

**Reliability based disaggregate stochastic process models with  
strict capacity constraints in congested transit networks**

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The candidate confirms that the work submitted is her own and that appropriate credit has been given where reference has been made to the work of others.

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## **Abstract**

Reliability is considered the most important attribute of transit service by passengers. There are congested transit environments wherein even if a transit service is perfectly on schedule it is termed unreliable from a passenger's perspective when they are unable to board the first service of their choice set. The phenomenon 'failure to board' arises in congested transit networks having strict physical capacity constraints wherein the transit service cannot take in passengers beyond its capacity. This results in some of the passengers being left waiting for the next service at the transit stops.

The existing transit assignment models; be it hyperpath based effective frequency models, Bureau of Public Roads (BPR) based route section models or aggregate stochastic process models with strict capacity constraints, all assume that the passengers have perfect knowledge of the network seldom discussing the sources of such information. In the current thesis this assumption is renounced and a reliability based disaggregate stochastic process model with strict capacity constraints (R-DSPM) using route section approach is proposed such that each passenger in the absence of information updates his/her route choice based on their individual experience. Though the aggregate stochastic process model implements the strict capacity constraint for each transit service generated; the model along with the assumption of perfect knowledge of the network assumes that the passengers are risk neutral. The proposed R-DSPM implements a strict capacity constraint for each transit service generated thereby accounting for failure to board situation in congested network. The proposed model differs from the existing aggregate stochastic process model in its assumption of risk averse passengers. Risk aversion in R-DSPM considers variance associated with:- interarrival times of transit service at the transit stop; the waiting time of passengers due to the 'failure to board' condition; the in-vehicle travel times of routes comprising of route sections containing more than one attractive line section and the variable demand generated for each day's travel. The risk aversion of each passenger is accounted for in R-DSPM through the linear combination of mean total travel time and total travel variance (mean-variance) and a linear combination of mean total travel time and expected lateness (mean-lateness). A generic day to day framework is developed with markovian properties such that it enables the integration of both mean-variance and mean-lateness costs with ease and results in a unique stationary distribution of costs and flows for each route.

The proposed R-DSPM thus accounts for: strict capacity constraints of transit vehicle, passengers learning process, risk aversion of passengers, differences in passenger perceptions, day to day variability in demand and supply of transit network. The micro simulation framework shows through implementation on example networks that while accounting for passenger's risk aversion the R-DSPM is able to arrive at a unique stationary distribution irrespective of its initial conditions. The sensitivity of the proposed R-DSPM with strict capacity constraint under different parameter assumptions has been carried out .

A calibration of the parameters involved in the route section based BPR styled cost function and the hyperpath based effective frequency cost function using the proposed R-DSPM indicates that different congestion function parameters are required for different sections of a transit network. It is also shown through implementation on an example network that the proposed R-DSPM framework enables the passengers to learn about the reliability of routes and strategies. At higher dispersion values R-DSPM assign risk averse passengers such that the standard deviation of flows and experienced total travel time on various routes and strategies are lesser than that obtained by accounting for risk aversion using the aggregate stochastic process models.

The impact of accounting for risk aversion on various policy measures that could be carried out by the operators to improve the waiting time reliability of passengers is also assessed using the proposed R-DSPM with strict capacity constraints. It is shown that for certain parameter assumptions and for certain policy measures the assumption of risk aversion in transit assignment could result in an entirely different reliability profile from that of an assignment process assuming risk neutral passengers. The implementation of the proposed R-DSPM with strict capacity constraints on a real network has been carried out on a section of London underground and several possible policy measures have been evaluated. The evaluation of policies has further emphasised the need to consider the risk aversion in passengers especially to account for the number of passengers preferring to make a transfer (in absence of transfer penalty) at the transfer stops to minimise their risk aversion costs.

## Table of Contents

<b>Acknowledgements</b> .....	<b>iii</b>
<b>Abstract</b> .....	<b>iv</b>
<b>Table of Contents</b> .....	<b>vi</b>
<b>List of Tables</b> .....	<b>x</b>
<b>List of Figures</b> .....	<b>xiv</b>
<b>Chapter 1 Introduction</b> .....	<b>1</b>
1.1 Modelling Operational Characteristics in transit networks .....	4
1.3 Congested Transit system and reliability .....	5
1.4 Variations in transit supply and demand , in passenger perceptions and in behaviour.....	5
1.5 Research Context.....	6
1.6 Gap in Literature .....	9
1.7 Research Objectives:.....	11
1.8 Thesis layout.....	12
<b>Chapter 2 Literature Review</b> .....	<b>14</b>
2.1 Introduction .....	14
2.2 Transit assignment models.....	16
2.2.1 Common lines problem:.....	17
2.2.2 Unconstrained transit assignment models:.....	18
2.2.2.1 Route – Section approach: .....	19
2.2.2.2 Hyperpath/ Strategy approach:.....	20
2.2.3 Capacity constrained models:.....	30
2.2.3.1 BPR models.....	30
2.2.3.2 ‘Effective frequency’ models .....	32
2.2.3.3 ‘Strict capacity’ models .....	34
2.2.3.4 Dynamic models.....	35
2.3 Random Utility Models (RUMs): .....	35
2.3.1 Probabilistic choice models – Multinomial Logit Model (MNL): .....	35
2.3.2 C-Logit and Path size logit:.....	36
2.3.3 Probit Models:.....	37
2.4 Reliability from passenger’s perspective: .....	38
2.4.1 Scheduling approach:.....	39

2.4.2 Mean – variance approach: .....	41
2.5 Stochastic process models (SPMs):.....	43
2.6 Summary .....	46
<b>Chapter 3 Disaggregate Stochastic Process Model formulation and implementation on risk neutral passengers .....</b>	<b>47</b>
3.1 Introduction to stochastic process models .....	47
3.1.1 Overview of aggregate stochastic process model in uncongested network.....	48
3.1.2 Disaggregate stochastic process model .....	53
3.2 Model description .....	59
3.2.1 Model Inputs.....	60
3.2.1.1 Network Supply .....	60
3.2.1.2 Transit Services.....	60
3.2.1.3 Passenger demand.....	62
3.2.1.4 Learning process Model: .....	62
3.2.1.5 Route Choice formulation: .....	64
3.3. Methodological framework:.....	64
3.4 Example Network 1 .....	67
3.4.1 Uncongested network:.....	68
3.4.2 Congested Network: .....	69
3.4.2.1 Autocorrelation .....	73
3.5 Initial conditions: .....	77
3.6 Sensitivity Analysis: .....	83
3.6.1 Autocorrelation .....	83
3.6.2 Partial Autocorrelation.....	85
3.6.3 Varying memory lengths.....	86
3.7 Summary .....	88
<b>Chapter 4 R-DSPM formulation for Mean – Variance and comparison with existing models .....</b>	<b>89</b>
4.1 Risk Averse vs Risk Neutral passengers (uncongested transit network): .....	90
4.1.1 Change in the shortest route for risk averse passengers:.....	91
4.1.2 Influence of interarrival reliability in the shortest route for risk averse passengers:.....	93
4.1.3 Need for a stochastic process model to analyse risk averse transit network.....	94
4.2 R-DSPM :- Mean –Variance cost for congested network: .....	95

4.2.1 Implementation on example networks:.....	96
4.2.2 Presence of stationary distribution:.....	101
4.2.3 Initial Conditions .....	102
4.2.4 Randomness test.....	105
4.2.4.1 Autocorrelation .....	105
4.2.4.2 Partial Autocorrelation .....	108
4.2.5 Failure to board:.....	108
4.2.6 Sensitivity tests: .....	110
4.2.6.1 Sensitivity analysis with Different shape factors: .....	110
4.2.6.2 Sensitivity analysis with different memory lengths ....	111
4.2.6.3 Sensitivity analysis with different $\beta$ values .....	113
4.2.7 Implications for general networks.....	114
4.3 Risk aversion models using BPR cost function, effective frequency cost function to account for congestion and mean- variance based aggregate stochastic process model. ....	115
4.3.1 Stochastic equilibrium model with BPR – type congestion function-Route section approach: .....	115
4.3.2 Deterministic user equilibrium with ‘effective frequency’ function - hyperpath approach:.....	120
4.3.3 Aggregate stochastic process model .....	122
4.4 Summary .....	124
<b>Chapter 5 R-DSPM formulation for Mean – Lateness cost and Policy Evaluation .....</b>	<b>125</b>
5.1 R-DSPM with strict capacity constraints using mean-lateness cost .....	126
5.2 Implementation on example network 2.....	129
5.3 Policy Interventions:.....	131
5.3.1 Giving information to the passengers: .....	139
5.5 Summary .....	143
<b>Chapter 6 London Underground – Case Study .....</b>	<b>145</b>
6.1 Introduction .....	145
6.2 Data Description.....	145
6.3 Demand and Supply Data: .....	147
6.4 Calibration: .....	149
6.4.1 Risk neutral:.....	149
6.4.2 Risk averse: .....	152
6.5 Validation:.....	152



6.6 Result discussion.....	153
6.7 Summary .....	159
<b>Chapter 7 Summary, Conclusion and Further Studies .....</b>	<b>161</b>
7.1 Summary: .....	161
7.2 Conclusions: .....	162
166	
7.3 Further research: .....	168
<b>List of References .....</b>	<b>173</b>
<b>Appendix A Formulation of Optimal strategy and assignment of flows on a test network as in Spiess and Florian, (1989) .....</b>	<b>182</b>
<b>Appendix B Route Choice of passengers at Transit stop with and without signs at stop information .....</b>	<b>187</b>
<b>Appendix C Markovian property Check for Mean-lateness model.....</b>	<b>189</b>
C.1 Converging irrespective of initial condition:.....	189
C.2 Sensitivity analysis :.....	191
C.2.1 Differing shape factors.....	191
C.2.2 Differing value of lateness.....	192
C.2.3 Differing memory lengths .....	193
C.3 Policy Interventions.....	194
C.3.1 Increasing capacity of line 2:.....	194
<b>Appendix D Publications .....</b>	<b>197</b>

## List of Tables

Table 2.1: Chen et al. (2002)'s Classification of route choice models in traffic assignment studies .....	14
Table 2.2: All possible Route Sections and all possible Strategies for the network in Fig 2.1 a. ....	19
Table 3.1 Waiting time realisations on Day 1 for route A .....	50
Table 3.2 Waiting time realisations on Day 2 for route A .....	51
Table 3.3 Waiting time realisations on Day 3 for route B .....	52
Table 3.4 Waiting time realisations on Day 4 on route B .....	52
Table 3.5: The enumerated 'sensible' routes .....	68
Table 3.6: Total Travel time obtained using a single realisation of micro simulation model for various routes over the simulation period of 700 days with $m = 1$ , $\alpha = 5$ and $\xi = 0.05$ .....	69
Table 3.7: Total Travel time obtained using De Cea and Fernandez L (1989) .....	69
Table 3.8: The congested costs for risk neutral passengers in network with shape factor $m = 1$ , $\alpha = 5$ and $\xi = 0.05$ .....	71
Table 3.9: The congested costs for risk neutral passengers in network with shape factor $m = 300$ , $\alpha = 5$ and $\xi = 0.05$ .....	75
Table 3.10 : route costs and flow distributions and sensitivity to initial conditions $m = 1$ , $\alpha = 5$ and $\xi = 0.05$ .....	78
Table 3.11 : route costs and flow distributions and sensitivity to initial conditions $m = 300$ , $\alpha = 5$ and $\xi = 0.05$ .....	79
Table 3.12: Wilcoxon Rank sum test results for different initial conditions for $m = 1$ , $\alpha = 5$ and $\xi = 0.05$ .....	81
Table 3.13: Wilcoxon Rank sum test results for different initial conditions for $m = 300$ , $\alpha = 5$ and $\xi = 0.05$ .....	81
Table 3.14: Two-sample Kolmogorov-Smirnov test results for different initial conditions for $m = 1$ , $\alpha = 5$ and $\xi = 0.05$ .....	82
Table 3.15: Two-sample Kolmogorov-Smirnov test results for different initial conditions for $m = 300$ , $\alpha = 5$ and $\xi = 0.05$ .....	83
Table 3.16: The average experienced travel time and the average flow on various routes for varying memory lengths when $m = 1$ and $\xi = 4$ .....	87
Table 4.1: Revised frequency for example network 1 .....	91
Table 4.2: The mean cost for routes between various OD pairs for modified example network 1 (table 4.1) with $m = 1$ .....	92
Table 4.3: The mean cost for the routes between various OD pairs for modified example network 1 (table 4.1) with $m = 300$ .....	93

Table 4.4 The experienced total travel time and flow values on various routes for risk neutral and risk averse passengers at $m = 1$ , $\xi=0.05$ , $\beta = 2.5$ and $\varrho=15$ days (scenario 2) .....	99
Table 4.5 The experienced total travel time and flow values on various routes for risk neutral and risk averse passengers at $m = 1$ , $\xi =0.05$ , $\beta = 2.5$ and $\varrho=15$ days (scenario 1).....	99
Table 4.6 The experienced total travel times and flow values on various routes for risk neutral and risk averse passengers at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ days (scenario 2) .....	100
Table 4.7 The experienced total travel times and flow values on various routes for risk neutral and risk averse passengers at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ days (scenario 1) .....	100
Table 4.8: Convergence to same distribution irrespective of its initial condition for risk averse passengers (example network 2) at $m = 1$ , $\xi=4$ , $\beta = 2.5$ , $\varrho=15$ days (scenario 2) .....	103
Table 4.9: Convergence to same distribution irrespective of its initial condition for risk averse passengers (example network 2) at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ days (scenario 1).....	103
Table 4.10 Wilcoxon rank sum test for risk averse passenger (example network 2) at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ (scenario 2).....	104
Table 4.11 Wilcoxon rank sum test for risk averse passenger (example network 2) at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ (scenario 1) .....	104
Table 4.12 Two sample Kolmogorov-Smirnov test for risk averse passenger (example network 2) at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\varrho=15$ (scenario 2).....	105
Table 4.13 : Experienced total travel time and flows along routes 2, 7,9 for various memory length (scenario 2) $m = 1$ at $\xi = 4$ .....	112
Table 4.14: The sensitivity of route experienced total travel times and flows to various $\beta$ values for $\xi = 4$ , $m = 1$ , $\varrho = 15$ (scenario 2).....	113
Table 4.15: The sensitivity of route flows to various $\beta$ values for $\xi = 0.05$ , $m = 1$ , $\varrho = 15$ (scenario 2).....	114
Table 4.16 : The calibration values of the parameters of various route sections of example network 2 .....	119
Table 4.17: Logit -SUE results for example network 2 using BPR -styled cost function for OD 1 - 400 and OD 2- 250. ....	120
Table 4.18: Calibration values of the parameters associated with example network 2. ....	121
Table 4.19 : Distribution of risk neutral passenger flows.....	122
Table 4.20 : Distribution of risk averse passenger flows .....	122
Table 4.21: Experienced total travel time and flows along various routes using aggregate stochastic process model with mean-variance cost (scenario 2) $\beta = 2.5$ , $\xi = 4$ , and $\varrho = 15$ .....	123

Table 5.1 <i>Boarding</i> Line loads on all lines without any policy implementation at $\xi=4, \varrho=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	134
Table 5.2: <i>Boarding</i> Line loads at $\xi=4$ for increased reliability of line 2, $\varrho=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	135
Table 5.3: <i>Boarding</i> Line loads at $\xi=4$ for increased capacity on line 2, $\varrho=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1)$ and $\mathcal{T}(Z = 2) = 32.5$ .....	137
Table 6.1 the in-vehicle travel time between the stations in the chosen case study section .....	146
Table 6.2 Uncongested travel time assuming risk neutral passengers - Case study 1 .....	150
Table 6.3: The calibration of on-board passengers in the transit service along the Jubilee and Metropolitan Line .....	151
Table 6.4 Mean Variance and Mean-lateness hypothesis testing for calibration .....	152
Table 6.5 Mean-Variance and Mean -lateness hypothesis testing for validation.....	153
Table 6.6 Variation in number of passengers <i>boarding</i> Baker Street and Finchley Road for the observed total travel time distribution ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4 \beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15, \theta_2/\theta_1 = 5$ ).....	154
Table 6.7 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for network with both lines having improved interarrival service reliability ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4 \beta = 2.25$ for mean-variance cost).....	156
Table 6.8 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for network with jubilee line having increased capacity ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4 \beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15, \theta_2/\theta_1 = 5$ ).....	158
Table 6.9 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for increased frequency of metropolitan line ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4 \beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15, \theta_2/\theta_1 = 5$ ).....	159
Table A.1: Possible Alternatives for travelling within the example network along with their costs .....	182

Table A.2: Finding Optimal Strategy for the Test Network .....	183
Table A.3: Assign Demand on Test Network.....	183
Table B.1: Deterministic headway: on-line information vs no information .....	188
Table C.1: The convergence of mean-latency model irrespective of its initial condition $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\xi = 4, \eta =$ $15, \theta_2/\theta_1 = 5$ .....	190
Table C.2: Wilcoxon rank sum test $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\xi = 4, \eta = 15, \theta_2/\theta_1 = 5$ .....	190
Table C.3: Two-sample Kolmogorov-Smirnov test $\mathcal{T}(Z = 1) =$ $48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\xi = 4, \eta = 15, \theta_2/\theta_1 = 5$ .....	191

## List of Figures

Fig 1.1 (a). Registered Vehicles under various categories in India (b).Trends in public transport demand in Great Britain 1970-2000 (Source: Balcombe et al., 2004) .....	2
Fig 2.1 . Different representations of a transit network (a) transit network (b) Hyperpath representation (c) Route section Representation .....	19
Fig 2.2: Frequency Relationship between G and H .....	26
Fig 2.3: Width Relationship between G and H .....	26
Fig 2.4 : Presence of multiple strategies as shortest hyperpath/strategy .....	34
Fig 2.5: Introduction on failure node and failure arc in hyperpath representation.....	35
Fig 2.6: Source: Ceder (2007): reliability attributes of concern to passengers .....	38
Fig 2.7 : Source : Small (1982) Disutility of schedule delay.....	40
Fig 3.1 Simplistic Network .....	49
Fig 3.2 The probability distribution of erlang headway of 10 services per hour for varying shape factors .....	62
Fig 3.3 (a) : R-DSPM:-passenger generation module .....	65
Fig 3.3 (b) : R-DSPM for risk neutral passengers.....	66
Fig 3.4: Example Network 1 .....	67
Fig 3.5 : Possible route sections in the network .....	67
Fig 3.6: The variation in the expectation of predicted costs of risk neutral passengers and the expectation of experienced costs in example network with $m = 1$ & 300 (Route 4).....	72
Fig 3.7: Histogram of risk neutral passenger flows along routes 4,6,8 (a)201-400 (b) 401-600 for $m=1$ $\gamma = 5$ and $\xi = 0.05$ .....	73
Fig 3.8: Autocorrelogram of flows in congested network with error bounds calculated using equation 3.18 along routes 4,6,8 and 10 for $m = 1$ , $\gamma = 5$ and $\xi = 0.05$ .....	74
Fig 3.9: Histogram of risk neutral passenger flows along routes 4, 6, 8 for (a) 201-400 days (b) 401-600 days $m=300$ , $\gamma = 5$ and $\xi = 0.05$ .....	76
Fig 3.10: Autocorrelogram of risk neutral passenger flows in congested network for routes 4,6,8 and 10 with error bounds computed using equation 3.18, for $m= 300$ , $\gamma = 5$ and $\xi = 0.05$ .....	77
Fig 3.11 The stationary distribution of flows on routes 4, 6 and 8 under (a) initial condition 1 (b) initial condition 2 and (c) initial condition 3 for $m = 1$ , $\gamma = 5$ and $\xi = 0.05$ . .....	79

Fig 3.12 The stationary distribution of flows on routes 4, 6 and 8 under (a) initial condition 1 (b) initial condition 2 and (c) initial condition 3 for $m = 300$ , $\gamma = 5$ and $\xi = 0.05$ .....	80
Fig 3.13 The autocorrelogram of flows along routes 4, 6 and 8 for $m = 1$ and $\gamma = 5$ (a) $\xi = 0.05$ (b) $\xi = 1.8$ (c) $\xi = 4$ .....	84
Fig 3.14 The autocorrelogram of flows along routes 4, 6 and 8 for $m = 300$ and $\gamma = 5$ (a) $\xi = 0.05$ (b) $\xi = 1.8$ (c) $\xi = 4$ .....	85
Fig 3.15 The partial - autocorrelogram of flows along routes 4, 6 and 8 with 95% confidence bounds for $m = 1$ and $\gamma = 5$ (a) $\xi = 0.05$ (b) $\xi = 1.8$ (c) $\xi = 4$ .....	86
Fig 3.16 The partial autocorrelogram of flows along routes 4, 6 and 8 with 95% confidence bounds for $m = 300$ and $\gamma = 5$ (a) $\xi = 0.05$ (b) $\xi = 1.8$ (c) $\xi = 4$ .....	86
Fig 4.1: The mean vs variances of shortest routes between Node 1 and Node 3 for modified example network 1 (table 4.1) with $m = 1$ .....	92
Fig 4.2: The mean vs variances of optimal routes between Node 1 and Node 3 for example network 1 (table 4.1) with $m = 300$ .....	94
Fig 4.3 Example network 2 (Teklu, 2008b) .....	97
Fig 4.4 (a) All the route sections for example network 2 and (b) possible routes .....	97
Fig 4.5: Evolution of costs during burn in period and stationary distribution of flows on routes 9,10,11 for risk neutral and risk averse passengers at $m = 1$ , $\xi=4$ , $\beta = 2.5$ and $\gamma=15$ (scenario 2).....	101
Fig 4.6 Stationary distribution for routes 9,10 and 11 (a) between days 201-400 (b) between days 401-600 at $m = 1$ , $\xi=4$ , $\beta = 2.5$ , $\gamma=15$ days (Scenario 2).....	102
Fig 4.7 Stationary distribution for routes 9,10 and 11, (a) between days 201-400 (b) between days 401-600 at $m = 1$ , $\xi=4$ , $\beta = 2.5$ , $\gamma=15$ days (Scenario 1).....	102
Fig 4.8 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 2) at $m = 1$ , $\xi = 4$ , $\beta = 2.5$ and $\gamma = 15$ .....	106
Fig 4.9 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 1) at $m = 1$ , $\xi = 4$ , $\beta = 2.5$ and $\gamma = 15$ .....	107
Fig 4.10 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 2) at $m = 1$ , $\xi = 0.05$ , $\beta = 2.5$ and $\gamma = 15$ .....	107
Fig 4.11 Partial Autocorrelation of flows on routes 2,7,9 with 95% confidence bounds (a) $m = 1$ (b) $m = 300$ (scenario 2) at $\xi = 4$ , $\beta = 2.5$ and $\gamma = 15$ . .....	108
Fig 4.12: Number of passengers experiencing failure to board on day 450 at $m = 1$ , $\xi = 4$ , $\beta = 2.5$ and $\gamma = 15$ . (example 2 - scenario 2).....	109
Fig 4.13: Risk neutral vs risk averse - shape factor $m = 300$ at $\xi = 4$ and $\gamma=15$ days (a) experienced route total travel times (b) flows on the route (scenario 2).....	110

Fig 4.14: Risk neutral vs risk averse ( $\beta = 2.5$ ) – shape factor $m = 1$ at $\xi = 4$ total experienced travel time distribution for various memory lengths on route 2, 7, 9 (scenario 2) .....	111
Fig 4.15: Risk neutral ( $\beta = 0$ ) vs risk averse ( $\beta = 2.5$ ) – shape factor $m = 1$ at $\xi = 4$ flow distribution for various memory lengths on route 2, 7, 9 (scenario 1).....	112
Fig 4.16 The sensitivity of route experienced total travel times and flows to various $\beta$ values for $\xi = 4, m = 1, \gamma = 15$ (scenario 2).....	113
Fig 4.17 The sensitivity of route flows to various $\beta$ values for $\xi = 0.05, m = 1, \gamma = 15$ (scenario 2) .....	114
Fig 4.18 : Calibration of the parameters of the route sections of example network 2. ....	118
Fig 4.19: Calibration of line segments for the example network 2 .....	121
Fig 5.1: The distribution of flows when the ‘acceptable total travel time’ $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\xi = 4, \gamma = 15, \theta_2/\theta_1 = 5$ (scenario 2) .....	130
Fig 5.2: The distribution of flows when the ‘acceptable total travel time’ $\mathcal{T}(Z = 1) = 123.5$ and $\mathcal{T}(Z = 2) = 107.5$ , $\xi = 4, \gamma = 15, \theta_2/\theta_1 = 5$ (scenario 2) .....	131
Fig 5.3 Waiting Time reliability for various cost without any policy implementation at $\xi=0.05, \gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	133
Fig 5.4 Waiting Time reliability for various cost without any policy implementation at $\xi=4, \gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	133
Fig 5.5 : Waiting Time reliability for various cost Line 2 $m=300$ at $\xi=4, \gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	135
Fig 5.6: Waiting Time reliability for various cost l2 cap=25 at $\xi=4, \gamma = 15, \beta = 5$ (mean-variance), $\theta_2/\theta_1 = 5, \gamma=15, \beta = 2.5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	136
Fig 5.7 Waiting Time reliability at $\xi=4$ for increased frequency of line 1, $\gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	138
Fig 5.8 Waiting Time reliability at $\xi=4$ changed dwell time line 2, $\gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	138
Fig 5.9: Waiting time reliability with and without information for various costs at stop 1 of example network 2, $\xi=4, \gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	140
Fig 5.10: Waiting time reliability with and without information for various costs at stop 2 of example network 2, $\xi=4, \gamma=15, \beta = 2.5, \theta_2/\theta_1 = 5, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ .....	141
Fig 6.1 OD pair 1 from Baker Street to Wembley park (a section of London Underground) OD pair 2 from Finchley Road to Wembley Park. (Line 1 – Jubilee line and Line 2 – Metropolitan Line). ....	146



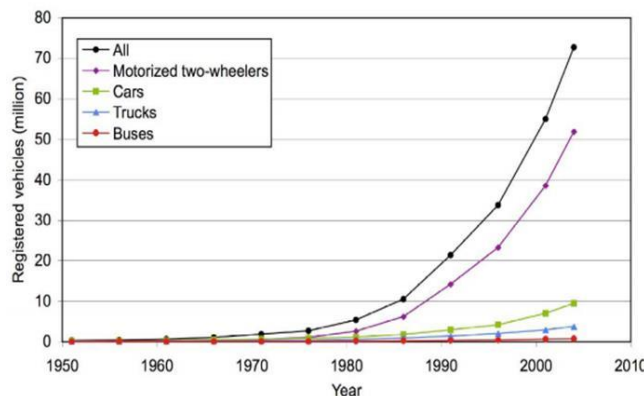
Fig 6.2 Interval distribution of Passengers (a) Baker street for OD1 (b) Finchley Road .....	148
Fig 6.3 Journey time distribution (a) From Baker Street to Wembley Park (b) From Finchley Road to Wembley Park.....	149
Fig 6.4 Calibration of the on-board loading already present within the transit service assuming risk neutral passengers at Baker street. ....	151
Fig 6.5 Validation of the R-DSPM assuming various cost passengers and using oyster card data between Finchley Road and Wembley Park .....	153
Fig 6.6: waiting time reliability along Jubilee and Metropolitan Line when the reliability of transit service is improved in both lines. ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4$ $\beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15$ , $\theta_2/\theta_1 = 5$ .....	156
Fig 6.7: Waiting time reliability along Jubilee and Metropolitan Line when the capacity of Jubilee line is increased. ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4$ $\beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15$ , $\theta_2/\theta_1 = 5$ ) .....	157
Fig 6.8: Waiting time reliability along Jubilee and Metropolitan Line when the frequency of Metropolitan line is increased. ( $\xi = 0.75$ for risk neutral passengers; at logit dispersion parameter of $\xi = 4$ $\beta = 2.25$ for mean-variance cost; a logit dispersion of $\xi = 0.75$ with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and $\varrho = 15$ , $\theta_2/\theta_1 = 5$ .....	158
Fig 7.1 Comparison of the salient features of the current R-DSPM with existing models available in literature.....	166
Fig 7.2 Comparison of the salient features of various models tested in the thesis. ....	167
Fig A.1: Test network.....	182
Fig A.2: Simplified Test network as in (Spiess and Florian, 1989) .....	183
Fig B.1: Test Network .....	187
Fig C.1: Stationary Distribution on routes 2,3 and 9 between days 201-400 and days 401-600 for $\xi = 0.05$ , $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\varrho = 15$ , $\theta_2/\theta_1 = 5$ .....	189
Fig C.2 the sensitivity of route flows to various shape factors for risk neutral and mean-lateness cost functions at $\xi = 0.05$ $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\varrho = 15$ , $\theta_2/\theta_1 = 5$ .....	191
Fig C.3 Flow and experienced total travel distribution on various routes for varying value of lateness at $\xi = 0.05$ , $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5$ , $\varrho = 15$ .....	192

Fig C.4 Flow and experienced total travel distribution on various routes for varying value of lateness at $\xi = 4, \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15$ .....	192
Fig C.5 : Waiting Time reliability for various cost functions l2 cap=25 at $\xi=0.05 \mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15, \theta_2/\theta_1 = 5, \beta = 2.5$ .....	194
Fig C.6 : Line loads at $\xi=0.05$ for increased capacity $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \xi = 4, \mathfrak{z} = 15, \theta_2/\theta_1 = 5$ .....	194
Fig C.7 Waiting Time reliability at $\xi=0.05$ for increased frequency of line $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15, \theta_2/\theta_1 = 5$ .....	195
Fig C.8: Line loads at $\xi=0.05$ for changed frequency $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15, \theta_2/\theta_1 = 5$ .....	195
Fig C.9 Waiting Time reliability at $\xi=0.05$ changed dwell time line 2 $\mathcal{T}(Z = 1) = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15, \theta_2/\theta_1 = 5$ .....	196
Fig C.10 : Line loads at $\xi=0.05$ for changed dwell time $\mathcal{T}Z = 1 = 48.5$ and $\mathcal{T}(Z = 2) = 32.5, \mathfrak{z} = 15, \theta_2/\theta_1 = 5$ .....	196

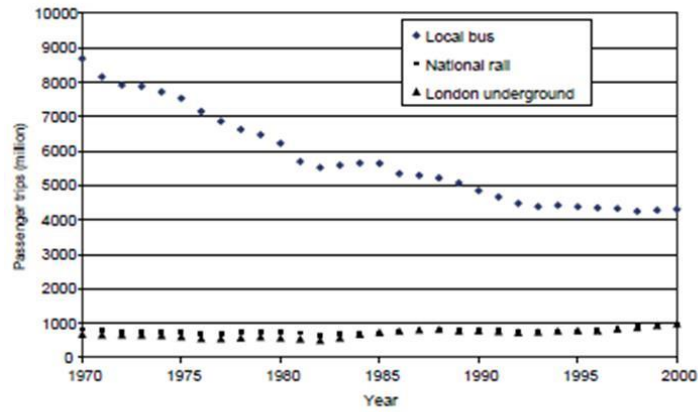
## Chapter 1 Introduction

Urbanisation around the world has intensified the need to travel. Countries, regardless of being developed or developing are faced with the problem of increasing number of vehicles. Transport planners have realised that a sustainable solution is required to deal with the increasing need for mobility. The solution to sustainability is envisioned through the promotion and improvement of public transit services.

The commuting pattern for cities around the world greatly varies from each other but all cities find a major part of the commuting population reliant on public transit. In India the commuting pattern of Delhi indicates that 36.2% of high income households and 31.43 % of the low income households use buses and a further 1.79% of low income households use rail as the commuting mode (Tiwari, 2002). In London around 27.8% of the low income group and 9.3% of high income group use Buses/trams whereas 3.9% of low income group and 12.2% of the high income group use Underground/DLR for daily commuting (TfL 2011). A look at the above percentages leads to a surmise that a growth in the transit network fleet size coupled with a growth in their patronage is the expected trend for the traffic sector. However historical evidence of the vehicle growth over the past years wherein public transit was still a predominant mode of travel indicates otherwise with a decreasing share in public transport patronage over the years (fig 1.1).



(a)



(b)

Fig 1.1 (a). Registered Vehicles under various categories in India <sup>1</sup> (b).Trends in public transport demand in Great Britain 1970-2000 (Source: Balcombe et al., 2004)

Several empirical studies (Balcombe et al., 2004; Peek and van Hagen, 2002; Jackson and Jucker, 1982) have tried to assess the attributes that would make public transit service attractive to commuters and find reliability the most weighed attribute of public transit services. The most common problems reported by transit users are overcrowding, particularly during peak hours, and the lack of service reliability (Badami and Haider, 2007; Ceder, 2007; Peek and van Hagen, 2002).

The main manifestations of public transport unreliability are excessive waiting times due to late arrival of transit services and excessive in-vehicle travel times, due to traffic or system problems (Paulley et al., 2006). Iles (2005) describes a typical scenario witnessed by public transit commuters in some cities of developing countries during peak hours as follows:

*'It will be several hours before all passengers reach their homes and many will walk for thirty minutes or more after leaving the bus. ....Many passengers have to transfer more than once from one bus to another during the course of their journeys, suffering yet another long wait and another scramble for a place.'*

The above description fits very well with the peak hour journey of passengers in some cities within India. Upon research it was realised that unreliability in transit services is a universal problem though of varying intensity from country to country.

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<sup>1</sup> [http://cpcb.nic.in/upload/NewItems/NewItem\\_157\\_VPC\\_REPORT.pdf](http://cpcb.nic.in/upload/NewItems/NewItem_157_VPC_REPORT.pdf) 1.2

It is amply evident that reliability is a feature which needs further investigation under public transit context because of the increased total travel times and waiting times of passengers associated with unreliable transit services (Paulley et al., 2006). However one needs to understand that a congested transit service which arrives perfectly as per schedule can also lead to unreliability associated with increased waiting times. The relationship between congested transit network and reliability is discussed in section 1.3 as it forms one of the key aspects of the current research. Ceder (2007) identifies that unreliability is an ambiguous term whose definition varies with the context:- as defined by operators or as defined by passengers. The key feature that integrates passenger reliability attributes and operator reliability attributes are the headways. Operators tend to fix the headways of various lines operating in system with a trade-off between increasing their revenue and minimising the waiting time of passengers (Fernandez and Marcotte, 1992; Li et al., 2008; Li et al., 2009; Seshagiri et al., 1969; Furth and Wilson, 1981). Hence the short term strategies adopted by operators to improve reliability of the lines serving a network predominantly include modification of headways or frequencies. Belmonte et al. (2005) describe several strategies an operator adopts to improve reliability:

- a. Change from time table to frequency regulation of lines.
- b. Change from frequency to time tabled regulation of lines.
- c. Change the frequency regulation or frequency distribution of the line.
- d. Increase or decrease speed of the individual bus
- e. Jumping of bus stops by buses
- f. Advance following service (a bus must over take the bus that is crowded)
- g. Advanced head service start (the bus at the head of the line should start ahead of its schedule.
- h. Time table rotation (each bus in a line adopts the scheduled time table of its successor)
- i. An additional bus is included in the line.

Reliability as defined by operators greatly varies from that defined by the passengers. In the current study we shall look upon reliability from passenger's perspective and assess the impact of certain operator implemented policies on passenger total travel times.

## 1.1 Modelling Operational Characteristics in transit networks

The operational characteristics of transit services vary around the world. The differences in operational characteristics are such that in some countries there are no timetables associated with transit services; in some countries time tables exist but are not adhered to and there are countries where services are run in accordance with timetable. Pritchard et al. (2014) illustrates that

*'Before 2009 London's buses ran to a timetable. Copies were displayed at bus stops and on TfL's website, listing the times a bus was scheduled to arrive at named locations. In 2009 the 'headway' system was adopted as a corrective measure to avoid bus bunching. Instead of publishing a specific time, the headway approach uses Location based services (LBS) to measure the distance between buses. Instead of a published timetable, notices now state the estimated time between services (e.g. 'services run every 5-7 minutes').'*

As a transport planner it is necessary to account for the operational characteristics while trying to model the arrival of transit services in the network. In networks with absence of timetables the services could be modelled based on *frequency based (headway based) approach* whereas in networks wherein a time tabled service exists the modellers could use *schedule based approach*.

The reliability measures often used by transit agencies to measure schedule adherence are 'on-time performance' and 'headway regularity'. One of the key measures to evaluate headway regularity by transit agencies is the coefficient of variation. Hunter-Zaworski (2003) in 'Transit Capacity and quality of service manual' indicate that different level of service have different coefficient of variation of headways. Headway variation is found to propagate delay to downstream stops where it is likely that additional passengers have arrived to board the bus (Abkowitz and Tozzi, 1987). Since it is found that headway variation influence the waiting time of passengers at downstream stops a relationship between coefficient of headway variation and passenger waiting time has been analytically derived in Osuna and Newell (1972) and utilised in several studies (eg: Marguier and Ceder, 1984). Kirnpel (2000) point out that while analysing transit service reliability the distinction between high frequency services (headways lesser than 10 min) and low frequency services needs to be made. Lines characterised by low frequency services should be concerned with schedule adherence whereas for lines with high frequency the headway variability needs to be the measure of reliability. Another characteristic of lines that impacts its reliability are the length of the line and

the number of stops in the line itinerary. The assumptions pertaining to each type of approach with respect to modelling reliability in service arrivals is dealt with in detail on chapter 2.

### **1.3 Congested Transit system and reliability**

Congestion in transit system is associated with the increased waiting time of passengers. In the context of transit assignment studies congestion is modelled in varying ways as discussed in detail in chapter 2. In real world congestion generally occurs due to a passenger being unable to board a service of his/her choice when the service is at full capacity (failure to board). Another definition of congestion would be the level of service being provided within the transit service. Passengers may perceive inability to get a seat as a form of congestion whereas some may consider being able to stand without bumping into each other as a relatively less congested ride. In these cases congestion then defines the level of comfort as perceived by the passengers.

*'During peak hours, stations can get so crowded that they need to be closed. Passengers may not get on the first train and the majority of passengers do not find a seat on their trains, some trains having more than four passengers every square metre. When asked, passengers report overcrowding as the aspect of the network that they are least satisfied with, and overcrowding has been linked to poor productivity and potential poor heart health. Capacity increases have been overtaken by increased demand, and peak overcrowding has increased by 16 per cent since 2004/5.'* (Wikipedia, the free encyclopedia, 2015)

A congested network defines reliability of the transit service from the passengers perspective. This is especially true in case of transit networks which has more demand than the supply during peak hours, such that the passengers often experience the 'failure to board' condition. In the event of failure to board the passenger perceives the system to be unreliable even though it may have been totally reliable in terms of its service operations.

### **1.4 Variations in transit supply and demand , in passenger perceptions and in behaviour**

Transit network is dynamic in nature. Not only do the supply and demand variations happen within a day but they also vary from day to day. Apart from the day to day variations there are variations within the working days of a week, between the weekends and seasonal variations as well. Balcombe et al. (2004); Abkowitz and Tozzi (1987); Kirnpel (2000) explicitly indicate these

variations in their report. It is hence necessary to capture these dynamics while modelling the transit network. Similar to variations associated with the supply and demand of transit network, variation in the perceptions of passengers with regard to their journey times is encountered. Different people tend to perceive their journey times differently than their actual experience- with some over estimating while some underestimating their journey times. These perception variations also are important in modelling terms, as different notion of perception may result in different flow patterns on various available lines in a transit network.

While assessing the impact of reliability on transit passengers, one needs to account for the heterogeneous nature of passengers. It would be erroneous to believe that all passengers travelling tend to minimise only his/her average journey time. There could be passengers who are risk averse and hence associate a degree of risk aversion towards variance associated with the total travel time experienced by them or passengers who are averse to total travel time exceeding a certain acceptable value. The current thesis shall deal with the route choices of risk averse as well as risk neutral passengers in a transit network.

## **1.5 Research Context**

A brief overview of the existing literatures in transit assignment and their tackling of the above mentioned problems is dealt with in this section. The detailed description of these literatures is given in Chapter 2. The aim of the current section is to introduce the level of research already done in the field of transit assignment and the existing standing of transit assignment studies in the field of reliability.

Most of the frequency based transit assignment models which follow either explicit path enumeration (De Cea and Fernandez L, 1989; De Cea et al., 1988) or implicit assignment of flows on various links (Spiess and Florian, 1989; Nguyen and Pallottino, 1988; Wu et al., 1994; Schmoecker, 2006; Cominetti and Correa, 2001; Cepeda et al., 2006) assume highly irregular interarrival of transit services (exponential interarrivals). Several other literatures (Marguier and Ceder, 1984; Bouzaïene-Ayari et al., 2001; Gentile et al., 2005) proposed an alternative inter arrival distribution of Erlang which provides the modeller with the flexibility of controlling the variance associated with the inter-arrival of services. These distributional assumptions hence help to model the service unreliability in the transit network at varying levels.



A hierarchy of studies on frequency based transit assignment pertain to accounting for congestion; a feature arising due to the physical capacity constraints of transit services and increased passenger demand. It is known that the transit assignment studies derive their complexity from asymmetric interaction of link flows in a congested network wherein the upstream flow has an influence on the costs/total travel time experienced by the downstream flows. It is also known that the flows on a strategy/route can influence the total travel time/ costs experienced on the other strategy/route. Hence transit assignment deals with cost functions that are not only influenced by its own flow but also by the flows on other links/routes. Assignment problems of these kind are termed asymmetric and keeping the asymmetric nature of transit assignment in context, the early stage models utilised BPR style function to model congestion. Spiess and Florian (1989); Nguyen and Pallottino (1988) utilised BPR type in-vehicle cost function to account for the 'discomfort' experienced by the passengers in event of congestion in a hyperpath/strategy based optimisation problem, whereas De Cea and Fernández (1993) introduced BPR type cost function in the waiting time of passenger's to depict the increased waiting time associated with higher congestion in a route section based assignment process.

Wu et al. (1994) utilised BPR styled cost function in both in-vehicle travel time function and in waiting time function. De Cea and Fernández (1993) account for the asymmetric interaction between the flows and use a 'diagonalization' algorithm for solution which they argue has good convergence properties even when monotonicity is not satisfied. Spiess and Florian (1989) acknowledge the limitations of their model wherein it is assumed that all passengers experience the same level of discomfort and waiting time. Similarly Wu et al. (1994) acknowledge the inability of their model to transform the hyperpath flows into an expression comprising of arc flows and also highlight the limiting presumption made that the hyperpath costs are strictly monotone in nature.

Modelling the effects of congestion was further improved upon by the introduction of 'effective frequencies'. The concept of 'effective frequency' was first defined in Spiess and Florian (1989) as the frequency of lines which are decreasing functions of the total volume aboard the transit service. The implementation of 'effective frequency' was achieved by De Cea and Fernández (1993) through 'equivalent average waiting time index' which is a line specific index common to all the passengers waiting for a line at the transit stop irrespective of the route section used by them. Effective frequency was computed as the inverse of 'equivalent average waiting time index' and the

boarding probability was computed as a function of these effective frequencies. The effective frequency was able to depict the decrease in boarding probability with the increase in waiting time realistically however at capacities the waiting time tends to infinity resulting in effective frequencies tending to zero. In such cases an upper limit of frequency was fixed which when reached the demand was assigned to strategies consisting of walking arcs, which were arcs with no waiting time and with infinite frequencies (Cominetti and Correa, 2001; Cepeda et al., 2006).

The introduction of strict capacity constraint was achieved by Schmoecker (2006) who using Bellman's dynamic equilibrium model proposed an alternative network layout consisting of failure arcs which were assigned the excess flows at a given time step. These flows were then reintroduced with the next time step generated flows to complete their journeys. The hyperpath based model of Schmoecker (2006)'s is a versatile within-day dynamic model which captures the congestion effects such as failure to board and excess waiting time with a great deal of success however as noted by the author it requires an experienced modeller to specify the time discretisation required for the dynamic framework. Schmoecker (2006) mentions that time interval duration should be longer than the time it takes to traverse an arc in the network; hence for trips which have longer travel time several arcs of shorter durations needs to be specified in the network design to capture the effect of congestion realistically. Adopting a dynamic simulation framework Cats et al. (2011) assessed the effect of information on the path choice of transit passengers. Trozzi et al. (2013) proposes a dynamic model which considers the FIFO principle of passengers at the bus stops and proposes a diversion probability which is time dependent and models the expected congestion at that time step. A day to day learning process model with strict capacity constraints with aggregate learning process was initially formulated by Teklu (2008a and 2008b) whose model has been furthered in the current study to account for reliability in a transit network.

The frequency based transit assignment models discussed above assumes random arrival of transit services coupled with various assumptions to account for congestion. The above mentioned models all assume that the passengers are risk neutral and hence associate no disutility towards the unreliability assumed in their models. To overcome this issue a series of attempts have been made in recent years (Yin et al., 2004; Yang and Lam, 2006; Szeto et al., 2011; Szeto et al., 2013) to account for reliability in congested transit assignment studies. All the existing reliability based transit assignment studies adopts the 'route

section' approach of frequency based transit assignment and deal with congestion by either assuming an 'overload delay' (Yin et al., 2004; Yang and Lam, 2006; Szeto et al., 2013) or a BPR styled increase in waiting time (Szeto et al., 2011). 'Overload delay models' (Lam et al., 1999; Lam et al., 2002; Li et al., 2009) computes the delay due to overloading of a line endogenously during the Stochastic User equilibrium (SUE) assignment. From the above discussion it can be deduced that an approach to modelling the unreliability associated with failure to boarding the first service of their choice (strict capacity constraint models) has not been dealt with so far in frequency based transit assignment. A schedule based approach using mean-variance model to account for unreliability based on disruptions was proposed by Hamdouch et al. (2014); wherein the disruptions were modelled by randomised in-vehicle travel times.

## **1.6 Gap in Literature**

As highlighted in Teklu (2008a) the above mentioned models do not account for the impact of strict capacity of the transit services on the waiting time of passengers (or the situation of failure to board the first service of their choice set) ; - though attempts have been made to address the issue of congestion by means of 'effective frequency'; 'overload delays' ; 'BPR functions' and 'dynamic models' (section 1.5). In hyperpath based models the priority of the passengers already in a transit service (those who boarded on the upstream transit stop) over the passengers boarding the service at the downstream stop is not observed. It is also realised that in hyperpath based formulation the decision of when to alight and when to continue a journey is as important as line choice (Nökel and Wekeck, 2009). Also with the exception of Trozzi et al. (2013) all the other hyperpath based transit assignment approach assume mingling of passengers at the transit stops which may not be necessarily true in certain transit stop layouts. The accounting for the interaction of passengers with different strategy choice ; the 'learning process' which a passenger would have gone through over his /her repeated travel and transit services having strict capacity constraint has been dealt with in Teklu (2008a and 2008b). Teklu (2008a and 2008b) made use of aggregate stochastic process model formulation based on day to day traffic assignment proposed by Cascetta (1989); Cascetta and Cantarella (1991). Watling (1996) elaborates the advantage of using stochastic process model which gives an unique stationary probability density function as output, analogous to unique equilibrium solution and can be argued as a much more realistic solution to assignment problems. The advantages of stochastic process models and its implementation as a

markovian process is dealt with in detail by Watling and Cantarella (2013); Watling and Cantarella (2012) through series of examples.

As mentioned, the implementation of a stochastic process model in transit network in order to understand the day-to-day evolution of flows with an exponentially distributed interarrival of supply and demand was done by Teklu (2008a and 2008b). Teklu (2008a and 2008b)'s work was able to establish a simulation framework for assessing the flow distribution as an aggregate stochastic process model. The presence of stationary distribution of flows irrespective of the initial conditions was proven under a Monte Carlo based Markovian framework (MCMC) for a strict capacity constrained network. However the assumption of aggregate learning in Teklu (2008a and 2008b) implies that the passengers have full 'information/awareness' of the network. It also assumes that in the aggregate learning process passengers overlook their own individual experiences to base their route choice on the predicted costs for a route. The predicted cost of the route is computed as the average of the experienced cost of all the passengers on the route. These assumptions in reality would mean that the passengers have an external information source which makes them aware of the experienced travel times of all the other passengers. Or that the passengers have full knowledge of the network and the expertise to derive the probability density function of the waiting times based on the current day's transit supply demand conditions. Such assumptions seem unrealistic and since reliability in itself is an individual's entity a disaggregate stochastic process model is proposed in the current research.

Though Teklu (2008a and 2008b)'s stochastic process model had modelled variations in transit interarrivals, passenger interarrivals and failure to board; it assumed that the passengers were risk neutral. The current research furthers the stochastic process model proposed in Teklu (2008a and 2008b)'s by accounting for unreliability associated with

- a. Varying interarrival times of transit service at the transit stop
- b. The variation in the waiting time of passengers due to the 'failure to board' condition. The condition arises as a result of strict capacity constraint enforced at disaggregate level which results in some passengers not being able to board the first transit service of their attractive line set.
- c. The variation associated with the in-vehicle travel times of routes comprising of route sections containing more than one attractive line section.
- d. The variation associated with the variable demand generated for each day's travel.

in the cost function of the passengers; thereby assuming that the passengers are risk averse. In the absence of 'information' the stochastic process model is formulated as a disaggregate process wherein each individual bases his/her route choice on his/her individual experience.

### **1.7 Research Objectives:**

Based on the discussions put forward in section 1.6 the current research aims to

- investigate the route choice variation of public transit users under the context of reliability as defined by the passengers. The model developed should be at disaggregate level enabling a micro level analysis of the impact of unreliability on each commuting passenger. The disaggregate aspect of the model is emphasised as in a congested network the passengers who are unable to board the first transit service of their choice set experience a different level of unreliability from those who are able to board the first transit service of their choice set. Also in the absence of 'external information' (as is the case in many developing countries and several smaller transit stops of developed countries) assumption of full awareness as proposed in aggregate models seems unrealistic. Thereby the aim is to account for the route choice of passengers who are averse to the variation in their total travel times. Since variance is associated with several aspects of transit modelling framework (section 1.6) a need to develop a holistic model accounting for all the aspects of variance arises.
- Upon understanding the impact of using variance of total travel time in the cost function of route choice model there is a need to address the issue of disutility associated with variation in the total travel time of passengers.
- Accounting for risk aversion in route choice of passengers ultimately needs to feedback to the operators looking to improve the transit service reliability. Thus there is a need to evaluate the possible policy measures a transit operator could implement to improve the reliability of the transit service while accounting for risk averse passengers.
- Implementation of the risk aversion models on a realistic network could help assess the actual implications of ignoring of risk aversion (as is done in almost all of the existing transit assignment models).

To achieve the set aims for the current research a series of objectives have been defined as follows:

1. To formulate a markovian disaggregate stochastic process framework which accounts for route choice of each passenger based on linear combination of mean total travel time and total travel time variance (mean-variance)cost. The route choice is assessed under variable transit supply and demand with strict capacity constraints. To test the developed framework on an example network and to carry out the sensitivity tests.
2. To implement the mean-variance cost function on existing equilibrium based transit assignment models and aggregate stochastic process model and compare the performance of these models with proposed reliability based disaggregate stochastic process model with strict capacity constraints.
3. To formulate a markovian disaggregate stochastic process framework which accounts for route choice of each passenger based on linear combination of mean total travel time and expected lateness (mean-lateness) cost. To test the developed framework on an example network along with carrying out various sensitivity tests.
4. To assess the behaviour of proposed mean-variance and mean-lateness models on policy interventions which could be made by network operators.
5. To validate the mean-variance and mean-lateness models with a real network data (The open source data on London underground from TfL is used) and to study the impact of certain policy interventions on waiting time reliability profile.

## **1.8 Thesis layout**

The thesis is structured such that chapter 1 deals with the motivation and the gaps in existing literature (with a brief introduction of the existing literatures) based on which the existing thesis objectives are framed.

Chapter 2 elaborates the state of art for the present study in terms of assumptions involved in frequency based transit assignment; dealing with capacity constraints; random utility models used for accounting for passenger perception variations and the choice variation of routes based on utility functions. The state of art on accounting for reliability of services on route choice of transit as well as traffic networks. The state of art on stochastic

process models on dealing with problems where the presence of multiple solutions cannot be ruled out.

Chapter 3 gives an overview about the stochastic process models together with the formulation of disaggregate stochastic process model used in the current study. The application of disaggregate stochastic process model on an example network for risk neutral passengers (as is assumed in most of the existing transit assignment model) together with various sensitivity tests and the tests to prove the markov property are carried out.

Chapter 4 introduces through some numerical tests the changes in the choice of shortest route when a network of risk averse passengers are considered. The application of disaggregate stochastic process model using mean-variance cost on an example network along with tests to prove its markov property and various sensitivity tests are carried out. A comparison of the disaggregate stochastic process model with a BPR styled Logit SUE model and an effective frequency styled hyperpath based DUE model is carried out.

Chapter 5 deals with a mean-lateness cost incorporation in disaggregate stochastic process models. The analysis is followed by a series of policy evaluation tests being carried out to show the impact of accounting for risk aversion in the waiting time reliability improvement.

Chapter 6 shows a case study of the London underground section implementing the mean-variance and mean-lateness models.

Chapter 7 summarises the finding of various chapters together with conclusions. Future directions of study are also highlighted.

## Chapter 2 Literature Review

### 2.1 Introduction

Reliability studies integrated with assignment process has long been studied. In traffic analysis the reliability studies has been embodied in the assignment process in order to evaluate the passenger's varying choices in event of unreliable service attributes. In these studies route choice models have accordingly been classified as shown in Table 2.1.

Table 2.1: Chen et al. (2002)'s Classification of route choice models in traffic assignment studies

		Perception Error	
		No	Yes
Network Uncertainty	No	DN-DUE	DN-SUE
	Yes	SN-DUE	SN-SUE

Where DN- Deterministic Network

DUE-deterministic user equilibrium

SN-Stochastic equilibrium

SUE- Stochastic user equilibrium

Similarly almost all transit assignment studies have integrated reliability aspects as well. The difference in the integration of reliability between traffic assignment studies and transit assignment studies lies in the fact that in transit assignment since the supply side of the assignment process is stochastic in nature an endogenous accounting for certain reliability issues by assuming varying headway distribution and random arrival of passengers results in embedded reliability analysis. The so called static approach to transit assignment modelling namely, the frequency based transit assignment accounts for the network uncertainty endogenously and often tends to minimize the average total travel time experienced by passengers thereby assuming that all passengers are risk neutral.

Transit assignment models deals with the route/strategy choice of passengers in a transit network and provides an estimate of the number of passengers travelling along the various routes/strategies in the network together with the estimated cost of travelling on these routes/strategies. In comparison to the



traffic networks the assignment problem of transit network is much more complex not only due to the presence of several stages (walking to the transit stop, waiting for the transit service, transferring to a different service, riding in a service etc), interaction between several entities (buses, information system, passenger heterogeneity) but also due to several difficulties in formulating it as a simple mathematical assignment model.

A transit network modeller needs to model the waiting time of passengers at a downstream transit stop which is affected by the number of passenger opting for the service at the upstream stop as the transit may be full when it reaches the downstream stop resulting in failure to board situation. Also within the transit service the passengers sitting enjoy a better level of comfort than those standing. Modelling the increased travel times of passengers due to above mentioned factors results in several complexities. When the costs of each alternative link/route can be expressed in terms of its own flow then they are

termed as symmetric problem as they have diagonal jacobians  $\left( \frac{\partial c_x}{\partial v_y} = 0, \forall x \neq y \right)$  where  $c_x$  is the cost along line segment  $x$  and  $v_y$  is the flow on the

competing line segment. Such an assumption is a common practice in traffic assignment. However in transit assignment such a symmetric jacobian cannot be assumed as the cost experienced by the passengers on one route/strategy is influenced by the passengers opting for routes sharing the same route sections. Also the cost experienced by the lower end transit stop passengers is influenced by the passengers already present within the transit service who have boarded the service at upper end transit stop.

As mentioned in Chapter 1 a transit modeller needs to understand the requirements of the network he/she models. In developed countries one finds the transit system evolution is advanced to a level such that the transit services run as per the given time tables and the issues of frequent non-availability of the scheduled services are minimal. They also have information dissemination systems such as 'signs at bus stops', 'online tracking of services' etc. However there are certain developing countries where the presence of time tables is negligent and the frequencies of services are completely random; fluctuating highly on day to day basis. On the other hand we also have countries where the transit network operation in spite of having a time table seldom follows them. These variations in supply side reliability are further complicated by each line within a transit network being associated with differing levels of unreliability.

Based on the various network characteristic to be modelled a transport planner has the option to choose between 'frequency based approach' or 'schedule based approach' to model a transit network. The following sections shall highlight the frequency based transit assignment models in detail as that is the approach followed in the current thesis. A discussion on use of Random utility models (RUM) for assignment process is also made. A review on reliability studies and the various methods of analysis namely, scheduling approach and mean-variance approach is also described along with a review of stochastic process models.

## **2.2 Transit assignment models**

In transit studies the modelling of the route choice decisions of a transit passenger can be achieved by two approaches

- Frequency based approach
- Schedule based approach

Transit assignment approaches mostly assume that the passengers have a good knowledge of the network in which they are travelling and hence often are modelled as passengers that make a 'pre-trip' choice. 'En-route' choice travellers are modelled as the 'clever' passengers or as the passengers who are provided with 'information' to make an en-route clever decision (Lam and Bell, 2003).

Earlier transit assignment approaches (Fearnside and Draper, 1971; Le Clercq, 1972) dealt with route choices similar to that of traffic networks wherein the 'strategy' concept was not implemented and the transit network was defined in terms of individual paths. Fearnside and Draper (1971) solve the transit assignment problem by associating walking time with the centroid connectors in the traffic network, travel time with the link length and the waiting time with frequencies that are associated with the turning penalty system. A distance dependent linear fare function was also included in the cost function. Le Clercq (1972) uses a 'once through' algorithm to find the shortest path in the transit network. The 'once through' algorithm searches for the shortest path by starting with the node having the least time and updating the time, if the time to reach that node from origin is lesser than the initial set time. Le Clercq (1972) also code transit network as traffic network. These studies were able to fulfil the transport planner requirements in earlier days as the demand for transit services had not exceeded the supply. However in the early 80's it was felt imperative to improve the existing transit assignment models to

incorporate the effect of physical capacity constraint of transit services to accurately model the total travel time/costs experienced by passengers.

### 2.2.1 Common lines problem:

The important principle which underlines the public transit assignment is the presence of multiple lines running between not only the adjacent bus stops and also between a pair of transfer stops. The earliest study to address the problem of common lines was Chriqui and Robillard (1975). The foundation of identifying the 'attractive line set' between two consecutive bus stops or identifying the attractive line set of route sections is based on the heuristic given by Chriqui and Robillard (1975). A route section is defined as a section of line which runs between two transit stops which are not necessarily consecutive and form a part of the route connecting an OD pair. A route section can be part of several routes and a route section can consist of one or more line sections. Chriqui and Robillard (1975) address the issue of identifying the line or route sections that can be chosen by the passenger as an optimisation problem by assuming that a transit user only chooses a subset of available lines between the bus stop and gets on the first bus that arrives in this subset of lines. The minimisation process derives a set of lines sections which when put together minimise the total travel time of the passengers. The algorithm defining the process of identifying the set of 'attractive lines' is as follows:

Arrange the common lines in ascending order of their in-vehicle travel times

Let  $\bar{S} = \{1,0, \dots, 0\}$  and  $S = \{1,1,0 \dots, 0\}$

Compare Total Travel Time (T.T) of  $\bar{S}$  with total travel time of S

If  $T.T(S) > T.T.(\bar{S})$  then  $\bar{S}$  is the solution set

Else

$\bar{S} = \{1,1, \dots, 0\}$  and  $S = \{1,1,1 \dots, 0\}$  and compare T.T of  $\bar{S}$  with T.T of S

Continue till  $T.T(S) > T.T.(\bar{S})$  then  $\bar{S}$  is the solution set else till  $\bar{S} = \{1,1, \dots, 1\}$  then  $\bar{S}$  is the solution set.

The above heuristic was applied for uniform and exponential headway distributions. The exponential headway assumption results in the following minimisation problem which can be solved by the above heuristic

$$\min \frac{1 + \sum_{l \in L} t^{in-veh}_l \varphi_l \bar{x}_l}{\sum_{l \in L} \varphi_l \bar{x}_l} \quad (2.1)$$

Subject to  $\bar{x}_l = 0,1 \forall l \in L$

Wherein  $t^{in-veh}_l$  is the in-vehicle travel time of line section l

$\phi_l$  is the frequency of line section l

Chriqui and Robillard (1975) argue that they have not been able to find a counter example for the above heuristic solution. However Marguier (1981) specify a set of conditions for deterministic headway distribution wherein the above heuristic fails. Marguier (1981) also mention that it is very rare that such set of conditions are met within the real network.

It becomes clear from the above paragraphs that the heuristics specified by Chriqui and Robillard (1975) works for most situations especially in the case of exponential headway distribution and is shown to not work for a specific condition of deterministic headway distribution. Gentile et al. (2005) proposed a straight forward modification to the existing heuristic wherein the sorting of lines currently based on in-vehicle travel time was replaced by sorting of lines based on total travel time in order to make the heuristic work for deterministic conditions as well.

### **2.2.2 Unconstrained transit assignment models:**

Transit assignment models initially assumed that the transit supply network was able to cater to the existing demand and hence were formulated as unconstrained models. These models were classified into two different approaches, namely:

- Hyperpath/ Strategy approach
- Route – Section approach.

The distinction between the approaches being that the route section approach enabled explicit enumeration of routes between an OD pair and defines the attractive line set between transfer stops where as hyperpath/strategy approach was formulated without explicit enumeration and defines attractive line set between each node. 'Strategy' is defined as the set of rules a passenger follows to reach their destination. The graphical representation of a strategy is hyperpath. The network structure in route section and hyperpath/strategy approaches differs from each other as shown in fig 2.1. The example network given in fig 1a. has several possible set of strategies (set of rules) to travel from various origin points in the network (S1 and S2) to the destination (S3). The table highlighting all the possible strategies and the possible route sections for the network is given in table 2.2. Fig 2.1 (b) gives the graphical (acyclic) representation of network in fig 2.1(a) and hence is called the hyperpath representation whereas fig 2.1(c) gives the route section representation of transit network in fig 2.1(a).

The table highlights that the alternatives A10 and A11 are absent in the route section approach. These approaches shall be further described in the following sections.

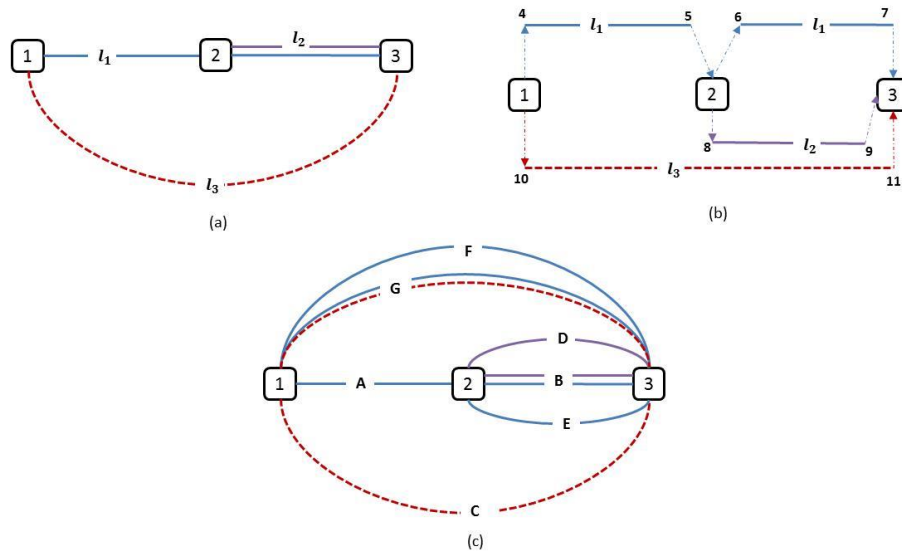


Fig 2.1 . Different representations of a transit network (a) transit network (b) Hyperpath representation (c) Route section Representation

Table 2.2: All possible Route Sections and all possible Strategies for the network in Fig 2.1 a.

	S1 - S2		S2 - S3		S1 - S3						
Strategies	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
Alternative	$l_1$	$l_1/l_2$	$l_1$	$l_2$	$l_1+l_1$	$l_1+l_2$	$l_1/l_3$	$l_3$	$(l_1+l_2/l_1)$	$(l_1+l_2)/l_3$	$(l_1+l_2/l_1)/l_3$
Routes	R1	R2	R3	R4	R5	R6	R7	R8	R9	-	-
Route sections	A	E	D	B	C	G	F	A+D	A+B	-	-

### 2.2.2.1 Route – Section approach:

Following the ‘common lines’ concept derived by Chriqui and Robillard (1975); De Cea and Fernandez L (1989) proposed a network representation based on line sections. This served the purpose of implementation of the ‘common lines’ concept onto a larger network. Line sections are defined as the lines joining two bus stops which are not necessarily consecutive. The network representation  $\mathcal{G}' = (\mathcal{N}, A)$  consists of  $\mathcal{N}$  as the nodes vector and  $A$  as the set of all possible line sections. ‘Route section’ is defined as a portion of the route between two consecutive transfer stops and each route section is associated with the set of attractive lines sections or common line sections. Let  $\mathcal{S}$  denote the set of

transfer stops,  $t^{in-veh}_l$  the in-vehicle travel time on the line section  $l$ ,  $V_{ij}$  is the total flow along the route section,  $v_l$  is the flow on line section  $l$ ,  $\varphi_l$  is the frequency of the line section,  $\mathfrak{x}_l$  indicates if the line section belongs to  $S_{ij}$  or not,  $d_i$  is the total demand from origin  $i$  to destination  $j$  and  $S_{ij}$  is the set of line sections directly connecting the nodes  $i, j$ . The model is formulated as minimisation problem wherein trips are assigned from origin to destination via route sections as follows:

$$\min_{(\mathfrak{x}_l, V_{ij})} \sum_{l \in A} v_l t^{in-veh}_l + \sum_{(i,j) \in \mathfrak{S}} \frac{V_{ij}}{\sum_{l \in S_{ij}} \varphi_l \mathfrak{x}_l} \quad (2.2)$$

Subject to

$$\sum_{l \in A^+} v_l + d_i = \sum_{l \in A^-} v_l \quad \forall i \in \mathcal{N}$$

$$v_l = \frac{\mathfrak{x}_l \varphi_l V_{ij}}{\sum_{l \in S_{ij}} \varphi_l \mathfrak{x}_l} \quad \forall (i,j) \in \mathfrak{S}, l \in S_{ij} \quad (2.3)$$

$$v_l > 0 \quad \forall l \in S_{ij}$$

$$\mathfrak{x}_l = 0, 1 \quad \forall l \in S_{ij}$$

The above model does not include capacity constraint. The solution to the above problem is achieved by means of a three step process where the first step involves the identification of the attractive line set using the heuristic provided by Chriqui and Robillard (1975). Since the heuristic algorithm does not consider capacity constraints, the attractive line set obtained is for an uncongested network. The assignment of flows to the route sections is done using all-or-nothing assignment process. From the route sections the flows are assigned to the line sections using eq(2.3).

### 2.2.2.2 Hyperpath/ Strategy approach:

#### (a) Optimal Strategy

'Strategy' is defined as the 'set of rules which a passenger follows to reach his/her destination'. The common lines problem was given a conceptual framework by 'Strategy' concept first introduced by Spiess and Florian (1989). The strategies when represented in graphical format were known as hyperpaths (Nguyen and Pallottino, 1988). The strategy/hyperpath based assignment assumes each line serving a bus stop as a separate arc. The strategy/hyperpath concept eliminates the explicit enumeration and hence proves advantageous for analysis of larger networks. In sync with the shortest path concept of traffic assignment, strategy/hyperpath approach introduces 'optimal strategy/hyperpath'. A detailed description of finding the optimal

strategy as given in Spiess and Florian (1989) shall be dealt with in this section. Spiess and Florian (1989) highlight that strategies can define several set of rules to reach the destination. In absence of information, passengers are unaware of the exact arrival of the lines serving a transit stop and when the transit services have capacity exceeding the demand, the passengers invariably choose the line which comes first amongst their set of attractive lines (lines which minimise the passengers total travel time). This is defined as 'take the first transit service' strategy and is represented by Spiess and Florian (1989) using the following algorithm:

Step 1: Choose an origin node and fix it as the STOP-NODE

Step 2: Board the vehicle that arrives first at the STOP-NODE from the predetermined set of attractive lines.

Step 3: Alight at the predetermined node.

Step 4. If the alighting node is not the destination then set the current node to STOP-NODE and return to step 2; else trip is completed.

From the above algorithm it is amply clear that the passengers choose their strategy based on a 'pre-trip' choice wherein the attractive line set of the passenger to reach his/her destination is defined before the journey. The alighting node is also predetermined based on the line boarded by the passenger hence the element of 'en-route' choice is involved only at the transit stops in the above strategy of 'take the first transit service' algorithm. An important aspect in the definition of strategy as mentioned in Spiess and Florian (1989) is that the origin node is not a fixed entity hence strategy defines the rules that enables a passenger to travel from any node to the destination node. A subtle difference between the strategy and hyperpath approaches lies in the solution approaches used. In case of strategy based models the minimisation problem is solved based on linear programming approach whereas hyperpath based models use dynamic programming.

The network  $\mathcal{G} = (\mathcal{N}, \mathcal{A})$  representation in Spiess and Florian (1989) characterises each arc  $a \in \mathcal{A}$  by  $(t^{in-veh}_a, \varphi_a)$  where  $t^{in-veh}_a$ -travel time associated with line segment and  $\varphi_a$ - frequency of line segment. The arcs which do not have in-vehicle travel times associated with it such as the waiting arc, boarding arc, alighting arc the value of  $t^{in-veh}_a$  is set to zero and for arcs such as in-vehicle arcs, alighting arcs, boarding arc the value of  $\varphi_a$  is set to zero. A strategy to reach destination stop  $j \in \mathcal{N}$  is represented by partial network  $\mathcal{G}^j = (\mathcal{N}^j, \mathcal{A}^j)$  where  $\mathcal{N}^j \in \mathcal{N}$  and  $\mathcal{A}^j \in \mathcal{A}$  consists of only attractive line set used as part of the strategy. Among the links included in the strategy a

passenger boards the first transit service that arrives. The attractive lines that make up a strategy are represented in terms of 0-1 variables  $\mathfrak{x}_a$ .

$$\mathfrak{x}_a = \begin{cases} 0, & \text{if } a \notin \mathcal{A}^j \\ 1, & \text{if } a \in \mathcal{A}^j \end{cases}$$

If the total demand from node  $i$  to  $j$  is denoted by  $d_j$  such that  $d_j = \sum_{vi \neq j} d_i$

Then the optimisation problem to identify the optimal strategy  $S^*$  is formulated as shown in Spiess and Florian (1989) :

$$\min \sum_{a \in \mathcal{A}} t^{in-veh}_a v_a + \sum_{i \in \mathcal{N}} \frac{V_i}{\sum_{a \in \mathcal{A}_i^{j+}} \varphi_a \mathfrak{x}_a} \quad 2.4$$

Subject to

$$\begin{aligned} v_a &= \frac{\mathfrak{x}_a \varphi_a}{\sum_{a \in \mathcal{A}_i^{j+}} \varphi_a \mathfrak{x}_a} \\ V_i &= \sum_{a \in \mathcal{A}_i^{j+}} v_a + d_i \\ V_i &\geq 0 \end{aligned}$$

Wherein  $v_a$  denotes the volume of passengers on the line segment  $a$  and  $V_i$  denotes the volume of passengers accumulated at the node  $i$  from various line segments preceding it,  $\varphi_a$  denotes the frequency of line segment  $a$  and  $t_a$  denotes the in-vehicle travel time of line segment  $a$ .

The above mentioned problem has a non-linear objective function with non-linear constraints which are converted into simpler linear programming problem and then solved by two step algorithm. The first step involves backward labelling of the shortest path algorithm to identify the shortest strategy from a destination to all other stops and the second step involves the assignment of flows/passengers onto the line-segments.

Part 1: finding the optimal strategy

1.1 Initialisation  $c_i = \infty; i \in (\mathcal{N} - j); c_j = 0$

$$\varphi_i = 0, i \in \mathcal{N}$$

$$\mathcal{S} = \mathcal{A}; S = \emptyset$$

1.2 getting the next link:

If  $\mathcal{S} = \emptyset$  then STOP else find  $a = (i, r) \in \mathcal{S}$  which satisfies  $t^{in-veh}_a + c_r \leq t^{in-veh}_{a'} + c_{r'}, a' = (i', r') \in \mathcal{S}$

Where  $c_r$  is the cost associated with node  $r$ .

$$\mathcal{S} = \mathcal{S} - \{a\}$$



### 1.3 updating node label

If  $c_i \geq t^{in-veh}_a + c_r$  then

$$c_i = \frac{\varphi_i c_i + \varphi_a (t^{in-veh}_a + c_r)}{\varphi_i + \varphi_a}$$

$\varphi_i = \varphi_i + \varphi_a$  and  $S = S + \{a\}$

Go to step 1.2

Part 2: assignment

2.2 (initialisation)  $V_i = d_i$

2.3 (Loading) for every link  $a \in \mathcal{A}$ , in decreasing order of  $t^{in-veh}_a + c_r$ :

If  $a \in S$  then

$$v_a = \frac{\varphi_a}{\varphi_i} V_i$$

Wherein  $V_i$  is the volume of flow accumulated at node  $i$

$$V_i = V_r + v_a$$

$$\text{otherwise } v_a = 0$$

Illustration of the implementation of the above algorithm for the network shown in fig 1(a) through the steps involved to get the optimal strategy and assign the flows are shown in **Appendix A**. The strategy formulation as proposed by Spiess and Florian (1989) forms the basis of the transit assignment software EMME-2 (Constantin and Florian, 1995).

(b) Hyperpath:

The 'strategy' concept was further enhanced by providing a graphical framework to the transit network in Nguyen and Pallottino (1988). The graphical approach enabled users to specify the network in a node-arc representation. The network therefore was represented as  $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$  with the bus stops  $\hat{s} \in \mathcal{N}$  and the directed boarding arcs  $a \in \mathcal{A}$ . A hyperpath which joins  $p^{\text{th}}$  OD pair is given as  $H_p = \{\mathcal{N}_p, \mathcal{A}_p, \pi_p\}$  where  $\mathcal{N}_p \subset \mathcal{N}$ ,  $\mathcal{A}_p \subset \mathcal{A}$  and  $\pi_p$  is the choice probability of hyperpath  $H_p$ .  $\mathcal{A}_p$  consists of several head and tail nodes denoted as  $\{i, j\} \in \mathcal{A}_p$  such that  $\{i\} \in \mathcal{N}_p$  and  $\{j\} \in \mathcal{N}_p$ . Each arc  $\{i, j\}$  is associated with as cost  $c_{ij}$  and from the tail node  $\{j\}$  there is a set of forward star nodes  $\{j^+\} \in E_j$ .  $E_j$  denotes the set of tails nodes which forms the attractive line set of travel from the  $\{i\}$  node. From each  $\{i\}$  there is a choice probability associated with accessing node  $\{j\} \in E_j$

$$\eta_{ij}^p = \frac{\varphi_j}{\sum_{a \in E_j^p} \varphi_a} \quad (2.5)$$

Where  $\varphi_j$  is the frequency of the line segment  $ij$

The choice probability of a path  $\check{r}$  within the hyperpath  $p$  is given as

$$\kappa_{\check{r}}^p = \prod_{(i,j) \in \check{s}} \eta_{ij}^p$$

Using the above specification the cost of the hyperpath  $p$  is computed as the sum of the costs of constituent nodes and arcs thereby resulting in the following formulation:

$$C^p = \sum_{\check{r} \in S^p} \kappa_{\check{r}}^p \left( \sum_{(i,j) \in \check{s}} t^{in-veh}_{ij} + \sum_{i \in \check{s}} W_i^p \right) \quad (2.6)$$

Where  $W_i^p$  is the waiting time associated with path  $p$  at node  $i$  and  $t^{in-veh}_{ij}$  is the in-vehicle travel time of the arc connecting nodes  $i$  and  $j$  and  $S^p$  is the set of paths within the hyperpath  $p$ . The proposed solution algorithm uses bellman's dynamic principle to solve the shortest hyperpath problem recursively from destination to the origin. Bellman's principle states that 'every node in the quickest path will have a unique back node'. Hence if node  $i$  is on the quickest path and node  $j$  forms its back node on that path; then any other quickest path from origin to destination via node  $i$  shall have node  $j$  as its back node (Bellman, 1956). The optimality principle required for recursive bellman's dynamic principle is further explained by Gentile et al. (2005) as the one where all the sub strategies of the optimal strategy are themselves optimal.

The methods reviewed above are based on several assumptions such as

- (a) Random arrivals of passengers
- (b) Exponential arrival of transit services
- (c) Independence of the lines serving the bus stop.
- (d) The passengers do not have passenger information at bus stops.

The assumption of exponential arrivals has been criticised by many studies such as (Gentile et al., 2005; Marguier and Ceder, 1984) and they have adopted Erlang headway distribution. Marguier (1981) was able to show that the common lines problem solved by the heuristic algorithm given in Chriqui and Robillard (1975) was applicable only in exponential interarrivals. A modified algorithm to solve the common lines for deterministic interarrivals was proposed by Gentile et al. (2005). Though the application of erlang interarrivals and its impact on the waiting time distributions has been dealt with in Gentile et al. (2005) and Marguier and Ceder (1984) it has not been explicitly understood. The relationship between waiting and interarrival distribution was

detailed in Larson and Odoni (1981). Larson and Odoni (1981) show that if  $R^{\text{th}}$  bus arrives at the bus stop,  $H_r$  units of time after the  $R-1^{\text{th}}$  bus arrival then  $H_r$  denotes the bus headway and that the probabilistic occurrence of the headway will decide the probability distribution of the waiting time of a passenger for that line.

It is assumed that  $H_r$  values are identically distributed though they are not independent. It is indicated that the 'random' arrival of the passengers at the bus stop is of significance as such an assumption indicates that the arrival of the next passenger at the bus stop cannot be determined using historical data of actual arrival times of the passengers which is obtained through surveys.

Having assumed that the passengers arrive at the bus stop at random, the derivation of probability law for  $W$  (waiting time; which is the duration between the time of the random incidence of passenger arrival at the bus stop and the time of next arrival of bus) is carried out. In order to achieve the probability law for  $W$  it is necessary to know the probability law on  $Y$  (the length of the inter arrival gap entered by random incidence). The length of this inter arrival gap can be split as (a). The time gap between arrival of the most recent bus and the arrival of passenger at the bus stop (b). The time gap between the arrival of the passenger at the bus stop and the arrival of the next bus.

The probability that a gap which a passenger arriving at random at the bus stop enters assumes a value between  $y$  and  $y + dy$  and is given as

$$P(y \leq Y \leq y + dy)$$

which is the p.d.f of  $Y$

$$f_Y(y)dy$$

The relationship between the random variables  $Y$  and  $H$  has to be ascertained to determine the probability of a random incidence entering a gap between  $y$  and  $y + dy$ . Given below (fig 2.2) is an example wherein it is assumed that the values of  $y$  and  $h$  are discrete and not continuous in order to explain the relationship between  $Y$  and  $H$ . It is understood that the logic applied to a discrete case will also hold true for a continuous case. Assuming that (as given in fig 1) the headway  $H$  has two values  $h_1 = 15\text{min}$  and  $h_2 = 60\text{ min}$

$$P(h_1) = 8/10 \text{ and } P(h_2) = 2/10$$

Now assuming that a passenger arrives at uniform intervals of 1 min we have 15 passengers in the interval of 15 minutes and 60 passengers in the interval of 60 minutes. Hence in the total time period of 4 hours, 120 passengers have entered the gap of 60 minutes width and 120 passengers entered the gap of 15

minutes width hence the probability of being incident on a gap of width 15 min and the probability of being incident on a gap of width 60 minutes is the same though the frequency of occurrence of these gaps varies.

$$P(y_1=15 \text{ min}) = 120/240=1/2$$

$$P(y_2=60 \text{ min}) = 120/240=1/2$$

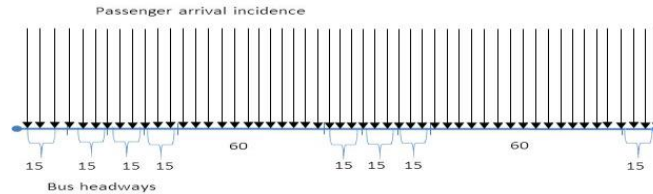


Fig 2.2: Frequency Relationship between G and H

Now let us assume that in an interval of 45 minutes, 15 minute headway and 30 minute headway have equal probability of occurrence as shown in fig 2.3.

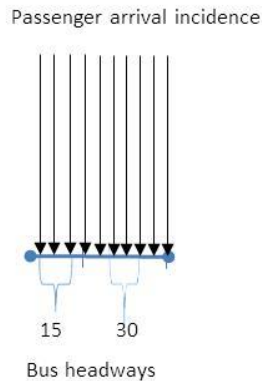


Fig 2.3: Width Relationship between G and H

It can be seen from fig 2 that  $p(h_1 = 15 \text{ min})=1/2$  and  $p(h_2= 30 \text{ min})=1/2$ , however as assumed in the previous example if a passenger arrives at intervals of 1 minutes then we have 15 passengers arriving in a 15 min gap and 30 passengers arriving in 30 minute gap.

$$P(y_1=15 \text{ min}) = 15/45=1/3$$

$$P(y_2=30 \text{ min}) = 30/45=2/3$$

Hence a passenger arriving at random is twice as likely to enter the gap of 30 min rather than in the gap of 15 min though the probability of occurrence of both the headways are the same. Therefore these examples highlight that the probability of a passenger arriving at random and entering a gap of width  $y$  and  $y + dy$  (continuous variable) is dependent on the frequency of occurrence of such gaps as well as the width of the gaps. We can also deduce from these

examples that Y is the headway as experienced by the passengers whereas H is the headway as set by the operator.

Mathematically

$$f_Y(y)dy \propto f_H(y) dy \cdot y$$

Where  $f_H(y) dy$  denotes the relative frequency of occurrence of gaps with length y to y+dy where y - is the length of the gap

Then the constant of proportionality is  $1/E(H)$  because  $\int_0^{\infty} f_H(y) dy \cdot y = E(H)$  hence

$$f_G(y)dy \propto E(H)$$

$$f_G(y)dy = kE(H)$$

where  $\int_0^{\infty} f_G(y)dy = 1$

Hence k (constant of proportionality) =  $1/E(H)$

Therefore  $f_Y(y)dy = yf_H(y)/E(H)$

Having found the probability of a passenger entering a gap of size [y ,y+dy] we now find the probability of an arriving passenger being on various location within the gap.

Let us assume that based on the p.d.f of Y we identify a gap of length y which is the gap in which the passenger is incident upon on random arrival i.e. the passenger arrives at the bus stop during the headway gap of Y. There is a constant probability of arriving passenger being in any interval § and §+h where [§,§+h] is fully contained in y.

Hence if the value of y is 15 min the probability that the passenger arrives in 1 min  $P(1) = P(2)=P(3)...=P(15)$  where  $P(1)=1/15 = 1/ y$ . This example assumes y is discrete, if g is continuous then the probability that a passenger arrives within an interval [§,§+h] conditional that the value of waiting time W does not exceed the value of gap Y will also  $1/ y$  and is denoted as

$$f_{W|Y}(w|y) = 1/y \quad 0 \leq w \leq y$$

The joint p.d.f for W and Y:

$$f_{W,Y}(w, y) = f_{W|Y}(w|y)f_Y(y)$$

$$= \frac{1}{y} \frac{y f_H(y)}{E(H)} \quad 0 \leq w \leq y \leq \infty$$

The marginal for W is formed as

$$f_W(w) = \int_w^{\infty} f_H(y) dy / E(H) \quad (2.7)$$

Or when representing the above equation as the probability density function of a line

$$f_l(w) = \int_w^{\infty} g_l(h) dh / E(h)$$

From the generic formulation shown in eq(2.7) it is possible to arrive at the p.d.f of the waiting time for various headway distributions.

For example when the line headway for line 'l' is exponentially distributed the probability density function for the waiting time of the line can be computed from equation (2.7) as

$$g_l(h) = \begin{cases} \varphi_l e^{-\varphi_l h}, & h_1 \geq 0 \\ 0, & h_1 < 0 \end{cases}$$

Wherein  $\varphi_l$ - frequency of the line l

And

$$E(h_l) = \frac{1}{\varphi_l}$$

$$f_l(w) = \frac{\int_w^{+\infty} \varphi_l e^{-\varphi_l h} dh}{\frac{1}{\varphi_l}}$$

Hence

$$f_l(w) = \varphi_l e^{-\varphi_l w} \quad (2.8)$$

In case of erlang distribution

$$g_l(h) = \frac{(e^{-m_l \varphi_l h} (m_l \varphi_l)^{m_l} h^{m_l-1})}{(m_l - 1)!}$$

Wherein m- is the shape factor of erlang distribution

$$f_l(w) = \int_w^{\infty} \frac{(e^{-m_l \varphi_l h} (m_l \varphi_l)^{m_l} h^{m_l-1})}{(m_l - 1)! E(h)} dh$$

$$\int_w^{\infty} \frac{(e^{-m_l \varphi_l h} (m_l \varphi_l)^{m_l} h^{m_l-1})}{(m_l - 1)!} dh = \int_w^{\infty} \frac{m_l \varphi_l e^{-m_l \varphi_l h} (m_l \varphi_l h)^{m_l-1}}{(m_l - 1)!} dh$$

$$\begin{aligned}
 &= \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m_1-1}}{(m_1-1)!} + \int_w^\infty \frac{(m_1\varphi_1)^{m_1-1} e^{-m_1\varphi_1 h} h^{m-2}}{(m_1-2)!} dh \\
 &= \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m_1-1}}{(m_1-1)!} + \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m-2}}{(m_1-2)!} \\
 &\quad + \int_w^\infty \frac{(m_1\varphi_1)^{m_1-2} e^{-m_1\varphi_1 h} h^{m-3}}{(m_1-3)!} dh \\
 &= \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m_1-1}}{(m_1-1)!} + \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m-2}}{(m_1-2)!} + \frac{e^{-m_1\varphi_1 w} (m_1\varphi_1 w)^{m-3}}{(m_1-3)!} \\
 &\quad + \int_w^\infty \frac{(m_1\varphi_1)^{m_1-2} e^{-m_1\varphi_1 h} h^{m-4}}{(m_1-4)!} dh \\
 &= e^{-m_1\varphi_1 w} \sum_{k=0}^{m-1} \frac{(m_1\varphi_1 w)^{\check{k}}}{\check{k}!} \\
 f_1(w) &= \frac{e^{-m_1\varphi_1 w} \sum_{k=0}^{m-1} \frac{(m_1\varphi_1 w)^{\check{k}}}{\check{k}!}}{E(h)} \\
 f_1(w) &= \frac{e^{-m_1\varphi_1 w} \sum_{k=0}^{m-1} \frac{(m_1\varphi_1 w)^{\check{k}}}{\check{k}!}}{\frac{1}{\varphi_1}}
 \end{aligned}$$

$$f_1(w) = \varphi_1 e^{-m_1\varphi_1 w} \sum_{k=0}^{m-1} \frac{(m_1\varphi_1 w)^{\check{k}}}{\check{k}!} \quad (2.9)$$

The assumption of no information at bus stops was overcome by Gentile et al. (2005) who showed that ‘signs at bus stops’ resulted in a lesser travel time in case of uncongested (demand not exceeding supply) transit networks. The choice probability at node  $j$  for line  $\ell$  within the hyperpath framework with information was therefore formulated as in eq 2.10.

$$\eta_{j,\ell}^p = \int_0^\infty f_\ell(w) \prod_{a \in A^* \setminus \ell} P(w_a \geq w_\ell + t^{in-veh}_\ell - t^{in-veh}_a) dw \quad (2.10)$$

Where  $f_\ell(w)$ - waiting time density function of line  $\ell$  (computed as given in eq 2.9 for erlang distribution and 2.8 for exponential distribution).

$P(w_a \geq w_\ell + t^{in-veh}_\ell - t^{in-veh}_a)$ - probability that the waiting time of line  $a$  ( $w_a$ ) (given by the 'sign at bus stop') is greater than or equal to the difference between the total travel time on line  $\ell$  ( $w_\ell + t^{in-veh}_\ell$ ) and the in-vehicle travel time of line  $a$  ( $t^{in-veh}_a$ ).

**Appendix B** highlights the difference in choice probabilities of a line with 'signs at bus stops' and without 'signs at bus stops' for an example network using Gentile et al. (2005) formulation. The application of the information scenario on the traffic network can be studied extensively; Henn and Ottomanelli (2006) assessed the impact of information when stochastic traffic generation is simulated.

### **2.2.3 Capacity constrained models:**

The previous section dealt with unconstrained models wherein it was assumed that the passenger was successful in boarding the first arriving transit service in his/her attractive line set. However several of the existing transit system seldom run with its demand being significantly lesser than the supply. Hence the need to model capacitated transit services arises.

#### **2.2.3.1 BPR models**

The earlier models of congestion described congestion by 'discomfort' cost. 'Discomfort cost' introduced in seminal paper of Spiess and Florian (1989) was meant to model the 'discomfort' associated with the exceeding passenger demand within the vehicle. The model did not explicitly account for the capacity constraints of transit vehicle as the passengers were allowed to board the first transit service arriving and queuing delays at the transit stop were not dealt with. The discomfort experienced by the passenger within the transit vehicle was modelled as a BPR function such that the experienced in-vehicle travel time 'increased' with the increase in passenger flow on the line. The model was able to propose an algorithm for an objective function which was separable by destination and the sub problem for each destination was equivalent to the problem with constant link travel times. Spiess and Florian (1989) however acknowledge that the proposed model had certain deficiencies such as (a) all the passengers suffered from the same degree of discomfort (b) the waiting times were not affected and the dwell time associated with the number of passengers boarding and alighting was also not considered. De Cea and Fernández (1993) introduced the increased waiting time associated with congestion by a BPR function. The BPR styled congestion function is then added to the in-vehicle travel time and the uncongested waiting time to obtain the



cost of the route section. In the congested model the attractive lines set were modified based on 'effective frequencies' (described in section 2.2.3.2). The network is thus defined initially as  $\mathcal{G} = (N, \mathcal{L})$  where  $\mathcal{L}$  consists of the set of attractive lines associated with uncongested network and the network is redefined as  $\mathcal{G} = (N, \mathcal{L}')$  wherein  $\mathcal{L}'$  consists of the attractive lines set which includes even the slow lines thereby expanding the attractive lines set to include all the lines of the network. De Cea and Fernández (1993) note that the congested waiting time experienced by a passenger boarding the route section  $s$  at its origin node  $i(s)$  shall depend on

- a.  $v^s$  the total number of passengers boarding the same route section at the origin
- b.  $v_{is}^+$  the number of passengers boarding other route sections that use the lines contained in route section  $s$
- c.  $\bar{v}_{is}$ , the number of passengers boarding the route section  $s$  before  $i(s)$  and alighting after  $i(s)$ .

The above definitions help define the capacity sharing and competition between passengers of the upstream transit stop with the downstream transit stop and the interaction between passengers trying to board the same line shared by different route sections. The congested waiting time formulation was thus efficient in capturing the asymmetric interaction of the costs between various route users. The waiting time in BPR styled models following the above specification is given as in eq 2.11:

$$E[W] = \frac{1}{\varphi_s} + \zeta \left( \frac{\bar{v}_{is} + v^s}{Cap_s} \right)^b \quad (2.11)$$

Where  $Cap_s$  – is the capacity of the route section which is defined as  $Cap_s = \sum_{l \in s} c_l \varphi_l$  with  $c_l$  and  $\varphi_l$  being the capacity and frequency of line  $l$ ,  $\varphi_s$  – is the frequency of the route section  $s$  and  $\zeta, b$  are the calibration parameters.

A hyperpath implementation of BPR styled cost function was done by Wu et al. (1994) whose waiting arcs had a BPR styled cost function depicting an increase in waiting time due to congestion in addition to waiting time experienced in an uncongested network, the in-vehicle arcs consisted of the in-vehicle travel costs as well as discomfort cost which was again a BPR styled function. The waiting arcs had cost function of the form:

$$c_{\bar{f}}(v_{\bar{f}}, v_d) = \alpha_2 \left( \frac{v_{\bar{f}} + \zeta_2 v_d}{\varphi_{\bar{f}} c_{\bar{f}}} \right)^{b_1}, \bar{f} \in E \text{ (waiting arcs)} \quad (2.12)$$

And the discomfort cost was denoted as

$$c_{\bar{x}}(v_{\bar{x}}, v_d) = \alpha_3 t^{in-veh}_{\bar{x}} + \zeta_3 \left( \frac{\gamma_3 v_{\bar{x}} + v_d}{\varphi_{\bar{x}} c_{\bar{x}}} \right)^{b_2}, \bar{x} \in I(in - vehicle arcs) \quad (2.13)$$

Where  $\bar{x}$  is the in-vehicle arc associated with waiting arc  $\bar{x}$ ,  $v_{\bar{x}}$  are the passengers waiting to board the transit service,  $v_b = v_{\bar{x}} + v_d$ ,  $v_d$  denotes the direct passenger flows,  $\alpha_1, \alpha_2, b_1, \alpha_3, \zeta_3, b_2, \zeta_2, \gamma_3$  are calibration parameters,  $t^{in-veh}_{\bar{x}}$  is the in-vehicle travel time of line  $\bar{x}$ .

The equations given in 2.12 and 2.11 are structurally same, however De Cea and Fernández (1993) utilize a 'diagonalization' algorithm to solve the asymmetric problem whereas Wu et al. (1994) utilise the 'symmetric linearization' method.

### 2.2.3.2 'Effective frequency' models

The notion of 'effective frequency' is first mentioned in Spiess and Florian (1989) however is mathematically formulated and implemented in De Cea and Fernández (1993). De Cea and Fernández (1993) define 'effective frequency' as inverse of 'equivalent average waiting time index' associated with a line section and is given as :

$$w_l^i = \frac{\alpha}{\varphi_l} + \mu_l \left( \frac{\bar{v}_l}{\varphi_l c_l} \right) \quad (2.14)$$

Where  $\bar{v}_l$  is the number of passengers taking the line section before and alighting after the transit stop,  $\mu_l$  is a monotonically increasing function of  $\bar{v}_l$  and takes the BPR formulation as  $\mu_l = \zeta_l \left( \frac{\bar{v}_l}{\varphi_l c_l} \right)^b$ ,  $c_l$  is the capacity of line section  $l$ . It is noted that 'equivalent average waiting time index'  $w_l^i$  at transfer stop  $i$  is the same for all passengers boarding at stop  $i$  irrespective of their route section choice (provided that the route section consists of line section  $l$  within its attractive line set). The boarding probabilities are therefore modified with respect to the 'effective frequency' over every iteration such that with increase in congestion the boarding probability for the line gets reduced. Hyperpath based implementation of effective frequency was also carried out along with parallel modifications on the waiting time cost function which modelled the residual capacities in congested conditions. Bouzaïene-Ayari et al. (2001) mention the modification of distribution models which were initially based on the nominal frequency into 'residual capacity models' (in case of congested networks) by Gendreau (1984). However Bouzaïene-Ayari et al. (2001) pointed out that the 'residual capacity models' produced better results only in congested condition and hence he proposed an 'adjusted residual capacity model' which were efficient at various levels of congestion. The 'adjusted residual capacity model' is given as follows

$$\eta^1_{\ell}(v_{\ell}) = \frac{\overline{Cap}_{\ell}}{Cap_{\ell}} \varphi_{\ell} \forall \ell \in A^* \quad (2.15)$$

Where  $Cap_{\ell}$  is the capacity of line  $\ell$ ,  $\overline{Cap}_{\ell}$  is the residual capacity (after boarding the stop)  $\varphi_{\ell}$  is the mean frequency of the line  $\ell$ ,  $v_{\ell}$  the aggregate passenger flow on the line  $\ell$ ,  $\eta^1_{\ell}(v_{\ell})$  is the distribution model.

The waiting time model proposed by Bouzaïene-Ayari et al. (2001) are of two different types wherein both the models have the following generic formulation:

$$W_{\delta}(v_{A^*}) = w_{\ell}(v_{\ell}) \frac{\mu_{\ell}(v_{\ell})}{\sum_{\ell' \in A^*_{\delta}} \mu_{\ell'}(v_{\ell'})} \quad (2.16)$$

Where  $\delta$  defines a specific strategy,  $W_{\delta}(v_{A^*})$ - is the waiting time of passengers (which is a function of flow  $v_{A^*}$ ) following strategy  $\delta$ ,  $\mu_{\ell}(v_{\ell})$ - attraction factor of line  $\ell$  and  $\mu_{\ell}(v_{\ell})$  is a strictly decreasing function of  $(v_{\ell})$ .

The first model with strict capacity constrains assumes that the line capacity are strict and therefore the aggregated passenger flows in different lines are not allowed to exceed the capacities. The generic equation given in eq 2.16 above is then reformulated as:

$$w_{\ell}(v_{\ell}) = \frac{1}{2\varphi_{\ell}} \left(1 + \frac{1}{m_{\ell}}\right) + \frac{\zeta_4}{2\varphi_{\ell}} \left(\frac{1}{m_{\ell}} + \frac{\varphi_{\ell}}{c_{\ell} + \varphi_{\ell}}\right) \left(\frac{v_{\ell}}{c_{\ell} - v_{\ell}}\right) \quad (2.17)$$

Where  $\zeta_4$  is a parameter that can be calibrated using real data or simulation results,  $c_{\ell}$ - capacity of line  $\ell$ ,  $\varphi_{\ell}$ - frequency of line  $\ell$  and  $m_{\ell}$ - integer shape factor of erlang distribution, . The second model was without strict capacity constraints and allowed for the aggregate flows to exceed the capacity of the line. This resulted in the formulation of the waiting time as follows:

$$w_{\ell}(v_{\ell}) = \frac{1}{2\varphi_{\ell}} \left[ \left(1 + \frac{1}{m_{\ell}}\right) + \left(\zeta_5 \frac{v_{\ell}}{c_{\ell}}\right)^{b_3} \right] \quad (2.18)$$

Wherein  $\zeta_5$  is a parameter that can be calibrated using real data or simulation results,  $c_{\ell}$ - capacity of line  $\ell$ ,  $\varphi_{\ell}$ - frequency of line  $\ell$  and  $m_{\ell}$ - integer shape factor of erlang distribution,  $v_{\ell}$ - aggregate passenger flow on line  $\ell$ .

Cominetti and Correa (2001) propose the usage of 'effective frequency' for each line segment which is differentiable such that the optimal decision of each passenger is affected by the choices of others. Hence they reformulate the common lines problem as an equilibrium problem. They prove the existences of multiple 'equilibrium cost strategies' in event of congestion for certain flow ranges and argue that an increase in flow during such situations does not increase the costs of these equilibrium strategies.

Consider the two line example as shown in fig 2.4:

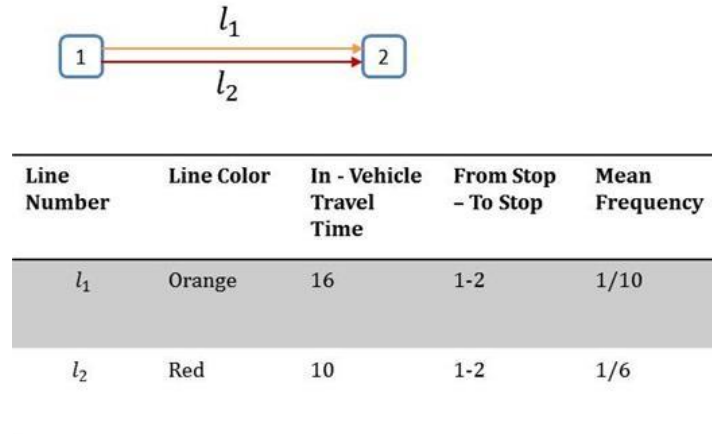


Fig 2.4 : Presence of multiple strategies as shortest hyperpath/strategy

The computation of shortest strategy between node 1 and node 2 results in {2} and {1,2} as candidates for shortest hyperpath/strategy, hence proving the presence of multiple solutions for a problem even in case of uncongested transit networks. However Cominetti and Correa (2001) did not specify a solution algorithm for the congested network with ‘effective frequency’. Cepeda et al. (2006) developed a MSA based solution algorithm for the ‘effective frequency’ model proposed by Cominetti and Correa (2001). The effective frequency formulation proposed by Cepeda et al. (2006) is of the form

$$\lambda'_a(v) = \begin{cases} \varphi \left[ 1 - \left( \frac{v_a}{\varphi c - v_a' + v_a} \right)^\beta \right] & \text{if } v_a' < \varphi c \\ 0 & \text{otherwise} \end{cases} \quad (2.19)$$

Where  $\varphi$  is the ‘nominal frequency’ of line segment a,  $c$  is the capacity of line segment a,  $v_a$  is the flow boarding at stop onto line segment a,  $v_a'$  is the flow immediately after the boarding stop,  $\beta$  is parameter.

### 2.2.3.3 ‘Strict capacity’ models

The above mentioned models worked under the surmise that the demand doesn’t exceed capacity. ‘Strict capacity Models’ handle cases where the demand exceeds capacity. In frequency based assignment models the strict capacity was explicitly dealt with by Schmoecker (2006). He proposed a dynamic framework to model congested network and introduced a ‘failure arc’ (fig 2.5) to account for the passengers who couldn’t board the service of their choice in a given time step. The algorithm was formulated such that these passengers were added onto the demand generated in the next time step.

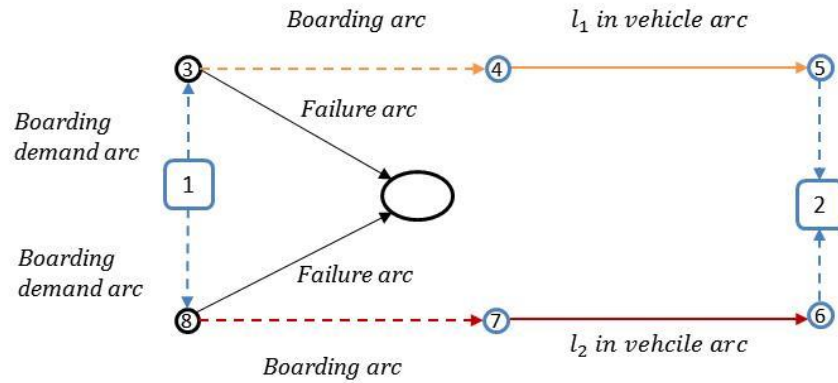


Fig 2.5: Introduction on failure node and failure arc in hyperpath representation

### 2.2.3.4 Dynamic models

The strict capacity models developed by Kurauchi et al. (2003), Schmöcker et al. (2008) are quasi dynamic in nature and assume a discrete division of time periods within which the flow assignment is considered static. Schmöcker et al. (2011) implemented the dynamic model to assess the route choice based on the seat availability at transit stop. Trozzi et al. (2013) proposes a dynamic model which considers the FIFO principle of passengers at the transit stops and proposes a diversion probability which is time dependent and models the expected congestion at that time step. The congestion effect is modelled as a 'Bottle neck queue model' which has a time varying exit capacity. Cats et al. (2011) used a simulation based framework to assess the impact of information provision in the path choice of Stockholm metro passengers. The model studied the effect of information on the total travel time at a microscopic as well as aggregate level.

## 2.3 Random Utility Models (RUMs):

Apart from the total journey time (in-vehicle travel time + waiting time) a passenger chooses his/her route choice based on several other factors with each factor being weighed differently by each passenger. These factors put together tend to form the dis-utility/cost associated with each route. Hence utility helps identifying the preference of a passenger when faced with several alternatives. The various forms of RUMs are discussed in the following sections:

### 2.3.1 Probabilistic choice models – Multinomial Logit Model (MNL):

Once the routes between the OD pair have been defined then based on the random utilities associated with each route the choice probability for that route can be computed. When routes are defined in terms of utility the passengers

tend to choose a route which maximises his/her utility if however the routes are defined based on generalised costs the passenger then tends to choose a route which minimises his/her generalised cost. As mentioned already in probabilistic choice models the utilities are assumed random hence they have a deterministic component and a stochastic component as shown in eq (2.20).

$$U_k^i = \mathcal{V}_k^i + \varepsilon_k^i \quad (2.20)$$

Wherein  $U_k^i$  denotes the utility of route  $k$  for individual  $i$ ;  $\mathcal{V}_k^i$  denotes the deterministic component and  $\varepsilon_k^i$  the stochastic component. It is noted that  $\varepsilon_k^i$  is independent and identically distributed for each individual  $i$  and the joint distribution of  $\varepsilon_k^i$  assumed over all  $k$  decides the choice probabilistic choice model (logit or probit).

In event of MNL,  $\varepsilon_k^i$  has a gumbel probability distribution with zero mean. The independence of  $\varepsilon_k^i$  results in covariance between any pair of residuals to be zero i.e.

$$Cov[\varepsilon_j^i, \varepsilon_h^i] = 0 \forall j, h \in k$$

As described in Cascetta (2001), Sheffi (1985) the multinomial logit suffers from ‘independence from irrelevant alternatives (IIA)’ wherein the choice probability ratio of two alternatives remains the same irrespective of the number and the utility of the other alternatives. A realistic approach to overcome this disadvantage is to allow for covariance to exist between the random residuals of the alternatives having overlapping links (section 2.3.2). Improvements to the MNL to deal with IIA is brought about by introduction of C-logit and Path-Size logit models.

### 2.3.2 C-Logit and Path size logit:

Cascetta et al. (1996) and Ben-Akiva and Bierlaire (1999) overcome the IIA problem of MNL by introducing a correction factor which accounts for the overlap between the alternatives. Cascetta et al. (1996) define that in C-Logit the commonality factor is described in several different ways giving rise to different C-Logit specifications. The commonality factor is accounted for by subtracting it from the deterministic part of the utility function as shown below

$$U_r = \mathcal{V}_r - CF_r$$

Wherein  $\mathcal{V}_r$ - is the deterministic component and  $CF_r$  – is the commonality factor for route .

One possible way is by using the length of the links common to paths wherein the lengths can be either physical or link additive part of the generalised cost and is given as specified below:

$$CF_r = \beta_0 \ln \sum_{h \in I_{rs}} \left( \frac{L_{hr}}{L_h^{1/2} L_r^{1/2}} \right)^\gamma \quad (2.21)$$

Where  $L_{hr}$  is the length of the links common to paths  $h$  and  $r$ ; while  $L_h$  and  $L_r$  are the overall length of paths  $h$  and  $r$ ;  $\gamma$  is a positive parameter and the summation is extended to all paths belonging to  $I_{rs}$  and  $\beta_0$  is a parameter. Alternative forms of commonality factor as specified in Cascetta et al. (1996) are :

$$CF_r = \beta_0 \ln \sum_{i \in r} w_{ir} \mathfrak{N}_i \quad (2.22)$$

$$CF_k = \beta_0 \sum_{i \in r} w_{ir} \ln \mathfrak{N}_i \quad (2.23)$$

Wherein the summation is extended over all the links  $i$  in path  $r$ ,  $\mathfrak{N}_i$  is the number of paths and  $w_{ik}$  is the proportional weight for link  $i$  in path  $r$ .

In case of path size logit a 'size' variable is introduced in the utility function as shown

$$U_r = \mathcal{V}_r + \ln s_r$$

Wherein

$$s_r = \sum_{a \in \Gamma_k} \frac{l_a}{L_r} \frac{1}{\sum_{h \in \text{Pod}} \delta_{ah}} \quad (2.24)$$

Where  $\Gamma_k$  is the set of links composing path  $r$ ,  $l_a$  is the length of link  $a$  and  $\delta_{ah}$  equals 1 if link  $a$  is in the path  $h$  or 0 otherwise.

### 2.3.3 Probit Models:

In probit models the random error term is assumed to have a normal distribution for each utility. Hence the joint density function of the random error term is assumed to follow a multivariate normal function as shown below

$$U_r \sim MNV[\mathcal{V}_r, \Sigma] \quad (2.25)$$

$$\text{Wherein } (\Sigma)_{rr} = \text{var}[\varepsilon_r] \forall r \text{ and } (\Sigma)_{rh} = \text{cov}[\varepsilon_r, \varepsilon_h] \forall r \neq h$$

Sheffi (1985) highlights that in case of probit models the choice probability cannot be expressed analytically since the cumulative normal distribution of the error terms cannot be evaluated in closed form. In case of binary alternative the choice probabilities can be computed by referring the cumulative normal tables. Sheffi (1985) indicates that in case of more than two alternative the choice probability can be computed using Monte carlo simulation or Clarks Method.

## 2.4 Reliability from passenger's perspective:

In this section the reliability aspect from passenger's perspective shall be focussed. The importance an operator associates to schedule adherence of a service percolates onto the reliability issues faced by passengers however the correlation between the two reliability issues needs a cautious approach. A service which is 2 minutes late everyday implies a deviation from its schedule to the operators however from passenger's perspective since the service is always 2 minutes late they associate service arrival time with the modified arrival times and hence tend to find the service reliable. Several attitudinal surveys (Bates et al., 2001, Jackson and Jucker, 1982, Noland and Polak, 2002) help define passenger's attitude towards reliability attributes. Bates et al. (2001) find reliability in a system synonymous to system's ability to be consistent and predictable. The definition put forth by Bates et al. (2001), Abkowitz (1978) helps deduce that an ideal measure of reliability could be one which measures the deviation of an attribute from its average value experienced by the passenger. This deduction is further emphasised by the fact that in a system where headway distributions are random, passengers often tend to base their journeys on their previous experience of the reliability attribute most weighted by them rather than base it on a specified time table.. Ceder (2007) define the various attributes of importance to the passengers(fig 2.6).

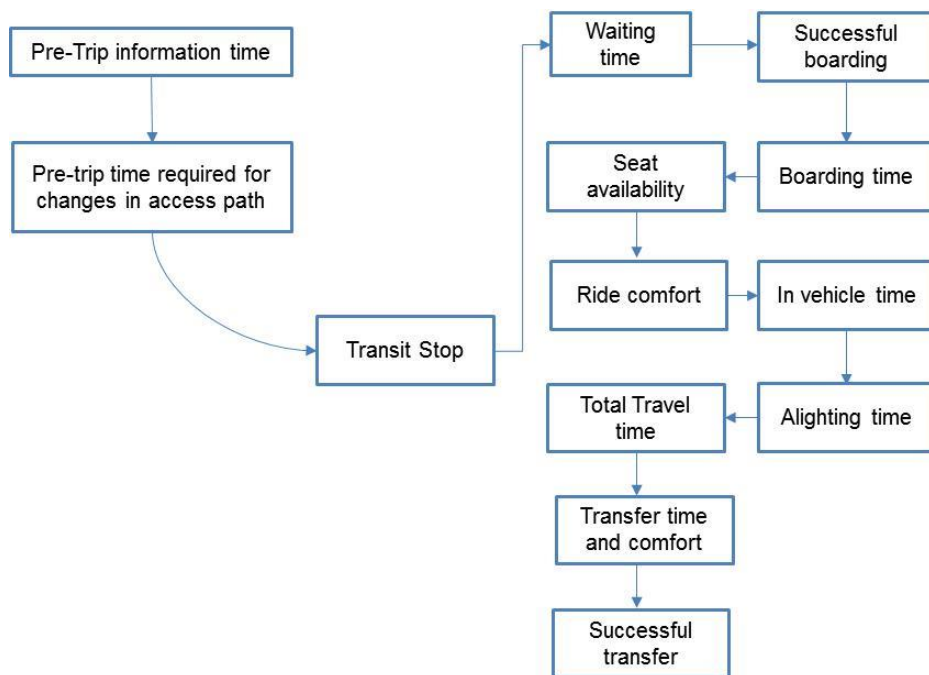


Fig 2.6: Source: Ceder (2007): reliability attributes of concern to passengers



Bates et al. (2001) identify that reliability has an impact on the route choice of passengers because of two reasons; first being that passengers are sensitive in consequences associated with travel time variability such as being late, missing connection etc. This can be modelled by the planners by assuming that each of the route choices available to passengers is associated with a distribution of consequences which is represented in terms of utility function and the passenger chooses a route that maximises his/her utility. The second reason is that passengers are sensitive to variability in itself due to the stress it causes and hence the route choice is modelled by adding an extra term of travel time variability or a dummy variable, to indicate the deviations in headways.

The first approach to modelling the impact of the consequences of unreliability is termed 'scheduling approach' whereas the second approach is given the term 'mean-variance' approach.

#### **2.4.1 Scheduling approach:**

The most important aspect of modelling reliability using schedule based approach is the determination of the departure time for various purposes of the trip. The central idea behind the optimisation of the departure time was the notion that each departure time is associated with a disutility function which not only consisted of the disutility from travel time but also the disutility associated with the early arrival or late arrival at the destination (Noland and Small, 1995; Small, 1982; Bates et al., 2001; Noland and Polak, 2002). The concept was formulated by Small (1982) based on earlier works of Vickrey (1969). With an assumption that the passengers have a PAT (preferred arrival time) associated with each purpose of the journey and departure time  $D$  the formulation of schedule based reliability approach essentially consists of choosing a departure time which maximises the utility:

$$\text{Max } U(D, PAT)$$

The above maximisation function is expanded to take the form given in Small (1982)

$$U(D) = \alpha T.T + \eta SDE + \gamma SDL + \partial D_1 \quad (2.26)$$

Where  $T.T$  -travel time

$SDE$ - schedule delay associated with early arrival at the destination  $\text{Max}(0, PAT - [D + T.T(D)])$

$SDL$  - scheduled delay associated with late arrival  $\text{Max}(0, [D + T.T(D)] - PAT)$

$D_1$  - Dummy variable equal to 1 if  $SDL > 0$ , 0 otherwise

$\hat{\alpha}, \eta, \gamma, \partial$  - Model parameters which is negative and depends on family status, occupation, choice of transport mode and employers policy towards work hour flexibility in case of journey to work models.

Equation 2.26 equates the additional costs incurred at destination plus the value of additional travel time due to a change in  $D$  to the value of utility gained both directly and indirectly through the additional time period associated with changing departure time (Small, 1982).

Fig (2.7) shows the shape of 'schedule disutility'(disutility computed without considering the disutility associated with travel time variation) as given in Small (1982) for work trip with varying flexibility in arrival times at the work (destination).

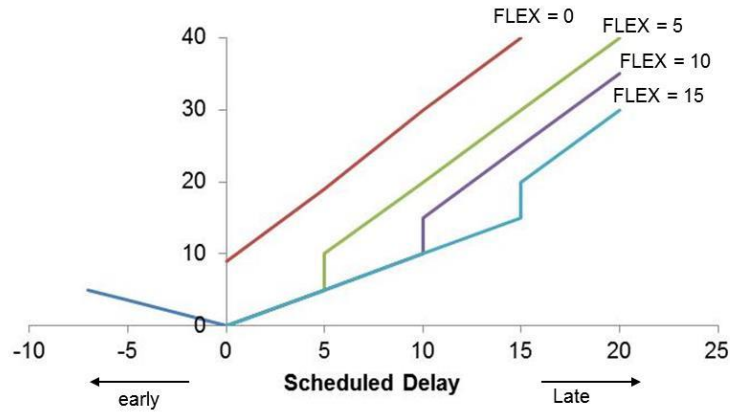


Fig 2.7 : Source : Small (1982) Disutility of schedule delay

In order to represent the stochastic nature of travel times, Noland and Small (1995) assume that the travel time has two components namely free flow travel time and extra travel time due to recurrent congestion and non-recurrent congestion.

$$T.T = T_f + T_{rc} + T_{nrc} \quad (2.27)$$

Where  $T_f$  - free flow travel time,

$T_{rc}$  - travel time recurrent congestion

$T_{nrc}$  - travel time due to non -recurrent congestion

Noland and Small (1995) also integrate non-recurrent congestion with the cost function specified by Small (1982), by assuming non-recurrent congestion to follow distributions, namely, uniform distribution and exponential distribution. Since  $T_{nrc}$  in the above formulation is stochastic in nature the cost function also takes a stochastic form as follows:

$$E[U(D)] = E[T.T] + \eta E[SDE] + \gamma E[SDL] + \partial E[P_{late}] \quad (2.28)$$

Where  $P_l$  describes the probability of arriving late.

$$E[U(D)]^* = \hat{\alpha}(T_f + T_{rc} + B) + \partial P_{late}^* + B \left\{ \eta \ln \left[ \frac{\partial + B(\eta + \gamma)}{B(\eta - \hat{\alpha}\Delta)} \right] - \frac{\partial(\eta - \hat{\alpha}\Delta)}{\partial + B(\eta + \gamma)} - \hat{\alpha}\Delta \right\} \quad (2.29)$$

$B$  – mean travel time due to non-recurrent congestion

$\Delta$  – the change in profile of recurrent congestion

$P_{late}^*$  the optimal probability of arriving late

Due to the stochastic nature of the above equation determining optimal departure time for passengers having a specified PAT, would require a trade-off between the  $E[SDE]$  and  $E[SDL]$  values. Some of the simplifying assumptions made in Noland and Small (1995) in order to arrive at optimal departure time is that the distribution of non-recurrent congestion ( $T_{nrc}$ ) is fixed with departure time and that the rate of change of the profile of recurrent delays is less than unity to ensure that the FIFO rule is observed.

A similar analysis involving the stochastic nature of travel times but including only the recurrent congestion within its formulation, Fosgerau and Karlström (2010) follow the standard Small (1982) approach to formulate the expected utility function assuming that the travel time associated with the journey and the preferred arrival time are both normalised to zero. The maximum expected utility is given as in eq 2.30

$$-E[U]^* = \hat{\alpha}\mu + (\eta + \gamma)\sigma \int_{\frac{\gamma}{\eta + \gamma}}^1 \Phi^{-1}(s) ds \quad (2.30)$$

The optimal departure time associated with the above utility function is derived as given in

$$D^* = -\mu - \sigma \Phi^{-1} \left( \frac{\gamma}{\eta + \gamma} \right) \quad (2.31)$$

Where  $\mu$  – mean travel time excluding non-recurrent congestion,  $D^*$  – is the optimal departure time,  $\Phi^{-1} \left( \frac{\gamma}{\eta + \gamma} \right)$  – inverse of the CDF which is the function of scheduled delay early and scheduled delay late parameters and  $\hat{\alpha}, \eta$  and  $\gamma$  are parameters.

#### 2.4.2 Mean – variance approach:

The alternative approach to the schedule based approach namely ‘mean-variance’ approach is based on the proposition that variability in travel time of and by itself results in disutility (Noland and Polak, 2002). The mean-variance approach ignores the effect of scheduling decisions, such as the selection of a

'safety margin' (Noland and Polak, 2002). The linear incorporation of variance in the utility function while using mean variance approach can be achieved by making simplifying assumption such as no lateness penalty and the change in recurrent congestion profile is zero to the scheduling approach given in eq 2.17 (Noland and Polak, 2002). Noland and Polak (2002) also indicate that though such simplifying assumptions may seem unrealistic; under certain trip conditions (where the transit services are not influenced by recurrent congestion,  $\Delta = 0$ ) or trips where arriving late doesnot result in a penalty ( $\partial = 0$ ) may justify a linear incorporation for variability leading to eq(2.32).

$$E[C]^* = \hat{\alpha}E[\hat{T}] + b^1\eta \ln\left(1 + \frac{\gamma}{\eta}\right) \quad (2.32)$$

Where  $\hat{T}$ - travel time without non-recurrent congestion;  $\hat{\alpha}, b^1, \eta, \gamma$  parameter values.

Empirical studies such as that by Jackson and Jucker (1982) reiterates the importance of inclusion of variability in cost function and specify a model where the passenger seeks to trade-off between travel time and travel time variance explicitly eq(2.33).

$$\text{minimize } E[T.T] + \beta_p \text{var}[T.T] \quad (2.33)$$

Where  $\beta_p$  is a non-negative parameter which represents the degree to which the variance of travel time is undesirable to traveller on path p.

The empirical study by Jackson and Jucker (1982) involved paired comparison wherein the participants are given the options between the expected travel time with no delays versus a shorter travel time with once a week delay ranging from 5 to 20 min.

The mean-variance model was furthered into the mean-lateness model to account for the penalty associated with arriving late. A LAPUE (Lateness Arrival Penalty User Equilibrium) model was proposed by Watling (1996) which followed the schedule delay approach proposed by Vickrey (1969) such that an individual considering to travel between an OD pair is associated with an acceptable total travel time beyond which he/she incur a penalty. The LAPUE model had a cost function as follows:

$$u_r = \theta_0 \text{voc}_r + \theta_1 t_r + \theta_2 \max(0, t_r - \mathcal{J}_r) \quad (2.34)$$

Where  $\mathcal{J}_r$  - is the acceptable total travel time for route r

$t_r$  - Total travel time experienced for route r

$\text{voc}_r$ - Composite of attributes independent of time (such as distance)

$\theta_0$  - Value of attributes

$\theta_1$  - Value of travel time

$\theta_2$  - Value of being one unit latter than acceptable.

Furth and Muller (2006) indirectly derive the importance of waiting time as a measure of reliability and as a measure of quality experienced by passengers and splits the waiting time of passengers into different components. They argue that the perceptions of passengers are based on the extreme values of waiting time and that variation in service reliability has a greater impact on the extreme values (90<sup>th</sup> or 95<sup>th</sup> percentiles) of waiting time than the expected value of waiting time. The 95<sup>th</sup> percentile is indicated as 'budgeted waiting time' wherein a passenger is aware that he/she shall have to face a maximum of 95<sup>th</sup> percentile waiting time and hence they often incorporate this waiting time by starting early from home and thereby reaching their destination earlier. This excess waiting time is called the 'potential waiting time'. The use of 'budgeted and potential' waiting time in cost formulations could also be tried in future analysis.

Integration of reliability studies in transit assignment has been achieved in recent years. Frequency based transit assignment studies have studied reliability in transit network through varying approaches (Yin et al., 2004; Yang and Lam, 2006; Szeto et al., 2013; Szeto et al., 2011; Zhang et al., 2009; Zhang et al., 2010) adopted to deal with congestion. Of particular distinction is the use of BPR styled congestion function in computing the cost of a route section by Szeto et al., 2011. Other studies (Yin et al., 2004; Yang and Lam, 2006; Szeto et al., 2013; Zhang et al., 2009; Zhang et al., 2010) utilise the overload delay to account for congestion.

## **2.5 Stochastic process models (SPMs):**

Stochastic process models excepting for the study by Teklu (2008a) has not been dealt with in transit assignment. However its application in traffic assignment studies has been immense. SPMs differ from the conventional equilibrium models (detailed in above sections) as it studies the evolution of the system on a sequence of time frame. The most common context under which SPMs are studied is day-to-day dynamics (Davis and Nihan, 1993; Cascetta, 1989; Cantarella and Cascetta, 1995). Some within-day stochastic process models have also been developed (Cascetta and Cantarella, 1991; Balijepalli et al., 2007). The advantage of using stochastic process models lies in its ability to be used to solve an asymmetric assignment problem for which solution uniqueness is not guaranteed. The study by Cascetta and Cantarella (1991) establish that unlike equilibrium models SPMs do not rely on the system

converging quickly to an equilibrium solution or the solution being unique and stable. SPMs also allows the model to capture the heterogeneity of traveller's behaviour in terms of their route choice, learning and perception differences along with the possibility of assessing the day-to-day and within-day variations in demand and supply. The use of Markov (memory less) property wherein the route choice at a given time period is based on the costs experienced in the previous time period has evolved since 1977 as quoted in Cascetta (1989). This memory less property is exploited in design of various learning process models which is based on the behavioural assumption that passengers tend to have a finite memory.

Cascetta (1989) showed that the mean route flows and link flows in case of SPMs are similar to the SUE flows for constant or separable linear link-cost function. Cascetta (1989) indicate that in systems where multiple equilibrium exists the SPMs and SUE values are significantly diverse. Watling (1996) emphasise that the use of SPMs for asymmetric problem requires that the system be tested for a range of initial conditions, random seed numbers and at least one extremely long simulation should be performed. The exhibition of markov property through the presence of stationary distribution and ergodicity of SPMs has been shown in Cascetta (1989). The conditions specified in Cascetta (1989) ensured that the SPM was m-dependent Markov chain with a time-homogeneous transition probability matrix. The necessary requirement for stationary and ergodic SPM is irreducibility and aperiodicity of the Markov chain. This is achieved when the route choice probabilities on each day are:

1. Time homogeneous i.e. the probabilities of transition from one state to another remained invariant given the set of previous states.
2. Positive on all the available routes
3. Depend on a finite memory length of the previous states.

Watling and Cantarella (2012) and Watling and Cantarella (2013) gives a detailed description of the various components involved in a SPM and their interactions through a series of illustrative examples. An implementation of their formulations for the current study is specified in Chapter 3.

An important feature of the SPMs is the ability of passengers to assimilate their experience and base their route choice on these experiences. The assimilation of experience is done over a fixed finite memory length. A learning process model is introduced to model the assimilation of experiences of the passengers. Different learning process models are available in literature and as quoted in Teklu (2008a) can be classified as

1. Weighted average approaches

2. Adaptive expectation approach
3. Bayesian approach

Weighted average approach used in traffic network assumes that the drivers tend to remember experiences over finite number of days and assign more weightage to their most recent experience. Thus at beginning of day  $\Omega$  the drivers update the route cost for each route based on a linear combination of the weighed experienced costs. The weights are determined by an appropriate weighing system. Since the assumption is that the drivers remember the recent experiences the most they are assigned a higher weightage thus the weighted average approach can be expressed as:

$$\check{C}^{\Omega} = \omega_1 G^{\Omega-1} + \omega_2 G^{\Omega-2} + \dots + \omega_{\Omega} G^{\Omega-\Omega} \quad (2.35)$$

Where  $C^{\Omega}$  is the mean perceived cost,  $G^{\Omega-1}$  is the cost experienced on day  $\Omega - 1$ ,  $\omega = \{\omega_r\}$  vector of weights such that  $\sum_{r=1}^{\Omega} \omega_r = 1$  and  $\Omega$  is the driver's memory length. This approach has been widely used (eg: Cascetta, 1989; Horowitz, 1984; Teklu, 2008a and 2008b).

Adaptive expectation approach was introduced due to wide criticism of weighted average approach for its inability to account for the 'regret' in the passengers' past decisions which could be an important parameter in the route choice decision (Iida et al., 1992). The adaptive expectation approach combines the perceived and actual costs from previous time period. The model in its simplest form is expressed as :

$$\check{C}^{\Omega} = \omega G^{\Omega-1} + (1 - \omega) \check{C}^{\Omega-1} \quad (2.36)$$

Where  $0 \leq \omega \leq 1$  and has been used in studies by Cascetta and Cantarella (1991), Iida et al. (1992).

The third approach namely the Bayesian approach was proposed by Jha et al. (1998) wherein the travellers cost perception were updated based on the previous experiences and information obtained from ATIS. The travellers chose the routes based on the probability distribution of the perceived travel time. This model assumes that the mean travel time doesn't change significantly during the simulation period and hence cannot model disruptions. Chen and Mahmassani (2004) apply a similar approach wherein the perception updating is done only when a certain amount of defined period is elapsed or after an experience very different from prior experiences has occurred.

In order to account for the varying perceptions of reliability amongst the travellers, stochastic cost/utility functions are introduced. A real life simulation was carried out on participants residing beside a four lane urban corridor for 24 days by Chang and Mahmassani (1988) in order to determine the day to day

departure time decisions of passengers. The results found were consistent with the fact that the most recent experience of passengers in terms of travel time and scheduled delay has a greater influence on the current perception of travel time than past experiences of these attributes which diminish over time due to factors such as memory loss and natural discounting. Iida et al. (1992); Mahmassani (1990) found that travellers with lesser information tend to choose suboptimal routes whereas travellers with more information (accumulation of information over all experiences) results in smaller variation.

## **2.6 Summary**

The chapter dealt with a brief description of various models existing in transit assignment and how these models were formulated to capture the real world transit network details. The literature review on reliability based models from passengers perspective is also presented along with description of benefits and uses of stochastic process models. The chapter was able to emphasise that a challenge still remains in dealing with reliability issues arising due to the failure to board conditions faced by transit service users in the congested environment. The chapter also emphasised that an equilibrium based approach in congested transit networks doesn't guarantee a unique solution. The following chapter aims to utilise the benefits of stochastic process models in assessing the route choice of passengers in a strict capacity constrained congested transit environment.



## **Chapter 3**

### **Disaggregate Stochastic Process Model formulation and implementation on risk neutral passengers**

The chapter shall introduce the concepts which shall form the background on which the reliability analysis will be carried out. The chapter begins with an introduction to the concepts involved in stochastic process modelling and shall further progress into the experimental setup for the current research. The chapter shall then test the experimental setup on an example network and the results obtained shall then be discussed.

#### **3.1 Introduction to stochastic process models**

A stochastic process model is adopted in networks which are under constant change over successive time periods due to several factors such as varying demand/supply, fluctuations in costs and of user's choices (Cascetta, 1989). Since transit network involves supply and demand aspects which are stochastic in nature not only within the day but on day to day basis; application of stochastic process model to simulate the same seems a natural way forward. A stochastic process in probability theory is defined as the evolution of system over time (considered discrete in the current model) wherein the system is a collection of random variables. Stochastic process models help define the high dimensional space from which the probability distributions which depict the simulated system can be sampled. Since transit assignment process involves interaction of many attributes (passenger arrivals and transit service arrivals) there is a high dimensional interaction of more than one probability distribution.

The Markov property is defined as the memory-less property which shows that the conditional probability distribution of the future states of the system depends on only the present state (or a finite number of previous states) (Watling and Cantarella, 2013).

In simpler words a markov based stochastic process model can be defined as a system wherein stochastic process models are used to model evolutionary interaction between several random variables; the results of which can be sampled to obtain a series of correlated random variables which obey the 'memory less' markovian property.

Frequency based transit assignment represent interaction of several stochastic variables wherein the transit headway and passenger arrival are assumed to have a predefined distribution and are often characterised by the mean value of the assumed distribution in transit assignment studies (chapter 2). These studies do not consider the evolution of the predefined distributions in day to day time frame and limit themselves to the within day interaction of transit supply and demand as highlighted in Chapter 2 excepting for the study done by Teklu (2008a and 2008b). The current study takes advantage of the inherent stochastic nature of transit network to model a day-to-day stochastic process framework which exhibits the memory-less markov property. The chapter shall describe the concept behind aggregate and disaggregate learning process for example networks followed by an implementation of the proposed disaggregate stochastic process assuming risk neutral passengers.

### **3.1.1 Overview of aggregate stochastic process model in uncongested network**

Watling and Cantarella (2013) and Watling and Cantarella (2012) define that the representation of time in a stochastic process can be discrete 'epochs' which could be individual days, weeks or years. These discrete time epochs are denoted by letter  $\Omega$  and the state vector describing the epoch  $\Omega$  is given as  $X^{(\Omega)}$ . In the current model the time epoch is individual day and henceforth shall be referred to as the same. The state vector  $X^{(\Omega)}$  describes the state of the system at day ( $\Omega$ ) and the state  $X^{(\Omega)}$  for a particular day  $\Omega$  is the resultant of combination of various attributes which define the system. The attributes which define the system in an aggregate model consists of; the parameters ( $\lambda$ ) assumed for the distribution of supply and demand and the logit dispersion parameter ( $\xi$ ) assumed.

Hence  $X^{(\Omega)}$  essentially consists of all information required for design for the markov process. In the aggregate stochastic process model  $X^{(\Omega)}$  is a vector which consists of the predicted total travel time over the specified memory period of  $\Delta$  days for each individual route/strategy. In event of risk averse passengers they consist of the predicted mean-variance or mean-lateness cost. The day to day evolution of the supply and demand probability densities results in a correlated joint probability distribution  $q^{\Omega}(X) : X \in \hat{Z}$  of the travel cost and flows for each route/strategy where  $\hat{Z}$  defines all possible combinations (states) that  $X$  can possibly take. The system evolves based on the notion that the passengers 'learn' from a sequence of past days wherein the sequence is over a finite history (denoted as memory length  $\Delta$ ) thereby satisfying the Markov property. This evolutionary rule wherein the

state of the system is defined by a finite history of states for a specific set of attributes is termed 'transition function' and is denoted as

$$\varnothing(X, Y; \lambda) : X, Y \in \hat{Z}$$

Where  $Y$  defines the state vector on  $\Omega - 1$  if  $\alpha = 1$  and  $\lambda$  defines the parameters.

The above highlighted concepts of markov property shall now be explained by means of a simple example. The example consists of a single OD pair having two transit lines and the choice of moving from one stop to the other is only using either one of the lines in an uncongested network. Fig 3.1 describes a simplistic network.

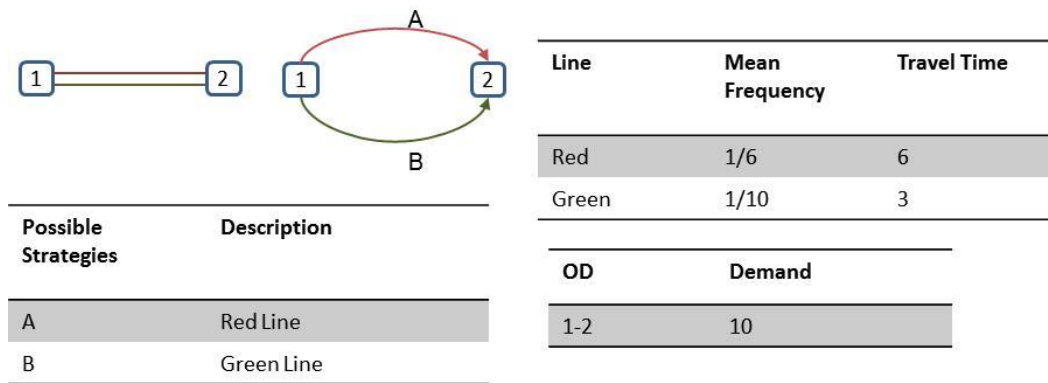


Fig 3.1 Simplistic Network

The average total travel time experienced along each of the route is the average of the sum of the waiting time as well as the in-vehicle travel time of passengers travelling along the routes. Assuming that the interarrival time (headway) of both the red and the green line is exponentially distributed the waiting time along each line for an uncongested network at stop 1 can be worked out as given in eq 3.1:

$$E[W_A] = \frac{1}{\varphi_A} = \frac{1}{6} = 6 \text{ min}$$

$$E[W_B] = \frac{1}{\varphi_B} = \frac{1}{10} = 10 \text{ min}$$

Wherein  $\varphi_A$  and  $\varphi_B$  are frequency of red and green line,  $E[W_A]$  and  $E[W_B]$  are the expected waiting times of strategy A and B .

As explained in chapter 2 the probability density function for the waiting time of randomly arriving passengers for exponential inter arriving transit services along the routes A and B is given as

$$f_A(w) = \varphi_A e^{-\varphi_A w} \tag{3.1}$$

$$f_B(w) = \varphi_B e^{-\varphi_B w} \quad (3.2)$$

The average total travel time /cost on routes A and B is given as

$$E[G_A] = 6 + 6 = 12$$

$$E[G_B] = 10 + 3 = 13$$

A logit choice formulation for the route selection on any particular day  $\Omega$  based on the above costs is given as below

$$P(A) = \frac{e^{-\xi G_A}}{e^{-\xi G_A} + e^{-\xi G_B}}$$

And

$$P(B) = \frac{e^{-\xi G_B}}{e^{-\xi G_A} + e^{-\xi G_B}}$$

For a  $\xi = 4$  the probability of choosing A would be

$$P(A) = 0.9820$$

Assuming *no* random draw for the current example, on day 1 all the passengers choose to travel along route A. Drawing the waiting time for the 10 passengers travelling from the probability density function of the waiting time given in eq 3.1 the average waiting time for day 1 along route A is computed<sup>2</sup>.

Table 3.1 Waiting time realisations on Day 1 for route A

S.No	Waiting time	Total Travel time/cost experienced
1	3.9	9.9
2	4	10
3	18.5	24.5
4	6.3	12.3
5	5.7	11.7
6	2.2	8.2
7	4.4	10.4
8	1.3	7.3
9	0.06	6.06
10	7.6	13.6
Average	<b>5.4</b>	<b>11.4</b>

<sup>2</sup> For explanation purposes the waiting time is drawn from the waiting time distribution given in eq 3.1. In disaggregate stochastic process model the waiting time is computed from interaction of transit supply and demand distribution.

Table 3.1 indicates that on day one the average experienced cost along route A is 11.4. Based on the cost experienced on day 1 the probability of passengers choosing route A between routes A and B is computed as

$$P(A) = 0.9983$$

Since all the passengers choose route A on day 2 the waiting time for each passenger is as shown in table 3.2.

Table 3.2 Waiting time realisations on Day 2 for route A

S.No	Waiting time	Total Travel time/cost experienced
1	12.7	18.7
2	23.4	29.4
3	5.8	11.8
4	1.3	7.3
5	1.7	7.7
6	7.4	13.4
7	7.6	13.6
8	0.53	6.53
9	0.53	6.53
10	23.2	29.2
Average	<b>8.4</b>	<b>14.4</b>

Similarly the choice probability of route A on day 3 is computed as

$$P(A) = 0.0037$$

Hence on day 3 none of the 10 passengers choose to travel along route A and all of the passengers travel on route B. The experienced waiting time along route B is as shown in Table 3.3.

Using the average cost experienced along route B the probability of choosing route A on day 4 is computed as

$$P(A) = 0.0000$$

Hence on Day 4 as well all the passengers travel on route B and the experienced waiting time is given as in Table 3.4.

Table 3.3 Waiting time realisations on Day 3 for route B

S.No	Waiting time	Total Travel time/cost experienced
1	0.59	3.6
2	13.2	16.2
3	2.0	5.0
4	0.4	3.4
5	16.3	19.3
6	4	7
7	13.9	16.9
8	3.8	6.8
9	6.5	9.5
10	2.3	5.3
Average	<b>6.3</b>	<b>9.3</b>

Table 3.4 Waiting time realisations on Day 4 on route B

S.No	Waiting time	Total Travel time/cost experienced
1	2.9	5.9
2	5.8	8.8
3	0.8	3.8
4	22.9	25.9
5	5	8
6	20.1	23.1
7	0.6	3.6
8	3.22	6.22
9	23.0	26.0
10	8.2	11.2
Average	<b>9.2</b>	<b>12.2</b>

The experienced cost at end of day 4 is 12.2; based on which the probability of choosing route A on day 5 is given as

$$P(A) = 0.690$$

Hence on day 5, 7 of the passengers shall choose route A and remaining shall choose route B.

From the above example it becomes clear that in an uncongested network the transition state of the flows on a route/strategy depends on

1. The waiting time experienced by the flows. This waiting time is in-turn dependent on the assumed headway distribution and the passenger arrival distribution.
2. The dispersion parameter ( $\xi$ ) value assumed for the choice of route which determines the level of 'belief' a passenger possess on the average travel costs computed. Hence a high  $\xi$  would imply a passenger's total belief in the average travel cost. This would result in almost all the passengers routing themselves in a similar manner. On the other hand a lower  $\xi$  value implies that the passengers perceive the average travel cost to be not very true to the actual value and hence route themselves more equally on the various available options.

Unlike the deterministic/stochastic equilibrium approach wherein a single unique solution defines equilibrium state; stochastic process model result is correlated joint probability distribution of the flows and costs on each route/strategy. A stochastic process model is said to be stationary if there is at least one stationary distribution. It is said to ergodic if the stationary distribution exhibited is the only one stationary distribution and it is regular if the stationary distribution converges to the same irrespective of its initial conditions.

### **3.1.2 Disaggregate stochastic process model**

The above formulation saw the evolution of system based on average costs experienced by all the passengers at end of each 'day'. However in disaggregate stochastic process model the evolution of the system is based on each individual passenger's travel experience.

Consider a network such that the OD demand is randomly varying from day-to-day, but that there is a fixed number of potential travellers  $d^z$  for each OD movement where  $Z = \{1,2,\dots,N\}$ ;  $N$  being the total number of ODs in the network. The simulation framework uses as input a rate for passenger arrivals from which the OD demand for each OD pair on any one day is generated. Let  $n^z$  be the number of routes within the OD pair  $Z$ .

Based on the specified passenger generation rates for each OD pair the number of passengers generated on each day varies. On each day, there are two important 'decision' elements for each passenger in an OD pair: whether they travel at all, and if they do travel which route they choose. Since the number of

passengers generated between each OD pair for travel on each day is random the difference between  $d^Z$  and the generated number of passengers are assumed to have not travelled.

The indicator variable  $\delta_{i^z}^Z$  takes the value 1 if individual  $i^z$  of OD pair Z travels on a given day, and takes the value 0 otherwise. For those that travel,  $f_{i^z}^Z$  denotes the route selected by individual  $i^z$  where  $f_{i^z}^Z \in \{1^Z, 2^Z, \dots, n^Z\}$ ,  $n^Z$  being the total number of routes between the OD pair denoted by Z. Collecting these two pieces of information together across all individuals between each OD pair Z, we have the pair of  $n^Z$ -vectors  $(\delta^Z, f^Z)$ . For the network with N OD pairs  $(\delta, f)$  shall be a large vector having a collection of vectors across all OD pairs within the network, that is  $(\delta, f) = (\delta^1, \delta^2, \dots, \delta^N, f^1, f^2, \dots, f^N)$ .

Once the distributions and parameters are specified for the supply and  $(\delta, f)$  of the demand model is drawn randomly for all N OD pairs, then an interaction with the 'supply model' results in the corresponding OD travel times that each individual will experience on their chosen alternative. In transit network since transit supply is characterised by capacity constraints and since passengers from different OD pairs may find the same route attractive, a non-separable problem arises. This results in passengers of an OD pair influencing the experienced travel times  $t_{i^z}(i^z)^z$  of another OD pair. Consider passenger  $i^1$  of OD pair 1 choosing a route  $f_{i^1}^1 \in \{1^1, 2^1, \dots, n^1\}$  such that  $f_{i^1}^1$  comprises of route section  $k$ . Let us assume that route section  $k$  forms route  $f_{i^2}^2$  in OD pair 2 chosen by passenger  $i^2$  of OD pair 2 to travel on the same day as passenger  $i^1$  of OD pair 1. Since both the OD pair passengers find route section  $k$  attractive they compete for the space within the transit services of route section  $k$  thereby influencing each other's experienced total travel time<sup>3</sup>. The step wise procedures involved in the current markovian framework for risk neutral passengers are specified in section 3.3.

The output of the supply and demand model interaction is a random variable of experienced travel time, and is used to determine the routes choice  $f_{i^z}^Z$  of each individual on subsequent day for each OD pair Z. The relationship between the output random variables (representing individuals' OD total travel times) and  $(\delta^Z, f^Z)$  is rather complex; hence the objective will be to make a Monte Carlo draw of the supply and demand distribution and allow for their interaction. It is

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<sup>3</sup> The experienced travel time  $t_{i^z}(i^z)^z$  should be written as a function of  $(\delta, f)$  however in the current formulation  $t_{i^z}(i^z)^z$  has not been mentioned as a function in order to avoid complex mathematical formulation which would make the current mathematical process difficult to understand.



more informative to understand the interaction by a ‘procedural’ definition (i.e. how to simulate from the supply, demand distribution) rather than through a definition in terms of compositions of probability distributions. Section 3.3 highlights the demand side distributions and parameters assumed in the current model and shows the process for determining  $(\delta^Z, f^Z)$  of the current model. Section 3.2 highlights various components involved in the current simulation process.

Since each individual  $i^Z$  has an experienced travel time  $t_{fz}(i^Z)^Z$  for a chosen route  $f_{i^Z}^Z$  and the uncongested travel time for non-chosen routes  $\bar{t}_{n^Z}(i^Z)^Z \forall n^Z \neq f_{i^Z}^Z$ , these together could be represented by a random variable  $T(i^Z)^Z = (t_{1^Z}(i^Z)^Z, \bar{t}_{2^Z}(i^Z)^Z, \dots, \bar{t}_{n^Z}(i^Z)^Z)$  where  $T(i^Z)^Z$  is a vector containing all the travel times associated with passenger  $i^Z$  on all routes between OD pair  $Z$  and  $T_{k^Z}(i^Z)^Z$  forms the updated travel time for passenger  $i^Z$  on route  $k^Z \in n^Z$  for the OD pair  $Z$ . A collection of these random variables is given together in a random vector  $T^Z = (T(1^Z)^Z, T(2^Z)^Z, \dots, T(d^Z)^Z)$ . This random vector is used to determine the average predicted total travel time for each route at the end of a day. The vector of average experienced travel times on all routes between the OD pair  $Z$  ( $t^Z$ ) will be continuous, and correlated. The joint pdf of  $t^Z$  depends on  $(\delta^Z, f^Z)$ , on the form of the distributions assumed for passenger/transit headways, and on the parameters assumed for these distributions. Suppose that the parameters are collected together in a vector  $\lambda^Z$ , then it is possible to reflect this dependence by saying that the joint pdf of  $t^Z$ , where  $t^Z$  denotes the vector consisting of average experienced total travel time, obtained by averaging the experienced total travel of all passengers on a route at end of each day, over all routes between the OD pair  $Z$ , is given by:

$$\psi(t^Z; \delta^Z, f^Z, \lambda^Z) \quad (t^Z \geq 0)$$

where  $\delta^Z$  and  $\lambda^Z$  are ‘parameters’ specific to OD pair  $Z$ .

An important aspect of the disaggregate model is that (in the absence of communicating with others or receiving information) when travellers learn, they learn only of the travel time for the route<sup>4</sup> they actually followed, whereas for the unchosen routes they assume an uncongested total travel time on the same. The assumption of uncongested total travel time for updating of non-

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<sup>4</sup> This assumption might be questionable as when a passenger has an experience of a route “choose first of line A or B” then they learn the waiting+in-vehicle time associated with such a route, but learn nothing about the waiting+in-vehicle time associated with the route “choose line A” or “choose line B”. But it is not easy to represent this kind of cross-route information transfer under no-information scenario, so on that basis it seems reasonable to assume that travellers only learn a route by actually following it themselves.

travelled route costs is based on the surmise that in the absence of external ‘information’ the passengers believe they will be able to board the first service of his/her choice set and hence update their experience matrix for that route based on the uncongested cost. Mathematically it also suits the weighted average formulation adopted for the study. If instead an assumption of the passengers updating their experience matrix with only their experience is adopted it would result in a breach of the implementation of the weighted average method adopted in the current model. The fact that individuals learn individually means that we must separately record/update the predicted and experienced travel time for each route, for each individual. The route choice of individual  $i^z$  is based on the cost predicted by individual  $i^z$  for various routes between the OD pair before the start of the trip/journey. It is to be noted that in mean-variance cost formulation the variance associated with only the experienced travel time is accounted for. Hence if a route is travelled only once within its memory length the variance associated with the route is assumed zero in spite of updating the experienced cost matrix with uncongested total travel time on the non-travelled days. In such situation the weighted average cost for the route will be computed based on the average of the uncongested travel cost and a single day’s experienced cost whereas the variance for the route as assumed by the passenger will be zero.

Suppose that the predicted OD travel times/costs of individual  $i^z$  for all routes in OD pair  $Z$ , prior to travelling on the given day, are contained in the vector  $\mathbf{g}(\mathbf{i}^z)^z = (g_{1^z}(\mathbf{i}^z)^z, g_{2^z}(\mathbf{i}^z)^z, \dots, g_{n^z}(\mathbf{i}^z)^z)$  for risk neutral or  $\hat{\mathbf{g}}(\mathbf{i}^z)^z = (\hat{g}_{1^z}(\mathbf{i}^z)^z, \hat{g}_{2^z}(\mathbf{i}^z)^z, \dots, \hat{g}_{n^z}(\mathbf{i}^z)^z)$  for risk averse wherein  $g_{n^z}(\mathbf{i}^z)^z$  is the predicted travel time for route  $n^z$  by risk neutral passenger  $i^z$  travelling between OD pair  $Z$  for the day and  $\hat{g}_{n^z}(\mathbf{i}^z)^z$  is the predicted travel cost for route  $n^z$  by risk averse passenger  $i^z$  travelling between OD pair  $Z$  for the day. Based on the notation already given,  $\delta_{i^z}^z$  denotes whether individual  $i^z$  travels between the OD pair  $Z$  and  $f_{i^z}^z$  the route chosen between OD pair  $Z$  if travelling. We denote by  $t_{f^z}(\mathbf{i}^z)^z$  the experienced travel time on route  $f_{i^z}^z$  and the updated OD travel time for individual  $i^z$  for that day is given as  $T(\mathbf{i}^z)^z = (t_{1^z}(\mathbf{i}^z)^z, \bar{t}_{2^z}(\mathbf{i}^z)^z, \dots, \bar{t}_{n^z}(\mathbf{i}^z)^z)$ . The memory length over which the individual  $i^z$  bases the predicted cost of a route is given by  $\Omega$  and is assumed the same between for all  $N$  OD pairs in the network. The learning process model then weighs the experienced and updated random travel times by a weighed averaging process for various assumed behaviour of passengers as follows:

$$g_{n^z}(\mathbf{i}^z)^z = \sum_{j=1}^{\Omega} \omega_j T_{n^z}(\mathbf{i}^z)^{z^{\Omega-j}} \quad \forall n^z \quad \text{risk neutral} \quad (3.3)$$

It is to be noted that when a passenger  $i$  travels more than once along the route  $k^z$  within his/her memory length  $\beth$  then equation 3.4a is applicable. In case the passenger travels the route only once or never within his/her memory length  $\beth$  then equation 3.4b is applicable.

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z + \beta \text{var} \begin{pmatrix} t_{k^z}(i^z)^{z\Omega-1} \\ \vdots \\ t_{k^z}(i^z)^{z\Omega-\beth} \end{pmatrix} \forall k^z = f_{i^z}^z, k^z \in n^z \text{ (mean - variance)} \quad (3.4a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z \forall k^z \neq f_{i^z}^z, k^z \in n^z \text{ (mean - variance)} \quad (3.4b)$$

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z + \theta_2 c_{k^z}^{i^z z} \forall k^z = f_{i^z}^z, k^z \in n^z \text{ (mean - lateness)} \quad (3.5a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z \forall k^z \neq f_{i^z}^z, k^z \in n^z \text{ (mean - lateness)} \quad (3.5b)$$

Where the weights can take a geometric progression of the form:

$$\omega_j = \frac{\rho^{j-1}}{\sum_{b=1}^{\beth} \rho^{b-1}} \quad \forall j \quad (3.6)$$

$$c_{k^z}^{i^z z} = \sum_{j=1}^{\beth} \omega_j \max(0, t_{k^z}(i^z)^z - \mathcal{T}_{k^z}(z))^{\Omega-j} \forall k^z = f_{i^z}^z, k^z \in n^z \quad (3.7)$$

$\Omega$ - denotes the current simulation day

$c_{k^z}^{i^z z}$  - weighed average lateness penalty associated with each individual  $i^z$  along route  $k^z$  between OD pair Z.

$\mathcal{T}_{k^z}(z)$  - 'Acceptable total travel time' for route  $k^z$  between OD pair Z.

$\beta$ - non-negative parameter which represents the degree to which the variance is undesirable to passengers (Jackson and Jucker, 1982) (kept constant for all N OD pairs in the network).

And  $\theta_1$  indicates the value of total travel time and  $\theta_2$  reflects the value of being one time unit later than expected (Watling, 2006). In the current study the value of  $\theta_2/\theta_1$  is assumed to be 5.

The learning model takes  $g_{n^z}(i^z)^z, \delta_{i^z}^z, f_{i^z}^z, \beth$  and  $t_{f^z}(i^z)^z$  and produces an updated vector of OD predicted travel times/costs for the  $n^z$  routes between OD pair Z for each individual  $i^z$ , for  $k^z = 1^z, 2^z, \dots, n^z$ ;  $\hat{g}_{k^z}(\delta_{i^z}^z, f_{i^z}^z, t_{f^z}(i^z)^z)^z = \hat{g}_{k^z}(i^z)^z$  predicted travel cost of individual  $i^z$  for route  $k^z$  between OD pair Z for

the day and  $\hat{\mathbf{g}}(\delta_{i^z}^z, f_{i^z}^z, t_{f^z}(i^z)^z)^z = \hat{\mathbf{g}}(i^z)^z$  is the vector of these updated predicted travel times across all routes for individual  $i^z$ .

Based on these predicted costs, each passenger independently and probabilistically chooses a route between each OD pair based on a random utility model.

The probabilities  $p(\hat{\mathbf{g}}(i^z)^z) = (p_{1^z}(\hat{\mathbf{g}}_{1^z}(i^z)^z), p_{2^z}(\hat{\mathbf{g}}_{2^z}(i^z)^z), \dots, p_{n^z}(\hat{\mathbf{g}}_{n^z}(i^z)^z))$  for the  $n^z$  routes between OD pair  $Z$ , are thus a function of the predicted OD travel times  $\hat{\mathbf{g}}(i^z)^z$  of individual  $i^z$ . A multinomial logit model, for example, would be a particular choice for these probability relationships, with:

$$p_{k^z}(\hat{\mathbf{g}}_{k^z}(i^z)^z) = \frac{\exp^{-\xi \hat{\mathbf{g}}_{k^z}(i^z)^z}}{\sum_{j=1}^{n^z} \exp^{-\xi \hat{\mathbf{g}}_j(i^z)^z}} \quad (\forall k^z = 1^z, 2^z, \dots, n^z) \quad (3.8)$$

where  $\xi > 0$  is a parameter of dispersion.

Assuming that the total number of OD pairs in the network is 1 i.e  $N = 1$ , then  $Z=1$  and passengers  $i$  of OD pair 1 is  $i^1$ . The above described modelling framework implies that in order to represent the dynamics of this model for a single OD pair network we need a state variable  $\mathbf{x} \in \mathcal{S}$  where:

$$\mathcal{S} = (\{0,1\} \times \{1^1, 2^1, \dots, n^1\} \times \mathbb{R}^2)^{d^1}$$

and where  $\mathbf{x}$  is stacked by individual (with the 'yesterday' state denoted by a tilde):

$$\mathbf{x} = \begin{pmatrix} \delta_{1^1}^1 \\ f_{1^1}^1 \\ t_{f^1}(1^1)^1 \\ \hat{\mathbf{g}}(1^1)^1 \\ \vdots \\ \delta_{d^1}^1 \\ f_{d^1}^1 \\ t_{f^1}(d^1)^1 \\ \hat{\mathbf{g}}(d^1)^1 \end{pmatrix} \quad \tilde{\mathbf{x}} = \begin{pmatrix} \tilde{\delta}_{1^1}^1 \\ \tilde{f}_{1^1}^1 \\ \tilde{t}_{f^1}(1^1)^1 \\ \tilde{\hat{\mathbf{g}}}(1^1)^1 \\ \vdots \\ \tilde{\delta}_{d^1}^1 \\ \tilde{f}_{d^1}^1 \\ \tilde{t}_{f^1}(d^1)^1 \\ \tilde{\hat{\mathbf{g}}}(d^1)^1 \end{pmatrix}$$

If there are two routes between an OD pair ( $n^z = 2$ ) and the memory length is assumed to be  $\beth=1$  then based on the above stated assumptions, the transition function follows the joint probability/probability-density function given by:

$$\begin{aligned}
 \phi(x, \tilde{x}) &= \phi \left( \begin{pmatrix} \delta_{1^1} \\ f_{1^1} \\ t_{f^1}(1^1)^1 \\ \hat{\mathbf{g}}(1^1)^1 \\ \vdots \\ \delta_{d^1} \\ f_{d^1} \\ t_{f^1}(d^1)^1 \\ \hat{\mathbf{g}}(d^1)^1 \end{pmatrix}, \begin{pmatrix} \tilde{\delta}_{1^1} \\ \tilde{f}_{1^1} \\ \tilde{t}_{f^1}(1^1)^1 \\ \tilde{\hat{\mathbf{g}}}(1^1)^1 \\ \vdots \\ \tilde{\delta}_{d^1} \\ \tilde{f}_{d^1} \\ \tilde{t}_{f^1}(d^1)^1 \\ \tilde{\hat{\mathbf{g}}}(d^1)^1 \end{pmatrix} \right) \\
 &= \prod_{i^1 \in \{1, 2, \dots, d^1\}} \left( \prod_{k^1=1}^2 p_{k^1}(\hat{\mathbf{g}}_{k^1}(i^1)^1)^{\Delta(k^1, f_{i^1}^1)} (1 - p_{k^1}(\hat{\mathbf{g}}_{k^1}(i^1)^1))^{1-\Delta(k^1, f_{i^1}^1)} \right) \\
 &\times \prod_{i^1 \in \{1, 2, \dots, d^1\}} \prod_{k^1=1}^2 \left( \hat{\mathbf{g}}_{k^1}(\tilde{\delta}_{i^1}^1, \tilde{f}_{i^1}^1, \tilde{t}_{k^1}(i^1)^1)^1 \right) \times \psi(t^1; \delta^1, f^1, \lambda^1) \tag{3.9}
 \end{aligned}$$

where the function  $\Delta(a, b) = 0$  unless  $a = b$ .

$\prod_{k^1=1}^2 (p_{k^1}(\hat{\mathbf{g}}_{k^1}(i^1)^1))^{\Delta(k^1, f_{i^1}^1)} (1 - p_{k^1}(\hat{\mathbf{g}}_{k^1}(i^1)^1))^{1-\Delta(k^1, f_{i^1}^1)}$  describes the probability of choosing route 1 or 2 (assuming only two routes are available between the OD pair Z which is equal to 1).

$\prod_{i^1 \in \{1, 2, \dots, d^1\}} \prod_{k^1=1}^2 \left( \hat{\mathbf{g}}_{k^1}(\tilde{\delta}_{i^1}^1, \tilde{f}_{i^1}^1, \tilde{t}_{k^1}(i^1)^1)^1 \right)$  gives the conditional probability density function of the predicted total travel costs on each route based on the learning process adopted for the experienced travel time on the two routes between the single OD pair,  $Z=1$ .

$\psi(t^1; \delta^1, f^1, \lambda^1)$  gives the probability density function of experienced travel times based on the individual specific systematic component for the single OD pair network where  $Z=1$ .

### 3.2 Model description

As per the mathematical model discussed in section 3.1 the current section shall detail the distributions and parameters assumed in the current study. In order to fulfil the set of objectives the model uses Monte carlo simulation for generation of distributions and the interaction between the distributions is set such that the strict capacity constraint is respected. The stochastic nature of demand is captured by varying rate of passenger arrivals and similarly the stochastic nature of supply is captured by line headway variation in the day-to-day micro simulation model. The 'service reliability' of a transit line shall be achieved by controlling the amount of variance in the interarrival time of the

services reaching the transit stop. It shall be denoted by the shape factor  $m$ . The detailed description of the various input components of the base model are described in the following section. All sections of the model are coded in Matlab.

### **3.2.1 Model Inputs**

This subsection describes the demand and supply specific inputs. The proposed model - Reliability based disaggregate stochastic process model- referred to from hereon as R-DSPM with strict capacity constraint is discussed in the following sections.

#### **3.2.1.1 Network Supply**

The network supply (i.e transit services, lines) are considered stochastic in nature as per the assumptions of frequency based assignment. The structure of the network consists of transit stops, and the arcs represent the line sections between the stops. A De Cea and Fernández (1993) based route section approach is adopted to model the enumeration of routes. In-vehicle travel times between transfer stops are assumed to be given. The in-vehicle travel time given for each line section is kept constant in the current study.

#### **3.2.1.2 Transit Services**

A transit service is characterised by the fixed subset of stop that the passenger encounters in his/ her trip and for every transit line the alighting stop is predefined. Each vehicle is characterised by the vehicle capacity and passenger volume dependent dwell time. The capacity of each transit vehicle is strict capacity beyond which the passengers are not allowed to board. As frequency based assignment approach is used each line is defined on line headways. As an input to the model average arrival rate of transit vehicles along with the shape factor associated with erlang distribution are given.

Each passenger in the network is characterised by the set of attractive lines which defines his/her transfer stop based on his/her chosen route. In case of risk averse passengers it is assumed that all the passengers in a simulation run are homogeneous in their choice of  $\beta, \mathcal{T}(Z)$  values and hence have the same degree of aversion associated with the variance of total travel time for all OD pairs and the acceptable total travel time for each OD pair. In the micro simulation model, for the specified headway rates the transit vehicles are generated. The arrival of the transit vehicles at the subsequent stops is derived by adding the average in-vehicle travel time to the departure time of the transit service. The departure times at subsequent stops are formulated by adding the dwell time to the in-vehicle travel time. The importance of dwell time at system,

route and point level has been emphasized indicating that excess dwell time forms one of the main reasons for non-adherence of schedule in transit services (Bertini and El-Geneidy, 2003).

The dwell time is taken as a function of number of passengers boarding and alighting at a transit stop and varies between different types of vehicle operated in each line. The dwell time function for the current model is given as follows:

$$DW(x_{\text{alighting}}, x_{\text{boarding}}) = B + \Delta_{\text{boarding}} \text{Boarding} + \Delta_{\text{alighting}} \text{Alighting} \quad (3.10)$$

Where,  $B=7$ ;  $\Delta_{\text{boarding}}= 5 \text{ sec}$  and  $\Delta_{\text{alighting}}= 3 \text{ sec}$ , Boarding - number of passengers boarding ; Alighting - number of passenger alighting from the bus (Ceder, 2007).

The variance associated with the interarrival times of each line can be modelled using the shape factor  $m$  of Erlang distribution which tends to reflect highly unreliable service arrivals (exponential) with a value of 1 and highly reliable service arrivals (deterministic) with a shape factor tending to infinity. The pseudo code utilized for generation of transit arrivals is as follows:

Step 1: Generate  $U_1, U_2, \dots, U_m$  as IID  $U(0,1)$

Step 2: Return  $Y = \frac{-h}{m} \ln(\prod_{i=1}^m U_i)$  Law and Kelton (1991)

Wherein  $h$  denotes the headway assumed for the transit service. The drawback of the above algorithm is pointed out that at large shape factors ( $m$ ) the value of  $(\prod_{i=1}^m U_i)$  tends to zero which makes computation of logarithm difficult.

The difference in reliability associated with arrival times can be seen from fig 3.2 which shows the inter arrival distribution of transit services at a transit stop for a mean frequency of 10 services/hr. A line having a specified average headway of 6 minutes, when modelled with a line shape factor of  $m = 1$  results in higher variability of interarrival times (Fig 3.2) and shall be classified as 'less reliable' than the same line when modelled with a shape factor of  $m = 300$  wherein the variability between the inter arrivals is reduced. In order to assess the varying behaviour of route choice between passengers who are highly risk averse to those who are risk neutral the model shall be run with passengers having varying  $\beta, \mathcal{T}(Z)$  values.

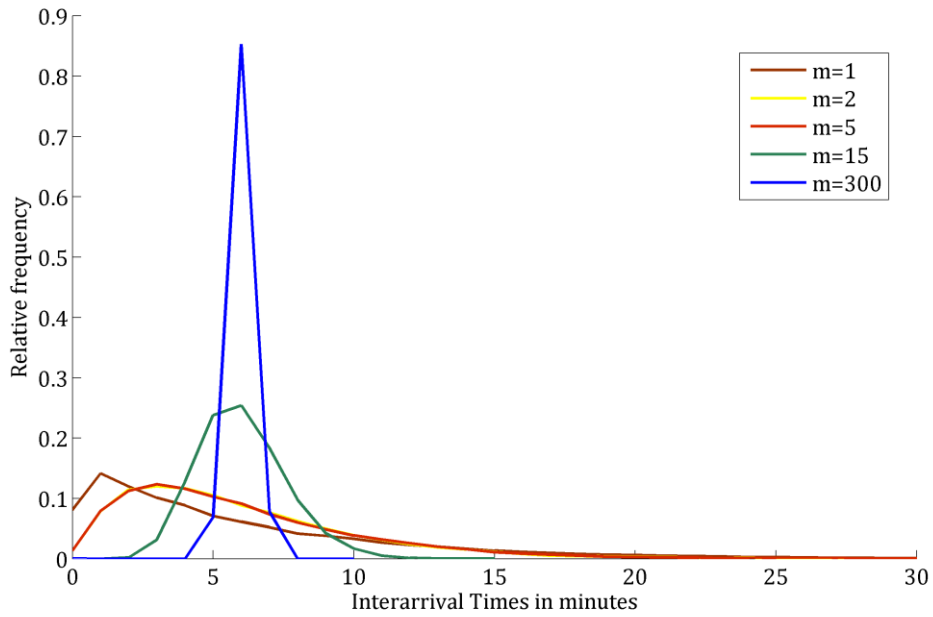


Fig 3.2 The probability distribution of erlang headway of 10 services per hour for varying shape factors

### 3.2.1.3 Passenger demand

Passenger demands across all OD pairs are simulated in the model through specified arrival rates. As described in section 3.1.2 each OD pair  $Z$  is assumed to have a population size of passengers  $d^Z$ . In order to simulate passenger arrivals; a fixed rate of passengers is fed as input for each OD pair. The number of passengers generated using the assumed rate of passenger arrivals for each OD pair  $Z$  are considered to be the passengers travelling for the day between the OD pair  $Z$ . The difference between the number of generated passengers and the population size  $d^Z$  are assumed to be not travelling for the day. The day to day passenger demand between OD pairs is assumed to be varying and there exists an indicator variable  $\delta_{i^Z}$  which takes the value 1 if individual  $i^Z$  travels on a given day, and takes the value 0 otherwise .

The portion of travel time which involves the walk from origin to the transit stop and the walk from the transit stop to destination is excluded. Passengers arrival is modelled as poisson arrivals with exponential inter arrival times. Passenger behaviour is constrained to changing of lines only at the predefined alighting stops decided upon by the passenger before boarding a line.

### 3.2.1.4 Learning process Model:

The route choice of the passengers is based on the costs experienced by the passengers over the memory length fed as input to the model. As indicated in Horowitz (1984) the weighted average learning process model is used. The



disaggregate modelling of route choice using weighed average learning process assumes that a passenger assigns weight to his/her experience over the memory length to predict the travel time. Similar to model 3 of Horowitz (1984) the disaggregate learning process model is formulated as shown below:

$$g_{n^z}(i^z)^z = \sum_{j=1}^{\Omega} \omega_j T_{n^z}(i^z)^{z\Omega-j} \quad \forall n^z \quad \text{risk neutral} \quad (3.11)$$

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z + \beta \text{var} \begin{pmatrix} t_{k^z}(i^z)^{z\Omega-1} \\ \vdots \\ t_{k^z}(i^z)^{z\Omega-2} \end{pmatrix} \quad \forall k^z = f_{i^z}^z, k^z \in n^z \text{ (mean - variance)} \quad (3.12a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z \quad \forall k^z \neq f_{i^z}^z, k^z \in n^z \text{ (mean - variance)} \quad (3.12b)$$

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z + \theta_2 c_{k^z}^{i^z z} \quad \forall k^z = f_{i^z}^z, k^z \in n^z \quad \text{(mean - lateness)} \quad (3.13a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z \quad \forall k^z \neq f_{i^z}^z, k^z \in n^z \text{ (mean - lateness)} \quad (3.13b)$$

Where the weights can take a geometric progression of the form:

$$\omega_j = \frac{\rho^{j-1}}{\sum_{b=1}^{\Omega} \rho^{b-1}} \quad \forall j \quad (3.14)$$

$$c_{k^z}^{i^z z} = \sum_{j=1}^{\Omega} \omega_j \max(0, t_{k^z}(i^z)^z - \mathcal{T}_{k^z}(z))^{\Omega-j} \quad \forall k^z = f_{i^z}^z, k^z \in n^z \quad (3.15)$$

$\Omega$ - denotes the current simulation day

$c_{k^z}^{i^z z}$  - weighed average lateness penalty associated with each individual  $i^z$  along route  $k^z$  between OD pair  $Z$

$\mathcal{T}_{k^z}(z)$  - 'Acceptable total travel time' for route  $k^z$  between OD pair  $Z$

$\beta$ - non-negative parameter which represents the degree to which the variance is undesirable to passengers (Jackson and Jucker, 1982).

And  $\theta_1$  indicates the value of total travel time and  $\theta_2$  reflects the value of being one time unit later than expected (Watling, 2006). In the current study the value of  $\theta_2/\theta_1$  is assumed to be 5.

In aggregate model (eg: Teklu, 2008b) transit assignment model it is assumed that the cost experienced by passengers travelling along various routes is

available to all the other passengers embarking on the journey the next day. It is assumed that passengers in spite of having travelled along different routes are fully aware (informed) of the costs associated with all the other routes enumerated for the OD pair. In disaggregate model this assumption is not applicable as the passengers route choice is solely based on his/her experienced costs on the routes in the network. The awareness of other passengers experience results in 'information' sharing model which is not dealt with in the current chapter.

### 3.2.1.5 Route Choice formulation:

The route choice is formulated as multinomial logit. The choice between the various routes is associated with the probability of a route having the total travel time/ cost lesser than the total travel time/ costs of other routes available for commuting between an OD pair. The logit choice model between routes is specified as shown below:

$$p_{k^z}(\hat{g}_{k^z}(i^z)^z) = \frac{\exp^{-\xi \hat{g}_{k^z}(i^z)^z}}{\sum_{j=1}^{n^z} \exp^{-\xi \hat{g}_j(i^z)^z}} \quad (k^z = 1^z, 2^z, \dots, n^z) \quad (3.16)$$

Where  $\xi$  – is the dispersion parameter ;  $\hat{g}_{k^z}(i^z)^z$  - is the cost of route  $k^z$  between OD pair Z obtained from the learning process for individual  $i^z$  ;  $n^z$ - is the total number of routes between the OD pair Z.

### 3.3. Methodological framework:

The combination of the above mentioned model inputs results in the framework for R-DSPM. The interaction of various highlighted components within the R-DSPM framework is shown in Fig 3.3 (a) and 3.3 (b).

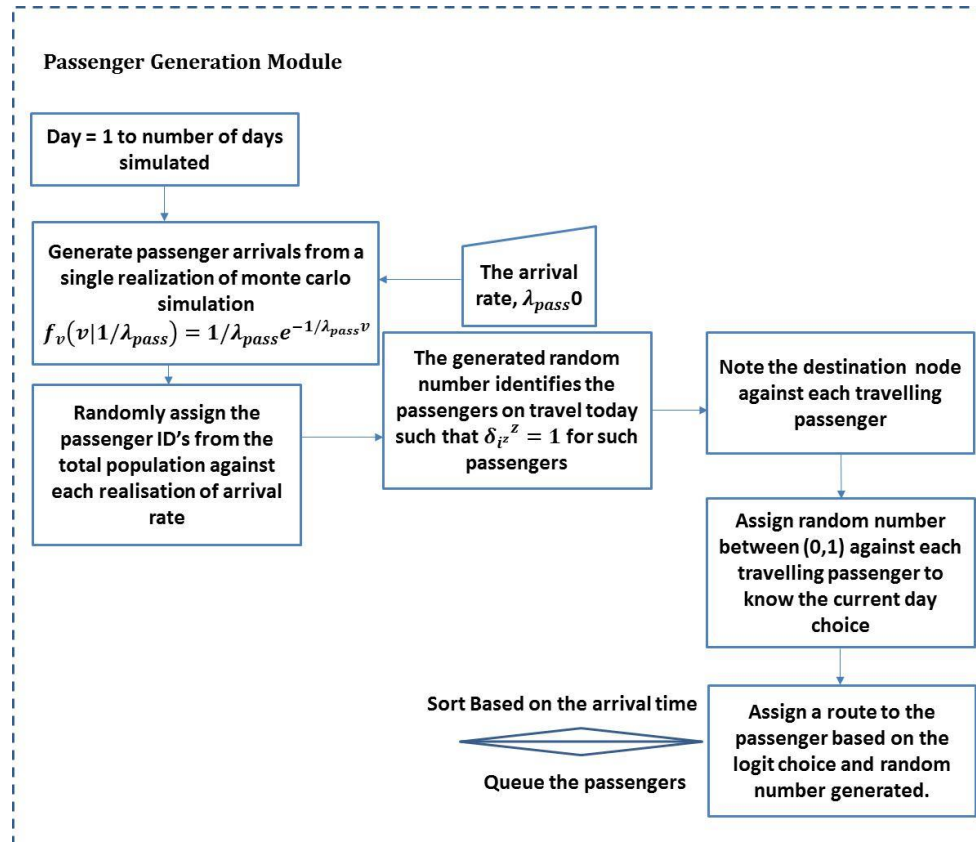


Fig 3.3 (a) : R-DSPM:-passenger generation module

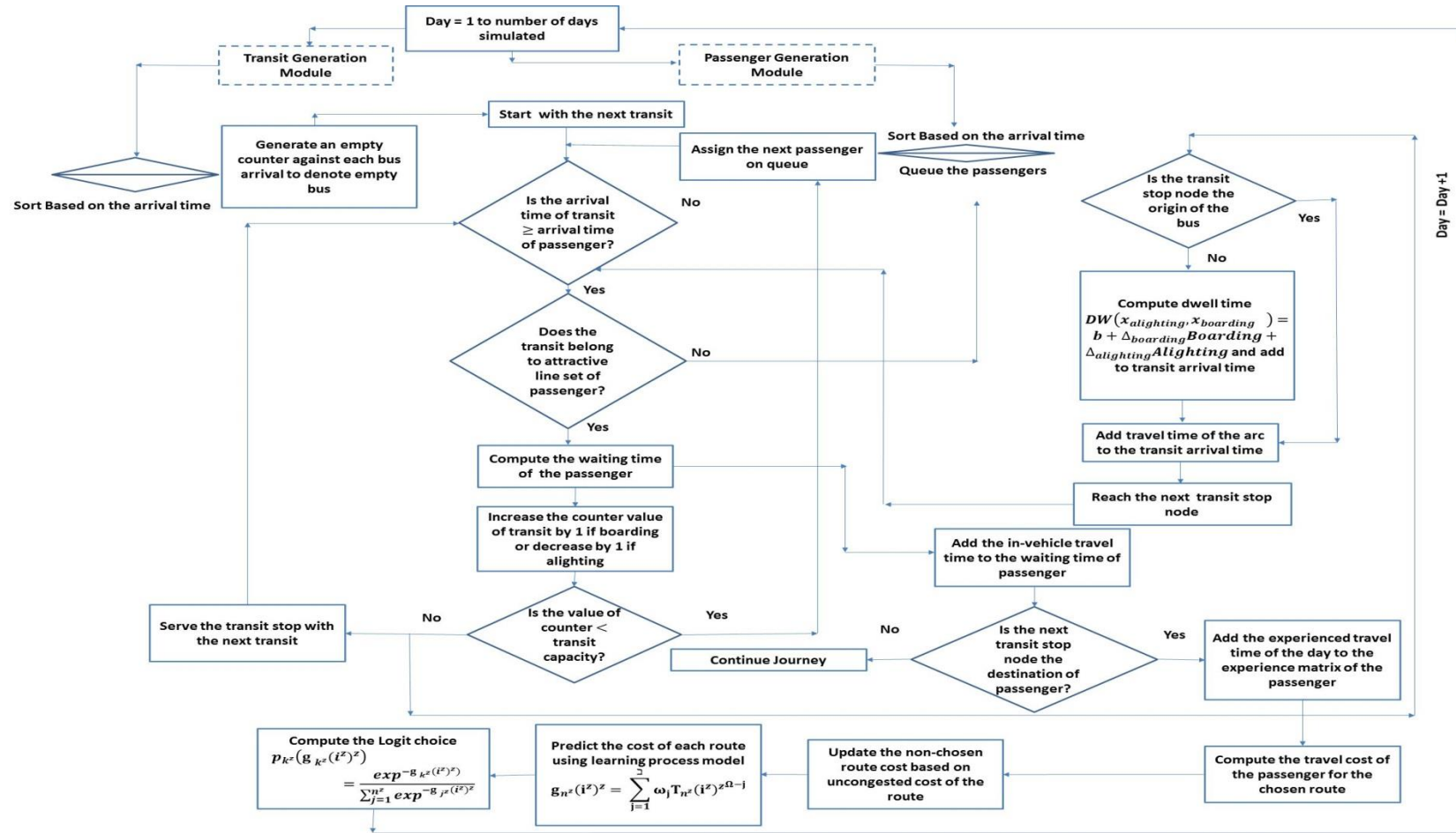
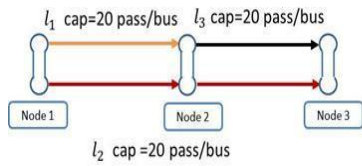


Fig 3.3 (b) : R-DSPM for risk neutral passengers

### 3.4 Example Network 1

Consider the network shown in fig 3.4.



Line Number	Line	In - Vehicle Travel Time in min	From Stop – To Stop	Services per hour
$l_1$	Orange	6	1-2	9
$l_2$	Red	9	1-2	10
$l_2$	Red	9	2-3	10
$l_3$	Black	10	2-3	10

	Node 2				Node 3			
	Uncongested		Congested		Uncongested		Congested	
	Population Size for 4 hours	Poisson rate of passenger arrivals/hr	Population Size for 4 hours	Poisson rate of passenger arrivals/hr	Population Size for 4 hours	Poisson rate of passenger arrivals/hr	Population Size for 4 hours	Poisson rate of passenger arrivals/hr
Node 1	59	10	846	190	67	11	1304	300
Node 2	-	-	-	-	110	20	1086	250

Fig 3.4: Example Network 1

Following De Cea and Fernández (1993) there are 12 different routes possible for travelling within the network. The routes for the network shown in the fig 3.5 are enumerated in table 3.5.

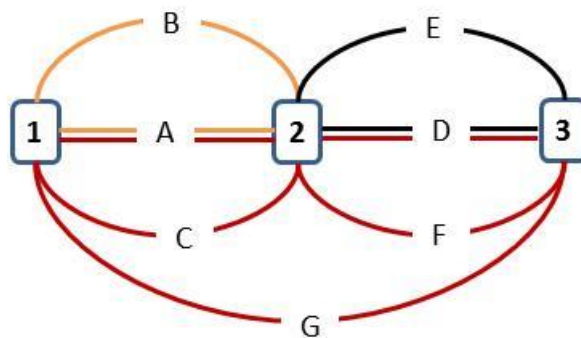


Fig 3.5 : Possible route sections in the network

Table 3.5: The enumerated ‘sensible’ routes<sup>5</sup>

OD Pair	Route Number	Route Sections
Node 1 – Node 2	1	B
	2	C
	3	A
Node 2 - Node 3	10	E
	11	F
	12	D
Node 1 to Node 3	4	G
	5	B+E
	6	B+F
	7	B+D
	8	A+E
	9	C+E

### 3.4.1 Uncongested network:

The test network was simulated using the framework given in fig 3.3(b). Having enumerated the routes the headway distribution is assumed to be exponential  $m = 1$ , the  $\alpha$  was kept as 5 days,  $\xi = 0.05$  unless otherwise specified and the simulation was run for a period of 4 hours over 700 days, without dwell time. The population size (population sampled from) as mentioned in fig 3.4 is for 4 hours between each OD pair. The demand (population size) in Fig 3.4 is such that for OD 1 it is taken as 59 passengers for 4 hours with an arrival rate of 10 passengers per hour, OD 2 is 67 passengers for 4 hours with an arrival rate of 11 passengers per hour and OD 3 is 110 passengers for 4 hours with an arrival rate of 20 passengers per hour. It is to be noted that the terms ‘population size’ and ‘constant demand/demand’ shall be used interchangeably throughout the thesis and shall denote the population size the rate of passenger arrivals is being sampled from. These assumptions

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<sup>5</sup> Route C+D, A+F and A+D has not been enumerated as a possible route because it involves getting down from red line and boarding the same red line. In case of uncongested network this would not be sensible. With congested network if there are passengers queued for the transit service before the arrival of the same at stop 2 a FIFO rule would ensure that these passengers are boarded first on to the transit service. Those who got down from the transit service would be allowed to board only if there is any capacity left within the service. In a realistic network such a possibility of alighting and boarding the same transit service is rare and has not been considered in example network 1. Example network 2 however looks at a scenario wherein routes consisting of route sections having same lines make alighting and boarding the same service possible.

have been used to run the uncongested R-DSPM. However it is to be noted here that on days that the passenger do not travel they update their costs based on the uncongested costs. The uncongested costs for the days not travelled is derived from De Cea and Fernandez L (1989) formulation. Table 3.6 shows the results obtained using the R-DSPM in uncongested condition and Table 3.7 shows the results obtained using De Cea and Fernandez L (1989).

Table 3.6: Total Travel time obtained using a single realisation of micro simulation model for various routes over the simulation period of 700 days with  $m = 1$ ,  $\alpha = 5$  and  $\xi = 0.05$

OD	Node 1 - Node 2 (Z=1)			Node 2 - Node 3 (Z=3)			Node 1 - Node 3 (Z=2)					
Route	1	2	3	10	11	12	4	5	6	7	8	9
$E(\mathbf{g}_k)$	12.58	15.1	10.7	16.1	15.1	12.6	24	29.1	27.5	25.1	26.7	31.2
Std <sup>6</sup>	1.25	0.9	0.5	1.4	1.1	0.47	1.3	3	2.6	2.0	2.1	2.7

Table 3.7: Total Travel time obtained using De Cea and Fernandez L (1989)

OD	Node 1 - Node 2 (Z=1)			Node 2 - Node 3(Z=3)			Node 1 - Node 3 (Z=2)					
Route	1	2	3	10	11	12	4	5	6	7	8	9
$E(\mathbf{g}_k)$	12.7	15	10.7	16	15	12.5	24	28.7	27.7	25.2	26.7	31

Tables 3.6 indicate that R-DSPM provides total travel time similar to that computed using De Cea and Fernandez L (1989) differing only by the fact that the current model uses a dynamic framework whereas De Cea and Fernandez L (1989) utilises static framework.

### 3.4.2 Congested Network:

The congested scenario of the test network is very similar in its input data and formulation to the uncongested scenario explained in the section 3.3.1., with almost the same input parameters excepting for the arrival rate of passengers and the population size between various OD pairs which was modified as given in fig 3.4. The modified population size between each OD pair is given for 4 hours such that the demand for Z=1 is 846 passengers for 4 hours with an arrival rate of 190 passengers per hour; Z=2 is 1304 passengers for 4 hours

<sup>6</sup> The standard deviation is between the expectation of predicted costs obtained as a result of weighted average learning process

with an arrival rate of 300 passengers per hour and  $Z=3$  is 1086 passengers for 4 hours with an arrival rate of 250 passengers per hour. Differing from the uncongested model is the introduction of dwell time function as given in section 3.2.1.2. and the introduction of strict capacity constraint in each transit service (assuming a capacity of 20 passengers/transit service). The strict capacity constraint ensures that each transit service at disaggregate level takes in a maximum of 20 passengers beyond which the passenger experiences failure to board. The memory length,  $\alpha$ , was kept as 5 days,  $\xi = 0.05$  unless otherwise specified and the simulation was run for a period of 4 hours over 700 days out of which the first 200 days were discarded as burn in period. It is noted that in a congested network passengers generated within the simulation duration are unable to reach their destination within the specified simulation period. Hence a buffer time is given at the end of the simulation period wherein the transit services are generated to enable the passengers queued up at the transit stops to reach their destination. It is brought to the attention of the readers that though transit services are generated during the buffer time passengers are not generated. It is also mentioned that the buffer time is kept for as long as the last passenger generated within the simulation period reaches their destination.

As mentioned in section 1.6 the distinction between the aggregate model in Teklu (2008b) and the current R-DSPM lies in the prediction of costs for propagation of flows in absence of non-selection of a route and the process involved in the prediction of the cost itself. In aggregate models the learning process of passengers is kept continuous during the non-selection of routes by assuming that the cost of the non-selected route (combination of route sections) is equal to cost experienced along the component route-section which now forms the part of the an used route. For eg if on a particular  $\Omega^{\text{th}}$  day route 5 doesn't get chosen then the waiting time associated with the route 5 at stop 1 is assumed to be equal to the waiting time experienced by users of route 1 and the waiting time at stop 2 is assumed to be equal to the waiting time for associated with the users of route 11. Also in aggregate process the predicted cost for each route is based on the average of the experienced cost of all passengers at end of each day. In the current R-DSPM the absence of external 'information' results in the costs of non-selected routes to be equivalent to the uncongested cost for the same and the predicted cost for each individual for each route is based on only his/her experience.

The R-DSPM was run for the passengers arrival rates, population size specified in Fig 3.4 and with strict capacity constraints for erlang shape factors of



both  $m = 1$  and  $m = 300$ . The results for  $m=1$  are tabulated in Table 3.8. From the table it can be deduced that the flow distribution on the routes between an OD pair are almost equal to each other especially between ODs  $Z=1$  and  $Z=3$ . This is due to the usage of  $\xi = 0.05$ . It is brought to the attention of the readers that in logit choice model a  $\xi$  value of zero would result in equal distribution of flows between the routes irrespective of the cost of the routes. Hence a value closer to zero for the current values of travel costs results in almost equal distribution of flows in the example network. The behavioural assumption is that as the  $\xi$  value tends to zero the passengers do not consider the costs experienced by them as the true values of the journey. They believe that the costs experienced by them have several unaccounted variance associated with it and base their route choice randomly.

Table 3.8: The congested costs for risk neutral passengers in network with shape factor  $m = 1, \alpha = 5$  and  $\xi = 0.05$

OD	Node 1 - Node 2 (Z=1)			Node 2 - Node 3 (Z=3)			Node 1 - Node 3 (Z=2)					
	1	2	3	10	11	12	4	5	6	7	8	9
$E(g_k)$	25.7	25.3	22.6	25.4	24.7	22.4	31.3	40.2	39.2	37.3	37.9	40.6
Std	2.4	2.1	2.3	2	2.0	2.1	1.4	1.4	1.5	1.5	1.5	1.4
Route	1	2	3	10	11	12	4	5	6	7	8	9
$E(X_k)$	242.5	243	275.5	312.6	323.4	359.9	254.3	178	189	206.5	199.5	171
Std	18.6	17	18.2	18.8	19.5	21.0	17	13.9	14.8	14.9	14.2	13.5

It is observed that irrespective of the network reliability the expectation of predicted costs for each route and the expectation of experienced costs on the same route are significantly different from each other (fig 3.6). This is because the predicted costs are obtained by averaging over the memory length the experienced costs when a passenger travels on a route as well as the uncongested route cost when a passenger doesn't travel on the same.

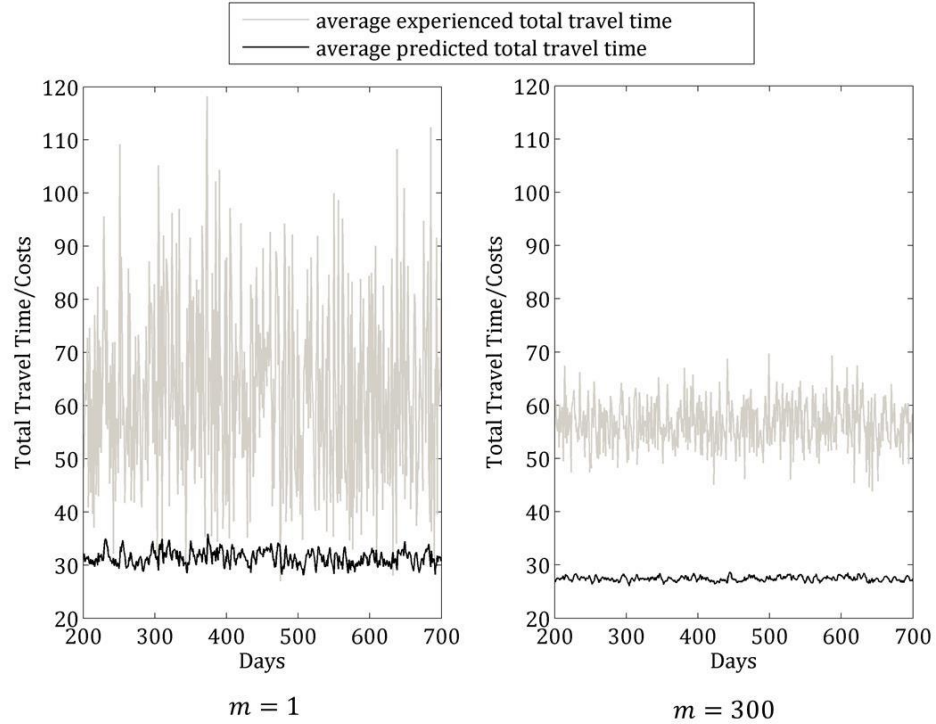


Fig 3.6: The variation in the expectation of predicted costs of risk neutral passengers and the expectation of experienced costs in example network with  $m = 1$  & 300 (Route 4).

One needs to note that in the current R-DSPM the evolution of flows on routes for either  $m=1$  or  $m=300$  does not depend on the expectation of predicted costs of the entire route but on individual's predicted costs for each route.

As explained in section 3.1.2 the current R-DSPM applies the Markov principles. One of the necessary condition but not a sufficient condition for stability of markov process is the stationary evolution of route flows and costs. It is expected that in a markov process the costs/total travel time or flows of the routes after the 'burn-in' period shall result in a stationary distribution independent of its initial condition. Fig 3.7 shows the histogram of flows on various routes for network with  $m = 1$ . The visual inspection of the histograms reveal that the distribution of flows along various routes for the time period between 201 to 401 and 402 to 602 are almost similar to each other. The mean and the standard deviation of the flows are almost identical indicating that the stochastic process being considered is stationary.

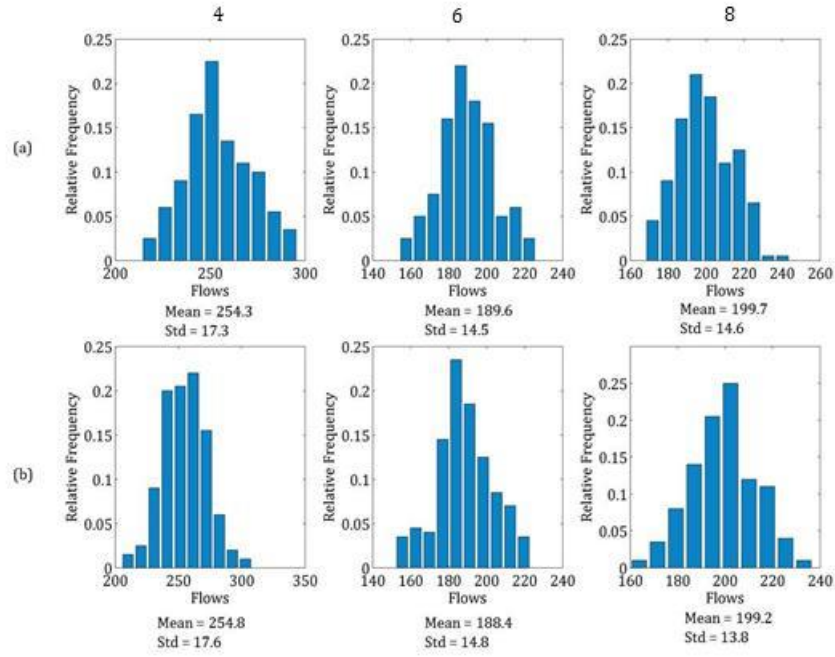


Fig 3.7: Histogram of risk neutral passenger flows along routes 4,6,8 (a)201-400 (b) 401-600 for  $m=1$   $\alpha = 5$  and  $\xi = 0.05$

### 3.4.2.1 Autocorrelation

Another approach to prove that the markov process is stationary can be by computing the ‘large lag standard error’ for the autocorrelograms of flows along various routes and assessing if the autocorrelations die down after a hypothesised lag of K days (Balijepalli et al., 2007). It is understood that an important measure to assess the persistence or the memory property of time series is autocorrelation. Autocorrelation measures the correlation between the observations at different times. The autocorrelation of observations separated by K time steps is given by

$$\rho_K = \frac{\sum_{j=1}^{Y-K} (x_j - \bar{x})(x_{j+K} - \bar{x})}{\sum_{j=1}^Y (x_j - \bar{x})^2} \quad (3.17)$$

Where

$x_j$ - observations such as average flow on each route and average experienced travel time on each route

$\bar{x}$ - average of observations

K- lag days

The 'large lag standard error' is given as follows:

$$\text{var}(\hat{Q}_K) \approx \frac{1}{Y} \left( 1 + 2 \sum_{j=1}^K Q_j^2 \right) \quad (3.18)$$

Where

$\hat{Q}_K$  is an estimator of autocorrelation at lag  $K$

$Q_j$  is the true theoretical autocorrelation at lag  $j$

$Y$  is the length of the time series.

Figure 3.8 gives the auto-correlogram for various routes for passengers under congested condition and with an erlang shape factor of  $m = 1$ ,  $\eta = 5$  and  $\xi = 0.05$ . The autocorrelation of flows with themselves at lag=0 is 1. From then on there is a decreasing positive correlation for route 4 flows. Beyond the memory length of 5 days the correlations die down with an isolated positive correlation happening on 14<sup>th</sup> day indicating that the samples are more independent beyond 5 days.

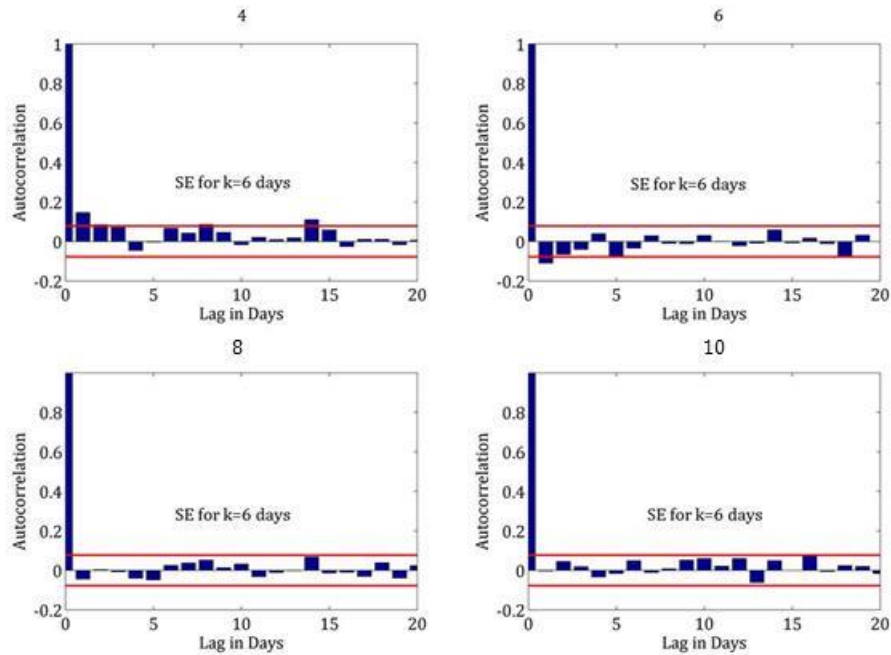


Fig 3.8: Autocorrelogram of flows in congested network with error bounds calculated using equation 3.18 along routes 4,6,8 and 10 for  $m = 1$ ,  $\eta = 5$  and  $\xi = 0.05$ .

Fig 3.8 also shows the error bounds as computed using eq 3.18 for a hypothesised lag of 6 days; beyond which it is assumed that the autocorrelation dies down. The error bounds are indicative of the significance of the correlation. It is seen that most of the correlations are insignificant as they lie

below the error bounds. In case of route 6,8,10 there is a presence of negative correlation within the memory period of 5 days however these correlations are seen to lie below the error bounds indicating that the correlations are not significant. Fig 3.8 indicates insignificant correlation of flows for routes 6,8 and 10 even within the memory period of 5 days. This could be due to the high stochasticity brought about by the random distribution of supply and demand; failure to board along with the perception error assumptions associated with the cost. The high level of stochasticity ultimately also results in the autocorrelations dying down at large lags. A detailed analysis of the autocorrelation and partial autocorrelation under varying  $m$ , and  $\xi$  is given in section 3.6.

Following the assumption of erlang shape factor  $m = 1$  the erlang shape factor is changed to  $m = 300$ . As explained in section 3.2.1.2 a shape factor of  $m = 300$  results in a reduced variability between the interarrival times of the transit line.

Table 3.9 shows a reduction in standard deviation of expectation of predicted costs  $E(g_k)$  between stops 1 and stop 3 for  $m = 300$  network when compared with the  $m = 1$  network. One could conclude that the reduction in cost's standard deviation is a reflection of a more stable evolution of the network.

Table 3.9: The congested costs for risk neutral passengers in network with shape factor  $m = 300$ ,  $\alpha = 5$  and  $\xi = 0.05$

Route	Node 1 – Node 2 (Z=1)			Node 2- Node 3 (Z=3)			Node 1 – Node 3 (Z=2)					
	1	2	3	10	11	12	4	5	6	7	8	9
$E(g_k)$	20.5	21.6	20.4	21.1	20.5	20	27.4	32.3	31.8	31.2	32.4	33.6
<b>Std</b>	0.74	0.72	0.73	0.66	0.68	0.67	0.44	0.65	0.6	0.58	0.59	0.56
Route	1	2	3	10	11	12	4	5	6	7	8	9
$E(X_k)$	258.4	244.2	258.4	321.8	332.5	341.5	232.6	192.9	199.1	203.5	192.7	177.5
<b>Std</b>	16.4	15.8	16	18.3	18.9	19.2	15.1	14.7	13.4	13.9	13.8	14.1

Similar to  $m = 1$  network, the tests to prove that the markov process is stationary using auto correlogram and histograms are shown in Fig 3.9 and Fig 3.10. From table 3.9 it can be seen that in a congested network the expectation

of predicted costs of a more reliable network service modelled here with erlang shape factor of  $m = 300$  are lesser than the expectation of predicted costs in an unreliable network which is modelled with shape factor of  $= 1$  . This is anticipated as the inter arrival of the services in a reliable network are more closer to the specified average headways thereby reducing the waiting times of passengers who could otherwise have experienced a larger inter arrival time. Since, in case of non-travelled routes the R-DSPM will use the uncongested costs of  $m=300$  to determine the prediction costs of each passenger the  $E(g_k)$  values are lesser in table 3.9 than those shown in table 3.8.

The histograms of flows shown in fig 3.9 indicate that between days 201 to 401 and 402 to 602 the flow distribution is almost similar. The mean and standard deviation of the two periods are also found to be almost same. As the stationary distribution between the two time intervals are similar it can be claimed that there is only one probability distribution of the flows for each route under the assumed conditions and hence the stochastic process is ergodic (Watling and Cantarella, 2012).

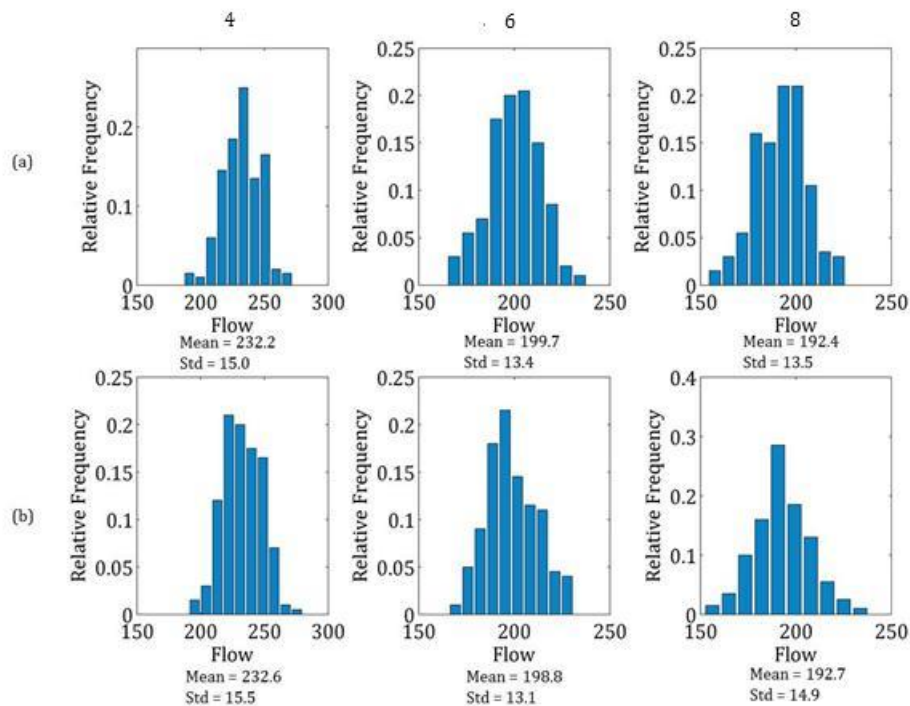


Fig 3.9: Histogram of risk neutral passenger flows along routes 4, 6, 8 for (a) 201-400 days (b) 401-600 days  $m=300$ ,  $\alpha = 5$  and  $\xi = 0.05$

The presence of insignificant autocorrelograms of the flows beyond the standard error bars computed for some hypothetical  $K^{\text{th}}$  day as shown in fig

3.10 is an indication that the flow on the routes do not depend on the flows on the same route beyond K days thereby implying that the process is stationary.

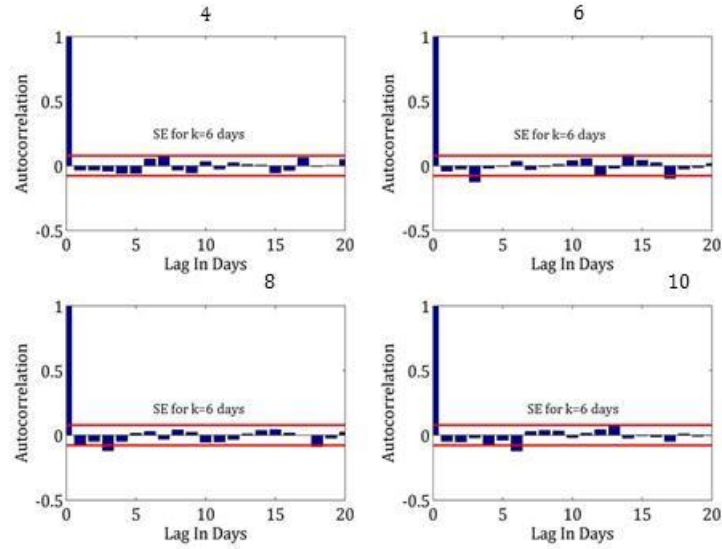


Fig 3.10: Autocorrelogram of risk neutral passenger flows in congested network for routes 4,6,8 and 10 with error bounds computed using equation 3.18, for  $m= 300$  ,  $\alpha = 5$  and  $\xi = 0.05$

### 3.5 Initial conditions:

Another essential condition to prove Markov property of the model is to see if the system converges to the same probability distribution irrespective of its initial conditions. To study the convergence different initial conditions were simulated. Initial condition was varied by changing the random number seed values of the R-DSPM framework and by varying the rate of Poisson passenger arrivals along with the population size between OD pairs for the first 80 days (Z=1- poisson rate of passenger arrivals-60/3600, population size (constant demand)-302; Z=2- poisson rate of passenger arrivals-60/3600, population size (constant demand)-298; Z=3- poisson rate of passenger arrivals-20/3600, population size (constant demand)-110). These two different initial condition results were compared with the model results specified earlier. It is expected that irrespective of the initial conditions the model shall converge to a stationary distribution as per the ergodic markovian property.

Table 3.10 : route costs and flow distributions and sensitivity to initial conditions  $m = 1, \alpha = 5$  and  $\xi = 0.05$

	Route	Node 1- Node 2 (Z=1)			Node 2 - Node 3 (Z=3)			Node 1 - Node 3 (Z=2)					
		1	2	3	10	11	12	4	5	6	7	8	9
Initial condition 2	$E(\mathbf{t}_k)$	25.9	25.4	22.7	25.3	24.5	22.2	21.4	40.3	39.2	37.4	37.9	40.7
	Std	2.9	2.6	2.8	2.2	2.3	2.4	1.7	1.9	2	2	2.1	1.9
	$E(X_k)$	240.4	243.1	277.5	311.6	323.8	360.4	254.4	178.7	188.1	206.4	199.2	171.5
	Std	18.0	16.9	17.8	18.8	19	19.4	17	13.3	14	14.3	15.4	13.8
	$E(\mathbf{t}_k)$	25.7	25.3	22.5	25.4	24.7	22.4	31.4	40.3	39.1	37.3	37.9	40.7
Initial condition 3	Std	2.3	2.1	2.3	2	2.1	2.1	1.5	1.4	1.4	1.5	1.6	1.4
	$E(X_k)$	241.6	242.4	277	312.2	323.2	360.4	255.4	179	188.2	206.2	198.4	171.4
	Std	17.8	17.7	16.7	19.6	20.4	19.3	17.1	13.6	13.7	14.4	14.3	12.8

From table 3.10 and table 3.11 we can see that for both the assumed shape factors the system converges to a stationary distribution irrespective of their initial conditions. The mean and standard deviation values of the average experienced costs and flows are similar irrespective of the initial conditions assumed for both  $m = 1$  &  $m = 300$ . Fig 3.11 and 3.12 shows the histogram of stationary distribution of flows on various routes which again are almost similar irrespective of the initial conditions.



Table 3.11 : route costs and flow distributions and sensitivity to initial conditions  $m = 300, \alpha = 5$  and  $\xi = 0.05$

	Route	Node 1 – Node 2 (Z=1)			Node 2- Node 3 (Z=3)			Node 1- Node 3 (Z=2)					
		1	2	3	10	11	12	4	5	6	7	8	9
Initial condition 2	$E(t_k)$	20.4	21.6	20.4	21.1	20.5	19.9	27.4	32.3	31.8	31.2	32.4	33.7
	Std	0.7	0.7	0.7	0.62	0.69	0.68	0.4	0.6	0.6	0.6	0.6	.6
	$E(X_k)$	258.6	243.4	259.0	321.8	332.3	341.8	231.7	193.5	199	203.8	191.6	178.6
	Std	15.9	16.7	17.1	17.8	19	18.8	15.2	13.8	14.4	14.6	14.0	13.7
Initial condition 3	$E(t_k)$	20.5	21.6	20.4	21.2	20.5	19.9	27.4	32.3	31.8	31.2	32.4	33.7
	Std	0.7	0.7	0.75	0.66	0.67	0.67	0.45	0.6	0.64	0.61	0.55	0.54
	$E(X_k)$	258.6	243.2	259.1	321.7	333.1	341	231.4	193.3	198.0	203.8	192.6	179.1
	Std	16.3	15.4	16.0	18.0	18.4	19.2	15.2	13.1	14.9	14.4	13.5	12.9

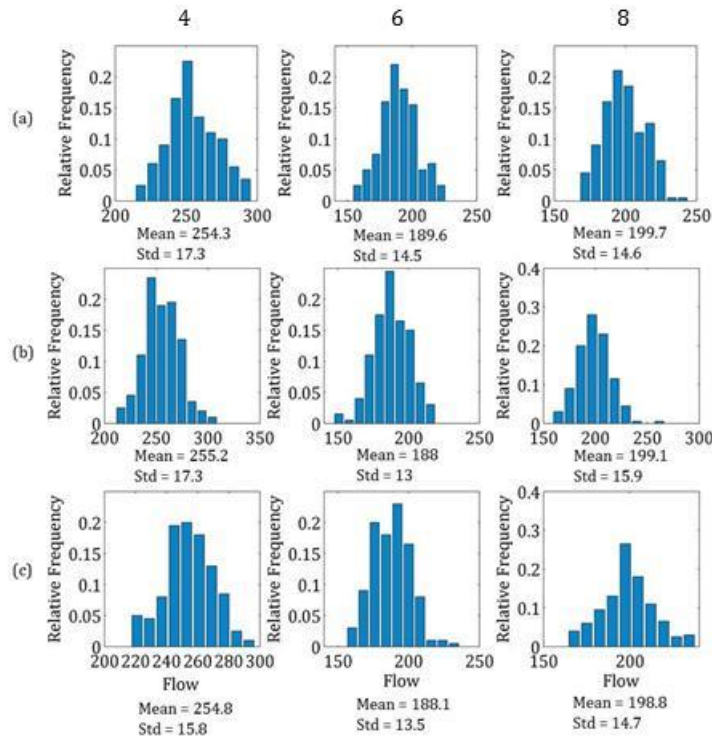


Fig 3.11 The stationary distribution of flows on routes 4, 6 and 8 under (a) initial condition 1 (b) initial condition 2 and (c) initial condition 3 for  $m = 1, \alpha = 5$  and  $\xi = 0.05$ .

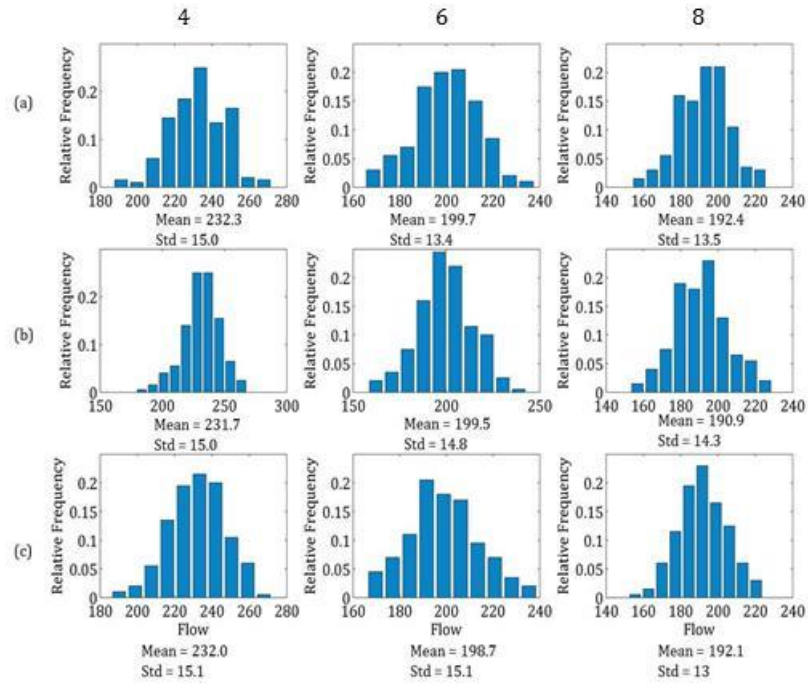


Fig 3.12 The stationary distribution of flows on routes 4, 6 and 8 under (a) initial condition 1 (b) initial condition 2 and (c) initial condition 3 for  $m = 300$ ,  $\eta = 5$  and  $\xi = 0.05$

Statistical testing of whether the markov process has converged irrespective of its initial conditions is done using Wilcoxon rank sum test and two-sample Kolmogorov-Smirnov test. These are non-parametric test for assessing if two samples of the observation come from the same continuous distribution. In case of Wilcoxon rank sum test the null hypothesis is that the two samples are independent samples from identical continuous distribution, with equal medians. The alternative hypothesis is that they do not have equal medians.

As can be seen from the autocorrelograms (fig 3.8 and 3.10) the route flows for each route are correlated as the evolution of the flows within the R-DSPM framework involves dependence of the current flow state on a finite memory period. Hence the correlation of the costs and the flows are expected to remain for a period of 5 days which is the memory length  $\eta$  assumed for the current test run. From the correlogram it can also be seen that beyond 10 days (arbitrarily chosen) the correlations die down hence a process of subsampling was carried out to perform the statistical test. In the process of subsampling every 10<sup>th</sup> element of  $E(g_k)$  and  $E(X_k)$  was used to derive a new set of elements to perform the statistical test. The necessary conditions for the Wilcoxon rank sum tests are that the two samples being tested are independent of each other and that the two samples have the similar distributions. The Wilcoxon rank sum test was carried out using Matlab. The results of the statistical test are as shown in Table 3.12 and Table 3.13.

Table 3.12: Wilcoxon Rank sum test results for different initial conditions for  $m = 1, n = 5$  and  $\xi = 0.05$

Routes	Node 1- Node 2 (Z=1)				Node 1 – Node 3 (Z=2)					Node 2 – Node 3 (Z=3)			
	1	2	3	4	5	6	7	8	9	10	11	12	
	$E(g_k)$												
1 vs 2	<b>0.03</b>	0.59	0.47	0.57	0.35	0.56	0.46	0.67	0.69	0.55	0.10	0.11	
2 vs 3	0.07	0.6	0.37	0.59	0.51	0.68	0.51	0.6	0.97	0.48	0.09	0.09	
1 vs 3	0.83	0.99	0.84	0.96	0.75	0.91	0.9	0.97	0.41	0.89	0.93	0.96	
$X_k$													
1 vs 2	<b>0.02</b>	0.07	0.2	0.89	0.99	0.25	0.88	0.73	0.56	<b>0.03</b>	0.1	0.4	
2 vs 3	<b>0.02</b>	0.05	0.75	0.33	0.35	0.94	0.33	0.22	0.8	0.83	1	0.7	
1 vs 3	0.7	0.7	0.29	0.28	0.25	0.24	0.43	0.34	0.75	0.05	0.09	0.86	

Table 3.13: Wilcoxon Rank sum test results for different initial conditions for  $m = 300, n = 5$  and  $\xi = 0.05$

Routes	Node 1 – Node 2 (Z=1)				Node 1 – Node 3 (Z=2)					Node 2- Node 3 (Z=3)			
	1	2	3	4	5	6	7	8	9	10	11	12	
	$E(g_k)$												
1 vs 2	0.84	0.61	0.73	0.81	0.79	0.84	0.80	0.89	0.83	0.95	0.93	0.74	
2 vs 3	0.96	0.65	0.91	0.85	0.92	0.65	0.67	0.61	0.59	0.94	0.82	0.9	
1 vs 3	0.99	0.82	0.61	0.99	0.73	0.42	0.99	0.54	0.67	0.83	0.97	0.57	
$X_k$													
1 vs 2	0.77	0.89	0.67	0.45	0.61	0.31	0.8	0.89	0.48	0.9	0.87	0.72	
2 vs 3	0.37	0.52	0.63	0.65	0.92	0.59	0.64	0.52	0.46	0.07	0.31	0.38	
1 vs 3	0.61	0.58	0.79	0.88	0.69	0.13	0.68	0.48	0.84	0.1	0.48	0.63	

The tables 3.12 and 3.13 results show that the null hypothesis cannot be rejected at the 5% significance level as the p-values are greater than 0.05 in



Table 3.15: Two-sample Kolmogorov-Smirnov test results for different initial conditions for  $m = 300$ ,  $\alpha = 5$  and  $\xi = 0.05$

Routes	Node 1- Node 2 (Z=1)				Node 1 – Node 3 (Z=2)				Node 2 – Node 3 (Z=3)			
	1	2	3	4	5	6	7	8	9	10	11	12
$E(g_k)$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	0	0	0	0	0	0	0	0	0	0	0
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0
$X_k$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	0	0	0	0	0	0	0	0	0	0	0
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0

### 3.6 Sensitivity Analysis:

#### 3.6.1 Autocorrelation

The sensitivity of the model to various  $\xi$  values is shown in fig 3.13 and fig 3.14. The anticipation is that as the  $\xi$  value increases the passengers become more sensitive to changes in the costs of the competing routes between an OD pair. In other words as  $\xi$  increases the random term in the predicted cost  $g_k$  is assumed to be fully explained and the cost value is assumed to reflect the true value of the journey. At higher  $\xi$  values the cost value becomes reflective of the actual travel costs and hence the passengers ‘believe’ the costs experienced by him/her. This results in all passengers formulating the same opinion about the route costs and thereby think alike or homogenously.

$m = 1$

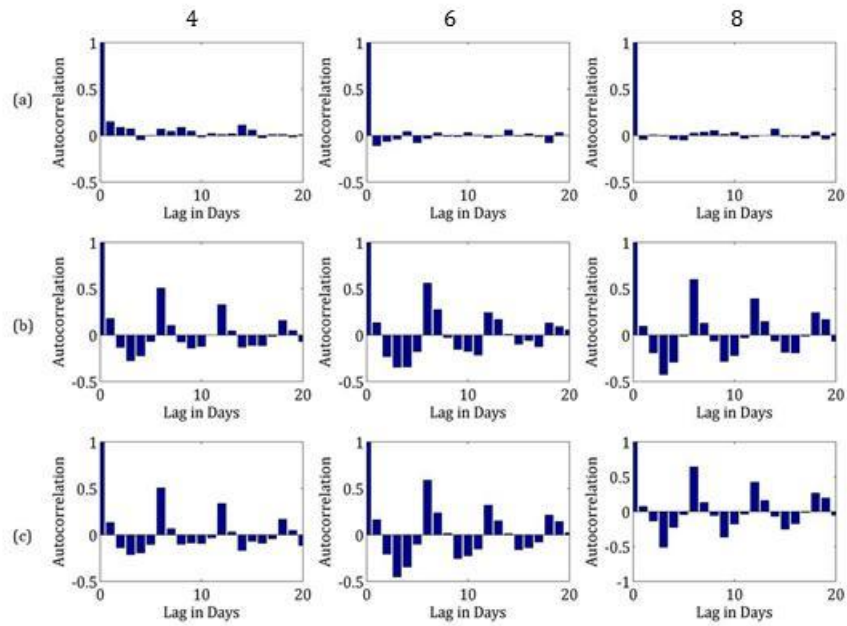


Fig 3.13 The autocorrelogram of flows along routes 4, 6 and 8 for  $m = 1$  and  $\tau = 5$  (a)  $\xi = 0.05$  (b)  $\xi = 1.8$  (c)  $\xi = 4$

The behaviour of flows when the logit dispersion parameter  $\xi$  is increased is assessed and is shown in fig 3.13 and fig 3.14. A  $\xi$  value closer to zero would result in equal distribution of flows between all the routes serving an OD pair. It would then mean that the passengers are not sensitive to the difference in costs of the routes due to the assumption that the component involving the unexplained factors of the journey are much more. The passengers make similar route choice decisions at higher  $\xi$  as they ‘believe’ the expected costs to be the true value of the journey. Hence on a given day almost all passengers choose the same route and since they experience higher costs on that day along their chosen route all the passengers end up choosing a different route the next day. Hence the route flows tend to be more negatively correlated as shown in fig 3.13 and 3.14. The periodic attractors are more visible at higher  $\xi$  values as can be seen from Fig 3.13 wherein strong negative correlations are observed within the memory period of 5 days punctuated with a high positive correlation at end of the memory period duration. The presence of periodic attractors is much more distinctly visible in case of higher reliable interarrival (fig 3.14) as the stochasticity associated with transit service interarrivals is reduced.

$m = 300$

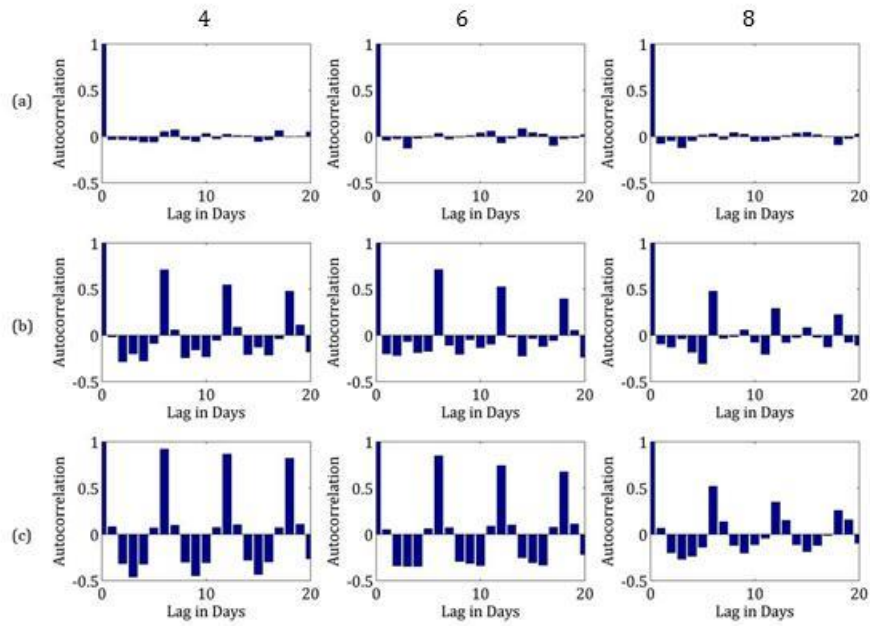


Fig 3.14 The autocorrelogram of flows along routes 4, 6 and 8 for  $m = 300$  and  $\tau = 5$  (a)  $\xi = 0.05$  (b)  $\xi = 1.8$  (c)  $\xi = 4$

### 3.6.2 Partial Autocorrelation

Partial autocorrelation gives the autocorrelation between  $x_{\Omega}, x_{\Omega-K}$  after removing the linear dependence between  $x_1, x_2, \dots, x_{\Omega-K+1}$  wherein  $x_{\Omega}$  is the average total travel time or average flow on a route at simulation day  $\Omega$  and  $K$  is the lag in days. In other words partial autocorrelation gives the partial correlation of the time series with its own lag values, controlling for the lag values of the time series at all shorter lags (Lee and Fambro, 1999). Partial autocorrelation helps to identify the possible order of autoregressive moving average (ARMA) time series models. The ARMA models are used for predicting the behaviour of time series through generation of similar time series having the same persistence structure which could be used for future policy evaluation in transport network. Fig 3.15 and 3.16 show the partial autocorrelogram of flows with 95% confidence band. Fig 3.15 indicates that at  $\xi = 0.05$  the partial correlation plot doesn't show a clear statistical significance beyond lag 1 (lag 0 is always 1). The next few lags are at border line statistical significance. As the  $\xi$  value is increased the partial autocorrelogram of flows shows clear statistical significance upto interval of  $\tau = 5$  days and next few lags at border line statistical significance. This clear show of statistical significance within the interval of memory length is indicative of the already mentioned fact that at higher  $\xi$  periodic attractors are observed in the network.

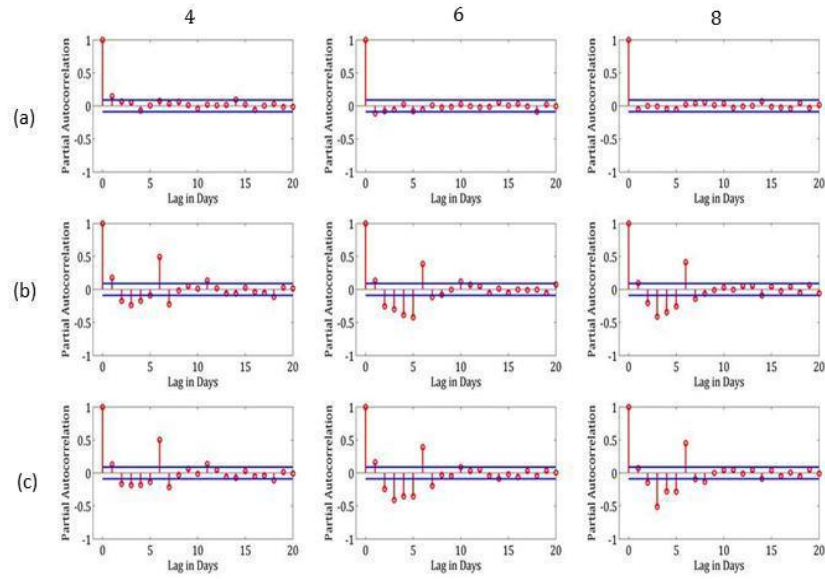


Fig 3.15 The partial - autocorrelogram of flows along routes 4, 6 and 8 with 95% confidence bounds for  $m = 1$  and  $\beth = 5$  (a)  $\xi = 0.05$  (b)  $\xi = 1.8$  (c)  $\xi = 4$

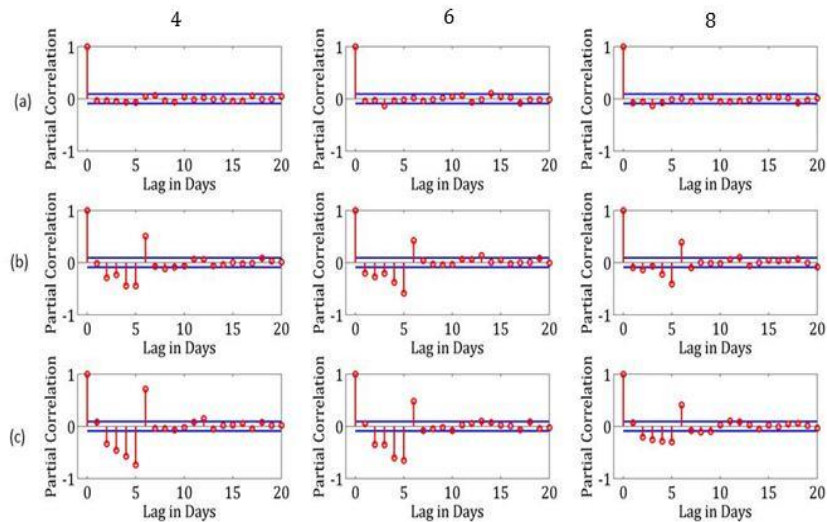


Fig 3.16 The partial autocorrelogram of flows along routes 4, 6 and 8 with 95% confidence bounds for  $m = 300$  and  $\beth = 5$  (a)  $\xi = 0.05$  (b)  $\xi = 1.8$  (c)  $\xi = 4$

A similar trend is observed with  $m = 300$  and as can be seen from fig 3.16 the negative partial correlations within the memory period of 5 days is much more pronounced in comparison with  $m = 1$  for higher  $\xi$  values.

### 3.6.3 Varying memory lengths

The sensitivity of risk neutral passengers to varying memory lengths ( $\beth$ ) was tested. Memory lengths ( $\beth$ ) of 5 days, 15 days and 30 days were used to test the difference in the flow and total experienced travel time distribution between risk neutral and risk averse passengers.



Table 3.16: The average experienced travel time and the average flow on various routes for varying memory lengths when  $m = 1$  and  $\xi = 4$ .

		Z=1		Z=2		Z=3	
Route		2	3	4	5	9	11
$\gamma=5$	$E(t_s)$	49.6	46.4	60.4	110.3	100.5	51.3
	Std	17.4	16.3	17.2	22.7	25.6	18.4
	$E(X_s)$	231.7	301.9	310.3	164.8	70.5	299.3
	Std	30.9	37.3	76.8	40.0	34.7	52.0
$\gamma=15$	$E(t_s)$	52.1	47.3	62.8	108.3	104.6	52.5
	Std	17.3	15.9	17.2	22.3	26.0	18.8
	$E(X_s)$	220.5	304.5	360.1	151.4	111.1	294.5
	Std	32.8	39.8	83.0	24.1	27.7	40.3
$\gamma=30$	$E(t_s)$	52.2	47.2	62.9	108.5	104.7	52.6
	Std	17.2	15.9	17.3	22.6	25.8	18.6
	$E(X_s)$	220.1	305.8	362.5	150.6	111.0	293.5
	Std	31.2	41.1	82.6	24.1	27.7	39.3

Table 3.16 indicates that for a change in memory length from 5 days to 15 days there is a slight difference in the flow values and the experienced travel time values. The change in flow and experienced travel times between the memory length values of 15 and 30 days is almost negligible indicating that for a set of stochastic demand and supply parameters of a network the evolution of passenger flow stabilises beyond a certain memory length for a given  $\xi$  value. This is expected as with lower memory length passenger's base their route choice on lesser past experiences. In higher memory lengths the passengers have a larger experience pool to base their route choice on. In a small network as with example network 1, for the current rate of passengers and the current number of potential passengers, it can be summarised that passengers travelling between an OD pair would have experienced most of the routes within the memory period of 15 days resulting in the flow distribution stabilising beyond a memory length of 15 days. The influence of larger number of potential passengers on the results of the model is dealt with section 5.4. Larger network with most of the passengers travelling on daily basis would require a higher memory length beyond which the route flow distribution would remain the same. The above results, apart from the influence of the supply parameters, are also influenced by the parameters assumed for the learning process. As mentioned in Teklu (2008b) a relationship between the rate of progression  $\rho$  and memory length  $\gamma$  is present. The relationship is such

that a lower value of  $\rho$  results in higher weightage to recent experience. A  $\rho$  value of 1 results in equal weightage to the past experiences within the memory length resulting in a weightage of 0.5 for each day if the memory length  $\Delta$  is chosen as 2 days. A smaller  $\rho$  value with higher memory length would result in older experiences having almost negligible weights.

### **3.7 Summary**

The chapter proposes a R-DSPM which is run under various reliability scenarios of interarrival of services on an example network. The highly disruptive scenario is modelled with exponential interarrivals and a more 'normal' service is modelled using higher shape factor of erlang distribution. The current chapter dealt with the behavioural analysis of risk neutral passengers who minimise only their average travel costs. It is seen that the passengers tend to have a higher average expected costs in an unreliable network than a reliable network. It is shown that the proposed stochastic process model has a stationary distribution and exhibits the markovian property which enables the system to converge to the same stationary distribution irrespective of the initial conditions. The R-DSPM has been successful in considering the interaction between the passengers assigned to different routes but having the same attractive line set. The FIFO arrangement of passengers in queue for a line at the transit stop ensures that the passenger who arrive first get the first opportunity to board the transit vehicle if the arriving vehicle is of his/her attractive line set. The passengers who have already boarded the transit service and are continuing the journey beyond the current transit stop remain inside the transit vehicle thereby ensuring that those boarding the line before have a priority over the passengers boarding latter at the transit stop. The strict capacity constraint of the transit vehicles ensures that a transit service is not loaded beyond its capacity.

Aggregate stochastic process model is based on the assumption that the flows at end of each day revise their routes for the next day based on the predicted costs which are the weighted average of the experienced travel times over the demand generated for the day. This however is counterintuitive as more often in the absence of any external information source the knowledge of the costs experienced by other passengers is not known to an individual while revising his/her route choice. The R-DSPM therefore provides a more realistic framework while evaluating the passenger route choice.

## **Chapter 4**

### **R-DSPM formulation for Mean – Variance and comparison with existing models**

A transit service experiences unreliability due to several factors as highlighted in chapter 2. A passenger travelling in a transit service apart from experiencing unreliability associated with the supply side of the network also experiences unreliability due to demand wherein a strict capacity constrained network would result in passengers often being unable to board the first transit service of their chosen attractive line set. In event of the inability to board the first transit service of their choice set the passengers inherently experience an unreliable service though the service may have been reliable from operator's perspective. The current chapter includes the variance experienced by the passengers due to the stochastic nature of demand and supply, in the cost of the passengers. The following sections shall examine the impact of considering the variance associated with the individual's total travel time on their route choice.

As highlighted in chapter 2 the existing stochastic process models (Teklu, 2008b) and equilibrium based models (De Cea and Fernández, 1993; Cominetti and Correa, 2001; Cepeda et al., 2006) are aggregate in nature and these models mostly assume that passengers are risk neutral. The aggregate models assume that the passengers are aware of the total travel time experienced by the others. As mentioned in chapter 1 in reality such an aggregate information wherein the total travel time experienced by others is known is not easily available and hence passengers tend to rely on individual experiences. Furthering the disaggregate model derived in chapter 3 (wherein a stochastic process model for risk neutral passengers was implemented) a risk averse learning process model is developed in the current chapter which essentially models variance as an individual  $i$ 's attribute.

Transit assignment models based on mean-variance approach has been, to the authors knowledge, dealt by Szeto et al. (2011) and Szeto et al. (2013). Szeto et al. (2011) developed a BPR congestion function based SUE model and Szeto et al. (2013) an overload delay based SUE model accounting for the variance associated with the in-vehicle travel time; the uncongested waiting time and the increased waiting time due to the congestion; using the route section approach. However Szeto et al. (2011) and Szeto et al. (2013) model reliability in transit network without considering the strict capacity constraints; day to

day evolution of supply and demand in the transit network or the effect of learning process on passengers route choices.

Chapter 3 saw the implementation of R-DSPM on example network 1. The mean-variance analysis was carried out for example network 1 and example network 2 (a network given in Teklu (2008a)). However only example network 2 results are discussed in the current chapter. It is noted that the interpretation of results for various parameters in both the networks remain the same.

The aim of the chapter is to show the need to consider the risk averseness of passengers while modelling the route flows in a transit network and the need to use stochastic process model to do the same. The chapter is organised such that initially uncongested transit network wherein the unreliability is only due to supply variations is considered. The change in the shortest route between risk neutral and risk averse passengers in an uncongested network is assessed in Section 4.1. The section establishes the shift in the shortest route for risk averse passengers in comparison with risk neutral passengers thereby asserting the difference in passengers route choice while considering risk aversion. Section 4.2 shall implement R-DSPM for risk averse passenger on a congested transit network (example network 2) with strict capacity constraint. The results obtained are discussed and several sensitivity tests are carried out to assess the performance of the model.

Section 4.3 shall implement a BPR congestion function for an SUE based assignment of risk averse flows on example network 2 followed by the 'effective frequency' congestion function based DUE on hyperpath representation of example network 2. The parameters of effective frequency and the BPR waiting time function shall be calibrated to make a comparison with R-DSPM possible.

#### **4.1 Risk Averse vs Risk Neutral passengers (uncongested transit network):**

Spiess and Florian (1989) and De Cea et al. (1988) define the strategy/route chosen by risk neutral passengers in an uncongested network as optimal strategy or optimal route. The optimal strategy/route essentially consists of line segments/sections which minimise the average costs experienced by passengers. The aim of the current section is to assess:

1. If the shortest strategy/route of risk averse passengers differs from that of risk neutral passengers.

2. If the interarrival reliability of lines (modelled using shape factor  $m$ ; which when equal to 1 depicts exponential interarrivals and when equal to 300 depicts a more reliable service) serving a transit stop has an influence on the shortest strategy/route of risk averse passengers.

The answers to these questions shall be explained through a series of simulation tests run on example network 1. The example network given in chapter 3 is modified to suite the current requirement with revised in-vehicle times and frequencies as shown in table 4.1.

Table 4.1: Revised frequency for example network 1

Line Number	Line Color	In - Vehicle Travel Time in min	From Stop – To Stop	Buses per hour
$l_1$	Orange	6	1-2	10
$l_2$	Red	4	1-2	6
$l_2$	Red	4	2-3	6
$l_3$	black	10	2-3	20

As explained earlier a risk neutral passenger is defined as a person who ignores the variance associated with his/her journey and minimises only his/her average costs. Whereas risk averse passengers assign a non-negative parameter to the variance ; which represents the degree to which the variance is undesirable to passengers (Jackson and Jucker, 1982). The weightage associated with the variance as given by risk averse passengers is symbolised by beta ( $\beta$ ) in the current model, which then means that a risk neutral passenger has a  $\beta$  value of 0 and passengers having  $\beta > 0$  are risk averse.

#### 4.1.1 Change in the shortest route for risk averse passengers:

The results obtained by running a monte-carlo simulation model of modified example network 1 in uncongested scenario shows that there is a change in shortest route between risk averse and risk neutral passengers. From Table 4.2 it is found that in a network with  $m = 1$  (exponential inter arrival of transit services) the shortest cost route at  $\beta = 0$  (risk neutral) is route 4 whereas at  $\beta \geq 0.094$  the shortest route shifts to route 8.

Table 4.2: The mean cost for routes between various OD pairs for modified example network 1 (table 4.1) with  $m = 1$

Routes	Z = 1				Z = 2				Z = 3			
	1	2	3	4	5	6	7	8	9	10	11	12
	12.1	14.0	<b>9.0</b>	<b>18</b>	25	26.1	23.0	22.0	27	13	14	<b>10.9</b>
0.028	13.1	16.7	<b>9.5</b>	<b>20.7</b>	23.3	30	24.4	22.7	30	13.2	16.7	<b>11.2</b>
0.05	14	18.9	<b>9.8</b>	<b>22.9</b>	27.3	33.1	25.5	23.2	32.2	13.4	18.8	<b>11.5</b>
0.094	15.6	23.1	<b>10.5</b>	27.1	29.4	39.2	27.7	<b>24.3</b>	36.9	13.8	23.1	<b>12.0</b>
0.194	19.4	32.8	<b>12</b>	36.8	34	53.1	32.6	<b>26.7</b>	47.4	14.7	32.8	<b>13.2</b>
0.5	31.1	62.5	<b>16.6</b>	66.5	48.1	95.7	47.7	<b>33.9</b>	79.6	17.4	62.4	<b>16.7</b>
2.25	97.7	232.1	<b>43</b>	236.1	129.0	339.2	134.3	<b>75.6</b>	263.8	<b>32.8</b>	231.7	36.8

From table 4.2 it can be concluded that for certain values of aversion the risk averse passengers have the same shortest route as risk neutral passengers. After a certain threshold value of risk aversion a shift in the shortest route is observed in the network. This outcome can be explained as the insensitivity of risk neutral passengers towards variance values hence the shortest route for a risk neutral passengers ( $\beta = 0$ ) has lower mean and higher variance whereas the shortest route for a risk averse passenger ( $\beta \geq 0.094$ ) has a higher mean and lower variance. This notion is emphasised by fig 4.1 wherein at  $\beta = 0$  one can see the mean value being lesser than the variance value and at  $\beta \geq 0.094$  a shift is observed such that the shortest cost route is now the route having higher mean but lower variance.

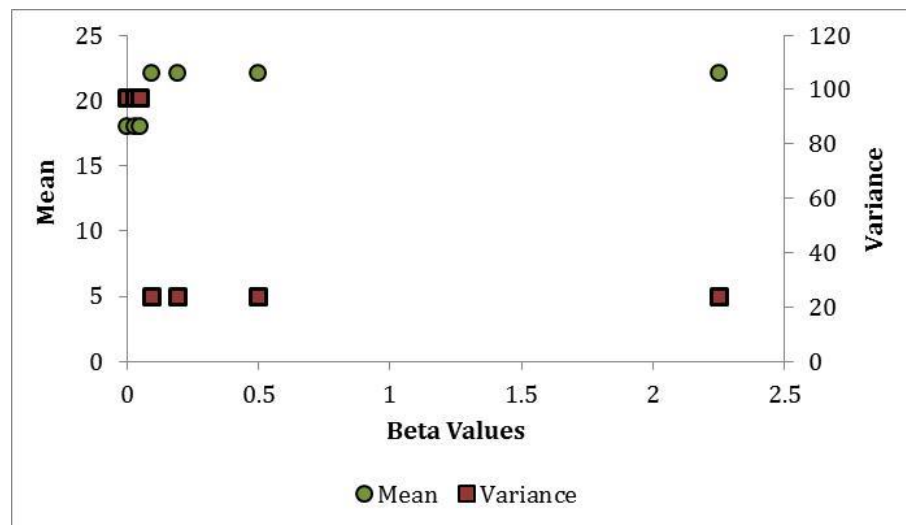


Fig 4.1: The mean vs variances of shortest routes between Node 1 and Node 3 for modified example network 1 (table 4.1) with  $m = 1$ .

#### 4.1.2 Influence of interarrival reliability in the shortest route for risk averse passengers:

Continuing with the uncongested network example given in section 4.1.1 here we shall look at, whether an improvement on the interarrival times of the transit services, the shortest route for risk averse passengers changes. The interarrival times are improved by using  $m = 300$ . The results of the analysis is shown in table 4.3.

Table 4.3: The mean cost for the routes between various OD pairs for modified example network 1 (table 4.1) with  $m = 300$

Routes	Z = 1				Z = 2				Z = 3			
	1	2	3	4	5	6	7	8	9	10	11	12
Beta Value												
0	9	9.0	<b>7.8</b>	<b>13.0</b>	20.5	18.0	19.5	19.3	20.5	11.5	<b>9.0</b>	10.4
0.028	9.1	9.3	<b>7.9</b>	<b>13.3</b>	20.6	18.4	19.7	19.5	20.8	11.5	<b>9.3</b>	10.6
0.05	9.2	9.5	<b>8.0</b>	<b>13.5</b>	20.7	18.6	19.9	19.6	21	11.5	<b>9.4</b>	10.7
0.094	9.3	9.8	<b>8.2</b>	<b>13.8</b>	20.9	19.1	20.3	19.8	21.4	11.6	<b>9.8</b>	11.0
0.194	9.6	10.7	<b>8.6</b>	<b>14.7</b>	21.3	20.3	21.3	20.2	22.3	11.7	<b>10.7</b>	11.6
0.5	10.5	13.3	<b>9.8</b>	<b>17.3</b>	22.4	23.9	24.0	21.7	25.2	<b>11.9</b>	13.3	13.5
2.25	<b>15.9</b>	28.3	16.7	32.3	<b>29.2</b>	44.3	40	30.0	41.6	<b>13.2</b>	28.2	24.0
5	<b>24.3</b>	51.8	27.7	55.8	<b>39.7</b>	76.3	64.8	43.1	67.4	<b>15.3</b>	51.7	40.6
10	<b>39.6</b>	94.5	47.5	98.5	<b>58.9</b>	134.6	110.1	66.9	114.2	<b>19.1</b>	94.3	70.7

Table 4.3 shows a similar shift in shortest cost route between risk neutral and risk averse passengers when the network runs transit services with a more reliable interarrival times (modelled with  $m = 300$ ). It is observed from the results that in a reliable network the variance associated with the inter arrival of services is not very large and hence the shift of shortest cost route takes place at a larger  $\beta$  value. Hence route 4 remains shortest uptill a  $\beta$  value of 0.5. At a  $\beta$  value of 2.25 the shortest route shifts to route 5. Fig 4.2 shows that the reliable network also exhibit the trend of the shortest route remaining the same beyond a certain  $\beta$  value.

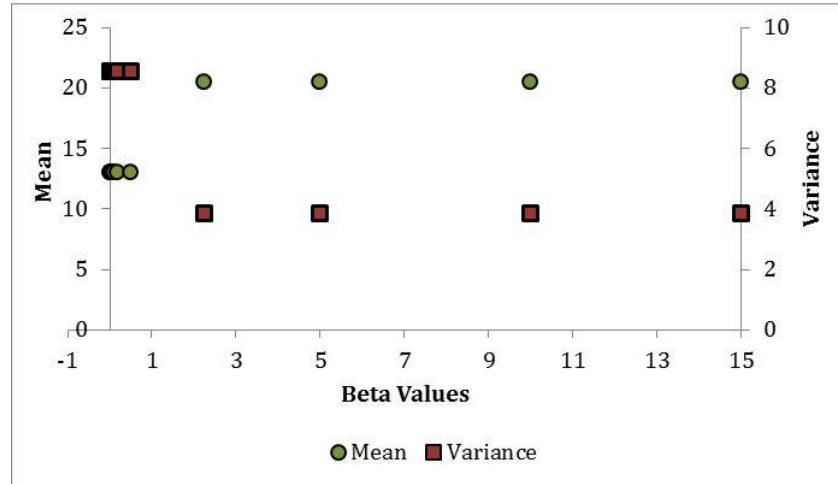


Fig 4.2: The mean vs variances of optimal routes between Node 1 and Node 3 for example network 1 (table 4.1) with  $m = 300$ .

The above analysis indicates that in uncongested reliable transit networks the passengers need to have a higher  $\beta$  value of risk aversion to have a significantly different route choice from that of risk neutral passengers.

#### 4.1.3 Need for a stochastic process model to analyse risk averse transit network

The previous sections highlighted that the route flow distribution for risk averse and risk neutral passengers would be considerably different if AON assignment rule is followed in an uncongested network. This distinction was only observed for certain values of aversion to variance of experienced total travel time. In the current section a look at the need to consider stochastic process model to assess the route choice of transit network passengers shall be explored. The transit assignment problem being asymmetric in nature is highlighted in De Cea and Fernández (1993) example network wherein the jacobian of the network is as shown below

$$\begin{bmatrix} 1/10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/10 & 0 & 0 & 1/10 & 0 \\ 0 & 0 & 1/14 & 0 & \mathbf{1/14} & 1/14 \\ 0 & 0 & 0 & 1/24 & 0 & \mathbf{1/24} \\ 0 & 1/10 & \mathbf{0} & 0 & 1/10 & 0 \\ 0 & 0 & 1/14 & \mathbf{0} & 0 & 1/4 \end{bmatrix}$$

The asymmetric flows are highlighted in bold wherein

$$\frac{\partial c_3(x)}{\partial(x_5)} \neq \frac{\partial c_5(x)}{\partial(x_3)}$$

And

$$\frac{\partial c_4(x)}{\partial(x_6)} \neq \frac{\partial c_6(x)}{\partial(x_4)}$$



This is as the result of the flow boarding at the higher end transit stops influencing the cost functions of the flow boarding at lower end transit stops. The presence of unique solution is possible when the jacobian is positive definite (Sheffi 1985). In situations such as that of multiuser class networks the dominance of one user class group over the other user class results in the positive definite condition being violated (Watling, 1996). In case of asymmetric jacobian the positive definite is assessed by finding the positive definite of a new matrix B which is equal to the existing matrix  $(A+A^T)/2$ . The advantages of stochastic process models in presence of multiple solutions is highlighted in Watling (1996). The advantages are enumerated as:

- The need for a unique solution is overridden by the presence of a unique stationary distribution of flows on the various routes.
- It is mentioned that in case of strictly convex functions the user equilibrium overestimates the costs in comparisons with the costs computed by the day-day models.

#### 4.2 R-DSPM :- Mean -Variance cost for congested network:

In the current section, R-DSPM with strict capacity constraint as described in Chapter 3 is used for assessing the route choice in risk averse passengers. The learning process model is modified such that each individual in the disaggregate model makes his/her route choice based on not only the average costs experienced by them but also on the variance experienced on the route over the memory length period  $\Omega$ . It is to be noted that when a passenger  $i$  travels more than once along the route  $k^z$  within his/her memory length  $\Omega$  then equation 4.1a is applicable. In case the passenger travels the route only once or never within his/her memory length  $\Omega$  then equation 4.1b is applicable. Hence the cost of each risk averse passenger  $i^z$  travelling between OD pair Z is given as

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z + \beta \text{var} \begin{pmatrix} t_{k^z}(i^z)^{z\Omega-1} \\ \vdots \\ \vdots \\ t_{k^z}(i^z)^{z\Omega-1} \end{pmatrix} \forall k^z = f_{i^z}^z, k^z \in n^z \quad (4.1a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = g_{k^z}(i^z)^z \forall k^z \neq f_{i^z}^z, k^z \in n^z \quad (4.1b)$$

Where

$$g_{k^z}(i^z)^z = \sum_{j=1}^{\beth} \omega_j T_{k^z}(i^z)^{z^{\Omega-j}} \quad \forall k^z = \{1^z, 2^z, \dots, n^z\} \quad (4.2)$$

Where  $\beta$ - non-negative parameter which represents the degree to which the variance is undesirable to passengers (Jackson and Jucker, 1982).

$t_{k^z}(i^z)^z$ - experienced travel time along route  $k^z$  by passenger  $i^z$  between OD pair Z

$\omega$  -weight associated with each day of the memory length  $\beth$ .

$T_{k^z}(i^z)^z$ - updated travel time for passenger  $i^z$  on route  $k^z$  between OD pair Z.

$\Omega$ - current simulation day.

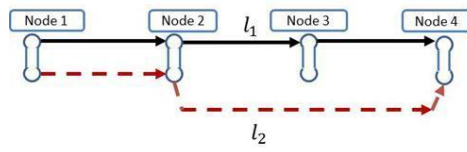
$n^z$ -total number of routes between OD pair Z.

It is to be noted that the obtained variance is only the variance of experienced travel cost of each individual. Hence if in a memory length  $\beth$  of 2 days a passengers  $i^z$  travels a route on day 1 and doesn't travel the same route on day 2 the variance is considered as zero in-spite of the passenger updating his/her experience cost matrix for the untraveled route on day 2 with the uncongested cost of that route. With the above mentioned modification the R-DSPM with strict capacity constraints is run for risk averse passengers under congested condition.

#### 4.2.1 Implementation on example networks:

The R-DSPM was run for example network 2 (Fig 4.3) under the congested demand given in fig 4.3. A  $\beth$  value of 15 days,  $\beta = 2.5$  was chosen and the simulation was run for 700 days. The initial 200 days were discarded as the burn-in period.

The network in-vehicle travel time and their frequencies are set as shown in fig 4.3. The demand matrix shown in the figure is that for the congested network.

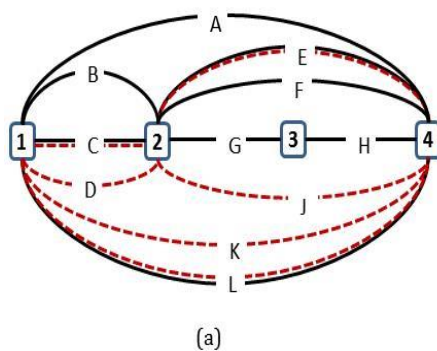


Node 4			
Uncongested		Congested	
Population Size for 1 hr	Poisson rate of passenger arrivals/hr	Population Size for 1 hr	Poisson rate of passenger arrivals/hr
Node 1	21	465	400
Node 2	24	307	250

Line Number	Line Color	In - Vehicle Travel Time in min	From Stop – To Stop	Buses per hour
$l_1$	Black	15	1-2	8
$l_1$	Black	20	2-4	8
$l_2$	Red	10	1-2	10
$l_2$	Red	12	2-4	10

Fig 4.3 Example network 2 (Teklu, 2008b)

The route sections and routes available for travel between the OD pairs in example network 2 are shown in Fig 4.4.



OD Pair	Route Number	Route Sections
Node 1 - Node 4	1	A
	2	K
	3	L
	4	B+J
	5	C+E
	6	C+F
	7	C+J
	8	D+F
Node 2 - Node 4	9	F
	10	J
	11	E

(b)

Fig 4.4 (a) All the route sections for example network 2 and (b) possible routes

The De Cea and Fernández (1993)'s route section approach assumes that the common lines exist between transfer stops, hence for example network 2, stop 2 is a transfer stop. In such a situation for routes 5,6,7 the passengers have to transfer at stop 2 though such a transfer on these routes would imply getting down from a transit service line and getting on another or possibly the same line. In real world alighting and boarding the same line may not seem realistic hence in this chapter an analysis is made with two possible scenarios.

Scenario 1: all the passengers on route 5,6,7 make a transfer at transit stop 2.

Scenario 2: the passengers on 6 and 7 do not alight at stop 2 if the line they choose to board at stop 1 continues till stop 4 and route 5 passengers who choose line 1 at stop 1 alight to board line 2 and vice versa at the transfer stop 2.

From Table 4.4 and table 4.5 one can see that at lower  $\xi$  the passengers route themselves such that in both the scenarios the average total travel time experienced on each route remains similar irrespective of the network being risk neutral or risk averse. Table 4.6 and table 4.7 shows the results for  $\xi=4$  and it is seen that at higher  $\xi$  values all risk neutral passengers find a particular route attractive and route themselves onto that route (as already discussed in chapter 3). The increased flow on a route results in higher experienced total travel time and due to this almost all the passengers on the subsequent day end up choosing a different route. This generalises the result obtained in chapter 3 for example network 1 wherein it was shown that at higher  $\xi$  values a network with risk neutral passengers sees periodic attractors and the flow distribution on attractive routes has higher variance. On some of the routes (scenario 1 - routes 1,4,6,8 (table 4.7) and scenario 2 - routes 1,8 (table 4.6)) risk neutral passengers find the uncongested costs to be higher than the experienced costs on the attractive routes for a particular day and hence these routes are not assigned any flows though the transition probability for these routes are greater than zero.

In both scenarios it is observed that the risk averse passengers at higher  $\xi$  values learn from the higher variance associated with all flows choosing the same route. Due to this learning process they tend to route themselves such that the flows are now assigned onto the routes which were found to be unattractive by risk neutral passengers. Fig 4.5 asserts this finding by showing the evolution of costs during burn-in period and the stationary probability distribution of flows along routes 9, 10 and 11 for risk neutral and risk averse passengers for a  $\xi$  value of 4 (scenario 2). In fig 4.5 it is seen that during the burn-in period the risk averse passengers learn from experiencing higher variance in initial days and after day 150 have almost stable evolution of costs. This phenomenon is absent in risk neutral passengers as the oscillations in the costs are observed to be still higher in comparison to risk averse costs.

Table 4.4 The experienced total travel time and flow values on various routes for risk neutral and risk averse passengers at  $m = 1$ ,  $\xi=0.05$ ,  $\beta = 2.5$  and  $\gamma=15$  days (scenario 2)

Route	Node 1 – Node 4 (Z=1)								Node 2 – Node 4 (Z=2)			
	1	2	3	4	5	6	7	8	9	10	11	
Risk Neutral	$E(t_k)$	52.3	38.2	40.9	92.8	93.8	73	62.4	94.4	63.8	56.8	53.2
	Std	8.8	8.5	7.6	20.4	19.4	19	18.4	21.5	14.9	13.8	12.6
	$E(X_k)$	39.5	77.8	67.1	37.3	40.1	41.3	62.2	33	68.3	91.8	90.8
	Std	6.3	8.4	8.0	5.9	6.1	6.7	8	6.2	8.4	10.3	9.7
Risk Averse	$E(t_k)$	53.0	37.7	40.3	87.8	93.2	76.7	57.7	95.8	64.3	50.3	48.9
	Std	9.1	8.3	7.5	19.9	19.8	20.5	17.5	22	14.7	12.8	12
	$E(X_k)$	48.4	57.8	55.3	46.3	46.9	46.9	52.6	44.1	71.6	85.5	93.9
	Std	7.1	7.9	7.1	6.2	6.8	6.8	7.3	6.4	15.1	13.4	13.4

Table 4.5 The experienced total travel time and flow values on various routes for risk neutral and risk averse passengers at  $m = 1$ ,  $\xi=0.05$ ,  $\beta = 2.5$  and  $\gamma=15$  days (scenario 1)

Route	Node 1 – Node 4 (Z=1)								Node 2 – Node 4 (Z=2)			
	1	2	3	4	5	6	7	8	9	10	11	
Risk Neutral	$E(t_k)$	52.3	38.4	41.2	91.7	80.9	90.7	85.2	91.2	59.5	52.7	49.7
	Std	8.8	8.5	7.4	19.9	18.4	19.6	19.5	20.5	13.2	12.6	11
	$E(X_k)$	40	84.9	74.1	34.5	40.0	35.4	57.7	31.6	66.1	92.8	92.0
	Std	6.2	8.9	8.4	6.2	6.0	5.9	7.8	5.8	8.2	10.7	9.9
Risk Averse	$E(t_k)$	53.1	38	40.5	86.5	76.6	92	78.6	91.7	59.4	46.4	45.3
	Std	9.1	8.3	7.6	19.3	17.9	20.2	18.5	20.8	13.0	11.1	10
	$E(X_k)$	49	59.4	56.6	45.2	47.2	45.3	51.7	43.8	72.2	84.3	94.4
	Std	7.2	8.3	7.5	6.8	6.6	6.6	7.5	6.8	15.3	14.7	12.5

Table 4.6 The experienced total travel times and flow values on various routes for risk neutral and risk averse passengers at  $m = 1$ ,  $\xi = 4$ ,  $\beta = 2.5$  and  $\gamma = 15$  days (scenario 2)

Route	Node 1 - Node 4 (Z=1)							Node 2 - Node 4 (Z=2)				
	1	2	3	4	5	6	7	8	9	10	11	
Risk Neutral	$E(t_k)$	0	42.3	42.5	0.21	19.5	1.45	89.1	0	70	92.9	67.2
	Std	0	10.7	6.5	4.7	42.6	10.1	24.4	0	20.6	21.1	19.3
	$E(X_k)$	0	197.3	133.2	0.002	0.716	0.034	67.0	0	67.4	80.5	103.03
	Std	0	48.0	34	0.04	2.9	0.25	24.1	0	20.0	22.7	23.5
Risk Averse	$E(t_k)$	53.1	37.6	40.1	90.2	93.2	76.6	57.9	94.7	64.5	53.1	50.9
	Std	9.1	8.2	7.5	19.8	19.8	20.1	17.3	22.3	14.7	13.2	12
	$E(X_k)$	43.2	63.2	61.1	45.4	52.2	51.4	59.0	22.7	75.0	82	94
	Std	8.6	13.1	15.4	8.4	9.5	8.9	78	7.4	14.8	14.5	12.3

Table 4.7 The experienced total travel times and flow values on various routes for risk neutral and risk averse passengers at  $m = 1$ ,  $\xi = 4$ ,  $\beta = 2.5$  and  $\gamma = 15$  days (scenario 1)

Route	Node 1 - Node 4 (Z=1)							Node 2 - Node 4 (Z=2)				
	1	2	3	4	5	6	7	8	9	10	11	
Risk Neutral	$E(t_k)$	0	42.9	42.4	0	1.8	0	108.7	0	72.0	88.7	68.3
	Std	0	11.0	6.6	0	13.4	0	22.8	0	20.8	22.2	19.3
	$E(X_k)$	0	203.8	149.8	0	0.028	0	44.7	0	61.9	84.8	104.3
	Std	0	51.8	38.1	0	0.23	0	26.9	0	23.2	22.0	22.9
Risk Averse	$E(t_k)$	53.2	37.8	40.2	88.9	78.4	91.7	80.9	91.3	60.4	48.3	46.8
	Std	9.2	8.3	7.4	19.6	18.1	20	19.2	20.9	13.6	11.8	10.2
	$E(X_k)$	53.2	64.1	62	40	54.1	43.3	57.6	24.0	74.4	82.9	93.6
	Std	7.9	13.3	8.8	7.8	9.7	7.8	11.6	7.2	15.1	14.7	13.8

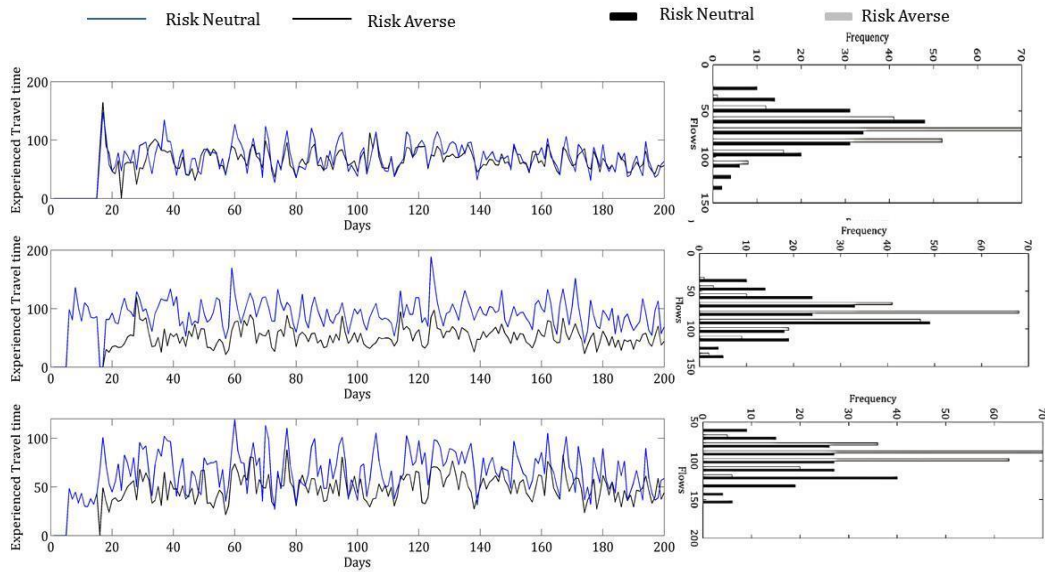


Fig 4.5: Evolution of costs during burn in period and stationary distribution of flows on routes 9,10,11 for risk neutral and risk averse passengers at  $m = 1$ ,  $\xi=4$ ,  $\beta = 2.5$  and  $\varrho=15$  (scenario 2)

The stationary distribution of flows (fig 4.5) also indicate that standard deviation of risk neutral flows on these routes is much higher than the standard deviation of risk averse flows.

#### 4.2.2 Presence of stationary distribution:

As discussed in Chapter 3 if the current disaggregate framework obeys the markovian property and results in a ergodic and regular distribution of flows it should fulfil the following conditions:

- The presence of a unique stationary distribution
- The convergence of the system to the same stationary distribution irrespective of its initial condition

In accordance with the markovian property's requirement the example network-2 was tested for the presence of a unique stationary distribution at higher  $\xi$  values for risk averse passengers. Fig 4.6 shows the distribution of flows between different days and indicates the presence of stationary distribution as noted by the similar mean and standard deviation of the flows for scenario 2. A similar analysis for scenario 1 shows the presence of almost the same stationary distribution of flows (fig 4.7) on routes 9,10,11.

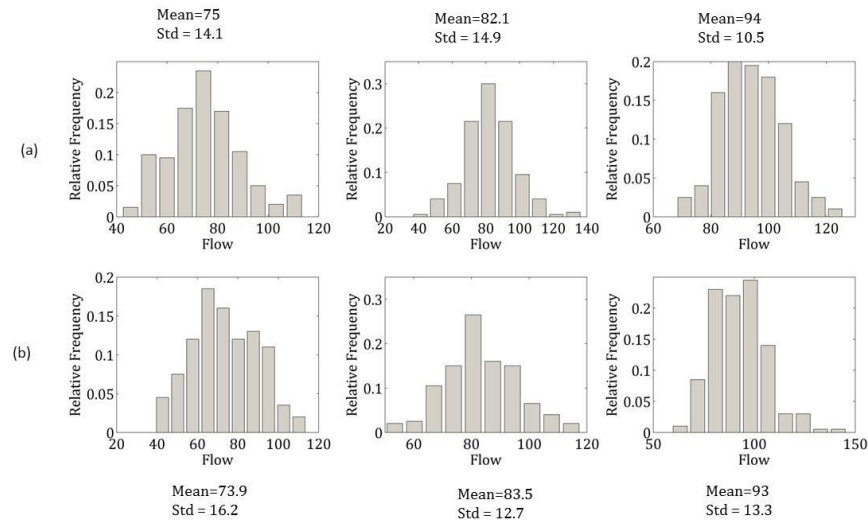


Fig 4.6 Stationary distribution for routes 9,10 and 11 (a) between days 201-400 (b) between days 401-600 at  $m = 1, \xi = 4, \beta = 2.5, \gamma = 15$  days (Scenario 2)

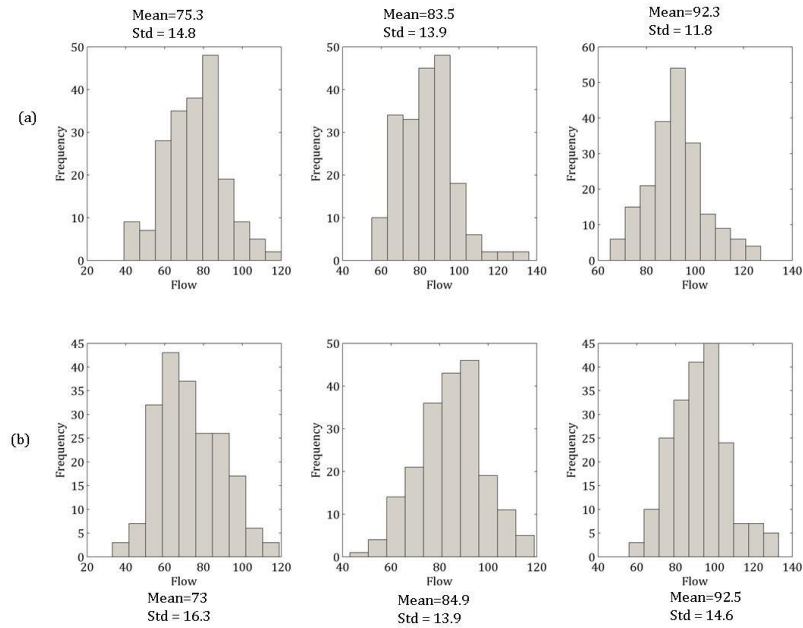


Fig 4.7 Stationary distribution for routes 9,10 and 11, (a) between days 201-400 (b) between days 401-600 at  $m = 1, \xi = 4, \beta = 2.5, \gamma = 15$  days (Scenario 1)

### 4.2.3 Initial Conditions

Similar to the analysis of risk neutral passengers in chapter 3 the convergence to the same distribution irrespective of its initial condition is checked for risk averse passengers (example network 2). The initial conditions were varied by varying the random number seed values of the R-DSPM framework (initial condition II) and by varying the rate of poisson arrivals and the population size between OD pairs for the first 80 days ( $Z=1$ - poisson rate of passenger arrivals-



400/3600, population size (constant demand)-83; Z=2-poisson rate of passenger arrivals-250/3600, population size (constant demand)-88) (Initial condition III). Table 4.8 shows the result of different initial conditions for scenario 2 and table 4.9 shows the result for scenario 1.

Table 4.8: Convergence to same distribution irrespective of its initial condition for risk averse passengers (example network 2) at  $m = 1$ ,  $\xi = 4$ ,  $\beta = 2.5$ ,  $\gamma = 15$  days (scenario 2)

Route		Node 1 - Node 4 (Z=1)								Node 2 - Node 4 (Z=2)		
		1	2	3	4	5	6	7	8	9	10	11
Initial condition II	$E(t_k)$	54.3	38.2	40.7	90.6	94.8	78.1	58.3	96.7	65.3	53.0	51.3
	Std	9.7	8.5	7.9	20.3	20.5	20.9	17.1	22.4	14.6	14.1	13
	$E(X_k)$	43.2	63.1	60.8	45.5	52.2	51.3	59	23.1	76.9	83.6	90.4
	Std	8.9	12.1	14.4	8.4	9.1	8.3	8.7	7.2	16.4	14	12.7
Initial condition III	$E(t_k)$	53.2	37.5	40.2	90.6	93.5	76.7	58	94.7	64.2	53.1	50.7
	Std	9.3	8.3	7.4	20	20	20.7	18	22.5	14.9	13.6	12.1
	$E(X_k)$	43.3	63.1	61	45.3	52.2	51.5	59.1	22.9	73.4	81.4	96.1
	Std	10.5	15.5	14.6	8.9	9.4	9.4	11.2	7.8	15.7	14.6	13.9

Table 4.9: Convergence to same distribution irrespective of its initial condition for risk averse passengers (example network 2) at  $m = 1$ ,  $\xi = 4$ ,  $\beta = 2.5$  and  $\gamma = 15$  days (scenario 1).

Route		Node 1 - Node 4 (Z=1)								Node 2 - Node 4 (Z=2)		
		1	2	3	4	5	6	7	8	9	10	11
Initial condition II	$E(t_k)$	54.3	38.2	40.9	89.8	79.1	93.3	81.5	92.7	60.7	48.4	47.2
	Std	9.6	8.3	8	19.7	19	20.3	19.6	20.7	13.1	12.0	10.9
	$E(X_k)$	53.1	63.8	61.7	40.3	54.1	43.1	57.7	24.5	75.7	84.1	91.1
	Std	8.1	12.9	8.7	8	8.7	7.9	11	7.1	15.7	13.3	12.7
Initial condition III	$E(t_k)$	53.3	37.7	40.3	88.9	78.2	91.8	80.8	91.1	60.2	48.3	46.8
	Std	9.2	8.2	7.5	19.5	18.1	20.1	19.1	20.8	13.6	11.6	10.3
	$E(X_k)$	53.1	64.1	62.1	39.9	54	43.2	57.9	23.9	74.6	82.6	93.7
	Std	9	14.4	8.5	7.6	9.3	8.3	12.5	7.3	16.2	15.4	13.7

The results of statistical test (Wilcoxon rank sum test) to check if the stationary distribution is from same distribution or not are shown in table 4.10 (scenario

2) and table 4.11 (scenario 1). Since the memory length is 15 days the sample for statistical tests contains every 20<sup>th</sup> element of  $E(t_k)$  and  $E(X_k)$ .

Table 4.10 Wilcoxon rank sum test for risk averse passenger (example network 2) at  $m = 1, \xi=4, \beta = 2.5$  and  $\mathfrak{N}=15$  (scenario 2)

Route		Node 1 - Node 4 (Z=1)							Node 2 - Node 4 (Z=2)			
		1	2	3	4	5	6	7	8	9	10	11
$E(t_k)$	1 vs 2	0.13	0.3	0.4	0.99	0.79	0.68	0.55	0.83	0.38	0.98	0.55
	2 vs 3	0.18	0.36	0.44	0.6	0.61	0.64	0.47	0.7	0.4	0.98	0.64
	1 vs 3	0.9	0.99	0.98	0.55	0.83	0.88	0.91	0.92	0.98	0.83	0.92
$E(X_k)$	1 vs 2	0.11	0.83	0.86	0.37	0.95	<b>0.02</b>	0.09	0.23	0.82	0.53	0.54
	2 vs 3	0.17	0.99	0.46	0.21	0.92	0.22	0.88	0.48	0.93	0.21	0.24
	1 vs 3	0.99	0.95	0.38	0.72	0.78	0.49	0.13	0.53	0.73	0.79	0.72

Table 4.11 Wilcoxon rank sum test for risk averse passenger (example network 2) at  $m = 1, \xi=4, \beta = 2.5$  and  $\mathfrak{N}=15$  (scenario 1)

Route		Node 1 - Node 4 (Z=1)							Node 2 - Node 4 (Z=2)			
		1	2	3	4	5	6	7	8	9	10	11
$E(t_k)$	1 vs 2	0.13	0.31	0.17	0.98	0.88	0.34	0.94	0.34	0.19	0.83	0.74
	2 vs 3	0.09	0.24	0.13	0.85	0.92	0.45	0.98	0.36	0.24	0.91	1
	1 vs 3	0.85	0.83	0.89	0.97	0.97	0.82	0.91	0.92	0.97	0.95	0.76
$E(X_k)$	1 vs 2	0.51	0.86	0.87	0.17	0.66	0.55	0.71	0.23	0.2	0.59	0.4
	2 vs 3	0.78	0.92	0.82	0.16	0.72	0.76	0.95	0.38	0.3	0.31	0.68
	1 vs 3	0.59	0.82	0.91	0.85	0.82	0.72	0.61	0.71	0.92	0.6	0.72

The table 4.10 and table 4.11 results show that the null hypothesis cannot be rejected at the 5% significance level as all the p-values are greater than 0.05 in all cases for  $m = 1$ . This shows that there is not sufficient evidence to show that the samples from the three realisations do not come from the same stationary distribution and do not have the same median.

Similar to Chapter 3 a two sample Kolmogorov-Smirnov test is carried out to assess if the two independent samples obtained by running different initial conditions are from same distribution or not. As indicated in chapter 3 the test is carried out using the 'kstest2' function in matlab wherein 0 indicates that the null hypothesis is true and 1 indicates that null hypothesis is rejected. The output of the test is given in table 4.12.

Table 4.12 Two sample Kolmogorov-Smirnov test for risk averse passenger (example network 2) at  $m = 1, \xi=4, \beta = 2.5$  and  $\gamma=15$  (scenario 2)

Route		Node 1 - Node 4 (Z=1)							Node 2 - Node 4 (Z=2)			
		1	2	3	4	5	6	7	8	9	10	11
$E(t_k)$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	0	0	0	0	0	0	0	0	0	0	0
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0
$E(X_k)$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	0	0	0	0	0	0	0	0	0	0	0
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0

Table 4.12 shows that the results of Wilcoxon rank sum test and the two-sample Kolmogorov-Smirnov test are almost similar with the implication that all the samples are from the same distribution.

#### 4.2.4 Randomness test

##### 4.2.4.1 Autocorrelation

The autocorrelation of the time series data are plotted to check the randomness of the generated data. As the correlation dies down with the lag in days it can be

said that the time series is indeed random. Fig 4.8 and fig 4.9 shows the autocorrelation diagram for the risk averse passengers and risk neutral passengers. It is observed that the autocorrelation dies down for  $\xi = 4$  at larger lag days for risk neutral passengers. At higher  $\xi$  the tendency to move towards periodic attractors is observed in risk – averse passengers as can be seen from the alternating negative and positive autocorrelations occurring over every 15 day memory length cycle. In case of route 2 (fig 4.8)in scenario 2 and routes 2 and 7 in scenario 1(fig 4.9) the periodic oscillations between the negative and positive autocorrelation decay down very slowly thereby indicating that a larger run time is required to get a random sample of flows for these routes.

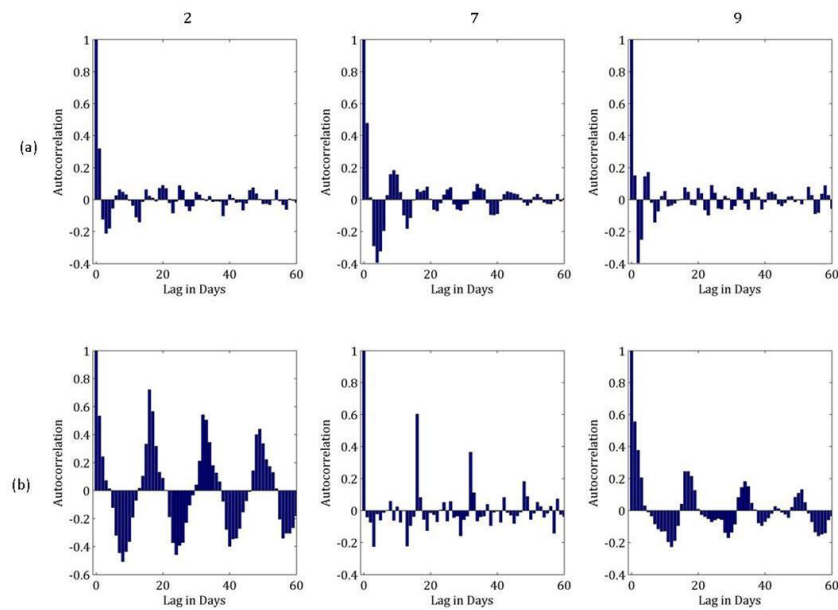


Fig 4.8 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 2) at  $m = 1, \xi = 4, \beta = 2.5$  and  $\gamma = 15$

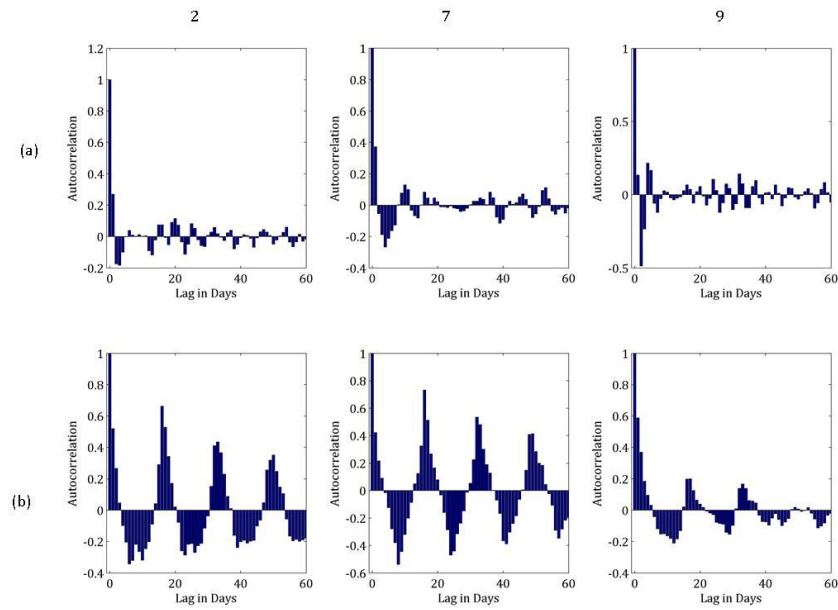


Fig 4.9 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 1) at  $m = 1, \xi = 4, \beta = 2.5$  and  $\varrho = 15$

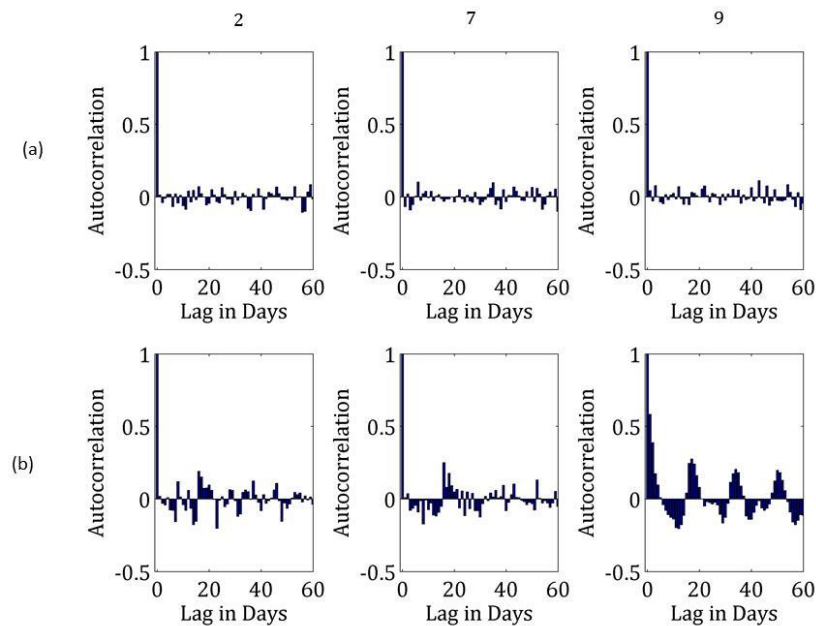


Fig 4.10 Autocorrelation of flows on routes 2,7,9(a) risk neutral (b) risk averse (scenario 2) at  $m = 1, \xi = 0.05, \beta = 2.5$  and  $\varrho = 15$

At lower  $\xi = 0.05$  (fig 4.10) one finds that again the risk averse passengers have the tendency to move towards periodic attractors especially on route 9 which caters to OD pair at the lower end transit stop of the example network 2. This is expected as at the lower end transit stop the transit services may already be full resulting in the lower end OD movement competing for space in the two line network. Due to this a larger number of passengers experience variance in travel times due to failure to board condition at the lower end transit stop (stop 2). Over larger lags the periodicity for risk averse passengers

appears to die down for routes 2 and 7 but decay at a slower rate for route 9. The results of the autocorrelation of flows for risk averse network indicate the presence of more than one attractor irrespective of the  $\xi$  value assumed for a  $\beta$  value of 2.5.

#### 4.2.4.2 Partial Autocorrelation

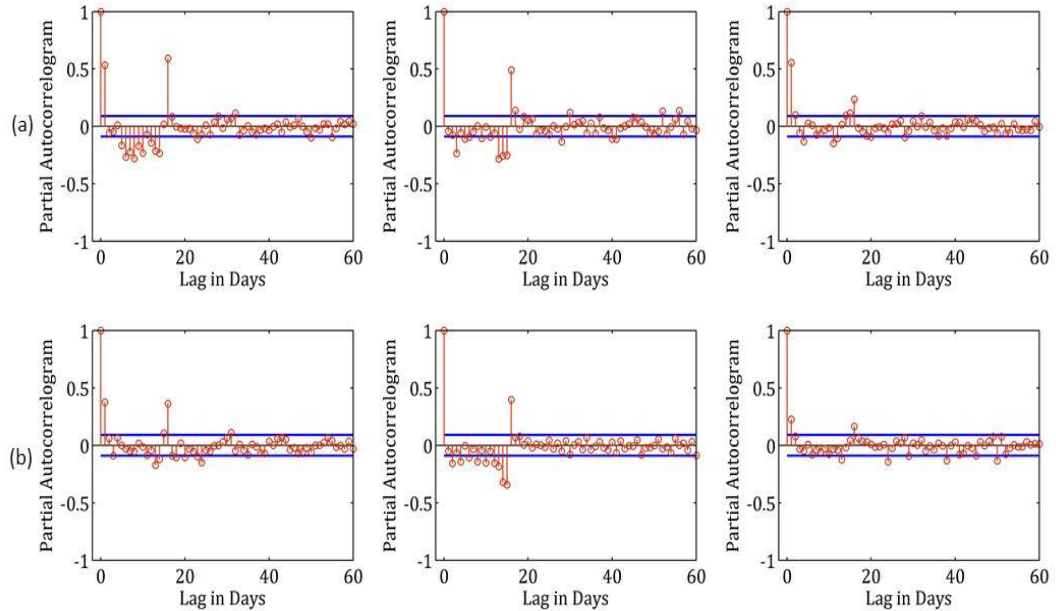


Fig 4.11 Partial Autocorrelation of flows on routes 2,7,9 with 95% confidence bounds (a)  $m = 1$ (b)  $m = 300$  (scenario 2) at  $\xi = 4, \beta = 2.5$  and  $\varrho = 15$ .

As highlighted in section 3.6.2 partial autocorrelation help identify the order of ARMA time series models. The partial autocorrelation of flows on routes 2,7, and 9 show a strong statistical significance within the memory period of  $\varrho = 15$  days. There is a strong significance on the 16<sup>th</sup> day after which the correlation are at borderline of statistical significance. The intermittent peaking of the positive correlation followed by negative correlations indicates the tendency of the model to move towards periodic attractors. Fig 4.11 indicate that the model exhibits persistent correlation within the memory period

#### 4.2.5 Failure to board:

The strict capacity constraint of the R-DSPM results in several passengers not being able to board the first arriving transit service of their choice set. This phenomenon is referred to as failure to board. Fig 4.12 indicates the number of passengers failing to board at various stops of the example network 2 on a randomly chosen day.

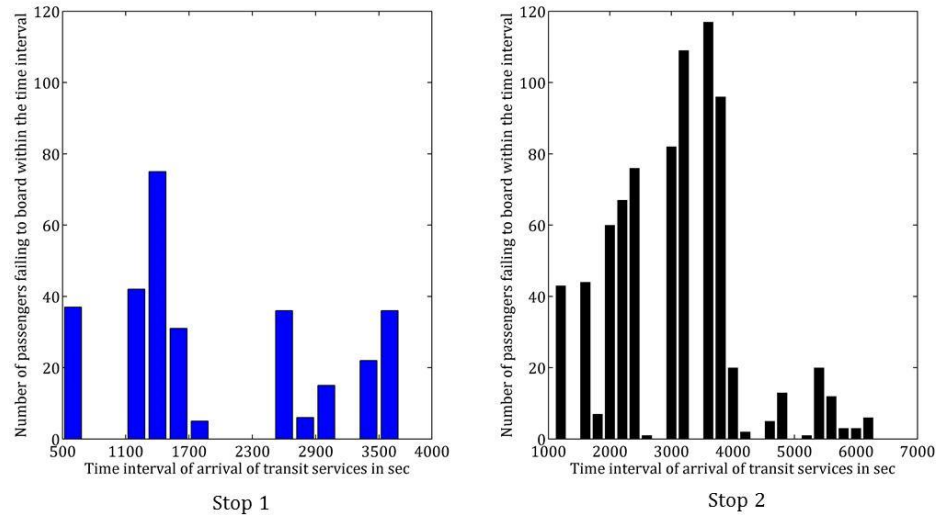


Fig 4.12: Number of passengers experiencing failure to board on day 450 at  $m = 1, \xi = 4, \beta = 2.5$  and  $\varrho = 15$ . (example 2 - scenario 2).

Figure 4.12 is generated by running the simulation for the duration of 1 hour and a buffer time till the last generated passenger reaches destination. On the departure of each transit service from a transit stop the number of passengers who find the exiting service attractive but are unable to board is counted. The count only includes the passengers who had arrived before the arrival of transit service of their choice. Hence if transit service 1 is the attractive service for 21<sup>st</sup> passenger in the queue at the transit stop; who happens to have arrived before the arrival of transit service 1; the passenger is counted as failure to board if all the previous 20 passengers fill the capacity of the transit service. If the passenger finds the next arriving transit service of his/her attractive line set to be full as well, the passenger is again counted as ‘failure to board’ and is included in the queue count for the arrived transit service. Figure 4.12 gives the arrival time of the transit service at the transit stop and the number of passengers failing to board the arrived service. Figure 4.12 indicates that within the simulation period several passengers are unable to board the transit service of their attractive line set. The built up of passengers at the transit stops indirectly implies these passenger experience an increased waiting time as a result of failure to board condition and hence find the service unreliable. As is expected the lower end transit stop (stop 2) has more number of passengers experiencing failure to board condition due to the transit services arriving almost full/full at stop 2. This results in OD movement 2 competing for space in the two lined network and hence experiencing more number of failure to board phenomenon.

#### 4.2.6 Sensitivity tests:

Since the conclusions derived from the results of both scenario 1 and scenario 2 are similar the following sections show only the sensitivity test results for scenario 2.

##### 4.2.6.1 Sensitivity analysis with Different shape factors:

As discussed in section 4.1.2 a shape factor of  $m = 300$  would result in a more reliable interarrivals. The current section shall compare the behaviour of risk averse and risk neutral passengers in a congested network with shape factor  $m = 300$ . It is found that similar to  $m = 1$  the lower  $\xi$  results in similar flow and experienced travel times between risk neutral and risk averse passengers. At higher  $\xi$  value the distribution of risk averse flows onto the routes found unattractive by risk neutral passengers is also observed. Only the result of  $\xi = 4$  is presented in Fig 4.13 which shows a comparison of experienced total travel time and flows on routes 2,7 and 9 for risk neutral and risk averse passengers. From the figure it can be observed that the risk neutral passengers find route 2 attractive and predominantly route themselves onto that route. Even though a large number of passengers choose route 2 the expectation of the experienced total travel time is lesser than the expectation of the experienced total travel time on route 7 which has comparatively lesser flows. In case of risk averse passengers we find that the flows assign themselves in such a way that the routes found unattractive by risk neutral passengers are also utilised and the standard deviation of the experienced total travel times and flows on various routes are lesser than those of risk neutral passengers.

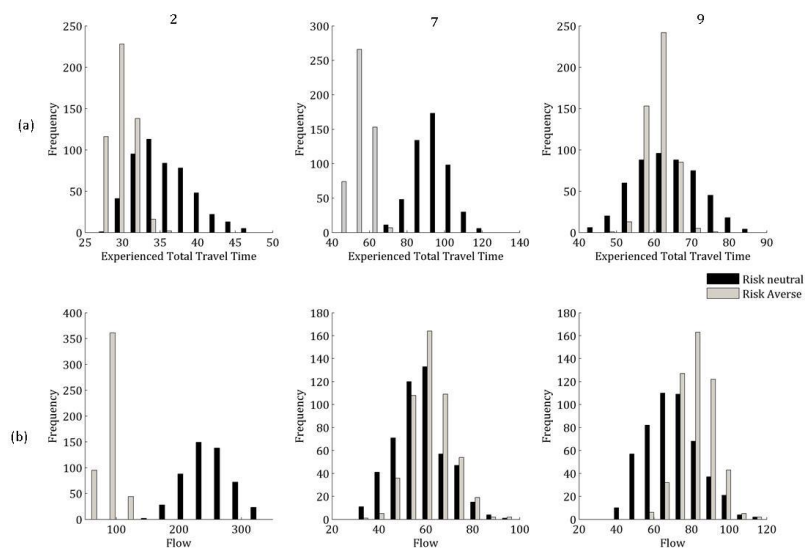


Fig 4.13: Risk neutral vs risk averse - shape factor  $m = 300$  at  $\xi = 4$  and  $\lambda = 15$  days (a) experienced route total travel times (b) flows on the route (scenario 2)



#### 4.2.6.2 Sensitivity analysis with different memory lengths

The sensitivity of the model to varying memory lengths ( $\tau$ ) was tested. Memory lengths ( $\tau$ ) of 5 days, 15 days and 30 days were used to test the difference in the flow and total experienced travel time distribution between risk neutral and risk averse passengers. Fig 4.14 shows the result of the comparison of total experienced total travel time between the risk neutral and risk averse passengers assuming  $\xi = 4$ . It is observed that as the memory length increases the standard deviation of the total experienced travel times of risk averse passengers reduces (Table 4.13). On the other hand the standard deviation of risk neutral passengers remains almost similar. This is because at higher  $\xi$  almost all the risk neutral passengers travel on the same route on a particular day leading to high average experienced total travel times. However this phenomenon is absent in risk averse passengers as when the memory length increases they learn about the variance associated with all possible routes and hence assign themselves such that the routes found unattractive by risk neutral passengers becomes attractive to risk averse passengers.

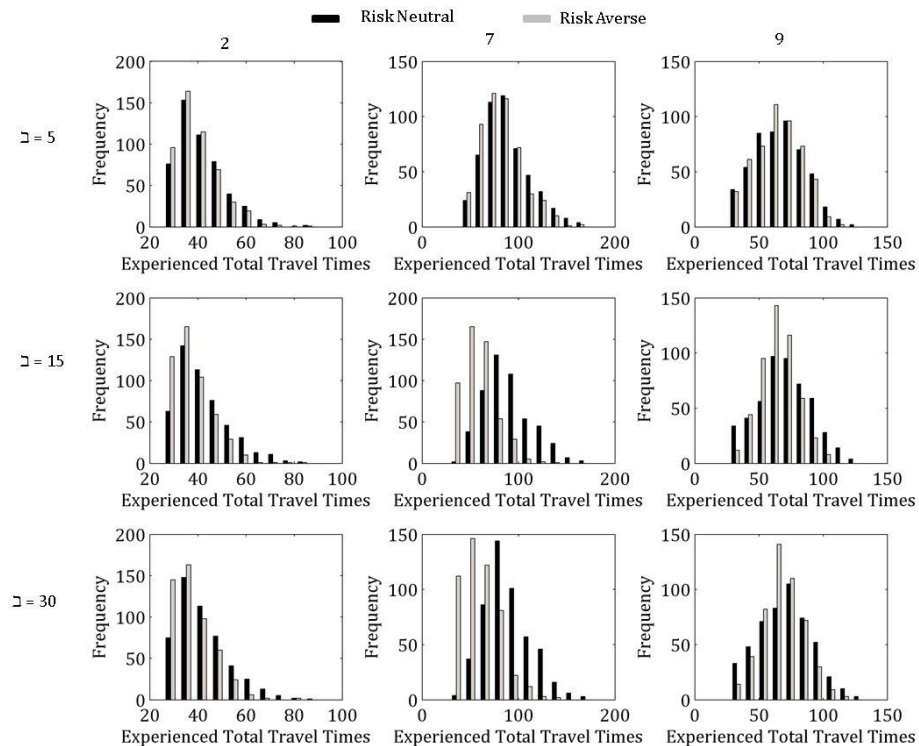


Fig 4.14: Risk neutral vs risk averse ( $\beta = 2.5$ ) – shape factor  $m = 1$  at  $\xi = 4$  total experienced travel time distribution for various memory lengths on route 2, 7, 9 (scenario 2)

Table 4.13 : Experienced total travel time and flows along routes 2, 7,9 for various memory length (scenario 2)  $m = 1$  at  $\xi = 4$

		Risk Averse ( $\beta = 2.5$ )			Risk Neutral ( $\beta = 0$ )			
		Route	2	7	9	2	7	9
$\kappa = 5$	$E(t_k)$		39.7	82.3	64.5	41.6	88.1	66.0
	Std		9.1	21.8	18	10.2	24.1	19.8
	$E(X_k)$		147.6	94.3	73.2	187.6	89.2	67.9
	Std		18	18.1	13.3	39.0	17.6	18.1
$\kappa = 15$	$E(t_k)$		37.6	57.9	64.5	42.3	89.1	70
	Std		8.2	17.3	14.7	10.7	24.4	20.6
	$E(X_k)$		63.2	59.0	75.0	197.3	67	67.4
	Std		13.1	8.8	14.8	48.0	24.1	20
$\kappa = 30$	$E(t_k)$		37.6	59.8	68.5	42.3	88.9	70.2
	Std		8.4	19.6	15.8	10.7	24.1	20.6
	$E(X_k)$		81.7	36.3	63.3	197.5	66	66.9
	Std		14.1	15.0	14.2	47.8	23.4	20.2

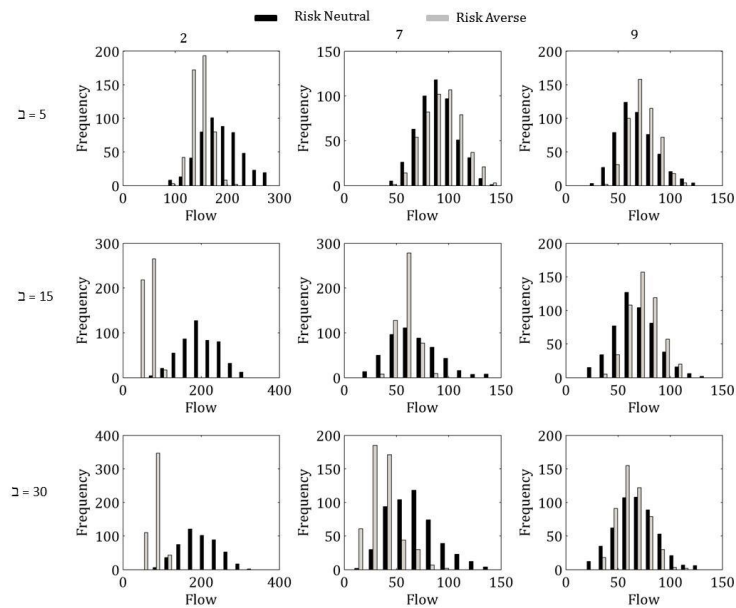


Fig 4.15: Risk neutral ( $\beta = 0$ ) vs risk averse ( $\beta = 2.5$ ) – shape factor  $m = 1$  at  $\xi = 4$  flow distribution for various memory lengths on route 2, 7, 9 (scenario 1)

A similar observation of an increase in the memory length resulting in risk averse passengers routing themselves such that the standard deviation of the flows on the routes are lesser than that of risk neutral passengers (fig 4.15) is

observed for scenario 1. It is also seen that irrespective of the scenario adopted at higher memory lengths the routes found unattractive by risk neutral passengers are found attractive by risk averse passengers (section 4.2.1).

#### 4.2.6.3 Sensitivity analysis with different $\beta$ values

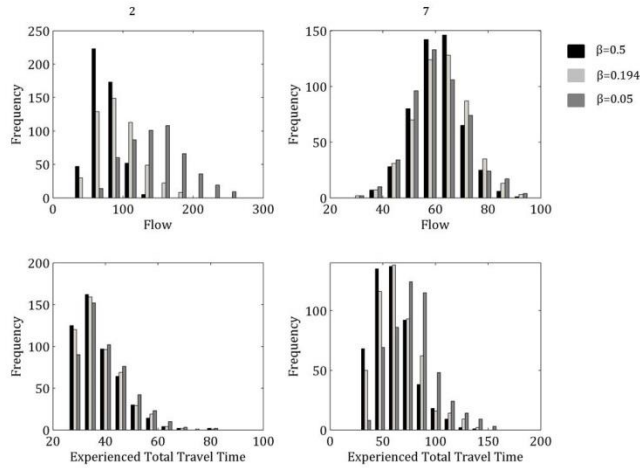


Fig 4.16 The sensitivity of route experienced total travel times and flows to various  $\beta$  values for  $\xi = 4, m = 1, \zeta = 15$  (scenario 2)

Table 4.14: The sensitivity of route experienced total travel times and flows to various  $\beta$  values for  $\xi = 4, m = 1, \zeta = 15$  (scenario 2)

	$\beta$ Values	Route 2		Route 7	
		Mean	Std	Mean	Std
Experienced total travel time	0.5	37.7	8.4	60.9	18.7
	0.194	38.1	8.6	65.3	20.7
	0.05	39.7	9.2	77.9	22.6
Flow	0.5	75	18.4	61.3	9.3
	0.194	93.4	29.9	62.4	10.5
	0.05	145.6	41.2	60.9	10.8

Fig 4.16 (table 4.14) shows the sensitivity of various  $\beta$  values for  $\xi = 4$  and  $\zeta=15$  days. The fig and table indicates that as the  $\beta$  values increase the standard deviation of the experienced total travel times and the flows along various routes decrease. A similar trend is visible at lower  $\xi$  values. However the distinction in the mean values of the flows (fig 4.17 and table 4.15) between various  $\beta$  values happens only when the  $\beta$  values are significantly different from each other. Another aspect to be noted at lower  $\xi$  values is that there is

still a difference in the route flow distribution between risk averse and risk neutral passengers at higher  $\beta$  values.

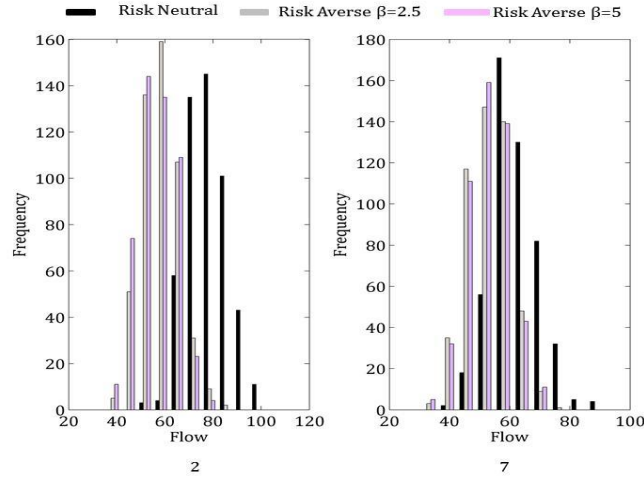


Fig 4.17 The sensitivity of route flows to various  $\beta$  values for  $\xi = 0.05, m = 1, \gamma = 15$  (scenario 2)

Table 4.15: The sensitivity of route flows to various  $\beta$  values for  $\xi = 0.05, m = 1, \gamma = 15$  (scenario 2)

$\beta$ Value	Route 2		Route 7	
	Mean	Std	Mean	Std
0	77.8	8.4	62.2	8
2.5	57.8	7.9	52.6	7.3
5	56.4	7.8	52.5	7.2

#### 4.2.7 Implications for general networks

The above analysis indicates that the behaviour of risk averse network will vary from risk neutral network under several conditions. At lower  $\xi, \beta$  and  $\gamma$  values the distinction in the flow distribution between risk neutral passengers and risk averse passengers is not significant and can be said to be almost similar. At higher  $\beta$  values with lower  $\xi, \gamma$  values, the difference becomes more pronounced. Determining if a  $\beta$  value is high enough to be able to observe a significant difference in route flows is an aspect which requires further research. It may be deduced that a high enough value of  $\beta$  depends on the network characteristics namely the number of available routes between an OD pair; the amount of variance associated with the network; the transfer penalty if any assumed for each transfer in the network; the socio-economic background of the transit users. The current study indicates that under certain

assumed parameter for the considered example network a significant difference in the risk averse and risk neutral flows is observed for higher  $\xi$ ,  $\beta$  and  $\gamma$  values.

### **4.3 Risk aversion models using BPR cost function, effective frequency cost function to account for congestion and mean-variance based aggregate stochastic process model.**

Section 4.2 dealt with the mean-variance analysis using R-DSPM with strict capacity constraints. The mean-variance cost used in section 4.2 was a linear combination of mean total travel time and variance associated with the experienced total travel time. In the current section an equilibrium based approach using route section based BPR cost function; hyperpath based effective frequency formulation; and aggregate stochastic model formulation shall be explored which will help establish the advantages of using R-DSPM with strict capacity constraint over the existing theoretical models. Sub section 4.3.1 shall implement a logit SUE to obtain the flows along various routes wherein the components of arc cost function are detailed and are similar to the ones proposed by Szeto et al. (2011). Sub section 4.3.2 shall implement a DUE based hyperpath formulation wherein the concept of effective frequency is used to deal with congested transit network. It is however noted that the non-additive nature of the mean-variance cost function (using standard deviation) makes a hyperpath based approach complex and shall require a separate study to make the cost function additive or may require a solution algorithm which solves for non-additive cost function. Example network 2 in the current thesis proves an ideal example to implement the mean-variance cost function as the current cost function uses variance instead of standard deviation and variance remains additive.

#### **4.3.1 Stochastic equilibrium model with BPR – type congestion function-Route section approach:**

In this section an insight into a BPR cost function based mean-variance logit SUE is given. The initial step involves computation of the cost function. The cost function of each route section comprises of various components and is highlighted below. The variance calculation for various components of cost function is similar to Szeto et al. (2011) excepting the in-vehicle travel time .

1. In-vehicle travel time:

Unlike Szeto et al. (2011) the in-vehicle travel time in the current research is considered constant. The variance of in-vehicle travel time arises in case of

route sections which consist of more than one line in their attractive line set. In such a case the variance is the result of the probability associated with which of these lines serves the transit stop first. Analytically the expected in-vehicle travel time for route section  $s$  ( $E[T^{in-veh}]_s$ ) is given as in eq(4.3).

$$E[T^{in-veh}]_s = \int_0^\infty \sum_{l \in A^*} a_l p_l(w) \prod_{\substack{i \in A^* \\ i \neq l}} \bar{P}_i(w) dw \quad (4.3)$$

Where  $a_l$ - is the in-vehicle travel time of line section  $l$ ;  $\bar{P}_i(w)$ - complementary cumulative distribution of the waiting time associated with transit services forming the attractive line set;  $p_l(w)$ - probability density function of the line arriving first at the transit stop;  $A^*$ - attractive line set;  $T^{in-veh}$ - in-vehicle travel time.

In case of exponential inter-arrivals the above equation reduces to

$$E[T^{in-veh}]_s = \sum_{l \in A^*} a_l Y_{jp}^l \quad (4.4)$$

Where

$$Y_{jp}^l = \frac{\varphi_l}{\sum_{l \in A^*} \varphi_l} \quad (4.5)$$

Wherein  $\varphi_l$ - frequency of the line section  $l$  and  $Y_{jp}^l$ - choice probability of line section  $l$  amongst the attractive line set between nodes  $j$  and  $p$ . It is known that

$$\text{Var}[T^{in-veh}]_s = E[T^{in-veh^2}]_s - E[T^{in-veh}]_s^2 \quad (4.6)$$

Hence for exponential inter-arrivals:

$$\text{var}[T^{in-veh}]_s = \sum_{l \in A^*} \frac{\varphi_l}{\sum_{l \in A^*} \varphi_l} \left( a_l - \sum_{l \in A^*} a_l \frac{\varphi_l}{\sum_{l \in A^*} \varphi_l} \right)^2 \quad (4.7)$$

## 2. The uncongested waiting time:

The average waiting time associated with being able to board the first arriving transit service in his/her attractive line set is derived in Chapter 2. Furthering the derivation of average waiting time, the variance of the waiting time is computed as given in eq 4.2:

$$E[W]_s = \frac{1}{\sum_{l \in A^*} \varphi_l} \quad (4.8)$$

$$\text{var}[W]_s = \left( \frac{1}{\sum_{l \in A^*} \varphi_l} \right)^2 \quad (4.9)$$

3. The congested waiting time :

The BPR function given in De Cea and Fernández (1993) and discussed in chapter 2 gives the excess waiting time due to congested condition. Eq (4.3) gives the formulation specified in Chapter 2

$$w_s = \zeta_s \left( \frac{\bar{v}_{ps} + v^s}{Cap_s} \right)^b \quad (4.10)$$

Where  $Cap_s$  – is the capacity of the route section which is defined as  $Cap_s = \sum_{l \in s} c_l \varphi_l$  with  $c_l$  and  $\varphi_l$  being the capacity and frequency of line section  $l$ ,  $\varphi_s$  – is the frequency of the route section  $s$  and  $\zeta, b$  are the calibration parameters,  $v^s$  the total number of passengers boarding the same route section  $s$  at the origin,  $\bar{v}_{ps}$  the number of passengers boarding the route section  $s$  before node  $\hat{l}(s)$  and alighting after  $\hat{l}(s)$ .

As already described  $\varphi_l$  is a random variable as the headway  $H_l$  associated with each line is a random variable. This makes  $Cap_s$  a random variable. Assuming that  $c_l$  is the same for all line sections i.e  $c_l = c$  the expected value of the BPR function given in eq(4.10) becomes

$$E[w]_s = \zeta_s \left( \frac{\bar{v}_{ps} + v^s}{c\kappa} \right)^b E[(h_s)^b] \quad (4.11)$$

$$E[w]_s = \zeta_s b! \left( \frac{\alpha(\bar{v}_{ps} + v^s)}{c\kappa\varphi_s} \right)^b \quad (4.12)$$

Where  $\alpha$  and  $\kappa$  are unit conversion factors.  $h_s$  is the combined headway of all the line sections within the route section  $s$ .

The variance associated with BPR function of excess waiting time is as given as Szeto et al. (2011):

$$Var[w]_s = \zeta_s^2 ((2b)! - (b!)^2) \left( \frac{\alpha(\bar{v}_{ps} + v^s)}{c\kappa\varphi_s} \right)^{2b} \quad (4.13)$$

Using the above mentioned components the cost of each route section was computed as follows.

$$C_s = E[T^{in-veh}]_s + E[W]_s + E[w]_s + \beta \left( var[T^{in-veh}]_s + var[W]_s + Var[w]_s \right) \quad (4.14)$$

The parameters  $\zeta_s, E[W]_s$  and  $b$  were calibrated by running the R-DSPM. The calibration was done by running the model at various demand levels for which different demand rates were specified. The rates were chosen such that there is

at least one day within the simulation period when all the demand between each OD pair chooses to travel. Each simulation run is made for 700 days of which the first 200 days are discarded as the burn-in period and the average waiting times associated with each route section at the end of each day is obtained and plotted. After each simulation run we get 500 days (500 data points) of average waiting times for each route section. The volume was determined by ascertaining the flow utilising a route section (including the passengers who complete their journeys in the buffer time) and the capacity was determined by counting the number of transit services that arrive at the transit stop within the simulation period of 4 hours.

The passengers are allowed to complete the journey during the buffer time. Buffer time is defined as the time period wherein the transit services get generated but the passengers are not generated. The capacity of the route section consists of only the transit services generated within the specified simulation period (in the current case 4 hours). These criteria for obtaining the volume and capacity results in the volume/capacity ratio exceeding 1 as shown in Fig 4.18. However it is to be noted that each transit service generated has a strict capacity constraint of 20 passengers. Fig 4.18 shows the calibrated function fitted onto the simulated data for example network 2. The route sections originating at the lower end transit stops see a slightly steeper curve indicating that the waiting time required for a flow by capacity ratio is higher than at the transit stop where line originates. It also implies the need to use different parameters for different sections of the same line.

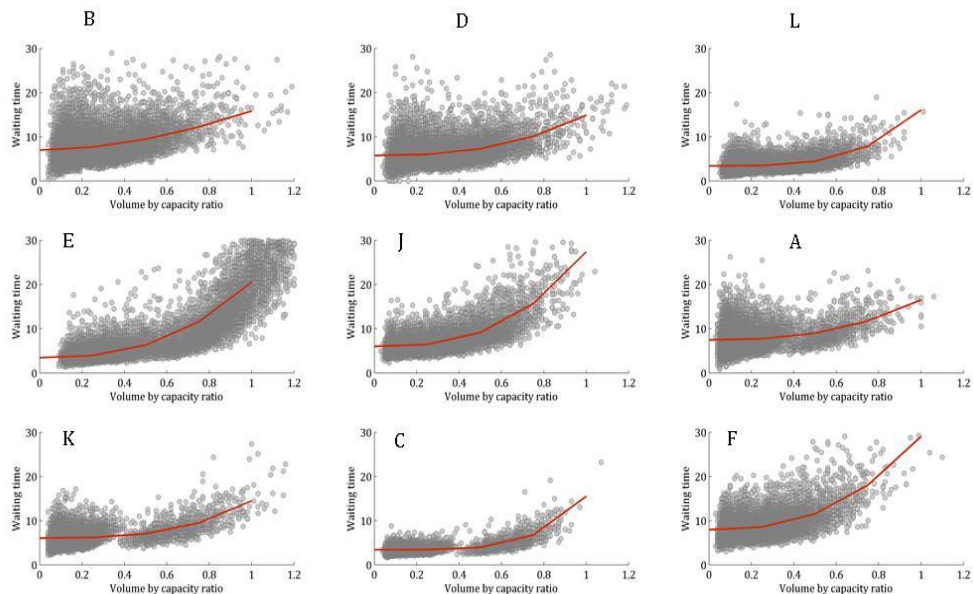


Fig 4.18 : Calibration of the parameters of the route sections of example network 2.



Table 4.16 : The calibration values of the parameters of various route sections of example network 2

Parameters	B	D	L	E	J	A	K	C	F
$E[W]_s$	6.979 (0.054)	5.755 (0.04)	3.403 (0.015)	3.456 (0.056)	5.987 (0.036)	7.463 (0.035)	6.056 (0.021)	3.415 (0.009)	7.928 (0.045)
$\zeta$	8.853 (0.222)	9.098 (0.225)	12.663 (0.414)	17.036 (0.093)	21.393 (0.33)	9.032 (0.442)	8.419 (0.485)	12.088 (0.255)	21.101 (0.549)
$b$	1.837 (0.055)	2.588 (0.073)	3.617 (0.074)	2.582 (0.014)	2.775 (0.042)	2.567 (0.123)	3.102 (0.087)	4.479 (0.073)	2.545 (0.055)

The parameters calibrated as given in table 4.16 are used in the BPR styled logit stochastic equilibrium model. The logit-SUE algorithm using BPR cost function specified above is as follows:

1. Initialisation: assume initial route section costs  $C^0$  route flows  $F^0$  and  $i = 1$ .
2. Route Choice : For each OD pair, find the auxiliary route flows vector  $\hat{F}^i$  by using the logit function

$$\frac{e^{-(\xi C_s)}}{\sum_{r=1}^S e^{-(\xi C_r)}}$$

3. MSA: set the route flows for iteration  $i$  and update the flows

$$F^i = F^{i-1} + \frac{(\hat{F}^i - F^{i-1})}{i}$$

4. Update Costs: Obtain the revised route section costs and set  $C^{i+1}$
5. Set counter  $i = i + 1$ , if  $i > \Gamma'$ , the maximum number of iterations, STOP , otherwise go to 2.

The equilibrium model was run for 3000 iterations. Table 4.17 shows the proportion of flows between each OD pair for the logit-SUE model with  $\xi=0.5 \cdot 10^{-8}$  and 4 and  $\beta= 2.5$ , a demand of OD1 =400 and OD2=250. The results show that most of the flows in case of risk neutral passengers at a low  $\xi$  value are split equally between the available routes. In case of risk averse passengers the flow is split between route 1 and 2 for OD1 and between 9 and 10 for OD 2. A similar analysis with a  $\xi$  value of 4 was carried out. At a higher  $\xi$  of value 4 there is not much difference in the proportion of flows on route be it risk averse or risk neutral passenger.

Table 4.17: Logit –SUE results for example network 2 using BPR –styled cost function for OD 1 – 400 and OD 2- 250.

$\xi$	Route	Node 1 – Node 4 (Z=1)							Node 2 – Node 4 (Z=2)			
		1	2	3	4	5	6	7	8	9	10	11
$0.5 \cdot 10^{-8}$	$\beta=0$	0.13	0.13	0.13	0.13	0.12	0.12	0.12	0.12	0.34	0.33	0.33
	$\beta=2.5$	0.5	0.49	0	0.01	0	0	0	0	0.63	0.37	0
0.001	$\beta=0$	0.44	0.41	0	0.07	0	0	0	0.08	0.58	0.42	0
	$\beta=2.5$	0.55	0.45	0	0	0	0	0	0	0.56	0.44	0
4	$\beta=0$	0.54	0.46	0	0	0	0	0	0	0.58	0.42	0
	$\beta=2.5$	0.55	0.45	0	0	0	0	0	0	0.56	0.44	0

An intermediate  $\xi$  value of 0.001 shows a slight distinction in the proportion of risk neutral and risk averse flows choosing each route. The above analysis indicates that since a logit-SUE doesn't comprise of a learning process the risk averse passengers route themselves similar to risk neutral passengers at higher  $\xi$  values however in R-DSPM (section 4.2) the risk averse passengers learn from their experiences of higher variance and thereby route themselves onto the routes which were found to be less attractive to the risk neutral passengers.

#### 4.3.2 Deterministic user equilibrium with 'effective frequency' function - hyperpath approach:

Example network 2 is tested for hyperpath based 'effective frequency' type modelling of congested transit network. As described in chapter 2 effective frequency concept was utilised in hyperpath context by Cepeda et al. (2006). The 'effective frequency' of each line segment is given as shown in eq 4.15:

$$\lambda'_a(v) = \begin{cases} \varphi_a \left[ 1 - \left( \frac{v_a}{\varphi_a c - v_a' + v_a} \right)^\ell \right] & \text{if } v_a' < \varphi_a c \\ 0 & \text{otherwise} \end{cases} \quad (4.15)$$

Where  $v_a$  –is the flow boarding the line segment a at the transit stop and  $v_a'$  is the flow immediately after the transit service leaves the transit stop.  $\varphi_a$  – nominal frequency of the line segment a  $c$ - capacity of each transit service of line segment a  $\lambda'_a$  – effective frequency of the line segment a. Waiting time of the line segments are considered as inverse of 'effective frequencies'.

Similar to the previous section the parameter  $\ell$  is calibrated by running R-DSPM for several demand values and finding the corresponding waiting time of each line segment at the origin stop. The calibration results are shown in Table

4.18 and the fit of the calibrated values with the simulated data is shown in fig 4.19.

The calibrated values indicate that the  $\beta$  values cannot be kept constant for a particular line and that for the same line it varies at different segments. The calibrated values were used to solve the deterministic equilibrium model using the hyperpath approach for the example network 2.

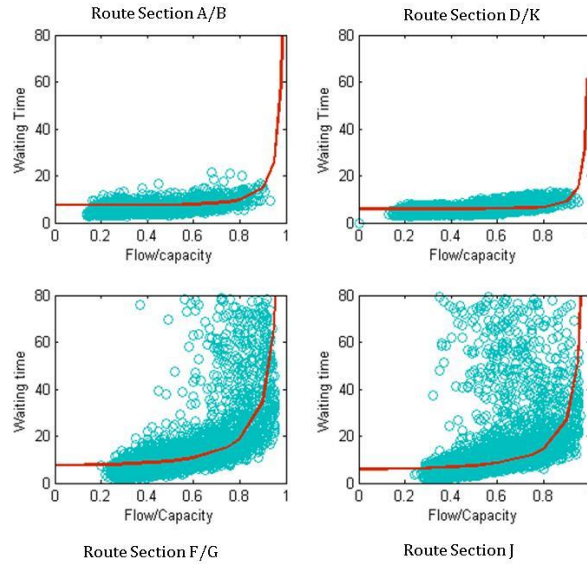


Fig 4.19: Calibration of line segments for the example network 2

Table 4.18: Calibration values of the parameters associated with example network 2.

Line segment	1	2	3	4
$\beta$	6.678	10.222	2.32	2.406
(standard error)	0.213	0.341	0.04	0.067

The deterministic hyperpath based user equilibrium for risk neutral and risk averse passengers

1. Initialisation: assume initial line segment flows  $F^0$  and  $i = 1$ .
2. Compute the line-segment effective frequency
3. Determine the shortest hyperpath.
4. Find the auxiliary line segment flows vector  $\hat{F}^i$  by AON on the shortest hyperpath.
5. MSA: set the line segment flows for iteration  $i$  and update the flows

$$F^i = F^{i-1} + \frac{(\hat{F}^i - F^{i-1})}{i}$$

6. Set counter  $i = i + 1$ , if  $i > \Gamma'$ , the maximum number of iterations, STOP, otherwise go to 2.

Table 4.19 : Distribution of risk neutral passenger flows

OD	Line segment	1	2	3	4
OD1-150	Flows	66.74	83.26	84	166
OD2-100					
OD1-400	Flows	200	200	325	325
OD2-250					

The deterministic hyper path model is run in case of risk averse passengers and resulted in the flow distribution as given in Table 4.20. The risk aversion in passengers is accounted for by adding the variance associated with the waiting time (inverse of effective frequency) and the variance associated with in-vehicle travel time at each node.

Table 4.20 : Distribution of risk averse passenger flows

OD	Line segment	1	2	3	4
OD1-150	Flows	66.74	83.26	112.26	137.74
OD2-100					
OD1-400	Flows	200	200	325	325
OD2-250					

Table 4.19 and 4.20 indicate a difference in the distribution of flows for risk averse and risk neutral passengers at the lower line segments of the network when the OD demand is lesser than the capacity of the lines serving the network. For higher OD demand the distribution of flows for risk averse and risk neutral passengers remains the same.

### 4.3.3 Aggregate stochastic process model

The aggregate stochastic process model assumes that the passengers revise their route choice based on the average of costs experienced by the total flow on a route. As explained in section 3.4.2 if a particular route is not chosen on a day the cost of the route is computed by adding the cost of the route sections (when the route section is shared by more than one route). Based on the above surmise the aggregate stochastic process model was run for the mean-variance cost assuming an OD demand given in fig 4.3. Since at the end of a day each route has a cost associated with it even when no passenger has chosen the particular route and since all passengers are aware of the costs associated with all the routes in the network, the mean-variance cost for aggregate stochastic process model is given as below

$$\hat{g}_{k^z}^z = \sum_{j=1}^{\omega} \omega_j \left( E \begin{bmatrix} t_{k^z}(1^z)^z \\ \vdots \\ t_{k^z}(d^z)^z \end{bmatrix} + \beta \text{var} \begin{pmatrix} t_{k^z}(1^z)^z \\ \vdots \\ t_{k^z}(d^z)^z \end{pmatrix} \right)^{\Omega-j} \quad (4.16)$$

Where  $t_{k^z}(i^z)^z$ - is the experienced total travel time along route  $k^z$  by passenger  $i^z$  between OD pair Z

$\hat{g}_{k^z}^z$ - the predicted total travel time for route  $k^z$  between OD pair Z.

$\Omega$ -current simulation day

$\omega$ -weight associated with the memory length

$\beta$ - non-negative parameter which represents the degree to which the variance is undesirable to passengers (Jackson and Jucker, 1982).

Table 4.21: Experienced total travel time and flows along various routes using aggregate stochastic process model with mean-variance cost (scenario 2)  $\beta = 2.5, \xi = 4, \text{ and } \omega = 15$

		Node 1 - Node 4 (Z=1)								Node 2 - Node 4 (Z=2)		
Route		1	2	3	4	5	6	7	8	9	10	11
$\xi=0.05$	$E(t_k)$	51.1	38.5	40.5	101.8	103.8	74.3	68.6	107.2	91.6	84	80
	Std	8.6	9	7.3	23.5	22.8	26.1	27.4	29.3	21.7	19.7	20.1
	$E(X_k)$	84.7	146.4	148.5	3.5	3.5	4.4	4.3	3.2	60.9	89.8	100.1
	Std	13.9	12.1	11.1	1.7	1.7	3.1	2.7	1.4	11.1	13.8	10.4
$\xi=1.5$	$E(t_k)$	0	32.7	4.6	0	0	0	0	0	45.6	49.1	54.9
	Std	0	20.5	7.2	0	0	0	0	0	56.7	52.6	40.9
	$E(X_k)$	0	117.4	280.9	0	0	0	0	0	64.2	61.2	125.5
	Std	0	129.0	130.1	0	0	0	0	0	105.1	93.2	110.1

Table 4.21 shows the results of implementing the mean-variance cost as an aggregate stochastic process model. It can be seen that a network consisting of fully aware passengers; as assumed in the aggregate stochastic process models, find route 2 and 3 attractive such that all passengers alternate between these routes for Z=1. The distinction between R-DSPM and the aggregate stochastic process model lies in the distribution of flows and thereby experienced total travel time on each route. The flows and the experienced total travel times have a large standard deviation at higher  $\xi$  values in aggregate stochastic process

models whereas the standard deviation of flows and experienced travel time is much lesser when adopting a disaggregate approach.

#### **4.4 Summary**

The current chapter dealt with the formulation of R-DSPM for risk neutral passengers. Numerical tests were carried out to show the changes observed in the route choice behaviour of risk averse passengers in comparison with risk neutral passengers. It is observed that the differences in the route choice behaviour of are not very pronounced at lower  $\xi$ ,  $\beta$  and  $\gamma$  values. It is only for higher  $\xi$  and  $\beta$  values that a marked difference is noticed. At a higher  $\beta$  value with a lower  $\xi$  there was a difference between risk averse and risk neutral passenger flow distribution, similarly a higher  $\beta$  value for a lower  $\gamma$  values resulted in a difference between risk neutral and risk averse passengers flow distribution. The determination if a  $\beta$  value is high enough to result in a marked difference between risk neutral and risk averse passenger flows is dependent on the network characteristics assumed such as the number of available routes between an OD pair; the amount of variance associated with the network; the transfer penalty if any assumed for each transfer in the network; the socio-economic background of the transit users. It is also shown that the variance at the lower end transit stops is comparatively higher to the variance at the transit service starting stops as the lower end stops experience more number of failure to board passengers.

It is also shown that the markovian properties of the mean-variance model are satisfied and hence a unique stationary distribution is present. A comparative study was also carried out between the existing transit assignment models and R-DSPM for risk averse passengers. The Chapter was able to note that the existing models excepting for the aggregate stochastic process model don't account for the learning process of an individual and hence are not able to route the passengers realistically as in a day to day framework. The aggregate stochastic process model on the other hand resulted in flow distribution observing periodic oscillations between a pair of routes at higher  $\xi$  values. Though similar periodic oscillations at higher  $\xi$  values were also observed in R-DSPM, the distribution of flow between the various available routes was such that the standard deviation of flows and experienced total travel times was significantly lesser. It was found that in R-DSPM the risk averse passengers route themselves even on those routes which were found unattractive by risk neutral passengers.

## **Chapter 5**

### **R-DSPM formulation for Mean – Lateness cost and Policy Evaluation**

Chapter 3 and Chapter 4 saw R-DSPM with mean-variance cost being formulated and implemented on various example networks along with the discussion of the results. The mean – variance model measures irregularity through dispersion of travel times and does not explicitly enumerate as to where the disutility associated with irregularity affects the passengers. One of the key assumptions associated with the mean-variance model is that the passengers are not affected by the time they arrive at the destination or the duration of the journey. In reality such an assumption may not be realistic especially for work based trips. In the current chapter a mean-lateness model which measures irregularity through the delay associated with the total travel time experienced by passengers exceeding their acceptable total travel time to the destination is proposed.

Adopting the ‘scheduling’ concept proposed by Small (1982) several studies such as Watling (2006), Noland et al. (1998) Arnott et al. (1990) have studied the mean-lateness model in a traffic network. Watling (2006) studied the effect of route choice under variable demand and supply conditions by solving the assignment problem as late arrival penalised user equilibrium (LAPUE). LAPUE used a modified cost function which considered only the lateness penalty associated with a total travel time exceeding beyond a predefined desired total travel time. A desired total travel time when exceeded mimics late arrival at the destination thereby resulting in a disutility. Most of these studies (Noland et al., 1998; Arnott et al., 1990; Watling, 2006) assume an identical desired total travel times for all commuters.

The current chapter follows the modified cost function proposed in Watling (2006) accounting for disutility associated with a passenger experiencing a total travel time being beyond his/her desired total travel time. The chapter is organised such that section 5.1 discusses the integration of the mean-lateness cost formulation with R-DSPM. Section 5.2 looks at the implementation of the mean-lateness model on a test network. Section 5.3 evaluates the policy measures that could be undertaken by the operating agencies and compares the performance of various cost assumptions on the line loads of the network.

## 5.1 R-DSPM with strict capacity constraints using mean-lateness cost

The current chapter introduces the mean-lateness cost for R-DSPM. As there is sufficient literature on the relationship between mean-standard deviation and mean-lateness it was felt imperative to study mean-lateness cost as a part of R-DSPM. 'Lateness' is modelled in several different ways in traffic assignment. The following section highlights some of the different ways in which lateness could be modelled. One way of modelling lateness is through the mean lateness factor,  $H'$ . A comprehensive description of the relationship between mean-standard deviation and mean lateness factor can be found in Franklin and Karlstrom (2008). In Franklin and Karlstrom (2008) the relationship between the mean-standard deviation approach and scheduling approach is first explored and eventually the relationship between mean-standard deviation and a 'mean lateness factor' is established. Franklin and Karlstrom (2008) highlight that the mean-standard deviation approach and the scheduling approach are the same when exponential distribution of travel time, together with no lateness penalty and travel time being independent of departure time is assumed. Bates et al. (2001) found that the expected scheduled lateness (eq 2.26) and expected scheduled early (eq 2.26) of the scheduling approach can be approximated by a constant  $H'$ . Hence

$$\eta SDE + \gamma SDL = H'(\eta, \gamma) \quad (5.1)$$

Wherein  $H'(\eta, \gamma)$  – is the mean lateness factor,  $SDE$ - schedule delay early,  $SDL$ - schedule delay late,  $\eta, \gamma$  are parameters. This relationship acts as a bridge between mean-standard deviation and mean lateness factor.

Fosgerau and Karlstrom (2010) and Fosgerau and Engelson (2011) derive the relationship for any assumed distribution of travel time by defining the travel time into a deterministic and stochastic component.

$$\bar{T} \text{ is the travel time such that } \bar{T} = \mu + \sigma \bar{X} \quad (5.2)$$

Where  $\bar{X}$  is standardised random variable with mean 0 and variance 1 with cumulative density .

$\mu$ - mean travel time

$\sigma$ - standard deviation of travel time

Assuming that the preferred arrival time is 0 such that the departure time is  $-D$ ; the utility function given in eq 2.26 (scheduling approach) is rewritten as eq 5.3

$$U = \alpha \bar{T} + \eta (\bar{T} - D)^- + \gamma (\bar{T} - D)^+ \quad (5.3)$$



Wherein  $\hat{\alpha}, \eta$  and  $\gamma$  are parameters

Wherein  $(T - D)^+$  is the scheduled delay.

By substituting eq (5.2) in eq(5.3) the optimal departure time is worked out as

$$D^* = \mu + \sigma \Phi^{-1} \left( \frac{\gamma}{\eta + \gamma} \right) \quad (5.4)$$

It is shown in Fosgerau and Karlstrom (2010) that the mean lateness factor  $H'(\eta, \gamma)$  is equal to  $\int_{\frac{\gamma}{\eta+\gamma}}^1 \Phi^{-1}(s) ds$  where  $\Phi$  cumulative distribution of  $\check{X}$ . The mean lateness factor hence denotes the average lateness associated with optimal departure time and can be derived for any assumed distribution of travel time.

The mean-lateness cost used in current R-DSPM follows the concept of 'acceptable total travel time' put forward by Watling (2006). The 'mean lateness factor' as derived by Fosgerau and Karlstrom (2010) differs from the mean-lateness model as defined by Watling (2006) wherein lateness is associated with exceeding an 'acceptable travel time' for a specific route within an OD pair. Hence in Watling (2006), lateness is modelled as a penalty which a traveller incurs when the total travel time experienced by them exceeds their acceptable travel time. Watling (2006) adopts the 'schedule delay' approach proposed by Small (1982) and illustrates that for normal distribution of travel time the generalised cost for user  $i$  could be written as

$$u_i = \theta_0 \text{voc}_i + \theta_1 \mu_i + \theta_2 \sigma_i L \left( \frac{T^i - \mu_i}{\sigma_i} \right) \quad (5.5)$$

Wherein  $\theta_1$  indicates the value of total travel time,  $\theta_2$  reflects the value of being one time unit later than expected,  $\text{voc}_i$ - is the vehicle operation cost for user  $i$ ,  $\mu_i$ - mean of total travel time of user  $i$ ,  $\sigma_i$ - standard deviation of the total travel time of user  $i$ ,  $T^i$ - total acceptable travel time of user  $i$ . Watling (2006) further describes that the term  $\theta_2 L \left( \frac{T^i - \mu_i}{\sigma_i} \right)$  could be separated out as it acts in place of the 'reliability ratio' used in mean-standard deviation formulation. Wherein 'reliability ratio' is defined as the ratio of the value of standard deviation of travel time to the value of time. Hence in case of normal distribution a direct relationship between mean-lateness model and the mean-standard deviation model is established. It therefore seems a natural extension to test the implementation of R-DSPM using mean-lateness cost.

Similar to the formulation in chapter 3 consider a single OD pair such that there are  $n^Z$  routes to choose between the OD pair Z. The OD demand is randomly varying from day-to-day, but that there is a fixed rate of potential travellers for

each OD movement, from which the demand for any particular day is derived. Let the number of potential travellers on each OD movement be denoted by an integer  $d^Z$ . On each day, there are two important ‘decision’ elements for each traveller: whether they travel at all, and if they do travel which route they choose. Supposing that the indicator variable  $\delta_{i^z}^Z$  takes the value 1 if individual  $i^z$  travels on a given day, and takes the value 0 otherwise. For those that travel,  $f_{i^z}^Z$  denotes the route selected by individual  $i^z$  between the OD pair  $Z$  where  $f_{i^z}^Z \in \{1^Z, 2^Z, \dots, n^Z\}$ ,  $n^Z$  describing the total number of routes between the OD pair  $Z$ . Each of the passenger  $i^z$  there exists a ‘acceptable total travel time’  $\mathcal{T}^{i^z}(z)$  which is OD pair specific.

For ease in the inference of the results the ‘acceptable total travel time’ is chosen such that it exceeds the longest route total travel time between an OD pair and is kept the same for all passengers. Hence for each O–D movement, there is assumed to be a  $d^Z$  number of ‘acceptable total travel time’ i.e

$$\mathcal{T}(Z) = \begin{pmatrix} \mathcal{T}^{1^Z} \\ \mathcal{T}^{2^Z} \\ \vdots \\ \mathcal{T}^{d^Z} \end{pmatrix} \text{ wherein } \mathcal{T}^{1^Z} = \mathcal{T}^{2^Z} = \dots = \mathcal{T}^{d^Z}.$$

Where  $\mathcal{T}(Z)$ - is the ‘acceptable total travel time’ for OD pair  $Z$   $\mathcal{T}^{1^Z}$ - is the acceptable total travel time for passenger 1 of OD pair  $Z$ .

Keeping the remaining formulation similar to that given in chapter 3 we define the cost used for mean-lateness model. The approach is based on Noland et al. (1998), Arnott et al. (1990), Watling (2006). In this a passenger  $i^z$  considering route  $k^z$  ( $k^z = 1^z, 2^z, \dots, n^z$ ) perceives a route cost  $\hat{g}_{k^z}(i^z)^z$  represented as

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z + \theta_2 c_{k^z}^{i^z} \forall k^z = f_{i^z}^z \quad (5.6a)$$

Else

$$\hat{g}_{k^z}(i^z)^z = \theta_1 g_{k^z}(i^z)^z \forall k^z \neq f_{i^z}^z \quad (5.6b)$$

Where

$$g_{k^z}(i^z)^z = \sum_{j=1}^{\omega} \omega_j T_{k^z}(i^z)^{z, \omega-j} \quad (5.7)$$

$g_{k^z}(i^z)^z$ - weighted average total travel time of individual (predicted)  $i^z$  along route  $k^z$

$$c_{k^z}^{i^z} = \sum_{j=1}^{\omega} \omega_j \max(0, t_{k^z}(i^z)^z - \mathcal{T}_{k^z}(z))^{\omega-j} \forall k^z = f_{i^z}^z \quad (5.8)$$

$c_{k^z}^{i^z}$  - weighted average lateness penalty associated with each individual  $i^z$  along route  $k^z$  between the OD pair Z

$\omega_j$ - weights associated with the memory length as specified in Chapter 3.

And  $\theta_1$  indicates the value of total travel time and  $\theta_2$  reflects the value of being one time unit later than expected (Watling, 2006);  $t_{k^z}(i^z)$ - is the experienced travel time on route  $k^z$  by passenger  $i^z$  on OD pair Z;  $\mathcal{T}_{k^z}(z)$  -‘acceptable total travel time’ for all passengers on route  $k^z$  between OD pair Z;  $\Omega$ - current simulation day and  $T_{k^z}(i^z)$ - updated travel time for route  $k^z$  by passenger  $i^z$  on OD pair Z. In the current study the value of  $\theta_2/\theta_1$  is assumed to be 5.

## 5.2 Implementation on example network 2

The R-DSPM was run for example network 2 under the congested demand given in Chapter 4. A  $\Delta$  value of 15 day is chosen. Watling (2006) indicates that the value of  $\theta_1, \theta_2$  should be greater than 0. In transit network the value of waiting time is considered to be higher than the value of in-vehicle travel time (Benezech and Coulombel, 2013) however such a distinction is not explored in the current study. The value of  $\theta_2/\theta_1$  is chosen to be 5. The sensitivity of the model to various assumed parameter values ( $\Delta, \theta_2/\theta_1$ ) is tested and results are shown in **appendix c**. The simulation was run for 700 days and the initial 200 days were discarded as the burn-in period.

It is expected that as the value of  $\mathcal{T}(Z)$  is increased the passengers tend to be more flexible with respect to the total travel time needed to reach the destination and hence essentially become risk neutral as the lateness penalty incurred in most cases would be zero. On the other hand a lower  $\mathcal{T}(Z)$  value implies a shorter desired total travel time preference resulting in a significantly varying flow distribution. In order to test the expectation it is assumed that all passengers have the same desired total travel time for an OD pair.

Fig 5.1 shows the distribution of flows when the ‘acceptable total travel time’ of all the passengers are the same with a low value of  $\mathcal{T}(Z = 1) = (48.5^{1^z}, 48.5^{2^z}, \dots \dots 48.5^{d^z})$  (5 minutes more than the largest uncongested total travel time associated with OD1) and  $\mathcal{T}(Z = 2) = (32.5^{1^z}, 32.5^{2^z}, \dots \dots 32.5^{d^z})$  (5 minutes more than the largest uncongested total travel time associated with OD2). A low acceptable total travel time results in a shift in the probability distribution of passengers associated with a mean-lateness cost when compared with the probability density function of risk neutral passengers. Similar to the mean-variance model in chapter 4 it is observed that at higher  $\xi$  values the routes which are found to be unattractive

by risk neutral passengers due to its high cost may become attractive to passengers having a low acceptable total travel time as travelling on these routes may reduce the penalty associated with delay.

Fig 5.2 shows the distribution of passenger flows when all the passengers have the same higher acceptable total travel time  $\mathcal{T}(Z = 1) = (123.5^{1^Z}, 123.5^{2^Z}, \dots, 123.5^{d^Z})$  (80 minutes more than the largest uncongested total travel time associated with OD1)  $\mathcal{T}(Z = 2) = (107.5^{1^Z}, 107.5^{2^Z}, \dots, 107.5^{d^Z})$  (80 minutes more than the largest uncongested total travel time associated with OD2). In such a scenario in spite of experiencing longer travel times the passengers minimise only their average total travel times as they have higher tolerance to delay. The distribution of flows being almost similar to that of risk neutral flows on all routes is an indication of such a phenomenon. A minor shift observed in some density functions is due to some passengers experiencing journey times greater than 123.5 minutes for OD1 and 107.5 minutes for OD2 in the considered example network.

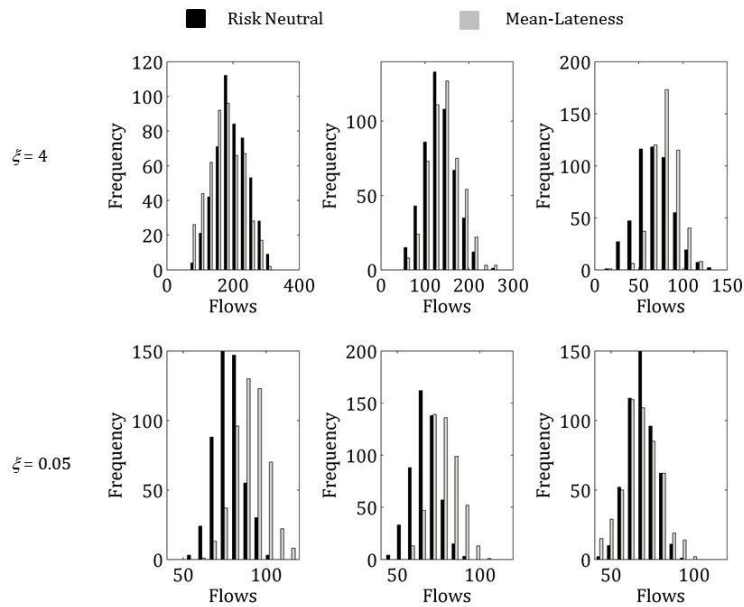


Fig 5.1: The distribution of flows when the ‘acceptable total travel time’  $\mathcal{T}(Z = 1) = 48.5$  and  $\mathcal{T}(Z = 2) = 32.5$ ,  $\xi = 4, \eta = 15, \frac{\theta_2}{\theta_1} = 5$  (scenario 2)

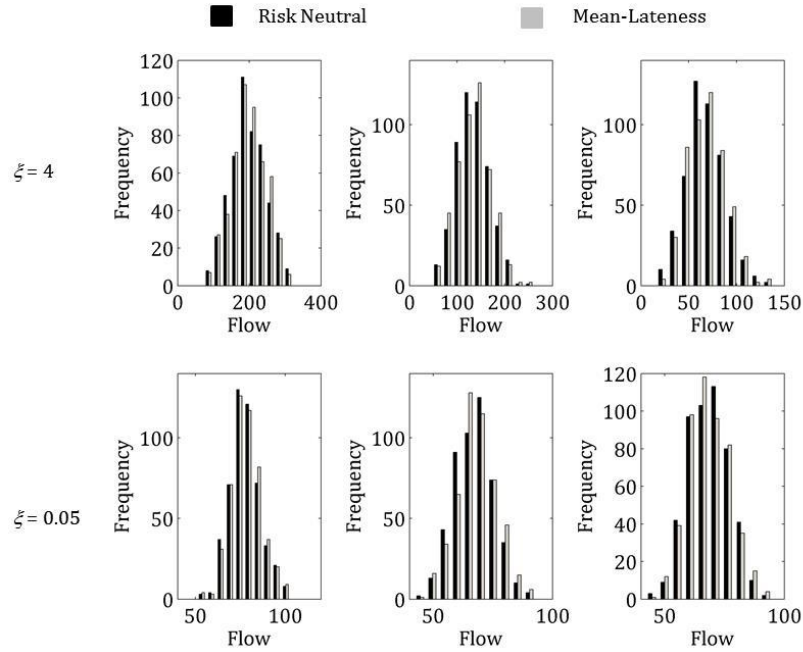


Fig 5.2: The distribution of flows when the ‘acceptable total travel time’  $\mathcal{T}(Z = 1) = 123.5$  and  $\mathcal{T}(Z = 2) = 107.5$ ,  $\xi = 4, \gamma = 15, \frac{\theta_2}{\theta_1} = 5$  (scenario 2)

The above analysis provides sufficient evidence that behaviour of passengers is similar to the expected results. The proposed model is able to capture the significant shift in the distribution of flows using the mean-lateness cost and provides sufficient evidence for further exploration of the same. The satisfaction of the markovian properties to prove that the mean-lateness R-DSPM is ergodic and regular are given in **Appendix C**.

### 5.3 Policy Interventions:

The performances of various R-DSPM considered in the thesis on policy interventions which a transit agency would carry out in a network is assessed in this section. The impact is tested on the example network 2 and the tests are carried out for various input parameters (the test results of  $\xi = 0.05$  are given in **appendix C**).

Similar to the policy evaluations made in Yin et al. (2004) four policy alternatives are considered to evaluate waiting time reliability. In addition to the policy initiatives specified by Yin et al. (2004) an additional policy on providing information to the passengers is being considered in the current section. Yin et al. (2004) presents a waiting time reliability measure for each line as

$$WTR_l^j = \Pr(w_l^j \leq \alpha w_{l_0}^j) \quad \forall l \in L \forall j \in N(l) \tag{5.9}$$

Wherein

$WTR_1^j$ -waiting time reliability of line l at stop j;

$w_1^j$  the actual waiting time at stop j for line l;

$w_{10}^j$  - the average waiting time for passengers boarding on line l at station j according to the nominal schedule under free-flow conditions.

$\alpha \geq 1$ - a predefined threshold value  $N(l)$ - the number of stops on line l. The value of  $w_{10}^j$  is chosen to be  $\frac{1}{\lambda_1}$  in accordance with the assumption that the example network before implementation of policies has interarrival times which are exponentially distributed.

The policies to be tested are

1. Increasing the shape factor  $m$  of line 2 to make the interarrival times of the transit service on line 2 more reliable. This improves the service reliability of the line by reducing the variance in the interarrival times.
2. Increasing the capacity of transit service from 20 pass/hr to 25 pass/hr of line 2.
3. The frequency for line 1 is increased from 8 to 15.
4. Changing the dwell time constant to from 7 sec to 20 sec for line 2.
5. Giving information to the passengers.

Before looking at the waiting time reliability improvement under various policy implementations Fig 5.3 and fig 5.4 shows the waiting time reliability profile for example network 2 without any policy implementations. From Fig 5.3 and fig 5.4 it can be discerned that the reliability profile for various cost evolves in a different manner.

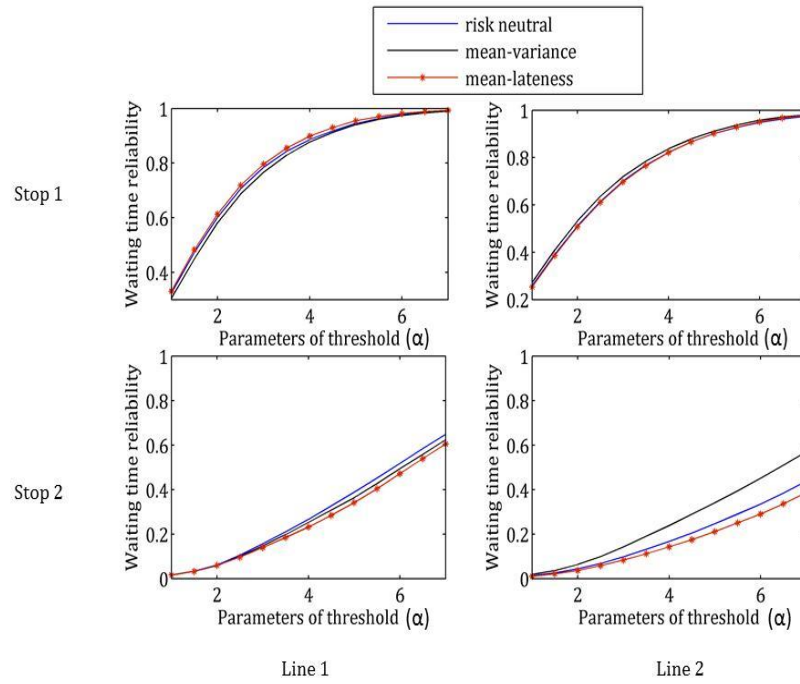


Fig 5.3 Waiting Time reliability for various cost without any policy implementation at  $\xi=0.05$ ,  $\varrho=15$ ,  $\beta = 2.5$ ,  $\frac{\theta_1}{\theta_2} = 5$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

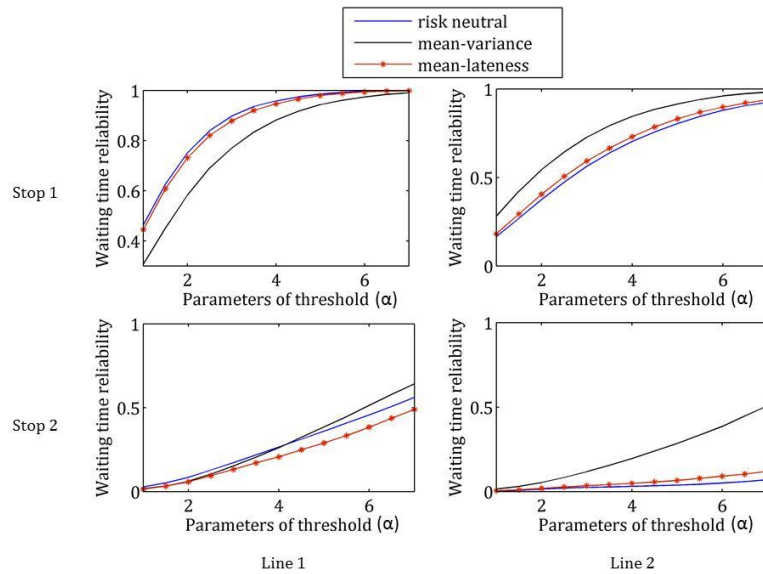


Fig 5.4 Waiting Time reliability for various cost without any policy implementation at  $\xi=4$ ,  $\varrho=15$ ,  $\beta = 2.5$ ,  $\frac{\theta_1}{\theta_2} = 5$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

The distribution of *boarding* flows at various stops and on various lines for  $\xi=4$  are shown in Table 5.1. As is expected table 5.1 indicates that the boarding flow distribution is quite varied for different cost assumed.

Table 5.1 *Boarding* Line loads on all lines without any policy implementation at  $\xi=4, \eta=15, \beta = 2.5, \frac{\theta_1}{\theta_2} = 5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

	Cost	Line 1		Line 2	
		Mean	Std	Mean	Std
Stop 1	Risk Neutral	142.8	39.8	255.5	39.2
	Mean-Variance	182.2	40.9	216.1	41.1
	Mean-Lateness	150.6	42.2	247.7	41.7
Stop 2	Risk Neutral	248.6	40.5	400.6	41
	Mean-Variance	216.1	22.9	248.2	20.8
	Mean-Lateness	266.0	42	383.2	42.2

The policy initiatives described above shall now be tested one by one and the results will be compared with the non-policy network results given in figure 5.4 and table 5.1.

Fig 5.5 shows the waiting time reliability profile for various threshold values when the inter-arrival reliability of line 2 is improved. From Fig 5.5 one can see that the improvement of reliability in inter-arrival times of line 2 sees a change in the profile of mean-variance cost at stop 1 for both lines 1 and line 2. It is seen that there is an increase in the probability of passengers being able to experience a waiting time lesser than the nominal waiting time at a threshold value of 1 ( $\alpha=1$ ). However for the mean-lateness cost and risk neutral cost, at stop 1, a drop in probability of passengers being able to experience a waiting time lesser than the nominal waiting time is observed for line 2.



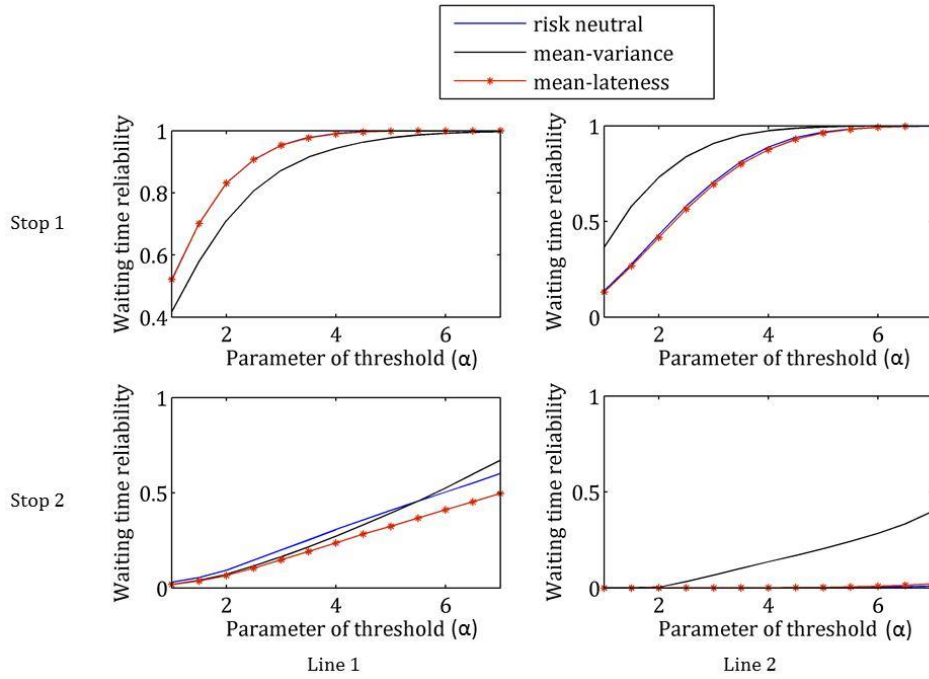


Fig 5.5 : Waiting Time reliability for various cost Line 2  $m=300$  at  $\xi=4, \varrho=15,$   
 $\beta = 2.5, \frac{\theta_1}{\theta_2} = 5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

No such distinction between policy implementation and without implementation is visible at stop 2 on line 1. In line 2 (Stop 2) one sees a drop in the probability of passengers experiencing waiting time lesser than various threshold values while assuming risk neutral or mean-latency costs. In case of mean-variance cost assumption the probability of passengers experiencing waiting time lesser than a value of twice the nominal waiting time is significantly different from that of no policy network.

Table 5.2: *Boarding* Line loads at  $\xi=4$  for increased reliability of line 2,  $\varrho=15$   
 $\beta = 2.5, \frac{\theta_1}{\theta_2} = 5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

	Cost	Line 1		Line 2	
		Mean	Std	Mean	Std
Stop 1	Risk Neutral	131.8	26	266.5	25.4
	Mean-Variance	165.8	29.2	232.4	28.3
	Mean-Latency	129.2	26.1	269.1	25.0
Stop 2	Risk Neutral	239.2	28.4	410	30
	Mean-Variance	206.5	19.5	246.5	22.1
	Mean-Latency	257.0	31.8	392.1	32.4

Though the waiting time reliability profile do not seem to project a major difference for mean-lateness and risk neutral cost after the improving the reliability of transit services on line 2 a look at the distribution of the boarding line loads on various lines given in Table 5.2 and comparing them with those on Table 5.1 indicate a difference in the standard deviation of flows. From the above results one can conclude that the profile variation would largely depend on the order in which a transit stop is visited between the OD pairs; with the origin stops (terminal stops) wherein the transit services are assumed empty experiencing a different reliability profile from that of transit stops at lower end of the journey. The profile variations are more pronounced for different costs at lower end transit stops of the network emphasising the need to consider the risk averseness of the passengers travelling in the network.

Similar to improving the interarrival reliability of line 2 the second policy of increasing the capacity of line 2 from 20 to 25 passengers is considered and the results are shown in Fig 5.6. Increasing the capacity of line 2 shows an improvement in reliability profile at both stop 1 and stop 2.

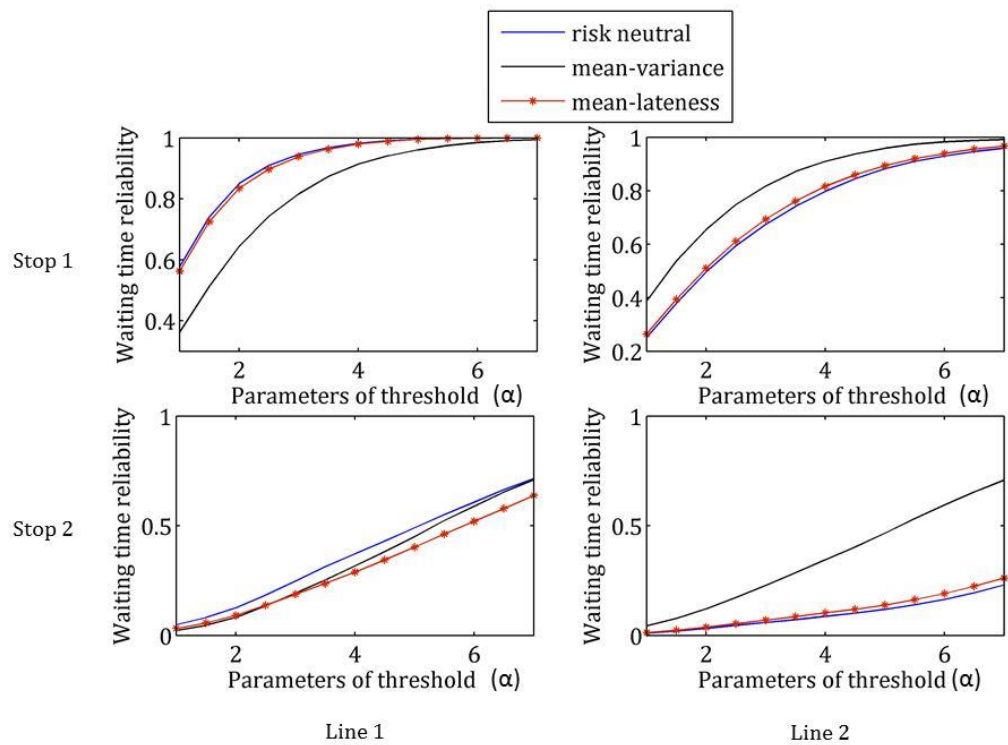


Fig 5.6: Waiting Time reliability for various cost  $l_2$  cap=25 at  $\xi=4, \gamma = 15, \beta = 5$  (mean-variance),  $\frac{\theta_1}{\theta_2} = 5, \gamma=15, \beta = 2.5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

Table 5.3: *Boarding* Line loads at  $\xi=4$  for increased capacity on line 2,  $\alpha=15$ ,  $\beta = 2.5$ ,  $\frac{\theta_1}{\theta_2} = 5$ ,  $\mathcal{I}(Z = 1)$  and  $\mathcal{I}(Z = 2) = 32.5$

Cost		Line 1		Line 2	
		Mean	Std	Mean	Std
Stop 1	Risk Neutral	119.6	42	278.7	41.9
	Mean-Variance	168.6	40.9	229.6	41.7
	Mean-Lateness	125.0	42.6	273.3	42.9
Stop 2	Risk Neutral	216.0	41.7	433.1	43.6
	Mean-Variance	203.9	23.2	254.8	22.4
	Mean-Lateness	233.1	44.6	415.3	46.4

A capacity increase of merely 5 passengers per transit service has provided a slight improvement in the reliability profile of the network for different cost at various transit stops. A look at the *boarding* loads given in Table 5.3 shows an increase in the loads on line 2. In spite of the increase there is a marginal improvement in the reliability profile of line 2 at various stops. Hence a capacity increase assuming that the OD demand rate remains the same could improve the waiting time reliability profile of the network in the long run.

A frequency increase of line 1 shown in fig 5.7 has a marked effect on the waiting time reliability. An improvement in the reliability profile of all the cost is seen across all the stops and lines of the network. A change in dwell times of line 2 (fig 5.8) indicates no change in the waiting time reliability profile from that of the reliability profile without policy improvements.

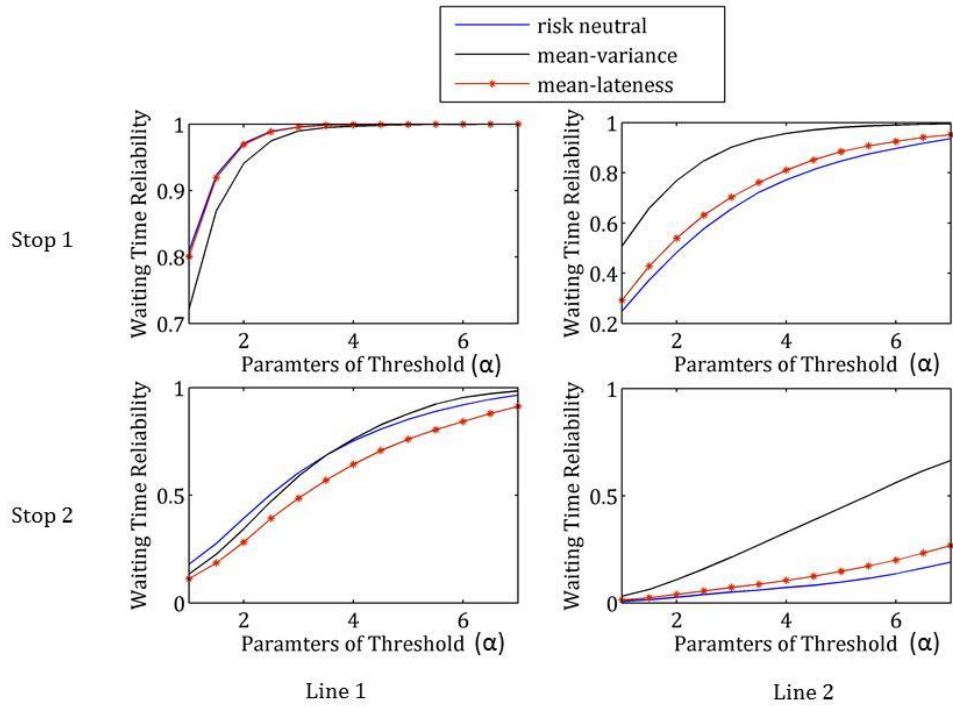


Fig 5.7 Waiting Time reliability at  $\xi=4$  for increased frequency of line 1,  $\lambda=15, \beta = 2.5, \frac{\theta_1}{\theta_2} = 5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

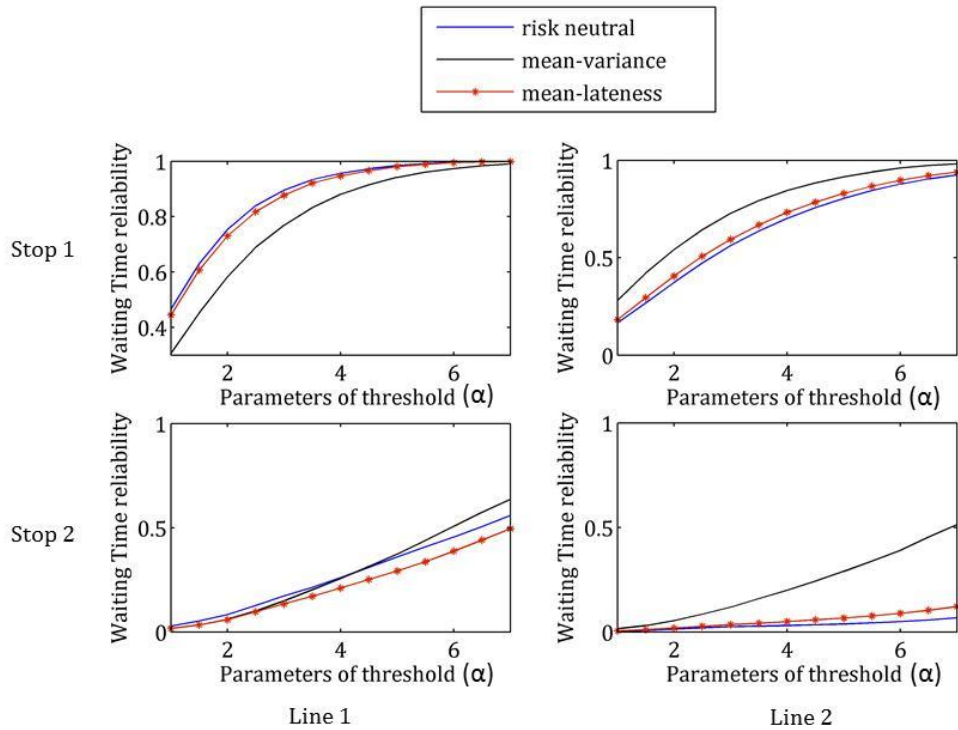


Fig 5.8 Waiting Time reliability at  $\xi=4$  changed dwell time line 2,  $\lambda=15, \beta = 2.5, \frac{\theta_1}{\theta_2} = 5, \mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

### 5.3.1 Giving information to the passengers:

The above policy measures are quite straight forward in their integration with the existing R-DSPM with strict capacity constraints. In this section we shall look upon the impact of possible provision of information to the passengers. As is mentioned in Chapter 3 the current R-DSPM assumes that the passengers revise their route choice based on his/her experience alone. In the 'information' scenario we shall assume that at the end of each day the passengers are aware of the average total travel times experienced by all the passengers on a particular route and the standard deviation of the same.

Based on this surmise at the end of each day each passenger will base his/her route choice for the next day using the formulation given in Jha et al. (1998) and shown below:

$t_k^{1,i^z,\Omega-1}$  mean perceived travel cost by individual  $i^z$  on day  $\Omega-1$  before receiving information and before the trip for  $k^{\text{th}}$  route between his/her origin and destination. In event of passenger not having travelled on the  $k^{\text{th}}$  route throughout his/her memory length then the term is assumed to have a value equal to mean informed total travel time of  $k^{\text{th}}$  route and the variance associated with the informed total travel time for the route.

$T_k^{2,i^z,\Omega}$  Updated distribution of  $t_k^{1,i^z,\Omega-1}$  in light of information (i.e. after pre-trip updating).

Similar to (Jha et al., 1998) updating the pre -trip travel time is done as follows:

Let the travel time provided to the passengers as information on day  $\Omega$  for  $k^{\text{th}}$  route be  $\hat{t}_k^{i^z,\Omega}$  wherein  $\hat{t}_k^{i^z,\Omega}$  is the average experienced total travel time for route  $k$  on day  $\Omega-1$ . Without loss of generality, it is assumed that the average total travel times provided by information do not vary across individuals. It is hypothesized that when users receive information, they modify it based on their perceptions of information. The modified information travel time can be expressed as:

$$\hat{t}_{pk}^{i^z,\Omega} = \hat{t}_k^{i^z,\Omega-1} + \epsilon_k^{i^z,\Omega-1} \quad (5.10)$$

Where  $\hat{t}_{pk}^{i^z,\Omega}$  is the perceived value of information total travel time by individual  $i^z$  for  $k^{\text{th}}$  route.  $\epsilon_k^{i^z,\Omega-1}$  is the perception error, which is due to the user's past experience with information, his/her attitude towards the information system, etc. The distribution of  $\epsilon_k^{i^z,\Omega-1}$  is assumed  $N(0, \sigma_k^{\Omega-1})$  wherein  $\sigma_k^{\Omega-1}$  is also known to the passengers and is equal to the standard deviation of total travel times experienced on route  $k$  at end of day  $\Omega-1$ .

The updated best estimate is given by the following Ang and Tang (1975):

$$E[T_k^{2,i^z,N}] = \frac{E(\hat{t}_{pk}^{i^z}) \text{var}(t_k^{1,i^z}) + E(t_k^{1,i^z}) \text{var}(\hat{t}_{pk}^{i^z})}{\text{var}(t_k^{1,i^z}) + \text{var}(\hat{t}_{pk}^{i^z})} \quad (5.11)$$

Where the values of  $\text{var}(t_k^{1,i^z}) = \text{var} \begin{pmatrix} t_k^{1,i^z,\Omega-1} \\ t_k^{1,i^z,\Omega-2} \\ \vdots \\ t_k^{1,i^z,\Omega-\beth} \end{pmatrix}$  and  $\text{var}(\hat{t}_{pk}^{i^z}) = \text{var} \begin{pmatrix} \hat{t}_{pk}^{i^z,\Omega} \\ \hat{t}_{pk}^{i^z,\Omega-1} \\ \vdots \\ \hat{t}_{pk}^{i^z,\Omega-\beth} \end{pmatrix}$  are obtained over the memory length of  $\beth$  days.

The updated variance of the mean perceived total travel time is given by

$$\text{Var}[T_k^{2,i^z,N}] = \frac{\text{var}(t_k^{1,i^z}) \cdot \text{var}(\hat{t}_{pk}^{i^z})}{\text{var}(\hat{t}_{pk}^{i^z}) + \text{var}(t_k^{1,i^z})} \quad (5.12)$$

Fig 5.9 and Fig 5.10 shows the results of implementation of information scenario on the reliability profile of waiting times at various stops in the example network 2. From the figure 5.9 it is seen that the information provision reduce the reliability of waiting times at the origin stop for line 1 wherein the transit services are assumed to empty however it has improved for line 2. At stop 2 the reliability profile is greatly improved for both the lines with information provision (fig 5.10).

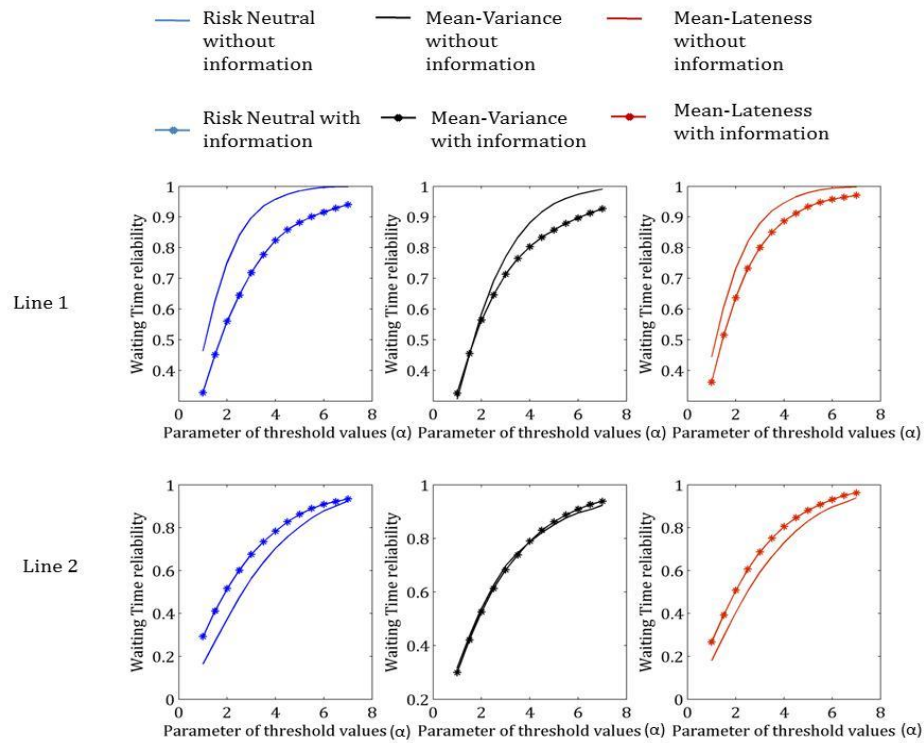


Fig 5.9: Waiting time reliability with and without information for various costs at stop 1 of example network 2,  $\xi=4$ ,  $\beth=15$ ,  $\beta = 2.5$ ,  $\frac{\theta_1}{\theta_2} = 5$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

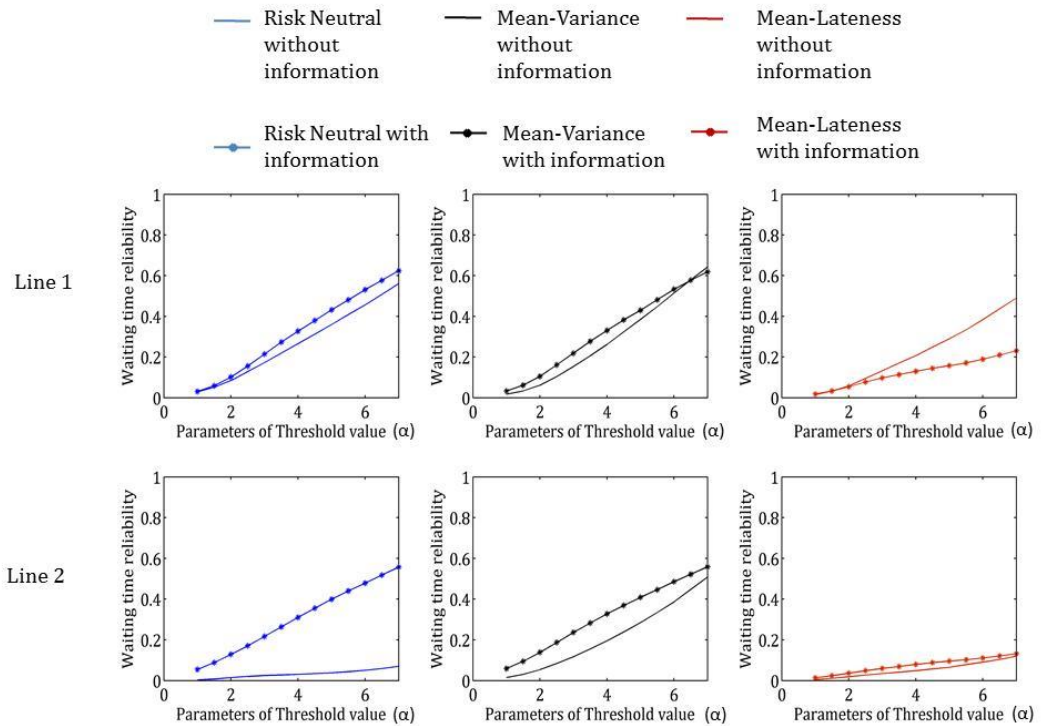


Fig 5.10: Waiting time reliability with and without information for various costs at stop 2 of example network 2,  $\xi=4$ ,  $\zeta=15$ ,  $\beta = 2.5$ ,  $\frac{\theta_1}{\theta_2} = 5$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

The above analysis has shown that information provision which accounts for the variance in total travel times experienced along the routes of the transit network could yield a significant improvement in the waiting time reliability profile of the network at lower end stops. This could be because the maximum variation is observed at stop 2 for the current network wherein apart from the service unreliability the passengers of the second OD pair are competing for space in the transit service at stop 2. The true impact of the information scenario however can be assessed only if it is implemented on a larger network.

#### 5.4 Impact of assumptions made in R-DSPM on the outcome:

To get the results as shown in the current chapter and in the previous chapters the R-DSPM is run on certain specific conditions. One of the main condition to achieve the above set of results is that the R-DSPM has a demand ( $d^Z$ ) such that there is at-least a day within the simulation period when all the passengers between the OD pairs make their journey and that most of the passengers between each OD pair choose to travel on any given day.

In event of the population size (demand ( $d^Z$ ) between each OD pair) being high but the rate of arrival of passengers remaining the same; that is when the

sample population from which the daily passengers are drawn is increased; it would result in the number of passengers making the journey on a given day becoming lesser than those not making the journey for the same day. In such a network most of passengers would assume uncongested total travel time for the untraveled routes. They would therefore base their route choice on the uncongested cost as most wouldn't have travelled within their memory period. This would result in risk averse passengers being unaware of the variance or lateness associated with their travel times. A suitable population size to be chosen for the current model to replicate the findings in chapter 4 and the findings in section 5.1 should be such that the number of passengers travelling on a particular day be closer to the rate of arrival of passengers assumed between each OD pair. For eg the poisson rate of passenger arrival in example network 2 for OD 1 is 400 passengers/hr (6.67 passengers/minute) and the population size is taken as 465 passengers/hr (7.75 passengers/minute). Therefore with the current rate the probability of the chosen population size of 7.75 passengers/minute arriving in one minute at the transit stop works out to be 0.61. On the other hand if the population size was increased to 1000 passengers/hour (16.67 passengers/minute) the probability works out as 0.003 which implies that almost on all days the number of passengers arriving may be much lesser resulting in most of the passengers deciding 'not to travel'. The situation of having a large population size to sample from; often arises in developed countries wherein in case of work trips the option to 'work from home' is feasible. 'Work from home' concept would result in the number of potential travellers between an OD pair being high without all the passengers travelling on each day. Such type of 'work from home' concept is still at its infancy in developing countries like India. Nevertheless R-DSPM model still could be applicable for large population size if the memory length of individual traveller is proportionately increased. Hence for the numeric example cited above if a population size of 1000 passengers between an OD pair exists then for each passenger to travel a route between the OD pair more than once requires a memory size larger than 15 days. A large memory length would ensure that the passenger remembers the experiences on the routes travelled more than once. Also a large memory length would increase the simulation period to arrive at a unique stationary distribution thereby increasing the probability of capturing a passenger having travelled more than once in a route. A large memory length would also let the passengers arrive at the variability associated with the experienced travel times for a route or to associate a lateness penalty with respect to an acceptable total travel time.



Table 5.4: Flow and experienced travel time distributions using mean-variance cost  $\beta=2.5$  and mean-lateness  $\frac{\theta_2}{\theta_1} = 5$  at  $\xi=4$ ,  $\eta = 15$ ,  $m = 1$ ,  $d^{Z=1} = 1465$  and  $d^{Z=2} = 1307$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$

Route		Z=1						Z=2				
		1	2	3	4	5	6	7	8	9	10	11
Mean-variance	$E(t_s)$	11.4	39.9	42.9	105.1	13.5	59.8	87.8	1.3	56.1	57.8	54.8
	Std	22.3	9.3	7.4	38.5	21.4	15.8	23.9	10.9	17.2	18.8	16.5
	$E(X_s)$	0.24	155.6	100.2	2.6	20.9	11.5	107.2	0.02	39.8	114.0	97.1
	Std	0.47	14.1	10.8	1.6	5.2	3.5	10.8	0.15	6.8	10.9	10.3
Mean-lateness	$E(t_s)$	0.1	43.7	42.5	4.9	61.8	15.1	96.5	0	62	96.8	60.8
	Std	2.2	11.6	6.3	21.7	55.5	28.2	26.5	0	19.1	21.7	18.5
	$E(X_s)$	.002	223.2	96.2	0.06	2.23	0.48	76.1	0	44.9	88.8	117.2
	Std	0.04	35.3	21.8	0.25	3.3	1.1	14.3	0	12.3	11.5	15

Table 5.4 shows the result of risk averse passengers being sampled from a large population. From table 5.4 it is observed that as the population size increases for a fixed rate of passenger inter-arrivals most of the passengers do not travel on a particular day. The likelihood of the same passenger deciding to travel on subsequent day also gets considerably reduced. Most of the passengers base their route choice on the uncongested travel times. The phenomenon observed in chapter 4 (table 4.6) of risk averse passengers finding the routes unattractive to risk neutral passengers attractive at higher  $\xi$  values becomes absent. It is found that with increased population size the routes which had lower uncongested total travel times became the attractive routes and those having higher uncongested travel times became less attractive. Similarly in case of mean-lateness when the acceptable total travel time for the passengers is kept  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$  at  $\xi=4$  and  $\frac{\theta_2}{\theta_1} = 5$  the routes with lesser uncongested travel time become attractive. As is observed in mean-variance cost with a large population to be sampled from, there is an absence of finding the risk neutral passenger's unattractive routes attractive (fig 5.1) in mean-lateness cost for a fixed rate of passenger arrivals.

## 5.5 Summary

The current chapter discussed the integration of the mean-lateness model in R-DSPM. The model was then implemented on the example network 2 and the results discussed. The markovian properties of the model have been proved.

The chapter also dealt with the waiting time reliability changes in a network without policy implementations and with policy implementation. It was shown that the boarding loads varied with the costs assumed reiterating the need to consider risk aversion in traffic assignment models. It is to be noted that the loads mentioned in tables 5.1, 5.2 and 5.3 are boarding loads and hence will not add up to form the demand between OD pairs.

The information scenario proposed in the current chapter assumes that the information provides the passengers with the average travel times experienced on all the routes between an OD pair at the end of the day and it is assumed that the passengers perceive the information such that the perception error is normally distributed with a standard deviation equal to the standard deviation of the total travel time distribution of the route at the end of the day. Such an assumption seems unrealistic but the operators could conceive a way of introducing such a system for better utilisation of the existing supply demand ratio.

## **Chapter 6**

### **London Underground – Case Study**

#### **6.1 Introduction**

The previous chapters illustrated the principles of R-DSPM on small example networks. In this chapter the assignment model is applied onto a section of larger network namely the London Underground. The main objective of the chapter is to show the practical implementation of the proposed model. The implications of ignoring the risk aversion of transit network passengers and its effect on policy decisions will be assessed.

The chapter is organised such that the first section introduces the London underground open source data base. The next section deals with the simulation being run under various parameter assumptions for the mean-variance R-DSPM with strict capacity constraint followed by the mean-latency model with strict capacity constraint and risk neutral passenger model with strict capacity constraint. The simulation results are then tested using a non-parametric test - Wilcoxon rank sum test - to check if the chosen parameters result in a total travel time distribution similar to the existing observed total travel time distribution. Having assessed the best fitting parameters certain policy evaluations are carried out to check the performance of the section under these policy scenarios.

#### **6.2 Data Description**

London underground is considered the oldest rapid transit system in the world and the system serves 270 stations and has 402 kilometres of track. Of the several sections within London underground the following section which does not fall within zone 1 of London underground was chosen mainly because the inner zone (zone 1) of London underground is so well connected that though there may be only two direct lines between the origin and destination stations it is always possible to reach any destination within zone 1 through several possible ways. For example if we consider the section between Edware road and Notting Hill gate though the direct lines are only the Circle line and the district line it is possible to reach Notting Hill from Edware road by travelling on either circle line or Hammersmith and City line getting down at Baker street and taking either Jubilee line or Bakerloo line then taking the Central line to reach Notting Hill. Though such a huge diversion does not seem reasonable it

however cannot be overlooked in the absence of data confirming the same. Another reason is that the oyster card data from the open source of TfL for the year 2009 has lesser number of entries for zone 1 ODs than for the currently considered section of London Underground. Fig 6.1 shows the section of London underground which is used as case study.

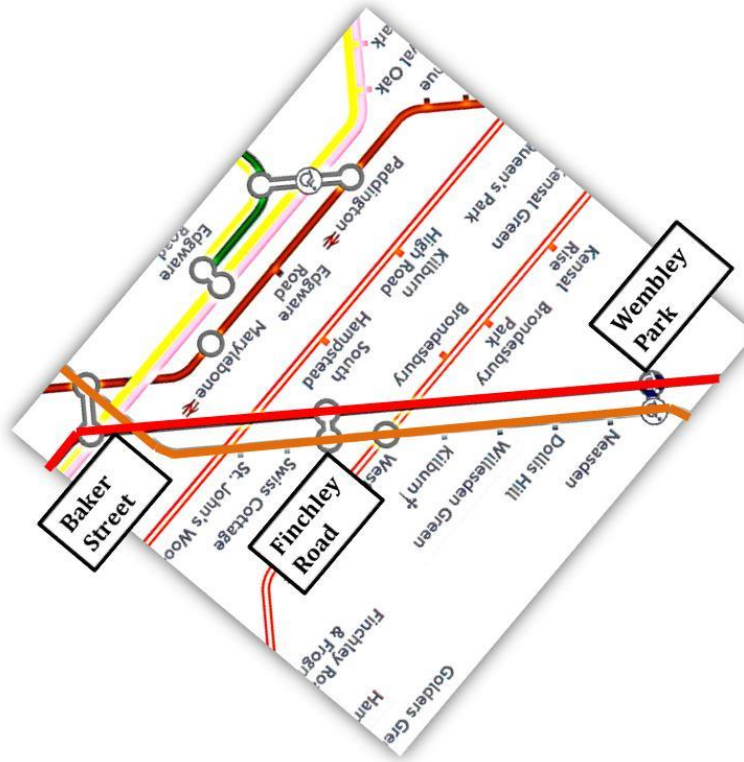


Fig 6.1 OD pair 1 from Baker Street to Wembley park (a section of London Underground) OD pair 2 from Finchley Road to Wembley Park. (Line 1 – Jubilee line and Line 2 – Metropolitan Line).

MySociety (2008) shows the in-vehicle travel time (without disruptions) between the stations on the Metropolitan line and the Jubilee line (table 6.1).

Table 6.1 the in-vehicle travel time between the stations in the chosen case study section

	Baker Street to St John's Wood	St John's Wood to Swiss cottage	Swiss Cottage to Finchley Road	Finchley Road to West Hampstead	West Hampstead to Kilburn	Kilburn to Willesden Green	Willesden Green to Dollis Hill	Dollis Hill to Neasden	Neasden to Wembley Park	Total
<b>Metropolitan Line</b>		5.55 min				7.05 min				12.06
<b>Jubilee Line</b>	2.85 Min	1.52 min	1.18 min	1.20 min	1.55 min	2.07 min	1.67 min	1.38 min	2.65 min	16.07

The capacity of service is difficult to ascertain as the train characteristics for each line is different however the Train Service Model (TSM – the simulation model of TfL) output given for Victoria Line indicates that the train along the line has a capacity of 1004. It is currently assumed that the Jubilee line and the Metropolitan Line all have a similar capacity of 1004 passengers per train.

### **6.3 Demand and Supply Data:**

The open data link on TfL website consists of several files such as a file on oyster card information; a file on the line section flows; a file on number of boarding and alighting passengers on each line; the rolling origin and destination survey results, demand profile at various stations. The oyster card data consists of details pertaining to weekdays and weekends for Nov 2009. The website mentions that the data represents 5% data of the total oyster card journey made in the said week. The Oyster card data from Open data source on TfL website consists of details such as entry time at the origin as well as the exit time at the destination of the passengers entering the station.

The current study utilises only the week day's data (taken from Monday to Friday). Upon filtering the oyster card data there were 77 passengers traveling from Baker Street to Wembley Park and 39 passengers travelling from Finchley Road to Wembley Park on weekdays. The interarrival times of passengers (derived from the entry times of the passengers given in oyster card data) for each day within the said week was plotted to assess the distribution of passenger arrivals. The plot of the interarrivals indicate exponential distribution (fig 6.2) for both the OD pairs. A chi-square goodness of fit test was carried out with

$H_0$  : The random variable follows exponential distribution

$H_1$ : The random variable does not follow exponential distribution (Washington et al., 2003)

The chi-square goodness of fit test using matlab's 'chi2gof' resulted in a value of 0 which indicates that the goodness of fit test does not reject the null hypothesis at 5% significance level for both the OD pairs. This indicates that the passenger arrival at both Baker street and Finchley Road follow exponential distribution.

The frequency of Jubilee line services is taken as 23 trains per hour whereas for the Metropolitan line it is taken as 21 trains per hour (taken from the current timetable of the lines). Since the chosen London Underground section is similar to example network 2 given in Chapter 4 the route sections and all the routes

enumerated for example network 2 are applicable for the current section of London Underground case study. Though the platforms are not shared between the metropolitan line and the jubilee line at Baker Street it is assumed that the passengers that make a route choice of 3,5,6,7 choose between the lines, just before starting their journey, based on the display boards at the entrance of the station. At Finchley road the platforms are shared and hence the passengers can choose between Metropolitan Line and Jubilee line.

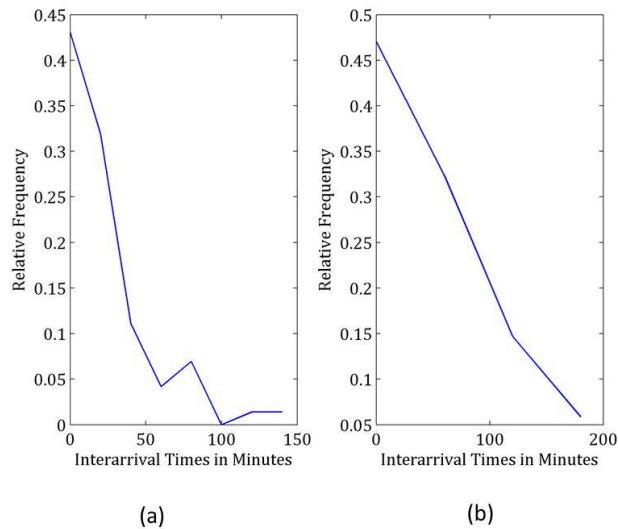


Fig 6.2 Interval distribution of Passengers (a) Baker street for OD1 (b) Finchley Road

The journey time distribution as obtained from the oyster card data for the two OD pairs of case study are shown in Fig 6.3. The Oyster card data showed a total of 3869 passengers entering the Baker street in a week of which 77 travelled to Wembley Park making a ratio of 0.02. The 'entry file' available on the TfL open source data- for the month of November 2012 -indicates a total of 15672 passengers entering Baker Street. The entry details also indicate that the P.M. peak is between 5:00 P.M to 10 P.M. Assuming the same ratio of passengers travelling from Baker street to Wembley Park as in Oyster card data we get an OD demand of 312 passengers during the peak evening hours (5 hours). Similarly the ratio of passengers travelling from Finchley Road to Wembley Park to the total number of passengers entering Finchley Road is 0.04 (according to the oyster card data). Similar to Baker Street the OD demand from Finchley Road is therefore computed as 131 passengers during the peak evening hours.

Ascertaining the OD demand from the 'entry file' as above gives an indication of the rate of passengers arriving at the origins. For the current study the rate of

passengers has been calculated as 60 passengers per hour from Baker street and 20 passengers per hour from Finchley Road.

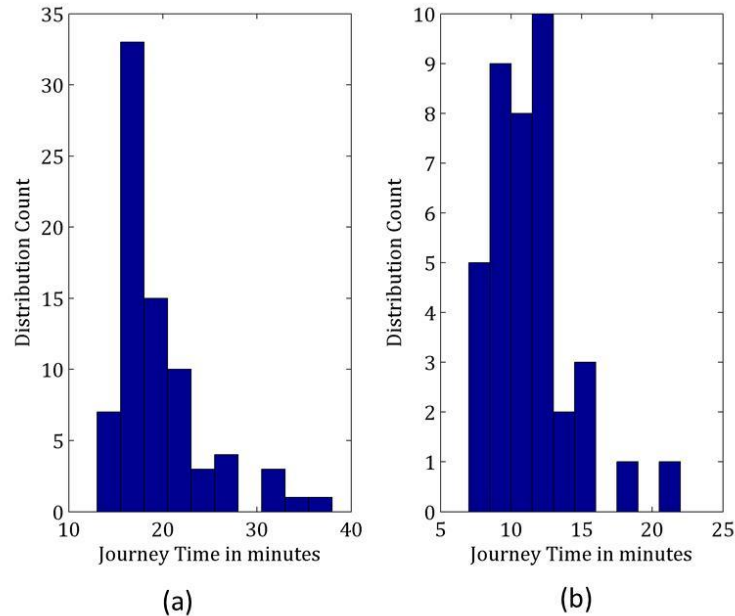


Fig 6.3 Journey time distribution (a) From Baker Street to Wembley Park (b) From Finchley Road to Wembley Park

## 6.4 Calibration:

The R-DSPM was calibrated using the oyster card journey times of OD1 (Baker Street – Wembley Park). The calibration was done for the number of on-board passengers, the dispersion parameters and the interarrival distribution shape factor. The calibration was carried out firstly assuming that all passengers are risk neutral; then it is assumed that all are risk averse to variance and finally the mean-latency model is calibrated.

### 6.4.1 Risk neutral:

A value of 0.05 was initially chosen for the logit dispersion parameter  $\xi$ . A series of runs were carried out for various combinations of line interarrivals shape factors  $m$  in order to best simulate the observed total travel times of the case study. It was found that a shape factor of  $m = 150$  for Jubilee line and a shape factor of  $m = 1$  for the metropolitan line were unable to reject the null hypothesis using Wilcoxon rank sum test. Hence these shape factors were assumed to be the representative shape factor for line interarrival distributions. These values can be corroborated from the online information obtained that the new signalling system was installed on Jubilee line only in 2011 and since the data is of 2009 the reliability of the line could be reasonably assumed as  $m = 150$  and since Metropolitan line is still undergoing installation

of new signalling system and since it is one of the oldest lines it seems reasonable to assume a less reliable service than Jubilee line with  $m = 1$ .

Based on the in-vehicle travel times (table 6.1), the frequencies assumed for the lines (section 6.3) and a risk neutral passenger cost, table 6.2 shows the simulated uncongested travel time for the various enumerated routes (chapter 4) using the aggregate stochastic process model. Comparing the total travel times from the oyster card data (Fig 6.3) with the simulated uncongested travel times shown in Table 6.2 it is found that 22.1% of the total passenger records in the current oyster card data experience congested total travel time between Baker Street and Wembley Park whereas 23.1% of the total records between Finchley Road to Wembley Park experience congested travel time.

Table 6.2 Uncongested travel time assuming risk neutral passengers - Case study 1

OD		Between Baker Street and Wembley Park								Between Finchley Road and Wembley Park		
Risk Neutral	Route	1	2	3	4	5	6	7	8	9	10	11
	$E(t_s)$	17.4	15.5	15.9	17.1	17.4	17.5	15.8	20.5	12.0	10.2	10.5
	Std	0.3	1.4	0.8	1.4	1.3	0.3	1.4	1.4	0.6	1.3	0.8

The distribution of journey times indicates that the section is not very congested with only a few passengers experiencing congestion during evening peak hours. In order to mimic the congested journey times experienced by a few passengers it is assumed that the arriving transit service at the origin stops already carry a certain number of passengers from the stations further up from Baker street such that a certain number of passengers boarding at Baker street experience failure to board and thereby an increased total travel time.

The stochastic process model was run for various on-board passenger assumptions and the probability density function of the total travel times as experienced by the risk neutral passengers was visually compared with the total travel time obtained for the passengers travelling from Baker Street to Wembley Park (Oyster Card data). A wilcoxon-ranksum test was conducted to determine if the simulated total travel times showed similar distribution and equal median values as the observed total travel times. Fig 6.4 shows the probability density function of the total travel time simulated using various on-board passengers with that of the oyster card journey times.



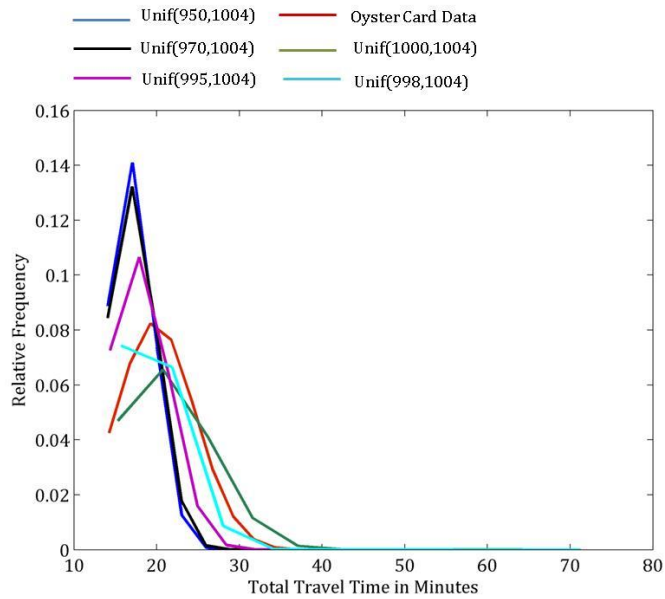


Fig 6.4 Calibration of the on-board loading already present within the transit service assuming risk neutral passengers at Baker street.

The probability distribution of the total travel time experienced by risk neutral passengers as simulated using R-DSPM is statistically compared with probability distribution of the total travel time from oyster card data. The ‘ranksum’ function of matlab works such that a value lesser than 0.05 rejects the null hypothesis at 5% significance level. The null hypothesis is that the probability distributions are from the same continuous distribution with equal medians.

Table 6.3: The calibration of on-board passengers in the transit service along the Jubilee and Metropolitan Line

Description	p-value	inference
Unif (950,1004)	6.02e-11	Rejects null hypothesis
Unif(970,1004)	6.2e-10	Rejects null hypothesis
Unif(995,1004)	1.1e-08	Rejects null hypothesis
Unif(1000,1004)	0.06	Null hypothesis cannot be rejected

Since the on-board passenger distribution of Unif(1000,1004) is unable to reject the null hypothesis it is assumed that the distribution best simulates the congestion already on board when reaching Baker street of the case study section. With the fixed set of on board passengers the simulation was again run for several logit dispersion parameters  $\xi$  and a value of 0.75 was chosen for risk neutral cost following the results of Wilcoxon rank sum test. For higher dispersion parameters the Wilcoxon rank test rejected the null hypothesis. For brevity the results are not included in the thesis.

### 6.4.2 Risk averse:

The mean-variance passenger model was run for different  $\beta$  values which represent the degree to which the variance of the total journey time is undesirable to passenger  $i$  and a mean lateness model was run for different  $\mathcal{T}$  value which indicates the acceptable total travel time of passenger between an OD pair. As in the previous section the transit services arriving at Bakers street were assumed to have on-board passengers which uniformly varied between 1000 and 1004.

Table 6.4 Mean Variance and Mean-lateness hypothesis testing for calibration

Mean - Variance		
logit dispersion parameter of $\xi = 4$		
$\beta$	p-value	Inference
2.25	0.17	Null hypothesis cannot be rejected
Mean -Lateness		
logit dispersion parameter of $\xi = 0.75$		
$\mathcal{T}$	p-value	Inference
40.5 and 30.5	0.06	Null hypothesis cannot be rejected
logit dispersion parameter of $\xi = 0.75$		
25.5 and 15.5	0.06	Null hypothesis cannot be rejected

In case of mean-variance cost for dispersion parameters higher than 4 also the Wilcoxon rank test could not reject the null hypothesis. However it is assumed that a value of 4 is sufficient enough to account for perception error in the experienced travel times and a risk aversion value of 2.25 is enough to exhibit the averseness of the passengers to travel time variations for the current section of London Underground. The results of the successful tests are shown in Table 6.4.

### 6.5 Validation:

The validation of risk neutral passengers travel time distribution is carried out using the total travel times observed between Finchley Road and Wembley Park. Assuming that at Finchley road 0.2% of the passengers alight, the simulation is run for the various parameters satisfying the necessary condition in the calibration section (section 6.4). The results obtained for validation are as shown below:

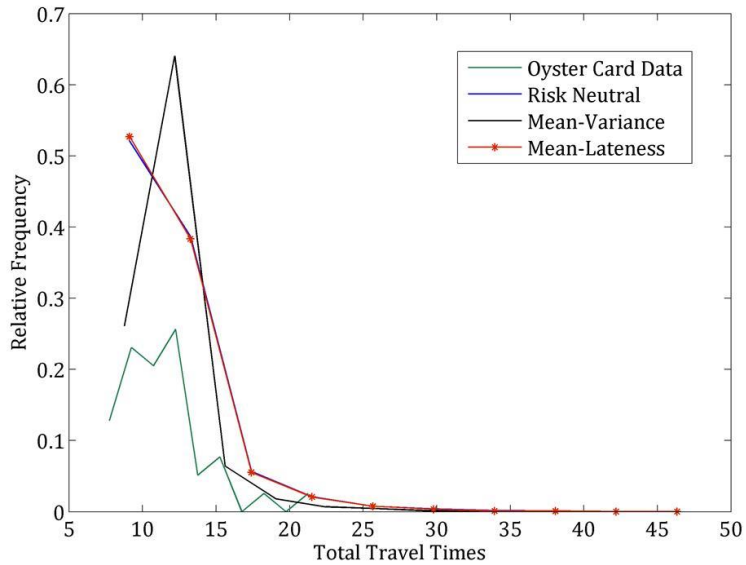


Fig 6.5 Validation of the R-DSPM assuming various cost passengers and using oyster card data between Finchley Road and Wembley Park

The Wilcoxon ranksum test yielded a value of 0.55 which is greater than 0.05 thereby not rejecting the null hypothesis at 5% significance level. This indicates that the model is able to simulate the total travel time of the risk neutral passengers to accuracy. For mean -lateness and mean-variance models the results are tabulated in 6.5.

Table 6.5 Mean-Variance and Mean -lateness hypothesis testing for validation

Mean - Variance		
logit dispersion parameter of $\xi = 4$		
$\beta$	p-value	Inference
2.25	0.65	Null hypothesis cannot be rejected
Mean -Lateness		
logit dispersion parameter of $\xi = 0.75$		
40.5 and 30.5	0.53	Null hypothesis cannot be rejected
logit dispersion parameter of $\xi = 0.75$		
25.5 and 15.5	0.48	Null hypothesis cannot be rejected

### 6.6 Result discussion

Tables 6.4 and 6.5 indicate that the simulation results fit the observed journey time distribution the best at logit dispersion parameter of  $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 40.5 and OD2 as 30.5 and a logit dispersion of  $\xi =$

0.75 with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5. Since such a wide range of measures are able to simulate the total travel time of case study; it would be erroneous to assume that all passengers are risk neutral (as is generally done in current transit assignment models). Such an assumption could lead to the policy measures being undertaken for the section giving an entirely different reliability profile from the models than the actual experienced reliability profile in the network. In the absence of empirical evidence of the degree of risk aversion of passengers or the possible number of risk averse passengers it is assumed that all passengers are risk averse to the same degree. Though the assumption is extreme the current aim is to understand the difference in the passenger loadings as a result of such risk averseness and since the simulation is done for evening peak of 1 hr one assumes on weekdays all passengers would be equally risk averse. Table 6.6 shows the variations in the number of passengers boarding the Jubilee line and Metropolitan line from Baker Street and Finchley Road using the current simulation parameters ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5).

Table 6.6 Variation in number of passengers *boarding* Baker Street and Finchley Road for the observed total travel time distribution ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\alpha = 15, \frac{\theta_2}{\theta_1} = 5$ )

Cost	Transit stop	Jubilee Line		Metropolitan Line	
		Mean	Std	Mean	Std
<b>Risk Neutral</b>	Baker Street	31.7	6.2	28.4	6
	Finchley Road	12.8	3.6	26.9	5.1
<b>Mean-variance</b>	Baker Street	34.5	6.1	25.6	6
	Finchley Road	18.6	4.7	24.5	4.8
<b>Mean-Lateness</b>	Baker Street	31.9	6.2	28.2	5.9
	Finchley Road	12.9	3.6	26.4	4.9

From table 6.6 one can see that the number of passengers boarding the jubilee line and the metropolitan line vary slightly based on the cost assumed by the passengers. Though the variation between mean-lateness and risk neutral cost is not much a slightly greater variation can be observed between risk neutral and mean-variance costs.

A study of the impact of various policy implementations on the number of passengers boarding at each station is carried out. The various policy evaluations to be undertaken are similar to those carried out in chapter 5 and are as shown below:

1. Changing the reliability of both Jubilee line as well as metropolitan line
2. Changing the frequency of the Metropolitan line
3. Increasing the capacity of Jubilee line

Fig 6.6, 6.7, 6.8 indicates the probability of number of passengers being able to experience a waiting time lesser than or equal to the threshold times of uncongested waiting time (assumed to be the inverse of frequency of the line per minute) under various policy measures. The waiting time reliability profile is computed using eq 5.4 in chapter 5. Fig 6.6 shows that the profiles of waiting time reliability for mean-lateness and risk neutral passengers are similar. However the profile of waiting time reliability assuming mean-variance cost is different from that of risk neutral and mean-lateness costs. A look at the number of passengers boarding at each station while considering mean-variance and risk neutral cost (Table 6.7) shows a significant difference in the number of passengers boarding from each transit stop on to each serving line.

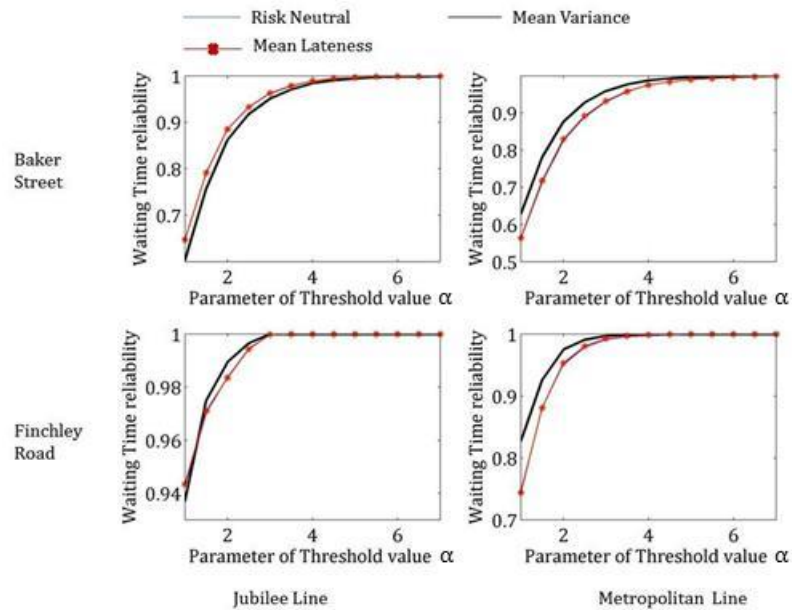


Fig 6.6: waiting time reliability along Jubilee and Metropolitan Line when the reliability of transit service is improved in both lines. ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\varrho = 15, \frac{\theta_2}{\theta_1} = 5$ )

Table 6.7 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for network with both lines having improved interarrival service reliability ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost)

Cost	Transit stop	Jubilee Line		Metropolitan Line	
		Mean	Std	Mean	Std
<b>Risk Neutral</b>	Baker Street	28.7	5.5	31.4	5.7
	Finchley Road	8.5	2.8	34.6	5.9
<b>Mean-variance</b>	Baker Street	31.7	5.6	28.4	5.8
	Finchley Road	14.7	4.1	31.5	5.8

A differing trend from that seen in Fig 6.6 is seen in Fig 6.7 which shows the profile of waiting time reliability when the capacity of jubilee line is increased. Fig 6.7 indicates that the risk averse passengers experience a very high waiting time reliability at Baker Street when commuting on the jubilee line.

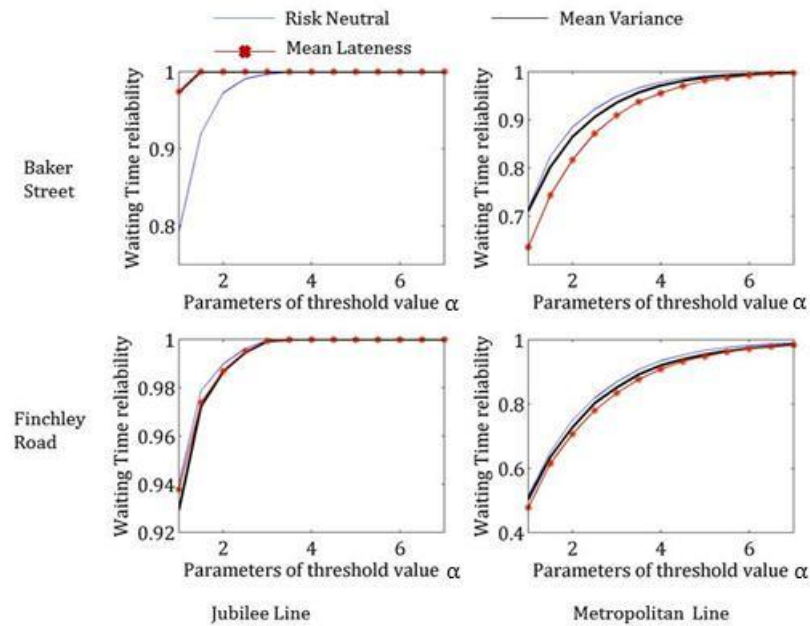


Fig 6.7: Waiting time reliability along Jubilee and Metropolitan Line when the capacity of Jubilee line is increased. ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\gamma = 15, \frac{\theta_2}{\theta_1} = 5$ )

From the reliability profile given in Fig 6.7 it is expected that the number of passengers boarding would be slightly different for differing costs on metropolitan line at both the stops. From Table 6.8 it is found that there is a slight difference in the boarding passengers on these lines. Also one would expect that the boarding flow on Jubilee line would be similar for mean-variance and mean-lateness cost at both the transit stops given that the reliability profiles are similar. However table 6.8 indicates a slight difference in the number of passengers boarding jubilee line.

Table 6.8 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for network with jubilee line having increased capacity ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\varrho = 15, \frac{\theta_2}{\theta_1} = 5$ )

Cost	Transit stop	Jubilee Line		Metropolitan Line	
		Mean	Std	Mean	Std
Risk Neutral	Baker Street	34.8	5.9	25.2	5.6
	Finchley Road	19.9	4.7	23.5	4.6
Mean-variance	Baker Street	39.7	7.1	20.3	4.6
	Finchley Road	16	3.9	25.9	5.5
Mean-Lateness	Baker Street	36.0	6.7	24.03	4.9
	Finchley Road	11.9	3.4	27.8	5.5

Upon increase of frequency in Metropolitan line (fig 6.8) the profile of waiting time reliability is almost similar for all the cost; at all stations and along all lines. However as shown in table 6.9 the routing options chosen and thereby the number of passengers boarding at each station vary slightly with the cost assumed.

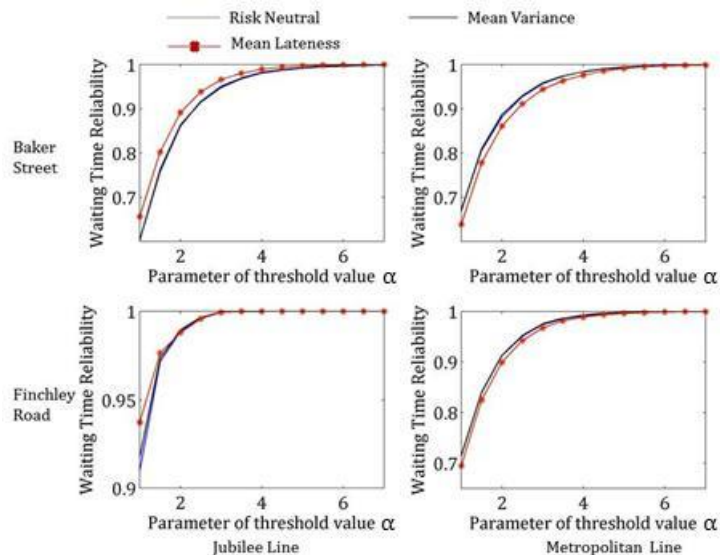


Fig 6.8: Waiting time reliability along Jubilee and Metropolitan Line when the frequency of Metropolitan line is increased. ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\varrho = 15, \frac{\theta_2}{\theta_1} = 5$ )



Table 6.9 Variation in number of passengers boarding at various stations on Jubilee and Metropolitan line for increased frequency of metropolitan line ( $\xi = 0.75$  for risk neutral passengers; at logit dispersion parameter of  $\xi = 4$   $\beta = 2.25$  for mean-variance cost; a logit dispersion of  $\xi = 0.75$  with mean-lateness acceptable total travel time between OD1 as 25.5 and OD2 as 15.5 and  $\alpha = 15, \frac{\theta_2}{\theta_1} = 5$ )

Cost	Transit stop	Jubilee Line		Metropolitan Line	
		Mean	Std	Mean	Std
Risk Neutral	Baker Street	30.7	5.7	29.4	6
	Finchley Road	19.5	4.8	23.9	4.7
Mean-variance	Baker Street	30.5	5.7	29.5	6.1
	Finchley Road	17.3	4.6	24.7	4.8
Mean-Lateness	Baker Street	27.6	5.7	32.5	6.1
	Finchley Road	11.2	3.4	27.3	5.4

## 6.7 Summary

The above analysis has indicated that modelling the flows as risk neutral passenger (as is currently done in most of the transit assignment studies) would yield a different set of route choices for a section from that of considering risk averse passengers. The boarding loads therefore vary based on the cost used. The above analysis shows that the number of passengers boarding jubilee line at Finchley Road were significantly different especially when the interarrival reliability of both the lines was improved for different cost confirming the expectation that risk averse passengers would be willing to make a transfer (in absence of transfer penalty) to avoid variations in their total travel time or to reach their destination within their accepted total travel time. Hence a policy decision to incentivise transfer options could be envisaged as a much more productive option for the current section if the considered level of risk aversion for the passengers is indeed true.

However with the current sample size of London Underground case study it is not possible to make any generic or concrete conclusions. It can only be deduced that there is an impact on the route choice based on the cost assumed as well as on the number of flows opting for a transfer at each transfer point in the network. It is also noted that other sources of variance such as the variation in walking times of the passengers within the transit station as well as the in-

vehicle travel time variations have not been considered in the current study. The current analysis shows almost similar trends on waiting time reliability values for various policy measures and only a slight variation in boarding flows, it could be because of capacity constraint not being realised in an extreme way (with just 22% of the passengers experiencing increased total travel time). As is seen in chapter 5 the impact of knowing the correct risk aversion coefficients (mean-variance or mean-lateness) on a highly congested network would definitely impact the results on route choice from that of assuming risk neutral passengers.

## **Chapter 7**

### **Summary, Conclusion and Further Studies**

#### **7.1 Summary:**

In this study a wide range of issues pertaining to reliability analysis in transit network have been addressed. Care has been taken to ensure that the disadvantages of using the existing theoretical models are overcome and a holistic framework flexible enough to run under varying cost assumptions is developed. The study was motivated by the unreliability associated not only with the transit service arrivals at transit stops but also the unreliability associated with failure to board situations of passengers – a common phenomenon in congested transit network. The aim of the study given in chapter 1 was:

- a. to specify the framework of strict capacity constrained frequency based transit assignment model which could assess the route choice variation of passengers in an unreliable transit network;
- b. to run numerical experiments on example networks to test the sensitivity of the model to various input parameters and assumptions;
- c. to study the impact of assessing unreliability using various costs on the possible policy decisions made by the operators.

Chapter 2 saw a general review of the literature associated with the various aspects of the current study namely; transit assignment models accounting for congestion; models accounting for reliability; stochastic process models dealing with day to day variations.

Based on the gaps identified from the literature review and highlighted in chapter 1, chapter 3 had successfully formulated R-DSPM with the following properties:

1. A strict capacity constraint at disaggregate level such that each transit service is not loaded beyond its capacity.
2. A disaggregate model at demand level wherein each passenger's route choice is based on his/her own experience and not on the aggregate experiences of all the users on a particular route.
3. A day to day variation of demand and supply with the demand having the flexibility of choosing not to travel on a particular day.
4. Difference in passenger's cost perceptions.
5. A weighed average learning process model which results in a more realistic evolution of flows.

6. Accounting for unreliability associated with
  - a. Varying interarrival times of transit service at the transit stop
  - b. The variation in the waiting time of passengers due to the 'failure to board' condition. The condition arises as a result of strict capacity constraint enforced at disaggregate level which results in some passengers being not able to board the first transit service of their attractive line set.
  - c. The variation associated with the in-vehicle travel times of routes comprising of route sections containing more than one attractive line section.
  - d. The variation associated with the variable demand generated for each day's travel.

The framework in Chapter 3 is run under passenger behaviour attribute assuming a risk neutral behaviour. The sensitivity of the model to various parameter values and the tests to show that the model obeys markovian properties is also discussed in chapter 3.

Chapter 4 shows the shift in the passenger's route choice behaviour when risk aversion in form of mean-variance cost is accounted for in both uncongested and congested networks. The chapter also establishes through numerical examples the need to use stochastic process model in assessing reliability of transit networks. Numerical tests were carried out on a simple example network and the sensitivity of the model to various input parameters is also carried out.

Chapter 5 applies the mean-lateness cost formulation in R-DSPM and runs it on an example network. The chapter also assesses the impact of using varying cost on the reliability profile of waiting time at various stops in the example network. The chapter also makes an assessment of the variation in reliability profile under various policy implementations which could be carried out by the operators.

Chapter 6 then deals with a small case study of a section in London underground to understand the practical implementation of the proposed R-DSPM with strict capacity constraints.

## **7.2 Conclusions:**

Based on the objectives and aims set out in chapter 1 and the analysis carried out in chapter 3,4 5, and 6 the following conclusions have been drawn:

- A generic R-DSPM framework was developed in chapter 3 which enabled the implementation of various cost accounting for risk aversion in route

choice of passengers in a transit network. Chapter 3 showed that the proposed framework produced a unique stationary distribution with ergodic and regular markovian properties for risk neutral passengers. The sensitivity of model to various parameters assumed in the framework was assessed with risk neutral cost. It was seen that at higher  $\xi$  values the risk neutral passengers tend to think alike and hence choose the same route to travel between an OD pair on a particular day.

- The generic framework provided in chapter 3 was used to assess the route choice of risk averse passengers in chapter 4. The risk aversion was accounted for in the cost of the passengers by introducing a non-negative parameter  $\beta$ , which denoted the degree to which the variance of total travel time is undesirable to the passenger (Jackson and Jucker, 1982). The implementation of the model on an example network showed that the parameter,  $\beta$  associated with the variance plays an important role in determining the route choice of passengers together with the memory length  $\gamma$  and the dispersion parameter,  $\xi$  adopted in the study. It was seen that for lower values of the mentioned parameters there was not a significant difference in the flow distribution on the various routes between a network with all risk neutral and all risk averse passengers. At higher values of the said parameters a marked difference could be observed between the network of all risk neutral and all risk averse passenger flows on various routes. The all risk averse network passengers, at higher  $\xi$  values, assigned themselves onto routes found unattractive by all risk neutral network passengers.
- The existing transit assignment models such as the BPR based, effective frequency based and the aggregate stochastic process model were discussed in chapter 4 with a mean-variance cost function. It was noted that all the existing transit assignment models did not assign flows onto routes found unattractive by all risk neutral network which was a noticeable phenomenon in the proposed R-DSPM with strict capacity constraints.
- Though mean-variance cost deals with the risk aversion by introducing a non-negative parameter  $\beta$  in the cost it does not account for how exactly this variance would affect the passengers in day to day context. In order to assess the disutility associated with the increased total travel time a mean-lateness cost is proposed in chapter 5. The cost includes a lateness penalty for passengers experiencing total travel times greater than the acceptable total travel time between an OD pair. The analysis on an example network showed that when the acceptable total travel time

value is large then the flow distribution is similar to that of assuming all risk neutral passengers. It is shown that the value of lateness  $\theta_1/\theta_2$  assumed plays a major role in determining the difference in flow distribution between all risk neutral and all mean-lateness passengers. When the acceptable total travel time is small and the value of lateness is large then the distribution of the flows between the all mean-lateness network and all risk neutral network is different.

- The policy implementation on the example network has varying effects on the reliability of waiting times (chapter 5). It is seen that some of the policy measures do not have any impact on the reliability profile whereas some have an impact at the lower end transit stops. Overall it is seen that the distinction in reliability profiles using various cost is more pronounced in higher  $\xi$  values.
- The essence of utilising reliability based cost to route the passengers in a realistic network is captured by implementing the framework in a section of London Underground (chapter 6). The results have shown that certain policy measures may result in a distinctly different performance profile for various cost in terms of waiting time reliability. It can successfully be concluded that not only the distribution of flows on various lines but also the number of passengers making transfer at various transfer stops (in the absence of transfer penalty) greatly varies between the cost chosen.
- In general it can be concluded that the current thesis is successful in the implementation of a holistic R-DSPM with strict capacity constraint. The model has successfully overcome the disadvantages of existing transit assignment models by:
  1. Maintaining passenger priority,
  2. the non-separable problem not being able to guarantee a unique solution is accounted for by the presence of a single unique stationary distribution for various reliability based cost
  3. The R-DSPM framework allows for strict capacity constraint being observed at each transit vehicle level.
  4. The R-DSPM framework enables the assessment of unreliability associated with failure to board the first service of a passengers choice set.
  5. The assumption of passengers being fully aware of the network is overcome by assuming that the passengers revise their route choice based only on their experienced costs. The knowledge of uncongested total travel times on various routes in the event of not travelling on a particular route seems realistic enough as such a knowledge can easily

be worked out from the assumption of waiting time being inverse of frequency of the lines within the attractive line set.

It is seen from the above paragraphs that the current reliability based stochastic process models with strict capacity constraint provides a framework bettering the existing transit assignment models at several levels (shown in Fig 7.1). The various models tested in the thesis through its implementation on various example networks is summarised in Fig 7.2. Fig 7.2 helps to compare the models with ease based on the assumptions made in each of these models.

It is also shown in the thesis that the risk aversion of passengers needs to be accounted for, as this would give a significantly different set of flows under certain conditions on each route from the conventional assumption of risk neutral passengers. This implies that the passengers willing to make transfers, in the absence of transfer penalties, would be significantly different while assuming risk averse than when assuming risk neutral passengers. This has a direct implication on the transit station design as well as policy settings adopted for the network. The stationary, ergodic and regular markovian framework provided in the thesis makes it possible to consider its integration with transit network design problems and problems wherein frequency optimisation is carried out by operators for the transit network.

Various models	Network State	Passenger OD demand and arrival at travel time	Waiting time/service headway	Dwell time/boarding time	Route choice	Learning Model	Costs	Updating of costs
Szeto et al., 2011	Congested	Constant demand – static model	BPR type congestion model for waiting time increase due to congestion- Exponential interarrivals	Not considered	Reliability based user equilibrium	No learning model	<ul style="list-style-type: none"> <li>• Mean-variance- (Risk Averse) Cost= Mean+<math>\beta</math>(std. dev)</li> </ul>	No updation
Szeto et al., 2013	Congested	Constant demand – static model	Overload delay model-exponential interarrivals	Not considered	Stochastic user equilibrium	No learning model	<ul style="list-style-type: none"> <li>• Mean-variance (Risk – Averse) cost = mean +<math>\beta</math>(std. dev)</li> </ul>	No updation
Teklu, 2008a,b	Congested	Variable demand- exponential interarrivals	Strict capacity constraint- exponential interarrivals	Linear Dwell time function	Probit choice model	Weighed average learning process model with markovian properties	<ul style="list-style-type: none"> <li>• Risk Neutral Cost= weighed mean</li> </ul>	Aggregate updation model – based on collective experience
<b>R-DSPM- Current Model</b>	Congested	Variable demand (sample from constant population)- exponential interarrivals	Strict capacity constraint-erlang interarrivals	Linear Dwell time function Dependent on the number of passengers boarding and alighting. Constant coefficients	Logit choice model	Weighed average learning process with markovian properties	<ul style="list-style-type: none"> <li>• Risk neutral Cost = weighed mean</li> <li>• Mean – variance (Risk –Averse) Cost=Weighed mean+<math>\beta</math>(variance over memory)</li> <li>• Mean- Lateness (Risk Averse) Cost= weighed mean + weighed latness</li> </ul>	Disaggregate updation model – based on individual’s experience- nontravelled route cost updated assuming uncongested travel times

Fig 7.1 Comparison of the salient features of the current R-DSPM with existing models available in literature.



Various models	Network State	Passenger OD demand and arrival at travel time	Waiting time/service headway	Dwell time/boarding time	Route choice	Learning Model	Costs	Updating of costs
Example network 2 – Chapter 4 Section 4.3.1	Congested	Constant demand – static model	BPR type congestion model for waiting time increase due to congestion-Exponential interarrivals	Not considered	Reliability based stochastic user equilibrium	No learning model	• Mean-variance- (Risk Averse) Cost= Mean+ $\beta$ (var)	No updation
Example network 2 – Chapter 4 Section 4.3.2	Congested	Constant demand – static model	Hyperpath based Effective frequency function	Not considered	deterministic user equilibrium	No learning model	• Mean-variance (Risk – Averse) cost = mean + $\beta$ (var)	No updation
Example network 1- Chapter 3 (R-DSPM)	Congested	Variable demand (sample from constant population)-exponential interarrivals	Strict capacity constraint-erlang interarrivals	Linear Dwell time function Dependent on the number of passengers boarding and alighting. Constant coefficients	Logit choice model	Weighed average learning process with markovian properties	• Risk Neutral Cost= weighed mean	Disaggregate updation model – based on individual's experience- nontravelled route cost updated assuming uncongested travel times
Example Network 2- Chapter 4 Section 4.2 (R-DSPM)	Congested	Variable demand (sample from constant population)-exponential interarrivals	Strict capacity constraint-erlang interarrivals	Linear Dwell time function Dependent on the number of passengers boarding and alighting. Constant coefficients	Logit choice model	Weighed average learning process with markovian properties	• Mean – variance (Risk –Averse) Cost=Weighed mean+ $\beta$ (variance over memory)	Disaggregate updation model – based on individual's experience- nontravelled route cost updated assuming uncongested travel times
Example Network 2- Chapter 5 Section 5.2 (R-DSPM)	Congested	Variable demand (sample from constant population)-exponential interarrivals	Strict capacity constraint-erlang interarrivals	Linear Dwell time function Dependent on the number of passengers boarding and alighting. Constant coefficients	Logit choice model	Weighed average learning process with markovian properties	• Mean- Lateness (Risk Averse) Cost= weighed mean + weighed latness	Disaggregate updation model – based on individual's experience- nontravelled route cost updated assuming uncongested travel times

Fig 7.2 Comparison of the salient features of various models tested in the thesis.

### 7.3 Further research:

The current thesis has shown through various example implementation the need to account for risk averseness while assigning passengers on the transit lines. The current R-DSPM has accounted for several variations possible in a transit network (as highlighted in section 7.1) however there are several other sources of variation which has been unaccounted for in the current study and could be accounted for in future research. These include the variations brought about by differing walking speeds from the ticketing kiosks to the platforms and the variation in the in-vehicle travel times. Accounting for these additional variations could have possibly resulted in the shift to unattractive routes at a lower  $\xi$  value for the given  $\alpha$  length.

There are several aspects of the current research which can be further investigated such as:

- The current R-DSPM with strict capacity constraints only considers the numerical experiments on small networks. The possibility of extending it to much larger network needs to be explored. The challenge associated with a larger network lies in the possibility of several probable transfer stops between an OD pair. The presence of several transfer stops makes it necessary for a choice model between these transfer stops to ascertain the best alternative (Guo and Wilson, 2004; Liu et al., 1997; Shafahi and Khani, 2010). It is also to be noted that the current thesis does not consider any transfer penalty at the transfer stops and hence there is a need to account for such a penalty for realistic modelling of transit network. Guo and Wilson (2004) highlight the importance of transfer penalties and indicate that absence of accurate assessment of transfer penalties could result in over estimation or under estimation of travel costs. Liu et al. (1997) carry out a stated preference survey to assess the transfer penalty values in terms of in-vehicle travel times whereas Shafahi and Khani (2010) develop a model to minimise the transfer time of passengers.
- The current model assumes that the passenger learns from their own travel experiences on a route. Each route however comprises of route sections. The current study ignores the fact that a passenger travelling on a route learns not only the total travel time of the route but also the total travel time of the route section comprising it. Hence in example network 2 a passenger experiencing the waiting time for route 8

experiences the waiting time for route section D which is similar to the waiting time associated with route section K. This information can be used by the passenger while updating their experience matrix for route 2. This phenomenon of acquiring indirect information is called 'cross learning'. The process of accounting for cross learning is straight forward for the current model wherein the in-vehicle travel time is assumed to be constant. The cross learning is however ignored in the current study. The assumption of in-vehicle travel time being constant is a limiting assumption. Though the current R-DSPM with strict capacity constraints accounts for in-vehicle travel time variance when a particular route section has more than one attractive line section; there is a need to account for the random in-vehicle travel times on each line segment especially while considering bus networks. In absence of a dedicated right of way for bus transit the in-vehicle travel time of each line segment of the network is randomised due to the interaction with other traffic in the network. In event of assuming random in-vehicle travel times, the waiting time associated with route section K would only give partial information on the total travel time of route 2. The modeller would then have to make a learned guess to model the in-vehicle travel time while accounting for cross learning. While using the model to assess the train networks apart from considering the variance associated with the in-vehicle travel time it is to be noted that the variance associated with walking to the platform; ticketing process also needs to be accounted for.

- The generalised cost function used in R-DSPM can be easily modified to account for approaches such as the 'regret theory' and the 'prospect theory'. 'Regret theory' works on the principle that a passenger is interested in reducing the likelihood of something bad from happening (Chorus et al., 2008; Chorus, 2012). Prospect theory works on the principle that the route choice is made based on the gains or losses made with respect to a reference point (de Moraes Ramos et al., 2011; Gao et al., 2010; Ben-Elia and Shiftan, 2010). de Moraes Ramos et al. (2011), compared the expected utility maximisation concept with regret theory and prospect theory and found that both the prospect theory and regret theory under-perform for reliability based route choice. Ben-Elia and Shiftan (2010) highlight that the prospect theory is difficult to apply analytically due to its ability to describe the outcomes in short number with particular probabilities instead of probability density function as is

obtained in the current R-DSPM. The ability of passengers to remember extreme negative incidences should be reflected in the weighed learning process model wherein an experienced travel time exceeding certain permissible limit of each individual could be provided a higher weightage within the memory length. The possibility of such a weighed learning process implementation could be explored.

- Another extension would be to consider the departure time of passengers within the day so as to follow a 'scheduling based approach' to account for the disutility associated with arriving early or late at the destination. This would require an actual time table for the network which is being modelled in order to associate a specific arrival time for the passengers. The specific arrival times would enable the modeller to associated a schedule delay late or early penalty at the origin and destination.
- The burn-in time of the current R-DSPM with strict capacity constraints and the number of days for which the simulation is run to obtain a stationary distribution has been chosen arbitrarily. A more detailed study into the determination of burn-in periods and the determination of the stopping time needs to be explored. A look into Gilks et al. (1996) could provide several possible ways of doing the same.
- Though a section of London Underground has been explored in the current thesis a need to calibrate the numerous parameters assumed in the model for a real world network is required thereby emphasising the need for an empirical study on a real world transit network to assess the risk averseness of transit passengers.
- The present R-DSPM with strict capacity constraints can be extended to consider multiple user class who would define their attractive line set based on their economic welfare. The degree to which variance of total travel time is considered undesirable to the passengers in the mean-variance model and the value of mean lateness in mean-lateness model shall be different for each economic group, purpose of trip etc.
- The current R-DSPM with strict capacity constraints utilises logit choice for route choice identification. The logit choice model suffers from the inherent drawback of ignoring the overlaps in between different route

sharing the same route section. The R-DSPM could be extended to include the probit based models; c logit or path size logit models which account for the overlapping route sections in different routes of the network (Nielsen, 2000; Teklu, 2008a; Teklu, 2008b; Vovsha and Bekhor, 1998; Zhou et al., 2012). Teklu (2008a and 2008b) use probit based transit assignment model in the day to day framework and Nielsen (2000) show the implementation of probit models in SUE. Zhou et al. (2012) utilise C-logit in SUE based traffic assignment whereas Vovsha and Bekhor (1998) indicate the drawback of using C-logit.

- The current R-DSPM assumes a linear learning process models for the passengers which could be modified to account for habits wherein the route choice of the passengers is not altered on a day to day basis. A possible way to do that would be by using continuous markov process model wherein each passenger would have an exponentially distributed time interval on a particular route before they decide to update their route choice.
- Transit system around the world doesn't work in isolation and has several feeder services and 'intermediate public transport' systems acting as arteries and providing access to the public transit network. A need to look at the possible integration of such modes to develop a framework with multimodal route choice assessment is needed. Verma and Dhingra (2006) develop a combinatorial optimisation problem with a train scheduling sub-model and a schedule coordination sub-model to integrate the train services with the feeder bus services. Shrivastava and O'Mahony (2006) use genetic algorithm to integrate the main transit service with feeder buses leading to an improved patronage of transit services.
- The integration of R-DSPM with strict capacity constraints as an initial planning tool for transit network design development or transit network frequency modifications needs to be explored. The use of bi-level optimisation process could be considered as one of the possibilities for such an exploration.
- The current R-DSPM with strict capacity constraints does not consider fare of the transit system as one of possible disutility's in the generalised cost function. Hence the impact of fare on the route choice of risk averse

passengers needs to be explored. The exploration should be such that elasticity of the demand with variations in fare structure is accounted for in the model.

- The current thesis assumes a FIFO principle of queuing being followed at each transit stop which in reality is not the case. Hence the need to explore the mingling of the passengers at the transit stops needs to be modelled This could be done by not sorting the passengers at the transit stop by their arrival times. Instead those in the queue for the arriving transit service could have a probability associated with their boarding resulting in the boarding process being randomised.

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## Appendix A

### Formulation of Optimal strategy and assignment of flows on a test network as in Spiess and Florian, (1989)

(Spiess and Florian, 1989) formulate an optimisation problem to arrive at a routing policy which produces the least generalised cost route. Optimal strategy is based on the minimisation of the travel time (generalised cost) and works on the concept that traveller chooses the first vehicle that arrives from the attractive lines set at each bus stop.

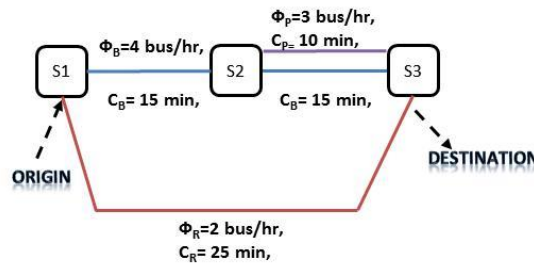


Fig A.1: Test network

For the test network in figure A.1, the alternative ways of travelling along the network are as shown in table A.1. A user is faced with these alternatives before or during his/her journey. These alternatives can be also called strategies. According to (Spiess and Florian, 1989) the optimum strategy is the strategy which the user perceives gives him/her the minimum generalised cost. Since it is assumed in (Spiess and Florian, 1989) that users have full knowledge of the frequency of services in a line and also have knowledge of the travel time involved, the ‘optimal strategy’ arrived by the algorithm in (Spiess and Florian, 1989) gives the path of minimum generalised cost of the network.

Table A.1: Possible Alternatives for travelling within the example network along with their costs

Stops	S1-S2	S2-S3			S1-S3			
Line	Blue	Blue	Purple	Blue/Purple	Blue	Red	Blue/Red	Blue & Purple (Blue/Purple)
travel time + waiting time (min)	30	30	30	17.15	45	55	38.33	39.65

The ‘optimal strategy’  $\bar{A}^*$  to travel from the origin to destination for the example network based on the optimisation algorithm and assignment algorithm defined by (Spiess and Florian, 1989) shall be obtained by minimising the following function

$$\min \sum_{l \in L} C_l v_l + \sum_{i \in I} \frac{v_i}{\sum_{l \in L_i^+} \phi_l x_l} \quad (A.1)$$



Subject to the conditions

$$v_l = \frac{x_l \varphi_l}{\sum_{n \in L_{i+}} \varphi_n x_n} v_i$$

$$v_i = \sum_{l \in L_{i-}} v_l + g_i$$

$v_i \geq 0$  and  $x_l = 0$  or  $1$  for  $i \in I$  and  $l \in \bar{L}$ .

In order to analyse the test network we shall simplify the network representation as in (Spiess and Florian, 1989) and the simplified network is given in fig A.2.

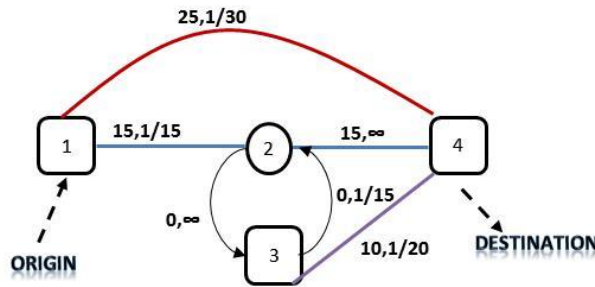


Fig A.2: Simplified Test network as in (Spiess and Florian, 1989)

The optimal strategy for the test network is given in table A.2 and the assignment of a unit flow from node 1 to node 4 or S1 to S3 is given in table A.3.

Table A.2: Finding Optimal Strategy for the Test Network

Iteration No:	Node Labels ( $u_i, \varphi_i$ )				Link with min $u_m + C_l$			
	1	2	3	4	$l = (i,m)$	$\varphi_l$	$u_j + C_l$	$l \in \bar{L}$
1	$\infty, 0$	$\infty, 0$	$\infty, 0$	$0, 0$	(3,4)	1/20	10	yes
2	-do-	-do-	30,1/20	-do-	(2,4)	$\infty$	15	yes
3	-do-	15, $\infty$	-do-	-do-	(3,2)	1/15	15	yes
4	-do-	-do-	21.4,0.117	-do-	(2,3)	$\infty$	21.4	no
5	-do-	-do-	-do-	-do-	(1,4)	1/30	25	yes
6	55,1/30	-do-	-do-	-do-	(1,2)	1/15	30	yes
	38.33,1/10	15, $\infty$	21.4,0.117	0,0				

Table A.3: Assign Demand on Test Network

Iteration No:	Link	
	(i,m)	Volume ( $v_l$ )
6	(1,2)	0.667
5	(1,4)	0.33
4	(2,3)	0.667
3	(3,2)	0.38
2	(2,4)	0.38
1	(3,4)	0.29

The step wise iterative process to arrive at the optimal strategy and the assignment of flows is as follows:

1<sup>st</sup> Iteration:

$$[(0 + 10), (0 + 15), (0 + 25)], (\infty + 0), (\infty + 15), (\infty + 0) \quad u_m + C_l = 10$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$   
 4 (4, 3) 4 (4, 2) 4 (4, 1) 3 (3, 2) 2 (2, 1) 2 (2, 3)

Start @ (4, 3)

$$\text{Update } u_3 \quad u_3 = 1 + \left(\frac{10}{20}\right) / \frac{1}{20} = 30$$

$$\varphi_3 = 1/20$$

Node 3 (30, 1/20) --- included in strategy - yes

2<sup>nd</sup> Iteration:

$$[(0 + 15), (0 + 25), (30 + 0)], (\infty + 15), (\infty + 0) \quad u_m + C_l = 15$$

4 (4, 2) 4 (4, 1) 3 (3, 2) 2 (2, 1) 2 (2, 3)

Start @ (4, 2)

$$\text{Update } u_2 \quad u_2 = 15$$

$$\varphi_3 = \infty$$

Node 2 (15,  $\infty$ ) ----- included in strategy - yes

3<sup>rd</sup> Iteration:

$$[(0 + 25), (30 + 0), (15 + 15), (15 + 0)] \quad u_m + C_l = 15$$

4 (4, 1) 3 (3, 2) 2 (2, 1) 2 (2, 3)

Start @ (2, 3)

$$u_3 = 30 \text{ From 1<sup>st</sup> iteration } \geq 15$$

$$\text{Hence, Update } u_3 \quad u_3 = \frac{30}{20} + \frac{15}{15} / \frac{1}{15} + \frac{1}{20} = 21.4$$

$$\varphi_3 = \frac{1}{15} + \frac{1}{20} = 0.117$$

Node 3 (21.4, 0.117) ----- included in strategy - yes

4<sup>th</sup> Iteration:

$$[(0 + 25), (21.4 + 0), (15 + 15),] \quad u_m + C_l = 21.4$$

4 (4, 1) 3 (3, 2) 2 (2, 1)

Start @ (3, 2)

$u_2 = 15$  From 2<sup>nd</sup> iteration  $< 21.4$

Hence, ----- included in strategy – no

5<sup>th</sup> Iteration

$$[(0 + 25), (15 + 15),] \quad u_m + C_l = 25$$

4   (4, 1)   2   (2, 1)

Start @ (4,1)

Update  $u_1 \quad u_1 = 1 + \frac{25}{30} / \frac{1}{30} = 55$

$$\varphi_1 = 0 + \frac{1}{30} = \frac{1}{30}$$

Node 1 (55, 1/30) ----- included in strategy - yes

6<sup>th</sup> Iteration

$$[(15 + 15),] \quad u_m + C_l = 30$$

2   (2, 1)

Start @ (2, 1),  $u_1 = 55$  From 5<sup>th</sup> iteration  $\geq 30$

Hence , Update  $u_1 \quad u_1 = \frac{55}{30} + \frac{25}{30} / \frac{1}{30} + \frac{1}{15} = 38.33, \quad \varphi_1 = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$

Node 1 (38.33, 1/10) ----- included in strategy – yes

Assignment:

Arrange the links in decreasing order of  $u_m + C_l$

$$(1,2) \rightarrow v_l = \varphi^l / \varphi_i v_i = \frac{1}{15} / \frac{1}{10} (1) = 0.667$$

$$(1,4) \rightarrow \frac{1}{30} / \frac{1}{10} (1) = 0.33$$

$$(2,3) \rightarrow 0.667$$

$$(3,2) \rightarrow \frac{1}{15} / 0.117 (0.667) = 0.38$$

$$(2,4) \rightarrow 0.38$$

$$(3,4) \rightarrow \frac{1}{20} / 0.117 (0.667) = 0.29$$

## Appendix B

### Route Choice of passengers at Transit stop with and without signs at stop information

Consider the test network given in Fig B.1. At bus stop S2 we have two lines (blue and purple) with the blue line having a frequency of 4 bus/hr and a travel time of 20 min whereas the purple line has a frequency of 3 bus/hr and a travel time of 10 min.

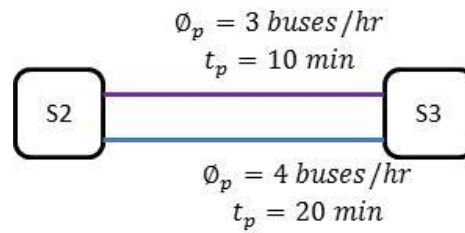


Fig B.1: Test Network

Assuming that the headway distribution is deterministic we get the probability of choosing a line that arrives first at bus stop S2 (in the absence of information) as given in equation below.

$$\eta_{l,j} = \int_0^{\infty} f_l(\partial') \prod_{n \in L^* \setminus \{l\}} \bar{F}_n(\partial') d\partial'$$

The p.d.f of waiting time of line given that the line has deterministic headway distribution can be formulated as

$$f_l(\partial) = \begin{cases} \varphi_l & \text{if } 0 \leq \partial' \leq 1/\varphi_l \\ 0 & \text{otherwise} \end{cases}$$

If  $\bar{u} = \min\{u_n | n\} = 1, 2, \dots, L$  then for deterministic headways

$$\eta_{l,j} = \int_0^{\bar{u}} \varphi_l \prod_{n \in L^* \setminus \{l\}} (1 - \varphi_n \partial') d\partial'$$

Hence for the example network in fig B.1 the conditional probability of choosing line 1 (purple) line is given as

$$\eta_{purple,j} = \int_0^{15} \frac{1}{20} * (1 - 1/15 x) dx$$

$$\eta_{purple,j} = 0.375$$

$$\eta_{blue,j} = \int_0^{15} \frac{1}{15} * (1 - 1/20 x) dx$$

$$\eta_{blue,j} = 0.625$$

With information the conditional probability if choosing a line is given as

$$\eta_{purple,j} = \int_0^{20} \frac{1}{20} * \overline{F_{2|1}} (\vartheta' - 10) dw$$

$$= \int_0^{10} 1/20 dx + \int_{10}^{15} \frac{1}{20} * \left( \frac{25-x}{15} \right) dx + \int_{15}^{20} 1/20 * \left( \frac{25-x}{15} \right) dx$$

$$\eta_{purple,j} = 0.833$$

$$\eta_{1,j} = \frac{3}{7} = 0.4286$$

$$\eta_{2,j} = \frac{4}{7} = 0.4286$$

$$\eta_{blue,j} = \int_0^{10} \frac{1}{15} * \overline{F_{1|2}} (\vartheta' + 10) dw$$

$$= \int_0^5 1/15 * \left( \frac{10-x}{20} \right) dx + \int_5^{10} \frac{1}{15} * \left( \frac{10-x}{20} \right) dx$$

$$\eta_{blue,j} = 0.167$$

Table B.1: Deterministic headway: on-line information vs no information

Information Scenario	$\eta_{purple,j} =$	$\eta_{blue,j} =$
Sign at stop	0.833	0.167
Without sign at stop	0.375	0.625

## Appendix C

### Markovian property Check for Mean-lateness model

The presence of single stationary distribution for the mean lateness model to prove the ergodic nature of the R-DSPM is shown in Fig C.1. From the visual inspection and from comparison of the mean and standard deviation of the flows between various day intervals it can be concluded that there exists a single stationary distribution. The presence of single stationary distribution implies that the R-DSPM for mean-lateness is ergodic.

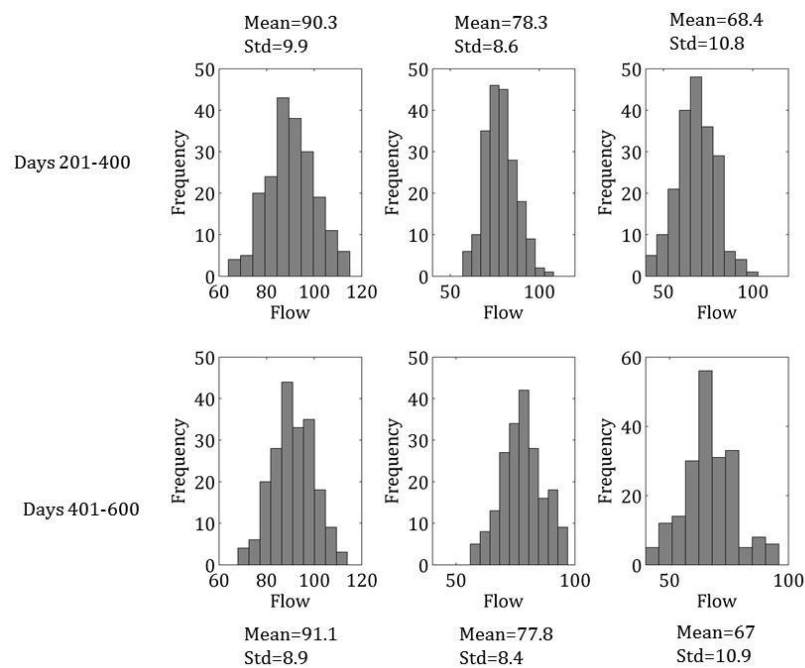


Fig C.1: Stationary Distribution on routes 2,3 and 9 between days 201-400 and days 401-600 for  $\xi = 0.05, \mathcal{T}(Z = 1) = 48.5$  and  $\mathcal{T}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

#### C.1 Converging irrespective of initial condition:

The convergence of the R-DSPM irrespective of its initial conditions to the same stationary distribution is a proof of that the current stochastic process is regular. To prove that the mean-lateness R-DSPM is regular different initial conditions were tested similar to those done in earlier chapters ( $Z=1$ - poisson rate of passenger arrivals-400/3600, population size (constant demand)-83;  $Z=2$ -poisson rate of passenger arrivals-250/3600, population size (constant demand)-88). The comparison of mean and standard deviation (Table C.1) indicates convergence to the same distribution.

Table C.1: The convergence of mean-lateness model irrespective of its initial condition  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\xi = 4, \eta = 15, \frac{\theta_2}{\theta_1} = 5$

Route		1	2	3	4	5	6	7	8	9	10	11
Initial condition II	$E(t_s)$	22.6	41.9	43.4	73.3	95.7	57.1	83.7	12.9	74.6	84.5	69.5
	Std	26.4	10	7.7	50.3	34.5	27.8	23.5	37.8	20.3	21.0	19.0
	$E(X_s)$	1.7	175.6	139.9	5.4	14.9	13.1	50.1	0.2	78.6	73.8	96.3
	Std	3.2	49.3	31.7	7.6	15.4	14.3	18.8	0.7	13.9	19	16.7
Initial condition III	$E(t_s)$	15.7	41.3	42.5	65.8	90.6	55	83.3	10.2	73.8	84.8	68.9
	Std	23.5	102	7	52.3	38.1	29.6	22.7	31.6	20	20.5	18.1
	$E(X_s)$	1.1	177.3	142.8	4.1	13.3	10.8	48.6	0.2	79.6	73.7	97.6
	Std	2.6	51.3	35	6.8	16	12.8	18.0	0.57	15.1	20.2	17.2

A statistical test of Wilcoxon rank sum test was carried out to check if the distributions obtained from various initial conditions were indeed similar and having the same mean or not. The test results are shown in Table C.2. The table C.2 results show that the null hypothesis cannot be rejected at the 5% significance level as all the p-values are greater than 0.05 in all cases for  $m = 1$ . This shows that there is not sufficient evidence to show that the samples from the three realisations do not come from the same stationary distribution and do not have the same median.

Table C.2: Wilcoxon rank sum test  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\xi = 4, \eta = 15, \frac{\theta_2}{\theta_1} = 5$

Route		1	2	3	4	5	6	7	8	9	10	11
$E(t_s)$	1 vs 2	0.15	0.85	0.83	0.1	0.53	0.45	0.79	0.15	0.48	0.86	0.3
	2 vs 3	<b>0.01</b>	0.66	0.68	0.21	0.71	0.76	0.79	0.14	0.44	0.94	0.36
	1 vs 3	0.38	0.86	0.79	0.66	0.76	0.35	0.99	1	0.92	0.97	1
$E(X_s)$	1 vs 2	<b>0.03</b>	0.92	0.33	<b>0.04</b>	0.33	<b>0.03</b>	0.39	0.16	0.71	0.55	0.12
	2 vs 3	0.01	0.69	0.83	0.07	0.44	0.07	0.42	0.16	0.89	0.38	0.06
	1 vs 3	0.55	0.9	0.62	0.7	0.72	0.64	0.95	1	0.82	0.88	0.65



Table C.3: Two-sample Kolmogorov-Smirnov test  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\xi = 4, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

Route		1	2	3	4	5	6	7	8	9	10	11
$E(t_s)$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	1	0	0	0	0	0	0	0	0	0	0
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0
$E(X_s)$	1 vs 2	0	0	0	0	0	0	0	0	0	0	0
	2 vs 3	0	0	0	0	0	0	0	0	0	0	1
	1 vs 3	0	0	0	0	0	0	0	0	0	0	0

The two sample Kolmogorov-Smirnov test gives the same result as wilcoxon rank sum test whereby it is observed that the samples are from the same distribution.

## C.2 Sensitivity analysis :

### C.2.1 Differing shape factors

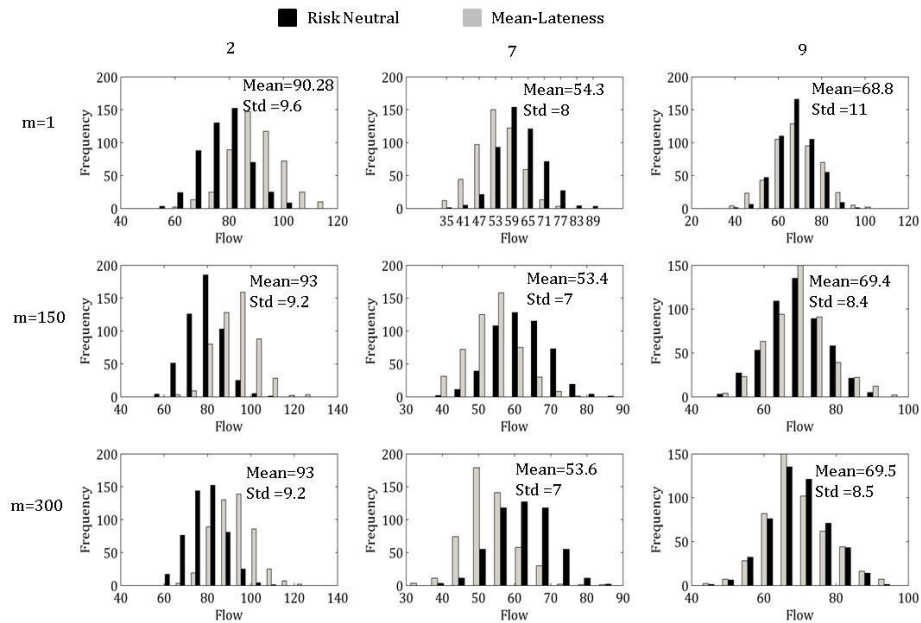


Fig C.2 the sensitivity of route flows to various shape factors for risk neutral and mean-lateness cost functions at  $\xi = 0.05$   $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

Figure C.2 indicates that at routes originating at transit stop 1 the flows distribution between the risk neutral and mean-lateness cost function is different however at the lower end transit stop 2 the distribution is almost similar. The distribution of route flows for various shape factor values at  $\xi = 0.05$  for mean-lateness cost function is also almost similar to each other.

### C.2.2 Differing value of lateness

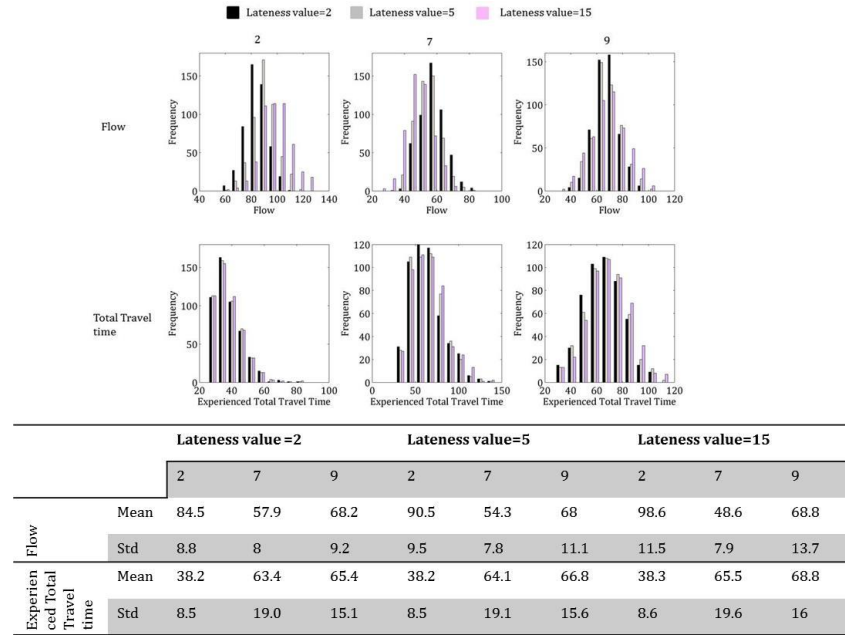


Fig C.3 Flow and experienced total travel distribution on various routes for varying value of lateness at  $\xi = 0.05$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\alpha = 15$

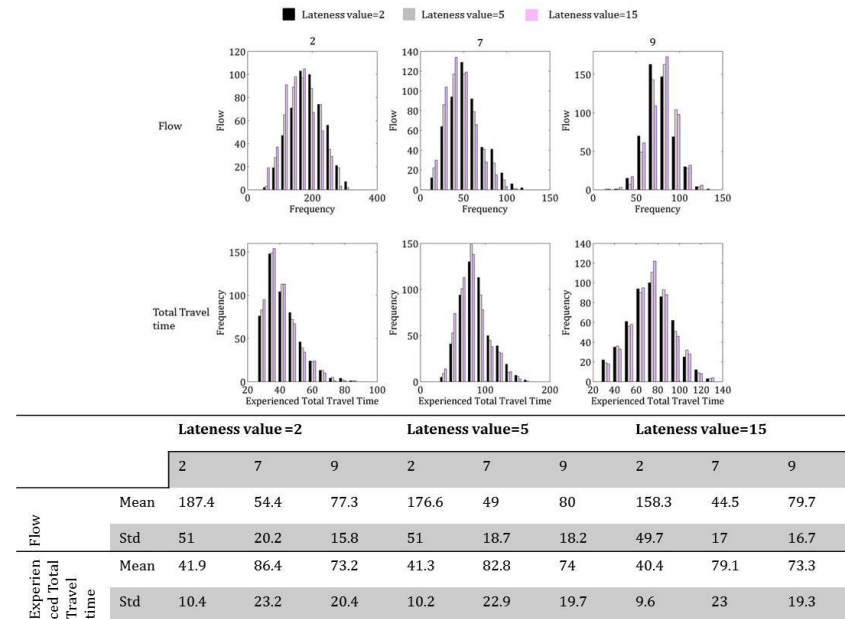


Fig C.4 Flow and experienced total travel distribution on various routes for varying value of lateness at  $\xi = 4$ ,  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\alpha = 15$

The flow distribution and its standard deviation vary significantly on various routes for both at  $\xi = 0.05$  and  $\xi = 4$ . However a decreasing trend in the mean values of the flows with increasing value of lateness is observed at higher  $\xi$  value.

### C.2.3 Differing memory lengths

Table C.4: sensitivity of experienced travel times and flows for differing memory lengths  $\mathcal{T}(Z = 1) = 48.5$  and  $\mathcal{T}(Z = 2) = 32.5$ ,  $\xi = 4, \frac{\theta_2}{\theta_1} = 5$

		Risk Averse			Risk Neutral		
Route		2	7	9	2	7	9
$\kappa=5$	$E(t_s)$	40.9	84	67.2	41.6	88.1	66.0
	Std	9.7	23.2	19	10.2	24.1	19.8
	$E(X_s)$	169.6	89.2	82.8	187.6	89.2	67.9
	Std	48.6	16.5	17.5	39.0	17.6	18.1
$\kappa=15$	$E(t_s)$	41.3	83.4	73.9	42.3	89.1	70
	Std	10.1	22.8	19.6	10.7	24.4	20.6
	$E(X_s)$	177.0	48.6	79.8	197.3	67	67.4
	Std	50.8	19.0	15	48.0	24.1	20
$\kappa=30$	$E(t_s)$	41.3	83.7	74.3	42.3	88.9	70.2
	Std	10.1	22.8	19.7	10.7	24.1	20.6
	$E(X_s)$	178.6	47.6	80.2	197.5	66	66.9
	Std	51.0	18.7	14.6	47.8	23.4	20.2

At lower memory lengths the distinction between risk neutral and risk averse passengers is still obvious for higher  $\xi$  values but the shift to routes found unattractive by risk neutral passengers by the risk averse passengers happens only at higher memory values. It is again seen that after certain memory length the expected travel time and flow values along the routes stabilises as passengers become aware of the complete network and hence there is not much of a distinction in these values between the memory length of 15 days and 30 days.

### C.3 Policy Interventions

#### C.3.1 Increasing capacity of line 2:

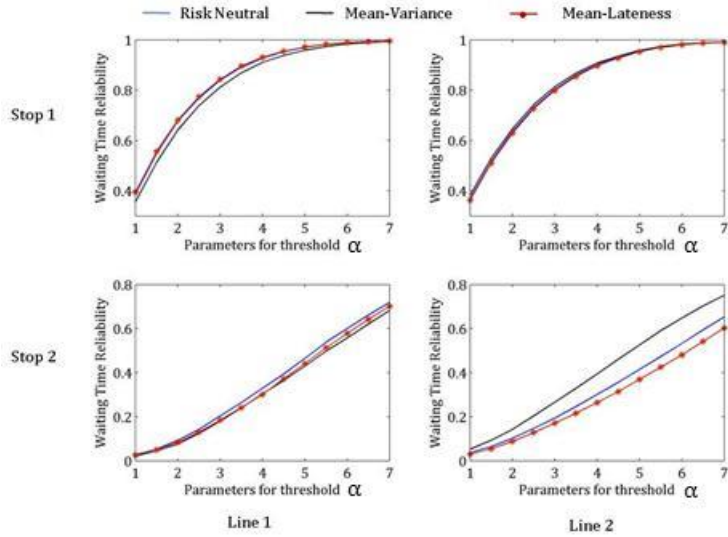


Fig C.5 : Waiting Time reliability for various cost functions  $l_2$  cap=25 at  $\xi=0.05$   
 $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\gamma = 15$ ,  $\frac{\theta_2}{\theta_1} = 5$ ,  $\beta = 2.5$

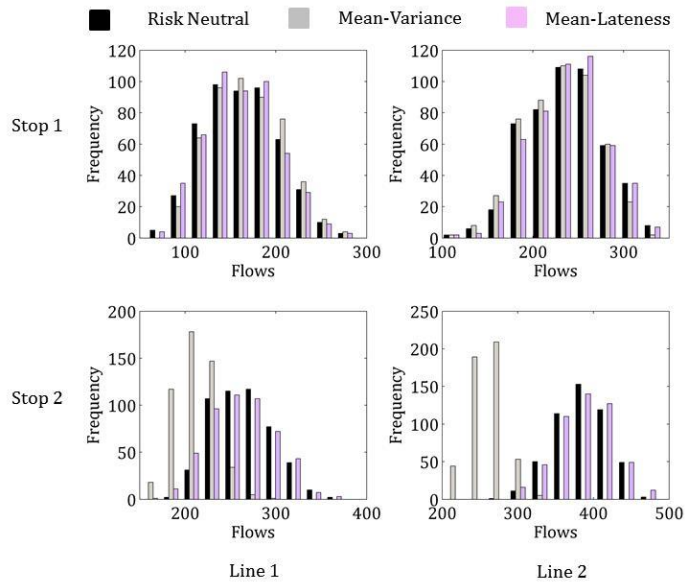


Fig C.6 : Line loads at  $\xi=0.05$  for increased capacity  $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5$ ,  $\xi = 4$ ,  $\gamma = 15$ ,  $\frac{\theta_2}{\theta_1} = 5$

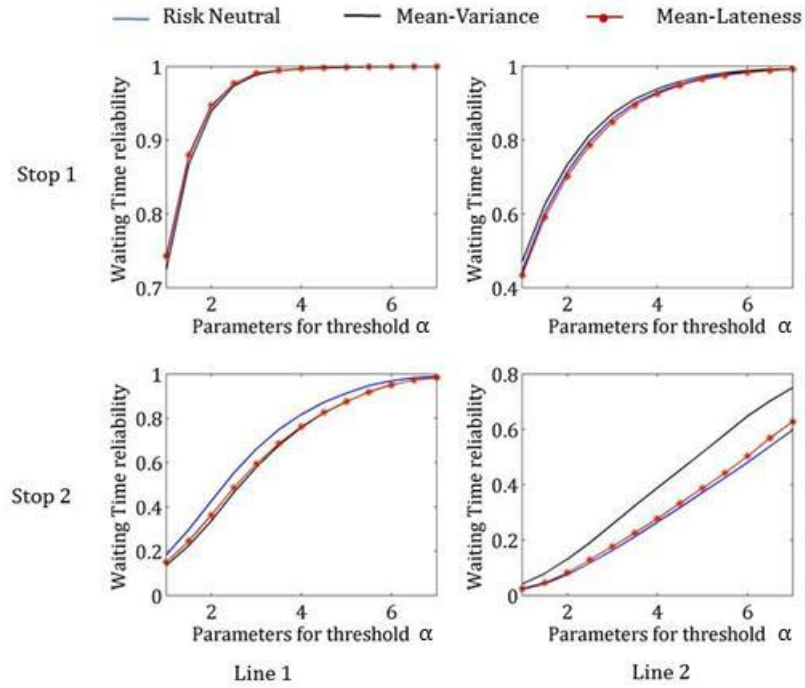


Fig C.7 Waiting Time reliability at  $\xi=0.05$  for increased frequency of line  $\mathcal{T}(Z = 1) = 48.5$  and  $\mathcal{T}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

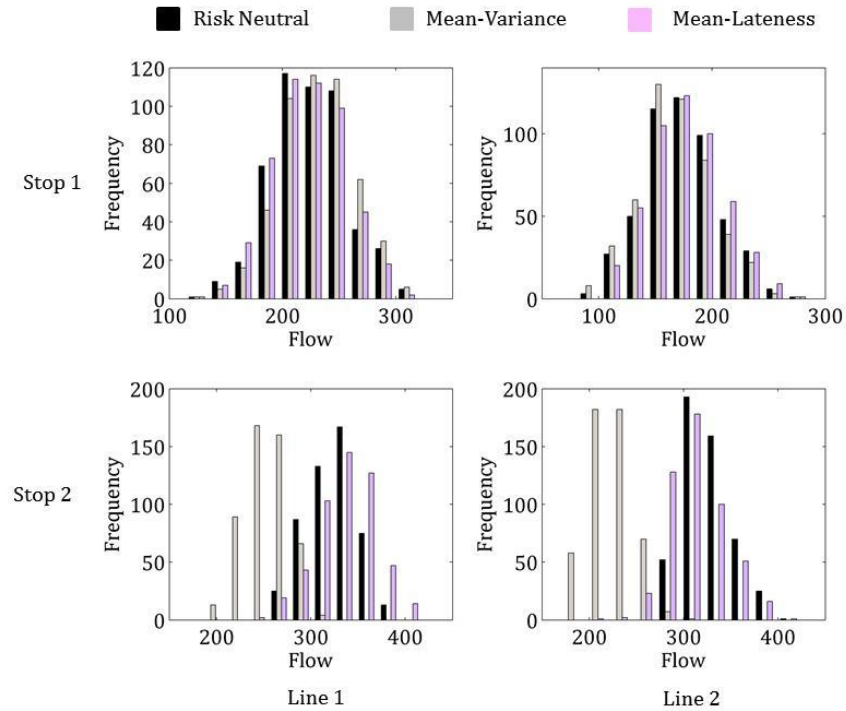


Fig C.8: Line loads at  $\xi=0.05$  for changed frequency  $\mathcal{T}(Z = 1) = 48.5$  and  $\mathcal{T}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

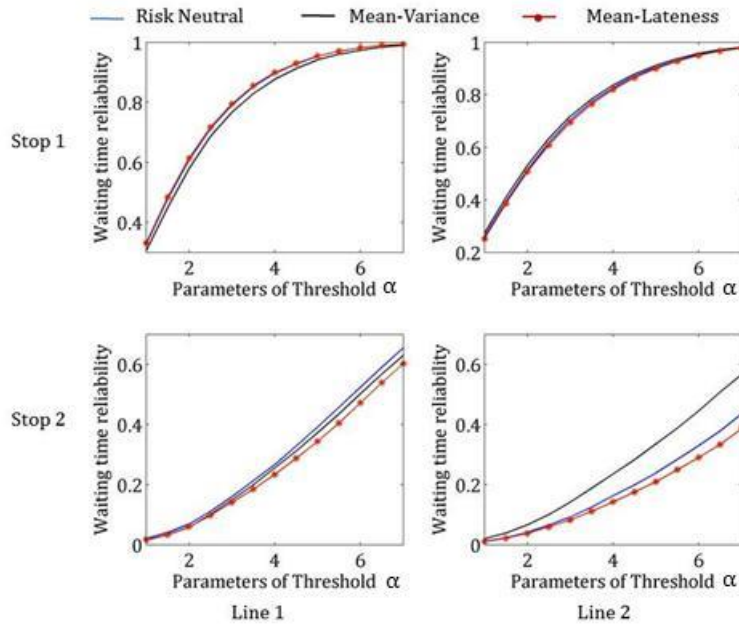


Fig C.9 Waiting Time reliability at  $\xi=0.05$  changed dwell time 2  
 $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

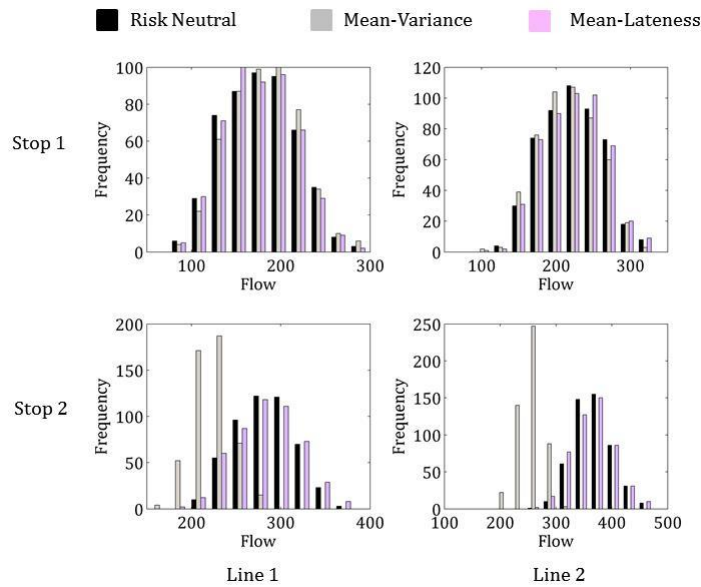


Fig C.10 : Line loads at  $\xi=0.05$  for changed dwell time  
 $\mathcal{J}(Z = 1) = 48.5$  and  $\mathcal{J}(Z = 2) = 32.5, \varrho = 15, \frac{\theta_2}{\theta_1} = 5$

The policy interventions at  $\xi=0.05$  indicate that there is very slight difference in the waiting time reliability profile of the cost functions at various stops of example network 2. This indicates that if the passengers perception error in the cost function is assumed large then the impact of policy interventions may seem almost nil.

## **Appendix D Publications**

Conference presentations:

1. Seetharaman. P , Watling. D and Balijepalli. N.C. (2013) 'Hyperpath Line Choice Probability during Bus Breakdowns in a Transit Network', in proceedings of the 45<sup>th</sup> Universities Transport Study Group Annual Conference , UTSG, Oxford, UK.
2. Seetharaman. P , Watling. D and Balijepalli. N.C. (2013) 'Modelling the impact of unreliability on passengers' strategy choice in public transport networks -a hyperpath approach', Symposium of the European Association for Research in Transportation hEART 2014 -2<sup>nd</sup> Symposium of the European Association for Research in Transportation, KTH University, Stockholm, Sweden.
3. Seetharaman. P , Watling. D and Balijepalli. N.C. (2014), 'Markovian framework to assess route choice in a transit network service influenced by endogenous and exogenous reliability factors', Symposium of the European Association for Research in Transportation hEART 2014 - 3<sup>rd</sup> Symposium of the European Association for Research in Transportation, University of Leeds, Leeds, UK.
4. Seetharaman. P , Watling. D and Balijepalli. N.C. (2015),' Reliability Based Disaggregate Stochastic Process Model in Congested Transit Network', 47<sup>th</sup> Universities Transport Study Group Annual Conference , UTSG, London, UK.