Estimation of Time-Varying Risk Premia on Stock Market Indices and Exchange Rates Pricing Macroeconomic Variables

—

A Multivariate GARCH-In-Mean Approach

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ABSTRACT

This thesis considers estimation of time-varying risk premia on broad stock market indices in the UK and US and on the UK-US exchange rate with covariances betwen macroeconomic variables and returns determining the time variation - if risk premia are varying over time this means covariances between asset returns and macroeconomic variables must vary over time.

The thesis discusses the Stochastic Discount Factor methodology and interpret various asset pricing models used in the literature as Stochastic Discount Factor models. An econometric model is proposed to estimate the time-varying risk premia on any asset - the econometric model proposed has the advantage that it can be used to interpret the relation between the business cycle and asset returns. Statistical properties and interpretation of the proposed econometric model are discussed.

Following the risk premium on broad stock market indices in the UK and US is estimated using very general consumption based asset pricing models. It is concluded that the risk premium varies significantly over time and the time-variation in the UK and US are rather different. Consumption and inflation are signficantly priced in the UK and US stock markets. The thesis then propose an econometric model to investigate the relation between the business cycle and stock returns. It is emphasised that such a model needs to allow for the possibilities of asymmetries. It is shown that asymmetries are indeed important in the US.

The thesis then discusses the theory of exchange rates and relates UIP violations to time-varying risk premia. The risk premium on the UK-US exchange rate is estimated pricing macroeconomic variables. A simplifying estimation method is proposed - several key macroeconomic variables are found priced in the FOREX market but the premium does not vary sufficiently to resolve the UIP Puzzle. Finally the thesis attempts to reconcile the results found previous modelling the time-varying risk premium in the FOREX and equity market. A test whether the FOREX and equity markets are integrated is proposed and it is found that the UK FOREX and equity markets are indeed integrated based on several well known models.

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DECLARATION

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A previous version of chapter 5 is forthcoming as a joint paper with Peter N. Smith and Mike R. Wickens [107] with the title "An Asset Market Integration Test Based on Observable Macroeconomic Stochastic Discount Factors" in "Exchange Rates, Capital Flows and Policy" edited by Peter Sinclair. A version of chapter 2 can be found as a discussion paper, joint with Peter N. Smith and Mike R. Wickens, at the University of York [106]. It has been submitted for publication at the Review of Economic Studies. The work in chapters 2 and 5 includes extensions of the work in those papers.

1. Introduction

A Framework For Estimating Time-Varying Risk Premia Using Observable Factors

It is commonly assumed in financial economics that expected returns or expected returns over a risk-free rate are constant over time and sometimes it is even assumed that the expectation of the latter, the exante risk premium, is equal to zero. This assumption is probably more based on failure, so far, to detect the sources of risk that reflect this time-variation than based on simple intuition. If we think of the expected returns of "experts" in the field like investors, portfolio managers etc. the picture that arises is very different - their expectation varies considerably over time and is the reason why they change their optimal portfolios over time. The common suggestion that the composition of the optimal portfolio depends on the investment horizon also reflects the fact that risk premia must vary depending on the horizon.

Back to the economic equilibrium models, one fundamental problem arises since models that dictate the risk premium could be time-varying are extraordinarily difficult to estimate - almost any valid equilibrium model of asset prices in economics tells us that the potential risk premium on an asset may be time-varying because the conditional covariance between the return on the asset and macroeconomic variables is varying over time. The vast majority of work in financial econometrics assumes that the risk premium, if varying over time, can be proxied by the fitted value of a return regression on past returns - the explanatory power of these type of regressions are often very low, leading researchers to conclude once again that risk premia are close to being constant. However, modelling the risk premium as a function of past returns is spurious since these lags can only be proxying for the actual risk premium and it should be of interest to attempt to model the conditional covariance between returns and the macroeconomic variables directly.

That the risk premium should be proportional to the conditional covariance between the asset return and macroeconomic variables (marginal utility) poses another problem. Macroeconomic data are rarely available with a frequency higher than monthly and in some cases it is only possible to obtain quarterly or annual data. However, more and more monthly macroeconomic data have become available in recent decades and this allows the researcher to attempt to model the conditional covariance matrix between financial returns and macroeconomic variables, since the number of months on which we have data is increasing. This motivates the first part of this thesis. Using monthly data in the UK and US this thesis attempts to answer the question whether the stock market risk premium in these countries is varying over time proportionally to the conditional covariance between the returns and key macroeconomic variables. In addition the thesis aims to look at the time-variation in the risk premium per unit of return standard deviation, often denoted the Sharpe Ratio, or the unit of risk premium per unit of variance.

It is important for the economist to understand whether the risk premium is varying over time because he or she has to make some assumption on the risk premia and its time-variation when they derive their equilibrium models. For the investor or portfolio manager it is more important whether the risk premium per unit of volatility is varying over time since this ratio determines the optimal proportion they have to invest in an asset - wrong assumptions on the constancy or computation of this ratio can lead to severe loss of money. The fact that most investors fail repeatedly to beat an investment strategy tracking a broad national stock market portfolio may suggest that more work could or should be devoted to understanding the computation of the Sharpe Ratio. This latter point has recently been emphasised in a survey by Lettau and Ludvigson [82]. They write

> In addition, the behaviour of the Sharpe ratio over time is fundamental for assessing whether stocks are safer in the long run than they are in the short run, as increasingly advocated by popular guides to investment strategy. Only if the Sharpe ratio grows more quickly than the square root of the horizon-so that the variance of the return grows more slowly than its mean-are stocks safer investments in the long run than they are in the short run.

The Sharpe Ratio is the expectations of the excess return relative to its standard deviation and hence if these moments are varying over time we need to find an appropriate way to model them jointly. In this thesis we model the expected excess return, the risk premium, joint with the time-varying variance of excess return.

The second part of this thesis is devoted to estimation of risk premia in the FOReign EXchange (FOREX) market where the risk premium equivalently should be proportional to the conditional covariance between innovations in the exchange rate and macroeconomic variables. A potential time-varying FOREX risk premium could resolve one of the big puzzles in financial economics, the FOREX puzzle. One way to state the puzzle is the observation that regressing FOREX excess returns on the forward premium, for many currencies, one obtains a significantly negative slope coefficient, though it is supposed to be equal to zero. This puzzle assumes that investors are risk neutral and have rational expectations. As mentioned, it seems inconsistent with intuition that risk premia are constant or zero - it can be shown that an omitted risk premium correlated with the forward premium could be a potential explanation. On the other hand, attempts to model the FOREX risk premium have failed to remove the negative bias in the estimate of the coefficient on the forward premium (as summarised in the surveys by, for example, Engel [44] and Lewis [83]). One potential reason for this could be the failure to model the conditional covariances between exchange rate innovations and macroeconomic variables directly. One could argue that the interest rate differential is just proxying for the omitted conditional covariance between exchange rate returns and macroeconomic variables (or marginal utility) and it is the aim of the second part of this thesis to investigate whether this is the case.

The above discussion motivates the topic of this thesis, which is to estimate potential time-varying risk premia in the UK and US stock markets and on the UK-US exchange rates. First we consider how far we can get with traditional consumption-based models allowing conditional covariances to be time-varying and second we consider alternative models, preferably with some theoretical justification, relating risk compensation to the movements in the macro economy. This thesis emphasises that risk premia are likely to be time-varying and we propose and implement an estimation method for estimating this time-variation.

In this introductory chapter we first discuss the risk premium, then we discuss the modelling of a risk premium, based on a no-arbitrage argument, introducing the Stochastic Discount Factor (SDF) model, then we discuss the modelling of the SDF and propose an econometric model capable of estimating SDF models (and the risk premium) allowing for time-variation in the conditional covariance matrix. Description of the proposed estimation method is the main aim of this chapter and we will repeatedly refer to this chapter throughout. The discussion in this chapter focuses solely on equity returns but the discussion is also applicable to exchange rates which will be shown in the relevant chapters on FOREX risk premia.

1.1 The Risk Premium

To establish notation used throughout this thesis we discuss the implication of investors with aversion to risk. Let $\mathcal{R}_{t+1}$ denote any net simple real return between time t and $t+1$, then

$$\begin{aligned} \mathcal{R}_{t+1} &= \underline{\phi}_t + \mathcal{R}_{f,t} + \epsilon_{t+1}, \\ \mathrm{E}_t(\mathcal{R}_{t+1}) &= \underline{\phi}_t + \mathcal{R}_{f,t} \\ \mathrm{E}_t(\mathcal{R}_{t+1}) - \mathcal{R}_{f,t} &= \underline{\phi}_t, \end{aligned} \tag{1.1}$$

where $\underline{\phi}_t$ is the risk premium, $R_{f,t}$ is the real net return on a risk-free asset between t and $t+1$ if such an asset exists, $\mathrm{E}_t(\cdot)$ denotes the expectation conditional on information available at time t. Empirically it is questionable whether a real risk-free asset exists and it is preferable to work with models where we do not need to assume this to be the case - however, inflation uncertainty is negatively correlated with the frequency of the data. ϵ is the noise component of the excess return - this residual is orthogonal to the risk premium, $\underline{\phi}_t$. Often in economic and financial models it is assumed that investors are risk neutral, that is the risk premium is identically zero. We can decompose the variance of the excess return as

$$1 = \frac{\mathrm{V}(\underline{\phi}_t)}{\mathrm{V}(\mathcal{R}_{t+1} - \mathcal{R}_{f,t})} + \frac{\mathrm{V}(\epsilon_{t+1})}{\mathrm{V}(\mathcal{R}_{t+1} - \mathcal{R}_{f,t})} \tag{1.2}$$

Throughout V(.) will be used to denote the variance of the variable in brackets. When a subscript t is added, it is variance conditional on the information set available at time t. The conditional variance of a risk-free return is zero. It is of interest to investigate whether it is the first or the second fraction on the Right Hand Side (RHS) which contribute to the empirically observed variability of asset returns. Much empirical research suggests that the second term dominates. We will investigate whether this is true when modelling the time-varying risk premia based on a no-arbitrage argument.

Many empirical studies have shown, inconsistent with risk neutrality, that returns are predictable (see for example Campbell, Lo and MacKinlay [29]), this particularly evident for long horizon returns - one reason for this may be that the expectation of returns over a risk-free rate is time-varying depending on the information available at the time when investors form expectations about future excess returns. To model risk premia we need a model to get some theoretical understanding of the time-variation in excess returns. In this thesis we attempt to model equity and FOREX risk premia directly rather than look for variables proxying for the premium.

1.2 The Stochastic Discount Factor Model

The Stochastic Discount Factor (SDF) model has been known for several decades[1], and many important asset pricing models used in the literature can be given an interpretation in terms of the SDF. The SDF is also sometimes referred to as the Pricing Kernel. We will derive the risk premium from this well-known model first and then consider the logarithmic version.

1.2.1 The Non-Transformed SDF Model

To be credible it is desirable that a risk premium model has a theoretical justification. The SDF model is simple and we need only assume the law of one price to hold. Otherwise no structure is imposed - this may also be considered a weakness. In this thesis, different additional structures and estimation methods will be considered on equity and FOREX.

When there is uncertainty in a financial market, an investor will require a premium to invest in this market - otherwise it would be preferable to invest in a risk-free asset. We decide, for simplicity, to discuss the model with only one asset but it is easily extended to the case with many assets. When there exist no arbitrage possibilities, and the holding period is one, a Law of one Price argument states

$$\mathcal{P}_t = \mathrm{E}_t \{\mathcal{M}_{t+1} \, \mathcal{X}_{t+1}\}, \tag{1.3}$$

where $\mathcal{X}_t$ is the real payoff of the asset, $\mathcal{M}_{t+1}$ is the real SDF and $\mathcal{P}_t$ is the real price. If for instance the asset is a stock then the real payoff is $\mathcal{P}_{t+1} + D_{t+1}$, where D is real dividends.

[1] For an early reference, see Lucas [85].

Defining gross real return as $1 + \mathcal{R}_{t+1} \equiv \frac{\mathcal{X}_{t+1}}{\mathcal{P}_t}$ and rearranging

$$1 = \mathrm{E}_t \{\mathcal{M}_{t+1} (1 + \mathcal{R}_{t+1})\} \tag{1.4}$$

As will be shown this is the standard Euler equation that arises as a first order condition in any model of asset prices. We can manipulate this equation to obtain a convenient expression for the conditional or unconditional expected return on an asset or portfolio

$$\mathrm{E}_t(1 + \mathcal{R}_{t+1}) = \frac{1}{\mathrm{E}_t(\mathcal{M}_{t+1})}[1 - \rho_t(\mathcal{R}_{t+1}, \mathcal{M}_{t+1})\sigma_t(\mathcal{R}_{t+1})\sigma_t(\mathcal{M}_{t+1})] \tag{1.5}$$

$\sigma_t()$ is the standard deviation conditional on available information at time t and $\rho_t(x, y)$ is the conditional correlation between x and y. If the real return is risk-free

$$\mathrm{E}_t(\mathcal{M}_{t+1}) = \frac{1}{1 + \mathcal{R}_{f,t}} \tag{1.6}$$

Combining these equations we obtain an expression for the risk premium, the expected excess return, on any financial asset will be

$$\begin{aligned} \mathrm{E}_t(\mathcal{R}^e_{t+1}) &= -\frac{\rho_t(\mathcal{R}_{t+1}, \mathcal{M}_{t+1})\sigma_t(\mathcal{R}_{t+1})\sigma_t(\mathcal{M}_{t+1})}{\mathrm{E}_t(\mathcal{M}_{t+1})} \\ &= -\rho_t(\mathcal{R}_{t+1}, \mathcal{M}_{t+1})\sigma_t(\mathcal{R}_{t+1})\sigma_t(\mathcal{M}_{t+1})(1 + \mathcal{R}_{f,t}) \equiv \underline{\phi}_t \end{aligned} \tag{1.7}$$

Superscript e denotes a return excess over the risk-free interest rate, $\mathcal{R}^e_{t+1} \equiv \mathcal{R}_{t+1} - \mathcal{R}_{f,t}$. The lower the correlation ($\rho_t < 0$) and the higher the standard deviations (provided $\rho_t \neq 0$), the higher the risk premium required for buying the specific asset or portfolio. $\underline{\phi}_t$ is the risk premium on a risky asset and it must be positive at all times. Correlations have a lower bound of -1, hence

$$\mathcal{SR}_t(\mathcal{R}^e_{t+1}) \equiv \frac{\mathrm{E}_t(\mathcal{R}^e_{t+1})}{\sigma_t(\mathcal{R}^e_{t+1})} \leq \sigma_t(\mathcal{M}_{t+1})(1 + \mathcal{R}_{f,t}) \tag{1.8}$$

The Left Hand Side (LHS) is the conditional Sharpe Ratio, that is the premium per unit of risk (volatility) and the RHS is an upper bound on the ratio[2]. Since the real risk-free rate (if existent) would presumably not be very volatile it is evident that a high Sharpe Ratio must imply a highly volatile Pricing Kernel. Several attempts have been made to construct a time-varying Sharpe Ratio (see for instance Lettau and Ludvigson [82] for a summary) - modelling factors and the excess return jointly, as will be done in this thesis, provides an alternative way to create a time-varying Sharpe Ratio.

An important puzzle in financial economics, the Equity Premium Puzzle, is the observation that the implied volatility of the SDF in consumption-based asset pricing models can only match the actual data on stock returns if investors have implausible high aversion towards risk. Risk aversion or neutrality implies

$$\rho_t(\mathcal{R}_{t+1}, \mathcal{M}_{t+1})\sigma_t(\mathcal{R}_{t+1})\sigma_t(\mathcal{M}_{t+1}) \leq 0, \tag{1.9}$$

Rearranging equation (1.5)

$$\mathrm{E}_t(\mathcal{R}^e_{t+1}) = \underbrace{\frac{\rho_t(\mathcal{R}_{t+1}, \mathcal{M}_{t+1})\sigma_t(\mathcal{R}_{t+1})\sigma_t(\mathcal{M}_{t+1})}{\mathrm{V}_t(\mathcal{M}_{t+1})}}_{\beta_t} \underbrace{\frac{-\mathrm{V}_t(\mathcal{M}_{t+1})}{\mathrm{E}_t(\mathcal{M}_{t+1})}}_{\lambda_t} \tag{1.10}$$

β_t is the quantity of risk and λ_t is the price of risk. The SDF model is not new but it is a very powerful equation. The SDF model holds for any time interval, whether a day, a week, a month, a quarter or a century. It incorporates all sorts of uncertainty that people consider in making investment decisions. The model, however, imposes very little structure and many difficult choices will have to be taken - most disturbingly we do not know what is the SDF and there may be many (Cochrane [37]) !

[2]This upper bound was first derived by Hansen and Jagannathan [72].

1.2.2 A Second Order Approximation To The SDF Model

It is convenient to assume a joint conditional log normal distribution between the Pricing Kernel and returns. This gives a more straightforward analytical expression for the risk premium. Defining the real log return as the natural logarithm of the real gross simple return, $\mathrm{r} \equiv \ln(1+\mathcal{R})$, and taking logs on both sides of equation (1.4) we obtain

$$0 = \ln\left[\mathrm{E}_t\left\{\mathcal{M}_{t+1}(1+\mathcal{R}_{t+1})\right\}\right] = \ln\left[\mathrm{E}_t\left\{\exp(m_{t+1}+r_{t+1})\right\}\right] \tag{1.11}$$

The multivariate moment generating function. Assuming joint log-normality yields

$$0 = \mathrm{E}_t(m_{t+1}+\mathrm{r}_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1}+r_{t+1}) \tag{1.12}$$

If the return is risk-less then

$$0 = r_{f,t} + \mathrm{E}_t(m_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1}) \tag{1.13}$$

Throughout $r_{f,t}$ denotes the log real gross return on a risk-free asset between period t and $t+1$ and m is the logarithm of the SDF. If an asset is risk-free then $r_{f,t} - \mathrm{E}_t(r_{f,t}) = 0$, implying a time t conditional variance equal to zero and no time t conditional covariance with any variables. Combining equation (1.12) and (1.13) yields the no-arbitrage condition using nominal returns
real ?

$$\mathrm{E}_t(r_{t+1} - r_{f,t}) + \frac{1}{2}\mathrm{V}_t(r_{t+1}) = -\mathrm{Cov}_t(m_{t+1}, r_{t+1}) = \ln\left\{1 + \frac{\phi_t}{1+\mathcal{R}_{f,t}}\right\} = \phi_t, \tag{1.14}$$

Throughout we define, since we always work with the logarithmic version, ϕ_t as the risk premium. Defining logarithmic excess return as $r^e_{t+1} \equiv r_{t+1} - r_{f,t}$ yields

$$\begin{aligned} \mathrm{E}_t(r^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(r_{t+1}) &= -\,\mathrm{Cov}_t(m_{t+1}, r_{t+1}) \\ &= -\,\mathrm{Cov}_t(m_{t+1} - \mathrm{E}_t(m_{t+1}), r_{t+1} - \mathrm{E}_t(r_{t+1})) \end{aligned} \quad (1.15)$$

We must model the covariance structure of the residuals to obtain an estimate of the risk premium. We use a variant of this log-normal SDF model throughout this thesis with some minor changes depending on the setup in each chapter.

Campbell and Shiller ([26],[27]) make a logarithmic linearisation of the equity return and obtain the approximation

$$r_{t+1} \approx k + \wp p_{t+1} + (1-\wp)d_{t+1} - p_t, \quad (1.16)$$

where d is the logarithm of the real dividend, p is the logarithm of the real stock price and $\wp$ is the linearisation constant, $\wp = \frac{1}{1+\exp(\overline{d-p})}$. k is a function of $\wp$. $\overline{d-p}$ is the average logarithm of the dividend yield. The risk premium can be decomposed into a dividend component and a capital gain component

$$\begin{aligned} \mathrm{E}_t(r^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(r^e_{t+1}) &= -\,\mathrm{Cov}_t(m_{t+1}, r_{t+1}) \\ &= -\,\wp\mathrm{Cov}_t(m_{t+1}, \Delta p_{t+1}) - (1-\wp)\mathrm{Cov}_t(m_{t+1}, \Delta d_{t+1}) \end{aligned} \quad (1.17)$$

Hence the stock market risk premium is a weighted average of the covariance of the log SDF and the capital gain and the covariance of the log SDF and dividend growth - the weight being determined by the coefficient of linearisation which is generally large and close to 1 (how close depends on the frequency of the data).

We define logarithmic nominal return as

$$\begin{aligned} i_{t+1} \equiv \ln(1+\mathcal{I}_{t+1}) &= \ln\left\{(1+\mathcal{R}_t)\frac{P_{t+1}}{P_t}\right\} \\ i_{t+1} &= r_{t+1} + \pi_{t+1}, \quad \pi_{t+1} \equiv \ln\left\{\frac{P_{t+1}}{P_t}\right\} \end{aligned} \tag{1.18}$$

and note that $i^e_{t+1} \equiv i_{t+1} - i^f_t = r^e_{t+1}$. The excess return is the same whether it is a real return over the real risk-free return or a nominal return over a nominal risk-free return. π_{t+1} is price inflation. With a monthly frequency inflation uncertainty is low, especially in the 1990s. In the UK and few other countries, index linked bonds have existed for a while and the real risk-free rate "is known". The nominal risk-free rate $\mathcal{I}_{f,t}$ is known at time t.

One may not like the assumption of joint log normality. However, the derivations, as mentioned in Wu [111], will hold as a second order approximation to any joint distribution. It looks simple but two problems occur. First the SDF is not observable and further there may be time-variation in the conditional covariance matrix. The aim of this thesis is to do some considerations on modelling the SDF and propose a method for estimating this time-varying premium.

If we consider FOREX, it is important to note that exchange rates are priced in two countries - the logarithmic SDF model for FOREX will be discussed in chapters 4 and 5.

1.3 Theoretical Modelling of the SDF

The SDF is not observable and we need to rely on a proxy. Ideally we would like to model the SDF using theory such as for example General or Partial Equilibrium models. Unfortunately many models derived from theory have failed to match the actual data which led researchers to consider multifactor models where factors are not necessarily theoretically justified but are chosen under the belief that they summarise general risk affecting the average investor. In this section different models considered in the literature will be outlined and discussed. Many traditional models can be given an interpretation as a SDF model. Research on SDF models is expanding fast and therefore it is not possible to discuss every attempt that has been made to match the actual data. This section should be considered illustrative of the various directions taken in modelling the SDF. Excellent surveys can be found in, for instance, Cochrane [37], Smith and Wickens [105], Söderlind [108] and Campbell, Lo and MacKinlay [29].

1.3.1 Traditional Pricing Kernels - Partial and General Equilibrium Models

Much work has been done on the Capital Asset Pricing Model (CAPM). It is a static model that can be derived from mean variance optimality conditions. It can be given a SDF model interpretation - in the CAPM the logarithmic Stochastic Discount Factor is approximately linear in the log return on a wealth portfolio.

$$m_{t+1} \approxeq a_t - b_t r_{w,t+1} = \alpha_t - \beta_t(i_{w,t+1} - \pi_{t+1}) \tag{1.19}$$

This version of the CAPM model, assuming a and b to be time independent, has failed repeatedly empirically and several explanations for this have been proposed. First, the CAPM is a static model, not taking into account that an investor is faced with a multi-period investment scheme - in other words it is a Partial Equilibrium model. Second, most empirical tests assume that the return on the wealth portfolio is equivalent to the return on a broad national stock market index. This may not be true (Roll's critique [96]) - it is more likely to be a good proxy, as discussed in Campbell [21], in countries where the stock market is big relative to the Gross Domestic Product (GDP) level of the country. Third many empirical tests have assumed the coefficients in the SDF to be constant, implying a constant relationship between the risk premium on an asset and the covariance between the asset return and the return on the market portfolio. Empirically it would be more likely that the coefficients are time-varying.

Alternatively we could rely on an inter-temporal model such as the inter-temporal model of Merton [90], often referred to as the Inter-temporal CAPM (ICAPM). Most widely used is the representative agent consumption-based model of Breeden [19]. If, in an economy, there exists a representative investor with a given preference maximising utility subject to a budget constraint, the Euler equation that arises from the optimisation problem is

$$1 = \mathrm{E}_t \left\{ \delta_t \frac{\frac{\partial \mathcal{U}(\mathcal{C}_{t+1})}{\partial \mathcal{C}_{t+1}}}{\frac{\partial \mathcal{U}(\mathcal{C}_t)}{\partial \mathcal{C}_t}} (1 + \mathcal{R}_{j,t+1}) \right\}, \tag{1.20}$$

subscript j referring to the financial asset or portfolio under consideration. Hence the SDF can be given an interpretation as the Inter-temporal Marginal Rate of Substitution (IMRS), that is

$$\mathcal{M}_{t+1} = \delta_t \frac{\frac{\partial \mathcal{U}(C_{t+1})}{\partial C_{t+1}}}{\frac{\partial \mathcal{U}(C_t)}{\partial C_t}}, \tag{1.21}$$

where $\mathcal{U}(.)$ is some function for the utility of a representative investor, C is real per capita consumption and δ_t is the subjective rate of time preference. Hence the SDF is related to marginal utility. If capital markets are complete, marginal utilities of all investors are perfectly correlated and the SDF is unique. If we have incomplete capital markets, there exists, several SDFs since there is idiosyncratic variation in investors' marginal utility. Testing a consumption-based asset pricing model, we have to specify some functional form for $\mathcal{U}(.)$ - a rejection of an empirical test is not a rejection of the consumption-based model per se, but a rejection of the consumption-based model under the assumption of a specific functional form for $\mathcal{U}(.)$. Often it has turned out to be convenient to assume that the utility function is a power utility function, implying

$$m_{t+1} = \ln(\delta_t) - \gamma_t \Delta c_{t+1}, \tag{1.22}$$

with γ_t being the coefficient of relative risk aversion. The no-arbitrage condition and the implied real risk-free rate, using this functional form, are given by

$$\mathrm{E}_t(r^e_{t+1}) + \frac{1}{2}V_t(r_{t+1}) = \gamma_t \,\mathrm{Cov}_t(r_{t+1}, \Delta c_{t+1}) \quad \text{and} \tag{1.23}$$

$$r^f_t = -\ln(\delta_t) + \gamma_t \mathrm{E}_t(\Delta c_{t+1}) - \frac{\gamma_t^2}{2} V_t(\Delta c_{t+1}) \tag{1.24}$$

Hence an inter-temporal substitution and a precautionary saving term - volatile consumption makes people worry about low consumption, causing an incentive to save, which in turn drives down the real risk-free rate. A representative agent economy with power utility preferences has led to what is called the Equity Premium (Mehra and Prescott [89]) and Risk-Free Rate Puzzle (Weil [109]). The Equity Premium Puzzle states that the coefficient of relative risk aversion required to fit the actual data, assuming it is constant, is high and inconsistent with most microeconomic studies - stated otherwise, the covariances between consumption growth and the returns on different assets is too low to fit the observed equity risk premium. Assuming that it is truly so that the coefficient of relative risk aversion is so large, the variability of the real risk-free rate implied by the model is much larger than the actual empirical variability - this is the Risk-Free Rate Puzzle of Weil.

The likely solution to the puzzle is that either the assumption of power utility or the assumption of a constant coefficient of relative risk aversion are wrong ! The power utility function implies that the coefficient of relative risk aversion is equal to the inverse of the elasticity of inter-temporal substitution. There is no reason to believe that this is true. One has to do with aversion towards substituting across states, the other with aversion towards substituting inter-temporally. It would be desirable to consider more advanced consumption-based models eventually separating the tight link between the two or modelling a time-varying coefficient of relative risk aversion.

Another problem with testing the consumption-based model is the choice of consumption data. First, it is not obvious what consumption data to use. Ideally non-durable consumption plus services would be the correct measure since they measure consumption flow during the month, but it is difficult to obtain these data with lower frequency than quarterly - one exception being the US. Another problem that arises when testing the model is that consumption (or macro data in general) is usually measured as a flow during the current month whereas return series are point in time, that is the difference between the end of month price and the end of previous month price. Many empirical studies (see Söderlind[108]) have found that the correlation, $\rho(r_{t+1}, \Delta c_{t+1})$ is low and argue that due to the recording of the data it is more appropriate to consider $\rho(r_{t+1}, \Delta c_{t+2})$, that is consumption growth in a month should be related to the return of the previous month. Using the latter method often implies a higher correlation and a less severe rejection of the consumption-based model - however, using this timing convention still does not save the empirical relevance of the model (Söderlind [108]). Most, if not all, tests and estimates of the consumption-based model have assumed the moments in (1.23) and (1.24) to be constant. One recent attempt to model such time-variation is Duffee [43].

Rejection of the consumption-based model with power utility and constant risk aversion suggests two directions. Either other or additional factors need to be priced than consumption or we need to think of methods of modelling the time-variation in the coefficient of relative risk aversion. We show, in a later chapter, that the two directions are not mutually exclusive.

1.3.2 General Multiple Factor Pricing Kernel

In a recent paper, Smith and Wickens[105] propose writing the logarithmic pricing kernel as a linear combination of observable macroeconomic variables[3] where a constant may be included

$$m_{t+1} = -\mathbf{a}_t^{\mathsf{T}}\mathbf{f}_{t+1} + \zeta_{t+1}. \tag{1.25}$$

$\mathbf{f}$ is a vector of factors and $\mathbf{a}_t$ is the equivalent vector of time-varying factor loadings with the same dimension. If the SDF includes a constant, the first element in $\mathbf{f}$ is 1. ζ is an error term, uncorrelated with the factors. If we do not have a theoretically derived model, then the choice of factors will always be subject to the criticism of adhoc selection and the question of mismeasurement of the SDF may be important. Though additional variables to be priced ought to be variables that affect the average investor that cannot be hedged against (Cochrane [37]) - such variables are likely to be macroeconomic (see also Shiller [104]). All it takes to be convinced that people care, on average, about macroeconomic depression and unemployment is to follow the daily news.

The risk premium model implied by general factor models becomes

$$\mathrm{E}_t(r^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(r_{t+1}) = \mathbf{a}_t^{\mathsf{T}}\mathrm{Cov}_t(\mathbf{f}_{t+1}, r_{t+1}), \tag{1.26}$$

where $\mathrm{Cov}_t(\mathbf{f}_{t+1}, r_{t+1})$ is an $(N \times 1)$ vector of covariances between the individual factors and excess returns. We assume that the covariance between the error term in the log SDF and excess return is zero. In general factor models, the implied real risk-free rate becomes

$$r_{f,t} = \mathbf{a}_t^{\mathsf{T}}\mathrm{E}_t(\mathbf{f}_{t+1}) - \frac{1}{2}\mathbf{a}_t^{\mathsf{T}}\mathrm{Cov}_t(\mathbf{f}_{t+1}, \mathbf{f}_{t+1}^{\mathsf{T}})\mathbf{a}_t + \frac{1}{2}\mathrm{V}_t(\zeta_{t+1}) \tag{1.27}$$

Only when the error terms are identically equal to zero can we recover the constant in the SDF. The factor loadings may be time-varying, but to model the time-variation in the parameters

[3] The assumption of a log linear SDF is innocuous since most theoretically derived models imply a log linear pricing kernel. Under the assumption that the SDF is log-normally distributed we can recover the expectations and the variability of the SDF as $\mathrm{E}_t(\mathcal{M}_{t+1}) = \exp\{\mathrm{E}_t(m_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1})\} - 1$ and $\mathrm{V}_t(\mathcal{M}_{t+1}) = \exp\{2\mathrm{E}_t(m_{t+1}) + \mathrm{V}_t(m_{t+1})\}\{\exp\{\mathrm{V}_t(m_{t+1})\} - 1\}$. See also Cochrane [37] for a proof that any SDF nonlinear in the factors under common assumptions imply existence of a log linear SDF.

we need a theory for doing so. If no theory is available, it may as well be desirable to assume constant parameters. We refer throughout to a conditional factor model as a SDF model with time-varying parameters and an unconditional model as a SDF model with constant parameters.

One of the first studies to make an empirical investigation of the relation between stock returns and macroeconomic variables was by Chen, Roll and Ross [32]. However, rather than modelling the risk premium directly they also relied on a proxy (macroeconomic variables) for the conditional covariance between the SDF and the stock returns. Among other variables they consider price inflation, oil price inflation, industrial production growth and consumption growth.

A widely used asset pricing model in empirical finance these days is the Fama and French [54] three factor model. It has been found that pricing the return on a broad stock market portfolio, CAPM, was inadequate in capturing different risk premia on various equity portfolios. Fama and French found that pricing two additional variables HML and SMB was better at capturing the cross sectional differences in returns - the variable HML is the return difference between stocks with high book to market value and SMB is the return difference between small companies and large companies. Although these two additional factors improved the CAPM, Fama and French [55] argue that 5 factors are necessary to characterise the cross sectional differences in excess returns, three factors related, as mentioned, to the stock market and two factors related to the bond market. The two additional bond factors are the term-spread and a default variable, the difference between return on a market portfolio of long term corporate bonds and the long term government bond return. The approach implies a logarithmic SDF given by

$$m_{t+1} \approx \alpha_1 - \alpha_2 r_{m,t+1} - \beta_1^{\intercal} \mathbf{x}_{t+1} - \beta_2^{\intercal} \mathbf{y}_{t+1} + \zeta_{2,t+1}, \tag{1.28}$$

where $\mathbf{x}$ is the vector of two equity related variables and $\mathbf{y}$ is a vector of bond related variables. It could, however, be that the risk captured by term structure (and even the two equity portfolio returns) variables captures general macroeconomic risk and it may be preferable to use macroeconomic variables directly. The risk premium model is thus given by

$$\begin{aligned} E_t(r^e_{t+1}) + \frac{1}{2}V_t(r_{t+1}) &= \alpha_2 \text{Cov}_t(r_{t+1}, r_{m,t+1}) + \beta_1^T \text{Cov}_t(r_{t+1}, \mathbf{x}_{t+1}) \\ &+ \beta_2^T \text{Cov}_t(r_{t+1}, \mathbf{y}_{t+1}), \end{aligned} \tag{1.29}$$

However, they do not model the risk premium directly but uses the factors in regression analysis to proxy for the conditional covariance between the stock returns and the SDF. It may be argued that the two portfolio returns, HML and SMB, summarises the current state of the economy and it could be that observable macroeconomic variables does a better job. He and Ng [74] challenge this by comparing the Fama and French three stock market factors with those considered by Chen, Roll and Ross [32] and conclude the HML factor dominates. However, none of these studies aim at modelling the risk premium directly. This may be considered a disadvantage.

1.3.3 Consumption-Based Pricing Kernel with Time-Varying Risk aversion

A growing literature has argued that the consumption-based model with a power utility maximising representative investor may be true, but its failure is due to an assumption of constant risk aversion (unconditional SDF model). Recently, models have been developed where the coefficient of relative risk aversion varies with the business cycle (a conditional SDF). In two papers, Lettau and Ludvigson [79] [80] argue that allowing for time-varying factor loadings in the SDF makes the consumption-based model work considerably better empirically. First, they note that the log consumption wealth ratio can be proxied as a linear relationship between log consumption, asset wealth and labour income,

$$c_t - w_t \simeq cay_t \equiv c_t - \omega a_t - (1-\omega)y_t \approx E_t \sum_{i=1}^{\infty} \rho_m^i (r_{m,t+i} - \Delta c_{t+1}) + (1-\omega)z_{t+1} \tag{1.30}$$

c_t is the logarithm of consumption, a_t is log asset wealth, w_t is log aggregate wealth, z_t is a mean zero random variable and y_t is log labour income. Subscript m indicates a market return and ω is the average asset share of wealth. They call this ratio cay_t and it can be seen that this variable summarises investors expectations about future logarithmic returns on the market, and hence about future excess returns and real risk-free interest rates, and future consumption expenditure. When consumption is high, wealth is low or labour income is low, the investor expects either higher future log returns on the market portfolio, lower consumption growth or changing return to human capital in the future. Since this variable reflects investors expectations, it should be a useful variable to summarise investors attitude towards risk. They reach the approximation, with several assumptions. Write the log consumption aggregate wealth ratio as

$$c_t - w_t = \mathrm{E}_t \sum_{i=1}^{\infty} \rho_m^i (r_{m,t+i} - \Delta c_{t+1}) \tag{1.31}$$

That is the LHS variable summarises the expectation of future market returns, consumption, excess returns and interest rates. ρ_m is a linearisation constant. The human component of wealth is, however, not observable and we cannot summarise this expectation perfectly. As an approximation

$$w_t \approx \omega a_t + (1-\omega)h_t, \tag{1.32}$$

$$r_{w,t} \approx \omega r_{m,t} + (1-\omega)r_{h,t}, \tag{1.33}$$

$r_{w,t}$ the log return on total wealth, $r_{m,t}$ is the log return on financial wealth and $r_{h,t}$ is the log return on human wealth. Assuming that log human capital can be written as.

$$h_t = \kappa + y_t + z_t, \tag{1.34}$$

where κ is a constant, y_t is log labour income and z_t is a mean zero stationary variable, one can obtain equation (1.30) by combining equations (1.31), (1.32) and (1.34). Concluding that cay_t summarises investors expectations, Lettau and Ludvigson consider a consumption-based asset pricing model where the representative investor has a power utility function allowing the coefficients in the SDF from the consumption-based model to be time-varying (a conditional SDF model) yielding the logarithmic SDF:

$$m_{t+1} = -(a_1 + a_2 f_t) - (b_1 + b_2 f_t)\Delta c_{t+1} \tag{1.35}$$

In their papers they consider, among other models, $f_t = cay_t$. They find, using quarterly data, that this model substantially outperforms the traditional CCAPM and CAPM models and the important term added is the $b_2 f_t \Delta c_{t+1}$ term - in many cases a_2 is estimated equal to zero. They conclude that their results are in favor of a habit persistence model, which will be discussed below, where the coefficient of relative risk aversion is time-varying. Further they consider the 25 portfolios constructed by Fama and French and conclude that a conditional CCAPM does a good job of explaining the value-premium, that is the observation that portfolios with higher book-to-market value tend to have higher excess returns and argue that the higher excess return on portfolios of firms with higher book-to market value are more exposed to idiosyncratic macroeconomic shocks. Assuming that it is the log SDF given by the RHS of the above specified Pricing Kernel the risk premium model proposed by Lettau and Ludvigson is

$$\mathrm{E}_t(\mathrm{r}^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(\mathrm{r}_{t+1}) = (b_1 + b_2 cay_t)\mathrm{Cov}_t(\Delta c_{t+1}, \mathrm{r}_{t+1}) \tag{1.36}$$

and the real risk-free rate is given by

$$\begin{aligned} r^f_t &= \not{-}\, a_1 + a_2 f_t + (b_1 + b_2 cay_t)\mathrm{E}_t(\Delta c_{t+1}) \\ &- \frac{(b_1 + b_2 cay_t)^2}{2} V_t(\Delta c_{t+1}) \end{aligned} \tag{1.37}$$

Hence there is an additional source to create time-variation in the risk premium and real risk-free rate. That it is not important to allow the constant in the SDF to be time-varying can better be understood from the above logarithmic model, equation (1.36), allowing for second order moments to be time-varying. Since a time-varying constant in the Pricing Kernel is known at time t, it does not affect the time-varying risk premium since the conditional covariance with a time t variable is equal to zero. Lettau and Ludvigson do not estimate the risk premium directly but rely as well on their factors proxying for the conditional covariance between returns and the SDF.

In a similar fashion Lettau and Ludvigson [81] construct an alternative approximation to the consumption aggregate wealth ratio given by

$$c_t - w_t \simeq c_t - \nu d_t - (1-\nu)y_t \approx \mathrm{E}_t \sum_{i=1}^{\infty} \rho_m^i (\nu \Delta d_{t+i} + (1-\nu)y_{t+1} - \Delta c_{t+1}), \quad (1.38)$$

where d is log real dividend and call this variable cdy_t. The variable is similar to cay_t except that asset wealth has been replaced by real dividend by expressing the market value of assets in terms of expected future returns and expected future income flows. They find that cay_t has forecasting power on the long horizon excess return and cdy_t has forecasting power on long horizon dividend growth. Since both dividend growth and excess return are found predictable by these two variables, this may explain why the dividend price ratio explains little of long horizon dividend growth and only the very long horizon excess return - because dividend growth and the excess return has a predictable component that is shared, offsetting each other in the dividend price ratio.

A similar conclusion is reached by Campbell and Cochrane [22]. However, their approach is different in many respects. They consider a representative investor economy where the representative investor has preferences described by the power utility function. In their model it is not consumption that matters for utility but consumption relative to habit. Specifying the utility function as a power utility function in consumption differing from habit

$$\mathcal{U}_t = \left\{ \frac{C_{t+1} - X_{t+1}}{C_t - X_t} \right\}^{1-\gamma} = \left\{ \frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right\}^{1-\gamma}, \quad (1.39)$$

with the definition of the consumption surplus ratio as

$$S_t = 1 - \frac{X_t}{C_t}, \quad (1.40)$$

where X_t is the habit level of consumption for the representative investor, implies the logarithmic SDF

$$m_{t+1} = \ln\delta - \gamma(\Delta c_{t+1} + \Delta s_{t+1}) \quad (1.41)$$

Assuming that consumption is an i.i.d. log-normal process and specifying the log surplus ratio as a mean reverting process

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g), \tag{1.42}$$

where g is the constant growth rate of consumption and $\lambda(.)$ is a sensitivity function specified as

$$\begin{aligned} \lambda(s_t) &= \frac{1}{\bar{S}}\sqrt{(1-2(s_t-\bar{s}))} - 1, && \text{if } s_t \leq s_{max} \\ &= 0, && \text{if } s_t \geq s_{max} \end{aligned}$$

$s_{max} \equiv \bar{s} + \frac{1}{2}(1-\bar{S}^2)$. $\bar{S}$ is defined as

$$\bar{S} \equiv \sigma\sqrt{\frac{\gamma}{1-\phi}}, \tag{1.43}$$

where σ is the standard deviation, assumed constant, of the unpredictable component of consumption growth. This specification of the sensitivity function satisfies some prior conditions, one of them being a constant real risk-free rate. The log SDF is given by

$$m_{t+1} = \ln(\delta) - \gamma g - \gamma\{(\phi-1)(s_t-\bar{s}) + (1 + \lambda(s_t))(\Delta c_{t+1} - g)\} \tag{1.44}$$

and the corresponding risk premium by

$$\mathrm{E}_t(\mathrm{r}^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(\mathrm{r}_{t+1}) = \gamma(1+\lambda(s_t))\mathrm{Cov}_t(\Delta c_{t+1}, \mathrm{r}_{t+1}) \tag{1.45}$$

with the real risk-free rate given by

$$r^f_t = -\ln(\delta) + \gamma g - \frac{\gamma}{2}(1-\phi) \tag{1.46}$$

In other words, the coefficient of relative risk aversion changes over time because the sensitivity function $\lambda(s_t)$ depends on the state variable, s_t, which could be time-varying. The implied risk-free rate is constant, due to the specification of the sensitivity function. Empirical evidence, however, shows some time-variation in the risk-free rate, though the variability is low. Since consumption, in the model, is assumed i.i.d, the time-variation in the risk premiums is driven solely by $\lambda(s_t)$.

One criticism of the Campbell and Cochrane model is an "ad hoc" choice of the function $\lambda(s_t)$. Guevenen [69] shows that a similar model arises with limited participation in the stock market. His model on the other hand does not imply a constant real risk-free rate but a risk-free rate with low variability. Another criticism of the Campbell and Cochrane model is that it is often calibrated with the assumption that the covariance between return and consumption growth is constant. In this case the risk premium is varying over time only because $\lambda(s_t)$ is varying over time.

Brandt and Wang [17] provide an alternative framework for estimating a time-varying coefficient of relative risk aversion. Their model assumes that risk aversion varies over time with unexpected consumption and inflation shocks (the Campbell and Cochrane model as special case) and show that a model pricing also inflation is superior to the Campbell and Cochrane model. The no-arbitrage condition implied by their model is given by

$$\mathrm{E}_t(i^e_{t+1}) + \frac{1}{2}\mathrm{V}_t(r_{t+1}) = a_t\mathrm{Cov}_t(r_{t+1}, \pi_{t+1}) + b_t\mathrm{Cov}_t(r_{t+1}, \Delta c_{t+1}) \tag{1.47}$$

They provide no estimate of the implied time-varying risk premium. We will show later that this no-arbitrage condition can be derived in the context of a consumption-based asset pricing model with no "ad-hoc" assumptions on sensitivity functions and we will estimate the implied time-varying risk premium.

1.4 Econometric Modelling of the Risk Premium - Our Approach

Another difficulty with the SDF model is the estimation method and the observability of conditional covariance matrices. Most preferably the estimation method should allow for time-variation in the conditional covariance matrix between returns and the macroeconomic variables.

Assets are riskier if their returns are more highly conditionally correlated with factors rather than unconditionally correlated. This section outlines a way to estimate alternative SDF models, allowing for time-varying first and second moments. The aim of subsequent chapters is to show how this method can be used for a variety of setups, estimating FOREX and equity risk premia. The SDF model tells us that conditional expectations, conditional covariances and conditional variances could be time-varying. Further, since the moments are conditional, we need to specify the information available to investors when forming their expectations. This is not an easy task. In general we, as researchers, cannot replicate the information available to investors. However, the more information we allow for in the estimation the more correct the estimation will be. Allowing for more conditioning information we obtain more precise estimates of the conditional covariance matrix and hence the risk premium.

Smith and Wickens [105] propose using a multivariate GARCH in mean (MGM) model which has the advantage that we allow the conditional covariance matrix to vary over time and allows for the possibility to condition these moments on additional variables. The main problem with adding conditioning variables is non-feasibility of estimation, since the number of parameters to be estimated rises dramatically with the addition of variables. It is essential that we allow as general dynamics in the conditional covariance matrix as possible, making the SDF model more flexible.

With the estimation method proposed by Smith and Wickens as a starting point this thesis considers different dynamic specifications of the time-varying moments and proposes an estimation technique capable of modelling the risk premium on several assets or portfolios jointly.

1.4.1 Specification of the Mean Equation

To estimate a SDF model using the multivariate GARCH in mean model we have to specify three sets of variables. $\mathcal{Z}_1$ is a $(N_1 \times 1)$ vector of assets on which we wish to estimate risk premia. $\mathcal{Z}_2$ is a $(N_2 \times 1)$ vector containing the set of variables that proxy for the SDF excluding variables that are contained in $\mathcal{Z}_1$. An example of a variable in $\mathcal{Z}_2$ is Δc_{t+1} in the power utility consumption-based asset pricing model. $\mathcal{Z}_3$ is a $(N_3 \times 1)$ vector containing additional variables that are not priced but variables to which unexpected shocks affect the covariance matrix of variables in $\mathcal{Z}_1$ and $\mathcal{Z}_2$. To test a SDF model we therefore need $N = N_1 + N_2 + N_3$ variables.

The choice of variables in $\mathcal{Z}_1$ is straightforward but a difficult choice is the variables in $\mathcal{Z}_2$ and $\mathcal{Z}_3$ because we do not know which model of the SDF is correct and we do not have a clear idea as to which variables can be used as additional conditioning variables. The approach outlined in Smith and Wickens specifies the mean equation of the data as

$$\mathbf{Y_{t+1}} = \mathbf{A} + \sum_{j=0}^{p} \mathbf{B_j Y_{t-j}} + \sum_{i=1}^{N_1} \mathbf{\Phi_i H_{[1:N,i],t+1}} + \sum_{k=1}^{d} \mathbf{\Theta_k \Upsilon_{k,t+1}} + \epsilon_{t+1} \quad (1.48)$$

$\mathbf{Y}$ is the vector of dependent variables with dimension $(N \times 1)$. Throughout we assume that the first N_1 variables of $\mathbf{Y}$ are the excess returns on which we wish to model risk premia. $\mathbf{A}$ is a $(N \times 1)$ vector, the matrices $\mathbf{B_j}$ are of dimension $(N \times N)$ - in the N_1 equations of excess return the corresponding row in the $\mathbf{B}$ matrices includes only zeros (from the no-arbitrage condition, equation (1.14)). The $\mathbf{\Phi_i}$ are $(N \times N)$ matrices - in rows corresponding to excess returns the parameters are restricted to obey the no-arbitrage conditions whereas for all other variables the rows include only zeros. $\mathbf{H_{[1:N,i],t+1}}$ is the i th column of the conditional covariance matrix of dimension $(N \times 1)$. In practice we may want to include dummy variables to account for defined extreme outliers - Υ_k is an indicator variable taking the value one if the event k occurs and zero otherwise and Θ_k are $(N \times 1)$ vectors containing parameters in equations with the extreme event and zeros elsewhere. ϵ is a heteroskedastic error term

$$\epsilon_{t+1} = \mathbf{H}_{t+1}^{\frac{1}{2}} \mathbf{u}_{t+1}, \qquad \mathbf{u}_{t+1} \sim \mathcal{D}(0, \mathbf{I}_N),$$

$\mathbf{I_N}$ being an identity matrix of dimension $(N \times N)$ and $\mathcal{D}$ could be any distribution. It has been chosen to specify the model as a Vector Auto Regression (VAR) of order p. In practice one would hope a VAR of order 1 would be sufficient to remove residual correlation since estimation of multivariate GARCH in mean models will be very highly parameterised using a high order of the VARs - we note, for now, that there is a tradeoff between ease of estimation and the choice of p.

1.4.2 Specification of the Conditional Covariance Matrix

We wish as general a specification of the conditional covariance matrix as possible. We assume throughout that multivariate GARCH in mean is an adequate specification of the dynamics in the conditional covariance matrix of the dependent variables. Moreover we assume that the conditional covariance matrix only depends on the first lag of the outer product of shocks to the dependent variables and on the first lag of the conditional covariance matrix. The main reason for this is not that we believe that higher order lags will not improve the estimation but is simply due to the difficulty in the estimation. A recent survey of multivariate GARCH models can be found in Bauwens, Laurent and Rombouts [9] - for the theory, see Comte and Lieberman [38].

One of the most general multivariate GARCH specifications is the vec model

$$\mathrm{vec}(\mathbf{H_{t+1}}) = \mathbf{C_0} + \overline{\mathbf{D}}\,\mathrm{vec}(\mathbf{H_t}) + \overline{\mathbf{E}}\,\mathrm{vec}(\epsilon_t\epsilon_t^{\mathsf{T}}) + \overline{\mathbf{G}}\,\mathrm{vec}(\eta_t\eta_t^{\mathsf{T}}), \tag{1.49}$$

where $\mathrm{vec}(\mathbf{H_{t+1}})$, $\mathrm{vec}(\epsilon_t\epsilon_t^{\mathsf{T}})$, $\mathrm{vec}(\eta_t\eta_t^{\mathsf{T}})$ and $\mathbf{C_0}$ are $(N^2 \times 1)$ vectors. $\eta_t = min(\epsilon_t, 0)$ - hence the specification allows positive and negative residuals to have different impacts on the conditional covariance matrix. $\overline{\mathbf{D}}$, $\overline{\mathbf{E}}$ and $\overline{\mathbf{G}}$ are $(N^2 \times N^2)$ matrices. The disadvantage of the model is that it is highly parameterised. In addition it is difficult, if not impossible, to impose the necessary restrictions to ensure the covariance matrix to be positive definite at all times[4]. A more convenient but more restricted specification, a special case of the model proposed in Kroner and Ng [77], is the extension of the BEKK model with the covariance matrix specified as

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}^{\mathsf{T}} + \mathbf{D}\mathbf{H}_t\mathbf{D}^{\mathsf{T}} + \mathbf{E}\epsilon_t\epsilon_t^{\mathsf{T}}\mathbf{E}^{\mathsf{T}} + \mathbf{G}\eta_t\eta_t^{\mathsf{T}}\mathbf{G}^{\mathsf{T}} \tag{1.50}$$

The BEKK model discussed in Engle and Kroner [50] is the special case where $\mathbf{G}$ is a matrix of zeros. That asymmetries may be important for generating a more dynamic conditional covariance matrix as will be shown in a later chapter. The above specification of the conditional covariance matrix has been used in Bekaert and Wu [10], Kroner and Ng [77] - it is the multivariate equivalent of the univariate GARCH model by Glosten, Jagannathan and Runkle [66]. Hansen and Lunde [73] show that, in a univariate context, asymmetric GARCH models often

[4]This could give numerical problems when estimating the model.

perform better in terms of forecasting than symmetric models and this is likely to be the case in a multivariate context as well[5].

$\mathbf{C}$ could be a lower triangular or symmetric matrix of dimension $(N \times N)$. $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{G}$ are $(N \times N)$ matrices - so is $\mathbf{H}$, $\epsilon_t\epsilon_t^{\mathsf{T}}$ and $\eta_t\eta_t^{\mathsf{T}}$. We refer to this model as the Asymmetric BEKK (ABEKK) model. It is a vec model imposing the restriction that $\overline{\mathbf{D}} = \mathbf{D} \otimes \mathbf{D}$, $\overline{\mathbf{E}} = \mathbf{E} \otimes \mathbf{E}$ and $\overline{\mathbf{G}} = \mathbf{G} \otimes \mathbf{G}$ with $\otimes$ being the Kronecker product. The model is interesting in that the conditional covariance matrix will be positive definite given few assumptions easily imposed during estimation (Engle and Kroner [50]).

The model does not rule out many interesting vec models and the number of parameters to be estimated is reduced. The model is identified if one assumes that the diagonal elements in $\mathbf{C}$ are positive and, for example, that the first element in the first row and first column in $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{G}$ is positive. The model should be easy to estimate relative to the vec model and numerical problems should be fewer. The model can be rewritten in Error Correcting form (see Flavin and Wickens [59] [60] for a model not allowing for asymmetries) as

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}^{\mathsf{T}} + \mathbf{D}(\mathbf{H}_t - \mathbf{C}\mathbf{C}^{\mathsf{T}})\mathbf{D}^{\mathsf{T}} + \mathbf{E}(\epsilon_t\epsilon_t^{\mathsf{T}} - \mathbf{C}\mathbf{C}^{\mathsf{T}})\mathbf{E}^{\mathsf{T}} + \mathbf{G}(\eta_t\eta_t^{\mathsf{T}} - \overline{\mathbf{C}\mathbf{C}^{\mathsf{T}}})\mathbf{G}^{\mathsf{T}} \quad (1.51)$$

The bar over $\mathbf{C}\mathbf{C}^{\mathsf{T}}$ indicates that the appropriate correction is made since $\mathrm{E}_t(\eta_t\eta_t^{\mathsf{T}}) \neq \mathbf{C}\mathbf{C}^{\mathsf{T}}$. One possibility is to replace $\mathbf{C}\mathbf{C}^{\mathsf{T}}$ with the average of $\eta_t\eta_t^{\mathsf{T}}$ across all observations during estimations. This is what we are doing in this thesis[6].

This specification, the Error Correction Model (ECM), has two advantages. 1) The model implies a long run covariance matrix given by $\mathrm{E}(\mathbf{H}_{t+1}) = \mathbf{C}\mathbf{C}^{\mathsf{T}}$. 2) It is a very convenient specification when one wants to estimate the BEKK model, since one could estimate the starting value instead of setting the starting values equal to the covariance matrix of the actual data or equivalent. The $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{G}$ matrices can now be given the interpretations as loadings, measuring the impact on the covariance matrix of short run shocks that differ from long run level. Flavin and Wickens further impose the restriction that the $\mathbf{D}$ and $\mathbf{E}$ matrices are symmetric, which reduces the number of parameters to be estimated.

[5] However, forecasting out of sample it may be a disadvantage that the model is highly parameterised.
[6] Capiello, Engle and Sheppard [31] are doing the same.

A possible variant of the vec model is the Diagonal BEKK model, equation (1.51), assuming the parameter matrices, except **C**, to be diagonal (see Ding and Engle [42]). Alternatively one could use a factor ARCH structure (see Engle, Ng and Rothschild[46]) but this would require an apriori assumption on which factors drive the conditional covariance matrix.

Another class of multivariate GARCH models model the conditional covariance matrix as

$$\mathbf{H}_{t+1} = \mathbf{S}_{t+1}\mathbf{R}_{t+1}\mathbf{S}_{t+1} \tag{1.52}$$

where **S** is a diagonal matrix containing the conditional standard deviations of the dependent variables and **R** is the conditional correlation matrix. Bollerslev [12] proposed this model assuming the conditional correlations to be constant (we refer to it as the CCC model) - when we have no relation between the mean of the dependent variables and the conditional covariance matrix we can "easily" estimate the model, first by estimating univariate GARCH processes for all dependent variables, then obtain a consistent estimate of the conditional correlations by computing the correlation matrix of the standardized residuals. Recently Engle [48] and Engle and Sheppard [45] have proposed a similar model allowing for dynamics in the conditional correlations (therefore called Dynamic Conditional Correlation (DCC) model). They model, for example, the conditional correlations as a diagonal BEKK specification. This model is convenient since it is not highly parameterised and when there is no relation between the conditional mean of the dependent variables and the conditional covariance matrix, we can estimate the conditional covariance matrix in two steps, estimating first the conditional variances and then the conditional correlations of the standardised residuals from the first step, allowing for time-variation in the correlations. Although simplifying, these models are mainly applicable when there is no relation between conditional mean and the conditional covariance matrix, and the estimation advantage disappears when estimating risk premia using a multivariate GARCH in mean model. Hence when risk premia are time-varying, depending on the conditional covariance matrix, the advantages using the two-step estimation disappear - therefore CCC and DCC models seem unlikely to prove useful for modelling financial returns if risk premia are varying over time.

1.5 The Estimation of Multivariate GARCH Models

Multivariate GARCH models are difficult to estimate (see Bollerslev [13]) because they easily get highly parameterised. We propose a method to estimate multivariate GARCH in mean models, in particular BEKK type of models. Using a series of steps, one can obtain good estimates of parameters in the model to use as starting values. The steps are the following:

- For each of the N equations in (1.48) estimate the parameters in $\mathbf{A}$ and $\mathbf{B}$ as if the $\mathbf{\Phi}$ matrix was a matrix of only zeros, using an unrestricted VAR, assuming, the covariance matrix of the residuals to be constant - we obtain $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}_1$ (in case $p = 1$). For these N series of residuals we compute the covariance matrix $\mathbf{H}_0$.

- For each residual series estimate a univariate GARCH(1,1) with, if necessary, variance targeting, and obtain the parameter estimates. By variance targeting we mean fixing the long run variance to equal the computed variance, $\mathbf{H}_0$.

- Use the square root of the absolute value of the estimated ARCH and GARCH parameters from the univariate GARCH(1,1) estimation as starting values and estimate a multivariate Diagonal BEKK model on the residual series. Fix the long run covariance matrix $\mathbf{CC}^{\intercal}$ equal to $\mathbf{H}_0$ or the covariance matrix of the actual data.

- Use the parameters estimated in the previous step together with $\hat{\mathbf{A}}_1$ and $\hat{\mathbf{B}}_1$ as starting values and estimate a diagonal BEKK model on the actual data - that is assuming that the parameter matrices $\mathbf{D}, \mathbf{E}$, and if allowing for asymmetries $\mathbf{G}$, to be diagonal.

- Use the parameters estimated in the previous step as starting values and perform the estimation allowing the ARCH and GARCH matrices to be symmetric if we consider the symmetric model. If allowing for parameterised matrices, it is sometimes useful to do this in two steps.

- Use the estimated parameters from the previous step as starting values and estimate, in addition, the $\mathbf{\Phi}$ matrix with the no-arbitrage condition imposed.

- Finally we want to estimate the full model including estimation of the long run matrix $\mathbf{CC}^{\intercal}$. Good starting values for $\mathbf{C}$ are obtained simply by taking the Cholesky Decomposition of $\mathbf{H}_0$.

During all the above steps stationarity conditions on the conditional variance covariance matrix could be imposed during estimation. Performing all these steps should ease the estimation considerably. All estimations are performed using Gauss 3.2 or 3.6. The multivariate GARCH in mean model is estimated using a quasi maximum likelihood estimator when we estimate the conditional covariance matrix recursively conditional on a starting value. In the above we could have used the covariance matrix of the dataset as $\mathbf{H}_0$.

1.5.1 Scaling and The No-Arbitrage Condition

One problem in modelling macroeconomic variables joint with financial returns, in particular equity returns, is that the variability of the two sets of variables can differ very much. This potentially creates numerical problems. In some cases it can improve estimations when scaling data. In this case one has to take care with interpretation of parameters in the excess return equation. For instance if we scale excess return by a factor μ_r then the appropriate Jensen correction is not $\frac{1}{2}\mathrm{V}_t(r_{t+1})$ but $\frac{1}{2\mu_r}\mathrm{V}_t(r_{t+1})$. Similarly if we scale a factor in the SDF (scale factor μ_f), other than the excess return of the asset on which we model the risk premium, the coefficients on the estimated conditional covariance in the mean equation of excess return is $\frac{1}{\mu_f}$ times the estimate that would appear had we not scaled the data.

Note, in cases where we assume the parameter matrices to be symmetric it matters if the actual data are scaled. Assumption of symmetric matrices with scaled data is not the same as assumption of symmetric matrices without scaling.

1.5.2 Starting Values

The multivariate models are estimated recursively subject to a starting value. There are several possibilities. One estimator proposed (see Engle and Mezrich [49]) is the sample covariance matrix of $\mathbf{Y}_{t+1}$. An alternative, and potentially better, estimator is to perform the multivariate vector auto regression in equation (1.48) and compute the covariance matrix of the residuals. However, this variance targeting procedure is only consistent if there is no relation between the mean of the dependent variables and the conditional covariance matrix. When risk premia are varying over time, the Information Matrix is no longer block diagonal and the variance target estimators are inconsistent. However, to date it is not known if it is a serious problem to use the variance targeting estimator when we have a relation between first and second moments.

Most likely this depends on the variability of the risk premium[7]. A final estimator is to set the starting value equal to the long run covariance matrix during estimation. We note that $E(\mathbf{H}_{t+1}) = \mathbf{CC}^{\intercal}$. This estimator is consistent.

1.5.3 The Likelihood Function

It is common to use the multivariate normal distribution with log-likelihood function given by

$$\begin{aligned} \ell_{t+1,nd} &= -\frac{1}{2}\left\{\ln\{2\pi\}\mathrm{N} + \ln\{|\mathbf{H}_{t+1}|\} + \epsilon_{t+1}^{\intercal}\mathbf{H}_{t+1}\epsilon_{t+1}\right\} \\ \ell_{nd} &= \sum_{t=1}^{T}\ell_{t+1,nd} \end{aligned} \tag{1.53}$$

However, assuming normality of the joint distribution may not be a good idea. Empirically it has been found that returns have more heavy tails than implied by the normal distribution - wrong assumption on the distribution may affect the estimated covariance matrix. A joint distribution with more heavy tails is the multivariate t-distribution (see Hafner [70] for discussion) with log-likelihood given by

$$\begin{aligned} \ell_{t+1,td} &= \ln\left\{\frac{\Gamma\left[\frac{\mathrm{N}+\nu}{2}\right]}{\Gamma\left[\frac{\nu}{2}\right]}\right\} - \frac{\mathrm{N}}{2}\ln\{\pi\,(\nu-2)\} - \frac{1}{2}\ln\{|\mathbf{H}_{t+1}|\} - \frac{\nu+\mathrm{N}}{2}\ln\left\{1 + \frac{\epsilon_{t+1}^{\intercal}\mathbf{H}_{t+1}\epsilon_{t+1}}{\nu-2}\right\} \\ \ell_{td} &= \sum_{t=1}^{T}\ell_{t+1,td} \end{aligned} \tag{1.54}$$

T is the sample size. $|\cdot|$ indicates the determinant of the matrix, $\Gamma()$ is the gamma function and ν is the degree of freedom parameter which will be estimated in all models. The multivariate t-distribution will be used throughout this thesis[8].

[7] It should be noted that it is common practice empirically to use an inconsistent estimator as starting value and fixing $\mathbf{CC}^{\intercal}$ to equal this inconsistent starting value.

[8] Quasi maximum likelihood estimation of parameter estimates are consistent even if assumed distribution is wrong.

1.6 Conditioning Information - An Example

The description in the previous section may seem abstract and it is useful to illustrate the approach for estimating a SDF model in the multivariate GARCH in mean framework. Consider a version of the CAPM for the market return itself, assuming that a real risk-free rate exists. The excess return equation is

$$r_{t+1}^{e} + \frac{1}{2}\mathrm{V}_t(r_{t+1}) = c\mathrm{V}_t(r_{t+1}) + \epsilon_{1,t+1} \tag{1.55}$$

In this case the set $\mathcal{Z}_1$ is the market excess return and the sets $\mathcal{Z}_2$, $\mathcal{Z}_3$ are empty. This model could be estimated using a univariate GARCH model with the conditional variance in mean. However, univariate GARCH models assume that no additional information is available. If unexpected shocks to a variable affect the conditional variance of the market excess return it is of interest to model this variable bivariate with excess return. Evidence that such variables exist will be shown in subsequent chapters - in particular, industrial production growth. For the moment assume that unexpected shocks to industrial production growth affect the conditional variance of the excess return. Then we obtain a more precise estimate of the variance in the market when modelling the two variables bivariate. If industrial production growth (or changes in the log of industrial production, $\Delta ln(Y)$) is modelled, for instance, as a vector auto regression of order one,

$$\Delta y_{t+1} = a + b_1 r_t^{e} + b_2 \Delta y_t + \epsilon_{2,t+1}, \tag{1.56}$$

we obtain the unexpected component as $\epsilon_{2,t+1}$. The set of conditioning variables, $\mathcal{Z}_3$, now contains Δy_{t+1}. The vector of unexpected shocks is

$$\epsilon_{t+1} = \begin{pmatrix} r_{t+1}^{e} - \mathrm{E}_t(r_{t+1}^{e}) \\ \Delta y_{t+1} - \mathrm{E}_t(\Delta y_{t+1}) \end{pmatrix} = \begin{pmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{pmatrix} \tag{1.57}$$

We wish to model the conditional variance covariance matrix of these residuals. Let the long run variance covariance matrix be, assuming $\mathbf{C}$ to be lower triangular, specified as

$$\mathbf{C}\mathbf{C}^{\mathsf{T}} = \begin{bmatrix} c_1^2 & c_1c_2 \\ c_1c_2 & c_2^2 + c_3^2 \end{bmatrix} = \begin{bmatrix} c_1^* & c_2^* \\ c_2^* & c_3^* \end{bmatrix}, \tag{1.58}$$

Expanding the conditional covariance matrix assuming, for simplicity, ARCH and GARCH parameter matrices to be symmetric with no allowance for asymmetries, the vech[9] of the conditional covariance matrix can be written as

$$\begin{aligned} \text{vech}(\mathbf{H}_{t+1}) &= \begin{bmatrix} h_{11,t+1} \\ h_{21,t+1} \\ h_{22,t+1} \end{bmatrix} = \begin{bmatrix} c_1^* \\ c_2^* \\ c_3^* \end{bmatrix} \\ &+ \begin{bmatrix} d_{11}(h_{11,t} - c_1^*) + 2d_{12}(h_{12,t} - c_2^*) + d_{22}(h_{22,t} - c_3^*) \\ d_{12}(h_{11,t} - c_1^*) + (d_{12} + d_{13})(h_{12,t} - c_2^*) + d_{23}(h_{22,t} - c_3^*) \\ d_{22}(h_{11,t} - c_1^*) + 2d_{23}(h_{12,t} - c_2^*) + d_{33}(h_{22,t} - c_3^*) \end{bmatrix} \\ &+ \begin{bmatrix} e_{11}(\epsilon_{11,t} - c_1^*) + 2e_{12}(\epsilon_{12,t} - c_2^*) + e_{22}(\epsilon_{22,t} - c_3^*) \\ e_{12}(\epsilon_{11,t} - c_1^*) + (e_{12} + e_{13})(\epsilon_{12,t} - c_2^*) + e_{23}(\epsilon_{22,t} - c_3^*) \\ e_{22}(\epsilon_{11,t} - c_1^*) + 2e_{23}(\epsilon_{12,t} - c_2^*) + e_{33}(\epsilon_{22,t} - c_3^*) \end{bmatrix}, \end{aligned} \tag{1.59}$$

with symmetric parameter matrices given by

$$\mathbf{D} = \begin{bmatrix} d_1 & d_2 \\ d_2 & d_3 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} e_1 & e_2 \\ e_2 & e_3 \end{bmatrix}$$

and $\xi_{ij} = \xi_i\xi_j$ with $\xi = e,\ d,\ \epsilon,\ h$. This example illustrates why it may be desirable to have a set of conditioning variables that is not empty. Unexpected shocks to the conditioning variable, ϵ_2 may affect the conditional variance of excess return, $h_{11,t+1} = \mathrm{V}_t(r_{t+1})$ and hence the conditional expectation of the excess return - the risk premium. Omitting variables to which unexpected shocks affect the conditional variance covariance matrix could be a potential reason why time-variation in the risk premium may be rejected.

The diagonal BEKK imposes $d_2 = e_2 = 0$ - adding extra informational variables have no

[9]The vech operator stacks the lower triangular part of a matrix as a vector.

effect on the conditional variances and covariances of variables already in the multivariate system except that additional restrictions are imposed across the existing parameters[10]. The diagonal BEKK model seems to be too restrictive to use for estimating a conditional moment SDF model. However, the diagonal BEKK model has the advantage that it is easy to estimate - even with $N = 10$ the model can be estimated relatively easily and in the case where the set $\mathcal{Z}_2$ is large this model, though restrictive, may be the best available with the current length of data sets. Further the diagonal BEKK model allow us to obtain a better representation of the residuals.

1.7 Data

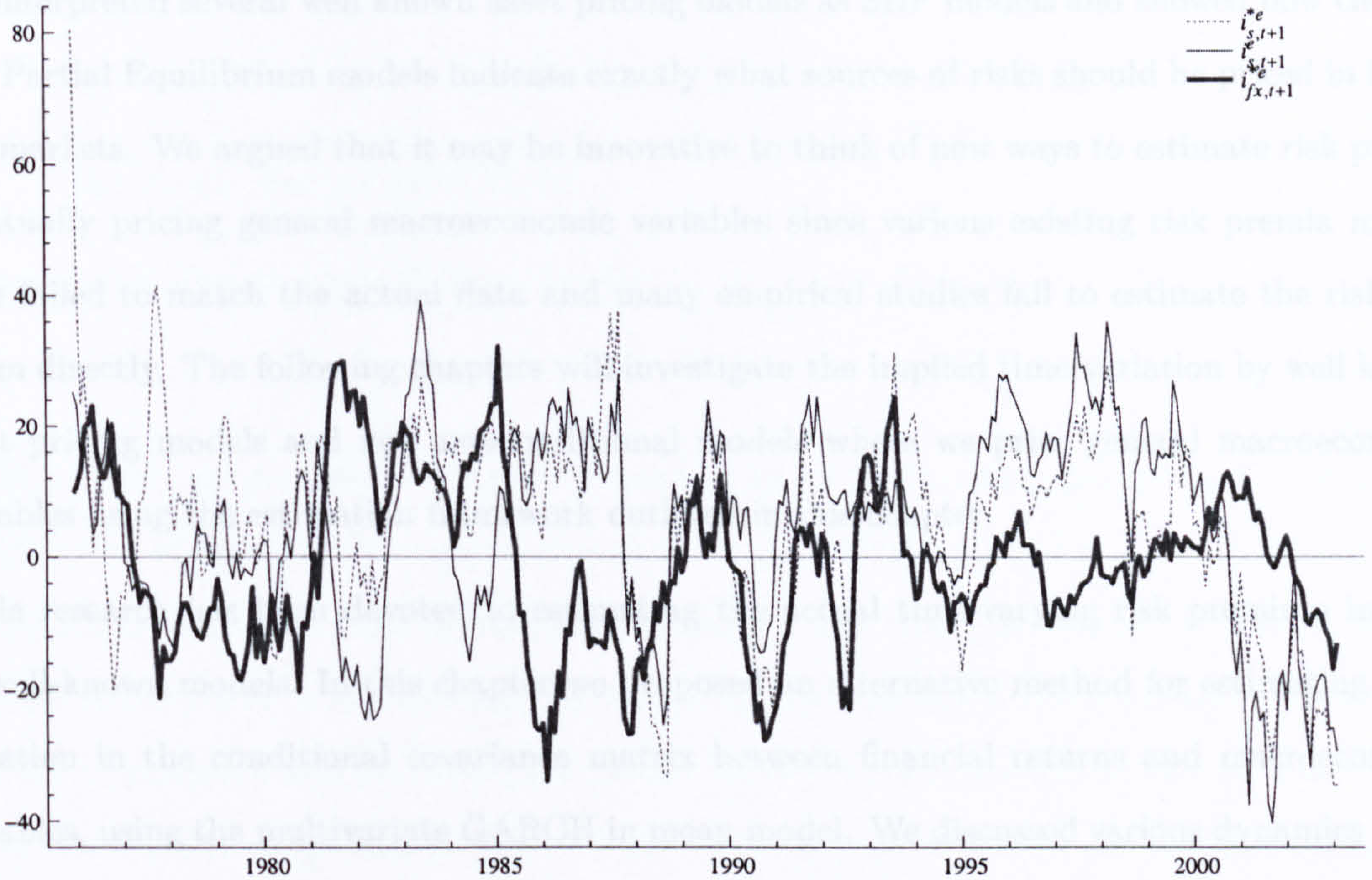
We have chosen to focus on UK and US data since much work has been done on these data. The estimation method requires many data points and we wish to obtain as large a sample as possible to obtain some better properties of the estimated parameters. Macroeconomic variables are usually available on a quarterly or annual basis - this frequency may not be high enough to get reasonable sample sizes to test any of the outlined theories. Moreover, the evidence of ARCH in data with quarterly frequency or lower is, to our knowledge, weak. In the UK and US a number of monthly macroeconomic data series are available and it is of interest whether innovations to these contain risk priced in financial markets

In chapter 4 and 5, considering joint estimation of the risk premium in equity and FOREX markets, we consider the US-UK exchange rate. The US-UK exchange rate is one, among several exchange rates, where the FOREX puzzle has been documented, and it is of interest to see if a solution to the puzzle could be omission of a time-varying risk premium. Including additional countries in the analysis could be done but we will leave it to future analysis.

In figure (1.1) we plot rolling twelve month moving averages of UK and US stock market log excess returns and UK FOREX log excess returns - this plot can be taken as a "crude" measure whether the means of the data are in fact time-varying. The plot is interesting. First we note that all three series appear to have a time-varying mean - hence the topic of the thesis is relevant. Second we note that the moving average of UK and US log excess returns (stock market) have a very strong comovement (correlation of 0.67). In addition it seems to be the case that the movement in the FOREX risk premium follows the risk premia in the stock market, most highly

[10]Hence, adding more variables, rather than improving the model fit, it may "destroy" the dynamics of the covariance matrix.

Figure 1.1: Moving Average of Stock- And FOREX Excess Return



12 month moving average of excess return. Each point in time measures the average log excess return in the previous 12 months. In annual percentages. Star as a superscript denotes that it is return for a UK investor, s as a subscript that it is excess return in the stock market and fx as subscript that it is excess return on FOREX. Note that $i^e_{fx,t+1} = \Delta\ln(S_{t+1}) - i_{f,t} + i^*_{f,t}$, where S is the exchange rate and $i_{f,t}$ is the nominal risk-free interest rate.

correlated with the logarithmic excess return in the UK (correlation is 0.17). It is worth noting that the potential comovement in the mean is higher towards the end of the sample.

We conjecture that covariances of the log excess return with macroeconomic and financial variables account for the time-variation in the means of the data. The aim of this thesis is to investigate whether our conjecture is correct.

1.8 Conclusion, Aims and Contributions

The aim of this introductory chapter was to make a broad introduction to the thesis and the estimation of time-varying risk premia. We argued first that risky financial returns must compensate investors for taking on risk buying such assets - most likely this risk premium is varying over time. We used the SDF model to derive the risk premium, implying that the time-varying risk premium is proportional to the conditional covariance between the risky asset return and the SDF. The logarithmic version of the SDF model will be used throughout the thesis.

We interpreted several well known asset pricing models as SDF models and showed how General and Partial Equilibrium models indicate exactly what sources of risks should be priced in financial markets. We argued that it may be innovative to think of new ways to estimate risk premia eventually pricing general macroeconomic variables since various existing risk premia models have failed to match the actual data and many empirical studies fail to estimate the risk premium directly. The following chapters will investigate the implied time-variation by well known asset pricing models and new non-traditional models where we price general macroeconomic variables using the estimation framework outlined in this chapter.

Little research has been devoted to estimating the actual time-varying risk premium implied by well-known models. In this chapter we proposed an alternative method for estimating time-variation in the conditional covariance matrix between financial returns and macroeconomic variables, using the multivariate GARCH in mean model. We discussed various dynamics of the conditional covariance matrix and proposed a step-wise estimation method for the multivariate GARCH in mean model - estimation of these models are not easy due to the high parameterisation. We argue that the BEKK specification of the conditional covariance matrix is attractive due to its interesting economic interpretation.

In chapter 2 we estimate the time varying risk premium implied by various well-known models of the equity risk premium, chapter 3 discusses a more novel SDF model pricing general macroeconomic variables in the stock market. In that chapter we propose a multivariate model capable of investigating the interaction between business-cycle variability, stock return variability and risk compensation. We argue that such a model need to allow for asymmetric transmission of macroeconomic and return shocks in the conditional covariance matrix. In chapter 4 and 5 we discuss the FOREX SDF model and consider joint estimation of time-varying risk premia in equity and FOREX markets and propose a test whether asset markets are integrated. Estimation of FOREX risk premia involves many variables and we propose an alternative method for estimating FOREX risk premia "easily" implemented in practice. Finally chapter 6 concludes and discusses future directions.

2. The Stock Market SDF Model

Consumption-Based Asset Pricing Models in the Stock Market

In this chapter we propose an alternative method to estimate consumption-based asset pricing models. The advantage of the approach is that we can estimate all of the preference parameters in the utility function of the representative investor and determine the proportion of wealth invested in risky and risk-free assets. We derive approximations to the standard errors of these estimated preference and portfolio parameters. This general representative agent asset pricing model has the advantage that several well known asset pricing models are special cases and we can test whether the more general model fits the data significantly better.

Whilst these deep parameters have been estimated previously by methods such as GMM the proposed estimation method has the advantage that we obtain an estimate of the risk premium implied by the various models and detect whether risk premia vary significantly over time. In the literature there are few studies considering estimation of the time-varying risk premia implied by various General Equilibrium models. In a recent survey of consumption-based asset pricing models (Campbell [21]), allowing for time-varying second moments is not considered. However, many studies recognise that it could be interesting to allow for time-varying second moments (see Attanasio, Banks and Tanner [3] and Attanasio and Vissing-Jørgensen [2]). However, they assume the moments to be constant and leave it for future work whether the assumption is valid or not.

Excess market returns in the UK and US are highly correlated, the correlation is 0.67 in the period 1975-2001. In this chapter we estimate various General and Partial Equilibrium Risk Premium models for the UK and US. From the estimation we can investigate whether expected excess returns, as implied by the various models, or risk premia have such a high correlation as in the actual excess return data. If not, the correlation of expost returns may simply reflect a high correlation of common shocks. The empirical application in the chapter is estimation of risk premia on broad stock market indices in the UK and US for the period 1975-2002. These broad stock market indices have been widely used in the literature (see for instance Fama and French [57] or Campbell [21]) - we discuss the implementation when using other assets as well.

Having estimated the various models we propose a test of whether the implied risk premium generated by the Power Utility Inter-temporal CCAPM and CAPM varies significantly over time. From the test we obtain an estimate of the coefficient of relative risk aversion in the long run and an estimate of the coefficient of relative risk aversion when the covariance between excess return and consumption growth differs from its long run level.

As proposed in the introductory chapter we wish to consider additional variables to which unexpected shocks can eventually affect the conditional covariance matrix between excess returns and the macroeconomic variables whose covariance with return is to determine the level of excess return implied by the various consumption based models - in particular, we consider industrial production growth. This may be an important variable since many studies have attempted to relate asset pricing models to the business cycle. Empirical evidence (Bollerslev, Chou and Kroner [14] and Schwert [98] among others) shows that there is some business cycle variation in stock market volatility - we have the possibility to check this when we model the variables from the most general asset pricing model jointly with industrial production growth. Cochrane [36] makes a bivariate study of GNP growth with consumption growth and stock return with dividend growth and concludes that there are many similarities between the two bivariate models - if this is true there could be a gain in modelling these variables jointly. Our framework allows us to test an alternative asset pricing model pricing industrial production growth in the stock market - significant pricing of this variable serves as a rejection of our most general consumption-based model.

Finally having estimated the models for both the representative UK and US investor we compare the results across the two countries and compare the development of the macroeconomic variables and risk premia across the two countries.

Campbell [21], in a recent survey, concludes that an important questions for students in macroeconomics and finance is:

- Why is the average real stock return so high in relation to the average short term real interest rate ?

This chapter attempts to explain this in the US and UK stock markets and analyses the time-variation in the expected return difference between risky stocks and a risk-free bond, the risk premium - a modified question we attempt to answer becomes:

- Why is the average real stock return so high in relation to the average short term real interest rate and why is the expected stock market return high at some points and low at others ?

Although we do not claim that the framework solves major puzzles in financial economics, the approach adopted will serve as an important benchmark for future analysis and serves as the first estimate of the time-varying risk premium implied by well-known asset-pricing models. If we can detect a significant time-varying risk premium it is important to incorporate this in economic models and optimal portfolio allocation.

The chapter is organised as follows. Section (2.1) discusses the implication of Generalised Isoelastic Preferences and its implication on the risk premium, section (2.2) describes the models we will estimate, section (2.3) discusses the US and UK dataset, section (2.4) discusses the estimation method, section (2.5) and (2.6) presents the results, in section (2.7) we propose a test for time-variation in risk premia implied by asset pricing models, section (2.8) looks at extreme events and section (2.9) concludes.

2.1 Generalised Isoelastic Preferences

Often it has been assumed that economies can be characterised by a representative agent with preferences described by the power utility function - using this preference specification has led to many puzzles in financial economics. One criticism of power utility is that it implies close ties between the coefficient of relative risk aversion and the elasticity of inter-temporal substitution. One has to do with substitution across states of nature and the other to do with substitution over time. Epstein and Zin [51][52] proposed a recursive utility function with no close ties between the two parameters. The utility function is given by

$$\mathcal{U}_t = \left\{(1-\delta)\{\mathcal{C}_t\}^{\frac{1-\gamma}{\theta}} + \delta\left\{\mathrm{E}_t\left[\{\mathcal{U}_{t+1}\}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \; \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{2.1}$$

γ being the coefficient of relative risk aversion, ψ is the elasticity of inter-temporal substitution, $\delta \in (0,1)$ the subjective discount factor and C_t is real consumption. The investor maximises the consumption path for this utility function subject to the budget constraint

$$W_{t+1} = (1+\mathcal{R}_{w,t+1})(W_t - C_t), \tag{2.2}$$

where W_t is the real wealth and $(1+\mathcal{R}_{w,t+1})$ is the real gross return on the wealth portfolio. For the moment we will leave out any considerations on human wealth. One can show (see for instance Campbell, Lo and MacKinlay [29]) that the log consumption wealth ratio is given by

$$c_t - w_t = \mathrm{E}_t\left\{\sum_{j=1}^{\infty}\rho^j(r_{w,t+j} - \Delta c_{t+j})\right\} + \frac{\rho k}{1-\rho} \tag{2.3}$$

$$= \mathrm{E}_t\left\{\sum_{j=1}^{\infty}\rho^j(r^e_{w,t+j} + r_{f,t+j-1} - \Delta c_{t+j})\right\} + \frac{\rho k}{1-\rho}, \tag{2.4}$$

$$\rho \equiv 1 - \exp(\overline{c-w}), \tag{2.5}$$

k being a function of ρ and $\overline{c-w}$ is the average log consumption aggregate wealth ratio. Consumption is relatively high to wealth when we expect, today, higher wealth, higher excess return, higher risk-free interest rates or lower consumption in the future. The logarithm of the consumption to wealth ratio summarises investors expectations about future returns, log excess returns, future real risk-free interest rates and consumption growth. Maximising the utility function (2.1) subject to the budget constraint (2.2) yields the Euler Equation

$$1 = \mathrm{E}_t\left[\left\{\delta\left\{\frac{C_{t+1}}{C_t}\right\}^{-\frac{1}{\psi}}\right\}^{\theta}\left\{\frac{1}{1+\mathcal{R}_{w,t+1}}\right\}^{1-\theta}(1+\mathcal{R}_{i,t+1})\right] \tag{2.6}$$

It appears clearly that Power Utility is a special case of Generalised Isoelastic Preferences when $\theta = 1$ and one version of the traditional CAPM when we assume that the representative agent is myopic, the special when $\gamma = 1$ (or $\theta = 0$). Since the real return on the wealth portfolio is a component of the real Pricing Kernel given by

$$\mathcal{M}_{t+1} = \left\{\delta\left\{\frac{C_{t+1}}{C_t}\right\}^{-\frac{1}{\psi}}\right\}^{\theta}\left\{\frac{1}{1+\mathcal{R}_{w,t+1}}\right\}^{1-\theta}, \text{ or} \tag{2.7}$$

$$\begin{aligned} m_{t+1} &= \theta\ln(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)\ln(1+\mathcal{R}_{w,t+1}) \\ &= \theta\ln(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)(i_{w,t+1} - \pi_{t+1}), \end{aligned} \tag{2.8}$$

with $m \equiv \ln(\mathcal{M})$, there may be some hope that we can generate more variability in the SDF if the return on the wealth portfolio has high variability. The signs on the three variables Δc_{t+1}, $i_{w,t+1}$ and π_{t+1} depend on specific parameter values - the signs can be positive or negative governed by the values of γ and ψ. An advantage of this model is that there is no apriori assumed relation between the preference parameters as in the inter-temporal Power Utility CCAPM model where $\gamma = \frac{1}{\psi}$. It may be that a consumption-based model assuming complete markets is not realistic - however, it gives a good guide when we have to search for factors that need to be priced in financial markets and it is difficult to compete with its sound economic intuition. The mean and the variance of the logarithmic Pricing Kernel are given by

$$\mathrm{E}_t(m_{t+1}) = \theta\ln\delta - \frac{\theta}{\psi}\mathrm{E}_t(\Delta c_{t+1}) - (1-\theta)\mathrm{E}_t(r_{w,t+1}) \tag{2.9}$$

$$\mathrm{V}_t(m_{t+1}) = \frac{\theta^2}{\psi^2}\mathrm{V}_t(\Delta c_{t+1}) + (1-\theta)^2\mathrm{V}_t(r_{w,t+1}) + \frac{2\theta(1-\theta)}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, r_{w,t+1}) \tag{2.10}$$

If the representative investor has Power Utility, the two latter terms on the right hand side in the last equation disappear. Assuming that the logarithmic SDF is normally distributed, the model implied real risk-free rate, when it exists, is given by

$$r_{f,t} = -\ln(\delta) + \frac{1}{\psi}\mathrm{E}_t(\Delta c_{t+1}) - \frac{(1-\theta)}{2}\mathrm{V}_t(r_{w,t+1}) - \frac{\theta}{2\psi^2}\mathrm{V}_t(\Delta c_{t+1}) \tag{2.11}$$

We note in the Power Utility CCAPM model only the consumption variance term enters whereas the variance of the return on the wealth portfolio does not determine the real risk-free rate. If $\theta \neq 1$ the conditional variance of the wealth portfolio could be important for determining the real risk-free rate[1].

2.1.1 The Wealth Portfolio

It is common to assume that $\mathcal{I}_{w,t+1} = \mathcal{I}_{s,t+1}$ ($\mathcal{I}$ now denotes a net simple nominal return), where subscript s throughout refers to a broad national stock index. This lead to the critique of Roll [96] - Roll points out that the return does not include the return on human wealth. Campbell [21] mentions it is likely to be a good proxy in countries where the stock market is large relative to the level of GDP. On the other hand it is not clear why an investor would want the financial wealth portfolio to consist only of domestic equities. There could be many reasons why an investor would want to diversify his or her portfolio abroad or in other domestic assets. Writing the return on the wealth portfolio as a linear combination of several returns on domestic and foreign assets, that is

$$
\begin{aligned}
1+\mathcal{I}_{w,t+1} &= \sum_{j=1}^{k_1}\omega_{j,t}(1+\mathcal{I}_{do,j,t+1}) + \sum_{l=1}^{n}\sum_{i=1}^{k_2}\omega_{il,t}(1+\mathcal{I}_{fo,il,t+1})\frac{\mathcal{S}_{l,t+1}}{\mathcal{S}_{l,t}} \\
1 &= \sum_{j=1}^{k_1}\omega_{j,t} + \sum_{l=1}^{n}\sum_{i=1}^{k_2}\omega_{il,t}, \qquad \text{for all t} \qquad (2.12)
\end{aligned}
$$

k_1 is the number of domestic assets, k_2 the total number of different foreign assets and n is number of foreign countries in which the domestic investor has assets. Subscript (do) refers to a return on a domestic asset and (fo) to a foreign asset return. Returns on foreign assets have to be converted back to national currency and investors face currency risk when investing abroad. $\mathcal{S}$ is the exchange rate, denoting the domestic price of foreign currency. Often it is preferable to work with logarithmic returns when testing an SDF model. It is not exactly the case that the log of the simple gross return on the financial wealth portfolio is equal to the sum of the logs weighted by the portfolio weights ω. However, as the frequency of the data becomes higher the approximation holds better - we will assume it to hold throughout this thesis. We assume

[1] If we estimate this model we can recover δ such that the implied real risk-free rate matches the mean of the expost real risk-free rate and compare the variability of the implied risk-free rate and the expost real rate. However, this is not the aim of the current chapter.

$$
\begin{aligned}
i_{w,t+1} &= \sum_{j=1}^{k_1} \omega_{j,t} i_{do,j,t+1} + \sum_{l=1}^{n}\sum_{i=1}^{k_2} \omega_{il,t}(i_{fo,il,t+1} + \Delta s_{l,t+1}) \\
1 &= \sum_{j=1}^{k_1} \omega_{j,t} + \sum_{l=1}^{n}\sum_{i=1}^{k_2} \omega_{il,t}, \qquad \text{for all t,} \qquad (2.13)
\end{aligned}
$$

where $i \equiv \ln(1+\mathcal{I})$. Portfolio weights have subscript t because they could be time-varying[2]. Hence there may be several sources of financial risk to be priced in the consumption-based model and it is of interest to see whether pricing additional financial risk factors generates additional time-variation in the implied risk premium.

In the current chapter we will consider a simple case where the return on the financial wealth portfolio is equal to the return on an investment in a broad domestic equity index and the return from an investment in a risk-free asset, $i_{w,t+1} = \omega_1 i_{f,t} + \omega_2 i_{s,t+1}$. Throughout, since we will estimate risk premia on broad stock market indices, we use subscript s on returns to denote that it is a return on a broad stock market index. Campbell, Viceira and White [30] use an equivalent assumption. In practice it may also be that a representative investor has bonds (see Attanasio and Vissing-Jørgensen [2]) in the wealth portfolio but we leave that possibility out in this chapter mainly since it would be difficult to estimate such a model due to dimensionality of the number of involved parameters.

2.1.2 The No-Arbitrage Condition In the General Consumption-Based Model

Recall the no-arbitrage condition for the return on a broad stock market index is given by:

$$
1 = \mathrm{E}_t \left\{ \mathcal{M}_{t+1} \frac{1+\mathcal{I}_{s,t+1}}{1+\pi_{t+1}} \right\}, \qquad (2.14)
$$

where $\mathcal{I}$ is the nominal net return. Taking the natural logarithms on both sides yields

[2] However, we assume the weights to be constant since the aim of this chapter is just to illustrate an alternative method for estimating consumption-based asset pricing models and no immediate theory is available as how to model the time-variation in the parameters.

$$
\begin{aligned}
0 &= \mathrm{E}_t(m_{t+1} + i_{s,t+1} - \pi_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1} + i_{s,t+1} - \pi_{t+1}) \\
0 &= \mathrm{E}_t(m_{t+1} + i_{s,t+1} - \pi_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1}) + \frac{1}{2}\mathrm{V}_t(i_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(\pi_{t+1}) \\
&+ \mathrm{Cov}_t(m_{t+1}, i_{s,t+1}) - \mathrm{Cov}_t(m_{t+1}, \pi_{t+1}) - \mathrm{Cov}_t(i_{s,t+1}, \pi_{t+1}) \qquad (2.15)
\end{aligned}
$$

For a risk-free nominal return, which obviously exists, subsequently denoted $i_{f,t}$ we note that

$$
\begin{aligned}
0 &= \mathrm{E}_t(m_{t+1} - \pi_{t+1}) + i_{f,t} + \frac{1}{2}\mathrm{V}_t(m_{t+1}) + \frac{1}{2}\mathrm{V}_t(\pi_{t+1}) \\
&- \mathrm{Cov}_t(m_{t+1}, \pi_{t+1}) \qquad (2.16)
\end{aligned}
$$

Combining these two equations yields the no-arbitrage condition for the excess return when logarithmic nominal returns and the logarithm of the SDF are jointly normally distributed:

$$
\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i_{s,t+1}) = -\mathrm{Cov}_t(m_{t+1}, i_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i_{s,t+1}) \qquad (2.17)
$$

The RHS is the risk premium[3]. Define $i^e_{s,t+1} \equiv i_{s,t+1} - i_{f,t}$ using that $i_{f,t}$ is known at time t

$$
\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) = -\mathrm{Cov}_t(m_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) = \phi_t \qquad (2.18)
$$

Combining equation (2.14) and (2.18) we note that

$$
\frac{\mathcal{M}_{t+1}}{1+\pi_{t+1}} = \frac{1}{1+i_{f,t}+\phi_t} + \epsilon_{t+1}, \qquad \mathrm{E}_t(\epsilon_{t+1}) = 0 \qquad (2.19)
$$

is a potential SDF consistent with joint log normality of the SDF and stock return. Estimating a model for the time-varying risk premium one could, if that is the aim, back out one estimate of the conditional expected implied SDF since we obtain an estimate of the risk premium and

[3]Sometimes it is preferable to consider $\bar{\phi}_t = -\frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) - \mathrm{Cov}_t(m_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1})$. This is the risk premium subtracting the Jensen term. This term is often left out in financial economics when working with logarithmic excess return.

the nominal risk free interest rate is known. We can write the nominal logarithmic stock returns as:

$$i_{s,t+1} = -m_{t+1} + \pi_{t+1} + \epsilon_{t+1} \tag{2.20}$$

Using the logarithmic SDF from the Epstein Zin model (equation 2.8) with our assumption on the financial wealth portfolio yields the no-arbitrage condition

$$\begin{aligned} \mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) &= (1-\theta)\mathrm{Cov}_t(i_{w,t+1} - \pi_{t+1}, i^e_{s,t+1}) \\ &+ \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \\ &= (1-\theta)\left[\omega_2 \mathrm{V}_t(i^e_{s,t+1}) - \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1})\right] \\ &+ \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \end{aligned} \tag{2.21}$$

$\frac{1}{2}\mathrm{V}_t(i^e_{s,t+1})$, on the LHS, is the Jensen correction from working with logarithmic returns. When $\omega_2 = 1$, the traditional version of the Epstein Zin model, the coefficient on the inflation covariance is equal to minus the coefficient on the conditional variance (on the RHS) of excess return plus one. With our specification of the wealth portfolio, inflation becomes a variable that should be priced unrestricted. Financial risk, nominal macroeconomic variables and real macroeconomic variables could all be significantly priced. Further there is an additional term in the risk premium, $\mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1})$, which appears since we use nominal returns and discount these using a nominal SDF.

When modelling the risk premium on any asset other than the market itself the risk premium would instead be proportional to the covariance between the excess return on that asset and the excess return on the market index. Hence we would need to model the risk premium on this asset jointly with the risk premium on the market portfolio and impose cross-equation restrictions in the no-arbitrage condition.

The Power Utility CCAPM is the special case with $\theta = 1$ where the coefficient on the consumption covariance is the coefficient of relative risk aversion - we would expect this coefficient (in the special case) to be positive since an asset that pays of well when consumption growth is high is undesirable - in the Epstein Zin model the sign is ambiguous depending on the magnitudes of the coefficient of relative risk aversion and the coefficient of inter-temporal substitution.

If on the other hand $\theta = 0$ the model can be thought of as a version of the CAPM and the conditional covariance between excess return and consumption growth is not important.

2.1.3 Recovering the Parameters of the Models

We estimate the general consumption-based model including industrial production growth in the multivariate model, with shocks to industrial production growth potentially affecting the conditional covariance matrix. In addition we test an alternative model with the no-arbitrage condition, in its most general form, given by[4]

$$\begin{aligned} \mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) &= \alpha_1 \mathrm{V}_t(i^e_{s,t+1}) + (\alpha_2+1)\mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \\ &+ \alpha_3 \mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \alpha_4 \mathrm{Cov}_t(\Delta y_{t+1}, i^e_{s,t+1}) \end{aligned} \quad (2.22)$$

$y_{t+1} \equiv \ln(Y_{t+1})$, where Y_{t+1} is industrial production. In the consumption-based models estimated we impose the restriction $\alpha_4 = 0$ - evidence for the null hypothesis that α_4 is significantly different from zero serves as a rejection of the consumption-based model. From each consumption-based model we can recover an estimate of the coefficient of relative risk aversion, the elasticity of inter-temporal substitution and the two portfolio weights (if we assume $\omega_1 + \omega_2 = 1$).

Using a Quasi Maximum Likelihood (QML) estimation method we obtain consistent estimates of the four parameters - α_1, α_2, α_3 and α_4. The coefficient of relative risk aversion, elasticity of inter-temporal substitution and the two portfolio weights will be given by

$$\hat{\gamma} = \hat{\alpha}_3 - \hat{\alpha}_2, \quad \hat{\psi} = \frac{1+\hat{\alpha}_2}{\hat{\alpha}_3}, \quad \hat{\omega}_2 = -\frac{\hat{\alpha}_1}{\hat{\alpha}_2}, \quad \hat{\omega}_1 = \frac{\hat{\alpha}_2 + \hat{\alpha}_1}{\hat{\alpha}_2} = 1 - \hat{\omega}_2, \quad (2.23)$$

and θ by

[4]In a later chapter we show that all general macroeconomic variables can be given an interpretation as a conditional CAPM with time-varying mean to variance relationship.

$$\hat{\theta} = 1 - \frac{\hat{\alpha}_1}{\hat{\omega}_2} = 1 + \hat{\alpha}_2 \tag{2.24}$$

The variances of the parameter estimates are given by

$$\begin{aligned}
\text{Var}(\gamma) &= \text{Var}(\alpha_3) + \text{Var}(\alpha_2) - 2\text{Cov}(\alpha_3, \alpha_2) && (2.25)\\
\text{Var}(\psi) &= \text{Var}\left\{\frac{1+\alpha_2}{\alpha_3}\right\} \\
&\cong \frac{1}{\hat{\alpha}_3^2}\text{Var}(\alpha_2) + \frac{(1+\hat{\alpha}_2)^2}{\hat{\alpha}_3^4}\text{Var}(\alpha_3) - 2\frac{(1+\hat{\alpha}_2)}{\hat{\alpha}_3^3}\text{Cov}(\alpha_3, \alpha_2) && (2.26)\\
\text{Var}(\omega_2) &\cong \frac{1}{\hat{\alpha}_2^2}\text{Var}(\alpha_1) + \frac{\hat{\alpha}_1^2}{\hat{\alpha}_2^4}\text{Var}(\alpha_2) - 2\frac{\hat{\alpha}_1}{\hat{\alpha}_2^3}\text{Cov}(\alpha_1, \alpha_2) && (2.27)\\
\text{Var}(\omega_1) &= \text{Var}(\omega_2) && (2.28)\\
\text{Var}(\theta) &= \text{Var}(\alpha_2) && (2.29)\\
& && (2.30)
\end{aligned}$$

The variance of the estimated coefficient of relative risk aversion and α_2, α_3 is exact, whereas for ψ and ω_2 it is only an approximation obtained taking the variance of a first order Taylor expansion of the ratio (in the case of ψ) $\frac{1+\alpha_2}{\alpha_3}$. The approximation around the maximum likelihood estimates, $(1+\hat{\alpha}_2, \hat{\alpha}_3)$, yields

$$\frac{1+\alpha_2}{\alpha_3} \cong \frac{1+\hat{\alpha}_2}{\hat{\alpha}_3} + \frac{1}{\hat{\alpha}_3}(\alpha_2 - \hat{\alpha}_2) - \frac{1+\hat{\alpha}_2}{\hat{\alpha}_3^2}(\alpha_3 - \hat{\alpha}_3), \tag{2.31}$$

equation (2.26) follows trivially from taking the variance of equation (2.31). It is not known how well a first order Taylor expansion works - it is conjectured that it works well and will be used subsequently.

In the following, eight models of the risk premium on a broad stock market index will be estimated - the aim is to see which fits the data best and which models generate time-variation, if any, in the expectation of excess return in the equity market. It is an important advantage of our estimation method that we can recover all parameters. This is not the case, for example, in Attanasio, Banks and Tanner [3]. The different models allow us to investigate whether financial risk, nominal macroeconomic risk and/or real macroeconomic risk contribute significantly to the stock market risk premium.

2.2 Special Cases of the No-Arbitrage Condition - Model Description

From equation (2.22) we note that several well known models appear as special cases. We briefly discuss the eight models that will be estimated in this chapter.

- The General Model

The most general model prices both the excess return in the stock market, inflation, consumption growth and industrial production growth. It is not theoretically justified but it can be used to check whether industrial production is a significant risk factor to be priced when we have priced the risk by the most general consumption-based model. If so, it serves as a rejection of the consumption model.

- The SDF Model

The SDF model prices macroeconomic variables only, that is $\alpha_1 = 0$ in equation (2.22). The model can be seen as a test whether industrial production growth contains significantly sources of risk to be priced after having priced the two macro variables, inflation and real consumption growth.

- The EZ2 Model

It is the commonly used version of the Epstein Zin model assuming that the return on the wealth portfolio is equivalent to the return a broad stock market index, that is $\omega_2 = 0$. In this model $\alpha_4 = 0$ and $\alpha_2 = -\alpha_1$.

- The PU-Nom-H0 Model

The power utility model, using nominal returns, is a widely used asset pricing model where the risk premium is proportional to the conditional covariance between stock market excess return and consumption growth - in this model $\alpha_1 = \alpha_2 = \alpha_4 = 0$ implying an inverse relationship between the coefficient of relative risk aversion and the elasticity of inter-temporal substitution.

- The PU-Real-H0 Model

If we were to use real returns then the correction to the risk premium, equal to one unit of the conditional covariance between inflation and excess return, would not be present - we assume that a real risk-free rate exists. The restrictions on the no-arbitrage condition, equation (2.22), is that $\alpha_1 = \alpha_4 = 0$ and $\alpha_2 = -1$.

- The PU-Nom-H1 Model

In this model $\alpha_1 = \alpha_4 = 0$. In addition to pricing consumption growth we price inflation. The finding of significant pricing of inflation after taking account for consumption risk would serve as a rejection of the Power Utility CCAPM. Often it has been shown, Campbell [21], that the implied coefficient of relative risk aversion implied by the Power Utility Inter-temporal CCAPM model is implausibly high. We noted that the coefficient of relative risk aversion is given by $\alpha_3 - \alpha_2$. Hence this model allows for the possibility that investors are more or less risk averse than implied by the Power Utility model. The coefficient of relative risk aversion is increasing the more negative is α_2. It is the general EZ1 model forcing the representative investor to invest all wealth in the risk-free asset and we would expect $\alpha_2 - \alpha_3$ to be large if we want to interpret the results in a representative investor framework. We note that this model is similar to the model of Brandt and Wang, see equation (1.47).

- The EZ1 Model

The PU-Nom-H1 model above can be seen as a special case of this Epstein Zin model, EZ1. In this model we allow for the possibility that some or all wealth comes from the return on a risk-free asset. If all wealth is invested in a risk-free asset we would expect the representative agent to be very risk averse and expect high estimates of the coefficient of relative risk aversion as mentioned above. The model implied portfolio weight on the risky asset is given by $\omega_2 = \frac{-\alpha_1}{\alpha_2}$. If α_1 is positive then the more negative[5] is the coefficient α_2 the less is invested in the risky stock and the more is invested in the risk-free asset. In the extreme case when α_2 is infinite investment in the risky asset is zero.

- The CAPM

The final model we estimate is a version of the CAPM commonly used in the literature to model risk premia on broad stock market indices. It assumes that the risk premium on the broad stock

[5] It makes sense that $\alpha_2 < 0$ since inflation tends to be high during recessions - an asset that pays off during recessions is desirable.

market index is proportional to the conditional variance of the return on that index. For our setup this model implies $\alpha_1 = -\alpha_2$ and $\alpha_3 = \alpha_4 = 0$. It is of interest to see whether any of our other models outperform this commonly used version of the CAPM on market portfolios. One test of the model was by Ng [91] including many different portfolios and a Constant Conditional Correlation multivariate GARCH-in-mean model and Bekaert and Wu [10] using Japanese data.

From the general consumption-based model, EZ1, there appears to be two special cases where the conditional variance of excess return is unimportant for determining risk premia, either $\theta = 1$ or all wealth is invested in the domestic risk-free asset, that is $\omega_2 = 0$. One may want to restrict the portfolio weights to be in the interval between 0 and 1. This can be done and amounts to the restriction

$$0 \leq -\frac{\alpha_1}{\alpha_2} \leq 1 \tag{2.32}$$

We do not impose this during estimation since it is of interest to obtain the most generality in the risk premium as possible. It may be that the consumption-based model is not correct but it has provided us with a very suitable theory for choosing factors (Cochrane [37]) - it may be questionable whether we should put much interpretation on the estimated preference parameters. All estimated models are essentially SDF models and markets may not be complete in which case the SDF is not unique and contains a residual, ζ_{t+1}. The logarithmic SDF will be given by

$$m_{t+1} = -\alpha_0 - \mathbf{a}^{\intercal}\mathbf{f}_{t+1} + \zeta_{t+1} \tag{2.33}$$

where the vector of factors are stock market return, inflation, consumption growth and industrial production growth.

Using either of the classical tests (Wald, LR or LM) we can evaluate whether pricing additional variables gives a significantly better fit to the data. We estimate all models and use the Likelihood Ratio (LR) test since it is easy to implement. The LR test is asymptotically $\chi^2(\xi)$ distributed where ξ is the number of restrictions. In many cases it suffices to use simple standard t-tests to test between different models. Presumably the most important test, whether we can reject the general consumption-based model, is the null hypothesis:

$$H_0 \quad : \quad \alpha_4 = 0, \tag{2.34}$$

in the general model. A rejection of the null tells us that the most general consumption-based model does not adequately capture stock market risk[6].

2.3 Data

We consider two samples - UK and US samples covering the period 1975-2002. We include the descriptive statistics of these datasets in table (2.14) in the appendix. The frequency of the data is monthly. The US data are log excess return - log return on a broad market index above the risk-free rate, as used in Fama and French [55]. Inflation is seasonally adjusted log Consumer Price Index (CPI) changes obtained from Datastream, seasonally adjusted real nondurable expenditure and seasonally adjusted industrial production are obtained from the Federal Reserve Bank of St. Louis[7]. In the UK we use the MSCI composite stock market index, the one month euro sterling risk-free rate, non seasonally adjusted Retail Price Index (RPI) and seasonally adjusted real industrial production growth - all obtained from Datastream. We use real nondurable consumption data, obtained from NIESR, as the measure of consumption[8]. The correct price deflator, according to the theory, would be nondurable CPI since nondurable consumption data are used - however, nondurable Consumer Price Indices are available with only one digit - this gives the problem, especially in the beginning of the sample, when the index is low, that the index remains constant and one spuriously computes inflation to equal zero. On the other hand the CPI and RPI series are available with two digits and allows us to compute more precise inflation rates - one would think that the two price deflators are highly correlated and not much is lost by using this measure. Another justification for doing so is that the definition of durable vs non-durable goods is rather arbitrary since most non-durable goods have some durability.

[6]It should be noted that the Classical tests are not valid if we assume a wrong joint distribution of the data but we assume throughout that this is not the case.

[7]From http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html the Fama and French data can be downloaded and the data from the Federal Reserve Bank of St. Louis can be found at http://research.stlouisfed.org/fred/.

[8]The construction of these data can be found in Salazar and Weale [97]

We choose monthly data since it is the highest frequency with which we can obtain macro data and we start in 1975 because we do not have real nondurable consumption data available for the UK before 1975 - we wish to compare the risk premium in the UK and US and start also in 1975 using the US dataset.

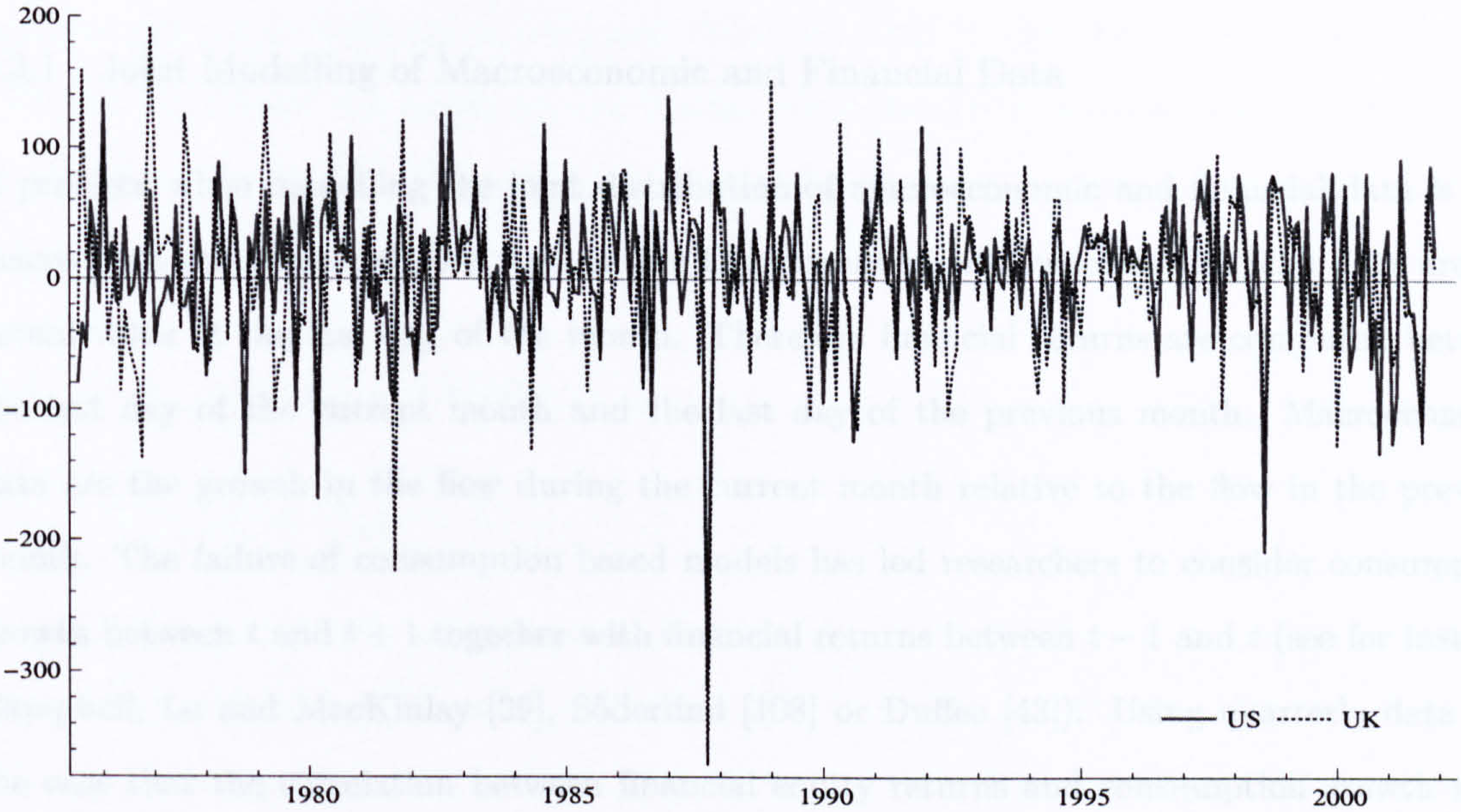
We denote UK data with a star as a superscript. We include also FOREX excess return, the US-UK risk-free interest rate differential and narrow money growth in the table of descriptive statistics - these variables will be used in a later chapter and will not be commented on until then. Here we comment only on the data from 1975 to 2002, for both countries, that are of interest in this chapter.

The average simple excess return in the US is 7.50 % and 6.84 % in the UK, with higher stock market variability in the UK. Skewness is about the same in the two countries (approx -1) and the excess kurtosis in the log excess return series is 3.98 and 4.58 respectively. The pair of US and UK macro data share many of the same properties - however, UK industrial production growth has been low and has almost twice as high a standard deviation as US industrial production. Moreover UK industrial production growth has much more excess kurtosis than the US data. We reject normality for all data except US real nondurable consumption growth. US inflation has higher correlation in the level and squares than the UK inflation rate and US industrial production growth has more correlation in the level than the corresponding UK variable.

In figure (2.10), (2.11) and (2.12) in the appendix we plot pairs of UK and US macroeconomic variables and the characteristics from the tables of descriptive statistics are apparent. In table (2.15) we tabulate the correlation matrix between UK and US variables[9]. The correlation between UK and US macro variables is highest for inflation, 0.45, and lowest for consumption, 0.12. The two industrial production growth series have a correlation of 0.23. US log excess return has a higher correlation with the US macro variables than does UK log excess return with UK macro variables. The correlation between consumption growth and log excess return and consumption growth is 0.15 in the US and 0.07 in the UK. It is curious that UK log excess return has twice as high a correlation with US consumption growth as with UK consumption growth. The highest correlation in the matrix is between the two log excess return series - the correlation is 0.62. We plot the two excess return series in figure (2.1)

[9] Again we include US FOREX excess return, the interest rate differential and narrow money growth for both countries.

Figure 2.1: US And UK Stock Market Excess Return



Logarithmic excess return in UK and US stock markets. In annual percentages.

Large declines in excess return usually occur simultaneously in both countries and negative excess returns tend to be higher in absolute value than positive excess return - one exception being the UK in 1976/1977. In common with much of the literature we treat the two markets as separable and estimate the US and UK risk premium individually (the US premium allowing only US variables in the information set and the UK premium allowing only UK variables in the information set). Modelling the risk premia independently it is of interest whether the implied expectation of log excess return, the risk premium, in the two countries is highly correlated - it may not be the case since excess return contains a predictable and unpredictable component as discussed in the introduction.

The largest negative excess return was during the stock market crash in October 1987. Due to the fact that the stock market crash in 1987 seems to be a relative extreme outlier we include a dummy variable in the estimations. This may have the advantage that some of the negative skewness in excess return may be removed. In the US it seems that August 1998 is a relatively large negative excess return and so is September 1981 in the UK. Many of the variables have some excess kurtosis and we estimate the models using the multivariate t-distribution which allows for more heavy tails in the joint distribution.

Finally we note from the plot of the macro variables in the appendix, that the variability of the macroeconomic data seems to be lower towards the end of the sample.

2.3.1 Joint Modelling of Macroeconomic and Financial Data

A problem when modelling the joint distribution of macroeconomic and financial data is that macro data are measured as a flow during the current month, whereas financial data are the actual value at the last day of the month. Therefore financial returns are computed between the last day of the current month and the last day of the previous month. Macroeconomic data are the growth in the flow during the current month relative to the flow in the previous month. The failure of consumption based models has led researchers to consider consumption growth between t and $t+1$ together with financial returns between $t-1$ and t (see for instance Campbell, Lo and MacKinlay [29], Söderlind [108] or Duffee [43]). Using quarterly data it is the case that the correlation between financial equity returns and consumption growth using that time convention is higher. For the current monthly dataset the correlations between the macroeconomic variables and excess return are tabulated in table (2.1).

Table 2.1: Correlation Between Stock Excess Return And Macroeconomic Variables

	UK			US		
	π	Δc	Δy	π	Δc	Δy
$\rho(i^e_{s,t+1}, x_t)$	-0.032	-0.092	0.009	-0.081	-0.017	-0.007
$\rho(i^e_{s,t+1}, x_{t+1})$	-0.004	0.070	0.003	-0.125	0.157	-0.079
$\rho(i^e_{s,t+1}, x_{t+2})$	-0.032	0.013	0.086	-0.050	0.065	0.044

The correlation, ρ, between log excess return and lead and lag of macroeconomic variables in UK and US. x refers to π, Δy or Δc.

The strongest US correlation in absolute value between $i^e_{s,t}$ and Δc_t or π_t is with the conventional timing of the macro data and hence we find no reason for leading or lagging the macro variables.

Testing consumption-based models one has to make a choice on the use of consumption data. As mentioned in Lettau and Ludvigson [79] much empirical work has used real expenditure on nondurable consumption and services. This can be justified since the consumption-based theories applies to a flow of consumption and therefore durable consumption should not be included - it represents a replacement and addition to the consumers stock and is therefore not a flow from existing stock. Non-durables and services is only a component of consumption and may be an imperfect measure of consumption. However, if we assume that it is a constant proportion of consumption and that true consumption is unobservable we can use nondurable and service expenditures as proxies for consumption. One may expect that there would be many similarities between real non-durable expenditure and real retail sales. In the following it is chosen to work with real nondurable consumption, the reason being that these are available for both the US and the UK. In many other countries real nondurable data are not available but real retail sales are and can be used as a proxy for real non-durables.

2.4 The Estimation Method

2.4.1 Alternative Ways to Estimate ψ

It is of interest to briefly consider how the elasticity of inter-temporal substitution has been estimated traditionally. Hansen and Singleton [71], Campbell and Mankiw [25] and others estimate the elasticity of inter-temporal substitution using an Instrumental Variable (IV) approach. They use the log Euler condition from the optimisation problem assuming homoskedasticity and that asset returns and consumption growth are conditional log-normally distributed - this assumption may be problematic especially for excess return as will be evident shortly. Under these assumptions, one gets

$$r_{i,t+1} = \mu_i + \frac{1}{\psi}\Delta c_{t+1} + \eta_{i,t+1}, \tag{2.35}$$

where $\eta_{i,t+1} = (r_{i,t+1} - \mathrm{E}_t(r_{i,t+1})) - \gamma(\Delta c_{t+1} - \mathrm{E}_t(\Delta c_{t+1}))$. Campbell [21] estimates $\frac{1}{\psi}$ for several countries in the post World War II period and the broad conclusion is that $\frac{1}{\psi}$ is imprecisely estimated; sometimes large and positive but often negative though in most cases never significantly different from zero. One of the problems with IV estimation is that the instruments are only weakly correlated with the regressor because consumption growth is hard to forecast and hence we may not want to rely on asymptotic theory. Reversing the regression as

$$\Delta c_{t+1} = \tau_i + \gamma r_{i,t+1} + \zeta_{i,t+1}, \tag{2.36}$$

and obtain estimates of γ the conclusions from this regression are more or less the same. The estimation method proposed in this chapter provides a better econometric framework with joint estimates of both ψ and γ from the same model allowing for Epstein Zin preferences of the representative investor. Another problem with the above regressions is that homoskedasticity is assumed. If, as is likely, that second order moments are time-varying then the intercept in the above regressions would be varying over time and hence could introduce a bias in the estimates of the elasticity of inter-temporal substitution. Attanasio and Vissing-Jørgensen [2] use a similar method for estimating preference parameters in the Epstein Zin model and acknowledge the possibility of time-varying intercept could be an interesting extension. We assume multivariate heteroskedasticity in our system of equations.

2.4.2 Estimating the Parameters using the Multivariate GARCH in Mean Model

In this section we propose an alternative econometric framework to estimate the risk premium implied by Partial and General Equilibrium models which allows us to estimate various preference parameters without the implausible assumption of a constant conditional covariance matrix. Recall from section (1.4) that we need to specify three sets to estimate the SDF model using the multivariate GARCH in mean model. The set, $\mathcal{Z}_1$, consists of the excess return on the market. The set, $\mathcal{Z}_2$, is the set of additional factors used to proxy the SDF. The return on the market is included in $\mathcal{Z}_1$ and is therefore not in $\mathcal{Z}_2$. In $\mathcal{Z}_2$ real non-durable growth and inflation (CPI or RPI) are included (depending obviously on the model under consideration). We estimate the consumption-based model joint with industrial production growth with the hope to get a more precise estimate of the conditional covariance matrix and get a better estimate of the unexpected component of inflation and consumption growth. Moreover, we test whether industrial production growth is an alternative additional variable to be priced in the UK and US stock market when first we have accounted for the risk implied by the consumption-based model - that is we can test whether industrial production growth belongs to $\mathcal{Z}_2$, $\mathcal{Z}_3$ or none of the two - if not, it is an irrelevant variable for testing the asset pricing model. The conditional covariance matrix is specified, in error correcting form

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}^{\intercal} + \mathbf{D}(\mathbf{H}_t - \mathbf{C}\mathbf{C}^{\intercal})\mathbf{D}^{\intercal} + \mathbf{E}\,(\epsilon_t\epsilon_t^{\intercal} - \mathbf{C}\mathbf{C}^{\intercal})\mathbf{E}^{\intercal} \tag{2.37}$$

where $\mathbf{D}$ and $\mathbf{E}$ are assumed to be symmetric. We assume, without lag of generality, that $\mathbf{C}$ is lower triangular. This model was written out specifically for the bivariate case in section (1.6). A sufficient condition for covariance stationarity of the conditional covariance matrix which can easily be imposed during estimation, is that the absolute value of the eigenvalues of

$$(\mathbf{D} \otimes \mathbf{D}) + (\mathbf{E} \otimes \mathbf{E}) \tag{2.38}$$

all be less than 1 in absolute value. The specification of the conditional mean of the dependent variables is given by

$$\mathbf{Y}_{t+1} = \mathbf{A} + \mathbf{B}\mathbf{Y}_t + \mathbf{\Phi}\mathbf{H}_{[1:4,1],t+1} + \Theta_1 \Upsilon_{1987:10,t+1} + \epsilon_{t+1}, \tag{2.39}$$

For both setups the no-arbitrage condition states that the first element in $\mathbf{A}$ is zero, the first row in $\mathbf{B}$ is only zeros and all rows, except the first, in $\mathbf{\Phi}$ contain zeros. $\mathbf{H}_{[1:4,1]}$ is the first column in the conditional covariance matrix. We decide to use a vector auto regression of order 1 and conjecture that this is sufficient to remove eventual residual correlation. For further description of this specification see section (1.4). $\Upsilon_{1987:10}$ is an indicator function in the excess return equation taking the value of one in October 1987 and zero otherwise. Introducing this dummy variable may have the effect that some of the skewness and excess kurtosis in the logarithmic return will be removed. We argue that the stock crash in October 1987 was extreme (see Schwert [100]) and may cause spurious estimates of the risk premium. We will estimate all models using the multivariate t distribution[10].

We estimate the risk premium in the UK and US in the period 1975-2002 and compare the implied conditional expectation of log excess return in the two countries. Since the sample size will be relatively short for multivariate GARCH estimation we have made the restrictive assumption that $\mathbf{D}$ and $\mathbf{E}$ are symmetric and conjecture that it is not too restrictive. We note that the Diagonal BEKK (as discussed in the previous chapter) appears as a special case and we allow for time-varying correlations. Evidence will be shown that the conditional correlations are indeed not constant and the off-diagonal elements in the ARCH and GARCH matrices are

[10]We also carried out estimation using the normal distribution and found the dynamics in the conditional covariance matrix to be slightly different - however, since the t distribution allows for potential higher kurtosis as evidenced in the descriptive data we report only results using that distribution.

significant.

2.4.3 The Mean Equation of Macroeconomic Variables and the Risk Premium

The vector of dependent variables will be given by $\mathbf{Y}_{t+1} = \{i^e_{s,t+1}, \pi_{t+1}, \Delta c_{t+1}, \Delta y_{t+1}\}^\intercal$ - hence $N = 4$. The conditional expectation of the dependent variables is given by

$$
\begin{aligned}
\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) &= \alpha_1 \mathrm{V}_t(i^e_{s,t+1}) + (\alpha_2 + 1)\mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \\
&+ \alpha_3 \mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \alpha_4 \mathrm{Cov}_t(\Delta y_{t+1}, i^e_{s,t+1}) \\
&+ \theta_1 \Upsilon_{1987:10,t+1} \\
\mathrm{E}_t(\pi_{t+1}) &= a_2 + b_{21} i^e_{s,t} + b_{22}\pi_t + b_{23}\Delta c_t + b_{24}\Delta y_t \\
\mathrm{E}_t(\Delta c_{t+1}) &= a_3 + b_{31} i^e_{s,t} + b_{32}\pi_t + b_{33}\Delta c_t + b_{34}\Delta y_t \\
\mathrm{E}_t(\Delta y_{t+1}) &= a_4 + b_{41} i^e_{s,t} + b_{42}\pi_t + b_{43}\Delta c_t + b_{44}\Delta y_t
\end{aligned}
$$

In all the consumption-based models $\alpha_4 = 0$ and in the General model the parameter is unrestricted. Restrictions on the parameters in the excess return equation depend on the model under consideration (recall section (2.2)).

2.5 Results

The main aim of this chapter is the modelling of the risk premium on broad UK and US stock market indices and it is of interest, to inspect each of the implied risk premia before discussing the estimates. Therefore we will first discuss the model implied risk premia, then consider the coefficient estimates on the conditional variances and covariances in mean and relate our results to the equity premium puzzle. Subsequently we will discuss other estimated parameters, possible non-stationarity of the conditional covariance matrix and consider the impact of modelling the variables jointly with output growth. Finally, we will consider the model implied conditional correlations between the macroeconomic variables and the excess return.

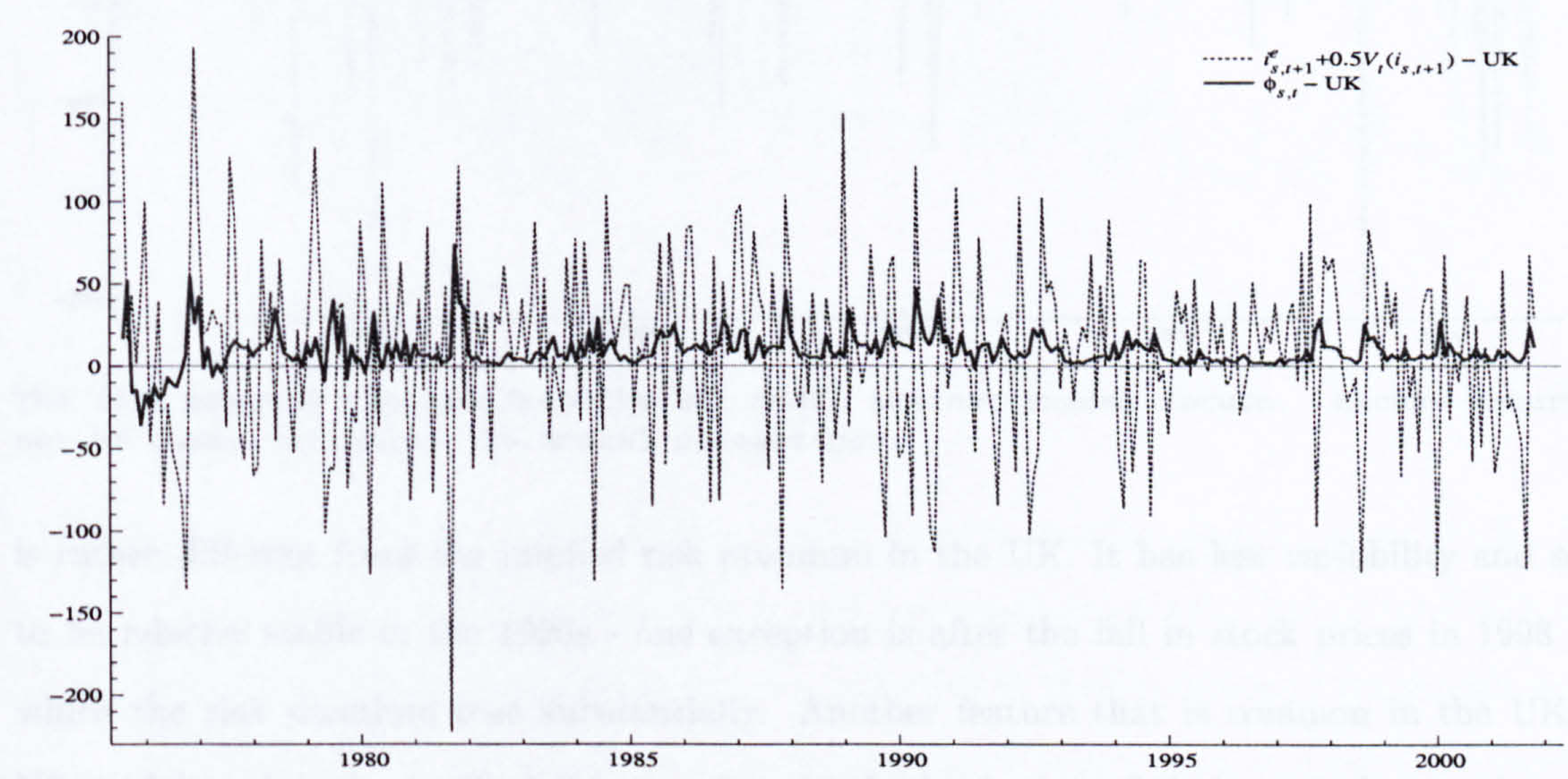
2.5.1 The Equity Risk Premium

It is useful to take a look at the model implied risk premia. Since presumably investors are risk averse or neutral we would expect a non-negative risk premium at any point in time. However, the risk premium has variation and therefore the estimated risk premium can be negative but should in principle not be statistically less than zero[11]. As discussed in the introductory chapter it is of interest to see how much of the variation in the excess returns is due to variation in the risk premium and how much is due to variability in the noise component. First we look at the risk premiums implied by the most general Epstein Zin model, EZ1. We do not plot the risk premium from the General model since it is, for both countries, indistinguishable from that implied by the EZ1 model. All plots of the risk premium are annualised, that is multiplied by 1200.

The EZ1 Model - UK and US

In figure (2.2) we plot the UK risk premium implied by EZ1 model against the excess return. The implied risk premium is positive over almost the entire sample with an exception in the

Figure 2.2: The EZ1 UK Stock Market Risk Premium



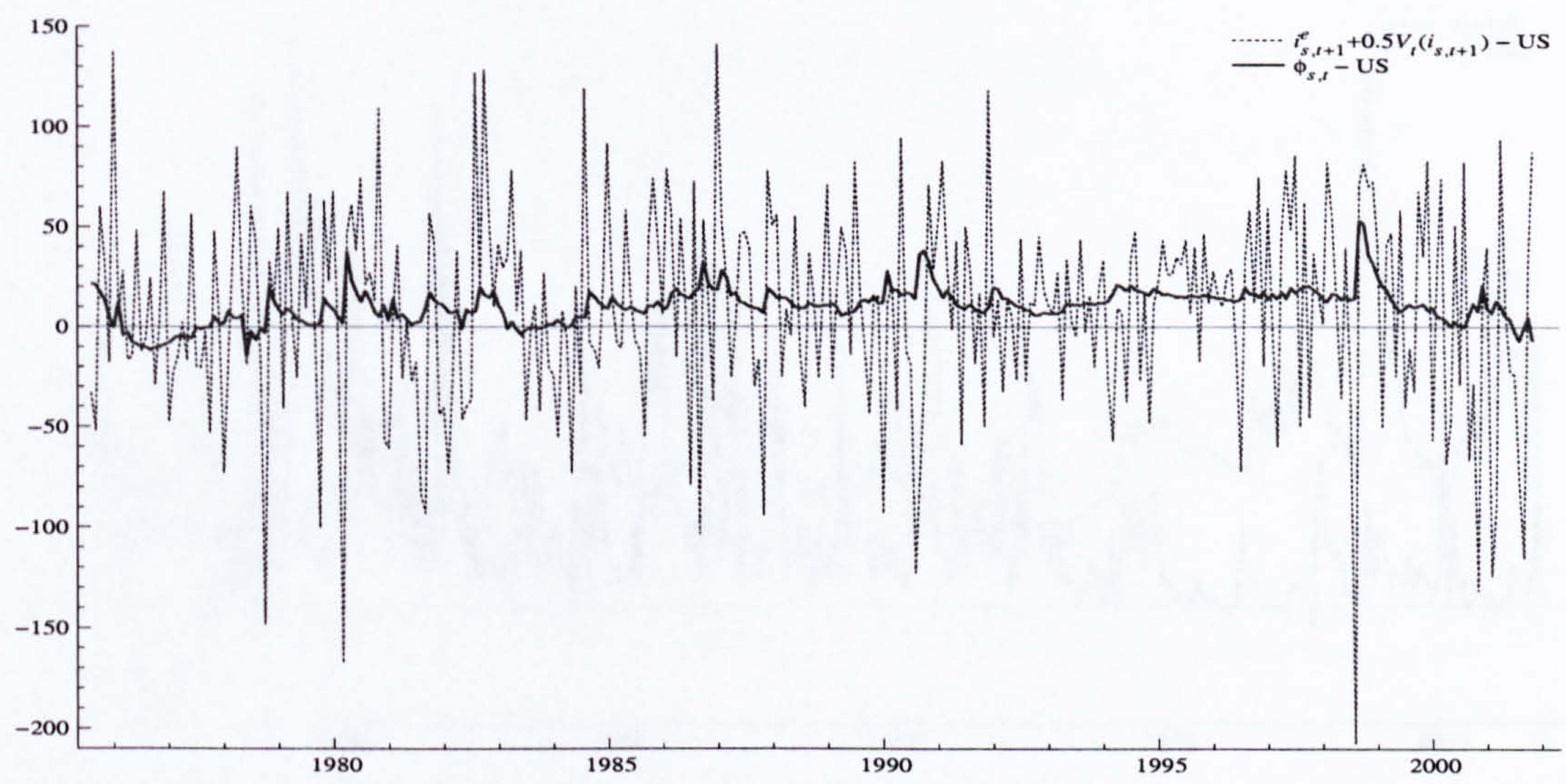
The risk premium implied from the EZ1 model against excess return. Excess return net of the dummy variable. In annual percentages.

[11]That is, it can take a negative value of 1.96 $\sigma(\phi_t)$ and still be insignificantly different from zero using a 95 % critical value.

beginning of the sample. One possible reason could be that in the period we start the estimation is a period with much macroeconomic variability following the oil price shocks in the mid 1970s. However, it is only a very short period over which it is negative. The UK risk premium was primarily variable in the beginning of the sample, a period with much macroeconomic variability (especially inflation), the variability tends to fall over the period up to 1983, stabilising at a relatively high level toward the end of the 1980s. After 1995 the model implied risk premium in the UK seems small with two increases following the price fall in the stock market in August 1998.

Next we consider the equivalent risk premium implied by the same consumption-based model in the US in figure (2.3). The implied risk premium from the general Epstein Zin model, EZ1,

Figure 2.3: The EZ1 US Stock Market Risk Premium



The risk premium implied from the EZ1 model against excess return. Excess return net of dummy variable. In annual percentages.

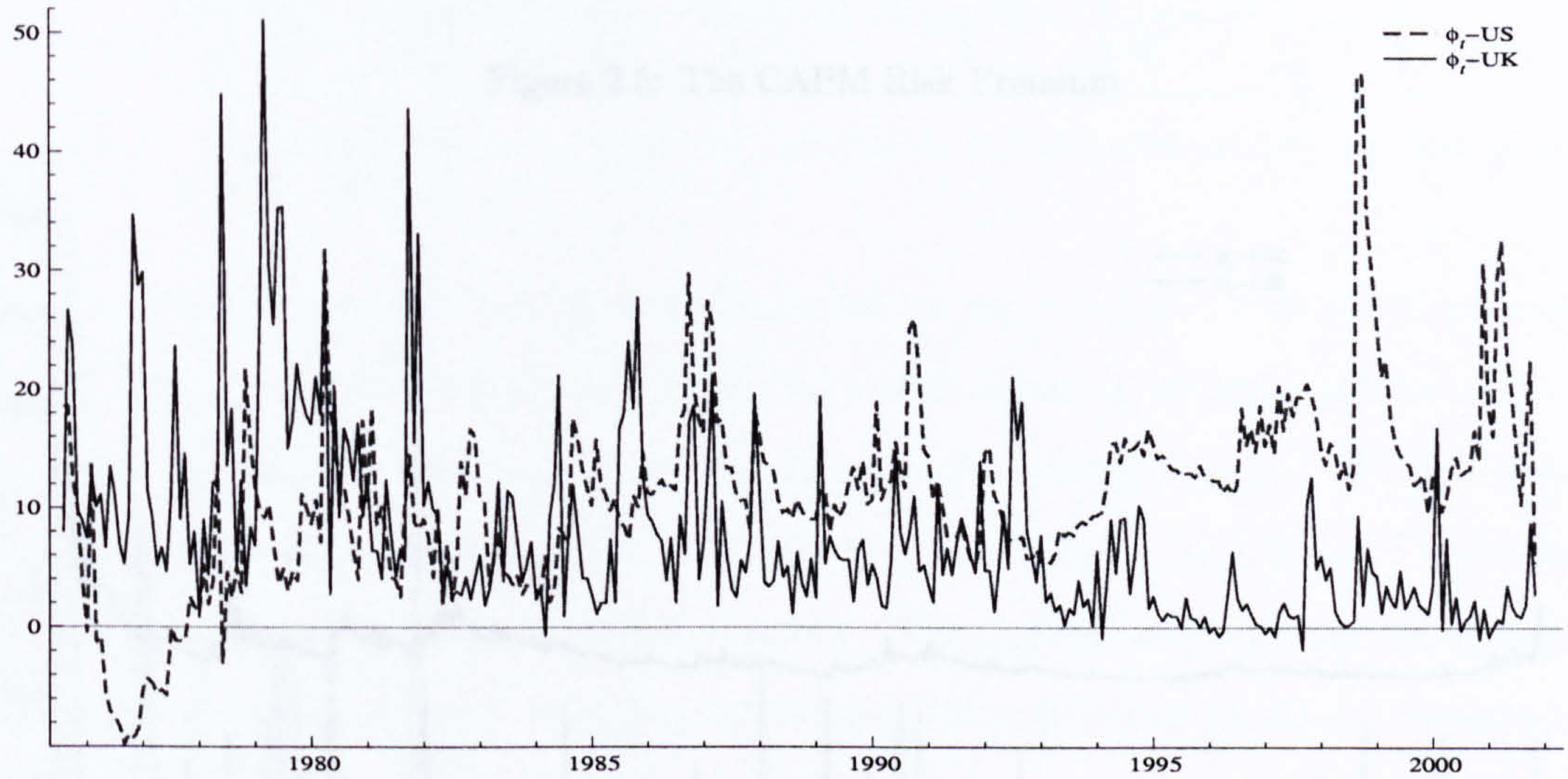
is rather different from the implied risk premium in the UK. It has less variability and seems to be relative stable in the 1990s - one exception is after the fall in stock prices in 1998 after which the risk premium rose substantially. Another feature that is common in the UK and US model is that the implied risk premium in the beginning of the sample is negative. The correlation between the UK and US risk premium implied by the general Epstein Zin model is 0.20 which is relatively low compared to the correlation of log excess return of 0.62. From the EZ1 model we have obtained a consistent estimate of the risk premium in the UK and US.

In general it is evident that the stock market risk premium increases following negative excess returns. It is curious that the implied risk premium in the US is relatively high in the 1990s. This is inconsistent with the common belief in the press and recent studies that stock prices were high in the 1990s (the "bubble") indicating low risk premia.

The Power Utility Inter-temporal CCAPM - UK and US

Next it is of interest to look at the implied risk premium from the traditionally used asset pricing model where the risk premium is proportional to the conditional covariance between excess return and consumption growth. In figure (2.4) we plot the US and UK risk premium implied by the Power Utility (PU) inter-temporal CCAPM.

Figure 2.4: The Power Utility CCAPM Risk Premium



The UK and US risk premium from the inter-temporal Power Utility CCAPM. In annual percentages.

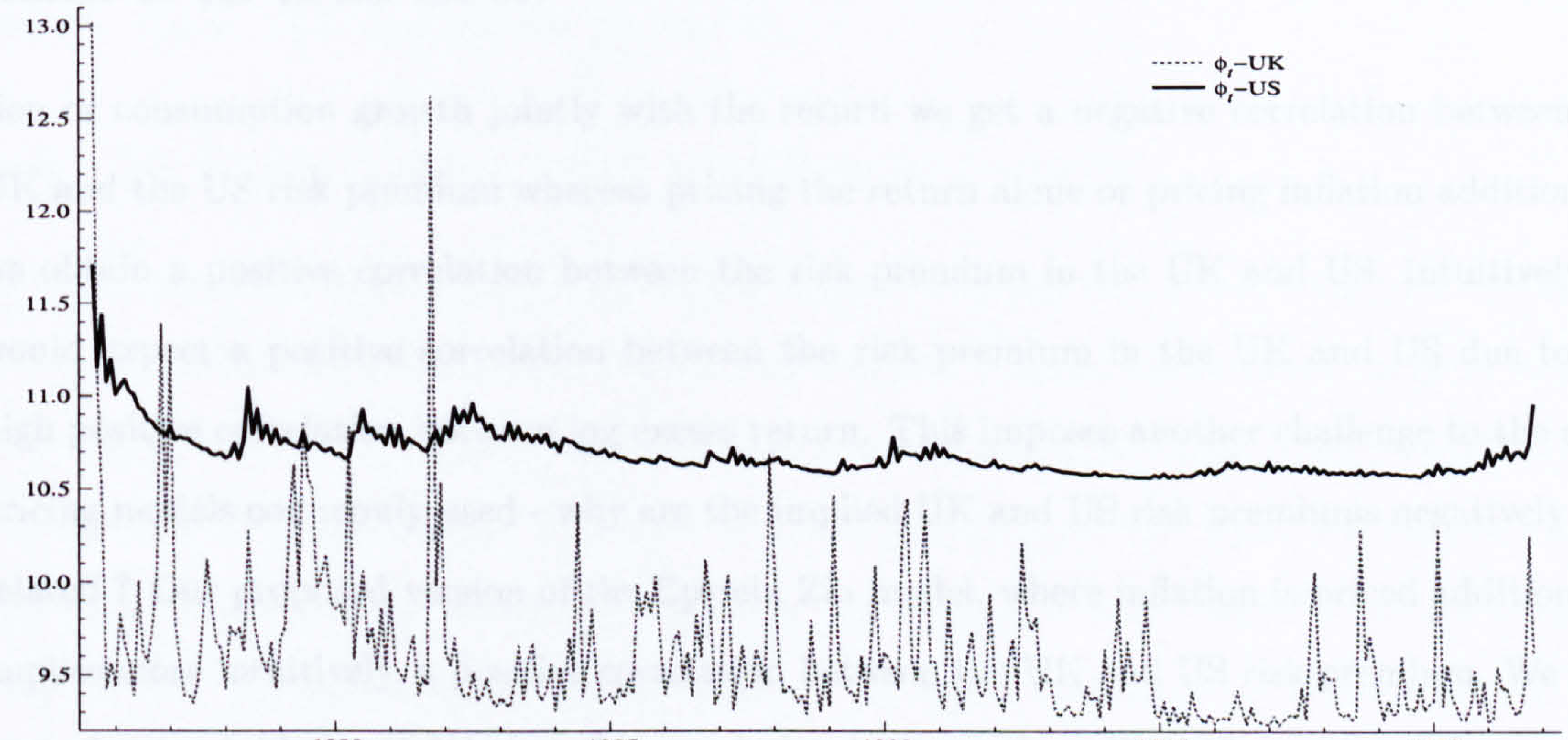
The implied UK risk premium has had a declining trend over the period due to the fall in the conditional covariance between excess return and consumption growth. The covariance between excess US return and consumption growth was negative in the beginning of the sample but has been increasing and mainly positive over the sample - after 1993 the implied risk premium in the US has been very high, above 10 %, rising considerably after the price fall in the stock market in August 1998. This plot imposes another challenge to the inter-temporal PU model - Why has the risk premium been declining in the UK while remained high in the US ? It is obvious that due to the higher level of covariance between consumption growth and returns the equity premium puzzle may be less severe in the US in the most recent period.

Much research has assumed that the conditional covariance between excess return and consumption growth is constant - the estimated models in this chapter show that this is simply not a valid assumption in the UK and US. We will later propose a test whether there is significant time-variation in the conditional covariance and hence expected excess returns.

The CAPM - UK and US

In figure (2.5) we plot the implied risk premium from the CAPM in the UK and US. The CAPM on the market portfolio implies that the risk premium is a constant proportion of the conditional variance of excess return on the market portfolio. This assumption has often been made in applied work (see Ng [91] or Bekaert and Wu [?]).

Figure 2.5: The CAPM Risk Premium



The UK and US risk premium implied by the CAPM. Series are annualised.

It is interesting that risk premia implied from this version of the CAPM are relatively constant - the implied risk premium in the US is on average higher than the risk premium in the UK. The model implies a slightly more volatile risk premium in the UK than in the US. However - one conclusion is clear - modelling risk premia proportional to the exante conditional variance of excess return we can only obtain a positive sign on the parameter determining the proportionality between the risk premium and the variance - since the average excess return is positive there must always be a positive relation between the risk premium and the variance in this model. The implied variability of the risk premium is almost non-existent. Hence we conclude that models of the risk premium pricing macroeconomic variables generates risk premia varying significantly more over time.

The Correlation Between UK and US Risk Premia

UK and US log excess returns were highly correlated. It is of interest to compute the correlation between the conditional expectation of UK and US excess return, risk premia, implied by the different models. The correlations are tabulated in table (2.2). In models pricing only consump-

Table 2.2: Correlation Between US and UK Risk Premia From Various Models

	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
$\rho(\phi^{US},\phi^{UK})$	0.162	0.122	0.204	0.189	−0.209	−0.208	−0.213	0.439

For each of the eight models we compute the correlation between the implied risk premium in the UK and the US.

tion or consumption growth jointly with the return we get a negative correlation between the UK and the US risk premium whereas pricing the return alone or pricing inflation additionally we obtain a positive correlation between the risk premium in the UK and US. Intuitively we would expect a positive correlation between the risk premium in the UK and US due to the high positive correlation between log excess return. This imposes another challenge to the asset pricing models commonly used - why are the implied UK and US risk premiums negatively correlated ? Our proposed version of the Epstein Zin model, where inflation is priced additionally, implies more intuitively, a positive correlation between the UK and US risk premium. We note from chapter 1 (figure (1.1)) that the correlation between the rolling 12 month moving average in the US and UK stock market log excess return had a correlation of 0.67. The Power Utility CCAPM fails to capture the comovement in the mean of excess return on UK and US stock market indices. The high correlation of the actual data, conditional on our assumed asset pricing model, is due to a high degree of common shocks.

Finally, it is of interest to ask whether the implied risk premia from different models within the two countries are highly correlated across models. We compute these correlations in table (2.3).

In the US we note that the implied risk premium from the general model is highly correlated with the implied risk premium in SDF, EZ1 and PU-Nom-H1. It has a negative correlation with the implied risk premium from the CAPM model. In the UK the risk premium from the General model is most highly correlated with the risk premium from the SDF model with a correlation of 0.78, it has less correlation with the EZ1 model and has a positive correlation with the risk premium implied by the CAPM. Further there appears to be no difference, on the implied risk premium in the Power Utility model, whether we use real or nominal returns.

Table 2.3: The Correlation Between US and UK Risk Premia In Different Models

	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
General	x	0.777	0.598	0.590	0.444	0.444	0.442	0.571
SDF	0.998	x	0.853	0.863	0.626	0.627	0.601	0.582
EZ1	0.996	0.990	x	0.997	0.513	0.514	0.501	0.651
PU-Nom-H1	0.996	0.993	0.999	x	0.551	0.553	0.537	0.646
PU-Nom-H0	0.839	0.819	0.856	0.841	x	1	0.995	0.751
PU-Real-H0	0.839	0.820	0.857	0.841	1	x	0.995	0.751
EZ2	0.841	0.820	0.859	0.842	0.999	0.999	x	0.777
CAPM	−0.294	−0.251	−0.325	−0.289	−0.334	−0.334	−0.362	x

The Correlation matrix between model implied risk premia. The lower part of the table contains the correlation between the different implied risk premia in the US and the upper half risk premia in the UK.

2.5.2 The Equity Premium Puzzle and Parameter Estimates

As discussed in the introductory chapter, the Equity Premium Puzzle has to do with the magnitude of the estimated coefficient of relative risk aversion in consumption-based models, which is often large. Microeconomic studies have found these high estimates implausible (Campbell [21]). One of the aims of this chapter is to allow for more general preference structure without making any restrictions across the coefficient of relative risk aversion and the elasticity of inter-temporal substitution. For each country we tabulate the estimated coefficient on the conditional variance and covariances in the excess return equation, see equation (2.22), for each model with t-statistics in parenthesis. The estimate of the conditional covariance between excess return and inflation excludes $\mathrm{Cov}_t(i^e_{s,t+1}, \pi_{t+1})$ from working with nominal returns. In other words the tables contain only the estimate of $\hat{\alpha}_2$ as can be seen from equation (2.22). The left column denotes which conditional variance or covariance the estimated parameter tabulated corresponds to. $\overline{\varepsilon}_{t+1}$ is a measure of the average of the residual in the excess return equation in annual percentages, $V(\phi_{t+1})$ is the variance of the model implied risk premium in annual percentages and $\frac{V(\phi_{s,t+1})}{V(i^e_{s,t+1}+\frac{1}{2}V_t(i^e_{s,t+1})-\widehat{\theta}_1\Upsilon_{1987:10,t+1})}$ is the variance of the risk premium relative to the variance of the dependent variable. $\phi_{longrun}$ is the average risk premium.

UK Estimates

In table (2.4) we tabulate the estimates of the UK parameters from the no-arbitrage condition.

Table 2.4: UK Parameter Estimates

UK: t-Distribution and dummy	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
$V_t(i^e_{s,t+1})$	3.3585 (0.74)		2.2125 (0.45)				**3.0451** **(2.03)**	**3.6978** **(3.24)**
$Cov_t(i^e_{s,t+1},\pi_{t+1})$	−490.1937 (1.72)	**−592.0490** **(2.06)**	**−690.2728** **(2.73)**	**−701.8789** **(2.75)**	0	−1	**−3.0451** **(2.03)**	**−3.6978** **(3.24)**
$Cov_t(i^e_{s,t+1},\Delta c_{t+1})$	**257.6526** **(1.96)**	230.2904 (1.85)	153.5293 (1.38)	169.5078 (1.57)	**213.4112** **(2.54)**	**202.9858** **(2.54)**	113.1139 (1.50)	
$Cov_t(i^e_{s,t+1},\Delta y_{t+1})$	−114.1241 (1.07)	−56.2950 (0.60)						
$\Upsilon_{1987:10,t+1}$	−0.3178 (1.86)	−0.3189 (1.80)	−0.3213 (1.65)	−0.3209 (1.70)	−0.3138 (1.53)	−0.3138 (1.53)	−0.3157 (1.54)	−0.3177 (1.45)
ν	**11.6281** **(3.32)**	**11.3645** **(3.37)**	**11.4693** **(3.42)**	**11.2838** **(3.43)**	**12.5356** **(3.16)**	**12.5348** **(3.16)**	**12.9027** **(3.05)**	**12.6660** **(3.06)**
Log likelihood	3912.2733	3912.0413	3912.0707	3911.9702	3907.8397	3907.8510	3909.1882	3908.3249
$\phi_{long\ run}$	10.54%	10.41%	10.81%	10.65%	7.23%	7.21%	9.76%	9.70%
$\lvert\lambda_{max}\rvert$	0.9755	0.9770	0.9787	0.9790	0.9717	0.9716	0.9705	0.9716
$\overline{\varepsilon}_{t+1}$	−1.4330	−0.9934	−1.4855	−1.1703	0.1451	0.1471	−2.1266	−1.7371
$V(\phi_{t+1})$	127.5257	139.9932	140.0820	146.0857	68.1886	68.1479	26.5122	0.1516
$\frac{V(\phi_{t+1})}{V(i^e_{s,t+1}+\frac{1}{2}V_t(i^e_{s,t+1})-\widehat{\theta}_1\Upsilon_{1987:10,t+1})}$	0.0395	0.0433	0.0434	0.0452	0.0211	0.0211	0.0082	0.00005

UK Results, 1975-2002. Using multivariate GARCH in mean model with symmetric ARCH and GARCH matrices. Numbers in parenthesis are absolute t-statistics and emphasised parameters are significant using a 95 % critical value. ν is the estimated degrees of freedom in the multivariate distribution.

Consumption growth is reasonably significant in all models pricing consumption. In the Power Utility model, PU-Nom-H0, the estimated coefficient of relative risk aversion is 213.41 which is unrealistically high. The lower 95 % confidence bound is 48.73 - we reject that the coefficient of relative risk aversion is below 48.73. In the more general consumption based models pricing additionally industrial production growth and inflation we see that the variability of the implied risk premia, consistent with the figures in section (2.5.1), are twice as high - the main contribution to the risk premium comes from inflation and we reject the standard Power Utility CCAPM model relative to our more general specification. It is interesting that we cannot reject that the coefficient on industrial production growth is zero and therefore, when we have accounted for all significant risk in our most general consumption-based model, industrial production includes no additional risk to be priced and we cannot reject the general consumption-based model, EZ1.

Table 2.5: US Parameter Estimates

US: t-Distribution and dummy	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
$V_t(i^{*e}_{s,t+1})$	−3.5442 (0.72)		−3.6152 (0.75)				−3.2256 (0.63)	**4.7268** **(3.60)**
$Cov_t(i^{*e}_{s,t+1}, \pi^*_{t+1})$	−434.5221 (1.58)	−460.6514 (1.68)	−423.2935 (1.65)	−421.4718 (1.69)	0	−1	3.2256 (0.63)	**−4.7268** **(3.60)**
$Cov_t(i^{*e}_{s,t+1}, \Delta c^*_{t+1})$	**356.6612** **(2.34)**	**294.3312** **(2.89)**	**358.7856** **(2.38)**	**296.5564** **(2.94)**	**328.6734** **(3.10)**	**328.5413** **(3.10)**	**393.5033** **(2.38)**	
$Cov_t(i^{*e}_{s,t+1}, \Delta y^*_{t+1})$	8.5397 (0.08)	28.9794 (0.29)						
$\Upsilon_{1987:10,t+1}$	**−0.2455** **(1.98)**	**−0.2458** **(2.02)**	−0.2458 (1.95)	−0.2475 (1.90)	−0.2501 (1.80)	−0.2501 (1.80)	−0.2480 (1.88)	−0.2540 (1.47)
ν	**13.8292** **(3.10)**	**13.8773** **(3.15)**	**13.7932** **(3.13)**	**13.7443** **(3.21)**	**13.5299** **(3.25)**	**13.5301** **(3.25)**	**13.5650** **(3.18)**	**12.6107** **(3.07)**
Log likelihood	4441.2744	4440.8900	4441.2708	4440.8430	4439.3653	4439.3728	4439.6839	4436.4261
$\phi_{long\ run}$	8.83%	9.75%	8.85%	9.99%	12.01%	12.00%	10.79%	10.66%
$\lvert\lambda_{max}\rvert$	0.9659	0.9660	0.9660	0.9663	0.9605	0.9606	0.9607	0.9824
$\bar{\varepsilon}_{t+1}$	−1.8426	−2.7531	−1.8281	−2.8124	−2.8527	−2.8535	−1.8960	−2.1086
$V(\phi_{t+1})$	93.9201	77.8848	94.0757	76.8676	58.8817	58.8811	76.4793	0.0174
$\frac{V(\phi_{t+1})}{V(i^e_{s,t+1}+\frac{1}{2}V_t(i^e_{s,t+1})-\hat{\theta}_1\Upsilon_{1987:10,t+1})}$	0.0358	0.0297	0.0359	0.0293	0.0225	0.0225	0.0292	0.000007

US Results, 1975-2002. Using multivariate GARCH in mean model with symmetric ARCH and GARCH matrices. Numbers in parenthesis are absolute t-statistics and emphasised parameters are significant using a 95 % critical value. ν is the estimated degrees of freedom.

We note that the CAPM and EZ2 models have the highest average residual implying that the model implied average level of risk premia are too high to fit the mean of excess return. This is less evident in the alternative specifications of the risk premium. The implied long run risk premia, $\phi_{long\ run}$, is higher in the more general models than in the PU-Nom-H0 and PU-Real-H0 models - though presumably the estimate from the latter two models is more plausible - in addition we note that the estimate of the long run covariance matrix is very variable and hence our estimate of the long run risk premium has a high variance. In the EZ1 model we explain 4.34 % of the variation in excess return, in the PU-Nom-H0 model 2.11 % and in the CAPM 0.005 %. Testing the CAPM model one would have problems rejecting the null hypothesis of a constant risk premium, as will be evident shortly, whereas this is not the case in the more general macroeconomic models. We conclude that the macroeconomic variables real consumption growth and, in particular, inflation are significant variables to be priced in the UK stock market.

US Estimates

The US estimates can be found in table (2.5). In the US a consistent picture emerges in that real consumption growth is significantly priced in all models where it is included. Inflation is significantly priced in some models using the 90 % critical value[12]. In the traditional PU-Nom-H0 model the estimated coefficient of relative risk aversion is 328.67 with a lower 95 % critical value bound of 120.87. Thus, if this is the true risk premium model, we conclude the lowest significant implied coefficient of relative risk aversion is higher for a US investor than for the UK investor. In terms of model rejection we cannot reject the PU-Nom-H0 model to any of the more general consumption-based models or the General model. However, we reject the CAPM relative to all other models. In the US case the implied long run risk premium is higher for the two null hypothesis Power Utility models whereas it is lower in the general consumption-based models and the CAPM. Different models of the risk premium tend to overstate the true risk premium. The variability of the US risk premium implied from the CAPM is only 0.0007 % relative to the variance of excess return whereas in the EZ1 model we explain 3.59 % of the variation. In general we conclude that the CAPM fails to account for variation in the expectation of excess returns. Moreover, we cannot reject the PU-Nom-H0 relative to any other model whereas in the UK we reject the PU-Nom-H0 model relative to our more general specification of preferences. UK inflation is always significant whereas US real consumption growth is the most significant factor !

That the CAPM implies a constant risk premium could be the reason why Glosten, Jagannathan and Runkle [66] and Scruggs [101] find the variance of excess return to be insignificant when including a constant in the mean equation.

Parameters in the Consumption-based Models - UK and US

Essentially all the estimated models are SDF models with the logarithmic SDF:

$$m_{t+1} \;=\; -\,a \;-\; \mathbf{b}^{\mathsf{T}}\mathbf{f}_{t+1} + \zeta_{t+1} \tag{2.40}$$

If markets are not complete then ζ_{t+1} is random and not identically equal to zero. In Partial and General Equilibrium models markets are assumed to be complete and $\zeta_{t+1} \equiv 0$. In this case we can interpret the preference parameters and the portfolio weights in the various models. In table (2.6) we compute the model implied parameters and associated standard errors, using

[12] It is an interesting finding that inflation is significant and may be related to the findings by Campbell and Vuolteenaho [28] and Brandt and Wang [17] that stock prices may be related to inflation in some way.

formulas in section (2.1.3), from the estimations in table (2.4) and (2.5).

Table 2.6: The Estimated Preference Parameters For Representative Investor

	UK						US					
Parameter	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
γ	843.8021 (2.79)	871.3867 (2.89)	213.4112 (2.54)	202.9858 (2.54)	116.5590 (1.56)	x	782.0791 (2.57)	718.0282 (2.60)	328.5413 (3.10)	328.5413 (3.10)	390.2778 (2.41)	x
ψ	−4.4895 (1.39)	−4.1347 (1.55)	0.0047 (2.54)	0.0049 (2.54)	−0.0181 (0.88)	x	−1.1770 (1.39)	−1.4178 (1.51)	0.0030 (3.10)	0.0030 (3.10)	0.0107 (1.08)	x
ω_1	0.997 (25.25)	1	x	x	0	0	1.0101 (3.72)	1	x	x	0	0
ω_2	0.003 (0.04)	0	x	x	1	1	−0.0101 (0.27)	0	x	x	1	1
θ	−689.2728 (2.73)	−701.8789 (2 75)	x	x	−2.0451 (2 03)	x	−422.2935 (1.65)	−420.4718 (1.69)	x	x	4.2256 (0.63)	x

US and UK implied parameters for the consumption-based model. x indicates that this parameter does not exist in model. Numbers in parenthesis are absolute t-statistics and emphasised parameters are significant using a 95 % critical value.

We note several interesting things. First, as we have discussed, the estimate of the coefficient of relative risk aversion in PU-Nom-H0 is implausibly high. In models pricing inflation the implied coefficient of relative risk aversion becomes even higher. In the most general consumption-based model, EZ1, the estimated coefficient of relative risk aversion is 843 for a UK investor and 782 for a US investor and the implied portfolio weight is 100 % in the risk-free asset. We note in addition that the coefficient of relative risk aversion is significantly estimated in all models - the estimate of the elasticity of inter-temporal substitution is most often negative, though never significantly negative. In a sense our specification of the market portfolio as an investment in both the risk-free asset and stock returns has allowed for the possibility that the representative investor is more risk averse than traditionally thought. This creates another puzzle, call this the "Extreme Equity Premium Puzzle". If we have a representative investor with a choice of investing both in a risk-free asset and a broad stock market index he (or she) would, since he/she is extremely risk averse, invest all wealth in a risk-free asset - this follows for both the US and UK investor.

The PU-Nom-H1 model, as mentioned, can be interpreted as the EZ1 model where we force the representative investor to invest all in the risk-free asset and hence the results of the two models are similar. When we allow for more general preference structure and broader wealth portfolio the model implies an estimate of the Elasticity of Inter-temporal Substitution (EIS) which is both negative and large in absolute value.

We conclude that consumption growth and inflation are significant factors to be priced in the

US and UK stock market - interpreting the results in terms of General Equilibria models the estimated parameters are implausible as has often been found in the literature. The results suggest that more likely a SDF model with the logarithmic SDF linear in macroeconomic variables is better able to capture the sources of risk to be priced in the UK and US stock market. Parker [93] argues that consumption risks in aggregate data are biased downwards by a factor of 6 due to, for instance, limitted participation in the stock market and this could be the reason why we obtain these unreasonable estimates of the preference parameters.

2.5.3 Other Estimated Parameters

The estimates of the constant vector and the parameters in the vector auto regression do not differ much between the estimates of the different models[13]. Therefore, it is chosen to report the estimates from the most general Epstein-Zin model, EZ1, in the UK and US only. Recall that $\mathbf{Y}_{t+1} = \{i^e_{t+1}, \pi_{t+1}, \Delta c_{t+1}, \Delta y_{t+1}\}^\top$. The estimates of the mean equation of the macroeconomic variables are tabulated in table (2.7).

Table 2.7: The Mean Equation of Macroeconomic Variables

	US					UK				
$i^e_{s,t+1}$	0	0	0	0	0	0	0	0	0	0
π_{t+1}	**0.0009**	0.0027	**0.6377**	**0.0396**	0.0079	**0.0025**	0.0021	**0.2838**	0.0327	0.0173
	(4.46)	(1.08)	(14.56)	(2.14)	(0.47)	(6.22)	(0.29)	(4.28)	(0.64)	(0.62)
Δc_{t+1}	**0.0043**	0.0147	**−0.3007**	**−0.4021**	0.0366	**0.0036**	0.0110	−0.0446	**−0.2248**	−0.0408
	(7.64)	(1.85)	(2.47)	(8.02)	(0.69)	(8.58)	(1.42)	(0.72)	(4.31)	(1.17)
Δy_{t+1}	**0.0032**	0.0068	−0.2206	−0.0825	**0.2906**	**0.0014**	0.0190	−0.0339	0.1389	**−0.2403**
	(5.89)	(0.98)	(1.85)	(1.46)	(4.95)	(2.14)	(1.82)	(0.40)	(1.50)	(4.45)

For each country the first column is the estimated constants in the mean equation, A, of the dependent variables whereas the other 4 × 4 matrix is the matrix from the vector auto regression, B. Absolute t-statistics in parenthesis. Emphasised parameters significant using the 95 % critical value.

The first column in the table indicates the dependent variable. For each country we have a 4×5 matrix where the first column is the estimate of $\hat{\mathbf{A}}$ and the 4×4 matrix in the bottom right corner is the estimate of the matrix $\hat{\mathbf{B}}_1$. In the US inflation lagged and consumption growth lagged predict changes in the inflation rate. Lagged stock market excess return, inflation and consumption growth predicts changes in consumption growth. In the UK the macroeconomic variables are mainly predictable from their own lag and log excess return predicts changes in industrial production growth using the 90 % asymptotic critical value. This is weak evidence

[13] All estimated parameters are available in a reasonably readable format upon request.

that asset prices are forward looking - they forecast changes in industrial production growth in the UK and changes in consumption growth in the US.

In table (2.8), we tabulate the lower triangular matrix in the long run covariance matrix from the EZ1 models. This matrix has an intuitive interpretation since it measures the responses to the dependent variables in the short run from shocks to the dependent variables. We note that in the UK only diagonal elements are significant meaning that it is only own shocks to the variables in the UK that affect the variables in the long run. In the US, additionally, positive shocks to excess return increases consumption growth in the long run.

Table 2.8: The Constant Part of The Covariance Matrix

	US				UK			
$i^e_{s,t+1}$	**0.0415** (23.93)	0	0	0	**0.04582** (22.57)	0	0	0
π_{t+1}	0.00004 (0.20)	**0.00270** (2.93)	0	0	−0.000024 (0.07)	**0.00481** (11.94)	0	0
Δc_{t+1}	**0.00096** (2.97)	−0.00015 (0.26)	**0.00565** (14.20)	0	0.00029 (0.59)	−0.00065 (1.30)	**0.00650** (4.62)	0
Δy_{t+1}	−0.0003 (0.62)	0.00090 (1.80)	**0.00133** (2.90)	**0.00666** (8.22)	0.00043 (0.53)	0.00080 (0.71)	0.0015 (0.90)	**0.01128** (3.93)

The lower triangular matrix from the estimated long run covariance matrix, **C**. Absolute t-statistics in parenthesis. Emphasised parameters significant using the 95 % critical value.

One can recover the long run variance covariance matrix, $\mathbf{CC}^\intercal$. It does not differ much between the models. The implied long run covariance matrix[14] in the UK and US are tabulated in table (2.9) below.

Table 2.9: The Long Run Covariance Matrix

	US				UK			
$i^e_{s,t+1}$	2704.93	-6.01	34.71	-15.79	3023.31	-1.60	19.23	28.48
π_{t+1}	-6.01	7.48	-2.38	3.48	-1.60	33.33	-4.50	5.53
Δc_{t+1}	34.71	-2.38	44.54	11.52	19.23	-4.50	61.58	13.29
Δy_{t+1}	-15.79	3.48	11.52	67.04	28.48	5.53	13.29	187.51

The estimated long run variance covariance matrix in annualised dataset.

We note that the UK variables have a higher long run variance, with industrial production in the UK being three times as variable as US industrial production growth. In both countries the estimated covariance between log excess return and inflation is negative, the covariance between consumption growth and log excess return is positive - the covariance between log excess return

[14]The covariance matrix is annualised, that is multiplied by 1200 squarred.

and industrial production is negative in the US and positive in the UK.

GARCH AND ARCH PARAMETER ESTIMATES

In table (2.16), (2.17), (2.18) and (2.19) in the appendix we tabulate the estimated parameters in the ARCH and GARCH matrices. It is seen that there is not much difference in the estimated parameters depending on which model is used for the risk premium. We note that several off diagonal elements, this as well in the ARCH as the GARCH matrices, are significant and hence we have obtained a gain relative using the diagonal BEKK specification.

2.5.4 Non-stationarity of the Conditional Moments

In table (2.10) we tabulate the two highest eigenvalues (in absolute value) for the estimated models, CAPM and General.

Table 2.10: The Eigenvalues of Covariance Matrix

	US		UK	
	General	CAPM	General	CAPM
$\|\lambda\|_{max}$	0.9659	0.9822	0.9756	0.9716
$\|\lambda\|_{max-1}$	0.9557	0.9718	0.9344	0.9280

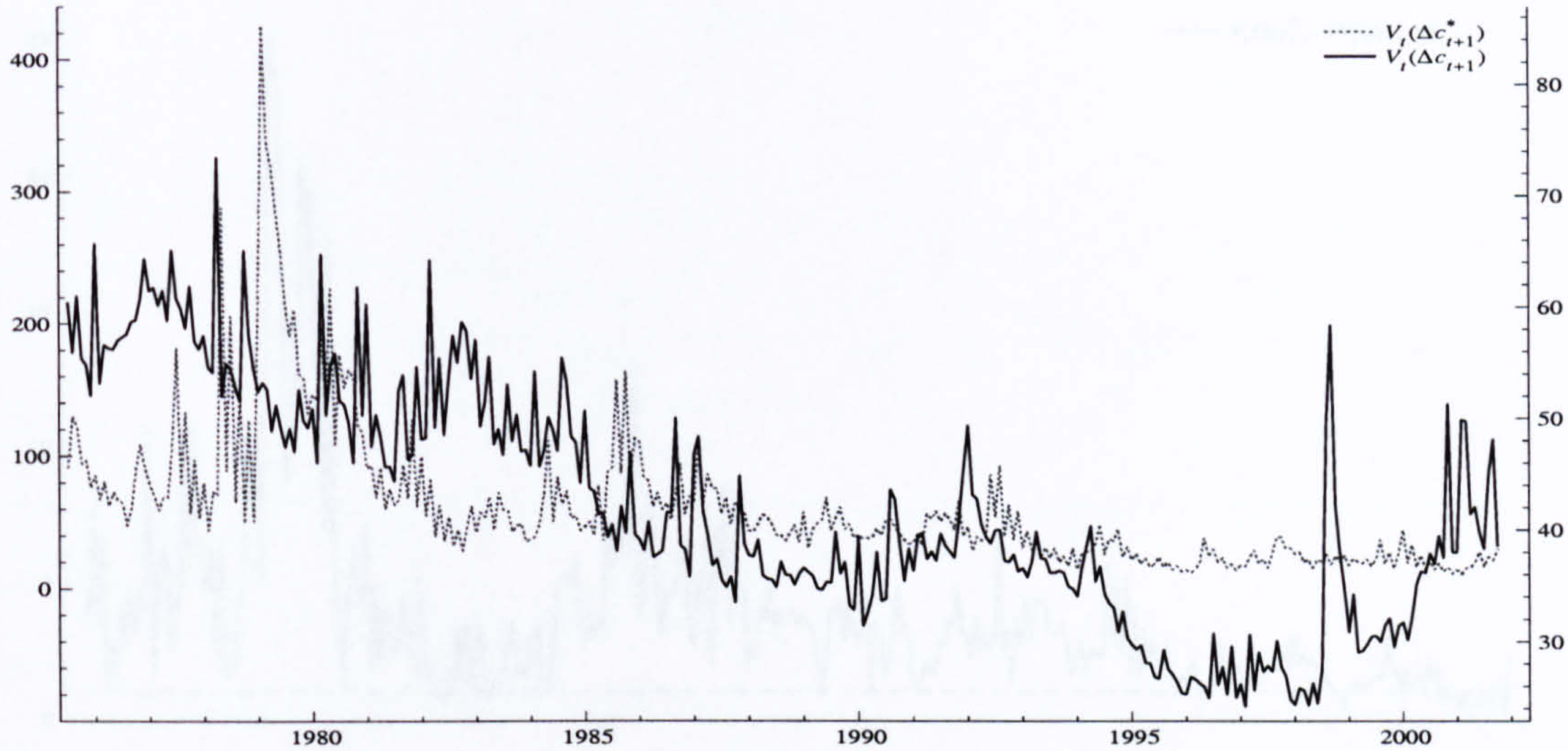
The two highest eigenvalues for covariance stationarity in the General model and CAPM in the UK and US.

The highest eigenvalues are not too close to 1. The variable that displays most non-stationarity is the conditional variance of consumption growth. To get a better understanding, the implied conditional variance of real non durable consumption growth is plotted, from the Power Utility CCAPM model, in the following figure (2.6).

The conditional variance of consumption growth has been declining over the sample period, the level of the conditional variance almost halved in both countries. Though there is some ARCH in the conditional variance series it is evident that there has been a downward sloping trend in the level of the conditional variances. The equity premium puzzle is that consumption is too smooth to fit the empirical Sharpe Ratio. This graph shows that the equity premium puzzle has become even more severe in the recent decade due to the decline in consumption variability (though consumption volatility in the US has increased more recently).

The estimated long run variance of industrial production growth was much higher in the UK than the US. In figure (2.7) we plot the conditional variance of output in the UK relative to the

Figure 2.6: The Conditional Variance of Consumption Growth



Plot of the conditional variance series of consumption growth implied by the Power Utility models. Star as a superscript indicates UK consumption. The conditional variance of US consumption growth has scale to the right. The conditional variance of annualised dataset.

US.

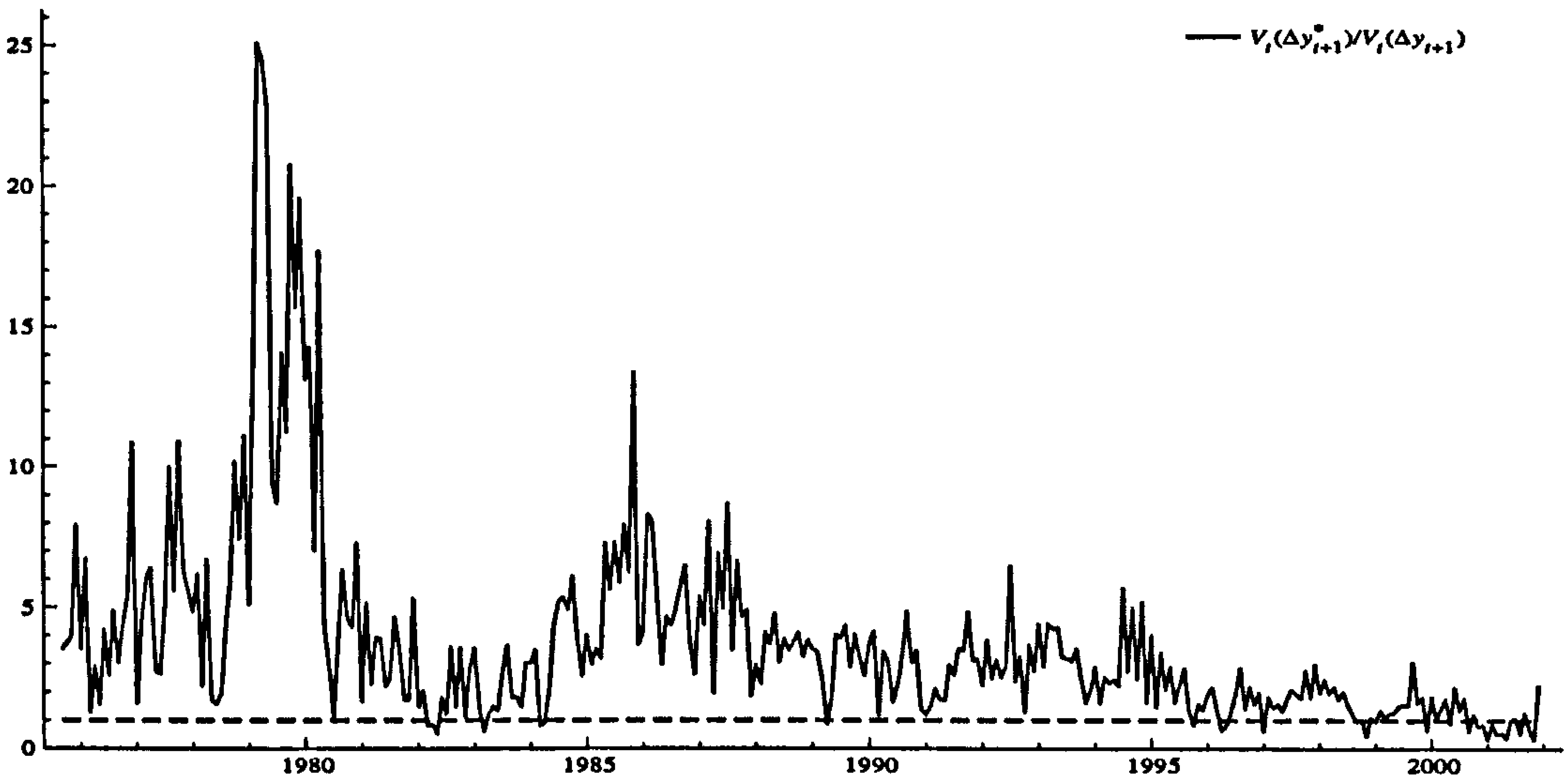
We see that the relative variance of UK and US industrial production growth has been declining very much and after 1999 it seems that the relative conditional variance of the two series has converged close to 1.

2.5.5 The Jensen Correction and the Correction Working with Nominal Returns

We have emphasised that a Jensen correction should be included in the model due to working with logarithmic returns and a covariance correction between log excess return and inflation should be included in the risk premium due to working with nominal returns. The economic importance of the two terms is of interest and we plot the correction term in the risk premium due to working with nominal return in figure (2.8), this correction term is one unit of the conditional covariance between log excess return and inflation, together with the Jensen correction from working with logarithmic returns, a half times the conditional return variance.

The correction term from working with nominal returns is neglible. In the UK it was highest in absolute value in the beginning of the sample, a period of high inflation variability, where it had the largest negative value of 0.20% whereas it is smaller in absolute value for both countries

Figure 2.7: The Relative Conditional Variance of UK and US Industrial Production Growth



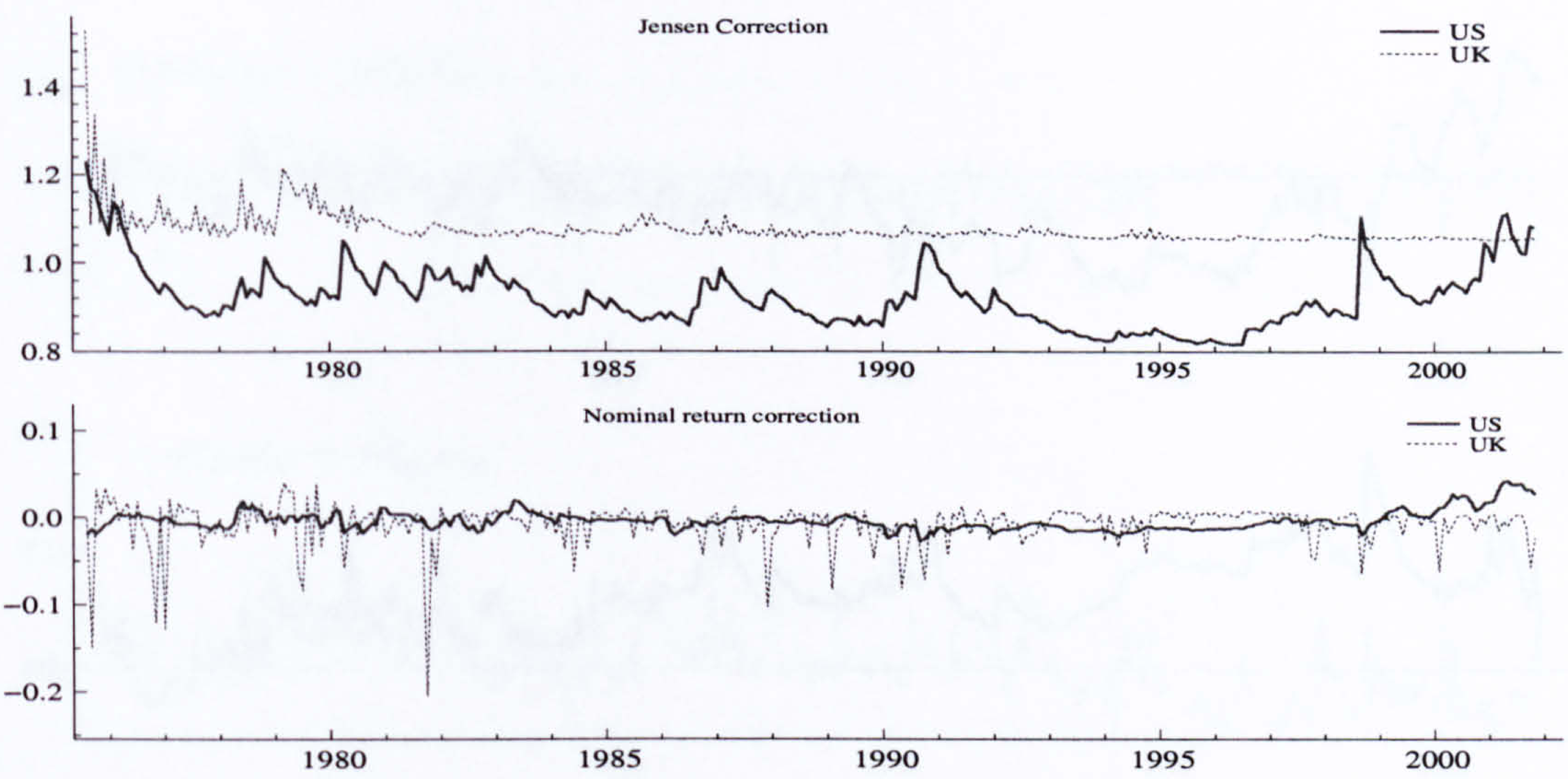
Conditional ratio of variabilities of industrial production growth in the general model. Star as a superscript indicates a UK variable.

in the rest of the sample. Wheras omission of this correction term does not matter is crucial that inflation is included as a variable in the SDF since it is significant in both the US and the UK. The Jensen correction is rather constant for both of the countries but has a level of around 1.08% in the UK and around 0.9% in the US. We note that the US and UK conditional return variances were at the same level in the beginning of the sample and the end of the sample whereas over the late 1970s, 1980s and 1990s the variance of US excess return is higher than UK excess return.

2.6 Time-Varying Correlations

In figure (2.9) we plot pairs of the conditional correlations between UK and US log excess return and the national macroeconomic variables implied by the most general Epstein Zin model. In this chapter three macroeconomic variables were priced. Consumption growth and inflation were often significant. An asset is riskier if its return is higher conditionally correlated with the factors and therefore it is of interest to plot the implied conditional correlations between excess return and the macroeconomic variables. In the US the conditional correlation between consumption growth and log excess return has been slowly growing over the sample whereas the correlation between log excess return and the two other macro variables has been declining over

Figure 2.8: The Correction Terms



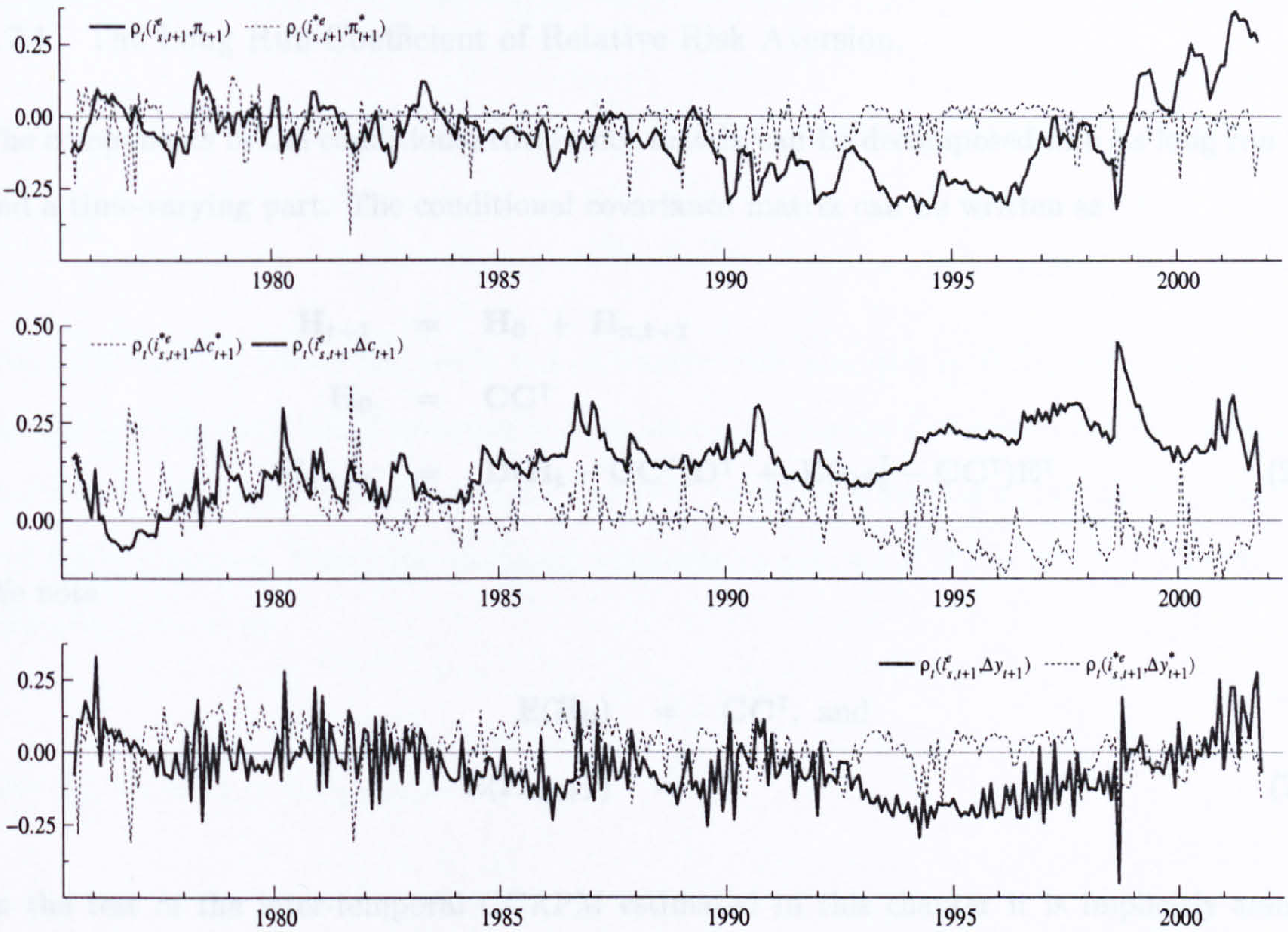
The Correction terms from the US and UK EZ1 models. In upper panel the Jensen correction, in panel below the correction from working with nominal returns. The corrections are annualised.

most of the sample with an increase in both correlations from 1997 onwards - it is interesting to note the comovement between the correlation of log excess return with inflation and industrial production growth.

The conditional correlation between UK stock returns and inflation has been negative over most of the sample and the recent increase in this correlation in the US is not evident in the UK. Similarly the correlation between UK stock returns and consumption growth has been positive over most of the sample but whereas the US correlation has been increasing in the most recent decade the UK correlation has been declining. Finally the conditional correlation between UK stock returns and industrial production growth has been fluctuating around zero in most of the sample whereas the US correlation is mainly negative.

It may seem puzzling that the conditional correlation between excess return and consumption growth becomes negative but we end this section with a warning. If one is interested in estimating the conditional correlation between the stock market and consumption growth the correlation depends on the way you model the risk premium. To illustrate this we plot the conditional correlation between UK log excess return and consumption growth implied by the General and PU-Nom-H0 models in figure (2.13) in the appendix. We note that in the PU-Nom-H0 model

Figure 2.9: Time-Varying Correlations Between Return And Macroeconomic Variables



Pairs of conditional correlations from the general Epstein Zin model between UK and US log excess return and the macroeconomic variables. 1975-2002. A star as superscript indicates a UK variable and $\rho_t(\cdot,\cdot)$ is the conditional correlation.

the correlation is always positive due to the fact that we have only one factor to model the risk premium and the risk premium seems positive at all times. When we have a different no-arbitrage condition the conditional correlations can be different and is an important point when we are interested in conditional correlations between financial returns and other variables.

2.7 Time-Varying Coefficients in the Inter-temporal CCAPM

In this section two alternative methods to test the inter-temporal CCAPM model will be discussed. First, an alternative method will be proposed to recover the long run coefficient of relative risk aversion in the inter-temporal CCAPM model which allows us to test whether the model implies a time-varying risk premium. We perform the test for the UK and US datasets. Second, it will be shown that the approach of Lettau and Ludvigson which allows for a time-varying coefficient of relative risk aversion, as outlined in chapter 1, can be estimated using the estimation framework provided in this chapter if monthly data were available.

2.7.1 The Long Run Coefficient of Relative Risk Aversion.

The components in the conditional covariance matrix can be decomposed into its long run level and a time-varying part. The conditional covariance matrix can be written as

$$\begin{aligned} \mathbf{H_{t+1}} &= \mathbf{H_0} + \mathbf{H_{x,t+1}} \\ \mathbf{H_0} &= \mathbf{CC^T} \\ \mathbf{H_{x,t+1}} &= \mathbf{D(H_t - CC^T)D^T} + \mathbf{E}(\epsilon_t \epsilon_t^T - \mathbf{CC^T})\mathbf{E^T} \end{aligned} \tag{2.41}$$

We note

$$\begin{aligned} \mathrm{E}(\mathbf{H_0}) &= \mathbf{CC^T}, \text{ and} \\ \mathrm{E}(\mathbf{H_{x,t+1}}) &= 0 \end{aligned} \tag{2.42}$$

In the test of the inter-temporal CCAPM estimated in this chapter it is implicitly assumed that the coefficient of relative risk aversion is the same in the long run as it is when short run movements were above or below the long run level. Using this multivariate GARCH in mean model allows us to estimate a more flexible inter-temporal CCAPM model allowing to estimate the coefficient of relative risk aversion in the long run and test whether time movements in the conditional covariance between log excess return and consumption growth generate significant time-variation in the risk premium. Write the conditional covariance between consumption growth and excess return as two components using the notation above.

$$\mathrm{Cov}_t(i^e_{s,t+1}, \Delta c_{t+1}) = \mathrm{Cov}_0(i^e_{s,t+1}, \Delta c_{t+1}) + \mathrm{Cov}_{x,t}(i^e_{s,t+1}, \Delta c_{t+1}), \tag{2.43}$$

where the first term on the RHS is the long run covariance between excess return and real consumption growth, hence no time t subscript, and the second term is a measure of the mean zero component that causes time-variation in the conditional covariance. If we model the excess return equation as

$$E_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) = \gamma_1 \text{Cov}_t(i^e_{s,t+1}, \Delta c_{t+1}) + \gamma_2 \text{Cov}_{x,t}(i^e_{s,t+1}, \Delta c_{t+1}) + \text{Cov}_t(i^e_{s,t+1}, \pi_{t+1}) \quad (2.44)$$

We note that the estimate of the coefficient of relative risk aversion in the long run is

$$\hat{\gamma}_{lr} = \hat{\gamma}_1 \quad (2.45)$$

The parameter γ_2 allows the coefficient of relative risk aversion to differ when there is time-variation in the conditional covariance. It is basically a measure of whether allowing for time-variation in the covariance improves the CCAPM or whether there is significant time-variation in the risk premium. Hence the coefficient determining whether it is important to allow for time variation is equal to

$$\hat{\gamma}_{tv} = \hat{\gamma}_1 + \hat{\gamma}_2, \quad (2.46)$$

with the variance of the parameter

$$V(\gamma_{tv}) = V(\gamma_1) + V(\gamma_2) + 2\text{Cov}(\gamma_1, \gamma_2), \quad (2.47)$$

Estimating the inter-temporal CCAPM model in this way we have a method to test whether the time-variation in the conditional covariance between consumption growth and log excess return is important determining time-variation in risk premia. In a similar fashion we can test whether the CAPM model implies time-variation in the risk premium. We call the estimations CCAPMA and CAPMA. The UK and US results are tabulated in table (2.11).

Table 2.11: Test For Time-Varying Risk Premium

CCAPM	UK	US	CAPM	UK	US
$Cov_t(\Delta c_{t+1}, i^e_{s,t+1})$	190.5146 (2.14)	288.3208 (2.31)	$V_t(i^e_{s,t+1})$	3.6643 (2.98)	4.7323 (3.58)
$Cov_{x,t}(\Delta c_{t+1}, i^e_{s,t+1})$	30.9329 (0.33)	64.7839 (0.38)	$V_{x,t}(i^e_{s,t+1})$	67.6234 (1.60)	−31.2241 (0.30)
$\hat{\gamma}_1 + \hat{\gamma}_2$	221.4475 (2.30)	353.1047 (2.52)	$\hat{\gamma}_1 + \hat{\gamma}_2$	71.2877 (1.69)	−26.4918 (0.25)

Estimates of CCAPMA and CAPMA, UK and US. Absolute t-statistics in parenthesis.
Emphasised parameters significant using the 95 % critical value.

We note that the estimated long run coefficient of relative risk aversion is lower, though not significantly different. In both countries we reject the null hypothesis of no significant time-variation in the risk premium implied by the PU-Nom-H0 model but on the contrary, in both countries, we accept the null hypothesis of no time-variation in the risk premium implied by the CAPM. This could be the reason why Glosten, Jagannathan and Runkle [66] and Scruggs [101] find the excess return to be insignificantly priced when including a constant in the mean excess return equation - the risk premium implied by the CAPM is constant whereas the risk premium implied by the CCAPM is not ! Time-variation in the conditional covariance between log return and consumption growth captures time-varying risk premia.

2.7.2 The Framework of Lettau and Ludvigson

An aim of this chapter is to propose an estimation method to estimate different consumption-based models. The estimation method proposed in this chapter can easily be extended to allow for time-varying coefficients in the SDF. In this case we have very realistic models with time-varying coefficients and a time-varying conditional covariance between returns and the macroeconomic variables. To illustrate the estimation allowing for time-varying coefficients, let us consider the Lettau and Ludvigson CCAPM. The equivalent SDF is given by equation (1.35) and the no-arbitrage condition given by equation (1.36). All we have to do is to include cay_t and model the coefficient of relative risk aversion as $(b_1 + b_2 cay_t)$ in the log excess return equation and estimate b_1, b_2. The main problem for us in evaluating the Lettau and Ludvigson model, is that monthly data are not available on cay_t and the highest frequency is quarterly. With quarterly data it is usually difficult to get more than 40 years of data. With such short samples along with the difficulty in identifying conditional heteroskedasticity in lower frequency data using the multivariate GARCH model may not be appropriate. It could be interesting to look for other variables that may capture some of the effects of cay_t using monthly data. Time-varying coefficients were not the aim of this chapter and therefore will not be discussed more detail. Duffee [43] estimate a model of this kind but assume that cay_t in first two months of a quarter is equal to the value of the previous quarter.

We have assumed that the return on the wealth portfolio is equal to a linear combination of return on equity and a risk-free investment. As pointed out in Restoy and Weil [95], the return to wealth is linked to consumption by

$$1 + \mathcal{R}_{w,t+1} = \frac{W_{t+1}}{W_t - C_t} \tag{2.48}$$

Restoy and Weil [95] show that log linearising the budget constraint one can show that the implication in an Epstein Zin model on the no-arbitrage condition is that

$$\begin{aligned} \mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) &= (\theta - 1)\mathrm{Cov}_t(i^e_{s,t+1}, c_{t+1} - w_{t+1}) \\ &+ \gamma \mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \end{aligned} \tag{2.49}$$

Hence we have conditional betas - one with the consumption aggregate wealth ratio and a conditional consumption beta. We still have the problem that the consumption aggregate wealth ratio is not observable but the above derivation gives another explanation as to the role of the approximate consumption aggregate wealth ratio derived by Lettau and Ludvigson - it should be included as an additional beta and perhaps not as a multiple with the covariance of consumption.

2.8 Robustness - Another Extreme Event ?

We have included a dummy variable in October 1987 - US and UK stock prices fell dramatically during this month and we found in the UK and US estimations that it appeared to be a significant outlier. Looking at the data in figure (2.1) we note another potential outlier in the two excess return series. In August 1998 log excess return in the US fell, annualised, by more than 200% - this was not so radical in the UK. On the other hand in September 1981 log excess return, annualised, in the UK fell by more than 200% which was not the case in the US. It is of interest to see whether the particular outlier in the US and UK individually affects the estimates in the risk premium models. We estimate all models with an additional dummy variable - August 1998 in the US and September 1981 in the UK. The UK results can be found in table (2.12).

In the UK we note that the dummy variable in September 1981 is insignificant but has a high absolute value. It is seen, in the general model, that inflation is borderline significant but consumption growth loses significance. The residual of the UK excess return has a mean closer to zero and industrial production growth is less significant. The relative variance of the implied risk premium to the actual data is lower than in estimations with only one dummy variable.

Table 2.12: UK Estimates With Two Extreme Events

UK: t-Distribution and dummy	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	CAPM
$V_t(i^e_{s,t+1})$	3.8532 (0.89)		3.3569 (0.77)				**3.3485** (2.21)	**3.8852** (3.27)
$C_t(i^e_{s,t+1}, \pi_{t+1})$	**−523.7383** (1.96)	**−639.2499** (2.32)	**−584.9641** (2.51)	**−619.7022** (2.56)	0	−1	**−3.3485** (2.21)	**−3.8852** (3.27)
$C_t(i^e_{s,t+1}, \Delta c_{t+1})$	179.5319 (1.44)	152.9439 (1.24)	144.6420 (1.33)	166.8318 (1.55)	**196.9726** (2.43)	**197.6131** (2.43)	103.2065 (1.39)	
$C_t(i^e_{s,t+1}, \Delta y_{t+1})$	−36.0812 (0.39)	11.2617 (0.13)						
$\Upsilon_{1987:10,t+1}$	−0.3185 (1.61)	−0.3195 (1.58)	−0.3195 (1.53)	−0.3191 (1.61)	−0.3138 (1.39)	−0.3138 (1.38)	−0.3159 (1.44)	−0.3176 (1.37)
$\Upsilon_{1981:09,t+1}$	−0.1884 (0.11)	−0.1886 (0.13)	−0.1891 (0.11)	−0.1883 (0.14)	−0.1895 (0.06)	−0.1896 (0.05)	−0.1948 (0.22)	−0.1949 (0.20)
Degrees of Freedom	**11.9640** (3.18)	**11.5856** (3.27)	**11.9086** (3.22)	**11.5879** (3.25)	**13.6179** (2.97)	**13.1096** (3.02)	**13.6496** (2.86)	**13.3190** (2.90)
Log likelihood	3919.0387	3918.7119	3918.9976	3918.7064	3914.9601	3914.9825	3916.6464	3915.8604
$\lvert\lambda\rvert_{max}$	0.980	0.981	0.980	0.980	0.973	0.973	0.972	0.973
$\bar{\varepsilon}_{t+1}$	-0.8425	-0.2653	-0.8179	-0.2275	1.1864	1.1284	-1.4927	-1.1723
$V(\phi_{t+1})$	87.9882	105.5663	92.5334	104.2556	57.4074	57.6479	20.3913	0.1477
$\frac{V(\phi_{t+1})}{V(i^e_{s,t+1}+\frac{1}{2}V_t(i^e_{s,t+1})-\theta_1\Upsilon_{1987:10,t+1}-\theta_2\Upsilon_{1981:09,t+1})}$	0.0287	0.0345	0.0302	0.0340	0.0187	0.0189	0.0067	0.00005

UK Results, 1975-2002. Using multivariate GARCH in mean model with symmetric ARCH and GARCH matrices. Absolute t-statistics in parenthesis. Emphasised parameters significant using the 95 % critical value. Two dummy variables.

In the US, table (2.13), inclusion of an additional dummy variable increases the significance of inflation in almost all models where it is priced and consumption growth less significant. The estimated degrees of freedom increases much suggesting a joint conditional distribution with less heavy tails, the relative variance of the implied risk premium to the variance of the actual data is smaller relative to inclusion of only one dummy variable - it is mainly the case in the Power Utility CCAPM - finally we note that the dummy variable is significantly estimated in all models using the 90% critical value.

Our results suggests that august 1998 may be a significant outlier in the US, the observation creating heavy tails in the joint distribution, whereas this is not the case for September 1981 in the UK. In both countries, however, including a dummy variable increases the precision in the estimate of the coefficient on the inflation covariance and decreases the significance of the consumption covariance.

2.9 Conclusion

In this chapter we propose a method to estimate the risk premium implied by various General and Partial Equilibrium models. The advantage of the approach is that all preference parameters and

Table 2.13: US Estimates With Two Extreme Events

US: t-Distribution and dummy	General	SDF	EZ1	PU-Nom-H1	PU-Nom-H0	PU-Real-H0	EZ2	Capm
$V_t(i^e_{s,t+1})$	−2.3888 (0.50)		−2.3818 (0.50)				−0.8443 (0.17)	**5.7715 (4.07)**
$C_t(i^e_{s,t+1}, \pi_{t+1})$	−598.0080 (1.88)	−590.9736 (1.89)	**−600.1251 (2.05)**	**−587.19 (2.02)**	0	−1	0.8443 (0.17)	**−5.7715 (4.07)**
$C_t(i^e_{s,t+1}, \Delta c_{t+1})$	293.7342 (1.92)	**239.3276 (2.47)**	**293.1344 (1.97)**	**240.16 (2.54)**	**292.1599 (2.96)**	**291.9976 (2.96)**	318.0805 (1.85)	
$C_t(i^e_{s,t+1}, \Delta y_{t+1})$	−1.5908 (0.02)	3.1585 (0.04)						
$\Upsilon_{1987:10,t+1}$	**−0.2509 (1.99)**	**−0.2512 (2.00)**	**−0.2508 (2.00)**	**−0.2514 (1.99)**	**−0.2550 (2.10)**	**−0.2550 (2.10)**	**−0.2548 (2.13)**	**−0.2530 (1.99)**
$\Upsilon_{1998:08,t+1}$	−0.1586 (1.68)	−0.1584 (1.68)	−0.1587 (1.69)	−0.1584 (1.70)	**−0.1819 (2.06)**	**−0.1819 (2.05)**	**−0.1828 (2.11)**	−0.1698 (1.55)
Degrees of Freedom	**18.3216 (2.23)**	**18.3282 (2.26)**	**18.3377 (2.23)**	**18.2998 (2.67)**	**16.9136 (2.40)**	**16.9168 (2.40)**	**16.8705 (2.39)**	**18.0802 (2.27)**
Log likelihood	4446.5249	4446.3543	4446.5247	4446.3536	4444.3740	4444.3807	4444.3913	4442.8355
$\lvert\lambda\rvert_{max}$	0.9607	0.9607	0.9607	0.9606	0.9571	0.9571	0.9570	0.9588
$\bar{\varepsilon}_{t+1}$	-2.3514	-2.8441	-2.3497	-2.8519	-2.2837	-2.2855	-2.0638	-2.5029
$V(\phi_{t+1})$	68.3968	56.7219	68.4090	56.6247	27.5993	27.5908	32.2909	0.1520
$\frac{V(\phi_{t+1})}{V(i^e_{s,t+1}+\frac{1}{2}V_t(i^e_{s,t+1})-\hat{\theta}_1\Upsilon_{1987:10,t+1}-\hat{\theta}_2\Upsilon_{1998.09,t+1})}$	0.0276	0.0229	0.0276	0.0229	0.0111	0.0112	0.0131	0.00006

US Results, 1975-2002. Using multivariate GARCH in mean model with symmetric ARCH and GARCH matrices. Absolute t-statistics in parenthesis. Emphasised parameters significant using the 95 % critical value. Two dummy variables.

portfolio weights can be recovered while getting an estimate of the time-varying risk premium. We derived approximate standard errors of the estimated preference parameters and portfolio weights.

We propose a general representative agent consumption-based asset pricing model - we assume that the representative investor has Generalised Isoelastic Preferences. We derive the no-arbitrage condition using nominal and real return and show that the difference in the risk premium when using nominal returns instead of real returns is equal to one unit of covariance between inflation and return on the asset - we show that the magnitude of this difference is very small when using monthly data.

Assuming that the wealth portfolio of the representative investor is a combination of an investment in a broad stock market index and a risk-free asset we show that this representative agent model implies that consumption growth, inflation and stock market return are all potential sources of risk to be priced unrestricted. We argued that all models could be given an interpretation as a Stochastic Discount Factor model. Interpreting the models in SDF framework we allow for the possibility that markets are not complete - different SDFs could be valid in different asset markets - the SDF may not be unique.

With an empirical application, 1975-2002 using monthly data, to broad stock market indices in the UK and US we show that real non-durable consumption growth and inflation both are significant sources of risk to be priced. In the UK inflation is most significant while in the US real non-durable consumption growth is most significant. Stock returns are not significantly priced in any of the two countries. We model the excess return, inflation and consumption growth joint with industrial production growth arguing that it may yield a better estimate of the conditional covariance matrix - interpreting our models as SDF models we tested a general model where we, in addition to other variables, priced industrial production growth. In neither of the countries do we find it to be significantly priced, having accounted for the sources of risk in the most general consumption based model, and the results suggest that a two factor SDF model, log-linear in inflation and consumption growth, may be an appropriate SDF model in the UK and US stock markets.

US and UK expost log excess return have a high correlation in the sample period under consideration. We show, on the contrary, that the expected excess return, the risk premium, implied by well known asset pricing models has a very low correlation - the traditionally used Power Utility inter-temporal CCAPM implies a risk premium in the UK and US with a negative correlation of -0.20. This is a puzzle - why should the risk premium in the UK and US be negatively correlated ? On the other hand it suggests that common shocks may be the reason for the high sample correlation.

We show that the conditional variance of consumption growth in the UK and US has been declining over the sample period. Since the Sharpe Ratio of the data has remained relatively high in the 1990s this makes the equity premium puzzle even stronger.

In addition we find that the volatility of industrial production growth has converged in the UK and US. In the beginning of the sample UK industrial production growth had a much higher variance relative to the US.

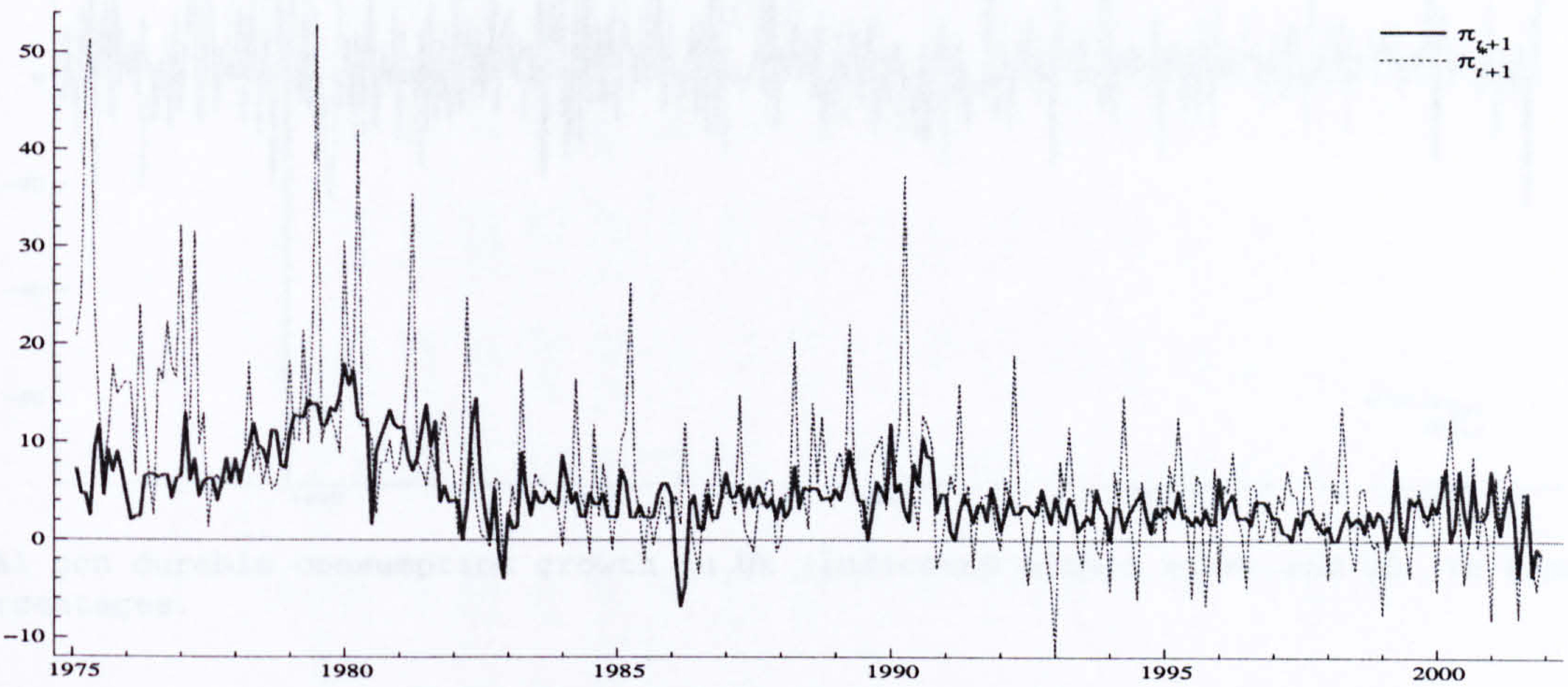
Finally we propose a test, using the multivariate GARCH in mean model, whether the Power Utility ICCAPM model and a traditional version of the CAPM implied a risk premium that had significant time-variation. The test allowed us to estimate the coefficient of relative risk aversion in the long run. We found that a traditional used version of the CAPM implied, as well in the UK as the US, a risk premium without significant time-variation whereas the Power

Utility ICCAPM implied a risk premium that is not constant.

In summary we conclude that an asset pricing model with macroeconomic sources of risk to be priced is adequate for modelling the time-varying risk premium in the UK and US. There seems to be benefits from estimating the stock market risk premium by modelling the joint distribution of macroeconomic variables with stock returns - instead of thinking of Partial and General Equilibrium models it may be of useful to think of SDFs logarithmic linear in macroeconomic variables. Macroeconomic risks seem to be priced in the US and UK stock markets and well-known asset pricing models imply a time-varying equity risk premium. US and UK risk premia varies considerably over time and varies considerably more than the conditional return variance.

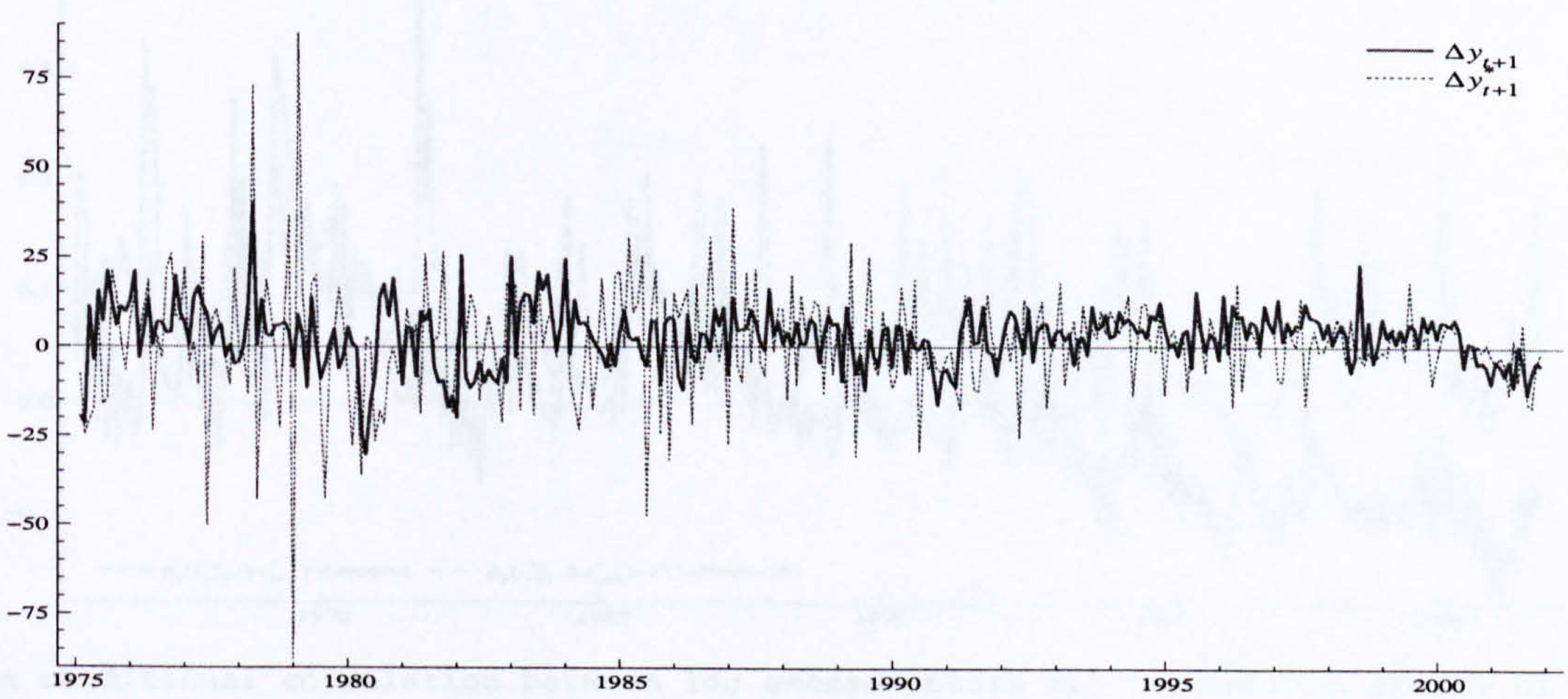
2.10 Appendix Chapter 2

Figure 2.10: UK and US Inflation Rates



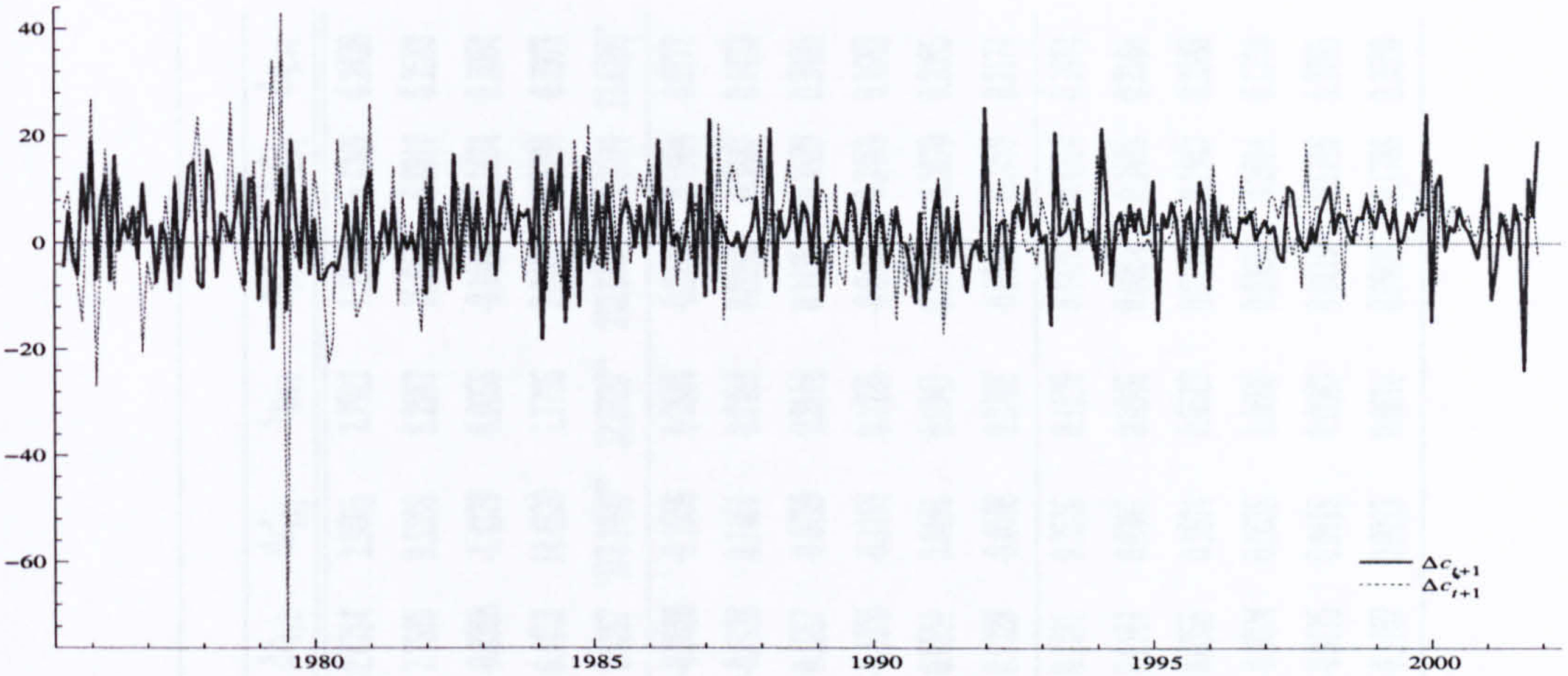
US (CPI log differences) and UK (RPI log differences) inflation in annual percentages. A star indicates a UK variable.

Figure 2.11: UK and US Industrial Production Growth



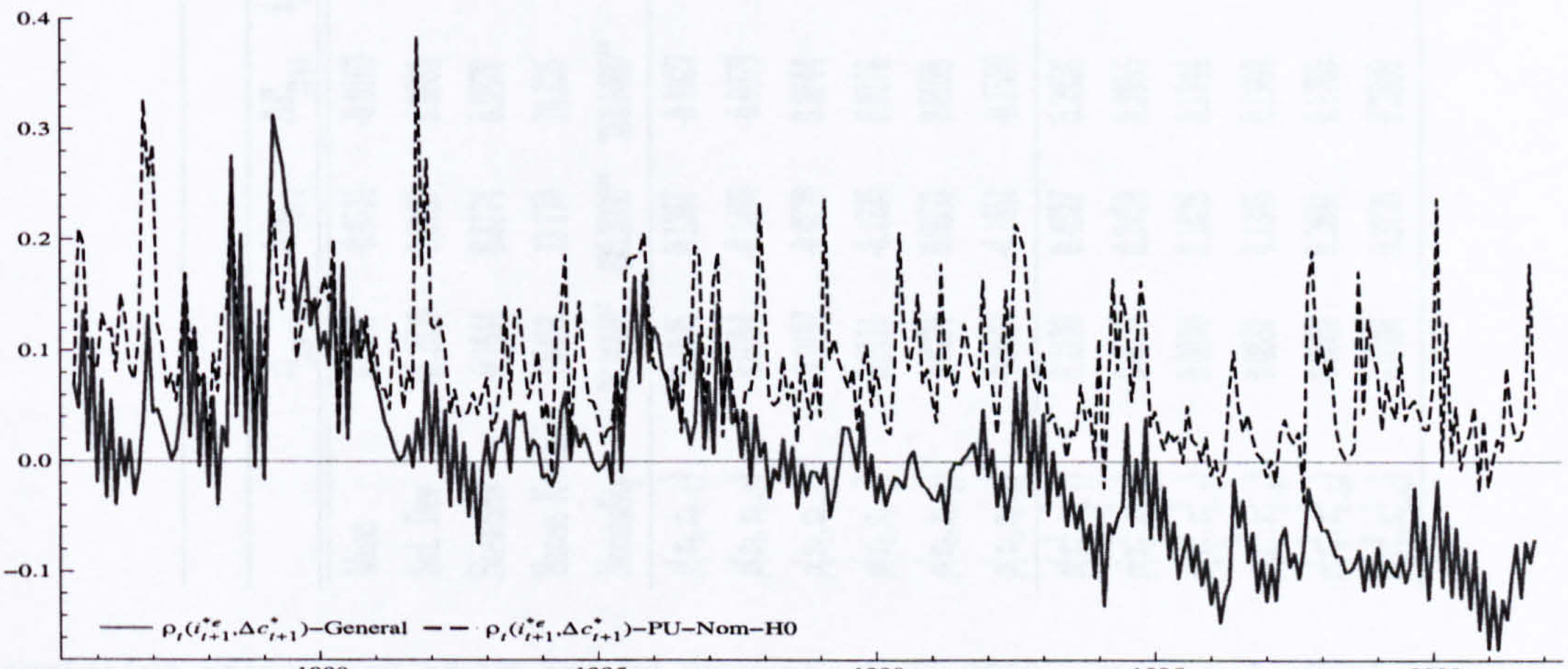
US and UK industrial production growth in annual percentages. A star indicates a UK variable.

Figure 2.12: UK and US Real Consumption Growth



Real non durable consumption growth in UK (indicated with a star) and US. In annual percentages.

Figure 2.13: Conditional Correlation Between Stock Return And Consumption Growth



The conditional correlation between log excess return and consumption growth in the UK in the General and PU-Nom-H0 model.

Table 2.14: Descriptive Statistics Of Dataset

	x													
	$i^e_{fx,t+1}$	$\Delta i_{f,t+1}$	$\Delta i^*_{f,t+1}$	$i_{f,t+1}-i^*_{f,t+1}$	$i^e_{s,t+1}$	$i^{*e}_{s,t+1}$	π_{t+1}	π^*_{t+1}	Δc_{t+1}	Δc^*_{t+1}	Δy_{t+1}	Δy^*_{t+1}	Δq^*_{t+1}	Δq_{t+1}
Mean	0.8457	-0.0131	-0.0163	-2.3749	6.2508	5.3242	4.5212	6.0657	2.6784	2.9605	2.7923	1.3879	5.4249	6.2428
Std. Dev	38.2830	0.8187	0.9885	2.9281	54.2440	60.4400	3.6252	7.3884	7.7575	9.5286	8.2967	15.644	6.8917	6.2583
Skewness	-0.1644	0.8174	0.9870	-0.5920	-1.0174	-0.9398	0.8340	1.8082	-0.0308	-1.4379	0.0055	-0.1004	0.1434	0.3064
Excess Kurtosis	1.6452	13.170	10.825	1.9679	3.9780	4.5808	1.0578	6.5753	0.4453	14.4820	1.7726	7.0462	0.3794	0.7973
Normality	28.0650**	490.3900**	353.1400**	27.5200**	50.4180**	65.9750**	37.0090**	117.0600**	3.6367	273.2900**	32.6320**	232.6600**	3.3024	13.6195*
$\rho(x_t,x_{t-1})$	0.0846	0.1261	-0.1027	0.9143	0.0339	0.0199	0.7233	0.3903	-0.3686	-0.1959	0.2884	-0.2978	0.5904	0.0777
$\rho(x_t,x_{t-2})$	0.0394	-0.1470	-0.0079	0.8261	-0.0874	-0.1147	0.5993	0.2793	-0.0178	0.1464	0.2483	0.0082	0.3847	0.1433
$\rho(x_t,x_{t-3})$	-0.0197	-0.0770	0.0844	0.7505	-0.0556	-0.0948	0.5921	0.2177	0.1587	-0.0759	0.2116	0.1176	0.3820	0.2402
$\rho(x_t,x_{t-4})$	0.0111	-0.1235	0.0178	0.6804	-0.0586	0.0229	0.5526	0.2123	-0.1305	-0.1168	0.1225	-0.0109	0.3489	0.1682
$\rho(x_t,x_{t-5})$	0.0294	0.0650	0.0028	0.6152	0.0751	-0.0030	0.5211	0.2749	0.0551	0.0496	0.0941	0.0495	0.3670	0.2085
$\rho(x_t,x_{t-6})$	-0.0504	-0.1651	-0.1550	0.5454	0.0112	-0.0967	0.5144	0.3893	0.0228	-0.0108	0.0798	-0.0989	0.3759	0.2174
$\rho(x_t^2,x_{t-1}^2)$	0.2130	0.4627	0.2958	0.8314	0.0225	0.0861	0.7879	0.1669	0.1011	0.2516	0.1529	0.4476	0.4014	0.1074
$\rho(x_t^2,x_{t-2}^2)$	0.0559	0.2439	0.3955	0.6857	0.0288	0.0731	0.7108	0.0972	0.0164	0.0301	0.0940	0.0284	0.2415	0.2168
$\rho(x_t^2,x_{t-3}^2)$	0.0070	0.1628	0.1344	0.5945	0.0150	-0.0189	0.6399	0.2107	0.0358	0.1624	0.0512	0.0125	0.1943	0.2583
$\rho(x_t^2,x_{t-4}^2)$	0.0881	0.1385	0.1903	0.5015	0.0061	-0.0252	0.5963	0.0460	-0.0024	0.0638	0.0698	0.0303	0.2014	0.1750
$\rho(x_t^2,x_{t-5}^2)$	0.0502	0.2681	0.1765	0.3885	-0.0054	0.0096	0.5897	0.0645	-0.0175	0.0115	0.0083	0.0219	0.1716	0.2765
$\rho(x_t^2,x_{t-6}^2)$	0.0304	0.3720	0.2880	0.3194	0.0070	-0.0491	0.5699	0.2097	-0.1152	0.0215	0.0876	0.0900	0.1788	0.3500

Descriptive statistics of UK and US Dataset, 1975:2002. Mean and standard deviation in annual percentages. Variables with a star as superscript indicate that the variable is a UK variable. Two stars in Normality row as superscript rejects hypothesis of normality using a 99% critical value and a star using the 95% critical value. $i^e_{fx,t+1}$ is FOREX logarithmic excess return, $\Delta i_{f,t}$ is innovations in the 1 month nominal interest rate, $i^e_{s,t+1}$ is logarithmic stock market excess return, π is first difference of logarithmic price level, Δc is first difference of logarithmic consumption, Δy is the first difference of the logarithm of industrial production and Δq is the first difference of the logarithm of money (M1 in US and narrow money in UK).

Table 2.15: Correlation And Covariance Matrix Of Dataset

	$i_{fz,t+1}$	$\Delta i_{f,t+1}$	$\Delta i^*_{f,t+1}$	$i_{f,t+1}-i^*_{f,t+1}$	$i^e_{s,t+1}$	$i^{*e}_{s,t+1}$	π_{t+1}	π^*_{t+1}	Δc_{t+1}	Δc^*_{t+1}	Δy_{t+1}	Δy^*_{t+1}	Δq^*_{t+1}	Δq_{t+1}
$i_{fz,t+1}$	1470.204	-7.015	-6.572	-21.930	-25.828	-188.125	2.651	27.641	-29.942	0.960	-35.767	2.429	-14.239	-13.473
$\Delta i_{f,t+1}$	-0.224	0.6702	0.079	0.401	-4.137	-0.5879	0.243	0.242	0.017	-0.594	1.194	-0.340	0.648	0.484
$\Delta i^*_{f,t+1}$	-0.173	0.097	0.9772	-0.3422	1.569	-12.3657	0.887	0.014	0.538	0.562	0.056	2.875	-0.390	0.522
$i_{f,t+1}-i^*_{f,t+1}$	-0.195	0.167	-0.118	8.601	-16.743	-2.334	0.153	-3.948	0.854	2.238	0.697	-2.324	-5.301	-0.138
$i^e_{s,t+1}$	-0.012	-0.093	0.029	-0.105	2951.662	2042.094	-24.659	-14.865	66.161	17.972	-35.565	-24.707	12.950	22.309
$i^{*e}_{s,t+1}$	-0.081	-0.012	-0.207	-0.013	0.621	3664.573	-24.833	-1.769	72.366	40.211	-12.603	3.085	24.283	0.119
π_{t+1}	0.019	0.082	0.121	0.014	-0.125	-0.113	13.183	12.256	-4.694	-3.405	0.319	-3.356	0.514	2.376
π^*_{t+1}	0.097	0.040	0.002	-0.182	-0.037	-0.004	0.456	54.761	-7.207	-18.324	-0.124	-1.166	4.021	6.687
Δc_{t+1}	-0.101	0.003	0.070	0.038	0.157	0.154	-0.166	-0.125	60.369	8.886	9.137	12.206	3.047	-0.539
Δc^*_{t+1}	0.003	-0.076	0.060	0.080	0.035	0.070	-0.098	-0.259	0.120	91.081	-0.209	23.130	0.621	-1.212
Δy_{t+1}	-0.112	0.281	0.007	0.029	-0.079	-0.025	0.011	-0.002	0.141	-0.003	69.326	30.777	-3.072	7.251
Δy^*_{t+1}	0.004	-0.027	0.186	-0.051	-0.029	0.003	-0.059	0.010	0.101	0.155	0.237	243.871	-1.425	-3.179
Δq^*_{t+1}	-0.054	0.115	-0.058	-0.264	0.035	0.059	0.021	0.079	0.057	0.009	-0.054	-0.013	46.910	5.748
Δq_{t+1}	-0.050	0.084	0.075	-0.007	0.059	0.0003	0.093	0.129	-0.010	-0.018	0.124	-0.029	0.120	49.115

Descriptive statistics of UK and US Dataset, 1975:2002. Correlation matrix is in the lower half of the matrix and the covariance matrix between the variables in the upper half. Covariance matrix of the annualised dataset. The diagonal contains variances of the variables (annualised data). Star as superscript for the variables indicates that the variable is a UK variable.

Table 2.16: Estimated parameters In GARCH Matrix, US

US:GARCH	11	21	22	31	32	33	41	42	43	44
General	**0.907** **(21.71)**	0.005 (0.37)	**0.918** **(28.96)**	0.001 (0.04)	0.030 (1.49)	**0.913** **(17.61)**	**0.234** **(2.62)**	0.011 (0.15)	−0.201 (1.25)	**−0.485** **(3.64)**
SDF	**0.904** **(21.10)**	0.006 (0.43)	**0.920** **(30.00)**	−0.002 (0.05)	0.030 (1.50)	**0.915** **(18.56)**	**0.244** **(2.73)**	0.007 (0.09)	−0.185 (1.14)	**−0.495** **(3.84)**
EZ1	**0.907** **(22.27)**	0.004 (0.33)	**0.917** **(29.01)**	0.002 (0.07)	0.031 (1.50)	**0.912** **(17.45)**	**0.232** **(2.60)**	0.014 (0.19)	−0.207 (1.38)	**−0.483** **(3.65)**
Pu-Nom-H1	**0.906** **(21.94)**	0.004 (0.32)	**0.919** **(29.91)**	0.001 (0.04)	0.0308 (1.53)	**0.911** **(17.46)**	**0.238** **(2.67)**	0.016 (0.22)	−0.203 (1.32)	**−0.490** **(3.80)**
Pu-Nom-H0	**0.913** **(24.34)**	0.003 (0.27)	**0.909** **(25.73)**	−0.002 (0.07)	0.039 (1.68)	**0.901** **(16.07)**	**0.207** **(2.38)**	0.035 (0.48)	−0.219 (1.44)	**−0.478** **(3.47)**
PU-Real-H0	**0.913** **(24.35)**	0.003 (0.27)	**0.909** **(25.74)**	−0.002 (0.07)	0.039 (1.69)	**0.901** **(16.08)**	**0.207** **(2.38)**	0.035 (0.48)	−0.219 (1.44)	**−0.478** **(3.47)**
EZ2	**0.913** **(24.66)**	0.003 (0.26)	**0.908** **(25.38)**	−0.001 (0.05)	0.038 (1.68)	**0.903** **(15.90)**	**0.203** **(2.34)**	0.036 (0.49)	−0.223 (1.50)	**−0.471** **(3.35)**
CAPM	**0.5635** **(2.78)**	**−0.188** **(2.61)**	**0.732** **(7.68)**	**0.462** **(4.22)**	**0.253** **(2.96)**	**0.456** **(2.03)**	**0.518** **(4.04)**	**0.278** **(3.72)**	**−0.612** **(6.52)**	0.276 (1.37)

US, 1975-2002: Estimates of parameters in GARCH matrix for all models. Top row indicates entry ij=ji in GARCH parameter matrix. Emphasised parameters significant using a 95% critical value. Numbers in parenthesis are absolute t-statistics.

Table 2.17: Estimated parameters In GARCH Matrix, UK

UK:GARCH	11	21	22	31	32	33	41	42	43	44
General	−0.010 (0.06)	**0.487** **(4.83)**	0.016 (0.05)	−0.243 (1.52)	**−0.679** **(9.14)**	−0.026 (0.09)	**0.613** **(5.98)**	0.020 (0.23)	**0.553** **(8.63)**	**0.389** **(2.64)**
SDF	0.004 (0.02)	**0.500** **(5.23)**	0.093 (0.31)	−0.197 (1.24)	**−0.678** **(8.67)**	−0.100 (0.36)	**0.603** **(5.90)**	0.001 (0.01)	**−0.557** **(9.09)**	**0.389** **(2.61)**
EZ1	0.021 (0.11)	**0.509** **(5.57)**	0.214 (0.68)	−0.134 (0.83)	**−0.659** **(6.60)**	−0.210 (0.76)	**0.589** **(5.77)**	−0.029 (0.31)	**0.561** **(9.17)**	**0.389** **(2.60)**
Pu-Nom-H1	0.021 (0.12)	**0.506** **(5.66)**	0.206 (0.67)	−0.139 (0.88)	**−0.661** **(6.80)**	−0.204 (0.74)	**0.589** **(5.82)**	−0.027 (0.29)	**0.560** **(9.35)**	**0.390** **(2.61)**
Pu-Nom-H0	−0.273 (1.41)	**0.385** **(2.69)**	**0.721** **(3.70)**	0.120 (1.31)	**−0.438** **(2.24)**	**−0.550** **(3.43)**	**0.445** **(3.51)**	0.008 (0.09)	**0.557** **(6.46)**	**0.548** **(4.77)**
PU-Real-H0	−0.273 (1.41)	**0.385** **(2.69)**	**0.721** **(3.70)**	0.120 (1.31)	**−0.438** **(2.24)**	**−0.550** **(3.43)**	**0.445** **(3.51)**	0.008 (0.09)	**0.557** **(6.46)**	**0.548** **(4.77)**
EZ2	−0.216 (1.01)	**0.370** **(2.80)**	**0.716** **(3.61)**	0.147 (1.63)	**−0.459** **(2.28)**	**−0.535** **(3.14)**	**0.486** **(3.72)**	0.003 (0.04)	**0.549** **(6.28)**	**0.493** **(3.70)**
CAPM	−0.081 (0.36)	**0.366** **(2.96)**	**0.693** **(3.23)**	0.161 (1.64)	**−0.486** **(2.37)**	**−0.519** **(2.78)**	**0.536** **(4.56)**	−0.022 (0.25)	**0.527** **(6.16)**	**0.414** **(2.73)**

UK, 1975-202: Estimates of parameters in GARCH matrix for all models. Top row indicates entry ij=ji in GARCH parameter matrix. Emphasised parameters significant using a 95% critical value. Numbers in parenthesis are absolute t-statistics.

Table 2.18: Estimated parameters In ARCH Matrix, US

US:ARCH	11	21	22	31	32	33	41	42	43	44
General	**0.107** (3.22)	0.001 (0.14)	**0.310** (5.20)	**0.033** (3.63)	0.027 (1.27)	−0.029 (0.52)	**0.036** (3.17)	0.040 (1.34)	0.045 (0.78)	**0.589** (6.47)
SDF	**0.104** (3.29)	0.0004 (0.10)	**0.307** (5.22)	**0.033** (3.68)	0.027 (1.26)	−0.036 (0.63)	**0.036** (3.26)	0.042 (1.38)	0.039 (0.68)	**0.591** (6.54)
EZ1	**0.107** (3.39)	0.0004 (0.13)	**0.310** (5.24)	**0.033** (3.75)	0.027 (1.29)	−0.027 (0.50)	**0.036** (3.17)	0.039 (1.31)	0.047 (0.87)	**0.589** (6.48)
Pu-Nom-H1	**0.106** (3.45)	0.0001 (0.04)	**0.308** (5.24)	**0.034** (3.85)	0.027 (1.30)	−0.033 (0.58)	**0.037** (3.27)	0.038 (1.27)	0.044 (0.81)	**0.590** (6.53)
Pu-Nom-H0	**0.110** (3.46)	−0.0003 (0.07)	**0.304** (4.90)	**0.032** (3.71)	0.036 (1.60)	−0.042 (0.77)	**0.034** (3.01)	0.031 (1.07)	0.048 (0.90)	**0.571** (6.36)
PU-Real-H0	**0.110** (3.46)	−0.0003 (0.07)	**0.304** (4.90)	**0.032** (3.71)	0.036 (1.60)	−0.042 (0.77)	**0.034** (3.01)	0.031 (1.07)	0.048 (0.90)	**0.571** (6.36)
EZ2	**0.111** (3.40)	−0.00002 (0.01)	**0.304** (4.90)	**0.031** (3.61)	0.036 (1.58)	−0.039 (0.74)	**0.034** (3.00)	0.032 (1.10)	0.050 (0.95)	**0.572** (6.34)
CAPM	−0.002 (0.11)	−0.002 (0.58)	**0.359** (4.77)	0.014 (1.83)	0.052 (1.89)	0.037 (0.71)	**0.041** (4.42)	−0.058 (1.86)	**0.165** (3.53)	**0.345** (5.23)

US, 1975-202: Estimates of parameters in ARCH matrix for all models. Top row indicates entry ij=ji in ARCH parameter matrix. Emphasised parameters significant using a 95% critical value. Numbers in parenthesis are absolute t-statistics.

Table 2.19: Estimated parameters In ARCH Matrix, UK

UK:ARCH	11	21	22	31	32	33	41	42	43	44
General	**0.062** (3.29)	**−0.034** (3.70)	**0.187** (2.48)	**0.023** (2.00)	0.060 (1.20)	**−0.128** (2.23)	**−0.036** (2.42)	0.022 (0.59)	−0.059 (1.46)	**0.422** (6.95)
SDF	**0.068** (3.59)	**−0.033** (3.61)	**0.182** (2.49)	**0.023** (2.06)	0.066 (1.35)	**−0.118** (2.06)	**−0.038** (2.46)	0.026 (0.70)	−0.066 (1.61)	**0.422** (6.77)
EZ1	**0.076** (3.85)	**−0.030** (3.46)	**0.172** (2.55)	**0.023** (2.04)	0.076 (1.69)	−0.097 (1.69)	**−0.038** (2.41)	0.031 (0.86)	−0.069 (1.65)	**0.422** (6.49)
Pu-Nom-H1	**0.076** (3.86)	**−0.030** (3.46)	**0.173** (2.55)	**0.023** (2.04)	0.075 (1.64)	−0.100 (1.74)	**−0.038** (2.44)	0.032 (0.88)	−0.070 (1.70)	**0.422** (6.53)
Pu-Nom-H0	**0.135** (2.68)	**−0.039** (4.05)	**0.115** (1.98)	**0.034** (3.34)	**0.102** (2.60)	−0.006 (0.12)	**−0.055** (3.19)	0.020 (0.59)	**−0.085** (2.10)	**0.399** (7.61)
PU-Real-H0	**0.135** (2.68)	**−0.039** (4.05)	**0.115** (1.98)	**0.034** (3.34)	**0.102** (2.60)	−0.006 (0.12)	**−0.055** (3.19)	0.020 (0.59)	**−0.085** (2.10)	**0.399** (7.61)
EZ2	**0.137** (2.68)	**−0.039** (3.93)	**0.120** (2.03)	**0.031** (2.85)	**0.105** (2.58)	−0.005 (0.10)	**−0.061** (3.58)	0.021 (0.59)	−0.081 (1.88)	**0.424** (6.91)
CAPM	**0.122** (2.85)	**−0.037** (3.55)	**0.128** (2.12)	**0.027** (2.29)	**0.109** (2.53)	−0.010 (0.18)	**−0.062** (3.65)	0.026 (0.69)	−0.081 (1.78)	**0.450** (6.17)

UK, 1975-2002: Estimates of parameters in GARCH matrix for all models. Top row indicates entry ij=ji in GARCH parameter matrix. Emphasised parameters significant using a 95% critical value. Numbers in parenthesis are absolute t-statistics.

3. Macroeconomic Sources of Equity Risk - An Alternative SDF Model

An Econometric Model to Investigate the Relation Between The Business Cycle and Stock Returns

3.1 Introduction

In the previous chapter we estimated the time-varying risk premium implied by several well-known asset pricing models and showed that consumption growth and inflation are potentially factors to be priced in the UK and US stock markets in the period 1975-2002. The nice thing about the work in the previous chapter was that we derived the factors (sources of risk) to be priced in asset markets using a consumption based asset pricing model. The consumption based model analysed is a version of the Inter-temporal CAPM (ICAPM) of Merton [90]. From the ICAPM we note that the market return should be priced together with further variables that may affect the average investor/consumer (their marginal utility). We showed that two such additional factors were consumption growth and inflation. It is commonly emphasised (Merton [90], Cochrane [35] [37]) that factors containing sources of risk affecting the average investor are likely to be macroeconomic variables - people dislike, on average, recessions and unemployment.

Although consumption-based asset pricing models are very interesting intuitively, some problems arises when one wishes to test/estimate these models. One, that was mentioned in the previous chapter, is the difficulty to agree on the correct consumption measure for testing a specific asset pricing model. Moreover, consumption data may be subject to measurement error which could be one reason for the rejection of the model. Probably the most important problem in testing a consumption-based asset pricing model, using the approach we outlined in chapter 1 and any other approach emphasising joint conditional moments of return and consumption, is that consumption data are available only in few countries with a monthly frequency and the length of the available time-series is often insufficient for estimating a multivariate GARCH in mean model. Further, in the US, we found little ARCH effects in the consumption growth series that we used[1]. This raises the question whether other key macroeconomic variables, more commonly available (than consumption) with a monthly frequency, can capture the short term risk in the US stock market or other financial markets. It is often argued that monetary policy authorities set interest rates based on expectations of monetary aggregates, inflation, stock returns and/or the output gap. As we saw in the introductory chapter, if a risk-free return is set according to the expectations of a set of macroeconomic variables and if markets are complete, then the same factors that determine the risk-free interest rate must determine the risk premium on other risky financial assets. Commonly cited targeted variables are monetary aggregates, inflation, stock market variables and output measures. Schwert [98] and Fama [58] (among others) recognise and test whether there is a relation between stock returns and measures of monetary aggregates, price inflation and output. The two authors pay little attention, if any, to the modelling of the stock market risk premium - it may well be that covariances between stock market return and inflation, money, stock return (variances) and/or output determine the expected risk premium in the US (and other countries) stock market. If so, these macroeconomic variables are available in most countries with a monthly frequency. This is clearly an advantage if one wishes to use a multivariate GARCH-in-mean model to estimate the risk premium.

That macroeconomic variables should be priced in the stock market is very intuitive since it is difficult to hedge (see Shiller [104]) against fluctuations in the business cycle, in other words shocks to the macro economy affect the average investor. If macroeconomic factors need to be priced in the stock market, it is necessary for policy makers to understand the interaction between macroeconomic and financial variables. Much empirical work has been using vector auto regressions to investigate how various shocks affect levels of variables - if it is the case that the conditional covariance matrix between shocks to macroeconomic variables and stock returns is varying over time it is also interesting to analyse how shocks are transmitted into the conditional covariance matrix.

In the previous chapter we estimated the multivariate GARCH-in-mean models assuming the parameter matrices in the conditional covariance matrix, governing deviations from long run levels, were symmetric. The diagonal BEKK is a special case of the model we estimated and

[1] This is consistent with Duffee [43] using alternative consumption data.

the model we proposed seemed useful and adequate for modelling the risk premium with short lengths of the datasets. However, if we wish to estimate a multivariate model analysing the interaction between return shocks and macroeconomic shocks allowing for more general dynamics is desirable. To analyse the interaction between macroeconomic shocks and return shocks we need the parameter matrices to be fully flexible - further it would be of interest to use a specification of the conditional covariance matrix that allow positive and negative shocks to macroeconomic variables and returns to be transmitted differently into the conditional covariance matrix - for instance it makes good sense that negative return shocks increase the variance of return more than positive shocks and it is intuitive that negative output shocks, for instance, have different impact than negative shocks - the latter being presumably more undesirable because negative output shocks are often associated with recessions and unemployment.

The aim of the current chapter is to build a multivariate econometric model of the joint distribution of key macroeconomic variables and the return on a broad stock market index. As in the previous chapter the stock market return must obey the no-arbitrage condition implied by the SDF model. We considered modelling stock returns jointly with consumption growth, inflation and industrial production growth. In this chapter we wish to construct an empirical model to investigate the interaction between the macro economy and the stock return considering a broader set of key macroeconomic variables, available in most countries, potentially capable of capturing the sources of risks affecting the average investor. In the previous chapter we showed that risk premia in the stock market tend to be higher following negative shocks. In this chapter we extend the econometric model to allow positive and negative return or macroeconomic shocks to have different impacts - this allows us to investigate whether negative shocks have differential impacts on the risk premium and it allows us to measure the economic importance of negative shocks.

Empirically it seems that the volatility of stock returns is unusually high during recessions (see Schwert [98] [99], French, Schwert and Stambaugh [63] and/or Lettau and Ludvigson [82]) raising the question whether nominal or real macroeconomic variables can help predict the conditional variance of excess return. On the other hand, changes in stock market variability can have important effects on investment, consumption expenditure, production and other business cycle related variables. We wish to understand whether uncertainty in the business cycle predicts[2]

[2]Some studies have also been looking at the relation between uncertainty of output and inflation such as

uncertainty in the stock market, whether uncertainty in the stock market predicts uncertainty in the macro economy or whether the two sets of variables predict one another. Schwert [98] investigates the relation between the volatility, as measured by the time-varying standard deviation of the excess return and the volatility of macroeconomic variables in a long monthly US dataset from 1857 to 1987. The macroeconomic variables he considers are Producer Price Index (PPI) inflation, growth in narrow money and industrial production growth. Schwert does not use a multivariate model for investigating the relation and he is not concerned with theoretical issues associated with modelling the risk premium in the stock market - this latter variable he assumes can be proxied by lags of returns - an assumption, evident from the previous chapter, clearly not valid.

The aim, and main contribution, of this chapter is to investigate whether stock market uncertainty (standard deviation and variance), macroeconomic uncertainty and risk compensation (variance and/or covariance) in the stock market move together over time using a multivariate model more capable of answering exactly the questions considered by Schwert. Following up on his work the macroeconomic variables we model jointly with the stock market excess return are industrial production growth, CPI inflation (we use CPI to make our results more comparable to those of the previous chapter) and money (M1) growth. The advantage of the multivariate model we propose is that it allows us to test whether macroeconomic sources of risk are significantly priced in the stock market. With the stock return obeying a no-arbitrage condition we obtain a potential better description of shocks to stock returns allowing us to get a better estimate of the conditional variance of return. We do not specify a complete model of preferences but ensure the absence of arbitrage and the estimated model can be given an interpretation as a conditional CAPM.

In a multivariate model we need to make potentially strong assumptions on the conditional covariance dynamics of the joint distribution of the variables - the one assumption in this chapter is to assume a joint multivariate GARCH-in-mean structure. Using the multivariate GARCH-in-mean model we assume further that the dynamics of the conditional covariance matrix can be described by an extended version of the BEKK allowing for asymmetric impact of shocks to the variables. The Asymmetric BEKK (ABEKK) model was discussed in chapter 1. In this

Fountas, Karanasos and Kim [62] and Henry, Olekalns and Shields [68]. Our study is in a sense an augmentation of these studies since we can investigate the relation between output, inflation and stock return uncertainty.

chapter we make no unnecessary strong assumptions on the parameter matrices since we have a long sample and we wish to know exactly what are the causation between the business cycle and the stock market as suggested by the parameter matrices.

Much research has focused on asymmetries in the conditional variance of the stock returns, that is negative unexpected shocks to excess return have a different impact on the variance of stock returns than positive unexpected shocks - three explanations have been suggested for this phenomenon. The first explanation relates the asymmetry to leverage. When prices decline, the leverage ratio of firms is higher. Some studies have found that leverage effects cannot account for the sort of asymmetries found in stock market returns (Campbell, Lo and MacKinlay [29] and Bekaert and Wu [10]). The second explanation is the volatility feedback hypothesis (see Campbell and Hentschel [23]) - if we get good news about future dividend then, since volatility is persistent, we would expect to get further good news about dividends. This increases expected future volatility and hence the risk premium which in return lowers stock prices now, offsetting the dividend news effect. If, on the other hand, releases of negative dividend news then the volatility of returns will increase, implying increased expected return and hence a fall in the stock price today and the dividend news effect will be amplified. This chapter considers a third explanation; that the asymmetry may be present due to misspecification of the risk premium. Incorrect modelling, as will be shown, of the risk premium could potentially create asymmetries in the conditional variance. We show this to be the case when modelling risk compensation in the US stock market, 1960-2003.

To our knowledge this is the first study to construct a joint model of the stock market return and macroeconomic uncertainty allowing for, and documenting, asymmetries not found in previous research relating the findings to the US business cycle. Lettau and Ludvigson [82] attempts in a rather "ad-hoc" way to determine the relation between the US stock market risk premium and the variance of US stock return relating both to a business cycle variable[3] - however, we show that this relationship is relatively easily analysed, and more correctly, within our multivariate model. The chapter can also be seen as an alternative way to estimate a time-varying relation between the risk premium and the conditional return variance, where the innovation is in terms of estimating the numerator of the Sharpe Ratio as a linear combination of conditional covariances between returns and macroeconomic variables.

[3] The proxy for the consumption aggregate wealth ratio discussed in the first chapter.

This chapter is organised as follows. In section (3.2) the stock market risk premium is discussed, describing the Stochastic Discount Factor model, the CAPM and the more general Inter-temporal CAPM of Merton [90] establishing the link between the work in the present chapter with the work in chapter 2. In section (3.3) we discuss the modelling of the macroeconomic variables and discuss some empirical characteristics. Section (3.4) discusses the econometric framework allowing us to investigate the comovement in second moments of macroeconomic and financial variables, section (3.5) describes the models and data, section (3.6) presents the results and section (3.7) concludes.

3.2 The Stock Market Risk Premium

The aim of this section is to build a bridge between the work in the previous chapter with the work in the current chapter. We show how all SDF models imply a potential inter-temporal relation between the risk premium and the variance of returns.

3.2.1 The No-Arbitrage Condition

With the assumption we made on the wealth portfolio in the previous chapter the implied SDF by the most general consumption-based model is given by[4]

$$m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)(\omega_1 i_{f,t} + \omega_2 i_{s,t+1} - \pi_{t+1}) \qquad (3.1)$$

Hence the model[5] implies a logarithmic SDF linear in the market return, inflation and consumption growth. What was interesting was the finding that inflation was a significant source of risk priced in the UK (most strongly) and US stock markets. Implausible parameter estimates of preference parameters when using actual data is a problem for the consumption-based models (see Campbell [20][21]). Recently it has been shown that habit persistence models are capable of solving some of these problems but as we mentioned in the introductory chapter habit persistence models are usually calibrated and do not deliver an estimate of the risk premium.

As mentioned in Mehra [88], the consumption-based model is appealing since it has a sound theoretical justification to which it is hard to argue against. Further it is nice to use a theoretical

[4] Note we continue to use subscript s when it is a broad stock market index.
[5] Recall discussion in equation 2.8 in chapter 2.

model when the choice of factors to be priced has to be taken. However, in this chapter we step back and attempt pricing three alternative key macroeconomic variables and leave the challenge for theory to explain our results.

In the previous chapter we showed that inflation turned out to be a significantly priced variable and we may have to think of a broader class of variables which generate priced risk in financial markets. It may well be that not only unexpected shocks to consumption and financial variables matter but unexpected macroeconomic shocks in general. Claiming, as in Smith and Wickens [105], that the logarithmic SDF is a linear combination of macroeconomic- and potentially financial variables we can model the logarithmic SDF as

$$m_{t+1} = -\alpha_t - \mathbf{b}_t^{\mathsf{T}}\mathbf{f}_{t+1} + \zeta_{t+1}, \qquad \zeta_{t+1} \sim \mathcal{D}(0, \sigma_m^2) \tag{3.2}$$

where $\mathbf{f}$ is a vector of macroeconomic and/or financial variables and $\mathcal{D}$ could be any distribution. The variables can both be foreign and domestic - especially if we consider non-US financial markets. If markets are complete the SDF is unique, $\zeta_{t+1} \equiv 0$, whereas if markets are not complete there can exist several SDFs for different financial markets (see Cochrane [37]).

If the market return is a factor this is essentially the Inter-temporal CAPM (ICAPM) of Merton [90] - in the ICAPM the additional factors are associated with states of nature that affect the average investor. Since most people dislike recessions, or low economic activity, it is likely that the additional set of variables, to the market return, are macroeconomic (see Cochrane [35][37]). The no-arbitrage condition, assuming the parameters in the SDF to be constant, will be

$$\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) = \mathbf{b}^{\mathsf{T}}\mathrm{Cov}_t(\mathbf{f}_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(i^e_{s,t+1}, \pi_{t+1}) = \phi_t \tag{3.3}$$

We note that the implied nominal risk-free interest rate by this model is given by

$$i_{f,t} = -\mathrm{E}_t(m_{t+1}) + \mathrm{E}_t(\pi_{t+1}) + \frac{1}{2}\mathrm{V}_t(m_{t+1} - \pi_{t+1}) \tag{3.4}$$

The nominal interest rate depends on the expectations of the logarithmic SDF. Given the ways in which US monetary policy has been fixed in the four decades we will consider in this chapter, it is not implausible to assume that the risk-free interest rate is set based on expectations of stock returns, inflation, money growth and output growth over a given period (in our case over the given month). It is often argued that monetary authorities are targeting exactly these variables. Since Fama and Schwert investigates the interaction between these variables it may be that exactly these four variables variables should be priced in the stock market. In order to investigate this we need a multivariate model to estimate the time-varying covariance matrix of these variables. Such a model will be proposed in this chapter.

3.2.2 The Relation Between Mean and Variance and the Sharpe Ratio

The traditional unconditional[6] CAPM of Sharpe [103] and Lintner [84] appears as a special case of the SDF model when we model the risk premium on the market return. To make this clearer we re-write the no-arbitrage condition, including inflation as a factor, and obtain the implied relation between the time-varying risk premium and conditional return variance as

$$\begin{aligned} \mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) &= \sum_{j=1}^{k} \frac{b_j \rho_t(f_{j,t+1}, i^e_{s,t+1})\sigma_t(f_{j,t+1})}{\sigma_t(i^e_{s,t+1})} \mathrm{V}_t(i^e_{s,t+1}) \\ &= \underbrace{\sum_{j=1}^{k} \gamma_{j,t}}_{\overline{\gamma}_t} \mathrm{V}_t(i^e_{s,t+1}) = \phi_t, \end{aligned} \tag{3.5}$$

where $\rho_t(\cdot,\cdot)$ is the conditional correlation, $\sigma_t(\cdot)$ the conditional standard deviation and k the number of factors to be priced. This is essentially a conditional CAPM (intertemporal relation between risk premium on market portfolio and the conditional return variance). The unconditional Sharpe-Lintner version implies a SDF given by $m_{t+1} = -\alpha - b_1 i_{s,t+1}$, when only the market return is priced. However, given that other variables than the market return are priced the unconditional Sharpe-Lintner version of the CAPM is the special case of the SDF model when $\sum_{j=1}^{k} \gamma_{j,t}$ is constant. When this sum is not constant there is an intertemporal relation between the risk premium on the market portfolio and the conditional return variance.

[6]By unconditional we mean that the risk premium on the market portfolio is perfectly correlated with the conditional variance of the market return.

As mentioned in Graham and Harvey [67] their survey indicates that three-fourth of US firms use the CAPM of Sharpe and Lintner to establish the cost of capital and hence our investigation whether there is an inter-temporal relation between mean and variance is certainly important.

Since the studies by Schwert and Fama investigates the relation between stock returns, inflation, monetary aggregates and industrial production growth without modelling the stock market risk premium it is of interest to model the premium in our multivariate model. Many studies using univariate and multivariate GARCH models have used the unconditional CAPM (see Campbell, Lo and MacKinlay [29] or Bekaert and Wu [10]) to model the time-varying risk premium. The aim of this chapter is to look at potential ways to model the risk premium of stock returns in a multivariate model of the joint conditional distribution of stock returns, inflation, narrow money growth and industrial production growth. We do this by including covariances between the market return and the macroeconomic variables in addition to the conditional variance of the market return in the mean equation of excess return on the market portfolio. Findings of any significant covariances between returns and a macroeconomic variable serves as a rejection of the unconditional CAPM.

Based on our results in the previous chapter it is obvious that it could be useful to have consumption in the model, additionally, but it is simply not feasible to estimate a multivariate GARCH model with five variables with the given dataset without making strong restrictive assumptions on the dynamics of the conditional covariance matrix[7]. We also saw in the previous chapter that many of the macroeconomic variables tend to imply conditional covariances with returns that were pretty highly correlated and it is conjectured that pricing the three macroeconomic variables that we do in this chapter capture most of the risk associated with consumption. In any case our specification of the mean return equation will be shown to be superior to that of empirical studies using the unconditional CAPM.

If we divide through the above equation by the conditional standard deviation of returns we obtain the Conditional Sharpe Ratio (CSR), a variable important for carrying out the optimal portfolio allocation. The CSR is the units of risk compensation per unit of volatility of the asset return, that is

[7]Further it was seen in the previous chapter that there is little ARCH effects in US consumption data which causes some difficulties with the estimations.

$$\mathcal{SR}_t(i^e_{s,t+1}, \mathrm{f}_{t+1}) = \sum_{j=1}^{k} \gamma_{j,t}\sigma_t(i^e_{s,t+1}) = \sum_{j=1}^{k} b_j\rho_t(f_{j,t+1}, i^e_{s,t+1})\sigma_t(f_{j,t+1}) = \frac{\phi_t}{\sigma_t(i^e_{s,t+1})}, \quad (3.6)$$

Much literature has been devoted to determine the relation between the time-varying risk premium and the conditional variance (or standard deviation) of the market return, $\rho(\phi_t, \mathrm{V}_t(i_{s,t+1}))$ (or $\rho(\phi_t, \sigma_t(i_{s,t+1}))$). Many of these studies do not pay much attention to the modelling of the risk premium which often turns out to be estimated in a rather ad-hoc way. A recent example, trying to model the relation between the risk premium and the return variance, is performed by Lettau and Ludvigson [82]. They conclude using quarterly data, that the relation is strongly negative. However, their modelling of the risk premium does not obey a no-arbitrage condition - they perform regressions of quarterly excess return on lagged values of cay_t, the approximation of the consumption aggregate wealth ratio discussed in chapter 1, and use the fitted value of this regression as a measure of the expected excess return. In a similar way they use a regression of realised volatility on forecasting variables of stock return volatility and use the fitted value from this regression as a measure of the conditional volatility. Having constructed the expected excess return and the expected volatility they conclude that there is a strong negative relation between the two. We believe that any modelling of the mean equation of expected excess return should take place within an arbitrage free world. We argue that conditional covariances between stock return and macroeconomic variables and not cay_t should be an estimate of the conditional expected excess return.

Other attempts to model the relation between the mean and the variance have used univariate GARCH in mean type of models (our estimated models includes the univariate GARCH in mean model as a special case). Bollerslev, Engle and Wooldridge [15] and French, Schwert and Stambaugh [63] (among others) conclude that there is a positive relation between expected stock return and exante volatility returns using US data. Glosten Jagannathan and Runkle [66], Whitelaw [110] and Brandt and Kang [18] (among others) conclude that the relationship is negative.

Hence the broad conclusion is mixed. In this chapter we present an alternative method, using the multivariate GARCH-in-mean model, to investigate the relation between the expected excess return and the return variance of returns in an arbitrage free world. We argue that the relation depends on how covariances between stock return and macroeconomic variables vary with the expected variance of stock returns and the relation depends on the position in the business cycle. We will discuss the implications of our findings of the previous chapter shortly together with a new set of results for this more general asset pricing model. Both the inter-temporal models with a time-varying relationship between mean and variance and the traditional model assuming the risk premium and the expected variance of return to be perfectly correlated will be estimated. We notice that the consumption-based models estimated in the previous chapter implied an inter-temporal relation between the risk premium and the variance of returns. The nature of these relations will become clearer in the results section.

3.3 Modelling the Macroeconomic Variables, Allowing For Asymmetry

3.3.1 Asymmetries in the Covariance between Macroeconomic and Stock Market Variables

Much work has documented that positive and negative shocks to stock market return have differing impacts on the conditional variance of return. Using weekly data Capiello, Engle and Sheppard [31] find evident asymmetries in the variance of most equity indices in the developed world.

Three explanations of the variance asymmetry in equity returns have been proposed - the leverage (Black [11] and Christie [34]) hypothesis states that when the total value of the levered firm falls, the value of its equity becomes smaller relative to the total value of the firm. If equity characterises the full risk of a firm, the variance of the equity return should rise. A price increase should have the opposite effect. Some studies have found this explanation to not fully account for the magnitude of return and volatility correlation found empirically (see Pindyck [94] and/or Campbell, Lo and MacKinlay [29] and Schwert [98]). The second explanation, the volatility-feedback hypothesis (Campbell and Hentschel [23]), claims that positive shocks to volatility drive down returns - the fundamental story put forward by these authors relies on an assumption that the CAPM is the true model for modelling the risk premium on the market portfolio. If there is a large piece of good news about future dividends, large news tend to be followed by large pieces of news since volatility is persistent, news increase expectations of future volatility which increase future expected excess return if the expectation is perfectly correlated with the conditional variance of this excess return therefore decreases the stock price today offsetting the positive dividend news. If, on the other hand, there is negative news about future dividends we would expect future bad news about dividend and hence expected higher volatility which would imply higher expected risk premia (CAPM argument) and the price of the stock will fall amplifying the negative news about future dividends. The volatility feedback story would require squared return innovations to be negatively correlated with future volatility.

A univariate GARCH model[8] allowing for this asymmetry was proposed by Glosten, Jagannathan and Runkle [66]. They specify the conditional variance of the excess return as

$$h_{t+1} = \omega + \alpha_{11}\epsilon_t^2 + \alpha_{12}\eta_t^2 + \alpha_{21}h_t, \tag{3.7}$$

where ϵ_t is stock return innovations and $\eta_t = \min(\epsilon_t, 0)$. We will use a multivariate extension of this model allowing macroeconomic shocks lagged to affect the conditional variance of excess return, as well as return shocks.

The third explanation for the asymmetry, which we will emphasise, is misspecification of the mean equation of excess return, the risk premium, or modelling the conditional variance univariate instead of as part of a multivariate system. We assume that this omitted variable is the conditional covariance between excess return and macroeconomic variables which may imply an inter-temporal relation between the variance of the excess return and the risk premium. Our model allows positive and negative shocks to the return to affect the risk premium over the subsequent period but the effect does not necessarily come from the conditional variance of the excess return.

We will investigate whether innovations to macroeconomic variables affect the conditional covariance matrix between the stock market excess return and macroeconomic variables. Allowing positive and negative macroeconomic and return innovations to be differently transmitted into the conditional covariance matrix. In this way we can see whether the position in the business cycle causes changes in the conditional covariance matrix between the return and the macro economy affecting the stock market risk premium.

[8]Other models allowing for asymmetry includes Engle [47] and Sentana [102]. Capiello, Engle and Sheppard [31] has a brief survey of asymmetric models. Van Dijk and Franses [41] may also be a good reference.

Covariance (risk compensation) asymmetry has previously been documented using multivariate GARCH models by Kroner and Ng [77] without focus on the risk premium and in Bekaert and Wu [10] with focus on the risk premium implied by the CAPM on different leveraged Japanese portfolios. They consider weekly data on the Japanese market portfolio and three leverage sorted portfolios and model the risk premium, under an assumption that the CAPM is the true model, on the four portfolios joint using a restricted version of the multivariate GARCH-in-mean specification proposed in this chapter. Additionally they use a risk-less debt model that implies that their specification can divide the potential asymmetry into leverage effect and volatility feedback effects. They find, for the Japanese market, that the leverage effect is not really important though they find strong evidence of variance asymmetry as well as covariance asymmetry. Bekaert and Wu conclude that their CAPM model generates a time-varying risk premium that they conjecture cannot be replicated by general equilibrium models.

It is of interest to investigate whether pricing of more general macroeconomic variables can generate time-varying expected excess returns and if an alternative way of modelling of the risk premium alters our conclusion on asymmetries in the conditional covariance matrix. We have already shown, in the UK and US, that General Equilibrium models imply a more variable risk premium than the CAPM.

3.3.2 Macroeconomic and Financial Uncertainty

Schwert [98] investigates the relation between stock return and macroeconomic uncertainty. From the Dynamic Gordon Model (Campbell and Shiller [26][27]) one can show that the log stock price reflects the expectation of future cash flows, future interest rates and future excess return - if macroeconomic data contains information about expected future cash flows or expected future discount rates it can potentially explain the time-variation in monthly stock market returns. Schwert uses data on the S&P composite portfolio from January 1928 to December 1987. He computes the variability of stock market excess return in two ways but in neither case does he offer any theoretical arguments on the modelling of the risk premium - this is one of the main contribution of this chapter. One of the main reasons that there is a controversy about the relation between the risk premium and the variance is that there is little agreement as to how one should estimate the conditional variance and the risk premium.

Schwert finds that the estimated stock market volatilities are roughly similar using either of the two methods though daily data produce standard deviations generally larger. He uses Producer Price Index (PPI) inflation and concludes that inflation volatility does not help predict future stock return volatility. Volatility of money growth can help predict stock market volatility using the measure of monthly stock market volatility based on daily returns but the predictability is not present when using monthly stock market volatility.

Since common stocks reflect claims on the future profits of corporations, it is plausible that the volatility of real economic activity is a major determinant of stock return volatility. However, Schwert does not find evidence that volatility of industrial production growth can help predict stock market volatility - on the contrary Schwert finds that stock market volatility can predict output volatility. His study may suffer from being univariate - as will be evident later our conclusions using a multivariate model of the variables are different and we find evident comovement between volatility in the stock market and the real economy.

Schwert concludes that stock market volatility is higher during recessions. Equivalently he shows that industrial production volatility is higher during recessions but it is not the case for PPI inflation and the evidence that money growth is more volatile during recessions is at best weak.

An aim of this chapter is to follow up on his work and investigate the relation in a multivariate model between the volatilities of the macroeconomic and stock market variables - the multivariate model allows for contagion from unexpected shocks to the conditional covariance matrix, modelling asymmetries and the risk premium. We consider several models of the time-varying stock market risk premium.

3.4 The Econometric Framework

To investigate the time-variation and comovements we need an assumption on the dynamics of the conditional covariance matrix. We assume that the conditional covariance matrix follows a multivariate GARCH process with in-mean covariance terms for the excess stock return.

We wish to model asymmetric effects in the conditional covariance matrix allowing positive and negative news to have different impacts. In this chapter we use an extension of the BEKK model, discussed in Engle and Kroner [50], which is a special case of the model proposed by

Kroner and Ng [77] as a starting point. This has the advantage that the variance of each of the dependent variables can be predicted by 1) lagged values of the conditional variances of all the variables and lagged covariances between all variables, 2) lagged squared residuals and cross products of residuals (variance and covariance news). We can allow many sources of shocks to affect the conditional variances (uncertainty of macroeconomic and financial variables) and covariances (that determines risk premia in the stock market).

There are potential advantages of using a multivariate relative to a univariate model. Some can be seen as expanding the information set. Estimating a risk premium model we face the problem that our information set is a subset of the information set used for forming expectations and estimates of the risk premium can be spurious as pointed out by Glosten, Jagannathan and Runkle [66]. In the framework of this chapter we assume that investors at time t have information set $\mathcal{F}_t$ and the econometrician has information set $\mathcal{L}_t$ where $\mathcal{L}_t \subseteq \mathcal{F}_t$. The SDF approach we follow generates the pricing equation for investors:

$$\mathrm{E}(i^e_{s,t+1} \mid \mathcal{F}_t) + \frac{1}{2}\mathrm{V}(i^e_{s,t+1} \mid \mathcal{F}_t) = \phi(\mathcal{F}_t), \tag{3.8}$$

Taking expectations of both sides conditional on the information set of the econometrician and using the law of iterated expectations yields

$$\mathrm{E}(i^e_{s,t+1} \mid \mathcal{L}_t) + \frac{1}{2}\mathrm{V}(i^e_{s,t+1} \mid \mathcal{L}_t) = \phi(\mathcal{L}_t), \tag{3.9}$$

which implies that

$$i^e_{s,t+1} + \frac{1}{2}\mathrm{V}(i^e_{s,t+1} \mid \mathcal{L}_t) = \phi(\mathcal{L}_t) + \epsilon_{t+1} \tag{3.10}$$

The error term $\epsilon_{t+1} = u_{t+1} + \xi_{t+1}$, where

$$u_{t+1} = \phi(\mathcal{F}_t) - \phi(\mathcal{L}_t) + \frac{1}{2}\mathrm{V}(i^e_{s,t+1} \mid \mathcal{L}_t) - \frac{1}{2}\mathrm{V}(i^e_{s,t+1} \mid \mathcal{F}_t), \tag{3.11}$$

ξ_{t+1}, a pure expectational error. Using a multivariate GARCH-in-mean model we assume that the variance of u_{t+1} is constant since the correlation between u_{t+1} and ξ_{t+1} is equal to zero. In cases where the information set of the econometrician is not identical to the information set of the economy we will get a bias in the estimate of the parameters in the risk premium. Using the multivariate BEKK model we allow for a broader information set. We assume that the information set consists of lagged values of the conditional variances and covariances and the outer product of shocks to the excess return in the stock market, inflation, money growth and industrial production growth. All of this takes place within an arbitrage-free model for the level of the stock return.

From the no-arbitrage condition, equation (3.3), we model the dependent variables as (see Smith and Wickens [105])

$$\mathbf{Y}_{t+1} = \mathbf{A} + \sum_{i=1}^{p} \mathbf{B}_i \mathbf{Y}_{t+1-i} + \sum_{j=1}^{N_1} \mathbf{\Phi}_j \mathbf{H}_{[1:N,j],t+1} + \sum_{k=1}^{d} \Theta_k \Upsilon_{k,t+1} + \epsilon_{t+1}, \qquad (3.12)$$

Recall definition of these vectors and matrices from equation (1.48) in the introductory chapter and the subsequent discussion in that chapter. To apply this setup in this chapter the vector of dependent variables is $\mathbf{Y}_{t+1} = \{i^{e}_{s,t+1}\ \pi_{t+1}\ \Delta q_{t+1}\ \Delta y_{t+1}\}^{\mathsf{T}}$, where the latter two variables are changes in the logarithm of money (Δq) and changes in the logarithm of industrial production (Δy) respectively. We use a vector auto regression of order 1 (p=1) - that is for $\mathbf{B}_1$ the first row is restricted to be zero and the parameters in all other equations are unrestricted. Since we model the risk premium on only one asset, the US stock market index, $N_1 = 1$ and Φ_1 has parameters in the first row satisfying the no-arbitrage condition, equation (3.3), and parameters in the other rows are restricted to equal zeros. We include an indicator variable to take account of the large negative outlier in excess return in the month of the stock crash (October) in 1987. Hence Θ_1 has a parameter in the first entry and zeros in all other entries and $\Upsilon_{1987:10,t+1}$ takes the value of 1 for $(t+1)$ equal to October 1987. ϵ_{t+1} is the heteroskedastic error term

$$\epsilon_{t+1} = \mathbf{H}^{\frac{1}{2}}_{t+1}\mathbf{u}_{t+1}, \qquad \mathbf{u}_{t+1} \sim \mathcal{D}(0, \mathbf{I}_4)$$

$\mathcal{D}$ could be any distribution and $\mathbf{I}_4$ is the identity matrix of dimension four. The specification of the dynamics of the multivariate GARCH process assumed in this chapter is the extension to the BEKK model allowing for asymmetries in the conditional covariance matrix and hence in the risk premium (ABEKK). The dynamics of the conditional covariance matrix are the asymmetric specification, see equation (1.51), as discussed in chapter 1.

Writing the conditional covariance matrix in Error Correcting Form as in equation (1.51) we note that there are three potential sources of time-variation in the conditional covariance matrix:

$$\mathbf{H}_{t+1} = \mathbf{H}_0 + \mathbf{H}_{1,t+1} + \mathbf{H}_{2,t+1} + \mathbf{H}_{3,t+1} \tag{3.13}$$

The unconditional expectation of the three latter terms on the right hand side equals zero and the first term, $\mathbf{H}_0$, is the long run variance covariance matrix. We can decompose the estimated covariance matrix and analyse which of the latter three terms makes the biggest contribution to time-variation. In the following we denote $H^{ij}_{t+1} = \mathrm{Cov}_t(Y_{i,t+1}, Y_{j,t+1})$.

Similarly we can decompose the estimated risk premium as

$$\phi_t = \phi_0 + \phi_{1,t} + \phi_{2,t} + \phi_{3,t} \tag{3.14}$$

ϕ_0 is the expected, or long run, risk premium. In a similar fashion it could be of interest to decompose the risk premium into different factor components, that is

$$\phi_t = \phi_{excess\ return,t} + \phi_{inflation,t} + \phi_{narrow\ money,t} + \phi_{ind.\ production,t} \tag{3.15}$$

where, for instance, $b_1 \mathrm{V}(i^e_{s,t+1})$ is the first term on the right hand side[9].

All estimations were done using either of the starting values discussed in section (1.5.2) in the introductory chapter - no significant differences were found !

[9] b_1 refers to the first element in $\mathbf{b}$. See equation (3.5).

3.5 Models and Data

3.5.1 Models

Without guidance of an equilibrium model for the risk premium we consider and estimate six multivariate models. The general no-arbitrage condition for all models is given by

$$\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) = \mathbf{b}^{\mathsf{T}}\mathrm{Cov}_t(i^e_{s,t+1}, \mathbf{f}_{t+1}) + \theta\Upsilon_{1987:10,t+1} + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \tag{3.16}$$

where $\mathbf{f}$ is the vector of the return and the three macroeconomic variables and $\Upsilon_{1987:10,t+1}$ is an indicator variable taking the value of one in October 1987 and zero otherwise. We include an indicator variable since it is a rather extreme event relative to the rest of the sample (see Schwert [100]).

Model 1-WA is the standard CAPM (since we assume that our return data is the market portfolio) - in this case the single factor causing time-variation in the SDF is the market return. The model assumes a constant relationship between the mean and the variance[10]

$$\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) = \gamma\mathrm{V}_t(i^e_{s,t+1}) + \theta\Upsilon_{1987:10,t+1} + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \tag{3.17}$$

Model 2-WA and Model 3-WA are more general with no particular theoretical justification. Model 2-WA is a version of the ICAPM pricing the market return and the three macroeconomic variables. Significant pricing of any of the macroeconomic variables serves as a rejection of model 1-WA (a version of the CAPM commonly used in financial economics). Model 3-WA prices only macroeconomic variables - this models investigates the relative benefit of having the variance in mean together with the covariances with the macroeconomic variables. Model 4-WA, 5-WA and 6-WA prices each of the macroeconomic variables individually and enable us to evaluate

[10]This model is the only theoretically justified model of the risk premium. However, as we showed in equation (3.5) all other models are special cases.

whether individual covariances generates a significant time-varying risk premium. This allows us, as well, to evaluate whether the signs of the parameters on the time-varying covariances follow economic intuition, as will be discussed shortly.

3.5.2 Data

We analyse monthly data for the United States. The stock market index is the value-weighted return on all NYSE, AMEX and NASDAQ stocks. We use a one-month Treasury Bill interest rate as the risk-free rate[11]. Real seasonally adjusted industrial production, seasonally adjusted CPI inflation and money (M1) growth are obtained from Datastream. The sample period is 1960:01 to 2003:12.

In table (3.3) in the appendix we tabulate the descriptive statistics of the dataset. The main characteristics of the dataset are that inflation is positively skewed, the excess return and industrial production growth have negative skewness; most negative for the excess return. Most variables do not have an extreme amount of excess kurtosis except industrial production growth and the excess return with excess kurtosis of 2.79 and 2.92, respectively - normality is rejected for all variables. There is little auto-correlation in the excess return but more in the squares. Inflation and money growth have substantial auto-correlation in both the level and squares. Industrial production growth has some first and second order auto-correlation in the level and in the squares Finally, there seems to be auto-correlation in the absolute value of all the variables supporting our specification allowing for asymmetries.

In the money data there are two extreme events. First in September 2001 money growth increases sharply whereas in October 2001 it falls again with a similar magnitude. We treat these as measurement errors and consider these extreme outliers and decide to replace both observations with the mean of the dataset. It would probably have been a better solution to include two indicator variables for the two events but it would give more parameters to be estimated which is not desirable since our model is already highly parameterised. We conjecture that the results would have been qualitatively the same.

[11] Available from the homepage of Kenneth French, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. Same data as used in chapter 2.

3.6 Results

3.6.1 The Parameters Governing The Risk Premium

Most investors would prefer an asset that pays off well during recessions, when times are bad. Everything else being equal, one would expect that the excess return on the market portfolio should be positively related to its own variance - if the variability of the stock market increases we would want a higher compensation for investing in the stock market relative to an investment in a risk-free asset. Inflation tends to be high during recessions (bad times) - if the covariance between inflation and return in the stock market increases we would, everything else being equal, expect a lower risk premium on the market portfolio and hence expect a negative coefficient on the conditional covariance between excess return and inflation. In recessions people tend to spend less and the money stock decreases - in this case when the covariance between the return and money increases the market index is more risky and we would expect a positive coefficient on the conditional covariance with money growth. Falling industrial production is a characteristic of bad times - when the conditional covariance between the market return and industrial production growth increases we would expect, everything else being equal, that the asset is more risky and hence we would expect a positive coefficient on the covariance with industrial production growth. When pricing several factors it is, however, difficult to draw such conclusions because of the role of the cross covariances. Below we will examine how shocks are transmitted in the conditional covariance matrix.

The estimated parameters from the models, presented in section (3.5), can be found in table (3.1). The estimates of model 1-WA yields a risk premium which explains a small proportion, 0.88%, of the variation in actual excess returns. The residuals of the excess return equation have an annualised mean considerably different from zero.

Next, model 2-WA, one version of the ICAPM shows that the macroeconomic variables inflation and industrial production growth are significantly priced and so is the stock market return. The variability of the implied risk premium is very high, more than 11 times as high as the implied risk premium in model 1-WA which assumed that the exante variance of returns is equal to the risk premium. The average residual in the excess return equation is considerably closer to zero than in model 1-WA. Next, in model 3-WA, we ask whether pricing the macroeconomic variables

Table 3.1: The Estimated Parameters In The Risk Premium

	Model 1-WA	Model 2-WA	Model 3-WA	Model 4-WA	Model 5-WA	Model 6-WA
$V_t(i^e_{s,t+1})$	**4.8264**	**11.6208**				
	(4.92)	(2.76)				
$Cov_t(i^e_{s,t+1}, \pi_{t+1})$	1	**809.2528**	**629.6142**	**−696.0830**		
		(3.02)	(2.15)	(4.24)		
$Cov_t(i^e_{s,t+1}, \Delta q_{t+1})$		−12.7946	**546.0069**		**1434.1984**	
		(0.11)	(2.72)		(2.84)	
$Cov_t(i^e_{s,t+1}, \Delta y_{t+1})$		**−305.3246**	**−362.3551**			2947.6613
		(3.67)	(3.31)			(1.40)
$\Upsilon_{1987:10,t+1}$	**−0.2590**	−0.2714	−0.2761	−0.2909	**−0.2830**	−0.2950
	(2.42)	(0.80)	(0.83)	(1.32)	(2.08)	(1.58)
ν	**10.8912**	**9.9502**	**9.8722**	**9.5470**	**9.5963**	**9.3484**
	(4.70)	(5.31)	(5.42)	(5.26)	(5.12)	(5.26)
Log Likelihood	−2125.579	−2104.649	−2107.980	-2116.5349	-2119.0168	-2111.7761
Mean residual (annualised)	−2.0760	−1.3340	−0.8577	-2.0613	-1.2646	-0.3450
Annualised average risk premium	11.2734	10.4343	9.9447	11.2577	10.3500	9.3484
$Var(\phi_t)/Var(i^e_{s,t+1} + \frac{1}{2}V_t(i^e_{s,t+1}) - \hat{\theta}\Upsilon_{1987:10,t+1})$	0.0088	0.1154	0.1207	0.0918	0.1135	0.1210

The estimated parameters in the models. Absolute t-statistics in parenthesis. $\Upsilon_{1987:10,t+1}$ is the coefficient on the dummy variable taking the value 1 in October 1987 and 0 otherwise. Emphasised parameters significant using a 95% critical value. ν is the estimated degrees of freedom in the multivariate t-distribution.

alone delivers a significant time-varying risk premium - it can be seen that all macroeconomic variables are significantly priced in this model. The reason for money being significant in this model is straightforward. Since we no longer have the variance of return in the mean equation the estimated covariance between money and return has an unconditional correlation with the variance of the stock market return of 0.65 suggesting that money is significant due to an omitted variable. It is interesting though that the variability of the implied risk premium in model 3-WA relative to the variability of the excess return is more than 12% and the residual in the excess return equation has the value, relative to other models, most close to zero.

The conclusion on the pricing of all variables are that inflation and industrial production growth are both significant variables but the signs are opposite the intuition that we gave above.

However, we emphasised that the intuition was everything else being equal. We have no particular model in mind and hence it is difficult to interpret the parameters. To make the point clearer in model 4-WA, 5-WA and 6-WA we price only single macroeconomic variables - the estimates have signs following economic intuition - money growth and inflation both individually strongly significant. The results are very interesting in that the implied variability in the risk premium implied by single covariances between return and macroeconomic variables is as high as in our most general model suggesting that covariances with macroeconomic variables generates a risk premium that is much more variable than pricing the market return only. Further the conditional covariances between excess return and the three macroeconomic variables are pretty highly correlated. In what follows we concentrate on the most general model, model-2WA.

3.6.2 Other Estimated Parameters

In table (3.6) and (3.7) in the appendix we tabulate all of the other estimated parameters. In the means of the vector auto regression we note the diagonal significance of all variables. The more interesting result is that lagged logarithmic excess return is significant in the money equation - this prediction is significant in all models. Further it is found that inflation in the previous period predicts changes in industrial production growth.

The diagonal elements in the GARCH parameter matrices are always strongly significant across all models - it is curious, though, that the estimated parameters become smaller and less significant when allowing for more general risk premia. From the off-diagonal elements it is interesting to note that increases in the variance of excess return predicts increases in the variance of output growth in the following period.

Model 2-WA and 3-WA broadly agrees with the significance of parameters in the GARCH matrix. When we have high variance in the stock market the variance of all the macroeconomic variables will increase in the following period (everything else being equal) and similarly when we have high variance of output the variance of all other variables will increase in the subsequent period. High inflation variance predicts higher future output variability.

In the ARCH matrices the diagonal elements in the inflation and money growth equations are significant across all models indicating that squared shocks to these variables individually, whether positive or negative, significantly increase the variance of inflation and money growth respectively. However, whereas positive squared shocks to excess return and industrial production growth increases the own variance respectively in model 1-WA this is not the case in model 2-WA where we have a much more variable risk premium. In all models we find that shocks to industrial production growth squarred significantly increase the variance of money. In model 2-WA and model 3-WA we find that squared shocks to the excess return increase the variance of output.

In the asymmetry parameter matrices we note the strong asymmetry in the variance of industrial production growth which is significant in all models allowing for asymmetries. In these models we also find that negative shocks to money growth have a different impact on the variance of excess return and hence, everything else being equal, the variance of stock returns increases when we have negative money shocks. In model 2-WA and model 3-WA where we allow for a more general risk premium we find that negative shocks to the excess return have a significantly different impact on the variance of inflation than positive shocks. Negative shocks to stock returns increase inflation volatility whereas positive shocks do not.

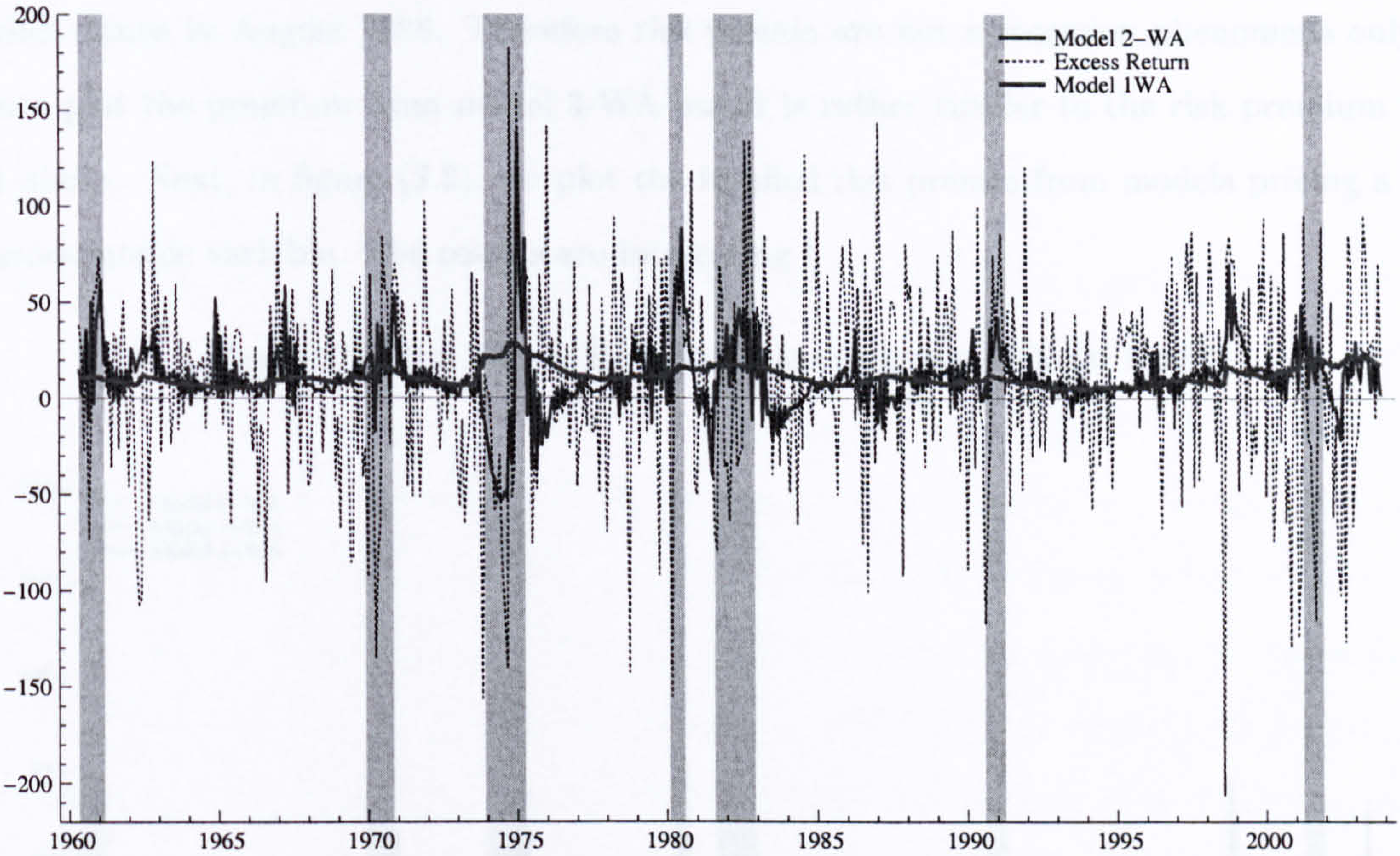
To conclude, we focus on the own asymmetry in the variance of the excess return. In model 1-WA, where we assume the risk premium to be a linear function of the variance of return, we note that the own asymmetry is significant - negative shocks to excess return increase the variance of excess return, everything else equal, significantly more than positive shocks. However, when we estimate a version of the ICAPM, in model 2-WA, we find the own asymmetry to be only borderline significant and positive shocks lose their significance in increasing return variability. Also, in model 3-WA, 5-WA and 6-WA, we find that the own asymmetry in the variance of excess return "disappears" in significance suggesting that the own asymmetry is strong when pricing the return in the stock market only - this single factor model may be far too restrictive. The extent of the own asymmetry in the variance of stock return is very sensitive to the modelling of the risk premium - this, to our knowledge, is the first study that shows this asymmetry to disappear when modelling the risk premium differently. From the six estimated models we note that significant asymmetries depend critically on the modelling of the risk premium.

3.6.3 The Risk Premium and its Components

There are many requirements that need to be met for a model of the risk premium. One criterion is that the risk premium should be non-negative at all times, or at least never significantly negative. In figure (3.1) we plot of the implied premium against excess return net of the indicator variable. The shaded areas in the figure, and subsequent figures, are recessions as defined by the NBER.

We plot the risk premium from model 1-WA and model 2-WA. We note that the model pricing macroeconomic variables, in addition to the market return, implies a risk premium which is

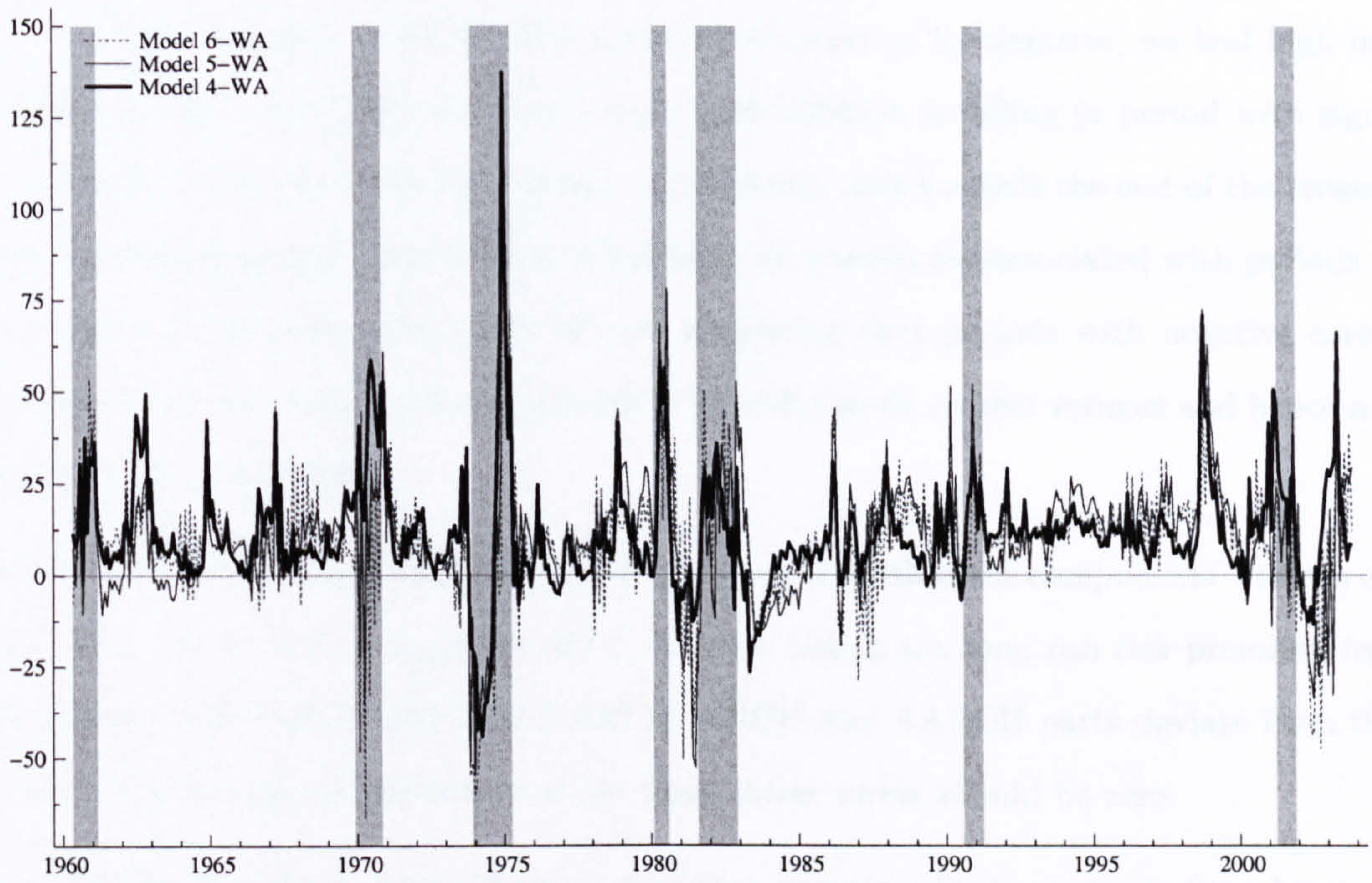
Figure 3.1: General And CAPM Risk Premium Against Excess Return



Log excess return inclusive of estimated Jensen term with dummy correction against estimated risk premium from model 1 - WA and model 2 - WA. Both series are annualised. Shaded areas are recessions as defined by the NBER.

considerably more variable over time than the model assuming that the risk premium is only a function of the conditional variance. This conclusion can also be reached from table (3.4) in the appendix which tabulates the auto correlation structure of the risk premium in the various models. From model 1-WA we would conclude that the risk premium has high and slowly decaying auto correlation. However, we would conclude differently in the more general, or alternative, model 2-WA, 3-WA, 4-WA, 5-WA and 6-WA. In these cases the risk premium is much less persistent (that is a lower first-order auto correlation). The model allowing for a time-varying relationship between mean and variance implies occasionally negative risk premia. This happens in particular during the recession in the mid 1970s. It is interesting to note that the risk premium tends to become negative only when recessions start and increases rapidly during the recession. Most likely large unfavourable economic shocks, relative to the rest of the sample period, are the reason for the risk premium becoming negative. It is also interesting to note that at the end of recession periods, and shortly after, the risk premium tends to decline implying that unfavourable economic conditions makes the stock market more risky and a premium is required. We note further that the risk premium is quite high in several periods that are not characterised as recessions - this in particular in the mid 1960s and following the negative stock market return in August 1998. Therefore risk premia are not a recession phenomena only. We do not plot the premium from model 3-WA but it is rather similar to the risk premium in the plot above. Next, in figure (3.2), we plot the implied risk premia from models pricing a single macroeconomic variable. The results are interesting !

Figure 3.2: Risk Premia From Single Macroeconomic Factor Models



Estimated risk premia from model 4-WA, 5-WA and 6-WA. In annual percentages. Shaded areas are recessions as defined by the NBER.

First, the risk premium implied by the 3 models is pretty similar with some important differences. The risk premium implied by the conditional covariance between inflation and return is very intuitive and is rarely as negative as implied by the two other covariances - in fact it is positive over almost the entire sample. This empasise the results in chapter 2 for stock returns and Balfoussia and Wickens [7] for bond returns that inflation is a significant variable priced in the US stock and bond markets.

In figure (3.16) in the appendix we plot the implied conditional correlations between the macroeconomic variables and excess return (model 2-WA) and note that there is much time-variation in these correlations and take this as evidence that the simplifying assumption of constant correlations is not adequate for modelling the joint distribution of the macro variables and excess return. It is worth noting that the correlation between money growth and the excess return is positive over most of the sample. We note as well the "high" comovement in the correlations in the 1990s.

In figure (3.9) in the appendix we plot the implied risk premium (model 2-WA) against the conditional correlations between the macroeconomic variables and in figure (3.7) the conditional standard deviation of the macroeconomic variables against the implied risk premium. During the 1973-1975 recession, when the risk premium appears to be negative, we had high inflation volatility and the correlation between output and inflation is falling (a period with significant supply shocks). Output volatility increase significantly only towards the end of the recession. It is also interesting to note that periods with high risk premia are associated with periods of very low correlation between money and output suggesting that periods with negative correlation between money and output shocks coincide with risky stock market returns and hence a higher compensation is required.

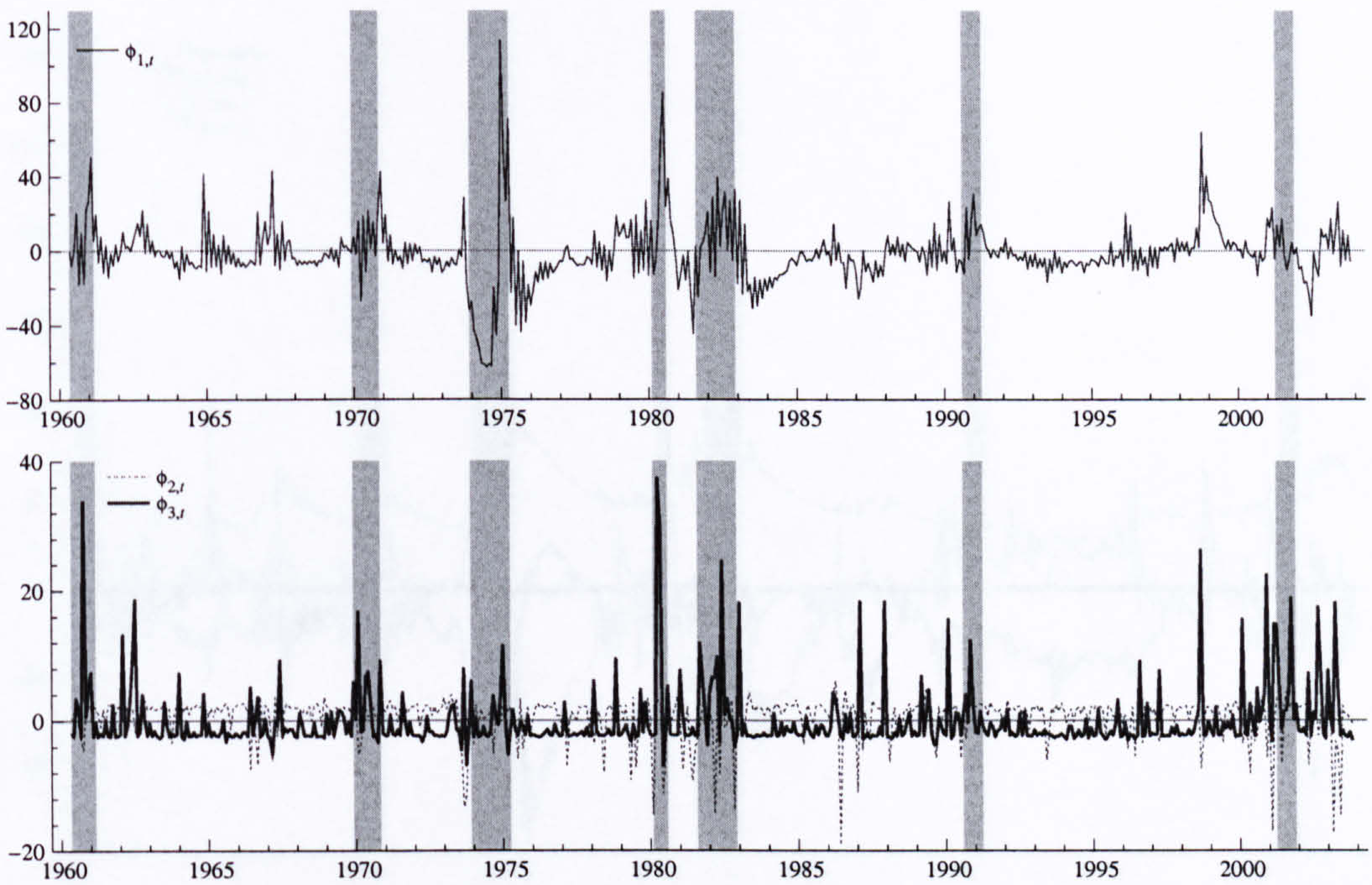
Next it is of interest to decompose the risk premium into different components - this is done in figure (3.3). As we saw in equation (3.14), we can isolate the long run risk premium from the time-varying contribution when the GARCH, ARCH and AARCH parts deviate from the long run level. The average contribution of the three latter terms should be zero.

An interesting picture emerges. First we note that negative shocks accounts for a large proportion of increases in the stock market risk premium, this in particular the case during recessions and towards the end of the sample - the majority of negative shocks toward the end of the sample are negative excess return shocks. There is little contribution from positive shocks. Since the overall implied risk premium is not very persistent, the increases in the risk premium are mainly for short periods of time - one exception is the period around 1980.

Next we look at the contribution from the individual risk factors to the risk premium, shown in figure (3.4) below.

It is interesting to note that all the factors signal that the stock market has been very risky over the time period considered. The variance of the stock return has been high and changing

Figure 3.3: GARCH, ARCH And AARCH In The Stock Market Risk Premium



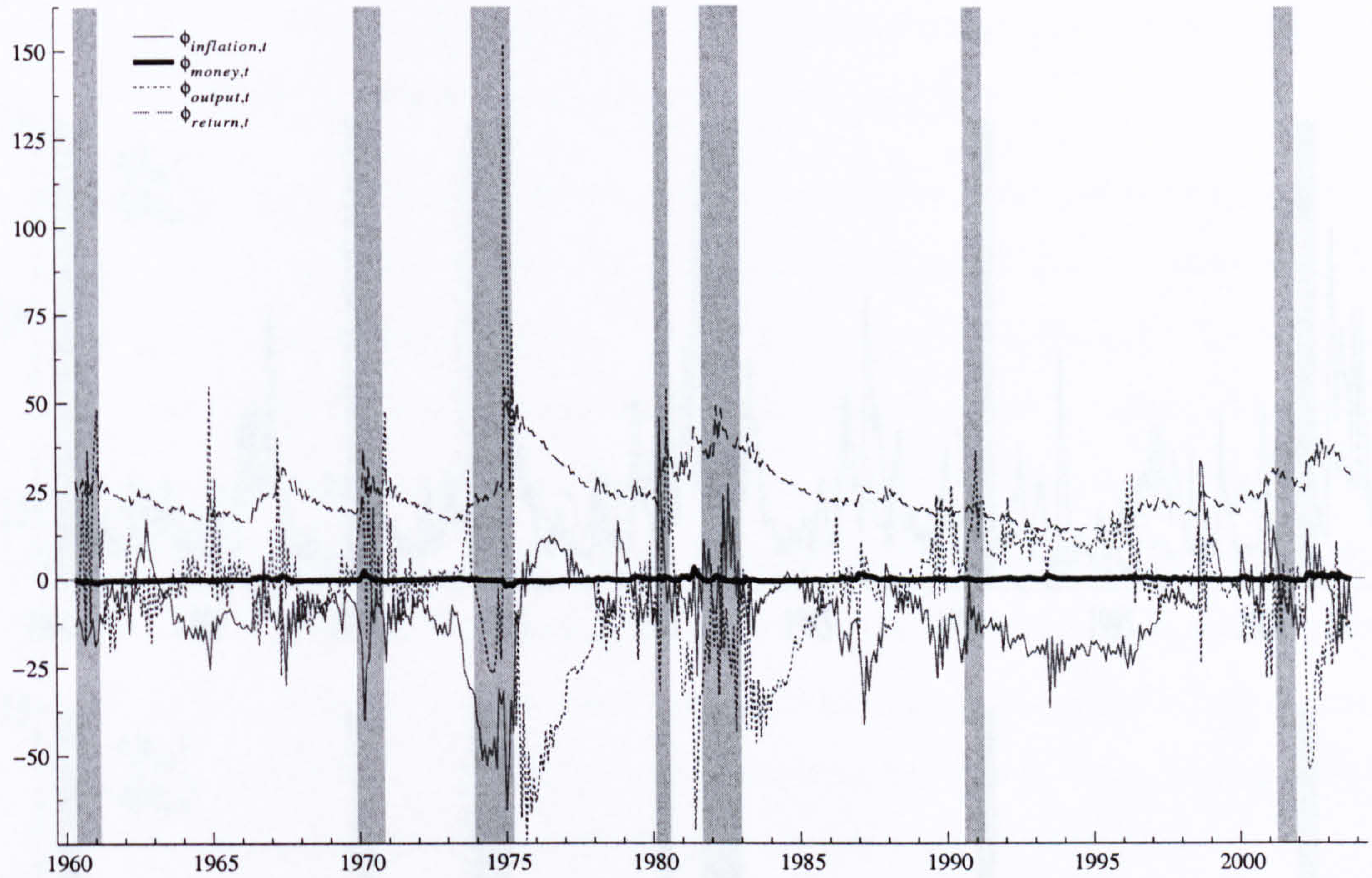
Risk premium (Model 2 - WA) decomposition into different components of the conditional covariance matrix (see equation (3.14)). Graphs from annualised dataset. Shaded areas are recessions as defined by the NBER.

over time, the stock return in the US market has a negative covariance with inflation and a positive covariance with industrial production growth indicating that the US stock market has paid off well exactly in states of nature that are not desirable for the average investor and is the reason why the stock market has paid of so well. It is curious that the two covariances seem to be capable of creating some short run variation in the stock market risk premium other than implied by the variance of the stock return. The variance of stock returns is simply too highly autocorrelated - the risk premium is clearly not. Macroeconomic risks, in particular inflation, are priced significantly in the US stock market in addition to a narrow measure of market risk.

3.6.4 Interaction Between Macroeconomic and Stock Market Uncertainty

Next we are interested in the comovement in the volatility of the dependent variables. According to the study by Schwert [98] there should be very little causation from the macro volatility to stock return volatility but it is of interest to see what would be the conclusions using our multivariate model. In figure (3.5) we plot combinations of the conditional standard deviations

Figure 3.4: Four Factor Contribution To The Risk Premium



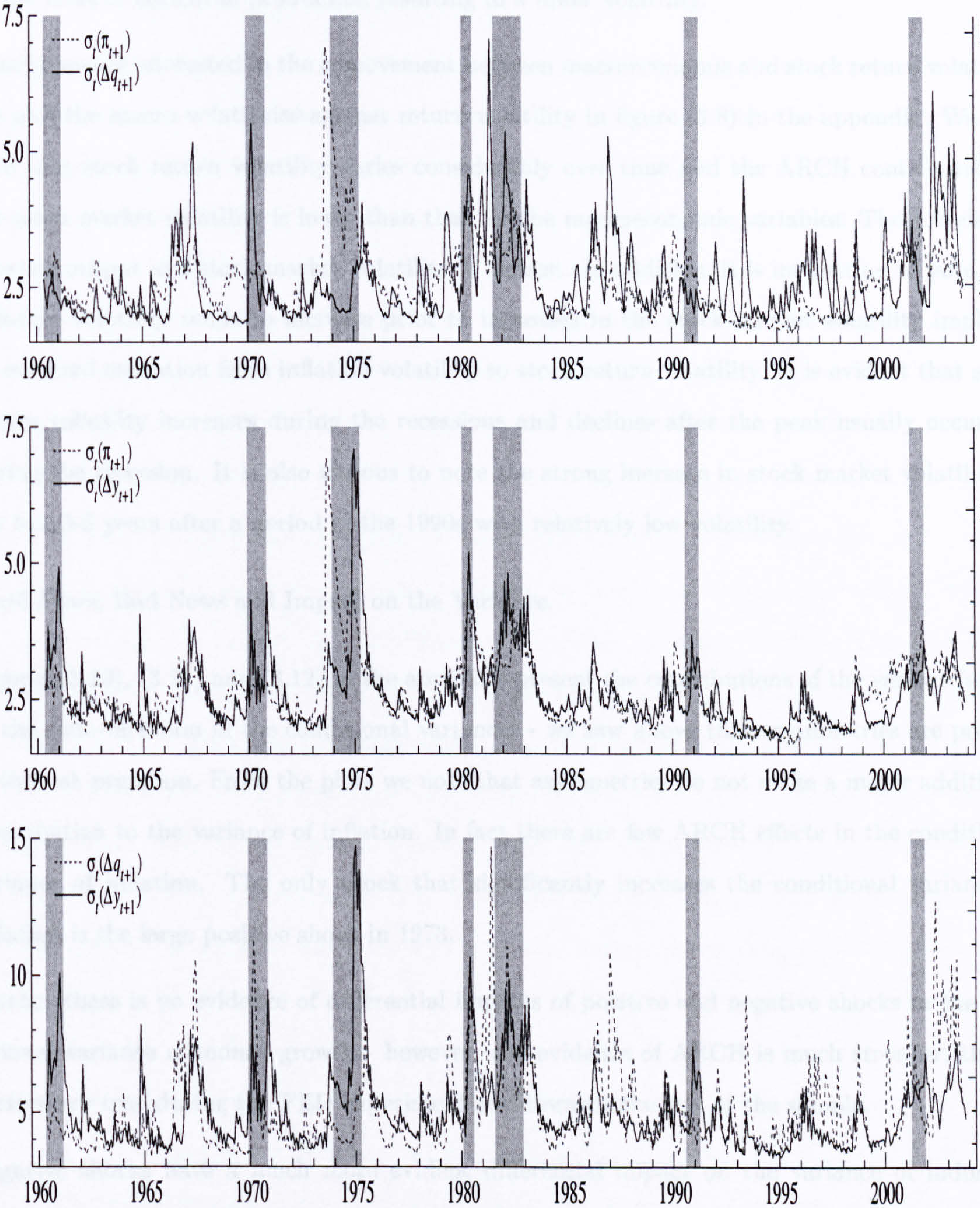
Risk premium decomposition (model 2 - WA) into different risk factors. Graphs from annualised dataset. See equation (3.15). Shaded areas are recessions as defined by the NBER.

of the three macroeconomic variables.

In 1973/1974, with a large increase in inflation volatility, it seems that output volatility increases sharply as well although with a lag. The same picture arises, though with smaller increases, in the early 1990s. Inflation- and output volatility have the highest correlation. We note that 2002, a period with high volatility in money growth, is also a period where the risk premium falls radically and becomes negative. In general we conclude that there is, though not perfect, comovement between the volatility of business cycle variables. It appears that the volatility of output rises during recessions reaching a maximum towards the end of the recessions after which volatility decreases sharply again. Inflation volatility, on the other hand, increases in the very early part of the recession whilst decreasing over the recession.

Another interesting finding is that the average standard deviations of inflation and industrial production growth are lower in the most recent period of the sample whereas the average standard deviation of the money supply is higher in this period. One explanation of the decline in

Figure 3.5: Macroeconomic Volatility



The volatility of macroeconomic variables in pairs implied from model 2 - WA. Variable above in label corresponds to scale on left y-axis. Standard deviations are from annualised dataset. The correlations between the series $\rho(\sigma_t(\pi_{t+1}), \sigma_t(\Delta q_{t+1})) = 0.29$, $\rho(\sigma_t(\pi_{t+1}), \sigma_t(\Delta y_{t+1})) = 0.57$ and $\rho(\sigma_t(\Delta y_{t+1}), \sigma_t(\Delta q_{t+1})) = 0.41$. Shaded areas are recessions as defined by the NBER.

the variability of industrial production growth is that the growth rate of output follows a square root process as emphasised by Maccini and Pagan [86]. Hence the reduction in the number of negative shocks to industrial production growth in the most recent period has contributed to a higher level of industrial production resulting in a lower volatility.

Finally, we are interested in the comovement between macroeconomic and stock return volatility. We plot the macro volatilities against return volatility in figure (3.8) in the appendix. We first note that stock return volatility varies considerably over time and the ARCH contribution to the stock market volatility is lower than that for the macroeconomic variables. The correlation between output and stock market volatility is highest. In addition, it is interesting to note that inflation volatility tends to increase prior to increases in the stock market volatility implying an eventual causation from inflation volatility to stock return volatility. It is evident that stock return volatility increases during the recessions and declines after the peak usually occurring during the recession. It is also curious to note the strong increase in stock market volatility in the last 3-5 years after a period in the 1990s with relatively low volatility.

Good News, Bad News and Impact on the Variance.

Figures (3.10), (3.11) and (3.12) in the appendix present the contributions of the various factors to the time-variation in the conditional variances - we saw above that asymmetries are present in the risk premium. From the plots we note that asymmetries do not make a major additional contribution to the variance of inflation. In fact there are few ARCH effects in the conditional variance of inflation. The only shock that significantly increases the conditional variance of inflation is the large positive shock in 1973.

Further there is no evidence of differential impacts of positive and negative shocks in the conditional variance of money growth - however, the evidence of ARCH is much stronger, this in particulary true during the FED experiment and towards the end of the sample.

Negative shocks have a much more evident differential impact on the variance of industrial production. In 1974-1975 negative news make a major contribution to the increase in the output variance (due to large negative shock to output) which is the case from the 1960s to the mid 1980s. It is very interesting to note that the negative shocks to the variables particularly increase the variance of output during recessions[12].

We have modelled the contribution to the conditional variance of excess return from negative shocks in model 1-WA and model 2-WA in figure (3.14). When modelling the risk premium as a constant multiple of the conditional variance of excess return there is a large differential impact from negative shocks - however, this large impact is smaller in magnitude in the model pricing the three macroeconomic variables in addition to the variance. This is a curious finding and shows that alternative modelling of the risk premium implies different transmission of unexpected shocks in the conditional covariance matrix. The asymmetries found in model 1-WA arise as a consequence of the fact that the conditional variance of returns has to little variability relative to that of the stock market risk premium. The stock market risk premium is not as highly correlated as the variance of stock returns. The asymmetry in the risk premium is a covariance asymmetry, not a variance asymmetry. Asymmetries can arise as a consequence of failure of the returns to obey a no-arbitrage condition !

From the table of parameter estimates we noted that the main additional contribution to the conditional return variance, due to negative shocks, is from return and money shocks. Next we ask another question: Assuming that there had been only negative shocks to a single variable (either return or money), what would be the impact on the return variance ? The answer can be seen directly in the table with parameters but the following plot, figure (3.15), additionally allows us to detect in which parts of the sample we have the contribution from the negative return shocks and in which period we have a contribution from the negative money shocks. Whereas the contribution from negative return shocks depend much on the choice of which model of the risk premium is used the contribution from negative money shocks does not. Negative money shocks increase the variance of return in the early 1970s, during the FED experiment and in 2002. The main contribution from negative return shocks in model 1-WA is in autumn 1998 but it is seen that this is a relative unimportant event in model 2-WA in the conditional return variance. The reason is, not that the risk premium does not increase following the large return fall, but because the risk premium increases due to a change in the covariance between return and inflation.

[12]Available upon request. We have carried out a similar decomposition of the conditional variance of excess return in model 2-WA but do not report it here.

3.6.5 The Parameters Governing the Asymmetries in the Variances and Risk Premium.

It is of interest to interpret the estimated asymmetries in the risk premium. We wish to answer whether positive and negative shocks are transmitted differently into the conditional covariances between the variables and in the risk premium. We have estimated many models - we decide to analyse the shocks transmitted in to the conditional variances of the variables in model 2-WA since this is the most general model. Model 1-WA is the traditionally used version of the CAPM with perfect correlation between the expected risk premium and the expected variance. For the estimate of the risk premium it is of interest to contrast these two models.

Table 3.2: A "Crude" Decomposition of Shocks In The Risk Premium and Conditional Variance

	$\widehat{E}(\varepsilon_t\varepsilon_t^{\intercal} - CC^{\intercal})\widehat{E}^{\intercal}$	$\widehat{G}(\eta_t\eta_t^{\intercal} - CC^{\intercal})\widehat{G}^{\intercal}$
$V_t(i^e_{s,t+1})^{Model-2WA}$	0	$0.0050\bar{\eta}_{11,t} + 0.01870\bar{\eta}_{13,t} + 1.7646\bar{\eta}_{33,t}$
$V_t(\pi_{t+1})^{Model-2WA}$	$0.0877\bar{\varepsilon}_{22,t} - 0.0172\bar{\varepsilon}_{24,t} + 0.0008\bar{\varepsilon}_{44,t}$	$0.0001\bar{\eta}_{11,t} - 0.0013\bar{\eta}_{14,t} + 0.0044\bar{\eta}_{44,t}$
$V_t(\Delta q_{t+1})^{Model-2WA}$	$0.2745\bar{\varepsilon}_{33,t} - 0.1058\bar{\varepsilon}_{34,t} + 0.0102\bar{\varepsilon}_{44,t}$	0
$V_t(\Delta y_{t+1})^{Model-2WA}$	$0.0003\bar{\varepsilon}_{11,t} + 0.0485\bar{\varepsilon}_{33,t} + 0.0079\bar{\varepsilon}_{13,t}$	$0.3215\bar{\eta}_{44,t}$
$\phi_t^{Model\text{-}1WA}$	$0.0535\bar{\varepsilon}_{11,t} + 2.1431\bar{\varepsilon}_{12,t} + 21.4382\bar{\varepsilon}_{22,t}$	$0.2135\bar{\eta}_{11,t} - 2.7805\bar{\eta}_{13,t} + 9.0547\bar{\eta}_{33,t}$
$\phi_t^{Model\text{-}2WA}$	0	$0.7055\bar{\eta}_{11,t} + 20.5037\bar{\eta}_{33,t} + 14.8784\bar{\eta}_{13,t} + 8.9454\bar{\eta}_{14,t} + 168.6291\bar{\eta}_{34,t}$

Expanding conditional variances as functions of shocks. Similarly expanding the risk premium as a function of shocks multiplied by the estimated parameters on the conditional covariances, see table (3.1). The outer product of the shocks are over lined indicating that the estimated long run level is subtracted. For instance, $\bar{\eta}_{11,t} = (\eta^2_{1,t} - \omega_{11})$, where ω_{11} is the element in column 1 (in first row 1) of $CC^{\intercal}$.

The results can be found in table (3.2). We have taken the estimate of the parameter matrices in table (3.6), from the estimated parameter matrices we have set all parameters not significantly different from zero (95 % critical value) equal to zero, then we have expanded the matrices. In the case of the risk premium we have multiplied the variances and covariances with the estimated parameters in table (3.1). One should note that this is a "crude" way to interpret the estimation results since even if some parameters are not individually significant different from zero that does not mean that they are not jointly different from zero. However, given the estimates in the tables we still use this approach arguing that it gives a good impression of the most important transmission of shocks in the conditional covariance matrix and risk premium.

First, positive shocks to returns and macroeconomic growth rates do not increase the variance of stock returns. On the other hand negative return shocks and money shocks do - if we have a negative money growth shock and a negative return shock in the same month the effect is amplified.

One reason for this could be that money growth falls unexpectedly, especially during recessions increasing the variance of stock returns. Shocks to inflation and output increase the variance of inflation - however, negative output shocks increases the variance of inflation considerably more than positive shocks. Periods with negative return shocks increase the variance of inflation as well. Shocks to money growth and output growth increases the variance of money growth but positive and negative shocks do not have differential impacts. Shocks to money growth and the stock market return, whether positive or negative, increases the variance of output and negative shocks to output increases the variance of output quite radically (this is consistent with the plot in figure (3.12)).

Finally for the comparison of how shocks in model 1-WA and Model 2-WA are transmitted differently into the risk premium we note that in model 1-WA shocks to stock returns and inflation increase the risk premium but negative return shocks implies a much higher increase in the risk premium than positive shocks (consistent with results in chapter 2). It is also interesting that negative money shocks increase the risk premium. Things look different in our more general model. First positive shocks do not affect the risk premium "at all" but negative return shocks, negative money shocks and negative output shocks all increases the risk premium - this is consistent with our previous plot, figure (3.3). However, negative output shocks increase the risk premium if we have a negative money or return shock in the same period only. Positive return and macroeconomic shocks do not increase the risk premium !

3.6.6 Volatility and Risk Premium Causation

One method to determine the causation between the variances and covariances is to use the estimated conditional standard deviations, for longer horizons than one month, of the variables and the risk premium and estimate a vector auto regression with all variables included to examine causation. Whilst the BEKK model itself can provide an answer to the causation between variances and covariances, unfortunately, due to the high number of parameters, we cannot allow the conditional covariance matrix to have higher order dynamics. Granger causality tests from the vector auto regression can be found in table (3.5) in the appendix. We perform the test using a vector auto regression of order 3 for the full sample and sub samples before and after the FED experiment (omitting the period 1979:01 to 1981:12). We use only a vector auto regression of order 3 to focus on the short run causation between macroeconomic volatility, stock return volatility and the risk premium.

It is first of all important to note that the diagonal effect is strongly significant for all variables in both the full sample and sub samples. The interesting finding, in contrast to the findings of Schwert, is that there is causation between macroeconomic volatility, stock return volatility and the stock market risk premium. Macroeconomic volatility and the stock market risk premium causes stock return volatility whereas stock return volatility does not predict changes in macroeconomic volatility as well as it does not predict changes in the risk premium. The conclusion is independent of the choice of sub sample.

The stock market risk premium is caused by macroeconomic volatility and not by stock return volatility (This is consistent with the model of Bansal and Yaron [8] where economic volatility determines time-variation in the risk premium). Further the stock market risk premium causes inflation and output volatility, and this causation is important only in the sample after 1982 - that is, a relatively recent phenomenon that we are not aware has been shown elsewhere.

Money and output volatility predicts changes in inflation volatility in the full sample whereas looking at sub samples we conclude the causation from output to inflation belongs only to the most recent period and the money to inflation causation is only a characteristic of the period of the FED experiment, which we have defined as 1979:01-1981:12. Inflation and money volatility significantly predict changes in output volatility, the predictability being stronger in the sample from 1982 onwards.

3.6.7 The Importance of Conditioning

We emphasised that failing to condition could have serious implications for our conclusions. In the worst case, failing to use the correct conditioning variables can lead one to draw wrong conclusions. In figure (3.17) in the appendix we plot the conditional variance of the the stock market excess return in three of the estimated models. The most general estimated model 2-WA[13] implies a conditional variance of the excess return that is substantially different from the one implied by the univariate GARCH-in-mean model and model 1-WA. It seems that the variance of the stock market excess return is smoother in the most general model. For instance in 1975 the implied variance of the excess return in the univariate model is almost twice as high

[13]We estimate also a univariate GARCH in mean model, allowing for positive and negative shocks to be transmitted differently into the conditional return variance, to obtain the variance of stock return for comparison.

as the implied variance in model 2-WA. We conclude that conditioning on macroeconomic variables and different modelling of the risk premium implies different estimates of the conditional variance of return. Stock market fluctuations may simply suggest time-varying risk premia (this is consistent with our findings in chapter 2 that conditional correlations between returns and macroeconomic variables depend critically on the modelling of the risk premium.).

3.6.8 Relation Between Mean and Variance

In figure (3.13) in the appendix we plot the risk premium relative to the conditional variance of the excess return, that is

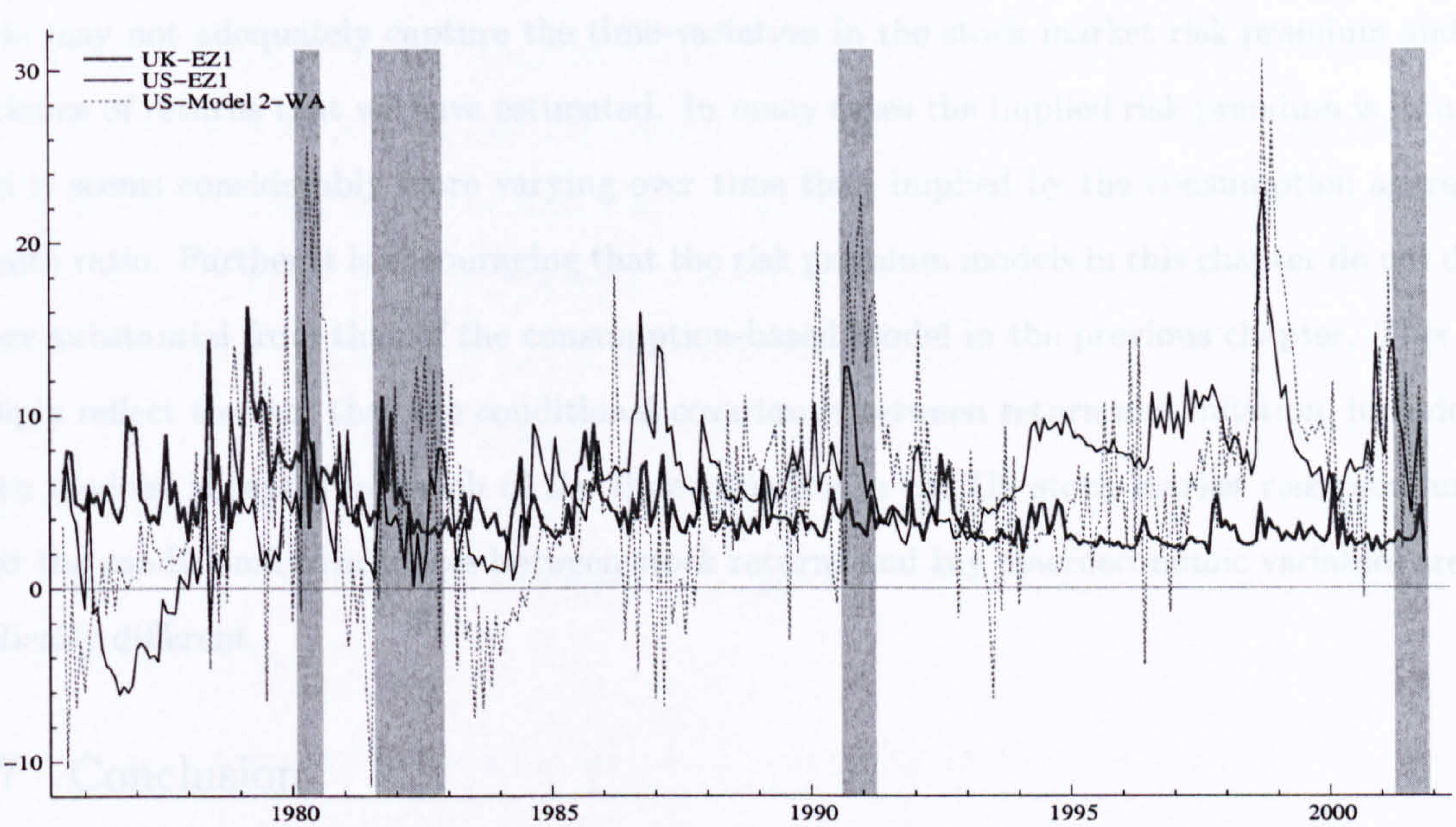
$$\overline{\gamma}_t = \sum_{j=1}^{k} \gamma_{j,t} = \frac{\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1})}{\mathrm{V}_t(i^e_{s,t+1})} \tag{3.18}$$

We note that the general model implies a time-varying relationship between the mean and the variance. Model 2-WA with an inter-temporal relation between the mean and the variance of return implies a $\bar{\gamma}_t$ which is occasionally negative but we have to keep in mind that the implied risk premium is more variable and the period under consideration has some large macroeconomic shocks which have a very high impact on the estimate of the conditional covariances. In model 2-WA the correlation between the conditional variance of return and the risk premium is 0.20 and shows, in contrast to Lettau and Ludvigson [82], that this correlation is positive and not even close the -0.59 they find in quarterly data. We believe that the method used in this chapter and in chapter 2 provides a superior econometric framework for computing such a correlation.

To end this section we compare the results from the previous chapter and the current chapter in the sample period that is overlapping. In figure (3.6) we plot the risk premium per unit of variance from model 2-WA together with the estimated relationship in the EZ2 model in chapter 2 for both the UK and US estimations. We plot the ratios shading the US recession periods only in order to compare the behaviour of the US model in this chapter and the US model in the previous chapter during recessions.

First we note the decline in the UK ratio over the whole sample period which appears not to be the case in the US models. The UK ratio was relatively high until the early 1980s. The US ratio is considerably more time-varying in both models. Although the trend in our alternative model from the current chapter and the consumption-based model from the previous chapter is the same there are some differences between the models. The correlation between the two US ratios is 0.39 and it appears to be highest towards the end of the sample. The more alternative model estimated in this chapter seems to imply a lower level of the risk premium during the early/mid 1990s - since stock prices were rising steadily most likely it is that risk premia are lower. The correlation between the risk premium per unit of variance in the UK and US implied by the consumption based models is -0.27. Although, as noted in the introductory chapter, the correlations between the UK and US datasets is high the investment strategies for people in the two markets seem rather different based on our estimates of the premium per unit of variance[14].

Figure 3.6: The Risk Premium Per Unit of Variance



The implied time-varying price of risk from model 2-WA and from the UK and US EZ1 models in chapter 2. We plot the series for overlapping sample only. Shaded areas are recessions as defined by the NBER.

3.6.9 A Relation to the Consumption Aggregate Wealth Ratio ?

We have emphasised, in chapter 1 and the discussion in chapter 2, the approximate consumption aggregate wealth ratio derived by Lettau and Ludvigson. The problem of compatibility is that Lettau and Ludvigson use quarterly data whereas we are using monthly data. To get an idea if

[14]Strictly speaking we should look at premium per unit of standard deviation but the implied dynamics are highly correlated.

any relation between our estimated risk premia and conditional variance of stock returns with the approximate consumption aggregate wealth ratio we plot in figure (3.18) and figure (3.19) in the appendix the consumption aggregate wealth ratio against the risk premium and conditional variance estimated in model 2-WA. From the figure we see that the correlation is far from perfect. There is a high comovement between the conditional variance of return and the consumption aggregate wealth ratio in the early part of the sample and towards the end. However, the low level of stock return variance in the 1990s has been a period with a high consumption aggregate wealth ratio. Plotting the wealth ratio against the risk premium in model 2-WA we note the tendency for the wealth ratio to increase prior to periods with higher risk premia but the relationship between the consumption aggregate wealth ratio and our risk premium seems to have broken down in the 1990s. Our general conclusion is that the consumption aggregate wealth ratio may not adequately capture the time-variation in the stock market risk premium and the variance of returns that we have estimated. In many cases the implied risk premium is intuitive and it seems considerably more varying over time than implied by the consumption aggregate wealth ratio. Further it is encouraging that the risk premium models in this chapter do not differ more substantial from that of the consumption-based model in the previous chapter. This may simply reflect the fact that the conditional covariance between return and inflation, included in both models, is capturing much of the time-variation in the US stock market risk premium or that the conditional covariances between stock returns and key macroeconomic variables are not radically different.

3.7 Conclusion

In this chapter we investigate the relationship between several US macroeconomic variables and the US stock market excess return - more specifically the relation between financial- and macroeconomic uncertainty (variance or standard deviation) and financial compensation for taking risk in the US stock market (variance and covariance). We proposed a multivariate model for this purpose that accounts for many characteristics of the data. The model allows positive and negative shocks to have a different impact on uncertainty and risk compensation. Modelling of risk compensation is often disregarded in empirical studies of financial returns with macroeconomic variables. The model can be applied to investigate the relation between financial returns and any key macroeconomic variables.

Long time series of consumption data are not available for most countries and we used this as a motivation for considering alternative key, at higher (monthly) frequency, macroeconomic variables that are commonly available for most countries. To model the risk premium on a broad stock market index in the US we relied on a version of the CAPM commonly used in empirical finance. Although we showed in the previous chapter that consumption growth was significantly priced in the US stock market it was of interest whether these alternative key macroeconomic variables were significantly priced in the US stock market after having accounted for the priced risk in the market return itself. Pricning just the market return is common practice in financial economics.

Examining a sample from 1960 to 2003 of US stock return and macroeconomic data we show that a model allowing a time-varying relationship between the risk premium and the excess return variance implies a risk premium that varies considerably over time. We document that CPI inflation, money- and industrial production growth are important macroeconomic sources of risk which are priced in the US stock market creating this time-varying relationship. A version of the Inter-temporal CAPM with a logarithmic SDF linear in the market return, inflation and industrial production growth is also able to generate a significant time-varying risk premium. Pricing only a single variable, inflation, we document that the implied risk premium and its time-variation is very intuitive - the stock market risk premium rises during recessions. This serves to further highlight our conclusion in chapter 2. Inflation is a significant source of risk priced in the UK and US.

Much of the literature has been devoted to investigating whether the asymmetry in the conditional variance of the excess return is due to the leverage effect or the volatility feedback hypothesis. Few studies have considered the possibility that the asymmetry arises from misspecification of the risk premium. Asymmetry has been documented in papers which primarily model the risk premium as a constant function of the variance of the excess return. We show, on the contrary, that when allowing for a time-varying relationship between the mean and the variance the asymmetry is much less significant and is only borderline significant - in some of the models it disappears ! We find an economically important and significant asymmetry in the variance of industrial production growth. We show that the importance of negative shocks is most evident during recessions.

From our model we obtain consistent estimates of the conditional correlation between the stock market excess return and the macroeconomic variables and conclude that the Constant Conditional Correlation model is not an adequate model for investigating the interaction between financial uncertainty, macroeconomic uncertainty and risk compensation because correlations between macroeconomic variables and US stock returns are clearly varying over time.

Risk premia in the US stock market generally follow the conditional correlation between industrial production growth and inflation (also the money and output correlation). When output and inflation have a positive correlation, that is periods where unexpected shocks are mainly demand shocks, risk premia tend to be higher and with negative correlation, that is supply shocks are prevalent, risk premia tend to be low - on several occasions when the correlation is strongly negative the model implied risk premium is negative and is closely related to the US economy going into a recession. The risk premium generally increases during recessions while falling immediately after.

The important message of the chapter, and the previous chapter, is that macroeconomic risks are priced in the US stock market in a way consistent with the ICAPM of Merton [90]. The implied risk premium varies considerably over time even though we analyse monthly data. Most encouraging it is that five of our estimated risk premium models imply risk premia that does not differ radically from one another. We show that findings of asymmetries in the conditional covariance matrix between excess return and the macroeconomic variables depend critically on the modelling of the risk premium.

Univariate GARCH in mean models are not suitable for modelling the conditional variance of monthly excess returns in the UK and US stock markets (at least on broad market indices). Having shown that asymmetries in the risk premium are economically significant we conclude that it could be interesting to extend the work in chapter 2 estimating a consumption-based asset pricing model with asymmetries. However, whereas it may be possible for the US it could prove difficult for the UK since data are only available from 1974. Moreover, there is less ARCH in consumption data than the macroeconomic variables used in this chapter.

3.8 Appendix Chapter 3

Table 3.3: Descriptive Statistics of Dataset

	$i^e_{s,t+1}$	π_{t+1}	Δq_{t+1}	Δy_{t+1}
Mean	7.0922	4.2691	5.0167	3.0398
Std. Dev	53.456	3.6192	6.0158	9.0017
Skewness	-0.7145	1.0313	0.1164	-0.5939
Kurtosis	5.7939	4.6984	4.0817	5.9231
Normality	59.907**	90.096**	21.523**	72.3600**
$\rho(x_t, x_{t-1})$	0.0707	0.6623	0.5204	0.3675
$\rho(x_t, x_{t-2})$	-0.0527	0.6030	0.3336	0.2935
$\rho(x_t, x_{t-3})$	-0.0050	0.5651	0.3324	0.2647
$\rho(x_t, x_{t-4})$	-0.0056	0.5435	0.3058	0.2102
$\rho(x_t, x_{t-5})$	0.0673	0.5415	0.3303	0.0819
$\rho(x_t, x_{t-6})$	-0.0307	0.5391	0.3358	0.0969
$\rho(x_t^2, x_{t-1}^2)$	0.0525	0.6610	0.5252	0.2686
$\rho(x_t^2, x_{t-2}^2)$	0.1189	0.6248	0.3429	0.1436
$\rho(x_t^2, x_{t-3}^2)$	0.1463	0.5872	0.3077	0.1354
$\rho(x_t^2, x_{t-4}^2)$	0.0847	0.5618	0.2319	0.0518
$\rho(x_t^2, x_{t-5}^2)$	0.1020	0.5723	0.2229	-0.0438
$\rho(x_t^2, x_{t-6}^2)$	0.0877	0.5783	0.2620	0.0672
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-1})$	0.0548	0.6287	0.4361	0.3137
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-2})$	0.0563	0.6112	0.2597	0.1301
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-3})$	0.0671	0.5431	0.2231	0.0970
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-4})$	0.0315	0.5222	0.2257	0.0521
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-5})$	0.0157	0.5473	0.2014	-0.0394
$\rho(\lvert x\rvert_t, \lvert x\rvert_{t-6})$	0.0237	0.5226	0.2130	0.0375

Descriptive statistics of dependent variables. $\rho()$ is the correlation and x refers to the variable in the first row in column 2-5. Mean and standard deviation of annualised dataset.

Table 3.4: The Correlation Structure of Risk Premia

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}
$\phi_t^{Model\ 1\text{-}WA}$	0.9803	0.9617	0.9343	0.9060	0.8755	0.8464	0.8166	0.7870	0.7568	0.7246	0.6890	0.6524
$\phi_t^{Model\ 2\text{-}WA}$	0.4291	0.5927	0.3147	0.2975	0.0926	0.0896	−0.0110	−0.0404	−0.0757	−0.0878	−0.0843	−0.0971
$\phi_t^{Model\ 3\text{-}WA}$	0.3324	0.5771	0.2586	0.2689	0.1077	0.0654	0.0001	−0.0519	−0.0665	−0.1142	−0.1265	−0.1222
$\phi_t^{Model\ 4\text{-}WA}$	0.7321	0.6082	0.4236	0.2315	0.0896	−0.0305	−0.1215	−0.1917	−0.2327	−0.2610	−0.2492	−0.2304
$\phi_t^{Model\ 5\text{-}WA}$	0.5723	0.5761	0.4136	0.3233	0.1959	0.1684	0.0922	0.0530	−0.0102	−0.0931	−0.0950	−0.1283
$\phi_t^{Model\ 6\text{-}WA}$	0.2648	0.6000	0.1819	0.3536	0.0509	0.2032	−0.0073	0.0856	−0.0615	−0.0341	−0.1334	−0.0684

The correlation structure of the risk premium. $\rho_j = \rho(\phi_t, \phi_{t-j})$.

Table 3.5: Granger Causality Tests Of Risk Premium And Volatility

Full Sample

	ϕ_t	$\sigma_t(r^2_{s,t+1})$	$\sigma_t(\pi_{t+1})$	$\sigma_t(\Delta q_{t+1})$	$\sigma_t(\Delta y_{t+1})$	R^2
ϕ_t	155.9350**	0.5006	$\boxed{31.8040^{**}}$	21.5820**	135.581**	0.7243
$\sigma_t(r^2_{s,t+1})$	$\boxed{13.8826^{**}}$	1308.33**	7.9013**	22.5505**	128.66**	0.9577
$\sigma_t(\pi_{t+1})$	13.5864**	0.1398	478.0280**	4.4917**	$\boxed{7.7517^{**}}$	0.8242
$\sigma_t(\Delta q_{t+1})$	1.9100	0.6077	2.4889	127.9240**	1.5452	0.5996
$\sigma_t(\Delta y_{t+1})$	4.6932**	0.8900	14.3416**	$\boxed{3.0156^{*}}$	97.1524**	0.7259

1960:01 to 1978:12

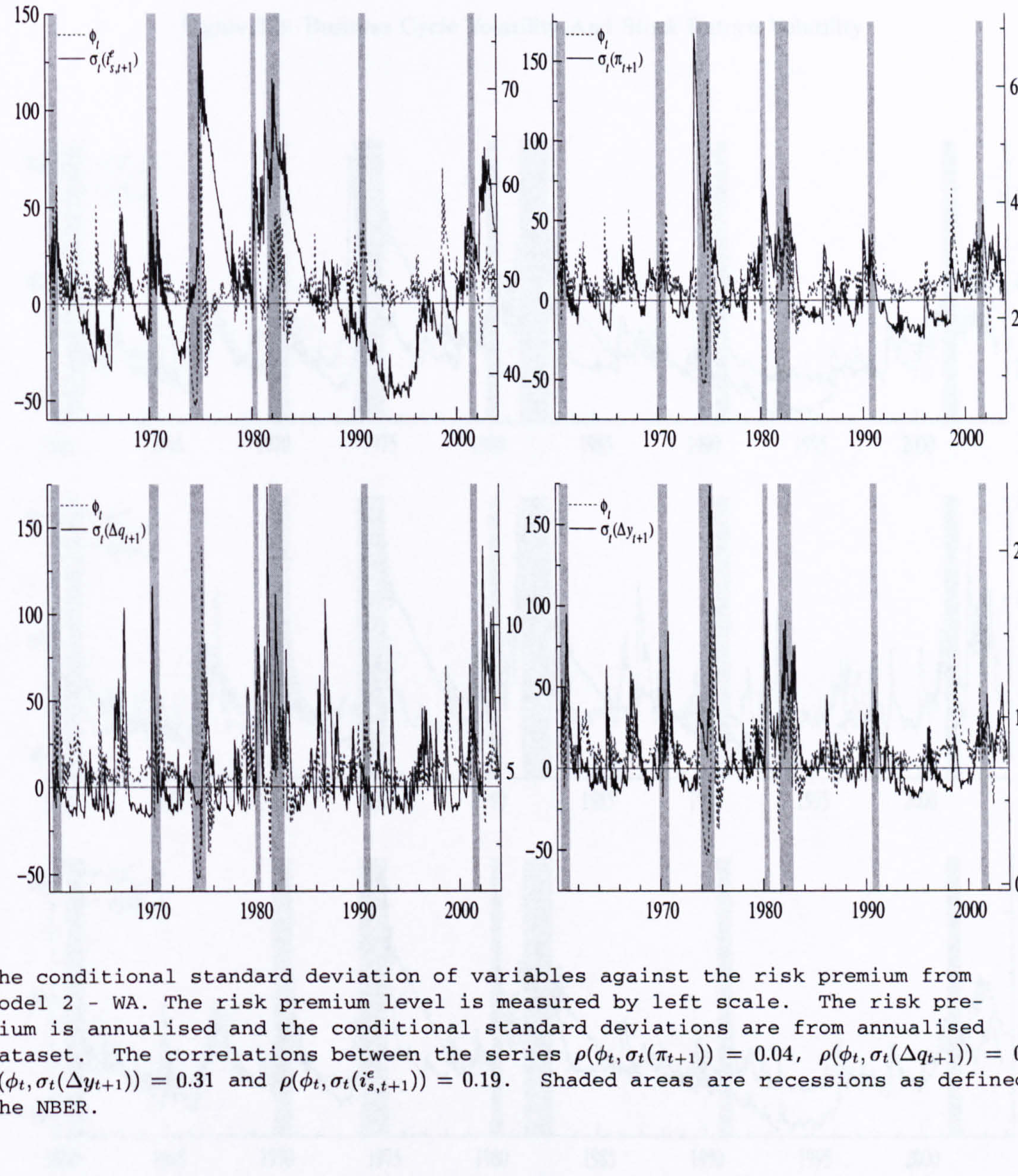
	ϕ_t	$\sigma_t(r^2_{s,t+1})$	$\sigma_t(\pi_{t+1})$	$\sigma_t(\Delta q_{t+1})$	$\sigma_t(\Delta y_{t+1})$	R^2
ϕ_t	66.2531**	0.1754	$\boxed{25.5524^{**}}$	14.6575**	142.6540**	0.8364
$\sigma_t(r^2_{s,t+1})$	$\boxed{6.3838^{**}}$	709.6300**	3.4006*	9.1776**	151.0227**	0.9701
$\sigma_t(\pi_{t+1})$	2.1842	1.1489	160.4240**	1.4050	1.7813	0.8214
$\sigma_t(\Delta q_{t+1})$	1.5718	0.3327	0.2103	58.9148**	0.7489	0.6218
$\sigma_t(\Delta y_{t+1})$	2.1835	1.4521	2.8380*	0.3108	42.8149**	0.7120

1982:01 to 2002:11

	ϕ_t	$\sigma_t(r^2_{s,t+1})$	$\sigma_t(\pi_{t+1})$	$\sigma_t(\Delta q_{t+1})$	$\sigma_t(\Delta y_{t+1})$	R^2
ϕ_t	66.7640**	1.0734	$\boxed{11.3491^{**}}$	7.5888**	$\boxed{18.4895^{**}}$	0.6396
$\sigma_t(r^2_{s,t+1})$	$\boxed{5.4203^{**}}$	615.1920**	4.4179**	9.7944**	15.9390**	0.9649
$\sigma_t(\pi_{t+1})$	15.2953**	0.3754	31.4020**	2.2766	5.3350**	0.8416
$\sigma_t(\Delta q_{t+1})$	0.5599	1.4656	2.3692	29.6139**	3.3761*	0.5663
$\sigma_t(\Delta y_{t+1})$	8.1768**	1.8979	6.9216**	$\boxed{5.7638^{**}}$	58.7001**	0.7974

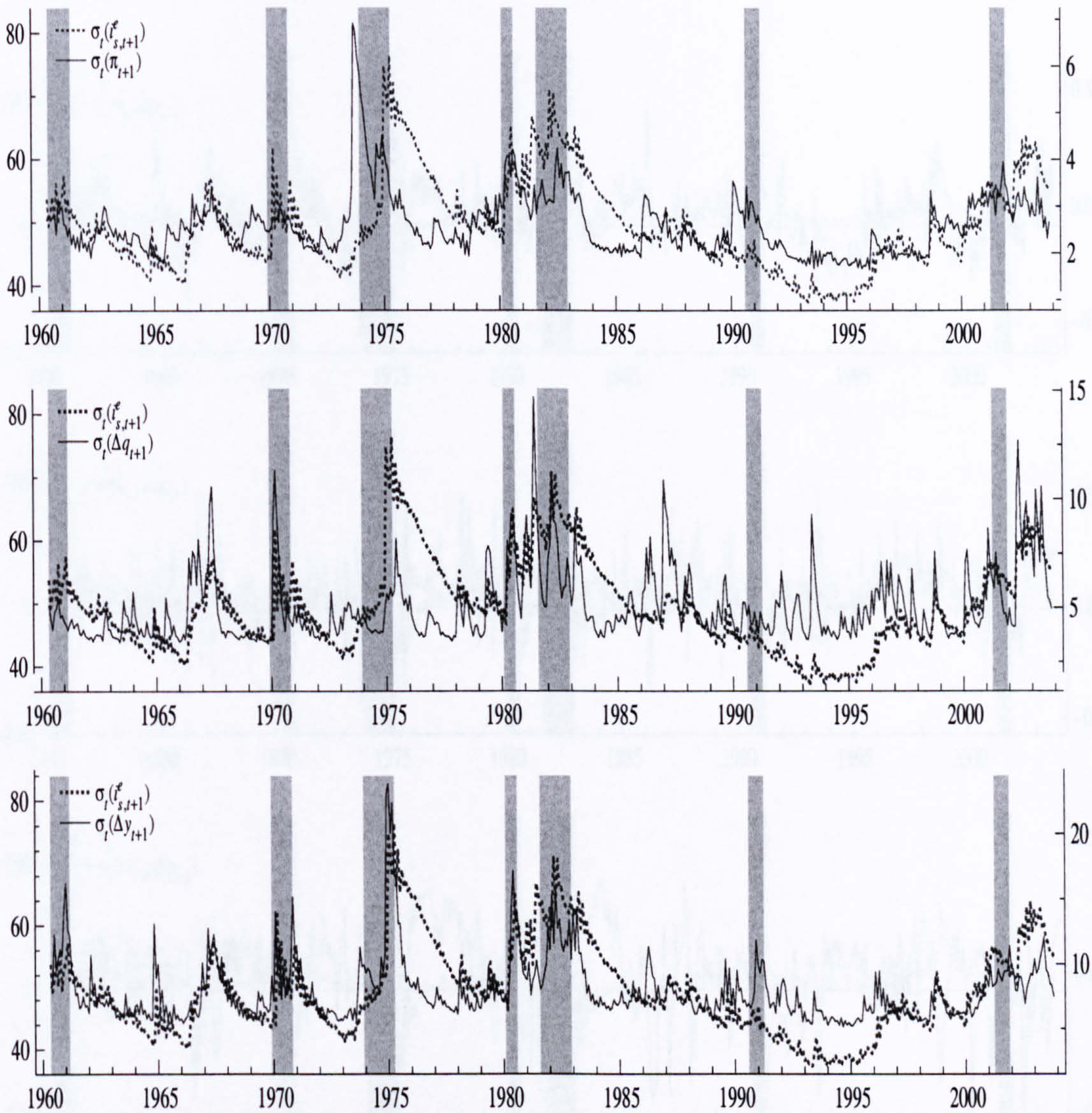
5 variable 3^{rd} order vector auto regression. F test in main table test joint significance of three lags of the column variable in the equation of the variable in the first column. A star indicates significance using the 95 % asymptotic critical value and two stars indicate joint significance using the 99 % critical value. We emphasise variables significant using the latter critical value. In each equation we underline the R^2. If a signigicant F-statistic has a box around it, it means that the sum of the coefficients on the three lags of the variable is negative.

Figure 3.7: Risk Premium Vs. Volatility



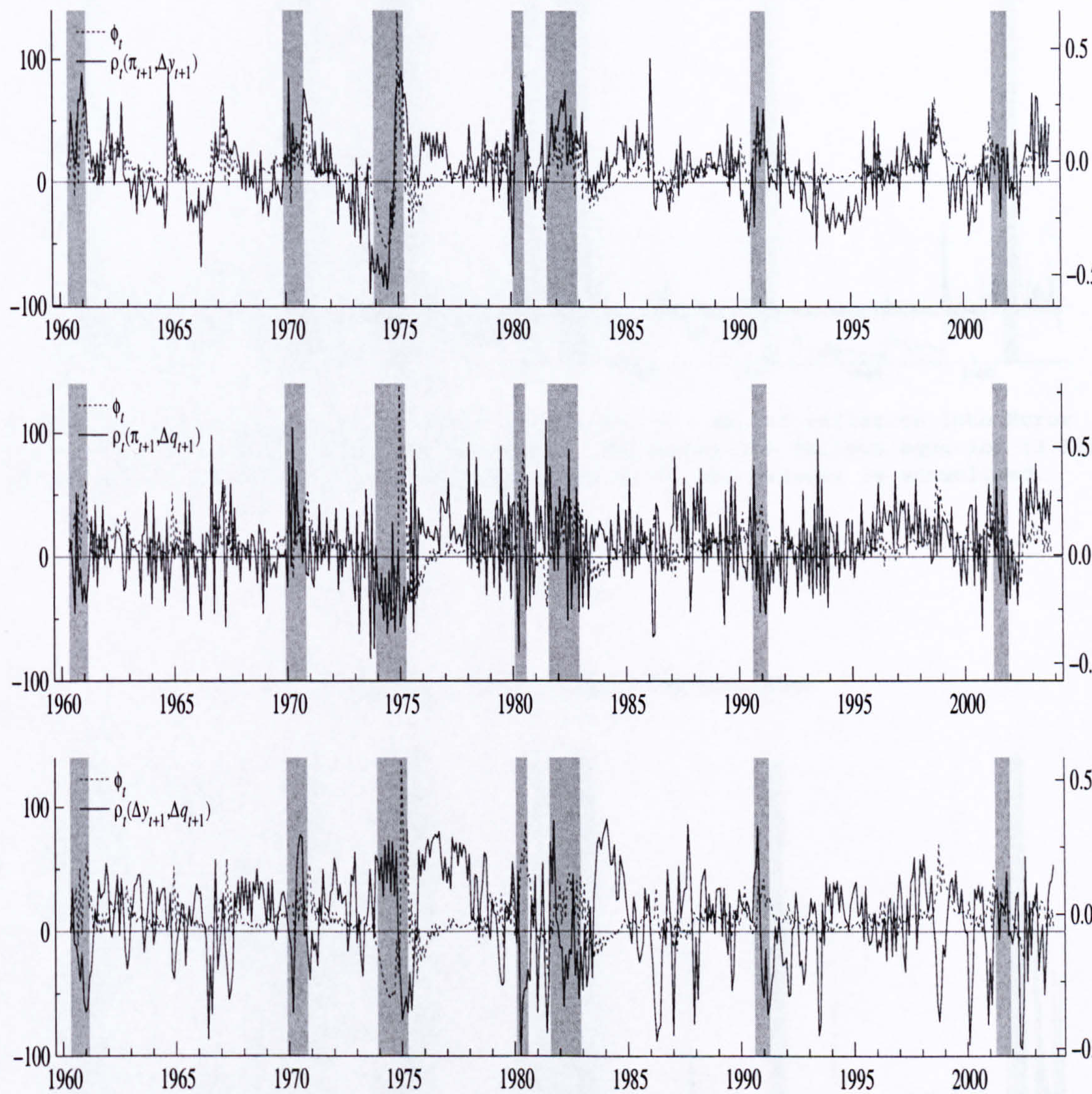
The conditional standard deviation of variables against the risk premium from model 2 - WA. The risk premium level is measured by left scale. The risk premium is annualised and the conditional standard deviations are from annualised dataset. The correlations between the series $\rho(\phi_t, \sigma_t(\pi_{t+1})) = 0.04$, $\rho(\phi_t, \sigma_t(\Delta q_{t+1})) = 0.07$, $\rho(\phi_t, \sigma_t(\Delta y_{t+1})) = 0.31$ and $\rho(\phi_t, \sigma_t(i^e_{s,t+1})) = 0.19$. Shaded areas are recessions as defined by the NBER.

Figure 3.8: Business Cycle Volatility And Stock Return Volatility



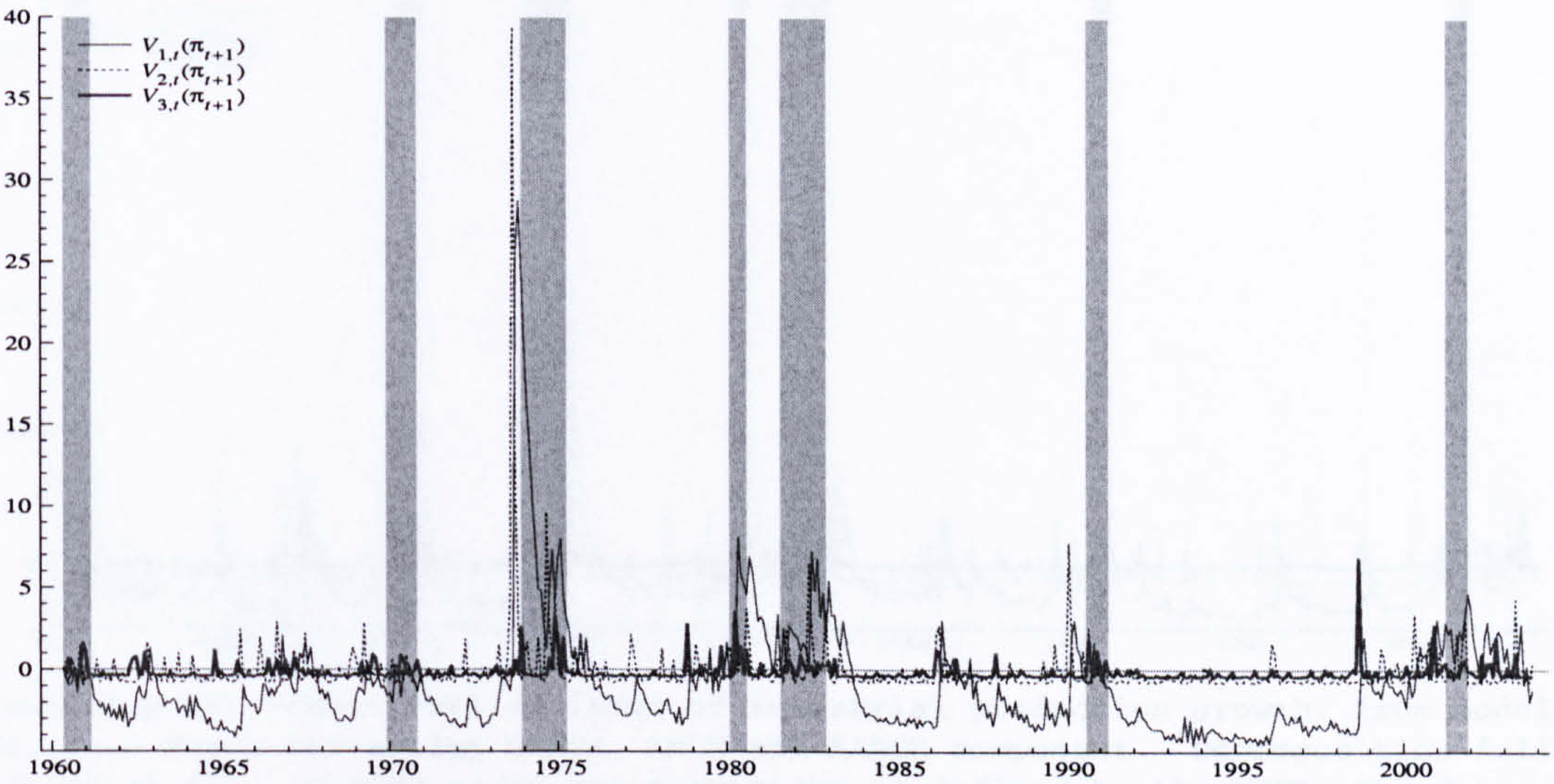
The conditional standard deviation of excess return against standard deviation of macro variables from model 2 - WA. Left y-scale measures the standard deviation of excess return and the right scale measures the macroeconomic standard deviations. Standard deviations of annualised dataset. The correlations between the series are $\rho(\sigma_t(i^e_{s,t+1}), \sigma_t(\pi_{t+1})) = 0.46$, $\rho(\sigma_t(i^e_{s,t+1}), \sigma_t(\Delta q_{t+1})) = 0.48$ and $\rho(\sigma_t(i^e_{s,t+1}), \sigma_t(\Delta y_{t+1})) = 0.61$. Shaded areas are recessions as defined by the NBER.

Figure 3.9: Risk Premium vs Conditional Correlations



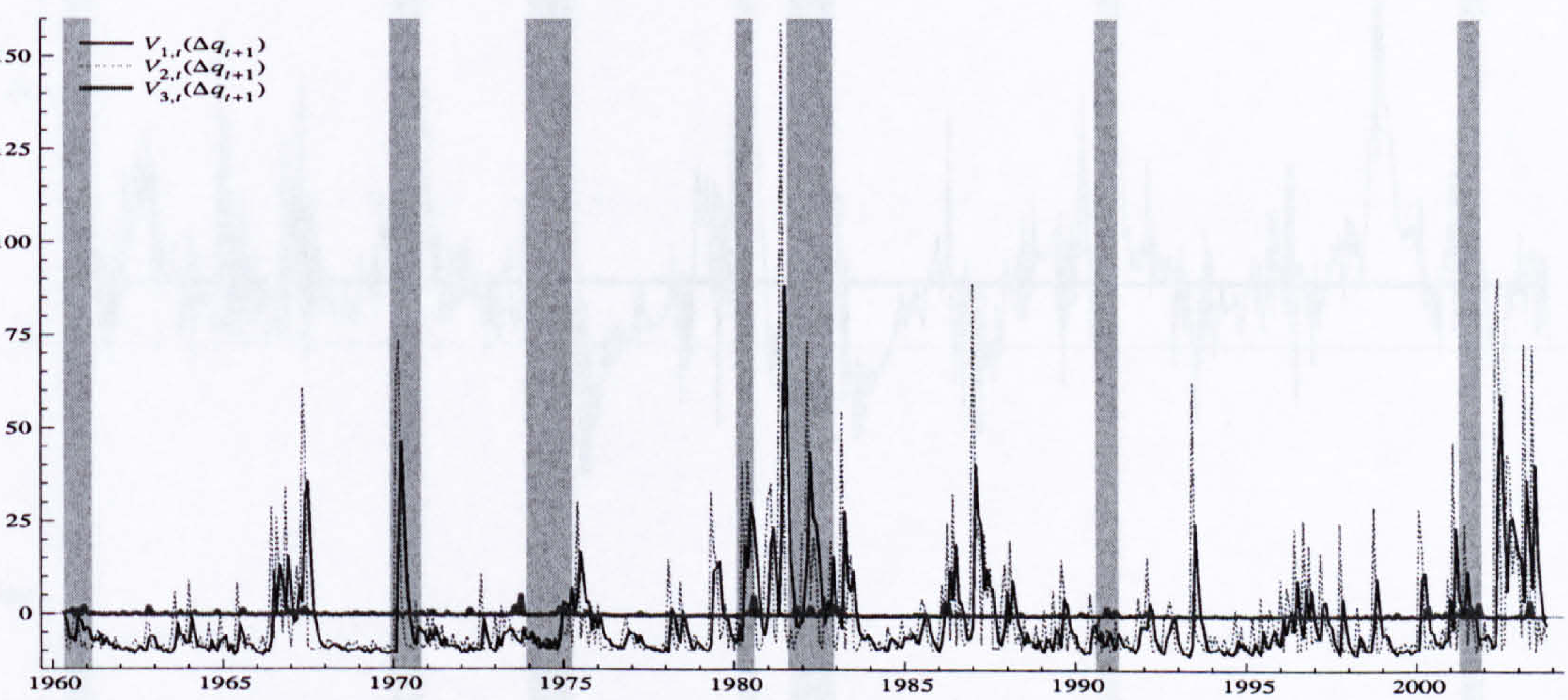
Risk premium against conditional correlations between macroeconomic variables in model 2 - WA. The risk premium has scale on left y-axis and the risk premium is annualised. Shaded areas are recessions as defined by the NBER.

Figure 3.10: Inflation Variance Decomposition



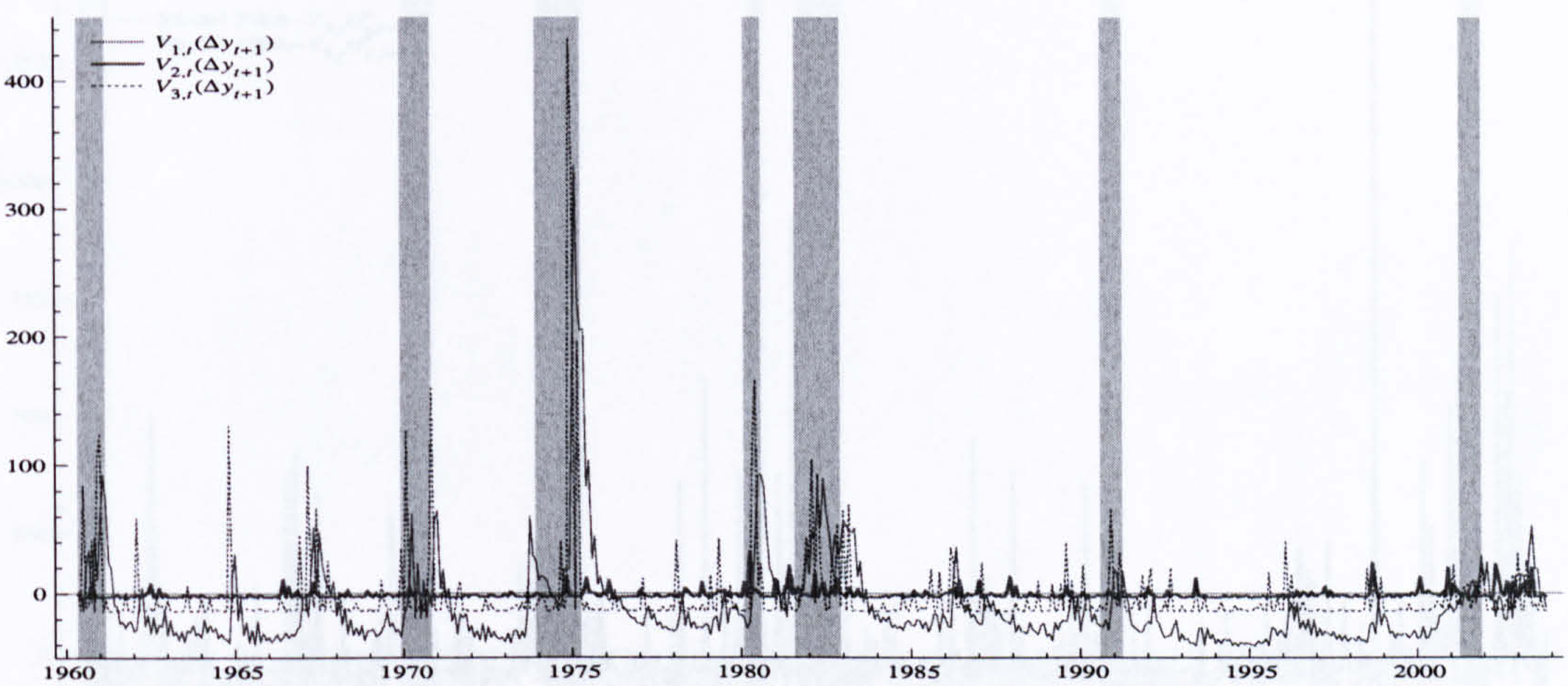
Decomposing the conditional variance, from model 2 - WA, of inflation into Error Correcting GARCH, ARCH and AARCH component. Decomposition follows equation (3.13). Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.11: Money Variance Decomposition



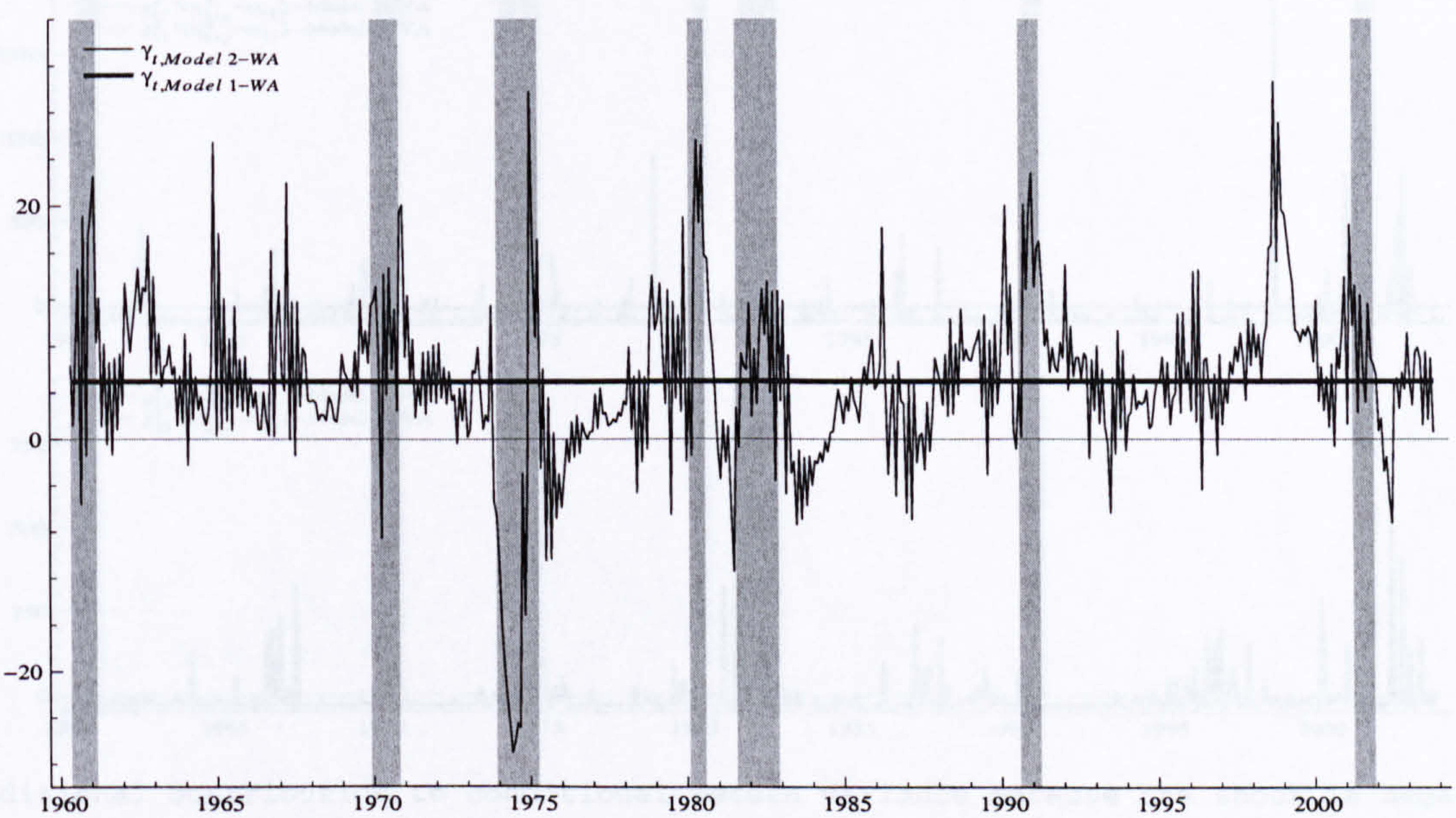
Decomposing the conditional variance of money growth, from model 2 - WA, into Error Correcting GARCH, ARCH and AARCH component. Decomposition follows equation (3.13). Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.12: Industrial Production Variance Decomposition



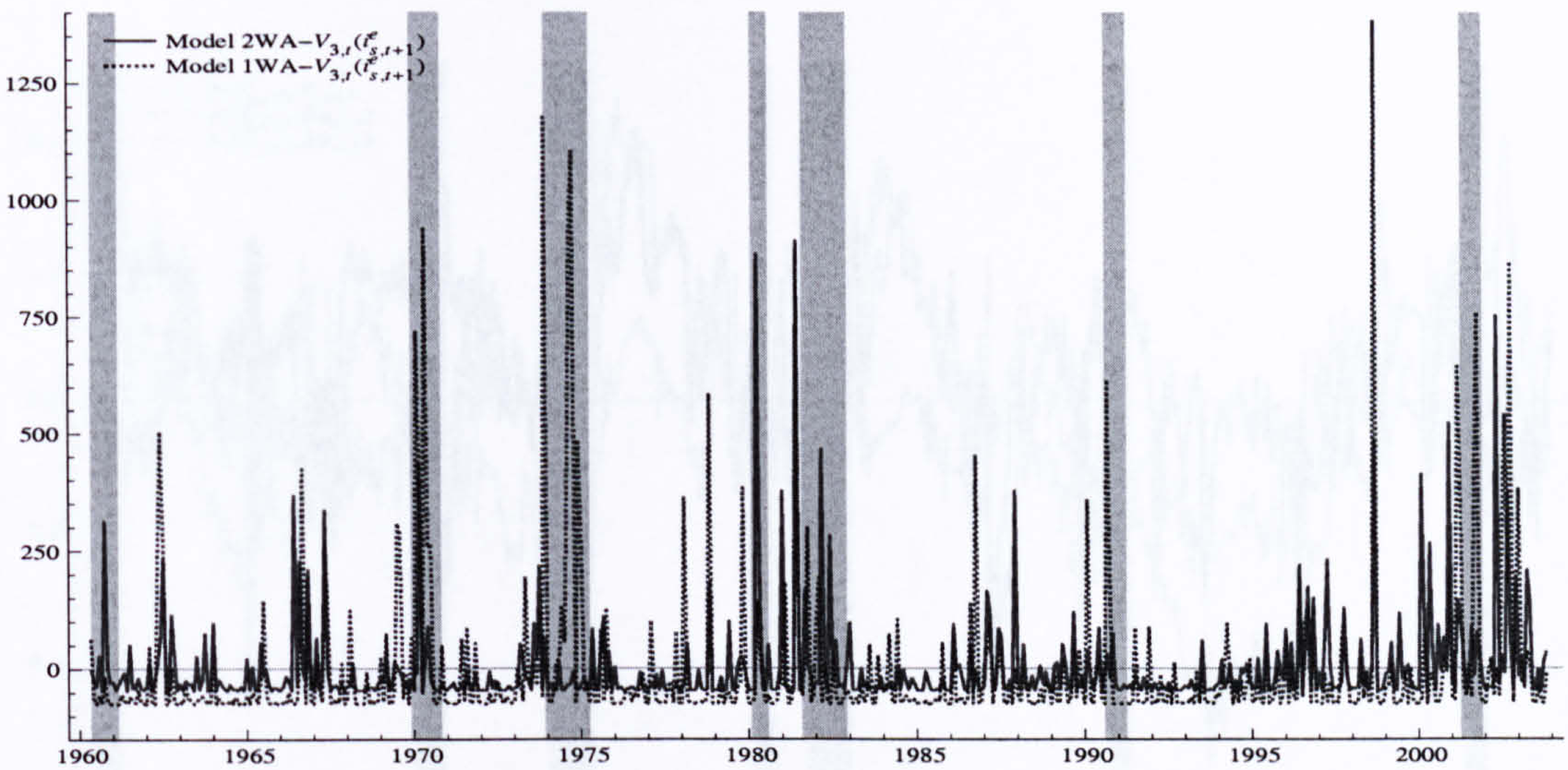
Decomposing the conditional variance of industrial production growth, from model 2 - WA, into Error Correcting GARCH, ARCH and AARCH component. Decomposition follows equation (3.13). Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.13: Risk Compensation Per Unit Of Variance



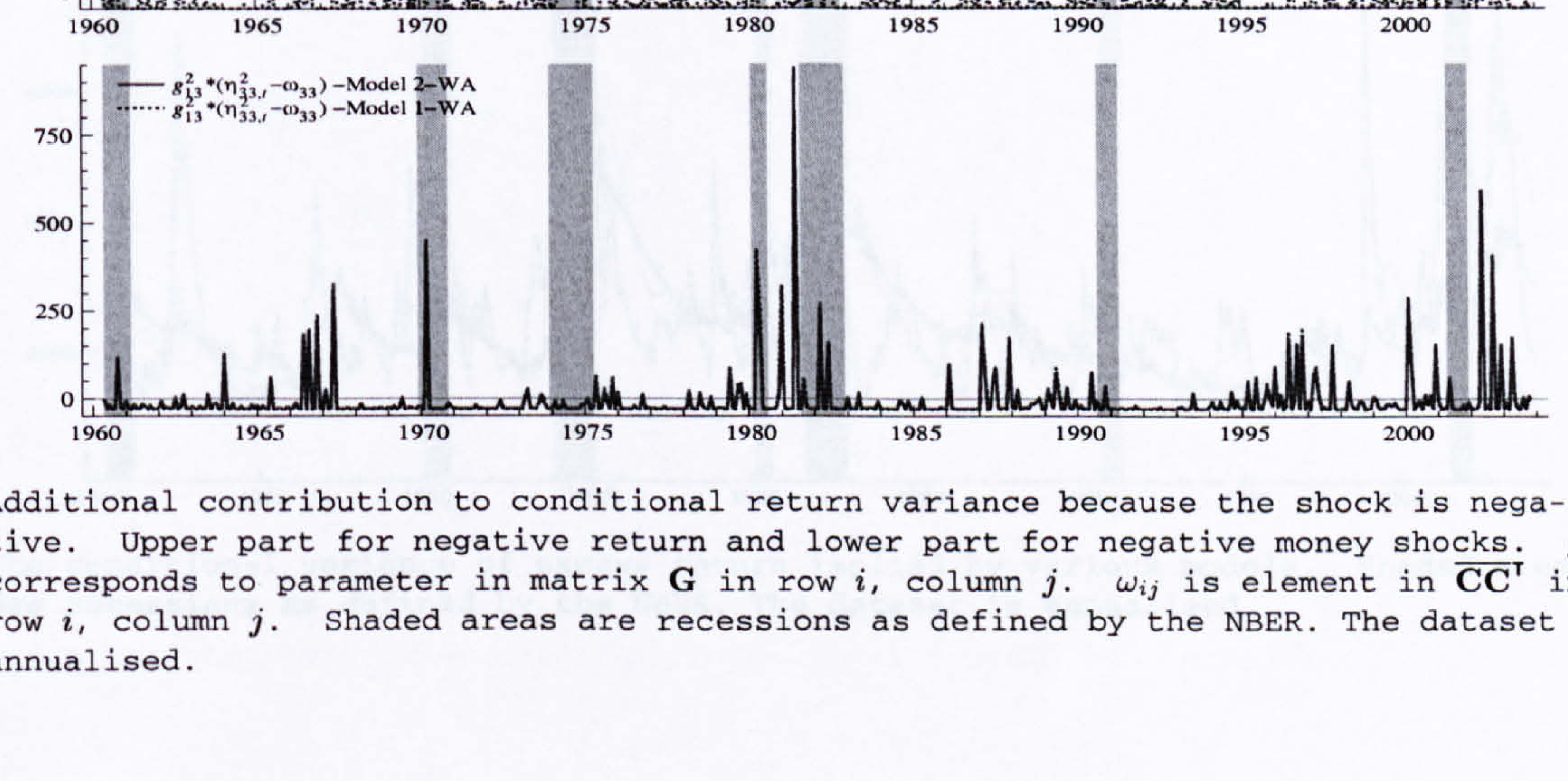
The implied time-varying and ''long run" price of risk in the CAPM, from model 1-WA and model 2 - WA. Shaded areas are recessions as defined by the NBER.

Figure 3.14: Negative Shocks In Conditional Return Variance



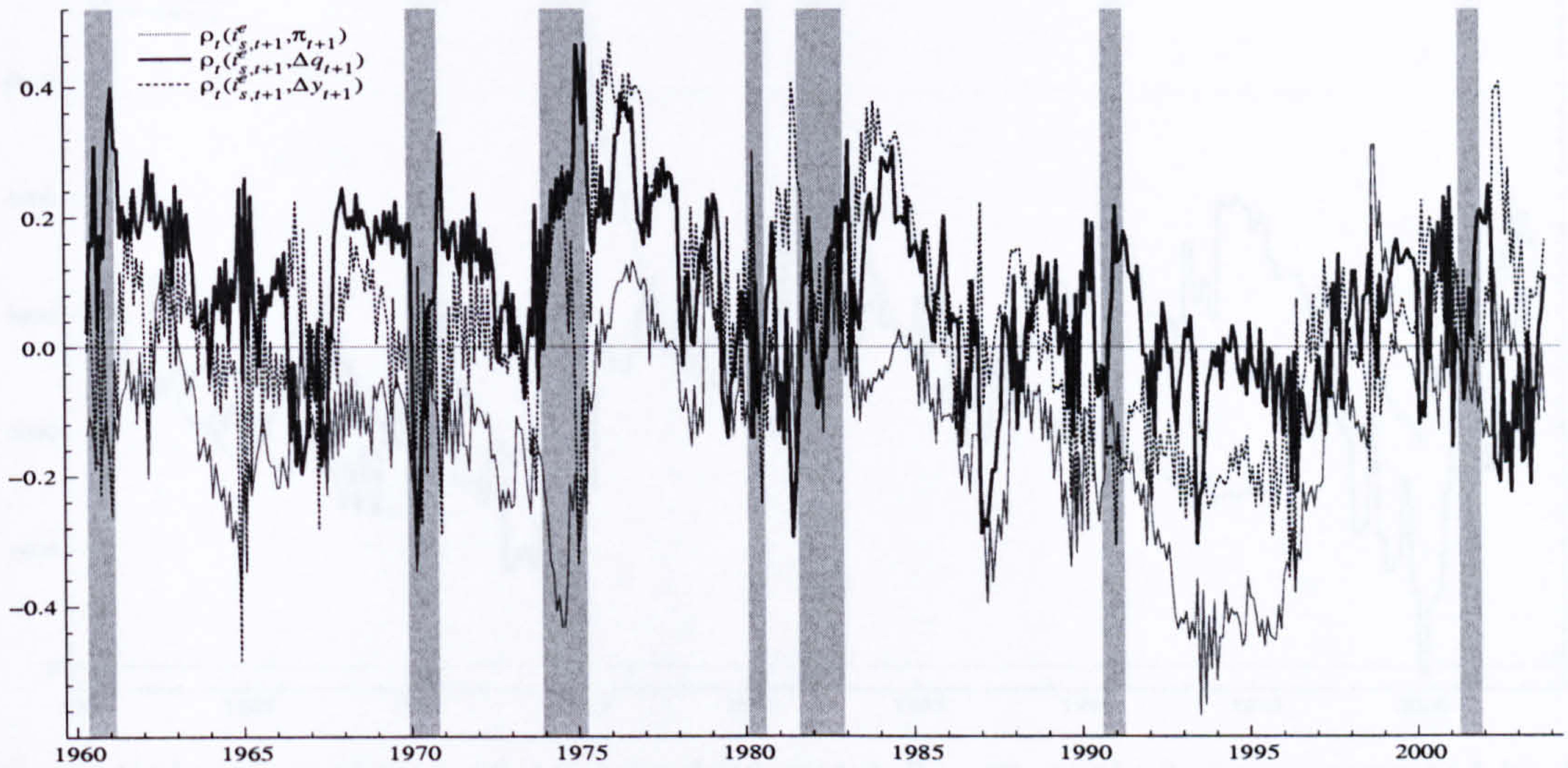
Contribution to variance of the excess return from negative shocks in two different risk premium models. Decomposition follows equation (3.13). Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.15: Negative Money or Return Shocks ?



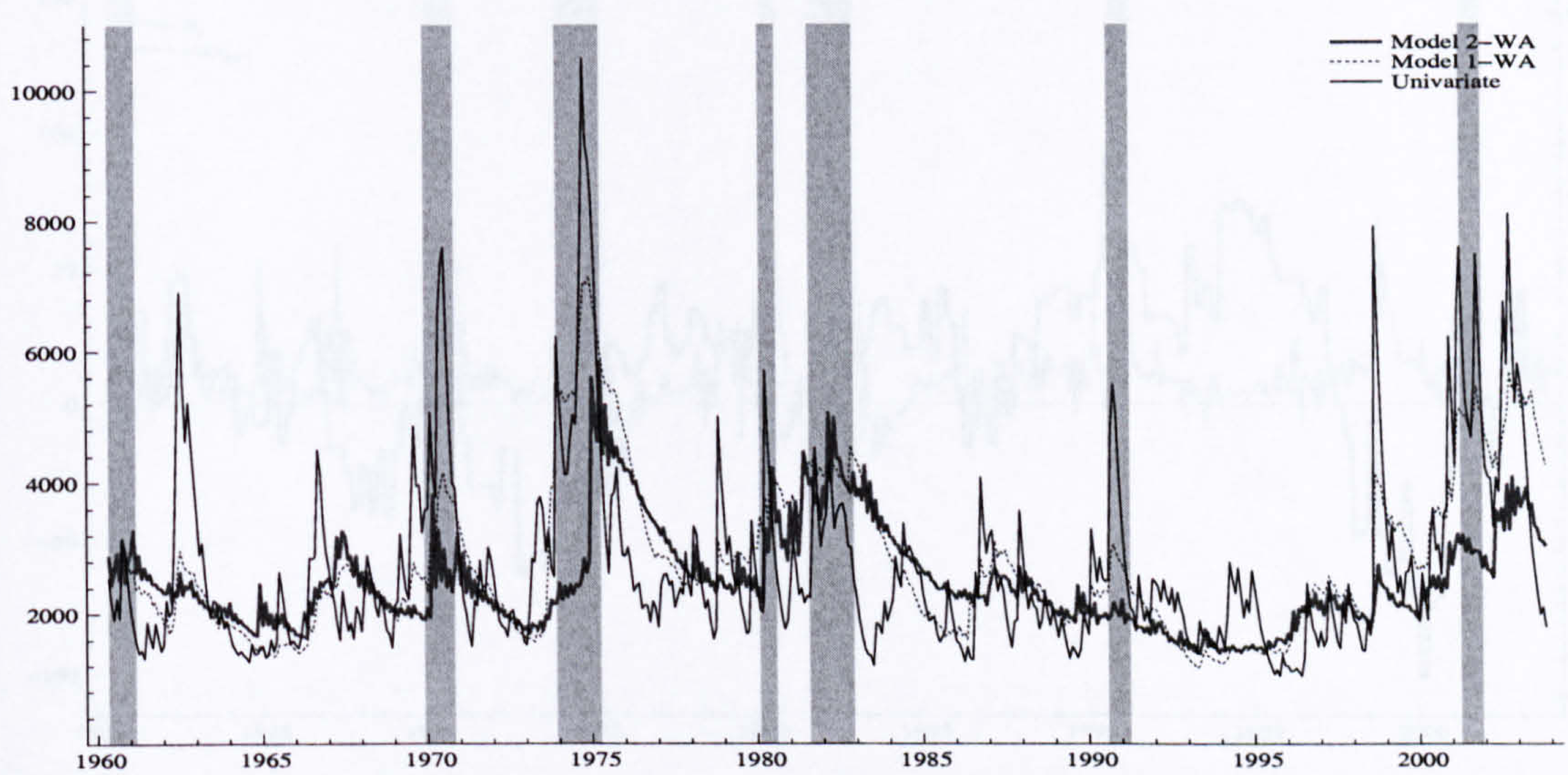
Additional contribution to conditional return variance because the shock is negative. Upper part for negative return and lower part for negative money shocks. g_{ij} corresponds to parameter in matrix $\mathbf{G}$ in row i, column j. ω_{ij} is element in $\overline{\mathbf{CC}^\intercal}$ in row i, column j. Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.16: Conditional Correlations Between Stock Return And Macroeconomic Variables



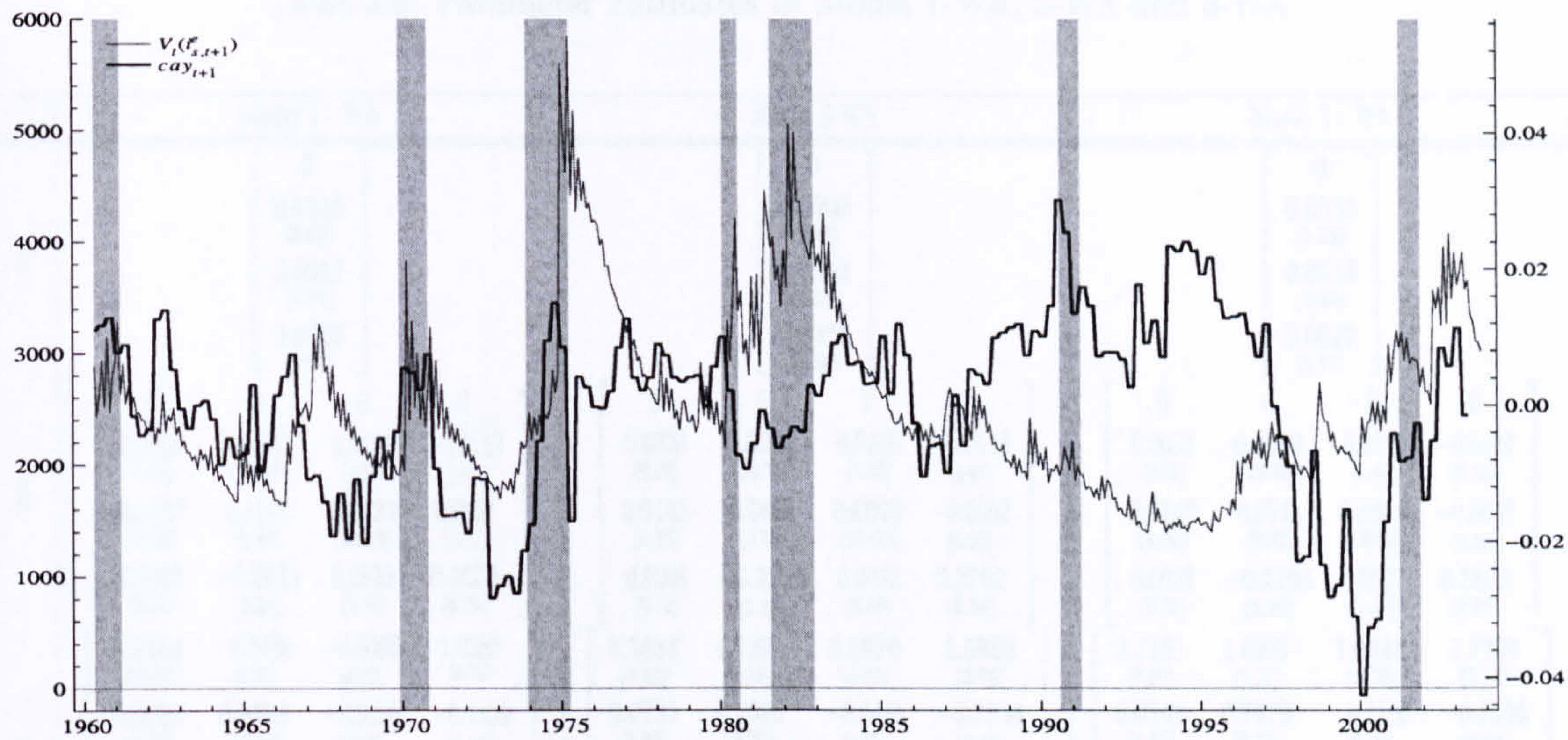
The conditional correlations, from model 2 - WA, between the log excess return and the macro variables. Shaded areas are recessions as defined by the NBER.

Figure 3.17: The Conditional Variance Of Stock Return



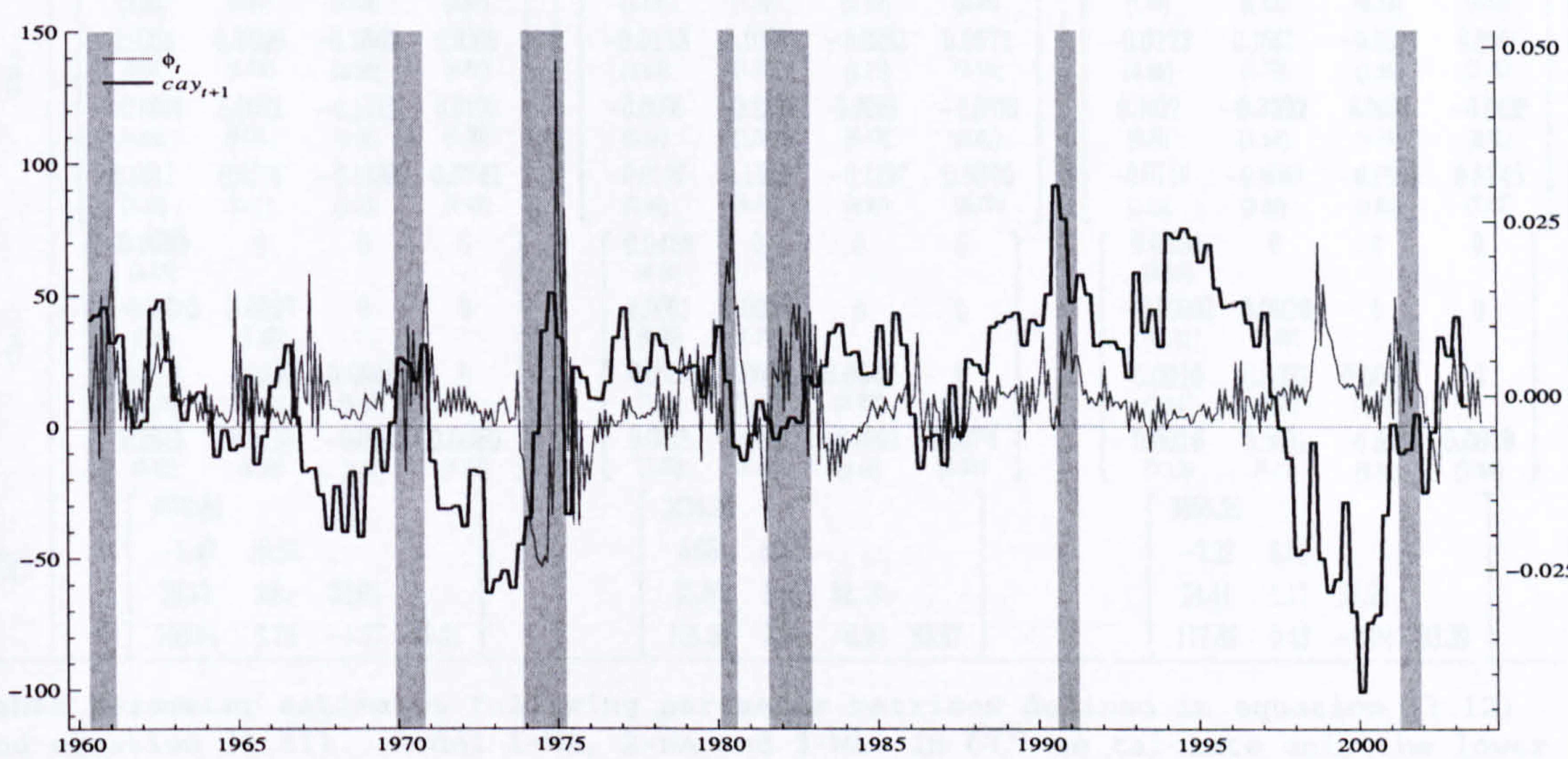
The conditional variance of excess return implied by various models. Shaded areas are recessions as defined by the NBER. The dataset is annualised.

Figure 3.18: Conditional Return Variance Related To Consumption Aggregate Wealth Ratio



The conditional variance of returns from model 2 - WA against cay_{t+1} computed by Lettau and Ludvigson. Since the data from Lettau and Ludvigson are quarterly we assume that this variable takes the same value each month within that quarter. Variance has scale to the left. Shaded areas are recessions as defined by the NBER. The conditional variance is of annualised data.

Figure 3.19: Risk Premium Related To Consumption Aggregate Wealth Ratio



The risk premium from model 2 - WA against cay_{t+1} computed by Lettau and Ludvigson. Since the data from Lettau and Ludvigson are quarterly we assume that this variable takes the same value each month within that quarter. Shaded areas are recessions as defined by the NBER. The risk premium is annualised and has left scale.

Table 3.6: Parameter Estimates In Model 1-WA, 2-WA and 3-WA

	Model 1 - WA				Model 2-WA				Model 3 - WA			
$\hat{A}$	0				0				0			
	0.0010 (6.09)				0.0009 (5.78)				0.0010 (5.99)			
	0.0011 (3.71)				0.0013 (4.37)				0.0013 (4.44)			
	0.0025 (4.97)				0.0028 (5.88)				0.0027 (5.77)			
$\hat{B}$	0	0	0	0	0	0	0	0	0	0	0	0
	0.0038 (1.73)	0.6605 (20.47)	0.0187 (1.05)	−0.0112 (0.91)	0.0020 (0.89)	0.6688 (20.79)	0.0205 (1.26)	−0.0116 (0.97)	0.0003 (0.13)	0.6562 (19.80)	0.0222 (1.40)	−0.0124 (1.05)
	0.0157 (4.15)	0.1165 (1.84)	0.5831 (14.35)	0.0097 (0.41)	0.0145 (4.28)	0.0480 (0.78)	0.6029 (15.62)	−0.0052 (0.22)	0.0140 (4.00)	0.0545 (0.92)	0.5986 (15.56)	−0.0049 (0.20)
	0.0009 (0.14)	−0.2641 (2.58)	0.0839 (1.47)	0.3058 (6.26)	0.0006 (0.12)	−0.2222 (2.32)	0.0452 (0.88)	0.2792 (6.84)	0.0022 (0.38)	−0.2165 (2.29)	0.0537 (1.06)	0.2888 (7.07)
$\hat{D}$	0.9144 (10.49)	−0.5460 (0.96)	−0.0066 (0.02)	1.0280 (0.76)	0.5684 (4.53)	2.1397 (1.50)	3.5926 (3.33)	2.5869 (3.78)	0.7251 (7.80)	1.0993 (1.27)	1.6848 (2.75)	1.7707 (3.17)
	0.0098 (2.15)	0.9308 (18.07)	−0.0035 (0.12)	−0.1139 (1.65)	0.0231 (3.05)	0.7626 (8.79)	−0.1445 (1.91)	−0.1324 (2.43)	0.0248 (2.61)	0.7679 (9.13)	−0.1156 (1.39)	−0.1536 (2.76)
	−0.0045 (0.59)	−0.0364 (0.37)	0.8068 (16.02)	0.1802 (1.73)	0.0408 (2.04)	−0.0194 (0.28)	−0.5724 (5.87)	0.1224 (1.96)	0.0538 (2.31)	0.0508 (0.17)	−0.6187 (8.57)	0.0881 (1.78)
	0.1009 (3.49)	0.2214 (0.35)	0.0504 (0.20)	−0.7475 (6.80)	0.1498 (4.20)	−1.1802 (2.42)	0.3261 (1.78)	−0.4083 (3.21)	0.1693 (3.74)	−1.2325 (2.58)	0.0633 (0.53)	−0.5059 (4.52)
$\hat{E}$	−0.1040 (2.29)	−2.0805 (2.65)	−0.7880 (1.53)	0.1841 (0.67)	−0.0074 (0.20)	−0.3696 (0.83)	−0.8104 (1.59)	0.0627 (0.29)	0.0002 (0.01)	−0.2058 (0.51)	0.0719 (0.52)	−0.0314 (0.25)
	0.0012 (0.39)	0.2354 (3.67)	0.0002 (0.01)	−0.0099 (0.57)	−0.0025 (0.96)	0.2959 (6.28)	−0.0069 (0.21)	−0.0291 (1.87)	−0.0009 (0.33)	0.3159 (6.02)	0.0252 (0.77)	−0.0264 (1.57)
	−0.0004 (0.06)	0.0004 (0.004)	0.4328 (6.97)	−0.0836 (2.29)	−0.0040 (0.81)	−0.0456 (0.41)	0.5239 (7.19)	−0.1010 (2.60)	−0.0030 (0.57)	−0.0453 (0.40)	0.5382 (7.58)	−0.1022 (2.79)
	0.0106 (1.23)	−0.1288 (0.65)	0.0752 (0.86)	0.1574 (2.04)	−0.0180 (2.19)	−0.0210 (0.14)	−0.2203 (2.29)	0.0071 (0.57)	−0.0145 (1.67)	−0.0531 (0.33)	−0.2312 (2.33)	−0.0326 (0.47)
$\hat{G}$	−0.2076 (4.25)	0.8581 (0.67)	1.3521 (2.36)	−0.3036 (0.54)	−0.0704 (1.97)	−1.2354 (1.19)	−1.3284 (3.04)	0.0774 (0.36)	−0.0340 (1.48)	−1.2550 (1.73)	−1.3945 (4.38)	0.1222 (0.82)
	0.0016 (0.39)	0.2509 (2.78)	−0.1662 (4.30)	0.0305 (0.87)	−0.0118 (3.64)	0.1060 (1.12)	−0.0283 (1.11)	0.0571 (2.19)	−0.0123 (3.65)	0.1667 (1.79)	−0.0321 (1.25)	0.0461 (1.76)
	−0.0008 (0.08)	0.0002 (0.001)	−0.1431 (0.98)	0.0196 (0.30)	−0.0005 (0.04)	−0.2339 (1.08)	0.0208 (0.13)	−0.0378 (0.61)	0.0022 (0.21)	−0.3222 (1.96)	0.0638 (0.78)	−0.0438 (0.85)
	0.0011 (0.10)	0.0714 (0.17)	−0.1830 (1.33)	0.5741 (6.42)	−0.0126 (0.96)	0.1344 (0.42)	−0.1197 (0.97)	0.5670 (6.72)	−0.0116 (1.03)	−0.0068 (0.02)	−0.0994 (1.05)	0.5745 (7.67)
$\hat{C}$	0.0563 (4.28)	0	0	0	0.0488 (6.39)	0	0	0	0.0504 (8.98)	0	0	0
	−0.00002 (0.05)	0.0027 (5.02)	0	0	0.0001 (0.28)	0.0024 (7.79)	0	0	−0.00003 (0.15)	0.0026 (7.49)	0	0
	0.0003 (0.64)	0.0005 (0.71)	0.0048 (5.87)	0	0.0005 (1.12)	0.0005 (1.11)	0.0047 (6.79)	0	0.0010 (2.51)	0.0003 (0.88)	0.0046 (8.97)	0
	0.0012 (1.17)	0.0010 (0.96)	−0.0004 (0.61)	0.0080 (5.44)	0.0015 (1.52)	0.0005 (0.60)	−0.0003 (0.56)	0.0074 (6.58)	0.0016 (2.12)	0.0001 (0.17)	−0.0004 (0.66)	0.0079 (7.69)
$\hat{C}\hat{C}^{\intercal}$	4570.84				3434.19				3655.29			
	−1.40	10.52			5.05	8.34			−2.22	9.59		
	23.12	2.01	33.05		35.30	1.65	32.18		74.41	1.17	32.24	
	100.84	3.70	−1.77	95.31	105.98	1.84	−0.90	83.67	117.89	0.43	−0.24	93.39

Other parameter estimates following parameter matrices defined in equation (3.12) and equation (1.51). Model 1-WA, 2-WA and 3-WA. In $\hat{C}\hat{C}^{\intercal}$ we tabulate only the lower triangular part. Absolute t-statistics in parenthesis. Emphasised parameters significant using 95 % critical value.

Table 3.7: Parameter Estimates In Model 4-WA, 5-WA and 6-WA

	Model 4 - WA				Model 5-WA				Model 6 - WA			
$\widehat{A}$	0				0				0			
	0.0886 (4.98)				0.0772 (4.60)				0.0813 (4.98)			
	0.1172 (3.78)				0.1265 (4.10)				0.1186 (3.93)			
	0.2348 (4.80)				0.2848 (5.54)				0.2576 (5.11)			
$\widehat{B}$	0	0	0	0	0	0	0	0	0	0	0	0
	0.0038 (1.81)	0.6899 (23.11)	0.0147 (0.83)	0.0033 (0.26)	0.0010 (0.50)	0.7191 (24.08)	0.0421 (2.56)	−0.0131 (1.22)	0.0020 (0.97)	0.7257 (23.75)	0.0335 (2.15)	−0.0086 (0.76)
	0.0197 (5.21)	0.1176 (1.96)	0.5601 (15.11)	0.0185 (0.79)	0.0154 (4.17)	0.0668 (1.09)	0.5890 (15.76)	0.0013 (0.05)	0.0171 (4.83)	0.0751 (1.25)	0.5880 (15.63)	0.0132 (0.57)
	−0.0018 (0.30)	−0.2170 (2.21)	0.0892 (1.62)	0.3180 (7.17)	0.0037 (0.59)	−0.2887 (3.18)	0.0063 (0.11)	0.3142 (7.09)	0.0030 (0.52)	−0.1988 (2.14)	0.0452 (0.81)	0.3000 (6.84)
$\widehat{D}$	0.8264 (10.52)	2.9620 (2.76)	−0.0306 (0.08)	1.8490 (2.20)	0.8127 (10.09)	2.4244 (2.60)	−0.1725 (0.66)	1.0559 (2.15)	0.8787 (14.84)	4.9665 (3.17)	0.3946 (1.08)	−0.5546 (2.22)
	0.0153 (2.48)	0.8221 (6.85)	0.0352 (0.88)	−0.2562 (0.26)	0.0273 (2.49)	0.4651 (4.28)	0.0531 (1.35)	−0.0827 (1.92)	0.0054 (1.29)	0.5544 (5.35)	0.0396 (1.37)	−0.1245 (2.89)
	−0.0040 (0.55)	−0.0509 (0.50)	0.9261 (31.34)	0.1538 (1.59)	−0.0212 (1.56)	0.0326 (0.20)	0.8808 (15.89)	0.2554 (3.34)	0.0028 (0.85)	−0.0646 (0.49)	0.8877 (27.88)	0.2266 (2.39)
	0.1001 (3.40)	−0.2108 (0.33)	−0.0863 (0.40)	−0.5217 (4.06)	0.1256 (2.01)	−0.5633 (1.01)	0.1217 (0.47)	−0.5385 (4.93)	−0.0076 (1.20)	−0.0837 (0.72)	0.1773 (1.35)	−0.7557 (18.10)
$\widehat{E}$	0.1441 (2.90)	0.3610 (0.69)	−0.8023 (1.91)	−0.1287 (0.44)	0.0077 (0.49)	−0.5055 (1.38)	−0.0781 (0.78)	0.1259 (1.25)	0.1096 (2.29)	−1.8144 (1.49)	−1.2997 (2.87)	0.1327 (0.41)
	−0.0033 (1.32)	0.1646 (3.07)	−0.0438 (1.59)	0.0072 (0.39)	0.0026 (0.80)	0.4162 (5.17)	0.0698 (2.34)	−0.0387 (1.73)	−0.0011 (0.35)	0.4814 (6.83)	0.0524 (1.87)	−0.0127 (0.56)
	0.0059 (1.18)	0.0692 (0.73)	0.2505 (4.19)	−0.0541 (1.83)	0.0007 (0.16)	−0.0312 (0.31)	0.3442 (7.08)	−0.0678 (2.19)	0.0116 (2.68)	0.0332 (0.28)	0.2975 (5.91)	−0.0744 (3.20)
	0.0215 (2.43)	0.0236 (0.11)	0.2344 (2.56)	0.1609 (2.19)	0.0292 (3.13)	−0.4032 (1.96)	−0.1512 (1.54)	−0.0467 (0.57)	0.0013 (0.84)	−0.0021 (0.07)	0.0223 (1.06)	−0.0002 (0.03)
$\widehat{G}$	−0.1657 (2.80)	−1.6260 (1.54)	−0.4074 (0.68)	−0.2959 (0.70)	0.0587 (1.85)	−2.7662 (2.49)	0.4549 (1.47)	−0.3476 (1.45)	−0.0125 (0.95)	0.2891 (0.93)	0.0940 (0.97)	0.0040 (0.15)
	0.0036 (1.16)	0.3019 (3.80)	0.0101 (0.27)	0.0590 (2.23)	0.0174 (3.66)	0.3386 (2.69)	0.0829 (1.67)	−0.0123 (0.42)	0.0009 (0.34)	0.0131 (0.22)	−0.0017 (0.07)	0.0566 (2.65)
	0.0058 (1.21)	−0.1504 (0.84)	0.2594 (3.53)	−0.0893 (1.72)	0.0126 (2.24)	−0.0638 (0.77)	−0.0734 (1.26)	−0.0782 (1.90)	−0.0037 (0.43)	−0.1890 (1.19)	0.1806 (2.09)	−0.0839 (2.12)
	−0.0316 (2.17)	0.3261 (0.91)	−0.0475 (0.35)	0.6577 (6.15)	−0.0234 (1.60)	0.2773 (0.73)	0.0892 (0.57)	0.6647 (6.43)	0.0066 (0.53)	0.1806 (0.57)	0.1738 (1.74)	0.5576 (6.71)
$\widehat{C}$	0.0506 (3.09)	0	0	0	0.0041 (12.99)	0	0	0	0.0050 (2.19)	0	0	0
	−0.0002 (0.60)	0.0025 (3.78)	0	0	−0.0002 (1.02)	0.0023 (8.02)	0	0	0.0001 (0.15)	0.0024 (5.28)	0	0
	0.0005 (0.50)	0.0001 (0.13)	0.0054 (2.50)	0	0.0001 (1.27)	0.0004 (1.05)	0.0043 (9.02)	0	0.0012 (0.60)	0.0010 (0.62)	0.0048 (3.22)	0
	0.0011 (0.82)	0.0009 (0.71)	−0.0005 (0.73)	0.0077 (4.76)	0.0001 (0.37)	−0.0003 (0.61)	−0.0005 (0.93)	0.0069 (8.83)	0.0001 (0.96)	−0.0002 (0.38)	0.0002 (0.42)	0.0062 (9.75)
$\widehat{C}\widehat{C}^{\intercal}$	3683.39				2470.67				3661.98			
	−12.34	9.38			−12.54	7.65			8.17	8.11		
	33.22	0.21	41.73		7.73	1.28	27.11		83.59	3.64	36.56	
	79.49	2.91	−2.76	89.05	8.58	−1.18	−3.06	69.08	3.84	−0.59	0.94	57.96

Other parameter estimates following parameter matrices defined in equation (3.12) and equation (1.51). Model 4-WA, 5-WA and 6-WA. In $\widehat{C}\widehat{C}^{\intercal}$ we tabulate only the lower triangular part. Absolute t-statistics in parenthesis. Emphasised parameters significant using 95 % critical value.

4. The FOREX SDF Model

An Alternative Method To Estimate Risk Premia on A Single Asset

4.1 Introduction

In chapter 2 we considered modelling and estimation of the risk premium in the UK and US stock markets when the economy could be represented by a single investor with GIS preferences and markets were complete. We concluded that consumption growth as well as inflation were significantly priced in both countries. In chapter 3 we considered an alternative SDF model pricing alternative key macroeconomic variables usually of concern for monetary authorities and showed that money growth, industrial production growth and inflation all had some impact on the US stock market risk premium - in chapter 2, as well as 3, the overall conclusion was that there is an inter-temporal relation between the stock market risk premium and the stock return variance and this inter-temporal relation could be modelled by pricing additional macroeconomic variables.

If financial markets are complete then it must be, as argued in the introductory chapter, that the sources of risk priced in the FOReign EXchange (FOREX) market should be the same as priced in the stock market, since the SDF will be unique, raising the natural question whether consumption growth, inflation, stock returns, growth in monetary aggregates and/or of output growth are significantly priced in the FOREX market. The aim of this chapter and the next is to answer this question using a similar framework as employed in the previous three chapters. We outline a method to estimate and obtain an inter-temporal relation between the FOREX risk premium and the conditional variance of FOREX excess return.

Estimation of risk premia on exchange rates using the multivariate-GARCH-in-mean model is not as straightforward as in the stock market[1]. The main difficulty arises since two sets of

[1] One could argue that estimating the models in the stock market is not straightforward.

investors invest in the FOREX market - domestic and foreign investors - and both domestic and foreign variables may need to be priced. As one can imagine, due to the dimension of the problem, the multivariate GARCH in mean model, as discussed in the introductory chapter, is practically impossible to estimate due to the high number of parameters. Except potentially the diagonal BEKK model most of the multivariate-GARCH-in-mean models discussed in the introductory chapter are impossible to estimate when more than four variables are included except if many data points, as is usually not the case, are available or we make some restrictive assumptions on the dynamics of the conditional covariance matrix.

In this chapter we discuss the FOREX SDF model on the exchange rate, deriving the single investor and two investor models of the FOREX risk premium. Subsequently we consider modelling of the SDF for both the domestic and the foreign investor, with main focus on the pricing of macroeconomic variables, and show that many variables may potentially be involved when estimating the FOREX risk premium. Due to the high number of variables we propose an alternative estimation method of estimating the risk premium in the FOREX market pricing multiple macroeconomic variables - which can easily be estimated with more than 10 variables. The estimation method we propose can be used to estimate risk premia on any single asset return - it has its advantage in that it can be estimated in two steps which makes estimation with many factors, contributing to FOREX risk, feasible.

A desirable feature of the estimation method is that we obtain a potential better representation of the residuals of the macroeconomic variables relative to the multivariate GARCH model used in chapter 2 and 3 while a disadvantage is that we will have to make some strong assumptions on the dynamics in the conditional covariance matrix that may not be desirable. We leave the question open as to whether the benefits from pricing several additional variables is greater than the loss from making potentially strong assumptions on the covariance dynamics between macroeconomic variables and FOREX returns. In our proposed estimation method no contagion is allowed for in the conditional covariance matrix - that is we do not allow shocks to the dependent variables to be transmitted into the conditional covariance matrix (only to the variance of the individual dependent variables and conditional covariances between dependent variables). Allowing shocks to be transmitted into the conditional covariance matrix will be considered in chapter 5.

The estimated models of the risk premium will be compared and related to the FOREX puzzle.

The possible solution to the FOREX puzzle we consider is the omitted time-varying risk premium solution. As mentioned in Backus, Gregory and Telmer [5] the puzzle in the FOREX market is not so much that the expected excess return is high as in the stock market, it is rather small, but that it must be varying considerably over time. Other potential explanations such as finite sample problems when testing deviations from the Uncovered Interest rate Parity (UIP) (see Baillie and Bollerslev [6]) will not be considered in this and the next chapter but it will be shown that the finite sample properties problem cannot fully account for the FOREX puzzle since if this was true, and the information set is the same for both investors, the conditional variance of FOREX excess return would have to equal zero if it is assumed that the exchange rate is log normally distributed.

Using either the estimation framework proposed in this chapter or the estimation method proposed in the next chapter one can relate the estimation results to the FOREX puzzle and give this puzzle an interpretation in terms of an omitted risk premium that is varying over time (see Engel [44] and Lewis [83]). It is of interest too see whether the implied time-varying risk premium, pricing macroeconomic variables, can contribute towards resolving the puzzle. In this chapter we outline the FOREX puzzle and consider its implication on the dataset for the UK-US exchange rate that will be used in this chapter and chapter 5. Afterwards we will investigate whether the risk premium implied by models, to be specified, do resolve the FOREX puzzle. The UK is one country where a considerable amount of research has been done on the FOREX puzzle.

The sort of macroeconomic variables we consider priced in this chapter are motivated by our work in chapter 2 and 3 - we consider key macroeconomic variables such as consumption growth, price inflation, money growth and industrial production growth. We do not consider the Epstein Zin model on the exchange rate (it is considered in chapter 5) in this chapter but consider among others the Power Utility CCAPM model and the Monetary model of Obstfeld and Rogoff [92] and Frenkel [64]. We propose several general alternative models.

As in the last chapters, on equity, we will derive the no-arbitrage conditions on the FOREX excess return without the assumption that a real risk-free asset exists. It gives a condition that must hold except if unlimited arbitrage possibilities exist. The no-arbitrage conditions for the domestic and foreign investor will first be derived and then they will be combined to get a no-arbitrage condition when two investors are exposed to FOREX risk. It will be emphasised that in order to estimate a two investor model, assuming that domestic and foreign information sets are equivalent simplifies matters a lot. The FOREX model will be highly parameterised in this framework using the estimation method proposed in chapter 1 and this is our reason for proposing an alternative estimation method. As in chapter 3 we recover the relation between the risk premium in the FOREX market and the conditional variance of innovations in the log exchange rate and show that there is a minimum and maximum bound on the expected FOREX excess return.

The chapter is organised as follows: In section (4.2) we briefly review the FOREX puzzle, in section (4.3) we discuss the single and two investor no-arbitrage conditions, in section (4.4) we outline the macroeconomic models of the risk premium, in section (4.5) we describe the data, in section (4.6) we propose a method to estimate multivariate-GARCH-in-mean models with multiple sources of risk priced on a single asset. Results are presented in section (4.7) and section (4.8) concludes.

4.2 Motivation for a FOREX Risk Premium - The FOREX Puzzle

Throughout we refer to a UK investor as a foreign investor and the US investor is considered a domestic investor. Let $\mathcal{S}$ denote the domestic price of foreign currency and let $i_{f,t}$ denote a domestic risk-free rate between t and $t+1$ and let a star as superscript denote a foreign (UK) variable. In a world where investors are risk neutral the Uncovered Interest Parity (UIP) for a domestic investor states that

$$\mathrm{E}_t\left\{\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\right\} = \frac{1+i_{f,t}}{1+i^*_{f,t}}, \tag{4.1}$$

or taking the natural logarithm to both sides, defining $i_{fx,t+1} \equiv \Delta\ln(\mathcal{S}_{t+1})$ and $\ln(1+x) = x$, assuming the exchange rate to be normally distributed then

$$\mathrm{E}_t(i_{fx,t+1}) + \frac{1}{2}V_t(i_{fx,t+1}) = i_{f,t} - i^*_{f,t} \tag{4.2}$$

This can be rearranged, since the nominal risk-free rates are known at time t, as

$$\mathrm{E}_t(i^e_{fx,t+1}) \;+\; \frac{1}{2}V_t(i^e_{fx,t+1}) \;=\; 0, \tag{4.3}$$

where $i^e_{fx,t+1} \;=\; i_{fx,t+1} \;-\; (i_{f,t} - i^*_{f,t})$. The variance correction is due to the fact that $\mathrm{E}_t(x) \neq \ln[\mathrm{E}_t(X)]$. The equivalent UIP condition for the foreign investor, assuming risk-neutrality states that

$$\mathrm{E}^*_t\left\{\frac{S_t}{S_{t+1}}\right\} \;=\; \frac{1+i^*_{f,t}}{1+i_{f,t}} \tag{4.4}$$

The equivalent to equation (4.3) for the foreign investor becomes

$$\mathrm{E}^*_t(i^e_{fx,t+1}) \;-\; \frac{1}{2}V^*_t(i^e_{fx,t+1}) \;=\; 0 \tag{4.5}$$

The star as a superscript on the conditional moments for the foreign investor indicates that the information set for a domestic and a foreign investor may differ. If, as is commonly assumed, the information sets of the two investors are equivalent then equation (4.3) and equation (4.5) can only be satisfied if $V_t(i^e_{fx,t+1}) \;=\; 0$. This is strongly rejected empirically. In continuous time, or when the time intervals are very small, the conditional variance term vanishes. On the other hand, if one thinks that the foreign and domestic investor have different information sets then it must follow from the UIP conditions that

$$\frac{\mathrm{E}^*_t(i^e_{fx,t+1}) \;+\; \mathrm{E}_t(i^e_{fx,t+1})}{2} \;=\; \frac{1}{4}\left[V^*_t(i^e_{fx,t+1}) \;-\; V_t(i^e_{fx,t+1})\right] \tag{4.6}$$

In other words since the variance of FOREX returns is empirically different from zero then log normality of the exchange rate is only consistent with risk neutrality and UIP if the conditioning information across the two investors differ - if information sets differ the average expectation of the domestic excess return on FOREX is greater than zero if the conditional variance for the foreign investor is greater than for the domestic investor given their potentially different information sets. In other words, average expectation of excess return on FOREX different from zero under risk neutrality is only consistent with differing foreign and domestic information sets. However, given the variance of the exchange rate it seems quite implausible that the differences in expectations should be so great - moreover there is not many good reasons to believe that the information sets differ much across domestic and foreign investors. In the next section it will be shown that abandoning the assumption of risk neutrality we obtain a more plausible no-arbitrage conditions.

One version of the FOREX puzzle states that performing the regression

$$i^e_{fx,t+1} = \alpha + \beta(i_{f,t} - i^*_{f,t}) + \epsilon_{t+1}, \tag{4.7}$$

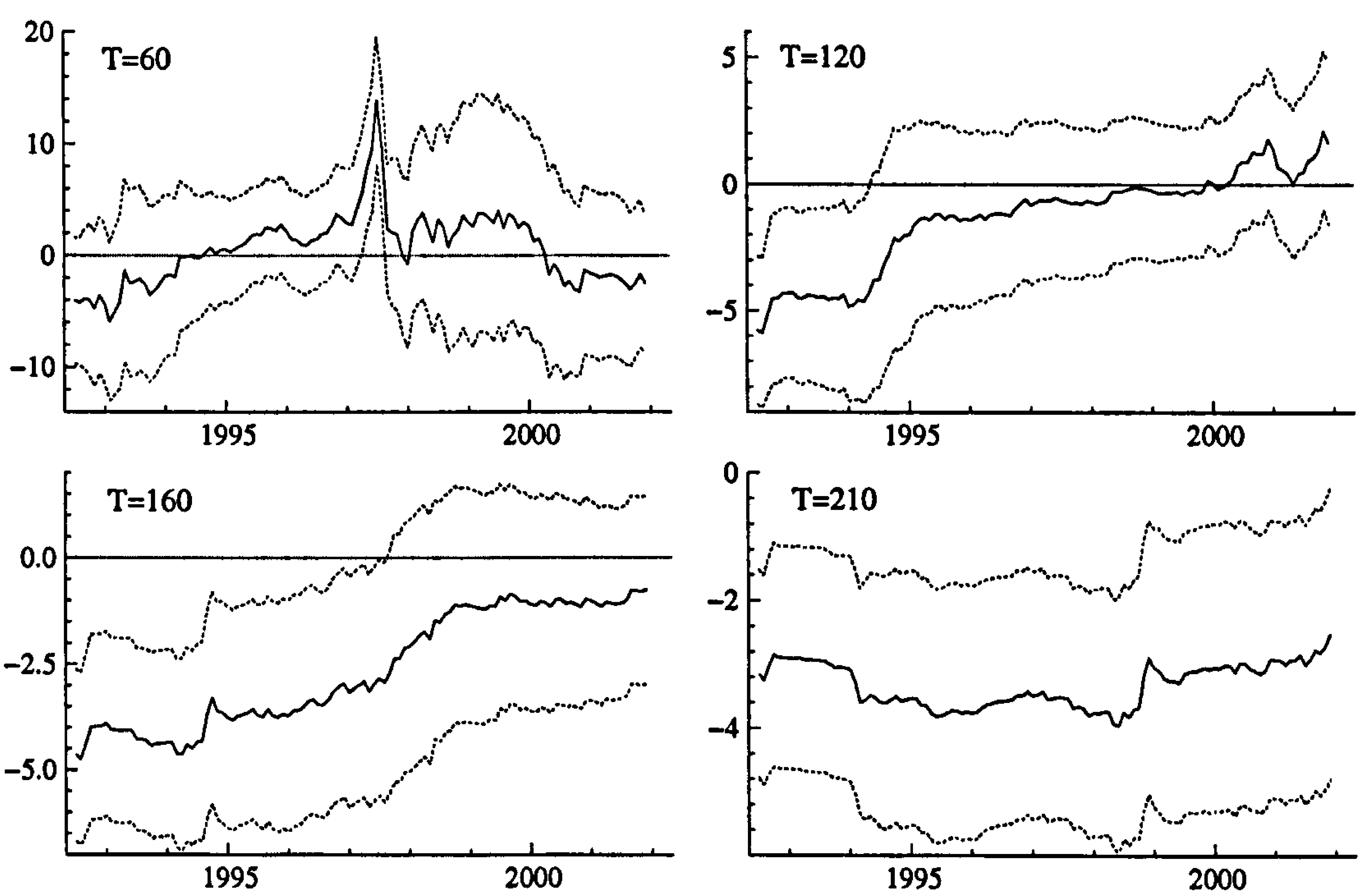
one obtains a significantly negative estimate of β though the above equation (4.3) and (4.5) tells us that the interest rate differential is not supposed to predict excess return on FOREX. There are problems with this estimation. First, the estimation assumes that the information sets of investors are equivalent and if they are risk neutral then the conditional variance of FOREX should equal zero at any point in time. Second, if we consider the case where two information sets differ, then the Jensen effect is left out. However, in this case estimating the equation alone is wrong - a multivariate model would be necessary since if the information sets differ then it must be true that he/she uses different variables to predict the conditional variance of FOREX excess return in which case the regression is valid. Potentially one could assume, which may or may not be true, that the conditional variance of FOREX excess return is constant and different from zero. In practice this is not a good assumption - the following will show that the conditional variance of FOREX excess return on the US-UK exchange rate appears to be varying considerably over time. Performing the above regression for the US-UK dataset starting in 1975 and ending in the end of 2001 (data will be explained in details shortly) one obtains

$$\mathrm{E}_t(i^e_{fx,t+1}) = \underset{(1.89)}{0.0043} - \underset{(3.28)}{2.3454}(i_{f,t} - i^*_{f,t}) \tag{4.8}$$

This is the classical example of the FOREX puzzle - when the interest rate differential is positive the above regression tells us that the domestic currency excess return will be less than zero though UIP tells us that there should be no predictability - in other words unlimited arbitrage possibilities exist. In figure (4.4) in the appendix we plot the fitted values from the regression against the actual values. The fitted values could be given an interpretation as proxying for the FOREX risk premium - it is seen that if interpreted as a proxy for the risk premium it has been slowly moving over time with a tendency to remain positive and relatively stable after 1993[2]. The standard misspecification tests reveals that the error term has significant ARCH effects and is non-normally distributed though the deviation from normality is not enormous. There is no sign of auto correlation in the residuals. If investors are not risk neutral we need theory to derive the risk premium that the investors require for taking on investments in the FOREX market. Such a theory will be considered in the next section using SDF methodology.

Another explanation of the FOREX puzzle that has been emphasised recently is by Baillie and Bollerslev [6]. They claim that the rejection of the "UIP" condition could be due to the fact that samples are finite and the negative bias from the above regressions should be explained by the fact that samples are too small. The following figure (4.1) sheds some light on the US-UK exchange rate when using small sample sizes.

Figure 4.1: The UIP Bias And Dependence On Sample Size



Rolling OLS estimates of β with 95 % confidence bounds (dotted lines) around it. See equation (4.7). T refers to the sample size.

The intuition in the above graph is the following: suppose we start at a particular date in July 1992 we take 4 samples back in time with different sizes, $T = 60$, $T = 120$, $T = 160$ and

[2]This is consistent with our findings later that the conditional variance of the log FOREX excess return has fallen radically after 1994.

$T = 210$. The first sample starts in 1987, the next in 1982, the third in 1979 and the final in 1975. For each sample we perform the OLS regression equation (4.7) and compute the OLS estimate on the coefficient on the interest rate differential and calculate the standard error of this estimate. For each sample we add one extra observation each month and remove the last observation and perform the OLS regression again and compute the standard errors. For each sample we can plot the OLS estimate with the confidence bounds on the estimate. The above graphs are pretty conclusive. Choosing a sample size of $T = 60$ we find that any point in time after May 1992 we accept the null hypothesis of UIP within a 95 % confidence interval. With a sample that is twice as long it is only for samples after May 1994 where we accept the null of UIP. When using sample sizes $T = 160$ it is only samples ending in March 1998 or later for which we accept the null of UIP and with any sample size of 210 ending in the period 1992-2002 we always reject the UIP condition within a 95 % confidence bound. Based on the above graphs there seems to be good reason to look for alternative explanations to the FOREX puzzle than finite sample problems. It is interesting that the longer the samples go back in time the more the data deviates from UIP suggesting that periods of the 1970s and 1980s were key periods rejecting the UIP - these decades are periods of high variability of the macroeconomic variables[3] suggesting that risk premia may have been fluctuating in the period due to macroeconomic uncertainty. This could be one explanation of the FOREX puzzle.

If we consider an omitted variable as explanation for the FOREX puzzle Fama [53] shows that in terms of an omitted variable bias in the OLS FOREX regression the requirement for the variance of this omitted variable, or risk premium (for the moment we think of it as a risk premium), is that

$$\mathrm{Var}(\overline{\overline{\phi}}_t) = \frac{\mathrm{Var}(i_{f,t} - i^*_{f,t})\mathcal{B}^2}{\rho(i_{f,t} - i^*_{f,t}, \overline{\overline{\phi}}_t)^2}, \tag{4.9}$$

where $\mathcal{B}$ is the bias of the estimate of the coefficient on the interest rate differential (equation (4.7)). For the moment we refer to $\overline{\overline{\phi}}_t$ as the risk premium which is not exactly true as will be discussed shortly. Knowing that the absolute correlation cannot exceed 1 using the estimated bias for the current dataset above, we are looking for a risk premium with variance

[3] And a period with an experimental monetary policy.

$$\mathrm{Var}(\overline{\overline{\phi}}_t) \geq 5.50\ \mathrm{Var}(i_{f,t} - i^*_{f,t}) \tag{4.10}$$

For the given dataset the variance of the interest differential is 8.64 in annual percentages and therefore we are looking for a model of the risk premium able to generate a risk premium with variance of a minimum of 47.52 where the risk premium is measured in annual percentages. Hence we have the requirement by the above minimum bound (equation (4.10)) and need an intuitive derivation of the risk premium from the no-arbitrage condition.

In a survey Lewis [83] conclude that

> No risk premium model with believed measures of risk aversion has yet been able to generate the variability in predictable excess returns that are observed in the data.

This chapter, and the next, takes up this challenge and compare the variability of the risk premium implied by several models of the time-varying risk premium.

4.3 The No-Arbitrage Condition

On the international transaction of domestic and foreign risk-free bonds one can consider either a single investor model (domestic or foreign investor) or a two investor model - in this section the single and two investor models will be derived. First the no-arbitrage condition for the domestic investor will be considered, then the no-arbitrage condition for the foreign investor and finally we use these conditions to characterise a two-investor model of the risk premium, which is only consistent when the information sets of domestic and foreign investors are identical.

4.3.1 The SDF Model for FOREX

The domestic investor has the option of investing 1 unit of national currency abroad at time t and receive return $\frac{\mathcal{S}_{t+1}(1 + i^*_{f,t})}{\mathcal{S}_t}$ at time $t+1$, where $\mathcal{S}$ is the nominal exchange rate - it is the price that the domestic investor has to pay for one unit of foreign currency. Using a law of one price argument

$$\mathcal{S}_t = \mathrm{E}_t\left\{\mathcal{M}_{t+1}\mathcal{S}_{t+1}\frac{1 + i^*_{f,t}}{1+\pi_{t+1}}\right\}, \tag{4.11}$$

where π is the domestic inflation rate. Taking the natural logarithm to both sides, assuming that $\ln(1+\pi) = \pi$ and a joint log normal distribution we obtain

$$\begin{aligned} 0 &= \mathrm{E}_t(m_{t+1} + i_{fx,t+1} - \pi_{t+1}) + i^*_t + \frac{1}{2}V_t(m_{t+1} + i_{fx,t+1} - \pi_{t+1}) \\ 0 &= \mathrm{E}_t(m_{t+1} + i_{fx,t+1} - \pi_{t+1}) + i^*_t + \frac{1}{2}V_t(m_{t+1}) + \frac{1}{2}V_t(i_{fx,t+1}) + \frac{1}{2}V_t(\pi_{t+1}) \\ &+ \mathrm{Cov}_t(m_{t+1}, i_{fx,t+1}) - \mathrm{Cov}_t(m_{t+1}, \pi_{t+1}) - \mathrm{Cov}_t(i_{fx,t+1}, \pi_{t+1}) \end{aligned} \tag{4.12}$$

If a domestic risk-free asset, in nominal terms, exists it follows that

$$- i_{f,t} = \mathrm{E}_t(m_{t+1} - \pi_{t+1}) + \frac{1}{2}V_t(m_{t+1}) + \frac{1}{2}V_t(\pi_{t+1}) - \mathrm{Cov}_t(m_{t+1}, \pi_{t+1}) \tag{4.13}$$

Combining these two equations, using the definition of FOREX excess return, $i^e_{fx,t+1} \equiv i_{fx,t+1} - i_{f,t} +$ we obtain the no-arbitrage condition for the domestic investor

$$\begin{aligned} \mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= - \mathrm{Cov}_t(m_{t+1}, i^e_{fx,t+1}) + \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \\ &= \phi_t \end{aligned} \tag{4.14}$$

The RHS is the risk premium[4]. Nominal risk-free interest rates between t and $t+1$ are always known at time t. The equivalent no-arbitrage condition for the foreign investor, by a symmetry argument[5], is

[4]Note, we use ϕ to denote the FOREX risk premium as we used for the equity risk premium. However, the abuse of notation does not matter since we do not consider equity risk premia in the current chapter.

[5]Note that in the foreign country the risk-free rate is related to the SDF and inflation by $1 = \mathrm{E}_t\left\{\mathcal{M}^*_{t+1}\frac{1+i^*_{f,t}}{1+\pi^*_{t+1}}\right\}$.

$$
\begin{aligned}
\mathrm{E}_t^*(i_{fx,t+1}^{*e}) + \frac{1}{2}\mathrm{V}_t^*(i_{fx,t+1}^{*e}) &= -\operatorname{Cov}_t^*(m_{t+1}^*, i_{fx,t+1}^{*e}) + \operatorname{Cov}_t^*(i_{fx,t+1}^{e*}, \pi_{t+1}^*) \\
&= \phi_t^*, \qquad (4.15)
\end{aligned}
$$

where $i_{fx,t+1}^{*e} = -i_{fx,t+1}^{e}$. Subsequently the following definitions will be used

$$
\begin{aligned}
\bar{\phi}_t &\equiv \phi_t - \frac{1}{2}V_t(i_{fx,t+1}^{e}) \\
\bar{\phi}_t^* &\equiv \phi_t^* - \frac{1}{2}V_t^*(i_{fx,t+1}^{e}) \qquad (4.16)
\end{aligned}
$$

If domestic risk neutrality or aversion then $\phi_t \geq 0$ and foreign risk neutrality or aversion implies $\phi_t^* \geq 0$. These variables, equation (4.16), are part of the omitted variables that may explain the FOREX puzzle. Asterisk as a superscript on the conditional expectation, variance and covariances indicates that these moments are conditional on the information of the foreign investor. Domestic and foreign information sets do not necessarily have to be the same. Combining the domestic (equation (4.14)) and the foreign (equation (4.15)) no-arbitrage conditions we obtain the two investor models as

$$
\begin{aligned}
\frac{\mathrm{E}_t(i_{fx,t+1}^{e}) + \mathrm{E}_t^*(i_{fx,t+1}^{e})}{2} &= \frac{1}{4}\left[\mathrm{V}_t^*(i_{fx,t+1}^{e}) - \mathrm{V}_t(i_{fx,t+1}^{e})\right] \\
&- \frac{1}{2}\left[\operatorname{Cov}_t^*(m_{t+1}^*, i_{fx,t+1}^{e}) + \operatorname{Cov}_t(m_{t+1}, i_{fx,t+1}^{e})\right] \\
&+ \frac{1}{2}\left[\operatorname{Cov}_t^*(\pi_{t+1}^*, i_{fx,t+1}^{e}) + \operatorname{Cov}_t(\pi_{t+1}, i_{fx,t+1}^{e})\right] \\
&= \frac{1}{2}\left[\bar{\phi}_t - \bar{\phi}_t^*\right] \qquad (4.17)
\end{aligned}
$$

If it happens to be the case that information sets are the same this simplifies to the two investor model.

$$\begin{aligned}\mathrm{E}_t(i^e_{fx,t+1}) &= \frac{1}{2}\left[-\mathrm{Cov}_t(i^e_{fx,t+1}, m^*_{t+1} + m_{t+1}) + \mathrm{Cov}_t(i^e_{fx,t+1}, \pi^*_{t+1} + \pi_{t+1})\right] \\ &= \frac{1}{2}\left[\bar{\phi}_t - \bar{\phi}^*_t\right] = \frac{1}{2}\left[\phi_t - \phi^*_t\right] \qquad (4.18)\end{aligned}$$

The Jensen terms cancel out[6]. From equation (4.18) we note an interesting thing. If risk premia (domestic or foreign) are identical, then $\mathrm{E}_t(i^e_{fx,t+1}) = 0$. Hence a test of Uncovered Interest Parity regressing FOREX excess return on the risk-free interest differential must have low power since risk netrality cannot be distinguished from identical domestic and foreign risk premia at any point in time.

The foreign investor model can also be expressed in terms of domestic excess return as

$$-\mathrm{E}^*_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}^*_t(i^e_{fx,t+1}) = \mathrm{Cov}^*_t(m^*_{t+1}, i^e_{fx,t+1}) - \mathrm{Cov}^*_t(i^e_{fx,t+1}, \pi^*_{t+1}) = \phi^*_t, \qquad (4.19)$$

which can be rearranged as[7].

$$\mathrm{E}^*_t(i^e_{fx,t+1}) - \frac{1}{2}\mathrm{V}^*_t(i^e_{fx,t+1}) = -\mathrm{Cov}^*_t(m^*_{t+1}, i^e_{fx,t+1}) + \mathrm{Cov}^*_t(i^e_{fx,t+1}, \pi^*_{t+1}) = -\phi^*_t \qquad (4.20)$$

Estimating this equation the RHS is the foreign risk premium on the domestic excess return. Hence we would expect the RHS to be negative at all times. The implication of these conditions is

$$\mathrm{V}_t(i^e_{fx,t+1}) = \phi_t + \phi^*_t, \qquad (4.21)$$

[6]Following discussion after equation (4.9), $\bar{\bar{\phi}}_t$ can be interpreted either as $\bar{\phi}_t$, $-\bar{\phi}^*_t$ or, if domestic and foreign information sets are the same, $\frac{1}{2}(\bar{\phi}_t - \bar{\phi}^*_t)$.

[7]We note that it must be fulfilled that $m^*_{t+1} - m_{t+1} - (\pi^*_{t+1} - \pi_{t+1}) + \eta_{t+1} = i_{fx,t+1}$, where η is a noise component.

Outlined above we have a model, consistent with a log normal distributed exchange rate, which does not imply the conditional variance of FOREX excess return to equal zero (or be constant) except in the special case of risk neutrality - the UIP is a special case.

The variance of the exchange rate is high at times where domestic and/or foreign risk premia are high and low when required risk premia are relatively low.

The above condition, as shown in Smith or Wickens or Backus, Foresi and Telmer [4], when markets are complete and Purchasing Power Parity (PPP) holds implies that $i_{fx,t+1} = m^*_{t+1} - m_{t+1}$.

The discussion above has two important implications. First if we have a complete market model of the exchange rate with common information sets to the two investors we need only consider either the domestic or foreign pricing kernel to back out both domestic and foreign risk premia since the risk premium in the two investor model is given by the RHS of[8]

$$\mathrm{E}_t(i^e_{fx,t+1}) = -\frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) + \phi_t, \quad \text{or} \tag{4.22}$$

$$\mathrm{E}_t(i^e_{fx,t+1}) = \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) - \phi^*_t \tag{4.23}$$

Hence we need to estimate a single investor domestic (foreign) model only to back out the foreign (domestic) risk premium or the risk premium in the two investor model. Second, if markets are not complete it is only feasible to estimate the two investor model if we believe that information sets are equivalent. Finally we can state a condition allowing for the possibility that FOREX investors are averse towards risk, consistent with log normality and time-varying risk premia, if investors have the same information set as:

$$\begin{aligned}
\mathrm{E}_t\left\{\frac{S_{t+1}}{S_t}\right\} &= \frac{1+i_{f,t}+\phi_t}{1+i^*_{f,t}} = \frac{\mathrm{E}_t\left\{\frac{\mathcal{M}^*_{t+1}}{1+\pi^*_{t+1}}\right\}}{\mathrm{E}_t\left\{\frac{\mathcal{M}_{t+1}}{1+\pi_{t+1}}\right\}} + \underline{\phi}_t \\
\mathrm{E}^*_t\left\{\frac{S_t}{S_{t+1}}\right\} &= \frac{1+i^*_{f,t}+\phi^*_t}{1+i_{f,t}} = \frac{\mathrm{E}^*_t\left\{\frac{\mathcal{M}_{t+1}}{1+\pi_{t+1}}\right\}}{\mathrm{E}^*_t\left\{\frac{\mathcal{M}^*_{t+1}}{1+\pi^*_{t+1}}\right\}} + \underline{\phi}^*_t
\end{aligned} \tag{4.24}$$

[8]This is easily seen by plugging $m^*_{t+1} = i_{fx,t+1} + m_{t+1} - \pi_{t+1} + \pi^*_{t+1} - \eta_{t+1}$ into equation (4.18).

This is the risk adjsuted UIP condition. Recall the definition of $\underline{\phi}$ and its relation with ϕ is discussed in section (1.2.1). We can verify that with common information sets and the conditional variance of FOREX excess return equal to the sum of the foreign and domestic risk premium the two conditions above are consistent with joint logarithmic normality between FOREX return and the SDF even with a conditional variance different from zero - the conditional variance is different from zero when investors are averse towards risk[9].

Finally we note that

$$E_t(m^*_{t+1} - m_{t+1} + \pi_{t+1} - \pi^*_{t+1}) = i_{f,t} - i^*_{f,t} + \frac{1}{2}(\phi_t - \phi^*_t). \tag{4.25}$$

The interest rate differential plus an average of the domestic and foreign risk premium, in domestic currency, reflects the expectation of the difference in the nominal foreign and domestic logarithmic Stochastic Discount Factor.

Using the above equations gives some intuition as to the composition of the unexpected component of $i_{fx,t+1}$, which we denote $\epsilon_{fx,t+1}$, when markets are complete, since

$$\epsilon_{fx,t+1} = \{m^*_{t+1} - E_t(m^*_{t+1})\} - \{m_{t+1} - E_t(m_{t+1})\} - \{\pi^*_{t+1} - E_t(\pi^*_{t+1})\} + \{\pi_{t+1} - E_t(\pi_{t+1})\} \tag{4.26}$$

News about the exchange rate reflects unexpected shocks to domestic and/or foreign inflation and unexpected shocks to the domestic and/or foreign real logarithmic Stochastic Discount Factors. If the logarithmic SDF is linear in macroeconomic variables then shocks to the FOREX excess return are purely macroeconomic shocks.

Next we consider the modelling of the domestic and foreign Stochastic Discount Factors and propose a new method to estimate the FOREX risk premium, or the risk premium on any single asset return.

4.3.2 The SDF - General Approach - Macroeconomic Sources of Risk.

Based on the findings in chapter 2 and 3, where we found all macroeconomic variables priced in one or the other models we consider the possibility that all macroeconomic variables are priced

[9] An example of this is the recent increased volatility of the EURO-Dollar exchange rate - basically the increase in the volatility must reflect changing risk premia !

in the FOREX market. It could be that industrial Production growth, money growth, inflation, consumption growth etc. are all significant sources of risks priced in the FOREX market. We can estimate the FOREX risk premium while modelling the SDF as a general function of macroeconomic factors. Let $\mathbf{f}$ denote a vector containing all variables in the domestic and foreign SDF. Assuming that the logarithmic SDFs of the foreign and domestic investor are linear combinations of macroeconomic (and eventually financial) variables, we can model the domestic and foreign logarithmic SDFs as

$$m_{fx,t+1} = -\alpha_t - \mathbf{b}_t^{\mathsf{T}}\mathbf{f}_{t+1} + \zeta_{fx,t+1}, \quad m^*_{fx,t+1} = -\alpha^*_t - \mathbf{b}_t^{*\mathsf{T}}\mathbf{f}_{t+1} + \zeta^*_{fx,t+1}, \quad (4.27)$$

where $\mathbf{f}$ and $\mathbf{b}$ are vectors of the same dimension. ζ is a noise components with standard properties. If, for example, domestic variables do not enter the foreign SDF the corresponding loading $b^*_{i,t} = 0$, where subscript i refers to the loading on variable i. Assume that the variables can be both foreign and domestic and assume that all relevant factors for domestic and foreign investor are in $\mathbf{f}$. fx as subscripts refer to SDF in the FOREX market[10].

The variables in the foreign SDF need not necessarily be equivalent to the variables in the domestic SDF. It will be assumed that the factor loadings in equation (4.27) are constant. If we assume that foreign and domestic information sets are equal the no-arbitrage conditions for the domestic, foreign and two investor models respectively[11] become

$$\begin{aligned} \mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \mathbf{b}^{\mathsf{T}}\mathrm{Cov}_t(\mathbf{f}_{t+1}, i^e_{fx,t+1}) + \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \\ \mathrm{E}_t(i^{*e}_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^{*e}_{fx,t+1}) &= \mathbf{b}^{*\mathsf{T}}\mathrm{Cov}_t(\mathbf{f}_{t+1}, i^{*e}_{fx,t+1}) + \mathrm{Cov}_t(i^{e*}_{fx,t+1}, \pi^*_{t+1}) \\ \mathrm{E}_t(i^e_{fx,t+1}) &= \frac{(\mathbf{b}+\mathbf{b}^*)^{\mathsf{T}}}{2}\mathrm{Cov}_t(i_{fx,t+1}, \mathbf{f}_{t+1}) + \frac{1}{2}\mathrm{Cov}_t(i^e_{fx,t+1}, \pi^*_{t+1} + \pi_{t+1}) \end{aligned} \quad (4.28)$$

With equivalent information sets the two investor model implies that the average loading on

[10]If markets are complete then $m_{fx,t+1} = m_{s,t+1}$ and $m^*_{fx,t+1} = m^*_{s,t+1}$, where subscript s indicates that it is the stock market SDF.

[11]We assume that the correlation between the error term in the log SDF and log FOREX excess return is zero.

the macro and financial variables that determines the importance of the individual factors. If it happens that UK variables are significantly priced in the US and vice versa then one should note that it is very difficult to interpret the coefficients in what follows subsequently.

4.3.3 The Relation Between FOREX Risk Premia and the Variance

We can rewrite the model above to recover the inter-temporal relation between the FOREX risk premium, including inflation rates as factors, and the conditional variance as:

$$\begin{aligned}
\mathrm{E}_t(i^e_{fx,t+1}) &= \frac{1}{2}\sum_{j=1}^{k}(b_j+b^*_j)\frac{\rho_t(i^e_{fx,t+1},f_{j,t+1})\sigma_t(f_{j,t+1})}{\sigma_t(i^e_{fx,t+1})}\mathrm{V}_t(i^e_{fx,t+1}) && (4.29)\\
&= \frac{1}{2}\sum_{j=1}^{k}(b_j+b^*_j)\frac{\rho_t(i^e_{fx,t+1},f_{j,t+1})\sigma_t(f_{j,t+1})}{\sigma_t(i^e_{fx,t+1})}(\phi_t+\phi^*_t) && (4.30)\\
&= \overline{\gamma}^{two}_t\mathrm{V}_t(i^e_{fx,t+1}) && (4.31)
\end{aligned}$$

where k is the number of factors priced, f_j is the j^{th} element of $\mathbf{f}$ and b_j is the j^{th} element of $\mathbf{b}$. Hence

$$\overline{\gamma}^{two}_t = \frac{1}{2}\sum_{j=1}^{k}(b_j+b^*_j)\frac{\rho_t(i^e_{fx,t+1},f_{j,t+1})\sigma_t(f_{j,t+1})}{\sigma_t(i^e_{fx,t+1})} = \frac{1}{2}\frac{(\phi_t-\phi^*_t)}{\phi_t+\phi^*_t} \qquad (4.32)$$

The above equations are useful if one wish to determine the inter-temporal relation between the conditional variance of FOREX excess return and the FOREX risk premium. Similarly we can rewrite the no-arbitrage condition for the domestic investor as

$$\begin{aligned}
\mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \sum_{j=1}^{k} b_j\frac{\rho_t(i^e_{fx,t+1},f_{j,t+1})\sigma_t(f_{j,t+1})}{\sigma_t(i^e_{fx,t+1})}\mathrm{V}_t(i^e_{fx,t+1}) && (4.33)\\
&= \underbrace{\frac{\phi_t}{\phi_t+\phi^*_t}}_{\overline{\gamma}^{domestic}_t}\mathrm{V}_t(i^e_{fx,t+1}) = [1-\underbrace{\frac{\phi^*_t}{\phi_t+\phi^*_t}}_{\overline{\gamma}^{foreign}_t}]\mathrm{V}_t(i^e_{fx,t+1}) && (4.34)
\end{aligned}$$

We can test directly whether the domestic and foreign investor are risk neutral. We can test the null hypothesis that the domestic investor is risk neutral at all times, in which case $\overline{\gamma}^{domestic}$ will be constant and amounts to the null hypothesis

- $H_0 : \overline{\gamma}^{domestic} = 0.$

Alternatively we can test the null hypothesis that the foreign investor is risk neutral at any point in time as

- $H_0 : \overline{\gamma}^{domestic} = 1$

The relation could be estimated over sub periods to see whether $\overline{\gamma}_t^{domestic}$ is changing over time. In any case, if we believe that $\overline{\gamma}_t^{domestic}$ is constant and time independent then an estimate:

- $0.5 < \overline{\gamma}^{domestic} \leq 1$

implies that the domestic investor is more risk averse than the foreign investor ! If the foreign investor is risk neutral then the domestic risk premium moves in 1:1 correspondence with the conditional variance of the FOREX excess return. We have estimated this univariate GARCH in mean model and obtained an estimate of 1.46 in the dataset to be described shortly (US-UK exchange rate). However, the estimated standard error on the coefficient is very large and we can neither reject the coefficient equal to zero or one.

We leave the discussion for now and focus on the estimation of the risk premium but conclude that developments of tests of whether there is time-variation in $\overline{\gamma}^{domestic}$ is an interesting topic for future research. Since, below, we use monthly macroeconomic data it is possible to estimate the risk premia directly. However, estimating the risk premium using higher frequency data, such as weekly or daily, may not be feasible since macroeconomic data are not available and it may be impossible to estimate the time-varying risk premium.

In the general single investor models estimated below it is assumed that foreign variables do not enter the domestic pricing kernel and domestic variables do not enter the foreign SDF. In the estimated two investor model we cannot exclude the possibility that domestic (foreign) variables enter the foreign (domestic) pricing kernel. Hence if we think of our general model as a SDF model and foreign (domestic) variables can enter the domestic (foreign) SDF then the loadings in the SDF should be interpreted with caution.

4.4 Data Generating Processes - The General Macroeconomic Factor Models

In this section we describe the general macroeconomic factor models that will be estimated. We estimate domestic, foreign and two investor models. We refer to the no-arbitrage conditions in equation (4.28) and describe, in the following, the variables to be included as factors in the individual models.

- The Benchmark Model (BM)

The model is derived from traditional tests of FOREX market efficiency based on the Uncovered Interest Parity. There are three independent sources of randomness (the exchange rate and the two risk-free rates). Hence the conditional covariance between the exchange rate excess return and the interest rates determine potential time-varying risk premia on FOREX. The vector of factors in this model is $\mathbf{f}_{t+1}^{BM} = (i_{f,t+1} - i_{f,t+1}^{*}, \Delta i_{f,t+1})$, where f as a subscript refers to a euro-sterling or a euro-dollar interest rate.

- The CCAPM (CC) Model

In the Power Utility CCAPM real consumption growth and inflation are the only sources of risk, recall discussion of the most general Epstein Zin model without pricing stock return in chapter 2. The vector of factors is given by $\mathbf{f}_{t+1}^{CC} = (\pi_{t+1}, \pi_{t+1}^{*}, \Delta c_{t+1}, \Delta c_{t+1}^{*})$. Hence, for instance, domestic consumption and domestic inflation are relevant in the domestic investor model.

- The Monetary Model (MM)

In the monetary model, the exchange rate is determined by future expected relative money supplies and output levels, see for example Frenkel [64] and Obstfeld and Rogoff [92]. The domestic SDF in this model is given by $m_{t+1} = -\alpha - \beta_1 \Delta q_{t+1} - \beta_2 \Delta y_{t+1}$. The model can also be seen as an alternative to the consumption-based model assuming money growth a proxy for consumption growth. The vector of factors in the monetary model is $\mathbf{f}_{t+1}^{MM} = (\Delta q_{t+1}, \Delta q_{t+1}^{*}, \Delta y_{t+1}, \Delta y_{t+1}^{*})$, where q is the logarithm of a narrow measure of money. We use M0 for the UK and M1 for the US (we use M1 since we found it "significant" in the equity model in chapter 3 - we tried also with narrow money but found the results to be similar). y is the logarithm of industrial production.

- Combined Model 1 (CM1)

The combined model is a general SDF model where the logarithm of the SDF is linear in all the macroeconomic variables suggested above. Hence the vector of factors is given by $\mathbf{f}_{t+1}^{CM1} = (\pi_{t+1}, \pi_{t+1}^{*}, \Delta c_{t+1}, \Delta c_{t+1}^{*}, \Delta q_{t+1}, \Delta q_{t+1}^{*}, \Delta y_{t+1}, \Delta y_{t+1}^{*})$ - we allow all macroeconomic variables to be priced in the two investor model.

- Combined Model 2 (CM2)

The second combined model is equivalent to the one above where we include, in addition, the variables from the Benchmark model as factors such that

$$\mathbf{f}_{t+1}^{CM2} = (i_{f,t+1} - i_{f,t+1}^{*}, \Delta i_{f,t+1}, \pi_{t+1}, \pi_{t+1}^{*}, \Delta c_{t+1}, \Delta c_{t+1}^{*}, \Delta q_{t+1}, \Delta q_{t+1}^{*}, \Delta y_{t+1}, \Delta y_{t+1}^{*}).$$

We note that these models include many variables but will show that it is feasible to estimate the models using an alternative method which we propose in section (4.6).

4.5 The Data

Most of the data used in this chapter are all described in chapter 2 (see appendix of that chapter). In this chapter we use additionally the excess return on the US-UK exchange rate, the US-UK risk-free interest rate differential, the changes in the UK and US risk-free rates and a measure of narrow money in both countries. The two additional macroeconomic variables that we did not use in the previous chapter, the money growth rates are plotted in figure (4.2) (first difference of the risk-free interest rates in figure (4.3)) in the appendix and the descriptive statistics and correlation with other variables can be found in table (2.14) and table (2.15) in the appendix to chapter 2. We use the MSCI exchange rate available from Datastream and the 1 month Euro sterling interest rates.

The sample period is from June 1975 to October 2002. In the US dataset there are two extreme outliers in money growth, as discussed in chapter 3 - we decide, as in chapter 3, to treat these shocks as extreme outliers and replace the two points with the mean of the rest of the sample. US data have higher correlation in the growth rates than do the UK data. Moreover, the US data has higher correlation in the squares of the growth rates. The standard deviation of the data are about the same and so is skewness and excess kurtosis - this in particular caused by the two extreme outliers. UK money has the highest (in absolute value) correlation with the forward premium (correlation -0.26) and US money growth has highest correlation in absolute value of 0.129 and 0.124 with UK inflation and US industrial production growth respectively.

The first difference of the UK and US risk-free rates have similar characteristics - first the variability of the two series has declined over the sample, the US risk-free rate differences being particularly variable during the FED. experiment in 1979-1982 whereas the UK rate was variable, as well, in the period 1975-1979. Both series exhibit a considerable amount of excess kurtosis. Changes in the US risk-free rate has the highest correlation with US industrial production growth (correlation 0.28) and further UK changes has a high correlation with UK industrial production growth (correlation is 0.19). In addition changes in the UK risk-free rates has a negative correlation with UK stock market return of -0.207.

4.6 An Alternative Method for Estimating "Multivariate GARCH-in-Mean" Models

The aim of this section is to outline an alternative estimation method that we propose for modelling a risk premium pricing general macroeconomic factors on a single asset. The proposed method belongs to the category of Constant Conditional Correlation multivariate-GARCH-in-mean models.

From the discussion in the chapters so far we have emphasised that estimation of multivariate GARCH models would require the use of many parameters - especially if we wish many variables in our multivariate model. To estimate the general macroeconomic factor model we specify the conditional covariance matrix, in accordance with equation (1.52), in the introductory chapter, as

$$\mathbf{H}_{t+1} = \mathbf{S}_{t+1}\mathbf{R}\mathbf{S}_{t+1}, \tag{4.35}$$

where $\mathbf{R}$ is the matrix containing the constant correlations and $\mathbf{S}$ is a diagonal matrix containing the conditional standard deviation of the variables to be specified shortly. The only asset on which we wish to model the risk premium is the exchange rate. We assume that only correlations between the factors and the log FOREX excess return are different from zero[12]. Hence $\mathbf{R}$ is a

[12]We note this may be a strong assumption since especially inflation and industrial production growth in the

symmetric matrix containing ones along the diagonal and correlations in the first row and the first column to be estimated. In this way we can proceed using a two step estimator of the model. Let the vector of variables be

$$\mathbf{F} = \{i^e_{fx,t+1}, i_{f,t+1} - i^*_{f,t+1}, \Delta i^*_{f,t+1}, \Delta i_{f,t+1}, \pi_{t+1}, \pi^*_{t+1}, \Delta c_{t+1}, \Delta c^*_{t+1}, \Delta q_{t+1}, \Delta q^*_{t+1}, \Delta y_{t+1}, \Delta y^*_{t+1}\}.$$

This vector has dimension (12×1).

- For each of the variables in $\mathbf{F}$ (except the log FOREX excess return), x_i, estimate a univariate GARCH(1,1) with the conditional variance specified as

$$h_{i,t+1} = \overline{\omega}_i + \alpha(h_{i,t} - \overline{\omega}_i) + \beta_i(\epsilon^2_{i,t} - \overline{\omega}_i), \qquad i = 2....12. \tag{4.36}$$

and the conditional mean of each factor specified as

$$x_{i,t+1} = a_i + \sum_{l=1}^{p} \sum_{j=1}^{12} a_{i,j,l} x_{j,t-l+1} + \epsilon_{i,t+1}, \qquad i = 2....12. \tag{4.37}$$

$x_{i,t+1}$ follows the ordering of the vector $\mathbf{F}$ - for example $x_{1,t+1}$ is the logarithmic FOREX excess return for the US investor, $i^e_{fx,t+1}$. We use a vector auto regression of order 1 ($p = 1$) for each variable - since we have many variables we get a potential better representation of the factor residuals, $\epsilon_{i,t+1}$ of which we wish to estimate the conditional variance. This may be considered a strict advantage of the estimation method we are proposing in this chapter.

If we use a vector auto regression of order 2 we note that it is necessary to impose a restriction, that the coefficient on the second lag of the interest rate differential is equal to zero - otherwise colinearity becomes a problem since $(i_{f,t} - i^*_{f,t}, \Delta i^*_{f,t}, \Delta i_{f,t})$ all are in the vector auto regression. All models that we estimate have been estimated using a vector auto regression of order 2 but results do not differ substantially[13].

- For each variable (factor), $x_{i,t+1}$ and $i = 2...12$, obtain the estimated variance series $\hat{h}_{i,t+1}$

In the second step we estimate the mean equation of FOREX excess return, the risk premium in the single or two investor models using estimated conditional variances (in first step) of the

UK and US have high correlation. Though we conjecture that the assumption is not extremely critical relative to the ease of the estimation of the models.

[13]Results are available upon request.

factors, as

$$\begin{aligned} i^e_{fx,t+1} + \psi h_{fx,t+1} &= \sum_{i=1}^{12} \gamma_i \sqrt{h_{fx,t+1}} \sqrt{\hat{h}_{i,t+1}} + \epsilon_{1,t+1} \\ &= \sum_{i=1}^{12} \bar{\gamma}_{\rho i} \mathrm{Cov}_t(i^e_{fx,t+1}, x_{i,t+1}) + \epsilon_{1,t+1}, \end{aligned} \tag{4.38}$$

ψ is determined whether the UK, US or two investor model. In the US model $\psi = \frac{1}{2}$, in the UK model $\psi = -\frac{1}{2}$ and in the two investor model $\psi = 0$. Each of the models outlined in section (4.4) imposes restrictions that some of the parameters, γ_i, are equal to zero[14]. $\hat{h}_{i,t+1}$ is obtained in step 1 but the conditional variance of FOREX, $h_{fx,t+1}$, is estimated simultaneously following the univariate GARCH(1,1) process

$$h_{fx,t+1} = \overline{\omega}_{fx} + \alpha_{1,fx}(h_{fx,t} - \overline{\omega}_{fx}) + \beta_{1,fx}(\epsilon^2_{fx,t} - \overline{\omega}_{fx}) + \beta_{2,fx} I_{t+1}(\epsilon^2_{fx,t} - \frac{\overline{\omega}_{fx}}{2}), \tag{4.39}$$

where I_{t+1} is an indicator function taking the value of one if $\epsilon_{fx,t}$ is less than zero and zero otherwise. In one version of the two investor and general model we allow for asymmetries in the conditional variance equation. In the other models we assume $\beta_{2,fx} = 0$.

We note that $\hat{h}_{1,t+1}$ is not estimated in step 1 but is estimated simultaneously in step 2 set equal to $h_{fx,t+1}$. In addition to the single and two investor models we estimate a general alternative model to see if, when modelling the two investor implied FOREX risk premium, the US-UK risk-free interest rate differential and lagged log FOREX excess return have predictive power on FOREX log excess return. The estimated general model is

$$i^e_{fx,t+1} = \beta_{fx} i^e_{fx,t} + \beta_{fp}(i_{f,t} - i^*_{f,t}) + \sum_{j=1}^{F+1} \gamma_i \sqrt{h_{fx,t+1}} \sqrt{\hat{h}_{i,t+1}} + \epsilon_{1,t+1}, \tag{4.40}$$

[14]For instance to test the CCAPM two investor model $i^e_{fx,t+1} = \gamma_6 \sqrt{h_{fx,t+1}} \sqrt{h_{\Delta c,t+1}} + \gamma_7 \sqrt{h_{fx,t+1}} \sqrt{h^*_{\Delta c,t+1}} + \gamma_4 \sqrt{h_{fx,t+1}} \sqrt{h_{\pi,t+1}} + \gamma_5 \sqrt{h_{fx,t+1}} \sqrt{h^*_{\pi,t+1}}$. Recall that $m_{t+1} = -\alpha - \beta_1 \pi_{t+1} - \beta_2 \Delta c_{t+1}$ in the most general consumption-based model discussed in chapter 2, pricing only macroeconomic variables.

If our estimated models of the risk premium has solved the FOREX puzzle we would expect $\hat{\beta}_{fx} = \hat{\beta}_{fp} = 0$. In this general model we leave the variance of FOREX log excess return unrestricted and γ_1 is therefore unrestricted.

We have proposed an alternative method to estimate risk premia on a single asset. We do not claim that the restrictions we have imposed are strictly correct. However, we hope that the benefit of estimating the risk premium using many factors is high and outweighs the empirical drawbacks of the assumptions we have made. Moreover, the method has the advantage that a better representation of the residuals of the macroeconomic variables is obtained.

As a final comment on the proposed estimation procedure we note that interpreting the estimates γ_i as the estimated parameters on the conditional covariances, $\gamma_{\rho i}$, would be wrong. We estimate the first equation in equation (4.38). From our estimated single investor models we back out $\gamma_{\rho i}$ from the estimates as $\hat{\gamma}_{\rho i} = \frac{\hat{\gamma}_i}{\rho(x_{i,t+1}, i_{fx,t+1})}$, where $\rho(x_{i,t+1}, i_{fx,t+1})$ is a consistent estimator of the conditional constant correlation between FOREX log excess return and the factor and in the two investor models as $\hat{\gamma}_{\rho i} = \frac{2\hat{\gamma}_i}{\rho(x_{i,t+1}, i_{fx,t+1})}$. We have several options of the correlation estimate. A first estimator is to use the correlations from the actual dataset. Another estimator can be obtained from the correlation matrix of the residuals from vector auto regression (where logarithmic FOREX excess return is modelled as a vector regression) in step 1. A third estimator computes the correlation matrix of the residual from FOREX excess return in the two investor models with the residuals from the univariate vector auto regressions of the other variables in the first step. This latter method is presumably most correct. We do not necessarily wish to put much interpretation into the parameter estimates but are more interested in the consistent estimate we obtain of the risk premium. We report all three sets of correlations to allow the reader to back out the implied parameters on the conditional covariances if it is of interest. All univariate models are estimated under the assumption of a conditional students t-distribution.

4.7 Results

The estimates modelling each of the variables (factors other than log FOREX excess return) as a vector auto regression of order 1 are included in table (4.1) in the appendix. These estimates are used in step 1 to create the conditional variance series $\hat{h}_{i,t+1}$. The results estimating the general macroeconomic factor models are included in table (4.2), (4.3), (4.4), (4.5) and (4.6) in the appendix. We comment briefly on the results

- Benchmark Model

The Benchmark model include the variables traditionally used in tests of the UIP condition. These variables are the forward premium (or the differential between domestic and foreign risk-free rates) and the US risk-free rate changes. Hence the estimate on the conditional covariance with the forward premium can be interpreted as the contribution to the risk premium from the covariance between FOREX log excess return and the UK risk-free rate (the risk-free rate between $t+1$ and $t+2$ is not risk-free at time t). The conditional covariance between log excess return and the forward premium and the US first difference of the short rate are both significant variables in the US, UK and two investor models. The coefficients on the covariance of the forward premium, using the computed correlations in table (4.7) in the appendix from Two-Benchmark, are -12327.1 and 3226.94 respectively. However, the Benchmark model does not resolve the FOREX puzzle and we reject the test of asymmetries in the conditional variance of FOREX excess return. In the two investor model the R^2 is 1.73 %.

- Inter-temporal CCAPM Model

In the consumption-based Power Utility model none of the covariances are significant for determining risk premia. This result is consistent in the single or two investor models. The estimated parameters on the conditional covariances, i.e. corrected estimate for constant correlation estimated in the two investor model in table (4.7), in the two investor model is 2542.78 on US consumption growth, 80.62 on UK consumption growth and 4207.71 and 6783.10 on US and UK inflation covariance respectively but none of them are significant. The FOREX puzzle is not resolved, we reject asymmetries in the conditional variance of log FOREX excess return and the R^2 in the two investor model is only 0.36 %.

- Monetary Model

In the monetary model we find that UK money and industrial production growth are significant both in the single and two investor models - in the two investor model the implied coefficient on the conditional covariance with US and UK money growth are 143.86 and 4146.00 respectively and on the conditional covariance with US and UK industrial production growth the estimates are 65.08 and -8878.70 respectively. The R^2 is relatively higher in the two-investor model with

3.22 % but the monetary model does not resolve the FOREX puzzle and we reject the test of asymmetries in the conditional variance of log FOREX excess return although the t-statistics is 1.55 in the general model on the asymmetry coefficient. That is, almost significant using a 90 % critical value.

- Combined 1 Model

In the first combined model we find that UK money and industrial production growth are significant. We find that UK consumption covariance is significant for determining risk premia when modelled joint with the other UK macroeconomic variables. The R^2 in the two investor model is 4.45 % but this is not enough to resolve the FOREX puzzle. We now accept the null of asymmetries in the conditional variance of log excess return using a 90 % critical value. It is interesting though that UK consumption growth is a significant variable suggesting the findings of its insignificance in the CCAPM is due to omitted variables.

- Combined 2 Model

In the last combined model we price the forward premium and the US risk-free rate first difference additionally. We see that in the two investor model, the US risk-free rate covariance is significant for determining risk premia and the conditional covariance with UK consumption growth loses its significance. The R^2 is now 6.65 % and the main contribution comes from the UK variables. However, even our most general model does not resolve the FOREX puzzle.

In the tables we have included summary statistics of the standardised residuals. Auto correlation and heteroskedasticity in the standardised residuals is rejected in all models. A general conclusion about the standardised residuals is that the null hypothesis of normality is always rejected justifying our assumption that the conditional distribution has higher excess kurtosis. All standardised residuals have a mean not significantly different from zero.

The conclusion on pricing of variables is that we find that the UK macroeconomic variables consumption growth, narrow money growth and UK industrial production growth are significant variables determining FOREX risk premia. The only US variable found significant is the US interest rate[15].

[15]However, this latter variable loses significance as soon as we include the lagged interest rate differential between domestic and foreign risk-free interest rates in the equation.

In figure (4.6), (4.7) and (4.8) in the appendix we plot the implied risk premium in the UK, the US and two investor models. We note several things - first in both the UK and US single investor models the implied risk premium is both positive and negative which may indicate that we have got the risk premium wrong. It may be a sign that UK variables enter in the US SDF and vice versa in which case the two investor models are more interesting. In the two investor model we note that the implied risk premium in the combined model was high prior to and in the beginning of the FED. experiment suggesting that US investors required high risk premia for taking on investments abroad in that period (which could be due to much more variable inflation in the UK), then a period with relatively stable and negative risk premia followed suggesting that during the FED. experiment UK investors required high risk premia for investing in the US bond. From 1985, the risk premium on FOREX for the US investor increased again relatively much (maybe suggesting more favourable economic conditions in the US) to the risk premium required by the UK investor. Since then the FOREX risk premium has been decreasing towards zero (with an increase in the risk premium for the US investor in the period where UK was leaving the Exchange Rate Mechanism (ERM)) being negative towards the end of the sample. In figure (4.9) we plot the implied risk premia from the two investor models against US FOREX excess return and note that in the period where actual log excess return was high the implied risk premium captures much of the positive part. However, though intuitive and variable risk premia, our approach does not resolve the FOREX puzzle suggesting either a time-varying risk premium is not the way to resolve the problem or that we have got the dynamics or the risk premium wrong.

4.7.1 Risk Sharing ?

To end this chapter we will consider the conditional variance of the FOREX excess return. In equation (4.21) we saw that when foreign and domestic information set is the same, the conditional variance should be equal to the sum of domestic and foreign risk premia. The plots in the appendix suggests that risk premia on the exchange rate have become smaller in the 1990s with considerably smaller fluctuations which suggest that the conditional variance (risk premia) of the exchange rate has fallen and is less volatile. From the plot in figure (4.5) in the appendix we see that this is indeed the case. It looks as if there is a "regime shift" in the conditional variance of the FOREX excess return from 1994 onwards. We conclude that, even if we have not got the risk premium model correct it seems that our estimates of the risk premium and the conditional variance of the FOREX excess return is consistent with theory outlined in the current chapter in that risk premia and the conditional variance has fallen in the most recent year but is inconsistent in that the variability of the implied risk premia are too volatile in the late 1970s and early 1980s.

Brandt, Cochrane and Santa-Clara[16] computes a risk sharing index in complete markets. This index can be related to the way we outlined the SDF theory and our results. In complete markets

$$V_t(i_{fx,t+1}) = V_t(\overline{m}^*_{t+1} - \overline{m}_{t+1}) = V_t(\overline{m}^*_{t+1}) + V_t(\overline{m}_{t+1}) - 2\mathrm{Cov}_t(\overline{m}^*_{t+1}, \overline{m}_{t+1}) = \phi_t + \phi^*_t, \quad (4.41)$$

where an over lined variable indicates that inflation is subtracted. Perfect risk sharing implies that the correlation between domestic and foreign logarithmic SDF is equal to 1. Their maximum level of risk sharing (maximum number is 1), when $\overline{m}^*_{t+1} = \overline{m}_{t+1}$, is when there is risk neutrality, that is $\phi_t + \phi^*_t = 0$. It is curious that they, using an alternative framework for computing the risk sharing index, find that risk sharing is extremely high between US and UK, Japan and Germany with values close to 0.98. From our estimates we get an estimate of the time-variation in risk sharing. Risk sharing between US and UK investors has increased very much in the last 5-6 years relative to the 1970s and 1980s - this is evident from the decline in the conditional variance of the exchange rate and/or from the smoothness of risk premia in the most recent decade. Increase in risk sharing may also be related to macroeconomic volatility as evidenced in the various plots in this chapter and chapter 2. The increased risk sharing could be due to decrease in macroeconomic volatility (recall that the ratio of the conditional variance of UK and US industrial production growth has declined towards 1 in the previous three decades). However, more research would be necessary to determine the relation between risk sharing and macroeconomic volatility and it is beyond the scope of this chapter.

We believe that an interesting direction for future research would be developing the simplifying estimation method that we have proposed in this chapter allowing for, in some way, more general dynamics in the conditional covariance matrix between the macroeconomic variables.

4.8 Conclusion

In this chapter we considered estimation of risk premia on the US-UK exchange rate using observable macroeconomic variables as considered in chapter 2 and 3, on stock market indices. The aim was threefold - first we developed a theory for modelling the risk premium for a single investor and discussed requirements necessary to develop a two investor model where both a domestic and foreign investor were exposed to FOREX risk - we used the SDF model to develop the risk premium on FOREX. The second aim was to propose a simplified method for testing two investor FOREX models allowing to test general SDF models where many factors could be significant sources of risk to be priced on the exchange rate - we apply it to the exchange rate for illustration purposes but the proposed method could also have been applied to the estimation of stock market risk premia in chapter 2 and 3 or on any other single assets. The final aim of the chapter was to investigate whether our proposed way of modelling the time-varying FOREX risk premium was capable of resolving the FOREX puzzle.

We stated the FOREX puzzle and related it to the dataset considered in this chapter which was the US-UK exchange rate, in the period 1975-2002, and showed that if information available to domestic and foreign investors is the same then the UIP condition (under risk neutrality) can only be consistent with joint log normality of the exchange rate provided the conditional variance of the exchange rate is zero when foreign and domestic investors have the same information sets. This suggests that finite sample problems may not be a good explanation for the FOREX puzzle - it seems more plausible that the solution to the FOREX puzzle is the failure to account for a time-varying risk premium or other explanations, for example irrational expectations, not considered in this chapter.

The chapter derived the no-arbitrage condition for the FOREX returns using SDF methodology while distinguishing between a single investor model for domestic and foreign investors and a joint two investor model. It was shown that the two investor model implied that the FOREX risk premium was an equally weighted average of the risk premium to foreign (in domestic currency) and domestic investor if we assumed that the foreign and domestic information sets were equal. If the information sets across investors is not equal then it is complicated, if not impossible, to estimate the two investor SDF model since one has to define what is domestic and foreign information sets. If information sets are equivalent across investors, which is probably not an unrealistic assumption, then the conditional variance of FOREX excess return is equal to the sum of the foreign and domestic risk premium - when the conditional variance of excess return is fluctuating it is because risk premia are changing.

We finally restated the UIP condition taking into account that the domestic investor requires a risk premium for investing in a foreign bond, as well as the foreign investor requires a risk premium for investing in the domestic bond, then log normality of the exchange rate is consistent with the UIP condition for both the domestic and the foreign investor even when the conditional variance of excess return is varying over time. Although it was not the aim of this chapter to consider the inter-temporal relation between the FOREX risk premium and the variance of the exchange rate we rewrote the single and two investor model such that it was easy to interpret the relation. The relation depends on both the domestic and foreign risk premium and the expectation is constrained to be in the interval between minus a half and a half times the conditional variance of innovations in the exchange rate.

We concluded that a SDF model was an appropriate model for modelling the FOREX risk premium and argued that many variables would potentially need to be priced in a two investor model. Therefore it may be a problem to estimate the two investor FOREX model with the model outlined in chapter 1 allowing for more general dynamics of the time-varying covariance matrix since large scale (by scale we mean more variables) multivariate-GARCH-in-mean models are difficult, if not impossible, to estimate.

We propose an alternative estimation method to price several macroeconomic variables on the exchange rate. The estimation method had the advantage that estimation of the two investor model becomes feasible even with many domestic and foreign variables. The drawback of the proposed estimation method is that we need to assume that correlations between the macroeconomic variables should equal zero. However, we conjecture that the potential empirically rejected assumption may not be serious relative to the gain from pricing many additional factors. In addition, one benefit from the proposed estimation method is that we obtain a better representation of the residuals of the dependent variables. The advantage of the estimation method is due to the assumption of zero correlation between the variables that are not log excess return which allows us to estimate the single and two investor models using a two step procedure.

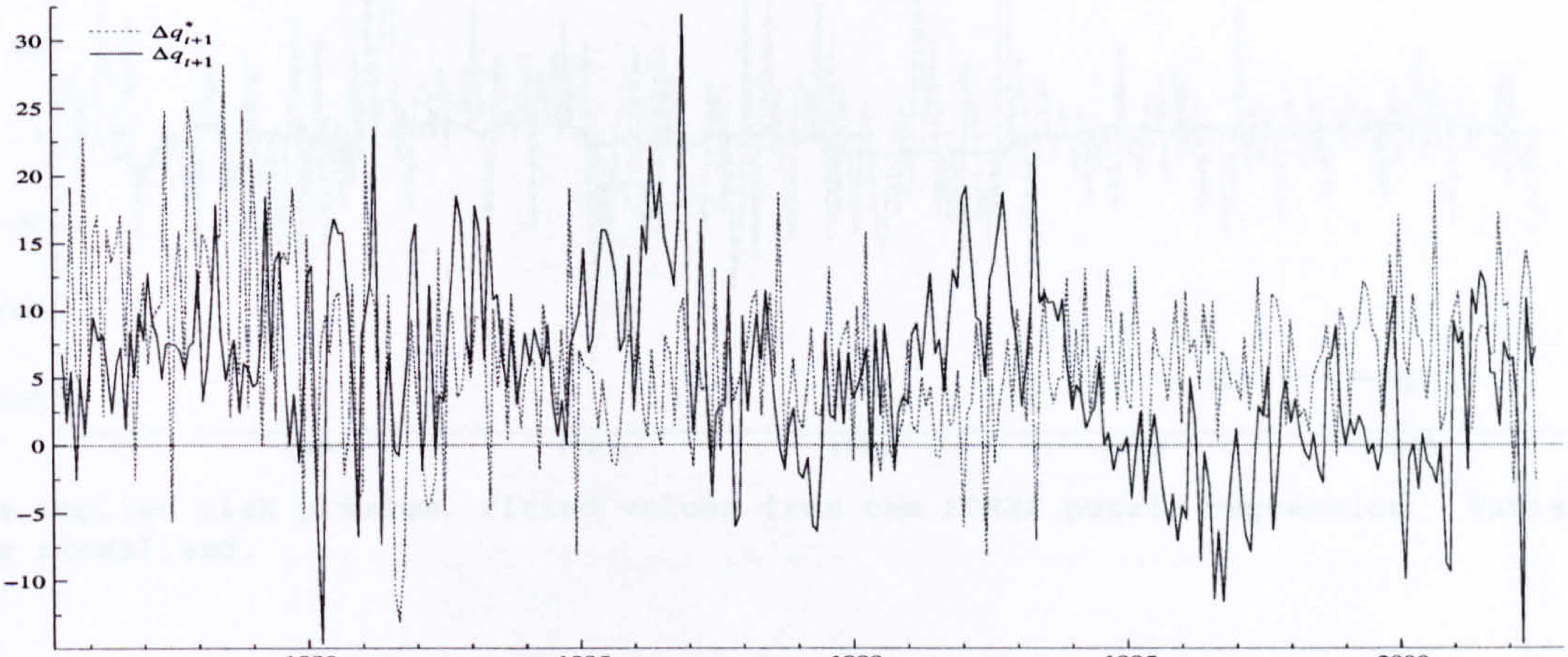
An empirical example was given with both a one single investor and a two investors (UK and US), using the proposed estimation method, modelling the risk premium on FOREX in the period 1975-2002. We test several macroeconomic models, some well known and others general alternatives not considered previously in the literature, and find that the UK macroeconomic variables consumption growth, narrow money growth and industrial production growth are all significant variables to be priced on the US-UK exchange rates. US macroeconomic variables are not significantly priced. We show that the proposed models generate risk premia that are varying over time but the general macroeconomic factor models that we estimate cannot resolve the FOREX puzzle - we conclude that in particular UK macroeconomic variables are significant variables to be priced in the UK-US FOREX market - it will be an interesting starting point for future work. Macroeconomic sources of risk seem priced in the US-UK FOREX market as it was in the US and UK stock markets. Though we find a time-varying risk premium we are left with the conclusion of Lewis, stated in section (4.2), that more work will need to be done along the lines of recovering the time-varying risk premium if this is the true solution to the FOREX puzzle.

We have proposed a method capable of estimating the risk premium on a single asset, pricing many variables - the method potentially gives a better estimate of the residuals of the macroeconomic variables. With the empirical example it was shown that the method was capable of estimating a risk premium that had significant time-variation.

We believe that the proposed method is a step in the direction estimating large scale multivariate GARCH in mean models. Our assumptions on the dynamics in the conditional covariance matrix may be too strong and we conclude that an interesting topic for future research would be to develop the proposed method considering how one can allow more general dynamics in the conditional covariance matrix while keeping the estimation relatively simple.

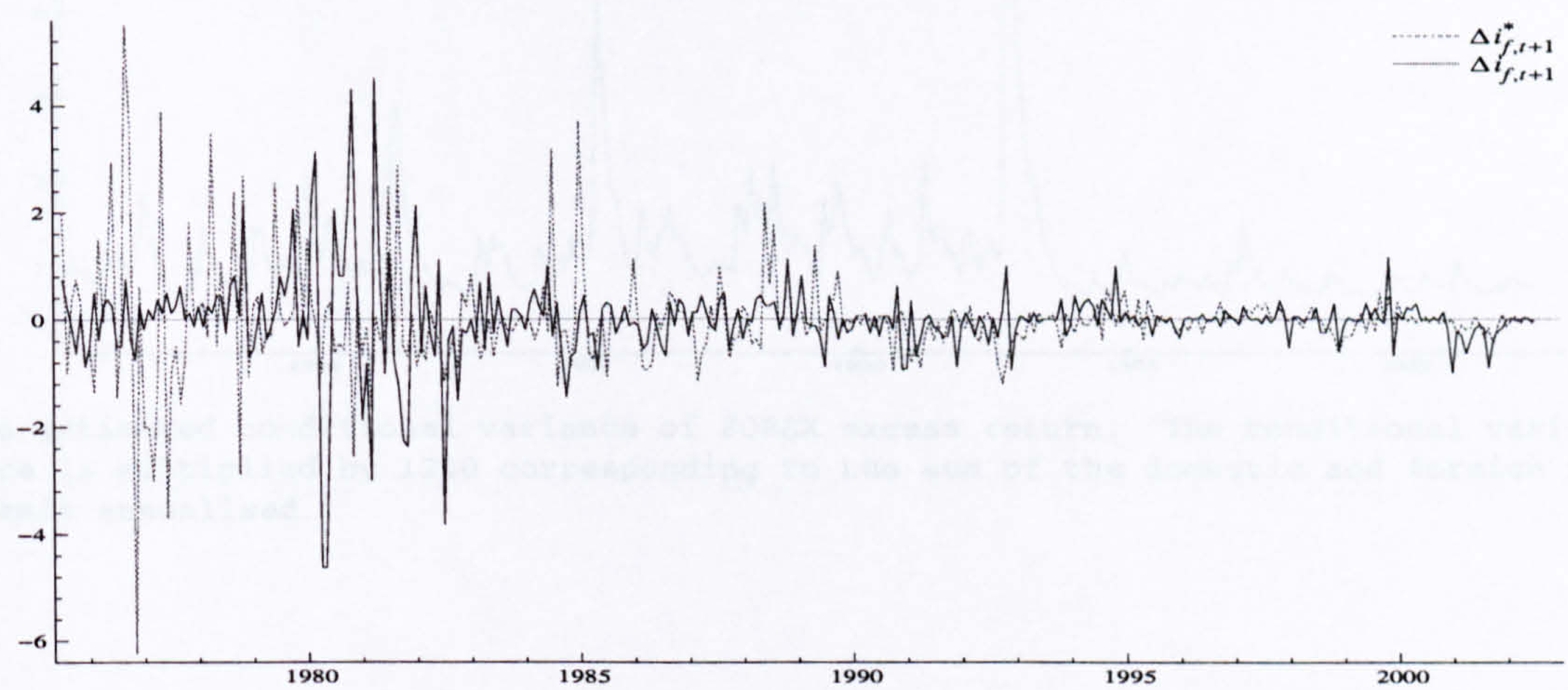
4.9 Appendix Chapter 4

Figure 4.2: Money Growth Rates, UK and US



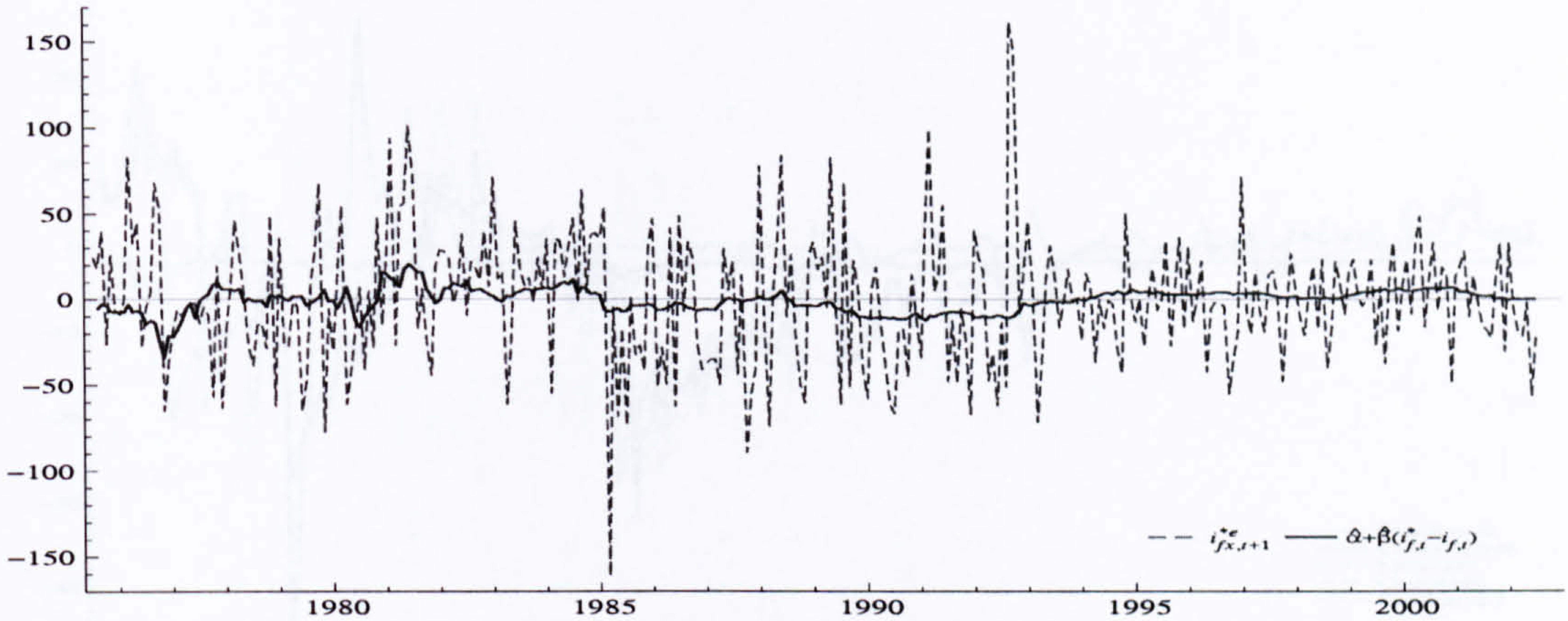
UK growth (indicated with a star) in M0 and US growth in M1. Note that two extreme outliers were present in the US dataset around September 2001 - we have replaced these with the mean of the data set. Data are annualised.

Figure 4.3: Changes In The Monthly Interest Rate, UK and US



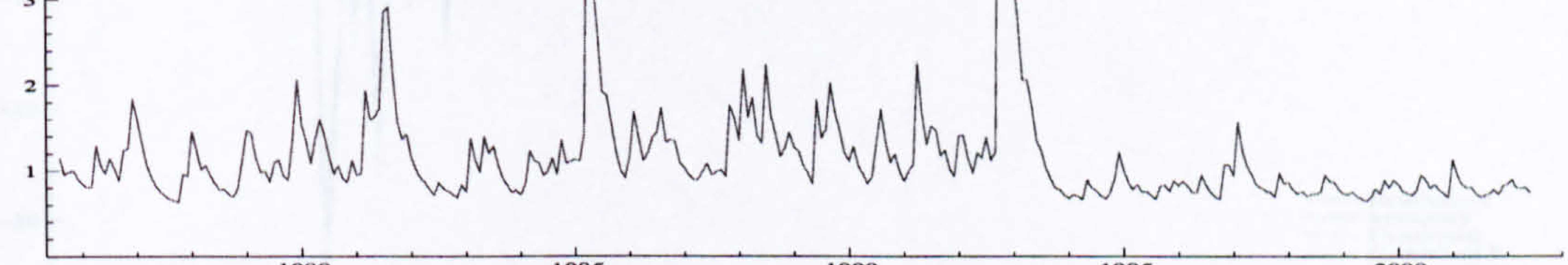
UK (Star as superscript) and US changes in the risk-free rate. Data are annualised.

Figure 4.4: Fitted Values From FOREX Puzzle Regression Against FOREX Excess Return



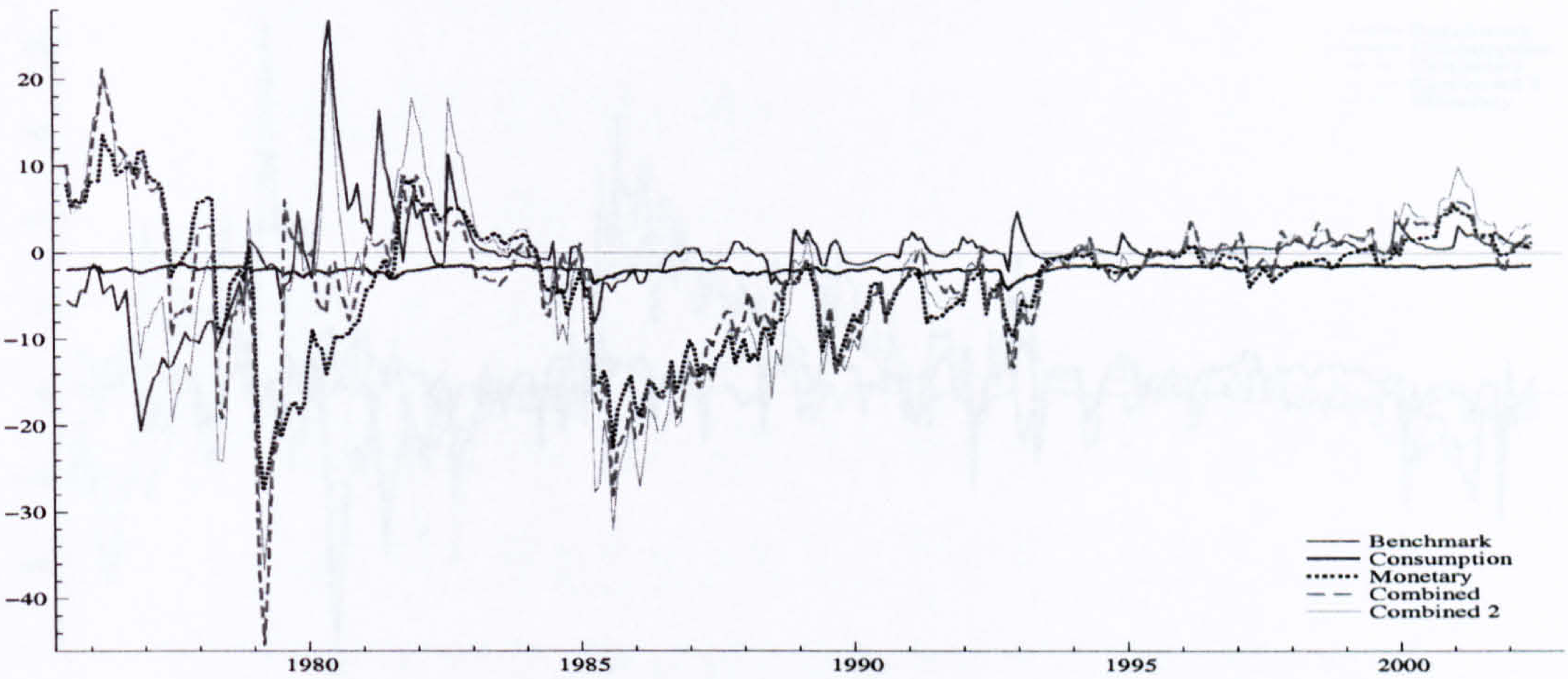
The implied risk premium, fitted values from the FOREX puzzle regression. Variables are annualised.

Figure 4.5: The Conditional Variance Of FOREX Excess Return



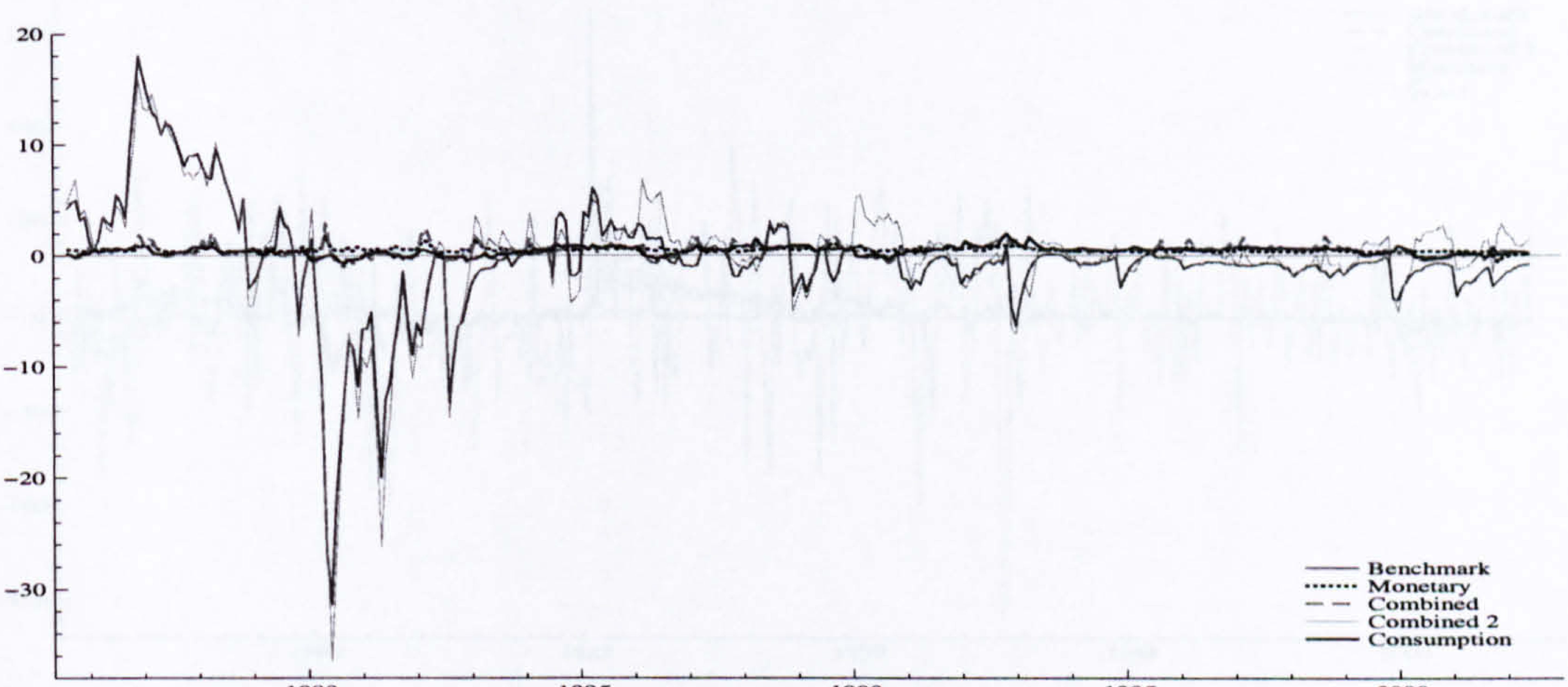
The estimated conditional variance of FOREX excess return. The conditional variance is multiplied by 1200 corresponding to the sum of the domestic and foreign risk premia annualised.

Figure 4.6: Risk Premium, UK Investor Models



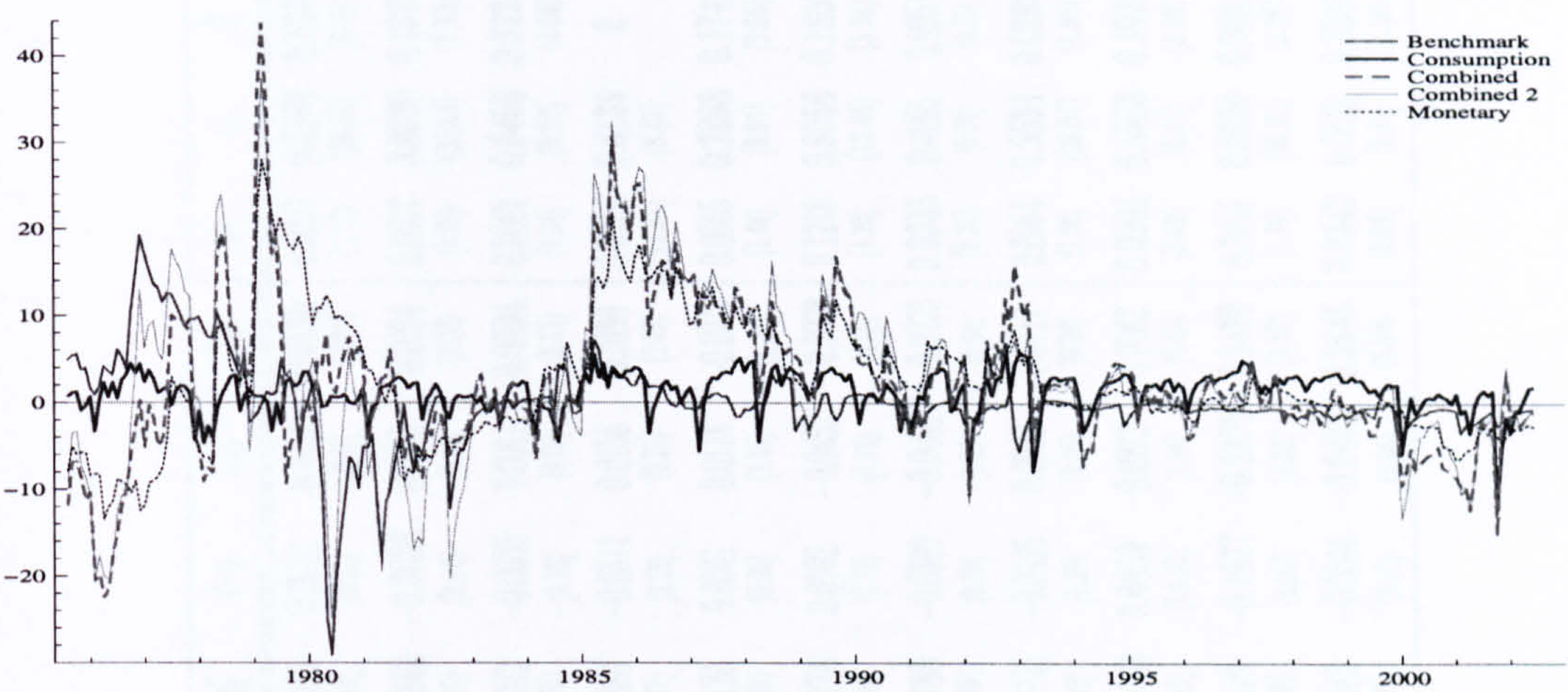
Implied risk premium from UK investor models. Note that we plot ϕ_t^* and not $-\phi_t^*$. The risk premia are annualised.

Figure 4.7: Risk Premium, US Investor Models



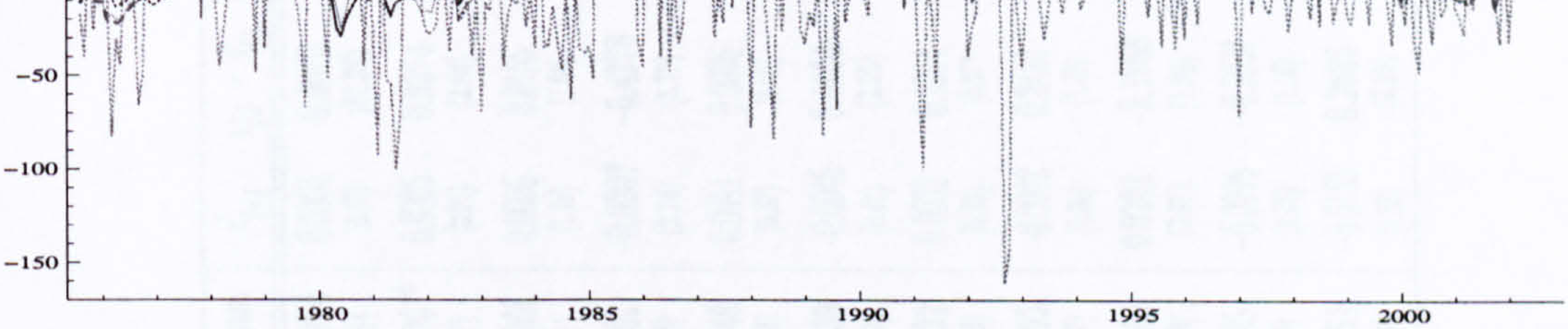
Implied risk premium from US investor models. The risk premia are annualised.

Figure 4.8: Risk Premium, Two Investor Models



Implied risk premium from two investor models. The risk premia are annualised.

Figure 4.9: Risk Premium, Two Investor Models VS Excess Return



Two investor against log excess return. All series are annualised.

Table 4.1: Estimates of Single Equation Vector Auto Regression with GARCH Effects

Univariate	Constant	$i^e_{f,t}$	$i_{f,t}-i^*_{f,t}$	$i^*_{f,t}-i^*_{f,t-1}$	$i_{f,t}-i_{f,t-1}$	π^*_t	π_t	Δc^*_t	Δc_t	Δq^*_t	Δq_t	Δy^*_t	Δy_t	ϖ	α	β
$i_{f,t}-i^*_{f,t}$	−0.0104 (2.01)	0.0004 (0.47)	0.9670 (61.99)	−0.0345 (0.48)	−0.0992 (1.20)	−0.0077 (1.41)	0.0080 (0.68)	−0.0018 (0.46)	0.0009 (0.22)	0.0036 (0.60)	0.0012 (0.27)	0.0018 (0.67)	0.0110 (2.71)	0.0022 (1.17)	0.8398 (30.34)	0.1593 (5.82)
$i^*_{f,t+1}-i^*_{f,t}$	$5.20*10^{-5}$ (0.01)	0.0005 (0.91)	0.0274 (2.94)	0.0788 (1.30)	0.0534 (1.07)	0.0042 (1.11)	0.0015 (0.20)	0.0009 (0.30)	0.0009 (0.35)	−0.0030 (0.85)	−0.0038 (2.14)	0.0005 (0.29)	0.0034 (1.32)	0.0017 (1.30)	0.8600 (36.51)	0.1397 (5.82)
$i_{f,t+1}-i_{f,t}$	−0.0066 (1.91)	0.0006 (1.18)	0.0155 (1.68)	−0.0105 (0.47)	−0.0219 (0.36)	$3.34*10^{-5}$ (0.01)	0.0167 (2.44)	0.0013 (0.58)	−0.0021 (0.80)	0.0023 (0.70)	−0.0005 (0.16)	0.0011 (0.73)	0.0094 (3.11)	0.0003 (1.24)	0.6493 (9.57)	0.3221 (4.64)
π^*_{t+1}	0.0102 (0.18)	−0.0207 (2.74)	−0.4278 (3.70)	−0.1183 (0.40)	−0.3705 (0.80)	0.1920 (3.27)	0.5570 (5.52)	0.0551 (1.51)	0.0461 (1.15)	0.0380 (0.77)	−0.0311 (0.76)	0.0156 (0.73)	0.0490 (1.05)	0.0200 (11.57)	0.6135 (8.47)	0
π_{t+1}	0.0748 (3.28)	0.0011 (0.37)	0.0036 (0.08)	−0.1250 (1.00)	0.3710 (1.68)	0.0524 (2.64)	0.6395 (14.18)	0.0040 (0.25)	0.0380 (2.35)	0.0128 (0.60)	0.0041 (0.25)	0.0118 (1.45)	−0.0217 (1.16)	0.0065 (1.44)	0.7898 (9.91)	0.1741 (2.55)
Δc^*_{t+1}	0.3539 (4.89)	−0.0045 (0.41)	0.4664 (2.95)	0.8453 (1.73)	−0.6009 (0.87)	−0.0360 (0.56)	−0.0542 (0.39)	−0.2360 (4.01)	0.1023 (2.09)	0.1278 (2.08)	0.0102 (0.22)	−0.0452 (1.74)	−0.0078 (0.15)	0.1218 (1.20)	0.8056 (13.45)	0.1808 (3.24)
Δc_{t+1}	0.4322 (6.10)	0.0032 (0.33)	0.1244 (0.87)	−0.2947 (0.78)	0.7667 (1.52)	−0.0208 (0.32)	−0.3842 (3.49)	−0.0896 (1.97)	−0.3728 (7.09)	0.1786 (2.99)	−0.0399 (0.70)	−0.0062 (0.25)	−0.0275 (0.54)	0.0335 (8.25)	0.4095 (0.74)	0.0951 (0.95)
Δq^*_{t+1}	0.5192 (8.57)	−0.0092 (1.00)	0.2420 (1.79)	0.3917 (0.91)	−0.9823 (1.52)	−0.0261 (0.47)	0.0879 (0.78)	0.0803 (2.06)	−0.0325 (0.83)	−0.0131 (0.22)	−0.0495 (1.39)	0.0037 (0.16)	0.0117 (0.24)	0.0964 (1.60)	0.9391 (29.67)	0.0593 (1.96)
Δq_{t+1}	0.1069 (2.20)	0.0201 (2.87)	−0.1968 (1.98)	−0.0910 (0.29)	−2.1168 (5.18)	0.0054 (0.13)	−0.0357 (0.41)	0.0083 (0.25)	0.0057 (0.16)	−0.0108 (0.25)	0.6659 (14.42)	0.0273 (1.44)	0.0162 (0.45)	0.0201 (5.49)	0.5958 (5.41)	0.1929 (3.08)
Δy^*_{t+1}	0.0095 (0.08)	−0.0298 (1.78)	−0.3033 (1.16)	0.7497 (0.91)	−1.3526 (1.09)	0.0500 (0.49)	−0.0847 (0.36)	0.1344 (1.56)	−0.0353 (0.45)	0.0479 (0.40)	−0.1677 (2.90)	−0.2379 (4.36)	0.2470 (2.82)	0.2016 (1.39)	0.8999 (25.21)	0.0931 (2.78)
Δy_{t+1}	0.1551 (2.44)	−0.0153 (1.53)	0.2685 (2.19)	0.0068 (0.02)	2.0792 (3.88)	−0.0855 (1.33)	−0.1143 (1.00)	0.0541 (1.31)	−0.0278 (0.58)	0.0585 (0.98)	−0.0044 (0.11)	−0.0472 (1.96)	0.3646 (5.94)	0.0343 (5.69)	0.0638 (0.31)	0.3281 (2.50)

Estimates of (1st step) univariate GARCH models with mean equation modelled as a vector auto regression. The estimated parameters in the conditional variance ϖ, α and β refers to equation (4.36) in the main text. The first column indicates the dependent variable that we model depending on the first lag of all variables. The first row of the table indicates which variable lagged the corresponding estimates in the table corresponds to. Absolute t-statistics in parenthesis. Emphasised parameters significant using a 95 % critical value. Note, we have imposed the restrictions α, $\beta \geq 0$.

Table 4.2: Estimates, Benchmark Model

Benchmark	US	UK	US and UK	General	US and UK	General
Model			Non switching model		Switching model	
$Constant * 100$				0.0226 (0.04)		0.0924 (0.17)
$\sigma_t^2(i^e_{fx,t+1})$	0.5	-0.5		−3.0867 (0.46)		−4.9885 (0.70)
$\sigma_t(i^e_{fx,t+1})\sigma_t(i_{f,t+1} - i^*_{f,t+1})$	206.0542 (1.90)	**224.1540 (2.07)**	**215.1080 (1.99)**	22.7889 (0.17)	**216.1835 (1.99)**	22.5898 (0.17)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta i_{f,t+1})$	**−372.4051 (2.41)**	**−356.7638 (2.29)**	**−365.1277 (2.35)**	−153.3586 (0.87)	**−369.4825 (2.42)**	−148.2996 (0.83)
$i^e_{fx,t}$				0.0142 (0.23)		0.0099 (0.14)
$i_{f,t} - i^*_{f,t}$				**−2.9408 (3.65)**		**−3.2782 (3.86)**
Conditional Variance						
df	11.3569 (1.86)	11.6618 (1.80)	11.5065 (1.83)	**7.8141 (2.76)**	10.9532 (1.93)	**8.0854 (2.95)**
$\overline{\omega}_{fx} * 1000$	**1.0137 (3.93)**	1.0139 (1.80)	**1.0143 (3.95)**	**0.9790 (4.92)**	**0.8011 (2.71)**	**1.2892 (2.18)**
$h_{fx,t}$	**0.8006 (8.60)**	**0.7897 (8.14)**	**0.7960 (8.41)**	**0.6292 (3.32)**	**0.8472 (11.12)**	**0.5711 (2.69)**
$\varepsilon^2_{fx,t}$	**0.1252 (2.37)**	**0.1312 (2.41)**	**0.1279 (2.39)**	**0.1662 (2.07)**	**0.1161 (1.98)**	0.0971 (1.02)
$I_t\varepsilon^2_{fx,t}$					−0.0283 (0.54)	0.1354 (1.09)
R^2	0.0174	0.0170	0.0173	0.0473	0.0148	0.0505
Standardised residuals						
Mean	0.0212	0.0303	0.0258	0.0247	-0.0147	-0.0043
Variance	1.0039	1.0054	1.0044	1.0001	1.0069	1.0121
Normality test	11.511	10.648	11.081	13.658	34.884	42.059

The Benchmark Model. $\sigma_t(xxx_{t+1})$ denotes the conditional standard deviation of variable xxx. For the estimated parameters in the conditional variance, recall equation (4.39). The standardised residuals are computed as $\frac{\epsilon_{fx,t+1}}{\sqrt{h_{fx,t+1}}}$. The estimated constant in the mean FOREX log excess return equation is multiplied by 100 and the estimate of the long run variance of FOREX excess return is multiplied by 1000. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value. The variance of the annualised FOREX excess return is 1465.36.

Table 4.3: Estimates, Inter-temporal CCAPM

Power Utility	US	UK	US and UK	General	US and UK	General
Model			Non switching model		Switching model	
$Constant * 100$				−0.5512 (0.14)		−7.9423 (0.83)
$\sigma_t^2(i^e_{fx,t+1})$	0.5	−0.5		−8.5132 (0.24)		−75.6745 (0.80)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c_{t+1})$	7.4955 (0.24)		−130.5718 (0.99)	−139.1145 (1.08)	−150.2881 (1.12)	−155.5814 (1.19)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c^*_{t+1})$		0.5778 (0.04)	2.3340 (0.16)	−9.5559 (0.62)	−5.3971 (0.37)	−14.5703 (0.98)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi_{t+1})$	−15.1341 (0.18)		−41.8667 (0.48)	−17.6731 (0.20)	−51.7989 (0.58)	−30.6551 (0.34)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi^*_{t+1})$		10.2788 (0.39)	190.9442 (1.13)	267.7971 (0.48)	231.7866 (1.36)	1305.9684 (0.97)
$i^e_{fx,t}$				0.0189 (0.31)		0.0093 (0.13)
$i_{f,t} - i^*_{f,t}$				**−3.1332 (4.01)**		**−3.3440 (4.10)**
Conditional Variance						
df	**11.2786 (1.96)**	**11.3421 (1.96)**	**10.7373 (2.02)**	**7.4052 (2.89)**	**10.3979 (2.30)**	**7.6805 (3.02)**
$\overline{\omega}_{fx} * 1000$	**1.0278 (4.28)**	**1.0290 (4.33)**	**1.0220 (4.42)**	**0.9816 (4.83)**	**0.6342 (3.08)**	**1.1704 (3.16)**
$h_{fx,t}$	**0.7514 (5.98)**	**0.7390 (5.54)**	**0.7281 (5.22)**	**0.6239 (3.21)**	**0.9019 (15.71)**	**0.5338 (2.56)**
$\epsilon^2_{fx,t}$	**0.1449 (2.28)**	**0.1504 (2.26)**	**0.1526 (2.23)**	**0.1644 (2.03)**	0.0868 (1.62)	0.0389 (0.48)
$I_t\epsilon^2_{fx,t}$					−0.0399 (0.85)	0.1524 (1.26)
R^2	0.0013	0.0026	0.0036	0.0442	0.0012	0.0819
Standardised Residuals						
Mean	0.0011	-0.0056	-0.0045	-0.0075	-0.0151	-0.0085
Variance	1.0070	1.0069	1.0068	1.0076	1.0081	1.0143
Normality test	12.955	12.685	13.653	25.728	39.140	43.352

The inter-temporal CCAPM. $\sigma_t(xxx_{t+1})$ denotes the conditional standard deviation of variable xxx. For the estimated parameters in the conditional variance, recall equation (4.39). The standardised residuals are computed as $\frac{\epsilon_{fx,t+1}}{\sqrt{h_{fx,t+1}}}$. The estimated constant in the mean FOREX log excess return equation is multiplied by 100 and the estimate of the long run variance of FOREX excess return is multiplied by 1000. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

Table 4.4: Estimates, Monetary Model

Monetary Model	US	UK	US and UK (Non switching model)	General (Non switching model)	US and UK (Switching model)	General (Switching model)
$Constant * 100$				0.3119 (0.45)		0.2774 (0.39)
$\sigma_t^2(i^e_{fx,t+1})$	0.5	-0.5		−7.8762 (1.02)		−9.7368 (1.19)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q_{t+1})$	−1.1608 (0.06)		−4.9705 (0.27)	−5.4125 (0.28)	−5.4295 (0.29)	−4.6866 (0.23)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q^*_{t+1})$		**−101.0164** (3.32)	**−96.8092** (2.89)	**−102.2896** (2.60)	**−103.8152** (3.05)	**−121.2887** (2.86)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y_{t+1})$	3.3460 (0.20)		−4.2334 (0.15)	−4.4595 (0.11)	−5.8998 (0.20)	1.5119 (0.04)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y^*_{t+1})$		**55.0072** (3.89)	**55.4919** (3.74)	**53.6435** (3.16)	**59.2614** (3.97)	**62.4195** (3.44)
$i^e_{fx,t}$				−0.0135 (0.22)		−0.0238 (0.34)
$i_{f,t} - i^*_{f,t}$				**−2.9898** (4.00)		**−3.2102** (3.81)
Conditional Variance						
df	**11.2281** (1.96)	**10.0854** (2.04)	**10.1747** (1.98)	**7.1359** (2.84)	**10.3079** (2.02)	**7.1148** (3.17)
$\overline{\omega}_{fx} * 1000$	**1.0280** (4.30)	**0.9868** (5.08)	**0.9875** (5.03)	**0.9743** (4.48)	**1.2612** (2.30)	1.5439 (1.59)
$h_{fx,t}$	**0.7476** (5.84)	**0.6415** (3.45)	**0.6389** (3.44)	**0.5752** (3.09)	**0.6075** (3.14)	**0.5991** (3.50)
$\varepsilon^2_{fx,t}$	**0.1463** (2.28)	**0.1769** (2.15)	**0.1792** (2.14)	**0.1926** (2.25)	0.1220 (1.23)	0.0556 (0.69)
$I_t\varepsilon^2_{fx,t}$					0.1061 (1.00)	0.1981 (1.55)
R^2	0.0019	0.0333	0.0322	0.0759	0.0297	0.0805
Standardised Residuals						
Mean	-0.0006	-0.0327	-0.0290	-0.0203	-0.0356	-0.0192
Variance	1.0071	1.0041	1.0048	1.0091	1.0029	1.0149
Normality test	13.070	13.747	13.300	15.600	33.374	46.408

The Monetary Model. $\sigma_t(xxx_{t+1})$ denotes the conditional standard deviation of variable xxx. For the estimated parameters in the conditional variance, recall equation (4.39). The standardised residuals are computed as $\frac{\epsilon_{fx,t+1}}{\sqrt{h_{fx,t+1}}}$. The estimated constant in the mean FOREX log excess return equation is multiplied by 100 and the estimate of the long run variance of FOREX excess return is multiplied by 1000. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

Table 4.5: Estimates, Combined Model 1

Combined 1 Model	US	UK	US and UK	General	US and UK	General
			Non switching model		Switching model	
$Constant * 100$				1.7469 (0.43)		−7.5416 (0.98)
$\sigma_t^2(i^e_{fx,t+1})$	0.5	-0.5		2.7248 (0.07)		−79.4083 (1.00)
$\sigma_t(i^e_{fx,t+1})\sigma_t(i_{f,t+1} - i^*_{f,t+1})$			79.1933 (0.53)	55.0843 (0.34)	77.0148 (0.51)	−7.2612 (0.04)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi_{t+1})$	−18.8205 (0.21)		−72.0960 (0.75)	−31.9173 (0.33)	−70.9041 (0.71)	−36.0531 (0.36)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi^*_{t+1})$		−5.4326 (0.15)	209.0469 (1.22)	6.5458 (0.01)	201.2598 (1.16)	1314.6153 (1.18)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c_{t+1})$	0.3744 (0.01)		−160.3286 (1.17)	−148.6009 (1.11)	−157.3742 (1.15)	−187.6202 (1.41)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c^*_{t+1})$		**−48.3729 (2.13)**	**−50.2310 (2.18)**	**−58.0079 (2.39)**	**−47.7295 (2.07)**	**−59.9259 (2.54)**
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q_{t+1})$	0.6715 (0.03)		8.8067 (0.40)	3.0872 (0.15)	8.5003 (0.39)	9.0380 (0.44)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q^*_{t+1})$		**−84.3306 (2.51)**	**−94.0233 (1.96)**	**−105.6916 (1.97)**	**−99.7858 (2.02)**	−108.1986 (1.92)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y_{t+1})$	7.9628 (0.19)		7.3974 (0.17)	3.0872 (0.15)	8.5003 (0.39)	13.5657 (0.33)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y^*_{t+1})$		**78.3044 (4.08)**	**79.9226 (4.10)**	**81.7563 (3.51)**	**82.2401 (4.26)**	**84.9152 (3.84)**
$i^e_{fx,t}$				−0.0117 (0.19)		−0.0202 (0.30)
$i_{f,t} - i^*_{f,t}$				**−3.0793 (4.22)**		**−3.1966 (3.88)**
Conditional Variance						
df	**11.1339 (1.97)**	**10.2960 (2.14)**	**9.7955 (2.04)**	**6.4402 (3.05)**	**9.9249 (2.09)**	**6.5801 (3.28)**
$\overline{\omega}_{fx} * 1000$	**1.0273 (4.25)**	**0.9614 (5.35)**	**0.9584 (5.23)**	**0.9351 (4.64)**	**1.1794 (2.41)**	**1.2746 (2.33)**
$h_{fx,t}$	**0.7523 (5.98)**	**0.6560 (3.50)**	**0.6372 (3.35)**	**0.6628 (3.72)**	**0.6027 (2.91)**	**0.6334 (3.97)**
$\epsilon^2_{fx,t}$	**0.1440 (2.26)**	**0.1643 (2.03)**	**0.1720 (2.09)**	**0.1431 (1.98)**	0.1219 (1.13)	0.0042 (0.07)
$I_t\epsilon^2_{fx,t}$					0.0952 (0.85)	0.1761 (1.62)
R^2	0.0014	0.0449	0.0449	0.0848	0.0416	0.1162
Standardised Residuals						
Mean	0.0012	-0.0199	-0.0201	-0.0148	-0.0215	-0.0228
Variance	1.0072	1.0066	1.0057	1.0093	1.0022	1.0144
Normality test	13.418	16.064	15.692	17.963	44.234	51.266

The Combined Model 1. $\sigma_t(xxx_{t+1})$ denotes the conditional standard deviation of variable xxx. For the estimated parameters in the conditional variance, recall equation (4.39). The standardised residuals are computed as $\frac{\epsilon_{fx,t+1}}{\sqrt{h_{fx,t+1}}}$. The estimated constant in the mean FOREX log excess return equation is multiplied by 100 and the estimate of the long run variance of FOREX excess return is multiplied by 1000. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

Table 4.6: Estimates, Combined Model 2

Combined 2 Model	US	UK	US and UK	General	US and UK	General
			Non switching model		Switching model	
$Constant * 100$				1.1047 (0.26)		−8.5735 (1.03)
$\sigma_t^2(i^e_{fx,t+1})$	0.5	-0.5		−1.1993 (0.03)		−88.5316 (1.03)
$\sigma_t(i^e_{fx,t+1})\sigma_t(i_{f,t+1} - i^*_{f,t+1})$	170.3006 (1.24)	325.3460 (1.66)	292.7861 (1.37)	121.6673 (0.56)	295.9657 (1.42)	94.4402 (0.43)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta i_{f,t+1})$	**−426.4527** (2.49)	**−396.2998** (2.21)	−412.0485 (1.95)	−137.7748 (0.62)	−413.9786 (1.95)	−193.7951 (0.88)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi_{t+1})$	53.2484 (0.55)		18.2793 (0.18)	0.9027 (0.01)	19.9443 (0.19)	10.6801 (0.10)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\pi^*_{t+1})$		23.9089 (0.62)	177.3703 (1.04)	78.0358 (0.13)	167.3156 (0.98)	1443.0368 (1.20)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c_{t+1})$	−22.9346 (0.45)		−153.2411 (1.11)	−151.6689 (1.13)	−149.2366 (1.09)	−186.2116 (1.41)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta c^*_{t+1})$		−31.0185 (1.28)	−32.3049 (1.28)	−50.8523 (1.91)	−49.4121 (1.17)	**−51.4504** (2.02)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q_{t+1})$	3.7045 (0.19)		9.1729 (0.43)	−151.6689 (1.13)	8.8357 (0.41)	9.6751 (0.48)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta q^*_{t+1})$		**−136.0886** (2.74)	**−128.2462** (2.41)	**−117.4743** (2.06)	**−134.5680** (2.51)	**−126.6842** (2.15)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y_{t+1})$	13.0327 (0.32)		15.2954 (0.36)	14.2581 (0.34)	15.1114 (0.35)	17.5601 (0.42)
$\sigma_t(i^e_{fx,t+1})\sigma_t(\Delta y^*_{t+1})$		**73.9488** (3.66)	**75.0016** (3.61)	**80.0470** (3.38)	**77.5795** (3.77)	**84.0679** (3.75)
$i^e_{fx,t}$				−0.0120 (0.20)		−0.0174 (0.26)
$i_{f,t} - i^*_{f,t}$				**−2.8476** (3.80)		**−2.9120** (3.57)
Conditional Variance						
df	10.9014 (1.95)	10.3913 (1.82)	**9.3926** (1.96)	**6.5020** (2.93)	**9.4963** (2.01)	**6.5681** (3.23)
$\overline{\omega}_{fx} * 1000$	**0.9977** (3.89)	**0.9686** (4.68)	**0.9518** (4.84)	**0.9291** (4.74)	**1.2107** (2.09)	**1.2855** (2.26)
$h_{fx,t}$	**0.8313** (10.17)	**0.6608** (3.97)	**0.6713** (3.96)	**0.6526** (3.38)	**0.6321** (3.39)	**0.6509** (4.18)
$\varepsilon^2_{fx,t}$	**0.1073** (2.26)	**0.1801** (2.22)	**0.1648** (2.12)	0.1438 (1.86)	0.1208 (1.21)	−0.0003 (0.001)
$I_t\varepsilon^2_{fx,t}$					0.0940 (0.86)	0.1737 (1.60)
R^2	0.0217	0.0610	0.0665	0.0895	0.0653	0.1206
Standardised Residuals						
Mean	0.0046	-0.0220	-0.0219	-0.0181	-0.0239	-0.0248
Variance	1.0061	1.0035	1.0033	1.0076	1.0008	1.0129
Normality test	14.366	11.2580	13.9050	15.9900	40.0110	48.3030

The Combined Model 2. $\sigma_t(xxx_{t+1})$ denotes the conditional standard deviation of variable xxx. For the estimated parameters in the conditional variance, recall equation (4.39). The standardised residuals are computed as $\frac{\epsilon_{fx,t+1}}{\sqrt{h_{fx,t+1}}}$. The estimated constant in the mean FOREX log excess return equation is multiplied by 100 and the estimate of the long run variance of FOREX excess return is multiplied by 1000. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

Table 4.7: Estimates Of Constant Conditional Correlations

	$\rho(i^x_{fx,t+1}, i_{f,t+1} - i^*_{f,t+1})$	$\rho(i^x_{fx,t+1}, \Delta i^*_{f,t+1})$	$\rho(i^x_{fx,t+1}, \Delta i_{f,t+1})$	$\rho(i^x_{fx,t+1}, \pi^*_{t+1})$	$\rho(i^x_{fx,t+1} \pi_{t+1},)$	$\rho(i^x_{fx,t+1}, \Delta c^*_{t+1})$	$\rho(i^x_{fx,t+1} \Delta c_{t+1},)$	$\rho(i^x_{fx,t+1}, \Delta q^*_{t+1})$	$\rho(i^x_{fx,t+1}, \Delta q_{t+1})$	$\rho(i^x_{fx,t+1}, \Delta y^*_{t+1})$	$\rho(i^x_{fx,t+1}, \Delta y_{t+1})$
Unrestricted VAR	−0.0791	−0.1300	−0.2762	0.0420	−0.0105	0.0518	−0.1036	−0.0955	−0.0656	0.0009	−0.1469
Dataset	−0.1920	−0.1647	−0.2062	0.0896	0.0216	0.0325	−0.0772	−0.0778	−0.0436	0.0173	−0.0715
Two-Benchmark	−0.0349	−0.1553	−0.2263	0.0350	−0.0203	0.0582	−0.1114	−0.0892	−0.0591	−0.0240	−0.1589
Two-Consumption	−0.0229	−0.1637	−0.2155	0.0563	−0.0199	0.0579	−0.1027	−0.0467	−0.0639	−0.0118	−0.1419
Two-Monetary	−0.0328	−0.1661	−0.2295	0.0529	−0.0401	0.0421	−0.0998	−0.0469	−0.0691	−0.0125	−0.1301
Two-Combined	−0.0301	−0.1629	−0.2249	0.0732	−0.0362	0.0118	−0.1116	−0.0637	−0.0829	−0.0338	−0.1342
Two-Combined 2	−0.0454	−0.1539	−0.2405	0.0651	−0.0297	0.0119	−0.1172	−0.0598	−0.0829	−0.0371	−0.1437

Different estimates of the constant conditional correlations. A star as a superscript indicates that it is a UK variable. Unrestricted VAR indicates that the correlations are obtained from the correlation matrix of an unrestricted Vector Auto Regression of all variables, Dataset indicates that the correlations are computed from the actual dataset, TWO-xxxx indicates that the correlations are obtained between the residual of the excess return equation in the two investor model xxxx and the residuals of the other variables as obtained from a VAR in step 1 of the estimation procedure. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

5. Epstein-Zin: The Joint FOREX and Equity Model

A Test For Asset Market Integration Based on Observable Stochastic Discount Factors

5.1 Introduction

In chapter 2 and 3 we showed that macroeconomic sources of risk were significantly priced in the stock markets and similar findings were obtained in chapter 4 for the FOREX market, this in particular UK macroeconomic variables. This raises a natural question whether there is a link between risk compensation in the national equity markets and the FOREX market or more specifically whether the macroeconomic risks are similarly priced in the two markets. Since we found that US variables were not priced at all in the FOREX market this chapter focuses on the UK stock market and the UK FOREX investor model only.

So far we have not claimed, strictly speaking, that our specification of the SDF is correct but we have attempted to rely on well-known asset pricing models. General Equilibrium models tell us that the logarithm of the SDF is linear in macroeconomic variables and we believe it is a natural starting point. We expanded our work in a different direction pricing other key macroeconomic variables than consumption growth and we showed that pricing these alternative variables implied significant time-variation in equity and FOREX risk premia. Most preferably we should price all potentially significant variables in which case one could avoid omitted variable biases - unfortunately with the estimation method proposed in this thesis it is not feasible. In the UK we found some role for consumption growth, money growth and industrial production growth in chapter 4 on the UK-US exchange rate whereas in chapter 2 we found consumption growth and inflation to be priced in the UK stock market. This raises the question whether there is a link between pricing of macroeconomic risk in the UK stock market and the UK-US FOREX market which we will investigate in this chapter - the aim is to propose a test whether the two markets are integrated based on observable Stochastic Discount Factors.

Many approaches have been taken to test for asset market integration and the current chapter will summarise some of these. Then we will propose an alternative method to test whether the FOREX and stock market are integrated based on observable Stochastic Discount Factors. We define two financial markets to be integrated in terms of the risk price of each factor in the Stochastic Discount Factor. If these estimated prices of risk are the same in the FOREX and stock market then we think of the markets as being integrated. Asset market integration is to be distinguished from complete markets - whereas asset market integration means that the expected Stochastic Discount Factors are the same across markets, market completeness implies that the SDFs across markets are unique, not only unique in terms of their expectation. If markets are complete then there is only one unique SDF whereas even if markets are integrated then there can exist several SDFs pricing an asset. The advantages of our test is that we can back out an estimate of the time-varying risk premium and the estimated conditional correlation between the two asset returns - conditional correlations between asset returns depend, as we have shown in other chapters, on the modelling of the risk premium.

Although we considered pricing of other variables than consumption in chapter 3 and 4 we feel it is necessary to return to models with stronger theoretical foundations when testing whether the two financial markets are integrated. We show, and argue, that any rejection of integration can simply be due to wrong modelling of the SDF and we believe it is a necessity, when one wishes to test for asset market integration using the approach proposed in this chapter, that we use a theoretical model for the choice of factors to be priced in the markets. Flood and Rose [61], for instance, use another approach to test for integration of financial markets (based on the Fama and French 3 factor model, see Fama and French [56]). Their model has less theoretical foundations than the one proposed in this chapter but it can be interpreted as a version of the inter-temporal CAPM of Merton. More specifically we consider the cross-equation restrictions imposed by the ICAPM across returns when modelling the dynamic risk premium joint on several assets.

The theoretical model we use in this chapter is an extended version of the Epstein Zin model proposed in chapter 2 - we show that the implication of the Epstein Zin model on the time-varying FOREX risk premium is that the FOREX risk premium needs to be modelled joint with the risk premium on the market portfolio. Hence when estimating an Epstein Zin model on

FOREX or bonds it is necessary that excess returns in these markets need to be modelled joint with the market return and a natural question arises whether the FOREX- and bond (we do not consider bonds in this thesis but it is currently under investigation) markets are integrated with the stock market - are macroeconomic variables and financial variables priced similarly in the FOREX and stock market ?

This chapter can be seen as the first estimate of the time-varying risk premium on the exchange rate implied by the Epstein Zin model[1]. The implementation in this chapter of the joint FOREX and equity Epstein Zin model is done assuming first equity and FOREX markets to be integrated and then not to be integrated - we test which of the assumptions is the better. Modelling the risk premium on the exchange rate and stock market joint we can extend the Epstein Zin model used in chapter 2 allowing the wealth portfolio of the representative investor to be more general - this chapter considers the extension that the representative investor invests part of his or her wealth in a foreign bond as well. The implications of this is that four variables need to be priced in each financial markets in the most general form. One could ask why FOREX and stock markets should be integrated ! We show that implementing the Epstein Zin model on FOREX it is a necessity to impose that these markets are integrated in order to obtain a correct measure of shocks in the stock market whose conditional covariance with shocks to FOREX excess return may determine the FOREX risk premium.

First we discuss the implications on the risk premium when two economies each can be characterised by a representative investor both with, though the preference parameters can differ, Epstein Zin utility. Unfortunately, we will show that a two investor Epstein Zin model on FOREX is difficult to estimate allowing for time-variation in the conditional covariance matrix with the available sizes of datasets. Using the estimation framework proposed in chapter 1 it is practically infeasible to estimate the two investor model except if one makes strong, empirically rejectable, assumptions on the dynamics of the conditional covariance matrix. Instead this chapter proposes a method for testing general consumption-based models using FOREX data - all we need is the single investor FOREX model considered in chapter 3. Rejection of equivalent prices of risk in the single investor domestic FOREX model and the domestic stock market

[1]Estimation of the Power Utility CCAPM on exchange rates has been done also by Kaminsky and Peruga [76]. However, they make very strong assumptions on the dynamics of the conditional covariance matrix which we will show are clearly not valid. They obtain implausible but imprecise estimates of the coefficient of relative risk aversion.

serves as a rejection of the consumption-based models since consumption-based models assume financial markets to be integrated. The test of integration and estimation approach proposed has the advantage that one obtains an estimate of the FOREX risk premium joint with the risk premium in the stock market. The important implication of Generalised Isoelastic Preferences, as adopted in this and chapter 2, is that part of the risk premium on FOREX is determined as the covariance between log excess return on domestic and foreign equities with log excess return on FOREX - this is the reason why we need to model the risk premium on equity and FOREX jointly.

Considering this asset pricing model one has an alternative way to distinguish whether a Partial or General Equilibrium model is most consistent with the actual data. An implication of the discussion in this chapter, together with our proposed method of estimating the risk premium, is that the Epstein Zin model gives us an intuitive answer as to why international market contagion occurs - because unexpected shocks in foreign countries may affect the conditional covariance matrix in domestic financial markets. More specific it could give an explanation as to why unexpected shocks in say the US equity market can affect variances and covariances, and hence risk premia, in other markets. In addition the model gives an explanation as to why domestic variables may need to be priced in different countries and vice versa - because the financial wealth portfolio can consist of investments in both domestic and foreign assets.

A single- and a two investor FOREX model will be considered - the no-arbitrage conditions for the single and two investor models, when a representative investor with Epstein Zin preferences in the two countries exists, will be derived. The empirical application in this chapter estimates a single investor FOREX model for a representative UK investor who invests part of his or her financial wealth portfolio in a domestic market portfolio, part in a domestic risk-free asset (in nominal terms) and part in a foreign risk-free asset (risk-free in foreign currency). We showed in the previous chapter that growth in narrow money as well as industrial production growth were significantly priced in the UK. Based on these results and our findings for the US stock market in chapter 3, we estimate an alternative joint FOREX and equity model pricing money- and industrial production growth in both the stock market and the FOREX market. Therefore we have two equilibrium asset pricing models to test whether the UK FOREX- and stock markets are integrated. We extend our discussion of the two asset pricing models to interpret them as the inter-temporal CAPM and the implications and cross return restrictions to be imposed when estimating the risk premium on several assets in the inter-temporal CAPM framework.

We will conclude the chapter by investigating whether the implied risk premium from various models can resolve the FOREX puzzle. It may be that these more general consumption-based models or the monetary model imply risk premia capable of resolving the puzzle - one reason that the models in chapter 4 could not resolve the puzzle could be the strong assumptions on the dynamics of the conditional covariance matrix.

The chapter is organised as follows. In section (5.2) we define asset market integration and relate the discussion to the no-arbitrage conditions in the stock- and FOREX markets considered in previous chapters, in section (5.3) we discuss the Epstein Zin model on FOREX and propose a test whether the stock- and FOREX markets are integrated based on this asset pricing model, we do this similarly for the monetary model in section (5.4). We summarise, interpret and attempt to justify the models to be estimated in section (5.5) and in section (5.6) we describe the data. Section (5.7) outlines the estimation method of risk premia when we model two asset returns jointly, in section (5.8) and (5.9) we report and discuss the results. Finally section (5.10) concludes.

5.2 Financial Market Integration: Review, Definition And a Test

5.2.1 Tests of Financial Market Integration - A Review

After the adoption of the single common currency in Europe it is of increasing interest whether financial markets are, and have become more, integrated. In the EU one argument for increased financial market integration is the elimination of currency risk. Also on a more general international basis it can be argued that financial markets have become more integrated in the most recent decade. In particular due to the increased possibility of, almost costless, financial transactions abroad and the speed of financial transactions with the IT inventions throughout the 1990s and potential "convergence" in the macro economy. We recall from chapter 2 that the ratio of the conditional macroeconomic variance of UK to US variables has declined towards 1 in the last three decades.

If we look at the stock market excess returns on the market portfolios considered in previous chapters from 1975 to 2002 the correlation is 0.6 - this even though the returns are denoted in different currencies. Recall figure (1.1), plotting a "crude" measure of the time-varying risk premium in the FOREX market and in the US and UK stock markets by computing 12 months moving averages, we found the correlation of the mean of the US and UK risk premium was 0.67 and between the UK stock market excess return and the FOREX excess return a correlation of 0.17 was obtained. However, using the general asset pricing models of chapter 3 we noticed that the correlation between the implied risk premia in the UK and US was low (a high of 0.18 with consumption based asset pricing models and 0.43 by the Sharpe-Lintner version of the CAPM) and in some cases negative !

In an interesting paper by Adam et all. [1] they report and propose various tests whether financial markets are integrated. They emphasise that correlations of ex-post returns is not a correct measure of financial market integration. The reason for this is that ex-post return can be decomposed into an expectation (the exante risk premium) and a residual (we emphasised this in the first section in the introductory chapter). If the residual of two asset returns are highly correlated if does not mean that the two financial markets are integrated - financial markets can be segmented even if a substantial part of returns is common shocks. Hence in order to test for integration of two financial markets one needs specify an asset pricing model and compare the estimated prices of risk in the two financial markets based on that asset pricing model. The point made by Adam et all. is that rejection of integration of two financial markets may simply reflect wrong choice of asset pricing model. This is a valid point but it is the type of approach that we will take in the current chapter using as general asset pricing models as possible. The advantage of our implementation of the test is that we obtain an estimate of the risk premium.

Relating this to our discussion above the high correlation in UK and US stock market returns, if the Epstein Zin model considered in chapter 2 is the "correct" model, simply reflects a high correlation of common shocks in the two markets and not high correlation between the UK and US risk premium.

Correlations are varying over time and we have shown that conditional correlations between returns and macroeconomic variables are varying considerably over time. It could be that return correlations are varying over time as well. Therefore it is of interest to see how the correlation of shocks to various returns vary over time - perfect correlation of shocks means that the financial markets are exposed to the same shocks. Obtaining an estimate of the time-varying correlation between returns can answer the question whether return shock correlations have increased in the recent decades. Capiello, Engle and Sheppard [31] compute conditional correlations of returns in the euro area and show that correlations between financial returns in the EURO area have increased following the adoption of the EURO in the beginning of 1999. The estimation framework we will discuss in the current chapter modelling two asset returns jointly can answer two questions - first whether the markets are integrated based on a particular asset pricing model and second whether the correlation of shocks to the returns has been varying over time.

As emphasised in Adam et all. much of the work on asset market integration has been devoted to international integration based on the CAPM or international CAPM (much in the framework of De Santis and Gerrard [40]). In this chapter we will take a different approach investigating whether the market return and macroeconomic variables are identically priced in two markets allowing for a time-varying risk premium [2]. There are very few studies, if any, considering asset market integration based on the pricing of macroeconomic variables.

In a recent paper Flood and Rose [61] uses a "SDF" approach. Recall the SDF model from chapter 1 the expected excess return on asset i is given by

$$\mathrm{E}_t(1 + \mathcal{R}_{i,t+1}) = \frac{1}{\mathrm{E}_t(\mathcal{M}_{i,t+1})}[1 - \mathrm{Cov}_t(\mathcal{M}_{i,t+1}, \mathcal{R}_{i,t+1})] \tag{5.1}$$

Flood and Rose assume that the conditional covariance can be modelled as a linear combination of a constant and three factors - the market return, the Small Minus Big stock (SMB) portfolio return and the High Minus Low (HML) book to market portfolio return. The latter two are the factors constructed by Fama and French [56], discussed in chapter 1. They rely on the assumption that

$$\mathrm{Cov}_t(\mathcal{M}_{t+1}, \mathcal{R}_{i,t+1}) = \beta_{0,i} + \beta_{i,m}\mathcal{R}_{market,t+1} + \beta_{i,hml}\mathcal{R}_{hml,t+1} + \beta_{i,smb}\mathcal{R}_{smb,t+1}, \tag{5.2}$$

[2] Our approach is slightly related to the work of Kaminsky and Peruga [76] although we use a considerably more general asset pricing model.

where i refers to the asset. Hence they can back out an estimate of the expected SDF for each asset and compare the implied risk-free rate, testing whether they are equivalent. Hence they assume that either the conditional covariance between the SDF and the asset returns is constant or the time-variation can be summarised by the three factors - we have shown in previous chapters that the conditional covariance may be varying significantly over time.

Their approach is closely related, but different, to our approach - instead of attempting to proxy for the conditional covariance between the financial return and the SDF we attempt to model the conditional covariance, based on well known asset pricing model, directly. Our approach based on the multivariate GARCH in mean model has the disadvantage that the number of assets we can model joint is limited whereas their approach can include several assets. However, we do no attempt to compare their approach with our approach.

5.2.2 An Alternative Test of Financial Market Integration With Observable Stochastic Discount Factors

When the stock market and FOREX markets are not integrated the SDF priced in the FOREX market and the SDF priced in the stock market will be different in the sense that $E_t(m_{fx,t+1}) \neq E_t(m_{s,t+}$
The no-arbitrage condition in the two markets, respectively, will be given by

$$\begin{aligned} E_t(i^e_{fx,t+1}) + \frac{1}{2}V_t(i^e_{fx,t+1}) &= -\ Cov_t(m_{fx,t+1}, i^e_{fx,t+1}) + Cov_t(i^e_{fx,t+1}, \pi_{t+1}) \\ E_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= -\ Cov_t(m_{s,t+1}, i^e_{s,t+1}) + Cov_t(i^e_{s,t+1}, \pi_{t+1}) \end{aligned}$$

The SDFs in the two markets will be given by

$$m_{fx,t+1} = -\ \mathbf{b}_{fx}^{\intercal}\mathbf{f}_{t+1} + \zeta_{fx,t+1} \qquad m_{s,t+1} = -\ \mathbf{b}_{s}^{\intercal}\mathbf{f}_{t+1} + \zeta_{s,t+1} \tag{5.3}$$

We distinguish between market integration and market completeness by the following two definitions:

Definition 5.1 Stock and FOREX Market Integration:

The UK stock and FOREX markets are integrated if $\mathbf{b}_{fx} = \mathbf{b}_s$. $\zeta_{fx,t+1}$ is an error term uncorrelated with the FOREX excess return and $\zeta_{s,t+1}$ is an error term uncorrelated with the stock market excess return on stock or portfolio s. Stated differently, integration between stock and FOREX markets require $\mathrm{E}_t(m_{fx,t+1}) = \mathrm{E}_t(m_{s,t+1})$.

Definition 5.2 Market Completeness:

Markets are complete when $\zeta_{s,t+1} = \zeta_{fx,t+1} \equiv 0$ and the Stochastic Discount Factor is unique, that is $\mathbf{b}_{fx} = \mathbf{b}_s$. This does not only hold for FOREX and Stock markets but any financial market. If markets are complete, $m_{fx,t+1} = m_{s,t+1} = m_{j,t+1}$, where j refers to any other financial market.

In other words, asset market integration is a necessary but not sufficient condition for complete markets. Market completeness is not a necessary condition for subsets of asset markets to be integrated but it is sufficient. It follows easily that it is practically impossible to test and reject market integration. The reason being that we do not know what is the Stochastic Discount Factor (or its expectation) and what factors determine the time-variation in the Pricing Kernel. What we can do is to pick a model for the SDF, test whether asset markets are integrated - if we reject a subset of financial markets to be integrated then we have to take care. It may be that markets are not integrated but it may also be that we have chosen the wrong model of the SDF ! Hence it is very difficult to reject that financial market integrated. This point is emphasised by Chen and Knez [33] - their paper also gives a much more detailed discussion of the notion of financial market integration.

Most importantly in connection with this chapter is to answer the question whether various asset pricing models imply the FOREX and stock market to be integrated. Let us for the moment assume that the logarithmic SDF includes a constant, then

$$m_{fx,t+1} = -b_{fx} - \mathbf{b}_{0,fx}^{\mathsf{T}}\mathbf{f}_{0,t+1} + \zeta_{fx,t+1} \qquad m_{s,t+1} = -b_s - \mathbf{b}_{0,s}^{\mathsf{T}}\mathbf{f}_{0,t+1} + \zeta_{s,t+1} \quad (5.4)$$

where 0 as a subscript indicates that a constant is no longer included in the vector of factors. $\mathbf{f}$ is a vector of factors including a constant and $\mathbf{f}_0$ is the same vector of factors excluding the constant. If the UK stock and FOREX markets are integrated then the no-arbitrage condition will be given by

$$\mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) = \mathbf{b}_0^T\mathrm{Cov}_t(\mathbf{f}_{0,t+1}, i^e_{fx,t+1}) + \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \quad (5.5)$$

$$\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{s,t+1}) = \mathbf{b}_0^T\mathrm{Cov}_t(\mathbf{f}_{0,t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(i^e_{s,t+1}, \pi^*_{t+1}) \quad (5.6)$$

In this respect when estimating the time-varying risk premium on the assets jointly one can test whether the parameter vector in the two equations are equal, that is whether $\mathbf{b}_{fx,0} = \mathbf{b}_{s,0}$. This will be the test we will perform in this chapter. It is clear that if this has to work as a test for financial market integration then it must be assumed that $b_{fx} = b_s$. Since we choose the factors to be priced from well-known asset pricing models this is not a strong assumption. The asset pricing models assume that the constants in the SDFs are the same. The test we perform is not suitable when the vector of factors is chosen in an "ad hoc" manner.

Another potential criticism of the test is that estimation of the conditional covariance adopting the approach outlined in chapter 1 we need many sample points - in this respect, the question we answer is whether financial markets are integrated over long periods of time and the test does not reveal whether financial markets are more integrated towards the end of the sample than in the beginning. However, we acknowledge this and conclude that potential sub sample integration could be an interesting topic for future research.

Using the relations stated in previous chapters the importance of testing for asset market integration becomes clearer. Recall that

$$i_{s,t+1} = -m_{s,t+1} + \pi_{t+1} + \epsilon_{1,t+1}$$

$$i_{fx,t+1} = m^*_{fx,t+1} - m_{fx,t+1} + \pi_{t+1} - \pi^*_{t+1} + \epsilon_{2,t+1}$$

Taking expectations to the above equations we see that our expectations of the foreign nominal

SDF depends on expectations of domestic stock returns and expectations of exchange rate appreciation. If markets are integrated then

$$\mathrm{E}_t(m^*_{fx,t+1} - \pi^*_{t+1}) = -\mathrm{E}_t(i_{s,t+1} - i_{fx,t+1}) \quad (5.7)$$

$$= \phi_{fx,t} - \phi_{s,t} - i^*_{f,t} \quad (5.8)$$

$$m^*_{fx,t+1} - \pi^*_{t+1} = -\, i_{s,t+1} + i_{fx,t+1} + \epsilon_{1,t+1} - \epsilon_{2,t+1} \quad (5.9)$$

Hence from our expectations of domestic returns and the movement in the exchange rate must be related to our expectation of the foreign nominal SDF. It is not the aim of the current chapter to go into detail with the above. Rather we will continue the discussion of equilibrium models and propose a test whether it is the case the stock- and FOREX markets are integrated.

5.3 Market Integration: Consumption-Based Asset Pricing Models

The first question to be addressed in this chapter is whether the UK FOREX market and stock market are integrated according to a more general version of the Epstein Zin model considered in chapter 2. It will be discussed in this section. In the previous chapter we referred to the US investor as the domestic investor and the UK investor as the foreign investor. Since in this chapter we estimate only the UK FOREX - and stock market risk premia, we refer to the UK investor as the domestic investor and the US investor as the foreign investor. Hence UK variables and conditional moments will not have a star as a superscript as in previous chapters.

5.3.1 Epstein Zin

We consider an extension of the Epstein Zin model to dictate which factors should have the same risk prices across the FOREX and stock markets. This asset pricing model tells us why it may be necessary to model FOREX and stocks jointly - moreover, from the Epstein Zin model it becomes clear that both financial and macroeconomic variables ought to be priced in the two markets.

We discuss the single and two investor Epstein Zin FOREX model and argue that estimation with a time-varying conditional covariance matrix would involve too many parameters with available sample sizes and this "justifies" our choice to focus on the single investor model.

If a representative agent with Generalised Isoelastic Preferences (GIS) exists, recall that the logarithmic SDF is given by

$$m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} - (1-\theta)(i_{w,t+1} - \pi_{t+1}), \tag{5.10}$$

where one has to make an assumption on the unobservable wealth portfolio. We will discuss it soon but we will always assume that $i_{s,t+1}$, denoting the return on a broad national equity index, is part of the wealth portfolio. We use the same notation as in chapter 2. Hence $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$, where γ is the coefficient of relative risk aversion and ψ is the elasticity of inter-temporal substitution. Combining this with the no-arbitrage condition, equation (5.5), yields

$$\begin{aligned} \mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{fx,t+1}) + \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \\ &+ (1-\theta)[\mathrm{Cov}_t(i^e_{fx,t+1}, i_{w,t+1}) - \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1})]. \\ &= \phi_{fx,t} \end{aligned} \tag{5.11}$$

The FOREX risk premium is given by the RHS of the above equation. The FOREX risk premium has three components, the covariance between FOREX return and consumption growth, the covariance between FOREX return and inflation and the covariance between FOREX return and the return on the wealth portfolio. The signs on the prices of risk are not unique - they can be either sign depending on the preference parameters of the representative agent. The single covariance term with inflation is also part of the risk premium and can be thought of as a correction term to the risk premium from working with nominal returns and pricing the nominal (instead of the real) wealth portfolio return. If, as mentioned in the previous chapter, the domestic investor invests part of his financial portfolio abroad then excess return on foreign equity would also appear in the no-arbitrage condition[3].

If we wish to model the time-varying risk premium on FOREX for the single investor we need to model it joint with the time-varying risk premium on the wealth portfolio - the reason for this

[3] However, we do not consider this possibility.

being that the conditional covariance between the residuals of the FOREX return and wealth portfolio return determines the risk premium in the FOREX market for the domestic investor ! According to the asset pricing model we will need to impose integration to obtain a correct measure of the residual.

We can reverse the above no-arbitrage condition for the foreign investor, using that the SDF of the foreign investor is given by

$$m^*_{t+1} = \theta^* \ln(\delta^*) - \frac{\theta^*}{\psi^*}\Delta c^*_{t+1} - (1-\theta^*)(i^*_{w,t+1} - \pi^*_{t+1}) \tag{5.12}$$

The equivalent no-arbitrage condition becomes

$$\begin{aligned} \mathrm{E}^*_t(i^{*e}_{fx,t+1}) + \frac{1}{2}\mathrm{V}^*_t(i^{*e}_{fx,t+1}) &= \frac{\theta^*}{\psi^*}\mathrm{Cov}^*_t(\Delta c^*_{t+1}, i^{*e}_{fx,t+1}) + \mathrm{Cov}^*_t(i^{*e}_{fx,t+1}, \pi^*_{t+1}) \\ &+ (1-\theta^*)[\mathrm{Cov}^*_t(i^{*e}_{fx,t+1}, i^e_{w,t+1}) - \mathrm{Cov}^*_t(i^{*e}_{fx,t+1}, \pi^*_{t+1})] \end{aligned} \tag{5.13}$$

We note that the foreign investor may have different preference parameters. Multiplying through by -1, and using $i^{*e}_{fx,t+1} = -i^e_{fx,t+1}$, yields

$$\begin{aligned} \mathrm{E}^*_t(i^e_{fx,t+1}) - \frac{1}{2}\mathrm{V}^*_t(i^e_{fx,t+1}) &= \frac{\theta^*}{\psi^*}\mathrm{Cov}^*_t(\Delta c^*_{t+1}, i^e_{fx,t+1}) + \mathrm{Cov}^*_t(i^e_{fx,t+1}, \pi^*_{t+1}) \\ &+ (1-\theta^*)[\mathrm{Cov}^*_t(i^e_{fx,t+1}, i^*_{w,t+1}) - \mathrm{Cov}^*_t(i^e_{fx,t+1}, \pi^*_{t+1})] \\ &= -\phi_{fx,t} \end{aligned} \tag{5.14}$$

The no-arbitrage conditions for the foreign and domestic investors can be combined under the assumption that the two investors have homogenous information. This yields the two investor Epstein Zin model

$$
\begin{aligned}
\mathrm{E}_t(i^e_{fx,t+1}) &= \frac{1}{2}\left[\frac{\theta^*}{\psi^*}\mathrm{Cov}_t(\Delta c^*_{t+1}, i^e_{fx,t+1}) + \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{fx,t+1})\right] \\
&+ \frac{1}{2}\left[(1-\theta^*)\mathrm{Cov}_t(i^e_{fx,t+1}, i^*_{w,t+1}) + \theta^*\mathrm{Cov}_t(i^e_{fx,t+1}, \pi^*_{t+1})\right] \\
&+ \frac{1}{2}\left[(1-\theta)\mathrm{Cov}_t(i^e_{fx,t+1}, i_{w,t+1}) + \theta\mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1})\right] \\
&= \frac{1}{2}\left[\phi_{fx,t} - \phi^*_{fx,t}\right] \qquad (5.15)
\end{aligned}
$$

The expected FOREX excess return, in domestic terms, depend on the average of conditional covariance between domestic FOREX excess return and consumption growth of the foreign and domestic investor, it depends as well on the average conditional covariance between FOREX excess return and domestic and foreign inflation and it depends similarly on the average covariance with domestic and foreign wealth portfolio return[4].

To estimate a two investor Epstein Zin FOREX model we will need at least seven variables in the multivariate model. The variables are $i^e_{fx,t+1}$, $i^e_{s,t+1}$, $i^{*e}_{s,t+1}$, π_{t+1}, π^*_{t+1}, Δc_{t+1} and Δc^*_{t+1}. First it is very difficult, if not impossible, to estimate such model using a multivariate GARCH-in-mean model without making strong and potentially unreasonable assumptions on the dynamics of the conditional covariance matrix. Second it is basically impossible to add any conditioning variables to obtain better estimates of conditional variances and covariances (and hence risk premia). Moreover, as discussed in the previous chapter, if $i_{s,t+1} \neq i_{w,t+1}$ then further variables may have to be included. Next, potential assumptions on $i_{w,t+1}$ will be discussed - the best we can hope for is to leave the conditioning set[5], $\mathcal{Z}_3$, empty if we want to consider the two investor model.

It is, however, feasible to estimate the single investor model using the multivariate GARCH-in-mean model since the single investor model will require only 4 variables such as FOREX excess return, stock market excess return, consumption growth and inflation. The no-arbitrage condition in the stock market, as we considered in chapter 2, will be given by

[4]Note that the Epstein Zin model does not say that perfectly correlated domestic and foreign consumption growth implies perfect risk-sharing. Hence the model tells us why it could be potentially wrong to conclude that foreign and domestic markets are not integrated because consumption is imperfectly correlated.

[5]Recall discussion of the set in chapter 1.

$$\begin{aligned} E_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= (1-\theta)[\mathrm{Cov}_t(i^e_{s,t+1}, i_{w,t+1}) - \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1})] \\ &+ \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \\ &= \phi_{s,t} \end{aligned} \quad (5.16)$$

We note that we will have to impose cross-equation restrictions on the FOREX and equity market excess return equations if we believe in a consumption-based model with complete markets or if we believe that the stock market and FOREX market are integrated - that the UK (for instance) FOREX and stock market are integrated, based on the Epstein Zin model, require that the prices of risk in the stock- and FOREX market are identical. We will test this in the current chapter together with various special cases of the Epstein Zin model. Acceptance of equal prices of risk means that the markets are integrated based on the particular asset pricing model but it is not sufficient to conclude that markets are complete !

5.3.2 The Financial Wealth Portfolio

In chapter 2 we assumed the wealth portfolio to be partly invested in a broad domestic stock market portfolio and partly in a domestic risk-free bond. We extend this in this chapter to allow the representative investor to invest in a foreign risk-free bond (risk-free in foreign currency). For the domestic investor this is not a risk-free asset since the investor faces currency risk. If the currency risk premium is positive the domestic investor will be compensated for the riskiness due to risks associated with exchange rate movements. Campbell, Viceira and White [30] argue that foreign currency is not necessarily a pure speculative asset - it can play an important role in the portfolios of long-term investors and a portfolio should not necessarily always be fully domestic. Return on the financial wealth portfolio for the domestic becomes a linear combination of several returns[6], that is

$$i_{w,t+1} = \omega_1 i_{f,t} + \omega_2(i^*_{f,t} + i_{fx,t+1}) + \omega_3 i_{s,t+1}$$

[6]Note warning on log return and simple return approximation from chapter 2.

One could assume that $\sum_{j=1}^{3} \omega_j = 1$ and that all the portfolio weights individually are in the interval between zero and one. If it had been possible to estimate a two investor model it could also have been of interest to consider the case where the domestic investor tracks a foreign equity market index with part of his portfolio and vice versa. However, it will be assumed that the only amount of the wealth portfolio not invested domestically is in a foreign risk-free asset. The portfolio weights are assumed constant though they may in practice be time-varying[7].

5.3.3 The No-Arbitrage Condition And Intuition

With our assumption of the composition of the financial wealth portfolio above we rewrite the no-arbitrage conditions, equation (5.11) and (5.16), as

$$\begin{aligned} \mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{fx,t+1}) + \theta \mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \\ &+ (1-\theta)[\omega_2 V_t(i^e_{fx,t+1}) + \omega_3 \mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1})] \\ \mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= \frac{\theta}{\psi}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \theta \mathrm{Cov}_t(\pi_{t+1}, i^e_{s,t+1}) \\ &+ (1-\theta)[\omega_2 \mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1}) + \omega_3 V_t(i^e_{s,t+1})] \end{aligned} \tag{5.17}$$

The Epstein Zin model with our assumption on the financial wealth portfolio implies that, both in the FOREX and stock market, the covariance between return and consumption growth determines risk premia - what is more interesting, since we use nominal return, is that the addition to the risk premium is a combination of the return-inflation covariance and the return-financial wealth portfolio return covariance. The relative importance of the two terms is determined solely by the preference parameters of the representative investor. If the coefficient of relative risk aversion is equal to the inverse of the elasticity of inter-temporal substitution then the wealth portfolio has no role but inflation has[8]. Hence the wealth portfolio is more important in the contribution to the risk premium when $\gamma > \frac{1}{2}\left\{1 + \frac{1}{\psi}\right\}$.

From the Epstein Zin model we can analyse when the variance of return on asset or portfolio

[7] If weights are time-varying it is not obvious how one could model and estimate the time varying portfolio weights. In many cases the portfolio weights would be some function of some lagged financial or macroeconomic variables used by the portfolio manager or representative investor (see for example Dahlquist and Harvey [39]).

[8] However, recall that we showed in chapter 2 that the covariance between inflation and stock market return is small in magnitude

i is important for determining its risk premium - the stock return variance is important for determining the stock market risk premium the higher is ω_3 and $(1-\theta)$ whereas the FOREX return variance is important to determine risk premia in the FOREX market the higher is ω_2 and $(1-\theta)$. If we assume there is a restriction that the sum of the portfolio weights should equal 1 then we note that there is a "trade-off" in their relative importance.

The aim of the current chapter is, however, not to interpret our result in terms of preference parameters though it is always interesting to note that an underlying equilibrium model exists justifying the choice for determining the factors priced in the two markets. Most likely, the representative agent models do not hold in practice but we cannot neglect the sound economic intuition.

Finally, since in previous chapters we have focused on the relation between the risk premium and the variance of returns, we note that the Epstein Zin model has implications on this relation across the assets in the wealth portfolio. If we have two risky returns in the wealth portfolio, denote them k and j, then the relation between their risk premium and their conditional variance is shown in equation (5.25) in the appendix.

5.4 Financial Market Integration: The Monetary Model and The ICAPM

The second question we aim to answer in this chapter is whether the equity and FOREX markets are integrated as implied by versions of the monetary model.

5.4.1 The Monetary Model

In the previous chapter we considered the monetary model of the exchange rate where UK narrow money growth and industrial production growth were found priced in the UK FOREX market. With our finding in chapter 3, that money and output seemed to have some role in the US stock market, it is of interest to see whether money and output are priced in the UK stock market and whether they are priced in the UK FOREX market with less restrictions on the dynamics in the conditional covariance matrix than considered in chapter 4. This also serves as a benchmark whether the simplifying estimation method from chapter 4 is capable of detecting potential significantly priced variables. Therefore we estimate another multivariate UK model with the four variables - these are $(i^e_{fx,t+1}, i^e_{s,t+1}, \Delta q_{t+1}, \Delta y_{t+1})$, where q is the logarithm of money and y is the logarithm of output. As mentioned in Lee [78] the monetary model can be seen as the power utility CCAPM where, in equilibrium, consumption is equal to output and money is equal to inflation.

5.4.2 The ICAPM

The inter-temporal CAPM (ICAPM) of Merton [90] does not impose the restriction that financial markets are integrated. Rather, the ICAPM tells us that the price of risk on the market portfolio are equal across assets - in addition to the common source of risk from the market portfolio, equally priced, each asset can have infinitely many sources of factors significantly priced. The ICAPM has recently been implemented on the UK exchange rate by Giurda and Tzavalis [65]. However, they assume rather restrictive that the correlation between returns are constant, an assumption that does not seem empirically justified (and will be shown is not valid for the UK FOREX and stock return) - as mentioned by Capiello, Engle and Sheppard [31] a common characteristic of conditional correlations between financial returns is that they increase sharply when markets go down and within the EURO area most return correlations are changing much in the past decade. The ICAPM implies a logarithmic SDF given by

$$m_{t+1} = -b_0 - b_1 r_{m,t+1} - \gamma^{\top} \mathbf{x}_{t+1}, \tag{5.18}$$

where $r_{m,t+1}$ is the real return on the market portfolio and $\mathbf{x}$ is a vector of variables that affect the average investor. As emphasised by Merton and reiterated strongly in Cochrane [37] these additional variables must affect the average investor - such variables are likely to be macroeconomic such as inflation, consumption, output etc. It is common to assume that the wealth portfolio is equal to the return on a broad stock market index - therefore we estimate two additional models each with the market return priced, one with inflation and consumption in addition and one with money growth and industrial production growth additionally. We estimate these models assuming the portfolio weights $\omega_1 = \omega_2 = 0$ (and $\omega_3 = 1$). Hence the model pricing inflation and consumption in addition is derived theoretically, as is desirable, whereas pricing money and output additionally has less theoretical justification.

5.5 Summary of Models To Be Estimated

In this section we describe and give the intuition of the models estimated in this chapter and describe how FOREX- and stock market integration affects the no-arbitrage condition in each market.

5.5.1 The Consumption-Based Asset Pricing Models

In the consumption-based models we have a vector of four variables when estimating the multivariate GARCH in mean model. This vector is given by $\mathbf{Y}_{t+1} = \{i^e_{fx,t+1}, i^e_{s,t+1}, \pi_{t+1}, \Delta c_{t+1}\}$, where c is the logarithm of the level of consumption and π is the first difference of the logarithm of the level of a retail price index.

Recalling the no-arbitrage condition on FOREX and equity in equation (5.17), we can estimate the following two equations by, for instance, a Quasi Maximum Likelihood estimator and recover, if desired, the preference parameters of the representative investor. The return equations to be estimated are the following

$$\begin{aligned}
\mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \alpha_{c,11}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{fx,t+1}) + \alpha_{c,12}V_t(i^e_{fx,t+1}) \\
&+ \alpha_{c,13}\mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1}) + (1+\alpha_{c,14})\mathrm{Cov}_t(i^e_{fx,t+1}, \pi_{t+1}) \\
\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= \alpha_{c,21}\mathrm{Cov}_t(\Delta c_{t+1}, i^e_{s,t+1}) + \alpha_{c,22}V_t(i^e_{s,t+1}) \\
&+ \alpha_{c,23}\mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1}) + (1+\alpha_{c,24})\mathrm{Cov}_t(i^e_{s,t+1}, \pi_{t+1})
\end{aligned} \tag{5.19}$$

Obtaining estimates of $\hat{\alpha}_{i,j}$, $i = 1,2$, $j = 1,..,4$ we can recover the parameters of the consumption-based models. While performing the estimations, one can impose some cross-equation restrictions. That is

- $\alpha_{c,11} = \alpha_{c,21}$, $\alpha_{c,13} = \alpha_{c,22}$, $\alpha_{c,12} = \alpha_{c,23}$, $\alpha_{14} = \alpha_{c,24}$

The above will, in general, constitute our test whether the stock market and FOREX market are integrated in the UK. Estimating the models both restricted or unrestricted we can use

Likelihood Ratio (LR) or other classical tests to test the restrictions. If we do not impose cross-equation restrictions on the FOREX and equity equations one also have the additional benefit that we can analyse the consumption-based model on equity and FOREX separately as general SDF models. In chapter 2 we discussed the derivation of the preference parameters and portfolio weights. In this chapter we do not focus much on the estimated parameters and interpreting estimates in terms of preference parameters. Since we just use the consumption-based model for deriving sources of risks to be priced we have left the derivation and discussion to section (5.11.2) in the appendix.

In terms of consumption-based models we estimate four models both assuming markets to be integrated and allow for markets not to be integrated. We discuss the models briefly and discuss their implications on market integration.

- Model 1 and Model 2

Model 1 is the standard inter-temporal CCAPM with a Power Utility function. In this model $\alpha_{c,k2} = \alpha_{c,k3} = \alpha_{c,k4} = 0$, where $k = 1,2$. Model 2 is similar to model 1 but we impose market integration - that is $\alpha_{c,11} = \alpha_{c,21}$. With the log-likelihood from the two models we can test whether markets are integrated according to the Power Utility model.

- Model 3 and Model 4

Model 3 and model 4 is based on our results in chapter 2 and 3 that inflation may be an additional source of risk to be priced (recall that according to our most general consumption-based models it should in fact be priced). In model 3 we impose the restriction $\alpha_{c,k2} = \alpha_{c,k3} = 0$ for $k = 1,2$ and model 4 imposes additionally the restriction that markets are integrated - $\alpha_{c,11} = \alpha_{c,21}$ and $\alpha_{c,14} = \alpha_{c,24}$.

- Model 5 and Model 6

Model 5 is our most general model pricing all four variables and model 6 imposes integration - that is model 6 imposes restriction $\alpha_{c,11} = \alpha_{c,21}$, $\alpha_{c,12} = \alpha_{c,23}$, $\alpha_{c,13} = \alpha_{c,22}$ and $\alpha_{c,14} = \alpha_{c,24}$.

- Model 7 and Model 8

In model 7 we assume that the representative investor does not invest part of his wealth in a foreign bond. Hence we impose restriction $\alpha_{c,12} = \alpha_{c,23} = 0$. Model 8 is the same as model 7 but with cross-equation restrictions imposed. We can interpret this model as the ICAPM where the return on wealth is equal to the return on a broad stock market index and two additional macroeconomic sources of risks are priced.

In total we have four special cases of the most general asset pricing model allowing us to test whether these four models implies that the UK stock and FOREX market are integrated.

5.5.2 The Monetary Models

In addition to the consumption-based asset pricing models we estimate four additional models to test whether the multivariate models including the macroeconomic variables, money growth and industrial production growth, implies that markets are integrated. The vector of dependent variables is given by $\mathbf{Y}_{t+1} = \{i^e_{fx,t+1}, i^e_{s,t+1}, \Delta q_{t+1}, \Delta y_{t+1}\}$ and the no-arbitrage condition by.

$$
\begin{aligned}
\mathrm{E}_t(i^e_{fx,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{fx,t+1}) &= \alpha_{m,11}\mathrm{Cov}_t(\Delta q_{t+1}, i^e_{fx,t+1}) + \alpha_{m,12}V_t(i^e_{fx,t+1}) \\
&+ \alpha_{m,13}\mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1}) + \alpha_{m,14}\mathrm{Cov}_t(i^e_{fx,t+1}, \Delta y_{t+1}) \\
\mathrm{E}_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= \alpha_{m,21}\mathrm{Cov}_t(\Delta q_{t+1}, i^e_{s,t+1}) + \alpha_{m,22}V_t(i^e_{s,t+1}) \\
&+ \alpha_{m,23}\mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1}) + \alpha_{m,24}\mathrm{Cov}_t(i^e_{s,t+1}, \Delta y_{t+1})
\end{aligned} \tag{5.20}
$$

- Model 9 and Model 10

First we estimate, model 9, the monetary model pricing money growth and industrial production growth unrestricted. This model imposes restrictions $\alpha_{m,12} = \alpha_{m,13} = \alpha_{m,22} = \alpha_{m,23} = 0$. Model 10 is the market integrated version imposing the cross-equation restrictions.

- Model 11 and Model 12

Model 11 is a version of the ICAPM assuming the wealth portfolio return to equal the return on the broad stock market index and two additional macroeconomic variables are priced. It is not theoretically justified but one can think of money and output to be proxies for inflation and consumption though it is likely not true. Finally model 12 is the same model imposing cross-equation restrictions.

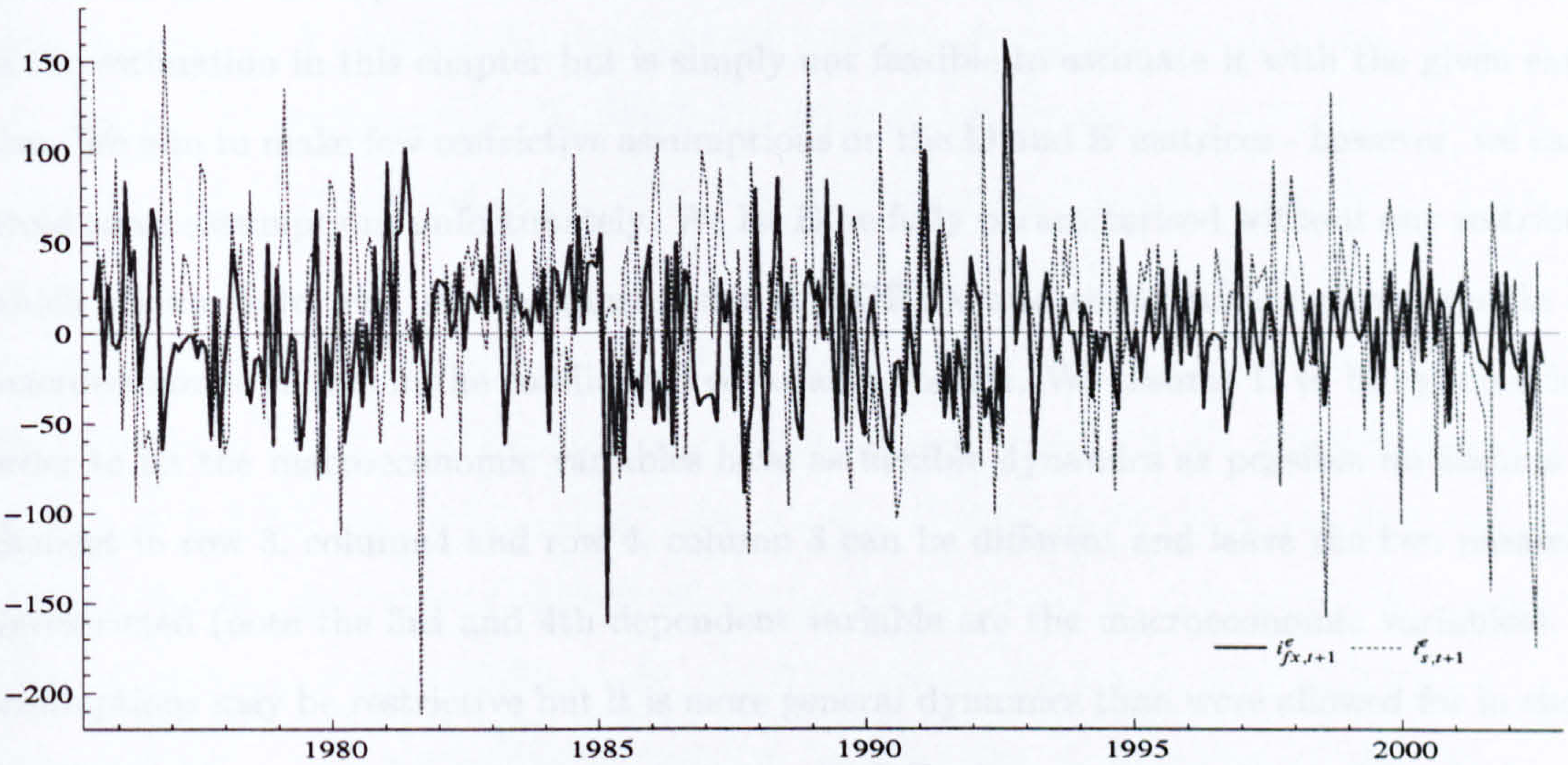
The two additional models estimated gives six tests whether UK FOREX and equity markets are integrated based on six different asset pricing model - five of them being special cases of the most general Epstein Zin model.

5.6 The Data

The data used in this chapter are all described in the previous chapters. Therefore we do not discuss them here but refer to the relevant tables and figures in other parts of the thesis. The descriptive statistics can be found in tables (2.14) and (2.15). Plots of the macroeconomic variables can be found in figure (2.10), (2.11), (2.12) and (4.2).

Since we want to model the FOREX and stock market risk premium joint in the multivariate models we finish the data description part by looking at a time-series plot of the two excess return series in figure (5.1).

Figure 5.1: UK FOREX And Stock Market Excess Return



UK Logarithmic excess return on FOREX and logarithmic excess return in the UK stock market. We have replaced October 1987 with its sample mean in the stock excess return equation. Data are annualised.

From the figure we note that FOREX excess return is less variable than UK stock market log excess return. A clear pattern between the two series above is difficult to find but if we look at the moving average plot of the two series in figure (1.1) and the moving average series of

US stock market log excess return in the introductory chapter there seems to be some evidence that the moving average of the series is correlated, to some extent. There may be a connection between the FOREX risk premium and the stock market risk premium !

5.7 The Estimation Method

This section describes the estimation method used of the joint FOREX and equity Epstein Zin or Monetary model. It can be applied to modelling the risk premium on any two assets jointly.

5.7.1 Estimation of the Epstein Zin FOREX Model

We estimate the joint FOREX and equity model using the multivariate GARCH-in-mean model as described in chapter 1. We specify the conditional covariance dynamics as the BEKK model, with the conditional covariance matrix

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}^{\intercal} + \mathbf{D}(\mathbf{H}_t - \mathbf{C}\mathbf{C}^{\intercal})\mathbf{D}^{\intercal} + \mathbf{E}\,(\epsilon_t \epsilon_t^{\intercal} - \mathbf{C}\mathbf{C}^{\intercal})\mathbf{E}^{\intercal}, \tag{5.21}$$

We do not allow for asymmetries in the conditional covariance matrix. That may be a criticism of the estimation in this chapter but is simply not feasible to estimate it with the given sample size. We aim to make few restrictive assumptions on the **D** and **E** matrices - however, we cannot avoid some assumptions unfortunately. We let **E** be fully parameterised without any restrictions which allows us to look at the transmission of FOREX and stock market return shocks with macroeconomic shocks in the conditional covariance matrix. We assume **D** to be symmetric - in order to let the macroeconomic variables have as flexible dynamics as possible we assume that element in row 3, column4 and row 4, column 3 can be different and leave the two parameters unrestricted (note the 3rd and 4th dependent variable are the macroeconomic variables). Our assumptions may be restrictive but it is more general dynamics than were allowed for in chapter 2 and with more parameters to estimate it is very difficult to estimate the models. As has been assumed throughout the thesis, we assume **C** to be lower triangular.

The no-arbitrage condition for the mean equation is given by

$$\mathbf{Y}_{t+1} = \mathbf{A} + \mathbf{B}_1\mathbf{Y}_t + \sum_{j=1}^{2}\mathbf{\Phi}_j\mathbf{H}_{[1:4,j],t+1} + \Theta_{1987:10}\Upsilon_{1987:10,t+1} + \epsilon_{t+1}, \tag{5.22}$$

$\mathbf{Y}$ is (4×1) vector of dependent variables with the first two variables being equity and FOREX log excess return respectively and the third and fourth variables being either inflation and consumption growth or money- and industrial production growth, $\mathbf{B}_1, \mathbf{\Phi}_j$ are (4×4) matrices and $\mathbf{H}_{[1:4,j]}$ refer to column j in the conditional covariance matrix and is of dimension (4×1). $\epsilon_{t+1} = \mathbf{H}_{t+1}^{\frac{1}{2}}\mathbf{u}_{t+1}$, where $\mathbf{u}$ is a vector of independent and identical multivariate t-distributed residuals with mean zero and covariance matrix equal to the identity matrix. We use the t-distribution, as we have done previously, to allow for eventual excess kurtosis in the conditional distribution of the variables. $\Upsilon_{1987:10,t+1}$ is an indicator function taking the value of 1 in October 1987 in the stock market excess return equation to account for the stock crash and zero otherwise - the indicator variable is not included in the FOREX excess return equation since there seem to be no significant abnormal increase or decrease in the FOREX excess return series in that particular month. Hence $\Theta_{1987:10}$ is a (4×1) parameter vector with an unrestricted parameter in the stock excess return equation and zeros elsewhere.

The first row in $\mathbf{\Phi}_1$ fulfills the no-arbitrage condition for equity and all other elements in it are zeros. The second row in $\mathbf{\Phi}_2$ fulfills the no-arbitrage conditions on FOREX and all other elements in this matrix are zeros. The first two elements of $\mathbf{A}$ are restricted to be zero and so are the first two rows in $\mathbf{B}$. All other elements in $\mathbf{A}$ and $\mathbf{B}$ are left unrestricted. We use a vector auto regression of order 1. To summarise - the vector of dependent variables is, for all estimations, $\mathbf{Y}_{t+1}^{CBM} = \{i_{fx,t+1}^e, i_{s,t+1}^e, \pi_{t+1}, \Delta c_{t+1}\}$ when considering the consumption-based models and $\mathbf{Y}_{t+1}^{MM} = \{i_{fx,t+1}^e, i_{s,t+1}^e, \Delta q_{t+1}, \Delta y_{t+1}\}$ when considering the Monetary models. The log-likelihood function (t-distribution), ℓ_{td}, can be found in equation (1.54) in chapter 1. With cross-equation restrictions on $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ we can test whether stock and FOREX market are integrated, in the sense defined in section (5.2.2), using the Likelihood Ratio test.

Finally we discuss a potential problem with the BEKK specification when several returns and in mean effects. As we showed in chapter 3, for the US stock market, various models of the risk premium implied more or less the same risk premium - this increases the possibility of the estimation to get stuck in a potential local maximum. This problem, as can be imagined, is even greater when two asset returns in the same model - therefore care is needed when estimating these and effort will need to be taken to insure that a global maximum of the likelihood function is reached.

5.8 Results

5.8.1 Consumption-Based Models

In table (5.1) and (5.2) we tabulate the estimates of the parameters in the consumption-based models, model 1-8. Each of the estimated models has two columns. The first reports the estimate of the coefficient on the conditional covariance reported in the first column of the table in the stock market equation and the other in the FOREX excess return equation. Further description of the reported statistics we recall from in chapter 2 in the results section.

First we see that both the conditional covariance between consumption growth and the two returns is significantly positive. Whereas the implied coefficient of relative risk aversion is 251 in the stock market it is 132 in the FOREX market and considerably more precisely estimated in the latter[9]. When imposing asset market integration the estimated coefficient is 160[10]. Judging from a simple Likelihood Ratio test we cannot reject that the UK FOREX and stock markets are integrated based on this model. Next, in model 3, pricing also inflation, the estimate on the consumption covariance increases - whereas inflation is not significant in the stock market it is significant in the FOREX market. The implied coefficient of relative risk aversion is 474.63 in the stock market and 68.08 in the FOREX market. In the stock market it confirms our results in chapter 2 for the UK that allowing the representative investor to invest in a risk-free domestic bond implies that he (or she) is much more risk averse whereas in the FOREX market the conclusion is opposite since the estimated coefficient of relative risk aversion is lower in the FOREX market. Finally imposing the cross-equation restrictions we note first that we cannot reject that the markets are integrated based on this model and second that the implied estimate of the coefficient of relative risk-aversion is negative. It is curious, however, that inflation is not significant in the stock return equation in model 3 since we found it strongly significant in

[9]We recall from Lewis [83] that most estimates of the coefficient of relative risk aversion are very imprecisely estimated.

[10]This results are consistent with the findings of Mark and Wu [87] and Kaminsky and Peruga [76] that the estimate is implausibly large. However the estimate is lower than that estimated in the stock market and it is interesting that our estimates are highly significant.

chapter 2. One possible explanation could be due to allowing for more general dynamics in the conditional covariance matrix or maybe, more likely, due to the fact that we no longer have industrial production in the model[11].

In model 5-8 we consider additional pricing of financial risks. First in the equity market we confirm our results from chapter 2 that financial variables are not priced in the UK stock market whereas in the FOREX market the FOREX return is significantly priced using a 90% critical value. The market return is neither priced in the stock nor the FOREX market. When imposing market integration FOREX return is found significantly priced (90% critical value) and we cannot reject the null hypothesis that the two markets are integrated. It is curious that pricing many variables in the stock and FOREX market many of the variables in the equity equation looses its significance though the relative variation of the equity risk premium relative to the equity return is pretty high. One reason could be that during the estimation the FOREX risk premium is relatively better at explaining the actual data of FOREX excess return. Similar findings were found in the general model in chapter 2. The four variables that we consider priced yields a FOREX risk premium that explains a higher proportion of the actual data than the equity premium - most variable in model 7.

5.8.2 The Monetary Model

Next, the estimate of the monetary models 9-12. In model 9 and 10 we see that output is borderline significant in the stock market and FOREX market but whereas money is borderline significant in the FOREX market it is not significant in the equity market. However, when imposing cross-equation restrictions they both become significant. Though the implied equity premium does not fit the data well in terms of its mean we cannot reject market integration based on the monetary model and it seems that our estimation method proposed in chapter 4 was capable of detecting the two variables significantly priced. Next we look at the models pricing also the market return - whereas the market return is not priced in the stock market it is significant in the FOREX equation (though borderline) while money and output keep their significance. In this model we cannot reject market integration either. One reason for the market return becoming significant though it was not significant in the consumption-based model could

[11]It could be modelling the joint distribution of the variables with industrial production growth would have implied a different estimate of the conditional covariance between inflation and stock market return.

be that either it is significant because we do not have consumption and inflation in the model or because money and industrial production affects (and gives a more precise estimate) the conditional return variance.

It is obvious that to give a better answer to such a question one would have to model all variables jointly but it is simply not feasible with the length of our sample. As will become evident shortly money shocks and output shocks certainly affects the conditional variance of the two returns.

5.8.3 Monetary or Consumption-Based Models ?

Ideally, as just mentioned, one should estimate the whole joint distribution of all the variables to answer this question. Most likely money has some similarities with inflation and industrial production growth with consumption. In chapter 2 we recall that industrial production was not significantly priced when pricing consumption as well which may suggest, at least for the equity market, that the consumption-based model is better of capturing time-varying risk in the UK stock market. We will shortly analyse the estimated models plotting the implied risk premia which may help answer the question. The important message of the above discussion is that macroeconomic variables seem to be priced in the UK stock and FOREX market and ALL asset pricing models imply that the two markets are integrated in the period 1975-2002.

Table 5.1: Estimate of UK FOREX And Equity Consumption-Based Models 1-4

UK	Model 1		Model 2		Model 3		Model 4	
	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$
$V_t(i^e_{s,t+1})$								
$Cov_t(i^e_{fx,t+1}, i^e_{s,t+1})$								
$V_t(i^e_{fx,t+1})$								
$Cov_t(i^e_{s,t+1}, \Delta c_{t+1})$	**251.6495** (2.85)		**160.6498** (4.04)		**476.1105** (2.00)		**452.1144** (3.30)	
$Cov_t(i^e_{fx,t+1}, \Delta c_{t+1})$		**132.4104** (3.49)		**160.6498** (4.04)		**603.1029** (3.07)		**452.1144** (3.30)
$Cov_t(i^e_{s,t+1}, \pi_{t+1})$					1.4839 (0.01)		**495.0191** (2.98)	
$Cov_t(i^e_{fx,t+1}, \pi_{t+1})$						**535.0259** (2.76)		**495.0191** (2.98)
$\Upsilon_{1987:10,t+1}$	**−0.3056** (3.58)		**−0.3065** (3.22)		**−0.2966** (4.40)		**−0.2009** (3.71)	
υ	**11.1921** (3.39)		**11.3847** (3.38)		**8.7206** (4.37)		**9.5461** (4.17)	
Log Likelihood	3710.9784		[3709.7779]		3713.7764		[3710.7236]	
$\|\lambda_{max}\|$	0.9851		0.9847		0.9880		0.9877	
Mean Residual (annualised)	0.8819	0.4675	2.5797	−0.0079	−1.8170	−0.0782	−0.4047	0.2109
Annualised average risk premium	7.2553	−0.3979	5.5653	0.0751	9.8719	0.1087	8.4532	−0.1797
$Var(\phi_t)/Var(i^e_{t+1} + \frac{1}{2}V_t(i^e_{t+1}) - \widehat{\theta}\Psi_{1987:10,t+1})$	0.0348	0.0551	0.0159	0.0767	0.0446	0.1100	0.0222	0.0816

Estimates of the UK FOREX and equity consumption-based asset pricing models 1-4. Emphasised parameters significant using 95 % critical value. A box around the log likelihood indicates that we cannot reject the null of market integration using a 99% critical value. Absolute t-statistics in parenthesis.

Table 5.2: Estimate of UK FOREX And Equity Consumption-Based Models 5-8

UK	Model 5		Model 6		Model 7		Model 8	
	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$
$V_t(i^e_{s,t+1})$	3.0415 (0.83)		1.3151 (0.32)		1.3883 (0.43)		2.9947 (0.89)	
$Cov_t(i^e_{fx,t+1}, i^e_{s,t+1})$	−17.9715 (0.67)	−15.5511 (0.71)	14.0322 (1.74)	1.3151 (0.32)		8.5834 (0.55)		2.9947 (0.89)
$V_t(i^e_{fx,t+1})$		18.7948 (1.77)		14.0322 (1.74)				
$Cov_t(i^e_{s,t+1}, \Delta c_{t+1})$	269.1705 (1.07)		**517.2810 (3.17)**		413.6846 (1.57)		**392.6679 (3.35)**	
$Cov_t(i^e_{fx,t+1}, \Delta c_{t+1})$		**587.2480 (3.05)**		**517.2810 (3.17)**		**578.6447 (3.06)**		**392.6679 (3.35)**
$Cov_t(i^e_{s,t+1}, \pi_{t+1})$	−203.6612 (0.71)		**537.1379 (2.79)**		−64.8129 (0.24)		**436.5229 (2.81)**	
$Cov_t(i^e_{fx,t+1}, \pi_{t+1})$		**551.4411 (2.76)**		**537.1379 (2.79)**		**506.4969 (2.73)**		**436.5229 (2.81)**
$\Upsilon_{1987:10,t+1}$	**−0.2926 (4.28)**		**−0.3012 (3.76)**		**−0.2970 (4.50)**		**−0.2995 (3.57)**	
υ	**8.1578 (4.64)**		**9.2174 (4.35)**		**8.7087 (4.27)**		**9.7026 (4.05)**	
Log Likelihood	3716.7405		3713.1250		3713.9141		3711.0624	
$\|\lambda_{max}\|$	0.9871		0.9879		0.9876		0.9877	
Mean Residual (annualised)	−2.3567	−0.6242	−1.1150	−0.4964	−1.9945	−0.2696	−0.7427	0.0780
Annualised average risk premium	10.4380	0.6441	9.1696	0.5104	10.0477	0.3007	8.0452	−0.0334
$Var(\phi_t)/Var(i^e_{t+1} + \frac{1}{2}V_t(i^e_{t+1}) - \hat{\theta}\Psi_{1987:10,t+1})$	0.0440	0.1152	0.0208	0.0836	0.0408	0.1126	0.0214	0.0795

Estimates of the UK FOREX and equity consumption-based asset pricing models 5-8. Emphasised parameters significant using 95 % critical value. A box around the log likelihood indicates that we cannot reject the null of market integration using a 99% critical value. Absolute t-statistics in parenthesis.

5.8.4 Other Estimated Parameters

In table (5.5), (5.6) and (5.7) in the appendix we tabulate other estimated parameters in the mean equation of the macroeconomic variables and parameters in the conditional covariance matrix.

First, in the consumption models, we note that lagged excess return is always borderline significant in the consumption growth equation - following intuition the sign is positive. Second in the monetary models we see that UK money growth lagged is borderline significant in the industrial production growth equation. It is interesting to note that in all models estimated in this thesis involving consumption growth, some lagged financial variables are always found capable of predicting changes in consumption and hence suggests that these are not independent and identically distributed variables as assumed in some asset pricing models.

On the estimates of the parameters in the conditional covariance matrix we conclude as in previous chapters that the significance of the parameters depend much on the assumed variables priced in return equations. As an example note that, from model 1 and 2, pricing only consumption growth, we conclude that past consumption variance increases the stock market and FOREX return variances significantly as well as the lagged return variance increases predicts

Table 5.3: Estimates of The Monetary Model

UK	Model 9		Model 10		Model 11		Model 12	
	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$	$i^e_{s,t+1}$	$i^e_{fx,t+1}$
$V_t(i^e_{s,t+1})$					0.1862 (0.04)		**3.9823** (2.64)	
$Cov_t(i^e_{fx,t+1}, i^e_{s,t+1})$						**26.6046** (2.03)		**3.9823** (2.64)
$V_t(i^e_{fx,t+1})$								
$Cov_t(i^e_{s,t+1}, \Delta q_{t+1})$	191.1337 (0.69)		**−123.5915** (1.97)		226.6220 (0.66)		−109.0796 (1.86)	
$Cov_t(i^e_{fx,t+1}, \Delta q_{t+1})$		−120.7616 (1.79)		**−123.5915** (1.97)		**−219.3749** (2.16)		−109.0796 (1.86)
$Cov_t(i^e_{s,t+1}, y_{t+1})$	262.1349 (1.55)		**84.2516** (2.59)		326.9014 (1.42)		55.9351 (1.81)	
$Cov_t(i^e_{fx,t+1}, y_{t+1})$		51.3717 (1.64)		**84.2516** (2.59)		**84.4601** (2.02)		55.9351 (1.81)
$\Upsilon_{1987:10,t+1}$	**−0.3023** (2.02)		**−0.2920** (2.30)		**−0.3132** (1.88)		**−0.2979** (2.29)	
v	23.7587 (1.78)		29.4841 (1.54)		26.7504 (1.69)		26.0289 (1.68)	
Log Likelihood	3543.3377		[3540.6012]		3546.8137		[3543.7512]	
$\|\lambda_{max}\|$	0.9876		0.9862		0.9874		0.9880	
Mean Residual (annualised)	0.6171	1.1004	4.4663	0.7113	−1.0747	−0.5586	−0.3159	0.2014
Annualised average risk premium	7.4364	−1.0535	3.5830	−0.6638	9.1318	0.5905	8.3555	−0.1458
$Var(\phi_t)/Var(i^e_{t+1} + \frac{1}{2}V_t(i^e_{t+1}) - \hat{\theta}\Psi_{1987:10,t+1})$	0.0232	0.0288	0.0260	0.0377	0.0213	0.0547	0.0176	0.0251

Estimates of the UK FOREX and equity consumption-based asset pricing models 5-8. Emphasised parameters significant using 95 % critical value. A box around the log likelihood indicates that we cannot reject the null of market integration using a 99% critical value. Absolute t-statistics in parenthesis.

significant increases in the consumption variance. However, when pricing more variables this pattern disappears - instead it seems that the symmetric (due to our assumption) pattern is between inflation variance and return variances. It is interesting to note that all consumption based models imply that lagged FOREX return variance and lagged stock return variances increases the variance of each-other in the following period - this is not the case in the monetary models ! Hence consumption-based models imply that the variances of FOREX return and stock return are more highly correlated (part of the explanation may obviously due to our assumed symmetric GARCH matrix). Finally we note that there may have been a great benefit from our assumption that past variances of inflation and consumption do not have symmetric impact on the conditional covariance matrix - similarly findings for money growth and industrial production growth.

We note that squared shocks to consumption, inflation and FOREX excess return all increases the conditional variance of the FOREX return variance (risk premia under the assumption that the FOREX return variance is the sum of foreign and domestic risk premia) - this being independent of estimated model. Further squared shocks to all variables increases the variance of inflation in the following period. This is the same for consumption growth. Much of this suggest that there is no business cycle (as measured by lagged squared shocks) in the variance of equity returns but there is in the variance of FOREX return. In addition return shocks increases the conditional variance of inflation and consumption growth.

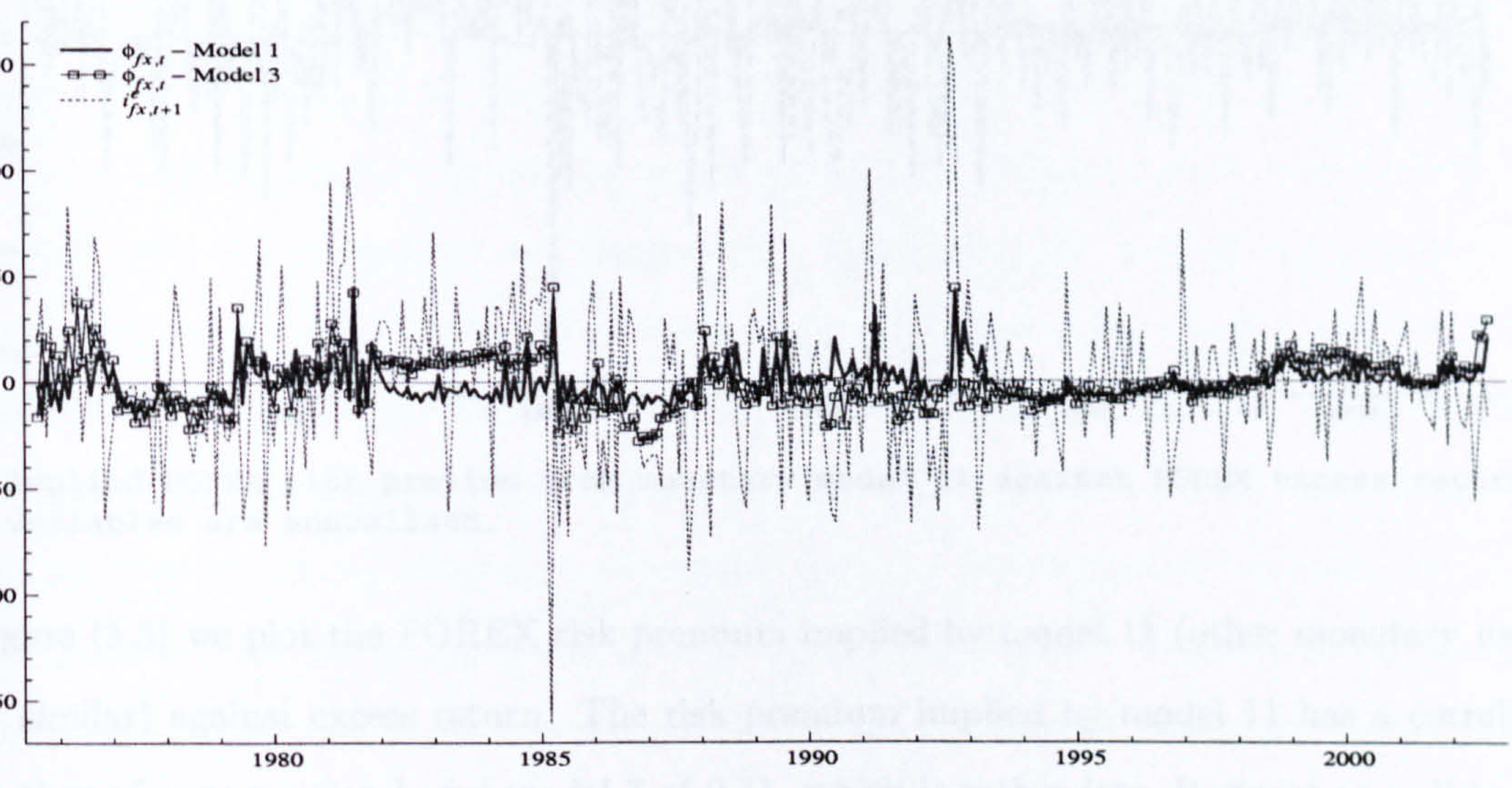
In the monetary models a different picture emerges. First there is much business cycle variability in the variance of stock return variance (as measured by lagged squared shocks to money growth and industrial production growth) and this is the case also for the conditional variance of FOREX excess return. There seem, as well, to be evidence that squared shocks to stock returns increase the variance of industrial production growth. In general there is less correlation between past variances and covariances and current variances and covariances in the Monetary model.

Looking at the parameter estimates in the conditional covariance matrix we note that more parameters seem to be significant in the consumption-based models which could suggest, potentially, that modelling the returns joint with consumption growth and inflation is "better" than modelling the returns joint with money growth and industrial production growth.

5.8.5 The FOREX Risk Premia

Next, we look at the implied FOREX risk premia from the various models. In figure (5.2) we plot the FOREX risk premium from model 1 and 3 together with actual FOREX excess return.

Figure 5.2: The FOREX Risk Premium Implied By Consumption-Based Asset Pricing Models

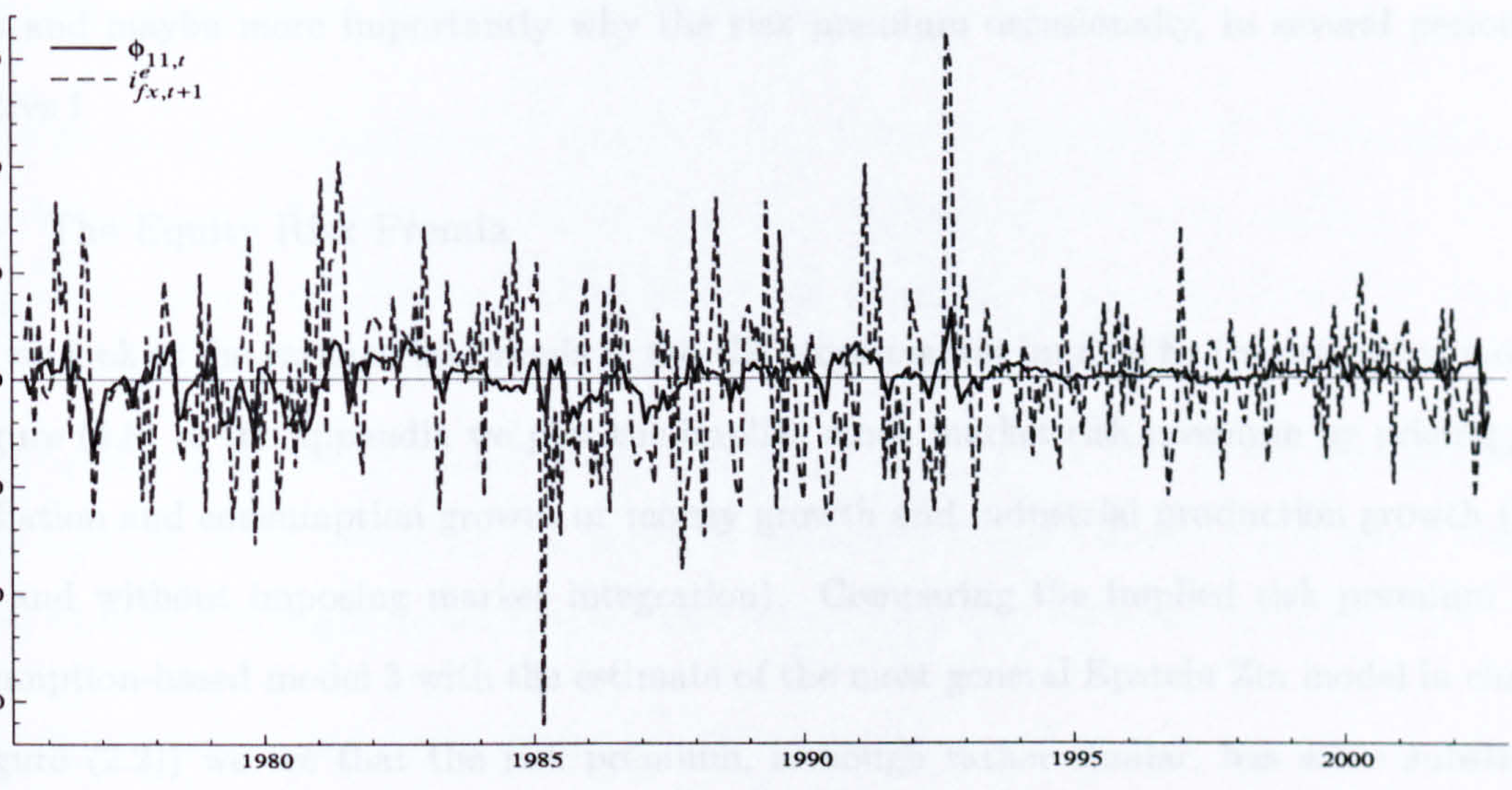


The implied FOREX risk premium from consumption-based model 1 and model 3 against FOREX excess return. All variables are annualised.

First we note that the risk premium is pretty variable. It is positive during a great part of the sample but it is also negative for substantial periods of time. It has been fluctuating mainly in the beginning of the sample and has tended to become much less variable and stable in the end of the sample.

In the previous chapter we concluded that if the variance of FOREX return was equal to the sum of foreign and domestic FOREX risk premia then risk premia must be quite smooth and never exceed 10 % and FOREX risk premia must have become much lower in the last part of the sample. The estimates of the risk premium from the consumption-based model are consistent with a declining conditional variance but risk premia in the early part of the sample are too variable and perhaps more damaging - risk premia becomes negative in some periods. Implied risk premia are low and quite smooth from 1998 onwards consistent with a falling conditional variance of FOREX return (provided that FOREX risk premia are also low) ! We do not plot the risk premia from other consumption-based models but they are very similar[12].

Figure 5.3: The FOREX Risk Premium Implied By The Monetary Model



The implied FOREX risk premium from monetary model 11 against FOREX excess return. All variables are annualised.

In figure (5.3) we plot the FOREX risk premium implied by model 11 (other monetary models very similar) against excess return. The risk premium implied by model 11 has a correlation with that of consumption-based model 7 of 0.11, which is rather low. It seems as well to have stabilised towards the end of the sample, maybe even more than implied by the consumption-based models. The risk premium tends to vary most in the early part of the sample and

[12]Available upon request.

occasionally the risk premium becomes more negative than implied by the consumption-based model, again eventually suggesting that the consumption-based multivariate model fits the data better.

It seems that the FOREX models implies risk premia that are far too volatile in the 1970s and 1980s and have declined very much over the sample period considered. If the conditional variance of FOREX return reflects the sum of foreign and domestic risk premia then the implied risk premia by our models are far too volatile in the early part of the sample but the smoothness of risk premia towards the end of the sample seem to be consistent with the strong decline in the conditional variance of FOREX return. Much of the reason is obviously the decline in the variance of macroeconomic variables. If it is the case that the conditional variance of FOREX return reflect foreign and domestic risk premia then the decline in the variance is consistent with a decline of macroeconomic variability and hence that macroeconomic variables are priced in the FOREX market. We fail to answer why the risk premia are too volatile in the 1970s and 1980s and maybe more importantly why the risk premium occasionally, in several periods, is negative !

5.8.6 The Equity Risk Premia

Next we look at the various risk premia in the UK stock market implied by the estimated models. In figure (5.8) in the appendix we plot the implied stock market risk premium by pricing pairs of inflation and consumption growth or money growth and industrial production growth (both with and without imposing market integration). Comparing the implied risk premium from consumption-based model 3 with the estimate of the most general Epstein Zin model in chapter 2 (figure (2.2)) we see that the risk premium, although rather similar, has some substantial differences. Most likely there are two reasons for this. First we have added an additional return in the multivariate model which is more variable than the macroeconomic variables and second we do not have industrial production in the model (simply not feasible to estimate the model with 5 variables). It may well be that the interaction in the conditional covariance matrix between industrial production, stock return, consumption growth and inflation is important - omission of industrial production growth may be the reason why we find inflation to be insignificantly priced in the stock market in these equity-FOREX models. This re-emphasise the discussion in the introductory chapter that, in addition to the returns on which we model the risk premium and the variables proxying for the SDF, it could be important to add an additional set of variables to obtain a more precise estimate of the conditional covariance matrix of the variables in the baseline model.

The estimate of the Monetary models implies a risk premium that looks different from that of the consumption- based model but similarly is most variable in the beginning of the sample (with higher macroeconomic volatility) - it is interesting to note, however, that imposing integration implies a risk premium that is more or less always positive whereas the model that does not impose integration implies a risk premium that is substantially negative at certain points.

The estimate of the FOREX and stock market risk premia in the UK tend to have one thing in common. The implied variability of the risk premia has fallen in the most recent decade mainly due to a fall in macroeconomic volatility. This is probably the reason that we fail to reject that the markets are integrated based on any asset pricing model pricing macroeconomic variables.

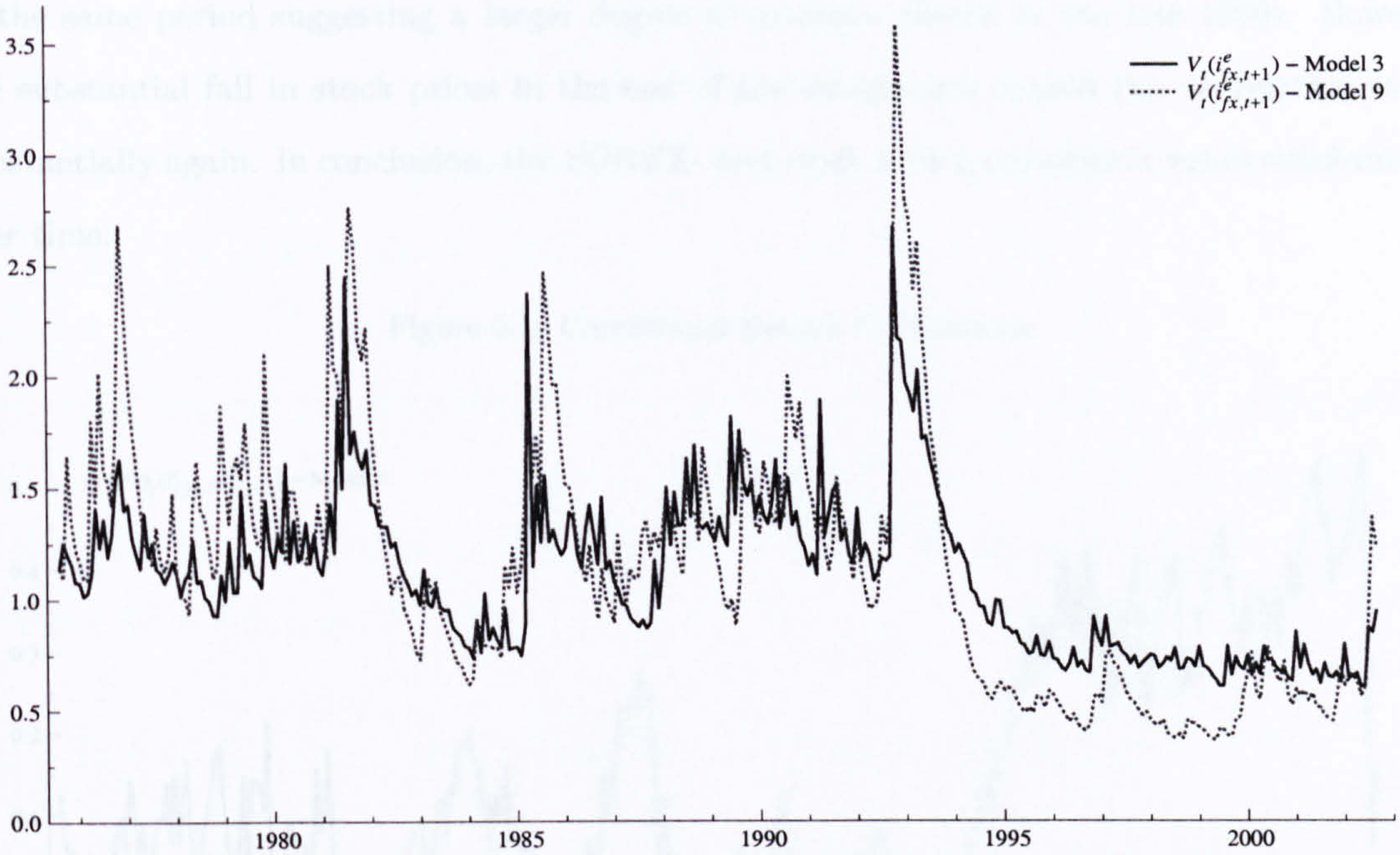
5.8.7 The Conditional Variance of FOREX Return

Finally, to re-emphasise our point of chapter 3, that different univariate and multivariate models imply different conditional return variances, we plot the implied conditional variance of FOREX excess return from a consumption-based model and a monetary model in figure (5.4). First we note that the series are rather different at certain points and second we note that the volatility of the exchange rate is considerably different than in the univariate context in the previous chapter (recall figure (4.5)) at certain points. Hence univariate and multivariate models give a different answer as to the degree of variability in sum of the foreign and domestic risk premium !

5.8.8 The Jensen- and Nominal Return Corrections.

In the chapters on FOREX, as well as in the equity chapters, we have emphasised the Jensen correction and the correction to the risk premium due to using nominal return. We showed in the UK and US stock markets that the magnitude of the Jensen correction was not neglible whereas the correction due to working with nominal returns was almost neglible. In figure (5.7) in the appendix we plot the same corrections for UK FOREX excess return and a similar picture arises. The Jensen correction is significant and has significant time-variation (it is the average foreign and domestic risk premium) - a high of 1.8% in 1993 and then decreasing ever since to a

Figure 5.4: The Conditional Variance of FOREX Return



`The conditional Variance of FOREX and stock market return. The conditional variances are from annualised data.`

level around 0.3%. The correction because returns are nominal has most variation in the early part of the sample but has similarly become neglible after 1993. The lowest value of −0.28% in 1985 and a high of 0.12% in 1993. Hence, it does not matter whether estimating stock market and FOREX risk premia using nominal returns rather than real returns[13].

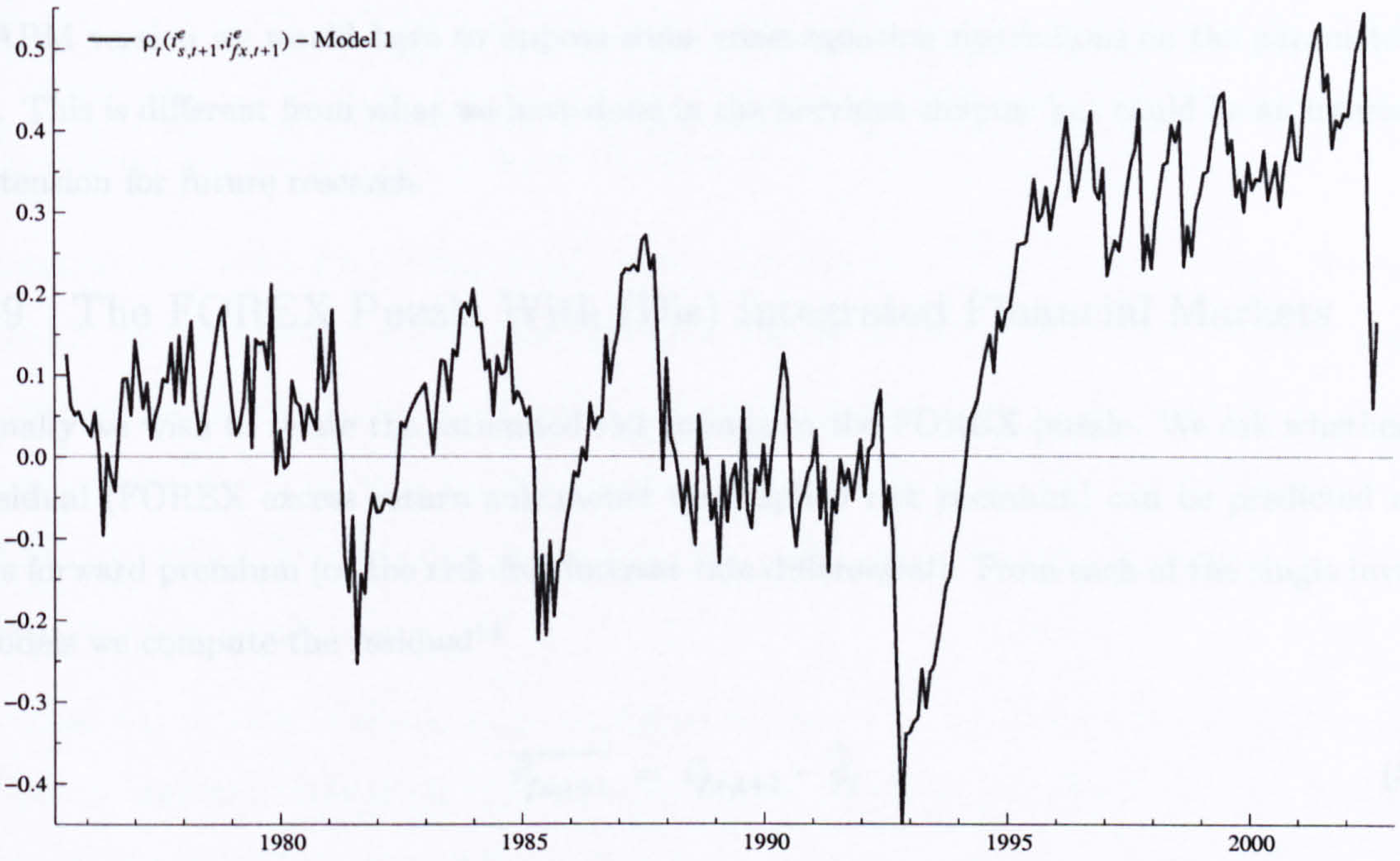
5.8.9 The Conditional Return Correlation - Common Shocks

An advantage of our test of market integration is that we obtain an estimate of the conditional return variance (FOREX and stock return) which measure whether the two markets have been more or less exposed to common shocks in the sample. In figure (5.5) we plot the implied correlation from a consumption-based model (similar picture using any other model). The plot is interesting, though maybe not surprising. Having varied between −0.2 and 0.2 from the start of the sample until 1992, where the EMS problems starts, the conditional correlation falls radically and becomes negative then rises very much again reaching a high in early 2002. Our analysis

[13]This correction term is related to the measure of inflation contribution studied in Hollifield and Yaron [75]. However, we have shown that inflation can be part of the real SDF as well - they omit this possibility.

suggest first that FOREX and stock market risk premia have fallen much in the 1990s (especially after 1992-1994) and the correlation of shocks in the two markets has increased substantially in the same period suggesting a larger degree of common shocks in the late 1990s. However, the substantial fall in stock prices in the end of the sample has caused this correlation to fall substantially again. In conclusion, the FOREX- and stock return correlation varies substantially over time.

Figure 5.5: Conditional Return Correlations



`Conditional correlation between stock return and FOREX return.`

5.8.10 Dynamic Financial Market Integration

As we have emphasised in this thesis, in particular chapter 3, all SDF models can be considered a conditional version of the CAPM where each model imply a time-varying relation between the risk premium on the market portfolio (assumed a broad stock market index) and the conditional variance of the market portfolio. We have not done this since we were interested to test whether the equity market and FOREX market were integrated based on an unconditional SDF model. Recall the discussion in chapter 3, in particular equation (3.18). If we assume the broad stock market index to be the market portfolio and we want to interpret the model as a conditional

CAPM then the correct method to implement this conditional CAPM with joint modelling of stock- and FOREX market risk premia would be

$$
\begin{aligned}
E_t(i^e_{s,t+1}) + \frac{1}{2}V_t(i^e_{s,t+1}) &= \overline{\gamma}_t V_t(i^e_{s,t+1}) \\
E_t(i^e_{fx,t+1}) + \frac{1}{2}V_t(i^e_{fx,t+1}) &= \overline{\gamma}_t \mathrm{Cov}_t(i^e_{fx,t+1}, i^e_{s,t+1})
\end{aligned}
$$

Performing this version of the CAPM imposes market integration directly. If we had to test this CAPM version we would have to impose some cross-equation restrictions on the parameters in $\overline{\gamma}_t$. This is different from what we have done in the previous chapter but could be an interesting extension for future research.

5.9 The FOREX Puzzle With (Dis) Integrated Financial Markets

Finally we wish to relate the estimated risk premia to the FOREX puzzle. We ask whether the residual (FOREX excess return subtracted the implied risk premium) can be predicted using the forward premium (or the risk-free interest rate differential). From each of the single investor models we compute the residual[14]

$$
\overline{i^e_{fx,t+1}} = i^e_{fx,t+1} - \hat{\overline{\phi}}_t \tag{5.23}
$$

and perform the regression

$$
\overline{i^e_{fx,t+1}} \equiv \alpha + \beta(i_{f,t} - i^*_{f,t}) + \eta_{t+1} \tag{5.24}
$$

where η is an error term. If the implied risk premia from the different models is "correct" then we would expect $\hat{\alpha} = \hat{\beta} = 0$. The regression results are in the following table (5.4).

First, we note that in most models there is still a significant bias in the estimate of β. However in the consumption-based models 3-7 we cannot reject the joint null hypothesis that $\alpha = \beta = 0$. In these models the explanatory power of the interest rate differential of the excess return residual is reduced much. We conclude that there seems to be some evidence that more general

[14] $\overline{\phi}_t$ is defined in chapter 4 and a hat indicates that it is the estimate from the model.

Table 5.4: The FOREX Puzzle Revisited

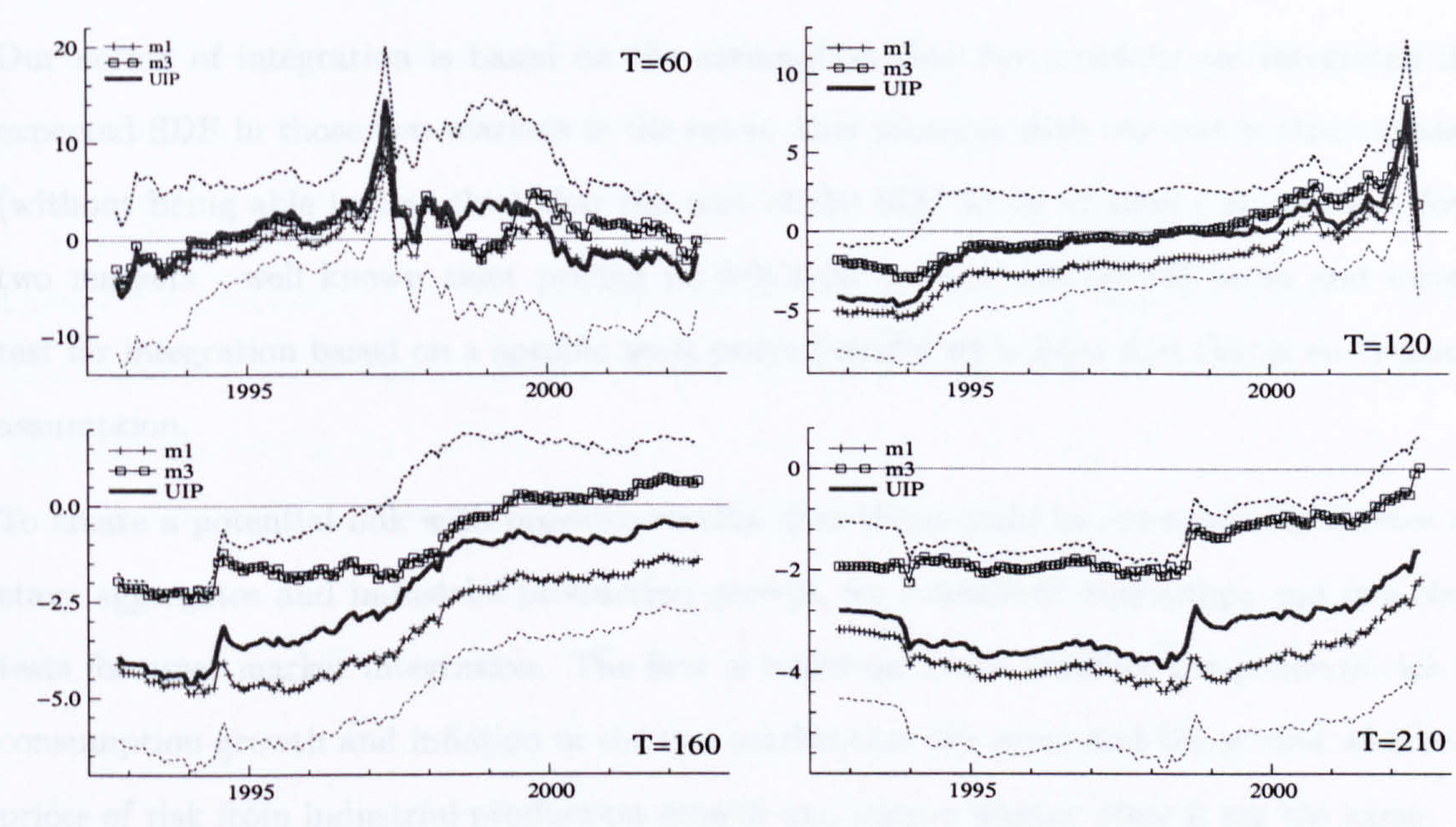
	Neutrality	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
$\hat{\alpha}$	**5.63**	**6.95**	**6.95**	3.58	4.00	2.86	3.38	3.25	3.9079	5.36	**5.32**	3.43	4.84
	(2.10)	(2.94)	(2.94)	(1.58)	(1.76)	(1.27)	(1.50)	(1.43)	(1.52)	(1.90)	(2.02)	(1.24)	(1.77)
$\hat{\beta}$	**−2.36**	**−2.72**	**−2.72**	**−1.53**	**−1.58**	−1.44	**−1.61**	−1.47	−1.83	−1.64	**−1.99**	−1.71	**−1.99**
	(3.28)	(3.35)	(3.35)	(2.03)	(2.12)	(1.82)	(2.11)	(1.94)	(1.89)	(1.50)	(2.14)	(1.76)	(2.09)
R^2	0.032	0.044	0.044	0.015	0.016	0.013	0.017	0.014	0.017	0.021	0.024	0.018	0.024
$F_{\alpha=\beta=0}$	5.38**	7.41**	7.43**	[2.47]	[2.66]	[2.21]	[2.76]	[2.27]	3.49*	3.62*	4.03*	3.09*	4.00*

The risk premium adjusted regression results. $F_{\alpha=\beta=0}$ is the F-test of joint insignificance of the intercept and the slope in the risk adjusted UIP regression - one star as superscript rejects the null using a 95% critical value and two stars using a 99%. Heteroscedastic and autocorrelation consistent standard deviations in parenthesis. There is a box around F-statistics where we cannot reject the null hypothesis of joint insignificance of intercept and slope. Absolute t-statistics in parenthesis and emphasised parameters significant using a 95 % critical value.

consumption-based asset pricing models, except the Power Utility CCAPM, do help resolve the FOREX puzzle on the UK-US exchange rate. This is not the case for the monetary models though we cannot reject the null hypothesis of insignificance of the parameters in the regression. These results combined with previous findings potentially suggest that the consumption-based models are better description of the actual data. Note, as well, that the F-statistic increases when imposing integration.

Another way to look at whether our estimated risk premia have resolved the FOREX puzzle is to plot rolling estimates of βs from the estimated equation (5.24). We focus only on model 1 and model 3. Recall our plot of the rolling slope coefficients from the standard UIP log normal regression in figure (4.1) in chapter 4. We plot these rolling betas for different sample sizes, in a similar style as in chapter 4, with a 95 % confidence bound around it, in figure (5.6). We plot also, without confidence bounds, the rolling betas from model 1 and model 3 from regression of FOREX excess return net of Jensen correction and estimated risk premium on the interest rate differential. This can potentially give new insights whether the estimated risk premia in this chapter help, and in what way, resolving the FOREX puzzle. The plot is interesting in that the Power Utility risk premium in all cases increases the negative bias in the OLS estimate but the risk premium model pricing also inflation implies a bias which is considerably smaller than in the standard UIP regression. In the samples with 210 observations the rolling beta is more or less following the 95 % confidence bound of the beta estimate from the standard UIP regression. In this case we almost conclude that, adjusting the FOREX excess return with the Jensen correction and risk premium from model 3 the bias in the OLS is significantly different at all times after 1992 than in the standard UIP regression assuming risk neutrality omitting a Jensen correction. We should note that with sample sizes greater or equal to 160 we accept the bias in the OLS estimate is significantly different using a 90 % critical value at all times.

Figure 5.6: UIP And Risk Adjusted UIP Bias And Dependence On Sample Size



Estimate of OLS slope coefficient on interest rate differential. Solid line is estimate from UIP regression in chapter 4 and the dotted lines are the 95 % confidence bound on this standard UIP estimate. We do not report confidence bounds on the slope coefficient in the risk adjusted regressions.

The broad conclusion is that it seems that it is not sufficient to price just consumption growth in the FOREX market to resolve the FOREX puzzle. It seems that there is a great improvement in resolving the puzzle by pricing also inflation. However, the puzzle is still not fully resolved and it is of interest to consider additional variables to be priced that could eventually remove the bias completely. Further we recall that our estimated risk premia are far too variable inconsistent with the fluctuations in the variance of the exchange rate.

5.10 Conclusion

In this chapter we discussed a single and two investor Epstein Zin Model on the UK-US exchange rate. We showed that modelling the FOREX risk premium allowing for time-variation one would need to estimate the FOREX risk premium joint with the risk premium on the wealth portfolio. If, as commonly assumed, a broad stock market index is included as part of wealth then we would need to model the risk premium on this stock index as well. Since the consumption-based model implies that the price of risk on assets is the same it was natural to test whether this was the case in the FOREX and stock market - we propose this as a test for integration between any two financial markets. The advantage of the test we propose is that we obtain an estimate of the risk premium in the two markets.

Our notion of integration is based on the assumption that two markets are integrated if the expected SDF in those two markets is the same. One problem with our test is that we assume (without being able to test this) that the part of the SDF know at time t is the same for the two markets - well known asset pricing models tells us that this should be so and when we test for integration based on a specific asset pricing model we believe that this is an "innocent" assumption.

To create a potential link with previous results, that there could be some role for narrow monetary aggregates and industrial production growth, we considered two setups and two class of tests for asset market integration. The first is based on a test whether the prices of risk from consumption growth and inflation in the two markets are the same and the second whether the prices of risk from industrial production growth and narrow money growth are the same.

We consider pricing the stock return and exchange rates, in addition to the macroeconomic variables, as well. In none of the estimated models we can reject that the UK FOREX and stock markets are integrated. That is whether pricing 1, 2, 3 or 4 factors in the two markets we always conclude that the markets are integrated - this whether the priced macroeconomic variables are money and industrial production or inflation and consumption.

The estimated risk premium in the stock market are intuitively positive over most of the sample (in particular the consumption-based models) but the estimated risk premia in the FOREX market are occasionally negative over some periods in the sample - we fail to give a good explanation for this but conclude that one possibility could be variable omission. In any case we show that adjusting the expost FOREX excess return for the estimated exante risk premium the residual, in all consumption-based models except the traditional power utility CCAPM, cannot be explained as strongly by the UK and US risk-free interest rate differential as when the risk premium is assumed equal to zero. This is not the case for the monetary model and we conclude that it is interesting that a theoretically justified model of the risk premium is capable of resolving the puzzle. Further developments of the consumption-based model could be a step in the right direction for solving the puzzle.

A problem with the consumption-based models, as with the monetary models, is that the implied risk premia are too volatile since, if the two investor model of the exchange rate is correct, the volatility of the exchange rate is too smooth ! Further investigations into that issue may be necessary. Comparing with the results in chapter 4, the estimate of the conditional FOREX return variance in a multivariate setting is more smooth than implied by the univariate estimates in chapter 4.

Finally it is interesting that the univariate estimate (and simplified approach) adopted in chapter 4 seems capable of detecting significant macroeconomic variables to be priced in the FOREX market. However, in the previous chapter we found no role for inflation whereas in the multivariate model we found inflation priced also in the FOREX market.

5.11 Appendix Chapter 5

5.11.1 The Relation Between Risk Premium And Variance of Risky Returns in Wealth Portfolio

Assume that there are two risky assets in the wealth portfolio, denoted with subscript j and k respectively. Then the risk premium/conditional variance relationship of asset j depends on the asset k return and portfolio weight in the following way:

$$\begin{aligned}\frac{\mathrm{E}_t(i^e_{j,t+1}) + \frac{1}{2}\mathrm{V}_t(i^e_{j,t+1})}{\mathrm{V}_t(i^e_{j,t+1})} &= \frac{\theta}{\psi}\frac{\rho_t(\Delta c_{t+1}, i^e_{j,t+1})\sigma_t(\Delta c_{t+1})}{\sigma_t(i^e_{j,t+1})} + \theta\frac{\rho_t(\pi_{t+1}, i^e_{j,t+1})\sigma_t(\pi_{t+1})}{\sigma_t(i^e_{j,t+1})} \\ &+ (1-\theta)\omega_j + (1-\theta)\omega_k\frac{\rho_t(i^e_{j,t+1}, i^e_{k,t+1})\sigma_t(i^e_{k,t+1})}{\sigma_t(i^e_{j,t+1})} \qquad (5.25)\end{aligned}$$

5.11.2 Recovering the Preference Parameters of The Representative Investor

Throughout we do no include the subscript c on the estimated parameters but note that, for instance, $\alpha_{12} = \alpha_{c,12}$. If one believes in the consumption-based model we may want the portfolio weights to lie in the interval between 0 and 1 and we may want the sum of the portfolio weights to sum to one (recall equation 5.19). This amounts to the following restrictions.

- $0 \leq -\frac{\alpha_{12}}{\alpha_{14}},\ -\frac{\alpha_{13}}{\alpha_{14}},\ -\frac{\alpha_{22}}{\alpha_{24}},\ -\frac{\alpha_{23}}{\alpha_{24}} \leq 1$
- $0 \leq -\frac{\alpha_{12}+\alpha_{13}}{\alpha_{14}},\ -\frac{\alpha_{22}+\alpha_{23}}{\alpha_{24}} \leq 1$

It is quite cumbersome to impose all these parameter restrictions and we decide to leave them unrestricted. From the restricted estimations one can recover the portfolio weights as

- $\hat{\omega}_2 = -\frac{\hat{\alpha}_{12}}{\hat{\alpha}_{14}} = -\frac{\hat{\alpha}_{23}}{\hat{\alpha}_{24}},\ \hat{\omega}_3 = -\frac{\hat{\alpha}_{13}}{\hat{\alpha}_{14}} = -\frac{\hat{\alpha}_{22}}{\hat{\alpha}_{24}}$
- $\hat{\omega}_1 = 1 - \hat{\omega}_3 - \hat{\omega}_2$

From an unrestricted estimation one could get two separate estimates of the portfolio weights, one set from the FOREX market and one set from the equity market which may be interesting though a consumption-based model with complete markets dictate that the two set of estimates ought to be the same. Using first order Taylor approximations we can recover the standard errors of the portfolio weights, as was done in chapter 2 for the coefficient of relative risk aversion and elasticity of inter-temporal substitution. Variance formulas for the parameters are given by

- $V(\omega_3) \simeq \frac{1}{\hat{\alpha}_{14}^2}V(\alpha_{13}) + \frac{\hat{\alpha}_{13}^2}{\hat{\alpha}_{14}^4}V(\alpha_{14}) - 2\frac{\hat{\alpha}_{13}}{\hat{\alpha}_{14}^3}\mathrm{Cov}(\alpha_{13},\alpha_{14})$, or
- $V(\omega_3) \simeq \frac{1}{\hat{\alpha}_{24}^2}V(\alpha_{22}) + \frac{\hat{\alpha}_{22}^2}{\hat{\alpha}_{24}^4}V(\alpha_{24}) - 2\frac{\hat{\alpha}_{22}}{\hat{\alpha}_{24}^3}\mathrm{Cov}(\alpha_{22},\alpha_{24})$
- $V(\omega_2) \simeq \frac{1}{\hat{\alpha}_{14}^2}V(\alpha_{12}) + \frac{\hat{\alpha}_{12}^2}{\hat{\alpha}_{14}^4}V(\alpha_{14}) - 2\frac{\hat{\alpha}_{12}}{\hat{\alpha}_{14}^3}\mathrm{Cov}(\alpha_{12},\alpha_{14})$, or
- $V(\omega_2) \simeq \frac{1}{\hat{\alpha}_{24}^2}V(\alpha_{23}) + \frac{\hat{\alpha}_{23}^2}{\hat{\alpha}_{24}^4}V(\alpha_{24}) - 2\frac{\hat{\alpha}_{23}}{\hat{\alpha}_{24}^3}\mathrm{Cov}(\alpha_{23},\alpha_{24})$
- $V(\omega_1) = V(\omega_3) + V(\omega_2) + 2\mathrm{Cov}(\omega_2,\omega_3)$

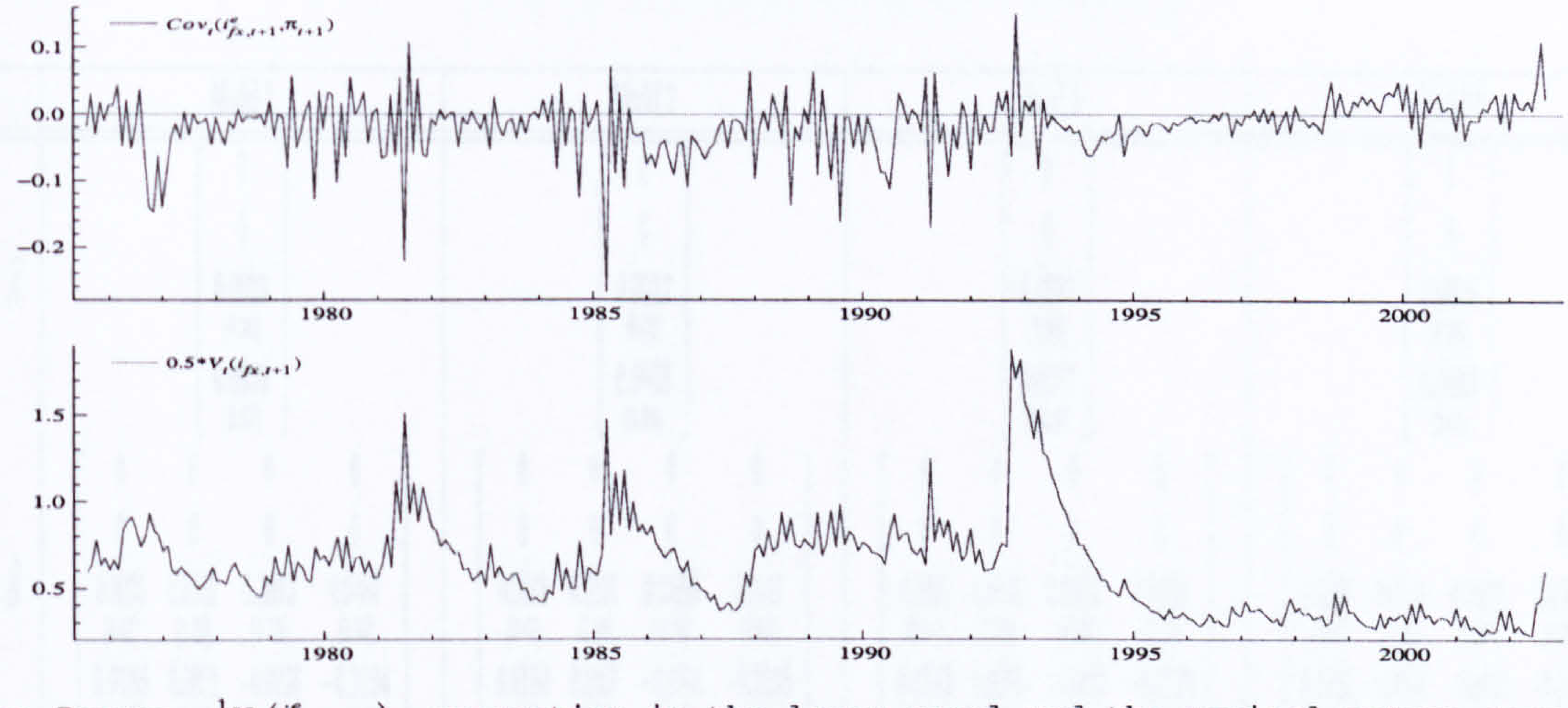
The latter formula for the weight on investment in domestic risk-free asset is problematic since an approximation to the covariance between the two other weights is not readily available. In the special case where the investor invests nothing in the foreign risk-free asset, then it follows trivially that $V(\omega_1) = V(\omega_3)$, and we have an approximation for this. Then of ultimate interest for the consumption-based model we can recover the coefficient of relative risk aversion and the elasticity of inter-temporal substitution as

- $\hat{\gamma} = \hat{\alpha}_{11} - \hat{\alpha}_{14} = \hat{\alpha}_{21} - \hat{\alpha}_{24}$
- $\hat{\psi} = \frac{1+\hat{\alpha}_{14}}{\hat{\alpha}_{11}} = \frac{1+\hat{\alpha}_{24}}{\hat{\alpha}_{21}}$
- $\hat{\theta} = 1+\hat{\alpha}_{14} = 1+\hat{\alpha}_{24}$

From the above estimates we can recover the standard error of the estimates, or an approximation, from the estimation by using a first order Taylor approximation taking the variance of it in a similar way as was done in chapter 2. For convenience we replicate them here as

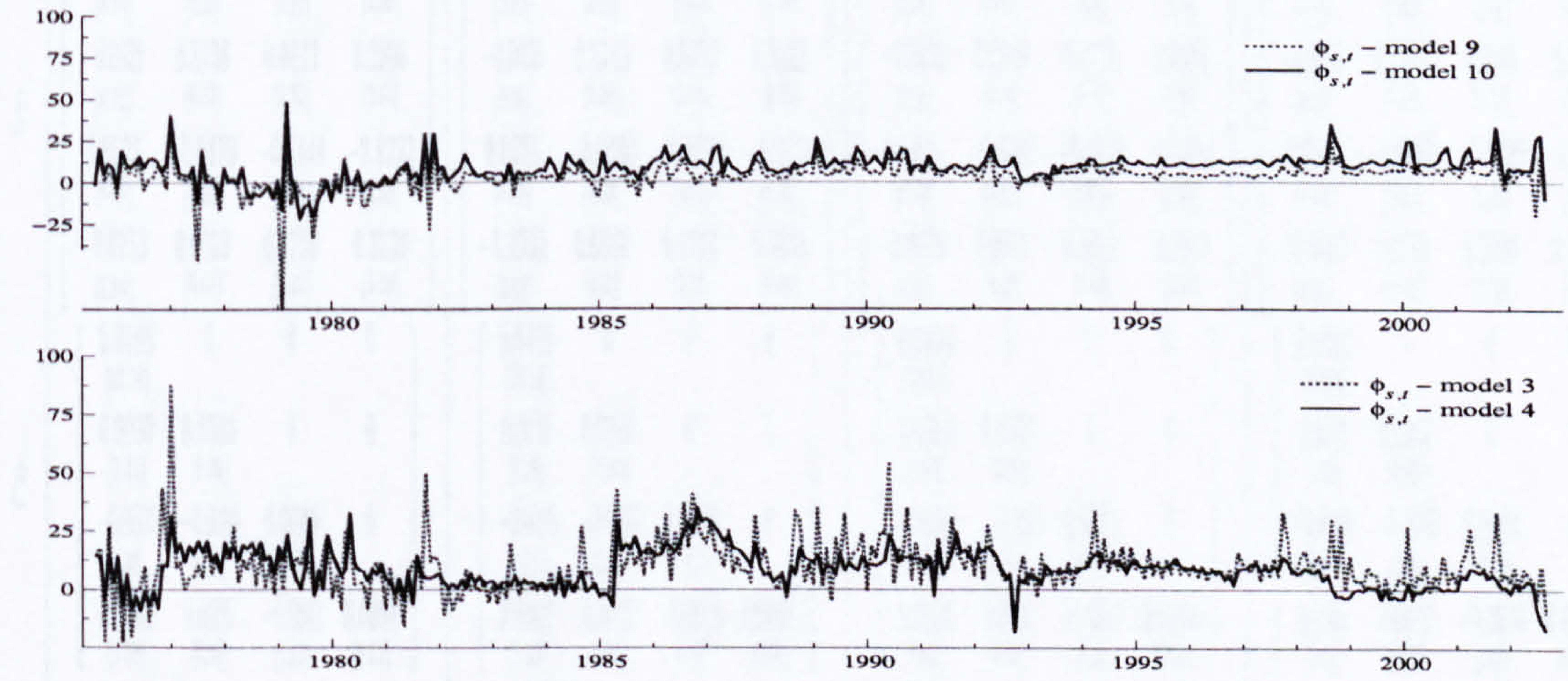
- $V(\gamma_{fx}) = V(\alpha_{11}) + V(\alpha_{14}) - 2\mathrm{Cov}(\alpha_{11},\alpha_{14})$, or
- $V(\gamma_s) = V(\alpha_{21}) + V(\alpha_{24}) - 2\mathrm{Cov}(\alpha_{21},\alpha_{24})$
- $V(\psi_{fx}) \simeq \frac{1}{\hat{\alpha}_{11}^2}V(\alpha_{14}) + \frac{(1+\hat{\alpha}_{14})^2}{\hat{\alpha}_{11}^4}V(\alpha_{11}) - 2\frac{(1+\hat{\alpha}_{14})}{\hat{\alpha}_{11}^3}\mathrm{Cov}(\alpha_{14},\alpha_{11})$, or
- $V(\psi_s) \simeq \frac{1}{\hat{\alpha}_{21}^2}V(\alpha_{24}) + \frac{(1+\hat{\alpha}_{24})^2}{\hat{\alpha}_{21}^4}V(\alpha_{21}) - 2\frac{(1+\hat{\alpha}_{24})}{\hat{\alpha}_{21}^3}\mathrm{Cov}(\alpha_{24},\alpha_{21})$
- $V(\theta_s) = V(\hat{\alpha}_{14})$
- $V(\theta_{fx}) = V(\hat{\alpha}_{24})$

Figure 5.7: The Jensen and Nominal Return Correction



The Jensen, $\frac{1}{2}V_t(i^e_{fx,t+1})$, correction in the lower panel and the nominal return correction, $Cov_t(i^e_{fx,t+1}, \pi_{t+1})$, in the upper panel. Both multiplied by 1200.

Figure 5.8: The Equity Risk Premium From The Consumption-Based And The Monetary Model



The implied Equity risk premia from UK model 3, 4, 9 and 10. All variables are annualised.

Table 5.5: Parameter Estimates in Model 1-4

	Model 1	Model 2	Model 3	Model 4
$\hat{A}$	0 0 0.0023 (6.20) 0.0039 (9.37)	0 0 0.0022 (6.09) 0.0038 (9.29)	0 0 0.0025 (7.36) 0.0037 (9.12)	0 0 0.0024 (6.80) 0.0038 (9.69)
$\hat{B}$	0 0 0 0 0 0 0 0 0.0035 (0.48) 0.0153 (1.70) 0.2873 (4.73) 0.0484 (0.93) 0.0155 (2.33) 0.0059 (0.55) −0.0736 (1.22) −0.2194 (4.25)	0 0 0 0 0 0 0 0 0.0033 (0.47) 0.0161 (1.80) 0.2895 (4.74) 0.0530 (1.03) 0.0156 (2.39) 0.0057 (0.54) −0.0800 (1.31) −0.2196 (4.16)	0 0 0 0 0 0 0 0 0.0036 (0.54) 0.0143 (1.74) 0.2577 (4.50) 0.0329 (0.72) 0.0142 (1.99) 0.0060 (0.55) −0.0438 (0.74) −0.1774 (3.50)	0 0 0 0 0 0 0 0 0.0006 (0.09) 0.0154 (1.76) 0.2806 (5.00) 0.0500 (1.09) 0.0168 (2.46) 0.0040 (0.36) −0.0438 (0.71) −0.1805 (3.44)
$\hat{D}$	−0.1917 (1.03) −0.7678 (9.78) 0.0357 (0.60) 0.2540 (3.83) −0.7678 (9.78) 0.4531 (2.62) 0.0145 (0.37) 0.1711 (3.25) 0.0357 (0.60) 0.0145 (0.37) 0.0755 (0.37) 0.5091 (6.72) 0.2540 (3.83) 0.1711 (3.25) 1.5385 (6.18) −0.0051 (0.02)	−0.2320 (1.16) −0.7425 (7.65) 0.0535 (0.84) 0.2460 (3.62) −0.7425 (7.65) 0.4900 (2.54) 0.0189 (0.47) 0.1684 (2.99) 0.0535 (0.84) 0.0189 (0.47) 0.1258 (0.69) 0.5161 (6.96) 0.2460 (3.62) 0.1684 (2.99) 1.5256 (5.80) −0.0721 (0.32)	0.0342 (0.16) −0.6401 (7.04) −0.1851 (2.61) −0.0412 (0.47) −0.6401 (7.04) 0.5301 (2.90) −0.1547 (3.64) −0.0083 (0.15) −0.1851 (2.61) −0.1547 (3.64) −0.8030 (4.41) 0.1383 (0.67) −0.0412 (0.47) −0.0083 (0.15) 1.2851 (4.74) 0.8826 (5.56)	0.0317 (0.12) −0.7675 (6.50) −0.2045 (3.31) 0.1276 (1.32) −0.7675 (6.50) 0.3216 (1.54) −0.1919 (3.60) 0.1339 (1.67) −0.2045 (3.31) −0.1919 (3.60) −0.2716 (0.89) 0.3015 (2.14) 0.1276 (1.32) 0.1339 (1.67) 1.6489 (5.91) 0.6110 (2.79)
$\hat{E}$	−0.1478 (3.79) −0.0183 (0.30) −0.0433 (0.21) 0.3374 (1.99) −0.0039 (0.12) 0.3136 (6.43) 0.6842 (3.22) 0.2996 (2.64) 0.0522 (6.65) −0.0479 (4.55) −0.1444 (2.58) −0.1222 (3.58) −0.0252 (2.94) 0.0603 (4.43) 0.1750 (2.47) 0.3139 (5.99)	−0.1507 (3.52) −0.0127 (0.18) 0.0150 (0.07) 0.4111 (1.96) −0.0030 (0.09) 0.3140 (6.30) 0.5902 (2.84) 0.2821 (2.56) 0.0505 (6.35) −0.0466 (4.49) −0.1319 (2.34) −0.1236 (3.50) −0.0232 (2.62) 0.0568 (4.35) 0.1789 (2.52) 0.3209 (6.04)	−0.0932 (2.91) 0.0206 (0.43) −0.0404 (0.26) 0.0792 (0.78) −0.0352 (1.15) 0.2860 (5.60) 0.8271 (4.23) 0.2674 (3.11) 0.0465 (6.09) −0.0587 (5.41) −0.1778 (2.94) −0.1314 (3.48) −0.0375 (3.94) 0.0627 (4.20) 0.1671 (2.43) 0.2100 (5.15)	−0.0052 (1.58) −0.1019 (1.66) −0.4055 (1.98) 0.0323 (0.26) −0.0055 (0.18) 0.2587 (5.14) 0.7186 (3.74) 0.2447 (2.65) 0.0366 (5.53) −0.0561 (5.05) −0.1739 (2.96) −0.1297 (3.51) −0.0383 (3.76) 0.0762 (4.53) 0.2280 (2.58) 0.2820 (5.14)
$\hat{C}$	0.0469 (22.78) 0 0 0 0.0030 (1.31) 0.0308 (8.09) 0 0 −0.0005 (1.20) −0.0003 (0.76) 0.0049 (10.19) 0 0.0006 (1.98) 0.0005 (0.94) −0.0007 (1.37) 0.0054 (5.32)	0.0471 (22.14) 0 0 0 0.0030 (1.29) 0.0310 (7.80) 0 0 −0.0005 (1.11) −0.0003 (0.68) 0.0048 (10.11) 0 0.0007 (1.96) 0.0005 (1.06) −0.0006 (1.28) 0.0055 (5.35)	0.0464 (23.44) 0 0 0 0.0051 (2.15) 0.0303 (8.27) 0 0 −0.0002 (0.62) −0.0003 (1.14) 0.0044 (8.07) 0 0.0004 (2.01) 0.0002 (0.37) −0.0011 (1.93) 0.0050 (3.61)	0.0458 (21.94) 0 0 0 0.0041 (1.63) 0.0303 (9.79) 0 0 −0.0001 (0.18) −0.0003 (1.01) 0.0043 (9.42) 0 0.0004 (1.32) 0.0003 (0.59) −0.0010 (2.03) 0.0049 (3.54)
$\hat{C}\hat{C}^{\intercal}$	3169.55 205.87 1382.34 −32.58 −16.74 34.77 39.62 24.30 −5.39 44.09	3192.12 202.22 1398.62 −31.03 −16.79 34.84 45.92 26.58 −5.15 44.51	3094.54 340.60 1358.26 −12.85 −16.24 28.72 29.95 12.37 −7.49 38.47	3019.92 269.88 1344.49 −3.17 −12.73 27.04 23.10 13.89 −6.11 35.69

Other parameter estimates (recall equations (5.21) and (5.22)). Model 1, 2, 3 and 4. We tabulate only the lower part of $\widehat{C}\widehat{C}^{\intercal}$. Absolute t-statistics in parenthesis. Emphasised parameters significant using 95 % critical value.

Table 5.6: Parameter Estimates in Model 5-8

	Model 5	Model 6	Model 7	Model 8
$\hat{A}$	0	0	0	0
	0	0	0	0
	0.0025 (7.49)	0.0024 (6.99)	0.0025 (7.35)	0.0023 (6.76)
	0.0037 (8.71)	0.0038 (9.56)	0.0037 (9.04)	0.0038 (9.74)
$\hat{B}$	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
	0.0041 (0.61) 0.0133 (1.59) 0.2539 (4.39) 0.0252 (0.54)	0.0013 (0.22) 0.0152 (1.73) 0.2686 (4.66) 0.0410 (0.90)	0.0034 (0.51) 0.0144 (1.74) 0.2584 (4.49) 0.0331 (0.71)	0.0003 (0.06) 0.0159 (1.80) 0.2825 (5.04) 0.0511 (1.12)
	0.0139 (1.89) 0.0081 (0.74) −0.0476 (0.77) −0.1858 (3.51)	0.0165 (2.47) 0.0061 (0.55) −0.0438 (0.71) −0.1824 (3.51)	0.0144 (2.01) 0.0062 (0.56) −0.0445 (0.75) −0.1789 (3.53)	0.0171 (2.43) 0.0038 (0.34) −0.0482 (0.78) −0.1820 (3.46)
$\hat{D}$	−0.0784 (0.35) −0.6278 (5.31) −0.1855 (2.40) −0.0391 (0.39)	−0.0885 (0.29) −0.8245 (7.35) −0.1954 (2.89) 0.0902 (0.98)	−0.0149 (0.06) −0.6377 (6.43) −0.1873 (2.57) −0.0491 (0.50)	0.0274 (0.09) −0.7722 (5.84) −0.2266 (3.46) 0.1238 (1.20)
	−0.6278 (5.31) 0.6037 (3.13) −0.1559 (3.24) 0.0130 (0.23)	−0.8245 (7.35) 0.3158 (1.42) −0.1783 (2.64) 0.1022 (1.46)	−0.6377 (6.43) 0.5530 (2.94) −0.1505 (3.39) −0.0119 (0.20)	−0.7722 (5.84) 0.3117 (1.49) −0.2083 (3.34) 0.1279 (1.52)
	−0.1855 (2.40) −0.1559 (3.24) −0.7042 (3.70) 0.2539 (1.39)	−0.1954 (2.89) −0.1783 (2.64) −0.2958 (1.04) 0.2459 (1.66)	−0.1873 (2.57) −0.1505 (3.39) −0.7993 (4.31) 0.1437 (0.69)	−0.2266 (3.46) −0.2083 (3.34) −0.1846 (0.58) 0.3331 (2.48)
	−0.0391 (0.39) 0.0130 (0.23) 1.3088 (5.07) 0.7810 (4.73)	0.0902 (0.98) 0.1022 (1.46) 1.5906 (5.84) 0.6964 (3.40)	−0.0491 (0.50) −0.0119 (0.20) 1.2624 (4.54) 0.8784 (5.57)	0.1238 (1.20) 0.1279 (1.52) 1.5561 (5.92) 0.5622 (2.67)
$\hat{E}$	−0.0850 (2.40) 0.0397 (0.69) 0.0517 (0.30) 0.1350 (1.14)	−0.0354 (1.16) −0.1005 (1.78) −0.3804 (1.99) −0.0380 (0.32)	−0.0911 (2.84) 0.0231 (0.45) −0.0314 (0.20) 0.0740 (0.73)	−0.0466 (1.37) −0.1161 (1.85) −0.4520 (2.17) 0.0153 (0.11)
	−0.0400 (1.49) 0.2514 (4.73) 0.7449 (3.81) 0.2488 (2.91)	0.0001 (0.002) 0.2406 (5.10) 0.6747 (3.66) 0.2035 (2.47)	−0.0299 (1.01) 0.2837 (5.66) 0.8266 (4.28) 0.2584 (3.11)	−0.0009 (0.03) 0.2629 (5.14) 0.7523 (3.78) 0.2599 (2.63)
	0.0475 (6.27) −0.0543 (5.23) −0.1764 (2.80) −0.1213 (3.18)	0.0373 (5.27) −0.0587 (5.19) −0.1916 (2.93) −0.1368 (3.45)	0.0459 (5.95) −0.0594 (5.36) −0.1796 (2.90) −0.1316 (3.43)	0.0365 (5.52) −0.0559 (4.90) −0.1706 (2.92) −0.1270 (3.41)
	−0.0384 (3.79) 0.0512 (3.53) 0.1275 (1.70) 0.2185 (5.05)	−0.0366 (3.66) 0.0680 (4.19) 0.2048 (2.35) 0.2626 (4.97)	−0.0354 (3.60) 0.0632 (4.16) 0.1711 (2.47) 0.2111 (5.12)	−0.0397 (3.70) 0.0801 (4.67) 0.2390 (2.64) 0.2977 (5.27)
$\hat{C}$	0.0468 (24.56) 0 0 0	0.0454 (20.10) 0 0 0	0.0463 (22.58) 0 0 0	0.0458 (21.47) 0 0 0
	0.0059 (2.78) 0.0283 (7.52) 0 0	0.0051 (2.30) 0.0282 (9.30) 0 0	0.0054 (2.25) 0.0302 (8.14) 0 0	0.0042 (1.56) 0.0307 (9.49) 0 0
	−0.0003 (0.88) −0.0004 (1.11) 0.0045 (8.02) 0	−0.0002 (0.57) −0.0004 (1.39) 0.0043 (10.22) 0	−0.0002 (0.55) −0.0004 (1.22) 0.0045 (8.17) 0	−0.0002 (0.52) −0.0003 (1.10) 0.0043 (8.64) 0
	0.0003 (0.91) −0.0004 (0.61) −0.0011 (2.14) 0.0053 (4.82)	0.0003 (0.84) −0.0005 (0.91) −0.0008 (1.90) 0.0045 (3.27)	0.0003 (1.06) 0.0001 (0.20) −0.0012 (2.03) 0.0050 (3.77)	0.0002 (0.65) 0.0003 (0.65) −0.0010 (2.05) 0.0051 (3.63)
$\hat{C}\hat{C}^{\intercal}$	3151.42	2963.11	3084.50	3026.85
	396.58 1202.40	335.29 1182.36	358.08 1352.10	275.67 1381.76
	−19.34 −17.45 29.00	−11.38 −18.00 26.42	−11.67 −17.26 29.09	−10.58 −15.27 26.77
	22.41 −15.42 −6.73 43.01	16.92 −17.54 −4.87 30.85	23.03 7.22 −7.67 38.82	13.46 14.56 −6.53 38.59

Other parameter estimates (recall equations (5.21) and (5.22)). Model 5, 6, 7 and 8. We tabulate only the lower part of $\widehat{C}\widehat{C}^{\intercal}$. Absolute t-statistics in parenthesis. Emphasised parameters significant using 95 % critical value.

Table 5.7: Parameter Estimates in Model 9-12

	Model 9	Model 10	Model 11	Model 12
$\hat{A}$	0 0 0.0052 (12.50) 0.0002 (0.21)	0 0 0.0052 (12.58) 0.0002 (0.22)	0 0 0.0051 (12.59) 0.0001 (0.09)	0 0 0.0052 (12.74) 0.0002 (0.23)
$\hat{B}$	0 0 0 0 0 0 0 0 −0.0043 (0.82) 0.0067 (0.72) −0.0012 (0.02) 0.0159 (0.67) 0.01771 (1.69) −0.0221 (1.19) 0.2200 (1.83) −0.2842 (5.41)	0 0 0 0 0 0 0 0 −0.0052 (1.00) 0.0070 (0.75) −0.0022 (0.04) 0.0231 (0.95) 0.0164 (1.47) −0.0247 (1.32) 0.2337 (2.05) −0.3075 (6.07)	0 0 0 0 0 0 0 0 −0.0045 (0.89) 0.0061 (0.66) 0.0067 (0.11) 0.0069 (0.30) 0.0179 (1.77) −0.0185 (0.99) 0.2017 (1.70) −0.2799 (5.23)	0 0 0 0 0 0 0 0 −0.0055 (1.04) 0.0064 (0.69) −0.0033 (0.06) 0.0193 (0.80) 0.0166 (1.50) −0.0230 (1.25) 0.2325 (2.01) −0.3059 (5.93)
$\hat{D}$	−0.0520 (0.60) 0.1567 (1.59) −0.0313 (1.05) −0.1608 (2.78) 0.1567 (1.59) 0.9014 (23.37) 0.0022 (0.31) 0.0334 (1.49) −0.0313 (1.05) 0.0022 (0.31) 0.9621 (57.19) 0.0013 (0.14) −0.1608 (2.78) 0.0334 (1.49) 0.0867 (1.93) 0.9436 (36.68)	−0.0712 (0.45) 0.1956 (1.53) −0.0296 (1.02) −0.1965 (2.95) 0.1956 (1.53) 0.8708 (14.60) 0.0003 (0.04) 0.0421 (1.43) −0.0296 (1.02) 0.0003 (0.04) 0.9629 (57.19) −0.0003 (0.04) −0.1965 (2.95) 0.0421 (1.43) 0.0580 (1.43) 0.9425 (32.02)	−0.0666 (0.98) 0.0946 (1.10) −0.0654 (1.99) −0.1505 (2.52) 0.0946 (1.10) 0.9147 (32.07) 0.0042 (0.59) 0.0203 (1.21) −0.0654 (1.99) 0.0042 (0.59) 0.9691 (73.85) −0.0063 (0.68) −0.1505 (2.52) 0.0203 (1.21) 0.0864 (2.08) 0.9472 (40.08)	−0.0893 (0.53) 0.1892 (1.48) −0.0403 (1.36) −0.1839 (2.57) 0.1892 (1.48) 0.8687 (15.01) 0.0026 (0.30) 0.0392 (1.34) −0.0403 (1.36) 0.0026 (0.30) 0.9618 (56.63) −0.0009 (0.10) −0.1839 (2.57) 0.0392 (1.34) 0.0705 (1.65) 0.9429 (30.37)
$\hat{E}$	0.2842 (3.65) −0.0571 (0.55) −1.5095 (2.82) 0.8946 (3.32) −0.0423 (1.14) 0.2725 (5.85) 0.9133 (3.28) 0.2977 (2.61) −0.0105 (1.29) 0.0067 (0.79) −0.1498 (3.03) 0.0880 (3.24) 0.0225 (1.81) −0.0161 (0.93) 0.1727 (1.46) −0.0749 (1.36)	0.2729 (3.32) −0.0359 (0.30) −1.6327 (2.99) 0.8570 (3.26) −0.0578 (1.42) 0.3065 (5.32) 0.9939 (3.04) 0.2852 (2.32) −0.0120 (1.52) 0.0147 (1.47) −0.1408 (2.91) 0.0877 (3.25) 0.0309 (1.96) −0.0121 (0.53) 0.0796 (0.60) 0.0122 (0.25)	0.2435 (3.63) −0.0074 (0.07) −1.7733 (3.67) 0.9543 (3.42) −0.0304 (1.00) 0.2826 (6.05) 0.7403 (2.91) 0.2964 (2.49) −0.0031 (0.39) 0.0077 (0.87) −0.1981 (3.75) 0.1017 (3.77) 0.0136 (1.33) −0.0108 (0.83) 0.1843 (1.48) −0.0691 (1.25)	0.2654 (3.19) −0.0111 (0.03) −1.6023 (3.04) 0.8943 (3.16) −0.0579 (1.43) 0.3120 (5.46) 1.0232 (2.97) 0.3038 (2.35) −0.0090 (1.16) 0.0133 (1.28) −0.1617 (3.16) 0.0990 (3.38) 0.0258 (1.58) −0.0061 (0.26) 0.1262 (0.88) −0.0263 (0.46)
$\hat{C}$	0.0459 (14.77) 0 0 0 0.0045 (2.85) 0.0287 (5.59) 0 0 0.0003 (0.72) 0.00003 (0.08) 0.0050 (4.37) 0 0.0002 (0.27) −0.0006 (0.67) −0.0013 (0.30) 0.0118 (3.35)	0.0463 (16.91) 0 0 0 0.0047 (2.67) 0.0300 (5.93) 0 0 0.00004 (0.15) 0.0002 (0.47) 0.0047 (6.85) 0 0.0010 (2.05) −0.0003 (0.30) 0.0003 (0.15) 0.0111 (5.07)	0.0450 (15.57) 0 0 0 0.0035 (2.36) 0.0295 (5.84) 0 0 0.0005 (0.93) 0.0003 (0.82) 0.0049 (4.72) 0 0.0002 (0.29) −0.0005 (0.49) −0.0010 (0.31) 0.0115 (3.47)	0.0458 (15.11) 0 0 0 0.0047 (2.67) 0.0299 (5.65) 0 0 0.0004 (0.87) 0.0002 (0.39) 0.0050 (4.40) 0 0.0003 (0.45) −0.0002 (0.21) −0.002 (0.34) 0.0114 (3.63)
$\hat{C}\hat{C}^{\intercal}$	3032.52 295.97 1212.77 22.29 3.61 36.60 14.27 −24.89 −9.06 201.92	3092.49 312.03 1325.32 2.73 9.60 31.75 69.07 −4.46 2.29 181.23	2917.22 226.83 1268.35 29.20 14.88 35.13 14.38 −18.12 −7.33 191.37	3018.66 313.15 1316.28 27.55 10.39 36.61 21.51 6.24 9.38 188.79

Other parameter estimates (recall equations (5.21) and (5.22)). Model 9, 10, 11 and 12. We tabulate only the lower part of $\widehat{C}\widehat{C}^{\intercal}$. Absolute t-statistics in parenthesis. Emphasised parameters significant using 95 % critical value.

6. Are Macroeconomic Risks Priced in Financial Markets ?

The Conclusion

Each of the chapters in the thesis has an extensive conclusion. The aim of this final chapter is to briefly summarise the results and contributions of this thesis. Then we briefly comment on potential future directions from this work.

6.1 Summary and Conclusion

The aim of the thesis was to investigate whether ex-ante risk premia in the UK and US stock markets and the UK-US FOREX market are varying over time and investigate the variability of expected returns when pricing macroeconomic variables. If so, it is a necessity for models in economics involving these risk premia to incorporate this potential time-variation. More important but since the Sharpe Ratios, are crucial for determining optimal portfolio composition (see for instance Campbell and Viceira [24]), significant time-variation in the risk premium and return volatility suggests that higher returns can be obtained by investors by developing models that take a stance on the modelling of this time-variation.

A difficulty with modelling time-variation in risk premia is that few econometric models allow us to back out a time-varying risk premium. Another problem is that the Stochastic Discount Factor is not observable and there may be many Stochastic Discount Factors pricing an asset since markets are most probably not complete. In the introductory chapter we discussed the Stochastic Discount Factor model and various asset pricing models developed in the literature telling us how to model the Pricing Kernel based on observable variables. Then we outlined an estimation framework, using the multivariate GARCH-in-mean model, capable of estimating risk premia on any asset. The chapter discussed the difficulty in estimating these model but we proposed a method to perform the estimations which should simplify matters much. The main advantage of the proposed method for estimating multivariate GARCH-in-mean models was that we estimate it in a series of steps, starting from the standard univariate GARCH model. The estimation method proposed was used throughout the thesis.

In the second chapter we used one of the most general consumption-based asset pricing models to estimate the risk premium in the UK and US stock market in the period 1975-2002. We showed that the risk premium in the two markets varies significantly over time - this is not, as has been commonly assumed, only because financial sources of risk are priced in the stock markets but also because macroeconomic variables (inflation and consumption growth) are significantly priced. We also tested an alternative model pricing industrial production growth additionally, but concluded that industrial production is not significantly priced when also pricing inflation and consumption growth as dictated by the consumption based asset pricing model. US and UK ex-post excess returns are highly correlated. Based on our estimates of the UK and the US risk premium, we conclude that the high correlation in UK and US excess returns reflects a high correlation of common shocks and not a high correlation of UK and US risk premia. Some of the estimated models of the risk premium imply that the risk premium in the two countries is negatively correlated. One of the problems with the consumption based model is that consumption growth has very little ARCH, in particular US consumption growth, while in many countries consumption is not even available at a monthly frequency.

This led us to consider pricing more general key macroeconomic variables in chapter 3. More specifically, in chapter 3, we proposed an econometric model to investigate the relation between stock returns and macroeconomic variables and their covariance matrix when enough data points are available. Monetary policy authorities are often said to be focusing on the macroeconomic variables, inflation, monetary aggregates, output growth or stock market related variables. This was our choice, together with some previous empirical studies, for the set of macroeconomic variables to model jointly with the stock market excess returns. We argued that one should allow for asymmetries in the conditional covariance matrix between the variables considered, simply for intuitive reasons. It makes sense that positive and negative macroeconomic and financial shocks are transmitted differently into the conditional covariance matrix. For instance, it makes alot of sense that negative output shocks increase the variance of output considerably, whereas positive shocks should not. This was confirmed in our data, a US sample from 1960-

2003. One of the main contributions of the proposed joint model of macroeconomic variables and financial returns was that we could model the financial return, obeying a no-arbitrage condition. In the GARCH literature it has often been emphasised that there is an asymmetry in the conditional variance of stock returns, that is negative shocks increase the variance of returns more than positive shocks. This finding is mainly found in models modelling the stock market risk premium proportional to the conditional return variance. We find that, when pricing several macroeconomic variables this asymmetry is considerably less significant and it almost disappears in significance. This suggests that the asymmetry found previously in the literature may be due to incorrect modelling of the stock market risk premium. Modelling the risk premium constant proportional to the conditional variance of returns we use a variable that is far too highly auto correlated and hence this creates the asymmetry. The stock market risk premium is simply not highly auto correlated and it is varying considerably over time. The advantage of the proposed econometric model in chapter 3 is that it can address many interesting questions in economics and finance, such as the relation between the risk premium and return variance (we obtain an estimate of both variables within the same model and can construct the potential Sharpe Ratio which tells us how much to invest in the stock index). The disadvantage is that many data points are necessary in order to estimate the model. Most importantly, the econometric model proposed can be used to investigate the relation between the business cycle and financial markets.

Among the several key puzzles in financial economics is the FOREX puzzle. One way to state this puzzle is that changes in the logarithm of the exchange rate subtracted the forward premium (excess return) can be explained by the forward premium. This should not be so, if investors are risk neutral. First we showed that the regression for testing the Uncovered Interest Rate Parity is not consistent with log normality of the exchange rate, since then the conditional variance of the exchange rate should be zero which is not empirically true - this is because Uncovered Interest Rate Parity assumes investors to be risk neutral and the conditional variance of the exchange rate reflects the sum of domestic and foreign risk premia when we assume the exchange rate to be log normally distributed. Assuming that investors may be risk averse we derived a two investor model using the Stochastic Discount Factor model and showed that the reason that the forward premium can explain the FOREX excess return could be the omission of a time-varying variable related to the foreign risk premium, the domestic risk premium and the conditional variance of innovations in the log exchange rate. A crucial assumption for deriving the two-investor FOREX model is that foreign and domestic investors have the same information set. If this is so, we showed that the regression test, often used to test the Uncovered Interest Rate Parity condition assuming a log normal distributed exchange rate, has low power since acceptance of the condition from such a regression is also consistent with risk averse foreign and domestic investors provided the foreign risk premium is equal to the domestic risk premium at every point in time. If the information sets of the domestic and foreign investor are the same then we showed that there is an upper bound on expected FOREX excess returns equal to half the conditional variance of the innovations to the log exchange rate, and a lower bound of minus half the conditional variance. The intertemporal relation between the expected FOREX excess return and the conditional variance of innovations to the log exchange rate is itself governed by foreign and domestic risk premia.

We showed that estimation of a two investor FOREX model of the risk premium involves many variables and it is very difficult, if not impossible, to estimate it using the multivariate GARCH-in-mean approach that we proposed in the introductory chapter. This led us to propose an alternative estimation method of the FOREX risk premium. The crucial assumption that makes it easy to estimate the FOREX model with many variables is that the correlation between all variables other than FOREX excess return is zero and this assumption allows us to estimate the FOREX model in two steps. That these variables are uncorrelated may be a strong assumption, but it has the advantage that we can price many domestic and foreign variables in the FOREX market and we can obtain a potentially much better representation of the residuals of the variables in the multivariate model.

We estimated the FOREX model pricing all UK and US variables considered in the chapters on stock returns and found in particular the UK variables money growth, output growth and to some degree consumption growth to be significantly priced. The implied domestic and foreign FOREX risk premia vary considerably over time. In particular we found that pricing of the UK variables were significant in generating time-varying risk premia. Nevertheless the models we estimated failed to resolve the FOREX puzzle. However, our estimated models implied that the FOREX risk premia were varying considerably in the 1970s and 1980s but have been declining much ever since consistent with the fall in the conditional FOREX return variance which reflects the sum of domestic and foreign risk premia. Despite the inevitable criticism that important factors may have been omitted from the risk premium specification we believe that the evidence in chapter 4 of the decline in the risk premium, shown by the pricing of the macro variables or the fall in the conditional variance of innovations to the log of the exchange rate show that the FOREX regression underlying the FOREX puzzle is not valid, since risk premia are neither constant nor zero.

The evidence that UK macroeconomic variables were significantly priced in the stock market and FOREX market in chapters 2 and 4 led us to test whether the UK FOREX- and stock markets are integrated in chapter 5. We discussed the risk premium in the FOREX market (single and two investor models) in an asset pricing model when the representative investor has Generalised Isoelastic preferences. The implication of this model was that it is necessary to model the FOREX excess return joint with the stock market return when modelling the FOREX risk premium. Hence this asset pricing model is a natural starting point when we wish to test whether the two markets are integrated. The test we conduct is whether the prices of risk of the variables priced in the FOREX and equity markets are the same. If this is the case, then the two markets imply that the expectation of the Stochastic Discount Factor is the same and hence one can conclude that they are integrated (a conclusion conditional on an assumed asset pricing model). Based on our results in chapter 4, we asked the same question in a multivariate model with money growth and output growth modelled jointly with the FOREX and stock market returns. The proposed test has the drawback that we do not test whether an eventual constant in the SDF is the same across the two assets, but since we rely on well known asset pricing models of the risk premium this problem can be considered less severe. In any case, the proposed test can be used to check whether the prices of risk in two financial markets are the same.

We estimated 6 different models with and without imposing market integration. Interestingly we concluded that all asset pricing models imply that the UK FOREX and stock markets are integrated or the prices of risk for the various factors are the same across UK FOREX and stock market excess returns. One of the potential reasons for this is that the UK stock market and FOREX risk premia have been declining in the last three decades and so has macroeconomic volatility. This suggests that our approach to pricing macroeconomic variables is in the correct direction. Subtracting the implied risk premia from the expost FOREX excess return we checked whether the residuals could be explained by the forward premium (or the UK-US risk-free interest rate differential) and we found that in all consumption-based asset pricing models, except the standard Power Utility inter-temporal CCAPM model, we reject the ability of the differential to explain the excess return residual, whereas this was not the case for the monetary model. This suggests, contrary to what has traditionally been argued, that a time-varying risk premium may be capable of explaining the FOREX puzzle. In particular we showed that pricing consumption growth only, the OLS estimate obtained when regressing the Jensen and risk adjusted FOREX excess return on the risk-free interest rate differential was even more negatively biased than in the traditional Uncovered Interest Parity regression. When we additionally priced inflation, the bias was substantially reduced, suggesting that it is necessary to price several variables, in addition to consumption, to remove *the bias completely.*

6.2 Future Directions

We have focused on "in-sample" estimation, and it is of interest whether the models we have estimated are also useful "out of sample". An interesting topic for future research could be to evaluate the capabilities of these models of the risk premium to forecast out of sample. One problem with our models for this purpose is that we have been using "final" data, as theory tells us that we should use these data, rather than "real" time data. "Final" data are revised data, and this is a fundamental problem with macroeconomic data. Very few "real" (by "real" we mean data that have not been revised after first announcement) are available at a monthly frequency. An interesting topic for future research would be to estimate the stock market and FOREX market risk premia using real data when these become available with a sample that is long enough to perform the estimations. If real data are useful for estimating risk premia, then this could potentially also be an advantage for forecasting out of sample but we leave this topic for future research.

Another potentially interesting topic could be developments of the consumption-based models in chapter 2, allowing for time-varying parameters. There is some literature, as discussed in the introductory chapter, that considers modelling the time-varying parameters depending on various lagged values that have been found capable of explaining a small proportion of the variability of stock returns. However, it seems rather "ad hoc" that this should be so and we believe that more appropriate theories on how to model this parameter time-variation (in other words how to model the conditional Stochastic Discount Factor) are needed.

A final area where we feel research is necessary is the development of other larger scale multivariate GARCH models. We took a first step in chapter 4 and our proposed estimation method was able to create a FOREX risk premium that varied considerably over time but we feel that our assumptions underlying the estimation were rather strong and it could be interesting to make further developments of the model without complicating estimation. Finally, a new class of multivariate GARCH model, the Dynamic Conditional Correlation model, seem to have become quite popular for empirical research. However, this model has its estimational "advantage", when applied to financial returns, in the assumption that risk premia are zero, constant or can be proxied by lagged returns (or excess returns). Future developments of this model, allowing returns to obey a no-arbitrage condition, may be a step in the right direction.

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