}\{i\}$. So, what the DPL theorem really says is that $s[\![\phi]\!] = (\bigcup_{i \in s}\{i\})[\![\phi]\!] = \bigcup_{i \in s}(\{i\}[\![\phi]\!])$, what justifies its name.

we get the desired result. □

Notice how distributivity is carried out in parallel throughout all dimensions of a DBPL text. The kind of "synchronization" required was provided by function $Proj$ which produced a presequence of all elements belonging to the same dimension index. This means, as might be expected, that distributivity is done in parallel for every dimensional index i for every multidimensional state s^n. And this is so by virtue of the impenetrable assumption. If we had adopted a different position with respect to the impenetrable hypothesis, for example, having made it weaker, we might have developed a completely different semantical system where distributivity, among many other results, would not be expected to hold. Carrying on properties from one dimension to another might remove the local nature of distributivity. But this is not the aim of the present work and this kind of logic must await future work.

6.5 Additional Remark

The literature on anaphoric pronouns all rests on identifying pronouns with variables (the polemic is about what kind of variables they are, i.e., if they are free or bound variables). So, if we allow our first-order language to include *functional symbols* then we run into a new problem, since, now, the class of "variable terms" includes not only "plain variables" but also "functional variables". Functional variables might be seen as a kind of "anonymous variable" which depend only on the input argument variable used. Therefore, all future references to such an indirectly determined element must be done through the use of its anonymous name, i.e., its functional term. Now, note that functions are likely to take us away from the anaphoric source of reference, as showed below:

Example 6.5 *A farmer's son owns a donkey. He got it for free.*

a. $\exists x(f(x) \wedge \exists y(d(y) \wedge o(son(x), y)))$. $gff(*he*, y)$.

b. $\exists x(f(x) \wedge \exists y(d(y) \wedge o(son(x), y)))$. $gff(son(x), y)$.

c. $\exists x(f(x) \wedge \exists w(son\_of(w,x) \wedge \exists y(d(y) \wedge o(w,y))))$. $gff(w,y)$.

Example 6.5.a makes clear that the place holder marked as ***he*** can not be identified to any variable: the pronoun clearly refers back to the son of a farmer (where *a farmer* has been associated to x). Two solutions are almost self-evident: (i) we might use the same functional term and therefore allowing pronouns to be identified to a broader class of terms, as in 6.5.b, or (ii) we might stick to the "conservative" hypothesis, as in 6.5.c, page 149 since, after all, it is theoretically possible to regard functional terms as special cases of relational predicative symbols (if the theory admits equality, what is always the case for logic systems dealing with anaphoric pronouns).

6.6 Summary

The semantic framework presented in this chapter was built up under the assumption that discourses are structured multidimensional objects, which could be analyzed in a *extended dynamic semantic framework* in tune with the one firstly developed by Groenendijk and Stokhof. The cornerstone of the Groenendijk and Stokhof theory is that sentences in a discourses behave like update functions over information states. Basic for Groenendijk and Stokhof's theory is: (i) their keeping to the meaning compositionality principle, (ii) their characterization of discourse as a sequence of sentences, and (iii) their characterization of information states as a set of assignment functions.

By keeping to the compositionality principle, Groenendijk and Stokhof's discussion strongly supported the view that meaning is a richer concept that should not be conflated to the traditional truth conditional semantics, since truth conditions are one, but only one, important aspect for characterizing meaning. In this sense, we agree that DPL was a first step on the right direction.

Independently of accepting (or not) the principle of compositionality, no framework (be it on the representationalist grounds of Kamps' DRT, be it on the grounds of Groenendijk and Stokhof semantics) attempted to go beyond the unidimensional character assigned to discourses. However, the discussion presented in chapter 3 gives support for the existence of multidimensional kinds of discourses.

In this research we made a step towards multidimensional discourse semantics analysis taking a closer look at hierarchically structured discourses dealing with interruption phenomena. We focused on interruptions exhibiting a co-routining behaviour. But to deal with such complex entities we had to make some decisions. So, we assumed that

1. all dimensions share the same domain of individuals,
2. the multidimensional information states are impenetrable,
3. the assignment functions are total.

As should be expected, the decisions made would affect the framework in many different ways. For example, the assumption that all dimensions share the same domain of individuals made easier the development of a formal system. The sameness domain is particularly emphasized in the conceptualization of multidimensional information state. Since we are now acquainted with the multidimensional information state concept, it seems natural to think of a more 'realistic' characterization for it. Abolishing the sameness domain assumption, we might have attached a specific domain of individuals for every discourse block (or, in our terminology, for every dimensional index) occurring at any discourse. So, the multidimensional information state might have been any tuple of sets of assignment functions from each set of variables in use in each dimensional index to the domains of each dimension. It would look like this $s^n = \langle \{g \mid g \in \mathcal{D}_1^{V_1}\}, \ldots, \{g \mid g \in \mathcal{D}_n^{V_n}\}, \ldots \rangle$. This new multidimensional information state definition would not pose any new problems for the framework developed except, maybe, some more philosophical questions such as cross-block identification (.i.e, which elements are shared between two discourse blocks and how could they be recognized as such).

In fact, assumptions 1 and 3 above do not change the main aspects and properties of the framework developed in this chapter; as we already said before, they only made easier the development of the framework. The same could not be said with respect to assumption 2. It is the impenetrability character assumed, that avoids phenomena occurring at one dimensional level being propagated to another dimension. Keeping dimensions independent of each other allowed us to preserve the pointwise character of updates. If we abolish the impenetrable assumption, then the local character for update computation would be lost since its computation would depend on entities, or even properties, inherited from ancestor blocks. It is true that inheritance patterns reflect a more realistic discourse modeling than the one presented in this chapter. But it is true, also, that they need a more sophisticated framework to cope with them (a suggestion of how we could develop such a multidimensional framework, without the impenetrable assumption rule, will be presented in chapter 8). And so, the framework developed in this chapter, under the impenetrable assumption rule, deserves a special place among the dynamic systems for being the first landmark into the multidimensional discourse space.

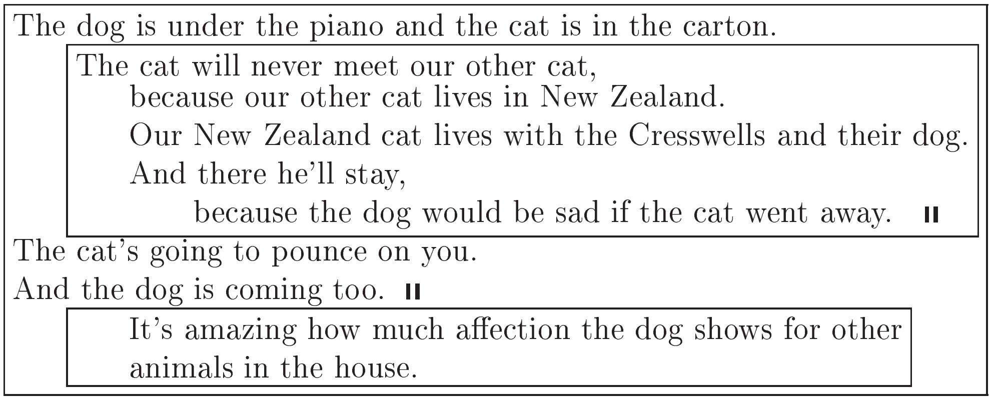
Chapter 7

Example of Application

7.1 Lewises' scenario II revisited

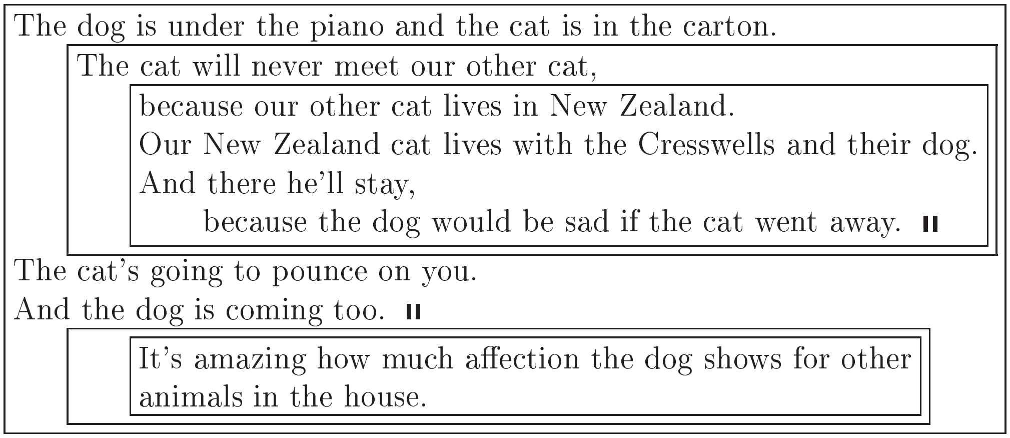
Chapter 5 presented us with examples of "sophisticated" discourses exhibiting a co-routining like interruption structure (which we have coined the Lewises' scenario I and II pages 95 and 97 resp.). In this short chapter we will show how an admittedly over-simplified version of the Lewises' scenario II would be handled in DBPL. Scenario II was chosen because it displays more clearly than scenario I the co-routining resuming character discussed in chapter 5.

Notice that there is a mismatch between the boxes drawn and the indented discourse blocks shown in the same picture on page 97 (repeated in figure 7.1 for convenience). The boxes were drawn in the attempt of characterizing the two distinctive situations, namely, the American household from the New Zealand one, in order to more clearly show the "leaving" and "resuming" of the NZ coroutine. However, the real NZ coroutine is the one displayed at the third level of indentation (as emphasized on figure 7.2). The mismatch is, of course, located at the second level of indentation. Indeed, the second level is a mediating discourse block between the USA and NZ situations. This block works in different ways: one of its roles is to (prepare to) introduce a new discourse block along with a new discourse referent, namely, the New Zealand cat. So, to be realistic, this example does not support the impenetrability hypothesis assumed in this research proposal. However, we did not claim that real discourses would always conform to the "impenetrability law." The proposal developed in chapter 5 is indeed the first step into the multidimensional "landscape" of structured discourses; a more elaborated step, abolishing the impenetrable assumption, has been already envisaged as indicated in chapter 8, section 8.4, page 164.



The dog is under the piano and the cat is in the carton.
The cat will never meet our other cat,
because our other cat lives in New Zealand.
Our New Zealand cat lives with the Cresswells and their dog.
And there he'll stay,
because the dog would be sad if the cat went away. ⏸
The cat's going to pounce on you.
And the dog is coming too. ⏸
It's amazing how much affection the dog shows for other animals in the house.

Figure 7.1: Lewises scenario II



The dog is under the piano and the cat is in the carton.
The cat will never meet our other cat,
because our other cat lives in New Zealand.
Our New Zealand cat lives with the Cresswells and their dog.
And there he'll stay,
because the dog would be sad if the cat went away. ⏸
The cat's going to pounce on you.
And the dog is coming too. ⏸
It's amazing how much affection the dog shows for other animals in the house.

Figure 7.2: NZ coroutine for Lewises scenario II

Taking these points into consideration, we feel justified to undertake further simplifications on the Lewises' scenario II as shown in example 7.1. Therefore, the over-simplified example below has the sole purpose of showing how DBPL copes with such kind of discourses.

Example 7.1 *Let us work on a slightly modified but heavily shrunken version of Lewises scenario II as displayed below.*

> *The dog is under the piano and the cat is in the carton.*
>
> > *I've got a cat that lives in New Zealand with the Cresswells and their dog.*
> >
> > *And there she will stay because the dog would be sad if the cat went away* ‖
>
> *The cat went out for his all night walk.* ‖
>
> > *It's amazing how much affection the dog shows for other animals in the house.*
> >
> > ⋮
>
> ⋮

This discourse could be translated to a DBPL*-like discourse as shown below.*

> $is\_under(y, z) \wedge is\_in\_the\_carton(x)$
>
> > $\exists x(cat(x) \wedge lives\_in\_with(x, NZ, Cresswells) \wedge is\_owned\_by(x, David))$
> >
> > $\exists y(dog(y) \wedge lives\_in\_with(y, NZ, Cresswells) \wedge is\_owned\_by(y, Cresswells))$
> >
> > $will\_stay\_in(x, NZ)$
> >
> > $went\_away(x) \rightarrow would\_be\_sad(y)$
>
> $went\_out\_for\_his\_all\_night\_walk(x)$
>
> > $\forall z(animal(z) \wedge lives\_in\_with(z, NZ, Creswells) \rightarrow shows\_affection\_for(y, z))$
> >
> > ⋮
>
> ⋮

To make shorter the notational representation above, let us assume that the predicate letters ϕ_1 through ϕ_{12} stand for the following natural language predicates:

ϕ_1 is for cat
ϕ_2 is dog
ϕ_3 is for is_under
ϕ_4 is for is_in_the_carton
ϕ_5 is for for lives_in_with
ϕ_6 is for is_owned_by
ϕ_7 is for will_stay_in
ϕ_8 is for went_out_for_its_all_night_walk
ϕ_9 is for animal
ϕ_{10} is for shows_affection_for
ϕ_{11} is for went_away
ϕ_{12} is for would_be_sad

and that the constant letters a and b stand for David Lewis and the Cresswells respectively. Due to DBPL's entailment definition and DBPL's text deduction theorem (theorem 6.12, page 140), we would accept that in a previous "page" of this scenario discourse referents for the American cat (x), dog (y), and the piano (z) have been introduced by existential formulas. Suppose also that s^n is the information state reached after processing the discourse with the "initial" pages and that the DBPL text Υ corresponds to the following scenario's page.

$\Upsilon = \blacktriangleright\ \phi_3(y,z) \wedge \phi_4(x) \bullet\ \blacktriangleright\ \exists x(\phi_1(x) \wedge \phi_5(x, NZ, b) \wedge \phi_6(x,a)) \bullet \exists y(\phi_2(y) \wedge \phi_5(y, NZ, b) \wedge \phi_6(y,b)) \bullet \phi_7(x, NZ) \bullet \phi_{11}(x) \rightarrow \phi_{12}(y)\ \blacktriangleleft\ \bullet \phi_8(x) \bullet\ \blacktriangleright\ \forall z(\phi_9(z) \wedge \phi_5(z, NZ, b) \rightarrow \phi_{10}(y,z))\ \blacktriangleleft\blacktriangleleft$

And since Υ above is not easily readable, for convenience, a more visually readable version is presented below.

$$
\begin{array}{llll}
\Upsilon = \blacktriangleright\ \phi_3(y,z) \wedge \phi_4(x) & & & \\
 & & \blacktriangleright & \\
 & & & \exists x(\phi_1(x) \wedge \phi_5(x, NZ, b) \wedge \phi_6(x,a)) \\
 & & & \exists y(\phi_2(y) \wedge \phi_5(y, NZ, b) \wedge \phi_6(y,b)) \\
 & & & \phi_7(x, NZ) \\
 & & & \phi_{11}(x) \rightarrow \phi_{12}(y) \\
 & & \blacktriangleleft & \\
 & \phi_8(x) & & \\
 & & \blacktriangleright & \\
 & & & \forall z(\phi_9(z) \wedge \phi_5(z, NZ, b) \rightarrow \phi_{10}(y,z)) \\
 & & \blacktriangleleft & \\
 & \blacktriangleleft & &
\end{array}
$$

We are now ready to compute the update of s^n operated by the DBPL text Υ. The update is calculated as follows.

$$s^n\llbracket\Upsilon\rrbracket \;=\; \langle\; s^n_1\llbracket\phi_3(y,z)\wedge\phi_4(x)\bullet\phi_8(x)\rrbracket,\; s^n_2\llbracket\exists x(\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a))\bullet \exists y(\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b))\bullet\phi_7(x,NZ)\bullet\phi_{11}(x)\to\phi_{12}(y)\bullet\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to \phi_{10}(y,z))\rrbracket,\ldots\rangle$$

Focusing now on s^n_2 we get

$$s^n_2\llbracket\exists x(\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a))\bullet\exists y(\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b))\bullet\phi_7(x,NZ)\bullet \phi_{11}(x)\to\phi_{12}(y)\bullet\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to\phi_{10}(y,z))\rrbracket =$$

$$s^n_2\llbracket\exists x(\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a))\rrbracket\llbracket\exists y(\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b))\rrbracket\llbracket\phi_7(x,NZ)\rrbracket \llbracket\phi_{11}(x)\to\phi_{12}(y)\rrbracket\llbracket\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to\phi_{10}(y,z))\rrbracket =$$

$$s^n_2[x]\llbracket\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a)\rrbracket\llbracket\exists y(\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b))\rrbracket\llbracket\phi_7(x,NZ)\rrbracket \llbracket\phi_{11}(x)\to\phi_{12}(y)\rrbracket\llbracket\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to\phi_{10}(y,z))\rrbracket =$$

$$s^n_2[x]\llbracket\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a)\rrbracket[y]\llbracket\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b)\rrbracket\llbracket\phi_7(x,NZ)\rrbracket \llbracket\phi_{11}(x)\to\phi_{12}(y)\rrbracket\llbracket\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to\phi_{10}(y,z))\rrbracket =$$

So, to the USA dimension s^n_1, the update $s^n_1\llbracket\phi_3(y,z)\wedge\phi_4(x)\bullet\phi_8(x)\rrbracket$, says that only assignments in s^n_1 which assign a cat to x which is required to be, firstly, in the carton and afterwards away, are preserved in the updated state. For the NZ dimension s^n_2, the update of s^n_2 with $\blacktriangleright\ \exists x(\phi_1(x)\wedge\phi_5(x,NZ,b)\wedge\phi_6(x,a))\bullet\exists y(\phi_2(y)\wedge\phi_5(y,NZ,b)\wedge\phi_6(y,b))\bullet\phi_7(x,NZ)\bullet\phi_{11}(x)\to\phi_{12}(y)\bullet\forall z(\phi_9(z)\wedge\phi_5(z,NZ,b)\to\phi_{10}(y,z))\ \blacktriangleleft$ says that only the assignments in s^n_2 which assign a cat to x and a dog to y such that the cat and the dog live in New Zealand with the Cresswells would survive in the updated state. And since anaphoric relationships are confined to their dimensional niches the information state s^n tells us that there are two different cats satisfying the conditions established in each dimensional index corresponding to the discourse blocks occurring at the scenario.

Finishing this section, notice that we did not take into account tense and mood since the framework developed does not deal with such points.

7.2 Summary

In this chapter we showed how the framework developed in this thesis deals with "real" discourses fitting the impenetrable hypothesis. Recalling that the framework is only the first step into the realm of complex, structured discourses, we would not have expected to find here a very sophisticated example. We expect to present better examples of application when the development of more sophisticated frameworks, such as the ones drafted in chapter 8, come in to existence.

Chapter 8

Discussion and Outlook

The aim of this chapter is to draw attention to the contributions and advantages of the new theory over the old ones. A critical discussion of the weak points are presented as well as an outlook on future research.

8.1 Last Comments on Dynamic Semantics

A great deal of work in formal semantics over the last two and a half decades has been dedicated to the analysis of particular constructions and semantic phenomena in natural language. This analysis, which has frequently been referred to as *dynamic semantics*, is based on the view that the meaning of a sentence does lie in the way it changes (the representation of) the information of the interpreter (Groenendijk and Stokhof (1991)). And naturally, the shift from traditional approaches based on truth conditions (since then, referred to as *static* semantics) to dynamic approaches has often involved the collaboration of linguists with logicians, philosophers, and mathematicians.

The roots of the dynamic view on meaning can be traced back to the works of Stalnaker (Stalnaker (1974)), Kamp (Kamp (1981)), and Heim (Heim (1982)). Kamp (1981), and Heim (1982) offer solutions to certain problems involving indefinite noun phrases and anaphora in multisentence discourses and in the so called *donkey sentences* of Geach (1962). In their systems, indefinite and definite noun phrases are interpreted as variables and conditions, i.e., open sentences, instead of quantifier phrases. In this unselective binding philosophy the puzzle about why an indefinite noun phrase seems to be interpreted as existential in simple sentences but universal in the antecedents of conditionals is no longer localized on the noun phrase itself. As Partee (1995, p. 30) points out "its apparently varying interpretations are explained in terms of the larger properties of the structures in which it occurs, which contributes explicit or implicit unselective binders which bind everything they find free within their scope."

Kamp's and Heim's work has led to a great deal of further research, applying it to other phenomena, extending and refining it in various directions, and even, challenging it. And, among several proposals, including the revival of a modified version of Evans (1980) E-type analysis (Neale (1990) and Heim (1990)), there is Dynamic Predicate Logic of Groenendijk and Stokhof (1991), developed in part in connection with a claim that Kamp's Discourse Representation Theory is not fully compositional.

8.2 The Multi-Dynamic Semantics

In almost all compositional approaches to anaphora, pronouns are reduced to variables. The underlying idea is that pronouns are *syntactically free* variables, although, somehow, *semantically bound* variables. By defining the interpretation process as a function updating information about possible values of these variables, the value of antecedents will be available for further occurrences of coindexed pronouns. This is achieved by DPL and its offspring by equating information states to sets of assignment functions. Nevertheless to say, this was (and it still is) an insightful breakthrough.

Together with DRT, DPL is a landmark in the *dynamic semantics* (or, *information states semantics*) *paradigm*. Both have been targeted by research programmes aiming to reduce or eliminate their idiosyncrasies. But no programme, to the best of my knowledge, has made or suggested the step toward embracing the analysis of hierarchically structured discourses. Not up to now!

The first step into this field was made by this research work. That means that the analysis of hierarchically structured discourses are the target. And information states the weapons directed to the target. It is clear that some kind of change in the information state character is needed since *sets do not have any internal interesting structure*. To grasp the multi-dimensionality character present in the recursive "definition" of discourse we have to search for more powerful weaponry; n-tuples provide an almost self-suggestive answer.

The new information state model is an n-tuple of sets of assignment functions. This model allows us to keep discourse components and the referential system represented by a set of assignment functions on a one to one basis. As a natural consequence, we could capture a *new scope dimension* responsible for important linguistic phenomenon such as for example preventing anaphoric reference between entities that do not belong to the same block. This confinement is trivially done in our work by adopting a strong assumption we coined the *impenetrable hypothesis.* But it also might have been *softened* in order to deal with a greater number of linguistic phenomena. For instance, one segment might have been paving the way for a subsequent *"non-ambiguous" double sense* discourse block. The intended sense might be a joke while the conveyed "main sense" would be a normal unsuspicious situation. So, intentions (or yet rhetorical relations) would "migrate" among discourse blocks and therefore the impenetrability of the *impenetrable hypothesis* is not so impenetrable after all. Or we might have expanded the information state model in order to cope with multi-agent discourses. Or ... Some of these points are, of course, left for future work (see future work below).

With the multi-dimensional approach a new dynamic has been superimposed to the dynamic semantics.

8.2.1 Pros and Cons over Other Approaches

DPL (and its offspring) and DRT are empirically equivalent. They are equivalent in the sense that both address the same phenomena and both theories achieve the same results, even though through different methodological approaches. While the first sticks to the compositionality criteria, the other does not so much (although several "fixes" have been developed addressing this particular DRT issue).

DPL and DRT come up with an impressive answer for the intra and inter-sentential anaphoric reference as well as the apparently varying behaviour of indefinite noun phrases. However, this is achieved only for simple discourses, discourses made of sentences. From an ontological point of view both theories are shallow.

Data presented in chapter 3 give support for a much richer ontology, since discourses are hierarchically structured entities. A clear recursive pattern might be inferred from the underlying overall structure. Discourses are made of sub-discourses which are in turn discourses. And only in the last instance they are made of sentences. And this is one of the building blocks of this research and one of its main contributions. For the first time, complex discourses have been tackled from the dynamic perspective. The answer provided sheds light onto the ontological nature of information states (as well as discourse ontology) and paves the way for new varieties of dynamic logics.

A criticism one can make of the approach presented in this research is its apparently parasitic character on DPL. So, one might expect to find here the same weak points emanating from DPL multiplied by all dimensions. I could have listed the criticisms of DPL found in the literature, but I won't. Indeed, a great deal of work on the DPL's weak points is being undertaken in Europe, and particularly in Amsterdam. Improved versions, that I have been calling DPL offspring, have been published in important journals and conferences; also the DIANA project has provided the community with electronic Internet sites in Europe where work on dynamic semantics can be found (the biggest repository sites for DIANA project as well as literature on dynamic semantics, be it seen from the DRT or DPL perspective, are located in

Edinburgh and Amsterdam, respectively. The http electronic addresses are provided in the references). But, if I am a parasite then all good “fixes” and improvements made on DPL will flow naturally into the one presented here. Therefore, the weak points are not so weak after all. But a word of warning is needed here. This is not a parasitic work, *it is, indeed, a symbiotic one*. After all no one has undertake the course I did and the results provided here are not only empirically important for strengthening Groenendijk and Stokhof’s view on semantics but also to push it even further as the future research work, below, points to.

Of course that the work presented here is not free of problems. One particularly odd problem is directly related to the *impenetrable assumption* made which allows us to confine any phenomena to the dimensional index they occur. The oddity is that this allows the discourse to be restructured in unnatural ways. For example, $\blacktriangleright \alpha_1 \bullet \alpha_2 \bullet \blacktriangleright \beta_1 \bullet \beta_2 \blacktriangleleft \bullet \alpha_3 \blacktriangleleft$, $\blacktriangleright \alpha_1 \bullet \alpha_2 \bullet \alpha_3 \bullet \blacktriangleright \beta_1 \bullet \beta_2 \blacktriangleleft\blacktriangleleft$, and $\blacktriangleright\blacktriangleright \beta_1 \bullet \beta_2 \blacktriangleleft \bullet \alpha_1 \bullet \alpha_2 \bullet \alpha_3 \blacktriangleleft$ will lead one to the same multidimensional states pattern although, as interruption cases make clear, not all are natural. Interruptions clearly show that the block-dimension shift is anchored to the point they occur. A fix for this problem is enrolled under the cover name “anchoring fix” in the future research work.

8.3 Contributions

The main contribution of this research work is to show that it is possible to undertake linguistically based discourse frameworks under the dynamic semantic paradigm. As we have showed in chapter 3, concepts which originated from computer programming were largely used to explain the linguistic phenomenon under analysis. And a stack processing was the prevalent model. Changing from stack based discourse processing to list (general list) processing wasn’t an easy step. Having done it, it seems very useful and one would wonder if the multidimensional processing couldn’t be pushed even further. An expressive affirmative answer to this question is given in the future research section. *And this is at least as expressive as the contribution emanating from the research work presented here.*

The future work is expressive because it proposes not only conservative "patch extensions" for dealing with the weak points of the present work but mainly new extensions coping with a new range of linguistic phenomena. Patch extensions are conservative since their goal is to correct "minor errors." So, in a sense, they are not important as an evaluation measure of the contributions made by any work. Under this category I would put any extension making an update a real update ($s^n[\![\Upsilon]\!] \leq s^n$), even knowing that this is not an easy task.

Another important contribution of this research work was to expand (and improve our knowledge of) the ontology underlying discourses. This research presents us with a simple one (yet more complex than any other presented in the dynamic semantics literature); we did not take into account any intention as a phenomenon attached to discourse blocks (as in Grosz and Sidner (1986)). If we did so, we would need to extend the ontology. An intentional system would provide the answer in a format of lambda abstraction along the line suggested in 8.4.3 (see future work section). This makes clear that $\blacktriangleright\blacktriangleleft$ is indeed a functor instead of a simple parenthetical notational device. And this is an important contribution to our understanding of the dynamics of semantic interpretation for natural languages.

Finally, this research may lead to the development of a metatheoretic study focusing all dynamics multidimensional settings already developed in this thesis and others (not yet in existence, but already envisaged by this author in the section below).

8.4 Future Research

The "multidimensional paradigm" introduced in this research work was developed in a way that allows the following "patches":

Update fix The update fix should try to make an updated information state more specific than its parent, i.e., make it eliminative. Eliminative updates show more clearly the real flow of information growth. In other words, the update fix ought to remove downdates from this system without losing its dynamics. This is not an easy task since we will be dealing with a combination of eliminative and distributive modalities of updates in a multidimensional setting. If not done carefully, the result will be a classical update which means that the semantic system will be equivalent to a static one (cf. Groenendijk and Stokhof (1990), Dekker (1993)). But this kind of "patch" is being accounted for by the research task force located mainly in Amsterdam. The ongoing research work on this topic is exemplified by Vermeulen (1993), and Dyana deliverables such as Dekker (1993), Dekker (1994), Does (1993), Groenendijk et al. (1994), Dekker (1995a), Dekker (1995b), and Groenendijk, Stokhof and Veltman (1995) (to cite a few). Therefore, this extension should not worry us since the results achieved by that task force might be incorporated into our framework. The really important fix that should concern us, since it is specific to the proposal developed in this work, is the issue of next item.

Anchoring fix The anchoring fix should try to remove the odd character emanating from the impenetrable hypothesis. As stated, the impenetrable hypothesis imposes a higher degree of parallelism (or independence) between dimensions than would pertain in a real situation.

Some possibilities for more creative future research are:

Multi-Agent Discourses The multidimensional paradigm developed in this research work could be used to model multi-agent discourse: discourses where more than one agent take part on it.

Adding Intentionality The multidimensional model presented in the present work might be extended in a way that it would be possible to account for the 'intensional' component of Grosz and Sidner (1986) system (the present one deals only with the attentional component). Analogously, rhetorical relations might be accounted for.

8.4.1 The Anchoring Fix

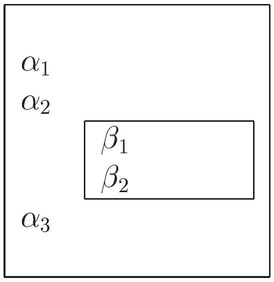
The anchoring fix is concerned with the problem imposed on DBPL by the impenetrable hypothesis. The DBPL version developed in this thesis allows us to take as "equivalent" DBPL texts such as

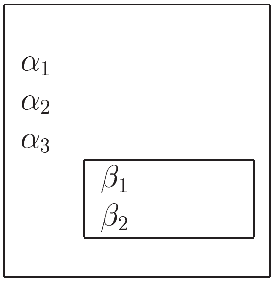
$$\blacktriangleright\ \alpha_1 \bullet \alpha_2 \bullet \blacktriangleright\ \beta_1 \bullet \beta_2 \blacktriangleleft \bullet \alpha_3 \blacktriangleleft$$

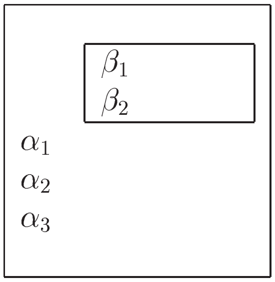
$$\blacktriangleright\ \alpha_1 \bullet \alpha_2 \bullet \alpha_3 \bullet \blacktriangleright\ \beta_1 \bullet \beta_2 \blacktriangleleft\blacktriangleleft$$

$$\blacktriangleright\blacktriangleright\ \beta_1 \bullet \beta_2 \blacktriangleleft \bullet \alpha_1 \bullet \alpha_2 \bullet \alpha_3 \blacktriangleleft$$

These DBPL texts might be thought as corresponding to the following natural language abstract discourse frames.







However, it does not seem natural to think of these discourses as equivalent (in the sense they lead us to the same output state when presented to the same input). The point the inner blocks occur at seems to have a role to play and therefore this role should be accounted for into the theory. Two possibilities could be attempted here: the first one would try to keep the impenetrable assumption as strong as stated in the present work. The second one would try to overcome the impenetrable assumption.

For the first case pointed out, a new class of anchor operators $\bullet_{a_i}$ would be envisaged along with a new type of information state where a new component would be present. The new multidimensional information state would look like a n-tuple of 2-tuples from $\mathcal{D}^V \times \mathcal{N}$ (for every dimensional component). As usual, $\mathcal{D}^V$ would be the set of assignments from the set of variables to the domain $\mathcal{D}$ while $\mathcal{N}$ would be the set of natural numbers; in this way, any update would have to handle the set of assignments as well as the anchor set $\mathcal{N}$. The second component, could be thought as a kind of anchor register, a register where the position of an anchored block is occurring at is recorded. Updates should then take care of these aspects and in this way the resulting state for all situations as the ones depicted above should be different. This seems to be specially relevant for coroutining blocks since the points of leaving a block and resuming "the same block" are relevant. But, of course, these are only rough ideas that need to be further investigated.

For the second case, the attachment point is thought of as indicating an underlying intention. If this is the case, then the impenetrable hypothesis should be somehow relaxed and a new intentional system along the lines proposed in section 8.4.3 would be an adequate tool to deal with this situation.

8.4.2 Multi-agent Discourses

Imagine, for example, a two person dialogue. An n-tuple of 2-tuples might suit this case. For each dimensional block, we might have distinctive updates for each participant, i.e., each sentence updates (*possibly in different ways*) each participant's information state. A typical information state would look like the one below.

$$s^n = \langle\langle sa^n_1, sb^n_1\rangle, \ldots, \langle sa^n_n, sb^n_n\rangle\rangle \tag{8.1}$$

The update of s^n with an atomic formula $Px_1 \ldots x_m$ in a dimensional index i would be characterized as

$$\langle\langle sa^n_1, sb^n_1\rangle, \ldots, \langle sa^n_n, sb^n_n\rangle\rangle[\![Px_1 \ldots x_m]\!]^i = \langle \ldots, \langle sa^n_i[\![Px_1 \ldots x_m]\!], sb^n_i[\![Px_1 \ldots x_m]\!]\rangle, \ldots\rangle$$

Analogously, we might straightforwardly transfer the results from DBPL to here. And, of course, the number of participants does not pose further concerns. All we have got to do is to extend the size of each tuple component of the multidimensional information state getting something looking like (8.2) below.

$$s^n = \langle\langle s_1a^n_1, \ldots, s_ma^n_1\rangle, \ldots, \langle s_1a^n_n, \ldots, s_ma^n_n\rangle\rangle \tag{8.2}$$

The multi-agent multidimensional information state proposed in (8.2) would be used to formalize the *common ground* concept. The common ground would be the intersection set of each $\langle s_1 a_i^n, \ldots, s_m a_i^n \rangle$ (where $1 \geq i \geq n$). The initial common ground configuration should correspond to $\langle s_1 a_1^n, \ldots, s_m a_1^n \rangle$. As we proceed evaluating the discourse, new configurations for the common ground will appear as updates of the previous configurations. Under this conception, every sentence would update (or, maybe, downdate) the common ground. And, maybe, some type of metric could be used to verify how big (or small) the disagreement becomes. Such a metric might trigger a *recovery procedure* in the hope of putting the discourse back in a "right direction." If, by some magic spell (or the like), all participants share the same initial 100% perfect matching common ground, then the model presented in (8.2) collapses to DBPL as the initial state would be the same and the semantic updates would follow the same pattern for all participants.

These conjectures pose us with interesting new possibilities and problems. I wonder what kind of system we might obtain by dropping: *(1) the sameness domain hypothesis*: the domain might be different not only for every discourse block component but also for every discourse participant. *(2) the impenetrable assumption*: what happens if we allow a cross-fertilization among blocks or even among participants?

The answers have to wait for the right time come.

8.4.3 Adding Intentionality

The multidimensional model of information states might be generalized in a way to deal with the intentional component of Grosz and Sidner (1986); similarly, rhetorical relations could be accounted for. The idea is that rhetorical relations (and intentions) would be update functions between dimensions. (So, the impenetrable hypothesis would not be valid any longer.) This move implies a further research development into the ontological nature of discourse components. But it seems clear that a new range of parenthetical $\blacktriangleright\blacktriangleleft$ functors would be necessarily one of the pillars for the theory to be developed.

At first glance, it seems that an intentional system would provide the answer to the points presented in previous paragraph. Lambda abstraction might work alongside the new class of $\blacktriangleright\blacktriangleleft$ functors (lambda abstraction would be the other pillar). Through lambda abstraction (seen as rhetorical relations) we would, for example, establish anaphoric links between pronouns and anaphoric constructions, occurring at an inner block, and their antecedent noun phrases, occurring at the outer parent block (see figure 5.2, page 95). For this example, the anaphoric *the cat* in *"The cat will never meet our other cat"* refers back to the American cat that inhabits the outer block. The link would be made through the application of $\lambda x.x[\![Cat]\!]$ to the initial configuration of the information state attached to the inner block. Lambda abstraction would therefore produce the intentional "transferring" process along the lines below.

$$\begin{aligned}
s^n[\![\blacktriangleright_{\lambda x.xP_i}\alpha_1 \bullet \ldots \bullet \alpha_n \bullet \blacktriangleright_{\lambda x.xP_j}\beta_1 \bullet \ldots \bullet \beta_m \blacktriangleleft \bullet \ldots \blacktriangleleft]\!] &= \\
&\langle s_1^n[\![\alpha_1 \bullet \ldots \bullet \alpha_n]\!]^{\lambda x.xP_i}, s_2^n[\![\beta_1 \bullet \ldots \bullet \beta_m]\!]^{\lambda x.xP_j}, \ldots\rangle \\
&= \langle(\lambda x.xP_i)s_1^n[\![\alpha_1 \bullet \ldots \bullet \alpha_n]\!], (\lambda x.xP_j)s_2^n[\![\beta_1 \bullet \ldots \bullet \beta_m]\!], \ldots\rangle \\
&= \langle(\lambda x.xP_i)s_1^n[\![\alpha_1]\!][\![\alpha_2]\!]\ldots[\![\alpha_n]\!], (\lambda x.xP_j)s_2^n[\![\beta_1]\!][\![\beta_2]\!]\ldots[\![\beta_m]\!], \ldots\rangle
\end{aligned}$$

Admitting that α_n has introduced $\exists xCatx$, the existential formula for the American cat, into the outer block, we have for granted that all subsequent uses of x, in the outer block, will be referring to the same unique cat. If we take $\lambda x.xP_j$, the rhetorical parameter for the second block, as $\lambda x.x[\![Cat]\!]$ then $(\lambda x.xP_j)s_2^n[\![\beta_1\bullet\ldots\bullet\beta_m]\!]$ will become $(\lambda x.x[\![Cat]\!].s_2^n)[\![\beta_1\bullet\ldots\bullet\beta_m]\!] = (s_2^n[\![Cat]\!])[\![\beta_1\bullet\ldots\bullet\beta_m]\!]$. Therefore, before we start processing the inner block, a new referent is already present in it. And, we would certainly deal with more complex phenomena along the same lines as the one just presented.

Note that $\lambda x.xP_i.s_1^n = s_1^nP_i$ where $s_1^nP_i$ is the update of s_1^n made by P_i (in a postfix notation). This formulation suggests that the intentional function P_i would work an update out only once and the updated state would serve as input state for the subsequent sentences in the block. Another possibility is to distribute the update throughout all subsequent sentences. This would look like

$$\begin{aligned}
& s^n\llbracket \blacktriangleright_{\lambda x.xP_i}\alpha_1 \bullet \ldots \bullet \alpha_n \bullet \blacktriangleright_{\lambda x.xP_j}\beta_1 \bullet \ldots \bullet \beta_m \blacktriangleleft \bullet \ldots \blacktriangleleft \rrbracket \;= \\
& \qquad \langle s_1^n\llbracket \alpha_1 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i}, s_2^n\llbracket \beta_1 \bullet \ldots \bullet \beta_m \rrbracket^{\lambda x.xP_j}, \ldots\rangle \\
= \;& \langle s_1^n\llbracket \alpha_1 \rrbracket^{\lambda x.xP_i}\llbracket \alpha_2 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i}, s_2^n\llbracket \beta_1 \rrbracket^{\lambda x.xP_j}\llbracket \beta_2 \bullet \ldots \bullet \beta_m \rrbracket^{\lambda x.xP_j}, \ldots\rangle \\
= \;& \langle (\lambda x.xP_i)s_1^n\llbracket \alpha_1 \rrbracket\llbracket \alpha_2 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i}, (\lambda x.xP_j)s_2^n\llbracket \beta_1 \rrbracket\llbracket \beta_2 \bullet \ldots \bullet \beta_m \rrbracket^{\lambda x.xP_j}, \ldots\rangle \\
= \;& \langle s_1^n P_i\llbracket \alpha_1 \rrbracket\llbracket \alpha_2 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i}, s_2^n P_j\llbracket \beta_1 \rrbracket\llbracket \beta_2 \bullet \ldots \bullet \beta_m \rrbracket^{\lambda x.xP_j}, \ldots\rangle
\end{aligned}$$

This latter formulation insists in keeping all updates processed so far re-updated by the intentional relations holding among discourse blocks. This is exemplified by the following close-up computation on the first component displayed on the penultimate line of the example above.

$$\begin{aligned}
(\lambda x.xP_i)s_1^n\llbracket \alpha_1 \rrbracket\llbracket \alpha_2 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i} &= s_1^n P_i\llbracket \alpha_1 \rrbracket\llbracket \alpha_2 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i} \\
&= (s_1^n P_i\llbracket \alpha_1 \rrbracket)\llbracket \alpha_2 \rrbracket^{\lambda x.xP_i}\llbracket \alpha_3 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i} \\
&= \lambda x.xP_i.(s_1^n P_i\llbracket \alpha_1 \rrbracket)\llbracket \alpha_2 \rrbracket\llbracket \alpha_3 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i} \\
&= ((s_1^n P_i\llbracket \alpha_1 \rrbracket)P_i)\llbracket \alpha_2 \rrbracket\llbracket \alpha_3 \bullet \ldots \bullet \alpha_n \rrbracket^{\lambda x.xP_i}
\end{aligned}$$

This would, for example, account for eliminate downdates that would eventually be produced previously. But these are only rough initial conjectures.

Once more, the dynamics of the multidimensional approach shows its powerful face.

8.5 A Final Word

DPL was the spaceship which took us to the new DBPL multi-dimensional world. The first steps into this world have been done in this thesis. To establish the geography of the new world is the task to be embraced by future generations.

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