Monetary Policies for Open Economies

Beatrice Pierluigi

Degree in Economics, Università Bocconi Milano, 1994

Submitted for the degree of Doctor of Philosophy

to the University of York

Department of Economics and Related Studies

February 2000

PAGE NUMBERING AS ORIGINAL

Abstract

International monetary policies are at the centre of the thesis. The investigation is both theoretical and empirical. The theory finds its roots within DSGE models, where money enters agents' utility function and is the object of policy concern because of imperfections in assets or goods markets. The empirical evidence focuses on the behaviour of the UK relative to its foreign counterparts: in turn the OECD aggregate, the US and the EU.

In the first part of the thesis the UK is interpreted as a small open economy whose cyclical behaviour is entirely determined by five exogenous driving forces, domestic and foreign. The presence of traded and non-traded goods leads to the failure of the purchasing power parity. Simulations of the model are able to produce a highly volatile real exchange rate. Estimated impulse response functions show overshooting of the exchange rate after a monetary shock. Foreign shocks can explain most of the volatility of the variables.

The second part deals with a two-country economy with pricing-to-market and price stickiness and where monetary shocks are the only source of fluctuations. The estimate of a VECM for the UK and the US economy uses theoretical long-run restrictions. Given the high degree of interdependence found between the two economies, we introduce a policy rule for the UK that takes into account US monetary shocks. The model is also used for investigating the behaviour of the UK against the EU countries and to test for the benefits and costs of having a Monetary Union. We then introduce productivity and preference shocks and simulate the model for different values of the rigidity parameter. The degree of price stickiness determines the size and the length of real effects after a monetary shock.

The third part of the thesis carries out optimal monetary policy exercises. First, a general rule for the nominal interest rate in the UK is estimated and compared to the optimal solution of the Central Bank problem. Second, we build a Central Bank problem whose preferences are subject to a constraint that relates inflation differentials between the two countries to the real exchange rate. This allows us to build a real exchange rate-inflation gap volatility frontier showing a trade-off between the two choice variables.

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Preface

'It is perfectly true, as philosophers say, that life must be understood backwards. But they forget the other proposition, that it must be lived forward', Kierkegaard, 1938[1].

Why do economies fluctuate? And why do we care? These questions have been at the centre of theoretical and empirical analysis since the beginning of macroeconomics as an autonomous discipline. Technological, environmental and institutional changes are constantly renewing those questions, over time. In the last twenty years changes in the world-wide economy have been coupled with a 'revolution' in economic thinking, modelling and testing. As a result new challenging answers have been proposed to the questions at the outset. In the first chapter we provide evidence of this threefold revolution by looking at international macroeconomic aspects. We will rephrase the previous questions in these terms:

1. Are there any empirical regularities in the economies' fluctuations?
2. Is there a preferred instrument for measuring them?
3. Is there a preferred model to explain them?

Questions 2 and 3 imply that we should try to find, empirically and theoretically, the sources responsible for the economies' fluctuations. Chapter 1 enumerates problems and obstacles in giving satisfactory answers. After providing a selective reading of the empirical and theoretical literature that has addressed these problems in the last twenty years, the chapter gives the motivation and the guide lines for the structure of the remaining parts of the thesis. At the centre

[1] The Journal of Soren Kierkegaard, 1938, Oxford: Oxford University Press.

of the analysis lies money as a policy issue, acting as source of and response to the business cycle.

This work finds its place within a research project[2] whose aim is to bridge the gap between the theoretical modelling typical of Dynamic Stochastic General Equilibrium (DSGE) models and the econometric approach of Vector Error Correction models (VECM), although with autonomous and original contributions. Grounded in a methodological approach that suggests how to use DSGE models for detrending the economy and interpreting its cyclical fluctuations, the thesis follows three directions and, thus, is divided into three parts:

1. Monetary shocks and the determinants of the real exchange rate and the current account fluctuations are analysed within a small open economy.

2. The international monetary transmission mechanism is then considered in a two-country economy.

3. Optimal monetary policy exercises are finally carried out in a two-country economy.

The first part consists of chapters 2, 3 and 4. Chapter 2 presents the small open economy model. The model is very stylized. Output is exogenous and divided between traded and non-traded goods. Under this distinction the law of one price holds for the traded but not for the non-traded sector. We can therefore meaningfully introduce the real exchange rate.

The foreign economy (the rest of the world) is taken as exogenous and is big enough with respect to the small open economy to determine the entire path of

[2]Prof. Mike Wickens suggested and guided the research project at the beginning of my D. Phil studies. The group was composed by Dr. P.N. Smith, R. Motto and F. Perrone.

the real interest rate. The Government, however, in a world where the nominal exchange rate is perfectly flexible, carries out an autonomous monetary policy and can affect the nominal interest rate. Given uncovered interest rate parity (UIP), the nominal exchange rate plays the role of an asset price. The money supply follows an autoregressive process. Thus, in total, five exogenous driving forces determine the cyclical behaviour of the economy. Two of them derive from abroad: the foreign price and the real interest rate, and three have a domestic origin: they come from non-traded and traded output and from money supply. We derive the steady-state solution. The long-run relationships are interpreted in terms of cointegration and used to detrend the economy.

Chapter 3 describes and implements a procedure for estimating the model. The small open economy under analysis is the UK. Data are quarterly and the sample period runs from 1969:3 to 1997:3. First, a VAR on theoretical cointegrating vectors and structural errors is estimated. Given that the cointegrating vectors originate in the model, the short-run impulse response functions of their residuals to the structural shocks have a meaningful interpretation. They provide evidence of the overshooting of the real exchange rate after a foreign interest rate shock. Second, a structural VECM based on the same theoretical cointegrating vectors and same exogenous variables is estimated. In this case impulse response functions describe the dynamic behaviour of the original variables to the structural errors. They give a long-run measure of the behaviour of the system, whereas the VAR impulse responses show the short-run path of the deviations of each variable from common stochastic trends. Traded and non-traded output shocks, respectively measuring global and country-specific productivity shocks, have a different impact on the behaviour of traded consumption. The former are more persistent than the latter. A shock to the foreign interest rate and to the money supply leads to the overshooting of the nominal exchange rate. The forecast error variance decomposition shows that foreign shocks (either prices and interest rates) have been the main determinants of the UK fluctuations over the past 30 years.

Chapter 4 concludes the analysis of the small open economy. The final form of the model is derived and the short-run behaviour of log-linear deviations from the steady state is simulated. Simulated impulse response functions to temporary shocks of the exogenous variables, derived from the restricted VAR, are compared with those obtained in the previous chapter from the unrestricted VAR. The simulations can, in most cases, replicate the patterns showed by the actual data, although only in the shape and not in the dimension of the responses. The comparison is then carried out by computing unconditional moments of the artificial data and of the actual data. The model can generate a highly volatile real exchange rate but, in general, its cyclical properties poorly replicate the cyclical properties of the data, especially those related to the domestic price level. These exercises show, therefore, that the model is better suited for providing robust long-run restrictions on the actual data than for generating short-run patterns from the simulated time-series.

The second part consists of chapters 5, 6 and 7. With respect to the previous framework, chapter 5 introduces some complications into the transmission mechanism on the one hand, but narrows the investigation of the sources of fluctuations on the other hand. The whole world economy is now treated as endogenous and is modelled within a two-country model. To keep things simple output is exogenous, although we demonstrate that by allowing for its endogenous determination the previous analysis holds. The unique source of fluctuations is the money supply, either domestic and foreign. The assumption of Pricing-to-Market allows us, then, to model the real exchange rate. The assumption of price stickiness, following Calvo's (1983) model, makes it possible to have real effects coming from monetary shocks. The model is solved by taking the difference between domestic and foreign variables. This leads to the creation of an eight-variable system that can be easily simulated and estimated. Simulations show that the degree of price stickiness determines the size of the autocorrelation coefficient in the price differential time-series. We also show that the size of the liquidity effect does not depend on the degree of price stickiness. Domestic and foreign monetary shocks have asymmetric effects.

A structural VECM is then estimated for the UK and the US data by imposing cointegrating vectors derived from the solution of the model. This leads to a new interpretation of nominal and real interdependence between the two economies and to test for the asymmetry of shocks. UK monetary policy heavily depends on US policy and this entails the estimation of a policy rule for the UK that takes into account US monetary shocks. At this stage of the thesis the policy rule does not have any 'normative' meaning, but it aims at capturing the 'systematic' part of the monetary policy, which depends on the economic relations derived from the solution of the model.

Chapter 6 offers an empirical analysis of the effects of monetary shocks in an European context. The two-country model is used for investigating the behaviour of the UK against the average of the EMU countries and to test for the benefits and costs of having a Monetary Union. Despite of the problem related to a forward looking interpretation of the estimates (because the data used correspond to a different monetary policy regime - Lucas' critique); the exercise leads to the finding that the UK would have been better off with a monetary policy EMU determined while facing US shocks because more insulated.

Chapter 7 extends the model of chapter 5 by introducing a production sector and shocks coming from technology and preferences. The log-linear equations derived from the solution of the model are estimated by Instrumental Variables (IV) and interpreted according to a 'two-country' AD-AS model. The 'two-country' AS curve is not standard because it does not relate the output gap to inflation, but it does describe a relation between the actual and expected inflation differential with the real exchange rate. Its estimation for the pair UK-EU leads to find a value for the length of stickiness of a year. In this model, where fluctuations suboptimally drive away the economy from the steady state, policy interventions can be justified.

Therefore, in chapters 8 and 9 we carry out optimal monetary policy exercises. Chapter 8 starts with a comparison between the equations derived from the two-country model of chapter 7 and those of the standard 'textbook' Dornbusch (1976) model. The two-country model delivers relationships among macro-variables with an intertemporal dimension, leads to a structural interpretation of the parameters and relieves expected inflation from the relation with the output gap (the Phillips curve). We take from the two-country model the domestic equation for the goods market, the uncovered interest parity and the two-country 'Aggregate Supply' curve. We complete the model by adding a policy rule for the UK, inspired by rules of the Taylor's (1993) type and incorporating future inflation and foreign variables. We estimate the rule by Instrumental Variables and we test its theoretical implications by studying the system's reaction to exogenous shocks. The performance of the estimated rule is better understood when compared with two well known rules, that have been shown to be robust to different models: a Taylor and a Forward rule. The estimated rule shares many properties with the Taylor rule. We show that a rule that does not take into account output gap (the Forward rule) leads to a more unstable system, when facing exogenous foreign shocks.

We then use the model to constrain the Central Bank in its optimal policy problem. We consider a one-period problem and we model Central Bank's preferences as quadratic deviations of output and inflation from their target values. The optimal policy implies to set the nominal interest rate equal to the foreign interest rate plus a 'correction factor' that depends on the size of the inflationary bias, on expected future inflation and on foreign inflation. We obtain a measure for the total loss. The optimal policy leads to lower volatility and lower loss than the estimated rule. We show that among rules, the Taylor rule produces the lowest loss.

In chapter 9 we let the UK Central Bank's preferences to depend on the inflation gap between the UK and the EU and on the real exchange rate. We find a direct relationship between inflationary bias and degree of stickiness. We then rule out the possibility of systematic inflationary bias and build the real exchange rate-inflation differential volatility frontier. There exists a trade-off for every value of the weight attached to the real exchange rate volatility in the Central Bank's preferences. When this weight is low we obtain a limited variability of the inflation gap. The same trade-off can be found in actual data when computing the real exchange rate and the inflation differential volatility between the UK and the EU.

Acknowledgment

I have been lucky during the time spent at the University of York because I have met extraordinary people, who have helped me in many ways. First of all, I am grateful to Professor Mike Wickens, for his invaluable teaching and guidance in Economics and Econometrics. I thank Dr. Peter N. Smith for his helpful discussions, suggestions and advice. I have benefited from comments and discussions with very good friends who have been fellow-students in Economics at the University of York: Francesca Perrone, Roberto Motto, Claudio Zoli.

I thank Emanuela, Maura, Teresa, Christine, Nicole, Susan, Shyamala, Shin-Chun, Andrea, Giovanni, my 'little sister' Francesca and my new 'conquest' Chiara, for their great friendship during my stay in York. They have helped me in many difficult situations and we have had so much fun together. I wish them all the best.

Many thanks to Professor Luigi Bernardi for having encouraged me in undertaking my D. Phil. studies at the University of York and for his invaluable advice. I thank Professor Roberto Artoni, under whom I graduated five years ago, for his generous help during all my postgraduate studies.

I thank Barbara and Nick, a splendid family, who took care of me in my last months in York.

Financial support from the ESRC for the academic year 1998-99 (scholarship no. R00429824620) is gratefully acknowledged.

I would like to dedicate this work to my parents and my brother, whose unconditional love gave me serenity and strength in this period lived apart.

Author's Declaration

The second chapter of the thesis is a simplified version of the joint work with Francesca Perrone presented at the European Summer School of the European Economic Association held in Paris (7-12 September 1998), with the title: 'Shocks and Monetary Policy Rules in a Small Open Economy'. The authors benefited from very helpful comments from F. Canova and V. Chari.

Mike Wickens wrote the program in Gauss (February 1999) for the VECM estimation used in chapters 3, 5, 6.

The simulations carried out in chapters 4, 5, 7 and 8 have been obtained with the program written by Uhlig (1997) in Matlab available on the web page:

http://cwis.kub.nl/~few5/center/STAFF/uhlig/toolkit.dir/toolkit.htm,

with some author's modifications (Appendix D).

1 Sources and International Propagation of Business Cycles

1.1 Introduction

The identification of the sources and the propagation mechanisms of business cycles is an empirical and theoretical matter of crucial importance. Once we have isolated structural shocks we should be able to provide a measure and a reliable interpretation of the economy's fluctuations. This should then lead to the formulation of useful advice for policy making.

The process of identifying structural shocks is not an exclusively econometric task. It requires, firstly, the construction of a model whose solution can be derived from an optimization problem specified at the level of preferences. In fact, if the model consisted of relations specified at the level of equilibrium conditions and demand functions it would be of little usefulness because it could not say anything about structural shocks. Any estimation of this kind of models would lead, at best, to the identification of a shock hitting each demand function, without the possibility of going back to its origin. One might argue that this should not be a problem if, say, the shock entering the money demand equation could be distinguished from a shock entering the IS equation. The problem is that if we do not know their nature we could well treat the two shocks differently, even though they were composed by common structural elements.

Moreover, if we do not know anything about the structure of the model we cannot use it for policy experiments because parameters will not be invariant to changes in regimes (Lucas, 1977). Therefore, the analysis of the thesis is carried out in an intertemporal, stochastic, general equilibrium (DGSE) model specified at the level of preferences.

The intertemporal approach to macroeconomics finds its roots in Lucas' work. In his 1977 article *'Understanding Business Cycle'*, Lucas put forward a program for empirical and theoretical research about business cycles. One of the main ideas was that *'business cycles are all alike'*, across countries and over time. This observation led him to think that business cycles were driven by a few important factors, perhaps just a single type of shock. Lucas suggested that *monetary shocks* were the most important factors behind aggregate fluctuations.

A second important idea was that formalizing economic agents expectations is crucial for understanding business cycles. Lucas' insistence on grounding policy analysis in actual forward-looking decision rules of economic agents suggested that open economy models might also yield more reliable policy conclusions if demand and supply functions were derived from optimization problems of households and firms, rather than specified to match reduced-form estimates based on ad hoc econometric specifications.

A third idea in Lucas' program was that business cycles should be explained in terms of equilibrium models. Five years after Lucas' (1977) thoughts, Kydland and Prescott (1982) started a research program for explaining business cycles in terms of neoclassical growth models. Real Business Cycle (RBC) models were built in the spirit of Lucas (1977) in the sense that they stressed a single type of shock (but in this case innovation in *technology*) as the source of macroeconomic fluctuations. A calibrated version of stochastic models driven by technological shocks was then used to derive theoretical stylized facts, in the form of variances, cross correlations and autocorrelations. These model-based statistics were finally compared with the corresponding population statistics. In this approach there was typically no estimation and no testing.

Contemporaneously to the RBC approach, an alternative approach used to investigate sources of business cycle has been carried out by means of vector autoregression (VAR) analysis. The seminal article belongs to Sims (1980) and expresses the dissatisfaction with the common practice of using strong overidentifying restrictions, sometimes based on theory of rather dubious empirical status, within models of simultaneous equations (SEM). The remedy suggested by Sims was to avoid overidentification and limit the role of theory to that of providing the set of assumptions necessary to pass from the residuals of vector autoregressions to structural shocks.

These two main approaches to macroeconomic research (RBC and VAR) are almost orthogonal in their view about the roles of economic theory and econometric methods. However, they share the general idea that the time pattern of macroeconomic variables (the business cycle) is the result of the interaction between a set of stochastic disturbances (the impulses) and a model describing the propagation mechanism. Under the RBC perspective, business cycle phenomena are the result of permanent real shocks to the economy supply side rather than transitory and demand side shocks. Instead early VAR models (Sims, 1980) did not allow for permanent (real) shocks. All shocks were transitory by assumption and changes in technology with permanent impact were captured by a deterministic trend. Those shocks were initially identified by using the Cholesky factorization, that is Sims imposed an atheoretical recursive triangular structure between dependent variables and error terms. Of course, there is no reason (from a theoretical or an empirical point of view) to believe that the structure of instantaneous shocks is recursive.

Other type of orthogonalization schemes, more general than the Cholesky factorization, have been proposed since 1980, claiming to be theoretically grounded. The new approach has been called the Structural VAR analysis. For example, Blanchard and Quah (1989) applied to the residuals long-run restrictions based on economic theory. Others, like Blanchard (1986) and Gali (1992), introduced interpretative schemes for the instantaneous correlations among the error terms of VAR guided by theoretical considerations.

Important ideas for the RBC and VAR models have been brought together by King, Plosser, Stocks and Watson (1991). Their paper takes its point of departure from a Solow-type model for a closed economy where technology follows a stochastic trend. King's et al. (1991) econometric procedure uses the long-run balanced growth of the model to isolate the permanent shock in productivity and, then, to trace out the short-run effect of this shock. This procedure relies on the fact that balanced growth under uncertainty implies that consumption, investment and output are cointegrated. In turn, this means that a cointegrated VAR nests log-linear approximations of all RBC models that generate long-run balanced growth. The common stochastic productivity trend is capable of explaining fluctuations in consumption, investment and output in a three variable reduced-form system. But the power of the common trend drops off sharply when money, the price level and the nominal interest rate are added to the system. Their evidence does not support the key implication of the standard RBC model, that permanent productivity shocks are the dominant source of economic fluctuations. Moreover, nominal shocks, identified by imposing long-run neutrality for output, explain little of the variability in the real variables.

These results suggest that models that uniquely rely on permanent productivity or long-run neutral nominal shocks are not capable of capturing important features of the postwar US experience.

The conclusion reached in King et al. (1991) paper's provides a critical starting point for investigating the role of money and, in general, of nominal variables in understanding the business cycle. The empirical and theoretical literature that undertook this investigation in the last ten years came across many obstacles and a number of 'puzzling' results.

In this chapter the reading of open economy models is done in the spirit of Lucas' (1977) work. Moreover, King's et al. (1991) work provides a challenge for a twofold investigation. On the one hand, the emphasis is on how international models can be extended to take into account monetary shocks. On the other hand, the analysis focuses on how the literature has dealt with the problem of mapping dynamic stochastic general equilibrium (DSGE) models into their estimated version.

The structure of this chapter is the following. In section 1.2 we describe the theoretical environment, characterized by the intertemporal approach to open economies (dynamic and stochastic). Within this environment, we look at the International Real Business Cycle (IRBC) literature by focusing on the directions in which it failed in explaining some empirical stylized facts. Thus, in this section we present several attempts to overcome the empirical puzzles.

In section 1.3 we introduce money and nominal variables in intertemporal open economy models. Modelling the impulse and the propagation mechanism of monetary shocks requires to add new assumptions to the IRBC approach. If, on the one hand, monetary models can help to solve some empirical puzzles, on the other hand, the identification of the exogenous source of a monetary shock is a matter of controversy. This is because once we allow for shocks coming from policy we need to make a clear distinction between the endogenous policy responses to changes in the economic environment and the exogenous policy change. In this last decade a great amount of empirical literature has focused on this second task (Christiano, Eichenbaum and Evans, 1998, provide an excellent survey).

Remaining aware of this identification problem, the theoretical literature that introduces money needs to consider some 'transmission mechanisms' as well. Thus, in section 1.3 we deal with short-run rigidities that allow money to have real effects.

The natural extension consists, then, of looking at optimal monetary policies in open economies. In section 1.4 we survey the existing approaches to the optimal policy literature. We can observe a distinction between empirical and theoretical approaches. The empirical approach looks for a rule able to capture the actual behaviour of the monetary authority, the theoretical approach aims at modelling the Central Bank behaviour and at suggesting optimal policy making.

In section 1.5 we draw some final conclusions. This three-steps survey creates the appropriate background to the thesis. Thus, we conclude by singling out the key concepts of the survey and we relate them to the key concepts over which the thesis is built.

1.2 Real Shocks under the International RBC Perspective

1.2.1 The Empirical 'Puzzles'

No more than ten years ago the RBC approach started to be extended to open economies. RBC theories have been criticized on the grounds that they require implausibly large and frequent technological shocks. While such a criticism may have some validity in a closed economy, neither large nor frequent shocks are required to generate business cycles in a trade-dependent economy. A country-specific disturbance can lead to strong, prolonged fluctuations because its initial impact can be amplified by a feedback effect operating through the trade account. Similarly, the possibility of imported business cycles can drastically reduce the frequency of required supply shocks for any individual country. These suggestions signalled that a big gain could have been made by extending the RBC theories to international models. But with the very first works on IRBC models this hope was easily cancelled out.

The typical international framework is a two-country model (Backus, Kehoe, Kydland, 1992, 1995) and the impulse of the motion comes from a common world exogenous shock to technology. The two economies are usually thought to be large, allowing for the interest rate to be endogenously determined.

This approach has been very useful for the explanation of some international properties of the data: the time series correlation of saving and investment (Baxter, Crucini, 1993), the countercyclical movements of the trade balance (Backus, Kehoe, Kydland, 1994) and the relation between the trade balance and the term of trade (Backus, Kehoe, Kydland, 1994). The ingredients for this success relied on the possibility of intertemporal substitution, which takes place through the current account, on the high degree of persistence of the forcing processes and on adjustment costs on the capital accumulation equation.

However, the theory remained significantly different from the data in three directions (Backus, Kehoe, Kydland, 1995 and Baxter, 1995): the first two are related to comovements problems, the latter to a volatility problem. The first discrepancy with the data, the *output-consumption anomaly*, concerns the relation between the business cycle of consumption and output across countries. While actual correlations of output across countries are larger than the analogous correlations for consumption and productivity, in the theoretical economy the opposite happens. In fact, according to the model, the ability to trade internationally affects the economy's behaviour by breaking the link between production and spending on consumption and investment. This permits a country to have a smoother consumption over time than in a closed economy and a greater investment response to changes in expected rates of return.

The second discrepancy, the *international comovement anomaly*, concerns the investment cross-country correlation. Despite the basic IRBC model's prediction that investment and labour are negatively correlated across countries these correlations are positive in the data. The reason is that the one good characteristic of the model, combined with the international mobility of physical capital, leads the capital to move to its most productive location in response to persistent productivity shocks. Given that capital moves to the more productive location, the returns to labour rise in the country with the investment boom, while they are low in the other country.

The third discrepancy, called the *price-variability anomaly,* concerns the volatility of the term of trade and of the real exchange rate that are much higher in the data than in the theoretical economies.

The IRBC models following the work of Backus, Kehoe and Kydland (1992) attempted to solve those puzzles, by extending the theory to include non-traded goods, by considering a higher number of sources of shocks, by introducing incomplete markets, imperfect competition and money. While the first two anomalies can be explained within a 'real' model that allows for some imperfections in trading goods or assets, to explain the third anomaly we need to introduce 'monetary' elements into the story.

The measure of fit of models relied on the comparisons between theoretical and actual moments (standard deviations, auto and cross-correlations).

1.2.2 Non-traded Goods and more than one Source of Shocks

One of the first attempts to overcome the empirical puzzles summarized above was the introduction of some complications in modelling the productive structure and the exogenous driving forces. Traded and non-traded goods and several sources of real shocks started to be introduced into IRBC models.

These two modifications are linked. The presence of traded and non-traded goods allows to consider global or common (generated or transmitted through traded goods) and idiosyncratic or country-specific shocks (generated or transmitted through non-traded goods) and thus to relax the assumption of a single worldwide shock.

Stockman and Tesar's (1995) paper deals with the output/consumption anomaly. Their model disaggregates the economy into internationally traded and non-traded sectors, both subject to random shocks to productivity. They succeed in matching some of the features of consumption and production data and in replicating the international correlations of aggregate output and consumption and the countercyclical behaviour of the trade balance.

Three problems remain. First, the model cannot account for the cross-country correlation of consumption of traded goods. In other words the consumption anomaly has been pushed onto the traded component of consumption. Second, it cannot explain the near zero correlation between the relative price of non-traded to traded goods with the relative consumption of those goods. Third, it understates the standard deviation of the trade balance. The model is then adjusted to include shocks to taste that, although improving its performance, do not completely solve all the previous problems.

Canova and Ubide (1997) succeed in solving the consumption puzzle and the international comovement puzzle. Their model introduces household production. Empirically, household production is an important feature of the real world economy, and theoretically, it provides a rationale for the presence of non-traded goods in an international business cycle model. In Canova and Ubide's (1997) two-country model each country produces one intermediate tradable good with a market technology and one final non-tradable good with a household technology. Since household goods can only be consumed, household production disturbances play the role of taste shocks. They change the composition of the bundle consumed by agents in equilibrium, the allocation of time between market and non-market activities and the composition of investments by sectors, therefore producing dynamics that are different from those generated by disturbances to the market technology. In addition, because household production is not part of measured GDP, disturbances to the household technology affect market output only to the extent that the elasticity of substitution between market and non-market goods is different from zero. Thus, in this model the degree of substitution between the two types of goods is crucial for solving the consumption puzzle. The higher is the substitutability between consumption of market and non-market goods, the lower are likely to be international market consumption correlations (relative to output correlations). Also the international comovement puzzle disappears in this model. When household production requires capital, negative shocks to market technology reduce investment in the domestic and foreign market sector. Capital is reallocated to household technologies and this leads to a cross-country correlation of market investment less negative than in traditional IRBC models. Similarly, with positive household technology shocks, investment in the market sectors of both countries tends to decline leading to a positive correlation of market investments. Therefore, a combination of technology disturbances in both sectors helps to generate cross-country investment correlations in the range of values observed in the data.

The model is not able to explain the variability of the term of trade. To make the relative price of exported goods as volatile as the data, it seems that some more dynamics (either endogenous or exogenous) is needed.

1.2.3 Incomplete Markets

The IRBC literature has assumed, so far, that individuals have access to complete international contingent-claim markets that permit them to pool all risks.

Conversely, the strand of literature that studies business cycles in small open economies, typically, restricts access to international risk sharing in ways that seem, empirically, to be more reasonable than the assumption of complete markets. But these analyses are necessarily silent on the factors affecting world interest rates and asset prices. Baxter and Crucini's (1995) paper develops a two-country, general equilibrium model where individuals have incomplete access to international risk sharing. The model evaluates the importance of financial market linkages for international business cycles by comparing its predictions to a model with complete markets. It provides two important results. The first is that restricting asset trade to non-contingent bonds does not alter the predictions of the standard, complete markets model, if the international productivity process is trend stationary with international spillover. If, however, productivity in each country follows a random walk without spillover the incomplete market version produces high output correlation and low consumption correlations, explaining in this way the 'consumption puzzle'[3]. This last result relies on different wealth effects under the two market structures. When productivity follows a random walk, under complete markets, individuals receiving a favourable productivity shock experience a negative wealth effect. This is because the optimal insurance character of equilibrium requires them to increase labour supply while transferring a large proportion of the additional output to residents of the other country. In the bond economy, however, individuals own all the risky claims to their country's output. Thus, individuals receiving a favourable productivity shock experience a positive wealth effect which induces them to increase consumption by more than in the complete markets economy and causes them to decrease labour input. This result helps to solve the international comovement puzzle.

Therefore, the empirical implications of Baxter and Crucini's (1995) model are very sensitive to the specification of the stochastic process for productivity. If productivity follows a trend-stationary process with highly persistent shocks and international transmission, the business cycle implications of the incomplete market economy are very similar to those of the complete markets economy. However, if productivity follows a random walk without transmission, the impli-

[3]Kollman (1996) as well demonstrates that an International RBC model with incomplete asset markets can generate cross-country consumption correlations that replicate the cyclical behaviour of the data.

cations of the alternative model are quite different. It is thus important that the exogenous characterization of the model should be based on the estimation of the forcing processes of the system. But this aspect is missing from the analysis of Baxter and Crucini (1995).

In this section we saw that an effective way for solving the two correlation puzzles has been to introduce some mechanisms for isolating one country from the other. These mechanisms work either through the goods or the assets market and they must be coupled with some asymmetry in the effects of the exogenous shocks to the economic system.

1.3 Monetary Shocks and their International Transmission Mechanism

The literature analysed so far is completely real. In an Arrow-Debreu framework without imperfections there is no reason for holding money. In a bond economy the introduction of money could instead be justified because of the limited possibility of risk-pooling future uncertainty (that is transferring a unit of consumption goods from one state of the nature to another).

In what follows we will try to understand how the introduction of money can help to explain the price variability puzzle, which could not be accounted for by the literature surveyed above. Specifically, we will look at the behaviour of the real exchange rate (the ratio of national price levels) instead of the term of trade (the relative price of exports in term of imports). Moreover we will give a general overview of the reasons why monetary policy shocks can contribute to a better understanding of international business cycles.

The introduction of monetary policy brings about a different dimension (or perspective) in our analysis. By introducing policy shocks we implicitly allow for policy intervention which can affect, in one way or another, the economic system according to the transmission mechanism designed. Thus, the peculiarity of dealing with monetary policy shocks concerns not only the 'nature' of the shock, but also the kind of monetary transmission mechanism adopted.

Typically the identification of the transmission mechanism is a theoretical issue whereas the identification of a monetary policy shock is an exclusively empirical issue. This mismatching of objectives arises because DSGE models are not directly estimated, but simulated. Therefore they do not need to ask whether the use of an unconstrained money supply mechanism leads to the actual identification of exogenous shocks to money supply. There exists convergence only when the econometric exercise, based on a specific identification procedure for the exogenous monetary shock, is used for assessing the empirical plausibility of theoretical models. As a result of this duality between theoretical modelling and econometric analysis, our survey will distinguish between the literature devoted to the investigation of the transmission mechanisms and that devoted to the identification of exogenous shocks to monetary policy. Within the first line of investigation the real exchange rate can potentially play a crucial role.

There are two main approaches to the study of the role of the real exchange rate in the transmission of monetary shocks. The first assumes price flexibility but allows for important distributional effects of monetary policy (Lucas, 1990). This is called the liquidity approach. The second approach assumes sticky nominal prices.

Obstfeld and Rogoff's (1995) paper introduces this second line of research. It explores the determination of exchange rates and the international monetary transmission mechanism in an intertemporal general equilibrium model with nominal price rigidity. Obstfeld and Rogoff (1995) show that monetary surprises have permanent welfare effects, because without complete risk pooling, a money shock affects the current account. Thus, their model generates ex-post long-run non-neutrality after a monetary shock. They find, however, contrary to the intuition of the Dornbusch (1976) model, that sticky prices do not give rise to the overshooting of the exchange rate. In fact, the response of the exchange rate to a money shock is identical to what it would be in a flexible price economy. Their model does not help in explaining the real exchange rate variability that we observe in international data. Given that preferences are identical across countries, all goods are freely traded and prices are set in the currency of the seller, purchasing power parity (PPP) holds at all times.

1.3.1 The Transmission Mechanism: Evidence and Theory on Pricing-to-Market

The evidence seems to suggest that a missing piece in the Obstfeld and Rogoff's (1995) model consists of some type of goods market frictions. Whether or not prices are sticky, the law of one price will hold when goods are freely traded and consumers are making unconstrained choices. Deviations from the law of one price in traded goods require that markets are segmented by countries in some way. In the last decade the empirical literature has attributed real exchange rate fluctuations to systematic failures of the law of one price among internationally traded goods industries (Engel and Rogers, 1996). Since the mid-1980s, many studies have shown extensive Pricing-to-Market (PTM) in traded good industries. That is, firms tend to set prices in local currencies of sale and not adjust prices to movements in the exchange rate[4]. Some recent papers have developed a segmented markets approach to real exchange rate determination. Kollman (1997) studies a dynamic optimizing open economy model in which nominal prices and wages are set in advance. The paper uses a Calvo (1983) type price and wage adjustment process by assuming that nominal prices and wages are changed after time intervals of random length[5]. A semi-small open economy[6] is modelled with four types of exogenous shocks: to the domestic money supply, to the domestic labour productivity, to the price level in the rest of the world and to the world real interest rate. Kollman (1997) succeeds in generating variability of nominal and real exchange rates consistent with that of quarterly G7 effective exchange rates during the post-Bretton Woods era. Money supply changes are the dominant source of exchange rate fluctuations. The model exhibits exchange rate overshooting in response to money supply shocks. It predicts that a positive shock to the domestic money supply lowers the domestic nominal interest rate, raises domestic output and leads to a nominal and real depreciation of the country's currency. Likewise, an increase in the foreign interest rate induces a

[4]Engel and Rogers (1996), found evidence that the failure of the law of one price depends on the distance among locations where goods are sold and on the presence of national borders that separate locations. They show that the distance is a significant determinant of price dispersion, but the major determinant is the effect of the border relative to the distance.

[5]At each period, a randomly selected fraction α of producers revise their nominal price according to the simple indexation rule: $P_{jt} = (1 + \overline{\pi})P_{jt-1}$, where $\overline{\pi}$ is the average inflation rate. The remaining 1-α choose P_{jt} so as to maximize the value of their profit. α reflects the extent of nominal price rigidity. At any given period, the probability of being allowed to reoptimize their price is 1-α for all firms, regardless of the time elapsed since the last price optimization. Thus the average duration of price fixity is given by $\tau = \frac{1}{1-\alpha}$.

[6]Semi-small economy: in the sense that the home country faces a downward-sloping aggregate export demand function, while import prices and the international interest rate are exogenous.

nominal and real depreciation of the country's currency. These predictions seem consistent with the empirical evidence on the effects of monetary shocks provided by Eichenbaum and Evans (1995) and Grilli and Roubini (1995).

In a similar paper, Chari, McGrattan and Kehoe (1997) examine a two-country model with prices also set in terms of local currency and complete assets market. They employ the staggered price model of Taylor (1979) to determine the rules for the price adjustment[7]. For their benchmark model with conventional assumptions about money demand elasticities and four-quarter price stickiness, they cannot match the variability and the persistence in observed real exchange rates. But with a very low consumption elasticity of money demand, and six-quarter price stickiness, they can get close to matching the data. In a more recent version of the paper, Chari, McGrattan and Kehoe (1998) rewrite a model with separable preferences over consumption and leisure. The model can account for essentially all the volatility in the real exchange rates. To match the persistence the model still requires a quite long price stickiness (12 quarters).

Finally, Betts and Devereux (1996, 1997) show that a segmented markets approach to the exchange rate determination can explain the high variability of the real exchange rate and of the term of trade. Their model has two characteristics:

[7]All nominal prices in the economy are assumed to be fixed by two-period nominal contracts. Agents negotiating in period t agree on nominal price z_t that will prevail in their trades during period t and $t+1$. Price contracts for half of the economy's output are alternately negotiated each period, so that in period t the log nominal price level p_t is the average of z_t and z_{t-1} :

$$p_t = 0.5(z_t + z_{t-1})$$

Thus in period t, half the economy's output is priced at z_t with the remaining half priced at z_{t-1}, while in period $t+1$, half of output remains priced at z_t and the rest of output changes prices from z_{t-1} to z_{t+1}.

first, nominal price stickiness is introduced through staggered price setting and second, local pricing is set by firms with monopoly power in the international trade of differentiated goods. The model suggests that local prices are sticky in response to money shocks although the exchange rate responds significantly, thus all the real exchange rate variability in the model comes from the existence of PTM.

1.3.2 Testing International Monetary Transmission Mechanisms with VARs: the Empirical 'Puzzles'

So far, the measurement of the business cycle has followed the method typical of the RBC theory. This approach relies on sample variances, covariances, and autocovariances between the variables involved, after some sort of detrending[8]. Indeed, while enumerating the empirical anomalies we have always referred to this kind of evidence.

The approach of vector autoregression (VAR) may be viewed as another technique of data presentation. It is based on multivariate rather than univariate time series methods and relies slightly more on theory and identifying assumptions in order to provide interpretation of the facts. The empirical evidence contained in the last three papers mentioned before combines the use of simple correlations with that of impulse response functions from VAR analysis.

VAR analysis has been widely used in the last decade as an instrument for interpreting the response of the economy to different shocks and for testing the theoretical predictions of IRBC models, with the result of accumulating some other international stylized facts.

[8]The most common detrending procedure consists of applying the Hodrick and Prescott filter to quarterly data.

In a series of recent papers Christiano et al. (1996) apply the VAR approach to derive stylized facts on the effects of a contractionary monetary policy shock and conclude that plausible models on the monetary transmission mechanism, in a closed economy, should be consistent with the following evidence: *i*) the aggregate price level initially responds very little; *ii*) interest rates initially rise; *iii*) aggregate output initially falls with a j-shaped response and with a zero long-run effect of the monetary impulse. Such evidence leads to the dismissal of traditional RBC models that are not compatible with the liquidity effect of monetary policy on interest rates. The evidence seems in line with monetary transmission mechanisms based on sticky-price and liquidity models.

The extension of the stylized facts to an open economy context has been more problematic.

In two papers that use very different methodologies but obtain very similar results, Eichenbaum and Evans (1995) and Clarida and Gali (1994) investigate the impact of monetary policy shocks on G7 real exchange rates, given particular assumptions for identifying innovations in monetary policy. Their results are consistent with the traditional link between monetary policy and exchange rates.

Eichenbaum and Evans (1995) identify a US monetary shock through interest rate innovations and through innovations in the ratio of non-borrowed reserves to total reserves. Looking at separate VARs for Japan, Germany, UK, France and Italy, they find that a US monetary policy contraction leads to a significant rise in US short-term interest rates and a significant real and nominal appreciation of the exchange rate, which is quite persistent. There is overshooting of the exchange rate. According to their estimate monetary policy shocks can explain 40% of the total variability in the real exchange rate, depending upon the currency.

Although these results might be thought to be very specific to the identification assumptions, Clarida and Gali (1994) find very similar results, using a completely different identification scheme. They estimate a VAR based on a structural sticky price model of the exchange rate, where long-run restrictions are used to identify shocks. They find strong support for the sticky price model in their estimates. A nominal shock causes a US real and nominal depreciation and a rise in relative US output. As in Eichenbaum and Evans (1995) the real exchange rate displays a hump-shaped response to money shocks.

The use of similar methods to identify monetary shocks in other countries has been less successful. For instance Sims (1992) shows that for non-US G7 countries a positive innovation in the short-term interest rate is associated with a persistent depreciation of the home currency. The explanation offered is that an interest rate increase in these countries is often a response to a previous increase in the US interest rate.

We can observe that, if on the one hand there is a sound agreement on the properties of the international monetary transmission mechanism a model must possess, on the other hand the schemes adopted to identify exogenous monetary shocks lack of a common ground.

1.3.3 The Identification of Monetary Policy Shocks

The problem of identifying monetary shocks has been a purely empirical matter. For understanding how monetary policy actions affect the economy it is crucial to identify those actions that reflect policy makers' responses to non-monetary developments in the economy. These responses are captured by the notion of policy feedback rule, that is the rule which relates policy makers' actions to the state of the economy. To the extent that a policy action is an outcome of the feedback rule, the response of economic variables reflects the combined effects of the action itself and of the variables that policy reacts to. To isolate the effect of the Central Bank policy action per se, one needs to identify the component of those actions that is not reactive to other variables, namely the exogenous monetary policy shock. Thus, monetary policy actions are the sum of two components: the endogenous part of policy, that is captured by a feedback rule, and the exogenous shock. The question: how does the economy respond to a monetary policy action? is interpreted as: how does the economy respond to an exogenous monetary policy shock? The answers to such questions depend on the identification assumption made to isolate monetary policy shocks and on the transmission mechanism designed. The latter has been analysed in the previous sub-section. The former problem is related to the following puzzles:

1. *The liquidity puzzle.* When monetary policy shocks are identified by innovations in monetary aggregates (such as M0, M1, M2, etc.), they appear to be associated to increases rather than decreases in nominal interest rates. Sims (1992) argued that innovations in broad monetary aggregates reflect other structural shocks (in particular money demand shocks) in addition to monetary policy shocks, and so they are not exogenous. Several researchers suggested other measures of monetary policy shocks that are less likely to reflect non-policy shocks. Bernanke and Blinder (1992) argue that over much of the past 30 years the Fed has implemented policy changes through changes in the federal fund rate. They conclude that the fund rate may therefore be used as an indicator of the policy stance. Christiano and Eichenbaum (1992) have made the case using the quantity of non-borrowed

reserves as the primary measure of monetary policy. Strongin (1995) proposed as a policy indicator the proportion of non-borrowed reserves growth that is orthogonal to total reserve growth.

2. *The price puzzle.* When monetary policy shocks are identified with innovations in interest rates, the output and money supply responses are correct, as a contractionary increase in interest rates is associated with a fall in the money supply and the level of economic activity. However, the response of the price level is wrong, as the monetary tightening is associated with a persistent increase in the price level rather than a decrease. Sims (1992) conjectured that some parts of innovations in interest rates are systematic responses to structural shocks generating inflationary pressures. After including some variables representing inflationary pressures such as the commodity price index in the monetary reaction functions, the price puzzle has been resolved.

Two other puzzles have been found in an open economy context (Grilli and Roubini, 1996):

3. *The exchange rate puzzle.* While a positive innovation in interest rates in the US is associated with an impact appreciation of the US dollar relative to other G7 currencies, such monetary contraction in the other G7 countries is often associated with an impact depreciation of their currency value relative to the US dollar. Two possible explanations have been given. The first is based on the idea that the US is the leader country in setting the monetary policy for the G7 area, while the other countries are followers. The second suggests that interest rate innovations in the non-US G7

countries are endogenous responses to inflationary shocks that cause an exchange rate depreciation.

4. *The forward discount bias puzzle.* If uncovered interest parity (UIP) holds, a positive innovation in domestic interest rates relative to foreign ones should be associated with a persistent depreciation of the domestic currency after the impact appreciation, as the positive interest rate differential leads to an expected depreciation of the currency. This is because, given uncovered interest parity, the positive interest rate differential has to be associated with an anticipation of a depreciation of the domestic currency in order for agents to continue to hold foreign assets. However, the data show that a positive interest differential is associated with a persistent appreciation of the domestic currency for periods up to two years after the initial monetary policy shock. A correct interpretation of the uncovered interest parity condition obtained by solving the equation for the nominal exchange rate forward shows that this last point cannot be considered a puzzle[9] (Wickens, 1999). Since markets' expectations of future interest rate differentials determine the current value of the exchange rate, it will remain appreciated until when those expectations are changed.

The literature has exploited three general strategies for isolating monetary policy shocks. The first involves making enough identifying assumptions to allow the analyst to estimate the parameters of the Central Bank's (usually the Fed's)

[9] If we solve forward the UIP we obtain:

$$s_t = E_t \sum_{k=0}^{\infty} (i^*_{t+k} - i_{t+k})$$

Expectations of future interest rate differentials determine the current value of the exchange rate.

feedback rule. The assumption on the interaction of the policy shock with the variables in the feedback rule is that of recursiveness. The economic content of this assumption is that the time t variable in the Fed's information set does not respond to time t realizations of the monetary policy shock.

The second strategy involves looking at data that signal exogenous monetary policy actions. For example, Romer and Romer (1989) examine records of the Fed's policy deliberations to identify times in which they claim there were exogenous monetary policy shocks.

The third strategy identifies monetary policy shocks by assuming that they do not affect economic activity in the long run.

The literature has finally suggested a threefold interpretation of policy shocks. The first is that they reflect exogenous shocks to the preferences of the monetary authority, perhaps due to stochastic shifts in the relative weight given to unemployment and inflation. A second source of exogenous variation in policy arises because of strategic considerations, developed in Ball (1995). This author argues that the Central Bank's desire to avoid the social costs of disappointing private agents' expectations can give rise to an exogenous source of variation in policy. Specifically, shocks to private agents' expectations about the Central Bank policy can be self-fulfilling and lead to exogenous variation in monetary policy. A third source of exogenous variation in policy could reflect measurement errors in the preliminary data available to the policy maker at the time he makes its decision.

1.4 Optimal Monetary Policy

The survey has made a clear cut between theoretical models that design plausible international monetary transmission mechanisms while leaving unresolved the identification of monetary shocks, and the empirical testing that necessitates to build a policy feedback rule for the identification of its exogenous part. Building a feedback rule implies making precise assumptions about the behaviour of the Central Bank. It implies the knowledge of the Central Bank's targets and of the instruments used to achieve them. If the Central Bank can commit to such a rule then the time inconsistency problem is resolved. Even if in practice monetary authorities are unwilling to renounce their discretionary power and strictly adhere to a rule, they could still benefit from using the rule as a guide for policy discussions. Under such an approach, the policy rule provides information that would help make short-run policy decisions consistent with long-run goals. This is the background of a great amount of literature aimed at designing and implementing monetary policy rules, that followed the work of Taylor (1993).

These rules are designed in a way such that the monetary authority does not need to rely on a specific model of the economy in order to implement them. They are based simply on the feedback mechanism, which specifies precise adjustments in the policy instrument when the nominal target variable deviates from its desired path. This property, in turn, implies that the rules must be robust, that is they have to produce a moderately good performance in a variety of macroeconomic models, rather than be 'optimal' in a single model (McCallum and Nelson, 1998).

The theory of optimal monetary policy is instead model dependent. It usually starts by modelling Central Bank's preferences within an economic model specified at the level of equilibrium conditions and behavioural equations (whose coefficients are some complicated and obscure combinations of 'structural' preferences). The Central Bank's preferences can be expressed in term of loss function or utility function. In general, the loss depends on deviations of output from the natural level and of inflation from the target. There exists a short-run trade-off between the two targets. The possibility of exploiting this trade-off, when the Central Bank's preferences are biased toward a higher level of employment, leads to an inflationary outcome. The optimal policy approach derives the discretionary solution and compares it with the commitment case (Walsh, 1999).

Since the work of Barro and Gordon (1983), who built on an example first presented by Kydland and Prescott (1977), the theory suggests that monetary policy tends to have an inflationary bias when is discretionary. This bias can be eliminated if the monetary authority is able and willing to precommit itself to a policy rule that would ensure price stability in the long run. But the precommitment solution leads to the problem of time inconsistency (i.e. optimal future plans set at time t are not any more optimal at time $t+i$, $i = 1, 2, ..$).

The literature on optimal monetary policy has put forward several solutions to the time inconsistency problem, based on different types of constraints to the monetary authority behaviour or preferences. A first type of solutions is based on the concept of reputation in a repeated game version of the basic framework. The idea is that succumbing to the temptation to inflate today worsens the Central Bank's reputation for delivering low inflation; as a consequence the public expects more inflation in the future and this response lowers the expected value of the Central Bank's objective function.

A second type of solutions is related to the concept of conservatorism (Rogoff, 1985). The Central Bank selects as a policy maker an individual who places a larger than normal weight on achieving low inflation and then gives that individual the independence to conduct policy.

A third type of solutions consists of restricting the policy flexibility by targeting rules. In this case the Central Bank is judged on its ability to achieve a prespecified value for some macro variables. Inflation targeting is currently the most commonly discussed form of targeting and, in different forms, it has been adopted in Canada, Sweden, Finland, the United Kingdom, and New Zealand. The mandate of the European Central Bank to pursue price stability can also be viewed as representing a form of inflation targeting. Fixed or target-zone exchange rate systems can also be interpreted as targeting regimes. The Central Bank's ability to respond to economic disturbances, or to succumb to the temptation to inflate, is limited by the need to maintain an exchange rate target. When the lack of credibility is a problem for the Central Bank, committing to maintaining a fixed nominal exchange rate against a low-inflation country can serve to import credibility. Giavazzi and Pagano (1988) provide an analysis of the advantages of 'tying one's hands' by committing to a fixed exchange rate.

Very recently a new optimal monetary policy approach considers the derivation of the optimal monetary policy rule in micro-founded models (Ireland, 1997, King and Wolman, 1998) by allowing the Central Bank to have the same preferences of the representative private agent. This coincidence of objectives leads the Central Bank to have, as a unique preference, price stability.

The theory of optimal monetary policy in open economies is a rather new object of investigation, as well as the extension to an open economy context of the 'monetary policy rules' approach. Along these two lines we will suggest, in chapters 8 and 9, some particular way of designing and measuring the Central Bank's preferences, targets and rules.

1.5 Conclusions

We conclude by enumerating three key concepts discussed in this chapter that are at the centre of the analysis of the whole thesis.

- *Identification of Real and Nominal shocks.* The need of introducing several sources of shocks within the IRBC literature derives from the inability of the first models to account for several 'stylized facts' in actual economies' fluctuations. Real and nominal shocks have been introduced via exogenous autoregressive processes. Their measurement requires identification assumptions on the specification of the errors. The major problem is that, when we deal with exogenous monetary shocks, which are policy induced, they need to be distinguished from endogenous responses of the monetary authority to the economic environment.

- *The monetary international transmission mechanisms.* Real and nominal imperfections allow for dynamic responses of the economic variables to exogenous shocks that can explain better the actual behaviour of the economy. Real imperfections are those related to the presence of non-traded goods, to the incomplete nature of the assets market and to the possibility of having local currency pricing (Pricing-to-Market). Nominal imperfections pass through some degree of price stickiness in a world inhabited by price setters. The assumption of a particular transmission mechanism leads to have different effects of monetary shocks to the economic system. The international dimension of the transmission mechanism is captured by the behaviour of the nominal and real exchange rate and of the balance of payments.

- *Optimal monetary policy.* Optimal feedback rules can be used for a positive or a normative purpose. From a positive point of view they help the understanding of actual policy making; from a normative point of view they suggest how actual policy making should behave. In an open economy context they can strengthen international interdependence and provide alternative mechanisms for the international transmission of monetary shocks.

2 Modelling a Small Open Economy

2.1 Introduction

This chapter studies the case of a 'small open economy'. In the open economy literature, a small open economy connotes an economy that is too small to affect world prices, interest rates and the economic activity, therefore what happens outside the borders is taken as exogenous from the small economy point of view. Since many countries are really 'small'[10] relative to the world economy, the small open economy model has been used as a framework relevant for studying policy issues. This chapter derives the theoretical model and builds the necessary environment for the estimation and simulation carried out in the next two chapters. The model only partially fulfils the typical properties of a IRBC model. This is mainly due to the fact that it does not describe a general equilibrium economy since it considers as a unique relevant perspective that of the domestic economy. A typical IRBC model would have instead described the world economy within a two-country framework.

The model presented in this chapter has four main characteristics:

1. The economy is characterized by a traded and a non-traded sector, both perfectly competitive. Households possess an exogenous endowment of output, which is distinguished between traded and non-traded goods. Purchasing power parity (PPP) holds for the traded good, but not for the domestic or foreign price index, that, in turn, implies a meaningful role for the real exchange rate.

[10] After the EMU one can argue that this is not any more true for the countries members.

2. There exists an incomplete assets market. The only asset available is a foreign bond indexed to the foreign general price level. The foreign bond market is used to balance the domestic budget constraint. The presence of a unique non-state-contingent risk-free bond, which 'clears' the net foreign position, introduces a kind of imperfection in the model. Therefore one cannot have risk pooling in this incomplete assets market.

3. Money enters the utility function and is used for the domestic purchasing of goods, it does not yield any return. Since holding real money balances yields direct utility and ensures a positive demand for money, in equilibrium money is held as a value. The Government prints money which transfers, in a lump-sum form, to households.

4. Agents are facing an exogenous real interest rate equal to the foreign one. The nominal domestic interest rate does not need to be equal to the foreign one since inflation differentials can arise. They are caused by the presence of different goods in the domestic and foreign price index.

The assumption of an exogenous foreign sector and of an endowment economy leads to have a model whose behaviour is heavily determined by the exogenous variables. The exogenous part of the model follows a vector autoregressive process. There are five types of exogenous shocks. Three of them have a domestic origin: to the traded and non-traded exogenous output and to the domestic money supply. Two have a foreign origin: to the price level in the rest of the world and to the world real interest rate.

Foreign shocks affect the small open economy via the balance of payments and the real exchange rate.

The balance of payments emphasizes the *intertemporal* dimension of individual agents' decision problem. Foreign trade and assets exchange open up a way for transferring resources over time that is not available in a closed economy. A temporary positive shock that raises current traded output relative to future output induces individuals to increase consumption both now and in the future as they try to smooth the path of consumption. Since domestic consumption rises less than domestic output, the economy increases its net exports, therefore accumulating claims against future foreign output. These claims can be used to maintain higher consumption in the future, after the temporary productivity increase has ended. The trade balance, therefore, plays an important role in facilitating the intertemporal transfer of resources.

The real exchange rate emphasizes the role of 'across-country' relative prices in determining agents' consumption path. It appears in the model because of the presence of two different goods in the domestic price index. Thus, the emphasis is directed toward the *intratemporal* dimension of individual agents' decision problem who allocate consumption spending between traded and non-traded goods. Indeed, the real exchange rate is playing a pivotal role in the model. Since all the exogenous shocks, either domestic and foreign, affect it directly and indirectly, the real exchange rate has a key role in the transmission mechanism and turns out to be highly volatile.

The structure of the chapter is the following. In section 2.2 we describe the model economy, the focus is on the interaction among the balance of payments, the real exchange rate and the exogenous sources of shocks. In section 2.3 we describe the VAR structure of the exogenous processes and provide an estimate of it. In section 2.4 we derive the steady state, necessary for the log-linear approximation. This allows us to write down the cointegrating vectors that describe the long-run behaviour of the economy. In section 2.5 we measure the deep parameters of the UK economy both by calibrating and by estimating the model's Euler equations. Section 2.6 concludes. The determination of the model's long-run behaviour has implications for both the estimation and simulation that are carried out in the next two chapters.

2.2 The Model Economy

A perfectly competitive economy is inhabited by consumers endowed with an exogenous income stream. The representative consumer's decision problem consists of maximizing an intertemporal utility function which depends on total consumption and real money balances. Total consumption is measured as a composed index of tradable and non-tradable goods. Real money balances allow agents to save time in their transactions. Money is supplied by the government and is used to finance the budget deficit.

The numeraire is the domestic price of traded goods, not nominal money. Therefore the focus is on relative prices of different goods, not on money prices. This requires to build a general price index.

In this section we will look at the household problem, at the government problem and we will then aggregate to find the market equilibrium.

2.2.1 The Household Problem

The representative household is assumed to maximize a discounted intertemporal utility function:

$$max E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}, \frac{M_{t+s}}{P_{t+s}}) \tag{2.1}$$

where β is the discount factor. The intratemporal utility function is equal to

$$u_t = \log c_t + \zeta \log \frac{M_t}{P_t}$$

i.e. is log-linear and separable in real consumption, c_t, and real money holding, M_t/P_t, with $u' > 0$, $u'' < 0$, and where ζ measures the relative importance of real balances in the utility function, $\zeta > 0$. Total real consumption is defined as a CES (constant elasticity of substitution) consumption index in traded c_{Tt} and non-traded c_{Nt} goods:

$$c_t = [\gamma^{\frac{1}{\theta}} c_{Tt}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} c_{Nt}^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{2.2}$$

γ is the share of consumption of tradables in total consumption and θ is the elasticity of intratemporal substitution between consumption of tradables and non-tradables.

The consumption-based price index P_t is obtained as the minimum expenditure $\{\min P_t c_t = c_{Tt} + q_t c_{Nt}\}$ such that $c_t = f(c_{Nt}, c_{Tt}) = 1$ given q_t. The solution of this problem leads to the following price index:

$$P_t = [\gamma + (1-\gamma) q_t^{1-\theta}]^{\frac{1}{1-\theta}} \tag{2.3}$$

where the price of traded goods has been normalized to 1 and q_t is the price of non-traded goods.

Domestic households do not derive utility from holding foreign real money balance.

Initially, we consider the household budget constraint in nominal terms. Net income from assets and flow endowment (y_{Tt}, y_{Nt}) is used for the purchase of traded and non-traded consumption goods, for holding internationally traded bonds and for buying new issued money, necessary for transactions.

Foreign bonds held by domestic households, B_t^*, are indexed to the ex-post foreign price level composite index of traded and non-traded goods, P_t^*. S_t is the nominal exchange rate, defined as the domestic price of foreign currency and i_t^* the nominal foreign interest rate. The nominal budget constraint is[11]:

$$y_{Tt} + q y_{Nt} + S_t(1+i_t^*)B_{t-1}^* + M_{t-1} = c_{Tt} + q_t c_{Nt} + M_t + S_t B_t^* \tag{2.4}$$

q_t and P_t have been previously defined as the relative price of non-traded goods and the composite price index, and $c_{Tt} + q_t c_{Nt} = P_t c_t$. In real terms the budget constraint is:

$$\frac{y_{Tt}}{P_t} + \frac{q_t}{P_t} y_{Nt} + \frac{S_t P_t^*}{P_t}\frac{B_{t-1}^*}{P_{t-1}^*}(1+i_t^*)\frac{P_{t-1}^*}{P_t^*} = c_t + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{S_t P_t^*}{P_t}\frac{B_t^*}{P_t^*} \tag{2.5}$$

and taking into account the definition of the domestic and foreign inflation rate ($\frac{P_t}{P_{t-1}} \simeq 1+\pi_t$; $\frac{P_t^*}{P_{t-1}^*} \simeq 1+\pi_t^*$) and the Fisher identity in its ex-post formulation $(1+i_t^*)\frac{P_{t-1}^*}{P_t^*} = (1+r_t^*)$, we can write the following equation for the household's budget constraint:

$$\frac{y_{Tt}}{P_t} + \frac{q_t}{P_t} y_{Nt} + \frac{S_t P_t^*}{P_t} b_{t-1}^*(1+r_t^*) = c_t + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{1}{1+\pi_t} + \frac{S_t P_t^*}{P_t} b_t^* \tag{2.6}$$

where we have indicated $b_t^* = \frac{B_t^*}{P_t^*}$. In the household's budget constraint the real exchange rate $\left(\frac{S_t P_t^*}{P_t}\right)$ multiplies real holdings of foreign bonds.

2.2.2 The Balance of Payments and the Real Exchange Rate

We now introduce the country net capital account position, defined as:

$$F_t = S_t B_t^* - B_t^F \tag{2.7}$$

[11]We use date $t-1$ rather than date t as subscript for bonds and money. This is just a notational difference but it will be particularly convenient once we need to solve for the dynamics. With this notation the date of a variable refers to the point in time when it is actually chosen. Put differently, it refers to the information, with respect to which a variable is observable.

where B_t^F is the domestic bond held by foreigners. The previous expression in real terms is equal to:

$$f_t = \frac{S_t P_t^*}{P_t} b_t^* - b_t^F \tag{2.8}$$

b_t^F is the real stock of domestic bonds held by foreigners and b_t^* is the real stock of foreign bonds held by domestic residents, multiplied by the real exchange rate.

Imports in nominal domestic prices are $P_{Tt}^* S_t z_t$ and exports to the rest of the world, expressed in domestic prices, are x_t. Recalling that the domestic price level of traded goods has been normalized to unity, P_{Tt}^* is the foreign price level of traded goods. The balance of payments in nominal terms is then equal to:

$$S_t B_t^* - B_t^F = x_t - S_t P_{Tt}^* z_t + [S_t B_{t-1}^* - B_{t-1}^F] + i_t^* S_t B_{t-1}^* - i_t B_{t-1}^F \tag{2.9}$$

and in real terms is:

$$\frac{S_t P_t^*}{P_t} \frac{B_t^*}{P_t^*} - \frac{B_t^F}{P_t} = \frac{x_t}{P_t} - \frac{P_{Tt}^* S_t}{P_t} z_t + \frac{S_t P_t^*}{P_t} \frac{B_{t-1}^*}{P_{t-1}^*} (1 + i_t^*) \frac{P_{t-1}^*}{P_t^*} - \frac{B_{t-1}^F}{P_{t-1}} (1 + i_t) \frac{P_{t-1}}{P_t}$$

or

$$\frac{S_t P_t^*}{P_t} b_t^* - b_t^F = \frac{x_t}{P_t} - \frac{P_{Tt}^* S_t}{P_t} z_t + \frac{S_t P_t^*}{P_t} b_{t-1}^* (1 + r_t^*) - b_{t-1}^F (1 + r_t) \tag{2.10}$$

By substituting (2.8) in (2.10) we obtain:

$$\begin{aligned}
f_t &= \frac{x_t}{P_t} - \frac{P_{Tt}^* S_t}{P_t} z_t + \left(\frac{S_t P_t^*}{P_t} / \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} \right) \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} b_{t-1}^* (1 + r_t^*) - b_{t-1}^F (1 + r_t) \\
&= \frac{x_t}{P_t} - \frac{P_{Tt}^* S_t}{P_t} z_t + (1 + r_t) f_{t-1} + \left[\frac{\frac{S_t P_t^*}{P_t}}{\frac{S_{t-1} P_{t-1}^*}{P_{t-1}}} (1 + r_t^*) - (1 + r_t) \right] \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} b_{t-1}^* \\
&= \frac{x_t}{P_t} - \frac{P_{Tt}^* S_t}{P_t} z_t + (1 + r_t) f_{t-1} + [\Delta rer_t + r_t^* - r_t] \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} b_{t-1}^* \qquad (2.11)
\end{aligned}$$

where we have indicated $\frac{S_t P_t^*}{P_t} / \frac{S_{t-1} P_{t-1}^*}{P_{t-1}} = 1 + \Delta rer_t$, i.e. equal to 1+ rate of change of the real exchange rate.

We can see from equation (2.11) that the sign of f_t depends on the relative importance of two terms: net exports, $(x_t - P^*_{Tt}S_t z_t)$, and capital account that is multiplying the differential between changes in the real exchange rate and in the real returns of domestic and foreign bonds, $(\Delta rer_t + r^*_t - r_t)$. We can be sure about the sign taken by f_t only in two cases. f_t will be positive in the case of a surplus of the net exports and of an expected real depreciation of the exchange rate (leading to a deficit of the capital account).

Since in this small open economy we allowed for perfect capital mobility we have $r_t = r^*_t$ at each time t, i.e. the real interest rate is completely determined by the foreign interest rate. And, from the uncovered interest parity condition the equality between the two real interest rates implies that $\Delta rer_t = 0$[12]. Therefore we can rewrite equation (2.11) as:

$$\begin{aligned} f_t &= \frac{x_t}{P_t} - \frac{P^*_{Tt}S_t}{P_t} z_t + (1 + r^*_t) f_{t-1} \\ &= nx_t + (1 + r^*_t) f_{t-1} \end{aligned} \tag{2.12}$$

where net exports have been indicated with $nx_t = \frac{x_t}{P_t} - \frac{P^*_{Tt}S_t}{P_t} z_t$.

Let us finally assume that $b^F_t = 0$ (i.e. foreigners do not hold domestic bonds), therefore $f_t = \frac{S_t P^*_t}{P_t} b^*_t$. We obtain:

$$\frac{S_t P^*_t}{P_t} b^*_t = nx_t + (1 + r^*_t) \frac{S_{t-1} P^*_{t-1}}{P_{t-1}} b^*_{t-1} \tag{2.13}$$

[12] We can rewrite the uncovered interest parity $i_t \simeq i^*_t + Es_{t+1} - s_t$, where $s_t = \log S_t$, in this way: $r_t + E_t p_{t+1} - p_t \simeq r^*_t + E_t p^*_{t+1} - p^*_t + E_t s_{t+1} - s_t$, where $p = \log P$, that is $r_t \simeq r^*_t + E_t \Delta rer_{t+1}$, where $rer \simeq s - p + p^*$ is the real exchange rate. Thus, since $r_t = r^*_t$ we have $\Delta rer = 0$.

We also notice that the impossibility for the small open economy to determine autonomously the real interest rate leads to have coincidence between the balance of payments and the current account.

By rewriting the balance of payments as in (2.13) we can extract a direct relationship between net exports and the real exchange rate. The balance of payments (2.13) corresponds to the domestic budget constraint of the representative household (2.6), since we have that $nx_t = \frac{y_{Tt}}{P_t} + \frac{q_t}{P_t} y_{Nt} - c_t - \frac{M_t - M_{t-1}}{P_t}$. Thus the real exchange rate is determined by the relative productivity in the traded and non-traded sector and by the accumulation of interest payments on real foreign assets.

Equations (2.13) and (2.6) show that the behaviour of the real exchange rate is affected by all the exogenous variables of the model. We can thus infer that if all the shocks are acting together they will generate a highly volatile real exchange rate. Indeed, as we will see in chapter 4, this is exactly what happens in our simulation. The interplay among a high number of shocks leads to the 'desirable' result of a highly volatile real exchange rate. But we can give another less 'desirable' reading of this result. Any unconditional measure of volatility is not informative at all because it derives from the sum of many sources of fluctuations. Thus, if the objective is to understand how the economy reacts to each shock, any model that contains a high number of exogenous components should not ground its explanatory power simply on unconditional second order moments, because the contribution of each shock is mixed with all the others.

The only shock that directly affects the real exchange rate (because it enters in its definition) comes from the foreign price level. A decrease in the foreign price level reduces the competitiveness of the domestic economy by a real appreciation of the exchange rate that leads net exports to decrease. This, in turn, implies an increase on traded goods domestic consumption spending. Productivity on traded and non-traded sectors and money supply shocks, instead, affects the real exchange rate through the net exports. A shock to the foreign interest rate affects the real exchange rate through the balance of payments.

We can obtain a final form for the stock of foreign bonds if we solve backward the balance of payments (2.13) and we assume that the real interest rate is constant:

$$b_t^* = \frac{P_t}{S_t P_t^*} \sum_{i=0}^{\infty} nx_{t-i} \left(1 + r^*\right)^i \tag{2.14}$$

This expression reveals that the stock of the foreign bonds will be permanently affected by unanticipated shocks in all the exogenous variables, since its pattern is determined by the real exchange rate and the entire dynamics of net exports.

2.2.3 The Government Problem

The nominal government budget constraint is assumed to be the following:

$$M_t - M_{t-1} = T_t \tag{2.15}$$

i.e. lump-sum transfers T_t are money financed. In real terms we have:

$$\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{1}{(1 + \pi_t)} = \tau_t \tag{2.16}$$

The government chooses $\{\tau, M\}$ to balance its budget constraint. It is assumed that the following *simple* monetary policy rule is adopted[13]:

$$\frac{M_t}{M_{t-1}} = e^{\mu + \varepsilon_t^M} \tag{2.17}$$

[13] *I should like to emphasize that I do not regard my particular proposal as a be-all and end-all of monetary management,as a rule which is somehow to be written in tablets of stones and enshrined for all future time. It seems to me to be the rule that offers the greatest promise of achieving a reasonable degree of monetary stability in light of our present knowledge. I would hope that as we operated with it, as we learned more about monetary matters, we might be able to devise still better rules, which would achieve still better results.* (Friedman, 1962, pag. 54-55)

where μ is the deterministic growth rate of the economy and ε_t^M is the stochastic term of the money growth, assumed to be *i.i.d.* $\sim (0, \sigma_M^2)$. If we take the logarithm of (2.17) and we solve backward we obtain:

$$m_t = m_0 + \mu t + \sum_{s=1}^{t} \varepsilon_{t-s}^M$$

where low case m indicates the logarithm of the nominal stock of money. According to this last equation money supply follows two trends: a deterministic trend (μt) and a stochastic trend ($\sum_{s=1}^{t} \varepsilon_{t-s}^M$).

2.2.4 The Market Equilibrium

In our economy there are in total four markets: foreign bonds, money, traded and non-traded goods. By Walras' law to obtain a competitive equilibrium we need only three out of the following four market clearing conditions:

1. the bond market

$$b_{t-1}^{*d} = b_{t-1}^{*s} = b_{t-1}^{*} \tag{2.18}$$

2. the money market

$$M_{t-1}^{d} = M_{t-1}^{s} \tag{2.19}$$

3. the goods market for traded and non-traded goods

$$c_{Nt} = y_{Nt} \tag{2.20}$$

$$c_{Tt} + x_t - P_{Tt}^* S_t z_t = y_{Tt} \tag{2.21}$$

Only relative prices can be determined in equilibrium, we remind that our normalization at the outset consisted in setting $P_T = 1$.

2.2.5 The Solution to the Problem

The solution to the problem can be obtained in two steps. We start by considering the intratemporal problem, which consists on households decisions on how to allocate total consumption expenditure between traded and non-traded goods. The demand functions for traded and non-traded goods are then[14]:

$$c_{Tt} = \gamma(\frac{1}{P_t})^{-\theta} c_t \tag{2.22}$$

$$c_{Nt} = (1-\gamma)(\frac{q_t}{P_t})^{-\theta} c_t \tag{2.23}$$

Once we have obtained the intratemporal optimal demands for traded and non-traded goods, the intertemporal problem can be solved.

The representative household maximizes equation (2.1) subject to (2.6) with respect to the following choice variables $\{c_t,\ b_t^*,\ M/P\}$. This involves writing down the Lagrangean and computing the first order conditions on that function. Throughout the analysis, for convenience, we assume certainty equivalence. This enables us to omit all conditional covariance terms and risk effects. Alternatively we can assume that they are small enough to be ignored.

1. The first order condition for c_t is:

$$\beta^s E_t \left(\frac{1}{c_{t+s}}\right) = E_t \lambda_s \;\; s = 0, 1, 2, 3.. \tag{2.24}$$

setting $s = 1$ leads to:

$$\beta \frac{1}{E_t c_{t+1}} = E_t \lambda_1 \tag{2.25}$$

where λ_s is the Lagrange multiplier of the constraint represented by (2.6);

[14]Equations (2.22) and (2.23) are obtained by maximizing the intratemporal utility function $\log c + \varsigma \log \frac{M}{P}$ with respect to $Pc = c_T + qc_N$ and to the constraint (2.2).

2. the first order condition for b_t^* is:

$$E_t\left[\lambda_{s+1}(1+r^*_{t+s+1})\frac{S_{t+s+1}P^*_{t+s+1}}{P_{t+s+1}}\right]-E_t[\lambda_s\frac{S_{t+s}P^*_{t+s}}{P_{t+s}}]=0 \tag{2.26}$$

setting $s=0$ and using equation (2.24) we obtain:

$$\beta E_t\left[\frac{c_t}{c_{t+1}}(1+r^*_{t+1})\frac{S_{t+1}P^*_{t+1}}{P_{t+1}}\frac{P_t}{S_tP^*_t}\right]=1 \tag{2.27}$$

or

$$E_t\left[\frac{c_t}{c_{t+1}}\frac{(1+r^*_{t+1})(1+\pi^*_{t+1})}{(1+\pi_{t+1})}\frac{S_{t+1}}{S_t}\right]=\frac{1}{\beta} \tag{2.28}$$

Recalling the definition of the real interest rate, another way of writing down the Euler equation (2.28) is the following:

$$E_tc_{t+1}=\beta E_t\left[(1+i^*_{t+1})\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}c_t\right] \tag{2.29}$$

By substituting, then, the two intratemporal conditions for consumption in the two sectors, (2.22) and (2.23), we have the following Euler equation for the traded sector:

$$E_tc_{Tt+1}=E_t\left[\left(\frac{P_t}{P_{t+1}}\right)^{1-\theta}\beta(1+i^*_{t+1})\frac{S_{t+1}}{S_t}c_{Tt}\right] \tag{2.30}$$

and for the non-traded sector:

$$E_tc_{Nt+1}=\left[\left(\frac{P_t}{P_{t+1}}\right)^{1-\theta}\left(\frac{q_{t+1}}{q_t}\right)^{-\theta}\beta(1+i^*_{t+1})\frac{S_{t+1}}{S_t}c_{Nt}\right] \tag{2.31}$$

3. The first order condition for the money demand is:

$$\beta^sE_t\frac{P_{t+s}}{M_{t+s}}\zeta-E_t\lambda_s+E_t\frac{\lambda_{s+1}P_{t+s}}{P_{t+s+1}}=0,\ s=0,1,2,3... \tag{2.32}$$

setting $s=0$ and recalling the expression for $E_t\lambda_s$ in (2.24)-(2.25) we obtain:

$$E_t\left[\frac{P_t}{M_t}\zeta-\frac{1}{c_t}+\beta\frac{1}{c_{t+1}}\frac{P_t}{P_{t+1}}\right]=0 \tag{2.33}$$

if we now divide (2.33) by $\frac{1}{c_t}$ we get:

$$E_t\left[\zeta\frac{P_t}{M_t}c_t - 1 + \beta\frac{c_t}{c_{t+1}}\frac{P_t}{P_{t+1}}\right] = 0 \tag{2.34}$$

and by substituting equation (2.29) into (2.34) we end up with:

$$\frac{M_t}{c_t P_t} = E_t\left[\frac{\zeta(1 + \Delta s_{t+1} + i^*_{t+1})}{\Delta s_{t+1} + i^*_{t+1}}\right] \tag{2.35}$$

where $\Delta s_{t+1} = s_{t+1} - s_t$, and $s_t = \log S_t$. From equation (2.35) we can see that the standard result of homogeneity of degree zero of real money holds. In other words, the model possesses the property of *long-run neutrality*, proportional changes in the level of M and P leave $\frac{M}{P}$ unaffected and have no real effects. This is an ex-ante property of the model. After a stochastic monetary shock the stock of bonds held by agents is permanently affected and thus we have ex-post non-neutrality of money.

In this model, where capital and investment do not enter we can consider total consumption c_t, as providing a satisfactory proxy for fluctuations in total output. Real money balances are positively related to an expenditure indicator and negatively related to an opportunity cost variable. Thus, we have a relationship expressing the end of period real money balances[15] as a function of current consumption spending, current nominal foreign interest rate and expected change in the exchange rate.

4. Finally we have to consider a debt limit which prevents the agent from borrowing and never paying back his debts (the no-Ponzi condition):

$$\lim_{s\to\infty} R_{t,t+s}\left(\frac{M_{t+s}}{P_{t+s}} + \frac{S_{t+s}P^*_{t+s}}{P_{t+s}}b^*_{t+s}\right) = 0, \text{ where } R_{t,s} = \frac{1}{\prod_{j=t}^{s-1}(1+r_j)} \tag{2.36}$$

[15] i.e. according to the formulation of the problem, in this model we need to acquire money before consumption can take place.

i.e. the variable $R_{t,s}$ is the date s value of consumption in period t. This transversality condition is derived by iterating forward the period budget constraint (2.6).

2.3 The Exogenous Processes

The theoretical framework is closed by considering the exogenous processes responsible for driving away the economy from the balanced growth path. Following a practice common to the RBC approach we model the exogenous variables as a first order vector autoregressive process (VAR). We have the following system of equations:

$$z_t = \mu + \Phi z_{t-1} + \varepsilon_t^z \tag{2.37}$$

where z_t is the vector of the exogenous driving forces of the model and μ is the common deterministic trend of the economy[16]. Φ is the matrix of the autoregressive coefficients. The diagonal parameters of Φ give a measure of persistence whereas the off-diagonal parameters of Φ measure the spillovers among the variables of the system. The VAR model for the forcing processes contains the following zero restrictions:

$$z_t = \begin{bmatrix} r_t^* \\ p_t^* \\ m_t \\ y_{Tt} \\ y_{Nt} \end{bmatrix} = \mu + \begin{bmatrix} \varphi_{r*} & 0 & 0 & 0 & 0 \\ 0 & \varphi_{p*} & 0 & 0 & 0 \\ 0 & 0 & \varphi_m & 0 & 0 \\ \varphi_{y_T r*} & \varphi_{y_T p*} & 0 & \varphi_{y_T} & \varphi_{y_T y_N} \\ \varphi_{y_N r*} & \varphi_{y_N p*} & 0 & \varphi_{y_N y_T} & \varphi_{y_N} \end{bmatrix} \begin{bmatrix} r_{t-1}^* \\ p_{t-1}^* \\ m_{t-1} \\ y_{Tt-1} \\ y_{Nt-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{r*} \\ \varepsilon_t^{p^*} \\ \varepsilon_t^m \\ \varepsilon_t^{y_N} \\ \varepsilon_t^{y_T} \end{bmatrix} \tag{2.38}$$

According to these restrictions, the foreign interest rate and the foreign price level are strictly exogenous with respect to the domestic variables of the system

[16]Solving backward the first order VAR we obtain $z_t = z_0 + \mu t + \sum_{s=1}^{t} \Phi^s \varepsilon_{t-s}^z$, where we can see that μ is the constant which multiplies the time trend.

(i.e. past values of the other exogenous variables do not Granger-cause r_t^* and p_t^*). This assumption depends on the original hypothesis we made about the size of the economy. The economy is too small to affect foreign variables, therefore the foreign price level does not depend on domestic monetary conditions or on the domestic supply of output.

The assumption of a first order autoregressive process for foreign prices can be interpreted as if they possessed some degree of stickiness in their adjustment (from the point of view of the small domestic economy).

As previously described, we are assuming that the policy rule for money supply does not take into account any other variable of the system but money itself. The system of equations (2.38) shows that output supply in the two sectors does not directly depend on money supply, this means that we allow for superneutrality of money, that is the real equilibrium is independent of the rate of growth of the nominal money supply.

We finally allow for spillover between y_N and y_T, and for Granger-causality going from r^* and p^* to y_N and y_T.

The vector of errors is assumed to be identically and independently distributed, $i.i.d. \sim (0, \Sigma)$. We do not impose any restriction on the covariance structure of the error terms. The estimation of the unrestricted VAR leads to an estimate of the off-diagonal elements of Σ with coefficients not significantly different from zero.

Our model economy fluctuates stochastically around a common deterministic trend, arising from the drift of the output process. This is assumed to be the same in the two sectors. The data for the UK economy suggest that we cannot reject the unit root hypothesis for y_T, y_N, p^*, m whereas the evidence is not conclusive about the presence of a unit root in r^* (see section 2.4.1 and Appendices A-B for data description and testing). We decide to treat r^* as a $I(1)$ variable[17]. This means that there are six forces determining the long-run behaviour of the economy: a deterministic one, given by the common constant μ which multiplies the time trend, and five stochastic paths. Three of them are dictated by the real side of the economy and two come from the nominal side of the economy.

We assume that our exogenous variables are not cointegrated, and we test for the absence of cointegration, which is not rejected. Given that we are in the presence of non-stationary exogenous variables (y_T, y_N, p^*, r^*, m), before the estimation we needed to take the difference of the non-stationary ones.

2.4 Steady State and Long-Run Relationships

After having completed the description of the model we can look at its behaviour in steady state. Owing to the neutrality of money, which is an ex-ante property, i.e. before the realization of shocks, the nominal and real side of the economy are independent from each other in the long run. But after the realization of shocks, the incomplete market assumption leads to have permanent effects on the net foreign assets position.

Given the assumption of a deterministic real trend, the economy is growing in steady state along a balanced growth path. The standard way in which neoclassical growth models deal with the presence of a deterministic trend is to divide all the variables by this trend (King, Plosser, Rebelo, 1988a)[18].

[17]We are aware of the theoretical problems related to this finding and we interpret the unit root in r^* as an approximation of a highly persistent process.

[18]A necessary condition for the use of linear methods is that the model should display a fixed point around which linearization makes sense.

Since we have modelled a deterministic trend common to both sectors of the economy which drives the balanced growth of real money balances, domestic and foreign bonds, we need, according to this procedure, to deflate the variables by their deterministic growth component. Once linearly detrended, the model is not yet stationary. In our model things are complicated by the presence of different types of stochastic trends. King, Plosser and Rebelo (1988b) consider also the case of a common stochastic trend which cannot be directly extended to the model we are dealing with because of the presence of five common stochastic trends, not just one.

We start from the definition of the steady state. If X_t is an endogenous variable in our system of equations, its steady state is determined by the sum of a deterministic trend and 5 common stochastic trends:

$$\overline{X}_t = \overline{X}e^{\mu t+\sum_{j=0}^{\infty}\varphi_{y_N}\varepsilon_{t-j}^{y_N}+\sum_{j=0}^{\infty}\varphi_{y_T}\varepsilon_{t-j}^{y_T}+\sum_{j=0}^{\infty}\varphi_{p^*}\varepsilon_{t-j}^{p^*}+\sum_{j=0}^{\infty}\varphi_{r^*}\varepsilon_{t-j}^{r^*}+\sum_{j=0}^{\infty}\varphi_{M}\varepsilon_{t-j}^{M}} \tag{2.39}$$

The steady state is not a fixed point in our economy and the deterministic growth path of all the variables ($e^{\mu t}$) is subject to permanent shifts induced by exogenous movements in the forcing processes. We have modelled five causes that lead to a permanent deviation of the economy from the deterministic growth path, which in (2.39) are shown in their moving average representation.

We rewrite now the first order conditions and the equations for the constraints in steady state, indicated by an upper bar. The steady-state intratemporal relationship between consumption goods is:

$$\overline{c}_t = \frac{\overline{q}_t}{\overline{P}_t}\overline{c}_{Nt} + \frac{1}{\overline{P}_t}\overline{c}_{Tt} \tag{2.40}$$

Since the Government is balancing its budget, in the non-traded sector we have:

$$\overline{y}_{Nt} = \overline{c}_{Nt} \tag{2.41}$$

And in the traded sector:

$$\overline{y}_{Tt} + \overline{r}_t^* \overline{P_t^* S_t b_t^*} = \overline{c}_{Tt} \tag{2.42}$$

From the balance of payments we obtain the long-run value of foreign debt held by domestic agents:

$$\frac{\overline{S_t P_t^*}}{\overline{P}_t} \overline{r}_t^* \overline{b}_t^* = \frac{\overline{P_t^* S_t}}{\overline{P}_t} \overline{z}_t - \frac{\overline{x}_t}{\overline{P}_t} \tag{2.43}$$

From the money demand equation we have:

$$\frac{\overline{M}_t}{\overline{c}_t \overline{P}_t} = \zeta \frac{(\overline{i}_t^* + 1)}{\overline{i}_t^*} \tag{2.44}$$

These steady-state relationships continue to be indexed by the time subscript t. This is because our steady state is not a stable fixed point, but varies stochastically.

From these conditions we can conclude that the model exhibits a property called superneutrality of money: the steady-state values of consumption and output are all independent of the money supply growth rate. That is, not only is money neutral ex-ante, so that anticipated proportional changes in the level of nominal money balances and prices have no real effects, but changes in the rate of growth of nominal money also have no ex-ante effects on the steady-state values of real variables.

Notwithstanding the general definition in (2.39) the steady-state relationships (2.40)-(2.44) together with (2.38) imply that not all the stochastic trends will affect the long-run behaviour of all the variables. In other words the model puts restrictions on the common trends driving each cointegrating vector. This point will be clarified shortly, when we will express (2.40)-(2.44) in a stationary fashion. Unanticipated changes in money supply will temporary affect all the real variables and permanently the stock of foreign bonds held by domestic households.

The long-run description of the variables makes then clear that all the endogenous variables inherit the unit root possessed by the exogenous one. Thus we are expecting to find 7 $I(1)$ variables: b^*, the real foreign bond; s, the nominal exchange rate; c_N, the real consumption of traded goods; c_T, the real consumption of non-traded goods; c, the total real consumption; P, the relative domestic price index and q, the relative price of non-traded goods with respect to traded goods.

2.4.1 The Cointegrating Vectors

In the previous sub-section we have seen that detrending the model for the deterministic growth component does not lead to have a stationary steady-state value for each variable of the economy. This is due to the presence of permanent stochastic shocks, as in (2.39). The fact that we have less shocks than integrated variables leads to introduce the concept of cointegration in our framework. The presence of cointegrated variables makes it possible a long-run interpretation of the model's behaviour consistent with the data. The solution of the model implies that we are in the presence of 7 endogenous integrated variables, therefore we should expect to find 7 cointegrating relationships[19]. These relationships are obtained from the long-run solution of the model, (2.40)-(2.44), and from the definition of the real exchange rate:

$$\frac{\overline{c}_{Nt}}{\overline{y}_{Nt}} = 1 \tag{2.45}$$

[19]Stock J. and Watson M., 1988, show that integrated processes follow a common stochastic trend if there exists a cointegration relationships among them. The behaviour of those processes can be decomposed in two distinct parts: a transitory component and a permanent component, this last is based on the common stochastic trend.

$$\frac{\overline{c}_{Tt}}{\overline{y}_{Tt}} = 1 + \overline{r}_t^* \overline{S}_t \overline{P}_t^* \frac{\overline{b}_t^*}{\overline{y}_{Tt}} \tag{2.46}$$

$$\frac{\overline{c}_t}{\overline{c}_{Nt}} = \frac{\overline{q}_t}{\overline{P}_t} + \frac{1}{\overline{P}_t}\frac{\overline{c}_{Tt}}{\overline{c}_{Nt}} \tag{2.47}$$

$$\frac{\overline{M}_t}{\overline{c}_t \overline{P}_t} \frac{\overline{i}_t^*}{(\overline{i}_t^* + 1)} = \zeta \tag{2.48}$$

$$\overline{rer}_t = \frac{\overline{S}_t \overline{P}_t^*}{\overline{P}_t} \tag{2.49}$$

These long-run relationships provide testable restrictions on the cointegrating vectors that we can rewrite in logarithmic form[20]: $[\log \overline{c}_{Nt} - \log \overline{y}_{Nt}]$, $[\log \overline{c}_{Tt} - \log \overline{y}_{Tt}]$, $[\overline{r}_t^* + \log \overline{S}_t + \log \overline{P}_t^* + \log \overline{b}_t^* - \log \overline{y}_{Tt}]$, $[\log \overline{c}_t - \log \overline{c}_{Nt}]$, $[\log \overline{q}_t - \log \overline{P}_t]$, $[\log \overline{M}_t - \log \overline{c}_t - \log \overline{P}_t - \log((\overline{i}_t^* + 1)/\overline{i}_t^*)]$, $[\log \overline{S}_t + \log \overline{P}_t^* - \log \overline{P}_t]$.

The analysis of the data starts by carrying out integration tests on each variable after having detrended for the linear trend (Appendix B). The sample is 1969:3 - 1997:3 and the description of the UK data is provided in Appendix A; the graphs of the cointegrating vectors are in Appendix B. The evidence is against the stationarity of each variable, although it is weak for the real foreign interest rate and for the relative price of non-traded goods.

We have then carried out two cointegration tests. The first is a unit root test for theoretical residuals of the previous cointegrating relationships. The second is a Johansen test based on the rank of the cointegrating vectors, after the estimation of a VAR in levels (Appendix B). For the first cointegration test we built the long-run relationships and the test for unit roots showed that we could reject a unit root in six out of the seven long-run relationships seen before. The only case where there is no strong evidence against unit root is for the

[20]Equation (2.47) contains three cointegrating vectors. Among those we do not need to use the relationship between the variables $[c_T, P, c_N]$ because the information relative to their relative behaviour is already contained in the other two cointegrating vectors $[c, c_N]$ and $[P, q]$.

cointegrating vector involving the velocity of money adjusted for the foreign nominal interest rate. If we look at equation (2.48) one possible correction to this apparently too simplified cointegrating vector is to make some assumptions on ζ, for example by letting it to be endogenous.

The Johansen test based on the rank of the cointegrating vectors, after the estimation of a 12 variables VAR in levels, reveals that we can accept that the rank of the cointegrating matrix, β, is seven.

The relationships (2.45)-(2.49) help in understanding the long-run movements of the model economy. (2.45) says that the long-run path of non-traded goods consumption is affected by non-traded output and since we allowed for spillover among y_N, y_T, r^*, p^* (see system (2.38)), it is also affected by the remaining three exogenous stochastic components. The right-hand-side of (2.46) suggests that also the long-run behaviour of traded consumption is affected by y_N, y_T, r^*, p^*. The left-hand-side of (2.46) describes a long-run relationship between capital gains in foreign bonds and traded output. It explicitly suggests that the driving forces of permanent movements are y_T, r^*, p^*. In (2.48) the long-run behaviour of the velocity of money is driven by shocks in the money supply and by the long-run determinants of total consumption, namely y_N, y_T, r^*, p^*. In (2.49) permanent shifts in the nominal exchange rate and in the domestic price index are determined by the foreign price index. It suggests that the real exchange rate is the result of nominal exchange rate deviations from its stochastic trend. The long-run stationarity of the real exchange rate is related to the long-run convergence of traded and non-traded goods prices, showed in the right-hand-side of (2.47).

2.5 The 'Deep Parameters' of the UK Economy

In this section we carry out the measurement of the model's deep parameters. Within the RBC literature the most common way of finding out 'values' for the model's structural parameters is by a calibration approach. Calibration exercises concern the assignment of parameter values, traditionally on the basis of other studies or evidence[21]. Information drawn from other studies usually is informal and does not include standard errors. Moreover, some method of aggregation is required in order to use parameters estimated from microeconomic panels in representative-agent models (i.e. to make sure that one is measuring the same thing). In some cases parameters are set so as to exactly match a statistic generated by the model with the one in the data. For example, Kydland and Prescott (1982) calibrated the coefficient of relative risk aversion in their business-cycle model by matching the variance of detrended output.

In this section we follow both a calibration and an estimation approach to the measurement of the parameter's values.

The steady-state value of the real interest rate $(1 + r) = R = 1.005$ is the average quarterly real interest rate over the period 1970-1997 in the UK, from this value we obtain $\beta = 0.98$.

We distinguish between traded and non-traded sectors using services as a proxy for non-traded goods. Similarly, for the price index of non-traded goods we use the services price index. This distinction leads to compute γ: $\gamma = 0.57$, i.e. the share of consumption on traded goods over total consumption (average over the sample period 1970-1997).

[21] Cooley and Prescott, 1995, 'Economic Growth and Business Cycle', in Frontiers of Business Cycle Research, T. Cooley Edition, 1995, Princeton.

We estimate the elasticity of substitution between traded and non-traded goods using equation (2.22), according to which the demand of traded goods is $c_T = \gamma(\frac{1}{P})^{-\theta}c$. This gives $\theta = 2.27$ [20.11]. This estimation has been carried out with Non-linear Least Squares on (2.22)[22].

The parameter ς, the weight in the utility function for real money balances, has been obtained directly from the steady-state values of the variables in (2.44) (see chapter 4, table 4.1).

Each variable of (2.38) has been linearly detrended (which made it possible to control for the deterministic trend) and differentiated since the purpose is that of simulating the short-run behaviour of our economy[23]. The estimation of the VAR consisted of two-steps. First, we estimated an unrestricted VAR made by the detrended 'exogenous' forcing processes, we then tested each theoretical restriction of the system (2.38) using Wald tests and finally we re-estimated the whole restricted system by (Full Information) Maximum Likelihood (FIML) (Appendix B). We should point out that this system method of estimation did not lead to very different results in terms of values and significance of the relevant parameters with respect to the OLS estimation equation by equation. We have obtained the following results:

$$z_t = \begin{bmatrix} r_t^* \\ p_t^* \\ m_t \\ y_{Tt} \\ y_{Nt} \end{bmatrix} = \begin{bmatrix} .20 & 0 & 0 & 0 & 0 \\ .26 & .85 & 0 & 0 & 0 \\ 0 & 0 & .35 & 0 & 0 \\ 0 & -.76 & 0 & -.21 & 0 \\ -.42 & 0 & 0 & .23 & -.06 \end{bmatrix} \begin{bmatrix} r_{t-1}^* \\ p_{t-1}^* \\ m_{t-1} \\ y_{Tt-1} \\ y_{Nt-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{r*} \\ \varepsilon_t^{p^*} \\ \varepsilon_t^{m} \\ \varepsilon_t^{y_T} \\ \varepsilon_t^{y_N} \end{bmatrix} \tag{2.50}$$

[22]See Appendix B.3, t-stat. within square brackets.

[23]Although the evidence is rather contrasting, in the estimation of the VAR we decided to treat r^* as $I(1)$ given that its autocorrelation coefficient is close to unity.

The system (2.50) shows that the data did not allow us to treat changes in p^* as strongly exogenous with respect to r^*. We also observe that changes in y_N and y_T are strongly mean reverting. y_T depends negatively on foreign inflation and y_N depends negatively on foreign interest rates and positively on y_T. The estimated correlation matrix $\widehat{corr}$ among residuals is equal to[24]:

$$\widehat{corr} = \begin{bmatrix} 1 & & & & \\ -.38 & 1 & & & \\ .01 & -.05 & 1 & & \\ .12 & -.04 & .11 & 1 & \\ .20 & -.03 & -.03 & -.19 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{r*} \\ \varepsilon_t^{p^*} \\ \varepsilon_t^{m} \\ \varepsilon_t^{y_T} \\ \varepsilon_t^{y_N} \end{bmatrix} \tag{2.51}$$

There are four cases where the correlation among residuals is relatively high. These are: $[\varepsilon_{r^*}, \varepsilon_{p^*}]$, $[\varepsilon_{y_T}, \varepsilon_{r^*}]$, $[\varepsilon_{y_N}, \varepsilon_{r^*}]$, and $[\varepsilon_{y_T}, \varepsilon_{y_N}]$.

The model needs now to be log-linearized around the steady state. It can, then, be used for computing artificial data and statistics. Before taking this step, in the following chapter we will estimate a Vector Error Correction Model (VECM) based on the long-run theoretical restrictions singled out in (2.45)-(2.49). In chapter 4 we will then solve the model for the log-linear deviations from the steady state and simulate the short-run reduced form of the system.

2.6 Conclusions

This chapter has dealt with a small open economy. The model has been built in the spirit of the DSGE approach by deriving the behavioural equations from an utility-maximization problem and by modelling the exogenous driving forces of

[24]and the estimated error covariance matrix is equal to:

$$\widehat{\Sigma} = \begin{bmatrix} .39 & -.08 & .01 & .16 & .22 \\ -.08 & .11 & -.02 & -.03 & -.02 \\ .01 & -.02 & 1.27 & .26 & -.06 \\ .16 & -.03 & .26 & 4.41 & -.68 \\ .22 & -.02 & -.06 & -.68 & 2.89 \end{bmatrix}$$

the system as autoregressive processes. The fact that we have modelled a very simple endowment economy does not prevent us from obtaining some interesting features from the model. Given that output is exogenous, consumption plays a key role in modelling spending decisions. We have an incomplete assets market that justifies the presence of money in the model. We also have two markets for traded and non-traded goods. Since PPP does not hold for the price index we could model the real exchange rate. The balance of payments shows that the real exchange rate depends on all the shocks hitting the economy (directly or indirectly). This is very likely to produce a highly volatile real exchange rate. Owing to the foreign determination of the real interest rate, the international transmission mechanism of domestic monetary shocks relies on the real exchange rate. By solving backward the balance of payments we can see that all the shocks affecting net exports and the real exchange rate will permanently change the stock of foreign bonds held by domestic household.

The behaviour of the exogenous part of the model has been estimated by FIML on a restricted first order VAR model. Given the high number of the exogenous variables, the way in which we decided to model their behaviour turns out to be essential for the dynamics of the whole system. The model suggested some zero restrictions among the diffusion parameters of the Φ matrix in (2.38), i.e. the real foreign interest rate and the foreign price level were assumed to be strictly exogenous with respect to the other variables. Conversely, we allowed for spillovers from the foreign variables to the growth rate of output in the traded and non-traded sector. Money growth has been assumed strictly exogenous with respect to all the other exogenous variables in the system.

The last step consisted of describing the long-run behaviour of the model by exploiting the notion of cointegration among variables.

The analysis now takes two directions. In chapter 3 the long-run restrictions (2.45)-(2.49) will be used to estimate a Vector Error Correction model (VECM) whose structural errors correspond to those estimated in this chapter. In chapter 4 the same restrictions will be used for detrending the economy and for simulating its cyclical behaviour. Finally, impulse responses of the artificial data will be compared with an estimated version of the same detrended reduced form VAR.

The work that follows is related to two lines of inquiry. The next chapter is devoted to the estimation of structural models. This leads to an assessment of the importance of different shocks (nominal and real, domestic and foreign) in propagating economic fluctuations in open economies. The second line of inquiry, pursued in chapter 4, is related to papers trying to test the model's performance using a simulation approach. This leads to computing volatilities and correlations of business cycles between our measure of real expenditure (total consumption) and the model's real and nominal variables.

To conclude, this chapter has provided the economic underpinnings of the structural long-run identifying restrictions imposed on the cointegrating vectors. Moreover it has supplied a theoretical framework useful for comparing its cyclical performance (obtained by simulating the short-run behaviour) with the cyclical performance of the UK economy.

3 Estimating a Structural VECM based on Theoretical Long-Run Restrictions: Application to the Small Open Economy

3.1 Introduction

This chapter applies a new method for estimating a VECM[25] built on the solution of the model presented in chapter 2. In that chapter we argued that the theoretical model provides long-run restrictions that can be used for its estimation. We interpreted the long-run relationships in terms of cointegration, we built the theoretical cointegrating residuals and we tested for their stationarity (Appendix B). Those tests suggested that the model is robust in its long-run behaviour. Now, the theoretical long-run restrictions will be used for estimating the model by leaving the short run unrestricted.

This chapter shows the results of two kinds of estimation. The first simply consists of estimating a short-run VAR composed by the theoretical cointegrating residuals and by the exogenous forcing variables expressed in first differences. We can interpret this exercise as the second step of the estimation procedure started in the previous chapter where we built the cointegrating vectors. The exogenous variables, estimated in the previous chapter, are added to this system and used to drive the economy temporarily away from the steady state. This will allow us to compute and study the impulse responses of the cointegrating vectors to the economy's exogenous shocks. This unrestricted short-run VAR will then be compared (in the next chapter) with the restricted short-run VAR obtained from

[25]The new methodology has been put forward by Wickens and Motto (1999).

the simulation of the model.

The second exercise, the estimation of a VECM, constitutes the core of the chapter. While the VECM is based on theoretical long-run restrictions, we will use the data to determine the short-run behaviour of the model. We will add to the VECM the system of exogenous variables responsible for the common stochastic trends and look at the dynamics of the variables hit by each shock and at the proportion of their volatility explained by each shock.

There are several advantages related to our estimation strategy. First of all the VECM is estimated in its structural form. Second, this estimation approach does not separate the long-run behaviour from the short run. This makes it possible to take into account spillover effects from the stochastic growing path to the stochastic cyclical movements of the economy. These spillovers are removed in a model that has been detrended before its estimation.

This approach bears some similarity to the work of Ahmed, Ickes, Wang, Yoo (1993). Their paper develops and estimates a multivariate structural two-country model of the world economy. They measure the relative contribution of different shocks in explaining movements in key macroeconomic variables for the US and a five-nation OECD aggregate. The empirical results enable them to assess whether the correlation of real GNP movements across countries is primarily due to a common world disturbance, or whether spillover effects of shocks moving from one country to the other play a major role. Their econometric methodology relies on long-run restrictions based on the theoretical model. The model does not restrict short-run dynamic interactions between the variables. There are five sources of shocks: (*i*) a worldwide labour-augmenting productivity shock, (*ii*) country-specific labour supply shocks, (*iii*) country-specific fiscal policy shocks,

(iv) country-specific monetary policy shocks, (v) relative demand disturbances (preference shocks). Ahmed et al. (1993) find that all the coefficients in the estimated structural VAR are invariant to exchange rate regimes. Thus, there is no evidence of differences in the transmission properties of economic disturbances across exchange rate regimes. Their findings, then, strongly suggest that supply shocks are very important in generating international economic fluctuations.

There are several remarkable distinctions between our VECM approach and the Ahmed's et al. (1993) VAR based on the long-run identification procedure. Our derivation of the long run does not depend on neutrality results but is expressed in terms of cointegration, which is a property of models driven by common stochastic trends. Based on theoretical cointegrating vectors, our estimation strategy relies upon an extra piece of information, theoretically founded, for understanding the behaviour of a small open economy, the UK.

Furthermore, by building up stationary relationships among variables that describe the steady state, we provide a natural way for extracting short-run information. We are not modelling a world economy, just the behaviour of a small open economy. Within this framework we will show how the economy's fluctuations rely primarily on shocks with a foreign origin. According to our findings, shocks coming from foreign prices and interest rates can explain almost all the variability of the system. The impulse response functions show that an increase in traded and non-traded productivity triggers a nominal and real depreciation of the country's currency. An increase in the price level in the rest of the world induces a nominal appreciation in the short run followed by a depreciation. Monetary shocks generate overshooting of the nominal exchange rate.

The chapter is organized as follows. In section 3.2 we introduce some general methodological definitions and issues related to the instruments used in this chapter and to the problem of identification. In section 3.3 we carry out the short-run VAR estimates of the theoretical cointegrating vectors and we interpret the impulse response functions. In section 3.4 we carry out the VECM estimates and we describe the results in terms of impulse responses and forecast error variance decomposition.

3.2 Methodological Issues

As we will see in the next chapter, the solution to the model presented in chapter 2 leads to a restricted reduced form VAR representation. In this section we start from a VAR representation of the actual data and we study the problem of obtaining a structural representation after having estimated the reduced form (i.e. the identification problem)[26]. The methodological approach follows Watson

[26] '*A linearized dynamic stochastic general equilibrium model becomes a VAR model, with a particular pattern of identifying restrictions on its coefficients. Since linearized DSGE models are generally much more strongly restricted that identified VAR models, there are many fewer parameters to estimate. However, the kinds of restrictions that are used to identify VAR models are often imposed as a subset of the restrictions used in DSGE models, so that identified VAR models can be thought of as weakly restricted linearized DSGE models. This is what, in fact, distinguishes the DSGE from the identified VAR modelling approach. The former begins with a complete interpretation of each source of stochastic disturbance in the model, invoking many conventional but arbitrary restrictions on functional forms of utility and production functions and on stochastic properties of disturbances. The fitted model can tell the full story about how, and by what means, each source of disturbance affects the economy. The identified VAR modelling approach, by contrast, begins with an unidentified time-series model of the economy and introduces identifying information cautiously. The fitted model then fits the data well, usually much better than the DSGE models of the same data, but tells only an incomplete story about*

(1994).

We start by considering the following VAR representation

$$\Pi(L)y_t = \varepsilon_t \tag{3.1}$$

where $\Pi(L) = I - \sum_{i=1}^{p} \Pi_i L^i$ and L is the lag operator such that $y_{t-1} = Ly_t$. y_t is an $n \times 1$ vector composed of $I(0)$ and $I(1)$ variables, and ε_t is an error vector which satisfies $\varepsilon_t \sim i.i.d.(0, \Sigma_\varepsilon)$. Since each variable in the system is $I(0)$ or $I(1)$, the determinental polynomial $\mid \Pi(L) \mid$ contains at most n unit roots. When there are fewer than n unit roots, then the variables are cointegrated, in the sense that certain linear combinations of the y_t's are $I(0)$.

3.2.1 The Vector Error Correction Model (VECM)

To derive the VECM we have to subtract y_{t-1} from both sides of (3.1) and rearrange the equation as:

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=0}^{p} \Phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.2}$$

where $\Pi = -(I - \sum_{i=1}^{p} \Pi_i) = -\Pi(1)$, and $\Phi_i = \sum_{j=i+1}^{p} \Pi_j$, $i = 1, ...p-1$ and with Π having rank r. Let now β denote an $n \times r$ matrix whose columns form a basis for the row space of Π, so that every row of Π can be written as a linear combination of the rows of β'. Thus we can write $\Pi = \alpha\beta'$, where α is an $n \times r$ matrix with full column rank. Therefore we can rewrite (3.2) as:

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{i=0}^{p} \Phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.3}$$

or

$$\Delta y_t = \alpha w_{t-1} + \sum_{i=0}^{p} \Phi_i \Delta y_{t-i} + \varepsilon_t \tag{3.4}$$

---

each source of disturbance' (Leeper E., Sims C., Zha T., 1996).

where $w_t = \beta' y_t$. If we solve (3.4) for w_{t-1} we obtain:

$$w_{t-1} = (\alpha'\alpha)^{-1}\alpha'[\Delta y_t - \sum_{i=0}^{p} \Phi_i \Delta y_{t-i} - \varepsilon_t]$$

so that w_t is $I(0)$. Thus, linear combinations of potentially $I(1)$ elements of y_t, formed by the columns of β are $I(0)$, and the columns of β are the cointegrating vectors.

The VECM imposes $k < n$ unit roots in the VAR by including first differences of all of the variables and $r = n - k$ linear combinations of the levels of the variables. The levels of y_t are introduced in a special way, as $w_t = \beta' y_t$, so that all of the variables in (3.4) are $I(0)$. The term 'error correction' was introduced in Davidson et al. (1978), who interpreted $\beta' y_t = 0$ as the 'equilibrium' of the dynamic system, w_t as the vector of 'equilibrium errors' and equation (3.4) as describing the self correcting mechanism of the system.

3.2.2 The Moving Average and the Common Trend Representations

We write the moving average representation of (3.2) as:

$$\Delta y_t = C(L)\varepsilon_t \tag{3.5}$$

There is a close relationship between $C(1)$ and the matrix of cointegrating vectors β. In particular we have that $\beta' C(1) = 0$.

The equivalence of vector error correction models and cointegrated variables with the moving average representation of the form (3.5) forms the basis of the Granger Representation Theorem (Engle and Granger, 1987).

The common trend representation follows directly from (3.5). Adding and subtracting $C(1)\varepsilon_t$ from the right hand side of (3.5) yields:

$$\Delta y_t = C(1)\varepsilon_t + [C(L) - C(1)]\varepsilon_t \tag{3.6}$$

and solving backwards for the level of y_t leads to obtain:

$$y_t = C(1)\xi_t + C^*(L)\varepsilon_t + y_0 \tag{3.7}$$

where we have defined $\xi_t = \sum_{s=1}^{t} \varepsilon_s$ and $C^*(L) = (1 - L)^{-1}[C(L) - C(1)] = \sum_{i=0}^{\infty} C_i^* L^i$, $C_i^* = -\sum_{j=i+1}^{\infty} C_j$, and where $\varepsilon_t = 0$ for $i \leq 0$ is assumed.

Equation (3.7) is the multivariate Beveridge-Nelson (1981) decomposition of y_t. It decomposes y_t into its 'permanent component', $C(1)\xi_t + y_0$, and its 'transitory component', $C^*(L)\varepsilon_t$.

Since $C(1)$ has rank k, we can find a non-singular matrix G, such that $C(1)G = [A \; 0_{n\times r}]$, where A is an $n \times k$ matrix with full column rank. Thus $C(1)\xi_t = C(1)GG^{-1}\xi_t$, so that:

$$y_t = A\tau_t + C^*(L)\varepsilon_t + y_0 \tag{3.8}$$

where τ_t denotes the first k components of $G^{-1}\xi_t$.

Equation (3.8) is the common trend representation of the cointegrated system. It decomposes the $n \times 1$ vector y_t into k permanent components, τ_t, and n transitory components, $C^*(L)\varepsilon_t$. The concept of common trend representation (3.8)[27] has been introduced in the previous chapter and it has been used to define

[27] To obtain the common trend representation in (3.8):

(*i*) estimate the VECM (3.3) imposing the cointegration restrictions;

(*ii*) invert the VECM to find the moving average representation (3.5);

(*iii*) find the matrix G (the matrix G is not unique. One way is to construct G from the eigenvectors of A. The first k columns of G are the eigenvectors corresponding to the non-zero eigenvalues of A and the remaining eigenvectors are the last $n - k$ columns of G);

(*iv*) and finally, construct τ_t recursively from $\tau_t = \tau_{t-1} + e_t$, where e_t is the first element of $G^{-1}\varepsilon_t$, and where ε_t denotes the vector of residuals from the VECM.

(Kim and Pagan, 1995)

the permanent component, i.e. the long run, of the endogenous variables of the system (2.39).

If, on the one hand, cointegration can be thought as an useful instrument to interpret long-run relations among data, on the other hand, it has been demonstrated (Wickens[28], 1995, 1996) to be a very dangerous tool of analysis if it is used without imposing theoretical restrictions. In other words, it is not possible to give any economic interpretation to estimated cointegrating vectors (if more than one) derived from the estimation of unrestricted vector error correction models. In our analysis we overcome this problem by using theoretically driven cointegrating vectors, whose interpretation is provided by the long-run solution of the model presented in chapter 2.

We now introduce the instruments for interpreting the estimations results and the problem related to the identification of the structural model.

3.2.3 Impulse Response Functions and Forecast Error Variance Decomposition

In this sub-section we introduce the structural moving average model and show that this model provides answers to the 'impulse' and 'propagation' questions asked at the outset of the thesis. We will then discuss the conditions under which the structural moving average polynomial can be inverted, so that the structural shocks can be recovered from a VAR. When this is possible, a structural VAR obtains, which can be interpreted as a dynamic simultaneous equations model.

The starting model looks like the system of equations (3.5) where y_t is an $n_y \times 1$ vector of economic variables and ε_t is an $n_\varepsilon \times 1$ vectors of shocks, where

[28] *'Far from recovering long-run relations, it can be shown that cointegration analysis is more likely to obscure them'* (Wickens, 1995, pag. 1645).

we allow $n_y \neq n_\varepsilon$. Therefore we call (3.5) structural moving average model, since the elements of ε_t are given a structural economic interpretation. This model is used for answering the following two questions:

1. how the endogenous variables respond dynamically to exogenous shocks?

2. which shocks are the primary causes of variability in the endogenous variables?

To answer the first question we need to look at the dynamic effects of the elements of ε_t on the elements of y_t, that are determined by the elements of the matrix lag polynomial $C(L)$. Letting $C(L) = C_0 + C_1 L + C_2 L^2 + ...$, where C_k, $k = 1, 2..p$, is an $n_y \times n_\varepsilon$ matrix with typical element $[c_{ij,k}]$, we compute the following partial derivative:

$$c_{ij,k} = \frac{\partial y_{i,t}}{\partial \varepsilon_{j,t-k}} = \frac{\partial y_{i,t+k}}{\partial \varepsilon_{j,t}} \tag{3.9}$$

where $y_{i,t}$ is the ith element of y_t, $\varepsilon_{j,t}$ is the jth element of ε_t and the last equality follows from the time invariance of (3.5). Viewed as a function of k, $c_{ij,k}$ is called the impulse response function of $\varepsilon_{j,t}$ for $y_{i,t}$, showing how $y_{i,t+k}$ changes in response to a unit impulse in $\varepsilon_{j,t}$.

To answer the second question, concerning the relative importance of shocks, we need to specify the probability structure of the model and we reformulate it in terms of the h-step-ahead forecast errors of y_t. We assume that shocks are $i.i.d. \sim (0, \Sigma_\varepsilon)$, so that any serial correlation in the exogenous variables is captured in the lag polynomial $C(L)$.

Let $y_{t/t-h} = E(y_t \mid \{\varepsilon_s\}_{s=-\infty}^{t-h})$ denote the h-step-ahead forecast of y_t made at time $t-h$, and let $a_{t/t-h} = y_t - y_{t/t-h} = \sum_{k=0}^{h-1} C_k \varepsilon_{t-k}$ denote the resulting forecast error. The importance of a specific shock can then be represented as the fraction of the variance in $a_{t/t-h}$ that is explained by that shock and it can be calculated for short-run and long-run movements in y_t by varying h. When the shocks are mutually correlated, there is no unique way to do this, since their covariance must somehow be distributed. However, when the shocks are uncorrelated the calculation is straightforward. Assume Σ_ε is diagonal with elements σ_j^2, then the variance of the ith element of $a_{t/t-h}$ is $\sum_{j=1}^{n_\varepsilon}\left[\sigma_j^2 \sum_{k=0}^{h-1} c_{ij,k}^2\right]$, so that we have:

$$R_{ij,h}^2 = \frac{\left[\sigma_j^2 \sum_{k=0}^{h-1} c_{ij,k}^2\right]}{\sum_{m=1}^{n_\varepsilon}\left[\sigma_m^2 \sum_{k=0}^{h-1} c_{im,k}^2\right]} \tag{3.10}$$

(3.10) shows the fraction of the h-step-ahead forecast error variance in $y_{i,t}$ attributed to $\varepsilon_{j,t}$. The set of n_ε values of $R_{ij,h}^2$ are called the variance decomposition of $y_{i,t}$ at horizon h. Impulse responses and forecast error variance decompositions will be used to interpret the dynamic behaviour of our economies in this chapter and in the rest of the thesis.

3.2.4 Identification of the Structural Errors

The structural VAR representation from (3.5) is obtained by inverting $C(L)$ to yield:

$$A(L)y_t = \varepsilon_t \tag{3.11}$$

where $A(L) = A_0 - \sum_{k=1}^{p} A_k L^k$. Since A_0 is not restricted to be diagonal, (3.11) is a dynamic simultaneous equation model. It differs from standard representations of simultaneous equation models because observable exogenous variables are not included in the equations of the system (3.11). The reduced form of (3.11) is

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \ldots + \Phi_p y_{t-p} + e_t \tag{3.12}$$

where $\Phi_i = A_0^{-1}A_i$, for $i = 1, ..p$ and $e_t = A_0^{-1}\varepsilon_t$. Consistent estimates of the Φ_i's can be obtained by running OLS equation by equation on (3.12). One can then estimate $\Sigma_e(= Ee_te_t')$ from the fitted residuals.

The issue concerns now the identifiability of the structural parameters in (3.11). In fact even if we new the Φ_i's, the e_t's and Σ_e it still would not be possible to compute the dynamic response function of y_t to the fundamental shocks in the economy. The basic reason is that e_t is the one-step-ahead forecast error in y_t. In general, each element of e_t reflects the effects of all the fundamental economic shocks. There is no reason to presume that any element of e_t corresponds to a particular economic shock, say for example a shock on foreign prices.

Thus, given the relation between structural and reduced form errors, $e_t = A_0^{-1}\varepsilon_t$, we need to know A_0 as well as the Φ_i's in order to compute the impulse response function. While the Φ_i's can be estimated via OLS regressions, getting A_0 is not so easy. The only information in the data about A_0 is that it solves the two equations:

$$1)\ e_t = A_0^{-1}\varepsilon_t \ \text{ and } 2)\ \Sigma_e = A_0^{-1}\Sigma_\varepsilon(A_0^{-1})'$$

Without restrictions on A_0 there are in general many solutions to these equations.

The traditional simultaneous equation literature places no assumptions on Σ_ε, so that the equations represented by $\Sigma_e = A_0^{-1}\Sigma_\varepsilon(A_0^{-1})'$ provide no information about A_0. Instead, that literature develops restrictions on A_i, $i = 0, ..p$, that guarantee a unique solution to $A_0\Phi_i = A_i$.

Conversely, the VAR literature always imposes the restriction that the fundamental economic shocks are uncorrelated (i.e. Σ_ε is a diagonal matrix), and places no restrictions on A_i. Without additional restrictions on A_0, we can set

$$\Sigma_\varepsilon = I$$

We can also note that if we do not impose restrictions on the A_i's, the equations represented by $A_0\Phi_i = A_i$ provide no information about A_0. All of the information about this matrix is contained in the relationship $\Sigma_e = A_0^{-1}(A_0^{-1})'$.

We now derive the order condition for the identification of the structural shocks. Since y_t is $n \times 1$, there are pn^2 elements in $(\Phi_1, \Phi_2, ..., \Phi_p)$ and $n(n+1)/2$ elements in $\Sigma_e = A_0^{-1}(A_0^{-1})'$, the covariance matrix of the reduced form disturbances. In the structural model (3.11) there are $(p+1)n^2$ elements in $(A_0, A_1, ..., A_p)$, whereas the elements in $\Sigma_\varepsilon = I$ are all known. Thus, there are $n(n-1)/2$ more parameters in the structural model, so that $n(n-1)/2$ restrictions are required for identification.

To know whether these restrictions are sufficient for the identification we need to look at the rank conditions as well. Restrictions on A_0 lead to have highly non-linear constraints on Σ_e, and this in general implies that we can, at most, obtain local identification instead of global identification (see Christiano, Eichenbaum, Evans, 1998).

A widely used way for identifying structural errors is to impose to the elements of the A_0 matrix a Cholesky factorization. This implies a triangular structure of the error terms. In this case, the model results exactly identified (or, in other words, the order condition is exactly satisfied). But, as we were arguing before that each element of e_t reflects the effects of all the fundamental economic shocks in the economy and thus it does not correspond to a particular structural shock, the same problem arises with this a-priori identification scheme. The Cholesky factorization results very arbitrary because it imposes a relation between reduced form errors without theoretical underpinning. Indeed, as long as we do not have a model that suggests a recursive or some other structure among fundamental errors, any a-priori assumption will look very arbitrary.

For the short-run VAR and the VECM estimates carried out in the next two paragraphs we use a different identification scheme. The root of the scheme lies on the distinction between endogenous and exogenous variables that we made at the outset, in the theoretical model. This is really what distinguishes our 'theoretically' driven VAR model from a 'data' driven VAR model. Having singled out the exogenous shocks allows us the identification, for each cointegrating vector, of the sources of the common stochastic trend. In the short-run VAR we do not impose the restriction $\Sigma_\varepsilon = I$, thus we require to find n^2 constraints on A_0. These constraints are obtained by imposing a block diagonal matrix between the endogenous and the exogenous variables, i.e. $A_0 = \left[\begin{array}{c|c} I_r & 0 \\ \hline 0 & I_{n-r} \end{array}\right]$, where r are the endogenous and $n-r$ the exogenous variables. We are only interested in giving a structural interpretation to the shocks multiplying the second block of A_0. This kind of restriction can be read as if we were treating reduced form errors coming from the exogenous variables as if they were structural errors. Indeed they are structural, since they correspond to the exogenous structural errors estimated in chapter 2, and used to complete the probabilistic structure of the model. Said in other words, our assumption is coherent with that made for the exogenous errors driving the fluctuations of the model. In this way we solve the problem of identifying cointegrating vectors and structural shocks prior to the estimation of the model. A similar argument applies to the estimate of the VECM. In section 3.4 we will see how it is possible to estimate the VECM in its structural form by introducing a procedure that solves the identification problem at the outset.

3.3 The Short-Run VAR

3.3.1 The Cointegrating Vectors

In this section we carry out the econometric analysis of the cyclical behaviour of 11 macroeconomic variables[29]: consumption of traded and non-traded goods and total consumption, output of traded and non-traded goods, the balance of payments, the domestic consumer price index, the foreign consumer price index, the foreign nominal interest rate, the nominal exchange rate and the nominal stock of money.

According to the model of the previous chapter, five variables are strongly exogenous with respect to the six endogenous one. The long-run path of the six endogenous variables is determined by the exogenous variables.

In this section we estimate a VAR model made of I(0) variables: the theoretical cointegrating residuals found in the previous chapter. We proceed according to the following steps. We build the 6 cointegrating relationships among the variables singled out while describing the steady state of the model economy in chapter 2. These are: $[c_T - y_T]$, $[c_N - y_N]$, $[c - c_N]$, $[s - p + p^*]$, $[b^* + s + i^* + p^* - y_T]$, $[p - m + c - i^*]$, where all the variables are expressed in logarithms except the nominal foreign interest rate.

[29]In this chapter we reduce a bit the analysis of the model by leaving apart the relative price of non-traded goods. In chapter 2 we argued that the information content of the cointegrating vector containing the relative price of non-traded goods and the price index is the same of that expressing long-run stationarity of the real exchange rate. Moreover, among the exogenous variables we consider the nominal foreign interest rate instead of the real foreign interest rate. Given that the real interest rate has been built by using ex-post inflation, it means that a shock coming from foreign inflation has already been realized. Therefore it is not a strong assumption to think that a shock to i^* has the same origin of a shock to r^*.

The first cointegrating vector identifies the average propensity to consume traded goods, the second the average propensity to consume non-traded goods. These two cointegrating vectors are pretty standard, although we are more used to see a long-run relation between total consumption and total income. The third cointegrating vector derives from the definition of total nominal consumption expenditure which has been rewritten in equation (2.47) as the ratio between total real consumption and non-traded goods consumption. This cointegrating vector, thus, says that there is a stable relationship between total and non-traded consumption or, in other terms, that preferences over the two categories of goods are stable over time. The fourth cointegrating vector defines the real exchange rate. The fifth expresses a relation between returns on foreign assets and traded goods output. We think of it as the average propensity to earn interest on foreign assets. Otherwise, this measure of capital gains on foreign assets can be thought to provide a description of the behaviour of the current account (which is defined, in the model, as the change in the net foreign assets). The sixth cointegrating vector expresses a relation between the velocity of money, where total consumption is replacing total income, and the nominal foreign interest rate.

We call the vector of those relationships Y_t. We then define X_t the vector of our five exogenous $I(1)$ processes: $X_t' = [y_{Tt}, y_{Nt}, i_t^*, p_t^*, m_t]'$.

We can now build the vector $W_t = \beta' Z_t$, where β is the $n \times r$ matrix containing the cointegrating vectors, with $n = 11$ and $r = 6$, and $Z_t = \begin{pmatrix} Y_t \\ X_t \end{pmatrix}$. According to the relationships written above β is equal to:

$$\beta = \begin{array}{cc} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} c_T \\ c_N \\ c \\ s \\ b^* \\ p \\ y_T \\ y_N \\ i^* \\ p^* \\ m \end{array} \end{array}$$

whose columns are formed by the 6 cointegrating vectors enumerated before.

3.3.2 The Methodology

We start by writing down the following VECM:

$$\Delta Z_t = \alpha W_{t-1} + A(L)\Delta Z_{t-1} + e_t \tag{3.13}$$

where $W_{t-1} = \beta' Z_{t-1}$ and $e_t = \begin{bmatrix} e_t^y \\ e_t^x \end{bmatrix}$ are reduced form shocks but with e_t^x having a structural interpretation because coming from the exogenous variables of the model.

The limit of carrying out the direct estimation of this model is that shocks to e_t cannot affect $\beta' Z_{t-1}$ because that vector has been introduced exogenously in the VECM. We overcome the problem by pre-multiplying the previous system of equations (3.13) by the block-diagonal matrix Q :

$$\Delta Q Z_t = Q\alpha W_{t-1} + QA(L)Q^{-1}\Delta Q Z_{t-1} + Qe_t \tag{3.14}$$

where $Q = \begin{pmatrix} \beta' & 0 \\ 0 & I_{n-r} \end{pmatrix}$, with $n - r = 6$. It is now convenient to rewrite the system (3.14) in this way:

$$\begin{pmatrix} \Delta W_t \\ \Delta X_t \end{pmatrix} = \begin{pmatrix} \beta' \alpha_1 \\ \alpha_2 \end{pmatrix} W_{t-1} + C(L) \begin{pmatrix} \Delta W_{t-1} \\ \Delta X_{t-1} \end{pmatrix} + u_t \tag{3.15}$$

where $u_t = Qe_t$, yielding:

$$\begin{pmatrix} W_t \\ \Delta X_t \end{pmatrix} = D(L) \begin{pmatrix} W_{t-1} \\ \Delta X_{t-1} \end{pmatrix} + u_t \tag{3.16}$$

We can estimate this unrestricted short-run VAR by OLS, which we use to interpret the impulse response of the stationary long-run relationships to structural shocks.

3.3.3 Impulse Response Functions

The estimation program has been written in Gauss[30]. The long-run cointegrating vectors are over-imposed by the theory whereas the short-run dynamics among the cointegrating residuals does not have any theoretical restriction (thus, the name unrestricted short-run VAR). The identification of the exogenous shocks is obtained by applying an orthogonal structure to the correlation matrix of the residuals, as we have seen in the previous paragraph. We will present the results by looking at the impulse response functions of each variable to the structural shocks. On the horizontal axes the time after the shock is measured in quarters. On the vertical axes it is shown the deviation from the steady state expressed in basis points. The data used have been described in the previous chapter, section 2.5. The small open economy under analysis is the UK and the rest of the world is represented by a weighted average of the OECD countries (Appendix A).

Figures 3.1 show two common features. The first is that impulse responses of the endogenous cointegrating vectors to the exogenous variables take a very

[30]The estimation results are presented in Appendix C.

long time to decline to zero, this is particularly true for the real exchange rate and for the cointegrating vector that we have called. for brevity, *c.a.* (current account) expressing the average gain or loss in trading foreign assets. The second characteristic is that deviations from equilibrium are very small in size, with an upper and lower bound that never exceeds 0.8%.

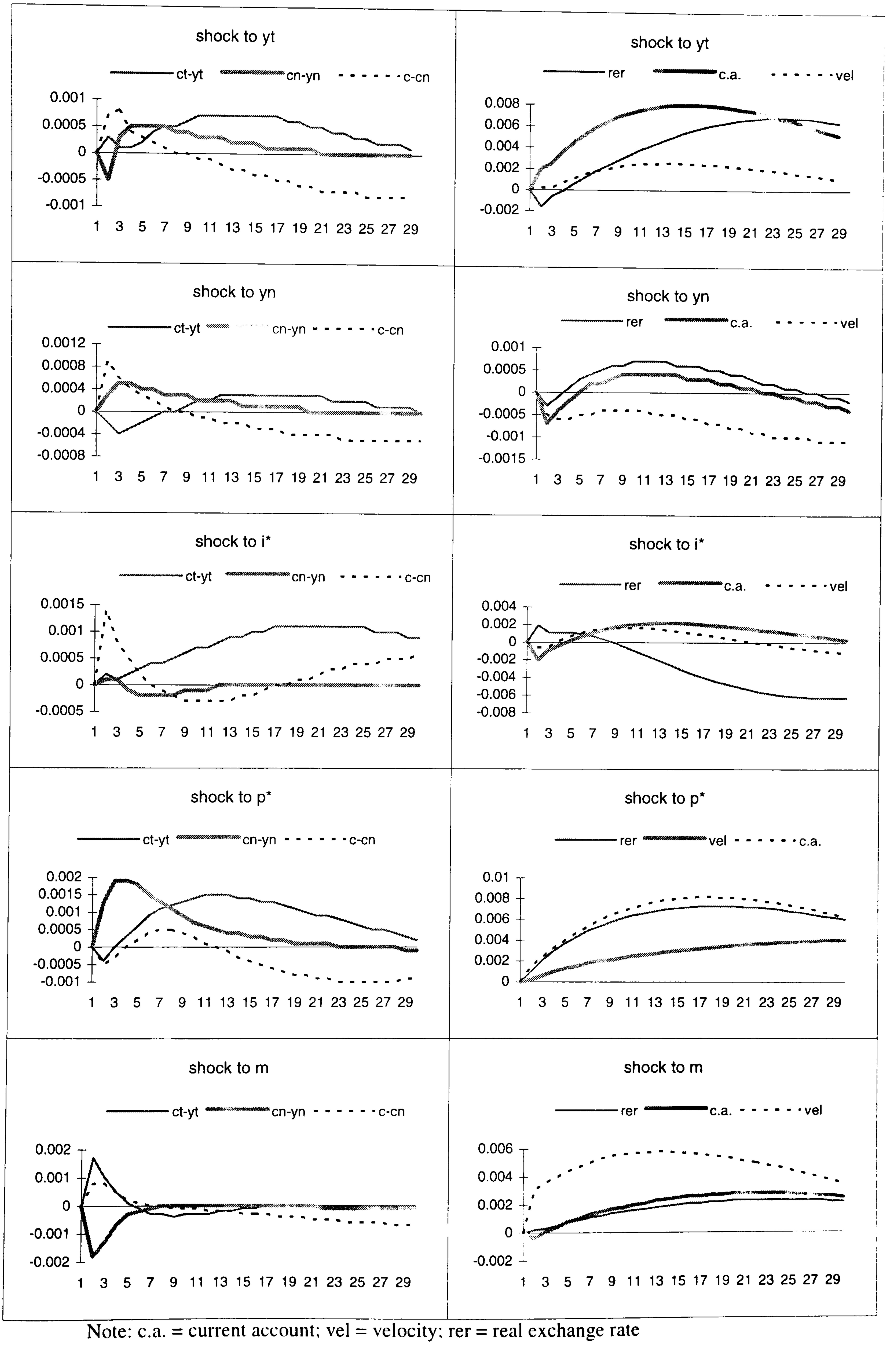
Starting from the first picture (top-left - fig. 3.1). a shock to traded output leads to a temporary increase of the average propensity to consume traded $(c_T - y_T)$ at the expense of non-traded goods $(c_N - y_N)$. Thus, we assist to a substitution of consumption expenditure towards traded goods. The same shock (top-right - fig. 3.1) leads to a real depreciation of the exchange rate (rer), to an increase of the velocity of money (vel) and to a surplus of the balance of payments then translated into higher income from foreign assets $(c.a.)$. This last pattern is consistent with the idea of consumption smoothing, that is consumption does not increase as much as output after a productivity shock.

A positive shock to non-traded output (second row - fig. 3.1) leads the average propensity to consume traded and non-traded goods to respond in an opposite way with respect to the previous case, whereas the response of total consumption does not change. Now consumers switch their spending decisions towards non-traded goods. As before, we can observe that productivity shocks coming from the non-traded sector lead to a depreciated real exchange rate. Conversely, the same shock leads to a negative reaction of the velocity and the current account. This means that a shock to non-traded productivity does not have an inflationary impact on the domestic price index in the short run. For the short-run behaviour of the current account we start from a situation of deficit followed by a surplus, this implies that at the beginning of the period consumption reacts more than output and that this position is reversed when output goes back to the steady state.

Shocks from abroad are captured in the model by the exogenous behaviour of the foreign price index (built as a weighted average of the price indices of the OECD countries - excluded highly inflationary countries, i.e. Turkey) and of the foreign nominal interest rate.

A positive shock to the foreign nominal interest rate (third row - fig. 3.1) leads to a positive reaction of total consumption which goes back to zero after two years. This shock causes a temporary deficit of the current account which is reversed after a year, a fall in the velocity of money (and thus in the price level) and the overshooting of the real exchange rate. More precisely, the exchange rate depreciates at the beginning of the period and it appreciates after two years. Under the hypothesis of a 'small open economy' we are expecting that the domestic interest rate follows the same path of the foreign real interest rate. A shock to the domestic nominal interest rate is then transmitted to the domestic real one. Thus, this leads, on the one hand, to substitute today consumption by tomorrow consumption (substitution effect) and, on the other hand, to increase today consumption because of the wealth effect. According to the estimated impulse responses the wealth effect dominates on impact and vanishes after 5 quarters. We can also note that the substitution effect is dominant for the consumption of traded goods. The dynamics of the real exchange rate and of the price level (deduced by that of the velocity) is also theoretically consistent: the induced increase of the domestic interest rate leads to an appreciated currency and reduces inflationary pressures.

Figures 3.1: Impulse response functions of the unrestricted VAR (1969:3-1997:3)



Note: c.a. = current account; vel = velocity; rer = real exchange rate

We finally notice that a positive shock to foreign prices (fourth row - fig. 3.1) leads to an increase of domestic prices (increase in the velocity of money), to a real depreciation and to a temporary surplus of the current account. The dynamic responses are again theoretically consistent. The domestic economy seems to follow the behaviour of the rest of the world. Higher inflation in the rest of the world translates into higher domestic inflation.

The short-run responses of the endogenous variables to a monetary shock (fifth row - fig. 3.1) witness the presence of positive real effects in traded and total consumption. This shock leads instead to a negative reaction of the non-traded goods consumption. Given that the real interest rate is exogenously determined, the degrees of freedom that the policy maker has in deciding the monetary policy are limited to the short-run. Despite this, monetary policy can determine the short-run path of the nominal domestic interest rate that needs not to be equal to the foreign one because of inflation differentials. A monetary shock causes a persistent increase in the velocity (and thus in the price level) and a persistent real depreciation.

We consider this exercise as a test on the reliability of the structural relationships among variables found in the previous chapter. We have found that the short-run dynamics of the variables is theoretically consistent.

Given that we are using cointegration to extract the long-run common trends, each residual expresses the deviation from the long run of the six variables appearing in each of the six cointegrating vectors. This enables us to compare the results of these short-run impulse response functions with those obtained in the following chapter, where we will simulate the short-run behaviour of the artificial data.

3.4 The VECM Estimation Procedure

3.4.1 The Methodology

In this section we describe the procedure used for the estimation of a structural VECM (Wickens and Motto, 1999). Differently from standard VECM (see equation 3.3), we allow for contemporaneous exogenous variables to enter in the short-run dynamics of the endogenous variables, as suggested by the model. The structural form VECM can be written in a more general form than equation (3.13), i.e.:

$$\begin{bmatrix} B_0 & C_0 \\ 0 & I \end{bmatrix} \Delta Z_t = -\begin{bmatrix} B(1) & C(1) \\ 0 & I \end{bmatrix} Z_{t-1} + A(L)\Delta Z_{t-1} + \varepsilon_t \qquad (3.17)$$

where Z_t has been previously defined to be the vector of all the variables of the system, $Z_t = \begin{pmatrix} Y_t \\ X_t \end{pmatrix}$: and $A(L) = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ 0 & A_{22}(L) \end{bmatrix}$, where $A_{21}(L) = 0$ implies that X_t is strongly exogenous. To get the reduced form we write (3.17) as:

$$\begin{aligned} F\Delta Z_t &= -\alpha W_{t-1} + RA(L)\Delta Z_{t-1} + e_t \\ e_t &= R\varepsilon_t \end{aligned} \qquad (3.18)$$

with $E\left[\varepsilon_t \varepsilon_t'\right] = \Sigma_\varepsilon$ (diagonal), and $F = \begin{bmatrix} I & B_0^{-1}C_0 \\ 0 & I \end{bmatrix}$, $R = \begin{bmatrix} B_0^{-1} & 0 \\ 0 & I \end{bmatrix}$, $\alpha = \begin{pmatrix} B_0^{-1}a \\ 0 \end{pmatrix}$. We then pre-multiply (3.18) by F^{-1} to give:

$$\begin{aligned} \Delta Z_t &= -\alpha\beta' Z_{t-1} + A(L)\Delta Z_{t-1} + u_t \\ u_t &= F^{-1}R\varepsilon_t \end{aligned} \qquad (3.19)$$

We thus estimate the system of equations:

$$\begin{aligned}\Delta Y_t &= -B_0^{-1}C_0\Delta X_t - B_0^{-1}aW_{t-1} + +B_0^{-1}\left[\begin{array}{cc} A_{11}(L) & A_{12}(L)\end{array}\right]\Delta Z_{t-1} + B_0^{-1}\varepsilon_{1t} \\ \Delta X_t &= A_{22}(L)\Delta X_{t-1} + \varepsilon_{2t} \end{aligned} \quad (3.20)$$

where we remind that $W_t = \beta' Z_t$; $\beta' = a\left[\begin{array}{cc} B(1) & C(1)\end{array}\right]$; $a = diag\{B(1)\}$. From the estimate of the first block of equations in (3.20) we can obtain B_0^{-1}, then C_0, thus we can estimate the structural form (3.17). We then use (3.18) to compute the impulse response functions.

3.4.2 Estimation Results

Each equation of the VECM has been estimated by OLS. We obtained the following results:

Table 3.1: OLS estimates of the VECM, 1969:3 - 1997:3, (t-stat between brackets)

	Δc_{Tt}	Δc_{Nt}	Δc_t	Δs_t	Δb_t	Δp_t
Δc_{Tt-1}	-.05 [-.26]	-.09 [-.58]	.06 [.36]	-.14 [-1.02]	-.09 [-.45]	.11 [3.2]
Δc_{Nt-1}	-.07 [-.60]	-.33 [-3.2]	-.005 [-.05]	.05 [.57]	.006 [.05]	.03 [1.6]
Δc_{t-1}	-.35 [-.71]	.66 [1.6]	-.24 [-.58]	-.14 [-.39]	.46 [.91]	-.13 [-1.5]
Δs_{t-1}	.19 [1.3]	-.14 [-1.5]	.09 [.69]	.34 [3.3]	-.05 [-.32]	-.002 [-.06]
Δb_{t-1}	-.22 [-.64]	.15 [.53]	-.17 [-.60]	-.18 [-.74]	.36 [1.04]	-.02 [-.06]
Δp_{t-1}	-1.14 [-2.1]	.17 [.39]	-.69 [-1.6]	.64 [1.7]	.76 [1.4]	.48 [5.2]
Δy_{Tt-1}	-.09 [-29]	-.23 [-.87]	.10 [.37]	.16 [.69]	-.26 [-.79]	.01 [.25]
Δy_{Nt-1}	.15 [.66]	-.12 [-.63]	.08 [.42]	.08 [.49]	-.16 [-.72]	.01 [.25]
Δi^*_{t-1}	.51 [.67]	.15 [.24]	1.05 [1.6]	.54 [1.0]	-.89 [-1.1]	.009 [.07]
Δp^*_{t-1}	-.70 [-.54]	2.1 [1.9]	.006 [.006]	-1.2 [-1.3]	-.09 [-.07]	.11 [.49]
Δm_{t-1}	.23 [2.2]	.11 [1.3]	.20 [2.3]	.18 [2.6]	-.22 [-2.1]	.04 [2.2]
ecm^1_{t-1}	.18 [3.1]	.17 [3.4]	.12 [2.4]	.04 [.97]	-.14 [-2.3]	-.02 [-2.2]
ecm^2_{t-1}	.02 [.95]	-.04 [-1.7]	.005 [.25]	-.013 [-.72]	-1.7e^{-4}[$-$.01]	2.6e^{-4}[.06]
ecm^3_{t-1}	.13 [1.15]	-.34 [-3.6]	.09 [.97]	-.17 [-2.1]	-.09 [-.81]	.03 [1.3]
ecm^4_{t-1}	-.23 [-2.9]	-.003 [-.05]	-.21 [-3.1]	-.16 [-2.9]	.23 [2.8]	.008 [.54]
ecm^5_{t-1}	.06 [1.0]	.22 [4.3]	.06 [1.1]	.10 [2.3]	-.07 [-1.1]	-.02 [-1.7]
ecm^6_{t-1}	-.07 [-2.3]	-.07 [-2.6]	-.04 [-1.6]	-.05 [-2.3]	.04 [1.43]	.008 [1.5]
Δy_{Tt}	.47 [9.4]	.08 [1.96]	.33 [7.7]	-.01 [-.33]	.56 [10.9]	-.03 [-3.7]
Δy_{Nt}	.27 [4.3]	.33 [6.5]	.29 [5.6]	-.003 [-.07]	.31 [5.0]	-.02 [-2.0]
Δi^*_t	.52 [.64]	1.43 [2.1]	.12 [.18]	-.15 [-.25]	-.12 [-.14]	-.19 [-1.4]
Δp^*_t	1.17[.95]	-2.8 [2.76]	-.15 [-.15]	-.73 [-.84]	.28 [.22]	.45 [2.1]
Δm_t	.18 [1.8]	-.11 [-1.3]	.11 [1.3]	.09 [1.3]	-.08 [-.82]	.03 [1.9]
$const$	4e^{-4}[.42]	6e^{-4} [.74]	-2e^{-4}[$-$.25]	-7e^{-4}[$-$1.01]	3e^{-4}[.33]	-2e^{-5}[$-$.15]

where $ecm_t^1 = [c_{Tt} - y_{Tt}]$, $ecm_t^2 = [c_{Nt} - y_{Nt}]$, $ecm_t^3 = [c_t - c_{Nt}]$, $ecm_t^4 = [s_t - p_t + p_t^*]$, $ecm_t^5 = [b_t + s_t + i_t^* + p_t^* - y_{Tt}]$, $ecm_t^6 = [p_t - m_t + c_t - i_t^*]$.

The OLS estimates of the exogenous variables produce:

Table 3.2: OLS estimates of the exogenous variables, 1969:3 - 1997:3, (t-stat between brackets)

	Δy_{Tt}	Δy_{Nt}	Δi_t^*	Δp_t^*	Δm_t
Δy_{Tt-1}	-.19 [-1.9]	.24 [3.07]	.008 [1.4]	3.3e^{-4} [.08]	.03 [.65]
Δy_{Nt-1}	.01 [.12]	-.06 [-.65]	.01 [1.5]	.006 [1.3]	.03 [.44]
Δi_{t-1}^*	.63 [.47]	-1.83 [-1.7]	.36 [4.1]	.2 [3.7]	-.15 [-.22]
Δp_{t-1}^*	-2.6 [-2.09]	-.29 [-.30]	-.10 [-1.2]	.79 [15.8]	1.6 [2.4]
Δm_{t-1}	.05 [.29]	.14 [1.0]	.03 [2.7]	.02 [2.4]	.31 [3.4]
const	1.5e^{-4}[-.07]	-.001 [-.68]	-7e^{-5}[$-$.5]	-6.9e^{-5}[$-$.8]	7e^{-4}[.64]

By looking at the system of the endogenous variables (table 3.1) we can notice, at first sight, that lagged endogenous variables do not enter with a significant coefficient in almost any of the equations. Lagged inflation is significant in the equation for traded consumption growth and in its own equation. Non-traded consumption growth and nominal exchange rate differentials significantly depend on their respective lagged values.

Among the error correction terms, in the equation for traded consumption only the forth (residual from the real exchange rate) and the sixth (residual from the money demand equation) have a significant and negative coefficient. The exogenous variables significantly related to traded consumption are the growth rate of traded and non-traded output. If we sum up the two significant coefficients of the error correction terms in the traded consumption equation we get a negative value, and this happens for all the equations of the endogenous variables. This implies that the system is returning to its steady-state path, with a speed of adjustment determined by the sum of the parameters multiplying past deviations from the steady state. But each of these coefficients does not contribute always in bringing back each growth rate of the endogenous variables toward the long-run path.

If we now look at the equation for non-traded consumption we find that it significantly depends on foreign variables. This is not surprising given that we observed spillovers between non-traded output and foreign variables. The dynamics of non-traded goods consumption positively depends on foreign interest rates and negatively on foreign inflation.

In the equation for the nominal exchange rate the only significant exogenous variable is the lagged money growth rate. Among lagged endogenous variables, changes in the nominal exchange rate and past domestic inflation enter significantly and with a positive coefficient. An important role in explaining the short-run dynamics of Δs_t is also played by the error correction terms: four out of six are significant, namely those identifying the long-run behaviour of total consumption, the exchange rate, the net foreign bonds position and the domestic price level.

The equation describing the short-run behaviour of the foreign assets position negatively depends on the lagged money growth rate and positively on contemporaneous traded and non-traded output growth rates.

Finally, the dynamics of the domestic price level looks quite complex. Lagged and contemporaneous money growth rates are positively related to current domestic inflation. Lagged inflation has, then, a quite high autoregressive coefficient. Current domestic inflation is positively related to foreign inflation.

The estimate of the correlation matrix of the structural errors is the following:

Table 3.3: The Correlation Matrix between Structural Errors

	c_T	c_N	c	s	b^*	p	y_T	y_N	i^*	p^*	m
c_T	1	.013	-.75	-.69	-.49	.38	0	0	0	0	0
c_N	.013	1	-.32	-.49	.67	.84	0	0	0	0	0
c	-.75	-.32	1	.83	.44	-.42	0	0	0	0	0
s	-.69	-.49	.83	1	.09	-.7	0	0	0	0	0
b^*	-.49	.67	.44	.09	1	.55	0	0	0	0	0
p	.38	.84	-.42	-.7	.55	1	0	0	0	0	0
y_T	0	0	0	0	0	0	1	-.18	.06	-.07	.11
y_N	0	0	0	0	0	0	-.18	1	.19	-.05	-.01
i^*	0	0	0	0	0	0	.06	.19	1	.22	.00
p^*	0	0	0	0	0	0	-.07	-.05	.22	1	-.07
m	0	0	0	0	0	0	.11	-.01	.00	-.07	1

From table 3.3 we notice that correlations among exogenous errors (second diagonal block) are very low, whereas the errors coming from the cointegrating relationships are highly correlated (first diagonal block).

3.4.3 Impulse Response Functions and Forecast Error Variance Decomposition

The analysis proceeds by answering the following two questions: how do endogenous variables respond dynamically to exogenous shocks? What are the important sources of fluctuations in the endogenous variables? As we saw before, the first question is answered by the moving average representation of the structural VECM and by its associated impulse response functions. The second question is answered by analysing the structural VECM forecast error variance decomposition.

In figures 3.2 we plot the long-run impulse response functions of each variable with respect to the exogenous shocks. Before commenting the graphs, it is important to clarify the difference between this analysis and that carried out in the previous paragraph. Here, we are analysing the long-run responses of the original endogenous variables to shocks in the level (permanent components) of the exogenous variables. Previously, we looked at the short-run responses of the cointegrating residuals to shocks coming from the cyclical (or temporary) components of the exogenous variables.

A permanent shock to traded output (top-panel - fig. 3.2 - pag. 104) leads to a permanent increase of traded consumption, and to a persistent increase of non-traded and total consumption. The long-run increase of consumption is lower than that of output and this leads to a surplus of the balance of payments. The nominal exchange rate depreciates.

A permanent shock to non-traded output (bottom-panel - fig. 3.2 - pag. 104) creates a wedge between traded and non-traded goods consumption: the former is declining toward negative values after a positive reaction, whereas the latter is permanently and positively affected. This opposite reaction is then consistent with an increase in the relative price of traded goods and a depreciation of the nominal exchange rate.

On impact a shock in the foreign nominal interest rate (top-panel - fig. 3.2 - pag. 105) leads to an increase of total, non-traded and traded goods consumption. This positive effect lasts two years and becomes then negative. To understand the dynamic behaviour of consumption we have to distinguish two effects. The substitution effect leads to a delay of consumption expenditures whereas the wealth effect leads to an increase of consumption expenditures. This second effect is higher on impact.

Given the assumption of a small open economy, an increase of foreign interest rates is translated to an increase of domestic interest rates, but this occurs with a delay. The explanation relies on the behaviour of the nominal exchange rate.

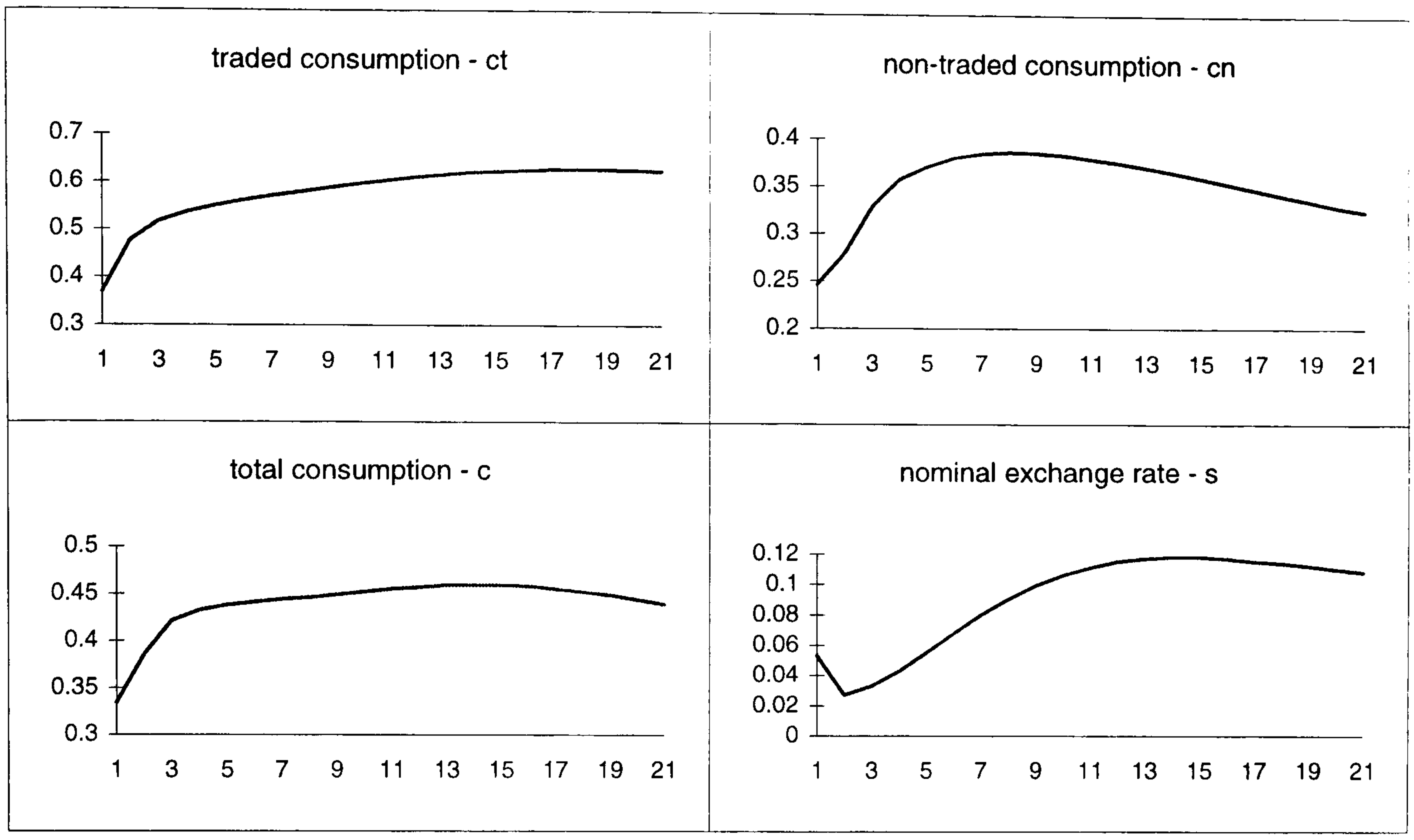
We have overshooting of the nominal exchange rate. It appreciates only after an impact depreciation. If we interpret the shock in the foreign interest rate as a foreign monetary policy shock, then, in our model, we do not have any exchange rate puzzle nor any forward discount puzzle (see section 1.1.3 in chapter 1 and Grilli, Roubini, 1996).

The reaction of consumption in both sectors to a shock in the foreign price level is negative (bottom-panel - fig. 3.2 - pag. 105). In this case the reaction depends on the anticipated increase of domestic inflation. We can observe, indeed, that a shock on the foreign price level leads to an increase of the domestic price of traded goods. The nominal exchange rate depreciates after an initial appreciation. Thus, we have undershooting of the exchange rate.

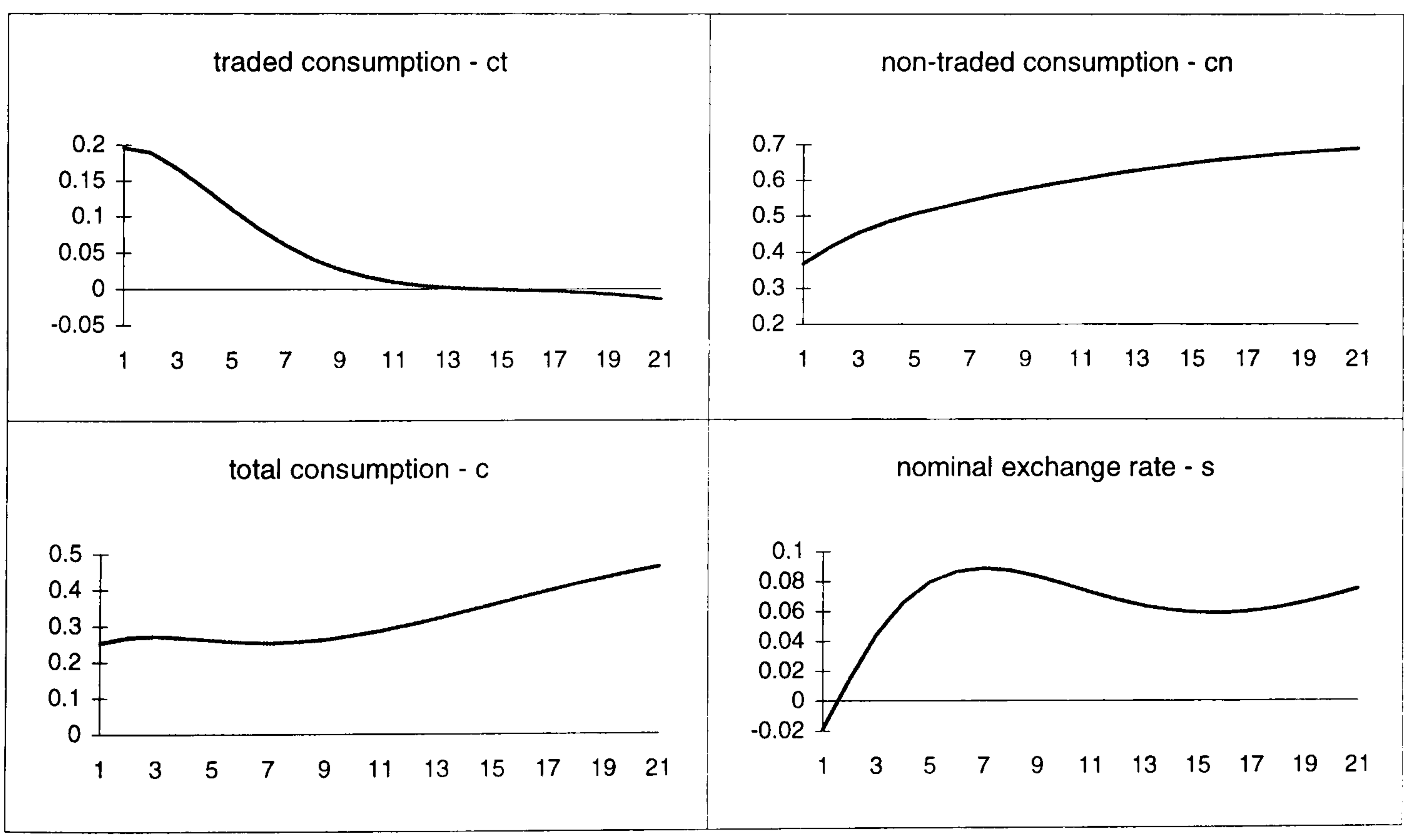
We now look at the effects of a monetary shock (top-panel - fig. 3.2 - pag. 106). We know that the reactions of the variables to a monetary shock serve as a test on the reliability of the identification scheme adopted. For example, when monetary policy shocks are identified as innovations in monetary aggregates (such as M0, M1, M2, etc.), such innovations appear to be associated with increases rather than decreases in nominal interest rates. In our model the exogenous nature of the interest rate (because determined by the foreign one) does not directly lead to address the liquidity puzzle problem. However, we can observe that a monetary shock has a positive effect on consumption of traded and non-traded goods that lasts one year followed by a negative adjustment path in the case of traded goods and by an oscillating path for the non-traded and total consumption. These impulse response functions support the idea of a liquidity effect in the short run and of an anticipated inflation effect in the longer term. The price level is positively and permanently affected.

Figures 3.2: Impulse response function of the VECM

Shock to yt

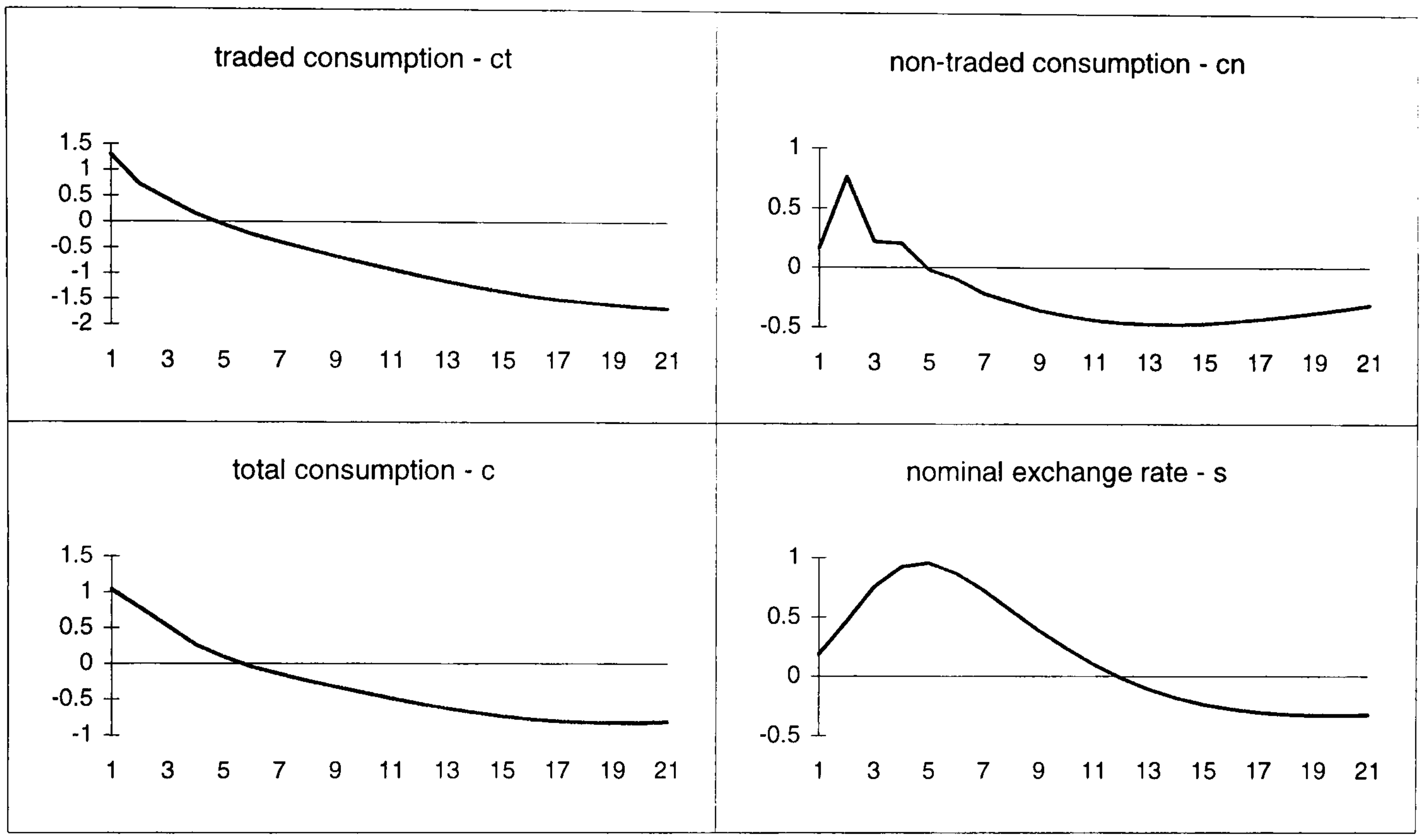


Shock to yn

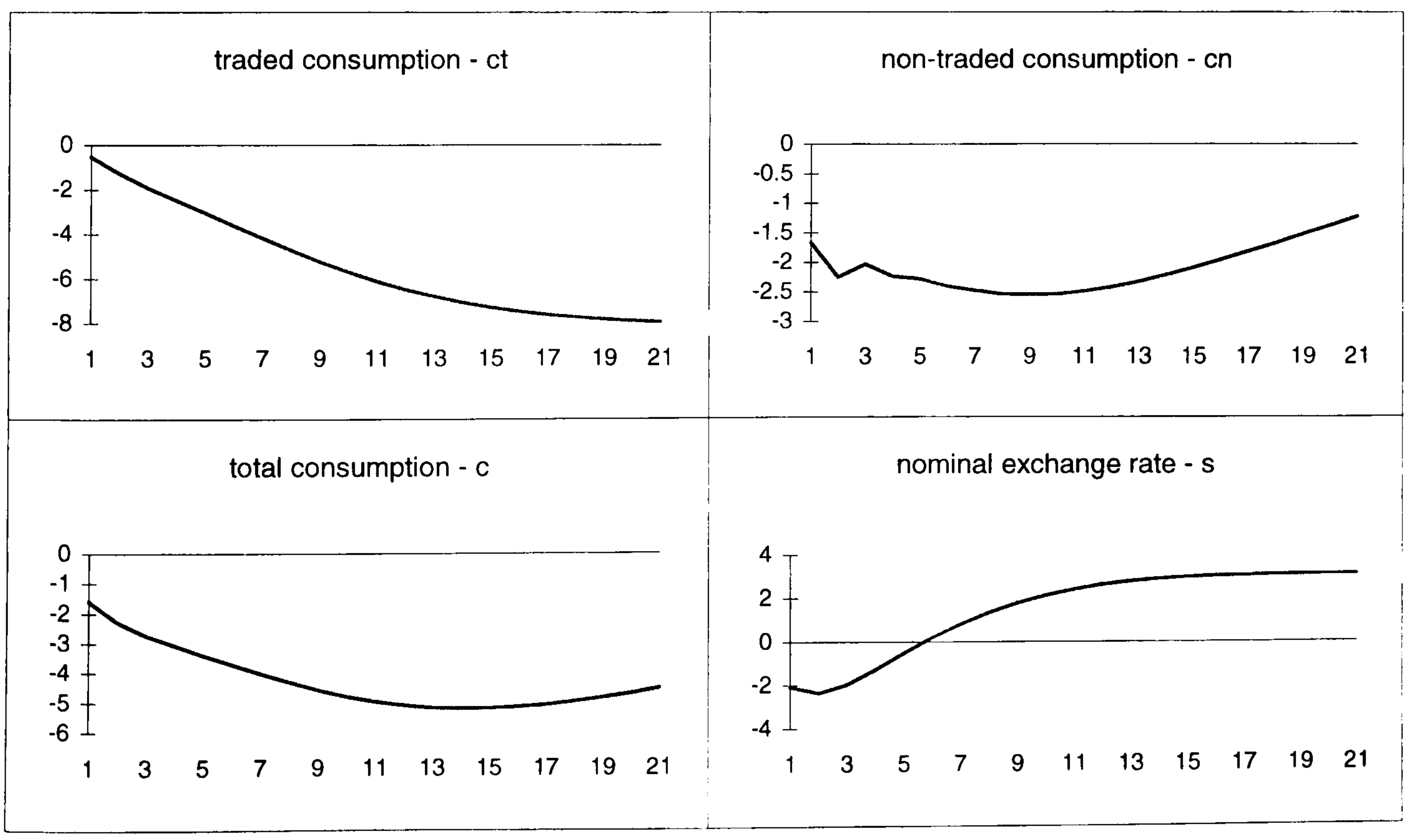


Figures 3.2: Impulse response function of the VECM

Shock to i*

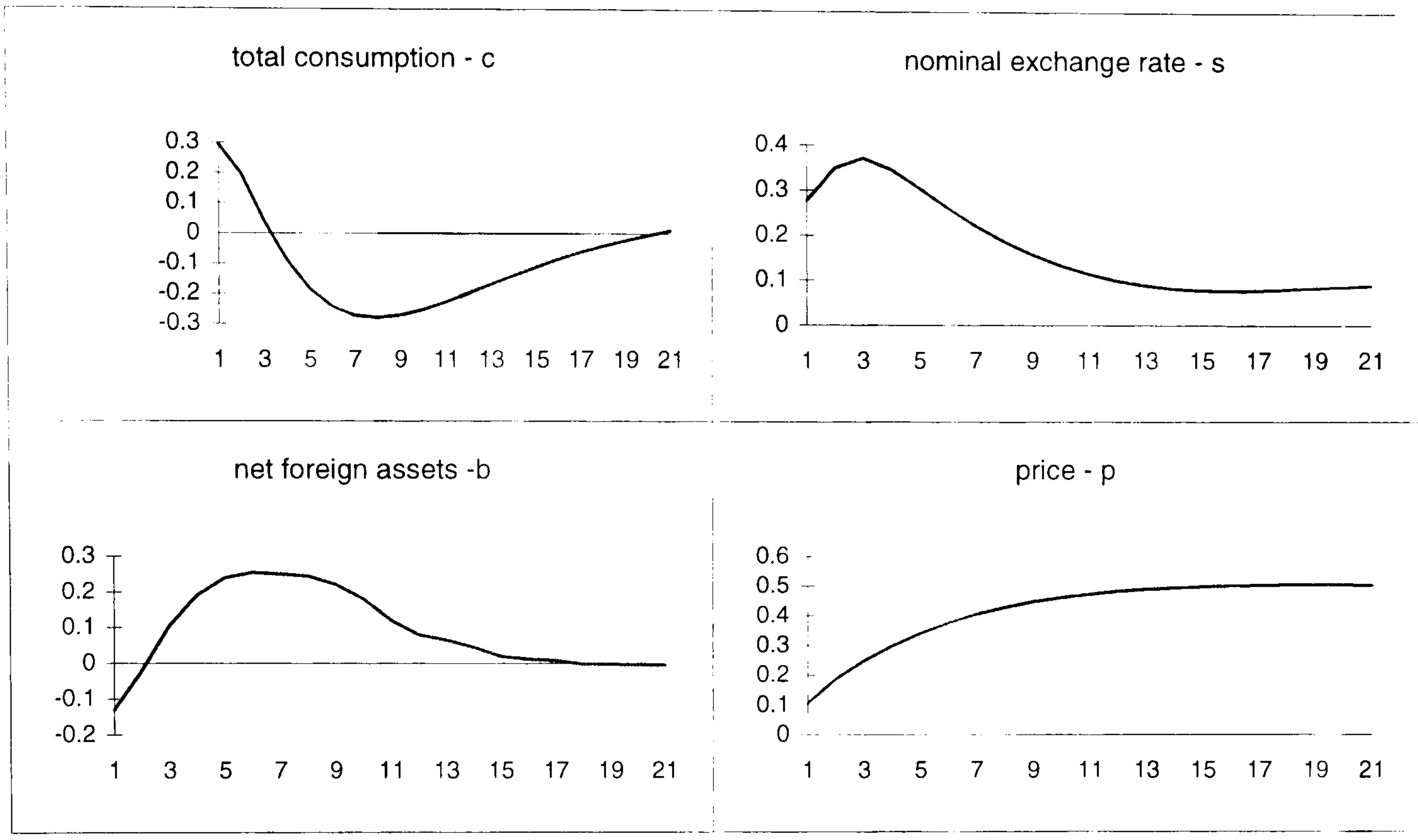


Shock to p*

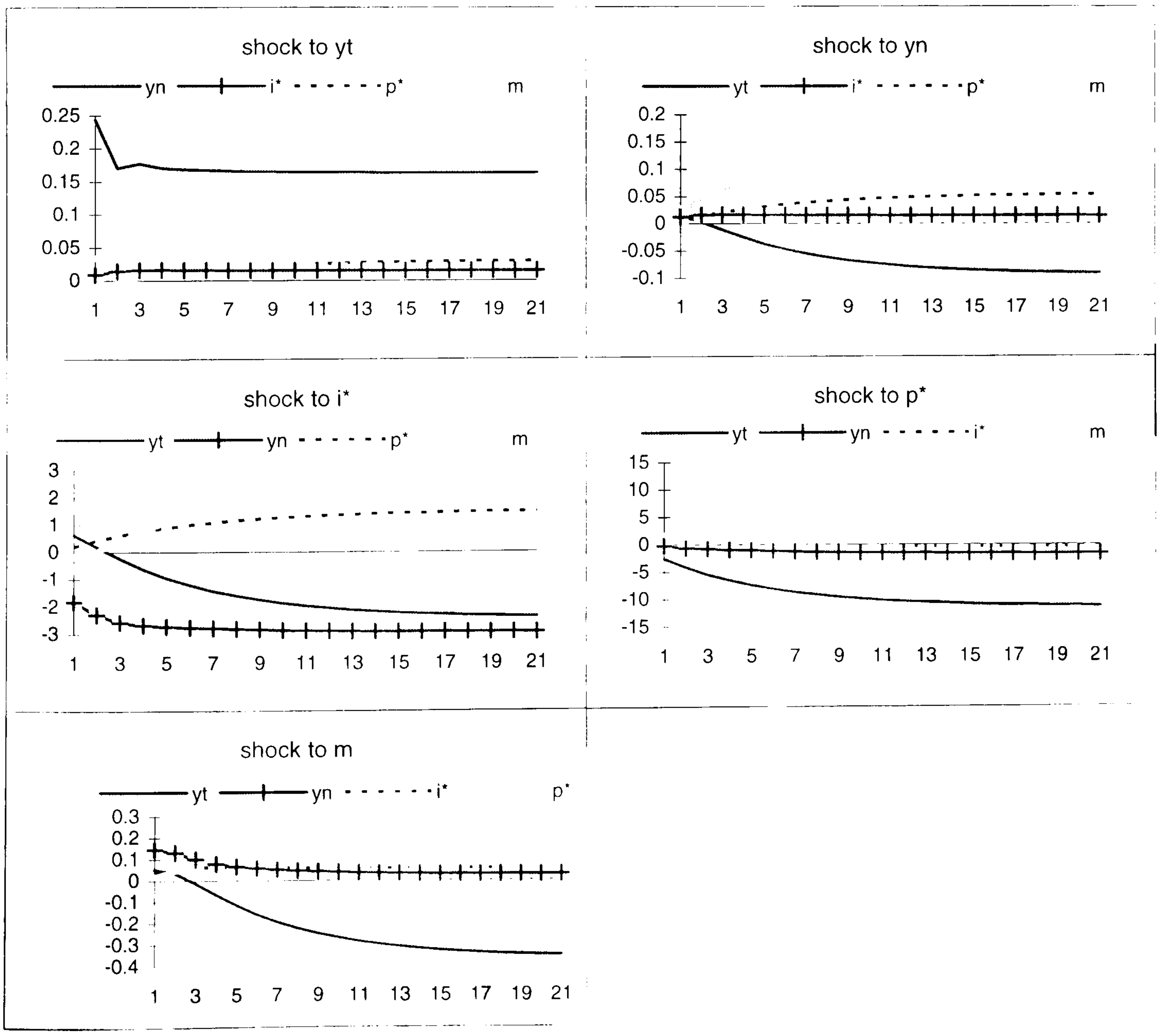


Figures 3.2: Impulse response function of the VECM

Shock to m



Exogenous Shocks



After a monetary shock the nominal exchange rate undergoes a persistent depreciation with overshooting, that is the impact depreciation greatly exceeds the final equilibrium level.

From the impulse responses of the exogenous variables (bottom-panel - fig. 3.2 - pag. 106) we notice that the money supply, as measured by M0 for the UK, does not result independent from real shocks. This means that the exogeneity assumption made for the monetary policy rule is too strong and that we should allow for a more complex 'systematic' behaviour of the monetary authority.

Table 3.4: Forecast Error Variance Decomposition

	c_T					c_N					c				
horizon	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m
1	.06	.02	.72	.13	.06	.02	.04	.01	.92	.00	.03	.02	.27	.65	.02
2	.07	.02	.46	.40	.04	.02	.03	.07	.87	.00	.03	.01	.17	.77	.01
5	.05	.01	.10	.84	.01	.02	.04	.03	.91	.00	.02	.01	.04	.93	.00
10	.02	.00	.03	.94	.01	.02	.04	.02	.91	.00	.01	.00	.02	.96	.00
20	.01	.00	.03	.95	.01	.02	.05	.03	.90	.00	.01	.00	.02	.97	.00

	s					b^*					p				
horizon	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m	ε_{y_T}	ε_{y_N}	ε_{i^*}	ε_{p^*}	ε_m
1	.00	.00	.01	.96	.02	.09	.04	.68	.16	.01	.00	.00	.01	.98	.01
2	.00	.00	.02	.95	.02	.05	.02	.56	.36	.00	.00	.00	.00	.99	.01
5	.00	.00	.13	.83	.03	.02	.01	.36	.61	.00	.00	.00	.01	.98	.01
10	.00	.00	.14	.83	.02	.01	.00	.30	.68	.00	.00	.00	.03	.97	.00
20	.00	.00	.06	.93	.01	.01	.00	.26	.73	.00	.00	.00	.04	.95	.00

We finish with the analysis of the forecast error variance decomposition. Table 3.4 shows how much variance of each endogenous variable can be explained by each exogenous shock for twenty-step-ahead. We notice, firstly, that the five exogenous sources of fluctuations cover all the variability of the endogenous variables. This means that the spectrum of exogenous shocks considered in the model is sufficient to generate all the variability observed in actual data. In this respect we can say that the probabilistic structure of the model has been well specified. From table 3.4 it clearly appears that shocks to foreign prices play the most important role in explaining the economy's fluctuations. The second important exogenous force is given by foreign interest rates. Therefore, fluctuations in our small open economy crucially depend on the behaviour of the foreign exogenous variables. A common characteristic among the volatilities of c, c_N ,c_T is that the contribution of foreign nominal interest rates declines to zero after 10-step-ahead. Conversely, the contribution of a shock to the foreign price level in explaining the volatility of the same set of variables is stable over the horizon considered. All the variability of b^* is explained by foreign shocks, with the explanatory power of p^* increasing and that of i^* decreasing for longer horizons.

3.5 Conclusions

In this chapter we estimated a stationary VAR among theoretical cointegrating residuals and a structural VECM based on the same long-run relationships. Our approach is innovative in two ways. First, we can give an economic interpretation to the long-run cointegrating vectors. Second, those cointegrating vectors are not reduced forms taken from traditional macroeconomics, but are derived from the solution of an explicit maximization problem. Four of these vectors are very well known: consumption and output in the two sectors, the velocity of money and the real exchange rate. However, the long-run relation based on the foreign bonds market is new. It relates the behaviour of real bonds and output to foreign interest rates and to the exchange rate in equilibrium. In terms of the small open economy model presented in the previous chapter, a positive value of the change in foreign bonds indicates a surplus of the balance of payments, and a negative value a deficit of the balance of payments. Therefore, the VECM estimation suggests that positive shocks to domestic output lead to a surplus of the balance of payments together with increased consumption and a depreciated real and nominal currency. Monetary shocks have an overshooting effect on the nominal exchange rate. A positive shock on foreign prices generates undershooting of the nominal exchange rate, whereas a positive shock on foreign nominal interest rates leads to the overshooting of the nominal exchange rate.

The forecast error variance decomposition obtained from the estimates of the VECM showed that foreign shocks to the price level and the interest rate have been the main determinants of the UK fluctuations over the period 1969-1997.

4 Short-run Dynamics of the Small Open Economy: Simulation Results

4.1 Introduction

In chapter 2 we derived the behavioural equations of a small open economy. We described the economy's steady-state and we used the steady-state information for calibrating and estimating the parameters measuring preferences for consumption goods and real money balances. In chapter 3 those steady-state conditions have been exploited for estimating an unrestricted VAR and a structural VECM. In this chapter we continue the analysis by deriving the solution of the model. The first step consists of log-linearizing each equation of our highly non-linear intertemporal stochastic model. The second step consists of simulating the model. The procedure is implemented by using the structural parameters measured in chapter 2. This will allow us to compute statistics on the simulated data. The problem of measuring the deep parameters is strictly related to that of testing the fit of the model. And this last problem can be accounted for only if the model possesses a fully specified probabilistic structure.

The probabilistic underspecification of DSGE models was first recognised by Hansen and Sargent (1979) who used a maximum likelihood approach for the estimation of the deep parameters of DSGE and for testing their validity. They concentrate on linear-quadratic specifications for the primitives of the model and linear processes for the exogenous variables and they augment their model with additional random components (measurement errors, errors in variables or unobserved components). Once a final form solution is obtained and there are enough shocks in the economy to make the model 'complete' in a probabilistic sense, one proceeds to identify and estimate the parameters. Instead of focusing on linear-quadratic specifications, Hansen (1982) proposed to estimate and test hypotheses on 'deep' parameters directly from the Euler equations using simple moment conditions. Although Hansen's Generalized Method of Moments (GMM) approach does not require a closed form solution for the endogenous variables, it still requires a fully specified probability structure for the model.

Contemporaneously to the work of Hansen (1982), Kydland and Prescott (1982) suggested an alternative procedure to tackle the problem of probabilistic underspecification. Rather than augmenting an artificial economy with extraneous random components to obtain a richer statistical structure, they start from the observation that the model, as a data generating process (DGP), is false. It is false because, as the sample size grows, the data generated by the model will be at greater and greater variance with the observed time-series. For Kydland and Prescott a model is only an approximation to the stochastic process generating actual data. Consequently, because the model is a false DGP for the actual data, classical estimation of the parameters is meaningless. In addition, classical hypothesis testing is inappropriate because a false model cannot be considered a null hypothesis to be statistically examined.

Researchers working in this area have adopted a two-step approach. First, they choose parameters so that the model replicates the data in some basic dimension of interest and, second, they evaluate the model on its ability to replicate variances and covariances of the cyclical component of macro variables.

Recently, Smith (1993) has suggested a VAR metric to judge the fit of the model. Its approach applies to both situations where the parameters are calibrated or estimated. According to Smith (1993) a model is regarded as appropriate if the distance between the unrestricted VAR representations of simulated and actual data is small either in absolute terms or relative to the distance of other models to the actual data.

In this chapter the problem of testing the fit of the model takes a step further than the RBC and VAR approach. We start from a model whose probabilistic structure, as shown in chapter 2, is determined by common stochastic trends responsible for long-run movements of the cointegrated variables. Within this framework, we describe a log-linearization procedure that makes it possible to distinguish between long-run and short-run components. Because we are interested in analysing the performance of the model at high frequencies, we isolate the short-run log-linear part of the model. By short-run we mean that each variable is evaluated in terms of its deviations from a common stochastic trend. Indeed, the steady-state relationships singled out in chapter 2 have been interpreted as cointegrating vectors.

It is very important to provide a clear derivation of the cyclical components of the variables because the comparison only makes sense if we are operating the same trend extraction between simulated and actual data. Residuals from the cointegrating vectors are therefore built from actual data and interpreted as short-run deviations from common stochastic trends. After having operated this trend extraction, unconditional moments of simulated and actual data can be compared.

The chapter is structured as follows. In section 4.2 we log-linearize the model around a common stochastic trend. In section 4.3 we present the simulated solution of the log-linear model, written in a restricted VAR form, and the impulse response functions of the artificial data. We then carry out a comparison with the impulse responses of the unconstrained short-run VAR estimated in the previous chapter. Section 4.4 compares the unconditional moments computed with the artificial data with those computed with the actual data.

4.2 The Problem of Log-linearizing a Stochastically Growing Model

We follow the procedure developed by Uhlig (1997) to solve, analyse and simulate our non-linear dynamic stochastic model. In chapter 2 we ended up with a global system of 12 variables for 12 equations: constraints, first order and equilibrium conditions. Two equations, (2.29) and (2.33), involve expectations, they will be solved forward; five, (2.2), (2.3), (2.6), (2.22), (2.23), are deterministic equations and five, the system (2.38), are the exogenous forcing variables.

We remind that these five exogenous state variables are: total output in each sector (y_N, y_T), the money supply (m), the foreign real interest rate (r^*), and the foreign price level (P^*). Among the remaining 7 endogenous variables we need to distinguish between state variables: foreign bonds (b^*), and control variables: consumption of traded and non-traded goods, total consumption, the relative price of traded and non-traded goods, and the nominal exchange rate.

In order to implement the simulation method of Uhlig (1997) we need, firstly, to log-linearize the model about its steady state. In chapter 2 we described the behaviour of the economy in steady state. We now describe a general log-linearization procedure that delivers the short-run equations of the log-linear model useful for the simulation carried out in Matlab.

We consider a general function $f(Y_t, X_t)$ with steady-state solutions $\overline{Y}_t$, $\overline{X}_t$. We then define the log-deviations from the steady state as $y_t = \ln Y_t - \ln \overline{Y}_t$ or $Y_t = \overline{Y}_t e^{y_t}$. Taking a first order Taylor expansion about $y_t = x_t = 0$ leads to

$$0 = f(\overline{Y}_t e^{y_t}, \overline{X}_t e^{x_t}) \simeq f(\overline{Y}_t, \overline{X}_t) + y_t \frac{\partial Y_t}{\partial y_t} |_{y_t=0} \frac{\partial f_t}{\partial Y_t} + x_t \frac{\partial X_t}{\partial x_t} |_{x_t=0} \frac{\partial f_t}{\partial X_t} \quad \text{(I)}$$

Reminding that the steady state is composed by a deterministic trend and by a stochastic trend, we can write it as:

$$\overline{Y}_t = \overline{Y} e^{\mu t + \sum_{j=0}^{\infty} \varphi^j \varepsilon_{t-j}} \text{ and } \overline{X}_t = \overline{X} e^{\mu t + \sum_{j=0}^{\infty} \varphi^j \varepsilon_{t-j}}$$

We then can expand about $\mu = \varphi = 0$ to obtain:

$$0 = f(\overline{Y}_t, \overline{X}_t) \simeq f(\overline{Y}, \overline{X}) + (\mu t + \sum_{j=0}^{\infty} \varphi^j \varepsilon_{t-j}) \left(\overline{Y} \frac{\partial f}{\partial \overline{Y}_t} + \overline{X} \frac{\partial f}{\partial \overline{X}_t} \right) \quad \text{(II)}$$

Combining (I) and (II), or expanding about $y_t = x_t = \mu = \varphi = 0$ leads to:

$$0 = f(\overline{Y}, \overline{X}) + (\mu t + \sum_{j=0}^{\infty} \varphi^j \varepsilon_{t-j}) \left(\overline{Y}_t \frac{\partial f}{\partial \overline{Y}_t} + \overline{X}_t \frac{\partial f}{\partial \overline{X}_t} \right) + y_t \overline{Y} \frac{\partial f}{\partial Y_t} + x_t \overline{X} \frac{\partial f}{\partial X_t} \quad \text{(III)}$$

In a static equilibrium $f(\overline{Y}, \overline{X}) = 0$ and $\mu = \varphi = 0$, hence we are left with

$$y_t \overline{Y} \frac{\partial f}{\partial Y_t} |_{y_t=0} + x_t \overline{X} \frac{\partial f}{\partial X_t} |_{x_t=0} = 0 \quad \text{(IV)}$$

Equation (IV) describes the short-run behaviour of the system. It suggests that to obtain a variable that is measured as deviation from the steady state is an independent problem from obtaining its steady-state behaviour. Thus, for whatever definition of the steady state, we can use (IV) to find the model's short-run deviations. A detrended economy can tell us something only about the short run. Therefore, the use of (IV) for detrending every model economy without knowing its steady-state solution, will not be by itself informative on the long-run behaviour. This leads to think that an indiscriminate use of (IV) (i.e. leaving aside the study of the economy in the long run) can be very dangerous.

A common characteristics of modern business cycle theory is its use for answering various quantitative questions about cyclical fluctuations, while typically taking cycles themselves as given. Researchers, generally, study macroeconomic time-series which have been detrended in some way (for example by linear detrending, calculation of growth rates, or application of the Hodrick-Prescott filter) without investigating the nature of the trend implicit in the model. The problem is that the low-frequency and non-stationary components of the shocks processes will generally have implications for the behaviour of the endogenous macro-variables at all frequencies (including business cycle frequencies).

In this chapter a cycle-trend decomposition follows from a theoretical model which includes stochastic growth trends. Thus, it is the theory that has been used to define and measure the business-cycle component of the series. The same kind of decomposition is then applied to actual data.

We now write down these log-linear short-run equations for the model of chapter 2. We distinguish between deterministic equations and equations with expectations.

The goods market. We describe the behaviour of the goods market by the following set of deterministic equations. We firstly have the log-linearized version of the budget constraint, equation (2.6):

$$
\begin{aligned}
& \frac{\overline{y}_T}{\overline{P}}(y_{Tt}-p_t)+\frac{\overline{q}}{\overline{P}}\overline{y}_N(y_{Nt}+q_t-p_t)+\overline{b}^*(1+\overline{r}^*)\frac{\overline{SP}^*}{\overline{P}}(b^*_{t-1}+s_t+p^*_t-p_t)+\overline{b}^*\frac{\overline{SP}^*}{\overline{P}}\widetilde{r}^*_t \\
= \;& \overline{c}c_t+\overline{b}^*\frac{\overline{SP}^*}{\overline{P}}(b^*_t+s_t+p^*_t-p_t) \qquad (4.1)
\end{aligned}
$$

We then consider the intratemporal equilibrium conditions and demand functions. Total consumption is:

$$
0=\overline{c}c_t-\frac{\overline{c}_T}{\overline{P}}(c_{Tt}-p_t)-\frac{\overline{qc}_N}{\overline{P}}(c_{Nt}+q_t-p_t) \qquad (4.2)
$$

The log-linear definition of the price index is:

$$0 = \overline{P}^{1-\theta} p_t - (1-\gamma)\overline{q}^{1-\theta} q_t \tag{4.3}$$

The log-linear intratemporal demand for consumption in the traded, (2.22), and non-traded, (2.23), sectors are:

$$0 = c_{Tt} - \theta p_t - c_t \tag{4.4}$$

$$0 = q_t - \theta y_{Nt} + \theta c_{Nt} \tag{4.5}$$

The two following log-linear equations differ from the formers because they are stochastic, in the sense that they involve expectations:

The money market. The log-linear version of (2.33) is[31]:

$$\begin{aligned} 0 &= c_t + p_t - m_t - \frac{1}{\overline{r}^*(1+\overline{r}^*)} E_t \widetilde{i}^*_{t+1} + \frac{1}{\overline{r}^*(1+\overline{r}^*)} (E_t s_{t+1} - s_t) \\ &\quad + \left(\frac{1}{\overline{r}^*} - \zeta \overline{r}^* \overline{c}\right) (E_t p^*_{t+1} - p^*_t) \end{aligned} \tag{4.6}$$

The Euler equation for total consumption. The log-linear version of (2.28) is:

$$0 = E_t \left(c_t - c_{t+1} + \widetilde{r}^*_{t+1} + s_{t+1} - s_t + p_t - p_{t+1} + p^*_{t+1} - p^*_t\right) \tag{4.7}$$

In the system of equations (2.38) we have already expressed the autoregressive processes as log-deviations from their steady-state values.

[31]Given the variable X_t, we substitute it with $X_t = \overline{X} e^{x_t}$ where x_t is a real number close to zero, $x_t = \log X_t - \log \overline{X}$, and $\overline{X}$ is its detrended steady-state value. The variable X_t can be approximated in this way: $X_t = \overline{X}(1 + x_t)$. Therefore all the variables in (4.1)-(4.7) express the log-deviations from the steady state. There is an exception represented by the interest rate. In this case we cannot apply the same kind of transformation, we thus rewrite it in this way: $i^*_t = \widetilde{i}^*_t + \overline{i}^*$ where $\widetilde{i}^*_t = i^*_t - \overline{i}^*$.

Our log-linear model is composed by two forward looking intertemporal equations (4.6) and (4.7), one backward looking equation (4.1), while the others are just static intra-temporal equations or identities.

4.3 Simulation of the Model

We rewrite the previous equations (4.1)-(4.7), (2.38) in a three matrix system. We indicate with x_t the endogenous state variables: $x_t = [b_t^*]$; with y_t all the endogenous other variables: $y_t = [c_t, c_{Nt}, c_{Tt}, s_t, q_t, p_t]$; and with $z_t = [\, y_{Nt}, y_{Tt}, p_t^*, r_t^*, m_t]$ the exogenous state variables.

The system of equations written in matrix form is then the following:

$$\begin{aligned} 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t \\ 0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right] \\ z_{t+1} &= Nz_t + \varepsilon_{t+1} \text{ with } E_t(\varepsilon_{t+1}) = 0 \end{aligned} \tag{4.8}$$

The first block collects the deterministic equations (4.1)-(4.5). The second block contains the two stochastic equations (4.6) and (4.7), and the third block the system of the exogenous variables (2.39).

The whole system is solved for the recursive equilibrium law of motion via the method of undetermined coefficients[32]. The solutions to the system are then analysed via impulse response analysis. The results obtained, i.e. the recursive law of motion, are:

$$\begin{aligned} x_t &= Px_{t-1} + Qz_t \\ y_t &= Rx_{t-1} + Sz_t \\ z_t &= Nz_{t-1} + \varepsilon_t \end{aligned} \tag{4.9}$$

[32]Blanchard, Fischer, 1989, Lectures on Macroeconomics, MIT Press, Chapter 5.

and they can be used to examine the model's implications. The model solved in this way can be regarded as a VAR in greater depth. The VAR that results is restricted, since the only lagged variables that enter are those of the states, whereas a proper VAR would have lags of all the variables entering each equation. Thus, the solution of the model produces restrictions upon the dynamics of the processes that are potentially testable.

We also remind that each variable is stationary because it represents a deviation from the steady state. The system (4.9) represents the reduced final form of (4.1)-(4.7). The errors (2.38) enter in their 'structural' form, they coincide with those estimated in the previous chapter.

Since x_t, y_t and z_t are log-deviations, the entries in P, Q, R, S and N can be understood as elasticities and interpreted accordingly. Impulse responses to a particular shock ε_1 are calculated by setting $x_0 = 0$, $y_0 = 0$, $z_0 = 0$, $\varepsilon_0 = 1$, $\varepsilon_t = 0$ for $t \geq 2$, and recursively calculating z_t and then x_t and y_t, given x_{t-1}, y_{t-1}, z_{t-1} and ε_t for $t = 1, ..T$, with the recursive equilibrium law of motion and the law of motion for z_t.

The matrices of the system of equations (4.8): $[A, B, C, D, F, G, H, J, K, L, M, N]$ have been filled up with the values of the deep parameters and the autoregressive coefficients measured in chapter 2 (see Appendix C and Appendix D for the complete derivation of the program used for simulations).

From the steady-state of equations (2.3), (2.22), (2.23), (2.41)-(2.45) we obtained the following values:

Table 4.1: The Model's Steady-State Values

$\overline{q} = \left(\frac{\gamma}{1-\gamma}\right)^{\theta} = 1.89$
$\overline{p} = \left[\gamma + (1-\gamma)\overline{q}^{1-\theta}\right]^{\frac{1}{1-\theta}} = 1.24$
$\overline{c}_N = \overline{y}_N = 1$
$\overline{c}_T = \overline{c}_N \left(\frac{\gamma}{1-\gamma}\right) \overline{q}^{\theta} = 5.664$
$\overline{c} = 6.09$
$\frac{\overline{m}}{\overline{p}} = 0.806$
$\zeta = \frac{\overline{r}^*}{1+\overline{r}^*} \frac{1}{\overline{c}\overline{p}} = 0.0013$
$\overline{b}_0^* = \overline{c} = 6.09$

According to our calibration on the UK data, table 4.1 shows that the steady-state price level of non-traded is higher than that of traded goods (i.e. the ratio is 1.89). The steady-state value of traded consumption is approximately five times bigger than that of non-traded consumption. As a result of our normalization of the nominal money stock to unity, we obtain an equilibrium value for real balances slightly less than unity. Table 4.1 then shows that we log-linearized the model around an initial value for foreign bonds held by domestic inhabitants equal to the value of total domestic consumption. We are now going to observe the dynamics of the system in response to temporary shocks in a neighbourhood of the initial steady state.

4.3.1 Simulation Results

The simulated reduced final form obtained for each detrended endogenous variable of the system (equations (4.9)) is:

$$
\begin{aligned}
b_t^* &= .98b_{t-1}^* - .001y_{Nt} + .008y_{Tt} + .97r_t^* + .001p_t^* + .01m_t \\
s_t &= .56b_{t-1}^* + .003y_{Tt} + .927r_t^* - 1.05p_t^* + .015m_t \\
c_{Tt} &= .68b_{t-1}^* - .21y_{Nt} + .005y_{Tt} + .82r_t^* - .066p_t^* + .018m_t \\
c_{Nt} &= -.16b_{t-1}^* + 1.29y_{Nt} - .001y_{Tt} - .19r_t^* + .016p_t^* - .004m_t \\
c_t &= .47b_{t-1}^* + .16y_{Nt} + .003y_{Tt} + .56r_t^* - .04p_t^* + .012m_t \\
q_t &= .37b_{t-1}^* - .66y_{Nt} + .003y_{Tt} + .45r_t^* - .036p_t^* + .01m_t \\
p_t &= .09b_{t-1}^* - .16y_{Nt} + .11r_t^* - .009p_t^* + .003m_t
\end{aligned}
$$

From the above system of log-linear equations we can see that, although money growth does not affect the steady-state values of the endogenous variables, it affects the dynamics toward the steady state. In particular, total consumption increases for an exogenous shift of the money supply growth and comes back to its previous steady state after a year. The consumption of traded goods displays the same dynamics. Consumption of non-traded goods has instead an opposite dynamics with respect to the behaviour of traded goods consumption. This result reflects the fact that traded and non-traded goods are substitutes in consumption. We can also observe that the short-run behaviour of total consumption is completely dominated by that of traded goods. On impact, the relative price of the traded goods is negatively affected by a shock in the foreign price level and positively by a money supply shock. Foreign interest rates positively affect the domestic price level.

We finally look at the dynamics of the current account, whose measure is given by changes in the net foreign bonds position. Positive shocks to traded output lead to an impact surplus of the current account. An increase of the world real interest rate, an increase in the foreign price level and in the money supply growth lead to an impact surplus of the current account.

4.3.2 Impulse Response Functions

The short and medium run reaction of these variables to exogenous shocks is described by the impulse response functions. Figures 4.1 show the dynamic behaviour of traded, non-traded and total consumption and of prices, real exchange rate and bonds to shocks in each of the five exogenous variables. On the horizontal axis, the time after the shock is measured in years; on the vertical axis it is shown the percentage deviation from the steady state.

Starting from a shock to y_N we can observe that its cyclical (or temporary) component is able to transmit a very short-lasting impulse. Total and non-traded consumption are positively affected by a shock to non-traded output. This shock leads to a negative response of prices and worsens the current account.

A positive shock to y_T increases total consumption in the short run and has a more persistent effect than the shock to y_N. The model suggests the presence of consumption smoothing in the traded sector. This leads to an improvement of the balance of payments. A positive shock to traded output leads then to a persistent depreciation of the real exchange rate.

A positive shock to the foreign interest rate (r^*) leads to a negative reaction of consumption in the short run that reverses its position after six months. We observe that the intertemporal substitution effect is dominant (with respect to the wealth effect) leading consumers to prefer to lower today consumption in favour of savings. Only in this case, i.e. when the economy is hit by a shock to foreign interest rates, total consumption is driven by the behaviour of non-traded goods consumption. The real exchange rate closely follows the behaviour of total consumption. The real exchange rate appreciates and comes back to the previous steady state after two years.

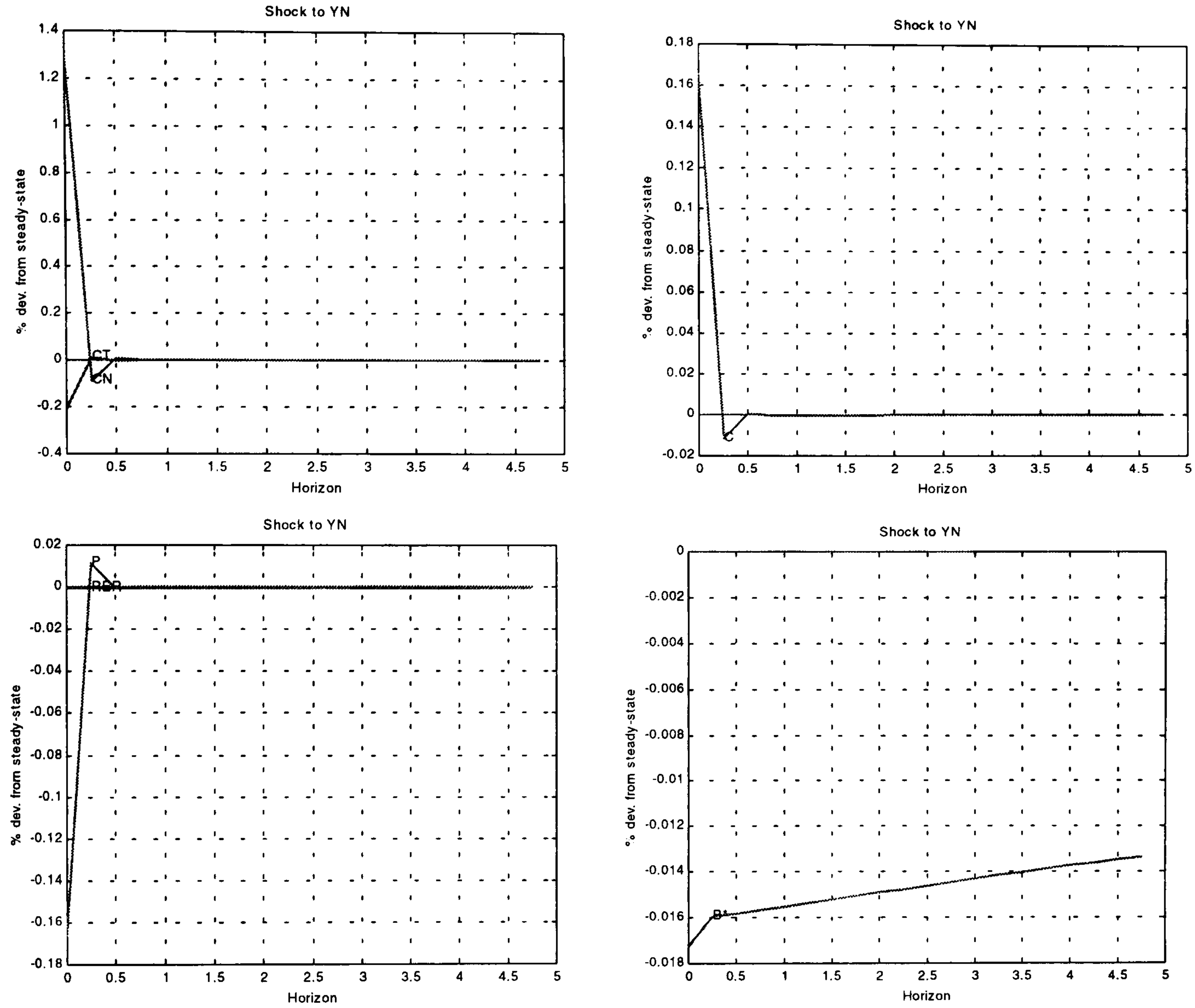
Also a positive shock to the foreign price level has a negative effect on total consumption although it is very small and lasts for two years and half. This negative reaction is driven by the indirect effect coming from output. In fact, the estimation of the exogenous processes led to find a negative relationship between traded output growth and foreign inflation. Although the shock on foreign prices leads to have higher domestic prices, the real exchange rate depreciates at impact and the current account improves.

The graphs show that each shock has a very long lasting effect on the behaviour of the net foreign bonds position.

A monetary shock has a positive impact effect on total consumption (although very small). The absence of neutrality in the adjustment path toward the initial steady state depends on the spillover effects that are present in the estimated errors covariance matrix. This shock leads then to a depreciation of the real exchange rate, to an increase of the domestic price level and to the improvement of the current account. The improvement is very persistent although very small.

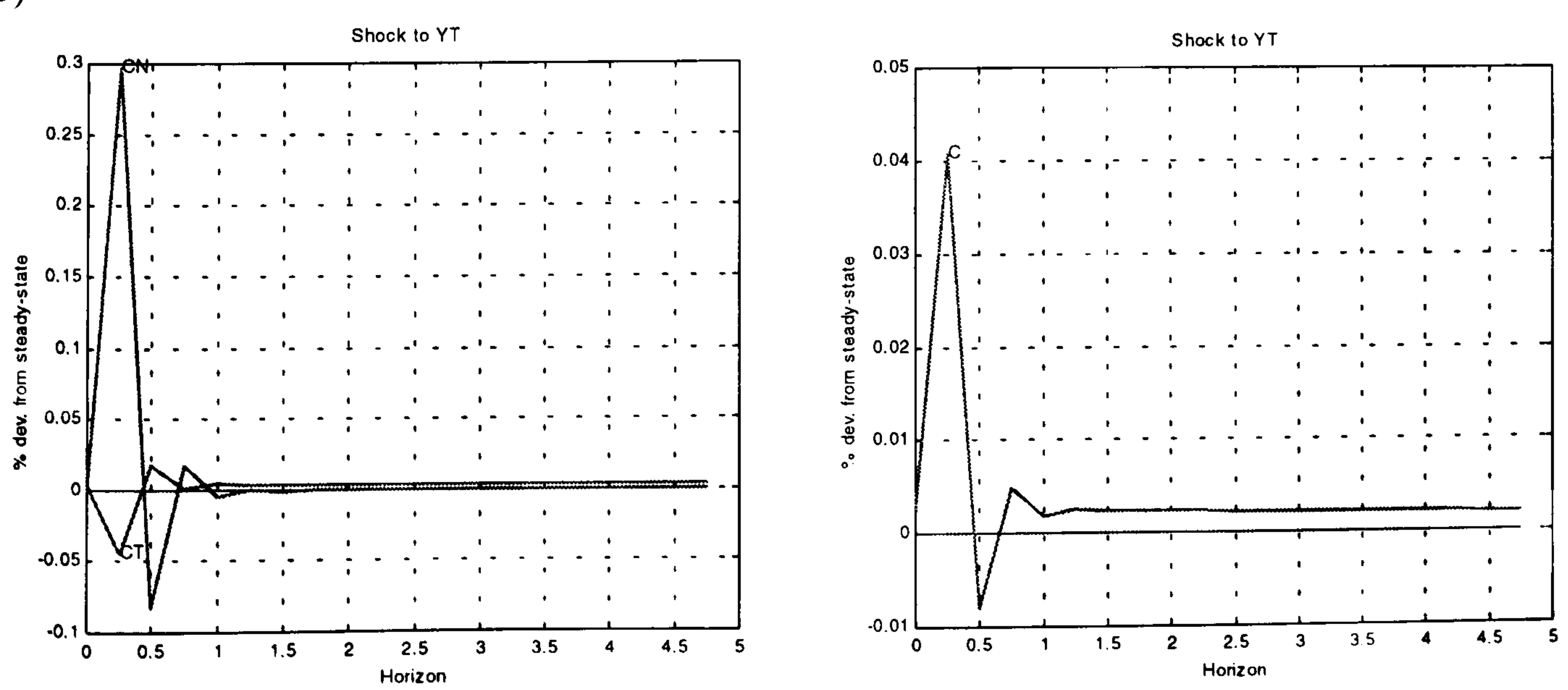
Figures 4.1: Impulse response functions of the simulated model

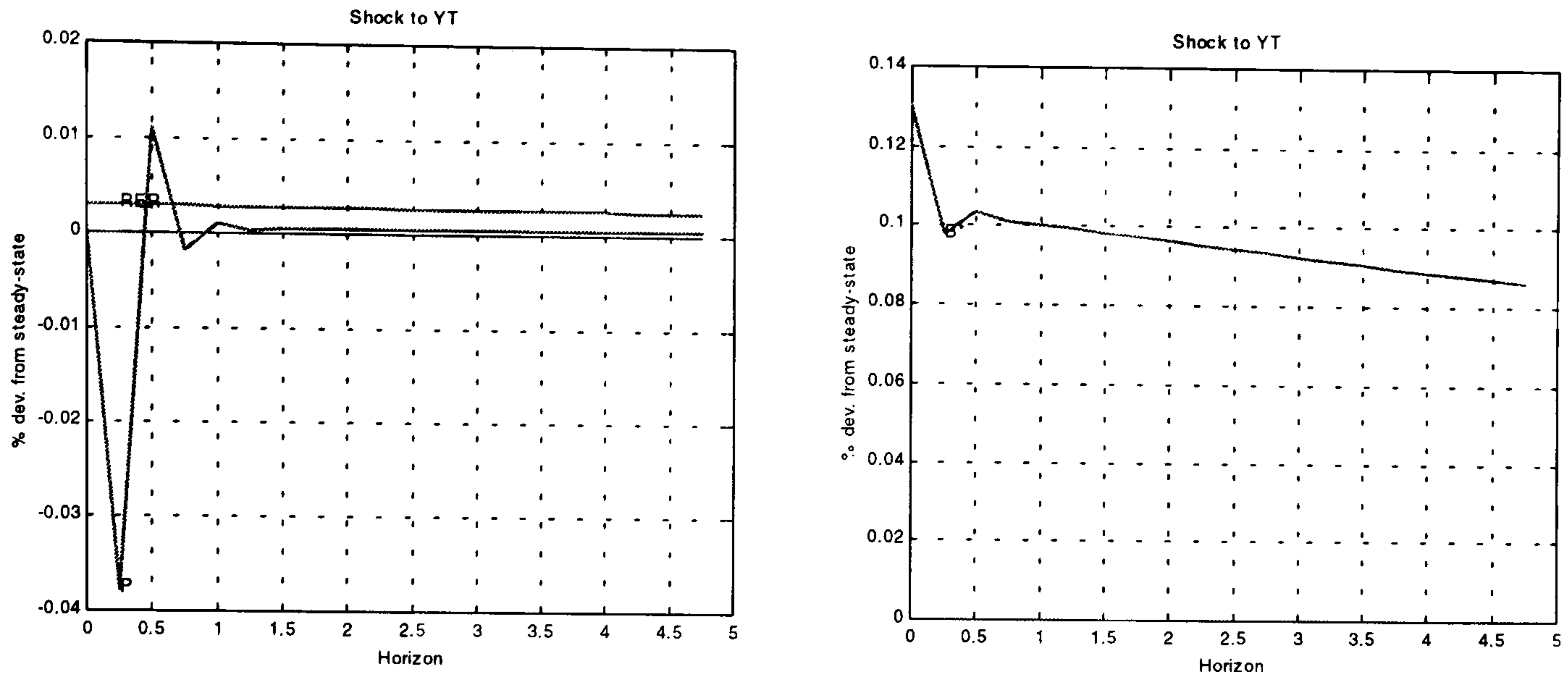
A)



Note: In order - first left: CT=traded consumption, CN=non-traded consumption; - first right: C= total consumption; -second left: P=price level, RER=real exchange rate; - second right: B* foreign assets. The same order is kept for the impulse response functions of all the shocks, where YN=non-traded output; YT=traded output; R*=foreign interest rate; P*=foreign price level; M=money supply.

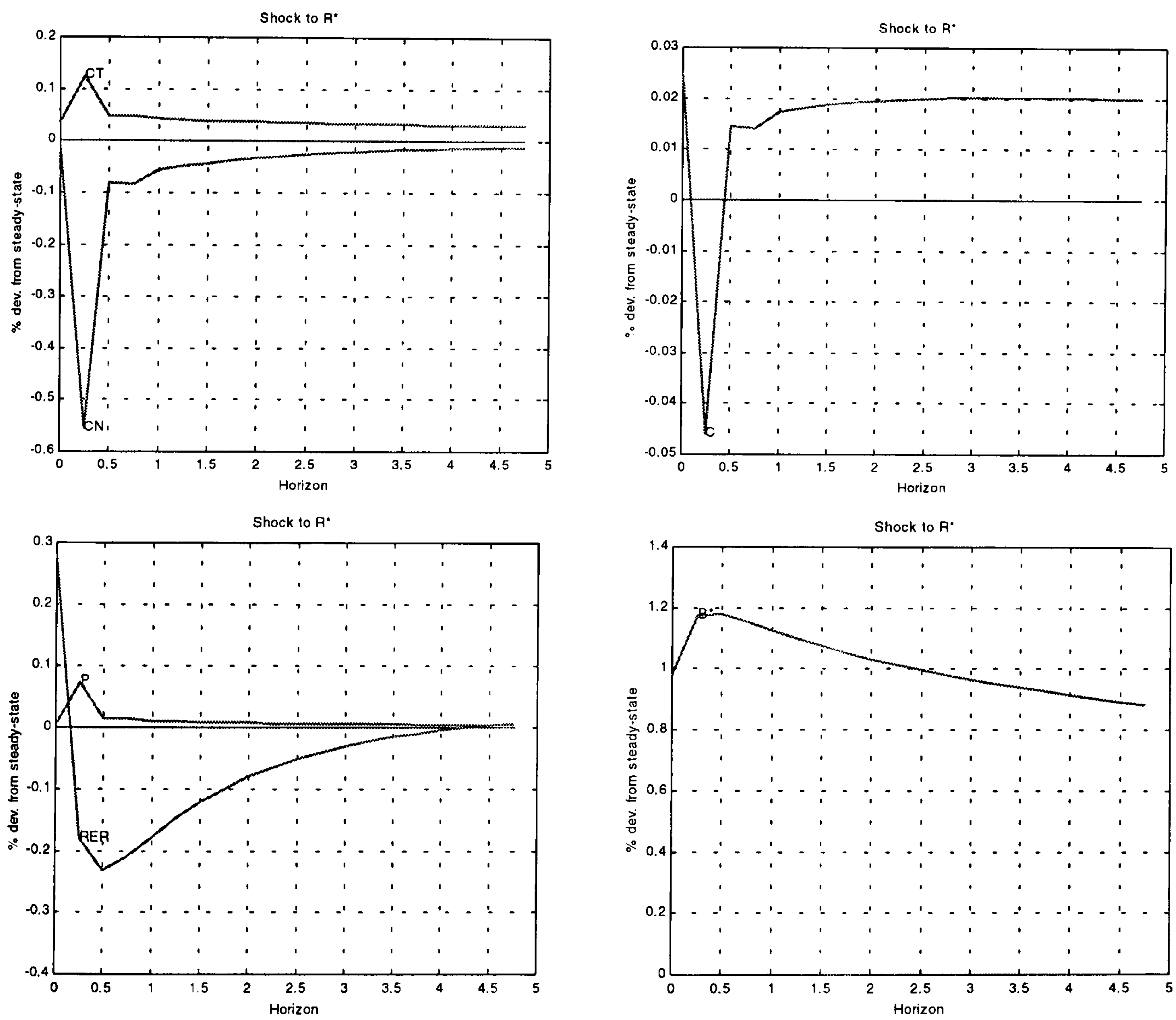
B)





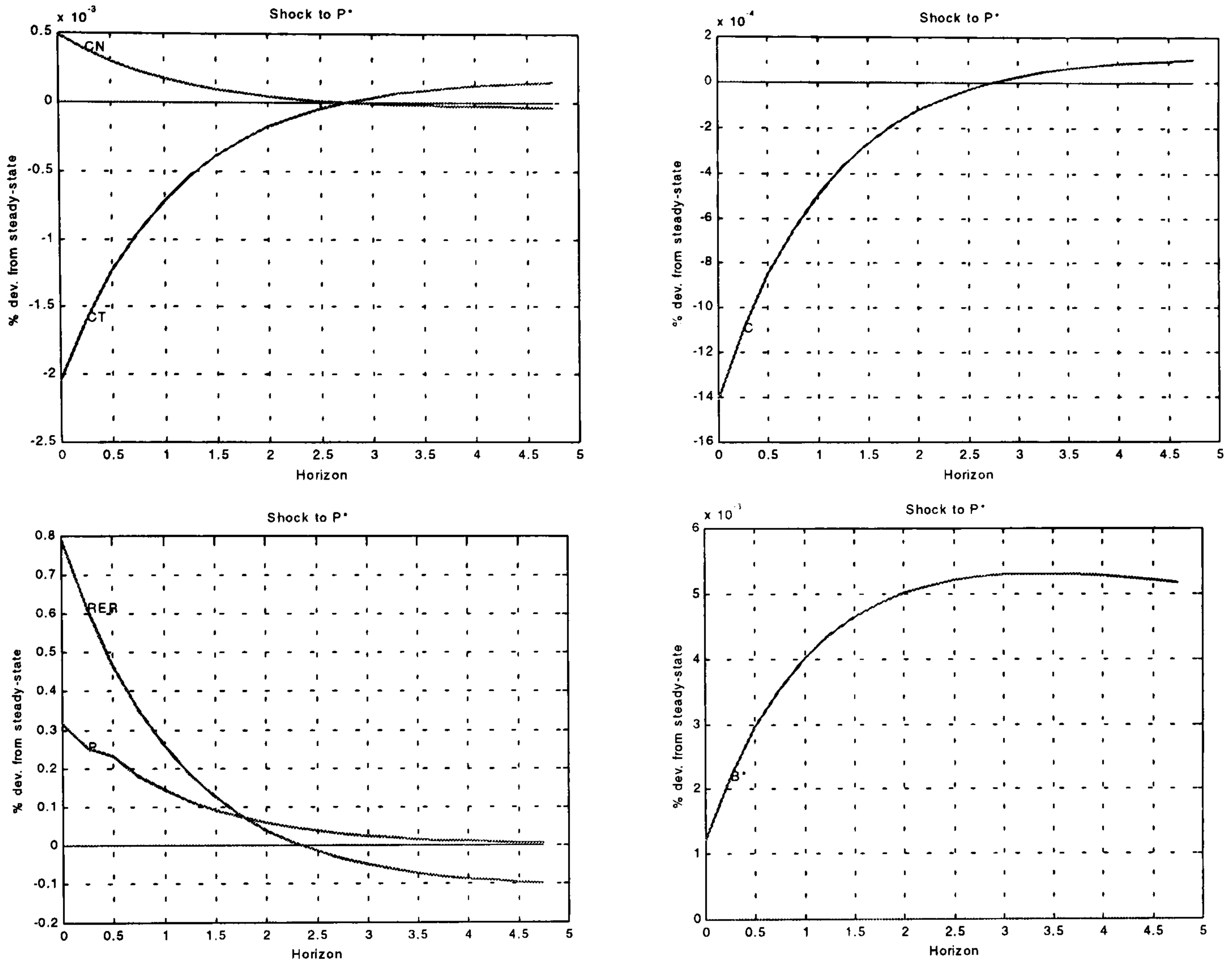
Shock to YT
% dev. from steady-state
RER
P
Horizon
Shock to YT
% dev. from steady-state
B
Horizon

C)

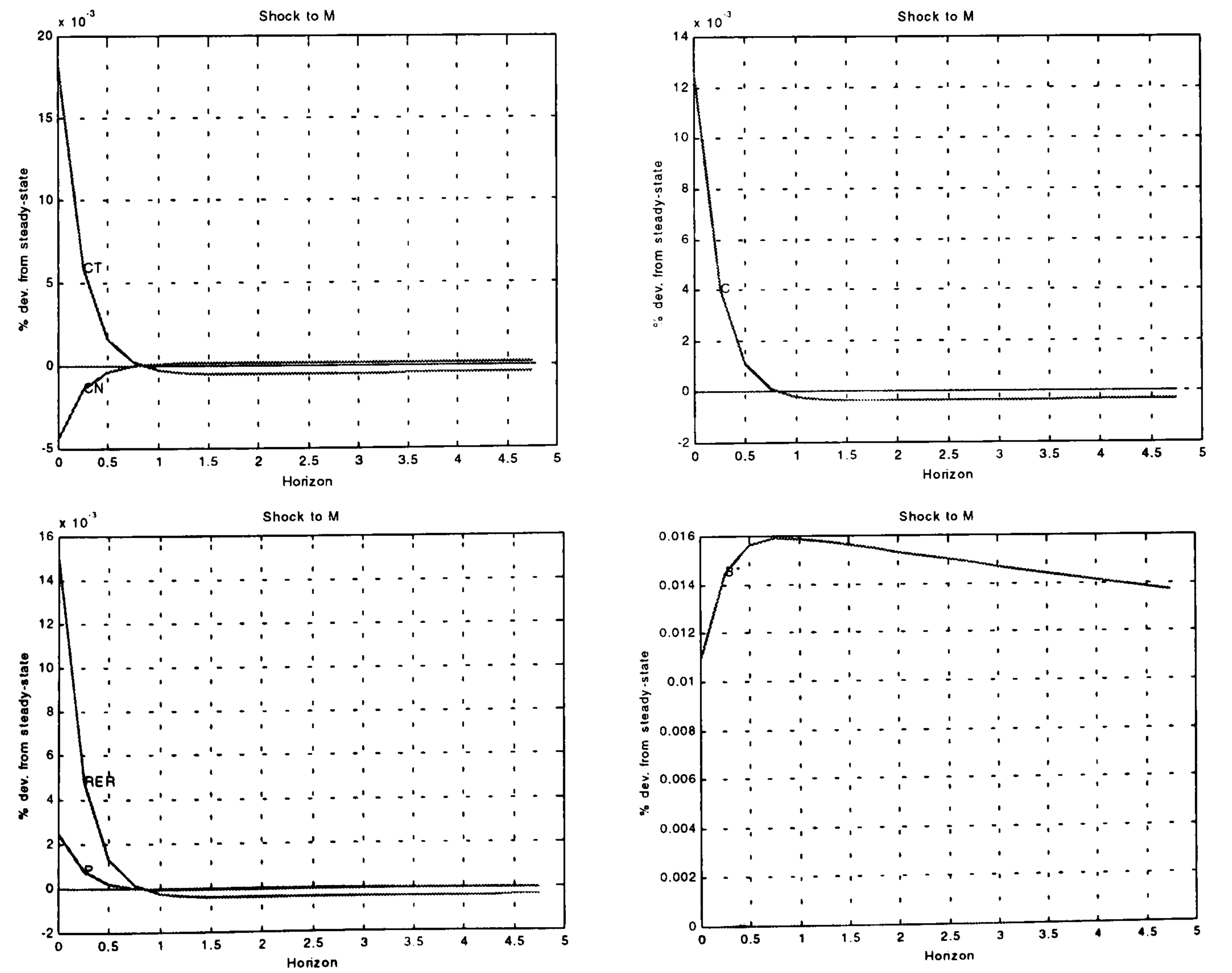


Shock to R*
% dev. from steady-state
CT
CN
Horizon
Shock to R*
% dev. from steady-state
C
Horizon
Shock to R*
% dev. from steady-state
P
RER
Horizon
Shock to R*
% dev. from steady-state
B
Horizon

D)


Shock to P*
x 10^-3
CN
CT
% dev. from steady-state
Horizon
Shock to P*
x 10^-4
C
% dev. from steady-state
Horizon
Shock to P*
RER
P
% dev. from steady-state
Horizon
Shock to P*
x 10^-3
B*
% dev. from steady-state
Horizon

E)


Shock to M
x 10^-3
CT
CN
% dev. from steady-state
Horizon
Shock to M
x 10^-3
C
% dev. from steady-state
Horizon
Shock to M
x 10^-3
RER
P
% dev. from steady-state
Horizon
Shock to M
B*
% dev. from steady-state
Horizon

4.3.3 Can we compare the Impulse Response Functions with those obtained in the previous chapter?

In the previous chapter we carried out the econometric analysis by estimating two systems of equations. We first built a reduced form short-run VAR among the cointegrating residuals and the exogenous processes in first differences. We then built a structural VECM imposing the same theoretical restrictions on the cointegrating vectors used for the VAR.

In this chapter we have simulated a short-run reduced form VAR obtained from the solution of the detrended model. The same estimated structural errors have been used for generating fluctuations of the endogenous variables. Although we have employed the same structural errors, the simulated endogenous variables contain different information, and provide thus different (but not contrasting) evidence on the behaviour of the small open economy with respect to the estimated VECM. But if the impulse responses of the VECM cannot be really compared with the simulated impulse responses, because the first are expression of the dynamics of the original variables, the VAR impulse responses can be taken into greater consideration for comparisons. We are reminded that the VAR model has been built among theoretical residuals of the cointegrating vectors, that can be interpreted as short-run deviations from the common stochastic trends. Thus, in this respect we can carry out a meaningful comparison. The main difference relies on the fact that the estimated VAR is unrestricted (i.e. lags of all the endogenous and exogenous variables appear in the reduced form) whereas the simulated VAR is restricted (i.e. only lagged state and exogenous variables appear in the reduced form).

By looking at figures 3.1 and 4.1 we can see that the dynamic response of traded and non-traded consumption is very similar when the system is hit by a shock in the non-traded sector, although the magnitude of the reaction is much higher in the restricted simulated VAR. Foreign shocks (to prices and interest rates) lead to quite similar reactions in the restricted and unrestricted VAR for the behaviour of real variables and we observe a good matching, at least in the shape, between the behaviour of the real exchange rate in the two cases.

We obtain the best matching between restricted and unrestricted impulse responses in the case of monetary shocks over traded, non-traded and total consumption, not only in the dynamics but also in the dimension (size) of the response.

The comparison between a restricted simulated VAR and an unrestricted estimated VAR can, thus, be used for testing the restrictions imposed in the reduced form of the simulated VAR. In general, we observe a good matching in directions and shapes of the impulse responses but not in the dimension of the responses. The restricted simulated VAR seems to produce a too high impact reaction of the endogenous variables after a shock.

4.4 Evaluating the Model by Comparing Theoretical and Actual Moments

In this section the evaluation of the model is carried out by comparing second order moments of the simulated and actual data. Among others, one of the peculiarity of the model is that total consumption represents the expenditure variable, whose role is usually played by total output. Therefore, in the analysis of covariances among variables, the cyclical behaviour of the series will be defined in terms of total consumption. The analysis of unconditional second order moments is the object of study of the modern theory of business cycles[33]. In particular, the aspects considered for each time-series are those of volatility, persistence and comovements with other time-series. In the time domain, these statistics are measured by the standard deviation, the autocorrelation and cross-correlation. For a given time-series the standard deviation (generally reported as a percentage of GDP) measures the variability of the series. The autocorrelation gives a measure of the persistence of a given series, and the cross-correlation measures the comovements among different series and, in particular, the cyclical behaviour of each variable with respect to a referring series. We report those moments for the simulated data and for the actual data referred to the UK economy over the period 1969-1997.

Table 4.2 shows the measures of volatility and comovements among the theoretical time series and table 4.3 the same measures for the actual detrended data, where we are reminded that the detrending procedure consisted of taking the theoretical residuals from the cointegrating vectors.

The five exogenous sources of shocks seem to generate too much variability for non-traded and traded consumption (respectively 2.4 for simulated data against 1.6 of the actual data, and 3.9 against 2.9), and for total consumption (2.6 of simulated data against 1.9 of the actual data). But if we make the comparison in relative terms, we obtain a much better matching of the volatility, namely: $\frac{\sigma_{c_N}}{\sigma_c} = 0.92$ for simulated data against $\frac{\sigma_{c_N}}{\sigma_c} = 0.84$ for actual data; and $\frac{\sigma_{c_T}}{\sigma_c} = 1.50$ for simulated data against $\frac{\sigma_{c_T}}{\sigma_c} = 1.52$ for actual data.

[33]'Modern theory' according to the meaning of business cycle given by Lucas (1977) as movements around the trend of GDP.

Table 4.2: Standard Deviations (%) and Cross-correlations of the Artificial Data

	$S.D.$	b^*	c_N	c_T	rer	c	p	y_N	y_T	r^*	p^*	m
b^*	5.6	1.0										
c_N	2.4	.00	1.0									
c_T	3.9	-.07	-.47	1.0								
rer	3.4	.98	.01	.01	1.0							
c	2.6	-.07	.05	.86	-.02	1.0						
p	.61	-.03	-.92	.79	.00	.36	1.0					
y_N	1.8	-.01	.99	-.34	-.01	.19	-.85	1.0				
y_T	2.2	-.04	-.23	-.01	-.05	-.15	.16	-.25	1.0			
r^*	.63	.06	.21	-.46	.03	-.40	-.36	.15	.13	1.0		
p^*	.66	-.06	-.20	.92	-.01	.92	.20	-.06	-.04	-.16	1.0	
m	1.2	.06	-.05	.07	.06	.03	.07	.00	.07	.01	.04	1.0

Too low is the volatility of the domestic goods price level. For the term expressing changes in the foreign assets position (b^*) we obtain a very high volatility in both cases (5.6 for the simulated time-series against 5.0 for the actual data). This model can generate a very highly volatile real exchange rate similar to that of the actual data (3.4 for the simulated data and 3.9 for the actual data).

If we look at the cross-correlations we notice that the model generates very low correlations among endogenous and exogenous variables. In general the sign of cross-correlations is correct but the values are lower than those computed for the actual data. For example, traded and non-traded consumption are negatively correlated in the artificial and actual data, but the correlation is too low in the artificial data (we have -0.47 in the actual data against -0.17 in the simulated data). b^* is highly correlated with the real exchange rate both in the artificial and simulated data. The correlation of the real exchange rate with traded and non-traded consumption is 0.01 in the artificial data and 0.09 in the actual data. The cross-correlation between the real exchange rate and the domestic price is insignificant in the model and equal to 0.67 in the data. Simulated data systematically mistake in generating the appropriate pattern of the domestic price time-series in terms of volatility and cross-correlations.

Table 4.3: Standard Deviation and Cross-correlations of the UK data (1969:4-1997:3)

	$S.D.$	b^*	c_N	c_T	rer	c	p	y_N	y_T	r^*	p^*	m
b^*	5.0	1.0										
c_N	1.6	-.17	1.0									
c_T	2.9	.11	-.17	1.0								
rer	3.9	.86	.09	.09	1.0							
c	1.9	-.19	-.09	.07	-.12	1.0						
p	6.9	.42	.11	.49	.67	-.18	1.0					
y_N	1.8	-.19	-.32	.06	.11	-.02	.11	1.0				
y_T	2.2	.10	.13	-.26	-.08	.32	-.02	-.20	1.0			
r^*	.60	-.02	-.05	-.12	.03	-.04	-.07	.19	.06	1.0		
p^*	.70	.27	.03	-.08	.37	.32	.09	-.09	-.15	.01	1.0	
m	1.2	-.25	-.11	-.20	-.18	.32	-.33	.02	.05	.15	.25	1.0

Table 4.4: Autocorrelation of the Artificial data: $corr(v_{t+j}, c_t)$.First row shows j.

	-4	-3	-2	-1	0	1	2	3	4
b^*	-.10	-.10	-.10	-.09	-.07	-.02	-.02	-.04	-.05
c_N	-.04	-.03	-.06	-.09	.05	-.12	-.03	-.03	.02
c_T	.25	.31	.39	.56	.86	.57	.37	.30	.22
rer	-.09	-.08	-.08	-.06	-.02	.01	.00	-.02	-.04
c	.26	.33	.41	.58	1.0	.58	.41	.33	.26
p	.14	.16	.22	.32	.36	.34	.19	.16	.09
y_N	.00	.02	.01	.03	.19	-.03	.03	.02	.05
y_T	-.05	-.08	-.02	.01	-.15	.04	-.02	.02	.04
r^*	-.01	-.01	-.04	-.11	-.40	.04	.17	.17	.15
p^*	.29	.36	.44	.62	.92	.68	.50	.42	.32
m	-.03	-.04	.00	.01	.02	-.03	-.02	-.02	-.03

Tables 4.4 and 4.5 show the comovements of each variable with lag/lead of total consumption, the first table is referred to simulated data and the second to actual data.

When describing the comovements we usually say that a variable is procyclical (countercyclical) if it has predominantly positive (negative) and statistically significant coefficient of correlation. Otherwise we call the variable acyclical. We say, then, that the variable displays cyclical behaviour (does not display) if its correlation coefficient has a pronounced peak. If such a peak occurs when the variable is lagged (led) relative to GDP, we refer to it as a leading (lagging) variable. The non-standard thing in this exercise is that the cycle is that of consumption.

In general, artificial and actual data show acyclical behaviour with respect to total consumption.

Table 4.5: Autocorrelation of the UK data, $corr(v_{t+j}, c_t)$. First row shows j.

	-4	-3	-2	-1	0	1	2	3	4
b^*	-.08	-.12	-.16	-.16	-.20	-.09	-.02	.06	.13
c_N	-.12	-.07	-.04	-.04	-.10	-.12	-.16	-.14	-.11
c_T	.18	.14	.08	.08	.07	-.13	-.19	-.21	-.32
rer	-.26	-.24	-.21	-.17	-.13	-.07	.00	.08	.14
c	.53	.60	.72	.77	1.0	.77	.72	.60	.53
p	-.31	-.27	-.24	-.23	-.20	-.20	-.19	-.17	-.17
y_N	-.15	.01	-.05	.02	-.03	.17	.03	.06	-.09
y_T	.09	.18	.20	.14	.32	-.01	.08	.05	.04
r^*	-.16	-.05	-.03	.07	-.05	.01	.11	.10	-.06
p^*	.07	.10	.14	.21	.31	.39	.45	.54	.55
m	.14	.20	.22	.27	.31	.30	.29	.30	.30

From table 4.4 and 4.5 we can see that the correlative behaviour of the net foreign bonds position and of the real exchange rate is very similar in the simulated and actual data. The same observation holds for the correlative properties of non-traded consumption, whereas the correlation of traded and total consumption is too high for every lead and lag in the simulated data.

The model generates wrong correlations between the behaviour of domestic prices and total consumption. Thus, the model results weak in replicating unconditional moments of domestic prices. They are not volatile, they are not sensible to changes in the exchange rate and they are positively correlated with total consumption. Actual data on domestic prices extracted from the cointegrating vector of the velocity of money result highly volatile, very sensible to changes in the exchange rate and negatively correlated with total consumption for every lead and lag.

4.5 Conclusions

This chapter concludes the first part of the thesis dedicated to the small open economy case. Here, the aim has been to provide an unbiased means of testing the fit of the model. The model has been formulated in its most simple version, consisting of exogenous endowment of output and log-linear utility function in consumption and real money holdings. The exogenous part has been assumed to follow a first order VAR process, which has been estimated according to the theoretical restrictions. Those exogenous processes are traded and non-traded output, foreign interest rates, foreign prices, and money supply. We solved the problem of identifying structural shocks by using the exogenous sources of shocks estimated for the theoretical model.

The long-run behaviour of the model has been exploited to single out cointegrating relationships among the variables and, thus, to detrend the economy. Once detrended, a calibrated version of the model has been simulated, and its evaluation has been carried out through impulse response functions and by computing second order moments. The presence of many exogenous shocks produces a highly volatile real exchange rate.

The comparison of the impulse response functions of the restricted reduced form VAR with those of an unrestricted VAR, carried out in chapter 3, showed that there is no divergence of information between the two exercises. But, overall, the evaluation of the model made in this chapter does not leave us completely satisfied. The model was better suited for suggesting long-run theoretical restrictions in the estimation of the VECM rather than in its short-run behaviour, which appeared to be too simplified for understanding the short-run behaviour of domestic prices. Cross-correlations and autocorrelations showed that the detrended model cannot explain some comovements among actual time-series, particularly those related to the behaviour of the domestic price level.

5 Interpreting the UK and US Monetary Shocks in a Two-Country model

5.1 Introduction

This chapter starts a new section of the thesis. In the former three chapters we have proposed a new set of methods, both theoretical and empirical, to aid the understanding of the dynamic behaviour of a small open economy. Many interesting results came out, concerning the distinction of short-run and long-run behaviour of the variables and the interplay between the dynamics of the balance of payments and the exogenous shocks hitting the economy. The UK business cycles showed a very high degree of dependence from foreign variables, taken as exogenous.

We now move from this framework and allow for the presence of an endogenous foreign sector. We build a two-country model following the theoretical structure used by Betts and Devereux[34] (1996, 1997).

There are several important reasons for considering the endogenous determination of the foreign sector. First, the two-country model has the advantage of capturing some of the important linkages between economies that have been missed in our previous work. Second, the small open economy case can be obtained by reducing the size of one of the two countries, without imposing the exogeneity of the rest of the world. This leads to a much lower number of the exogenous variables in the model and thus to make a better distinction between

[34]Whose famous precedent is Obstfeld and Rogoff's (1995) Redux model.

the dynamics generated by each shock. Third, owing to different country sizes, the same type of shock will have asymmetric effects according to the country where it originated.

In this part of the thesis we will emphasize the importance of monetary policy interdependence between countries. The possibility of modelling the world economy enables us to consider how policy actions in one economy can potentially affect the equilibrium in other economies. Thus, spillovers can occur in this framework. Policy actions in one country may also depend on the response of monetary policy in the other.

Although it will appear as a general equilibrium model, the analysis of this chapter will take a partial equilibrium view, because we will develop only a demand side story, without explicitly modelling the production sector.

We start the analysis with a very simple two-country model. All goods are traded and preferences are identical across countries, so that the composition of the consumption bundle is common to home and foreign consumers. The focus is on a unique source of fluctuations, coming from money supply.

The monetary transmission mechanism works according to these two assumptions:

1. Firms can charge different prices for the same good in home and foreign markets, so that the law of one price needs not to hold. Prices of imported goods are temporarily rigid in the importing country's currency, thus, a change in the exchange rate does not immediately affect either the domestic price index or the foreign price index, even though both indices contain the price of imported goods. This allows us to introduce the real exchange

rate in a different way with respect to our previous analysis[35]. Pricing-to-Market (PTM) in combination with local-currency sticky prices allows the real exchange rate to fluctuate and unties home and foreign price levels.

2. Only a fixed fraction of firms will adjust their price each period. The sticky price rule is taken from Calvo (1983), who solved the problem of aggregating among staggering firms by formalizing a set up where, randomly, a proportion of firms cannot adjust their price.

Models similar to the one presented below have been demonstrated to match well the unconditional second moments of the data. Betts and Devereux (1997), although with a more sophisticated version of the two-country model here presented, that assumes that firms can fix price in local currencies, show that the theoretical responses to a money shock are very similar to the empirical features of the international monetary transmission mechanism in the G7 countries. The analysis of this chapter is instead based on 'conditional' moments of the data.

By innovating with respect to the previous literature, we will use impulse response functions to show how the model's properties change while changing the parameter governing the degree of price stickiness. We will then use the long run properties of the model for estimating a structural VECM for the UK and US data. We will analyse the response of the economy to monetary shocks, and how much those shocks can explain the total variability of the system.

The presence of a unique source of shock (coming from money) facilitates its

[35]According to Chari, Kehoe and McGrattan (1998) the evidence in all the G7 countries is against the fact that variations in the relative price of non-traded to traded goods across countries account for any of the variability of the real exchange rate. So, it seems more realistic to have a model where real exchange rate movements arise from movements in relative prices of traded goods across countries.

identification and its transmission mechanism, and allows us to address several interesting questions.

1. How are monetary shocks transmitted to the economic system?
2. How strong and effective are monetary policy spillovers between the UK and the US?
3. How asymmetric is the response of the UK with respect to the US economy to monetary shocks?

We will see how the answers to these questions will naturally lead to formulate a 'two-country' monetary policy rule and to test for its plausibility.

The chapter is organized as follows. Section 5.2 derives the model, the steady-state and the log-linear equations. The system consists of eight log-linear equations in eight variables, expressed in differences between the home (UK) and the foreign (US) countries. Section 5.3 introduces short-run stickiness in the pricing mechanism. Section 5.4 describes the exogenous processes for the money supply in the UK and the US and derives the long-run values of the two economies. Section 5.5 presents the simulations of the model under different degrees of price-stickiness. Section 5.6 shows the estimate of the VECM based on the long-run relations derived from the steady-state solution of the model. Impulse response functions and forecast error variance decomposition are derived. Section 5.7 introduces an estimated 'two-country' monetary policy rule into the VECM. Impulse response functions and forecast error variance decomposition are finally derived under the rule.

5.2 The Model Economy

The model is essentially an extension of the Obstfeld and Rogoff's (1995) model which allows for Pricing-to-Market (PTM). We consider two countries, home and foreign and we denote the foreign country variables with an asterisk. Residents of each country value the consumption of the composite traded good produced (domestically and abroad) and real money balances. The composite good is made by a group of differentiated goods of total measure of unity. Of these goods a fraction n is produced by the home country, and $1 - n$ is produced in the foreign country. We also let n and $1-n$ represent the population of the home and foreign country respectively.

We finally assume that each good is sold exclusively by a price-setting firm that can price discriminate across countries. The structure underlying the goods market behaviour is that of monopolistic competition, where firms can separate their domestic and foreign markets and fix different prices in each local market. While underpinning in this way the productive structure of the economy, final aggregate output is taken as exogenous by the 'international' consumer, who simply perceives that there are differences in goods related to the border that has to be crossed to buy them. We will relax this assumption in chapter 7.

5.2.1 Households Choice and the World Economy Problem

The representative consumer in each country is assumed to choose consumption and real money balances to maximize the intertemporal utility function:

$$U = E_t \sum_{s=0}^{\infty} \beta^s u \left(c_{t+s}, \frac{M_{t+s}}{P_{t+s}} \right) \tag{5.1}$$

$$U^* = E_t \sum_{s=0}^{\infty} \beta^s u^* \left(c^*_{t+s}, \frac{M^*_{t+s}}{P^*_{t+s}} \right) \tag{5.2}$$

where the one-period utility functions, u and u^* for the home and foreign country respectively, are specialized to have a constant relative risk aversion form[36]:

$$u\left(c_t, \frac{M_t}{P_t}\right) = \log c_t + \gamma \log\left(\frac{M_t}{P_t}\right) \tag{5.3}$$

$$u^*\left(c_t^*, \frac{M_t^*}{P_t^*}\right) = \log c_t^* + \gamma^* \log\left(\frac{M_t^*}{P_t^*}\right) \tag{5.4}$$

Here c_t represents a composite consumption good, i.e. $c_t = \left(\int_0^1 c_t(i)^{\frac{\rho}{\rho-1}} di\right)^{\frac{\rho}{\rho-1}}$, $\rho > 0$, so that there is a fixed unit measure of differentiated goods, where $c_t(i)$ is the consumption of good i. $\frac{M_t}{P_t}$ is real money balances. P_t is the home country consumer price index[37], defined as

$$P_t = \left[\int_0^n p_t(i)^{1-\rho} di + \int_n^1 q_t^*(i)^{1-\rho} di\right]^{1/(1-\rho)}$$

where $p_t(i)$ is the home currency price of the home produced good, $q_t^*(i) = S_t p_t^*(i)$ is the domestic currency price of a foreign PTM good i sold in the home market (i.e. the domestic price of the imported good). P_t^* is the consumer price index of the foreign country. We let S_t be the nominal exchange rate (price of foreign currency).

The representative domestic consumer will pick consumption, money holdings, holdings of internationally traded bonds and output to maximize intertemporal expected utility subject to the budget constraint. This can be written in

[36] Whenever possibile, the notation will have the same meaning as in chapter 2.

[37] The foreign price index is then

$$P_t^* = \left[\int_0^{1-n} q_t(i)^{1-\rho} di + \int_{1-n}^1 p_{ft}^*(i)^{1-\rho} di\right]^{1/(1-\rho)}$$

where $q_t(i) = \frac{p_{ft}(i)}{S_t}$ is the foreign price of the domestic good sold in the foreign country (i.e. the domestic price of the exported good) and $p_t^{*f}(i)$ the foreign price of the foreign good sold in the foreign country.

nominal terms as:

$$P_t c_t + M_t + B_t + P_t \tau_t = P_t y_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} \tag{5.5}$$

A symmetric constraint holds for the representative consumer in the foreign country:

$$P_t^* c_t^* + M_t^* + B_t^* + P_t^* \tau_t^* = P_t^* y_t^* + (1 + i_{t-1}^*) B_{t-1}^* + M_{t-1}^* \tag{5.6}$$

One period bond, B_{t-1}, purchased at time $t-1$ yields a nominal return i_{t-1}. Thus from equations (5.5) and (5.6) we see that in addition to the market for final goods, nations are also linked by financial markets. We assume that individuals may trade only a risk-free real bond, therefore markets are incomplete. τ_t represents total real taxes less transfers. The domestic budget constraint, in real terms, is:

$$c_t + \frac{M_t}{P_t} + b_t + \tau_t = y_t + (1 + r_{t-1}) b_{t-1} + \frac{M_{t-1}}{P_{t-1}} \frac{1}{1 + \pi_t} \tag{5.7}$$

where π_t is the inflation rate from time $t-1$ to t and b_t is the real stock of domestic bonds $\left(\frac{B_t}{P_t}\right)$, r_t is the real interest rate: $(1 + r_t) = \frac{(1+i_t)}{(1+E_t \pi_{t+1})}$. A similar expression holds for the foreign country (with asterisks).

The optimal intratemporal consumption allocation between each differentiated good is given by[38]:

$$c_t(i) = \left(\frac{v_t(i)}{P_t}\right)^{-\rho} c_t$$

where $v_t(i)$ is equal to either $p_t(i)$ or $q_t^*(i)$, depending upon which category the good i falls within.

[38]The solution of the intratemporal problem is obtained by maximizing (5.3) with respect to the equation for the composite consumption good.

The first order conditions derived from the individual home consumer's decision problem are[39]:

$$E_t c_{t+1} = c_t \beta (1 + r_t) \tag{5.8}$$

$$\frac{M_t}{P_t} = \gamma c_t \frac{1 + i_t}{i_t} \tag{5.9}$$

together with the budget constraint (5.4) and the transversality condition:

$$\lim_{i \to \infty} \prod_{s=0}^{i} (1 + r_{t+s})^{-1} \left(b_{t+i} + \frac{M_{t+i}}{P_{t+i}} \right) = 0 \tag{5.10}$$

which prevents people from borrowing and never paying back and is obtained by iterating the budget constraint (5.7).

The representative agent of the foreign country will solve the same decision problem.

We now define the world real consumption c^W expressed in terms of the domestic consumer price index. It is equal to the weighted sum of domestic and foreign consumption, where the weights are given by the fraction of the country population n and $1 - n$.

$$c_t^W \equiv n c_t + (1 - n) \frac{S_t P_t^*}{P_t} c_t^* \tag{5.11}$$

In the aggregate global equilibrium, the domestic nominal money supply must equal domestic nominal money demand in each country, and global net foreign assets must be zero. In nominal terms (and in terms of the domestic consumer price index) the bond-market clearing condition is:

$$n B_t + (1 - n) S_t B_t^* = 0 \tag{5.12}$$

[39]The solution of the intertemporal problem is obtained by maximizing (5.1) with respect to (5.7).

which in real terms is:

$$nb_t + (1-n)\frac{S_t P_t^*}{P_t} b_t^* = 0 \tag{5.13}$$

Given this assets market clearing condition, we can derive the aggregate global goods market clearing condition. By taking a population-weighted average of the budget constraints (5.5)-(5.6) across home agents and foreign agents, by imposing condition (5.13) and the home and foreign budget constraint we obtain:

$$c_t^W \equiv nc_t + (1-n)\frac{S_t P_t^*}{P_t} c_t^* = ny_t + (1-n)\frac{S_t P_t^*}{P_t} y_t^* \equiv y_t^W \tag{5.14}$$

Equation (5.14) states simply that world real consumption equals world real income.

5.2.2 The Symmetric Steady State

In order to log-linearize the system we need to find a well-defined flexible-price steady-state solution around which to approximate[40]. Given that consumption and output are constant in steady state, the equilibrium real interest rate is obtained by the consumption Euler equation (5.8) and is given by:

$$r^* = r = \frac{1-\beta}{\beta} \tag{5.15}$$

Steady-state consumption must equal steady-state real income in both countries, so that:

$$c = rb + y \tag{5.16}$$

$$c^* = -\left(\frac{n}{1-n}\right) rb \frac{P}{SP^*} + y^* \tag{5.17}$$

[40]In this chapter and in the following we assume that each variable has been detrended by the deterministic time trend possessed by the exogenous output and inherited by all the variables in the system. Thus, without other sources of stochastic trend, the system in steady-state results constant.

where the corresponding foreign condition is derived by expressing b^* in terms of b, using the equilibrium condition (5.13). Equation (5.17) is the only one where relative country size considerations enters in steady state. It expresses a very simple and straightforward relationship between foreign consumption and relative country-size, i.e. foreign consumption is negatively related to the ratio between domestic and foreign country size.

Steady-state money demand must equal steady-state money supply (for the moment taken simply as a constant) in both countries[41]:

$$\frac{\overline{M}}{P} = \gamma c \frac{1+i}{i} \tag{5.18}$$

$$\frac{\overline{M}^*}{P^*} = \gamma c^* \frac{1+i^*}{i^*} \tag{5.19}$$

In general, there is no simple closed-form solution for the steady state. But when initial foreign asset is assumed to be zero, $B_0^* = 0$, the equilibrium is completely symmetric across the two countries implying[42]

$$c_0 = c_0^* = y_0 = y_0^*$$

With this assumption we can now log-linearize the model around the symmetric steady-state we have just characterized.

5.2.3 The Log-linear Approximation

We now develop a log-linear version of the model's equilibrium conditions. The general log-linearization approach has been explained in chapter 4[43].

[41]Imposing the no speculative bubbles assumption and zero money and consumption growth, the steady-state nominal interest rate equals the steady-state real interest rate.

[42]Obstfeld, Rogoff, 1995, Foundation of International Macroeconomics, MIT Press, chapter 10.1.

[43]From this section onward we will deal with variables expressed in logarithms. Now, low case letters indicate log-deviations from the steady state of the previously defined variables, which

We start from the definition of uncovered interest rate parity, which, expressed in log-linear form, says that the nominal interest rate differential equals the expected depreciation of the nominal exchange rate:

$$i_t = i_t^* + E_t(s_{t+1} - s_t) \tag{5.20}$$

The log-linear approximation of the world good markets equilibrium condition is:

$$c_t^W \equiv nc_t + (1-n)c_t^* = ny_t + (1-n)y_t^* \equiv y_t^W \tag{5.21}$$

while the consumption Euler equations in the two countries take the form:

$$E_t c_{t+1} = c_t + r_t \tag{5.22}$$

$$E_t c_{t+1}^* = c_t^* + r_t^* \tag{5.23}$$

so that the consumption differential is equal to:

$$E_t(c_{t+1} - c_{t+1}^*) = c_t - c_t^* + (r_t - r_t^*) \tag{5.24}$$

The log-linear versions of the money demand equations are:

$$m_t - p_t = c_t - \nu i_t \tag{5.25}$$

$$m_t^* - p_t^* = c_t^* - \nu^* i_t^* \tag{5.26}$$

where $\nu = \left[\frac{1}{(1+\bar{i})\bar{i}}\right]$, and $\nu^* = \left[\frac{1}{(1+\bar{i}^*)\bar{i}^*}\right]$. We make the assumption that, in the long run, the nominal interest rate in each country converges to the same value, implying that $\nu^* = \nu$, i.e. inflation differentials disappear in steady state.

should be distinguished from the previous notation where the low case letters indicated real variables. Exceptions are low case nominal and real interest rates that have the same meaning as before.

We can now subtract equation (5.25) from equation (5.26) and by using the uncovered interest parity condition we obtain:

$$m_t - m_t^* - (p_t - p_t^*) = c_t - c_t^* - \nu E_t(s_{t+1} - s_t) \tag{5.27}$$

Ceteris paribus, this gives us a relationship between the real exchange rate and the national interest rate differentials, which becomes clearer if we add s_t to both sides of (5.27):

$$m_t - m_t^* + (s_t - (p_t - p_t^*)) = c_t - c_t^* - \nu(i_t - i_t^*) + s_t \tag{5.28}$$

We observe that a positive interest rate differential in favour of the domestic economy leads to an appreciated nominal and real exchange rate.

5.3 The Pricing Mechanism

So far we have not included any price rigidity. We now introduce the assumption that the domestic consumer price index cannot adjust immediately after a monetary shock.

We adopt Calvo's (1983) approach, that assumes that individual firms set prices for some uncertain time interval and face a constant probability of price adjustment thereafter. We let each individual firm to have a probability $(1-\alpha)$ of changing its price in any given period. When a firm adjusts its price, it will set its price $\widetilde{p}_t$ so as to equate it with the discounted expected nominal marginal costs adjusted for a mark-up. Price dynamics in the home country can then be described by two equations in the price level p_t, and the new price set by the adjusting firms, $\widetilde{p}_t$. These equations are:

$$p_t = (1-\alpha)\widetilde{p}_t + \alpha p_{t-1} \tag{5.29}$$

$$\widetilde{p}_t = (1-\beta\alpha)[n(p_t + mc_t) + (1-n)(p_t^* + s_t + mc_t^*)] + \beta\alpha E_t \widetilde{p}_{t+1} \tag{5.30}$$

Equation (5.29) shows how the nominal home price level adjusts when only a share $(1-\alpha)$ of firms adjusts it every period. Equation (5.30) shows how home firms determine their price when it is adjusted[44][45]. The aggregate home price

[44] Calvo's price adjustment model has been justified by Rotemberg (1987) as follows. Although $\widetilde{p}_t$ would be charged in t by the typical firm if there were no adjustment costs, in the presence of such costs (assumed quadratic) the producer will instead choose p_t to minimize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[(p_t - \widetilde{p}_t)^2 + \alpha(p_{t+j} - p_{t+j-1})^2 \right]$$

where $\alpha > 0$ reflects the cost of price changes in relation to the opportunity cost of setting a price different from $\widetilde{p}_t$. The first order optimality condition leads to

$$\Delta p_t = \beta E_t \Delta p_{t+1} - \frac{1}{\alpha}(p_t - \widetilde{p}_t)$$

[45] We give a short analytical demonstration for the derivation of the first part of equation (5.30). We start by considering the prices set by firms in each country:

$$p(i) = \mu MC(i) \text{ and } p_f(i) = \mu MC(i)$$

$$p^*(i) = \mu MC^*(i) \text{ and } p_f^*(i) = \mu MC^*(i)$$

where $p_f(i)$ is the domestic price set by the domestic firm in the foreign country, $p_f^*(i)$ is the foreign price set by the foreign firm in the foreign country and μ is the cost- price mark-up. We then divide both expressions by their respective price indices and we indicate with $mc(i) = \frac{MC(i)}{P}$, $mc^*(i) = \frac{MC^*(i)}{P^*}$ the marginal costs in real term:

$$\frac{p(i)}{P} = \mu mc(i)$$

$$\frac{p^*(i)}{P^*} = \mu mc^*(i)$$

We can sum up for all the firms in each country and take the logarithm:

$$\log\left(\frac{\int_0^n p(i)di}{P}\right) = \log \int_0^n \mu mc(i)di$$

$$\log\left(\frac{\int_{1-n}^1 p^*(i)di}{P^*}\right) = \log \int_{1-n}^1 \mu mc^*(i)di$$

level p_t of the composite traded goods depends on both home goods and imported goods prices. Home firms adjust their price so as to make it equal to the discounted value of expected nominal marginal cost, which is $p_t + mc_t$, where the notation mc_t denotes real marginal cost. Symmetrically, foreign firms selling in the home market will set their price on the basis of expected marginal cost, which, in home currency terms, is $p_t^* + s_t + mc_t^*$. Thus, $\widetilde{p}_t$ is a weighted average (where the weights are given by the country's population equal to the number of goods produced in each country) of these two prices. This gives equation (5.30).

Similarly to (5.29)-(5.30), the dynamics of p_t^* and $\widetilde{p}_t^*$ in the foreign economy is:

$$p_t^* = (1-\alpha)\widetilde{p}_t^* + \alpha p_{t-1}^* \tag{5.31}$$

$$\widetilde{p}_t^* = (1-\beta\alpha)[n(p_t + mc_t - s_t) + (1-n)(p_t^* + mc_t^*)] + \beta\alpha E_t \widetilde{p}_{t+1}^* \tag{5.32}$$

The difference between the equations for the home and the foreign economies is:

$$p_t - p_t^* = (1-\alpha)(\widetilde{p}_t - \widetilde{p}_t^*) + \alpha(p_{t-1} - p_{t-1}^*) \tag{5.33}$$

$$\widetilde{p}_t - \widetilde{p}_t^* = (1-\beta\alpha)s_t + \beta\alpha E_t(\widetilde{p}_{t+1} - \widetilde{p}_{t+1}^*) \tag{5.34}$$

Equations (5.28), (5.33), (5.34) together with the two Euler equations for consumption in home and foreign countries, represent the system of equations

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The previous two expressions can be rewritten in this form

$$\log \int_{1-n}^{1} p(i)di = p + mc \tag{I}$$

$$\log \int_{1-n}^{1} p^*(i)di + s = p^* + mc^* + s \tag{II}$$

where $p = logP$, $p^* = logP^*$, $mc = \log\int_0^n \mu mc(i)di$, $mc^* = \log\int_{1-n}^1 \mu mc^*(i)di$ and in (II) we have added to both sides the logarithm of the nominal exchange rate.

Equation (5.30) follows directly by taking the arithmetic average (with the weights given by their respective country size) of the two prices above.

that governs the dynamics of s_t, $p_t - p_t^*$, $\widetilde{p}_t - \widetilde{p}_t^*$.

The production side of the story has not been considered. But, if we assume the same technology, the marginal rate of transformation of home goods into foreign goods is unity. As we will see in chapter 7 technological considerations do not contribute in determining the real exchange rate and deviations from PPP arise only because of nominal price rigidity in each local currency[46].

The real exchange rate has a short-run dimension and can be modelled only in the log-linear representation of the model that is meant to capture short-run deviations from the equilibrium.

We can solve equation (5.33) for $\widehat{\widetilde{p}}_t = \widetilde{p}_t - \widetilde{p}_t^*$, we take expectation of the same equation one period ahead and we solve it for $E_t\widehat{\widetilde{p}}_{t+1}$. We finally substitute the two expressions for $\widehat{\widetilde{p}}_t$ and $E_t\widehat{\widetilde{p}}_{t+1}$ in equation (5.34) obtaining the following equation:

$$\widehat{p}_t(1+\beta\alpha^2) - \alpha\widehat{p}_{t-1} - (1-\beta\alpha)(1-\alpha)s_t - \beta\alpha E_t\widehat{p}_{t+1} = 0 \tag{5.35}$$

where $\widehat{p}_t = p_t - p_t^*$. We can observe that for $\alpha \to 0$, i.e. the flexible price solution, (5.35) converges to the PPP relationship. For $\alpha \to 1$, i.e. the fixed price solution, (5.35) implies:

$$\widehat{p}_t(1+\beta) - \widehat{p}_{t-1} - \beta E_t\widehat{p}_{t+1} = 0$$

which can be rewritten in terms of inflation differentials:

$$E_t\widehat{\pi}_{t+1} = \frac{1}{\beta}\widehat{\pi}_t$$

[46]To make the point clearer: the assumption of pricing-to-market allows us to write equations (5.29) and (5.30) where the pricing behaviour of the foreign firm in the home country depends on the adjusting mechanism of the home country. But since for the foreign (and domestic) firm is optimal to set the same price in both countries deviations from PPP arise only because of nominal price rigidity in local currency.

where $\widehat{\pi}_t = \widehat{p}_t - \widehat{p}_{t-1}$. Thus, with fixed prices, the expectation on the future inflation differential is equal to present inflation differential divided by the discount rate. Given that $\frac{1}{\beta} > 1$ the fixed price solution delivers a non stationary first order autoregressive process.

5.4 Specification of the Money Supply Mechanism

We assume that τ_t allows for variation in the nominal supply of money, with $P_t \tau_t = M_t - M_{t-1}$. We assume then that money supply in each country follows a vector autoregressive process, given by:

$$\begin{bmatrix} m_t \\ m_t^* \end{bmatrix} = (I - \Phi) \begin{bmatrix} \mu \\ \mu^* \end{bmatrix} + \Phi \begin{bmatrix} m_{t-1} \\ m_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^{m*} \end{bmatrix} \tag{5.36}$$

where ε_t^m and ε_t^{m*} are *i.i.d.* $\sim (0, \Sigma_m)$ and $\sum_m = \begin{bmatrix} \sigma_m^2 & \sigma_{mm^*} \\ \sigma_{mm^*} & \sigma_{m^*}^2 \end{bmatrix}$.

The UK data for the money time series come from M0 whereas we used the Monetary Base for the US (that is the narrowest definition of the money supply in the US which we think is the closest definition to the UK M0). The sample period covers quarterly observations from 1969:3 to 1998:2. Each variable, expressed in per-capita terms, has been linearly detrended and differentiated before the estimation of the VAR[47].

[47] We need to differentiate the variables in order to have a stationary VAR system. The reason is that, as we have seen in chapter 2, the log-linear model expresses the short-run deviations from a stochastic trend. In this economy stochastic non-stationarity comes from money supply. The unit root in the money supply time-series affects only the long-run behaviour of nominal variables. Indeed, long-run money neutrality implies that real and nominal equilibria are completely separated. This ex-ante property needs not to hold when we will analyse the short-run deviations of the system from the steady state. Out of equilibrium a monetary shock can have real effects.

OLS estimation of (5.36) led to the following result:

$$\begin{bmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{bmatrix} = \begin{bmatrix} .0011\ [.16] \\ .0002\ [.28] \end{bmatrix} + \begin{bmatrix} .162\ [1.76] & .454\ [3.28] \\ -.007\ [-.16] & .695\ [9.64] \end{bmatrix} \begin{bmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{bmatrix}$$

where inside the squared brackets are t-statistics. The residual variance-covariance matrix is:

$$\widehat{\Sigma} = \begin{bmatrix} .013 & .004 \\ .004 & .007 \end{bmatrix}$$

From the estimates we see that the constant terms are insignificant once the variables have been linearly detrended. More interestingly, we have found that spillovers coming from the foreign economy (the US) are important and significant for modelling the behaviour of the UK money supply, whereas domestic money does not enter with a significant coefficient in the equation for the foreign money process. From the residual variance-covariance matrix we can observe that domestic monetary shocks are much more volatile (twice) than foreign monetary shocks. Furthermore, their covariance is very small.

5.4.1 Calibration of the Model

The measurement of the deep parameters of the two economies consists, here, of a simple calibration exercise. We calibrate the model by looking at the long-run properties of the time series for the UK and the US. Starting with the real interest rate, for each of these two economies we took the average quarterly interest rate over the period 1969-1998. The real interest rate has been built as an ex-post value, by subtracting to the nominal interest rate the current value of inflation.

The value of α has been set, for the moment, equal to 0.75, which implies that the average length of price adjustment is four quarters[48]. The country size

[48]Calvo's (1983) setup implies that the lenght of price adjustment is equal to $\frac{1}{1-\alpha}$.

is given by the country population. Each variable is therefore normalized by the number of inhabitants in each country.

After having computed the steady-state equilibrium values of the nominal interest rates in the two countries we could finally get the value of ν[49]. In the following table are reported the equilibrium values of the variables and of the deep parameters for quarterly data.

Table 5.1: The deep parameters of the UK and US economies (sample 1969:1-1998:2)

	UK	US
i	.024	.019
α	.75	.75
γ	1.027	.998
ν	9.5	12.2

For the sample period considered, the evidence of a long-run coincidence between the UK and the US nominal interest rate is rather weak. If we consider a much shorter time period (1994 - 1998) we have a significant convergence between the two nominal interest rates toward the long-run value of the US economy. Therefore for the simulation of the model we decided to take as common parameter ν that corresponding to the US economy.

5.5 The Solution of the Model in Differences

In this section we present the solution of the model. We look at the dynamic solution of a reduced size model, obtained by taking the country differential of each variable. We start by rewriting the previous system of equations in a more compact form:

$$E_t\widehat{c}_{t+1} = \widehat{c}_t + \widehat{r}_t \tag{5.37}$$

[49] $\nu's$ for the UK and the US are computed by considering the annualized steady-state values of i and i^*.

$$m_t - m_t^* + (s_t - \widehat{p}_t) = \widehat{c}_t - \nu E_t s_{t+1} + \nu s_t \tag{5.38}$$

$$\widehat{i}_t = E_t(s_{t+1} - s_t) \tag{5.39}$$

$$\widehat{r}_t = \widehat{i}_t - (E_t\widehat{p}_{t+1} - \widehat{p}_t) \tag{5.40}$$

$$\widehat{p}_t(1 + \beta\alpha^2) - \alpha\widehat{p}_{t-1} - (1 - \beta\alpha)(1 - \alpha)s_t - \beta\alpha E_t\widehat{p}_{t+1} = 0 \tag{5.41}$$

where $\widehat{c}_t = c_t - c_t^*$, $\widehat{p}_t = p_t - p_t^*$, $\widehat{r}_t = r_t - r_t^*$, $\widehat{i}_t = i_t - i_t^*$.

We also consider the system of equations for the money supply (5.36) and the equation defining the real exchange rate:

$$rer_t = s_t - \widehat{p}_t \tag{5.42}$$

Therefore we have a system of six endogenous variables: $\widehat{c}_t$, $\widehat{p}_t$, $\widehat{r}_t$, $\widehat{i}_t$, s_t, rer_t, in six equations combined with the bivariate VAR for m_t and m_t^*.

5.5.1 Simulations for Different Values of α

We now describe the method used for obtaining the final reduced form of the system. The first step consists in writing down the 'guess' of the solution of the system (5.36)-(5.42) obtained applying the method of undetermined coefficients.

We are interested in analysing the behaviour of this economy where all endogenous variables depend on the lagged price differential assumed to be a state variable, and on the exogenous domestic and foreign money supplies.

The system of the solution guessed for each equation is therefore:

$$\widehat{c}_t = \eta_{cp}\widehat{p}_{t-1} + \eta_{cm}m_t + \eta_{cm^*}m_t^* \tag{A}$$

$$\widehat{p}_t = \eta_{pp}\widehat{p}_{t-1} + \eta_{pm}m_t + \eta_{pm^*}m_t^* \tag{B}$$

$$s_t = \eta_{sp}\widehat{p}_{t-1} + \eta_{sm}m_t + \eta_{sm^*}m_t^* \tag{C}$$

$$rer_t = \eta_{rerp}\widehat{p}_t + \eta_{rerm}m_t + \eta_{rerm}m_t^* \tag{D}$$

$$\widehat{r}_t = \eta_{rp}\widehat{p}_{t-1} + \eta_{rm}m_t + \eta_{rm^*}m_t^* \tag{E}$$

$$\widehat{i}_t = \eta_{ip}\widehat{p}_{t-1} + \eta_{im}m_t + \eta_{im^*}m_t^* \tag{F}$$

We then equate the coefficients of the system's solutions (A)-(F) to those of the original system (5.37)-(5.42). By substituting (A)-(F) into (5.37)-(5.42) we have to take into account that $E_t\widehat{p}_{t+1} = \eta_{pp}\widehat{p}_t + \eta_{pm}\varphi_m m_t + \eta_{pm^*}\varphi_{m^*}m_t^*$, and the same for $E_t\widehat{c}_{t+1}$, $E_t s_{t+1}$[50].

The system (A)-(F) shows that the short-run form of each variable depends only on nominal variables: the price differential and the money supply differential. Because prices are sticky in the short run we have treated them as state variables, their past values enter therefore in each equation.

Table 5.2: The roots of η_{pp} for different values of α

α	η_{pp}^1	η_{pp}^2
.99 (25 years)	1.08	.503
0.917 (3 years)	1.078	.417
0.875 (2 years)	1.078	.415
0.75 (1 year)	1.077	.405
0.5 (6 months)	1.076	.352
0.2 (4 months)	1.074	.186
0.05 (3 months)	1.072	.049

Table 5.2 shows the values taken by the two roots of η_{pp} for different values of α. We are interested in investigating the behaviour of the system for different values of η_{pp}^2 (the stable root) when we change α. We notice, firstly, that there exists a saddle path for η_{pp} for any value in the range considered and that the stable root fluctuates for only about 50 basis points within a very large interval

[50] We use a modified version of the Uhlig's (1997) program implemented in Matlab to obtain the results.

of stickiness (from 3 months to 25 years). Having chosen a Calvo-type stickiness leads, thus, to a highly stable behaviour of the price differential time-series.

We concentrate our attention on the most realistic time period of price stickiness, that goes from 3 months to 3 years. Table 5.3 shows the different values of the elasticity taken by the variables in the system (A)-(F) within that interval. η^2_{pp}, the elasticity of the price level to its past value, decreases as we reduce the length of the period over which prices are fixed. The significant jump is when the stickiness is reduced to 4 months ($\alpha = 0.2$). According to our finding, a very low degree of stickiness implies a behaviour of the price differential well represented by a white noise.

Table 5.3: The elasticities of the reduced form equations for different values of α

α	η^2_{pp}	η_{cp}	η_{ip}	η_{rp}	η_{sp}	η_{rerp}
.916	.417	-.417	0	.243	0	-.417
.87	.415	-.415	0	.243	0	-.415
.75	.405	-.405	0	.241	0	-.405
.20	.186	-.186	0	.151	0	-.186
.05	.049	-.049	0	.047	0	-.049

α	η_{pm}	η_{pm^*}	η_{cm}	η_{cm^*}	η_{im}	η_{im^*}	η_{rm}	η_{rm^*}	η_{sm}	η_{sm^*}	η_{rerm}	η_{rerm^*}
.916	.0001	-.0001	.028	-.036	-.024	.024	-.024	.024	.028	-.035	.028	-.036
.87	.0002	-.0003	.028	-.035	-.024	.024	-.024	.024	.028	-.035	.028	-.036
.75	.001	-.001	.027	-.034	-.024	.024	-.023	.023	.028	-.035	.027	-.034
.20	.017	-.02	.012	-.015	-.024	.024	-.013	.014	.028	-.035	.012	-.013
.05	.025	-.032	.003	-.005	-.024	.024	-.004	.004	.028	-.035	.003	-.004

η_{cp} takes the same values of η_{pp} but with opposite sign. Thus, the consumption differential has always a negative elasticity with respect to the price gap which is decreasing, in absolute values, with α. This means that the intertemporal substitution effect is higher when the degree of price stickiness is high. Without stickiness the consumption differential does not depend on the price differential. In other words, only when there exists some rigidity in the system can changes in prices directly affect consumption decisions.

The elasticity of the real interest rate to a price change, η_{rp}, is positive and increasing with the size of α. The real exchange rate elasticity to a price change, η_{rerp}, takes the same values of the consumption elasticities. Thus, a price increase leads to an increase of the real interest rate and to a consequent real appreciation of the domestic currency (given uncovered interest parity). We, then, notice that the elasticities of the nominal interest rate, η_{ip}, and of the nominal exchange rate, η_{sp}, to a price change are always zero. This means that, according to their final reduced form, these two variables are only determined by movements in the exogenous domestic and foreign money supplies.

If we now observe the second panel of table 5.3 we see that, as long as the time period of fixed prices decreases, the elasticity of a price change to a domestic money change, η_{pm}, increases. Moreover, we can observe that the liquidity effect is present in the model and it is irrespective of the length of price rigidity for the nominal interest rate differential. For the real interest rate differential the liquidity effect is decreasing for lower values of α. We notice that the behaviour of the nominal variables is not affected by changes in the degree of price stickiness.

5.5.2 Impulse Response Functions for different values of α

The dynamic pattern of the endogenous with respect to the exogenous variables is described by impulse response functions. Impulse responses are computed for shocks to m and m^* (which are temporary since we extracted the trend by taking the first difference), for the two extreme cases of the price rigidity considered: $\alpha = .916$ (3 years) and $\alpha = .05$ (3 months) (figures 5.1)[51]. The first observation is that all the simulations show that a US monetary shock is dominant and the

[51] We are remined that on the horizontal axis, the time after the shock is measured in years; on the vertical axis it is shown the percentage deviation from the steady state.

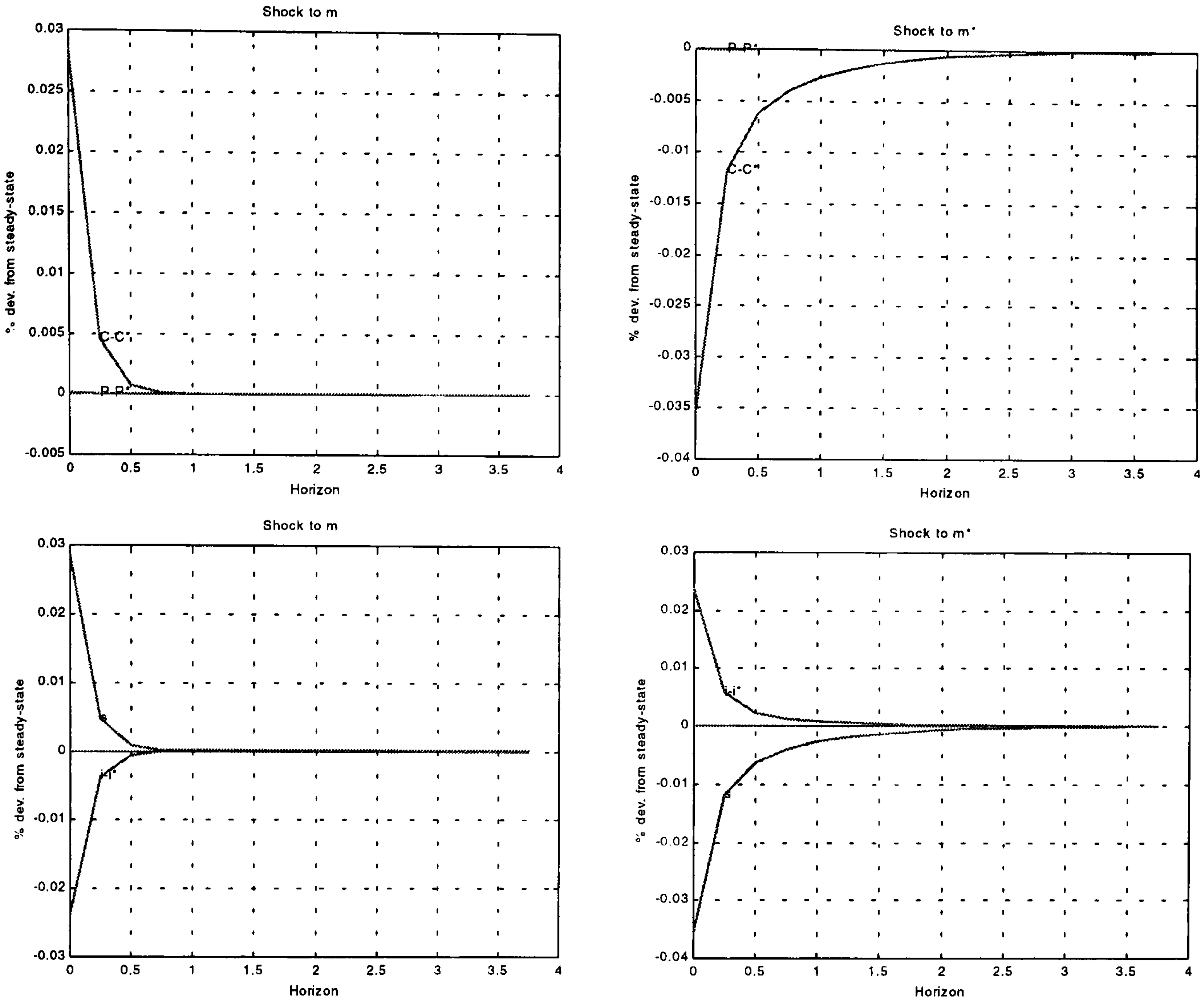
UK monetary policy cannot contrast the pattern determined by the US policy.

We start by looking to the case of $\alpha = 0.916$ (three years of price rigidity, panel A). The first graph to the left shows the reaction of the UK-US consumption and price differential to a shock in m. All the adjustment goes through consumption and the shock is fully absorbed after eight months. The first graph to the right shows the reaction of the UK-US consumption and price differential to a shock in m^*. The behaviour of the consumption differential is now opposite with respect to the previous case and the impact of the shock lasts for a much longer period, almost three years. The nominal exchange rate depreciates after a shock to m (second graph to the left) and the nominal interest rate becomes negative, but the effect is not persistent. The overreaction on impact (overshooting) characterizes the pattern of the nominal exchange rate. Given price rigidity, the behaviour of the real exchange rate follows that of the nominal exchange rate. A US money supply shock leads to an appreciation of the nominal exchange rate that lasts for three years (second graph to the right). Movements in the nominal exchange rate are not reflected in movements of prices.

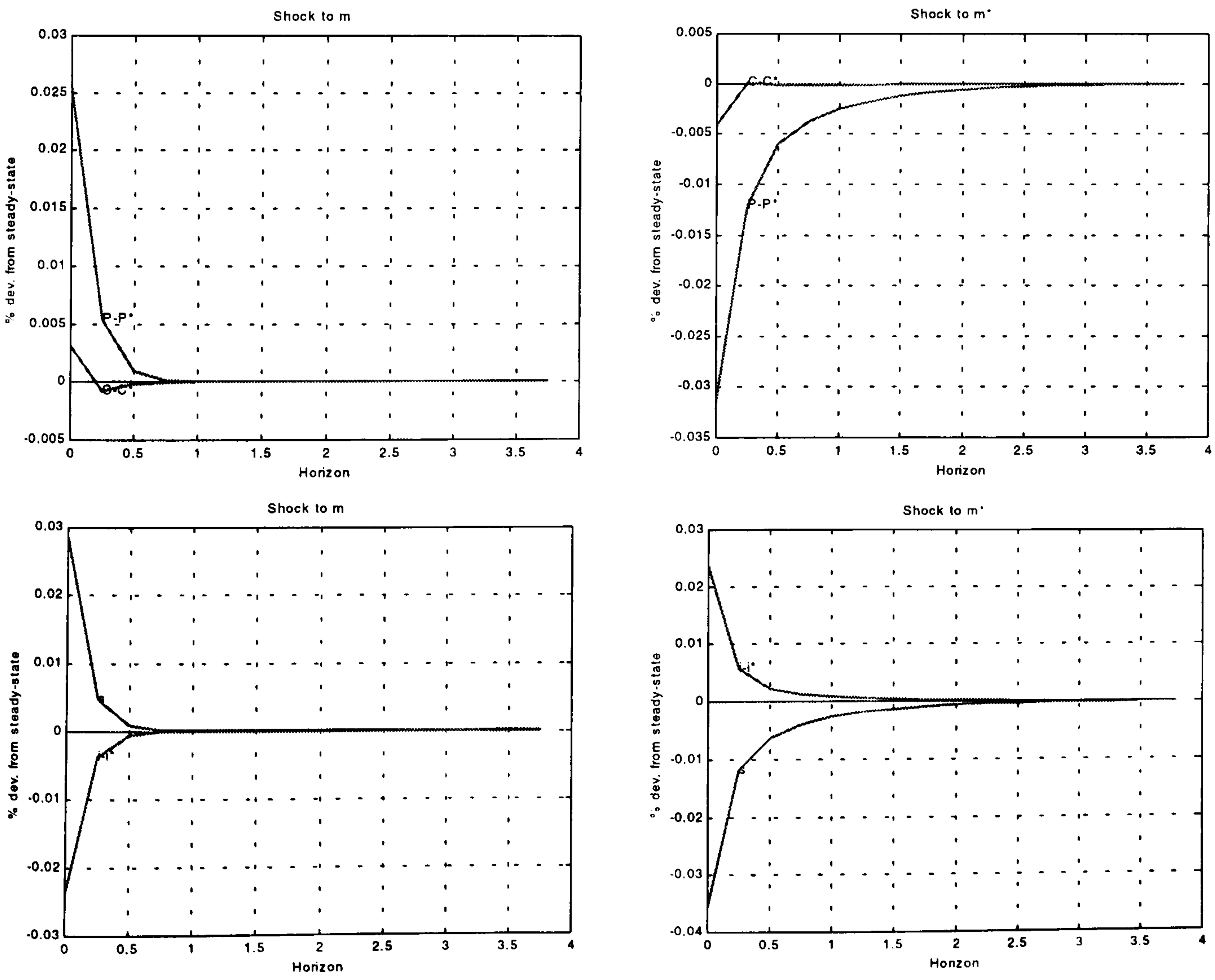
When we set $\alpha = 0.05$ (i.e. three months of price rigidity, panel B) we observe that the biggest amount of adjustment after a money supply shock (either domestic or foreign) goes through movements of relative prices. Conversely, there is no difference in the pattern of the interest and exchange rate responses to domestic and foreign money supply shocks with respect to the previouse case.

Figures 5.1: Impulse response functions: A) $\alpha = 0.916$; B) $\alpha = 0.05$

A)



B)



5.5.3 Results from the Simulations

In this section we have shown that by varying the parameter governing price rigidity we can characterize the autocorrelative properties of the price differential time-series. According to our findings, price flexibility corresponds to a white noise behaviour of the price differential.

We have shown that the degree of price stickiness determines the size of real effects of monetary shocks. Whereas, the asymmetric effects of domestic and foreign shocks determine the length of real effects. The size of the liquidity effect depends on the degree of price stickiness.

Finally, we have shown that temporary shocks to the domestic money supply lead to the overshooting of the nominal exchange rate. It overreacts on impact and comes back to the original level after one year.

5.6 Estimation of the UK-US Difference Variables Model

The analysis proceeds now with the estimation of a structural VECM based on the long-run behaviour of the model. The data used for the estimation correspond to that used for the calibration of model (paragraph 5.4.1). Before performing the estimation we carried out preliminary analysis of the data with unit-root tests[52]. In doing this we are reminded that the data are expressed in difference between the UK and the US. The absence of any limit in trading goods and assets leads to a common path for consumption in the two countries, therefore we expect to find a cointegration relation between UK and US per-capita consumption. The

[52] Unit-root tests and the graphs of the cointegrating vectors are reported in Appendix B.

long-run equality between domestic and foreign consumption suggests stationarity of the real interest rate differential. Only in the short run movements in the nominal exchange rate shift the real exchange rate and untie home and foreign consumption growth. Long run purchasing power parity expressed in first difference implies stationarity of the inflation differential and via the Fisher equation of the nominal interest rate differential. The unit-root tests suggest that we can reject the presence of a unit-root in the three time-series of real, nominal interest rate and consumption differential (Appendix B).

5.6.1 The Cointegrating Vectors and the Structural VECM

The estimation procedure follows that outlined in chapter 3. Therefore we present the results without going through the method. We have estimated a model composed of three $I(0)$ endogenous variables: $\widehat{c}_t = c_t - c_t^*$, $\widehat{r}_t = r_t - r_t^*$, $\widehat{i}_t = i_t - i_t^*$, two $I(1)$ endogenous variables $\widehat{p}_t = p_t - p_t^*$, and s_t, and two $I(1)$ exogenous variables: m_t, m_t^*.

The cointegrating vectors obtained by expressing in logarithmic form the model long-run solutions are contained in the following β matrix:

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \widehat{r} \\ \widehat{i} \\ \widehat{c} \\ \widehat{p} \\ s \\ m^* \\ m \end{bmatrix}$$

In the model real and nominal interest rate differentials are stationary, therefore the first two columns of the β matrix have just a one in the cells corresponding to the two stationary variables. The third column captures the long-run relation between real consumption differentials in the two countries. In the long run we assume that each individual in each country can pool all the risks associated with future consumption and perfectly smooth its consumption decisions so that the equality of consumption growth rates leads to the equality of consumption levels. Therefore we estimate a model with three $I(0)$ variables.

The fourth column shows equation (5.38) for the money market in the long run, it suggests that UK-US money and price differentials are stationary. The fifth column expresses the long-run purchasing power parity conditions, whose stationary error term is therefore the real exchange rate.

The results of the estimation of the endogenous variables are:

Table 5.4: OLS estimates of the VECM

	$\Delta\widehat{r}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{c}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{r}_{t-1}$	.05 [.66]	.12 [1.38]	-.70 [-2.42]	.039 [.94]	.22 [.85]
$\Delta\widehat{i}_{t-1}$	.57 [6.1]	-.12 [-1.2]	.032 [.09]	-.08 [-1.54]	-1.4 [-4.3]
$\Delta\widehat{c}_{t-1}$	-.01 [-.39]	.06 [2.01]	-.23 [-2.23]	.03 [1.98]	.21 [2.28]
$\Delta\widehat{p}_{t-1}$	-1.1 [-6.1]	.26 [1.3]	-1.15 [-1.7]	.45 [4.63]	.44 [.74]
Δs_{t-1}	-.03 [-1.2]	.006 [.20]	-.076 [-.71]	.008 [.49]	.22 [2.31]
Δm^*_{t-1}	.02 [.45]	-.12 [-2.25]	-.13 [-.71]	-.12 [-4.37]	-.23 [-1.4]
Δm_{t-1}	-.07 [-2.5]	.031 [1.0]	.106 [1.0]	.043 [2.79]	.014 [.15]
ecm^1_{t-1}	-.32 [-6.5]	-.09 [-1.67]	.31 [1.7]	-.068 [-2.5]	.18 [1.15]
ecm^2_{t-1}	.23 [3.3]	-.18 [-2.38]	.14 [.52]	.08 [2.07]	.37 [1.58]
ecm^3_{t-1}	-.007 [-.52]	.02 [1.51]	-.13 [-2.48]	.017 [2.26]	-.05 [-1.2]
ecm^4_{t-1}	.003 [.47]	-.003 [-.37]	-.04 [-1.36]	.005 [1.4]	-.009 [-.37]
ecm^5_{t-1}	-.01 [-1.0]	-.02 [-1.24]	.06 [1.44]	-.18[-2.65]	-.108 [-2.6]
Δm^*_t	-.06 [-1.2]	.08 [1.55]	-.26 [-1.47]	.075 [2.86]	.095 [.59]
Δm_t	.09 [3.38]	.05 [1.81]	.22 [2.39]	-.07 [-5.0]	.044 [.52]
const	2.6e^{-4}[-.89]	-5.8e^{-5}[-.19]	-5.1e^{-4}[$-$.5]	1.3e^{-5}[.07]	2.4e^{-4}[.23]

Note: sample 1969:4 -1998:2; t-statistics between brackets.

where $ecm^1_t = [\widehat{r}_t]$, $ecm^2_t = [\widehat{i}_t]$, $ecm^3_t = [\widehat{c}_t]$, $ecm^4_t = [-\widehat{p}_t + m_t - m^*_t]$, $ecm^5_t = [\widehat{s}_t - \widehat{p}_t]$. The estimates of the exogenous variables processes are:

Table 5.5: OLS estimates of the exogenous variables processes:

	Δm^*_t	Δm_t
Δm^*_{t-1}	.69 [9.6]	.45 [3.2]
Δm_{t-1}	-.008 [-.18]	.16 [1.7]
const	1.2e^{-4}[.18]	.001 [.82]

Note: sample 1969:4 -1998:2; t-stat. between brackets.

Finally, the estimate of the structural error correlation matrix leads to the following result:

$$\begin{array}{c|ccccc|cc|}
 & \widehat{r} & \widehat{i} & \widehat{c} & \widehat{p} & s & m & m^* \\
\hline
\widehat{r} & 1 & .57 & .49 & -.27 & .37 & 0 & 0 \\
\widehat{i} & .80 & 1 & .96 & -.89 & .79 & 0 & 0 \\
\widehat{c} & .49 & .96 & 1 & -.92 & .78 & 0 & 0 \\
\widehat{p} & -.27 & .-89 & -.92 & 1 & -.62 & 0 & 0 \\
s & .37 & .79 & .78 & -.62 & 1 & 0 & 0 \\
\hline
m & 0 & 0 & 0 & 0 & 0 & 1 & .14 \\
m^* & 0 & 0 & 0 & 0 & 0 & .14 & 1 \\
\hline
\end{array}$$

The first three error correction terms of the β matrix suggest stationary relationships between country-differentials of the same variables and they appear as if they were a single variable. Therefore the stability of the system requires that for the first three corresponding equations the sign of their autoregressive coefficient is negative, and this is what we observe on table 5.4.

Table 5.4 shows that changes in real interest rate differentials are negatively related to inflation differentials and to past domestic money growth rates, but positively related to contemporaneous domestic money growth rates. A change in the nominal interest rate is mainly determined by past foreign money growth differentials, consumption growth differentials and by its past level.

Changes in consumption differentials negatively depend on past inflation differentials and past real interest rate differentials. The dynamics of the inflation differential is more complex, although the major influence (in terms of size of the coefficients) comes from its past value and from the money supply: it is positively affected by changes in the domestic money supply and negatively by changes in the foreign money supply. Much simpler looks, instead, the dynamics of the change in the bilateral nominal exchange rate between the UK and the US. A depreciation of the nominal exchange rate is related to an increase of the consumption differential in favour of the UK consumption and to a decrease of the nominal interest rate differential (i.e. an increase of the foreign interest rate relative to the domestic one). Table 5.5 has been commented when we have estimated the VAR for the exogenous variables used for the simulations.

5.6.2 Impulse Response Functions and Forecast Error Variance Decomposition

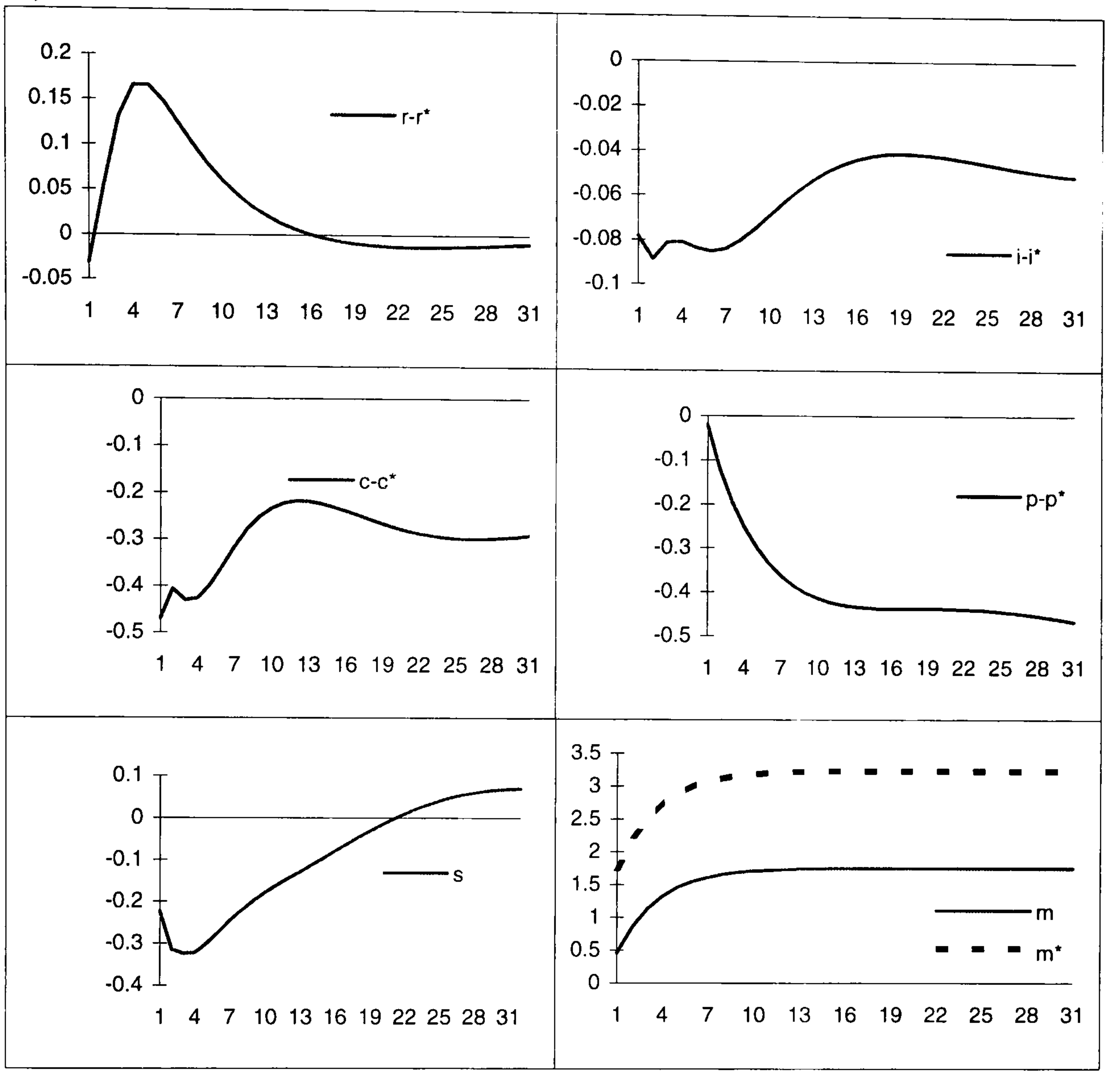
The second step of the analysis consists in investigating the impulse response functions of each variable to the exogenous shocks and in computing the forecast error variance decomposition.

Starting from a shock to the foreign (US) money supply (figures 5.2 panel A), we can observe an impact negative reaction of the real interest rate differential which becomes positive after a quarter and goes back to the previous steady state after two years. The impact on the nominal interest rate differential is negative and it remains negative afterwards. The interpretation is not straightforward. In this case we expect that a foreign money supply shock leads to a decrease of the foreign nominal interest rate relative to the domestic one and therefore to a persistent increase of the domestic/foreign interest rate differential. The problem is that domestic monetary conditions (M0 in the UK) are not independent from foreign monetary policy decisions, as can be seen from the estimate of the two exogenous monetary processes. Thus, an increase of the US money supply leads to a decrease of the domestic interest rate as well, which, eventually, overcomes that of the foreign interest rate.

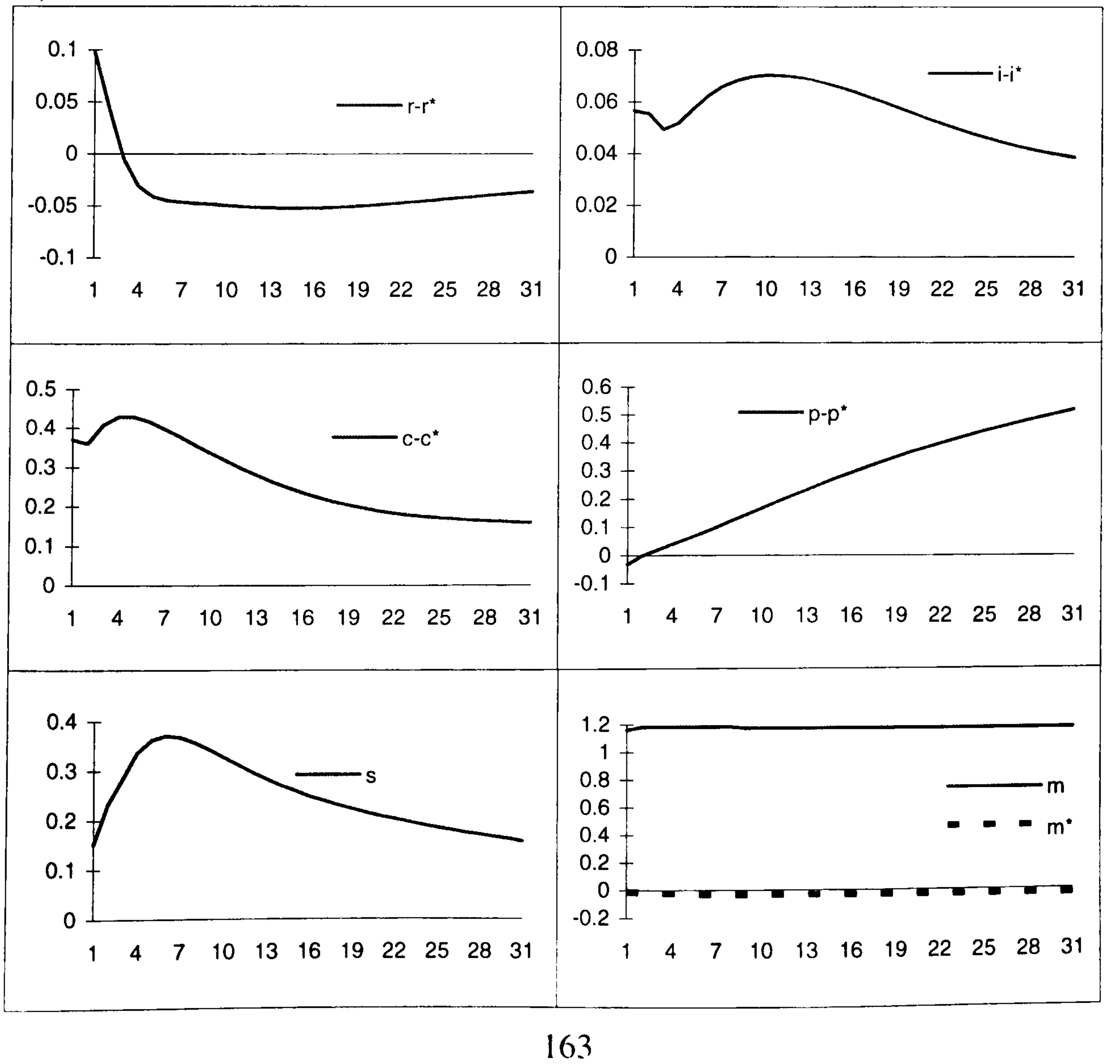
A positive shock to m^* leads, then, to a persistent decrease of domestic consumption relative to foreign consumption, to a persistent decrease of the domestic price level relative to the foreign one.

Figures 5.2: Impulse response function from the VECM estimate

A) Shock to m*



B) Shock to m



The same shock generates an impact appreciation of the UK pound against the US dollar, followed by a depreciation. Thus, the nominal exchange rate undershoots after a foreign increase of the money supply.

The general impression obtained by this first round of impulse response functions is that there is a high degree of spillovers between the two economies. A US monetary expansion strongly affects the UK nominal variables and creates a very long lasting unfavourable effect on the UK consumption differential. These patterns of the effects of m^* and m make clear that there are strong asymmetries coming from monetary shocks in the two countries.

A domestic (UK) money supply shock (figures 5.2 panel B) leads to a persistent increase of domestic consumption relative to foreign consumption, a persistent increase of domestic price level relative to the foreign price level and to an impact depreciation of the bilateral exchange rate followed by an appreciation. The dynamics of the nominal exchange rate shows overshooting. We end up with the so called liquidity puzzle, given by the increase of the nominal interest rate differentials after a money supply shock, this increase is driven by the very persistent increase of the price differential.

A different but complementary perspective on the relative importance of each shock for the evolution of the variables is provided by computing the forecast error variance decomposition. Table 5.6 presents the variance decomposition for forecast errors (ε_k, $k = \widehat{r}, \widehat{i}, m^*, m$) at different horizons. In the computation of the forecast error variance decomposition we decided to take into consideration $\widehat{r}$ and $\widehat{i}$. Although we did not attach an independent error structure to their behaviour in the model, they seem to be responsible for a great amount of volatility of each variable in the system. At long forecast horizons, these variance decompositions can be interpreted as the unconditional variance shares attributable to different shocks. On table 5.6 we have only reported values higher than the third decimal order.

Table 5.6: Forecast error variance decomposition

	$\widehat{r}$				$\widehat{i}$				$\widehat{c}$			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.74	.08	.02	.16	.19	.46	.00	.33	.27	.03	.40	.27
2	.68	.14	.02	.17	.05	.56	.10	.28	.11	.03	.43	.39
5	.58	.15	.18	.14	.05	.37	.31	.27	.06	.01	.38	.52
10	.44	.13	.30	.12	.03	.20	.39	.37	.03	.01	.39	.56
25	.44	.13	.30	.12	.02	.15	.41	.42	.02	.00	.43	.54

	$\widehat{p}$				s			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.24	.53	.04	.19	.06	.86	.04	.01
2	.11	.22	.53	.11	.05	.79	.10	.02
5	.02	.05	.88	.04	.06	.61	.18	.10
10	.01	.01	.96	.02	.07	.52	.17	.19
25	.00	.00	.94	.05	.06	.47	.20	.23

Note: sample 1969:3 -1998:2

In explaining the variance of the real interest rate differential at different horizons the major role is played by its own error. The same behaviour characterizes the variance decomposition of the nominal interest rate differential, although in this case domestic monetary shocks can explain more than a third of its variation. At the beginning of the horizon $\widehat{r}$ and $\widehat{i}$ can explain almost 80% of the variability of $\widehat{p}$, but after two quarters and for all the remaining horizon considered, foreign monetary shocks are the main determinant of the price differential variability. Shocks coming from the nominal interest rate equation can explain almost all the variation in the nominal exchange rate.

Movements in relative consumption are completely determined by the two exogenous monetary shocks. This finding is consistent with the impulse response functions obtained before, but it is not very plausible that almost 90% of the forecast error variance of consumption differentials can be explained exclusively by monetary shocks (domestic and foreign).

We can see, in general, that at the beginning of the forecasted period domestic monetary shocks are predominant and their importance decreases through time, the opposite behaviour is followed by foreign monetary shocks, whose importance increases at longer forecast horizons.

The analysis of the forecast error variance decomposition suggests that monetary shocks are able to explain a limited fraction of the total variability of interest rates differentials and of the exchange rate but can explain a consistent fraction of the variability of consumption and price differentials.

5.7 Changing Monetary Policy

5.7.1 Adding a Feedback Rule to the Model

So far, we have focused the analysis on the economy's reaction to exogenous shocks coming from money supply, measured by some narrow measures (M0 in the UK and the Monetary Base in the US). We have made the simplest assumption on the behaviour of the monetary authority by considering a first order autoregressive process for the money supply. But, using M0 to identify monetary policy shocks led to some puzzling results, mainly related to the dynamic pattern of the nominal and real interest rate differential.

The problem related to the use of any money supply measure (the Monetary Base, M0, M1 or M2) is that it captures all the information that concerns the money market, thus not only the supply but also the demand side.

Moreover, the previous exercise, as well as the simulations carried out with the artificial data, showed also that there are important spillover effects, that are transmitted from the US to the UK monetary policy and in general to the UK economic system. Thus, on the one hand, we are facing the potential problem of endogeneity of the UK money supply and, on the other hand, the need to incorporate international spillovers in a more structural way. These two aspects lead us to formulate a more sophisticated behaviour of the UK monetary authority, that could be captured by a feedback rule.

In this section we decided therefore to account for this feedback. We introduce a rule where the nominal interest rate is the policy instrument. Once established a feedback between macroeconomic variables and the monetary instrument, money supply cannot be considered exogenous any more. Since the focus is on a different policy instrument than money supply, instead of using the long-run restriction implied by equation (5.38) among the cointegrating vectors of the VECM, we will introduce an estimated policy rule. The monetary instrument depends on the economy's behaviour formalized in the rule. In this new set up we assume that the foreign nominal interest rate plays the role of the only exogenous variable in the model, with an attached autoregressive process and an $i.i.d. \sim (0, \sigma^2_{i*})$ shock. The rule aims at capturing the information contained in equation (5.41) that can be expressed in terms of a relation between the inflation differential (current and expected) and the real exchange rate (see chapter 6). For the model developed in this chapter, equation (5.41) is the relation formalizing the trade-off between the goal of price stability and the possibility of real gains in the short run. Thus, the rule is designed to capture what the model suggests to be the important characteristics of our two-country economy.

In line with the other equations of the model the rule is not expressed as deviations from the steady-state values but from the corresponding values of the foreign country. We have carried out OLS estimation of the equation for the domestic nominal interest rate (UK overnight interest rate) against the foreign interest rate (US federal fund rate), which is now assumed to be the unique exogenous driving force of the system, the real interest rate differential and the real exchange rate. The estimated monetary policy rule is[53]:

$$i_t = \underset{[7.2]}{0.94}\, i_t^* + \underset{[6.4]}{0.65}\, \widehat{r}_t + \underset{[3.8]}{0.05}\, rer_t \tag{5.43}$$

The three coefficients are highly significant. The coefficient on the foreign nominal interest rate is close to unity, whereas that on the real exchange rate is close to zero. At this point of the thesis we have not yet reached a formal treatment of the Central Bank behaviour. In equation (5.43) we are modelling the systematic reaction of the monetary authority to the endogenous changes in the economic system and to the exogenous foreign nominal interest rate (foreign policy shock). The rule wants simply to test the plausibility of a constraint such as (5.41) for the determination of the behaviour of the domestic nominal interest rate.

[53]We have also carried out the estimation of a slightly different monetary policy rule, where we have used the price differential in place of the real exchange rate:

$$i = \underset{[7.1]}{0.98}\, i_t^* + \underset{[5.1]}{0.48}\, \widehat{r}_t + \underset{[2.9]}{0.11}\, \widehat{p}_t$$

Figures 5.4 show the graphs with the two rules and the actual behaviour of the nominal interest rate in the UK between 1978-1998.

Appendix E.2 shows the results of other sets of estimates of monetary policy rules for the UK. We consider different specifications of the Taylor rule (1993) that take into account also US variables.

We proceed thus in investigating how the introduction of that rule affects our estimates. We use (5.43) instead of (5.38) as long-run relation in the estimation of the VECM. And now the only exogenous shock is represented by the foreign interest rate (the US Federal Fund Rate). The β matrix is thus given by:

$$\beta = \begin{bmatrix} 1 & 0 & 0.65 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -0.05 & -1 & 0 \\ 0 & 0 & 0.05 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \widehat{r} \\ \widehat{c} \\ \widehat{p} \\ s \\ i \\ i^* \end{bmatrix}$$

The third column of the β matrix contains the feedback rule (5.43). It expresses a stationary relationship among the variables of the rule, in this vector we restricted the coefficient of i^* to be unity. Given that there are no other shocks than i^*, the job of isolating the exogenous policy shock is straightforward in this environment. According to the rule, thus, the domestic monetary policy reacts to changes in the economic system and to changes in the exogenous foreign interest rate without any degree of freedom (that is we are not modelling unexpected changes in the policy rule other than the shock coming from abroad). This is probably a strong assumption for the behaviour of the domestic monetary authority, but it perfectly captures the idea of interdependence.

5.7.2 The Results of the New Estimation

Table 5.7 reports the OLS estimates of the VECM for the endogenous and the exogenous variables.

Table 5.7: OLS estimates of the VECM with the interest rate rule

	$\Delta\widehat{r}_t$	$\Delta\widehat{c}_t$	$\Delta\widehat{p}_t$	Δs_t	Δi_t	Δi^*_t
$\Delta\widehat{r}_{t-1}$	-.27 [-1.1]	-.61 [-.79]	-.54 [-1.3]	.24 [.31]	-.36 [-1.7]	
$\Delta\widehat{c}_{t-1}$	-.02 [-.61]	-.19 [-1.6]	.13 [2.1]	.27 [2.25]	.04 [1.23]	
$\Delta\widehat{p}_{t-1}$	-.09 [-.88]	.04 [.13]	.04 [.21]	.07 [.21]	.04 [.46]	
Δs_{t-1}	.01 [.27]	-.05 [-.43]	-.06 [-1.0]	.12 [1.06]	.005 [.15]	
Δi_{t-1}	.19 [.79]	1.23 [1.6]	.25 [.61]	.74 [.98]	.25 [1.15]	
Δi^*_{t-1}	-.26 [-1.1]	-.44 [-.57]	-.64 [-1.6]	.87 [1.2]	-.19 [-.91]	-.26 [-2.3]
ecm^1_{t-1}	-.57 [-2.31]	1.35 [1.7]	.67 [1.64]	-.43 [-.55]	-.10 [-.47]	
ecm^2_{t-1}	-.02 [-.96]	-.21 [-3.2]	.05 [1.47]	-.08 [-1.2]	-.004 [-.21]	
ecm^3_{t-1}	.022 [.62]	-.17 [-1.5]	-.09 [-1.6]	-.03 [-.32]	-.007 [-.23]	
ecm^4_{t-1}	-.023 [-1.4]	.12 [2.26]	-.01 [-.36]	-.08 [-1.6]	-.02 [-1.3]	
ecm^5_{t-1}	.46 [1.34]	-1.99 [-1.8]	-.76 [-1.3]	.83 [.78]	-.09 [-.30]	
Δi^*_t	-.9 [-7.91]	-.64 [-1.8]	.004 [.02]	.03 [.08]	.07 [.67]	
const	5.6e^{-5}[.13]	.002 [1.2]	-1.8e^{-4}[$-$.2]	-8e^{-5}[$-$.06]	9e^{-5}[.26]	2e^{-4}[.54]

Note: sample 1978:3 -1998:2; t-statistics between brackets.

where $ecm^1_t = [\widehat{r}_t]$, $ecm^2_t = [\widehat{c}_t]$, $ecm^3_t = [-0.65\widehat{r}_t - 0.05(s_t - \widehat{p}_t) + i_t - i^*_t]$, $ecm^4_t = [s_t - \widehat{p}_t]$, $ecm^5_t = [i_t - i^*_t]$.

Before commenting the results we have to make some observations on the data and the sample period used for the estimation. In this case the sample period is much shorter than the sample period used previously, it starts in 1978:3 instead of 1969:3. This is due to the use of a different definition for the domestic short-term interest rate, which corresponds to the overnight interest rate, whose series is available since 1978. We also used a different definition of the foreign nominal interest rate, the US federal fund rate, suggested by a number of papers aimed at the identification of exogenous shocks to the US monetary policy (see Christiano, Eichenbaum and Evans, 1998).

The first column of table 5.7 shows that very few variables significantly contribute to a change in relative real interest rates. The first significant component is $ecm^1_{t-1}\{= \widehat{r}_{t-1}\}$, which means that the past value of the real interest rate gap is an important determinant of the current real interest rate differential. The second significant element is Δi_t^* that seems to influence the entire behaviour of $\Delta \widehat{r}_t$. It enters with a negative sign.

The second equation, for $\Delta \widehat{c}_t$, shows a significant contribution of ecm_{t-1}^2, ecm_{t-1}^4 and ecm_{t-1}^5. The first term corresponds to $\widehat{c}_{t-1}$, thus, again, past values determine the behaviour of the current consumption differential. If we rewrite the equation in levels we obtain that the autoregressive coefficient for the consumption differential is equal to 0.60. The second error correction term, ecm_{t-1}^4, that is relevant in explaining the behaviour of relative consumption differentials, identifies the real exchange rate. We obtain that a deviation of $\widehat{c}_t$ from its equilibrium value is strengthened by a real depreciation. In other words, positive movements in the real exchange rate increase the consumption gap in favour of the domestic economy. Finally, ecm_{t-1}^5 that corresponds to the nominal interest rate differential, affects the consumption differential with a negative sign. Also Δi_t^* enters significantly and with a negative sign in the consumption differential equation.

$\Delta \widehat{p}_t$ depends positively on past real interest rates (to see this effect we have to sum up the coefficient on $\Delta \widehat{r}_{t-1}$ and that on ecm_{t-1}^1), on the past consumption growth differential and negatively on the past nominal interest rate differential.

The nominal exchange rate positively depends on the past change in consumption differential which is the only significant variable. Finally, changes in the nominal interest rate are related almost significantly and with a negative coefficient only to the past real interest rate differential.

The estimate of the structural error correlation matrix leads to the following result:

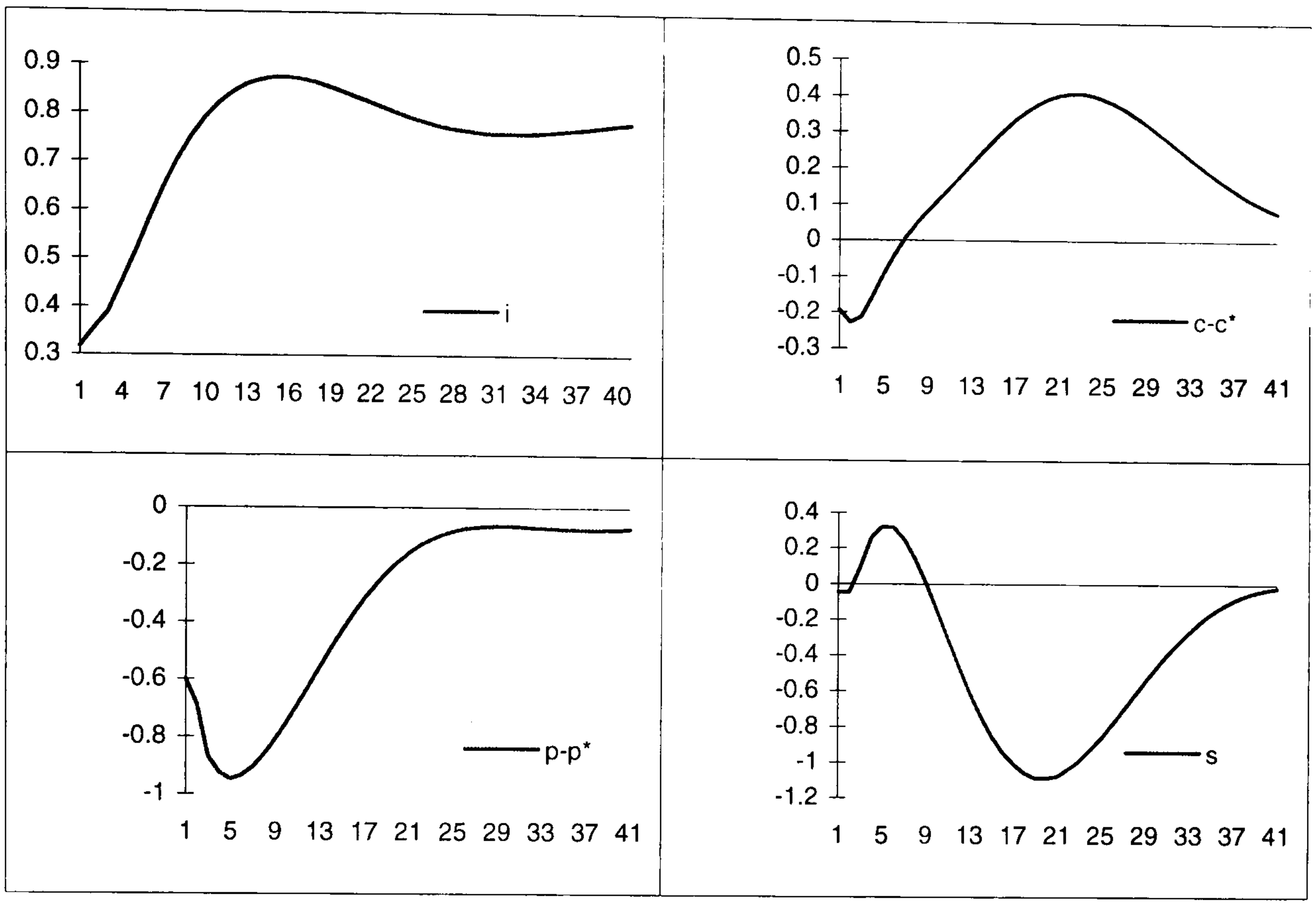
	$\widehat{r}$	$\widehat{c}$	$\widehat{p}$	s	i	i^*
$\widehat{r}$	1	-.39	.24	-.32	.89	0
$\widehat{c}$	-.39	1	-.63	.54	-.15	0
$\widehat{p}$	.24	-.63	1	-.50	-.14	0
s	-.32	.54	-.50	1	.02	0
i	.89	-.15	-.14	.02	1	0
i^*	0	0	0	0	0	1

There exists a high and positive degree of correlation between residuals of nominal and real interest rate differentials, whereas the degree of correlation is much lower among residuals of the nominal interest rate differential with all the other variables.

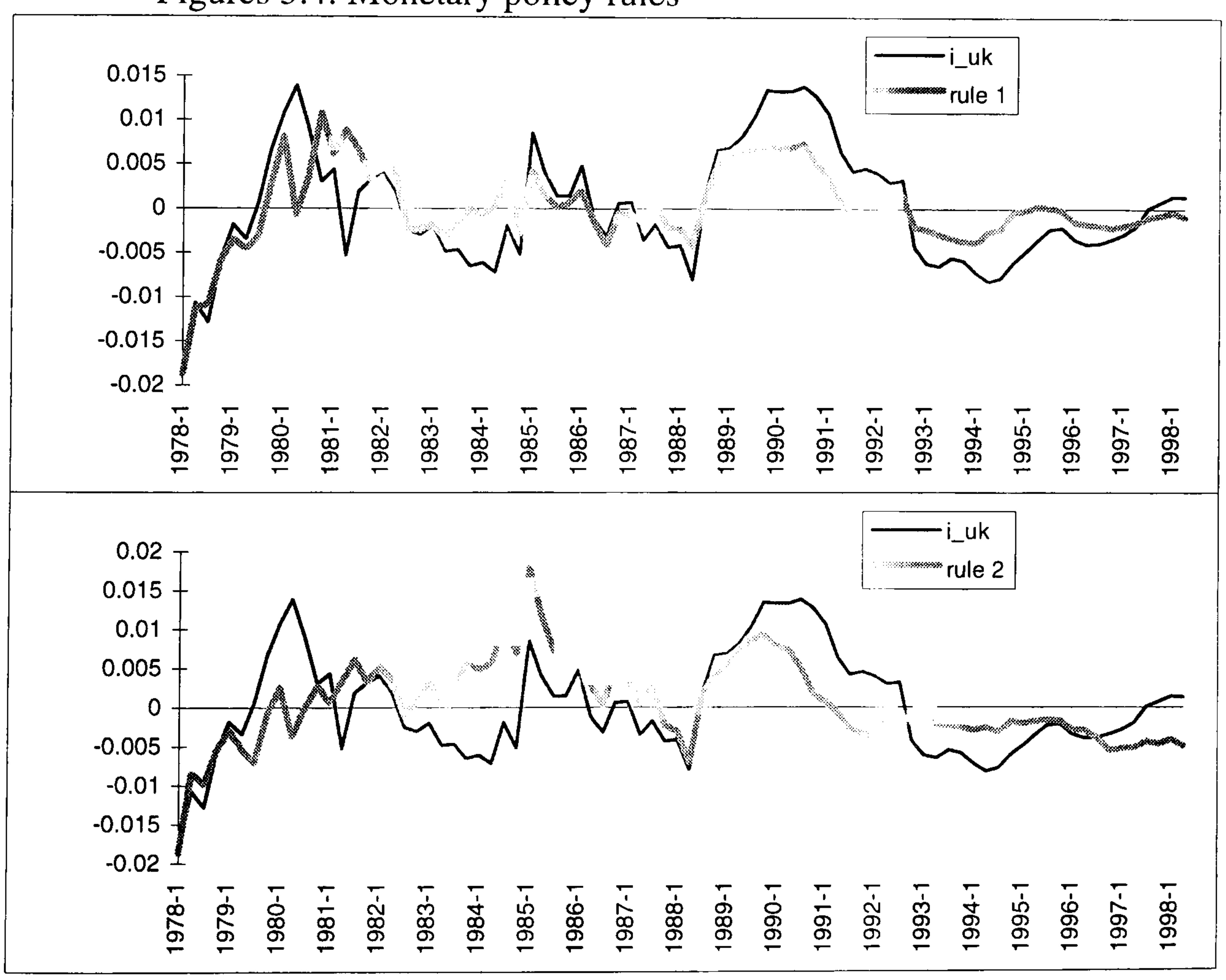
5.7.3 Impulse Response Functions and Forecast Error Variance Decomposition

In figure 5.3 we show the system impulse response functions to a shock in i^*. An unforecastable change in the foreign nominal interest rate has a permanent effect on the domestic nominal interest rate. This shock leads then to an impact decrease of the consumption differential which lasts for six quarters, followed by a persistent increase. It seems, according to that pattern, that in the short term domestic consumption is affected more negatively than foreign consumption, whereas things change for a longer time period. The impact response of the price differential is negative.

Figures 5.3: Impulse response functions to a shock to i*



Figures 5.4: Monetary policy rules



rule 1: $i=i^*+.5(r-r^*)+.1(p-p^*)$

rule 2: $i=i^*+.055(rer)+.65(r-r^*)$

sample 1978-1: 1998-2

The response of the nominal exchange rate shows an opposite behaviour with respect to that of the consumption differential. We have an initial depreciation followed by a persistent appreciation of the exchange rate.

Finally, we comment on the forecast error variance decomposition. Table 5.8 shows that the foreign interest rate can explain most of the variability of the real interest rate differential.

Table 5.8: Forecast error variance decomposition

	$\widehat{r}$			$\widehat{c}$				$\widehat{p}$		
horizon	ε_r	ε_i	ε_{i*}	ε_r	ε_c	ε_i	ε_{i*}	ε_r	ε_i	ε_{i*}
1	.08	.07	.85	.30	.09	.40	.19	.37	.45	.18
2	.05	.09	.85	.28	.12	.31	.28	.32	.46	.21
5	.03	.10	.87	.17	.14	.36	.31	.22	.42	.36
10	.05	.13	.82	.10	.07	.64	.16	.15	.38	.46
25	.07	.20	.73	.07	.04	.47	.42	.13	.35	.51

	s			i		
horizon	ε_r	ε_i	ε_{i*}	ε_r	ε_i	ε_{i*}
1	.15	.82	.00	.17	.64	.19
2	.19	.79	.00	.16	.60	.24
5	.20	.78	.00	.10	.43	.47
10	.19	.79	.01	.05	.19	.76
25	.16	.70	.13	.02	.08	.89

Note: sample 1978:3 -1998:2

In the case of the consumption differential we observe that the explanation of its variability is equally distributed among the real interest rate differential, domestic and foreign nominal interest rates. The importance of foreign interest rate shocks is increasing through time. The variability in the price differential can be attributed, again, to the real interest rate differential and to domestic and foreign nominal interest rates. The major determinant of the variability in the nominal exchange rate is the domestic nominal interest rate.

Therefore, although a shock to the federal fund rate seems to isolate quite well exogenous shocks to the foreign monetary policy, it does a poor job in explaining the variability of our endogenous system of equation, suggesting that, also in this case, we are only explaining a bit of the whole story. Said in other words, having a unique source of shock is not enough to complete the probabilistic structure of the model.

5.8 Conclusions

The aim of this chapter has been to combine a theoretical explanation of the monetary transmission mechanism with an estimation approach based on the model's long-run relationships. The model is oversimplified and it describes the behaviour of home and foreign variables expressed in differences. In the introduction of the chapter we listed several questions related to the dynamics of the international monetary transmission mechanism. In this conclusion we summarize the answers found by carrying out the analysis. Firstly, the model helps in understanding how monetary shocks are transmitted to the economic system. Since prices are sticky we could use them as a state variable in the final form of the system and, thus, relate all the variables of the model to their behaviour. A high degree of price stickiness leads real variables to benefit longer from monetary shocks on the one hand (i.e. the real effect is higher with higher α), and to suffer longer from a price change on the other hand (i.e. the elasticity of consumption to a price change is more negative with higher α). Secondly, the model's simulations show a dominant effect of a US money shock with respect to a UK money shock on the domestic/foreign economic system. They support the evidence of spillover and asymmetric effects of the two shocks.

The VECM estimation relies upon long-run restrictions suggested by the model. Monetary shocks (domestic and foreign) turn out to generate very long-lasting shifts of real and nominal variables and to make a great contribution in explaining the total variability of consumption and price differentials. Conversely, their contribution in explaining the volatility of the exchange rate and the interest rate differential is minimal. The evidence that foreign monetary shocks seem to determine the path of domestic monetary shocks and the fact that the measure used for the domestic money supply contains information also about money demand, led us to build a domestic monetary policy rule for the nominal interest rate based on the feedback coming from abroad. This rule is based on the constraint that relates price differentials to the real exchange rate. Specifically, the domestic nominal interest rate depends on the foreign nominal interest rate, on the gap between domestic and foreign real interest rates and on the real exchange rate. We isolated the foreign monetary policy shock (to the federal fund rate) and we found that this shock leads to an increase of the domestic nominal interest rate, to an impact deterioration of domestic consumption, to a fall of domestic prices and to a long-run appreciation of the nominal exchange rate.

6 The UK and the EMU: Toward a Single Currency

6.1 Introduction

This chapter deals with empirical exercises, whose theoretical underpinning lies on the two-country model presented in the previous chapter. The two economies under investigation are now the UK and a weighted average of the European Union (EU) countries. We will try to understand the relative performance of these two economies and compare them with the behaviour of the US economy. These comparisons will suggest a way of measuring costs and benefits related to the European Monetary Union (EMU).

On 1 January, 1999, the third and final stage of the EMU has begun with the establishment of a currency union encompassing 11 of the 15 members countries of the EU - Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. On that date, these countries locked their exchange rates and adopted the Euro as their common currency, with monetary and exchange rate policies determined by area-wide institutions. Thus, each country has given up the possibility of independent monetary and exchange rate policies.

The general area that studies EMU monetary policies is large and growing, both in empirical and in theoretical terms. A problem common to the empirical analysis is the lack of an appropriate data base for the EMU monetary variables and, thus, all the analysis is referred to some kind of aggregation among countries members that can be referred back until the beginning of the European Monetary System[54].

[54] *An unavoidable problem in studying monetary relationships in the Euro area is the fact that*

The most common and recent empirical endeavour is to find a monetary policy reaction function that could represent the behaviour of the European Central Bank (ECB).

Clarida, Gali, Gertler (1998) provide estimates of monetary policy reaction functions among the European economies. Although the original Taylor (1993) rule was based on the actual inflation rate, Clarida et al. (1998) give a forward looking perspective to the estimated feedback rule for the Bundesbank since the mid-1970s. Their estimates indicate that the Bundesbank has typically pushed up by about 130 basis points for every 1% point rise in expected inflation one year ahead, holding the output gap constant, and that it has reduced the nominal short rate by about 25 basis points for every 1% point shortfall in output relative to potential, holding expected inflation constant.

These estimates encompass the parameters used in the original Taylor (1993) rule, describing the behaviour of the US Federal Reserve Bank[55]. According to Clarida's et al. (1998) reaction function the Bundesbank monetary policy is characterized by a high degree of interest rate smoothing.

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there is virtually no data from the period after the introduction of the Euro, on the 1 January, 1999. One possible reaction to this is to conclude that empirical work had better wait until more data is available, perhaps 10 years or so from now. Alternatively, one can construct measures of aggregate money, prices, output and interest rates in the 11 countries forming the Euro area for the period before the introduction of the Euro, and use these to study the information content of money. Of course, in doing so one must hope that the dynamic relationships between the data will remain broadly stable, even after the introduction of the Euro, in defiance of the Lucas' critique. (Gerlach and Svensson, 1999).

[55]The original Taylor (1993) rule assumed that nominal interest rates are increased by 150 basis points for every 1% point increase in actual inflation and are reduced by 50 basis points for every 1% point shortfall in output relative to potential.

As far as theoretical studies are concerned, the analysis is usually carried out within the IS-LM-AS model. There are two general area of investigation: on the one hand, the focus is on the tasks of the new ECB; on the other hand, the interest is toward relative gains and costs of a common monetary policy for each country member.

Along the first area of investigation, Svensson (1999) suggests inflation targeting as a monetary targeting for the Eurosystem. The constitution of the ECB, and in particular the primacy it accords to price stability, is similar to that of the Bundesbank. Svensson (1999) describes the Bundesbank's policy strategy as 'pragmatic' monetary targeting, in the sense that it is an inflation target in action and a monetary target in words. It would be better for the ECB to be more transparent in this respect.

Along the second area of investigation Leichter and Walsh (1999) suggest that the presence of a common currency does not eliminate the potential role for real exchange rate adjustments due to deviations from the law of one price. They investigate the role of national differences in the monetary transmission mechanism under a common monetary policy and find that non-German aggregate economies may prefer that the common monetary policy responds more strongly to inflation than the Bundesbank.

The aim of the chapter is to provide together a new evidence and a new theoretical framework for analysing the implications of the EMU. The theoretical framework relies on behavioural equations that, although they can be interpreted in the light of the IS-LM framework, are explicitly derived from an intertemporal maximization problem. The empirical analysis does not try to build a feedback rule for the EMU, but to capture the monetary transmission mechanism under a 'neutral' perspective. By 'neutral' we mean that money supply is governed by an exogenous autoregressive process. Firstly, we analyse the relationships between the EU aggregate and the UK, and between the EU and the US from 1979 to 1998. The focus is on the reciprocal monetary transmission mechanisms over real and nominal variables. Secondly, we carry out some experiments which allow us to evaluate the effects on the UK of having a monetary policy determined by the EU, and therefore the potential consequences for the UK of moving toward a single currency.

The chapter is structured as follows. Section 6.2 outlines the theoretical framework by further developing the log-linear equations of the model seen in the previous chapter. Section 6.3 presents the results of the VECM estimation between the UK and the EU. Section 6.4 considers the EU and the US. Finally, section 6.5 presents the results of a VECM estimation between the UK and the US, with the UK monetary policy determined by the EU.

6.2 Outline of the Model

We are reminded that, although the following system of log-linear equations is specified at the level of equilibrium conditions (describing the relationships between various aggregate variables), it has been derived after having constructed a DGSE model, specified at the level of preferences. The equations are, thus, the solution of the optimizing behaviour of households and firms that populate the economy.

$$E_t\widehat{c}_{t+1} = \widehat{c}_t + \widehat{r}_t \tag{6.1}$$

$$\widehat{r}_t = \widehat{i}_t - E_t\widehat{\pi}_{t+1} \tag{6.2}$$

$$E_t\widehat{\pi}_{t+1} = E_t\widehat{p}_{t+1} - \widehat{p}_t \tag{6.3}$$

$$\widehat{i}_t = E_t s_{t+1} - s_t \tag{6.4}$$

$$m_t = \widehat{c}_t + \widehat{p}_t + m_t^* - \nu \widehat{i}_t \tag{6.5}$$

$$E_t \widehat{p}_{t+1} = \frac{1}{\beta\alpha} \left[\widehat{p}_t (1 + \beta\alpha^2) - \alpha \widehat{p}_{t-1} - (1 - \beta\alpha)(1 - \alpha) s_t \right] \tag{6.6}$$

where, as in chapter 5, the hat $\widehat{}$ indicates that we are taking the difference between home an foreign country; i.e. $\widehat{c}_t = c_t - c_t^*$, etc. Equation (6.1) is the differential between home and foreign country consumption Euler equations; (6.2) is the Fisher relation written in terms of differential between the two countries; (6.3) is the definition of the inflation gap and (6.4) of the uncovered interest rate parity. Equation (6.5) is the differential between domestic and foreign money demand. It has been built under the assumption that inflation rates in the two countries converge to the same value in the long run.

We can rewrite equation (6.6) in a way that makes clear the relationship between the dynamics of the inflation differentials and the real exchange rate, i.e. we subtract from both sides of (6.6) $\widehat{p}_t$:

$$\beta E_t \widehat{\pi}_{t+1} - \widehat{\pi}_t = \frac{(1 - \beta\alpha)(1 - \alpha)}{\alpha} (\widehat{p}_t - s_t) \tag{6.7}$$

Equation (6.7) says that the (one-period) discounted expected inflation differential is proportional to the current discounted real exchange rate. If we combine equations (6.1) and (6.2) we obtain the following:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \widehat{i}_t + E_t \widehat{\pi}_{t+1} \tag{6.8}$$

Equation (6.8) can be interpreted as a two-country IS curve (McCallum, Nelson, 1996) with the characteristics of incorporating forward looking terms in consumption and inflation differentials. Equation (6.5), instead, does not involve future variables and it can be interpreted as a two-country LM equation. Equation (6.7) is the result of the assumptions made for the pricing mechanism of the model. It cannot literally be taken as an Aggregate Supply equation, given that output affects this equation only indirectly, through the aggregate demand system. Equation (6.7) plays a crucial role in the monetary transmission mechanism. Monetary shocks affect the real exchange rate given price stickiness. And its behaviour affects, in turn, the differential between future and actual inflation in the two countries.

The system of equations is then completed by the vector autoregressive process followed by the exogenous money supply of the two countries:

$$\begin{bmatrix} m_t^* \\ m_t \end{bmatrix} = \begin{bmatrix} 1-\varphi_{m^*} & 1-\varphi_{m^*m} \\ 1-\varphi_{mm^*} & 1-\varphi_m \end{bmatrix} \begin{bmatrix} \overline{m}^* \\ \overline{m} \end{bmatrix} + \begin{bmatrix} \varphi_{m^*} & \varphi_{m^*m} \\ \varphi_{mm^*} & \varphi_m \end{bmatrix} \begin{bmatrix} m_{t-1}^* \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m^*} \\ \varepsilon_t^{m^*} \end{bmatrix} \tag{6.9}$$

In the introduction we have emphasized that the first requirement for carrying out a meaningful econometric analysis is that monetary policy actions, that represent endogenous responses to current developments in the economy, are separated from exogenous policy actions. Only when the exogenous actions are identified the dynamic analysis of the implied (by the theoretical model) VAR system may yield reliable information on the monetary transmission mechanism. This result in our model is obtained a-priori, given that we do not allow for other elements than money itself to determine monetary policies.

6.3 The Monetary Transmission Mechanism between the UK and the Euro countries

The model (6.1)-(6.5), (6.7) is used to estimate a structural VECM whose variables are expressed in differences between the UK and the EU. Building a model in first differences has the limitation that we cannot isolate movements of one variable while taking the other fixed. Thus, each time that we are analysing the behaviour of one economy what we are really doing is to observe the relative dynamic behaviour of both the economies. In other words, the interpretation of the business cycle can be given only in relative terms.

But, if one the one hand, we can consider this aspect a limitation, on the other hand, this makes it possible to have a relatively high number of variables in a very small system of equations and to emphasize the model's prescriptions about the relative long-run behaviour of the variables.

According to our model, with identical preferences and without limit in trading goods, any consumption differentials should disappear in the long run. The hypothesis of free movements of capitals coupled with that of equal initial wealth in the two countries leads also to have coincidence of real interest rates. Thus, nominal interest rates differ only because of the inflation differentials. The stationarity of the inflation differential derives, then, from that of the real exchange rate (equation 6.7), given that we are assuming purchasing power parity and price flexibility in the long run. But this implies also that inflation differentials should disappear in the long run.

All this information has been used for the estimation of the VECM and the stationarity of the vectors: $[r_t - r_t^*]$, $[i_t - i_t^*]$, $[c_t - c_t^*]$, $[y_t - y_t^*]$, $[s_t - p_t + p_t^*]$, $[\pi_t - \pi_t^*]$ has been tested by unit-roots tests (Appendix B). The data are generally consistent with the model prescriptions.

We have carried out two sets of estimations involving each time 3 $I(0)$ endogenous variables: $\widehat{c}_t = c_t - c_t^*$, $\widehat{r}_t = r_t - r_t^*$, $\widehat{i}_t = i_t - i_t^*$, and $\widehat{y}_t = y_t - y_t^*$, $\widehat{\pi}_t = \pi_t - \pi_t^*$, $\widehat{i}_t = i_t - i_t^*$, 2 $I(1)$ endogenous variables $\widehat{p}_t = p_t - p_t^*$, s_t, and 2 $I(1)$ exogenous variables: m_t, m_t^*. The first substitution ($\widehat{y}_t$ in place of $\widehat{c}_t$) is underpinned by the idea that consumption spending plays the role of an expenditure variable in the model and thus can be replaced by real income. The second substitution ($\widehat{\pi}_t$ in place of $\widehat{r}_t$) is justified by the definition of the real interest rate that allows us to extract information on real interest rate differentials by looking at nominal interest rate and inflation differentials. Therefore the information content of the two sets of estimations is the same.

The cointegrating vectors obtained by expressing in logarithmic form the model long-run solutions are contained in the following β matrix:

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \widehat{\pi} \\ \widehat{i} \\ \widehat{y} \\ \widehat{p} \\ s \\ m^* \\ m \end{bmatrix}$$

where the β matrix in the second round of the estimation imposes exactly the same restrictions but with $\widehat{r}_t$ and $\widehat{c}_t$ replacing $\widehat{\pi}_t$ and $\widehat{y}_t$ respectively.

Since the model is suggesting that real and nominal interest rate differentials are stationary, the first two columns of the β matrix have just a one in the cells corresponding to the two stationary variables. The third column expresses the long-run relation between real income differentials in the two countries. We assume that income differentials follows the same path followed by consumption differentials. Therefore we will estimate a VECM containing three $I(0)$ variables. The fourth column shows the Euler equation for the money market in the two countries, that relates domestic and foreign real money balances. The fifth expresses the long-run purchasing power parity conditions, whose stationary error term is the real exchange rate.

6.3.1 Estimation Results between the UK and the EU

Table 6.1: OLS estimates of the VECM containing $\widehat{c}_t$, $\widehat{r}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^*.

	$\Delta\widehat{r}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{c}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{r}_{t-1}$	-.179[-1.6]	.0299[1.1]	-.54[-2.13]	.216[1.86]	-.54[-2.29]
$\Delta\widehat{i}_{t-1}$	.776[1.43]	.164[1.25]	.21[.17]	-.394[-.696]	.52[.45]
$\Delta\widehat{c}_{t-1}$	.474[2.31]	.022[.438]	.287[1.05]	-.244[-1.14]	.59[1.37]
$\Delta\widehat{p}_{t-1}$	-.106[-.38]	.13[1.93]	-.24[-.38]	.292[1.00]	-.55[-.92]
Δs_{t-1}	-.351[-1.6]	-.034[-.63]	-.029[-.059]	.149[.647]	-.13[-.28]
Δm_{t-1}^*	.099[.717]	.003[.104]	.55[1.75]	-.071[-.49]	.57[1.96]
Δm_{t-1}	-.21[-1.41]	-.035[-.98]	-.076[-.625]	.005[.032]	.26[.82]
ecm_{t-1}^1	-.875[-2.93]	.11[1.59]	-.05[-.078]	.038[.124]	-.18[-.29]
ecm_{t-1}^2	-.427[-1.14]	-.32[-3.49]	.015[.017]	.979[2.5]	.54[.67]
ecm_{t-1}^3	-.219[-3.9]	-.002[-.13]	-.12[-.93]	.182[3.12]	-.11[-.93]
ecm_{t-1}^4	-.061[-1.65]	-.017[-1.96]	.023[.272]	.069[[1.8]	.046[.59]
ecm_{t-1}^5	.206[3.83]	.015[1.14]	-.076[-.625]	-.19[-3.33]	-.069[-.61]
Δm_t^*	.051[.38]	-.043[-1.31]	-.056[-.185]	-.07[-.52]	-.079[-.28]
Δm_t	-.04[-.29]	.08[2.19]	-.146[-.399]	-.02[-.126]	-.28[-.828]
$const$	-.0012[-.63]	.0003[.73]	.0004[.098]	-.0014[-.74]	2e^{-5}[$-$.83]

Note: sample 1979:3 -1998:2; t-statistics between brackets[56].

We have indicated with: $ecm_t^1 = [\widehat{r}_t]$, $ecm_t^2 = [\widehat{i}_t]$, $ecm_t^3 = [\widehat{c}_t]$, $ecm_t^4 = [-\widehat{p}_t + m_t - m_t^*]$, $ecm_t^5 = [s_t - \widehat{p}_t]$.

As we have seen in the previous chapter, the first three error correction terms do not literally express the equilibrium errors, they appear in the β matrix already as stationary variables. Since they are not related to other variables, the requirement of having a negative coefficient in each of the estimated error correction equation is not stringent. In fact, provided that they have a negative sign in their respective equations, the presence of a positive sign in other equations does not preclude the system from being stable.

Changes in the real interest rate differential are significantly related to past consumption growth differentials and to the three cointegrating vectors ecm_{t-1}^1,

[56]The estimate of the model containing $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^* are presented in Appendix F.

ecm^4_{t-1}, ecm^5_{t-1}, i.e. the past real interest rate differential, the real money differential and the real exchange rate. Changes in nominal interest rates are significantly related to: past inflation differential, past nominal interest rate differential, ecm^2_{t-1}, ecm^4_{t-1}, i.e. the past nominal interest rate differential and the real money differential, and to the current domestic money growth rate. In the equation for $\Delta\widehat{c}_t$ the only two significant explanatory variables are the change in the past real interest rate differential and the growth rate of the past foreign money supply. Changes in past real interest rates enter with a negative sign in the equation for $\Delta\widehat{c}_t$, this means that the intertemporal substitution effect between consumption decisions dominates. Interestingly, we obtain a positive sign for the elasticity of the UK consumption growth with respect to the EU money supply growth, which means that there are positive spillovers from the EU to the UK.

The inflation differential positively depends on the change in the past real interest rate differential. We can observe that the coefficient multiplying ecm^4_{t-1}, although very small, is positive, thus, it does not lead the system back to the equilibrium, after a shock. In the equation for the change in the nominal exchange rate the only two significant values are those multiplying past changes in the real interest rate differential and past changes in the foreign money supply.

The estimate of the exogenous variables gives:

Table 6.2: OLS estimates of the exogenous variables

	Δm_t^*	Δm_t
Δm_{t-1}^*	-.132[-1.11]	-.0065[-.06]
Δm_{t-1}	.085[.77]	.557[5.61]
$const$	-.00015[-.09]	-.0004[-.285]

Note: sample 1979:3 -1998:2; t-stat. between brackets.

We can observe that the EU money growth seems to be a white noise, thus the EU money supply behaves like a random walk. The UK money growth rate shows instead a positive and significant first order autocorrelation coefficient. The coefficient of the EU money growth does not significantly enter in the equation for the UK money growth. Thus, if we stop our analysis to the relative behaviour of money supplies it seems that monetary conditions in the Euro countries do not affect at all the UK money market. We have obtained a different result in the previous chapter, where we analysed the reciprocal behaviour of the UK and US economies. In that context we found that the US money supply is very important in determining the behaviour of the UK money supply, i.e. US money supply Granger-causes UK money supply whereas EU money supply does not Granger-cause UK money supply.

Finally, the estimate of the structural error correlation matrix leads to the following result:

	$\widehat{r}$	$\widehat{i}$	$\widehat{c}$	$\widehat{p}$	s	m	m^*
$\widehat{r}$	1.0	.99	-.95	-.99	-.97	0	0
$\widehat{i}$	.99	1.0	-.94	-.99	-.97	0	0
$\widehat{c}$	-.95	-.94	1.0	.94	.97	0	0
$\widehat{p}$	-.99	-.99	.94	1.0	.98	0	0
s	-.97	-.97	.97	.98	1.0	0	0
m	0	0	0	0	0	1.0	.19
m^*	0	0	0	0	0	.19	1.0

We can observe that the residuals of the endogenous variables are highly correlated among them, whereas the correlation among the residuals of the exogenous variables is very low.

6.3.2 Impulse Response Functions and Forecast Error Variance Decomposition

Figures 6.1 show the impulse response functions of the system to shocks in m^*(EU). They contain the results of both sets of estimation (which resulted consistent between them), where we remind that the first set contains $[c_t - c_t^*]$ and $[r_t - r_t^*]$, and the second $[y_t - y_t^*]$ and $[\pi_t - \pi_t^*]$.

EU money supply shocks improve the competitiveness of the UK economy. Although for a very small amount (7 basis points) the real interest rate differential becomes negative (i.e. $r_t < r_t^*$) after a year and tends to disappear after two years. The nominal interest rate differential results negative as well, but smoothed because of a lower inflation differential. A relatively less inflationary domestic currency is reflected in a short-run appreciation of the nominal exchange rate, that reaches its maximum after a year, with an improvement of six basis points.

A positive shock to $m^*(EU)$ not only is not inflationary for the UK, at least in the very short run, but it also positively affects consumption and output differential in favour of the UK economy.

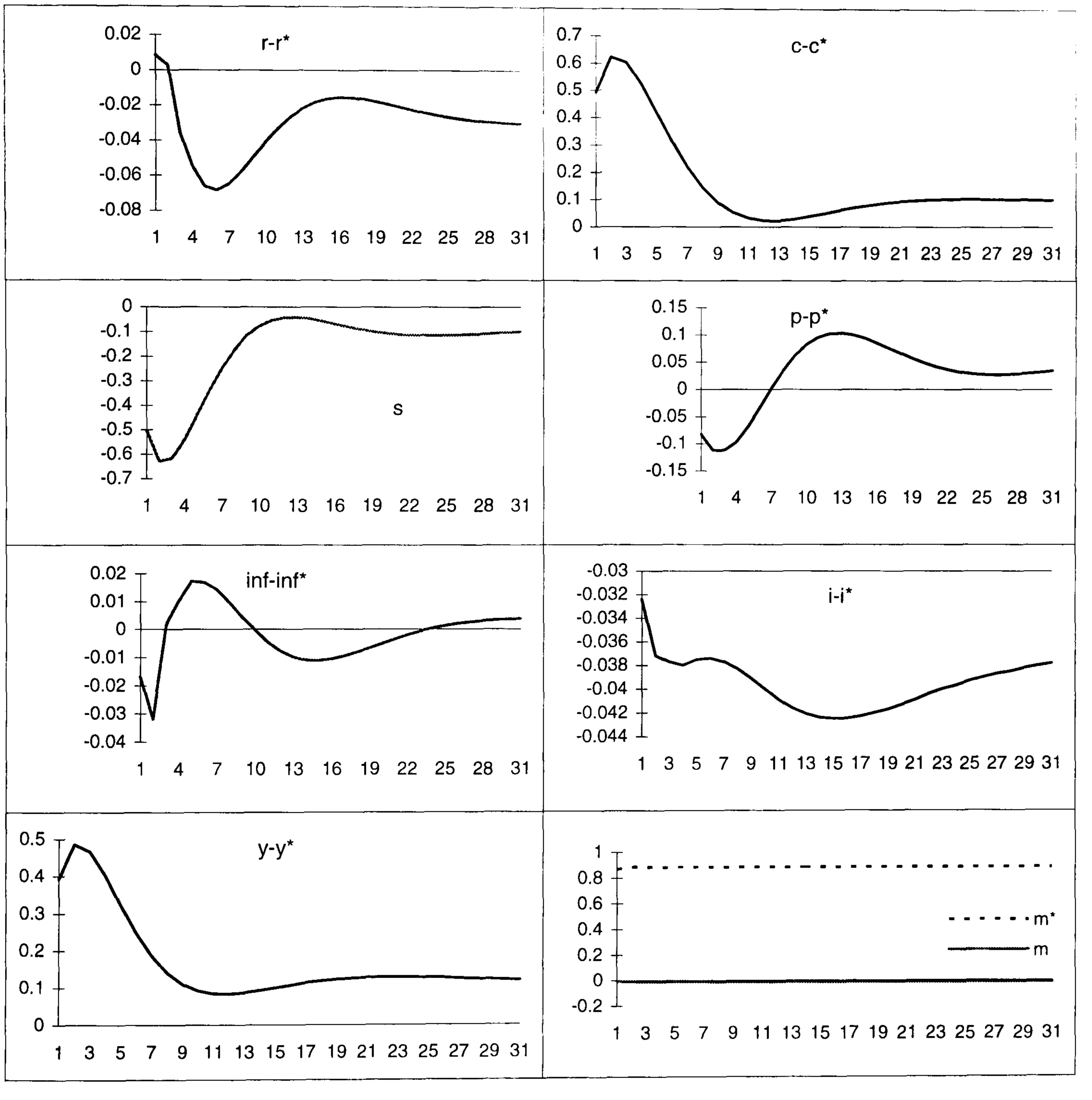
The UK seems to benefit from a positive spillover when the overall Euro area stimulates the economy by an impulse to the money supply.

In figures 6.2 we gathered the impulse response functions to a shock in $m(UK)$ corresponding to the two sets of estimation, as before. The general impression is that, in this case, a domestic monetary shock leads to a more inflationary outcome and to weaker real effects. If we observe the consumption differential we can notice that the positive effect of a domestic monetary expansion lasts a shorter period of time with respect to an expansionary monetary policy in the Euro-wide area. In the case of the income differential a domestic monetary expansion does not succeed in leading to any short-run benefit. We then observe a liquidity effect for the real interest rate but not for the nominal interest rate because of the contemporaneous increase of the domestic inflation with respect to the EU inflation.

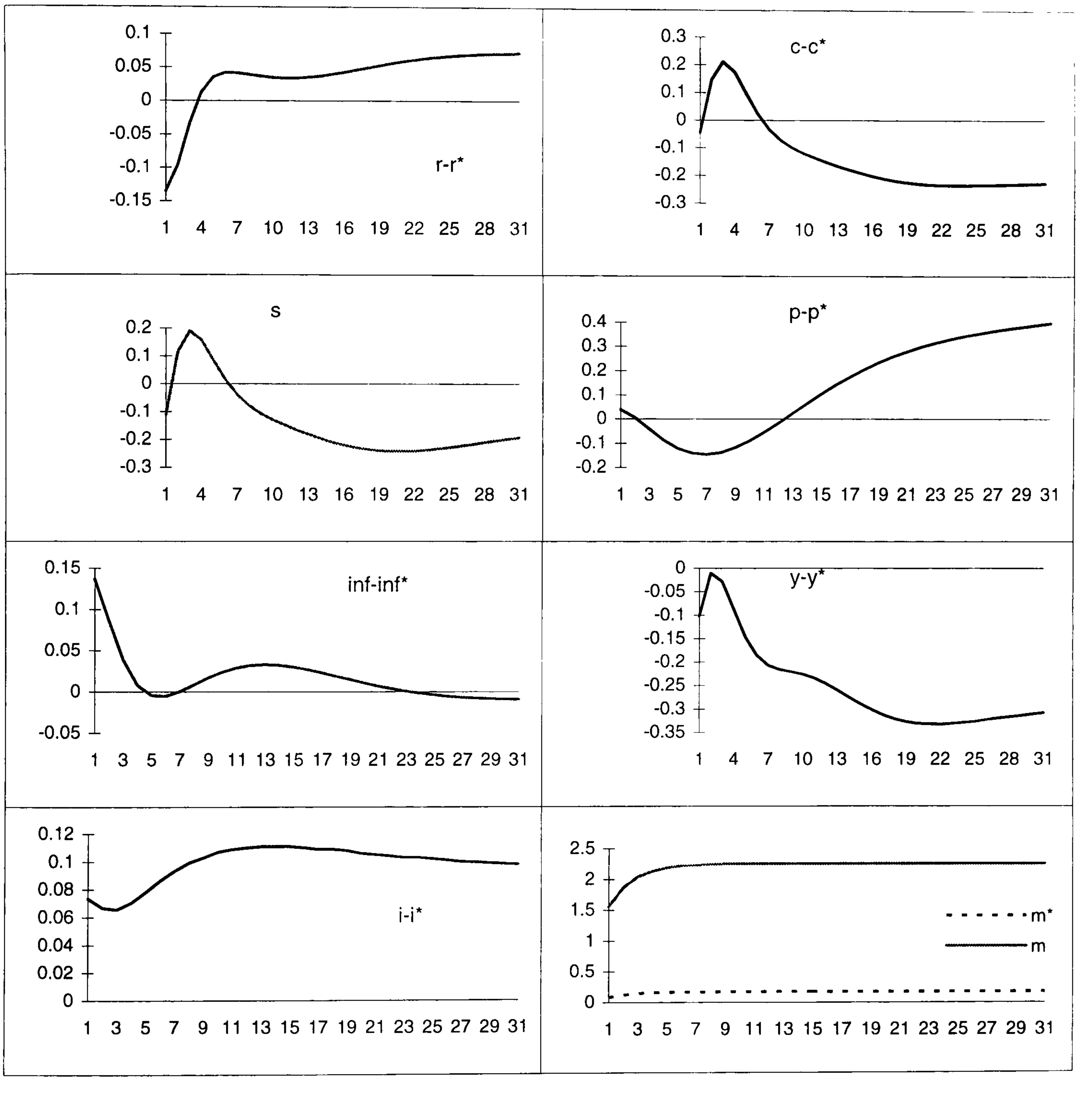
From our estimation it seems that the UK economy is better off after a monetary expansion coming from all the EU countries than after a domestic monetary expansion. Thus, a synchronized monetary policy seems to deliver better results than an autonomous monetary policy. Moreover, coming back to figures 6.1, we can observe that $m(UK)$ does not react at all to a shock on $m^*(EU)$. Thus, the effects of a shock on $m^*(EU)$ on the UK economy do not pass through the domestic money supply. This point has been already noticed before, when looking at the estimation results of the exogenous variables.

We can now emphasize the differences between this exercise and the one carried out in the previous chapter, where we analysed the UK and US economies. In that case the UK money supply resulted endogenous with respect to the US monetary policy. But a US monetary expansion led the UK economy to be worse off. Conversely, in this case we have seen that Euro-wide monetary shocks seem to deliver positive effects to the UK economy.

Figures 6.1: Impulse response functions to a shock in m* (EU)



Figures 6.2: Impulse response functions to a shock in m(UK)



Surely, all these results deserve further investigation. This chapter, therefore, proceeds with two other estimation exercises. The first studies the international monetary transmission mechanism between the EU and the US and the second reconsiders the UK against the US but with the domestic monetary policy determined by the EU countries.

Before carrying out the two estimations we comment on table 6.3, which shows the forecast error variance decomposition obtained with the first set of estimation.

Table 6.3: Forecast error variance decomposition

	$\widehat{r}$				$\widehat{i}$				$\widehat{c}$			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.23	.34	.001	.41	.008	.79	.04	.14	.43	.04	.46	.004
2	.27	.63	.002	.10	.011	.77	.06	.15	.22	.11	.58	.02
5	.21	.68	.024	.08	.013	.65	.09	.23	.11	.25	.55	.04
10	.19	.64	.061	.09	.008	.42	.11	.45	.09	.27	.53	.044
30	.15	.56	.072	.19	.003	.14	.13	.72	.08	.23	.40	.24

	$\widehat{p}$				s			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.013	.91	.009	.003	.50	.036	.40	.019
2	.016	.85	.018	.010	.29	.083	.56	.022
5	.016	.69	.033	.10	.14	.18	.58	.036
10	.009	.62	.022	.24	.13	.24	.53	.039
30	.009	.59	.035	.29	.09	.24	.41	.22

Note: sample 1979:3 -1998:2

The forecast error variance decomposition of each endogenous variable shows that both domestic and foreign monetary shocks give a limited contribution to the explanation of the variance of the real interest differential and the price differential, for the whole horizon considered. Domestic monetary shocks have, instead, an increasing explanatory power for the variance of the nominal interest rate differential, covering 70% of its total variance after 30 quarters. The forecast error variance decomposition of consumption differential and of the nominal exchange rate relies on foreign monetary shocks, that cover more than 50% of their total variance after 10 quarters.

6.4 The Monetary Transmission Mechanism between Two Big Economies: the EU and the US

6.4.1 Estimation Results

We study now the second pair of countries: the EU against the US. In this case we treat the EU as the domestic country. We use the previous model but, clearly, we are now dealing with two big economies. Like in the previous case we have carried out the estimation with the two sets of variables, the first contains $\widehat{c}_t$ $\widehat{r}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^* and the second $\widehat{y}_t$ $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^*. We report the results of the OLS estimates of the VECM containing the first set of variables[57].

Table 6.4: OLS estimates of the VECM containing $\widehat{c}_t$, $\widehat{r}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^*

	$\Delta\widehat{r}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{c}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{r}_{t-1}$	-.27[-2.58]	-.007[-1.1]	-.47[-1.24]	.24[2.46]	.46[1.26]
$\Delta\widehat{i}_{t-1}$	-2.29[-1.56]	.134[1.1]	9.06[1.32]	2.88[1.62]	-7.18[-1.1]
$\Delta\widehat{c}_{t-1}$	.24[1.24]	-.016[-1.24]	.29[.41]	-.27[-1.46]	-.293[-.43]
$\Delta\widehat{p}_{t-1}$	-.14[-.49]	.016[.87]	-.07[-.06]	1.16[4.23]	.29[.29]
Δs_{t-1}	.16[.75]	-.018[-1.3]	-.087[-.11]	-.189[-.95]	.066[.09]
Δm_{t-1}^*	.40[1.09]	-.013[-.54]	-1.48[-1.11]	-.39[-1.12]	1.26[.99]
Δm_{t-1}	-.038[-.32]	.01[1.41]	-.65[-1.48]	.04[.35]	.62[1.48]
ecm_{t-1}^1	-.869[-2.88]	.018[.93]	.09[.08]	.868[3.03]	-.006[-.0056]
ecm_{t-1}^2	4.4[2.19]	-.71[-5.42]	.31[.04]	-4.78[-2.51]	1.07[.15]
ecm_{t-1}^3	-.21[-4.0]	.012[3.6]	.017[.088]	.213[4.28]	-.082[-.45]
ecm_{t-1}^4	-.06[-1.88]	.005[2.47]	-.103[-.85]	.06[2.04]	.049[.43]
ecm_{t-1}^5	-.259[-3.64]	.013[2.89]	.22[.86]	.26[3.87]	-.31[-1.28]
Δm_t^*	-.19[-.52]	-.02[.99]	2.1[1.54]	.15[.438]	-2.23[-1.73]
Δm_t	-.26[-3.64]	.013[1.62]	-.62[-1.42]	.14[3.87]	.59[1.43]
$const$	.0004[-1.16]	1.6e^{-6}[.14]	-.002[-.33]	-.0006[-.4]	.0013[.22]

Note: sample 1979:3 -1998:2; t-statistics between brackets.

[57]The estimates of the VECM with the second set of variables are presented in Appendix F.

where $ecm_t^1 = [\widehat{r}_t]$, $ecm_t^2 = [\widehat{i}_t]$, $ecm_t^3 = [\widehat{c}_t]$, $ecm_t^4 = [-\widehat{p}_t + m_t - m_t^*]$, $ecm_t^5 = [s_t - \widehat{p}_t]$.

The equation for the change in the real interest differential shows a very complex dynamics. Almost all the explanatory variables are significant. On the contrary, in the case of the equation for the change in the nominal interest rate differential the only significant variables are the last four *ecm* terms. None of the explanatory variables significantly affects the change in consumption differential. The inflation differential positively depends on past inflation differentials and negatively on past nominal interest rate differentials. It is also positively related to past changes in consumption differentials and to contemporaneous changes in the domestic money supply. The only variable that appears to be significant in explaining changes in the nominal exchange rate is the contemporaneous change in foreign money supply, that enters with a negative sign.

The estimate of the exogenous variables gives:

Table 6.5: OLS estimates of the exogenous variables

	Δm_t^*	Δm_t
Δm_{t-1}^*	.656[7.13]	.165[.63]
Δm_{t-1}	-.032[-.77]	-.126[-1.07]
const	-.0003[-.48]	-.0001[-.06]

Note: sample 1979:3 -1998:2; t-stat. between brackets.

We have obtained a white noise behaviour for the growth rate of the EU money supply and a significant first order autocorrelation value for the growth rate of the US money supply. Spillover effects do not significantly contribute either in explaining EU or US money growth.

6.4.2 Impulse Response Functions and Forecast Error Variance Decomposition

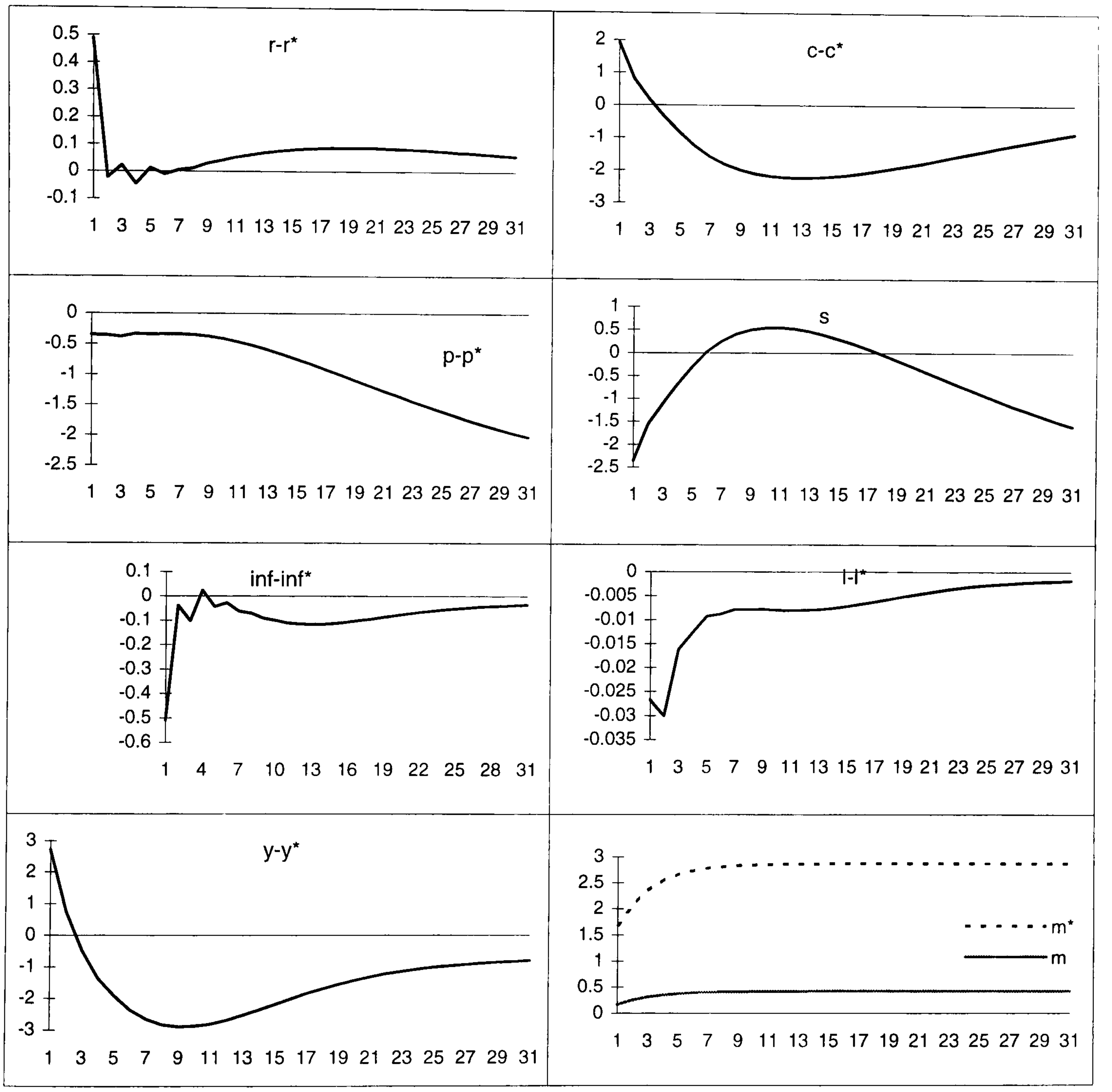
We now study the dynamic behaviour of the system by looking at the impulse response functions of the economy to exogenous monetary shocks. Starting from a shock to m^*(US) - figures 6.3 - we can observe an impact increase of the real interest rate differential (i.e. $r > r^*$) of about 40 basis points. The real interest rate differential returns to its steady-state level after a quarter. We, then, notice that the same shock leads to a decrease of the inflation differential (i.e. $\pi < \pi^*$) of about 45 basis points. As a result we obtain a decrease of the nominal interest rate differential (i.e. $i < i^*$) of about 3 basis points.

Although consumption and income differentials improve at impact, they became negative after a quarter and they adjust toward the previous steady state from below.

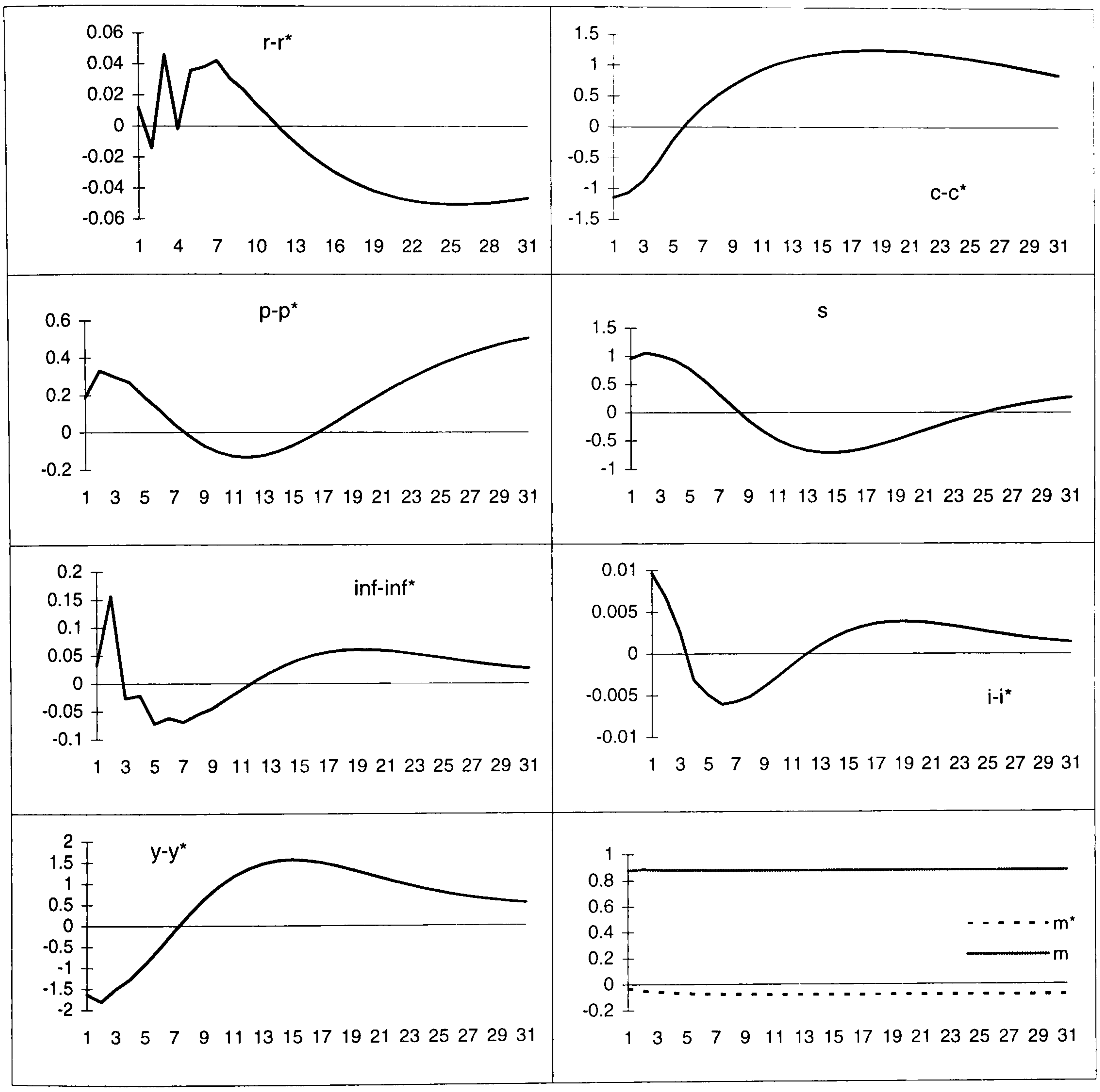
Figures 6.4 show the impulse response functions to a shock to $m(EU)$. In this case we observe a more instable reaction of the real interest rate differential. It jumps between positive and negative values for a year, it is positive in the second year and persistently negative thereafter. A domestic money supply shock leads to an immediate increase in domestic inflation relative to abroad and to a depreciated currency. Consumption and output differentials become positive only after a year.

Thus, there is some inertial behaviour of the system before observing a positive reaction of the real variables to a monetary shock.

Figures 6.3: Impulse response functions to a shock in m*(US)



Figures 6.4: Impulse response functions to a shock in m(EU)



The forecast error variance decomposition, presented on table 6.6 for the first set of estimates, reveals that foreign and domestic monetary shocks can explain very little of the total variability of the system. Their explanatory power improves for farther horizons. The major role in explaining the variability of all the variables is played by the nominal interest rate differential.

Table 6.6: Forecast error variance decomposition

	$\widehat{r}$				$\widehat{i}$				$\widehat{c}$			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.01	.93	.05	.00	.00	.99	.01	.00	.00	.87	.09	.03
2	.00	.97	.02	.00	.00	.98	.02	.00	.01	.83	.10	.06
5	.00	.97	.02	.00	.00	.98	.02	.00	.03	.88	.05	.03
10	.00	.97	.02	.00	.00	.98	.02	.00	.06	.73	.16	.04
30	.00	.96	.03	.00	.00	.97	.02	.00	.06	.43	.37	.13

	$\widehat{p}$				s			
horizon	ε_r	ε_i	ε_{m*}	ε_m	ε_r	ε_i	ε_{m*}	ε_m
1	.05	.93	.01	.00	.00	.68	.25	.06
2	.09	.87	.02	.00	.02	.61	.27	.09
5	.20	.74	.05	.00	.05	.80	.11	.04
10	.29	.61	.08	.01	.10	.75	.10	.04
30	.11	.20	.67	.02	.12	.62	.17	.08

Note: sample 1979:3 -1998:2

6.5 The Effect of having an 'European' Currency for the UK

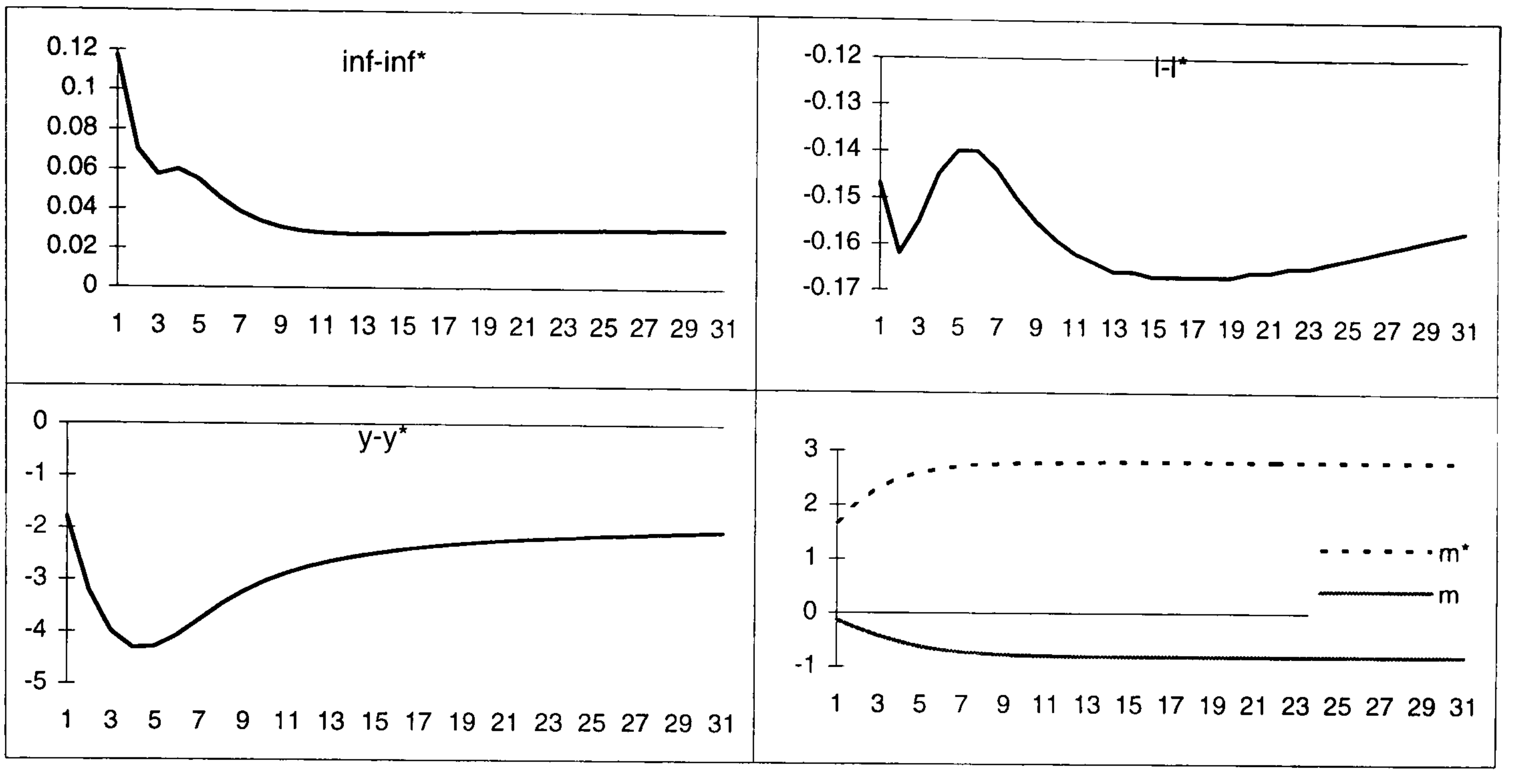
In this section we carry out the last estimation of the chapter. The experiment that follows has the objective of studying the 'hypothetical' monetary transmission mechanism between the UK and the US by using for the UK the money supply time-series of the European Union. The econometric exercise is interesting because it examines how a country (the UK) would have fared if its monetary policy had been dictated by another country (the Euro-wide area)[58]. In this sec-

[58]We are aware of the fact that the experiment can be read only in backward looking terms.

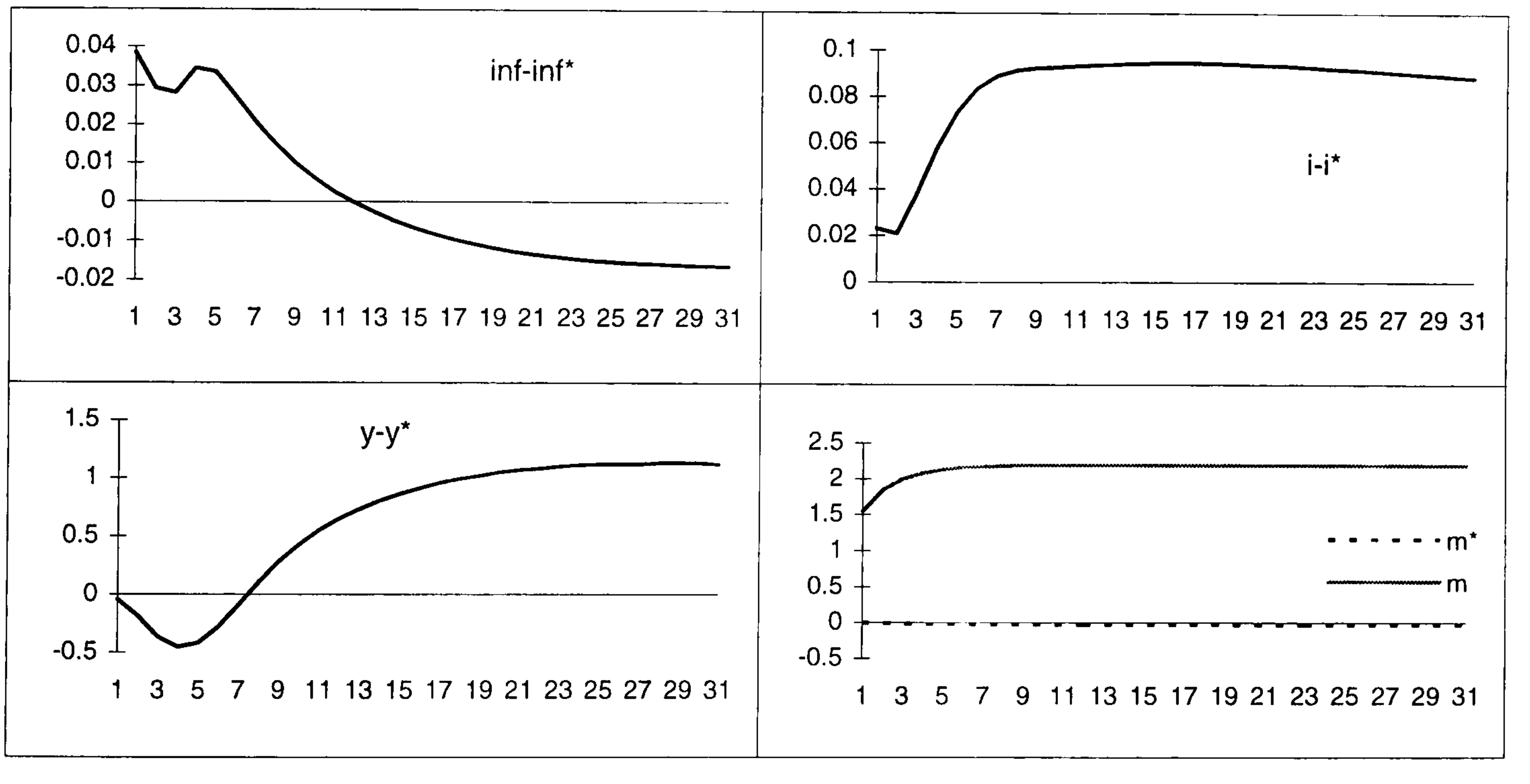
tion we do not report the estimations of the VECM (showed in Appendix F) and we concentrate our attention on the set of variables that involves $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^* in the two cases where the UK monetary policy is 'independent' (figures 6.5) and where the UK monetary policy is determined by the Euro countries (figures 6.6). To carry out the second experiment we replace the UK monetary aggregate by the EU monetary aggregate.

Starting from a shock on foreign money supply (US) the comparison of figures 6.5 and 6.6 (panel A) reveals that a EU-driven UK monetary policy leads to have an economic system overall more insulated from US monetary shocks. In this sense we have a clear benefit for the UK of having a monetary policy determined by the European Union. This benefit is measured in terms of less inflationary pressures derived from foreign shocks and of a lower output gap with respect to the US. But, when we turn to domestic monetary shocks it is not any more clear whether there exists a true benefit for the UK economy of giving up an independent monetary policy. In the two cases (figures 6.5 and 6.6 panel B) we observe an impact inflationary pressure that can explain the increase in the nominal interest rate differential and that is higher when the EU determines the UK monetary policy.

Figures 6.5: Impulse response functions of the VECM (UK-US)
Shock to m* (US)

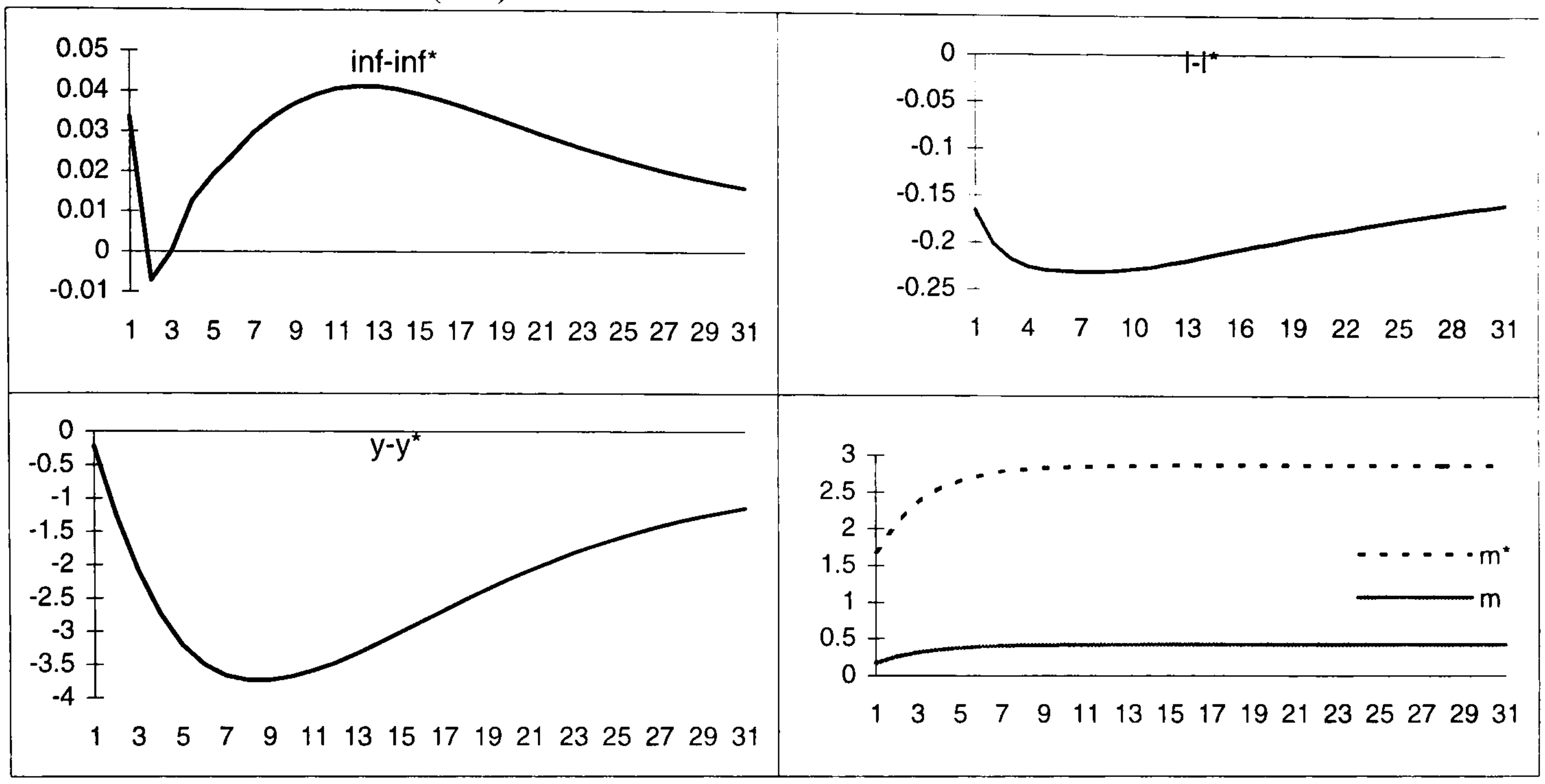


Shock to m (UK - UK)

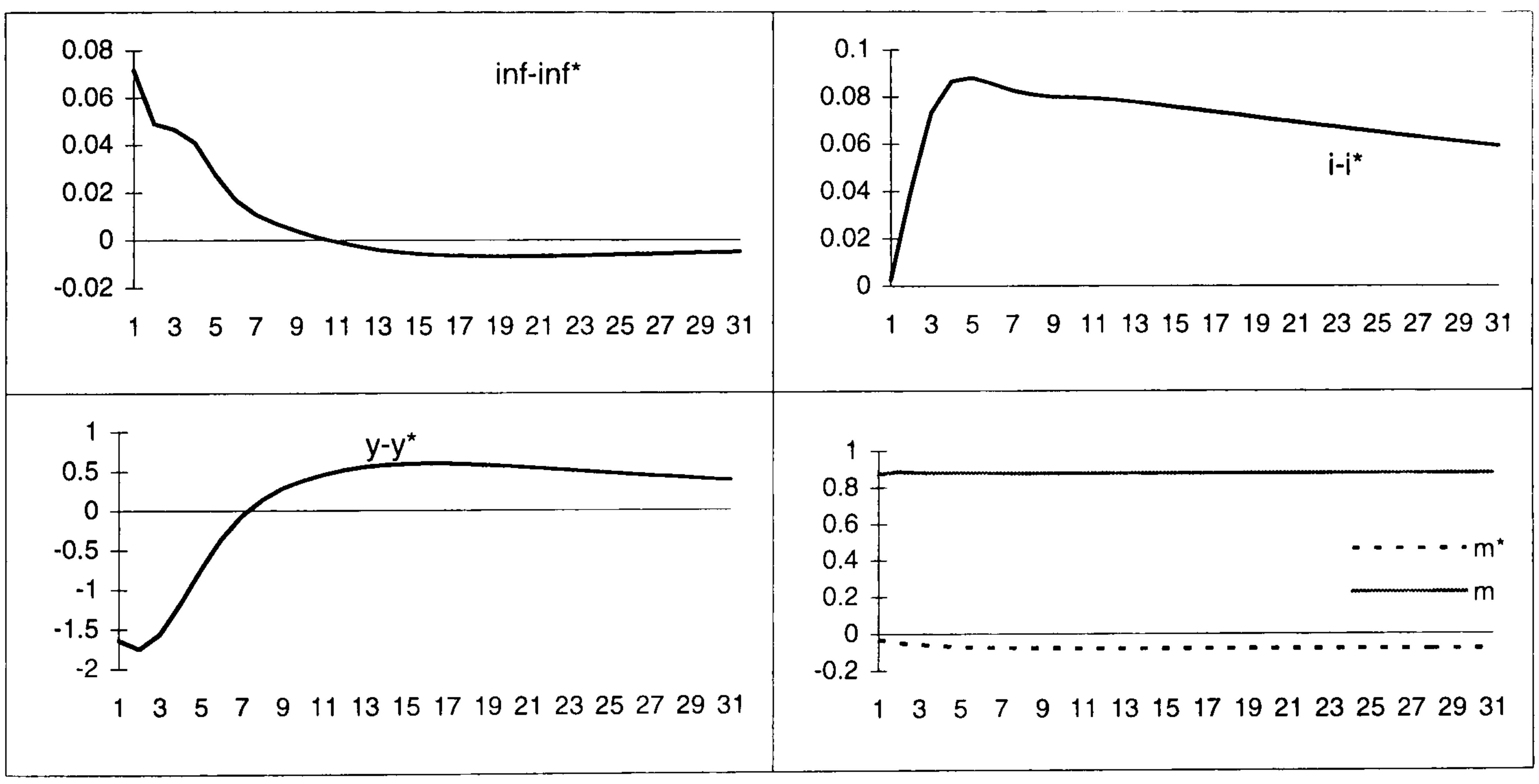


Figures 6.6: Impulse response functions of the VECM (UK/EU - US)

Shock to m* (US)



Shock to m (UK - EU)



6.6 Conclusions

In this chapter we have carried out three estimation exercises designed to better understanding of the effects of having a single monetary policy in the Euro-area for the international monetary transmission mechanism. The exercises suffer from the absence of a proper Euro-area data base, like many previous exercises on the EMU. We did not try to find an optimal monetary policy for the Euro countries and we did not estimate any hypothetical feedback rule for the ECB. We asked which advantages we could expect from having a new-bigger-country (the EU) in the determination of the monetary policy with respect to another big country, the US.

The estimation strategy consisted again of OLS estimates of a structural VECM, whose long-run restrictions are obtained from the model of the previous chapter. In the outline of the model, we took a step further by introducing inflation (equation 6.5) and by replacing consumption differential with income differential. This last action is a strategy commonly adopted (see McCallum, Nelson, 1997) but not very precise, at least in our context. Income differentials can replace consumption differentials only in the initial symmetric steady state. In fact, we have to take into account that in the budget constraint the income differential is equal to the consumption differential plus the reciprocal capital gains in foreign assets. Thus, the replacement that we have made holds just as a first approximation and in the neighbourhood of the initial symmetric steady state (where foreign bonds have been assumed to be zero).

The first exercise of this chapter has suggested that the UK could gain from entering the monetary union since there are positive effects from EU-monetary shocks on the UK real economy.

The second exercise investigated the international transmission mechanism between the EU and the US. The use of the money supply as a monetary instrument did not lead to any puzzling results in terms of liquidity effects. Dealing with two big countries enabled us to avoid the problem of domestic monetary policy dependence on the foreign monetary policy.

The third exercise compared the UK performance with respect to the US performance if the monetary policy were determined by the Euro countries. Although with some *caveat*, we found that the UK could benefit from entering the Monetary Union. Having a Monetary Union provides a more insulated system against foreign shocks (coming from the US) for the participating countries. We are reminded again that these results come from a backward-looking estimation that relied on data prior to the change in the monetary policy regime. We fully recognise the problem implicit in this strategy, that is that any forward looking interpretation is subject to the Lucas' critique.

7 The Two-Country Model with Preference and Productivity Shocks: Estimation of the Euler equations

7.1 Introduction

This chapter completes the theoretical framework presented in chapter 5. The model is extended in three directions: first, a money demand shock is derived. It is obtained by modelling a preference shock to the utility of real money balances. Second, we introduce leisure in the utility function. Third, we explicitly model a production function with a stochastic term describing the behaviour of the technology. These three complications complete the analysis of the monetary transmission mechanism and they allow to carry out optimal monetary policy exercises (in the last two chapters of the thesis). In fact, the presence of shocks different from that coming from the exogenous money supply combined with the suboptimality of outcomes in an imperfectly competitive world can justify policy intervention.

Many authors (Svensson (1998, 1999), Taylor (1993, 1999), Ball (1998), Haldane and Batini (1998)) have analysed the economic performance of models under alternative policy rules by working with models that are specified at the level of equilibrium conditions, i.e. that describe the relationships between various aggregate variables using a small set of log-linear equations.

The present analysis differs from those previous works because the model is specified at the level of preferences and technologies. The equations of the model, therefore, explicitly describe the optimizing behaviour of households and firms that populate the economy. As the model's parameters ultimately describe agent's preferences and technologies, they ought to be invariant with respect to changes in the monetary policy regime. Thus, there is the hope that the model is, in fact, truly structural and useful for policy evaluation.

This chapter serves as a bridge between the second and the third part of the thesis. We follow the same approach used in chapter 5 for studying the theoretical properties of the model. The major difference with our previous analysis relies on the measurement of the 'deep' parameters. In this chapter they are derived from the estimation of the Euler equations.

The structure of the chapter is the following. Section 7.2 derives the model by specifying households, firms and monetary authority behaviour in a world economy. Section 7.3 presents the steady-state solution and the log-linear approximation of the model. Section 7.4 describes the pricing mechanism. Section 7.5 shows the solution of the simulated model. Section 7.6 describes the estimation results of the model's structural parameters. We employed Instrumental Variables (IV) for the Euler equations and we used data on the UK and the average of the EU countries. Section 7.7 presents the simulations for different values of the parameter governing the adjustment of prices. It is shown how the findings of chapter 5 hold also in this more general context.

7.2 The Model

Although the model (in its demand-side specification) has already been presented in chapter 5, we need to reconsider it again. The model is essentially an extension of the Obstfeld and Rogoff's (1995) model to allow for Pricing-to-Market (PTM) and price stickiness in local currency. There are two countries, home and foreign.

We denote the foreign country variables with an asterisk. Residents of each country value consumption of the composite traded good produced, real money balances and leisure. The household purchases a composite good and supplies labour to his own firm. The composite good is made by a group of differentiated goods of total measure of unity. Of these goods, a fraction n is produced by the home country, and a fraction $1-n$ is produced in the foreign country. We also let n and $1-n$ represent the population of the home and foreign country respectively. Each good is sold exclusively by a price-setting firm. Firms in each country can price-discriminate across countries.

7.2.1 Households

A representative consumer in each country chooses consumption, real money balances and leisure to maximize expected lifetime utility, taking prices and wages as given:

$$U = E_t \sum_{s=0}^{\infty} \beta^s u\left(c_{t+s}, \frac{M_{t+s}}{P_{t+s}}, 1 - h_{t+s}\right) \tag{7.1}$$

$$U^* = E_t \sum_{s=0}^{\infty} \beta^s u^*\left(c^*_{t+s}, \frac{M^*_{t+s}}{P^*_{t+s}}, 1 - h^*_{t+s}\right) \tag{7.2}$$

where the period utility functions u, u^*, for home and foreign consumers, are specialized to have a constant relative risk aversion form (CRRA)[59]:

[59] $\sigma = -c\frac{u''_c}{u'_c}$ and $\varsigma = -\left(\frac{M}{P}\right)\frac{u''_{m/p}}{u'_{m/p}}$ measure the household's attitude toward risk. In a CRRA utility function they are constant and thus respectively independent of c and $\frac{M}{P}$. But they also measure the household's willingness to shift consumption between different periods. The smaller are σ and ς the more slowly marginal utility falls as consumption and real money balances rise, and so the more willing the household is to allow its consumption/real money balances to vary over time. Specifically, one can show that the elasticity of substitution between consumption/real money balances at any two points in time is $1/\sigma$, $1/\varsigma$. We assume that they are equal in the two countries.

$$u\left(c_t, \frac{M_t}{P_t}, 1-h_t\right) = \frac{1}{1-\sigma}c_t^{1-\sigma} + \frac{\gamma}{1-\varsigma}\left(\frac{M_t}{P_t}\right)^{1-\varsigma} e^{\chi_t} + \eta \log(1-h_t) \quad (7.3)$$

$$u^*\left(c_t^*, \frac{M_t^*}{P_t^*}, 1-h_t^*\right) = \frac{1}{1-\sigma}c_t^{*1-\sigma} + \frac{\gamma}{1-\varsigma}\left(\frac{M_t^*}{P_t^*}\right)^{1-\varsigma} e^{\chi_t^*} + \eta^* \log(1-h_t^*) \quad (7.4)$$

Here c_t represents a composite consumption good, i.e. $c = \left(\int_0^1 c(i)^{\frac{\rho}{\rho-1}} di\right)^{\frac{\rho}{\rho-1}}$, ρ is the elasticity of substitution between any two goods produced within a country, which we assume to be greater than one; so there is a fixed unit measure of differentiated goods, where $c(i)$ is the consumption of good i.

h_t represents total hours worked by the domestic household. They enter in the utility function with weights equals to η, η^* respectively. $\frac{M_t}{P_t}$ is the real money balance, whose weight is γ. P_t is the home country consumer price index, defined as

$$P_t = \left[\int_0^n p_t(i)^{1-\rho} di + \int_n^1 q_t^*(i)^{1-\rho} di\right]^{1/1-\rho} \quad (7.5)$$

where $p(i)$ is the home currency price of the home produced good, $q_t^*(i) = S_t p_t^*(i)$ is the domestic currency price of a foreign PTM good i sold in the home market (i.e. the domestic price of imported goods). P_t^* is then the foreign country consumer price index. We let S_t be the nominal exchange rate (price of foreign currency). Home-country agents own home-country firms (and similarly foreign-country agents). Each consumer receives a share of profit from every firm in which he owns shares. Moreover the home and foreign representative household choice of h_t, h_t^* must satisfy: $h_t = \int_0^n h_t(i)di$, $h_t^* = \int_{1-n}^1 h_t^*(i)di$ for all $t = 0, 1, 2..$

We allow for preference shocks e^{χ_t}, $e^{\chi_t^*}$ in both countries, which are translated into shocks to money demand. We clearly need to find a justification for the form assumed by these preference shocks. In fact, the introduction of real money balances in the utility function itself (Sidrauski, 1967) has been the object of many controversial positions. The original motivation relied on the fact that money helps in facilitating transactions of goods and services. This is like to say that preferences for having real balances in the utility function do not have an independent nature (because they are related to consumption decisions). This aspect could lead to think that they are more subject of being uncertain, given their dependence on spending decisions. This uncertainty in our model is additive with respect to real money balances. e^{χ_t} and $e^{\chi_t^*}$ follow an autoregressive process, that can be written in logarithmic form:

$$\chi_t = (1-\varphi_\chi)\chi + \varphi_\chi \chi_{t-1} + \varepsilon_t^\chi \tag{7.6}$$

$$\chi_t^* = (1-\varphi_{\chi^*})\chi^* + \varphi_{\chi^*} \chi_{t-1}^* + \varepsilon_t^{\chi^*} \tag{7.7}$$

with $\begin{bmatrix} \varepsilon_t^\chi \\ \varepsilon_t^{\chi^*} \end{bmatrix}$ $i.i.d. \sim (0, \begin{bmatrix} \sigma_{\varepsilon\chi}^2 & 0 \\ 0 & \sigma_{\varepsilon\chi^*}^2 \end{bmatrix})$. We do not allow for spillovers between domestic and foreign money demand shocks.

We concentrate on the domestic consumer since the foreign consumer will solve the same problem. The representative agent of the domestic country will pick consumption, money holdings, holdings of internationally traded bonds and labour to maximize his utility subject to the following budget constraint, which has been written in nominal terms:

$$P_t c_t + M_t + B_t = W_t h_t + \Pi_t + (1+i_{t-1})B_{t-1} + M_{t-1} + P_t \tau_t \tag{7.8}$$

Namely, we assume that households receive income from wages, $W_t h_t$, profits on their ownership of domestic firms, Π_t, and interests from bonds $(1+i_{t-1})B_{t-1}$. The nominal rate of interest is denoted by i_t, thus bonds purchased at time $t-1$ yields nominal return i_{t-1}. τ_t represents real transfers minus taxes. In real terms the budget constraint is:

$$c_t + \frac{M_t}{P_t} + b_t = \frac{W_t}{P_t} h_t + \frac{\Pi_t}{P_t} + (1 + r_{t-1}) b_{t-1} + \frac{M_{t-1}}{P_{t-1}} \frac{1}{1+\pi_t} + \tau_t \tag{7.9}$$

where π_t is the inflation rate from time $t-1$ to t and b_t is the real stock of domestic bonds $\left(\frac{B_t}{P_t}\right)$.

The optimal intratemporal consumption allocation between each differentiated good is given by:

$$c_t(i) = \left(\frac{v_t(i)}{P_t}\right)^{-\rho} c_t \tag{7.10}$$

where $v_t(i)$ is equal to either $p_t(i)$ or $q_t^*(i)$, depending upon which category the good i falls within.

In addition, the following first order conditions can be derived from the individual intertemporal consumer's decision problem:

$$E_t c_{t+1}^{\sigma} = c_t^{\sigma} \beta (1 + r_t) \tag{7.11}$$

$$\left(\frac{M_t}{P_t}\right)^{\varsigma} = \gamma c_t^{\sigma} \frac{1 + i_t}{i_t} e^{\chi_t} \tag{7.12}$$

$$\frac{\eta}{1 - h_t} = \frac{W_t}{P_t c_t^{\sigma}} \tag{7.13}$$

together with the budget constraint and the transversality condition:

$$\lim_{i \to \infty} \prod_{s=0}^{i} (1 + r_{t+s})^{-1} \left(b_{t+i} + \frac{M_{t+i}}{P_{t+i}}\right) = 0 \tag{7.14}$$

which prevents agents from borrowing without paying back. The representative agent of the foreign country will solve the same decision problem.

7.2.2 Firms

We assume that there is international trade in output, so that all goods are traded, but markets are segmented by country. Firm managers make production and pricing decision, they may price-to-market.

Let consider the home firm i, $i = 1, ..n$. It operates with a simple technology $y_t(i) = A_t h_t^{\theta}(i)$, where $\theta > 0$[60]. $h_t(i)$ are units of labour hired by each firm.

Symmetrically, the foreign firm i^*, $i^* = 1-n, ..1$, has a production function equal to $y_t^*(i^*) = A_t^* h_t^{*\theta}(i^*)$. Technology shocks A_t and A_t^* follow the vector autoregressive process:

$$\begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} 1-\varphi_A \\ 1-\varphi_A^* \end{bmatrix} \begin{bmatrix} \log A \\ \log A^* \end{bmatrix} + \begin{bmatrix} \varphi_A & \varphi_{AA^*} \\ \varphi_{AA^*} & \varphi_{A^*} \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{A_t} \\ \varepsilon_{A_t^*} \end{bmatrix} \tag{7.15}$$

where $-1 \leq \varphi_A, \varphi_A^* \leq 1$ and we assume that the two shocks $\begin{bmatrix} \varepsilon_{A_t} \\ \varepsilon_{A_t^*} \end{bmatrix}$ are *i.i.d.* $\sim (0, \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_{A^*}^2 \end{bmatrix})$.

We now concentrate on the decision problem of the domestic firm i. We divide total output produced by each firm between a fraction x sold domestically, given by $xc_t(i)$, and a fraction $1-x$ sold abroad, given by $(1-x)c_t^*(i)$. The firm hires labour domestically and chooses $p(i)$ and $p_f(i)$, the nominal price for the home and foreign market (both expressed in domestic currency), respectively. Profits of the PTM firm are:

$$\Pi_t(i) = p_t(i)xc_t(i) + p_{ft}(i)(1-x)c_t^*(i) - W_t \left(\frac{xc_t(i) + (1-x)c_t^*(i)}{A_t}\right)^{\frac{1}{\theta}} \tag{7.16}$$

where $p_{ft}(i)$ is the domestic currency price of a good sold in the foreign market. Thus the intertemporal problem of each firm is given by:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \frac{\Pi_{t+s}(i)}{P_{t+s}} \tag{7.17}$$

[60] i.e. we are simply ruling out the possibility that output is exogenously determined, otherwise we would come back to the results obtained in chapter 5.

The firm sets $p_t(i)$ and $p_{ft}(i)$ separately to maximize profits. It faces the demand schedules of the domestic and foreign consumers which are given by $c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\rho} c_t$ and $c_t^*(i) = \left(\frac{p_{ft}(i)/S_t}{P_t^*}\right)^{-\rho} c_t^*$. We can explicitly write down the problem by substituting in (7.16) the domestic and foreign demand schedules:

$$\max_{\{p(i),\, p_f(i)\}} E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{P_{t+s}} \begin{array}{l} \{p_{t+s}(i)x\left(\frac{p_{t+s}(i)}{P_{t+s}}\right)^{-\rho} c_{t+s} + p_{ft+s}(i)(1-x)\left(\frac{p_{ft+s}(i)/S_{t+s}}{P_{t+s}^*}\right)^{-\rho} c_{t+s}^* \\ -W_{t+s}(\frac{1}{A_t})^{\frac{1}{\theta}}\left(x\left(\frac{p_{t+s}(i)}{P_{t+s}}\right)^{-\rho} c_{t+s} + (1-x)\left(\frac{p_{ft+s}(i)/S_{t+s}}{P_{t+s}^*}\right)^{-\rho} c_{t+s}^*\right)^{\frac{1}{\theta}} \} \end{array} \tag{7.18}$$

We then maximize with respect to $p_t(i)$ and $p_{ft}(i)$ whose first order conditions for $s = 0$ are:

$$\frac{\partial \Pi}{\partial p_t(i)} = 0 \Rightarrow p_t(i) = \frac{\rho}{\rho-1} W_t \frac{1}{\theta}\frac{h_t(i)}{y_t(i)} A_t^{\frac{\theta-1}{\theta}} \tag{7.19}$$

$$\frac{\partial \Pi}{\partial p_{ft}(i)} = 0 \Rightarrow p_{ft}(i) = \frac{\rho}{\rho-1} W_t \frac{1}{\theta}\frac{h_t(i)}{y_t(i)} A_t^{\frac{\theta-1}{\theta}} \tag{7.20}$$

Thus, we obtain $p_t(i) = p_{ft}(i)$. Since elasticities of demand are the same in each market, the firm will set the same price in both countries, therefore the law of one price holds with flexible prices even when there is perfect PTM. We can rewrite the previous expression in this way

$$p_t(i) = p_{ft}(i) = \mu MC_t(i) A_t^{\frac{\theta-1}{\theta}} \tag{7.21}$$

where $\mu = \frac{\rho}{\rho-1}$ is the price-cost mark-up for a domestic firm selling either in the domestic or the foreign market and $MC(i) = W_t \frac{h_t(i)}{\theta y_t(i)}$ is the nominal marginal cost given by the ratio between the wage rate and the labour productivity. A similar expression holds for the foreign firm:

$$p_t^*(i) = p_{ft}^*(i) = \mu MC_t^*(i) A_t^{*\frac{\theta-1}{\theta}} \tag{7.22}$$

where $p_t^*(i)$ and $p_{ft}^*(i)$ are the foreign nominal price of a good sold in the home and foreign country (both expressed in foreign currency).

7.2.3 The World Economy

We now define world real consumption c^W in terms of the domestic consumer price index. It is equal to the weighted sum of domestic and foreign consumption, where weights are given by the fraction of the country population n, $1-n$.

$$c_t^W \equiv nc_t + (1-n)\frac{S_t P_t^*}{P_t} c_t^* \tag{7.23}$$

In the aggregate global equilibrium, the domestic nominal money supply must equal domestic nominal money demand in each country, and global net foreign assets must be zero. In nominal terms (and in terms of the domestic consumer price index) the bond-market clearing condition is:

$$nB_t + (1-n)S_t B_t^* = 0 \tag{7.24}$$

and in real terms is:

$$nb_t + (1-n)\frac{S_t P_t^*}{P_t} b_t^* = 0 \tag{7.25}$$

Given the assets market clearing condition we can derive the aggregate global goods market clearing condition. By taking a population-weighted average of the budget constraint (7.9) across home and foreign agents, by imposing condition (7.25) and the home and foreign budget constraint we obtain:

$$c_t^W \equiv nc_t + (1-n)\frac{S_t P_t^*}{P_t} c_t^* = ny_t + (1-n)\frac{S_t P_t^*}{P_t} y_t^* \equiv y_t^W \tag{7.26}$$

Equation (7.26) states simply that world real consumption equals world real income.

7.2.4 The Monetary Authority

The Monetary Authority in each country manages nominal money supply M_t, M_t^* by making lump sum transfers (minus taxes) τ_t, τ_t^* to the representative household such that: $P_t\tau_t = M_t - M_{t-1}$ and $P_t^*\tau_t^* = M_t^* - M_{t-1}^*$.

We assume that in conducting its operations, each monetary authority uses a simple policy rule where the current value of the domestic money supply depends on its past and on the past foreign money supply decisions. Thus, the VAR representation of the money supply is given by:

$$\begin{bmatrix} \log M_t \\ \log M_t^* \end{bmatrix} = \begin{bmatrix} 1-\varphi_M \\ 1-\varphi_{M^*} \end{bmatrix} \begin{bmatrix} \log M \\ \log M^* \end{bmatrix} + \begin{bmatrix} \varphi_M & \varphi_{MM^*} \\ \varphi_{MM^*} & \varphi_{M^*} \end{bmatrix} \begin{bmatrix} \log M_{t-1} \\ \log M_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^{M^*} \end{bmatrix} \tag{7.27}$$

where we assume that $\begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^{M^*} \end{bmatrix}$ are *i.i.d.* $\sim \left(0, \begin{bmatrix} \sigma_M^2 & 0 \\ 0 & \sigma_{M^*}^2 \end{bmatrix}\right)$.

7.3 The Steady State

For obtaining the solution to the problem (i.e. the final form of each control variable in terms of the state variables) we log-linearize the model. In equilibrium most of the model's real variables inherit a deterministic trend from the constant rate of technological progress. Therefore, first of all, we consider a linearly detrended economy, whose steady-state solution is stationary. Since consumption and output are constant in steady state, from the consumption Euler equation (7.10) we obtain the equilibrium value of the real interest rate:

$$r^* = r = \frac{1-\beta}{\beta} \tag{7.28}$$

Steady-state consumption must equal steady-state real income in both countries, so that:

$$c = rb + y \tag{7.29}$$

$$c^* = -\left(\frac{n}{1-n}\right) rb \frac{P}{SP^*} + y^* \tag{7.30}$$

where the corresponding foreign condition is derived by expressing b^* in term of b by using the equilibrium condition (7.25). Steady-state money demand must equal steady-state money supply (for the moment taken simply as a constant) in both countries[61]:

$$\left(\frac{\overline{M}}{P}\right)^{\varsigma} = \gamma c^{\sigma} \frac{1+i}{i} \tag{7.31}$$

$$\left(\frac{\overline{M}^{*}}{P^{*}}\right)^{\varsigma} = \gamma^{*} c^{*\sigma} \frac{1+i^{*}}{i^{*}} \tag{7.32}$$

In the symmetric equilibrium we have $p_t(i) = P_t$, $p_t^*(i) = P_t^*$ for all $i \in [0,1]$ and $t = 0, 1, 2...$. For the labour market of the two countries we then obtain the steady-state condition after having equalized labour demand and supply:

$$\frac{h^{1-\theta}}{1-h} = \frac{\theta}{\eta} \frac{\rho - 1}{\rho} \frac{A^{1/\theta}}{c^{\sigma}} \tag{7.33}$$

$$\frac{h^{*1-\theta^{*}}}{1-h^{*}} = \frac{\theta^{*}}{\eta^{*}} \frac{\rho - 1}{\rho} \frac{A^{*1/\theta}}{c^{*\sigma}} \tag{7.34}$$

By Walras law, in equilibrium we need to consider just two markets out of the three.

As in chapter 5 we further assume that at the outset $B_0^* = 0$. This condition allows us to obtain a symmetric steady state. Namely, we have $c_0 = c_0^* = y_0 = y_0^*$ and $\frac{SP^*}{P} = 1$. We can now log-linearize the model around the symmetric steady state we have just characterized.

7.3.1 The Log-linear Approximation

As in chapter 5, we develop the log-linear version of all the model's equilibrium conditions[62].

[61] Imposing the no speculative bubbles assumption and zero money and consumption growth steady-state nominal interest rate equals the steady-state real interest rate.

[62] As in chapter 5, in the log-linearized model low case letters indicate the log-deviations from the steady state, which should be distinguished from the previous notation where low case letters have been used to indicate real variables.

We start from the definition of the uncovered interest rate parity, which, expressed in log-linear form, says that the nominal interest rate differential equals the expected depreciation of the nominal exchange rate:

$$i_t = i_t^* + E_t(s_{t+1} - s_t) \tag{7.35}$$

The log-linear approximation of the world goods market equilibrium condition is:

$$nc_t + (1-n)c_t^* = ny_t + (1-n)y_t^* \tag{7.36}$$

where, since $b_0^* = 0$ in the initial symmetric steady state we log-linearize b_t and b_t^* around the initial home consumption steady-state level (c_0). Thus the log-linear expression for the bonds market clearing condition is:

$$nb_t + (1-n)(b_t^* + s_t + p_t^* - p_t) = 0 \tag{7.37}$$

and those for the home and foreign budget constraints are:

$$c_t + b_t = y_t + (1+r)(r_{t-1} + b_{t-1}) \tag{7.38}$$

$$c_t^* + b_t^* = y_t^* + (1+r)(r_{t-1}^* + b_{t-1}^*) \tag{7.39}$$

We can now combine equations (7.36) and (7.39) to obtain, after some algebraical manipulations, the following expression:

$$\overline{c}(c_t - c_t^*) + \frac{1}{n}(1+\overline{r})(r_{t-1} + b_{t-1}^*) - \frac{1}{n}b_t^* = y_t - y_t^* \tag{7.40}$$

This equation expresses the world log-linear equilibrium condition. It is the only equation where the relative country size plays a role.

The log-linear consumption Euler equations in the two countries take the following form:

$$\sigma E_t c_{t+1} = \sigma c_t + r_t \tag{7.41}$$

$$\sigma E_t c^*_{t+1} = \sigma c^*_t + r^*_t \tag{7.42}$$

so that the consumption differential is equal to:

$$\sigma E_t (c_{t+1} - c^*_{t+1}) = \sigma (c_t - c^*_t) + (r_t - r^*_t) \tag{7.43}$$

The log-linear versions of the money demand equations are:

$$m_t - p_t = \frac{\sigma}{\varsigma} c_t - \frac{\nu}{\varsigma} i_t + \frac{\sigma}{\varsigma} \chi_t \tag{7.44}$$

$$m^*_t - p^*_t = \frac{\sigma}{\varsigma} c^*_t - \frac{\nu^*}{\varsigma} i^*_t + \frac{\sigma}{\varsigma} \chi^*_t \tag{7.45}$$

where $\nu = \frac{1}{(1+\bar{i})\bar{i}}$, $\nu^* = \frac{1}{(1+\bar{i}^*)\bar{i}^*}$. Our symmetric steady state implies that $\bar{i}^* = \bar{i}$ and thus $\nu = \nu^*$. We can now subtract equation (7.44) from equation (7.45) and by using the uncovered interest parity condition we obtain:

$$m_t - m^*_t - (p_t - p^*_t) = \frac{\sigma}{\varsigma}(c_t - c^*_t) - \frac{\nu}{\varsigma} E_t (s_{t+1} - s_t) + \frac{\sigma}{\varsigma}(\chi_t - \chi^*_t) \tag{7.46}$$

We can easily obtain a relationship between the real exchange rate and national interest rate differentials, which becomes clearer if we add s_t to both sides of (7.46):

$$m_t - m^*_t + (s_t - (p_t - p^*_t)) = \frac{\sigma}{\varsigma}(c_t - c^*_t) - \frac{\nu}{\varsigma}(i_t - i^*_t) + s_t + \frac{\sigma}{\varsigma}(\chi_t - \chi^*_t) \tag{7.47}$$

The log-linear equations for the labour market in the two countries are:

$$h_t = \frac{h^{1-\theta}}{\Lambda} \frac{1}{\theta} a_t - \frac{h^{1-\theta}}{\Lambda} \sigma c_t \tag{7.48}$$

$$h^*_t = \frac{h^{1-\theta}}{\Lambda} \frac{1}{\theta} a^*_t - \frac{h^{1-\theta}}{\Lambda} \sigma c^*_t \tag{7.49}$$

where $\Lambda = \frac{(1-h)(1-\theta)+h^{2-\theta}}{1-h}$ and we have assumed that hours worked in steady state are equals in the two countries. By taking the difference we obtain:

$$h_t - h^*_t = \frac{h^{1-\theta}}{\Lambda} \frac{1}{\theta}(a_t - a^*_t) - \frac{h^{1-\theta}}{\Lambda} \sigma (c_t - c^*_t) \tag{7.50}$$

this last equation can be rewritten in terms of output differentials by using the relationship between hours of work and output given by the production functions of the two countries:

$$y_t - y_t^* = \Gamma_0(a_t - a_t^*) - \Gamma_1(c_t - c_t^*) \tag{7.51}$$

where $\Gamma_0 = \left[\frac{h^{1-\theta}}{\Lambda} + 1\right]$ and $\Gamma_1 = \theta\frac{h^{1-\theta}}{\Lambda}\sigma$. This is the only equation where technological (supply side) considerations play a role. The equation makes clear that there exists a trade-off between output demand determined, on the one hand, and supply determined, on the other hand. The degree of this trade-off is given by the parameter Γ_1 which depends on the degree of returns to scale, on the equilibrium level of hours of work and on the intertemporal substitution between different consumption levels. This trade-off disappears only if we assume $h_t = 1$, i.e. fixed labour supply. Thus, the analysis carried out in the previous chapter is still valid in a model where the production side is explicitly considered, but only if leisure time does not enter the utility function. Without this assumption, the previous equation suggests that it is not very precise to approximate output decisions with consumption decisions.

In the following analysis we take a_t and a_t^* as a measure of potential output. Thus, the differences $(y_t - a_t)$ and $(y_t^* - a_t^*)$ express domestic and foreign output gap. This implies a rather peculiar interpretation and use of the production function. Namely, we are interested in knowing the supply-side mechanism because it measures how much hours of work are needed to fill the gap between actual (y) and potential (a) output.

7.4 The Pricing Mechanism

This section corresponds to that of chapter 5. We use Calvo's (1983) approach to formalize the assumption that the domestic consumer price index cannot adjust immediately after a monetary shock. Calvo's (1983) framework assumes that individual firms set prices for some uncertain time interval and face a constant probability of price adjustment thereafter. Thus, we assume that each individual firm has a probability $(1-\alpha)$ of changing its price in any given period. The firm adjusting its price will set $\widetilde{p}_t$ so as to equate the price with the discounted expected marginal costs, adjusted for a mark-up. Price dynamics in the home country may then be described by two equations in the price level p_t, and the new price set by the adjusting firms, $\widetilde{p}_t$. These equations are:

$$p_t = (1-\alpha)\widetilde{p}_t + \alpha p_{t-1} \tag{7.52}$$

$$\widetilde{p}_t = (1-\beta\alpha)[n(p_t + mc_t + (1-\frac{1}{\theta})a_t) + (1-n)(p_t^* + s_t + mc_t^* + (1-\frac{1}{\theta})a_t^*] + \beta\alpha E_t\widetilde{p}_{t+1} \tag{7.53}$$

Equation (7.52) shows the relation between the home price index and the rigidity parameter α.

Equation (7.53) shows how domestic firms determine their price when they can adjust it[63]. Given the assumption of a composite traded good, the home price level $\widetilde{p}_t$ contains the price of both, home and imported goods. Home good firms adjust their price so as to equate the discounted value of the nominal expected marginal cost, which is $p_t + mc_t + (1-\frac{1}{\theta})a_t$, where the notation mc_t denotes the real marginal cost. This expected cost depends also on the technological parameter as long as we allow for $\theta \neq 1$, i.e. we leave out the possibility of

[63]See notes 43 and 44 in chapter 5 for the derivation of (7.53).

having constant return to scale[64]. Symmetrically, the foreign firm selling in the home market will set its price based on expected marginal cost, which, in home currency terms, is $p_t^* + s_t + mc_t^* + (1 - \frac{1}{\theta})a_t^*$. Thus, $\widetilde{p}_t$ in the home country is the weighted average of these two prices, which gives equation (7.53).

Symmetrically, in the foreign country p_t^* and $\widetilde{p}_t^*$ are given by:

$$p_t^* = (1-\alpha)\widetilde{p}_t^* + \alpha p_{t-1}^* \tag{7.54}$$

$$\widetilde{p}_t^* = (1-\beta\alpha)[n(p_t + mc_t - s_t + (1-\frac{1}{\theta})a_t) + (1-n)(p_t^* + mc_t^* + (1-\frac{1}{\theta})a_t^*)] + \beta\alpha E_t \widetilde{p}_{t+1}^* \tag{7.55}$$

Taking the difference between the equations for the home and the foreign economies we obtain:

$$p_t - p_t^* = (1-\alpha)(\widetilde{p}_t - \widetilde{p}_t^*) + \alpha(p_{t-1} - p_{t-1}^*) \tag{7.56}$$

$$\widetilde{p}_t - \widetilde{p}_t^* = (1-\beta\alpha)s_t + \beta\alpha E_t(\widetilde{p}_{t+1} - \widetilde{p}_{t+1}^*) \tag{7.57}$$

In section 7.2.2 we have seen that, with identical preferences across countries, even PTM firms will optimally select home and foreign currency prices as a constant mark-up over marginal costs, and hence the law of one price will be satisfied ex-ante (i.e. in equilibrium). In the event of a shock, however, prices that are sticky in each local currency imply that exchange rate movements will cause ex-post deviations from the law of one price.

We solve equation (7.56) for $\widehat{\widetilde{p}}_t = \widetilde{p}_t - \widetilde{p}_t^*$, we take expectation of the same equation one period ahead and we solve it for $E_t\widehat{\widetilde{p}}_{t+1}$. We finally substitute the

[64]Most of the literature which relies on monopolistic competition usually assumes for each firm a costant return to scale (CRS) production function. Under this assumption, one is building a peculiar enviroment where price decisions are made on the basis of a mark-up over nominal wages on the one hand, and where firms face the impossibilty (because of the CRS assumption) of having a well defined profit maximization solution in any period of time, on the other hand.

two expressions for $\widehat{\widetilde{p}}_t$ and $E_t\widehat{\widetilde{p}}_{t+1}$ in equation (7.57) to obtain the following:

$$\widehat{p}_t(1+\beta\alpha^2) - \alpha\widehat{p}_{t-1} - (1-\beta\alpha)(1-\alpha)s_t - \beta\alpha E_t\widehat{p}_{t+1} = 0$$

where $\widehat{p}_t = p_t - p_t^*$.

As all the other log-linear equations, this last equation has a short-run dimension. It expresses the log-deviation from the equilibrium characterized by PPP. Only in the short run, thus, the model allows the real exchange rate to fluctuate.

7.5 The Solution to the Model

As in chapter 5 we study the dynamic solution obtained by taking the country differential of each variable. We rewrite the previous system of equations in a more compact form:

$$E_t\widehat{c}_{t+1} = \widehat{c}_t + \frac{1}{\sigma}\widehat{r}_t \tag{7.58}$$

$$m_t - m_t^* - \widehat{p}_t = \frac{\sigma}{\varsigma}\widehat{c}_t - \frac{\nu}{\varsigma}\widehat{i}_t + \frac{\sigma}{\varsigma}\widehat{\chi}_t \tag{7.59}$$

$$\widehat{p}_t(1+\beta\alpha^2) - \alpha\widehat{p}_{t-1} - (1-\beta\alpha)(1-\alpha)s_t - \beta\alpha E_t\widehat{p}_{t+1} = 0 \tag{7.60}$$

$$\widehat{i}_t = E_t(s_{t+1} - s_t) \tag{7.61}$$

$$\widehat{y}_t = \Gamma_0\widehat{a}_t - \Gamma_1\widehat{c}_t \tag{7.62}$$

$$\widehat{c}_t + \frac{1}{n}(1+\overline{r})(r_{t-1}^* + b_{t-1}^*) - \frac{1}{n}b_t^* = \widehat{y}_t \tag{7.63}$$

$$rer_t = s_t - \widehat{p}_t \tag{7.64}$$

where $\widehat{c}_t = c_t - c_t^*$, $\widehat{p}_t = p_t - p_t^*$ $\widehat{r}_t = r_t - r_t^*$, $\widehat{i}_t = i_t - i_t^*$, $\widehat{y}_t = y_t - y_t^*$, $\widehat{a}_t = a_t - a_t^*$ $\widehat{\chi}_t = \chi_t - \chi_t^*$.

We have a system composed by two parts. The first consists of seven endogenous variables: $\widehat{y}$, $\widehat{c}_t$, $\widehat{p}_t$, $\widehat{r}_t$. $\widehat{i}_t$, s_t, rer_t, in seven equations (7.58)-(7.64); the second consists of six exogenous variables: a_t, a_t^*, m_t, m_t^*, χ_t, χ_t^* in six equations that we rewrite in the following compact first order VAR form:

$$Z_t = (I - \Phi_Z)Z + \Phi_Z Z_{t-1} + \varepsilon_{Zt} \tag{7.65}$$

with $Z_t' = [a_t \; a_t^* \; m_t \; m_t^* \; \chi_t \; \chi_t^*]$ and where:

$$\Phi_Z = \begin{bmatrix} \varphi_a & \varphi_{a^*a} & 0 & 0 & 0 & 0 \\ \varphi_{aa^*} & \varphi_{a^*} & 0 & 0 & 0 & 0 \\ 0 & 0 & \varphi_m & \varphi_{m^*m} & 0 & 0 \\ 0 & 0 & \varphi_{mm^*} & \varphi_{m^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \varphi_\chi & 0 \\ 0 & 0 & 0 & 0 & 0 & \varphi_{\chi^*} \end{bmatrix}$$

and the errors term are uncorrelated, i.e. ε_{Zt} are $i.i.d. \sim (0, \Sigma_z)$, with:

$$\Sigma_z = \begin{bmatrix} \sigma_a^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{a^*}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_m^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{m^*}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\chi^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\chi^*}^2 \end{bmatrix}$$

The matrix Φ_Z is divided in three blocks. We do not allow for spillover effects among real and nominal shocks neither among money supply and money demand shocks. We allow, instead, for technological spillovers and money supply spillovers between the two countries.

7.6 The Estimation of the Model's Structural Parameters

In this section we present the estimation of the Euler equations. We measured the model's parameters by using the time-series for the UK and the average of the EU-15 countries from 1979 to 1998. The EU countries considered are all the European Union countries (i.e. the 11 countries that participate to the EMU from the beginning of the period and the 3 that do not, i.e. UK, Sweden and Greece). This means that informations on the UK economy are contained also in the EU data base.

For the real interest rate we took the average quarterly interest rate over the period 1979-1998, that implies a value for the two economies equal to 4% per year.

The country-size is given by the country population. Each variable is therefore normalized by the number of inhabitants in each country.

We first rewrite equation (7.60) in this way:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\alpha)(1-\alpha)}{\alpha}(s_t - \widehat{p}_t) \tag{7.66}$$

We estimated equations (7.58), (7.59), (7.66) by using Instrumental Variables. This estimation method allows us to deal with the problem of having a forward looking term and therefore expectations for consumption and inflation, and with the problem of endogeneity of the explanatory variables. The IV estimation of (7.58) leads to:

$$\widehat{c}_t = -\underset{[-1.9]}{2.47}\,\widehat{r}_t - \underset{[-1.7]}{1.4}\,\Delta\widehat{c}_t + \underset{[3.36]}{1.5}\,\Delta\widehat{c}_{t-1} + \underset{[1.29]}{0.46}\,\Delta\widehat{c}_{t-2} + \underset{[1.99]}{0.61}\,\Delta\widehat{c}_{t-4} \tag{I}$$

Note: Sample: 1980:2-1998:2. t-stat. between brackets. $\Delta\widehat{c}_t$, $\Delta\widehat{c}_{t-1}$, $\Delta\widehat{c}_{t-2}$, $\Delta\widehat{c}_{t-4}$, $\widehat{c}_{t-4}$ used as instruments for $E_t\widehat{c}_{t+1}$.

Before carrying out the estimation of (7.59) we used the equation coming out from the difference between the log-linear versions of (7.31) and (7.32), i.e. the steady-state expressions of the money market, to obtain a time series for χ_t and χ_t^*. We then used Instrumental Variables on (7.59)[65]:

[65]The estimation of this Euler equation follows the procedure suggested by Wickens and Breuch (1988). They showed how it is possible to obtain robust estimates of the long-run coefficients by estimating each equation in levels and by adding lags of first differences among explanatory variables and instruments.

$$m_t - m_t^* - \widehat{p}_t = \underset{[5.13]}{.72}\ \widehat{c}_t - \underset{[-9.18]}{32.49}\ \widehat{\imath}_t + \underset{[11.0]}{.69}\ \widehat{\chi}_t + \underset{[1.83]}{.45}\ \Delta(m - m^* - \widehat{p})_{t-1} + \underset{[2.34]}{.58}\ \Delta(m - m^* - \widehat{p})_{t-2} \tag{II}$$

Note: Sample 1980:2-1998:2, t-stat. between brackets. Additional instruments used: $(m - m^* - \widehat{p})_{t-4}$, $(m - m^* - \widehat{p})_{t-5}$, $\widehat{c}_{t-4}$, $\widehat{c}_{t-5}$, $\Delta\widehat{\imath}_{t-1}$, $\Delta\widehat{\imath}_{t-2}$, $\Delta\widehat{\imath}_{t-3}$, $\widehat{\imath}_{t-4}$, $\widehat{\imath}_{t-5}$.

The IV estimation of (7.66) leads to:

$$\widehat{\pi}_t = \underset{[2.65]}{0.98}\ E_t\widehat{\pi}_{t+1} + \underset{[3.55]}{0.108}\ (s_t - \widehat{p}_t) \tag{III}$$

Note: Sample 1980:2-1998:2. t-stat. between brackets. Instruments used for $E_t\widehat{\pi}_{t+1}$: $\widehat{\imath}_t$. Additional instruments used $\widehat{\imath}_{t-1}$, $\widehat{\imath}_{t-2}$, $(s - \widehat{p})_{t-1}$, $(s - \widehat{p})_{t-2}$, $(s - \widehat{p})_{t-3}$, $\widehat{\pi}_{t-1}$, $\widehat{\pi}_{t-2}$, $\widehat{\pi}_{t-3}$.

We used equation (7.58) to obtain an estimate of σ, from (I) we obtained $\frac{1}{\sigma} = 2.47$ or $\sigma = 0.40$. The estimate has been obtained under the restriction that the relative risk aversion coefficient was the same for the UK and the EU. We then used equation (7.59) to estimate $\frac{\sigma}{\varsigma}$ and $\frac{\nu}{\varsigma}$. For the first term we obtain a coefficient close to unity (Wald tests have been carried out for the coefficients of $\widehat{c}_t$ and of $\widehat{\chi}_t$, see Appendix F.4), this means that we cannot reject the restriction $\sigma = \varsigma$. We then obtain a value for $\frac{\nu}{\varsigma}$ equals to 32.49 which implies $\nu = 12.99$ and thus an annual nominal interest rate of 7.5%. This long-run value for the nominal interest rate is far too high with respect to the current values in the UK and in the other EU countries. But it captures quite well the past history of the nominal interest rate since the beginning of the sample period considered, the 1980s.

Equation (III) is the IV estimate of (7.66). It shows very interesting results. We obtain an estimate of the discount factor β equal to 0.98 and an estimate of the coefficient multiplying the real exchange rate. $\frac{(1-\beta\alpha)(1-\alpha)}{\alpha} = 0.108$, which implies two possible values for $\alpha : \alpha_1 = 0.75$ and $\alpha_2 = 1.23$. We are interested in the root smaller than unity: $\alpha_1 = 0.75$ which corresponds to a one year of staggered price rigidity.

Table 7.1 shows that in steady state the only difference between the two economies lies on the inflation rates. In the UK the average annual inflation rate is equal to 6% whereas in the average of the EU countries it is equal to 4%.

Table 7.1: The deep parameters of the UK and EU economies (sample 1979:1-1998:2)

	UK	EU
n	.16	.84
$\overline{r}$	1.01	1.01
α	.75	.75
$\overline{i}$	.024	.021
π	.015	.01
θ	.71	.71
σ	.40	.40
ς	.55	.55
h	1/3	
Λ	0.65	
Γ_0	2.12	
Γ_1	0.318	

The exogenous part of the model has been estimated by carrying out OLS regression on the VAR system (7.66). The estimation led to the following result:

$$\widehat{\Phi}_Z = \begin{bmatrix} .47[4.6] & 0 & 0 & 0 & 0 & 0 \\ 0 & .46[3.8] & 0 & 0 & 0 & 0 \\ 0 & 0 & .50[4.8] & 0.27[2.0] & 0 & 0 \\ 0 & 0 & 0 & .08[.11] & -.11[-2.5] & -.05[-2.6] \\ -1.6[-2.2] & 0 & 0 & 0 & -.18[-1.5] & 0 \\ 0 & .48[1.5] & 0 & -1.3[-1.5] & 0 & -.5[-3.8] \end{bmatrix}$$

where $Z_t = a_t,\ a_t^*,\ m_t,\ m_t^*,\ \chi_t,\ \chi_t^*$, and

$$\widehat{\sum}_Z \times 100 = \begin{bmatrix} .003 & 0 & 0 & 0 & 0 & 0 \\ 0 & .096 & 0 & 0 & 0 & 0 \\ 0 & 0 & .0146 & 0 & 0 & 0 \\ 0 & 0 & 0 & .018 & 0 & 0 \\ 0 & 0 & 0 & 0 & .158 & 0 \\ 0 & 0 & 0 & 0 & 0 & .675 \end{bmatrix}$$

Clearly, not all the restrictions imposed on Φ_Z have been accepted by the data. We did not find spillover effects between the two technological shocks. m_t^* is negatively related to the two demand shocks χ_t and χ_t^*, although with very small coefficients. χ_t depends negatively on a_t, and χ_t^* depends positively on a_t^* and negatively on m_t^* (although the two coefficients are not highly significant).

7.7 Simulation Results for Different Values of α

In this paragraph we present the solution of the model under the parameter restriction $\sigma = \varsigma$, which implies that the intertemporal substitution of consumption is equal to that of real money balances.

As showed in chapter 5, we 'guess' the solution by applying the method of undetermined coefficients to the system of equations (7.58)-(7.64).

We then analyse the behaviour of the economy determined by the elasticity of each variable to the price differential which is treated as a state variable. Equation (7.63) reveals that we should consider as state variables also r^* and b^*. To keep only $\widehat{p}$ in the reduced final form of the system we decided to treat the term $\left[b_t^* - (1+\overline{r})(r_{t-1}^* + b_{t-1}^*)\right]$ as a unique expression that we call ca^*. Since $\overline{r} = 0.01$ this expression can be rewritten as $\left[ca^* \simeq \Delta b_t^* - (r_{t-1}^* + \overline{r}b_{t-1}^*)\right]$ and, thus, interpreted as the foreign current account plus capital gains on foreign assets.

We start by writing down the 'guessed' solutions, the system $(A)-(H)$, we then equalize the coefficients of the solutions for each equation of the system $(A)-(H)$ to those of the original system (7.58)-(7.64).

The system of the guessed solutions is:

$$\widehat{c}_t = \eta_{cp}\widehat{p}_{t-1} + \eta_{cm}m_t + \eta_{cm^*}m_t^* + \eta_{ca}a_t + \eta_{ca^*}a_t^* + \eta_{c\chi}\chi_t + \eta_{c\chi^*}\chi_t^* \qquad (A)$$

$$\widehat{p}_t = \eta_{pp}\widehat{p}_{t-1} + \eta_{pm}m_t + \eta_{pm^*}m_t^* + \eta_{pa}a_t + \eta_{pa^*}a_t^* + \eta_{p\chi}\chi_t + \eta_{p\chi^*}\chi_t^* \qquad (B)$$

$$s_t = \eta_{sp}\widehat{p}_{t-1} + \eta_{sm}m_t + \eta_{sm^*}m_t^* + \eta_{sa}a_t + \eta_{sa^*}a_t^* + \eta_{s\chi}\chi_t + \eta_{s\chi^*t}\chi_t^* \qquad (C)$$

$$\widehat{r}_t = \eta_{rp}\widehat{p}_{t-1} + \eta_{rm}m_t + \eta_{rm^*}m_t^* + \eta_{ra}a_t + \eta_{ra^*}a_t^* + \eta_{r\chi}\chi_t + \eta_{r\chi^*}\chi_t^* \qquad (D)$$

$$\widehat{i}_t = \eta_{ip}\widehat{p}_{t-1} + \eta_{im}m_t + \eta_{im^*}m_t^* + \eta_{ia}a_t + \eta_{ia^*}a_t^* + \eta_{i\chi}\chi_t + \eta_{i\chi^*}\chi_t^* \qquad (E)$$

$$rer_t = \eta_{rerp}\widehat{p}_{t-1} + \eta_{rerm}m_t + \eta_{rerm^*}m_t^* + \eta_{rera}a_t + \eta_{rera^*}a_t^* + \eta_{rer\chi}\chi_t + \eta_{rer\chi^*}\chi_t^* \quad (F)$$

$$\widehat{y}_t = \eta_{yp}\widehat{p}_{t-1} + \eta_{ym}m_t + \eta_{ym^*}m_t^* + \eta_{ya}a_t + \eta_{ya^*}a_t^* + \eta_{y\chi}\chi_t + \eta_{y\chi^*}\chi_t^* \qquad (G)$$

$$ca_t^* = \eta_{cap}\widehat{p}_{t-1} + \eta_{cam}m_t + \eta_{cam^*}m_t^* + \eta_{caa}a_t + \eta_{caa^*}a_t^* + \eta_{ca\chi}\chi_t + \eta_{i\chi^*}\chi_t^* \quad (H)$$

The solution is obtained by substituting $(A)-(H)$ into (7.58)-(7.64), taking into account that $E_t\widehat{p}_{t+1} = \eta_{pp}\widehat{p}_t + \eta_{pm}\varphi_m m_t + \eta_{pm^*}\varphi_{m^*}m_t^* + \eta_{pa}\varphi_a a_t + \eta_{pa^*}\varphi_{a^*}a_t^* + \eta_{p\chi}\varphi_\chi\chi_t + \eta_{p\chi^*}\varphi_{\chi^*}\chi_t^*$, and the same for $E_t\widehat{c}_{t+1}$, $E_t s_{t+1}$.

Table 7.2: The roots of η_{pp} for different values of α

α	η_{pp}^1	η_{pp}^2
0.92 (3 years)	1.024	.415
0.75 (1 year)	1.02	.404
0.05 (3 months)	1.02	.050

We are interested in investigating the behaviour of the system for different values taken by η_{pp}^2 (the stable root) when we change α. The values that we consider cover a range of staggered prices that goes from 3 months to 3 years. Table 7.2 shows that by decreasing α, the roots of η_{pp}^2 become smaller. As happened in chapter 5, there is a very smooth change in η_{pp}^2 for high degrees of price stickiness and a sharp change for low degrees of price stickiness. Without price stickiness the price differential time-series cannot be characterized by an autoregressive process. The introduction of the equation describing the production sector and the world economy constraint does not change the results obtained in chapter 5.

Table 7.3 (panel 1) shows that η_{cp}, the consumption elasticity to a price change, is negative for every value of α and decreasing in absolute values when α is increasing.

The real interest rate elasticity to a price change is always positive and decreasing for lower values of α. The nominal interest elasticity to a price change is negative but almost insignificant for every value taken by α.

The nominal exchange rate elasticity to a price change is positive and decreasing while α is decreasing, but very small. The income elasticity to a price change goes in a smoothed and opposite direction with respect to the consumption elasticity. The trade-off between demand side and supply side determinants of income, that has been evidenced before, is weaker for low values of α.

The elasticities of the real exchange rate to a price change are of the same magnitude but take the opposite sign of η^2_{pp}.

We observe that ca^* is extremely highly reactive to a change in the relative price when the degree of price stickiness is high.

We can conclude that the system is more sensitive to changes in the state variable when the price rigidity lasts longer. Given that $\widehat{p}$ has been treated as a state variable, this result simply confirms that the size of real effects coming from a change in the price differential depends on the time length of the staggering price contract.

The analysis of the elasticities of the endogenous variables with respect to the exogenous variables reveals some symmetric behaviour between the UK and the EU. This is true for technological and preference shocks and to a lesser extent for money supply shocks. Starting from technological shocks (panels 4 and 5 of table 7.3) we can notice that changes in the parameter α do not affect their impact elasticities, and that only income differentials and the foreign current account paths depend on them. The fact that the consumption differential is not affected by technological shocks is not surprising since the model implies perfect consumption smoothing. We observe that technological shocks heavily determine the behaviour of the net foreign position. Domestic technological shocks worsen the foreign current account and the opposite happens with foreign technological shocks.

Table 7.3: The elasticities of the reduced form equations for different values of α

α	η_{pp}^{2}	η_{cp}	η_{ip}	η_{rp}	η_{sp}	η_{rerp}	η_{yp}	η_{cap}
.92	.415	-1.01	-.006	.236	.01	-.405	.322	-20.8
.75	.404	-.986	-.005	.235	.009	-.394	.314	-20.8
.05	.05	-.124	-.001	.047	.001	-.049	.039	-2.61

(1)

η_{pm}	η_{cm}	η_{im}	η_{rm}	η_{sm}	η_{rerm}	η_{ym}	η_{cam}
.00	.047	-.009	-.009	.018	.019	-.014	.985
.00	.045	-.009	-.009	.019	.018	-.014	.96
.017	.005	-.009	-.002	.019	.002	.00	.124

(2)

η_{pm^*}	η_{cm^*}	η_{im^*}	η_{rm^*}	η_{sm^*}	η_{rerm^*}	η_{ym^*}	η_{cam^*}
.00	-.012	.010	.009	-.005	-.005	.004	-.27
.00	-.012	.010	.009	-.005	-.005	.004	-.26
-.005	.00	.009	.001	-.005	.00	.00	-.016

(3)

η_{pa}	η_{ca}	η_{ia}	η_{ra}	η_{sa}	η_{rera}	η_{ya}	η_{caa}
0	0	0	0	0	0	2.12	-33.9
0	0	0	0	0	0	2.12	-33.9
0	0	0	0	0	0	2.12	-33.9

(4)

η_{pa^*}	η_{ca^*}	η_{ia^*}	η_{ra^*}	η_{sa^*}	η_{rera^*}	η_{ya^*}	η_{caa^*}
0	0	0	0	0	0	-2.12	33.9
0	0	0	0	0	0	-2.12	33.9
0	0	0	0	0	0	-2.12	33.9

(5)

$\eta_{p\chi}$	$\eta_{c\chi}$	$\eta_{i\chi}$	$\eta_{r\chi}$	$\eta_{s\chi}$	$\eta_{rer\chi}$	$\eta_{y\chi}$	$\eta_{ca\chi}$
0	-.02	.009	.009	-.008	-.008	.006	-.429
0	-.02	.009	.009	-.008	-.008	.006	-.415
0	-.007	.009	.011	-.008	0	0	-.401

(6)

$\eta_{p\chi^*}$	$\eta_{c\chi^*}$	$\eta_{i\chi^*}$	$\eta_{r\chi^*}$	$\eta_{s\chi^*}$	$\eta_{rer\chi^*}$	$\eta_{y\chi^*}$	$\eta_{ca\chi^*}$
0	.016	-.009	-.009	.006	.006	-.005	.338
0	.015	-.009	-.009	.006	.006	-.005	.325
0	.006	-.009	-.011	.007	0	0	.071

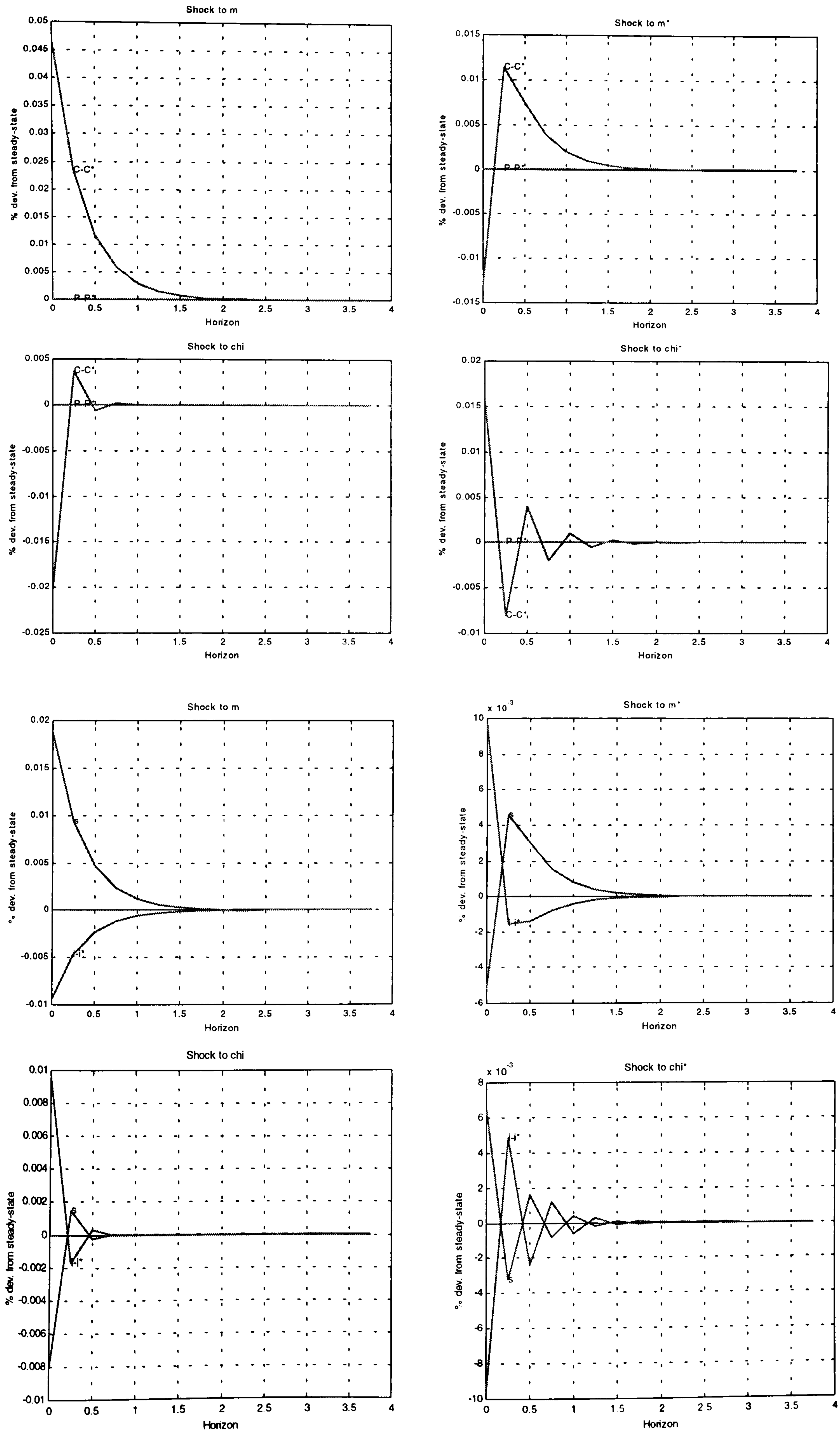
(7)

If we now move to the impact elasticities of the endogenous variables to the UK and EU money supply (panels 2 and 3 of table 7.3) we can observe that the size of α matters for the length of the period that each variable employs to die out towards the previous steady state. For a high degree of price stickiness all the adjustment is done by the consumption differential, for a low degree of price stickiness we can see that the consumption differential does not significantly react.

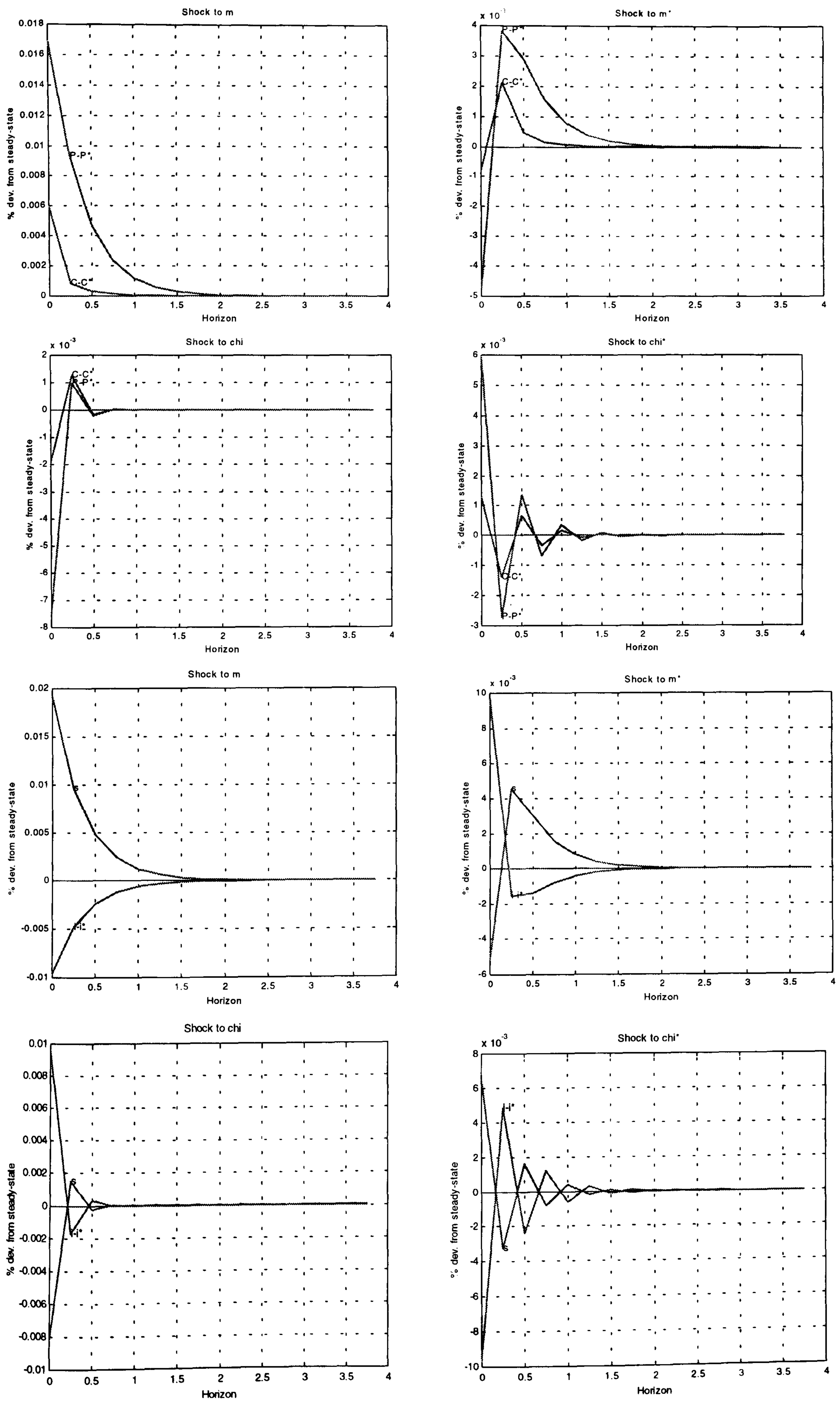
A UK money supply shock has a negative impact on nominal and real interest rate differentials regardless of the values taken by α. The impact is also negative on the income differential. This result needs an explanation. In our model output is affected by exogenous shocks that pass through the labour market and then go to the production function. Therefore, a fall in output is the result of a fall in hours worked after a positive money supply shock. This, clearly, reveals that a push to the economy coming from an expansionary monetary policy acts through output demanded, not supplied. In our model the equilibrium level of output demanded needs not to coincide with output supplied in each country because of the presence of the bonds market, as can be seen from equation (7.64). The equality between output demanded and supplied holds only at a world level.

The impact effect of a UK money demand shock (panels 6 and 7 of table 7.3) is negative on the domestic consumption, on the price differential and on the nominal and real exchange rate. It is positive on the nominal and real interest rate and on the output differential. The model shows a clear trade-off between shocks coming from money demanded and supplied. Money demand shocks have a negative effect on output demanded because there is a shift from consumption preferences to real money balance preferences. They have instead a positive effect on output supplied because of the increased hours of work.

Figures 7.1: Impulse response functions: $\alpha = 0.75$



Figures 7.2: Impulse response functions: $\alpha = 0.05$



7.7.1 Impulse Response Functions for Different Values of α

The last step consists on the analysis of the system's impulse response functions to shocks coming from money demand and supply (figures 7.1 and 7.2).

We first consider the case of $\alpha = 0.75$ (one-year stickiness, figures 7.1). A shock to the domestic money supply has a real effect that lasts two years. A foreign money supply shock has a negative impact effect on domestic consumption, it becomes positive after three months and declines toward the previous steady state within two years. This dynamics is the result of a positive spillover coming from EU monetary shocks to the UK economy. It is consistent with the result obtained in the previous chapter, where we have estimated the VECM containing UK and EU monetary shocks.

A domestic money supply shock leads, then, to a negative response of the interest rate differential which dies out after two years and to a depreciation of the nominal exchange rate, whose dynamics results opposite to that of the interest rate differential. A foreign supply shock has a positive impact effect on the domestic interest rate differential that overshoots, i.e. the impact increase is absorbed after three months. The dynamics of the nominal exchange rate is opposite to that of the domestic interest rate differential.

A domestic money demand shock (chi) has a very short lasting effect on the consumption and income differential. The impact effect on the consumption differential is negative and becomes positive after three months. A foreign money demand shock (chi^*) leads to a more instable outcome for the consumption differential.

If we now look at the impulse responses when $\alpha = 0.05$ (3 months price stickiness, figures 7.2) the major difference with respect to the previous case is not on the shape of the dynamics but on the period of time that real variables take for going back to the original steady state. Now money supply shocks and money demand shocks have a much shorter effect on consumption and income differentials.

7.8 Conclusions

In this chapter we extended the model presented in chapter 5 in several directions. We modelled the supply side and we demonstrated that the results obtained in chapter 5 hold even in a general equilibrium model, under the assumption that leisure does not appear in the utility function, i.e. workers cannot choose their hours of work. We enriched the structure of preferences by introducing an additive shock to the money demand, and we allowed for relative risk aversion coefficients for consumption and real money balances to be different from 1. The Instrumental Variables estimation of the economy's deep parameters has been carried out under the hypothesis that preferences were the same for the UK and the average Euro country. We succeeded in finding out a significant estimate of α, the parameter that governs the timing of price rigidity, which resulted to be 0.75.

We simulated the model for different values of the price stickiness parameter. Having a complete model did not change the main results obtained in chapter 5. We have used technological shocks to measure output gap. We have shown how consumption decisions are negatively related to stochastic shifts to the money demand.

The measurement of the structural parameters used for preferences, technology and policies, led to different results with respect to those obtained in chapter

5. In this chapter the monetary transmission mechanism between the UK and the EU acted through spillover and asymmetry like in chapter 5, but shocks coming from the EU led to potential gain for the UK and not losses as it happened with shocks coming from the US. This result is not only related to a different money supply time-series used for the foreign country, but also to the fact that shocks to the money supply could be distinguished from shocks to the money demand.

This chapter has prepared the ground for our final investigation. The model economy has been subject to supply and demand shocks that can justify some policy intervention. We decide, therefore, to explicitly model the monetary authority behaviour. This will be done by building and solving, over the economic structure here derived, the Central Bank's decision problem, as its objective consists of reducing the variability of inflation and output gap around their potential values.

8 Monetary Policy Rules and the Optimal Monetary Policy in the UK

8.1 Introduction

This chapter, that starts the third section of the thesis, aims at modelling the 'systematic' behaviour of monetary policies. While keeping the two-country model for the pair UK - EU developed in the last three chapters, we relax the assumption that there is an exogenous autoregressive process for the money supply[66]. The behaviour of the Monetary Authority is no longer taken to be extraneous (or exogenous) to the economy. The task is first accomplished by looking at the model's implications of having a feedback policy rule and second by deriving an optimal rule within the optimal control theory.

The literature on determining the best monetary policy rule for the Central Bank is large and growing. There exist two general approaches.

The first consists of choosing a policy instrument (usually the monetary base, M1, M2 or a short-term interest rate), a target variable and a rule. After having made this choice, the exercise proceeds by examining, within a model of the economy, how the economy would have behaved under the rule in comparison with how it in fact behaved. The rule typically expresses the policy instrument as a function of the deviation of the target variable from its target value. Usually, actual values of the variances of key macroeconomic variables, like the real growth rate and the rate of inflation, are compared to the values of the variances that would have occurred had the rule been followed. The aim is to find the rule

[66]The assumed exogeneity of the monetary variables makes the model invalid for policy analysis if monetary policy reacts endogenously to the macroeconomic variables.

(including the choice of the target variable) that gives (in some sense) the best overall performance of the economy (see amongst others, Haldane and Batini, 1998).

The second general approach derives the best monetary policy rule using optimal control techniques. In this case, a particular loss function is chosen under the assumption that every one (including the Central Bank) agrees that this is the loss function whose expected value the Central Bank should minimize. Then, the approach consists of choosing a policy instrument and of using this instrument to minimize the expected value of the 'true' loss function. The exercise is then either to use the instrument to minimize the expected value of other loss functions and to compare the different outcomes, or to compare the optimal rule derived from the former minimization problem with other instrument rules (Svensson, 1996, 1999).

In this chapter we consider the two approaches in turn. We start by rewriting the fundamental behavioural equations of our two-country model in a way that highlights the differences between our and the standard 'textbook' Dornbusch (1976) model. We complete the model with an estimated General monetary policy rule. The rule generalizes to an open economy context the Taylor (1993) rule and is estimated by Instrumental Variables on the UK data during the period 1979-1998. We consider three different sub-samples related to the UK monetary regimes and for each of those we simulate the model and study the economy's reactions to exogenous shocks. We repeat the exercise by changing the parameters in the rule according to two well known policy rules (of the Taylor-type). In the second part of the chapter we consider the Central Bank optimal problem within a two-country economy. We set up a one-period discretionary problem, where the Central Bank takes as given private expectations on future inflation and output gap. The solution to the problem leads to find an optimal rule for the nominal interest rate. The estimates of the Euler equations carried out in the previous chapter allow us to quantify the elasticity of the optimal interest, equilibrium inflation and output gap to the exogenous state variables. We finally measure the total utility loss. The loss derived from the optimal policy can be compared to that obtained under the estimated rule.

The structure of the chapter is the following. Section 8.2 introduces the economy's constraints to the Central Bank's problem. They are derived from the model of the previous chapter. In this section we will emphasize the differences between our behavioural equations and those that constitute the Dornbusch (1976) model. Section 8.3 introduces the policy rule and solves the model under the rule. We present its IV estimates in section 8.4. In section 8.5 we simulate the model for different parameters of the rule. Section 8.6 considers the optimal problem for the Central Bank. The policy maker trades off between fluctuations of output gap around its target (greater than potential) and inflation around its target. The total loss and the volatility of output gap and inflation is computed. Total loss and volatility are then used to compare the performance of the rules. The optimal policy delivers the lowest volatility and loss. According to the General rule the higher loss to the UK Central Bank has occurred during the monetary targeting regime.

8.2 The Economy's Constraints to the Central Banker: How they differ from the 'Textbook' Dornbusch Model

The study of optimal monetary policy rules in a two-country model, where home and foreign country behaviour has been endogenously determined, is a new task, whereas there is some work on optimal monetary policies in small open economies (where all the foreign variables are exogenous) and many in closed economies (Svensson, 1997, Ball, 1998). Very little of this work derives from a DSGE framework. Usually the economic environment is taken from the Dornbusch (1976) model that extends the AS-AD equations to open economies. The economy is assumed to be small and all the foreign variables are treated as exogenous.

In this chapter we will use, instead, as constraints to the monetary authority actions the behavioural equations obtained from the maximization problem of the previous chapter. To help the understanding of the key differences between our and the Dornbusch model we outline the structural equations of the two models. This comparison is done in two steps. We will first look at the behaviour of the domestic economy in isolation, since this is what the Dornbusch model does. We will then look at the relative behaviour of the domestic and foreign country by taking the difference of the respective variables since this is what our model does.

I) The home country perspective

A) The Two-country Model	B) The Dornbusch (1976) Model
A_1) $\widehat{i}_t = E_t s_{t+1} - s_t$	B_1) $\widehat{i}_t = E_t s_{t+1} - s_t$
A_2) $m_t - p_t = \frac{\sigma}{\zeta} c_t - \frac{\nu}{\zeta} i_t + \frac{\sigma}{\zeta} \chi_t$	B_2) $m_t - p_t = \phi y_t - \eta i_t$
A_3) $x_t = E_t x_{t+1} + \frac{\Gamma_1}{\sigma}(i_t - E_t \pi_{t+1}) + (\Gamma_0 - 1)(1 - \varphi_a) a_t$	B_3) $x_t = \delta(s_t - \widehat{p}_t)$
A_4) $\pi_t = \beta E_t \pi_{t+1} + \Omega \left[(1 - n)(s_t - \widehat{p}_t) + f(n, mc^i, a^i)\right]$	B_4) $\pi_t = \varphi x_t$

In the table we have called $f(n, mc^i, a^i) = n\left[mc_t + (1-\frac{1}{\theta})a_t\right] + (1-n)\left[mc_t^* + (1-\frac{1}{\theta})a_t^*\right]$, and $\Omega = \frac{(1-\beta\alpha)(1-\alpha)}{\alpha}$. The output gap x_t in the Dornbusch model is equal to $y_t - \overline{y}$ and $\overline{y}$ is the constant long-run value of output, whereas in our model it is equal to $y_t - a_t$ and a_t is an exogenous productivity parameter that follows an autoregressive process. All the other variables and parameters have been introduced in the previous chapter.

The comparison of the two models under the home country perspective leads to the following observations:

- $(A_1) = (B_1)$: Uncovered interest parity holds in both models, thus we start with the same equation that relates nominal interest rate differentials with the expected depreciation of the currency.

- In the Dornbusch model the domestic monetary equilibrium is characterized by equation (B_2), where $\phi > 0$ and $\eta > 0$ are reduced form parameters. The monetary equilibrium in the model of chapter 7 is given by equation (A_2). There are three main differences with respect to (B_2) : the expenditure measure is total consumption; we can give a structural interpretation to the parameters of the behavioural equation because they express preferences in the optimization problem; there is uncertainty related to the preference shock χ_t.

- The Dornbusch model assumes that PPP needs not to hold, so that the real exchange rate can vary. It assumes also that aggregate demand for home country output, y_t, is an increasing function of the home real exchange rate. It is a deliberately simplified aggregate demand schedule. The inclusion of other components does not add any insight to the Dornbusch model. Equation (A_3) is derived by substituting the Euler equation for the goods market into the equilibrium equation for the labour market. x_t expresses the output gap, that is the difference between actual output produced and potential output, measured by the exogenous productivity term in the model, a_t. This equation has an intertemporal dimension linked to the behaviour of expected future output gap, which depends on the current real interest rate and on productivity shocks. Thus in our model there is not a direct relationship between domestic output and the real exchange rate.

- The Dornbusch model is completed by a price adjustment equation (Phillips curve) (B_4). It says that the wider the gap between demand and capacity output, the higher is the rate of inflation. This Phillips curve, like the rest of the model, is not derived from an optimization process but it can be obtained (McCallum, Nelson, 1998) from the solution to a cost minimization problem which links the process of adjusting prices to that of changing the amount of output produced.

  Our aggregate supply curve looks quite different from (B_4). In our case each monopolistic PTM firm adjusts its price according to its nominal marginal cost. Thus, in the aggregate, the price index will contain information on the domestic and foreign marginal costs and on the nominal exchange rate. This leads to a relation between expected domestic inflation, real marginal costs and the real exchange rate. In our model the real exchange rate affects aggregate demand only indirectly, whereas a direct effect is assumed in the Dornbusch model. There are two other key differences between our model and the Dornbusch model that are brought out in equation (A_4). The first is related to the meaning of a small open economy: in our model it is captured by the country size n without imposing exogeneity on the behaviour of the foreign variables; in the Dornbusch model it is captured by an exogenously determined interest rate and constant foreign prices. The second difference is related to the presence of future expected inflation in the aggregate supply equation of our model.

Our two-country model assumes symmetric behaviour of foreign and home countries. This leads to a natural way of reducing the system of equations by taking the difference between the two economies. This procedure is not so 'natural' in the case of the Dornbusch model, since the foreign economy is left unmodelled. We will now try to look at the two models in differences, this step implies further assumptions on the behaviour of the foreign country not originally stated in the Dornbusch model.

II) The difference between home and foreign country

A) The Two-County Model	B) The Dornbusch (1976) Model
A_5) $\widehat{m}_t - \widehat{p}_t = \frac{\sigma}{\zeta}\widehat{c}_t - \frac{\nu}{\zeta}\widehat{i}_t + \frac{\sigma}{\zeta}\widehat{\chi}_t$	B_5) $\widehat{m}_t - \widehat{p}_t = \phi\widehat{y}_t - \eta\widehat{i}_t$
A_6) $\widehat{x}_t = E_t\widehat{x}_{t+1} + \frac{\Gamma_1}{\sigma}(\widehat{i}_t - E_t\widehat{\pi}_{t+1}) + (\Gamma_0 - 1)(1-\varphi)\widehat{a}_t$	B_6) $\widehat{x}_t = \delta(s_t - \widehat{p}_t) - x_t^*$
A_7) $\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \Omega(s_t - \widehat{p}_t)$	B_7) $\widehat{\pi}_t = \varphi\widehat{x}_t$

- We assume that the monetary equilibrium of the foreign country in the Dornbusch model can be represented in the same way as that of the domestic economy. We assume also the same parameters for the two economies, so that equation (B_5) can be obtained by taking the difference between the corresponding equations for each country. For the aggregate demand of the rest of the world we decided not to make any assumption about the foreign measure, thus we subtract x_t^* from both sides of (B_3). The justification for not modelling x_t^* is that it is very unlikely that a big country would care about its real exchange rate with respect to a small open economy. Finally, we model for the foreign country an equivalent Phillips curve to that of the domestic economy, the country-difference leads to (B_7).

- The aggregate demand in our two-country model has an intertemporal dimension. Given the assumption of symmetry between the domestic and the foreign country, (A_6) looks very similar to (A_3). In the derivation of (A_6) we assumed that the two exogenous processes for technology have the same autoregressive coefficient, i.e. $\varphi_a = \varphi_{a*} = \varphi$.

- In the two-country model the difference between the aggregate supply equations results very simplified. Owing to the fact that there are no technological differences between countries and that domestic and foreign price indices are built symmetrically, the terms multiplying the marginal costs disappear. This leads to an equation that relates current and expected inflation differentials to the real exchange rate.

For both models we can reduce the system and obtain two final form equations. To simplify matters we also take income fixed at the 'full-employment' level:

A) The Two-county Model	B) The Dornbusch (1976) Model
A_8) $\widehat{p}_t = \widehat{m} - \frac{\nu}{\sigma}(s_t - E_t s_{t+1}) - \frac{\Gamma_0 - 1}{\Gamma_1}\widehat{a}_t - \widehat{\chi}_t$	B_8) $\widehat{p}_t = \widehat{m} - \eta(s_t - E_t s_{t+1}) - \phi\widehat{\overline{y}}$
A_9) $\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \Omega(s_t - \widehat{p}_t)$	B_9) $\widehat{\pi}_t = \varphi\delta(s_t - \widehat{p}_t)$

In deriving (A_8) we use the restriction $\frac{\varsigma}{\sigma} = 1$, that has not been rejected by the data on the UK and EU over the sample period 1980:2-1998:2 (see previous chapter and Appendix F).

As a consequence of these assumptions, the two models share many properties. Equations (A_8) and (B_8) reveal that, given short-run price rigidity, a monetary shock produces overshooting of the nominal exchange rate. Equation (A_8) then differs from (B_8) because of the presence of two other sources of shocks: productivity and preferences. In our two-country model we end up with a unique 'reduced form' equation (A_8) containing all of the relevant shocks hitting the two economies: a money demand shock and a technological shock. The money demand shock derives from uncertainty in preferences for liquidity. The technology shock appears in the production function. We use the technological shock a_t as a measure of potential output, and thus we interpret $\widehat{x}_t$ as the relative output gap of the domestic with respect to the foreign economy. In the previous chapter we showed how fluctuations in the output gap are reflected in fluctuations in employment.

We can further notice that equation (A_8) can be related to the standard (textbook) way of writing the economic relation between income and the interest rate, i.e. we can rewrite (A_8) in this way:

$$\widehat{y}_t = -\Gamma_1 \frac{\nu}{\sigma} \widehat{\imath}_t - \Gamma_1 (\widehat{m}_t - \widehat{p}_t - \widehat{\chi}_t) + \Gamma_0 \widehat{a}_t$$

where we recognize something like a 'two-country' AD curve subject to real and nominal uncertainty. As we have shown before, this curve is the result of the aggregation of three optimality conditions: the Euler equation for consumption, the

Euler equation for money demand and the labour market equilibrium condition. This kind of aggregation shows, clearly, the difficulty of interpreting reduced form equations when we do not know the structural model from which they are derived.

Equations (A_9) and (B_9) differ because of the presence of expected future inflation in the two-country model. Equation (B_9) says that the rate of inflation is positive whenever the real exchange rate is above its equilibrium level (which is assumed to be zero). Equation (A_9) adds future expected inflations to (B_9). This equation describes the presence of a short-run trade-off between inflation differentials and real exchange rate movements. When the actual inflation differential exceeds the discounted future inflation differential the real exchange rate depreciates. Thus, a more competitive domestic economy is consistent with expectations of future low inflation. If we now solve equation (A_9) forward we obtain the following expression:

$$\widehat{\pi}_t = E_t \sum_{s=1}^{T} \beta^s \widehat{\pi}_{t+s} + \Omega E_t \sum_{s=1}^{T} \beta^s (s_{t+s} - \widehat{p}_{t+s})$$

By taking the limit for $T \to \infty$, we obtain

$$\widehat{\pi}_t = \Omega E_t \sum_{s=1}^{\infty} \beta^s (s_{t+s} - \widehat{p}_{t+s})$$

Current inflation therefore depends on the future path of the real exchange rate. Relative inflation today is affected not just by the current real exchange rate, but by the expected future real exchange rate too.

8.2.1 The Model's Economy

We consider the following set of equations of the two-country model that as been previously described:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \Omega (s_t - \widehat{p}_t) \tag{8.1}$$

$$i_t = i_t^* + E_t s_{t+1} - s_t \tag{8.2}$$

$$E_t \Delta \widehat{x}_{t+1} = -\frac{\Gamma_1}{\sigma}(\widehat{i}_t - E_t \widehat{\pi}_{t+1}) + (\Gamma_0 - 1) E_t \Delta \widehat{a}_{t+1} \tag{8.3}$$

As in the previous chapter the home country is taken to be the UK and the foreign country is the European Union.

The model is completed by a forth equation, that we will call the General rule. Given that money supply is endogenous, the rule is required to complete the model by providing a nominal anchor.

There exists a very large and growing literature that describes the Central Bank behaviour by using operational rules based on the feedback between the policy instrument (the nominal interest rate) and the targeting variables (output gap and inflation). The general endeavour of this modelling procedure is to look at the robustness of rules within a wide class of models and to study the implications in terms of volatility of macro-variables. As far as empirical testing is concerned, estimated policy rules for different countries aim at capturing the systematic behaviour of the Central Bank (Clarida, Gali, Gertler, 1988).

Starting from Taylor (1993) who put forward a well known rule for the US Federal Reserve Bank based on the feedback between the nominal interest rate and output gap and inflation, the rule has been complicated in two ways: 1. by adding lagged nominal interest rates among the independent variables (Levin, Wieland, Williams, 1998); 2. by introducing a forward term in place of current inflation (Clarida, Gali, Gertler, 1998).

We will now introduce some modifications to this type of rules by allowing for foreign terms as well.

We will call the rule that we are going to estimate the *General* policy rule. It will be added to equations (8.1) - (8.3) to provide a link between the policy instrument and the relevant variables of the economic system. The rule has the objective of describing how in practice the monetary authority operates.

The *General* rule extends the Taylor rule by introducing the real exchange rate and the foreign interest rate and by substituting current inflation with expected future inflation:

$$i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \theta_2 (s_t - \widehat{p}_t) + \theta_3 i_t^* + \varepsilon_{it} \tag{8.4}$$

If we substitute (8.1) in (8.4) we obtain an equation for the domestic nominal interest rate that depends on domestic output gap, domestic expected inflation, current and expected inflation gap and foreign interest rates, that is:

$$i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \frac{\theta_2}{\Omega} \widehat{\pi}_t - \frac{\theta_2 \beta}{\Omega} E \widehat{\pi}_{t+1} + \theta_3 i_t^* + \varepsilon_{it} \tag{8.5}$$

In this chapter we will consider the home country perspective and take foreign variables as exogenous. This implies that we need to make some assumptions on the exogenous variables. We assume that the foreign nominal interest rate and the inflation rate follow an autoregressive process with uncorrelated error terms, together with the technological progress:

$$\begin{bmatrix} i_t^* \\ \pi_t^* \\ a_t \end{bmatrix} = \begin{bmatrix} \varphi_{i^*} & 0 & 0 \\ 0 & \varphi_{\pi^*} & 0 \\ 0 & 0 & \varphi_a \end{bmatrix} \begin{bmatrix} i_{t-1}^* \\ \pi_{t-1}^* \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i^*t} \\ \varepsilon_{\pi^*t} \\ \varepsilon_{at} \end{bmatrix}$$

i.e. $\begin{bmatrix} \varepsilon_{i^*t} \\ \varepsilon_{\pi^*t} \\ \varepsilon_{at} \end{bmatrix} \; i.i.d. \sim (0, \begin{bmatrix} \sigma_{i^*}^2 & 0 & 0 \\ 0 & \sigma_{\pi^*}^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix}).$

Similarly, the error term appearing in equation (8.4) is $i.i.d. \sim (0, \sigma_i^2)$. This means that we can rewrite (8.5) in this way:

$$i_t = \theta_0 x_t + \left(\theta_1 - \frac{\theta_2 \beta}{\Omega}\right) E_t \pi_{t+1} + \frac{\theta_2}{\Omega} \pi_t + \frac{\theta_2}{\Omega} (\beta \varphi_{\pi^*} - 1) \pi_t^* + \theta_3 i_t^* + \varepsilon_{it} \tag{8.6}$$

8.3 Solution to the Model

In this section we present the solution to the problem. We consider equation (8.1) at time t and $t+1$ and take the difference between t and $t+1$. Under the assumption made on the foreign inflation rate we have:

$$(1+\beta+\Omega)E\pi_{t+1} = \beta E_t\pi_{t+2} + \pi_t + \Omega E_t\Delta s_{t+1} + \pi_t^* \left[(1+\beta+\Omega)\varphi_{\pi^*} - \beta\varphi_{\pi^*}^2 - 1\right] \tag{8.7}$$

We now substitute the uncovered interest rate parity condition in (8.7):

$$E\pi_{t+1} = \frac{\beta E_t\pi_{t+2}}{(1+\beta+\Omega)} + \frac{\pi_t}{(1+\beta+\Omega)} + \frac{\Omega(i_t - i_t^*)}{(1+\beta+\Omega)} + \pi_t^* \frac{[(1+\beta+\Omega)\varphi_{\pi^*} - \beta\varphi_{\pi^*}^2 - 1]}{(1+\beta+\Omega)} \tag{8.8}$$

To characterize our model economy we will use equation (8.8) instead of (8.1).

We now step back for a moment and without making any explicit assumptions on π^* we rewrite equation (8.1) at time $t+1$ and take the difference between $t+1$ and t:

$$E_t\Delta\widehat{\pi}_{t+1} = \beta E_t\Delta\widehat{\pi}_{t+2} + \Omega E_t\Delta(s_{t+1} - \widehat{p}_{t+1})$$

Since

$$\widehat{i}_t - E_t\widehat{\pi}_{t+1} = E_t\Delta(s_{t+1} - \widehat{p}_{t+1}) \tag{8.9}$$

we obtain:

$$E_t\Delta\widehat{\pi}_{t+1} = \beta E_t\Delta\widehat{\pi}_{t+2} + \Omega(\widehat{i}_t - E_t\widehat{\pi}_{t+1}) \tag{8.10}$$

We then substitute (8.3) in (8.10). This leads to:

$$E_t\Delta\widehat{\pi}_{t+1} = \beta E_t\Delta\widehat{\pi}_{t+2} - \frac{\Omega}{\Gamma_1}\sigma E_t\Delta\widehat{x}_{t+1} + \frac{\Omega}{\Gamma_1}\sigma(\Gamma_0 - 1)E_t\Delta\widehat{a}_{t+1} \tag{8.11}$$

We get rid of the difference operator and consider equation (8.11) at time t :

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \frac{\Omega}{\Gamma_1}\sigma\widehat{x}_t + \frac{\Omega}{\Gamma_1}\sigma(\Gamma_0 - 1)\widehat{a}_t$$

which can be written as:

$$\widehat{x}_t = \frac{\Gamma_1}{\Omega\sigma}\beta E_t\widehat{\pi}_{t+1} - \frac{\Gamma_1}{\Omega\sigma}\widehat{\pi}_t + (\Gamma_0 - 1)\widehat{a}_t$$

Since we are interested in analysing the home country perspective we rewrite the previous equation by considering only the home country variables:

$$x_t = \frac{\Gamma_1}{\Omega\sigma}\beta E_t\pi_{t+1} - \frac{\Gamma_1}{\Omega\sigma}\pi_t + (\Gamma_0 - 1)a_t \tag{8.12}$$

Therefore we end up with a system of three equations in three endogenous variables, (8.6), (8.8) and (8.12) that can be solved 'by hand'. By combining (8.6), (8.8) and (8.12) we obtain the following equation for π_t:

$$\begin{aligned}
&\left(\frac{1+\beta+\Omega}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma}\beta - \theta_1 + \frac{\theta_2\beta}{\Omega}\right)E_t\pi_{t+1} = \frac{\beta}{\Omega}E_t\pi_{t+2} + \\
&\left(\frac{1}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma} + \frac{\theta_2}{\Omega}\right)\pi_t + \frac{(1+\beta+\Omega)\,\varphi_{\pi^*} - \beta\varphi_{\pi^*}^2 - 1 + \theta_2(\beta\varphi_{\pi^*} - 1)}{\Omega}\pi_t^* \\
&+(\Gamma_0 - 1)\theta_0 a_t + (\theta_3 - 1)i_t^* + \varepsilon_{it}
\end{aligned} \tag{8.13}$$

We now rewrite equation (8.13) one period before[67]:

$$\begin{aligned}
&\left(\frac{1+\beta+\Omega}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma}\beta - \theta_1 + \frac{\theta_2\beta}{\Omega}\right)\pi_t \\
= \; &\frac{\beta}{\Omega}E_t\pi_{t+1} + \left(\frac{1}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma} + \frac{\theta_2}{\Omega}\right)\pi_{t-1} \\
&+\frac{(1+\beta+\Omega)\,\varphi_{\pi^*} - \beta\varphi_{\pi^*}^2 - 1 + \theta_2(\beta\varphi_{\pi^*} - 1)}{\Omega}\pi_{t-1}^* \\
&+(\theta_3 - 1)i_{t-1}^* + (\Gamma_0 - 1)\theta_0 a_{t-1} + \varepsilon_{it-1} \\
&+\left(\frac{1+\beta+\Omega}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma}\beta - \theta_1 + \frac{\theta_2\beta}{\Omega}\right)\varepsilon_{\pi t}
\end{aligned} \tag{8.14}$$

By using a simplified notation for equation (8.14) we have obtained the following second order differential equation:

$$a_1\pi_t = a_2E\pi_{t+1} + a_3\pi_{t-1} + b_1\pi_{t-1}^* + b_2i_{t-1}^* + b_3a_{t-1} + \varepsilon_{it-1} + a_1\varepsilon_{\pi t} \tag{8.15}$$

[67]We are assuming that $E_t\pi_{t+1}$ can be written at time t as $E_{t-1}\pi_t$ with $\pi_t = E_{t-1}\pi_t + \varepsilon_{\pi t}$, $\varepsilon_{\pi t} \sim i.i.d.(0, \sigma^2)$.

where

$$a_1 = \left(\frac{1+\beta+\Omega}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma}\beta - \theta_1 + \frac{\theta_2\beta}{\Omega}\right)$$
$$a_2 = \frac{\beta}{\Omega}$$
$$a_3 = \left(\frac{1}{\Omega} - \theta_0\frac{\Gamma_1}{\Omega\sigma} + \frac{\theta_2}{\Omega}\right)$$
$$b_1 = \frac{(1+\beta+\Omega)\varphi_{\pi^*} - \beta\varphi^2_{\pi^*} - 1 + \theta_2(\beta\varphi_{\pi^*} - 1)}{\Omega}$$
$$b_2 = (\theta_3 - 1)$$
$$b_3 = (\Gamma_0 - 1)\theta_0$$

Equation (8.15) can be solved by hand. We guess its final form:

$$\pi_t = \eta_\pi \pi_{t-1} + \eta_{\pi^*}\pi^*_{t-1} + \eta_{i^*} i^*_{t-1} + \eta_a a_{t-1} + \eta_{\varepsilon_\pi}\varepsilon_{\pi t-1} + \eta_{\varepsilon_i}\varepsilon_{it-1} \tag{8.16}$$

In (8.16) π_{t-1} is treated as state endogenous variable and all the others variables appearing in the right-hand-side are state exogenous variables. We substitute (8.16) into (8.15), we equalize the coefficients and obtain two roots for η_π :

$$\eta_\pi^{1,2} = \frac{a_1 \pm \sqrt{a_1 - 4a_2a_3}}{2a_2}$$

and then the following roots for the exogenous variables:

$$\begin{aligned}
\eta_{\pi^*} &= \frac{b_1}{a_1 - a_2\eta_\pi - \varphi_{\pi^*}} \\
\eta_{i^*} &= \frac{b_2}{a_1 - a_2\eta_\pi - \varphi_{i^*}} \\
\eta_a &= \frac{b_3}{a_1 - a_2\eta_\pi - \varphi_a} \\
\eta_{\varepsilon_\pi} &= \frac{a_1}{a_1 - a_2\eta_\pi} \\
\eta_{\varepsilon_i} &= \frac{1}{a_1 - a_2\eta_\pi}
\end{aligned}$$

The only unknown coefficients are the parameters of the rule that we are going to estimate in the following section.

8.4 Estimation of the Monetary Policy Rule

We estimated equation (8.4) by Instrumental Variables. The two countries are taken to be the UK and the average of the EU countries. The data used are quarterly, deseasonalized and detrended and they cover the period 1979:4-1998:2.

Useful instruments for $E_t\pi_{t+1}$ are suggested by equations (8.1) and (8.2). The IV estimation led to the following results:

Table 8.1 : Estimates of equation (8.4) by IV

$$i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \theta_2 (s_t - \widehat{p}_t) + \theta_3 i_t^* + \varepsilon_{it}$$

	1979-85	1979-92	1979-98	1985-92	1992-98
θ_0	0.27 [3.69]	0.28 [4.63]	0.21 [4.07]	0.69 [4.60]	0.82 [4.11]
θ_1	0.05 [0.92]	0.066 [2.55]	0.09 [3.89]	0.063 [2.75]	0.22 [10.98]
θ_2	−0.029 [−1.55]	0.019 [1.41]	0.004 [0.42]	0.02 [1.27]	−0.015 [−1.70]
θ_3	0.39 [2.22]	0.32 [2.14]	0.19 [1.68]	−0.55 [−2.23]	−0.87 [−2.76]

Note: t-stat between brackets. Instruments used for $E_t\pi_{t+1}$: a_t,a_{t-3},$i{-}i*_{t-2}$,$i{-}i*_{t-3}$,π_{t-1},$const$.

We have divided the sample period into three sub-samples, according to the institutional changes that have occurred to the Bank of England. Until 1985 the UK Central Bank pursued a policy of monetary targeting. From 1985 to 1992 it adopted exchange rate targeting. During the period 1990-1992 the pound entered the strict band of the Exchange Rate Mechanism (ERM). After 1992 the UK Central Bank moved toward inflation targeting using the interest rate as the policy instrument. In 1997 the Bank of England was given operational independence and an explicit inflation target of 2.5% based on the RPIX[68] index.

[68]Retail Price Index excluding mortgage interest payments.

From table 8.1 we notice that the estimates of the sub-samples differ from those of the whole sample. The robustness of the coefficients in the shortest sample periods is doubtful since we have only a small sample of observations.

Table 8.1 shows that in each sub-sample the estimated coefficients of θ_0 and θ_1 are very different from those obtained by Clarida, Gali and Gertler (1997) who proposed a forward Taylor rule. We did not find empirical support for the UK that the coefficients of the Taylor rule are robust when we allow for foreign variables.

In the whole sample, and in each sub-sample, the output gap plays an important role in explaining movements of the domestic nominal interest rate. Its coefficient is always significant and positive. We notice a difference in the size of the coefficient before and after 1985. In the first sub-period the interest rate elasticity to a change in output gap is nearly 0.3% in the last two sub-samples it is about 0.7-0.8%.

The foreign interest rate is an important factor in explaining movements of the domestic interest rate. Its coefficient is positive and significant in the first sub-sample but becomes negative in the last two sub-periods. The negative coefficient suggests that under inflation targeting the Bank of England has not responded to the potential capital movements arising from a widening interest differential.

Expected future inflation appears to have played an increasing important role, and is highly significant in the period of inflation targeting.

The real exchange rate seems to be the least important factor and it is unstable across sub-samples. In the first and last sub-period it is negatively related to the domestic interest rate, whereas it is positively related during the exchange rate targeting regime.

Under inflation targeting, a real depreciation seems to have led to a fall in interest rates whereas one might have expected an increase in order to offset the inflationary consequences.

8.4.1 Adding a smoothing term to the rule

We consider a small variation of the general rule (8.4) by adding the past nominal interest rate among the explicative variables. This modification is made in the spirit of an emerging literature on policy rules (Peersman, Smeetz, 1998; Clarida, Gali, Gertler, 1998) claiming that by allowing for the smoothing behaviour of the Central Bank we can capture its actual decision process. Therefore (8.4) becomes:

$$i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \theta_2 (s_t - \widehat{p}_t) + \theta_3 i_t^* + \theta_4 i_{t-1} + \varepsilon_{it} \tag{8.17}$$

The results of the IV estimates of (8.17) are:

Table 8.2: Estimates of equation (8.17) by IV

$$i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \theta_2 (s_t - \widehat{p}_t) + \theta_3 i_t^* + \theta_4 i_{t-1} + \varepsilon_{it}$$

	1979-85	1979-92	1979-98	1985-92	1992-98
θ_0	0.20 [3.20]	0.12 [2.75]	0.079 [2.34]	0.15 [0.98]	−0.19 [−1.16]
θ_1	0.021 [0.54]	0.036 [2.20]	0.04 [2.92]	0.02 [1.15]	0.02 [1.08]
θ_2	−0.049 [−3.17]	−0.021 [−2.34]	0.006 [1.11]	0.0045 [0.43]	0.014 [2.25]
θ_3	0.39 [2.84]	0.26 [2.67]	0.17 [2.53]	−0.087 [−0.32]	0.46 [1.78]
θ_4	0.56 [3.25]	0.68 [8.23]	0.71 [10.26]	0.75 [6.27]	0.82 [11.28]

Note: t-stat between brackets. Instruments used for $E_t\pi_{t+1}$: a_t,a_{t-3},$i-i*_{t-2}$,$i-i*_{t-3}$,π_{t-1},$const$.

Table 8.2 shows very different results from the previous set of estimates. The major difference relies on the behaviour of the output gap. Adding the smoothing term reduces the explicative power of the output gap in explaining the systematic component of the monetary policy. As we can see from the diagnostic tests (Appendix G) in the previous set of estimates there is always a problem of autocorrelated residuals, that can be reduced but not eliminated by augmenting the number of instruments in the estimation. This problem disappears once we introduce a lag in the domestic interest rate among the explicative variables. Another way of tackling the problem has been to allow for autocorrelated errors in the estimation of equation (8.4). We reestimated the general rule with Residual Autocorrelated Least Squares (Appendix G), we did not found significant differences with the value obtained in table 8.1.

In our theoretical environment there is no explicit justification for introducing the smoothing term in the rule. From table 8.2 we can compute the long-run elasticities[69] and notice, for the whole sample and for the first two sub-samples, that the results get closer to those obtained in table 8.1.

8.4.2 Estimation of the Exogenous Processes

We represented the exogenous processes of the model in a first order vector autoregressive form that we have estimated by OLS. We have obtained the following

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Implied long-run elasticities of table 8.2

	1979-85	1979-92	1979-98	1985-92	1992-98
x	0.45	0.38	0.27	0.60	−1.05
π	0.05	0.11	0.012	0.08	0.11
$s-\widehat{p}$	−0.11	−0.07	0.021	0.02	0.08
i^*	0.88	0.81	0.59	−0.35	2.56

results[70]:

$$\begin{bmatrix} i_t^* \\ \pi_t^* \\ a_t \end{bmatrix} = \begin{bmatrix} \underset{[11.05]}{0.81} & \underset{[0.82]}{0.016} & \underset{[0.32]}{0.004} \\ \underset{[1.75]}{0.70} & \underset{[3.50]}{0.45} & \underset{[-0.093]}{-0.007} \\ \underset{[-1.03]}{-0.19} & \underset{[-2.39]}{-0.12} & \underset{[26.2]}{0.919} \end{bmatrix} \begin{bmatrix} i_{t-1}^* \\ \pi_{t-1}^* \\ a_{t-1} \end{bmatrix}$$

The estimates show that foreign inflation has the lowest autoregressive coefficient and the technology parameter the highest. Among the off-diagonal elements there are no significant values in the equation for i_t^*. Conversely, π_t^* depends on i_{t-1}^* as well as on its own past value. Changes in π_{t-1}^* negatively determine the behaviour of a_t.

8.5 Simulation Results

We are now supplied of all the values necessary to quantify the elasticities, thus we can proceed with the derivation of the solution of the model. We compute the roots of the equation (8.16) for each sub-sample, for the long-run smoothing rule and for two 'nested' rules, whose formulation is well known in the existing literature on policy rules. Namely, starting from $i_t = \theta_0 x_t + \theta_1 E_t \pi_{t+1} + \theta_2(s_t - \widehat{p}_t) + \theta_3 i_t^*$ we consider 2 cases and we impose for both of them $\theta_2 = \theta_3 = 0$ (Clarida, Gali and Gertler, 1998 and Haldane and Batini, 1998):

	θ_0	θ_1
Taylor	0.5	1.5
Forward	0	1.5

The first rule, that we have called Taylor, differs from that put forward by Taylor (1993) because expected inflation replaces current inflation. The second rule imposes the zero restriction on the output gap term.

[70] t-stat. between brackes. Sample period runs from 1979:2 to 1998:1. The variables have been linearly detrended before the estimation.

For each estimated sub-sample of the General rule, for the long-run values of the smoothing rule and for the two rules above mentioned we obtained the following roots in the final form of (8.16):

Table 8.3: The roots of equation (8.16)

	η_π^1	η_π^2	η_{π^*}	η_{i^*}	η_a	η_{ε_i}	η_{ε_π}
1980-85	1.33	0.57	-0.018	-0.042	0.021	0.069	1.524
1985-92	1.099	0.43	-0.035	-0.131	0.066	0.086	1.504
1992-98	1.148	0.29	-0.025	-0.151	0.075	0.082	1.354
Smoothing	1.155	0.72	-0.031	-0.033	0.025	0.081	1.736
Taylor	1.106	0.55	-0.027	-0.084	0.048	0.085	1.62
Forward	1.237	0.82	-0.035	-0.075	0.00	0.075	1.78

Our model leads to a saddle path solution in each case considered. If we concentrate our attention to the General rule we observe that the stable root is decreasing in the most recent years. η_{π^*}, that measures the elasticity of equilibrium inflation to foreign inflation, is always negative and very low. η_{i^*}, the elasticity to foreign interest rates, is negative and increasing in absolute value during the three sub-samples. η_a, the elasticity to technology shocks, is positive and increasing over time. η_{ε_i} measures the elasticity of inflation to a policy shock. The size of the reaction of equilibrium inflation does not vary very much over time.

The elasticities obtained for the smoothing rule show a much higher value for η_π with respect to the general rule, whereas the other terms behaves very similarly. The Forward rule shows the highest value for η_π, it implies no reaction of inflation to technology shocks whereas it behaves very similarly to the Taylor rule for the other variables.

8.5.1 Impulse Response Functions

For studying the dynamic behaviour of the system we simulate the model (8.6), (8.8), (8.12) with Matlab[71]. This allows us to compute the impulse response functions of the system to exogenous shocks. The information we can gather from this exercise can be used to evaluate the performance of each rule when the system is facing exogenous shocks.

Figures 8.1 show the responses of inflation (inf), interest rate (i) and output gap (x) to a technology shock under different rules. The estimated General rule delivers very similar responses in each of the three sample periods. We can observe a higher response of all the variables (especially output gap and interest rate) in the last sub-sample, i.e. during the inflation targeting regime. During the monetary targeting regime (first sub-sample) the interest rate seems to be less sensitive to a change in a_t. Adding a smoothing term to the rule leads to a behaviour of the system very similar to that corresponding to the first sub-sample. The two calibrated forward rules produce a very different response of the system after a technology shock. The forward Taylor rule implies a highly positive reaction of all the variables, whereas the simple Forward rule implies only a positive reaction of the output gap but not of the inflation and the interest rate.

Figures 8.2 show the system's reactions to a shock in foreign inflation. In the three sample periods we assist to an impact negative reaction of output gap and interest rates that becomes positive after six months. The dimension of the reaction of the interest rate is lower in the first sample period. Under the forward Taylor rule the system's reaction to a shock in foreign inflation is very

[71] We used Uhlig's (1997) codes, like in the previous chapters, whose explanation is in Appendix D.

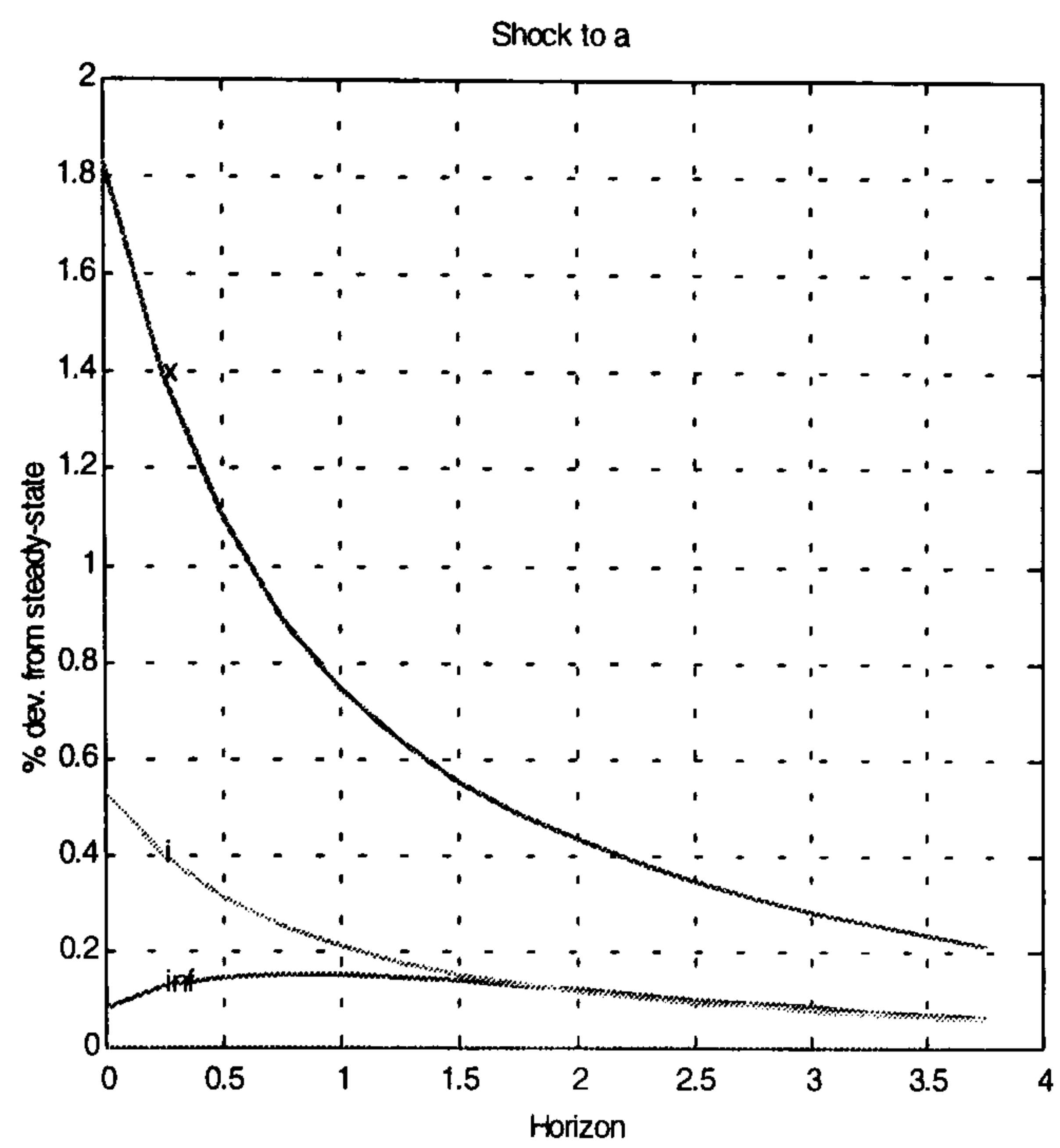
similar to that obtained with the General rule. Conversely, the simple Forward rule produces a very different response of all the variables. Output gap does not undershoot, as happens in the other cases. Moreover, the shock does not lead the domestic interest and inflation rate back to equilibrium.

Figures 8.3 show the system's reactions after a shock to the foreign interest rate. The general rule produces different outcomes according to the monetary regime considered with respect to the behaviour of the domestic interest rate. It reacts positively on impact only in the first sub-sample, whereas in the other two sub-samples the reaction is negative and becomes positive after six months. This result is in line with the estimated elasticity of the domestic interest rate to a change in the foreign rate in the last two sub-samples. While the pattern of the economy under the Taylor rule is similar to that under the General rule, we obtain different results under the simple Forward rule. Like a shock to π_t^*, a shock to i_t^* leads to a negative reaction of all the variables and to a destabilizing effect on π_t and i_t.

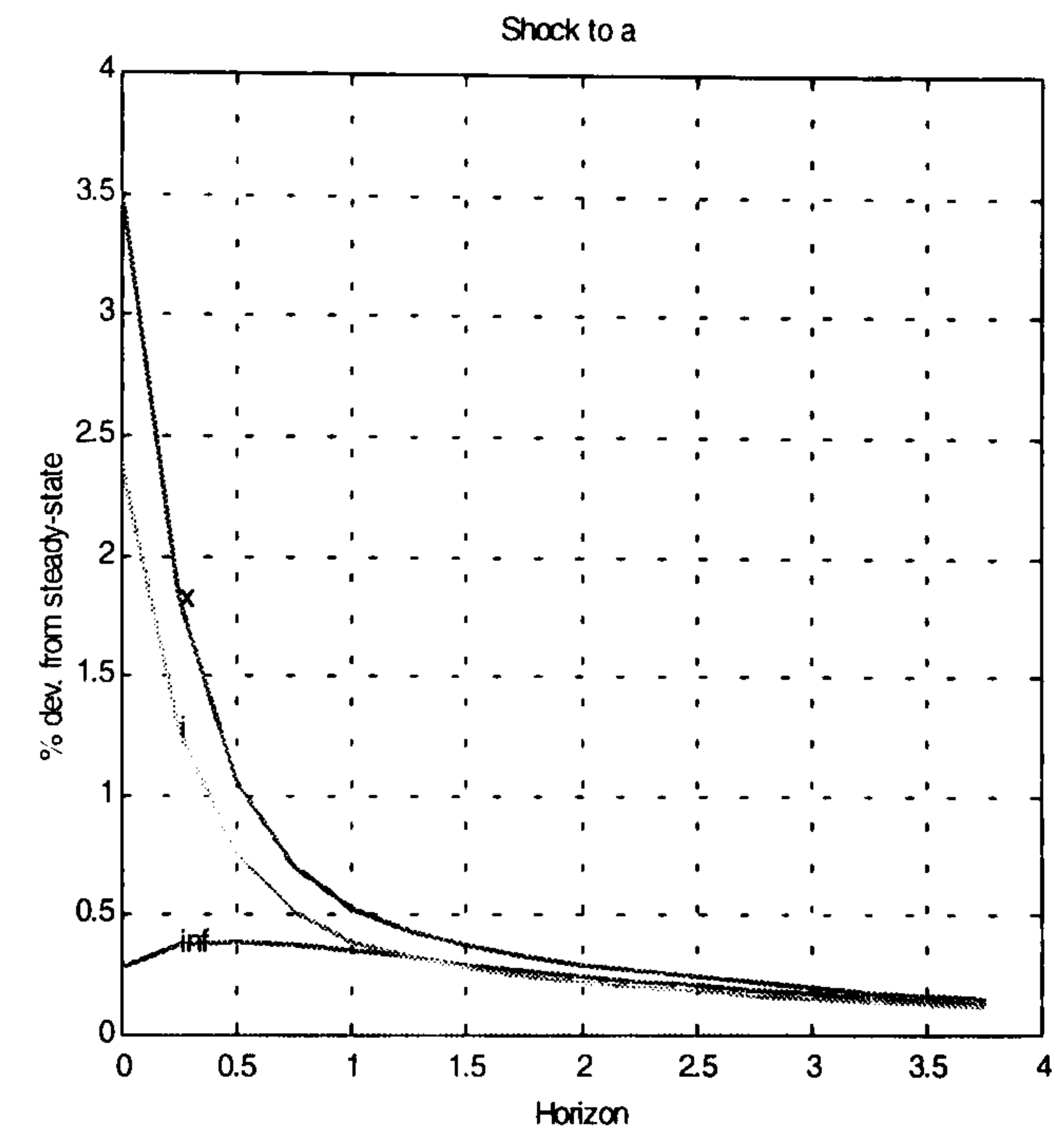
Figures 8.4 show the system's responses to an exogenous policy shock. An *i.i.d.* shock to the policy rule increases on impact the nominal interest rate. In general, the reaction of inflation and output gap is negative and lasts for almost two years. In each of the three sample periods we can observe some differences in the dynamics of the output gap and inflation. In the first sub-sample the impact reaction is positive and becomes negative after two months. Under the Forward rule we observe a positive (but very small) reaction of inflation. The response of the output gap is negative (after a positive impact) and very long lasting.

Figures 8.1: Impulse response functions to a shock in a

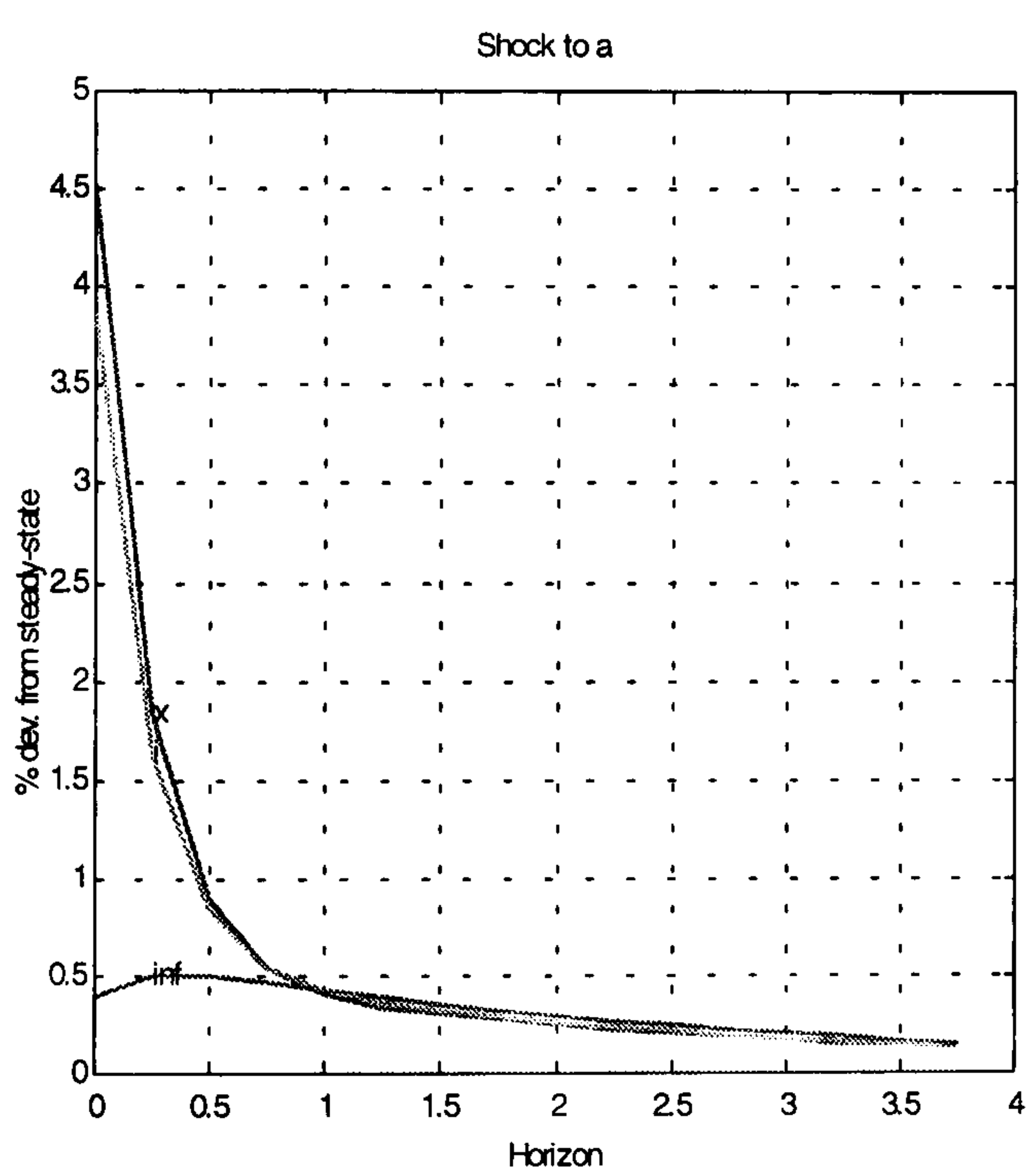
1) 1980-85



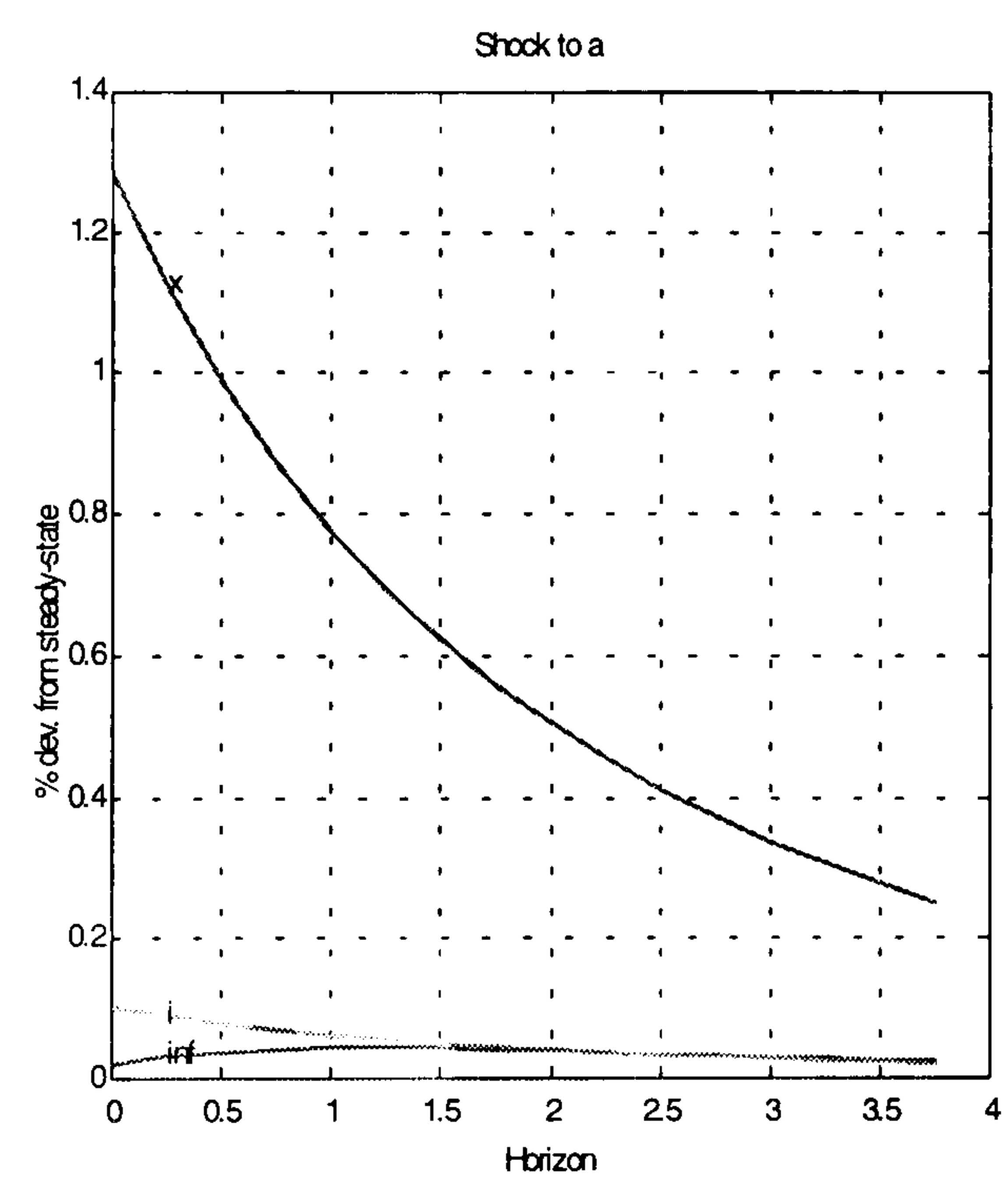
2)1985-92



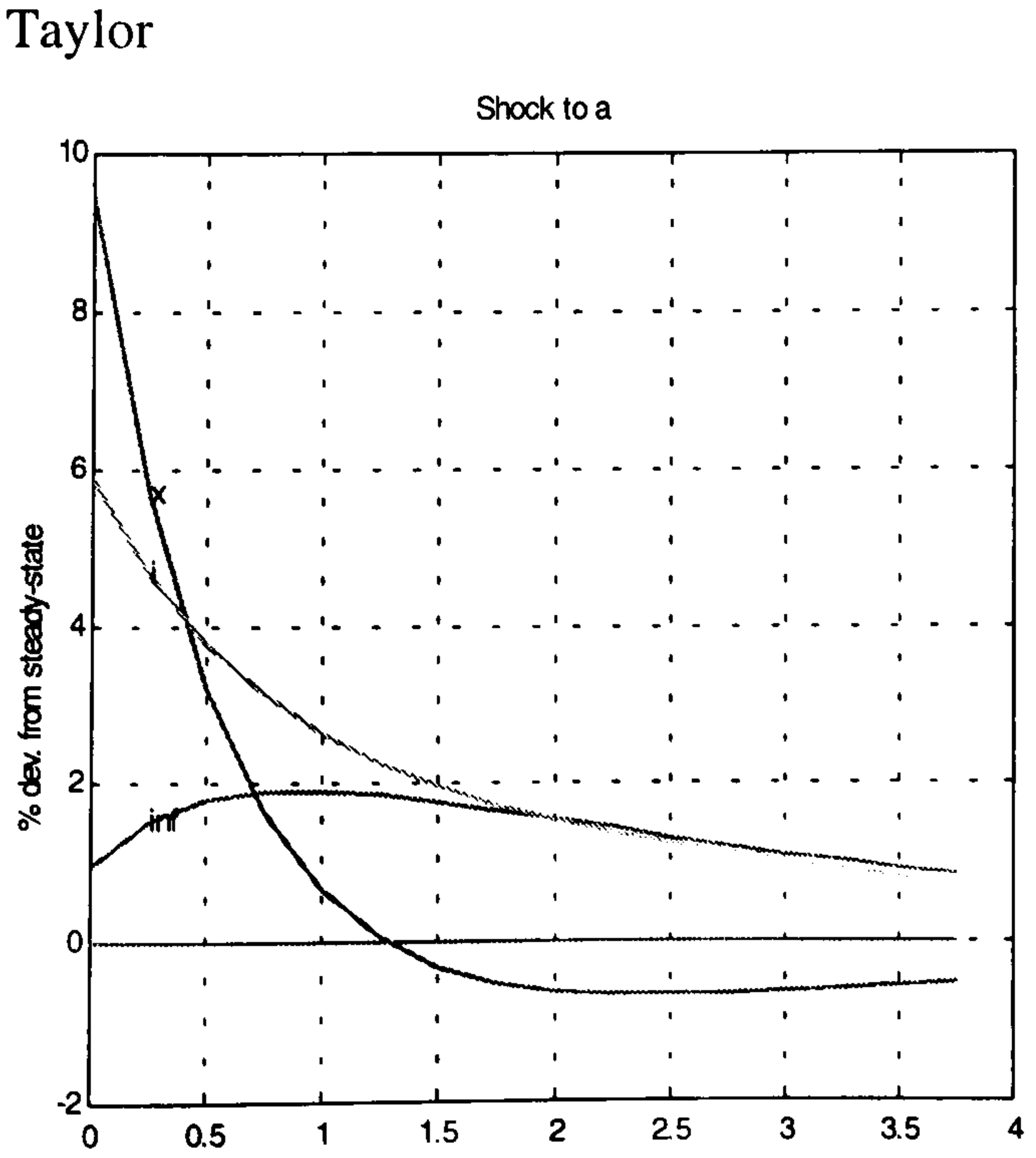
3) 1992-98



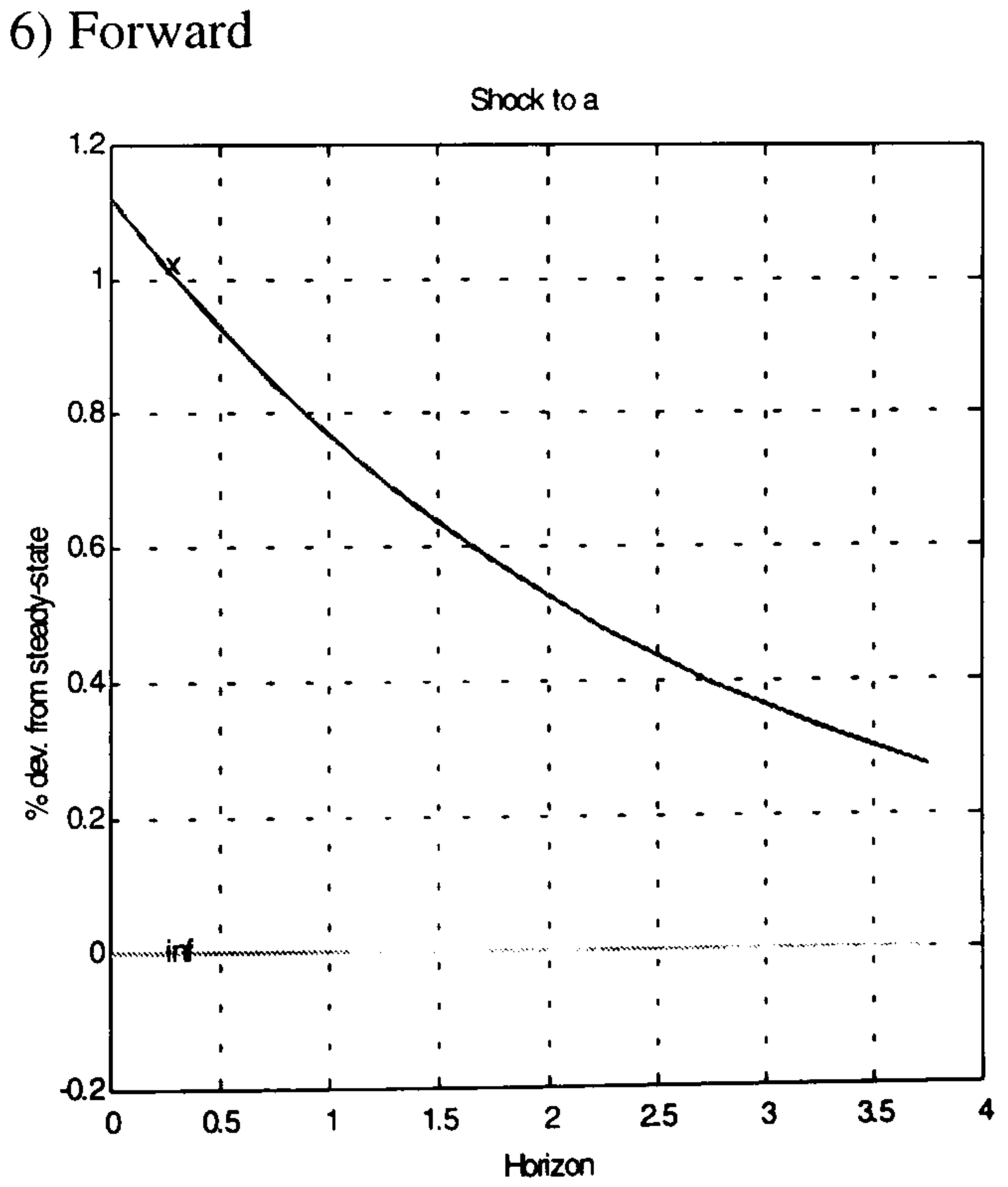
4) Smoothing



5) Taylor

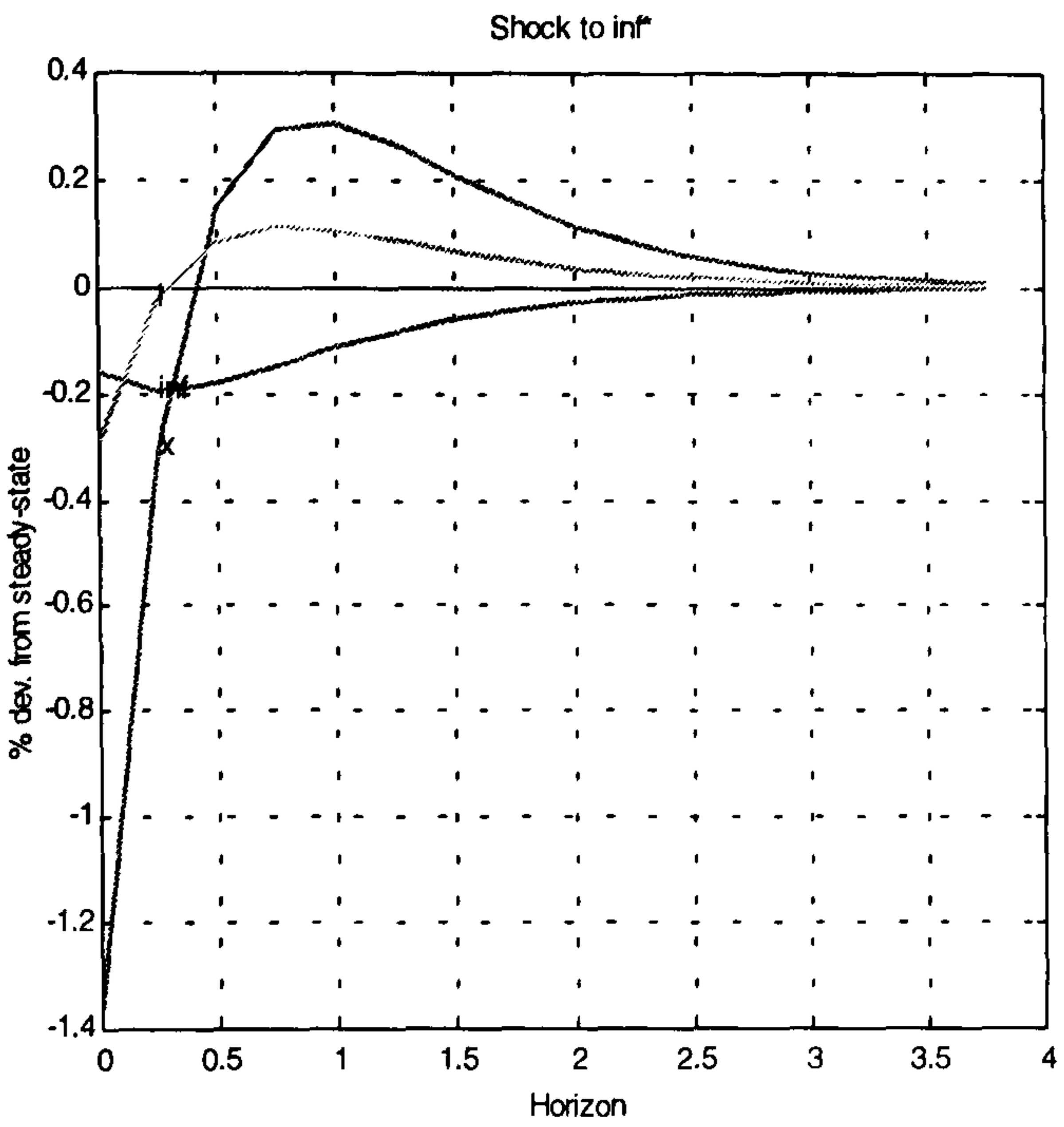


6) Forward

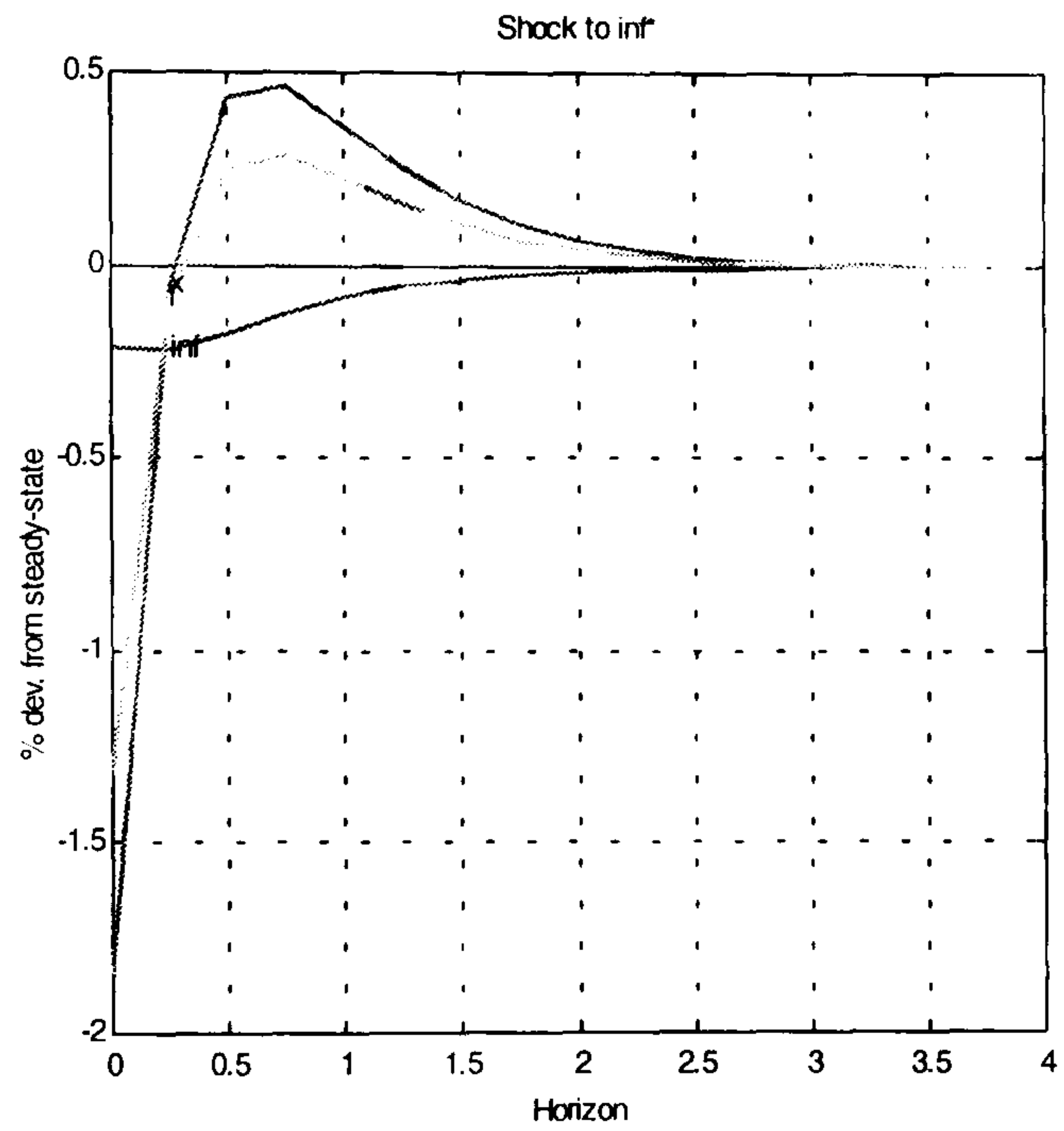


Figures 8.2: Impulse response functions to a shock in π^*

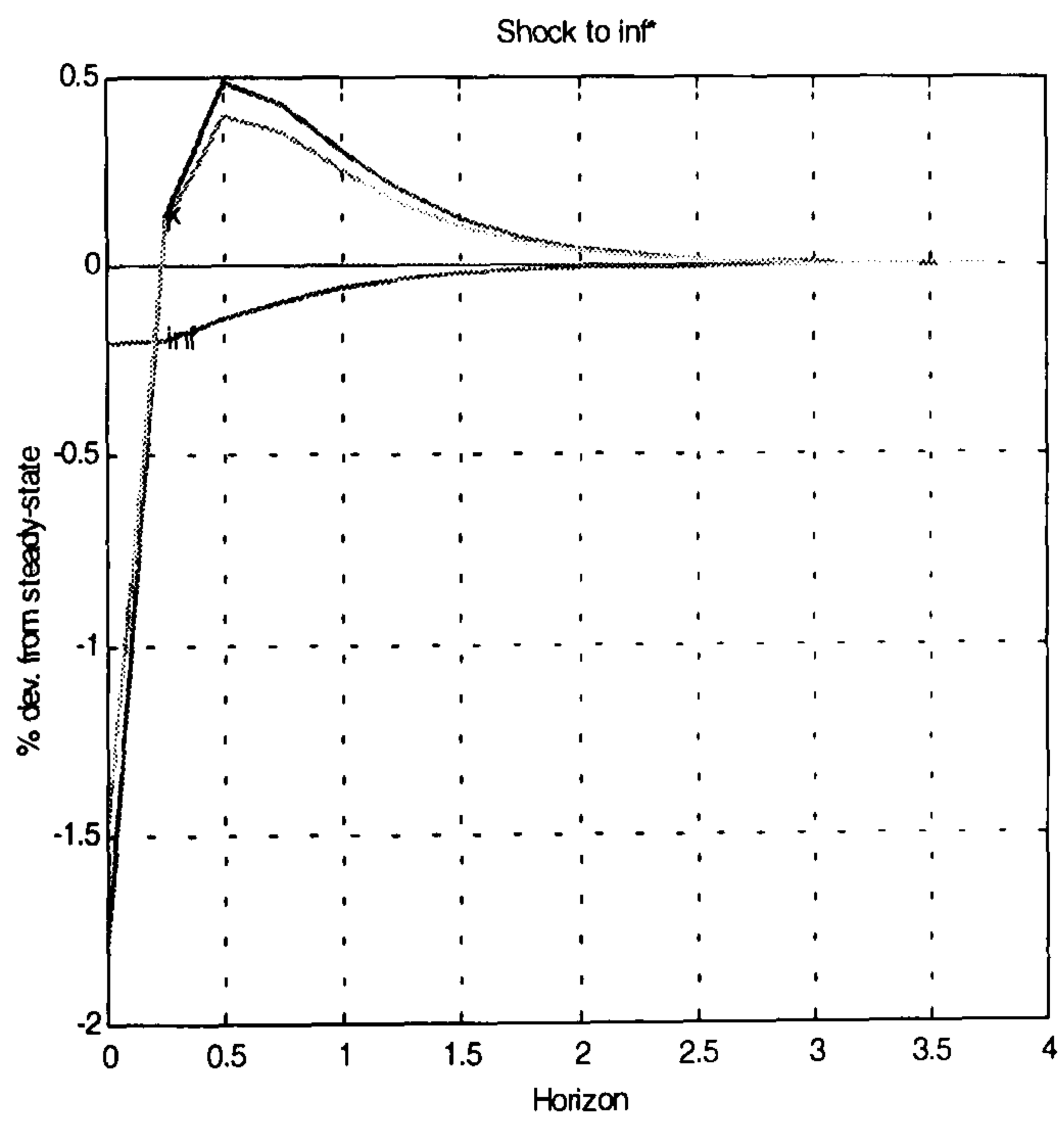
1) 1980-85



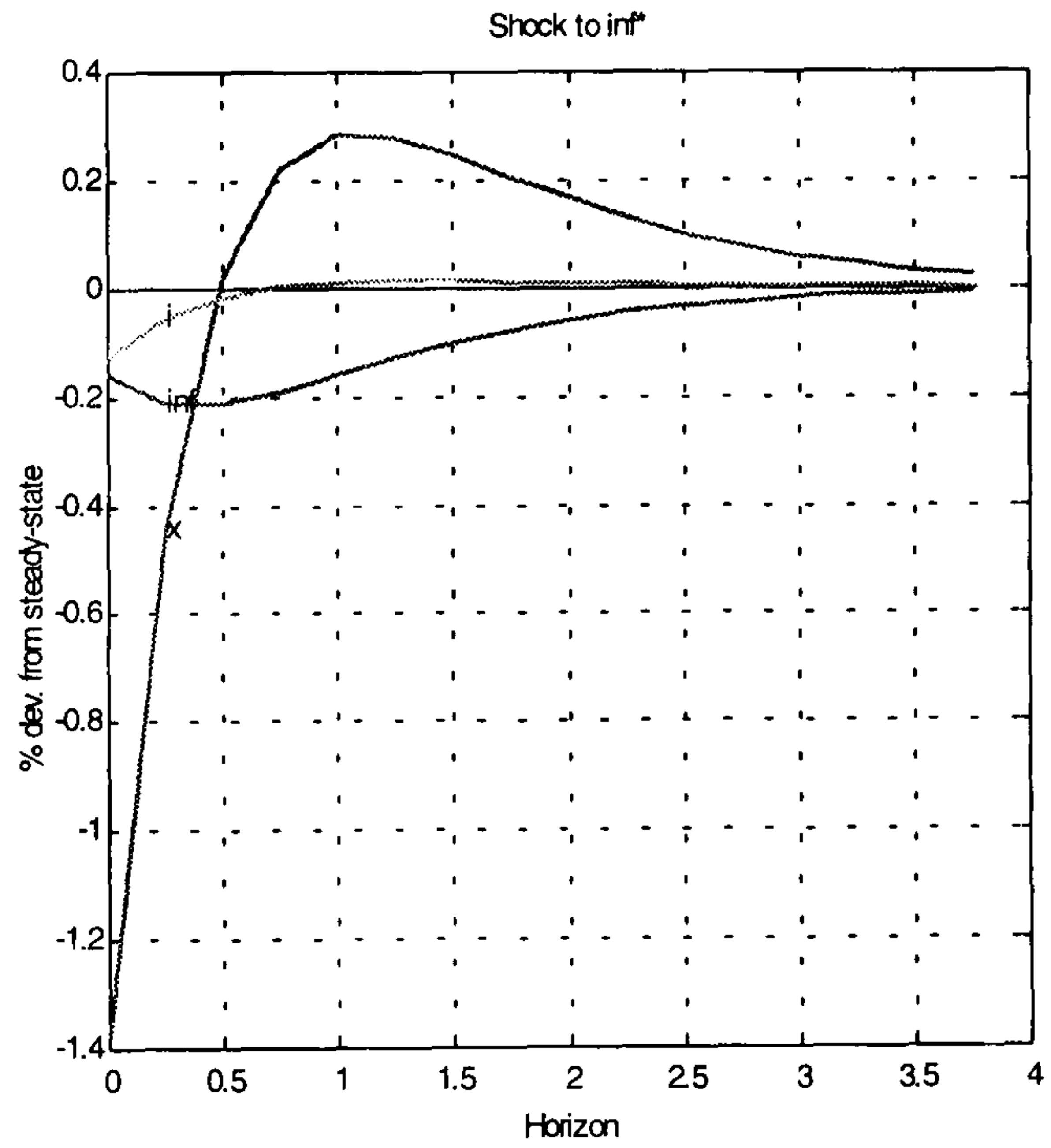
2)1985-92



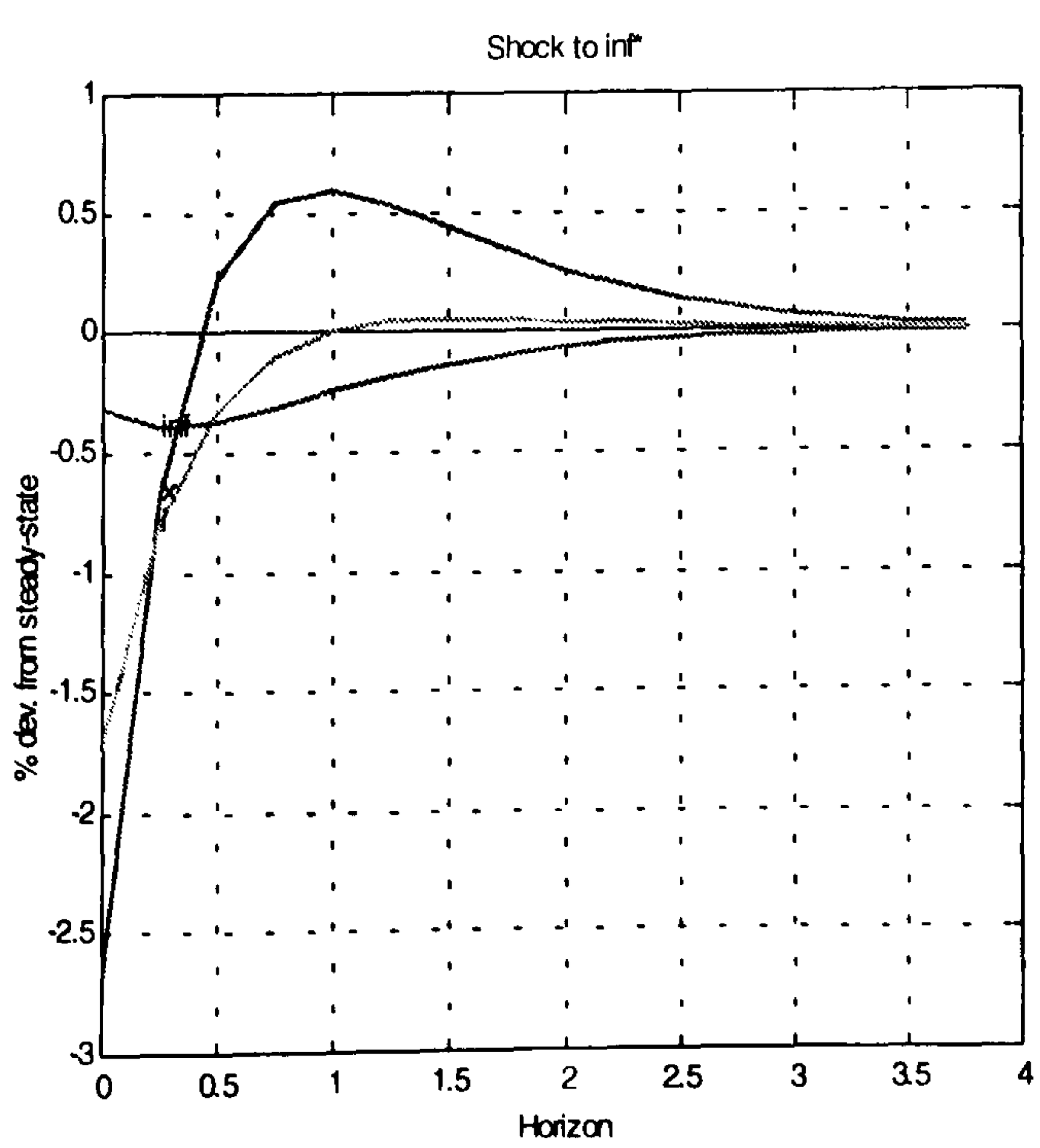
3) 1992-98



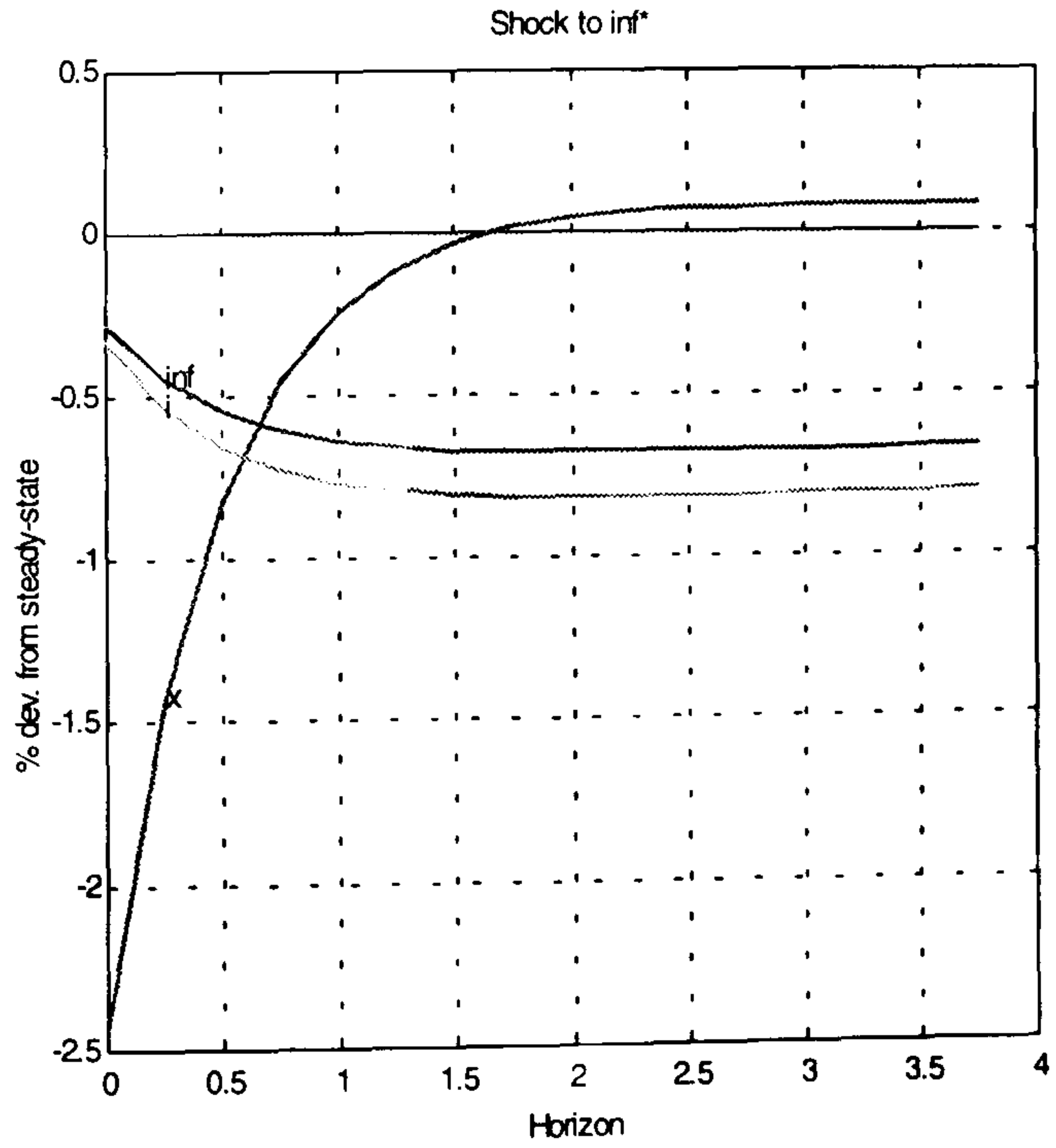
4) Smoothing



5) Taylor

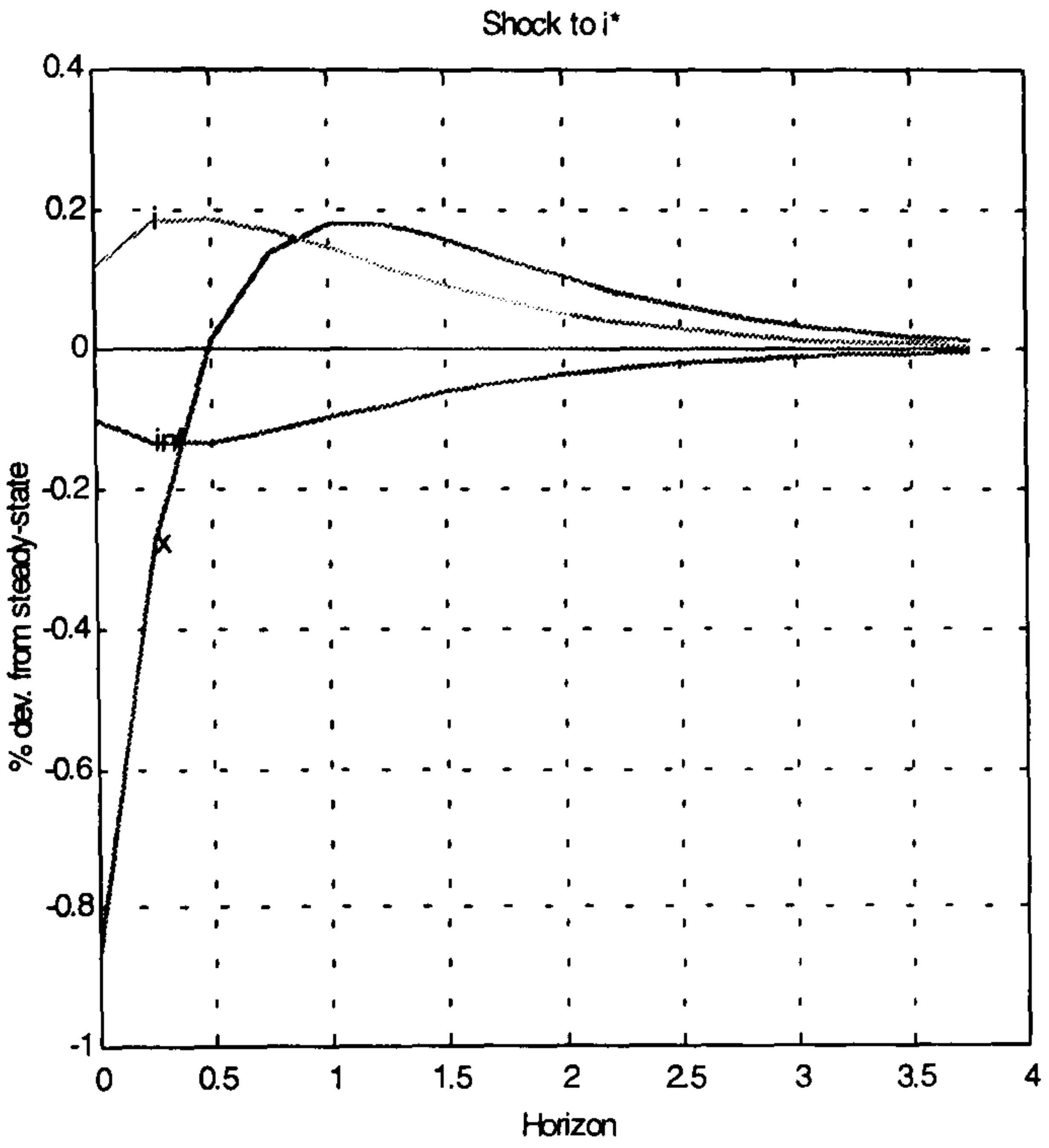


6) Forward

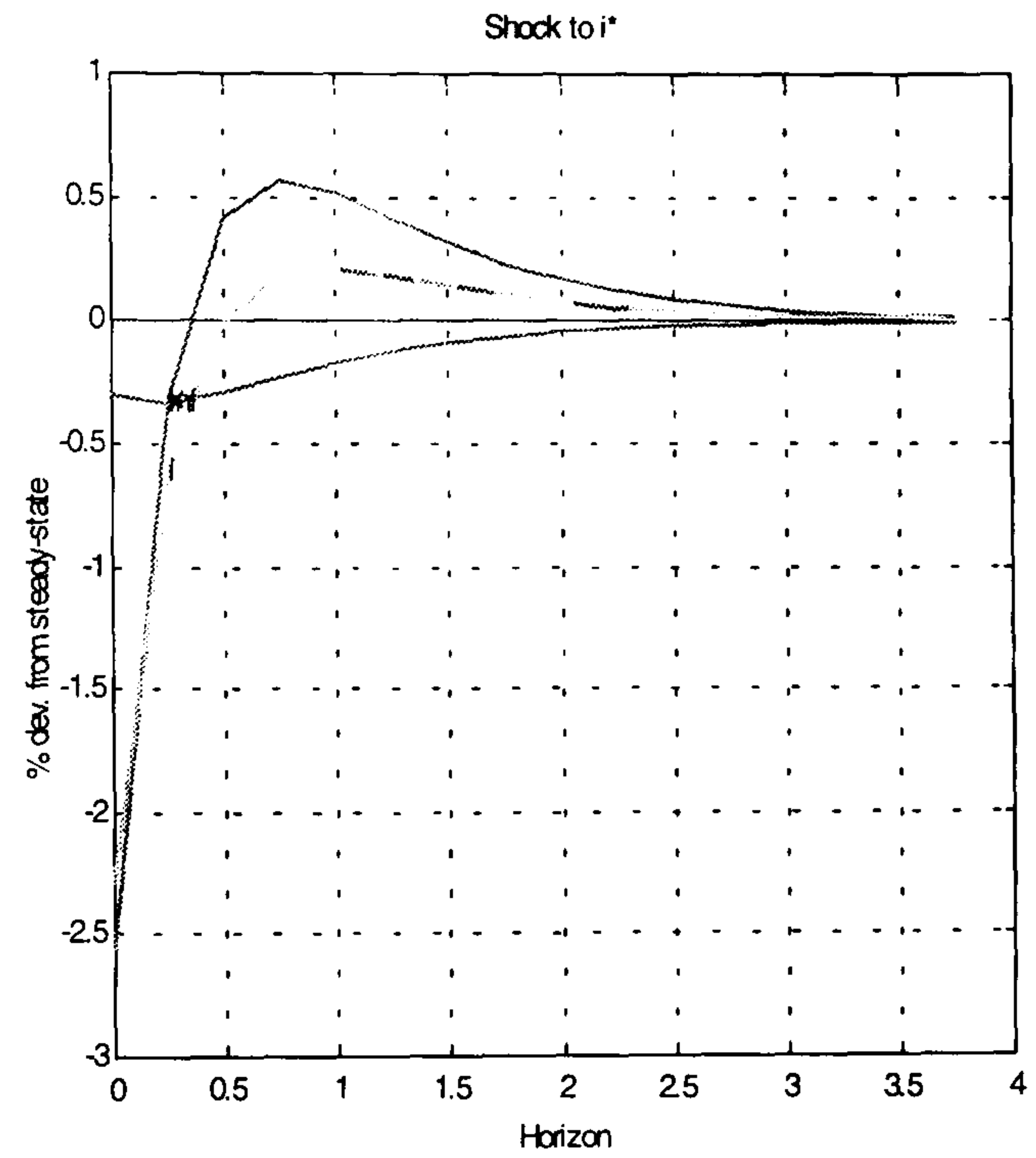


Figures 8.3: Impulse response functions to a shock in i^{*}

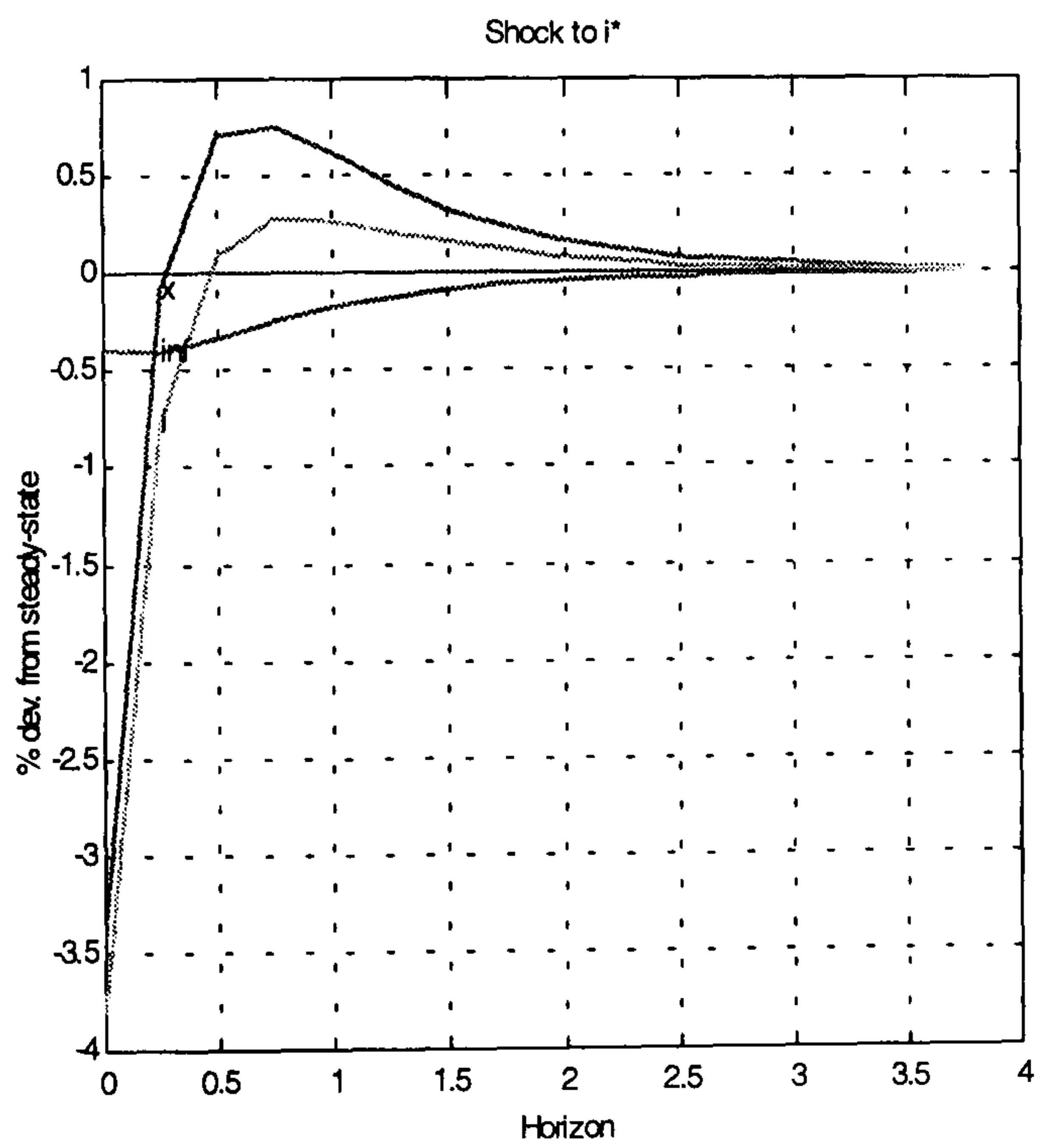
1) 1980-85



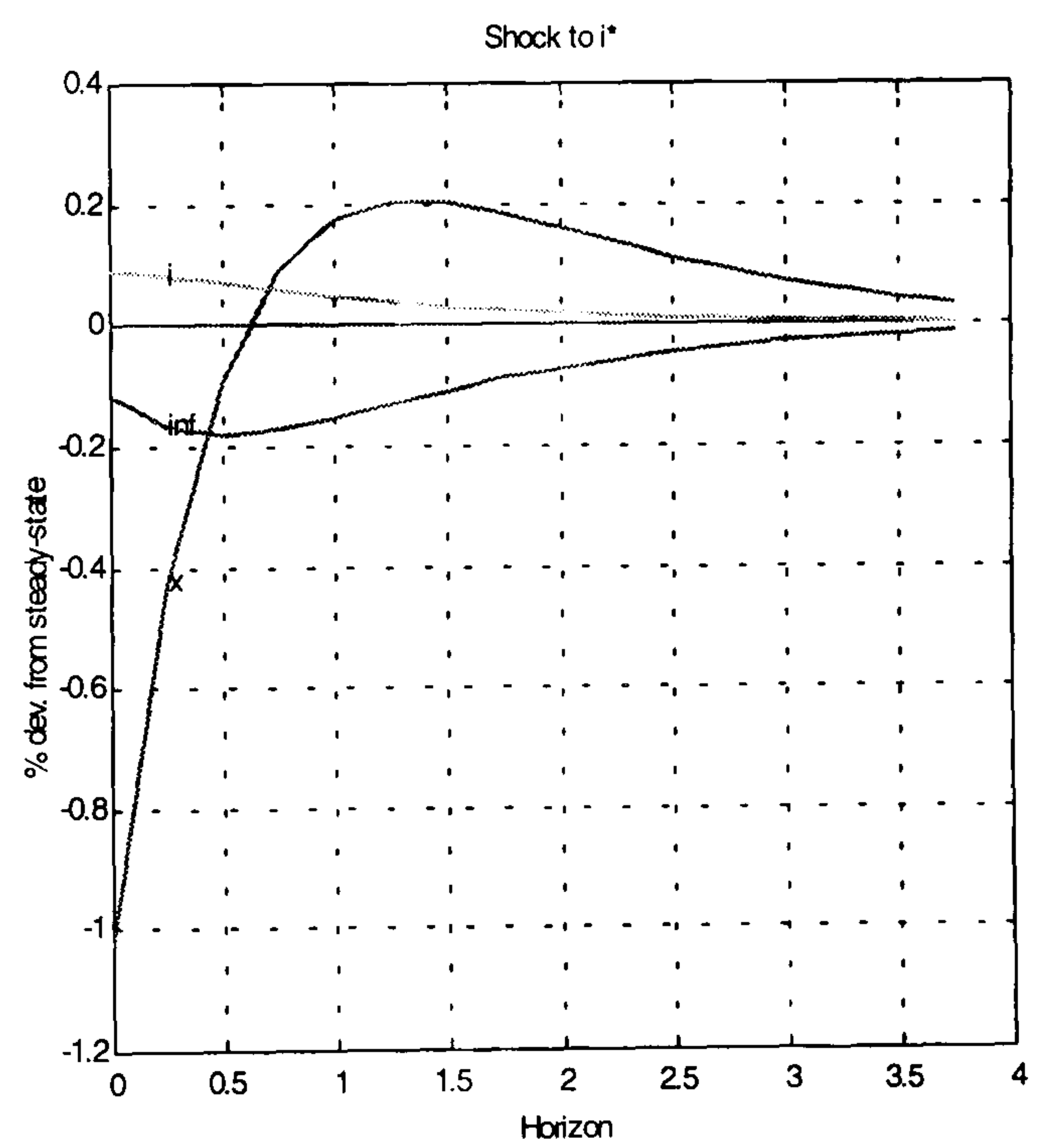
2)1985-90



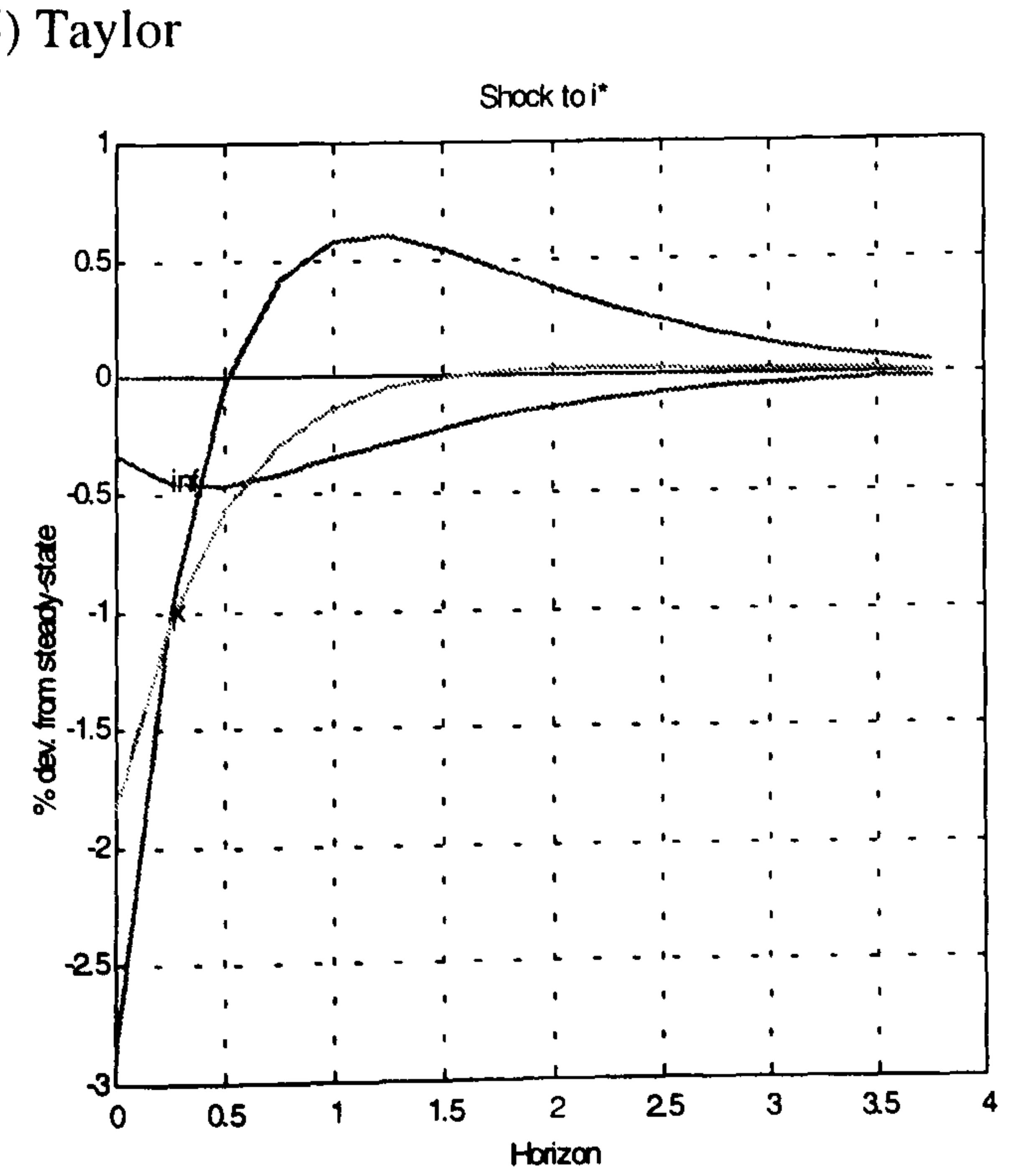
3) 1992-98



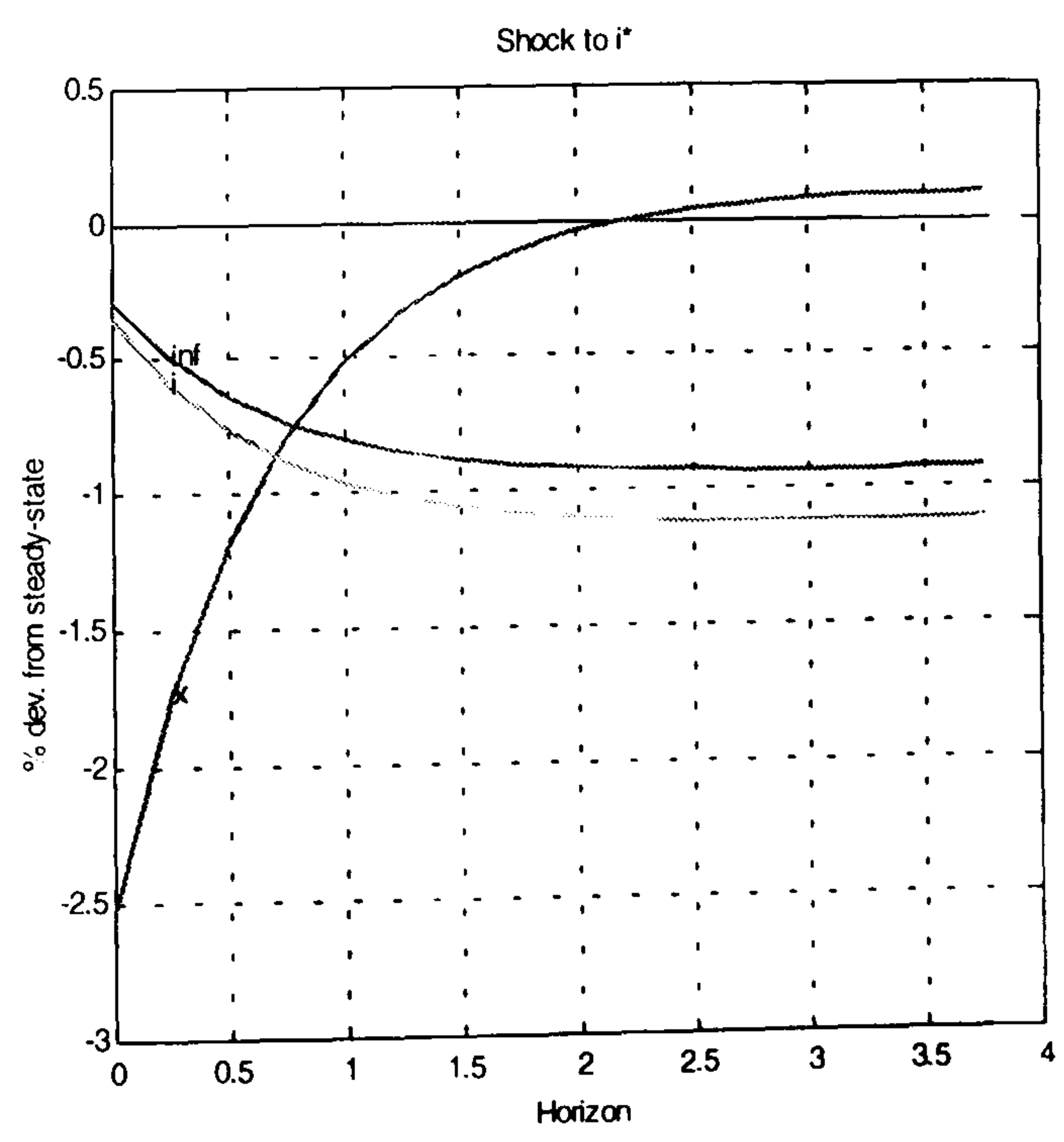
4) Smoothing



5) Taylor

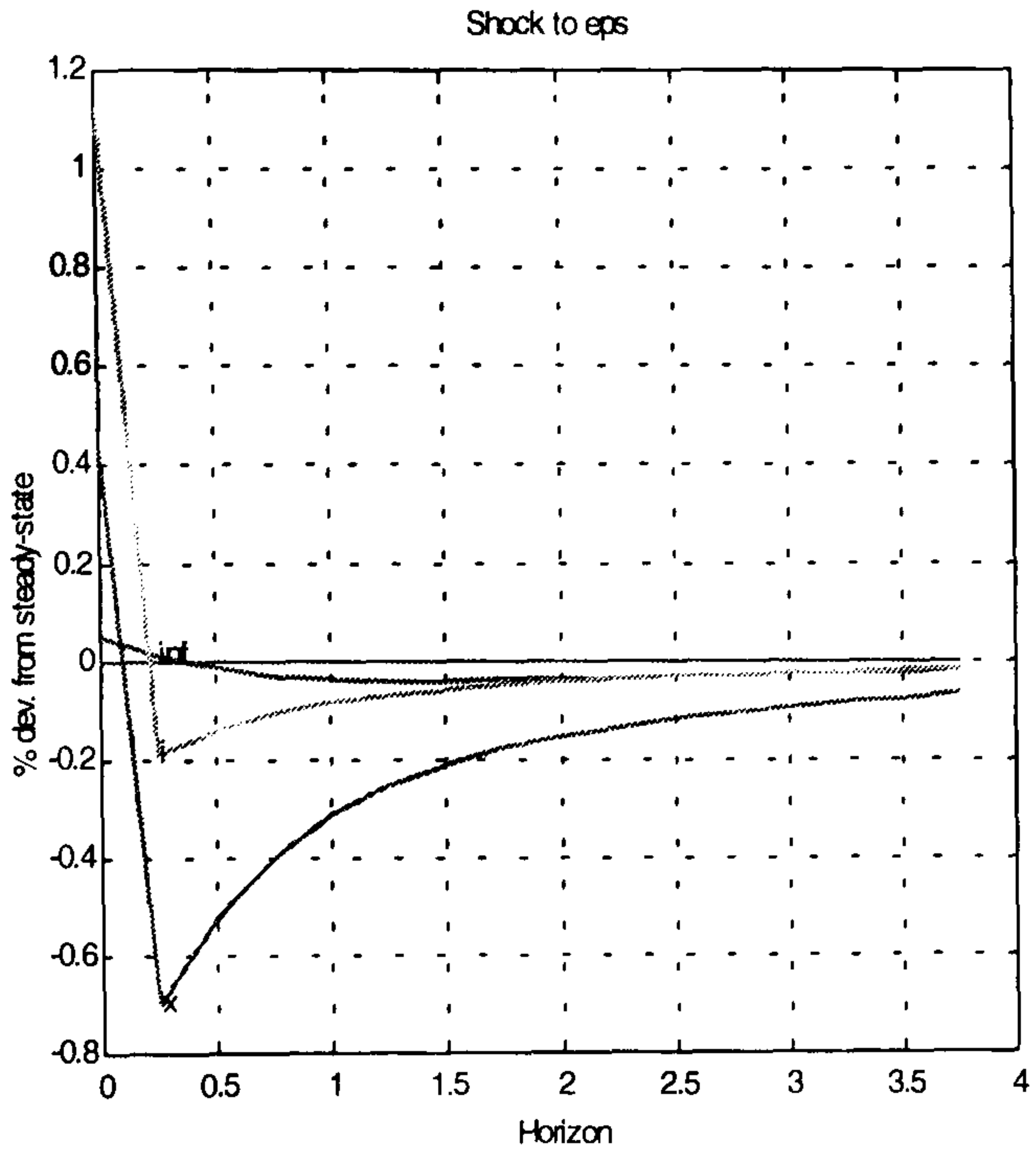


6) Forward

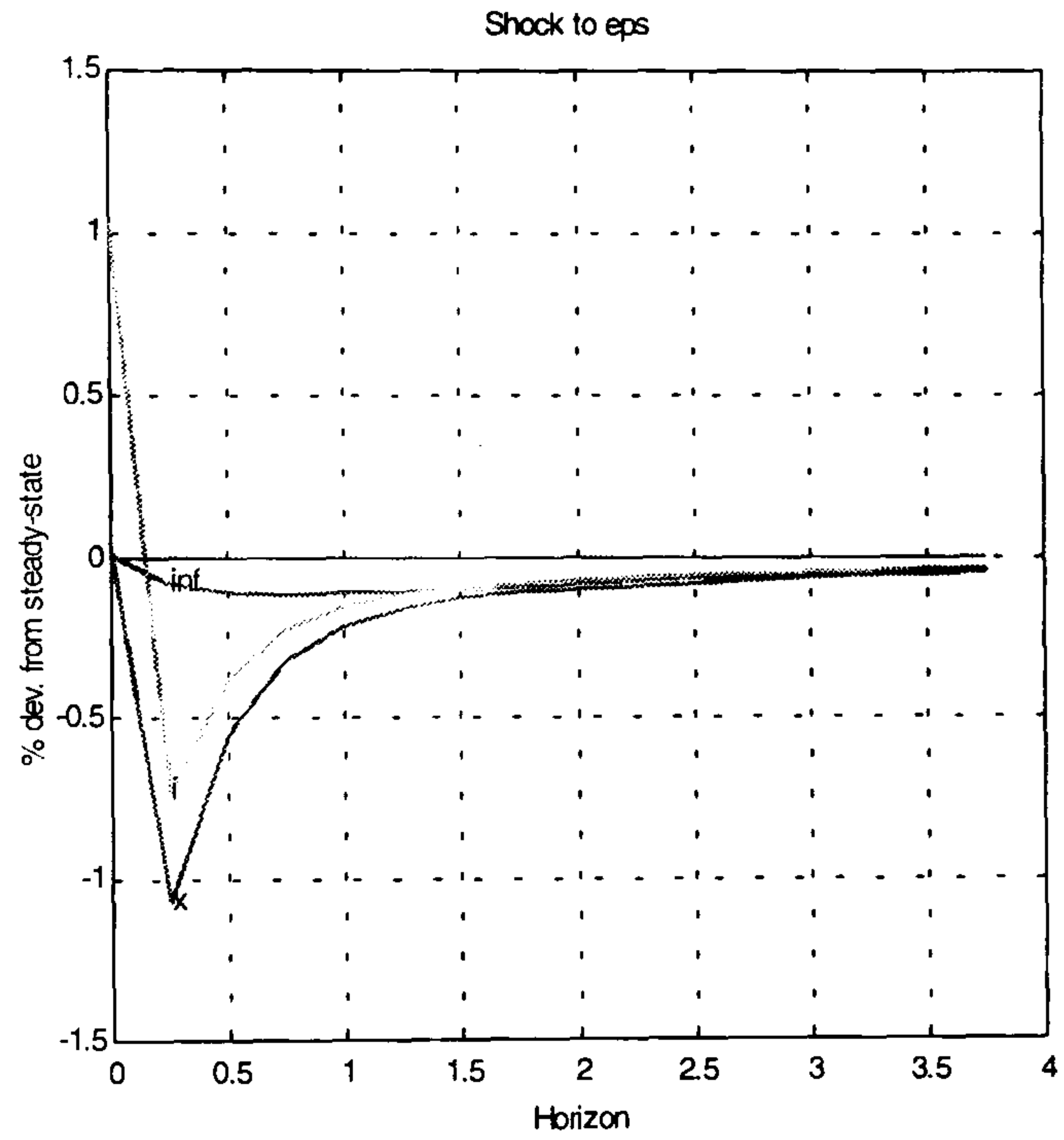


Figures 8.4: Impulse response functions to a shock in ε^{j}

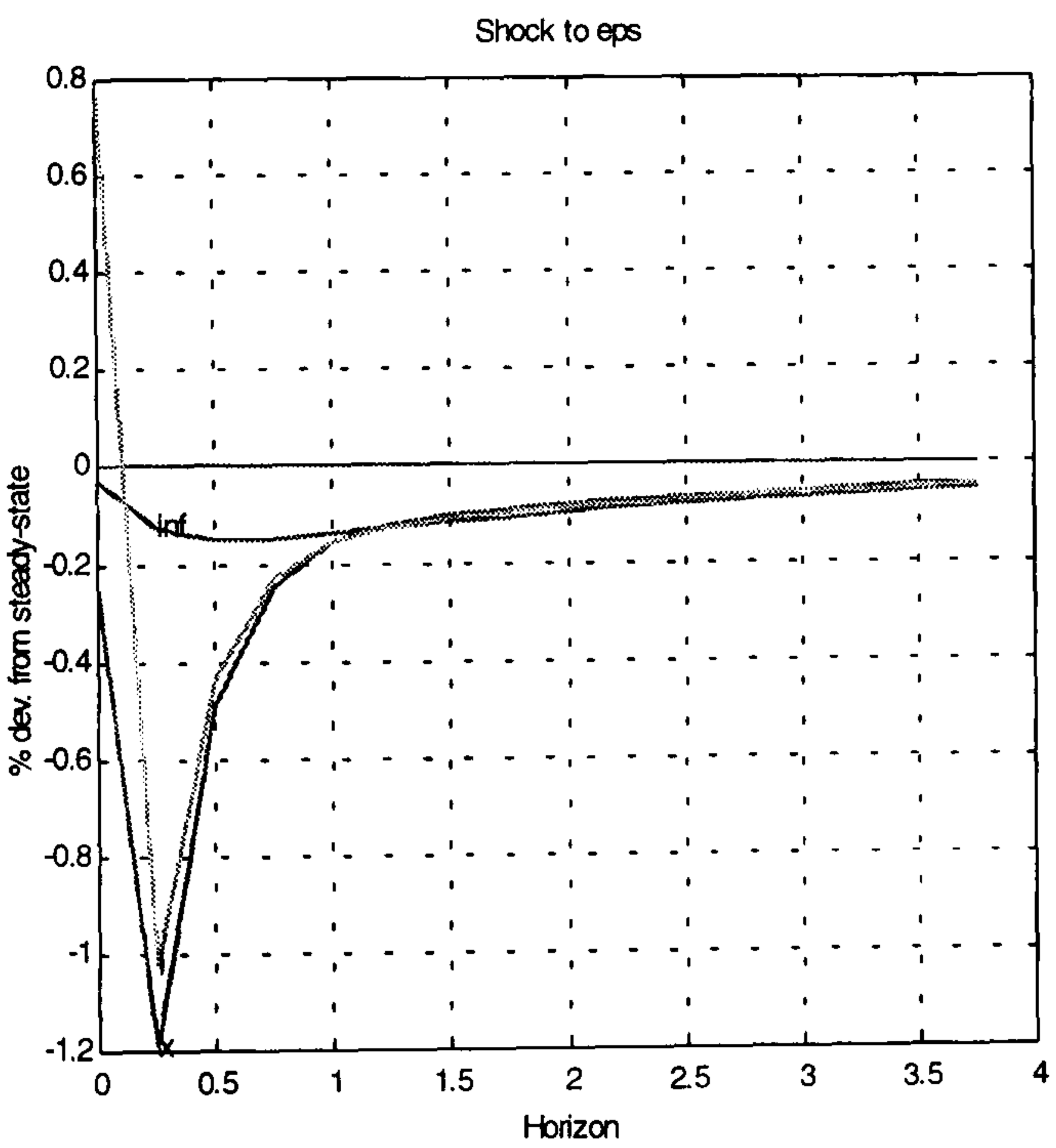
1) 1980-85



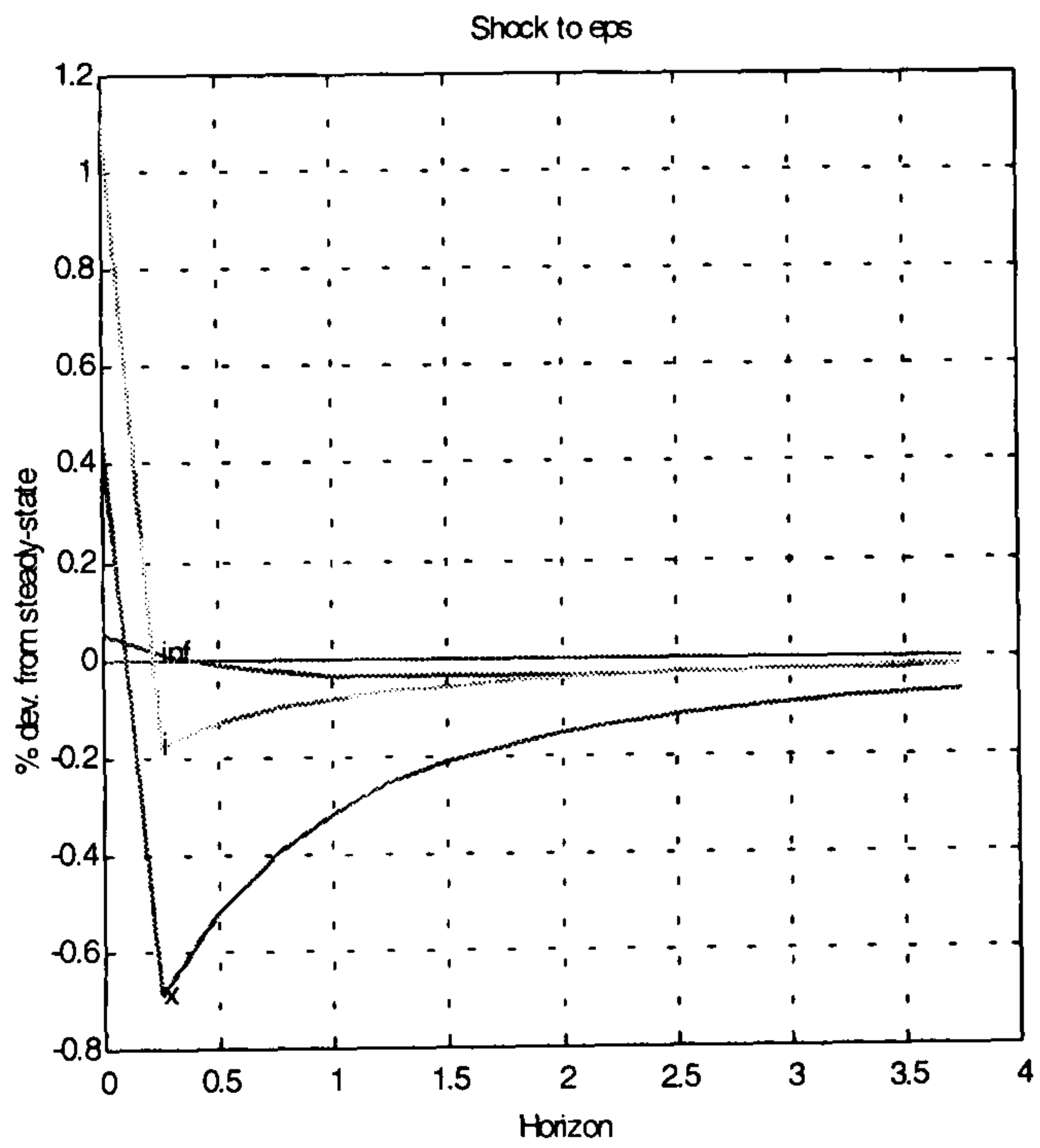
2)1985-90



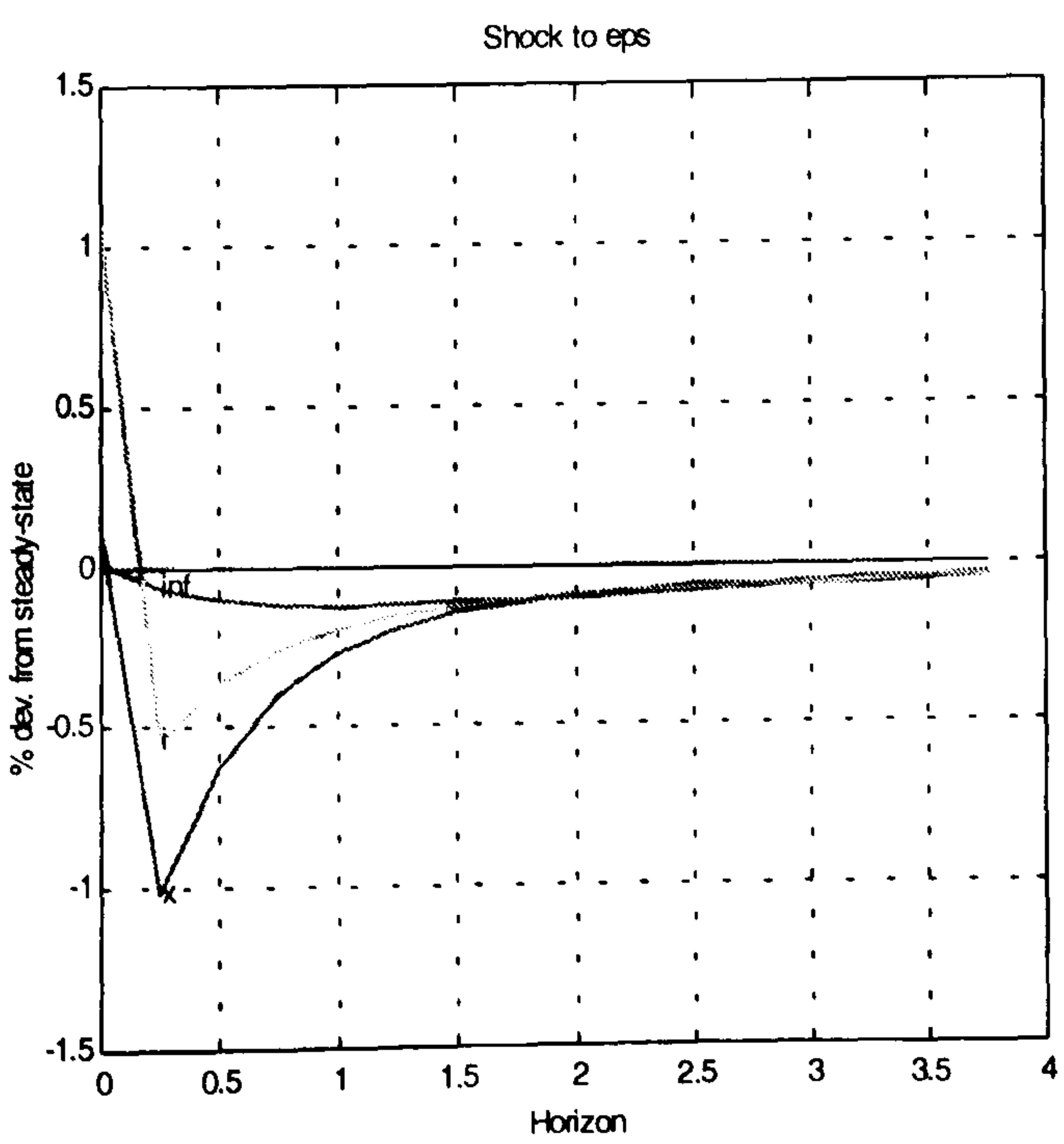
3) 1992-98



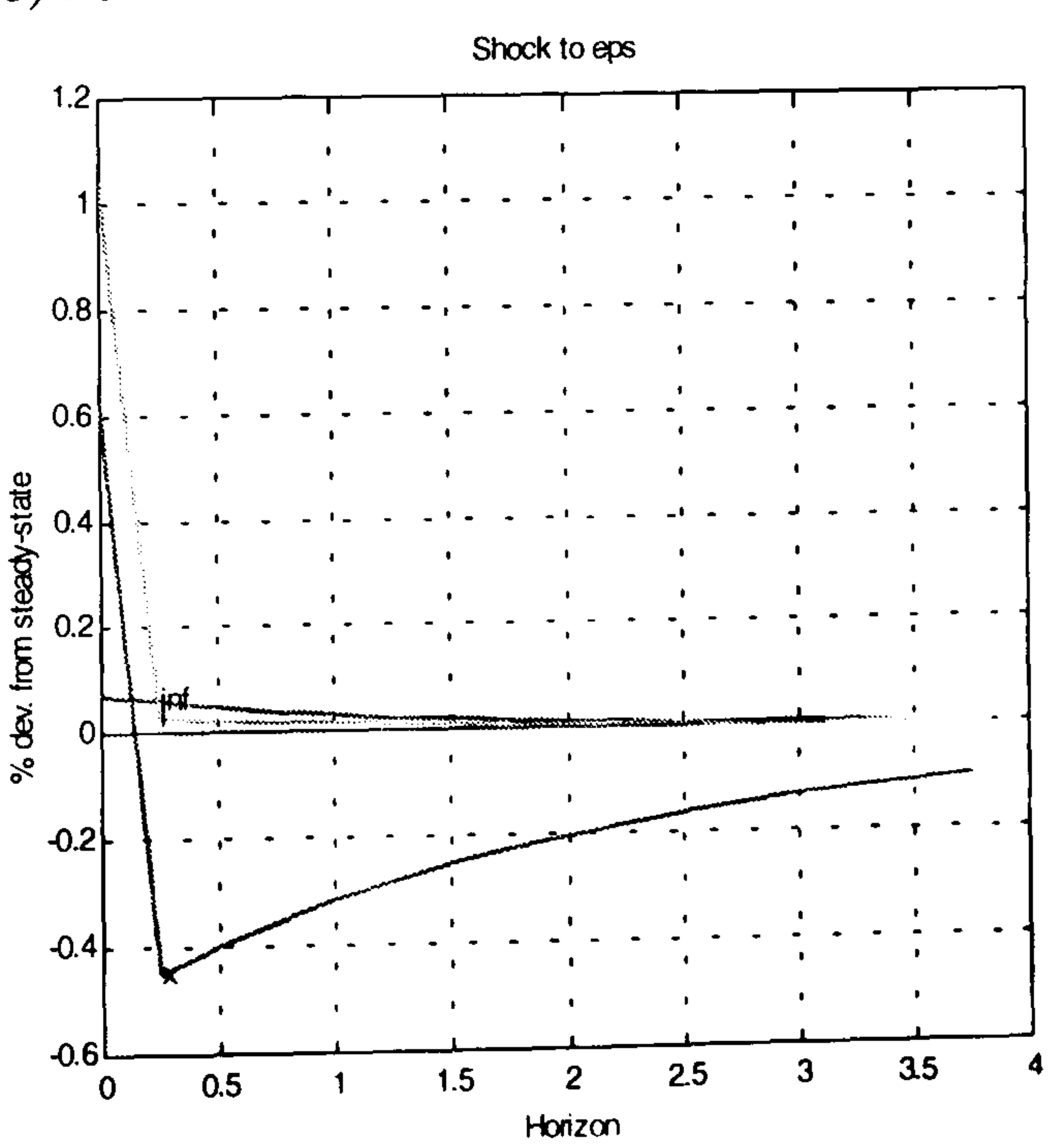
4) Smoothing



5) Taylor



6) Forward



8.5.2 Results from the Simulations

The simulations carried out in this section showed that under the monetary targeting regime the General rule led to a smoother response of the output gap and the interest rate when the economy was hit by shocks coming from technology and foreign nominal variables with respect to the other two regimes.

We have seen that a Taylor rule with a forward term behaves very similarly to our estimated General rule when the system is hit by exogenous shocks. We showed that a rule that does not take into account output gap (the simple Forward rule) leads to a more unstable system, when facing exogenous foreign shocks.

In general, *i.i.d.* shocks to the domestic interest rate lead to a negative reaction of output and inflation that lasts for almost two years.

8.6 Optimal Central Bank's Policy

In this section we take a further step and endogenize the behaviour of the Central Bank. This allows us to derive the optimal policy and to conduct some 'welfare' analysis of the rules. Namely, we will compare the Central Bank loss implied by our estimated rules to that derived from the optimal policy.

We assume that the Central Bank knows the model of the economy and builds its preferences upon that knowledge. Therefore, we are going to carry out a second stage optimal problem. In this respect we can find some similarities between this problem and the optimal taxation theory. An optimal tax scheme is found by maximising a welfare function that depends on agents' indirect utility functions. Similarly, we assume that the Central Bank welfare function depends on agents' aversion to stochastic fluctuations. Uncertainty in our model is related to the presence of expected values in output and inflation and to shocks in preferences and technology. While the Central Bank cannot do much ex-ante to stabilize exogenous variations in preferences and technology, its action can be used to minimize variations in output and inflation after having known the model of the economy.

The Central Bank desires to stabilize both the output gap and inflation. We assume that the Bank has a target for domestic output ($k > 0$) that might differ from potential output (a). We also assume that the authority compares the UK inflation performance to that of the EU. It is taken for granted that each European Central Bank cannot simply care about its own inflation level without looking at the inflation performance of its neighbours[72]. Thus, we assume that the inflation gap between domestic inflation and the target, given by foreign average inflation (π^*) adjusted for a domestic target ($\overline{\pi}$), appears in the loss function. The adjustment is such that the UK Central Bank would have a preference for low inflation even when the average of the European Countries is experiencing high inflation.

We let money supply to be endogenously determined and we assume that the Central Bank can affect the economy through the nominal interest rate.

The Central Bank's Objective. We start by reminding the two equations obtained by combining equations (8.1)-(8.3).

$$x_t = \frac{\Gamma_1}{\Omega\sigma}\beta E_t\pi_{t+1} - \frac{\Gamma_1}{\Omega\sigma}\pi_t + (\Gamma_0 - 1)a_t \tag{8.12}$$

$$\pi_t = \Omega(i_t^* - i_t) + (1 + \beta + \Omega)E_t\pi_{t+1} - \beta E_t\pi_{t+2} - \gamma\pi_t^* \tag{8.18}$$

[72] At least because of the constraint in the Maastricht Treaty.

where (8.18) derives from (8.8). We substitute (8.18) in (8.12) and obtain:

$$x_t = -\frac{\Gamma_1}{\Omega\sigma}(1+\Omega)\, E_t\pi_{t+1} - \frac{\Gamma_1}{\sigma}(i_t^* - i_t) + \frac{\Gamma_1}{\Omega\sigma}\beta E_t\pi_{t+2} + \gamma\frac{\Gamma_1}{\Omega\sigma}\pi_t^* + (\Gamma_0 - 1)a_t \quad (8.19)$$

where we have indicated $\gamma = (1+\beta+\Omega)\varphi_{\pi^*} - \beta\varphi_{\pi^*}^2 - 1$.

We analyse the simplest case in which the Central Bank takes as given private sector expectations on future output. This implies a period by period reoptimization.

The Central Bank's objective consists of minimizing the following loss function with respect to the economy's constraints (8.18)-(8.19).

$$\mathfrak{L}_t = \frac{1}{2}\left[\lambda(x_t - k)^2 + (\pi_t - \pi^* + \overline{\pi})^2\right] \quad (8.20)$$

As a result of our assumptions, the loss function depends on quadratic deviations of output gap from its target and of expected inflation from its target[73], with λ measuring their relative weights.

We substitute equations (8.18)-(8.19) into the loss function (8.20), which can now be minimized with respect to the nominal interest rate:

$$\min_{\{i_t\}} \frac{1}{2}\left[\begin{array}{c} \lambda\left\{-\frac{\Gamma_1}{\Omega\sigma}(1+\Omega)\, E_t\pi_{t+1} - \frac{\Gamma_1}{\sigma}(i_t^* - i_t) + \frac{\Gamma_1}{\Omega\sigma}\beta E_t\pi_{t+2} + \gamma\frac{\Gamma_1}{\Omega\sigma}\pi_t^* + (\Gamma_0 - 1)a_t - k\right\}^2 \\ +(\Omega(i_t^* - i_t) + (1+\beta+\Omega)E_t\pi_{t+1} - \beta E_t\pi_{t+2} - \gamma\pi_t^* - \pi^* + \overline{\pi})^2 \end{array}\right]$$

We then solve the first order condition for i_t which gives:

$$\begin{aligned} & \lambda\frac{\Gamma_1}{\sigma}\left\{-\frac{\Gamma_1}{\Omega\sigma}(1+\Omega)\, E_t\pi_{t+1} - \frac{\Gamma_1}{\sigma}(i_t^* - i_t) + \frac{\Gamma_1}{\Omega\sigma}\beta E_t\pi_{t+2} + \gamma\frac{\Gamma_1}{\Omega\sigma}\pi_t^* + (\Gamma_0 - 1)a_t - k\right\} \\ & -\Omega(\Omega(i_t^* - i_t) + (1+\beta+\Omega)E_t\pi_{t+1} - \beta E_t\pi_{t+2} - \gamma\pi_t^* - \pi^* + \overline{\pi}) \\ = {} & 0 \end{aligned}$$

where we remind that the Central Bank is taking as given private sector expectations on future inflation. The first order condition leads to the following optimal

[73] i.e. target inflation is equal to $\pi^* - \overline{\pi}$. In other words the domestic Central Bank has preferences for low inflation independent from the EU inflation target.

choice for the interest rate:

$$
\begin{aligned}
i_t = i_t^* &+ \frac{1}{\Psi}\left(\Omega(1+\beta+\Omega) + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega)\right) E_t\pi_{t+1} \\
&- \frac{\beta}{\Psi}\left(\Omega + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}\right) E_t\pi_{t+2} \\
&- \frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)a_t + \frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}k \\
&- \frac{1}{\Psi}\left(\Omega\gamma + \Omega + \gamma\left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}\right)\pi_t^* + \frac{\Omega}{\Psi}\overline{\pi}
\end{aligned} \tag{8.21}
$$

where we have called $\left(\left(\frac{\Gamma_1}{\sigma}\right)^2 \lambda + \Omega^2\right) = \Psi$. (8.21) is the optimal policy rule. The presence of expected future inflation gives a forward looking dimension to the rule. The optimal interest rate must be equal to i_t^*, the foreign nominal interest rate and to some 'correction factors' depending on future expected inflation, on the inflationary bias and on the exogenous shocks hitting the economy (to i^*, π^* and a).

Given this policy actual inflation will be equal to:

$$
\begin{aligned}
\pi_t = & \left[(1+\beta+\Omega) - \frac{\Omega}{\Psi}\left(\Omega(1+\beta+\Omega) + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega)\right)\right] E_t\pi_{t+1} \\
& + \left(\frac{\Omega}{\Psi}\left(\Omega + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}\right) - 1\right)\beta E_t\pi_{t+2} - \gamma\pi_t^* \\
& + \Omega\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)a_t - \Omega\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}k
\end{aligned} \tag{8.22}
$$

From (8.22) we can observe that the inflationary bias term does not imply higher inflation today, but it implies higher inflation at time $t+1$ and $t+2$.

Expectations are very important in determining the value of current inflation. We have set the discretionary case, where the Central Bank takes as given private sector expectations. If expected future inflation were equal to its target value, i.e. $E_t\pi_{t+1} = E_t\pi_{t+2} = \pi_t^* - \overline{\pi}$, then the optimal choice of interest rate and the actual inflation would have been:

$$i_t = i_t^* + \frac{1}{\Psi}\left(\Omega^2 + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega-\beta)\right)(\pi_t^* - \overline{\pi})$$
$$-\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)a_t + \frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}k$$
$$-\frac{1}{\Psi}\left(\Omega\gamma + \gamma\left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}\right)\pi_t^* \tag{8.23}$$

$$\begin{aligned}\pi_t &= \left[(1+\Omega) - \frac{\Omega}{\Psi}\left(\Omega(1+\Omega) + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega-\beta)\right)\right](\pi_t^* - \overline{\pi}) \\ &\quad -\gamma\pi_t^* + \Omega\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)a_t - \Omega\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}k \end{aligned} \tag{8.24}$$

Under the same assumption the output gap would be equal to:

$$\begin{aligned} x_t &= \left[\frac{\beta - 1 - \Omega}{\Omega} + \frac{1}{\Psi}\left(\Omega^2 + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega-\beta)\right)\right]\frac{\Gamma_1}{\sigma}(\pi_t^* - \overline{\pi}) \\ &\quad + \left(1 - \frac{\lambda}{\Psi}\left(\frac{\Gamma_1}{\sigma}\right)^2\right)(\Gamma_0 - 1)a_t + \frac{\lambda}{\Psi}\left(\frac{\Gamma_1}{\sigma}\right)^2 k \\ &\quad + \frac{\Gamma_1}{\sigma}\left(\frac{\gamma}{\Omega} - \frac{1}{\Psi}\left(\Omega\gamma + \gamma\left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}\right)\right)\pi_t^* \end{aligned} \tag{8.25}$$

Given that we know all the coefficients in (8.23)-(8.25), these three equations can be used to measure the volatility of the variables and the total loss.

By assuming independent errors of the exogenous variables, from (8.23)-(8.25) we obtain the expressions for the variances of i_t, π_t and x_t; i.e.:

$$\begin{aligned} Var(i_t) &= Var(i_t^*) + \frac{1}{\Psi^2}\left(\Omega^2 + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega-\beta-\gamma) - \Omega\gamma\right)^2 Var(\pi_t^*) \\ &\quad + \left(\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)\right)^2 Var(a_t) \end{aligned}$$

$$\begin{aligned} Var(\pi_t) &= \left[(1+\Omega) - \frac{\Omega}{\Psi}\left(\Omega(1+\Omega) + \left(\frac{\Gamma_1}{\sigma}\right)^2 \frac{\lambda}{\Omega}(1+\Omega-\beta)\right) - \gamma\right]^2 Var(\pi_t^*) \\ &\quad + \left[\Omega\frac{\lambda}{\Psi}\frac{\Gamma_1}{\sigma}(\Gamma_0 - 1)\right]^2 Var(a_t) \end{aligned}$$

$$
\begin{aligned}
Var(x_t) &= \left[\frac{\beta-1-\Omega}{\Omega}+\frac{1}{\Psi}\left(\Omega^2-\Omega\gamma+\left(\frac{\Gamma_1}{\sigma}\right)^2\frac{\lambda}{\Omega}(1+\Omega-\beta-\gamma)\right)+\frac{\gamma}{\Omega}\right]^2\left(\frac{\Gamma_1}{\sigma}\right)^2 Var(\pi_t^*) \\
&+\left(1-\frac{\lambda}{\Psi}\left(\frac{\Gamma_1}{\sigma}\right)^2\right)^2(\Gamma_0-1)^2Var(a_t)
\end{aligned}
$$

8.6.1 Measuring the Volatility and the Total Loss

The coefficients in (8.23)-(8.25) are known because functions of the economy's parameters estimated in the previous chapter, that are reported in table 8.4.

Table 8.4: The economy's parameters

$\beta =$	0.99	$\varphi_a =$	0.956
$\Omega =$	0.086	$\varphi_{i*} =$	0.835
$\sigma =$	0.40	$\sigma_a =$	0.027
$\Gamma_1 =$	0.318	$\sigma_{i*} =$	0.0054
$\Gamma_0 =$	2.12	$\sigma_i =$	0.023
$\pi^* =$	0.013	$\overline{\pi} =$	0
$\varphi_{\pi^*} =$	0.45	$\gamma =$	-0.266
$\sigma_{\pi^*} =$	0.023	$\lambda =$	0.5

In this exercise we have assumed that the weight of the output gap in the loss function is $\lambda = 0.5$, meaning that the distribution of weights is 35% for output and 65% for inflation[74]. Since average inflation in the UK has been slightly above the average inflation of the EU countries we assume that the correction factor $\overline{\pi}$ is equal to zero. We are also reminded that the values in the table have been obtained for quarterly data, from 1979 to 1998.

Table 8.5: The coefficients for the equilibrium inflation, output gap and interest rate

	k	π^*	i^*	a	$S.D.$ %
π	-0.106	1.233	–	0.118	2.86
x	0.977	-0.002	–	0.026	0.07
i	1.229	4.209	1	-1.376	10.3

[74]A more symmetric distribution of weights does not change the results.

Table 8.5 shows the equilibrium elasticities of inflation, output gap and interest rate to the exogenous state variables of the Central Bank problem. According to our measurement we have found that the elasticity of equilibrium inflation to foreign inflation is 1.2. We have approximately obtained a one to one relationship between the output gap, x_t, and the inflationary bias term, k. Output gap does not depend on foreign inflation.

The optimal interest rate set by the Central Bank is deviating from the foreign interest rate because of the presence of k, π^*, i_t^* and a_t. The highest influence on i_t is given by the coefficient multiplying π_t^* and on the inflationary bias. The optimal interest rate is negatively related to shocks in technology.

The values reported in table 8.5 can then be used to give a measure of the total loss. We normalize to unity the equilibrium values of the exogenous processes. We assume that k is 0.025, i.e. 1% higher than potential on a year basis. We replace π^* with the average EU inflation during the period 1979 - 1998. These numbers lead to obtain a value for the total loss equal to:

$$\mathfrak{L} = 0.015$$

The next step consists of computing the total loss to the Central Bank when it is committed to follow a rule.

Figure 8.5: The volatility of inflation and output gap

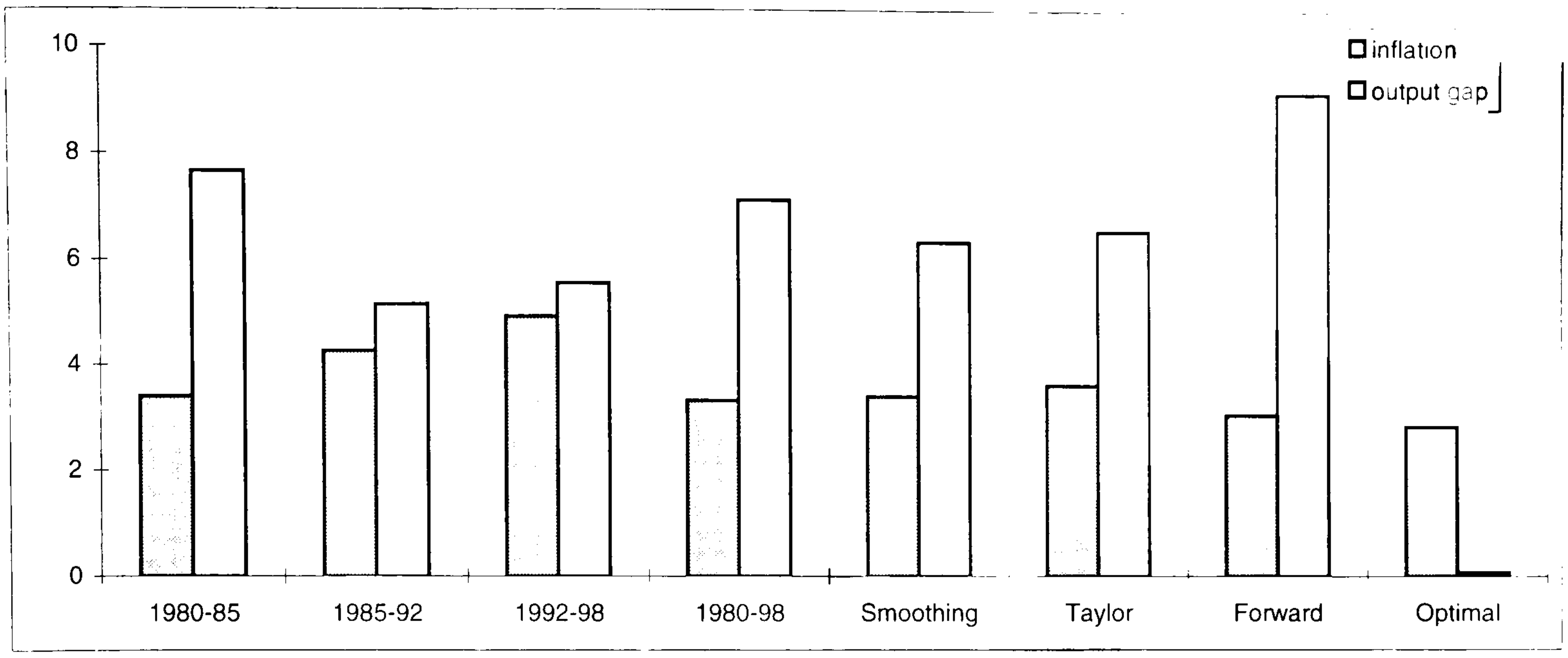


Figure 8.6: The Total Loss

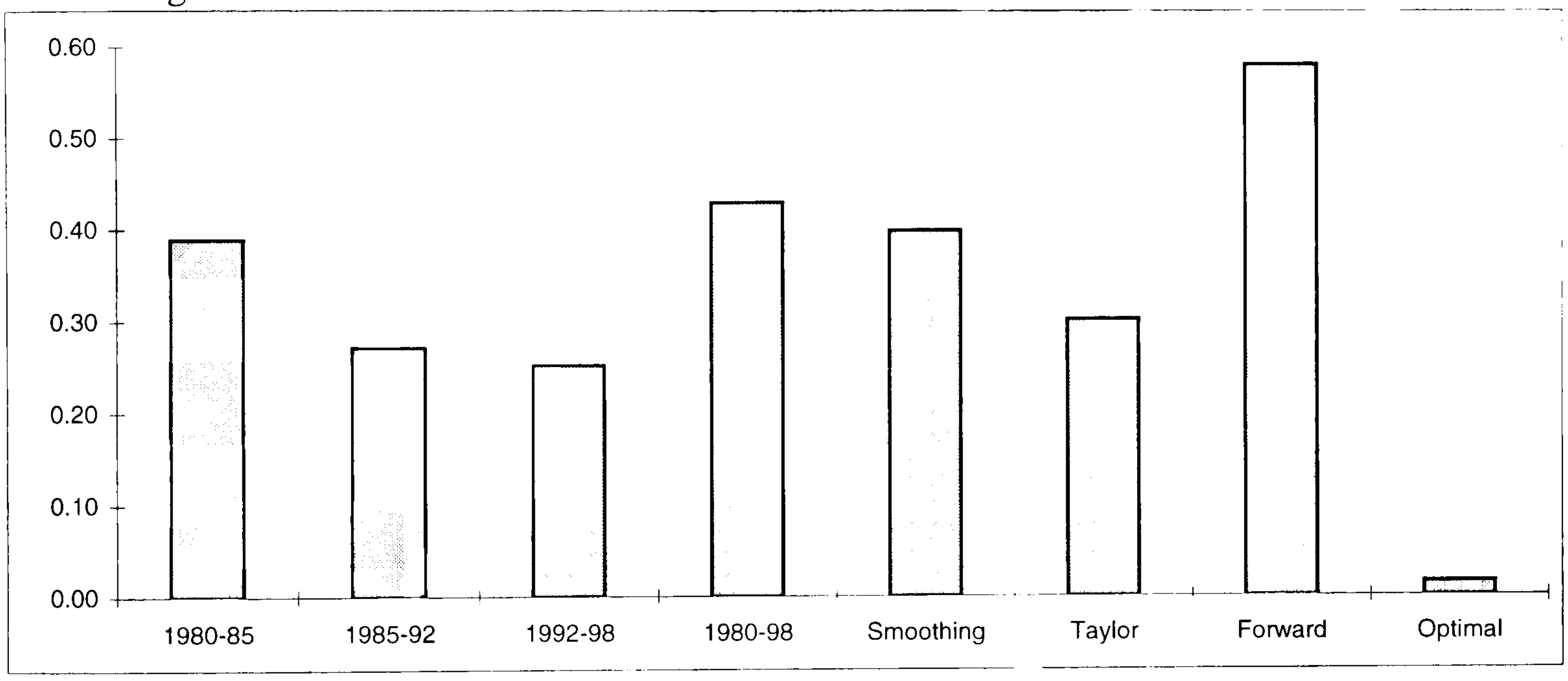
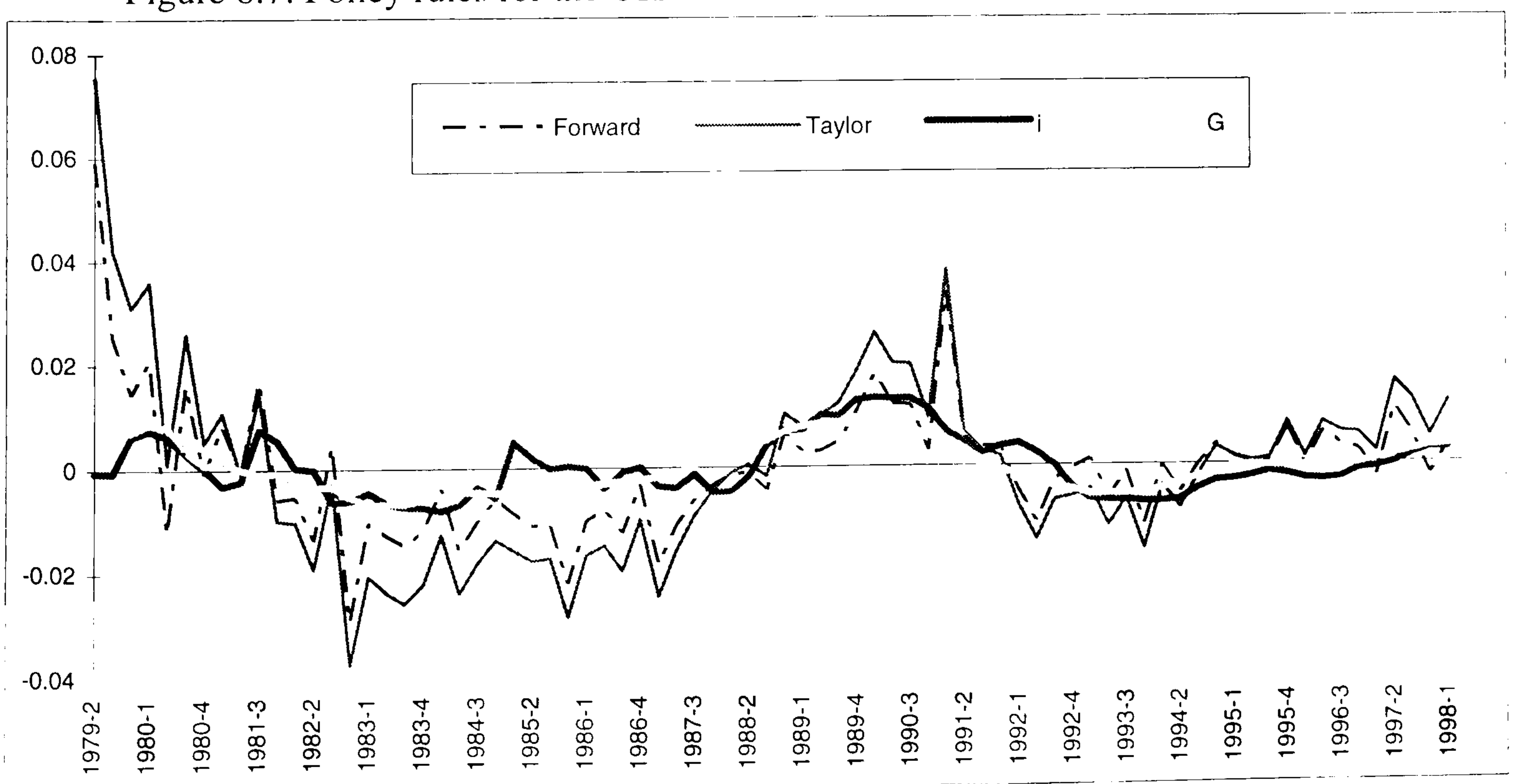


Figure 8.7: Policy rules for the UK



Note: i=nominal interest rate; G= general rule

8.7 Volatility and Total Loss with Policy Rules: a Comparison

To compute the volatility and the total loss under the policy rules analysed in section 8.5 we considered equations (8.6), (8.8) and (8.12) with the expected future inflation replaced by its equilibrium value. Under the previous assumption on the equilibrium values of k, π, i, we replaced (8.6), (8.8) and (8.12) into the loss function (8.20) for measuring the loss implied by each policy rule.

The results of the exercise are shown in figures 8.5 and 8.6. Figure 8.5 shows the volatility of inflation and output gap. The optimal policy delivers the lowest volatility of the two variables.

Amongst the rules we observe that the General rule produces higher inflation and lower output gap volatility in the last sub-period. If we look at the whole sample period there are no significant differences among the performance of the General rule, the long-run smoothing and the Taylor rule. The simple Forward rule leads to the highest volatility of the output gap.

Figure 8.6 shows the total loss. The optimal policy delivers the lowest loss. Among rules, the Forward rule produces the highest loss. According to our estimates the UK Central Bank has diminished its loss by moving to an inflation targeting regime.

8.8 Conclusions

In this chapter we tried to combine an estimation approach to monetary policy rules with the theoretical appeal of the optimal monetary policy theory. We started with a two-country model economy whose reduced form equations look very similar to the traditional Dornbusch model. We pointed out that the differences between the two approaches rely upon the possibility of identifying the structural parameters, upon the intertemporal dimension given to the AD and AS curves and upon the relation between the country differential of inflation and the real exchange rate. We took from the two-country model the domestic equation for the goods market, the uncovered interest parity and the two-country 'Aggregate Supply' curve. We completed the model by adding a policy rule for the UK, that has been inspired by rules of the Taylor's (1993) type, and that is incorporating future inflation and foreign variables. We estimated the rule by Instrumental Variables and we tested its theoretical implications within our two-country model. This test has been based on a study of the system's reaction to exogenous shocks. The performance of the estimated rule is better understood when compared with two well known rules that have been shown to be robust to different models: a forward Taylor and a simple Forward rule. The estimated rule shares many properties with the forward Taylor rule. Conversely, we showed that a rule that does not take into account the output gap leads to a more unstable system, when facing exogenous foreign shocks.

We then used the model to bind the Central Bank's problem for the derivation of an optimal policy. We consider the discretionary case after modelling Central Bank's preferences as quadratic deviations of output and inflation from their target. The optimal policy implies that the nominal interest rate is equal to the foreign interest rate plus a 'correction factor' that depends on the size of the inflationary bias, on expected future inflations and on foreign inflation. Given the estimates of the Euler equations obtained in the previous chapter, we could compute the elasticity of equilibrium inflation, output and interest rate with respect to changes in all the exogenous state variables of the problem. We also obtained a measure for the total loss.

The optimal policy leads to lower volatility and lower loss than the estimated rule. We showed that among rules, the Taylor rule produces the lowest loss.

This chapter has reformulated a very well known optimal monetary policy exercise within a new framework. Our approach is innovative in two ways. On the one hand, we use an estimation approach to policy rules to study the implied reaction of the system to exogenous shocks. On the other hand, optimal policy making is carried out in a micro-founded open economy model with the consequent optimal rule depending on foreign variables (foreign interest rate and inflation).

9 Inflation Gap, Real Exchange Rate and the Optimal Monetary Policy

9.1 Introduction

This chapter carries out the last analysis of the thesis. We reconsider the optimal control problem under the economy constraint derived from the two-country pricing mechanism, that is equation (8.1) of the previous chapter. This equation expresses a short-run relationship between the current and expected inflation gap and the real exchange rate. Given its empirical relevance for the pair UK-EU (see chapter 7) we decided to extract from equation (8.1) the policy objectives of a monetary authority who cares about monetary interdependence with the foreign counterpart. Thus, in this chapter we put forward a new policy objective for the Central Banker, whose preferences depend upon fluctuations in the inflation gap (i.e. domestic inflation relative to the foreign inflation) and in the real exchange rate. In this way we emphasise the international monetary transmission mechanism that could arise from the specification of the Central Bank's preferences. Technically, the choice depends on the fact that our AS curve does not show a trade-off between inflation and output gap but between the current and expected inflation gap and the real exchange rate. When the actual inflation differential exceeds the discounted future inflation differential the real exchange rate depreciates. Thus, a more competitive domestic economy is consistent with expectations of future low inflation, but at the price of higher actual inflation. In our model economy this constraint is the result of the assumptions on the pricing mechanism and on the technology in the two countries. In the previous chapter, while comparing the two-country model with the Dornbusch (1976) model, we have also seen that (8.1) is an implicit measure of the trade-off between inflation and output gap in an open economy context. In this chapter we will investigate how the Central Bank could deal with this trade-off by incorporating the real exchange rate and the inflation gap in its preferences.

We will first consider a problem where inflationary bias arises and see how the size of the bias depends on the degree of rigidity in the pricing system. We will then rewrite the problem without bias and build a frontier of the volatility of the real exchange rate and the inflation differential between England and the European Union. This frontier turns out to be downward sloping. Low volatility of the inflation gap is obtained at the price of a higher volatility of the real exchange rate.

The structure of the chapter is the following. Section 9.2 describes the policy objective and the economic environment. Section 9.3 derives the equilibrium level of inflation under discretion and shows the relationships between inflationary bias and price stickiness. Section 9.4 considers a specification of the Central Bank preferences without inflationary bias. Section 9.5 presents, under these new preferences, the real exchange rate - inflation differential frontier.

9.2 Policy Objective and the Constraint to the Central Bank Problem

We start by specifying the preferences of the Central Bank. It is standard to assume that the Central Bank's objective function involves output and inflation. We deviate from that assumption according to our economic constraint that relates current and expected inflation differential between the two countries to the real exchange rate. Thus, the Central Bank's objective consists of maximizing the following utility function:

$$u_t = \left[\lambda(s_t - \widehat{p}_t) - \frac{1}{2}\widehat{\pi}_t^2\right] \tag{9.1}$$

where we remind that $\widehat{p}_t = p_t - p_t^*$ and $\widehat{\pi}_t = \pi_t - \pi_t^*$ and the asterisk denotes the foreign country.

This formulation looks like that one put forward by Barro and Gordon (1983) where the real exchange rate is replacing deviation of output from the natural rate. This specification expresses a trade-off between the gains obtainable by having a more competitive economy with a depreciated real exchange rate and the loss coming from a higher inflation differential with respect to the foreign country.

The Central Bank desires to stabilize both the real exchange rate and the inflation differential around zero. While it is standard to find domestic inflation in the Central Bank's preferences we can have doubts about the presence of the foreign inflation term. Domestic and foreign inflation appears as a differential, thus deviations from the zero level of inflation are important not per se but in relative terms. As we observed in the previous chapter the empirical context in which we insert the analysis provides the justification for this specification. Namely, we are comparing the UK inflation performance with respect to the EU-wide inflation performance and as a matter of fact each European Central Bank cannot simply care about its own inflation level without looking at the inflation performance in its neighbourhood.

We then need to explain the presence of the real exchange rate in the Central Bank preferences. One the one hand, we can think that the endeavour of minimizing fluctuations of the real exchange rate is related to the desire of getting rid of those imperfections that arise among neighbouring countries and that lead to the failure of the law of one price. The real exchange rate enters in our model because of the existence of goods market segmentation combined with sticky prices. Prices do adjust over time, however, and the real exchange rate converges back to its long-run equilibrium level. The possibility of introducing these short-run imperfections is justified because of firms' price-making behaviour. Thus, we are in a framework where fluctuations in the real exchange rate reflect the general suboptimal level of output produced. In that respect we can think of a positive role for the Central Bank in stabilizing the volatility of the real exchange rate. But why would the Central Bank prefer to have a depreciated real exchange rate? Because, according to our model this would imply short-run gains in terms of future output differentials[75].

[75]This point can be proved analytically. Recalling equations (7.70) and (7.77) we rewrite (7.70), i.e. the Euler equation for consumption differential:

$$E_t\widehat{\pi}_{t+1} = \widehat{i}_{t+1} - \sigma(E_t\widehat{c}_{t+1} - c_t)$$

and (7.77), the world budget constraint

$$\widehat{y}_t = \widehat{c}_t - \frac{ca_t^*}{n}$$

where ca^* gives a gross measure of the foreign current account, i.e. $ca_t^* = \Delta b_t^* - (\overline{r}b_{t-1} + r_{t-1}^*)$. The two equations so rewritten can be then used in (8.1) to obtain a direct relationship between next period output differentials and the real exchange rate

$$E_t\widehat{y}_{t+1} = \widehat{y}_t + \frac{1}{\sigma}\widehat{r}_t + \frac{\Lambda}{\beta\sigma}rer_t - \frac{(E_tca_{t+1}^* - ca_t^*)}{n}$$

9.2.1 The Economy's Constraint

The specification of the economy is extremely simple. We assume that all the relevant information for the Central Bank is contained in the following equation, derived in chapter 6:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\alpha)(1-\alpha)}{\alpha}(s_t - \widehat{p}_t) \tag{8.1}$$

This equation describes a trade-off between inflation differentials and real exchange rate movements. When actual inflation gap (between home and foreign country) exceeds the discounted future inflation gap we observe a depreciated real exchange rate.

For the formulation of the problem it is convenient to write $\widehat{\pi}_t^e = \widehat{\pi}_{t+1} - e_t$, with $\widehat{\pi}_t^e = E_t \widehat{\pi}_{t+1}$, i.e. we exploit the definition of rational expectations, with $e_t \sim$ *white noise* $(0, \sigma_e^2)$, and thus

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \frac{(1-\beta\alpha)(1-\alpha)}{\alpha}(s_t - \widehat{p}_t) - \beta e_t$$

In our model e_t measures the forecasting mistake over the inflation differential due to an unforeseen event occurring at the end of time t.

The rest of the model is then a link between the inflation differential and the policy authority's actual policy instrument. We assume that the Central Bank's policy instrument is the nominal interest rate i_t:

$$\widehat{\pi}_t = i_t - \xi_t \tag{9.3}$$

where ξ_t is the stationary error term[76] $\xi_t \sim i.i.d.(0, \sigma_\xi^2)$.

The private sector's expectations are assumed to be determined prior to the Central Bank's choice of the nominal interest rate. Thus, in setting i_t the Central

[76] We need to justify the error term appearing in (9.3). Recalling the optimized IS equation

Bank will take $\widehat{\pi}_t^e$ as given. We will also assume that the Central Bank can observe e_t prior to setting i_t. Finally, we assume that e_t and ξ_t are uncorrelated.

We now describe the sequence of events. In the first stage $\widehat{\pi}_t^e$ is set. Then the exogenous shock e_t is realized. Because expectations have already been determined, they do not respond to the realization of e_t. However, policy can respond and the policy instrument is set after the Central Bank has observed e_t. The aggregate demand shock ξ_t is then realized, and actual inflation and output are determined. In chapter 7 we assumed that private agents, i.e. firms, are committed to price rigidity, now the further assumption is that this commitment has been made before the Central Bank sets the nominal interest rate. This means that the Central Bank has the opportunity to surprise the private sector by acting in a manner that differs from what private agents had expected when they choose not to change the price. Moreover, the information advantage over e_t on the part of the Central Bank introduces a role for stabilization policy and

obtained in the previous chapter we have

$$E_t\widehat{\pi}_{t+1} = i_t - i_t^* - \Phi_a\widehat{a}_t + \Phi_x(E_t\widehat{x}_{t+1} - \widehat{x}_t)$$

where Φ_a and Φ_x depend on the parameters characterizing preferences and exogenous variables. $\widehat{a}_t$ is the differential of the exogenous technology parameter and $\widehat{x}_t$ is the differential of the output-gap. We can then assume that i_t^* is exogenous and driven by a first order autoregressive process and that the difference $E_t\widehat{x}_{t+1} - \widehat{x}_t$ between expected and actual output gap is measured by a stationary exogenous error term. We make the same assumption for the difference $E_t\widehat{\pi}_{t+1} - \widehat{\pi}_t$ that allows us to replace $E_t\widehat{\pi}_{t+1}$ with $\widehat{\pi}_t + \varepsilon_{\pi t}$. Thus $\xi_t = i_t^* + \Phi_a\widehat{a}_t - \Phi_x(E_t\widehat{x}_{t+1} - \widehat{x}_t) - \varepsilon_{\pi t}$ is an exogenous and stationary error term, which is a complicated combination of real, nominal, domestic and foreign shocks. We assume that all these shocks in average cancel out and that the distribution is $i.i.d \sim (0, \sigma_\xi^2)$. This expression tells us how careful we must be in interpreting shocks that appear in any of the reduced form equations used. Usually this type of shock is called 'Aggregate Demand' shock.

is meant to capture the fact that policy decisions can be made more frequently than can most price (and wage) decisions.

9.3 Equilibrium Inflation

Since we are assuming that the Central Bank acts before observing the disturbance ξ_t, its objective will be to maximize the expected value of the utility function, where the Central Bank's expectations are defined over the distribution of ξ_t. Substituting (8.1) and (9.3) into (9.1) yields

$$\left[\lambda\frac{1}{\Omega}\left(i_t-\xi_t-\beta\widehat{\pi}_{t+1}+\beta e_t\right)-\frac{1}{2}\left(i_t-\xi_t\right)^2\right] \tag{9.4}$$

where we remind that we have defined $\Omega=\frac{(1-\beta\alpha)(1-\alpha)}{\alpha}$. The first order condition for the optimal choice of i_t, conditional on e_t and taking $\widehat{\pi}^e_t$ as given, is

$$i_t=\frac{\lambda}{\Omega} \tag{9.5}$$

Given this policy, actual inflation will equal $\widehat{\pi}_t=\frac{\lambda}{\Omega}-\xi_t$. Because private agents are assumed to understand the incentives facing the Central Bank they use (9.5) in forming their expectations about inflation. With private agents forming expectations prior to observing e_t, (9.3) and (9.5) imply

$$E\widehat{\pi}_{t+1}=E(\frac{\lambda}{\Omega})=\frac{\lambda}{\Omega}$$

Thus average inflation differential is fully anticipated. Substituting now in (8.1) we obtain a value for the real exchange rate:

$$s_t-\widehat{p}_t=\frac{\lambda}{\Omega}\frac{1-\beta}{\Omega} \tag{9.6}$$

The equilibrium, when the Central Bank acts with discretion in setting i_t, produces a positive average rate of inflation differential equal to $\frac{\lambda}{\Omega}$. There are no gains in term of competitiveness by exploiting a depreciated real exchange rate, since the private sector completely anticipates inflation, the domestic economy suffers from positive average inflation higher than foreign inflation without any benefit, i.e. $\pi_t = \pi_t^* + \frac{\lambda}{\Omega}$.

The size of the bias depends on Ω, i.e. on $\frac{(1-\beta\alpha)(1-\alpha)}{\alpha}$. We can differentiate $1/\Omega$ with respect to α:

$$\frac{\partial(1/\Omega)}{\partial\alpha} = \frac{1-\beta\alpha^2}{(1-\alpha)^2(1-\beta\alpha)}$$

since $0 < \alpha < 1$ we obtain a positive value. There exists a positive relation between α and the size of the bias. If we let the private discount rate to be $\beta = 0.99$, we can give a measure to the relation between the bias and the parameter α governing the timing of the change in price.

Table 9.1: Inflationary Bias and Stickiness

α	$1/\Omega$
0.87 (2 years)	48.25
0.75 (1 year)	11.65
0.5 (6 months)	1.98
0.2 (4 months)	0.31
0.15 (3 months)	0.21

Higher values of α imply a greater inflation bias. The more the economy is subject to rigidities the more it is suffering inflation bias in equilibrium and the increase in inflation is exponential. According to this framework, therefore, Central Bank's preferences not 'neutral' with respect to the real exchange rate will produce a higher loss to the society the more prices are staggered and take longer to adjust. The larger is α, the greater is the Central Bank incentive to produce inflationary bias. Recognizing this fact, private agents anticipate a higher rate of inflation.

The inflation bias is also increasing in the weight the Central Bank places on its real exchange rate objective, λ. A small λ implies that the gain from a more competitive exchange rate are low relative to achieving inflation objectives, so the Central Bank has less of an incentive to generate inflation.

Under this discretionary policy outcome, the expected utility of the Central Bank is equal to

$$\begin{aligned} E_t[u^d] &= E_t\left[\frac{\lambda}{\Omega}\left(\frac{\lambda}{\Omega}(1-\beta)+\xi_t(1-\beta)+\beta e_t\right)-\frac{1}{2}\left(\frac{\lambda^2}{\Omega^2}-2\frac{\lambda}{\Omega}\xi_t+\xi_t^2\right)\right]= \\ & \frac{\lambda^2}{\Omega^2}(\frac{1}{2}-\beta)-\frac{1}{2}\sigma_\xi^2 \end{aligned} \tag{9.7}$$

The expected utility is decreasing in the variance of the error ξ_t. The loss due to this error is unavoidable, whereas the loss due to the inflation bias could be avoided if, for example, the monetary authority was able to commit to setting the nominal interest rate equal to zero. In this case expected utility would equal:

$$E_t[u^c] = -\frac{1}{2}\sigma_\xi^2 > E_t[u^d] \tag{9.8}$$

(where the subscript c stands for commitment and d for discretion). So the Central Bank would be better off if it were possible to commit to a rule.

9.4 Ruling out the Inflation Bias with an Alternative Policy Objective

We focus now the attention to an alternative specification of the Central Bank's objective based on the loss associated to inflation and real exchange rate fluctuations around desired values. With this formulation we rule out the possibility of inflationary bias at the outset.

$$L_t = \frac{1}{2}\left[\lambda(s_t-\widehat{p}_t)^2+\widehat{\pi}_t^2\right] \tag{9.9}$$

By keeping the previous assumption on the economy's constraints and on the timing of the private sector and Central Bank's actions we can derive the first order conditions for the optimal choice of i_t, conditional on e_t and taking $\widehat{\pi}_t^e$ as given:

$$i_t = \frac{\Omega^2}{\Omega^2 + \lambda^2}\beta(\widehat{\pi}_{t+1} - e_t) \tag{9.10}$$

The optimal monetary policy under discretion implies to set i_t conditional to the expected inflation differential[77]. The most important difference between (9.5) and (9.10) is related to the presence of the term $(\widehat{\pi}_{t+1} - e_t)$. The white noise shock e_t creates a gap between future realized inflation differential and the inflation differential expected by the private sector. The Central Bank can choose i_t after observing the shock e_t. Because the Bank wants to minimize the variance of the real exchange rate, it will make policy conditional on the realization of that shock. Thus, an explicit role for stabilization policies arises that will involve trading off some inflation volatility for reduced real exchange rate volatility. But this possibility depends on private sector expectations about inflation.

Private agents are assumed to understand the incentives facing the Central Bank, so again they use (9.10) in forming their expectations about inflation. With expectations formed prior to observing the shock e_t, (8.1) and (9.10) imply

$$E_t\pi_{t+1} = E_t\left[\frac{\Omega^2}{\Omega^2 + \lambda^2}\beta(\widehat{\pi}_{t+1} - e_t)\right] = \frac{\Omega^2}{\Omega^2 + \lambda^2}\beta E_t\widehat{\pi}_{t+1}$$

solving for $E_t\pi_{t+1}$ yields $E_t\pi_{t+1} = 0$. Substituting this back into (9.10) and using (8.1) gives an expression for the equilibrium rate of inflation

$$\widehat{\pi}_t^d = -\frac{\Omega^2}{\Omega^2 + \lambda^2}\beta e_t - \xi_t \tag{9.11}$$

[77]This is the way in which Svensson (1996, 1997, 1998, 1999) obtains a forward looking optimal monetary policy rule.

We have found a substantial difference between this solution and the one obtained with the semi-quadratic Central Bank's utility function. Namely, if the Central Bank does not have biased preferences toward a higher degree of competitiveness then the average rate of inflation is zero.

9.5 The Real Exchange Rate - Inflation Gap Variability Frontier

In this section we consider the Central Bank problem which consists of minimizing the quadratic loss function (9.10) in an intertemporal framework. The economy is modelled by taking into consideration the unique equation (8.1). Minimizing the loss function subject to that constraint leads to a rule for the real exchange rate that is contingent to the size of the inflation rate. The problem can be rewritten in these terms:

$$\min_{\{\pi\}} E_t \sum_{s=0}^{\infty} \delta^s L_{t+s} \tag{9.12}$$

or

$$\min_{\{\pi\}} E_t \sum_{s=0}^{\infty} \delta^s \left[\frac{1}{2}\lambda \left(\frac{\widehat{\pi}_{t+s}}{\Omega} - \beta \frac{\widehat{\pi}_{t+s+1}}{\Omega} + \beta \frac{e_{t+s+1}}{\Omega} \right)^2 + \frac{1}{2}\widehat{\pi}_{t+s}^2 \right] \tag{9.13}$$

where δ is the discount factor of the central banker. The first order condition for the inflation differential is equal to:

$$E_t \left\{ \delta^s \frac{\lambda}{\Omega} \left(\frac{\widehat{\pi}_{t+s}}{\Omega} - \beta \frac{\widehat{\pi}_{t+s+1}}{\Omega} + \beta \frac{e_{t+s+1}}{\Omega} \right) + \delta^s \widehat{\pi}_{t+s} - \delta^{s-1} \frac{\lambda}{\Omega} \beta \left(\frac{\widehat{\pi}_{t+s-1}}{\Omega} - \beta \frac{\widehat{\pi}_{t+s}}{\Omega} + \beta \frac{e_{t+s}}{\Omega} \right) \right\} = 0 \tag{9.14}$$

for $s = 0$ and after some simplification we obtain:

$$\left(\frac{\widehat{\pi}_t}{\Omega} - \beta \frac{E_t \widehat{\pi}_{t+1}}{\Omega} \right) + \frac{\Omega}{\lambda} \widehat{\pi}_t - \delta^{-1} \beta \left(\frac{\widehat{\pi}_{t-1}}{\Omega} - \beta \frac{\widehat{\pi}_t}{\Omega} + \beta \frac{e_t}{\Omega} \right) = 0$$

The previous equation can be written in this way:

$$\widehat{\pi}_t \left(\frac{1}{\Omega} + \frac{\Omega}{\lambda} + \frac{\beta^2}{\delta\Omega} \right) = \frac{\beta}{\Omega} \left(E_t \widehat{\pi}_{t+1} + \frac{1}{\delta} \widehat{\pi}_{t-1} + \frac{\beta}{\delta} e_t \right) \tag{9.15}$$

We now consider (9.15) one period ahead and take the expectations. We also assume that the best forecast for the inflation gap at time $t+2$ is equal to that at time $t+1$, i.e. $E_t\widehat{\pi}_{t+1} = E_t\widehat{\pi}_{t+2}$ and reminding that $E_t e_{t+1} = 0$ we end up with:

$$\left(\frac{1}{\Omega} + \frac{\Omega}{\lambda} + \frac{\beta^2}{\delta\Omega} - \frac{\beta}{\Omega}\right) E_t\widehat{\pi}_{t+1} + \frac{\beta}{\delta\Omega}\widehat{\pi}_t = 0$$

or

$$E_t\widehat{\pi}_{t+1} = \frac{\beta\lambda\widehat{\pi}_t}{\lambda\delta + \delta\Omega^2 + \lambda\beta^2 - \lambda\beta\delta} \tag{9.16}$$

Substituting back into equation (9.15) we obtain

$$\widehat{\pi}_t\frac{\lambda\delta + \delta\Omega^2 + \lambda\beta^2}{\lambda\delta} - \frac{\beta\lambda\widehat{\pi}_t}{\lambda\delta + \delta\Omega^2 + \lambda\beta^2 - \lambda\beta\delta} - \frac{\beta}{\delta}\widehat{\pi}_{t-1} - \frac{\beta^2}{\delta}e_t = 0$$

If we assume $\beta = \delta$ we have the following equilibrium level of inflation gap:

$$\widehat{\pi}_t\frac{(\lambda + \Omega^2)^2 + \Omega^2\lambda\delta}{\lambda(\lambda + \Omega^2)} = \widehat{\pi}_{t-1} + \delta e_t$$

We can now compute the unconditional variance of the inflation gap, by assuming that $cov(\widehat{\pi}_t, e_t) = 0$:

$$\sigma^2_{\widehat{\pi}} = \frac{\delta^2}{A^2 - 1}\sigma^2_e \tag{9.17}$$

where $A = \frac{(\lambda+\Omega^2)^2+\Omega^2\lambda\delta}{\lambda(\lambda+\Omega^2)}$. We use equation (8.1), after having substituted (9.16) in (8.1), to compute the unconditional variance of the real exchange rate:

$$\sigma^2_{s-\widehat{p}} = \left[\frac{\lambda(1-\delta) + \Omega^2}{\Omega\,(\lambda + \Omega^2)}\right]^2 \sigma^2_{\widehat{\pi}} \tag{9.18}$$

We are now ready to build a frontier between the inflation differential volatility and the real exchange rate volatility for different values of the parameter λ. We first write down the relevant values:

$\delta = 0.99$	$\beta = 0.99$	$\alpha = 0.75$	$\Omega = 0.086$	$\sigma_e = 1$

These values can be substituted in (9.17) and (9.18). Thus we obtain:

$$\sigma^2_{\widehat{\pi}} = \frac{0.98\lambda^2(\lambda + 0.0074)^2}{(\lambda^2 + 0.022\lambda + 0.00006)^2 - \lambda^2(\lambda + 0.0074)^2}$$

$$\sigma^2_{s-\widehat{p}} = \left[\frac{0.01\lambda + 0.0074}{0.089\,(\lambda + 0.0074)}\right]^2 \sigma^2_{\widehat{\pi}}$$

We remind that the parameter λ governs the relative weight that the Central Bank places on fluctuations of the real exchange rate around the desired value with respect to fluctuations in the inflation rate differential.

From (9.17) and (9.18) we can see that for $\lambda = 0$ the Central Bank minimizes fluctuations of both the inflation differential and the real exchange rate. Here the Bank optimizes by fixing inflation at its target in every period. There is no inflation variability and no inflation uncertainty.

Figure 9.1 shows the real exchange rate -inflation frontier obtained by changing the values of λ. The frontier has been drawn for $0.01 < \lambda < 0.5$, i.e. for very small values of λ. In that interval, for $\lambda = 0.01$ we get the highest volatility of the real exchange rate and the lowest volatility of inflation. We can notice that the trade-off is much more intense for low values of λ.

Figure 9.2 shows the inflation differential - real exchange rate volatility between the UK and the EU from 1980 to 1998. It has been obtained by computing the standard deviations for sub-samples of three years. A comparison between the graphs 9.1 and 9.2 shows that simulated and actual frontier have a similar shape. But, in general, the actual frontier is drawn for higher values of the volatility of the real exchange rate $\sigma(rer)$. According to our model small values of λ imply a lower weight to the real exchange rate in the Central Bank loss function and thus a higher volatility. In conformity to the simulations' results, the data reveal that lower volatility for the inflation gap is obtained when the degree of volatility in the real exchange rate is higher.

Figure 9.1: The real exchange rate - inflation differential
frontier for different values of λ

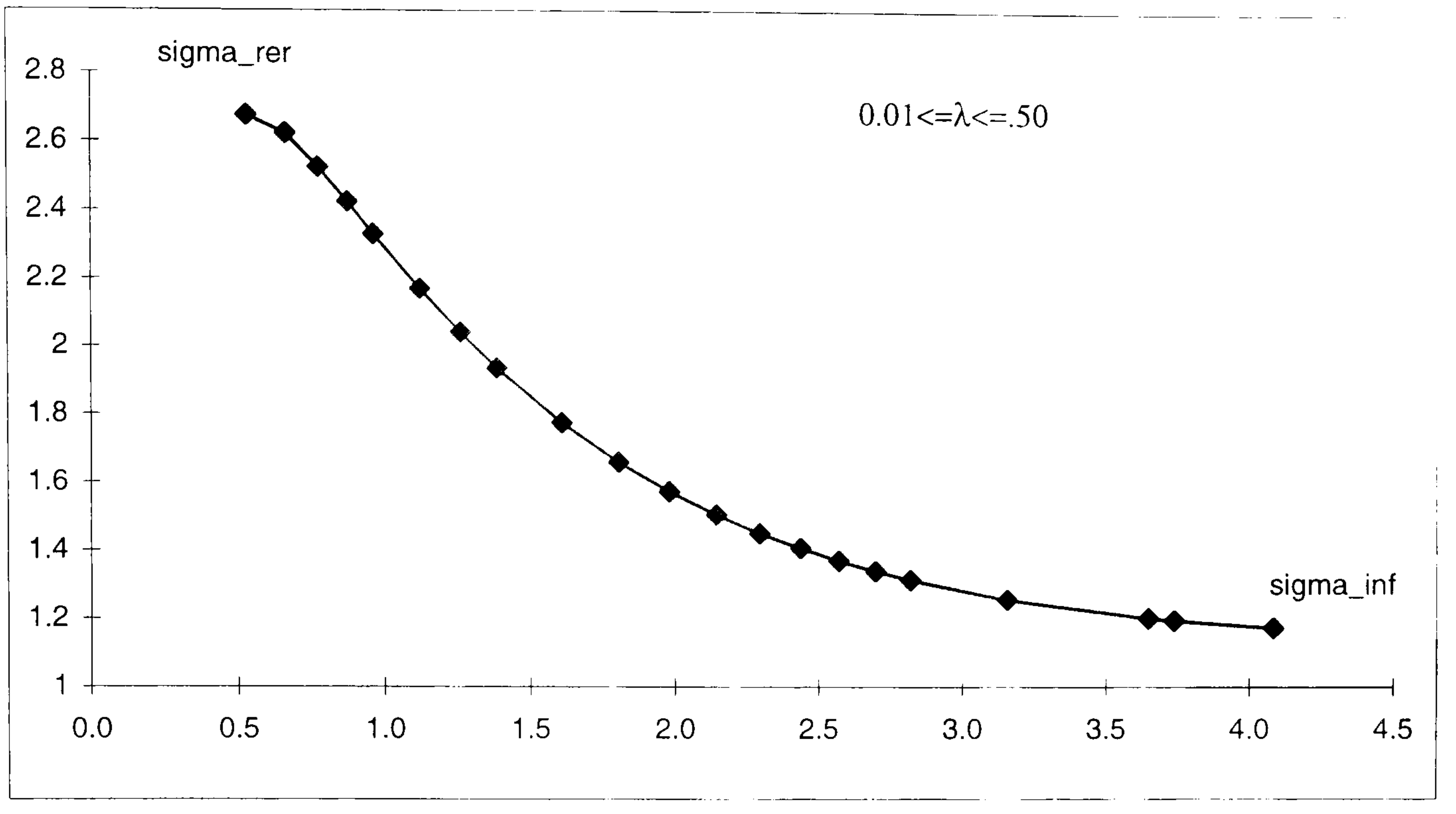
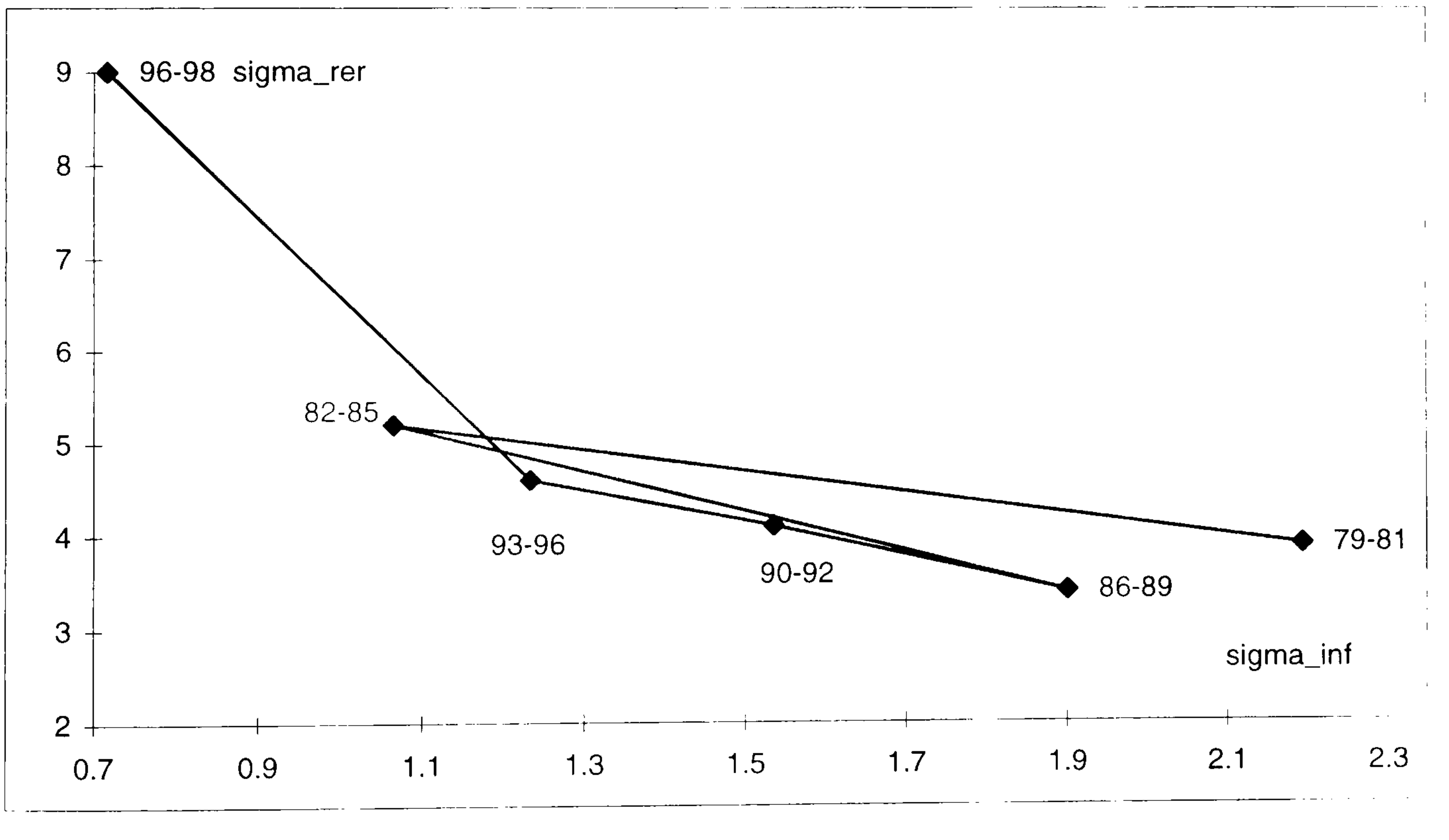


Figure 9.2: The inflation differential-real exchange rate volatility: 1980-1998



Note: The frontier in fig. 9. 2 has been built by computing standard deviations of the real exchange rate and the inflation differential every three years.

9.6 Conclusions

This chapter concludes the analysis of monetary policies in open economies. We constructed the Central Bank's problem following the 'optimal control' theory. We deviated from the classical approach by modelling preferences on the inflation differential and the real exchange rate volatility. The discretionary case delivers inflationary bias with respect to the commitment case when the Central Bank's preferences have a semi-quadratic form and depend linearly on the real exchange rate.

We have found a direct relationship between the inflation bias and α, the parameter that governs price rigidity in the model. There exists an incentive for the Central Bank to exploit the trade-off in favour of a more competitive economy when the price mechanism has a high degree of stickiness, but this implies a higher equilibrium level of inflation and thus a higher loss for the society.

We then ruled out the inflationary bias. With an intertemporal quadratic loss function built on the real exchange rate and the inflation gap, we derived the real exchange rate-inflation gap volatility frontier. There exists a trade-off between the two volatilities for any values of λ, the weight attached to the real exchange rate. The trade-off is stronger for very low values of λ, i.e. when the Central Bank cares more about inflation volatility. We found that the Central Bank can pursue the objective of minimizing the variability of inflation only when λ is given very low values.

10 Conclusions

The thesis has combined a theoretical and empirical investigation of international monetary policies.

The UK economy has been at the centre of the analysis. We started by thinking of it as a small open economy. In chapter 2 we designed the simplest model able to capture the behaviour of the real exchange rate and the balance of payments. This model distinguishes between traded and non-traded goods and characterises foreign variables as an exogenous autoregressive process. We used its long-run behaviour to detrend actual and artificial data and to conduct different types of experiments. In chapter 3 we estimated a structural VECM based on theoretical long-run restrictions. Impulse response functions showed that positive shocks to domestic output, traded and non-traded, lead to higher consumption and to a depreciated domestic currency. A positive shock to the foreign interest rate and to the domestic money supply leads to the overshooting of the nominal exchange rate. Money shocks have an undershooting effect on the balance of payments, implying an impact deterioration and an improvement after three quarters. Shocks to foreign prices have an undershooting effect on the nominal exchange rate. The forecast error variance decomposition shows that foreign shocks to the price level and the interest rate have been the main determinant of the UK fluctuations over the period 1969-1997.

In chapter 4 we simulated the short-run behaviour of the model and compared the results with those obtained from an unrestricted short-run cointegrated VAR with exogenous shocks. The comparison of the impulse response functions of the restricted simulated VAR with those of the unrestricted estimated VAR showed that there is no divergence of information between the two exercises. The theoretical model seems to be better suited for providing long-run theoretical restrictions for the empirical VECM rather than for its short-run behaviour, as it seems too simplified to give understanding on the short-run behaviour of domestic prices. Simulated data can produce a highly volatile real exchange rate as observed in the actual data; they also show correct comovements with consumption expenditures in different goods.

The second step consisted of modelling the UK monetary policy within a two-country economy. Chapter 5 outlines an oversimplified model and describes the behaviour of home and foreign variables expressed in differences. The foreign country is the US. Price stickiness and Pricing-to-Market are the main ingredients of the model. The model helps in understanding how monetary shocks are transmitted to the economic system. Since prices are sticky, we could use them as a state variable in the final form of the system and, thus, relate all the variables of the model to prices' behaviour. A high degree of price stickiness leads real variables to benefit longer from monetary shocks on the one hand (because of a higher real effect), and to suffer longer from a price change on the other hand (because of a more negative consumption elasticity with respect to a price change). The model's simulations show a dominance of US money shocks with respect to UK money shocks on the domestic and foreign economic system. They witness the presence of spillovers and asymmetric effects of the two shocks.

For the two-country model we also carried out the structural VECM estimation, which relies on long-run restrictions. Monetary shocks (domestic and foreign) turned out to determine very long-lasting shifts in real and nominal variables and to make a great contribution in explaining the total variability of consumption and price differentials. Conversely, their contribution to explaining the volatility of exchange rate and interest differential is minimal. Two pieces of evidence led us to construct a domestic monetary policy rule for the nominal interest rate based on the feedback coming from abroad. These are that foreign monetary shocks seemed to determine the path of domestic monetary shocks, and that the measure used for the domestic money supply also contains information about money demand. This rule is used to interpret a fundamental relation between the real exchange rate and inflation differentials in the model. It suggests that during the period 1979-1998 the behaviour of the nominal UK interest rate can be greatly explained by that of the federal fund rate, by the real interest rate differential and by the real exchange rate between the UK and the US.

In chapter 6 we estimated the VECM by using the average of the EU countries as a foreign economy. We have carried out three estimation exercises to understand the effect on the international monetary transmission mechanism of having a single monetary policy in the Euro area. The exercises suffer from the absence of a proper Euro-area data base, like many previous exercises on the EMU. We asked which advantages we could obtain from a new big country (the EU) in the determination of the monetary policy with respect to another big country, the US.

The first exercise suggested that the UK can gain from entering the monetary union, given the positive effects of the EU-monetary shocks on the UK real economy.

The second exercise investigated the international transmission mechanism between the EU and the US. The use of the money supply as a monetary instrument did not lead to any puzzling results in terms of liquidity effects. Dealing with two big countries allowed us to avoid the problem of having domestic monetary policy dependence on foreign monetary policy.

The third exercise compared the performance of the UK with respect to the US, if the UK monetary policy were determined by the Euro countries. Having a Monetary Union leads to a better-insulated system against foreign shocks (US) for the participating countries; in this respect the UK would be better off for joining the EU. We are reminded that these results come from a backward-looking estimation that relied on data prior to the change of the monetary policy regime. We fully recognise the problem implicit in this strategy, that is that any forward looking interpretation is subject to the Lucas' critique.

Chapter 7 extended the model presented in chapter 5 in several directions. We modelled the supply side and we demonstrated that the results obtained in chapter 5 hold even in a general equilibrium model, under the assumption that leisure does not appear in the utility function, that is workers cannot choose their hours of work. We enriched the structure of preferences by introducing an additive shock to the money demand and we allowed for relative risk aversion coefficients for consumption and real money balances to be different from 1. The IV estimation of the economy's deep parameters has been carried out under the hypothesis that the structural preference parameters were the same for the UK and the average Euro countries. We succeeded in estimating a significant value for α, the parameter that governs the timing of price rigidity, which turned out to be 0.75, a value that corresponds to a one year length of price rigidity. We simulated the model for different values of the price stickiness parameter. Having a complete model did not change the main results obtained in chapter 5. Conversely, the measurement of the structural parameters used for preferences, technology and policies led to different results with respect to those obtained in chapter 5. In this chapter the monetary transmission mechanism between the UK and the EU acted through spillover and asymmetry as it did in chapter 5, but shocks coming from the EU led to potential gains for the UK and not losses as happened with shocks coming from the US.

This chapter prepares the ground for the final investigation. The model economy has been subject to supply and demand shocks that can justify some policy intervention in a world characterized by suboptimal outcomes.

In chapter 8 we decided, therefore, to explicitly model the monetary authority's behaviour. From the two-country model we took the domestic equation for the goods market, the uncovered interest rate parity and the two-country 'Aggregate Supply' curve. We completed the model by adding a policy rule for the UK, inspired by rules of the Taylor (1993) type and incorporating future inflation and foreign variables. We estimated the rule by Instrumental Variables and we tested its theoretical implications by studying the system's reaction to exogenous shocks. The performance of the estimated rule has been compared to two well known rules, that have been shown to be robust within different models: a Taylor and a Forward rule. The estimated rule has many of the properties of the Taylor rule. We showed that a rule that does not take into account the output gap leads to a more unstable system, when facing exogenous foreign shocks.

We then used the model to bind the Central Bank's problem for the optimal policy. We considered the discretionary case after modelling Central Bank preferences as quadratic deviations of output and inflation from their target. The optimal policy implies that the nominal interest rate is equal to the foreign interest rate plus a 'correction factor' that depends on the size of the inflationary bias, on expected future inflation and on foreign inflation. Given the estimates of the Euler equations obtained in the previous chapter, we could compute the elasticity of equilibrium inflation, output and interest rate to changes in all the exogenous state variables of the problem. We also obtained a measure for the total loss. The optimal policy leads to lower volatility and lower loss than the estimated rule. We showed that of the rules, the Taylor rule produces the lowest loss.

In chapter 9 we deviated from the classical Central Bank's problem by modelling preferences on the inflation gap between the UK and the EU and on the real exchange rate volatility. The discretionary case delivers inflationary bias with respect to the commitment case when the Central Bank's preferences have a semi-quadratic form and depend linearly on the real exchange rate. There exists a positive relationship between inflation bias and α, the parameter that governs price rigidity in the model. We then eliminated the bias. With a quadratic loss function we derived the real exchange rate-inflation volatility frontier. We found that only for very low values of λ can the Central Bank pursue the objective of minimizing inflation volatility at the expense of a higher real exchange rate volatility. The trade-off is confirmed by the data on the UK and EU between 1980 and 1998.

The thesis did not put forward a new model of the economy, but created new windows within the intertemporal stochastic approach to open economies. These new windows, in turn, showed the possibility of using theoretical restrictions for interpreting actual data, of extracting short-run behavioural equations whose fluctuations could be simulated, of estimating 'deep' parameters, of deriving new AS-AD curves usable for policy analysis. Much work can still be done both on a theoretical and an empirical level. We used extremely simplified models that can be made more realistic by formalizing other channels of monetary transmission mechanisms, and by introducing risk (i.e. by ruling out the assumption of certainty equivalence). As a short-run objective, instead of imposing theoretical restrictions on the estimated VECM, the simplicity of the model derived in chapter 5 could allow us to estimate it directly. In so doing, we could make a comparison with the results obtained from the estimate of the VECM.

Models specified at the level of preferences open up an avenue for reinterpreting and redesigning many traditional macroeconomic relationships. We tried to derive results which are logically consistent and at the same time not devoid of common sense. Our analysis required coupling theoretical reasoning with empirical estimation and testing. My hope is that this thesis will give me the opportunity to continue this so stimulating, combined analysis.

The end

A Data sources and Definitions

Data are quarterly and seasonally adjusted, they have been transformed to constant price (=1990). We have taken the logarithm of all the variables except for the trade balance (which has been divided by GNP), interest rates and inflation rates.

A.1 UK data

Variable	Definition	Sample	Source
c_N	consumption of services, prices 1990	69:1-97:4	Datastream
c_T	consumption of non durables, prices 1990	69:1-97:4	Datastream
c	total private cons. expenditure, prices 1990	69:1-97:4	Datastream
y_N	GNP, services, prices 1990	69:1-97:4	Datastream
y_T	GNP, non durables, prices 1990	69:1-97:4	Datastream
P	CPI all item index, 1990=100	69:1-97:4	Datastream
p	traded price index, 1990=100	69:1-97:4	Datastream
q	non-traded price index, 1990=100	69:1-97:4	Datastream
i	3 months T.Bill rate %, end of period	69:1-97:4	Datastream
r	$i_t - (\ln p_{t+1} - \ln p_t)$	69:1-97:4	Datastream
s	£ effective exchange rate, index 1990=100	69:1-97:4	Datastream
m	M0	69:2-97:4	Datastream
b	recursively on nx	69:1-97:4	Datastream
nx	trade balance (Exp.-Imp.), prices 1990	69:1-97:4	Datastream

Note: We distinguish total consumption expenditure by type: durables, non durables and services, and by object: food, beverage and tobacco; clothing and footwear; gross rent, fuel and power; transport and communications, furniture and household operation; other good and services. And we take as non-traded: services plus gross rent, fuel and power; transport and communications. The relative price of non traded has been taken from the price deflator of services.

A.2 US data

Variable	Definition	Sample	Source
c	total private cons. expenditure, prices 1990	69:1-98:2	Datastream
y	GNP, prices 1990	69:1-98:2	Datastream
m	Monetary Base	69:1-98:2	Datastream
i	3 month T.Bill rate % (end of period)	69:1-98:2	Datastream
p	CPI all items (index, 1990=100)	69:1-98:2	Datastream
r	$i_t - (\ln p_{t+1} - \ln p_t)$	69:1-98:2	Datastream
b	recursively from nx (same def. UK)	69:1-98:2	Datastream
s	US$ to UK£	69:1-98:2	Datastream

A.3 European Union data

Variable	Definition	Sample	Source
c	Private final cons. expend., curr. prices. const. PPP	79:1-98:3	OECD econ.trend
y	GNP, curr. prices. const. PPP	79:1-98:3	OECD econ.trend
m	Narrow M1 (index 1990=100) - EU11	79:1-98:3	OECD econ.trend
i	short-run nominal interest rate	79:1-98:3	OECD econ.trend
p	CPI all items (index 1990=100)	79:1-98:3	OECD econ.trend
r	$i_t - (\log p_{t+1} - \log p_t)$	79:1-98:3	OECD econ.trend
b	recursively from nx (same def. UK)	79:1-98:3	OECD econ.trend
s	ECU against \$US	79:1-98:3	OECD econ.trend

Note: The variables for EU have been obtained by averaging over the 15 members countries. Aggregation for the EMU area is provided by the OECD database. c and y are in million of dollars.

A.4 Other data

Variable	Definition	Sample	Source
n^{UK}	UK total population, annual	1969-1997	Datastream
n^{US}	US total population, annual	1969-1997	Datastream
n^{EU}	EU 15 total population, annual	1979-1998	EU- econ. trend
P^*	average OECD price index (1990=100)	69:1-97:4	Datastream
i^*	average OECD short-run nominal interest rate	69:1-97:4	Datastream
r^*	obtained as before	69:1-97:4	Datastream

Except interest rates, all the data have been initially linearly detrended. In the OECD data are excluded the highly inflationary economies: Turkey.

B Data Analysis

In this appendix we collect the results of the descriptive tests carried out on the data used in the whole thesis. The definitions of the variables are those given before and can be find all over the thesis. We then show the graphs of the theoretical cointegrating vectors.

B.1 Unit Root tests

UK-Ch.3	ADF(4)	UK-US Ch.5	ADF(4)	EU-Ch.6	ADF(4)
c_T	-2.58	c_{US}	-2.39	c	-0.72
c_N	-2.29	y_{US}	-3.37	y	-1.04
c	-2.05	s_{UK-US}	-2.29	s	-1.5
y_T	-2.35	r_{US}	-1.66	π	-3.3*
y_N	-1.87	p_{US}	-.95	p	-1.74
y	-2.37	π_{US}	-3.44*	r	-3.72**
s	-1.61	m_{US}	-2.84	m	-1.59
b	-1.75	i_{US}	-2.96*	i	-2.02
p	-2.30				
P	-2.58	r_{UK}	-2.43		
q	-2.98*	i_{UK}	-2.39		
m	-1.69	π_{UK}	-3.06*		
P^*	-.633				
r^*	-3.01*				
i^*	-2.82				
C.V. (5%)	-2.88	(1%) -3.49			

Note: UK and US sample: 1970:3 to 1997:3. EU sample 1980:2 to 1998:3. Critical Value for ADF with constant but without trend. For r^* ADF(3) sample: 1974:2-1997:3, for i^* ADF(3) sample 1972:2-1997:3. The unit root tests are carried out over linearly detrended variables.

UK	ADF(4)	US	ADF(4)	EU	ADF(4)
dy_T	-4.59**	dm	-3.03**	dm	-2.26
dy_N	-4.87**			dy	-3.72**
dm	-2.14**	$d\lambda$	-4.4**	$d\lambda$	-3.25**
dP^*	-2.29**				
di^*	-3.86**				
C.V.	-1.94(5%)		-2.58(1%)		

Note: Sample: 1970:4 to 1998:3. Critical value for ADF without constant and trend.

B.2 Tests on cointegrating vectors

ADF test on residuals

chapter 3	ADF(4)	chapter 5	ADF(4)	chapter 6	ADF(4)
$c_T - y_T$	-2.59**	$c_{UK} - c_{US}$	-3.66**	$c_{UK} - c_{EU}$	-2.67**
$c_N - y_N$	-3.67**	$p_{UK} - p_{US}$	-1.98*	$p_{UK} - p_{EU}$	-2.57**
$c - c_N$	-3.40**	$r_{UK} - r_{US}$	-3.20**	$r_{UK} - r_{EU}$	-3.33**
rer	-2.30*	$i_{UK} - i_{US}$	-2.38**	$i_{UK} - i_{EU}$	-2.65**
$b + i^* + p^* + s - y_T$	-2.25*	$\pi_{UK} - \pi_{US}$	-2.91**	$\pi_{UK} - \pi_{EU}$	-2.09*
vel	-1.92	rer	-2.29*	rer	-2.35*
		$\widehat{vel}$	-2.22*	$\widehat{vel}$	-2.104*
C.V.	-1.94 (5%)	-2.58 (1%)			

Note: Chapter 3: sample: 1970:4 to 1997:3. Chapter 5 and 6: sample 1970:4 to 1998:2. CV for ADF without constant and trend. ** indicate that the null of unit root is rejected with 99% of significance, * with 95% of significance. $vel = p + c - m - i$; $\widehat{vel} = -\widehat{c} - \widehat{p} - m^* + m$; $rer = s - p + p^*$. For $b + i^* + p^* + s - y_T$ sample 1973:3 -1997:3 and for vel sample 1973:3-1994:3. We remind also that the ^ over the variables means that we are taking the difference between the UK variable and the corresponding foreign variable (in chapter 5 is the US and in chapter 6 is the average of the EU countries).

Johansen's rank test

- Chapter 3

$Ho: rank = r$	$Max\ eig.$	$using(T-nm)$	95%	$Trace$	$using(T-nm)$	95%
$r = 0$	174	155.5		623.4	557.2	
$r \leq 1$	107.9**	96.47**	68.8	449.4**	401.7**	277.7
$r \leq 2$	80.12**	71.61**	62.8	341.5**	305.2**	233.1
$r \leq 3$	65.14**	58.22*	57.1	261.4**	233.6**	192.9
$r \leq 4$	49.78	44.5	51.4	196.2**	175.4**	156.0
$r \leq 5$	43.06	38.48	45.3	146.4**	130.9*	124.2
$r \leq 6$	35.39	31.63	39.4	103.4**	92.4	94.2
$r \leq 7$	30.26	27.04	33.5	67.99	60.77	68.5

Note: this is Johansen (1988, 1991) maximum likelihood approach used to estimate the number of common trends in the set of data. Reported are the maximum eigenvalue statistics: $-Tlog(1-\widehat{\lambda}_r)$ and $-(T-nm)\log(1-\widehat{\lambda}_r)$, $r = 1, ..n$, $n =$ number of variables and $m =$ number of lags in the VAR. This tests $H_0 : r$ cointegrating vectors against $H_1 : r{+}1$ cointegrating vectors. So the first row tests $H_0 : r = 0$ against $H_1 : r = 1$. If this is significant H_0 is rejected. Also reported are the trace statistics: $-T\sum_{i=r+1}^{n} log(1-\widehat{\lambda}_i)$ and $-(T-nm)\sum_{i=r+1}^{n} log(1-\widehat{\lambda}_i)$. This tests $H_0 : r$ cointegrating vectors against $H_1 :> r$ cointegrating vectors. So the first raw tests $H_0 : r = 0$ against $H_1 : r > 0$. If this is significant H_0 is rejected and the next raw tests $H_0 : r = 1$ against $H_1 : r > 1$. The second form of both tests uses a small-sample correction obtained by replacing T with $T - nm$. ** indicates the 99% significance and * the 95% significance. Reported are only the critical values for the 95% significance.

B.2.1 Wald test on the theoretical restrictions

- Chapter 2

Modelling c_T by IV. The present sample is: 1970:3 to 1997:3

Variable	y_{Tt}	dc_{Tt}	dy_{Tt}	dc_{Tt-1}	dc_{Tt-2}	dy_{Tt-1}	$const.$
Coefficient	0.63	-4.10	7.01	-2.06	-1.47	1.88	0.009
t-value	2.03	-1.19	1.37	-1.06	-0.94	1.198	0.682
σ	.137						
D.W.	2.05						
RSS	1.93						

Additional Instrument Used: y_{Tt-4}, c_{Tt-3}, y_{Tt-3}, c_{Tt-4} for 7 variables and 109 observations 4 endogenous and 4 exogenous variables with 8 instruments.

Reduced Form $\sigma = 0.0144$ Specification $Chi^2(1) = 0.059$ [0.807]

Testing $\alpha = 0$: $Chi^2(6) = 6.385$

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1,102) = 1.412 [0.237]

R -matrix							r vector
y_T	dc_T	dy_T	dc_{T-1}	dc_{T-2}	dy_{Tt-1}	$const$	
1.00	0	0	0	0	0	0	1.00

$\Longrightarrow$ The restriction cannot be rejected.

Modelling c_N by IV. The present sample is: 1970:3 to 1997:3

Variable	y_{Nt}	dc_{Nt}	dy_{Nt-2}	dc_{Nt-1}	dc_{Nt-2}	dy_{Nt-1}	dy_{Nt-2}	$const.$
Coefficient	0.93	-4.73	.315	-0.79	0.279	0.476	0.584	-0.001
t-value	4.39	-.896	1.76	-.838	0.195	0.773	0.942	-.239
σ	.053							
D.W.	2.01							
RSS	0.28							

Additional Instruments used: y_{Nt-4}, c_{Nt-4}, c_{Nt-3}, y_{Nt-3}, for 8 variables and 109 observations 4 endogenous and 5 exogenous variables with 9 instruments.

Reduced Form $\sigma = 0.0117$ Specification $Chi^2(1) = 0.203$ [0.651]
Testing $\alpha = 0$: $Chi^2(7) = 94.42$ [0.000]**.

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1, 101) = 0.0972 [0.755]

R-matrix								r vector
y_N	dc_N	dy_N	dc_{N-1}	dc_{N-2}	dy_{Nt-1}	dy_{Nt-2}	$const$	
1.00	0	0	0	0	0	0	0	1.00

$\Longrightarrow$ The restriction cannot be rejected.

Modelling c by IV. The present sample is: 1970:3 to 1997:3

Variable	c_{Nt}	dc_t	dc_{Nt-1}	dc_{Nt-2}	dc_{t-1}	dc_{t-2}	$const.$
Coefficient	0.89	-2.01	1.00	-0.32	-0.79	0.139	0.003
t-value	9.74	-1.99	1.56	-0.61	-1.18	0.336	0.799
σ	.004						
D.W.	1.9						
RSS	0.16						

Additional Instruments used: c_t, c_{Nt-3}, c_{Nt-4}, c_{t-3} for 8 variables and 109 observations 4 endogenous and 5 exogenous variables with 9 instruments.
Reduced Form $\sigma = 0.0108$ Specification $Chi^2(1) = 3.677$ [0.055]
Testing $\alpha = 0$: $Chi^2(7) = 128.02$ [0.000]**.

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1 ,101) = 1.634 [0.204]

R-matrix							r vector
c_{Nt}	dc_t	dc_{Nt-1}	dc_{N-2}	dc_{t-1}	dc_{t-2}	$const$	
1.00	0	0	0	0	0	0	1.00

$\Longrightarrow$ The restriction cannot be rejected.

Modelling s by IV. The present sample is: 1970:3 to 1997:3

Variable	p^*_t	dp^*_t	p_t	dp_t	ds_t	dp_{t-1}	$const$
Coefficient	-1.29	-14.7	0.96	-5.47	-4.52	10.4	-.021
t-value	-1.67	-1.13	1.91	-.63	-2.03	1.67	-.83
σ	.204						
D.W.	1.11						
RSS	4.24						

Additional Instruments used: $p*_{t-4}, s_{t-3}, p_{t-4}, s_{t-4}, p^*_{t-3}$, for 7 variables and 109 observations 6 endogenous and 2 exogenous variables with 8 instruments.

Reduced Form $\sigma = 0.059$ Specification $Chi^2(1) = 1.995$ [0.157]
Testing $\alpha = 0$: $Chi^2(6) = 12.217$ [0.057]

Wald test for linear restrictions ($R\alpha = r$) LinRes F(2 ,102) = 0.588 [0.557]

R-matrix							r vector
p_t^*	dp_t^*	p_t	dp_t	ds_t	dp_{t-1}	$const$	
-1.00	0	0	0	0	0	0	1
0	0	1.0	0	0	0	0	1

$\Longrightarrow$ The restriction cannot be rejected.

Modelling b by IV. The present sample is: 1970:2 to 1997:3

Variable	p_t^*	s_t	y_{Tt}	dy_{Tt}	dp_t^*	i_t^*	dy_{Tt-1}	dp_{t-1}^*	dp_{t-2}^*	di_{t-1}^*	$const.$
Coeff.	.225	-.18	.985	3.1	15.6	-.52	.666	-10.2	3.25	-4.28	.029
t-value	1.07	-.82	1.98	1.5	1.8	-.39	1.14	-1.31	1.03	-1.17	2.91
σ	.096										
D.W.	1.67										
RSS	.929										

Additional Instruments used: $b_{t-3}, p_{t-4}^*, b_{t-4}, y_{Tt-3}, s_{t-3}, i_{t-3}^*, i_{t-4}^*, s_{t-4}^*, p_{t-3}^*, y_{Tt-4}$, for 11 variables and 110 observations 7 endogenous and 5 exogenous variables with 15 instruments.

Reduced Form $\sigma = 0.0222$ Specification $Chi^2(4) = 8.722$ [0.0684]
Testing $\alpha = 0$: $Chi^2(10) = 22.63$ [0.012]*

Wald test for linear restrictions ($R\alpha = r$). LinRes F(4, 99) = 39.46 [0.00]**

R-matrix											r vector
p_t^*	s_t	y_{Tt}	dy_{Tt}	dp_t^*	i_t^*	dy_{Tt-1}	dp_{t-1}^*	dp_{t-2}^*	di_{t-1}^*	$const$	
-1	0	0	0	0	0	0	0	0	0	0	1
0	-1	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	-1	0	0	0	0	0	1

Wald test for linear restrictions ($R\alpha = r$). LinRes F(1, 99) = 0.0008 [0.977]

R-matrix											r vector
p_t^*	s_t	y_{Tt}	dy_{Tt}	dp_t^*	i_t^*	dy_{Tt-1}	dp_{t-1}^*	dp_{t-2}^*	di_{t-1}^*	$const$	
0	0	1	0	0	0	0	0	0	0	0	1

Wald test for linear restrictions ($R\alpha = r$). LinRes F(2, 99) = 3.799 [0.025]*

R-matrix											r vector
p^*_t	s_t	y_{Tt}	dy_{Tt}	dp^*_t	i^*_t	dy_{Tt-1}	dp^*_{t-1}	dp^*_{t-2}	di^*_{t-1}	$const$	
0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	0	0	-1	0	0	0	0	0	1

$\Longrightarrow$ For the cointegrating vector $(b + s + p^* + i^* - y_T)$ only the third and fourth restrictions cannot be rejected.

Modelling p by IV. The present sample is: 1970:3 to 1997:3.

Variable	q_t	dp_t	dq_t	dp_{t-1}	dp_{t-2}
Coefficient	.652	-2.22	1.01	.422	-.297
t-value	7.8	-.97	1.12	1.46	-1.05
σ	.012				
D.W.	1.76				
RSS	.016				

Additional Instruments used: $const.$, $p_{t-3}, q_{t-3}, p_{t-4}, q_{t-4}$ for 5 variables exogenous variables with 7 instruments.

Reduced Form $\sigma = 0.0064$ Specification $Chi^2(2) = 5.202$ [0.074]

Testing $\alpha = 0$: $Chi^2(5) = 237.06$ [0.00]**

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1,104) = 3.124 [0.08]

R-matrix					r vector
q_t	dp_t	dq_t	dp_{t-1}	dp_{t-2}	
1	0	0	0	0	.8

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1,104) = 8.783 [0.004]**

R-matrix					r vector
q_t	dp_t	dq_t	dp_{t-1}	dp_{t-2}	
1	0	0	0	0	.9

Wald test for linear restrictions ($R\alpha = r$) LinRes F(1,104) = 17.303 [0.00]**

R-matrix					r vector
q_t	dp_t	dq_t	dp_{t-1}	dp_{t-2}	
1	0	0	0	0	1

$\Longrightarrow$ From the sets of Wald tests we observe that the only restriction that we cannot reject is $(q - .8p)$ and not $(q - p)$.

B.3 Non-linear Least Squares

The equation is $\frac{c_T}{c} = \gamma \left(\frac{1}{p}\right)^{-\theta}$. Set actual $= \frac{c_T}{c}$ and fitted $= \gamma \left(\frac{1}{p}\right)^{-\theta}$
Modelling actual by NLS. The present sample is 1970:2-1997:3π

	γ	θ
	.567	-2.27
t-value	223.4	-20.39
$\sigma =$	0.022	
D.W.=	.117	
RSS =	.0566	
F(1,108)=	409.95	[.00]

B.4 The IV estimate of the 'deep parameters'

- The simulation carried out in chapter 5 relied on calibrated values for the deep parameters. In this section we show some estimates of the Euler equation for real money balances in the UK and US.

Modelling $(m-p)_{UK}$ by IV. The present sample is: 1970:3 to 1998:3.

Variable	$d(m-p)_t^{UK}$	c_t^{UK}	i_t^{UK}	dc_t^{UK}	di_t^{UK}	$d(m-p)_{t-1}^{UK}$	const.
Coefficient	-14.46	3.65	-.497	-4.04	-.528	5.73	.027
t-value	-2.15	2.21	-4.47	-.253	-.883	1.92	.603
σ	0.4508						
D.W.	2.04						
RSS	21.138						

Additional Instruments used: $c_{t-3}^{UK}, c_{t-1}^{UK}, (m-p)_{t-4}^{UK}, di_{t-1}^{UK}, di_{t-2}^{UK}, i_{t-3}^{UK}, i_{t-4}^{UK}$, for 7 variables and 113 observations 4 endogenous and 4 exogenous variables with 9 instruments.

Reduced Form $\sigma = 0.041$.	Specification $Chi^2(2) = 0.598$ [0.741].
Testing $\alpha = 0$: $Chi^2(6)$=28.23 [0.00]**	Error Autocorr. 5 lags $Chi^2(5) = 10.95$[0.05]
ARCH 4 F(4, 96) = 0.723 [0.578]	Normality $Chi^2(2)$= 3.842 [0.146].
X_i^2 F(16, 87) = 7.987 [0.00]**	$X_i X_j$ F(44, 59) = 14.32 [0.00]**

Modelling $(m-p)_{US}$ by IV. The present sample is: 1970:3 to 1998:3

Variable	$d(m-p)_t^{US}$	c_t^{US}	i_t^{US}	dc_t^{US}	di_t^{US}	$d(m-p)_{t-1}^{UK}$	dc_{t-1}^{US}	const.
Coeff.	-10.20	.052	-.223	6.08	-.204	6.95	-5.59	-.002
t-value	-1.51	.206	-4.18	.949	-.82	1.42	-1.20	-.076
σ	0.2717							
D.W.	1.98							
RSS	7.68							

Additional Instruments used: $di_{t-2}^{US}, dc_{t-2}^{US}, (m-p)_{t-3}^{US}, c_{t-3}^{US}, c_{t-4}^{US}, (m-p)_{t-4}^{US}, i_{t-3}^{US}, i_{t-4}^{US}$, for 9 variables and 113 observations 6 endogenous and 4 exogenous variables with 12 instruments.

Reduced Form $\sigma = 0.0496$	Specification $Chi^2(3) = 3.982$ [0.263]
Testing $\alpha = 0$: $Chi^2(8)$=37.78[0.00]**	Error Autocorr. 5 lags $Chi^2(5) = 244.2$[0.0]**
ARCH 4 F(4, 96) = 0.563 [0.69]	Normality $Chi^2(2)$= 0.399 [0.819].
X_i^2 F(16, 87) = 9.5 [0.00]**	X_iX_j F(44, 59) = 20.75 [0.00]**

B.5 The exogenous processes

- Estimating the unrestricted reduced form VAR by OLS. The present sample is: 1969:4 to 1997:3 (chapter 2).

	dm_t	$dr^*{}_t$	dy_{Tt}	dy_{Nt}	dp_t^*
dm_{t-1}	0.311 [3.40]	0.041 [0.81]	0.06 [0.34]	0.168 [1.21]	0.059 [2.17]
dr_{t-1}^*	0.186 [-1.09]	0.155 [1.61]	-0.116 [-0.35]	-0.457 [-1.75]	0.254 [4.9]
dy_{Tt-1}	0.04 [0.78]	0.04 [1.39]	-0.182 [-1.84]	0.238 [3.04]	0.0012 [0.08]
dy_{Nt-1}	0.047 [0.76]	0.03 [0.86]	0.034 [0.28]	-0.059 [-0.63]	0.017 [0.93]
dp_{t-1}^*	0.393 [2.49]	0.135 [1.53]	-0.621 [-2.05]	-0.158 [-0.66]	0.83 [17.4]
$const.$	.0008 [0.76]	7.3e^{-5}[0.11]	-.0001 [-0.06]	-.0012 [-0.76]	-.0003 [-0.80]
σ	0.0113	0.00637	0.0218	0.0173	0.0034
RSS	0.013	0.0043	0.0508	0.032	0.0013

Note: t-values between brackets.

- Correlation of URF residuals

	dm	dr^*	dy_T	dy_N	dp^*
dm	1.0				
dr^*	0.073	1.0			
dy_T	0.102	0.108	1.0		
dy_N	-0.022	0.209	-0.199	1.0	
dp^*	-0.053	-0.403	-0.044	-0.031	1.0

loglik = 2617.09; log$|\widehat{\sum}|$ = -46.73; $|\widehat{\sum}|$ = 5.05e^{-21}; T = 112: log|Y'Y/T| = -44.6: Rα(LR) = 0.88; Rα(LM) = 0.224.

F-test against unrestricted regressors, F(25, 380) = 11.77 [0.00]**, variables entered unrestricted: Constant.

- F-tests on retained regressors, F(5, 102):

dm_{-1}	4.031 [0.0022]**
dr^*_{-1}	9.257 [0.0000]**
dy_{T-1}	2.497 [0.0355]*
dy_{T-1}	0.780 [0.5659]
dp^*_{-1}	79.61 [0.0000]**

- Wald test for general restrictions GenRes $Chi^2(10)$ = 13.618 [0.191]

  C1eq1=0; C2eq1=0; C3eq1=0; C4eq1=0; C5eq1=0; C1eq2=0; C3eq2=0; C4eq2=0; C5eq2=0; C1eq3=0. These restrictions correspond to those imposed on the matrix Φ of equation (2.39); where C1eq1 means the first coefficient of equation 1 and so on.

- Estimating the model by FIML The present sample is: 1969 (4) to 1997 (3)

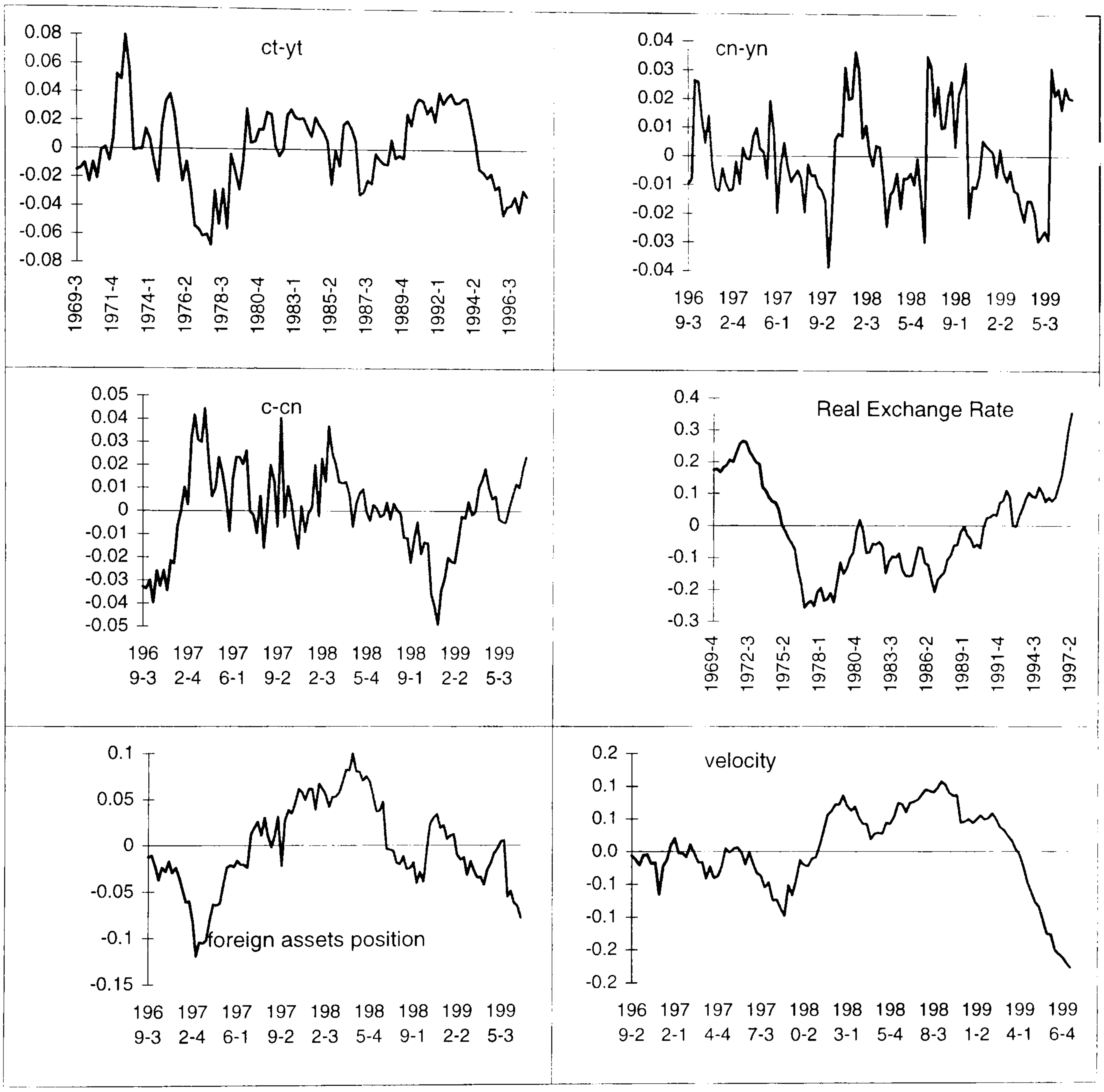
	dm_t	$dr^*{}_t$	dy_{Tt}	dy_{Nt}	dp^*_t
dm_{t-1}	0.29[3.3]				.07[2.74]
dr^*_{t-1}		0.19[2.1]		-.42[-1.7]	.26[5.15]
dy_{Tt-1}			-.21[-2.25]	.23[3.1]	
dy_{Nt-1}				-.06[-.74]	
dp^*_{t-1}	0.34[2.3]		-.71[-2.57]		.85 [20.1]
$const.$	.0008[0.72]	-9e^{-5}[-.15]	-.00023[-.11]	-0.0009[-.61]	-.00025[-.77]
σ	0.0113	0.00644	0.02163	0.0172	0.0034

- loglik = 2611.34 log$|\hat{e}|$ = -46.63 $|\hat{e}|$ = 5.6e^{-21} T = 112
- LR test of over-identifying restrictions: $Chi^2(14)$ = 11.498 [0.646]
- Correlation of URF residuals

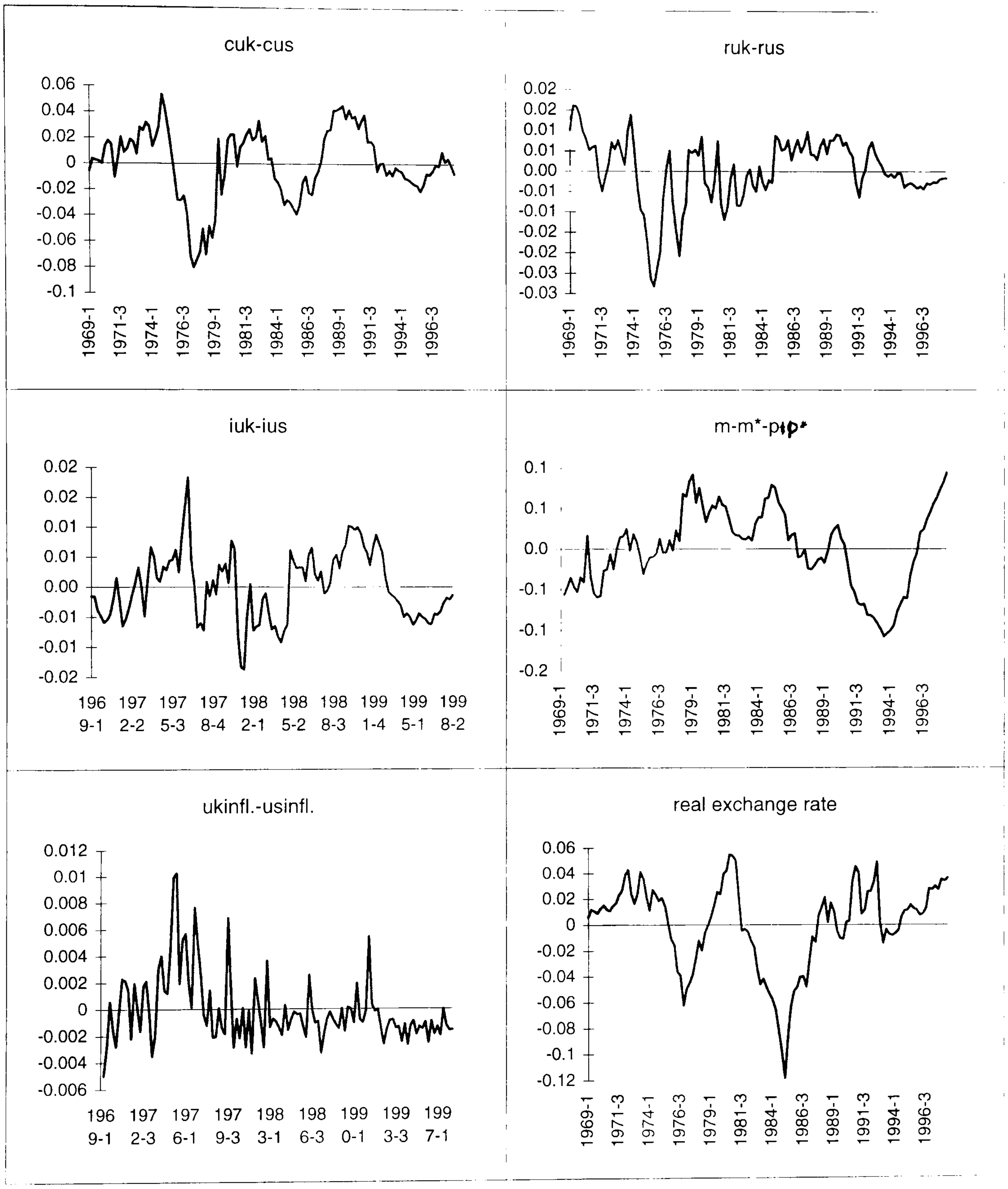
	dm	dr^*	dy_T	dy_N	dp^*
dm	1.0				
dr^*	0.087	1.0			
dy_T	0.107	0.118	1.0		
dy_N	-0.019	0.208	-0.194	1.0	
dp^*	-0.048	-0.396	-0.045	-0.031	1.0

B.6 Long-run behaviour of the variables

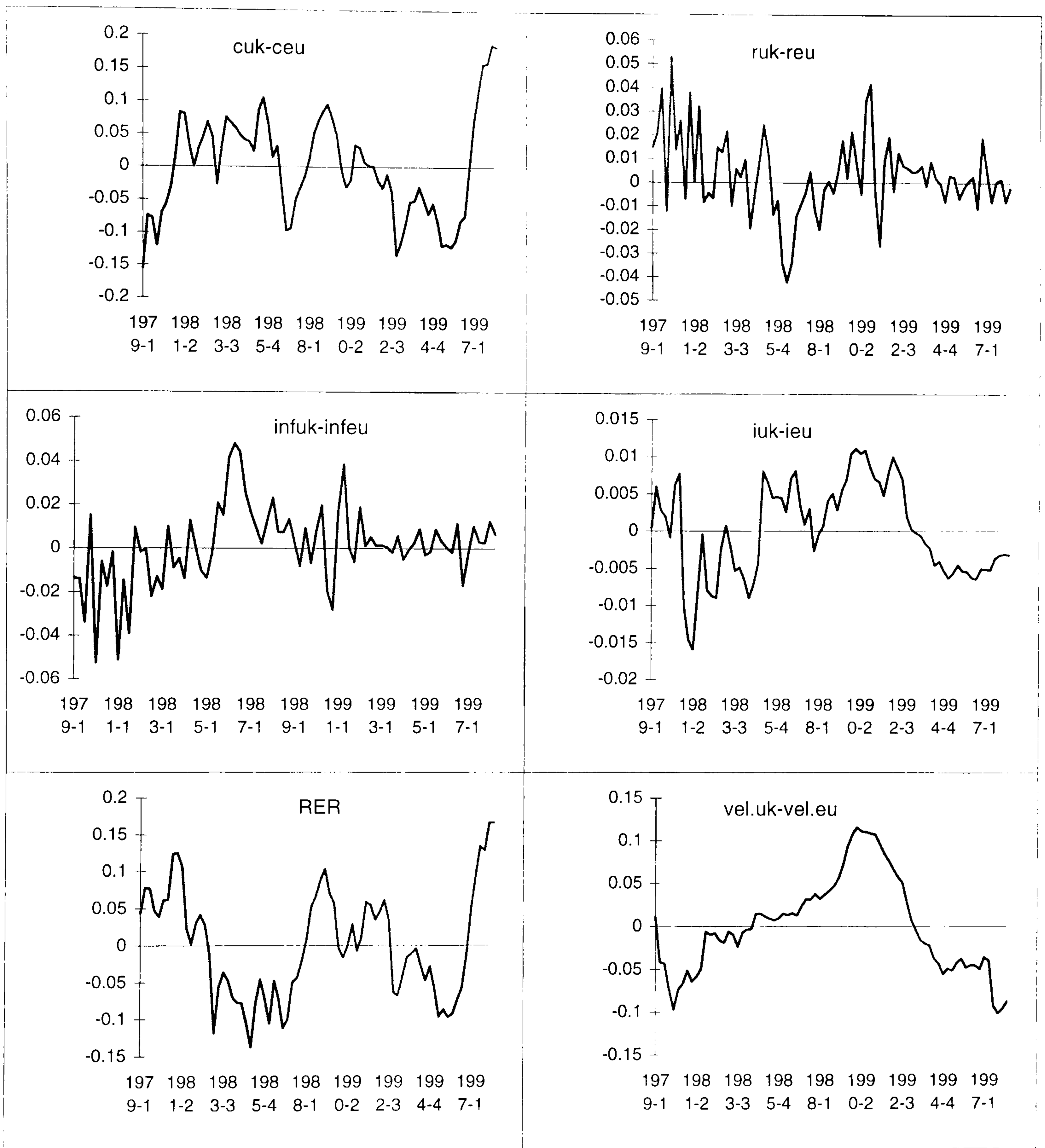
I) The cointegrating vectors of chapter 2



II) The cointegrating vectors of chapter 5



III) The cointegrating vectors of chapter 6



C Estimates of Chapter 3

OLS estimates of the short-run VAR with diagonal error covariance matrix, sample 1969:3-1997:3.

	$LR1_t$	$LR2_t$	$LR3_t$	$LR4_t$	$LR5_t$	$LR6_t$
$LR1_{t-1}$	.72[10.2]	-.034[-.61]	.056[1.19]	.098[.71]	-.001[-.01]	.08[2.07]
$LR2_{t-1}$	-.007[-.07]	.578[6.6]	.124[1.65]	-.143[-.68]	.099[.89]	.04[.69]
$LR3_{t-1}$	-.09[-.75]	.021[0.21]	.76[8.64]	.628[2.54]	.106[.82]	-.11[-1.58]
$LR4_{t-1}$	-.06[-2.7]	.0003[.01]	-.017[-1.1]	.982[22.8]	.007[.29]	.01[.81]
$LR5_{t-1}$	.055[2.11]	.0139[.65]	-.025[-1.4]	.126[2.45]	.985[36.7]	-.015[-1.08]
$LR6_{t-1}$	.024[.64]	-.036[-1.2]	.054[2.05]	-.076[-1.0]	.008[.19]	.98[47.0]
dy_{Tt-1}	.029[.33]	-.063[-.87]	-.092[-1.5]	-.333[-1.9]	.185[2.02]	.003[.05]
dy_{Nt-1}	-.027[-.27]	-.023[-.28]	.045[.66]	-.176[-.92]	.044[.43]	-.038[-.71]
$dr^*{}_{t-1}$	-.073[-.28]	.213[1.03]	.274[1.53]	.204[.407]	.102[.39]	-.032[-.22]
$dp^*{}_{t-1}$	-.121[-.39]	.33[1.33]	-.204[-.95]	-.651[-1.1]	1.26[4.03]	.557[3.29]
dm_{t-1}	.166[1.14]	-.18[-1.5]	.077[.75]	-.634[2.21]	.11[.76]	.323[3.98]
const.	-.001[-.74]	.0006[.41]	.0009[.69]	-.006[-1.8]	-.001[-.73]	.001[1.38]
R^2	0.6745	0.3566	0.6120	0.9531	0.9825	0.9915
$\overline{R}^2$	0.6384	0.2851	0.5689	0.9479	0.9805	0.9906
S.D.	0.0164	0.0133	0.0114	0.0321	0.0168	0.0091

	dy_{Tt}	dy_{Nt}	dr_t^*	dp_t^*	dm_t
$LR1_{t-1}$	.138 [1.46]	-.093 [-1.32]	.002 [.07]	-.002 [-0.17]	-.026 [-.57]
$LR2_{t-1}$	.024 [.163]	.357 [3.29]	-.01 [-.24]	.004 [.20]	-.035 [-.48]
$LR3_{t-1}$	.18 [1.06]	.231 [1.83]	-.09 [-2.03]	.07 [2.71]	.09 [1.17]
$LR4_{t-1}$	.046 [1.55]	.0005 [.02]	-.006 [-.73]	.001 [.29]	.02 [1.41]
$LR5_{t-1}$	-.02 [-.579]	.037 [1.44]	-.02 [-2.0]	.007 [1.26]	-.03 [-1.8]
$LR6_{t-1}$	-.024 [-.47]	-.0008 [-.02]	.028 [1.95]	-.003 [-.49]	-.025 [-1.01]
dy_{Tt-1}	-.22 [-1.88]	.114 [1.28]	.07 [1.94]	-.022 [-1.18]	-.019 [-.32]
dy_{Nt-1}	-.005 [-.04]	-.019 [-.19]	.042 [1.11]	.007 [.375]	.035 [.54]
$dr^*{}_{t-1}$	.37 [.108]	-.457 [-1.78]	.119 [1.22]	.272 [5.15]	-.11 [-.64]
$dp^*{}_{t-1}$	-.79 [-1.93]	-.77 [-2.53]	.24 [2.05]	.72 [11.5]	.56 [2.81]
dm_{t-1}	-.006 [-.03]	.24 [1.65]	.049 [.88]	.05 [1.68]	.155 [1.61]
const.	-.0002 [-.1]	-.003 [-1.72]	.0006 [.82]	-.0006 [-1.55]	.002 [1.46]
R^2	0.0165	0.2039	0.049	0.7867	0.2326
$\overline{R}^2$	-0.0928	0.1155	-0.0566	0.763	0.1474
S.D.	0.022	0.0164	0.0063	0.0034	0.0108

where $LR1 = c_T - y_T$, $LR2 = c_N - y_N$, $LR3 = c - c_N$, $LR4 = s - p + p^*$, $LR5 = b + s + i^* + p^* - y_T$, $LR6 = p - m + c - i^*$

Sigma :

$$\begin{bmatrix}
.0003 & -.0001 & .00 & .00 & .0001 & .00 & -.0003 & .0001 & .00 & .00 & .00 \\
-.0001 & .0002 & .00 & -.0001 & .0001 & .00 & .0001 & -.0002 & .00 & .00 & .00 \\
.00 & .00 & .0001 & .00 & -.0001 & .00 & .0001 & .00 & .00 & .00 & .00 \\
.00 & -.0001 & .00 & .001 & .0001 & .00 & .00 & .00 & .00 & .00 & .00 \\
.0001 & -.0001 & -.0001 & .0001 & .0003 & .00 & -.0002 & .0001 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .0001 & -.0001 & .00 & .00 & .00 & .00 \\
-.0003 & .0001 & .0001 & .00 & -.0002 & -.0001 & .0005 & -.0001 & .00 & .00 & .00 \\
.0001 & -.0002 & .00 & .00 & .0001 & .00 & -.0001 & .0003 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .0001
\end{bmatrix}$$

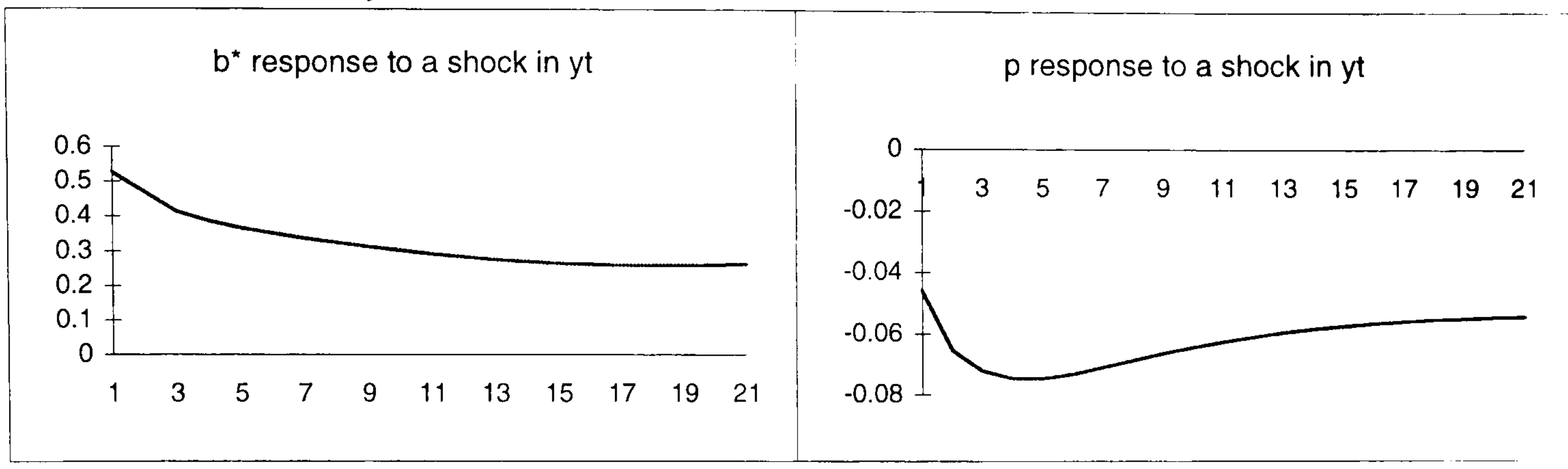
J :

$$\begin{bmatrix}
.0164 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & .0133 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .0114 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & .0321 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & .0168 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & .0091 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & .022 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & .0164 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0063 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0034 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0108
\end{bmatrix}$$

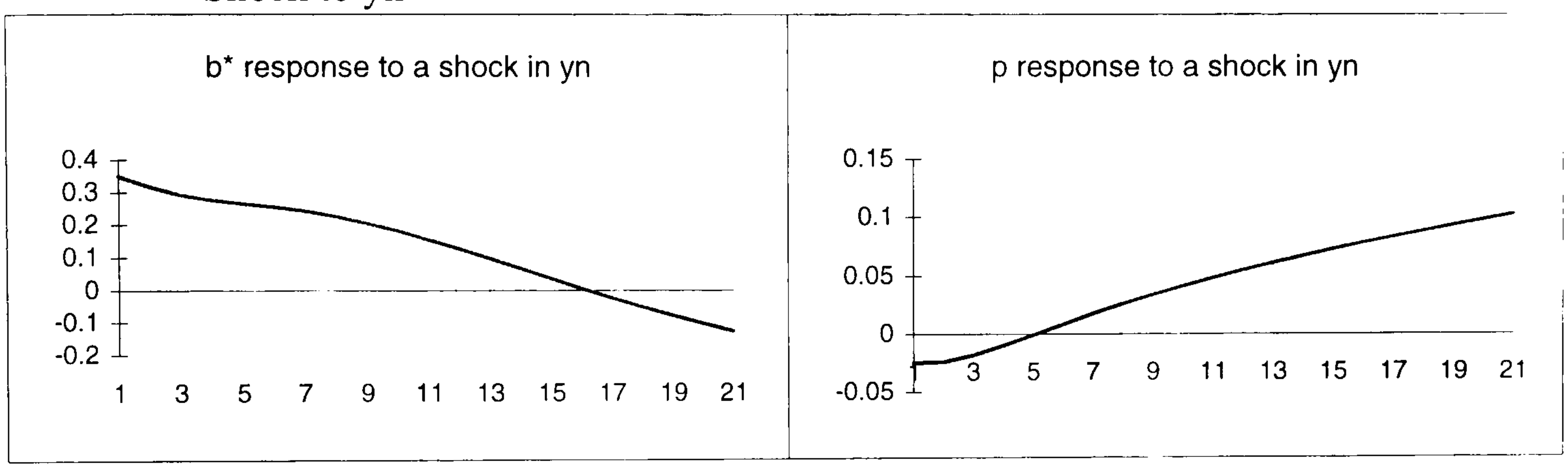
J is the matrix that transforms reduced form errors in structural errors.

C.1 Impulse response functions of the VECM

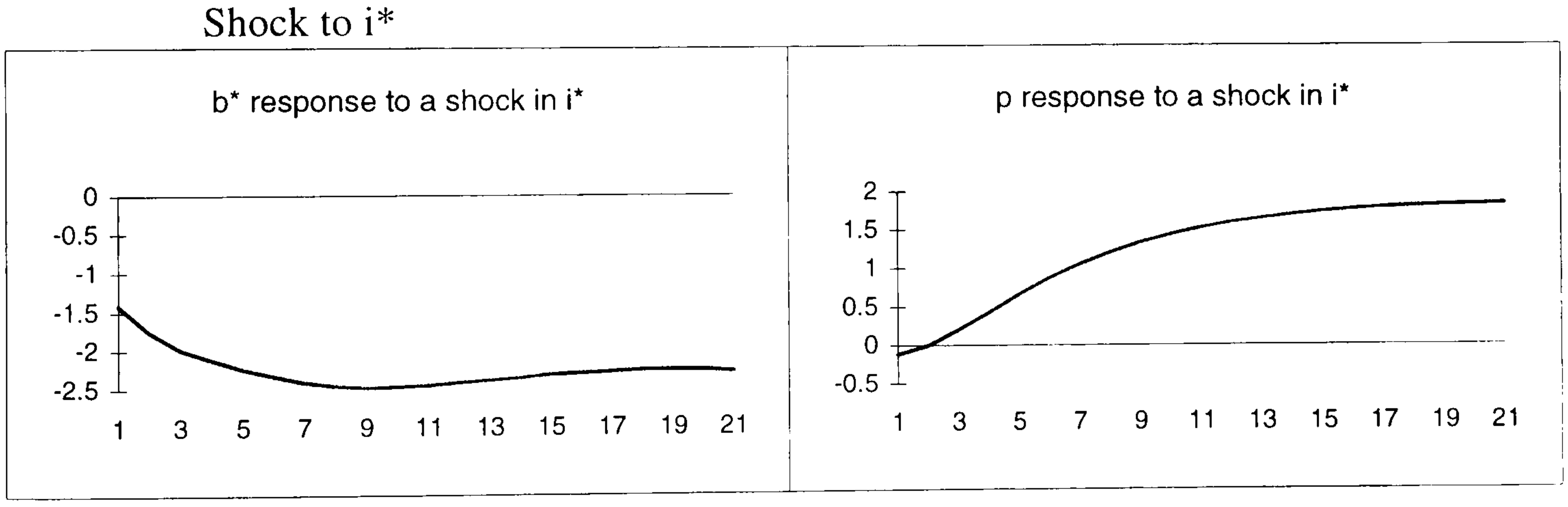
Shock to yt



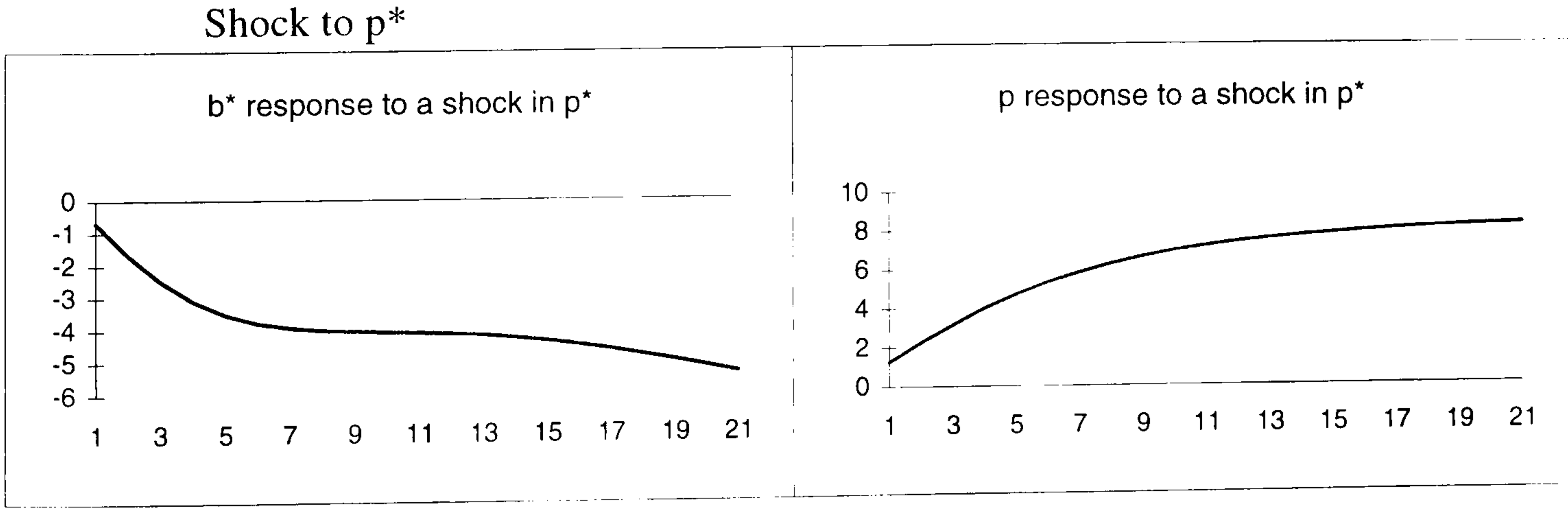
Shock to yn



Shock to i*



Shock to p*



D The Program for the Simulations

We describe the program used for simulating the model of chapter 2. Results from the simulations are in chapter 4.

a) The first part of the program consists of setting the parameters and the steady-state relationships. See chapter 4 for the values of the parameters:

% Setting parameters in steady-state:

$\overline{y}_T = 1$; % normalization for traded-output in steady state

$\overline{y}_N = 1$; % normalization for non-traded-output in steady state

$\overline{S} = 1$; % normalization for the nominal exchange rate in steady state

$\overline{M} = 1$; % normalization for nominal money supply in steady state

$\overline{R}$ =1.01; % steady-state quarterly real interest rate (see chapter 4)

$\overline{P}^* = 1.00$; % normalization for foreign price index in steady state

$\overline{rer}$ =1; % normalization for the real exchange rate in steady state

gamma = 0.57; % consumption of traded over total consumption

theta = 2.4; % elasticity of substitution between traded and non-traded

z=0.0013; % (ς : weight on real balances in the utility function)

psi$_j$ = ..; % $j = y_N$, y_T, p^*, i^*, m; autocorrelation of shocks, values in App. B.

sigma$_j$=..; % $j = y_N$, y_T, p^*, i^*, m; standard deviation of shocks, unit: %,values in App. B.

corr$_{jk}$ = ..; % $j, k = y_N$, y_T, p^*, i^*, m; correlation between shocks, values in App. B.

SV$_{jk}$=corr$_{jk}$*sigma$_j$*sigma$_k$; % $j, k = y_N$, y_T, p^*, i^*, m

% Calculating the steady state:

beta = $1/\overline{R}$;

$\overline{q}$ = (gamma/(1-gamma))^theta;

$\overline{p}$ = [(gamma+(1-gamma)*Q_bar^(1-theta))^1/(1-theta))];

$\overline{c}_N = \overline{y}_N$;

$\overline{c}_T = \overline{c}_N$*[(gamma*$\overline{q}$)*$\overline{p}$^(theta-1)/(1-gamma*$\overline{p}$^(theta-1))];

$c = (\overline{q}^*\overline{c}_N+\overline{c}_T)/\overline{p}$;

$\overline{b} = c$;

% Definitions:

$mp = \overline{M}/\overline{p}$;

$ctp = \overline{c}_T/\overline{p}$;

$cnpq = \overline{c}_N * \overline{q}/\overline{p}$;

$y_N = \overline{y}_N * (\overline{q}/\overline{p})$;

$y_T = \overline{y}_T/\overline{p}$;

$br = \overline{R} * \overline{b}$;

$brr = br * \overline{S} * \overline{P}^*/\overline{p}$;

$bbr = \overline{b} * \overline{S} * \overline{P}^*/\overline{p}$;

$p$theta=$\overline{p}$^(1-theta);

$q$theta= (1-gamma)*$\overline{q}$^(1-theta);

bR=$\overline{R}$*($\overline{M} - \overline{p} * c * z$);

bRR=$\overline{R}$*($\overline{M}$−2*$\overline{p} * c * z$);

b) The second part of the program consists of writing down in matrix form the log-linear system of equations:

VARNAMES=['b^*', 'c_N', 'c_T', 'S', 'c', 'q', 'p', 'rer', 'y_N', 'y_T', 'R^*', 'P^*', 'M']:

% Translating into coefficient matrices - The loglinearized equations are ordered:

% 1) $0 = y_T*(y_{Tt} - p_t) + brr*(s_t + p_t^* + r_t - p_t + b_{t-1}) - mp*(m_t - m_{t-1}) - c*c_t - bbr*(s_t + p_t^* - p_t + b_t) + y_N - bbr*(s_t + p_t^* - p_t + b_t) + y_N(y_{Nt} + q_t - p_t)$;

% 2) $0 = c*c_t - ctp*(c_T - p_T) - cnpq*(q_t + c_{Nt} - p_t)$;

% 3) $0 = c_{Tt}$-theta*$p_t - c_t$;

% 4) $0 = q_t$-theta*y_{Nt}+theta*c_{Nt};

% 5) $0 =$ $p$theta*p_t- $q$theta*q_t;

% 6) $0 = rer_t - s_t - p_t^* + p_t$;

% E1) $0 = E_t[(c_t - c_{t+1} + r_{t+1} + s_{t+1} - s_t + p_t - p_{t+1} + p_{t+1}^* - p_t^*]$;

% E2) $0 = E_t[-\overline{p}cz*c_t + \overline{M}*(\overline{R}-1)*m_t + p_t\overline{R}*(\overline{M} - 2\overline{p}c) - \overline{M}s_t - z\overline{R}s_t - \overline{M}p_{t+1} - \overline{M}p_t^* + z\overline{R}p_{t+1}^* + z\overline{R}r_{t+1}]$

% 9-13) j(t) = psi$_j$j(t-1)+epsilon$_j$(t), $j = y_N, y_T, p^*, i^*, m$.

% Switch to that notation. Find matrices for format

% 0 = AA x_t + BB x_{t-1} + CC y_t + DDz_t

% 0 = E_t[FFx_{t+1}+GGx_t+HHx_{t-1}+JJy_{t+1}+KKy_t+LLz_{t+1}+MMz_t]

% z_{t+1}=NNz_t+epsilon$_{t+1}$ with E_t[epsilon$_{t+1}$]=0,

% Endogenous state variables x_t: b_t, c_{Nt}

% Endogenous other variables y_t: c_t,, c_{Tt},q_t, s_t,p_t, rer_t

% Exogenous state variables z_t: y_{Nt}, y_{Tt}, r_t,p_t^*, m_t

% for b_t and c_{Nt}:

$$AA = \begin{bmatrix} -bbr, & 0 \\ 0, & -cnpq \\ 0, & 0 \\ 0, & theta \\ 0, & 0 \\ 0, & 0 \end{bmatrix};$$

% for b_{t-1} and c_{Nt-1}:

$$BB = \begin{bmatrix} bbr, & 0 \\ 0, & 0 \\ 0, & 0 \\ 0, & 0 \\ 0, & 0 \\ 0, & 0 \end{bmatrix};$$

for c_{Tt}, s_t,c_t,q_t,rer_t, p_t

$$\text{CC} = \begin{bmatrix} 0, & bbr*(\overline{R}-1), & -c, & yn, & -bbr*(\overline{R}-1)-yt-yn, & 0 \\ -ctp, & 0, & c, & -cnpq, & ctp+cnpq, & 0 \\ 1, & 0, & -1, & 0 & -theta, & 0 \\ 0, & 0, & 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & -qtheta, & ptheta, & 0 \\ 0, & -1, & 0, & 0, & 1, & 1 \end{bmatrix};$$

$$\text{DD} = \begin{bmatrix} yn & yt, & bbr, & bbr*(\overline{R}-1), & psi_m - 1 \\ 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0 \\ -theta, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & -1 & 0 \end{bmatrix};$$

FF = zeros(2,2);

GG = zeros(2,2);

HH = zeros(2,2);

$$\text{JJ} = \begin{bmatrix} 0, & 1, & -1, & 0, & -1, & 0 \\ 0, & z\overline{R}, & 0, & 0, & -\overline{M}, & 0 \end{bmatrix};$$

$$\text{KK} = \begin{bmatrix} 0, & -1, & 1, & 0, & 1, & 0 \\ 0, & -M\_bar, & -\overline{p}*c*b, & 0, & bbr, & 0 \end{bmatrix};$$

$$\text{LL} = \begin{bmatrix} 0, & 0, & 1, & 1, & 0 \\ 0 & 0, & br, & br, & \overline{M}*(\overline{R}-1), \end{bmatrix};$$

$$\text{MM} = \begin{bmatrix} 0, & 0, & 0, & -1, & 0 \\ 0, & 0, & 0, & -\overline{M} & 0 \end{bmatrix};$$

$$\text{NN} = \begin{bmatrix} psi_{y_N}, & .23, & -.42, & 0, & 0 \\ 0, & psi_{y_T}, & 0, & -.76, & 0 \\ 0, & 0, & psi_{r^*}, & 0, & 0 \\ 0, & 0, & .26, & psi_{p^*}, & 0 \\ 0, & 0, & 0, & .34, & psi_m \end{bmatrix};$$

$$\text{Sigma} = \begin{bmatrix} sigma^2_{y_N}, & SV_{y_N y_T}, & SV_{y_N r^*}, & SV_{y_N p^*}, & SV_{y_N m} \\ SV_{y_N y_T}, & sigma^2_{y_T}, & SV_{y_T r^*}, & SV_{y_T p^*}, & SV_{y_T m} \\ SV_{y_N r^*}, & SV_{y_T r^*}, & sigma^2_{r^*}, & SV_{r^* p^*}, & SV_{r^* m} \\ SV_{y_N p^*}, & SV_{y_T p^*}, & SV_{r^* p^*}, & sigma^2_{p^*}, & SV_{p^* m} \\ SV_{y_N m}, & SV_{y_T m}, & SV_{r^* m}, & SV_{p^* m}, & sigma^2_m \end{bmatrix}$$

do it;

% This command recalls the procedures used to compute roots; impulse responses functions and moments.

This program has then been modified and used to simulate the models in chapter 5, 7 and 8.

E Estimates of Chapter 5

E.1 The exogenous processes

Estimating the unrestricted reduced form VAR by OLS. The present sample is: 1969:4 to 1998:3, for UK and US data. US variable are indicated with an asterisk.

	dm_t	dm^*_t
dm_{t-1}	0.162[1.75]	0.695[9.64]
dm^*_{t-1}	0.455[3.28]	-0.008 [-.16]
$const.$	.0011 [0.91]	.0002[0.28]
σ	0.0133	0.0069
RSS	0.0201	0.0054

Correlation of URF residuals:

	dm_t	dm^*_t
dm_{t-1}	1.0	
dm^*_{t-1}	0.147	1.0

loglik = 1081.69 log|ê| = -18.649 |ê| = $7.95e^{-9}$ T = 116 log|Y'Y/T| = -17.93;
R^2(LR) = 0.509 R^2(LM) = 0.2615
F-test against unrestricted regressors, F(4, 224) = 23.96 [0.00]**.
Variables entered unrestricted: constant.
F-tests on retained regressors, F(2, 112): dm_{t-1} 1.617 [0.203] dm^*_{t-1} 47.86[0.00]**

correlation of actual and fitted:

dm_t	dm^*_t
0.388	0.692

E.2 The policy rules

In this section we report the estimate of alternatives Taylor rules for the UK that take into account US variables:

A) $i^{UK} = f(\pi^{UK}, y_t^{UK}, i_t^{US})$. Modelling i^{UK} by OLS. Sample: 1978:2 to 1998:2

Variable	π^{UK}	y_t^{UK}	i_t^{US}	const.
Coeff.	0.655	0.129	0.192	0.0001
t-value	1.87	5.64	1.86	0.175
σ	0.0051			
D.W.	0.532			
RSS	0.002			

R^2= 0.429; F(3, 77) = 19.33 [0.00]; AR 1- 5 F(5, 72) = 15.793 [0.00]**;
ARCH 4 F(4, 69) = 2.426 [0.056]; Normality $Chi^2(2)$= 0.852 [0.652];

X_i^2 F(6, 70) = 4.66 [0.00]**; X_iX_j F(9, 67) = 3.962 [0.00]**;
RESET F(1, 76) = 0.787 [0.377].

B) $i^{UK} = f(\pi^{UK}, y_t^{UK}, i_t^{US}, s)$. Modelling i^{UK} by OLS. Sample: 1978:2 to 1998:2:

Variable	π^{UK}	y_t^{UK}	i_t^{US}	s	const.
Coeff.	0.668	0.124	0.155	0.037	0.0003
t-value	1.968	5.545	1.529	2.339	0.424
σ	0.005				
D.W.	0.529				
RSS	0.0019				

R^2= 0.468; F(4, 76) = 16.71 [0.00]; AR 1- 5 F(5, 71) = 15.814 [0.00]*;
ARCH 4 F(4, 68) = 2.114 [0.088]; Normality $Chi^2(2)$= 0.062 [0.969];
X_i^2 F(8, 67) = 2.22[0.036]*; X_iX_j F(14, 61) = 1.44 [0.159];
RESET F(1, 75) = 0.20 [0.656].

C) $i^{UK} = f(p^{UK}, y_t^{UK}, i_t^{US}, s)$. Modelling i^{UK} by OLS. Sample: 1978:2 to 1998:2

Variable	p^{UK}	y_t^{UK}	i_t^{US}	s	const.
Coeff.	.081	.122	.22	.038	-.0003
t-value	2.21	5.44	2.04	2.41	-.588
σ	.0049				
D.W.	.577	R^2	.474		
RSS	.0018	F(4,76)	17.17	[0.00]	

D) $i^{UK} = f(\pi^{UK}, p^{UK}, y_t^{UK}, i_t^{US}, s)$. Modelling i^{UK} by OLS. Sample: 1978:2 to 1998:2

Variable	p^{UK}	y_t^{UK}	i_t^{US}	s	π^{UK}	const.
Coeff.	0.099	0.11	0.267	0.039	0.854	0.0004
t-value	2.763	4.98	2.535	2.565	2.567	0.631
σ	.0047					
D.W.	.607	R^2	.517			
RSS	.0017	F(5,75)	16.06	[0.00]		

AR 1-5 F(5, 70)=13.49 [0.00]**; ARCH 4 F(4, 67) = 1.55 [0.196];
Normality $Chi^2(2)$= 0.209[0.90]; X_i^2 F(10, 64) = 1.50 [0.15];
X_iX_j F(20, 54) = 1.101 [0.37]; RESET F(1, 74) = 1.02 [0.31].

E) $i^{UK} = f(p^{UK} - p^{US}, r^{UK} - r^{US}, i^{US})$. Modelling i^{UK} by OLS. Sample:1978:1 to 1998:2

Variable	$p^{UK} - p^{US}$	$r_t^{UK} - r_t^{US}$	i_t^{US}	const.
Coeff.	0.107	0.478	0.976	-0.00005
t-value	2.91	5.075	7.082	-0.088
σ	.0052			
D.W.	.477	R^2	.402	
RSS	.0021	F(3,78)	17.53	[0.00]

AR 1-5 F(5, 73) = 25.13 [0.00]**; ARCH 4 F(4, 70) = 3.14 [0.019]*;
Normality $Chi^2(2)$= 10.61[0.00]**; X_i^2 F(6, 71) = 1.77 [0.12];
X_iX_j F(9, 68) = 1.26 [0.27]; RESET F(1, 77) = 0.242 [0.623]

This equation has been used in chapter 5 to describe the actual behaviour of the UK Central Bank.

F) $i^{UK} = f(p^{UK}-p^{US}, r^{UK}-r^{US}, i^{US}, i_{t-1}^{UK})$. Modelling i^{UK} by OLS. Sample: 1978:3 to 1998:2

Variable	$p_t^{UK} - p_t^{US}$	$r_t^{UK} - r_t^{US}$	i_t^{US}	i_{t-1}^{UK}	const.
Coeff.	-0.009	0.169	0.274	0.76	0.0001
t-value	-0.340	2.495	2.357	11.0	0.356
σ	.0033				
D.W.	1.83	R^2	.74		
RSS	.0008	F(4. 75)	52.6	[0.00]	

G) $i^{UK} = f(rer, r^{UK} - r^{US}, i^{US})$. Modelling i by OLS. Sample: 1978:3 to 1998:2

Variable	rer	$r_t^{UK} - r_t^{US}$	i_t^{US}	const.
Coeff.	0.055	0.647	0.937	0.0005
t-value	3.79	6.446	7.17	0.913
σ	.005			
D.W.	.357	R^2	.44	
RSS	.002	F(3.78)	20.50	[0.00]

AR 1-5 F(5, 73) = 38.34 [0.00]**; ARCH 4 F(4, 70) = 6.02 [0.00]**;
Normality $Chi^2(2)$= 33.7 [0.00]**; X_i^2 F(6, 71) = 3.56 [0.003]**;
X_iX_j F(9, 68) = 3.364 [0.001]**; RESET F(1, 77) = 2.01 [0.16].

F Estimates of Chapters 6 and 7

F.1 OLS estimates of the VECM - chapter 6

A) UK relative to EU data. Sample 1979:3 -1998:2 (t-stat between brackets)

OLS estimates of the VECM containing $\widehat{y}_t$ $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t. m_t, m_t^*.

	$\Delta\widehat{\pi}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{y}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{\pi}_{t-1}$	-.216[-1.97]	-.043[-1.63]	.44[1.90]	-.209[-1.83]	.53[2.29]
$\Delta\widehat{i}_{t-1}$	-.45[-.77]	.237[1.70]	-.015[-.012]	-.267[-.44]	-.22[-.17]
$\Delta\widehat{y}_{t-1}$	-.58[-2.06]	-.051[-.765]	1.03[1.74]	-.33[-1.13]	1.2[2.0]
$\Delta\widehat{p}_{t-1}$	.034[.116]	.125[1.78]	.073[.12]	.45[1.47]	.008[.01]
Δs_{t-1}	.45[1.56]	.044[.641]	-.60[-.99]	.22[.75]	-.74[-1.22]
Δm_{t-1}^*	-.06[-.44]	.0139[.416]	.47[1.58]	-.04[-.29]	.50[1.69]
Δm_{t-1}	.088[.585]	-.037[-1.02]	.298[.93]	.005[.035]	.39[1.22]
ecm_{t-1}^1	-.75[-2.52]	-.093[-1.31]	-.34[.55]	-.186[-.60]	-.44[-.70]
ecm_{t-1}^2	.96[2.64]	-.28[-3.24]	-.051[-.065]	1.07[2.82]	.385[.49]
ecm_{t-1}^3	.20[3.56]	-.005[-.372]	-.09[-.76]	.17[2.94]	-.07[-.59]
ecm_{t-1}^4	.044[1.42]	-.016[-2.1]	.27[.42]	.047[1.46]	.055[.828]
ecm_{t-1}^5	-.20[-3.97]	.011[.939]	-.08[-.80]	-.17[-3.23]	-.09[-.85]
Δm_t^*	-.05[-.37]	-.035[-1.07]	-.11[-.38]	-.057[-.40]	-.14[-.48]
Δm_t	.068[.44]	.069[1.90]	-.205[-.64]	.019[.12]	-.21[-.66]
$const$	.0004[.45]	.0003[.56]	.0017[.43]	-.001[-.55]	.002[.58]

OLS estimates of the exogenous variables

	Δm_t^*	Δm_t
Δm_{t-1}^*	-.132[-1.11]	-.006[-.06]
Δm_{t-1}	.085[.767]	.56[5.61]
$const$	-.0001[-.09]	-.0004[-.28]

Note: sample 1979:3 -1998:2; t-stat between brackets.

B) EU relative to US data. Sample 1979:3 -1998:2 (t-stat between brackets)

OLS estimates of the VECM containing $\widehat{y}_t$ $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^*.

	$\Delta\widehat{\pi}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{y}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{\pi}_{t-1}$	-.35[-3.3]	.002[.37]	1.15[2.53]	-.327[-3.19]	-.43[-1.24]
$\Delta\widehat{i}_{t-1}$	.81[.42]	-.02[-.16]	13.8[1.66]	.72[.38]	-7.09[-1.11]
$\Delta\widehat{y}_{t-1}$	-.11[-2.2]	-.008[-2.4]	.63[2.93]	-.107[-2.2]	-.15[-.93]
$\Delta\widehat{p}_{t-1}$	.08[.28]	.014[.75]	.06[.049]	1.07[3.88]	.289[.30]
Δs_{t-1}	.001[.02]	-.009[-1.91]	.25[.83]	.002[.033]	.167[.72]
Δm^*_{t-1}	-.37[-.99]	-.015[-.58]	-1.65[-1.02]	-.347[-.95]	1.24[1.0]
Δm_{t-1}	.016[.13]	.005[.68]	-1.41[-2.69]	.009[.079]	.57[1.44]
ecm^1_{t-1}	-.705[-2.3]	-.011[-.54]	-.39[-.302]	-.687[-2.35]	-.094[-.095]
ecm^2_{t-1}	.638[.34]	-.44[-3.53]	3.15[.39]	.73[.406]	-.87[-.14]
ecm^3_{t-1}	.087[3.35]	.0016[.92]	-.07[-.68]	.084[3.35]	-.12[-1.41]
ecm^4_{t-1}	.01[.42]	.0003[.22]	-.155[-1.47]	.009[.41]	.03[.40]
ecm^5_{t-1}	.13[2.76]	.0003[.12]	.137[.657]	.12[2.75]	-.40[-2.53]
Δm^*_t	.36[.95]	-.015[-.60]	2.49[1.52]	.349[.94]	-2.18[-1.74]
Δm_t	.17[1.35]	.008[.96]	-.473[-.87]	.16[1.33]	.39[.97]
const	-.0009[-.51]	-3.33e^{-5}[$-$.29]	-.004[-.48]	-.001[-.66]	.0006[.12]

OLS estimates of the exogenous variables

	Δm^*_t	Δm_t
Δm^*_{t-1}	.656[7.13]	.165[.626]
Δm_{t-1}	-.032[-.77]	-.126[-1.07]
const	-.0003[-.48]	-.0001[-.063]

Note: sample 1979:3 -1998:2; t-stat between brackets.

C) The UK relative to US data. Sample 1979:4 -1998:2

OLS estimates of the VECM containing $\widehat{y}_t$ $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t, m_t, m_t^*.

	$\Delta\widehat{\pi}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{y}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{\pi}_{t-1}$	.208[1.63]	-.07[-.80]	-1.05[-.75]	.404[1.97]	-.78[-.57]
$\Delta\widehat{i}_{t-1}$	.295[1.45]	.205[1.48]	-2.24[-1.01]	.038[.12]	-2.76[-1.27]
$\Delta\widehat{y}_{t-1}$	-.03[-.51]	.108[2.68]	.88[1.37]	.085[.90]	.73[1.16]
$\Delta\widehat{p}_{t-1}$	-.233[-2.4]	.193[2.95]	1.49[1.43]	-.016[-.104]	.95[.93]
Δs_{t-1}	.034[.57]	-.099[-2.45]	-.82[-1.26]	-.09[-1.01]	-.68[-1.08]
Δm_{t-1}^*	.048[.42]	.015[.19]	-1.5[-1.19]	.07[.38]	-1.52[-1.22]
Δm_{t-1}	.004[.08]	-.05[-1.92]	-.134[-.285]	-.01[-.14]	-.15[-.33]
ecm_{t-1}^1	-.69[-4.05]	-.165[-1.41]	-2.18[-1.17]	-.41[-1.5]	-1.84[-1.01]
ecm_{t-1}^2	-.199[-1.3]	-.47[-4.54]	6.4[3.86]	-.009[-.037]	7.01[4.32]
ecm_{t-1}^3	.011[.43]	-.026[-1.47]	.26[.90]	.064[1.52]	.392[1.39]
ecm_{t-1}^4	-.015[-2.01]	-.024[-4.13]	.206[2.61]	-.016[-1.39]	.22[2.95]
ecm_{t-1}^5	-.021[-.73]	.024[1.25]	-.469[-1.49]	-.072[-1.56]	-.60[-1.95]
Δm_t^*	.127[1.06]	-.09[-1.18]	.209[.16]	.198[1.03]	.39[.31]
Δm_t	.018[.44]	.044[1.55]	.06[.13]	.007[.105]	-1.8e^{-5}[$-4.2e^{-5}$]
const	.0002[.49]	-.0001[-.46]	-.0012[-.23]	-8.2e^{-6}[$-.01$]	-.0009[-.18]

OLS estimates of the exogenous variables

	Δm_t^*	Δm_t
Δm_{t-1}^*	.64[6.84]	-.13[-.56]
Δm_{t-1}	-.0048[-.12]	.54[4.43]
const	-.0003[-.48]	-.0005[-.34]

Note: sample 1979:3 -1998:2; t-stat between brackets.

D) UK relative to US data with the UK monetary policy driven by the EU - Sample 1979:3-1998:2

OLS estimates of the VECM containing $\widehat{y}_t$ $\widehat{\pi}_t$, $\widehat{i}_t$, $\widehat{p}_t$, s_t. m_t, m_t^*.

	$\Delta\widehat{\pi}_t$	$\Delta\widehat{i}_t$	$\Delta\widehat{y}_t$	$\Delta\widehat{p}_t$	Δs_t
$\Delta\widehat{\pi}_{t-1}$	.20[1.6]	-.087[-1.02]	-.91[-.69]	.41[2.05]	-.659[-.51]
$\Delta\widehat{i}_{t-1}$	.31[1.58]	.208[1.55]	-2.03[-.97]	.087[.27]	-2.51[-1.22]
$\Delta\widehat{y}_{t-1}$	-.028[-.48]	.103[2.6]	.84[1.37]	.09[.98]	.70[1.17]
$\Delta\widehat{p}_{t-1}$	-.23[-2.48]	.199[3.12]	1.63[1.64]	-.007[.049]	1.07[1.10]
Δs_{t-1}	.03[.54]	-.095[-2.39]	-.74[-1.19]	-.099[-1.07]	-.61[-1.01]
Δm_{t-1}^*	.0069[.059]	.026[.328]	-.75[-.61]	.063[.34]	-.76[-.62]
Δm_{t-1}	.024[.59]	-.032[-1.11]	-.33[-.74]	-.007[-.10]	-.28[-.65]
ecm_{t-1}^1	-.699[-4.26]	-.19[-1.75]	-2.6[-1.5]	-.44[-1.7]	-2.25[-1.32]
ecm_{t-1}^2	-.20[-1.31]	-.54[-5.18]	5.6[3.45]	-.128[-.52]	6.14[3.86]
ecm_{t-1}^3	-.0005[-.016]	-.02[-1.21]	.51[1.75]	.05[1.25]	.62[2.16]
ecm_{t-1}^4	-.03[-2.0]	-.049[-4.8]	.33[2.05]	-.048[-2.0]	.36[2.33]
ecm_{t-1}^5	-.007[-.26]	.02[1.02]	-.75[-2.35]	-.06[-1.28]	-.85[2.73]
Δm_t^*	.09[.77]	-.101[-1.25]	.77[.61]	.178[.94]	1.0[.81]
Δm_t	.05[1.37]	-.005[-.21]	-1.1[-2.7]	.016[.27]	-1.13[-2.84]
const	.0002[.52]	-7.3e^{-5}[$-$.21]	-.001[-.25]	4.5e^{-5}[.06]	-.0009[-.19]

OLS estimates of the exogenous variables

	Δm_t^*	Δm_t
Δm_{t-1}^*	.65[7.13]	.165[.62]
Δm_{t-1}	-.03[-.77]	-.12[-1.07]
const	-.0002[-.77]	-.0001[-.06]

Note: sample 1979:3 -1998:2; t-stat between brackets.

F.2 The exogenous processes - chapter 7

Estimating the unrestricted reduced form (URF) by OLS. Sample: 1980:4-1998:2.

	da_t	$da^*{}_t$	dm_t	dm_t^*	$d\chi_t$	$d\chi_t^*$
da_{t-1}	.48[4.9]	-.48[-.83]	.24[1.09]	-.048[-.19]	-1.62[-2.2]	.57[.37]
da_{t-1}^*	.02[1.01]	.46[3.84]	-.002[-.04]	.034[.65]	.001[.01]	.48[1.53]
dm_{t-1}	.07[1.51]	.12[.44]	.50[4.81]	.14[1.28]	.344[1.0]	-.13[-.18]
dm_{t-1}^*	.017[.27]	-.25[-.72]	.27[2.0]	.087[.57]	-.35[-.79]	-1.35[-1.4]
$d\chi_{t-1}$	-.06[-3.47]	-.28[-2.84]	-.003[-.08]	-.11[-2.54]	-.18[-1.46]	-.23[-.87]
$d\chi_{t-1}^*$	-.006[-.81]	-.07[-1.37]	-.05[-2.86]	-.05[-2.56]	.018[.30]	-.50[-3.87]
$const.$	.0003[.44]	-.0009[-.25]	-.0007[-.46]	.0004[.24]	-.0003[-.06]	.006[.58]
σ	.0055	.0031	.0121	.0135	.0397	.0822
RSS	.0019	.0627	.0094	.01177	.10117	.4325

Correlation of URF residuals

	da_t	$da^*{}_t$	dm_t	dm_t^*	$d\chi_t$	$d\chi_t^*$
da_{t-1}	1.0					
da_{t-1}^*	.22	1.0				
dm_{t-1}	.03	.21	1.0			
dm_{t-1}^*	-.24	.03	.14	1.0		
$d\chi_{t-1}$	.32	.33	.44	-.05	1.0	
$d\chi_{t-1}^*$	-.29	-.03	.009	.66	-.18	1.0

loglik=1704.83; log$|\widehat{\sum}| = -48.023$; $|\widehat{\sum}| = 1.39e^{-21}$; T=71.
log$|Y'Y/T| = -45.69$; $R^2(LR) = .902$; $R^2(LM) = .289$
F-test against unrestricted regressors, F(36,261)=5.098[0.00]**
variables entered unrestricted: constant
F-test on retained regressors, F(6,59):

da_{t-1}	7.637[0.00]**
da_{t-1}^*	3.068[0.01]*
dm_{t-1}	4.638[0.00]**
dm_{t-1}^*	2.356[0.04]*
$d\chi_{t-1}$	4.39[0.00]**
$d\chi_{t-1}^*$	4.64[0.00]**

Correlation of actual and fitted:

da	da^*	dm	dm^*	$d\chi$	$d\chi^*$
.615	.492	.629	.399	.345	.628

F.3 Diagnostic tests on equations (I), (II) and (III) - chapter 7

- Equation (I) - for 6 variables and 73 observations 3 endogenous and 4 exogenous variables with 9 instruments.

$\sigma = .0912$	DW = 1.58
Reduced form $\sigma = .0428$	RSS =0.55735
Testing $\beta = 0 : Chi^2(5) = 27.76[.00]^{**}$	Specification $Chi^2(3)$=7.049[.07]
Normality $Chi^2(2)$=1.205[0.547]	ARCH4 $F(4.59) = 0.76[0.549]$
$X_i^2 X_j^2$ $F(20, 46)$=7.701[.00]**	X_i^2 $F(10.56) = 4.805[.0001]^{**}$

- Equation (II) - for 6 variables and 73 observations, 4 endogenous and 6 exogenous variables with 13 instruments.

$\sigma = .0487$	DW = 0.51
Reduced form $\sigma = .032$	RSS = 0.152
Testing $\beta = 0 : Chi^2(5) = 171.4[.00]^{**}$	Specification $Chi^2(4)$=7.098[.13]
Normality $Chi^2(2)$=0.25[0.88]	ARCH 4 $F(4.59) = 8.56[0.00]$
$X_i^2 X_j^2$ $F(20, 46)$=3.90[.00]**	X_i^2 $F(16, 47) = 2.57[.01]^{**}$

- Equation (III) - for 3 variables and 75 observations, 3 endogenous and 1 exogenous variables with 10 instruments.

$\sigma = .0173$	DW = 1.24
Reduced form $\sigma = .0015$	RSS = 0.02179
Testing $\beta = 0 : Chi^2(2) = 19.39[.00]^{**}$	Specification $Chi^2(7)$=13.41[.06]
Normality $Chi^2(2)$=4.02[0.13]	ARCH 4 $F(4, 64) = 3.08[0.02]$
$X_i^2 X_j^2$ $F(5, 66)$=1.39[.24]	X_i^2 $F(4.67) = .83[.51]$

F.4 Wald tests for linear restrictions - equation (II) - chapter 7

Wald test for linear restrictions ($R\alpha = r$). LinRes F(1, 67) = 4.48 [0.038]*

R-matrix							r vector
	$\widehat{c}$	$\widehat{i}$	$\widehat{\lambda}$	$\widehat{d(m-p)}_{t-1}$	$\widehat{d(m-p)}_{t-2}$	$const$	
0	1	0	0	0	0	0	1

Wald test for linear restrictions ($R\alpha = r$). LinRes F(2, 67) = 0.016 [0.98]

R-matrix							r vector
	$\widehat{c}$	$\widehat{i}$	$\widehat{\lambda}$	$\widehat{d(m-p)}_{t-1}$	$\widehat{d(m-p)}_{t-2}$	$const$	
0	1	0	0	0	0	0	.70
0	0	0	1	0	0	0	.70

G Diagnostic Tests for Chapter 8

- Equation (8.11) table 8.1 chapter 8: The present sample is: 1979 (4) to 1998 (1) IV regression for 4 variables and 74 observations 2 endogenous and 3 exogenous variables with 6 instruments.

$\sigma = 0.00406$	DW $= 0.435$
Reduced form $\sigma = 0.0038$	RSS $= 0.001153$
Testing $\beta = 0 : Chi^2(2) = 80.89[.00]^{**}$	Specification $Chi^2(2)$=3.91[.142]
Normality $Chi^2(2)$=2.09[0.352]	ARCH 4 $F(4, 64) = 6.74[0.00]^{**}$
$X_i^2 X_j^2$ $F(14, 55)$=2.176[0.021]	X_i^2 $F(4, 67) = 1.193[0.318]$

Testing for Error Autocorrelation from lags 1 to 5 $Chi^2(5) = 112.28$ $[0.00]^{**}$.

- Modelling i by RALS. The present sample is: 1980 (2) to 1998 (1).

variable	coefficient	t-stat.
$E\pi_{t+1}$	0.0618	1.189
i^*	0.1859	0.973
x	0.2340	1.713
rer	0.0035	0.352
$\widehat{u}_{t-1}$	0.7984	5.604
$\widehat{u}_{t-2}$	-0.043	-0.246
$\widehat{u}_{t-3}$	0.1403	0.791
$\widehat{u}_{t-4}$	-0.166	-0.932
$\widehat{u}_{t-5}$	0.1761	1.256

Normality $Chi^2(2)$=19.42[0.00]**	ARCH 4 $F(4, 56) = 0.035[0.99]$
$X_i^2 X_j^2$ $F(14, 49)$=2.26[0.018]*	X_i^2 $F(4, 67) = 2.26[0.018]^*$

- Equation (8.18), table 8.3 chapter 8. The present sample is: 1979(4) to 1998(1). IV regression for 5 variables and 73 observations 2 endogenous and 4 exogenous variables with 11 instruments.

$\sigma = 0.0024$	DW $= 1.94$
Reduced form $\sigma = 0.0025$	RSS $= 0.0038$
Testing $\beta = 0 : Chi^2(5) = 343.6[.00]^{**}$	Specification $Chi^2(6)$=0.92[.988]
Normality $Chi^2(2)$=20.1[0.00]**	ARCH 4 $F(4, 60) = 0.08[0.98]$
$X_i^2 X_j^2$ $F(10, 57)$=1.302[0.25]	X_i^2 $F(10, 57) = 1.30[0.251]$

Testing for Error Autocorrelation from lags 1 to 5 $Chi^2(5) = 4.1566$ [0.5271].

References

[1] Ahimed S., Ickes B., Wang P., Yoo B.S., 1993, 'International Business Cycles', American Economic Review, 83, 335-359.

[2] Backus D., Kehoe P., 1992, 'International Evidence of Historical Properties of Business Cycle', American Economic Review, 82, 864-888.

[3] Backus D., Kehoe P., Kydland F., 1992, 'International Business Cycle' Journal of Political Economy, 100, 735-775.

[4] Backus D., Kehoe P., Kydland F., 1994,'The Dynamics of the Trade Balance and of the Term of Trade: the J-curve?', American Economic Review, 84, 84-103.

[5] Backus D., Kehoe P., Kydland F., 1995, 'International Business Cycle: Theory and Evidence' in Frontiers of Business Cycle Research, T.F. Cooley (ed.), Princeton NJ: Princeton University Press.

[6] Bagliano F., Favero C., 1998, 'Measuring Monetary Policy with VAR models: An evaluation' European Economic Review, 42, 1069-1112.

[7] Ball L., 1995, 'Time-Consistent Policy and Persistent Changes in Inflation', Journal of Monetary Economics, 36, 329-350.

[8] Ball L., 1998, 'Policy Rules for Open Economy', Research Discussion Paper 9806, Reserve Bank of Australia.

[9] Barro R., Gordon D.B., 1983, 'Rules, Discretion and Reputation in a Model of Monetary Policy', Journal of Monetary Economy, July 1983, 101-122.

[10] Batini N., Haldane A., 1998, 'Forward-looking Rules for Monetary Policy', NBER working paper n. 6543.

[11] Baxter M., 1995, 'International Trade and Business Cycle', in Handbook of International Economics, vol. III, G.M. Grossman and K. Rogoff (eds.), Amsterdam: North-Holland.

[12] Baxter M., Crucini M., 1993, 'Explaining Savings and Investment Correlations', American Economic Review, 83, 416-436.

[13] Baxter M., Crucini M., 1995, 'Business Cycle and the Asset Structure of Foreign Trade', International Economic Review, 36, 821-854.

[14] Baxter M., Stockman A., 1989, 'Business Cycles and the Exchange Rate Regime: Some International Evidence', Journal of Monetary Economics, 23, 377-400.

[15] Beaudry P. Devereux M., 1995, 'Money and the Real Exchange Rate with Sticky Prices and Increasing Returns', Carnegie Rochester Conference Series on Public Policy, 43, 55-101.

[16] Bernanke B.S., Blinder A.S., 1992, 'The Federal Fund Rate and the Channels of Monetary Transmission', American Economic Review, 82, 901-921.

[17] Bernanke B.S., Mihov I., 1998, 'Measuring Monetary Policy', Quarterly Journal of Economics, 869-902.

[18] Bernanke B.S., Woodford R., 1997, 'Inflation Forecast and Monetary Policy', Journal of Money Credit and Banking, 29, 653-84.

[19] Betts C., Devereux M., 1996, 'The Exchange Rate in a Model of Pricing-to-Market', European Economic Review, 40, 1007-1021.

[20] Betts C., Devereux M., 1997, 'The International Monetary Transmission Mechanism: A Model of Real Exchange Rate Adjustment under Pricing-to-Market', Manuscript, University of British Columbia.

[21] Betts C., Devereux M., 1998, 'Exchange Rate Dynamics in a Model of Pricing-to-Market', Manuscript, University of British Columbia.

[22] Beveridge S., Nelson C., 1981, 'A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle', Journal of Monetary Economics, 7, 151 - 174.

[23] Blanchard O., 1989, 'A Traditional Interpretation of Macroeconomic Fluctuations', American Economic Review, 79, 1146-1164.

[24] Blanchard O. J. Fisher S., 1989, Lectures on Macroeconomics, MIT press: Cambridge MA.

[25] Blanchard O., Khan C., 1980, 'The Solution of Linear Difference Models under Rational Expectations', Econometrica, 48, 1305-1311.

[26] Blanchard O., Quah D., 1989, 'The Dynamic Effects of Aggregate Demand and Supply Disturbances', American Economic Review. 79/4, 655-673.

[27] Burnside C., 1998, 'Detrending and Business Cycle Facts: A Comment', Journal of Monetary Economics, 41, 513-532.

[28] Calvo G.A., 1983, 'Staggered Pricing in a Utility Maximizing Framework', Journal of Monetary Economics, 12, 383-398.

[29] Calvo G.A., Reinhart C.M., Végh C.A., 1995, 'Targeting the Real Exchange Rate: Theory and Evidence', Journal of Development Economics, 47, 97-133.

[30] Campbell J., 1994, 'Inspecting the Mechanism: an Analytical Approach to the Stochastic Growth Model', Journal of Monetary Economics, 33, 463-506.

[31] Canova F., 1998, 'Detrending and Business Cycle Facts', Journal of Monetary Economics, 41, 475-512.

[32] Canova F., 1998, 'Detrending and Business Cycle Facts: A user's guide', Journal of Monetary Economics, 41, 533-540.

[33] Canova F., 1995, 'Sensitivity Analysis and Model Evaluation in Simulated Dynamic General Equilibrium Economies', International Economic Review, 36, 477-501.

[34] Canova F., 1995, 'Vector Autoregressive Models: Specification, Estimation, Inference and Forecasting', in Handbook of Applied Econometrics (Macroeconomics), M.H. Pesaran and M.R. Wickens (eds.), 73-138.

[35] Canova F., Finn M., Pagan A., 1993, 'Evaluating a Real Business Cycle Model', in Non Stationary Time-Series and Cointegration, C. Hargraves (ed.), Oxford: Oxford University Press, 225-255.

[36] Canova F., Ubide, 1998, 'International Business Cycle, Financial Markets and Household Production', Journal of Economic Dynamics and Control, 22, 545-572.

[37] Canzoneri M., Dellas H., 1995, 'Monetary Integration in Europe: Implications for Real Interest Rates and Stock Markets'. Scandinavian Journal of Economics, 98, 541-557.

[38] Cardia E., 1991, 'The Dynamics of a Small Open Economy in Response to Monetary, Fiscal and Productivity Shocks', Journal of Monetary Economics, 28, 411-434.

[39] Chari V., Kehoe J.,1998, 'Optimal Fiscal and Monetary Policy', Federal Reserve Bank of Minneapolis, Research Department Staff Report, 251.

[40] Chari V., Kehoe J., McGrattan E., 1998, 'Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?' Federal Reserve Bank of Minneapolis, Staff Report, 223.

[41] Chari V., Kehoe J., McGrattan E., 1997, 'Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles', NBER working paper n. 5876.

[42] Christiano L., Eichenbaum M., 1995, 'Liquidity Effects, Monetary Policy and the Business Cycle', Journal of Money, Credit and Banking, 27, 1113-1136.

[43] Christiano L., Eichenbaum M., 1992, 'Liquidity Effects and the Monetary Transmission Mechanism', American Economic Review, 82, 346-352.

[44] Christiano L., Eichenbaum M., Evans C., 1996, 'The Effect of Monetary Policy Shocks: Evidence from the Flow of Funds', Review of Economics and Statistics, 78, 16-34.

[45] Christiano L., Eichenbaum M., Evans C., 1998, 'Modelling Money', NBER working paper n. 6371.

[46] Christiano L., Eichenbaum M., Evans C., 1998, 'Monetary Policy Shocks: What Have We Learned and to What end?', Manuscript, Northwestern University.

[47] Christiano L., Eichenbaum M., Evans C., 1997, 'Sticky price and Limited Participation Models: a Comparison', European Economic Review, 41, 1201-1249.

[48] Clarida R. ,Gali J., 1994, 'Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks?'. Carnegie Rochester Conference Series on Public Policy, 41. 1-56.

[49] Clarida R., Gali J., Gertler M., 1997. 'Has the Bundesbank Followed a Taylor Rule? A New Test and some Implications', IMF Seminars Series 1997-10A.

[50] Clarida R., Gali J., Gertler M., 1998, 'Monetary Policy Rules in Practice: some international evidence', European Economic Review, 42. 1033-48.

[51] Clarida R., Gali J., Gertler M., 1999, 'The Science of Monetary Policy: A New-Keynesian Perspective', Manuscript.

[52] Clark P. Goodhart C., Huang H.. 1999. 'Optimal Monetary Policy Rules in a Rational Expectations Model of the Phillips curve', Journal of Monetary Economics, 43, 497-520.

[53] Cochrane J., 1994, 'Shocks', NBER, working paper n. 4698.

[54] Cogley T., Nason J., 1995, 'Output Dynamics in a Real Business Cycle Models', American Economic Review, 85, 492-511.

[55] Cogley T., Nason J., 1995, 'Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time-Series, Implications for Business Cycle Research', Journal of Economic Dynamics and Control, 19, 253-278.

[56] Cooley T.F., 1995, Frontiers of Business Cycle Research, Princeton NJ, Princeton University Press.

[57] Cooley T.F., Hansen G., 1997, 'The Role of Monetary Shocks in Equilibrium Business Cycle Theory: Three Examples'. Manuscript, University of Rochester.

[58] Cooley T.F., Hansen G., 1997. 'Unanticipated Money'. Institute for Advanced Studies, Economic Series, 42.

[59] Correia I., Neves J., Rebelo S., 1995, 'Business Cycle in a Small Open Economy', European Economic Review, 39, 1098-1113.

[60] Cushman D., Zha T., 1997, 'Identifying Monetary Policy in a Small Open Economy under Flexible Exchange Rates', Journal of Monetary Economics, 39, 433-448.

[61] Davidson J.E.H., Hendry D.F., Srba F., Yeo S.. 1978, 'Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom', Economic Journal, 88, 661-692.

[62] Devereux M.B., 1997, 'Real Exchange Rates and Macroeconomics: Evidence and Theory', Canadian Journal of Economics, 30, 773-808.

[63] Dornbusch, R., 1976, 'Expectations and Exchange Rate Dynamics', Journal of Political Economy, 84, 1161-1176.

[64] Eichenbaum M. Evans C., 1995, 'Some Empirical Evidence on the Effects of Monetary Shocks on Exchange Rates', Quarterly Journal of Economics, 110, 975-1010.

[65] Engel C., Rogers J.H., 1996, 'How Wide is the Border?', American Economic Review, 86, 1112-1125.

[66] Engle R.F., Granger C.W.J., 1987, 'Cointegration and Error Correction: Representation, Estimation and Testing', Econometrica, 55, 251-276.

[67] Erceg C., 1997, 'Nominal Wage Rigidities and the Propagation of Monetary Disturbances', International Finance discussion paper n. 590, Board of Governors of the Federal Reserve System.

[68] Estrella A., Mishkin F.S., 1997, 'Is there a Role for Monetary Aggregates in the Conduct of Monetary Policy?', Journal of Monetary Economics, 40, 279-304.

[69] Fair R.C., Howrey, 1996, 'Evaluating Alternative Monetary Policy Rules', Journal of Monetary Economics, 38, 173-193.

[70] Farmer R., 1997, 'Money in a Real Business Cycle Model', Journal of Money Credit and Banking, 29, 568-611.

[71] Fuhrer J.C., 1997, "The (Un)Importance of Forward-looking Behaviour in Price Specifications', Journal of Money Credit and Banking, 29, 338-50.

[72] Fuhrer J.C., Moore G.R., 1995, 'Forward-looking behaviour and the stability of a conventional Monetary Policy Rule', Journal of Money Credit and Banking, 27, 1060-70.

[73] Fuhrer J.C., Moore G.R., 1995, 'Inflation Persistence', Quarterly Journal of Economics, 110, 127-29.

[74] Fuhrer J.C., Moore G.R., 1995, 'Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Output', American Economic Review, 85, 219-39.

[75] Gali J., 1992, 'How well does the IS-LM Model fit postwar data?', Quarterly Journal of Economics, 106, 709-738.

[76] Giavazzi F., Pagano M., 1988, 'The Advantage of Tying One's Hand: EMS discipline and Central Bank Credibility', European Economic Review, 32, 1055-1082.

[77] Gerlach S., Svensson L., 1999, 'Money and Inflation in the Euro area: a case for Monetary Indicators', Manuscript, in http://www.iies.su.re/leosven/.

[78] Goodfriend M., King R.G., 1997, 'The New Neoclassical Synthesis and the Role of monetary policy', NBER Macroeconomic Annual 1997, 231-83.

[79] Gordon D., Leeper E., 1994, 'The Dynamic Impacts of Monetary Policy: an exercise in tentative identification', Journal of Political Economy, 102, 1228-1247.

[80] Gregory A.W., Smith G.W., 1991, 'Calibration as Testing: Inference in Simulated Macroeconomic Models', Journal of Business and Economic Statistics, 9, 297-303.

[81] Grilli V., Roubini N., 1992, 'Liquidity and Exchange Rates', Journal of International Economics 33, 339-352.

[82] Grilli V., Roubini N.,1996, 'Liquidity Models in Open Economies: Theory and Empirical Evidence', European Economic Review, 40, 847-859.

[83] Hairault J.O., Portier F., 1993, 'Money, New Keynesian Macroeconomics and the Business Cycle', European Economic Review, 37, 1533-1568.

[84] Hasen L., 1982, 'Large Sample Properties of Generalized Method of Moments Estimators', Econometrica, 50, 1029-1054.

[85] Hansen L., Sargent T., 1979, 'Formulating and Estimating Dynamic Linear Rational Expectations Models', Journal of Economic Dynamic and Control, 2, 7-46.

[86] Hodrick R., Prescott E., 1997, 'Post-war US business cycles: An empirical investigation', Journal of Money, Credit, and Banking, 29, 1-16.

[87] Ireland P.N., 1997, 'A Small Structural Quarterly Model for Monetary Policy Evaluation', Carnegie-Rochester Conference series on Public Policy, 47, 83-108.

[88] Jeanne O., 1998, 'Generating real persistent effects of monetary shocks: how much nominal rigidity do we really need?', European Economic Review, 42, 1009-1032.

[89] Kasa K., 1992, 'Adjustment costs and Pricing-to-Market: theory and evidence', Journal of International Economics, 32, 1-30.

[90] Kydland F., Prescott E., 1977, 'Rules rather than discretion: the inconsistency of optimal plans', Journal of Political Economy, 85, 473-491.

[91] Kim K., Pagan A., 1995, 'The Econometric Analysis of Calibrated Macroeconomic Models', in Handbook of Applied Econometrics (Macroeconomics) M.H. Pesaran and M.R. Wickens (eds.).

[92] Kimball M.S., 1996, 'The quantitative analytics of the basic neo-monetarist model', Journal of Money, Credit and Banking, 27, 1247-77.

[93] King R.G., Watson M.W., 1996, 'Money, Prices, Interest Rates and the Business Cycles', Review of Economics and Statistics, 78, 35-53.

[94] King R.G., Wolman, 1998, 'What Should the Monetary Authority Do When Prices are Sticky?', Manuscript, University of Virginia, to appear in Monetary Policy Rules, J.B. Taylor (ed.), NBER, 1998.

[95] Kollman R., 1997, 'The Exchange Rate in a Dynamic Optimizing Current Account Model with Nominal Rigidities: a Quantitative Investigation', IMF working paper, WP/97/7.

[96] Kollman R., 1996, ' Incomplete asset markets and the cross-country consumption correlation puzzle', Journal of Economic Dynamic and Control, 20, 945-961.

[97] Knetter M., 1993. 'International Comparisons of Price-to-Market behaviour', American Economic Review, 83, 473-86.

[98] Kumah F., 1996, 'The effect of Monetary Policy on Exchange Rates: How to solve the Puzzles', Tilburg University, Center for Economic Research, discussion paper n. 70.

[99] Kydland F., Prescott E., 1990, 'Business Cycles: Real Facts and a Monetary Myth', Federal Reserve bank of Minneapolis Quarterly Review, 14, 3-18.

[100] Kydland F., Prescott E.,1982, 'Time to Build and Aggregate Fluctuations', Econometrica, 50, 1345-1370.

[101] Lane P., 1999, 'The New Open Economy Macroeconomics: A survey', CEPR discussion paper n. 2115 (march 1999).

[102] Leeper E., Sims C., 1994, 'Toward a Modern Macroeconomic Model Usable for Policy Analysis', in S. Fisher and J.J. Rotemberg (eds.), NBER Macroeconomics Annual. MIT Press, 81-118.

[103] Leeper E., Sims C., Zha T., 1996, 'What does monetary policy do?', Brookings Paper on Economic Activity 2, 1-63.

[104] Leichter J., Walsh C.E., 1998, 'Different Economies, Common Policy: Policy trade-offs under the European Central Bank', Manuscript.

[105] Levin A., Wieland V., Williams J., 'Robustness of Simple Monetary Policy Rules under Model Uncertainty', Manuscript, Federal Reserve System, Washington D.C.

[106] Levin J.H., 'Stabilization Policy, Exchange Rate Expectations and International Transmission,' Journal of Policy Modelling, 19, 19-40.

[107] Lucas R.E., 1972, 'Expectations and the Neutrality of Money', Journal of Economic Theory, 4, 103-124.

[108] Lucas R.E., 1990, 'Liquidity and interest rates', Journal of Economic Theory, 50, 237-64.

[109] Lucas R.E., 1977, 'Understanding Business Cycles', Journal of Monetary Economics, 22, 3-42.

[110] McCallum B., 1999, 'Analysis of the Monetary Transmission Mechanism: Methodological Issues', NBER working paper 7395.

[111] McCallum B., 1997, 'Issues in the Design of Monetary Policy Rules', NBER working paper n. 6016.

[112] McCallum B., 1999, 'Recent Developments in Monetary Policy Analysis: the Roles of Theory and Evidence', NBER working paper n. 7088.

[113] McCallum B., Nelson E., 1999, 'An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis', Journal of Money Credit and Banking, 31, 296-316.

[114] McCallum B., Nelson E., 1998, 'Nominal Income Targeting in an Open Economy Optimizing Model', NBER working paper n. 6675.

[115] McCallum B., Nelson E., 1998, 'Performance of Operational Policy Rules in a Estimated Semi-Classical Structural Model', NBER working paper n. 6675.

[116] Meliz J., Weber A., 1996, 'The cost/benefit analysis of a common monetary policy in France and Germany and possible lesson for monetary union', CEPR discussion paper 1374.

[117] Mishkin F., 1999, 'International Experiences with Different Monetary Policy Regimes', NBER working paper n. 7044.

[118] Obstfeld M., Rogoff K., 1995, 'Exchange Rate Dynamics Redux', Journal of Political Economy, 103, 624-660.

[119] Obstfeld M., Rogoff K., 1996, Foundation of International Macroeconomics, MIT Press.

[120] Obstfeld M., Rogoff K., 1999, 'New Direction for Stochastic Open Economy Models', NBER working paper n. 7313.

[121] Peersman G., Smets F., 1998, 'The Taylor Rule: A useful monetary policy guide for the ECB?', Manuscript, BIS.

[122] Pagan A., Robertson J., 1995, 'Resolving the liquidity effect', Federal Reserve Bank of St. Louis Review, 77, 33-54.

[123] Prasad E., Kumar M., 1997, 'International Trade and Business Cycle', Manuscript, Research Department, IMF, Washington DC.

[124] Ravn M.O., 1997, 'International Business Cycle in Theory and Practice', Journal of International Money and Finance, 16, 255-283.

[125] Rogoff K., 1995, 'The Optimal Commitment to an Intermediate Monetary Target', Quarterly Journal of Economics, 100, 1169-1189.

[126] Romer C., Romer D., 1989, 'Does Monetary Policy Matter? A new test in the spirit of Friedman and Schwartz' NBER Macroeconomic Annual 1989, Cambridge, MIT Press, 121-170.

[127] Rotemberg J.J., 1987, 'The New Keynesian Microfoundations', NBER Macroeconomic Annual, MIT Press: Cambridge MA, 63-129.

[128] Rotemberg J.J., Woodford M., 1997, 'An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy', in O. Blanchard and S. Fisher (eds.), NBER Macroeconomic Annual, MIT Press: Cambridge MA.

[129] Rotemberg J.J., Woodford M., 1998, 'Interest-rate rules in an Estimated Sticky Price Model', NBER working paper n. 6618.

[130] Rudebusch G.D., 1998, 'Do Measures of Monetary Policy in VAR make sense?', International Economic Review, 39, 907-948.

[131] Rudebusch G.D., March 1999, 'Is the Fed too timid? Monetary policy in an uncertain world', Manuscript, Federal Reserve Bank of San Francisco.

[132] Sims C., 1992, 'Interpreting the Macroeconomic Time Series Facts: the Effects of Monetary Policy', European Economic Review, 36, 975-1000.

[133] Sims C., 1996, 'Macroeconomic and Methodology', Journal of Economic Perspectives, 10, 105-120.

[134] Sims C., 1980, 'Macroeconomy and Reality', Econometrica, 48, 1-48.

[135] Sims C., Zha T., 1996, 'Does Monetary Policy Generate Recessions?', Manuscript, Yale University.

[136] Smith T., 1993, 'Estimating Non-linear Time Series Models Using Simulated VAR: Two Approaches', Journal of Applied Econometrics, 8, S63-S84.

[137] Stockman A., Tesar L., 1995, 'Tastes and technology in a two country model of the business cycle: explaining international comovements', American Economic Review, 85, 168-185.

[138] Strongin S., 1995, 'The identification of monetary policy disturbances: Explaining the liquidity puzzle' Journal of Monetary Economics, 34, 463-497.

[139] Svensson L.E.O., 1999, 'Inflation Targeting as a Monetary Policy Rule', Journal of Monetary Economics, 43, 607-654.

[140] Svensson L.E.O., 1999, 'Inflation Targeting: Some extensions', Scandinavian Journal of Economics, 101, 337-361.

[141] Svensson L.E.O., 1999, 'Monetary Policy Issues for the Eurosystem', Manuscript, Stockholm University, available at http://www.iies.su.re/leosven/.

[142] Svensson L.E.O., 1999, 'Price-level Targeting versus Inflation Targeting: A Free Lunch?', Journal of Money Credit and Banking, 31, 277-295.

[143] Svensson L.E.O., 1998, 'Open Economy Inflation Targeting', NBER working paper n. 6545.

[144] Taylor, J.B., 1998, 'An Historical Analysis of Monetary Policy Rules', NBER working paper n. 6768.

[145] Taylor J.B., 1993, 'Discretion Versus Policy Rules in Practice', Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

[146] Taylor J.B., 1998, 'The robustness and efficiency of monetary policy rules as guidelines for interest rate setting by the European Central Bank', Institute for international economic studies, seminar paper n. 649, August 1998.

[147] Taylor J.B., 1979, 'Staggered Price Setting in a Macro Model', American Economic Review, 69, 108-113.

[148] Uhlig H., 1997, 'A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily', Manuscript, CentER, University of Tilburg, (version 2). published in Computational Methods for the Study of Dynamic Economies, Marimon R., Scott A. (eds.), Oxford University Press, March 1999.

[149] Walsh C.E., 1998, Monetary Theory and Policy, MIT Press.

[150] Watson M., 1993, 'Measures of fit for Calibrated Models', Journal of Political Economy, 101, 1011-1041.

[151] Watson M., 1994, 'Vector Autoregression and Cointegration' in Handbook of Econometrics, vol. 4, Engle R.F. and McFadden D.L. (eds.), North Holland, 2843-2915.

[152] Wickens M.R., Breush T.S., 1998, 'Dynamic Specification, the long run and the estimation of transformed regression models', Economic Journal, 98 (Conference 1988), 189-205.

[153] Wickens M.R., 1996, 'Interpreting Cointegrating Vectors and Common Stochastic Trends', Journal of Econometrics, 74, 255-271.

[154] Wickens M.R., 1995, 'Real Business Cycle Analysis: a Needed Revolution in Macroeconometrics', Economic Journal, 105, 1637-1648.

[155] Wickens M.R., 1999, 'The transmission mechanism of monetary policy using interest rates', Manuscript, University of York, July 1999.

[156] Wickens M.R., Motto R., 1999, 'Using a VECM to identify impulse response functions', Manuscript, University of York, February 1999.

[157] Wickens M.R., Motto R., 2000, 'Estimating shocks and impulse response functions', Manuscript, University of York, January 2000, revised version of 'Using a VECM to identify impulse response functions', February 1999.

[158] Wright S., 1998, 'Monetary policy, nominal interest rates and long-horizon inflation uncertainty', University of Cambridge, D.A.E. working paper n. 9820.