Essays on Growth, Productivity and Public Capital

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ABSTRACT

This thesis collects theoretical and empirical work related to two fields of research. First, the literature on fiscal policy in endogenous growth models. Second, the empirics of productivity growth using macro-level applications of Data Envelopment Analysis.

As for the theoretical part of the research, attention has been paid to Barro-type models of endogenous growth driven by public investment. The endogenous growth model presented in Chapter 3 extends the Barro model to the case of finite lives. The main innovation is the conclusion that the assumed demographic structure affects both the level of long-run growth and the optimal provision rule of public capital.

The empirical part of the research deals with two applications of the DEA approach to the measurement of productivity growth to the case of Italian regions over the period 1970-95. Departing from existing literature on the Italian case, TFP growth is decomposed in technical efficiency change and technological progress in order to study the contribution of public infrastructure provision to both of them (section 5.3, Chapter 5). The second empirical contribution (section 5.4, Chapter 5) reconciles traditional approaches to the analysis of economic growth determinants and convergence patterns with the frontier productivity measurement literature. Efficiency change and technological progress are interpreted as proxies of catching-up and innovation respectively, in order to test the convergence hypothesis within Italian regions. It is concluded that Italian regions have diverged at a decreasing rate.

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DECLARATION

An earlier version of Chapter 3 has been circulated under the title *An Endogenous Growth Model with Productive Public Spending and Uncertain Lifetime Consumers* (Discussion Paper 03/10, Department of Economics and Related Studies, University of York). It has also been presented in revised form at the 57^{th} International Atlantic Economic Society Conference held in Lisbon (Portugal) in March 2004.

A preliminary version of the empirical study collected in section 5.3 of Chapter 5 has been presented at the 1^{st} Hellenic Workshop on Productivity and Efficiency Measurement held in Patras (Greece) in December 2001. The revised version entitled *Total Factor Productivity Growth and Public Capital: The Case of Italy* has appeared on Studi Economici (2002), 78(3):65-92 (Italian refereed journal).

The work collected in section 5.4 of Chapter 5 has been written jointly with Leone Leonida (Queen Mary University of London) and Luis Murillo-Zamorano (University of Uxtremadura, Spain). It has been published under the title *Total Factor Productivity and the Convergence Hypothesis in the Italian Regions* in Applied Economics (2004), 36(19):2187-2193.

1. INTRODUCTION

Original work presented in this thesis is related to two fields of research. First, the literature on fiscal policy in endogenous growth models. Second, the empirics of productivity growth using macro-level applications of Data Envelopment Analysis.

Chapters 2 and 3 collect work within the literature on fiscal policy in endogenous growth models. Chapter 2 is devoted to review the theory of the productive role of public capital. The first part of the Chapter focuses on the relationship between public capital, aggregate output and productivity. In particular, it is argued that public capital may contribute to aggregate output either as a direct unpaid input or as a productivity enhancing environmental variable. The second part of the review focuses on endogenous growth models dealing with public investment. Starting from the well known model of Barro (1990), recent contributions dealing with endogenous growth and public investment are grouped and discussed according to their main departures from the Barro model.

The research question under investigation in Chapter 3 is:

- How does the assumption of finite lives affect the Barro rule for the provision of public capital?

In order to provide an answer, I develop an endogenous growth model where sustained long-run growth is due to investment in public capital, the government provides lump-sum transfers, public consumption, and investment subsidies, and consumers have uncertain lifetimes. A flexible framework capable of analysing the growth effects of fiscal policy in both infinite and finite horizons cases is provided. The Barro rule is extended to the finite horizons case and it is shown to be dependent of the finite horizon index. In particular, the growth maximizing income tax rate is lower in the latter scenario and decreasing in the probability of death parameter. The growth hampering effect of unproductive public spending is depicted in the finite horizons as well as in the infinite horizons case. Increases in either public consumption or lump-sum transfers reduce long-run economic growth less in the former than in latter case. Furthermore, the relationships relating the growth maximizing level of public investment and each of the other categories of government expenditure are derived. Finally, an optimal rule for investment subsidies provision is analytically derived.

The empirical part of my research focuses on the Italian regions case over the period 1970-95 and it is presented in Chapter 5, in which I explore two issues in sections 5.3 and 5.4, respectively:

- The empirical study of public capital as a productivity enhancing externality to regional economics;
- Testing of the catching-up hypothesis across Italian regions, shedding some insight on the determinants of convergence/divergence patterns.

In Italy, the provision of productive infrastructures has historically been a policy instrument aimed at reducing the development gap between northern and southern regions. Indeed, since the 1950s government policy has driven public investment towards the South which, however, is still lacking in terms of per capita GDP with respect to the regions in the North of the country. Moreover, despite the huge amount of resources devoted to the provision of new infrastructure services, the South is still characterized by a lower endowment of infrastructures, such as transport and communication networks, compared to the North of the country. This is the case of public works which have either been completed with strong delay, have not been completed at all or have ended up with under-utilized structures because of a failure to correspond to the actual needs of a specific area.

Table 1.0.1: Structural Fund Commitments in Objective 1 regions (% of nominal values)

Country	1989-93				1994-99			
	A	B	H	I	A	B	H	I
Austria	-	-	-	-	15.0	68.7	16.3	0.0
Belgium	-	-	-	-	0.0	66.2	17.2	16.6
France	28.6	15.9	10.1	45.4	9.6	32.7	18.7	39.0
Greece	11.2	18.4	16.6	53.8	18.7	13.4	13.6	54.3
Ireland	14.7	33.7	26.4	25.2	0.0	54.7	3.9	41.4
Italy	14.4	35.0	1.9	48.8	21.0	21.3	27.0	30.7
Netherlands	-	-	-	-	22.2	20.4	21.0	36.4
Portugal	11.5	6.1	35.3	47.2	0.0	15.2	8.6	76.1
Spain	26.7	13.2	8.8	51.4	0.6	14.3	7.5	77.6
U.K.	10.5	38.1	20.9	30.4	12.2	25.0	33.1	29.7
Total	17.6	21.1	16.3	45.0	7.0	24.0	12.1	56.8

Source: Rodríguez-Pose and Fratesi (2003). Notes: A = Support to agriculture and rural promotion; B = Business and tourism support; H = Investment in education, re-qualification and all measures targeting the human capital of the region; I = Investment in infrastructure, transport and environment.

A further motivation for this study is the relevance attributed by the policy maker to infrastructure investment in less developed regions within the European Union. Table 1.0.1 (from Rodríguez-Pose and Fratesi, 2003) gives a measure of such emphasis. In the Table, resources allotted by the Community Support Framework to Objective 1 Regions during the first two periods of implementation (1989-93 and 1994-99) are disaggregated with respect to their allocation to four priority axes: 1) Support to agriculture and rural promotion (A); 2) Business and tourism support (B); 3) Investment in education, re-qualification and all measures targeting the human capital of the region (H); 4) Investment in infrastructure, transport and environment (I). The figures clearly show that a consisted share of the resources are devoted to the implementation of infrastructure projects.

In the light of the emphasis put by the European policy maker on policies oriented to the improvement of infrastructure endowment in less developed areas of the Union, empirical support to the thesis of the productive enhancing role played by public capital has great policy implications.

Chapter 5 contributes to the debate on the relationship between public capital and productivity in the Italian regions (see section 5.3). Previous works on Italy have mainly exploited the *production function* approach (Picci, 1999), the *growth accounting* approach (La Ferrara *et al.*, 2000) and the *growth* approach (Acconcia and Del Monte, 2000). These recent contributions have enhanced the literature on the topic, allowing for taking into account heterogeneity across Italian regions by means of panel data estimation techniques. However, I argue that their conclusions can suffer from an important shortcoming: they do not take account of the *inefficiency issue.* This is due to the implicit assumption that the production process is fully efficient, which implies that the estimates of average production functions will be biased in the presence of inefficiency. Furthermore, if such an assumption does not hold, total factor productivity growth will be identified with technological progress, while another source of productivity growth — technical efficiency change — will be neglected (Grosskopf, 1993).

The main novelty of the present contribution rests on the decomposition of productivity growth into technical efficiency change and technological progress by implementing a non-parametric frontier approach to the measurement of productivity. The empirical analysis proceeds in two parts. In the first part, a DEA model is implemented under the two alternative assumptions of two (labour and private capital) and three (labour, private and public capital) inputs. A test of the significance of public capital as an additional input in the DEA model does not support the view of public capital affecting aggregate output as a direct unpaid factor. In the second part, an econometric analysis of the linkage between productivity gains and the provision of public capital is performed with the aim of assessing the role of public capital as a positive environmental variable. The results show the positive impact of public capital on both technical efficiency change and technological progress, especially in the southern regions. In view of these results, policies aimed at increasing the endowment of infrastructure services maintain their relevance despite the fact that public capital is not a statistically significant direct input in aggregate production.

Most of existing applied work on Italian regions agree on the finding of conditional convergence: Italian regions tend to converge to different steady state levels of per capita GDP. A less investigated topic remains the issue of the determinants of such convergence/divergence pattern. Based on the results obtained in the first part of the Chapter, section 5.4 is aimed at reconciling traditional approaches to the analysis of economic growth determinants and convergence patterns with the frontier productivity measurement literature. The empirical estimation process is developed in two steps. The first involves the decomposition of TFP growth on the basis of considering GDP as output, and capital and labour as the relevant productive inputs. In the second one, the convergence issue is analysed by means of panel data estimation techniques. Estimated technological progress and technical efficiency change are interpreted, respectively, as innovation and catching-up measurements and the catching-up hypothesis is tested for the Italian regions. The analysis leads to a conclusion that regional economies diverge at a decreasing rate.

Chapter 6 reports concluding remarks and some ideas for future research.

2. MODELS OF PUBLIC CAPITAL AND GROWTH: A LITERATURE REVIEW

2.1 Introduction

This Chapter does not aim at exhaustively reviewing theoretical models incorporating public investment for productive uses. The aim is rather to introduce the aspects of the literature most related to the model presented in Chapter 3. In particular, more emphasis will be placed on the expenditure side of fiscal policy, although I will deal to some extent also with the problems related to different systems of taxation. On the other hand, the issue of government deficits will not be covered.

Section 2.2 focuses on the relationship between public capital, aggregate output and productivity. In particular, it is argued that public capital may contribute to aggregate output either as a direct unpaid input or as a productivity enhancing environmental variable.

The overview of the theory on public capital and long-run growth follows three steps. First, the main features of the neoclassical view on fiscal policy and growth are introduced in section 2.4. Section 2.5 deals with the endogenous growth model developed by Barro in 1990, regarded as the path-breaking paper in the literature. Some recent works based on Barro (1990) are grouped in section 2.6 according to their main departures from the original model. In particular, subsection 2.6.1 reviews models extending Barro (1990) on the production side, with particular focus on the works by Futagami *et al.* (1993) and Barro and Sala-i-Martin (1992, 1999). Subsection 2.6.2 deals with extensions to the Barro model interested in analyzing the composition of public expenditure (de la Fuente, 1997; Greiner, 1999). Finally, in subsection 2.6.3, I focus on the model by Mourmouras and Lee (1999), that modifies the consumption side of Barro (1990) and in section 2.7 it will be introduced the motivation behind the model presented in Chapter 3.

2.2 Public Capital, Output and Productivity

The public provision of productive services (infrastructures) may affect economic activity through different channels, which can be studied starting from the following aggregate production function[1]:

$$Y_t = A_t(G_t) \cdot f(K_t, L_t, G_t) \tag{2.2.1}$$

where, at each moment in time t, Y_t is a measure of real aggregate output of the private sector, K_t is the aggregate stock of private capital, L_t represents aggregate employment, G_t is the aggregate stock of public capital and A_t is a measure of the level of technology. Given this general specification of the aggregate technology, the effects of G_t on output and productivity can be studied under two alternative assumptions:

(a) public services represent a direct input of production and influence both production *directly* and productivity *indirectly*;

(b) public services influence productivity indirectly without entering the aggregate production function as a direct input, but rather being a source of externalities.

The debate on whether (a) or (b) is the prevailing avenue through which public capital is related to economic activity is not new, dating back to

[1] Examples of public infrastructures include highways, airport, harbors, communication networks, electric and gas facilities.

Meade (1952) who analysed both possibilities. More recently, the two alternative views on the productive role of public capital can be found in the works by Aschauer (1989a) and Hulten and Schwab (1984, 1991) respectively. These works provide the theoretical background on which is based the wide empirical literature on the impact of public capital on output and productivity growth following the so-called *production function* and *growth accounting* approaches. This section deals with the key features of such theory, while the empirical literature on the productive role of public capital will be reviewed in Chapter 4.

Aschauer (1989a), assumes that G_t enters the aggregate production function as a direct input:

$$Y_t = A_t \cdot f(K_t, L_t, G_t) \tag{2.2.2}$$

Under this assumption, public capital contributes to output:

(1) *directly*;

(2) *indirectly* by enhancing the productivity of other inputs of production.

The *direct* impact (1) will depend on whether or not the marginal product of G_t is positive and, as a consequence, its output elasticity is positive as well. Examples of categories of public capital which are likely to directly contribute to national output as productive inputs of the private economy are all those belonging to the transport and communication network of a country. For instance, roads, highways, ports and all other public transport facilities can be thought as productive inputs of private providers of transport services.

On the other hand, the *indirect* effect (2) will actually arise only if private and public inputs are in a relationship of complementarity, in the sense that $Y_{LG} > 0$ and/or $Y_{KG} > 0$, where Y_K and Y_L are marginal products of capital and labour respectively. For instance, thinking about the simple example of the national transport network, a higher endowment of highways is likely to be positively linked to the productivity of drivers and trucks employed by firms that provide transport services.

Under the assumption that public capital enters the aggregate production function as in (2.2.2) — in Cobb-Douglas form — the effects (1) and (2) can be evaluated using two alternative measures of productivity. Hence, behind both these potential effects there is the assumption that public capital is a direct factor of production and, once we make such assumption, one effect is implied by the other as for any other direct productive input. Indeed, Aschauer (1989a) does not explicitly model the effect of government intervention in enhancing total factor productivity using some specification of $A_t(G_t)$. However, he derives the relationship relating public capital to total factor productivity implied by the assumption that G_t is a direct factor of production. He assumes that the government provides the private sector of the economy with a flow of services free of user charge. If such flow is proportional to the national infrastructure network, it will be equivalent to the stock of public capital G_t. For a Cobb-Douglas functional form of (2.2.2), Aschauer obtains the following logarithmic version of aggregate production:

$$\ln(Y_t) = \ln(A_t) + \alpha \cdot \ln(K_t) + \beta \cdot \ln(L_t) + \gamma \cdot \ln(G_t) \qquad (2.2.3)$$

where α, β and γ are elasticity values of private capital, labour and public capital respectively and can be empirically estimated. The factors of production are exogenous[2] and it is implicitly assumed that they are paid their respective marginal products. It is important to notice that a further restriction imposed by the Cobb-Douglas specification in (2.2.3) is that the substitution elasticities of the production inputs are equal to one by definition.

Once one assumes the public provision of productive services with no user charge, it is necessary to make clear how the government finances such

[2] In the sense that aggregate output is related to available inputs by a unilateral relationship: Y is produced for given levels of K, L and G.

a provision. In the easiest case, it is possible to assume a one sector economy where the government purchases a share of total output and uses it to provide these services to the private sector. The basic assumption in Aschauer's analysis is that the provision of public services G_t is financed by income taxation at the flat rate τ, which equates the public capital-output ratio at each moment in time: $\tau = G_t/Y_t$. The rationale behind the public provision of productive services is that — given their nature of pure public goods — it would be difficult to allocate resources to their most efficient uses in any private market for G_t.

Further to the basic assumption of public provision with no user charge, Aschauer considers the two cases of (i) *public production* of G_t and (ii) *congestion* in the usage of G_t. These two cases coincide with two alternative restrictions made on returns to scale in aggregate production: (i) constant returns to scale (CRTS from now on) to private inputs and (ii) CRTS to all factors of production.

(i) Starting from the first case, Aschauer argues that if the existence of significant economies of scale in the *public production* of G_t is considered plausible, then the aggregate production function could show CRTS to private inputs L_t and K_t and increasing returns to scale to L_t, K_t and G_t together (*i.e.*, $\alpha + \beta = 1$ and $\alpha + \beta + \gamma > 1$). It is obvious that this way of modelling the public provision of G_t implies a theoretical framework more elaborated than a simple one sector economy. For instance, the assumption of increasing returns to scale could be due to the fact that G_t is produced by the government. Hence, we should assume a two sectors economy, with a public sector using its own capital and labour force to produce G_t. In the real world, this is the case of the distribution of water, electricity and other public utilities for which larger levels of production imply decreasing costs and the most efficient form of production are natural monopolies.

Under the assumption of CRTS in L_t and K_t, profit maximization in perfect competition implies that private inputs earn exactly their marginal products. Hence, the two corresponding measures of productivity derived from (2.2.3) will be:

$$\ln\left(\frac{Y_t}{K_t}\right) = \ln(A_t) + \beta \cdot \ln\left(\frac{L_t}{K_t}\right) + \gamma \cdot \ln(G_t) \qquad (2.2.4)$$

and

$$tfp_t = \ln(Y_t) - s_K \cdot \ln(K_t) - s_L \cdot \ln(L_t) \rightarrow tfp_t = \ln(A_t) + \gamma \cdot \ln(G_t) \quad (2.2.5)$$

where s_K and s_L are output shares of K_t and L_t respectively; (2.2.4) relates output per unit of capital to the labour-capital ratio and the *absolute level* of public capital[3], whereas (2.2.5) expresses the increasing relationship between total factor productivity and the *absolute level* of public capital.

(ii) The alternative restriction on returns to scale is based on the presence of *congestion* effects in the usage of G_t. This will make the assumption of increasing returns to scale less attractive, leading to prefer the assumption of CRTS to all factors of production, which will imply decreasing returns to scale over private inputs (*i.e.*, $\alpha + \beta < 1$ and $\alpha + \beta + \gamma = 1$). Since public capital is freely available to producers, if the aggregate production function exhibits CRTS to all inputs, the output will not vanish when L_t and K_t are paid their marginal products. Hence, it is necessary to make some hypothesis on the distribution of the rents for public services amongst producers. In particular, Aschauer assumes that the output shares of L_t and K_t — s_L and s_K — are proportional to their respective marginal productivity values: $s_L = \theta \cdot \beta$ and $s_K = \theta \cdot \alpha$, with $\theta > 1$. If this is true, then the following measures of productivity can be derived:

$$\ln\left(\frac{Y_t}{K_t}\right) = \ln(A_t) + \beta \cdot \ln\left(\frac{L_t}{K_t}\right) + \gamma \cdot \ln\left(\frac{G_t}{K_t}\right) \qquad (2.2.6)$$

[3] We should refer to the flow of government services but, under the assumption that these are proportional to public capital, the two definitions are equivalent.

and

$$tfp_t = \ln(A_t) + \gamma \cdot \ln(G_t - I_t) \tag{2.2.7}$$

where $I_t = s_K \cdot \ln(K_t) + s_L \cdot \ln(L_t)$ and equations (2.2.4) and (2.2.5) are special cases of (2.2.6) and (2.2.7), respectively.

So far we have regarded G_t as a direct input in (2.2.1), however, as we said at the beginning of this section — see point (b), page 8 — public capital can play the role of an externality. In this case G_t contributes

(3) *indirectly* to economic performance by enhancing total factor productivity without being a direct input of production:

$$Y_t = A_t(G_t) \cdot f(K_t, L_t) \tag{2.2.8}$$

In this case, the productive role of public capital is defined as an *external* effect on total factor productivity. This view goes back to the work by Meade (1952) and it has been adopted by Hulten and Schwab (1984, 1991).

According to this view, public capital may act like an *environmental* factor which enhances the productivity of productive inputs like an externality in the sense of Romer (1986) and Lucas (1988).

To see how G_t could affect private output via A_t, let us first totally differentiate (2.2.8) under the assumption that A_t is not a function of G_t:

$$\dot{Y}_t = \dot{A}_t + \alpha \cdot \dot{K}_t + \beta \cdot \dot{L}_t \tag{2.2.9}$$

where dots over variables indicate time derivatives, $\alpha = A \cdot (Y_K K/Y)$ and $\beta = A \cdot (Y_L L/Y)$ are output elasticity values[4]. Thus, the rate of growth of output is given by the rate of growth of the technological factor plus the weighted sum of the growth rates of labour and capital inputs, where the weights are their respective output elasticity values. Rearranging (2.2.9)

[4] Y_K and Y_L are partial derivatives of Y with respect to K and L respectively.

properly, we obtain the following measure of total factor productivity (TFP) growth (Solow, 1957):

$$\dot{A}_t = \dot{Y}_t - \alpha \cdot \dot{K}_t - \beta \cdot \dot{L}_t \tag{2.2.10}$$

Since $\dot{A}_t$ is computed as a residual, this measure of TFP growth will capture any sources of bias due to errors in measurements and/or the omission of relevant factors of production. For instance, if one thinks that G_t should enter (2.2.8) as a direct input, the measurement of $\dot{A}_t$ in (2.2.10) will suffer from the bias due to the omission of G_t[5].

Let us now assume that public capital is an environmental factor affecting TFP growth as defined in (2.2.8). Under this assumption, (2.2.10) becomes:

$$\dot{Y}_t = \dot{A}_t + \underbrace{A_t(G_t) \cdot (Y_K K_t/Y_t)}_{\alpha} \cdot \dot{K}_t + \underbrace{A_t(G_t) \cdot (Y_L L_t/Y_t)}_{\beta} \cdot \dot{L}_t \tag{2.2.11}$$

where a rise in public capital yields a proportional change in the marginal products of both private inputs and, as a consequence, raises both output elasticity values α and β.

Since we are not assuming that G_t is a factor of production, the rate of growth of the technological factor A_t is still correctly defined by equation (2.2.10). However, total factor productivity is now assumed to depend on G_t and, for $A_t(G_t) = A_t G_t^{\gamma_A}$, it will be defined by two parts:

$$\dot{A}_t = \gamma_A \cdot \dot{G}_t + \hat{\dot{A}}_t \tag{2.2.12}$$

where γ now indicates the elasticity of the measured residual with respect to public capital and $\hat{\dot{A}}_t$ is the growth rate of the "true" shift in the production function. It is clear that the relationship in (2.2.12) is equivalent to the indirect effect of public capital on total factor productivity as derived by Aschauer in (2.2.5) expressed in terms of growth rates.

[5] Since the elasticity values are not directly observed, this equation is not adaptable to the empirical analysis. However, equation (2.2.10) becomes a suitable empirical specification under the assumption that private inputs are paid their respective marginal products.

2.3 Public Capital and Long-run Economic Growth

Regardless of which of the effects (1)-(3) — see pages 9 and 13 — is the prevailing one, the fact that the provision of public capital enhances economic activity through one of these channels does not imply the existence of a positive relationship between public investment in infrastructures and long-run economic growth. In general, the latter will arise depending on whether a permanent increase in public investment yields a permanent (Barro, 1990) or simply a temporary (Solow, 1956) effect on economic growth in the long-run. The latter is the conclusion reached within *exogenous growth* models *à la* Solow described below in section 2.4.

The standard neoclassical prediction is that a permanent change in the saving rate has solely a transitory effect on economic growth. Indeed, long-run growth is only determined by population growth and exogenous technological progress. Under the assumption of decreasing returns to private capital, an increase in the provision of public capital will only temporarily induce higher investment. Therefore, in the long-run the level of output will be higher but its rate of growth will not be affected by the increase in public investment.

Abandoning the basic neoclassical assumption that production solely requires direct investment in physical capital and labour, the literature on *endogenous growth* or *new growth* has placed emphasis on other forms of capital provided through private investment in research, knowledge and human capital or public investment in infrastructure. As a consequence, it has been possible to relax the assumption of diminishing returns to capital in favor of the hypothesis of constant (or increasing) returns to scale in the accumulating factors of production. The important implication is that the long-run rate of growth of the economy becomes dependent on factors endogenously determined in the model, rather than being merely determined by exogenous variables such as population growth and technological change as in Solow-type models.

Fiscal policy (changes in the structure of taxation and public expenditure) has no permanent impact on long-run growth in the standard neoclassical model. On the other hand, the long-run rate of growth is dependent on the investment decisions endogenously determined in endogenous growth models and, as a consequence, fiscal policy can influence the incentive to invest in private capital, permanently affecting long-run growth.

As for endogenous growth models dealing with public investment in infrastructure, the seminal contribution is provided by Barro (1990), who developed an endogenous growth model in which publicly provided productive services are the source of sustained long-run growth. This model will be discussed in section 2.5.

However, although under certain conditions public investment can positively affect economic growth, more sceptical economists argue that increasing public expenditure will generally tend to *crowd out* private investment by reducing private disposable income and incentive to save, leading to a reduction in the level of productivity. This view gives particular relevance to the *negative externality* effects arising from various kinds of distortions induced by government intervention in private markets as a buyer, seller or regulator (de la Fuente, 1997). It is argued, for instance, that the government activity as a regulator may affect the efficient allocation of resources in private markets. In general, the same type of inefficiency is considered to be caused by all the categories of government expenditure financed by distortionary taxation.

2.4 Neoclassical View on Fiscal Policy and Growth

The standard neoclassical model of growth was developed during the late 1950s and the early 1960s and became in the following years the starting point of the debate on the determinants of growth. This section introduces the standard model proposed by Solow (1956) and Swan (1956) showing the ineffectiveness of fiscal policy on the long-run rate of growth. On the supply side of the model, the homogeneous good Y_t is produced through the following aggregate production function:

$$Y_t = F(K_t, A_t L_t) \tag{2.4.1}$$

where A_t is the labour-augmenting (Harrod-neutral) technological factor, L_t is labour factor and K_t is capital input. Both A_t and L_t are assumed to grow at a constant exogenous rate: $A_t = A_0 e^{\gamma t}$, $L_t = L_0 e^{\eta t}$. It is assumed that the function F is twice differentiable and linearly homogeneous, *i.e.*, the production of Y_t exhibits CRTS in K_t and L_t. Furthermore, both L_t and K_t are assumed to have positive but declining marginal products. Under these assumptions it is possible to express Y_t in effective units of labour, $y_t \equiv Y_t / A_t L_t$:

$$\frac{1}{A_t L_t} \cdot F(K_t, A_t L_t) = F\left(K_t / A_t L_t, 1\right) = f(k_t) \tag{2.4.2}$$

where $f(k_t)$, defined as the production function in intensive form, is assumed to satisfy the so-called Inada conditions — $f(0) = 0$, $\lim_{k_t \to \infty} f'(k_t) = 0$ and $\lim_{k_t \to 0} f'(k_t) = \infty$ — and k_t has a positive but declining marginal product, with $f'(k_t) > 0$ and $f''(k_t) < 0$.

On the demand side, the economy is closed and without government. Total saving is assumed to be the constant fraction s of total income (that equates total output Y_t): $\dot{K}_t = sY_t - \delta K_t$, where δ is the depreciation rate of capital.

The above set-up implies the well-known fundamental differential equation of the Solow model, which states that the rate of change of the capital stock (in units of effective labour) is given by the difference between actual investment $sf(k)$ and the so-called *break even* investment $(\eta + \gamma + \delta)k$, that is to say, the investment that must be done in order to prevent k from falling over time[6]:

$$\dot{k} = sf(k) - (\eta + \gamma + \delta)k \tag{2.4.3}$$

According to this differential equation, k rises or falls over time whether $sf(k)$ is greater or smaller than $(\eta + \gamma + \delta)k$. Only if investment exactly equates the resources needed to prevent capital from falling, k will be constant overtime at the value k^*. Regardless of the initial level of k — provided that the Inada conditions and the law of diminishing returns to capital are satisfied and that $k_0 > 0$ — the economy will always converge to its balanced growth path (BGP from now on) where k is constant at the value k^* and all variables grow at a constant rate.

The unique and stable BGP is obtained by setting (2.4.3) equal to zero. The solution to the model gives the value k^* at which all variables are constant and the economy grows at the exogenously given rate $(\eta + \gamma)$. On the BGP, since L and A grow at rates η and γ respectively, the capital stock K grows at the rate $\eta + \gamma$[7]. This implies that — under the assumption of CRTS — total output also grows at the rate $\eta + \gamma$. Output per capita Y/L is equal to $Af(k^*)$ on the BGP, where k^* is constant and A grows at the rate γ. Hence, long-run per capita growth is driven by the exogenous and constant rate of the technological progress, independently of the saving ratio s.

Within this framework, any government policy aimed at influencing the private decisions on the allocation of resources between consumption and investment, will not have any permanent effect on long-run growth. Indeed,

[6] Two assumptions of the model make necessary the *break even* investment in order to keep k constant over time: 1) capital depreciates at the constant rate δ and, as a consequence, in each period the fraction δ of the capital stock K must be replaced in order to keep k constant; 2) the quantity of effective labour AL grows at the constant rate $\eta + \gamma$, which implies that the capital stock K must grow at the same rate in order to keep k constant.

[7] This is due to the fact that $K = ALk^*$, where k^* is constant, hence, $\dot{K}/K = \eta + \gamma$.

an increase in the saving rate s has a *level* effect, but not a *growth* effect on output. This means that a change in s influences the level of per capita output, leaving unchanged its growth rate on the BGP.

As we said above, this basic model does not assume the presence of a government. However, we can analyse the steady state effects of fiscal policy changes, assuming that the saving rate s is the parameter most likely to be influenced by changes in public expenditure and/or taxation (Romer, 1996).

For instance, let us consider a permanent increase in government expenditure in enhancing the national transport network. Under the assumption that the availability of a higher endowment of transport infrastructure presumably increases the marginal product of k, this policy will cause a permanent increase in the saving rate. As a consequence, actual investment will be higher in (2.4.3), exceeding the level of break even investment, and k will start growing ($\dot{k} > 0$) from its initial steady state value k^*. While k is increasing, per capita output $Y/L = Af(k)$ will increase at a higher rate than its balanced rate of growth γ. However, k will rise until actual investment equates again break even investment on a higher BGP. The new steady state value of capital k^{**} will be higher than the initial one, but the rate of growth of per capita output will be again determined by γ. This example shows the null impact of increases in public expenditure on long-run economic growth in Solow-type models.

2.5 The Barro Model

Barro (1990) incorporates a public sector in a constant-returns model of endogenous growth. As pointed out by the author, the model is based on two aspects of the *new growth* literature: models in which private returns to scale may be diminishing but social returns to scale can be constant or increasing, reflecting spillovers of knowledge or other externalities (Romer, 1986), and models without externalities but constant returns to private inputs of production (Rebelo, 1991).

The Barro model leaves room for a new macroeconomic role of public investment, challenging the neoclassical wisdom that fiscal policy has no permanent impact on the long-run rate of growth. Although the most famous result of this model is the derivation of an optimal rule for the provision of productive public spending (the so-called *Barro Rule*), Barro provides many other insights in the debate on the relevance of taxes and government spending on economic growth. In particular, the model considers the cases of distortionary and non-distortionary taxation and the two types of productive (infrastructure spending) and unproductive (public consumption) expenditure, deriving the growth effects of alternative combinations of taxes and expenditure.

2.5.1 Decentralized Economy and Income Taxation

A representative, infinitely lived household maximizes her lifetime utility in a closed economy, given a constant relative risk aversion (CRRA) instantaneous utility function:

$$U = \int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{2.5.1}$$

where $\sigma > 0$, $\rho > 0$ is the constant rate of time preference and marginal utility has a constant elasticity, $cu''(c)/u'(c) = -\sigma$. Population is constant, there is no labour-leisure choice and each household-producer has access to the same technology $y = f(k)$, where both output y and capital k are expressed in per capita terms. Under these assumptions, the solution to the lifetime utility maximization problem yields the following Euler equation:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (f' - \rho) \tag{2.5.2}$$

where f' is the marginal product of capital. Assuming CRTS to a broad concept of private capital (including both human and physical capital), the individual production function can be written as in a standard Ak model:

$$y = Ak \tag{2.5.3}$$

where $A > 0$ stands for the constant net marginal product of capital. Substituting $f' = A$ into (2.5.2), Barro obtains the following expression for the long-run per capita growth rate[8]:

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (A - \rho) \tag{2.5.4}$$

The economy is always at a position of steady state growth in which per capita consumption, capital and output grow at the same rate γ shown in (2.5.4). Given the initial value of capital stock, $k(0)$, the levels of c and y are also determined, with:

$$c(0) = k(0) \cdot (A - \gamma) \tag{2.5.5}$$

Within this framework, the role of the public sector is modelled extending (2.5.3) in order to include an additional input of production. The government is supposed to provide a flow of public services g, made available to each household-producer in the economy, with neither user charge nor congestion effects. Hence, the production function in (2.5.3) is modified accordingly:

$$y = \Phi(k, g) \Rightarrow y = k \cdot \Phi\left(\frac{g}{k}\right) \tag{2.5.6}$$

with $\Phi' > 0$ and $\Phi'' < 0$. Production is characterized by CRTS in k and g taken together and decreasing returns to scale in k separately. The intuition behind the extension of the production function in order to include the flow of public services is that private and public inputs are complementary or, at least, they are not close substitutes. Hence, private capital (even if it includes both physical and human capital) will show diminishing returns, if

[8] If the inequalities $A > \rho > A(1 - \sigma)$ hold, then the steady state rate of growth is positive — since $A > \rho$ implies $\gamma > 0$ in (2.5.4) — and utility is bounded.

the government does not increase the provision of g in a parallel manner. For a Cobb-Douglas production function, (2.5.6) becomes:

$$y = Ak^{1-\alpha}g^{\alpha} \tag{2.5.7}$$

where $0 < \alpha < 1$. The government does not own capital in this model and, as a consequence, does not contribute to production in the private sector of the economy. Indeed, the publicly provided services g appearing in (2.5.6) and (2.5.7) are purchased by the government as a share of the output produced in the economy and made available free of charge to the representative household-producer. Hence, to keep the analysis as simple as possible, such public services are assumed to have the nature of a pure public good: g is a non-rival and non-excludable good and represents the *total* amount of government purchases.

The government is assumed to run a balanced budget constraint at each moment in time:

$$g = T = \tau \cdot y = \tau \cdot k \cdot \Phi\left(\frac{g}{k}\right) \tag{2.5.8}$$

where total expenditure g equates total revenue T collected by levying a flat-rate income tax τ. Due to the absence of congestion effects, each producer's choice on output does not influence the amount of g that he can use. Hence, given the production function in (2.5.6), the after-tax marginal product of capital is computed by letting vary k, holding g fixed:

$$\frac{\partial y}{\partial k} = (1-\tau) \cdot \Phi\left(\frac{g}{k}\right) \cdot \left(1 - \Phi' \cdot \frac{g}{y}\right) = (1-\tau) \cdot \Phi\left(\frac{g}{k}\right) \cdot (1-\eta) \tag{2.5.9}$$

where $0 < \eta < 1$ is the output elasticity of g.

Since the provision of g is not assumed to affect overall utility, the inclusion of the public sector in the model leaves unchanged the Euler equation in (2.5.2). However, the production side of the model changes for the presence of the public input and the long-run rate of growth in (2.5.4) has to be modified in order to take into account the new marginal product of k shown in (2.5.9):

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[\underbrace{(1-\tau) \cdot \Phi\left(\frac{g}{k}\right) \cdot (1-\eta)}_{(\partial y/\partial k)} - \rho \right] \tag{2.5.10}$$

The dynamics of the model do not change after the inclusion of the public sector. The economy is always at the position of steady state, where c, k and y grow at the same constant rate γ, and for a given initial value of capital stock — $k(0)$ — c and y are also determined, with:

$$c(0) = k(0) \cdot \left[(1-\tau) \cdot \Phi\left(\frac{g}{k}\right) - \gamma)\right] \tag{2.5.11}$$

The main result of the model is contained in (2.5.10) and can be summarized as follows. Different sizes of the government are represented by different values of $\tau = g/y$. Letting vary the income tax rate τ — or equivalently the share of private output purchased by the government g/y — the growth rate is affected in two directions.

A rise in g/y implies that the government makes available a higher level of productive services. Under the assumption that these services are positively linked to the marginal product of capital, there will be an increase in the incentive to save, with a beneficial effect on long-run growth[9]. However, the provision of productive services is financed by the income tax and — provided that the government runs a balanced budget constraint — τ will also increase, contracting disposable income, reducing private investment and, as a consequence, hampering growth.

Thus, the net effect of public investment on long-run growth is ambiguous and will be positive or negative whether or not the growth enhancing effect due to higher productive services will dominate the growth hampering effect caused by higher taxation. The first effect prevails when the government is small, whereas the second will dominate when the government is large.

[9] See the positive effect of $\partial y/\partial k$ on γ in (2.5.10).

Hence, the long-run rate of growth is increasing in τ up to a point, reaching a maximum at that point, after which it declines for further increases in τ.

This analysis implies the existence of a growth maximizing level of income taxation τ_{max}, where the value of τ_{max} depends on the level at which the two effects discussed above offset one each other.

For the production function (2.5.6), it is complicated to study the net effect on γ because η is a function of g/k in (2.5.10). Hence, τ_{max} is not a constant and it becomes difficult to define the growth maximizing level of taxation. However, for the easy case of the Cobb-Douglas production function (2.5.7), the output elasticity of g is constant by definition and equal to α. When η is constant, the derivative of γ with respect to g/y is given by:

$$\frac{\partial \gamma}{\partial (g/y)} = \frac{1}{\sigma} \cdot \Phi\left(\frac{g}{k}\right) \cdot (\Phi' - 1) \tag{2.5.12}$$

Thus, γ will be increasing or decreasing in g/y whether or not Φ' is greater or smaller than 1 and long-run growth will be maximized for $\Phi' = 1$. Since $\alpha = \eta = \Phi' \cdot \; g/y$, it follows that:

$$\tau_{max} = \alpha = \frac{g}{y} \tag{2.5.13}$$

This latter result is known in the literature as the *Barro Rule*, which states that it is optimal for the government to set its share of national output equal to the share that it would get if public services were a competitively supplied input of production[10].

2.5.2 Central Planner Solution and Consumption Taxation

Barro defines the result obtained in the previous section as the solution to a second-best policy problem. Indeed, he recognizes that his model suffers

[10] A further conclusion regards the fact that τ_{max} turns out to be also the welfare maximizing level of taxation: for a benevolent central planner the maximization of households' lifetime utility is equivalent to the maximization of long-run economic growth.

from the standard externalities implied by public expenditure and taxation. Thus, in the following step of his analysis, he compares the decentralized outcome of the model with the solution to the corresponding planning problem in order to assess the relevance of such external effects. In the central planner scenario the rate of growth of consumption is derived as:

$$\gamma_P = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[\left(1 - \frac{g}{y}\right) \cdot \Phi\left(\frac{g}{k}\right) - \rho \right] \tag{2.5.14}$$

and the derivative of γ_P with respect to g/y is given by:

$$\frac{\partial \gamma_P}{\partial (g/y)} = \frac{\Phi(g/k) \cdot (\Phi' - 1)}{\sigma(1 - \eta)} \tag{2.5.15}$$

where η is defined between 0 and 1 and, as a consequence, the efficiency condition $\Phi' = 1$ — which ensures that g/y is fixed at its growth maximizing level — holds *regardless of the functional form of the production function.*

Let us now compare the long-run growth rate in the decentralized economy γ in (2.5.10) with the one of the central planner case γ_P in (2.5.14). In (2.5.10), the term in brackets and to the left of the minus sign is:

$$(1 - \tau) \cdot \Phi\left(\frac{g}{k}\right) \cdot (1 - \eta) = \textit{private} \text{ marginal return on capital;} \tag{2.5.16}$$

whereas the corresponding term in (2.5.14) is given by:

$$\left(1 - \frac{g}{y}\right) \cdot \Phi\left(\frac{g}{k}\right) = \textit{social} \text{ marginal return on capital.} \tag{2.5.17}$$

Thus, given a proportional tax $\tau = g/y$, decentralized and command solutions differ for the term $(1 - \eta)$ and $\gamma_P > \gamma$ for any value of τ. The presence of the income tax causes the decentralized economy to have lower long-run growth with respect to the central planned economy (see the lowest curve in Figure 2.5.1).

In order to assess the presence of external effects in the model, Barro checks whether the central planner solution can be obtained by replacing the income tax with a lump-sum tax in the decentralized version of the model.

Under the assumption of a lump-sum tax[11], the long-run rate of growth becomes:

$$\gamma_L = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[\Phi\left(\frac{g}{k}\right) \cdot (1-\eta) - \rho\right] \tag{2.5.18}$$

which is the same as (2.5.10), except that for the absence of the term $(1-\tau)$. The growth rate now is monotonically increasing in the ratio g/y, because after increasing g/y, the marginal product of capital $\partial y/\partial k$ will increase and consumption will grow at a higher rate.

Comparing the central planner solution in (2.5.14) with the decentralized solution with lump-sum taxes in (2.5.18), it is easy to verify that they coincide only if g/y is set at its optimum. Indeed, γ_L depends on $(1-\eta)$, whereas γ_P depends on $(1-g/y)$, where $\eta = \Phi' \cdot (g/y)$, which implies that $\eta = (g/y)$ when $\Phi' = 1$. The fact that the decentralized solution with lump-sum taxes lies on the central planner solution only under the condition $\Phi' = 1$, leads Barro to conclude that in any other case — *i.e.*, for $\Phi' \neq 1$ — the income tax is not the only distortion in the model.

The existence of an external effect other than income taxation is due to the interaction between the private output (and capital) decision and the public investment decision.

As we said above, each producer computes $\partial y/\partial k$ holding g fixed. In other words, *individual* producers do not expect their output (and capital) decisions to affect the total amount of g available for their use. However — provided that the equality $\tau = g/y$ holds — any unit increase in *aggregate* output, will force the government to raise its provision of public services by g/y units. Hence, the individual choice on output indirectly influences fiscal policy and its effect on public decision will depend on whether or not the government has set g/y optimally.

Under the optimal policy $\Phi' = 1$, public spending is worthless because

[11] Since labour supply is assumed to be constant, a consumption tax is equivalent to a lump-sum tax.

a marginal increase brings a revenue exactly equal to its cost. Thus, the distortion induced by the public sector is null and the lump-sum solution is identical to the central planner solution. On the other hand, if the government is too large, (*i.e.*, if $\Phi' < 1$ or, equivalently, $g/y > \alpha$), when the individual producer expands his own output, the government has to increase in a parallel manner the provision of public services, in order to keep g/y constant. Hence, each producer has an incentive to increase his own production because this behavior will lead the government to expand the provision of public services. Such an expansion in the provision of g due to the individual producer's decision represents a *negative externality*, which causes the decentralized solution with lump-sum taxes to be higher than the solution of the central planner: $\gamma_L > \gamma_P$. The opposite will be true when the government size is too little (in this case, $\Phi' > 1$ or equivalently $g/y < \alpha$).

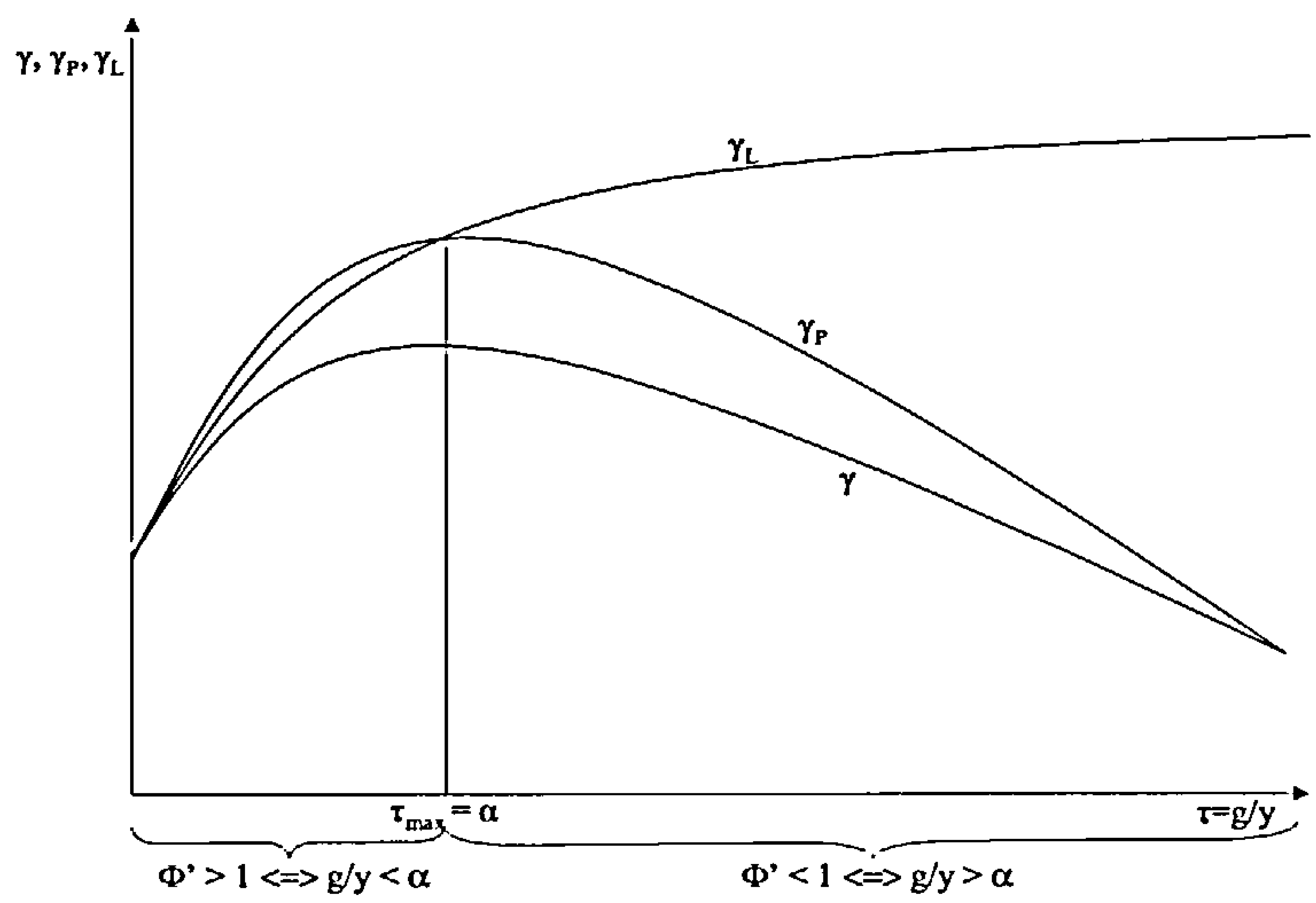

Figure 2.5.1: Barro model - Growth and Public Services, Cobb-Douglas Case

Figure 2.5.1 allows to compare the outcomes of distortionary and non-distortionary taxes in the Cobb-Douglas case. For $\tau_{max} = g/y = \alpha$, the lump-sum tax gives the command optimum and, as a consequence, it is superior to the income tax. The lump-sum tax has to be preferred to the income tax also when $g/y < \alpha$. However, if $g/y > \alpha$, the comparison of the two cases becomes ambiguous because γ_L becomes too large, γ becomes too small and the solution with income tax can be a superior to the solution with the lump-sum tax. This is due to the fact that income taxation is not an appropriate solution to the "over-provision" of public services induced by the producers' increase of y. This distortion could be internalized by imposing the income tax $\tau = g/y$, (*i.e.*, when $\Phi' = 1$ and the government choices a growth maximizing income tax rate). As g/y becomes larger than its optimal value α, the marginal return on public spending (Φ') becomes smaller. Hence, moving away from the point $g/y = \alpha$ to the right, the income tax becomes more and more the adequate way of correcting for the external effect in the economy and, as a consequence, the value of γ gets closer and closer to the value of γ_P[12].

2.6 Extensions to the Barro Model

Table 2.6.1 reports the key features of the Barro model with respect to its production and consumption sides and to the assumptions made on the government. Starting from this framework, many authors have succeeded in providing more insights on the issue of the relationship between public investment and economic growth within more elaborated models.

A summary of the literature dealing with Barro-type models is reported in Table 2.6.2, where models are grouped on the basis of their main departures from Barro (1990). Many models are characterized by more than one departure; this explains why some authors have been cited more than once in the Table[13].

[12] Barro also analyses the case in which the government, together with productive spending, provides consumption services h which enter the household's instantaneous utility function $u(c, h)$. This case is not covered in the present review.

[13] A further group of works extend the Barro model to the open economy case in order to

Table 2.6.1: Key Features of the Barro Model

Production side:	g is a flow variable
	g is a pure public good
	CRTS in k and g
	One sector economy
Consumption side:	Representative Agent model
	Infinite Horizons
	No labour-leisure choice
Government:	Balanced Budget Constraint
	Public Investment is the only category of public spending
	Distortionary/Non-distortionary Taxation

The first set of models includes those interested in modifying the structure of the production side of the model. Within this group of works, some authors argue that the appropriate input to enter the Ak standard production function in order to account for the public sector is the stock of public capital, rather than the flow of public services (Futagami *et al.*, 1993). The main implication of such an assumption is the presence of two state variables (private and public capital) in the model and transitional dynamics (TD). Other researchers — following Barro and Sala-i-Martin (1992, 1999) — have abandoned the way of modelling g as a pure public good, basing their analysis on publicly provided rival and excludable private goods and publicly provided goods (rival but not excludable) subject to congestion.

Regarding the behavior of the government, Barro-type models have been implemented complicating two aspects: the composition of public expendi-

account for the relationship between fiscal policy and exchange rates and terms of trade. See, for instance, Turnovsky (1997), Heijdra and Meijdam (2002) and Kalyvitis (2003).

Table 2.6.2: Extensions to the Barro Model

Extensions	*Authors*
Production Side	
g as a stock and TD	Futagami *et al.*(1993), Cashin (1995), Turnovsky (1997)
	Aschauer (1997), de la Fuente (1997), Greiner (1999)
	Dasgupta (2001), Cassou and Lansing (1998)
	Ghosh and Roy (2002), Heijdra and Meijdam (2002)
	Kalyvitis (2003), Sanchez-Robles (2003)
	Park and Philippopoulos (2003)
Congestion	Barro and Sala-i-Martin (1992, 1999)
	Cassou and Lansing (1998)
	Gloom and Ravikumar (1994, 1997)
	Turnovsky (1996, 1997), Bajo-Rubio (2000)
	Fisher and Turnovsky (1998), Acconcia (2000)
	Rauirich-Puigdevall (2000), Piras (2001)
Consumption Side	
OGs models	Mourmouras and Lee (1999)
	Ghosh and Mourmouras (2002)
	Heijdra and Meijdam (2002)
End. L. S.	Cassou and Lansing (1998), Greiner (1999)
	Turnovsky (2000), Dasgupta (2001)
Government Side	
Debt	Turnovsky (1997), Aschauer (1997)
	Greiner and Semmler (2000)
	Heijdra and Meijdam (2002)
Other kinds of PPS	de la Fuente (1997), Greiner (1999)
UPS	Turnovsky *et al.* (1995), Cashin (1995)
	Devarajan *et al.* (1996)
	de la Fuente (1997), Greiner (1999)
	Park and Philippopoulos (2003), Bajo-Rubio (2000)
	Cassou and Lansing (1998), Acconcia (2000)

ture and the channels of financing public investment (taxes and debt). In particular, authors interested in the first aspect have attempted to extend the results obtained by Barro in the presence of other categories of productive (PPS) and unproductive (UPS) public spending.

Extensions regarding the consumption side have been the less investigated in the literature. There have been two major departures from the original model: the analysis of finite horizons and the hypothesis of endogenous labour supply. This review focuses on the first extension, presenting a summary of the perpetual youth overlapping generations model provided by Mourmouras and Lee (1999) and based on Blanchard (1985). The particular focus on this paper is motivated by the fact that its results are questioned in the model presented in Chapter 3.

The second group of works which complicates the Barro model on the consumption side, relaxes the assumption of no labour-leisure choice in favor of endogenous labour supply. Some authors regard the assumption that households offer a fixed amount of labour as a limitation of the Barro model. It seems too unrealistic to these authors abstracting from the analysis of the private decision of allocating time between work and leisure. Hence, they have devoted their interest to the analysis of fiscal policy under the more realistic assumption of elastic labour supply. The most instructive and comprehensive model dealing with this issue is provided by Turnovsky (2000).

2.6.1 Production Side

Public Services as a Stock Variable

Futagami *et al.* (1993) assume that the stock of public capital enters the aggregate production function, instead of the flow of public services[14]. The

[14] Arrow and Kurz (1970) make the same assumption within a model of exogenous growth.

main reason for making this assumption is that most of publicly provided services (such as, highways, airports and so on) are stock variables in nature. Hence, the aggregate production function is still defined by (2.5.7) — see section 2.5, page 22 — the only difference being that g now represents a stock variable. This assumption allows to study not only the steady state effects of a change in the income tax rate, but also its impact along the path converging to the steady-growth equilibrium.

Under the assumption that the stock of public capital enters the aggregate production function, the government budget constraint becomes:

$$\dot{g} = T = \tau \cdot k \cdot \Phi\left(\frac{g}{k}\right) \tag{2.6.1}$$

Although (2.6.1) represents the only departure from Barro (1990), the assumption of two state variables (private and public capital stocks) implies the important feature that the model has transitional dynamics.

The solution to the representative agent lifetime utility maximization problem — given the lifetime utility in (2.5.1) at page 20 — subject to the inter-temporal consumer constraint $\dot{k} = (1-\tau) \cdot f(k,g) - c$, gives the same Euler equation as in (2.5.10) at page 23.

The joint consideration of the government budget constraint, the usual profit maximizing condition and the Euler equation resulting from the lifetime utility maximization problem, gives a system of two differential equations describing the overall behavior of the economy:

$$\frac{\dot{x}_1}{x_1} = \frac{\dot{g}}{g} - \frac{\dot{k}}{k} = \tau\left(\frac{\Phi}{x}\right) - (1-\tau)\Phi + x_2 \tag{2.6.2}$$

$$\frac{\dot{x}_2}{x_2} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{1}{\sigma} \cdot [(1-\tau) \cdot (1-\eta)\Phi - \rho] - (1-\tau)\Phi + x_2 \tag{2.6.3}$$

where x_1 and x_2 are the public to private capital and the private consumption to private capital ratios respectively, and η is the output elasticity of public capital as defined in section 2.5 (see page 22).

Given this system, it is shown that the economy is characterized by a unique steady-growth equilibrium (defined by $\dot{x_1} = \dot{x_2} = 0$) and there exists a unique stable path converging to such equilibrium[15].

As for the steady state effects of an increase in the income tax rate, the authors are able to replicate the result of the Barro rule within their framework. Hence, they find that — for η constant and equal to α — the growth maximizing income tax rate equates the output elasticity of public capital.

Regarding the study of the transitional dynamics after a change in τ, their results are the following. The analysis refers to the case $\eta \leq \tau$, that is to say, for values of τ on the decreasing part of the Barro curve, where the growth hampering effect of higher income taxation dominates the growth enhancing effect due to a higher steady state value of the public to private capital ratio. Thus, the government size is too large and the net effect of higher income taxation on the steady-growth equilibrium is negative.

Assuming that the economy is initially at its steady-growth equilibrium, if a unanticipated increase in τ occurs at time 0, Futagami *et al.* (1993) distinguish two cases characterized by a different initial response of the consumption to private capital ratio x_2. Indeed, their result is such that:

$$\frac{\partial x_2(0,\tau)}{\partial \tau} > (\leq) 0 \quad \text{when} \quad \rho + \eta < (\geq) 1 \tag{2.6.4}$$

where (2.6.4) states that the short run effect of a change in τ on x_2 is positive or negative according to whether $\rho + \eta$ is less or greater than 1.

Figure 2.6.1 shows the graphical analysis of the unanticipated rise in τ in the case of $\rho + \eta > 1$. The initial steady-growth equilibrium lies in A. When τ rises at time 0, both the locus of $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ shift downward

[15] Since the economy in the model presented in Chapter 3 will be described by a system of differential equations similar to (2.6.2)-(2.6.3), this result is not discussed further here. The properties of the BGP implied by such system will be studied in sections 3.B.1 and 3.B.2 of the Mathematical Appendix (see pages 92-95).

in the upper diagram of Figure 2.6.1.

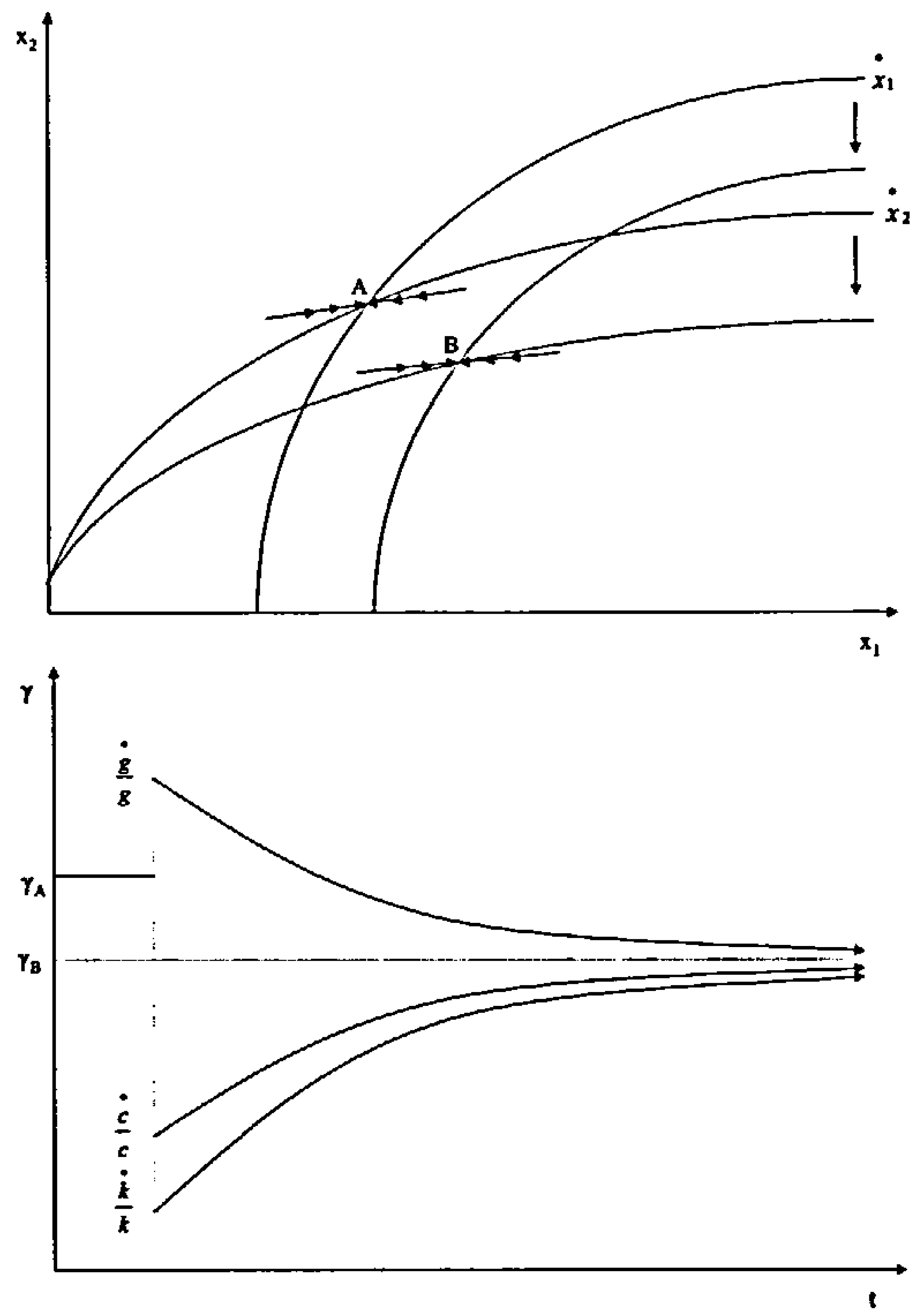

Figure 2.6.1: Futagami Model, Transitional Dynamics ($\rho + \eta > 1$)

According to (2.6.4), x_2 initially jumps downward and then increases, while x_1 gradually increases. This implies that $\dot{x}_1/x_1 = \dot{g}/g - \dot{k}/k > 0$ and $\dot{x}_2/x_2 = \dot{c}/c - \dot{k}/k > 0$ on the transitional path and consumption decreases. The initial impact of the rise of τ on the rate of growth of public capital is derived from the definition of the government budget constraint in (2.6.1):

$$\frac{\partial(\dot{g}(0,\tau)/g(0,\tau))}{\partial\tau} = \frac{\Phi}{x_1} > 0 \tag{2.6.5}$$

hence, public capital initially increases and then decreases gradually. On the other hand, from the Euler equation it results that:

$$\frac{\partial(\dot{c}(0,\tau)/c(0,\tau))}{\partial\tau} = -\frac{\Phi - \Phi' x_1}{\rho} < 0 \tag{2.6.6}$$

which tells that consumption initially decreases and then increases gradually. The rate of growth of consumption is greater than the rate of growth of private capital (see the lower diagram in Figure 2.6.1). In the long run, the economy converges to the lower steady-growth equilibrium B. The new steady state value of x_2 is lower than the initial one. On the other hand, x_1 ends up with a higher steady state value.

Figure 2.6.2 depicts the case $\rho+\eta < 1$. After the increase in τ, the locus of $\dot{x_1} = 0$ shifts downward, while the locus of $\dot{x_2} = 0$ shifts upward in the upper graphic. According to (2.6.4), x_2 initially jumps upward and then decreases and x_1 increases.

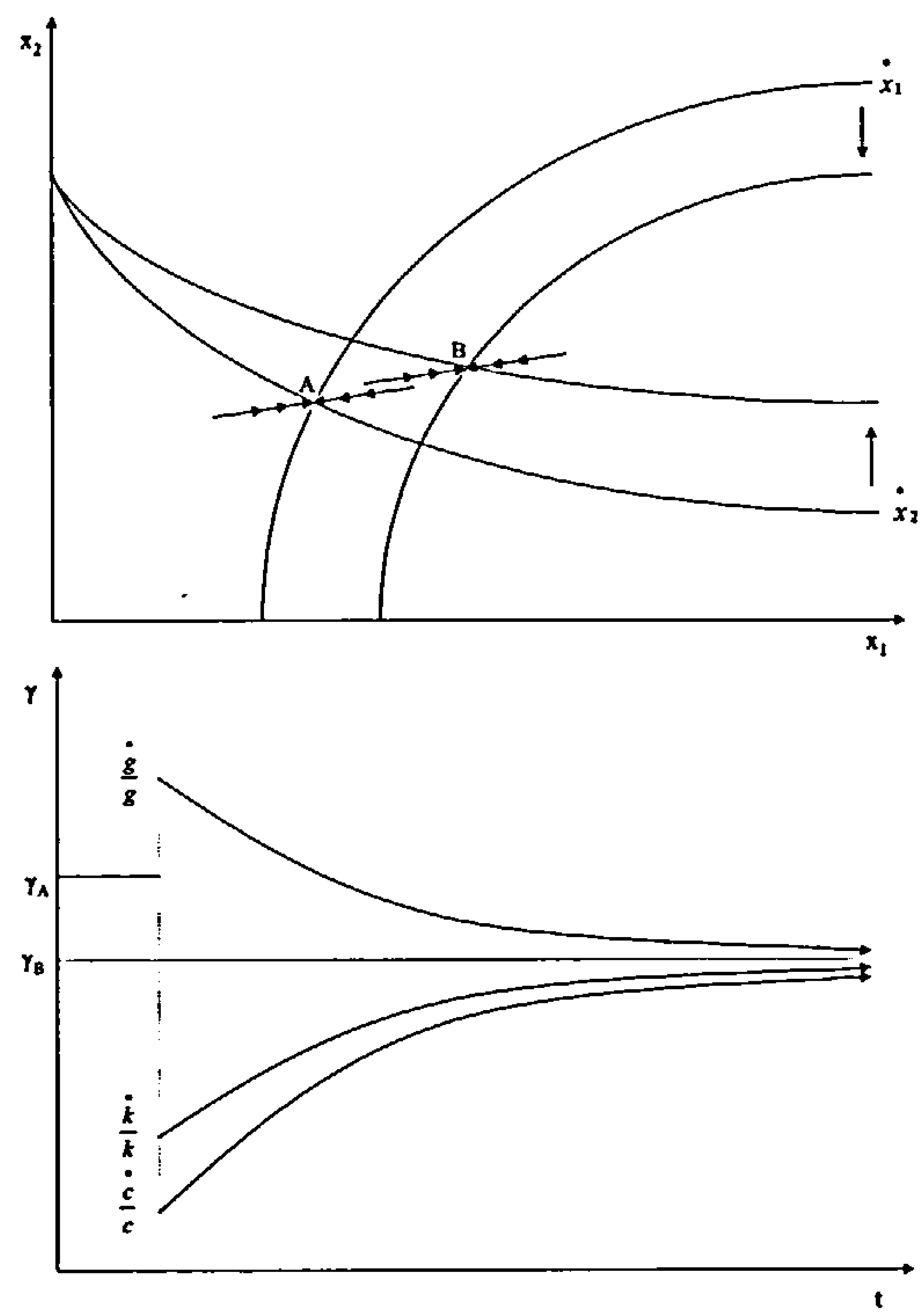

Figure 2.6.2: Futagami Model, Transitional Dynamics ($\rho + \eta < 1$)

Thus, contrary to the previous case, $\dot{x}_1/x_1 = \dot{g}/g - \dot{k}/k > 0$ and $\dot{x}_2/x_2 = \dot{c}/c - \dot{k}/k < 0$ on the transitional path. The initial impacts of a higher τ on

c and g are the same as before. From the consumer budget constraint and equation (2.6.2), the initial impact on private capital is defined by:

$$\frac{\partial(\dot{k}(0,\tau)/k(0,\tau))}{\partial\tau} = -\Phi - \frac{\partial x_2(0,\tau)}{\partial\tau} < 0 \qquad (2.6.7)$$

and, contrary to the previous case, the rate of growth of private capital is greater than the rate of growth of consumption (see the lower diagram in Figure 2.6.2). In the new steady state equilibrium B, both x_1 and x_2 have values higher than before the rise of τ occurred.

The difference between the two cases is the short run behavior of consumption. Indeed, any change in τ generates two effects of opposite directions on the level of present consumption: the (inter-temporal) substitution effect and the income effect. The income effect negatively affects present consumption, while the substitution effect yields an increase in current consumption at the expense of future consumption. The relative magnitude of the time preference parameter will determine which of the two effects dominates the other. Increasing τ causes the interest rate to jump downward at time 0 (see the Euler equation) and then to increase gradually up to the new steady state value, which will be lower than the original one. When $\rho + \eta < 1$, the elasticity of inter-temporal substitution is large and, as a consequence, the substitution effect will be stronger than the income effect, yielding an increase in current consumption.

Following Futagami *et al.* (1993), other authors have considered an aggregate production function extended to include the stock of public capital introducing, by doing so, transitional dynamics in their models. For instance, Ghosh and Roy (2002) include both public services and public capital into the aggregate production function in the attempt of reconciling Barro's and Futagami *et al.*'s views. The aim of these authors is to study the trade-off of the decision faced by the government of either providing public services or accumulating public capital. They work out a model able to include Barro (1990) and Futagami *et al.* (1993) as special cases, using the following production function:

$$y = ({g_s}^{\alpha} \cdot {g_f}^{1-\alpha})^{1-\beta} \cdot k^{\beta} \tag{2.6.8}$$

where for $\alpha = 0$ and $\alpha = 1$ the model collapses to the models by Barro and Futagami *et al.* respectively.

Public services are not pure public goods

Barro and Sala-i-Martin (1992) discuss two extensions to Barro (1990), based on alternative definitions of the degree of publicness of g[16], assumed to be either (i) *rival and excludable* or (ii) *rival but not excludable.*

(i) According to the first definition of the flow variable g, each individual producer has a property right on a specific share of the total amount of public services. Under this assumption, this share of public services matters for the individual production function, rather than its total amount. Leaving unchanged the notation, now g becomes a *rival and excludable* good: each individual uses his own share of public services and can not use or congest the services provided to others. For instance, the government could purchase any input needed by producers and allocate it equally across firms without direct charges. The production function (2.5.7) at page 22 is modified in order to include $g = G/n$:

$$y = Ak^{1-\alpha}\left(\frac{G}{n}\right)^{\alpha} \tag{2.6.9}$$

where n is the number of producers, $0 < \alpha < 1$, and the technology shows diminishing returns to k separately and constant returns to k and g taken together. The main difference with the model discussed in section 2.5 is

[16] The authors provide three versions of the Barro model, each of them characterized by an alternative definition of public services: 1) publicly provided rival and excludable private goods; 2) publicly provided non-rival and non-excludable public goods (that is to say, the Barro model); 3) publicly provided goods (rival but not excludable) subject to congestion.

that, each producer computes the marginal product of the capital employed in her firm, holding fixed $g = G/n$, that is to say, the part of public services in her availability, rather than the total amount of public services.

Leaving unchanged the assumptions of a government running a balanced budget constraint and levying the income tax at the rate $\tau = g/y$, the optimality condition for the provision of public spending is still the same: $\Phi' = \partial y/\partial g = 1$, which implies (from (2.6.9) — $g/y = \alpha$.

The marginal product of private capital is:

$$\frac{\partial y}{\partial k} = (1-\tau)(1-\alpha) \cdot A^{1/(1-\alpha)} \cdot (g/y)^{\alpha/(1-\alpha)} \tag{2.6.10}$$

where (2.6.10) is identical to (2.5.9)[17], the only difference being that now g is an individual share, instead of the total amount of public services. Once again, if government investment is financed by a lump-sum tax, private and social marginal returns on capital — (2.5.16) and (2.5.17), page 25 — equate and the rate of growth is optimal as in the Barro model. On the other hand, given a proportional income tax at rate τ, social and private returns on investment in private capital will equate only if $\tau = g/y = 0$. For a positive τ, the private return is lower than the social one, private producers will have a disincentive to invest and long-run growth will be negatively affected.

(ii) In the second version of the model, the flow of public services is a *rival but not excludable* good:

$$y = Ak^{1-\alpha} \cdot \left(\frac{G}{Y}\right)^{\alpha} \tag{2.6.11}$$

where for a given amount of total government purchases G, the quantity of public services freely available to each firm is decreasing in the usage of other producers. Each individual firm's decision to expand its own output congests the quantity of public services provided by the government to all other firms. Hence, the usefulness of a certain category of public capital

[17] (2.6.10) gives the solution to (2.5.9) in the Cobb-Douglas case.

to each individual declines as more producers use the facility. Real world examples of public services of this sort would be transportation facilities as well as public utilities.

With this specification of the technology, production can exhibit constant returns to the two inputs k and g, only if the government is able to keep constant the degree of congestion (G/Y). Given the assumption of congestion effects, it becomes more relevant the issue of whether or not producers realize that their individual output decisions affect the provision of G. Indeed, since each producer takes G as given while deciding the scale of his production, there still exists an incentive to increase output and, by doing so, to increase congestion costs. The only instrument that the government can use in order to make producers aware of the congestion costs that they cause is a proportional user fee at the rate $\tau = G/Y$. As a consequence, in contrast with the previous case the optimal policy is given by a proportional tax on income, rather than by lump-sum taxation.

2.6.2 Government Side

As discussed above, the Barro model is able to predict the growth enhancing effect of public investment in infrastructure. This result, however, is derived abstracting from any other type of government expenditure and under the assumption of a balanced government budget constraint. The interest of many researchers has been devoted to relax both assumptions, laying emphasis on aspects related to the composition of government expenditure, the mix of expenditure and taxation and the presence of public deficits.

Table 2.6.3 summarizes the effects of alternative mixes of public expenditure and taxation on growth. Government expenditures are divided in productive and unproductive categories. Unproductive public spending includes any kind of government expenditure entering the consumers' utility function and, as a consequence, influencing their welfare without affecting aggregate

Table 2.6.3: Growth Effects of the mix of Public Expenditure and Taxation

		Public Spending	
		Productive	Unproductive
Taxes	Distortionary	+/-	-
	Non-Distortionary	+	0

production. For instance, public consumption and lump-sum transfers to households are commonly considered unproductive[18]. On the other hand, the productive categories of public expenditure enter the aggregate production function and, by doing so, affect private production efficiency. As we will see later in this section, the categories of public spending classified as productive are essentially investment in public capital and subsidies to private investment[19].

Examples of distortionary taxation are proportional taxes on labour income and profits, social security taxes, property taxes and so on. Lump-sum taxes are non-distortionary and equivalent to taxes on consumption of domestic goods and services under the assumption that labour is supplied inelastically.

Table 2.6.3 shows that the expected growth effect of a given category of public expenditure depends upon the kind of taxation used to finance it. From the Barro model, we know that the long-run rate of growth is monotonically increasing in productive public spending financed by non-distortionary taxation. On the other hand, the growth effect of productive public spending financed by distortionary taxation is ambiguous and depends on the level of expenditure. This is the result summarized by the

[18] Further examples of categories of unproductive public spending are social security and welfare expenditures and expenditures on recreation or other social activities.

[19] Government expenditures on Education and Health are classified by some authors as productive. This choice is justified by the fact that this expenditure enhances the productivity of human capital.

Barro curve for public investment and, as explained below, a similar result is obtained by Greiner (1999) for investment subsidies. Unproductive public spending categories do not influence aggregate production efficiency. When the government finances an increase in unproductive public spending through distortionary taxation, long-run growth is hampered. On the other hand if such an increase is financed by some form of non-distortionary taxation, long-run growth will be unaffected.

A good starting point for reviewing the part of the literature interested in the composition of public expenditure is the work by de la Fuente (1997). This is mainly an empirical work, but it is based on a version of the Barro model, extended in order to include other kinds of government expenditure.

This model modifies Barro (1990) in two respects. The first departure is the assumption that households instantaneous utility function (in additive form) is dependent on per capita private consumption c and per capita total government expenditure E:

$$U(c, E) = \mu \cdot \ln(c) + (1 - \mu) \cdot \ln(E) \tag{2.6.12}$$

where E is assumed to encompass any kind of public spending, including investment in infrastructures. As a second departure, de la Fuente captures alternative channels through which government activities affect economic performance, together with the productivity enhancing effect induced by public investment in infrastructure. This is done in a very simple way, by assuming that aggregate output in the economy is given by:

$$Y = \theta^{\gamma} \cdot K^{\alpha} \cdot G^{\beta} \cdot (AL)^{1-\alpha-\beta} \tag{2.6.13}$$

where the parameter θ^{γ} captures all the possible effects that the government can generate on aggregate output, in addition to the one induced by the provision of public capital in infrastructure G. The main point of the production function in (2.6.13) is that the inclusion of θ^{γ} allows us to encompass the wide range of effects that government activities can produce in the economy. The parameter θ is a proxy of the degree of public intervention in the private sector of the economy, and its exponent γ represents a *government externality* coefficient.

Since every single government activity can either enhance or hamper productivity, the net effect of the overall government size can be either positive or negative and the sign of γ is unknown on *a priori* grounds. For instance, the government could contribute to enhance productivity of human capital by raising expenditure on health services and education. In general, public consumption in defence, police protection or court system might be thought to have positive effects on productivity through different channels. On the other hand, other government activities could work in the opposite direction. This happens, for instance, when taxation is distortionary or public regulation activities lead to the inefficient allocation of resources in private markets. The net effect of the complete set of government activities is captured by the sign of γ, which will be positive according to whether growth enhancing effects will dominate growth hampering effects.

The government is assumed to finance public expenditure with a proportional tax on income at the rate τ and to run a balanced budget constraint. Total public spending is composed by transfers payments, public consumption and public investment in G, with the government devoting a constant fraction of total output to each of these categories. Consequently, the government budget constraint is:

$$\theta = \theta_T + \theta_P + \theta_C = \tau \tag{2.6.14}$$

where $\theta_T = T_P/Y$ = transfers payments, $\theta_P = G/Y$ = directly productive expenditures and $\theta_C = C_P/Y$ = public consumption.

De la Fuente distinguishes three channels through which the growth effect of changing fiscal policy parameters takes place within his framework. Total private investment I_p are defined as the product of the average propensity to save s and (after transfers and taxes) disposable income

$(1 - \theta_C - \theta_P)Y(\theta)$:

$$I_p = s[(1 - \tau)R(\theta)] \cdot (1 - \theta_C - \theta_P) \cdot Y(\theta) \qquad (2.6.15)$$

where s is a function of the after tax rate of return on private capital $(1 - \tau)R(\theta)$, and both the interest rate and the aggregate output depend on the level and the composition of government expenditure θ.

First, fiscal policy has a direct effect on productivity through the aggregate production function in (2.6.13). Second, given the presence of the term $(1-\tau)$, public expenditures (with the exception of lump-sum transfers) will tend to crowd out private investment[20]. Finally, the rate of return on private capital is also a function of θ[21]. As a consequence, the distortionary effect of taxation can be either reinforced or offset by the impact of fiscal policy on the marginal product of private capital.

Greiner (1999) extends the Barro model by dividing productive government spending in investment in public capital and subsidies to private investment and considering the same categories of unproductive public expenditure as de la Fuente (1997). Furthermore, in contrast with Barro (1990), the different effects of both variations in income and consumption taxation are also investigated under the alternative assumptions that labour is supplied either inelastically or elastically.

On the production side of the model, it is assumed that the stock of public capital enters the aggregate production function as in Futagami *et al.* (1993) and the economy has transitional dynamics. However, the author focuses solely on the steady state effects of fiscal policy, without analysing its impact along the path converging to the equilibrium.

The government budget constraint (2.5.8) at page 22 is modified in order

[20] This effect will disappear under the assumption of lump-sum taxation.

[21] This is simply the positive effect of public services provision on the marginal product of private capital that we studied in the Barro model, see section 2.5, page 23.

to account for alternative government expenditures:

$$T = \dot{G} + \varphi_1 T_p + \varphi_2 C_p + \theta_S \dot{K} \tag{2.6.16}$$

where φ_1 and φ_2 are the shares of total tax revenues devoted to lump-sum transfers to households and public consumption respectively ($\varphi_1 + \varphi_2 < 1$), and $\theta_S \dot{K}$ represents investment subsidies. Under the assumption of no depreciation of capital as in the Barro model, the consumer budget constraint is also modified accordingly:

$$C + \dot{K} = (w + rK + \pi)(1 - \tau) + \theta_S \dot{K} + T_p \tag{2.6.17}$$

where w is labour income, r the real interest rate, π are profits and both τ and θ_S are defined between 0 and 1.

Within this framework, the overall behavior of the economy turns out to be described by a system of two differential equations similar to (2.6.2)-(2.6.3) — see subsection 2.6.1, page 32 —, the only difference being the inclusion of additional fiscal policy parameters. The main results obtained in this model are related to:

(i) the effects of each category of expenditure on long-run economic growth.

(ii) the relationship relating each category of alternative government expenditures to the Barro rule.

As for the effects (i), the long-run rate of growth of the economy is monotonically decreasing in both categories of unproductive public spending. After a rise of either φ_1 or φ_2, less resources will be used for public investment in G, with a decelerating effect on long-run growth.

On the other hand, the growth effect of an increase in subsidies to private investment is ambiguous. Indeed, by increasing θ_S, the government will reduce the investment in public capital, which implies a negative effect on growth. However, a rise of θ_S has also the direct effect of increasing the opportunity cost of consumption, leading to a reallocation of private resources from consumption to investment and, as a consequence, positively affecting growth. The net growth effect of raising θ_S will be positive only if the positive effect induced by the higher incentive to save will be stronger than the negative effect caused by the reduction in public investment. The analytical condition under which this will occur is derived as:

$$\frac{\partial \gamma}{\partial \theta_S} > (\leq) 0 \quad \text{if} \quad \frac{\partial x_2}{\partial \theta_S}\frac{\theta_S}{x_2} > (\leq) - \frac{\theta_S}{\alpha\left(1-\theta_S\right)} \tag{2.6.18}$$

where x_2 defines the public to private capital ratio as in the Futagami *et al.* model. Hence, the growth effect of raising θ_S depends on the value taken by the elasticity of the public to private capital ratio with respect to θ_S.

Finally, Greiner (1999) modifies the optimal provision rule for public investment as follows:

$$\tau_{max} = \alpha\left(1 + \frac{\theta_S}{x_2}\right) \tag{2.6.19}$$

where (2.6.19) equates the Barro rule in (2.5.13) only for a null θ_S whereas, in the presence of other categories of government expenditure, the required growth maximizing rate of the income tax will be higher[22].

The results in (ii) are concerned with the relationships linking the optimal provision of public investment to transfers to households, public consumption and investment subsidies. From (2.6.19), it is clear that τ_{max} is increasing in θ_S.

A similar relationship is formally derived for the shares φ_1, φ_2 devoted to the two categories of unproductive expenditure. The conclusion is that given higher levels of unproductive public spending and/or generous investment subsidies, the government will be forced to increase productive investment in order to compensate for the negative effect on long-run growth.

[22] The numerical solution to this model allows Greiner to compute $\tau_{max} = 0.38 > \alpha$, where $\alpha = 0.30$.

2.6.3 Consumption Side

Mourmouras and Lee (1999) analyse the effects of productive government expenditure on economic growth, combining Blanchard-type consumers with endogenous growth driven by productive government spending. By doing so, the framework of an economy populated by the standard representative household-producer adopted by Barro is abandoned, and the relationship between public services and long-run growth is assessed within a perpetual youth overlapping generations (PY-OLGs) model *à la* Blanchard (1985). The aim of the authors is to extend the results obtained by Barro to the case of uncertain lifetime consumers. Since the consumption side of their model is a version of the PY-OLGs model by Blanchard (1985), I will briefly introduce this manner of modelling finite horizons, before discussing the main results obtained by Mourmouras and Lee (1999).

It is well known that when agents have finite horizons it is complicated to aggregate consumption. Indeed, agents alive at a given moment in time differ from each other for two fundamental reasons. First, agents of different ages are characterized by different levels and compositions of wealth. Second, agents with different life horizons have different propensity to consume out of wealth. These two features make impossible to aggregate consumption over agents alive at the same instant in time.

Standard OLGs models adopt simple population structures in order to avoid the aggregation problem, by aggregating consumption over "types" of consumers (for instance "old people" and "young people" in two periods OLGs models). On the other hand, Blanchard (1985) succeeds in overcoming the aggregation problem by making two crucial assumptions on the probability of death faced by consumers and, as consequence, on the population structure: (1) agents face a constant instantaneous probability of death throughout their life, and (2) there exist life insurance companies.

(1) The main assumption imposed by Blanchard is that consumers face a

constant instantaneous probability of death λ throughout their life[23]. There is no population growth and, at any time t, a new cohort of large size λ is born. Since λ is constant, the expected remaining life of any agent — regardless of her age — is equal to $\int_0^\infty t\lambda e^{-\lambda t}dt = \lambda^{-1}$. The ratio $1/\lambda$ is an horizon index: as λ approaches zero, $1/\lambda$ goes to infinity and agents have infinite lives. Each agent is uncertain about the instant in time when she will die, but the size of a cohort declines deterministically through time. A cohort born at time zero has size $\lambda e^{-\lambda t}$ at time t and, as a consequence, the population size at any point in time t is normalized to 1: $\int_{-\infty}^{t} \lambda e^{-\lambda(t-s)}ds = 1$.

(2) Under the assumption of uncertain lifetime, an agent could leave positive assets or debts when she dies. However, if the probability of death faced by the large number of consumers in the economy is constant, private markets can provide full life insurances risklessly. Hence, it is convenient to assume that there exist perfectly competitive insurance companies providing agents with one good contingent to their life at the rate λ. If there is no bequest motive and it is forbidden to leave debts when they die, agents will contract a full life insurance. They will insure all their wealth a, receiving λa if they do not die and paying a if they die.

Assumptions (1) and (2) allow to aggregate consumption across agents. In general, if $x(s, t)$ denotes an individual variable, its aggregate counterpart

[23] Even if it might sound unrealistic, this assumption implies that consumers of different ages have the same propensity to consume, allowing to overcome the aggregation problem of standard overlapping generations models. In support of this assumption, Blanchard cites some evidence on the mortality rates, according to which people aged between 20 and 40 face approximately the same probability of death.

will be given by:

$$X(t) = \int_{-\infty}^{t} x(s,t) \underbrace{\lambda e^{-\lambda(t-s)}}_{\text{population size}} ds. \tag{2.6.20}$$

This way of modelling finite lives is adopted by Mourmouras and Lee (1999), who modify the Barro model on the consumption side. Assuming a CRRA instantaneous utility function with $\sigma = 1$, they define the lifetime utility of the individual i, born at time s:

$$U_i(s,t) = \int_{t}^{\infty} \ln c_i\,(s,v)\, e^{(\rho+\lambda)} dv \tag{2.6.21}$$

and the following dynamic budget constraint:

$$\dot{a}_i\,(s,t) = [r(t) + \lambda]\, a_i\,(s,t) + \omega_i\,(t) - c_i\,(s,t) \tag{2.6.22}$$

where a stands for asset wealth, and ω is non asset income of the household. Maximizing (2.6.21) subject to (2.6.22), yields the standard individual Euler equation:

$$\dot{c}_i(v) = [r(v) - \rho]c_i(v) \tag{2.6.23}$$

and the individual consumption function:

$$c_i(t) = (\rho + \lambda)[a_i(t) + h_i(t)] \tag{2.6.24}$$

where individual consumption is proportional to human wealth $h_i(t)$ and non human wealth $a_i(t)$, and the propensity to consume $(\rho+\lambda)$ is constant across generations. Using the aggregation procedure suggested by Blanchard, the authors derive the rate of growth of aggregate consumption:

$$\dot{C}(t) = [r(t) - \rho] \cdot C(t) - \lambda(\rho + \lambda) \cdot A(t) \tag{2.6.25}$$

This Euler equation captures the dynamics of consumption under the assumption of finite lives. If consumers live infinite lives — *i.e.*, $\lambda = 0$ — consumption increases (declines) over time whether the interest rate is higher (lower) than the subjective discount rate. On the other hand, under the assumption of finite lives ($\lambda > 0$), the rate of growth of consumption is decreasing in the probability of death parameter λ and depends on asset wealth[24]. A higher value of λ implies a higher propensity to consume out of wealth and, as a consequence, a lower consumption growth rate: $(\dot{C}/C)_{\lambda>0} < (\dot{C}/C)_{\lambda=0}$.

This model departs from Barro (1990) solely on the consumption side, whereas production strictly follows the original model. Thus, aggregate production is given by (2.5.7), page 22 — the only difference being a positive depreciation rate of capital — and the government budget constraint is (2.5.8). Given the following definition for total investment:

$$\dot{K} = (1-\tau)K^{1-\alpha}G^{\alpha} - C - \delta K \qquad (2.6.26)$$

the economy reaches its steady state equilibrium and it is described by the following system of equations[25]:

$$c = \frac{\lambda(\lambda+\rho)k}{r-\rho-\gamma} \qquad (2.6.27)$$

$$\gamma = \dot{Y}/Y = -\delta + \frac{1-g-c}{k} \qquad (2.6.28)$$

$$r = (1-\tau)(1-\alpha)\,(g/k)^{\alpha} - \delta \qquad (2.6.29)$$

$$k = g^{\frac{\alpha}{\alpha-1}} \qquad (2.6.30)$$

where lower cases now denote aggregate variables in per-output terms: $c = C/Y$, $g = G/Y$ and $k = K/Y$. Equation (2.6.27) shows that the consumption to output ratio depends on g for two reasons. First, a change in g affects c through its effect on the long-run rate of growth γ in (2.6.28). Second, a change in g affects c through the production function in (2.6.30).

Given this framework, the authors do not derive analytically the steady state effects of varying g in the finite horizon scenario. They first note that

[24] In the model, however, the only asset is physical capital.

[25] Equation (2.6.27) is obtained by setting $\dot{c} = 0$. The definition of the long-run rate of growth in (2.6.28) is obtained from the equilibrium condition (2.6.26) in per-output terms.

an increase in g financed by a proportional income tax, will have the effect of crowding out both c and k, as disposable wealth will be reduced. The effect of increasing g on γ, however, becomes ambiguous as in the Barro model. Indeed, from equation (2.6.28), it can be noted that an increase in g generates two effects of opposite signs on γ: a negative direct effect and a positive indirect effect through the reduction in k and c. Then, the comparison between the infinite and the finite horizons scenario is pursued by using a numerical simulation of the model (2.6.27)-(2.6.30). They find the *Barro rule* to be satisfied both in finite and infinite lives scenarios, pointing out that the Barro rule is only determined by the production side of the economy, which is modelled as in Barro (1990) in their framework. In contrast with Barro (1990), instead, they depict the existence of the Barro curve in the finite horizons case even in the case of government expenditure financed by lump-sum taxes.

The main result of this paper is the finding of a τ_{max} independent of the probability of death parameter λ. In other words, the optimal level of public investment provision is not found to be dependent of the consumption externality due to the uncertain lifetime hypothesis.

2.7 Conclusions

The conclusion reached by Mourmouras and Lee (1999) will be questioned in Chapter 3, where I will develop a representative agent model with time preferences augmented by a probability of death λ and endogenous growth due to government investment. This model criticizes the approach followed by Mourmouras and Lee (1999), on the basis of the limitations of the Blanchard model. Indeed, according to Blanchard (1985) the main drawback of his way of modelling consumers' lifetime is that *"it captures the finite horizons of life but not the change in behavior over time, the life-cycle aspect of life"*. Hence the framework of the perpetual youth overlapping generations model *"is well adapted to issues where the finite horizons aspect is important, such as debt or deficit but poorly adapted to issues where differences in propensity to consume across agents are potentially important"*. As Mourmouras and Lee (1999), I will refer to Blanchard (1985) on the consumption side of the model. However, the optimal lifetime consumption plan will be determined within a standard representative agent model, the only difference being a rate of time preference augmented by λ.

3. FISCAL POLICY, ENDOGENOUS GROWTH, AND FINITE HORIZONS

3.1 Introduction

This Chapter focuses on an endogenous growth model in which sustained long-run growth is due to investment in public capital, the government provides lump-sum transfers, public consumption and investment subsidies, and consumers have uncertain lifetimes. The aim is to analyse the growth effects of varying fiscal policy parameters in infinite as well as finite horizons scenarios, reducing some recent theoretical contributions on this branch of the literature to special cases of a more general framework.

Barro (1990) predicts the existence of an optimal level of public investment financed by a flat rate income tax[1]. Greiner (1999) provides an extension by dividing productive government spending between investment in public capital and subsidies to private investment and including in his theoretical framework lump-sum transfers to households and public consumption[2]. His main findings are such that the growth maximizing income tax rate τ_{max} is monotonically increasing in the levels of public consumption, lump-sum transfers to households and subsidies to private investment.

On the other hand, Mourmouras and Lee (1999) analyse the effects of

[1] On the other hand, with lump-sum taxes, the long-run rate of growth is monotonically increasing in government provision of infrastructure.

[2] This author, in contrast with Barro, also analyses the different effects of both variations in income and consumption taxation under the alternative assumptions that labour is supplied either inelastically or elastically within an economy populated by an infinitely-lived representative agent.

productive government expenditure on growth — abstracting from any other type of government expenditure — combining Blanchard-type consumers with endogenous growth. Long-run growth is always found to be lower under the assumption of finite lives compared to the infinite horizons case, but the assumption of uncertain lifetime does not affect the Barro rule. Indeed, the Barro curve is obtained for both the finite and the infinite horizons cases[3], with the optimal level of government investment on infrastructure equating the share of public services in the aggregate production function in both cases.

In this Chapter, I develop a Ramsey-type model with endogenous growth due to government spending in public capital. As Mourmouras and Lee (1999), I refer to Blanchard (1985) to model consumers' lifetime horizons. However, the optimal lifetime consumption plan is determined as in a standard representative agent model, the only difference being a rate of time preference augmented by a probability of death λ. Such a device makes it possible to build a general framework collapsing to the infinite horizons scenario by simply setting to zero the parameter λ. Furthermore, this allows to analytically derive the role of finite lives in affecting the optimal provision of government spending whereas, in Mourmouras and Lee (1999), λ affects the long-run rate of growth but τ_{max} is independent of λ.

For a null λ, the model departs from Barro (1990) solely for the presence of fiscal policy parameters, others than government expenditure on infrastructure. Namely, lump-sum transfers to households φ_1, public consumption φ_2 and investment subsidies θ_S.

For a positive λ, the model is populated by uncertain lifetime consumers *à la* Blanchard, departing from Mourmouras and Lee (1999) for the fact that I explicitly take into account the effect of a positive λ not only on the

[3] In contrast with Barro, instead, they depict the existence of the Barro curve in the finite horizons case even in the case of government expenditure financed by lump-sum taxes.

long-run rate of growth, but also on the optimal income tax rate. Thus, the Barro rule is extended to the case of finite-lived consumers and it turns out to be dependent on the horizon index. I will refer to such an extension as a *modified Barro rule*: for $\lambda = 0$, the optimal provision of public investment equates the share of public capital in the aggregate production function. However, if consumers live finite lives, such an optimal level will be lowered by the consumption externality due to λ.

The assumption of uncertain lifetime consumers also affects the relationships relating other fiscal policy parameters to long-run economic growth and, as a consequence, their respective impacts on the optimal public investment provision rule.

As for the growth effects of other fiscal policy tools, the long-run rate of growth γ is lowered by either higher lump-sum transfers to households or public consumption, regardless of the value of λ. However, increases in either φ_1 or φ_2 of the same amounts reduce γ less for $\lambda > 0$ than for $\lambda = 0$.

On the other hand, the growth effect of increasing investment subsidies is ambiguous for any value of λ. Under the assumption of finite lives, the growth maximizing level of θ_S is negatively related to λ: as the consumer life horizon increases, the optimal value of θ_S is reached before.

As for the relationships linking τ_{max} to other categories of public expenditure, for $0 \leq \lambda \leq 1$, τ_{max} is increasing in φ_1, φ_2 and θ_S.

The remainder of the Chapter is organized as follows. The model will be built in section 3.2, introducing the behavioral assumptions imposed upon households, firms and the government. The two differential equations describing the overall behavior of the economy will be derived, depicting the role played by the uncertain lifetime hypothesis in decelerating long-term economic growth. Section 3.3 is devoted to the analysis of fiscal policy in the model. The growth effects of fiscal policy tools are derived analytically, with particular attention paid to the definition of the *modified Barro rule* and its relationships with other fiscal policy tools. Some conclusions will be drawn in section 3.4. Detailed tables showing the results of numerical solutions are reported in Appendix 3.A. All analytical results are derived and shown in detail in Appendix 3.B.

3.2 The Model

The economy is composed of consumers who maximize their lifetime utilities, profit-maximizing competitive firms and the government. Consumers supply labour inelastically and — for a positive λ — can have their savings insured by an insurance company. The aggregate production function shows diminishing returns to scale in private and public capital separately and constant returns to scale in the two forms of capital taken together. The government runs a balanced budget constraint, financing investment in infrastructure through a flat income tax rate, and providing public consumption, investment subsidies to firms and lump-sum transfers to households.

3.2.1 Firms

The production side of the economy is described by a Cobb-Douglas production function in private capital K and productive public services G. Following Barro (1990), the government purchases a share of private output and uses these purchases to provide free public services to the private sector. These services are assumed to be non-rival and non-excludable. Since the use of G by a firm does not prevent other users from benefiting from them, it is the total amount of publicly provided services that matters for the firms and enters the production function. This assumption is useful to model a broad concept of public capital, which can be thought as the infrastructure network of a country. Under the assumption that all the services belonging to G are publicly provided with no user fees, G represents an unpaid input of production and, indeed, it plays the role of a positive externality in enhancing the marginal product of private capital[4]. Given these assumptions, the aggregate production function is:

$$Y = K^{1-\alpha}G^{\alpha} = K\left(\frac{G}{K}\right)^{\alpha}; \quad 0 < \alpha < 1 \tag{3.2.1}$$

This production function shows diminishing returns to scale in G and K separately, and constant returns to scale in K and G taken together:

$$Y_K > 0; \quad Y_{KK} < 0 \tag{3.2.2}$$

An increase in G leads to an increase in the marginal product of private capital, which implies $Y_{KG} > 0$.

Assuming competitive markets, the first order condition for the firms' profit maximization problem requires the real interest rate to equalize the physical marginal product of private capital. This condition is expressed by:

$$r = (1-\alpha)\left(\frac{G}{K}\right)^{\alpha} \tag{3.2.3}$$

From the definition of the production function (3.2.1) and the first order condition (3.2.3), the following condition is derived:

$$rK = (1-\alpha)Y < Y; \quad 0 < \alpha < 1 \tag{3.2.4}$$

Therefore, this model allows the output of the economy (Y) to be larger than the payments to the owners of private capital (rK). This circumstance is due to the additional income induced by public spending through the positive effect on the marginal product of private capital.

The following conditions are introduced and they will be used later in order to obtain the dynamic expression describing the evolution in time of private capital:

$$w + rK + \pi = K^{1-\alpha}G^{\alpha} \tag{3.2.5}$$

$$T_p = \varphi_1 T = \varphi_1 \tau K^{1-\alpha}G^{\alpha} \tag{3.2.6}$$

[4] This concept of external economy due to G dates back to Meade (1952).

where w is labour income, π are profits.

Condition (3.2.5) simply states that total income must equate total output of the economy, while (3.2.6) comes from the definition of government lump-sum transfers to households T_p, as clarified below in section 3.2.3.

3.2.2 Consumers

Households are assumed to have uncertain lifetimes according to the model by Blanchard (1985). Hence, we shall assume that they face a constant instantaneous probability of death λ throughout their life. Their expected remaining life is $1/\lambda$ and it is constant throughout their life. Agents are of different ages and have different levels of wealth, but they all have the same propensity to consume. This approach allows for flexibility: the expected life $1/\lambda$ is interpreted as an horizon index that can be chosen anywhere between 1 and infinity to study the effects of the horizons of agents on the behavior of the economy. The limiting case of infinite horizons will occur by letting λ go to zero since this implies that $1/\lambda$ tends to infinity. It is assumed that there is no inter-generational bequest motive which, together with the assumption of uncertain lifetime, implies — as we will see later — a role for an insurance market.

Each consumer does not consider any choice regarding the allocation of her time endowment between labour and leisure. In other words, labour is inelastically supplied and the consumer supplies a constant amount of labour. The expected lifetime utility of the individual i is given by:

$$U^i = \int_t^{\infty} \ln C^i e^{-(\rho+\lambda)t} dt \tag{3.2.7}$$

The instantaneous utility function is assumed to have a logarithmic form. The rate of time preferences ρ is increased by the probability of death λ. The higher the probability of death, the more heavily consumers discount the future[5] and given the assumption that λ is constant throughout consumers'

[5] Cass and Yaari (1967) provide a theoretical proof of the fact that the effect of the

life, it is possible to assume a constant propensity to consume as well. This way of modelling the case of finite horizons can be regarded as an application of Blanchard (1985) to a standard Ramsey-type model with endogenous growth.

The Blanchard model has the merit of allowing for aggregation in OLGs models. On the other hand, it suffers from the drawback of abstracting from the life-cycle aspect of the individual consumption behavior. This limitation can be overcome by combining the Blanchard-type consumer with the standard representative agent model. Indeed, by doing so, no aggregation procedure is needed and the relationship between finite horizons and the evolution in time of consumption can be analytically determined by referring the analysis to the representative agent.

The inter-temporal budget constraint faced by the consumer must take into consideration the role played in the economy by the government. For this reason, the consumer budget constraint proposed by Greiner (1999) — see (2.6.17) in section 2.6.2, page 44 — is modified in order to adapt it to our framework[6]:

$$\dot{K} = \{[w + (r + \lambda) K + \pi] (1 - \tau) + T_p - C\} \left(\frac{1}{1 - \theta_S}\right) \tag{3.2.8}$$

The rationale behind this budget constraint is that the insurance covers only asset wealth: the consumer receives (pays) λK for every period of her life from (to) the insurance company and the amount K is paid to (cancelled by) the insurance company when the consumer dies. By using the conditions (3.2.5) and (3.2.6) and solving the budget constraint for $\dot{K}/K$ we obtain:

$$\frac{\dot{K}}{K} = K^{-\alpha} G^{\alpha} \frac{1 - \tau (1 - \varphi_1)}{1 - \theta_S} + \frac{\lambda (1 - \tau)}{1 - \theta_S} - \frac{C}{K (1 - \theta_S)} \tag{3.2.9}$$

This expression generalizes the dynamic equation of private capital in

probability to death is to raise the individual rate of time preference.

[6] Notice the analogy with the budget constraint used by Mourmouras and Lee (1999) shown in equation (2.6.22) (see subsection 2.6.3, page 48).

Greiner (1999), differing from the latter for the term that includes the parameter λ. We could then apply this expression to both cases of infinitely lived and uncertain lifetime consumers by simply imposing this parameter to equal to some value between 0 and 1.

The existence of a unique solution to the households' optimization problem is subject to the condition that K and G are bounded by the increasing function $e^{\gamma t}$, where $0 < \gamma < (\rho + \lambda)$. Provided that such a condition holds, the Pontryagin's maximum principle can be used to derive an optimal solution to (3.2.7) subject to (3.2.8), to which is associated the following Hamiltonian[7]:

$$H = \ln Ce^{-(\lambda+\rho)t} + \psi\frac{1}{1-\theta_S}\left\{[w + (r+\lambda)K + \pi](1-\tau) + T_p - C\right\} \tag{3.2.10}$$

and the following necessary optimality conditions:

$$\frac{\partial H}{\partial C} = 0 \Rightarrow \frac{1}{C}e^{-(\lambda+\rho)t} = \frac{\psi}{1-\theta_S} \tag{3.2.11}$$

$$\frac{\partial H}{\partial K} = -\dot{\psi} \Rightarrow \psi\frac{(r+\lambda)(1-\tau)}{(1-\theta_S)} = -\dot{\psi} \tag{3.2.12}$$

$$\frac{\partial H}{\partial \psi} = \dot{K} \Rightarrow \frac{1}{1-\theta_S}\left\{[w + (r+\lambda)K + \pi](1-\tau) + T_p - C\right\} = \dot{K} \tag{3.2.13}$$

The first order conditions (3.2.11)-(3.2.13) are also sufficient if the following transversality condition is satisfied:

$$\lim_{t\to\infty} e^{-(\rho+\lambda)t}\psi(K - K^*) \geq 0 \tag{3.2.14}$$

Substituting (3.2.11) into (3.2.12), we obtain:

$$(1-\theta_S)\frac{1}{C}e^{-(\lambda+\rho)t}\frac{(r+\lambda)(1-\tau)}{(1-\theta_S)} = -\dot{\psi} \tag{3.2.15}$$

where:

$$-\dot{\psi} = \left\{(1-\theta_S)\left[-\frac{1}{C^2}\dot{C}e^{-(\lambda+\rho)t} - \frac{1}{C}(\lambda+\rho)e^{-(\lambda+\rho)t}\right]\right\} \tag{3.2.16}$$

[7] Where ψ is the shadow price of capital.

Substituting (3.2.16) into (3.2.15), the Euler equation is derived:

$$\frac{\dot{C}}{C}=\frac{r(1-\tau)-\lambda(\tau+\theta_S)-\rho(1-\theta_S)}{(1-\theta_S)} \tag{3.2.17}$$

Imposing $\tau = \theta_S = \lambda = 0$, it is possible to obtain $\dot{C} = (r-\rho)C$, which gives the Euler equation for the case of a CRRA instantaneous utility function with elasticity of substitution equal to 1, when no government issues are considered and consumers live infinite lives.

We can now substitute the profit maximizing condition in the Euler equation in order to derive the equation of motion for consumption:

$$\frac{\dot{C}}{C}=\frac{(1-\tau)}{1-\theta_S}(1-\alpha)\left(\frac{G}{K}\right)^{\alpha}-\frac{\lambda(\tau+\theta_S)}{1-\theta_S}-\rho \tag{3.2.18}$$

where (3.2.18) states that consumption is decreasing over time in the subjective rate of discount as well as in the probability of death parameter. A higher value of the discount factor ρ will reduce consumption growth and this effect will be even stronger in the presence of a positive probability of death. The role played by λ in decelerating consumption growth over time can also be seen from the expression for the dynamics of private capital (3.2.9) where such a parameter enters with a positive sign.

3.2.3 Government

The government collects taxes T from total income produced in the economy and uses taxes to finance public consumption C_p, lump-sum transfers to households T_p, investment in public capital G and investment subsidies to firms $\theta_S\dot{K}$. No public debt issues are considered in the model and the government budget constraint is the same as in Greiner (1999) — see equation (2.6.16) in subsection 2.6.2, page 44.

Recalling the definition of the production function in (3.2.1) and assuming that the government uses shares φ_1 and φ_2 of tax revenue for lump-sum transfers to households and public consumption respectively (with φ_1 and

φ_2 defined between 0 and 1; $\varphi_1 + \varphi_2 < 1$), the budget constraint is written as follows:

$$\tau K^{1-\alpha} G^{\alpha} = \dot{G} + (\varphi_1 + \varphi_2)\, \tau K^{1-\alpha} G^{\alpha} + \theta_S \dot{K} \qquad (3.2.19)$$

Substituting $\dot{K}$ from the consumer budget constraint, we derive the dynamic equation of public capital:

$$\begin{aligned} \frac{\dot{G}}{G} &= K^{1-\alpha} G^{\alpha-1} \left\{ \tau (1 - \varphi_1 - \varphi_2) - \frac{\theta_S}{1-\theta_S} \left[1 - \tau (1 - \varphi_1)\right] \right\} \\ &- \frac{\theta_S}{1-\theta_S} \left[\frac{\lambda K (1-\tau)}{G} - \frac{C}{G} \right] \qquad (3.2.20) \end{aligned}$$

This equation differs from the one in Greiner (1999) for the term $\theta_S/(1-\theta_S) \cdot \lambda K(1-\tau)/G$, which is equal to zero for a null λ. Thus, as for the dynamic equations of consumption and private capital, we have an expression capable of treating the infinite horizon scenario as a limiting case.

3.2.4 The economy

The economy is described by the system of the differential equations (3.2.9), (3.2.18) and (3.2.20). We express public capital and private consumption in terms of private capital and define $\dot{x}/x = \dot{G}/G - \dot{K}/K$; $\dot{c}/c = \dot{C}/C - \dot{K}/K$. By doing so, the system (3.2.9)-(3.2.18)-(3.2.20) is reduced to[8]:

$$\begin{aligned} \frac{\dot{x}}{x} &= x^{\alpha-1} \left\{ \tau (1 - \varphi_1 - \varphi_2) - \frac{\theta_S}{1-\theta_S} \left[1 - \tau (1 - \varphi_1)\right] \right\} \\ &- x^{\alpha} \frac{1 - \tau (1 - \varphi_1)}{1-\theta_S} + \left(1 + \frac{\theta_S}{x}\right) \left[\frac{c}{1-\theta_S} - \frac{\lambda (1-\tau)}{1-\theta_S} \right] \qquad (3.2.21) \end{aligned}$$

$$\frac{\dot{c}}{c} = \left[x^{\alpha} \frac{(1-\tau)}{1-\theta_S} (1-\alpha) - \frac{\lambda (1+\theta_S)}{1-\theta_S} - \rho \right] - x^{\alpha} \frac{1 - \tau (1-\varphi_1)}{1-\theta_S} + \frac{c}{1-\theta_S} \qquad (3.2.22)$$

Proposition 1: *There exists a Balanced Growth Path (BGP) with endogenous growth for the economy described by (3.2.21)-(3.2.22) and such a BGP is unique.*

[8] It has to be noted that the two variables x and c are defined as x_1 and x_2 in the Futagami *et al.* model (see section 2.6.1, page 32).

Proof 1: see Appendix 3.B.1, page 92

The system (3.2.21)-(3.2.22) will have a steady state solution which will correspond to the balanced growth path (BGP) of the original system (3.2.9)-(3.2.18)-(3.2.20). In such a steady state the variables in the model will grow at the same rate and the long-run growth rate of the economy will be given by:

$$\gamma = \frac{\dot{C}}{C} = \frac{(1-\tau)}{1-\theta_S}(1-\alpha)\left(\frac{G}{K}\right)^{\alpha} - \frac{\lambda(\tau+\theta_S)}{1-\theta_S} - \rho \qquad (3.2.23)$$

Hence, the long-run rate of growth is decreasing both in the rate of time preferences ρ and in the probability of death parameter λ. Thus, we are able to capture the decelerating effect on economic growth caused by λ. From (3.2.23) it is clear that:

$$\left.\frac{\dot{C}}{C}\right|_{(\lambda=0)} > \left.\frac{\dot{C}}{C}\right|_{(\lambda>0)} \qquad (3.2.24)$$

Consumers with infinite lives are willing to postpone consumption in the future and to increase current saving. This behavior leads to a higher long-run growth rate. An increase in λ, *ceteris paribus*, is always associated with a lower long-run rate of growth of the economy.

The system (3.2.21)-(3.2.22) is similar to the one in Greiner (1999), who in turn refers to the model by Futagami *et al.* (1993). The system that we have produced departs from Greiner (1999) due to the inclusion of the probability of death and then it can be easily reduced to that form in order to make our results comparable with his conclusions. Moreover, our system collapses to that used by Futagami *et al.* (1993) when $\theta_S = \varphi_1 = \varphi_2 = \lambda = 0$.

The economy in Futagami *et al.* (1993) is characterized by saddle-path stability but it is assumed that tax revenues are used for public investment only. On the other hand, Greiner (1999) proves that the model is both locally and globally determinate, arguing that: "With inelastic labour supply there exists at most one BGP with endogenous growth and the Jacobian matrix of the system has one positive and one negative real root, *i. e.*, the rest point of the system is the saddle path". This implies that there exists a unique value for the initial level of consumption, which can be chosen freely by the household, such that the economy converges to the stable BGP in the long-run.

Proposition 2: *The Jacobian matrix of the system (3.2.21)-(3.2.22) has one positive and one negative real root, which implies that the unique BGP is saddle path.*

Proof 2: see Appendix 3.B.2, page 93

The numerical solution of the system allows to compare the steady state solutions across alternative life horizons scenarios. The numerical values of all the parameters involved in the analysis have been chosen to make the results as much as possible comparable with the ones obtained in previous studies. The initial values used for φ_1 and φ_2 are 0.35 and 0.40 respectively. Greiner (1999) uses 0.30 and so we choose to adopt this value[9]. With regard to the rate of time preferences ρ, the range of the values commonly used is between 0.01 and 0.04, which implies that the consumer is assumed to use an annual discount rate varying between 1% and 4%. Following Greiner (1999) and recalling that the model is concerned with the behavior of the economy in the long-run, we assume that one time period includes a spell of five years and we set the annual discount rate at 0.04, which will imply imposing $\rho = 0.2$. Finally, the income tax rate and the investment subsidies parameters are initially set at 0.15 and 0.10 respectively.

By imposing $\dot{c}/c = \dot{x}/x = 0$, and solving for x and c, we find the steady state solutions of the two variables. When the analysis is carried out in the finite horizons case, one would expect the steady state solutions to change.

[9] The value of the public capital share α usually used in the literature is around 0.30. Barro (1990) assumes $\alpha = 0.25$.

This is due to the fact that the steady state values for x and c are affected by the probability of death: the higher the probability of death, the higher they will be. Since for a higher probability of death, households have a disincentive to postpone consumption in the future, the steady state level of consumption to private capital ratio is increasing in λ. The steady state solution for x provides the value of the ratio G/K at which consumption is constant over time. As the probability of death increases, x will increase, since a larger amount of government spending is required in order to promote economic growth, thus compensating for the negative effect on growth caused by the higher level of current consumption.

The solution of the model for $\lambda = 0$ is ($x = 0.0711$, $c = 0.3191$). When the probability of death is set at 0.03 the new steady state solution is given by $x = 0.0759$ and $c = 0.3549$; when $\lambda = 0.06$, we find $x = 0.0808$ and $c = 0.3906$. The balanced growth rate is $\gamma = 0.01983$ in the infinite horizons

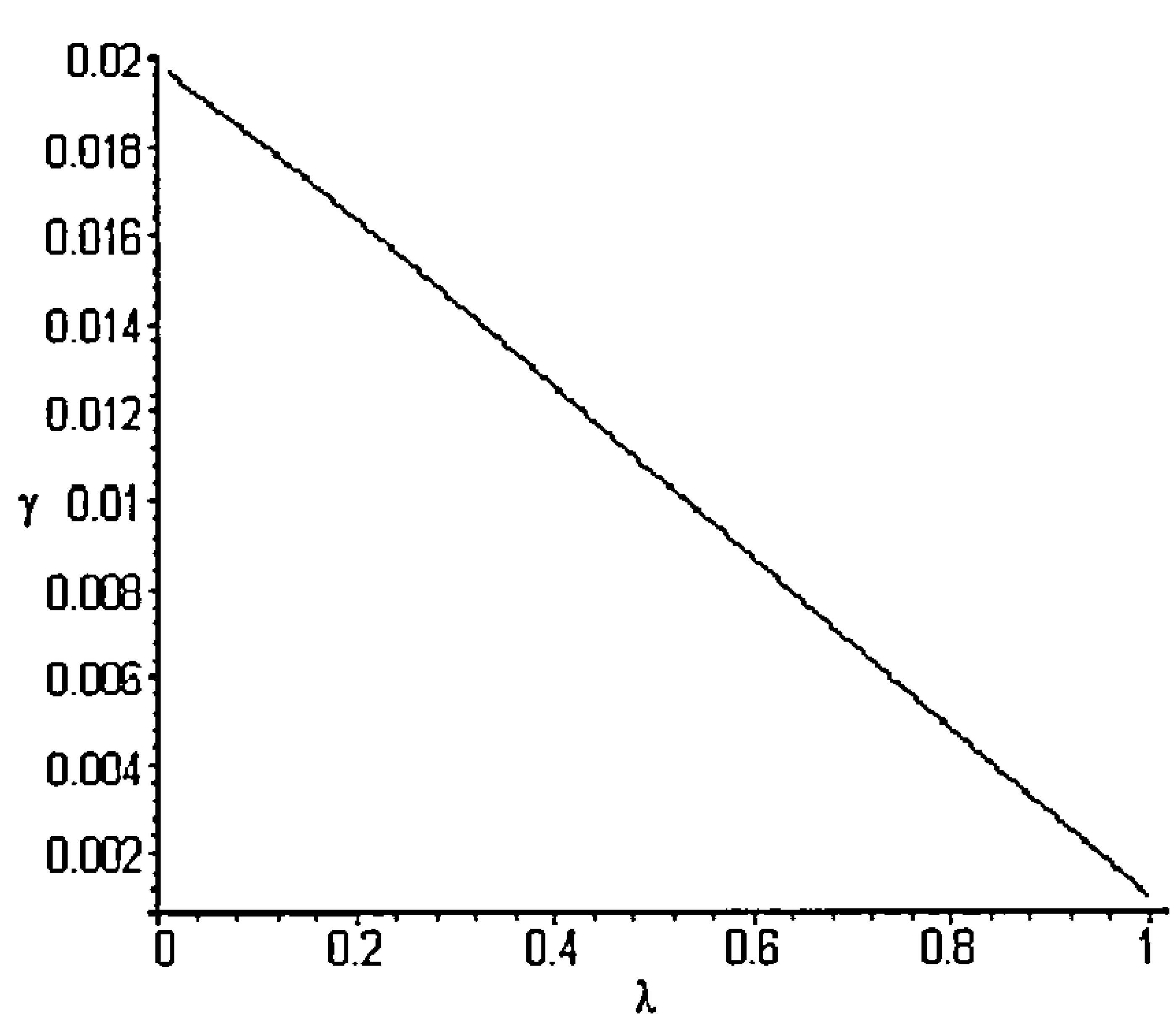

Figure 3.2.1: The Relationship between γ and λ

case, $\gamma = 0.01934$ for $\lambda = 0.03$, and $\gamma = 0.01883$ when $\lambda = 0.06$. Thus, the balanced growth rate is lower under the uncertain lifetime assumption and is decreasing in the probability of death parameter.

The role played by λ is to increase the propensity to consume: the higher the probability of death, the higher the willingness of consuming today, and this circumstance negatively affects the long-run rate of growth. In the general case $0 \leq \lambda \leq 1$, γ is linked to λ by the linear relationship shown in Figure 3.2.1.

3.3 Fiscal Policy

Given the theoretical framework provided above, it is possible to analyse the growth effects of changes in fiscal parameters, depicting the role played by the uncertain lifetime hypothesis. The following subsections 3.3.1-3.3.3 deal with the relationships between fiscal policy tools and long-term economic growth. In particular, the growth hampering effect of a rise in either public consumption or lump-sum transfers is described in subsection 3.3.1. In subsection 3.3.2, it is derived the growth maximizing income tax rate and how its value is influenced by the presence of public consumption and transfers to households. Finally, subsection 3.3.3 focuses on the ambiguous effect of investment subsidies on growth and their relationship with the growth maximizing income tax rate.

3.3.1 Public consumption and lump-sum transfers

The share of government expenditure devoted to public consumption has been modelled with the parameter φ_2. Starting with the infinite horizons case, an increase in public consumption implies that more resources will be devoted to unproductive purposes as opposed to public investment and private investment subsidies. As a direct consequence, productive public expenditure will decrease, which in turn will negatively affect growth. The expected effect of an increase in the parameter φ_2 is hence a decline in the balanced growth rate γ. Such a decelerating effect on long-term economic growth will be reflected by a smaller steady state value of $x = G/K$, as shown in equation (3.2.23).

Given a positive probability of death, let us consider the consequences of increasing public consumption. The key feature to be taken into account is that, compared to the infinite horizons scenario and other things being equal, the economy will always grow at a lower rate in the long-run. An increase in φ_2, is still expected to impact negatively long-term economic growth, but will such an impact be more or less effective than in the infinite horizons scenario?

Proposition 3: *The long-run rate of growth of the economy γ is decreasing in public consumption φ_2. Increases in φ_2 of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$:*

$$\frac{\partial \gamma}{\partial \varphi_2} < 0 \tag{3.3.1}$$

$$\left.\frac{\partial \gamma}{\partial \varphi_2}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_2}\right|_{(\lambda=0)} \tag{3.3.2}$$

Proof 3: see Appendix 3.B.3, page 95

Hence, the decrease in the long-run rate of growth caused by an increase in φ_2 will be lower when $\lambda > 0$.

After increasing φ_2 from 0.40 to 0.45, the balanced growth rate is lowered as expected in the three scenarios considered ($\lambda = 0$, $\lambda = 0.03$ and $\lambda = 0.06$, see Table 3.3.1). A higher share devoted to public consumption causes a decline in the share of government expenditure devoted to productive purposes and, as a consequence, economic growth is negatively affected. The peculiar property of the finite horizons case is the following: given the same increase in φ_2, the negative effect on growth is smaller than in the infinite horizons case. The decline in γ is denoted with γ_1 in Table 3.3.1. Starting from the initial steady state, after an increase in public consumption the growth rate decreases less in the finite horizon case than in the infinite one.

Increasing transfers to households (a higher value for the parameter φ_1) will lead to two opposite effects. On one side, a smaller share of total government expenditure will be devoted to productive uses, implying a reduction in the balanced growth rate. On the other hand, an income effect will take place, making consumers richer than before. However, provided that transfers are lump-sum, they will not affect decisions concerning the allocation of private resources between consumption and savings. Thus, the only expected effect will be the first one and a lower long-run growth rate is predictable. This is also true in the presence of a positive λ, but once again the question of interest is whether or not an increase in households transfers will affect economic growth in the same way.

Table 3.3.1: An increase in public consumption

	$\lambda = 0$		$\lambda = 0.03$		$\lambda = 0.06$	
	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$
x	0.0711	0.0576	0.0759	0.0620	0.0808	0.0665
c	0.3191	0.3106	0.3583	0.3465	0.3941	0.3824
γ	0.01983	0.01617	0.01934	0.01575	0.01883	0.01531
γ_1	-0.00366		-0.00359		-0.00352	
$\varphi_1 = 0.35, \alpha = 0.3,\ \rho = 0.2, \tau = 0.15, \theta = 0.1$						

Proposition 4: *The long-run rate of growth of the economy γ is decreasing in lump-sum transfers φ_1. Increases in φ_1 of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$:*

$$\frac{\partial \gamma}{\partial \varphi_1} < 0 \tag{3.3.3}$$

$$\left.\frac{\partial \gamma}{\partial \varphi_1}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_1}\right|_{(\lambda=0)} \tag{3.3.4}$$

Proof 4: see Appendix 3.B.3, page 95

Table 3.3.2 refers to a fiscal policy experiment similar to the one described above, the only difference being the increase in φ_1 instead of φ_2. Thus, the objective of the analysis is to describe the impact of increasing lump-sum transfers to households on long-term economic growth, other things being equal. Letting φ_1 varying from 0.35 to 0.40, the long-run rate of growth is lowered both when $\lambda = 0$ and for a positive probability of death, the latter case being characterized by a smaller reduction in γ (see γ_1 in Table 3.3.2).

Table 3.3.2: An increase in lump-sum transfers

	$\lambda = 0$		$\lambda = 0.03$		$\lambda = 0.06$	
	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$
x	0.0711	0.0576	0.0759	0.0620	0.0808	0.0665
c	0.3191	0.3138	0.3549	0.3498	0.3906	0.3857
γ	0.01983	0.01617	0.01934	0.01575	0.01883	0.01531
γ_1	-0.00366		-0.00359		-0.00352	
$\varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2, \tau = 0.15, \theta = 0.1$						

Hence, increases in lump-transfers and public consumption are both less effective in lowering the long-run rate of growth of the economy under the assumption of uncertain lifetime than in the infinite horizon scenario. We note that the only distinguishing feature is that when the government switches resources from public consumption to lump-sum transfers, the steady state value of c becomes higher.

Figure 3.3.1 illustrates the relationship between γ and φ_i $(i = 1, 2)$, in a more general case. Indeed, the picture is obtained by letting vary φ_i between 0 and 0.60, holding fixed the share devoted to the other category of unproductive government expenditure. In both cases, the three top curves

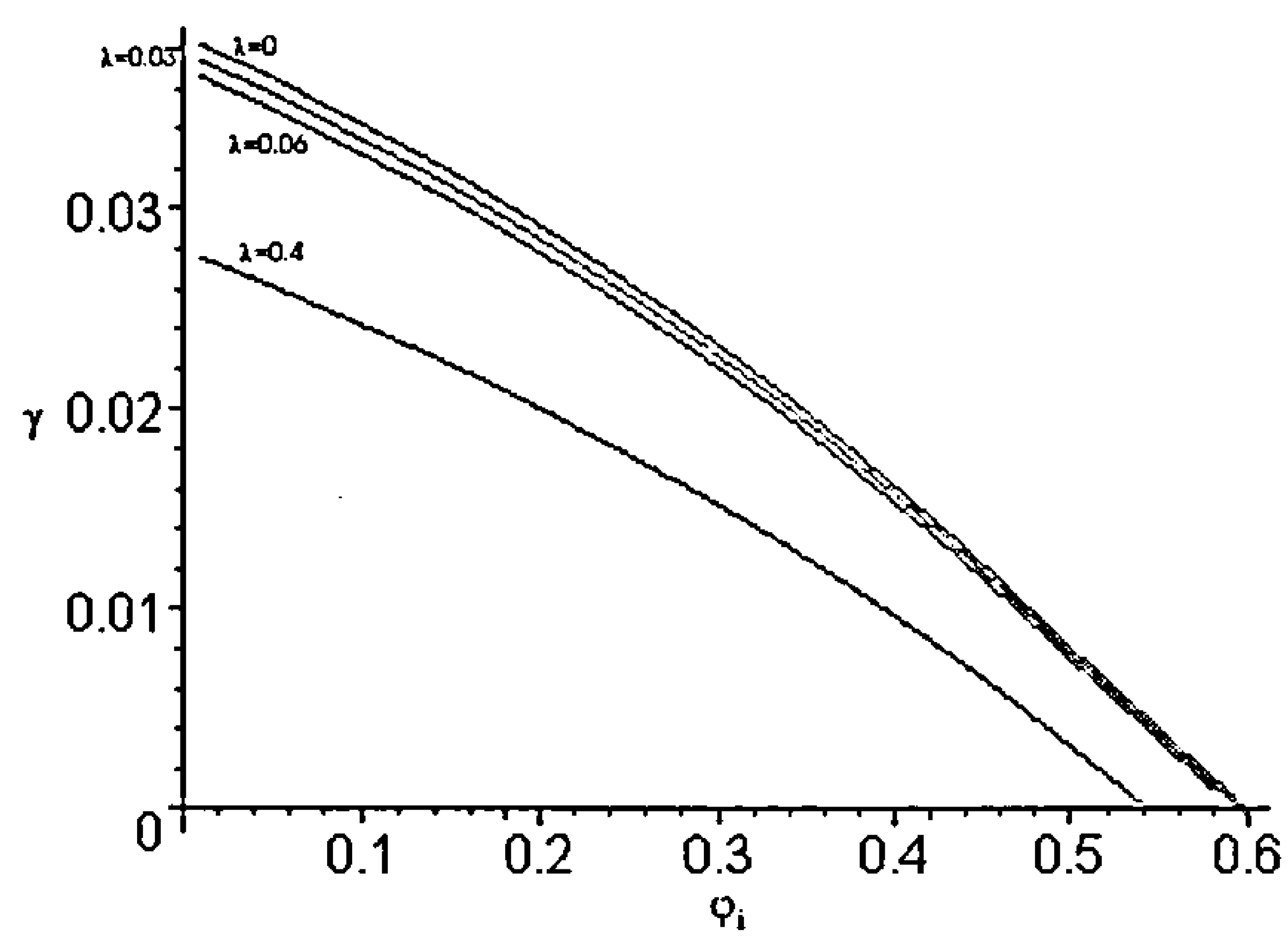

Figure 3.3.1: The relationship between γ and φ_i

(starting with the highest one) refer to values for λ of 0, 0.03, and 0.06 respectively. The lowest and flattest curve represents the limiting case of $\lambda = 0.4$[10]. As pointed out in propositions 3 and 4, an increase in either φ_1 or φ_2 leads to smaller and smaller reductions in the balanced growth rate, as the probability of death parameter is set to a higher value.

3.3.2 *Public investment*

Increasing the income tax rate τ yields two effects operating in opposite directions. In the infinite horizons case, given an increase in τ, the first effect to be taken into consideration is the higher taxation on returns on capital, which implies a disincentive to save and, as a consequence, a reduction in private investment with the effect of lowering long-term economic growth. However, an opposite effect will take place: for a given level of income, a higher income tax rate implies higher tax revenues which in turn leads to higher investment in public capital and accelerates economic growth. Thus, the net effect of an increase in τ might be either positive or negative,

[10] Both maximum values of φ_i and λ have been chosen to satisfy the condition $0 < \gamma < (\rho + \lambda)$.

depending on whether the second effect offsets the first one. The well-known *Barro rule* states that the optimal provision of public investment implies that a unit increase in government spending implies a unit increase in output. With a Cobb-Douglas production function, this means that the optimal government spending is equal to its share in the production function.

In section 2.6.2, we have studied that the extension of the Barro rule provided by Greiner (1999) — see equation (2.6.19) — implies that higher levels of unproductive public spending and/or generous investment subsidies, will force the government to increase productive investment in order to compensate for the negative effect on the long-run rate of growth.

The model presented in the previous section assumes the same fiscal policy tools proposed by Greiner (1999), generalizing his framework to include the case of finite horizons. Hence, the question of how the optimal level of τ might be affected by the uncertain lifetime hypothesis arises. Mourmouras and Lee (1999) find τ_{max} to be independent of the probability of death parameter. In other words, the optimal level of public investment provision does not depend on the consumption externality due to the uncertain lifetime hypothesis. They find the *Barro rule* to be satisfied both in finite and infinite lives scenarios, pointing out that this rule is only determined by the production side of the economy, which they model as in Barro (1990). On the other hand, we expect that the new growth maximizing level of τ will be affected by the different effect on the disincentive to save caused by the higher taxation on returns on capital. Indeed, once the probability of death is introduced in the model, the growth-maximizing income tax rate becomes:

$$\tau_{max}|_{(\lambda>0)} = \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \qquad (3.3.5)$$

Proposition 5: *There exists a growth maximizing income tax rate* τ_{max} *both in the infinite and the finite horizons scenarios, the first one being higher than the latter one:*

$$\tau_{max}|_{(\lambda>0)} < \tau_{max}|_{(\lambda=0)} \tag{3.3.6}$$

Proof 5: see Appendix 3.B.3, page 96

Hence, the growth maximizing level of the income tax will be reached before if consumers have uncertain lifetime than in the infinite horizons case. In contrast with Mourmouras and Lee (1999), optimal public investment turns out to be dependent on the horizon index. Such a relationship is depicted in (3.3.5), but it can be easily seen by directly compare our set up with the model provided by Barro (1990). This can be done by simply deleting θ_S in (3.3.5). By doing so, a *modified Barro rule* is obtained:

$$\tau_{max}|_{(\lambda>0,\theta_S=0)} = \frac{x^\alpha \alpha}{x^\alpha + \lambda} \tag{3.3.7}$$

For $\lambda = 0$, (3.3.7) is equivalent to (2.5.13) — see subsection 2.5.1, page 24 — and gives the *Barro rule*: the optimal provision of public investment is given by the share of public capital in the aggregate production function. However, if consumers live finite lives, such an optimal level will be lowered by the consumption externality due to λ.

In order to simulate an increase in public investment in infrastructure services, we let vary the income tax rate parameter τ between 0.15 and 0.50. The main interest of this experiment is to analyse the differences that emerge by assuming alternative values of λ. Namely, we want to test the existence of the Barro curve. This result is non trivial for two reasons. First, the model includes additional categories of expenditures with respect to Mourmouras and Lee (1999). Second and more importantly, the consumption externality due to the finite lives assumption is likely to affect the determination of τ_{max}. Holding fixed the values of all other parameters at their respective starting levels, the model is solved for values of τ varying from 0.15 to 0.50 (see Table 3.3.3).

Starting from the infinitely lived consumers case, γ increases for higher values of τ up to a point, after which it starts falling. The balanced growth rate reaches its maximum value for $\tau = 0.383$. Thus, this is the optimal value of the income tax rate. A similar behavior can be depicted under the uncertain lifetime hypothesis. For increasing values of the income tax rate, the balanced growth rate increases up to a point and then it goes down: the relationship between γ and τ takes the form of a hump-shaped curve in the finite horizons case.

Table 3.3.3: An increase in public investment

	$\lambda = 0$			$\lambda = 0.03$			$\lambda = 0.06$		
τ	0.15	0.383	0.50	0.15	0.359	0.50	0.15	0.335	0.50
x	0.071	0.360	0.667	0.0760	0.328	0.691	0.081	0.298	0.716
c	0.319	0.414	0.468	0.355	0.441	0.504	0.391	0.468	0.54
γ	0.0198	0.0306	0.0289	0.0193	0.0283	0.0256	0.188	0.0261	0.224
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2, \theta = 0.1$									

In Figure 3.3.2, the values of γ are plotted against the values taken by τ when $\lambda = 0$, $\lambda = 0.03$ and $\lambda = 0.06$. The highest curve refers to the infinite horizon case, the lowest one to a probability of death equal to 0.06. As argued above, the finite horizon case is always characterized by a lower balanced growth rate and this implies a Barro curve closer to the x-axis. For each given value of τ, γ is decreasing in λ for the role played by the probability of death in reducing economic growth. Hence, the higher λ, the lower the Barro curve. This result is in contrast with Mourmouras and Lee (1999), who find the optimal role for public investment provision to be independent of λ, due to the fact that the Barro rule only arises from the production side of the economy. The present framework, instead, captures the consumption externality effect of λ, explicitly accounting for its impact on optimal fiscal policy.

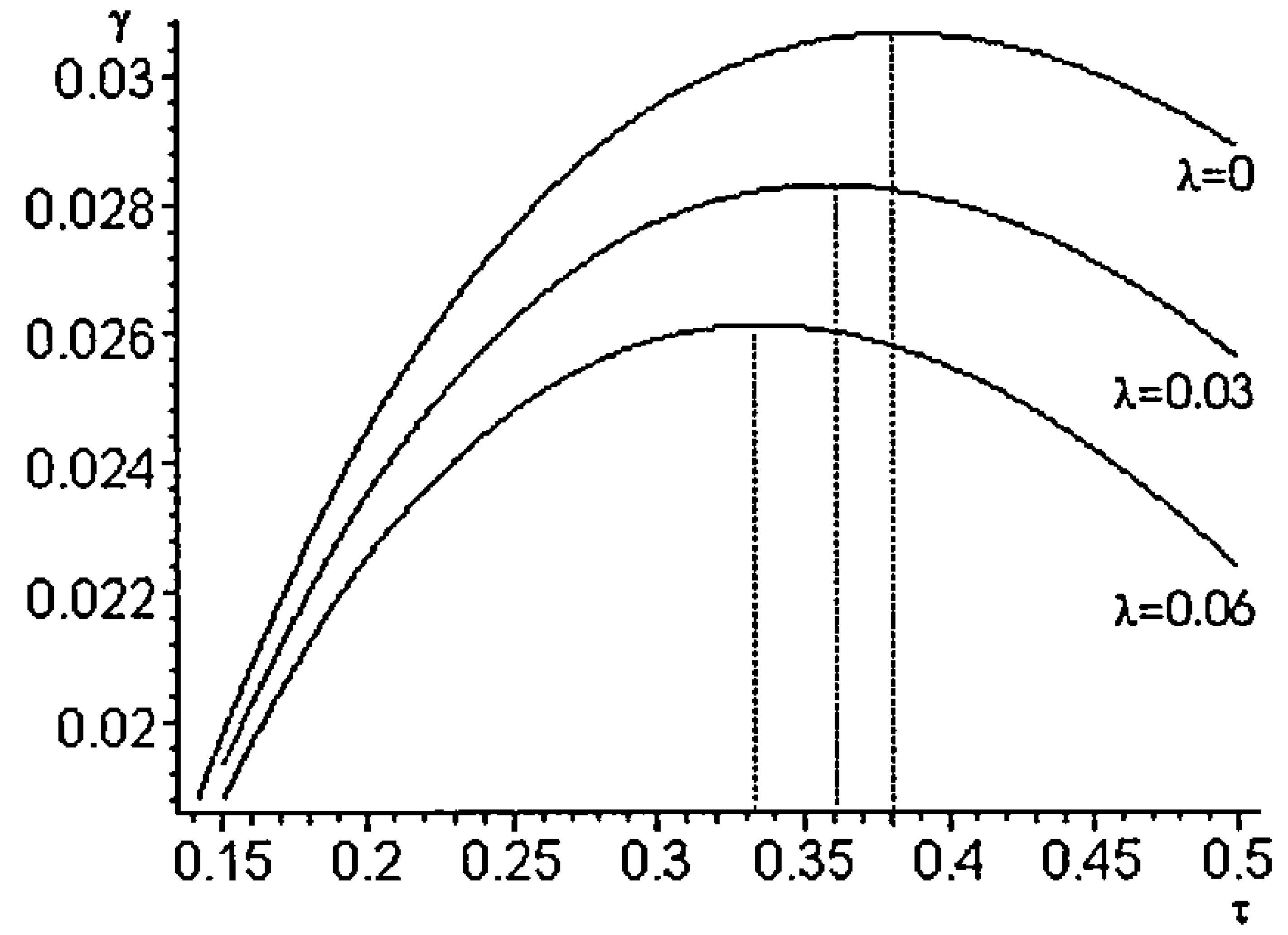

Figure 3.3.2: Barro Curve, Finite and Infinite Horizons

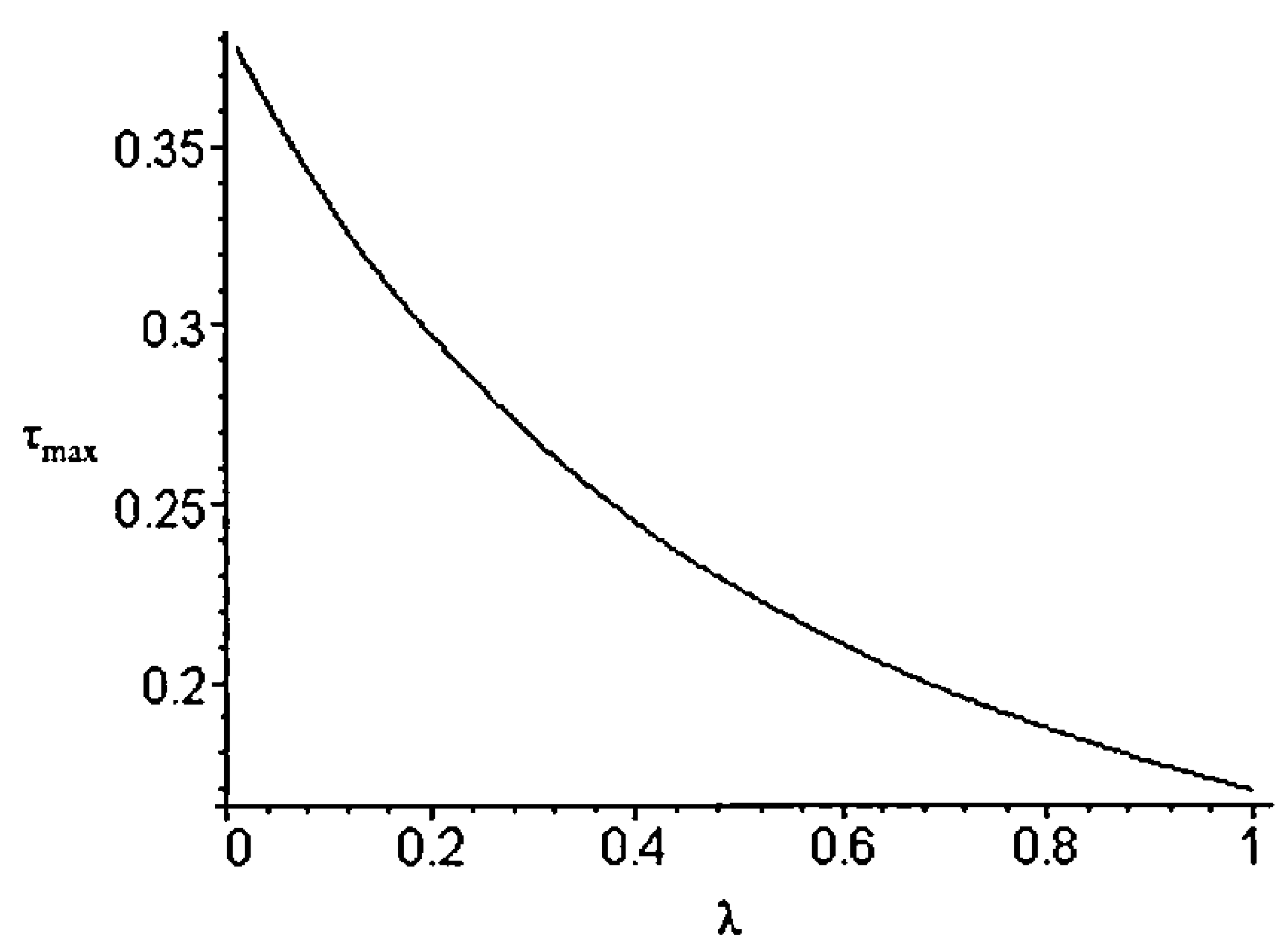

Figure 3.3.3: The relationship between τ_{max} and λ

Indeed, as shown in Table 3.3.3, when the probability of death is fixed at 0.03 the growth maximizing income tax rate is lower with respect to the case of $\lambda = 0$ and it is even lower for $\lambda = 0.06$. In the first case, the maximum value of γ (0.02830) is achieved when $\tau_{max} = 0.359$, while the growth maximizing income tax rate becomes 0.335 when λ is set at 0.06

(with $\gamma = 0.02614$).

This is coherent with the result summarized in proposition 5. The negative relationship between λ and τ_{max} is shown in Figure 3.3.3 for the more general case $0 \leq \lambda \leq 1$.

Public investment with higher public consumption or higher lump-sum transfers

Under the hypothesis of uncertain lifetime consumers, the relationships linking such an optimal rule for public investment to other fiscal policy tools will be also affected. As for unproductive public expenditure our results are summarized in proposition 6.

Proposition 6: *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in both public consumption φ_2 and lump-sum transfers to households φ_1.*

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(0\leq\lambda\leq 1)} > 0, \quad i = 1, 2 \tag{3.3.8}$$

Proof 6: see Appendix 3.B.3, page 96

Hence, regardless of the value of λ, in the presence of higher unproductive public expenditures, the need for a higher provision of public investment turns out to be always necessary.

When the government provides the economy with higher shares of either transfers to households or public consumption, it will also need to increase public investment in order to offset the growth hampering effect caused by the higher unproductive use of its resources.

In order to consider this fact, we solve the model assuming that the government devotes a share $\varphi_2 = 0.45$ to public consumption, comparing the outcome with the original scenario in which φ_2 was set at 0.40 (Table

3.3.3). The expected outcome will be a higher optimal income tax rate and the results of this experiment are shown in Table 3.3.4. In the infinite horizons case τ_{max} is 0.394, which confirms our expectation. For values of the probability of death equal to 0.03 and 0.06, the same behavior is observed: τ_{max} increases to 0.368 and 0.342 respectively.

Table 3.3.4: An increase in public investment with higher public consumption

	$\lambda = 0$			$\lambda = 0.03$			$\lambda = 0.06$		
τ	0.15	0.394	0.50	0.15	0.368	0.50	0.15	0.342	0.50
x	0.058	0.317	0.562	0.062	0.289	0.584	0.066	0.257	0.608
c	0.314	0.419	0.474	0.350	0.445	0.511	0.386	0.468	0.547
γ	0.016	0.0270	0.025	0.016	0.024	0.022	0.015	0.022	0.019
$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3, \rho = 0.2, \theta = 0.1$									

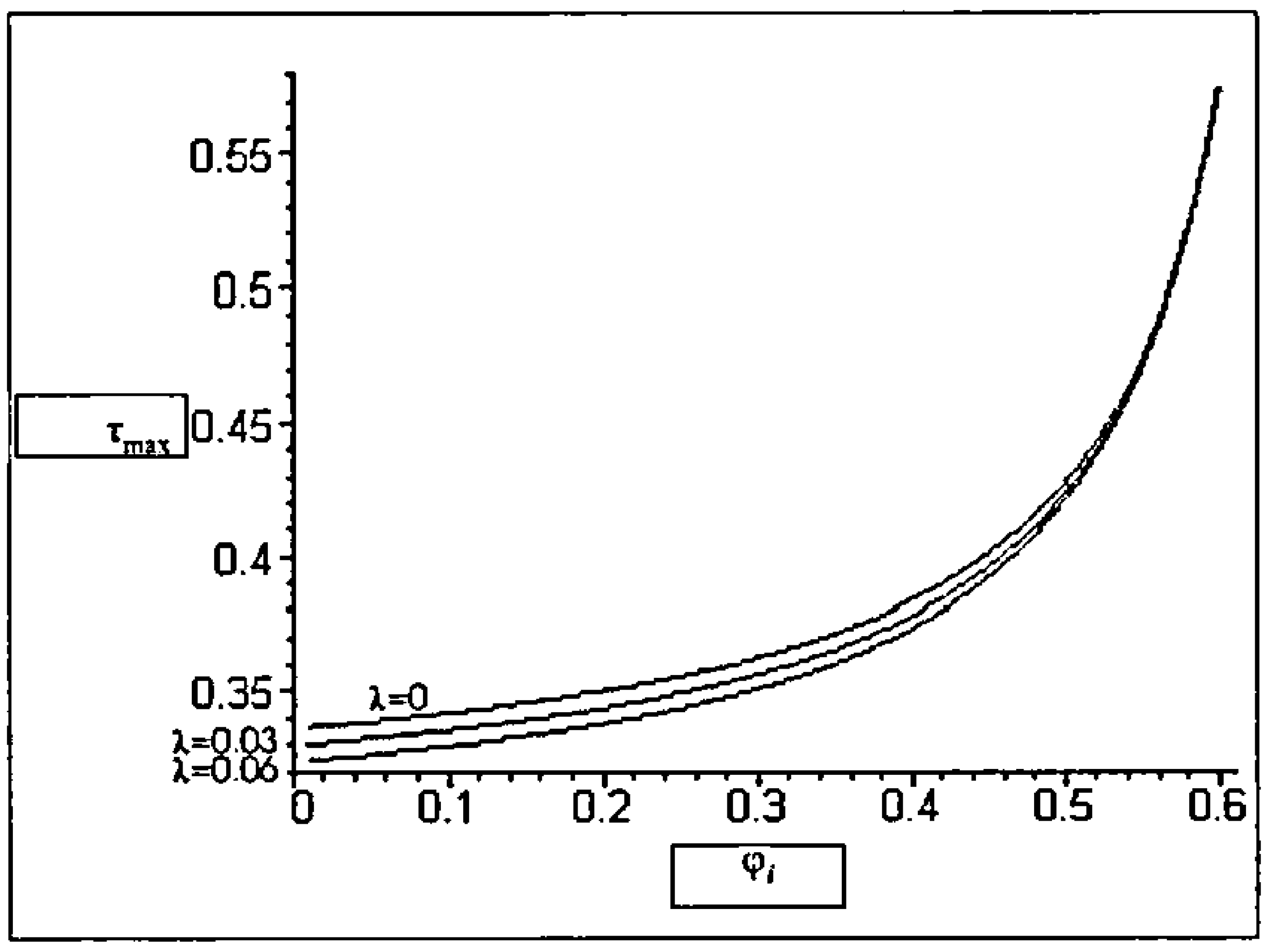

Figure 3.3.4: The relationship between τ_{max} and φ_i

An increase of the same amount either in φ_2 or in φ_1 will have an identical impact on the balanced growth rate. As a consequence, after an increase of either φ_2 or φ_1, τ_{max} will be affected in the same way. For this reason, the results of an increase in φ_1 are not shown.

Figure 3.3.4 shows the increasing relationship between unproductive uses of government resources and the optimal income tax rate.

3.3.3 Investment subsidies

The last fiscal policy tool to be considered are investment subsidies, represented by the parameter θ_S. As for the two categories of unproductive public spending, let us consider the growth effect of varying θ_S and the relationship linking this parameter to τ_{max}.

It is evident from the F.O.C. (3.2.11) that a change in θ_S affects consumers' marginal utility: higher investment subsidies lead to a reduction in the marginal utility for each given level of consumption. Moreover, from the consumer budget constraint it turns out that an increase in the parameter θ_S causes private investment to be cheaper. These two effects combined together will shift resources from consumption to investment in the private sector by increasing the opportunity cost of consumption. Thus, one will expect that the rate of growth of the economy will increase after the government's decision to provide the private sector with higher investment subsidies.

On the public side of the economy, however, devoting more resources to investment subsidies implies a depletion of resources from investment in public capital and thus a lower long-run rate of growth. As a consequence, the net effect resulting from the combination of the two effects in the private and in the public sectors is ambiguous. However, Greiner (1999) claims that there exists a growth-maximizing value for investment subsidies and that if it is in the interior $(0, 1)$ it will be determined by the elasticity of x with respect to θ_S on the balanced growth rate (see equation (2.6.18), page 45). The analysis provided by Greiner (1999) refers to the infinitely lived representative consumer case; what if we impose a positive probability of death? After the decision of the government to increase θ_S, the two opposite effects described above will occur again. On the public side of the economy, the decline in the share of public spending devoted to productive use will be the same as in the infinite lives scenario. On the other hand, we expect that the growth enhancing effect due to the decline in the marginal utility for each level of consumption will be larger than in the former case because of the presence of the probability of death. Hence, the growth maximizing level of θ_S will have a smaller value: it should be reached earlier than in the infinite horizons case.

Proposition 7: *There exists a growth maximizing value for investment subsidies, and such a value is decreasing in the probability of death parameter* λ*:*

$$\left.\frac{\partial \gamma}{\partial \theta_S}\right|_{(\lambda=0)} > (\leq) 0 \quad \text{when} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha\,(1-\theta_S)} \tag{3.3.9}$$

$$\left.\frac{\partial \gamma}{\partial \theta_S}\right|_{(\lambda>0)} > (\leq) 0 \quad \text{when} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha\,(1-\theta_S)} + \frac{\lambda\,(1+\tau)}{(1-\theta_S)^2} \tag{3.3.10}$$

Proof 7: see Appendix 3.B.3, page 99

The existence of a growth maximizing value of investment subsidies stated in proposition 7 is assessed by the simulation, whose results are summarized in Table 3.3.5[11]. The optimal value of θ_S in the infinite horizon case is found to be 0.113. A slightly lower θ_{max} (0.012) is obtained by re-running the experiment for a value of λ equal to 0.03. By setting a probability of death at 0.06, θ_{max} becomes 0.104.

The impact of public investment with higher investment subsidies

As for the impact of higher investment subsidies on the growth maximizing income tax rate, in analogy with public consumption and lump-sum transfers to households, our result is summarized in the following proposition.

[11] For this simulation, for comparability purposes it has been chosen the same value of the income tax rate (0.425) used by Greiner (1999).

Table 3.3.5: Growth maximizing level of investment subsidies

	θ	x	c	γ
$(\lambda = 0)$	0.07	0.4995	0.4469	0.03029
	0.113	0.4339	0.4281	0.03041
	0.15	0.3705	0.4085	0.03031
$(\lambda = 0.03)$	0.07	0.5115	0.4808	0.02759
	0.112	0.4520	0.4644	0.02767
	0.15	0.3944	0.4475	0.02752
$(\lambda = 0.06)$	0.07	0.5236	0.5148	0.02490
	0.104	0.4838	0.5044	0.02494
	0.15	0.4196	0.4866	0.02487
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \tau = 0.425$				

Proposition 8: *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in investment subsidies.*

$$\left.\frac{\partial \tau_{max}}{\partial \theta_S}\right|_{(0 \leq \lambda \leq 1)} > 0 \qquad (3.3.11)$$

Proof 8: see Appendix 3.B.3, page 99

Table 3.3.6 reports the results of an increase in θ_S from its starting value of 0.10 to 0.12, when λ is set to 0, 0.03 and 0.06 respectively. As stated before, increasing investment subsidies leads to the need for a higher optimal provision of investment in infrastructure. Such an impact on τ_{max} is evident by comparing the results in Table 3.3.6 with the ones reported in Table 3.3.3: in the infinite horizons scenario, τ_{max} increases from 0.383 to 0.399; for $\lambda = 0.03$, τ_{max} increases from 0.359 to 0.374; when $\lambda = 0.06$, τ_{max} increases from 0.335 to 0.35.

In the more general case of θ_S ranging from 0 to 0.15, such an increasing relationship between τ_{max} and θ_S is shown in Figure 3.3.5.

Table 3.3.6: An increase in public investment with higher investment subsidies

	$\lambda = 0$			$\lambda = 0.03$			$\lambda = 0.06$		
τ	0.15	0.399	0.50	0.15	0.374	0.50	0.15	0.35	0.50
x	0.0592	0.362	0.626	0.0648	0.330	0.655	0.070	0.302	0.685
c	0.308	0.412	0.458	0.345	0.438	0.496	0.382	0.465	0.533
γ	0.018	0.030	0.029	0.018	0.028	0.026	0.017	0.026	0.022
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2, \theta = 0.12$									

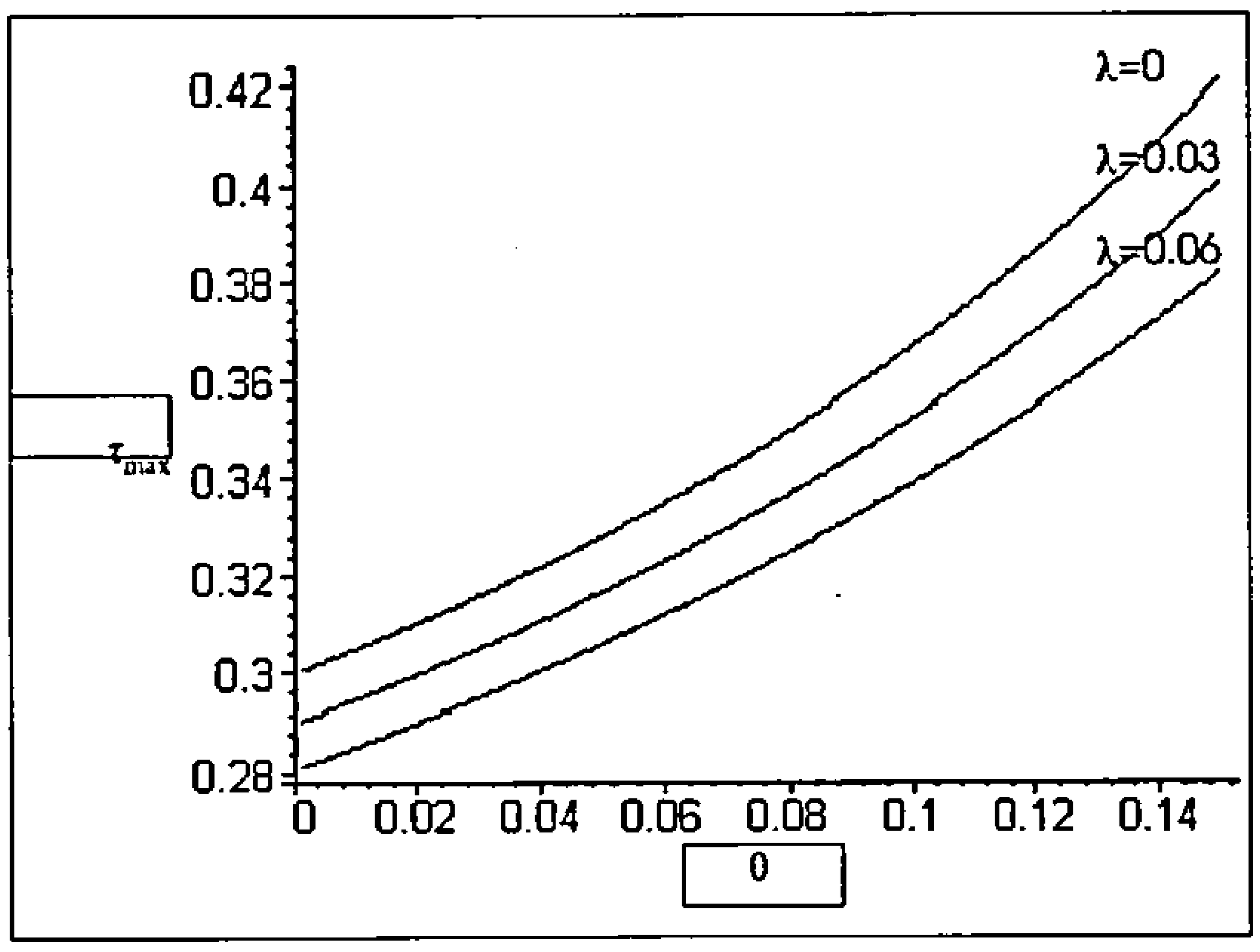

Figure 3.3.5: The relationship between τ_{max} and θ_S

3.4 Conclusions

This Chapter was aimed at studying the growth effects of fiscal policy in a Barro-type endogenous growth model with finite horizons. The model provides a flexible framework capable of studying the growth effects of fiscal policy both in infinite and finite horizons scenarios and reducing to limiting cases some recent Barro-type models. The optimal lifetime consumption plan has been determined within a standard representative agent model, the only difference being a rate of time preference augmented by a positive probability of death parameter. The government was assumed to run a balanced budget constraint, equating total expenditures to total revenues collected by levying a flat-rate income tax. I have distinguished between productive and unproductive categories of government expenditures. Productive public spending includes investment in public capital and private investment subsidies. On the other hand, public consumption and lump-sum transfers to households were assumed to be unproductive.

Comparing the two alternative scenarios of finite and infinite horizons, I have obtained results on (i) the growth effects of each category of government expenditures on long-run economic growth, and (ii) the relationships relating the Barro rule to the other categories of government expenditure.

Regarding the first set of conclusions, both categories of unproductive government spending are shown to have a decelerating effect on long-run growth. This result is in line with the existing literature and verified regardless of the assumption of uncertain lifetime. However, a rise of either lump-sum transfers to households or public consumption reduces the long-run rate of growth less in the finite than in the infinite horizon scenario. On the other hand, the growth effects of the two categories of productive expenditures are ambiguous, and for both I have derived a growth maximizing value. As for public investment, the Barro rule still holds in the infinite horizon scenario but, in contrast with the existing literature, is negatively linked to the probability of death parameter. This implies that the growth maximizing level of public investment is lower under the assumption of uncertain lifetime. Similarly, the growth maximizing level of private investment subsidies is reached earlier in the finite than in the infinite horizons scenario.

Relative to the second set of conclusions, the effects of public consumption, lump-sum transfers to households and investment subsidies on the optimal provision of public investment are similar. Indeed, it is shown that the growth maximizing level of public investment tends to increase in the presence of higher levels of other categories of expenditures. This result takes place regardless of the assumption on uncertain lifetime.

The model is based on some restrictive hypothesis and, as a consequence, can be extended along a number of directions. For instance, the assumption of no labour-leisure choice could be relaxed in favor of the assumption of endogenous labour supply, providing an extension on the consumption side. Moreover, it is assumed the absence of public debt and it is solely considered the case of proportional income taxation. Hence, a further extension of the model could cover the analysis of alternative mixes of different categories of expenditures and different structures of taxation — distortionary and non-distortionary — and/or alternative sources of financing — deficit or taxation.

3.A Fiscal Policy Experiment

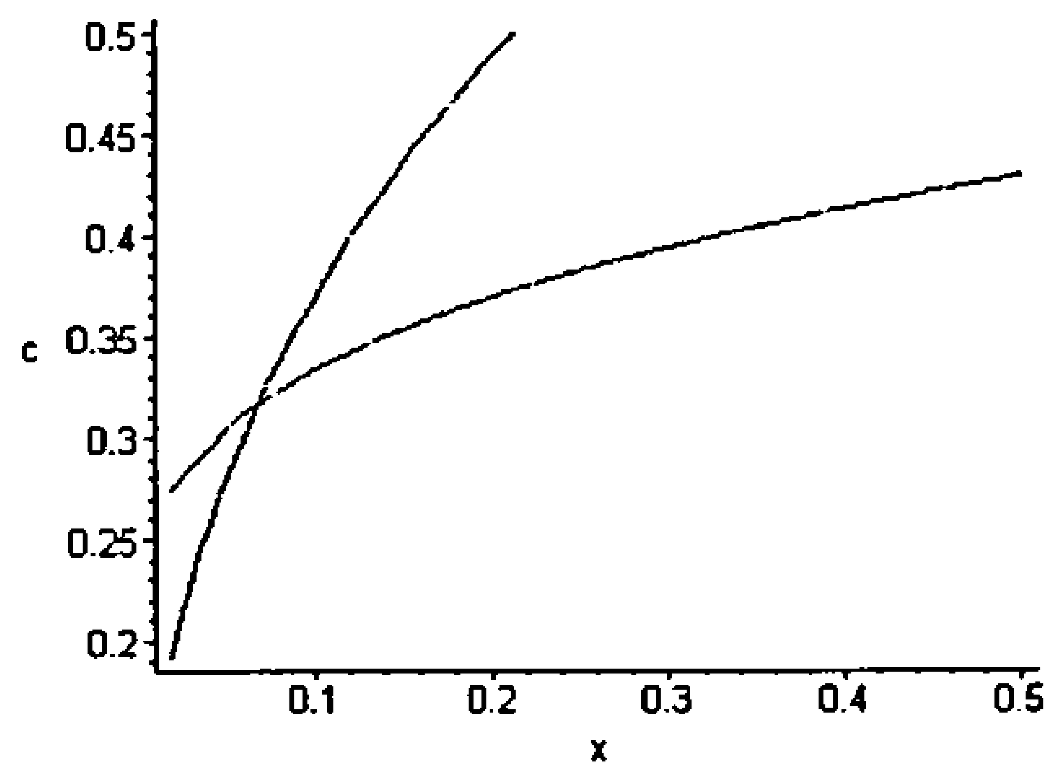

Figure 3.A.1: Steady State solution ($\lambda = 0$)

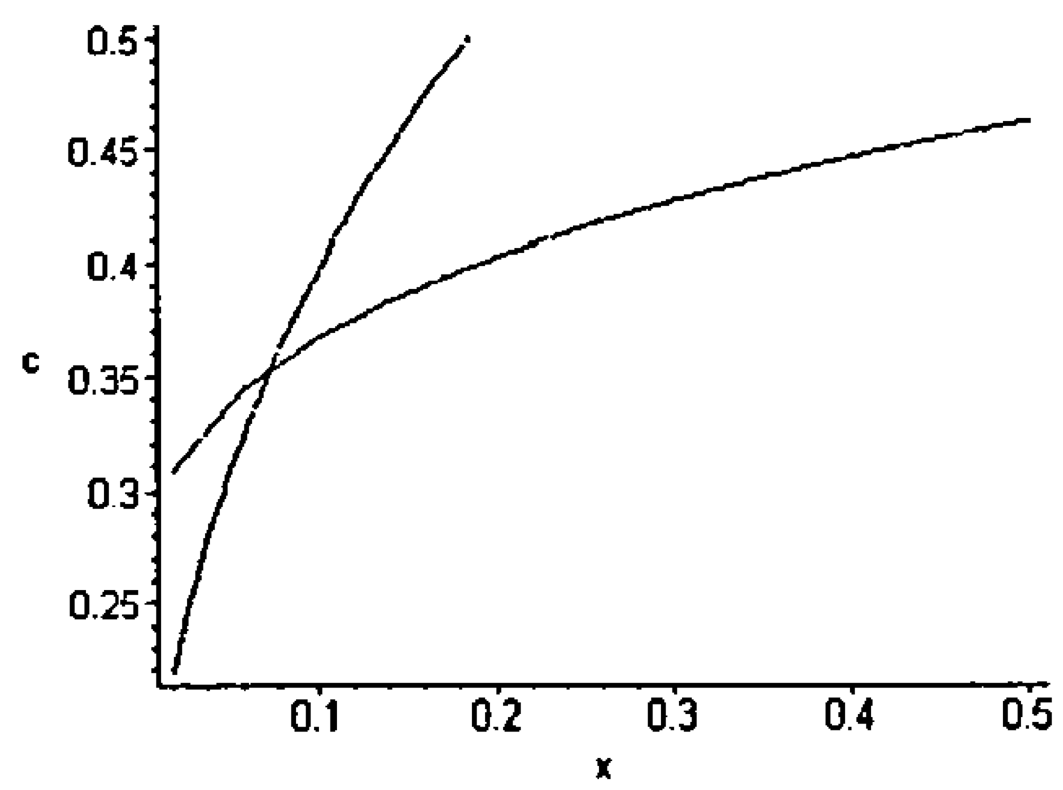

Figure 3.A.2: Steady State solution ($\lambda = 0.03$)

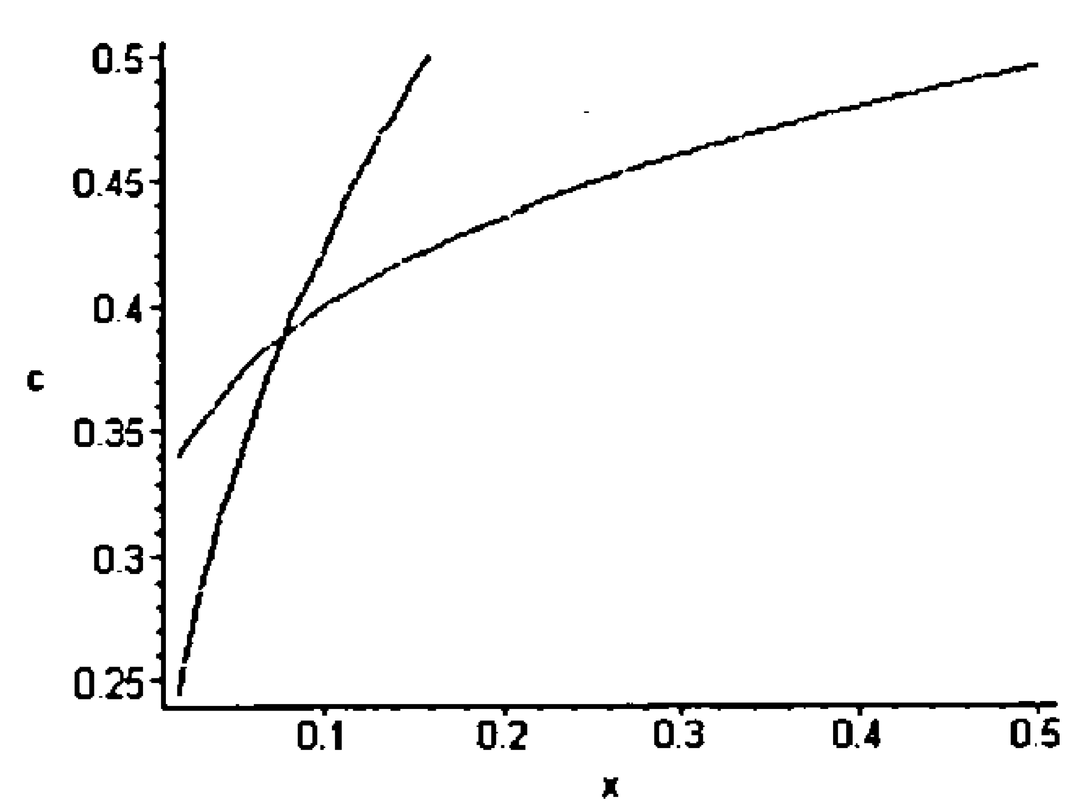

Figure 3.A.3: Steady State solution ($\lambda = 0.06$)

Table 3.A.1: An increase in public investment ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.0711	0.0784	0.0861	0.0942	0.1027	0.1116	0.1209	0.1306	0.1408
c	0.3191	0.3235	0.3279	0.3321	0.3364	0.3406	0.3445	0.3489	0.353
γ	0.01983	0.02089	0.02188	0.02281	0.0236	0.0244	0.0252	0.0258	0.025
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1513	0.1623	0.1737	0.1856	0.198	0.218	0.2243	0.2382	0.2528
c	0.3571	0.3611	0.3651	0.3692	0.3732	0.3772	0.3812	0.3852	0.3892
γ	0.027	0.02761	0.02809	0.02852	0.0289	0.02924	0.02954	0.02979	0.03002
τ	0.33	0.34	0.35	0.36	0.37	0.383	0.39	0.40	0.41
x	0.2675	0.2837	0.3002	0.3174	0.3354	0.3599	0.3738	0.3943	0.4159
c	0.3932	0.3972	0.4013	0.4054	0.4095	0.4136	0.4178	0.422	0.4263
γ	0.0302	0.03035	0.03047	0.03056	0.03063	0.03064	0.03063	0.0361	0.03054
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4385	0.4621	0.487	0.5131	0.5407	0.5697	0.6002	0.6325	0.6667
c	0.4307	0.435	0.4395	0.444	0.4486	0.4532	0.458	0.4628	0.4678
γ	0.03045	0.03033	0.0302	0.03003	0.02985	0.02963	0.0294	0.02915	0.02887

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \ \rho = 0.2, \theta = 0.1$

Table 3.A.2: An increase in public investment ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.07590	0.08347	0.09145	0.09982	0.1086	0.1178	0.1273	0.1373	0.1477
c	0.3549	0.3592	0.3635	0.3678	0.372	0.372	0.3803	0.3844	0.3885
γ	0.01934	0.02030	0.02120	0.02203	0.02279	0.02350	0.02415	0.02475	0.02529
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1586	0.1699	0.1816	0.1939	0.2066	0.2198	0.2337	0.2480	0.2630
c	0.3925	0.3967	0.4006	0.4046	0.4086	0.4126	0.4166	0.4207	0.4247
γ	0.02577	0.02621	0.02660	0.02695	0.02725	0.02751	0.02773	0.02791	0.02806
τ	0.33	0.34	0.35	0.359	0.37	0.38	0.39	0.40	0.41
x	0.2786	0.2949	0.3119	0.3278	0.3482	0.3676	0.3878	0.4091	0.4313
c	0.4287	0.4323	0.4368	0.4409	0.4451	0.4492	0.4535	0.4577	0.462
γ	0.02817	0.02824	0.02829	0.02830	0.02828	0.02823	0.02815	0.02805	0.02791
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4546	0.4791	0.5049	0.5319	0.5604	0.5905	0.6222	0.6557	0.6911
c	0.4664	0.4708	0.4753	0.4799	0.4845	0.4892	0.494	0.499	0.5039
γ	0.02775	0.02757	0.02736	0.02713	0.02687	0.02659	0.02629	0.02596	0.02562

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$

Table 3.A.3: An increase in public investment ($\lambda = 0.06$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.08082	0.08865	0.09689	0.1055	0.1146	0.1240	0.1339	0.1442	0.1549
c	0.3906	0.3949	0.3991	0.4034	0.4076	0.4117	0.4158	0.4199	0.4240
γ	0.01883	0.01970	0.02050	0.02124	0.02191	0.02253	0.02309	0.02360	0.02405
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1660	0.1776	0.1898	0.2023	0.2155	0.2291	0.2433	0.2581	0.2736
c	0.4281	0.4321	0.4361	0.4401	0.4441	0.4481	0.4521	0.4562	0.4602
γ	0.02445	0.02481	0.02512	0.02538	0.02560	0.02578	0.02592	0.02603	0.02610
τ	0.33	0.335	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2896	0.2979	0.3239	0.3422	0.3613	0.3813	0.4023	0.4242	0.4472
c	0.4642	0.4683	0.4724	0.4765	0.4807	0.4849	0.4891	0.4934	0.4978
γ	0.02613	0.02614	0.02610	0.02604	0.02595	0.02582	0.02567	0.02550	0.02529
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4731	0.4967	0.5233	0.5513	0.5808	0.6120	0.6449	0.6796	0.7164
c	0.5021	0.5066	0.5111	0.5157	0.5204	0.5252	0.53	0.5345	0.54
γ	0.02506	0.0248	0.02453	0.02423	0.02390	0.02355	0.02318	0.02279	0.02237

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$

Table 3.A.4: An increase in public investment with higher public consumption ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.05763	0.06340	0.06953	0.07602	0.08267	0.09007	0.09763	0.1057	0.1139
c	0.3138	0.3181	0.3225	0.3268	0.3311	0.3354	0.3397	0.3440	0.3483
γ	0.01617	0.01712	0.01802	0.01888	0.01969	0.02044	0.02115	0.02181	0.02242
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1225	0.1317	0.1411	0.1510	0.1613	0.1721	0.1833	0.1951	0.2073
c	0.3526	0.3569	0.3612	0.3654	0.3697	0.3740	0.3784	0.3827	0.3871
γ	0.02298	0.02350	0.02397	0.02440	0.02479	0.02514	0.02546	0.02573	0.02598
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.394	0.40	0.41
x	0.2201	0.2335	0.2474	0.2620	0.2773	0.2933	0.3169	0.3275	0.3459
c	0.3915	0.3959	0.4003	0.4048	0.4093	0.4139	0.4186	0.4232	0.4280
γ	0.02618	0.02636	0.02650	0.02661	0.02670	0.02675	0.02678	0.02677	0.02675
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3652	0.3855	0.4068	0.4293	0.4530	0.4780	0.5043	0.5322	0.5617
c	0.4328	0.4380	0.4426	0.4470	0.4528	0.4581	0.4634	0.4688	0.4744
γ	0.02669	0.02661	0.02651	0.02639	0.02624	0.02606	0.02587	0.02566	0.02542

$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3, \ \rho = 0.2, \ \theta = 0.1$

Table 3.A.5: An increase in public investment with higher public consumption ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06200	0.06802	0.07440	0.08114	0.08824	0.09570	0.1035	0.1117	0.1203
c	0.3498	0.3541	0.3584	0.3227	0.3670	0.3713	0.3756	0.3798	0.3841
γ	0.01575	0.01661	0.01742	0.01818	0.01889	0.01955	0.02017	0.02073	0.02126
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1293	0.1387	0.1484	0.1587	0.1693	0.1805	0.1921	0.2042	0.2168
c	0.3884	0.3927	0.3969	0.4012	0.4055	0.4098	0.4142	0.4185	0.4229
γ	0.02173	0.02216	0.02255	0.02290	0.02321	0.02348	0.02371	0.02391	0.02407
τ	0.33	0.34	0.35	0.36	0.368	0.38	0.39	0.40	0.41
x	0.2301	0.2439	0.2583	0.2734	0.2892	0.3057	0.3230	0.3412	0.3602
c	0.4273	0.4317	0.4362	0.4407	0.4453	0.4499	0.4545	0.4592	0.4640
γ	0.02420	0.02430	0.02436	0.02440	0.02441	0.02439	0.02434	0.02426	0.02416
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3803	0.4013	0.4234	0.4468	0.4713	0.4973	0.5247	0.5537	0.5845
c	0.4689	0.4738	0.4788	0.4839	0.4891	0.4944	0.4998	0.5053	0.5109
γ	0.02404	0.02389	0.02371	0.02352	0.02330	0.02306	0.02280	0.02251	0.02220

$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3, \ \rho = 0.2, \ \theta = 0.1$

Table 3.A.6: An increase in public investment with higher public consumption ($\lambda = 0.06$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06654	0.07281	0.07944	0.08643	0.09379	0.1015	0.1096	0.1181	0.1270
c	0.3857	0.3899	0.3943	0.3985	0.4028	0.4071	0.4114	0.4156	0.4199
γ	0.01531	0.01607	0.01679	0.01746	0.01808	0.01865	0.01918	0.01966	0.02009
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1362	0.1459	0.1560	0.1666	0.1776	0.1891	0.2011	0.2136	0.2267
c	0.4242	0.4284	0.4327	0.4370	0.4413	0.4456	0.4499	0.4543	0.4587
γ	0.02048	0.02083	0.02113	0.02140	0.02162	0.02181	0.02196	0.02208	0.02217
τ	0.33	0.342	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2403	0.2575	0.2695	0.2851	0.3015	0.3186	0.3365	0.3553	0.3751
c	0.4631	0.4676	0.4472	0.4766	0.4812	0.4858	0.4905	0.4953	0.5001
γ	0.02222	0.02224	0.02223	0.02219	0.02212	0.02203	0.02191	0.02176	0.02159
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3958	0.4177	0.4406	0.4649	0.4904	0.5174	0.5459	0.5761	0.6081
c	0.5050	0.5099	0.5150	0.5202	0.5254	0.5307	0.5362	0.5418	0.5475
γ	0.02139	0.02117	0.02092	0.02066	0.02037	0.02005	0.01972	0.01937	0.01899

$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3, \ \rho = 0.2, \ \theta = 0.1$

Table 3.A.7: An increase in public investment with higher investment subsidies ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.5924	0.06566	0.07251	0.07978	0.08748	0.09561	0.1042	0.1131	0.1225
c	0.3077	0.3121	0.3164	0.3207	0.3250	0.3293	0.3335	0.3377	0.3419
γ	0.01792	0.01903	0.02009	0.02110	0.02204	0.02293	0.02376	0.02454	0.02526
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1324	0.1427	0.1534	0.1647	0.1764	0.1886	0.2014	0.2147	0.2286
c	0.3461	0.3502	0.3544	0.3585	0.3625	0.3667	0.3707	0.3748	0.3789
γ	0.02592	0.02653	0.02709	0.02760	0.02807	0.02848	0.02886	0.02919	0.02948
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.399	0.41
x	0.2430	0.2582	0.2739	0.2904	0.3077	0.3257	0.3445	0.3622	0.3849
c	0.3830	0.3872	0.3913	0.3954	0.3996	0.4038	0.4081	0.4119	0.4167
γ	0.02973	0.02994	0.03012	0.03026	0.03037	0.03045	0.03049	0.03050	0.03049
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4066	0.4294	0.4533	0.4784	0.5049	0.5327	0.5621	0.5932	0.6260
c	0.4210	0.4255	0.4299	0.4345	0.4391	0.4438	0.4485	0.4534	0.4584
γ	0.03044	0.03037	0.03026	0.03012	0.02998	0.02980	0.02960	0.02937	0.02912

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \ \rho = 0.2, \theta = 0.12$

Table 3.A.8: An increase in public investment with higher investment subsidies ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06477	0.07152	0.07869	0.08630	0.09434	0.1028	0.1117	0.1210	0.1308
c	0.3449	0.3492	0.3535	0.3578	0.3620	0.3663	0.3705	0.3746	0.3788
γ	0.01765	0.01866	0.019612	0.02051	0.02135	0.02214	0.02286	0.02354	0.02416
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1410	0.1517	0.1628	0.1745	0.1866	0.1993	0.2125	0.2263	0.2407
c	0.3829	0.3871	0.3912	0.3953	0.3994	0.4035	0.4075	0.4116	0.4157
γ	0.02472	0.02524	0.02571	0.02613	0.02650	0.02683	0.02712	0.02736	0.02757
τ	0.33	0.34	0.35	0.36	0.374	0.38	0.39	0.40	0.41
x	0.2557	0.2714	0.2878	0.3049	0.3302	0.3415	0.3611	0.38162	0.4032
c	0.4116	0..4240	0.4281	0..4323	0..4382	0.4407	0.4450	0.4493	0.4536
γ	0.02773	0.02787	0.02796	0.02803	0.02806	0.02805	0.02802	0.02795	0.02786
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4257	0.4494	0.4744	0.5006	0.5282	0.5573	0.5880	0.6204	0.6547
c	0.4580	0.4625	0.4670	0.4716	0.4763	0.4810	0.4859	0.4908	0.4958
γ	0.02774	0.02759	0.02741	0.02721	0.02698	0.02673	0.02645	0.02615	0.02583

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.12$

Table 3.A.9: An increase in public investment with higher investment subsidies ($\lambda = 0.06$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.0705	0.0776	0.08514	0.0931	0.1014	0.1103	0.1195	0.1292	0.1393
c	0.3820	0.3863	0.39053	0.3948	0.3989	0.4032	0.4074	0.4115	0.4157
γ	0.0173	0.01823	0.01911	0.01990	0.02064	0.02132	0.02195	0.02253	0.02305
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1499	0.1610	0.1726	0.1847	0.1973	0.2104	0.2241	0.2384	0.2533
c	0.4198	0.4239	0.4279	0.4321	0.4361	0.4402	0.4443	0.4484	0.4525
γ	0.02352	0.02394	0.02432	0.02465	0.02493	0.02517	0.02538	0.02554	0.02566
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2689	0.2852	0.3022	0.3199	0.3386	0.3580	0.3784	0.3997	0.4221
c	0.4566	0.4608	0.4649	0.4691	0.4733	0.4776	0.4819	0.4862	0.4906
γ	0.02574	0.02579	0.02581	0.02579	0.02574	0.02566	0.02555	0.02541	0.02524
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4456	0.4703	0.4963	0.5236	0.5524	0.5828	0.6149	0.6488	0.6847
c	0.4951	0.4996	0.5042	0.5088	0.5135	0.5183	0.5232	0.5282	0.5333
γ	0.02504	0.02482	0.02457	0.02429	0.02399	0.02366	0.0233	0.02294	0.02255

$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \ \rho = 0.2, \ \theta = 0.12$

3.B Mathematical Appendix

3.B.1 Uniqueness

Proof of proposition 1 *There exists a BGP with endogenous growth for the economy described by (3.2.21) - (3.2.22) and such a BGP is unique.*

In order to prove proposition 1, we first set $\dot{c} = 0$,

$$cx^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - c\frac{\lambda(1+\theta_S)}{(1-\theta_S)} - c\rho - cx^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} + \frac{c^2}{(1-\theta_S)} = 0 \tag{3.B.1}$$

Solving (3.B.1) for c and substituting the result in (3.2.21) — see page 61 — yields

$$\begin{aligned} F(x,\cdot) &= x^{\alpha}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- x^{\alpha+1}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + (x+\theta_S)\left[\rho + \frac{\lambda(\tau+\theta_S)}{(1-\theta_S)}\right] \end{aligned} \tag{3.B.2}$$

A solution to $F(x,\cdot) = 0$ gives a BGP for the economy. For $x = 0$ we have $F(0,\cdot) > 0$

$$F(0,\cdot) = \theta_S\left[\frac{\lambda(\tau+\theta_S)}{(1-\theta_S)} + \rho\right] > 0 \tag{3.B.3}$$

We now calculate the sign of $\partial F(x,\cdot)/\partial x$:

$$\begin{aligned} \frac{\partial F(x,\cdot)}{\partial x} &= \alpha x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- (\alpha+1)x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + \frac{\lambda(\tau+\theta_S)}{(1-\theta)} + \rho \end{aligned} \tag{3.B.4}$$

where

$$\gamma = \frac{\dot{C}}{C} = x^{\alpha}\frac{(1-\alpha)(1-\tau)}{(1-\theta_S)} - \frac{\lambda(\tau+\theta_S)}{(1-\theta_S)} - \rho$$

Thus, by substituting this result into (3.B.4):

$$\begin{aligned} \frac{\partial F(x,\cdot)}{\partial x} &= \alpha x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- \alpha x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \gamma \end{aligned} \tag{3.B.5}$$

From

$$\gamma = \frac{\dot{G}}{G} = x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} - \frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} + \frac{c\theta_S}{x(1-\theta_S)}$$

It follows that

$$\alpha x^{\alpha-1}\tau\left(1-\varphi_1-\varphi_2\right)=\alpha\left\{\gamma+x^{\alpha-1}\frac{\theta\left[1-\tau\left(1-\varphi_1\right)\right]}{\left(1-\theta_S\right)}+\frac{\lambda\theta_S\left(1-\tau\right)}{x\left(1-\theta_S\right)}-\frac{c\theta_S}{x\left(1-\theta_S\right)}\right\}$$

Thus, by substituting this expression in (3.B.5), we obtain

$$\begin{aligned}\frac{\partial F\left(x,\cdot\right)}{\partial x} &= \left(\alpha-1\right)\gamma+\alpha x^{\alpha-1}\frac{\theta_S\left[1-\tau\left(1-\varphi_1\right)\right]}{\left(1-\theta_S\right)}-\alpha x^{\alpha}\frac{\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)}\\ &+ \frac{\alpha\lambda\theta_S\left(1-\tau\right)}{x\left(1-\theta_S\right)}-\frac{\alpha c\theta_S}{x\left(1-\theta_S\right)}-\alpha x^{\alpha-1}\frac{\theta_S\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)} \qquad (3.B.6)\end{aligned}$$

From (3.B.1), it follows that

$$\begin{aligned}\frac{\alpha c\theta_S}{x\left(1-\theta_S\right)} &= \frac{\alpha\theta_S\lambda\left(1+\theta_S\right)}{x\left(1-\theta_S\right)}+\frac{\alpha\rho\theta_S}{x}-\alpha x^{\alpha-1}\frac{\theta_S}{\left(1-\theta_S\right)}\\ &\quad\left\{\left(1-\tau\right)\left(1-\alpha\right)-\left[1-\tau\left(1-\varphi_1\right)\right]\right\} \qquad (3.B.7)\end{aligned}$$

Inserting this result in (3.B.6) yields

$$\begin{aligned}\frac{\partial F\left(x,\cdot\right)}{\partial x} &= \left(\alpha-1\right)\gamma-\alpha x^{\alpha}\frac{\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)}\\ &+ \frac{\alpha\theta_S\lambda\left(1-\tau\right)}{x\left(1-\theta_S\right)}-\frac{\alpha\theta_S\lambda\left(1+\theta_S\right)}{x\left(1-\theta_S\right)}-\frac{\alpha\rho\theta_S}{x} \qquad (3.B.8)\end{aligned}$$

Using again the definition of $\gamma=(\dot{C}/C)$ leads to

$$\frac{\partial F\left(x,\cdot\right)}{\partial x}=-\gamma-\left[\alpha\rho+\frac{\alpha\lambda\left(\tau+\theta_S\right)}{\left(1-\theta_S\right)}\right]\left(1+\frac{\theta_S}{x}\right) \qquad (3.B.9)$$

From (3.B.9) it follows that $\partial F\left(x,\cdot\right)/\partial x<0$ always holds on the BGP and, as a consequence, $F\left(x,\cdot\right)$ can not cross the horizontal axis from below. Since $F\left(0,\cdot\right)>0$ and $F\left(x,\cdot\right)$ is a continuous function the BGP is unique.

3.B.2 Stability

Proof of proposition 2 *The Jacobian matrix of the system (3.2.21) - (3.2.22) has one positive and one negative real root, which implies that the unique BGP is a saddle path.*

In order to prove proposition 2 we need to evaluate the partial derivatives of (3.2.21)-(3.2.22) at the steady state.

$$\begin{aligned}\frac{\partial\dot{x}}{\partial x} &= \alpha x^{\alpha-1}\left\{\tau\left(1-\varphi_1-\varphi_2\right)-\frac{\theta_S\left[1-\tau\left(1-\varphi_1\right)\right]}{\left(1-\theta_S\right)}\right\}\\ &- \left(\alpha+1\right)x^{\alpha}\frac{\left[1-\tau\left(1-\varphi_1\right)\right]}{\left(1-\theta_S\right)}+\frac{c}{\left(1-\theta_S\right)}-\frac{\lambda\left(1-\tau\right)}{\left(1-\theta_S\right)} \qquad (3.B.10)\end{aligned}$$

Setting $\dot{x} = 0$ implies that

$$\begin{aligned}\frac{c}{(1-\theta_S)} &= -x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} \\ &+ x^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} - \frac{c\theta_S}{x(1-\theta_S)} \\ &+ \frac{\lambda(1-\tau)(1+\theta_S)}{(1-\theta_S)} \end{aligned} \tag{3.B.11}$$

By substituting this result in (3.B.10), we obtain

$$\begin{aligned}\frac{\partial \dot{x}}{\partial x} &= (\alpha-1)x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} \\ &- \alpha x^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} - \frac{c\theta_S}{x(1-\theta_S)} + \frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} \end{aligned} \tag{3.B.12}$$

$$\frac{\partial \dot{x}}{\partial c} = \frac{x+\theta_S}{1-\theta_S} \tag{3.B.13}$$

$$\frac{\partial \dot{c}}{\partial x} = c\alpha x^{\alpha-1}\left\{\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \frac{[1-(1-\varphi_1)]}{(1-\theta_S)}\right\} \tag{3.B.14}$$

$$\frac{\partial \dot{c}}{\partial c} = x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \frac{\lambda(1+\theta_S)}{(1-\theta_S)} - \rho - x^{\alpha}\frac{[1-(1-\varphi_1)]}{(1-\theta_S)} + \frac{2c}{(1-\theta_S)} \tag{3.B.15}$$

Setting $\dot{c} = 0$ implies that

$$\frac{c}{(1-\theta_S)} = \frac{\lambda(1+\theta_S)}{(1-\theta_S)} + \rho - x^{\alpha}\left\{\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \frac{[1-(1-\varphi_1)]}{(1-\theta_S)}\right\}$$

By substituting this result in (3.B.15), we obtain

$$\frac{\partial \dot{c}}{\partial c} = \frac{c}{(1-\theta_S)} \tag{3.B.16}$$

Thus, the Jacobian matrix of the system (3.2.21)-(3.2.22) is given by

$$J = \begin{bmatrix} (\alpha-1)x^{\alpha-1}\phi_1 - \alpha x^{\alpha}\phi_2 - \frac{c\theta_S}{x(1-\theta_S)} + \frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} & \frac{x+\theta_S}{1-\theta_S} \\ c\alpha x^{\alpha-1}\phi_3 & \frac{c}{(1-\theta_S)} \end{bmatrix} \tag{3.B.17}$$

where

$$\phi_1 = \left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} \tag{3.B.18}$$

$$\phi_2 = \frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} \tag{3.B.19}$$

$$\phi_3 = \left\{ \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \frac{[1-(1-\varphi_1)]}{(1-\theta_S)} \right\} \tag{3.B.20}$$

The determinant of the Jacobian matrix is:

$$\begin{aligned} Det\ J &= \frac{cx}{(1-\theta_S)} \left\{ (\alpha-1)\, x^{\alpha-2} \tau (1-\varphi_1-\varphi_2) - \frac{c\theta_S}{x^2(1-\theta_S)} \right\} \\ &+ x^{\alpha-1} c \frac{\theta_S [1-\tau(1-\varphi_1)]}{(1-\theta_S)^2} - (x+\theta_S)\, c\alpha x^{\alpha-1} \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)^2} \\ &+ \frac{c\lambda\theta_S(1-\tau)}{x(1-\theta_S)} \end{aligned} \tag{3.B.21}$$

From the definition of $\gamma = (\dot{G}/G)$ it follows that

$$\begin{aligned} cx^{\alpha-1} \frac{\theta_S [1-\tau(1-\varphi_1)]}{(1-\theta_S)^2} &= -\frac{c\gamma}{(1-\theta_S)} + cx^{\alpha-1} \frac{\tau(1-\varphi_1-\varphi_2)}{(1-\theta_S)} \\ &- \frac{c\lambda\theta_S(1-\tau)}{x(1-\theta_S)^2} + \frac{c^2\theta_S}{x(1-\theta_S)^2} \end{aligned} \tag{3.B.22}$$

Substituting this result into (3.B.21)

$$\begin{aligned} Det\ J &= \frac{c\alpha}{(1-\theta_S)} x^{\alpha-1} \left\{ \tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)^2} \right\} \\ &- \frac{c\gamma}{(1-\theta_S)} - c\alpha x^{\alpha} \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)^2} \end{aligned} \tag{3.B.23}$$

From (3.B.4), it follows that

$$\begin{aligned} &\frac{c\alpha}{(1-\theta_S)} x^{\alpha-1} \left\{ \tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)^2} \right\} = \\ &c\alpha x^{\alpha} \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)^2} - \frac{c\alpha}{x(1-\theta_S)}(x+\theta_S)\left[\rho + \frac{(\tau+\theta_S)}{(1-\theta_S)}\right] \end{aligned} \tag{3.B.24}$$

Substituting this result in (3.B.23)

$$Det\ J = -c\left\{ \frac{\gamma}{(1-\theta_S)} + \frac{\alpha(x+\theta_S)}{x(1-\theta_S)} \left[\rho + \lambda \frac{(\tau+\theta_S)}{(1-\theta_S)}\right] \right\} \tag{3.B.25}$$

Since $Det\ J < 0$, proposition 2 is proved.

3.B.3 Fiscal Policy

Public consumption and lump-sum transfers

Proof of propositions 3 and 4 *The long-run rate of growth of the economy γ is decreasing in public consumption φ_2 and lump-sum transfers φ_1.*

Increases in φ_i ($i = 1, 2$) of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$.

The impact of public consumption and lump-sum transfers to households is derived by differentiating the long-run rate of growth γ with respect to φ_i, $i = 1, 2$

$$\frac{\partial \gamma}{\partial \varphi_i} = \frac{\partial \gamma}{\partial x}\frac{\partial x}{\partial \varphi_i} = \alpha \frac{(1-\tau)}{1-\theta_S}(1-\alpha)x^{\alpha-1}\frac{\partial x}{\partial \varphi_i}, \quad i = 1, 2 \tag{3.B.26}$$

where

$$\frac{\partial x}{\partial \varphi_i} = -\frac{\partial F(x,\cdot)/\partial \varphi_i}{\partial F(x,\cdot)/\partial x} = \frac{\tau x^{\alpha}}{\partial F(x,\cdot)/\partial x} < 0, \quad i = 1, 2 \tag{3.B.27}$$

From the proof of proposition 1, we know that $\partial F(x,\cdot)/\partial x < 0$. Hence, $\partial x / \partial \varphi_i < 0$ and

$$\frac{\partial \gamma}{\partial \varphi_i} < 0, \quad i = 1, 2 \tag{3.B.28}$$

Moreover — from (3.B.9) — $|\partial F(x,\cdot)/\partial x|_{(\lambda>0)} > |\partial F(x,\cdot)/\partial x|_{(\lambda=0)}$. Thus $|\partial x/\partial \varphi_i|_{(\lambda>0)} > |\partial x/\partial \varphi_i|_{(\lambda=0)}$ and

$$\left.\frac{\partial \gamma}{\partial \varphi_i}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_i}\right|_{(\lambda=0)}, \quad i = 1, 2 \tag{3.B.29}$$

Hence, propositions 3 and 4 are proved.

The growth maximizing income tax rate

Proof of proposition 5 *There exists a growth maximizing income tax rate τ_{max} both in the infinite and the finite horizons scenarios, the first one being higher than the latter one.*

In order to calculate the growth maximizing level of income taxation in the finite horizons case, the derivative of (3.2.23) with respect to τ is evaluated as follows

$$\frac{\partial \gamma}{\partial \tau} = x^{\alpha}\frac{(1-\alpha)}{(1-\theta_S)}\left[-1 + \frac{\alpha(1-\tau)}{\tau}\frac{\partial x}{\partial \tau}\frac{\tau}{x}\right] - \frac{\lambda}{(1-\theta_S)} \tag{3.B.30}$$

where

$$\left.\frac{\partial x}{\partial \tau}\right|_{F(x,\cdot)=0} = -\frac{\partial F(x,\cdot)/\partial \tau}{\partial F(x,\cdot)/\partial x} = \frac{-x^{\alpha}\left[(1-\varphi_1-\varphi_2)+\frac{(\theta_S+x)(1-\alpha)}{(1-\theta_S)}\right]-\frac{\lambda(\theta_S+x)}{(1-\theta_S)}}{-\gamma-\left[\alpha\rho+\frac{\alpha\lambda(\theta_S+\tau)}{(1-\theta_S)}\right]\left(1+\frac{\theta_S}{x}\right)} \tag{3.B.31}$$

Solving $F(x,\cdot)$ for ρ yields

$$\rho = x^{\alpha}\left[\frac{(1-\alpha)(1-\tau)}{(1-\theta_S)} - \frac{\tau(1-\varphi_1-\varphi_2)}{(\theta_S+x)}\right] - \frac{\lambda(\theta_S+\tau)}{(1-\theta_S)} \tag{3.B.32}$$

Let us now substitute (3.B.32) into (3.B.31) and the resulting expression for $\partial x/\partial \tau$ back into (3.B.30). By doing so, it is possible to solve $\partial \gamma/\partial \tau = 0$ for τ and to obtain the growth maximizing level of income taxation in the presence of a positive λ:

$$\left.\tau_{max}\right|_{(\lambda>0)} = \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \tag{3.B.33}$$

which for $\lambda = 0$ simplifies to

$$\left.\tau_{max}\right|_{(\lambda=0)} = \alpha\left(1+\frac{\theta_S}{x}\right) \tag{3.B.34}$$

where[12]

$$\alpha\left(1+\frac{\theta_S}{x}\right) > \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \tag{3.B.35}$$

Hence

$$\left.\tau_{max}\right|_{(\lambda>0)} < \left.\tau_{max}\right|_{(\lambda=0)} \tag{3.B.36}$$

And proposition 5 is proved.

Proof of proposition 6 *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in both public consumption φ_2 and lump-sum transfers to households φ_1.*

For $\lambda = 0$, τ_{max} is given by (3.B.34).

[12] This inequality has been evaluated using Maple 7.0 for the following values of the parameters: $x > 0, 0 \leq \alpha \leq 1, 0 \leq \theta_S \leq 1$ and $0 \leq \lambda \leq 1$.

Let us define the implicit function Γ:

$$\Gamma\left(x\left(\tau,\theta_S,\varphi_i\right),\tau,\theta_S,\varphi_i\right)\equiv\tau_{max}-\alpha\left(1+\frac{\theta_S}{x}\right)\equiv\tau_{max}-\bar{\Gamma}=0 \quad (3.B.37)$$

Totally differentiating (3.B.34) with respect to φ_i, $i=1,2$ and applying the implicit function theorem to (3.B.37):

$$\begin{aligned}\left.\frac{\partial\tau_{max}}{\partial\varphi_i}\right|_{(\lambda=0)} &= -\frac{\partial\Gamma/\partial\varphi_i}{\partial\Gamma/\partial\tau}=-\frac{\frac{\partial\bar{\Gamma}}{\partial x}\frac{\partial x}{\partial\varphi_i}}{1-\frac{\partial\bar{\Gamma}}{\partial x}\frac{\partial x}{\partial\tau}}=-\frac{\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\varphi_i}}{1+\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\tau}}= \\ &= -\frac{\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\varphi_i}}{\frac{\alpha\theta_S}{x^2}\left(\frac{x^2}{\alpha\theta_S}+\frac{\partial x}{\partial\tau}\right)}=-\frac{\frac{\partial x}{\partial\varphi_i}}{-\left(\frac{\partial\bar{\Gamma}}{\partial x}\right)^{-1}+\frac{\partial x}{\partial\tau}}, \quad i=1,2\end{aligned} \quad (3.B.38)$$

where from (3.B.27) $\partial x/\partial\varphi_i<0$ and from (3.B.31), $\partial x/\partial\tau>0$. Hence, we obtain:

$$\left.\frac{\partial\tau_{max}}{\partial\varphi_i}\right|_{(\lambda=0)}=-\frac{\partial x/\partial\varphi_i}{x^2/\alpha\theta_S+\partial x/\partial\tau}>0, \quad i=1,2 \quad (3.B.39)$$

For $0<\lambda\leq 1$, τ_{max} is defined by (3.B.33).

Let us define the implicit function Γ_2:

$$\begin{aligned}\Gamma_2\left(x\left(\tau,\theta_S,\varphi_i\right),\tau,\theta_S,\varphi_i\right) &\equiv \tau_{max}-\frac{\alpha x^{\alpha}\left(\alpha-1\right)\left(x+\theta_S\right)}{x^{\alpha+1}\left(\alpha-1\right)-\lambda x+\lambda\alpha\left(x+\theta_S\right)}\equiv \\ &\equiv \tau_{max}-\bar{\Gamma}_2\equiv \\ &\equiv \tau_{max}-\frac{A}{B}=0\end{aligned} \quad (3.B.40)$$

Totally differentiating (3.B.33) with respect to φ_i, $i=1,2$ and applying the implicit function theorem to (3.B.40):

$$\left.\frac{\partial\tau_{max}}{\partial\varphi_i}\right|_{(0<\lambda\leq1)}=-\frac{\partial\Gamma_2/\partial\varphi_i}{\partial\Gamma_2/\partial\tau}=-\frac{\frac{\partial\bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial\varphi_i}}{1-\frac{\partial\bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial\tau}}=-\frac{\frac{\partial x}{\partial\varphi_i}}{-\left(\frac{\partial\bar{\Gamma}_2}{\partial x}\right)^{-1}+\frac{\partial x}{\partial\tau}}, \quad i=1,2$$

where $\partial x/\partial\varphi_i<0$, $\partial x/\partial\tau>0$, and — for $x>0, 0\leq\alpha\leq1, 0\leq\theta_S\leq1$ and

$0 \leq \lambda \leq 1$ — the sign of the term $\left(\frac{\partial \bar{\Gamma}_2}{\partial x}\right)^{-1}$ is negative[13]:

$$\left(\frac{\partial \bar{\Gamma}_2}{\partial x}\right)^{-1} = \left\{\left[\alpha(\alpha-1)(\alpha+1)x^{\alpha} + \alpha^2\theta_S(\alpha-1)x^{\alpha-1}\right]B^{-1}\right\} - \left\{B^{-2}\left[(\alpha-1)(\alpha+1)x^{\alpha} - \lambda(\alpha+1)\right]A\right\} < 0 \quad (3.B.41)$$

Therefore, for all the values of the parameters coherent with our theoretical model:

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(0<\lambda\leq 1)} = -\frac{\partial x/\partial \varphi_i}{-\left(\frac{\partial \bar{\Gamma}_2}{\partial x}\right)^{-1} + \partial x/\partial \tau} > 0, \quad i = 1, 2 \qquad (3.B.42)$$

Investment subsidies

Proof of proposition 7 *There exists a growth maximizing value for investment subsidies, and such a value is decreasing in the probability of death parameter λ.*

Differentiating γ with respect to θ_S leads to

$$\frac{\partial \gamma}{\partial \theta_S} = \frac{(1-\alpha)(1-\tau)}{(1-\theta_S)^2}x^{\alpha}\left[1 + \frac{\alpha(1-\theta_S)}{\theta_S}\frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x}\right] - \frac{\lambda(1+\tau)}{(1-\theta_S)^2} \qquad (3.B.43)$$

Since from (3.B.47) $\partial x/\partial \theta_S < 0$, for $\lambda = 0$

$$\left.\frac{\partial \gamma}{\partial \theta_S}\right|_{(\lambda=0)} > (\leq) 0 \quad \text{if} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} \qquad (3.B.44)$$

However, for a positive λ we obtain

$$\left.\frac{\partial \gamma}{\partial \theta_S}\right|_{(\lambda>0)} > (\leq) 0 \quad \text{if} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} + \frac{\lambda(1+\tau)}{(1-\theta_S)^2} \qquad (3.B.45)$$

Proof of proposition 8 *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in investment subsidies.*

Differentiating (3.B.33) with respect to θ_S

$$\frac{\partial \tau_{max}}{\partial \theta_S} = -\frac{\frac{\partial \bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial \theta_S}}{1 - \frac{\partial \bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial \tau}} = -\frac{\frac{\partial x}{\partial \theta_S}}{-\left(\frac{\partial \bar{\Gamma}_2}{\partial x}\right)^{-1} + \frac{\partial x}{\partial \tau}} > 0 \qquad (3.B.46)$$

[13] This inequality has been evaluated using Maple 7.0 for the following values of the parameters: $x > 0, 0 \leq \alpha \leq 1, 0 \leq \theta_S \leq 1$ and $0 \leq \lambda \leq 1$.

where $\left(\frac{\partial \bar{\Gamma}_2}{\partial x}\right)^{-1} < 0$ is defined in (3.B.41) and $\partial x/\partial \theta_S$ is derived by totally differentiating $F(x, \cdot) = 0$ and using $\gamma = \dot{C}/C$:

$$\frac{\partial x}{\partial \theta_S} = -\frac{\partial F/\partial \theta_S}{\partial F/\partial x} = -\frac{-\gamma - x^{\alpha} \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)^2} (x + \theta_S) + \frac{\lambda \tau (1+\theta_S)}{(1-\theta_S)^2}}{-\gamma - \left[\alpha\rho + \frac{\alpha\lambda(\tau+\theta_S)}{(1-\theta_S)}\right] \left(1 + \frac{\theta_S}{x}\right)} < 0 \quad (3.B.47)$$

Hence, for $0 \leq \lambda \leq 1$, $\partial \tau_{max}/\partial \theta_S > 0$.

4. PUBLIC CAPITAL AND PRODUCTIVITY : A SURVEY ON EMPIRICAL EVIDENCE

4.1 Introduction

The aim of this Chapter is to survey the empirical literature on the productive role of public capital. Since the late 1980s, applied work on the relationship between public capital and economic performance has been a widely analysed and controversial issue in the economic literature, the debate being characterized by alternative underlying theories, the employment of a variety of estimation techniques and leading to a wide range of results.

The task of surveying this branch of the literature exhaustively is somehow ambitious and this Chapter is not aimed at this, rather intending to discuss the part of the literature most related to the present research.

On one hand, this survey represents the empirical counterpart of the theoretical concepts introduced in Chapter 2. On the other hand, it provides the necessary introduction to the state of the art of the Italian literature, before discussing my empirical contribution on Italian regions. In the light of this, the survey proceeds in two steps.

The first part of the review — section 4.2 — deals with the international literature, mainly interested in the U.S. economy and in cross-country comparisons. In particular, I overview studies that use three alternative methodological approaches: the *growth* approach — subsection 4.2.1 —, the *production function* and *growth accounting* approaches — subsection 4.2.2. Then, in the second part of the survey — section 4.3 — I will discuss in more detail the most relevant applied works to the case of Italy.

4.2 Empirical Evidence on the Productive Role of Public Capital

This section discusses the literature on the empirical linkage between public capital and economic performance.

Subsection 4.2.1 focuses on the *growth* approach. This brief survey distinguishes empirical works belonging to this field of research in two generations, where the turning point is given by the reviewed interest stimulated by the emerging of *new growth* models. To the first generation belong works published in the 1980s within the broader literature on the determinants of growth and mainly interested in the definition of the optimal government size in the economy. These early studies do not have a proper theoretical background, in the sense that they do not properly test analytically derived relationships. They rather study partial correlations between output growth and alternative measures of fiscal policy. On the other hand, during the 1990s, the development of new growth models has spurred a second generation of empirical investigations aimed at testing "structural" equations derived from the solution of growth models, including fiscal variables.

The first part of subsection 4.2.2 deals with the so-called *production function* approach applied by Aschauer (1989a) and boosted by the interest of researchers in analysing the productivity slowdown experienced by the United States in the 1970s and the 1980s. The methodology employed by this author has been criticized by many researchers and his results questioned with respect to many statistical problems. However, the paper by Aschauer (1989a) is still regarded as the seminal contribution within this empirical approach. After having described the main results obtained by Aschauer, I will discuss the main criticisms developed by other authors who have applied the same methodology to the American economy and, more recently, to other countries or samples of countries.

The assumption behind the implementation of the production function approach is that the beneficial effect on economic activity of public capital is due to its role of direct productive input. The second part of the subsection overviews works which do not make this assumption, analysing its impact on total factor productivity as an environmental variable. I will refer to this part of the literature as the traditional *growth accounting* approach, whose theoretical background is provided by Hulten and Schwab (1984, 1991).

4.2.1 Growth Regressions

The first generation of empirical studies on public spending in growth regressions has been undertaken at the beginning of the 1980s, within the literature on the determinants of growth. The crucial interest of this broader field of research is the explanation of differentials in growth rates across countries in terms of various macroeconomic indicators. The works belonging to this empirical literature that are more interesting for our analysis are those that view differences in fiscal policy (such as government expenditures, public consumption or tax measures) as the primary source of growth differentials across countries.

Typically, the government size is measured by the level of public expenditure and used as a regressor in cross-country regressions on growth rates of the following general type:

$$\gamma_{i,[0,t]} = \sum_{j=1}^{k} a_j \cdot X_{i,j} + \sum_{j=1}^{m} b_j \cdot Y_{i,j} + \sum_{j=1}^{n} c_i \cdot Z_{i,j} + \epsilon_{i,t} \qquad (4.2.1)$$

where γ is the average annual change in per capita GDP in country i over the period [0, t], X are k averaged fiscal policy explanatory variables, Y are m averaged non-fiscal policy explanatory variables (various macroeconomic indicators) and Z are n conditioning variables[1].

[1] Most commonly used conditioning variables are the initial level of per capita GDP, the initial private investment to GDP ratio and some measures of demography or education (such as the averaged annual population growth rate or the initial level of investment in human capital).

A number of studies have estimated alternative specifications of equation (4.2.1) with the aim of assessing the empirical linkage between fiscal policy and growth. In particular, the most common used explanatory variables X have been measures of the overall size of the government, disaggregated measures of various categories of government expenditures (or measures of their growth rates), measures of government deficits and disaggregated measures of government taxation (Levine and Renelt, 1991).

The results obtained by some works within the first generation of growth regressions are reported in Table 4.2.1. These are the most cited works in the literature: for a complete and exhaustive survey, see Levine and Renelt (1991).

Landau (1983), Kormendi and Meguire (1985) and Grier and Tullock (1989) retrieve data from the Summers and Heston data base and use the ratio of total government expenditures to GDP as a proxy of the size of the government. They share the common conclusion that governments of smaller size are associated with higher growth rates. For instance, Landau (1983) estimates the effect of two categories of government expenditures on per capita GDP growth: public consumption and total investment in education (both expressed as shares of per capita GDP). His analysis covers the period 1961-76 for a very heterogeneous sample of 104 countries and concludes for a positive impact of investment in education and a negative impact of public consumption on growth. However, the hampering effect of public consumption on growth is found to be not statistically significant for a sub-sample of poorest countries.

Landau (1986) extends the analysis to Less Developed countries, assessing the role of transfers to households, educational expenditure and public investment. The coefficient of all these categories of public spending are found to be statistically insignificant. Kormendi and Meguire (1985) estimate a non statistically significant coefficient of public consumption expen-

Table 4.2.1: Summary of Growth Regressions (1)

Authors	Data[a]	Estimations[b]	Results[c]
Landau (1983)	104 c.,1961-76	CS	PC (-), EDU (+)
Landau (1985)	16 OECDc,1952-76	PD,CS	T (not sig.), PI (-)
Landau (1986)	96 LDCs,1960-80	CS	T (not sig.), PI (not sig.), PC (not sig.)
Kormendi and Meguire (1985)	47 c.,1950-77	CS	PC (not sig.)
Ram (1986)	115 c., 1960-80	ATS,CS	PC (+)
Grier and Tullock (1989)	115 c., 1951-80	CS,ATS	PC (-)
Barro (1991)	98 c.,1960-85	CS	PI (not sig.), PC (-)
Engen and Skinner (1992)	107 c.,1970-85	IV	PC (-), TR (-)

[a] OECDc = OECD countries; LDCs = Less Developed countries.

[b] CS = Cross Section; PD = Panel Data; ATS = aggregate time series; IV = Instrumental Variables.

[c] Sign of statistically significant estimates in parenthesis. PC = Total Public Consumption; EDU = Public Expenditure in Education; T = Transfers to Households; PI = Total Public Investment; TR= Total Tax Revenues; not sig. = not statistically significant estimates.

diture as a share of GDP in a cross-section regression of 47 countries.

Barro (1991) attempts to distinguish the effect of government expenditures on growth depending on their allocation amongst alternative uses. In order to do so, he regresses average growth rates on the ratio of government consumption expenditure to GDP[2] and on a measure of public investment. His data are on a sample of 98 countries over the period 1960-85 and the estimated coefficients of the two explanatory variables are equal to -0.12 and 0.10 respectively, but only the first one is statistically significant.

A general drawback of these early studies is that their results are difficult to interpret from a theoretical point of view. In particular, the negative estimated effect of public consumption on growth is difficult to reconcile with the neoclassical prediction of no permanent effect of fiscal policy on long-run economic growth. Engen and Skinner (1992), for instance, argue that the negative coefficient of the share of government expenditure in growth regressions can be interpreted as a signal of spurious correlation between the dependent and the independent variables. This authors use Instrumental Variables for the changes in the government share of GDP in order to correct for this bias.

The evidence of a negative impact of government expenditure on growth, however, is commonly interpreted in support of "liberal" view according to which the taxes needed to finance government expenditure distort incentives in private markets and prevent the efficient allocation of resources, causing a fall in the level of output (de la Fuente, 1997).

Levine and Renelt (1992), Levine and Zervos (1993) and Easterly and Rebelo (1993) question the reliability of the above studies suggesting that their results can be sensitive to alternative specifications of the regression equation (4.2.1).

[2] Excluding expenditures in defence and education, which he considers to be growth promoting categories of public spending.

For instance, Levine and Renelt (1992) work on a data set of 106 countries over the period 1960-1986 in order to assess the robustness of the results obtained in previous empirical studies. They state: *"Given that over 50 variables have been found to be significantly correlated with growth in at least one regression, readers may be uncertain as to the confidence they should place in the findings of anyone study"*. Hence, their aim is to examine if partial correlations estimated in previous studies are robust to small changes in the list of right-hand side variables in equation (4.2.1).

In general, most of the partial correlations relating economic growth to each of the explanatory variables in (4.2.1) are found to be not robust to small changes in other explanatory variables. As for the fiscal policy explanatory variables X, none of the measures either of overall government size or disaggregated measures of public spending[3] are found to be robust to alternative specifications of (4.2.1).

The development of the new growth theory and its prediction of a permanent effect of fiscal policy on long-run growth has renewed the interest in the empirical investigation of the growth effect of government spending. This has given rise to a new generation of studies characterized by the estimation of reduced equations derived as solutions of endogenous growth models.

Amongst these works summarized in Table 4.2.2, those interested in the empirical test of endogenous Barro-type models deserve a particular attention. In particular, some recent studies have been aimed at empirically testing the prediction of Barro-type models regarding the composition of government expenditure and the form of taxation. Devarajan *et al.* (1996) have dealt with the first issue, whereas Kneller *et al.* (1999) and Bleany *et al.* (2001) have deepened the analysis in order to account for alternative

[3] They test the robustness of measures of public consumption, government capital formation, government expenditures on education and government expenditures on defence.

Table 4.2.2: Summary of Growth Regressions (2)

Authors	Data[a]	Estimations[b]	Results[c]
Devarajan *et al.* (1996)	43 Dc,1970-90	PD	HS (+), TC (+), D (-), EDU (-)
Kocherlakota and Yi (1997)	N,U.K.,1831-1991	ATS	PI (+), PI with T (not sig.)
	N,US, 1891-1991	ATS	PI (+), PI with T (not sig.)
de la Fuente (1997)	19 OECDc,1965-95	PD	PC (not sig.), T (+), SUB (+), PI (+)
Miller and Russek (1997)	39 c.,1975-84	PD	(*)
Kneller *et al.* (1999)	22 OECDc,1970-95	OLS,GLS,IV	DT (-), NDT (0), PE (+), UPE (0)
Bleany *et al.* (2001)	17 OECDc,1970-94	DPD	DT (-0.41), NDT (0), PE (0.39), UPE (0)
Démurger (2001)	P,China,1985-98	PD	C (+), Tr (+)
Kalyvitis (2003)	N,Canada,1955-99	ATS	PC (-), TR (-), IF (+)

[a] N = National Data; P = Data on Provinces; Dc = Developed countries; OECDc = OECD countries

[b] GLS = Generalized Least Squares; IV = Instrumental Variables; ATS = Aggregate Time Series; DPD = Dynamic Panel Data; PD = Panel Data.

[c] Sign of statistically significant estimates in parenthesis; not sig. = not statistically significant estimates. HS = Health Services; TC = Public Expenditure in Transports and Communications; D = Defence; EDU = Public Expenditure in Education; PI = Total Public Investment; PI with T = Public investment included in the regression equation together with the source of financing; PC = Public Consumption; SUB = Subsidies to Firms; (*) = In general, results depend on the source of financing public expenditure; PE = Productive Expenditures (Education, Health, Law and Order, General Public Services, Housing, Transports and Communication); UPE = Unproductive Expenditures (Welfare security and Welfare, Recreation and Economic Services); DT = Distortionary Taxation (Income and profits taxes, Social security taxes, payroll and manpower taxes); T = Transfers to Households; NDT = Non-distortionary Taxation (Domestic Goods and Services taxes); IF = Infrastructure Formation; Tr = Public Expenditure in Transports; TR = Total Tax Revenues; C = Public Expenditure in Communications.

structures of taxation[4].

The prediction of the growth effect of fiscal policy depends crucially on the source of financing. A rise of productive public spending financed by non-distortionary taxation raises the long-run rate of growth. On the other hand, the growth effect of a rise of productive public spending financed by distortionary taxation will depend on the ongoing level of expenditure. Increases in unproductive public spending through distortionary taxation negatively affect long-run growth, whereas if such increases are financed by some form of non-distortionary taxation, long-run growth will be unaffected.

Kneller *et al.* (1999) note that the estimation of regressions of the general type in (4.2.1) are unable to test appropriately such predictions[5]. They argue that empirical works belonging to — what we have above defined — the first generation of growth regression are only *"partial"* studies of the growth effects of fiscal policy, in the sense that they only consider one side of the government budget constraint, abstracting from the other. Let us suppose, for instance, that the fiscal variable X, whose growth impact we want to study, is public investment. If we omit from the estimated equation the tax used to finance it, the assumption about the source of financing of public investment will be *"implicit"*. This is likely to produce a biased estimate for the coefficient of X, unless the omitted variable has a null effect on growth. Indeed, the estimate will be biased if the implicit source of financing has a non neutral impact on growth.

For instance, Kocherlakota and Yi (1997) estimate the growth effect of public investment — both in U.K. and U.S. economies over very long time periods — finding the attached coefficient positive and significant when they include a tax measure in the regressed equation, and insignificant otherwise.

[4] See also Kneller and Gemmell (2002) for a study on European countries.

[5] A previous study by Miller and Russek (1997) also finds growth effects of public expenditure to vary according to the source of financing, abstracting from the various type of expenditure and taxation suggested in Barro-type endogenous growth models.

In general, if all the elements of the government budget constraint were included in (4.2.1), we would have:

$$\sum_{j=1}^{k} X_{i,j} = 0 \tag{4.2.2}$$

and, in order to avoid perfect collinearity, one of the X should be omitted from the regression equation. By omitting X_k for each country i, equation (4.2.1) becomes:

$$\gamma_{i,[0,t]} = \sum_{j=1}^{k-1} (a_j - a_k) \cdot X_{i,j} + \sum_{j=1}^{m} b_j \cdot Y_{i,j} + \sum_{j=1}^{n} c_j \cdot Z_{i,j} + \epsilon_{i,t} \tag{4.2.3}$$

This new regression equation implies that the correct estimate of the coefficient of each X does not indicate the growth impact of a unit change in X; it rather represents the growth effect of a unit change in X *"offset by a unit change in the omitted variable, which is the implicit financing element"*. As a consequence, if the omitted element of the government budget constraint changes, the coefficient of each fiscal variable entering the estimated equation will change too. In particular, this will not happen only if the omitted variable has a null impact on growth.

From equation (4.2.3), it follows that it is only possible to test the null hypothesis that the difference $(a_j - a_k)$ is equal to zero and not the null hypothesis that each a_j is equal to zero individually. However, it is still possible to test whether two a_j $(j \neq 1, 2)$ are equal to each other.

If $H_0 : a_1 = a_2 = 0$ can not be rejected, more accurate estimated coefficients of the included fiscal variables can be obtained by omitting both X_1 and X_2. Hence, the appropriate way of proceeding suggested by Kneller *et al.* (1999) is *"to test down from the most complete specification of the government budget constraint to less complete specifications, taking care to omit only those elements which theory suggests will have negligible growth effects"*.

In order to show how the mis-specification of the government budget constraint can lead to misleading results and does not allow to test properly the prediction of Barro-type models, Kneller *et al.* (1999) compare results obtained by estimating two equations of the types in (4.2.1) and (4.2.3), using data on OECD countries over the period 1970-95. The fiscal variables of interest are distortionary taxation (DT), non-distortionary taxation (NDT), productive expenditure (PE) and unproductive expenditure (UPE). In general, their results confirm that the sign of the coefficients of the variables of interest change widely if the government budget constraint is mis-specified.

In coherence with their suggested procedure, Kneller *et al.* (1999) estimate equation (4.2.3) including the fiscal variables PE and DT. They first omit the variables NDT and UPE individually. Then, they omit both variables and do not reject the null hypothesis that they have a common coefficient. Hence, they rely on the results obtained in this last case, where productive expenditure (included) is implicitly financed by non-distortionary taxation (omitted) and distortionary taxation (included) implicitly finances unproductive public spending (omitted). As predicted in the theory, the coefficients attached to DT and PE are estimated to have negative and positive signs respectively and are both statistically significant.

4.2.2 The Production Function and The Growth Accounting Approaches

In section 2.2, we have studied the productive role of public capital from a theoretical point of view. We concluded that if public capital enters the aggregate production function as an unpaid direct input, it will affect economic activity through a direct effect on output and an indirect effect on the productivity of other inputs — see effects (1) and (2), page 9. We now deal with the empirical implementation of this theory, known in the literature as the production function approach.

Aschauer (1989a) estimates alternative specifications of the following two general equations:

$$\ln\left(\frac{Y_t}{K_t}\right) = a_0 + a_1 \cdot t + a_2 \cdot \ln\left(\frac{L_t}{K_t}\right) + a_3 \cdot \ln\left(\frac{G_t}{K_t}\right) + a_4 \cdot CU_t + u_t \quad (4.2.4)$$

and

$$tfp_t = b_0 + b_1 \cdot t + b_2 \cdot \ln(G_t - I_t) + b_3 \cdot CU_t + e_t \quad (4.2.5)$$

where t is a linear time trend and the capacity utilization rate CU_t controls for the business cycle. Equations (4.2.4) and (4.2.5) are the empirical counterparts of equations (2.2.4) and (2.2.5) at page 12. Data on the "net stock of non-military public structure and equipments"are employed as a proxy of the variable G_t and OLS estimations are based on U.S. aggregate national time series data over the period 1949-1985.

The main purpose of the author is to figure out to which extent the accumulation of public capital explains the behavior of productivity over the sample period and, in particular, the productivity slowdown experienced by the U.S. economy in the 1970s and the 1980s.

Aschauer's results can be summarized as follows. In a first step of his analysis, the OLS point estimates for the coefficients of the labour to capital ratio and the public to private capital ratio in equation (4.2.4) — a_2 and a_3 — are positive and statistically significant (0.35 and 0.39 respectively). This means that under the assumption of CRTS, when either the labour to private capital ratio or the public to private capital ratio rises by 10%, private capital productivity increases by 3.5% or 3.9%, respectively. In a second step, equation (4.2.4) is estimated allowing for separate coefficients of the two private inputs in order to test the implicit restriction of CRTS in (4.2.4). The value of the relevant F test does not allow to reject such a restriction. In particular, when public capital is omitted, the coefficients of both private inputs become negative and significant (-0.48 and -0.66 respectively) and the DW test is very low in value, supporting the evidence of the presence of serial correlation in the residuals u_t. Hence, Aschauer concludes that adding G_t allows private inputs to get positive and significant elasticities and to eliminate serial correlation.

As for the estimation of equation (4.2.5), the general conclusion is that there exists a strong empirical linkage between the provision of public capital stock and total factor productivity.

The above analysis refers to the aggregate stock of non-military public capital, whose output elasticity is estimated to be 0.39. To some extent, the analysis is carried out also at a higher level of disaggregation. In particular, the author provides estimated values of output elasticities for the following categories of non-military public capital: *Core public capital*[6] (0.24), *Educational Buildings* (-0.01), *Other Buildings*[7] (0.04), *Hospitals* (0.06) and *Conservation and Development* (0.02)[8]. In general, the overall set of results leads to conclude that the decline in public investment in infrastructure has been an important determinant of the productivity slowdown experienced by the U.S. economy in the 1970s and the 1980s.

A similar methodology is applied by Aschauer (1989b) to the Group of Seven over the period 1966-85, with results supporting the evidence of the U.S. case: a 10% increase in public investment in infrastructure yields a 4% gain in labour productivity. Furthermore, Munnell (1990a) and Munnell (1990b) confirm the results obtained by Aschauer by estimating Cobb-Douglas production functions using aggregate time series for the period 1970-86 and pooled state data over the period 1949-87 respectively[9].

[6] This category is given by the sum of the stocks of highways, airports, electrical and gas facilities, water systems and sewers and mass transit.

[7] This category includes office buildings, police and fire stations, courthouse, garages, and passenger terminals.

[8] Garcia-Milà and McGuire (1992) estimate output elasticities of 0.04 and 0.07 for the categories *Highways* and *Education* respectively.

[9] Munnell (1990b) makes the alternative assumptions of no constraints, CRTS to private inputs and CRTS in the threes inputs. Regardless of the restrictions imposed, public capital results to be highly productive, with elasticity values ranging from 0.31 to 0.39.

Although Aschauer's (and Munnell's) findings are the most cited in the literature, a few authors followed similar methodologies in earlier works. For instance, Mera (1973) estimates Cobb-Douglas production functions for the primary, the secondary and the tertiary sectors of the Japanese economy using OLS and concluding that "social capital" had a positive and significant impact on output growth of the three sectors. Ratner (1983) uses pooled time series level data on the U.S. economy over the years 1949-73, and finds an output elasticity of public capital of 0.06. Costa *et al.* (1987) estimate public capital to have significantly contributed to manufacturing output in United States in the year 1972. Less optimistic are the conclusions reached by Eberts (1986) who estimates the values of output elasticity of various public capital categories[10] to be well below those of private capital and labour input in U.S. metropolitan areas.

Tables 4.2.3-4.2.6 provide a summary of the literature following the production function approach, by distinguishing a number of studies according to Data on which they base their analysis, implemented Estimation Techniques and Results that they obtain. Tables 4.2.3 and 4.2.4 collect empirical works on the U.S. economy.

Looking at these Tables, it seems that regional panel data (R, PD) studies are characterized more than others either by statistically insignificant or very low in magnitude coefficients[11]. On the other hand, aggregate time series (ATS) studies generally lead to more optimistic results. Moreover, as originally noted by Holz-Heakin (1994), within regional studies the more optimistic results are produced by those neglecting state specific effects, whereas when unobserved state specific effects are taken into account, output elasticity of public capital declines. In general, the major feature of this vast scientific production is the extremely wide range of results, which makes the literature finally inconclusive.

[10] Highways, public hospitals, sewage and sanitation facilities.

[11] In one case — Pinnoi (1994) — the point estimate of output elasticity is even negative.

Table 4.2.3: Summary of the Production Function Approach (United States)

Authors	Data[a]	Estimations[b]	Elasticities[c]
Ratner (1983)	N,1949-73	ATS,L	0.06
Eberts (1986)	MA,1958-78	PD,L	0.03/0.04
Costa *et al.* (1987)	R,1972	TL,CS,L	0.19/0.26
Aschauer (1989a)	N,1949-85	ATS,L	0.39
Ram and Ramsey (1989)	N,1949-85	ATS,L	0.24
Aaron (1990)	N,1952-85	ATS,L	not robust
Merriman (1990)	R,1972	TL,CS,L	0.20
Aschauer (1990)	R,1965-83	PD,L	0.11
Munnell (1990a)	R, 1970-86	PCS,L	0.06/0.15
Munnell (1990b)	N,1949-87	ATS,L,FD	0.31/0.39
Munnell and Cook (1990)	R,1970-86	PCS,L	0.15
Tatom (1991)	N,1949-89	ATS,FD,L	0.04
Ford and Poret (1991)	N,1957-89	ATS,L,FD	0.39/0.54
Eisner (1991)	R,1970-86	PCS,L	not sig.

[a] N = National Data; R = Regional Data; MA = Metropolitan Areas.

[b] PD = Panel Data; ATS = aggregate time series; L = (log) Levels; TL = Translog production function; CS = Cross Section; PCS = Pooled Cross Section; FD = (log) First Differences; SE = Simultaneous Equations.

[c] Figures listed are statistically significant point estimates of output elasticity of public capital; not sig. = not statistically significant estimates.

Most papers on the U.S. economy reported in Tables 4.2.3 and 4.2.4 are aimed at re-examining Aschauer's results by overcoming what they claim to be some methodological limitations of his work. Indeed, Aschauer's estimation strategy has been criticized along many directions, and his point estimates of γ — see "Elasticities"in Table 4.2.3 — has been often judged implausibly high due to some methodology drawbacks.

Table 4.2.4: Summary of the Production Function Approach (United States, 2)

Authors	Data[a]	Estimations[b]	Elasticities[c]
Duffy-Deno and Eberts (1991)	R,1969-82	SE,FD	0.08
Garcia-Milà and McGuire (1992)	R,1969-82	TL,PD,L	0.04/0.07
Munnell (1993)	R,1970-86	PCS,L	0.14/0.17
Holtz-Heakin (1994)	R,1969-86	PD,L,FD	not sig.
Eisner (1994)	N,1961-91	ATS,L	0.27
Pinnoi (1994)	R,1970-86	TL,PD,L	-0.11/0.08
Evans and Karras (1994a)	R,1970-86	PD,L,FD	not sig.
Baltagi and Pinnoi (1995)	R,1970-86	PD,L	not sig.
Andrews and Swanson (1995)	R,1970-86	PCS	0.11
Sturm and De Hann (1995)	N,1949-85	ATS, L,FD	0.41/not sig.
Ai and Cassou (1995)	N,1947-89	ATS, FD	0.15
Holtz-Heakin and Schwartz (1995)	R,1970-86	PD,FD	0.1
Garcia-Milà *et al.* (1996)	R,1970-83	PD,FD	not sig.
Crowder and Himarios (1997)	N,1947-89	ATS,L	0.17/0.38

[a] N = National Data; R = Regional Data.

[b] FD = (log) First Differences; L = (log) Levels; SE = Simultaneous Equations; TL = Translog production function; PD = Panel Data; PCS = Pooled Cross Section; ATS = aggregate time series; CS = Cross Section.

[c] Figures listed are statistically significant point estimates of output elasticity of public capital; not sig. = not statistically significant estimates.

Amongst the works that have applied Aschauer's analysis to other countries — see Tables 4.2.5 and 4.2.6 — the most interesting ones are those that have tried to correct some defects of his methodology, implementing more sophisticated estimation techniques and using alternative data sets. On the other hand, other researchers have merely replicated Aschauer's work to other countries by strictly following his estimation strategy and, as a consequence, their results are subject to similar criticisms.

In one of the first papers interested in cross-country comparisons, Ford and Poret (1991) analyse 11 OECD countries and make the very restrictive assumption that private capital and labour form a unique private sector's input[12]. They study the impact of non-military public capital on TFP growth, finding output elasticity values ranging from 0.15 (France) to 0.70 (Australia)[13]. However, their results are mixed and, under alternative specifications of the production function, the enhancing effect of infrastructure on TFP growth is never significant in the U.K., Norway and Australia, always significant in U.S.A., Canada, Germany, Belgium and Sweden, and only sometimes significant in France, Japan and Finland.

The overview of some of the contributions whose analysis are summarized in the Tables, can use as a guide line the objections that they move to earlier works and how their results have improved the existing literature accordingly. The most common criticisms moved to Aschauer regard (i) spurious correlation problems due to the mis-specification of the estimated equation (omission of relevant variables) (ii) non-stationarity of the data, (iii) the reserve causality issue, and (iv) the use of aggregate time series data.

(i) Tatom (1991) claims the *mis-specification* of the production function estimated by Aschauer, noting that the fall in public investment in infrastructure coincided with the sharp rise of oil prices in the 1970s. Hence, he extends the regression equation in (4.2.4) — see page 112 —, adding a vari-

[12] Toen-Gout and Jongeling (1994), who study the case of the Netherlands, impose a similar restriction.

[13] They use two definitions of public capital; one including electricity, water and gas suppliers, transport and communications and defence, the other excluding the defence.

able as a proxy of the energy prices. This leads to a relevant reduction in the estimated coefficient of γ, which goes down to 0.04[14]. Further problems of mis-specification of the production function regard the omission of other relevant variables. For instance, Duggal *et al.* (1999) argue that, since the production function is a Cobb-Douglas, the capacity utilization index *CU* should enter multiplicatively the estimated equation (4.2.4).

(ii) A second problem detected in Aschauer's work is the *non-stationarity of the data*, which may lead to spurious relationships between public capital and the relevant dependent variable. The variables used by Aschauer are indeed found to be neither stationary nor cointegrated in the following-up literature, which makes it necessary to use first differences (Tatom, 1991).

Sturm and De Hann (1995) argue that Aschauer's results do not hold if the stationarity issue is taken into account given that none of the series included in the regression is stationary in levels. Their conclusion is that if stationarity is tested for, the model must be estimated in first differences.

Previous studies estimating equations in first differences are characterized by mixed results. Hulten and Schwab (1991) fit in first differences the model of Aschauer, getting sometimes negative coefficients. Evans and Karras (1994a) and Garcià-Mila *et al.* (1996) also use first differences, but they do not find a statistically significant relationship between public capital and productivity. Munnell (1992) argues that estimations in first differences generate results which are difficult to interpret. Indeed, first differences specifications destroy the long-run relationship in the data removing all trend components and leading to analyse the impact of public capital growth in one year on the productivity growth experienced in the same year.

Amongst studies that use cointegration techniques in order to avoid spurious regressions, Bajo-Rubio and Sosvilla-Rivero (1993) use aggregate time

[14] However, for their nature, energy prices should be included in a cost function and the choice on including them in the aggregate production function has been criticized by some authors; see, for instance, Duggal *et al.* (1999).

Table 4.2.5: Summary of the Production Function Approach (Other Countries)

Authors	Data[a]	Estimations[b]	Elasticities[c]
Mera (1973)	Japan, 3 sectors	OLS	(+)
Aschauer (1989b)	G7,1966-85	PD,FD	0.34/0.73
Merriman (1990)	R,Japan,1954-63	TL,PD,L	0.43/0.58
Ford and Poret (1991)	11 OECDc	ATS,FD	sig. in 5 countries
Berndt and Hansson (1992)	Sweden,1960-88	ATS,L	not sig.
Bajo-Rubio and Sosvilla Rivero (1993)	Spain,1964-1988	C,ATS,L	0.19
Otto and Voss (1994)	Australia,1966-90	ATS,L	0.4
Evans and Karras (1994b)	7 OECDc, 1963-88	PD,FD	not sig.
Toen-Gout and Jongeling (1994)	Netherlands,1960-90	ATS,FD	0.37
Mas *et al.* (1996)	R,Spain,1980-89	PD,L	0.08
Sturm and De Hann (1995)	N,Netherlands,1960-90	C,ATS,L,FD	0.26/0.41
Dalamagas (1995)	Greece,1950-92	ATS,FD	0.53

[a] N = National Data; R = Regional Data; OECDc = OECD countries.

[b] PD = Panel Data; FD = (log) First Differences; TL = Translog production function; L = (log) Levels; ATS = aggregate time series; C = Cointegration Analysis.

[c] Sign of statistically significant point estimates of output elasticity of public capital in parenthesis. Figures listed are statistically significant point estimates of output elasticity of public capital; not sig. = not statistically significant estimates.

Table 4.2.6: Summary of the Production Function Approach (Other Countries, 2)

Authors	Data[a]	Estimations[b]	Elasticities[c]
Wylie (1996)	N,Canada,1946-91	ATS,L	0.11/0.52
Otto and Voss (1996)	N,Australia,Q,1959-92	ATS	0.17
Ramirez (1998)	N,Mexico,PI,1950-90	ATS,FD	0.12
de Frutos *et al.* (1998)	N,Spain,1964-92	C,ATS,L	0.21
Mamatzakis (1999)	N,Greece,1959-93	ATS,L	0.25
Sturm *et al.* (1999)	N,Netherlands,1853-1913	VAR	(+)
Dessus and Herrera (2000)	28 LDCs, 1981-91	PD,SE	0.13
Ligthart (2000)	N,Portugal,1965-95	C	0.20/0.35
Everaert and Heylen (2001)	N,Belgium,1953-96	C, ECM	0.29
Kemmerling and Stephan (2002)	German cities,1980/86/88	SE	(*)

[a] N = National Data, LDCs = Less Developed Countries.

[b] C = Cointegration Analysis; ECM = Error Correction Model; SE = Simultaneous Equations; VAR= Vector AutoRegression; FD = (log) First Differences; L = (log) Levels; PD = Panel Data; ATS = aggregate time series.

[c] Sign of statistically significant point estimates of output elasticity of public capital in parenthesis. Figures listed are point estimates of output elasticity of public capital; impl. = implausible values; (*) = Strong effect of public capital on output, negligible feedback effect from output to public capital and weak simultaneity between output and public capital.

series for Spain and estimate the same production function as in Aschauer (1989a) under the two alternative restrictions of CRTS in private inputs and CTRS over all inputs. They do not reject the assumption of CRTS in the three inputs and find an output elasticity of public capital of 0.19.

(iii) A further source of criticism is whether a positive and significant correlation indicates that infrastructures raise output (and productivity) or it means instead that output (and productivity) positively affects the demand for public infrastructure. In other words, the problem is the direction of causality: does it run from infrastructures to output (and productivity) or in the opposite direction? Eisner (1991) puts under question the results obtained by Aschauer, noting that the productivity slowdown may have the effect of reducing the demand for public capital. In this case, the positive and significant coefficient for the public input into the production function should be interpreted as the extent to which an increase in national income yields a higher level of government intervention in the economy.

To address the issue of causality, some authors have followed a simultaneous equations approach. Amongst these authors, Duffy-Deno and Eberts (1991), Cadot *et al.* (1999), Kemmerling and Stephan (2002).

Other authors have estimated Error-Correction Models instead. For instance, Everaert and Heylen (2001) study the impact of public capital on total factor productivity in Belgium over the period 1953-96. Within a cointegration framework[15], they analyse the direction of causality. They strictly follow Aschauer (1989a), the only exception being the use of "patent statistics" as a proxy of technological progress. The estimated output elasticity that they find is around 0.29.

If the production function is part of a system in which both inputs and output are endogenously determined, the estimation of such a production

[15] In a preliminary step of their analysis, they find that all variables included in their regressions exhibit a unit root. Instead of taking fist differences, they analyse the existence of a long-run equilibrium relationship.

function alone might be subject to possible *simultaneous equation bias*[16]. Dessus and Herrera (2000) study the panel of 28 Developing countries in the 1980s. They address the methodological problem of simultaneity bias by developing a framework in which public and private capital are both endogenous[17]. They estimate a simultaneous equations model composed by the production function in labour, private capital and public capital and the demand equations for private and public capital, in which the two forms of capital are assumed to depend on the level of output, an indebtedness index, their own one period lagged values and a linear time trend. The system is estimated using three-stage least squares method and the coefficient of public capital in the production function is found to be around 0.13.

(iv) The implicit assumption behind the use of aggregate time series is that marginal productivity and the rate of technological change do not vary across states. In other words, it is assumed the absence of any source of *heterogeneity* either across countries or across regions within a country. The limitation of this assumption has led progressively to prefer panel data techniques of estimation considering both state and time specific effects. In order to do so, given a panel of i states over t years, it is necessary to assume in equation (4.2.4) an error structure of the type $u_{i,t} = f_i + \delta_t + \mu_{i,t}$, where f_i is a time invariant state specific effect, δ_t is a common linear trend across states and $\mu_{i,t}$ is an idyosincratic error term.

[16] In order to overcome this defect, Everaert and Heylen (2001) use fully modified least squares estimation procedure.

[17] A further peculiarity in this study is that the lack of data on stocks forces the authors to distinguish between private and public capital on the basis of a property criterion. In particular, they disaggregate total investment in public and private capital, assuming that the stock of public capital in the middle of their sample period is equal to total capital stock multiplied by the average share of public investment over the full sample period. Then, they extrapolate backward and forward from the middle year using the permanent inventory method and obtain stocks data for the two forms of capital for every year of the sample period.

The first studies that take into account cross-state heterogeneity are by Holz-Heakin (1994) and Evans and Karras (1994a, 1994b). Criticizing the use of aggregate time series made by Aschauer, they argue that it is necessary to consider the terms $f_i's$ and $\delta_t's$ in the error structure to account for heterogeneity across United States. In particular, they indicate as sources of heterogeneity the differences in climate and topography and the systematic improvement in technologies which leads to systematic differences across states and over time. The authors motivate their choice of treating both $f_i's$ and $\delta_t's$ as fixed — and not as random terms — for three reasons. First, $f_i's$ can not be considered as a sample of realizations from a population because the panel is composed by the entire population of United States. Second, the technology improves over the entire sample period. Third — as long as the errors $\mu_{i,t}$ are uncorrelated with the explanatory variables — generalized least square estimation of the production function leads to consistent estimates when $f_i's$ and $\delta_t's$ are treated as fixed effects but would yield inconsistent estimates if the two effects were ignored or treated as random terms.

Using the same data as Munnell (1990a), Evans and Karras (1994a) compare OLS and Panel Data estimations of a production function similar to (4.2.4) — see page 112 —, the only difference being the inclusion of the unemployment rate instead of CU. The main results of their regressions is that the positive and significant OLS point estimate of the coefficient of public services, disappears when they control for country and time specific effects. This result seems to be robust to different specifications of the error term $u_{i,t}$, to the alternative specification of a translog production function and to different levels of disaggregation of public services. In particular, *Educational Services* is the only category of public services to get positive and significant coefficient when panel data techniques of estimations are employed[18]. However, Ai and Cassou (1997) demonstrate the presence of multicollinearity in the data used by Holtz-Heakin (1994) and Evans and Karras (1994a). Using the same data set employed by Holtz-Heakin (1994), they regress public capital on state and time dummies, obtaining a correlation index close to 1. A similar regression is carried out on data used by Evans and Karras (1994a). Even in this case, total public capital — and the disaggregated categories "highways", "water and sewers" and "others" — turns out to be highly correlated with state and time dummies.

All the above studies estimate average aggregate production functions augmented to include public capital. However, public capital can be productive without being an unpaid factor, but rather acting like an environmental variable to the private sector of the economy. Going back to Meade (1952), this view stands behind the empirical studies following the growth accounting (or sources of growth) approach. In general, empirical studies within this approach reach less optimistic conclusions on the productiveness of public capital. The most influential paper of this part of the literature is by Hulten and Schwab (1991), whose main results are as follows.

Recalling the concepts introduced in subsection 2.2 (see pages 13-14), TFP growth is decomposed in two parts:

$$\dot{A}_t = \gamma_A \cdot \dot{G}_t + \dot{\hat{A}}_t \qquad (4.2.6)$$

where γ_A is the elasticity of measured TFP growth with respect to public capital, $\dot{\hat{A}}_t$ is the "true" Solow residual, and $\dot{A}_t$ is measured from the data using $\dot{A}_t = \dot{Y}_t - s_K \cdot \dot{K}_t - s_L \cdot \dot{L}_t$. Within this framework, the external effect of public capital on private output is studied for the manufacturing sector of the U.S. economy during the period 1970-86.

The main interest of Hulten and Schwab (1991) is the role of public

[18] In order to overcome the problems related with the estimation longitudinal data using OLS, Andrews and Swanson (1995) use the same data employed by Munnell (1990b) and find a less optimistic value of 0.11 for the output elasticity of public capital.

capital as a source of externalities, but their analysis does not exclude the view of public capital as unpaid factor of production. Indeed, referring to Meade (1952), they estimate the following equation:

$$\dot{A}_t = (\gamma_A + \gamma) \cdot \dot{G}_t + (i-1)\dot{K}_t + \hat{\dot{A}}_t + u_t \qquad (4.2.7)$$

where γ_A is defined as in equation (4.2.6), γ is the elasticity of output with respect to public capital and $i = \alpha+\beta+\gamma$. Hence, measured TFP growth is regressed on the rate of growth of public capital, the rate of growth of private capital and the "true" Solow residual. The parameter $(\gamma_A+\gamma)$ relates public capital to TFP growth, embodying both its direct effect as unpaid factor — *i.e.*, effect (1) at page 9 — and its indirect as environmental variable — *i.e.*, effect (3) at page 13. Indeed, $(\gamma_A+\gamma)$ is interpreted as the overall elasticity of output with respect to public capital. The coefficient of K_t is given by the elasticity of scale minus one and it is used to test the restriction of CRTS to all inputs. The "true" residual is measured by a constant or alternatively by a dummy for each of the years included in the sample period.

In general, estimating alternative versions of equation (4.2.7), Hulten and Schwab (1991) find little evidence of a positive relationship between public capital and TFP growth. Similar weak relationships are also estimated for some disaggregated categories of public capital such as "Roads" and "Water and Sewer".

4.3 Empirical Evidence on the Italian Case

The investigation of the empirical linkage between public capital and economic performance has received an increasing interest in Italy. The issue is indeed particularly attractive in the case of Italy given the relevance attributed by policy-makers to infrastructure policies aimed at filling the development gap between northern and southern regions. In the light of this, empirical studies — summarized in Table 4.3.1 — have tried to measure the power of public capital in enhancing economic performance with the aim of providing recommendations for the implementation of such policies.

The empirical literature has progressively moved towards more sophisticated estimation techniques. At the same time, more disaggregated analyses have been carried out as the availability of regional data on public capital has allowed this to be done.

Early applied studies employed cross-sectional data. Brancalente and Di Palma (1982) use infrastructure indexes for Italian regions as calculated for the year 1977. The results obtained by applying OLS regressions and rotated-factor analysis support the idea of a positive relationship between infrastructure endowment and regional development.

The first empirical work using aggregate time series was by Jappelli and Ripa di Meana (1990), who point out that policies aimed at reducing public debt should carefully consider the growth effect of investment in infrastructure. As argued by the authors, the lack of data on public capital stocks precluded to strictly follow the methodology suggested by Aschauer (1989a). As a consequence, they assume the following reduced form equations for aggregate supply and demand, with the objective of evaluating the impact of public investment on aggregate supply[19]:

$$y^s = a_0 + a_1 \cdot i_g + a_2 \cdot c_g + a_3 \cdot m + a_4 \cdot cu + a_5 \cdot t \tag{4.3.1}$$

$$y^d = b_0 + b_1 \cdot i_g + b_2 \cdot c_g + b_3 \cdot m + b_4 \cdot cu + b_5 \cdot t \tag{4.3.2}$$

where all variables are expressed in terms of shares of private capital stocks, y^s is GDP, c_g is public consumption, i_g is public investment, m is the stock of money, cu is a capacity utilization index and t represents a linear time trend.

[19] The authors, however, claim the lack of reliability of the employed data set and, consequentially, consider their results as giving only preliminary evidence on the issue. The difference between (4.3.1) and (4.3.2) is interpreted as the trade surplus.

Table 4.3.1: Summary of the Empirical Literature on Italy

Authors	Approach[a]	Time Period	Data and Estimations[b]	Results[c]
Brancalente and Di Palma (1982)	GR	1977	R,PhI,OLS,RFA	(+)
Jappelli and Ripa di Meana (1990)	GR	1882-1960 and 1962-88	N,PI,OLS	3.08-6.67
Biel *et al.* (1990)	GR	1970 and 1987	R,CS,PhI	(+)
Picci (1997)	PF	1960-92	N,PK,OLS	0.35-0.85
Di Palma *et al.* (1998)	GR	N,1987-95	PhI,OLS	(+)
Picci (1999)	PF	1970-95	R,PK,OLS,PD	0.32-0-58
Cosci *et al.* (1999)	GR	1983-94	R,PI,PD	CEDU(-),CTER(-),CHS(-)
				IEDU(+),ITER(+),IHS(+)
Peroni and Picci (2000)	VAR	1970-96	P,PK	(*)
Acconcia and Del Monte (2000)	PF	1963-93	R,PK	0.1-0.22
	GR	1965-93	R,PI,OLS,IV	0.001-0.021
Bonaglia *et al.* (2000)	PF	1976-91	R,PK,PD	not sig.
	GA	1976-91	R,PK,PD	0.45
Di Palma and Mazziotta (2002)	GR	1997	PhI	(+)

[a] GA = Growth Accounting; GR = Growth; PF = Production Function

[b] RFA = Rotating Factor Analysis; PhI = Physical Index; PI = Public Investment; PK = public capital stocks; IEDU = PI in Education; ITER = PI in Housing and Territory; IHS = PI in Health Services; CEDU = PC in Education; CTER = PC in Housing; CHS = PC in Health Services

[c] Sign of statistically significant point estimates of output elasticity of public capital in parenthesis. Figures listed are: statistically significant point estimates of output elasticity of public capital for authors following the PF approach; statistically significant point estimates of the coefficient of PI in regressions on per capita GDP for those following the GR approach; statistically significant point estimates of the coefficient of PK in regressions on TFP growth for those following the GA approach; (*) = mixed results on the relationship between PK and income, ambiguous results on the inverse relationship; not sig. = not statistically significant estimates.

The coefficient of i_g is expected to be positive, capturing both the direct impact of public investment on output and its indirect positive effect in enhancing private inputs productivity. Since an increase in public consumption c_g may yield a reduction of resources in the private sector and an increase in labour supply, the expected sign of c_g is positive as well. However, the impact of c_g on output is decreasing in the degree of substitutability between c_g and private consumption. Hence, a rise of c_g should impact more on y_s than a rise of c_g of the same amount.

Both c_g and i_g also have a direct impact on y^d. Nevertheless, as for their impact on y^s, the effects of the two categories of government expenditure on aggregate demand are expected to be different in magnitude. Indeed, if c_g is a substitute for private consumption, an increase in c_g should lead to a less than proportional change in y^d. On the other hand, if public and private capital are complements, any increase in i_g leads to a higher marginal productivity of capital and higher private investment, implying a more than proportional impact on y^d.

The estimations are carried out for two different time periods 1882-1960 and 1961-88. For the period 1961-88, the authors use two alternative definitions of public investment: Public Administrations' investment and Public Sector's investment (where the Public Sector includes Public Administrations and Autonomous Administrations of the State and Public Corporations). As expected, a_1 is found to be positive and significant in most cases. The statistically significant point estimates range from 3.08 (over the period 1882-1960) to 6.67 (over the period 1970-88, for the tighter definition of public investment). The estimate of the coefficient a_2 is generally positive and very low in magnitude as expected, but never significantly different from zero.

Since these early studies, progress has been made thanks to the availability of regional data on public capital stocks which has allowed recent empirical investigations to improve the evidence on the Italian case. The first version of regional data on public capital stocks was provided by Picci (1995a) for the period 1970-91, lately extended by Bonaglia and Picci (2000) to cover the years 1992-96. Recently, Picci (2002) has disaggregated regional stocks of public capital for the 103 Italian provinces, following the earlier work performed by Peroni and Picci (2000) for the provinces of Emilia Romagna.

The reconstruction of regional data on public capital stocks has been based on a procedure which can be summarized as follows (Bonaglia and Picci, 2000).

(1) First, public investment aggregate time series provided by Rossi *et al.* (1993) for the period 1890-1992, have been extended to the year 1996.

(2) Then, the series of regional public investment have been obtained on the basis of the information retrieved from the annual surveys (from 1954 to 1996) provided by the National Institute of Statistics (ISTAT)[20]. These surveys are performed by means of the distribution of questionnaires to local public officers and allow to collect data on the amount of public investment, the type of good and the administration responsible for its realization and for its financing[21]. The categories of goods considered in the surveys are nine: Roads and Airports (GRA), Railways and Subways (GRS), Marine (ports, lake and river navigation) (GM), Water (river planning, etc.) and Electrical Lines (GWE), Public Buildings and Schools (GPBS), Sanitation (hospitals, water filtering, sewers) (KS), Land Reclamation and Irrigation (GLRI), Telecommunications (GT) and Other types of

[20] Istat (1954-1996), *Opere Pubbliche*, Roma.

[21] Public works are those financed by the State (including Ministries and Cassa per il Mezzogiorno), Public Administrations (Regions, Provinces and Municipalities) and Public Companies.

works (such as pipelines, infrastructures for tourism etc.) (GO). The sum (GRA+GRS+GM+GWE+GT) defines the "core"component of public capital and includes the categories which are more likely to directly affect productivity. The sum of the remaining categories (GWE+GPBS+GO) defines to the so-called "non-core" public capital.

(3) The regional time series obtained in (2) have been made coherent with data aggregated at a national level calculated in (1). This is done by splitting the aggregate time series in (1) amongst the 20 Italian regions — for each of the nine categories of public goods — in such a way that the ratios of all the components of these disaggregations equate the ones of the data contained in (2).

(4) Finally, regional capital stocks (for each of the nine categories of public goods) have been obtained by means of the permanent inventory (PI) method applied to each of the 180 time series obtained in (3).

The major limitation of the use of the PI method in order to derive stock data is that this procedure is likely to overestimate the endowment of infrastructure in the regions that are least efficient in using public funds. Indeed, stock data reconstructed on the basis of the PI method are representative of the amount of financial resources employed over time to constitute the stocks themselves, whereas physical measures are representative of the actual endowment of a give infrastructure service[22]. The ideal measure of the stock of regional public capital would be some physical index of the endowment of infrastructure in each region. However, such indexes, are available only for a few years, whereas data on stocks reconstructed from public investment are available for long series and are well adaptable to econometric

[22] For a comparison between these two alternative measures of data on stocks, see Picci (1995b).

techniques of estimation.

The availability of data on the stock of public capital has allowed to apply the methodologies suggested by Aschauer (1989a) and Hulten and Schwab (1984, 1991) to the case of Italy.

Applications to the Italian case of the production function approach are performed by Picci (1997 and 1999). Picci (1997) uses aggregate data at a national level from Rossi *et al.* (1993) and Picci (1995a). Following essentially the same methodology suggested by Aschauer (1989a), he estimates the following equations:

$$y_t - l_t = b_0 + b_1 \cdot t + b_2 \cdot k_t + (1 - b_3) \cdot l_t + b_4 \cdot g_t + b_5 \cdot cu_t + u_t \quad (4.3.3)$$

$$tfp_t = c_0 + c_1 \cdot t + c_2 \cdot g_t + c_3 \cdot cu_t + e_t \quad (4.3.4)$$

where all variable are expressed in logarithms and g_t is the stock of public capital. The OLS estimate of the output elasticity of g_t implied by equation (4.3.3)[23] — under alternative restrictions on the returns to scale[24] — is found to be statistically significant with values varying from 0.35 to 0.85. This results support the view of the determinant role of public capital in affecting economic performance over the sample period. The same conclusion is achieved by estimating equation (4.3.4) and reinforced when the "core"component of public capital is used as a regressor in both (4.3.3) and (4.3.4).

Picci (1999) uses panel data estimation techniques in order to overcome criticisms about the use of aggregate data (Holtz-Heakin, 1994), the relevance of the use of such techniques being noticeable in their ability to reflect heterogeneity across regions. Using data on regional public capital stocks in Bonaglia and Picci (2000), a Cobb Douglas production function is estimated

[23] Given (4.3.3), the output elasticity of public capital is defined as $(b_4 + b'_4)/(1 - \alpha)$, where b'_4 is the coefficient of g_{t-1} and $(1 - \alpha)$ is the coefficient of $(y_{t-1} - l_{t-1})$.

[24] Only in one case the restriction of CRTS can be rejected.

for each of the 20 Italian regions over the period 1970-95 according to the following model:

$$y_{i,t} = b_0 + b_1 \cdot l_{i,t} + b_2 \cdot k_{i,t} + b_3 \cdot g_{i,t} + \epsilon_{i,t} \tag{4.3.5}$$

where i and t denote cross-sectional and temporal dimensions of the data respectively, and $\epsilon_{i,t} = f_i + \delta_t + \mu_{i,t}$, where f_i is a time invariant region specific effect, δ_t is a time specific effect (invariant across regions) and $\mu_{i,t}$ is an *i.i.d.* idiosyncratic error term. Hence, the model gives the options of estimating a pooled OLS regression — for $f_i = 0$ — a random effects model or a fixed effects model.

The results of the analysis regard the whole country and the four macro-regions (North-East, North-West, Center and South) over the overall sample period and for two sub-periods (1970-1982 and 1983-1995).

Regarding the results for Italy as a whole, the pooled OLS estimate of b_3 is found to be negative and statistically significant. However, the null hypothesis that all fixed effects do not vary across regions is rejected, leading to rely on the fixed effects estimate of b_3, whose magnitude is estimated around 0.36[25]. Furthermore, equation (4.3.5) is re-estimated splitting the stock of public capital in its "core"and "non-core"components, which turn out to be characterized by output elasticity values of 0.50 and −0.05 respectively. The performing of the adequate tests allows to reject the restriction of CRTS to the three inputs, but not the analogous restriction on the two private inputs alone. Then, the model is estimated in some alternative specifications (first differences, second order translog specification and long differences) in order to check for the robustness of the results.

When the analysis is disaggregated at a macro-regional level, the estimated values of b_3 are found to be 0.21 in the North-East, 0.15 in the North-West, 0.89 in the Center and 0.60 in the South over the full sample

[25] The random effects point estimates are also positive and significant but the Hausman test leads to rely on the results of the fixed effects model.

period. As done for the whole country, the output elasticities of the "non-core"and "core"components of g are estimated for the four macro-regions, confirming the higher productivity enhancing effect of the latter, especially in the North-East. Moreover, the coefficient of the "non-core"component is found to be negative only in the North-West, leading to conclude that the result obtained for the whole country is only due to this macro-region.

Finally, the results obtained in the two sub-samples periods 1970-82 and 1983-95 show a mixed picture. For Italy as a whole, the estimate of b_3 is higher in the first period (0.48 versus 0.42). However, at the macro-regional level of the analysis the same is true only for the central regions (0.83 versus 0.56). The same evidence arises for the "core"component of g and the South of the country.

Bonaglia *et al.* (2000) evaluate the impact of productive infrastructures on economic performance following both the growth accounting and the production function approaches[26]. Results are presented both for the whole Italian economy and separately for sub-periods and macro-regions. When the authors implement the growth accounting approach, a Cobb-Douglas production function is considered as including private capital, labour and public capital. Thus, public capital is assumed both to influence TFP and to enter the production function as a direct unpaid input. Their main findings are the following: on the pooled sample, public capital seems to have contributed positively to TFP growth (the share of TFP growth attributed to public capital accumulation is estimated to be 0.45). On the other hand, the positive role of public capital as an input in the production function is not found to be significant at the national level. In terms of regional disaggregation, poorer regions seem to have benefited more from public capital provision than northern regions[27].

[26] They also estimate cost functions.

[27] Their analysis is limited to the manufacturing sector, whereas some of the categories included in the stock of public capital are not likely to affect the productivity of this

The work by Acconcia and Del Monte (2000) has the aim of verifying if public spending has played a role in determining differentials in terms of labour productivity across Italian regions during the period 1963-93. However, they are also able to study the impact of public capital on output in terms of an elasticity in a preliminary step of their analysis.

They first estimate the distribution of steady state levels of output per worker, using an AR(2) model for each Region during the sample period and then regress the resulting cross-sectional distribution on an infrastructure index[28]. Under the assumption of a Cobb-Douglas production function with CRTS and perfect capital mobility across regions, they find an estimate of 0.32, which implies a value of the output elasticity of public capital varying between 0.1 and 0.22 (for values of output elasticity of private capital ranging from 0.7 to 0.3).

The main objective of the authors, however, is to study public spending as a determinant of productivity differentials across regions. With this purpose in mind, they estimate the following model both for the whole economy and the "manufacturing and energy"sector:

$$y_{i,t} - y_{i,t-\tau} = \eta \cdot y_{i,t-\tau} + \sum \varphi_i \cdot X_{i,j} + \epsilon_{i,t} \qquad (4.3.6)$$

where differentials in productivity growth across regions are explained by relative initial conditions and a set of explanatory variables X including cg = public consumption, ig = government spending for investment in infrastructure and dl = sum of the depreciation rate of public capital and the growth rates of labour force and technology. Two versions of equation (4.3.6) are estimated under alternative assumptions on τ and $\epsilon_{i,t}$.

The first version of the model stresses the cross-sectional dimension of the data. Hence, variables are averaged assuming $\tau = 5$ and the error term

sector.

[28] This index of infrastructure endowment is measured at 1995 and includes roads, railways, energy, telephone and sewers.

$\epsilon_{i,t}$ is assumed to be composed by a region specific effect f_i invariant with time plus the idiosyncratic term $v_{i,t}$. Given the further assumption that f_i depends on an index (gh) of infrastructure and education, the error term becomes $\epsilon_{i,t} = b \cdot gh_{i,70} + \eta_i + v_{i,t}$ and (4.3.6) is re-written as:

$$y_{i,t} - y_{i,t-5} = a + b \cdot gh_{i,70} + \eta \cdot y_{i,t-5} + \varphi_1 \cdot ig_{i,t} + \varphi_2 \cdot cg_{i,t} + \varphi_3 \cdot dl_{i,t} + \xi_{i,t} \quad (4.3.7)$$

which is estimated using OLS and IV. The OLS estimate of the coefficient φ_1 is estimated to be positive and statistically significant for both the whole economy (0.004) and the manufacturing sector (0.012). The opposite is true for φ_2 (-0.035), whereas the coefficient of gh is positive and significant (0.013 for all sectors and 0.011 for the manufacturing sector). When (4.3.7) is re-estimated using IV, the results about the point estimates of φ_1 are mixed: the estimated coefficient is positive and significant for the manufacturing sector (0.021), but statistically insignificant for the whole economy. On the other hand, the negative impact of public consumption on growth is confirmed by the magnitude of the estimates (-0.010 and -0.020 respectively) for both the whole economy and the manufacturing sector.

In order to exploit the time dimension of the data, the second version of model (4.3.6) assumes $\tau = 1$ and $\epsilon_{i,t} = f_{1,i} + f_{2,i} \cdot t + v_{i,t}$:

$$y_{i,t} - y_{i,t-1} = \eta \cdot y_{i,t-1} + \Phi_1(D) \cdot ig_{i,t} + \Phi_2(D) \cdot cg_{i,t} + \Phi_3(D) \cdot dl_{i,t} + \epsilon_{i,t} \quad (4.3.8)$$

where $\Phi_r(D)$, $r = 1, 2, 3$, is a polynomial in the lag operator D, $f_{1,i}$ is a region specific effect and $f_{2,i} \cdot t$ a region specific time trend. The objective of the authors is to test the relevance of the long-run effects of ig and cg and whether, eventually, the two effects cancel out. For this reason they choose a number of five and six lags for both variables[29].

[29] Two lags are chosen for the initial level of productivity and dl. In order to check for the robustness of the results, equation (4.3.8) is also re-estimated under alternative assumptions on the numbers of lags.

Estimation results confirm the positive effect of public investment on productivity for the whole economy — $\Phi_1(5) = 0.013$ — and the manufacturing sector — $\Phi_1(6) = 0.032$ — whereas public consumption gets insignificant coefficients in all cases. Equation (4.3.8) is also re-estimated for the two groups of low-income (southern) regions and high-income (northern) regions and considering three disaggregated categories of infrastructure capital separately. Results are such that public investment seems to have a relevant impact on productivity in the manufacturing sector of the low-income regions — with estimates $\Phi_1(5) = 0.048$ and $\Phi_1(6) = 0.072$ — but not statistically significant effect in the high-income regions.

This evidence is motivated by the fact that infrastructure services may have decreasing returns, so that marginal increases in their endowment yields only to negligible productivity enhancing effects in regions that are already well endowed. The analysis of data on public investment in transport infrastructure (roads, airports and railways), buildings (public buildings, schools, and public spending devoted to private building) and SER (sanitation, energy and reclamation) allow to find a confirmation to this argument. Indeed, none of these categories explain productivity differentials across high-income regions. On the other hand, the point estimates of transport and SER in the low-income regions are 0.048 and 0.043, respectively.

5. FRONTIER APPROACH TO THE MEASUREMENT OF TFP GROWTH IN THE ITALIAN REGIONS

5.1 *Introduction*

This Chapter focuses on two empirical studies on Italian Regions over the period 1970-1995. Both contributions will follow the so-called *frontier approach to the measurement of TFP growth* by means of Data Envelopment Analysis. The methodology employed in both studies is reviewed in section 5.2, paying attention on the relevant departures from the existing literature about the explored topics.

The aim of the Chapter is to address the two empirical research questions raised in Chapter 1 (see page 2): first, the study of the impact of public capital as a productivity enhancing externality to regional economies (section 5.3); second, the testing of the catching-up hypothesis across Italian regions (section 5.4).

In section 5.3 regional productivity growth is decomposed into technical efficiency change and technological progress by implementing a non-parametric frontier approach. The empirical analysis involves two steps. First, we implement a DEA model under the two alternative assumptions of two (labour and private capital) and three (labour, private and public capital) inputs and test for the statistical significance of public capital as an additional input. Second, an econometric analysis of the linkage between productivity gains and the provision of public capital is performed with the aim of assessing the role of public capital as a positive environmental variable.

Based on the results obtained in the first part of the Chapter, section 5.4 is aimed at shedding some light on the determinants of convergence/divergence patterns across Italian regions and follows two stages. First, we consider the decomposition of TFP growth obtained in the two inputs scenario. Second, the convergence issue is analysed by means of panel data estimation techniques. Estimated technological progress and technical efficiency change are interpreted, respectively, as innovation and catching-up measurements and the catching-up hypothesis is tested for the Italian regions.

The remainder of the Chapter is organized as follows. The theoretical background of the methods that I employ is introduced in section 5.2. Section 5.3.1 deals with the description of the data set and the descriptive analysis of the variables involved in the study. Subsection 5.3.2 reports the results of the DEA model and those of the econometric analysis of public capital (and nine disaggregated categories of public capital) on TFP growth (and its two components technological progress and technical efficiency change). Some conclusions for this study are drawn in section 5.3.3. As for the second study of the Chapter, results and conclusions are reported in sections 5.4.1 and 5.4.2, respectively.

5.2 Frontier Approach to the Measurement of TFP Growth

As we have seen in Chapter 4, empirical studies within the production function and growth accounting approaches (section 4.2.2) share the common assumption that the aggregate production of observed units is fully efficient. As for studies on Italian regions (see section 4.3), this implies that regional economies are all assumed to be fully efficient: observed output in each region is equal to the potential level of production at each moment in time. From the empirical point of view this assumption is reflected by the estimations of *averaged production functions* within the production function approach and by the *identification of TFP growth with technological progress* within the growth accounting approach. The main motivation of the contribution in section 5.3 is to apply the so-called *frontier approach to the measurement of TFP growth* to the analysis of the productive role of public capital in Italian regions, in the attempt of overcoming the limitation of the existing literature (*i.e.*, neglecting the inefficiency issue).

The analysis of inefficiency in aggregate production is a subject which has recently become of great concern in the empirical literature comparing productivity growth across countries. In the remainder of this section, I present the key features of the theoretical background of my empirical work, which is mainly due to Färe *et al.* (1994a, 1994b).

The productivity measurements carried out within the non-frontier approach assume that the production process is fully efficient. This assumption leads to identify TFP growth with technological progress (Grosskopf, 1993). Denoting input and output quantities at time t and $t+1$ as x^t, x^{t+1}, y^t and y^{t+1}, respectively, the production functions at time t and $t+1$ will be respectively represented by:

$$y^t = A(t)\, f\left(x^t\right) \tag{5.2.1}$$

$$y^{t+1} = A(t+1)\, f\left(x^{t+1}\right) \tag{5.2.2}$$

The TFP index at time t will be given by the ratio of produced output and total inputs employed:

$$TFP(t) = \frac{y^t}{f\left(x^t\right)} = A(t) \tag{5.2.3}$$

As a consequence, TFP growth between time t and $t+1$ will be evaluated using the following expression:

$$\frac{TFP(t+1)}{TFP(t)} = \frac{A(t+1)}{A(t)} \tag{5.2.4}$$

It is clear that TFP growth is explained only in terms of technological progress. In fact, (5.2.4) is equivalent to the formulation originally introduced by Solow (1957), who provided the original analysis of the growth accounting approach. Let us consider the Cobb-Douglas production function in N inputs:

$$y^t = A\left(t\right)\prod_{1}^{N}\left(x_n^t\right)^{\alpha_n} \tag{5.2.5}$$

Taking first derivatives, dividing by y^t and using observed factor shares as proxies for the α_n terms, TFP growth between time t and $t+1$, will be evaluated as:

$$\frac{\dot{A}}{A} = \frac{\dot{y}}{y} - \sum_{1}^{N}\alpha_n\frac{\dot{x}_n}{x_n} \tag{5.2.6}$$

The observed output is hence assumed to be equal to the frontier output and the growth accounting measurement of TFP growth will capture shifts in the technology, *i.e.*, technological change. However, such an estimate will be biased in the presence of inefficiency. Note that (5.2.6) is the general version of the equation (2.2.10) — see section 2.2, page 14 — which defines the Solow residual for the case of two inputs (labour and private capital).

On the other hand, the frontier approach allows for the analysis of technical efficiency change. Technological progress is assumed to push the frontier of potential production upward, while efficiency change will reflect the capability of productive units to improve production with a set of given inputs and available technology. Assuming the presence of technical inefficiency in productive processes leads to a discrepancy between observed output and maximum efficient potential output:

$$y^t < A\left(t\right)f\left(x^t\right) \tag{5.2.7}$$

$$y^{t+1} < A\left(t+1\right)f\left(x^{t+1}\right) \tag{5.2.8}$$

The concept of distance function (Malmquist, 1953) is introduced into the analysis in order to bring observed output up to its efficient level. The output distance function D_0^t is given by:

$$D_0^t\left(x^t, y^t\right) = \inf\left\{\theta : \left(x^t, \frac{y^t}{\theta}\right) \in S^t\right\} = \left(\sup\left\{\theta : \left(x^t, \theta y^t\right) \in S^t\right\}\right)^{-1} \tag{5.2.9}$$

where θ is a scalar and S^t represents the production technology. The output distance function is hence defined as the reciprocal of the maximum expansion in production — given available inputs — such that output is still feasible, *i.e.*, $\left(x^t, \theta y^t\right) \in S^t$ (Farrell, 1957).

The definition of the distance function in (5.2.9) completely characterizes the technology. Indeed, the following is true:

$$D_0^t\left(x^t, y^t\right) \leq 1 \quad \text{if and only if} \quad \left(x^t, \theta y^t\right) \in S^t \tag{5.2.10}$$

and the value taken by the distance function will be 1 if and only if production is technically efficient.

From the concept of distance function it follows that:

$$D_0^t\left(x^t, y^t\right) = \frac{y^t}{A(t)\, f\left(x^t\right)} \tag{5.2.11}$$

$$D_0^{t+1}\left(x^{t+1}, y^{t+1}\right) = \frac{y^{t+1}}{A(t+1)\, f\left(x^{t+1}\right)} \tag{5.2.12}$$

where, at each moment in time, in the presence of technical inefficiency, maximum potential output $A(t)\, f\left(x^t\right)$ will be equal to the observed output y^t corrected for the output distance function $D_0^t\left(x^t, y^t\right)$.

The TFP indexes at time t and $t+1$ will be given respectively by:

$$TFP(t) = \frac{y^t}{f\left(x^t\right)} = A(t)\, D_0^t\left(x^t, y^t\right) \tag{5.2.13}$$

and

$$TFP(t+1) = \frac{y^{t+1}}{f\left(x^{t+1}\right)} = A(t+1)\, D_0^{t+1}\left(x^{t+1}, y^{t+1}\right) \tag{5.2.14}$$

which yields the following expression for the TFP growth index between the two periods:

$$\frac{TFP(t+1)}{TFP(t)} = \frac{A(t+1)}{A(t)} \frac{D_0^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_0^t\left(x^t, y^t\right)} \tag{5.2.15}$$

The measurement of TFP growth obtained in (5.2.15) will be equivalent to the one obtained following the growth accounting approach in (5.2.4) only in the absence of inefficiency (*i.e.*, only if TFP change can be explained solely in terms of technical change). On the other hand, in the presence of inefficiency, measurements of TFP growth based on non-frontier methods will lead to biased results.

Both non-parametric linear programming techniques (Data Envelopment Analysis, DEA) and the stochastic frontier approach (SFA) have been employed within the frontier approach. The implementation of both techniques suffers from limitations and implies advantages. The main weakness of non-parametric techniques when compared to SFA is related to the fact that the estimated gap between actual and potential output may result in upward bias since inefficiency scores may be influenced by other factors such as unobserved measurement errors.

On the other hand, SFA is able to distinguish between inefficiency and other possible causes of the discrepancy between observed and maximum potential output. However, this is made possible by separating two components of the error term in the stochastic production function and the distributional assumptions may significantly affect the results. Moreover, the need to specify a functional form for the production frontier together with the assumption of a common technical change across production units represent two important limitations which make us prefer the use of non-parametric techniques. In fact, DEA does not require the imposition of any functional form for the technology set and allows technical change to vary across decision-making units.

The implementation of non-parametric techniques implies the use of the output-oriented[1] CCD Malmquist productivity index introduced by Caves

[1] As pointed out by Caves *et al.* (1982), productivity differences over time may be interpreted in two ways: by interpreting productivity changes as changes in maximum output conditional on a given level of inputs (output-oriented productivity indexes) or by

et al. (1982) using the two distance functions defined above in (5.2.11) and (5.2.12) and the two following mixed period distance functions:

$$D_0^t\left(x^{t+1}, y^{t+1}\right) = \frac{y^{t+1}}{A(t)\, f\left(x^{t+1}\right)} \tag{5.2.16}$$

$$D_0^{t+1}\left(x^t, y^t\right) = \frac{y^t}{A(t+1)\, f\left(x^t\right)} \tag{5.2.17}$$

On the basis of the above output distance functions, Caves *et al.* (1982) define their output oriented Malmquist productivity indexes for period t and $t+1$ as:

$$M_0^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_0^t\left(x^{t+1}, y^{t+1}\right)}{D_0^t\left(x^t, y^t\right)} \tag{5.2.18}$$

evaluated with respect to technology at time t; and:

$$M_0^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_0^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_0^{t+1}\left(x^t, y^t\right)} \tag{5.2.19}$$

evaluated with respect to technology at time $t+1$.

In order to avoid the subjective choice of the reference technology, an additional productivity index is defined as the geometric mean of (5.2.18) and (5.2.19):

$$M_0^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left[\frac{D_0^t\left(x^{t+1}, y^{t+1}\right)}{D_0^t\left(x^t, y^t\right)} \frac{D_0^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_0^{t+1}\left(x^t, y^t\right)}\right]^{\frac{1}{2}} \tag{5.2.20}$$

Färe *et al.* (1994a, 1994b) decompose (5.2.20) into two components in order to account for both technical efficiency change and technological change (under the assumption of CTRS):

$$M_{0c}^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_{0c}^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_{0c}^t\left(x^t, y^t\right)} \left[\frac{D_{0c}^t\left(x^{t+1}, y^{t+1}\right)}{D_{0c}^{t+1}\left(x^{t+1}, y^{t+1}\right)} \frac{D_{0c}^t\left(x^t, y^t\right)}{D_{0c}^{t+1}\left(x^t, y^t\right)}\right]^{\frac{1}{2}} \tag{5.2.21}$$

where the first ratio represents the change in technical efficiency between period t and period $t+1$ and the term in brackets measures the shift in

interpreting them as changes in minimum input requirements, conditional on a given level of outputs (input oriented productivity indexes). We will focus our attention on the first possibility.

technology between the two periods. M_{oc} greater than 1 indicates that productivity has risen between period t and $t+1$ and this can be explained in terms of technical efficiency improvement and/or technological progress. A value of the index smaller than 1, will indicate a TFP slowdown between the two periods. It is important to notice that the two components may move in opposite directions. For instance, if neither input ($x_t = x_{t+1}$) nor output ($y_t = y_{t+1}$) change between the two periods, M_{oc} will be equal to 1 and technical change and efficiency change will be reciprocal but not necessarily both equal to 1.

The graphical example in Färe *et al.* (1994b) for the CRTS case provides the key intuition of the decomposition of the Malmquist index in (5.2.21).

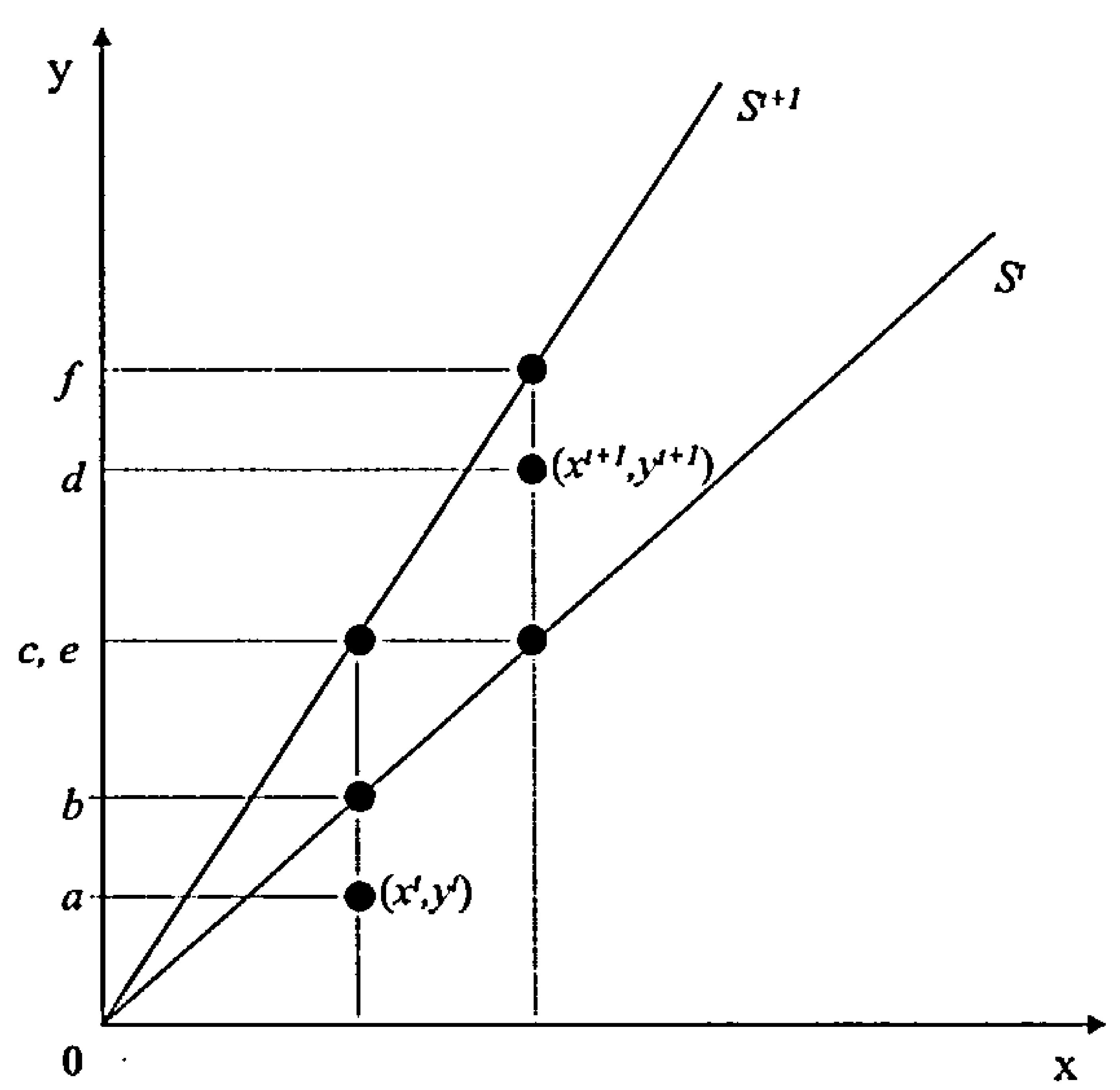

Figure 5.2.1: The Decomposition of the Malmquist Index

In Figure 5.2.1, S^t and S^{t+1} represent the production frontiers at time t and $t+1$, respectively. In this simple graphical example, the level of production observed at time t is not efficient. Indeed, observed output $y = a$ is below the frontier and maximum potential output is given by $y = b$. According to the definition in (5.2.11), the output distance function is defined by the ratio $0a/0b < 1$. Since $S^t \subset S^{t+1}$, a technical advance has taken place between time t and $t+1$. However, observed production at time $t+1$ ($y = d$) is still inefficient and the output distance function of the new period is equal to $0d/0f < 1$.

Given the two distance functions $0a/0b$ and $0d/0f$ and recalling the decomposition of the Malmquist productivity index in (5.2.21), we can define the ratio:

$$\frac{0d/0f}{0a/0b} = \text{efficiency change} \tag{5.2.22}$$

which for values greater than one will indicate that production is closer to its efficient level in period $t+1$ than in period t (*i.e.*, an efficiency improvement occurred between the two periods). From Figure 5.2.1, we can also derive the graphical counterparts of the two mixed output distance functions in (5.2.16) and (5.2.17), which are necessary to obtain the decomposed Malmquist index of our example:

$$D_0^t\left(x^{t+1}, y^{t+1}\right) = \frac{y^{t+1}}{A(t)\, f\left(x^{t+1}\right)} = \frac{0d}{0e} \tag{5.2.23}$$

$$D_0^{t+1}\left(x^t, y^t\right) = \frac{y^t}{A(t+1)\, f\left(x^t\right)} = \frac{0a}{0c} \tag{5.2.24}$$

where the ratio in (5.2.23) represents the highest proportional change in output requirements to make (x^{t+1}, y^{t+1}) feasible in relation to the technology at time t. On the other hand, the ratio defined in (5.2.24) indicates the highest proportional change in output requirements to make (x^t, y^t) feasible in relation to the technology at time $t+1$.

Finally, the Malmquist productivity index can be expressed as:

$$M_{0c}^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{0d}{0f} \Big/ \frac{0a}{0b} \left[\frac{0d/0e}{0d/0f}\, \frac{0a/0b}{0a/0c}\right]^{1/2} = \frac{0d}{0f} \Big/ \frac{0a}{0b} \left[\frac{0f}{0e}\, \frac{0c}{0b}\right]^{1/2} \tag{5.2.25}$$

In applied work considering suitable panel data, the Malmquist productivity index may be calculated by using DEA linear programming techniques. In the present application, I will use the panel of the 20 Italian regions over the period 1970-95. The availability of such a panel allows to locate the best practice frontier in each year of the sample period. The region (or regions) characterized by the most efficient aggregate production will lie on the frontier.

The frontier technology with CTRS at time t is constructed from the data as:

$$S^t = \left\{ (x^t, y^t) : y^t_m \leq \sum_{k=1}^{K} z^{k,t} y^{k,t}_m ; \sum_{k=1}^{K} z^{k,t} x^{k,t}_n \leq x^t_n ; z^{k,t} \geq 0 \right\} \quad (5.2.26)$$

where $k = 1,..K$ are regions, $x = 1,..,N$ are inputs, and $y = 1,..,M$ are outputs. In our case only one output is considered (regional GDP), and the terms $z^{k,t}$ stand for weights on each region. The assumption of CRTS can be relaxed by imposing the restriction $\sum_{k=1}^{K} z^{k,t} = 1$, which will give the case of variable returns to scale (VRTS).

In order to calculate the Malmquist productivity index for each region at each time t, the distance functions illustrated in (5.2.11), (5.2.12), (5.2.16) and (5.2.17) have to be evaluated. Following Färe *et al.* (1994b), we will refer to the case of CRTS. The distance function in (5.2.11) is evaluated by solving the following linear programming problem[2] for each region k':

$$\left[D^t_0 \left(x^{k',t}, y^{k',t} \right) \right]^{-1} = \max \theta^{k'} \quad (5.2.27)$$

subject to

$$\sum_{k=1}^{K} z^{k,t} y^{k,t}_m \geq \theta^{k'} y^{k',t}_m \quad (5.2.28)$$

$$\sum_{k=1}^{K} z^{k,t} x^{k,t}_n \leq x^{k',t}_n \quad (5.2.29)$$

$$z^{k,t} \geq 0 \quad (5.2.30)$$

The evaluation of $D^{t+1}_0(x^{t+1}, y^{t+1})$ will imply solving a linear programming problem such as the one in (5.2.27)-(5.2.30), transposing superscripts

[2] Charnes *et al.* (1978, 1981).

t with $t+1$. The mixed distance function in (5.2.16) is obtained by solving the following problem:

$$\left[D_0^t\left(x^{k',t+1}, y^{k',t+1}\right)\right]^{-1} = \max \theta^{k'} \tag{5.2.31}$$

subject to

$$\sum_{k=1}^{K} z^{k,t} y_m^{k,t} \geq \theta^{k'} y_m^{k',t+1} \tag{5.2.32}$$

$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leq x_n^{k',t+1} \tag{5.2.33}$$

$$z^{k,t} \geq 0 \tag{5.2.34}$$

The solution to the linear programming problem in (5.2.31)-(5.2.34) with superscripts t and $t+1$ transposed will give $D_0^{t+1}(x^t, y^t)$.

The Malmquist index as decomposed in (5.2.21) was applied for the first time in Färe *et al.* (1994b) to the comparison of productivity growth in a sample of 17 OECD countries over the period 1979-88. In this study, the authors provide a new explanation of the convergence process, interpreting technical efficiency change and technological progress as measurements of catching-up and innovation respectively. Since then, these techniques have received increasing attention in analysing the convergence issue and the impact on productivity growth of factors others than private capital and labour units.

For instance, Taskin and Zaim (1997) test the catching-up hypothesis for a group of 23 countries including both low-income and high-income economies over the period 1975-90. Their findings are such that innovation turns out to be the main source of productivity growth in high-income countries, while low-income economies are characterized by a higher technical efficiency change, which allows them to approach the best practice frontier at a faster rate. However, only private capital and labour are assumed to affect the production correspondence.

Some recent studies have made attempts of considering additional factors that might influence both technical efficiency change and technological progress patterns by following Färe *et al.* (1994a, 1994b). Maudos *et al.* (1999) and Murillo-Zamorano and Vega-Cervera (2001) use data on OECD countries and consider human capital and energy as additional productive factors. Puig-Junoy (2001) uses data on the U.S. economy and considers public capital as a productive input in addition to private capital and labour. In this paper both DEA and SFA are implemented in order to decompose productivity growth in technical efficiency change and technological progress. Boisso *et al.* (2000) also deal with data on the U.S. economy and test for the significance of public infrastructure investment in affecting technical efficiency levels. Pedraja Chaparro *et al.* (2000) use data on the Spanish regions and follow a two-step procedure. They first implement DEA, assuming that private capital and labour are the only productive inputs, and obtain technical efficiency change and technological progress scores. In a second step, productivity growth and its two components are regressed on human capital and public capital, finding a positive and statistically significant correlation in both cases.

5.3 TFP Growth and Public Capital: The Case of Italy

The first objective of this Chapter is to contribute to the debate on the empirical linkage between public capital and productivity in the Italian regions. The main novelty will be the decomposition of productivity growth into technical efficiency change and technological progress by means of Data Envelopment Analysis (DEA) and Malmquist productivity indexes, leading to the main conclusion of the significant role played by public capital as a productivity enhancing externality to regional economics.

Existing applied work on Italy has focused on the *production function* approach (Picci, 1997 and 1999), the *growth accounting* approach (La Ferrara *et al.*, 2000), and the *growth* approach (Acconcia and Del Monte, 2000). These recent contributions have enhanced the literature on the topic, allowing for taking into account heterogeneity across Italian regions by means of panel data estimation techniques. However, their conclusions can suffer from a common shortcoming: they do not take account of the *inefficiency issue*.

This is due to the implicit assumption that the production process is fully efficient, which implies that the estimates of average production functions will be biased in the presence of inefficiency. Furthermore, if such an assumption does not hold, TFP growth will be identified with technological progress, while another source of productivity growth — technical efficiency change — will be neglected (Grosskopf, 1993).

In the attempt of overcoming the limitations of the existing literature by implementing a non-parametric frontier approach to the measurement of productivity, my contribution departs from previous studies in three main aspects. First, I account for both technical efficiency change and technological progress, checking how the empirical evidence is modified accordingly. Second, since both technical efficiency change and technological progress are not only assumed to be time varying but also evaluated for each regional economy, an additional source of heterogeneity across regions is introduced. Third, the productive process is not restricted to being described by any functional form and the maximum degree of flexibility is allowed by the use of non-parametric analysis.

The DEA procedure is first implemented assuming a non-parametric production function in private capital and labour and then considering public capital as an additional input. The significance of public capital as a productive input is tested empirically using the Banker test (1996), which allows to compare these two alternative scenarios. Indeed, the null hypothesis that public capital does not directly affect the production correspondence as a productive factor can not be rejected.

An econometric analysis of the contribution of public capital to productivity gains is then performed. The main finding of such an analysis is the positive and statistically significant relationship between infrastructure services and both the mutually exclusive and exhaustive components of TFP growth. Our results suggest that public capital indirectly affects productivity, rather than directly enhancing it as an additional input in the production function. In other words, it seems that during the period 1970-95 public capital has played the role of a positive externality to regional economies. Finally, a detailed analysis of the productivity effects of each of the categories of public capital is provided for the whole country and for the two macro-regions (Center-North and South) with the aim of verifying our conclusions at higher level of territorial disaggregation.

5.3.1 Data

I use data on the 20 Italian regions for the period 1970-95. Observed output is regional GDP. Available inputs are represented by units of labour, private capital and public capital. Data on GDP and units of labour are retrieved from CRENOS NewRegioIt60-96[3]. Data on public and private capital stocks are those calculated by Bonaglia and Picci (2000)[4], whose statistical work has been described in section 4.3, pages 129-130[5].

Table 5.3.1 shows average annual growth rates of GDP, private capital and labour for the North-East, the North-West, the Center and the South of Italy in the overall sample period and during the three sub-periods 1970-79, 1980-89 and 1990-95. Italian regions belong to the four macro-regions as follows: North-West = Piemonte (PIE), Lombardia (LOM), Valle

[3] http://www.crenos.it.

[4] http://www.spbo.unibo.it/picci/indexkp.html.

[5] GDP, private capital and public capital values are expressed at constant prices 1990; units of labour are expressed in thousands.

Table 5.3.1: Average Growth Rates (%) of GDP, Labour and Private Capital (1970-95)

Macro-Regions	GDP	L	K
	1970-79		
North-East	4.38	1.44	4.96
North-West	2.66	0.42	3.70
Center	4.02	1.34	5.76
South	3.67	0.72	8.93
Italy	3.68	0.93	6.46
	1980-89		
North-East	2.11	0.52	2.73
North-West	1.85	0.06	1.91
Center	2.03	0.63	3.46
South	2.24	0.73	1.93
Italy	2.09	0.53	2.39
	1990-95		
North-East	2.10	-0.67	1.63
North-West	0.78	-0.93	0.17
Center	1.31	-0.73	1.05
South	0.99	-1.65	1.63
Italy	1.23	-1.12	1.22
	Full sample		
North-East	3.00	0.62	3.36
North-West	1.93	-0.02	2.23
Center	2.65	0.61	3.84
South	2.52	0.21	4.60
Italy	2.53	0.33	3.73

d'Aosta (VDA), Liguria (LIG); North-East = Trentino Alto Adige (TAA), Veneto (VEN), Friuli Venezia Giulia (FVG), Emilia Romagna (ER); Center = Toscana (TOS), Umbria (UMB), Marche (MAR) and Lazio (LAZ); South = Abruzzo (ABR), Molise (MOL), Campania (CAM), Puglia (PUG), Basilicata (BAS), Calabria (CAL), Sicilia (SIC) and Sardegna (SAR). In the whole sample period, the regions in the North-East had the best economic performance (+3%), followed by those in the Center (2.65%), in the South (2.52%) and in the North-West (1.93%). GDP has been growing at decreasing rates over time in the whole country. Labour input averaged a quite flat performance, with increases larger than 1% only in the Center and in the North-East during the 1970s. Labour units decreased during the 1990s in the whole country, particularly in the South (-1.65%). On the other hand, the accumulation of private capital was higher in the regions of the South, mostly due to the performance of the 1970s.

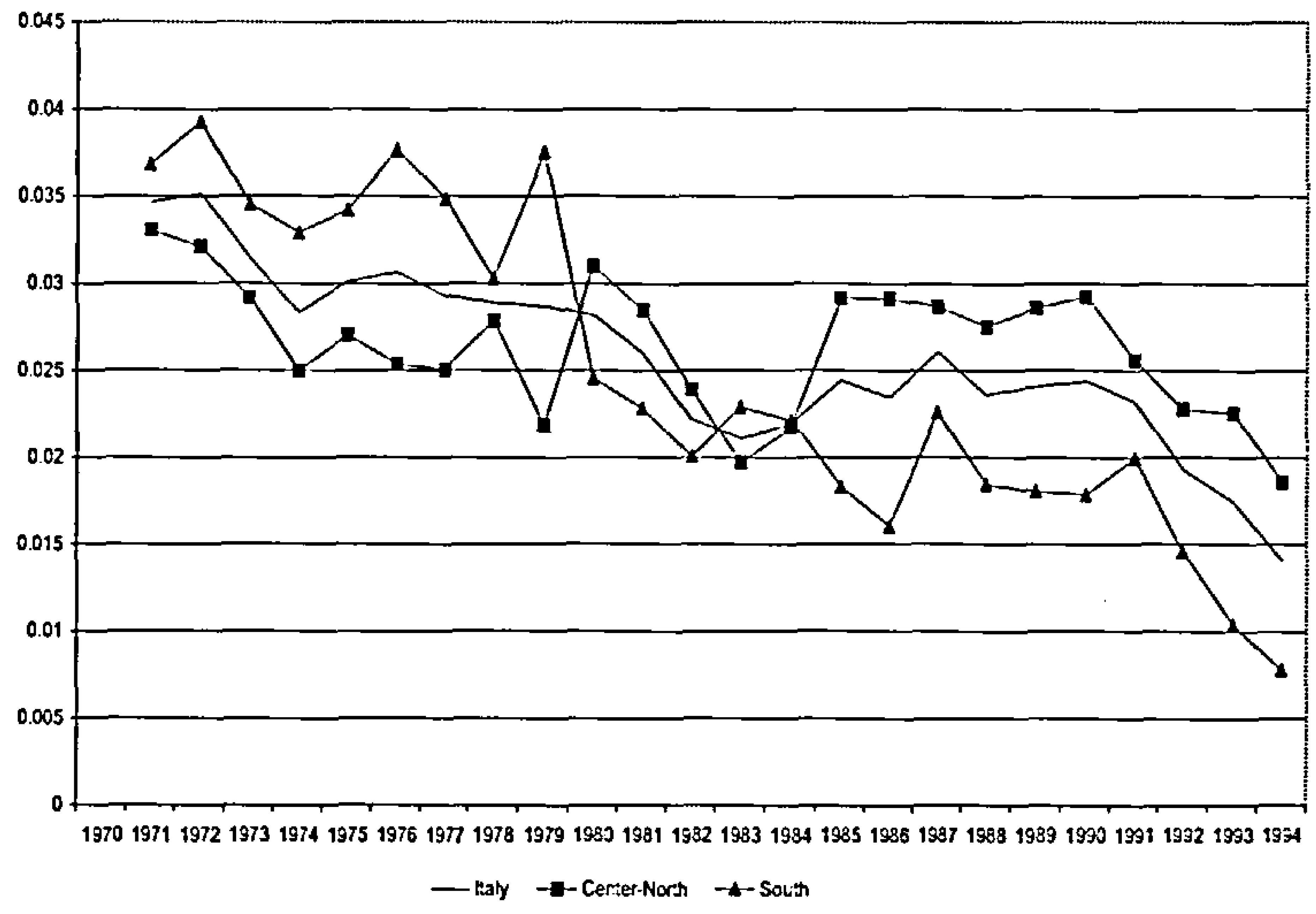

Figure 5.3.1: Public Capital Stocks, Annual Growth Rates (1970-95)

As shown in Figure 5.3.1 the provision of public capital grew at decreasing rates between 1970 and 1995. This was mainly due to the performance of

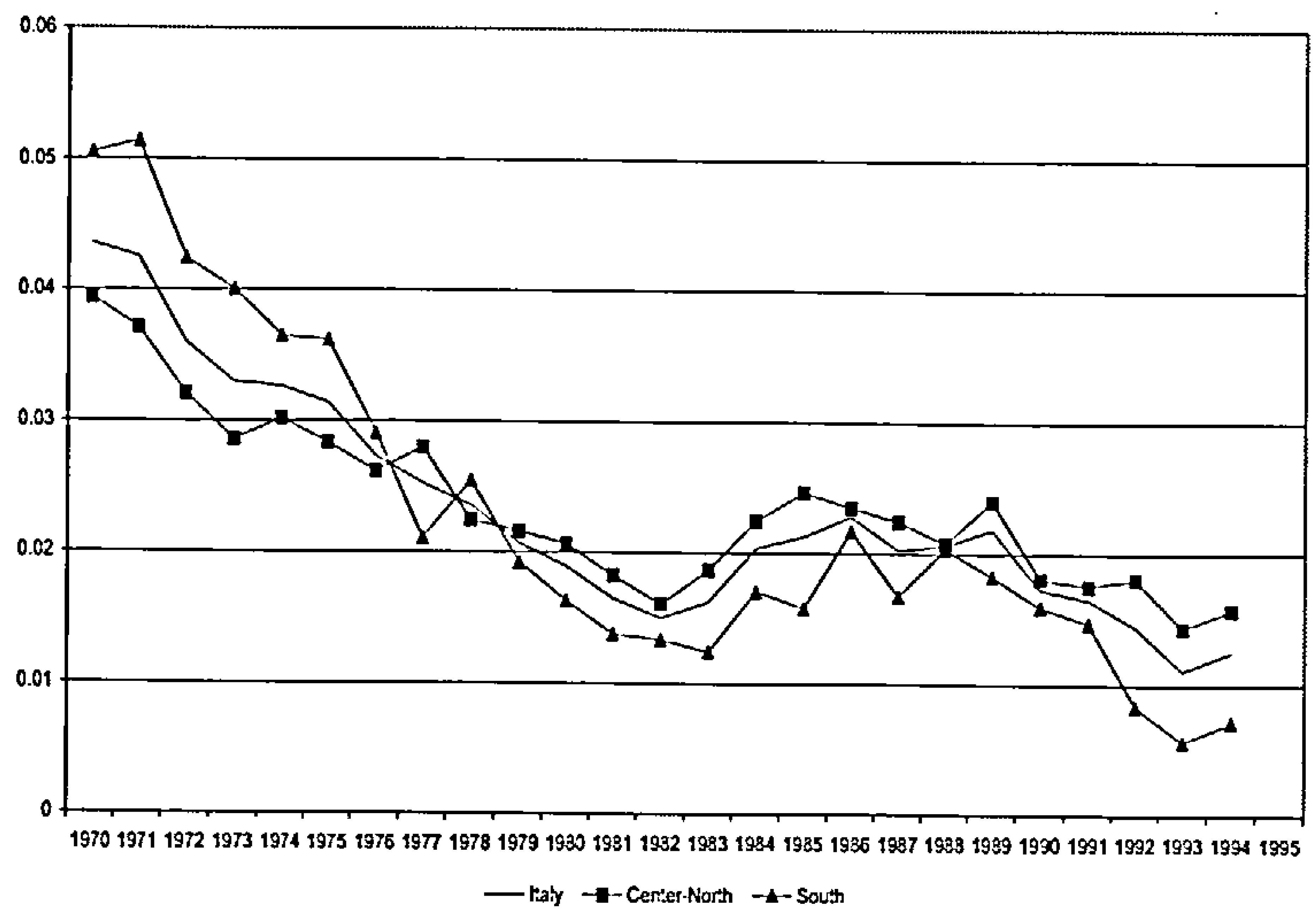

Figure 5.3.2: "Core"Public Capital Stocks, Annual Growth Rates (1970-95)

its "core"component[6] (Figure 5.3.2), whereas "non-core"public capital appears to have a less well-depicted decreasing behavior (Figure 5.3.3). Public capital grew more in the South than in the Center-North during the 1970s, the period with the highest accumulation of both public and private capital in the South. The following decade was characterized by a slowdown in public capital provision in the area, mainly due to the end of the state agency "Cassa del Mezzogiorno"for the development of the southern regions. Similarly, northern regions suffered from a decline in the rate of growth of public capital, which decreased by less than in the South though. In the early 1990s, the decreasing trend of the previous years did not come to an end. Figure 5.3.4 shows the time path of the "core/non-core"public capital ratio. The ratio increased up to the end of the 1970s in the Center-North, while it started to decline from 1976 in the southern regions. These figures seem to

[6] The "core"component of public capital includes the categories which are more likely to directly affect productivity: Roads and Airports, Railways and Subways, Marine (ports, lake and river navigation), Water (river planning, etc.) and Electrical Lines and Telecommunications

show that productive spending only in the first half of the period characterized the allocation of public capital towards the southern regions, whereas it continued up to the end of the 1970s in the Center-North. Afterwards the ratio shows a declining pattern in both the macro-regions.

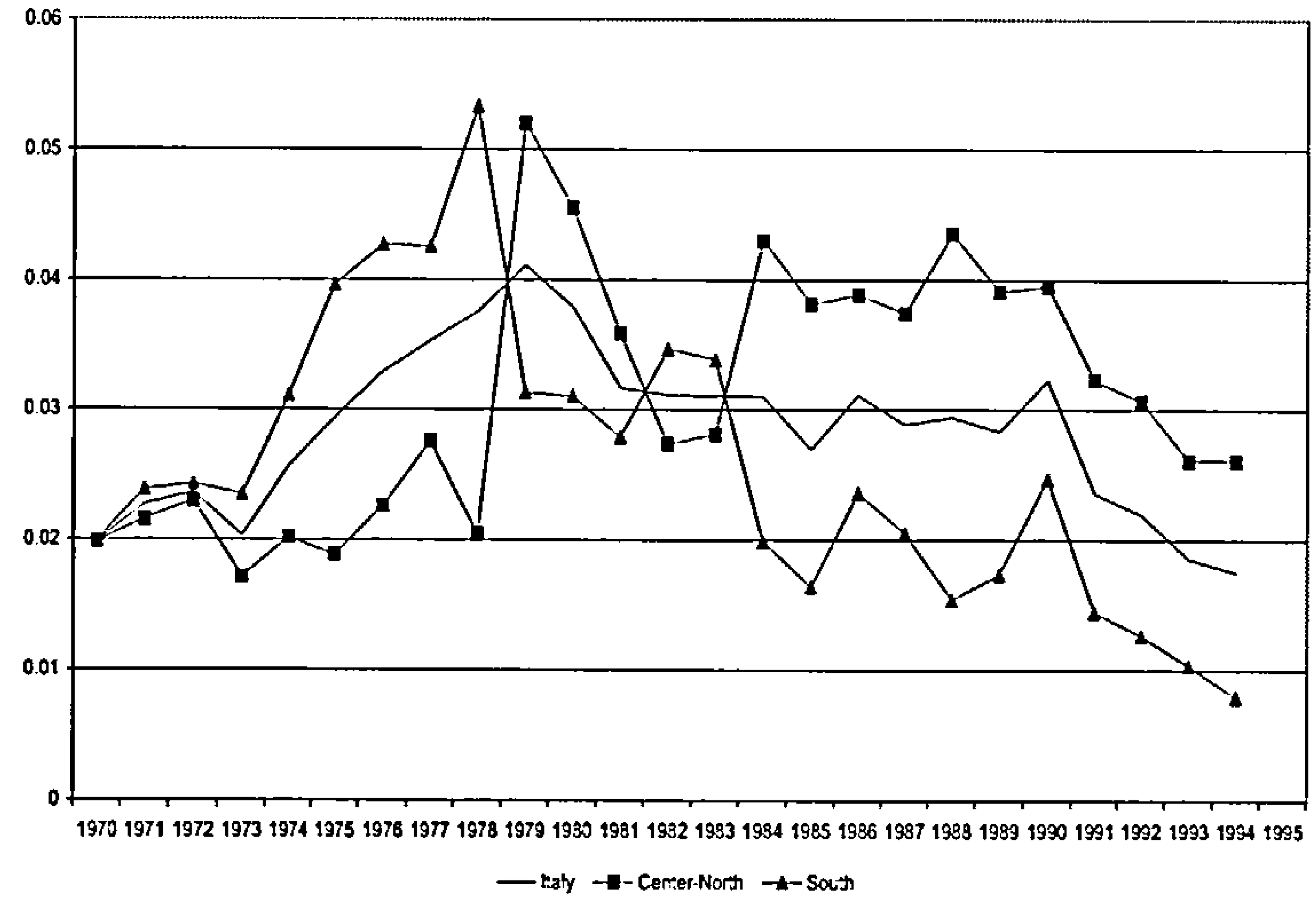

Figure 5.3.3: "Non-Core" Public Capital Stocks, Annual Growth Rates (1970-95)

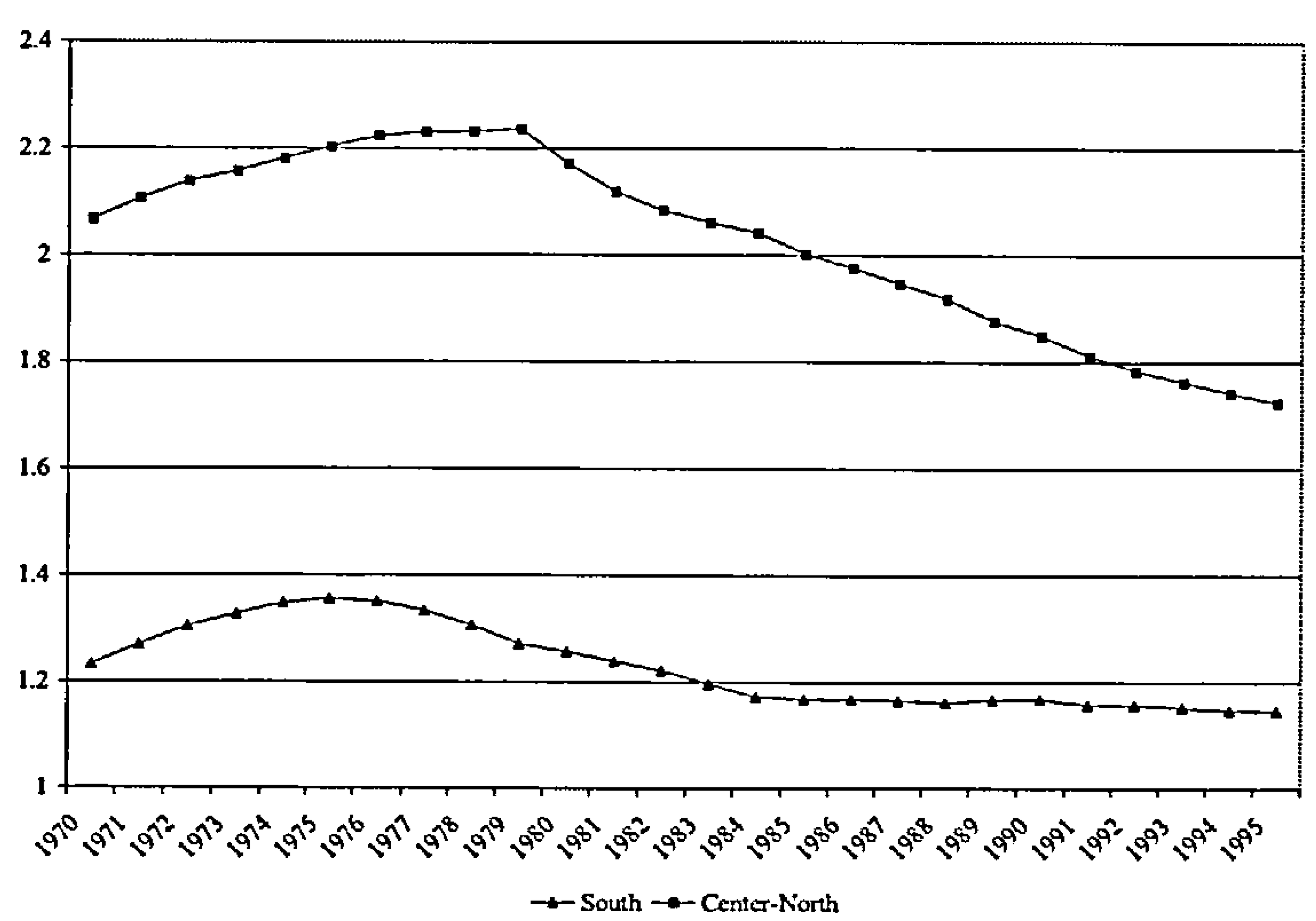

Figure 5.3.4: "Core/Non-Core" Public Capital Ratio (1970-95)

5.3.2 Results

The empirical analysis proceeds in two parts. In the first part of the analysis, I apply the DEA procedure to the measurement of TFP growth. First, I assume that non-parametric regional production functions include private capital and labour only. Then, the DEA model is constructed under the alternative scenario that public capital enters as an additional input the non-parametric production function of each Italian region. Finally, a test of the relevance of public capital as a direct input of production is performed on the basis of the comparison between the two scenarios. This part of the analysis can be thought as testing the power of public capital in producing the direct effect on output — see effect (1) at page 9 — with the advantage with respect of previous studies of taking into account inefficiency in production.

The second part of the analysis uses econometric techniques of estimation in order to assess the role of public capital in determining TFP growth in Italian regions over the sample period. Hence, the aim on this part of the analysis is the assessment of public capital in generating the indirect effect on TFP growth described by the effect (3) in section 2.2 at page 13. This investigation of the role of public capital as an environmental variable takes advantage of accounting for inefficiency, improving the evidence provided by the traditional growth accounting approach. The analysis considers the impact of public capital on TFP growth and both its components, technological progress and efficiency change. The same investigation is carried out for the "core" component of public capital and for the nine categories Roads and Airports (GRA), Railways and Subways (GRS), Marine (ports, lake and river navigation) (GM), Water (river planning, etc.) and Electrical Lines (GWE), Public Buildings and Schools (GPBS), Sanitation (hospitals, water filtering, sewers) (GS), Land Reclamation and Irrigation (GLRI), Telecommunications (GT) and Other types of works (such as pipelines, infrastructures for tourism etc.) (GO).

The DEA Model

The DEA procedure[7] is implemented under two alternative assumptions:

(a) two inputs: private capital and labour;

(b) three inputs: private capital, public capital labour.

Thus, it is not simply assumed on a priori grounds that public capital enters the aggregate production function, testing instead whether infrastructure services have a direct impact on output by the implementation of the Banker test (1996). Table 5.3.2 reports annual mean values of the Malmquist index and its mutually exclusive and exhaustive components for Italy during the three sub-periods 1970-79, 1980-89 and 1990-95 and in the two alternative scenarios (a) and (b).

Starting with the two inputs scenario (a) and looking at , we note that productivity — *tfp ch.* in column 4 — increased by 1.5%, 1.1% and 1.7% per annum averaged over the 1970s, the 1980s and the early 1990s, respectively. The poorest performance took place during the years 1974, 1975, 1981 and 1982. The highest values belong to 1976, to the last observation of the 1970s and to the late 1990s.

As for the decomposition of the productivity change index, our results show how during the sample period productivity growth was due mainly to gains in innovation — *tech. ch.* in column 3 — rather than technical efficiency change — *eff. ch.* in column 2. However, technical efficiency change prevails over technological progress when TFP slows down, namely after the two oil shocks.

When public capital is assumed to be an additional input in model (b), TFP change follows a pattern similar to one observed in the two inputs scenario, generally showing lower values for productivity growth though. The average change in productivity, however, is found to be less than in the

[7] DEA models are performed using DEAP 2.1 (Coelli, 1996).

Table 5.3.2: Technical Efficiency Change, Technological Progress and TFP Growth - Italy (1970-95)

(a) Inputs: L and K				(b) Inputs: L, K and G			
Year	*eff. ch.*	*tech. ch.*	*tfp ch.*	Year	*eff. ch.*	*tech. ch.*	*tfp ch.*
70-71	0.999	1.006	1.005	70-71	1.003	0.996	0.999
71-72	0.981	1.039	1.019	71-72	0.998	1.027	1.015
72-73	1.009	1.023	1.032	72-73	1.014	1.017	1.031
73-74	1.023	0.980	1.002	73-74	1.005	0.996	1.001
74-75	1.005	0.969	0.973	74-75	0.998	0.972	0.971
75-76	0.990	1.046	1.036	75-76	0.986	1.047	1.033
76-77	1.003	1.012	1.015	76-77	1.000	1.012	1.012
77-78	1.000	1.022	1.022	77-78	0.999	1.019	1.018
78-79	0.996	1.036	1.032	78-79	0.995	1.034	1.029
Mean	1.001	1.014	1.015	Mean	0.999	1.013	1.012
80-81	1.003	0.991	0.995	80-81	1.000	0.992	0.992
81-82	0.994	1.003	0.998	81-82	0.999	0.996	0.995
82-83	1.008	0.994	1.003	82-83	1.005	0.996	1.000
83-84	1.005	1.011	1.016	83-84	1.004	1.008	1.012
84-85	1.011	1.003	1.015	84-85	1.009	1.003	1.012
85-86	0.990	1.022	1.012	85-86	0.991	1.021	1.012
86-87	0.996	1.021	1.018	86-87	0.997	1.020	1.017
87-88	1.010	1.015	1.025	87-88	1.010	1.014	1.024
88-89	0.999	1.023	1.022	88-89	0.999	1.021	1.020
Mean	1.002	1.009	1.011	Mean	1.002	1.008	1.009
90-91	0.998	1.007	1.005	90-91	0.999	1.004	1.003
91-92	1.007	1.003	1.010	91-92	1.006	1.002	1.008
92-93	1.008	1.002	1.010	92-93	1.007	0.999	1.006
93-94	0.995	1.038	1.032	93-94	0.998	1.032	1.030
94-95	1.007	1.022	1.030	94-95	1.009	1.019	1.028
Mean	1.003	1.014	1.017	Mean	1.004	1.011	1.015

former scenario (-0.3% in the first sub-period and -0.2% both in the 1980s and in the first half of the 1990s).

Tables 5.3.3-5.3.5 report DEA output disaggregated by regions and macro-regions in the three sub-periods.

Looking at Table 5.3.3, with regard to the two inputs scenario (a), the average performances at a macro-regional level during the 1970s are the following: the North East achieves the highest TFP growth (2.8%), followed by the North-West (2.2%), the Center (1.5%) and the South (0.6%). Thus, southern regions suffered from low productivity growth, despite the high rate of accumulation of both public and private capital experienced during the period. On the other hand, the North of the country witnessed a productivity gain about four times as much as the southern regions did, due to technological progress more than technical efficiency change. Similarly, central regions found technological progress to be the main source of TFP enhancing, performing relatively well, but below the average of the North.

Once we estimate the DEA model under the assumption (b), TFP change in the South is the same as in the previous scenario[8]. The regions of the North-East show a similar behavior: an unimportant decrease in the technical efficiency change score and unaffected TFP growth. On the other hand, both the North-West and the central regions are characterized by a lower average TFP change with respect to the scenario (a) (-1.2% and -0.3% respectively), as a result of a decline in both of its components.

The pattern of productivity growth depicted by the DEA model for the 1980s (Table 5.3.4) is the following. In the two inputs scenario, the highest TFP growth is achieved by the North West (1.7%), followed by the South (1.2%), the North-East (1.0%) and the Center (0.5%). Once public capital is considered as an additional input in the non-parametric production function,

[8] The technological progress component is unaffected, while technical efficiency change is lower by merely 0.1%.

Table 5.3.3: Technical Efficiency Change, Technological Progress and TFP Growth - Italian Regions (1970-79)

	(a) Inputs: L and K			(b) Inputs: L, K and G		
Region	*eff. ch.*	*tech. ch.*	*tfp ch.*	*eff. ch.*	*tech. ch.*	*tfp ch.*
PIE	1.004	1.021	1.025	0.993	0.998	0.991
VDA	0.991	1.020	1.011	0.991	1.021	1.011
LOM	1.007	1.021	1.028	1.000	1.016	1.016
TAA	1.005	1.020	1.025	1.005	1.020	1.025
VEN	1.004	1.020	1.024	1.001	1.023	1.024
FVG	1.012	1.020	1.032	1.011	1.021	1.032
LIG	1.002	1.021	1.023	1.002	1.021	1.022
ER	1.009	1.020	1.029	1.008	1.021	1.029
TOS	1.005	1.020	1.025	1.005	1.021	1.026
UMB	1.006	1.020	1.026	1.006	1.020	1.026
MAR	0.996	1.005	1.001	0.991	1.002	0.993
LAZ	1.000	1.006	1.006	1.000	1.004	1.004
ABR	1.007	1.016	1.023	1.007	1.016	1.023
MOL	0.960	0.952	0.914	0.960	0.952	0.914
CAM	0.992	1.016	1.007	0.992	1.015	1.007
PUG	1.014	1.020	1.034	1.010	1.021	1.031
BAS	1.003	1.021	1.024	1.003	1.021	1.024
CAL	0.984	1.012	0.995	0.984	1.012	0.995
SIC	1.010	1.019	1.029	1.010	1.020	1.030
SAR	1.004	1.021	1.025	1.004	1.021	1.025
North-West	1.001	1.021	1.022	0.997	1.014	1.010
North-East	1.008	1.020	1.028	1.006	1.021	1.028
Center	1.002	1.013	1.015	1.001	1.012	1.012
South	0.997	1.010	1.006	0.996	1.010	1.006
Italy	1.001	1.014	1.015	0.999	1.013	1.012

Table 5.3.4: Technical Efficiency Change, Technological Progress and TFP Growth - Italian Regions (1980-89)

	(a) Inputs: L and K			(b) Inputs: L, K and G		
Region	*eff. ch.*	*tech. ch.*	*tfp ch.*	*eff. ch.*	*tech. ch.*	*tfp ch.*
PIE	1.002	1.017	1.020	0.996	1.008	1.004
VDA	0.999	1.017	1.015	0.999	1.017	1.015
LOM	1.003	1.019	1.022	1.000	1.009	1.009
TAA	1.005	0.991	0.996	1.005	0.991	0.996
VEN	1.002	1.015	1.016	1.004	0.999	1.003
FVG	1.001	1.013	1.014	1.000	1.013	1.014
LIG	0.998	1.013	1.011	0.998	1.013	1.010
ER	1.000	1.015	1.015	0.999	1.015	1.014
TOS	1.000	1.014	1.014	0.999	1.015	1.014
UMB	1.000	1.013	1.013	0.999	1.014	1.013
MAR	1.006	1.001	1.007	1.006	1.000	1.006
LAZ	1.000	0.987	0.987	1.000	0.998	0.998
ABR	1.001	1.010	1.011	1.001	1.010	1.012
MOL	1.009	1.010	1.019	1.009	1.010	1.019
CAM	1.023	0.980	1.002	1.023	0.980	1.002
PUG	1.007	1.014	1.022	1.006	1.015	1.022
BAS	0.995	1.014	1.009	0.995	1.014	1.009
CAL	1.000	1.012	1.012	1.000	1.012	1.012
SIC	1.003	1.011	1.014	1.003	1.011	1.014
SAR	0.988	1.020	1.007	0.988	1.020	1.007
North-West	1.001	1.017	1.017	0.998	1.012	1.010
North-East	1.002	1.009	1.010	1.002	1.005	1.007
Center	1.002	1.004	1.005	1.001	1.007	1.005
South	1.003	1.009	1.012	1.003	1.009	1.012
Italy	1.002	1.009	1.011	1.002	1.008	1.009

Table 5.3.5: Technical Efficiency Change, Technological Progress and TFP Growth - Italian Regions (1990-95)

	(a) Inputs: L and K			(b) Inputs: L, K and G		
Region	*eff. ch.*	*tech. ch.*	*tfp ch.*	*eff. ch.*	*tech. ch.*	*tfp ch.*
PIE	0.999	1.015	1.014	0.999	1.015	1.015
VDA	0.992	1.021	1.012	0.992	1.021	1.012
LOM	1.000	1.016	1.016	1.000	1.000	1.001
TAA	1.002	1.000	1.002	1.002	1.000	1.002
VEN	1.006	1.021	1.027	1.013	0.993	1.005
FVG	1.014	1.021	1.035	1.014	1.014	1.035
LIG	1.000	1.021	1.021	1.000	1.000	1.021
ER	1.004	1.020	1.024	1.006	1.016	1.021
TOS	0.993	1.022	1.015	1.001	1.014	1.014
UMB	1.000	1.021	1.021	1.002	1.017	1.018
MAR	1.010	1.003	1.013	1.010	1.003	1.013
LAZ	1.000	1.004	1.004	1.000	1.001	1.001
ABR	1.002	1.021	1.023	1.002	1.021	1.023
MOL	1.008	1.021	1.030	1.008	1.021	1.030
CAM	0.992	0.996	0.988	0.992	0.996	0.988
PUG	1.004	1.021	1.025	1.008	1.016	1.024
BAS	1.020	1.022	1.042	1.020	1.022	1.042
CAL	1.007	1.008	1.015	1.007	1.008	1.015
SIC	0.998	1.000	0.998	0.998	1.000	0.998
SAR	1.008	1.016	1.024	1.008	1.016	1.024
North-West	0.998	1.019	1.016	0.998	1.015	1.012
North-East	1.007	1.016	1.022	1.009	1.008	1.016
Center	1.001	1.013	1.013	1.003	1.009	1.012
South	1.005	1.013	1.018	1.005	1.013	1.018
Italy	1.003	1.014	1.017	1.004	1.011	1.015

only the North-West and the North-East get lower TFP change scores (-0.7% and -0.3% respectively).

The analysis of the results obtained for the early 1990s — see Table 5.3.5 — leads to similar conclusions: TFP growth and the performance of its two components in the South and in the Center are unaffected by the inclusion of public capital in the model, influencing DEA scores in the rest of the country. Namely, TFP growth declines by -0.4% and -0.6% for the North-West and the North-East respectively in model (b) with respect to model (a).

Hence, in each of the considered sub-periods, the results obtained at the national level do not change widely whether or not public capital enters the non-parametric regional production functions. Moreover, the lower productivity performance depicted in the three inputs scenario is mostly due to a reduction in technological progress whereas technical efficiency change is almost always found to be unaffected (with the exception of the 1970s).

As for the analysis disaggregated at a macro-regional level, comparing the two models (a) and (b), only the North of the country is characterized by lower TFP growth in the latter, whereas the performance of the Center and the South is left mostly unchanged.

Given these results, we now turn to the test of the significance of public capital as a productive input in the DEA model by implementing the Banker test (1996).

In general, this test is based on the comparison between a basic DEA model including inputs X and outputs Y and a model with the same outputs Y, and including additional inputs Z. The significance of the additional inputs Z is tested on the basis of their asymptotic properties. If inefficiency (θ) has a half-normal distribution, the Banker test (T_{HN}) is distributed as an $F_{N,N}$, with N indicating the number of observations:

$$T_{HN} = \frac{\sum_{j=1}^{N} [\theta(X_j; Y_j) - 1]^2}{\sum_{j=1}^{N} [\theta(X_j; Y_j; Z_j) - 1]^2} \sim T_{N,N} \tag{5.3.1}$$

where, in our case, the only output in both DEA models (a) and (b) is GDP and the additional input in model (b) is public capital.

Under the null hypothesis that public capital does not affect the production correspondence, we get an F of 1.04, thus we do not reject the null hypothesis. Carrying out the same test for the "core"component of public capital as a possible additional input, the F test becomes equal to 1.05, leading us again to conclude that there is non productive role for public capital. According to this result, public capital should not be considered as a direct productive factor. Thus, we rely on the results obtained in the two inputs scenario and move to the analysis of the role of public capital as a positive externality to regional economics.

The Impact of Public Capital on TFP Growth

In the second part of the analysis, our aim is to verify whether the endowment of public capital has enhanced the performance of regional economies with respect to both efficiency and technological aspects of production over the period 1970-95. With this purpose in mind, an econometric analysis of the impact of public capital on productivity gains (and on both of its components) is performed using Limdep 7.0. In general, the estimated equation is given by:

$$y = \gamma \cdot x + u \tag{5.3.2}$$

where the dependent variable y is either total factor productivity, or technical efficiency or technological progress and the independent variable x is either public capital, or its "core"component or one of its nine disaggregated categories.

When efficiency is regressed on public capital a censored Tobit model is implemented, due to the fact that efficiency scores from DEA can not be negative.

On the other hand, technological progress and TFP are expressed in terms of 1970 (first observation in time) = 100. Therefore, we refer to them as cumulative variables and we use fixed effects and random effects panel data estimation techniques when these two variables are regressed on public capital[9].

Table 5.3.6 shows the results of the regression of the cumulative performance of TFP on total public capital and "core"public capital individually[10]. A highly statistically significant positive relationship arises in both cases. Thus, public capital seems to have positively contributed to TFP gains over the sample period, this being more noticeable for its "core"component, as shown by the higher magnitude of the estimated coefficients (0.39 compared to 0.21).

This result is consistent with the fact that "core"public capital is indeed more likely to have productivity enhancing effects. Furthermore, estimated coefficients turn out to be noticeably higher in the South of the country than in the Center-North, which confirms the results of previous studies.

In Table 5.3.7 are reported the results obtained by considering each of the categories of public capital as the independent variable in the estimated equation[11]. All the categories are positively correlated with the cumulative performance of TFP in the whole country, with the exception of "Land reclamation and Irrigation"(GLRI). Regarding the magnitude of the estimated parameters, we note that the category "Water and Electrical Lines"(GWE)

[9] OLS procedure has also been applied and results are not shown since the LM test always led to prefer either the fixed or the random effects models to the pooled estimator.

[10] t-ratios are given in parentheses, the high value of the Hausman test leads us to rely on the results of the fixed effects model in all cases.

[11] t-ratios are given in parenthesis; critical value of $\chi^2(1) = 3.84$ (5%).

Table 5.3.6: TFP Growth and Public Capital

Italy	Fixed Effects	Random Effects	LM test	Hausman test
Public capital	0.21	0.19		
	(10.1)	(9.4)	1842.2	14.6
Core Public capital	0.39	0.33		
	(10.1)	(9.3)	1877.4	15.6
N=520				
Center- North	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
Public capital	0.18	0.15		
	(7.4)	(6.9)	535.3	8.2
Core Public capital	0.33	0.26		
	(7.7)	(6.9)	515.3	12.5
N=312				
South	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
Public capital	0.38	0.35		
	(8.7)	(7.9)	1144.9	8.1
Core Public capital	0.77	0.68		
	(7.9)	(7.3)	1040.2	9.7
N=208				

y = TFP scores (1970=100); x= per capita stock of public capital

gets the highest coefficients followed by "Roads and Airports"(GRA), "Marine"(GM), and "Telecommunications"(GT). Such ranking does not change widely across the two macro-regions.

The breaking down of productivity growth into its two mutually exclusive and exhaustive components enables us to analyse the role of the endowment of infrastructure services in explaining the dissimilar pattern of

Table 5.3.7: TFP Growth and the Categories of Public Capital

Italy	Fixed Effects	Random Effects	LM test	Hausman test
GRA	0.54 (5.6)	0.32 (4.0)	1713.0	16.0
GRS	0.30 (13.9)	0.29 (13.6)	2336.0	5.97
GM	0.47 (8.1)	0.43 (7.8)	1941.7	5.88
GWE	0.73 (8.7)	0.70 (8.5)	2045.9	2.54
GPBS	0.13 (11.2)	0.13 (11.1)	2243.4	2.11
GS	0.10 (9.3)	0.09 (8.8)	1736.8	12.8
GLRI	0.97 (0.5)	-0.09 (-0.6)	1781.3	0.68
GT	0.35 (7.1)	0.34 (7.1)	2155.3	0.31
GO	0.13 (6.7)	0.12 (6.6)	1968.8	0.90
Center-North	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
GRA	0.56 (5.0)	0.3 (3.5)	459.3	13.2
GRS	0.28 (11.5)	0.27 (11.4)	691.1	4.06
GM	0.78 (7.2)	0.69 (6.8)	550.6	5.54
GWEL	0.61 (6.8)	0.56 (6.5)	559.9	4.52
GPBS	0.11 (7.4)	0.11 (7.3)	620.0	2.13
GS	0.87 (6.7)	0.80 (6.3)	541.7	5.98
GLRI	0.12 (2.4)	0.76 (1.7)	433.0	3.28
GT	0.25 (4.6)	0.23 (4.4)	551.5	2.39
GO	0.09 (4.5)	0.09 (4.4)	515.2	1.58
South	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
GRA	0.50 (2.5)	0.30 (1.7)	1022.5	4.29
GRS	0.35 (7.6)	0.34 (7.5)	1191.3	3.35
GM	0.34 (5.6)	0.33 (5.6)	1235.3	0.36
GWE	0.27 (9.4)	0.27 (9.3)	1371.0	1.20
GPBS	0.17 (9.4)	0.17 (9.3)	1379.7	1.94
GS	0.15 (7.5)	0.14 (7.2)	1159.9	3.48
GLRI	-0.15 (-0.07)	-0.01 (-0.06)	1150.4	0.02
GT	0.10 (8.3)	0.10 (8.4)	1380.3	0.11
GO	0.04 (8.2)	0.04 (8.2)	1332.9	0.31

y = TFP scores (1970=100); x= per capita stock of the relevant category of G

TFP growth across Italian regions in terms of both efficiency and technological change during the period 1970-95. This aim is pursued by regressing first technical efficiency and then technological progress on public capital (its "core"component and each of its categories).

Table 5.3.8: Technical Efficiency and Public Capital

	Public capital	Core Public capital
Italy	0.27	0.47
	(36.0)	(42.3)
Center-North	0.31	0.44
	(30.7)	(28.7)
South	0.26	0.54
	(28.1)	(37.3)

y = Efficiency scores; x= per capita stock of public capital

As shown in Table 5.3.8, the endowment of public capital seems to significantly explain the differences across Italian regions in terms of efficiency in production[12]. Total public capital shows an estimated coefficient of lower magnitude than its "core"component (0.27 compared to 0.47), for which a higher coefficient is estimated in the southern regions (0.54 compared to 0.44). This evidence is confirmed by the results obtained for the nine categories of public capital in Table 5.3.9.

Tables 5.3.10 and 5.3.11 contain the estimation results of the regression of technological progress on public capital, its "core"component and each of its categories individually[13]. Given the different estimation procedures, the comparison of the magnitude of the estimated coefficients with the ones reported in Tables 5.3.8 and 5.3.9 does not look feasible. However, our results

[12] t-ratios are given in parenthesis.

[13] Critical value for $\chi^2(1) = 3.84(5\%)$; the high value of the Hausman test leads us to rely on the results of the fixed effects model in all cases in Table 5.3.10.

Table 5.3.9: Technical Efficiency and the Categories of Public Capital

Categories of Public capital	Italy	Center-North	South
GRA	0.65 (27.7)	0.62 (19.2)	0.71 (22.2)
GRS	0.42 (43.5)	0.42 (40.1)	0.42 (22.6)
GM	0.58 (12.4)	0.14 (11.2)	0.55 (10.5)
GWE	0.18 (43.4)	0.15 (27.3)	0.29 (71.9)
GPBS	0.44 (24.5)	0.40 (19.6)	0.51 (10.5)
GS	0.18 (31.3)	0.25 (29.5)	0.17 (71.9)
GLRI	0.56 (14.5)	0.39 (16.6)	0.63 (13.0)
GT	0.19 (29.3)	0.16 (24.0)	0.27 (19.2)
GO	0.48 (28.2)	0.39 (18.2)	0.76 (39.3)

y = Efficiency scores; x= per capita stock of relevant category of G

Table 5.3.10: Technological Progress and Public Capital

Italy	Fixed Effects	Random Effects	LM test	Hausman test
Public capital	0.11 (11.0)	0.19 (10.3)	1332.0	15.5
Core Public capital	0.38 (10.7)	0.32 (9.9)	1318.4	17.3
Center-North	Fixed Effects	Random Effects	LM test	Hausman test
Public capital	0.18 (8.3)	0.16 (7.8)	426.2	7.4
Core Public capital	0.32 (8.6)	0.26 (7.8)	402.4	12.3
South	Fixed Effects	Random Effects	LM test	Hausman test
Public capital	0.40 (9.4)	0.35 (8.7)	829.3	12.0
Core Public capital	0.80 (8.7)	0.68 (7.9)	753.2	13.3

y = Tech. ch. scores (1970=100); x= per capita stock of public capital

show the significance of the impact of infrastructure services on technological progress in the South as well as in the North of the country. As in previous estimations, the core component of public capital is characterized by higher coefficients in the whole country, especially in the South.

Table 5.3.11: Technological Progress and the Categories of Public Capital

Italy	Fixed Effects	Random Effects	LM test	Hausman test
GRA	0.54 (6.2)	0.32 (4.6)	1165.8	17.4
GRS	0.24 (11.3)	0.23 (10.9)	1451.8	6.9
GM	0.47 (8.9)	0.42 (8.5)	1283.4	7.4
GWE	0.69 (9.2)	0.73 (9.3)	1322.4	3.0
GPBS	0.13 (11.8)	0.13 (11.7)	1514.5	1.0
GS	0.10 (10.4)	0.95 (9.8)	1310.6	12.8
GLRI	0.16 (0.9)	0.49 (0.66)	1132.8	0.7
GT	0.34 (7.9)	0.34 (7.8)	1346.6	0
GO	0.13 (7.1)	0.12 (7.1)	1217.3	0.53
Center-North	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
GRA	0.57 (5.7)	0.32 (4.4)	339.8	13.1
GRS	0.22 (9.5)	0.29 (9.19)	493.4	6.5
GM	0.66 (6.6)	0.56 (6.0)	381.9	7.2
GWE	0.64 (8.0)	0.59 (7.8)	437.8	4.1
GPBS	0.11 (8.3)	0.11 (8.2)	453.2	0.9
GS	0.82 (7.6)	0.83 (7.3)	456.8	4.3
GLRI	0.13 (2.8)	0.70 (1.8)	281.1	5.7
GT	0.26 (5.5)	0.25 (5.4)	408.9	0.4
GO	0.10 (5.5)	0.98 (5.4)	370.4	1.7
South	**Fixed Effects**	**Random Effects**	**LM test**	**Hausman test**
GRA	0.49 (2.6)	0.25 (1.6)	662.0	4.7
GRS	0.31 (6.8)	0.29 (6.6)	803.6	2.7
GM	0.40 (7.2)	0.39 (7.3)	793.7	0.6
GWE	0.23 (8.3)	0.23 (8.2)	959.2	1.3
GPBS	0.16 (9.2)	0.16 (9.1)	1005.3	0.6
GS	0.15 (8.4)	0.15 (8.1)	862.1	4.7
GLRI	0.05 (0.02)	0.45 (0.3)	708.1	0:15
GT	0.99 (8.6)	0.10 (8.7)	865.4	0.4
GO	0.35 (6.6)	0.35 (6.7)	790.4	0.03

y = Tech. ch. scores (1970=100); x= per capita stock of public capital

5.3.3 Conclusions

The results reported in subsection 5.3.2 show how the implementation of a DEA approach allows accounting for inefficiency in the measurement of TFP growth in the Italian regions over the period 1970-95. The Banker test has been used to test empirically the significance of public capital in the DEA model, concluding that it would not be correct to consider it as a direct productive input. Partly in disagreement with the existing literature, this evidence suggests that the Italian case does not fit the view of public capital as producing the direct effect (1) at page 9.

However, the econometric analysis of the relationship between productivity gains and public capital has led to the conclusion that public capital plays a significant role of a positive externality in enhancing both technical efficiency in production and technological progress in regional economies. Such a result has turned out to be particularly evident for the "core" component of public capital. Moreover, the geographical disaggregation of our results has allowed us to depict a more significant role of infrastructure services in the southern regions rather than in the North of the country. Finally, the impact of public capital on TFP — and its two components — has been measured for each of the nine categories of public capital, underlying their own contribution to the enhancing of economic performance.

The whole set of results shows that the endowment of public capital acts as an environmental variable to regional economics with the effect of improving both technical efficiency change and technological progress. In view of this, policies aimed at increasing the endowment of infrastructure services maintain their relevance despite the fact that according to our results public capital is not a statistically significant direct input in aggregate production.

A first extension to this empirical work could be the implementation of the Stochastic Frontier Approach to the same data set in order to compare the results obtained under the two alternative approaches. Second, given the recent availability of data on stocks of public capital for the Italian provinces, future empirical investigation could cover a deeper level of territorial disaggregation. Third, an analysis similar to the one carried out for public capital could be repeated for other additional inputs in aggregate production such as, for instance, human capital.

5.4 *Total factor productivity and the convergence hypothesis in the Italian regions*

The classical approach to the analysis of the sources of economic growth and of the convergence patterns proposed by Abramowitz (1986) and Baumol (1986) has been extensively applied in empirical literature on convergence across countries (e.g. Barro and Sala-i-Martin, 1991, 1992, 1996; De la Fuente, 1997). As a result of this wide literature, the most agreed finding is that world economies are converging at a stable speed. This approach has also been employed to examine the convergence issue regarding a set of regions within a country. The underlying idea supporting this type of analysis is the fact that if regions within the same country converge, they are also likely to converge to the same GDP level as much as they are sharing similar economic fundamentals. Therefore, as shown in Barro and Sala-i-Martin (1996) and Terrasi (1999), within a regional framework, there seems no need to control the estimates for growth determinants. Following this scheme, the focus here is on the economic performance of the Italian regions. Barro and Sala-i-Martin (1991) find positive evidence on the convergence of the sample of Italian regions considered in their study. They conclude for a 2% speed of convergence and the highest dispersion in the log of per capita GDP across European economies. However, later research such as Mauro and Podrecca (1994); Del Monte and Giannola (1997); Paci and Saba (1998) and Terrasi (1999) put into doubt the optimistic view of Barro and Sala-i-Martin (1991) not only on the convergence speed but also on the existence of convergence itself. Mauro and Podrecca (1994) claim the results of Barro and Sala-i-Martin (1991) to be biased due to the lack of (time) homogeneity of the data employed in their estimations. They estimate the β-convergence parameter on three different time spans separately. As a result, they find that the dispersion of per capita income to not decline over time, if data are not homogenized over these three time spans. On the basis of a homogeneous time series analysis, Del Monte and Giannola (1997) and Paci and Saba (1998) show a clear divergence pattern arising around the mid-1970s across Italian regions. The estimate of the coefficient attached to the gap variable is found to be negative and statistically significant, with this relationship driven by a strong catching-up process over the 1960-75 period.

Finally, by using a set of more sophisticated econometric tools than the simple estimation of the β-convergence parameter, Terrasi (1999) also finds support for the reversal in the convergence path of the Italian regions. Further research on the economic performance of Italian regions has also dealt with the discussion of whether or not Italian regions are converging each other. In this perspective, most of the previous literature as Mauro and Podrecca (1994); Cellini and Scorcu (1997) and Di Liberto and Symons (1998) have found conditional convergence. In other words, Italian regions tend to converge to different steady state levels of per capita GDP. Despite all this effort, a research topic still remains to be explored, namely the determinants of such convergence/divergence process within a regional framework as the one implemented in the present study.

In an attempt to achieve a better understanding of this critical issue, the main contribution of the present work is to reconcile traditional approaches to the analysis of economic growth determinants and convergence patterns with the frontier productivity measurement literature. The use of Malmquist productivity indices allow previous research to be broadened by decomposing productivity growth into technological progress and technical efficiency change as well as analysing the influence of these productivity growth sources on the observed development gap across Italian regions. In doing so, the decomposition of productivity growth helps the divergence pattern experienced by Italian regions since the 1970s to be understood. Thus, the results suggest that productivity gains are mostly achieved by innovation. Moreover, the speed at which northern regions innovate seems to overcome the velocity of catching up of the southern regions. In sum, Italian regions are found to diverge at a decreasing rate.

5.4.1 Data and Results

The dataset employed for this study is the same as the one considered in section 5.3. The units considered are the 20 Italian regions taken as individuals and also aggregated within North-West, North-East, Centre and South regions over the period 1970-1995. The aggregate output of each region is measured by its Gross Domestic Product. The total capital represents the stocks of private capital. Both capital stock and GDP variables are expressed in 1990 constant prices as retrieved from CRENoS and Bonaglia and Picci (2000) respectively. The labour variable, also retrieved from CRENoS, indicates total employment in thousands.

The empirical estimation process will be developed in two steps. The first involves the decomposition of TFP growth on the basis of considering GDP as output, and capital and labour as the relevant productive inputs. In the second, the convergence issue is analysed by means of panel data estimation techniques where the explanatory variable in all regressions is real GDP per worker and its square. Then initially, a set of Malmquist productivity indices are calculated[14]. As Färe *et al.* (1994) note, since this is

[14] Linear programming problems required to implement the Malmquist productivity indices can be solved using any of a variety of computer programs. DEAP Version 2.1 issued here. A detailed description of the computer program is provided in Coelli (1996).

an index based on discrete time, each region will have an index for every pair of years. This entails calculating the component distance functions using linear programming methods such as those described in the previous section. Instead of presenting the disaggregated results for each region and year, the average annual rates for TFP growth (Tfpch), technological progress (Techch), and efficiency change (Effch) are collected in Table 5.4.1. The results show there to be a major variability of TFP growth rates across regions. Thus, Friuli V.G. attains the highest productivity growth rate (2.7%), followed by Puglia (2.5%), Lombardia and Emilia Romagna (2.3%). A lower number of regions experienced, on average, productivity declines, with Molise (1.9%) and Lazio (0.1%) having lowest average TFP growth rates.

On the basis of the above decomposition of TFP growth into technological change and technical efficiency change, the convergence patterns followed by the Italian regions are next explored over the sample period of interest. As is known, the Convergence Hypothesis is related to the idea that economies starting with a lower level of income or productivity should experience, on average, a higher growth rate. If this is the case, they catch up with richer economies. This hypothesis has been usually tested regressing the growth rates of all the economies against a proxy of the wealth of a region per worker or per capita income, or their logs. A negative estimated coefficient for this variable will be interpreted in favour of the convergence hypothesis (e.g. Barro and Sala-i-Martin, 1991; Bairam and McRae, 1999).

Following Taskin and Zaim (1997), the methodology used here allows one to test for the Convergence Hypothesis from a different and more accurate perspective. The Malmquist index decomposition permits the evaluation of whether the two components of productivity gains do show convergence, and so be able to shed light on the determinants of the divergence pattern experienced by Italian regions since the 1970s. As a result, it is discussed whether

Table 5.4.1: TFP growth decomposition. Average annual changes (1970-95)

Region	Effch	Techch	Tfpch
Piemonte	1.001	1.019	1.020
Valle DAosta	0.995	1.020	1.015
Lombardia	1.002	1.020	1.023
Trentino	1.002	1.006	1.008
Veneto	1.001	1.019	1.020
Friuli VG	1.008	1.019	1.027
Liguria	1.002	1.020	1.021
Emilia Romagna	1.004	1.019	1.023
Toscana	0.999	1.020	1.018
Umbria	1.002	1.019	1.022
Marche	1.000	1.005	1.005
Lazio	1.000	0.999	0.999
Abruzzo	1.004	1.016	1.020
Molise	0.988	0.992	0.981
Campania	1.001	1.000	1.000
Puglia	1.006	1.020	1.025
Basilicata	1.002	1.020	1.022
Calabria	0.995	1.013	1.008
Sicilia	1.001	1.014	1.015
Sardegna	0.996	1.020	1.016

the development gap across Italian regions has increased due to differences in terms of efficiency change or innovation processes. Hence, the estimated equations in the analysis depict the relationships linking both technological progress and technical efficiency change with an index for the initial level of the per worker income. Following Bairam and McRae (1999), an empirical framework that predicts linear convergence is compared with a set up where convergence is allowed to be non-linear, and which of the two prevails explicitly tested, using the former as the null hypothesis. Moreover, because of the small number of regions, but especially since the OLS framework would overestimate the effect of the dependent variable on the index, panel data estimation techniques are adopted. The panel is obtained according to Islam (1995) by averaging the indices over five years of non-overlapping panels; it is believed that these spans represent a good compromise in order to obtain a robust relationship between growth rates and initial levels — which forces to have time spans as big as possible — with the use of a panel data framework — which leads to reduce these spans. Finally, the hypothesis that the panel data framework encompasses the OLS pooled estimator is also tested, resulting in more reliable estimated coefficients (Quah, 1997).

Table 5.4.2 reports the results for the set of alternative linear convergence (a) models and non-linear convergence (b) models. Strong evidence against a simple OLS pooled estimator approach is found in all estimated models: the Lagrange multiplier test clearly shows that this model should not be regarded as the favourite empirical framework. Moreover, the fixed effects model encompasses the random effects, as shown by results of the Hausman test. This type of model is preferred because other control variables are not used, and the fixed effects framework is helpful in controlling for their effects on the growth rates. Finally, it is found that the two-ways estimator encompasses the one-way set up[15].

The results are listed in Table 5.4.3. The hypothesis that the productivity gain is achieved by innovation is confirmed by the positive coefficient estimated for the gap variable for the technological progress. However, this process is found to be non linear: the model allowing for non lin-

[15] The particular way used to build the panel has avoided autocorrelation and heteroscedasticity in all models (not reported).

Table 5.4.2: Model Testing

	Effch		Techch		Tfpch	
	(a)	(b)	(a)	(b)	(a)	(b)
Chi-sq. stat. for exclusion (Chi-S.)						
Model (a) versus model (b) (p-value)	0.0045					
	(0.946 79)					
Model (a) versus model (b) (p-value)			20.3881			
			(0.0000)*			
Model (a) versus model (b) (p-value)					10.8195	
					(0.0010)*	
Pooled versus panel model (LM)	13.570	13.570	178.360	187.580	10.780	16.060
Fixed effects versus random effects (Haus)	6.570	6.410	1.450	8.840	0.940	4.446
Two-ways versus one-way (Chi-S)	31.324	31.320	96.616	115.483	21.725	26.797

T-statistics are given in parentheses; * * (*) indicates statistical significance at 1% (5%)

Table 5.4.3: TFP growth decomposition and initial per worker income level

	Dependent variables					
	Effch		Techch		Tfpch	
	(a)	(b)	(a)	(b)	(a)	(b)
Independent variables intercept	1.081	1.080	0.956	0.779	1.039	0.821
	(36.425)**	(35.993)**	(33.602)**	(16.673)**	(22.822)**	(10.414)**
(YLt_0)	$-1.825*10^{-3}$	$-1.172*10^{-3}$	$1.318*10^{-3}$	$8.230*10^{-3}$	$-0.551*10^{-3}$	$7.949*10^{-3}$
	(−2.870)*	(2.393)*	(2.050)*	(5.035)**	(-0.593)	(2.881)*
$(YLt_0)^2$		$-1.191*10^{-3}$	-	$-6.346*10^{-3}$	-	$-7.803*10^{-3}$
		(-0.085)		(−4.515)**		(−3.289)**
R^2	0.367	0.367	0.717	0.778	0.455	0.524
Adjusted R^2	0.154	0.142	0.622	0.699	0.270	0.355

Note: YLt_0 = Initial per worker income; $(YLto)^2$ = square in per-worker income.

T-statistics are given in parentheses: $**$ $(*)$ indicates statistical significance at 1% (5%)

ear convergence/divergence encompasses the model imposing linear divergence/convergence process. The squared value of the initial level of initial productivity is positive and robustly correlated with innovation processes.

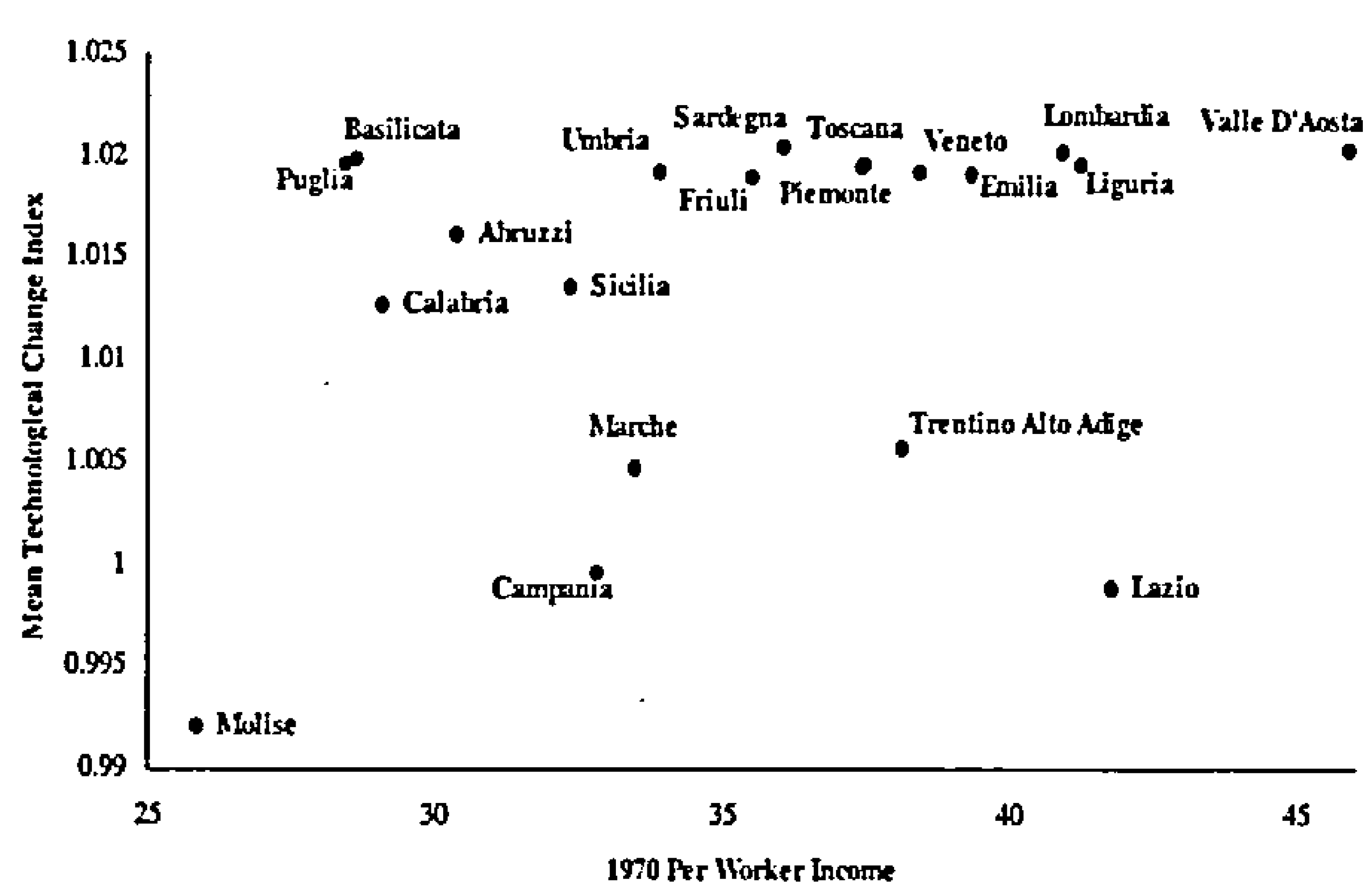

Figure 5.4.1: Technological progress and initial per worker income level

This also follows from the intuition behind Figure 5.4.1. The opposite is true for the technical efficiency change component. The model nonlinear convergence parameter is not significant, and with a slightly negative correlation between efficiency change and initial level of per worker GDP. The results plotted in Figure 5.4.2 also seem to confirm this. Finally, the strength at which northern regions innovate overcomes the velocity of catching up of the southern regions; this, in turn, drives the convergence process. Indeed, the relation between the productivity growth and initial level is driven from the former[16].

[16] All models have been re-estimated adding two slope dummies of the form $(south * YL)$ and $south * YL^2$, where south takes value 1 if the region belongs to the southern group of regions, and 0 otherwise. These models were all encompassed by the relative unrestricted

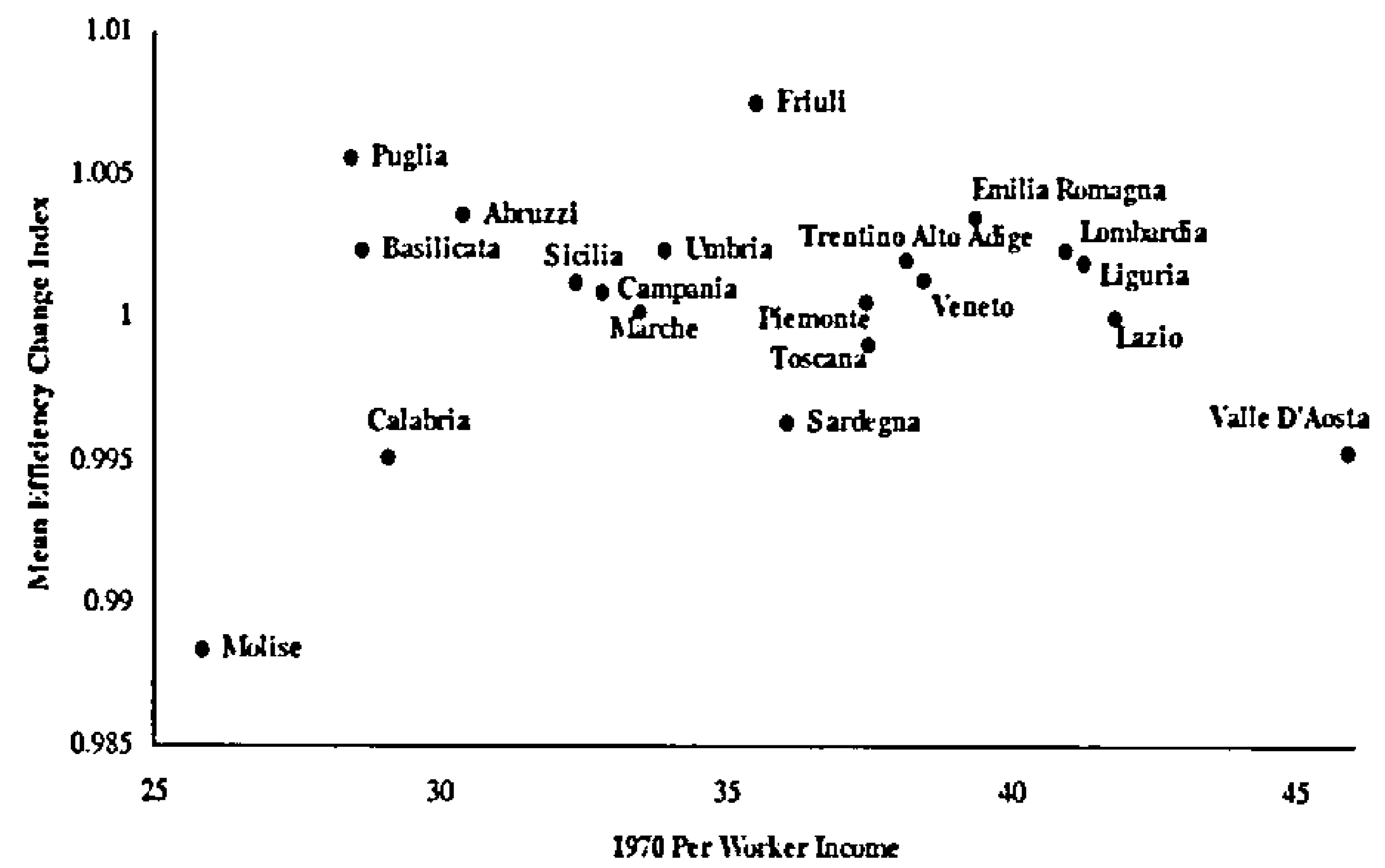

Figure 5.4.2: Technical efficiency change and initial per worker income level

5.4.2 Conclusions

This study had the aim of contributing to the debate on the catching-up hypothesis for the Italian regions over the period 1970–95. Data Envelopment Analysis has been used in order to decompose total factor productivity growth in technological progress and technical efficiency change, interpreting the former as a proxy of innovation and the latter as a measurement of catching up. According to the results, productivity gains experienced by regional economies during the sample period are mostly due to innovation, rather than improvements in productive efficiency. Furthermore, the econometric analysis shows that the northern regions tend to innovate at a speed higher than the velocity at which southern regions catch up. As a consequence, it is concluded that Italian regions have diverged at a decreasing rate over the sample period.

models without these dummies, rejecting the hypothesis that the southern regions follow a different convergence path to their own steady state with respect to northern economies.

6. CONCLUSIONS

The model built in Chapter 3 provides a flexible framework capable of studying the growth effects of fiscal policy both in infinite and finite horizons scenarios and reducing to limiting cases some recent Barro-type models. The optimal lifetime consumption plan has been determined within a standard representative agent model, the only difference being a rate of time preference augmented by a positive probability of death parameter. The government was assumed to run a balanced budget constraint, equating total expenditures to total revenues collected by levying a flat-rate income tax. I have distinguished between productive and unproductive categories of government expenditures. Productive public spending includes investment in public capital and private investment subsidies. On the other hand, public consumption and lump-sum transfers to households were assumed to be unproductive.

Comparing the two alternative scenarios of finite and infinite horizons, I have obtained results on (i) the growth effects of each category of government expenditures on long-run economic growth, and (ii) the relationships relating the Barro rule to the other categories of government expenditure.

Regarding the first set of conclusions, both categories of unproductive government spending are shown to have a decelerating effect on long-run growth. This result is in line with the existing literature and verified regardless of the assumption of uncertain lifetime. However, a rise of either lump-sum transfers to households or public consumption reduces the long-run rate of growth less in the finite than in the infinite horizon scenario.

On the other hand, the growth effects of the two categories of productive expenditures are ambiguous, and for both I have derived a growth maximizing value. As for public investment, the Barro rule still holds in the infinite horizon scenario but, in contrast with the existing literature, is negatively linked to the probability of death parameter. This implies that

- The growth maximizing level of public investment is lower under the assumption of uncertain lifetime.

which provides an answer to the first research question raised in the Chapter 1. Similarly, the growth maximizing level of private investment subsidies is reached earlier in the finite than in the infinite horizons scenario.

Relative to the second set of conclusions, the effects of public consumption, lump-sum transfers to households and investment subsidies on the optimal provision of public investment are similar. Indeed, it is shown that the growth maximizing level of public investment tends to increase in the presence of higher levels of other categories of expenditures. This result takes place regardless of the assumption on uncertain lifetime.

In section 5.3, the aim of contributing to the debate on the empirical linkage between public capital and economic activity has been accomplished by implementing a frontier approach to the measurement of TFP growth.

In the first part of the analysis, I have used non-parametric techniques to decompose productivity growth in technical efficiency change and technological progress in a DEA model. Comparing the two alternative scenarios of two (labour and private capital) and three (labour, private capital and public capital) inputs, the analysis has led to the conclusion that including public capital in the non-parametric production function, leaves mostly unchanged the productivity growth pattern experienced in the sample period, especially in the southern regions. Given this evidence, the significance of public capital as additional input in the DEA model has been tested using the Banker test. The null hypothesis that public capital does not directly affect output as a direct factor of production could not be rejected. This evidence does not support the view of public capital affecting aggregate output as a direct unpaid input.

The second part of the analysis has been devoted to the study of public capital as a source of a growth promoting externality. Using econometric techniques, I have studied the impact of public capital — and various disaggregated categories of public capital — on total factor productivity, technological progress and technical efficiency change scores obtained in the DEA model. In general, the conclusion has been that public capital has significantly contributed to productivity gains, and both its components over the sample period. In line with the previous studies, such a result has turned out to be particularly evident in the southern regions, compared to the North of the country and for the "core"categories of public capital. In sum:

- Public capital has played the significant role of a positive externality in enhancing both technical efficiency in production and technological progress in regional economies.

Empirical work of section 5.4 has also followed a two stage procedure. First, the decomposition of TFP growth obtained in the two inputs scenario in section 5.3.2 has been used in order to attain proxies of innovation and catching up within Italian region. Second, an econometric analysis has been carried out, concluding that:

- Italian regions have tended to diverge at a decreasing rate over the sample period 1970-95.

Both the theoretical and the empirical work of the present thesis should provide more insights into the respective fields of research. Nevertheless, they are not free of limitations, which make them admitting of developments and extensions to be attained in future research.

Barro (1990) and the following literature have revalued the growth enhancing effect of fiscal policy, within the new growth theory underlining the role of policy externalities in generating endogenous growth. Typically, the simultaneous presence of opposite policy externalities allows the determination of the optimum level of productive public services supply. This prediction crucially depends on the assumption that the externality associated with the provision of public productive services in taken as purely exogenous by the representative firm. Relaxing this assumption would make it possible to introduce into the analysis the strategic aspect of interactions between the private and the public side of the economy. Our attempt in future research will be to analyse the relationship between fiscal policy and growth in a framework where investment decisions are linked by strategic interactions — Vencatachellum (1998), Shibata (2002) and Philippopoulus and Economides (2003).

Furthermore, most of the recent literature on fiscal policy and growth assumes a balanced government budget constraint, neglecting the analysis of government deficits and public debt. This lack of attention does not fit the relevance that fiscal policy rules currently have in practice as designed either on the basis of instruments to be used or in terms of targets to be fulfilled, like specific values for the ratios of deficit and/or debt to GDP. As shown in recent empirical work — Gemmell *et al.* (1999); Bleaney *et al.* (2001) — in exploring the expansionary effect of alternative categories of public expenditure, it is relevant to specify their own source of financing (distortionary, non-distortionary taxation, deficit). The so-called "golden rule of fiscal policy"postulates that the government is allowed to run a budget deficit so long as this is used to finance productive public spending and — given the composition of fiscal policy on both taxation and expenditure sides — can be regarded as the benchmark with respect to modulate more or less binding budget rules (Greiner and Semmler, 2000). Future research will aim at verifying whether and at which extent the predicted growth enhancing/growth hampering effect of fiscal policy might be affected under alternative budgetary regimes.

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