Applications of non-linear time series models on finance and macroeconomics

Jinki Kim

University of York

Department of Economics and Related Studies

PhD

February 2003

to our Lord

Abstract

1) **Inflation uncertainty, output growth uncertainty, and macroeconomic performance.** The causality relationship among nominal uncertainty, real uncertainty, and macroeconomic performance measured by the inflation and output growth rate is examined for G7 countries. The application of a GARCH model leads to a number of interesting conclusions: (1) Inflation causes negative welfare effects, both directly and indirectly. (2) More inflation uncertainty provides an incentive to Central Banks to surprise the public by raising inflation unexpectedly. (3) More variability in the business cycle leads to more output growth (Chapter 2). Additionally, the trade-off between inflation and output variability is considered for G3 countries. Using a two-step procedure this study finds that (1) The nominal uncertainty significantly affects real uncertainty in all three countries. (2) A trade-off between nominal and real uncertainty in the pre 1980 period and a positive correlation in the post 1980 period are found (Chapter 3).

2) **Moments of the EGARCH and A-PARCH models.** We consider the moment structure of the general ARMA-EGARCH model and derive the autocorrelation function of any positive integer power of the squared errors. In addition, we obtain the autocorrelations of the squares of the observed process (Chapter 4). In the analysis of the Asymmetric Power ARCH model we obtain the existence condition for a certain fractional moment of the absolute observations and the autocorrelation function of the power-transformed absolute returns (Chapter 5). The practical implications of the results are illustrated empirically using daily data on five East Asia stock indices.

3) **The volume-volatility relationship in the Korean stock market.** This research examines whether the financial market opening to foreign investors affects the dynamic interaction between volume and volatility in the Korean stock market around the financial crisis in 1997. The evidence from causality tests suggests that (1) There is a strong positive bidirectional feedback between volume and volatility for the entire sample. (2) Foreign investors' trading volume caused the stock market volatility after the crisis. (3) Domestic investors' trading volume had a mixed effect on volatility before the crisis and the effect disappears in the post-crisis period (Chapter 6).

Contents

List of Figures

List of Tables

Acknowledgments

Firstly, I would like to thank Dr. Menelaos Karanasos for his excellent supervision. Without his support, I could not have finished my work. I am also grateful to Professor Leslie Godfrey and Dr. Giovanni Forchini for their comments and encouragement at various stages of my research.

I would like to thank Dr. Malcolm Wren not only for his proofreading but also for his encouragement to overcome my difficulties as a non-native speaker.

On a more personal note, I should firstly thank my parents, who have always supported me throughout my life. I wish to express my gratitude to my wife, Sooyoung Lee, and our lovely son, Hyungu, for their support with great patience over the long time of this journey.

Another enormous thank you should go to all my friends. I would like to thank Kwangnam Choi, Junsik Bae, Sungeun Lee, Hyuksoo Suh, Hyunghoon Lee, Jinkwon Lee and Gyujin Hwang for their friendship and help.

Author's Declaration

Chapter 2 and 3 are mainly due to Kim, although joint papers with Karanasos are expected. Chapter 4 and 5 are joint work with Menelaos Karanasos. Specifically, the section 4.3 and 5.3 are due to Kim. Chapter 6 and all other material in the thesis is sole-authored by Kim.

A version of Chapter 4 is accepted for publication in Econometrics Journal, forthcoming (2003).

An early version of Chapter 5 was presented at International conference on the Econometric of Financial Markets. 22-25, May 2001 in Delphi, Greece.

Chapter 1

Introduction

This thesis is a collection of independent essays. The detailed structure and introduction to each chapter is presented at the beginning of each chapter. However, this chapter is intended to introduce a brief picture of what each chapter is about.

Chapter 2 analyses the empirical relationship among four macroeconomic variables: inflation, output growth, inflation uncertainty and output growth uncertainty. A bivariate GARCH (Generalised Autoregressive Conditional Heteroschedasticity) model is used to obtain the conditional variances of inflation and output growth as proxies of inflation and output growth uncertainty, respectively and the Granger causality tests are performed for G7 countries. This study also attempts to consider all testable hypotheses regarding bidirectional causality among these four variables.

Taylor (1979) argues that the trade-off between the variability of inflation and output can explain the absence of the long-run trade-off between inflation and output. In contrast, Logue and Sweeney (1981) suggest that nominal uncertainty and real uncertainty are positively correlated and can move same directions.

Chapter 3 focuses on the relationship between inflation uncertainty and output growth uncertainty for three main economies: US, Japan and Germany. The Matrix-diagonal (MD) model introduced by Engle et al. (1994) and the BEKK model by Engle and Kroner (1996) are used to generate the conditional variance of inflation and output growth.

Chapter 4 and Chapter 5 are about the autocorrelation functions of ARCH type models. We study the EGARCH (Exponential GARCH) model of Nelson (1991) in Chapter 4 and the autocorrelation structure of the general A-PARCH (Asymmetric Power ARCH) model of Ding et al. (1993) in Chapter 5. A comparison between the estimated theoretical autocorrelations and the sample autocorrelations helps us to choose the best model that replicates certain stylised facts of the data. The theoretical results apply to four Asian countries' stock indices (Korea, Japan, Taiwan and Singapore) in Chapter 4 and five Asian countries' indices (the four Asian countries in Chapter 4 and Hong Kong) in Chapter 5. Using a maximum likelihood estimation method we estimate models and present graphical comparisons between the theoretical autocorrelations of fitted values and sample values.

Chapter 6 explores the relationship between the financial market opening and Korean stock market volatility. The East Asian financial turmoil in 1997 forced many Asian countries to make a fundamental choice in respect of the financial liberalisation request from the outside world. Many of them have retreated from the liberalisation process whereas Korea seemed to have no choice but open their market wide under the IMF bailout program. There are studies about the emerging countries which delayed or blocked the financial market opening. However, research about the countries which took further liberalisation is difficult to find. Chapter 6 studies the Korean stock market volatility after the Asian crisis, which is closely related to its financial market opening. Fractionally integrated GARCH type models are used in the estimation of the volatility of stock returns.

Finally, Chapter 7 presents concluding remarks.

Chapter 2

Inflation uncertainty, output growth uncertainty, and macroeconomic performance

2.1 Introduction

Since the early 1980s, there has been a significant improvement in macroeconomic performance in industrialised and developing countries. Krause (2001) reports that in a cross-section of 63 countries, mean inflation fell from 7.04% in the pre-1995 period to 2.97% in the latter half of the 1990s. Furthermore, both inflation and output growth have become more stable. Cecchetti and Krause (2001) report that in a sample of 23 industrial and developing countries the average country experienced a decline in both inflation and output variability in the 1990s compared to the 1980s. A second fact reported in the Cecchetti and

Krause (2001) study is that there seems to exist a trade off between inflation and output variability. A number of issues arise from the above findings: First, is the reduction in average inflation related to the reduction in inflation uncertainty, and if so, is the causality between the two variables bi-directional ? Second, is it true that a reduction in inflation and inflation uncertainty can have a favourable impact on the rate of economic growth as predicted for example by Friedman (1977)? Third, can a more stable and less volatile output growth lead to more output growth ?

This study analyses the empirical relationship among four important macroeconomic variables: average inflation, output growth, nominal (inflation) uncertainty and real (output growth) uncertainty. In this regard, current research examines all possible effects among these four variables using time-series data for the G7 to attempt to provide answers to the above three questions. To test the empirical relevance, several theories have been advanced on the relationship between inflation, output growth, real and nominal uncertainty. These theories include: First, the Cukierman and Meltzer (1986) hypothesis that Central Banks tend to create inflation surprises in the presence of more inflation uncertainty, second, the Black (1987) hypothesis that increasing output uncertainty leads to more output growth, and third, the Taylor effect, which predicts a trade off between inflation and output variability and hence uncertainty.

These issues will be examined with the use of a bivariate GARCH model that allows the measurement of uncertainty about inflation and output growth by the respective conditional variances. This approach has been recently applied by, among others, Caporale and McKiernan (1996, 1998), Grier and Perry (1998, 2000), Fountas (2001) and Henry and Olekalns (2001). However, these studies suffer from two disadvantages: First, they focus almost exclusively on the empirical relationship between either (i) inflation and inflation uncertainty or (ii) output growth and output growth uncertainty (a notable exception is Grier and Perry (2000), which examines a richer, though not complete, set of hypotheses). Second, the majority of these studies (an exception being Grier and Perry (1998)) employ only US or UK data. To cover these gaps in the existing literature the current study uses monthly data on the G7 to examine the relationships among inflation, output growth and the respective uncertainties.

This chapter is structured as follows: Section 2.2 presents the theoretical macroeconomic implications concerning the relationship among the four variables of interest. Section 2.3 summarises the empirical literature to date. Section 2.4 lays out an econometric model and section 2.5 reports and discusses results. The last section contains the main conclusions and draws some policy implications.

2.2 Theory

2.2.1 The effect of macroeconomic uncertainty on inflation and output growth

Macroeconomists have placed considerable emphasis on the impact of economic uncertainty on the state of the macroeconomy. The profession seems to agree that the objectives of monetary policy are inflation and output stabilisation around some target levels. Exogenous shocks to the economy that generate uncertainty about the inflation rate and output (or its growth rate) tend to cause a deviation of these variables from their desired values and hence necessitate some policy response. Friedman (1977) argues that inflation uncertainty causes an adverse output effect. This outcome is based on the idea that inflation uncertainty distorts the allocative efficiency aspect of the price mechanism. More specifically, inflation uncertainty affects both the intertemporal (through its effect on the interest rate) and intratemporal (through its effect on relative prices in the presence of nominal rigidities) allocation of resources. Lucas (1973), in his price-misperceptions theory, shows that, in an imperfect information setting, more inflation uncertainty obfuscates the distinction between real and nominal shocks, thus leading to welfare-reducing economic activities.

The effect of inflation uncertainty on output growth works also through its impact on investment. Recent theoretical literature on investment (Pindyck, 1991) focuses on the irreversibility aspect of investment and considers current investment as giving up the option to invest in the future; hence, the value of this lost option represents the opportunity cost of an investment project. Inflation uncertainty increases uncertainty regarding the potential returns on investment projects and therefore provides an incentive to delay these projects, thus contributing to lower investment and output growth[1]. Finally, Dotsey and Sarte (2000), using a cash-in-advance framework, obtain a rather puzzling result: more inflation uncertainty can increase output. This result is based on a precautionary motive.

Uncertainty about the inflation rate also affects the average rate of inflation.

[1] Evidence of a negative impact of inflation uncertainty on primarily irreversible investment like R&D is provided by Goel and Ram (2001).

However, the direction of the effect is ambiguous from a theoretical point of view. Cukierman and Meltzer (1986) assume that agents face uncertainty about the rate of money supply growth and hence inflation. In a Barro-Gordon set up, the monetary authority surprises the agents by setting an unexpectedly high money supply growth rate. This argument predicts a positive effect of inflation uncertainty on inflation. In contrast, Holland (1995) claims that the monetary authority, when faced with more inflation uncertainty in the economy, will contract the growth rate of the money supply and hence reduce inflation (and the associated uncertainty) in order to counteract the negative welfare effects of inflation uncertainty on the economy. This is the so-called "stabilizing Fed hypothesis" and postulates a negative effect of inflation uncertainty on inflation.

Real uncertainty, measured for example by the variability of output, may also affect the rate of inflation and output growth. Deveraux (1989) extends the Barro-Gordon model by introducing endogenous wage indexation. He considers the impact of an exogenous increase in real (output) uncertainty on the degree of wage indexation and the optimal inflation rate delivered by the policymaker. He shows that more real uncertainty reduces the optimal amount of wage indexation and induces the policymaker to engineer more inflation surprises in order to obtain favourable real effects. Hence, the testable implication of the model is that more output growth uncertainty should lead to a higher rate of inflation[2].

A number of theories have been put forward to examine the impact of output uncertainty on output growth. In a nutshell, the sign of such an effect is ambigu-

[2]It is possible that more uncertainty about output growth leads to a lower inflation rate. Higher output uncertainty implies lower inflation uncertainty (the Taylor effect discussed below) and hence a lower inflation rate (the Cukierman and Meltzer hypothesis).

ous. First, there is the possibility of independence between output variability and growth occuring when the determinants of the two variables are separate. For example, business cycle models predict output will fluctuate around its natural level arising from price misperceptions. In contrast, output growth is affected by real factors such as technological changes. Black (1987) argues for a positive effect of output growth uncertainty on output growth. Investors are only willing to invest in riskier technologies if the expected return on these investments (i.e., the average rate of growth) is sufficiently large to offset the extra risk. Given the time aspect of investment, this effect would be captured with a long lag. Finally, the idea of a negative impact of output growth uncertainty on output growth goes back to Keynes (1936), who argued that in the presence of more uncertainty about the return on investment (which is positively correlated with output uncertainty), entrepreneurs will demand less investment, thus lowering output growth. Ramey and Ramey (1991) show that in the presence of commitment to technology in advance, higher output volatility can lead to suboptimal ex post output levels by firms (due to uncertainty-induced planning errors) and hence, lower mean output and growth.

2.2.2 The relationship between inflation and output growth

Mean inflation and output growth are interrelated. The traditional short-run Phillips curve implies that an increase in output above its natural level would result in inflationary pressures, hence a positive causal effect of output growth on the rate of inflation. Modern sticky-price New Keynesian models predict that the long-run Phillips curve might also be downward sloping (Calvo, 1983; Walsh,

1998) implying that higher output growth increases the inflation rate. A recent study by Deveraux and Yetman (2002) shows that in the presence of endogenous frequency of price adjustment. this negatively sloping long-run Phillips curve applies only for very low inflation rates, a result similar to Akerlof et al. (2000).

Much more controversy surrounds the causal effect of inflation on output growth. Economic theory predicts a positive, zero, or negative effect of inflation on output growth depending on how money enters the analysis. In a classic article Tobin (1965) focuses on the role of inflation as an engine of economic growth and considers money as a substitute for capital. He shows that inflation, by reducing the real return on money balances, leads to the substitution of capital for money, thus encouraging investment and growth. Brock (1974), using a money-in-the-utility function model, derives the result that money is superneutral and hence the effect of inflation on output growth is zero. Stockman (1981), using a cash-in-advance framework where money is required to buy capital goods, shows that anticipated inflation leads to a lower demand for real money balances, hence a lower capital stock and growth, i.e., a reversed Tobin effect. Such an effect is also obtained by Zhang (2000) using a transactions costs approach.

Inflation, even if predicted, would also affect output growth adversely by impairing the effectiveness of the financial markets to channel funds from surplus to deficit units. The recent theoretical literature on the importance of informational asymmetries in credit markets shows that higher inflation worsens credit market frictions by reducing the real return on all financial assets. The deterioration in credit market frictions arises from the reduced availability of credit and the worsening of the average quality of borrowers (Huybens and Smith, 1999).

The reduction in loan supply and the lower intermediation activity causes a more inefficient credit allocation, less capital investment and a lower long-run output growth rate[3].

2.2.3 Output uncertainty and inflation uncertainty

There is a consensus among macro theorists to express the ultimate objectives of the monetary authority in terms of deviations of inflation and output from their target levels. Nevertheless, one may argue that Central Banks are also interested in minimising the variability of inflation and output around their target levels (see for example, Cecchetti and Krause, 2001). Taylor (1979) shows that a trade off between the variabilities of inflation and output exists, it is consistent with rational expectations and sticky prices, and implies no long-run trade off between the levels of inflation and unemployment (the Taylor effect). Clarida et al. (1999) derive a short-run inflation-output variability trade off that represents an efficient frontier, i.e., a policymaker can enjoy more output stability only at the expense of more inflation variability. Fuhrer (1997) employs a structural model of optimal monetary policy chosen by minimising a loss function that depends on the variances of inflation and output (expressed as deviations from their targets) and derives the variance trade off. Finally, cross-country evidence by Cecchetti and Krause (2001) shows a variability trade off for a cross section of 23 countries

[3]Boyd et al. (2001) provide cross-country empirical evidence that predictable (sustained) inflation adversely affects various indicators of financial sector performance, such as financial sector lending to the private sector and the volume of bank assets. Given the well-established negative effect of financial development on real growth, this evidence points to the adverse effects of inflation on growth.

that seem to have improved during the 1990s in comparison with the 1980s.

In contrast to the Taylor effect, Logue and Sweeney (1981) claim that inflation uncertainty can have a positive impact on output uncertainty. A higher inflation rate makes it more difficult for producers to distinguish between nominal and real demand shifts, thus leading to more relative price variability. Assuming this relative price variability leads to more producer uncertainty, the upshot will be more variability in real investment and economic activity.

2.2.4 The impact of output growth and inflation on the uncertainty about inflation and output growth

The causal effects of inflation and output growth changes on nominal (inflation) and real (output) uncertainty can be examined according to the theories outlined in sections 2.2.1-2.2.3 above. Friedman (1977) argues that during high-inflation periods it is more likely the monetary authority will instigate an erratic policy response, and therefore, uncertainty about the future inflation rate increases (the so-called Friedman hypothesis). The informal argument presented by Friedman (1977) was subsequently formalised by Ball (1992), who analyses an asymmetric information game where the public faces uncertainty about the type of policymaker in office. Policymakers alternate stochastically in office and can be of two types: a weak type that is unwilling to disinflate and a tough type that is prepared to bear the costs of disinflation. In periods of high inflation, uncertainty about the type of policymaker that will be in office in the next period causes uncertainty about the rate of money growth and hence the future inflation rate. In periods of low inflation, such uncertainty does not arise.

Opposite to the Friedman-Ball hypothesis, Ungar and Zilberfarb (1993) show that in the presence of increasing inflation, agents may invest more resources in forecasting inflation, thus curtailing inflation uncertainty. In summary, theoretically speaking, the effect of inflation on inflation uncertainty is ambiguous. Similarly, the effect of inflation on output uncertainty is also ambiguous. First, a rising inflation rate would be expected to have a negative impact on output uncertainty via a combination of the Friedman and Taylor effects. However, this effect could be positive: higher inflation reduces inflation ucnertainty (Ungar and Zilberfarb, 1993) and increases output uncertainty (Taylor effect).

The sign of the effect of output growth on nominal and real uncertainty is also ambiguous. Consider first the effect of higher output growth on nominal uncertainty. A higher output growth rate will raise inflation according to the short-run Phillips curve and therefore nominal uncertainty, as predicted by the Friedman hypothesis. Hence, the impact of output growth on nominal uncertainty is positive. Several theories predict that this effect will be negative. First, the increased inflation rate arising from more output growth might reduce rather than increase inflation uncertainty (Ungar and Zilberfarb, 1993). Second, Brunner (1993) claims that a decline in economic activity generates uncertainty about the response of the monetary authority and hence the average rate of inflation. Third, if more output growth leads to less inflation (due to the inflation-stabilizing actions of the monetary authority), inflation uncertainty also falls (Friedman hypothesis).

The sign of the effect of real growth on real uncertainty is also ambiguous. An increase in real growth (assuming the Phillips curve and Friedman effects) pushes nominal uncertainty upward and real uncertainty downward (the Taylor effect). However, if the impact of inflation on nominal uncertainty is negative, the opposite conclusion applies.

2.3 The empirical evidence

Early empirical studies on the relationship between inflation and its uncertainty used the variance (or standard deviation) as a measure of uncertainty and hence measured inflation variability as opposed to uncertainty. Following the development of the ARCH approach by Engle (1982) several studies measured inflation uncertainty using the conditional variance of the inflation process. The findings of most of these studies are summarised in Holland (1993b) and Davis and Kanago (2000). In general, the majority of these studies find evidence supporting the first part of the Friedman hypothesis that more inflation leads to more inflation uncertainty. Similar evidence obtains in more recent studies that use GARCH measures of inflation uncertainty, as in Caporale and McKiernan (1997), Grier and Perry (1998, 2000), and Fountas (2001). The second part of the Friedman hypothesis is examined in a number of studies using various measures of inflation variability (see Holland, 1993b). GARCH studies of this issue that represent a more accurate test of the hypothesis that inflation uncertainty has negative welfare effects are much more limited and only include US data (e.g., Coulson and Robins, 1985; Jansen, 1989; Grier and Perry, 2000). Only Grier and Perry (2000) obtain evidence in support of the negative output effects of inflation uncertainty.

The causal impact of inflation uncertainty on inflation is tested empirically using the GARCH approach in Baillie et al. (1996). Grier and Perry (1998, 2000), and Hwang (2001). Many of these studies employ US data, the only exceptions being Baillie et al. (1996) and Grier and Perry (1998). In general, the evidence is mixed. Baillie et al. (1996) find evidence supporting the Cukierman-Meltzer hypothesis for the UK and some high-inflation countries, whereas Grier and Perry (1998) in their G7 study find evidence in favour of the Cukierman-Meltzer hypothesis for some countries and in favour of the Holland hypothesis for other countries.

The empirical evidence to date on the association between output variability and output growth is mixed. Early studies employed cross section (Kormendi and Meguire, 1985) or pooled data (Grier and Tullock. 1989) and found evidence for a positive association. Ramey and Ramey (1995) use a panel of 92 countries and a sample of OECD countries (for the 1960-1985 period) and find strong evidence that countries with higher output variability have lower growth. A similar result is obtained by Zarnowitz and Moore (1986), who divide the 1903-1981 period into 6 subperiods and compare high and low growth periods in terms of output growth variability (measured by the standard deviation of the annual growth rate in real GNP). Empirical evidence on the causal effect of output growth uncertainty (as opposed to variability) on output growth has appeared only recently. Caporale and McKiernan (1996. 1998) obtain evidence of a positive causal effect using UK and US data, respectively, supporting, among others, the Black hypothesis. Speight (1999) finds no relationship between output growth uncertainty and output growth and Henry and Olekalns (2001) find evidence of a negative effect.

In theory. there are potential causal effects of output growth on output variability, for example, an increase in output above its natural level would lead to higher inflation (the short-run Phillips curve) and this higher inflation could induce more inflation variability (the Friedman hypothesis). Hence, the sign of the effect of real growth on real uncertainty can be negative (the Taylor effect) or positive (the Logue and Sweeney hypothesis). However, little empirical evidence for this causal effect has been found.

The existing evidence the effects of inflation on the output growth is mostly of the cross-section and panel type and is mixed (Grier and Tullock, 1989; Fischer, 1993; Clark, 1997). These cross-section and/or panel studies are subject to two criticisms: First, as they include a group of countries where the relationship between inflation and output growth could differ, it would be inappropriate to estimate a single set of regression coefficients using a panel estimation. Second, most of these studies do not control for the effects of inflation uncertainty on output, and hence cannot separate the effects of inflation from those of inflation uncertainty on output. Grier and Perry (2000) provide time-series evidence in favour of a positive effect of inflation on output growth using a bivariate GARCH-M approach.

Finally, with the exception of the effect of output on inflation (the short-run Phillips curve effect), the empirical evidence on the rest of the testable hypotheses discussed above is limited. Grier and Perry (2000) test for the Deveraux hypothesis and find no supporting evidence and Lee (1999) provides some weak evidence for the Taylor effect. Finally, Logue and Sweeney (1981), using a cross-section approach, find that higher inflation variability (measured by the standard deviation) leads to more output growth variability.

2.4 A bivariate GARCH model of inflation and output growth

A bivariate GARCH model is used to estimate simultaneously the conditional means, variances, and covariances of inflation and output growth. Let π_t and y_t denote the inflation rate and real output growth respectively, and define the residual vector ε_t as $\varepsilon_t = (\varepsilon_{\pi t}, \varepsilon_{yt})'$. Note that a general bivariate VAR(p) model can be written as

$$x_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t, \tag{2.1}$$

with

$$\Phi_0 = \begin{bmatrix} \phi_{\pi 0} \\ \phi_{y0} \end{bmatrix}, \quad \text{and} \quad \Phi_i = \begin{bmatrix} \phi_{\pi\pi,i} & \phi_{\pi y,i} \\ \phi_{y\pi,i} & \phi_{yy,i} \end{bmatrix},$$

where x_t is a 2×1 column vector given by $x_t = (\pi_t \ y_t)'$, Φ_0 is the 2×1 vector of constants and Φ_i, $i = 1, \ldots, p$, is the 2×2 matrix of parameters. The Akaike (AIC) and Schwarz (SIC) information criteria are utilised to determine the order of the VAR process in estimation order up to 12. Regarding ε_t it is assumed that it is conditionally normal with mean vector 0 and covariance matrix H_t. That is $(\varepsilon_t | \Omega_{t-1}) \sim N(0, H_t)$ where Ω_{t-1} is the information set up to time $t-1$. We also estimate VAR models where the Φ_i matrix is either lower triangular ($\phi_{\pi y,i} = 0$), or upper triangular ($\phi_{y\pi,i} = 0$) or diagonal ($\phi_{y\pi,i} = \phi_{\pi y,i} = 0$). The best model is chosen on the basis of Granger-causality tests. Following Bollerslev (1990), we perform the Granger-causality tests by imposing the constant conditional correlation (ccc) GARCH(1,1) structure on the conditional covariance matrix H_t:

$$h_{\pi t} = \omega_\pi + \beta_\pi h_{\pi,t-1} + \alpha_\pi \varepsilon^2_{\pi,t-1}, \quad (2.2a)$$

$$h_{yt} = \omega_y + \beta_y h_{y,t-1} + \alpha_y \varepsilon^2_{y,t-1}, \quad (2.2b)$$

$$h_{\pi y,t} = \rho\sqrt{h_{\pi t}}\sqrt{h_{yt}}. \quad (2.2c)$$

where $h_{\pi t}$, h_{yt} denote the conditional variances of the inflation rate and output growth, respectively, and $h_{\pi y,t}$ is the conditional covariance between $\varepsilon_{\pi t}$ and ε_{yt}. It is assumed that ω_i, $\alpha_i > 0$, $\beta_i \geq 0$, for $i = \pi, y$, and $-1 \leq \rho \leq 1$.

Bollerslev (1990) states that the constant correlation model is computationally attractive. His argument is that the correlation matrix can be concentrated out from the log-likelihood function, resulting in a reduction in the number of parameters to be optimized. Moreover, it is relatively easy to control the parameters of the conditional variance equations during the optimization so that h_{it} is always positive.

The system of equations (2.1) and (2.2) are estimated using the Berndt et al. (1974) numerical optimization algorithm (BHHH) to obtain the maximum likelihood estimates of the parameters. Bollerslev (1990) shows that under the assumptions of our model, the BHHH estimate of the asymptotic covariance matrix of the coefficients will be consistent. Given relatively large sample sizes (472-523 observations), the estimated asymptotic t- statistics should be sufficiently accurate.

To measure inflation and output uncertainty, the conditional variances of inflation and output growth are estimated, respectively. Then Granger causality tests are performed to examine the bidirectional causal relationships between the four variables[4]. The choice of the Granger causality approach (see also Grier and Perry, 1998) over the simultaneous-estimation approach has 3 reasons: (1) It allows us to capture the lagged effects between the variables of interest. (2) The simultaneous approach is subject to the criticism of the potential negativity of the variance. (3) The Granger causality approach minimises the number of estimated parameters.

2.5 Empirical results

2.5.1 Data and empirical approach

We use monthly data on the WPI (Wholesale Price Index) and the IPI (Industrial Production Index) as proxies for the price level and output, respectively for the US. The data range from 1957:02 to 2000:08 and cover 523 usable observations. Inflation is measured by the annualized monthly difference of the log WPI [$\pi_t = \log(\frac{\text{WPI}_t}{\text{WPI}_{t-1}}) \times 1200$]. Real output growth is measured by the annualized monthly difference in the log of the IPI [$y_t = \log(\frac{\text{IPI}_t}{\text{IPI}_{t-1}}) \times 1200$]. Table 2.1 summarises the data for the other six countries.

Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are conducted for a unit root test in both the inflation and output growth rates. The results of these tests are reported in Table 2.2. Both tests reject the null hypothesis of a unit root at the 0.01 significance level, implying that the inflation rate and the

[4] The heteroschedasticity consistence (HC) covariance matrix (White, 1980) and the heteroschedasticity and autocorrelation consistence (HAC) covariance matrix (Newey and West, 1987) are used when heteroschedasteity and autocorrealation are detected.

Country[a]	Price Data[b]	Output Data[c]	Sample period	Number of Observations
US	WPI	IPI	1957:02- 2000:08	523
UK	IOP	IPI	1957:02- 2000:07	522
GER	PPI	IPI	1958:02-2000:07	510
FRA	CPI	IPI	1961:02- 2000:06	473
ITA	CPI	IPI	1961:02- 2000:05	472
CAN	AISP	IPI	1957:02- 2000:07	522
JAP	WPI	IPI	1957:02- 2000:08	523

Notes: The price data for France and Italy were obtained from OECD

The rest of the data were obtained from the Institute for Fiscal Studies (IFS).

[a] US: United States, UK: United Kingdom, GER: Germany, FRA: France ITA: Italy, CAN: Canada, JAP: Japan.

[b] WPI: Wholesale price index; IOP: Industrial output price; PPI : Producer price index, CPI : Consumer price index; AISP: Aggregate industrial selling price.

[c] IPI : Industrial production index.

Table 2.1: Price and output data for G7 countries

	US	UK	GER	FRA	ITA	CAN	JAP
π_t^a							
ADF[b]	−6.33	−7.44	−6.01	−3.79	−3.86	−5.29	−5.52
PP[b]	−17.05	−21.92	−14.22	−9.73	−8.20	−16.20	−9.05
y_t^a							
ADF	−9.00	−10.68	−10.43	−12.77	−10.72	−8.03	−6.28
PP	−14.84	−27.92	−38.58	−31.71	−32.41	−27.32	−26.31

Notes: [a] π_t and y_t denote the inflation rate and output growth rate, respectively.

[b] ADF and PP are the Augmented Dickey Fuller and Phillips-Perron test statistics, respectively.

A constant and four lagged difference terms are used for the Augmented Dickey-Fuller test.

The MacKinnon critical value for rejection of the hypothesis of a unit root at 1% significance level is -3.45.

Table 2.2: Unit root tests

growth rate of industrial production can be treated as stationary processes in all countries[5].

[5] Note that the results from the ADF tests are not sensitive to the number of lagged differenced terms. Likewise, the results from the PP tests are not sensitive to the choice of truncation lag.

Estimates of the inflation rate and the real output growth are based upon the following bivariate VAR(12) model:

$$\begin{aligned} \pi_t &= \phi_{\pi 0} + \sum_{i=1}^{12} \phi_{\pi\pi,i}\pi_{t-i} + \sum_{i=1}^{12} \phi_{\pi y,i} y_{t-i} + \varepsilon_{\pi t}, \\ y_t &= \phi_{y0} + \sum_{i=1}^{12} \phi_{y\pi,i}\pi_{t-i} + \sum_{i=1}^{12} \phi_{yy,i} y_{t-i} + \varepsilon_{yt}, \end{aligned}$$

where the conditional covariance matrix H_t follows the ccc GARCH(1,1) model defined in (2.2). We assume that ε_t is conditionally normal[6]. Table 2.3 reports estimates of the bivariate GARCH model for the US. According to the Granger-causality test results reported in Table 2.7 the Φ_i matrix is lower triangular[7].

The estimated parameters of the conditional mean and variance equations for inflation are reported in equations (1) and (2) of Table 2.3. The sum of lagged inflation coefficients is 0.791. The ARCH and GARCH parameters are significant at the 0.01 level. Equations (3) and (4) in Table 2.3 report estimates of the conditional mean and variance of output growth. The sum of lagged output coefficients is 0.42. Both the GARCH and ARCH parameters are highly significant. The sum of the ARCH and GARCH parameters are 0.94 and 0.939 for inflation and output growth, respectively. That is, for both series, current information remains important for the forecasts of the conditional variances for long time horizons. Table 2.4 reports the estimated ARCH and GARCH parameters

[6] For completeness, we have also estimated our bivariate VAR(12)-constant correlation GARCH(1,1) models assuming conditionally t-distributed errors. Results from these models (not reported) are quite similar to those reported in the text using the normal distribution.

[7] In particular, according to the Granger-causality test results reported in Table 2.7, the Φ_i matrix for the UK, France, Italy and Japan is lower triangular whereas for Canada it is upper triangular. Finally, for Germany the full Φ_i matrix is used.

(1) $\pi_t = \underset{(2.11)}{0.593} + \underset{(5.28)}{0.282}\pi_{t-1} + \underset{(0.8)}{0.042}\pi_{t-2} + \underset{(1.24)}{0.072}\pi_{t-3} - \underset{(1.21)}{0.054}\pi_{t-4}$

$+ \underset{(0.79)}{0.04}\pi_{t-5} + \underset{(3.05)}{0.153}\pi_{t-6} - \underset{(1.45)}{0.071}\pi_{t-7} + \underset{(1.78)}{0.082}\pi_{t-8}$

$+ \underset{(0.98)}{0.046}\pi_{t-9} + \underset{(1.47)}{0.070}\pi_{t-10} + \underset{(0.39)}{0.018}\pi_{t-11} + \underset{(2.4)}{0.111}\pi_{t-12} + \varepsilon_{\pi t}$

(2) $h_{\pi t} = \underset{(3.08)}{2.35} + \underset{(5.92)}{0.217}\varepsilon^2_{\pi,t-1} + \underset{(16.91)}{0.723}h_{\pi,t-1}$

(3) $y_t = \underset{(4.84)}{2.98} + \underset{(3.56)}{0.211}y_{t-1} + \underset{(1.29)}{0.081}y_{t-2} + \underset{(1.85)}{0.103}y_{t-3} + \underset{(0.55)}{0.033}y_{t-4}$

$- \underset{(0.48)}{0.028}y_{t-5} - \underset{(0.86)}{0.044}y_{t-6} + \underset{(0.58)}{0.031}y_{t-7} + \underset{(0.93)}{0.045}y_{t-8}$

$+ \underset{(0.56)}{0.025}y_{t-9} + \underset{(0.65)}{0.035}y_{t-10} - \underset{(0.23)}{0.01}y_{t-11} - \underset{(1.46)}{0.062}y_{t-12}$

$+ \underset{(1.21)}{0.075}\pi_{t-1} - \underset{(0.04)}{0.002}\pi_{t-2} - \underset{(0.55)}{0.028}\pi_{t-3} - \underset{(2.08)}{0.102}\pi_{t-4}$

$- \underset{(0.33)}{0.017}\pi_{t-5} - \underset{(1.04)}{0.078}\pi_{t-6} - \underset{(0.3)}{0.024}\pi_{t-7} - \underset{(0.02)}{0.001}\pi_{t-8}$

$+ \underset{(0.91)}{0.053}\pi_{t-9} - \underset{(1.59)}{0.098}\pi_{t-10} - \underset{(0.43)}{0.031}\pi_{t-11} + \underset{(0.2)}{0.013}\pi_{t-12} + \varepsilon_{yt}$

(4) $h_{yt} = \underset{(3.33)}{6.22} + \underset{(4.67)}{0.169}\varepsilon^2_{y,t-1} + \underset{(17.5)}{0.77}h_{y,t-1}$

(5) $h_{\pi y,t} = \underset{(0.21)}{0.011}\sqrt{h_{\pi t}}\sqrt{h_{yt}}$

Notes: Table 2.3 reports parameter estimates of the bivariate AR(12)-GARCH(1,1) model for the US data.

π_t is the inflation rate calculated from the Wholesale Price Index.

y_t is the growth rate calculated from the Industrial Production Index.

$h_{\pi t}$ is the inflation uncertainty.

h_{yt} is the output growth uncertainty.

The numbers in parentheses are absolute t-statistics.

Table 2.3: Bivariate AR(12)- Constant conditional correlation GARCH(1,1) Model (US)

and the conditional correlations for all other G7 countries.

Table 2.5 presents Ljung-Box Q statistics at twelve lags for the levels, squares, and cross-equation products of the standardized residuals for the estimated bivariate GARCH system. The results, reported in Table 2.5, show that the time series models for the conditional means and the GARCH(1,1) model for the residual conditional variance-covariance adequately capture the joint distribution of the disturbances. The conditional correlation coefficient is close to zero, suggesting that the residual covariance between equations is not statistically significant.

Next, Granger-causality tests are reported in Tables 2.6, 2.7 and 2.8 providing the F statistics of Granger-causality tests using four, eight, and twelve lags, as well as the sign of the sums of the lagged coefficients in case of statistical significance. The following subsection presents and discusses these results.

2.5.2 The impact of macroeconomic uncertainty on inflation and output growth

The Granger causality test results of macroeconomic uncertainty on inflation and output growth are given in Table 2.6. Friedman's hypothesis regarding the negative output effects of inflation uncertainty receives support in all countries, except France and Italy. The evidence is stronger in Canada, Japan and the US where it applies to the majority of the chosen lags, and weaker in Germany and the UK, where it applies to only one of the chosen lags. Table 2.6 also reports the results of the tests of the causal effect of inflation uncertainty on inflation. Strong evidence in favour of the Cukierman-Meltzer hypothesis applies in Canada, France, UK, and US. Evidence in favour of the Holland hypothesis applies in Italy

	UK	GER	FRA	ITA	CAN	JAP
$h_{\pi t}$						
ω_π	11.73 [1.57]	2.823 [2.38]	1.205 [2.70]	0.073 [1.05]	1.020 [2.00]	11.59 [3.70]
a_π	0.106 [1.06]	0.168 [3.19]	0.121 [4.06]	0.107 [4.68]	0.105 [4.19]	0.223 [4.13]
β_π	0.797 [6.35]	0.597 [4.49]	0.731 [11.28]	0.890 [48.10]	0.865 [28.31]	0.376 [2.76]
h_{yt}						
ω_y	31.18 [2.33]	54.75 [2.19]	73.58 [4.37]	221.6 [3.64]	6.193 [1.62]	11.46 [1.40]
a_y	0.101 [4.31]	0.057 [2.47]	0.489 [5.73]	0.177 [3.48]	0.068 [3.23]	0.056 [2.14]
β_y	0.797 [13.38]	0.800 [10.55]	0.336 [4.55]	0.497 [4.30]	0.903 [30.01]	0.895 [16.86]
ρ	0.169 [1.75]	0.060 [0.97]	0.063 [1.23]	0.020 [0.36]	0.008 [0.16]	0.025 [0.47]

Notes: Absolute t-statistics are given in brackets. ω_π is the constant term in the conditional variance of inflation. α_π denotes the ARCH parameter in the conditional variance of inflation. β_π denotes the GARCH parameter in the conditional variance of inflation. ω_y is the constant term in the conditional variance of output growth. α_y denotes the ARCH parameter in the conditional variance of output growth. β_y denotes the GARCH parameter in the conditional variance of output growth. ρ is the constant conditional correlation.

Table 2.4: Constant conditional correlation GARCH(1.1) models

	US	UK	GER	FRA	ITA	CAN	JAP
$h_{\pi t}$							
Q_{12}	5.23	6.66	5.71	3.30	13.67	10.19	19.37
Q^2_{12}	11.42	0.30	8.23	18.30	8.97	21.00	14.21
h_{yt}							
Q_{12}	4.00	1.37	0.42	9.59	3.97	1.11	4.55
Q^2_{12}	16.18	0.66	18.04	3.18	19.94	14.42	11.54
Cross equation							
Q_{12}	18.58	0.06	6.59	11.11	9.93	11.94	5.23

Notes: Q_{12} is the 12th order Ljung-Box test for standardised residuals.
Q^2_{12} is the Ljung-Box test for squared standardised residuals.
The critical value at 5% significance level is 21.02.

Table 2.5: Residual Diagnostics

and Japan. None of the two theories is supported in Germany, where inflation is independent of changes in inflation uncertainty.

The results reported in Table 2.6 show strong support for the Black hypothesis that uncertainty about output growth positively affects the rate of output growth in all countries, except Japan and the US. In the US, there is considerable evidence suggesting a negative impact, as hypothesized by Keynes (1936). The lack of any effect of output uncertainty on output growth in Japan squares with the proposition of independence between output growth variability and economic growth outlined in section 2.2.

Finally, Table 2.6 reports mixed evidence on the impact of output growth uncertainty on inflation. Evidence for Deveraux's (1989) theory is provided for France (4 and 8 lags) and Italy (8 lags). In three of the seven countries, namely Japan, UK and US, we find no effect of output uncertainty on inflation. This lack of a direct effect is in agreement with the absence of an indirect effect that takes place via changes in inflation uncertainty[8]. Finally, in Canada and Germany there is evidence of a negative impact consistent with the Taylor effect and the Cukierman-Meltzer hypothesis.

2.5.3 The relationship between inflation and output growth

Table 2.7 reports the results of Granger-causality tests on the relationship between inflation and output growth. These results indicate strong evidence in favour of Stockman (1981): that higher inflation has a negative impact on output in most

[8] Note that in these three countries, output growth uncertainty does not affect inflation uncertainty in Table 2.7.

countries. For Japan, Germany and the US, this finding of a negative direct impact of inflation on output growth squares with an indirect impact: as Tables 2.6 and 2.8 indicate, inflation affects inflation uncertainty positively (the first part of the Friedman hypothesis) and inflation uncertainty affects output growth negatively (the second part of the Friedman hypothesis). In the UK, the direct effect is negative and the indirect effect is positive. Relatively weak evidence for Stockman applies for France. Finally, for Canada we find that inflation has no output effects, supporting Brock (1974). Regarding the impact of output on inflation, Table 2.7 shows a lack of such an effect in all countries except Canada, France, Germany and the UK. In three countries, France, Germany, and the UK, there is weak evidence in favour of a Phillips curve effect, whereas in Canada the evidence is stronger.

2.5.4 The relationship between inflation uncertainty and output uncertainty

Table 2.7 also reports the results on the Granger-causality tests between uncertainty about inflation and output growth. Taylor effect is supported only in Japan. In the rest of the countries, inflation uncertainty does not Granger-cause output uncertainty, except perhaps in Germany, where some weak evidence for the Taylor effect applies. The reverse type of causality (from output uncertainty to inflation uncertainty) exists in two of the seven countries, namely Canada and Germany. The Taylor effect is supported by the Canadian data but rejected by the German data.

2.5.5 The effects of output growth and inflation on macroeconomic uncertainty

The Granger-causality test results on the impact of changes in inflation and output growth on uncertainty is reported in Table 2.8. The results are as follows:

(a) Inflation affects its uncertainty positively as predicted by Friedman (1977) and Ball (1992) in most countries. The evidence is strong in Canada, Italy, and Japan and weaker in France, Germany and the US. In the UK, the Ungar and Zilberfarb (1993) view that inflation reduces inflation uncertainty finds strong support, whereas in France there seems to be no effect.

(b) Inflation has a mixed impact on output uncertainty. The impact is positive in Italy and Japan, weakly negative in Germany and the UK, and zero in the rest of the countries. Recall that, theoretically speaking, the effect of inflation on output uncertainty is ambiguous and is based on the interaction of the Friedman (or Ungar and Zilberfarb, 1993) effect and the Taylor (or Logue-Sweeney, 1981) effect. Therefore, the absence of evidence for the Taylor effect in Canada, France, the US and the UK, as reported in Table 2.7, is consistent with the absence of any effect from inflation on output uncertainty in the first three countries and the very weak effect in the last country. Equivalently, the direct effect of inflation on output uncertainty is in line with the indirect effect that works through the inflation uncertainty channel.

In contrast, in Germany, the evidence for the Friedman and Taylor effects confirms the negative impact of inflation on output uncertainty, i.e., direct and indirect effects point to the same conclusion. Finally, in Japan and Italy, the evidence obtained for the Friedman and Taylor effects points towards a negative indirect effect, whereas the direct effect is positive. Hence, some other mechanism must be at work to explain such a direct effect.

(c) Output has a negative (in Germany, Japan and US), or zero (in Canada. France. Italy, and UK) effect on output uncertainty. Theoretically speaking, the impact of output on output uncertainty depends on the interaction of three effects: The Phillips curve effect. the Friedman (or Ungar and Zilberfarb) effect, and the Taylor effect. The negative effect in Germany is consistent with the weak evidence for the Phillips curve effect. the evidence for the Friedman effect, and the (weak) evidence for the Taylor effect. For Japan and US some other mechanism must be at work to explain the negative impact of output on its uncertainty. Finally, the lack of evidence of an output effect on output uncertainty in Canada, France, Italy, and the UK squares with the lack of an effect of inflation uncertainty on output uncertainty in these countries (see Table 2.7).

(d) Overall, output has a rather weak or zero impact on inflation uncertainty. The impact is weakly negative in Germany, and UK, and weakly positive in Italy. In the rest of the countries. the effect is zero. This result is, in general, consistent with the theoretical underpinnings that predict an ambiguous relationship between the two variables due to the interaction of the Phillips curve effect with, either the Friedman effect. or the Ungar and Zilberfarb effect. The negative impact of output growth on inflation uncertainty for the UK agrees with the evidence on the Phillips curve and the Friedman hypothesis. The lack of an effect in the US is consistent with the lack of an inflationary impact on output growth. In the rest of the countries. namely. Canada. France, Germany. Italy, and Japan. some other mechanism must be at work as the evidence on the effects of output growth on inflation uncertainty (Table 2.7) does not square with the evidence on the Phillips curve and the Friedman hypothesis (Tables 2.6 and 2.8).

2.6 Conclusion

This research has examined the relationship among the uncertainty about the inflation rate and the rate of output growth, and macroeconomic performance, measured by the average rate of inflation and the average rate of economic growth for the G7 countries. Using the GARCH methodology to measure uncertainty, and the constant conditional correlation GARCH(1,1) model to estimate the conditional variances, this study has derived several important conclusions for the majority of countries: First, inflation is a negative determinant of growth. This effect takes place both directly (supporting Stockman, 1981) and indirectly, via the inflation uncertainty channel, as put forward by Friedman (1977). Second, inflation uncertainty affects inflation positively as predicted by Cukierman and Meltzer (1986). Third, uncertainty about the growth rate of output is a positive determinant of the rate of output growth. This result implies that macro theorists should consider simultaneously the analysis of the business cycle and the growth rate of the economy.

Panel A	US	UK	GER	FRA	ITA	CAN	JAP
H0: $h_{\pi t} \rightarrow \pi_t$							
4 lags	4.58***(+)	12.4***(+)	0.98	2.55**(+)	1.84▲(+)	3.35***(+)	32.1***(-)
8 lags	3.60***(+)	5.70***(+)	1.16	2.10**(+)	1.92*(-)	2.34**(+)	18.6***(-)
12 lags	3.55***(-)	5.49***(+)	1.03	2.40***(-)	1.44▲(-)	2.47***(-)	9.69***(-)
H0: $h_{\pi t} \rightarrow y_t$							
4 lags	4.50***(-)	0.65(+)	2.61**(-)	0.39	1.03	1.42(-)	8.59***(-)
8 lags	2.45***(-)	2.56***(-)	1.38(-)	0.47	0.75	1.72*(-)	7.04***(-)
12 lags	3.89***(-)	0.91(-)	1.22(-)	0.58	1.28	2.24***(-)	3.93***(-)
H0: $h_{yt} \rightarrow y_t$							
4 lags	3.28***(-)	8.85***(+)	2.40**(+)	19.5***(+)	1.11(+)	2.40**(+)	0.56
8 lags	2.08**(-)	6.52***(+)	14.7***(+)	10.9***(+)	1.63▲(+)	1.50▲(+)	0.81
12 lags	1.55*(+)	5.66***(+)	36.6***(+)	7.97***(+)	1.99**(+)	1.28(-)	0.88
H0: $h_{yt} \rightarrow \pi_t$							
4 lags	0.80	0.55	6.26***(-)	7.91***(+)	0.72(+)	0.63(-)	0.42
8 lags	0.91	0.40	5.36***(-)	4.90***(+)	2.52***(+)	1.69*(-)	0.44
12 lags	0.90	0.51	2.12***(+)	3.47***(-)	2.62***(-)	2.13***(-)	1.03

Notes: $h_{\pi t} \rightarrow \pi_t$: Inflation uncertainty does not Granger-cause inflation.

$h_{\pi t} \rightarrow y_t$: Inflation uncertainty does not Granger-cause output growth.

$h_{yt} \rightarrow y_t$: Output growth uncertainty does not Granger-cause output growth.

$h_{yt} \rightarrow \pi_t$: Output growth uncertainty does not Granger-cause inflation.

In panel A, a +(-) indicates that the sum of the lagged coefficients is positive(negative).

***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 2.6: Bivariate Granger-causality tests from uncertainty about inflation and output growth to inflation and output growth

Panel B	US	UK	GER	FRA	ITA	CAN	JAP
H$_0$: $\pi_t \rightarrow y_t$							
4 lags	2.56**(-)	8.32***(-)	0.48(-)	0.06(-)	2.18*(-)	0.63	2.30*(-)
8 lags	1.74*(-)	5.77***(-)	1.53$^{\blacktriangle}$(-)	2.03**(-)	1.81*(-)	1.11	2.40**(-)
12 lags	1.59*(-)	5.16***(-)	1.72*(-)	1.19(-)	1.45$^{\blacktriangle}$(-)	0.78	2.37***(-)
H$_0$: $y_t \rightarrow \pi_t$							
4 lags	0.27	4.48***(+)	0.27(+)	1.31(+)	0.11	1.94*(+)	0.61
8 lags	0.67	0.65(+)	0.53(+)	2.45**(+)	0.29	1.64$^{\blacktriangle}$(+)	0.48
12 lags	0.88	0.76(+)	1.52$^{\blacktriangle}$(+)	1.23(+)	1.26	1.54$^{\blacktriangle}$(+)	0.55
H$_0$: $h_{\pi t} \rightarrow h_{yt}$							
4 lags	1.09	0.69	0.33(+)	1.03	1.17	0.36	1.88$^{\blacktriangle}$(-)
8 lags	1.29	0.77	1.33(-)	0.60	1.35	0.74	2.82***(-)
12 lags	1.26	0.50	1.63*(-)	0.45	1.09	0.71	2.20***(-)
H$_0$: $h_{yt} \rightarrow h_{\pi t}$							
4 lags	0.78	1.14	2.92**(+)	1.15	0.94	2.09*(-)	0.34
8 lags	0.27	1.08	1.71*(+)	0.64	0.95	1.61$^{\blacktriangle}$(-)	0.22
12 lags	0.32	0.95	0.77(-)	1.12	0.76	0.86(-)	0.36

Notes: $\pi_t \rightarrow y_t$: Inflation does not Granger-cause output growth.

$y_t \rightarrow \pi_t$: Output growth does not Granger-cause inflation.

$h_{\pi t} \rightarrow h_{yt}$: Inflation uncertainty does not Granger-cause output growth uncertainty.

$h_{yt} \rightarrow h_{\pi t}$: Output growth uncertainty does not Granger-cause inflation uncertainty.

In panel B, a +(-) indicates that the sum of the lagged coefficients is positive(negative).

***,**,* and $^{\blacktriangle}$ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 2.7: Bivariate Granger-causality tests between (i) inflation and output growth and (ii) nominal and real uncertainty

Panel C	US	UK	GER	FRA	ITA	CAN	JAP
H_0: $\pi_t \nrightarrow h_{\pi t}$							
4 lags	$1.75^{\blacktriangle}$(+)	3.45***(-)	$1.83^{\blacktriangle}$(+)	1.37(+)	6.38***(+)	2.59**(+)	8.42***(+)
8 lags	1.35(+)	2.04**(-)	1.71*(+)	1.09(+)	3.58***(+)	1.69*(+)	4.81***(+)
12 lags	$1.52^{\blacktriangle}$(+)	$1.42^{\blacktriangle}$(-)	1.61*(+)	$1.45^{\blacktriangle}$(+)	1.90**(+)	1.19(+)	3.93***(+)
H_0: $\pi_t \nrightarrow h_{yt}$							
4 lags	0.99	2.23*(-)	0.47(-)	0.82	$1.68^{\blacktriangle}$(+)	1.38	0.28(+)
8 lags	0.93	1.22(-)	1.77*(-)	0.64	1.08(+)	1.38	1.76*(+)
12 lags	1.03	0.91(-)	$1.46^{\blacktriangle}$(-)	0.45	1.73*(+)	1.08	$1.42^{\blacktriangle}$(+)
H_0: $y_t \nrightarrow h_{yt}$							
4 lags	2.21*(-)	1.15	6.08***(+)	1.60	0.69	1.07	0.14(-)
8 lags	2.28**(-)	0.59	5.40***(-)	1.21	1.03	1.42	$1.53^{\blacktriangle}$(-)
12 lags	1.28(+)	0.52	1.72*(-)	0.81	0.94	1.00	2.35***(-)
H_0: $y_t \nrightarrow h_{\pi t}$							
4 lags	0.29	2.11*(-)	$1.89^{\blacktriangle}$(-)	1.13	2.34*(+)	0.20	1.64
8 lags	0.25	1.23(-)	1.28(-)	0.40	1.47(+)	0.38	1.28
12 lags	0.42	0.89(+)	1.22(-)	0.64	1.05(+)	0.65	1.14

Notes: $\pi_t \nrightarrow h_{\pi t}$: Inflation does not Granger-cause inflation uncertainty.

$\pi_t \nrightarrow h_{yt}$: Inflation does not Granger-cause output growth uncertainty.

$y_t \nrightarrow h_{yt}$: Output growth does not Granger-cause output growth uncertainty.

$y_t \nrightarrow h_{\pi t}$: Output growth does not Granger-cause inflation uncertainty.

In panel C, a +(-) indicates that the sum of the lagged coefficients is positive(negative).

***,**,* and $^{\blacktriangle}$ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 2.8: Bivariate Granger-causality tests from inflation and output growth to uncertainty about inflation and output growth

Chapter 3

The inflation-output variability trade-off and the monetary policy in G3

3.1 Introduction

One of the most actively debated issues in macroeconomics is the nature of the trade-off between the levels of inflation and output or unemployment. Taylor (1994) analysed the long-term relationship between inflation and output in a different way. He claims that there is a trade-off relationship between the variability of inflation and output, which implies no long-run trade-off between the levels of inflation and unemployment. His argument is in line with the Friedman-Phelps hypothesis that there is no long-run Phillips curve trade-off. Such a variability trade-off is also consistent with sticky prices and rational expectations.

However, Logue and Sweeney (1981) find that nominal uncertainty and real uncertainty are positively correlated and can move in the same directions. They suggest that a high level of price variability induces more uncertainty about future investment and produce. This controversial issue is closely related to the efficiency of monetary policy.

Cecchetti and Ehrmann (1999) point out that in the 1990s throughout the world, central banks moved away from focusing on intermediate objectives, such as money, toward the direct targeting of inflation. Although inflation targeting has been successful since its inception, there still remain questions as to the costs it may entail. One of the main criticisms against inflation targeting is that inflation targeting can lead to undesirable outcomes, such as excessive output variability, as predicted by Taylor. Clarida et al. (2001) point out that the empirical evidence of the trade-off relationship between the two variables is an important guiding principle in many applied studies of monetary policy.

The trade-off relation between inflation and output variability is an issue that cannot be resolved on merely theoretical grounds. This chapter explores the trade-off between inflation and output variability in the G3 using bivariate GARCH models. Most of the empirical studies have used either stochastic optimal control techniques or sensitivity analysis of dynamic general equilibrium models to investigate the trade-off in variances. Only the studies of Lee (1999), Arestis et al. (2001) and Fountas et al. (2002) attempt to investigate the variability trade-off using measures of conditional volatilities.

The current study adopts a two-step procedure. First, we estimate conditional variances from a bivariate GARCH model as the statistical measures of inflation and output variability. Using the estimated time series of nominal and real uncertainty, in the second part the Granger causality tests are performed. Grier and Perry (1998) also employed the two-step approach. This approach provides a simple method to illustrate the existence or absence of a trade-off.

Several results stand out for the entire sample period. First, nominal uncertainty significantly affects real uncertainty in all three countries but not all in the same manner. In Japan and the USA increased inflation uncertainty does lead to an increased output uncertainty. This is in line with the hypothesis advanced by Logue and Sweeney (1981). By contrast, in Germany there is mild evidence that increased nominal uncertainty lowers real uncertainty, confirming the theoretical predictions made by Taylor (1979). Second, in the USA and Germany real uncertainty has a negative effect on nominal uncertainty. The evidence is mild in the USA and weak in Germany. In Japan real uncertainty does not Granger-cause nominal uncertainty. These results are not qualitatively altered by changes in the measures of volatility. The results are supportive of a bidirectional feedback between nominal and real uncertainty, but with the line of causation running from the former to the latter being the stronger of the two.

This research also examines whether the changes in monetary policy around 1980 affect the trade-off in variances by dividing the whole sample period into two sub-periods and conducting causality tests for each sub-period separately. The effect of nominal uncertainty on real uncertainty is negative, supporting the Taylor hypothesis in the sixties and seventies but turns to positive in the 1980s and 1990s in favour of Logue and Sweeney's hypothesis. On the other hand real uncertainty has a significant causal effect on nominal uncertainty in the eighties and nineties but the effect does not appear in the 1960s and 1970s.

The structure of the chapter is as follows. Section 3.2 provides a background discussion of the debate over the inflation/output variability trade-off. Section 3.3 describes the theoretical model used for estimation. In addition to the BEKK model, the vector-diagonal model is also considered for parameterisation. Section 3.4 presents the empirical analysis and the results from the Granger causality tests. Section 3.5 discusses some monetary policy issues and conducts subsample analyses. Concluding remarks are in section 3.6.

3.2 Prior research

3.2.1 Theory

Taylor (1979, 1994) argues that the existence of the short-run output/inflation trade-off implies a long-run trade-off in variances. In other words, if the policy-makers wish to reduce nominal uncertainty in the face of demand and supply shocks they must vary real output a great deal in order to stabilize inflation. On the other hand, in order to lower the variability of output the policy-makers must allow shocks that affect inflation to persist, thus increasing the nominal uncertainty.

However. Logue and Sweeney (1981) argue that there are two reasons to suspect that greater uncertainty of inflation leads to greater uncertainty in production, investment, and marketing decisions, and greater variability in real growth. One reason is that relative price variation creates additional uncertainty of produce. The real growth in investment and all other economic activity will be more variable because of the inability to distinguish real shifts in demand from nominal shifts. Second, models with a stable inflation-unemployment trade-off imply a positive relationship between the variability of inflation and the variability of real activity.

More recently, the relationship between nominal and real uncertainty has been analysed by using intertemporal general equilibrium models. The models developed by Goodfriend and King (1997), King and Wolman (1998) and Rotemberg and Woodford (1998) show that inflation targeting, by keeping the inflation rate constant, also minimises the output gap variability. Bean (1998) questions the existence of a variance trade-off. He develops a model in which an optimal monetary policy is defined as the policy that minimizes the variances of output and inflation.

However, Erceg et al. (1998) demonstrate the existence of an inflation/output variance trade-off using a dynamic general equilibrium model which incorporates reasonable wage inertia. In their model, when nominal wages are sticky, the trade-off between output-gap variability and inflation variability exists regardless of the degree of price stickiness. However, when wages are perfectly flexible the trade-off disappears. Svensson (1998) also analyses a model in which the inflation/output variance trade-off arises because of cost-push supply shocks whereas Jadresic (1999) argues that targeting headline inflation, when prices are sticky and price shocks are anticipated, can severely destabilise the output gap (for a recent discussion see Arestis et al., 2001). Clarida et al. (2001) illustrate the trade-off in variances by constructing the efficient policy frontier corresponding to their baseline model. They also emphasise that the trade-off emerges only if cost push inflation is present. In the absence of cost inflation there is no trade-off.

3.2.2 Empirical Evidence

Taylor (1979, 1980) was the first to define and estimate a long-run trade-off between nominal and real uncertainty. In sharp contrast, Logue and Sweeney (1981), using cross-sectional tests and data from 24 countries that are members of the OECD, found that the variability in real growth is strongly and positively related to the variability in inflation. Subsequently, Taylor (1993) and Fuhrer and Moore (1993), using stochastic optimal control techniques, estimated a new trade-off. Taylor (1994) revisited the trade-off between the variability of inflation and of output. Using a series of simple diagrams and graphs he demonstrated that the trade-off exists because of the slow adjustment of prices. He also argued that monetary policy can determine where on the trade-off curve the economy lies.

Fuhrer (1997) estimates an efficient set of weighted combinations of inflation and output variance and finds that when monetary policy attempts to make output (inflation) variation too small there is a dramatic increase in inflation (output) variances. Cecchetti and Ehrmann (1999) suggest that the standard deviation of inflation falls less in the non-inflation targeting countries than in the inflation targeting countries. They observe that output variability (as measured by the standard deviation) falls far more in the latter than in the former. Clarida et al. (1999) show that a trade-off in variances exists and is less favourable the higher the degree of inflation persistence. Arestis et al. (2001) utilise a stochastic volatility model to analyse the possible effects of inflation targeting on the trade-off between output-gap variability and inflation variability. They find that the adoption of inflation targets, in countries like Australia, Canada and the UK, results in a more favourable monetary policy trade-off.

On the other hand, Batini and Haldane (1998) for the UK and Amano et al. (1999) for Canada show that inflation targeting lowers both real and nominal uncertainty. Lee (1999), using US data, estimates the BEKK parameterisation of a bivariate GARCH process and finds a positive relationship between nominal and real uncertainty. The evidence is stronger for the post-1979 period than in the pre-1979 period. Fountas et al. (2002) employ a constant conditional correlation bivariate GARCH model and find no evidence of a trade-off in variances for Japan.

3.3 The models

This research uses bivariate VAR models to estimate the conditional means of the rates of inflation and output growth. Let π_t and y_t denote the inflation rate and real output growth respectively, and define the residual vector ε_t as $\varepsilon_t = (\varepsilon_{\pi t}, \varepsilon_{yt})'$. Note that a general bivariate VAR(p) model can be written as

$$x_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t, \tag{3.1}$$

with

$$\Phi_0 = \begin{bmatrix} \phi_{\pi 0} \\ \phi_{y0} \end{bmatrix}, \quad \text{and} \quad \Phi_i = \begin{bmatrix} \phi_{\pi\pi,i} & \phi_{\pi y,i} \\ \phi_{y\pi,i} & \phi_{yy,i} \end{bmatrix},$$

where x_t is a 2×1 column vector given by $x_t = (\pi_t \; y_t)'$, Φ_0 is the 2×1 vector of constants and Φ_i, $i = 1, \ldots, p$, is the 2×2 matrix of parameters. The Akaike information criterion (AIC) is utilised to determine the optimal lag order of the VAR process. On the basis of the minimum AIC and the requirement of white residuals 12 lags is chosen in the VAR process. It is assumed that ε_t, is conditionally normal with mean vector 0 and variance-covariance matrix H_t, where $\text{vech}(H_t) = (h_{\pi t}, h_{\pi y,t}, h_{yt})'$. That is, $(\varepsilon_t|\Omega_{t-1}) \sim N(0, H_t)$, where Ω_{t-1} is the information set up to time $t-1$. This study also estimated VAR models where the Φ_i matrix was either lower triangular ($\phi_{\pi y,i} = 0$), or upper triangular ($\phi_{y\pi,i} = 0$), or diagonal ($\phi_{y\pi,i} = \phi_{\pi y,i} = 0$). The choice between the three models was based on the results of Granger-causality tests. Following Engle and Kroner (1995), these Granger causality tests were performed under the assumption that the conditional covariance matrix follows the BEKK representation[1]. That is, H_t is parametrized as

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BH_{t-1}B', \tag{3.2}$$

with

$$C = \begin{bmatrix} c_{\pi\pi} & c_{\pi y} \\ c_{y\pi} & c_{yy} \end{bmatrix}, \quad A = \begin{bmatrix} \alpha_{\pi\pi} & \alpha_{\pi y} \\ \alpha_{y\pi} & \alpha_{yy} \end{bmatrix}, \quad B = \begin{bmatrix} \beta_{\pi\pi} & \beta_{\pi y} \\ \beta_{y\pi} & \beta_{yy} \end{bmatrix}$$

Because of the presence of a paired transposed matrix factor for each of these three matrices non-negative definiteness of the conditional matrix is assured. Also,

[1] In the presence of conditional heteroskedasticity, Vilasuso (2001) investigates the reliability of causality tests based on least squares. He demonstrates that when conditional heteroskedasticity is ignored, least squares causality tests exhibit considerable size distortion if the conditional variances are correlated. In addition, inference based on a heteroskedasticity and autocorrelation consistent covariance matrix constructed under the least squares framework offers only slight improvement. Therefore, he suggests that causality tests be carried out in the context of an empirical specification that models both the conditional means and conditional variances.

in the above BEKK model. $\{\varepsilon_t\}$ is covariance stationary if and only if all the eigenvalues of $A \otimes A + B \otimes B$ (where $\otimes$ stands for Kronecker product) are less than one in modulus (see Engle and Kroner, 1995). The system of equations (3.1) and (3.2) is estimated using the Berndt et al. (1974) numerical optimization algorithm (BHHH) to obtain the maximum likelihood estimates of the parameters.

The bivariate GARCH(1,1) system using the vector-diagonal (VECD) model by Bollerslev et al. (1994) is also estimated as an alternative of the BEKK approach. The form of H_t for this model is

$$H_t = CC' + A_m A_m' \odot \varepsilon_{t-1}\varepsilon_{t-1}' + B_m B_m' \odot H_{t-1}, \tag{3.3}$$

with

$$A_m = \begin{bmatrix} \alpha_{\pi\pi} \\ \alpha_{yy} \end{bmatrix}, \quad B_m = \begin{bmatrix} \beta_{\pi\pi} \\ \beta_{yy} \end{bmatrix},$$

where the symbol $\odot$ stands for Hadamard product (that is element-by-element multiplication), and the C is defined in (3.2).

3.3.1 Commonality in volatility movements

The notion of "persistence" of a shock to volatility within the GARCH class of models is considerably more complicated than the corresponding concept of persistence in the mean for linear models. One definition of persistence would be that shocks fail to persist when $\{h_{it}\}$ $(i = \pi, y, \pi y)$ is stationary and ergodic. The persistence of shocks can also be defined in terms of forecast moments; i.e. to say that shocks to h_{it} fail to persist if and only if for every s, $E_s(h_{it})$ converges, as $t \to \infty$, to a finite limit independent of time s information. In this study we will adopt the latter definition.

Note that the two conditional (co)variances in equation (3.2) can be expressed as

$$\begin{aligned}
h_{\pi,t} &= c_{\pi\pi}^2 + c_{\pi y}^2 + \alpha_{\pi\pi}^2\varepsilon_{\pi,t-1}^2 + 2\alpha_{\pi\pi}\alpha_{\pi y}\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{\pi y}^2\varepsilon_{y,t-1}^2 + \\
&\quad +\beta_{\pi\pi}^2 h_{\pi,t-1} + 2\beta_{\pi\pi}\beta_{\pi y}h_{\pi y,t-1} + \beta_{\pi y}^2 h_{y,t-1}, && (3.4a)\\
h_{y,t} &= c_{yy}^2 + c_{y\pi}^2 + \alpha_{y\pi}^2\varepsilon_{\pi,t-1}^2 + 2\alpha_{y\pi}\alpha_{yy}\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{yy}^2\varepsilon_{y,t-1}^2 + \\
&\quad +\beta_{y\pi}^2 h_{\pi,t-1} + 2\beta_{y\pi}\beta_{yy}h_{\pi y,t-1} + \beta_{yy}^2 h_{y,t-1}, && (3.4b)\\
h_{\pi y,t} &= c_{\pi\pi}c_{y\pi} + c_{yy}c_{\pi y} + \alpha_{\pi\pi}\alpha_{y\pi}\varepsilon_{\pi,t-1}^2 + (\alpha_{\pi\pi}\alpha_{yy} + \alpha_{y\pi}\alpha_{\pi y})\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \alpha_{yy}\alpha_{\pi y}\varepsilon_{y,t-1}^2 \\
&\quad +\beta_{\pi\pi}\beta_{y\pi}h_{\pi,t-1} + (\beta_{\pi\pi}\beta_{yy} + \beta_{y\pi}\beta_{\pi y})h_{\pi y,t-1} + \beta_{yy}\beta_{\pi y}h_{y,t-1} && (3.4c)
\end{aligned}$$

The cross-equation restrictions implied by (3.4) make it difficult to link the persistence in a particular component of the conditional variance-covariance matrix to particular parameters. Following Engle and Kroner (1995), this research presents the largest eigenvalue of $A \otimes A + B \otimes B$ as a measure of persistence.

From the expressions in (3.4) it is easily seen that the off-diagonal elements of the matrix A (B) depict how the past squared error (conditional variance) of one variable affects the conditional variance of another variable. In other words, $\alpha_{\pi y}$, $\alpha_{y\pi}$, $\beta_{\pi y}$ and $\beta_{y\pi}$ can be viewed as providing information on the correlation between real and nominal uncertainty.

Moreover, the conditional (co)variances in equation (3.3) may be written as

$$\begin{aligned}
h_{\pi,t} &= c_{\pi\pi}^2 + c_{\pi y}^2 + \alpha_{\pi\pi}^2\varepsilon_{\pi,t-1}^2 + \beta_{\pi\pi}^2 h_{\pi,t-1}, && (3.5a)\\
h_{y,t} &= c_{yy}^2 + c_{y\pi}^2 + \alpha_{yy}^2\varepsilon_{y,t-1}^2 + \beta_{yy}^2 h_{y,t-1}, && (3.5b)\\
h_{\pi y,t} &= c_{\pi\pi}c_{y\pi} + c_{yy}c_{\pi y} + \alpha_{\pi\pi}\alpha_{yy}\varepsilon_{\pi,t-1}\varepsilon_{y,t-1} + \beta_{\pi\pi}\beta_{yy}h_{\pi y,t-1} && (3.5c)
\end{aligned}$$

Hence, this model clearly does not allow for commonality in volatility movements and for causality in variance. The sum of $\alpha_{\pi\pi}^2$ and $\beta_{\pi\pi}^2$ in (3.5a) reflects the level of persistence in the conditional variance of inflation whereas α_{yy}^2 and β_{yy}^2 are linked to the persistence in the real uncertainty.

3.4 Empirical Analysis

3.4.1 Data

The current research uses monthly data on the WPI (Wholesale Price Index) and the IPI (Industrial Production Index) for the US and Japan as proxies for the price level and output, respectively. The data ranges from 1957:02 to 2000:08 with 523 usable observations. For Germany PPI (Producer Price Index) and the IPI (Industrial Production Index) are used. The data covers from 1958:02 to 2000:07 with 510 observations. Inflation is measured by the annualized monthly difference of the log WPI [π_t =log($\frac{\text{WPI}_t}{\text{WPI}_{t-1}}$) × 1200] (PPI for Germany). Output growth is measured by the annualized monthly difference in the log of the IPI [y_t =log($\frac{\text{IPI}_t}{\text{IPI}_{t-1}}$) × 1200][2].

3.4.2 Empirical Results

Table 3.1 reports parameter estimates for the BEKK(1,1) and DVEC(1,1) models[3]. The parameter $\alpha_{\pi y}$ suggests a cross-effect running from the lagged output

[2]The data were obtained from the Institute for Fiscal Studies (IFS).

[3]The BEKK and DVEC estimates of the inflation and output uncertainty are based upon a bivariate VAR(12) model. On the basis of the AIC and the requirement of white residuals we have decided to include twelve lags in the VAR.

error to the inflation variance whereas the parameter $\alpha_{y\pi}$ depict a cross-effect in the opposite direction. The off diagonal elements in B depict the extent to which the conditional variance of one variable is correlated with the lagged conditional variance of the other variable.

To test for volatility transmissions between inflation and output we perform joint tests under the null hypothesis that $a_{ij} = \beta_{ij} = 0$ for $i \neq j$. Based on the likelihood ratio test statistic the null hypothesis of no cross effects is accepted. In other words, the likelihood ratio test shows the dominance of the DVEC model in all three countries. Alternatively, the Akaike or Schwarz information criteria (AIC, SIC respectively) can be applied to rank the two alternative bivariate GARCH models. These model selection criteria check the robustness of the log-likelihood ratio. Specifically, according to both the AIC and SIC, the optimal GARCH type model for all countries was the DVEC one. In all three cases the statistical insignificance of the estimates of $\alpha_{\pi y}$, $\beta_{\pi y}$, $\alpha_{y\pi}$ and $\beta_{\pi y}$ shows the lack of any association between the variability of inflation and output growth. Clearly, there is no support for any relationship between real and nominal uncertainty. Finally, with all countries, the hypothesis of uncorrelated standardized and squared standardized residuals is well supported.

Table 3.2 reports the persistence for the BEKK(1,1) and DVEC(1,1) model. For the BEKK model as a measure of persistence in the two volatilities the largest eigenvalue of $A \otimes A + B \otimes B$ is used (see column 2). The estimated eigenvalue for Germany is markedly lower than the corresponding values for Japan and the USA. These two countries generated very similar persistence parameters (0.97 and 0.96 respectively). In the DVEC model the sum of the estimated $\alpha^2_{\pi\pi}(\alpha^2_{yy})$ and

	USA		JAPAN		GERMANY	
	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)
ω_{11}	1.505 (6.22)	1.527 (6.25)	3.637 (10.12)	3.551 (9.55)	1.916 (4.94)	2.275 (5.17)
ω_{12}	0.912 (0.65)	−0.169 (0.19)	0.573 (0.29)	0.164 (0.31)	0.744 (0.27)	0.911 (1.16)
ω_{22}	6.139 (12.37)	6.490 (14.66)	2.454 (1.72)	2.415 (2.55)	6.162 (3.74)	6.922 (4.90)
α_{11}	0.478 (11.97)	0.483 (11.52)	0.564 (11.21)	0.552 (13.41)	0.401 (6.41)	0.432 (5.95)
α_{21}	0.042 (0.51)	-	0.038 (0.22)	-	0.176 (0.46)	-
α_{12}	−0.024 (0.68)	-	−0.027 (1.03)	-	−0.006 (0.45)	-
α_{22}	0.724 (13.13)	0.713 (12.89)	0.200 (4.73)	0.173 (4.26)	0.304 (4.85)	0.315 (6.07)
β_{11}	0.852 (36.85)	0.848 (34.67)	0.531 (5.21)	0.558 (5.78)	0.733 (6.84)	0.619 (3.68)
β_{21}	−0.069 (0.88)	-	0.048 (0.28)	-	0.245 (0.58)	-
β_{12}	0.005 (0.12)	-	−0.009 (0.28)	-	−0.005 (0.39)	-
β_{22}	0.395 (5.43)	0.355 (4.41)	0.965 (52.13)	0.972 (66.61)	0.903 (24.0)	0.891 (23.97)
Likelihood	-3489.96	-3491.17	-3800.34	-3800.87	-3602.49	-3605.01.42
LR test	2.42	-	1.06	-	5.04	-
SIC	7286.6	7264.0	7585.3	7565.4	7907.4	7883.4
AIC	7077.9	7072.3	7327.0	7324.0	7698.7	7691.8
$Q_{\pi_t}(10)$	1.36	1.54	4.81	5.66	9.23	13.34
$Q_{y_t}(10)$	8.04	7.01	0.65	0.92	4.96	4.89
$Q_{\pi\, y_t}(10)$	12.22	12.79	9.60	8.18	9.63	7.60
$Q^2_{\pi_t}(10)$	8.25	8.35	3.40	3.34	14.35	13.85
$Q^2_{y_t}(10)$	16.63	17.28	17.06	16.93	16.00	17.94

Notes: This table reports parameter estimates for the BEKK(1,1) and DVEC(1,1) models with data for USA, Germany and Japan. The numbers in parentheses are t-statistics. Schwarz (SIC) and Akaike information criteria (AIC) are presented.
Likelihood ratio (LR) test is conducted as follows :

$$LR=2 \times \left[ML_U - ML_R\right]$$

where ML_U and ML_R denote the maximum loglikelihood values of the unrestricted (BEKK(1,1)) and restricted (DVEC(1,1)) models respectively. The critical value at 5% significance level is 9.49.
Q(10) and $Q^2(10)$ are the Ljung-Box statistics for tenth-order serial correlation in the residuals and in the squared residuals respectively. The critical value at 5% significance level is 18.30.

Table 3.1: Parameter estimates for the BEKK(1,1) and DVEC(1,1) models (Entire sample)

	BEKK(1.1)	BEKK(1,1) AR(1)		DVEC(1.1)	
	$h_{\pi,t}, h_{y,t}$	$h_{\pi,t}$	$h_{y,t}$	$h_{\pi,t}$	$h_{y,t}$
USA	0.955	0.900	0.343	0.952	0.634
JAPAN	0.968	0.475	0.978	0.616	0.975
GERMANY	0.876	0.744	0.917	0.570	0.893

Notes: The second column reports the largest eigenvalue of $A \otimes A + B \otimes B$ as a measure of persistence for BEKK model. The column 3 and 4 show the estimated AR (1) coefficients of the two conditional variances in the BEKK model. The last column reports the sum of the estimated $\alpha^2_{\pi\pi}(\alpha^2_{yy})$ and $\beta^2_{\pi\pi}(\beta^2_{yy})$ in the DVEC model.

Table 3.2: Persistence for the BEKK(1,1) and DVEC(1,1) models

$\beta^2_{\pi\pi}(\beta^2_{yy})$ reflects the level of persistence in the conditional variance of inflation (output) (see columns 5 and 6). A simple way to compare the persistence in the two conditional variances in the BEKK model is to regress $\widehat{h}_{it}$ $(i = \pi, y)$ on a constant and $\widehat{h}_{i,t-1}$ (see columns 3 and 4). In the USA it is clear that inflation volatility is more persistent than output volatility. However, for Japan and Germany real uncertainty is more persistent than nominal uncertainty.

3.4.3 Granger-causality tests

In the previous section the relationship between nominal and real uncertainty was estimated in a simultaneous approach using the BEKK model which allows each h_{it} ($i = \pi, y$) to depend on lagged squared residuals and past variances of both variables in the system. The simultaneous approach suffers from the disadvantage that it does not allow the testing of a lagged effect of inflation uncertainty on output uncertainty (and vice versa), which would be expected in a study that employs monthly data. The trade off between variability in output and variability in inflation takes time to materialize and cannot be fairly tested in a model that restricts the effect to be in one month only. In this section we employ a two-step approach where the estimates of the two conditional variances are first obtained from our bivariate GARCH models and causality tests are then run to test for bidirectional effects.

Table 3.3 reports the results of causality between nominal and real uncertainty for both bivariate GARCH models. Panel A tests the null hypothesis that inflation uncertainty does not cause output uncertainty. The BEKK columns show that increased nominal uncertainty significantly affects real uncertainty in all three countries, but not all in the same manner. In Japan and the USA increased inflation uncertainty does lead to an increased output uncertainty and this effect is statistically significant at the 1% level. These results strongly support the hypothesis advanced by Logue and Sweeney (1981). By contrast, in Germany there is mild evidence (at lag 12) that increased nominal uncertainty lowers real uncertainty, confirming the theoretical predictions made by Taylor. In other words the Taylor hypothesis is verified by the Granger-causality tests only for Germany.

Panel B tests the null hypothesis that output uncertainty does not cause inflation uncertainty. The BEKK columns show that in Germany and Japan at each lag length the null hypothesis that real uncertainty does not Granger-cause nominal uncertainty is accepted at the 0.01 level. Hence, for these two countries an insignificant negative relationship between nominal uncertainty and past real uncertainty is found. In the USA the null hypothesis is rejected at the 0.15 level using 4 lags. There is very weak evidence in favour of the Taylor theory since the sum of the coefficients on lagged real uncertainty in the nominal uncertainty equation is negative.

The above conclusion on the relationship of inflation uncertainty and output growth uncertainty is not altered as a result of using DVEC models versus BEKK models. When we use the DVEC model to obtain estimates of the two conditional variances, the null hypothesis in Panel B is now accepted(rejected) at the 0.15(0.10) level using twelve (four) lags for the USA(Germany).

3.5 Monetary policy and sub sample analysis.

The full sample, which runs from 1957:02 through 2000:08, has been broken into two sub samples, corresponding to assumed shifts in the monetary policy regime. In particular, we have examined the pre- and post- 1980 periods. The reasons why we chose 1980 as a breakpoint are: (1) there was important monetary policy change in central banks around 1980 (2)1980 is the mid point between the high inflation of the sixties and seventies and the low inflation of the 1980s and 1990s (Krause, 2001). The first sample period covers the period between February 1957-and December 1979 (afterwards sample A). The second sub-sample covers the period from January 1980 to August 2000 (afterwards sample B). In addition, we examine the breakpoint of 1975 focusing on central bank reformation in Japan and Germany. In the following analysis, sample C and sample D correspond to the pre-1975 and post-1975 periods, respectively.

3.5.1 Monetary policy

United States: The primary instrument of monetary policy in the U.S. up to 1979 was the federal funds rate -the interbank lending rate. The Fed placed greater weight on reducing unemployment than meeting money growth targets. A commitment to reducing inflation was signalled by a change in Fed operating procedures in October 1979. The funds rate targeting regime came to an end and the new regime that followed was described by the Fed as targeting non borrowed bank reserves. As Bernanke and Blinder (1992) point out, the change in the operating procedures seems to have been accompanied by a decision by the Fed to place greater weight on monetary targets and to tolerate high and volatile interest rates in order to bring down inflation. One more key objective with lower inflation during the latter part of the 1980s was exchange rate stabilization. Beginning in early 1985, the Fed attempted to bring down the dollar by driving up both M1 and M2 growth rates.

Japan: The increase in oil prices in late 1973 was a major shock for Japan, with substantial adverse effects on inflation, economic growth, and the government's budget. In response to an increase in the inflation rate to a level above 20% in 1974 the Bank of Japan, like the other central banks we have considered, began to pay more attention to money growth rates. In addition, the liberalization of financial markets, which started in Japan around 1975, resulted ultimately in a weaker tie between bank lending and economic activity and the introduction of new markets and financial instruments. The Bank of Japan moved gradually to a system emphasising the use of interbank interest rates as the primary instruments of monetary control, open-market operations in the interbank market, and more attention to money growth.

Prior to 1978 the Bank of Japan was committed only to monitoring rather than to controlling money growth. However, after 1978 there did appear to be a substantive change in policy strategy, in the direction of being more "money-focused". Particularly, there was a different response of monetary policy to the second oil price shock in 1979. Instead of allowing extremely high money growth as occurred in 1973, the Bank of Japan quickly reduced M2+Cds growth in 1979 and 1980 to quite a low level. The difference in the inflation outcome in this episode was striking, as inflation increased only moderately with no adverse effects on the unemployment situation (Bernanke and Mishikin, 1997).

Also in parallel to the United States, ultimately financial innovation and deregulation in Japan began to reduce the usefulness of the broad money target. In particular, the introduction of money market certificates and large time deposits in 1985 led to increases in the demand for M2. Beginning in 1989 asset prices came down as money growth slowed, economic activity weakened and there was a slow down in lending by Japanese banks. In responding to these developments the Bank of Japan permitted a considerable increase in the variability of broad money growth after late 1980.

Germany: The Bundesbank set a monetary target in 1975, after the break up of Bretton Woods. The Bundesbank originally targeted a construct of monetary aggregate it termed Central Bank Money (CBM). CBM is roughly the monetary base minus excess reserves. Originally, a fixed money target was announced but after two years this was changed to a fixed range. Like many other central banks, the Bundesbank translated its main policy goals (e.g. controlling inflation) into near term interest rate objectives. It in turn supplied bank reserves to meet these objectives. Until the mid-1980s, the Bundesbank manipulated short term market interest rates (and bank reserves) via discount window lending to commercial banks. After 1985 the Bundesbank supplied banks with reserves mainly via repurchase agreements. Reunification introduced new complexities for monetary management. The one-for-one currency exchange with East Germany led to a 13 percent increase in the M3 aggregate within a single month. The jump complicated monetary targeting. Given the large implicit subsidy in the currency swap, the possible consequences for inflation (which accelerated above target in 1991) were another concern of the Bundesbank.

3.5.2 Sub samples analyses

Table 3.5 reports the results of Granger causality test for the pre- and post- 1980 periods. First, the results for the pre-1980 period are presented in Panel A and B. For Japan and Germany we find strong evidence of a negative unidirectional inflation/output variability relationship with the line of causation running from nominal uncertainty to real uncertainty. No effect in either direction is presented for the USA. The results of applying the Granger causality tests for the post-1980 period are reported in Panel C and D. The picture is different from that of the pre-1980 period. In Japan there is a lack of a causal effect of nominal uncertainty on real uncertainty whereas in the other two countries the effect is positive. The evidence is strong in the USA and weak in Germany. Panel D also shows that in all three countries real uncertainty Granger causes nominal uncertainty. The effect is positive in Japan and Germany and negative in the USA. In general, the empirical evidence shows the trade-off relation between nominal uncertainty and real uncertainty in the 1960s and 1970s and the positive correlation in the eighties and nineties.

Comparing the results of the entire period with those of the pre- and post- 1980 periods the following observations are noted. The evidence in Germany (Japan) that the Taylor (Logue-Sweeney) hypothesis holds for the entire sample reflects the pre (post)-1980 period. In the USA the extensive evidence of bidirectional feedback between real and nominal uncertainty reflects the post-1980 period. That is, the results for the USA after the changes in operating procedures in 1979 support both the Taylor and the Logue-Sweeney hypotheses. Unlike before the changes in operating procedures, there is no causal relation between nominal and real uncertainty.

Table 3.6 presents the results of the Granger causality tests for the pre- and post- 1975 periods. The results for the pre-1975 period shows similar results for the pre-1980 period for Japan. For the USA, a negative causal effect from inflation uncertainty to real uncertainty appears and the sign of causality is changed from negative to positive in the BEKK model for Germany. The results of the Granger causality test for post-1975 is unchanged for the USA. In Japan, a causal effect of nominal (real) uncertainty on real (nominal) uncertainty emerges (disappears) in the comparison with post-1980 period. The causal effect of nominal uncertainty on real uncertainty of post-1980 is no longer apparent in the post-75 analysis in Germany. However, the general picture of the post-1980 period holds for the analysis of the post-1975 periods.

3.6 Conclusions

This study has found that inflation uncertainty affects real uncertainty positively in the USA and Japan (Logue and Sweeney, 1981) but negatively in Germany (Taylor, 1979) for the entire sample period. The sub sample analysis suggests that, in the sixties and seventies, the effect of nominal uncertainty on real uncertainty is negative in Japan and Germany, supporting the Taylor hypothesis but turns positive in the 1980s and 1990s, supporting Logue and Sweeney's hypothesis. The results are mixed in the USA case. Hence, this evidence seems not to justify the criticism that inflation targeting can lead to undesirable output variability.

	USA		JAPAN		GERMANY	
	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)
Panel A	H_0: $h_{\pi t} \rightarrow h_{yt}$					
4 lags	1.09(+)	1.10(+)	3.24***(+)	2.16*(+)	0.35(-)	1.47(-)
8 lags	2.38**(+)	3.33***(+)	3.74***(+)	3.56***(+)	0.75(-)	1.68*(-)
12 lags	2.11**(+)	2.04**(+)	2.69***(+)	2.84***(+)	3.00***(-)	2.25***(-)
Panel B	H_0: $h_{yt} \rightarrow h_{\pi t}$					
4 lags	1.73▲(-)	0.28	1.24	0.90	1.48	2.22**(-)
8 lags	0.72(-)	0.50	1.18	0.91	0.58	0.89(-)
12 lags	2.11**(-)	0.64	0.69	0.59	0.38	0.40(-)

Notes: $h_{\pi t} \rightarrow h_{yt}$: Inflation uncertainty does not Granger-cause output growth uncertainty.

$h_{yt} \rightarrow h_{\pi t}$: Output growth uncertainty does not Granger-cause inflation uncertainty.

***, **, *and ▲denote significance at the 0.01, 0.05, 0.10 and 0.15 levels, respectively.

A + (-) indicates that the sum of the lagged coefficients is positive (negative).

Table 3.3: Granger-causality tests between inflation uncertainty and output growth uncertainty (Entire sample)

	USA		JAPAN		GERMANY	
	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)
Sample A.						
α_{11}	0.513 (8.26)	0.536 (7.95)	0.604 (4.91)	0.584 (9.21)	0.425 (4.29)	0.555 (5.20)
α_{21}	0.033 (0.19)	-	0.310 (0.95)	-	0.233 (0.50)	-
α_{12}	0.028 (0.60)	-	0.044 (1.10)	-	0.001 (0.03)	-
α_{22}	0.718 (7.52)	0.691 (7.56)	0.133 (1.11)	0.082 (1.25)	0.336 (3.38)	0.340 (4.54)
β_{11}	0.811 (16.08)	0.803 (15.24)	0.663 (4.82)	0.642 (5.04)	0.756 (5.69)	0.517 (2.55)
β_{21}	-0.159 (0.94)	-	-0.242 (1.08)	-	0.324 (0.73)	-
β_{12}	-0.054 (0.83)	-	-0.036 (0.78)	-	-0.009 (0.77)	-
β_{22}	0.316 (2.16)	0.298 (2.03)	0.964 (22.57)	0.990 (97.70)	0.941 (21.70)	0.915 (22.93)
Sample B.						
α_{11}	0.423 (4.24)	0.409 (4.01)	0.169 (1.90)	0.321 (3.18)	-0.061 (0.20)	0.006 (0.02)
α_{21}	0.235 (2.50)		0.191 (0.82)		-0.159 (0.08)	
α_{12}	-0.070 (0.81)		-0.107 (2.55)		0.013 (0.37)	
α_{22}	0.197 (2.07)	0.132 (2.54)	0.309 (2.34)	0.302 (2.33)	0.368 (2.06)	0.408 (2.97)
β_{11}	0.825 (12.78)	0.855 (15.12)	0.930 (8.49)	0.802 (8.00)	0.783 (3.35)	0.994 (160.34)
β_{21}	-0.139 (1.62)		-0.401 (1.00)		-2.172 (0.79)	
β_{12}	0.017 (0.45)		0.086 (2.48)		-0.062 (0.76)	
β_{22}	0.958 (23.26)	0.984 (77.03)	0.819 (7.00)	0.882 (8.13)	0.346 (0.36)	0.379 (0.59)

Notes: Sample A covers from Feb 1957 to Dec 1979 with 275 observations for USA and Japan (For Germany it covers from Feb. 1958 to Dec. 1979)
Sample B extends from Jan. 1980 to Aug. 2000 with 248 observations for USA and Japan. (For Germany it covers from Jan. 1980 to Jul. 2000)
This table reports parameter estimates for the BEKK(1,1) and DVEC(1,1) models with data for USA, Germany and Japan for Sample A and B.
The numbers in parentheses are t-statistics.

Table 3.4: Parameter estimates for the BEKK(1,1) and DVEC(1,1) models (Sample A and B)

	USA		JAPAN		GERMANY	
	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)
Sample A						
Panel A (H_0: $h_{\pi t} \rightarrow h_{yt}$)						
4 lags	0.67	1.02	15.53***(-)	1.58(+)	2.71**(-)	2.18*(-)
8 lags	0.82	1.39	9.39***(-)	3.46***(-)	1.93*(-)	1.71*(-)
12 lags	0.60	0.89	4.94***(-)	0.26(-)	3.14***(-)	2.40***(-)
Panel B (H_0: $h_{yt} \rightarrow h_{\pi t}$)						
4 lags	0.72	0.95	0.56	0.52	0.17	0.12
8 lags	0.44	0.69	0.70	0.65	0.14	0.14
12 lags	0.49	0.51	0.23	0.34	0.26	0.13
Sample B						
Panel C (H_0: $h_{\pi t} \rightarrow h_{yt}$)						
4 lags	3.00**(+)	1.16	0.76	1.25	2.10*(+)	0.44
8 lags	2.20**(+)	1.42	1.17	1.24	1.05(+)	0.70
12 lags	2.90***(+)	1.31	1.05	1.19	0.75(+)	0.60
Panel D (H_0: $h_{yt} \rightarrow h_{\pi t}$)						
4 lags	5.64***(-)	1.40	3.59***(+)	0.63	46.88***(+)	0.30
8 lags	3.23***(-)	1.15	2.01**(+)	0.50	24.03***(+)	0.47
12 lags	2.58***(-)	0.85	1.77**(+)	0.63	15.43***(+)	0.43

Notes: Sample A covers from Feb 1957 to Dec 1979 for USA and Japan.
(For Germany it covers from Feb. 1958 to Dec. 1979)
Sample B extends from Jan. 1980 to Aug. 2000 for USA and Japan.
(For Germany it covers from Jan. 1980 to Jul. 2000)
$h_{\pi t} \rightarrow h_{yt}$: Inflation uncertainty does not Granger-cause output growth uncertainty.
$h_{yt} \rightarrow h_{\pi t}$: Output growth uncertainty does not Granger-cause inflation uncertainty.
***, **, *and ▲denote significance at the 0.01, 0.05, 0.10 and 0.15 levels, respectively.
A + (-) indicates that the sum of the lagged coefficients is positive (negative).

Table 3.5: Granger-causality tests between inflation uncertainty and output growth uncertainty BEKK (1,1) and DVEC(1,1) models (Sample A and B)

	USA		JAPAN		GERMANY	
	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)	BEKK(1,1)	DVEC(1,1)
Sample C						
Panel A (H_0: $h_{\pi t} \rightarrow h_{yt}$)						
4 lags	0.14	0.76(-)	12.94***(-)	1.01(+)	2.34*(+)	1.42(-)
8 lags	0.08	2.64***(-)	7.23***(-)	0.89(-)	2.43**(-)	2.73***(-)
12 lags	0.09	1.73*(-)	2.05**(-)	2.45**(+)	2.16**(+)	2.88***(-)
Panel B (H_0: $h_{yt} \rightarrow h_{\pi t}$)						
4 lags	1.29	0.71	0.38	0.84	0.30	0.20
8 lags	0.56	0.49	0.43	0.87	0.16	0.19
12 lags	0.37	0.26	0.26	0.47	0.49	0.22
Sample D						
Panel C (H_0: $h_{\pi t} \rightarrow h_{yt}$)						
4 lags	3.46***(+)	0.12	2.02*(+)	0.53	0.62	0.49
8 lags	2.84***(+)	0.21	1.72*(+)	1.12	0.61	0.71
12 lags	2.50***(+)	0.44	1.33(+)	1.00	0.72	0.48
Panel D (H_0: $h_{yt} \rightarrow h_{\pi t}$)						
4 lags	1.11(-)	1.15	0.70	0.95	165.52***(+)	0.49
8 lags	1.70*(-)	0.96	0.72	0.66	82.12***(+)	0.65
12 lags	1.12(-)	0.72	1.01	1.08	52.62***(+)	0.59

Notes: Sample C covers from Feb 1957 to Dec 1974 for USA and Japan.
(For Germany it covers from Feb. 1958 to Dec. 1974)
Sample D extends from Jan. 1975 to Aug. 2000 for USA and Japan.
(For Germany it covers from Jan. 1975 to Jul. 2000)
$h_{\pi t} \rightarrow h_{yt}$: Inflation uncertainty does not Granger-cause output growth uncertainty.
$h_{yt} \rightarrow h_{\pi t}$: Output growth uncertainty does not Granger-cause inflation uncertainty.
***, **, *and ▲denote significance at the 0.01, 0.05, 0.10 and 0.15 levels, respectively.
A + (-) indicates that the sum of the lagged coefficients is positive (negative).

Table 3.6: Granger-causality tests between inflation uncertainty and output growth uncertainty BEKK (1,1) and DVEC(1,1) models (Sample C and D)

Chapter 4

Moments of the ARMA-EGARCH model

4.1 Introduction

One of the principal empirical tools used to model volatility in asset markets has been the ARCH class of models. Following Engle's (1982) ground-breaking idea, several formulations of conditionally heteroscedastic models (e.g. GARCH, Fractional Integrated GARCH, Switching GARCH, Component GARCH) have been introduced in the literature (see, for example, the survey of Bollerslev et al. 1994). These models form an immense ARCH family. Many of the proposed GARCH models include a term that can capture correlation between returns and conditional variance. Models with this feature are often termed asymmetric or leverage volatility models[1]. One of the earliest asymmetric GARCH models

[1] The asymmetric response of volatility to positive and negative shocks is well known in the finance literature as the leverage effect of the stock market returns (Black, 1976). Researchers

is the EGARCH (Exponential generalized ARCH) model of Nelson (1991). In contrast to the conventional GARCH specification, which requires nonnegative coefficients, the EGARCH model does not impose nonegativity constraints on the parameter space by modeling the logarithm of the conditional variance.

Although the literature on the GARCH/EGARCH models is quite extensive, relatively few papers have examined the moment structure of models where the conditional volatility is time-dependent. Karanasos (1999) and He and Teräsvirta (1999a) derived the autocorrelations of the squared errors for the GARCH(p,q) model, while Karanasos (2001) obtained the autocorrelation function of the observed process for the ARMA-GARCH-in-mean model. Demos (2001) studied the autocorrelation structure of a model that nests both the EGARCH and stochastic volatility specifications. He et al. (2001) considered the moment structure of the EGARCH(1,1) model.

This chapter focuses solely on the moment structure of the general ARMA(r,s)-EGARCH(p,q) model. It would be useful to know the properties of the autocorrelation function of power-transformed observations when comparing the EGARCH model with the standard GARCH model. In particular, possible differences in the moment structure of these models may shed light on the success of the EGARCH model in applications.

We contribute to the aforementioned literature by deriving (i) the autocorrelation function of any positive integer power of the squared errors, (ii) the cross correlations between the levels and the squares of both the observed process and

have found that volatility tends to rise in response to "bad news" (excess returns lower than expected) and to fall in response to "good news" (excess returns higher than expected).

the squared errors, and (iii) the autocorrelations of the squared observations. To obtain the theoretical results and to carry out the estimation, we assume that the innovations are drawn from either the normal, double exponential, or generalized error distributions. To facilitate model identification, the results for the autocorrelation function of the power-transformed errors can be applied so that properties of the observed data can be compared with the theoretical properties of the models.

The derivation of the autocorrelations of the fitted power-transformed values and their comparison with the corresponding sample equivalents will help the investigator (a) to choose, for a given estimation technique, the model (e.g. Asymmetric power ARCH (APARCH), EGARCH) that best replicates certain stylized facts of the data and, (b) in conjuction with the various model selection criteria, to identify the optimal order of the chosen specification.

This chapter is organized as follows. Section 4.2.1 presents the ARMA(r, s)-EGARCH(p, q) process. Section 4.2.2 investigates the autocorrelation function of any positive integer power of the squared errors for the EGARCH model. Section 4.2.3 derives the cross correlations between the levels and the squares of the ARMA-EGARCH process. Section 4.2.4 provides the autocorrelation function of the squared observations. Section 4.3 discusses the data and presents the empirical results. In the conclusions we suggest future developments. Proofs are found in an early version of this work (Karanosos and Kim. 2000).

4.2 ARMA-EGARCH Model

4.2.1 ARMA(r,s)-EGARCH(p,q) process

Of the many different asymmetric GARCH specifications the EGARCH model has become one of the most common. Here we examine the general ARMA(r, s)-EGARCH(p, q) model. The stochastic process $\{y_t\}$ is assumed to be a causal ARMA(r,s) process satisfying

$$\Phi(L)y_t = b + \Theta(L)\varepsilon_t, \tag{4.6a}$$

where

$$\Phi(L) \equiv \prod_{l=1}^{r}(1 - \phi_l L). \tag{4.6b}$$

$$\Theta(L) \equiv 1 + \sum_{l=1}^{s} \theta_l L^l \tag{4.6c}$$

Further, let $\{\varepsilon_t\}$ be a real-valued time stochastic process generated by

$$\varepsilon_t = e_t h_t^{\frac{1}{2}}, \tag{4.7}$$

where $\{e_t\}$ is a sequence of independent, identically distributed random variables with mean zero and variance 1. h_t is positive with probability one and is a measurable function of Σ_{t-1}, which in turn is the sigma-algebra generated by $\{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$. That is, h_t denotes the conditional variance of the errors $\{\varepsilon_t\}$, $(\varepsilon_t|\Sigma_{t-1}) \sim (0, h_t)$. As regards h_t we assume that it follows an EGARCH(p,q) process

$$B(L)\ln(h_t) = \omega + C(L)z_t, \tag{4.8a}$$

$$z_{t-l} \equiv d\frac{\varepsilon_{t-l}}{\sqrt{h_{t-l}}} + \gamma\left[\left|\frac{\varepsilon_{t-l}}{\sqrt{h_{t-l}}}\right| - \mathsf{E}\left|\frac{\varepsilon_{t-l}}{\sqrt{h_{t-l}}}\right|\right], \tag{4.8b}$$

where

$$C(L) \equiv \sum_{l=1}^{q} c_l L^l, \tag{4.8c}$$

$$B(L) \equiv \prod_{l=1}^{p} (1 - \beta_l L) \tag{4.8d}$$

Various cases of the EGARCH(p,q) model have been applied by researchers. More specifically, Donaldson and Kamstra (1997) found that the optimal EGARCH specification for the NIKKEI stock index was a flexible 3,2. Hu et al. (1997) found that in the pre-EMS period, the majority of the European currencies followed an AR(5)-EGARCH(4,4) model.

4.2.2 Higher-order moments of the squared errors

Although the EGARCH model was introduced over a decade ago and has been widely used in empirical applications, its statistical properties have only recently been examined. Engle and Ng (1993) artificially nested the GARCH and EGARCH models, estimated this nested specification, and then applied likelihood ratio tests (see also Hu et al. 1997). Hentschel (1995) developed a family of asymmetric GARCH models that nests both the asymmetric power ARCH (A-PARCH) model and the EGARCH model.

In this section we focus solely on the moment structure of the general EGARCH(p,q) model.

Assumption *4.1.* All the roots of the autoregressive polynomial $B(L)$ lie outside the unit circle (covariance-stationarity condition).

Assumption *4.2.* The polynomials $C(L)$ and $B(L)$ in (4.8c) and (4.8d) respectively, have no common left factors other than unimodular ones (irreducibility condition). Moreover, $\beta_p, c_q \neq 0$.

In what follows we examine only the case where all the roots of the autoregressive polynomial $B(L)$ in (4.8d) are distinct. The following proposition establishes the lag-m autocorrelation of the kth power of the squared errors $\{\varepsilon_t^{2k}\}$, $\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k}) \equiv \mathsf{Corr}(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k})$.

Proposition 4.1. *Let assumptions 1 and 2 hold. Suppose further that* $\mathsf{E}(e_t^{4k}) < \infty$, $\mathsf{E}[\exp(2kz_t)] < \infty$ *and* $\mathsf{E}[e_t^{2k}\exp(kz_t)] < \infty\ \forall\ t$, *for any finite positive scalar* k. *Then the autocorrelation of the* k*th power of the squared error* $\{\varepsilon_t^{2k}\}$, *at lag* m $(m \in \mathbb{N})$, *has the form*

$$\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k}) = \mathsf{E}\left(e_t^{2k}\right)\left\{\prod_{i=0}^{m-2}\left[\mathsf{E}\left(\exp\left(\xi_{0,\frac{k}{2},i}z_{t-i-1}\right)\right)\right]\mathsf{E}\left[e_{t-m}^{2k}\exp\left(\xi_{0,\frac{k}{2},m-1}z_{t-m}\right)\right]\times\right.$$
$$\left.\prod_{i=0}^{\infty}\left[\mathsf{E}\left(\exp\left(\xi_{m,k,i}z_{t-m-i-1}\right)\right)\right] - \mathsf{E}\left(e_t^{2k}\right)\left[\prod_{i=0}^{\infty}\left[\mathsf{E}\left(\exp\left(\xi_{0,\frac{k}{2},i}z_{t-i-1}\right)\right)\right]\right]^2\right\}\times$$
$$\left\{\mathsf{E}(e_t^{4k})\prod_{i=0}^{\infty}\left[\mathsf{E}\left(\exp\left(\xi_{0,k,i}z_{t-i-1}\right)\right)\right] - \left[\mathsf{E}(e_t^{2k})\right]^2\left[\prod_{i=0}^{\infty}\left[\mathsf{E}\left(\exp\left(\xi_{0,\frac{k}{2},i}z_{t-i-1}\right)\right)\right]\right]^2\right\}^{-1}, \tag{4.9a}$$

where

$$\xi_{m,k,i} \equiv k\sum_{f=1}^{p}\zeta_f(\lambda_{f,m+i+1} + \lambda_{f,i+1}). \tag{4.9b}$$

with

$$\lambda_{fi} \equiv \begin{cases} \sum\limits_{n=0}^{i-1} c_{i-n}\beta_f^n, & \text{if} \quad i \leq q, \\ \lambda_{fq}\beta_f^{i-q}, & \text{if} \quad i > q \end{cases}, \tag{4.9c}$$

$$\zeta_f \equiv \frac{\beta_f^{p-1}}{\prod\limits_{\substack{n=1 \\ n \neq f}}^{p} (\beta_f - \beta_n)} \tag{4.9d}$$

Note that, when $m = 1$, the first product term in the right hand side of (4.9a) is replaced by 1.

He et al. (2001) derived the autocorrelations of positive powers of the absolute-valued errors of the EGARCH(1,1) model.

In the following theorem we provide the autocorrelations of the kth power of the squared errors $\{\varepsilon_t^{2k}\}$ of the EGARCH(p, q) model.

Theorem 4.1. *Let k be any finite positive integer. Then, when the distribution of $\{e_t\}$ is generalized error, the second moment and the autocorrelation function of $\{\varepsilon_t^{2k}\}$ are given by*

$$\mathsf{E}(\varepsilon_t^{4k}) = \exp\left(\frac{2k\left[\omega - \gamma\frac{\Gamma(\frac{2}{v})\lambda 2^{\frac{1}{v}}}{\Gamma(\frac{1}{v})}\sum_{l=1}^{q} c_l\right]}{\prod_{f=1}^{p}(1-\beta_f)} \right) \mu_{4k}^{(g)} B_{0,k}^{(g)}. \tag{4.10a}$$

$$\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k}) = \frac{\mu_{2k}^{(g)}\left[A_{m-1,k}^{(g)} D_{m-1,k}^{(g)} B_{m,k}^{(g)} - \mu_{2k}^{(g)}\left(B_{0,\frac{k}{2}}^{(g)}\right)^2\right]}{\left[\mu_{4k}^{(g)} B_{0,k}^{(g)} - \left(\mu_{2k}^{(g)} B_{0,\frac{k}{2}}^{(g)}\right)^2\right]}, \quad (A_{0,k}^{(g)} \equiv 1), \tag{4.10b}$$

where

$$A_{m,k}^{(g)} \equiv \prod_{i=0}^{m-1}\left\{\sum_{\tau=0}^{\infty}\left[2^{\frac{1}{v}}\lambda\xi_{0,\frac{k}{2},i}\right]^{\tau}\left[(\gamma+d)^{\tau} + (\gamma-d)^{\tau}\right]\frac{\Gamma(\frac{1+\tau}{v})}{2\Gamma(\frac{1}{v})\Gamma(1+\tau)}\right\}, \tag{4.10c}$$

$$B_{m,k}^{(g)} \equiv \prod_{i=0}^{\infty}\left\{\sum_{\tau=0}^{\infty}\left[2^{\frac{1}{v}}\lambda\xi_{m,k,i}\right]^{\tau}\left[(\gamma+d)^{\tau} + (\gamma-d)^{\tau}\right]\frac{\Gamma\left(\frac{1+\tau}{v}\right)}{2\Gamma\left(\frac{1}{v}\right)\Gamma(1+\tau)}\right\}. \tag{4.10d}$$

and

$$D^{(g)}_{m,k} \equiv 2^{\frac{2k}{v}}\lambda^{2k}\sum_{\tau=0}^{\infty}\left(\lambda 2^{\frac{1}{v}}\xi_{0,\frac{k}{2},m}\right)^{\tau}\left[(\gamma+d)^{\tau}+(\gamma-d)^{\tau}\right]\frac{\Gamma\left(\frac{\tau+2k+1}{v}\right)}{2\Gamma\left(\frac{1}{v}\right)\Gamma(1+\tau)}, \quad (4.10e)$$

$$\mu^{(g)}_{2k} \equiv \frac{\left[\Gamma\left(\frac{1}{v}\right)\right]^{k-1}\Gamma\left(\frac{2k+1}{v}\right)}{\left[\Gamma\left(\frac{3}{v}\right)\right]^{k}}, \quad (4.10f)$$

with

$$\lambda \equiv \left\{2^{\frac{-2}{v}}\Gamma\left(\frac{1}{v}\right)\left[\Gamma\left(\frac{3}{v}\right)\right]^{-1}\right\}^{\frac{1}{2}},$$

where $\xi_{m,k,i}$ is *defined in proposition 1, v are the degrees of freedom of the generalized error distribution and $\Gamma(\cdot)$ is the Gamma function. When $v > 1$, the summations in (4.10c), (4.10d) and (4.10e) are finite; when $v < 1$, the three summations are finite if and only if $\xi_{0,\frac{k}{2},i}\gamma + |\xi_{0,\frac{k}{2},i}d| \leq 0$, $\xi_{m,k,i}\gamma + |\xi_{m,k,i}d| \leq 0$ and $\xi_{0,\frac{k}{2},m}\gamma + |\xi_{0,\frac{k}{2},m}d| \leq 0$, respectively (see Nelson, 1991).*

One of the most widely used models in financial economics to describe a time series r_t, of the returns from some asset, is the martingale process

$$r_t \equiv \varepsilon_t = e_t\sqrt{h_t},$$

where e_t is i.i.d (0, 1) and h_t is a GARCH type process.

In many applications in financial economics, it is not reasonable to assume the normality of e_t, because of the substantial excess kurtosis present in the conditional density of returns. Hence investigators often use maximum likelihood estimation (MLE) by assuming some fat-tailed conditional density such as generalized error. Therefore, when it comes to model identification, practitioners in this area may find the results in theorem 4.1 quite useful.

In the following proposition, when the innovations are drawn from the double exponential distribution, we provide the autocorrelations of $\{\varepsilon_t^{2k}\}$ for any finite positive integer k.

Proposition 4.2. *When the distribution of $\{e_t\}$ is double exponential, the autocorrelation function of the kth power of the squared errors is given by*

$$\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k}) = \frac{\mu_{2k}^{(d)}\left[A_{m-1,k}^{(d)} D_{m-1,k}^{(d)} B_{m,k}^{(d)} - \mu_{2k}^{(d)}\left(B_{0,\frac{k}{2}}^{(d)}\right)^2\right]}{\left[\mu_{4k}^{(d)} B_{0,k}^{(d)} - \left(\mu_{2k}^{(d)} B_{0,\frac{k}{2}}^{(d)}\right)^2\right]}, \quad (A_{0,k}^{(d)} \equiv 1), \tag{4.11a}$$

with

$$A_{m,k}^{(d)} \equiv \prod_{i=0}^{m-1} \frac{2 - \sqrt{2}\xi_{0,\frac{k}{2},i}\gamma}{2 - \sqrt{2}\xi_{0,k,i}\gamma + \xi_{0,\frac{k}{2},i}^2(\gamma^2 - d^2)}, \tag{4.11b}$$

$$B_{m,k}^{(d)} \equiv \prod_{i=0}^{\infty} \frac{2 - \sqrt{2}\xi_{m,k,i}\gamma}{2 - 2\sqrt{2}\xi_{m,k,i}\gamma + \xi_{m,k,i}^2(\gamma^2 - d^2)}, \tag{4.11c}$$

and

$$D_{m,k}^{(d)} \equiv 2^{-(k+1)}\Gamma(2k+1)\times \left\{F\left[2k+1; \frac{\xi_{0,\frac{k}{2},m}(\gamma + d)}{\sqrt{2}}\right] + F\left[2k+1; \frac{\xi_{0,\frac{k}{2},m}(\gamma - d)}{\sqrt{2}}\right]\right\}, \tag{4.11d}$$

$$\mu_{2k}^{(d)} \equiv \frac{\Gamma(2k+1)}{2^k}, \tag{4.11e}$$

where $F(\cdot)$ is the hypergeometric function (see Abadir, 1999) and $\xi_{m,k,i}$ is *defined in proposition 4.1. Expressions (4.11b) and (4.11c) hold if and only if $\xi_{0,\frac{k}{2},i}\gamma + |\xi_{0,\frac{k}{2},i}d| < \sqrt{2}$* and *$\xi_{m,k,i}\gamma + |\xi_{m,k,i}d| < \sqrt{2}$, respectively; the right hand side of (4.11d) converges if and only if $\xi_{0,\frac{k}{2},m}\gamma + |\xi_{0,\frac{k}{2},m}d| < \sqrt{2}$ (see Nelson, 1991).*

Also note that the coefficients of the Wold representation of the kth power of the conditional variance are needed for the computation of the autocorrelations of the kth power of the squared errors (see Demos, 2001).

In the following proposition, when the errors are conditionally normal, we derive the autocorrelation function of the kth power of the squared errors $\{\varepsilon_t^{2k}\}$, $k \in \mathbb{N}$.

Proposition 4.3. *When the distribution of $\{e_t\}$ is normal, the autocorrelations of the kth power of the squared errors are given by*

$$\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k}) = \frac{\mu_{2k}^{(n)} \left[A_{m-1,k}^{(n)} D_{m-1,k}^{(n)} B_{m,k}^{(n)} - \mu_{2k}^{(n)} \left(B_{0,\frac{k}{2}}^{(n)} \right)^2 \right]}{\left[\mu_{4k}^{(n)} B_{0,k}^{(n)} - \left(\mu_{2k}^{(n)} B_{0,\frac{k}{2}}^{(n)} \right)^2 \right]}, \quad (A_{0,k}^{(n)} \equiv 1), \quad (4.12a)$$

with

$$A_{m,k}^{(n)} \equiv \prod_{i=0}^{m-1} \left\{ \exp\left(\frac{(\gamma+d)^2 \xi_{0,\frac{k}{2},i}^2}{2} \right) \frac{1}{2} \left[1 + \exp\left(-2\gamma d \xi_{0,\frac{k}{2},i}^2 \right) \right] + \frac{(\gamma+d)\xi_{0,\frac{k}{2},i}}{\sqrt{2\pi}} \times \right.$$
$$\left. F\left(1; \frac{3}{2}; \frac{(\gamma+d)^2 \xi_{0,\frac{k}{2},i}^2}{2} \right) + \frac{(\gamma-d)\xi_{0,\frac{k}{2},i}}{\sqrt{2\pi}} \times F\left(1; \frac{3}{2}; \frac{(\gamma-d)^2 \xi_{0,\frac{k}{2},i}^2}{2} \right) \right\},$$

(4.12b)

$$B_{m,k}^{(n)} \equiv \prod_{i=0}^{\infty} \left\{ \exp\left(\frac{(\gamma+d)^2 \xi_{m,k,i}^2}{2} \right) \frac{1}{2} \left[1 + \exp\left(-2\gamma d \xi_{m,k,i}^2 \right) \right] + \frac{(\gamma+d)\xi_{m,k,i}}{\sqrt{2\pi}} \times \right.$$
$$\left. F\left(1; \frac{3}{2}; \frac{(\gamma+d)^2 \xi_{m,k,i}^2}{2} \right) + \frac{(\gamma-d)\xi_{m,k,i}}{\sqrt{2\pi}} \times F\left(1; \frac{3}{2}; \frac{(\gamma-d)^2 \xi_{m,k,i}^2}{2} \right) \right\},$$

(4.12c)

and

$$D_{m,k}^{(n)} \equiv \frac{1}{2}\left\{\frac{\partial}{\partial\left[\xi_{0,\frac{k}{2},m}(\gamma-d)\right]^{2k}}\left\{\exp\left(\frac{\xi_{0,\frac{k}{2},m}^{2}(\gamma-d)^2}{2}\right)\left[1+\Phi\left(\frac{\xi_{0,\frac{k}{2},m}(\gamma-d)}{\sqrt{2}}\right)\right]\right\}\right.$$
$$\left.+\frac{\partial}{\partial\left[\xi_{0,\frac{k}{2},m}(\gamma+d)\right]^{2k}}\left\{\exp\left(\frac{\xi_{0,\frac{k}{2},m}^{2}(\gamma+d)^2}{2}\right)\left[1+\Phi\left(\frac{\xi_{0,\frac{k}{2},m}(\gamma+d)}{\sqrt{2}}\right)\right]\right\}\right\}, \tag{4.12d}$$

$$\mu_{2k}^{(n)} \equiv \prod_{j=1}^{k}[2k-(2j-1)], \tag{4.12e}$$

where ∂ denotes partial derivative, $\Phi(\cdot)$ is the error function of the standard normal distribution and $\xi_{m,k,i}$ is given by (4.9b).

Several previous articles dealing with financial market data-e.g. Dacorogna et al. (1993), Ding et al. (1993) and Muller et al. (1997)- have commented on the behaviour of the autocorrelation function of positive powers of the squared returns, and the desirability of having a model which comes close to replicating certain stylized facts in the data (abstracted from Baillie and Chung, 2001). In this respect, one can apply the results in this section to check whether the EGARCH model can effectively replicate the observed pattern of autocorrelations of power-transformed returns.

4.2.3 Dynamic Asymmetry

In this section we examine the cross correlations between the levels and the squares of the ARMA-EGARCH process in (4.6)-(4.8).

Proposition 4.4. *Let the distribution of $\{e_t\}$ be generalized error and k a finite positive integer. Suppose further that $\mathsf{E}(e_t^{4k}) < \infty$, $\mathsf{E}[e_t^{2k-1}\exp(kz_t)] <$*

∞ and $\mathsf{E}[\exp(2kz_t)] < \infty \ \forall \ t$. *Then, if assumptions 1 and 2 hold, the cross correlations between the 2kth and* $(2k-1)$th *powers of* $\{\varepsilon_t\}$ *are given by*

$$\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k-1}) = \frac{\mu_{2k}^{(g)} A_{m-1,k}^{(g)} D_{m-1,(k)}^{(g)} B_{m,(k)}^{(g)}}{\sqrt{\left[\mu_{4k}^{(g)} B_{0,k}^{(g)} - (B_{0,\frac{k}{2}}^{(g)})^2\right] \mu_{4k-2}^{(g)} B_{0,(\frac{4k-1}{4})}^{(g)}}}, \quad (m \in \mathbb{N}), \tag{4.13a}$$

with

$$D_{m,(k)}^{(g)} \equiv 2^{\frac{2k-1}{v}} \lambda^{2k-1} \sum_{\tau=0}^{\infty} \left(\lambda 2^{\frac{1}{v}} \varphi_{0,\frac{2k+1}{4},m}\right)^{\tau} [(\gamma+d)^{\tau} - (\gamma-d)^{\tau}] \frac{\Gamma\left(\frac{\tau+2k}{v}\right)}{2\Gamma\left(\frac{1}{v}\right)\Gamma(1+\tau)}. \tag{4.13b}$$

$$B_{m,(k)}^{(g)} \equiv \prod_{i=0}^{\infty} \left\{ \sum_{\tau=0}^{\infty} \left[2^{\frac{1}{v}} \lambda \varphi_{m,k,i}\right]^{\tau} [(\gamma+d)^{\tau} + (\gamma-d)^{\tau}] \frac{\Gamma\left(\frac{1+\tau}{v}\right)}{2\Gamma\left(\frac{1}{v}\right)\Gamma(1+\tau)} \right\}, \tag{4.13c}$$

and

$$\varphi_{m,k,i} \equiv \sum_{f=1}^{p} (k\lambda_{f,m+i+1} + (k-0.5)\lambda_{f,i+1})\zeta_f,$$

where $A_{m,k}^{(g)}$, $B_{m,k}^{(g)}$ *and* $\mu_{4k}^{(g)}$ *are defined in theorem 4.1. Note that, when* m *is a negative integer,* $\rho(\varepsilon_t^{2k}, \varepsilon_{t-m}^{2k-1}) = 0$.

Demos (2001) derived the cross correlations between the levels and the squares of an observed series, under the assumption that the mean parameter is time-varying and the conditional variance follows a flexible parameterization which nests the autoregressive stochastic volatility and the exponential GARCH specifications[2]. In the same spirit, the following theorem obtains the cross correlations between the levels and the squares of the ARMA(r,s)-EGARCH(p,q) process in (4.6)-(4.8).

Assumption *4.3.* All the roots of the autoregressive polynomial $\Phi(L)$ lie outside the unit circle.

[2]Demos called this model time varying parameter generalized stochastic volatility in mean (TVP-GSV-M).

Assumption *4.4.* The polynomials $\Phi(L)$ and $\Theta(L)$ are left coprime.

Theorem 2.2. *Let assumptions 1-4 hold. Suppose further that* $\mathsf{E}(e_t^4) < \infty$, $\mathsf{E}[e_t \exp(z_t)] < \infty$ *and* $\mathsf{E}[\exp(2z_t)] < \infty \; \forall \; t$. *Then the cross correlations between the squares and the levels of* $\{y_t\}$ *are given by*

$$\rho(y_t^2, y_{t-m}) = \left(\frac{\eta - 1}{\kappa - 1}\right)^{\frac{1}{2}} (F_m + H_m), \quad (m \in \mathbb{N}), \tag{4.14a}$$

where

$$F_m \equiv \frac{\sum_{i=0}^{j+m-1} \sum_{j=0}^{\infty} \delta_i^2 \delta_j \rho\left(\varepsilon_t^2, \varepsilon_{t-(j+m-i)}\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}, \tag{4.14b}$$

$$H_m \equiv \frac{2\sum_{i=j+m+1}^{\infty} \sum_{j=0}^{\infty} \delta_i \delta_j \delta_{j+m} \rho\left(\varepsilon_t^2, \varepsilon_{t-(i-j-m)}\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}, \tag{4.14c}$$

Furthermore

$$\rho(y_t, y_{t-m}^2) = \left(\frac{\eta - 1}{\kappa - 1}\right)^{\frac{1}{2}} (K_m + L_m), \quad (m \in \mathbb{N}), \tag{4.15a}$$

with

$$K_m \equiv \frac{\sum_{i=j+m+1}^{\infty} \sum_{j=0}^{\infty} \delta_i \delta_j^2 \rho\left(\varepsilon_t^2, \varepsilon_{t-(i-j-m)}\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}, \tag{4.15b}$$

$$L_m \equiv \frac{2\sum_{i=j+1}^{\infty} \sum_{j=0}^{\infty} \delta_i \delta_j \delta_{j+m} \rho\left(\varepsilon_t^2, \varepsilon_{t-(i-j)}\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}, \tag{4.15c}$$

where η *and* κ *denote the kurtosis of* ε_t and y_t *respectively;* δ_i *is the ith coefficient in the Wold representation of the ARMA(p, q) process in (4.6). Note that, when the distribution of* $\{e_t\}$ *is generalized error,* $\rho\left(\varepsilon_t^2, \varepsilon_{t-m}\right)$ *is given in proposition 4.4, and* η, κ *are given in proposition 4.5 below.*

Also observe that when there is no leverage effect ($d = 0$), the $D_{m,(k)}^{(g)}$ term in (4.13b) is zero and hence the cross correlations between the levels and the squares of both the errors and the observed process are zero. Demos (2001), for the TVP-GSV-M model, does not need the asymmetric EGARCH effect to obtain dynamic asymmetry even under the assumption of conditional normality.

4.2.4 Autocorrelations of the squared observations

In this section, we establish the autocorrelation properties of the squares of the ARMA-EGARCH process in (4.6)-(4.8). Demos (2001) obtained the autocorrelation function of the squares of the observed series for the TVP-GSV-M model.

The result presented in the following proposition, which is a special case of theorem 2.2 in Palma and Zevallos (2001), is higly relevant since it helps to identify the nature of the process. By analyzing the autocorrelation function of the squared series it is possible to discard those theoretical models which are incompatible with the data under study.

Proposition 2.5. *Let assumptions 1-4 hold. Suppose further that* $\mathsf{E}(e_t^4) < \infty$, $\mathsf{E}[\exp(2z_t)] < \infty$ *and* $\mathsf{E}[e_t^2 \exp(z_t)] < \infty \; \forall\, t$. *Then, when the distribution of* $\{\varepsilon_t\}$ *is generalized error, the autocorrelation function of* $\{y_t^2\}$ *is given by*

$$\rho\left(y_t^2, y_{t-m}^2\right) = \frac{2\left[\rho\left(y_t, y_{t-m}\right)\right]^2}{\kappa - 1} + \frac{(\kappa - 3)\Gamma_m}{(\kappa - 1)} + \frac{\eta - 1}{\kappa - 1}\left[G_m + 2\Delta_m - 3\Delta_0\Gamma_m\right], \quad (m \in \mathbb{N}), \tag{4.16a}$$

where

$$\Gamma_m \equiv \frac{\sum_{i=0}^{\infty} \delta_i^2 \delta_{i+m}^2}{\sum_{l=0}^{\infty} \delta_l^4}, \tag{4.16b}$$

$$\Delta_m \equiv \frac{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \delta_i \delta_j \delta_{i+m} \delta_{j+m} \rho\left(\varepsilon_i^2, \varepsilon_{t-(i-j)}^2\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}, \tag{4.16c}$$

$$G_m \equiv \frac{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \delta_i^2 \delta_j^2 \rho\left(\varepsilon_t^2, \varepsilon_{t-(m+j-i)}^2\right)}{\left(\sum_{l=0}^{\infty} \delta_l^2\right)^2}. \tag{4.16d}$$

and

$$\kappa \equiv 3 - \frac{2\eta\left(\sum_{i=0}^{\infty}\delta_i^4\right)}{\left(\sum_{l=0}^{\infty}\delta_l^2\right)^2} + 3(\eta - 1)\Delta_0, \tag{4.16e}$$

$$\eta \equiv \frac{\mathsf{E}\left(\varepsilon_l^4\right)}{\left[\mathsf{E}\left(\varepsilon_l^2\right)\right]^2}, \tag{4.16f}$$

where $\rho\left(\varepsilon_t^2, \varepsilon_{t-(i-j)}^2\right)$ *and* $\mathsf{E}\left(\varepsilon_t^4\right)$ *are given in theorem 4.1;* δ_i *is defined in theorem 4.2 and* $\rho\left(y_t, y_{t-m}\right)$ *is the lag-m autocorrelation of* $\{y_t\}$.

The significance of the above result is that it allows us to establish whether the ARMA-EGARCH model in (4.6)-(4.8) is capable of reproducing key features exhibited by the data. These features include, for example, time series with very little autocorrelation but with strongly dependent squares. Another potential motivation for the derivation of the results in theorem 4.2 is that the autocorrelations of the squared process in (4.16) can be used to estimate the ARMA and GARCH parameters in (4.6) and (4.8) respectively. The approach is to use the minimum distance estimator (MDE), which estimates the parameters by minimizing the mahalanobis generalized distance of a vector of sample autocorrelations from the corresponding population autocorrelations (see Baillie and Chung, 2001).

The following proposition provides the lag-m autocorrelation of $\{h_t^k\}$, $k \in \mathbb{R}_+$.

Proposition 4.6. *Let assumptions 4.1 and 4.2 hold. Suppose further that* $\mathsf{E}[\exp(2kz_t)] < \infty \ \forall \ t$. *Then, when the distribution of* $\{e_t\}$ *is generalized error, the autocorrelation function of the kth power of the conditional variance is given by (4.10b) where now the terms* $D^{(g)}$ *and* $\mu^{(g)}$ *are replaced by 1 and* $A_{m-1,k}^{(g)}$ *is replaced by* $A_{m,k}^{(g)}$.

Demos (2001) derived the autocorrelations of the kth power of the conditional variance for the GSV model under the assumption of conditional normality.

Next, consider a process y_t governed by

$$y_t = \mathsf{E}(y_t|\Sigma_{t-1}) + \varepsilon_t$$

Further, suppose that the conditional mean of y_t, given information through time $t-1$, is

$$\mathsf{E}(y_t|\Sigma_{t-1}) = \delta h_t^k, \quad (k > 0)$$

The results in proposition 4.6 can be used to derive the autocorrelation function of the above process. Mean equations of this form have been widely used in empirical studies of time-varying risk premia. Demos (2001), in the TVP-GSV-M model, allowed the conditional variance to affect the mean with a possibly time varying coefficient.

4.3 Empirical results

4.3.1 Data Selections

We use four daily stock indices: the Korean stock price index (KOSPI), the Japanese Nikkei index (NIKKEI) and the Taiwanese Se weighted index (SE) for the period 1980:01-1997:04 , and the Singaporean Straits Times price index (ST) for the period 1985:01-1997:04. The reason why we choose this period is to avoid the structual change after the Asian financial crisis in 1997. The daily observations for each country are extracted from the 'Datastream' database. In each case, the index return is the first difference of log prices.

4.3.2 Estimation Results

In order to carry out our analysis of stock returns, we have to select a form for the mean equation. Scholes and Williams (1977), Ding et al., (1993), and Ding and Granger (1996) suggested an MA(1) specification for the mean equation. Lo and MacKinlay (1988), Akgiray (1989), Nelson (1991) and Hafner and Herwartz (2001) used an AR(1) form, while Hentschel (1995) modeled the index return as a white noise process. In practice, there is little to differentiate an AR(1) and an MA(1) model when the AR and the MA coefficients are small, and the autocorrelations at lag one are equal, since the higher order autocorrelations die out very quickly in the AR model (Nelson, 1991). We therefore model the stock returns as MA(1) processes. The MA(1) model is

$$y_t = b + (1 + \theta L)\varepsilon_t, \tag{4.17}$$

To select our 'best' EGARCH specification[3], we begin with high order models (e.g., EGARCH(4,4)) and follow a 'general to specific' modelling approach to fit the data. The general EGARCH(p,q) specification that we estimate is

$$\varepsilon_t = e_t h_t^{\frac{1}{2}}, \quad e_t \sim \text{i.i.d } (0,1), \tag{4.18a}$$

$$\prod_{i=1}^{p}(1 - \beta_i L)\ln(h_t) = \omega + \sum_{i=1}^{q} c_i(|e_{t-i}| + d_i e_{t-i}) \tag{4.18b}$$

We estimate EGARCH models of order up to 4,4 for the returns on the four stock indices using three alternative distributions: the normal, double exponential and generalized error. The Akaike Information Criterion (AIC) chose high order

[3]We define 'best' as the specification chosen by the Akaike Information Criterion (AIC) or the Schwarz Information Criterion (SIC).

EGARCH specifications for all indices. In contrast, in most of the cases, the Schwarz Information Criterion (SIC) chose the EGARCH(1,1) model. In addition, we use the likelihood ratio (LR) test to show the performance of the high order models over the EGARCH(1,1) model. The test results in Table 4.3 show the dominance of the high order models.

For all the stock returns, parameter estimation is conducted jointly on an MA(1) mean specification[4] and the appropriate EGARCH model for the conditional variance. Table 4.1 and Table 4.4 report the results for the period 1980-1997 and present parameter estimates along with t-statistics. For two out of the four indices the AIC and the SIC are minimized when the double exponential distribution is used, while for the KOSPI and ST indices, the information criteria choose the generalized error distribution. The parameters b and θ are the intercept and MA(1) coefficient respectively for the return equation (5.31). The remaining parameters are from the EGARCH model (5.33a). Not surprisingly, for all the EGARCH specifications, most of the moving average, leverage and autoregressive parameters are significantly different from zero. The estimated values of degrees of freedom in the generalized error distribution, for the KOSPI and ST indices, are 1.07 and 1.13 respectively in the models in table 1 and 1.058 and 1.13 in Table 4.4.

Researchers have found that nontrading periods contribute much less than do

[4]The only exceptions are the Taiwanese Se weighted index, where the white noise specification is used for the generalized error and double exponential distributions, and the Japanese index where the white noise specification is used when the errors are drawn from the double exponential distribution. The estimated MA(1) coefficients of these cases were statistically insignificant.

	KOSPI MA(1)-EGARCH(3,3) (GEN ERROR)	NIKKEI WN-EGARCH(1,3) (DOUBLE EXP)	SE WN-EGARCH(1,3) (DOUBLE EXP)	ST MA(1)-EGARCH(2,1) (GEN ERROR)
b	-0.0002 (1.32)	$6E-10$ (0.00)	$7E-08$ (0.00)	0.0002 (1.15)
θ	0.059 (4.57)	-	-	0.200 (11.99)
ω	-2.180 (9.88)	-0.384 (7.07)	-0.273 (6.85)	-1.294 (7.16)
c_1	0.258 (12.93)	0.193 (5.38)	0.145 (3.82)	0.349 (11.05)
c_2	0.374 (12.90)	0.149 (2.95)	0.105 (2.06)	-
c_3	0.254 (12.66)	-0.146 (3.37)	-0.065 (1.65)	-
β_1^*	-0.508 (84.56)	0.973 (206.00)	0.983 (260.60)	0.620 (5.89)
β_2^*	0.387 (40.51)	-	-	0.267 (2.67)
β_3^*	0.949 (157.92)	-	-	-
d_1	-0.124 (2.31)	-1.000 (3.85)	-0.713 (2.88)	-0.196 (3.29)
d_2	-0.099 (1.87)	0.050 (1.84)	0.118 (3.18)	-
d_3	-0.122 (2.28)	-0.637 (2.29)	-1.000 (1.24)	-
v	1.076 {0.02}	1	1	1.134 {0.02}

For each of the four stock indices, table 4.1 reports parameter estimates for the 'best' EGARCH model chosen by AIC. The general MA(1)-EGARCH(3,3) model is

$$y_t = b + (1 + \theta L)\varepsilon_t,$$
$$\varepsilon_t = \sqrt{h_t}e_t, \quad e_t \sim \text{i.i.d } (0,1),$$
$$(1 - \sum_{j=1}^{3} \beta_j^* L^j)\ln(h_t) = \omega + \sum_{i=1}^{3} c_i(d_i e_{t-i} + |e_{t-i}|).$$

The numbers in parentheses are t-statistics.
v are the degrees of freedom of the generalized error distribution.
Standard errors are reported in brackets.

Table 4.1: Parameter Estimates for the 'best' EGARCH model (1)

trading periods to market variance (see Nelson, 1991). Therefore, the selected specifications reported in Table 4.1and Table 4.4 have been reestimated taking into account the number of nontrading days between day t and $t-1$. That is ω in (4.13b) is replaced by $\omega_t = \omega + \ln(1 + \delta N_t)$. In all cases the estimated δ's were statistically significant and less than unity (the results for these cases are not reported here).

For all four indices, parameter estimates are consistent with those generally reported in the literature. The roots of the autoregressive parts of the conditional variances are reported in the first column of Table 4.2 and of Table 4.5. Table 4.2 reports the figures of the high order models chosen by the AIC. In particular, for the KOSPI index volatility appears nearly integrated (the values of the two complex roots are -0.72$\pm$0.68i). For the ST index there is one positive and one negative root with values 0.91 and -0.29 respectively. Fiorentini and Sentana (1998) used a measure of persistence of shocks for stationary processes based on the impulse response function, which captures the importance of the deviations of a series from its unperturbed path following a single shock. Accordingly, the persistence of a shock to z_t on $\ln(h_t)$ is $P_\infty[\ln(h_t)|z_t] = \sum_{l=1}^{\infty}\sum_{f=1}^{p}(\zeta_f\lambda_{fl})^2$, where ζ_f and λ_{fl} are defined in proposition 4.1. This measure is the ratio of the variance of $\ln(h_t)$ to the variance of z_t. The second column of Table 4.2 and Table 4.5 reports a measure for the persistence of a (positive) shock to e_t on $\ln(h_t)$. Most noteworthy is the observation that in all EGARCH models, the product of the moving average parameter and the leverage coefficient for the first lagged error is negative (see column 3, Table 4.2 and Table 4.5). In addition, the sum of these products, over all the lagged errors, is also negative (see column 4, Table 4.2).

	β_j	Persistence	$c_1 d_1$	$\sum_{j=1}^{q} c_j d_j$
KOSPI (GEN ERROR) MA(1)-EGARCH(3,3)	$\beta_1 = 0.95$ $\beta_2 = -0.72 + 0.68i$ $\beta_3 = -0.72 - 0.68i$	0.554	-0.032	-0.100
NIKKEI (DOUBLE EXP) WN-EGARCH(1,3)	$\beta_1 = 0.973$	0.209	-0.193	-0.092
SE (DOUBLE EXP) WN-EGARCH(1,3)	$\beta_1 = 0.983$	0.745	-0.103	-0.026
ST (GEN ERROR) MA(1)-EGARCH(2,1)	$\beta_1 = -0.29$ $\beta_2 = 0.91$	0.298	-0.068	-0.068
The second column of this table reports a measure for the persistence of a (positive) shock to e_t on $\ln(h_t)$.				

Table 4.2: Persistence of EGARCH mdoel (1)

4.3.3 Autocorrelation structure of the estimated models

For each of the four indices, Figure 4.1 plots the estimated theoretical autocorrelations[5] of the squared observations of the 'best' EGARCH model chosen by the AIC. Specifically, for Korea and Singapore we use the EGARCH(3,3) and EGARCH(2,1) specifications respectively, with innovations that are drawn from the generalized error distribution. Further, for Japan and Taiwan we use the EGARCH(1,3) process with the double exponential distribution. Figure 4.1 also plots the estimated theoretical autocorrelations of the 'best' EGARCH model chosen by the SIC. Note that all the 'best' EGARCH models have been estimated without (see Figure 4.1) the inclusion of the no-trade dummy.

[5]We used Maple to evaluate the autocorrelations.

	Likelihood Ratio	Critical Value at 5% significant level
KOSPI (GEN ERROR) MA(1)-EGARCH(3,3)	18.78	12.60
NIKKEI (DOUBLE EXP) WN-EGARCH(1,3)	37.14	9.49
SE (DOUBLE EXP) WN-EGARCH(1,3)	24.80	9.49
ST (GEN ERROR) MA(1)-EGARCH(2,1)	3.28	2.71*

Table 4.3 reports the value of the following likelihood ratio (LR) test: LR=$2\times\left[ML_U - ML_R\right]$, where ML_U and ML_R denote the maximum log likelihood values of the unrestricted and restricted [EGARCH(1,1)] models respectively.

* indicates the critical value at 10% significance level.

Table 4.3: Likelihood Ratio test

	KOSPI MA(1)-EGARCH(1,1) (GEN ERROR)	SE WN-EGARCH(1,1) (DOUBLE EXP)	ST MA(1)-EGARCH(1,1) (GEN ERROR)
b	-0.0002 (1.34)	$-3E-09$ (0.00)	0.0002 (1.23)
θ	0.057 (4.41)	-	0.199 (12.13)
ω	-0.712 (8.42)	-0.273 (7.37)	-1.365 (8.38)
c_1	0.265 (11.71)	0.180 (12.45)	0.315 (12.82)
β_1^*	0.942 (113.48)	0.982 (270.2)	0.877 (51.51)
d_1	-0.118 (2.19)	-0.10 (1.82)	-0.233 (3.82)
υ	1.058 {0.02}	1	1.130 {0.02}

Table 4.4 reports parameter estimates for the 'best' EGARCH model chosen by SIC. For the NIKKEI the 'best' EGARCH model chosen by SIC is EGARCH (1,3) in Table 4.1.

The numbers in parentheses are t-statistics.

υ are the degrees of freedom of the generalized error distribution.

Standard errors are reported in brackets.

Table 4.4: Parameter Estimates for the 'best' EGARCH model (2)

	β_j	Persistence	$c_1 d_1$
KOSPI (GEN ERROR) MA(1)-EGARCH(1,1)	$\beta_1 = 0.942$	0.485	-0.031
SE (DOUBLE EXP) WN-EGARCH(1,1)	$\beta_1 = 0.982$	0.739	-0.018
ST (GEN ERROR) MA(1)-EGARCH(1,1)	$\beta_1 = 0.877$	0.251	-0.073
Table 4.5 reports the estimated autoregressive root and the persistence of the EGARCH models in table 4.4.			

Table 4.5: Persistence of EGARCH model (2)

Firstly, we analyse the high order models (see dark columns in Figure 4.1). The estimated autoregressive coefficient for the SE index is 0.983. As a result the estimated autocorrelations of the squared observations start at lag one 0.136 and decrease very slowly. Observe that the autocorrelation at lag ten, twenty and thirty is 0.107, 0.082 and 0.065 respectively. As with the SE index the 'best' EGARCH model for the NIKKEI index is of order 1,3 and has errors that are drawn from the double exponential distribution, but the estimated autoregressive coefficient is lower (0.973). Thus, although the estimated autocorrelations start at lag one 0.165 they decrease more rapidly. The autocorrelation at lag ten, twenty and thirty is 0.077, 0.050 and 0.034 respectively. For the KOSPI and ST indices the distribution of the innovations is generalized error. However the value of the highest root of the autoregressive polynomial for the ST index is 0.913. Therefore, although the autocorrelations start very high, at lag one 0.226, they decrease very quickly. For the KOSPI index, the autocorrelation at lag ten, twenty and thirty is 0.082, 0.044 and 0.026, respectively, whereas for the ST index it is 0.050, 0.017 and 0.006 respectively. For the EGARCH(1,1) models chosen by SIC, the estimated theoretical autocorrelations are not much different from those for the high order models. The estimated autoregressive coefficient for the SE index is 0.982 over against 0.983 in EGARCH(1,3) model. The estimated autocorrelations of the squared observations are 0.135 at lag one, and 0.105, 0.081 and 0.064 at lag ten, twenty and thirty respectively. For the ST index the estimated autoregressive coefficient of EGARCH(1.1) is lower (0.877) than that of EGARCH(2,1). The autocorrelations begins with 0.218 and decrease quicker than in the case of EGACH(2,1). For the KOSPI the estimated autoregressive coefficient of EGARCH(1,1) is 0.942 and also the autocorrelation at lag one is 0.211.

It is useful to uncover the properties of the autocorrelation function of the squared observations, when comparing the EGARCH model with the standard GARCH model family. Possible differences in the moment structure of these models may shed light on the success of the EGARCH model in applications. To facilitate model identification, the results for the autocorrelations of the power-transformed observations can be applied so that the properties of the observed data can be compared with the theoretical properties of the models. For each of the four stock indices, Figure 4.1 plots the sample autocorrelations of the squared observations. It also plots the estimated theoretical autocorrelations of the squared observations of the 'best' EGARCH specification and of the GARCH(1,1) model with conditionally normal errors[6]. For all three indices, the autocorrela-

[6]The GARCH(1,1) specification that we estimate is $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$. In order to obtain the estimated theoretical autocorrelations of the squared errors of the above model we

tions of the EGARCH model are closer to the sample autocorrelations than those of the GARCH model[7]. For the KOSPI index, the autocorrelation of the squared observations of EGARCH(3,3) specification, at lag two, four and twenty is equal to the corresponding sample autocorrelation. For the ST index, Figure 4.1d also plots the autocorrelations of the squared errors of the GARCH(1,1) model with innovations drawn from the generalized error distribution[8]. Observe that these autocorrelations are much higher than those obtained with conditionally normal errors. For the SE index it can be seen that the fitted squared returns from the GARCH model generally have autocorrelations that are substantially too high when compared with the corresponding sample equivalents. In fact, they generally exceed the corresponding sample autocorrelations by a factor of two. In contrast, the EGARCH specification does a good job of replicating the observed pattern of autocorrelations of the squared returns. It generates a model where the autocorrelations of the fitted squared values are relatively 'close' to those of the population equivalents. The autocorrelation of the squared returns, at lag eleven, twelve, sixteen, and twenty six is equal to the corresponding sample autocorrelation. In other words, the EGARCH model can more accurately reproduce

use the following formula

$$\rho(\varepsilon_t^2, \varepsilon_{t-k}^2) = \frac{(a+\beta)^k\{1+\beta^2-\beta[(a+\beta)+(a+\beta)^{-1}]\}}{1+\beta^2-2\beta(a+\beta)}.$$

[7] We also estimate a GARCH(1,1) model with conditionally normal errors for the NIKKEI index. The sum of the ARCH and GARCH coefficients is greater than one.

[8] For all indices, we also estimated GARCH(1,1) models with innovations drawn from either the double exponential or t distributions. In all cases, the condition for the existence of either the first moment ($a + \beta < 1$) or the second one ($\beta^2 + 2a\beta + E(\epsilon_t^4)a^2 < 1$) was violated.

the nature of the sample autocorrelations of squared returns than the GARCH model.

Finally, for the four selected specifications, when the no-trade dummy enters in the conditional variance, Figure 4.2 plots the estimated theoretical autocorrelations of the squared observations and their corresponding sample equivalents. Except for the KOSPI case the estimated theoretical autocorrelations of the EGARCH model are closer to the sample autocorrelations than those of the GARCH(1,1) models.

4.4 Conclusion

In this chapter we have obtained a complete characterization of the moment structure of the general ARMA(r,s)-EGARCH(p,q) model. In particular, we provided the autocorrelation function of any positive integer power of the squared errors. Additionally, we derived the cross correlations between the levels and squares of the observed process. To obtain our results, we assumed that the error term is drawn from either the normal, double exponential or generalized error distributions. The results of the chapter can be used to compare the EGARCH model with the standard GARCH model or the Asymmetric power ARCH model. They reveal certain differences in the moment structure between these models. Further, to facilitate model identification, the results for the autocorrelations of the squared observations can be applied so that the properties of the observed data can be compared with the theoretical properties of the models. Finally, the techniques used in this paper can be employed to obtain the moments of more complex EGARCH models, e.g. EGARCH-in-mean, the Component EGARCH, and the Fractional Integrated EGARCH models. The derivation of the moment structure of these models is left for future research.

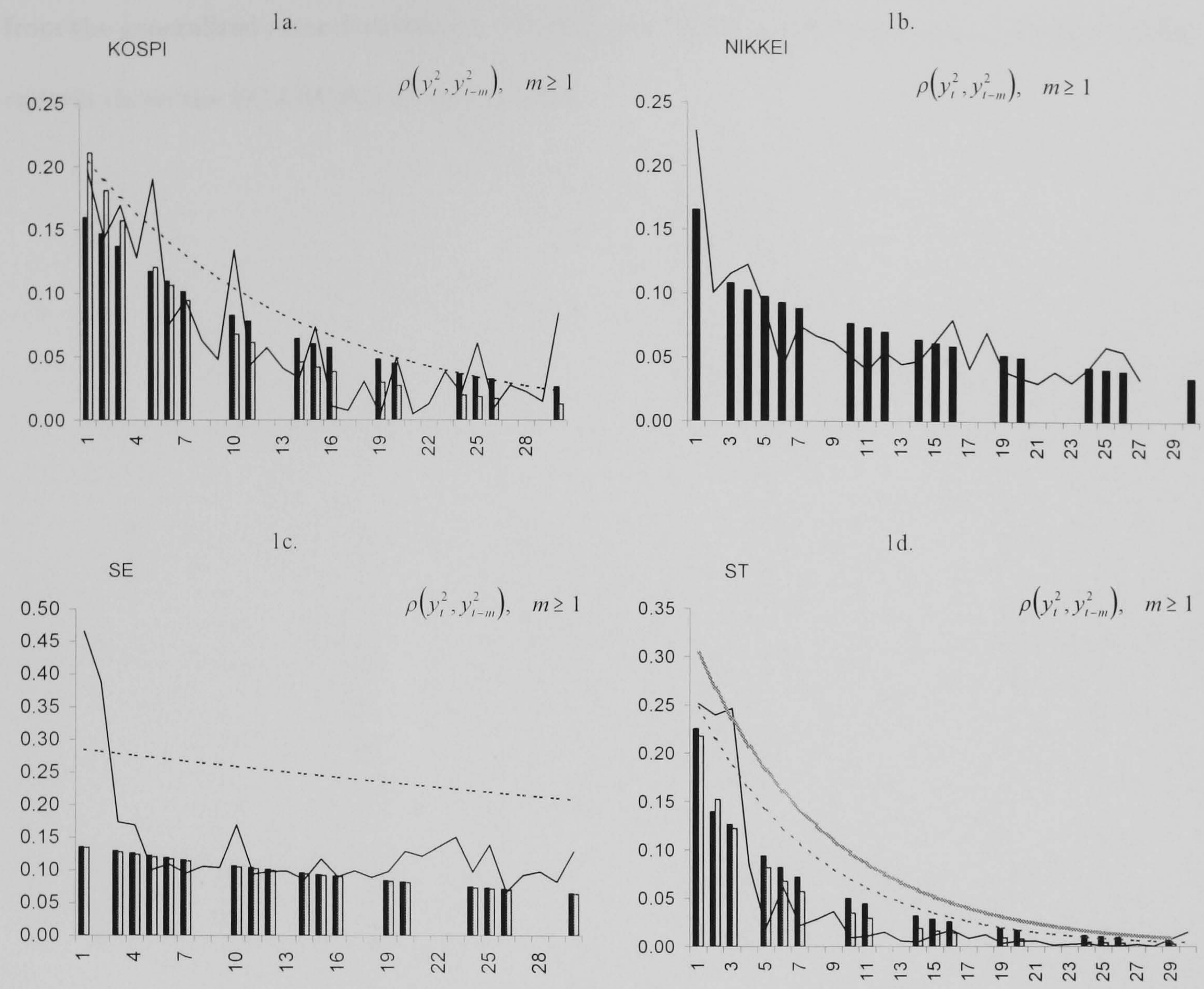


Figure 4.1: The estimated theoretical autocorrelations of EGARCH model (1)

For each of the four indices, Figure 4.1 plots the sample autocorrelations of the squared observations (solid line). It also plots the estimated theoretical autocorrelations (ETA) of the squared observations for the 'best' EGARCH specification chosen by AIC (dark columns) and by SIC (light columns). All the 'best' EGARCH models have been estimated without the inclusion of the no-trade dummy. Dotted lines represent the ETA of the squared observations for the GARCH(1,1) model with conditionally normal errors. Moreover, for the ST index, the grey line represents the ETA of the squared observations for the GARCH(1,1) model with errors drawn from the generalized error distribution. Finally, note that for the Nikkei index both information criteria chose the EGARCH(1,3) specification.

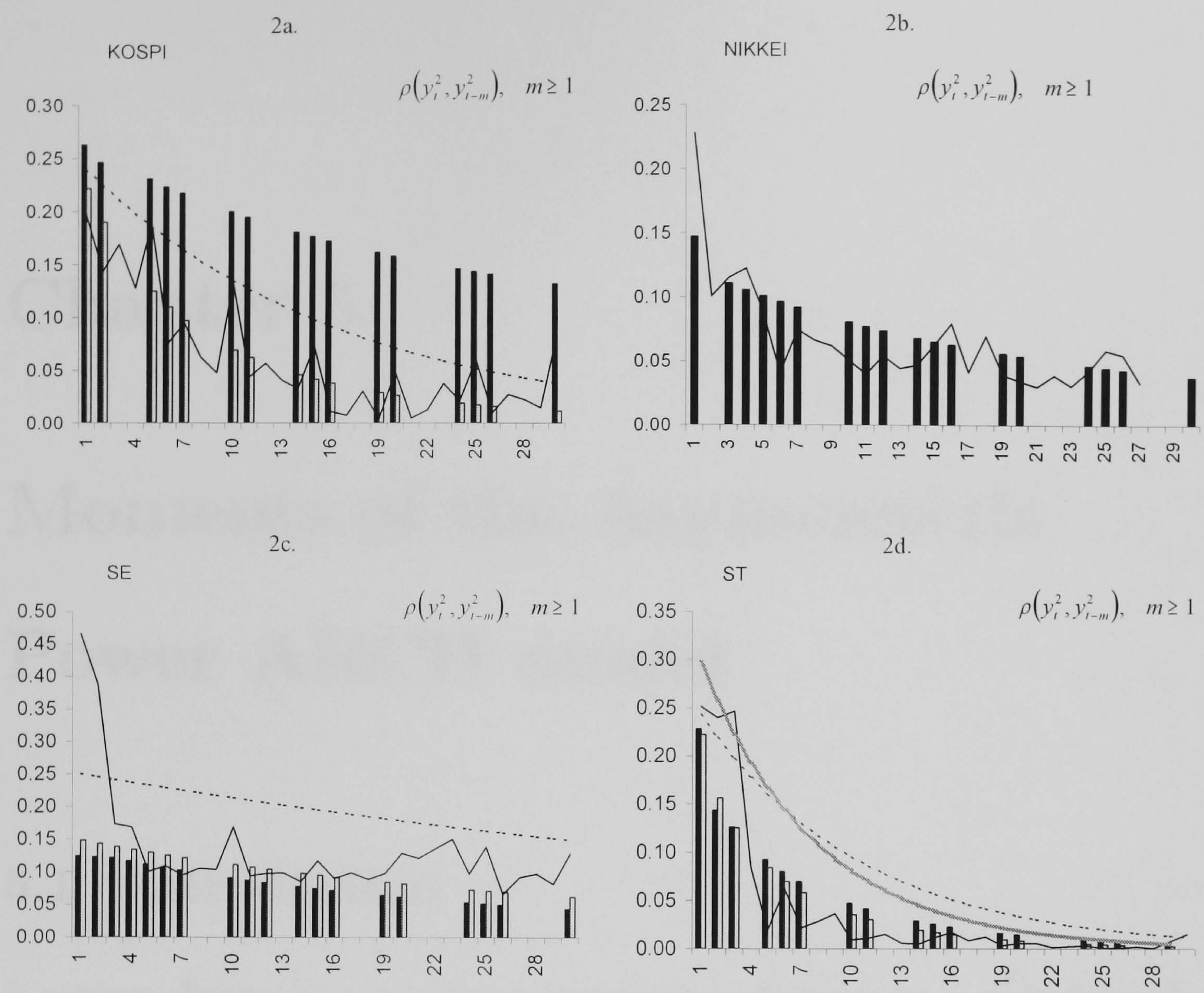


Figure 4.2: The estimated theoretical autocorrelations of EGARCH model (2)

For each of the four indices, Figure 4.2 plots the sample autocorrelations of the squared observations (solid line). It also plots the estimated theoretical autocorrelations (ETA) of the squared observations for the specifications used in Figure 4.1. All these EGARCH models have now been estimated with the inclusion of the no-trade dummy. Dotted lines represent the ETA of the squared observations for the GARCH(1,1) model with conditionally normal errors. Moreover, for the ST index, the grey line represents the ETA of the squared observations for the GARCH(1,1) model with errors drawn from the generalized error distribution.

Chapter 5

Moments of the Asymmetrtic Power ARCH model

5.1 Introduction

A common finding in much of the empirical finance literature is that although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated (see, for example, Taylor, 1986; Ding et al., 1993; Granger and Ding, 1995a, b; Ding and Granger, 1996). In particular, Ding et al., (1993) investigate the autocorrelation structure of $|r_t|^d$, where r_t is the daily S&P 500 stock market returns, and d is a positive number. They found that $|r_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which they call the Asymmetric Power ARCH (A-PARCH) model. In addition, they show that the A-PARCH model comprises seven other models in the literature.

Since its introduction, the A-PARCH model has been frequently applied. For example, He and Teräsvirta (1999b) illustrate how the A-PARCH model may also be viewed as a standard GARCH model for observations that have been transformed by a sign-preserving power transformation implied by a (modified) A-PARCH parameterization.

The purpose of this research is to study the autocorrelation structure of the general A-PARCH(p, q) model. He and Teräsvirta (1999b), using the sign-preserving transformation, obtained the autocorrelation function of the power-transformed absolute errors for the first-order A-PARCH model. Despite this progress, the moment structure of the A-PARCH(p, q) model has not been fully worked out yet. It would be useful to know the properties of the autocorrelation function of the power-transformed observations when comparing the A-PARCH model with the Exponential or the standard GARCH models.

In this chapter we view the A-PARCH model from a different angle, and provide a comprehensive methodology for the analysis of the general A-PARCH(p, q) process. First, we give the ARMA representations of the power transformations of the conditional variance and the absolute returns. Next, we derive an existence condition for a certain fractional moment of the absolute observations. The practical significance of the existence condition for a fractional moment is that when it is satisfied, then all lower-order moments exist as well. In contrast, violation of the above condition implies that no higher-order moments exist. Further, we obtain the autocorrelation function of the power-transformed absolute returns. Estimates of the autocorrelations of power-transformed observations can be of great importance. By comparing these estimates to those obtained directly by the data, one can have a clear indication of how well the estimated model fits the data.

Our results on the moment structure of the general A-PARCH(p, q) model extend the results in He and Teräsvirta (1999b) on the first-order A-PARCH model, and Karanasos (1999) and He and Teräsvirta (1999a) on the GARCH(p, q) model.

The practical implications of the results are illustrated empirically using daily data on five East Asia stock indices. To obtain the theoretical results and to carry out the estimation we assume that the innovations are drawn from either the normal, student-t, generalized error, or double exponential distributions. In most cases, likelihood ratio testing procedures choose high order A-PARCH specifications. Additionally, in the majority of the cases, model selection criteria support the general power ARCH model, as against Bollerslev's (1986) GARCH and Taylor/Schwert models[1]. These findings highlight the need to have analytical expressions for the moment structure of the general A-PARCH(p, q) model in addition to those for the GARCH(p, q) and A-PARCH(1,1) models.

The remainder of the chapter is organized as follows. Section 5.2 investigates the autocorrelation functions of the power-transformed conditional variance and absolute returns. Section 5.3 discusses the data and presents the empirical results. Section 5.4 concludes the analysis. Proofs are found in an early version of this work (Karanosos and Kim, 2001).

[1] Taylor (1986) and Schwert (1990) have suggested that the conditional standard deviation obeys a GARCH specification.

5.2 A-PARCH Model

5.2.1 A-PARCH(p,q) process

One of the most common models in finance and economics to describe a time series r_t, of the returns from some asset, is the martingale process

$$r_t = e_t h_t^{\frac{1}{2}}, \tag{5.19}$$

where $\{e_t\}$ are independent, identically distributed random variables with $E(e_t) = E(e_t^2 - 1) = 0$. h_t is positive with probability one and is a measurable function of Σ_{t-1}, which in turn is the sigma-algebra generated by $\{r_{t-1}, r_{t-2}, \ldots\}$. That is h_t denotes the conditional variance of the returns $\{r_t\}$, $(r_t|\Sigma_{t-1}) \sim (0, h_t)$. In addition h_t is specified as an A-PARCH(p,q) process

$$h_t^{\frac{\delta}{2}} = \omega + \sum_{l=1}^{p} \beta_l h_{t-l}^{\frac{\delta}{2}} + \sum_{l=1}^{q} a_l h_{t-l}^{\frac{\delta}{2}} f_l(e_{t-l}), \tag{5.20a}$$

with

$$f_l(e_{t-l}) \equiv [|e_{t-l}| - \gamma_l e_{t-l}]^{\delta}, \quad (l = 1, \ldots, q), \tag{5.20b}$$

where a_l and β_l are the ARCH and GARCH parameters respectively, γ_l $(-1 < \gamma_l < 1)$ is the leverage parameter and δ is the parameter for the power term. Further, to guarantee the nonnegativity of the conditional variance we assume that the GARCH and ARCH parameters satisfy the sufficient conditions given in Nelson and Cao (1992). Within the A-PARCH model, by specifying permissible values for a, β, γ, and δ in (5.20), it is possible to nest a number of the more standard ARCH and GARCH specifications (see Ding et al., 1993, Hentschel, 1995, and Brooks et al., 2000).

Since its introduction by Ding et al., (1993), the A-PARCH model has been frequently applied. For example, Hentschel (1995) defined a parametric family of asymmetric GARCH models that nests the EGARCH and A-PARCH models. Hagerud (1997a) investigated the extent to which seven asymmetric GARCH models (including the A-PARCH) have been the data generating process for 45 Nordic stocks, while Hagerud (1997b) presented two new Lagrange Multiplier test statistics that can be used to detect asymmetries generated by the A-PARCH and six other asymmetric GARCH models. Moreover, He and Teräsvirta (1999c) considered a family of first-order asymmetric GARCH processes which includes the A-PARCH as a special case. Finally, Brooks et al., (2000) analysed the applicability of the power ARCH models to national stock market returns for ten countries.

It is also worth noting that Fornari and Mele (1997) showed the usefulness of the A-PARCH scheme in approximating models developed in continuous time as systems of stochastic differential equations. This feature of GARCH schemes has usually been overshadowed by their well-known role as simple econometric tools providing reliable estimates of unobserved conditional variances (Fornari and Mele, 2001).

In order to distinguish the general model in (5.20) from a version in which $\beta_j = 0$ $(j = 1, \ldots, p)$, we will hereafter refer to the former as A-PGARCH and the latter as A-PARCH.

For the subsequent development of our theory, it is useful to write the $\frac{\delta}{2}$th power of the conditional variance in an ARMA form. Hence, from the right hand side of (5.20a) we add and subtract $a_l k_l h_{t-l}^{\frac{\delta}{2}}$ $(l = 1, \ldots, q)$, in order to obtain the ARMA representation of $h_t^{\frac{\delta}{2}}$

$$h_t^{\frac{\delta}{2}} = \omega + \sum_{l=1}^{\tilde{p}} \widetilde{\beta}_l h_{t-l}^{\frac{\delta}{2}} + \sum_{l=1}^{q} a_l v_{l,t-l}. \tag{5.21a}$$

with

$$v_{l,t-l} \equiv h_{t-l}^{\frac{\delta}{2}} [f_l(e_{t-l}) - k_l], \tag{5.21b}$$

where $\tilde{p} \equiv \max(p, q)$, $\widetilde{\beta}_l \equiv a_l k_l + \beta_l$ and k_i $(i = 1, \ldots, q)$ denotes the expected value of $[f_i(e_t)]$ and is given by

$$k_i \equiv \mathsf{E}[f_i(e_t)] \equiv \begin{cases} \frac{1}{\sqrt{\pi}} \left[(1-\gamma_i)^{\delta} + (1+\gamma_i)^{\delta}\right] 2^{\left(\frac{\delta}{2}-1\right)} \Gamma\left(\frac{\delta+1}{2}\right), & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{N}(0,1), \\ \frac{r^{\left(\frac{\delta}{2}-1\right)} (r-2) \Gamma\left(\frac{r-\delta}{2}\right) \Gamma\left(\frac{\delta+1}{2}\right)}{\Gamma\left(\frac{r}{2}\right) 2\sqrt{\pi}} [(1-\gamma_i)^{\delta} + (1+\gamma_i)^{\delta}], & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{t}_r(0,1), \\ [(1-\gamma_i)^{\delta} + (1+\gamma_i)^{\delta}] \frac{\Gamma\left(\frac{\delta+1}{v}\right) \lambda^{\delta} 2^{\left(\frac{\delta}{v}-1\right)}}{\Gamma\left(\frac{1}{v}\right)}, & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{GE}_v(0,1), \\ \left[(1-\gamma_i)^{\delta} + (1+\gamma_i)^{\delta}\right] \Gamma(\delta+1) 2^{-\left(\frac{\delta}{2}+1\right)}, & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{DE}(0,1) \end{cases}, \tag{5.22}$$

where $\lambda \equiv \left\{2^{\frac{-2}{v}} \Gamma\left(\frac{1}{v}\right) \left[\Gamma\left(\frac{3}{v}\right)\right]^{-1}\right\}^{\frac{1}{2}}$ and N, t_r, GE_v, and DE denote the normal, student-t, generalized error and double exponential distributions respectively. Moreover, r are the degrees of freedom of the student-t distribution, v is the tail thickness parameter of the generalized error distribution, and $\Gamma(\cdot)$ is the Gamma function.

Note that $v_{l,t-l}$ in (5.21b) is defined as the difference between $f_l(\varepsilon_{t-l})$ and its conditional expectation. Thus, $v_{l,t-l}$ is a serially uncorrelated process with zero mean.

Expression (5.21) will be used in the derivation of the autocorrelation function of the $\frac{\delta}{2}$th power of the conditional variance (see theorem 1 below). It can also be used (employing the methodology in Karanasos, 2001) to obtain the optimal predictor (and the corresponding forecast error and forecast error uncertainty) of the future values of $h_t^{\frac{\delta}{2}}$.

Assumption. 5.1 All the roots of the autoregressive polynomial $[\widetilde{B}(L) \equiv 1 - \sum_{l=1}^{\widetilde{p}} \widetilde{\beta}_l L^l \equiv \prod_{l=1}^{\widetilde{p}}(1 - \lambda_l L)]$ lie outside the unit circle (covariance-stationarity condition).

Assumption. 5.2 The polynomials $\widetilde{B}(L)$ and $A(L) \equiv \sum_{l=1}^{q} \alpha_l L^l$ have no common left factors other than unimodular ones (irreducibility condition).

Finally, for future development, it is helpful to note that $|r_t|^{\delta}$ may be expressed as

$$|r_t|^{\delta} = k_0 h_t^{\frac{\delta}{2}} + v_{0,t}, \tag{5.23}$$

where $v_{l,t-l}$ is defined by (5.21b) and k_0 is given by (5.22) with $\gamma_0 \equiv 0$.

5.2.2 Autocorrelation functions

In this section we present the autocorrelation functions of the power transformations of the conditional variance and the absolute-valued observations. We examine only the case where the roots of $\widetilde{B}(L) = 0$ are simple. That is $\lambda_i \neq \lambda_j$, for all $i, j \in \{1, \ldots, \widetilde{p}\}$ such that $i \neq j$.

From (5.23) one readily obtains

$$\mathsf{Cov}\left(|r_t|^\delta, |r_{t-m}|^\delta\right) = k_0^2\mathsf{Cov}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right) + k_0\mathsf{Cov}\left(h_t^{\frac{\delta}{2}}, v_{0,t-m}\right), \quad (m \in \mathbb{N}) \quad (5.24)$$

The derivation of the autocorrelations of the fitted power-transformed values and their comparison with the corresponding sample equivalents are useful to decide the appropriate method of estimation, the model and the optimal order of the chosen specification as described in section 4.1 in the previous chapter.

It is clear from the above expression that the autocovariances of $h_t^{\frac{\delta}{2}}$ are needed for the computation of the autocovariances of the power transformed absolute observations. Thus our first theorem establishes the lag-m autocorrelation of $h_t^{\frac{\delta}{2}}$, $\rho_m\left(h_t^{\frac{\delta}{2}}\right) \equiv \mathsf{Corr}\left(h_t^{\frac{\delta}{2}}, h_{t-m}^{\frac{\delta}{2}}\right)$, $m \in \mathbb{N}$.

Theorem 5.1. *Suppose that* $0 < \mathsf{E}\left[f_i(e_t) f_j(e_t)\right] < \infty \; \forall \; t \; (i, j = 1, \ldots, q)$. *Then, under assumptions 1 and 2, the autocorrelation function of* $h_t^{\frac{\delta}{2}}$ is

$$\rho_m\left(h_t^{\frac{\delta}{2}}\right) = \frac{\gamma_h^m}{\gamma_h^0}, \tag{5.25a}$$

with

$$\gamma_h^m \equiv \sum_{l=1}^{\tilde{p}} \zeta_{lm}\pi_{lm}, \tag{5.25b}$$

and

$$\zeta_{lm} \equiv \frac{\lambda_l^{\tilde{p}-1+m}}{\prod_{n=1}^{\tilde{p}}(1-\lambda_l\lambda_n)\prod_{\substack{n=1\\n\neq l}}^{\tilde{p}}(\lambda_l-\lambda_n)}, \tag{5.25c}$$

$$\begin{aligned}\pi_{lm} \equiv & \sum_{n=1}^{q} a_n^2(k_{nn}-k_n^2) + \sum_{d=1}^{m}\sum_{n=1}^{q-d} a_n a_{n+d}(k_{n,n+d}-k_n k_{n+d})(\lambda_l^d+\lambda_l^{-d}) + \\ & + \sum_{d=m+1}^{q}\sum_{n=1}^{q-d} a_n a_{n+d}(k_{n,n+d}-k_n k_{n+d})(\lambda_l^d+\lambda_l^{d-2m}).\end{aligned} \tag{5.25d}$$

where k_i $(i = 1, \dots, q)$ *denotes the expected value of the* $f_i(e_t)$ *and is given by (5.22). Moreover,* λ_j *is the inverse of the jth root of the autoregressive polynomial* $\widetilde{B}(L)$, and k_{ij} $(i, j = 1, \dots, q)$ *denotes the expected value of* $f_i(e_t)\times$ $f_j(e_t)$ *and is given by*

$$k_{ij} \equiv \mathsf{E}\left[f_i(e_t)f_j(e_t)\right] \equiv$$
$$\begin{cases} \frac{1}{\sqrt{\pi}}\left\{[(1-\gamma_i)(1-\gamma_j)]^{\delta}+[(1+\gamma_i)(1+\gamma_j)]^{\delta}\right\}2^{\delta-1}\Gamma\left(\frac{2\delta+1}{2}\right), & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{N}(0,1), \\ \frac{r^{(\delta-1)}(r-2)\Gamma\left(\frac{r}{2}-\delta\right)\Gamma\left(\delta+\frac{1}{2}\right)\{[(1-\gamma_i)(1-\gamma_j)]^{\delta}+[(1+\gamma_i)(1+\gamma_j)]^{\delta}\}}{\Gamma\left(\frac{r}{2}\right)2\sqrt{\pi}}, & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{t}_r(0,1), \\ \frac{\{[(1-\gamma_i)(1-\gamma_j)]^{\delta}+[(1+\gamma_i)(1+\gamma_j)]^{\delta}\}\Gamma\left(\frac{2\delta+1}{v}\right)\lambda^{2\delta}2^{\left(\frac{2\delta}{v}-1\right)}}{\Gamma\left(\frac{1}{v}\right)}, & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{GE}_v(0,1), \\ \{[(1-\gamma_i)(1-\gamma_j)]^{\delta}+[(1+\gamma_i)(1+\gamma_j)]^{\delta}\}\Gamma(2\delta+1)2^{-(\delta+1)}, & \text{if } e_t \overset{\text{(i.d)}}{\sim} \mathrm{DE}(0,1), \end{cases} \tag{5.26}$$

In addition, the δ*th moment of the conditional variance is*

$$\mathsf{E}(h_t^{\delta}) = \frac{\left[\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right)\right]^2}{1-\gamma_h^0}, \tag{5.27a}$$

with

$$\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right) = \frac{\omega}{1-\sum_{l=1}^{\widetilde{p}}\widetilde{\beta}_l}, \tag{5.27b}$$

where γ_h^0 *is defined by (5.25b).*

Remark 5.1. The condition for the existence of the $\frac{\delta}{2}$th and δth moments of the conditional variance are $\sum_{l=1}^{\widetilde{p}}\widetilde{\beta}_l < 1$ and $\gamma_h^0 < 1$ respectively[2].

[2]Note that the autocorrelation function of $h_t^{\frac{\delta}{2}}$ exists if and only if the $\frac{\delta}{2}$th and δth moments of the conditional variance exist.

Remark 5.2. The result in (5.27a) is very important because $\mathsf{E}(h_t^{\delta})$ will be used in the derivation of the 2δth moment of the absolute returns (see theorem 5.2 below).

The practical significance of the existence condition for a fractional moment is that when it is satisfied, then all lower-order moments also exist. On the other hand, violation of the above condition implies that no higher-order moments exist.

Now suppose that the conditional mean of r_t, given information through time $t-1$, is governed by

$$\mathsf{E}(r_t|\Sigma_{t-1}) = cg(h_t)$$

Mean equations of this form have been widely used in empirical studies of time varying risk premia. Various specifications for the functional form of the risk premium $cg(h_t)$, have appeared in the empirical literature, most commonly imposing $g(h_t) = \sqrt{h_t}$, $g(h_t) = \ln(h_t)$ or $g(h_t) = h_t$ (see, for example, Engle et al., 1987, Duan, 1995, and Härdle and Hafner, 2000). The results in theorem 1 can be used to derive the autocorrelations of r_t, when $g(h_t) = h_t^{\frac{\delta}{2}}$.

Next, we examine the moment structure of the power-transformed absolute returns. Estimates of the autocorrelations of power transformations of the absolute observations are critical. By comparing these estimates to those obtained directly by the data, one can have a clear indication of how well the estimated model fits the data.

Theorem 5.2. *Suppose that* $0 < \mathsf{E}\left[f_i(e_t)f_j(e_t)\right] < \infty \ \forall \ t \ (i,j = 0,\ldots,q)$. *Then, under assumptions 5.1 and 5.2, the autocorrelation of* $|r_t|^{\delta}$ *in (5.19) and*

(5.20), at lag m $(m \in \mathbb{N})$, *is given by*

$$\rho_m\left(|r_t|^\delta\right) \equiv \frac{\mathsf{Cov}(|r_t|^\delta, |r_{t-m}|^\delta)}{\mathsf{Var}(|r_t|^\delta)}$$

$$= \frac{k_0^2\gamma_h^m + k_0 \sum_{f=1}^{\tilde{p}} \zeta_{f0}^* \sum_{l=1}^{\min(m,q)} a_l(k_{0l} - k_0 k_l)\lambda_f^{m-l}}{k_0^2\gamma_h^0 + (k_{00} - k_0^2)}, \quad (m \geq 1) \tag{5.28a}$$

with

$$\zeta_{f0}^* \equiv \frac{\lambda_f^{\tilde{p}-1}}{\prod_{\substack{n=1\\ n\neq f}}^{\tilde{p}}(\lambda_f - \lambda_n)}, \tag{5.28b}$$

Moreover, the 2δ*th moment of the absolute returns is*

$$\mathsf{E}\left(|r_t|^{2\delta}\right) = k_{00}\mathsf{E}\left(h_t^\delta\right),$$

where γ_h^m, k_l, *are defined by (5.25b) and (5.22), respectively,* k_{0l} *is given by (5.26) with* $\gamma_0 \equiv 0$ *and* $\mathsf{E}\left(h_t^\delta\right)$ *is given in theorem 1.*

Example. Suppose that $r_t = \sqrt{h_t}e_t$, where e_t ~i.i.d $(0,1)$ and h_t obeys an A-PGARCH(3,1) specification

$$(1 - \beta_1 L - \beta_2 L^2 - \beta_3 L^3)h_t^{\frac{\delta}{2}} = \omega + a_1 f_1(e_{t-1})h_{t-1}^{\frac{\delta}{2}},$$

where

$$f_1(e_{t-1}) \equiv [|e_{t-1}| - \gamma_1 e_{t-1}]^\delta,$$

and $-1 < \gamma_1 < 1$.

The ARMA representation of $h_t^{\frac{\delta}{2}}$ is

$$(1 - \tilde{\beta}_1 L - \beta_2 L^2 - \beta_3 L^3)h_t^{\frac{\delta}{2}} = \omega + a_1 v_{1,t-1},$$

with

$$v_{1,t-1} \equiv [f_1(e_{t-1}) - k_1]\, h_{t-1}^{\frac{\delta}{2}},$$

where $\widetilde{\beta}_1 \equiv a_1 k_1 + \beta_1$ and k_1 denotes the expected value of $f_1(e_{t-1})$. Further, let λ_i $(i = 1, 2, 3)$ denote the inverse of the ith root of the third order autoregressive polynomial $(1 - \widetilde{\beta}_1 L - \beta_2 L^2 - \beta_3 L^3)$. It is assumed that $|\lambda_i| < 1$ and $\lambda_i \neq \lambda_j$ for all $i, j \in \{1, 2, 3\}$ such that $i \neq j$.

Moreover, the variance of $|r_t|^\delta$ is

$$\mathsf{Var}(|r_t|^\delta) = [k_0^2 \gamma_h^0 + (k_{00} - k_0^2)]\mathsf{E}(h_t^\delta),$$

where k_0 is given by (5.22) with $\gamma_0 = 0$, and k_{00} is defined in (5.26) with $\gamma_0 = 0$.

Further, the expected value of $|r_t|^{2\delta}$ is

$$\mathsf{E}\left(|r_t|^{2\delta}\right) = \frac{k_{00}\left[\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right)\right]^2}{1 - \gamma_h^0},$$

with

$$\mathsf{E}\left(h_t^{\frac{\delta}{2}}\right) = \frac{\omega}{(1 - \widetilde{\beta}_1 - \beta_2 - \beta_3)},$$

where γ_h^0 is given below.

Finally, the autocovariance function of $|r_t|^\delta$ is

$$\begin{aligned}\mathsf{Cov}(|r_t|^\delta, |r_{t-m}|^\delta) &= \left\{k_0^2\gamma_h^m + k_0 a_1 (k_{01} - k_0 k_1)\left[\frac{\lambda_1^{1+m}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}\right.\right. \\ &\left.\left. + \frac{\lambda_2^{1+m}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} + \frac{\lambda_3^{1+m}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}\right]\right\} \mathsf{E}(h_t^\delta) \quad (m \geq 1),\end{aligned}$$

where

$$\begin{aligned}\gamma_h^m \equiv\ & a_1^2(k_{11} - k_1^2)\left[\frac{\lambda_1^{2+m}}{(1 - \lambda_1^2)(1 - \lambda_1\lambda_2)(1 - \lambda_1\lambda_3)(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}\right. \\ & + \frac{\lambda_2^{2+m}}{(1 - \lambda_2^2)(1 - \lambda_2\lambda_1)(1 - \lambda_2\lambda_3)(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \\ & \left. + \frac{\lambda_3^{2+m}}{(1 - \lambda_3^2)(1 - \lambda_3\lambda_1)(1 - \lambda_3\lambda_2)(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}\right] \quad (m \geq 0),\end{aligned}$$

and k_{ij} $(i, j = 1, 2, 3)$ is defined by (5.26).

The significance of the results in this section is that they allow us to establish whether the A-PGARCH model in (5.19)-(5.20a) is capable of reproducing key features exhibited by the data. These features include, for example, time series with very little autocorrelation but with strongly dependent squares.

5.3 Empirical Analysis

5.3.1 Data

Daily stock price index data for five East Asia countries were sourced from the Datastream database for the period January 1980 to April 1997, giving a total of 4,518 observations. The five countries and their respective price indices are : Korea (KOSPI), Japan (NIKKEI), Taiwan (SE), Singapore (Straits-Times)[3], which were also used in the previous chapter, and Hong Kong (Hang Seng). For each national index, the continuously compounded return was estimated as r_t =log(p_t)−log(p_{t-1}) where p_t is the price on day t. Figure 5.1 plots the daily returns on the five stock indices.

5.3.2 Estimation Results

We model all the five stock returns as MA(1) processes following the same reason for modelling in the previous chapter. The MA(1) model is

[3] We use the daily returns for the Straits-Times index for the period January 1985 to April 1997 (3,213 observations) because the data is available only from January 1985.

$$r_t = b + (1 + \theta L)\varepsilon_t. \tag{5.31}$$

To select our best A-PGARCH specification, we begin with low order models (e.g., A-PGARCH(1,1)) and work upward as required to fit the data. The general A-PGARCH(4,4) specification is given by

$$\varepsilon_t = e_t h_t^{\frac{1}{2}}, \quad e_t \overset{\text{(i.i.d)}}{\sim} (0, 1), \tag{5.32a}$$

$$h_t^{\frac{\delta}{2}} = \omega + \sum_{l=1}^{4} \beta_l h_{t-l}^{\frac{\delta}{2}} + \sum_{l=1}^{4} a_l f_l(\varepsilon_{t-l}). \tag{5.32b}$$

where

$$f_l(\varepsilon_{t-l}) \equiv h_{t-l}^{\frac{\delta}{2}} f_l(e_{t-l}), \tag{5.32c}$$

and

$$f_l(e_{t-l}) \equiv [|e_{t-l}| - \gamma_l e_{t-l}]^{\delta} \quad (l = 1, \ldots, 4) \tag{5.32d}$$

We estimate A-PGARCH models of order up to A-PGARCH(4,4) for the returns on the five stock indices using four alternative distributions: the normal, student-t, double exponential and generalized error.

Table 5.1 reports the selected specifications. In most of the cases, the Akaike Information Criterion (AIC) (see Table 5.2) and the likelihood ratio (LR) test (see Table 5.4) choose high order A-PGARCH models. When the errors ε_t in (5.32a) are conditionally normal, the A-PGARCH(3,1) was chosen for two out of the five indices and the A-PGARCH(3,2) model was chosen for the SE index. When the innovations e_t in (5.32a) are t-distributed, the A-PGARCH(3,4) and A-PGARCH(3,3) specifications were chosen for the KOSPI and NIKKEI indices respectively. Further, the A-PGARCH(4,2), A-PGARCH(2,2) and A-PGARCH(2,1) specifications were chosen for the Hang-Seng, SE and Straits-Times indices respectively. When e_t is drawn from the generalized error distribution, the A-PGARCH(4,1) specification was chosen for the NIKKEI and Straits-Times indices, whereas the A-PGARCH(1,3), and A-PGARCH(3,2) specifications were chosen for the KOSPI and SE indices respectively.

We encountered many instances of negative estimated α's and β's. In all of these cases, the ARCH and GARCH coefficients satisfy the set of sufficient conditions to guarantee the nonnegativity of the conditional variance (see equation 29 in Nelson and Cao, 1992).

5.3.3 Tests of power term parameters in A-PGARCH models

Table 5.3 reports the estimated power terms for the A-PGARCH models fitted to each of the five national indices. When the errors are conditionally normal, the maximum power term was 3.62 for Singapore. Beyond this extreme case, the remainder of the estimated power terms were between 1.00 and 2.00.

The power terms estimated using the double exponential distribution were very similar to those obtained with the generalized error distribution for all five indices: KOSPI (1.49, 1.48), Hang Seng (1.52, 1.41), NIKKEI (1.22, 1.20), SE (1.34, 1.32) and Straits-Times (2.13, 2.02). When the innovations were t-distributed, all the estimated power terms were much lower-between 1 and 1.5, with the exception of Singapore (1.89).

	KOSPI	NIKKEI	Hang Seng	SE	Straits-Times
MA(1)-A-PGARCH (Cond.distribution)	(1,3) (Gen Error)	(3,3) (Student-t)	(4,2) (Student-t)	(3,2) (Gen Error)	(2,1) (Student-t)
b	−0.0001 (0.56)	4E-4 (4.02)	0.001 (5.60)	7E-11 (0.00)	3E-11 (1.53)
θ	0.058 (4.31)	0.016 (1.09)	0.068 (4.44)	2E-07 (0.00)	0.221 (12.05)
ω	2E-05 (1.19)	2E-05 (1.33)	2E-04 (1.57)	8E-05 (1.39)	9E-06 (5.56)
α_1	0.172 (5.76)	0.069 (2.33)	0.102 (6.67)	0.070 (3.42)	0.128 (6.25)
α_2	0.010 (0.22)	0.057 (2.77)	0.051 (1.85)	0.100 (2.48)	-
α_3	−0.089 (2.81)	−0.070 (2.40)	-	-	-
β_1	0.916 (88.59)	0.818 (5.85)	0.260 (1.39)	0.472 (1.36)	0.472 (3.03)
β_2	-	0.293 (1.41)	0.338 (2.34)	0.155 (0.41)	0.170 (1.37)
β_3	-	−0.181 (1.52)	−0.200 (1.56)	0.227 (1.25)	-
β_4	-	-	0.359 (3.65)	-	-
δ	1.488 (9.15)	1.260 (9.15)	1.280 (8.98)	1.338 (9.96)	2.000
γ_1	−0.207 (2.01)	−0.989 (1.70)	−0.178 (2.00)	−0.427 (2.30)	−0.167 (3.20)
γ_2	−0.480 (0.15)	−0.374 (1.64)	−0.609 (1.76)	0.138 (8.10)	-
γ_3	−0.391 (1.78)	−0.975 (1.89)	-	-	-
$v^\diamond$	1.07 (42.80)	5.05 (14.60)	(4.78) (15.83)	(1.01) (38.85)	(4.67) (14.59)

For each of the five stock indices, table 5.1 reports parameter estimates for the 'best' A-PGARCH model.

* The numbers in the parentheses are absolute t-statistics.

$\diamond$ v are the degrees of freedom of the conditional distribution.

Table 5.1: MA(1)-A-PGARCH ML estimation

	KOSPI	NIKKEI	Hang Seng	SE	Straits-Times
NORMAL					
A-PGARCH	(1,1)	(3,1)	(3,1)	(3,2)	(1,1)
AIC	-27499.6	-29560.0	-25544.1	-25087.9	-20396.6
LOG LIKEL	13755.8	14789.0	12781.1	12554.9	10205.3
DOUBLE EXP					
A-PGARCH	(1,3)	(4,1)	(1,1)	(1,1)	(3,2)
AIC	-28082.8	-30050.0	-26101.5	-25607.3	-20863.0
LOG LIKEL	14052.4	15035.0	13057.8	12810.7	10442.5
GEN ERROR					
A-PGARCH	(1,3)	(4,1)	(1,1)	(3,2)	(4,1)
AIC	-28083.1	-30055.9	-26106.9	**-25607.6**	-20879.7
LOG LIKEL	14053.6	15038.9	13061.4	12815.8	10449.8
STUDENT-t					
A-PGARCH	(3,4)	(3,3)	(4,2)	(2,2)	(2,1)
AIC	**-28083.3**	**-30075.3**	**-26130.6**	-25526.5	**-20959.4**
LOG LIKEL	14057.6	15051.7	13078.3	12774.3	10487.7

For each of the four distributions. Table 5.2 reports the Akaike Information Criterion (AIC) and the maximum log likelihood value of the preferred model.

The bold numbers indicate the minimum value of the AIC.

Table 5.2: Akaike Information Criterion

	KOSPI	NIKKEI	Hang Seng	SE	Straits-Times
NORMAL	1.86	1.47	1.88	1.92	3.62
DOUBLE EXP	1.48	1.20	1.41	1.32	2.02
GEN ERROR	1.49	1.22	1.52	1.34	2.13
STUDENT-t	1.33	1.26	1.28	1.16	1.89
For each of the four distributions, Table 5.3 reports the estimated power terms for the A-PGARCH models fitted to each of the five national stock indices.					

Table 5.3: Estimated power terms

The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis. To accommodate the presence of such leptokurtosis, one should estimate the A-PGARCH models using non-normal distributions. Accordingly, for four out of the five indices the AIC is minimized when the student-t distribution is used, while for the SE index, it chooses the generalized error distribution (see Table 5.2).

A series of tests in which the restricted case is either the Bollerslev or the Taylor/Schwert model were performed (see appendix). When the innovations are t-distributed, the LR tests provide evidence in support of the general power ARCH model, as three of the countries tested generate significant test statistics. In only three out of the twenty cases does the LR test produce insignificant calculated values, indicating an inability to reject the Bollerslev model over the power ARCH model. The three cases are Singapore, when the distribution of the innovations is either student-t or generalized error, and Japan, when the errors are conditionally normal. Likewise, when the conditional distribution of the errors is double exponential, the outcome of the likelihood ratio tests provides a clear rejection of both the Taylor/Schwert and the Bollerslev models against the power ARCH model. The Taylor/Schwert model cannot be rejected against the power ARCH model for the SE index with t-distributed innovations.

Further, the AIC chooses the power ARCH model instead of the Bollerslev and Taylor/Schwert models for three (Japan, Hong Kong and Taiwan) out of the five indices, regardless of the distributional assumptions. By and large, these findings support the conclusion that the power ARCH model is preferred.

5.3.4 Correlation Structure Results

The condition for the existence of the δth moment of the conditional variance (or the 2δth moment of the absolute errors) for the general A-PGARCH(p, q) model is $\gamma_h^0 < 1$, where γ_h^0 is defined in theorem 5.1. The practical significance of the existence condition for a fractional moment is that when it is satisfied then all lower-order moments exist as well. On the other hand, violation of the above condition implies that no higher-order moments exist.

For the A-PGARCH(4.4) model the estimated γ_h^0 ($\widehat{\gamma}_h^0$) is

$$\widehat{\gamma}_h^0 \equiv \sum_{l=1}^{4} \widehat{\zeta}_{l0} \widehat{\pi}_{l0}, \tag{5.33a}$$

where

$$\widehat{\zeta}_{l0} \equiv \frac{\widehat{\lambda}_l^3}{\prod_{n=1}^4 (1-\widehat{\lambda}_l\widehat{\lambda}_n)\prod_{\substack{n=1\\ n\neq l}}^4(\widehat{\lambda}_l-\widehat{\lambda}_n)}, \qquad (5.33b)$$

$$\widehat{\pi}_{l0} \equiv \sum_{n=1}^4 \widehat{a}_n^2(\widehat{k}_{nn}-\widehat{k}_n^2) + 2\sum_{d=1}^4\sum_{n=1}^{4-d}\widehat{a}_n\widehat{a}_{n+d}(\widehat{k}_{n,n+d}-\widehat{k}_n\widehat{k}_{n+d})\widehat{\lambda}_l^d \qquad (5.33c)$$

In (5.33c) the estimated values of k_n and k_{nn}, ($\widehat{k}_n$. and $\widehat{k}_{nn}$) are obtained using formulae (5.22) and (5.26).

The estimated δth moment of the conditional variance and 2δth moment of $|\varepsilon_t|$ are

$$\mathsf{E}(\widehat{h}_t^\delta) = \frac{\left[\mathsf{E}\left(\widehat{h}_t^{\frac{\delta}{2}}\right)\right]^2}{1-\widehat{\gamma}_h^0}, \qquad (5.34a)$$

$$\mathsf{E}(|\widehat{\varepsilon}_t|^{2\delta}) = \widehat{k}_{00}\mathsf{E}(\widehat{h}_t^\delta), \qquad (5.34b)$$

where

$$\mathsf{E}\left(\widehat{h}_t^{\frac{\delta}{2}}\right) = \frac{\widehat{\omega}}{1-\sum_{l=1}^4(\widehat{a}_l\widehat{k}_l+\widehat{\beta}_l)} \qquad (5.34c)$$

In (5.34b) the estimated value of k_{00} ($\widehat{k}_{00}$) is obtained using (5.26) with $\gamma_i = \gamma_j \equiv 0$.

Table 5.5 reports the sum of the estimated $\widetilde{\beta}_l$ ($l = 1, 2, 3, 4$) coefficients[4], $\widehat{\gamma}_h^0$ and the estimated $\frac{\delta}{2}$th and δth moments of the conditional variance for all five 'best' A-PGARCH specifications[5]. Table 5.7 reports $\widetilde{\beta}_1$, $\widehat{\gamma}_h^0$, $\mathsf{E}(h_t^{\frac{\delta}{2}})$ and $\mathsf{E}(h_t^\delta)$

[4]The persistence of a volatility shock for the general A-PGARCH(p, q) process is considered to be the sum of the $\widetilde{\beta}_j$ ($j = 1, \ldots, \widetilde{p}$) coefficients, as defined in (5.21a).

[5]We define 'best' as the specification chosen by the AIC.

	Likelihood Ratio	Critical Value (5% significant level)
KOSPI MA(1)-A-PGARCH(1,3) (Generalized Error)	14.06	9.49
NIKKEI MA(1)-A-PGARCH(3,3) (Student-t)	45.04	12.60
Hang Seng MA(1)-A-PGARCH(4,2) (Student-t)	11.82	11.10
SE MA(1)-A-PGARCH(3,2) (Generalized Error)	10.38	9.49
Straits-Times MA(1)-A-PGARCH(2,1) (Student-t)	2.74	2.71*

Table 5.4 reports the value of the following likelihood ratio (LR) test: $LR = 2 \times [ML_U - ML_R]$, where ML_U and ML_R denote the maximum log likelihood values of the unrestricted and restricted [A-PGARCH(1,1)] models respectively.

* Critical value at 10% significance level.

Table 5.4: Likelihood Ratio tests

	$\sum_{l=1}^{\widetilde{p}} \widetilde{\beta}_l$	$\mathsf{E}(h_t^{\frac{\delta}{2}})$	γ_h^0	$\mathsf{E}(h_t^{\delta})$
KOSPI MA(1)-A-PGARCH(1,3) (Generalized Error)	0.9889	0.0018	0.6689	1E-05
NIKKEI MA(1)-A-PGARCH(3,3) (Student-t)	0.9663	0.0006	0.1256	4E-07
Hang Seng MA(1)-A-PGARCH(4,2) (Student-t)	0.8548	0.0014	0.0737	2E-06
SE MA(1)-A-PGARCH(3,2) (Generalized Error)	0.9844	0.0051	0.6089	7E-05
Straits-Times MA(1)-A-PGARCH(2,1) (Student-t)	0.7736	0.00004	0.8406	1E-08
Table 5.5 reports the estimated values of the $\frac{\delta}{2}$-th and δ-th moments of the conditional variance. The conditions for the existence of the $\frac{\delta}{2}$th and δth moments of the conditional variance are $\sum_{l=1}^{\widetilde{p}} \widetilde{\beta}_l < 1$ and $\gamma_h^0 < 1$ respectively.				

Table 5.5: δ-th moments of the conditional variance (1)

for the five A-PGARCH(1,1) models chosen by SIC. Note that for the Straits-Times index the condition for the existence of the δth moment of the conditional variance is violated.

In order to obtain the estimated theoretical autocorrelations of the power-transformed conditional variance $\left[\widehat{\rho}\left(h_t^{\frac{\delta}{2}}\right)\right]$ and absolute errors $[\widehat{\rho}(|\varepsilon_t|^{\delta})]$, we use the estimated parameters and the formulae in theorems 5.1 and 5.2. For each of the five stock indices, Figure 5.2 plots the estimated theoretical autocorrelations of the 'best' A-PGARCH specification. Specifically, we use the power GARCH process, with conditionally t-distributed errors for Japan and Hong Kong, and innovations that are drawn from the generalized error distribution for the KOSPI[6] and SE indices. Finally, we use Bollerslev's GARCH model with t-distributed innovations for the Straits-Times index.

The estimated power GARCH model for the KOSPI index exhibits the highest persistence (0.99). As a result the estimated autocorrelations of the power-transformed absolute errors start high, at lag three 0.29, and decrease very slowly and the autocorrelation at lag 150 is 0.04. The estimated model for the Hang Seng index has t-distributed innovations and exhibits lower persistence (0.85). Thus, the estimated autocorrelations start considerably lower, at lag two 0.07, and decrease more rapidly. The autocorrelation at lag 120 is 0.002. The estimated models for the KOSPI and SE indices have innovations that are drawn from the generalised error distribution and demonstrate similar persistence (0.99 and 0.98 respectively). However, in the case of the SE index, the power term is much lower (1.34). Note that the autocorrelation at lag two is 0.32 and decreases very slowly. The autocorrelation at lag 120 is 0.08. The estimated models for the NIKKEI and Hang Seng indices have t-distributed innovations and very similar estimated power terms (1.26 and 1.28 respectively). However, the NIKKEI index demonstrates higher persistence (0.97). Note that the autocorrelation at lag three is 0.06 and decreases more slowly. Finally, the estimated model for the Straits-Times index exhibits the lowest persistence. Therefore, the autocorrelations decrease the fastest.

It is useful to uncover the properties of the autocorrelation function of the

[6]In the case of KOSPI we select the model with generalised error distribution althogh the AIC number is slightly less (-28083.1) than that of model with student-t distribution (-28083.3) for illustrative purpose.

power transformed absolute errors, when we investigate whether the Asymmetric power GARCH model can replicate the serial correlation of the power transformed sample data. Figure 5.2 also plots the sample autocorrelations of the δth power of the absolute-valued observations. Only for the SE index the estimated theoretical autocorrelations are close to the sample autocorrelations.

Unlike the Akaike Information Criterion (AIC), the Schwarz Information Criterion (SIC) chooses the A-PGARCH(1,1) model in all the cases. Table 5.6 and Table 5.7 report the estimated parameters a δ-th moment of the conditional variance of the selected A-PGARCH(1,1) models .

Figure 5.3 plots the estimated theoretical autocorrelations (and the corresponding sample equivalents) of the four A-PGARCH(1,1) models. For the Hang Seng index, it can be seen that the fitted power-transformed returns from the A-PGARCH(4,2) model generally have autocorrelations that are substantially lower than the corresponding sample equivalents (see Figure 5.2c). In contrast, the A-PGARCH(1,1) model does a good job of replicating the observed pattern of autocorrelations of the power-transformed returns (see Figure 5.3c).

For the Hang Seng index, table 5.8 reports Quasi-Maximum Likelihood parameter estimates with the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [7] for the A-PGARCH(4,2) model with t-distributed innovations. The autocorrelations of the model estimated by the Quasi-Maximum Likelihood method with the BFGS algorithm replicates closer to the sample autocorrelations than the corresponding of the model by MLE with BHHH (see Figure 5.4)

[7] The Berndt et al. (1974) numerical optimization algorithm (BHHH) is used to obtain the maximum likelihood estimates of the parameters.

	KOSPI	NIKKEI	Hang Seng	SE
MA(1)-A-PGARCH (Con.distribution)	(1.1) (Gen Error)	(1.1) (Student-t)	(1,1) (Student-t)	(1,1) (Gen Error)
b	−5E-05 (0.45)	0.0004 (4.29)	0.0010 (5.66)	1E-11 (0.00)
θ	0.053 (4.02)	0.016 (1.09)	0.067 (4.52)	6E-08 (0.00)
ω	4E-05 (1.26)	7E-05 (1.44)	2E-04 (1.62)	6E-05 (1.52)
a_1	0.147 (10.08)	0.115 (10.95)	0.116 (9.37)	0.103 (10.92)
β_1	0.856 (65.94)	0.891 (101.46)	0.869 (71.39)	0.915 (110.30)
δ	1.532 (9.65)	1.226 (8.65)	1.303 (8.94)	1.274 (10.42)
γ_1	−0.140 (3.09)	−0.469 (7.73)	−0.281 (4.66)	−0.094 (1.94)
$v^{\diamond}$	1.07 (42.80)	4.81 (14.94)	4.74 (16.02)	1.01 (39.76)

For each of the four stock indices, table 5.6 reports parameter estimates for the A-PGARCH(1,1) model.

* The numbers in the parentheses are absolute t-statistics.

$^{\diamond}$ v are the degrees of freedom of the conditional distribution.

Table 5.6: MA(1)-A-PGARCH(1,1) ML estimation

	$\widetilde{\beta}_1$	$\mathsf{E}\left[h_t^{\frac{\delta}{2}}\right]$	γ_h^0	$\mathsf{E}\left[h_t^{\delta}\right]$
KOSPI MA(1)-A-PGARCH(1,1) (GEN ERROR)	0.9759	0.0017	0.7352	1E-05
NIKKEI MA(1)-A-PGARCH(1,1) (Student-t)	0.9632	0.0019	0.3178	5E-06
Hang Seng MA(1)-A-PGARCH(1,1) (Student-t)	0.9430	0.0030	0.2094	1E-05
SE MA(1)-A-PGARCH(1,1) (GEN ERROR)	0.9914	0.0070	0.5689	1E-04
Straits-Times MA(1)-A-PGARCH(1,1) (Student-t)	0.8861	0.0001	4.2841	-

Table 5.7 reports the estimated values of the $\frac{\delta}{2}$-th and δ-th moments of the conditional variance.The conditions for the existence of the $\frac{\delta}{2}$th and δth moments of the conditional variance are $\widetilde{\beta}_1 < 1$ and $\gamma_h^0 < 1$ respectively.

Table 5.7: δ-th moments of the conditional variance (2)

5.3.5 Conclusions

In this chpter we have illustrated how the A-PGARCH model may also be expressed as an ARMA process. Further,we used this ARMA representation to derive results concerning the moments of the general asymmetric power GARCH(p, q) specification. In particular, we obtained the autocorrelation function of the power-transformed absolute errors. Since the A-PGARCH model includes the Bollerslev, Taylor/Schwert and five other models as special cases our theoretical results provide a useful tool which facilitates comparison between all these major classes of GARCH model.

It is worth noting that our results on the moment structure of the general A-PARCH(p, q) model extend the results in He and Teräsvirta (1999b) on the first-order A-PARCH model, and Karanasos (1999) and He and Teräsvirta (1999a) on the GARCH(p, q) model. We should also mention that the methodology used in this chapter can be applied to obtain the moments of more sophisticated asymmetric power GARCH models, e.g. the A-PGARCH-in-mean, the multivariate A-PGARCH and the fractional integrated A-PGARCH models.

Figure 5.1: Daily returns on the five East Asia Stock Indices

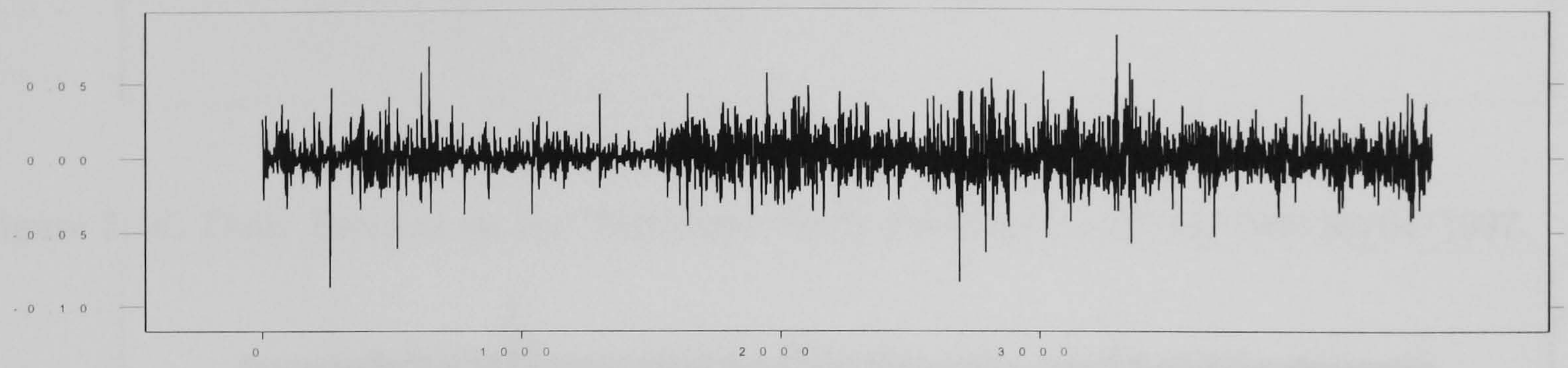


Figure 5.1a. Daily Returns on the Korean Stock Price Index, 07/01/1980-30/04/1997.

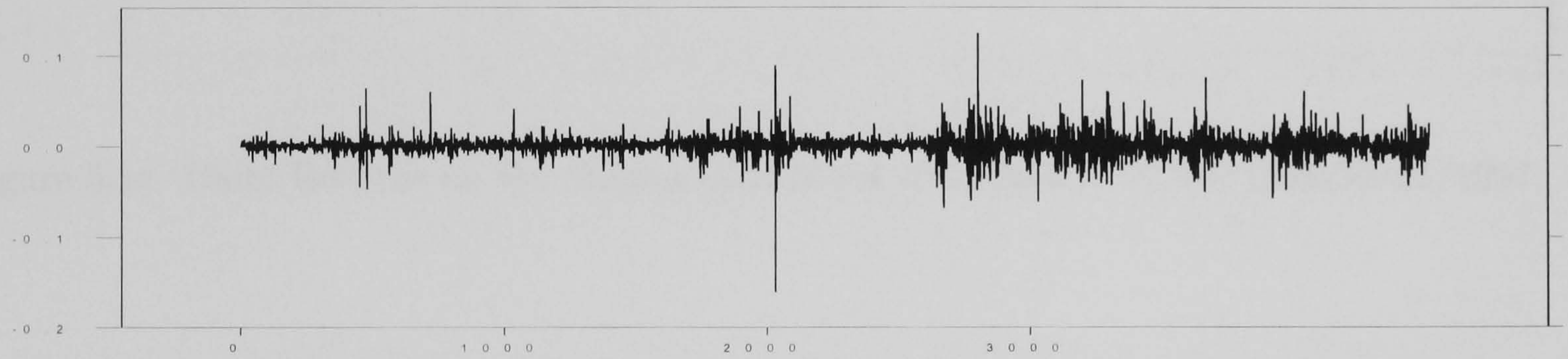


Figure 5.1b. Daily Returns on the Japanese Stock Price Index, 07/01/1980-30/04/1997.

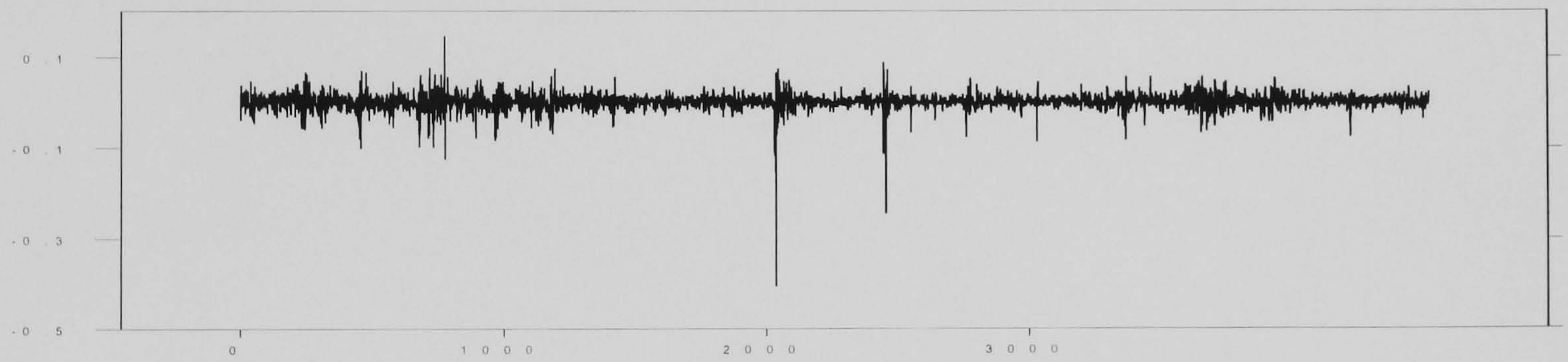


Figure 5.11c. Daily Returns on the Hang Seng Stock Price Index, 07/01/1980-30/04/1997.

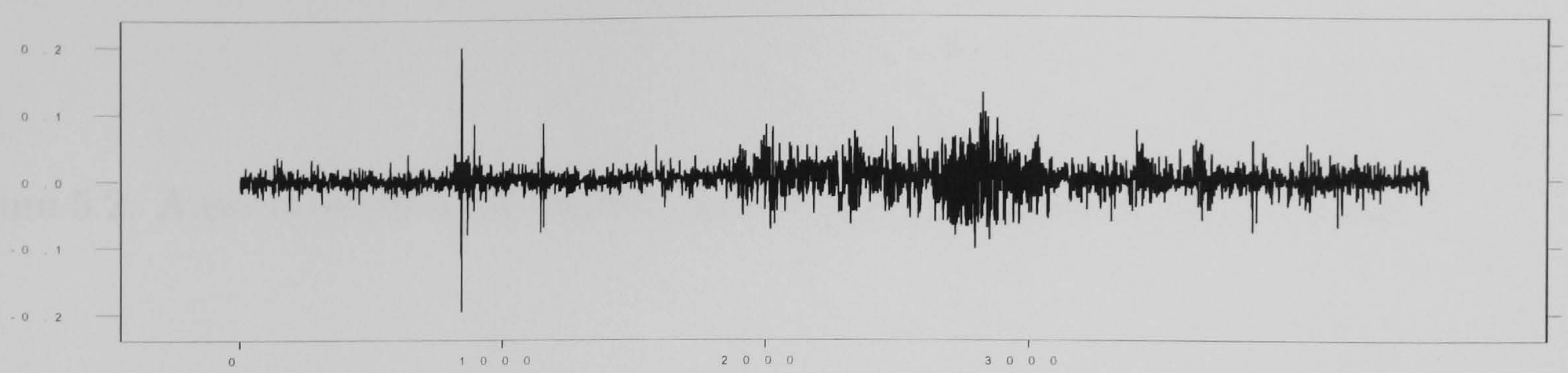


Figure 5.1d. Daily Returns on the Taiwanese Stock Price Index, 07/01/1980-30/04/1997.

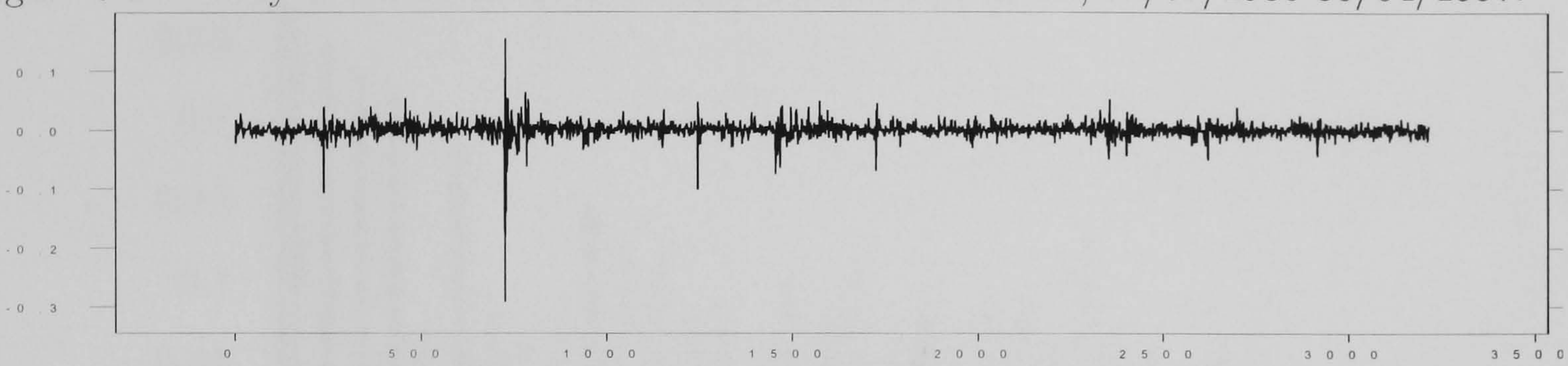


Figure 5.1e. Daily Returns on the Singaporean Stock Price Index, 07/01/1985-30/04/1997.

Figure 5.2: Autocorrelations of the δth power of the observations $\rho(|r_t|^{\delta}, |r_{t-m}|^{\delta})$ (1)

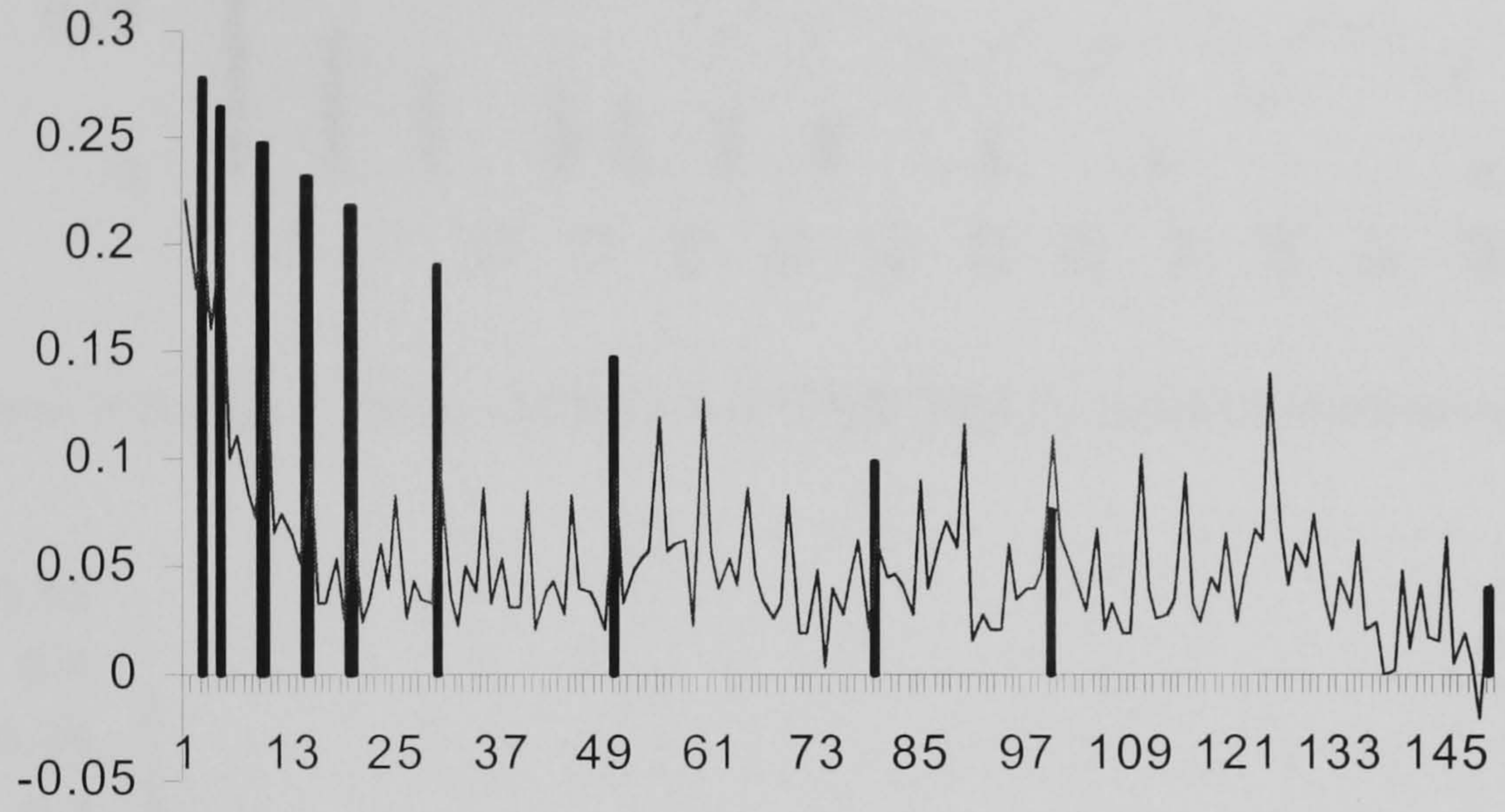


Figure 5.2a: KOSPI. MA(1)-A-PGARCH(1,3) model (Generalized Error).

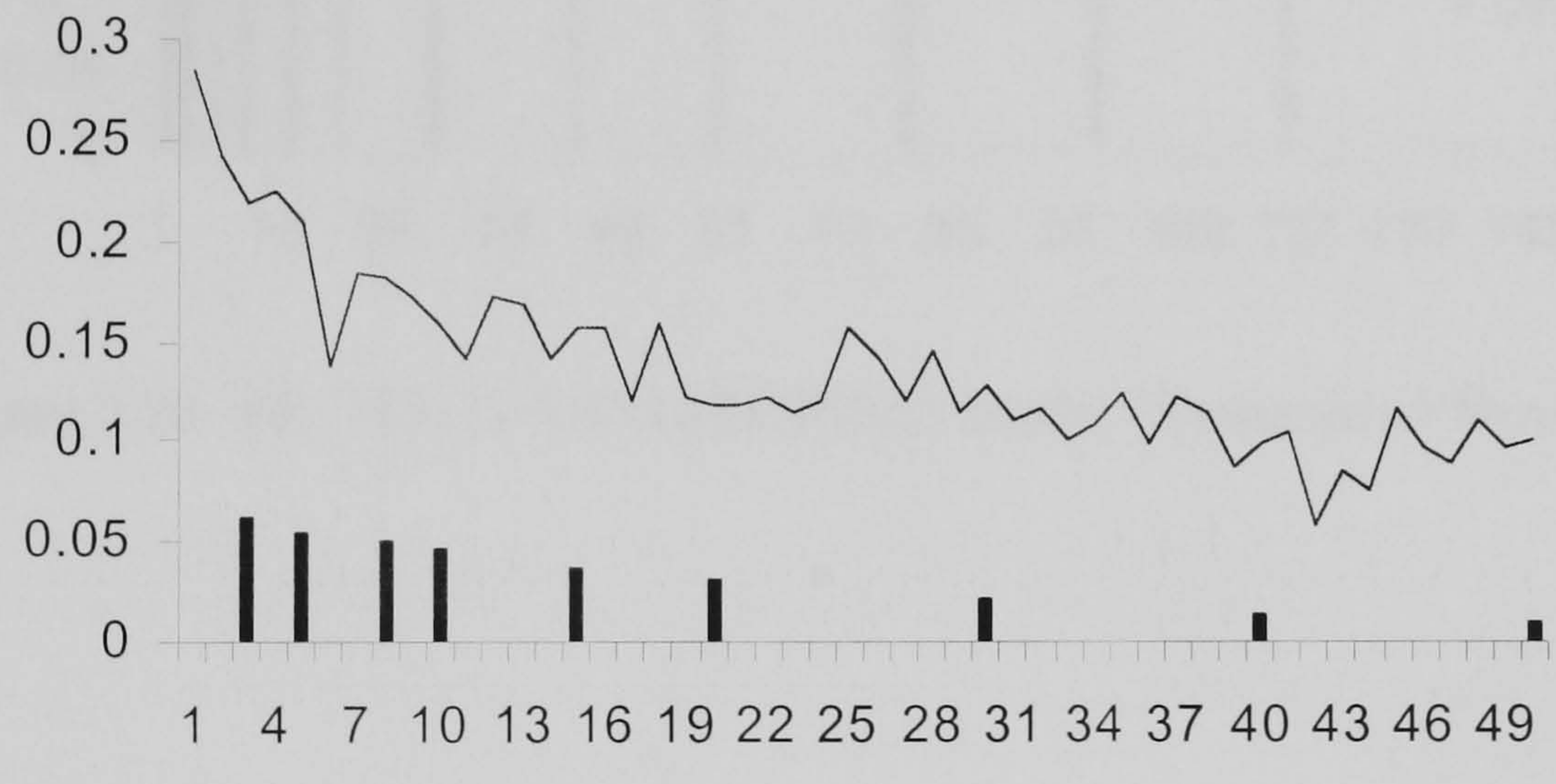


Figure 5.2b: NIKKEI. MA(1)-A-PGARCH(3,3) model (Student-t).

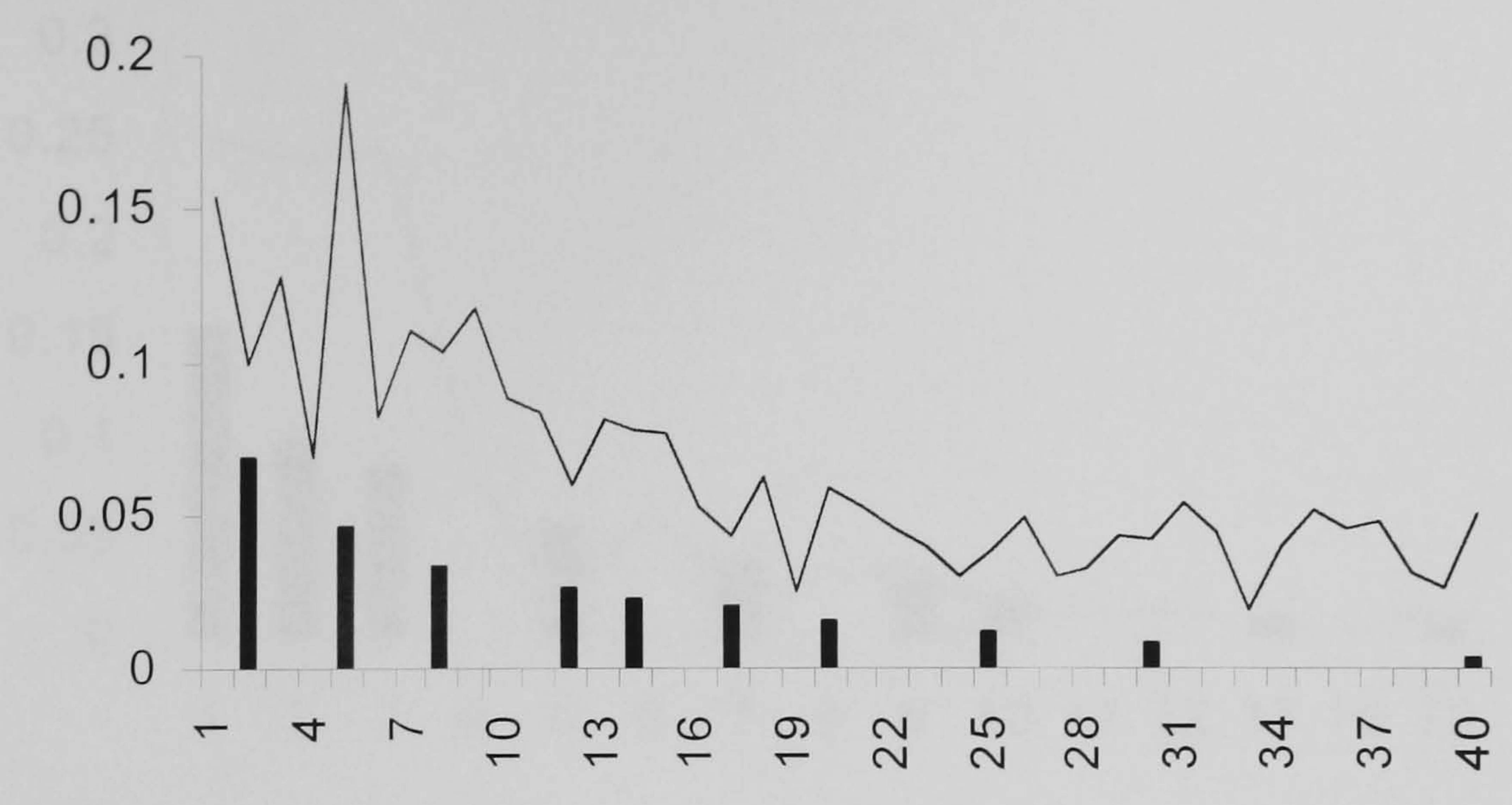


Figure 5.2c: Hang Seng. MA(1)-A-PGARCH(4,2) model (Student-t).

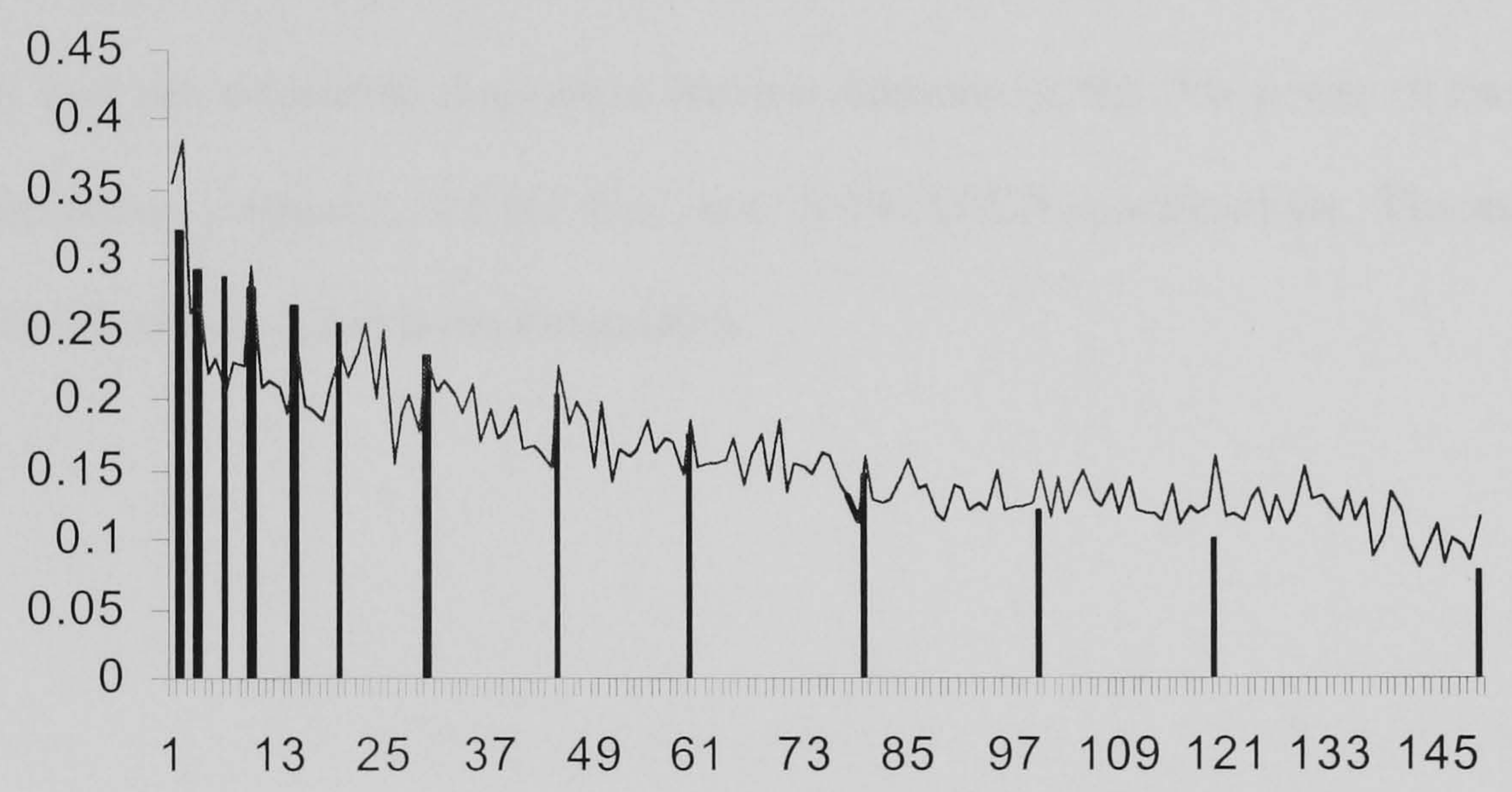


Figure 5.2d: SE. MA(1)-A-PGARCH(3,2) model (Generalized Error).

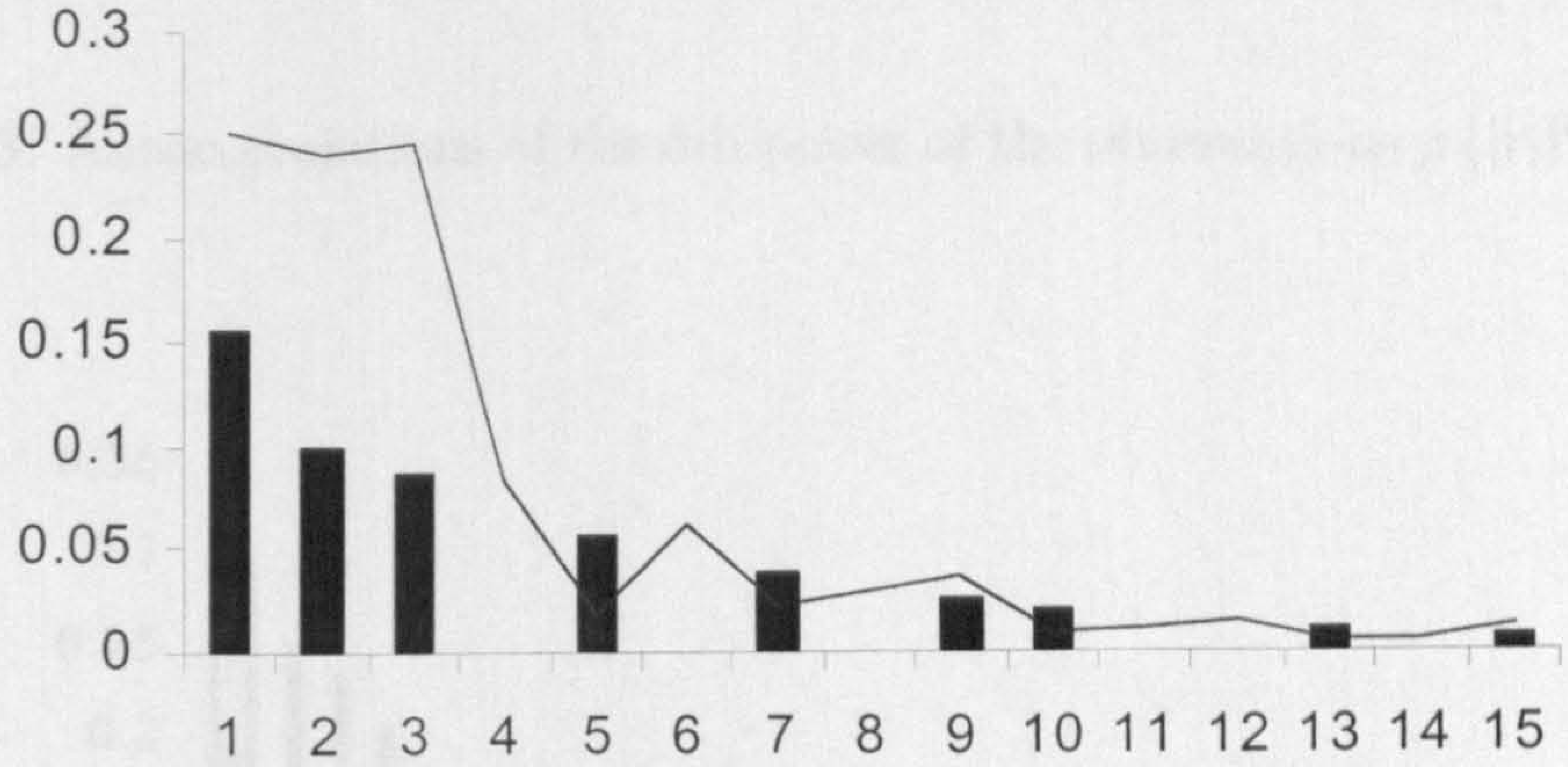


Figure 5.2e: Straits-Times. MA(1)-A-GARCH(2,1) model (Student-t).

Figure 5.2 plots the sample autocorrelations of the δth absolute power of the observations (solid line), and the estimated theoretical autocorrelations of the δth power of the absolute-valued observations (columns), for the five 'best' A-PGARCH specifications. The models were estimated by Maximum Likelihood Estimation.

Figure 5.3: Autocorrelations of the δth power of the observations $\rho\left(|r_t|^\delta, |r_{t-m}|^\delta\right)$ (2)

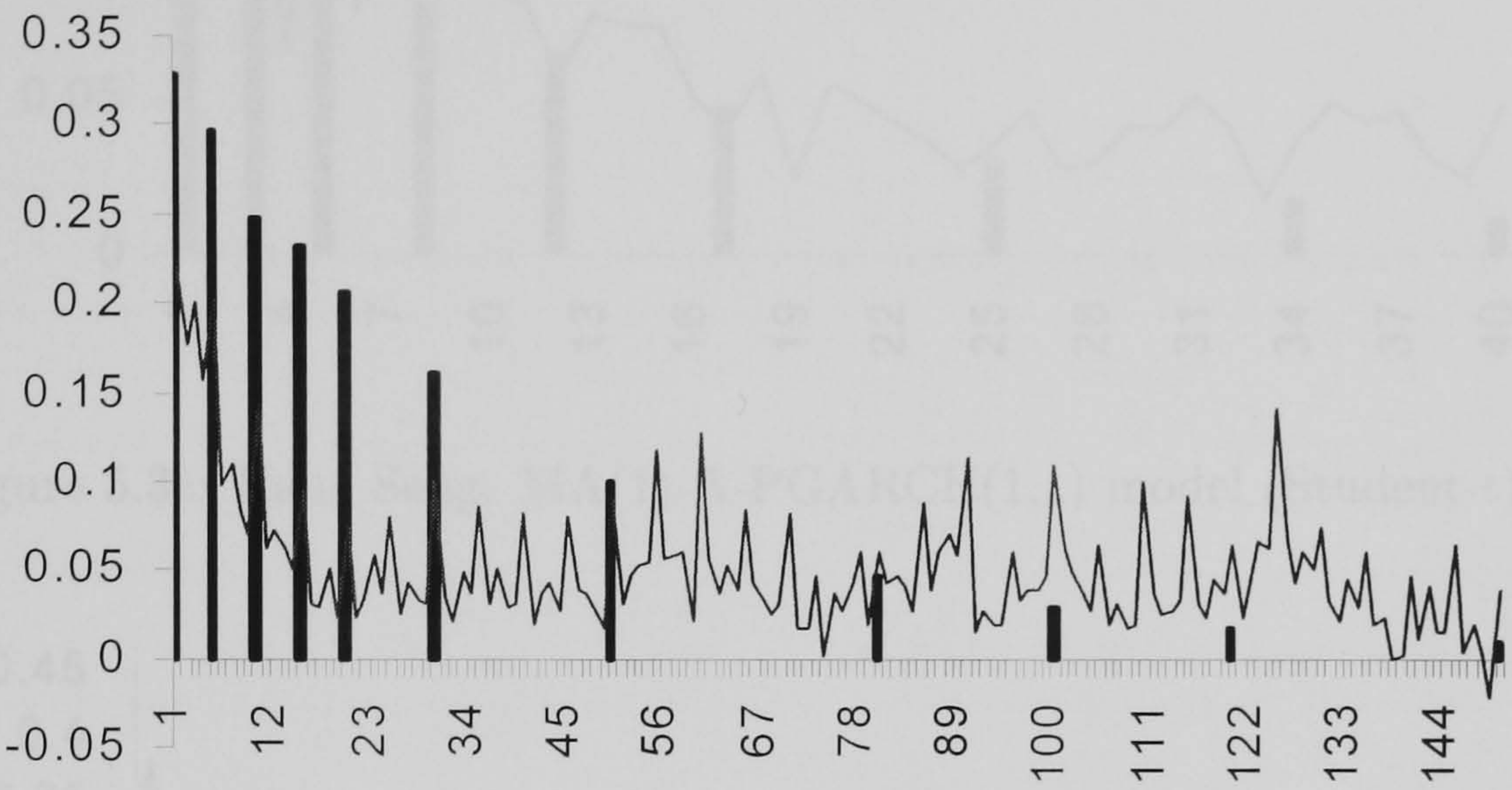


Figure 5.3a: KOSPI. MA(1)-A-PGARCH(1,1) model (Generalized Error).

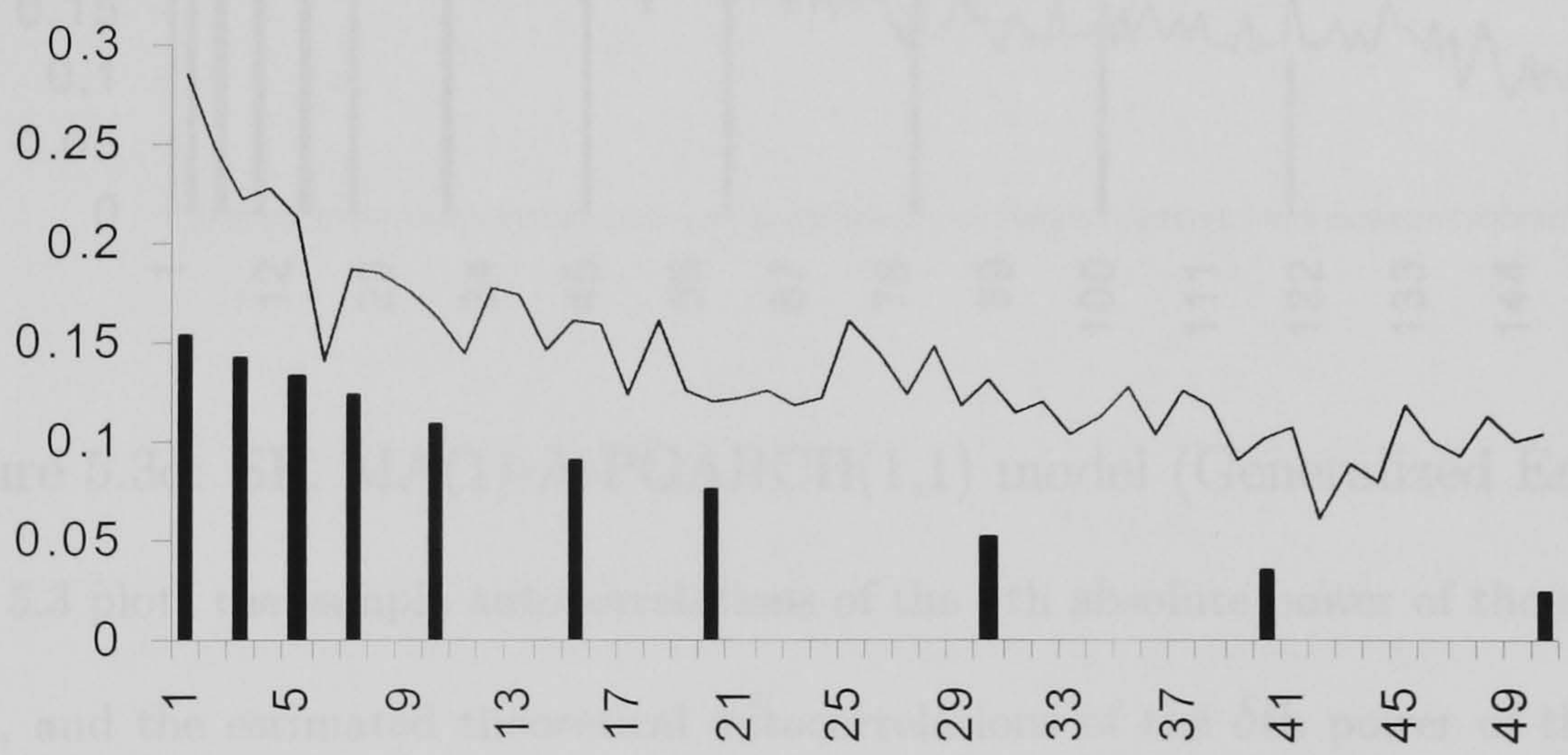


Figure 5.3b: NIKKEI. MA(1)-A-PGARCH(1,1) model (Student-t).

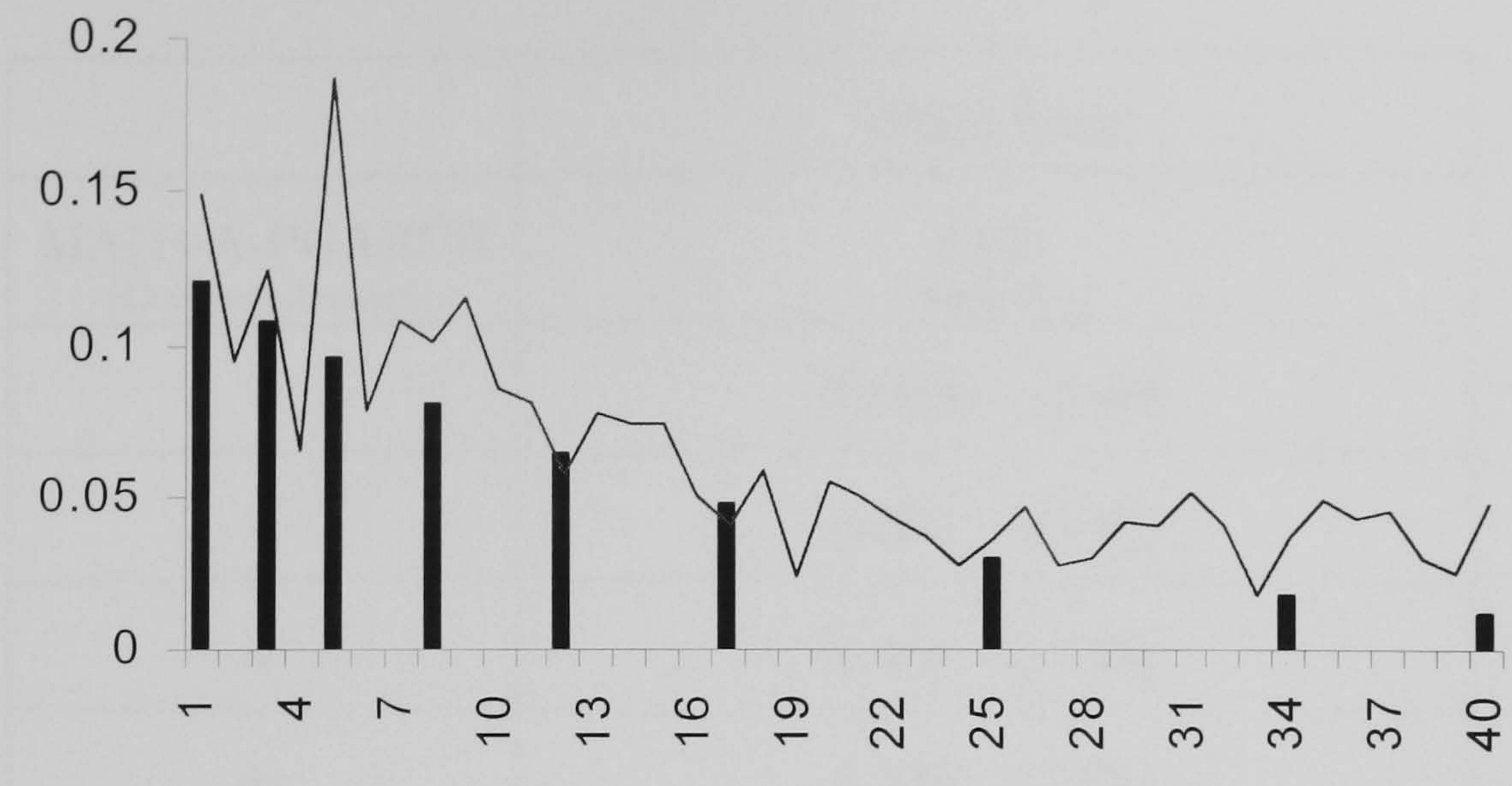


Figure 5.3c: Hang Seng. MA(1)-A-PGARCH(1,1) model (Student-t).

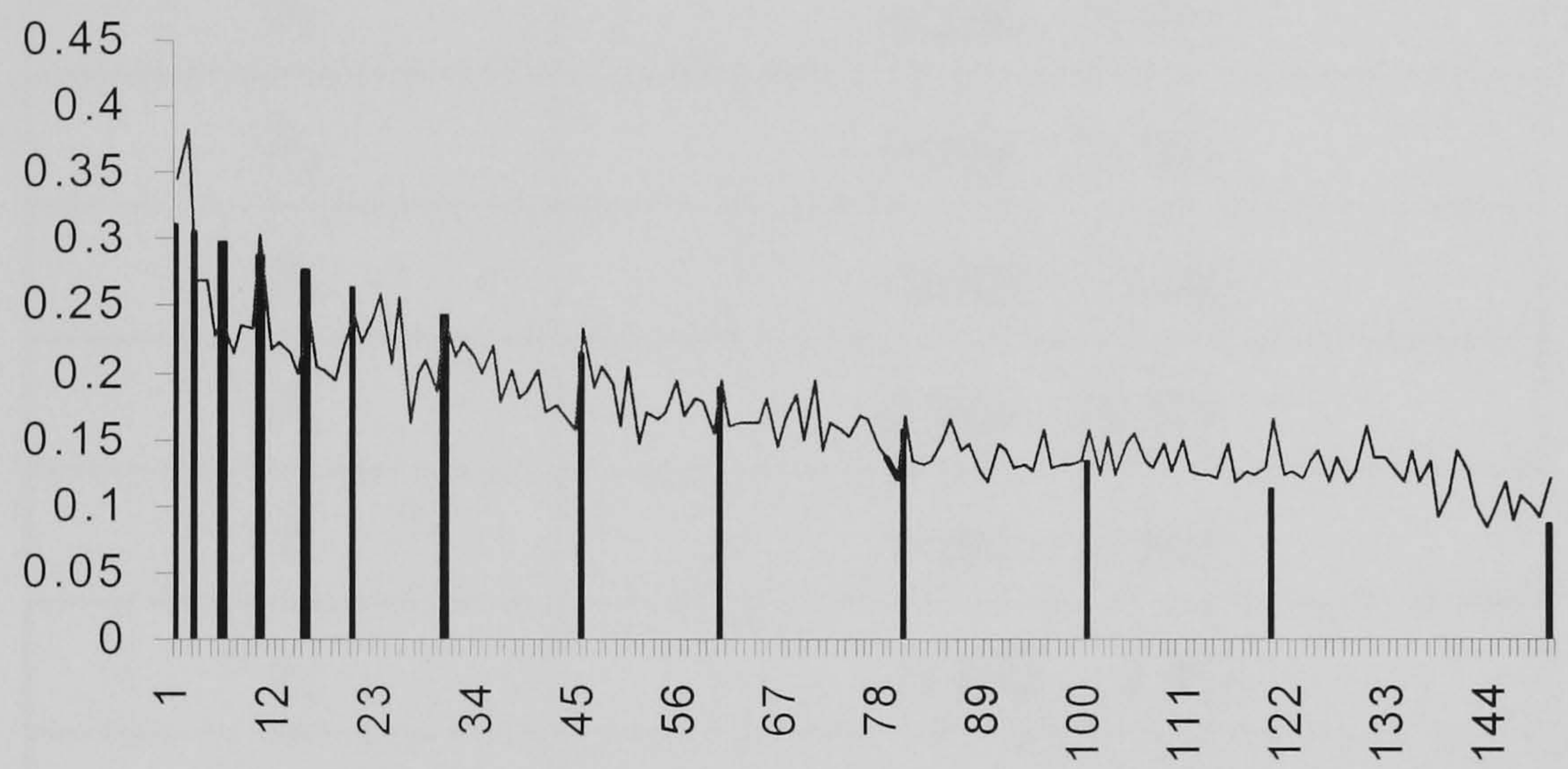


Figure 5.3d: SE. MA(1)-A-PGARCH(1,1) model (Generalized Error).

Figure 5.3 plots the sample autocorrelations of the δth absolute power of the observations (solid line), and the estimated theoretical autocorrelations of the δth power of the absolute-valued observations (columns), for the four A-PGARCH(1,1) specifications chosen by SIC. The models were estimated by Maximum Likelihood Estimation.

	Hang Seng
MA(1)-A-PGARCH (Con.distribution)	(4,2) (Student-t)
b	0.1029 (5.63)
θ	0.071 (4.45)
ω	0.135 (3.33)
a_1	0.146 (5.71)
a_2	0.073 (2.29)
β_1	0.276 (2.01)
β_2	0.355 (2.29)
β_3	−0.225 (1.96)
β_4	0.340 (2.57)
δ	1.392 (8.00)
γ_1	−0.180 (1.95)
γ_2	−0.566 (2.45)
$v^{\diamond}$	4.74 (11.97)

For the Hang Seng index, Table 5.8 reports (Quasi-Maximum Likelihood) parameter estimates for the A-PGARCH(4,2) model.

For this case the stock returns have been multiplied by 100

* The numbers in the parentheses are absolute t-statistics.

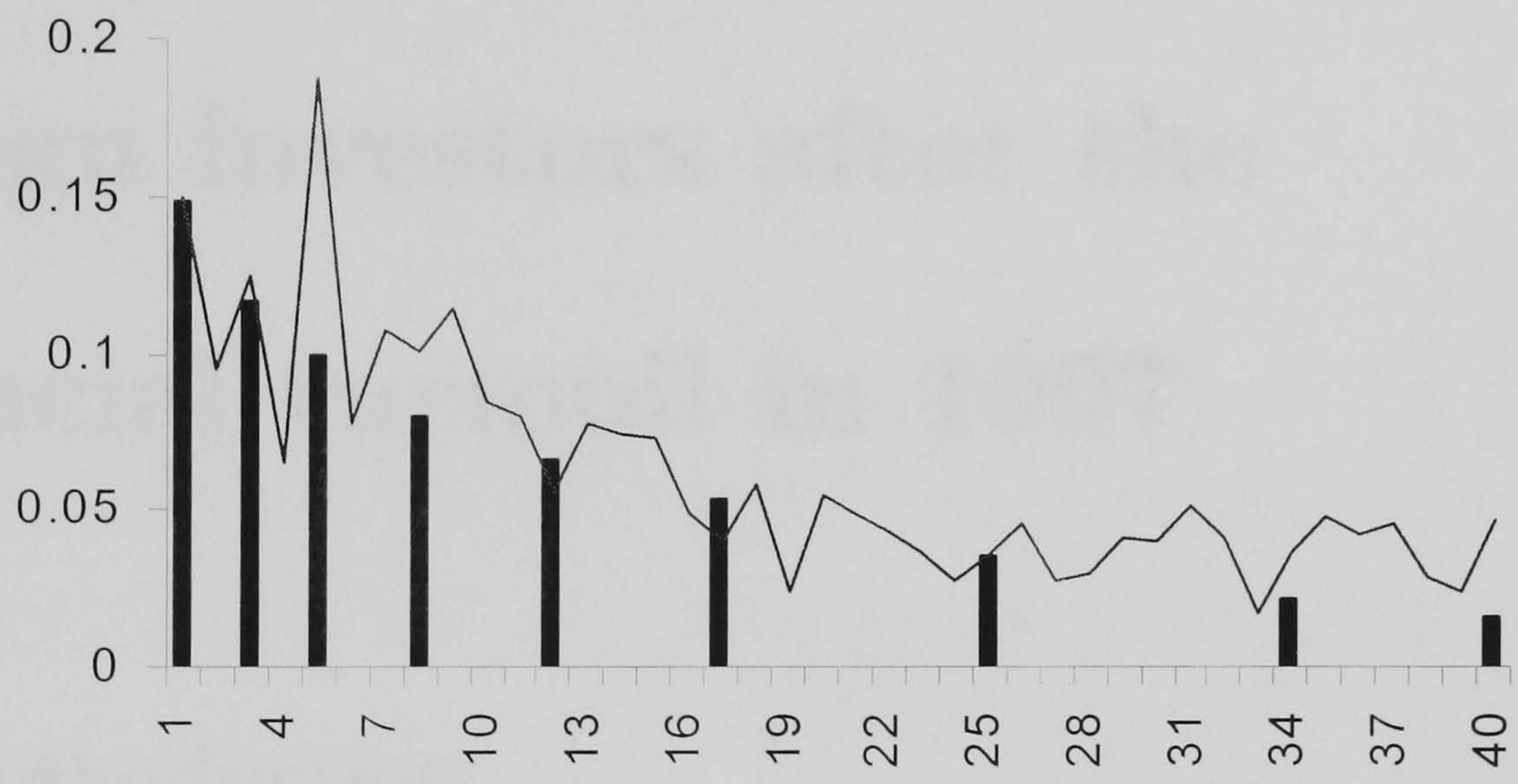
$^{\diamond}$ v are the degrees of freedom of the student-t distribution.

Table 5.8: MA(1)-A-PGARCH QML estimation

	$\sum_{\ell=1}^{\widetilde{p}} \widetilde{\beta}_\ell$	$\mathsf{E}\left[h_t^{\frac{\delta}{2}}\right]$	γ_h^0	$\mathsf{E}\left[h_t^{\delta}\right]$
Hang Seng MA(1)-A-PGARCH(4,2) (Student-t)	0.8950	1.2850	0.2931	2.3356
Table 5.9 reports the estimated values of the $\frac{\delta}{2}$-th and δ-th moments of the conditional variance.The conditions for the existence of the $\frac{\delta}{2}$th and δth moments of the conditional variance are $\sum_{l=1}^{\widetilde{p}} \widetilde{\beta}_l < 1$ and $\gamma_h^0 < 1$ respectively.				

Table 5.9: δ-th moments of the conditional variance (3)

Figure 5.4: Autocorrelations of the δth power of the observations $\rho\left(|r_t|^\delta, |r_{t-m}|^\delta\right)$ (3)



Hang Seng. MA(1)-A-PGARCH(4,2) model (Student-t).

Figure 5.4 plots the sample autocorrelations of the δth absolute power of the observations (solid line), and the estimated theoretical autocorrelations of the δth power of the absolute-valued observations (columns), for the A-PGARCH(4,2) model with t-distributed errors. The model was estimated by Quasi-Maximum Likelihood Estimation.

Chapter 6

The volume-volatility relationship and the opening of the Korean stock market to foreign investors after the financial turmoil in 1997

6.1 Introduction

Some researchers have carried out studies about the effect of capital controls introduced by emerging countries around the financial crisis in 1997 (see, for example, Edison and Reinhart, 2001). However, studies for countries which took further liberalisation after the crisis are difficult to find. This research investigates the

Korean stock market volatility after the crisis and hence contributes to the study of emerging markets' liberalisation after the crisis. Although there is a warning from some researchers that the stock market development and liberalisation in developing countries could dampen the country's long term economic growth[1] (see Singh, 1997; Singh and Weisse, 1998; Stiglitz, 2002), most of the previous empirical studies found that the market opening was favourable to emerging countries' economies (e.g., Bekaert and Harvey, 2000; Henry, 2000; Kim and Singal, 2000).

In developing countries, the empirical research on financial liberalisation suggested that the stock market opening to foreign investors did not increase the stock market volatility. However, these studies are not enough to explore the case of the Korean stock market because they analysed data only for periods before the crisis. In fact the crucial measures of the liberalization were introduced after the crisis under the IMF program. In other words, the previous studies examined the impact of liberalisation on the Korean stock market up to the period before the crisis although the Korean stock market abolished the foreign ownership limit immediately after the crisis and at the same time introduced measures to induce foreign capital. The IMF bailout program resulting from the financial crisis initiated the fundamental reformation of the Korean financial system. One of the major features of the reformation was the financial market opening to foreign investors. The opening included the abolition of the foreign ownership ceiling in the stock market, the free movement of the investment profit, the providing of transparent financial reports and so on.

The crisis in 1997 seems to have brought a different era in Korean stock market

[1] Singh (1997) suggests several reasons including the excess stock market volatility.

history. Four years after the crisis the stock market return series still showed much higher variability than ever before. The Korean economy has recovered rapidly after the financial turbulence, recording 10.7% and 8.8% of GDP growth rate in 1999 and 2000 respectively over against -6.7% in 1998. However, the stock market volatility has not returned to the level that it had before the crisis. This research examines the relationship between the market opening after the crisis and the sustained high degree of volatility of the Korean stock market.

This study makes four contributions. First, it investigates the stock price-volume relation in the Korean market. In particular, we use Granger causality tests to examine the dynamic relation between daily stock price volatility and trading volume. Causality tests can provide useful information on whether knowledge of past trading volume movements improves short-run forecasts of current and future movements in stock price volatility, and vice versa (see Lee and Rui, 2002). Although there have been numerous empirical studies that have examined the relationship between trading volume and stock returns (and volatility), these studies have focused almost exclusively on the well-developed financial markets, usually the US market. There is relative a scarcity in the literature investigating this relationship in fast-growing stock markets in emerging economies. Only Silvapulle and Choi (1999) and Pyun et al. (2000) attempt to examine the relation in the Korean market. However, both studies use data based on time series of stock returns up to 1994.

Second, unlike all previous studies, which used data only up to the period before the crisis, this study investigates the volume-volatility relationship for the period from 1995 to 2001. We examine whether the financial crisis affects the dynamic interaction between volume and volatility by dividing the whole sample period into two sub-periods and conducting causality tests for each sub-period separately. Third, in this research the 'total' trading volume is separated into the domestic investors' and the foreign investors' volume (hereafter 'domestic' and 'foreign' volume respectively) whereas all previous research investigated 'total' volume. By doing this the information used by two different groups of traders can be separated. Finally, in addition to the two most commonly used measures of stock volatility-that is the absolute value of the returns and their squares- we use the conditional volatilities from three alternative GARCH-type models. These models can mimic three stylized empirical facts about stock market volatility: (i) volatilities are highly persistent, (ii) there are different volatility components that will dominate different time periods, and (iii) volatility responds to price movements asymmetrically.

This study provides strong empirical support for the argument made, among others, by Brook (1998) that daily stock price volatility and trading volume are inter-temporally related. Hence, instead of focusing only on the univariate dynamics of stock price volatility one should study the joint dynamics of stock price volatility and trading volume. Moreover, as Bessembinder and Seguin (1993) and Lee and Rui (2002) point out, an important distinction in investigating the trading volume and volatility relation is to distinguish between expected and unexpected trading volume. The current study shows that it is also important to distinguish between domestic and foreign investors' trading volume.

The following observations, among other things, are noted about the volume-volatility causal relationship. First, for the entire period there is a strong positive bidirectional feedback between volume and volatility. In most cases this causal relationship is robust to the measures of volume and volatility used. Second, before the crisis there is no causal relation between 'foreign' volume and volatility whereas after the crisis a negative feedback relation begins to exist. In other words, 'foreign' volume tends to have more information about volatility in recent years, which suggests an increased importance of 'foreign' volume as an information variable. It turns out that using any of the five alternative measures of volatility results in exactly the same causal relation between 'foreign' volume and volatility. Third, the effect of volatility on 'domestic' volume is positive in the pre-crisis period but turns to negative after the crisis. On the other hand, 'domestic' volume has a mixed effect on conditional volatility before the crisis but the effect disappears in the post-crisis period. In both sub-periods absolute/squared returns are independent of changes in 'domestic' volume.

The remainder of this chapter is organized as follows. Section 6.2 presents a brief description of the Korean market liberalisation, and the next Section provides a summary of existing theories and empirical evidence. Section 6.4 outlines the data which are used in the empirical tests of this paper. Section 6.5 lays out our econometric model and reports and discusses our results. Section 6.6 contains summary remarks and conclusions.

6.2 The Korean market

The Korean market is classified as one of the emerging markets as it has experienced significant economic growth and development in the past years. The economic growth and development of the Korean market has been accompanied by financial liberalisation i.e. a series of important legislative and structural changes (Silvapulle and Choi, 1999). This section provides a brief description of the organizational and institutional factors of the Korean market.

6.2.1 Liberalisation date

The decision of the liberalisation date is important for understanding the effect of financial liberalisation and capital inflow on an emerging stock market, because researchers compare the two periods before and after the liberalisation date to study the effect. Various liberalisation dates are suggested and examined , including the date of government announcement of the stock market opening to foreign investors. Bekaert and Harvey (2000) and Kim and Singal (2000) used the same liberalisation date for Korea, that is January 1992. Authors generally agree that foreign capital flows do not increase emerging stock market volatility despite their differences in liberalisation dates and sample periods. Table 6.1 reports the sample period and the results of the previous research.

According to the above studies Asian emerging markets were liberalised mostly in the late 1980s and in the early 1990s. However, when emerging stock markets were liberalised the levels of foreign ownership were significantly different from country to country. Foreign ownership of domestic firms may not be a sufficient measure of stock market openness. Emerging countries have various barriers to hinder international portfolio investment. However, the lifting of the foreign investment ceiling is a necessary condition for the participation of foreign investors and therefore the foreign ownership limit is the crucial indicator of stock market

Authors	Number of countries[a]	Sample data[b]	Volatility after liberalisation[c]
Bekaert and Harvey (2000)	20	Jan. 1976 - Sep. 1996	decreased
Kim and Singal (2000)	18	Jan. 1976 - Dec. 1995	unchanged
Spyrou and Kassimatis (1999)	8	Jan. 1988 - Feb. 1998[d]	decreased or unchanged
Grabel (1995)	6	1956-1990	increased

Note: a. All these four studies include Korea.

b. The sample period depends on the country. The period represents here the earliest date and the last date of whole sample data.

c. There are exceptional cases among sample countries but these are the general conclusion of the research.

d. The financial crisis, which covers the period Sep. 1997- Feb 1998, is excluded for Korea and Pakistan.

Table 6.1: Impact of liberalisation on emerging stock market volatility

openness.

Noticeably Korea had a strict limitation of foreign investment in its stock markets at the 10% level. Korea pledged to increase these ceilings step by step in the future. However, the speed of this process was remarkably slow. It took more than five years that the foreign ownership limit of the Korean stock market reached only 23% in May 1997 (see Table 6.2). The aforementioned studies seem not to take into account fully the slow phase of the Korean liberalisation process when they simply investigated a period of three or five years after the liberalisation

Date	3 Jan 92	1 Dec 94	1 Jul 95	1 Apr 96	1 Oct 96
Collective ceiling(%)	10	12	15	18	20
Individual investor(%)	3	3	3	4	5
Date	2 May 97	3 Nov 97	11 Nov 97	30 Dec 97	25 May 98
Collective ceiling(%)	23	26	50	55	100
Individual investor(%)	6	7	50	50	100
Source: Korean Financial Supervisory Services					

Table 6.2: Ceiling of Foreign ownership in the Korean Stock Exchange

date. Moreover, they missed the most important period of liberalisation of Korea after the crisis. For example, the Korean stock market opened wide to foreign investors without any ownership ceiling in May 1998, eight months after the crisis (see Table 6.2).

This radical financial reform was implemented owing to the IMF, which has had a great role in Korean financial liberalisation after the crisis in 1997. The reform program of the Korean government under IMF supervision has managed to recover market confidence. The response of the Korean government to the IMF program had to be urgent. It abandoned step by step liberalisation and opened the stock market immediately. The Korean authority altered the foreign ownership ceiling three times from 26% to 55% in the two months of November and December 1997 and finally removed the limit in May 1998. It only took 6 months to change the ceiling from 26% to 100%, whereas it had taken more than five and half years to move from 0% to 26%.

Because of the financial crisis all the stock markets in East Asia became highly volatile so it is difficult to parse what is due to the financial crisis and what is owing to the ongoing liberalisation if the crisis period is included in the sample. This is a possible reason why the previous studies limited their sample periods to before the crisis. The current research may allow us to throw light on this latter problem, which is indeed of major concern. Studying whether the financial liberalisation caused the financial crisis is not the purpose of this paper.[2] The aim of this research is to study the effect of liberalisation on the stock market volatility. Hence, even if it is true that the financial liberalisation did not lead to the crisis this does not mean that the financial liberalisation does not make the financial market more volatile at all because in the middle of and after the crisis the financial liberalisation continued. Especially in Korea the liberalisation was accelerated and reached close to the goal of liberalisation in the middle of and after the crisis. Therefore, the extension of the period after the crisis seems to be natural to evaluate the effect of the financial liberalisation. This seems more appropriate when we consider that the IMF program not only brought about the abolition of the foreign investment limit but more profoundly changed the financial system itself.

[2] Unlike the aforementioned empirical research, Stiglitz (2002, p. 99) argues that capital account liberalisation was 'the single most important factor' leading to the crisis.

6.2.2 The informational change of the stock market after the crisis

One of the main features of the economic transformation after the crisis is that the Korean economy has created a climate favorable to foreign investors' activity. This was vital in attracting foreign capital. The IMF led the Korean government to revise laws and regulations for further free capital inflow. The foreign investors' shareholding in the Korean Stock Exchange had increased to 30.1% of total market capitalisation by the end of 2000 from 14.6% at the end of 1997. In manufacturing industries foreign controlling companies' sales grew up to 18.5% of total revenue in 1999 from 5.5% in 1996. Also in the financial industry foreign capital advanced. At the end of 1999 the market share of banks in which foreign investors are the first majority shareholders amounted to 41.7% in terms of deposits and lendings.

The securities companies of which the majority shareholders are foreigners increased their market share to 20.9% in 2000 from 3.9% in 1997. During the same period the market share of foreign insurance companies reached 9.6% from 1.3%. The number of listed companies that give stock options to their employees also increased to 105 in 2000 from only 2 in 1997 (Kim ed., 2001).

Table 6.3 reports the daily trading volumes of domestic investors and foreign investors in the Korean stock market. The fourth column shows the increased proportion of foreign investors' trading since 1995. Although the proportion of foreigners trading was under 11% in 2001 their shareholding was already over 30% at the end of 2000.

The obvious increase of foreign shares in the Korean companies has been

	Foreign investor (Trillion won)	Domestic investor (Trillion won)	$\frac{\text{Foreign}}{\text{Foreign+Domestic}} \times 100$
1995	23.7	464.4	4.86 %
1996	29.3	457.5	6.02 %
1997	37.2	518.6	6.69 %
1998	49.3	611.1	7.47 %
1999	179.5	3302.0	5.16 %
2000	238.5	2363.7	9.16 %
Jan - Sep 2001	198.9	1628.9	10.89 %

Note: Table 6.3 presents average daily trading volumes of the foreign and domestic investors from 1995 to Sep. 2001.

Source: Korean Stock Exchange

Table 6.3: Average daily trading volumes in the Korean Stock Market

supported by government regulations and the practice of firms. Put differently, the significant increase in foreign investors' stock trading volume can also be explained by the investment information changes in the Korean stock market. Even after foreign investment was allowed in 1992, external investors may have been uncomfortable trading because they did not have proper investment 'information'. Providing a transparent financial status can induce foreign capital inflow and activate foreign investors' trading. To assess the effect of stock market liberalisation the change in the informational environment should be considered. Therefore, the effect of Korean stock market liberalisation would be more clear if the period after the crisis is investigated. Afterwards in this research the word 'after the crisis' is used to focus on the aspect of the liberalisation of the Korean stock market after the crisis in 1997.

6.3 Prior research

6.3.1 The stock volatility-trading volume relation

This section reviews previous research on the relation between stock price changes and trading volume. Karpoff (1987) gives four reasons why the price-volume relation is important: (i) it provides insight into the structure of financial markets, (ii) it is important for event studies that use a combination of price and volume data from which to draw inferences, (iii) it is critical to the debate over the empirical distribution of speculative prices and, (iv) it has significant implications for research into futures markets.

There are several explanations for the presence of a causal relation between stock price volatility and trading volume. According to various mixtures of distributions models there is a positive relation between current stock return variance and trading volume. For example, Epps and Epps (1976) present a model which suggests a positive causal relation running from trading volume to absolute stock returns. The sequential information arrival models also suggest a positive causal relation between stock prices and trading volume in either direction (Copeland, 1976). Due to the sequential information flow, lagged absolute stock returns could have predictive power for current trading volume and vice versa. These theoretical models imply bidirectional causality between volume and volatility and hence provide motivation for empirical research into this relationship (see Hiemstra and Jones, 1994; Brooks, 1998, and the references therein).

Karpoff (1987) proposes a model which links trading volume, returns and volatility and predicts a positive but asymmetric relationship between trading volume and the absolute value of returns. Other researchers have developed models that are based on information economics and link information arrival with trading, price changes and price volatility. One such model suggests that trading volume and the variance of price changes move together, while another one suggests that there is no relationship between stock price volatility and trading volume (see Brailsford, 1996, and the references therein). Harris and Raviv (1993) assume that traders receive common information but differ in the way in which they interpret it. Their model predicts that absolute price changes and trading volume are positively correlated. Wang (1994) develops an equilibrium model of stock trading in which investors are heterogeneous in their information and the positive correlation between trading volume and absolute price changes increases with information uncertainty.

Brock (1993) develops a heterogeneous agent trading model, which implies a nonlinear stock price-volume relationship. Campbell et al. (1993) present a model of noninformational trading, which implies that the serial correlation in stock returns is a nonlinear function of the trading volume. Brailsford (1996) points out that a positive correlation between the trading volume, returns and variance may be inferred from the fact that the trading volume and both the level and variance of returns exhibit similar U-shaped patterns during the trading day.

6.3.2 A brief survey of the empirical literature

This section summarizes several empirical studies that investigate the relationship between stock price and trading volume or between volatility and volume. In a survey paper Karpoff (1987) finds that 18 of the 19 empirical investigations that examine the relationship between absolute price change and volume report a positive correlation. Harris (1987) documents a positive correlation between changes in volume and changes in squared returns for individual NYSE stocks. Smirlock and Starks (1988) provide strong evidence for a positive lagged relation between volume and absolute price changes. Gallant et al. (1992) using nonlinear impulse response functions find evidence of a strong nonlinear impact from lagged S&P 500 stock returns to current and future NYSE trading volume but only weak evidence of a nonlinear impact from lagged trading volume to current and future stock returns. Campbell et al. (1993), using regression models, provide statistically significant evidence of nonlinear interactions between stock returns and trading volume in the US market. Subsequently, Hiemstra and Jones (1994) indicate the presence of bidirectional nonlinear Granger causality between daily Dow Jones stock returns and changes in the NYSE trading volume. After controlling for volatility effects, their modified Baek and Brock (1992) test continues to provide evidence of significant causality running from trading volume to stock returns. Bhagat and Bhatia (1994) test for causality in both the mean and the variance and suggest that price changes lead volume. Brooks (1998) employing both linear and non linear Granger-causality tests, provides extensive evidence of bidirectional feedback between volume and volatility. He used the square of the day's return as a measure of the Dow Jones stock returns volatility. Lee and Rui (2002) show that there exists a positive feedback relationship between trading volume and return volatility in the three largest stock markets.

At the same time a parallel literature has developed which employs GARCH models to describe stock return volatility. Lamoureux and Lastrapes (1990) find that the inclusion of contemporaneous trading volume in the conditional variance equation eliminates the persistence in the volatility. However, as noted by Lamoureux and Lastrapes (1990), if trading volume is not strictly exogenous, then there is possibly simultaneity bias. One potential solution to this problem is to use lagged measures of volume, which will be predetermined and therefore not subject to the simultaneity problem. Lamoureux and Lastrapes (1990) find that lagged volume is insignificant. Brooks (1998) uses various GARCH-type models to forecast volatility out-of-sample, and considers their augmentation to allow for lagged values of market volume as predictors of future volatility. Chen et al. (2001) find that the persistence in EGARCH volatility remains even after incorporating contemporaneous and lagged volume effects.

Although there has been extensive research into the empirical and theoretical aspects of the stock price volatility-volume relation. most of this research has focused on the well-developed financial markets, usually the US market. However. some studies have examined the volatility-volume relation in markets outside of the United States. In particular, Tse (1991) examines the relations between volume and the absolute value of returns for different indices in the Tokyo Stock exchange and he finds mixed results. Brailsford (1996) uses both the squared returns and the absolute value of the returns as measures of volatility. He provides support for a positive relationship between trading volume and volatility for the Australian stock market. Saatcioglou and Starks (1998) employ Latin America stock data and document a positive relation between volume and both the price changes and their magnitude. Chen et al. (2001) find a positive correlation between trading volume and the absolute value of the stock price change for nine major stock markets.

Two recent studies have examined the price-volume relation in the Korean stock market. Silvapulle and Choi (1999) examine the dynamic relationship between daily aggregate Korean stock returns and trading volume. After controlling for volatility persistence in both series and filtering for linear dependence they find evidence of nonlinear bidirectional causality between stock returns and volume series. Pyun et al. (2000) examine the relationship between information flows and return volatility for individual companies actively traded in the Korean stock exchange. They find that adding the current trading volume into the conditional variance equation reduces the volatility persistence of returns and conclude that the Mixture of Distribution hypothesis is relevant in the Korean stock market. However, they also find that lagged volume has no effect on the conditional volatility of individual stocks (similar results have been reported by Brailsford, 1996 for the Australian stock market).

6.4 Measurement issues

6.4.1 Data and sample periods

The data set used in this study comprises 1844 daily trading volume and closing prices of the Korea Composite Stock Price Index (KOSPI), running from 3 January 1995 to 30 September 2001.

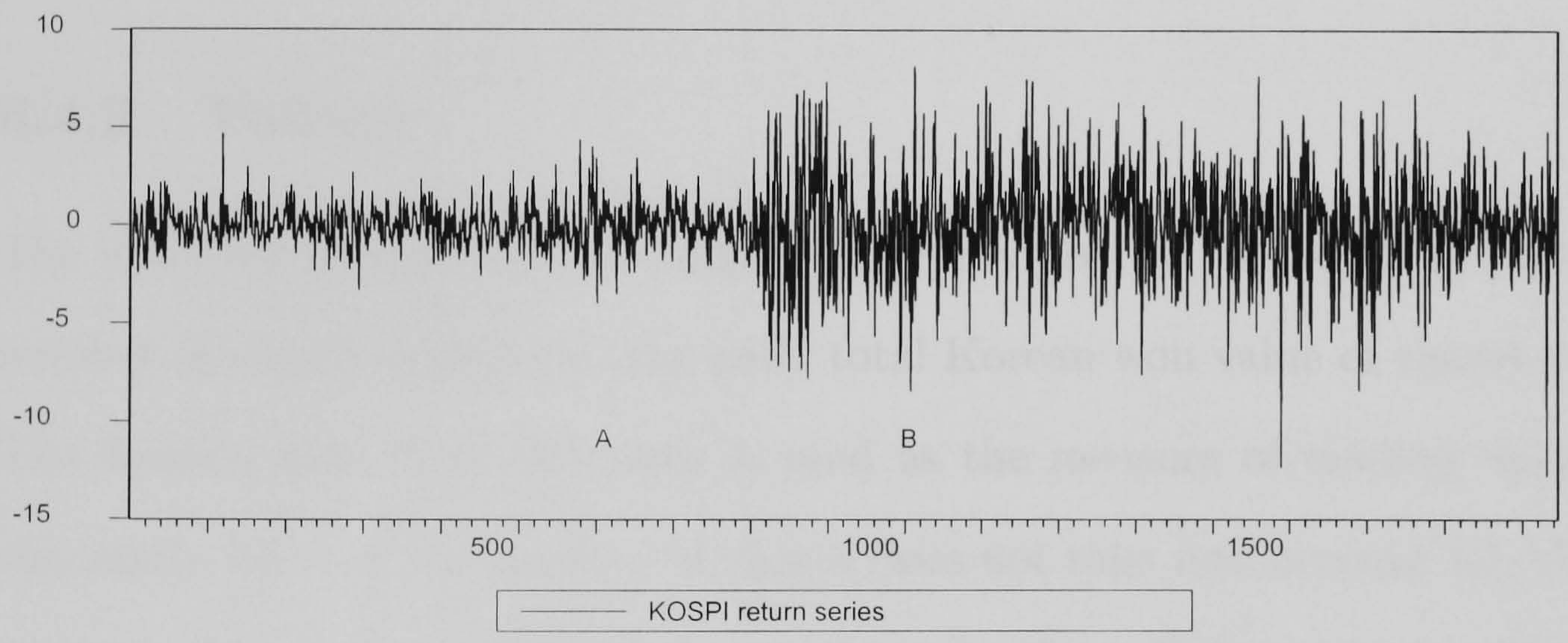


Figure 6.1: The daily KOSPI return series from Jan. 1995 to Sep. 2001.

Figure 6.1 plots the daily Korean Composite Stock Price Index (KOSPI) return series from January 1995 to September 2001. Sample A denotes the period from January 1995 to September 1997 (before the crisis). Sample B covers from October 1997 to September 2001 (after the crisis).

The data were obtained from the Korean Stock Exchange (KSE). The KOSPI is a market value weighted index for all listed common stocks in the KSE since 1980. Daily stock returns are measured by the daily difference of the log KOSPI [$r_t = \ln(\frac{\text{KOSPI}_t}{\text{KOSPI}_{t-1}}) \times 100$].

The whole sample period is divided into two sub-periods to investigate informational change after the financial crisis in 1997. The first sample period covers the period between January 1995-which is the first month from which categorical volume data are available-and September 1997 with 804 observations (afterwards sample A). The second sub-sample covers the period from October 1997-from which the KOSPI returns show dramatic change due to the crisis-to September 2001 with 1040 observations (afterwards sample B) (see Figure 6.1).

6.4.2 Volume

The available measures of trading volume provided by the KSE are the daily number of shares traded and the daily total Korean won value of shares traded. The Korean won value of shares is used as the measure of trading volume in this study because the number of shares does not take into account the relative market value of the individual shares. Among others, Gallant et al. (1992) and Silvapulle and Choi (1999) use the number of shares as a measure of trading volume. Brailsford (1996) employs three different measures of trading volume (number of transactions, number of shares traded and value of shares traded) and argues that the number of shares traded is the least preferred measure of trading volume and should be avoided in future research. Other researchers use the turnover (the ratio of the number of shares traded to the number of shares outstanding) as a measure of trading volume (see Campbell et al., 1993; Brooks, 1998).

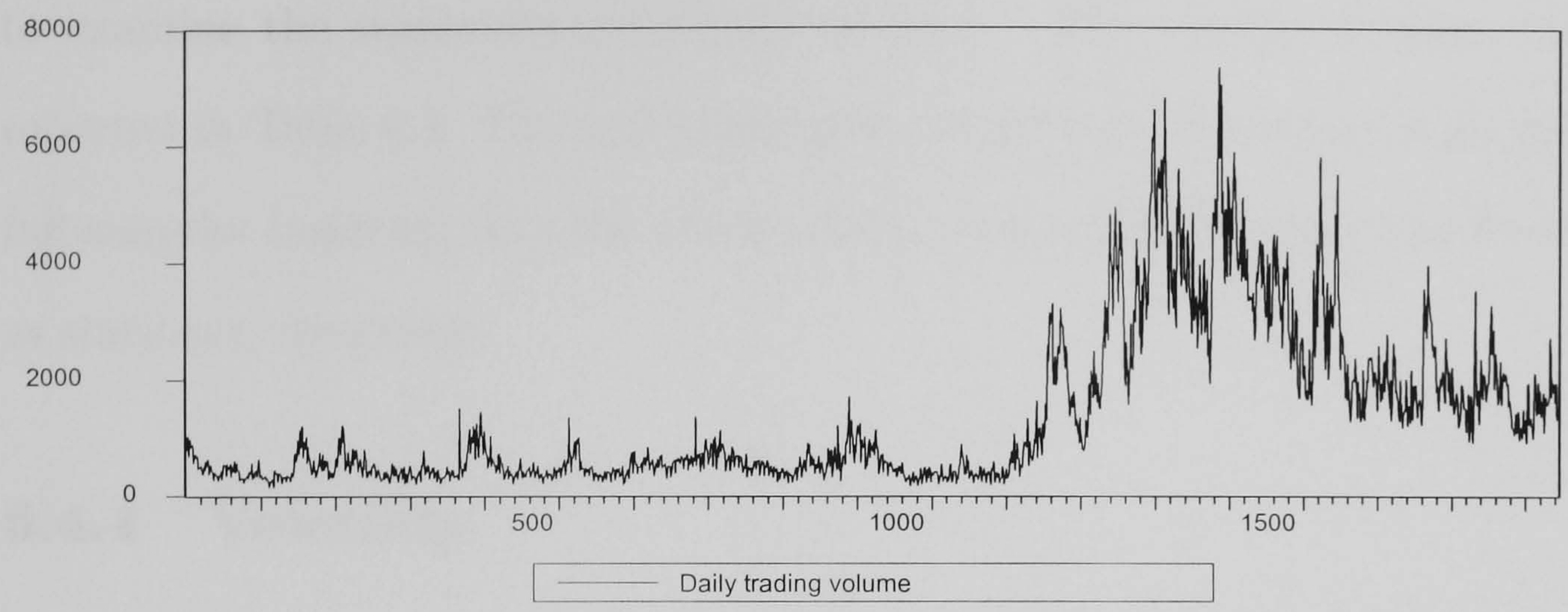


Figure 6.2: Korean stock market trading volume

Figure 6.2 plots the daily total Korean won value of traded shares of the Korean stock market from Jan 1995 to Sep 2001. The unit of the vertical axis is trillion Korean Won. The shaded area covers the period from Jan 1995 to Sep 1997 with 804 observations and non shaded part is the period from Oct 1997 to Sep 2001 with 1040 observations.

From January of 1995 the Korean Stock Exchange has recorded the daily trading volume of foreign investors and of 8 different domestic investors, including financial institutions, pension funds, individuals and so on. The domestic investors' trading volume is constructed by adding all the different domestic investors' trading[3]. Figure 6.2 plots the daily total Korean won value of traded shares.

[3]Due to the categorical trading volume records of the KSE this research can use the different investors' trading volumes to study the relationship between the trading volume and the volatility of the stock market. Further research could be done using all 9 different investors' trading volumes to find out investors' trading behavior in the stock market.

6.4.3 Unit root tests

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to examine the stationary properties of data . The results of these tests are reported in Table 6.4. The null hypothesis of unit root is rejected in all sub- and full-samples implying that the stock returns and trading volume can be treated as stationary processes.

6.4.4 Volatility

The Korean stock market after the crisis is more volatile than it was before the crisis according to Figure 6.1 and the standardised deviation of returns series (see Table 6.5). This is probably due to the crisis. However, the standardised deviation of stock return series and Figure 6.1 indicate that this higher volatility had become a normal feature of the Korean stock market even in 2001. Does this higher volatility have no connection with the financial liberalisation after the crisis? To answer this question the current research examines Granger causal relation between volatility and volume as a proxy of information flow. If the external information through the foreign investors' trading affects the higher volatility after the liberalisation the causality between volume and volatility can be established.

Table 6.5 presents summary statistics on the continuously compounded percentage KOSPI return series. The two return series show non-normality with leptokurtosis. The standard deviation for period B is almost 2.5 times as great as that of period A indicating much higher return volatility in period B.

The standardised deviations of the KOSPI returns before the crisis are 1.021,

	Augmented Dickey-Fuller test statistic	Phillips-Perron test statistic
Entire Sample		
KOSPI returns	−20.72	−38.13
Total trading volume	−3.30	−4.36
Domestic trading volume	−3.38	−4.35
Foreign rading volume	−4.38	−10.56
Sample A		
KOSPI returns	−11.59	−23.38
Total trading volume	−8.44	−11.94
Domestic trading volume	−5.30	−10.55
Foreign trading volume	−16.65	−22.05
Sample B		
KOSPI returns	−15.85	−28.86
Total trading volume	−3.08	−4.06
Domestic trading volume	−3.59	−4.01
Foreign trading volume	−3.00	−10.38
Note: A constant and four difference terms are used for the augmented Dickey-Fuller test. In the Phillips-Perron test 7 truncation lags are used for the Bartlett kernel. Critical value at 5% significant level is 2.86.		

Table 6.4: Unit root test

	Mean	Maximum	Minimum	Standard Deviation	Skewness	Kurtosis
Sample A[a]	-0.056	4.660	-3.963	1.116	0.315	3.946
Sample B[b]	-0.029	8.161	-12.804	2.755	-0.138	3.973
Note: [a] Sample A covers the period from January 1995 to September 1997 (804 obs). [b] Sample B covers the period from October 1997 to September 2001 (1040 obs).						

Table 6.5: Summary statistics for the KOSPI stock returns

	1995	1996	1997	1998	1999	2000	2001(Jan.-Sep.)
Std.deviation	1.021	1.089	2.218*	2.838	2.503	2.879	2.171
Average	-0.047	-0.104	-0.188	0.138	0.242	-0.295	-0.027
Note: * The Standardised deviation excluding the period of the crisis (Oct-Dec, 1997) is 1.266.							

Table 6.6: Standard Deviation of KOSPI stock returns

1.089 and 1.266 in 1995. 1996 and 1997 (Jan -Sep) respectively (see Table 6.6). The somewhat high figure 1.266 in the period from January to September in 1997 before the crisis might be due to turmoil in other East Asian countries, which had already begun in April. 1997. After the crisis all figures are far greater than those in the pre-crisis period. In 2001 the standardised deviation recorded 2.171 and is still twice as large as those in 1995 and 1996 although other economic indicators show the recovery from the crisis as pointed out by Kim ed.(2000, p.33).

In what follows, we use three different measures of return volatility. The most commonly used measure is the squared return series (see Brooks, 1998, and the references therein). Second. we use the absolute value of the return series (see Saatcioglou and Starks. 1998). Brailsford (1996) uses both the absolute value of the returns and their squares as a measure of volatility. Lee and Rui (2002) point out that the results from their causality tests between trading volume and volatility measured by a GARCH(1,1) model were very similar to those with squared returns. Hence, as a third measure we use the estimated volatility from three alternative GARCH-type models. These are the two component asymmetric GARCH (2C-AGARCH) models introduced by Engle and Lee (1999), the fractional integrated asymmetric power ARCH (FIAPARCH) model proposed by Tse (1998) and the fractionally integrated exponential GARCH (FIEGARCH) model defined in Bollerslev and Mikkelsen (1996).

6.4.5 GARCH models

In this section we denote the stock returns by r_t and define its mean equation as

$$r_t = c + \varepsilon_t + \theta\varepsilon_{t-1},$$

That is stock returns follow an MA(1) specification. We also assume that ε_t is conditionally normal with mean zero and variance h_t. Put differently, $\varepsilon_t|\Omega_{t-1} \sim N(0, h_t)$, where Ω_{t-1} is the information set up to time $t-1$.

Next, following Engle and Lee (1999), we specify the dynamic structure of the conditional volatility as the sum of a short-run (s_t) component and a long-run (q_t) component

$$h_t - q_t = \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \gamma D_{t-1}(\varepsilon_{t-1}^2 - q_{t-1}) + \beta(h_{t-1} - q_{t-1}), \quad (6.35)$$

with

$$q_t = \omega + \varphi(q_{t-1} - \omega) + \rho(\varepsilon_{t-1}^2 - h_{t-1})$$

where $\omega > 0$ and D_t is a dummy indicating the direction of the shock: $D_t = 1$ if $\varepsilon_t < 0$ and $D_t = 0$ if $\varepsilon_t > 0$. In other words, the treatment of Glosten et al. (1993) is used to allow shocks to affect the temporary volatility component asymmetrically. In addition, it is assumed that $0 < a + \beta + \gamma < \varphi < 1$. That is the permanent component is more persistent than the temporary component. We refer to the model in (6.35) as the 2C-AGARCH(1,1) model.

Second, we use the FIEGARCH(1,1) model, proposed by Bollerslev and Mikkelsen (1996)

$$\ln(h_t) = \omega + \frac{(1 + aL)}{(1 - \beta L)(1 - L^d)}[\gamma e_t + \eta(|e_t| - \mathsf{E}|e_t|)]$$

Finally, we use the FIAPARCH(1,1) model introduced by Tse (1998)

$$h_t^\delta = \omega + \left[1 - \frac{(1 - aL)}{(1 - \beta L)(1 - L)^d}\right](|\varepsilon_t| + \gamma\varepsilon_t)^\delta,$$

where $\delta, \omega > 0$, and $|\gamma| < 1$.

Maximum likelihood estimates of the various GARCH models for the entire period and the two sub-periods (before and after the crisis) are shown in Table 6.7, 6.8 and 6.9. Several findings emerge from this table. The estimated long memory parameter ($\widehat{d}$) is higher in sample A than in sample B. In particular, for the FIAPARCH and FIEGARCH models the values of the coefficients in sample B (0.216, 0.213, 0.347) are lower than the corresponding values in sample A (0.259, 0.226, 0.678). Further, negative shocks predict higher volatility than positive shocks, since in most cases the estimated asymmetry coefficient ($\widehat{\gamma}$) is significant and negative. In addition, in both sub-samples the value of the power coefficient is less than but not significantly different from one. Thus, it seems that the conditional standard deviation is a linear function of lagged absolute residuals. In sharp contrast, for the whole sample the estimated power term is very close to two (2.05). That is, the conditional variance is a linear function of lagged squared residuals. Generally speaking , the parameter estimates support the idea that long memory effects are present in stock volatility. The results also show strong evidence of asymmetry in the conditional variance.

6.5 Granger causality tests

The following bivariate autoregression is used to test for causality between the two variables among trading volume, and stock return volatility

$$
\begin{aligned}
x_t &= \sum_{i=1}^{m} a_i x_{t-i} + \sum_{i=1}^{n} b_i y_{t-i} + e_t, \\
y_t &= \sum_{i=1}^{m} c_i x_{t-i} + \sum_{i=1}^{n} d_i y_{t-i} + \nu_t,
\end{aligned} \tag{6.36}
$$

The test of whether $y(x)$ strictly Granger causes $x(y)$ is simply a test of the joint restriction that all the $b_i(c_i)$, $i = 1, \ldots, n(m)$, are zero. In each case, the null hypothesis of no Granger causality is rejected if the restriction is rejected. Bidirectional feedback exists if all the elements b_i, c_i, $i = 1, \ldots, n(m)$, are jointly significantly different from zero.

Next the results of Granger causality tests are reported to provide some statistical evidence on the nature of the relationship between trading volume and stock volatility. In Table 6.10, the F statistics of Granger-causality tests are presented for the entire sample using four, eight, and twelve lags, as well as the sign of the sums of the lagged coefficients in case of statistical significance. Panel A considers Granger-causality from trading volume to stock volatility. Panel B reports the results of the causality tests where causality runs from the stock volatility to the trading volume. There is strong evidence of a positive bidirectional feedback between volume and volatility. In most cases this causal relationship is robust to the measures of volume and volatility used. However, 'domestic' and 'total' volumes have no causal effect on squared/absolute returns while squared returns have a negative impact on 'foreign' volume.

6.5.1 Sub-sample analyses

This section examines whether the informational change after the crisis affects the dynamic interactions by dividing the whole sample period into two sub-periods and conducting causality tests for each sub-period separately. Tables 6.11 and 6.12 report the results of the Granger causality tests between volume and volatility for the two sub-periods. Panels A and B correspond to the panels that report the results for the whole sample.

First, the results for the pre-crisis period is discussed. When the absolute returns or their squares as a measure of stock volatility is used, the trading volume does not Granger-cause stock volatility. This confirms the difficulty of improving the predictability of volatility by adding public information about trading volume. However, volatility affects trading volume positively. Strong evidence is reported for the 'domestic' volume and weak evidence for the 'total' volume. Not surprisingly, 'foreign' volume is independent of changes in volatility. It turns out that using the 'FIEGARCH' volatility results in exactly the same causal relation between volume and volatility. On the contrary, when the estimated conditional variance from the FIAARCH (with $\delta = 1$) model as a measure of stock volatility is used, a feedback relation between volume and volatility is found. Panel A shows a significant positive impact of either 'domestic' or 'total' volume on volatility. Panel B shows a significant negative effect of volatility on either 'domestic' or 'total' volume. No evidence for an effect of volatility on 'foreign' volume is also noticed. The last column of the table reports the results of the causality tests when the estimated conditional variance from the 2C-AGARCH model is used as a measure of stock volatility. As in the FIAARCH model there is a feedback relation between either 'domestic' or 'total' volume and volatility. However, the effect of volatility on volume is now positive, while volume has a negative impact on volatility. The relationship is stronger for 'domestic' than for 'total' volume.

The evidence from the Granger causality tests suggests that the dynamic relation between 'total' volume and volatility reflects the dynamic relation between 'domestic' volume and volatility. In other words, the statistical evidence suggests that volatility is related only to the domestic investors' volume before the crisis, which is in line with the results of the previous work. Sample A covers the period from January 1995 to September 1997, that is three years after the 'liberalisation date' of the previous research (see Table 6.1). Some part of this period overlaps with that in Bekaert and Harvey (2000) and Spyrou and Kassimatis (1999). Hence, their conclusion that the nature of volatility has not changed dramatically after the ' liberalisation in 1992 ', is, in the case of the Korean stock market, probably because of no serious amount of information inflow from the outside world. That is, even after the 'liberalisation in 1992' it was the domestic rather than foreign investors' information or trading that affected the stock market volatility as it had before.

The results of applying the Granger causality tests for the period after the financial crisis in 1997 are reported in Table 6.12. The picture is different from that of the period before the crisis: that is there is extensive evidence of a negative bidirectional feedback between 'foreign' volume and volatility. This finding has an important implication. The evidence of causality running from 'foreign' volume to volatility suggests that it may be possible to use lagged values of 'foreign' volume to predict volatility. Regarding the 'domestic' and 'total' volume, Panel A shows that they do not have a significant causal effect on volatility, whereas according to Panel B, there is strong evidence that volatility has a negative effect on either the 'domestic' or the 'foreign' volume. These results are not qualitatively altered by changes in the measure of volatility.

In sum, before the crisis the 'domestic'/'total' volume-volatility relationship is altered by changes in the measure of volatility. That is, volume has a positive (negative) effect on 'FIAARCH' ('2C-AGARCH') volatility, while 'FIAARCH' ('2C-AGARCH') volatility has a negative (positive) effect on volume. There is also a lack of a causal effect from volume to either the absolute value of the returns or their squares. Moreover, after the crisis the 'domestic'/'total' volume-volatility relationship is robust to the measure of volatility used. There is strong evidence of causality running only from volatility to volume. In particular, increased volatility lowers volume. It also should be mentioned that before (after) the crisis the volume-volatility relationship is stronger (weaker) for 'domestic' volume than for 'total' volume. Finally, before the crisis there is no dynamic causal relation between 'foreign' volume and volatility, whereas after the crisis there is a strong negative bidirectional feedback between volatility and 'foreign' volume. These results are not qualitatively altered by changes in the measure of volatility.

In order to ensure that the results of this study are not unduly influenced by the financial crisis in 1997, the Granger causality tests are recalculated disregarding all data from January 1997 to December 1999. This leaves sample A running from January 1995 to December 1996 (hereafter sample A1) and sample B running from October 1997 to September 2001 (hereafter sample B1). In the pre-crisis sample when the period January 1997-September 1997 is excluded, the 'foreign' volume in four out of the five cases is independent of changes in volatility and vice versa (see Table 6.13). On the other hand, a feedback relation between 'FIEGARCH' volatility and 'foreign' volume is founded. In particular, volume has a negative impact on volatility, whereas the effect of volatility on volume is mixed. Moreover, the results indicate the lack of an effect of either 'domestic' or 'total' volume on volatility. There is also mixed evidence on the impact of volatility on volume. The effect of absolute/squared returns ('FIEGARCH' volatility) on 'domestic'/'total' volume is initially positive (negative) but turns to negative (positive). Comparing the results of the whole sample A, the following observations are noted. In the entire pre-crisis period absolute/squared returns and 'FIEGARCH' volatility have a positive effect on 'domestic'/'total' volume whereas when the period January 1997-September 1997 is excluded the effect becomes mixed. In addition, the effect of 'domestic'/'total' volume on '2C-AGARCH' volatility is negative but becomes negligible when the period of the crisis is removed. Finally, in sample A1 there is a strong feedback relation between 'FIEGARCH' volatility and 'foreign' volume whereas in sample A there is no causal relation between the two.

The following observations, among other things, are noted about the volume-volatility relationship for the second sub-period that excludes the crisis period. In most cases the results from the causality tests between volatility and 'total' volume are very similar to those between volatility and 'domestic' volume (see Table 6.14). Neither 'domestic' nor 'total' volume has a significantly causal effect on either squared returns or conditional volatility However, the 'domestic'/'total' volume has a positive impact on absolute returns. There is also a lack of a causal effect of absolute returns on 'domestic'/'total' volume. On the other hand, either 'FIAPARCH' volatility or squared returns have a positive impact on 'domestic'/'total' volume whereas 'FIEGARCH' or 2C-AGARCH' volatility affects it negatively. Moreover, there is a strong bidirectional feedback between 'foreign' volume and volatility (except for 'FIEGARCH' volatility). In particular, 'foreign' volume has a positive impact on volatility whereas volatility affects 'foreign' volume negatively.

Comparing the results of sample B1 with those of sample B, the following observations are noted. In the entire after-crisis period the (negative) effect of volatility on 'domestic'/'total' volume is robust to the measure of volatility whereas when we exclude the period October 1997-December 1999 the sign of the effect is altered by changes in the measure of volatility. Further, in sample B 'foreign' volume has a negative impact on volatility whereas in sample B1 in four out of the five cases it has a positive impact. In addition, in sample B causality runs from absolute returns to 'domestic'/'total' volume whereas in sample B1 it runs from 'domestic'/'total' volume to absolute returns. Finally, in sample B there is a negative bidirectional feedback between 'foreign' volume and 'FIEGARCH' volatility but it disappears when we exclude the period October 1997-December 1999.

6.6 Conclusions

This research has examined the dynamic causal relations between stock volatility and trading volume for the Korean market. For the overall period from 1995 to September 2001 we found a positive bidirectional feedback between volume and volatility. In general this causal relationship was robust to five alternative measures of volatility. However, absolute returns and their squares were independent of changes in 'domestic'/'total' volume.

In addition, by conducting sub-sample analyses this study shows that there are structural shifts in causal relations, and also that it is important to distinguish between domestic and foreign investors' volume. Specifically, before the financial crisis in 1997 there was no causal relation between foreign investors' volume and stock volatility whereas after the crisis a negative feedback relation began to exist. Further, the effect of volatility on domestic investors' volume was positive in the pre-crisis period but turned to negative after the crisis. In contrast, absolute returns and their squares were independent of changes in 'domestic' volume.

This study also found that some of these results are influenced by the financial crisis in 1997. For example, in the entire pre-crisis period absolute/squared returns and 'FIEGARCH' volatility had a positive effect on 'domestic'/'total' volume whereas when the period of the crisis excluded the effect became mixed.

Model:	(1a)	(1b)	(2)	(3)
c	−0.054 [1.43]	−0.055 [1.44]	0.022 [0.55]	−0.034 [882.0]
θ	0.141 [5.30]	0.141 [5.33]	0.152 [6.28]	0.148 [6.22]
ω	0.023 [0.69]	0.009 [0.16]	0.489 [2.44]	2.465 [0.77]
α	0.133 [1.50]	0.128 [1.44]	1.481 [7.07]	0.032 [1.46]
β	0.532 [4.70]	0.519 [4.47]	−0.897 [15.81]	0.861 [18.86]
ρ				0.014 [1.17]
φ				0.999 [482.7]
γ	−0.233 [3.03]	−0.227 [3.03]	−0.093 [2.31]	0.069 [2.22]
η			0.297 [2.46]	
d	0.445 [7.38]	0.436 [6.69]	0.735 [12.92]	
δ	2.00	2.049 [13.63]		

Notes: This table reports QML parameter estimates for various GARCH models for the entire sample period. The four alternative GARCH models are: (1a) FIAARCH(1,1) (δ=2), (1b) FIAPARCH(1,1), (2) FIEGARCH (1,1), (3) 2C-AGARCH(1,1) Absolute t-statistics are given in brackets.

Table 6.7: Four alternative GARCH models (Entire sample)

Model:	(1a)	(1b)	(2)	(3)
c	−0.066 [1.60]	−0.073 [1.92]	−0.046 [4.94]	−0.033 [0.78]
θ	0.184 [5.23]			0.205 [5.79]
ω	0.395 [7.17]	0.442 [4.98]	−0.002 [0.02]	1.312 [6.82]
α	−0.270 [4.98]	−0.234 [4.89]	0.489 [0.15]	−0.438 [1.08]
β				0.994 [1.72]
ρ				0.459 [1.12]
φ				0.801 [7.63]
γ	−0.652 [4.31]	−0.688 [3.62]	−0.215 [0.41]	−0.027 [0.42]
η				
d	0.259 [5.21]	0.226 [5.13]	0.678 [7.07]	
δ	1.00	0.974 [3.35]		

Notes: Table 6.8 reports QML parameter estimates for various GARCH models.

The four alternative GARCH models are:

(1a) FIAARCH(0,1) (δ=1), (1b) FIAPARCH(0,1), (2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)

Absolute t-statistics are given in brackets.

Table 6.8: Four alternative GARCH models (Sample A)

Model:	(1a)	(1b)	(2)	(3)
c	−0.065 [0.75]	−0.067 [0.82]	0.100 [1.27]	−0.054 [0.58]
θ	0.110 [3.38]	0.109 [3.56]	0.063 [1.46]	0.134 [4.31]
ω	1.092 [4.98]	0.946 [3.11]	2.307 [13.91]	7.119 [9.32]
α	−0.163 [2.49]	−0.162 [2.58]	2.245 [1.67]	−0.011 [0.25]
β				0.951 [17.66]
ρ				0.047 [1.11]
φ				0.857 [6.80]
γ	−0.377 [1.97]	−0.404 [1.86]	−0.234 [1.50]	0.058 [1.77]
η			0.945 [4.66]	
d	0.216 [3.65]	0.213 [3.88]	0.347 [5.16]	
δ	1.00	0.821 [2.44]		

Notes: Table 6.9 reports QML parameter estimates for various GARCH models.
The four alternative GARCH models are:
(1a) FIAARCH(0,1) (δ=1), (1b) FIAPARCH(0,1),
(2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)
Absolute t-statistics are given in brackets.

Table 6.9: Four alternative GARCH models (Sample B)

	$\|r_t\|$	r_t^2	(1a)	(2)	(3)
Panel A (H_0: V_{it} →Vol_t)					
$i = D$					
4 lags	0.91	1.43	2.60***(+)	1.74*(+)	3.54***(+).
8 lags	0.79	1.11	1.85**(+)	1.14	2.41***(+)
12 lags	0.90	1.00	1.59*(+)	0.98	2.06***(+)
$i = F$					
4 lags	2.34**(+)	3.93***(+)	6.30***(+)	2.21**(+)	6.74***(+)
8 lags	1.53▲(+)	2.61***(+)	4.22***(+)	1.86**(+)	4.52***(+)
12 lags	1.58*(+)	2.15***(+)	3.49***(+)	2.06***(+)	3.64***(+)
$i = T$					
4 lags	0.98	1.37	2.57***(+)	1.79*(+)	3.61***(+)
8 lags	0.84	1.04	1.81**(+)	1.16	2.41***(+)
12 lags	0.98	0.96	1.57*(+)	1.04	2.07***(+)
Panel B (H_0: Vol_t → V_{it})					
$i = D$					
4 lags	1.33	1.49	0.64	0.88	0.69
8 lags	1.60*(+)	1.53*(+)	1.72*(+)	1.62*(+)	1.54*(+)
12 lags	1.65**(+)	1.56*(-)	1.83**(+)	1.69**(+)	1.93***(+)
$i = F$					
4 lags	3.82***(+)	3.00***(-)	2.05**(+)	2.85***(+)	1.63▲(+)
8 lags	3.72***(+)	3.04***(-)	1.64*(+)	2.71***(+)	1.52▲(+)
12 lags	3.39***(+)	2.55***(-)	1.47*(+)	1.84**(+)	1.38▲(+)
$i = T$					
4 lags	1.69*(+)	1.81*(+)	0.78	1.03	0.88
8 lags	2.08**(+)	1.82**(+)	1.80**(+)	1.79**(+)	1.65*(+)
12 lags	2.18***(+)	1.77**(-)	1.90**(+)	1.77**(+)	2.09***(+)

Notes: V_{it} →Vol_t: Trading volume does not Granger-cause return volatility.
Vol_t → V_{it}: Volatility does not Granger-cause trading volume.
V_{it} is either domestic (i=D), foreign (i=F) or total (i=T) investors' trading volume respectively.
Vol_t is the stock volatility as measured by either the absolute value of returns ($|r_t|$), or their squares (r_t^2) or the estimated conditional variance from one of the three alternative GARCH type models.
The GARCH models are: (1a) FIAARCH(1,1) (δ=2),
(2) FIEGARCH (1,1), (3) 2C-AGARCH(1,1)
A +(-) indicates that the sum of the lagged coefficients is positive (negative).
***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 6.10: Granger causality tests between trading volume and stock volatility (Entire sample)

	$\lvert r_t \rvert$	r_t^2	(1a)	(2)	(3)
Panel A (H_0: $V_{it} \rightarrow Vol_t$)					
$i = D$					
4 lags	1.23	1.38	2.16**(+)	1.13	2.22**(-)
8 lags	1.24	1.03	2.08**(+)	1.04	1.62*(-)
12 lags	0.97	0.74	1.69**(+)	1.10	1.51*(-)
$i = F$					
4 lags	1.17	0.60	1.71*(+)	1.21	1.11
8 lags	0.98	0.45	1.41	1.07	0.85
12 lags	0.87	0.43	1.15	1.23	0.92
$i = T$					
4 lags	1.37	1.38	2.19**(+)	0.92	2.04**(-)
8 lags	1.20	0.98	2.10***(+)	0.91	1.53▲(-)
12 lags	0.92	0.71	1.70**(+)	1.06	1.43▲(-)
Panel B (H_0: $Vol_t \rightarrow V_{it}$)					
$i = D$					
4 lags	2.20**(+)	2.31**(+)	3.03***(-)	4.45***(+)	2.35**(+)
8 lags	1.85**(+)	1.94**(+)	1.85**(-)	3.54***(+)	1.74**(+)
12 lags	1.51*(+)	1.67**(+)	1.44▲(-)	3.02***(+)	1.98***(+)
$i = F$					
4 lags	0.81	0.63	0.87	0.78	0.41
8 lags	0.93	0.58	0.77	0.80	0.36
12 lags	0.80	0.54	0.69	0.86	0.46
$i = T$					
4 lags	1.85*(+)	1.69*(+)	2.56***(-)	5.07***(+)	1.64▲(+)
8 lags	1.60*(+)	1.34	1.72*(-)	3.77***(+)	1.13
12 lags	1.34	1.20	1.23	2.90***(+)	1.25

Notes: $V_{it} \rightarrow Vol_t$: Trading volume does not Granger-cause return volatility.
$Vol_t \rightarrow V_{it}$: Volatility does not Granger-cause trading volume.
V_{it} is either domestic (i=D), foreign (i=F) or total (i=T) investors' trading volume respectively.
Vol_t is the stock volatility as measured by either the absolute value of returns ($\lvert r_t \rvert$), or their squares (r_t^2) or the estimated conditional variance from one of the three alternative GARCH type models.
The GARCH models are: (1a) FIAARCH(0,1) (δ=1), (2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)
A +(-) indicates that the sum of the lagged coefficients is positive (negative).
***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 6.11: Granger causality tests between trading volume and stock volatility (Sample A)

	$\|r_t\|$	r_t^2	(1.b)	(2)	(3)
Panel A (H$_0$: V_{it} $\rightarrow$Vol$_t$)					
$i = D$					
8 lags	0.62	0.83	1.90*(-)	3.16***(-)	1.52▲(-)
12 lags	0.63	0.63	1.36	2.08**(-)	1.14
16 lags	0.77	0.64	1.09	0.56	1.01
$i = F$					
8 lags	2.38**(-)	2.18**(-)	1.93**(-)	6.19***(-)	3.19***(-)
12 lags	1.36	1.55*(-)	2.26***(-)	4.07***(-)	2.24***(-)
16 lags	1.35	1.33	2.57***(-)	0.67	1.90**(-)
$i = T$					
4 lags	0.60	0.76	1.60▲(-)	3.43***(-)	1.48
8 lags	0.63	0.59	1.17	2.25***(-)	1.11
12 lags	0.78	0.62	0.99	0.59	0.99
Panel B (H$_0$: Vol$_t$ $\rightarrow$ V_{it})					
$i = D$					
4 lags	1.43	1.65▲(-)	1.86*(-)	14.08***(-)	1.59▲(-)
8 lags	1.78**(-)	1.79**(-)	2.07**(-)	5.49***(-)	1.82**(-)
12 lags	1.88**(-)	1.85**(-)	1.79**(-)	60.59***(-)	2.10***(-)
$i = F$					
4 lags	4.03***(-)	2.97***(-)	2.75***(-)	15.82***(-)	2.34**(-)
8 lags	4.28***(-)	3.20***(-)	3.00***(-)	13.42***(-)	1.91**(-)
12 lags	3.29***(-)	2.60***(-)	2.40***(-)	32.15***(-)	1.75**(-)
$i = T$					
4 lags	1.81*(-)	2.01**(-)	2.16**(-)	16.46***(-)	1..84*(-)
8 lags	2.24***(-)	2.11***(-)	2.35***(-)	7.74***(-)	2.04**(-)
12 lags	2.16***(-)	2.07***(-)	1.99***(-)	68.20***(-)	2.30***(-)

Notes: V_{it} $\rightarrow$Vol$_t$: Trading volume does not Granger-cause return volatility.
Vol$_t$ $\rightarrow$ V_{it}: Volatility does not Granger-cause trading volume.
V_{it} is either domestic (i=D), foreign (i=F) or total (i=T) investors' trading volume respectively.
Vol$_t$ is the stock volatility as measured by either the absolute value of returns ($|r_t|$), or their squares (r_t^2) or the estimated conditional variance from one of the three alternative GARCH type models.
The GARCH models are: (1b) FIAPARCH(0,1), (2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)
A +(-) indicates that the sum of the lagged coefficients is positive (negative).
***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 6.12: Granger causality tests between trading volume and stock volatility (Sample B)

	$\lvert r_t \rvert$	r_t^2	(1.a)	(2)	(3)
Panel A (H$_0$: V_{it} $\rightarrow$Vol$_t$)					
$i = D$					
4 lags	1.06	0.88	1.35	0.87	1.51▲(+)
8 lags	0.91	0.71	1.48▲(+)	1.25	1.08
12 lags	0.84	0.65	1.26	1.28	0.95
$i = F$					
4 lags	1.36	1.02	0.66	1.51▲(-)	0.74
8 lags	0.97	0.82	0.74	1.77**(-)	0.72
12 lags	0.83	0.72	0.64	1.85**(-)	0.59
$i = T$					
4 lags	1.19	0.96	1.50▲(+)	0.69	1.39
8 lags	0.93	0.76	1.59*(+)	1.30	1.04
12 lags	0.86	0.67	1.32	0.91	0.92
Panel B (H$_0$: Vol$_t$ $\rightarrow$ V_{it})					
$i = D$					
4 lags	1.91*(+)	1.80*(+)	2.34**(-)	7.53***(-)	2.30**(-)
8 lags	1.73*(-)	1.78**(-)	1.77**(-)	5.25***(+)	1.79**(-)
12 lags	1.54	1.53*(-)	1.56*(-)	3.87***(+)	1.68**(-)
$i = F$					
4 lags	1.32	0.61	1.64▲(-)	1.66▲(-)	0.63
8 lags	1.31	0.65	1.15	1.48▲(+)	0.56
12 lags	1.13	0.60	0.94	1.47*(+)	0.46
$i = T$					
4 lags	1.69*(+)	1.39	2.24**(-)	6.39***(-)	1.62▲(-)
8 lags	1.55*(-)	1.24	1.71*(-)	4.86***(+)	1.22
12 lags	1.26	1.01	1.32	3.81***(+)	1.06

Notes: Sample A1 covers the period from January 1995 to December 1996.
V_{it} $\rightarrow$Vol$_t$: Trading volume does not Granger-cause return volatility.
Vol$_t$ $\rightarrow$ V_{it}: Volatility does not Granger-cause trading volume.
V_{it} is either domestic (i=D), foreign (i=F) or total (i=T) investors' trading volume respectively.
Vol$_t$ is the stock volatility as measured by either the absolute value of returns ($\lvert r_t \rvert$), or their squares (r_t^2) or the estimated conditional variance from one of the three alternative GARCH type models.
The GARCH models are: (1b) FIAARCH(0,1) (δ=1),
(2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)
A +(-) indicates that the sum of the lagged coefficients is positive (negative).
***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 6.13: Granger causality tests between trading volume and stock volatility (Sample A1)

	$\lvert r_t \rvert$	r_t^2	(1.b)	(2)	(3)
Panel A (H_0: V_{it} $\rightarrow$Vol$_t$)					
$i = D$					
8 lags	1.55▲(+)	1.04	1.64▲(+)	0.83	0.80
12 lags	2.05**(+)	1.36	1.12	0.58	1.13
16 lags	1.71**(+)	1.10	1.06	0.83	1.02
$i = F$					
8 lags	2.23**(+)	2.21**(+)	2.19**(+)	0.30	2.45***(+)
12 lags	1.39	1.47▲(+)	2.41***(+)	0.57	1.78**(+)
16 lags	1.55*(+)	1.18	2.44***(+)	0.61	1.44▲(+)
$i = T$					
4 lags	1.74*(+)	0.95	1.72*(+)	0.72	0.77
8 lags	1.85**(+)	1.24	1.18	0.50	1.06
12 lags	1.59*(+)	1.01	1.08	0.18	0.96
Panel B (H_0: Vol$_t$ $\rightarrow$ V_{it})					
$i = D$					
4 lags	1.35	1.84*(+)	1.94**(+)	8.31***(-)	1.16
8 lags	1.20	1.73*(+)	1.91**(+)	4.15***(-)	1.68*(-)
12 lags	1.30	1.52*(-)	1.54*(+)	0.65	1.84**(-)
$i = F$					
4 lags	3.47***(-)	3.69***(-)	3.31***(-)	0.49	2.32**(-)
8 lags	2.71***(-)	2.80***(-)	2.77***(-)	0.37	2.02**(-)
12 lags	2.10***(-)	2.17***(-)	2.29***(-)	0.36	1.70**(-)
$i = T$					
4 lags	1.56▲(+)	2.07**(+)	2.37**(+)	9.72***(-)	1.38
8 lags	1.34	1.87**(+)	2.14***(+)	5.34***(-)	1.78**(-)
12 lags	1.33	1.63*(+)	1.71**(+)	0.65	1.93**(-)

Notes: Sample B1 covers the period from January 2000 to September 2001.
V_{it} $\rightarrow$Vol$_t$: Trading volume does not Granger-cause return volatility.
Vol$_t$ $\rightarrow$ V_{it}: Volatility does not Granger-cause trading volume.
V_{it} is either domestic (i=D), foreign (i=F) or total (i=T) investors' trading volume respectively.
Vol$_t$ is the stock volatility as measured by either the absolute value of returns ($\lvert r_t \rvert$), or their squares (r_t^2) or the estimated conditional variance from one of the three alternative GARCH type models.
The GARCH models are: (1b) FIAPARCH(0,1), (2) FIEGARCH (0,1), (3) 2C-AGARCH(1,1)
A +(-) indicates that the sum of the lagged coefficients is positive (negative).
***,**,* and ▲ denote significance at the 0.01, 0.05, 0.1 and 0.15 levels, respectively.

Table 6.14: Granger causality tests between trading volume and stock volatility (Sample B1)

Chapter 7

Conclusions

This chapter summarises the main results of the previous chapters.

Chapter 2 explored the causal relationship among four macroeconomic variables: inflation, output growth, nominal uncertainty and real uncertainty. Unlike previous studies which focus on the empirical relationship between either inflation and inflation uncertainty or output growth and output growth uncertainty (a notable exception is Grier and Perry (2000)), this research examines the complete set of hypotheses to test. In addition, it employed G7 countries' data to test the relationship among variables whereas the majority of studies in this area investigate only US or UK data (an exception being Grier and Perry (1998)). The application of the constant conditional correlation GARCH(1,1) model provides a number of interesting conclusions. First, inflation does cause negative welfare effects, both directly (supporting Stockman, 1981) and indirectly, i.e., via the inflation uncertainty channel as predicted by Friedman (1997). Second, more inflation uncertainty leads to a higher level of inflation supporting Cukierman and Meltzer (1986), who suggest that Central Banks tend to surprise the public by raising inflation unexpectedly in the presence of more inflation uncertainty. Third, output growth uncertainty causes output growth positively. In contrast to the assumptions of macroeconomic models, the business cycle and the rate of economic growth are related. Hence, macroeconomics should consider the business cycle and the growth rate simultaneously.

Chapter 3 examined the relationship between inflation variability and output variability for G3 countries. Using a two-step procedure this study finds that nominal uncertainty significantly affects real uncertainty in all three countries. In Japan and the USA nominal uncertainty does Granger-cause real uncertainty positively. By contrast, in Germany there is a negative effect of inflation uncertainty on real uncertainty, supporting the theoretical predictions made by Taylor (1979). In the sub sample analysis a trade-off relationship between nominal and real uncertainty is observed in the sixties and seventies in Japan and Germany. However, it turns to a positive relationship in the 1980s and 1990s (supporting Logue and Sweeney, 1981). This evidence does not support the criticism that inflation targeting policy can increase undesirable output growth uncertainty.

Chapter 4 considered the moment structure of the general ARMA-EGARCH model. In particular, this study derives the autocorrelation function of any positive integer power of the squared errors. In addition, it obtains the autocorrelations of the squares of the observed process and cross correlations between the levels and the squares of the observed process. The purpose of Chapter 5 was to provide a comprehensive methodology for the analysis of the Asymmetric Power ARCH (A-PARCH) model. First, it gave the ARMA representations of a power transformation of the conditional variance and the absolute returns. Second, it derived a certain fractional moment of the absolute observations. Third, it obtained the autocorrelation function of the power-transformed absolute returns. In both Chapters 4 and 5 the estimates of the autocorrelations of power transformations of the absolute observations are critical. By comparing these estimates to those obtained directly by the data, one can have a clear indication of how well the estimated model fits the data.

Chapter 6 examined the relationship between the stock market volatility and volume in the Korean stock market around the financial crisis in 1997. The evidence from causality tests suggests that there is a strong positive bidirectional feedback between volume and volatility for the entire sample. In the sub sample analysis foreign investors' trading volume caused the stock market volatility after the crisis. However, domestic investors' trading volume had a mixed effect on volatility before the crisis and the effect disappears in the post-crisis period. This empirical evidence suggests that the financial market opening to foreign investors had an effect on Korean stock market volatility after the crisis.

Appendix to Chapter 4

The followings tables report the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC) numbers of the estimated EGARCH models for four Asian countries.

KOSPI

MA(1) (Gaussian)		p 1	2	3	4
q 1	AIC	-27437.01	-17244.49	-27335.14	12313.51
	SIC	-27398.51	-17199.59	-27283.81	12371.24
	likelihood	13724.50	8629.25	13675.57	-6147.75
2	AIC	-27433.78	-17530.59	28548.97	-26327.75
	SIC	-27382.45	-17472.86	28613.14	-26167.19
	likelihood	13724.89	8774.30	-14264.49	13129.88
3	AIC	-27434.73	-17808.98	27192.58	-23313.76
	SIC	-27370.58	-17738.41	27269.57	-23230.35
	likelihood	13727.37	8915.49	-13584.29	11669.88
4	AIC	-27439.35	-18079.72	25849.76	113691.70
	SIC	-27362.37	-17996.31	25939.58	113787.94
	likelihood	13731.68	9052.86	-12910.88	-56830.85

MA(1) (Double exp)		p 1	2	3	4
q 1	AIC	-28065.35	-28069.98	-28050.82	-28061.53
	SIC	-28021.02	-28018.03	-27999.49	-28003.80
	likelihood	14038.67	14041.99	14033.41	14039.77
2	AIC	-28059.32	-24687.78	-28052.14	-28058.49
	SIC	-28008.01	-24630.04	-27987.98	-27987.93
	likelihood	14037.67	12352.89	14036.07	14040.25
3	AIC	-28062.00	-24769.72	-27351.83	-28065.90
	SIC	-27997.84	-24699.15	-27274.85	-27982.49
	likelihood	14041.00	12395.86	13687.92	14045.95
4	AIC	-28066.47	-24849.08	-27953.76	-27379.76
	SIC	-27989.49	-24765.67	-27863.44	-27283.52
	likelihood	14045.24	12437.54	13990.63	13704.88

MA(1) (Gen error)		p 1	2	3	4
q 1	AIC	-28066.04	-22607.71	-23199.40	-25726.29
	SIC	**-28021.16**	-22556.37	-23141.66	-25662.14
	likelihood	14040.02	11311.85	11608.70	12873.15
2	AIC	-28058.40	-22754.31	-25327.00	-3463.19
	SIC	-28001.58	-22690.16	-25256.43	-3386.21
	likelihood	14038.66	11387.16	12674.50	1743.60
3	AIC	-28059.64	-22896.42	**-28072.82**	-23233.22
	SIC	-27989.07	-22819.43	-27989.41	-23143.40
	likelihood	14040.82	11460.21	14049.41	11630.61
4	AIC	-28065.84	-23033.94	-27231.53	-9576.97
	SIC	-27982.43	-22944.12	-27135.28	-9474.33
	likelihood	14045.92	11530.97	13630.76	4804.49

NIKKEI

MA(1) (Gaussian)		p 1	2	3	4
q 1	AIC	-29500.83	-20084.59	-22539.61	-23064.78
	SIC	-29462.33	-20039.77	-22488.27	-23007.04
	likelihood	14756.41	10049.34	11277.80	11541.39
2	AIC	-29345.24	-20353.23	14766.69	52518.08
	SIC	-29293.91	-20295.48	14830.84	52588.65
	likelihood	14680.62	10185.61	-7373.34	-26248.04
3	AIC	-29348.91	-20610.92	13705.01	-23053.82
	SIC	-29284.76	-20540.35	13782.01	-22970.41
	likelihood	14684.46	10316.46	-6840.51	11539.91
4	AIC	-29379.39	-20860.72	12669.13	-20996.19
	SIC	-29302.41	-20777.31	12758.94	-20899.96
	likelihood	14701.70	10443.36	-6320.56	10513.10

WN (Double exp)		p 1	2	3	4
q 1	AIC	-30035.62	-26601.88	-30022.67	-30020.99
	SIC	-30003.54	-26563.39	-29977.77	-29969.65
	likelihood	15022.81	13306.94	15018.34	15018.49
2	AIC	-30036.75	-26685.66	-30046.28	-29889.98
	SIC	-29991.85	-26694.33	-29988.54	-29825.82
	likelihood	15025.38	13380.83	15032.14	14954.99
3	AIC	**-30064.75**	-26766.57	-18082.93	-29999.36
	SIC	**-30007.01**	-26702.42	-18012.35	-29922.37
	likelihood	15041.38	13393.29	9052.46	15011.68
4	AIC	-30047.51	-26845.22	-18303.59	-30033.79
	SIC	-29976.93	-26768.23	-18220.17	-29943.96
	likelihood	15034.75	13434.61	9164.79	15030.89

MA(1) (Gen error)		p 1	2	3	4
q 1	AIC	-30036.78	-24732.99	-29224.46	-26000.95
	SIC	-29991.87	-24681.65	-29166.72	-25936.78
	likelihood	15025.39	12374.49	14621.23	13010.47
2	AIC	-29915.24	-24876.63	-26398.11	-24220.67
	SIC	-29857.50	-24812.46	-26327.55	-24143.67
	likelihood	14966.62	12448.31	13210.06	12122.33
3	AIC	-29879.86	-25014.72	-26429.19	-15392.71
	SIC	-29809.29	-24937.73	-26345.77	-15302.90
	likelihood	14950.93	12519.36	13227.59	7710.36
4	AIC	-29926.48	-25148.65	-29517.29	-24321.90
	SIC	-29843.07	-25058.84	-29421.06	-24219.25
	likelihood	14976.24	12588.33	14773.65	12176.95

SE

MA(1) (Gaussian)		p 1	2	3	4
q 1	AIC	-25010.01	-17947.37	-24968.51	32791.12
	SIC	-24971.51	-17902.45	-24917.17	32848.86
	likelihood	12511.00	8980.68	12492.25	-16386.56
2	AIC	-25017.97	-18169.84	13616.74	54343.39
	SIC	-24966.65	-18112.10	13680.90	54413.97
	likelihood	12516.99	9093.92	-6798.37	-27160.70
3	AIC	-25025.11	-18383.94	12645.38	-18060.76
	SIC	-24960.94	-18313.27	12722.37	-17977.35
	likelihood	12522.55	9202.92	-6310.69	9043.38
4	AIC	-25028.84	-18590.85	11701.14	-17973.67
	SIC	-24951.85	-18507.43	11790.96	-17877.44
	likelihood	12526.42	9308.42	-5836.57	9001.84

WN (Double exp)		p 1	2	3	4
q 1	AIC	-25596.00	-22852.24	-25578.23	-25571.86
	SIC	**-25563.91**	-22813.75	-25533.31	-25520.53
	likelihood	12803.01	11432.12	12796.11	12793.93
2	AIC	-25597.16	-22925.21	-25551.67	-25596.05
	SIC	-25552.25	-22873.89	-25493.92	-25521.90
	likelihood	12805.58	11470.61	12784.83	12803.03
3	AIC	**-25612.81**	-22995.82	-25197.37	-24134.45
	SIC	-25555.08	-22931.66	-25126.79	-24057.45
	likelihood	12815.41	11507.91	12609.68	12079.22
4	AIC	-25612.58	-23064.38	-25184.41	-24625.68
	SIC	-25542.01	-22987.39	-25101.01	-24535.86
	likelihood	12817.29	11544.19	12605.21	12326.84

WN (Gen error)		p 1	2	3	4
q 1	AIC	-25597.03	-21393.77	-25429.39	-17341.16
	SIC	-25558.55	-21348.87	-25378.05	-17283.42
	likelihood	12804.52	10703.89	12722.69	8679.58
2	AIC	-25598.24	-21516.57	-19503.10	-21024.03
	SIC	-25546.91	-21458.84	-19438.94	-20953.47
	likelihood	12807.12	10767.29	9761.55	10523.02
3	AIC	-25573.03	-21635.00	-23489.53	-15653.21
	SIC	-25508.86	-21564.43	-23412.55	-15569.81
	likelihood	12796.51	10828.50	11756.77	7839.61
4	AIC	-25361.65	-21749.63	-22632.86	-12774.58
	SIC	-25284.65	-21666.23	-22543.04	-12678.34
	likelihood	12692.82	10887.82	11330.43	6402.29

ST

MA(1) (Gaussian)		p 1	2	3	4
q 1	AIC	-20323.68	-20064.60	-20283.18	-16180.99
	SIC	-20287.23	-20022.08	-20234.58	-16126.33
	likelihood	10167.84	10039.30	10149.59	8099.50
2	AIC	-20007.96	-13423.10	8161.60	-18161.20
	SIC	-19959.36	-13368.43	8222.35	-18094.38
	likelihood	10011.98	6720.55	-4070.80	9091.60
3	AIC	-19871.76	-13605.81	-19731.14	-14633.56
	SIC	-19811.01	-13538.98	-19658.24	-14554.59
	likelihood	9945.88	6813.90	9877.57	7329.78
4	AIC	-19868.49	-13784.41	1315.05	30455.24
	SIC	-19795.58	-13705.43	1400.11	30546.36
	likelihood	9946.24	6905.20	-643.53	-15212.62

MA(1) (Double exp)		p 1	2	3	4
q 1	AIC	-20847.32	-20848.32	-20847.63	-20847.28
	SIC	-20810.93	-20805.80	-20799.04	-20792.61
	likelihood	10429.69	10431.16	10431.82	10432.64
2	AIC	-20850.96	-18506.45	-19790.31	-20171.00
	SIC	-20802.36	-18451.77	-19729.57	-20104.18
	likelihood	10433.48	9262.22	9905.16	10096.50
3	AIC	-20765.63	-18559.83	-20079.62	-20847.99
	SIC	-20704.89	-18493.00	-20006.72	-20769.01
	likelihood	10392.82	9290.91	10051.81	10436.99
4	AIC	-20847.72	-18611.68	-20691.03	-20846.40
	SIC	-20774.82	-18532.71	-20606.77	-20755.28
	likelihood	10435.86	9318.84	10359.91	10438.20

MA(1) (Gen error)		p 1	2	3	4
q 1	AIC	-20857.81	**-20859.01**	-18141.94	-18174.97
	SIC	**-20815.29**	-20810.48	-18087.27	-18114.23
	likelihood	10435.90	10437.54	9079.97	9097.49
2	AIC	-20574.96	-17143.00	-18822.60	-19490.09
	SIC	-20520.29	-17082.25	-18755.78	-19417.18
	likelihood	10296.48	8581.50	9422.30	9757.04
3	AIC	-20651.60	-17235.69	-19808.16	-19148.10
	SIC	-20584.78	-17162.80	-19729.19	-19063.05
	likelihood	10336.80	8629.85	9917.08	9588.05
4	AIC	-20858.78	-17325.78	-12482.67	-15091.45
	SIC	-20779.81	-17240.73	-12391.54	-14994.26
	likelihood	10442.39	8676.89	6256.33	7561.73

Appendix to Chapter 5

The following tables report the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC) of order up to A-PGARCH(4,4,) for five Asian countries. The blanks indicate the failure of estimation due to 'no convergence'.

KOSPI

MA(1) (Gaussian)		p1	p2	p3	p4
q1	AIC	-27496.00	-27491.49	-27490.71	-27491.97
	SIC	-27451.09	-27440.16	-27432.97	-27427.81
	likelihood	13755.00	13753.74	13754.36	13755.99
q2	AIC	-27487.42	-27379.36	-27386.87	-27480.24
	SIC	-27429.68	-27315.21	-27316.30	-27403.25
	likelihood	13752.71	13699.68	13704.44	13752.12
q3	AIC				
	SIC				
	likelihood				
q4	AIC			-27244.05	
	SIC			-27147.81	
	likelihood			13637.02	

MA(1) (Double exp)		p1	p2	p3	p4
q1	AIC	-28076.48	-28074.99	-28075.91	-28081.47
	SIC	-28031.57	-28023.66	-28018.17	-28017.32
	likelihood	14045.24	14045.49	14046.96	14050.74
q2	AIC		-27889.50	-27889.91	
	SIC		-27825.34	-27819.24	
	likelihood		13954.75	13955.91	
q3	AIC	-28082.76			
	SIC	-28012.19			
	likelihood	14052.38			
q4	AIC		-27982.19		
	SIC		-27892.37		
	likelihood		14005.10		

MA(1) (Ged)		p1	p2	p3	p4
q1	AIC	-28077.03	-28074.89	-28076.23	-28080.55
	SIC	-28025.71	-28017.15	-28012.07	-28009.98
	likelihood	14046.52	14046.45	14048.12	14051.28
q2	AIC		-27859.30	-28066.65	
	SIC		-27788.73	-27989.66	
	likelihood		13940.65	14045.33	
q3	AIC	-28083.11			
	SIC	-28006.11			
	likelihood	14053.55			
q4	AIC				
	SIC				
	likelihood				

MA(1) (t)		p1	p2	p3	p4
q1	AIC	-28071.42	-28069.73	-28069.75	-28077.53
	SIC	-28020.09	-28011.98	-28005.60	-28006.96
	likelihood	14043.71	14043.86	14044.88	14049.77
q2	AIC			-27951.97	-28069.32
	SIC			-27874.98	-27985.92
	likelihood			13987.98	14047.66
q3	AIC	-28071.32			
	SIC	-27994.33			
	likelihood	14047.66			
q4	AIC			-28083.26	
	SIC			-27980.61	
	likelihood			14057.63	

NIKKEI

MA(1) (Gaussian)		p1	p2	p3	p4
q1	AIC	-29519.95	-29528.91	-29560.02	-29557.00
	SIC	-29475.04	-29447.58	-29502.27	-29492.84
	likelihood	14766.98	14772.45	14789.01	14788.50
q2	AIC		-28776.44		
	SIC		-28712.29		
	likelihood		14398.22		
q3	AIC			-29521.74	-29537.70
	SIC			-29438.34	-29447.87
	likelihood			14773.87	14782.85
q4	AIC				
	SIC				
	likelihood				

MA(1) (Double exp)		p1	p2	p3	p4
q1	AIC	-30042.22	-30040.52	-30049.15	-30050.02
	SIC	-29997.31	-29989.19	-29991.41	-29985.87
	likelihood	15028.11	15028.26	15033.58	15035.01
q2	AIC				-30050.13
	SIC				-29973.14
	likelihood				15037.07
q3	AIC				
	SIC				
	likelihood				
q4	AIC			-29515.00	
	SIC			-29418.76	
	likelihood			14772.50	

MA(1) (Ged)		p1	p2	p3	p4
q1	AIC	-30047.38	-30044.82	-29996.14	-30055.88
	SIC	-29996.05	-29987.08	-29931.99	-29985.30
	likelihood	15031.69	15031.41	15008.07	15038.94
q2	AIC		-30043.71	-30054.17	-29488.80
	SIC		-29973.14	-29977.18	-29405.39
	likelihood		15032.86	15039.09	14757.40
q3	AIC			-29101.81	
	SIC			-29011.98	
	likelihood			14564.90	
q4	AIC		-29696.97	-29086.75	-30000.98
	SIC		-29600.73	-28984.10	-29891.91
	likelihood		14863.48	14559.38	15017.49

MA(1) (t)		p1	p2	p3	p4
q1	AIC	-30042.30	-30040.85	-30050.72	-30052.02
	SIC	-29990.98	-29983.10	-29986.56	-29981.45
	likelihood	15029.15	15029.42	15035.36	15037.01
q2	AIC		-30040.24	-30048.96	-30049.14
	SIC		-29969.66	-29971.97	-29965.74
	likelihood		15031.12	15036.48	15037.57
q3	AIC	-30074.21		-30075.33	
	SIC	-29997.22		-29985.51	
	likelihood	15049.11		15051.67	
q4	AIC		-29044.34		-29376.76
	SIC		-28948.11		-29267.69
	likelihood		14537.17		14705.38

HANGSENG

MA(1) (Gaussian)		p1	p2	p3	p4
q1	AIC	-25523.31	-25521.03	-25544.11	-25531.08
	SIC	-25478.40	-25469.71	-25486.36	-25466.92
	likelihood	12768.66	12768.52	12781.05	12775.54
q2	AIC				
	SIC				
	likelihood				
q3	AIC				
	SIC				
	likelihood				
q4	AIC			-24509.08	
	SIC			-24412.85	
	likelihood			12269.54	

MA(1) (Double exp)		p1	p2	p3	p4
q1	AIC	-26101.52	-26098.13	-26098.14	-26098.32
	SIC	-26056.61	-26046.80	-26040.40	-26034.16
	likelihood	13057.76	13057.06	13058.07	13059.16
q2	AIC	-26097.69			-26101.07
	SIC	-26039.95			-26024.08
	likelihood	13057.84			13062.54
q3	AIC				
	SIC				
	likelihood				
q4	AIC			-25744.84	
	SIC			-25648.61	
	likelihood			12887.42	

MA(1) (Ged)		p1	p2	p3	p4
q1	AIC	-26106.87	-26103.70	-26101.61	-26102.46
	SIC	-26055.55	-26045.96	-26037.46	-26031.88
	likelihood	13061.44	13060.85	13060.81	13062.23
q2	AIC	-26101.31			
	SIC	-26037.15			
	likelihood	13060.65			
q3	AIC				
	SIC				
	likelihood				
q4	AIC			-25727.48	
	SIC			-25169.83	
	likelihood			12652.24	

MA(1) (t)		p1	p2	p3	p4
q1	AIC	-26128.80	-26125.89	-26126.44	-26129.27
	SIC	-26077.48	-26068.15	-26062.28	-26058.69
	likelihood	13072.40	13071.95	13073.22	13075.63
q2	AIC		-26126.19	-26124.83	-26130.61
	SIC		-26055.61	-26047.84	-26047.21
	likelihood		13074.09	13074.41	13078.31
q3	AIC				
	SIC				
	likelihood				
q4	AIC				
	SIC				
	likelihood				

TAIWAN

MA(1) (Gaussian)		p1	p2	p3	p4
q1	AIC	-25084.19	-25079.89	-25070.57	-25074.36
	SIC	-25039.28	-25028.56	-25012.82	-25010.20
	likelihood	12549.09	12547.95	12544.28	12547.18
q2	AIC			-25087.89	-24624.80
	SIC			-25017.31	-24547.81
	likelihood			12554.94	12324.40
q3	AIC			-24652.74	
	SIC			-24569.34	
	likelihood			12339.37	
q4	AIC		-24632.67	-24596.35	
	SIC		-24542.85	-24500.11	
	likelihood		12330.34	12313.17	

MA(1) (Double exp)		p1	p2	p3	p4
q1	AIC	-25607.33	-25603.77	-25601.54	-25600.92
	SIC	-25562.42	-25552.44	-25543.80	-25536.76
	likelihood	12810.67	12809.88	12809.77	12810.46
q2	AIC		-25165.67	-25598.53	-25049.67
	SIC		-25101.51	-25527.95	-24972.68
	likelihood		12592.83	12810.26	12536.83
q3	AIC				
	SIC				
	likelihood				
q4	AIC		-25205.65		
	SIC		-25115.82		
	likelihood		12616.82		

MA(1) (Ged)		p1	p2	p3	p4
q1	AIC	-25605.22	-25600.80	-25600.20	-25599.23
	SIC	-25553.90	-25543.06	-25536.04	-25528.66
	likelihood	12810.61	12809.40	12810.10	12810.62
q2	AIC			-25607.59	-25567.77
	SIC			-25530.60	-25484.37
	likelihood			12815.80	12796.89
q3	AIC		-24984.80		
	SIC		-24901.40		
	likelihood		12505.40		
q4	AIC				
	SIC				
	likelihood				

MA(1) (t)		p1	p2	p3	p4
q1	AIC	-25520.40	-25516.94	-25513.69	-25512.07
	SIC	-25469.08	-25459.19	-25449.53	-25441.49
	likelihood	12768.20	12767.47	12766.84	12767.03
q2	AIC		-25526.50	-25525.35	-25523.66
	SIC		-25455.92	-25448.36	-25440.26
	likelihood		12774.25	12774.67	12774.83
q3	AIC		-24526.69		
	SIC		-24443.29		
	likelihood		12276.35		
q4	AIC				
	SIC				
	likelihood				

SINGAPORE

MA(1) (Gaussian)		p1	p2	p3	p4
q1	AIC	-20396.63	-20392.45	-20429.72	-20424.29
	SIC	-20354.11	-20343.85	-20375.05	-20363.54
	likelihood	10205.32	10204.22	10223.86	10222.15
q2	AIC	-20395.65		-20422.64	-19667.10
	SIC	-20340.98		-20355.81	-19594.20
	likelihood	10206.83		10222.32	9845.55
q3	AIC				-20333.34
	SIC				-20248.30
	likelihood				10180.67
q4	AIC				
	SIC				
	likelihood				

MA(1) (Double exp)		p1	p2	p3	p4
q1	AIC	-20861.44	-20862.01	-20862.17	-20862.95
	SIC	-20818.92	-20813.41	-20807.50	-20802.20
	likelihood	10437.72	10439.00	10440.09	10441.48
q2	AIC	-20863.01		-20863.03	
	SIC	-20808.33		-20796.20	
	likelihood	10440.50		10442.51	
q3	AIC				
	SIC				
	likelihood				
q4	AIC				-20438.88
	SIC				-20341.68
	likelihood				10235.44

MA(1) (Ged)		p1	p2	p3	p4
q1	AIC	-20874.82	-20875.05	-20875.66	-20877.57
	SIC	-20826.22	-20820.37	-20814.91	-20810.75
	likelihood	10445.41	10446.52	10447.83	10449.78
q2	AIC			-20526.92	
	SIC			-20454.02	
	likelihood			10275.46	
q3	AIC				
	SIC				
	likelihood				
q4	AIC				-20064.17
	SIC				-19960.89
	likelihood				10049.08

MA(1) (t)		p1	p2	p3	p4
q1	AIC	-20956.58	-20957.33	-20956.70	-20954.73
	SIC	-20907.98	-20902.66	-20895.95	-20887.91
	likelihood	10486.29	10487.67	10488.35	10488.36
q2	AIC	-20957.05	-20955.15	-20956.53	-20957.06
	SIC	-20896.30	-20888.33	-20883.63	-20878.08
	likelihood	10488.52	10488.58	10490.26	10491.53
q3	AIC		-20940.33		
	SIC		-20861.36		
	likelihood		10483.17		
q4	AIC				-19700.50
	SIC				-19597.23
	likelihood				9867.25

The following two tables show the estimated parameters of the selected model by the AIC. The models in Table 5.1 are not included here. * and ** denote the Bollerslev's GARCH and Taylor/Schwert models respectively.

A-PGARCH Models	b	theta	w	a_1	a_2	a_3	a_4	$beta_1$
KOSPI								
MA(1)- A-GARCH(1,1)*	0.0002	0.061	1E-05	0.122				0.801
(gaussian)	(1.04)	(3.57)	(15.07)	(14.12)				(69.07)
MA(1)- A-PGARCH(1,3)	-3E-05	0.055	2E-05	0.175	0.011	-0.097		0.922
(double exp)	(0.29)	(4.32)	(1.13)	(5.42)	(0.19)	(2.82)		(90.66)
MA(1)- A-PGARCH(3,4)	-0.0001	0.070	2E-05	0.116	0.044	-0.023	-0.080	0.686
(t)	(0.99)	(4.66)	(1.14)	(6.46)	(1.21)	(0.73)	(3.81)	(3.19)
NIKKEI								
MA(1)- A-PGARCH(3,1)	0.0004	0.044	3E-05	0.156				0.734
(gaussian)	(3.31)	(2.72)	(2.67)	(18.49)				(14.30)
MA(1)- A-PGARCH(4,1)	7E-06	0.001	1E-04	0.147				0.851
(double exp)	(0.08)	(0.07)	(1.30)	(7.71)				(4.91)
MA(1)- A-PGARCH(4,1)	0.0002	0.008	9E-05	0.137				0.914
(gen error)	(1.91)	(0.61)	(1.39)	(8.25)				(5.51)
Hang Seng								
MA(1)- A-PGARCH(3,1)	0.0009	0.088	2E-05	0.142				0.469
(gaussian)	(4.45)	(5.41)	(4.17)	(13.90)				(10.96)
MA(1)- A-PGARCH(1,1)	0.0003	0.032	1E-04	0.125				0.861
(double exp)	(1.87)	(2.63)	(2.46)	(8.08)				(61.55)
MA(1)- A-PGARCH(1,1)	0.0004	0.045	8E-05	0.123				0.855
(gen error)	(2.70)	(3.34)	(2.45)	(8.54)				(65.35)
SE								
MA(1)- A-PGARCH(3,2)	0.0004	0.029	9.E-06	0.057	0.078			0.521
(gaussian)	(1.96)	(1.84)	(2.27)	(6.28)	(3.27)			(1.90)
MA(1)- A-PGARCH(1,1)	-6.E-11	4.E-07	6.E-05	0.103				0.909
(double exp)	(0.00)	(0.00)	(1.55)	(1.06)				(1.01)
MA(1)- A-GARCH(2,2)**	5.E-04	0.017	1.E-04	0.039	0.047			0.797
(t)	(3.13)	(1.29)	(2.65)	(2.92)	(1.71)			(2.25)
Straits-Times								
MA(1)- A-PGARCH(1,1)	0.0004	0.228	1.E-08	0.130				0.564
(gaussian)	(1.88)	(11.20)	(0.83)	(8.68)				(32.78)
MA(1)- A-PGARCH(3,2)	0.0002	0.170	3.E-05	0.269	0.164			-0.311
(double exp)	(1.19)	(10.59)	(0.63)	(6.58)	(2.95)			(2.27)
MA(1)- A-GARCH(4,1)*	0.0002	0.191	2.E-05	0.240				0.536
(gen error)	(1.06)	(10.87)	(5.91)	(7.52)				(3.70)

A-PGARCH Models	beta$_2$	beta$_3$	beta$_4$	delta	γ_1	γ_2	γ_3	γ_4
KOSPI								
MA(1)- A-GARCH(1,1)*				2.000	-0.150			
(gaussian)					(5.24)			
MA(1)- A-PGARCH(1,3)				1.480	-0.215	-0.686	-0.383	
(double exp)				(8.68)	(1.95)	(0.15)	(1.78)	
MA(1)- A-PGARCH(3,4)	0.413	-0.167		1.330	-0.189	-0.215	-0.445	-0.212
(t)	(1.33)	(1.06)		(9.30)	(2.00)	(0.71)	(0.62)	(1.61)
NIKKEI								
MA(1)- A-PGARCH(3,1)	-0.264	0.371		1.470	-0.501			
(gaussian)	(3.58)	(7.53)		(20.08)	(15.43)			
MA(1)- A-PGARCH(4,1)	-0.143	-0.028	0.187	1.199	-0.527			
(double exp)	(0.49)	(0.10)	(1.40)	(7.89)	(7.65)			
MA(1)- A-PGARCH(4,1)	-0.290	0.107	0.139	1.218	-0.510			
(gen error)	(1.05)	(0.43)	(1.20)	(8.56)	(8.28)			
Hang Seng								
MA(1)- A-PGARCH(3,1)	0.655	-0.303		1.881	-0.270			
(gaussian)	(22.75)	(8.17)		(32.51)	(10.47)			
MA(1)- A-PGARCH(1,1)				1.410	-0.314			
(double exp)				(15.01)	(4.79)			
MA(1)- A-PGARCH(1,1)				1.518	-0.300			
(gen error)				(15.80)	(5.12)			
SE								
MA(1)- A-PGARCH(3,2)	0.173	0.149		1.924	-0.318	0.068		
(gaussian)	(0.58)	(1.51)		(22.21)	(4.49)	(0.89)		
MA(1)- A-PGARCH(1,1)				1.317	-0.103			
(double exp)				(1.05)	(2.08)			
MA(1)- A-GARCH(2,2)**	0.107			1.000	-0.723	0.392		
(t)	(0.33)				(2.13)	(1.05)		
Straits-Times								
MA(1)- A-PGARCH(1,1)				3.624	-0.247			
(gaussian)				(13.96)	(8.82)			
MA(1)- A-PGARCH(3,2)	0.278	0.390		2.018	-0.124	-0.239		
(double exp)	(3.37)	(4.93)		(5.83)	(1.95)	(2.56)		
MA(1)- A-GARCH(4,1)*	-0.134	0.073	0.123	2.000	-0.197			
(gen error)	(0.76)	(0.46)	(1.44)		(3.79)			

In addition to Table 5.7 this table reports the moments of the conditional variance of the selected model in the previous tables.

A-PGARCH Models	$\sum_{i=1}^{\tilde{p}} \tilde{\beta}_i$	$E\left(h_t^{\frac{\delta}{2}}\right)$	γ_h^0	$E\left(h_t^{\delta}\right)$
KOSPI				
MA(1)- A-GARCH(1,1)* (gaussian)	0.9257	0.0001	0.2457	2E-08
MA(1)- A-PGARCH(1,3) (double exp)	0.9916	0.0023	0.8794	5E-05
MA(1)- A-PGARCH(3,4) (t)	0.9666	0.0006	0.1469	4E-07
NIKKEI				
MA(1)- A-PGARCH(3,1) (gaussian)	0.9860	0.0021	1.0685	-
MA(1)- A-PGARCH(4,1) (double exp)	0.9775	0.0044	0.4827	4E-05
MA(1)- A-PGARCH(4,1) (gen error)	0.9763	0.0038	0.3916	2E-05
Hang Seng				
MA(1)- A-PGARCH(3,1) (gaussian)	0.9654	0.0006	0.7060	1E-06
MA(1)- A-PGARCH(1,1) (double exp)	0.9596	0.0024	0.3247	9E-06
MA(1)- A-PGARCH(1,1) (gen error)	0.9579	0.0019	0.3587	6E-06
SE				
MA(1)- A-PGARCH(3,2) (gaussian)	0.9797	0.0004	0.5801	5E-07
MA(1)- A-PGARCH(1,1) (double exp)	0.9861	0.0043	0.3940	3E-05
MA(1)- A-GARCH(2,2)** (t)	0.9484	0.0029	0.0479	9E-06
Straits-Times				
MA(1)- A-PGARCH(1,1) (gaussian)	0.9549	2E-07	22.060	-
MA(1)- A-PGARCH(3,2) (double exp)	0.8086	0.0002	1.4910	-
MA(1)- A-GARCH(4,1)* (gen error)	0.8473	0.0001	0.8280	1E-07

This table presents the value of the likelihood ratio (LR) test of high order models chosen by the AIC. This table excluded the models in Table 5.4.

A-PGARCH Models	Likelihood Ratio	Critical value (at 5% significant level)
KOSPI		
MA(1)- A-GARCH(1,3) (double exp)	14.28	9.49
MA(1)- A-PGARCH(3,4) (t)	27.84	15.50
NIKKEI		
MA(1)- A-PGARCH(3,1) (gaussian)	44.06	5.99
MA(1)- A-PGARCH(4,1) (double exp)	13.80	7.81
MA(1)- A-PGARCH(4,1) (gen error)	14.50	7.81
Hang Seng		
MA(1)- A-PGARCH(3,1) (gaussian)	24.78	5.99
SE		
MA(1)- A-PGARCH(3,2) (gaussian)	11.70	9.49
MA(1)- A-GARCH(2,2) (t)	12.10	7.81
Straits-Times		
MA(1)- A-PGARCH(3,2) (double exp)	9.58	9.49
MA(1)- A-GARCH(4,1) (gen error)	10.48	7.81

The following table reports the AIC numbers and Log likelihood numbers of the Bollerslev model and the Taylor/Schwert model along with those of the A-PGARCH model. The underlining indicates the minimum value of the AIC and the preferred model by the LR test among the three models.

A-PGARCH Models

KOSPI

MA(1)- A-PGARCH(1,1) (gaussian)	delta=1	delta=1.86	delta=2
AIC	-27425.98	-27496.00	-27499.63
Log Likelihood	13718.99	13755.00	13755.82

MA(1)- A-PGARCH(1.3) (double exp)	delta=1	delta=1.48	delta=2
AIC	-28070.99	-28082.76	-28013.55
Log Likelihood	14045.50	14052.38	14016.77

MA(1)- A-PGARCH(1,3) (gen error)	delta=1	delta=1.49	delta-2
AIC	-28071.32	-28083.10	-28077.29
Log Likelihood	14046.66	14053.55	14049.64

MA(1)- A-PGARCH(3.4) (t)	delta=1	delta=1.33	delta=2
AIC	-28073.72	-28083.26	-27394.29
Log Likelihood	14051.86	14057.63	13712.15

NIKKEI

MA(1)- A-PGARCH(3,1) (gaussian)	delta=1	delta=1.47	delta=2
AIC	-29525.85	-29560.02	-29559.81
Log Likelihood	14770.93	14789.01	14787.91

MA(1)- A-PGARCH(4.1) (double exp)	delta=1	delta=1.20	delta=2
AIC	-30047.56	-30050.03	-30033.50
Log Likelihood	15032.78	15035.01	15025.75

MA(1)- A-PGARCH(4,1) (gen error)	delta=1	delta=1.22	delta=2
AIC	-30051.52	-30055.88	-30038.33
Log Likelihood	15035.76	15038.94	15029.16

MA(1)- A-PGARCH(3.3) (t)	delta=1	delta=1.26	delta=2
AIC	-29974.74	-30075.33	-30059.31
Log Likelihood	15000.47	15051.67	15042.66

Hang Seng

MA(1)- A-PGARCH(3.1) (gaussian)	delta=1	delta=1.88	delta=2
AIC	-25415.76	-25544.11	-25525.73
Log Likelihood	12715.88	12781.05	12770.86

MA(1)- A-PGARCH(1.1) (double exp)	delta=1	delta=1.41	delta=2
AIC	-26089.04	-26101.52	-26098.77
Log Likelihood	13050.52	13057.76	13055.38

MA(1)- A-PGARCH(1,1) (gen error)	delta=1	delta=1.52	delta=2
AIC	-26088.29	-26106.87	-26105.38
Log Likelihood	13050.52	13061.44	13059.69

MA(1)- A-PGARCH(4,2) (t)	delta=1	delta=1.28	delta=2
AIC	-26126.64	-26130.61	-26117.25
Log Likelihood	13075.32	13078.31	13070.63

SE

MA(1)- A-PGARCH(3,2) (gaussian)	delta=1	delta=1.92	delta=2
AIC	-24967.87	-25087.89	-25079.78
Log Likelihood	12493.94	12554.94	12549.89

MA(1)- A-PGARCH(1,1) (double exp)	delta=1	delta=1.32	delta=2
AIC	-25602.19	-25607.33	-25595.11
Log Likelihood	12807.10	12810.67	12803.56

MA(1)- A-PGARCH(3.2) (gen error)	delta=1	delta=1.34	delta 2
AIC	-25603.12	-25607.59	-25448.46
Log Likelihood	12812.56	12815.80	12735.23

MA(1)- A-PGARCH(2.2) (t)	delta=1	delta=1.16	delta=2
AIC	-25525.87	-25526.50	-24379.38
Log Likelihood	12772.94	12774.25	12199.69

Straits-Times

MA(1)- A-PGARCH(1,1) (gaussian)	delta=1	delta=3.62	delta 2
AIC	-20278.87	-20396.63	-20369.24
Log Likelihood	10145.43	10205.32	10190.62

MA(1)- A-PGARCH(3.2) (double exp)	delta=1	delta=2.02	delta=2
AIC	-20627.87	-20863.03	-20786.26
Log Likelihood	10324.94	10442.51	10403.13

MA(1)- A-PGARCH(4.1) (gen error)	delta=1	delta=2.13	delta=2
AIC	-20681.42	-20877.57	-20879.68
Log Likelihood	10353.71	10449.78	10449.84

MA(1)- A-PGARCH(2,1) (t)	delta=1	delta=1.89	delta=2
AIC	-20938.26	-20957.33	-20959.44
Log Likelihood	10479.13	10487.67	10487.72

Bibliography

[1] Abadir, K. M.,1999. An introduction to hypergeometric functions for economists. Econometric Review, 18, 287-330.

[2] Akerlof, G.A., Dickens, W.T., Perry, G.L. 2000, Near-rational wage and price setting and the long-run Phillips curve, Brookings Papers on Economic Activity, 1-60.

[3] Akgiray, V., 1989. Conditional heteroskedasticity in time series of stock returns: evidence and forecasts. Journal of Business 62, 55-80.

[4] Amano, R., Coletti. D., Macklem. T., 1999. Monetary rules when economic behaviour changes. Bank of Canada, Working Paper 99-8.

[5] Arestis, P., Caporale. G. M., Cipollini, A., Does inflation targeting affect the trade-off between output-gap and inflation variability? South Bank University, unpublished manuscript.

[6] Baek. E., Brock. W., 1992. A general test for nonlinear Granger causality: bivariate model. working paper. University of Wisconsin, Madison.

[7] Baillie, R., Chung, H., 2001. Estimation of GARCH models from the autocorrelations of the squares of a process. Journal of Time Series Analysis 6, 631-650.

[8] Baillie, R., Chung, C., Tieslau, M., 1996. Analyzing inflation by the fractionally integrated ARFIMA-GARCH model, Journal of Applied Econometrics 11, 23-40.

[9] Ball, L., 1992. Why does high inflation raise inflation uncertainty?, Journal of Monetary Economics 29, 371-388.

[10] Batini, N., Haldane, A., 1998. Forward looking rules for monetary policy. Bank of England discussion paper.

[11] Bean, C., 1998. The new UK monetary arrangements: a view from the literature. Economic Journal 108, 1795-1805.

[12] Bekaert, G., Harvey, C. R., 2000. Foreign speculators and emerging equity markets. Journal of Finance 55, 565-613.

[13] Bera, A. K., Kim, S., 1996. Testing Constancy of Correlation with an Application to International Equity Returns, Mimeo, Center for International Business Education and Research (CIBER), University of Illinois, Urbana-Champaign, Working Paper 96-107.

[14] Bernanke, B., 1983. Irreversibility, uncertainty, and cyclical investment, Quarterly Journal of Economics 98, 85-106.

[15] Bernanke, B., Blinder, A., 1992. The Federal fund rate and the transmission of monetary policy. American Economic Review 82, 901-921.

[16] Bernanke, B., Mishkin, F., 1997. Inflation targeting: A new framework for monetary policy? Journal of Economic Perspectives 9, 97-116.

[17] Berndt, E., Hall, B., Hall, R., Hausman, J., 1974. Estimation and inference in nonlinear structural models. Annals of Economic and Social Measurement 3, 653-665.

[18] Bessembinder, H., Seguin, P.J., 1993 Price volatility, trading volume, and market depth: Evidence from futures markets. Journal of Financial and Quantitative Analysis 28, 21-39.

[19] Black, F., 1976. Studies of stock price volatility changes. Proceedings of the 1976 meetings of the Business and Economics Statistics Section, American Statistical Association, pp. 177-181, Alexandria VA: American Statistical Association.

[20] Bhagat, S., Bhatia, S., 1994. Trading volume and price variability: evidence on lead-lag relationships from Granger-causality tests. Graduate School of Business Administration, University of Colorado, unpublished manuscript.

[21] Black, F., 1987. Business Cycles and Equilibrium. Basil Blackwell, New York.

[22] Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307-327.

[23] Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. Review of Economics and Statistics 72, 498-505.

[24] Bollerslev, T., Engle, R., Nelson, D., 1994. ARCH models. in Engle, R., McFadden, D., (ed.). Handbook of Econometrics Vol 4, North Holland.

[25] Bollerslev, T., Engle, R., Wooldridge, M., 1988. A capital asset pricing model with time-varying covariances, Journal of Political Economy 96, 116-131.

[26] Bollerslev, T., Mikkelsen, H. O. 1996. Modeling and pricing long memory in stock market volatility. Journal of Econometrics 73, 151-184.

[27] Bollerslev, T., Wooldridge, J. M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances, Econometric Reviews 11, 143-172.

[28] Boyd, J., Levine, R., Smith, B. 2001. The impact of inflation on financial sector performance, Journal of Monetary Economics, 47, 221-248.

[29] Brailsford, T. J., 1996. The empirical relationship between trading volume, returns and volatility. Accounting and Finance 35, 89-111.

[30] Brock, W., 1974. Money and growth: the case of long-run perfect foresight, International Economic Review 15, 750-777.

[31] Brock, W., 1993. Pathways to Randomness in Economy: Emergent nonlinearity and chaos in economics and finance. Estudios Economicos 8, 3-28.

[32] Brooks, C., 1998. Predicting stock index volatility: Can market volume help? Journal of Forecasting 17, 59-80.

[33] Brooks, R.D., Faff. R.W., McKenzie. M.D., 2000. A multi-country study of power ARCH models and national stock market returns. Journal of International Money and Finance 19. 377-397.

[34] Brunner, A., 1993. Comment on Inflation regimes and the sources of inflation uncertainty. Journal of Money, Credit, and Banking 25. 512-514.

[35] Calvo, G.A. 1983. Staggered prices in a utility maximizing framework. Journal of Monetary Economics 12, 383-398.

[36] Campbell, J. Y., Grossman. S. J., Wang, J., 1993. Trding volume and serial correlation in stock returns, Quarterly Journal of Economics 108, 905-939.

[37] Caporale, T., McKiernan. B., 1996. The relationship between output variability and growth: evidence from post war UK data. Scottish Journal of Political Economy 43, 229-236.

[38] Caporale, T., McKiernan. B., 1997. High and variable inflation: further evidence on the Friedman hypothesis, Economics Letters 54, 65-68.

[39] Caporale, T., McKiernan. B., 1998. The Fischer Black hypothesis: some time-series evidence, Southern Economic Journal 64, 765-771.

[40] Cecchetti, S., Ehrmann. M., S., 1999. Does inflation targeting increase output volatility? An international comparison of policy makers' preferences and outcomes. NBER Working Paper 7426.

[41] Cecchetti, S., Krause. S., 2001. Financial structure, macroeconomic stability and monetary policy. NBER Working Paper 8354.

[42] Cecchetti, S., Krause, S., 2002. Central Bank structure, policy efficiency, and macroeconomic performance: Exploring empirical relationships, Federal Reserve Bank of St. Louis Review 84, 47-60.

[43] Chen, G., Firth, M., Rui, O. M., 2001. The dynamic relation between stock returns, trading voulme, and volatility. The Financial Review. 38, 153-174.

[44] Choi, S., Smith, B, Boyd, J. 1996. Inflation, financial markets, and capital formation. St. Louis Review 78, 9-35.

[45] Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: a New Keynesian perspective, Journal of Economic Literature 37, 1661-1707.

[46] Clark, T., 1997. Cross-country evidence on long-run growth and inflation. Economic Inquiry 35, 70-81.

[47] Copeland, T. E., 1976. A model of asset trading under the assumption of sequential information arrival. Journal of Finance 31, 1149-1168.

[48] Coulson, E., Robins, R., 1985. Aggregate economic activity and the variance of inflation: another look, Economics Letters, 71-75.

[49] Cukierman, A., Meltzer, A., 1986. A theory of ambiguity, credibility, and inflation under discretion and asymmetric information, Econometrica 54, 1099-1128.

[50] Dacorogna, M.M., Müller, U.A., Nagler, R.J., Olsen, R.B., Pictet, O.V., 1993. A geographical model for the daily and weekly seasonal volatility in the FX market. Journal of International Money and Finance 12, 413-438.

[51] Davis. G., Kanago. B., 2000. The level and uncertainty of inflation: results from OECD forecasts. Economic Inquiry 38, 58-72.

[52] Demos, A., 2001. Moments and dynamic structure of a time-varying-parameter stochastic volatility in mean model. Department of International and European Economic Studies, Athens University of Economic and Business, unpublished manuscript.

[53] Deveraux, M., 1989. A positive theory of inflation and inflation variance. Economic Inquiry 27, 105-116.

[54] Deveraux, M., Yetman, J. 2002. Menu costs and the long-run output-inflation trade off. Economics Letters 76, 95-100.

[55] Ding, Z., Granger, C.W.J., 1996. Modeling volatility persistence of speculative returns: a new approach. Journal of Econometrics 73, 185-215.

[56] Ding, Z., Ganger C.W.J., Engle, R., 1993. A long memory property of stock market returns and a new model. Journal of Empirical Finance 1, 83-106.

[57] Donaldson, R. D., Kamstra, M., 1997. An artificial neural network-GARCH model for international stock return volatility. Journal of Empirical Finance 4, 17-46.

[58] Dotsey, M., Sarte, P., 2000. Inflation uncertainty and growth in a cash-in-advance economy. Journal of Monetary Economics 45, 631-655.

[59] Duan, J.C., 1995. The GARCH option pricing model. Mathematical Finance 5, 13-32.

[60] Edison, H.. Reinhart. C.M.. 2001. Stopping hot money. Journal of Development Economics 66, 533-553.

[61] Engle. R., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987-1007.

[62] Engle, R., Kroner. K. F.. 1995. Multivariate simultaneous generalised ARCH. Econometric Theory 11, 122-150.

[63] Engle, R., Lee. G..1999. A long-run and short-run component model of stock return volatility. in Engle. R., White. H. (ed.), Cointegration, causality, and forecasting. Oxford, New York.

[64] Engle, R., Ng, V.. 1993. Measuring and testing the impact of news on volatility. Journal of Finance 48, 1749-1778.

[65] Engle, R., Lilien. D.. Robbins, R.. 1987. Estimating time varying risk premia in the term structure: the ARCH-M model. Econometrica. 55, 391-407.

[66] Epps, T., Epps, M. L.. 1976. The stochastic dependence of security price changes and transaction volumes: implications for the mixture-of-ditributions hypothesis. Econometrica 44, 305-321.

[67] Erceg, C., Henderson, D.W.. Levin, A.T.. 1998. Tradeoffs between inflation and output gap variances in an optimising agent model, International Finance Discussion papers, Washington: Board of Governors of the Federal Reserve System. No 627.

[68] Fiorentini. G., Sentana. E.. 1998. Conditional means of time series processes and time series processes for conditional means. International Economic Review 39. 1101-1118.

[69] Fischer, S.. 1993. The role of macroeconomic factors in growth, Journal of Monetary Economics 32, 485-511.

[70] Fornari, F., Mele. A., 1997. Weak convergence and distributional assumptions for a general class of nonlinear ARCH models. Econometric Reviews 16, 205-229.

[71] Fornari, F.. Mele. A.. 2001. Recovering the probability density function of asset prices using GARCH as diffusion approximations. Journal of Empirical Finance 8, 83-110.

[72] Fountas, S., 2001. The relationship between inflation and inflation uncertainty in the UK: 1885-1998. Economics Letters 74, 77-83.

[73] Fountas. S., Karanasos. M., Kim. J..2002. Inflation and output growth uncertainty and their relationship with inflation and output growth. Economics Letters 75. 293-301

[74] Friedman, M., 1968. The role of monetary policy, American Economic Review 58, 1-17.

[75] Friedman. M., 1977. Nobel lecture: Inflation and unemployment, Journal of Political Economy 85. 451-472.

[76] Fuhrer, J., 1997. Inflation/output variance trade-offs and optimal monetary policy. Journal of Money. Credit. and Banking 29. 214-234.

[77] Fuhrer, J., Moore, G., 1995. Monetary policy trade-offs and the correlations between nominal interest rates and real output. American Economic Review 85,.219-239

[78] Gallant, A.R.. Rossi. P.. Tauchen, G., 1992. Stock prices and volume. Review of Financial studies 5. 199-242.

[79] Goel, R., Ram, R.. 2001. Irreversibility of R&D investment and the adverse effect of uncertainty: Evidence from OECD countries. Economics Letters 71, 287-291.

[80] Goodfriend. M., King, R.. 1997. The new neoclassical synthesis and the role of monetary policy. NBER Macroeconomics annual 1997. MIT press, Cambridge. 233-283.

[81] Grabel, I., 1995. Assessing the impact of financial liberalisation on stock market volatility in selected developing countries. The Journal of Development Studies 31, 903-917.

[82] Gradshteyn, I., Ryzhik. I.. 1994. Table of Integrals. Series. and Products. London: Academic Press.

[83] Granger, C.W.J.. Ding, Z., 1995a. Some properties of absolute returns: an alternative measure of risk. Annales d'Economie de Statistique 40, 67-95.

[84] Granger, C.W.J.. Ding. Z.. 1995b. Stylized facts on the temporal and distributional properties of daily data from speculative returns. Department of Economics. University of California San Diego. unpublished manuscript.

[85] Grier. K., Perry. M.. 1998. On inflation and inflation uncertainty in the G7 countries, Journal of International Money and Finance 17, 671-689.

[86] Grier, K., Perry. M.. 2000. The effects of real and nominal uncertainty on inflation and output growth: Some GARCH-M evidence. Journal of Applied Econometrics 15. 45-58.

[87] Grier, K., Tullock. G.. 1989. An empirical analysis of cross-national economic growth: 1951-1980. Journal of Monetary Economics 24, 259-276.

[88] Hafner, C.M., Herwartz, H., 2001. Option pricing under linear autoregressive dynamics, heteroskedasticity. and conditional leptokurtosis. Journal of Empirical Finance 8, 1-34.

[89] Hagerud, G.E., 1997a. Modeling nordic stock returns with asymmetric GARCH models. Department of Finance, Stockholm School of Economics, working paper series in economics and finance, No 164.

[90] Hagerud, G.E., 1997b. Specification tests for asymmetric GARCH models. Department of Finance. Stockholm School of Economics, unpublished manuscript.

[91] Härdle, W.. Hafner, C.M.. 2000. Discrete time option pricing with flexible volatility estimation. Finance and Stochastics 4. 189-207.

[92] Harris, L.. 1987. Transaction data tests of the mixture of distributions hypothesis. Journal of Financial and Quantitative Analysis. 22, 127-141.

[93] Harris, M.. Raviv. A.. 1993. Differences of opinion make a horse race. Review of Financial studies 6.473-506.

[94] He, C., Teräsvirta, T., 1999a. Fourth moment structure of the GARCH(p,q) model. Econometric Theory 15, 824-846.

[95] He, C., Teräsvirta, T., 1999b. Statistical properties of the asymmetric power ARCH model, in Engle, R., White, H. (ed.), Cointegration, causality, and forecasting. Oxford, New York.

[96] He, C., Teräsvirta, T., 1999c. Properties of moments of a family of GARCH processes. Journal of Econometrics 92, 173-192.

[97] He, C., Teräsvirta, T., Malmsten, H., 2001. Fourth moment structure of a family of first-order exponential GARCH models. Econometric Theory, forthcoming.

[98] Henry, O., Olekalns, N., 2001. The effect of recessions on the relationship between output variability and growth, Southern Economic Journal 68, 683-692.

[99] Henry, P. B., 2000. Stock market liberalisation, economic reform, and emerging market equity prices. Journal of Finance 55, 529-564.

[100] Hentschel, L., 1995. All in the family. Nesting symmetric and asymmetric GARCH models. Journal of Financial Economics 39, 71-104.

[101] Hiemstra, C., Jones, J. D., 1994. Testing for linear and nonlinear Granger causality in the stock-volume relation. Journal of Finance 49, 1639-1664.

[102] Holland, S., 1993a. Uncertain effects of money and the link between the inflation rate and inflation uncertainty. Economic Inquiry 41, 39-51.

[103] Holland, S., 1993b. Comment on 'Inflation regimes and the sources of inflation uncertainty'. Journal of Money, Credit, and Banking 25, 514-520.

[104] Holland, S., 1995. Inflation and uncertainty: Tests for temporal ordering, Journal of Money, Credit, and Banking 27, 827-837.

[105] Hu, M. Y., Jiang, C. X., Tsoukalas, C., 1997. The European exchange rates before and after the establishment of the European monetary system. Journal of International Financial Markets, Institutions and Money 7, 235-253.

[106] Huybens, E., Smith, B. 1999. Inflation, financial markets, and long-run real activity, Journal of Monetary Economics 43, 283-315.

[107] Hwang, Y., 2001. Relationship between inflation rate and inflation uncertainty, Economics Letters 73, 179-186.

[108] Jadresic, E., 1999. Inflation targeting and output stability. IMF Working Paper, No.61.

[109] Jansen, D., 1989. Does inflation uncertainty affect output growth? Further evidence. Federal Reserve Bank of St. Louis Review, 43-54.

[110] Karanasos, M., 1999. The second moment and the autocovariance function of the squared errors of the GARCH model. Journal of Econometrics 90, 63-76.

[111] Karanasos, M., 2001. Prediction in ARMA models with GARCH-in-mean effects. Journal of Time Series Analysis 5, 555-576.

[112] Karanasos, M., Kim, J., 2000. Moments of the ARMA-EGARCH model. Department of Economics and Related Studies, University of York, unpublished manuscript.

[113] Karanasos, M., Kim, J., 2001. A re-examination of the asymmetric power ARCH model. Department of Economics and Related Studies, University of York, unpublished manuscript.

[114] Karpoff, J. M., 1987. The relation between price changes and trading voulme: a survey. Journal of Financial and Quantitative analysis 22, 109-126.

[115] Keynes, J. M., 1936. The General Theory of employment, interest, and money, London: Macmillan.

[116] Kim, E. H., Singal, V., 2000. Stock market openings: experience of emerging economics. Journal of Business 73, 25-66.

[117] Kim, K.W. (ed.). 2000. Two years after the IMF bailout: A Review of the Korean Economy's Transformation, Sam-Sung Research Economic Institution Report, Seoul: Sam-Sung Research Economic Institution

[118] Kim, K.W. (ed.). 2001. Three years after the IMF bailout: A Review of the Korean Economy Transformation Since 1998, Sam-Sung Research Economic Institution Report, Seoul: Sam-Sung Research Economic Institution.

[119] King, R.G., Wolman, A.L. 1996. Inflation targeting in a St. Louis model of the 21st century, NBER Working Paper No. 5507.

[120] King, R.G., Wolman, A.L. 1998. What should monetary authority do when price are sticky? in Taylor, J (ed.). Monetary Policy Rules, University of Chicago Press, Chicago.

[121] Kormendi, R., Meguire, P., 1985. Macroeconomic determinants of growth: cross-country evidence, Journal of Monetary Economics 16, 141-163.

[122] Krause, S., 2001. Measuring monetary policy efficiency in European Union countries, unpublished manuscript.

[123] Lamoureux, C. G., Lastrapes, W. D., 1990. Heteroskedasticity in stock return data: volume versus GARCH effects. Journal of Finance 45, 221-229.

[124] Lee, J., 1999. The inflation and output variability tradeoff: evidence from a GARCH model. Economics Letters 62, 63-67.

[125] Lee, B. S., Rui, O. M., 2002. The dynamic relationship between stock returns and trading volume: Domestic and corss-country evidence. Journal of Banking and Finance 26, 51-78.

[126] Logue, D., Sweeney, R., 1981. Inflation and real growth: some empirical results. Journal of Money, Credit, and Banking 13, 497-501.

[127] Lo, A.W., MacKinlay, A.C., 1988. Stock market prices do not follow random walks: evidence from a simple specification test. Review of Financial Studies 1, 41-66.

[128] Lucas, R., 1973. Some international evidence on output-inflation trade-offs, American Economic Review 63, 326-334.

[129] Mirman, L., 1971. Uncertainty and optimal consumption decisions. Econometrica 39, 179-185.

[130] Müller, U.A., Dacorogna, M.M., Davé, R.D., Olsen, R.B., Pictet, O.V., Von Weizsäcker, J.E., 1997. Volatilities of different time resolutions-analyzing the dynamics of market components. Journal of Empirical Finance 4, 213-239.

[131] Nelson, D., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica 59, 347-370.

[132] Nelson, D., Cao, C. Q., 1992. Inequality constraints in the univariate GARCH model. Journal of Business and Economic Statistics 10, 229-235.

[133] Newey, W. K., West, K., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703-708.

[134] Palma, W., Zevallos, M., 2001. Analysis of the correlation structure of square time series. Departmento de Estadustica, Pontificia Universidad Catulica de Chile, unpublished manuscript.

[135] Pindyck, R., 1991. Irreversibility, uncertainty, and investment, Journal of Economic Literature 29, 1110-1148.

[136] Prudnikov, A. P., Brychkov, Yu. A., Marichev, O. I., 1992. Integrals and Series. Glasgow: Bell and Bain Ltd.

[137] Pyun. C. S., Lee, S. Y., Nam. K., 2000. Volatility and information flows in emerging equity market a case of the Korean stock exchange. International Review of Financial Analysis 9, 405-420.

[138] Ramey, G., Ramey, V., 1991. Technology commitment and the cost of economic fluctuations. NBER Working Paper No. 3755.

[139] Ramey, G., Ramey, V., 1995. Cross-country evidence on the link between volatility and growth. American Economic Review 85, 1138-1151.

[140] Rotemberg, J., Woodford, M., 1998. An optimisation based econometric framework for the evaluation of monetary police, in NBER Macroeconomics Annual, MIT press, 297-346.

[141] Saatcioglu, K., Starks, L. T., 1998. The stock price-volume relationship in emerging stock markets: the case of Latin America. International Journal of Forecasting 14, 215-225.

[142] Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of Financial Economics 5, 309-327.

[143] Schwert, W., 1990. Stock volatility and the crash of '87. Review of Financial Studies 3, 77-102.

[144] Silvapulle, P., Choi, J., 1999. Testing for linear and nonlinear Granger causality in the stock price-volume relation: Korean evidence. The Quarterly of Economics and Finance 39, 59-76.

[145] Singh, A., 1997. Financial liberalisation, stock markets and economic development. The Economic Journal 107, 771-782.

[146] Singh, A., Weisse, B. A., 1998. Emerging stock markets, portfolio capital flows and long-term economic growth: micro and macroeconomic perspectives. World Development 26, 607-622.

[147] Smirlock, M., Starks, L. T., 1988. An empirical analysis of the stock price-volume relationship. Journal of Banking and Finance 12, 31-41.

[148] Speight, A.,1999. UK output variability and growth: some further evidence. Scottish Journal of Political Economy 46, 175-184.

[149] Spyrou, S. I., Kassimatis, K., 1999. Did equity market volatility increase following the opening of emerging markets to foreign investors? Journal of Economic Development 24, 30-51.

[150] Stiglitz, J., 2002. Globalisation and its discontents. Penguin Books, London.

[151] Stockman, A., 1981. Anticipated inflation and the capital stock in a cash-in-advance economy. Journal of Monetary Economics 8, 387-393.

[152] Svensson L.E.O 1998. Open-economy inflation targeting, National Bureau of Economic Research, Working Paper No. 6545.

[153] Taylor, J., 1979. Estimation and control of a macroeconomic model with rational expectations. Econometrica 47, 1267-86.

[154] Taylor, J., 1993. Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy, No. 39, 195-214.

[155] Taylor, J., 1994. The inflation-output variability trade-off revisited. in Goals, Guidelines and Constraints Facing Monetary Policymakers. Federal Reserve Bank of Boston, Conference series, No. 39, 152-214.

[156] Taylor. S., 1986. Modeling Financial Time Series. Wiley. New York.

[157] Tse, Y. K., 1991. Price and volume in the Tokyostock exchange. in Ziemba, W. T., Bailey, W. Hamao, Y. (ed.). Japanese Financial Market Research. Elsevier Science. Amesterdam.

[158] Tse, Y. K., 1998. The conditional heteroscedasticity of the Yen-Dollar exchange rates. Journal of Applied Econometrics. 13, 49 - 55.

[159] Tobin, J., 1965. Money and economic growth. Econometrica 33, 671-684.

[160] Ungar, M., Zilberfarb. B., 1993. Inflation and its unpredictability- Theory and empirical evidence. Journal of Money, Credit, and Banking 25, 709-720.

[161] Vilasuso, J., 2001. Causality tests and conditional heteroscedasticity: Monte Carlo evidence. Journal of Econometrics 101, 25-35.

[162] Walsh, C. 1998. Monetary Theory and Policy, MIT Press, Cambridge.

[163] Wang, J., 1994 A model of competitive stock trading volume, Journal of Political Economy 102.127-168.

[164] White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48, 817-838.

[165] Zarnowitz, V., Moore, G.. 1986. Major changes in cyclical behaviour, in Gordon, R. (ed.). The American Business Cycle, University of Chicago Press.

[166] Zhang, J., 2000. Inflation and growth: pecuniary transactions costs and qualitative equivalence, Journal of Money, Credit, and Banking 32, 1-12.