

BOUNDED RATIONALITY IN SAVINGS DECISIONS

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Abstract

This thesis investigates the decisions made by people or households about how much to save out of income. The literature survey examines and criticises the life cycle savings literature and summarises some of the alternative approaches to decision making that have been proposed. Then, a theory of savings behaviour is developed which makes more realistic assumptions about how people take savings decisions than the conventional models. The basis of this theory is Simon's concept of "Bounded Rationality". The simulation of this theory shows that it can generate some observed behaviour that is not explained by the conventional models; in particular, there is a high level of saving early on in the life cycle. Then an experiment is described which examines behaviour. Subjects solved a savings problem and performed trials of savings strategies using a calculation facility. Various different ways of thinking about the problem (or decision strategies) were identified. The experiment was then developed to examine the effects of parameter variations in the problem and the possibility that subjects used the method of backward induction to solve the savings problem. These experiments demonstrated the existence of a simplified decision strategy, named a "rolling strategy". There was little evidence of the use of backward induction. Simulations of various decision strategies were then performed for the types of income stream and discount rate used in the experiments. It was found that for most parameter values, in particular those used in the experiment, the rolling strategy performed nearly as well as the optimal strategy.

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Accompanying Material

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3 1/2 floppy disc containing savings experiment program SAV3.EXE, instruction file INSTRUCT.TXT and input files INC9999 and SAVDAT2

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1 Introduction

The main objective of this research was to investigate the psychology of economic decision makers. From my previous studies in economics it had become clear that, although there has been a considerable intellectual effort dedicated to the analysis of the production and exchange of goods and the attendant interactions between people, the science of economics has only reached a limited understanding. While the theoretical properties of markets and interactions between perfectly optimising economic agents are (I suggest) well understood, the actual decisions and decision making processes are much less clear. This is not surprising. Since every individual has their own unique personality, they also have their own unique desires and their own unique ways of deciding how to achieve those desires. There is only a hazy knowledge of the details of these cognitive processes, which implies that it is not practical or indeed possible to include a complete, accurate specification of people's behaviour in economic models. Economic theory has tended to the other extreme: it is often assumed that economic agents optimise perfectly, given the environment in which they find themselves. Given the modern trend of basing macroeconomics on microeconomic foundations, this is even more influential than before. The Experimental work in economics has shown that this is often a poor approximation to actual behaviour and this means that the results of the models themselves must be suspect. Econometric testing has, in general, given mixed evidence partly due to the need for more detailed data, but also due to problems of aggregation given the differences between individuals. Therefore, there seems to be a possibility for a slightly more realistic specification of behaviour to try and explain the anomalies between the theory and the empirical data. This has long been known among decision theorists and many alternative models have been proposed, especially in the field of risk and uncertainty (see e.g. Camerer and Weber 1992 for a survey).

The area of savings decisions was chosen because of the particularly appropriate characteristics of these decisions for a study of human decision making under uncertainty. Decisions about how much to consume or save are made by everybody and are perceived to be important. A corollary of this is that in general people try to behave rationally, in the sense that they try to direct their resources to achieve their consumption goals. At the same time, such decisions are characterised by massive uncertainty because of the intertemporal nature of the decision, especially the need to plan over the whole of a lifetime. As the person looks further into the future, their estimates of income, interest rates and even their own

preferences become more and more vague. Hence a perfectly optimal decision which takes into account the various distributions of the parameters becomes impractical. If perfect optimisation is impractical, people must in some (relatively) consistent way simplify the problem to be able to take these decisions: the question is, how?

This thesis investigates this question. In chapter 2 various strands of the literature are identified, firstly the current state of the 'conventional' literature and the empirical evidence, and the development of non-expected utility theories and what has been called 'near rational behaviour'. These are still grounded in the idea of optimising economic agents. Then the issue of the definition of rationality is examined. The psychology literature is considered next, leading into studies of the cognitive process. Theories of motivation, both general and with specific application to savings behaviour are considered and then the contribution from economic psychology is surveyed. The contribution of consumer research to consumption decisions is identified and finally the ideas of G.L.S. Shackle are summarised.

From these diverse areas, chapter 3 tries to combine some of the more important ideas in a new theory of life cycle consumption. The idea of bounded rationality from Herbert Simon is applied and the utility function of the conventional model is developed to include additional contributions to utility as well as the act of consumption. The main ideas are that there is a utility associated with a wealth stock per se and also with the act of saving; a bequest motive is also included. The decision process allows for habit formation and, following the direction of Pemberton (1993), the problem is simplified into a trade-off between two variables, current utility and a measure of future utility. The trade-off is between levels of utility, rather than the first order condition or marginal utilities. Also, two different classes of savings instrument are incorporated, a typical instrument as used in the conventional theory with a certain rate of interest and a pension instrument. Chapter 4 contains a simulation of the theory and presents the results. It is found that agents build up a stock of wealth early on and that the level of consumption is strongly influenced by current income. Bequests are also significant.

Chapters 5 and 6 describe two savings experiments. The first study at EXEC, University of York, in which various decision strategies were identified is described in chapter 5. Chapter 6 proceeds to the experiment carried out at CentER, University of Tilburg in which

parameter variations were tried and the possible use of backward induction was investigated. The most significant result of these experiments is that a new decision making strategy, which has been named a 'rolling' strategy, was identified. In this strategy, subjects limit the amount of information that they consider by only looking a few periods ahead. They act as if life is going to continue, for sure, for a fixed number of periods - and continue to act in this way. They repeat their calculations every period or two periods so that as they proceed through the periods in the experiment, they gradually cover all periods in the experiment. Chapter 7 performs a simulation to investigate the theoretical performance of this type of strategy against the optimal strategy for the income distributions used in the experiment. It is found that for the parameters used in the experiment, a rolling strategy performs worse, but not much worse than the optimal strategy. Chapter 8 draws conclusions and suggests directions for further research.

2 A Selective Survey of The Literature

2.1 Introduction

This section will explain the reasoning that has led to the particular form of literature survey that has emerged. It is intended to act as an introduction to the main body of the literature survey and provide the motivation for the diverse nature of the reading I have undertaken. My objective in undertaking this survey is to scan the different literatures so that I can discover the current state of knowledge about savings behaviour. At this stage of the thesis, I will not be describing models in detail. Since I wish to construct a model of the savings behaviour of economic agents, not only savings behaviour will be examined but also ideas about the determinants of economic behaviour in relation to economic choices in general. The final section of the survey will summarize the arguments, in order to provide a basis for the development of a model of savings behaviour.

The first question to be answered is: why are savings of interest in economic analysis? There are three points to be made. Savings can be defined as consumption foregone at the present time, to increase consumption in the future. As such, it is one of the main factors in determining consumption and hence demand. Personal savings are a significant part of economic decision making for many people. This type of decision is readily seen as being one in which it might be possible to make a 'rational' decision i.e. one based on expected future returns, assessment of risk, etc. Savings provide the funds for investment and government borrowing, both of which are important variables in determining the path that the economy takes. Firms and government have savings as well as individuals or households, but this work concentrates attention on the personal sector.

This view of the importance of savings, as a determinant of economic performance, has motivated an interest in predicting savings behaviour at the macroeconomic level of aggregation. This requires an understanding of consumers behaviour that can not be obtained by econometric analysis alone. This is because even the most sophisticated statistical analysis, leading to a convincing identification of the significant variables in the consumption function is based on a limited amount of data. Even with modern micro

panels, the different situations and motivations of each individual or household generate variability that makes it difficult to include all the variables that may conceivably be significant and then identify those that actually are significant.

Therefore, it is necessary to select variables for inclusion in the analysis. I see the process of selection as the task of theory, which should provide an internally consistent and plausible explanation of observed data. Of course, there are often many different theories that explain the same regularities in the data and it is then the task of empirical analysis to identify those theories that best explain the observations. In order to allow consideration of different theoretical approaches, this thesis is based on the viewpoint that there are no normative grounds for preferring one theory over another e.g. a theory that is based on the axioms underlying expected utility theory versus a theory of 'irrational' behaviour. Consequently, many different types of theory in addition to the solely economic theories of behaviour have been considered.

It is also necessary to select the level of aggregation for analysis. This thesis follows thinking that sees aggregate movements as the result of the behaviour of many individual agents, whose actions combine in some way to produce the overall effect. It could be assumed that things cancel out, but this is a restrictive and arbitrary assumption. Hence it is necessary to understand microeconomic factors and also the process by which individuals' actions aggregate to the macroeconomic level. In accordance with these ideas, the aim is to identify a theory that explains individual agent's observed savings behaviour.

Deaton (1992) provides a very good and up-to-date survey of the state of economists thinking about savings and consumption. What might be called conventional theoretical economic analysis starts from a treatment of consumption as an intertemporal optimization problem, where an economic agent chooses consumption levels through their lifetime, given an income flow and a set of preferences. In this context, the preferences map the utility of a given level of consumption now compared with that level of consumption at different times in the future.

This approach contains (as special cases) the two theories that were predominant in the literature until approximately 1980, the life cycle model (LCM) of through-life

consumption and the permanent income hypothesis (PIH). Early versions of the LCM show that, given a constant income distribution etc., an optimal consumption pattern over an agent's lifetime will keep consumption levels constant (see Deaton (1992) fig 2.1 for a plot of a typical result). Agents will save during their working life and dissave during retirement. Since income tends to increase with age until retirement, the amount saved per time period may well increase. In particular, the LCM predicts that accumulated wealth will increase until retirement and decrease thereafter. It also predicts that consumption is decoupled from income during the lifetime, while assuming that total lifetime consumption equals the sum of inherited assets and through life income - the lifetime budget constraint. The theory implicitly assumes that people can borrow as much as they want and, in the earlier versions, that intergenerational transfers are not significant. The PIH proposes that consumption is equal to an agent's permanent income, the income received from the agent's stock of wealth. This implies that the change in consumption from period to period depends on the interest rate.

Together, the LCM and the PIH predict that agents smooth consumption profiles over their lifetimes compared to their income profiles. The theories were often used to predict macroeconomic behaviour by the method of assuming that an economy could be treated as if there was one 'representative' consumer, whose behaviour could be multiplied by the population level to find the overall consumption path for the economy. The applications of these theories took no account of uncertainty in the analysis, since the theory of behaviour under uncertainty was not sufficiently developed at that time.

These theories have been found not to be consistent with empirical evidence. Deaton summarizes both the micro and macroeconomic data¹. At the micro level, surveys of households are now more widely available over a time period long enough to make time series analysis feasible and cohort data of individuals expenditure at differing ages can also be obtained: some cohort series are now available and pseudo cohorts can also be constructed. Consumption has been found to track income over agent's lifetimes - people do not smooth their consumption profiles - indeed, many households have little or no wealth, retired people often save rather than dissave and savings are not substitutes

¹ A recent paper Kim (1996) shows that consumption in the USA has deviated by only 4% from the PIH over the period 1953-1993.

for other forms of wealth, e.g. housing asset values. It is also intuitively obvious that uncertainty is a significant factor in peoples economic decisions, particularly in an intertemporal context and it is at least plausible that some agents (e.g. the unemployed) do face limits on the amount they can borrow.

Macroeconomic analysis also provides important evidence against these versions of the LCM. It is well known that there has been a fall in savings ratios, in particular in the last decade or so. Modigliani (1990) presents statistics that show that savings ratios have declined in many different countries in the period 1961-87 and Kenally (1985) presents typical evidence for the U.K. Bosworth et. al. (1991) perform a microeconomic analysis of the drop in savings ratios in the U.S., Canada and Japan in the 1980s and conclude that the underlying factors are macroeconomic. Carroll & Summers (1991) demonstrate that, if consumption depends on lifetime resources, the distribution of consumption in countries with higher rates of growth is more heavily weighted towards young people. This turns out not to be true, both for first and third world countries. The LCM does not explain these macroeconomic changes within countries e.g. the drop in the saving rate in the U.S. is not a result of a redistribution of aggregate income towards the old, but an increase in consumption over all ages. Neither does the LCM provide an explanation for the variations in savings ratio across countries. The values of the ratios differ considerably and perhaps the differences are due only to national variations in preferences, but the theory does not explain why these values all change in the same direction simultaneously.

The theoretical approach which involves the least change of the economic behaviour assumed by the theories is to remove some of the restrictions on the intertemporal optimization problem implied by the LCM and the PIH. The predictions from the LCM come to a large extent from the (often implicit) form of the utility function and from the assumption of certainty. Modigliani called this the stripped down life cycle model, with a quadratic or Cobb-Douglas utility function, which have an intertemporal elasticity of unity. Less restrictive forms of the utility function, such as iso-elastic forms incorporating uncertainty by means of the expectation operator, enable precautionary saving to be modelled and the presence of liquidity constraints and a bequest motive for saving have been incorporated into the analysis. The results of the developments in

expected utility theory have provided a method for modelling uncertainty. These extensions to the theory enable many of the empirical results to be explained and allow for a wide range of savings behaviour. Savings can be made to track income, several of these models show that it is possible for people to continue saving in their retirement and savings increase with an increase in uncertainty about future income levels.

However, some serious problems remain with this approach, as some important empirical observations remain unexplained. Carroll (1992) found that the expectation of future income has no effect on current consumption above that of current income and the modifications do not address the aggregate level observations discussed previously. Furthermore, there is a large and growing body of evidence that people do not behave as expected utility maximizers in the face of uncertainty. The axioms of rational behaviour underlying expected utility theory have been tested under laboratory conditions and it has been found that a significant proportion of subjects do not obey these axioms. Hey (1991) ch.6 gives a summary of the evidence. The behaviour associated with Investment Retirement Accounts has also left some unanswered questions. The introduction of these accounts in the U.S. increased the overall level of savings, rather than just diverting funds from other savings accounts. Carroll & Summers (1987) report similar behaviour in Canada. Contributions dropped considerably when the tax deductability was removed, even though tax levels were probably lower than they would reasonably be expected to be in the future. Also, contributions to these accounts were often made just before the tax filing deadline and not as early as possible, as would be optimal if the reason for contributing was the maximization of the return on a high return investment.

There are also problems with the methodology adopted for these developments. The concern is that they include only the value of consumption bundle in the utility function. This leaves out any behavioural variables outside the desire to purchase consumption goods. From the study of econometrics, we know that the omission of relevant variables leads to bias in all the estimators for the coefficients. Consequently, it is necessary to work from a general model and eliminate insignificant variables. By arguing for the continued emphasis on the life cycle model with the minimum of modification, Deaton is overlooking this consideration. Even given the lack of empirical data discussed above, it should be possible to include a few more variables in the analysis of current data, or

generate new data to test alternative models.

An experimental test of life cycle theory was performed by Hey & Dardanoni (1988a, b). This experiment will be described in some detail, as it forms the basis for the experiments in chs. 5 and 6. The experiment tested Hall's stochastic Euler equation for optimal consumption under uncertainty (Hall (1978)). This was achieved by implementing a problem with the same structure as the life cycle consumption problem. The experiment consisted of a random number of periods. In each period, subjects received an income in 'tokens', which were added to tokens accumulated from previous periods. This income stream was stochastic, being drawn from a normal distribution in each period². Subjects then chose how many tokens to convert to money³. Borrowing was not permitted. The payoff was the money converted from tokens in the final period; any money from conversion in previous periods was lost. After the experiment, subjects were given a questionnaire, asking them to state age, sex, income, the maximum they would be prepared to pay then and there for the chance of winning £10 on the toss of a coin, their strategy, how they took the rate of return into account and how they took the income distribution into account. In order to compare subjects' choices to the optimum, it was necessary to estimate the subjects' utility functions. They were assumed to have CARA form functions, so it was only necessary to estimate the one risk aversion parameter. This was done in two ways. The gamble question in the questionnaire provided one point estimate and at the end of the experiment, without advance warning, the computer offered the subject the chance to play a gamble with their earnings. The response to this offer provided another point estimate. Behaviour was found to be significantly time dependent, contradicting the theory, but the comparative static implications of actual behaviour agreed in qualitative terms with those of optimal behaviour.

There are a wide variety of approaches which have been used to tackle these problems. The evidence of non-expected utility maximizing behaviour under uncertainty has been

² Subjects could use the computer to find out the income distribution, but only a very few subjects did so.

³ The conversion function had the Constant Absolute Risk Aversion (CARA) form
Money = $30(1 - e^{-0.022314T})$ T = tokens converted.
Tokens saved received interest at 12% per period.

addressed by Epstein and Zin. They use recursive utility functions to maintain dynamic consistency with non-expected utility maximizing utility functions. Machina (1989) shows that non-EU preferences can be dynamically consistent, if individuals take past uncertainty into account in a manner consistent with their original preferences⁴.

Problems such as those found with the behaviour of people with regard to Investment Retirement Accounts have led to the behavioural life cycle hypothesis proposed by Shefrin and Thaler (1988).

Others have begun to investigate the implications of non-optimal behaviour e.g. Cochrane (1989). This direction might be called near rational behaviour, as it looks at topics such as the utility loss of deviations from the optimal through-life consumption path. Pemberton (1993) also looks at non-optimal behaviour, but using optimization as a solution method. The unifying idea behind these papers is that people wish to behave as rational economic agents, but do not perform constrained optimization calculations in the way assumed by conventional microeconomic analysis. This may be due to agents having insufficient cognitive capability or perhaps to the lack of knowledge about the future which is an inevitable aspect of intertemporal choice. It is also possible that people have a different concept of what the 'best' choice is compared with microeconomists.

The limitations on people's ability to behave rationally were considered by Simon, who developed the concept of bounded rationality. Given the emphasis that economists place on the assertion that people can and do act rationally, it is useful to look at the debate about what is rational behaviour. This has been the subject of fierce debate, especially among workers on the boundaries of economics with other disciplines - in particular psychology. Lea, Tarry & Webley (1987) argues that the debate is sterile, but I wish to argue that consideration of what constitutes rationality may provide insights into people's behaviour. 'Rational' behaviour has a normative appeal for many people outside economics. Indeed, the idea of a 'scientific' approach is based on what is thought to be rational (or logical) analysis and this includes many mental processes besides constrained optimization problems. This must surely imply that people have a view of what constitutes

⁴ Another approach to utility is to regard preferences as fuzzy, enabling the mathematical apparatus of fuzzy sets to be used. See Billot (1991) for an introduction to fuzzy sets in utility theory.

rational decision making and how this ought to be applied to economic choices. Consequently, a careful definition of rational behaviour might provide insights into what people are trying to do when they make economic decisions.

Seen in broad terms, the logical development of the above argument is to study people's decision making processes. It would then be possible to move in to the decision theory literature, but I suspect that this literature does not address the issues I have raised. This literature is intended to provide assistance to people taking decisions and given the prevalence of the rationality paradigm, takes statistical theory and optimization as its fundamental assumptions. The question of how people actually make decisions is not relevant to this literature and is not discussed within it, since it is not trying to explain or predict behaviour but is trying to improve it .

A desire for an understanding of how people think when they are making decisions leads to the discipline of psychology - the study of behaviour - and its theories of human thought processes. Unlike economics with constrained optimization, psychology has not developed a single dominant paradigm. In addition, Lea, Tarpy & Webley (1987) points out that psychologists as a group see their discipline as empirically grounded. This means that they see their science as starting from the collection of empirical data. This can then be examined for regularities which can be used as the foundations for theorizing. The contrast with economics is that economists have a widely accepted theory of behaviour, which is then utilized to provide predictions of what economic agents do. These predictions can then be tested against empirical data. The consequence is that psychology appears to be a very fragmented discipline. However, there have been trends in thinking with a perceptible direction over the last two decades. The movement has been from psychology as a science of cognition, seeing behaviour as a product of the introspective process and hence determined by the structure of the brain system to what is termed the behaviourist paradigm, which places less emphasis on the thought process and considers the main determinant of behaviour to be the environment in which organisms find themselves. Skinner (1985) serves as an introduction to the debate.

The study of cognition has taken computer software methods as a fruitful analogy for the problem solving process. The usefulness of this analogy has advanced significantly with

the development of the field of artificial intelligence. Thinking about these issues might provide understanding of peoples decision making processes and hence the behaviour that results. There are also some careful empirical analyses of cognitive processes e.g. Grunert (1989).

Behaviour in response to external factors is determined by motivation, i.e. motivation identifies the goals and hence the appropriate behavioural response to a particular set of circumstances. The study of cognition has not yet provided a satisfactory conceptualization of how the external factors are taken as inputs and operated upon to determine behaviour that tries to satisfy the needs suggested by motivation theory. Therefore, environmental and motivational factors will be treated together. There are several motivational theories, summarized in Lea, Tarpy & Webley (1987) of which Maslow's hierarchy of needs is probably the best known. Some empirical work has been undertaken by psychologists on savings behaviour and attitudes towards saving e.g. Lunt & Livingstone (1991). Economists have also considered motives for saving, as these have an obvious influence on how people save. I have already mentioned precautionary and bequest motives in the above discussion from Deaton (1992). Keynes (1936) wrote down a list of motivations as a part of his analysis of saving and consumption.

As economics is a social science, it is of necessity a science of human behaviour and a model of behaviour is the basis for all economic theory. Consequently, there is a common interest with psychology and recently a discipline of economic psychology has emerged. This has set itself the objective of using the methods of psychology to address economic problems. Lea, Tarpy & Webley(1987) gives an introduction to the field, including ideas about savings behaviour. The boundaries between economic psychology and behavioural economics are, unsurprisingly, very blurred.

An alternative route within the overlap between economics and psychology is the consumer choice literature. The literature has grown as an adjunct to marketing science, which has a tradition of using the methods of psychology in the analysis of economic decisions e.g. advertising. Consumer research has produced working theories of how people make decisions in economic situations and there has been a lot of empirical work to test these theories. Because of the marketing orientation of this branch of economics it has

concentrated on discrete choice problems, particularly with regard to choices between different consumer products with more than one attribute or dimension of comparison. Foxall (1986) relates this literature to the movements within psychological theory. In my view, the strength of this literature lies in its firm empirical base, grounded in the function of the discipline as an aid to marketing decisions.

Finally, Shackle's view of the decision process, described in Shackle (1979), has been applied to consumer choice. His theory emphasizes the nature of choice as an act of imagining different futures and hence a decision is a choice between these imagined futures. Ford (1987) uses Shackle's ideas to develop a new theory of choice under uncertainty. Earl (1983) combines Shackle's theory with lexicographic discrete choice theory to explain choice decisions. He also incorporates a theory of motivation from Kelly.

2.2 Conventional Economic Models

Following the structure of the Introduction, I shall start by considering the 'conventional' literature. A standard exposition of consumer theory is Deaton and Muellbauer (1989), which gives a comprehensive account of intertemporal optimization, including an analysis of uncertainty using expected utility theory. Increased uncertainty may increase rather than decrease current consumption, since increased saving reduces risk, but the benefits of intertemporal optimization are reduced. The econometric evidence from tests on the LCM and PIH are presented - it is described as "mixed" - and the issue of aggregation is discussed. Increased uncertainty about unemployment prospects will mainly affect poorer consumers and so will decrease consumption by more than would be anticipated from the decline in aggregate wealth alone. In contrast, increased uncertainty about e.g. tax on high incomes may increase rather than decrease aggregate consumption.

Deaton (1992) brings the theory up to date and gives a critical appraisal of the available evidence. In his exposition of the LCM, he indicates that convexity of marginal utility provides a rationale for precautionary saving, which is excluded by constant consumption models that are grounded in quadratic utility functions. Also, if preferences are not intertemporally additive, a wider range of consumption profiles can be generated.

Inclusion of aggregation in a model is necessary to allow for differing information between consumers and population changes over time. The empirical evidence suggests that there is a link between consumption and income over a shorter period than the complete life and that aggregation is significant, which implies that the rate of interest may not be as important as the LCM indicates. The PIH is also contradicted by most empirical data (c.f. Kim (1996) - the changes in non-durable goods and services are 30-40% of changes in current income. Representative agent models are criticized in the following way: if consumers cannot distinguish idiosyncratic from aggregate shocks, they will make small mistakes and if the correct part of their response is largely idiosyncratic, aggregation can lead to a representative agent who makes large mistakes and responds very slowly to innovations in aggregate income. Muellbauer (1994) also provides a summary of the limitations of the life cycle model.

Models incorporating precautionary saving, bequests and liquidity constraints can explain much more of the data than the older models, especially once certainty equivalence is abandoned, but it is difficult to distinguish between them econometrically. Kimball (1990) introduces an analysis of precautionary saving, using Pratt (1964) as an analogy. The mathematical theory of risk aversion developed by Arrow and Pratt is reinterpreted for a utility function with an additional independent choice variable, giving non-zero third derivatives. This can be simplified to a form for precautionary saving such that the Arrow-Pratt results apply. He defines prudence and relative prudence as mathematically analogous concepts to risk aversion and relative risk aversion, where prudence is "the propensity to prepare and forearm oneself in the face of uncertainty". From this he defines the precautionary saving motive as "risk aversion of the negative of marginal utility". An index of prudence is introduced and some predictions for behaviour are derived. If absolute prudence is decreasing, labour income uncertainty will raise the marginal propensity to consume and adjustment of risky security holdings as wealth varies tends to reduce the marginal propensity to consume⁵.

2.3 Empirical Evidence

⁵ Roy also introduced a model of 'Safety First' behaviour (Roy (1952)), where the upper bound of the chances of a disastrous event are minimised. Expected utility is maximised, assuming the utility function takes only two values e.g. = 1 if the disaster does not occur, = 0 if it does.

There has been a large amount of work on aggregate consumption functions; so much so that Deaton (1992) argues that the data has been exhausted and research should now concentrate on the micro level data that are becoming available. Kennally (1985) looks at the evidence from the U.K. for savings behaviour. There was a decline in the savings ratio from more than 15% to 11.5% in the period 1973-1984. Much of household saving is committed; discretionary saving is much smaller and more variable than committed saving. From 1980, there was a 40% increase in housing debt and a 45% increase in consumer debt, which coincided with an easing of credit controls. It is possible that the relaxation was followed by a change in the equilibrium stock of durables. He makes the proposition that if restraints are lifted, behaviour is more sensitive to changes in prices and interest rates. His conclusions are that the fall in the savings ratio was due mainly to changes in household sector discretionary saving and that the change from a high level of saving to dissaving was financed largely by borrowing, especially through the 'mortgage leak' and consumer credit. The data are consistent with a period of stock adjustment following the easing of borrowing restrictions.

Bosworth, Burtless & Sabelhaus (1991) analyse a similar pattern of behaviour in the U.S., Canada and Japan. They find that savings ratios moved together over virtually all age and income groups and there was no support for the idea that the reduction in savings was concentrated among holders of financial assets. However, there was a big decline among homeowners relative to renters, so capital gains on housing may have displaced saving. Since the change happened among all groups of the population, macro factors must be the cause. The commentators suggest that people may have had less to worry about, e.g. L H Summers draws attention to Slemrod (1986) who relates saving to the fear of nuclear war, which together with increases in medical insurance, social security and private pension plans would reduce precautionary saving. They point out that better capital and credit markets will ease liquidity constraints and reduce target borrowing. The change may be partly an artifact of data definitions; if income increased from sales of assets, this would not be treated as a change in income in the accounts.

2.4 Non-expected Utility Models

Epstein (1990) describes recent developments in this area. The method adopted is to apply

non-expected utility models within a dynamic, intertemporally consistent framework. The problem of dynamic consistency is discussed by Machina (1989). A criticism of non-EU models has been that they may be dynamically inconsistent, as they imply that when facing an intertemporal problem, non-EU agents will make initial plans, but may change their decisions when they arrive at the actual point of decision. Machina shows that this argument depends on the assumption that decisions are independent of previous events or outcomes. In Epstein & Zin, dynamic consistency is achieved by the use of recursive utility functions. So far, these have only been applied in the complete markets case with constant tastes. Two papers by Epstein and Zin (1989, 1991) describe the theory and the econometric application. This theoretical approach brings several advances: it enables risk aversion to be separated from the degree of intertemporal substitutability, it puts atemporal non-expected utility theories into a temporal framework and succeeds in generating testable implications for the temporal behaviour of consumption and asset returns. Different specifications for the recursive utility functions are examined: expected utility, Kreps/Porteus (Kreps & Porteus (1978)) and Chew/Dekel (Dekel (1986)). In the empirical tests, expected utility was not consistent with the data, but there was no conclusive evidence for choosing between the non-EU theories. Non-EU dynamics emphasizes the importance of the structure of the representation of decision problems, as the results obtained from non-EU optimization vary if the method of structuring (or thinking about) the problem changes.

2.5 The Behavioural Life Cycle Hypothesis

This theory is an attempt to provide a more sophisticated psychological model than the conventional economic model of behaviour, with which optimization methods can then be used. The ideas arose from the realization that a large quantity of evidence, both statistical and anecdotal, contradicts the LCM⁶. Cardes (1990) discusses these anomalies, in particular the idea of bequests. If bequests are included in a savings model, the factors entering the utility function will need to be changed. There are various views of bequests: parents and children may share the risk that parents will live longer than expected or

⁶ One important psychological effect is the frame in which decisions are taken (see e.g. Puto (1987)) for a discussion and experimental investigation of these effects).

parents may use bequests to extract services from children, in which case bequests give services during retirement. If, however, bequests are altruistic, the children's utility enters into the parents utility function. Given considerations such as these, savings will be dependent on the family structure. This is supported by a comparison of Japan with the U.S. Japan has a higher savings rate while 80% of elderly live with their children, compared to 10% in the U.S.

Shefrin and Thaler (1988) is a description of the behavioural life cycle hypothesis, which explains many of the anomalies in the evidence. Thaler (1990) details some of the behavioural implications of the hypothesis. The idea is that money is not fungible i.e. money from one source is not regarded in the same way as money from other sources. They suggest that people have three mental accounts: a current income account, an asset account and a future income account and that these accounts have different marginal propensities to consume. The MPC for current income is 1, for assets is 0 and for future income is between these two extremes. Also, people have two contradictory elements in their utility: a desire to consume now ("impatience") and a desire to save for the future (which is also true for conventional models that model assign utility to current and future consumption). Impatience, combined with differing MPCs implies that people will act as if their temporal discount rate exceeds the interest rate. At the same time, their desire to save causes them to regret their lack of self-control; they are, in fact, dynamically inconsistent. They therefore overcome this lack by making savings decisions that are difficult to reverse e.g. joining pension plans. In this way, they voluntarily constrain future choices and try to remove or reduce the effect of their dynamic inconsistency. It is also suggested that people adopt heuristics for saving. They do not borrow from assets or future income to finance current consumption except in emergencies such as unemployment and even then consumption is cut as far as possible. This means that consumption tracks income. People engage in precautionary saving, which means that the asset account is kept small and they save for retirement in ways that require little self control, which implies that most retirement saving will be non-discretionary.

A similar philosophy addressing a different aspect of economic behaviour is Akerlof and Dickens (1982), who also developed a behavioural theory to explain apparent anomalies in people's economic decisions. They took the idea of cognitive dissonance from

psychology, which proposes that people have preferences over their own beliefs about the state of the world as well as over the different states and that people can manipulate their own beliefs by selecting sources of information likely to confirm desired beliefs. Furthermore, once acquired, beliefs persist over time. Thus people may have and maintain beliefs which are not an accurate view of the state of the world. An implication of this theory is that people, if left alone, may purchase too little old age insurance if they prefer not to contemplate a time when they are old. This is a justification for social security legislation.

2.6 Near Rational Behaviour

This literature takes the conventional models described by Deaton (1992) as the point of departure and looks at small deviations from 'optimal' behaviour. There are several strands of thinking in this area. Campbell & Mankiw (1991) have a model where some fraction of aggregate income is received by agents who follow the simple rule of thumb of consuming their current income and the remainder is received by forward looking optimizing agents. They show that the utility loss relative to an all optimizing economy is quite small if all agents have an equal share of consumption, but much larger if some agents are consuming the current aggregate income while others consume aggregate permanent income. They find that variables that predict income growth also predict consumption growth. There is no evidence that real interest rates or changes in nominal interest rates have direct effects on consumption growth once the effects of current income are taken into account and also error correction terms do not improve the model if current income is included. There is no evidence that the fraction of optimizing agents has changed in the period 1970-1988.

In his comments on this paper, Cochrane (1991) makes an important point: showing that a decision rule is optimal isolates the invariant aspect of behaviour, but showing that it is near rational does not, as there are many near optimal rules. He proposes that research should proceed using the Euler equation. This has the advantage that the Euler equation describes a partial equilibrium that is independent of the underlying production technology and can therefore be studied to yield information about preferences without having to fully specify a general equilibrium model. However, Muellbauer (1994)

exposes the limitations of the Euler equation approach.

Cochrane (1989) investigates the utility lost by not quite following an optimal consumption path, using various decision rules. He finds that the cost of setting consumption equal to income rather than following the PIH is only between \$0 and \$1 per quarter, for a non-durable consumption of \$3,500 per year. He tests a simple upper bound on consumption changes, the cost of excess sensitivity and smoothness of the consumption path, tolerating predictable errors in the Euler conditions and the cost of following a moving average process rather than meeting the Euler conditions. The largest loss he finds is \$1.65 per quarter. He points out that since these costs are so small, behaviour may be dramatically changed by other factors e.g. costs of acquiring information. He also argues that it is impractical to allow for these simply by adding a few extra costs, as that would be an endless task.

Pemberton (1993) sets out a proposal for near rational behaviour. He accepts Hey's (1983) argument that people cannot perform the backward induction solution to their through life consumption problem because most people cannot solve the mathematical problem and also because of the large degree of uncertainty involved. As an alternative, they have forward looking models instead and he develops a relatively simple forward looking model. The model has two periods, the present and the future and various random income processes are examined. The consumption path is found to depend significantly on the nature of the income process.

2.7 The Issue of Rational Behaviour



The concept of rationality is at the core of most economic analysis. A good example of an economists view of rationality is the group of axioms underlying the EU model of choice under uncertainty, which enable behaviour to be modelled in an analytically tractable and consistent manner. Generally, the principle is that agents maximize the benefit to themselves of their actions in a consistent manner i.e. consumers maximize utility in all their decisions and their utility functions are fixed, at least for some significant length of time. Given that economics is a study of how to allocate scarce resources this is a very sensible position to take. A broader definition is given by Harsanyi (1986): rationality means that human behaviour is goal directed in some

consistent manner.

Lea, Tarry & Webley (1987) considers the status of economic rationality. Psychological experiments demonstrate that the principles of rationality on which most economics is grounded (constrained optimization and the axioms underlying EU theory) are untrue. Unfortunately, the implications of these experiments for real economic behaviour are unclear and the framework of rationality can always be maintained by making additional assumptions. Consequently, they consider that asking the question "do people behave rationally?" will not provide any useful insights and that work should concentrate on the content of behaviour. They identify two senses of rationality - 'sensible' behaviour and conscious reasoning. Alternatives to rational behaviour are random behaviour, learning/habit and satisficing. Rationality may vary, depending upon the importance of the decision.

Schoemaker (1991) discusses the use of the optimality paradigm. An essential feature is that it involves mathematical as well as empirical content. It can be misleadingly clear, since any behaviour can be modelled as optimal, given sufficient degrees of freedom. People cannot objectively perceive the whole of reality, so everyone has his or her own reality and consequently there is no single idea of what is rational. Optimality principles may be postulated that do not add up to a coherent whole. He contrasts purposive (teleological) things to which intention is ascribed, with non-teleological things, which have causal explanations. His worries about the use of the concept are expressed as possible biases: if the data fits optimality, this does not necessarily mean that the agent is optimizing, researchers may look harder for confirming than disconfirming evidence, there may be a tendency to rationalize away anomalies and the use of the paradigm may create an illusion of understanding by describing rather than explaining. A most acerbic comment comes from Bookstein (1991); optimality principles are only mathematical reformulations of the postulate.

Sugden (1991) reminds us that rational choice theory describes certain regularities in human behaviour and that these choices are consistent, but only from the perspective of a particular theory. He argues that Savage's axioms of choice under uncertainty are much stronger than can be justified merely by an appeal to an instrumental conception of

rationality and that, in a game context, rational choice theory is self-defeating - individuals who are known to be rational in the conventional sense may be less successful than they would have been if they were known to be less rational.

Simon's views on economic rationality are summarized in Simon (1987). Psychology and economics both accept that people are rational in the sense of having reasons for what they do. However, neo-classical economics differs from other social sciences by not stating the goals and values, by postulating global consistency and by postulating that behaviour is objectively rational in relation to its total environment.

March (1978) identifies different concepts of rationality. The most important is limited rationality: people simplify problems as they are incapable of anticipating and considering all possibilities and information. Instead, they develop 'reasonable' responses: rules that are not optimal but do solve the problem. These may be manifest as step-function tastes, simple search rules, organizational slack, 'incrementalism' and 'muddling through' and uncertainty avoidance. He defines process rationality where decisions are reasonable in terms of the decision process rather than the outcomes. System rationality occurs where knowledge is formed over time within a system so that optimal action is taken by agents without a full comprehension of its justification. Adaptive rationality captures the idea that learning from experience is retained in a large memory store, but experiences are not explicitly retrievable so that current reasons may be separated from current actions. Rules of behaviour may come from not from calculations, but from survival and growth of social institutions where the rules are followed. Choice is then dominated by standard operating procedures and social regulation of social roles. This is Selected rationality. Posterior rationality is a process whereby actions are exogenous. These lead to experience, which is then evaluated in terms of preferences. These preferences are generated by the actions and consequences themselves and the choices are justified by posterior consistency with goals developed through a critical interpretation of the choice.

The idea that people choose how rational to be is taken up by Devinney (1989). Originally formulated by Etzioni (1988), the concept is that rational behaviour is difficult and therefore costly. Rationality can be thought of as a variable determined by the

cognitive work required, personality and societal foundations and the maintenance and adaption of the level of rationality over time. Personality and society are exogenous, but cognitive work and maintenance can be chosen. By including a cost of effort function in an expression for expected utility, Devinney derives a first order condition for the optimal level of rationality.

The idea of limitations on people's rationality finds its most clear and comprehensive exposition in the work of Simon. A summary of the basic idea is given in Simon (1983). Rationality involves axioms and rules of inference, brought together in a theory to reach conclusions. Since normative outputs cannot be obtained from descriptive inputs, our view of reason and hence tests for rational behaviour necessarily rest on value judgments. The implication of this is that we cannot claim that a theory is better because it describes rational behaviour, but can only say that it fits the data. Furthermore, any theory of rational behaviour embodies its judgments. He disagrees with the judgments incorporated in subjective expected utility, claiming that humans do not have the facts, reasoning powers or the consistent structure of values assumed by SEU theory.

His alternative is bounded rationality. Peoples decisions are about specific matters, assumed to be independent of other areas of life. They do not work out a complete probability distribution over all possibilities, but have a general idea with a few contingencies worked out. Attention is focused on one immediate issue, with values and aspects of that issue. Different decision domains evoke particular values and so inconsistencies in choice may result from fluctuating attention. A large part of the effort in making a decision is devoted to gathering facts; the actual choice may take little time. In accordance with this view, one of the principal functions of emotions is to focus attention on e.g. physiological needs. People have mechanisms to generate alternatives and a capability to acquire facts, together with a modest ability to draw inferences. The consequences are that bounded rationality does not optimize and it does not guarantee consistency - choices may often depend on the order in which attention is drawn to them. In order to model decision making, we need to find the givens of the decision making situation, the focus of attention, the way in which the problem is represented and the process used to specify alternatives, estimate consequences and choose among possibilities (Simon(1987)).

2.8 The View from Psychology

Psychologists have, to a limited extent, considered problems of economic behaviour. Because of its nature as the science of behaviour, it has inevitably tended to question and often challenge the relatively simple concepts of behaviour that are often used in economics. An introduction to the discipline of psychology, written for economists, is chapter 1 of Lea, Tarpy & Webley (1987). The conclusions from this overview were described in the introduction. The different paradigms that have been prevalent in psychology are contrasted in Skinner (1985). These are cognitive science and behaviourism. Cognitive science postulates that the causes of behaviour come from within the organism - people think and then act. Behaviourists stress the role of antecedent events in the environment and the environmental histories of the species and the individual i.e. the environment selects behaviour. In cognitive science, the direction of the causal link is from the individual to the environment, but in behaviourism this is reversed. He also analyses subjective expected utility theory from a psychologists viewpoint. In behavioural terms, people act according to rules that describe contingencies under which behaviour is reinforced. People do most often those things which have been abundantly reinforced without making subjective estimates of the probabilities of enforcement and these reinforcers are real, not expected. Because contingencies are more effective than the rules derived from them, violations of the axioms of SEU are unsurprising. However, people are still 'rational' in the sense that contingencies of reinforcement are effective. A corollary of this is that the description of contingencies and actions based on the description can obscure the effect of the contingencies themselves.

Foxall (1986) describes this debate from consumer psychologist's outlook. The idea underlying cognitive processing is that a change in behaviour cannot be conceived without previous, intrapersonal change. As such, the process depends on personality traits. This sequence of cognition - affect - conation has been shown not to be the most accurate way of describing the consumer choice process. Behaviourists explain behaviour with reference to extrapersonal events alone; the radical behaviouralist paradigm being the study of behaviour in the context of other behaviour. Following this paradigm, an important determinant of behaviour is the resulting effects on the environment.

2.9 Studies of the Cognitive Process

A description of the brain for application to AI is given in Marris (1992). His general conclusion is that economic systems are networks of organizations, which are networks of brains and the brain itself is a neural supernet so that the brain (and perhaps economies) can be modelled as parallel networks. He also gives a description of features of the brain. There is a deep memory, which is a large network, whose access works from addressing content and from then association. It is inaccurate, but able to work from incomplete or inaccurate instructions and has an almost infinite capacity. In contrast to this deep memory, the brain has a sequential mode for logical operations and what is called 'thinking' generally. When the sequential mode is active, we are in the conscious state and the short term memory does logical work, interacting with the long term (deep) memory. Processing uses both sequential and parallel modes. The short term processor is severely limited. Access is addressed by content and is accurate so that logical operations can be performed, but they are constrained by the capacity limitation. Marris's model of learning

is of a change in the deep memory, in particular the mental models that are stored there. The processes of learning do not always pass through the short term memory. Preferences are mostly learned, with a few primitive desires being equivalent to a random access memory. They are stored in the deep memory and are activated when the mind is conscious. Marris's definition of rationality is behaving in accordance with one's preferences, which leads him to the conclusion that although processing capacity is bounded, if the brain has methods of converging to consistency with preferences in the deep memory, bounded rationality may be less significant than is often assumed. Against this, given that the deep memory has no internal requirement for consistency and the limitations of conscious logic mean that consistency cannot be imposed, the idea of rationality is called into question.

Another facet of this model of memory is the brain's ability to make decisions in seemingly new situations. The whole of the memory can be applied to any one problem to some extent so that the brain can find analogies in apparently remote areas, using both qualitative and quantitative information. This comes from the network structure of the deep memory. The formation of expectations is dependent on this ability.

Macro-expectations are the product of a social net of brains and since expectations of an individual can be very fragile, overall expectations will be subject to epidemics. These may occur without any statistical basis, but as a result of just one or two events, often without any clear previous parallels.

The possible potential of the computer analogy is illustrated by Fox et. al. (1990). While considering a way of integrating classical decision theory and qualitative reasoning, they suggest a list of elements that may be necessary for decision making. These are: initializing and terminating conditions, relevant theories, relations for the generation of options, associations, input and output parameters and subprocedures.

Moss (1992) describes processes for AI inductive learning. The two directions are generalization and specialization. He describes experience as incremental improvements in understanding the relations between actions and outcomes, leading to a better general understanding of the environment. When a set of initial conditions becomes familiar, the specialized appropriate actions become routine and the action rule is relied upon without using the full decision-making process.

Nilsson (1979) proposes a system for problem solving behaviour with the aim of providing methods for AI. He takes goal-seeking as a basic attribute of the human brain. This involves a model of the world, constructed of propositional statements that use predicate calculus and a hierarchy of routines to elicit behaviour. The top level routine specifies actions for each proposition and state of the world. A behaviour interpreter checks each proposition in turn until it finds one that is true, when it executes an action (which may itself be a routine) and starts from the top again. The scheme lacks flexibility to allow for changes in states and going back to the top each time can be wasteful. Planning is required, in which possible chains of events are simulated and he has a concept of hierarchical planning with reasoning proceeding in different levels to reduce the number of combinations that have to be considered. Deductions are 'if...then' statements with certainty, while inferences are similar statements with probabilities. Inferences involve propositional knowledge and ways of making inferences in an inference net, starting from certain premises and using deduction rules whose conclusions are uncertain.

2.10 The Human Decision-making Process

Following on from computing analogies to the brain, I will now consider descriptions of human problem solving and decision making. A description of problem solving is given in Bench-Capon (1990). Problems are decomposed into a sequence of sub-problems which may be further decomposed. There is a search for patterns, so that familiar patterns, to which the response is known, can be identified. Humans use rules of thumb: actions that are often, but not always successful in certain circumstances. This is similar to pattern seeking, but is applied opportunistically and is known to be fallible. Rules of thumb are used in an effort to discern a familiar pattern if these cannot be seen.

Hogarth (1987) gives a relatively comprehensive account of the human decision making process and discusses methods for decision-making. He describes the bases of judgment emphasizing the influence of people's limited information processing capacity, which means that their perception of information is selective, not comprehensive. An aspect of this is that anticipations play a large part in what is seen. Processing is sequential and people use heuristics rather than optimizing calculations to reduce mental effort. People have limited memory which works by an active process of reconstruction, so that memory can change depending on which association is used. People attach meaning to information which will change with context. Most judgments are the outcome of a number of comparisons with points of reference.

Although the world may be deterministic, people's imperfect knowledge makes it uncertain to them. They have emotional difficulty accepting uncertainty and therefore tend to deny it. This is manifest as a tendency to seek patterns where none exist and to make unjustified causal attributions. People do not accept statistical independence and believe that observed data are a true reflection of the data generating process. They also have problems combining information. Inconsistent and disconfirming data are not sought and if found are downplayed, prior probabilities are not assessed and when updating beliefs they do not evaluate new evidence against all other alternatives simultaneously. This means that, in contrast to probability theory, absence of an alternative does not imply strong belief. Frequency is judged by the number of instances experienced, without allowing for sample size and the order of presentation influences the weight attached to

data. Either the most recent or the first data to be received may be over-weighted.

There are costs and benefits of mental effort. The benefits are: it assists in controlling actions, helps clarify goals and preferences, creates the habit of thinking and may identify actions that are not immediately obvious. The costs are: uncertainty is highlighted, trade-offs are made explicit and demands are made on limited cognitive resources.

The limitations of information processing capacity mean that strategies have to be adopted to simplify judgment and choice, otherwise information overload will prevent people from discerning the significant variables. These strategies are often effective, but can lead to systematic biases. Various strategies for choice are listed, divided into compensatory (linear, additive difference, ideal point) and non-compensatory (conjunctive, disjunctive, lexicographic, elimination-by-aspects) methods. Payne found that when little information was available, all the information was used and the more comprehensive compensatory strategies were adopted and in more complex situations people selected information and there was more variation in search strategies with more intra-dimensional processing.

The structure of memory causes bias in judgment, in particular the way in which recall functions. People use ease of recall as a cue for frequency and information working on several senses is easier to recall. Emotional associations also aid memory. There is hindsight bias, where decisions are judged relative to the outcome and not the initial information, so that people are not as surprised by outcomes as they should be. Another source of bias is that knowledge of outcomes may unduly influence explanations of the past, so that necessary (but not sufficient explanations) are accepted too easily. These beliefs are then difficult to modify from feedback. Hogarth concludes that the most important information processing bias is an unwillingness to use mental energy to imagine different possible outcomes i.e. uncertainties that accompany such thought trials outweigh the benefits.

The consumer choice literature has also made suggestions about the nature of the human

decision process, often arguing from empirical observation of consumption choices. Olshavsky & Granbois (1979) makes the proposal that purchase decisions may not be preceded by a decision process, but may be just an expression of predetermined needs/tastes so that there is no evaluation.

Johnson & Formell (1987) argue that consumers representations of a product choice will depend on the attributes of the choice. More abstract attributes are likely to resemble continuous dimensions, for which dimensional analysis can be used and more concrete attributes will resemble discrete features, where feature-based methods are more appropriate.

An alternative to maximizing called matching is discussed in Miller, Heiner & Manning (1987). The idea is that in equilibrium subjects divide their behaviour so that the distribution of responses matches the distribution of the reinforcement produced by the responses. Whereas maximizing assumes agents have a single scale of value on which all activities can be found, matching postulates that there are many scales and agents are trying to equalize the value on each scale. Agents might also choose a combination of maximizing and matching, trying to extract the largest possible value out of different activities.

2.11 Empirical Analyses of Cognitive Processes

Bettman & Sujen (1987) tested the effects of framing on consumption decisions made by students. They found that framing influenced the decisions for expert and novice consumers when alternatives were non-comparable and novices when alternatives were comparable. They suggest that the fundamental distinction between sets of comparable and non-comparable alternatives may be the availability of criteria, as opposed to the need to construct them and not any inherent difference.

An experiment to investigate links in the cognitive process was performed by Grunert (1989). He has a model of the cognitive processing structure of the brain as a network of uses, alternatives and knowledge in which there are two types of cognitive process: automatic (largely unconscious) and strategic (conscious; altered at will). External

stimuli and internal motivation result in various parts of the network being activated and the activation spreads rapidly through the network along associative links. In spreading along these links, some activation is lost in inverse proportion to the strength of the link. If, as a result of the spreading process, activation reaches a threshold value the process becomes conscious. The activation level thus governs information retrieval and the level is used as a heuristic for the importance of the aspect for the problem so that the association strength gives hints on what to do with the information. Familiarity depends on the total extent of links to the rest of the network. The strength of the links relating a product category to an attribute category determines the probability that an attribute or an attribute value comes to mind when a product category is activated. The weight of an attribute in an evaluation is based on the strength of the link between an attribute/value category and a use category.

Analysing the transcripts of interviews with consumers who had made a complex choice (selection of central heating), he found that the number of concepts that become activated through the spreading process at any one time is 5-10, because of the limited capacity of working memory. Retrieval of additional concepts requires a new spreading pattern, based on a different start activation pattern, so that categories become conscious in clusters of 5-10. Within a cluster, categories are highly associated, but this is not always the case across clusters.

Payne et. al. (1990) performed a Monte-Carlo simulation of the relative performance of different solution methods to a multi-attribute choice problem. They divided each solution method into steps (such as read a value into working memory or compare two values in working memory) and used the number of steps for a solution as a measure of the effort required. The performance of each procedure was measured in the range of a full weighted additive method to random choice. The comparisons were made for an unlimited and a limited number of steps to simulate stress/time pressure. The following procedures were considered: equal weight method (EQM) in which attributes are not weighted, elimination by aspects (EBA) where the most important attribute (or the state of the world with the highest probability) is found, a cutoff value for that attribute is retrieved, options below the cutoff are eliminated and the process is repeated in order of importance of attributes until only one option remains, satisficing (SAT) where

alternatives are considered one at a time until an option is found that has no attribute values below the cutoff levels, lexicographic (LEX) in which the most important attribute is found and the option with the highest value of that attribute is selected and if there is a tie, the next most important attribute is compared and the majority of confirming decisions (MCD) where pairs of options are compared and the option with the largest number of higher attribute values is retained and compared to the next alternative.

Simple decisions enable people to use more comprehensive and consistent methods, where all attributes are considered simultaneously. Complicated or limited duration problems force simpler methods with selective use of the available information. When there was no time pressure, it was found that the LEX had 90% accuracy vs. the weighted additive method for 40% of the effort and EQM 89% accuracy for 30% effort. The most efficient heuristic varied with the number of options and attributes. With processing limitations, the results were more variable, with LEX and EBA best overall. It is worth noting that LEX may be intransitive.

2.12 Motivational Factors

This section brings together a series of suggestions for psychological factors that influence people's savings decisions. This level of analysis is of a more general nature than the cognitive theories and analogies just discussed and the limited understanding of the structure of human thought processes inevitably means that the contributions to this literature are very disparate, often dealing with parts of the problem. The basic ideas of motivation theory are surveyed in Lea, Tarpy & Webley (1987), grounded in activation and drive theory in which basic drives combine with knowledge or habit to produce responses. Drive motivation is learned from stimuli (positive or negative) and organisms are motivated to maintain an optimum level of activation, rather than trying to reduce a drive (or need) to the lowest possible level. The most famous theory of needs was developed by Maslow (1943). He postulated five levels of need that are ordered in a hierarchy in which needs of a level only affect behaviour when the needs of the lower levels have been satisfied. The levels are: physiological, safety, love affection and acceptance, social status and esteem and self-fulfilment. Deprivation of a need leads to dominance of that need until it is gratified, when the next higher level is activated.

Herzberg et.al. (1959) divided needs into hygiene (e.g. pay, recognition) and motivator (e.g. achievement, responsibility, exercise of skill) needs. If hygiene needs are not met, dissatisfaction is felt, but they cannot provide positive satisfaction and this is only provided by satisfying the motivator needs. In contrast to Maslow, they argue that hygiene needs are insatiable, so that if positive satisfaction is to be found, both categories of need must be satisfied simultaneously. A useful theory of motivation cannot be static; it must allow variation with circumstances.

At various times economists have also considered motivation in order to appreciate the factors that determine economic behaviour. They have not tended to develop theories of motivational structures, but have concentrated on direct consideration of the drives that activate behaviour.

In the General Theory, Keynes (1936) listed factors affecting his propensity to consume. The objective factors are: changes in the real wage, changes in the difference between net and gross income (though he dismisses this as not practically significant), windfall changes in capital values, the rate of time discounting, fiscal policy and expectations of the relation of present and future income. Subjective factors are: precaution, foresight - provision for an anticipated change in the relation between income and need, to enjoy interest and appreciation i.e. a preference for larger real consumption in the future rather than a smaller current consumption, to gratify an instinct to look forward to a gradually improving standard of living (even though the capacity for enjoyment may be diminishing), independence, for entrepreneurship, pride - to bequeath a fortune and avarice - unreasonable but insistent dislike of expenditure as such.

Among economists, Etzioni has been a strong proponent of considering psychological motivations for economic actions. In Etzioni (1988) he argues that individuals are governed by normative commitments and affective (i.e. emotional) involvements, both in choosing goods and the ways in which the goods are obtained. Normative/affective factors such as conforming to socially acceptable behaviour can exclude logical/empirical considerations and can even make an actor perceive only one course of action. These factors also colour interpretations of facts and the weight attached to information. They may also disrupt sequential logical thinking, causing steps in the logical process to be

skipped or terminated while incomplete, e.g. collecting a lot of facts, but then jumping to a conclusion. High stress levels increase random behaviour, the error rate and there is a regression to simpler responses, culminating in rigidity. Normative/affective factors shape the inner self, defining specific areas as demanding a decision on logical/empirical grounds. There is an emotional commitment to decisions once they are made.

From Pieters and van Raaij (1988), Etzioni discussed four functions of affect. It interprets and organizes information, influences the mobilization of a person's physical and mental resources, will control sensation seeking or avoiding to reach optimal arousal (e.g. boredom vs. stress) and is part of the process of communication.

Van Witteloostuijn (1988) argues that the major result of the behaviouralist framework is the introduction of routinised behaviour - as long as aspiration levels are satisfied, routines are maintained. In proposing that maximizing and satisficing are effectively equivalent, he claims that for the individual, chance and aspiration levels influence the comparability of results between maximizing and satisficing decision making.

Kagel & Green (1987) discuss intertemporal inconsistencies, such as the phenomenon of preference reversal, found in experiments with animals as well as humans. They consider the problem of self-control and suggest that this behaviour in animals will be caused by evolutionary pressures in their natural habitat.

The idea that savings behaviour in humans can be seen as analogous to hoarding behaviour in animals is explored in Lea & Tarpy (1990). They find that the economics literature agrees with Logue's synthesis (Logue (1988)) in its rejection of any simple statement about tendencies for impulsiveness vs. self-control.

Warneryd (1989) surveys various economists approaches to savings behaviour. Fisher claimed that society can affect the general level of 'impatience' and that if this goes up, interest rates will rise. Katona identified three classifications of saving: contractual, discretionary and residual (saving by default), all of which are affected by habit lags. He also developed a methodology which combined savings /consumption as a function of disposable income (statistics) with surveys to determine the willingness to save.

Consumer confidence is stressed by van Raaij & Gianotten (1990). Confidence is affected by expectations of the household's financial situation and by expectations of the general economic situation. Actual saving is largely determined by the evaluation of the household financial situation. Credit is seen as a luxury service, so that households with favourable income developments tend to borrow more.

Slemrod (1986) performed an analysis that demonstrated that saving was affected by peoples feeling of security. In the U.S. fear of nuclear war reduced savings rates. Fethke (1989) pointed out that marriage dissolution may decrease a family's savings rate, cause shifts in the portfolio to assets with lower interest rates and destroy or deplete assets.

One of the few empirical analyses of general factors that affect savings behaviour is Lunt & Livingstone (1991). They surveyed 279 subjects and performed two OLS regressions with recurrent and total savings as the dependent variables. Recurrent saving was measured as recurrent commitments to discretionary savings and total saving was measured as liquid assets in banks and building societies. The results are shown in table 1. The figures in the table are the coefficients of the explanatory variables that were found to be significant. Spending on clothes (a form of durable goods) is associated behaviour to saving, whereas pursuit of enjoyment involves consumption and the loss of ability to save. A saver does not spend to enjoy, but to possess goods of value at home and in the bank. Older people may have more total savings from redundancy payments, retirement, lump sum insurance payments and because they are more likely to have inherited money from the previous generation. In total saving, economic variables accounted for 42% of variance, demographic variables 11% and psychological variables 5%. For recurrent saving, demographic variables were insignificant and psychological variables accounted for 17%.

Dahlback (1991) tested the hypothesis that risk preferences affect saving. Cautious subjects tended to have a lower level of debt and more money in bank accounts, while there was no relationship between the propensity to take risks and total net capital or subjects ability to manage sudden extra expenditures. Furnham (1987) investigated peoples attitudes to saving as a part of their structure of economic beliefs. In order of importance, the significant attitudes to saving were: the pointlessness of saving, saving being

beneficial, saving leading to wealth, saving as denial, importance of investments in determining the best way to save. He analysed people along two dimensions: tough/tender minded and collectivist/individualist and associated attitudes with points in this space. Benefits and

Table 1

Regression analysis results from Lunt & Livingstone (1991)

Dependent variable - Recurrent Saving

Explanatory variable	coefficient
disposable income	0.29
spending on clothes	0.74
total savings	0.22
value of enjoyment	-0.23
spending on food	-0.26
attitude to debt as bad management	0.18
argue with partner	0.22
discuss with friends	0.20
use 'for sale' columns	-0.20
shop for best buy	0.15

No. of observations = 92 $R^2 = 0.86$ total variance explained = 65%

Dependent variable - Total Saving

Explanatory variable	coefficient
age	0.22
sex (male +)	0.14
no. of children	-0.24
disposable income	0.22
total investments	0.54
spending on insurance	0.16
value of achievement	0.13
attribution of financial problems to luck	-0.19
attribution of financial problems to unexpected repairs	0.15

No. of observations = 193 $R^2 = 0.78$ total variance explained = 57%

saving through investment were associated with tough individualists, saving leading to wealth with tough collectivists and saving as denial with tough mindedness.

2.13 Economic Psychology

This strand of the literature is self-consciously multi-disciplinary, stemming from a belief that the understanding of economic behaviour can be improved by tackling some of the behavioural issues raised by psychologists and by applying some of the methods used in that science to economic problems. In their introduction to this discipline, Lea, Tarpy & Webley (1987) describe both economics and psychology from which economic psychology arises as a synthesis of the two. In the section on attitudes and behaviour, they point out that attitudes must be at the same level of generality as behaviour if they are to predict behaviour, in combination with personality and situational factors. Social comparisons are important; people use the behaviour of others to interpret or define an ambiguous situation and have a notion of equity in which they try to keep their net rewards proportional to their inputs. They state that most behaviour, including economic behaviour, is learned. Learning may be classical/Pavlovian in which the meaning of a stimulus is acquired from its relationship to an already meaningful stimulus, learning of responses to affect outcomes, or imitative.

They also consider the psychology of saving. Laymen have a different definition of saving to economists; for most people, durable purchases and repayment of loans are not savings. In western society, there is a perceived morality of thrift. Katona noted that people plan to save more than they actually do, although conspicuous consumption confers status. They contend that the moral status of saving comes from the puritan culture and the realization in society that saving is difficult. This issue is emphasized by experimental results that find that people tend to choose a small reward now, rather than a larger reward later. Duesenberry (1949) argued that consumption depends on consumers relative income, so that preferences are interrelated, not independent. Saving up to buy a particular good is discussed; although few people regard this as an important motivation for saving, it accounts for about 1/3 of all saving. This form of saving will result in

irregularly alternating saving/dissaving behaviour over a relatively short time frame compared to the life cycle.

Considering the factors that affect saving, psychological data suggest that interest payments are too small and remote for the rate of interest to much affect savings behaviour, which is consistent with the idea of impatience. Higher inflation leads to higher saving in contradiction to typical economic analysis, usually at the expense of durable expenditure. Durables purchase is associated with striving after social status; which is consistent with Maslow's hierarchy of needs, but can also be seen as illiquid saving, so that shifts to bank deposits in times of difficulty are a move to liquidity, rather than a reduction in savings. However, durables purchase has different psychological properties to saving; people typically buy more durables than they predict but save less, although both increase disproportionately in response to windfalls and tend to have positive income elasticities. Possible explanations for saving among the elderly are an increased interest in bequests, or possibly a shift in tastes with higher risk aversion. Children are also observed to save a lot, although results of laboratory experiments show children being unable to save.

In their concluding chapter, a new paradigm of dual causation is proposed. Individuals display economic behaviour, which should be predictable to some extent by psychologists. Individuals behaviour combines to determine aggregate behaviour. At the same time, the economy in general is an important influence on individuals behaviour. They see attitudes as intervening variables in this process which may be the root cause and best predictive level of behaviour, even if some attitudes may be predictable from economic variables.

2.14 Consumer Research

In the context of this study, the usefulness of the consumer choice literature is its study of how consumers make decisions and as such, this section is related to the earlier section on decision making. However, it is also a field in applied psychology since it is undertaken to enable consumer's behaviour to be manipulated. Both the distribution and level of consumption may be affected.

One recent theory is that of Lehmann & Moore (1991). This is a combination of the Luce choice model (Luce (1959)) and hierarchical models, as suggested by Tversky & Sattath (Tversky & Sattath (1979)). There is a tree structure of attributes, with a value function similar in form to the Luce model including differentiation by physical attributes and by product image. The model nests the tree-structure and the Luce simply scalable models and so is a combination of hierarchical and brand based choice.

Haines & Ratchford (1987) propose a model allowing for intransitivity. They suggest four factors that affect consumer choice: commodities, wants, the want satisfying powers of the commodities and consumer knowledge. They define a preference function as a difference between the attributes of two products, defined as a weighted sum of the perceived instrumentality of the attributes, where the weights are the value importance of the attributes. Variations in the weights and instrumentality values can produce intransitivity. They suggest that the attributes could be grouped into subsets.

There is an extensive literature devoted to empirical studies of consumer choice. Puto (1987) studied the way in which frames of reference changed in a two stage buyers experiment. Expectations, the buyer's objective and the sales message were controlled and the initial and final frames of reference were measured. The frames were defined as the focus that the buyer had on the problem when they made the choice of purchase contract. The experiment demonstrates that experiments can suggest the frames that people will use in making decisions and how available information affects the frame.

Walker & Olsen (1991) performed a means/ends analysis. Means represent product knowledge/attributes and the ends represent aspects of the consumers self-knowledge that may vary with the context. Content analysis of a survey of card purchases showed that a thinking-of-you situation activated receiver related goals while a wedding situation activated goals related to self-expression. An analysis of the means-ends linkages indicated that differences in activated goals affected the meaning of the attributes to which the goals were connected. Values are only related to behaviour when those aspects of self represented by the values are activated in a given situation.

Navon (1987) studied effects on performance of doing two tasks simultaneously.

Subjects tend to adjust the level of performance of a task to the level being required, without affecting the performance of the concurrent task. If performance of the concurrent task is being maximized, it will be kept high regardless of how well the other task is being performed. This suggests that processing resources are not necessarily limited.

Solomon & Buchanan (1991) surveyed 1197 'yuppies' and found that there was a correlation between products purchased in terms of a symbolic relation rather than a functional relation e.g. gourmet ice-cream and foreign cars in the U.S.

Hutchinson & Alba (1991) used stereo choices to study the use of irrelevant information. Three attributes were used, of which only one was relevant. It is possible that learning of simple relationships may frequently fail in multiple aspect decisions. Increases in memory load will inhibit and intentional processing goals enhance analytic processing. They found that attention tended to be directed to a single attribute, which was only sometimes the relevant one. The choice of focus depended on the context. They conclude that beliefs are significantly affected by irrelevant data that should be ignored, such as advertising, packaging or brand loyalties. There was evidence that information was unequally weighted: non-analytic subjects were found to rely heavily on a limited amount of salient attribute information.

Javalgi (1988) made a comparison of a hierarchical choice model (based on a threshold concept) to a multinomial logit model (based on utility maximization). He used data on farmers choices of retail outlets to measure the models predictive performance and found that the hierarchical model was less affected by the number of attributes in the decisions. The logit model assumed simultaneous evaluation of all attributes.

2.15 Choice As Imaginative Creation - the Ideas of G L S Shackle

Shackle is known for his distinctive views on the nature of the decision making process. His ideas are presented in Shackle (1979). Direct experience is the transience of thought which is the basic meaning of time. Choice is the choice of thought of actions and as thought, choice is also a transient. There are two possible philosophies of choice: determinism/inevitability and decisionism. Shackle follows the latter. He sees the

thinking, imaginative being as a "continuous originator of history". If choice is effective its results are uncertain: what can happen may be bounded but not prescribed, if not there is no real choice and purpose is passive and illusory. The imagination of different possibilities determines the choice set and this is a process of original thought. "The question of choice is not: Which of these given, listed things shall we do? but: What shall exist? What history shall be made in the world?" Shackle (1979 p.92). The act of choosing results in a private moral commitment to take some form of action.

Ford (1987) bases his new theory of choice under uncertainty on Shackle's ideas. Shackle sees probability as an inappropriate measure of uncertainty⁷⁸, as many decisions are non-repeatable so that probability theory is not relevant and the subjective assessment of the chances of the imagined possibilities occurring may not sum to 1. An agent may think of a new situation, but not have changing beliefs about other possibilities. Shackle introduced the concept of potential surprise: a measure of how surprised an agent will be if an event happens. This has a lower bound of 0 and an upper bound depending on temperament. A degree is assigned to each imagined outcome and is separate for gains and losses. These levels assume an ascendancy in the mind; they maximize an ascendancy function. Only monetary outcomes are compared, ranked by an indifference map of gains/losses vs. the potential surprise variable. The process of choice collapses potential surprise value to a monetary value, thus losing a degree of freedom.

In Ford's perspective theory, the axiomatic foundations are stochastic dominance, risk aversion in such a way that choices are evaluated by weighing up gains against losses, there is no expectation principle since people are imagining the expectational elements of choice actions and a new measure of belief that does not just act as a weighting in some form of averaging process. There are four postulates: gains and losses are segregated, outcomes are assigned an ascendancy value, whose function has monetary

⁷ Carter (1972) also discusses measures of uncertainty.

⁸ Arrow and Hurwicz have also contributed to these ideas. In Arrow & Hurwicz (1972) they show that "a plausible set of desirable properties for a rational criterion of choice under complete ignorance" leads to a special case of Shackle's theory (Shackle (1949)) in which "the standardized focus gain and focus loss become simply the maximum and minimum payoff to a given action".

outcomes and an uncertainty variable as its arguments, the perspective index P of a choice action is a balancing of the best on the gain side vs the worst on the loss side and P is evaluated for each prospect. This is intended to highlight extreme outcomes. The uncertainty variable is an alternative to probability called a degree of belief. It has a range from 0 to some positive number θ , which may differ between individuals. The degrees of belief of potentially competing hypotheses sum to θ and for at least one hypothesis, θ is greater than 0.

Earl (1983) also develops a theory of choice based on Shackle's rejection of probabilities. He argues that observed economic systems function because of the bounds imposed by imperfections of knowledge, rather than despite them. He uses Lancaster's proposition that people buy goods for the sake of their characteristics rather than commodities. Commodities can be thought of as bundles of characteristics and preference maps can be drawn over characteristics rather than goods. Instead of continuous preferences, there are hierarchical priority systems. There are two alternatives for the choice process: compensatory methods, where the number of product attributes is restricted so that they can be considered for all options simultaneously or lexicographically, where attributes are evaluated sequentially, allowing more attributes to be considered, but eliminating trade-offs between attributes. Attributes can be ranked in importance, so the option that dominates on the most important attribute receives the highest rating. Also, target levels are set for each attribute, and an attempt is made to independently satisfy as many of the target levels as possible.

The evaluation process uses Shackle's potential surprise curves, with a dual filter. There is a neutral aspiration level, where the choice does not seem hazardous. If the person can conceive of an outcome lower than the neutral level, the chooser is taking a gamble and must have the possibility of a gain to make the gamble worthwhile. Each attribute is examined in order of importance to see if the potential surprise curve is acceptable. All schemes that are not acceptable are eliminated and the next attribute is then considered. If more than one scheme satisfies all levels, the most neutral scheme is selected. Gains are separated from losses. He uses Kelly's theory of motivation (Kelly (1969)) and considers possible biases that people may have in the evaluation process. The shape of the potential surprise curves may be fudged to favour the option that already has the best rating

or if attainment levels are threatened and attributes with ambiguous priorities are given priorities such that no compromises are necessary. For example, the distant future may not be traded off against the present, but given a lower priority in the hierarchy.

This school of thought also considered the limiting of the number of states of nature considered in the decision process. Carter (1972) made one proposal for simplifying decision problems in which nearly all the possible combinations of surrounding circumstances are ignored by restricting the number of factors taken into account and by restricting the number of values for each factor or alternative.

One final point is that behaviour is social and consequently so is choice. Hence there are social pressures on choice e.g. to conform.

2.16 Summary

This section will recap the main themes that I have covered. Conventional models of intertemporal optimization with liquidity constraints, precautionary saving and bequests can explain much, but not all of the data. Non-EU models allow for non-EU behaviour, maintaining dynamic consistency by using recursive utility functions. The behavioural life cycle model, with different marginal propensities to consume for different mental accounts can also explain many of the empirical anomalies of the older versions of the life cycle model. The near-rational literature finds that deviations from the optimal consumption path may have negligible utility penalties, even for very different behaviour.

There are many different ideas of rationality, among which Simon's bounded rationality stands out as a clear account of the limitations of the human brain. Analogies from computing of the human brain have led to useful descriptions of how the brain works, contrasting the limited access, but nearly unlimited capacity and parallel processing capability of the deep memory with the sequential logic, limited capacity short term memory. Descriptive analyses of human decision-making such as Hogarth emphasize the differences between peoples actual decision processes to the rational theoretical models. Factors such as an emotional fear of uncertainty and the structure of memory cause

systematic biases in peoples decisions.

The consideration of motivational factors is a necessary tool in constructing a model of behaviour, suggesting arguments for a more general utility function than is normally analysed. Economic psychology suggests ways in which more comprehensive behavioural models can be developed and analysed, so that the more comprehensive ideas about human behaviour can be fruitfully applied to economic problems.

Consumer research provides many useful insights into human behaviour in economic situations and has developed empirical methods for analysis of the choice process.

The school of thought originated by Shackle has generated some interesting alternatives to conventional theory. Alternatives to probability measures such as Ford's degree of belief and Shackle's potential surprise curves have been developed. Ford's theory also allows for differences between attitudes to gains and losses and highlights extreme outcomes. Earl's theory combines potential surprise curves with a lexicographic choice system to make choices based on the characteristics of a good, rather than the bundle of characteristics represented by a whole good.

3 A Theory of Saving with Bounded Rationality

3.1 Introduction

The following two chapters develop a theory of life cycle saving incorporating the idea of bounded rationality. This chapter contains the theoretical development and chapter 4 contains the computer simulation of the theory. An important result of Cochrane (1989) is that wide deviations from the optimal savings/consumption path through life do not lead to large losses in utility, when analysed using a conventional life-cycle model. Therefore, it is important to understand the decision-making process; if the penalties for deviating from optimal behaviour are negligible and people do not optimize, then a very wide range of behaviours is possible which will generate different savings patterns. Thus, the observed savings patterns will depend on the decision-making process. Therefore, the objective of this theoretical work is to develop a model of savings with a more realistic psychological specification than “conventional” models. The methodology to be followed in these two chapters consists of the following steps:

Chapter 3

- i) State the 'stylised facts' that the model should explain.**
- ii) Write down a description of how people take savings decisions.**
- iii) Translate the descriptive model into a mathematical model.**

Chapter 4

- iv) Test the model by means of a computer simulation.**
- v) Compare the results of the simulation to the ‘stylised facts’ and generate new predictions of the theory which can then be tested.**

Section 2 sets out the stylised facts that the theory will try to explain, section 3 describes the main concepts incorporated in the model which are developed in section 4 into a descriptive theory and section 5 translates this description into a formal model.

3.2 Stylized Facts

These are divided into two groups: facts about savings inferred from empirical analysis and facts about how people take decisions.

3.2.1 Stylized Facts About Savings⁹

In contrast with the predictions of the simpler versions of the life-cycle model, consumption tracks income over people's lifetime (Deaton 1992). Young people save more in countries with higher growth rates (Carroll & Summers (1991)). There is no clear evidence that people dissave during retirement, indeed some people increase savings in this part of their life and people often have a significant level of savings when they die, which are bequeathed to the next generation (Deaton 1992). It is known that there are significant levels of residual and committed savings (Katona 1975). Further, there are constraints on the amount that most people can borrow.

3.2.2 Stylized Facts about Decision-Making

First, some general points about people's decision making. People have reasons for what they do, derived from goal-oriented behaviour (Simon 1983) and behaviour is dependent upon previous experience of the overall environment, not just the cognitive decision-making process (Skinner 1985). In contrast to the assumptions of life-cycle savings theory, people are not capable of performing backward induction (Simon 1983, March 1978), a corollary of which is that people do not act as Expected Utility maximizers when they are taking decisions with uncertain outcomes (for evidence, see Hey 1991). They do not take account of all the relevant information when taking complex decisions, but instead act in a 'bounded rational' manner (Schoemaker 1991, March 1978, Simon 1983). In general, people use simple rules of

⁹ Cyclical and trend factors such as the ratchet effect (Duesenberry 1949) may also be significant but, to assist comparison with conventional models, these will be left for future investigation.

thumb to guide behaviour e.g. targets against which achievement can be measured (March 1978). Another simplification of the decision process is that people exhibit habitual behaviour (Lea et. al. 1987). However, people do trade off expenditures on different goods, taking into account their financial resources. One important motive for saving is to reduce the downwards variability of future consumption (the precautionary motive) which implies a utility function of consumption with a positive 3rd derivative.

3.3 Main Concepts Incorporated in the Model

The Problem

The person's objective is to try and ensure an adequate standard of living now and in the future, allowing for predicted changes in circumstances and the financial resources available.

The particular features of this problem are its perceived importance and people's belief that this type of decision should be taken 'rationally'. Also, these decisions are characterised by a large element of uncertainty because people are often looking forward over the whole of their possible lifetimes.

3.3.1 Rationality of Savings Decisions

Savings decisions are regarded as important, so that when they are being taken, the full capability of the person will be utilised and they may take advice from others. This perceived importance implies that people believe that financial decisions should be taken rationally, to obtain the greatest possible benefit from their limited resources. This rationality is manifest as taking into consideration as much information as is feasible given their mental capacity and allocating savings in accordance with the goal stated above.

Bounded Rationality

However, as Pemberton 1993 points out, people's mental capacity is far too limited for them to behave in accordance with the assumptions of economic models such as the life cycle hypothesis. As a more reasonable alternative Simon's idea of 'bounded rationality', summarised in Simon 1983, is utilised. Attention is focused on one single issue at a time, which is assumed to be independent of all other aspects of a person's life. Not only does this lead to habitual behaviour, but it is proposed that this also means that particular savings decisions are taken independently of other savings decisions. Furthermore, once the particular decision to be taken is identified, people do not trade off by an exact process of optimization, but (as is implied by the statement of the problem above) they exhibit satisficing behaviour; they try and achieve satisfactory levels of savings and consumption. A further simplification is that people do not consider a continuum of future states of the world¹⁰, they only evaluate two or three different situations e.g. staying in employment or being made redundant. The model also uses the concept of 'rules of thumb', as detailed in Bench-Capon (1990). These are implemented as simple decision rules that are not expected to generate a completely accurate answer, but provide an estimate that is considered reasonable without requiring an involved calculation procedure.

Habitual Behaviour

Because savings decisions are viewed as important by many people, they have considerable mental effort associated with them. Since people's mental capacity is limited, they concentrate attention on one issue (e.g. savings) at a time, rather than considering all

¹⁰ The Shackle school has made several suggestions about how decisions are based on only a few possible states of nature. Shackle (1969) has a chapter on the theory of Ascendancy in which the decision maker focuses on the best and worst outcomes. Levi (1966) argues that "In attempting to make predictions about the future via induction, investigators do not, as a rule, attempt to forecast the future condition in as complete detail as the evidence might warrant". This makes inductive conclusions dependant on the evidence used. He has a notion of caution dependence where some hypotheses of a discrete set are rejected, depending on that exogenous value of a behavioural parameter. See also section 2.15 p46 on Carter (1972).

case under consideration, the rate of savings) is maintained until attention is drawn to this particular aspect of behaviour¹¹. It is proposed that people monitor their financial situation and their expenditures until they perceive a significant change in their financial situation. The change draws attention to a particular aspect of (saving) behaviour. They examine their decision and make changes that they feel are both feasible and desirable. They then stop paying attention to their (savings) decisions until their attention is again drawn to this aspect of their life. The implication of this is that there is no continuous decision process; a savings decision is followed by a period of constant behaviour, whose length is determined by the person's perception of their environment.

3.3.2 The Fuzziness of the Future

The way in which people save is heavily influenced by uncertainty about the future. My hypothesis is that, from a point in time at which a savings decision is taken, there is a short period of time over which fairly precise predictions can be made about the future. The length of time being considered will be dependent on the particular purpose of the saving. This means that precise levels of desirable savings can be identified. As the length of time into the future being considered increases the future becomes much vaguer; precise plans become irrelevant because people have little idea of what their situation will be and so the process of taking savings decisions also becomes less exact (Hey 1983 quoted in Pemberton 1993). However, there are two situations where the distant future can be considered in detail: retirement and death. These situations give rise once more to definite plans for the future and closely determined patterns of saving.

3.3.3 Taxonomy of Savings

Katona (e.g. Katona 1975)'s categorisation of savings into committed, discretionary and residual savings will be used. Committed and discretionary instruments are employed as is felt

¹¹ The incorporation of habit implies a departure from the now established tradition of economic theory e.g. life cycle theory of Modigliani (1980), in which the consumption, savings and asset-liability portfolio is chosen simultaneously. This is not, however, a process of recursive choice.

appropriate by the person in the context of the particular decision being taken.

Commitment

It is important to differentiate between committed and discretionary savings because they reflect different aspects of savings behaviour and generate different allocations of resources among the available savings instruments. People do not enter into contracts for committed savings simply because they will receive a slightly higher rate of interest. Committed savings impose an increased cost of intertemporal inconsistency. People are aware that their preferences may exhibit intertemporal inconsistency. Often, this inconsistency may only be present for a short period of time: e.g. when someone is drunk, they may spend all the money they have available, although they will not have enough money for food the next day and will then wish that they had not spent all their money.

Preferences may change, but current preferences, including the current view of preferences in the future have priority. People commit to ensure that future behaviour will reflect their current view of their preferences through their life; they pay in terms of reduced flexibility of current behaviour to prevent loss of utility from undesirable future behaviour, viewed from the perspective of the current preference structure.

The various forms of committed savings also enable people to simplify their savings decisions by reducing the number of options readily available; they know that they are saving through their pension scheme, life insurance etc. and they can then concentrate on 'fine tuning' their future versus their current consumption through their discretionary savings decision.

Also, there are advantages to financial institutions and the government from committed savings, as these are more predictable, thus allowing for better financial planning. Therefore, there are financial incentives offered for committed savings.

Residual Savings

Two aspects of this model will lead to residual, or unplanned savings. There may be changes in the financial situation of the person which are not perceived to be important enough for the savings decisions to be retaken e.g. from errors in the predictions of values used to take the decisions or slight changes in preferences from e.g. advertising. The trade-off process may incorporate errors from approximation to reduce the cognitive effort involved in taking the decision.

3.4 A Description of the Model

3.4.1 Overall Structure

The model can be divided into two aspects: monitoring and savings decisions. Most of the time, savings behaviour is constant. The individuals continuously monitor their financial situation, predictions and preferences to check that the current behaviour is adequate, given their consumption goals. When the situation is perceived to significantly change, the savings decision is brought to the attention of the person and the pattern of savings is reexamined. The nature of the perceived change will determine which decisions are reconsidered. The model has been abstracted from a 'realistic' savings decision situation by reducing the number of savings instruments available to two: a general discretionary savings instrument through which a wealth stock is built up and a pensions instrument which gives a flow of income in retirement. The pensions instrument contains an element of commitment in that there is a much reduced rate of return if contributions are reduced.

The savings decisions are drastically simplified in comparison with constrained optimizing life-cycle models with uncertainty. Attention is concentrated on one form of savings, other behaviour being assumed to be exogenous to the current decision i.e. constant. The actual process by which decisions are taken will be satisficing rather than optimizing. The influence of the fuzziness of the future discussed in section 3.3.3 above is reflected by the operation of the two different savings instruments. The discretionary savings instrument is used for

relatively short term saving in which future values of variables are either assumed to be similar to current variables or change in a predictable way. The pension instrument is used for long term smoothing of consumption between work and retirement.

Consideration of uncertainty is simplified by only considering a few cases that are perceived to be important e.g. for precautionary saving a worst-case scenario might be considered, along with the predicted scenario if the worst case cannot be dealt with. If more than one future state of the world is examined, the predicted utilities in each future state will be combined in some simple way.

Preferences are assumed to be such that current consumption is more heavily weighted than future consumption (noting that some people do not save at all). Committed savings are assigned priority over discretionary savings within the monitoring process.

A further simplification is that only one type of decision is taken in a single time period. However, since all types of saving are monitored, if a decision for one instrument implies that the saving through another instrument becomes different from the rate desired, the behaviour can be corrected in the next period.

3.4.2 The Monitoring Process

The monitoring takes place as follows:

Economic variables that the person considers significant are monitored. The monitoring can be assumed to be effectively continuous. When one or more variables are perceived to have changed significantly, the savings decision is retaken. This is modelled by having a threshold for the difference of each continuous variable, beyond which the change is regarded as significant. The (subjective) difference is found as the realisation of a normally distributed random variable, with a mean of the objective difference and variance taken as a parameter of the model. Savings behaviour may also be changed by the occurrence of a discrete event, which the person assumes will affect the decision process e.g. a change of job, a child being born, etc.

Implications of the Monitoring Process

The monitoring and consequent intermittent nature of the decision process have important implications for the decision procedure itself. Both now and at the point in the future being considered, the periods subsequent to those under consideration are assumed to be the same in all respects. Individuals further assume that if a satisfactory outcome is attained at the times under consideration, subsequent periods will also be satisfactory. The corollary of this is that it is assumed to be possible to take a decision that will hold for an indefinite period of time. However, if the individuals situations change they know that they can retake the decision. Therefore, they do not have to worry about not taking accurately into account the complete future; they can use ad hoc simplistic assumptions such as 'the situation will not change' and can concentrate on the aspect that has been brought to their attention.

Cues for Taking a Decision

The first set of factors are those that determine the person's preference structure i.e. the values of parameters in the utility functions and desired levels of saving and consumption. Changes in the financial parameters such as interest rates or new instruments (which for the current analysis will be modelled as changes in the interest rates for discretionary or committed savings) could also be significant, although this is not a particularly strong effect (Lea, Tarpy & Webley 1987). Any new information that changes the predicted values of decision variables in the future may cause a change in behaviour. Events in the life of the person may affect any of these categories, but since there are innumerable individual events that may occur, each of which may have a different effect, it is not useful at this stage to try and map all the different feasible events onto parameter or variable changes in the model.

The final cue is the person's budget constraint. What this means is that individuals know how much they are spending and how much they want to save. If they find that they do not have enough money to do both, or that they have (much) more money than they thought they would have, they may also recheck their savings behaviour.

3.4.3 The Saver's View of Life

This is the situation at a point in time at which a decision is being taken: there is a fixed past, a fairly certain immediate future, a vague distant future but definite ideas about what will happen during retirement and at death. Retirement can be seen as an environmental change which will affect all other savings decisions via a change in the budget constraint as labour income and pensions contributions disappear and pensions payments start. As the person gets older, the certain and vague periods will maintain their relative times to the person's present; they move later in time. When retirement approaches, the certain and vague futures still exist, but the budget constraint may change during the period of saving being considered. The probable point of death, in contrast, approaches. If individuals become very old, but not terminally ill, their prediction of how long they will probably live might mean that they base their plans on dying soon (e.g. concentrating on bequests rather than precautionary savings for the distant, vague future), although they will perhaps also consider what will happen if they live on for the foreseeable future. Thus the near & distinct vs. distant & vague separation may break down, since the motivation to consider the distant future will be weaker.

3.4.4. The Decision Process

Once someone departs from rigorous through-life optimization, there are lots of different ways of trading off that are possible. The suggestions that are made here are not intended to be a definitive statement of how people behave, but only a set of rules that seem plausible given the cognitive limitations of the human brain. Discovering the actual rules that people use is a matter for empirical investigation.

There are some general comments that can be made. The nature of the decision process depends on how precisely the goal of savings can be defined by the person. It is proposed that it will be satisficing (rather than maximizing) and will take into account the financial resources available. There are several reasons why people may not exhibit full optimizing behaviour. They may be unsure of their preferences. They may also feel that the large level

of effort to reach an optimum (involving many recalculations) is not worth the small gain in utility (this follows from Cochrane 1989). They may be incapable of performing the optimization calculation, even in its simplified form or they may not have sufficient patience to complete a complex decision process (Etzioni 1988).

A similar cognitive decision process is assumed for both short term and retirement saving. The trading-off process is along the lines proposed in Pemberton 1993. It is proposed that the person simplifies the problem until there are only two variables, which are then traded off against each other. Other financial expenditures and incomes are assumed to be constant for the length of time being considered. This can be justified because individuals know that, if their assumptions are significantly wrong, they will usually be able to replan. The utility over the current time period associated with a given behaviour is traded off against a representative time period at a selected point in the future, a few years hence for short term saving and the first year of retirement for pensions. There is no summation of utilities over time, because the large amount of uncertainty involved about the lengths of time over which saving and retirement will take place and the large possible changes in the values of variables in the future make the extra calculation meaningless for practical purposes. The implicit assumption is made that if these typical time periods are financially satisfactory, other periods in the same region will also be adequate. Individuals believe that if they change their opinion at some point, the process of monitoring will enable them to identify a significant change and they can then retake the decision.

The trade-off process that is used is a form of 'matching'; saving is adjusted until the utility of the current period is equal to the predicted utility in the future period multiplied by some subjective discount factor. The idea of matching is discussed in Miller et. al. (1987). It is considered that the concept of thinking about how satisfied one is with (say) a particular consumption rate now compared to (say) a rate of consumption in the future has considerable intuitive appeal. The typical CARA or CRRA functional forms incorporate diminishing returns to scale, thus allowing for the idea of an 'adequate' rate by having very low levels of utility for low values of the argument.

The discount factor is dependent on the length of time into the future being considered. It can be assumed to be a linear function of time for small changes in the time period, but this will not hold for larger changes. In the current case, consumption in retirement might reasonably be assumed to be discounted to a lesser degree than short term saving, even though the short term discount function is a linear or exponential function of the length of time. This is an instance of a rule of thumb being applied; although retirement is a long way off, individuals may well feel that provision for retirement income is important because by then their earning potential will very probably be much reduced. They will not be able to reverse their pensions decisions during retirement so not saving is a form of commitment to current consumption.

The limitation on the person's resources is modelled by including a liquidity restraint: no borrowing is permitted. There is an obvious requirement for there to be some constraint and it is unnecessary to make the strong psychological assumption that people see borrowing simply as negative consumption. Borrowing may introduce different psychological factors and motivations which can only be adequately addressed in a separate decision process. The restriction is incorporated through the budget identity, which is described in detail in the next section.

Expectations Formation

In this problem, the household has to make estimates of several variables in the future e.g. wages, rates of return, their own utilities etc. How are these expectations formed? One method often used is adaptive expectations, which is a weighted extrapolation of the outcome in the previous period. A more general version of this idea is that of the distributive lag model, often used in time series econometrics. This can be viewed as a distribution of the variable, with weights being probabilities derived from observations in previous periods. There is no consideration of any predictable changes in the future. Rational expectations is the idea that predictions made by individuals use all relevant information, both economic theory and available data. This is, however, too complex for most people to use in their personal decision making. Another consideration might be the ratchet effect, mentioned in Duesenberry (1949),

which indicates the direction of the general economic trend. This is information about the macro economy and is therefore difficult to translate into a method for microeconomic decisions.

There are probably rules of thumb that people use to generate expectations, such as extrapolating the current trend. i.e. a subjective OLS estimation. An appealing idea is that people assume either that they will stay in the same job, with a known structure of pay increases. Given peoples' tendency to imagine trends where there is no real justification and to concentrate on actual outcomes rather than the distribution of possible states of the world (Hogarth 1987), some form of extrapolation based on the recent outcomes of a variable seems to be a reasonable assumption. It is also considered that people do not have a continuous probability distribution over all possible future outcomes. Instead, they will look at a few cases only and generate some subjective measure of the likelihood of the different cases.

Different possible states of the world will be accounted for by using the expectation of utility in the future.

3.4.5 General/Precautionary Saving

This form of saving is intended to generate a stock of wealth over the short to medium term. A simple savings instrument is assumed with a constant interest rate, no risk and perfect liquidity i.e. savers can withdraw as much of their wealth stock without delay or loss of interest income. An illustrative time period for the decision of 5 years has been chosen i.e. the decision process will assume that the rate of saving will be maintained for 5 years, after which the decision will be retaken. In accordance with the matching principle described above, the utility in the current period is equated to the discounted utility 5 years hence, the person trading off within the limitations imposed by the liquidity constraint. Other savings (retirement savings) are assumed to held constant at the same rate as in the previous time period. It is further assumed that the person's situation will remain constant over the time period being considered i.e. wage income, wealth innovations, the interest rate, subjective parameters of the person's preferences and the savings rate once decided will remain

unchanged for 5 years. It is then assumed that the situation may change after the fifth year, so that the saving is an allowance for possible changes at the point in the future being considered.

Utility will incorporate the benefits both of consumption and the act of saving itself.

The Utility of Wealth

Wealth cannot only be considered as generating consumption. It provides security and the ability to leave bequests as well. Therefore, the utility of a given wealth stock might incorporate other psychological effects in addition to the discounted flow of consumption that the wealth stock can generate. Wealth can provide utility through giving security. Security is provided by increasing the most likely predicted level and reducing the downwards variability of future consumption possibilities, since some future wealth stock can be made to be virtually certain. This is what is known as precautionary saving in the literature, where a non-zero 3rd derivative of the utility function allows the consumption/savings choice to affect the variance of the future utility stream, given income uncertainty.

Given the strong cultural bias of western society towards thrift associated with work as responsible and morally laudable behaviour (Weber 1930), there is a strong bias towards building up a stock of wealth. People who succeed in building up large stocks of wealth are considered to be successful. Etzioni emphasized the influence of social norms on peoples behaviour (Etzioni 1988). A further consequence of this work ethic is the avoidance of debt if at all possible, which suggests that the constraints that people see themselves as facing exclude borrowing possibilities, except in the very short term (e.g. the tendency to completely pay off credit card bills before they are due).

A further effect is discussed in Loewenstein 1987. This is the possible utility from the anticipation of expenditure, which he sees as a possible justification for Moore's proposal that people do get utility from wealth (Moore 1978). Other subjective factors mentioned by

Keynes 1936 are: independence, avarice and pride (as an explanation for bequests).

These considerations mean that the utility associated with a wealth stock will not only depend on the future consumption flow attainable from that stock, but will also be affected by these other psychological effects.

The Components of the Utility of a Stock of Wealth

The psychological effects associated with saving will now be detailed. These effects are:

- 1) The consumption flow that can be generated by a stock of wealth.
- 2) The increase in expected future consumption, allowing for uncertainty (the precautionary motive).
- 3) Feelings of achievement and identity: wealth may give a sense of having achieved something and possibly a certain status in society therefore contributing to a sense of self-worth.
- 4) Independence and control: wealth gives people freedom to act as they wish, without dependence on others and enables people to exercise power over others.
- 5) Entrepreneurship: wealth can be used as a capital stock to found and develop businesses.
- 6) Avarice: people may just enjoy having a stock of money entirely for its own sake, without considering the uses of money.
- 7) The Protestant work ethic and associated social approbation of saving may give utility through other people's approval.
- 8) Anticipation: people may gain utility from imagining the benefits that they expect to get in the future from saving.
- 9) Health: the guarantee of affording health care in old age (a form of precautionary saving)
- 10) Leisure: the ability to work less for a given consumption flow as a stock of wealth builds up. Since the consumption/leisure/income choice is not being specifically considered here, leisure effects will be subsumed in the utility of consumption.
- 11) Bequests: the ability to contribute to other people's consumption when one dies.

The timing of these effects and the relevant variable(s) for each effect can now be identified.

The immediate choice is between current consumption c_t and future wealth w_f , which given the assumption of constant behaviour determines w_{t+1} . This assumption implies that the difference between w_{t+1} and w_f is arbitrary in the sense that the length of time period is chosen arbitrarily i.e. one could be written as the other if the length of each time period is changed.

Consumption gives utility at the time at which it is effected. Precaution can be incorporated in the utility function of consumption at the time of the associated consumption. Achievement, independence, entrepreneurship, avarice and bequests all give utility through having a stock of wealth available. This implies that these effects give utility when the money has been saved. They are all a function of wealth w , with a subjective probability of death weighting the utility of bequests. The simplest way that bequests can be incorporated into this model is to follow Cardes, Deaton 1992 etc. and incorporate the predicted wealth stock at the predicted time of death in the utility of savings. However, this implies that there is a (conscious) division of the wealth stock into wealth allocated to bequests and wealth allocated to other purposes. Since the same instruments will be used for both, this division is entirely notional and can be changed at any time without affecting observable behaviour. It is proposed instead to treat bequests in the same way as other benefits of a wealth stock; the ability to leave bequests will contribute to the utility of wealth. The utility derived from social approval of thrift comes from the act of saving rather than the holding of a wealth stock, so this gives utility as soon as the person starts saving (at time t). Since the utility comes from saving, it is consistent to make this a function of the rate of saving. Defining the rate of saving as the change in wealth/unit time, this model measures the rate as $w_{t+1} - w_t$. Finally, anticipation gives utility at t , dependent on the predicted level of wealth in the future w_f . The utility from the anticipation of consumption will not be separately considered in this model, since it is difficult to distinguish between the consideration of utility in the future and the psychological satisfaction of anticipating utility in the future. Eliminating anticipation means also that it is not necessary to predict the anticipation in the future from saving in the future, which preserves the idea of comparing a single present period with a single future period.

3.4.6 Saving for Retirement

This type of saving has several unique features. Firstly, it is viewed as very important, indeed some people may view it as more important than current consumption, implying a negative discount rate. Since consumption now is traded off against consumption in retirement they can be assumed to have similar utility functions. The period in the future being considered may well be many years away when the decision is being taken. This means that the decision spans the fuzzy future described in section 3.3.2. It does not, therefore, make sense to take detailed account of the length of time being considered, either for current consumption or for the length of retirement, which is even more vague. Utility functions with rates of consumption as their arguments will be used.

It will be assumed that there is a single pension instrument available to a person. It will have the features typical of a British contributory pension: there is a regular rate of deduction from the person's salary which provides an income in perpetuity on retirement. The pension will depend on the savings rate and the length of time over which contributions are made. There is also an element of commitment in this instrument; if contributions are reduced at any stage, the return on contributions is significantly decreased. However, the psychology of commitment is not considered in detail at this stage.

The utility of consumption in the current period is traded off using the matching procedure against predicted discounted utility of consumption in the first period of retirement. As in the general savings decision, it is assumed that both external factors such as income and interest rates etc. and subjective parameters remain constant until the future period, when the new situation is assessed. It is also assumed that wealth will be held constant during retirement.

3.4.7 General Applicability of the Model

This model has been developed specifically to address one particular set of economic decisions

made by households¹². However, some of the concepts introduced here may be useful in analysing other personal economic decisions. The defining characteristics of savings decisions that can be observed when individuals take these decisions are:

- i) the possible courses of action are quantifiable
- ii) the decisions involve a large degree of uncertainty
- iii) there are many different possible courses of action
- iv) there is a massive amount of information available that is relevant to the decision
- v) people often believe that they should behave rationally

These characteristics combine to make these decisions very complex, but people do try and tackle them in an organised manner. They do this by treating the problem as one of constrained optimization, but they simplify the decision so that they can bring this process to bear. They edit the information flow and only change their decision when they think the situation has changed. They only consider one part of their decision at a time and impose a structure on the problem to enable them to decide which part to concentrate on. Finally, they group different periods together, assuming that periods within the groups are the same. By these means, a decision is only taken occasionally and when it is taken, it is simplified to a trade-off between just two variables.

This approach can be applied to other decisions that also have some of the above characteristics. In particular, complex decisions where people are trying to be rational may be subject to a similar process of simplification. Possible areas could be personal investment portfolio choice, insurance, durables purchase, labour supply decision, firms' investment decisions, firms' factor choice etc. etc.

¹² Models of switching with inertia have been used in the monetary economics literature, see e.g. Akerlof & Yellen (1985).

3.5 The Formal Model

3.5.1 The Monitoring Procedure

Firstly, if either savings instrument has not been examined for a long time, then whatever the situation that savings decision is reexamined.

Then, the change in various variables is examined to see whether they have changed significantly from the last period. This difference (DIFF) is thus the signal for retaking a savings decision. The general rule for deciding whether a variable or parameter of the model has changed significantly is to check the change from the previous period to generate a difference. The value of DIFF is determined as follows: if the order of magnitude of the value last period is the same in the current period,

$$\text{DIFF} = (\text{value}_t - \text{value}_{t-1}) / \text{value}_{t-1}$$

otherwise if the order of magnitude last period is smaller

$$\text{DIFF} = \text{value}_t - \text{value}_{t-1} \quad 3.1$$

This prevents a value that changes to zero or nearly zero from an already small value from generating an infinite or very large value of DIFF.

To allow for the possibility that people may not observe DIFF accurately, or may not act on what might be considered by the individual to be a significant level of DIFF, the change in a variable perceived by the person is assumed to be normally distributed about a mean of DIFF. If the magnitude of the subjective difference is greater than the threshold at which the person believes that a change is significant, the relevant decision is reexamined. This threshold value is an exogenous behavioural parameter in the model, in the same way as the parameters of the utility functions are exogenous. The value of DIFF is taken as an instantaneous value and is independently determined in each period.

Information relevant to pension saving is checked before general/precautionary saving. The anticipated return on pensions is checked and then changes in wage income are examined. If wages significantly increase and retirement saving is a small proportion of general saving or if wages decrease and retirement saving is a large proportion of retirement saving, then the pensions decision is retaken. For general saving, the liquidity constraint applied to wealth next period ($w > 0 \forall t$), the savings rate assuming constant behaviour and the parameters of the utility functions are checked.

The result of this monitoring process is either that one of the two savings decisions is considered or behaviour (i.e. consumption and pension contributions) is simply continued from the previous period with the same values.

3.5.2 General/Precautionary Saving

The decision rule selected for this model is that utility now (at time t) is set equal to discounted predicted future utility (at time f). This matching procedure allows for the possibility that utility in the future might be measured on a different psychological scale to current utility. So from section 3.4.5 above, the matching equation for general saving can be written as:

$$E[\text{utility at } t] = E[\text{utility at } f]D \quad 3.2$$

The components of utility are assumed to be additively separable to assist comparisons with conventional models with bequests such as Cardes (1990), where additive separability is also assumed. It is intended to concentrate on the structure of the savings decision, rather than the form of the utility function and it is not practicable to incorporate all the alternative forms surveyed in ch.2. For the same reasons, the expected utility form is used to allow for two possible states of nature. The subjective probability of death by the end of the period is used to weight the two states at the end of the time period - living (with probability $1-p$) and consuming and saving or dying (with probability p) and having utility for offspring from the bequests. Writing the components of the utility functions explicitly:

$$(1-p)\{U(c_t) + TH(s_t) + V(w_t)\} + pB(w_t) = \{(1-p)(E[U(c_f)] + E[TH(s_f)] + E[V(w_f)]) + pE[B(w_f)]\}D \quad 3.3$$

where

$U(.)$ is the utility of consumption

$TH(.)$ is the utility of social approval of thrift

$V(.)$ is the utility of achievement, independence, avarice and entrepreneurship

$B(.)$ is the utility of bequests

p is the subjective probability of death, assumed not to change between t and f

$D = D(\delta, f-t)$ is the discount factor looking from time t to time f

E is the expectation operator

The equation can be solved for c_t by using the budget identity (explained in section 3.5.4 below) to relate c_t to saving and hence to the wealth stock at a specified time in the future. All values and subjective parameters are assumed to remain constant for the number of time periods over which the saving takes place, but values at time f in the future may vary.

3.5.3 Saving for Retirement

The matching rule for retirement saving is rather more simple. The utility of the current rate of consumption is compared to the discounted utility of the rate of consumption in the first period of retirement. The difference in the two situations is that current income is wage income h_t whereas income in retirement i_{ret} is dependent on pension contributions sp_t . So, the matching rule is:

$$U(c_t) = U(c_{ret})D \quad 3.4$$

where the symbols are defined in the previous section and δ has the same form. sp_t maps onto i_{ret} , i.e. $i_{ret} = i(sp_t)$ and c_{ret} can be found from i_{ret} by applying the budget identity.

3.5.4 The Budget Identity

The budget (accounting) identity can be used to relate the different variables i.e. s_t , c_t and w_t to c_t . The budget identity for some time period t is:

$$w_{t+1} = (w_t + g_t + h_t - c_t - sp_t)(1+r_t) \quad 3.5$$

where

g = an exogenous wealth innovation

h = wage income

sp = pensions contributions

r = interest rate

In contrast to some other savings/consumption models, saving is then defined as the rate of change of wealth:

$$s = w_{t+1} - w_t \quad 3.6$$

The form of this budget identity is dependent on the assumptions made about the sequence of events in any one period. At the start of a period, w_t is known and is a consequence of the decisions made in the previous period. The wage income and any wealth innovations are then received. The monitoring and decision process is then undertaken, identifying sp_t and c_t . w_{t+1} is then found as the wealth stock at the end of the current time period t .

w_t is the predicted level of wealth that will result from a particular rate of consumption and therefore saving; it is the sum of the geometric progression of the constant savings rate s_t over the planned number of periods and the current wealth stock, so w_t and s_t can be calculated from each other. s_t can be < 0 , if w_t is relatively large, depending upon tastes, of course. The rate of return, r has to be estimated, as it is an expected rate over the length of time being planned for. w_t can be found by summing the budget identities over the planned period of saving ($f-t$):

$$w_f = (w_t + g_t)(1+r)^{\delta(f-t)} + (h_t - c_t - sp_t)GP\Sigma_{f-t} \quad 3.7$$

where

$GP\Sigma_{f-t}$ is the geometric sum multiplier from 1 to f-t

This equation also assumes a constant income h_t , no wealth innovations $g_{t+1}\dots g_f$, a constant rate of pension contributions sp_t and a constant interest rate r over the period of saving.

3.6 Summary

This chapter has developed a theory of savings behaviour using the idea of bounded rationality. A form of the utility function has been proposed that incorporates contributions to utility from factors in addition to current and (discounted) future consumption. A simplified decision process, compared to conventional life cycle models, has been suggested and two types of savings instrument, a general savings instrument and a pension instrument incorporating an element of commitment are included.

4 Simulation

4.1 Introduction

This chapter contains the details of the solution of the model, the simulation of the theory and the results obtained. The general method and overall structure of the simulation program is introduced and in section 2 the monitoring procedure is described. Sections 3 provide details for the general and retirement savings solutions and section 4 contains the results. Section 5 contains the discussion and conclusions.

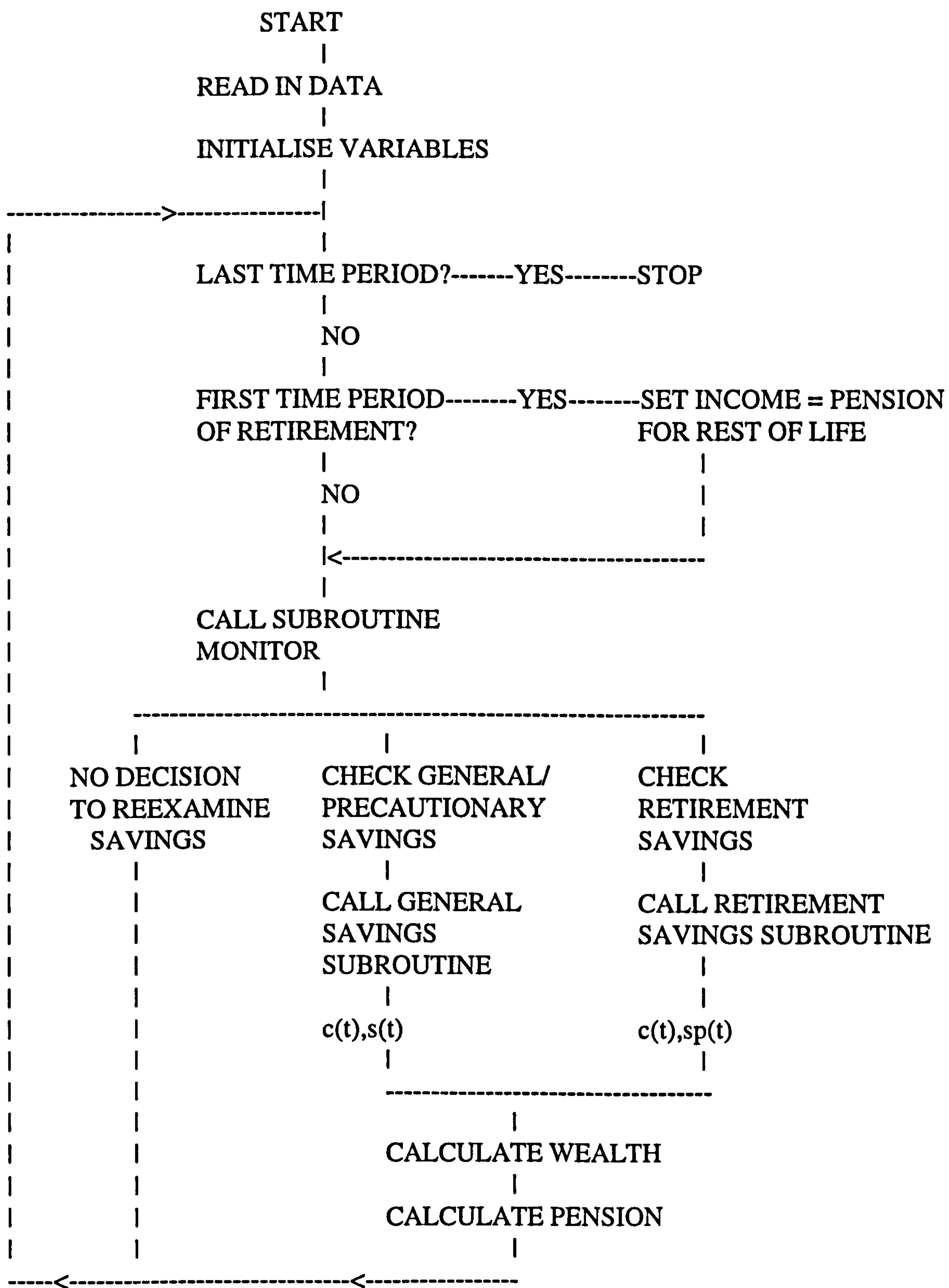
The point that the details of the decision rules that are set out here are not the most important part of the theory should be emphasized. Not only do people use different rules under different circumstances, but that different people will use different rules under the same circumstances. Finding a set of rules that usefully approximate a wide range of people's behaviour while being concise and plausible; in short, that obey the tenets of economic methodology is a matter for empirical investigation. The particular rules that are detailed are essentially illustrative. The important ideas that form the basis of the model have been set out in section 3.3 above.

The model was tested by writing a computer program to generate results for a range of parameters. There were two objectives of the simulation: first, to investigate whether the model generates consumption/savings paths that are consistent with the stylised facts set out in section 3.2.1. In particular, does saving track income over the life cycle and is there a significant level of bequests? Second, does the model generate predictions that are testable versus conventional models i.e. are consumption/savings paths generated that are significantly different to those predicted by conventional models?

The operation and combination of the various elements of the model can be explained with reference to the flow chart for the simulation program SIMUL7, figure 1. The program structure is the same as that of the model. Initially, the data for the parameters of the individual's decision process and for the income process are read in. Then the program loops

Figure 1 FORTRAN 77 program SIMUL7

Flow chart for overall structure



through all the periods for which there is data (i.e. until the individual dies). Note that the model assumes that the individual does not know how long they will live. In each period, the savings decisions (general and pension savings) are initially set equal to the values from the last period. This is the default, the application of the idea of habitual behaviour. Next, the income for the period is generated using a random number generator. There are three subroutines which form the decision process. Subroutine MONITOR checks whether a savings decision will be taken in the current period. The details of the subroutine are given in section 4.2 below. If no savings decisions are to be taken, the program moves to the next period and the default savings values used to calculate the consumption and savings paths. If MONITOR indicates that general savings should be considered, the subroutine GENPRE generates the new level of general saving, with pension saving set equal to the default value. The equations used are explained in section 4.3 below. If MONITOR indicates that pension savings should be considered, the subroutine RETIRE generates the new level of pension saving, with general saving set equal to the default value. The equations used are explained in section 4.4 below.

In order to generate a solution of the model, it was necessary to specify functional forms for the various utility functions. The CRRA form was used in all cases, because it is commonly used in consumer economics and has diminishing returns to scale with a marginal utility tending to infinity at zero consumption. Also, it has non-zero 2nd and 3rd derivatives, thus allowing for precautionary saving. Blanchard and Fischer(1989) argue that the CRRA function is more plausible than CARA because it prevents consumption from falling below zero along the optimal path.

The form of the discounting function for utility predicted to be enjoyed in the future was exponential, but altered from the normal function to account for the effects of matching as opposed to maximising. In particular the function should have a value greater than 1.0 to indicate a lower weight applied to utility in the future.

The function used was:

$$D(\delta, f-t) = 2.0 - e^{-(1-\delta)(f-t)} \quad 4.1$$

The program is structured in the same way as the formal model. (see flow chart fig.1). In each time period from the beginning of the person's (financial decision-making) life to death, the main routine calls the monitoring sub-routine, which determines which decision, if any, will be examined. The main decisions are incorporated in separate sub-routines. For details of the program, see the listing at Appendix 1. The program is written in FORTRAN77.

4.2 The Monitoring Procedure

See the flow chart for the subroutine MONITOR (Appendix 1) for details of the order in which different variables and parameters are checked. Firstly, if pensions have not been checked in the last ten periods (years), the pensions decision is retaken. Then, if general savings have not been checked in the last five years, the general savings decision is retaken. This is done to reflect the fact that people occasionally examine their affairs for various reasons. Next, possible triggers for the pension decision are checked. The return on pensions is examined and the size of pension contributions relative to general savings are also considered. The liquidity constraint is checked by looking at the wealth at $t+1$, given constant behaviour and the current values of exogenous variables. If it is predicted that the liquidity constraint will be violated i.e. $w_{t+1} < 0$, then the general savings decision is retaken. Also, the current rate of savings is compared to the rate of savings last period. Finally, the values of the subjective parameters of the utility functions are checked to see if they have changed significantly from the last period.

4.3 The General Savings Solution

From equation (3.3),

$$U(c_t) + TH(s_t) + V(w_t) + p/(1-p)B(w_t) = \{E[U(c_t)] + E[TH(s_t)] + E[V(w_t)] + p/(1-p)E[B(w_t)]\}D \quad 4.2$$

The CRRA functions for utility are:

$$U(c) = (1/(1-\alpha))c^{1-\alpha} \quad 4.3$$

$$\begin{aligned} V(w) &= (1/(1-\beta))w^{1-\beta} & w > 0 \\ &= (1/(1-\beta))(-w)^{1-\beta} & w < 0 \end{aligned} \quad 4.4$$

$$\begin{aligned} TH(s) &= (1/(1-\gamma))s^{1-\gamma} & s > 0 \\ &= (1/(1-\gamma))(-s)^{1-\gamma} & s < 0 \end{aligned} \quad 4.5$$

$$\begin{aligned} B(w) &= (1/(1-\lambda))w^{1-\lambda} & w > 0 \\ &= (1/(1-\lambda))(-w)^{1-\lambda} & w < 0 \end{aligned} \quad 4.6$$

where α , β , γ and λ are the parameters of the utility functions.

To solve this part of the model, an expression for c_t is found using eqn.(4.2). Some further assumptions are made in order to find a solution. Saving at the point in the future f is assumed to be zero. This implies:

$$w_{f+1} = w_f \quad 4.7$$

This avoids consideration of the general savings decision at time f , which would immediately require consideration of more than the two time periods t and f . It is also assumed that there will be no exogenous wealth innovations:

$$g_{t+1}, g_{t+2}, \dots, g_f = 0 \quad 4.8$$

and that wage income h and the interest rate r will remain at their current level and pensions contributions sp will remain at their previously decided level. From eqn.4.7 and using the budget identity (eqn. 3.5),

$$c_f = w_f(r/1+r) + h_f - sp_f \quad 4.9$$

Rules of thumb are required to generate the values of variables at the future time f . w_f can now be found as a function of c_t . Note that w_f is deterministic, being determined by current savings and consumption, both of which are assumed to be constant. Uncertainty is introduced in the form of uncertainty of future income. For the purposes of this simulation, it is assumed that the person considers up to 3 possible future incomes and associates a subjective probability to each possible income. Writing

$$a = w_f(r / 1+r) - sp_f \quad 4.10$$

The expected utility of future consumption is:

$$E[U(c_f)] = (1-p_h-p_l)*U(a+h_f) + p_h*U(a+h_h) + p_l*U(a+h_l) \quad 4.11$$

where

h_h is the highest possible future income

h_l is the lowest possible future income

h_f is the most likely future income

p_h is the subjective probability of receiving h_h

p_l is the subjective probability of receiving h_l

From eqn.3.6 and the budget identity,

$$\begin{aligned} s_t &= (w_t + g_t + h_t - c_t - sp_t) *(1+r) - w_t \\ &= (g_t + h_t - c_t - sp_t)*(1+r) + rw_t \end{aligned} \quad 4.12$$

So in eqn.(4.2), w_f and hence c_f and s_t can be found from c_t , while w_t is already known.

Eqn.(4.2) can be rearranged to form a transcendental equation of the form:

$$f(c_t, s_t, w_t) - D(\delta, f-t) * f(c_t, w_t) = 0 \quad 4.13$$

which is the form suitable for use in the solution routine to generate a value of c_t to satisfy the matching condition.

The solution routine used is the modified linear interpolation routine detailed on p11. of Gerald and Wheatley (1989).

4.4 The Retirement Savings Solution

The decision rule is given by eqn.(3.4) with the discount factor for predicted utility in the future given by eqn.(4.1). Since other decision variables are kept constant, the savings rate is assumed constant at the same rate as in the last period:

$$s_t = s_{t-1} \quad 4.14$$

Wealth innovations (g) are assumed to be zero and wage income (h) and the pension return factor (rp) and the interest rate (r) on any wealth stock are assumed to remain constant during the period of pension contributions. Again, this is not proposed as assumptions that are expected to be fulfilled, they just enable the calculation to be performed in the knowledge that if they prove to be incorrect, the decision can be retaken.

From the budget identity (eqn.3.5):

$$w_t + s_t = w_{t+1} = (w_t + g_t + h_t - c_t - sp_t)(1+r) \quad 4.15$$

$$sp_t = w_t + h_t - c_t - (w_t + s_t)/(1+r) \quad 4.16$$

for notational convenience, write:

$$c_{11} = w_t + h_t - (w_t + s_t) / (1+r) \quad 4.17$$

so

$$sp_t = c_{11} - c_t \quad 4.18$$

It is assumed that the pension income in the first period of retirement i_{ret} is a linear factor of the total pension contributions up to retirement. Assuming a constant rate of contributions,

$$i_{ret} = h_{ret-1} / (rp * h_t) * sp_t * (ret-1-t) \quad 4.19$$

Typically, a rate of contribution of 5% of salary for n years will give n 80ths of salary on retirement i.e. $rp = 4$.

write

$$i_{ret} = c_{10} * sp_t * (ret-1-t) \quad 4.20$$

To find h_{ret-1} the same expectations rule as for generating future values of variables in the general savings decision is used. It is necessary to allow for withdrawal from the committed pensions saving scheme. This was done by freezing the pension scheme if contributions are not maintained. If individuals decide to reduce the rate of contribution, they terminate their scheme and the pension from previous contributions is calculated at the lower rate of return. The person retains the pension income from this previous scheme which is paid out in retirement. The person then starts a new scheme with the new rate of contribution which will receive the higher rate of return unless this new scheme is also stopped before retirement. A record is kept of the pension income of all old schemes. Thus the predicted pension income p_{ret} will be:

$$p_{ret} = \sum i_{old} + \sum sp_{old} + i_{ret} \quad 4.21$$

where

Σi_{old} is the sum of pensions from old schemes

Σsp_{old} is the pension income from previous contributions to the current scheme

Consumption in retirement is found by applying the budget constraint.

$$w_{ret+1} = (w_{ret} + g_{ret} + p_{ret} - c_{ret})(1+r) \quad 4.19$$

A further rule of thumb is used to guess at w_{ret} . w_{ret} is assumed to be some linear multiple of w_t . As for general saving, it is assumed that the rate of saving in the future and wealth innovations will be zero and sp will be zero in retirement, so:

$$c_{ret} = p_{ret} + w_{ret} * r_{ret} / (1+r) \quad 4.22$$

write

$$c_{ret} = c13 - c10 * (ret-1-t) * c_t \quad 4.23$$

From the matching equation (eqn.3.4),

$$U(c_t) = U(c_{ret}) * D \quad 3.4$$

$$(1/(1-\alpha))c_t^{1-\alpha} = (1/(1-\alpha))(c13 - c10 * (ret-1-t) * c_t^{1-\alpha}) * D \quad 4.24$$

from eqn.4.1 write

$$D^{1/(1-\alpha)} = [2.0 - \exp(-(1-\delta) * (ret-t))]^{1/(1-\alpha)} = c12 \quad 4.25$$

so

$$c_t = c13 * c12 / (1 + c10 * (ret-1-t) * c12) \quad 4.26$$

which gives a direct solution for c_t and hence for sp .

4.5 Results

First, the general pattern of the results will be described, then the stability and sensitivity of the results to changes in parameter values will be examined and the comparative statics analysed.

Some typical results are at figure 2¹³. These curves are averages of realisations over 1000 lifetime simulations for an income distribution with 3 possible outcomes in each time period. Reading time in years, the curves run from an age of 20 to 90 years with retirement at 65. It can be seen that the consumption path is significantly different to conventional life-cycle models, even those with precautionary saving or liquidity constraints where saving tracks consumption. Until retirement, the consumption path is determined mainly by the level of income, but the path is also significantly affected by the pattern of savings. There is a short initial time span up to period 4 where pension savings are initiated, as pensions are checked first in the monitoring process. As the expectation of future income rises and the income trend is detected, general savings are started. The consequence of this is that in the initial 4 periods the changes in the two different savings instruments cancel out so that the amount of total saving is constant and changes in consumption follow the movement of income almost exactly. Consumption then declines sharply until period 10 as general saving takes over and a wealth stock is built up. From this relatively low level consumption then increases steadily and at a greater rate than income until period 40; after period 23 consumption is greater than income. At period 40, the sharp reduction in income due to retirement is taken into account (remember that the general savings decision looks forward for 5 periods only) and consumption

¹³ The various utilities incorporated in the utility functions (equation 4 for general savings and equation 6 for retirement saving) were all of the CRRA form

$$U(c) = (1/(1-\alpha)) * c^{1-\alpha}, TH(s) = (1/(1-\beta)) * s^{1-\beta}, V(w) = (1/(1-\gamma)) * w^{1-\gamma}, B(w) = (1/(1-\lambda)) * w^{1-\lambda}$$

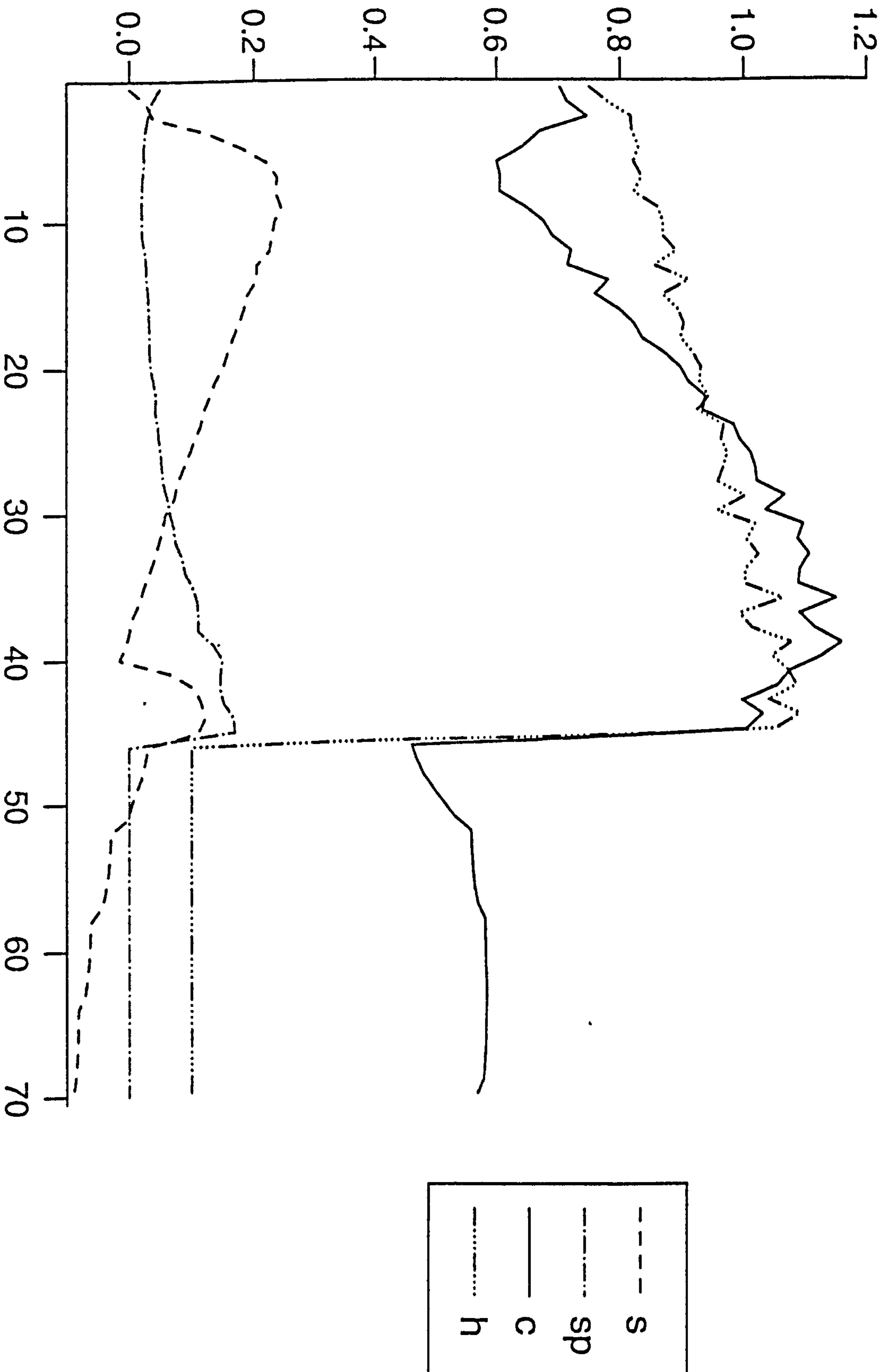
In figure 2, $\alpha = 0.8$, $\beta = 0.05$, $\gamma = 0.05$, $\lambda = 0.05$, r (rate of interest on general savings) = 5%, r_p (return parameter for pension contributions) = 10.0. The subjective probability of death p increased from 0.05 at period 1 to 0.5 at period 70. The income distribution was initially 0.1 with probability 0.3, 1.0 with probability 0.7, 1.5 with probability 0.1, increasing to 0.14 with probability 0.3, 1.4 with probability 0.7, 2.1 with probability 0.1 at retirement. After retirement, there was a certain exogenous (state) pension of 0.1 in addition to any private pension resulting from contributions before retirement.

These values of the parameters were also used as the baseline for the comparative statics analysis.

See Appendix 2 for a data listing for the simulation results of figure 2.

Figure 2

Consumption c , saving s and pension saving sp with income h



reduces to renew the build up of wealth. During retirement, consumption is roughly constant, being financed partly by dissaving and the state pension but also by a considerable private pension (0.53 or half the average income at retirement). Consumption can thus be seen to approximately track income, but there is smoothing between the working life and retirement and a development of a large wealth stock which results in a steeper rise in consumption than in income over time before retirement. It should also be noted that over the very short term once a trend in saving has been established, the movements in consumption follow the movements in income from period to period very closely, unless there is another reason to change savings behaviour.

The savings behaviour reflects this consumption pattern. At no point is the liquidity constraint binding; in all periods the wealth stock is greater than 0. There is an initial period of high (up to 30% of income) general saving in order to generate a wealth stock. As the level of wealth increases, saving is reduced and eventually (at period 36) becomes negative (dissaving), but as retirement is approached, general savings start again to allow for the decrease in income after retirement. The wealth saved up is considerable, reaching a local maximum of 4.5 (about 3*annual income at that period) in period 35. Then there is a further increase as retirement with a lower income becomes a factor in the general savings decision to attain the overall maximum of 5.2 (about 3*annual income at retirement) just after retirement at period 50; there is some saving from the pension income for a few periods. Pension saving starts off higher than general saving but is then reduced as general saving takes priority. Due to the exponential discounting of the first period of retirement in the pension decision, pension saving accelerates gradually as retirement comes nearer and the discount factor of consumption in retirement reduces. At period 30, as general saving becomes small, pension saving becomes greater than general saving and this remains the case until retirement. There are thus two aspects of this savings behaviour which are in contrast to a conventional life-cycle model. Firstly, there is extensive short to medium term saving when young and secondly there is a small 'boom' in saving, both general and pension, as retirement approaches.

The path of the wealth stock following from the savings behaviour is also somewhat unusual. Wealth increases steadily and quite rapidly until a high wealth stock is reached, when dissaving occurs. As retirement approaches, the wealth stock is increased still further to a maximum just after retirement and then the wealth stock is gradually run down. However, this process is quite slow and

there is still considerable wealth left in the final period. This is to be expected, partly due to the bequest motive, but also from the assumption that the person may live on (for at least another 5 years in this simulation).

Habit forming and the monitoring process play a minor part in the results. The priority of pension saving over general saving causes pension contributions in the first 4 periods to be higher than general saving and habit forming can be seen clearly after retirement, where there are changes in behaviour every 5 years after period 53. This means that as the situation is constant, the decisions are not retaken until the decisions are rechecked simply because a long time has passed since the level of savings was checked. Before retirement, the variability in the income process ensures that savings are checked regularly.

Turning to the stability of the results, it will be seen in the comparative statics analysis that the patterns of behaviour shown in fig.2 are maintained for moderate variations in all the main parameters of the model. This is to be expected, since variations in interest rates and income distribution have simple effects on the savings/consumption possibilities. A more severe test is the sensitivity of the results to the form of the utility function. In fig.3 the parameter α of the CRRA utility function for consumption $U(c) = (1/(1-\alpha)) * c^{1-\alpha}$ is varied and the resulting consumption and general savings paths plotted. α was varied from 0.05 (virtually a straight line) to 0.9, where the curvature is extreme. The forms of the consumption and savings paths are seen to be stable throughout this range, with only the magnitude of the values varying. Thus it can be seen that the general nature of the results is independent of the specific parameter values used; the saving in early periods and the saving before retirement, together with the form of the through-life consumption path are generally valid.

The comparative statics of general and pension savings and consumption as the interest rate on general saving varies are shown in fig.4. Consumption increases with increasing interest rate, as the consumption/saving frontier is opened out. The effect on general saving is the opposite to that on consumption. As the interest rate increases, less is saved early on when the stock of wealth is developed, as it is easier to build up an adequate level of wealth with a higher interest rate. Later on,

Figure 3

Consumption c and savings s as utility function varies

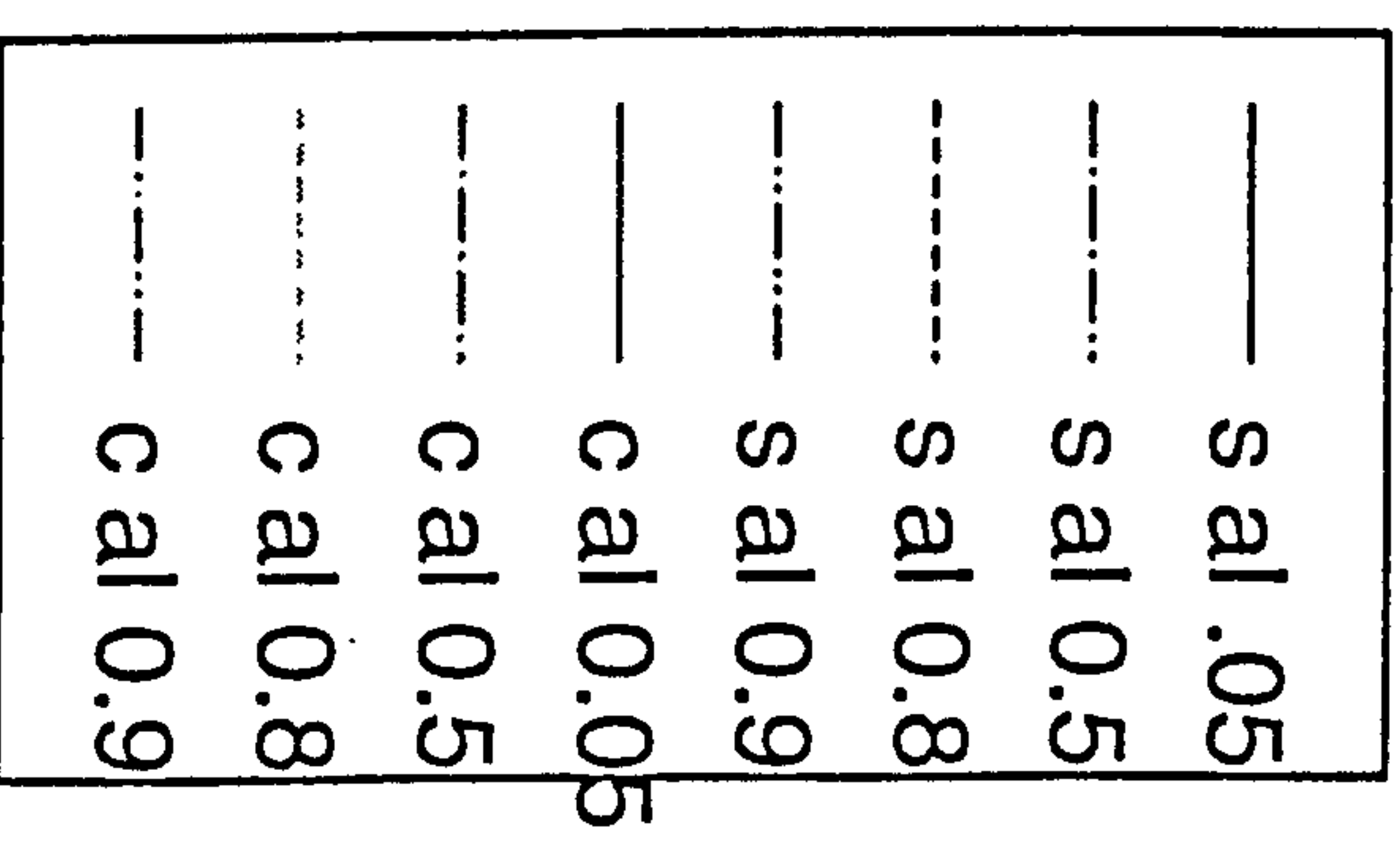
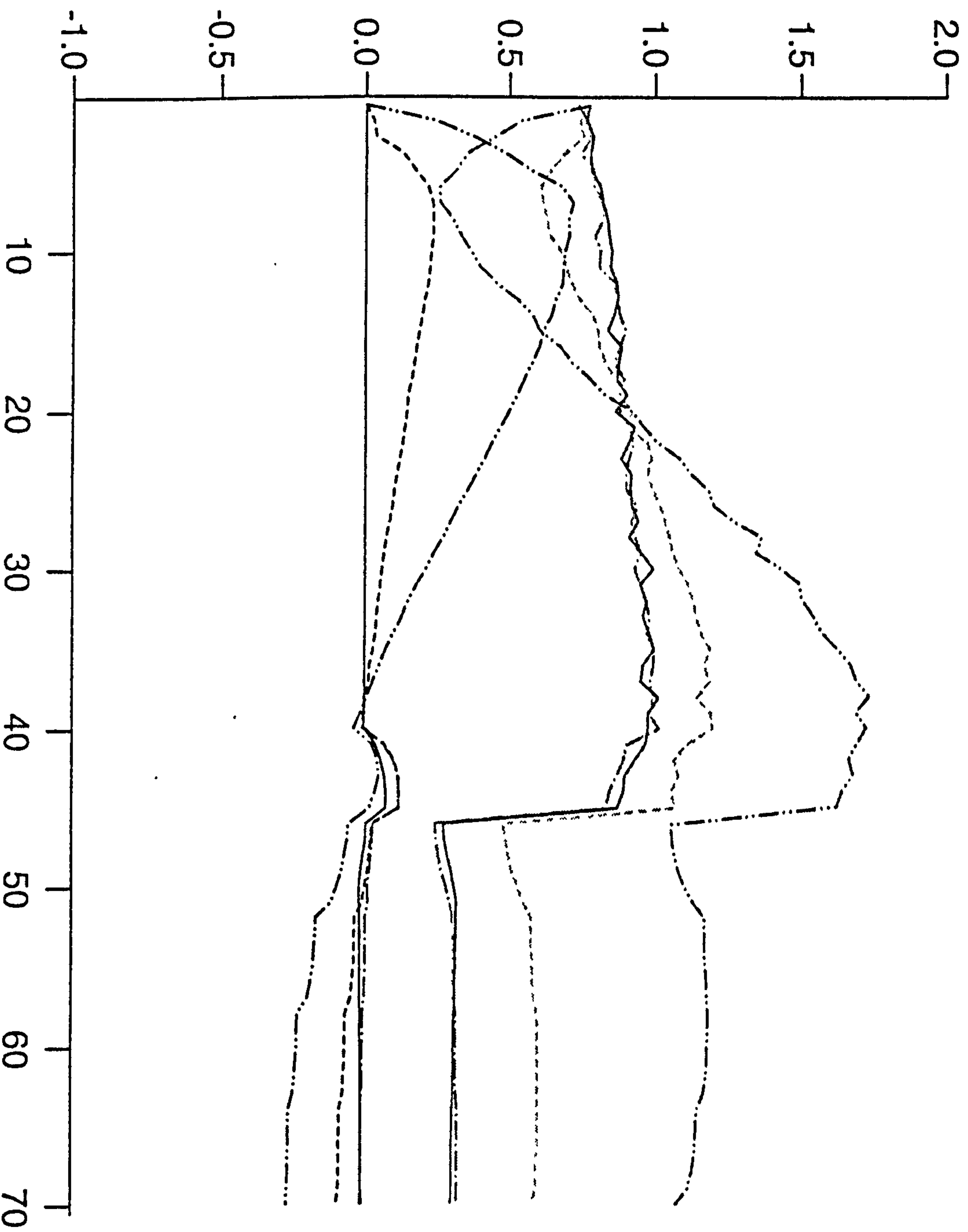


Figure 4

Consumption c , saving s , pension saving sp vs. interest rate r

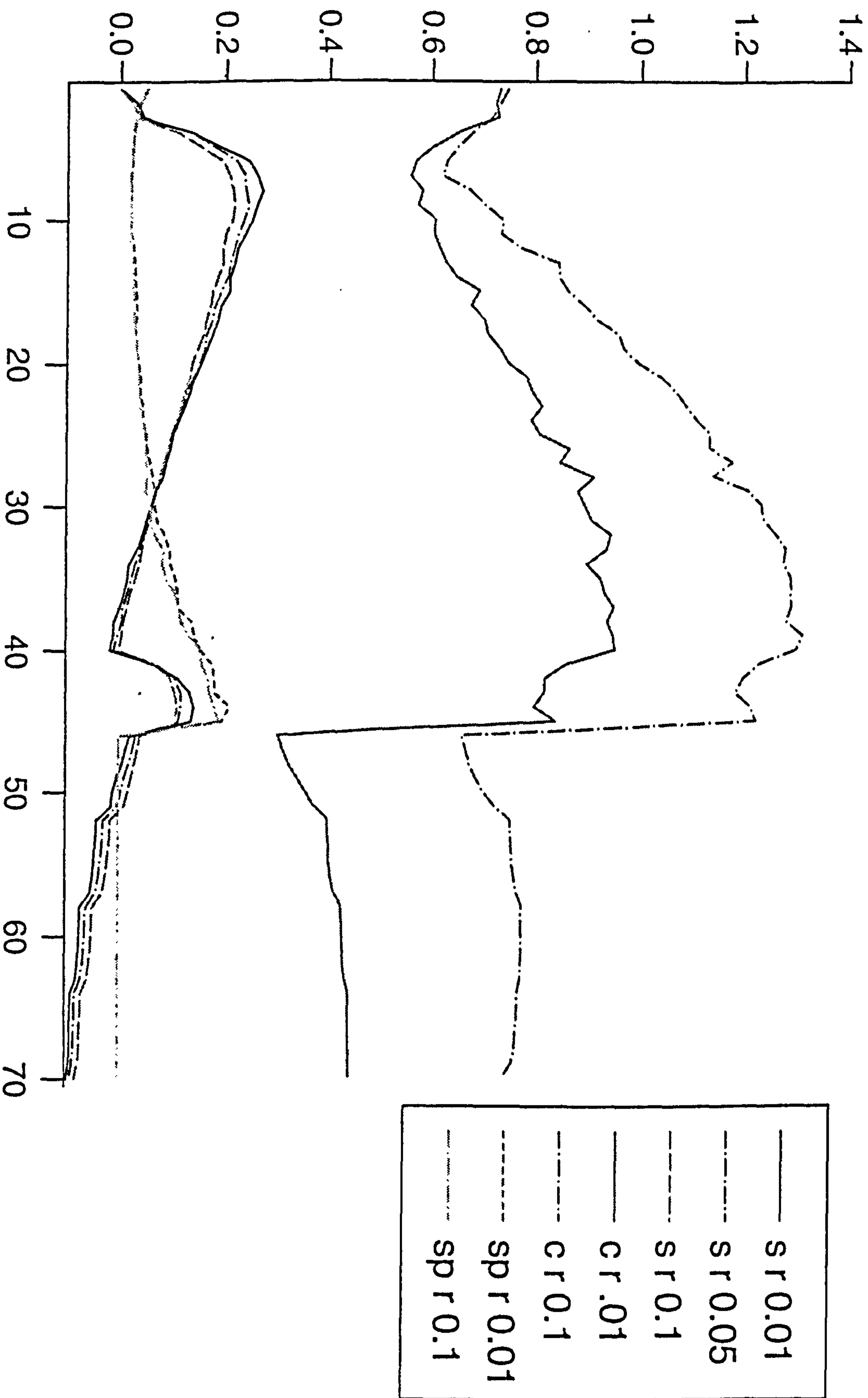


Figure 5
Consumption c as pension return rp varies $ur3,1,2$

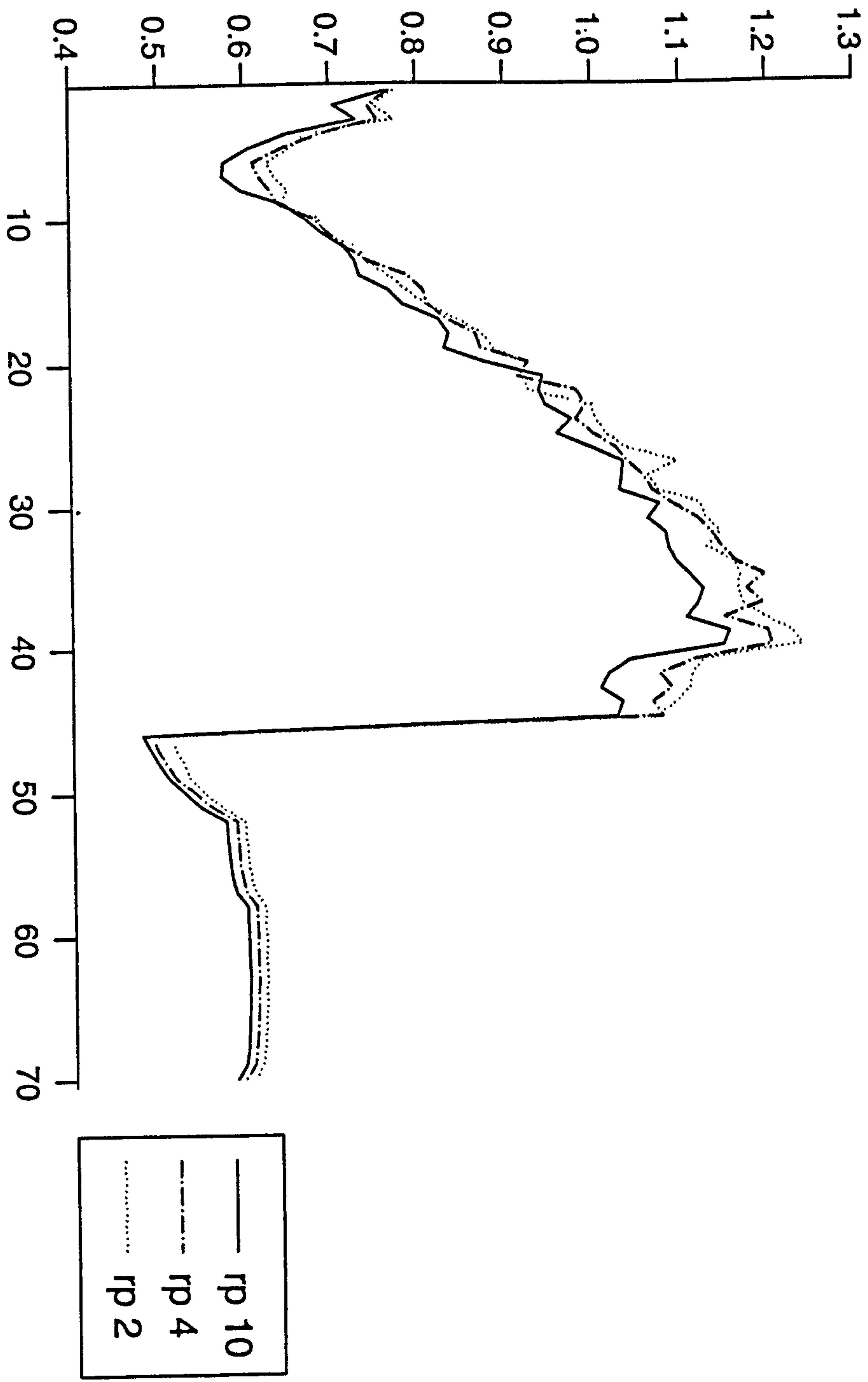


Figure 6
Consumption c and general savings s as income increases

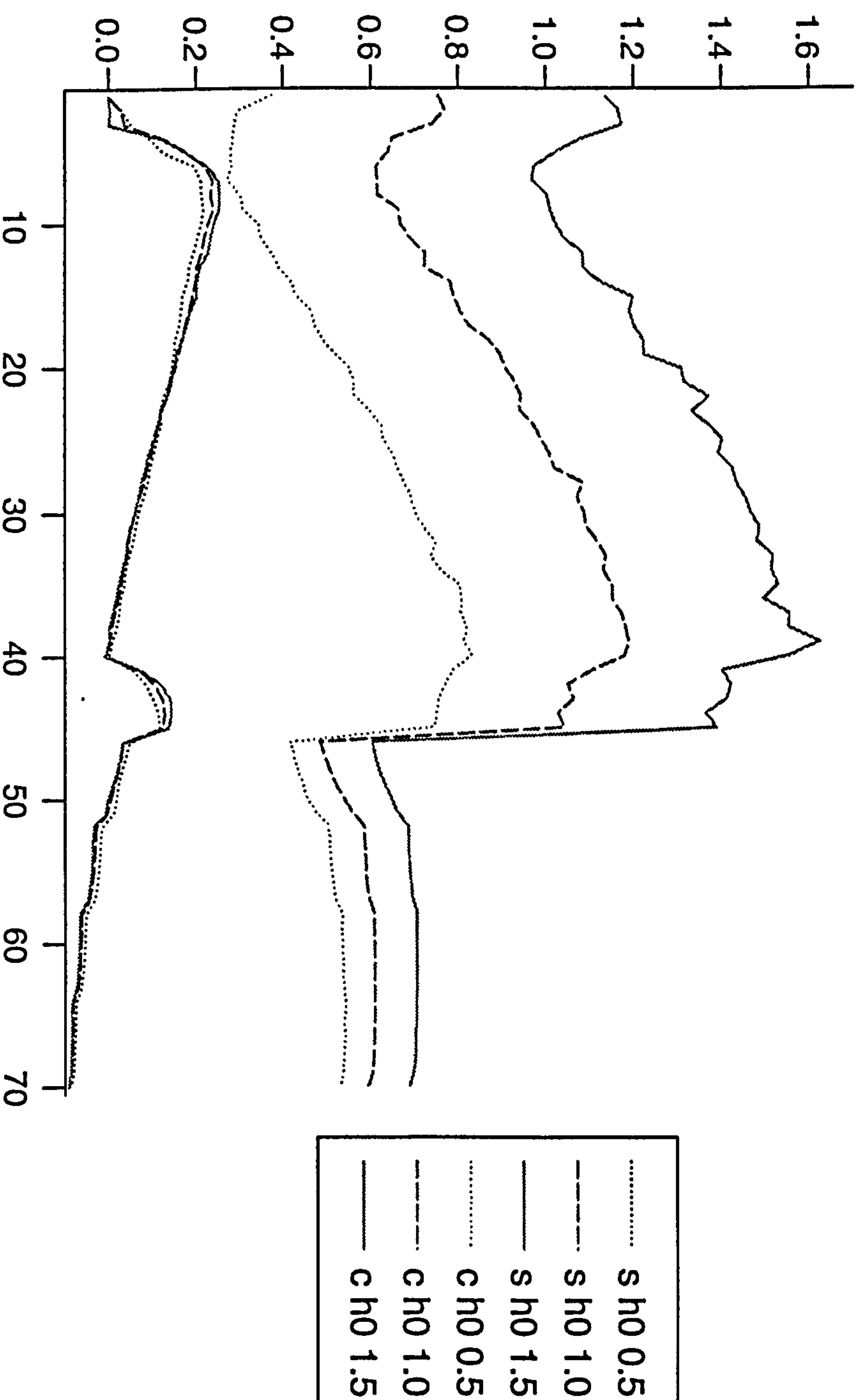
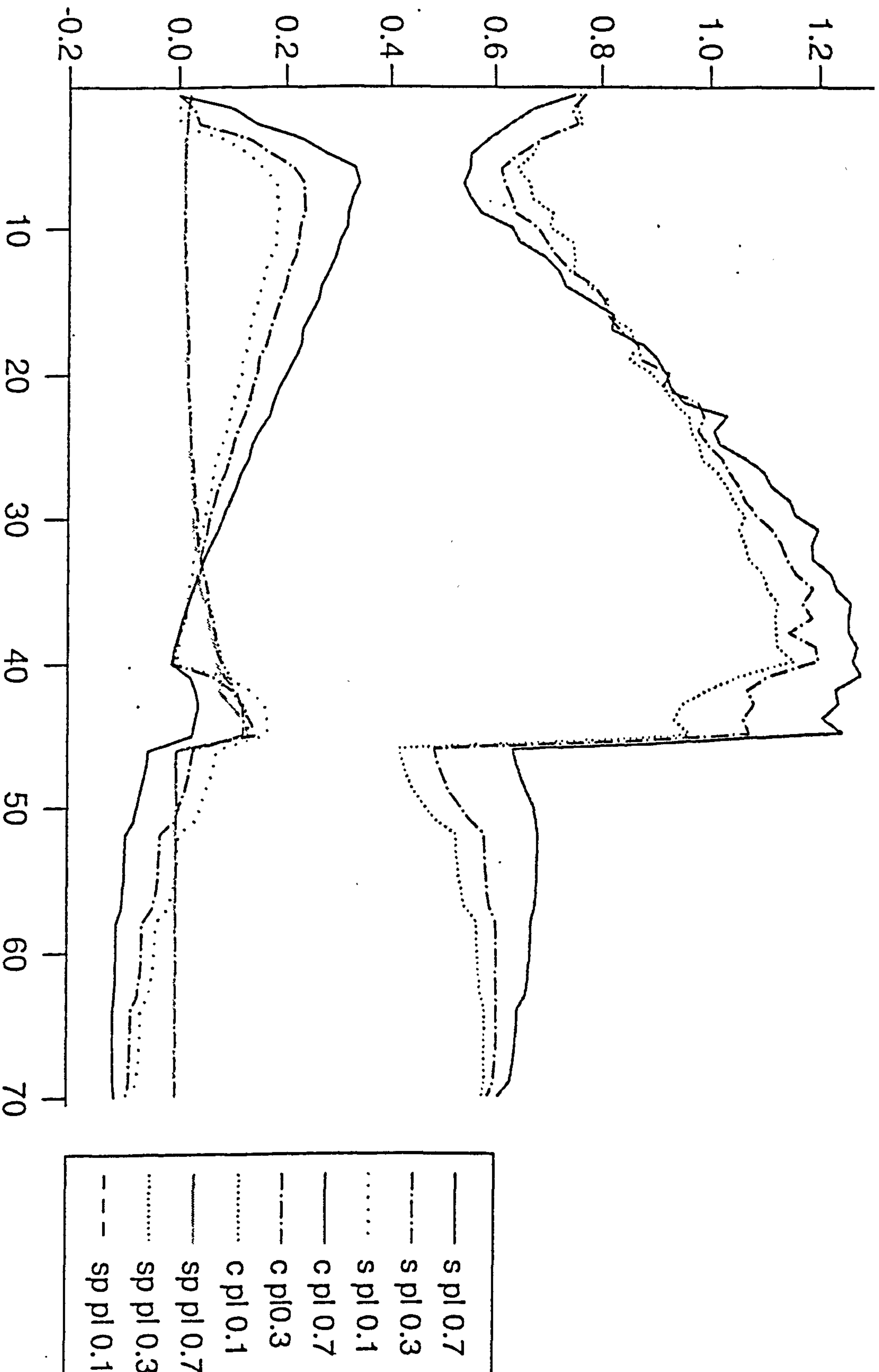


Figure 7

Consumption c , saving s , pension saving sp with prob. of low income



the dissaving and then the burst of saving before retirement, as well as the dissaving after retirement are all more pronounced with lower interest rates, again to compensate for the reduced effectiveness of saving. Pension saving increases as the general savings interest rate decreases, as pension saving becomes more attractive relative to general saving. The effect of changes in r_p , the return parameter for pension contributions, as shown in fig.5 is not very strong¹⁴. This is because pension saving takes place slowly over a long period of time so that small changes in consumption and hence pension saving can allow for large changes in the return parameter.

Finally, variations in the income distribution are considered. Fig.6 shows the variation in consumption and general saving as the income distribution before retirement is shifted. The baseline initial median value of income (h_0) is 1.0 (as in fig.2) and curves for initial median values of 0.5 and 1.5 are plotted. In these curves, the low income and high income levels change proportionately, as does the increase in the income over time. The rates of consumption and saving increase as the resources available increase with increased income. In fig.7 the changes in behaviour with a changing probability of the low value of income being received are shown. The influence of the precautionary motive is seen to be strong, which is to be expected with a value of the curvature parameter in the utility function for consumption of 0.8. A higher probability of the low income (p_l) means that the early saving to build up a stock of wealth is emphasised and increased to compensate for the increased probability, even though the average income decreases, so that a larger wealth stock is generated relative to lower p_l . This high level of assets reduces the necessity for the pre-retirement saving and it can be seen that after retirement, the larger wealth stock causes a higher rate of dissaving. Pension saving is seen to increase as p_l decreases; there is not so much general saving and there is more income available on average allowing a higher level of pension saving as well as consumption.

¹⁴ Note that, from eqn.4.19, r_p is an inverse parameter so the return on pension saving increases as r_p decreases.

4.6 Summary and Conclusions

A theory of saving and consumption over a life has been developed which has several new features, compared with previous life cycle models. The process of decision making is simplified in several ways. Firstly, only one 'representative' period in the future is considered in the decision. This point is taken to be representative of further periods in the future. Savings and consumption from the present until this future period are assumed to be constant, so that there is a trade off between two periods only. Two forms of saving are incorporated, of which the pension saving instrument incorporates commitment. Separate utilities of wealth stocks, savings rates and bequests have been included which enable the holding of wealth stocks to be modelled in addition to the utility of deferred consumption. Habit formation has been included and a trade off of levels of utilities instead of marginal utilities was incorporated, although these features of the model did not significantly affect the results. The income path was variable enough so that savings were regularly rechecked and the trade off of levels did not change the nature of the results compared to a trade off of marginal utilities.

The results show some new features, compared to conventional life cycle models. The results of the model are in accordance with the stylised facts about savings described in section 3.2. Consumption tracks income (Deaton (1992)) and there is a high level of wealth during retirement with an associated significant level of bequests (Deaton (1992)). Katona's committed savings (Katona (1975)) in the form of pensions play a significant role in generating a satisfactory retirement income. Most significantly, there are two humps in saving, one early on in life - an alternative explanation for Carroll & Summers (1991) result that the young may save more than the old - and the second as retirement approaches. Thus consumption is low early on in the life, but increases at a higher rate than income for most of the working life. Pension saving is enough to generate a considerable private pension and consumption in retirement is also increased by slowly running down a large wealth stock. There is a significant level of bequests, partly due to the utility of bequests incorporated in the decision process, but also because of the assumption that the person will continue to live for several years and therefore still wishes to enjoy an adequate level of consumption in the future. In the short term from one period to the next, consumption changes closely follow income changes and in general over the life cycle, consumption can be said to track

income quite closely. There are two important elements of actual savings behaviour which are not included in this model which make a very detailed comparison of the results with empirical data difficult. Firstly, there is no account taken of mortgage saving, so that the boost to available income in middle age when the mortgage is paid off, an important form of committed saving, is not allowed for. This means that data from the rental sector would have to be used, but in the U.K. the population that is wealthy enough to have significant savings are almost all home owners and they devote a large proportion of committed savings to mortgages. The model does attempt to allow for the institutional environment for savings, by including a pension instrument in addition to the savings instrument found in conventional life cycle models. The inclusion of mortgage saving was considered, but this would have caused several problems. Mortgages provide rental services in addition to the ability to change the time path of consumption. Therefore, the utility of consumption would have to be divided into housing and other goods. Also, there might be other arguments in the utility function (e.g. utility associated with the act of ownership) which would require another series of behavioural variables in the same way as was undertaken for saving. The inclusion of those effects would have made comparison with the results of conventional life cycle models, which do not allow for these institutional factors, very difficult. The main objective of the model and simulation was to compare the results with conventional models and so the inclusion of mortgages would be disadvantageous. Furthermore, mortgages are borrowing and the inclusion of borrowing for houses would be inconsistent with the liquidity constraint of no borrowing explained in section 3.4.4. Also, there is no allowance for short term saving up for large purchases such as cars or holidays, so very short term behaviour over time spans of less than 1 year is not modelled.

If some of the ideas about decision making incorporated in this theory can be confirmed by empirical evidence, it may have important implications for predictions about savings behaviour. Cochrane (1989) has shown that if the assumption of wholly optimising behaviour is dropped, very different patterns of savings can result in similar levels of utility. It is therefore necessary to examine the decision processes themselves to generate predictions about savings. Current versions of conventional life cycle models can be made to fit some empirical observations, but it is unlikely that theory based on unrealistic assumptions about the nature of the decision processes involved will be able to accurately predict detailed patterns of saving. The rest of this thesis begins this task, by looking experimentally at the cognitive processes that people use in taking savings decisions.

5 An Experimental Investigation of Savings Decision Making

5.1 Introduction

The primary aim of this experiment was to investigate the way in which people tackle dynamic decision problems; using the savings decision as an illustration. In particular, the experiment looked at timing in people's thinking, comparing the hypothesis of chapter 3 against the assumptions of conventional life cycle models of saving. These models assume that individuals consider the whole of their possible life, summing expected discounted utility from all future periods. In contrast to this, the hypothesis of chapter 3 is that people only look forward a little way for a few periods (apart from future events assumed to be known with certainty i.e. retirement). Do people use backward induction or do they use a forward-looking approach?

The secondary aim was to investigate savings behaviour and compare it to the optimal strategy for the savings problem with income uncertainty and an unknown length of life.

This chapter is organised as follows: section 5.2 describes the experiment, explaining the reasons for the particular features of the experiment and comparing the design to the savings experiment of Hey & Dardanoni (1988a). Section 5.3 describes problems that arose with the experimental design and section 5.4 contains the analysis of the results obtained from the trials data (to identify strategies) and the econometric analysis of the actual savings/conversion results. Section 5.5 summarises the results and draws conclusions.

5.2 Description of the Experiment

The concept of the experiment was to have subjects solve a savings problem, giving them a trial calculation facility with which subjects could try out different savings strategies. The trial facility was designed to provide information about people's decision processes: the trials that subjects performed could be examined to identify different patterns of thinking about savings with regard to the range of time considered, the consumption patterns tried and income streams assumed. The savings problem was arranged to have the same theoretical structure as a savings/consumption decision with one certain savings instrument and one

form of consumption (see Hey & Dardanoni 1988a for details). The savings decisions could then be analysed to identify the relationships between the experimental parameters and 'consumption' decisions of subjects, including their performance compared to the optimum strategy.

The experiment was run as follows (the details of the design are given in section 5.2.2 below). Subjects were given an audio-visual presentation of the instructions (see Appendix 3 for the instructions). The experiment was divided into periods. In each period the subjects received an income in tokens and they decided how much of their available resources (wealth accumulated from previous periods plus income in the current period) to convert to points. Subjects were told the income distribution, the (certain) interest rate on tokens saved and the (publicly constant) probability of the experiment finishing at the end of each period. They were also given a graph of the CARA conversion function from tokens to points. Before they input their consumption/savings choice, the subjects had a trial calculation facility with which they could try out different savings strategies (see section 5.2.1.1 below for details). Subjects first played the experiment for two practice periods, which did not contribute to their payoff, to familiarise them with the running of the program, the calculations involved and the trial calculation facility. They then performed the experiment for money.

The experiment itself was conducted in two parts: in the main experiment subjects used an interactive computer program to solve a savings problem and perform trials of different strategies and then it was necessary (see section 5.2.3) to include a separate payoff mechanism to control the incentives in the experiment. In the program, subjects played a savings game over a number of time periods in which they received income which could be converted to points using a CARA formula or saved for conversion in the future. The accumulated points were then used in the payoff mechanism to play a lottery between a high prize and a low prize.

The experiment was performed at EXEC, University of York, with four groups of 10

subjects¹⁵ (two groups each day over two days) so that the laboratory was not too crowded: subjects could not see each others' problems and decisions. After the first two groups had performed the experiment, various problems came to light and to overcome some of these problems some changes were made to the experimental program and the parameters of the experiment (see section 5.3 for a discussion).

Although the problem is greatly simplified in comparison to a real life savings problem for a 'representative' consumer, it should be noted that even this problem cannot be solved analytically for the optimal consumption strategy for a given income stream because of the presence of a liquidity constraint and the calculation must be performed using a numerical solution.

5.2.1 The User Interface, Instructions and the Trial Calculation Facility

Since the subjects were required to solve an analytically complex problem (if they used all the information available to them) and the experiment contained a complicated calculation facility, it was considered important to give the subjects every opportunity to learn and understand the rules of the game and the facilities provided by the experimental program. When subjects registered to take part in the experiment, they were given a set of instructions with examples of different strategies so that they could learn how to perform the experiment and think about the problem beforehand. At the start of the experiment, subjects were given a presentation about the experiment and a demonstration of the various facilities of the computer program. The presentation was based on the written instructions and the introduction was also repeated in the program. The importance of finding the best strategy was emphasised by means of examples. These examples were selected to emphasise the potential difference in possible strategies and the implications of stopping probabilities and

¹⁵ Unlike Hey and Dardanoni's experiment, a questionnaire was not given to the subjects at the end of the experiment. It was felt that subjects would not concentrate on such a questionnaire after an hour of the experiment itself and also, the subjects were not drawn from a random population so it would not be possible to try and draw general conclusions about the relationship between subjects' characteristics and their decision processes.

the income distribution, without giving direct hints about the optimal strategy¹⁶. In order to make the presentation as clear as possible and to ensure that different groups of subjects received the same information in the same manner, the presentation was recorded on an audio cassette. This was played to the subjects, coordinated with viewgraphs and the relevant screens from the computer program. The written instructions are in appendix 3. The subjects then went through a short practice run of the program, so that they could get used to the various screens and facilities and then went on to the main run of the program.

The program repeated the instructions (without the examples) and then proceeded to the first period. In each period, there was first a screen to say which period had been reached, with the subject responding to continue. This was included to avoid confusion about which period had been reached and to ensure that the subject realised that a new period was being started. Then there was a screen to emphasise the random and variable nature of the income stream. This had a rapidly changing number displayed, the numbers being those of the income distribution and finally showed the income for the period. The program next displayed an information screen, giving details of the current situation (see appendix 3 for details of the "current information screen"). At the bottom of this screen the subject was invited to try out some savings strategies. For the first 20 subjects, the subjects were given a choice of performing a trial or choosing their conversion, but most of these subjects ignored the trial so on the second day the program was modified to take them automatically to the trial facility when they had finished reading their current situation (see section 5.3).

5.2.1.1 The Trial Calculation Facility

Subjects could try out different strategies in each period of the savings problem. They selected the period in which their calculation would start and then highlighted the range of periods that they wished to examine on a 'range screen', illustrated in fig.8. This displayed the periods and scrolled forwards and backwards automatically as the subject varied the range. The subject chose an assumed initial wealth and an income stream and then entered

¹⁶ The patterns of saving/conversion in the examples were: spending all income as soon as it is received, saving all income and spending all the accumulated wealth in 1 period, having the same income and conversion paths over two different numbers of periods (see instructions in appendix 3 for the examples).

a conversion stream. The program displayed the resulting levels of wealth and the total number of points that the subject would have at the end of the range of periods used for the trial. The subject's actual level of wealth was displayed as a default value for the current period and values used in a trial could be retained for further trials if desired if the range of periods for the calculation was the same. The consequences of changes to a previous trial could then be immediately displayed, without having to reenter the values from the previous trial. It was possible to quit from any stage of the trial.

Figure 8 Range screen in the savings experiment

period	7	8	9	10	11	12	13	14	15	16	17	18	19
income	0	20	10	2	20	10	2	20	0	0	0	0	0
convert tokens	0	7	4	10	14	17	15	35	0	0	0	0	0
wealth	10.0	25.3	34.4	29.1	38.6	34.7	23.9	9.8	0.0	0.0	0.0	0.0	0.0

Input the number of tokens you wish to convert in
period 12 and press ENTER

Press the spacebar to keep the same value from previous trials
Press Q to quit

The range of time periods you highlighted was 8 to 14
You would have 46 points if the game ended in period 14
(not including any points you had before this range starts)
You would have 9.8 tokens remaining that you had not converted

When the subjects had finished their trials, the information screen was displayed again and the subjects chose how much to of their current wealth (wealth from the previous period + income) to convert. If the subjects tried to convert more than their current wealth, the computer gave an error message and they had to input a permissible value. They next had the opportunity to change their decision. The program then had a screen to emphasise the uncertainty of continuing to another period. The phrase 'checking to see if the experiment continues' was printed and underneath this a rapidly changing number, finally showing a number less than the stopping probability.

If the experiment continued, the program then moved to the next period and the procedure was repeated. After the last period, a screen was displayed showing the amount of points that had been earned. The subject then performed the payoff mechanism.

5.2.2 The Specification of the Savings Problem

The savings experiment ran in discrete time. Subjects started off with zero wealth in the first period and received a randomly drawn income in tokens. They then decided how much of their income to convert¹⁷ to points in the period, performing trial calculations if they wished. After they had input the number of tokens to be converted, the wealth at the beginning of the next period was calculated (the interest rate was 2% per period) and then the experiment proceeded to the next period. They were informed of their income in the new period and then the decision process was repeated until the end of the final period. Points from each period were added up at the end of the final period, for use in the payoff mechanism. This is different to Hey & Dardanoni (1988b), where subjects accumulated saved tokens, but only the tokens converted in the final period counted as points for the payoff. This meant that there was an increased incentive to spend all the available tokens compared to the current experiment where all conversion decisions contributed to the points total. Uncertainty was incorporated in two ways: the exact future income was not known and the subjects did not know how long the first stage of the experiment would last. They knew that their income would be one of three values and they knew the probability of each value occurring.¹⁸ A discrete income distribution was used in contrast to the continuous distribution in Hey & Dardanoni. This was because subjects who were not familiar with statistical theory would not understand the properties of a continuous distribution and most subjects would not bother to discover the details of a continuous distribution. Hey & Dardanoni found that very few of their subjects looked at more than the mean and the range of the income distribution.

¹⁷ The conversion function from tokens to points was a CARA function $\text{Points} = A(1 - e^{-B \cdot \text{tokens}})$ $A = 30$ for subjects 1-20; $A = 15$ for subjects 21-40. This was designed to give an expected payoff of approximately £15. $B = 0.03$ for all subjects. This value was chosen to give a reasonably curved function and thus significantly decreasing returns to reduce the likelihood that subjects would adopt boundary solutions.

¹⁸ The income distribution was: income 20 with probability 10%, income 10 with probability 80%, income 2 with probability 10%. A low income of 0 would give a symmetric and therefore simpler distribution, but it was considered that there should be a non-zero value for the low income so that the subjects still had a decision to make about income as well as previously accumulated wealth. A non-zero value would also generate more information for the econometric analysis.

The subjects were also informed that there was a chance of the experiment ending of 10% at the end of each period.

In fact, the number of periods and the income stream were read in from input files that had previously been generated at random. The range of lengths of life was restricted to between 8 and 12 periods, so that all subjects had the chance to play a reasonable number of periods and generate a useful amount of data, but did not have to think for so long that their concentration waned¹⁹. The slowest subject took 1 1/2 hours to complete the experiment. The income streams were also restricted so that there were no streams with a long series of low income, as this would greatly reduce the possibilities for different strategies, given that the low income was only 2 and that subjects could not borrow. The subjects knew with certainty the rate of interest on wealth.

A liquidity constraint was imposed - subjects could not borrow - to avoid the trivial solution that in the absence of a liquidity constraint the optimal strategy is to borrow an infinite amount, convert it all and die in debt. This would happen if borrowing were costless to the subjects. Within the experiment borrowing without any enforcement of repayment upon the savings problem ending would mean that people would be able to borrow without having to repay the loan. Enforcement of repayment at the completion of the savings problem would greatly complicate the problem and would require further controls. For example, subtracting borrowing from the points accumulated in the experiment would require a further control mechanism to ensure that people did not accidentally die in debt which they could not repay. Not allowing borrowing removed the need for these extra complications in what was already a complicated problem.

5.2.3 The Payoff Mechanism

Hey & Dardanoni (1988b) assumed that subjects had a CARA utility function and used two methods to find the parameter of a subject's utility function. However, neither method used was satisfactory. One relied on a question at the end of the experiment, before the payoff,

¹⁹ Hence the actual probability of the savings problem finishing was 0% for periods 1-7, 10% for periods 8-11 and 100% for period 12. However, some subjects may have guessed the true probability of the first stage finishing. See section 5.3 for further discussion.

in which the subjects valued a lottery between their winnings and another gamble between $\pounds(2 \cdot \text{winnings} - 2)$ and $\pounds 2$ (see Hey & Dardanoni (1988b) for details). This could give rise to possible regret and framing effects, since subjects already knew their payoff and were then faced with the possibility of losing their gains. Subjects might think about their regret at losing their gains if they lost and their frame of reference at the end would include their gains in the experiment which might now be lost, as opposed to the beginning of the experiment when they had no gains and would increase their wealth with certainty. These effects might change the subjects' attitude to risk, compared with their preferences in the savings problem itself and the elicited attitude to risk would then be incorrect for the savings decisions. Also, for the subjects to behave consistently with their preferences they would have to understand the theory of optimisation. Most subjects would not have this understanding. The second was a questionnaire filled in at the end of the experiment which did not affect payment which included a question about risk attitude (see Hey & Dardanoni (1988b) for details). The problem with a questionnaire is that some people say what they think the experimenter wants to hear and subjects may modify their strategy to be consistent with what they say (if they know that there will be a questionnaire) or will modify what they say to be consistent with what they did. This ex-post justification of actions is a commonly observed psychological phenomenon. Of course, many economists would argue that some people will lie without a (financial) incentive to tell the truth.

In order to avoid the need to measure utility functions so that people's optimal strategies can be calculated, the binary lottery mechanism was used to control the form of the utility function in this experiment. Subjects' incomes were denominated in tokens. It was necessary to have a utility function with decreasing returns to scale because a linear function means that the optimal strategy is to save everything or convert everything. To simulate risk averse consumption, tokens were converted into points according to a CARA function. Points were calculated in each period and at the end the points from all the periods were added up to make a total. Subjects then played a lottery, dependent in a linear way on their points total. They threw a dice and if the dice score was lower than their points total they won a large prize; if it was higher they won a small prize. Roth and Malouf (1979) shows that (assuming that subjects are von Neumann - Morgenstern utility maximisers over monetary payoffs) this procedure makes utility linear and monotonically increasing in points i.e. whatever their utility function, the subject should optimally try to get as many points as

possible. The idea that people will try and do as well as they can by trying to get as many points as possible is also realistic since it is a goal that people will understand.

5.3 Problems with the Experimental Design

In the first two groups of subjects, it was found that the trials facility was not used by the majority of the subjects; only 9 out of 20 subjects performed trials. Since the main aim of the experiment was to gather information about the subjects' trials strategies, it was necessary to ensure that all subjects performed a trial. This was achieved by modifying the software so that after their current financial situation was presented to the subjects, they were given the instruction 'press T to perform a trial' instead of being given a choice between performing a trial and choosing an amount to convert. Thus they had to enter the trial facility to proceed further in the experiment. Even though the subjects could still quit out of the trial without performing any calculations, all but one of the subsequent subjects did perform at least one trial, compared to less than half the subjects when the trial was obviously optional.

Subjects were not told how long the experiment would take as this would give them information about how many periods there would be. However, imposing a restricted stopping probability with a limited number of periods caused problems, because the results indicated that people assumed that the experiment would last at least a few periods with probability 1 and would not go on for 'too long' i.e. they actually guessed something close to the true stopping probability. Although they were told that the stopping probability was constant, they guessed correctly or assumed that the number of periods would fall within a limited range that they considered reasonable. Such a range would be of limited length and would have a minimum of at least a few periods before the savings problem finished, as it would not be useful to have an experiment about life-cycle saving behaviour just over one or two periods. This could have been reinforced by seeing other people play the game for some time and then leave, as the subjects started the experiment together in groups of 10 subjects. If this is true, the optimal strategy is changed and the ex-ante optimal strategy for a constant stopping probability is no longer the best that subjects can do (on average). This effect could be reduced by starting subjects in small groups and having examples with only 1 or 2 periods. This complicates the analysis of subjects' performance, because it is now

unclear what information they were using in the decision process. For this reason, subjects performance was assessed against the ex-ante optimal strategy (given the information in the instructions) and against the ex-post optimal strategy (assuming the subject knew the actual income stream and stopping period).

The fact that subjects appeared to ignore what they were told about the stopping probability raises the question of how such information should be presented. The successful guessing of the stopping probability by the subjects can be regarded as a common framing effect, as in Schelling (1960). Both the experimenter and the subjects had the same opinion about what was a reasonable length for the experiment, independent of the theoretical distribution. People also find it difficult to accept the implication of a constant stopping probability; that in each period the expected number of periods still to be played is always the same. Their understanding of a discrete income distribution is better, possibly because there are only three distinct states of the world to consider.

The parameters of the experiment (apart from the income levels) were chosen to provide realistic numbers so that people would find it easier to make 'realistic' decisions based on their experience of saving and consuming in: a 2% (real) rate of interest, a moderate chance of the savings problem ending each period and a CARA function with significant curvature for wealth levels of 30+. This resulted in the payoff function being quite flat and so there was no great penalty for being away from the optimal path. The strategy of spending all income as soon as it was received resulted in a points total virtually the same as the ex-ante optimal strategy and only 2% worse than the ex-post optimal strategy (see section 5.4.2.1 for definitions of ex-ante and ex-post optimal strategies). Another problem is that people often think in nominal terms, so a rate of 2% would appear small. Trials would show wealth only increasing by very small numbers for typical values of wealth (a wealth of less than 20 was assumed in the majority of cases, giving an increase of less than 1.0 token). Although this is a realistic interest rate, the numbers involved in real life decisions are usually of at least two orders of magnitude greater, making the return seem more significant. Also, an interest rate of 2% was small enough that a large proportion of subjects (10 out of 36) acted as if they set it equal to zero as an approximation and did not save at all. However, there was still a wide variation in subjects' performance (see fig.9) and it is considered that even with a limited penalty for small deviations from the optimum, most people will want

to do as well as they can and are prepared to think hard about the problem.

It was found that with the first two groups, a large proportion of subjects seemed to accumulate a lot of points without thinking for a long time about the problem, thus giving them a good chance of winning the high prize - subsequent analysis showed that many subjects came close to or performed better than the ex-ante optimum - (see discussion in the results section), so for the second two groups, the CARA conversion function was halved. A calibration of the experiment was performed by using these parameters to calculate the ex-ante optimal strategy; it was found that the number of points gained using the optimal strategy for the initial parameters was 93, but this was only correctly calculated after the experiment had been completed.

5.4 Results

This section describes and analyses the results of the experiment. Section 5.4.1 looks at the trials results, identifying the different trials strategies used and considering the evidence for the presence of backward induction calculations and Pemberton type calculations. The patterns of conversion in the trials are also examined. Section 5.4.2 analyses the actual savings/conversion choices made, comparing subjects performance relative to the ex-ante and ex-post optimal strategies, describing the actual strategies employed and finally reporting the results of a panel regression analysis.

5.4.1 Trials Strategies²⁰

There are two sets of results from the experiment; the trials data and the actual conversion/savings choices. The trials data were analysed first, so that subjects could be grouped by the different trials strategies that they used. Given the large difference between trials carried out by subjects 1-20 (who had to choose to enter the trials facility) and those who were automatically put in the trials facility (subjects 21-40), it is necessary to distinguish between the two sets of trials results. It is also important to consider learning

²⁰ Tables of the ranges of trials performed are at appendix 4 and descriptions summarising the trials behaviour and actual savings decisions are at appendix 5.

effects in trials. The trials behaviour of most subjects had an element of getting a feel for the magnitudes involved, as well as seeing the effects of different strategies, so it is to be expected that there will often be more trials early on in the experiment. Trials performed by subjects 21-40 included a large number of one-period trials. These subjects had a different conversion function to subjects 1-20, but were accidentally given the conversion schedule for the original conversion function. This mistake was noticed quite quickly by a subject who was performing trials to check the graph and the rest of the subjects in that group and the last group of 10 subjects were informed of the error. Therefore, they did not have a graph or table of the correct conversion function and had to use the trial function to calculate the amount of points for a given amount of tokens converted. Hence these one period trials were not trials of a savings strategy since they involve no consideration of the future, but were mainly performed because the subject was already in the trials facility and wanted to know how many points they would get for a given amount converted. Consequently, these single period trials are discounted in the trials analysis.

Of the 28 meaningful trials output files (other files had either all zeros or only one-period trials) 24 subjects started all their trials at the current (21 subjects) or the next period (1 subjects) or used a combination of these (2 subjects) i.e. a large majority of subjects looked ahead from the present period continuously into the future and did not project themselves forward to making a decision at some future time. Subjects automatically put into the trial option had a mean range of trial periods 3.49 with standard deviation 2.30. Those subjects who were not automatically put in trial option (1-20) had a mean range 5.54 with standard deviation 2.72. The difference in means is to be expected, as people who deliberately choose to perform a trial are probably more forward looking than the average subject, while people who don't bother to perform any trials will be less forward looking but do not show up in the data. Thus the mean for subjects 1-20 is biased upwards relative to the mean for the whole of this sample. The most important implication of these statistics is that subjects looked forward over a limited length of time, compared to the expected length of 10 periods.

5.4.1.1 Taxonomy of Trials Strategies

Perhaps the most effective trials strategy, given patience on the part of the subject, would

be to perform trials over a range of 15 to 20 periods, putting in the distribution of income i.e. including a few 2 and 20 incomes and trying a series of different savings patterns and choosing the pattern that gave the best expected payoff, allowing for the payoff for various finishing periods. No subjects did this. It is possible to identify 3 types of strategy with regard to the ranges of trials performed and the periods in which trials are performed; a learning strategy, a rolling strategy and a fixed end point strategy. See table 2 for the distribution of the various strategies.

Table 2. Distribution of trials strategies in experiment at EXEC

The distribution of trials strategies was as follows:

Of subjects 1-20: 5 learning, 1 learning and rolling, 1 fixed endpoint, 2 of 1 trial carried out in first period

Of subjects 21-40: 4 learning, 1 rolling, 10 learning and rolling, 1 fixed endpoint, 1 learning with fixed endpoint, 2 fixed endpoint and rolling

Overall proportions: 39% rolling and learning, 32% learning, 7% fixed end point, 7% fixed end point and rolling, 3.5% rolling, 3.5% learning with a fixed end point, 7% other.

The 'learning' trials strategy is a strategy in which several trials were performed in one or two consecutive periods. The subject was trying out different patterns of saving/conversion to get a feel for the numbers involved and to identify the results of various different strategies. Both static and dynamic aspects were considered; the tokens converted in a particular period were varied and patterns of conversion/saving over a number of periods were varied. Defining a learning trials strategy as a pattern of trials in which there were at least two trials performed in one of the first two periods of the experiment, 5 of subjects 1-20 and 4 of subjects 21-40 displayed learning trials strategies. The vast majority of these trials have ranges between 4 and 10 periods; with a mean range of 5.06 and standard deviation 3.07 periods for pure learning strategies. Two examples of this strategy are subjects 03 and 30. Subject 03 performed four trials in the first period, all looking forwards as far as period 8 and no trials subsequently. Subject 30 performed 11 trials in the first

period, 4 looking as far forwards as period 6 and 7 looking as far as period 10, also with no trials subsequently.

This learning behaviour usually took place in periods 1 and 2 of the experiment, with a few subjects learning in period 3 as well. Thereafter there were very few trials; the subject had decided on their strategy. It should be noted that these subjects, who took a definitive look forward at the beginning of the experiment, did not look that far ahead (5.06 periods on average), given that the expected length of the experiment at each period is 10 periods.

In contrast to the first strategy, some subjects employed a 'rolling' or continuous replanning strategy. They performed trials all the way through the experiment, usually with a relatively small number of trials in any one period, looking only a few periods ahead. The pattern of trials might be intermittent or continuous; e.g. subject 23 used a combination of rolling and learning, with trials in periods 1, 2, 4, 6, 9, 10, 12 and subject 34 performed three trials in every period. There was a mean range of 2.79 and standard deviation of 1.36 periods for the pure rolling strategy. Only subject 35 used a pure rolling strategy but among subjects 21-40 a combination of a rolling strategy with learning was the most common strategy, being used by 10 of these subjects. Subject 4 also used this combination of strategies and subjects 21 and 36 used a combination of a fixed end point for part of the experiment with a rolling strategy.

Why might people use such a strategy? The advantage of this strategy is that it dramatically simplifies the problem. The number of periods to be considered at any one time is greatly reduced, while allowing for an indefinite number of periods. If only a few periods ahead are considered, it is a reasonable approximation to assume that the experiment will not end over the periods being considered (given the stopping probability of 0.1) and hence it is possible to remove one source of uncertainty. The implications of this type of strategy are considered in chapter 3 and its performance compared to the optimum explored in the simulation of chapter 5, where it can be seen that this strategy does not involve a great penalty compared to the ex-ante optimal strategy.

It could be argued that the common use of the rolling strategy was an artefact of the experimental arrangement. This argument is supported by the behaviour of the first 20

subjects, who mostly only performed a few trials at the beginning, as they had to choose to perform a trial. Subjects 21-40 were automatically put into the trial option and had to take a positive decision to quit out of the trial to avoid performing any trials. Therefore, the 'default' action would be to perform a trial and then proceed to choose the amount to be converted. This would appear to be a rolling strategy as subjects would perform a single trial in each period. This is only true for subject 35, however. All other subjects that exhibited the characteristics of a rolling strategy performed more than one trial in a single period, or showed a fixed end point strategy as well. The implication of the common use of these combinations of strategies is that even if people were performing trials because this was the most obvious thing to do, they were taking these trials seriously, thinking about how they performed the trials and what range of periods they should consider. They were not simply performing a single trial, just because they were automatically put into the trial screens and had to get through the trial option in the course of going on to make their conversion choices.

Some subjects employed a 'fixed end point' strategy, where the subject chose a single period in the future as the end point for a series of trials that they performed over several periods. This would be one way of enabling a limited backward induction calculation to be performed (since only one endpoint is ever used a single length of the experiment is being assumed), as the endpoint of the experiment is assumed fixed for the purposes of the trials. Subjects 12 and 29 that had such a strategy chose the end periods 5 & 6 respectively. Subjects 21 and 36, who combined a fixed end point early in the experiment with a rolling strategy later on, had period 4 as the end point and subject 28 combined a fixed endpoint with a learning strategy with an end point at period 10. However, the use of this type of strategy was relatively uncommon; it was used only by 5 of the 28 subjects for which there is trials data.

Many subjects (half of those who performed meaningful trials) combined two of the above strategies. By far the most common combination and the most common strategy overall was combined rolling and learning, used by 11 subjects. These trials had a mean range of 3.31 periods with a standard deviation of 1.52. This would enable subjects to perform simple trials, while trying out the implications of the latest situation. It is possible that the combination of a rolling strategy combined with a fixed end point strategy would reflect someone who performed a series of trials early on over a limited time span and then realised

that the experiment would last longer than they had assumed.

5.4.1.2 Backward Induction and the Pemberton Strategy

The patterns of conversion in the trials can be examined for evidence of backward induction as a method of solving the savings problem. There are only three subjects who performed fixed end point trials without combining this with a rolling strategy. Of these, subject 12 performed trials in which all income was spent in all periods, subjects 28 performed trials in which all income was spent in most periods and subject 29 performed trials over three periods, of which the last trial over 2 periods converted all income in both periods. The large number of periods in which all income was spent indicates that these subjects were not following an optimal strategy for a fixed end point. Another possible strategy is that of Pemberton (1993). This would be characterised by a conversion strategy in which there was a constant conversion in all future periods being considered in the trial. There were no trials for which this was true, except for trials in which all income was converted which would not be true for a Pemberton-type strategy with a non-zero interest rate and a stopping probability of 0.1.

Hence it can be seen that there were no subjects that used the backward induction method of calculation or the Pemberton calculation to solve the savings problem.

5.4.1.3 Conversion Patterns in the Trials

When analysing the patterns of conversion tried in the trials, it is important to remember that many trials had at least an element of learning in them. Consequently, there is a lot of noise in these data; there will be many trials performed which do not reflect the subject's final decision and the way in which they reach that decision will be both obscure and variable between subjects given the limited nature of the data available about subjects' trials processes. However, the trials can shed light on the range of conversion patterns that were considered and the relationship between trials and actual strategies. 16 of the 28 subjects performed trials in which at least one was obviously similar to their actual conversion for that period.

Many subjects simplified the problem. Besides restricting the range of periods considered, 11 subjects assumed for most or all of their trials that the income would be constant at 10. 11 subjects²¹ tried a strategy of converting 0 for several periods than converting all or most of $y+w$, possibly because this strategy was illustrated in one of the examples. Subjects 21, 31 and 32 actually demonstrated this pattern over 3 or 4 periods during the experiment, but of these three subjects only subjects 31 had performed such a trial; subjects 21 and 32 were probably thinking in a different way. Trials by most of the subjects included a pattern of building up a stock of wealth over a number of periods and then reducing the wealth stock to 0 in the last period of the trial. The need to define an end point changes the structure of the problem, giving rise to this end point effect. However, it is possible that subjects used the trials to work out a strategy and then used the first period of their trials as their actual conversion. For people who were performing trials throughout the experiment, this implies constant replanning which would be consistent with the rolling strategy discussed above.

5.4.2 Analysis of Savings and Conversion Choices

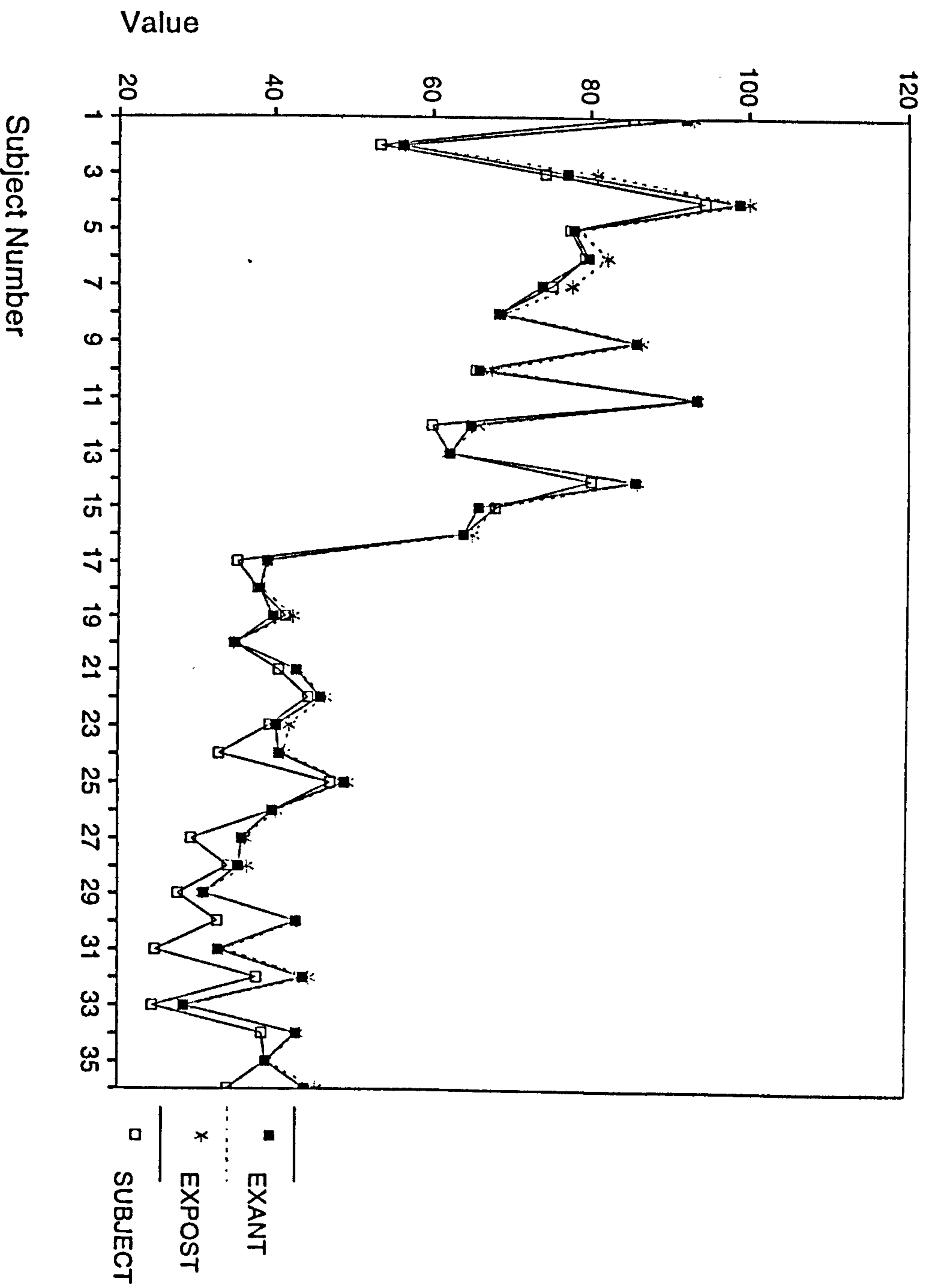
5.4.2.1 Performance relative to Optimal Strategies

There are two approaches to the calculation of the optimal strategy: the first is the ex-ante approach, where the optimum conversion for a given level of current wealth (wealth at beginning of period + income) maximises the expected value function, given the CARA conversion function, the stopping probability, the interest rate and the income distribution. The ex-post optimum is the optimal conversion strategy knowing the actual income path (and hence the length of the experiment). A graph of subjects' performance is shown at fig.9. It can be seen that some subjects performed better than the ex-ante optimum, although it should be remembered that the ex-ante optimum is only the best strategy **on average**. In fact, subjects did very nearly as well as the ex-ante optimum overall. The low interest rate of 2% per period in combination with the other parameters leads to a very flat distribution²², but given the wide variety of conversion strategies used, this is not a sufficient explanation. Another part of the explanation for this surprisingly good performance can be found by

²¹ subjects 3, 18, 23, 24, 27, 30, 31, 34, 36, 38, 40

²² See ch.7 for an analysis of the performance of different savings strategies given the income streams used in this experiment.

Figure 9 Performance of subjects in Experiment at EXEC
subjects 1 to 16 group 1, 17 to 36 group 2 (CARA multiplier halved)



considering the fact that subjects were not told the true stopping probability (i.e. 0 up to period 8, 0.1 period 8-12, 1 period 13). It is possible that the subjects subconsciously assumed that the experiment would continue for several periods with certainty but would not last too long i.e. that it would stop with certainty after 10-15 periods. This view is supported by the trials data, where trials had a mean range of 5.54 for subjects 1-20 and 3.49 for subjects 21-40. Thus the subjects were actually assuming a distribution quite close to the actual distribution, rather than the constant stopping probability that was part of the instructions. Given this assumption, it is then possible to outperform the ex-ante optimum which is based on an inaccurate stopping distribution. If the true distribution is guessed, savings can be greater than the ex-ante optimum in periods 1-7 (if income is such as to generate a high enough wealth for the optimal strategy to contain some saving), as it is believed that the experiment will not finish and wealth can be run down gradually over periods 8-12 to 0 at the end of period 12 since it is believed that the experiment will finish at the end of period 12. Thus less wealth will be left when the experiment finishes on average than with the ex-ante optimal strategy and interest from saving in the earlier periods will be higher.

5.4.2.2 Conversion Strategies

There was a wide variation in the actual savings patterns employed, from spending all income (10 subjects) to saving a large proportion of income and building up a large stock of wealth (e.g. subject 40 built up a stock of 35 by the end of the experiment; this was possibly someone who felt that having a stock of savings was good in itself and thought that the chance of stopping was small or failed to realise that having a large stock of wealth when the experiment finished was wasteful. There were also conversion patterns in which wealth cycles. Overall, it can be said that the amount of saving was much higher than the optimal strategy, given the interest rate of only 2% and the liquidity constraint. The optimal pattern (see fig.17 for a plot of optimal consumption for a given level of accumulated wealth + income for similar parameter values) would be to convert most or all of the income in most circumstances faced by the subjects since an income of 2 or 10 is not big enough to build up a large stock of wealth that is worth keeping. One general conclusion is that it is not helpful to try and approximate people's decision rules by one single rule because the wide variation in behaviour would mean that any single rule would be very inaccurate. It is more

sensible to try and define a few different rules, which will improve the accuracy of predictions of behaviour while also greatly reducing the different strategies that have to be allowed for when analysing peoples' behaviour.

5.4.2.3 Quantitative Analysis

The object of the quantitative analysis is to identify which conversion rules subjects actually used. It takes the form of a panel regression using LIMDEP. Firstly, however, there is one strategy which can easily be identified by inspection of the data: where the subject spends all income as soon as it is received. This strategy was used by 10 of the 36 subjects for which there is savings data²³. These subjects were not included in the analysis. The methodology used for this analysis was the general to specific method. All possible relevant variables were initially included, using plots of the data to look for breaks or other general patterns in the data to suggest relevant functional forms etc. Then insignificant variables were successively eliminated until the significant variables could be identified and the reduced number of variables gave stronger tests. This methodology has the advantage that it does not leave out variables that may be significant, thus reducing the possibility of misspecification and biased estimators.

The analysis checked for the possibility that subjects followed something close to the optimal ex-ante strategy. The optimal ex-ante strategy for similar parameter values is plotted in fig.17. Given that the rate of saving increases with wealth, the shape of the curve can be reasonably approximated by a log function so the log combined income and wealth should be considered. This is only approximate, as the optimal strategy is $c=w+y$ for $w+y$ up to 18. It is also possible that conversion is a linear function of wealth and income. Another possibility is that people build up a stock of wealth, implying that the rate of conversion increases once a certain level of wealth (considered adequate) has been reached. This can be captured by a wealth slope dummy (which would also allow for the change in the slope of the optimal consumption at $y+w=18$) but since people started out with zero wealth, there may also be a correlation of saving with time, particularly in the first few periods of the experiment. This implies that time may be a significant determinant of conversion and that

²³ 4 results files were accidentally lost

there should be a time dummy. However, scattergrams of consumption against wealth, income + wealth and consumption against did not show any significant breaks in the data, so these dummies are not included in the regression (to reduce the number of variables, hence increasing the power of the tests). Lagged consumption should also be included, which would indicate habitual behaviour. Dummies for the different trials strategies can be tried and as a check, a dummy for subjects 1-20. These are alternatives to individual subject dummies. It is also possible that responses to a large income or wealth could be delayed, so lagged wealth and income should also be included.

Alternatively, there could be a rule of thumb based on income so that an income dummy might be incorporated (e.g. if $y=2$ spend all income and if $y=20$ save a large proportion of income) However, since a scattergram of conversion against income shows considerable variation in conversion for each of the three incomes, an income dummy would not appear to yield useful information.

The regression could be run with wealth as the dependent variable. This would be useful assuming that subjects calculate the wealth that they want, given their current situation. they would then be thinking in terms of 'what wealth stock do I want?' rather than 'how much of my resources do I want to convert?'. However, this calculated wealth has a direct negative relationship with consumption, so subjects probably switch between the two and think of both variables simultaneously. The idea of subjects choosing consumption is a more direct approach to the causal relationships (and is easier to present in a meaningful way).

So, the initial regression is:

$$c_t = K_{11}c_{t-1} + K_{21}t + K_{31}y_t + K_{41}w_t + K_{51}\ln(y_t+w_t+1) + K_{61}y_{t-1} + K_{71}w_{t-1} + \text{subject dummies} + \epsilon_t$$

where ϵ_t is the usual (normal i.i.d) error term and there is a dummy for each subject provided automatically in LIMDEP.

y_t+w_t+1 was used instead of w_t to ensure that the log function started at 0 for $y+w=0$.

This was compared with:

$$c_t = K_{12}c_{t-1} + K_{22}t + K_{32}y_t + K_{42}w_t + K_{52}\ln(y_t+w_t+1) + K_{62}y_{t-1} + K_{72}w_{t-1} \\ + K_{82}dumr + K_{92}dum20 + \epsilon_t$$

dumr is a dummy indicating the use of a rolling strategy in trials and dum20 is a dummy for the first 20 subjects. A much greater part of the variation can be explained by including individual dummies for each subject, but this is not very informative: it only shows that each individual subject had a different strategy (remembering that people who converted all income are excluded). However, excluding these dummies that are significant could introduce misspecification. An F-test of the set of restrictions of the final regression compared to the same regression with individual subject dummies was significant at the 1% level, showing that it is necessary to include the subject dummies.

It was found that K_{11} , the coefficient of c_{t-1} and K_{51} , the coefficient of $\ln(y_t+w_t+1)$ were insignificant. Dropping these, $\ln y_t$ is still insignificant.

Thus the final regression was:

$c_t =$	0.249t	+	0.755 y_t	+	0.393 w_t	+	0.200 y_{t-1}	-	0.0566 w_{t-1}	
										+ subject dummies + ϵ_t
R^2	(2.55)		(10.39)		(7.40)		(2.68)		(-1.066)	
prob. that coefficient = 0	1.1%		≈0		≈0		0.7%		28.7%	

The coefficients of y_t and w_t are different, showing that subjects do not base their decisions simply on their total current available resources even though the information screen told the subjects that they could convert up to the total wealth $y+w$. This is in accordance with results from previous empirical studies showing that consumption is more responsive to income innovations than to permanent income (see e.g. Deaton (1992), Bosworth, Burtless & Sabelhaus (1991) p226). Overall, a high proportion of income is converted, as would be expected with a rate of interest of 2%. The lower coefficient for wealth compared with income indicates that in general a stock of wealth is being maintained, although the coefficient is still positive as a higher level of wealth increases the feasible set of savings/conversion pairs. Remember, however, that this excludes the 10 subjects who did

not save at all. The coefficient of w_{t-1} is so small as to be indistinguishable from 0, but that of y_{t-1} is significant. This can be seen in the savings choices, where the response to a high income of 20 is often to increase conversion over 2 periods, the increase in the next period giving rise to a lagged effect of the high income. Conversion also increases as time elapses in the experiment. In the first few periods there is an increase in wealth as the initial wealth is 0 and most people save in order to build up a wealth stock which restricts conversion in these periods. In fact, some subjects increase their wealth stock throughout the experiment, steadily increasing the feasible set of savings and conversion. This could also account for the dependence on t , but it should be noted that if this were the only reason, there would be multicollinearity between t and w . This is not evident; both variables are significant. There is some evidence of end effects. The pattern of reducing wealth to 0 in the final period was very common in the trials that were performed and some subjects also reduced wealth at the end of the experiment. Subjects 02, 03, 16, 21, 23, 31, 32 all reduced their wealth markedly and maintained the lower level for between 1 and 3 periods to the end of the experiment. This implies an increased rate of conversion in the last few periods which will also lead to a positive dependence on t .

Two rules seem to be common:

1. Saving is considered to be good, whatever the rates of interest and the effective rate of discount.
2. Smoothing income is considered to be good: the small income of 2 is not important, but an income of 20 makes a significant difference and the increased wealth is often converted over two periods e.g. subjects 06 and 11 who both converted all income otherwise, spread the extra conversion from an income of 20 over the current period and the next.

5.5 Summary and Conclusions

The subjects performed trials that could be categorised into three types: a learning strategy where there were several trials to find out about the relative performance of different strategies, a rolling strategy where there were a few trials in each period of the experiment looking between 2 and 4 periods ahead and a fixed end point strategy where a single end point was used for trials in several consecutive periods of the experiment.

Analysis of the trials strategies shows that there were no trials conducted which were consistent with the use of backward induction or the simplified strategy proposed by Pemberton (1993).

There was a wide variation in subjects' performance compared to the optimal strategy, with some subjects performing better than the theoretical optimum ex-ante strategy on average. This can be explained by the fact that the subjects guessed or assumed that the stopping probability was not constant and made assumptions that were closer to the actual probability (the experiment lasted between 8 and 12 periods) than a constant 10% chance of finishing.

Regression analysis of the actual conversion strategies shows that there is a strong dependence of conversion on wealth and income, but the coefficients are significantly different implying that subjects do not simply add up their wealth and income to determine their current resources. The coefficient for income is significantly higher than that of wealth, suggesting that subjects condition their conversion fairly strongly on the income they receive. This finding agrees with the econometric evidence that consumption is more sensitive to current income than to wealth.

6 The Savings Experiment in Holland

6.1 Introduction

This chapter describes a savings experiment carried out in September 1995 at CentER, University of Tilburg, Holland. The experiment in Holland was a development with minor changes of the savings experiment at the University of York. The same experimental program was used with subjects having to use the trial facility, but there were several different treatments, enabling different aspects of the savings decision to be emphasised and some comparative statics analysis to be undertaken. The method of analysis was also similar, being divided again into the (qualitative) analysis of the decision processes used by the subjects and the (quantitative) econometric analysis of the savings decisions. In addition, scatter plots of average values against time provided some useful insights. Section 2 describes the experiment, section 3 contains the analysis of the trials data, section 4 the analysis of the savings and consumption data and section 5 draws conclusions and compares the results of the analyses to the experiment in York.

6.2 Experimental method

6.2.1 Organisation and changes from the experiment at EXEC

The experiment was undertaken at the University of Tilburg. While the structure of the experiment was the same as the experiment at EXEC (c.f. section 5.2 for a description of the experiment), some minor changes were made to the running and design of the experiment. There were several organisational issues to be addressed. Firstly, the subjects would be Dutch and the interfaces were in English. It was decided not to translate the instructions and presentation into Dutch, because this would have required rather more time and the experimenter did not speak Dutch. Also, Dutch students and academics speak excellent English. However, this did mean that the subjects would be receiving instructions in a foreign language. Hence it was important to make the explanation of the experiment as clear as possible. It was therefore decided to present the instructions in the form of a video, as well as retaining the 2 practice periods.

A further organisational change was that the subjects were divided into groups of around five. If subjects are members of a large group they can easily get information about other participants' problems, in particular how long they can expect the savings game to last. To avoid this problem, Hey & Dardanoni (1988a, 1988b) had individual starting times for each subject. However since each subject had to watch a video for 15 minutes this was not practical for a large number of subjects. Another complication was that Tilburg does not have a dedicated laboratory; it was necessary to use the terminal rooms of the computer department. This meant that it was not possible to run the experiment continuously. The time was limited to 3-5 hours per day and each subject required at least 1 hour to complete the experiment, so it was necessary to process subjects in groups to reduce the time required to perform the experiment. Since the computing centre's computer network was used, detailed knowledge of the operation of the network was not available. Therefore, it was necessary to treat each terminal as an individual PC, which required that the program had to be individually loaded and the results files individually stored onto floppy discs. Just before the experiment started, it was decided that it would be useful to store the results of the trials in the practice sessions, which meant that each practice results file had to be individually saved onto floppy disc, as well as the main session. Thus for each group of subjects, the experimenter had to start and stop the video, start the practice run, start the main run, play the lottery with each subject when they had finished and answer any questions. Given a group size of 5, a group each half hour was the highest frequency that the experimenter could deal with.

The experiment was run over 6 days and there were 124 participants. Subjects were recruited mainly from the student population, both from economics courses and by talking to students in the cafeteria at lunchtime. Some of the administrative and academic staff of CentER also took part.

6.2.2 Treatments

At CentER, the experiment was run with many more subjects than at EXEC (124 vs. 40). This enabled some of the parameters of the problem to be varied and some different situations to be examined. In order to effectively perform an econometric analysis, it was judged necessary to have groups of about 20 subjects for each parameter variation or

treatment. This number allowed for a wide variation in the income streams and length of the savings problem, while still providing enough data points for the econometric tests to have reasonable power. The actual number in each group was determined by the number of subjects in the different sessions, as the subjects in a group had to be given the same instructions. Since there was a wide variation in the different treatments, a control treatment was used as a baseline for comparison. This control had more subjects and a wider range of income streams to improve the chance of the control group being representative. On the advice of CREED, Amsterdam, the payments were calculated so that the subjects received 20 DFL per hour (their normal expected rate of pay for students performing experiments). The high prize was fixed at 25 DFL and the low prize at 4 DFL (the same numbers as at EXEC). However, a subject in the first group pointed out that 4 DFL for an hour's thinking was very mean, so after the first 10 subjects the low prize was increased to 10 DFL. This maintained a significant differential between the high and the low prizes, so that the incentive mechanism still functioned correctly i.e. there was a perceived advantage in getting as many points as possible to maximise the chance of getting the high prize. Since the incentive mechanism was designed to induce risk neutral behaviour, the change in payments would not change the optimal behaviour and would probably not affect the actual behaviour. The treatments were as follows:

Treatment 1.

The control treatment. There were 34 subjects with a random income stream (10% chance of an income of 2, 80% chance of an income of 10, 10% chance of an income of 20) and the number of periods was taken at random from a distribution between 3 and 25 periods with a 10% stopping probability each period. The interest rate was 10% per period.

For the rest of the treatments, the range of the number of periods was restricted to 5 to 15, to ensure that each subject generated a reasonable amount of data, but did not take too long to complete the experiment (which would have led to organisational problems, given the limited availability of the computer rooms). Also, income streams with more than 50% low income were not used, to ensure that all subjects had sufficient income to have to make significant savings/consumption choices.

Treatment 2.

The 'Backward Induction' treatment. This treatment had 17²⁴ subjects, who knew that the experiment would last for 15 periods for sure. The aim of this treatment was to investigate whether subjects would perform trials over the 15 periods and thus perform something similar to a backward induction calculation.

Treatment 3.

The 'Pension' treatment. This treatment had 18 subjects, who knew that from period 10 onwards, their income would be 2 for sure. The length of the experiment was still randomly varied between 5 and 15 periods. The idea behind this treatment was to see if the knowledge that they would face a low income stream in the future would affect savings behaviour.

The rest of the treatments were variations for the purpose of comparative statics analysis.

Treatment 4.

Subjects were endowed with an initial wealth of 10. There were 19 subjects in this treatment.

Treatment 5.

Subjects were endowed with an initial wealth of 20. There were 18 subjects in this treatment.

Treatment 6.

²⁴ The numbers in the groups varied because the individual sessions were not all fully booked and it was not practicable to have different subjects doing different treatments in the same session, because of the changes in the instructions.

The interest rate was increased to 20% per period. There were 18 subjects in this treatment.

6.3 Qualitative Analysis of the Trials Data

The objective of the qualitative analysis was to examine the trials that subjects performed and to draw conclusions about some basic characteristics of the cognitive processes being employed, in particular with regards to the timing in the decision process. The method of analysis used the results of the EXEC experiment; various possible strategies were defined, based on the strategies identified by inspection of the trials data in the EXEC experiment. Due to the greater number of subjects, it was not practicable to manually examine each subjects' trials. The incidence of these strategies was determined by a computerised search of the data and descriptive statistics calculated for the number of periods over which trials were performed (the 'range' of a trial). The trials performed are summarised in table 3.

As in the EXEC experiment, trials which only last for a single period are ignored. In the EXEC experiment, the reason for this was that the graph of points gained against tokens converted was wrong, so the subjects had to use the trial facility to find the true relationship. However, even in the experiment in Holland, some subjects performed series of trials that only lasted for a single period. These trials do not reflect the intertemporal nature of the decision being taken i.e. the subjects would not get a useful solution from such trials. Since virtually all subjects understood that there was a high probability of the experiment lasting for more than one period, they were probably doing something other than trying out a strategy. The obvious explanation is that they were finding out the numbers generated by the program or familiarising themselves with the program. If this is the case, the inclusion of these one-period trials in the analysis would bias the results for the trial ranges downwards. Since the object of the experiment is to show that people perform calculations over a shorter range of time periods than is commonly assumed, it is better to be conservative in the analysis and remove the possible downward bias; the results are then robust to this possibility.

The trials data from all groups was amalgamated, as all groups apart from the backward

induction group faced an uncertain length of experiment. The results for the 'backward induction' treatment are analysed independently, as well as being included in the analysis of all subjects because the introduction of a known end point gives an obvious target for the range of the trials and so the incentives to perform trials are different to the other groups.

6.3.1 Numbers of trials performed

As discussed in section 6.2.3, many subjects had difficulty in distinguishing between the practice run and the run for real. Hence many subjects performed trials in the practice periods in which they developed a strategy (i.e. performed the learning function). Some subjects did this deliberately; the trial facility was the same in both the practice run and the run for real for all groups apart from group 6 which had a higher rate of interest (the trial facility allowed the initial wealth to be adjusted, except in period 1, which made trials for the groups with an initial endowment more difficult: they had to choose a later start period). This means that there was a large proportion of trials performed in the practice period and it is therefore informative to look at the ranges and number of trials performed in the practice periods. This can be seen in table 3. There were 559 trials in the practice periods and 1066 trials in the run for real. Also, the average number of trials per subject was 3.07 in the first period of the practice run, compared to 2.30 in the first period of the run for real. Given that a large proportion of subjects performed trials all the way through the experiment, this demonstrates that some subjects performed most or all of their trials during the practice run.

A plot of the average number of trials performed per subject in each period and the number of subjects in each period is shown at figure 10 (note that after period 15, only a few members of the control group were still in the experiment, so these values cannot really be regarded as an average over the whole population of subjects). There is a strong downward trend, both in the practice run and the run for real, suggesting that there is a strong element of learning about the experiment and possible strategies. As subjects became more used to the experiment or developed their strategies, they often stopped performing trials. However, there was a large proportion of subjects who performed trials all the way through the experiment in connection with a rolling trials strategy, discussed in section 6.3.2 below.

Figure 10 Average trials/subject in expt. at CentER

Main expt up to 26 periods, Practice 2 periods only

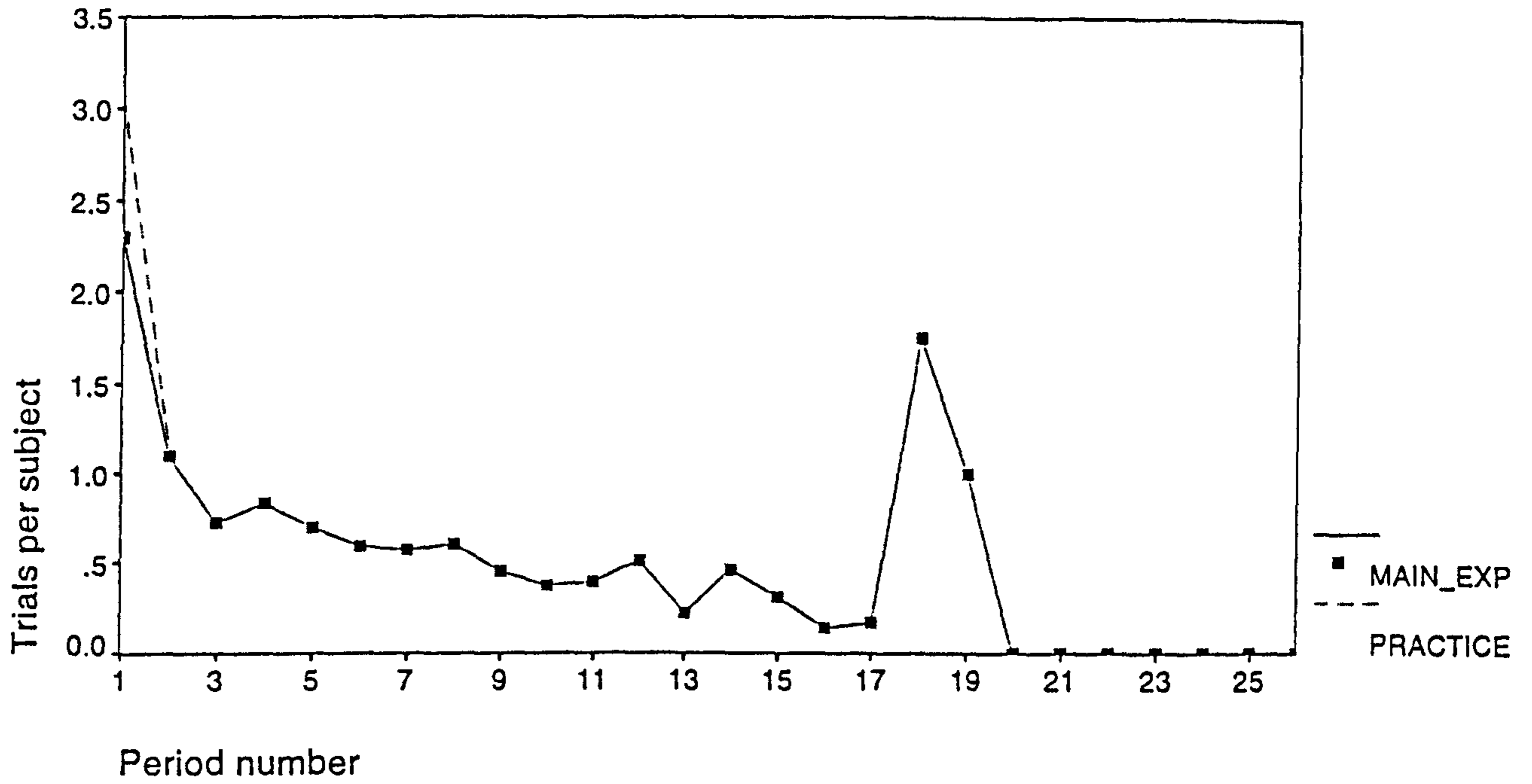
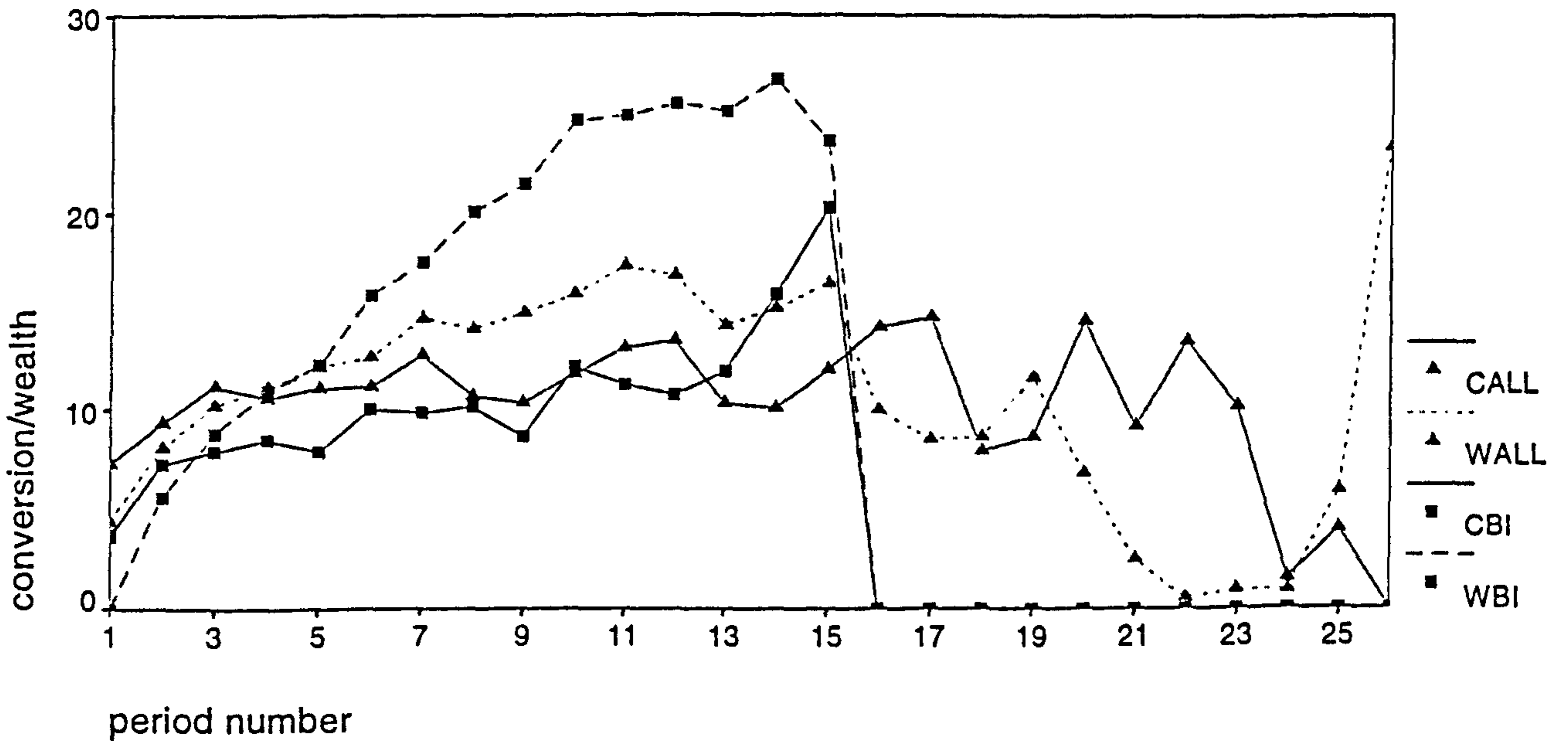


Figure 11 Average conversion, wealth over time in expt at CentER

CALL, WALL all subjects CBI, WBI backward induction treatment 2

All subjects up to 26 periods, Backward induction 15 periods



6.3.2 Trials strategies

6.3.2.1 Definitions of strategies

Since it was not practicable to look through all the results for such a large number of trials and subjects, it was necessary to precisely define each trials strategy and automatically check the output files for these strategies. This implies that certain possible patterns of trials behaviour must be assumed. These patterns were taken from the results of the EXEC experiment, in which every trial output file was inspected in order to see patterns of trials. The three trials strategies that were identified in the EXEC experiment were:

- i) a learning strategy in which several trials were performed in one or two periods,
- ii) a rolling strategy where there were trials all the way through the experiment, usually with a relatively small number of trials in any one period looking only a few periods ahead and
- iii) a fixed end point strategy where the subject chose a single period in the future as the end point for a series of trials that they performed over several periods.

The adaptations of these definitions for the automated search were deliberately made quite broad, to allow for the fact that there was a very large individual variation of patterns in the trials performed within the above classifications. However these adapted definitions maintain the essential features of the three strategies with regard to the pattern of trials through the 'lifetime' of the experiment. The broadness of the definitions is due to the requirements for numbers and lengths of trials. From the results of the EXEC experiment, it can be observed that within a particular pattern of trials over time, there may be a wide variation in the number and length of trials in any one period. In many cases, trials of different lengths were performed in the same period as part of a single trials strategy and the number of trials performed varied from period to period. Thus it was not possible to develop a clear definition of the number and length of trials associated with a particular strategy. Instead, a low requirement for numbers of trials was set and statistics calculated for the lengths of trials associated with each strategy. The three strategies were defined as

follows:

i) Learning strategy

The subject performed more than 1 trial in period 1 and/or period 2.

This definition reflects the fact that most subjects who wanted to learn about the results of different savings patterns would do their learning at the beginning of the experiment; there is no point in learning when the decisions have already been taken!

ii) Rolling strategy

The subject performed at least one trial in at least every other period all the way through the experiment i.e. if trials were not performed in 2 consecutive periods, then there was no rolling strategy.

Although this definition is very loose with respect to the range of trials performed, there is a requirement that the subject perform trials all the way through the experiment. In the sense of the timing of trials through the experiment, the definition is rigorous.

iii) Fixed end strategy

At some point in the experiment, the subject performed trials in three consecutive periods, all with the same end point.

Some subjects concentrated trials in the practice rounds. It is not possible to identify through-life trials strategies in the practice run, as there were only 2 rounds. The effect on trials in the run for real is to reduce the number of trials in the run for real and hence the number of subjects who employed identifiable strategies. This effect cannot be quantified, but it means that the strategy count in the run for real is an underestimate of the frequency with which people use the strategies. However, many of the trials in the practice run will have served the function of giving the subjects a feel for the numbers involved, which was indeed part of the purpose of including a practice run. This means that the learning behaviour in the run for real may be underestimated, but the underestimate for the other strategies will be less significant. It should also be pointed out that the presence of learning does not tell us very much about the savings decision process. It is mainly a function of the

inexperience of the subjects in the experiment.

Learning was very common: 63% of all subjects who performed 96% of all trials had identifiable strategies with some element of learning. Rolling was also very common: 51% of subjects who performed 88% of all trials had identifiable strategies with some element of rolling. By far the most common strategy was a combination of learning and rolling: 40% of all subjects used this combination of trials strategies and performed 72% of all trials. This strategy was also the most common strategy in the backward induction group. The only other trials strategy that was at all frequent was a simple learning strategy, used by 19% of all subjects to perform 16% of all trials.

Trials with a fixed end point were only frequent in the backward induction group and even then more subjects performed trials without a fixed end strategy than with. Of the (107) subjects apart from the backward induction group, only 4 were identified as employing a fixed end strategy. This is not surprising, as when facing an uncertain length of life, the only anchor point is the expectation of 10 periods. Given the average range of 4.1 periods, 10 is too far ahead for almost everybody. Thus the choice of a fixed end point will appear almost random. People who perform trials were thinking logically in that they were using the facilities of the program to help them achieve the goal of getting as high a number of points as reasonably possible and therefore a random trials strategy would be inconsistent with their decision process.

Much more common was a spread of ranges for a subject, some short trials and some long. In combination with the low average range, this suggests that there is considerable cognitive cost in performing long trials.

6.3.3 Ranges of trials (see table 3)

In the practice runs, over all groups there were 559 trials with an average range of 4.38 periods. The backward induction group performed 95 trials with an average range of 5.19 periods, so the groups other than the backward induction group performed 464 trials with an average range of 3.85 periods.

In the runs for real, over all groups there were 1066 trials with an average range of 4.10 periods. The backward induction group performed 270 trials with an average range of 5.84

Table 3 Trials statistics

Trials	no. of subjects using this strategy	average range of trials	standard deviation	no. of trials²⁵ performed
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All Subjects: practice runs

all trials		4.38	3.64	559
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All Subjects: runs for real

all trials		4.10	3.74	1066
learning	23	3.88	4.07	169
rolling	7	4.63	3.30	59
fixed end	0			
learning+rolling	50	3.96	3.66	772
learning+fixed end	5	6.13	4.75	83
rolling+fixed end	6	6.67	6.67	107

Subjects with known length of experiment (backward induction trial): practice runs

all trials		5.19	5.25	95
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Subjects with known length of experiment (backward induction trial): runs for real

all trials		5.84	4.55	270
learning	1	11.00	3.68	13
rolling	1	2.50	1.05	14
fixed end	0			
learning+rolling	7	5.68	4.65	191
learning+fixed end	3	7.14	5.05	59
rolling+fixed end	4	7.64	6.84	87

²⁵ The number of trials is the sum over all subjects of the number of trials made by each subject for each category of strategy.

periods, so the groups other than the backward induction group performed 796 trials with an average range of 3.51 periods.

The trials performed during the practice run and the run for real had similar ranges (4.38 periods for practice trials, 4.10 periods in trials during the run for real). The decrease in the average range between the practice run and the run for real may be due to the cognitive costs of performing long trials. Another possibility is that the subjects were bored, but there was no sign of this among the subjects; just some signs of impatience for a few of the subjects who had large numbers of periods. The 10% chance of ending gives an expectation of 10 for the number of periods (apart from the backward induction group). Thus the trials performed during both the practice and the run for real are short compared to the expected length of the experiment: overall, the 1066 trials had an average range of 4.10 periods, of which the backward induction group performed 270 trials with an average range of 5.84 periods. (Thus the backward induction group exercises a disproportionate influence on the overall figures: their 25% of trials is rather more than the proportion $17/124 = 14\%$ of subjects in this group).

On average, the trials performed by the backward induction group were longer and the trials with a fixed end point in the backward induction group were significantly longer. However, only 7 subjects (4,5,7,8,11,13,14) of 17 in the backward induction group performed trials with a fixed end point at 15 in the practice session; a further 4 subjects performed trials either finishing at period 15 or with a range of 15 periods.

The backward induction group performed proportionally more trials in the run for real and longer trials in both the practice periods and the run for real. Even then, the overall average range for all groups is only 4.10 periods in the run for real and 4.38 in the practice rounds.

The overall result is that when subjects knew the length of life, 65% of subjects performed at least some trials over the whole length of life. For all subjects, trials only looked a relatively short period into the future. **Thus the assumption about how far people look forward for general savings in the theory of chapter 2 is confirmed.**

6.3.4 Jumping into the future (see table 4)

Using the trials data, it is also possible to see if people look forward from the present time when performing trials, or whether they jump ahead to some period in the future. Table 4 shows that the majority - 80% - of all trials start from the current period; only in the first two periods of the experiment is there a large proportion of trials that jump ahead - 35% in the first period and 26% in the second period. When trials do start from some point in the

Table 4 Starting Period of Trials

time period	0	difference counts				total number of trials performed	proportion of trials starting from the current period (difference count = 0)
		1	2	3 or 4	5+		
1	184	22	23	28	28	285	0.65
2	101	7	8	13	7	136	0.74
3	72	9	2	3	3	89	0.81
4	84	8	5	2	2	101	0.83
5	75	2	1	3	2	83	0.90
6	61	1	1	1	0	64	0.95
7	54	3	0	2	0	59	0.92
8	50	1	2	1	1	55	0.91
9	33	2	3	1	0	39	0.85
10	28	0	0	0	1	29	0.97
11	25	0	4	0	0	29	0.86
12	27	5	0	0	1	33	0.82
13	13	0	0	0	0	13	1.00
14	18	2	0	1	2	23	0.78
15	14	0	0	0	1	15	0.93
16	1	0	0	0	0	1	1.00
17	1	0	0	0	0	1	1.00
18	7	0	0	0	0	7	1.00
19	1	3	0	0	0	4	0.25
20	0	0	0	0	0	0	

no trials after period 20

totals	848	65	49	55	49	1066	
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future, there is a spread of periods from which the trials start. Note that some subjects performed both trials that started from the current period and trials that started from some period in the future. Although it could be suggested that subjects were learning how to use the trials facility in the first two periods, this is unlikely to have a strong effect given that subjects had two practice periods.

The difference counts are the numbers of trials starting from a given number of periods in the future, compared to the actual period in the experiment (run for real). Thus a difference count of 0 means that the trial started from the current period, a count of 1 means that the trial started 1 period in the future compared to the current period etc.

6.4 Quantitative analysis of savings/conversion data

The quantitative analysis of the savings and conversion data was also performed in a similar way to the EXEC experiment: by means of a panel data analysis using LIMDEP. The same variables were used as in the EXEC experiment (see section 5.4.2.3 for discussion). Since there were several different treatments, Chow tests were conducted to test the hypothesis that the different treatments gave different values of the coefficients of the regressors. The Chow tests showed that for all groups except group 05, the hypothesis that the coefficients have the same values across different groups was not supported by the data. These results justified the running of a separate regression for each group of subjects. An analysis of all subjects together was also performed, so that the results could be compared to the individual group regressions.

In contrast to the EXEC experiment, only 1 subject converted all income in every period, therefore the issue of excluding such subjects does not arise.

6.4.1 Chow tests for the significance of group regressions

As in the analysis of the savings decisions in the EXEC analysis, it was not possible to use group dummies to distinguish between the different groups of subjects in the panel analysis. This was because the panel analysis provides dummies for each subject. As in the EXEC experiment, these individual dummies were (usually) found to be significant at the 1% level.

This means that these dummies had to be included, otherwise the model would be misspecified. This then implies that dummies for groups of subjects would be perfectly correlated with the individual dummies and the model cannot then be estimated. Hence an alternative method for testing for group effects was required.

The adapted Chow test (Kennedy (1992)) was used to test for the similarity of parameter coefficients between the individual groups and all subjects together. A separate test was run for each group of subjects. The constrained SSE was generated by running a regression on all subjects. This constrained the parameter coefficients to be the same across all groups. The unconstrained SSE was generated by allowing the parameter values for the group being tested to be different to those of other groups. Two regressions were run, one for the group and a separate regression for the rest of the subjects. The sum of the SSEs from these two regressions was then the unconstrained SSE. To ensure compatibility of the regressions, all regressions were run with the same explanatory variables, using the random coefficients model available in LIMDEP version 6.0 (see section 6.4.2 for details of this specification). The tests were conducted as follows:

H0: all parameter coefficients in an individual group are the same as for all other subjects (not including individual subject dummies).

H1: parameter coefficients vary between the group and all other subjects.

Under H0, the test statistic

$$\frac{[\text{SSE}(\text{constrained}) - \text{SSE}(\text{unconstrained})]/K}{\text{SSE}(\text{unconstrained})/(T1+T2-2K)}$$

is distributed as $F(K, T1+T2-2K)$, where

$\text{SSE}(\text{unconstrained}) = \text{SSE}(\text{group}) + \text{SSE}(\text{rest of the subjects})$

$\text{SSE}(\text{constrained}) = \text{SSE}(\text{all subjects})$

K (number of variables) = 7 (lagged consumption, income, lagged income, wealth, lagged wealth, $\log(\text{income}+\text{wealth})$, time)

$T1$ = number of observations in the group

$T2$ = number of observations for the rest of the subjects

so $T1+T2$ = total number of observations over all subjects = 1409

The results are shown in table 5. The values of the test statistic are higher than the 1% values for all groups except group 05, so H0 is not accepted and the values of the parameter coefficients must be treated as having different values for each group.

Table 5 Results of the Chow tests to test for common parameter coefficients across groups in the experiment at CentER

Regressions are panel regressions with GLS estimation of a random coefficients model (individual subject error terms)

Group	SSE Unconstrained *10 ⁻⁵	SSE Constrained all subjects *10 ⁻⁵	F value F(7,1409) under H0
01	0.537453	0.975144	89.45
02	0.947882	„	5.571
03	0.647504	„	61.44
04	0.942095	„	6.754
05	0.971509	„	0.7429
06	0.940625	„	7.055

At 5% level of significance, upper F value (7,1409) taken from $F(7,\infty) = 2.01$

At 1% level of significance, upper F value (7,1409) taken from $F(7,\infty) = 2.64$

6.4.2 Results of the Panel analysis

The savings decisions were analysed using panel regression methods, as available in LIMDEP version 6 (Greene (1992)). LIMDEP provides test statistics to indicate whether the subject dummies are significant in an OLS regression. If they are, a panel regression is performed and a Hausman statistic calculated to indicate whether a random effects model is significantly different to a fixed effects model (the fixed effects model is an OLS regression with a dummy for each group of subjects and a normal iid error term. In the random effects model, an individual specific error is added to the overall error. A GLS estimator is then used). A general to specific methodology was used for the analysis of each group/treatment. All possible variables of interest were initially included and variables that were obviously insignificant (i.e. insignificant at a generous level of significance of 15%) were removed and the regression performed again. The variable that was then least significant was excluded and the regression repeated. This procedure was continued until all variables that were not significant at the 5% level of significance were excluded.

The results are shown at table 6.

For the control group 01, there was a probability of only 0.185 that subject effects were different from 0; there were no large differences between the fixed and random coefficient regressions. For group 02 (backward induction) there was a 0.60 probability that subject effects were not different from 0 and an OLS regression was used. With group 03 (a known income (pension) of 2 from period 10 onwards) there is a structural split in the data at period 10. It is therefore not reasonable to fit a linear relationship of consumption vs. time for all periods. An alternative way to allow for effects over time would be to include a time dummy, but then there is possible multicollinearity with income, as income undergoes the structural change at period 10. A further disadvantage is that there may be a structural relationship between consumption and time. Therefore, a regression over the first 10 periods was performed to see if there was a significant relationship with t . If not, a time dummy would be included.

Regression over first 10 periods

$$c_t = 0.609y_t + 0.506w_t + 0.140t + v(i,t)$$

	(10.9)	(8.65)	(1.16)	
	≈ 0	≈ 0	0.245	

Random effects model. Figures in brackets are t-statistics. Figures below the t-statistics are the probability that the coefficient is different to 0.

So, there is no significant relationship (at a 10% level of significance) with t in periods 1 to 10 and a time dummy was incorporated in the regression over all periods.

Groups 04 and 05 had an initial endowment of wealth, but otherwise the same parameters as the control group. However, the distribution of the length of the experiment was altered: the number of periods was taken from a distribution with bounds 5 and 15 periods (with a 10% probability of stopping at the end of each period), compared to a distribution with bounds 3 and 25 periods (with a 10% probability of stopping at the end of each period) for the control group. Note that the subjects were told that there was a 10% probability of stopping in all periods i.e. the distribution was unbounded. Unfortunately, it is not possible to include initial wealth as variable if subject dummy variables are significant, as there would then be perfect multicollinearity between the initial wealth variable (which changes only between subjects) and subject dummies. Chow tests showed that the coefficients were significantly different between the groups. Therefore, it was necessary to perform two separate regressions. When these regressions are compared to the control group, the results can be seen to be similar; there is no obvious distinguishable effect of a wealth endowment (see the next section for a discussion of the paths over time, however). The regression for group 06 (interest rate of 20% per period) is characterised by a unusually strong positive correlation of conversion with time i.e. conversion increased over the course of the experiment. This is probably due to the high interest rate, which makes it possible to maintain a level of conversion similar to the other groups while building up a large wealth stock. This stock was then run down to increase conversion later on in the experiment.

Considering the results shown in table 6, y_t and w_t were by far the most significant

Table 6**Panel analysis of subjects' savings decisions: coefficients significant at the 5% level**

Group (treatment)	variables						
	c_{t-1}	y_t	y_{t-1}	w_t	w_{t-1}	$\log(y_t + w_t + 1)$	t
All subjects	.16 (4.52)	.55 (7.53)	-	.38 (8.19)	.085 (-1.64)	-	-
	Random effects model		$R^2 = 0.226$ Hausman statistic = 70.69				
Control (01)	.13 (1.93)	.44 (4.65)	-	.21 (2.96)	-	2.8 (2.50)	-
	Random effects model		$R^2 = 0.443$ Hausman statistic = 11.83				
Backward induction (02)	.44 (4.98)	.75 (5.52)	-.31 (-5.87)	.22 (3.36)	-	-	.41 (3.48)
	OLS		$R^2 = 0.414$				
Pension (03)	-	.74 (10.5)	-	.58 (12.0)	-	-	1.5(dummy) (1.82)
	Random effects model		$R^2 = 0.583$ Hausman statistic = 25.22				
Endowment of 10 (04)	.15 (2.20)	.68 (8.13)	-	.22 (4.85)	-.16 (-2.33)	-	-
	Random effects model		$R^2 = 0.176$ Hausman statistic = 53.03				
Endowment of 20 (05)	.17 (2.58)	.59 (4.75)	-.16 (-2.36)	.39 (8.35)	-	-	-
	Random effects model		$R^2 = 0.142$ Hausman statistic = 11.35				
Interest rate of 0.2 (06)	-	.47 (3.97)	-	.34 (8.74)	-	-	-
	Random effects model		$R^2 = 0.301$ Hausman statistic = 23.30				

The type of regression model used is stated for each group regression. Figures in brackets are t statistics.

The time dummy for group 03 is a pension dummy: $dum = 0$ for $t < 10$, $dum = 1$ for $t \geq 10$.

variables. As in other studies, the coefficient of y_t was larger than that of w_t . Consumption in the previous period had a significant positive correlation in 5 of these 7 regressions, giving some evidence that subjects maintained a smooth conversion strategy through time or had a constant strategy over several periods. Most regressions gave no evidence of strong effects over time (but see the discussion of the scatter plots in the next section). In

the combined regression, t was significant at the 10% level, but the coefficient is small (0.09) compared to the magnitude of the other significant coefficients and it cannot be claimed that this is weak evidence of a strong trend. Time was significant in the backward induction group, showing some tendency for subjects to concentrate their consumption towards the end of the known length of 15 periods. The strong effect of lagged consumption in this regression also provides evidence for increasing conversion through time. The time dummy ($= 1$ for $t \geq 10$) was also significant for the pensions group (03), but this is probably due to the fact that the income process changed at the same time as the dummy.

To summarise, the regressions do not really tell us much that is new about savings behaviour. y_t and w_t were the most significant explanatory variables of conversion and there is weak evidence of an increasing trend of conversion over time. There is evidence against the hypothesis that people take optimal savings decisions, since this would require that people treated wealth and current income equally, which they obviously do not in this experiment. The approximation of the form of an optimal strategy ($y_t + w_t + 1$) was only significant in the control group and in this group the coefficients of y_t and w_t were different also.

6.4.3 Scatter plots

Scatter plots of average values display some interesting features. In particular, the plot for all subjects of average wealth against time at fig.11 shows a strong non-linear trend over time, approximately a negative quadratic. There is a clear single peak at period 11. Thereafter, wealth decreases to 0 at around period 23. The pattern of consumption over time is less clear, but there is an increase in the first few periods and a step decrease to low levels of wealth after period 23 (note that for these periods there is data from 2 subjects only and these 'averages' should therefore not be regarded as a general trend).

Scatter plots for the control group (01) have similar features. The pattern of consumption over time is very similar to the pattern for all subjects. The plot of wealth against time is much less clear, but a significant increase in wealth over the first 4 periods and a step decrease after period 19 can be seen. The 'backward induction' group (02) displays

average patterns of consumption and wealth over time that are similar to the optimal paths. There is no discount rate over time, so the benefit of saving with a positive interest rate would be optimally traded off against the decreasing returns to scale in the conversion 'utility' function. In comparison to the other groups, there is more saving early on in the experiment with a building up a wealth stocks and a lot of dissaving in the last few periods of the experiment.

The 'pension' group (03) can be divided into two time ranges: up to period 10 before the 'pension' and thereafter with the certain low income. In the first range, a stock of wealth is built up and consumption lies above wealth. By period 10, wealth is approximately equal to consumption. Thereafter, wealth, income and consumption are roughly constant, all at a value of around 2 tokens. Given that average wealth is only 2, there is not much possibility to increase consumption above income by running down a wealth stock i.e. dissaving, so consumption would be expected to follow income closely in these periods given the savings behaviour before period 10.

The endowment effect on wealth holdings can be seen by comparing the average wealth of groups 01(no endowment), 04 (endowment of 10) and 05 (endowment of 20). There is no obvious linear progression as endowment increases. In groups 01 and 04, there is an initial increase in wealth and thereafter wealth is held constant in group 01. In group 04, wealth is variable, but at a much higher average than group 01 (16 for group 04 vs. 6 for group 01). The difference in levels is the initial endowment. After period 10, wealth then increases dramatically in group 04. Group 05 displays different behaviour. Wealth decreases in the first few periods to the same level as for group 04, but declines over periods 12-15 to approximately the same level as group 01.

This could be interpreted as evidence for a hierarchical decision process as follows: the holding of a stock of wealth is important and the size of the holding depends on overall combined income and wealth. There is also an aspiration level for the wealth stock of 16 (say). Thus in the early periods, a wealth stock of maximum 16 is created; if the initial endowment is greater than 16, then the wealth stock is reduced. If the initial endowment is less than 16, wealth stock is traded off against consumption. After periods 10-12, people's beliefs about the end point begin to affect their strategy. The expectations diverge, people

either believe that the experiment must finish soon and so run down their wealth stock or they believe that the game could last for much longer (because it has gone on for so long already) and build up their wealth stock.

The patterns in consumption are again much less clear than those in wealth. All 3 groups have very variable average consumption over time and there is no clear pattern of consumption smoothing over time. However, when consumption is compared to income over time, it can be seen that in all 3 groups, consumption roughly tracks income over time, apart from the first 2 periods when a stock of wealth is built up. This is similar to the results obtained in the simulation. This effect in the simulations was obtained by including an inherent utility of wealth so that people obtain utility from having a stock of wealth, independent of the altered pattern of consumption. Therefore, this empirical observation gives some support to the idea that people have a utility associated with holding wealth independent of the increase in future consumption possibilities. The tracking of income by consumption is in accordance with empirical results and the modern versions of the life-cycle theory with uncertain future income and either precautionary saving or liquidity constraints.

The results for a change in interest rate are less clear. All that can be definitely said is that consumption increases in the first 5 periods, with almost exactly the same (average) values as the control group. Thereafter, consumption is extremely variable, even when averages are taken. There is also no obvious conclusion about the relative statics over time to be drawn. The one reasonably clear pattern in the results is that the wealth holdings in group 06 follow a similar path over time to the overall averages, with a single peak somewhere around period 8. This discussion illustrates the difficulties associated with regression equations. In order to perform the regressions it is necessary to guess the functional form of the relationship between the variables. This makes it very difficult to allow for non-linear and discontinuous functions.

6.4.3.1 Conversion compared to the optimal strategy

The scatter plot of conversion against income + wealth (i.e. total assets) shows no particular pattern that could be regarded as close to the optimal strategy. Even allowing for

the variation in parameters among the groups, there is only a general upward trend as identified in the regression analysis. There is no obvious sign of tailing off at higher values of total assets, as would be seen if the subjects were following the optimal strategy.

6.5 Performance of the subjects

The performance of the subjects relative to two baseline strategies, the optimal strategy and the simple rule of consuming all income as soon as it is received, is shown in fig.12²⁶. This is for the baseline group 1, with the largest variation in income streams. It can be seen that the subjects' performance was often very close to the optimum. Of the 34 subjects for which there is data, only 4 - 23, 29, 31 and 35 - are more than 5% below the optimal strategy. However, this is not surprising as the strategy of consuming all income as soon as it is received is also very close to the optimal strategy (remember that only one subject used this strategy in this experiment). So there is a wide variation of strategies which produce very similar results. Fig.13 compares the subjects' performance to 3 period and 5 period rolling strategies. It can be seen that the rolling strategies are very similar to the subjects' choices. Given the similar performance of different strategies in fig.12, this is not strong evidence that subjects made choices consistent with a rolling strategy. It can be seen that the 5 period rolling strategy is noticeably further away from the subjects' choices than the 3 period rolling strategy.

6.7 Conclusions and comparison of the results with the experiment at EXEC

The trials data for the EXEC experiment and the main experiment are similar; in the EXEC experiment there is an average range of trials performed of 3.49 periods for subjects put automatically into the trial option in the run for real, compared with an average range of 3.85 in the run for real in the Dutch experiment (excluding those trials performed by subjects in the backward induction group 2). On average, the Dutch subjects performed trials that were a little bit longer. The EXEC subjects performed more trials than the Dutch subjects (average 14.7 vs. 8.6 trials performed by each subject over the whole experiment).

²⁶ There is no data for subjects 15 and 21.

Figure 12 Performance of subjects in experiment at CenterER

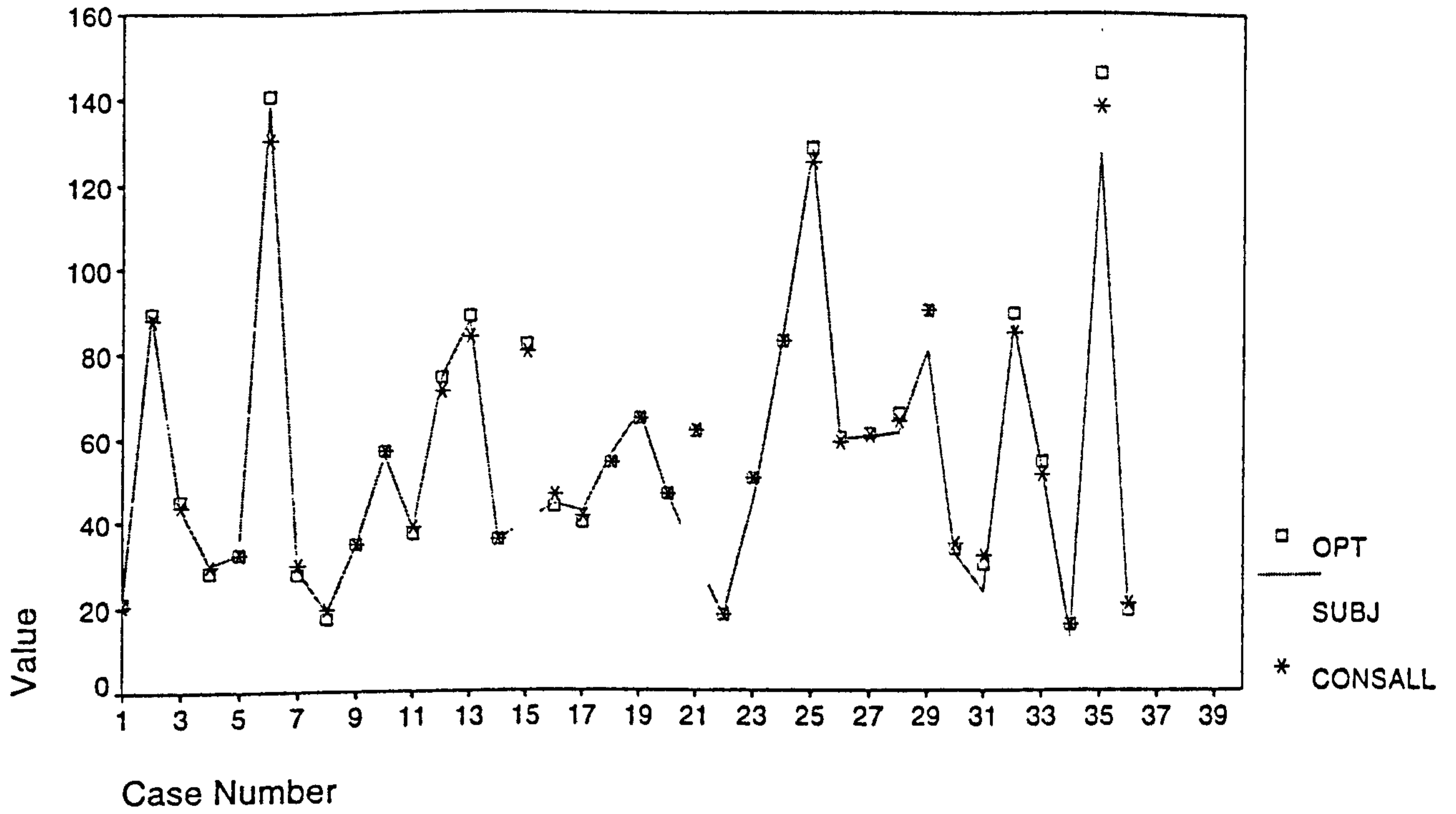
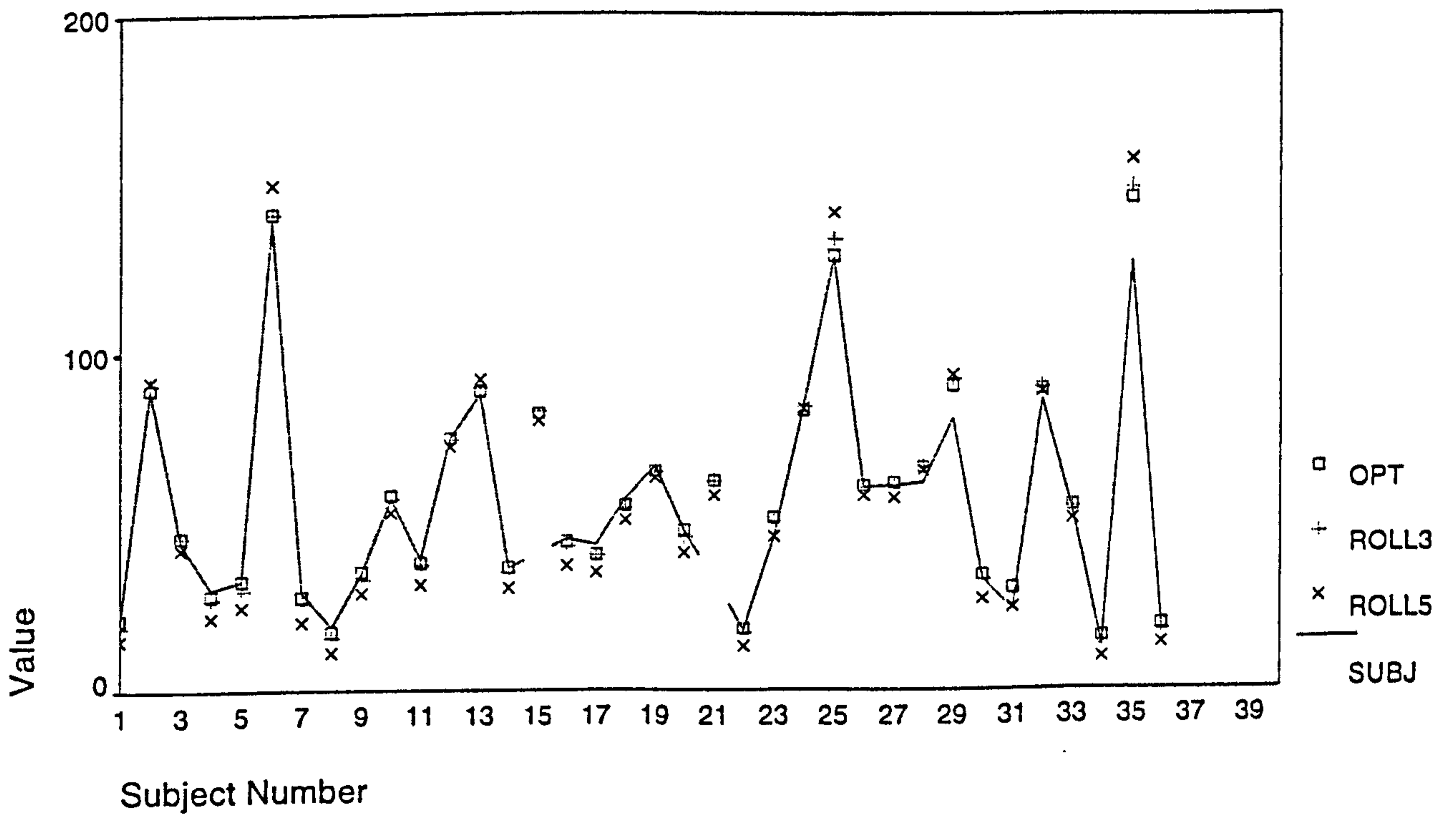


Figure 13 Subjects performance vs. rolling strategies CenterER



The results of the regression analysis are also similar in that consumption depends most strongly on current income and current wealth. In comparison with the control group the EXEC subjects consumed a higher proportion of both income (coefficients .76 EXEC, .55 Dutch control) and wealth (coefficients .39 EXEC, .21 Dutch control). This is to be expected given the much higher interest rate (2% per period in the EXEC experiment, 10% per period for the Dutch control group) which gave a greater incentive to save for the Dutch group.

The other coefficients bear little resemblance to each other. Given the non-linearities associated with the behaviour over time and the high variance of the data i.e. the very different patterns of behaviour in different subjects, it is misleading to try and draw firm conclusions about lags from the regression analysis.

The subjects made choices that gave them almost as many points as the optimal ex-ante strategy. However, this is true for a wide range of strategies, including that of consuming all income as soon as it is received, so this is not a strong result.

There was one obvious difference between the EXEC subjects and the Dutch subjects. In the EXEC experiment, 10 of 36 subjects simply converted all their income every period. This only happened with 1 of the 124 Dutch subjects. In this sense, the increase in the interest rate from 2% to 10% may have made a significant difference. Or the Dutch population may be more disposed towards trying to work out an optimal strategy, rather than taking a simple heuristic.

Overall, the results of this experiment confirmed the results of the EXEC experiment. The subjects performed many trials and these trials had an average range much below the expected length of the experiment. In addition to the expected learning pattern, a rolling trials strategy was very common, being used by 51% of all subjects. **This limited range of trials and the use of a rolling decision strategy confirms the assumption about timing in savings decisions of the theory of chapter 3.** The regression analysis showed a strong dependence of consumption upon current income and wealth, with a higher coefficient for income. This is evidence that, under the conditions of uncertain future income and a liquidity constraint, consumption tracks income.

In addition, scatter plots of wealth over time show a strong non-linear trend over time; approximately a negative quadratic $w = f(t, -t^2)$ with a single peak around period 10.

Finally, the scatter plots of wealth against time for different initial endowments show a building up of wealth in the early periods of the experiment, giving some support to the idea that people have a utility associated with holding wealth independent of the increase in future consumption possibilities.

7 Rolling Strategy Simulation

7.1 Introduction

The experiment described in chs. 5 and 6 demonstrated the use of a rolling decision strategy. To recap, in a 'rolling' strategy the person performs trials all the way through the experiment, usually with a relatively small number of trials in any one period, looking only a few periods ahead. This justifies one of the main assumptions of the theory of ch. 3, that people look ahead for a limited length of time, but may reconsider their saving decision quite often throughout their life. An obvious question is then: given that these people are not optimising, how much do they lose?

This chapter answers this question by performing simulations to compare a rolling strategy against the optimal strategy for the income distribution used in the experiment. It is found that over all parameter values a rolling strategy occasionally performs quite badly, but with the short income streams and the parameter values of the experiment, the rolling strategy is very close to the optimum.

Section 7.2 discusses the rolling strategy in more detail and suggests an explanation for this behaviour. Section 7.3 describes the simulation method and section 7.4 the results. Section 7.5 concludes.

7.2 The Rolling Strategy and why people use it

The rolling savings decision strategy is a form of continuous replanning for saving/consumption decisions under income uncertainty and an unknown length of life. In this strategy, decision makers deliberately ignore some of the information that is available to them. Instead of considering the whole of their possible life span (i.e. all possible future periods in the experiment) they restrict their attention to a few periods into the future (the number of periods ahead that are considered is a constant in all periods). Therefore, when savings are considered, there is a 'gap' in the decision maker's thinking - the distant future is ignored. They then reconsider their decision in each period, to check their assumptions and that their consumption choice is still their preferred level given the income that they receive in

the current period. All periods in the lifetime are eventually considered as the range or number of periods into the future that are considered is constant. Therefore, in each successive period another period forward in the future is included in the decision process. This idea of a rolling strategy was modelled in the theory of ch. 3 by having the decision maker assume a constant income and then choosing a constant level of savings/consumption for a fixed length of time (5 periods in the simulation of ch. 4). Utility in the current period was traded off against the utility in the period at the end of this fixed length of time. In this chapter, there is a smaller departure from the assumptions of conventional life-cycle theory: within the timing restrictions of the rolling strategy, the decision maker is assumed to optimise.

The reason why such a decision strategy should be considered is that it simplifies the decision process. The justifications for such a simplification are discussed in Pemberton 1993 and in ch. 2 section 3, where the consequences for the decision process are detailed following Simon's idea of bounded rationality. The idea is that the calculation of the optimal path under income uncertainty is a very complex calculation, requiring a computer simulation to be performed. Most people are not capable of performing such a calculation and even if they could, the investment in cognitive effort and time is not worthwhile given the large range of possible situations and parameter changes e.g. What positions and therefore incomes will the decision maker have in the future? Will they marry, have children? What might the future path of interest rates be? etc. etc. The adoption of a rolling strategy has the significant advantage that the area of largest uncertainty - the 'distant' future is eliminated from the calculation. Decision makers therefore find it easier to make a reasonable estimate of their utility, their future income and future interest rates. In turn, this enables the calculation to be made more specific, giving a guide for action instead of the feeling that there are so many different circumstances to be taken into account that it is not worth the cognitive effort of going through them all. This procedure is also in accordance with the emphasis of Simon on the fact that the attention of people is concentrated on immediate problems, while distant problems tend to be left until they become more immediate (Simon 1983).

7.3 Simulation Method

The performance of the rolling strategy against the optimal strategy was calculated for the income streams actually used in the experiment and the distribution that the students were

informed was being used (the ‘theoretical distribution). The theoretical distribution was:
 y (income)=2 probability = 0.1, $y = 10$ probability = 0.8, $y = 2$ probability = 0.1,
 Stopping probability at the end of each period = 0.1.

The actual distributions used did not include income streams with a sequence of income with a income level of 2 in a high proportion of periods. The stopping probability used was such that at EXEC, the experiment lasted between 8 and 12 periods, for the CentER control group between 3 and 35 periods and for CentER groups 3-6 between 5 and 15 periods (see ch. 5 section 2.2 and ch. 6 section 2.2 for the discussion).

7.3.1 Calculation of the optimal consumption strategy by dynamic programming

The subjects maximises the discounted expected value of utility over the whole of the life, subject to the liquidity constraint that wealth ≥ 0 . Given the constant discount rate, this is an infinite sum.

$$E(U_t) = E_t \sum_{s=t}^{\infty} \delta^{s-t} u(c_s) \quad 7.1$$

In the experiment, $u(c_t)$ was given by a CARA function and δ is the stopping probability described above. This problem can then be solved using a dynamic programming method. This is adapted to the simulation by the following steps²⁷:

i) for the final period of the life f , all wealth is spent. Therefore the expected maximum value function, v_f is the CARA utility function for the sum of accumulated savings in the last period and income in the last period. Since the accumulated savings are not known, it is necessary to calculate the value function for all possible levels of savings + income. Given the nature of the computer as a digital machine, the function must be stored in discrete steps of wealth. The range of wealth considered was 0 to 100 in steps of 0.1 (since income is received at the beginning of the period, this wealth includes the income for the period).

²⁷ See Appendix 6 for a listing of the simulation program

ii) then the expected value function in the next to last period is calculated. This is

$$E(v_{f-1}) = u_{f-1}(c_{f-1}) + E(v_f(w_{f-1})) \quad 7.2$$

w_{f-1} and v_f are connected by the level of c_{f-1} through the liquidity constraint and the income distribution is used to calculate the expected utility in the next period. v_f is known for all levels of assets at f , so if the level of assets at $f-1$ is known the problem can be solved. However, this level is dependent on previous periods which have not yet been calculated, so it is necessary to calculate the whole value function v_{f-1} for all possible levels of wealth (including income for the period). For each level, the maximising value of c is found and $\max_c(v_{f-1})$ calculated (in the experiment, only whole numbers for c were allowed, so only whole numbers for c were used in the simulation). The values of v_f that have been stored are used in the calculation. A linear interpolation routine is used to interpolate between adjacent values of the v function²⁸.

iii) The values of $\max_c(v_{f-1})$ for the (1000) different levels of initial wealth and the associated consumption are stored. Thus the maximum value and optimal consumption function are known as functions of initial wealth + income.

iv) this procedure is repeated for the previous period ($f-2$), using v_{f-1} for the expected value function.

In this way, the maximum value functions as a function of wealth for all periods can be calculated. To solve for a specific wealth and income in the current period, this asset level would be input as the value of wealth at the start of the current period. In order to calculate the expected total value over a lifetime with no restriction on the number of periods (thus giving the infinite sum), a 2-stage numerical approximation is used:

i) Starting from a notional final period of a life, with each period backward in time, the difference between the in-period value functions v_{t+1} and v_t becomes smaller and approaches a

²⁸ Since the v function is concave, a linear interpolation routine will consistently underestimate the value of the function. However, because of its simplicity linear interpolation is very robust for any function, even if it is highly non-linear. The number of values of the v function stored (1000 for steps in wealth of 0.1) was chosen to be sufficiently large that the interpolation errors are very small. Some runs were executed with 10000 steps of 0.01, but this made no discernable difference to the results. For the highest value of curvature in the CARA function (0.9) the difference in values between consecutive points at the point of highest curvature is 0.08, so the difference between the exact curve and the linear estimate is of the order of 0.01 at the value of 198 or 0.005%. Note also that since the same routine was used for all strategies, which are all concave, the difference in bias between the different strategies will be even smaller.

limiting form. This is because, given the constant stopping probability or discount rate and the constant income distribution, in the new (previous) period the process of calculation replaces v_{t+1} by v_t so that the v function for any period a long time before the end of the life is found by recursion. For a long expected length of life, the expected value function changes insignificantly and in the limit is constant. Therefore, the difference between functions v_{t+1} and v_t is found and the calculation repeated until this difference is sufficiently small²⁹. The expected value for any initial wealth can then be read off from the value function v .

ii) For a specific income stream the optimal consumption from the c function for the first period is read off, the resulting wealth at the beginning of the next period calculated and the procedure repeated going forward through all periods.

The lifetime value is then calculated as the sum of the CARA utilities from the consumption values.

7.3.2 Calculation of the rolling consumption strategy

The calculation of the rolling strategy is based on the method of calculating the optimal strategy for a specific income stream over a known number of periods using backward induction. A specific (constant) number of periods ahead is assumed and the max v and optimal c functions for each period are found as a function of wealth at the beginning of the period, working backwards from the last period being considered. The initial wealth and income in the current period is then used to find the consumption for the first period, the wealth for the next period is then calculated and the consumption for the next period found and the procedure repeated until the final period being considered. The rolling strategy always assumes that the experiment (or life) will last until the end of the final period being considered, so there is no discounting over these periods. The optimal consumption for a given wealth + income is found from the optimum consumption function for the first of the periods being considered.

²⁹ This difference was found as the sum of all differences for each level of wealth between functions v_{t+1} and v_t . The calculation was stopped when the difference sum was less than 10^{-4} .

7.4 Results

Two sets of simulations were performed: the first with the actual income stream and parameter values that were used in the experiment for the EXEC groups and the CentER control group and the second with the theoretical distribution and a wide range of different parameter values.

7.4.1 Results with the actual income streams used in the experiment

(see Figures 14 to 16)

These results are a comparison of what subjects in the experiment would have obtained by following various strategies, given the actual income streams they faced. In these figures, the values ('points' in the experiment) of four strategies are shown: the ex-ante optimal strategy, rolling strategies covering 3 and 5 periods (and thus looking forward 2 and 4 periods into the future respectively) and the optimal 'ex-post' strategy i.e. the optimal consumption (conversion) when the actual income stream is known and there is no uncertainty.

For the EXEC groups it can be seen that the rolling strategies perform worse than the ex-ante optimal strategy, but the difference in value is small showing that there is not much difference between the two strategies for these parameters and income streams. The rolling strategy over 5 periods performs slightly worse than the rolling strategy over 3 periods. This is because, given the convexity of the CARA points function, there is more incentive to save with the 5 period rolling strategy³⁰ and the 5 period strategy builds up a larger wealth stock and so loses more when the experiment ends. The same pattern is repeated for the CentER control group; there is not much difference in performance between the optimal and the rolling strategies. There is, however, a slight variation in that for income streams that last a large number of periods and have

³⁰ The CARA function is $A(1-e^{-Bc})$. A, the multiplier, was 30 in EXEC group 1, 15 in EXEC group 2 and 20 in the CentER group. This determines the limiting number of points that can be obtained in any one period. The average income was 10.2 per period, so saving all income could generate a wealth stock of 100+. However, the marginal utility of conversion, given these values of A, is small when a large stock is converted in one period. Hence conversion is more efficiently spread over several periods and the more periods that are considered, the larger the wealth stock that can be built up and still be efficiently converted.

Figure 14 Value of optimal & rolling strategies

EXEC group 1

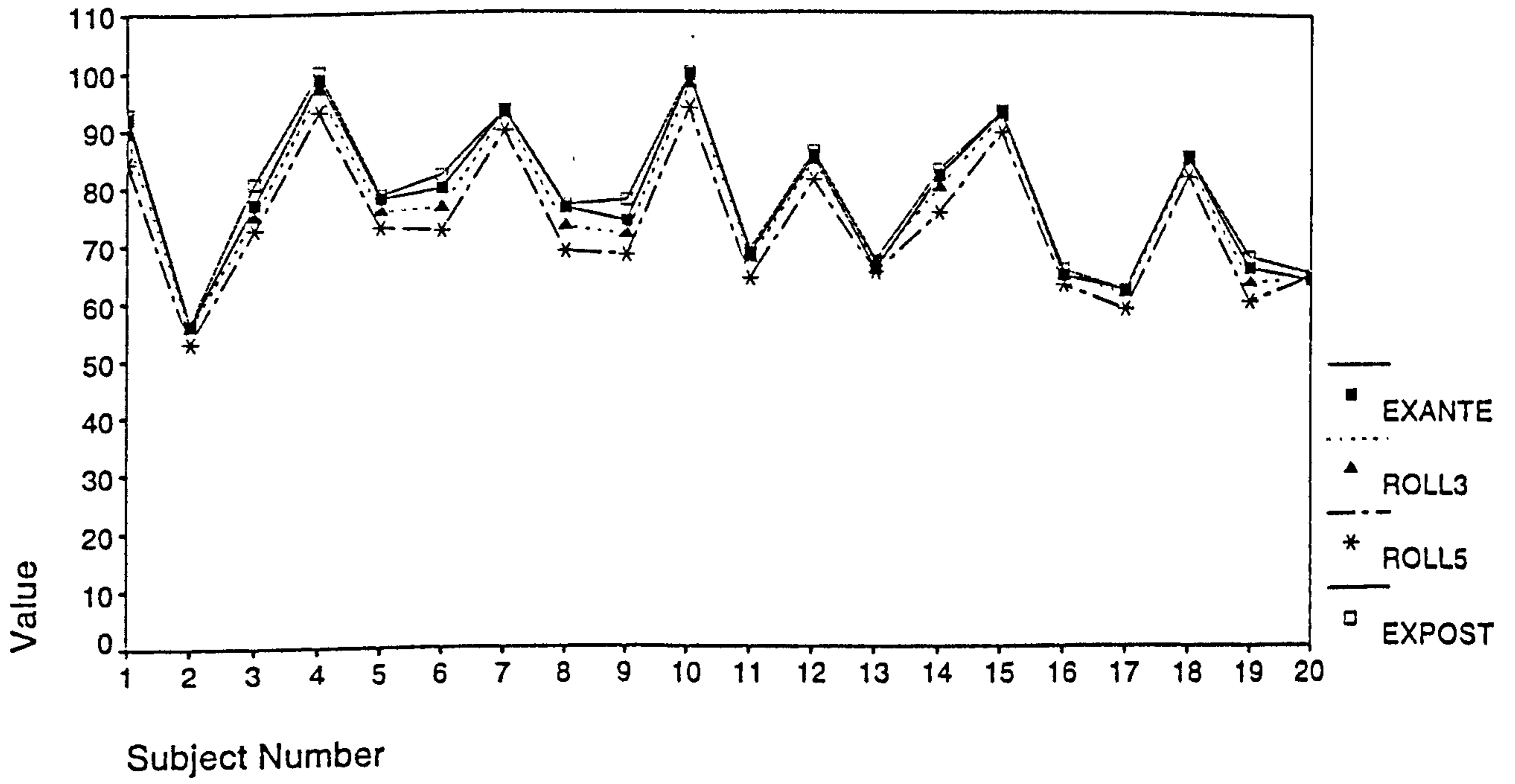


Figure 15 Value of optimal & rolling strategies

EXEC group 2

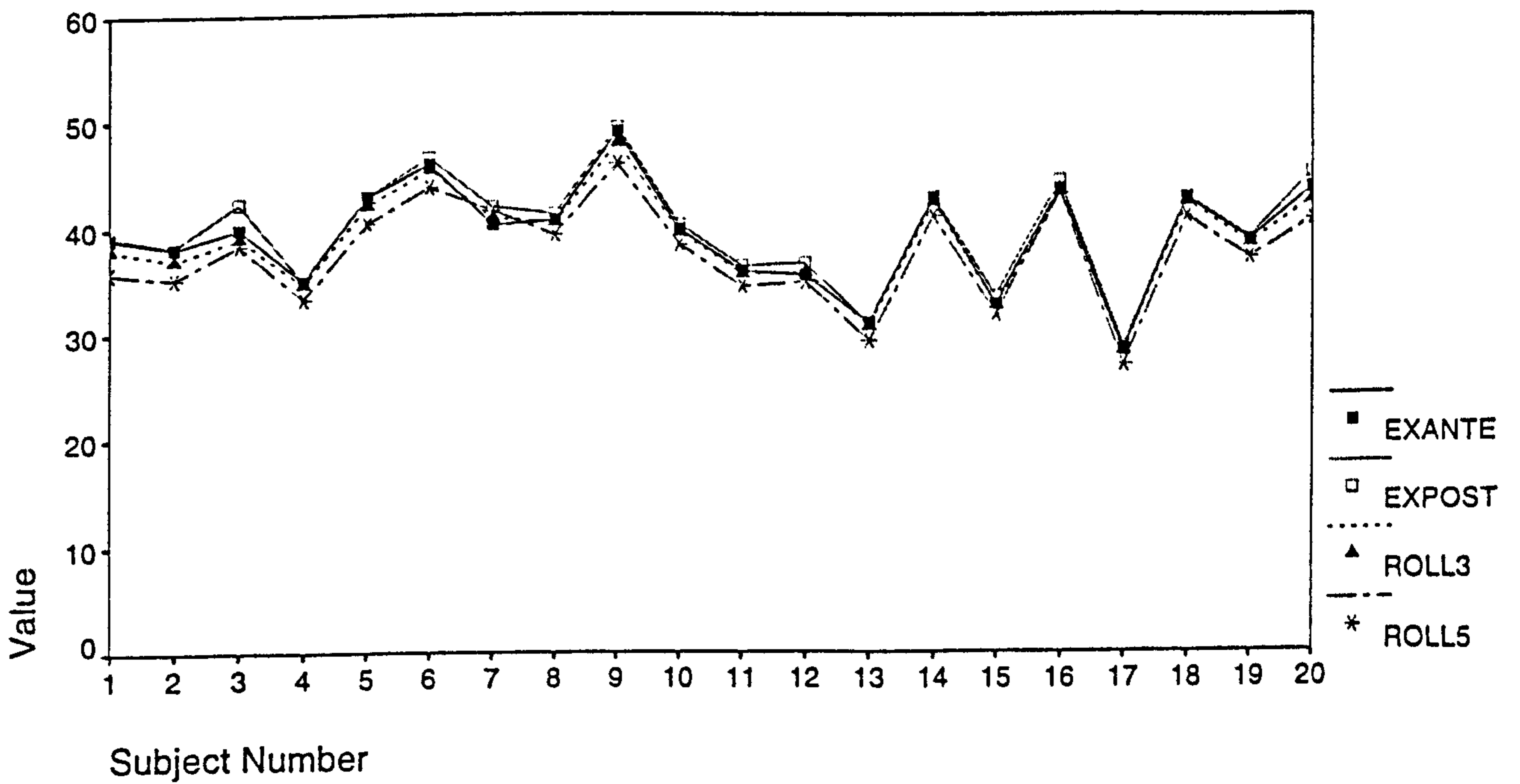


Figure 16 Value of optimal & rolling strategies

CentER control group 01

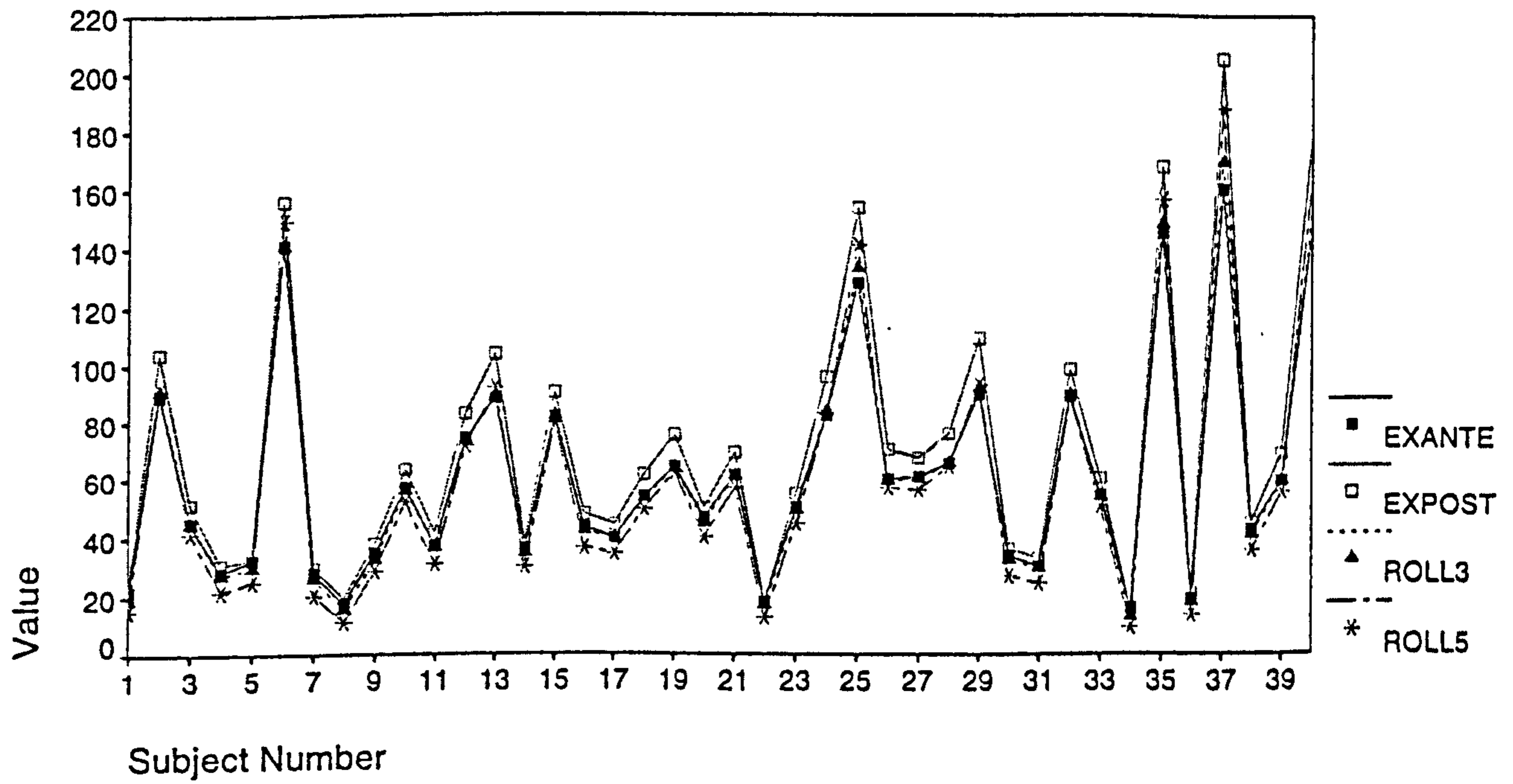
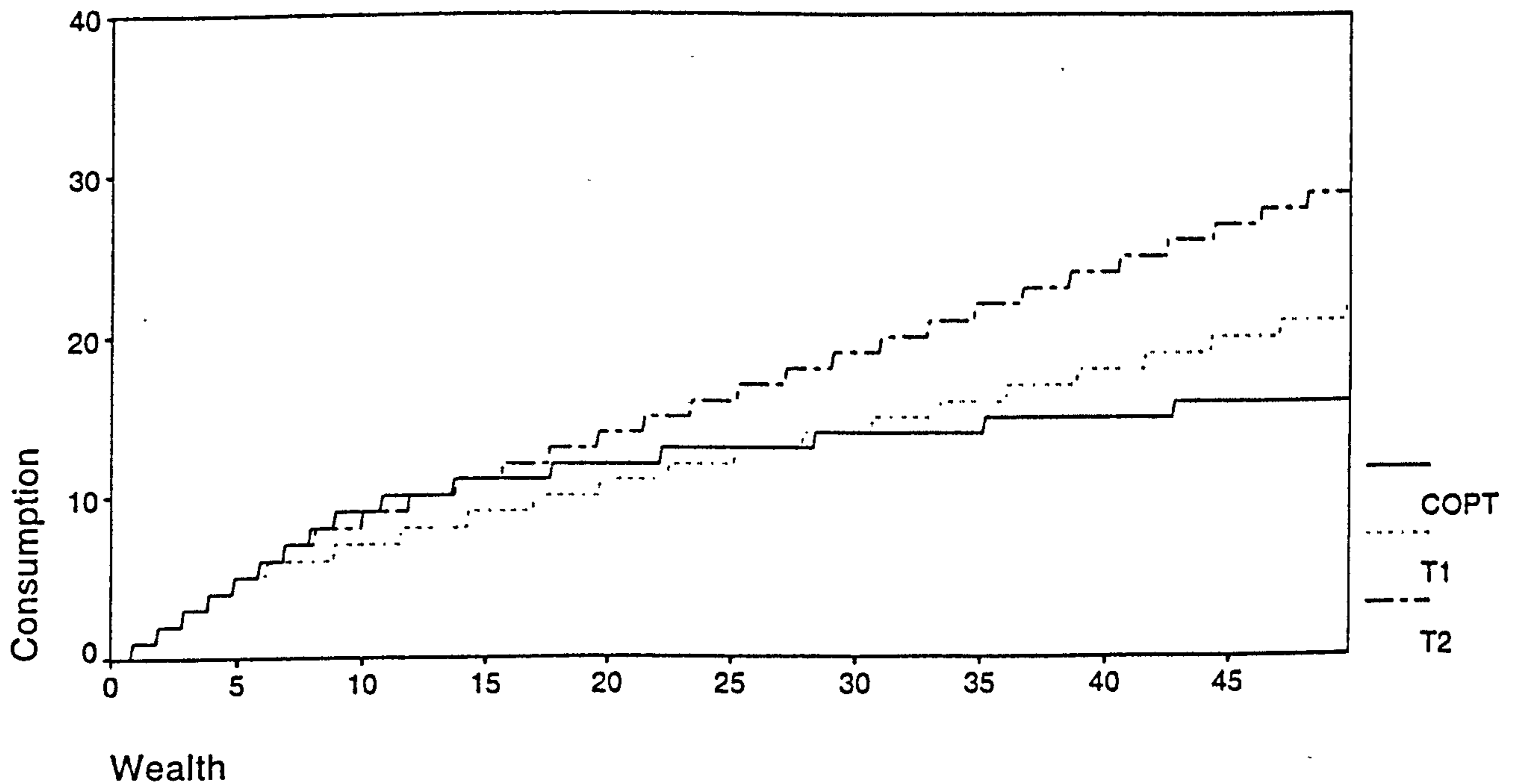


Figure 17 Consumption for given wealth

Copt: optimal T1, T2: 1st, 2nd Periods of roll3 strategy



relatively high incomes (in particular CentER subject numbers 6, 25, 35, 37, 40) the rolling strategies actually perform better than the ex-ante optimal strategy. This is because for high levels of income over many periods, relatively high levels of saving are generated. However, the assumption of the rolling strategy that the experiment will only last a few periods, in contrast to the optimal strategy which allows for the possibility that the experiment will continue for a long time, becomes more realistic towards the end of the experiment. Thus when the experiment actually ends, the rolling strategy loses less wealth stock than the optimum.

7.4.2 Results with the theoretical distribution

These results generalise the simulation in three ways. Firstly, the expectation of the value of the strategies, rather than the outcome for specific income streams, was calculated and the income distribution without restrictions on the number of periods or the frequency of low incomes (the theoretical distribution) was used. Secondly, different parameter values were examined. Thirdly, a wider range of rolling strategies - over 3 periods, 5 periods and 10 periods - was examined. For these simulations, the expected value of the (ex-ante) optimal strategy was calculated as described above³¹. The expected value of the rolling strategy was found by randomly generating an income stream, finding the value of the rolling strategy for the income stream and repeating the process until the average value changed by less than 10^{-4} . The multiplier in the CARA function was set at 20 (as in the CentER experiment). From these parameters and the fixed income distribution, a wide range of interest and discount rates³² per period (R, D) and curvature or risk aversion of the CARA function (B) was examined.

³¹ The conversion/consumption values were still restricted to whole numbers and the liquidity constraint still applied.

³² The discount rate is the probability of the income stream finishing at the end of the current period.

The values used were:

$$R = 0.5, D = 0.01, R/D = 50.0$$

$$R = 0.1, D = 0.01, R/D = 10.0$$

$$R = 0.1, D = 0.1, R/D = 1.0 \text{ (value in the experiment)}$$

$$R = 0.01, D = 0.1, R/D = 0.1$$

$$R = 0.01, D = 0.5, R/D = 0.02$$

$$R = 0.001, D = 0.5, R/D = 0.002$$

$$B = 0.001, 0.03 \text{ (value in the experiment)}, 0.1, 0.9$$

In figure 17, the optimal consumption levels for levels of wealth + income up to 50 are plotted for $R/D=1$ and $B=0.03$. The two functions for the first period (T1) and the second period (T2) of a rolling strategy covering 3 periods (in the final period, all wealth + income is consumed) are contrasted with the optimal strategy. The interesting aspect of these results is that the optimal consumption crosses the rolling consumption in the first (current) period. This is particularly important because it is the consumption level for the current period (T1) that is actually enacted. The second period of the calculation (T2) always lies in the future. The optimal strategy spends everything over a larger range of low levels of assets, but saves more at high levels of assets. This is because, at low levels of assets all the assets can be spent over a few periods with high marginal utility. The rolling strategy saves more, because there is no discounting over the first three periods and assets can be built up to take advantage of the positive rate of interest and the resulting higher level of wealth can then be spent in the last period. For high levels of assets, the assessment of the expected length of life takes effect. The rolling strategy assumes that life ends at the end of the third period, and the decreasing returns to scale of the utility function means that for high levels of wealth it is better to spread the consumption over the three periods. The optimal strategy takes into account the actual stopping probability (with an expected life of 10 periods) and so there are more periods in which assets can be spent and so a higher level of assets can be consumed at the best marginal utility.

The overall influence of these effects can be seen in Table 7. The table gives the expected utility of the optimal strategy (for an uncertain income and length of life) and three

different rolling strategies. These rolling strategies are the optimal strategies under income uncertainty, assuming the individual will live for 3, 5 and 10 periods. Two summary statistics are given; the average % difference for all simulations over a central (relatively realistic) range of parameter values and a weighted average % difference over all parameter values with the differences over the central range weighted at 1.0 and the differences outside the central range weighted at 0.1. The startling aspect of this table is that for most parameter values, the expected utility of the two strategies is practically the same. The average difference over the central range of parameters is 3.73% and a weighted average difference over all parameters is 2.48%. Only at high levels of interest and an expected life of 100 periods ($R/D=50$ and 10) does the optimal strategy perform markedly better, due to the higher saving associated with this strategy when high levels of wealth can be usefully built up. Not only that, but the difference in range considered for the rolling strategy does not make much difference either. The difference in performance between the 10 period and the 3 period rolling strategies is only large for $R/D=50$ and 10. This is emphasised in figures 18 and 19 where the variation in expected utility is plotted for the fixed values of B (fig.18) and R/D (fig.19) used in the experiment. The utility from the simple rule of consuming all income as soon as it is received is also plotted. It can be seen that the expected utility is insensitive to changes in saving strategy, to such an extent that for values of R/D of 0.002, 0.02, 0.10 and 1.0 there is no significant difference between the optimum and consuming everything immediately. This is true for all values of B . This is, however, not surprising given the parameter values used in the experiment. In particular, if a subject starts out with 0 wealth and the expected income per period is 10.2, it can be seen from fig.17 that for both the optimal and rolling strategies the best thing to do is to save little. Note that the consumption used in the rolling strategy is T_1 , the function for the current period. For the most common income of 10 and low levels of wealth, the difference in consumption is only 1 unit. Not only that, but for an income of 20 generating a possible asset level of 25 to 30, the consumption curves cross and there is no difference in saving at all. Therefore consumption is similar for both strategies and therefore the expected utility is about the same. The differences will only appear when the level of saving becomes high.

Table 7 Expected Utility of Optimal and Rolling Strategies

R = interest rate/period D = discount rate - (constant) probability of death at the end of the period
 B = risk aversion parameter in the individual's CARA utility function.

R/D	B 0.001				B 0.03				B 0.1				B 0.9			
	opt	roll3	roll5	roll10	opt	roll3	roll5	roll10	opt	roll3	roll5	roll10	opt	roll3	roll5	roll10
.002	.40	.13	.01	0.0	10.35	9.91	9.53	8.53	24.86	24.50	23.75	22.89	39.93	39.73	39.73	39.73
.02	.40	.40	.40	0.28	10.35	10.35	9.46	9.47	24.86	24.50	23.75	22.87	39.93	39.73	39.73	39.73
.10	2.02	1.69	.94	0.93	51.79	51.39	50.53	48.06	124.0	118.44	119.2	118.25	199.80	199.22	199.24	199.2
1.00	2.03	1.78	1.78	1.78	52.21	51.65	50.57	46.53	125.9	124.0	125.0	1120.6	199.90	199.26	199.76	199.7
10.0	20.29	20.83	20.85	21.85	526.5	385.2	438.4	522.3	1275.1	888.8	915.2	1017.0	1999.6	1379.6	1379.9	1380.0
50.0	252.85	51.89	51.91	51.91	1336.9	827.6	916.8	916.9	1915.9	1124.8	1301.5	1309.4	1999.9	1380.15	1380.17	1380.1

Average % differences in expected utility from optimum

Central range of parameters (R/D = 0.1, 1.0 B = 0.03, 0.1) 3.73%

Weighted average for all parameter values (weight 0.1 outside central range) 2.48%

Figure 18 Expected Utility of Strategies as R/D varies

B = 0.03 for all points

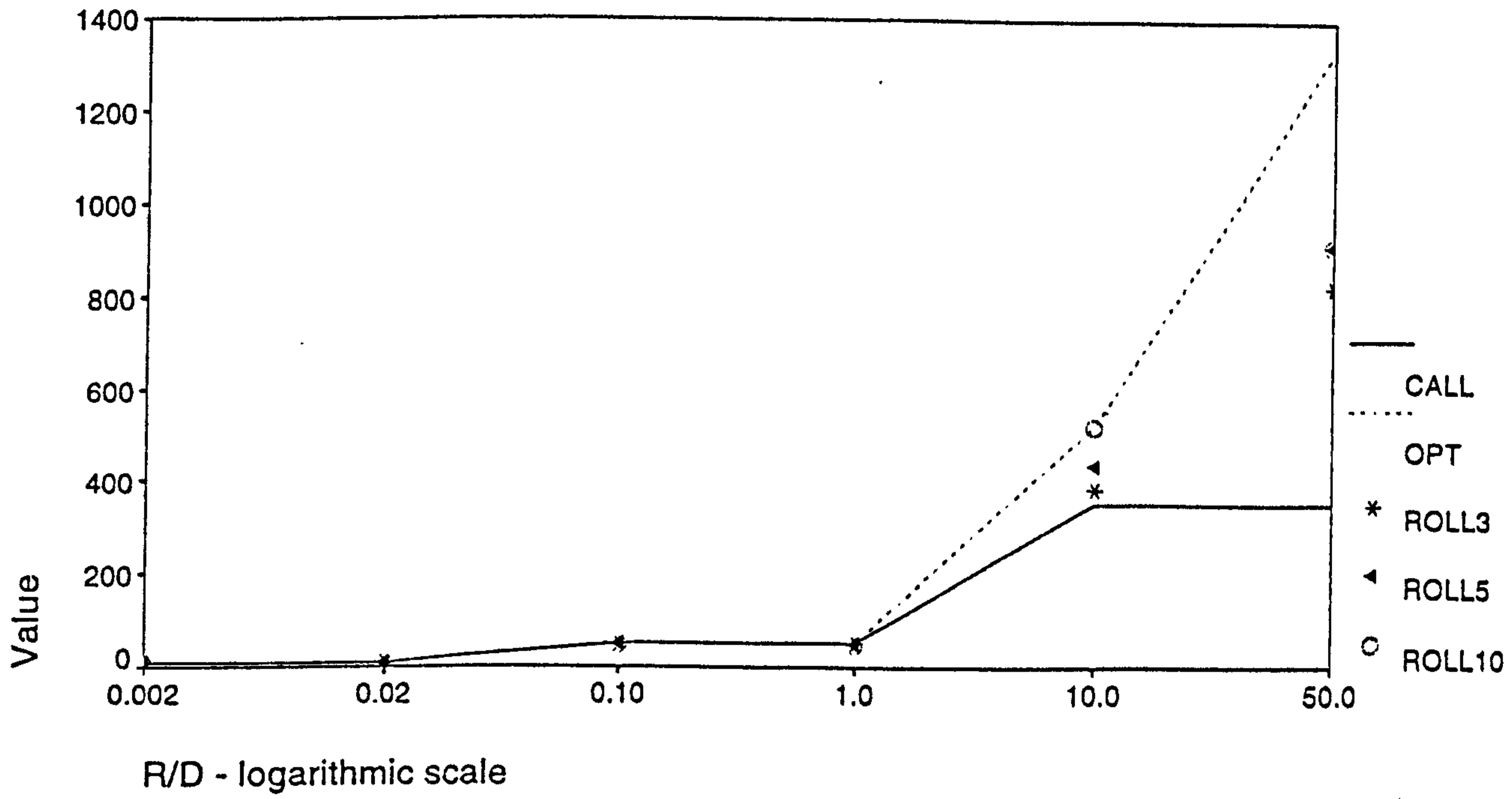
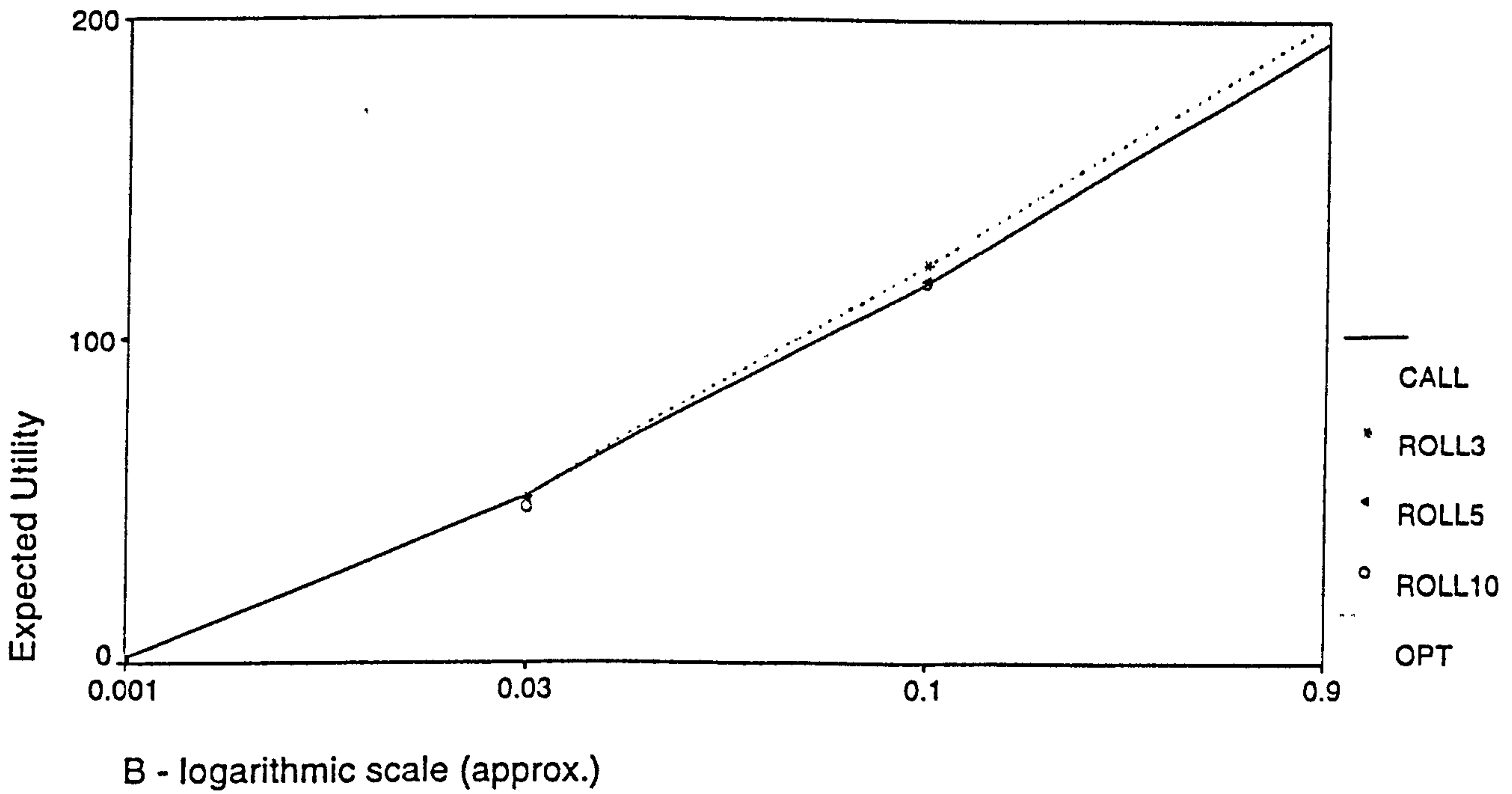


Figure 19 Expected Utility of Strategies as B varies

R/D = 1.0 for all points



For $R/D = 10$ and 50 (fig.19) it can be seen that the performances of the rolling strategies are spread out between the upper bound formed by the optimal strategy and the lower bound formed by consuming everything immediately. Of the three ranges, the 10 period rolling strategy performed the best for high R/D values. At these values, there is a high benefit from saving, as can be seen in fig.19 from the large difference in values between the optimum and consuming everything immediately. Fig.17 shows that for large values of wealth, as would happen with these R/D values and their low discount rate of 0.01, the 3 period rolling strategy undersaves compared to the optimum. This effect is reduced as the range of the rolling strategy increases; a 10 period strategy spreading consumption over 10 periods will save more than a 3 period strategy which assumes that consumption must be spread over 3 periods. The same effect makes the 10 period strategy perform worse than the 3 period strategy when the discount rate is high (0.5 for $R/D = 0.002$ and 0.02), since then the expected length of life is only 2 periods and spreading consumption over 10 periods means that a lot of wealth will be lost. This only has a slight effect on the relative values because there is very little difference between any of the strategies at these values of R/D .

One aspect of the results is that the expected value increases as curvature B increases for both strategies and all values of R/D . It might be expected that the risk-neutral (linear) points function (which is close to $B=0.001$) has the highest value as the subject does not save to avoid risk. However, this is in terms of points gained (or consumption) and not utility. When the CARA utility function is considered, as the curvature parameter increases the whole CARA function is raised giving a higher utility for a given consumption, so the expected utility must increase.

7.5 Conclusions

The performance of a rolling strategy was compared to the optimum for the income streams used in the experiment and, a little more generally, for a constant stopping probability or discount rate and a range of parameter values and rolling strategy ranges. It was found that for the relatively realistic parameter values used in the experiment, the rolling strategy was almost as effective as the optimal strategy - for the central range of parameters close to those used in the experiment, the rolling strategy was only 3.73%

worse than the optimum. If the subject was lucky and had a long and high income stream, the rolling strategy could actually outperform the optimum. The more general simulation showed that for parameter values apart from high values of R/D , the rolling strategy was virtually as effective as the optimum for the low levels of accumulated assets which are typical for the experiment. These results demonstrate that people can use a rolling decision strategy to simplify savings problems - indeed it enables them to perform some form of logical calculation - while losing hardly any utility (as long as they are not rich!).

The rolling strategies examined here are specific to the particular situation of the savings experiments of chs. 5 and 6. However, the concept can easily be applied to other intertemporal decision situations. As was shown in the theory of ch.3, in the context of a general savings decision, people can be assumed to apply the idea of a rolling strategy to a life cycle savings decision. It could also be used for investment decisions by firms, indeed the idea of a fixed payback time which is sometimes used in investment analysis is an application of a rolling strategy - the firm decides to look ahead for a fixed length of time and ignore the distant future. The idea could also be used to generate strategies for investment in financial or commodity markets, in fact the existence of a limited number of futures instruments only can be seen as a rolling strategy with some possible futures being ignored e.g. there are no futures for the price of oil in 1000 years time, to take an extreme example.

8 Summary and Conclusions

This final chapter draws together the different elements of this work and takes a general view of what this thesis has achieved. In the introduction, the objective of the thesis was described as the improvement of the psychological specification of people's decision making behaviour in economic decisions taken under conditions of uncertainty.

The area of individual (or household) savings behaviour was identified as having characteristics that are particularly suitable for this investigation. Savings is regarded as being an important economic decision which suggests that people will try to behave 'rationally'. This does not imply rational behaviour in the sense of constrained optimisation normally used in conventional economic analysis, but more generally the idea that people determine goals and have some decision rules which they apply consistently in order to try and achieve these goals. This definition of rationality is taken from Harsanyi (1986). The other aspect of saving decisions that suggests their usefulness for this analysis is the large degree of uncertainty associated with them. To make a fully optimal decision, predictions of tastes and parameters have to be made over the whole of a lifetime and their distributions used in the analysis. For a realistic specification of these distributions, this is extremely difficult and essentially impracticable for most people. In spite of the difficulties involved, the importance of these decisions means that they must be taken somehow, even though people are in general incapable of identifying the optimal strategy. This suggests that they simplify their decisions in some way (or even, perhaps do not even decide).

The literature survey in ch.2 identified the inadequacies of life cycle savings theory in its current state of development. This theory has been developed from the intertemporal analysis with quadratic utility functions used by Modigliani in his development of the Life Cycle Model. The achievements of the life cycle model are that utility functions with positive 3rd derivatives and liquidity constraints have enabled the theory to explain precautionary saving and the empirical evidence that consumption tracks current income. The inclusion of a bequest motive has allowed saving in retirement to be modelled. However, there are several features of savings behaviour that are still not explained. The fall in savings rates in the 1980s, the results of Carroll & Summers (1991) that the young save more than the old in countries with higher growth rates, the phenomenon of committed saving from Katona

(1975) and Lea, Tarpy and Webley (1987)'s evidence that saving increases in periods of high inflation (with lower real rates of return) are either not considered or generate savings patterns that are not predicted by life cycle theory.

Cochrane (1989) demonstrates the importance of a correct behavioural specification. He finds that deviations from fully optimising behaviour have only 2nd order effects on utility i.e. the penalties for not perfectly optimising are small. This implies that people may well save in ways that are considerably different from the optimum and it is therefore necessary to find out what rules they do use, as these may generate widely differing savings patterns from the optimum. Consequently, a more sophisticated specification of people's decision processes and their utilities in comparison with the extremely simple assumptions of the life cycle model - that agents optimise fully and derive utility solely from their personal in-period consumption (with the addition of utility from bequests) - may generate results that explain some of the behaviour noted above.

Simon's concept of bounded rationality (e.g. Simon (1987)) has several suggestions for improvements in the psychological specification of decision rules which have proved fruitful. His emphasis is on the concentration of attention on one problem at a time in isolation from other decisions and the simplification of the decision by concentrating attention on a few possibilities instead of working with the complete distributions of variables.

Chapter 3 developed a theory of savings behaviour along the lines suggested by Simon and chapter 4 describes the results of a computer simulation of the theory. The theory incorporates a more comprehensive specification of the benefits that people gain from saving by allowing for utility from wealth stocks themselves and from the act of saving as well as from consumption and bequests. The idea of commitment in saving was applied by including a pension instrument as well as a 'general' savings instrument. The decision process was drastically simplified by reducing the periods considered to the present period and a single representative period in the future (5 periods in the future for general saving and the first period of retirement for pensions), in contrast to the conventional theory in which all periods in the projected lifetime are taken into account.

These features generated results in the simulation which showed some new patterns of

behaviour compared with the conventional life-cycle model. There was a high level of saving early on in the lifetime in order to generate a stock of wealth and a bulge in saving as retirement approached. The early saving suggests an explanation for the evidence reported by Lea, Tarry & Webley that people save more with higher inflation: they may be trying to maintain the value of a wealth stock that they have accumulated. This early saving is also an alternative explanation for Carroll & Summers (1991) result. Committed saving played an important part in behaviour; it was used to generate a significant retirement income in addition to a state pension and income from accumulated wealth. In common with the modern life cycle model, consumption was found to track behaviour and there was a considerable level of bequests. Habit formation and concentration of attention on only one of the instruments at a time, together with a matching trade-off procedure instead of a maximising trade-off were also included. Overall, the theory has succeeded in explaining some of the stylised facts that are not in accordance with or are not considered by the conventional model in addition to including the stylised facts that have already been explained.

The rest of the thesis is more narrowly directed. It concentrates attention on the simplification of the savings decision process; in particular on the timing involved. The postulate of the theory of ch.2 that people only look a few periods into the future when they are taking general savings decisions was tested experimentally. This was done by having subjects take savings decisions, while a calculation facility was provided so that they could try out different savings paths and see how well these paths performed.

The experiments included some significant innovations in experimental method. Given the difficulty of the experiment for the subjects, considerable effort was devoted to giving them plenty of opportunity to learn about the problem and the computer interface, while making sure that all subjects received the same information. The subjects were given a presentation on the experiment, including instructions on how to use the calculation facility and examples showing the variability in performance of different strategies, in order to emphasise the uncertain end point. The presentation was recorded and played back from an audio cassette, coordinated with a display of the relevant screens from the experimental program and for the experiment at CentER, this presentation was shown as a video. This ensured that all subjects received exactly the same initial information even though the instructions and

examples were quite complex. These instructions and examples were repeated, both in written form and in the experimental program itself. The calculation facility provided a visual display of the problem through time by the incorporation of a range screen, where the subject could scroll through all possible time periods and see up to 13 periods displayed at once. The trials performed by the subjects using this calculator provided indirect information on the subjects' thought processes since the trials that they performed would have been a reflection of the decision process that they were using.

The first experiment at EXEC, University of York demonstrated the existence of three types of trials strategy:

i) a **learning strategy** in which several trials were performed in the first one or two periods of the main experiment. The subject was trying out different patterns of saving/conversion to get a feel for the numbers involved and to identify the results of various different patterns.

ii) a **rolling strategy** where there were trials all the way through the experiment, usually with a relatively small number of trials in any one period looking only a few periods ahead. The advantage of this strategy is that it dramatically simplifies the problem. The number of periods to be considered at any one time is greatly reduced, while allowing for a large and indefinite number of periods over the whole length of the experiment. If only a few periods ahead are considered, it is a reasonable approximation to assume that the experiment will not end over the periods being considered (given the stopping probability of 0.1) and hence it is possible to ignore one source of uncertainty.

iii) a **fixed end point strategy** where the subject chose a single period in the future as the end point for a series of trials that they performed over several periods.

These strategies were then identified in the experiment at CentER, University of Tilburg, in which the experiment was developed to examine the effect of variations in the interest rate, initial endowment and a greater variation in the income stream. The effects of a pension and a known length of life were also examined, the latter being a test for backward induction behaviour. The trials data for the EXEC experiment and the experiment at CentER were similar; in the EXEC experiment there was an average range of trials performed of 3.49 periods and the average range of trials was 3.85 in the experiment at CentER (excluding those trials performed by subjects in the backward induction treatment). This can be compared with the expected length of the experiment of 10 periods. There was no large

difference between the two sets of subjects. A combination of rolling and learning was by far the most common trials strategy (39% of subjects in the EXEC experiment and 40% of subjects in the experiment at CentER). The results of the regression analysis were also similar for the two experiments in that consumption depended most strongly on current income and current wealth. This is evidence that, under the conditions of uncertain future income and a liquidity constraint, consumption tracks income. In addition, scatter plots of wealth over time showed a strong non-linear trend over time; subjects built up wealth early on in the experiment when they had little wealth and dissaved later on when they believed that the experiment would soon finish. This reinforces the results of the theoretical model reported in ch.4 and is in contrast to the results of the conventional life cycle models.

The subjects performed a large number of trials and these trials had an average range much below the expected length of the experiment. By far the most common trials strategy was a rolling strategy in which the subject performed trials throughout the experiment, looking only a few periods ahead. This is strong evidence that people simplify savings decisions by restricting the amount of information that they take into account: they only look ahead for a short span of time, rather than the whole of a lifetime.

The last chapter contains a simulation of the performance of the rolling strategy in comparison with the optimal strategy. The rolling strategy was compared to the optimum for the income streams used in the experiment and, more generally, for a constant stopping probability or discount rate and a range of parameter values and rolling strategy ranges. It was found that for the relatively realistic parameter values used in the experiment, the rolling strategy was almost as effective as the optimal strategy. The more general simulation showed that for parameter values apart from high ratios of the interest to discount rate, the rolling strategy was virtually as effective as the optimum for the low levels of accumulated assets which are typical for the experiment. These results demonstrate that people can use a rolling decision strategy to simplify savings problems - indeed it enables them to perform some form of logical calculation - with virtually no loss of utility if there is no great incentive to building up a large wealth stock, which is a confirmation of Cochrane (1989)'s results.

To conclude, this thesis has been successful in developing a theory of savings at the microeconomic level which incorporates a more realistic specification of individual

behaviour than the conventional life cycle models. Together with a better institutional specification, allowing for pensions, this has generated results which demonstrate phenomena which have been observed, but which the conventional models cannot explain. The model has results which may be econometrically tested, if sufficient data is available. The age structure of saving is notably different from conventional models. A considerable rate of saving is predicted in the early part of people's working lives, very approximately between the ages of 20 and 30 years. After this, consumption increases at a greater rate than income until shortly before retirement, when there is again a high rate of saving until retirement. If data about savings and consumption, together with income and wealth of a cohort at different ages are available, they could be used to investigate whether these phenomena actually occur. The wider implications of this model are that overall saving is dependent on the age structure of the population; if the population has a high proportion of young working adults, or has a lot of people close to retirement, savings rates will be higher. Also, if people are more worried about the future, they will increase saving to try and compensate for their belief that a low income in the future is more likely.

The experimental work has demonstrated the existence of a rolling strategy which confirms one of the main assumptions of the theory, that only a limited period into the future is considered. This is in strong contrast to the conventional life cycle models, whose assumptions about the ranges of time considered by people are shown to be false.

This work is capable of considerable further development. The performance of the rolling strategy should be examined under a more general specification, especially for the income distribution. A simulation of the savings/consumption behaviour over the whole of the life cycle should be performed, to investigate whether the rolling strategy itself generates econometrically testable implications, without the various additional effects incorporated in the model of chs. 3 & 4. A theory which explains the motivation for committed saving needs to be written, incorporating the idea that preferences may change cyclically over time. This would give people an incentive to precommit to ensure that the decisions that they take would not be overturned in the future if their preferences change. Finally, the simulation of the experiments showed that the strategy used was more or less irrelevant in determining subjects' performance because the parameters were such that the optimal strategy was to consume most of income as soon as it was received. It would therefore be useful to conduct

a savings experiment in which subjects had a genuine incentive to build up a large wealth stock and to examine the patterns of trials and expenditure which would then result. This could be achieved by giving subjects a high expected income or a high rate of interest, together with a conversion function that had only slightly decreasing returns to scale.

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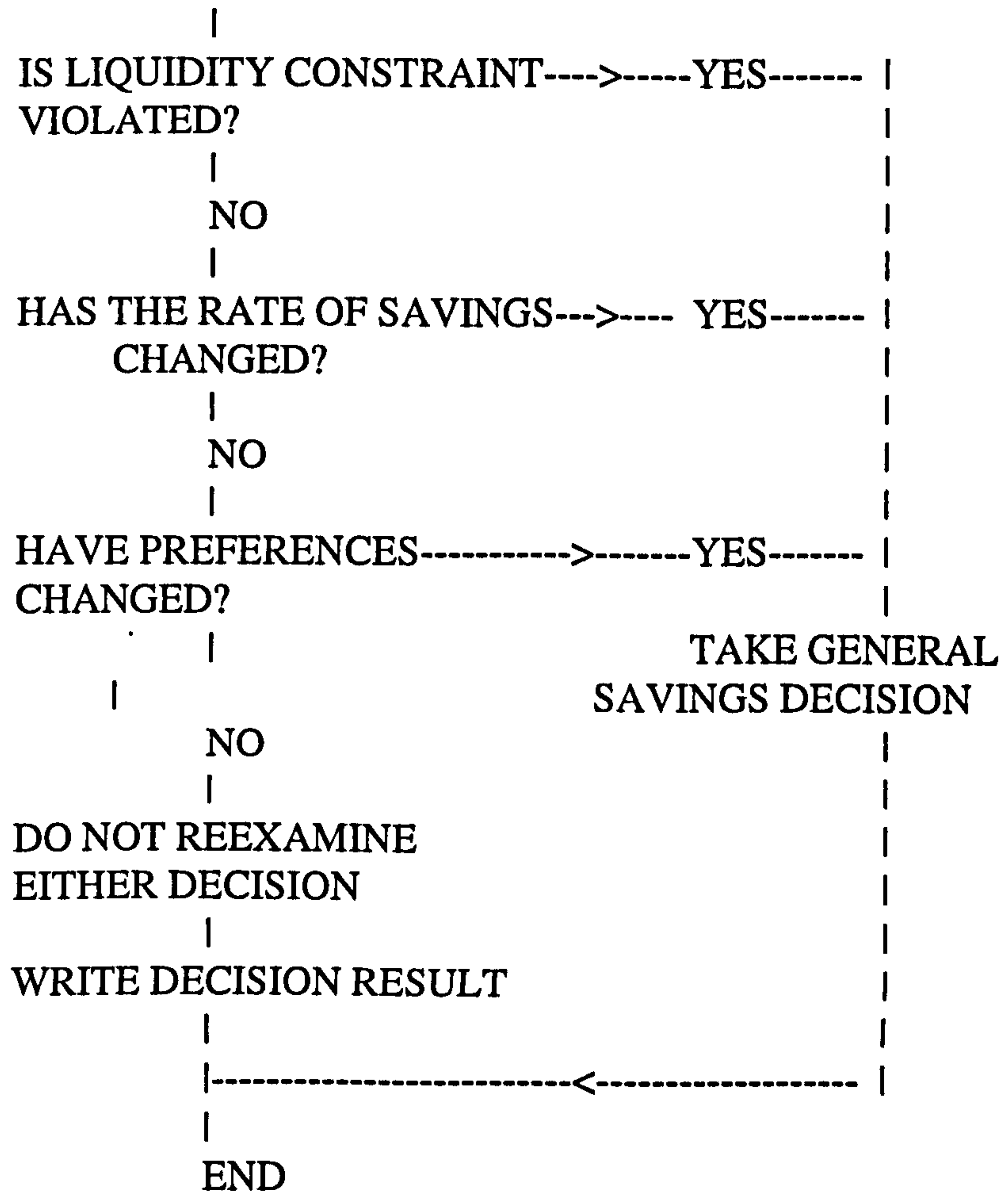
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CHECK GENERAL/
PRECAUTIONARY SAVING



Appendix 1 FORTRAN 77 program SIMUL7.FOR

```
PROGRAM SIMUL7

IMPLICIT INTEGER(T)

C SET NUMBER OF ITERATIONS
  PARAMETER (NAV=1000)
C SET NO OF TIME PERIODS
  PARAMETER (tlife=70, texPECT=5, TRET=45, Tstart=1)
C LIFE STARTS AT 20!!!
C NON DIMENSIONALISE BY HAVING Y1=1.0

C OBJ , SUBJ AND PATH
  PARAMETER (NSUBJ=6, NOBJ=4, NP=6)
  COMMON /ALPHA/ SUBJ(NSUBJ,tlife), OBJ(NOBJ,tlife)
  REAL PATH(NP,TLIFE),PATHAV(NP,TLIFE),X,G05CAF
  REAL H(TLIFE),HAV(TLIFE)
  EXTERNAL G05CAF, G05CCF
  CHARACTER*4 DEC, DECA(tlife)

C SUBJ IS AL:C BE:V GA:TH RLA:BEQ D P
C CRRAS U V TH B DISC (<1 FOR MATCHING) DEATH
C LOW VALUES OF AL BE GA RLA INCREASE VALUES
C OBJ IS DW H RW RPUP
C H IS INCOME DW IS WEALTH INNOVATION
C PATH IS S SP C W SIGPEN RETI
C GENSAV;W(T+1)-W(T) RETSAV CONS WEALTH

C SUBJECTIVE CONSTANTS TFG TIME PERIOD OF S UP D IS TIME
DISCOUNT RATE

C READ MONITORED VARIABLES INTO ARRAYS

  open(11,file='subj70.dat',status='old')
  open(12,file='obj70.dat',status='old')

19 FORMAT(4F8.3)
20 FORMAT(6F8.5)

  DO 25 K=1,TLIFE
  READ(11,20) (SUBJ(I,K),I=1,NSUBJ)
  READ(12,19) (OBJ(J,K),J=1,NOBJ)
25 CONTINUE
```


C SET SEED FOR RANDOM NO.GENERATOR

CALL G05CCF

C TAKE AVERAGE OF LOTS OF RUNS WITH RANDOM H

DO 210 IAV = 1,NAV

print*,'IAV=',IAV

C MAIN ROUTINE TO CALC PATH OVER LIFE

DO 200 T = TSTART, TLIFE

C PRINT*, 'T=',T

C SET DEFAULT UNCHANGED VALUES IN PATH

DO 50 J=1,4

PATH(J,T)=PATH(J,T-1)

50 CONTINUE

PATH(6,T) = PATH(6,T-1)

C GENERATE RANDOM H

IF (T .LE. TRET) THEN

P = G05CAF(X)

IF (P .LT. 0.3) THEN

WH=0.1

ELSE IF (P .GT. 0.9) THEN

WH=1.5

ELSE

WH=1.0

END IF

H(T) = OBJ(2,T)*WH

ELSE

H(T) = OBJ(2,T)

END IF

C STOP PENSION CONTRIBUTIONS AND CALC RETI AT TRET

IF (T .EQ. TRET+1) THEN

PATH(6,T) = PATH(6,T-1) + 1/OBJ(4,T)*PATH(5,T-1)

PATH(2,T) = 0.0

END IF

```

C SEE IF DECISION IS TO BE TAKEN
  CALL MONITOR(DEC, DECA, T, PATH, NP, WH)
c  PRINT *, 'DEC',DEC
  DECA(T) = DEC

C PERFORM TRADE OFF

  IF (DEC .EQ. 'GENP') THEN

    CALL GENPRE(T, C, S, PATH, NP, WH)
    PATH(3,T)=C
    PATH(1,T)=S

  ELSE IF (DEC .EQ. 'RETR') THEN

    CALL RETIRE(T, C, SP, PATH, NP, WH)
    PATH(3,T)=C
    PATH(2,T)=SP

  END IF

C CALCULATE WEALTH WT+1

  PATH(4,T) = PATH(4,T-1) + PATH(1,T)

C CALCULATE TOT PEN CONT. & SET RETI

C  PRINT*, 'SPT SPT-1 TOTSP', PATH(2,T), PATH(2,T-1), PATH(5,T)

  IF ((PATH(2,T) .LT. PATH(2,T-1)) .AND. (T .LE. TRET)) THEN
    PATH(6,T) = PATH(6,T-1) + 1/(OBJ(4,T)+8.0)*PATH(5,T-1)
    PATH(5,T) = PATH(2,T)

C  PRINT*, 'IF180 P5 P6', PATH(5,T), PATH(6,T)
  ELSE
    PATH(5,T) = PATH(5,T-1) + PATH(2,T)
C  PRINT*, 'ELSE180 P5 P6', PATH(5,T), PATH(6,T)
  END IF

  DO 95 KL = 1, NP
c  PRINT*, 'PATH', PATH(KL,T)
    PATHAV(KL,T) = PATHAV(KL,T) + PATH(KL,T)/NAV
  95 CONTINUE

C UPDATE AV H(T)

  HAV(T) = HAV(T) + H(T)/NAV

```



```

200     CONTINUE

210 CONTINUE

C OUTPUT RESULTS TO FILE
300 OPEN(10,FILE='path.res',status='old')
      OPEN(13,FILE='dec.res')
      OPEN(15,FILE='H.RES',STATUS='OLD')

305     FORMAT(6F10.5)

      DO 310 IJ=1,tlife
      WRITE(10,305) (PATHAV(IK,IJ),IK=1,NP)
C     WRITE(13,*) (DECA(IJ))
      WRITE(15,*) (HAV(IJ))
310 CONTINUE

      CLOSE(10)
      CLOSE(11)
      CLOSE(12)
      CLOSE(13)
      CLOSE(15)
      END

      SUBROUTINE MONITOR(DECI, DECAM, TT, PATHM, NPM, WHM)

      IMPLICIT INTEGER(T)

      REAL RJUDGE
      EXTERNAL RJUDGE

C SET NO OF MONITORED ECONOMIC VARIABLES
      PARAMETER (NSUBJ=6, NOBJ=4, TLIFE=70, TRET=45)

C BEHAVIOURAL MONITORING PARAMETERS
      PARAMETER (DTHRESH=0.500)

      COMMON /ALPHA/ SUBJ(NSUBJ,tlife), OBJ(NOBJ,tlife)

      REAL PATHM(NPM,TLIFE)
      CHARACTER*4 DECI, DECAM(TLIFE), DECC, DECG

C     print*,'IN MONITOR'

C IF PEN NOT CHECKED FOR LAST 10 YEARS

```

```
IF ((TT .GT. 10) .AND. (TT .LT. TRET)) THEN
DECC = 'ZERO'
DO 400 MI = 1,10
  IF (DECAM(TT-MI) .EQ. 'RETR') DECC = 'RETR'
400 CONTINUE
```

```
IF (DECC .NE. 'RETR') THEN
DECI = 'RETR'
GO TO 550
END IF
```

```
END IF
```

```
C IF SAV NOT CHECKED FOR LAST 5 YEARS
```

```
IF (TT .GT. 5) THEN
DECG = 'ZERO'
DO 405 MJ = 1,5
  IF (DECAM(TT-MJ) .EQ. 'GENP') DECG = 'GENP'
405 CONTINUE
```

```
IF (DECG .NE. 'GENP') THEN
DECI = 'GENP'
GO TO 550
END IF
```

```
END IF
```

```
H = OBJ(2,TT)*WHM
```

```
C SET SP = 0 IF IN RETIREMENT
```

```
IF (TT .GT. TRET) THEN
PATHM(2,TT) = 0.0
H = PATHM(6,TT)+OBJ(2,TT)
GO TO 475
END IF
```

```
C CHECK PENSION
```

```
DIFF1 = (OBJ(4,TT) - OBJ(4,TT-1))/OBJ(4,TT-1)
D = RJUDGE(DIFF1)
IF (D .GT. DTHRESH) THEN
C PRINT*, 'DIFF1', DIFF1, 'D', D
DECI = 'RETR'
GO TO 550
END IF
```


DIFF2 = (H - OBJ(2,TT-1))/OBJ(2,TT-1)

C H UP SP SMALL

IF ((RJUDGE(DIFF2) .GT. DTHRESH) .AND.
+ (H .GT. OBJ(2,TT-1)) .AND. (PATHM(2,TT) .LT. SAV*0.2)) THEN
C PRINT*, 'H UP SP SMALL'
DECI = 'RETR'
GO TO 550
END IF

C H DOWN SP LARGE

IF ((RJUDGE(DIFF2) .GT. DTHRESH) .AND.
+ (H .LT. OBJ(2,TT-1)) .AND. (PATHM(2,TT) .GT. SAV*0.1)) THEN
C PRINT*, 'H DOWN SP LARGE'
DECI = 'RETR'
GO TO 550
END IF

C CHECK WEALTH PATH

475 SAV = (PATHM(4,TT) + OBJ(1,TT) + H - PATHM(3,TT)
+ - PATHM(2,TT))*(1.0+OBJ(3,TT)) - PATHM(4,TT)

C PRINT*, 'P4O1HP3', PATHM(4,TT), OBJ(1,TT), H, PATHM(3,TT)
C PRINT*, 'P2O3SAV', PATHM(2,TT), OBJ(3,TT), SAV

C CHECK LIQUIDITY CONSTRAINT W>0

IF (PATHM(4,TT)+SAV .LT. 0.0) THEN
DECI = 'GENP'
C PRINT*, 'W CONST'
GO TO 550
END IF

C CHECK BUDGET IDENTITY

C IF (PATHM(1,TT-1) .NE. 0.0) THEN
C DIFF3 = (SAV - PATHM(1,TT-1))/PATHM(1,TT-1)
C ELSE
DIFF3 = SAV - PATHM(1,TT-1)
C END IF

IF (RJUDGE(DIFF3) .GT. DTHRESH) THEN
C PRINT*, 'SAV CHANGE', 'DIFF3', DIFF3
DECI = 'GENP'
GO TO 550

END IF

C CHECK SUBJECTIVE PARAMETERS

DO 500 IM=1,NSUBJ

IF (SUBJ(IM,TT-1) .EQ. 0.0) THEN

DIFF4 = SUBJ(IM,TT) - SUBJ(IM,TT-1)

ELSE

DIFF4 = (SUBJ(IM,TT) - SUBJ(IM,TT-1))/SUBJ(IM,TT-1)

END IF

IF (RJUDGE(DIFF4) .GT. DTHRESH) THEN

DECI = 'GENP'

C PRINT*,'SUBJ',IM,'CHANGE'

GO TO 550

END IF

DECI = 'ZERO'

500 CONTINUE

550 END

FUNCTION RJUDGE(DIF)

REAL G05DDF

EXTERNAL G05DDF

C USE SD OF 0.05 AS THRESH SO DIFF OF 0.165 > DEC 95% OF THE TIME
PARAMETER (SIG=0.03)

X=G05DDF(DIF,SIG)

RJUDGE=ABS(X)

OPEN(14,FILE='judge.res')

C WRITE(14,*) RJUDGE

END

FUNCTION GP1(T1, R1)

C FUNCTION TO CALC CONST OF GP SUM 1 TO N

INTEGER T1

REAL R1,A

A=1.0+R1

$GPI = A*(1.0-(A**T1))/(1.0-A)$

END

REAL FUNCTION CRRA(A,B)

C CRRA UTILITY FN.

REAL A,B

IF (B .GE. 0.0) THEN

CRRA = 1.0/(1.0-A)*B**(1.0-A)

ELSE IF (B .LT. 0.0) THEN

CRRA = -1.0/(1.0-A)*(-1.0*B)**(1.0-A)

END IF

END

REAL FUNCTION UMATCH(C1U,C2U,C3U,C4U,GPU,VBU,CU,TU)

C TRANSCENDENTAL MATCHING EQN.

IMPLICIT INTEGER(T)

REAL C1U, C2U, C3U, C4U, GPU, VBU, CU, CRRA

EXTERNAL CRRA

C SET NO OF MONITORED ECONOMIC VARIABLES

PARAMETER (NSUBJ=6, NOBJ=4, TLIFE=70, TRET=45, TF=5)

C EXPECTATIONS PARAMETERS

PARAMETER (DHFH=1.5, PH=0.1, DHFL=0.1, PL=0.3)

COMMON /ALPHA/ SUBJ(NSUBJ,tlife), OBJ(NOBJ,tlife)

R = OBJ(3,TU)

AL = SUBJ(1,TU)

BE = SUBJ(2,TU)

RLAM = SUBJ(4,TU)

P = SUBJ(6,TU)

C FUTURE VALUES

ALF = AL*0.5

BEF = BE

RLAMF = RLAM

WF = C3U - CU*GPU

CF = C2U + R/(1.0+R)*WF

```

      IF (TU .LE. TRET-TF) THEN
      CFH = C2U*DHFH + R/(1.0+R)*WF
      CFL = C2U*DHFL + R/(1.0+R)*WF
      UCF = (1.0-PH-PL)*CRRA(ALF,CF) + PH*CRRA(ALF,CFH)
+      + PL*CRRA(ALF,CFL)
      ELSE
      UCF = CRRA(ALF,CF)
      END IF

C   PRINT*,'WFCFCTC4VB',WF,CF,CU,C4U,VBU,'P',P
C   PRINT*,'ALFBEFRLAMF',ALF,BEF,RLAMF

      UMATCH = CRRA(AL,CU) + VBU -
+ C4U*(UCF + CRRA(BEF,WF) + P/(1.0-P)*CRRA(RLAMF,WF))

C   PRINT*,'CRRA(AL,CU)',CRRA(AL,CU),'ALFCF',CRRA(ALF,CF)
C   PRINT*,'CR(WF)CR(BEQ)',CRRA(BEF,WF),P/(1.0-P)*CRRA(RLAMF,WF)

      S = CIU - CU*(1.0+R)

      IF (S .GT. 0.0) UMATCH = UMATCH + CRRA(GAM,S)

C   PRINT*,'CU:U',CU,UMATCH

      END

      subroutinE GENpre(tg, CTG, STG, PATHG, NPG, WHG)

      IMPLICIT INTEGER(T)

      REAL GP1, CRRA, UMATCH
      EXTERNAL GP1, CRRA, UMATCH

C SET NO OF TIME PERIODS
      PARAMETER (TRET=45, TLIFE=70)

      REAL CTG, STG, PATHG(NPG, TLIFE)

C SET NO OF MONITORED ECONOMIC VARIABLES
      PARAMETER (NSUBJ=6, NOBJ=4, TFG=5)

      COMMON /ALPHA/ SUBJ(NSUBJ,tlife), OBJ(NOBJ,tlife)

C EXPECTATIONS PARAMETERS

```


PARAMETER (EINCH=0.05)

C ITERATION TOLERANCE

PARAMETER (ACC=0.01)

C PRINT*, 'IN GEN'

C INCOME BEFORE OR IN RETIREMENT

IF (TG .LE. TRET) THEN
H = OBJ(2,TG)*WHG
ELSE

C INCLUDE STATE PENSION

H = PATHG(6,TG)+OBJ(2,TG)
END IF

C GENERATE FUTURE VARIABLES

C IF (TG .LE. TFG) THEN
C HF = OBJ(2,TG)*(1.0+EINCH)*TFG
C SPF = PATHG(2,TG)
C ELSE IF ((TG .GT. TFG) .AND. (TG .LE. TRET-TFG)) THEN
C DH=0
C
C DO 590 JJ=1,TFG
C DH = DH + OBJ(2,TG+1-JJ) - OBJ(2,TG+1-JJ)
C 590 CONTINUE
C
C A=TFG
C DH=DH/A
C HF = OBJ(2,TG)+DH*TFG
C SPF = PATHG(2,TG)
C
C ELSE IF ((TG .GT. TRET-TFG) .AND. (TG .LE. TRET)) THEN
C HF = PATHG(6,TG) + 1/OBJ(4,TG)*(PATHG(5,TG-1)
C + PATHG(2,TG)*(TRET-TG))
C SPF = 0.0
C
C ELSE
C HF = PATHG(6,TG)
C SPF = 0.0
C

C END IF

C GENERATE FUTURE VARIABLES

IF (TG .LE. 3) THEN
HF = OBJ(2,TG)*(1.0+EINCH)*TFG
SPF = PATHG(2,TG)

ELSE IF ((TG .GT. 3) .AND. (TG .LE. TRET-TFG)) THEN
DH = 0.5*(OBJ(2,T-1) - OBJ(2,T-2))
HF = OBJ(2,TG) + TFG*DH
SPF = PATHG(2,TG)

ELSE IF ((TG .GT. TRET-TFG) .AND. (TG .LE. TRET)) THEN
HF = PATHG(6,TG) + 1/OBJ(4,TG)*(PATHG(5,TG-1)
+ PATHG(2,TG)*(TRET-TG))
SPF = 0.0

ELSE
HF = PATHG(6,TG)
SPF = 0.0

END IF

C GENERATE FUTURE VARIABLES

C

C

C IF (TG .LE. TRET-TFG) THEN

C HF = OBJ(2,TG)

C SPF = PATHG(2,TG)

C ELSE IF ((TG .GT. TRET-TFG) .AND. (TG .LE. TRET)) THEN

C HF = PATHG(6,TG) + 1/OBJ(4,TG)*(PATHG(5,TG-1)

C + PATHG(2,TG)*(TRET-TG))

C SPF = 0.0

C ELSE

C HF = PATHG(6,TG)

C SPF = 0.0

C END IF

R = OBJ(3,TG)

C TRANSCENDENTAL EQN FOR CT FROM MATCHING AND BUD IDENTITY
C ASSUMING WF CONSTANT I.E. SF =0 FOR CF PF=PT GF=0 SPF=SPT

C PRINT*,'HHFRW',H,HF,R,PATHG(4,TG-1),'SP',PATHG(2,TG)

C PRINT*,'CR(S(2,TG)P(4,TG-1))', CRRA(SUBJ(2,TG),PATHG(4,TG-1))


```

GP = GP1(TFG,R)
VB = CRRA(SUBJ(2,TG),PATHG(4,TG-1))
+ + SUBJ(6,TG)/(1.0+SUBJ(6,TG))*CRRA(SUBJ(4,TG),PATHG(4,TG-1))

C1 = (OBJ(1,TG) + H - PATHG(2,TG))*(1.0+R)
+ + R*PATHG(4,TG-1)

```

```

C CF ASSUMING WF = WF+1

```

```

C2 = HF - SPF
C3 = (PATHG(4,TG-1) + OBJ(1,TG))*(1.0+R)**TFG
+ + (H - PATHG(2,TG))*GP1(TFG,R)

```

```

C DISCOUNT= 2 - EXP(-(1-D)TFG) SO D REP DECREASE/YEAR DISC=0 FOR
T=0

```

```

C4 = 2.0 - EXP(-1.0*(1.0-SUBJ(5,TG))*TFG)

```

```

C PRINT*,'VB',VB,'GP',GP
C PRINT*,'C1',C1,'C2',C2,'C3',C3,'C4',C4

```

```

C SOLVE TRANSCENDENTAL EQN BY MODIFIED LINEAR INTERP APPLIED
NUM ANAL

```

```

C BY C F GERALD P O WHEATLEY P11

```

```

C START VALUES C=0 BOTTOM C SO WF = 0 OR WT+1=0 WHICHEVER IS
C SMALLER TOP (LQUIDITY CONSTRAINT)

```

```

CWZERO = PATHG(4,TG-1) + OBJ(1,TG) + H - PATHG(2,TG)
CWFZERO = C3/GP

```

```

C PRINT*,'CW10 CWF0',CWZERO, CWFZERO

```

```

CL = MIN(CWZERO,CWFZERO)

```

```

C PRINT*,'CL',CL

```

```

C CHECK BDY SOLN.

```

```

IF (UMATCH(C1,C2,C3,C4,GP,VB,CL,TG)*
+ UMATCH(C1,C2,C3,C4,GP,VB,0.0,TG) .GT. 0.0) THEN
C PRINT*,'FCWF0',UMATCH(C1,C2,C3,C4,GP,VB,CFW0,TG)
C PRINT*,'F0',UMATCH(C1,C2,C3,C4,GP,VB,0.0,TG)
C PRINT*,'BDY SOLN'
CTG = CL
GO TO 650
END IF

```

```

C INTERP ROUTINE

```

```

C   PRINT*,'CL',CL
    X1 = 0.0
    X2 = CL
    SAV = UMATCH(C1,C2,C3,C4,GP,VB,X1,TG)
    F1 = UMATCH(C1,C2,C3,C4,GP,VB,X1,TG)
    F2 = UMATCH(C1,C2,C3,C4,GP,VB,X2,TG)

C   PRINT*,'CF=X2F1F2',X2,F1,F2

    ICOUNT = 0

600  X3 = X2 - F2*(X2-X1)/(F2-F1)

    ICOUNT = ICOUNT+1
    IF (ICOUNT .GT. 50) GO TO 640
    IF ((ABS(X3-X2) .LT. 0.00001) .AND.
+ (ABS(X2-X1) .LT. 0.00001)) GO TO 640

C   PRINT*,'X123',X1,X2,X3

    F3 = UMATCH(C1,C2,C3,C4,GP,VB,X3,TG)

C   PRINT*,'F123',F1,F2,F3

    IF (F3*F1 .LT. 0.0) THEN
      X2 = X3
      F2 = F3

      IF (F3*SAV .GE. 0.0) THEN
        F1=F1/2.0
      END IF

    ELSE
      X1 = X3
      F1 = F3

      IF (F3*SAV .GE. 0.0) THEN
        F2=F2/2.0
      END IF

    END IF

    SAV = F3

    IF (ABS(SAV) .GT. ACC) THEN
C   PRINT*,'SAV',SAV
    GO TO 600
    ELSE
640  CTG = X3

```



```

        END IF

650  WT1 = (PATHG(4,TG-1) + OBJ(1,TG) + H - CTG
      +   - PATHG(2,TG))*(1.0+R)

C    PRINT*,'WT1 WT G',WT1,PATHG(4,TG-1),OBJ(1,TG)
C    PRINT*,'H SP R',H,PATHG(2,TG),R

      STG = WT1 - PATHG(4,TG-1)

C    PRINT*,'STG',STG,'CTG',CTG

655  END

subroutine RETIRE(TR, CTR, SPT, PATHR, NPR, WHR)

      IMPLICIT INTEGER(T)

      REAL GP1
      EXTERNAL GP1

C SET NO OF TIME PERIODS
      PARAMETER (tlife=70, TEXPECT=5, TRET=45)

      REAL CTR, SPT, PENR, PATHR(NPR, TLIFE)

C EXPECTATIONS PARAMETERS DELD = LONG TERM/SHORT TERM DISC
      PARAMETER (EINCH=0.05, EXW=1.0, DELD=1.05)

C SET NO OF MONITORED ECONOMIC VARIABLES
      PARAMETER (NSUBJ=6, NOBJ=4)

      COMMON /ALPHA/ SUBJ(NSUBJ,tlife), OBJ(NOBJ,tlife)

C    PRINT*,'IN RET'

C TRADE OFF ROUTINE: MATCHING WITH UCT=UCR*DISCOUNT

C CALC FUTURE VARIABLES HR = INC. AT RET

      YNP = TRET-TR+1.0
      HR = OBJ(2,TR)*(1.0+EINCH*YNP)

```

WR = PATHR(4,TR-1)*EXW

SR=0

RR=0

DO 675 II=0,TEXPECT-1

RR = OBJ(3,TR-II)/(TEXPECT-1) + RR

675 CONTINUE

C PENSION RETURN PARAMETER = PROPORTION OF FINAL H
RECEIVED/UNIT CONT/YEAR

RP = OBJ(4,TR)

RPDOWN = RP + 8.0

c initialise j

J=0

C EQN FOR CT FROM MATCHING AND CR GIVEN FUTURE VARIABLES &
DISCOUNTING

685 C10 = HR/RP/OBJ(2,TR)

C C11=SPT+CT FROM BUD ID WITH ST=ST-1

C11 = PATHR(4,TR-1) + OBJ(2,TR)*WHR

+ - (PATHR(4,TR-1) + PATHR(1,TR))/(1.0+OBJ(3,TR))

C print*,'w',pathr(4,tr-1),'h',obj(2,tr)*whr,'s',pathr(1,tr)

C12 = (2.0 - EXP((SUBJ(5,TR)*DELD-1.0)*YNP))

+ **(1.0/(1.0-SUBJ(1,TR)))

C PRINT*,'S5DEL YNP S3',SUBJ(5,TR)*DELD, YNP, SUBJ(1,TR)

C CRET = -CTC10YNP + C11C10YNP + PREV PENS + RWRET - SRET

C <-----C13----->

C13= C11*YNP*C10 + PATHR(6,TR) + PATHR(5,TR-1)*C10 +RR*WR-SR

C PRINT*,'C1011213J',C10, C11, C12, C13, J

CTR = C13*C12/(1.0+YNP*C10*C12)

SPT = C11 - CTR

C PRINT*,'SP CT',SPT,CTR

C USE LOW RP IF SP IS DECREASED


```
IF ((SPT .LT. PATHR(2,TR-1)) .AND. (J .NE. 1)) THEN
  RP = RPDOWN
  J=1
  GO TO 685
END IF
```

C IF SP DECREASED BUT UP WITH LOW SP SP = SPT-1

```
IF ((SPT .GE. PATHR(2,TR-1)) .AND. (J .EQ. 1)) THEN
  SPT = PATHR(2,TR-1)
  CTR = C11 - SPT
END IF
```

C CAN'T BORROW IN PENSION FUND

```
IF (SPT .LT. 0.0) THEN
  SPT = 0.0
  CTR = C11
END IF
```

C PRINT*,'SP CT',SPT,CTR

END

Appendix 2 Simulation Results for figure 2

Parameter values

Threshold difference for taking a decision	0.5
Standard deviation of subjective difference distribution	0.03
Discount rate	0.05 in period 1, increasing linearly to 0.5 in period 70
Interest rate on general savings	0.05
Return parameter for pension savings	4.0
Income Process: Discrete random process with 3 possible income levels	
Central value 1.0 in period 1 increasing to 1.4 in period 45	
Low income = 0.1*central value with probability of 0.3	
high income = 1.5*central value with probability of 0.1	
Preference parameters in CRRA utility functions $U(s) = (1/(1-\alpha))s^{1-\alpha}$ see equation 4.2	
Consumption $U(c)$	$\alpha = 0.8$
Utility of thrift $TH(s)$	$\alpha = 0.05$
Utility of a stock of wealth $V(w)$	$\alpha = 0.05$
Utility of bequests $B(w)$	$\alpha = 0.05$

General Saving	Pension Saving	Consumption	Wealth	Pension Contributions	Retirement Income	Period
0.00000	0.02104	0.76924	0.00000	0.02104	0.00000	1
0.02752	0.01621	0.74414	0.02752	0.03075	0.00054	
0.03657	0.01421	0.75510	0.06409	0.03832	0.00109	
0.13003	0.01308	0.68941	0.19411	0.04346	0.00176	
0.17290	0.01224	0.64489	0.36702	0.04698	0.00248	
0.21578	0.01218	0.61139	0.58280	0.05341	0.00296	
0.23458	0.01108	0.61809	0.81738	0.05323	0.00390	
0.23555	0.01118	0.63114	1.05293	0.05694	0.00452	
0.23619	0.01098	0.64048	1.28912	0.06065	0.00513	
0.22749	0.01113	0.68059	1.51662	0.06353	0.00582	10
0.22497	0.01084	0.69741	1.74159	0.06359	0.00671	
0.21936	0.01178	0.71711	1.96095	0.05842	0.00813	
0.20690	0.01210	0.74361	2.16785	0.06169	0.00886	
0.19992	0.01376	0.78868	2.36777	0.06496	0.00974	
0.19091	0.01516	0.80661	2.55867	0.06590	0.01092	
0.18099	0.01626	0.81191	2.73967	0.07105	0.01185	
0.17189	0.01565	0.83368	2.91156	0.07362	0.01294	
0.16409	0.01663	0.86683	3.07565	0.07496	0.01421	
0.14998	0.01737	0.87273	3.22563	0.07658	0.01552	
0.14689	0.01850	0.92612	3.37252	0.08333	0.01650	20
0.13796	0.01866	0.91381	3.51048	0.08114	0.01824	
0.13154	0.02173	0.98062	3.64201	0.08875	0.01942	
0.12193	0.02323	0.99157	3.76394	0.09379	0.02093	
0.11082	0.02321	0.98011	3.87476	0.09649	0.02264	
0.10272	0.02440	0.99952	3.97748	0.10136	0.02427	

General Saving	Pension Saving	Consumption	Wealth	Pension Contributions	Retirement Period Income	
0.09624	0.02522	1.02505	4.07373	0.09959	0.02652	
0.08775	0.02679	1.03888	4.16148	0.10354	0.02842	
0.07547	0.02854	1.05639	4.23695	0.10903	0.03034	
0.06757	0.03108	1.06692	4.30452	0.11529	0.03241	
0.05990	0.03564	1.08934	4.36442	0.12593	0.03449	30
0.05490	0.03573	1.11821	4.41932	0.13108	0.03704	
0.04760	0.03881	1.13530	4.46692	0.13965	0.03956	
0.04160	0.04404	1.14590	4.50852	0.15051	0.04233	
0.03623	0.04884	1.16223	4.54475	0.16283	0.04537	
0.02980	0.05396	1.19396	4.57454	0.16696	0.04952	
0.01894	0.06030	1.17504	4.59349	0.19200	0.05246	
0.01626	0.06815	1.19187	4.60974	0.19944	0.05752	
0.00368	0.06960	1.14941	4.61343	0.21065	0.06239	
-0.00207	0.07861	1.19910	4.61135	0.24385	0.06617	
-0.00680	0.08495	1.20348	4.60456	0.25592	0.07224	40
0.06889	0.10033	1.11476	4.67345	0.30208	0.07676	
0.10419	0.10553	1.07367	4.77764	0.33851	0.08252	
0.12175	0.12549	1.08663	4.89939	0.37843	0.08965	
0.12356	0.13410	1.06582	5.02295	0.39325	0.09959	
0.12331	0.13202	1.07651	5.14626	0.38973	0.11088	45 retirement
0.03004	0.00000	0.49067	5.17629	0.38973	0.20832	
0.02587	0.00000	0.49732	5.20217	0.38973	0.20832	
0.02035	0.00000	0.50740	5.22253	0.38973	0.20832	
0.01396	0.00000	0.51922	5.23648	0.38973	0.20832	
0.00384	0.00000	0.53962	5.24033	0.38973	0.20832	50
-0.00249	0.00000	0.55909	5.23784	0.38973	0.20832	
-0.02992	0.00000	0.58706	5.20792	0.38973	0.20832	
-0.03099	0.00000	0.58799	5.17694	0.38973	0.20832	
-0.03247	0.00000	0.58923	5.14446	0.38973	0.20832	
-0.03418	0.00000	0.59051	5.11028	0.38973	0.20832	
-0.03790	0.00000	0.59321	5.07238	0.38973	0.20832	
-0.04329	0.00000	0.59724	5.02910	0.38973	0.20832	
-0.06209	0.00000	0.60914	4.96701	0.38973	0.20832	
-0.06278	0.00000	0.60941	4.90423	0.38973	0.20832	
-0.06373	0.00000	0.60972	4.84050	0.38973	0.20832	60
-0.06478	0.00000	0.60988	4.77571	0.38973	0.20832	
-0.06707	0.00000	0.61017	4.70864	0.38973	0.20832	
-0.07044	0.00000	0.61073	4.63820	0.38973	0.20832	
-0.08165	0.00000	0.61003	4.55655	0.38973	0.20832	
-0.08201	0.00000	0.60979	4.47454	0.38973	0.20832	
-0.08250	0.00000	0.60939	4.39204	0.38973	0.20832	
-0.08300	0.00000	0.60872	4.30905	0.38973	0.20832	
-0.08404	0.00000	0.60716	4.22500	0.38973	0.20832	
-0.08560	0.00000	0.60504	4.13941	0.38973	0.20832	
-0.09035	0.00000	0.59499	4.04906	0.38973	0.20832	70

APPENDIX 3

Instructions for the Savings Experiment

This experiment is an attempt to find out about how people take savings decisions. It simulates a life in which people receive income and then spend and/or save it. There is a savings experiment and then a payoff mechanism. In the savings experiment, you take savings decisions over a number of periods and try out different strategies. In each period, you receive an income in tokens. These tokens can be either converted into points, or they can be saved for conversion in a later period. Tokens that are saved have interest paid on them. The aim is to get the maximum possible number of points.

To find your payoff, the points that you have earned are used in a lottery between a high prize and a low prize. The more points you have, the greater your chance of winning the high prize.

The Saving Experiment

You use a PC to take saving decisions. At the beginning of each period your income is randomly chosen and shown on the screen. Then your current situation is displayed and you can try out different savings strategies on the computer before you choose how many tokens to convert to points. You can explore as many different possibilities as you wish. The current information screen and instructions for the trials are shown in the handout.

Tokens are converted into points as shown in the graphs in the handout. You may convert as many tokens as you wish up to the total of your income plus your wealth. Wealth is tokens that you have saved in previous periods plus interest. After the last period, the points that you have converted in all the periods are added up.

There are two elements of uncertainty in this experiment:

1. You do not know how many periods there will be. At the end of each period, there is a chance that the savings experiment will finish. Any tokens that you have saved up and not converted will not be worth anything.
2. You do not know exactly what your income is going to be in the future. In each period, the income will be one of 3 possible numbers.

The chance of the savings experiment ending at the end of a period and the possible incomes are shown in the table at the end of the handout.

The Payoff Mechanism

To get your payoff, you play a lottery. The points that you have accumulated will be a number between 1 and 100. You throw a pair of percentage dice (1 to 100). If the number on the dice is lower than your points total, you receive £25. If the number on the dice is higher than your points total, you receive £4.

At the end of the payoff instructions, press C to start the experiment.

USING THE TRIAL FACILITY TO TRY OUT DIFFERENT SAVINGS STRATEGIES

Spending all income as it is received.

period	3	4	5	6	7
income	10	10	2	20	10
convert	10	10	2	20	10
wealth	0.0	0.0	0.0	0.0	0.0

total points at period 5 = 39

You could choose to save all income and convert all your wealth in one period. Total points at the end if the savings experiment finishes at period 5 is changed to 24 points for the same run of incomes and starting wealth.

Saving all income and converting all wealth in one period.

period	11	12	13	14	15
income	10	10	2	20	10
convert	0	0	0	0	56
wealth	10.5	21.5	24.7	46.9	1.0

total points at period 5 = 24

But, remember that there is a 10% chance that the experiment may stop at the end of any period, so it may last for a large number of periods or a small number of periods. Over a longer number of periods, you could have the following result, earning 88 points:

period	1	2	3	4	5	6	7	8	9	10	11	12
income	10	10	2	20	10	2	10	10	10	20	2	10
convert	5	7	4	15	8	5	17	12	9	20	4	14
wealth	5.3	8.7	7.0	12.6	15.3	12.9	6.2	4.5	5.7	6.0	6.3	2.4

total points at period 12 = 88

Over a shorter number of periods, the same income stream and savings strategy will give you a different number of points. Here, the total at the end is 35 points.

period	1	2	3	4	5	6
income	10	10	2	20	10	2
convert	5	7	4	15	8	5
wealth	5.3	8.7	7.0	12.6	15.3	12.9

total points at period 6 = 35

This is the information about your current situation that is displayed each turn:

THIS IS YOUR CURRENT SITUATION

This is period ____

Your income this turn is ____ tokens

Your wealth at the beginning of the period is ____

You may convert up to ____ tokens this period

You have so far earned ____ tokens from earlier periods

The wealth you have not converted increases by ____ percent at the beginning of the next period

There is a ____ percent chance that there will not be a next period

Table

When does the saving stage finish?

The end of the saving stage is determined by chance. At the end of each period, there is a 10% chance of the current period being the final period, so the savings stage may last for a small or large number of periods.

What income will you receive?

In each period, your income is picked at random from 3 numbers.

There is a 10% chance that your income will be 20 tokens

There is a 80% chance that your income will be 10 tokens

There is a 10% chance that your income will be 2 tokens

Appendix 4 Trial Ranges for Each Subject

TABLE OF NUMBER AND RANGES OF TRIALS BY EACH SUBJECT

SUBJECTS NOT AUTOMATICALLY PUT IN TRIAL OPTION

subject no.	range of periods considered by each subject (1period trials ignored)												
	2	3	4	5	6	7	8	9	10	11	12	14	52
2			1	2				1					
3							4						
4		2	2	1	1			2					
6	2												
11		1											
12			1	1									
16				1							1		
17	1												
18			2										

MEAN OF Range 5.54

STD. DEVIATION 2.72

SUBJECTS AUTOMATICALLY PUT IN TRIAL OPTION: SUBJECTS HAVE TO USE TRIAL FACILITY TO TO FIND CONVERSION FROM TOKENS TO POINTS

21	7	1	2										
23	9	4	6										
24		1	5	5	5	4	1						
25	7			1	2								
26	15	3											
27		14	13		1								
28							1	3				1	
29	1	1	1										1
30					4				7				
31	8	3	4	1									
32	6	8											
33	5			6									
34	56	6											
35	5	4											
36	4	3	1										
37			4										
38		13		11	2								
39	2						2						
40	2	5	4	2									

MEAN OF Range 3.49

STD. DEVIATION 2.30

TABLE OF TRIALS IN EACH PERIOD BY EACH SUBJECT

trials with a range of only one period are ignored

the numbers in the table are the ranges of each trial carried out in that period

- means that the experiment had finished

0 means that no trials were carried out

SUBJECTS NOT AUTOMATICALLY PUT IN TRIAL OPTION

subject no.	period of experiment											
	1	2	3	4	5	6	7	8	9	10	11	12
02	1-5;1-5;1-9	1-4	0	0	0	0	0	0	-	-	-	-
03	1-8;1-8;1-8;1-8	0	0	0	0	0	0	0	0	0	0	-
04	1-9;1-5;1-9;1-4	2-4;2-4	3-6	4-9	0	0	0	0	0	0	0	-
06	5-6;5-6; 2-4	0	0	0	0	0	0	0	0	-	-	1-
12	0	2-6	3-6	0	0	0	0	0	0	0	0	-
16	5-12;1-12	2-6	0	0	0	0	0	0	0	0	0	-
17	5-6	0	0	0	0	0	0	0	-	-	-	-
18	0	2-5;2-5	0	0	0	0	0	0	0	0	0	-

SUBJECTS AUTOMATICALLY PUT IN TRIAL OPTION SUBJECTS HAVE TO USE TRIAL FACILITY TO TO FIND CONVERSION FROM TOKENS TO POINTS

trials with a range of only one period are ignored

the numbers in the table are the ranges of each trial carried out in that period

- means that the experiment had finished

0 means that no trials were carried out

subject no.	period of experiment											
	1	2	3	4	5	6	7	8	9	10	11	12
21	1-4;1-4	2-4	3-4	4-5	5-6	6-7	0	8-9	9-10	10-11	-	-
22	0	0	0	0	0	0	0	0	0	-	-	-
23	1-4;1-4;1-4 1-4;1-4	2-3; 2-3	0	4-6	0	6-7	0	0	9-10; 9-10; 9-12	10-11 10-11	0	12-14
24	1-6;1-6;1-5;1-5	2-8; 2-8; 2-5	3-11; 3-6 3-6	4-10 4-8	5-10 5-10	6-12 6-8	7-10 7-10	8-12 8-12	9-14	-	-	-
25	1-5	2-7; 2-7	0	4-5 4-5 4-5	0	6-7 6-7 6-7 4-5	0	0	0	0	0	-
26	1-3;1-3;1-3;1-2; 1-2;1-2;1-2;	2-3; 10 trials	0	0	5-6	0	0	0	0	0	0	-
27	1-4;1-4;1-4;1-4 1-4;1-4;1-4;1-6	2-4; 4 trials	3-5; 3-5	4-6 4-6 4-6	5-8 5-8 5-8	6-8 6-8	7-10 7-10	0	0	10-12 10-12 10-12	0	-
28	2-15;2-10;2-10	2-10	3-10	0	0	0	0	0	0	0	0	0
29	1-25	2-5	3-5	4-5	0	0	0	0	0	0	0	-
30	1-6;1-6;1-6;1-6 1-10;1-10;1-10 1-10;1-10;1-10 1-10	0	0	0	0	0	0	0	0	0	0	-

subject no.	period of experiment											
	1	2	3	4	5	6	7	8	9	10	11	12
31	1-5	2-4; 2-4 2-4	3-6; 3-6 3-6 3-6	4-5	5-6	6-7	7-8 7-8 7-8	8-9	0	10-11	-	-
32	3-5;2-4;2-4	3-5;	3-4 3-5 3-5	4-7	5-7	7-8	7-8	8-9	9-10	10-11	-	-
33	1-5;1-5;1-5;1-5	2-5; 2-6	0	4-5	5-6	6-7	7-8	8-9	-	-	-	-
34	1-3;1-3;1-3 1-3;1-3;1-3	2-3; 2-3 2-3	3-4; 3-4 3-4 3-4	4-5 4-5 4-5 4-5 4-5	5-6 5-6 5-6 5-6 5-6	6-7 6-7 6-7 6-7	7-8 7-8 7-8 7-8 7-8	8-9 8-9 8-9 8-9	9-10 9-10 9-10 9-10	10-11 10-11 10-11 10-11 10-11	11-12 11-12 11-12 11-12 11-12	-
35	1-3	2-4	3-5	5-6	5-7	6-7	7-8	8-9	9-10	0	-	-
36	1-4	2-4	3-4	0	0	6-7	7-9 7-9	0	9-10	10-11	0	0
37	1-4;1-4;1-4;1-4	0	0	0	0	0	0	0	-	-	-	-
38	1-5;1-5;1-5;1-5 1-5;1-5;1-5;1-5 1-5	2-11	0	0	5-10	6-10	0	8-10 6-10	9-11	10-12	0	-
										8-10	9-11	
										8-10	9-11	
										8-10	9-11	
										9-11		
										9-11		
39	1-8;1-8;1-2;1-2	0	0	0	0	0	0	0	0	0	-	-
40	2-5;1-5	2-5	3-6	4-8	5-8	6-7 6-7	7-9	8-10	9-11 9-11	10-12	-	-

Appendix 5 Summaries of Individual Trials and Comparison with Actual Strategies Used

STRATEGIES

learning: more than 1 trial in one of the first two periods

rolling: trials all the way through the experiment, looking forwards the same or a similar number of periods

02 learning, 03 learning, 04 learning + rolling, 06 learning,
12 learning+ fixed endpoint, 16 learning, 18 learning

21 early fixed end point + rolling, 23 rolling + learning, 24 rolling + learning, 25 rolling + learning,

26 learning, 27 learning + rolling, 28 learning+ fixed endpoint, 29 learning+ fixed endpoint, 30 learning,

31 learning + rolling, 32 learning + rolling, 33 rolling + learning, 34 learning + rolling, 35 rolling,

36 early fixed end point + rolling , 37 learning, 38 learning + rolling, 39 learning, 40 learning + rolling

income = y , wealth = w

SUBJECTS NOT AUTOMATICALLY PUT IN TRIAL OPTION

Subject 02

Trials in periods 1 and 2, range 5 and 9.

Trial strategy: Period 1: chooses reasonable distance into the future (range 5), with variations in income (y) and variable levels of wealth (w). Does definitive trial range 9, keeping wealth within a fairly narrow band (2.5-8.4) apart from final period. Quick check in period 2 range 4, similar trial. Strategy is to keep a constant wealth level or learning to keep wealth low (between 2 and 5 in later trials).

Actual strategy: high saving in first period, then w varies between 1 and 9.

Subject 03

period 1. 4 trials range 1-8 with same varying y flow in all. Different consumption patterns including save all y until period 8 to convert all income.

Trial strategy: chooses reasonable distance to look ahead, then tries out different consumption strategies.

Actual strategy: alternates between no saving and high saving. Trigger level of wealth, beyond which dissaves?

No correlation between trials and actual strategy.

Subject 04

Trials in periods 1 to 4 looking up to 8 periods ahead.

Trial strategy: Period 1. Different strategies tried: save 50% of income unless there is high wealth and income, alternate between consumption of 1 or 2 and 20 as a lagged response to high income. Convert $y-1$ also tried.

Periods 2-4, look ahead 2 to 5 periods, save early in the trial and dissave in final period (once in last two periods). Wealth reduced to low level in all final periods

Actual strategy: w cycles.

Subject 06

Trial in period 1. 2 trials range 5-6, smooths in first converts all y in second. Variable y assumed.

Trials find difference between these two approaches.

Actual strategy: convert all y . Lagged response to high y .

Subject 11

Trial in period 1. 1 trial range 2-4, smooths and goes to wealth of 0. Variable y assumed.

Actual strategy: convert all y . Lagged response to high y .

Subject 12

Trials in periods 1 and 2 looking up to period 6. Converts almost all $y+w$. Variable y assumed.

Actual strategy: convert all y . Lagged response to high y .

Subject 17

1 trial in period 1. 1 trial range 5-6, converts all y . $y = 10$ assumed.

Actual strategy: convert all y

Subject 18

Trials in periods 2. Range 4, save all then high conversion in 3rd period. In 2nd trial the subject has learned how to reduce wealth to 0. $y = 10$ assumed.

Actual strategy: alternates saving all and converting all, after period 4 converts all y .

SUBJECTS AUTOMATICALLY PUT IN TRIAL OPTION SUBJECTS HAVE TO USE TRIAL FACILITY TO TO FIND CONVERSION FROM TOKENS TO POINTS

Subject 21

Trials in all periods, looking one period ahead, with alternating strategies between saving all and converting all. Variable y assumed.

Up to period 3 looks forward to period 4 and from period 3 has rolling trials strategy, looking forward 2 periods mostly. Trials have varying savings strategies, between save all and convert all y .

Actual strategy: initially saves all in response to low income, then keeps w between 0 and 5.

Subject 22

All 1 period trials to find conversion from tokens to points.

Actual strategy: converts all.

Subject 23

5 trials in period 1 looking to period 4, all except one (when all y always converted) with saving until last period and then converting all. Income of 10 assumed in all but one trial.

Periods 2-8, trials over 1 or 2 periods with low saving

Period 9 3 trials looking to period 10 or 12, differing strategies.

Period 10 and 11 1 trial of range 1 then 2. Period 12 3 trials with different strategies.

Later trials practiced keeping w low.

Trials strategy: serious trial in period 1, limited checks in later periods.

Actual strategy: alternates between high and low w ; smooths. Did not choose best of trials for actual strategy.

Subject 24

High and low levels of saving tried with different income streams assumed and variable levels of wealth.

Trials strategy: rolling strategy: several trials in each period mostly looking ahead 4-7 periods. Some smoothing?: build up wealth early on then dissave.

Actual strategy: convert almost all y ; keeps wealth at 1 mostly.

Subject 25

Trial strategy: Apart from finding conversions, limited trials performed intermittently, with saving early on and then wealth maintained at a moderate level. Multiple period trials mostly learning. $y = 10$ assumed.

Lots of trials to find conversion from tokens to points. 2 trials in period 2 range 2-7, 1 finding conversions, 1 where all y converted. Some trials with range of 2 periods with low

saving.

Actual strategy: convert most of y , w varies between 2 and 6.

Subject 26

Trials strategy: worked out in periods 1 and 2, with a single check in period 5.

Lots of trials in each period to find conversion from tokens to points. Lots of short trials in periods 1 and 2 and one trial (a check?) in period 5 range 2. Some with low saving, others with high saving then dissaving. Variable y assumed.

Actual strategy: convert most of y , w varies between 1 and 3 except for period 2 where w set to 8 for next period.

Subject 27

Trials strategy: rolling

Trials range 3 or 4 in most periods, more trials early on. High early saving at various rates in some

trials with dissaving in last period of trial so that $w < 1.0$, more variation in strategies in later trials. $y = 10$ assumed.

Actual strategy: Some saving, w varies between 5 and 10 until run of low income causes w to decrease to 3 and all y converted.

Subject 28

Long trials in periods 1 to 3, starting next period in period 1 only goes up to period 10 after first trial, but with variable conversion strategies. range between 14 and 8 with low saving. Income of 10 assumed.

Actual strategy: Converts 5 of 10 in first period, otherwise mostly high conversion, w kept between 0 and 5 apart from last 2 periods.

Subject 29

Trial in period 1 range 24, period 2 range 4, period 3 range 3, period 4 range 2. Conversion tracks wealth with low saving. i.e. One serious trial in which subject learns how to keep wealth low as income varies, and a few checks where wealth also kept low, by converting most of y . Variable y assumed.

Actual strategy: convert all y except for first two periods when convert 8 of 10.

Subject 30

11 trials in period 1, range 6 and 10, lots of different savings strategies tried, varying between saving all y for conversion in the last period of a trial and just converting all y . Same 'typical' income stream assumed in all trials.

Actual strategy: convert all y .

Subject 31

Rolling trials strategy.

Trials in all periods, lots in periods 2 and 3, range 5 decreasing to 2 by period 6, with varying strategies, often maintaining a wealth of 10. Two types of early trials: save all y for conversion in last period or convert all y . Later trials keep wealth low or save all y . Income of 10 assumed.

Actual strategy: alternates between 0 conversion and converting all $y+w$.

Subject 32

Rolling strategy:

Trials in all periods, range 3 early on later 2, varying income assumptions and savings strategies mostly with significant saving in at least the first period. Some trials have end effect: wealth reduced to 0 in last period, from period 8 trials have low wealth as in actual strategy: expecting experiment to finish? 2 period trials practice actual conversion in first period of trial. Income of 10 assumed.

Actual strategy: early saving, threshold wealth of 10 triggers dissaving?.

Subject 33

Trials strategy: learning in period 1 + rolling strategy

Trials of range 4 or 5 in periods 1 and 2, income assumed constant at 10, strategies all with higher saving early on lower later. Trials of range 1 or 2 later, with low saving then dissaving.

Actual strategy: Saving in first two periods, then convert most y , w maintained between 8 and 12.

Subject 34

Trials strategy: rolling + learning in each period

Lots of trials in all periods with range 2 assuming $y = 10$ always. Different savings strategies, but after first period pattern is moderate saving then dissaving, keeping $w < 1.0$ or reducing w to $w < 1.0$. First period of trials often keeps wealth high, as in actual conversion strategy.

Actual strategy: Some saving throughout, w increases continuously to 30.

Subject 35

Trials strategy: rolling

Trials in all periods range 3 decreasing to 2. High levels of w (10+) built up and maintained as go through periods. Different patterns of y assumed.

Actual strategy: Strategy variable, saves in most periods.

Subject 36

Trials strategy: rolling, variable wealth sometimes kept low, sometimes high.

1 trial of range 4 in 1st period, range 3 in second range 2 both with high initial saving in 3rd range 1 with dissaving. Income of 10 assumed almost always.

Actual strategy: Alternates between saving and dissaving as a result of short range thinking?.

Subject 37

Trials strategy: learning in first period, to maintain a moderate level of wealth.

4 trials of range 4 in period 1, assuming y of 10 2 2 10, moderate saving strategies tried with one trial where y smoothed for conversion .

Actual strategy: Low saving except a period of high income used to increase w from 1 - 3 in early periods to 7 - 13 in later periods. c tracks y .

Subject 38

Trials strategy: Period 1, learning assuming period 5 is end, all with same income flow. Period 2 longer trial, possibly also learning how to keep wealth to 3 approximately. After period 5, trials to period 10, most trials have significant saving then dissaving. Period 8 on rolling strategy.

later trials assumed $y = 10$, mainly high early saving with conversion of $w+y$ in last period of trial.

Actual strategy: Wealth cycles.

Subject 39

Trials strategy: Learning in period 1. Compares early saving then converting all to converting all.

Actual strategy: convert all y .

Subject 40

Trials strategy: rolling

1 or 2 trials in each period range 3 or 4 mainly. High wealth in first 1 or 2 periods of trial, then dissaving. Variable y assumed. First period of trial similar to actual conversion, but trials reduce wealth more.

Actual strategy: Cyclical from save all y to convert y to convert $y+ 10$. Wealth of 20 built up in first two periods and maintained and increased.

Appendix 6 FORTRAN 77 program EXSIM.F

```
program exsim

c program to generate ex ante optimal c for initial wealth at
c beginning of period excluding in period income w= 0.1 to 100
c y distribution and stopping prob known

parameter n=1000

parameter (y1=20,y2=10,y3=2,p1=0.1,p2=0.8,r=0.1,pd=0.9)

c y1-3 = incomes p = income probabilities
c pd = chance of death r = int rate

double precision w,v(n),rhs(n),points(n)

c w(n) is beginning of period wealth including any income
c w form 0.1 to 100.0 in steps of 0.1
c v(n) is maximum value function for wealth 100/n
c rhs(n) is expected value function for the next period
c points(n) is optimal consumption for output file: not used in calc

double precision aaa,rhsa1,rhsa2,rhsa3,ww,diff,rhsmax,ca

c aaa is counter for v(n) initialisation
c rhsa1-3 and ww are intermediate variables for rhs
c diff is difference between current iteration and last iteration
c of overall calc. used to determine convergence of calculation
c rhsmax is current maximum value of value function for the c's
c that have been tried so far

double precision c,maxc,copt(n),cara,vv

c c is current value of consumption copt(n) is value of c that
c maximises rhs for each n i.e. w
c cara is cara utility function, vv is interpolation of v fn.

c initialise v(n)

do 10 i=1,n
  aaa=i*100.0/n
  rhs(i)=cara(aaa)
c  print*,'rhs=',rhs(i),'cara=',cara(aaa)
c  print*,'i ca rhs =',i,ca,rhs(1),rhs(101),rhs(1000)
  10 continue

c conditional loop if difference large
```

```

15  do 30 ii=1,n
    v(ii)=rhs(ii)
30  continue

```

c loop through w from 0.1 to 100.0

```

    do 200 jj=1,n
c   print*,'(jj= ',jj)
    w=jj/10.0

```

c loop through c and find maximising value of c for current n(=w*10)

```

    rhs(jj) = 0.0
    rhsmax = 0.0
    maxc = 0.0
    kk = int(w)

    do 100 ic = kk,0,-1
    c = ic
c   print*,'wc= ',w,c

```

```

    ww=(w-c)*(1.0+r)
    rhsa1 = ww + y1
    rhsa2 = ww + y2
    rhsa3 = ww + y3

```

C IS THIS CORRECT OR IS IT RHSA3=WW+Y3 - NO 1+R?

C no 1+r ans y is for beginning of next period

C and w includes y for this period

```

c   print*,'wcww',w,c,ww
c   print*,'rhsa',rhsa1,rhsa2,rhsa3
c rhs = u from this period consumption + expected value next period

```

```

    rhs(jj) = cara(c) + pd*(p1*vv(rhsa1,v,n) + p2*vv(rhsa2,v,n)
*   + (1.0-p1-p2)*vv(rhsa3,v,n))

```

```

c   print*,'ccarajj rhs',c,cara(c),jj,rhs(jj)
c   print*,'vv',vv(rhsa1,v,n),vv(rhsa2,v,n),vv(rhsa3,v,n)
c check for maximising c

```

```

    if (rhs(jj) .gt. rhsmax) then
    rhsmax = rhs(jj)
    maxc = c
    end if

```

```

c   print*,rhsmax, maxc,w

```

```

100 continue

```



```
rhs(jj) = rhsmax
copt(jj) = maxc
```

```
200 continue
```

```
c check to see if current iteration significantly different
c over all w from last iteration and repeat calc if needed
```

```
diff=0.0
Do 220 l=1,n
diff = abs(v(l)-rhs(l)) + diff
220 continue
```

```
print*, 'diff=',diff
if (diff .gt. 0.0001) then
goto 15
end if
```

```
print*, 'v102 ',v(102)
```

```
c write results to file
```

```
open(10,file='copt.res')
open(11,file='v.res')
12 format(2f8.3,I3,f8.3)
14 format(f12.8)
16 format(f8.3)
do 223 ic=1,1000
write(10,16)copt(ic)
223 continue
c write(11,*)('r=.01 pd=.99 e=.9 n=1000')
do 225 iw=1,1000
write(11,16)(v(iw))
225 continue
c do 230 ll=1,n
c aa=ll/10.0
c points(ll)=cara(copt(ll))
c write(10,12) (aa,v(ll),copt(ll),points(ll))
```

```
c 230 continue
```

```
close(10)
c close(11)
```

```
end
```

double precision function cara(a)

c cara utility function

double precision a,f
parameter (d=20.0,e=-0.1)

f=e*a
cara = d*(1.0-exp(f))

end

double precision function vv(a,b,m)

c interpolate/extrapolate from v fn.

integer m,k
double precision b(m),a,c

c=100.0/m
k=int(a/c)

c if a le m then interpolate

if (a .le. m*c) then
vv=(a/c-k)*(b(k+1)-b(k)) + b(k)
else

c if a gt m then extrapolate

vv = b(m)
end if

end

Appendix 7 FORTRAN 77 program ROLLSIM.F

PROGRAM ROLLSIM

C PROGRAM TO GENERATE EX ANTE OPTIMAL C FOR INITIAL WEALTH AT
C BEGINNING OF PERIOD EXCLUDING IN PERIOD INCOME W= 0.1 TO 100
C CALCS POINTS ON AVERAGE FROM RANDOM INCOME FLOW ASSUMES Y
C DISTN KNOWN AND PROB OF DEATH=0 FOR ROLL PERIODS INTO FUTURE

PARAMETER (N=1000,NPMAX=200,R=0.1,PD=0.99,NROLL=10)

PARAMETER (Y1=20,Y2=10,Y3=2,P1=0.1,P2=0.8)

C Y = INCOME P = PROBABILITY PD = CHANCE OF DEATH R = INT RATE

DOUBLE PRECISION W,RESW(NPMAX),V(NROLL,N)
DOUBLE PRECISION DIFF,POINTS,AVPOINTS,AVPOINTSN
DOUBLE PRECISION AAA,RHS,RHSA1,RHSA2,RHSA3,WW,RHSMAX
double precision rhs1,rhs2
DOUBLE PRECISION C,MAXC,COPT(NPMAX,N),RESC,Y(NPMAX),YY
DOUBLE PRECISION CARA,VV
INTEGER T,TT,K,KK,KA

C NOTE ROLL INCLUDES FINAL PERIOD SO ROLL= 2 DOES CALC
C OVER 2 PERIODS AND THEREFORE LOOKS 1 PERIOD AHEAD

DOUBLE PRECISION X,Z,G05CAF
EXTERNAL G05CAF,G05CBF,G05CCF

OPEN(14,FILE='rl.res')

C CALC MAX VAL FUNCTION AND C OPT FUNCTION

C INITIALISE V

DO 85 I=1,N
AAA=I/10.0
V(NROLL,I)=CARA(AAA)
85 CONTINUE

C LIMITED LIFE VS INFINITE BY PERFORMING THE ITERATION ROLL TIMES

DO 250 T = NROLL-1,1,-1
print*,'t',NROLL,t
C LOOP THROUGH W
DO 200 JJ=1,N
W=JJ/10.0

C LOOP THROUGH C AND FIND MAXIMISING VALUE OF C

V(T,JJ)=0.0
RHSMAX=0.0
RHS=0.0
MAXC=0.0
KK=INT(W)

DO 100 IC = KK,0,-1

C=IC
WW=(W-C)*(1.0+R)
RHSA1 = WW + Y1
RHSA2 = WW + Y2
RHSA3 = WW + Y3
RHS = CARA(C) + P1*VV(RHSA1,V,NROLL,N,T+1)
* + P2*VV(RHSA2,V,NROLL,N,T+1) +
* + (1.0-P1-P2)*VV(RHSA3,V,NROLL,N,T+1)

C CHECK FOR MAXIMISING C

IF (RHS .GT. RHSMAX) THEN
RHSMAX = RHS
MAXC = C
END IF

100 CONTINUE

V(T,JJ) = RHSMAX
COPT(T,JJ) = MAXC

200 CONTINUE

C print*,t,jj,copt t,jj-1',t,jj,copt(t,jj-1)

250 CONTINUE

c OPEN(15,file='rd03.res')

255 FORMAT(2F8.3)

c do 257 iout=1,n

c write(15,255)copt(nroll-2,iout),copt(nroll-1,iout)

C * copt(nroll-2,iout)

c 257 continue

c close(15)

c go to 1051

C print*, 'finished'

C FIND END PERIOD AND Y

CALL G05CBF(0)

AVPOINTS=0.0

KA=0


```

400 IEND=0

540 IEND=IEND+1
C GENERATE Y
  Y(IEND)=10.0
  X=G05CAF(X)
C   print*,'x',x
  IF (X .LE. P1) Y(IEND)=20.0
  IF (X .GT. P1+P2) Y(IEND)=2.0
C CHECK FOR DEATH
  Z=G05CAF(Z)
C   print*,'z',z
  IF ((Z .LE. PD) .AND. (IEND .LT. NPMAX)) GO TO 540
  IF (IEND .GE. NPMAX) GO TO 400

  KA=KA+1
c   IF (KA .GT. 2000) GO TO 1050

C CALC PATH GIVEN INCOME STREAM

  RESW(1)=0.0
  POINTS=0.0

  DO 750 LJ=1,IEND

    LJW = INT((RESW(LJ)+Y(LJ))*10)
    IF (LJW .GT. N) LJW = N
    RESW(LJ+1) = (RESW(LJ) - COPT(1,LJW) + Y(LJ))*(1.0+R)
    POINTS = POINTS + CARA(COPT(1,LJW))
  750 CONTINUE

C   print*,'iend points',iend,POINTS

C CHECK TO SEE IF REPEAT CALC
  950 AVPOINTSN=(AVPOINTS*(KA-1)+POINTS)/KA
  WRITE(14,*)AVPOINTSN,POINTS,KA
C   PRINT*,POINTS,AVPOINTS
  DIFF=AVPOINTSN-AVPOINTS

  IF ((AVPOINTSN .LT. 0.0) .AND. (KA .GT. 10)) THEN
    KA=1
    WRITE(14,*)'AV',AVPOINTS,POINTS,KA
    GO TO 400
  END IF

  IF (ABS(DIFF) .GT. 0.0001) THEN
    AVPOINTS=AVPOINTSN
    GO TO 400
  END IF

```

```

1050 WRITE(14,*)'AV DIFF',AVPOINTS,DIFF
      WRITE(14,*)'Nroll KA RESW(LJ) ',nROLL,KA,RESW(LJ)
      WRITE(14,*)'N=1000 r=.01 pd=.5 e=.03'
      print*,'finish', avpoints

1051 CLOSE(14)
      END

```

DOUBLE PRECISION function CARA(A)

C CARA UTILITY FUNCTION

```

DOUBLE PRECISION F,A
PARAMETER (D=20.0,E=-0.03)

```

```

F=E*A
IF (F .EQ. 0.0) THEN
  CARA=0.0
ELSE
  CARA = D*(1.0-EXP(F))
END IF

```

```

END

```

DOUBLE PRECISION FUNCTION VV(A,B,NP,M,NT)

C INTERPOLATE/EXTRAPOLATE FROM V FN.

```

INTEGER NP,M,NT,K
DOUBLE PRECISION A,B(NP,M),C,F,G

```

```

C=100.0/M
K=INT(A/C)

```

C IF A LE M THEN INTERPOLATE

```

IF (A .LE. M*C) THEN
  VV=(A/C-K)*(B(NT,K+1) - B(NT,K)) + B(NT,K)
ELSE

```

C IF A GT M THEN EXTRAPOLATE

```

VV = B(NT,M)
END IF
END

```