MODELLING AND SIMULATION OF PRODUCTION-MARKETING SYSTEMS

by

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To my parents, Omotayo and Oreitan, and to Teri, Toju, Osaro, Sarah-Frances and Marie-Rose:

'The best is yet to come.'

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O. O. Apampa

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Summary

Three aspects of complexity in a Production-Marketing System are identified, namely the Model, Decision-Making and Behavioural complexities. Control theoretic and other techniques are reviewed in the context of their contribution to the resolution of these complexities and the modelling and simulation of the PMS is also viewed in this light.

Several analytical models of a consumer-durable marketing system are developed, reflecting various assumptions of market conditions ranging from the single-product constant-decision marketing system to the multi-product variable-decision marketing system, the latter explicitly accounting for price, advertizing, distribution and quality decisions for each product, repeat purchase dynamics and the tastes, income and population of consumers. A production system model is also developed involving a multiple final product, multi-stage production process, permitting the backlogging of demand, variation of production rate by variation in work-force levels and overtime and subcontracting of manufacture of intermediate products.

Computer simulations of the marketing system based on the models developed and using assumed data are carried out. An optimization routine is used to generate the variable decisions. The variable-decision marketing model is combined with the production model and the resulting limited capacity PMS is simulated using assumed data.

The simulation results are presented graphically and attention drawn to their realism. The use of the PMS simulation programme as a nucleus for a comprehensive PMS simulation and control package is commented upon.

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CHAPTER I

INTRODUCTION

1.1 THE PRODUCTION-MARKETING SYSTEM

The Production-Marketing System (PMS) is a complex socio-economic system, by this is meant that it is a human organization set up to achieve various social and economic goals.

Production has been defined [1] as "the intentional act of creating something useful." Implied in this definition are two fundamental characteristics of Production:

(i) Production involves decision-making,

(ii) Production involves a process of transforming a set of inputs into a set of outputs (products) which has value placed on it either by the production system and/or its environment.

Marketing has been described [2] as "The human activity directed at satisfying needs and wants through exchange processes." This in turn gives rise to the following characteristics of Marketing.

(i) Marketing involves decision-making,

(ii) Marketing involves the estimation of "needs and wants" of its environment and the acquisition, distribution (in space and time) and exchange of "things" (products) capable of satisfying these "needs and wants."

The activities of Production and Marketing meet in the entity called the product. Production activities create the product while Marketing activities exchange it for entities of value.

1.1.1 The Production Process

The production process is a multi-stage resource transformation system, converting input resources in the form of raw material inventories into output resources in the form of finished goods inventories thereby creating value. The value created by the process derives from the form, place and time utilities created as a result of its activities. These activities are of three kinds:

(i) Conversion;

(ii) Transportation;

(iii) Storage.

Conversion is the activity that transforms input inventory items (raw materials and work-in-process), by utilizing the production resources of labour (skill), capital (machines and equipment) and energy, into output items of inventory (work-in-process and finished goods) having a greater utility (of form) as a result. Conversion activity carried out at workstation level is called operation. (A workstation is the elemental centre of conversion capable of independent conversion activity scheduling). Operations are of two types:

(i) Assembly operations - where two or more units of an item (or one or more each of several items) of input inventory are operated upon together to form one unit of output inventory;

(iii) Unit operations - in which one unit of one item of input inventory is converted into one or more units of one item of output inventory.

Transportation (also called Materials Handling) is the activity of moving inventory between workstations and generates place utility within the factory. Lastly, storage is a function of elapsed time accompanied by no change of form and place of inventory. It confers time utility on the items of inventory stored and plays a significant part in production smoothing [3].

1.1.2 The Marketing Process

The Marketing Process consists of four kinds of activity:

(i) Communication - advertizing, the activity of informing and influencing buyers through advertizing media;

(ii) Transit - the activity of transporting goods to the buyers from the producers thus imparting place utility on the product;

(iii) Storage - the activity of carrying an inventory of products out of which orders are filled, conferring time utility on product;

(iv) Contact - the activity of searching for and negotiating with buyers over terms giving rise to possession utility of the product.

1.1.3 Market Demand

Market demand (hereafter referred to as demand) is defined as the volume of the set of products of a given seller (or set of sellers) that would satisfy a given market segment under given market conditions in a given period of time; a market segment is a set of consumers (buyers) considered together with a given set of its wants and a product is something that is capable of satisfying this set of wants. Thus a market segment simultaneously defines a product class and a set of consumers, and demand measures the intensity of the wants of this market segment in terms of aggregate volume of a set of products in this product class under specified market conditions.

In a PMS, the role of demand is difficult to over-emphasize as it serves four basic purposes:

(i) A measure of current market potential;

(ii) An aggregate measure of the environmental consequences of marketing activities;

(iii) A measure of the intensity of want of a set of consumers;

(iv) A measure of the volume of output desired from the production system.

Thus, to a seller, demand represents the sales potential under current marketing conditions (which he can influence) and an aggregate variable representing the impact of his activities on his environment; to the consumer (buyer) demand is a measure of his state of felt deprivation in terms of volume of product of the given seller, and to the producer it is a measure of the required volume of his output.

1.1.4 Time scale of Production-Marketing Decision-Making

Decision-making in a PMS is required to provide an economic balance between production output and demand. It is a complex process requiring interaction between many levels of the decision-making hierarchy and on widely different time scales. Figure 1.1 shows the main elements of the decision-making hierarchy and the typical time scale on which each level operates.

Figure 1.1 Time decomposed Production-Marketing Decision-Making Structure

TIME SCALE	PRODUCTION DECISION FUNCTION	MARKETING DECISION FUNCTION
2-10 years (Very long-term)	Production System Development: (i) Product Development (ii) Production Process Development (iii) Production Centre (Factory) Development	Marketing System Development: (i) Product Development (ii) Market Development (iii) Marketing Channel Development (iv) Sales Outlet Development
6 months-2 years (Long-term)	Capacity Planning: (i) Modifications in Work Methods (ii) Modifications in Production Facilities: (a) Modifications to Workstations (b) Modification to spatial arrangement of Workstations (Plant Layout)	Planning of Marketing Programmes: (i) Planning Product Line for each Market, ie. combination of Products to be sold in each market (ii) Planning of aggregate Marketing mix in each market, eg. Price and Quality levels of each product, Advertizing Strategy, Marketing Channel, Distribution Strategy in each Market (iii) Estimation of long-term Market Demand for each product
1-6 months (Medium-term)	1. Aggregate Resource Allocation and Scheduling: (i) Allocation and Scheduling of Labour (ii) Allocation and Scheduling of Workstations (iii) Allocation and Scheduling of Raw Materials (iv) Process Maintenance Scheduling 2. Aggregate Output Control (Inventory Management) (i) Quantity control (ii) Quality control	1. Detailed Implementation of Marketing Policies: (i) Reacting to variations in marketing environment (ii) Estimation of Sales of each product in each market 2. Distribution logistics management
1 day-1 week (Short-term)	Production Process Control; that is the detailed assignment of Labour, Production Inventory (Raw Materials and Work-in-Process) and Workstations to meet short-term output requirements	Exchange Process Control; that is order taking, and assignment of marketing resources to marketing outlets

1.2 PROBLEMS INVOLVED IN THE STUDY AND OPERATION OF A PMS

A PMS given a feedback control system representation is shown in Figure 1.2.

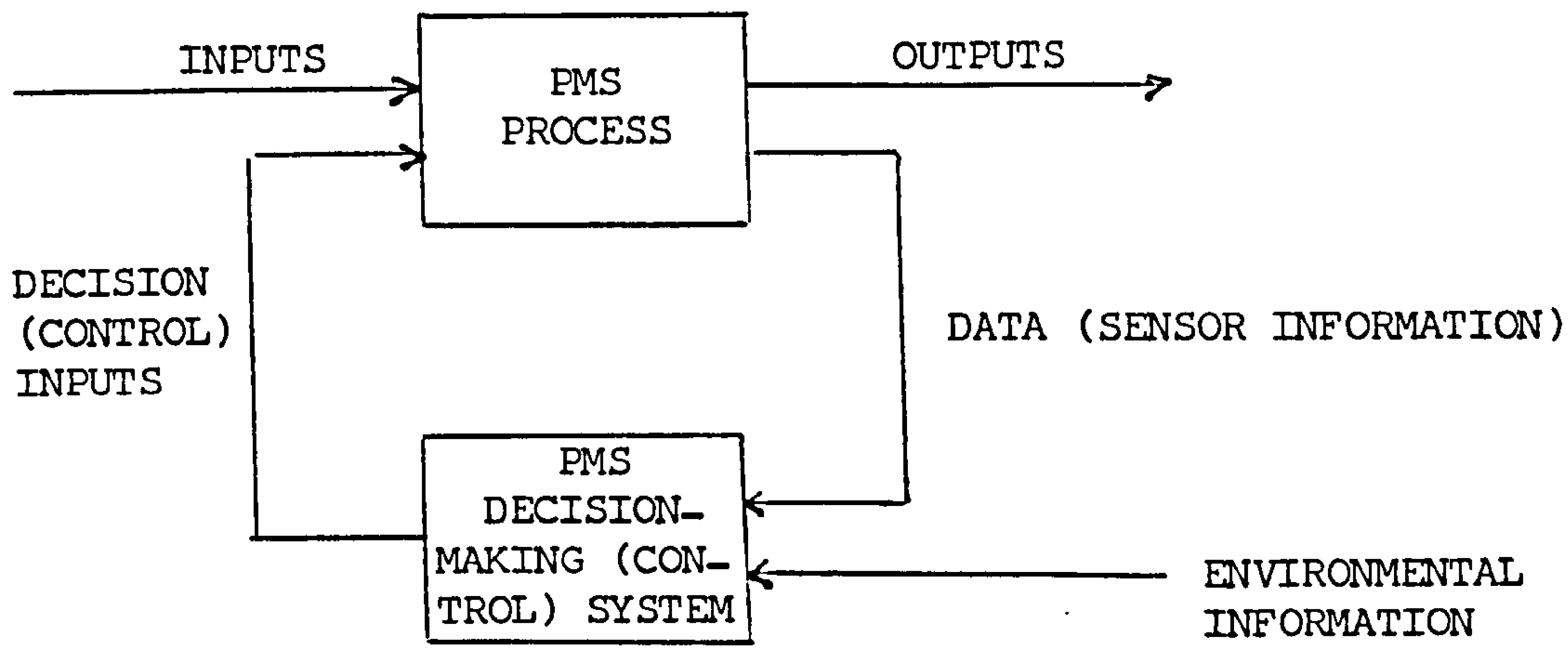

FIGURE 1.2 : THE PMS AS A FEEDBACK CONTROL SYSTEM

Mathematically, the PMS process can be conceptualized as a non-linear, multivariable dynamic system with time-delays and time-varying parameters with some of the variables manipulable by the PMS decision-making system.

A basic problem in the study and operation of a PMS arises from the difficulty involved in deriving a model or set of models detailed enough to represent the process adequately (at least to the extent that it relates measurable variables to each other) yet simple enough to permit its use for decision-making purposes. The fact that decision-making is carried out over different time-scales and with respect to different aspects of the process (as mentioned in 1.1.4 above) necessitates that, in general, a set of models is required and these models taken together should represent the process adequately. This basic problem is summarily referred to as the problem of Model Complexity.

The large number of manipulable process variables, the uncertainties characterizing the process environment, the time delays involved in obtaining information about current process behaviour, and the multiplicity of objectives pursued are all features of the second basic problem, that of Decision-Making Complexity. To give an idea of the scale of the problem, a PMS may involve the production and marketing of a number of products running into thousands [2,p.185], the objectives of the PMS are usually such that the attainment of some preclude the attainment of others, i.e., they conflict with each other [4,p.195], typically the behaviour of the PMS environment is variable, thus in addition to operating the PMS the decision-making system must incorporate strategies to adapt the PMS to its variable and uncertain environment.

In general, the set of models describing the PMS process is not available centrally, and decisions based on each of these models tend to be made in quasi-independence of each other because of the limited information processing and communication capacity of the decision-making system. The third basic problem then results: what are the effects of these quasi-independent decisions, based on a multiplicity of process models, on PMS behaviour (system dynamics [5]) and what are the qualitative properties (controllability, observability, stability, etc.) of this behaviour? This problem is summarily referred to as the Behavioural Complexity problem.

1.3 SUMMARY OF WORK REPORTED IN THE THESIS

The aims of the work reported in this thesis are two-fold:
(1) to review and assess the efficacy of control theoretic and other techniques to the solution of the three basic problems involved in the study and operation of a PMS as described in 1.2 above and
(2) to contribute to the understanding of the system dynamics aspect of the behavioural complexity of a PMS by the development of an interactive computer simulation package, simulating the dynamic behaviour of a PMS under conditions that can be varied by the operator. The package includes models of different aspects of the PMS operating on different time-scales (1.1.4 above) and driven by simulated quasi-independent decisions. The approach to PMS modelling embodied in this package represents a significant departure from the usual approach [5,6] where a single time-scale is used and therefore the effect of PMS adaptation cannot be included in the resulting simulation.

A description of the organization of the rest of the thesis now follows. Chapter II reports the work done in pursuance of the first aim described above and includes a review of previous work done on the modelling and simulation of a PMS. In Chapter III the models developed for the production aspect of a PMS are described while the models for the marketing aspect are discussed in Chapter IV. Chapter V describes the models of the quasi-independent decisions developed for both production and marketing aspects and comments upon the results of the various computer simulations. Chapter VI concludes the thesis and in it an evaluation of the work done is carried out, and its limitations and scope for future enhancement commented upon.

CHAPTER II

A REVIEW OF CONTROL THEORETIC AND OTHER TECHNIQUES AVAILABLE FOR THE RESOLUTION OF PRODUCTION-MARKETING SYSTEM COMPLEXITIES

2.1 PREAMBLE

A variety of techniques have been described in the control literature and elsewhere for analyzing complex dynamic systems and designing control strategies for them. In the main, these techniques have been based on the presumption of centrality; all the information available about the system, and the calculations based upon this information (information processing) are centralized, i.e., take place at a single location. Two kinds of available information are distinguishable:

(i) A priori or "off-line" information about the system, e.g., system model, objectives, constraints, etc.

(ii) Sensor information about actual system behaviour, i.e., at each time instant, the set of all measurements on the system made up to that time.

When dealing with Production-Marketing systems the presumption of centrality fails to hold due to the lack of centralized information or the lack or inadequacy of centralized information processing facility. Thus economic cost and reliability of communication links and information processing facilities have to be included in a PMS analysis or control strategy design.

This chapter discusses techniques which have been developed for the resolution of model and decision-making complexitites in large-scale systems where the presumption of centrality does not hold, and it reviews previous work done with regard to production-marketing systems on the question of the systems dynamics aspect of behavioural complexity. Accordingly, the chapter divides into three parts; in sections 2.2 and 2.3 techniques aimed at resolving model and decision-making complexities respectively are discussed while section 2.4 reviews previous work done on the modelling and simulation of the PMS.

2.2 TECHNIQUES DEALING WITH MODEL COMPLEXITY

2.2.1 Model Types

Ultimately, analyses of dynamic phenomena result in the formulation of (dynamic) mathematical models which belong to any one of three categories.

(1) Deterministic models - characterized by

(a) Partial differential equations [7], where continuums of space and time are presumed, or,

(b) Ordinary differential equations [8], where discrete space and continuous time are presumed, or,

(c) Difference equations [9], where discrete space and time are presumed

and for (a), (b) or (c) above, system elements are assumed to be deterministic time functions.

(2) Stochastic Models - characterized by differential or difference equations with random elements [10,11,12]. A key notion of wide applicability is that, though only known probabilistically, the state at time t+1 depends only on the state at time t. This property, the Markov property, means that the transition probability between state i at time t and stage j at time t+1 is all that needs to be known. If it is independent of t, the process is stationary. Time series models [13], i.e., models of outputs of stochastic processes, play a key role in many applications.

(3) Fuzzy Models - characterized by differential or difference equations with fuzzy elements [14,15,16]. The concept of fuzzy sets [20] has been developed via the theory of fuzzy statistics [21,22] to yield

the framework for fuzzy dynamical systems described in [15,16].

Fuzzy dynamical systems theory is still at a conceptual stage of development thus all dynamic models of the PMS at present are either of the deterministic or stochastic type. It is, however, worth noting that the progression from deterministic to stochastic and thence to fuzzy types reflects the increasing complexity, uncertainty, or vagueness associated with the dynamic phenomenon under study.

2.2.2 Model Simplification

Often, the available information processing capacity is inadequate and thereby forces the use of mathematical models that are simpler but less accurate than the best available model of the given dynamical system. Model simplification techniques outlined below provide simplified versions of a given model that satisfy certain criteria. Reference [23] contains an excellent survey of these techniques.

Model simplification techniques can be divided into two classes. The first class, called aggregation techniques simplifies a given model in state-space form by aggregating (forming a set of new variables fewer than those existing before from linear combinations of existing variables) the state variables and perhaps the input or output variables as well. This is done in such a way that the "errors" between the aggregated model and the best available model are within certain bounds. The definitions of these "errors" and linear combinations differentiates the various aggregation techniques from each other.

The second class, called perturbation techniques, consists of procedures in which certain dynamic interactions are ignored (and can thus be viewed as approximate aggregation techniques) and are of two kinds. Weak coupling (or non-singular perturbation) methods in which a perturbation term is interchanged in the right hand side of the equations of state as shown

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & \epsilon A_{12} \\ \epsilon A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

and ϵ is a small positive parameter. One approach is to set $\epsilon = 0$, thus decomposing the system into 2 lower dimensional independent systems with greatly reduced information processing requirements, and quantifying errors when in actual fact $\epsilon \neq 0$. A second approach is to exploit the weak coupling structure to obtain an iterative algorithm.

Strong coupling (singular perturbation) methods [81] in which perturbation terms are introduced in the left hand side of the state equations, i.e.

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \qquad \text{(slow dynamics)}$$

$$\epsilon\dot{x}_2 = A_{21}x_1 + A_{22}x_2 \qquad \text{(fast dynamics)}$$

where ϵ is a small positive parameter and A_{22} is a stable matrix. Setting $\epsilon = 0$ in (ii) yields

$$\dot{x}_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_1$$

$$x_2 = -A_{22}^{-1}A_{21}x_1$$

Note the separation of the system into slow and fast dynamics components and the attendant reduction of system order when $\epsilon = 0$.

Recently a new approach based on internally balanced principal axis state-space representations which contain equal amounts of information about controllability and observability has been advanced by Moore [24]. Such balanced representations are useful for model simplification since equal amounts of information about controllability and observability can be neglected without causing any imbalance in controllability or observability properties. This approach has been applied to continuous and discrete time systems using the ideas of both weak [24,25] and strong [26,27] couplings.

2.2.3 Decentralized Modelling

The model simplification techniques described in 2.2.2 above have presumed the centralized knowledge of the system model. Decentralized modelling techniques are applicable in situations where this presumption is invalid, and the idea is that while the "best available" system model may be too complex for practical use, a non-singleton set of simpler models, which need not be available centrally, may be used instead. The key aspect of the decentralized approach is the coordination of the individual models so that, taken together, they describe the overall system adequately. Tenney and Sandel [28] have considered this in a single-level setting.

A framework for multi-level (hierarchical), multi-model structures was outlined by Mesarovic et al in [29] with the resulting hierarchy of models, called a multi-strata hierarchy, having the following properties:

(i) Higher level models are more aggregated than lower level ones (2.2.2 above);

(ii) Decision periods of decision systems using higher level models are longer than those of lower units;

(iii) Higher level models are concerned with slower aspects of system behaviour (c.f. strong coupling in 2.2.2);

(iv) Higher level models contain more uncertainties in both their structure and quantitative representation of system variables (2.2.1 above).

Such a structure has been utilized in the hierarchical modelling of water resource systems [30]. It has been recognized [31] that different descriptions of a system can lead to different sets of models and that these descriptions and model hierarchies must be coordinated to make the best use of the information contained therein [32]. The different descriptions can be summarized as follows:

(i) Temporal description - based on the ideas of singular perturbation where the system is described in terms of its dynamics, e.g., a set of models describing the fastest dynamics form the lowest level of the temporal hierarchy and a set of models describing the slowest dynamics form the highest level;

(ii) Attribute description - loosely based on the ideas of model aggregation, where the system is described in terms of sets of system attributes ordered by set inclusion, and such that if attribute set A_i is contained in A_j, then the model of the system based on attribute set A_i is at a lower level than the model based on attribute set A_j;

(iii) Goal-functional description - where the decision-making goals of the system are arranged in a hierarchical manner (the method of Interpretive Structural Modelling described in [33] is a suitable means for effecting this arrangement) and this hierarchy induces the hierarchy of system models when there is a one-to-one correspondence between the goals and the models.

2.3 TECHNIQUES DEALING WITH DECISION-MAKING COMPLEXITY

We differentiate between decision-making and control systems as follows. In Figure 2.1, the decision agent and the model together form the decision-making system (DMS). When the decision, (output from the decision agent) is coupled to an actual process via an actuator

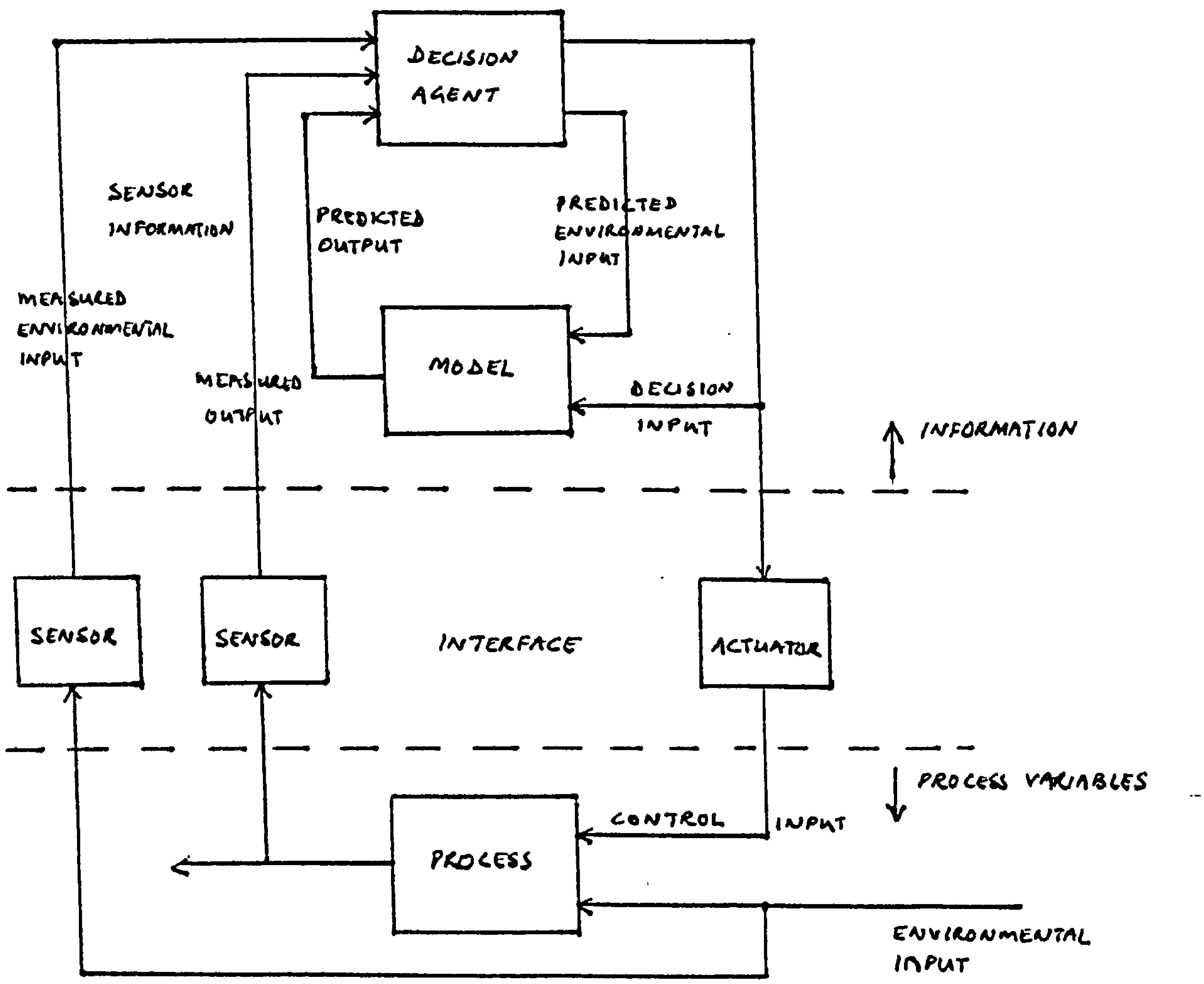

FIGURE 2.1. Relationship between a process and a decision-making system

then the DMS becomes a control system. Thus a DMS need not refer to any actual physical system. Of course, our interest in this section is on the control connotation of decision-making.

2.3.1 Elements of a decision-making situation

A decision-making situation is characterized by the following elements.

(i) A set $X = \times \{X_i\}$, $i \varepsilon \{1,2,...,I\}$, of decision alternatives where $\times \{X_i\}$ denotes the Cartesian product of the sets indexed by the set $\{1,2,...,I\}$. X_i is called an attribute set of X.

(ii) A set $E = \times \{E_j\}$, $j \varepsilon \{1,2,...,J\}$ of "states of nature" affecting outcomes of decisions.

(iii) A set $Y = \times \{Y_k\}$, $k \varepsilon \{1,2,...,K\}$ of decision outcomes (system output vectors in control theoretic language) where the set Y_k is called an attribute set of the set Y.

(iv) A model, i.e., a function $m: X \times E \rightarrow Y$. When m is a dynamic model then it may be thought as made up of the two functions,

$m_s: S \times X \times E \rightarrow S$ (state transition function)

(where $S = \times \{S_l\}$, $l \varepsilon \{1,2,...,L\}$)

and $m_o: S \times X \times E \rightarrow Y$ (output function)

(v) A set of evaluation functions (objective, utility, or performance functions)

$f_n: Y \rightarrow Z_n$, $n \varepsilon \{1,2,...,N\}$ where each Z_n is an ordered set called a <u>score</u> set.

(vi) A set of N problem statements, each statement defined with respect to a unique score set.

The set of problem statements may belong to one of three types according to the nature of its solution:

(a) it (the set of problem statements) may require a unique alternative, $\hat{x} \varepsilon X$ as solution, i.e., an optimization problem;

(b) it may require a unique subset $\hat{X} \subseteq X$ (the "preferred" set) as solution;

(c) it may require a unique ordering of the alternatives in X as solution.

Obviously by judicious re-definition of the decision situation, types (b) and (c) can be transformed to type (a); hence without loss of generality, it will be assumed that problem statements are defined as optimization problems.

Four categories of decision situations are identifiable [33]

(i) Decisions may be made under certainty where each alternative results in one and only one known outcome, the states of nature are known and the model, m, is deterministic.

(ii) Decision making under risk is a category in which each alternative may result in more than one outcome, the probabilities of these outcomes are known and the model, m, is stochastic.

(iii) Decision making under uncertainty involves making decisions when, not only do the alternatives result in more than one outcome, but the probabilities of these outcomes are unknown, i.e., the probabilities of the states of nature are unknown. It may be argued that the real-life situation is rare where there is no idea as to the likelihood of a particular outcome. Thus subjective probabilities may be assigned to the outcomes and Bayesian techniques applied. On the other hand, it may well be that the uncertainty stems from the fuzziness associated with the decision situation, i.e., the model, evaluation functions or problem statements are fuzzily defined thus calling for fuzzy decision-making techniques [34].

(iv) In the preceding categories, it had been assumed that the occurrence of a particular state of nature was independent of the alternative selected. In decision-making under conflict, the states of nature are determined in part by the alternative selected and the activities of a not necessarily hostile environmental "opponent" such that the state of nature resulting maximizes the performance measure of the nature states. This type of situation is resolved using game theory [35,36].

2.3.2 Aggregation Techniques: Multiple Criteria Decision-Making

A multiple criteria decision-making (MCDM) situation exists whenever in 2.3.1 above

(a) each alternative is characterized by more than one set of attributes and/or

(b) each outcome is evaluated by more than one evaluation function.

There are three kinds of MCDM situations

(a) Single decision-maker, single problem statement, multi-attribute outcome (SDSP) situation.

(b) Single decision-maker, multiple problem statement (SDMP) situation.

(c) Multiple decision-maker (MDM) situation.

In the "rational" approach to decision making, it is assumed that full information concerning the decision situation is available at the outset of the decision making process. Consequently, the axioms associated with the rational actor model of Von Neumann and Morgenstern [33,36] may be stated and applied under conditions of certainty, risk, uncertainty (after assigning subjective probabilities to the outcomes) or conflict. The axioms allow proof of the existence of cardinal utility functions and their uniqueness up to positive linear transformations. The existence of these functions (also known as value, worth, benefit or social welfare functions [33]) form the theoretical basis of a host of decision-making techniques. The fundamental problem in the "rational" approach is the synthesis of such a utility function in terms of the sets of attributes or scores from information obtained from a human decision-agent(s) (called the decision maker(s), DM). Once this function has been obtained (as is assumed in control theory) the set of alternatives can be mapped into an arbitrary totally ordered set, reducing the decision-making process to a search process [37] that could be carried out by non-human decision agents.

In the SDSP situation, the evaluation and utility functions are identical. MCDM techniques in this situation reduce to techniques for eliciting properties of the utility function from information given by the DM. Since the range of the utility function is a totally ordered set, the problem statement is defined in terms of the maximization of utility.

For the SDMP situation, the strategy is to transform it into an SDSP situation and then use the well developed SDSP techniques. Now in SDMP, it often happens that the various problem statements (objectives) conflict. By synthesizing a utility function based on the multiple score sets and structuring the individual problem statements into an overall or aggregate problem statement whose solution is implied by the maximization of this utility function, the conflicts can be removed and the situation transformed into an SDSP situation simultaneously. The Interpretive Structural Modelling Technique discussed in [33] is applicable to a multi-level (hierarchical) structuring of problem statements.

In an analogous manner, the MDM situation is first transformed into an SDMP situation and thence to an SDSP situation. This is done by synthesizing an overall or aggregate function, called the social welfare function, from the individual utility functions of the multiple decision-makers. However, there are some significant difficulties involved in the passing from MDM to SDMP. Principally, we have to ensure that, under given conditions, the social welfare function so defined is a utility function, i.e., it satisfies the Von Neumann and Morgenstern rational actor model axioms. Arrow has shown that under apparently mild conditions [39] no social welfare function exists that is also a utility function [33,36]. Thus the main theoretical thrust in this regard is to weaken Arrow's conditions and thereby prove the existence of a social welfare function having the properties of a utility function. These other conditions have been extensively discussed in [33,36].

A review of MCDM techniques according to the amount of information available (or assumed to be obtainable) from the DM now follows.

(1) The utility function U is unknown and implicit, but is assumed to be a monotone, non-decreasing and concave function. The task is to identify a set of nondominated alternatives (also known as Pareto optimal, non-inferior or preferred alternatives). Let the outcome vector y(x) corresponding to the alternative $x \in X$ be such that its k'th component is $y_k \in Y_k$, $k \in \{1,2,\ldots,K\}$ (see 2.3.1 above). A particular alternative $\hat{x} \in X$ is nondominated if, and only if, there exists no other alternative $x \in X$ such that (assuming maximization) $y_k(x) \geq y_k(\hat{x}), \forall k \in K$ and $y_k(x) > y_k(\hat{x})$ for at least one $k \in \{1,2,\ldots,K\}$.

Computerized linear programming algorithms have been developed to identify all non-dominated extreme points of X including Evans and Steuer's Revised Simplex Method [40] and Zeleny's Multicriteria Simplex Method [41].

(2) The utility function U is unknown and implicit but it can be made partially explicit via man-machine interactive dialogue. It is assumed that the DM is capable of providing information on local (incremental) properties of U, partial trade-offs and simple preference statements. These interactive programming methods include those of Geoffrion, Dyer and Feinberg [42], Zionts and Wallenius [66] and the surrogate worth trade-off method of Haimes and Hall [30,43]. See also the review in [43].

(3) U is unknown but can be revealed explicitly as a model of the DM's preference structure. In this context we speak of multi-attribute utility theory. The work of Keeney and Raiffa [44], Farquhar [45], Fishburn [46], Yutemma and Torgerson [48] are significant in this regard. Sage [33] has investigated multi-level (hierarchical) attribute structures and their applications to utility and worth (utility under conditions of certainty) assessment, and Farquhar [49,50] has surveyed in detail various types of multi-attribute utility models which range from the simpler unit weight linear models to the complex multiplicative ones. Johnson and

Huber [51] have reviewed a number of procedures that can be used to elicit utility functions.

The techniques discussed above owe their theoretical basis to the Von Neuman and Morgenstern axioms of rationality. Other axiom systems exist and SDSP techniques based on these axiom systems have been developed. Sage [52] has discussed some of them which include Handa's certainty equivalence theory [53], Karmarkar's subjectively weighted utility theory [54,55] and prospect theory due to Tversky and Kahneman [56,57]. Still other techniques exist that do not derive theoretical justification from any axiom system as such and are straight forward mathematical programming methods in which the DM supplies the attributes of the ideal alternative and parameters of a distance function measuring the distance between the available alternatives and the ideal. The available alternatives are then ranked in order of how close they approach the ideal. These techniques include goal programming [58,59], goal achievement method [60] and discrepancy analysis [61]. Heuristic techniques which involve comparisons of one alternative with another, generally within a restricted set of alternatives and sets of attributes, are available [52]. In general, these methods can result in intransitive choices [62]. Finally, Starr and Zeleny [37] have reviewed techniques (which they called decision process-oriented approaches) wherein the DM can incorporate additional information concerning the decision situation as it becomes available. Thus information gathered during the decision process can be used to clarify or describe the decision situation in greater detail. These techniques include those in which (as the dialogue with the DM progresses) the weights associated with a linear multi-attribute model vary in sympathy with successive reductions of the set of alternatives [41,63,64,65] and those wherein the ideal alternative mentioned above varies with successive reduction of the set of alternatives [67,68,70].

2.3.3 Disaggregation techniques

There are four aspects to the decision-making process:

(1) gathering information about the decision situation;

(2) synthesis of a decision rule;

(3) execution of a decision (search) algorithm to select a solution from a set of alternative solutions;

(N.B. In some MCDM techniques aspects (2) and (3) are indistinguishable)

(4) solution implementation.

The MCDM techniques described in 2.3.2 above have assumed that:

(i) information about the decision situation is given a priori (with the possible exceptions of the decision-process oriented techniques described in [37]) and hence the decision-making process commences de facto, with aspect (2) above;

(ii) this information is available at a central location, alternatively, it may be communicated to a central location in zero time, without error and without cost;

(iii) the information processing capacity at this location is adequate

(a) to store the a priori decision-situation information and

(b) to cope with the extra information (if needed) and processing requirements of aspects (2) and (3) above.

In a non-repetitive (single-stage, one-shot) decision-making situation, it is reasonable to suppose that the time taken to gather information about the decision situation, to elicit the utility function(s) from the DM(s) and for the algorithm to yield the solution, is insignificant. In a repetitive (multi-stage) decision-making situation, this supposition may be difficult to justify because of the following factors

(i) the finite time between decisions

(ii) the finite time required for information transfer

(iii) the finite time required for information processing

as has been noted by Mesarovic et al [29,p.43] and Findeisen et al [71, chapt.5]. Disaggregated (decentralized) decision-making (control) structures can be viewed as a means of coping with these factors.

Decentralized decision-making in the SDSP context

Decentralization of decision-making in the SDSP context may be considered in two mathematically identical ways:

(1) single-level decentralization of processing requirements of the decision algorithm among multiple processors;

(2) single-level decentralization of decision rule among multiple decision agents each working towards a common goal. Now in the formulation of 2.3.1 above, a decision alternative $x \varepsilon X$ was characterized by an I-tuple i.e. $x = (x_1, x_2, \ldots, x_I)$. Suppose the index set $\{1,2,\ldots,I\}$ were partitioned into Q non-empty, non-intersecting subsets indexed by the set $\{1,2,\ldots,Q\}$. Let $I_q \subseteq \{1,2,\ldots,I\}$ be such a subset, $q \varepsilon \{1,2,\ldots,Q\}$ and $\bar{X}_q = x\{X_i\} : i \varepsilon I_q$. Thus $X = x\{\bar{X}_q\}$, $\bar{x}_q \varepsilon \bar{X}_q$ can be viewed as being derived from the q'th decision agent where the Q agents together form a team.

The methods of team theory [72] and decentralized control theory [23] are applicable for the derivation of the decentralized decision rules for each of the decision agents. In fact, a static team is equivalent to decentralized open-loop stochastic control. Sandel et al [23,73] and Singh [89] have commented extensively on the problems involved when members of a dynamic team do not possess identical information about decision outcomes (non-classical information pattern). The case where the decision-model is decentralized as well is discussed in [74]. In general a price has to be paid for decentralizing information; in a static team it is suboptimal performance (compared with performance achievable in the centralized case) and in a dynamic team instability may also result [75].

Hierarchical (multi-level decentralized) decision-making

A hierarchical decision-making structure may come about in three ways:

(1) by hierarchical structuring of the decision-making process resulting in multi-layer hierarchies;

(2) by hierarchical structuring of decision makers in an MDM context or equivalently by hierarchical structuring of problem statements in an SDMP context, resulting in multi-echelon hierarchies;

(3) by processing of the decision algorithm in a hierarchy of processor units utilizing either a multi-layer or multi-echelon approach.

The fundamental dilemma of real-life decision making is that on one hand there is a need to act without delay, while on the other, there is an equally great need to understand the situation better [29] since full information about the decision situation is rarely available at the outset of decision activities. To resolve this, the original decision situation is structured into a hierarchical sequence of decision situations in the sense that the implementation of a decision at one level determines and fixes some parameters of the decision situation in the next lower level so that the latter is completely specified and the selection of its own decision may be attempted. The selection of the decision of the original decision situation is achieved when all decisions in the hierarchy of decision situations have been selected. This type of hierarchy is referred to as a multi-layer hierarchy [29,30,71]. The advantage of this structure is that the original decision situation is not dealt with at once, rather it is dealt with in phases, thus new information received concerning the original decision situation can be exploited in the execution of subsequent phases. Obviously decisions resulting from such a structure are sub-optimal [76] when full information about the decision situation is unavailable at the outset; however the structure permits learning and adaptation [29,71] and in repetitive decision situations where the environmental behaviour is unchanging, the decisions tend to the optimal ones in the limit.

Remarks:

(1) Since the decision situation in each layer differs, it implies that the decision models also differ, hence the overall decision situation is characterized by a hierarchy of decision models, i.e., a multi-strata hierarchy (see 2.2.3 above);

(2) the PMS decision-making hierarchy described in 1.1.4 above is a multi-layer hierarchy, see also [71];

(3) this sequential approach to decision-making is reminiscent of Simon's "Bounded rationality" [77,78] and Lindblom's "Muddling through" [79,80] characterizations of the decision process in organizations [52,76] and the decision process-oriented approaches [37] in an MCDM setting.

In a multiechelon* hierarchy [29,30,71] it is assumed that the decision situation is fully defined at the onset of the decision process; however, the problem statements in an SDMP situation are assumed to be goals or objectives of a corresponding number of decision agents (and the decision model is decentralized among them) which are organized in a hierarchical manner as are the decision makers in an MDM situation. Each decision-agent in the hierarchy is goal-seeking and conflicts between decision-agents on one echelon are resolved by higher echelon decision-agents. Coordination, the resolution of conflicts, is accomplished by intervention which occurs by including certain parameters in each decision-agents problem statement that are manipulable by higher echelon decision-agents. Intervention may be of three kinds:

(a) goal-intervention, which affects goal-related factors [29,71],

(b) information, which affects model-related factors [29,71],

(c) constraint intervention, which affects elements of the set of alternatives (controls) [71].

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* Also known as multi-level hierarchies in the literature, e.g., [71].

It is assumed that the solution to the overall problem statement (2.3.2 above) is achieved when all the decision-agents together obtain the solutions to their individual problem statements within their prescribed operating constraints. Conflict resolution is the raison-d'etre of multi-echelon hierarchies. If there were no conflicts as in the single-level decentralized case above where all the decision agents worked together towards a common goal or where a decision situation can be decomposed into a set of disjoint (unrelated) decision situations then the need for a multi-echelon hierarchy disappears. The mathematical theory of coordination has been extensively studied in [29,71] and applied to the hierarchical control of large-scale systems in [71,30]. The problem of information structure design in this context has been studied in [71] and the extension of hierarchical techniques to the case where consistency of interests between levels cannot be assumed is discussed in [75].

The selection of a decision can always be viewed as the solution of an optimization problem. In a complex decision-making situation it is often desirable to employ a multi-level (multi-layer or multi-echelon) decomposition of the optimization problem to permit the reduction of processing requirements of the resulting solution algorithm. This decomposition may be obvious or natural in the SDMP or MDM situations but may be arbitrary in the SDSP context. A great deal of effort has been focussed on the development of decomposition techniques and algorithms for solving decomposed optimization problems. References [23,30,71,75,82-88] have discussed these multilevel optimization techniques in great detail.

2.3.4 A brief note on fuzzy decision-making

In real-life, there exist a number of occasions where the decision-situation is not precisely defined. The decision process-oriented MCDM techniques described in section 2.3.2 above and multi-layer hierarchical techniques discussed in 2.3.3 above deal with such decision-situations by making a series of partial decisions, evaluating their consequences, and then utilizing this information to define the decision-situation more precisely and make further partial decisions.

A different approach to the problem is offered by the application of fuzzy set theory [14,17,18,19] originated by Zadeh [20]. Now, the usual definition of a subset A of U can be formalized in terms of the characteristic function V(x) where $x \varepsilon U$, $V(x) = y \varepsilon \{0,1\}$. The concept of a fuzzy subset replaces the characteristic function V(x) by a membership function $\mu(x)$ with the closed interval (0,1) of real numbers as range. (A later development [21] considers the range to be a set of linguistic truth values). Thus $\mu(x)$ is the degree of membership of x in A. In its current state of development [34], the fuzziness of the decision situation is assumed to derive from the fuzziness in the problem statements only; thus all aspects connected with the decision model, i.e., the sets of alternatives, states of nature, and outcomes are precise, the membership functions now play the role of evaluation functions. This formulation therefore permits the use of static or dynamic, deterministic or stochastic, decision models and has been applied to the control of physical processes [90], operations research problems [91], MCDM [92,93] and a host of other decision situations [94].

2.4 REVIEW OF PREVIOUS WORK ON MODELLING AND SIMULATION OF THE PMS

The problem of understanding and accounting for the behavioural complexity of a PMS has engaged the attention of researchers for a number of years. Model-building and simulation* have been the major tools in this endeavour. Forrester [5] used his systems dynamics methodology [5,6] to trace the effects of the interaction between decision systems and process dynamics on overall PMS dynamic behaviour. Because his primary aim was to show the destabilizing effect of time delays and information dynamics on PMS behaviour, his models of the production and marketing processes were highly aggregated, and thus could not account for individual products, workstations, skill (labour) or the multi-stage nature of production processes, etc., in the case of production, or product prices, qualities, and competition in the case of marketing. The fact that the models were derived on one time-scale only, meant that the effect of multi-layered decisions, especially the scope for PMS adaptation could not be included in the analysis.

Forrester's work was influential in the microanalytic marketing simulation model developed for the marketing game TOMES (Total Market Environmental Simulation) [95] and applied to a consumer product market [96] and an ethical drug market [97]. The models provided detailed specifications of the decision processes of individual consumers and market demand was synthesized by aggregating the decisions of these individual consumers. However, the level of disaggregation did not permit direct evaluation of the effect of aggregate marketing variables such as advertizing, price, quality and distribution strategy on market demand. In contrast, the MATE (Marketing Analysis Training Exercise)

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* The main model-building goal for the simulation models described here is to postulate a general structure that would account for the dynamic behaviour of the PMS; their applications to specific situations are secondary.

simulation [98,99] is an aggregate simulation, i.e., individual consumers are not explicitly modelled. Here total market demand is modelled as a multiplicative function involving a growth term, a seasonal term, a marketing effort term (price and advertizing expenditure) and an aggregate consumer income term. The Kotler simulation model [100] is similar except for the lack of the aggregate consumer income term. Other aggregate models include the Vidale-Wolfe model [101] which relates the rate of change of sales per period to current advertizing expenditure and sales per period; the Lanchester-type models [102,103] which can be viewed as generalizations of the Vidale-Wolfe model to allow for competitive effects [104]; the Nerlove-Arrow model [105] wherein current sales per period is a non-linear function of "goodwill" which is described by a first order differential equation with current advertizing expenditure per period as input; and the Brandaid model [106]. The Brandaid model represents a significant departure from other aggregate models in that it explicitly considers (in a modular fashion) the effects of advertizing, promotion, product price, salesmen, distribution, and competition in terms of their respective sales influence index, which are then combined multiplicatively with each other and a reference sales rate to give the overall sales per period.

Aside from the pioneering effort of Forrester [5] and the work of other system dynamics devotees [6,107], there is very little information in the literature regarding the modelling and simulation of the dynamic behaviour of a production system that explicitly considers the dynamic interactions among the functional areas of production, or the dynamics of the multi-layered decision structure. Rather, the role of simulation in production is, almost exclusively, to trace the possible consequences of heuristic decisions in a given functional area under various conditions where the analytical or computational problems associated with the dynamical nature of the situation are intractable as in job-shop scheduling simulation [108,109].

2.5 SUMMARY AND CONCLUSIONS

Control theoretic and other techniques available for the resolution of model, decision-making and behavioural complexities in a PMS have been discussed. The model simplification techniques (aggregation and perturbation methods) described have been well developed for linear systems. The PMS is inherently non-linear with time-delays and time-varying parameters thus further work is still required before wholesale adoption of these techniques may be attempted. More importantly, the ideas behind these techniques have been exploited intuitively in the modelling of the PMS, e.g., in the derivation of aggregate dynamic marketing behaviour from microanalytical models (section 2.4 above) and exposure to the model simplification techniques can greatly enhance the present intuitive approach. Similarly, the approach to decentralized modelling at present consists of building various models of different aspects of the PMS with very little regard for the coordination of such models or anticipation of effects of interactions of decisions based on different models. Active consideration of multi-strata modelling utilizing any of the three descriptive categories mentioned in 2.2.3 above can greatly improve the quality of the resulting models.

The normative "rational" approach to decision-making in complex situations has come under fire from several quarters lately. For instance, Keen [76] has argued that since the decision situation is rarely well-defined and the costs of information gathering and processing are quite significant, the normative concern with optimality as the quality of the solution should be replaced by the concept of optimality as the quality of the behaviour of the decision-making system. In fact, a simulation study of an industrial market [111] has shown that cost of information is a significant determinant of the size of a firm. Echoing this theme but in a different context, Sandel et al [23] have argued that it is not reasonable to seek a single best decision-making structure, rather decision-making structures that are preferable to others should be identified and decision-making in such structures should explicitly account for costs of communication, reliability, incomplete and/or delayed information, and a formal measure of systems complexity. As such, different definitions of optimality, principles of optimality and notions of optimal solutions may have to be developed. These observations taken together with those of Simon [77,78] and Lindblom [79,80] suggest that in complex decision situations, it may be more fruitful to strive for incremental improvements in decision-making rather than seeking the globally "best." It is thus apparent that a decision-making structure embodying both multi-layer (decision) and multi-strata (modelling) hierarchies represents a good starting point for this endeavour.

More theoretical and practical work is required for the understanding of multi-layer, multi-strata structures and to permit the exploitation of their potential for adaptation and self-organization. The modelling and simulation work on the PMS reported in this thesis is this author's contribution to this venture.

CHAPTER III

ANALYSIS AND MODELLING OF PRODUCTION SYSTEM DYNAMICS

3.1 PREAMBLE

The type of production system being considered in this chapter is one characterized by the following properties

1. A number of different finished products are manufactured;
2. The product line is constant;
3. Backlogging of demand is permissible;
4. Non-zero manufacturing lead-time;
5. Non-zero purchasing lead-time;
6. Processing routes for finished products share common machinery and facilities;
7. Production capacity is limited;
8. Subcontracting of the manufacture of intermediate products is permissible (with non-zero purchasing lead-time);
9. More than one labour shift is permissible.

The production system is composed of the production process (material conversion aspect) and its associated management system (decision-making aspect). The management system has been described with respect to its long-, medium- and short-term characteristics in Figure 1.1 above. The scope of the work reported in this chapter is however restricted to the medium-term (timescale of one month) behaviour of the production system; in particular the analysis and modelling of the effects of medium-term production scheduling decisions on the dynamical behaviour of the production system. Obviously, such schedules depend on forecasted sales of each product in the product line per medium-term period in the scheduling horizon and the distribution and customer order-processing dynamics (marketing system considerations) as well as the constraint imposed by the limitation of the available financial resources over the scheduling horizon.

3.2 PRODUCTION PROCESS ELEMENTS

3.2.1 Let there be:

F items of finished goods (FG) inventory

W items of work-in-process (WP) inventory

R items of raw material (RM) inventory

and thus a total of $I = F+W+R$ items of inventory.

Note: (1) FG inventory items are those whose demand comes from outside the factory, i.e., they are the products that are shipped to customers.

(2) WP inventory items are those that are produced (or capable of being produced) in the factory and are used wholly in the production of other inventory items.

(3) RM inventory items are those that are used in the production of other inventory items but, unlike WP items, they cannot be produced within the factory and hence must be wholly obtained from outside sources.

Assumption P1

(a) There is no internal demand for FG items.

(b) A factory does not obtain FG items from outside.

(c) There is no external demand for WP items.

(d) There is no external demand for RM items (thus factory activities exclude those of storage of goods for resale).

3.2.2 Let i, $1 \le i \le I$, be a general inventory index such that

$1 \le i \le F$	implies that inventory item i is a FG item
$F+1 \le i \le F+W$	" " " " " " " WP "
$F+W+1 \le I \le F+W+R$	" " " " " " " RM "

Let UM be an I X I matrix called the Inventory Utilization Matrix. Each element $U_{i,p}$ of UM is such that

(i) $U_{i,p} \ge 0$ for all i,p where $1 \le i,p \le I$

(ii) $U_{i,p} = 0$ for i such that $F+W+1 \le i \le F+W+R$

(a consequence of Note 3 in 3.2.1 above)

(iii) $U_{i,p} = 0$ for p such that $1 \leq p \leq F$

(a consequence of Assumption P1 in 3.2.1 above)

In other words, for each unit of inventory item i produced, $U_{i,p}$ units of inventory item p are required as input to the workcentre* producing it.

Assumption P2

The elements of UM are constant for given sets of FG, WP and RM inventories.

3.2.3 Let there be J workcentres. A workcentre is a group of workstations with the following properties:

(i) identical inventory items can be produced on every workstation in the group;

(ii) identical input inventory items are required for the production of a given inventory item on each workstation in the group;

(iii) each workstation in the group utilizes the same skill for its operation;

(iv) the items of inventory produced in a workcentre are unique to the workcentre.

These properties enable the workcentre to be regarded as an aggregate workstation; in the medium-term, the workcentre is the focus of conversion activity scheduling.

3.2.4 1. Let MPV, the <u>Workcentre Production Vector</u>, be an N-component vector (N=IFG+IWP) and $MPV_n = j$ implies that inventory item n $(1 \leq n \leq N)$ is produced in workcentre j $(1 \leq j \leq J)$.

2. Let STV, the <u>Inventory Set-up Vector</u>, be an N-component vector and $STV_n = s$ implies that inventory item n requires s labour-hours to set-up for production.

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* The workcentre is defined in 3.2.3 below.

3. Let EFV, the Efficiency Vector, be an N-component vector where $EFV_n = e$ implies that e units of inventory item n are produced per labour-hour in the workcentre producing item n.

4. Let each workstation in the j'th workcentre require ℓ workers for its operation, let there be a total of k workstations in the workcentre. This information is contained in the Workcentre Manpower Matrix, MMM, a 2 x J matrix

where $MMM_{1,j} = \ell$

and $MMM_{2,j} = k$

5. Let ILV, the Inventory Level Vector, be an I-component vector. ILV is derived from the inventory utilization matrix UM such that $ILV_i = \ell$ indicates that inventory item i is an ℓ'th level inventory item.

Definition: Inventory item p is a k'th level item if

(a) it requires at least one item of level k-1 inventory and

(b) no higher than level k-1 inventory items for its production.

Raw material items are defined to be zero-level inventory items.

3.3 MODELLING OF MEDIUM-TERM PRODUCTION PROCESS DYNAMICS

The models in this section describe the effects of aggregate resource allocation, scheduling, and conversion decisions and external demand for finished goods on the levels of the various production resources (stock levels of inventory items and available amount of labour-hours of each skill) in the medium-term.

3.3.1 Inventory Dynamics

Let instant m mark the beginning of medium-term period M; there are MINT medium-term periods in each long-term period and m = M-1.

3.3.1.1 For FG items ($1 \leq i \leq F$)

$$BORD_i(m+1) = (1-b_i)(BORD_i(m) - FGSR_i(m)) + DEM_i(m) \quad (3.1a)$$

$$(\text{if } BORD_i(m) > FGSR_i(m))$$

$$= BORD_i(m) + DEM_i(m) - FGSR_i(m) \quad (3.1b)$$

$$(\text{if } BORD_i(m) \leq FGSR_i(m))$$

and

$$SLVL_i(m+1) = SLVL_i(m) + PRUN_i(m) - FGSR_i(m) \quad (3.2)$$

$$FGSR_i(m) = \min(SLVL_i(m) + PRUN_i(m), BORD_i(m) + DEM_i(m)) \quad (3.3)$$

(i.e., the smaller of the two quantities separated by the comma)

where

$BORD_i(m)$ is the number of units of FG item i on backorder at instant m

b_i is the constant fraction of unfilled backorders that would be lost in period M

$DEM_i(m)$ is the external demand for FG item i over period M

$FGSR_i(m)$ is the number of units of FG item i shipped to customers over period M

$SLVL_i(m)$ is the number of units of FG item i in stock at instant m

$PRUN_i(m)$ is the number of units of FG item i processed in the last stage of manufacture over period M; it is a decision variable

3.3.1.2 For WP and RM items ($F+1 \leq i \leq F+W+R$)

$$SLVL_i(m+1) = SLVL_i(m) + PRUN_i(m) + BRUN_i(m) - URUN_i(m) \quad (3.4)$$

$$URUN_i(m) = \sum_{p=1}^{I} PRUN_p(m).U_{p,i} \quad (3.5)$$

with the constraint

$$SLVL_i(m) + PRUN_i(m) + BRUN_i(m) \geq URUN_i(m) \quad (3.6)$$

where

$SLVL_i(m)$ is the number of units of inventory item i in stock at instant m

$PRUN_i(m)$ a decision variable, is the number of units of inventory item i processed in the last stage of manufacture over period M; $PRUN_i(m) = 0$ for all m if $F+W+1 \leq i \leq F+W+R$

$URUN_i(m)$ is the number of units of inventory item i required for the manufacture of other items of inventory over period M; $URUN_i(m) = 0$ for all m if $1 \leq i \leq F$

$U_{p,i}$ is the relevant element of UM. The Inventory Utilization Matrix

$BRUN_i(m)$ is a decision variable; it is the number of units of inventory item i delivered from outside sources in period M; $BRUN_i(m) = 0$ for all m if $1 \leq i \leq F$

3.3.2 Labour Dynamics

3.3.2.1 Labour Demand

Let the subscript j refer to the j'th workcentre

$$WLBH_j(m) = \sum_{i=1}^{F+W} a_{j,i} \left(\frac{PRUN_i(m)}{EFV_i} + STV_i\right) \qquad (3.7)$$

where

$WLBH_j(m)$ is the number of labour-hours required for total production of inventory items in workcentre j over period M

$a_{j,i}$ $= 1$ if $MPV_i = j$

$= 0$ otherwise

3.3.2.2 Labour Supply

There are various options available for the provision of labour to meet the requirements of each workcentre given by equation (3.7) above. These options include

(1) hiring or laying-off of workers; i.e., variation in the number of workers on the payroll;

(2) allocation of overtime;

(3) increasing or decreasing the number of shifts worked (consequent to option (1) above).

Accordingly for the j'th workcentre

$$MEN_j(m+1) = MEN_j(m) + DMEN_j(m) \qquad (3.8)$$

$$ALBH_j(m) = NLBH_j \times MEN_j(m) + OLBH_j(m) \qquad (3.9)$$

subject to the constraints

$$MMM_{1,j} \times MMM_{2,j} \times KSHIFT \geq MEN_j(m) \qquad (3.10a)$$

$$ALBH_j(m) \geq WLBH_j(m) \qquad (3.10b)$$

$$\theta_j \times NLBH_j \times MEN_j(m) \geq OLBH_j(m) \qquad (3.10c)$$

where

$MEN_j(m)$ is the number of workers on the payroll at workcentre j over period M

$DMEN_j(m)$ a decision variable; it is the number of workers hired (if $DMEN_j(m) > 0$) or laid-off (if $DMEN_j(m) < 0$) at the <u>end</u> of period M

$ALBH_j(m)$ is the available number of labour-hours at workcentre j over period M

$NLBH_j$ a constant; it is the number of normal work hours per worker at workcentre j per medium-term period

$OLBH_j(m)$ a decision variable; it is the number of hours of overtime assigned over period M at workcentre j

θ_j a constant; it is the ratio of the maximum permissible overtime hours to normal work hours at workcentre j.

3.4 MODELLING OF DECISION-MAKING SYSTEM DYNAMICS

The production process described in sections 3.2 and 3.3 above is a multiple-(final)product, multistage non-linear assembly tree process.* The only inherent medium-term process dynamics is that due to the loss of backorders over time (see equation (3.1a)) which, strictly speaking, should be considered as part of the decision-making system as backorders are not material entities being operated upon on the shop-floor. Thus the medium-term dynamic behaviour of production systems can be said to derive wholly from the dynamics of its decision-making system.

In the analysis, the medium-term interaction between the various distribution levels, connecting the factory with the ultimate consumers of its output, and the demand felt at the factory is not discussed. Rather, it is assumed that perfect information concerning the demand for factory output for each medium-term period in the decision horizon is available. Thus the dynamic complexities involved in the estimation of factory demand as a derived demand have been avoided. Even with this simplifying assumption there are other sources of dynamic complexity to contend with:

(1) Production lead-time from raw-material to finished product stage. By modelling the production process as a parallel process rather than as a sequential one, this problem has been reduced significantly to the point where one needs consider only the number of units of each item of inventory produced in its final stage of manufacture in any given medium-term period - the stock level of this item of inventory at the beginning of the period being sufficient to account for its history up to that instant. By so doing, one has converted a multistage sequential system into a set of parallel single-stage systems.

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* A non-linear assembly tree process is one where each item of WP inventory may be used in producing one or more items of inventory; a linear tree process is one whereby only one item of inventory can be produced from a given item of WP inventory.

(2) Customer order processing lead-time: The simplifying assumption of a known factory demand per medium-term period for each period in the decision-horizon has effectively resulted in a production system that produces to stock with customer orders being filled from stock or else back-ordered. Thus the significant dynamic consequences of customer order-processing lead-time [5,113] have been avoided.

(3) Purchasing, Hiring/Laying-off lead times. The problem formulation has permitted the evasion of considerable dynamic complications so far, however it is felt that explicit inclusion of these three lead-times is necessary to give the production system some recognizable dynamic features. Accordingly, the following assumption results:

Assumption P3 : There is a lead-time of

(a) 3 medium-term periods between placing an order for RM or WP items purchased from outside and accepting delivery of same;

(b) 1 medium-term period between taking a hiring/laying off decision and the hired/laid-off workers joining/leaving the workforce.

Finally, before the decision cost models are described, the following assumption is made:

Assumption P4 : Production system decisions are made over a 12-period (one year) decision horizon.

3.4.1 Inventory Cost Model

(i) Holding Cost

$$HLDCST(m) = \sum_{i=1}^{I} C_{1,i}[SLVL_i(m) + 0.5(PRUN_i(m) + BRUN_i(m) - FGSR_i(m) - URUN_i(m))]$$

N.B.

$$PRUN_i(m) = 0 \quad \text{for} \quad i > F+W$$

$$BRUN_i(m) = 0 \quad \text{for} \quad i \leq F$$

$$URUN_i(m) = 0 \quad \text{for} \quad i \leq F$$

$$FGSR_i(m) = 0 \quad \text{for} \quad i > F$$

(ii) Setup Cost

$$SUPCST(m) = \sum_{i=1}^{I} C_{2,i}\ a_i(m)$$

where $a_i(m) = 1$ if $PRUN_i(m) > 0$

$= 0$ otherwise

(iii) Ordering Cost (for RM and WP inventories purchased from outside)

$$ORDCST(m) = \sum_{i=1}^{I} C_{3,i}\ b_i(m)$$

where $b_i(m) = 1$ if $BRUN_i(m+3) > 0$

$= 0$ otherwise

(iv) Purchase Cost (for RM and WP inventories purchased from outside)

$$PCHCST(m) = \sum_{i=1}^{I} (C_{4,i} + C_{5,i} BRUN_i(m)) BRUN_i(m)$$

$C_{4,i}$ = price per unit of item i

$C_{5,i} BRUN(m)$ = shipping cost per unit item i

3.4.2 Workcentre Cost Model

Energy and Maintenance Cost

$$ENMCST(m) = \sum_{j=1}^{J} [C_{6,j} + (C_{7,j} + C_{8,j} WLBH_j(m)) WLBH_j(m)]$$

3.4.3 Manpower Cost Model

(i) Regular time Payroll Cost

$$RTPCST(m) = \sum_{j=1}^{J} C_{9,j} \times NLBH_j \times MEN_j(m)$$

(ii) Hiring Cost (for $DMEN_j(m) > 0$)

$$HIRCST(m) = \sum_{j=1}^{J} C_{10,j} \times DMEN_j(m)$$

(iii) Laying-off Cost (for $DMEN_j(m) < 0$)

$$LAYCST(m) = \sum_{j=1}^{J} C_{11,j} \times (-DMEN_j(m))$$

(iv) Shift startup, shutdown cost

$$SSSCST(m) = \sum_{j=1}^{J} C_{12,j} \times d_{j,1}(m) + C_{13,j} \times d_{j,2}(m)$$

where $d_{j,1}(m) = 1$ if a new shift is to be added at workcentre j at end of period M

$= 0$ otherwise

and $d_{j,2}(m) = 1$ if a shift at workcentre j is to be closed down at end of period M

$= 0$ otherwise

(v) Overtime Cost

$$OVTCST(m) = \sum_{j=1}^{J} C_{14,j} \times OLBH_j(m)$$

3.4.4 Statement of Objective

"Maximize contribution (revenue less production costs) over the 12-period decision horizon"

i.e. $$\text{Max} \sum_{m=t}^{t+11} [REV(m) - PERCST(m)]$$

where

$$REV(m) = \sum_{i=1}^{F} PRICE_i(m) \times FGSR_i(m)$$

and
$$PERCST(m) = HLDCST(m) + SUPCST(m) + ORDCST(m) + PCHCST(m) + ENMCST(m) + RTPCST(m) + HIRCST(m) + LAYCST(m) + SSSCST(m) + OVTCST(m)$$

Subject to the 12-period financial resource constraint

$$\sum_{m=t}^{t+11} \text{PERCST}(m) \leq \text{FINRES}$$

N.B. (1) A benefit function could be included in the Statement of Objective, converting it to a multiple objective decision situation, to recognize the value of having non-zero finished goods and work-in-process inventories at the end of the horizon.

(2) Storage space constraints could be introduced to take cognizance of the limited storage area in any production facility (thus putting a limit to the amount of inventory in stock)

3.5 SUMMARY AND DISCUSSION

The production process modelled is a multiple-(final)product, multi-stage, non-linear assembly tree process where production capacity is limited (by way of limit to the number of workers at each workcentre), processing routes for finished products share common machinery and facilities, and more than one labour shift is permissible at each workcentre. The associated decision-making system makes allowance for the backlogging of demand, non-zero manufacturing and purchasing lead-times and the sub-contracting of manufacture of intermediate products.

The decision-making system performs different functions according to the timescale of interest. In the long-term these functions are summarily referred to as production planning functions and include production process planning, production capacity planning and financial resource planning. Interest in this Chapter is on the effects of these planning outputs on medium-term production system behaviour and hence it is assumed that process and capacity decision outputs are constant with the result that elements and dimensions of the vectors/matrices describing the process (3.2 above) are constant. Long-term production smoothing is achievable by suitable allocation of financial resources to both the production and marketing systems in such a way that supply and sales are equalized and supply is achieved at an acceptable cost (to the production-marketing system). This gives rise to the fundamental area of interaction between the work reported in Chapter 4 and that reported here.

The analysis and modelling of medium-term production system behaviour is based on the assumption of perfect sales forecast information for each product in each medium-term period in the decision horizon. Obviously one should be careful in the use of the sales/demand information derived from marketing models in Chapter 4 for medium-term scheduling purposes as medium-term dynamics of consumer demand, distribution and customer order-processing are not accounted for. However, this does not necessarily imply a significant disadvantage. Firstly, the allowance for backordering and non-zero horizon-end inventories may be sufficient to absorb the medium-term sales dynamics. In addition there is the second and more potent possibility that alarm conditions are built into the decision-making system so that whenever current conditions deviate significantly from the desired conditions a new schedule is calculated using a moving 12-period horizon. In fact, the economics of decision-making may well be such as to permit the trade of accuracy and precision in schedule outputs for greater frequency in scheduling updates, taking advantage of the facts that in real production systems, schedules need not be derived once and for all and the available data is rarely accurate enough for the degree of accuracy/precision in schedule outputs desired.

The influence of medium-term decisions on short-term behaviour is similarly by means of resource constraint. For instance, the medium-term decision outputs of the amount of overtime, the number of workers and the number of shifts attached to a given workcentre determines the upper limit on the usable amount of labour resource (labour-hours) available in each medium-term period. Short-term scheduling decisions (obtained using a decision horizon of one or more medium-term periods) which assign workers and workstations to operations and sequence the production of inventory in a workcentre, must recognize this and, additionally, treat medium-term production rates as targets and purchase (delivery) decisions as external inputs that can vary in amount and time of application from their nominal values.

CHAPTER IV

ANALYSIS AND MODELLING OF MARKETING SYSTEM DYNAMICS

In the analyses to be conducted, attention is focussed exclusively on the long-term activities of the marketing system. This is because the primary objective is to investigate the effects of marketing decisions made by sellers on marketing system dynamics and hence their effect on the demand and sales rate of products of a given producer which constitute the fundamental marketing system inputs to the production system in the long- and medium-terms.

The work in this chapter proceeds in steps from the analysis of an idealized marketing system where the marketing conditions include singleton seller and product sets, full awareness of the product and constant selling effort, to the more realistic marketing system where the marketing conditions include multiple sellers each with multiple products and variable marketing decisions. The effects of the functional relationship between marketing expenditures and marketing decisions on certain features of the marketing system are also studied.

Throughout, it is assumed that the number of consumers is constant, the products are consumer durables and the sellers are also producers.

4.1 SINGLE-SELLER, SINGLE-PRODUCT MARKETING SYSTEM

Assumptions: (defining marketing conditions)

M1 No product repurchasability - once the requisite volume of product has been purchased by a consumer, his needs are satisfied for all time.

M2 Awareness of seller's product is instantaneous.

M3 Constant marketing decision variables - i.e., product price, quality, selling effort and advertizing effort are constant.

M4 The volume of product, δ, required by each consumer to satisfy his needs for all time is constant.

4.1.1 Let ℓ be the instant of time denoting the beginning of the L'th long-term period (the current period). Thus at instant ℓ let

$N_B(\ell)$ be the number of consumers who have purchased the requisite volume of product, δ.

$D(\ell)$ be the demand per long-term period.

$S(\ell)$ be the sales per long-term period, i.e., the sales rate.

k be the selling effort factor (assumed constant as a consequence of M3) representing the effect of selling effort of the seller in converting demand to sales in each long-term period. k is always less than unity because the product will always face generic competition* and functional, place and time utilities of product will always be less than ideal. k is a marketing decision variable.

N_o be the number of consumers in the market segment.

Then

$$D(\ell) = \delta(N_o - N_B(\ell)) \tag{4.1}$$

$$S(\ell) = k\,D(\ell) = k\,\delta(N_o - N_B(\ell)) \tag{4.2}$$

$$N_B(\ell+1) = N_B(\ell) = \frac{S(\ell)}{\delta}$$

i.e.,

$$N_B(\ell+1) = (1-k)N_B(\ell) + k\,N_o \tag{4.3}$$

(4.1), (4.2) and (4.3) imply

$$D(\ell+1) - (1-k)D(\ell) = 0 \tag{4.4}$$

$$S(\ell+1) - (1-k)S(\ell) = 0 \tag{4.5}$$

Thus, under marketing conditions described by M1 - M4, demand and sales rate are represented by first-order homogeneous linear difference equations.

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* Generic competition - competition from other product categories that might satisfy some of the same consumer wants.

4.1.2 Suppose marketing conditions are such that product awareness is not instantaneous, i.e., M2 above is relaxed. It is assumed that the effect of the seller's advertizing is

(i) to make aware in the current period a fraction of those who were unaware at the beginning of the period (the communication function) and

(ii) to increase the number of aware consumers who would purchase the product in the current period (the promotion function).

Let $A(\ell)$ be the number of consumers aware of the product at instant ℓ

α ≤ 1, be the advertizing effort factor which is constant by virtue of M3. α is another marketing variable and $\alpha = 1$ corresponds to the case of instantaneous product awareness discussed in 4.1.1 above.

Then
$$A(\ell+1) = A(\ell) + \alpha(N_o - A(\ell))$$
$$= (1-\alpha)A(\ell) + \alpha N_o \qquad (4.6)$$

To account for the promotion effect of advertizing, the selling effort factor is modified as follows

$$k = \alpha k_o \qquad (4.7)$$

(Thus when $\alpha = 1$, i.e., for instantaneous awareness, $k = k_o$)

$$N_B(\ell+1) = N_B(\ell) + k(A(\ell) - N_B(\ell))$$
$$= (1-\alpha k_o)N_B(\ell) + \alpha k_o A(\ell) \qquad (4.8)$$

Now
$$D(\ell) = \delta(A(\ell) - N_B(\ell)) \qquad (4.9a)$$
and
$$S(\ell) = k\delta(A(\ell) - N_B(\ell)) \qquad (4.9b)$$

(4.6) - (4.9) imply that

$$D(\ell+2) - [(1-\alpha k_o) + (1-\alpha)]D(\ell-1) + (1-\alpha k_o)(1-\alpha)D(\ell) = 0 \qquad (4.10)$$

$$S(\ell+2) - [(1-\alpha k_o) + (1-\alpha)]S(\ell-1) + (1-\alpha k_o)(1-\alpha)S(\ell) = 0 \qquad (4.11)$$

Under these conditions, demand and sales rate are represented by second order, linear, homogeneous difference equations.

4.1.3 Suppose now that conditions are such that product repurchasability is non-zero, i.e., M1 is relaxed in addition to M2. Let

Υ denote the repurchasability factor ≤ 1. It is assumed for the moment to be independent of the age of the product and represents the fraction of products in use at instant ℓ that would require repurchase over the period L.

Now the volume of product in use at instant ℓ is $\delta N_B(\ell)$ hence

$$N_B(\ell+1) = N_B(\ell) - \frac{\Upsilon\delta N_B(\ell)}{\delta} + k\,[A(\ell) - N_B(\ell)]$$

$$= [1 - (k+\Upsilon)]N_B(\ell) + k\,A(\ell) \tag{4.12}$$

Equations (4.6) and (4.9) are unaffected by Υ hence these equations together with (4.12) imply that

$$D(\ell+2) - [(1-\alpha) + (1-(k+\Upsilon))]D(\ell+1) + (1-\alpha)(1-(k+\Upsilon))D(\ell) = \delta\alpha\Upsilon N_o \tag{4.13}$$

$$S(\ell+2) - [(1-\alpha) + (1-(k+\Upsilon))]S(\ell+1) + (1-\alpha)(1-(k+\Upsilon))S(\ell) = k\delta\alpha\Upsilon N_o \tag{4.14}$$

with equilibrium values of demand and sales rate given respectively by

$$D_E = \frac{\delta\Upsilon}{k+\Upsilon}N_o = \frac{\delta\Upsilon}{\alpha k_o + \Upsilon}N_o \tag{4.15a}$$

and $$S_E = \frac{k\delta\Upsilon}{k+\Upsilon}N_o = \frac{\alpha k_o\delta\Upsilon}{\alpha k_o + \Upsilon}N_o \tag{4.15b}$$

since $k = \alpha\,k_o$ (4.7).

4.1.4 Remarks

(1) At this point it is useful to compare the models derived in 4.1.3 with some of the popular ones in the marketing literature as described by Little [104].

Suppose the market segment is fully informed about the product, i.e., $A(\ell) = N_o$. We thus consider the promotion effect of advertizing only. Accordingly, $N_B(\ell+1) = [1 - (k+\Upsilon)]N_B(\ell) + k\,N_o$

and $D(\ell) = \delta[N_o - N_B(\ell)]$

$S(\ell) = k\,D(\ell) = k\delta[N_o - N_B(\ell)]$

thus $S(\ell+1) = [1 - (k+\Upsilon)]S(\ell) + k\delta\Upsilon\, N_o$

Now $S(\ell+1) - S(\ell) = \dot{S} = -(k+\Upsilon)S + k\delta\Upsilon\, N_o$.

i.e., $\dot{S} = k\delta\Upsilon\, N_o[1 - S/(\delta\Upsilon N_o)] - \Upsilon S$

But $k = \alpha k_o$ (4.7)

hence $\dot{S} = \alpha\, k_o\delta\Upsilon N_o[1 - S/(\delta\Upsilon N_o)] - \Upsilon S$ (4.16)

The Vidale-Wolfe model as described in [104] is given by

$$\dot{S} = \rho x[1 - S/m] - \lambda S \qquad (4.17a)$$

where ρ , m and λ are constants

with equilibrium value of S for constant expenditure rate x given by

$$S(x) = m(\rho x/\lambda m)/[1 + (\rho x/\lambda m)] \qquad (4.17b)$$

(4.17a) is identical with (4.16) when

$$\alpha = hx, \quad \rho = hk_o\delta\Upsilon N_o, \quad m = \delta\Upsilon N_o \text{ and } \lambda = \Upsilon$$

h is a constant such that $hx < 1$

and the corresponding equilibrium sales rate $S(\alpha)$, given in (4.15b) is identical with $S(x)$.

(2) Similarly, Little's Brandaid model [104] can be derived as a special case of the model in 4.1.3. Again, considering the promotion effect of advertizing only we have that

$$S(\ell+1) = [1 - (k+\Upsilon)]S(\ell) + k\delta\Upsilon N_o$$

$$= [1 - (\alpha k_o+\Upsilon)]S(\ell) + (\alpha k_o+\Upsilon).\frac{\alpha k_o\delta\Upsilon N_o}{(\alpha k_o+\Upsilon)}$$

which is identical to the Brandaid model with the carryover constant given by $[1-(\alpha k_o+\Upsilon)]$ and the long-term advertizing response given by $\alpha k_o\delta\Upsilon N_o/(\alpha k_o+\Upsilon)$.

(3) The advantages of the model derived in 4.1.3 over the Vidale-Wolfe and Brandaid models can be briefly summarized as follows:

(a) Communication effect of advertizing is modelled.

(b) The marketing system dynamics have been described in terms of the marketing decision variables α and k_o, respectively the advertizing and selling effort factors, and the parameters δ, γ and N_o, respectively the required volume of product per consumer, the product repurchasability factor and the market segment population. Thus it is straight forward to account for the effects of variations of these decisions/parameters on marketing system behaviour.

4.2 MARKETING SYSTEM WITH TWO SELLERS AND ONE PRODUCT PER SELLER

In this section the models developed in 4.1 for a monopoly market are generalized to the duopoly case, i.e., the effects of competition are explicitly considered. An analysis of the factors contributing to the advertizing effort factor is carried out and includes the "word-of-mouth" phenomenon where the consumers themselves contribute to the spreading of product awareness. The situation where the repurchasability factor is variable and is a function of the age of the product and the general case of variable marketing decisions on the part of either seller are also analyzed.

Assumptions:

M1 Zero product repurchasability (see Note 1 below).

M2 Instantaneous awareness of each seller's product.

M3 Marketing decision variables of each seller are constant.

M4 δ and γ are constants (see Note 2).

M5 Equal product prices and qualities are extant (see Note 3).

Notes

1. When M1 is relaxed, it will still be assumed that Υ is independent of individual products. This is not a restrictive assumption because differences in repurchasability factor due to different rates of wear, mal-function or other product quality-related parameters could be accounted for in the product quality rating.

2. δ and Υ are constants because they depend on parameters that are assumed constant. Some of these parameters are determined by marketing decision variables which are constant by virtue of M3. Hence M4 in effect is a consequence of M3.

3. M5 implies that (i) k, the selling effort factor for each seller effectively depends on distribution effort only, and (ii) the products are homogeneous, i.e., for those consumers aware of both, they are indistinguishable with respect to functional, price, and value-for-money properties.

4.2.1 For the marketing system described by M1 - M5

$N_o - N_B(\ell)$ is the number of consumers at instant ℓ yet to purchase any product and

$$[N_o - N_B(\ell+1)] = (1-k_1)(1-k_2)[N_o - N_B(\ell)]$$

where the suffices 1 and 2 respectively refer to sellers 1 and 2.

Let $$k_{12} = 1 - (1-k_1)(1-k_2) \tag{4.18a}$$

i.e., $$1 - k_{12} = (1-k_1)(1-k_2) \tag{4.18b}$$

then $$N_o - N_B(\ell+1) = (1-k_{12})[N_o - N_B(\ell)]$$

Hence $$N_B(\ell+1) = (1-k_{12})N_B(\ell) + k_{12}\, N_o \tag{4.19a}$$

$$D(\ell) = \delta[N_o - N_B(\ell)] \tag{4.19b}$$

$$S(\ell) = k_{12} \cdot \delta[N_o - N_B(\ell)] \tag{4.19c}$$

Equations (4.19) imply that

$$D(\ell+1) - (1-k_{12})D(\ell) = 0 \tag{4.20a}$$

$$S(\ell+1) - (1-k_{12})S(\ell) = 0 \tag{4.20b}$$

Equations (4.20) represent demand and sales rate as first-order, homogeneous linear difference equations (c.f. single-seller single-product case in 4.1.1). The overall (industry) sales effort has increased relative to what it would have been had either seller not participated since equations (4.18) can be rewritten as

$$k_{12} = k_1 + k_2(1-k_1) = k_2 + k_1(1-k_2) \tag{4.21}$$

Now $\quad S(\ell) = S_1(\ell) + S_2(\ell)$

<u>Assumption</u>, M6: The sales rate of each product is given respectively by

$$S_1(\ell) = \frac{k_1(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)} S(\ell) = \frac{k_1(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)} \cdot k_{12} D(\ell) \tag{4.22a}$$

and

$$S_2(\ell) = \frac{k_2(1-k_1)}{k_1(1-k_2)+k_2(1-k_1)} k_{12} D(\ell) \tag{4.22b}$$

Hence the respective market share of each product is given by

$$K_1 = \frac{k_1(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)} \tag{4.22c}$$

and

$$K_2 = \frac{k_2(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)} \tag{4.22d}$$

Note that for $k_2 < 1$, as $k_1 \to 1$, $K_1 \to 1$ and vice versa.

Also for $k_2 > 0$, $K_1 \to 0$ as $k_1 \to 0$.

Equations (4.21) and (4.22) together imply (with a little algebraic manipulation) that

$$S_1(\ell) = k_1[1 - k_2K_2]D(\ell)$$

and
$$S_2(\ell) = k_2[1 - k_1K_1]D(\ell)$$

The effect of competition is to reduce the selling effort factor of each seller by the fraction k_1K_1 or k_2K_2 as the case may be.

4.2.2 Let M2 be relaxed allowing for non-instantaneous product awareness. Let, at instant ℓ,

$A(\ell)$ be the total number of consumers aware of either or both products;

$N_1(\ell)$ be the total number of consumers aware of product 1 <u>only</u> but are yet to purchase it;

$N_2(\ell)$ be as for $N_1(\ell)$ but in respect of product 2;

$N_{12}(\ell)$ be the number of consumers aware of <u>both</u> products but are yet to purchase either;

$N_B(\ell)$ be the total number of consumers who have purchased either product in the requisite amount.

Then

$$N_B(\ell) = A(\ell) - [N_1(\ell) + N_2(\ell) + N_{12}(\ell)] \tag{4.23}$$

$$D(\ell) = \delta[N_1(\ell) + N_2(\ell) + N_{12}(\ell)] \tag{4.24a}$$

$$S(\ell) = \delta[k_1N_1(\ell) + k_2N_2(\ell) + k_{12}N_{12}(\ell)] \tag{4.24b}$$

$$A(\ell+1) = (1-\alpha_{12})A(\ell) + \alpha_{12} N_o \tag{4.25}$$

where
$$k_{12} = 1 - (1-k_1)(1-k_2) \tag{4.26a}$$

and
$$k_1 = \alpha_1 k_{10}, \quad k_2 = \alpha_2 k_{20} \tag{4.26b}$$

$$\alpha_{12} = 1 - (1-\alpha_1)(1-\alpha_2) \tag{4.27}$$

$$N_1(\ell+1) = (1-\alpha_2)[(1-k_1)N_1(\ell) + \alpha_1[N_o - A(\ell)]] \tag{4.28}$$

$$N_2(\ell+1) = (1-\alpha_1)[(1-k_2)N_2(\ell) + \alpha_2[N_o - A(\ell)]] \tag{4.29}$$

$$N_{12}(\ell+1) = (1-k_{12})N_{12}(\ell) + \alpha_2(1-k_1)N_1(\ell) + \alpha_1(1-k_2)N_2(\ell) + \alpha_1\alpha_2[N_o - A(\ell)] \qquad (4.30)$$

Hence

$$D(\ell+4) + a_1D(\ell+3) + a_2D(\ell+2) + a_3D(\ell+1) + a_4D(\ell) = 0 \qquad (4.31a)$$

$$S(\ell+4) + a_1S(\ell+3) + a_2S(\ell+2) + a_3S(\ell+1) + a_4S(\ell) = 0 \qquad (4.31b)$$

where $a_1 = -[(1-k_{12})+(1-\alpha_1)(1-k_2)+(1-\alpha_2)(1-k_1)+(1-\alpha_{12})]$ (4.32a)

$$a_2 = (1-k_{12})[(1-\alpha_1)(1-k_2)+(1-\alpha_2)(1-k_1)+(1-\alpha_{12})] + (1-\alpha_{12})[(1-k_{12})+(1-\alpha_1)(1-k_2)+(1-\alpha_2)(1-k_1)] \qquad (4.32b)$$

$$a_3 = -(1-\alpha_{12})(1-k_{12})[(1-k_{12})+(1-\alpha_1)(1-k_2)+(1-\alpha_2)(1-k_1)+(1-\alpha_{12})] \qquad (4.32c)$$

$$a_4 = [(1-\alpha_{12})(1-k_{12})]^2 \qquad (4.32d)$$

It is tempting to conclude from equations (4.31) that

$$S(\ell) = k_{12}D(\ell)$$

but equations (4.24) show that this is not the case. If the error of such a conclusion, $k_{12}D(\ell) - S(\ell)$, is denoted by $E(\ell)$ then

$$E(\ell+3) + b_1\,E(\ell+2) + b_2\,E(\ell+1) + b_3\,E(\ell) = 0 \qquad (4.33)$$

where $b_1 = a_1 + (1-k_{12})$ (4.34a)

$$b_2 = a_2 + (1-k_{12})b_1 \qquad (4.34b)$$

$$b_3 = a_3 + (1-k_{12})b_2 \qquad (4.34c)$$

and since $E(\ell) \geq 0$, it implies that

$$k_{12}D(\ell) \geq S(\ell)$$

Now $S(\ell) = S_1(\ell) + S_2(\ell)$

where $S_1(\ell) = \delta[k_1N_1(\ell) + \dfrac{k_1(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)}\,k_{12}N_{12}(\ell)]$ (4.35) and market shares are now

$$K_1(\ell) = \frac{S_1(\ell)}{S(\ell)} \quad \text{and } K_2(\ell) = 1 - K_1(\ell)$$

which are no longer constant.

4.2.3 Let M1 be relaxed allowing for non-zero product repurchasability in addition to non-instantaneous product awareness. For the moment it is assumed that γ is independent of the age of the product.

At instant ℓ, let

$A_1(\ell)$ denote the number of consumers aware of product 1

$A_2(\ell)$ " " " " " " " " 2

$A_{12}(\ell)$ " " " " " " " both products.

Then

$$A_1(\ell+1) = (1-\alpha_1)A_1(\ell) + \alpha_1 N_o \tag{4.36a}$$

$$A_2(\ell+1) = (1-\alpha_2)A_2(\ell) + \alpha_2 N_o \tag{4.36b}$$

$$A_{12}(\ell+1) = (1-\alpha_{12})A_{12}(\ell) + \alpha_2(1-\alpha_1)A_1(\ell) + \alpha_1(1-\alpha_2)A_2(\ell) + \alpha_1\alpha_2 N_o \tag{4.36c}$$

$$\text{and } A(\ell+1) = A_1(\ell) + A_2(\ell) - A_{12}(\ell) \tag{4.36d}$$

Equations (4.36) imply that

$$A(\ell+1) = (1-\alpha_{12})A(\ell) + \alpha_{12}N_o \quad \text{(as before).}$$

Now, $A_1(\ell) - A_{12}(\ell) - N_1(\ell)$ represents the number of consumers who have purchased product 1 and are still unaware of product 2 at instant ℓ.

Hence

$$\begin{aligned} N_1(\ell+1) &= (1-\alpha_2)[(1-k_1)N_1(\ell)+\alpha_1[N_o-A(\ell)]+\gamma[A_1(\ell)-A_{12}(\ell)-N_1(\ell)]] \\ &= (1-\alpha_2)[(1-k_1-\gamma)N_1(\ell)+\alpha_1[N_o-A(\ell)]+\gamma[A_1(\ell)-A_{12}(\ell)]] \end{aligned} \tag{4.37a}$$

and similarly for product 2

$$N_2(\ell+1) = (1-\alpha_1)[(1-k_2-\gamma)N_2(\ell)+\alpha_2[N_o-A(\ell)]+\gamma[A_2(\ell)-A_{12}(\ell)]] \tag{4.37b}$$

For those aware of both products but are yet to purchase either

$$N_{12}(\ell+1) = (1-k_{12}-\Upsilon)N_{12}(\ell) + \alpha_1(1-k_2-\Upsilon)N_2(\ell) + \alpha_2(1-k_1-\Upsilon)N_1(\ell)$$
$$+ \alpha_1\alpha_2[N_o-A(\ell)] + \Upsilon(1-\alpha_1-\alpha_2)A_{12}(\ell) + \alpha_2\Upsilon A_1(\ell) + \alpha_1\Upsilon A_2(\ell) \quad (4.38)$$

Hence

$$N_1(\ell+1) + N_2(\ell+1) + N_{12}(\ell+1)$$
$$= (1-k_{12}-\Upsilon)[N_1(\ell)+N_2(\ell)+N_{12}(\ell)]+k_2(1-k_1)N_1(\ell)+k_1(1-k_2)N_2(\ell)$$
$$+ \alpha_{12}[N_o-A(\ell)] + \Upsilon A(\ell) \quad (4.39)$$

$$D(\ell+4) + d_1D(\ell+3) + d_2D(\ell+2) + d_3D(\ell+1) + d_4D(\ell) = d_5N_o \quad (4.40a)$$

$$S(\ell+4) + d_1S(\ell+3) + d_2S(\ell+2) + d_3S(\ell+1) + d_4S(\ell) = k_{12}d_5N_o \quad (4.40b)$$

where

$$d_1 = -[(1-k_{12}-\Upsilon) + (1-\alpha_1)(1-k_2) + (1-\alpha_2)(1-k_1) + (1-\alpha_{12})] \quad (4.41a)$$

$$d_2 = (1-k_{12}-\Upsilon)[(1-\alpha_1)(1-k_2) + (1-\alpha_2)(1-k_1) + (1-\alpha_{12})]$$
$$+ (1-\alpha_{12})[(1-\alpha_1)(1-k_2) + (1-\alpha_2)(1-k_1) + (1-k_{12})] \quad (4.41b)$$

$$d_3 = -(1-\alpha_{12})[(1-k_{12}-\Upsilon)[(1-k_{12}) + (1-\alpha_1)(1-k_2) + (1-\alpha_2)(1-k_1)]$$
$$+ (1-\alpha_{12})(1-k_{12})] \quad (4.41c)$$

$$d_4 = (1-\alpha_{12})^2(1-k_{12}-\Upsilon)(1-k_{12}) \quad (4.41d)$$

$$d_5 = [1-(1-\alpha_1)(1-k_2)][1-(1-\alpha_2)(1-k_1)]\delta\alpha_{12}\Upsilon \quad (4.41e)$$

At equilibrium (i.e., as $\ell \to \infty$)

$$D_E = \frac{\delta\Upsilon}{(k_{12}+\Upsilon)}N_o \quad , \quad S_E = \frac{k_{12}\delta\Upsilon}{k_{12}+\Upsilon}N_o \quad (4.42)$$

(Compare equations (4.42) with (4.15) above)

and the market shares $K_1(\ell)$ and $K_2(\ell)$ approach, in the limit,

$$\frac{k_1(1-k_2)}{k_1(1-k_2)+k_2(1-k_1)} \quad \text{and} \quad \frac{k_2(1-k_1)}{k_1(1-k_2)+k_2(1-k_1)} \quad \text{respectively.}$$

Remark

Equations (4.37) and (4.38) imply the possibility of oscillatory behaviour whenever $k_1+\Upsilon >1$ or $k_2+\Upsilon >1$ or $k_{12}+\Upsilon >1$ as is indeed the case in the analogous situation described in 4.1.3 above. However, the structure of the marketing system model is such that these oscillations are transient and die out eventually. The period of oscillation, whenever Υ is assumed to be independent of the age of the product, is always 2 long-term periods.

4.2.4 We now consider the case where the repurchasability factor, Υ, varies with the age of the product. We shall need to modify the formulation of section 4.2.3 as follows:

Let

$B_1(i,\ell)$ be the number of consumers at instant ℓ who are aware of product 1 alone, have purchased it in the requisite amount, δ, and have used it for between i-1 and i long-term periods without repurchase.

$B_2(i,\ell)$ analogous to $B_1(i,\ell)$ but in respect of product 2.

$B_{12}(i,\ell)$ be the number of consumers at instant ℓ who are aware of both products, have purchased them in the requisite amount, and have used them for between i-1 and i long-term periods without repurchase.

For brevity, we shall consider 3 product age-groups only; the extension of the analysis to a greater number of age-groups is straight forward.

The age groups are

(i) one year and under, i = 1

(ii) between one year and 2 years, i = 2

(iii) over 2 years, i = 3.

Suppose the respective repurchasability factors for each of these age groups are $\Upsilon(1)$, $\Upsilon(2)$ and $\Upsilon(3)$.

Then

For the first age group,

$$B_1(1,\ell+1) = (1-\alpha_2)k_1N_1(\ell) \tag{4.43a}$$

$$B_2(1,\ell+1) = (1-\alpha_1)k_2N_2(\ell) \tag{4.43b}$$

$$B_{12}(1,\ell+1) = k_{12}N_{12}(\ell) + \alpha_1k_2N_2(\ell) + \alpha_2k_1N_1(\ell) \tag{4.43c}$$

For the second age group

$$B_1(2,\ell+1) = (1-\alpha_2)(1-\Upsilon(1))B_1(1,\ell) \tag{4.44a}$$

$$B_2(2,\ell+1) = (1-\alpha_1)(1-\Upsilon(1))B_2(1,\ell) \tag{4.44b}$$

$$B_{12}(2,\ell+1) = (1-\Upsilon(1))[B_{12}(1,\ell) + \alpha_1B_2(1,\ell) + \alpha_2B_1(1,\ell)] \tag{4.44c}$$

For the third age group

$$B_1(3,\ell+1) = (1-\alpha_2)[(1-\Upsilon(2))B_1(2,\ell) + (1-\Upsilon(3))B_1(3,\ell)] \tag{4.45a}$$

$$B_2(3,\ell+1) = (1-\alpha_1)[(1-\Upsilon(2))B_2(2,\ell) + (1-\Upsilon(3))B_2(3,\ell)] \tag{4.45b}$$

$$\begin{aligned} B_{12}(3,\ell+1) = {} & (1-\Upsilon(2))[B_{12}(2,\ell) + \alpha_1B_2(2,\ell) + \alpha_2B_1(2,\ell)] \\ & + (1-\Upsilon(3))[B_{12}(3,\ell) + \alpha_1B_2(3,\ell) + \alpha_2B_1(3,\ell)] \end{aligned} \tag{4.45c}$$

And the other state-equations are:

$$A(\ell+1) = (1-\alpha_{12})A(\ell) + \alpha_{12}N_o \tag{4.46a}$$

$$\begin{aligned} N_1(\ell+1) = {} & (1-\alpha_2)[(1-k_1)N_1(\ell) + \alpha_1(N_o-A(\ell)) \\ & + \Upsilon(1)B_1(1,\ell) + \Upsilon(2)B_1(2,\ell) + \Upsilon(3)B_1(3,\ell)] \end{aligned} \tag{4.46b}$$

$$\begin{aligned} N_2(\ell+1) = {} & (1-\alpha_1)[(1-k_2)N_2(\ell) + \alpha_2(N_o-A(\ell)) \\ & + \Upsilon(1)B_2(1,\ell) + \Upsilon(2)B_2(2,\ell) + \Upsilon(3)B_2(3,\ell)] \end{aligned} \tag{4.46c}$$

$$\begin{aligned} N_{12}(\ell+1) = {} & (1-k_{12})N_{12}(\ell) + \alpha_1\alpha_2(N_o-A(\ell)) + \alpha_2(1-k_1)N_1(\ell) + \alpha_1(1-k_2)N_2(\ell) \\ & + [\Upsilon(1)B_{12}(1,\ell) + \Upsilon(2)B_{12}(2,\ell) + \Upsilon(3)B_{12}(3,\ell)] \\ & + \alpha_1[\Upsilon(1)B_2(1,\ell) + \Upsilon(2)B_2(2,\ell) + \Upsilon(3)B_2(3,\ell)] \\ & + \alpha_2[\Upsilon(1)B_1(1,\ell) + \Upsilon(2)B_1(2,\ell) + \Upsilon(3)B_1(3,\ell)] \end{aligned} \tag{4.46d}$$

If we let $$\Upsilon' = \frac{\Upsilon(3)}{1+(\Upsilon(3)-\Upsilon(1))+(\Upsilon(3)-\Upsilon(2))(1-\Upsilon(1))} \qquad (4.47)$$

then the equilibrium values of demand and sales rate are given respectively by

$$D_E = \frac{\delta\Upsilon' N_o}{k_{12} + \Upsilon'} \qquad (4.48a)$$

and $$S_E = \frac{k_{12}\delta\Upsilon' N_o}{k_{12} + \Upsilon'} \qquad (4.48b)$$

(compare with equations (4.42) above).

In general, age-dependent repurchasability factor results in an increase in the period of oscillation of the system and decreases the steady-state demand (or sales rate) relative to what it would have been if Υ was constant at the maximum value of $\Upsilon(i)$. In other words

$\Upsilon' \leq \max(\Upsilon(1),\Upsilon(2),\Upsilon(3))$, equality obtains only when $\Upsilon(1) = \Upsilon(2) = \Upsilon(3)$.

4.2.5 Let assumption M5 be relaxed, i.e., the marketing system where differing product prices and product qualities (though constant) exist in addition to non-zero product repurchasability and non-instantaneous consumer awareness, is now considered.

Let q be the product quality rating (a marketing decision variable, thus q_1 and q_2 respectively for products 1 and 2)

p be the product price (another marketing decision variable, thus p_1 and p_2 respectively for products 1 and 2)

v be the "value-for-money" rating (thus v_1,v_2)

Since assumption M3 still holds, then q_1,q_2,p_1,p_2 are constants.

Because of the relaxation of M5, the parameter k_{12} needs to be redefined as follows:

$$(1-k_{12}) = (1-\nu_1 k_1)(1-\nu_2 k_2) \tag{4.49}$$

$$(k_1 = \alpha_1 k_{10} \ , \ k_2 = \alpha_2 k_{20})$$

where ν_1 and ν_2 are product parameters such that

(i) $\nu_1 = \nu_2 = 1$ whenever $v_1 = v_2$

where $v_1 = \dfrac{q_1}{p_1}$ and $v_2 = \dfrac{q_2}{p_2}$

(ii) $m_1 k_{12} \leq k_1 \ , \ m_2 k_{12} \leq k_2$

$$\text{where } m_1 = \frac{\nu_1 k_1(1-\nu_2 k_2)}{\nu_1 k_1(1-\nu_2 k_2)+\nu_2 k_2(1-\nu_1 k_1)} \ , \ m_2 = 1-m_1 \tag{4.50}$$

Sufficient conditions to ensure that (i) and (ii) above are satisfied are

(a) $\nu_1 \ , \ \nu_2 \leq 1$

(b) $\nu_2 = \dfrac{v_2}{v_1}\nu_1$

Thus if for $v_1 \geq v_2$, ν_1 and ν_2 are chosen as

$$\nu_1 = 1, \quad \nu_2 = \frac{v_2}{v_1}$$

or if for $v_2 > v_1$, ν_1 and ν_2 are chosen as

$$\nu_1 = \frac{v_1}{v_2} \ , \quad \nu_2 = 1$$

then (i) and (ii) above are satisfied.

Without loss of generality, it shall be assumed that $\nu_1 \geq \nu_2$

$$\text{i.e.,} \quad \nu_1 = 1, \quad \nu_2 = \frac{v_2}{v_1} \tag{4.51}$$

The state and output equations now read (assuming that repurchasability factor is independent of product age for brevity)

$$A_1(\ell+1) = (1-\alpha_1)A_1(\ell) + \alpha_1 N_o \tag{4.52a}$$

$$A_2(\ell+1) = (1-\alpha_2)A_2(\ell) + \alpha_2 N_o \tag{4.52b}$$

$$A_{12}(\ell+1) = (1-\alpha_{12})A_{12}(\ell) + \alpha_1(1-\alpha_2)A_2(\ell) + \alpha_2(1-\alpha_1)A_1(\ell) + \alpha_1\alpha_2 N_o \tag{4.52c}$$

$$N_1(\ell+1) = (1-\alpha_2)[(1-k_1-\Upsilon_1)N_1(\ell) + \alpha_1[N_o-A(\ell)] + \Upsilon_1[A_1(\ell)-A_{12}(\ell)]] \tag{4.53a}$$

$$N_2(\ell+1) = (1-\alpha_1)[(1-k_2-\Upsilon_2)N_2(\ell) + \alpha_2[N_o-A(\ell)] + \Upsilon_2[A_2(\ell)-A_{12}(\ell)]] \tag{4.53b}$$

$$\begin{aligned} N_{12}(\ell+1) = {} & (1-k_{12}-\Upsilon_{12})N_{12}(\ell) + \alpha_1(1-k_2-\Upsilon_2)N_2(\ell) + \alpha_2(1-k_1-\Upsilon_1)N_1(\ell) \\ & + \alpha_1\alpha_2[N_o-A(\ell)] + \Upsilon_{12}A_{12}(\ell) + \alpha_2\Upsilon_1[A_1(\ell)-A_{12}(\ell)] \\ & + \alpha_1\Upsilon_2[A_2(\ell)-A_{12}(\ell)] \end{aligned} \tag{4.53c}$$

$$A(\ell) = A_1(\ell) + A_2(\ell) - A_{12}(\ell) \tag{4.54}$$

$$D(\ell) = \delta_1 N_1(\ell) + \delta_2 N_2(\ell) + \delta_{12}N_{12}(\ell) \tag{4.55a}$$

$$S(\ell) = k_1\delta_1 N_1(\ell) + k_2\delta_2 N_2(\ell) + k_{12}\delta_{12}N_{12}(\ell) \tag{4.55b}$$

It will be noticed in equations (4.53) that instead of one parameter, Υ, there are three Υ_1, Υ_2, Υ_{12}, and in equations (4.55) there are three parameters δ_1, δ_2, δ_{12}. The reason for this situation is that thus far the products have been homogeneous - there has been no difference in price or quality of the products and hence the consumer could only achieve seller differentiation. In effect, what had been analyzed was a marketing system with two sellers both selling the same product. As a result, the parameters Υ and δ were the same for all populations creating demand, i.e., N_1, N_2 and N_{12}. With the relaxation of M5, the products are no longer homogeneous, hence each population (N_1, N_2, N_{12}) "sees" a different "product" (in terms of price and quality) and thus it can be expected that Υ and δ should vary with respect to the "product" seen by these groups.

Let the following postulates define Υ and δ respectively.

MP1: $\Upsilon = \Upsilon_o + (1-\Upsilon_o)rv$ i.e. $(1-\Upsilon) = (1-\Upsilon_o)(1-rv)$ (4.56a)

MP2: $\frac{p\delta}{v} = I'$ (4.56b)

where Υ_o, r and I' are constants for the market segment, p is product price and v is product value-for-money rating.

MP1 in effect states that Υ is made up of a constant component Υ_o (reflecting repurchasability due to normal wear, malfunction, usage rate, etc.) and a factor related to the value-for-money rating of the product.

MP2 results from the following argument. Consider a group of N consumers. These consumers belong to various market segments depending on the product class under discussion. Let the set $\{1,2,...,S\}$ index a set of mutually exclusive and exhaustive product classes and hence index a set of S market segments that these N consumers belong to. The amount spent by the N consumers in a given market segment s per period is

$\bar{p}_s.\bar{\delta}_s.\bar{k}_s N_s$ where $\bar{p}_s$, $\bar{\delta}_s$ and $\bar{k}_s$ are respectively the aggregate price, per unit demand and selling effort in segment s and N_s is the number of consumers contributing to demand in segment s ($N_s \leq N$).

Let I be the per capita income per period of this group of N consumers (assumed constant), f_s the fraction of per capita income devoted to purchases in market segment s, and $n_s = \frac{N_s}{N}$

Hence $f_s I N = \bar{p}_s \bar{\delta}_s \bar{k}_s n_s N$

If we assume that f_s is proportional to the marketing effort, i.e., $\bar{v}_s.\bar{k}_s$, and the relative value of products, i.e., w_s, in segment s, then

$$f_s = \frac{w_s \bar{v}_s \bar{k}_s n_s}{\sum_{r=1}^{S} w_r \bar{v}_r \bar{k}_r n_r \frac{(1-w_s\bar{v}_s\bar{k}_s n_s)}{(1-w_r\bar{v}_r\bar{k}_r n_r)}} = \frac{\bar{p}_s.\bar{\delta}_s.\bar{k}_s n_s}{I} \qquad (4.57)$$

hence
$$\frac{\bar{p}_s\ \bar{\delta}_s}{\bar{v}_s} = \frac{w_s I}{\sum_{r=1}^{S} w_r \bar{v}_r \bar{k}_r n_r \frac{(1-\hat{w}_s \bar{v}_s \bar{k}_s n_s)}{(1-w_r \bar{v}_r \bar{k}_r n_r)}} = I' \qquad (4.58)$$

I' is constant if

(i) w_s is constant

(ii) variations in $\bar{v}_s$,$\bar{k}_s$ and n_s do not affect the sum in equation (4.58).

Now MP1 and MP2 together imply that

$$\frac{(1-\Upsilon_1)}{(1-rv_1)} = \frac{(1-\Upsilon_2)}{(1-rv_2)} = \frac{(1-\Upsilon_3)}{(1-rv_3)} = (1-\Upsilon_o) \qquad (4.59a)$$

and

$$\frac{p_1\delta_1}{v_1} = \frac{p_2\delta_2}{v_2} = \frac{p_{12}\delta_{12}}{v_{12}} = I' \qquad (4.59b)$$

where

$$v_{12} = v_1 m_1 + v_2 m_2 \qquad (4.60a)$$

$$\delta_{12} = \delta_1 m_1 + \delta_2 m_2 \qquad (4.60b)$$

$$p_{12} = \frac{v_{12}}{\delta_{12}} \cdot I' = \frac{v_1 m_1 + v_2 m_2}{\frac{v_1}{p_1} m_1 + \frac{v_2}{p_2} m_2} \qquad (4.60c)$$

$$m_1 = \frac{\frac{v_1}{v} k_1}{\frac{v_1}{v} k_1 + \frac{v_2}{v} k_2 \frac{(1 - \frac{v_1}{v} k_1)}{(1 - \frac{v_2}{v} k_2)}} \qquad (4.61a)$$

(v is the larger of v_1 and v_2)

$$m_2 = 1 - m_1 \qquad (4.61b)$$

Sales rate of product 1 is

$$S_1(\ell) = k_1 \delta_1 N_1(\ell) + m_1 k_{12} \delta_1 N_{12}(\ell) \qquad (4.62a)$$

and of product 2 is

$$S_2(\ell) = k_2\delta_2N_2(\ell) + m_2k_{12}\delta_2N_{12}(\ell) \qquad (4.62b)$$

Aggregate marketing effort variables for the segment are

$$\bar{k}(\ell) = \frac{S(\ell)}{D(\ell)} \qquad (4.63a)$$

where $S(\ell)$ and $D(\ell)$ are given in equations (4.55).

Let $\bar{m}_1(\ell) = \dfrac{S_1(\ell)}{S(\ell)}$ and $\bar{m}_2(\ell) = \dfrac{S_2(\ell)}{S(\ell)}$

then $$\bar{v}(\ell) = v_1\bar{m}_1(\ell) + v_2\bar{m}_2(\ell) \qquad (4.63b)$$

$$\bar{\delta}(\ell) = \delta_1\bar{m}_1(\ell) + \delta_2\bar{m}_2(\ell) \qquad (4.63c)$$

and $$\bar{p}(\ell) = \frac{v_1\bar{m}_1(\ell) + \bar{v}_2\bar{m}_2(\ell)}{\dfrac{v_1}{p_1}\bar{m}_1(\ell) + \dfrac{v_2}{p_2}\bar{m}_2(\ell)} \qquad (4.63d)$$

4.2.6 Consider the marketing conditions where "word-of-mouth" contribution to advertizing is significant, i.e., where consumers already aware of a product help to create awareness in those as yet unaware in addition to the sellers' advertizing effort.

Let the market segment parameter λ (a constant) be such that for any two populations, X and Y in the market segment

$$Z = (\lambda X).Y$$

is the number of converts from population Y to population X in a given period.

Let $$a_1(\ell) = \alpha_1 + (1-\alpha_1)\lambda A_1(\ell) \qquad (4.64)$$

$$a_2(\ell) = a_2 + (1-\alpha_2)\lambda A_2(\ell) \qquad (4.65)$$

$$a_{12}(\ell) = a_1(\ell) + a_2(\ell) - a_1(\ell)a_2(\ell) \qquad (4.66)$$

Then the analysis is identical with that of 4.2.5 above with $a_1(\ell)$, $a_2(\ell)$ and $a_{12}(\ell)$ replacing respectively α_1, α_2 and α_{12}.

Note also that k_1 and k_2 become respectively

$$k_1(\ell) = a_1(\ell).k_{10} \quad , \quad k_2(\ell) = a_2(\ell).k_{20} \qquad (4.67)$$

Thus the effect of "word-of-mouth" process is to convert a set of linear difference state equations to a set of non-linear ones and to speed up the dynamics of the marketing system.

The model structure used for the "word-of-mouth" process is suitable for the modelling of the processes giving rise to the advertizing effort factor, α, itself. These processes are

(i) Media advertizing;

(ii) Salesmen advertizing;

(iii) Advertizing effect of distribution effort.

Let Z be the volume of media messages per long-term period and β be a factor related to media efficiency and copy effectiveness such that $\beta Z \leq 1$. Then $\beta Z(N_o - A(\ell))$ is the number of consumers made aware through media advertizing. In a multi-media situation, $z(N_o - A(\ell))$ is the number of consumers made aware where

$$z \leq 1$$

$$\text{and} \qquad (1-z) = \prod_{m=1}^{M} (1-\beta_m Z_m) \qquad (4.68)$$

Z and β are marketing decision variables.

Let Y be the number of salesmen available per long-term period and σ is a factor reflecting salesmen effectiveness.

$$\sigma Y \leq 1$$

$$\text{and} \qquad \sigma Y(N_o - A(\ell)) \qquad (4.69)$$

is the number of consumers made aware by this process.

Y and σ are also marketing decision variables.

Now, even in situations where a seller does not invest in media or salesmen advertizing, the fact that the product is available at retail outlets is sufficient to arouse consumer interest and spread consumer awareness. Let κ be a factor reflecting the contribution of retail outlets to advertizing.

i.e., $\kappa \leq 1$,

and $\kappa k_o(N_o - A(\ell))$ is the number of consumers made aware by this process.

Since these three processes are independent of each other and a given consumer may be exposed to them simultaneously, then α is defined as follows

$$(1-\alpha) = (1-z)(1-\sigma Y)(1-\kappa k_o) \quad (4.70)$$

4.2.7 Let assumption M3 now be relaxed to allow for variable marketing decisions (z, σ, Y, κ, k_o, p, q) of each seller. As a result, and by virtue of postulates MP1 and MP2, M4 is simultaneously relaxed, i.e., δ and Υ are no longer constant.

Thus
$$a_1(\ell) = \alpha_1(\ell) + (1-\alpha_1(\ell))\lambda A_1(\ell) \quad (4.71a)$$

$$a_2(\ell) = \alpha_2(\ell) + (1-\alpha_2(\ell))\lambda A_2(\ell) \quad (4.71b)$$

$$a_{12}(\ell) = a_1(\ell) + a_2(\ell) - a_1(\ell).a_2(\ell) \quad (4.71c)$$

$$\frac{p_1(\ell).\delta_1(\ell)}{v_1(\ell)} = \frac{p_2(\ell)\delta_2(\ell)}{v_2(\ell)} = \frac{p_{12}(\ell)\delta_{12}(\ell)}{v_{12}(\ell)} = I' \quad (4.72a)$$

$$\frac{\Upsilon_1(\ell)}{1+rv_1(\ell)} = \frac{\Upsilon_2(\ell)}{1+rv_2(\ell)} = \frac{\Upsilon_{12}(\ell)}{1+rv_{12}(\ell)} = \Upsilon_o \quad (4.72b)$$

And the analysis of 4.2.5 is again applicable with $a_1(\ell)$, $a_2(\ell)$ and $a_{12}(\ell)$ defined in equations (4.71) above replacing α_1, α_2 and α_{12} respectively and equations (4.72) replacing equations (4.59). The effect of variable marketing decisions of the sellers is to vary the parameters of the marketing system, thus transforming the marketing system model into a bilinear-type model [110].

4.3 MARKETING SYSTEM WITH MULTIPLE SELLERS EACH WITH MULTIPLE PRODUCTS

In the analyses of the marketing system with two sellers and one product per seller (4.2 above) it has been shown that marketing decision variables (parameters) of individual sellers (products) can be aggregated into decision variables (parameters) of a set of sellers (products) (Equations (4.63)). Thus, the analyses of 4.2 can be validly extended to the conditions in this section. However, there are certain features of the analyses of 4.2 that require attention before wholesale generalization is attempted. These are

1. Dimensionality - let i be the total number of products marketed in the given market segment and I(i) be the dimensionality of the marketing system, i.e., the number of independent states required to characterize system dynamics or, equivalently, the order of the demand/sales rate equations. Then, assuming that repurchasability factor is independent of product age

$$I(i) = (2^i - 1)2 \tag{4.73}$$

i.e., the order of the demand equation increases exponentially with the number of products marketed.

2. Diminishing effect of past values of demand (sales rate) on current values - consider the difference equation for demand

$$D(\ell) + d_1 D(\ell-1) + d_2 D(\ell-2) + \ldots\ldots + d_I D(\ell-I) = d_{I+1} N_o$$

as $i \to \infty$, $I(i) \to \infty$ (very rapidly) and $d_j \to 0$, $1 \leq j \leq I$, since d_j is the sum of products of terms that are all less than unity and tend to zero as $i \to \infty$. Thus, in order to describe the system adequately it is not necessary to consider all the terms in the equation.

3. From practical considerations, the increased accuracy that could result from considering all products marketed explicitly may not be justified because

(a) as the number of products increases, interactions among products, consumers and sellers may occur that were not considered in previous analyses;

(b) the market conditions may be such that it is possible to make some simplifying assumptions and/or the object of the analysis may be such that only aggregate variables need be considered.

As a result, a direct generalization of the analyses of 4.2 will not be attempted. Rather the multi-seller, multi-product case would be analyzed in the 2-product, 2-seller framework of 4.2. In particular, interest is focussed on two aspects of the multiple-seller, multiple-product case:

(i) Analysis of the effect of competition from products of other sellers on the demand and sales of the product line of a given seller in a given market segment (4.3.1 below), and

(ii) Analysis of the effect of introduction of new product items by a given seller on his existing product line and on his competition (4.3.2).

4.3.1 Let the suffix 1 refer to the given seller and the suffix c refer to all other sellers (the given seller's competition). With respect to the i'th product item of the given seller's product line, at instant ℓ, let

$k_{1i}(\ell)$ refer to the selling effort

$q_{1i}(\ell)$ " " " product quality

$p_{1i}(\ell)$ " " " " price

Assumption:

M7 All product items in the given seller's product line are advertized simultaneously, thus any consumer aware of one product item is aware of each and every item in the product line.

Then, as usual,

$$a_1(\ell) = \alpha_1(\ell) + (1-\alpha_1(\ell)\lambda A_1(\ell)$$

Let PL be the total number of products in seller 1's product line, $a_1(\ell)$ can then be viewed as resulting from the advertizing effort factors for the products in the product line which are all equal as a consequence of assumption M7. Let $a_{10}(\ell)$ be the value of the individual product advertizing effort factor. Then

$$a_{10}(\ell) = 1 - (1-a_1(\ell))^{1/PL} \tag{4.74}$$

For each i and r such that $1 \leq i,\ r \leq PL$

$$k_{1i}(\ell) = a_{10}(\ell) k_{1io}(\ell)$$

$$v_{1i}(\ell) = \frac{q_{1i}(\ell)}{p_{1i}(\ell)}, \quad w_{1i}(\ell) = \frac{v_{1i}(\ell)}{v_{1j}(\ell)}, \quad v_{1j}(\ell) \geq v_{1i}(\ell)\ \forall\ i$$

$$\delta_{1i}(\ell) = \frac{v_{1i}(\ell)\ I'}{p_{1i}(\ell)}$$

$$m_{1i}(\ell) = \frac{k_{1i}(\ell)}{\sum_{r=1}^{PL} k_{1r}(\ell) \dfrac{w_{1r}(\ell)}{w_{1i}(\ell)} \dfrac{(1 - w_{1i}(\ell).k_{1i}(\ell))}{(1 - w_{1r}(\ell).k_{1r}(\ell))}} \tag{4.75}$$

$$v_1(\ell) = \sum_{i=1}^{PL} v_{1i}(\ell)\ m_{1i}(\ell) \tag{4.76}$$

$$\delta_1(\ell) = \sum_{i=1}^{PL} \delta_{1i}(\ell)\ m_{1i}(\ell) \tag{4.77}$$

$$p_1(\ell) = \frac{v_1(\ell)\ I'}{\delta_1(\ell)} \tag{4.78}$$

$$k_1(\ell) = 1 - \prod_{i=1}^{PL} [1 - w_{1i}(\ell)\ k_{1i}(\ell)] \tag{4.79}$$

Thus $\alpha_1(\ell), v_1(\ell), \delta_1(\ell), p_1(\ell)$ and $k_1(\ell)$ represent the aggregated marketing decision variables/parameters of seller 1 with respect to the whole of his product line, and, $\alpha_c(\ell), v_c(\ell), \delta_c(\ell), p_c(\ell)$ and $k_c(\ell)$ respectively represent the aggregated marketing decision variables/parameters of the competition. The problem is now in the required 2-seller, 2-product framework and the analyses of 4.2.7 and 4.2.5 above may be applied to yield

$$S_1(\ell) = k_1(\ell)\delta_1(\ell)N_1(\ell) + m_1(\ell)k_{12}(\ell)\delta_1(\ell)N_{12}(\ell)$$

where (assuming $v_1(\ell) \geq v_c(\ell)$)

$$w_1(\ell) = 1 , \quad w_c(\ell) = \frac{v_c(\ell)}{v_1(\ell)}$$

$$m_1(\ell) = \frac{k_1(\ell)}{k_1(\ell) + k_c(\ell)w_c(\ell)\dfrac{(1-k_1(\ell))}{(1-w_c(\ell)k_c(\ell))}}$$

$$k_{12}(\ell) = 1 - (1-k_1(\ell))(1-w_c(\ell)k_c(\ell))$$

and the sales rate of the I'th item in seller 1's product line is given by

$$S_{1i}(\ell) = m_{1i}(\ell)\frac{\delta_{1i}(\ell)}{\delta_1(\ell)} S_1(\ell)$$

4.3.2 Finally, the case where the given seller, seller 1 has introduced new product items into his product line is considered. The analysis conducted here investigates the effects of the new set of product items on

(i) total sales rate of seller 1's products

(ii) total market share of seller 1's products

(iii) market share of new product items relative to the share of seller 1's product line (including the new items).

Assumption

In addition to M7, the following assumption is made

M8 All consumers in the market segment are aware of all products marketed prior to the introduction of the new product set.

The implication of this assumption is that what is, in essence, a 3-product marketing system (the new product set, the existing product line of seller 1 and the product line of the competition) reduces to a 2-product marketing system and hence the analyses of 4.2.7 and 4.2.5 are applicable. Equivalently, instead of considering $(2^3-1)2 = 14$ state equations, only $6 = (2^2-1)2$ state equations are required.

Let ℓ_o mark the beginning of the period L_o in which the new product set was introduced.

At each instant $\ell \geq \ell_o$

$A(\ell)$ is the number of consumers aware of any product item in any product set.

$A_e(\ell)$ is the number of consumers aware of existing products of seller 1 and his competition.

$N_e(\ell)$ is the number of consumers aware of existing products only but are yet to purchase.

$\alpha_e(\ell)$, $k_e(\ell)$, $p_e(\ell)$, $q_e(\ell)$ are the aggregated decision variables for the existing product set (of both seller 1 and his competition).

$A_n(\ell)$ is the number of consumers aware of the new product set (similarly $N_n(\ell)$, $\alpha_n(\ell)$, $k_n(\ell)$, $p_n(\ell)$, etc.).

$A_{ne}(\ell)$ is the number of consumers aware of both existing and new product sets (similarly $N_{ne}(\ell)$, $\alpha_{ne}(\ell)$, $k_{ne}(\ell)$, etc.).

From M8,

$$A(\ell) = A_e(\ell) = N_o \tag{4.80a}$$

hence $$A_n(\ell) = A_{ne}(\ell) \tag{4.80b}$$

and $$N_n(\ell) = 0 \tag{4.80c}$$

The other equations are:

$$A_{ne}(\ell+1) = [1-a_n(\ell)]A_{ne}(\ell) + a_n(\ell)\, N_o \tag{4.81}$$

where $$a_n(\ell) = \alpha_n(\ell) + (1-\alpha_n(\ell))\lambda\, A_n(\ell)$$

$$N_e(\ell+1) = [1-a_n(\ell)][[1-k_e(\ell)]N_e(\ell) + \gamma_e(\ell)[N_o-A_{ne}(\ell)]] \tag{4.82}$$

$$N_{ne}(\ell+1) = [1-k_{ne}(\ell)-\gamma_{ne}(\ell)]N_e(\ell) + \gamma_{ne}(\ell)A_{ne}(\ell) + a_n(\ell)[1-k_e(\ell)-\gamma_e(\ell)]N_e(\ell) + \gamma_e(\ell)[N_o-A_{ne}(\ell)] \tag{4.83}$$

$$D(\ell) = \delta_e(\ell)\, N_e(\ell) + \delta_{ne}(\ell)\, N_{ne}(\ell) \qquad (4.84)$$

$$S(\ell) = k_e(\ell)\, \delta_e(\ell)\, N_e(\ell) + k_{ne}(\ell)\, \delta_{ne}(\ell)\, N_{ne}(\ell) \qquad (4.85)$$

The total sales rate of seller 1's current product line (consisting of the existing product line and new product items) is

$$S_1(\ell) = m_{1e}(\ell)k_e(\ell)\delta_e(\ell)N_e(\ell) + [m_n(\ell) + [m_n(\ell)\delta_n(\ell) + m_{1e}(\ell)m_e(\ell)\delta_e(\ell)] e_{ne}(\ell)N_{ne}(\ell)] \qquad (4.86)$$

where (assuming $v_n(\ell) \geq v_e(\ell)$)

$$w_n(\ell) = 1 , \quad w_e(\ell) = \frac{v_e(\ell)}{v_n(\ell)}$$

and

$$m_n(\ell) = \frac{k_n(\ell)}{k_n(\ell)+k_e(\ell).w_e(\ell).\dfrac{(1-k_n(\ell))}{(1-w_e(\ell)k_e(\ell))}} , \quad m_e(\ell) = 1 - m_n(\ell)$$

$$k_{ne}(\ell) = 1 - [1-k_n(\ell)][1 - \frac{v_e(\ell)}{v_n(\ell)}\, k_e(\ell)]$$

$$v_{ne}(\ell) = v_n(\ell)m_n(\ell) + v_e(\ell)m_e(\ell)$$

$$m_{1e}(\ell) = \frac{k_{1e}(\ell)}{k_{1e}(\ell) + k_{ce}(\ell).w_{ce}(\ell)\, \dfrac{(1-k_{1e}(\ell))}{(1-w_{ce}(\ell)k_{ce}(\ell))}}$$

$$k_e(\ell) = 1 - [1-k_{1e}(\ell)][1 - w_{ce}(\ell)\, k_{ce}(\ell)]$$

$v_{1e}(\ell)$ and $v_{ce}(\ell)$ are respectively the aggregate value for money rating of the existing product sets of seller 1 and his competition. $k_{1e}(\ell)$ and $k_{ce}(\ell)$ are similarly the respective aggregate selling effort factors and $w_{ce}(\ell) = \dfrac{v_{ce}(\ell)}{v_{1e}(\ell)}$ (assuming $v_{1e}(\ell) \geq v_{ce}(\ell)$).

The market share of seller 1's current product line is

$$K_1(\ell) = \frac{S_1(\ell)}{S(\ell)}$$

and the market share of the new product set relative to the current product line is

$$K'_n(\ell) = \frac{m_n(\ell)k_{ne}(\ell)\delta_{ne}(\ell)N_{ne}(\ell)}{S_1(\ell)}$$

4.4 MARKETING EXPENDITURE MODELS

There is a fundamental difference between the marketing 'process' dynamics modelled in 4.1 to 4.3 above and production process dynamics modelled in Chapter 3. In a production process of the type described in 3.1 above, process dynamics are determined by current decision outputs only whereas marketing process dynamics are determined by both current and past marketing decision outputs. As a result, in a production system it is straight-forward to relate current production expenditure to current production outputs. This is not so in a marketing system where the carry-over effects of marketing expenditure in one period theoretically can affect marketing response in all subsequent periods.

In an analysis of the carryover effects of marketing expenditure, it is useful to regard carryover effects as being made up of two distinct elements [4] i.e.,

(i) delayed response effect

(ii) customer holdover effect.

Kotler [4] has identified four aspects of the delayed response effect as follows:

(a) execution delay - the time lag between the expenditure of marketing funds and the appearance of the marketing stimulus (for instance increased number of advertizing messages per period);

(b) noting delay - time lag between appearance of stimulus and noting by prospective buyers;

(c) purchase delay - time lag between noting of stimulus and effecting purchase;

(d) recording delay - time lag between effecting purchase and recognition by decision-making system that purchase has been made, in effect a measurement delay.

When considering long-term marketing system dynamics these effects are, in most cases, insignificant as they are essentially medium- or even short-term phenomena. It is therefore to the customer holdover effect, where marketing expenditure in a given long-term period affects sales in that and subsequent long-term periods, that attention is directed.

According to Little [104], empirical evidence suggests that at least the following factors should be considered when building dynamic models of response to current advertizing expenditure:

1. Sales respond dynamically upward and downward to increases and decreases in advertizing expenditure and frequently do so at different rates.
2. Steady-state response can be concave or S-shaped and will often have positive sales at zero advertizing expenditure.
3. Competitive advertizing affects sales.
4. Monetary effectiveness of advertizing can change over time as the result of changes in media, copy, etc.
5. Products sometimes respond to increased advertizing with a sales increase that falls off even as advertizing is held constant.

A brief review of the marketing models described so far shows that factors 1, 2 (concave steady-state response to advertizing expenditure), 3 and 5 are already accounted for as long as the advertizing effort factor expenditure function, $\alpha = C_A(X_A)$, relating advertizing expenditure per long-term period, X_A, to advertizing effort factor α satisfies the following criteria:

(1) $C_A(X_A)$ is a monotonically non-decreasing non-negative function of X_A

(2) as $X_A \rightarrow \infty$, $C_A(X_A) \rightarrow 1$

Two examples of such a function are

$$\text{(a)} \quad C_A(X_A) = h_m(1 - be^{-CX_A}) \tag{4.87a}$$

and

$$\text{(b)} \quad C_A(X_A) = h_m\left[1 - \frac{r}{f(X_A,n)}\right] \tag{4.87b}$$

where h_m $(0< h_m \leq 1)$ summarizes the effects of relative copy and media effectiveness thus explicitly incorporating factor 4 in the expenditure function; $b : 0 < b < 1$, $C > 0$; and $f(X_A,n)$ is a monotonically increasing polynomial (for $X_A \geq 0$) of order n such that $0< r< f(0,n)$

Remarks

(1) To account for possible non-zero sales at zero current advertizing expenditure, marketing models that incorporate the word-of-mouth effect (structurally equivalent to the 'goodwill' effect considered in the Nerlove-Arrow model [104,105]) have to be utilized (refer to section 4.2.6 above).

Let X_A denote current advertizing expenditure per period

X_A^E denote current equivalent advertizing expenditure per period

X_A^W denote current monetary worth of word-of-mouth contribution to the advertizing effort factor

α denote current advertizing effort factor

α_A denote component of α due to current expenditure

α_W denote component of α due to current word-of-mouth

Thus

$$\alpha = \alpha_A + (1 - \alpha_A)\alpha_W \quad \text{and} \quad \alpha_W = \lambda A(\ell)$$

But $\alpha = C_A(X_A^E)$ and $\alpha_A = C_A(X_A)$

hence word-of-mouth at current advertizing expenditure rate, X_A, is worth $X_A^E - X_A$ and when

$X_A = 0$, $X_A^E = X_A^W$ where

$$\alpha_W = C_A(X_A^W)$$

Note: The basic difference between the Nerlove-Arrow 'goodwill' formulation [105] and the word-of-mouth formulation is that Nerlove and Arrow explicitly consider a 'forgetting' factor to account for leakage of 'goodwill' in the absence of fresh marketing stimuli. In the word-of-mouth formulation, α_W is net of this 'forgetting' factor, i.e., it is assumed that advertizing expenditure always includes an amount sufficient to maintain α_W at its current level.

(2) To account for the possibility of S-shape steady state response, the cost function $C_A(X_A)$ must satisfy in addition the following conditions where

$$C_A'(X_A) = dC_A(X_A)/dX_A$$

(a) $C_A'(X_A) > 0$ for all X_A ; $0 \leq X_A < \infty$

(b) $C_A'(X_A)$ increases monotonically to a maximum value from $C_A'(0)$ after which it decreases monotonically to zero as X_A tends to infinity.

Two examples are

(i) $C_A'(X_A) = h_m C_A(X_A)[1 - C_A(X_A)]$, $C_A(0) > 0$

which results in the <u>logistic</u> function

$$\alpha = C_A(X_A) = \frac{h_m}{1 + be^{-CX_A}}$$

where $b = \dfrac{1 - C_A(0)}{C_A(0)}$ and $C = 1$

and

(ii) $$C_A'(X_A) = \frac{h_m \, rf'(X_A,n)}{[f(X_A,n)]^2} \left[1 - \frac{S}{[f(X_A,n)]^P}\right] \qquad (4.88a)$$

where r and $f(X_A,n)$ are as in equation (4.87b) above, $0 \leq S < [f(0,n)]^P$; $f'(X_A,n) = df(X_A,n)/d X_A$; P is a positive integer whence

$$C_A(X_A) = h_m [1 - \frac{r}{f(X_A,n)} [1 - \frac{S/(P+1)}{[f(X_A,n)]^P}]] \tag{4.88b}$$

The structure of the advertizing expenditure function can be utilized in both the distribution and quality expenditure functions - it is assumed that no expenditure is incurred directly in maintaining or varying a given price level. Considering for instance the example of equation (4.87a) the distribution and quality expenditure functions can be described respectively by

$$k_o = C_K(X_K) = h_d(1 - ve^{-WX_K}) \tag{4.89}$$

and

$$q = C_Q(X_Q) = (1 - ye^{-ZX_Q}) \tag{4.90}$$

where h_d accounts for relative distribution channel and retail outlet effectiveness.

4.5 SUMMARY AND DISCUSSION

The marketing aspect of a PMS has been analyzed and its dynamics modelled as a set of difference equations of state and a set of output equations relating the marketing decisions of sellers to market demand and sales rate of the sellers' products in a given market segment. The formulation that results is such that the marketing decisions affect the marketing system indirectly by variation of its parameters rather than as direct inputs to either its state or output equations.

The analysis is conducted on a long-term time scale, where each long-term period is of a year's duration. Throughout, the products marketed were assumed to be consumer durables and the population of the market segment was assumed constant. Different models of the marketing system were derived reflecting various assumptions of market conditions from the simple single-product, single-seller, constant-decision marketing system, through the detailed study of the two-seller, two-product (one product per seller) marketing system with either constant or variable decisions, to the general multi-product, multi-seller, variable-decision marketing system.

The models obtained in this chapter are analytical models based on "first principle" analysis of the marketing system. This is in contrast to empirical models of marketing system dynamics (see [104, 112] for reviews) which are usually derived thus:

(i) Observe the qualitative and quantitative behaviour of the marketing system over a period of time;

(ii) postulate a structure for the demand function such that either

(a) it involves current marketing effort (price, advertizing expenditure), a saturating growth term, and a term reflecting seasonal variations, or

(b) it involves current and past marketing efforts; and

(iii) use statistical analysis for parameter estimation.

There are obvious disadvantages to this approach, in that

(i) the estimated parameters have been averaged over the record length of the data involved and hence variations of these parameters with marketing effort or over time cannot be accounted for;

(ii) the resulting models apply only to a given product set which is assumed to remain constant - thus effects of introduction of new products cannot be estimated;

(iii) the models are valid only for a limited prediction period. On the other hand, the notable analytical (a priori) models described in the literature [104] are incremental in nature - that is they describe the response of the marketing system to variations in marketing decisions about a reference point and are therefore applicable only to established products in an established market. The models developed in this chapter have definite advantages over other a priori models in that they allow for

(i) the introduction of new products,

(ii) the tracing of the product (item, line or class) life-cycle,

(iii) the determination of market shares,

(iv) the variation of market shares, sales rate, demand (i.e., sales potential) over time as a consequence of either constant or variable marketing effort (decisions),

(v) the contribution of "word-of-mouth", salesmen, distribution effort, and media advertizing to the advertizing effort,

(vi) explicit inclusion of individual marketing effort variables i.e., advertizing, distribution, product price and quality,

(vii) explicit inclusion of the number of consumers in the market, their tastes and per capita income, and, product repurchasability and "value-for-money."

Finally, the analyses presented have shown that in general a marketing system can be adequately represented by a 2-product structure resulting in six state equations (or a 6th-order difference equation for demand/sales-rate) except, perhaps, in the specific instance of the introduction of new products into the market where a 3-product structure may be necessary resulting in 14 state equations, and that part of the observed marketing system response to current marketing expenditure derives from the functional relationship existing between marketing expenditures and marketing decision outputs (α, k, q) and are therefore not intrinsic to the marketing process itself.

CHAPTER V

SIMULATION OF MARKETING AND PRODUCTION SYSTEM DYNAMICS

5.1 SIMULATION OF A CONSTANT-DECISION MARKETING SYSTEM

In this section, the results of the simulation of a constant-decision marketing system, based on the models developed in Sections 4.1 and 4.2 of Chapter 4 are presented and discussed. Unless otherwise stated, the following market parameters are assumed:

1. Market population, No $= 10^6$
2. Word-of-mouth factor, λ, $= 0.4 \times 10^{-6}$
3. Product age-dependent repurchasability factor:

   For products aged between

   (i) 0 and 1 year, $\Upsilon_1 = 0.054$

   (ii) 1 and 2 years, $\Upsilon_2 = 0.072$

   (iii) 2 and 3 years, $\Upsilon_3 = 0.102$

   (iv) 3 and 4 years, $\Upsilon_4 = 0.156$

   (v) 4 and 5 years, $\Upsilon_5 = 0.278$

   (vi) 5 and 6 years, $\Upsilon_6 = 0.532$

   (vii) 6 and 7 years, $\Upsilon_7 = 0.776$

   (viii) 7 and 8 years, $\Upsilon_8 = 0.878$

   (ix) 8 and 9 years, $\Upsilon_9 = 0.922$

   (x) 9 years or more, $\Upsilon_{10} = 0.945$

4. Product age-independent repurchasability factor obtained from the age-dependent factors as follows

$$\Upsilon = \frac{\Upsilon_{10}}{1 + (\Upsilon_{10}-\Upsilon_1) + \sum_{i=2}^{9} [(\Upsilon_{10}-\Upsilon_i) \prod_{r=1}^{i-1} (1 - \Upsilon_r)]} \qquad (5.1)$$

which is the ten age-group equivalent of equation (4.47).

5. A 30-period (30-year) simulation horizon is used.

5.1.1 Single-product, constant-decision, marketing system simulation*

The single-product marketing system of Section 4.1 of Chapter 4 is simulated under conditions of

(i) Absence/presence of word-of-mouth effect, see sub-section 4.2.6 above;

(ii) Zero; non-zero, age-independent; non-zero, age-dependent product repurchasability factor (refer to sub-sections 4.1.2, 4.1.3 and 4.2.4 above). The six conditions are considered for four combinations of the advertizing effort factor (AEF) and the distribution effort factor (DEF). The simulation results are displayed in four diagrams, Figures 5.1 to 5.4, each containing six Sales (in units) - Time curves where curve

1 shows the zero repurchasability, no word-of-mouth case;

2 shows the zero repurchasability, word-of-mouth case;

3 shows the age-independent repurchasability, no word-of-mouth case;

4 shows the age-dependent repurchasability, no word-of-mouth case;

5 shows the age-independent repurchasability, word-of-mouth case;

6 shows the age-dependent repurchasability, word-of-mouth case.

It is assumed that the per unit demand, i.e., volume of product desired per consumer, $\delta = 1$.

Figures 5.1 to 5.4 show that the word-of-mouth effect is most significant at low levels of the applied AEF. At low levels of the AEF-DEF product, the difference between the non-zero age independent and the non-zero age-dependent repurchasability curves is insignificant; it is at high levels of AEF-SEF product, especially in markets where the word-of-mouth factor λ is large, that possible age-dependency of the product repurchasability factor must be seriously investigated. Figure 5.4 shows clearly that the transient oscillations have an 8-period cycle and can deviate by as much as 30% from the corresponding age-independent case. In markets where bulk replacement of the product after a fixed number of years of service is common (as in some

* Simulation results obtained using the simulation programme M22.FTN

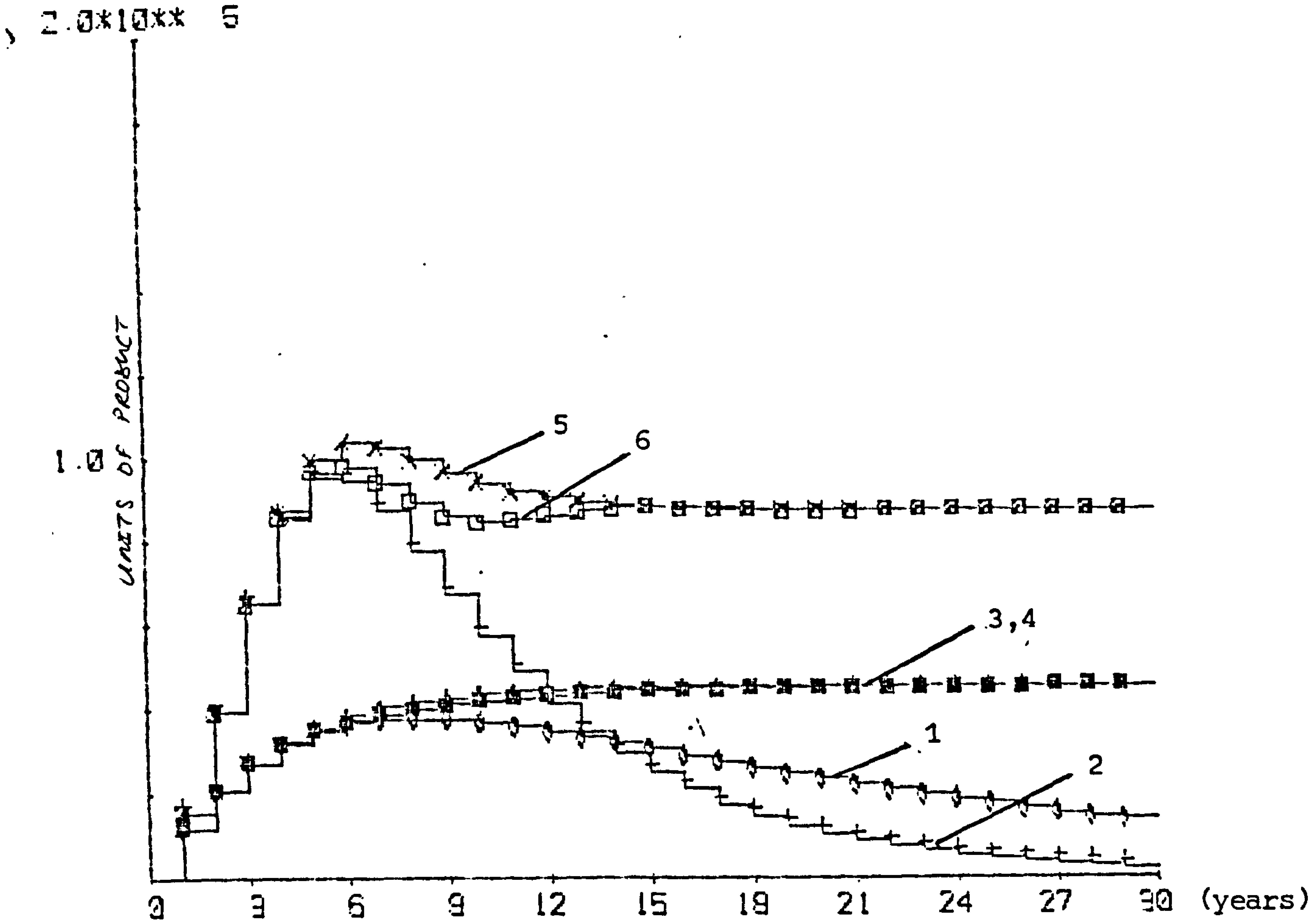

FIGURE 5.1 Sales-Time Curves: AEF = 0.2, SEF = 0.3

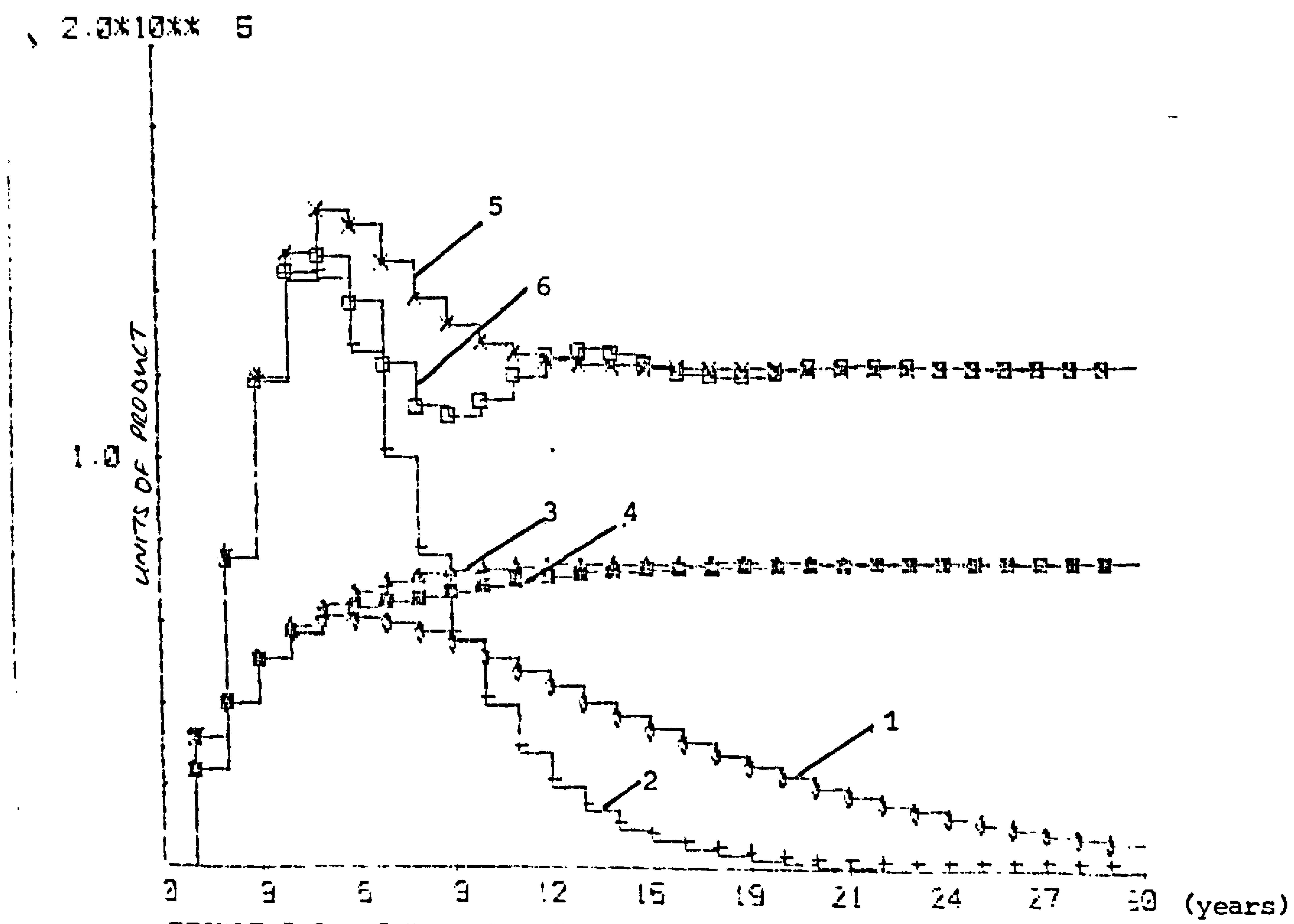

FIGURE 5.2 Sales-Time Curves: AEF = 0.2, SEF = 0.6

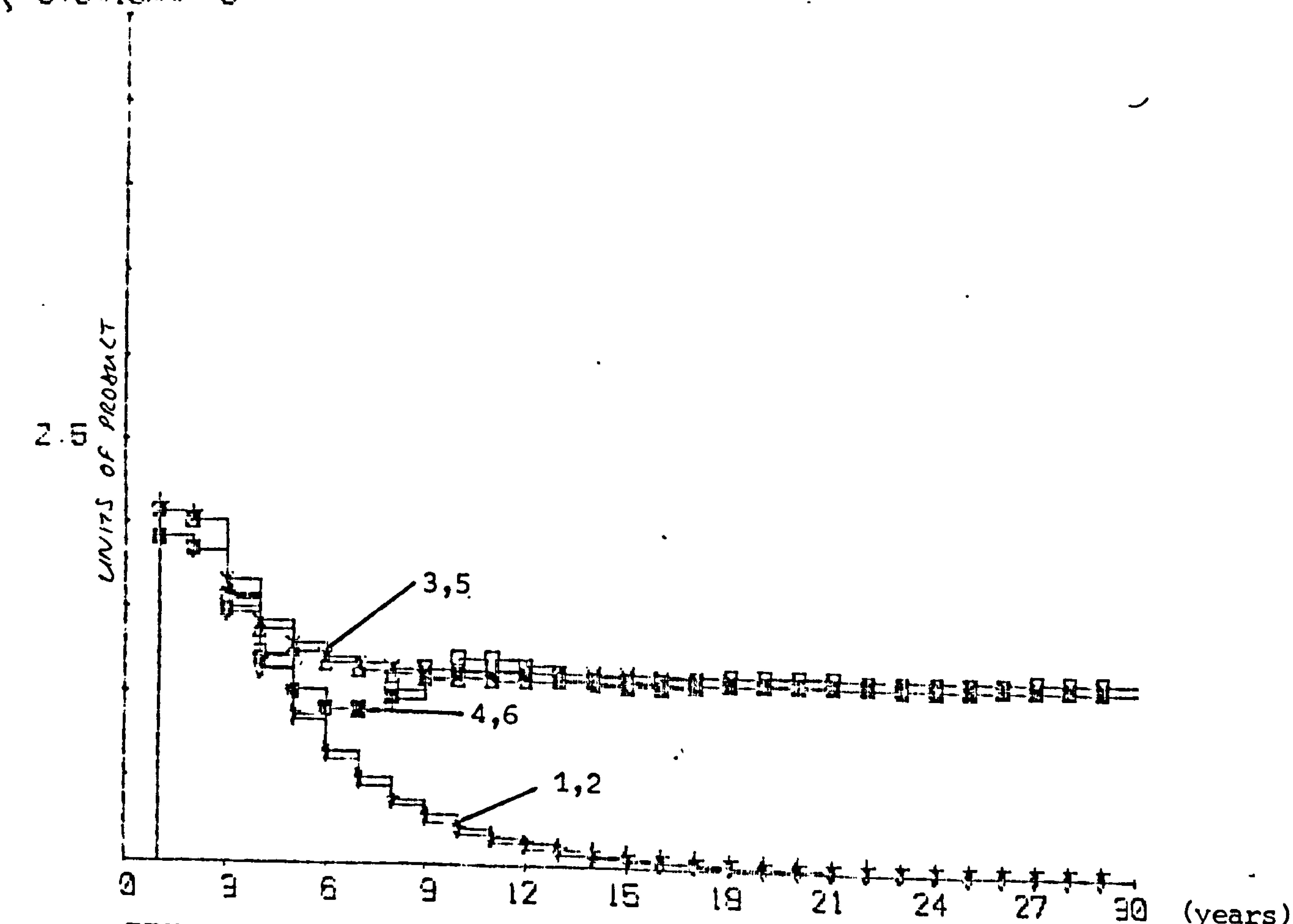

FIGURE 5.3 Sales-Time Curves: AEF = 0.8, SEF = 0.3

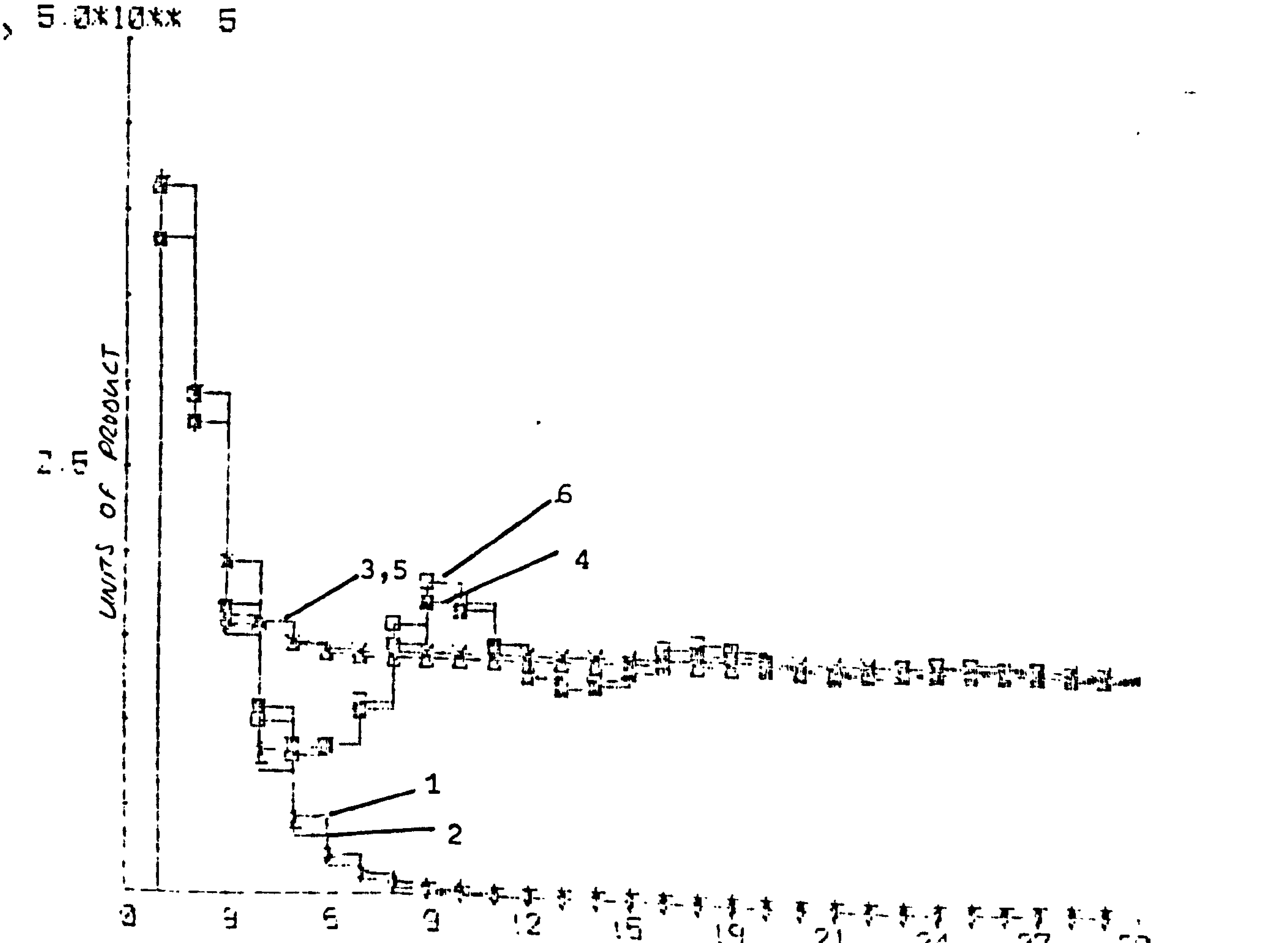

FIGURE 5.4 Sales-Time Curves: AEF = 0.8, SEF = 0.6

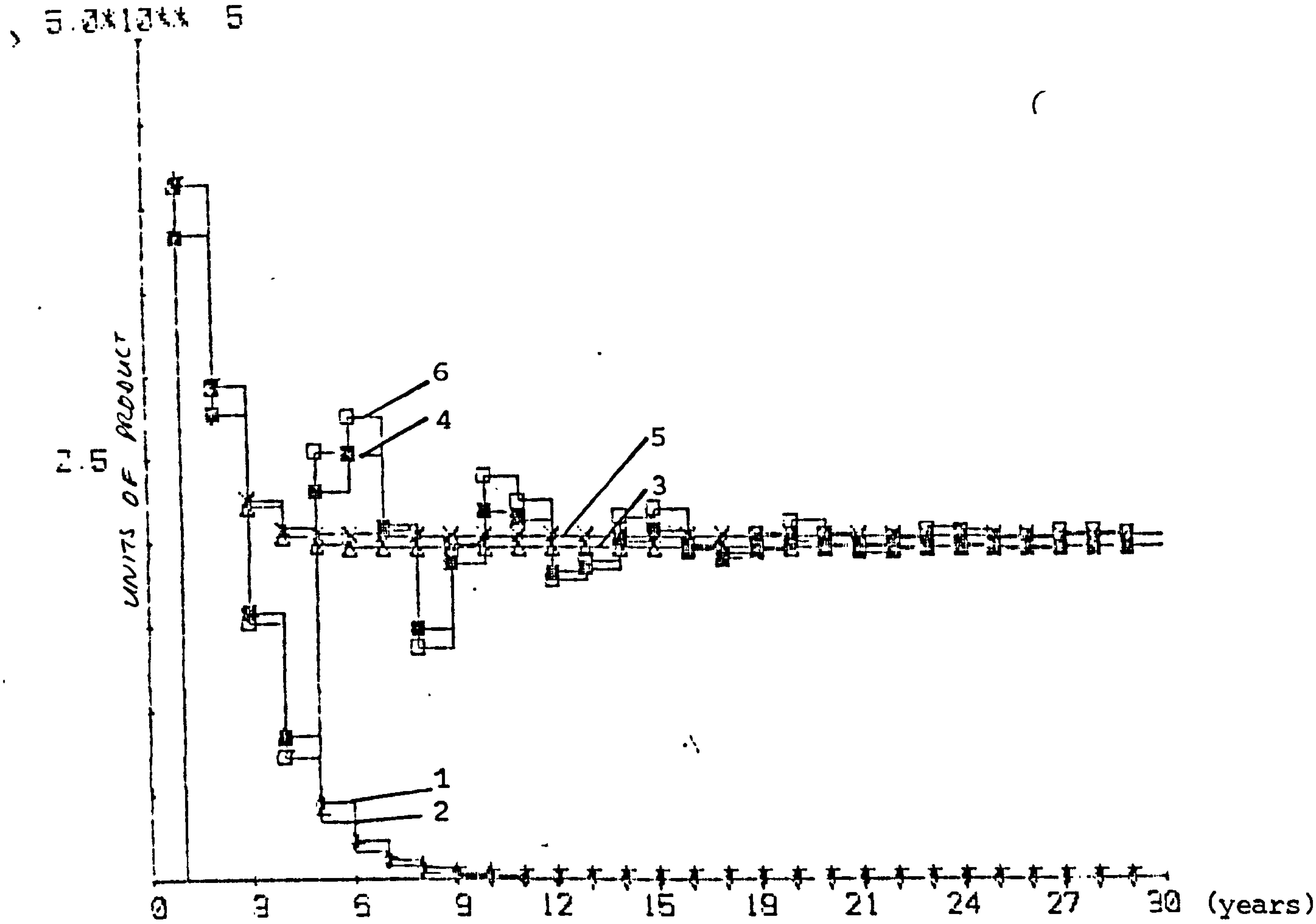

FIGURE 5.5 Sales-Time Curves: AEF = 0.8, SEF = 0.6
Replacement in 4th Year

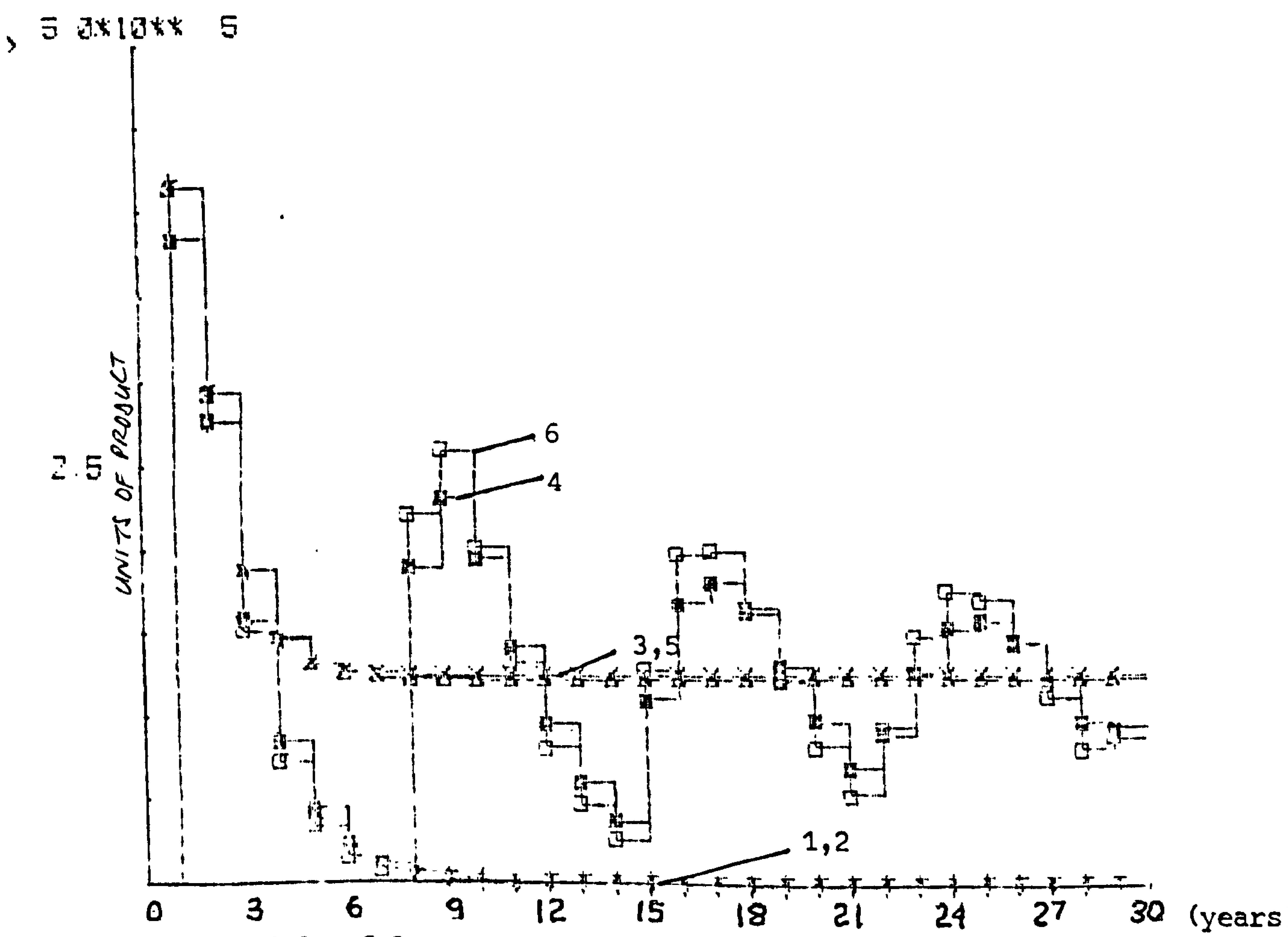

FIGURE 5.6 Sales-Time Curves: AEF = 0.8, SEF = 0.6
Replacement in 7th Year

industrial markets where the replacement policy is such that the product is replaced as soon as it enters a given age-group) these transient oscillations can be much more pronounced and decay at a slower rate as evidenced by the Sales-Time curves of Figures 5.5 and 5.6 where all products currently aged between 2 and 3 years are replaced in the following year and all products currently aged between 5 and 6 years are replaced in the following year respectively. (Figures 5.5 and 5.6 are to be compared with Figure 5.4).

5.1.2 2-product, homogeneous, constant-decision, marketing system simulation*

The 2-product, constant-decision, marketing system of section 4.2 above is simulated using the same marketing parameters as in 5.1.1 above but with no word-of-mouth and both products have equal prices and qualities, i.e., they are homogeneous. The constant decisions AEF2 = 0.5 and DEF2 = 0.3 are assumed for product 2 while four combinations of AEF1 and DEF1, identical to those in 5.1.1 above, are chosen for product 1, thus permitting easy comparison of results. The simulation results for each combination are displayed in Figures 5.7 to 5.10. The (a) part of each figure consists of six Sales-Time curves where curves

1,2 show the total market sales of products 1 and 2 assuming respectively age-independent and age-dependent repurchasability

3,4 show corresponding sales of product 1 under the respective repurchasability conditions.

5,6 similarly show sales of product 2.

The (b) part also consists of six curves where curves

1,2 respectively show market penetration (total sales divided by total demand) over time for age-independent and age-dependent repurchasabilities;

3,4 respectively show product 1 market share over time for the two repurchasability conditions;

---

* Simulation results obtained using M22.FTN.

5,6 correspondingly show product 2 market shares over time.

A glance at the (a) part of Figures 5.7 to 5.10 show that the differences between the age-independent and age-dependent repurchasability cases are accentuated for product 1 in the 2-product simulation (curves 3 and 4) as compared with the analogous single-product simulation (curves 3 and 4 of Figures 5.1 to 5.4). This is due to the fact that overall system AEF-DEF product has increased and as such the system transient oscillations are greater than for the analogous single-product simulation previously described and has affected product 1 sales dynamics in turn. However, the (b) parts show that the market penetration and market share curves for either product are relatively insensitive to the repurchasability assumptions. This is because while market penetration and market share are ratios of two quantities that are sensitive to the product repurchasability assumptions, the ratios themselves are relatively insensitive. Thus in a product market share study, the assumption of constant (age-independent) repurchasability need not introduce any significant error.

Comparing the Sales-Time performance of product 1 in the 2-product and single-product simulations (respectively curves 3 and 4 in (a) parts of Figures 5.7 to 5.10 and curves 3 and 4 in Figures 5.1 to 5.4) we note that aside from the accentuated transient dynamics, the steady state sales values of product 1 in the 2-product simulation are only slightly depressed relative to their corresponding values in the single product simulation. It can therefore be said that the sales of product 2 were derived almost wholly from the expansion of the market and only slightly at the expense of product 1 sales. This statement is confirmed by comparison of the market penetration curves (curves 1 and 2 of (b) part of Figures 5.7 to 5.10) with the market penetration curves for the single-product simulation shown in Figure 5.11 (note the change of scale) where curve

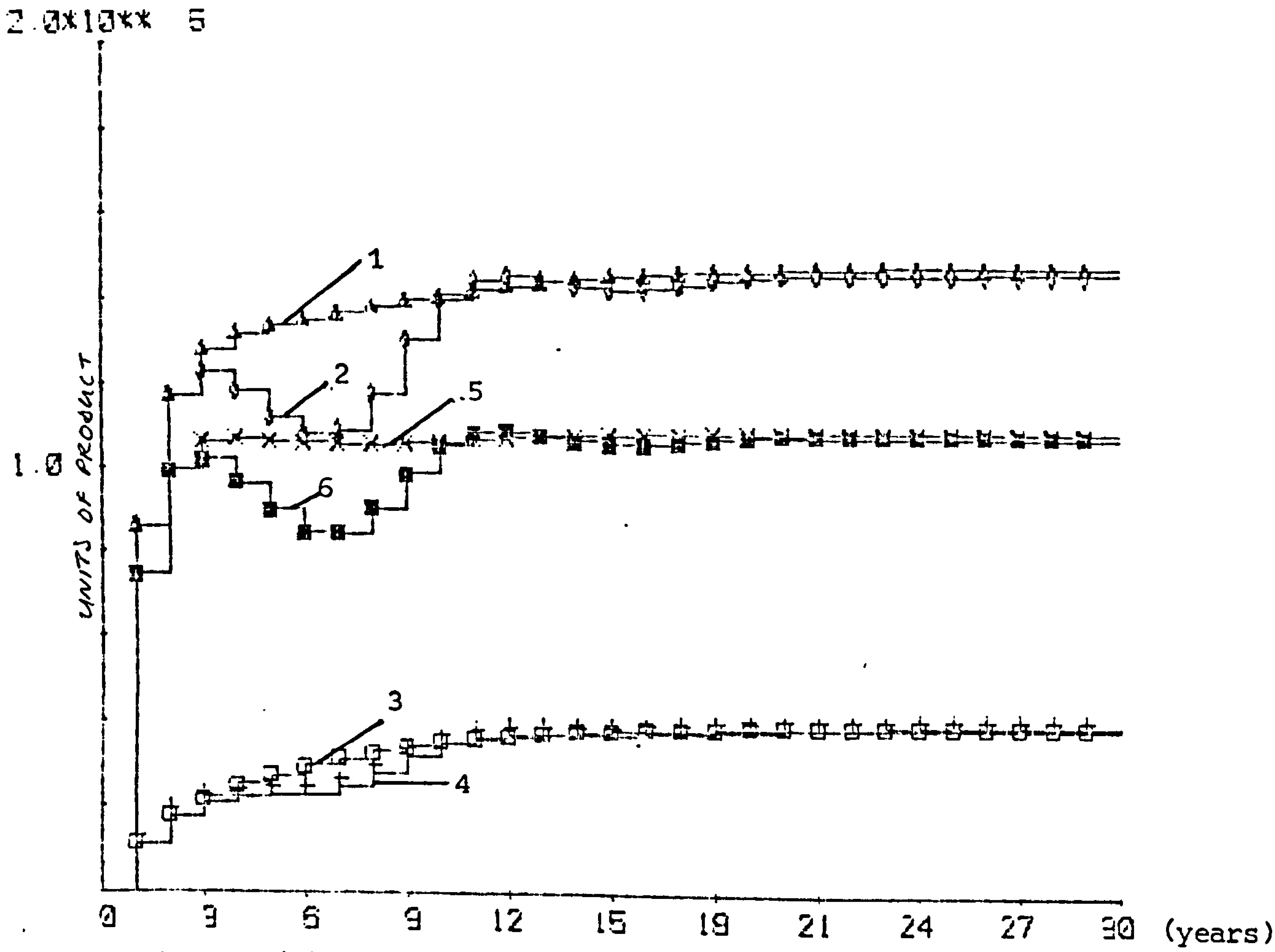

FIGURE 5.7(a) Sales–Time Curves: AEF1 = 0.2, SEF1 = 0.3

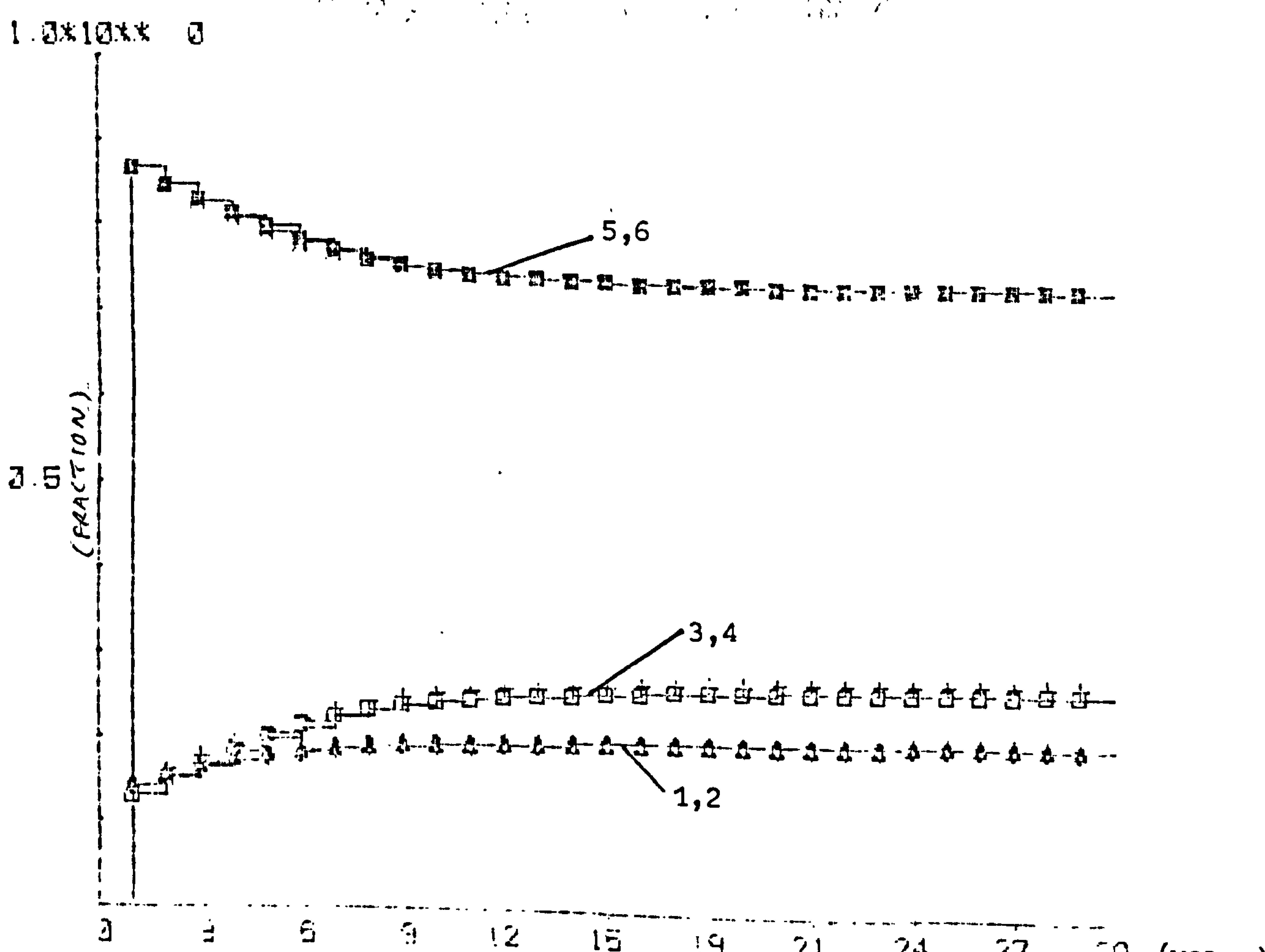

FIGURE 5.7(b) Market Penetration/Shares–Time Curves: AEF1 = 0.2, SEF1 = 0.3

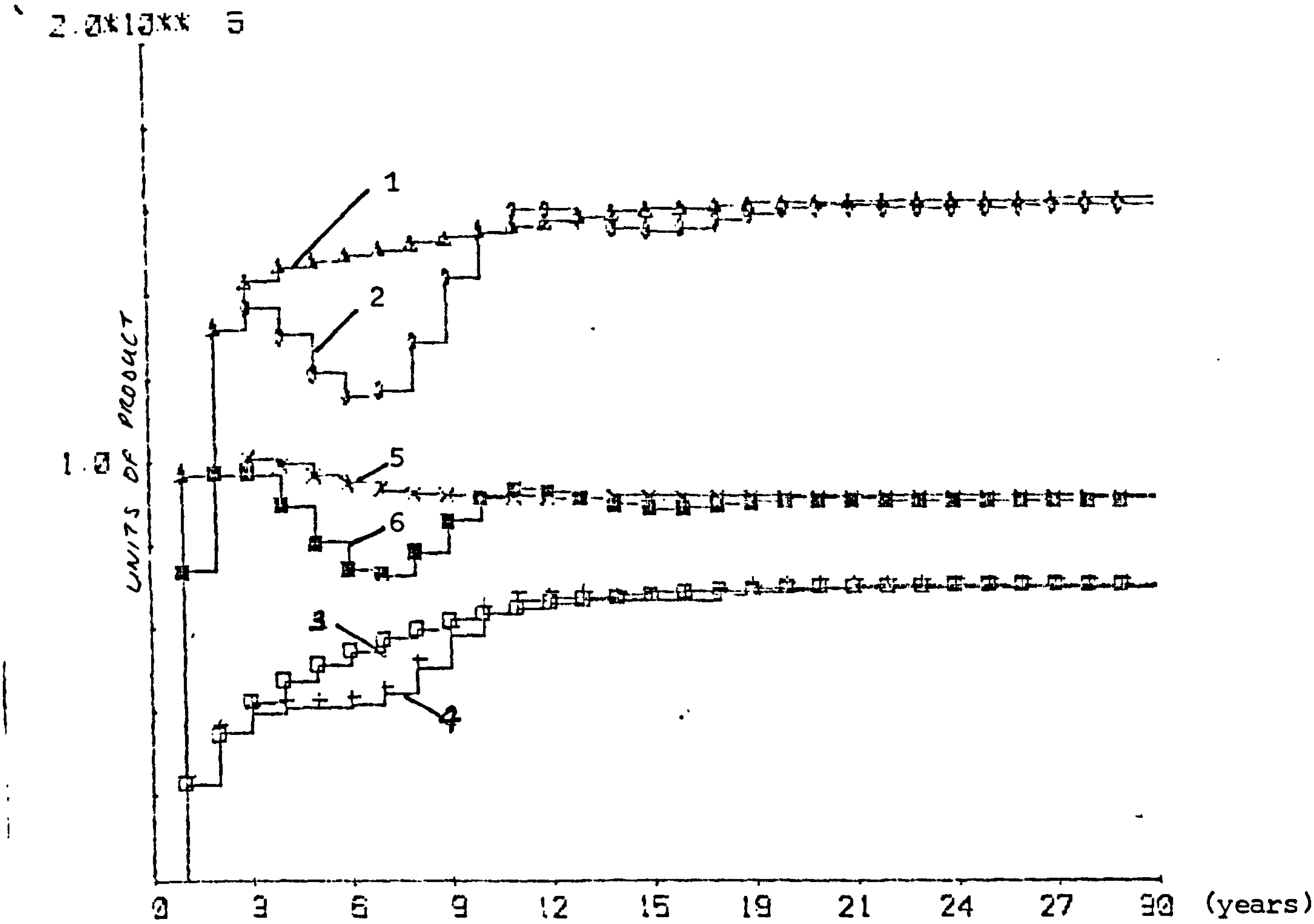

FIGURE 5.8(a) Sales-Time Curves: AEF1 = 0.2, SEF1 = 0.6

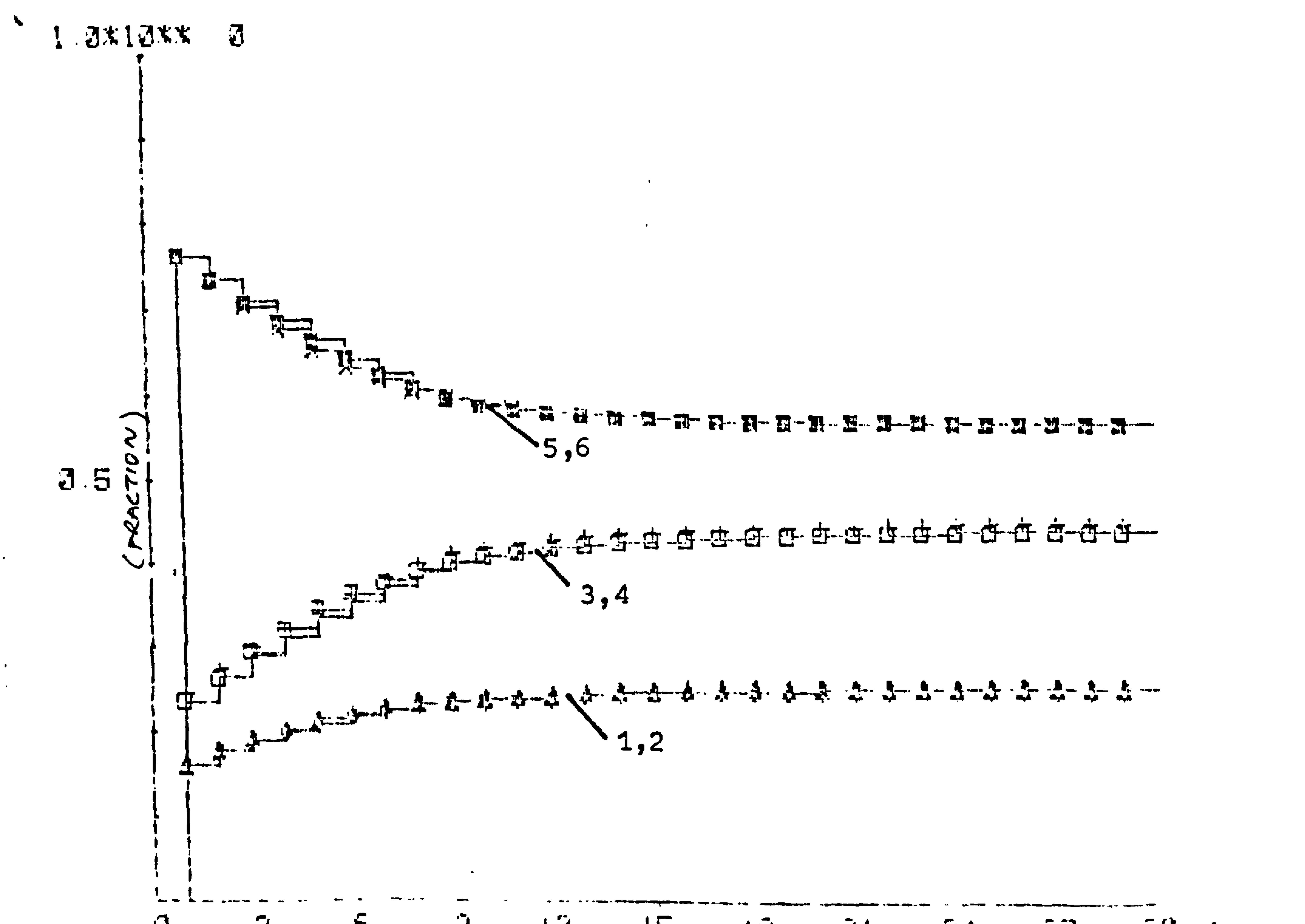

FIGURE 5.8(b) Market Penetration/Shares-Time Curves: AEF1 = 0.2, SEF1 = 0.6

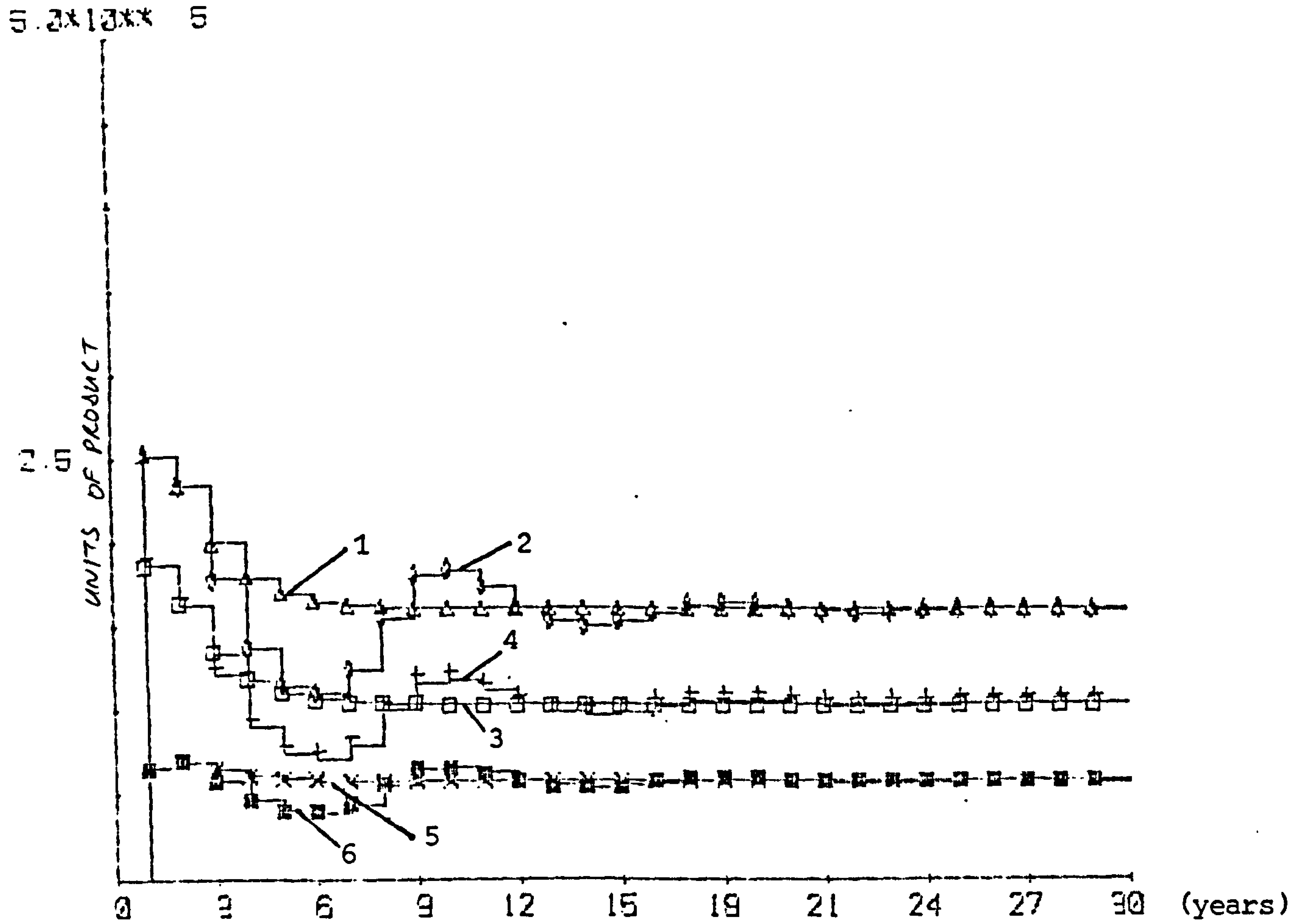

FIGURE 5.9(a) Sales-Time Curves: AEF1 = 0.8, SEF1 = 0.3

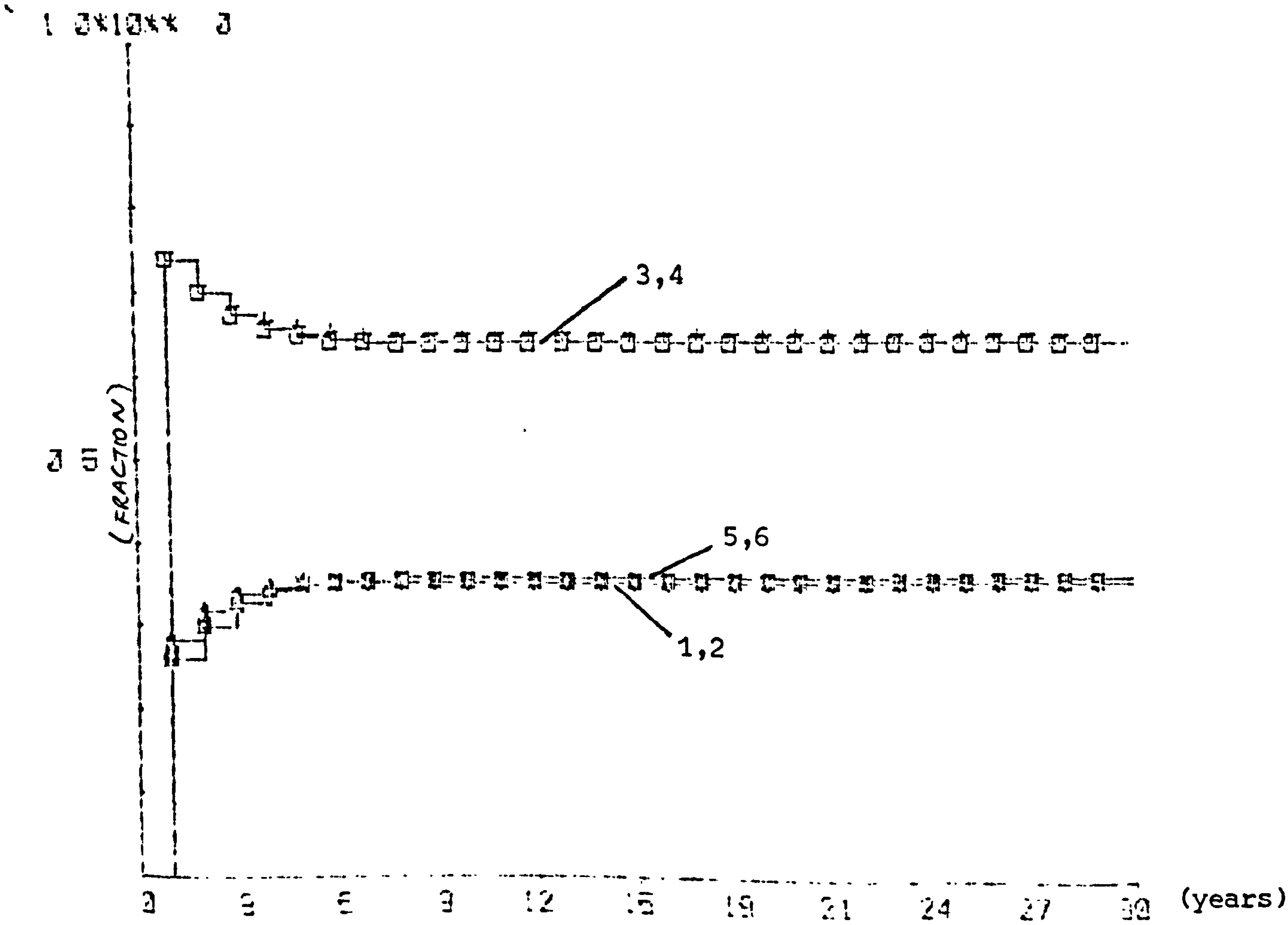

FIGURE 5.9(b) Market Penetration/Shares-Time Curves: AEF1 = 0.8, SEF1 = 0.3

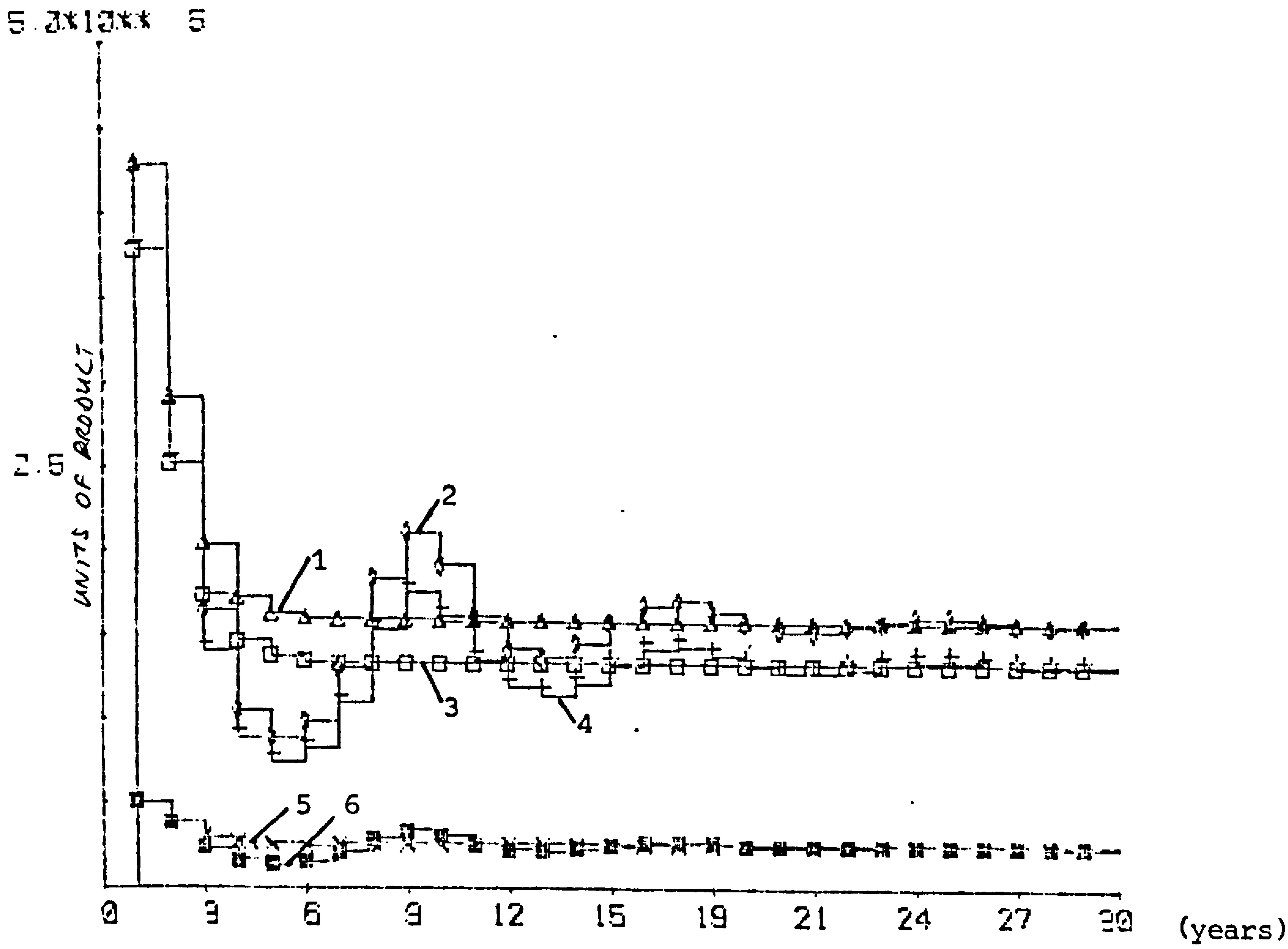

FIGURE 5.10(a) Sales-Time Curves: AEF1 = 0.8, SEF1 = 0.6

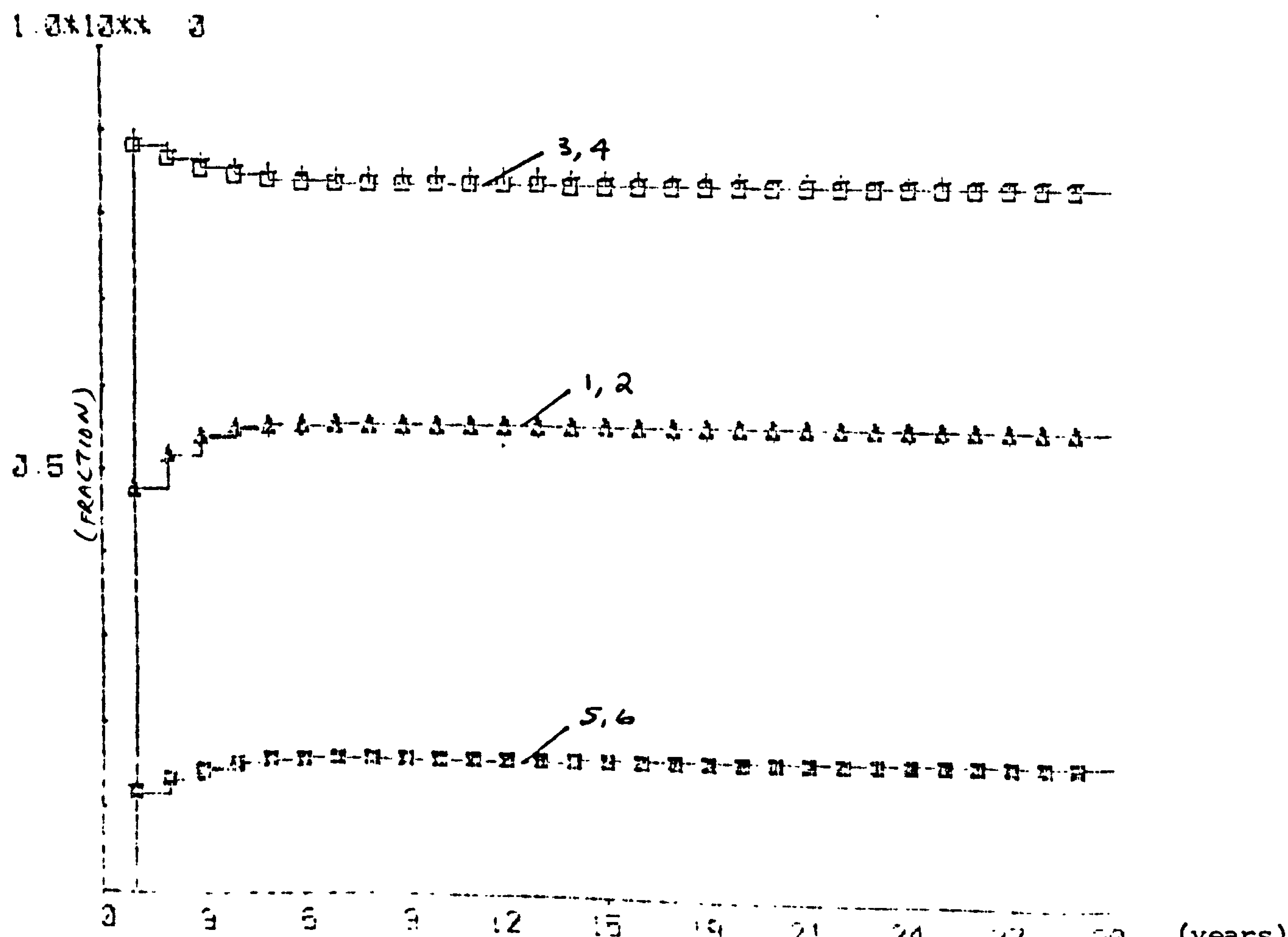

FIGURE 10(b) Market Penetration/Shares-Time Curves: AEF1 = 0.8, SEF1 = 0.6

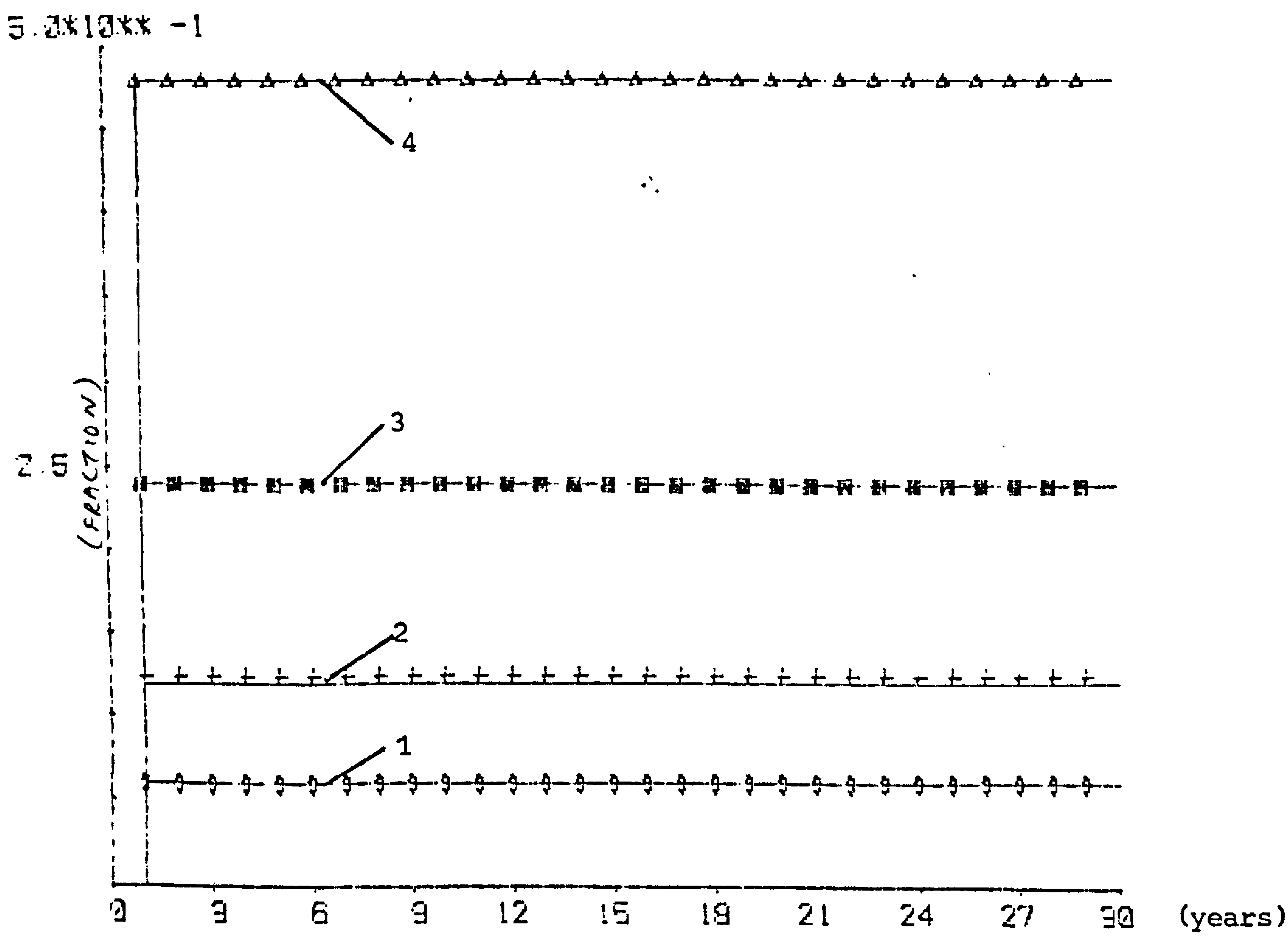

FIGURE 5.11 Market Penetration-Time Curves

1 shows the market penetration over time for AEF = 0.2, DEF = 0.3

2 " " " " " " " AEF = 0.2, DEF = 0.6

3 " " " " " " " AEF = 0.8, DEF = 0.3

4 " " " " " " " AEF = 0.8, DEF = 0.6

under conditions of age-dependent repurchasability and no word-of-mouth.

5.1.3 <u>2-product, non-homogeneous, constant decision, marketing system simulation</u>*

This simulation was conducted using the following data:

1. Market population, No $= 10^6$
2. Word-of-mouth factor, $\lambda = 0$
3. Market segment income factor (see equation (4.56) above), $I' = 9 \times 10^5$. I' was chosen such that when product quality rating $q = 0.4$ and product price $p = 600$, then per-unit demand, $\delta = 1$.
4. The constant, r, (see equation (4.55) above) and the age-dependent repurchasability factors have been chosen such that when $q = 0.4$ and $p = 600$, the age-dependent repurchasability factors described in the beginning of Section 5.1 result.

Thus $r = 52.5$ and

$\gamma_1 = 0.0197$

$\gamma_2 = 0.0383$

$\gamma_3 = 0.0694$

$\gamma_4 = 0.1254$

$\gamma_5 = 0.2518$

$\gamma_6 = 0.5158$

$\gamma_7 = 0.7679$

$\gamma_8 = 0.8736$

$\gamma_9 = 0.9192$

$\gamma_{10} = 0.9430$

---

* Simulation results obtained using the programme, MARK2.FTN

The simulation reported here considered two price levels and two quality ratings for product 1, 495 and 825, and 0.33 and 0.55 respectively, yielding four price/quality combinations for product 1. Two of these combinations, (495, 0.33) and (825,0.55) result in identical value-for-money rating as for product 2 where product 2's price/quality combination is given by (600, 0.4); the combination (495, 0.55) results in a superior value-for-money rating over product 2 and the combination (800, 0.33) results in an inferior value-for-money rating. Only one advertizing effort/distribution effort combination is considered for either product; (0.2, 0.6) for product 1 and (0.5, 0.3) for product 2.

The (a) parts of Figures 5.12 to 5.15 are Sales-Time curves where curves 1,2 and 3 respectively show the total, product 1, and product 2 sales over time while the (b) parts are Market Penetration/Share-Time curves with curves 1, 2 and 3 showing respectively market penetration, product 1 and product 2 market shares over time. On comparing the six curves in each of Figures 5.12 to 5.15 with each other and with curves 2, 4 and 6 of Figures 5.8(a) and (b) for the analogous homogeneous product case we note the following:

1. Whenever product 1 has the same value-for-money rating as product 2 (Figures 5.12 and 5.15), the steady-state market penetration/share values are as for the homogeneous case though overall and individual product sales are reduced relative to the homogeneous case when product 1 price is greater than product 2 price (Figure 5.12(a)) and increased all round when product 1 price is less than product 2 price (Figure 5.15(a)).
2. When product 1 has a higher value-for-money rating than product 2, the steady-state market penetration, product 1 market share, overall sales and product 1 sales increase relative to the case, at the same product 1 price, where both products enjoy the same value-for-money rating. Product 1's higher value-for-money rating can be regarded as being derived from the increase in quality rating over that required to equalize value-for-money (with that of product 2) at the given product 1 price; the sales benefit

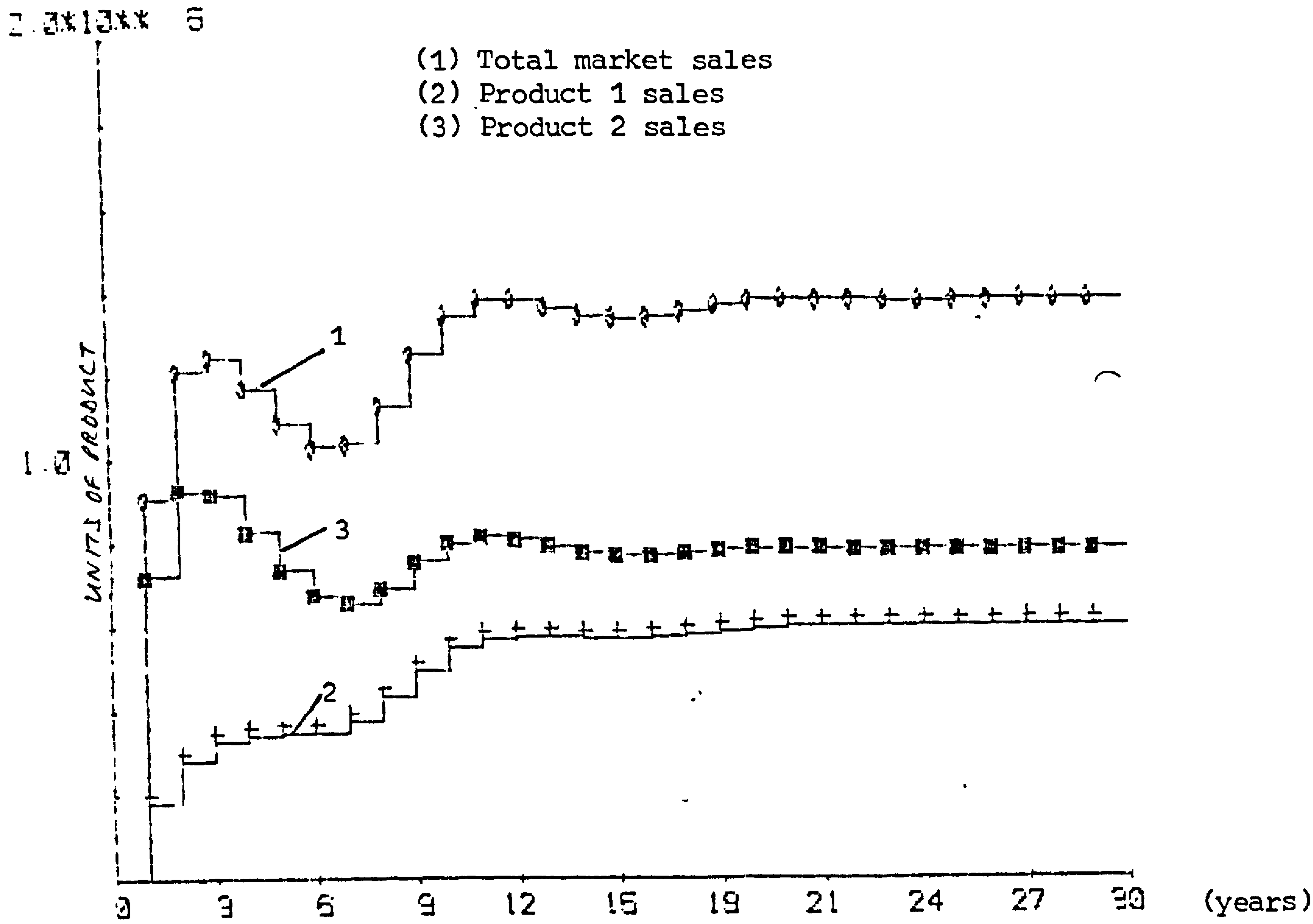

FIGURE 5.12(a) Sales-Time Curves: P1 = 825, Q1 = 0.55

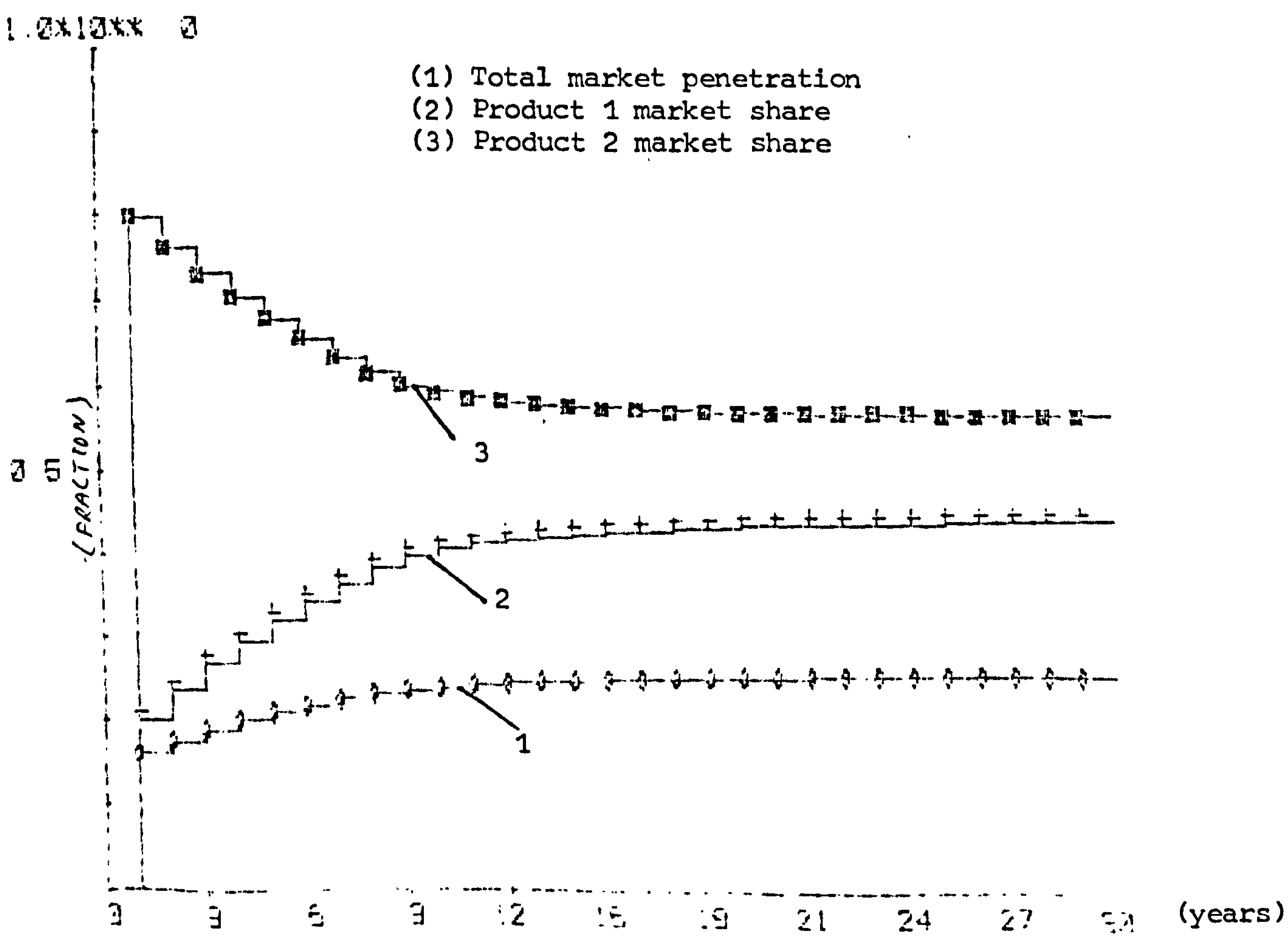

FIGURE 5.12(b) Market Penetration/Shares-Time Curves: P1 = 825, Q1 = 0.55

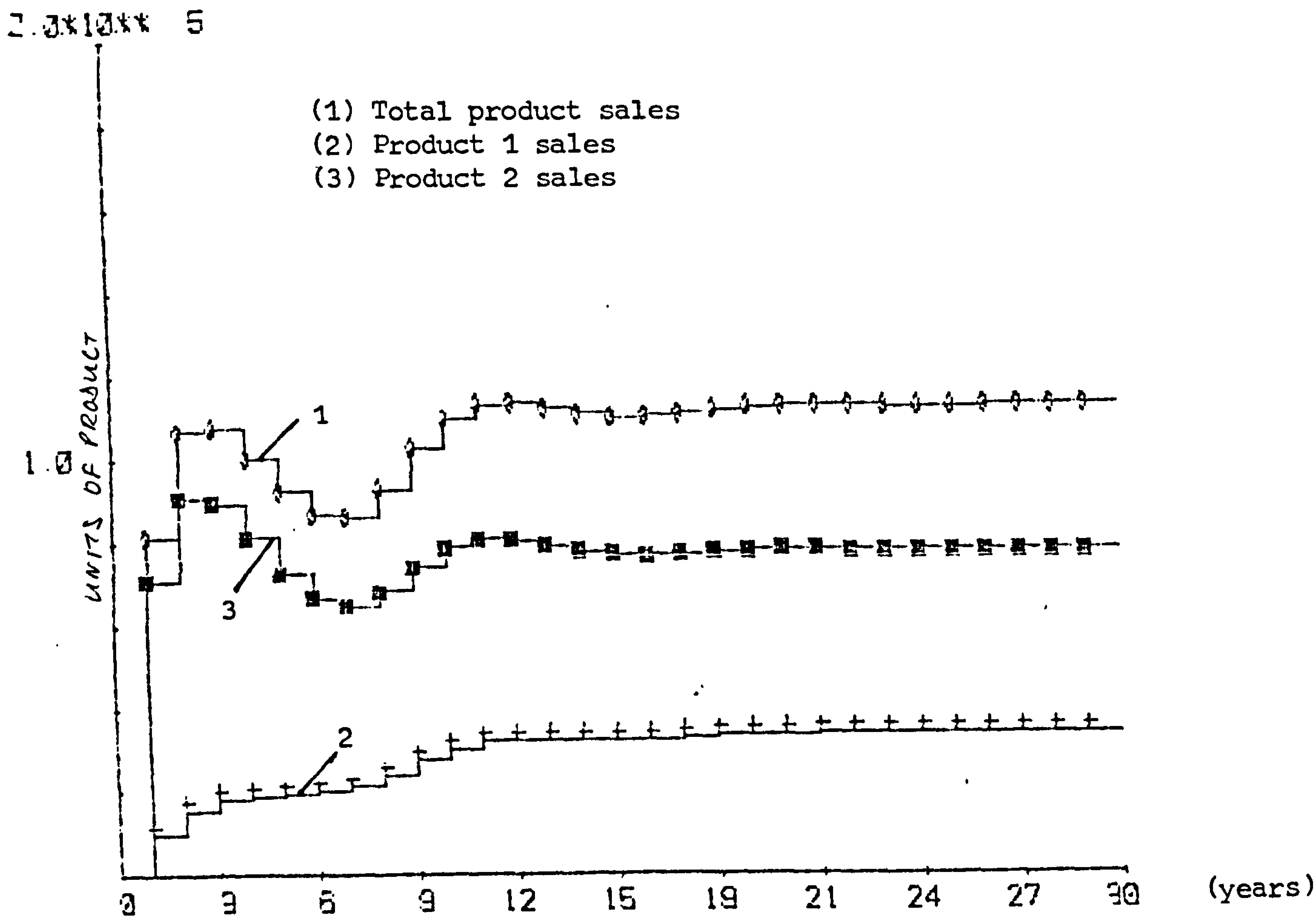

FIGURE 5.13(a) Sales-Time Curves: P1 = 825, Q1 = 0.33

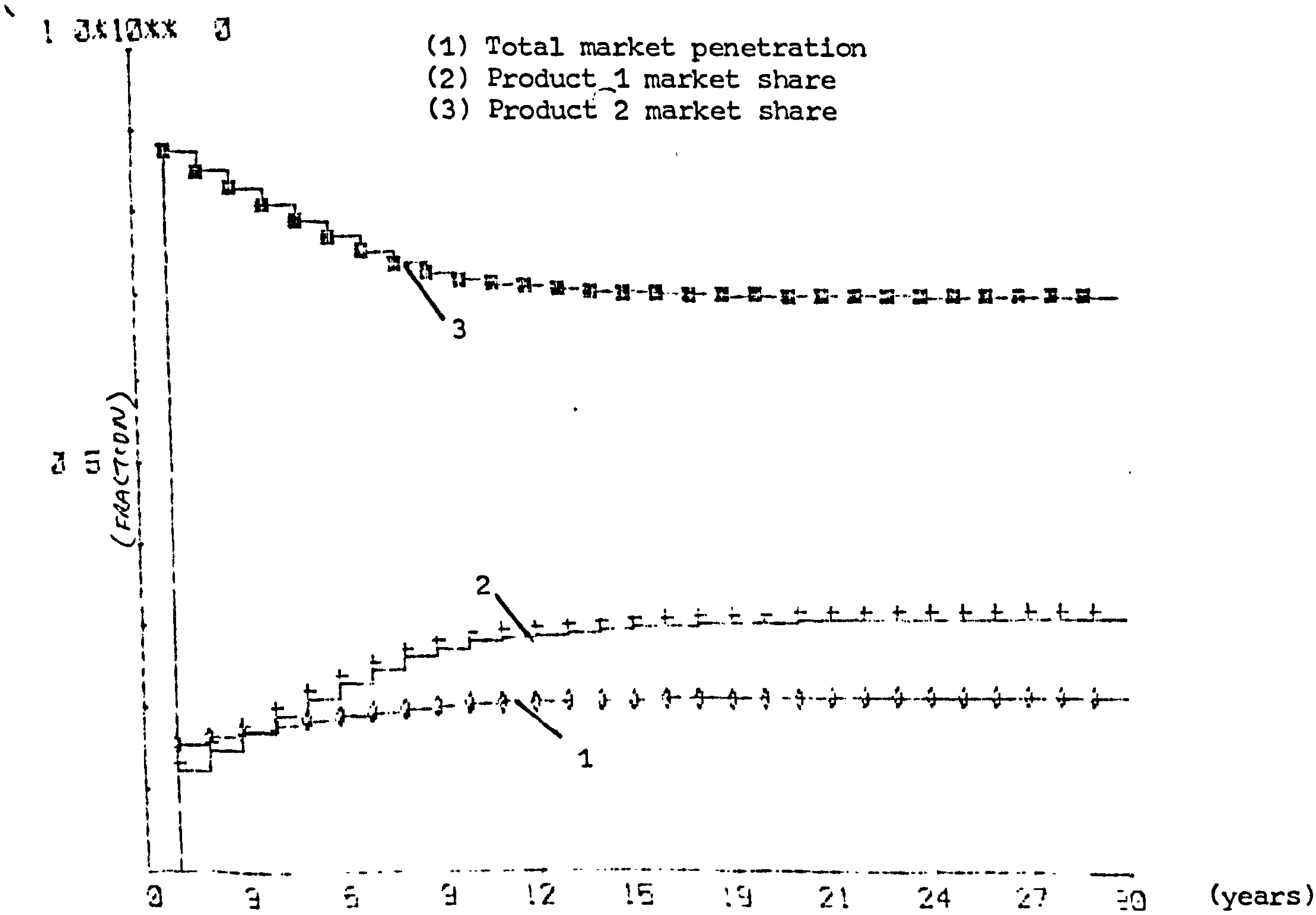

FIGURE 5.13(b) Market Penetration/Shares-Time Curves: P1 = 825, Q1 = 0.33

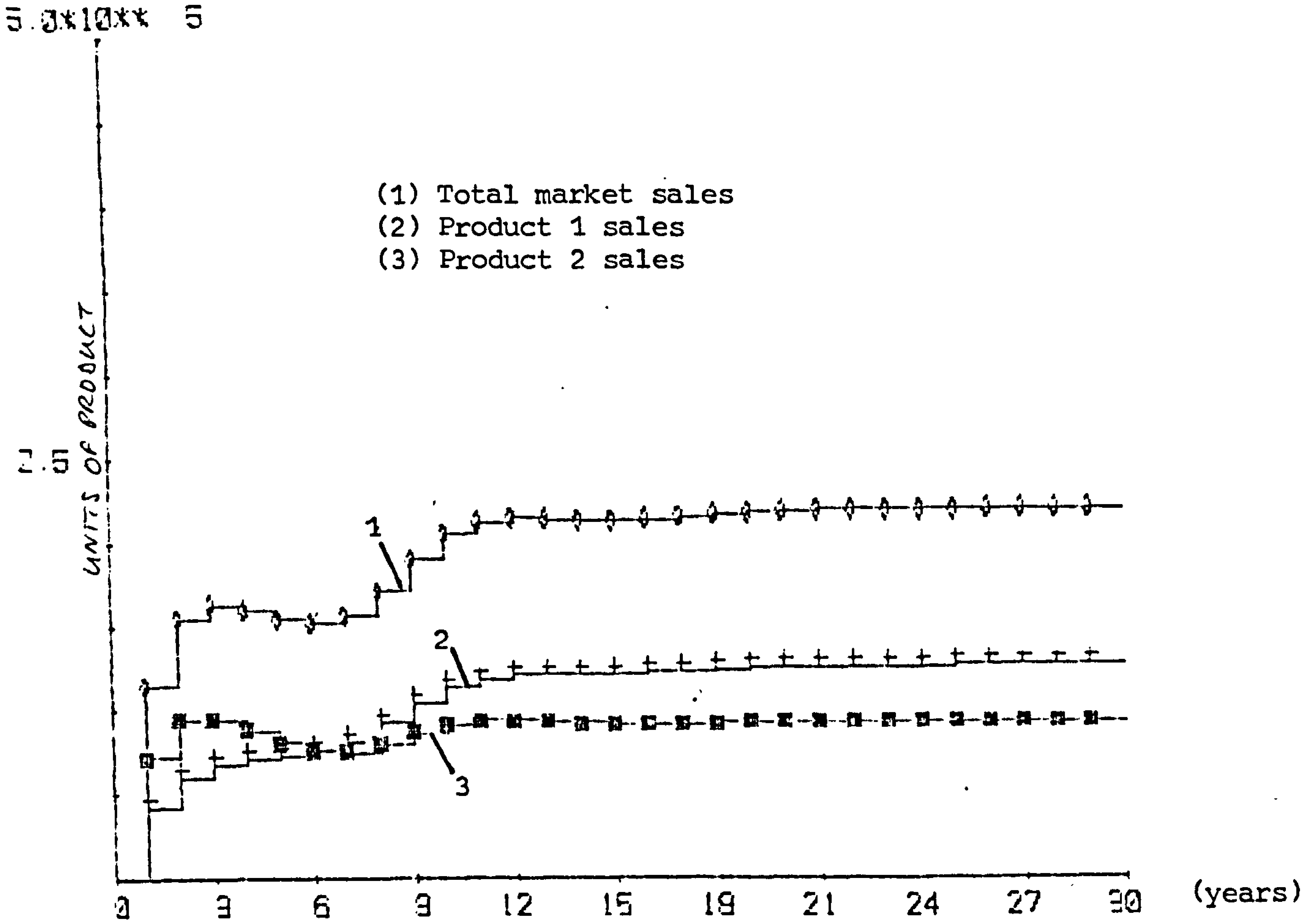

FIGURE 5.14(a) Sales-Time Curves: P1 = 495, Q1 = 0.55

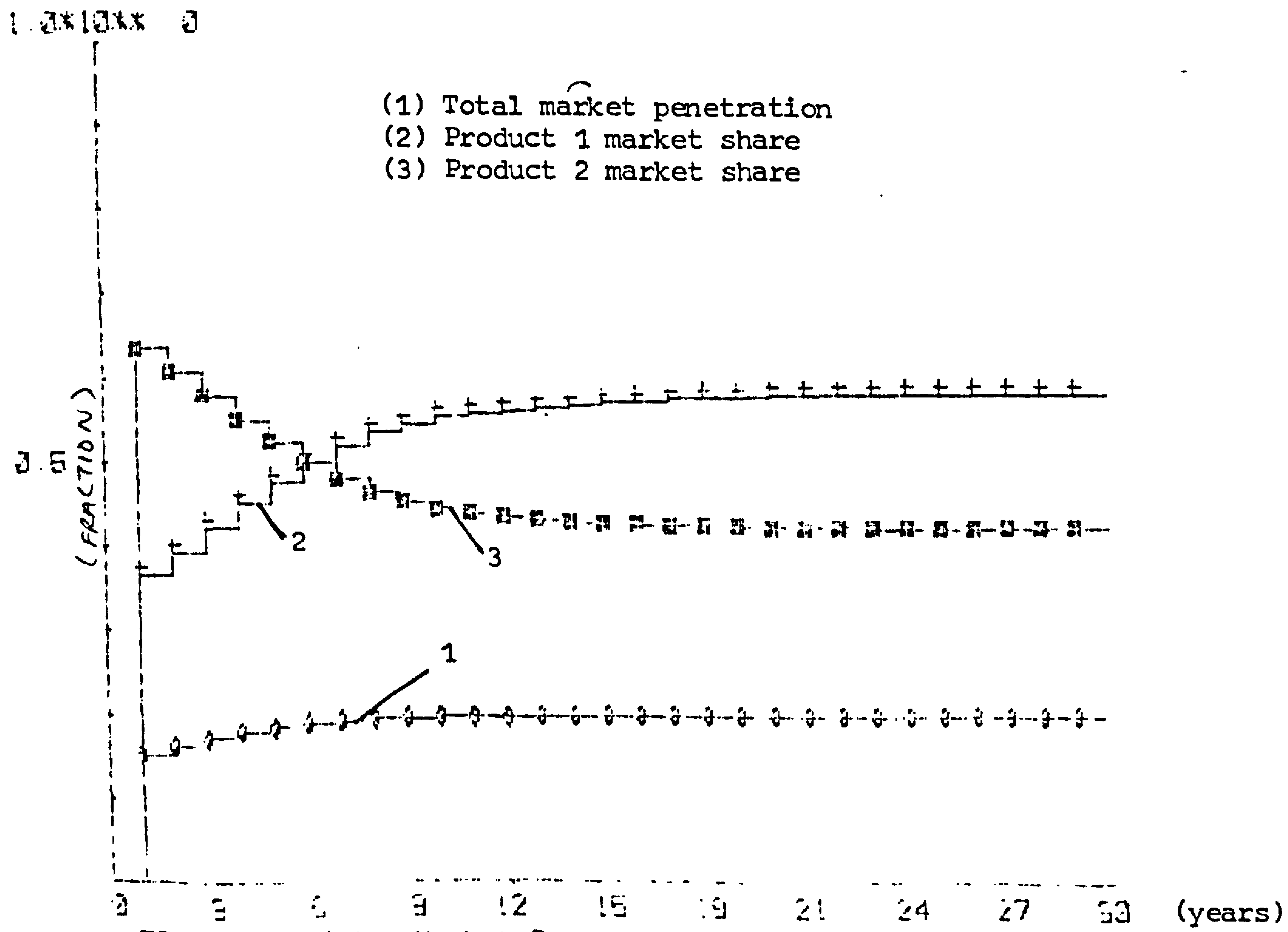

FIGURE 5.14(b) Market Penetration/Shares-Time Curves: P1 = 495, Q1 = 0.55

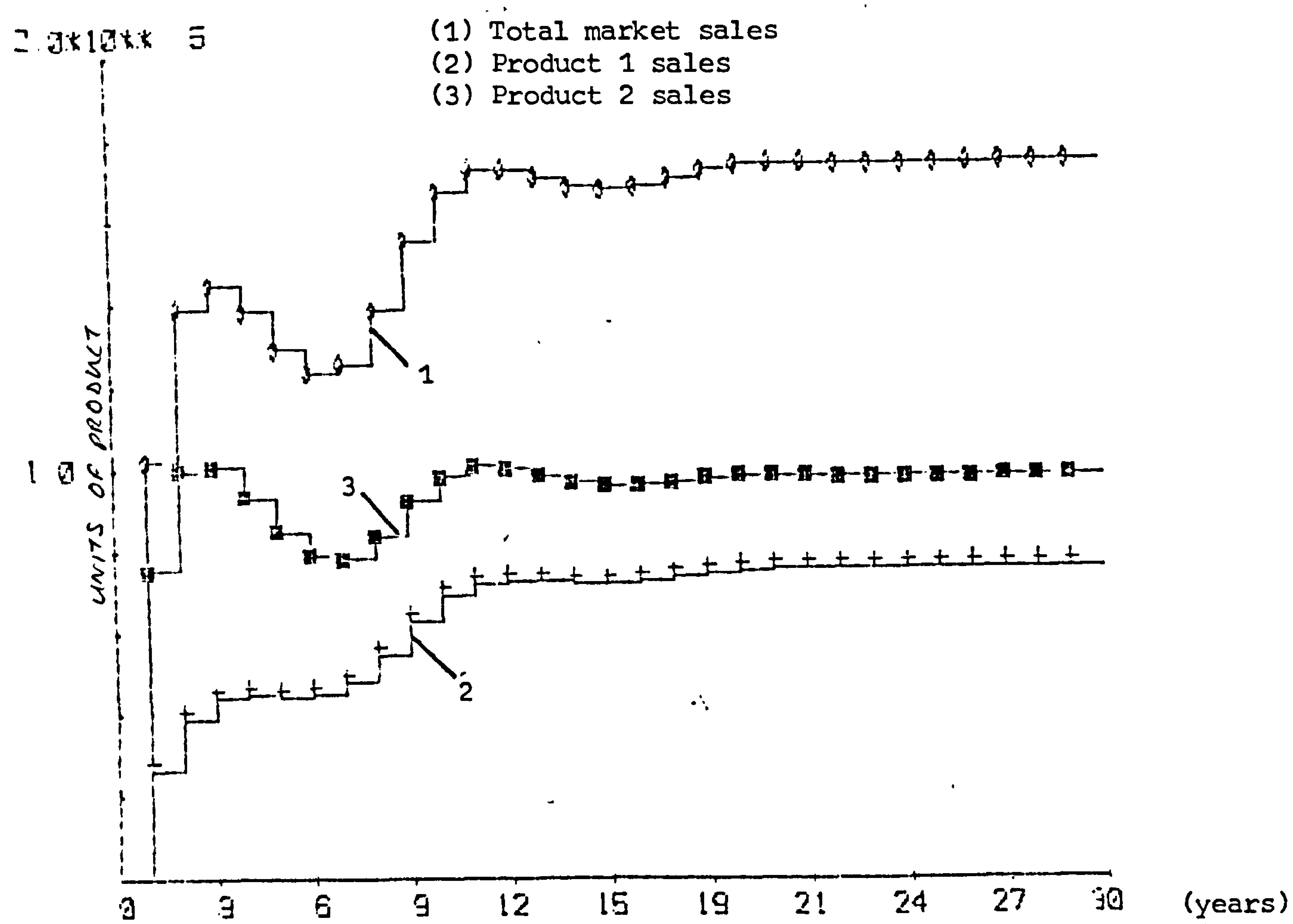

FIGURE 5.15(a) Sales-Time Curves: P1 = 495, Q1 = 0.33

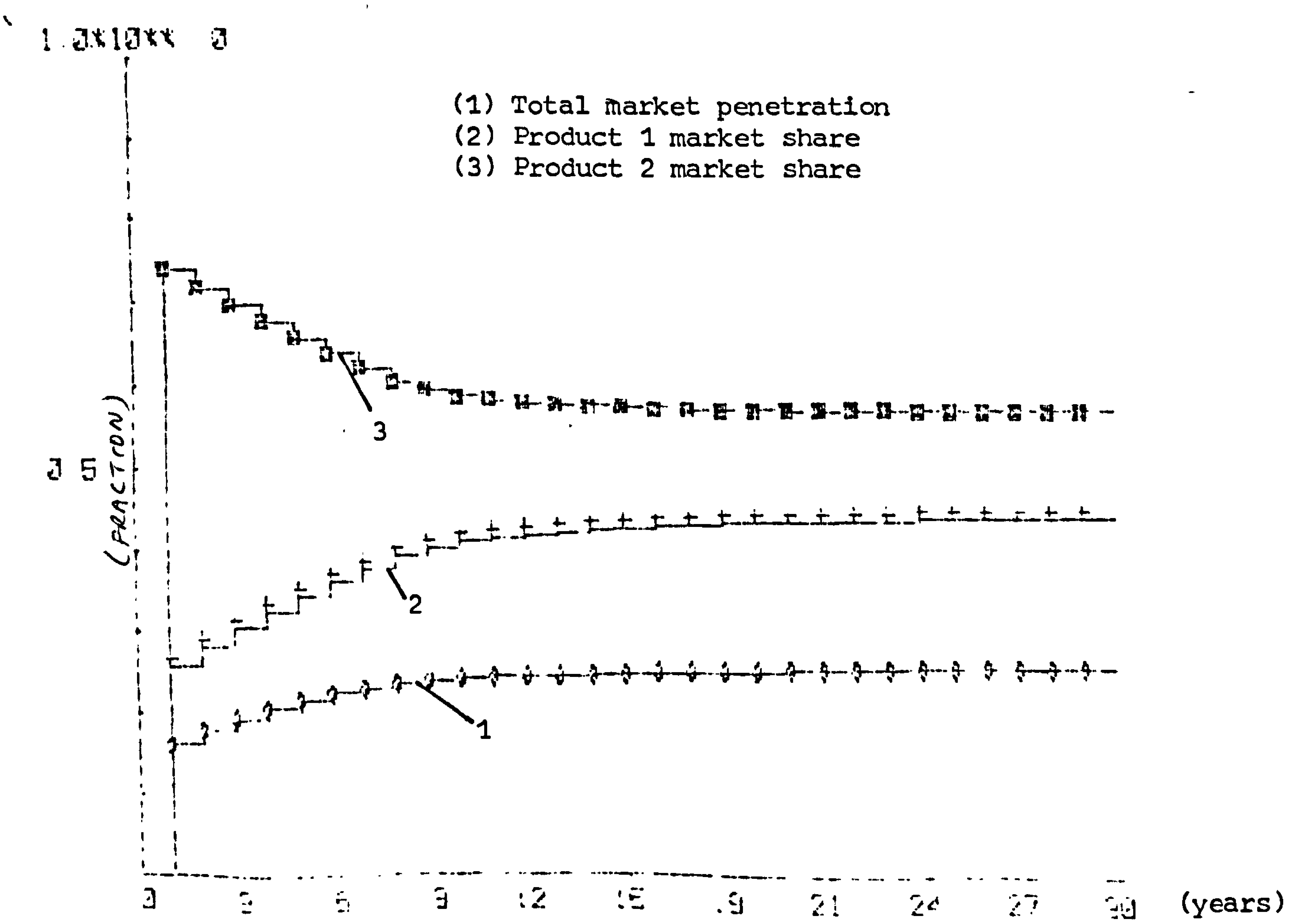

FIGURE 5.15(b) Market Penetration/Shares-Time Curves: P1 = 495, Q1 = 0.33

of the increase in market penetration attendant to this increase in quality rating accrues to product 1 entirely as does part of product 2's sales. This is shown in Figures 5.14 (for superior product 1 value-for-money) and Figures 5.15 (for equal value-for-money at same product 1 price). It is observed that the increase in product 2 sales in Figure 5.15(a) relative to that in 5.8(a) (curve 6) has been wiped out entirely in Figure 5.14(a).

3. An inferior product 1 value-for-money rating results in the steady-state market penetration, product 1 market share, overall sales and product sales all being decreased relative to the case, at the same product 1 price where both products are similarly rated. This time, product 2 sales are unaffected hence the reduction in overall sales is due entirely to the reduction in product 1 sales. This is demonstrated in Figures 5.13 (for inferior product 1 value-for-money) and Figures 5.12 (for equal value-for-money at same product 1 price).

The Revenue (Price x Sales) - Time curves of Figures 5.16 to 5.19 (curves 1,2 and 3 respectively show total market, product 1 and product 2 revenues over time) highlight some interesting aspects of market behaviour. For instance:

1. Whenever product value-for-money ratings are equal, the total market revenue curves are identical (compare curve 1 in Figures 5.16 and 5.19);

2. Depending on marketing system economics

(a) it may be more profitable to adopt a marketing strategy that calls for going 'up-market' with a product, i.e., increase both price and quality such that value-for-money is maintained or improved when facing a competing product;

(b) when facing a competing product with superior value-for-money rating, reducing the price of own product not only improves own product revenue but also that of competitor, but maintaining or even increasing own product price and increasing product quality such that product value-for-money is improved increases own revenue without increasing that of

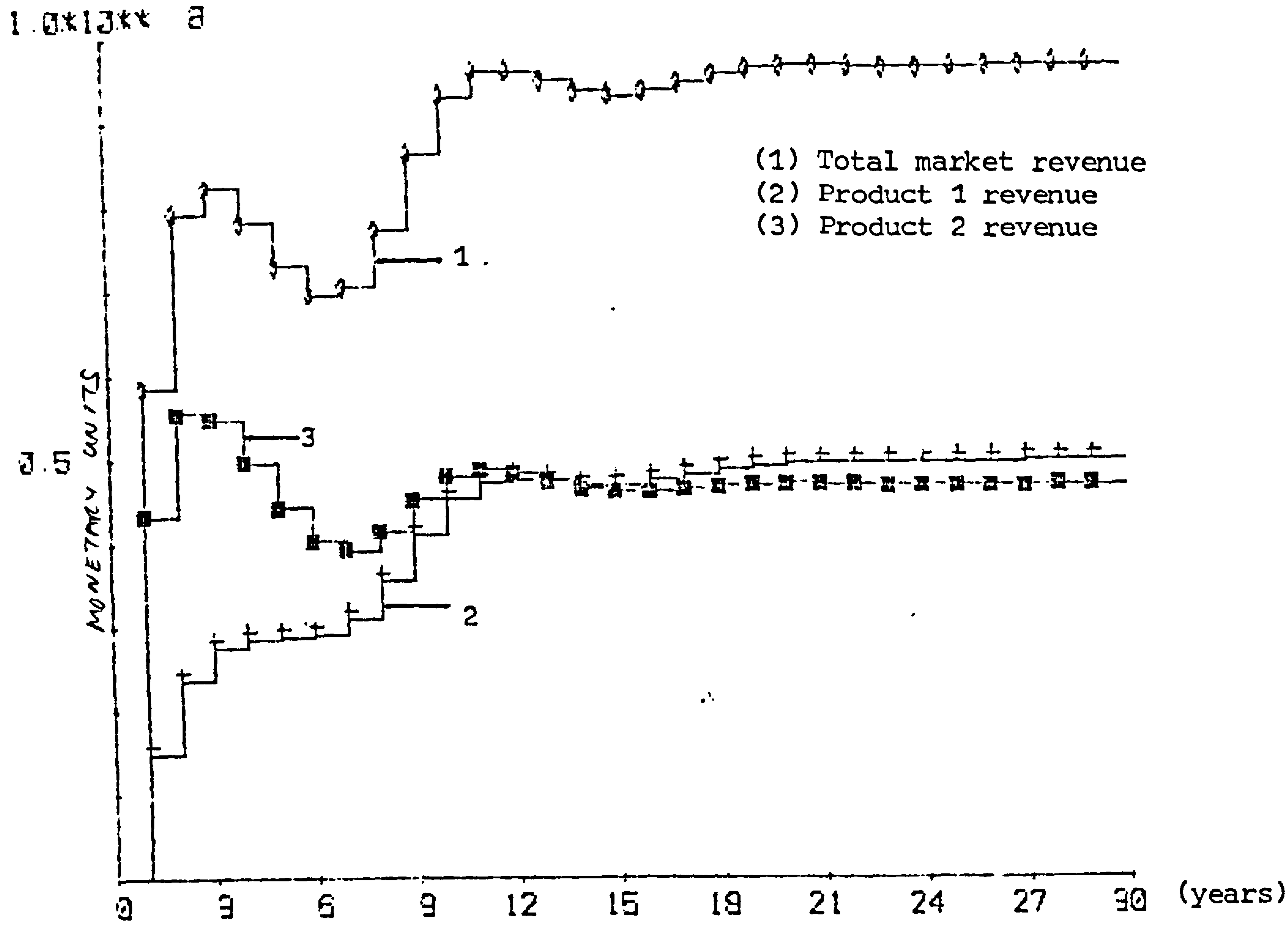

FIGURE 5.16 Revenue-Time Curves: P1 = 825, Q1 = 0.55

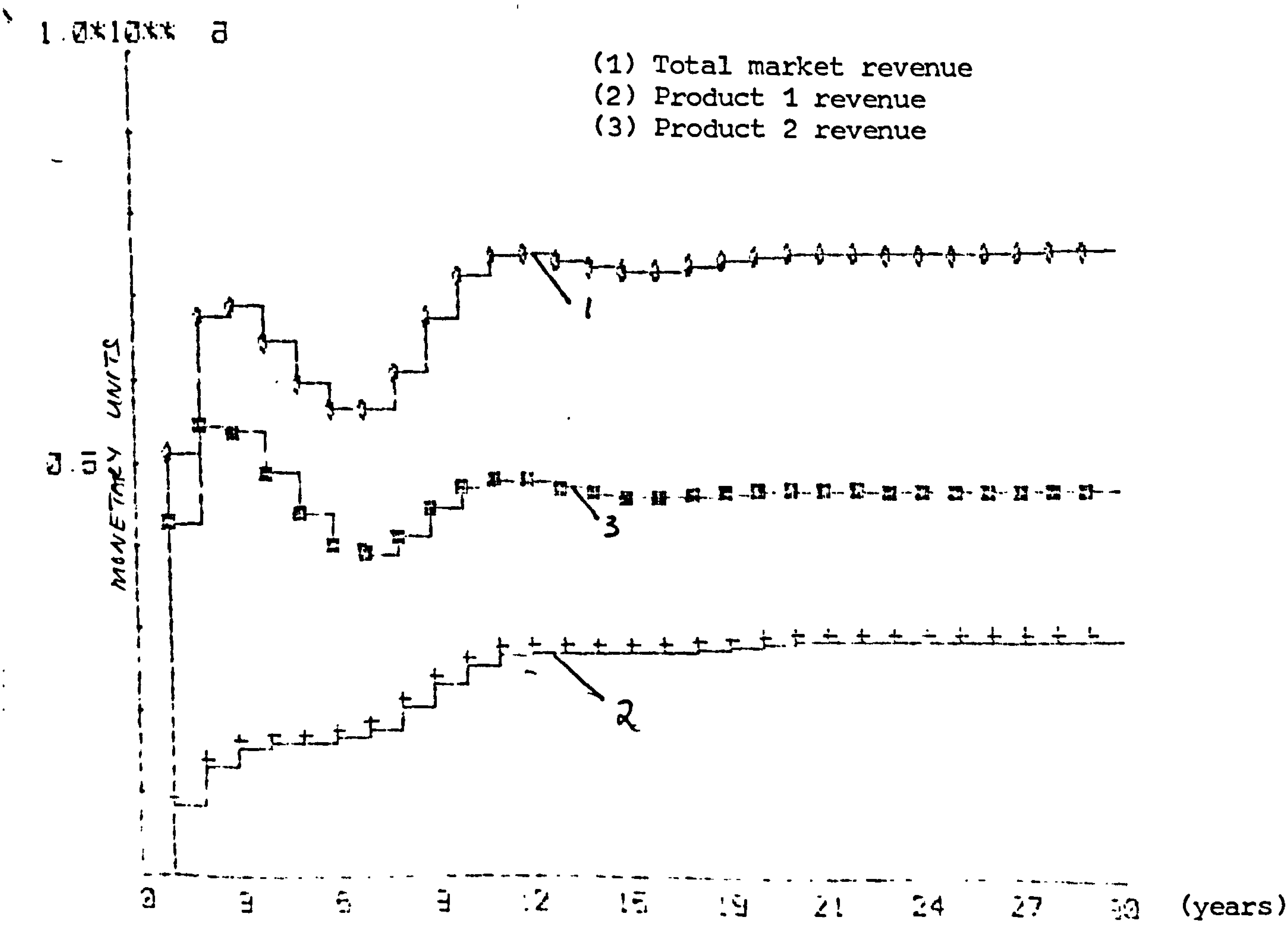

FIGURE 5.17 Revenue-Time Curves: P1 = 825, Q1 = 0.33

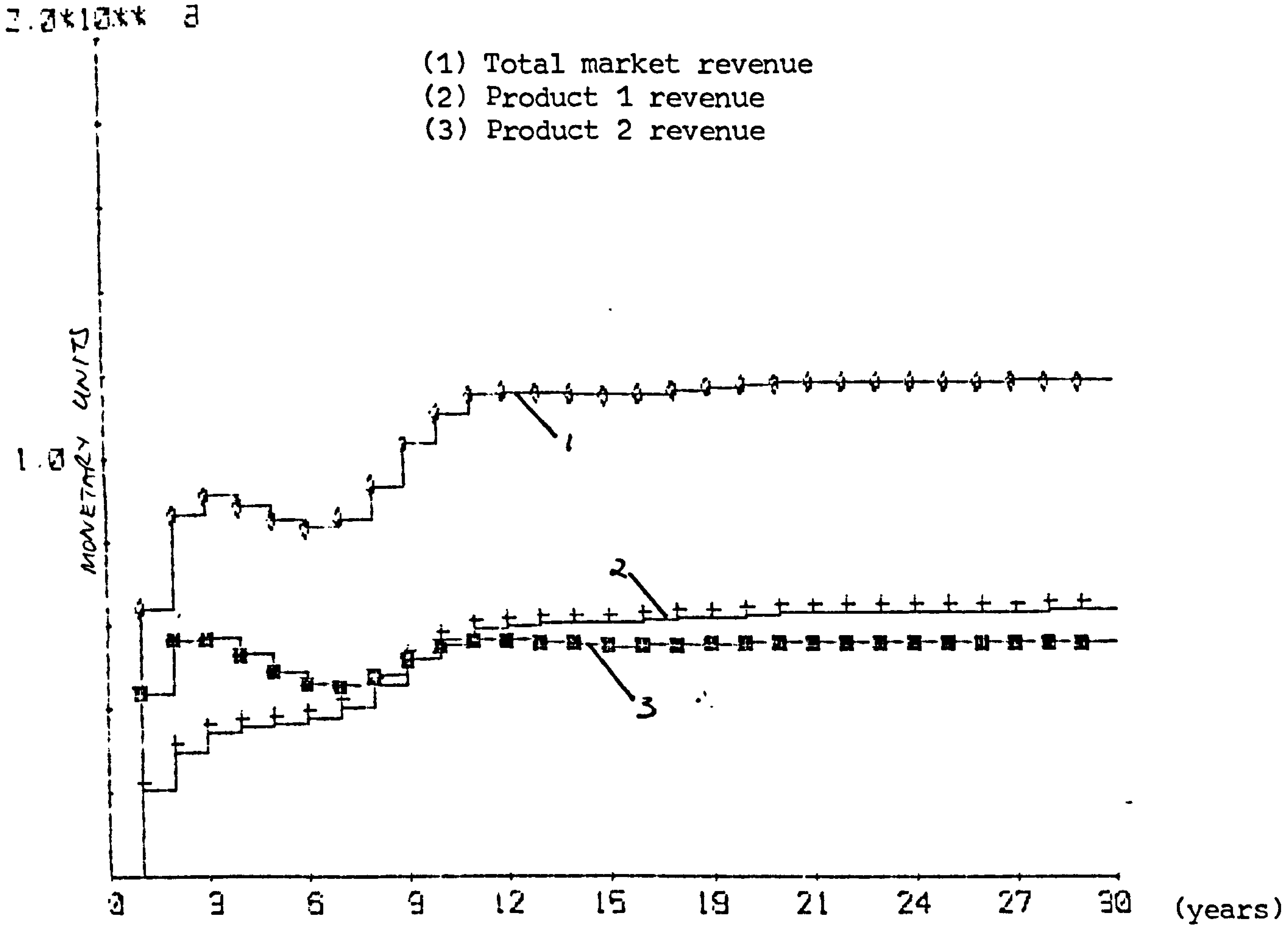

FIGURE 5.18 Revenue-Time Curves: P1 = 495, Q1 = 0.55

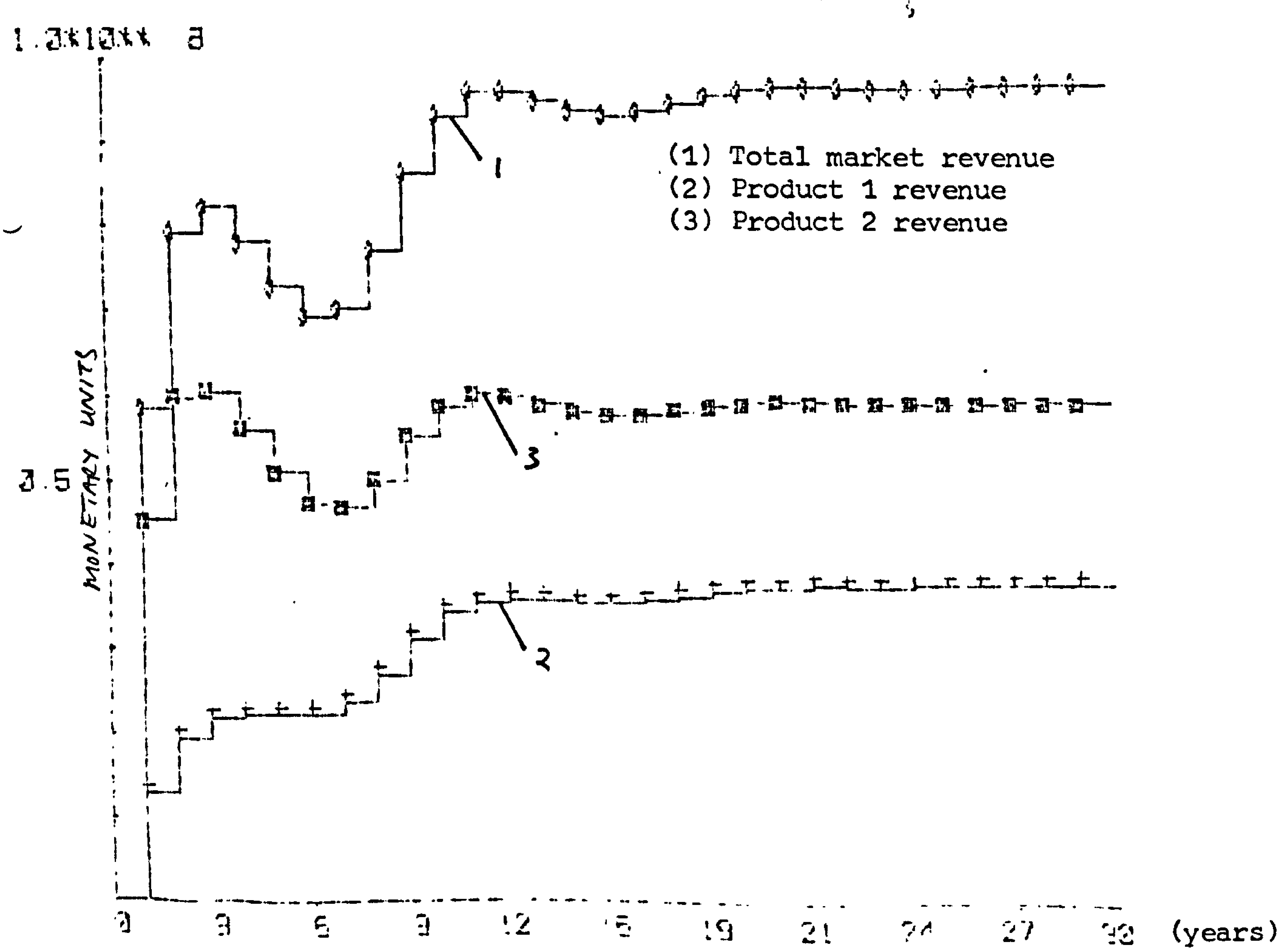

FIGURE 5.19 Revenue-Time Curves: P1 = 495, Q1 = 0.33

competitor, however unless marketing system economics are favourable (i.e., costs of maintaining new price and quality levels as well as costs of changing these levels must be considered) this policy will not necessarily be more profitable.

5.2 SIMULATION OF A 2-PRODUCT, VARIABLE DECISION MARKETING SYSTEM*

5.2.1 Preliminaries

The marketing parameters described in 5.1.2 above are used in obtaining the simulation results discussed here except for the word-of-mouth factor, λ, which is now given by $\lambda = 0.15 \times 10^{-6}$. Expenditure functions are defined relating marketing expenditure on advertizing, distribution and product quality to the resulting advertizing effort factor, distribution effort factor and product quality rating respectively, for each product. These functions are of the following generic form

$$x = \frac{X + Y_1}{X + Y_2}$$

where x is the resulting factor or rating, X is the expenditure per period and Y_1 and Y_2 are constants, $0 \leq Y_1 < Y_2$. In the simulations described, it is assumed that both competitors have identical expenditure functions for advertizing, for distribution and for quality rating. Both advertizing and distribution effort factors are zero at zero expenditure but quality rating is non-zero at zero expenditure because of the non-zero intrinsic product quality (i.e., the basic functional utility of the product). Thus

(i) for advertizing

$$Y_1 = 0, \quad Y_2 = 9 \times 10^5$$

where Y_2 was chosen such that for an expenditure of 10^5 per period, advertizing effort factor, $\alpha = 0.1$.

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* Simulation results obtained using programme MTEST2.FTN.

(ii) for distribution

$$Y_1 = 0, \quad Y_2 = 1.9 \times 10^5$$

where Y_2 was chosen such that for an expenditure of 10^4 per period, distribution effort factor, $k_o = 0.05$.

(iii) for quality rating

$$Y_1 = 0.75 \times 10^5, \quad Y_2 = 2.5 \times 10^5$$

where the constants have been chosen such that at zero expenditure, $q = 0.30$ (the intrinsic product rating) and for an expenditure of 10^6 per period this rises by 0.56 to $q = 0.86$.

Now α, k_o, q and product price, p, for each product are decision outputs from the respective product decision making system (DMS). Each product DMS selects the values of these variables for each period in an N-period decision horizon such that these values extremize a given objective function over this horizon. In the simulations, $N = 4$, and each DMS is required to maximize expected profit, i.e., revenue less cost over the decision horizon after discounting future profit by a given discounting factor. Period revenue is the product of price and sales rate; period cost is the sum of marketing expenditures, production (purchase) cost of product where unit production or purchase cost for each product is assumed constant and interest cost is assumed to be 15% of preceding period loss (if any). Hence

Period profit = Period revenue - period cost

and

Expected profit over 4-period decision horizon

$$= \sum_{i=1}^{4} (\text{Period Profit})_i \times \text{DFACT}^{i-1}$$

where the discounting factor, DFACT, is assumed to be 0.15. It is recognized that the requirement that the DMS considers an N-period decision horizon necessitates that the DMS possesses the facility to predict its competitor's decisions in each of the periods of the decision horizon. This facility is included in the simulation programme.

N.B. (1) Only the current period's decisions are applied; at the beginning of every period, a new decision-making exercise is carried out.

(2) It is assumed that there are no limitations to the supply of either product or of financial resources to acquire the products and effect their sale.

5.2.2 Simulation Results

The results of three simulation runs are described. The first, hereafter referred to as the monopoly simulation, is a single-product variable-decision simulation to provide a monopoly product comparison with the two 2-product simulations. The monopoly product (product 2) has a unit production cost of 450. The second is the first of the two 2-product simulations, in which both products have identical expenditure functions for advertizing, for distribution and for quality as well as facing identical unit production (purchase) cost of 450. This simulation is referred to as the equal parameter simulation. In the third simulation, the unequal unit cost simulation, product 1 unit production (purchase) cost is 400 while that of product 2 remains at 450.

The simulation results are displayed in Figures 5.20 to 5.27 which are respectively Price-Time, Advertizing Effort Factor-Time, Quality Rating-Time, Distribution Effort Factor-Time, Sales-Time, Revenue-Time, Market Penetration/Shares-Time, and Profit-Time curves. There are three parts to each figure, labelled (a) to (c), and referring respectively to the corresponding results for the monopoly, equal parameter and unequal unit cost simulations. The (a) part of each figure therefore contains only one curve, (the single curve of Figure 5.26(a) refers to market penetration over time) while in the (b) and (c) parts, curve 1 refers to product 1, curve 2 to product 2 and where there is a third curve, curve 3 refers to the combined value for products 1 and 2 (curves 1 and 2 in Figure 5.26(b) and (c) refer to the market shares of products 1 and 2 respectively and curve 3 refers to the market penetration achieved by both products together).

Figure 5.20 shows that market prices are lower for the 2-product simulations than for the monopoly simulation. In Figure 5.20(a) we observe that there was an initial sharp price rise followed by oscillation about and convergence to 850. This contrasts with the 2-product simulations where product prices lay between a minimum of 400 and a maximum of 820 with an average market price of 640 (product 2 prices in periods 15 to 20 inclusive in Figure 5.20(c) may be ignored as it will be shown that product 2 was not marketed in that interval). These results are in agreement with the popular observation that competition tends to drive down market prices. In the 2-product simulations we observe a repeated pattern involving an initial sharp price rise followed by a gradual decline.

Figure 5.21 shows that competition has also resulted in higher levels of the advertizing effort factor (AEF) significantly higher than 0.8 most of the time, than occur in the monopoly simulation where the AEF oscillates about and eventually stabilizes at the 0.8 level. Figure 5.21(b) shows that both products were not advertized at all in the 19th period and Figure 5.21(c) shows that product 2 only was not advertized in the interval containing periods 15 to 20 inclusive. Presumably in these intervals the affected product(s) were withdrawn from the market altogether.

This hypothesis is strengthened when reference is made to the Quality Rating-Time curves of Figure 5.22. We notice that in the unequal unit cost simulation (Figure 5.22(c)) the product qualities are very closely matched except in the six-period interval when product 2 quality rating drops to its intrinsic value of 0.30 indicating that there was no expenditure on product quality in that interval. Similarly in period 19 the quality ratings of both products drop to their intrinsic values. On comparing the quality ratings of the 2-product simulations with those of the monopoly simulation we notice that these quality levels are very similar. It thus appears that the optimal policy is the one in which quality rating of one product is matched with that of the competition and that this is not dissimilar to the quality rating that

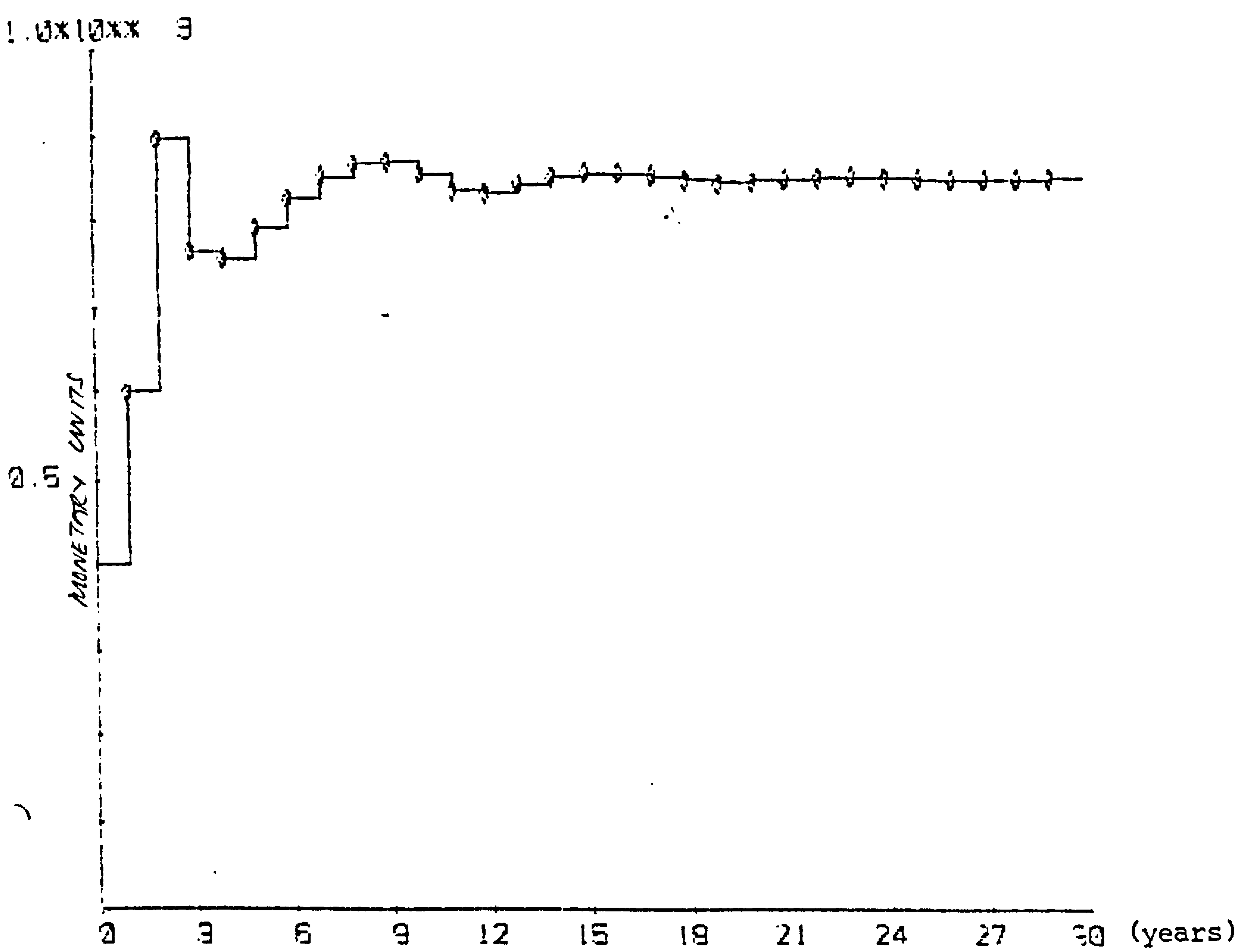

FIGURE 5.20(a) Price-Time Curve: Monopoloy Simulation

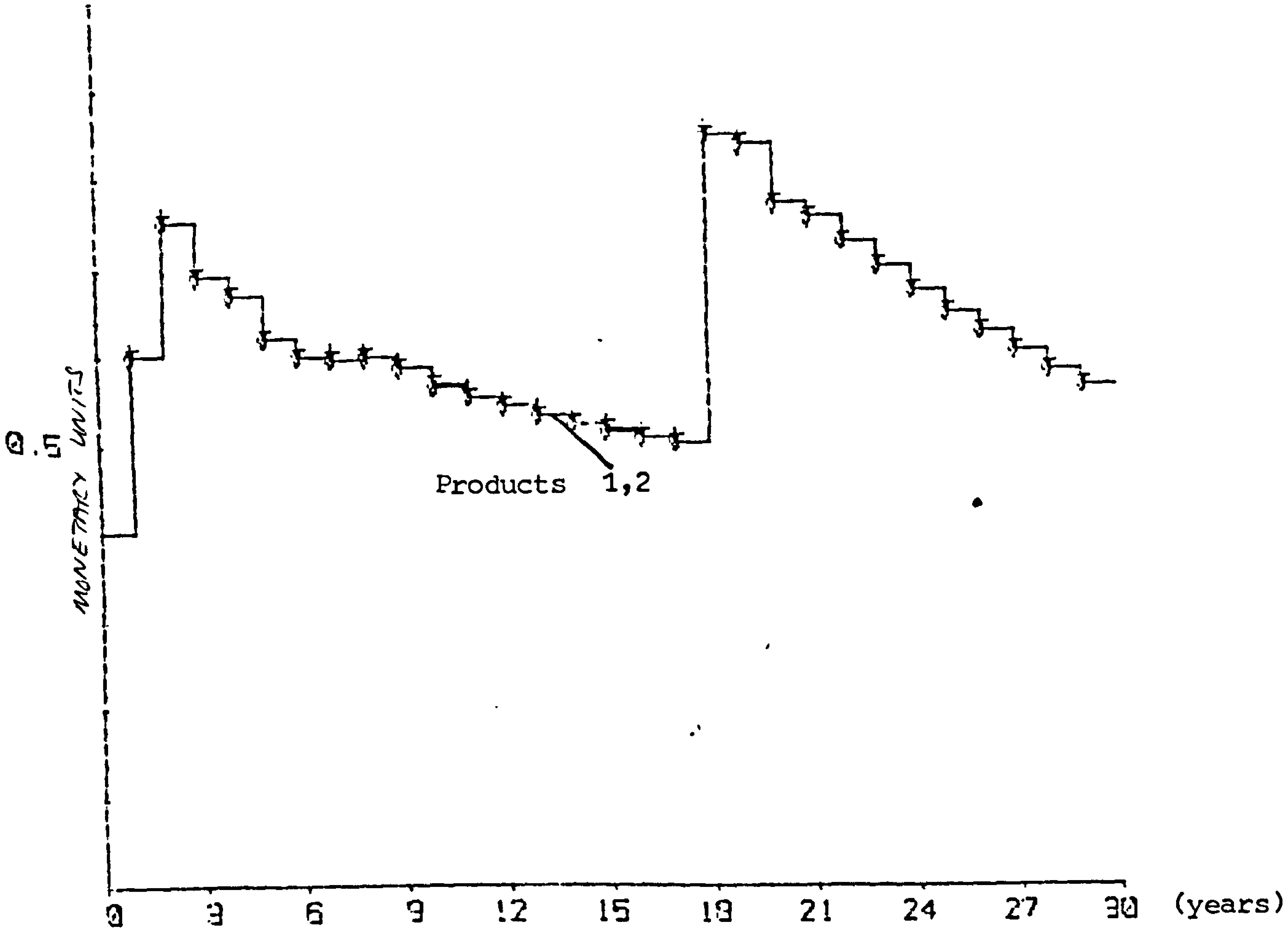

FIGURE 5.20(b) Price-Time Curves: Equal Parameter Simulation

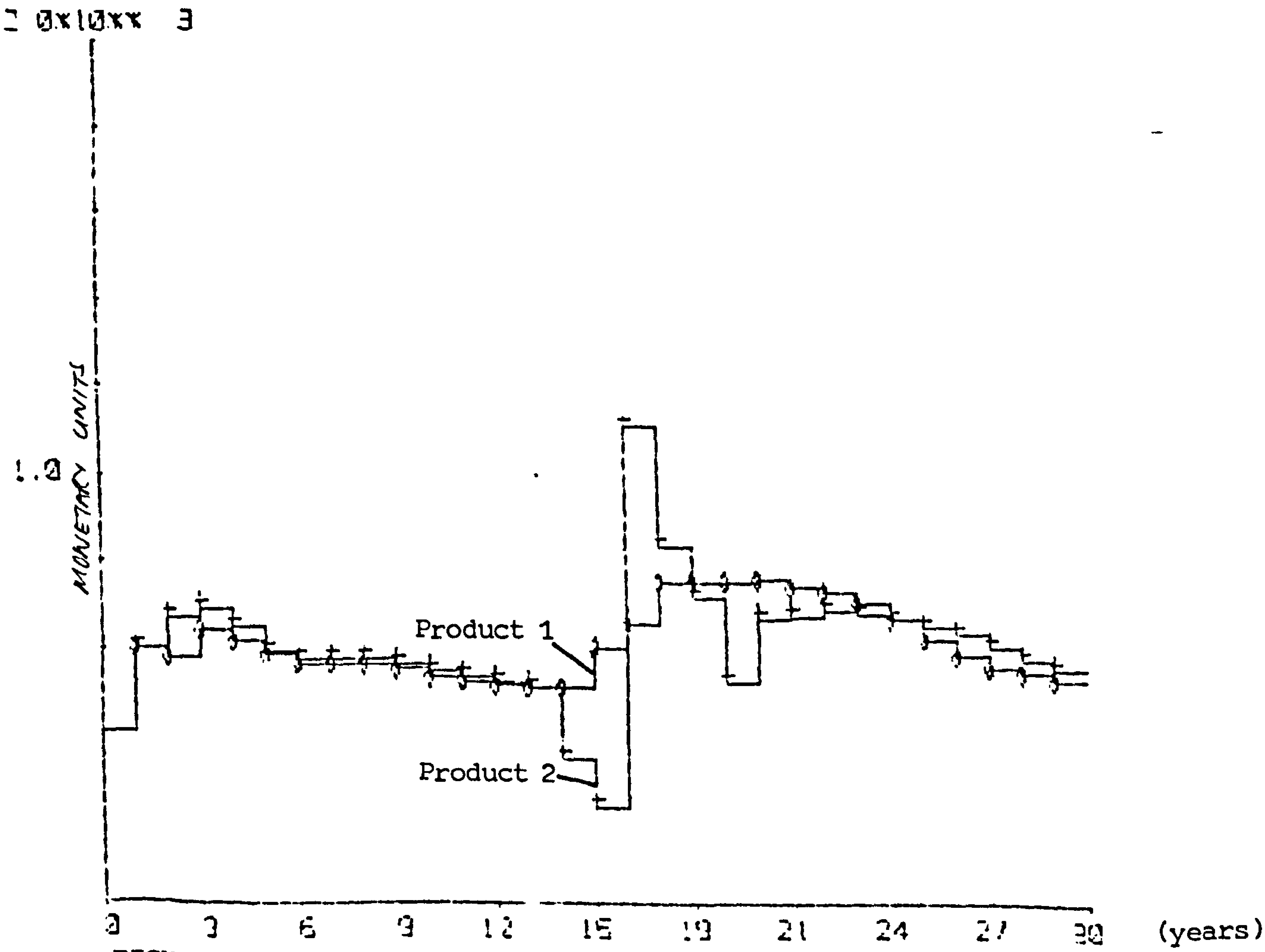

FIGURE 5.20(c) Price-Time Curves: Unequal Unit Cost Simulation

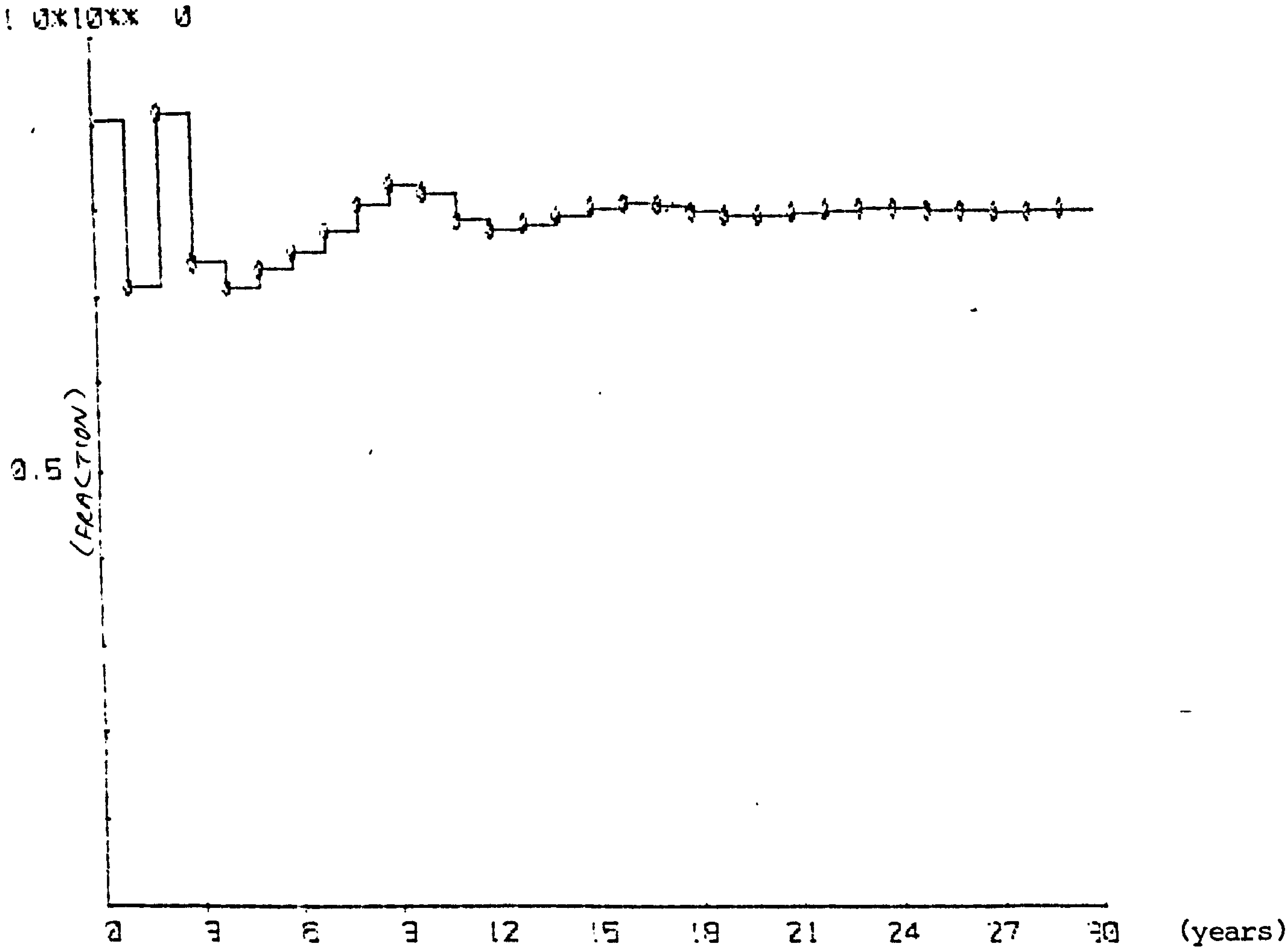

FIGURE 5.21(a) Advertizing Effort Factor-Time Curve: Monopoly Simulation

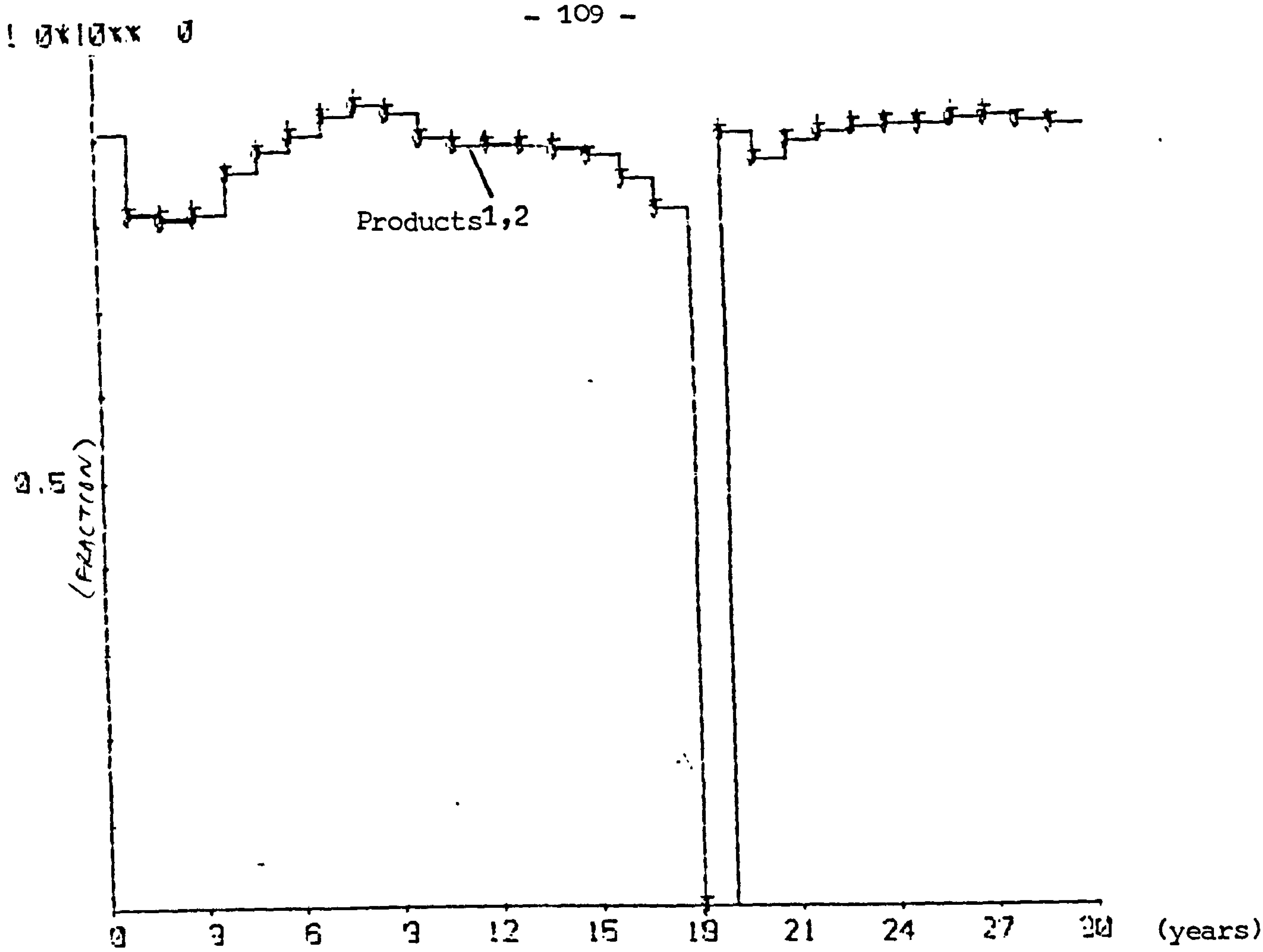

FIGURE 5.21(b) Advertizing Effort Factor-Time Curve: Equal Parameter Simulation

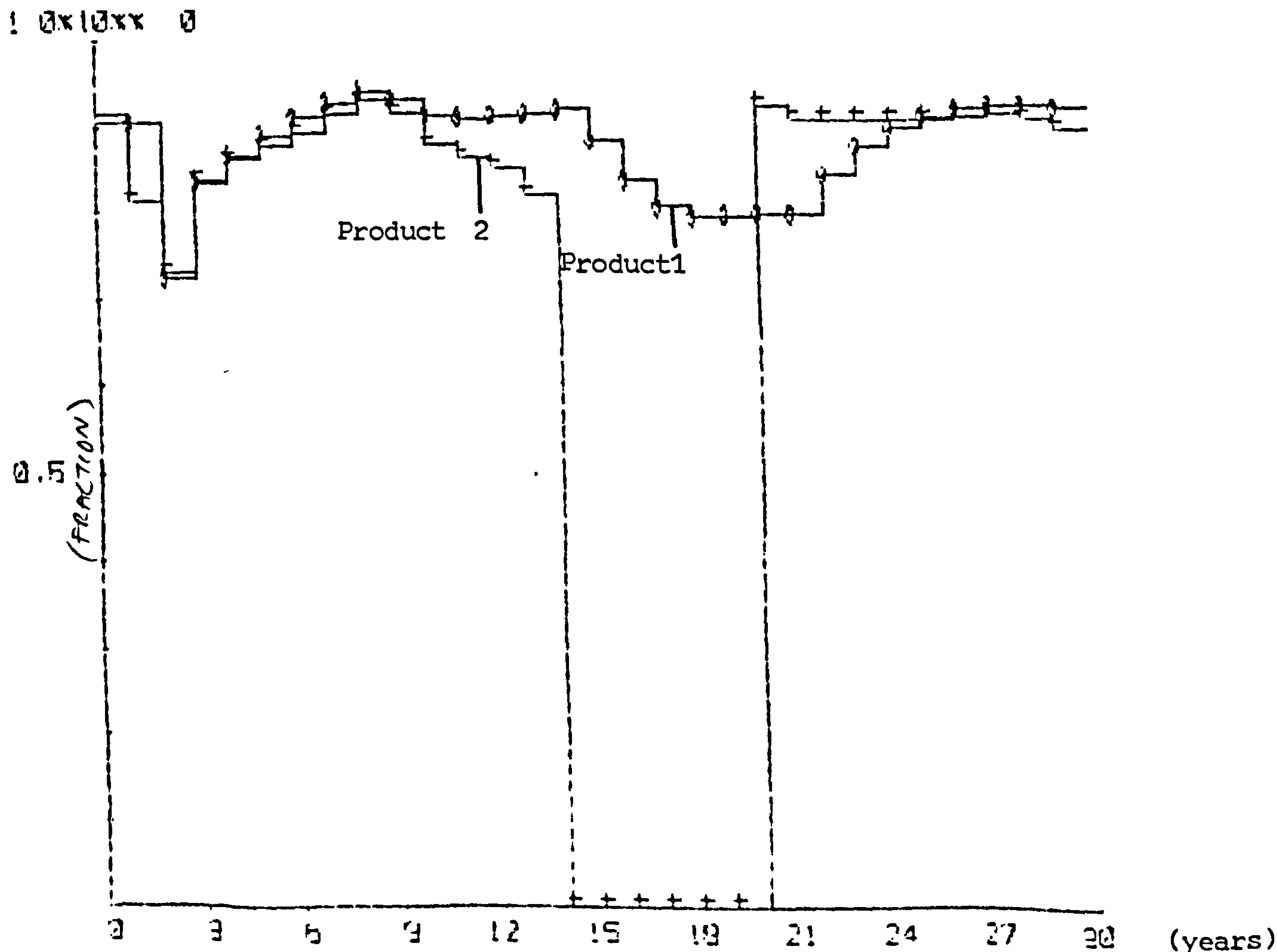

FIGURE 5.21(c) Advertizing Effort Factor-Time Curves: Unequal Unit Cost Simulation

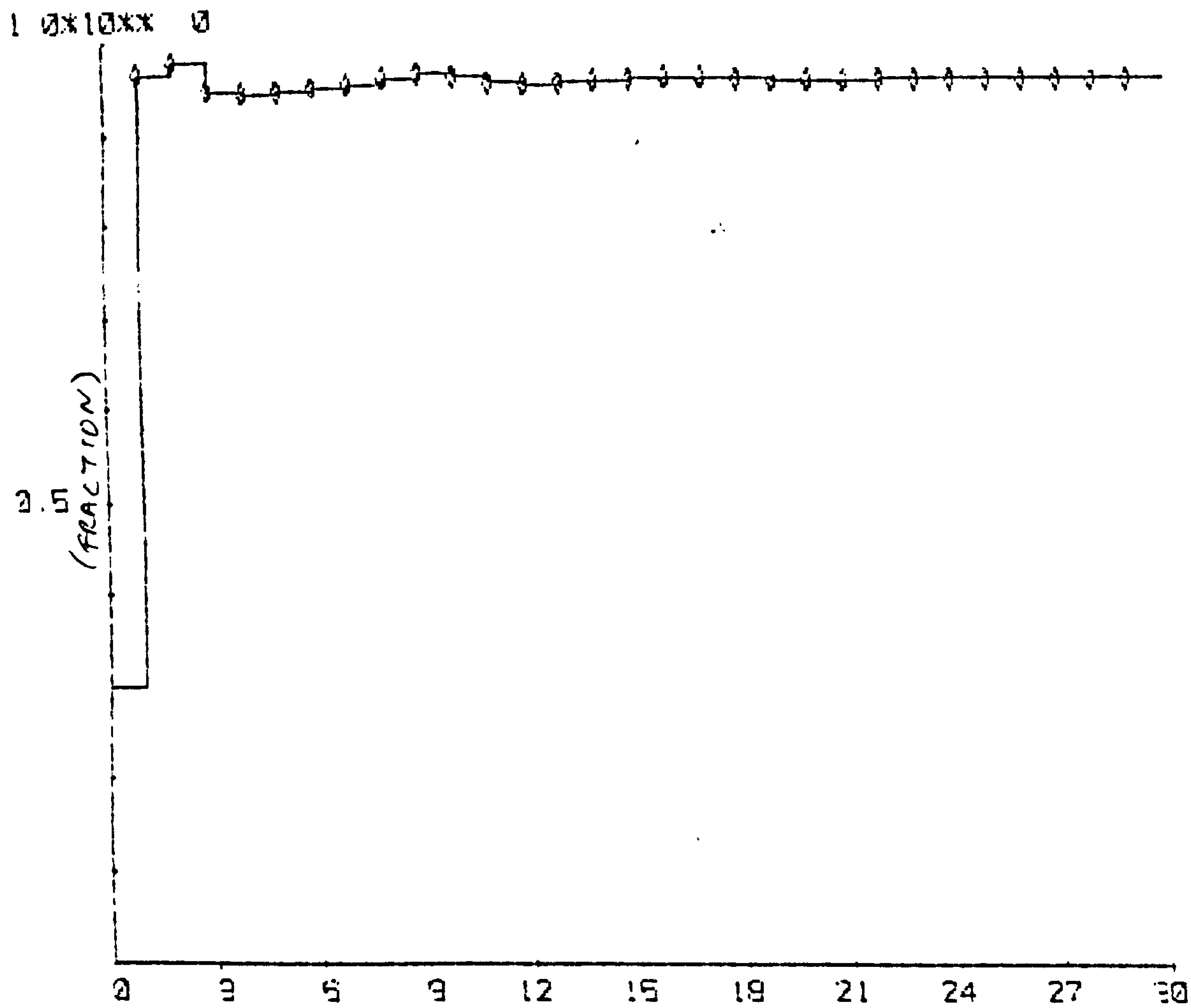

FIGURE 5.22(a) Quality Rating-Time Curve: Monopoloy Simulation

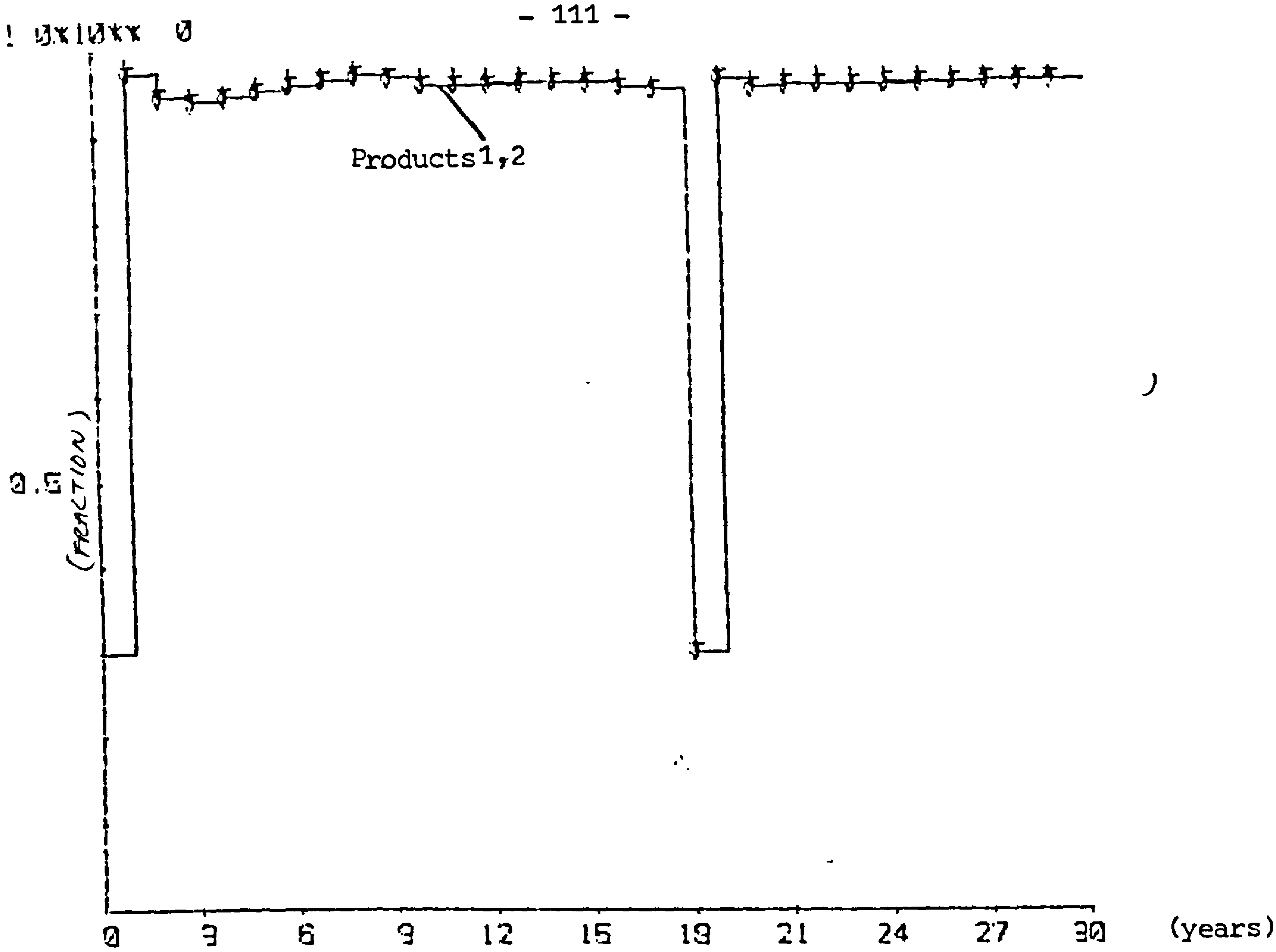

FIGURE 5.22(b) Quality Rating-Time Curves: Equal Parameter Simulation

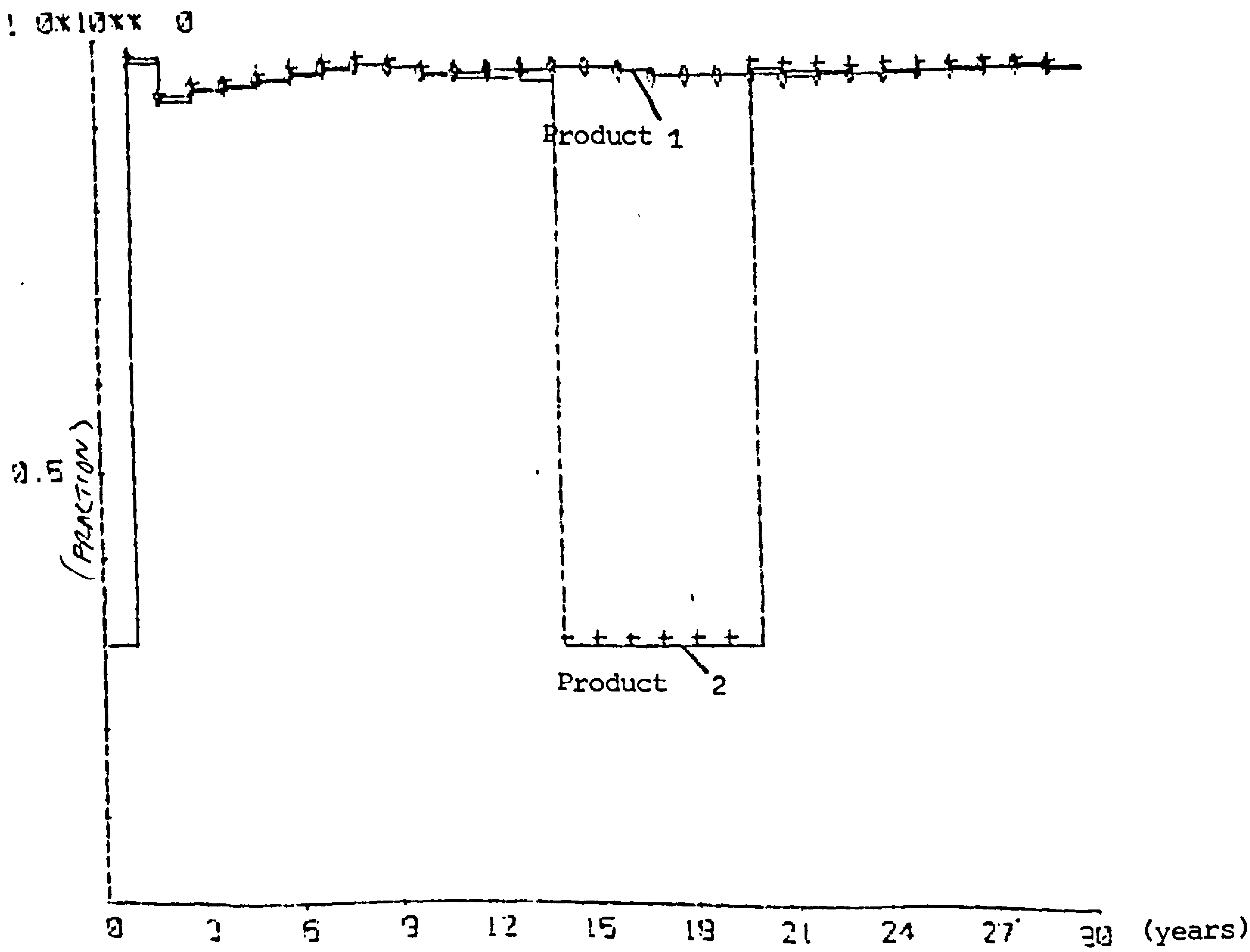

FIGURE 5.22(c) Quality Rating-Time Curves: Unequal Unit Cost Simulation

would obtain in a monopoly situation. If reference is made, in addition, to Figure 5.20 (Price-Time curves) it will be observed that competition has resulted in a marked increase in the value-for-money received by the consumers. The optimal policy appears to involve combating competition with successive price reductions while maintaining product quality at monopoly levels even though such a policy might lead to a temporary (or possibly permanent) withdrawal of either or both products from the market-place. This policy is to be compared with that suggested at the end of Section 5.1.3 above where competition was to be met by simultaneous increases in both prices and quality ratings such that the resulting value-for-money is superior to that of the opposition. It is now apparent that the latter policy can only confer a temporary benefit, it will be successful only insofar as the competition is unable to change prices and/or quality as fast - in other words the dynamics of the decision-making system itself is of importance (we note that in the simulations described here, the levels of the decision variables are changed without cost and without time lags, costs are attached only to the levels themselves).

The Distribution Effort Factor-Time curves of Figure 5.23 confirm the observations and conclusions made previously. The monopoly simulation distribution effort factor stabilizes at the 0.9 level while for the 2-product simulations, the distribution effort factors were greater than 0.9 with the obvious exceptions of period 19 in Figure 5.23(b) and periods 15 to 20 inclusive in Figure 5.23(c) where the distribution effort factors were at zero level for both products and product 2 respectively.

The constant decision simulations of Section 5.2 above have indicated that market system dynamics are such as to give rise to 8-period oscillations in marketing outputs. In the monopoly simulation of the present section, we note that after an initial 3-period 'priming' interval, the 8-period oscillations were clearly reflected in the marketing decision. In the 2-product simulations, the influence of market dynamics on the decisions is not as

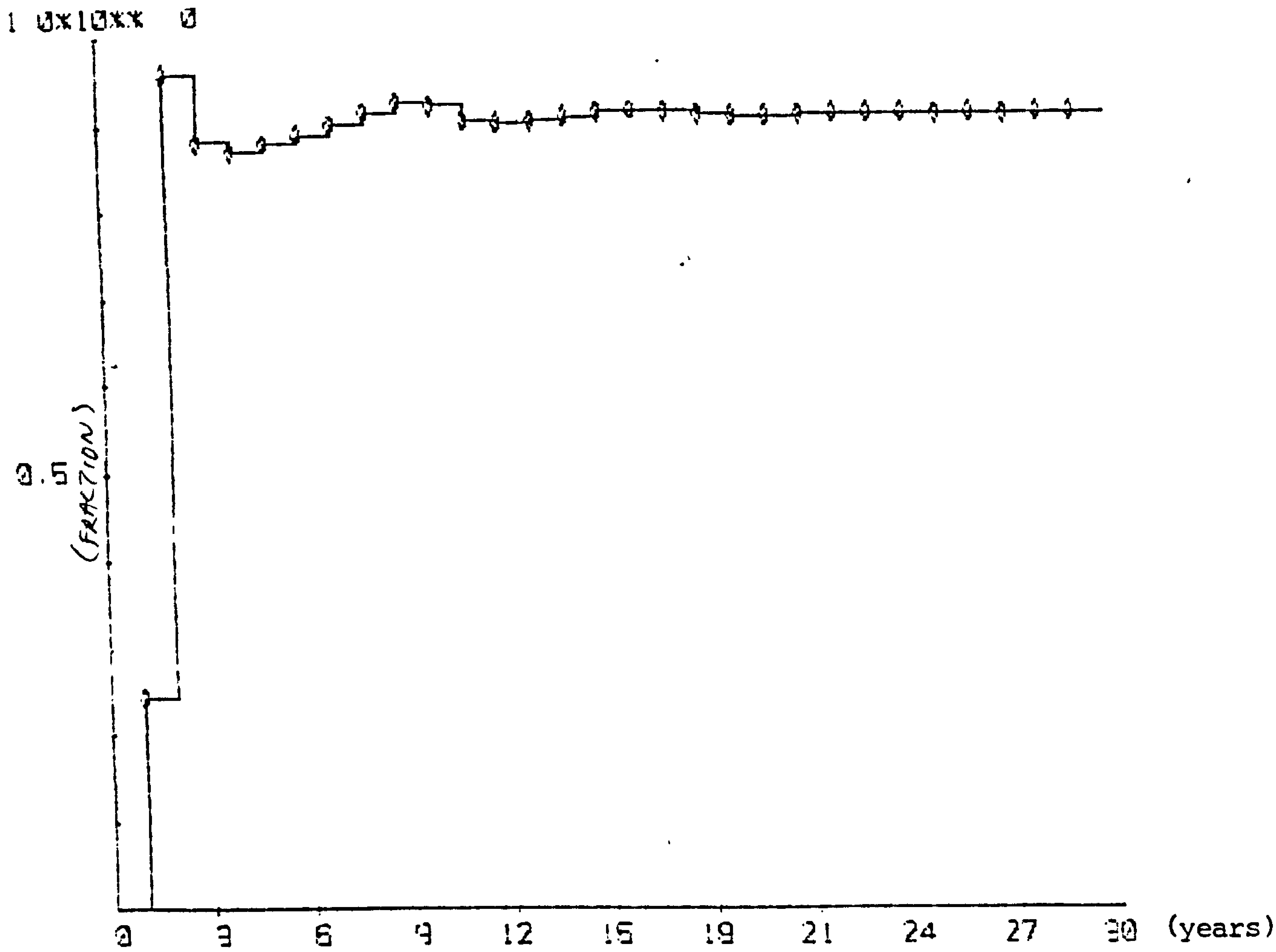

FIGURE 5.23(a) Distribution Effort Factor-Time Curve: Monopoly Simulation

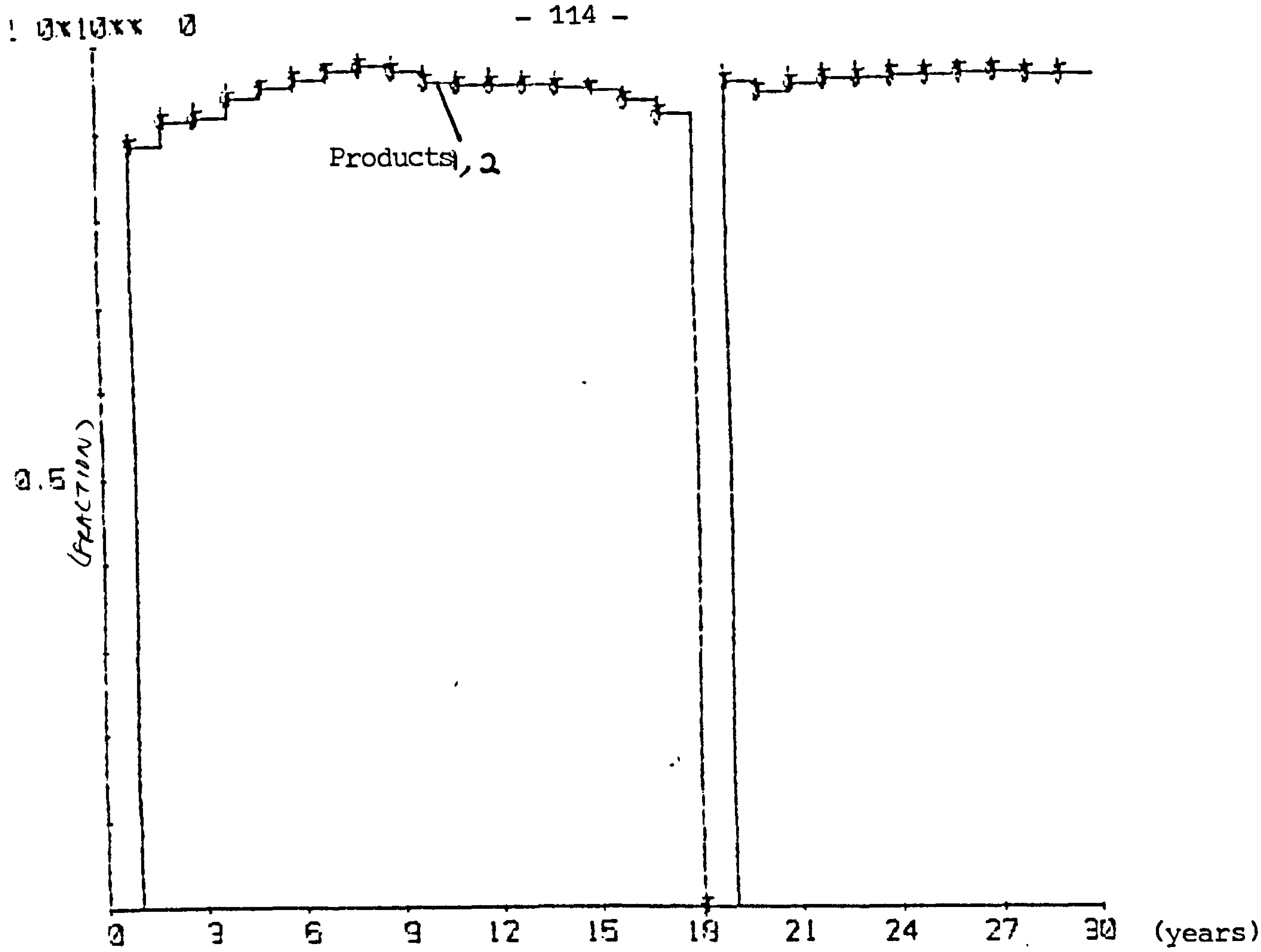

FIGURE 5.23(b) Distribution Effort Factor-Time Curve: Equal Parameter Simulation

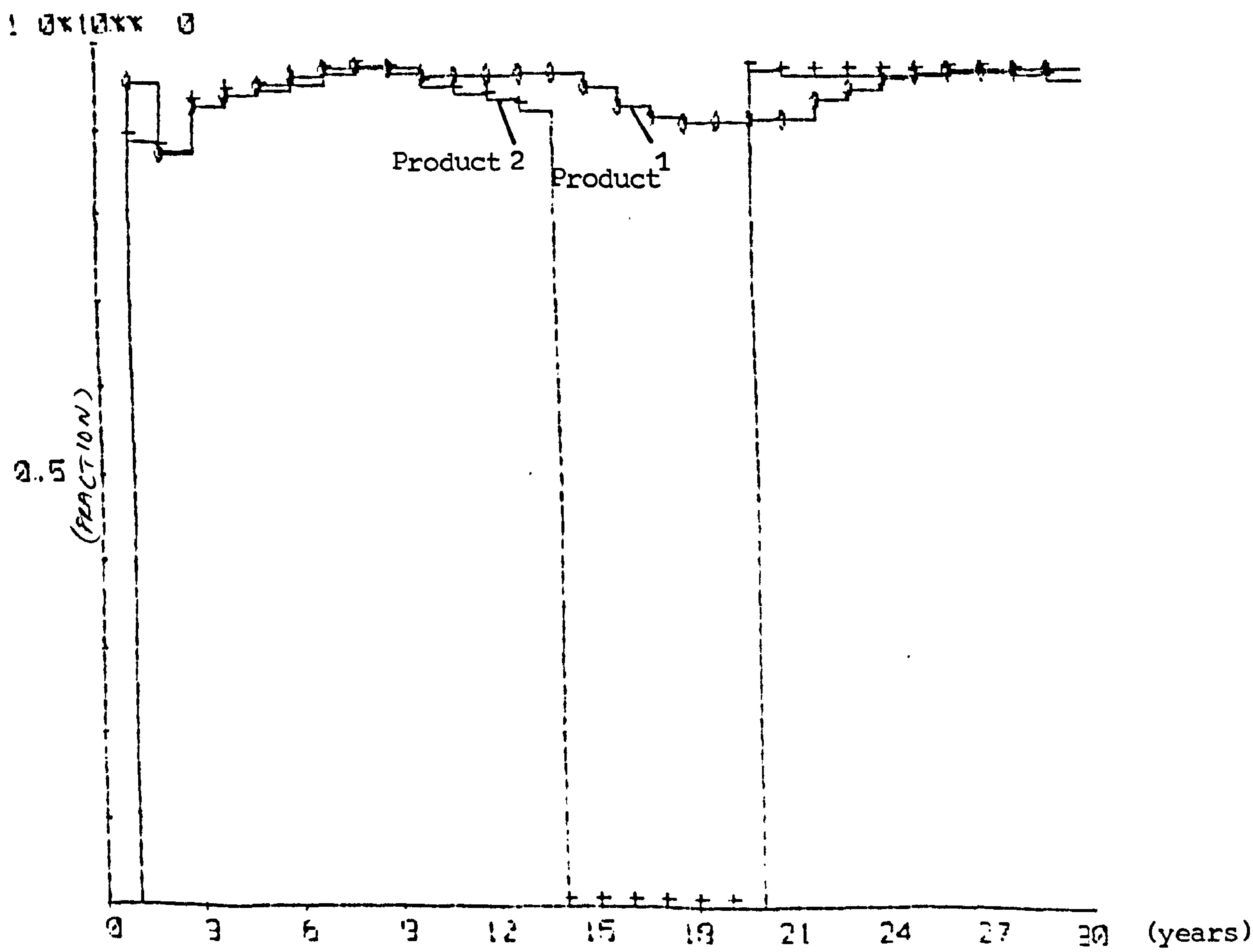

FIGURE 5.23(c) Distribution Effort Factor-Time Curves: Unequal Unit Cost Simulation

obvious but is present nevertheless. We note that period 19 is the last period in the second 8-period cycle after the first 3 'priming' periods and this period heralds the lowest point of the next cycle. It is significant that in the equal parameter simulation, period 19 is the period in which both products are withdrawn and the six-period interval in which product 2 is withdrawn in the different unit cost simulation also includes period 19. It can therefore be concluded that the timing of the withdrawal of the product(s) from the market is influenced significantly by the marketing system dynamics. The simulations have shown that even in situations where there are unlimited financial resources and where the competing products have no marketing advantages over each other, the pursuit of unbridled profit maximization inevitably results in a 'price war' and it is therefore not surprising that in practice such behaviour augurs well for the formation of cartels where the use of price as an instrument for competition is discouraged. Conversely, in markets where one product has significant marketing advantages over its competitors, 'price wars' would be the order of the day and would make the formation of cartels more difficult.

The combination of lower prices, higher expenditures on advertizing and distribution and maintenance of quality at monopoly levels lead to the conclusion that competition would result in higher total market sales and revenues (due to both higher market penetration and market expansion) and lower product profits than obtains for the monopoly simulation. Figures 5.24 to 5.27 confirm these conclusions. The total market sales, revenues and penetration are highest for the equal parameter simulation followed by the different unit cost simulation and then the monopoly simulation as shown respectively in Figures 5.24, 5.25 and 5.26. On the other hand, profits increase in precisely that order as shown in Figure 5.27. In fact profits for the monopoly simulation are greater on average than the sum of profits of both products in either 2-product simulation. Concentrating on the 2-product simulations, we note that the 11% unit cost advantage enjoyed by

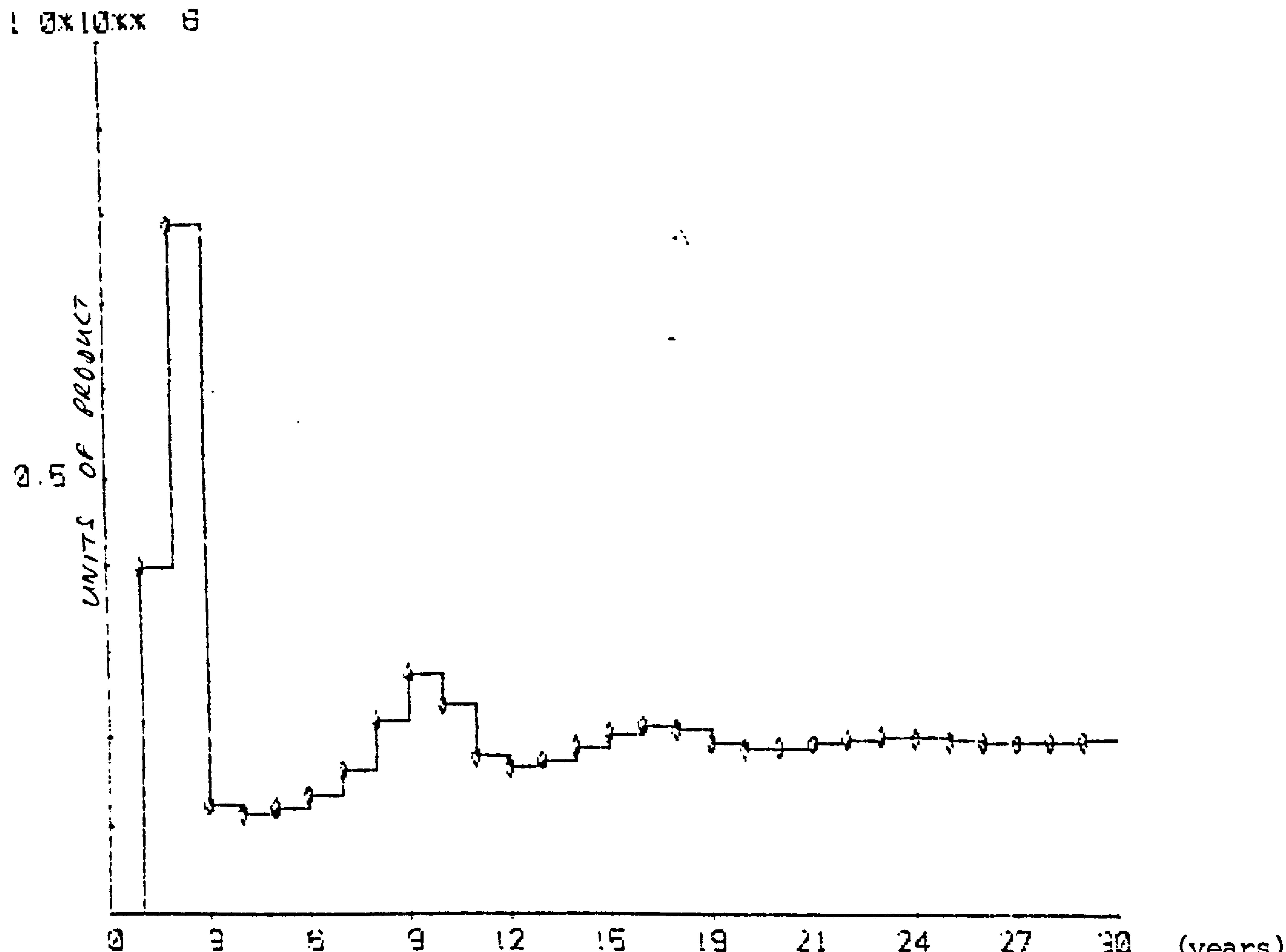

FIGURE 5.24(a) Sales-Time Curve: Monopoly Simulation

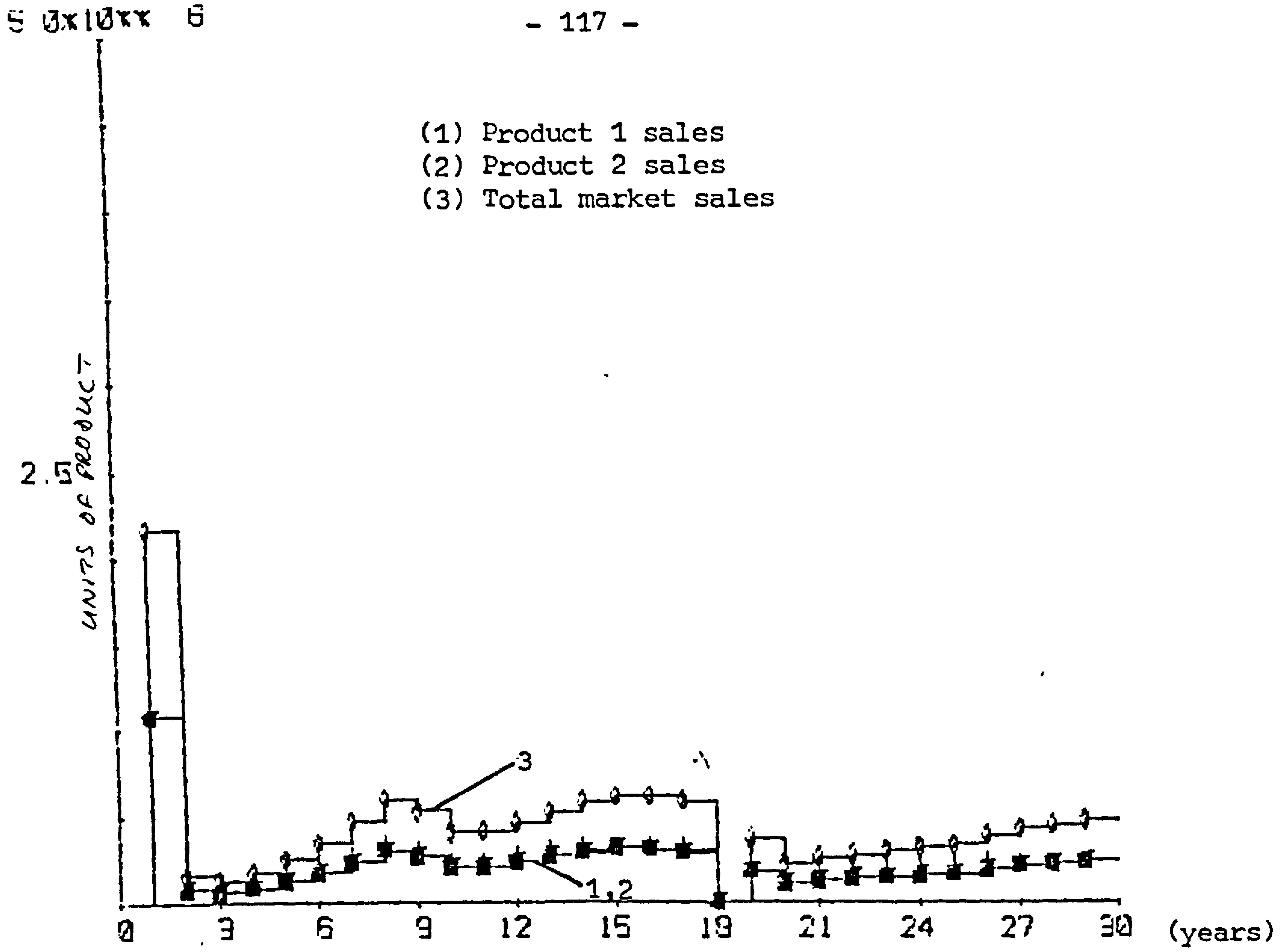

FIGURE 5.24(b) Sales-Time Curves: Equal Parameter Simulation

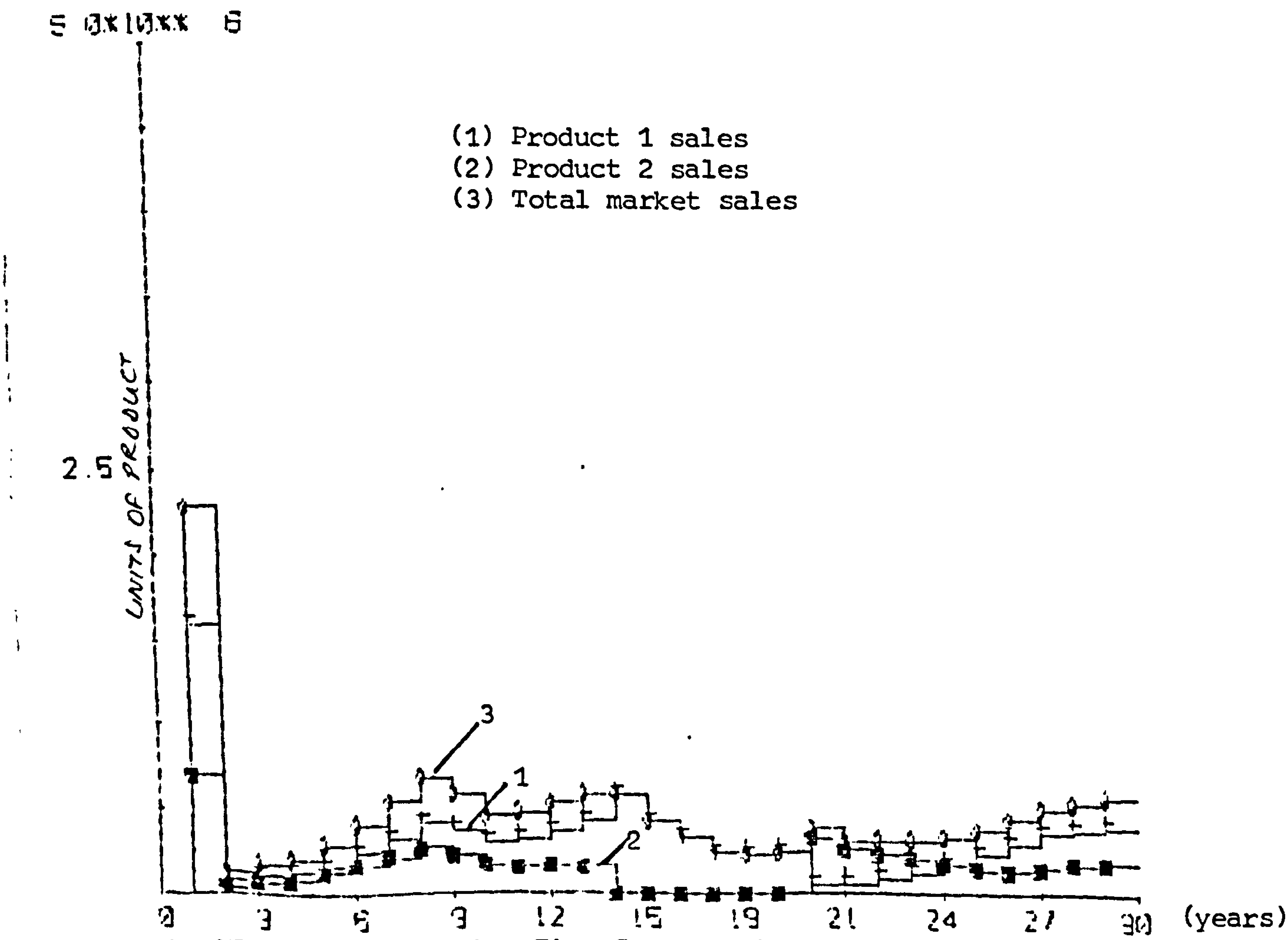

FIGURE 5.24(c) Sales-Time Curves: Unequal Unit Cost Simulation

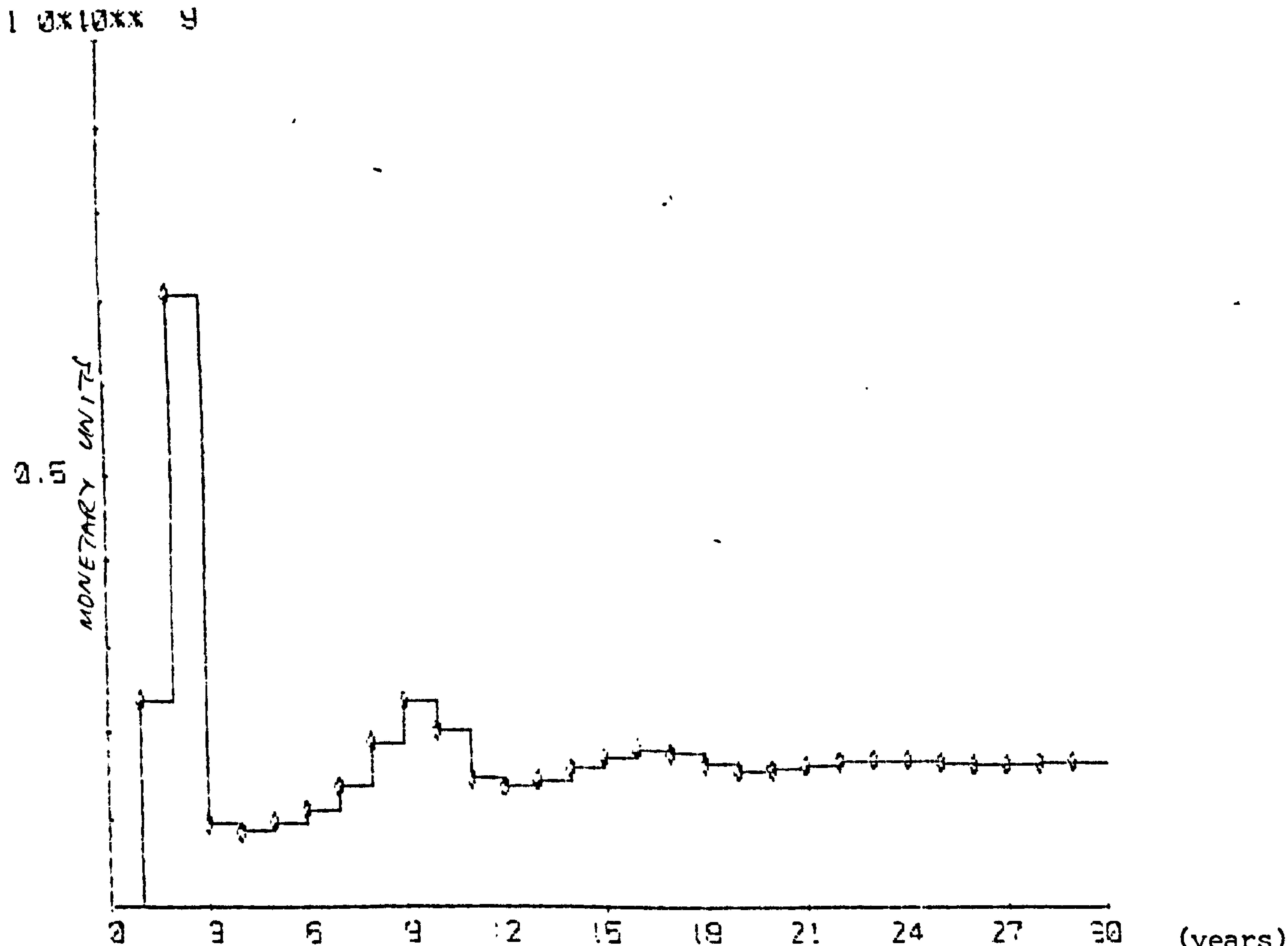

FIGURE 5.25(a) Revenue-Time Curve: Monopoly Simulation

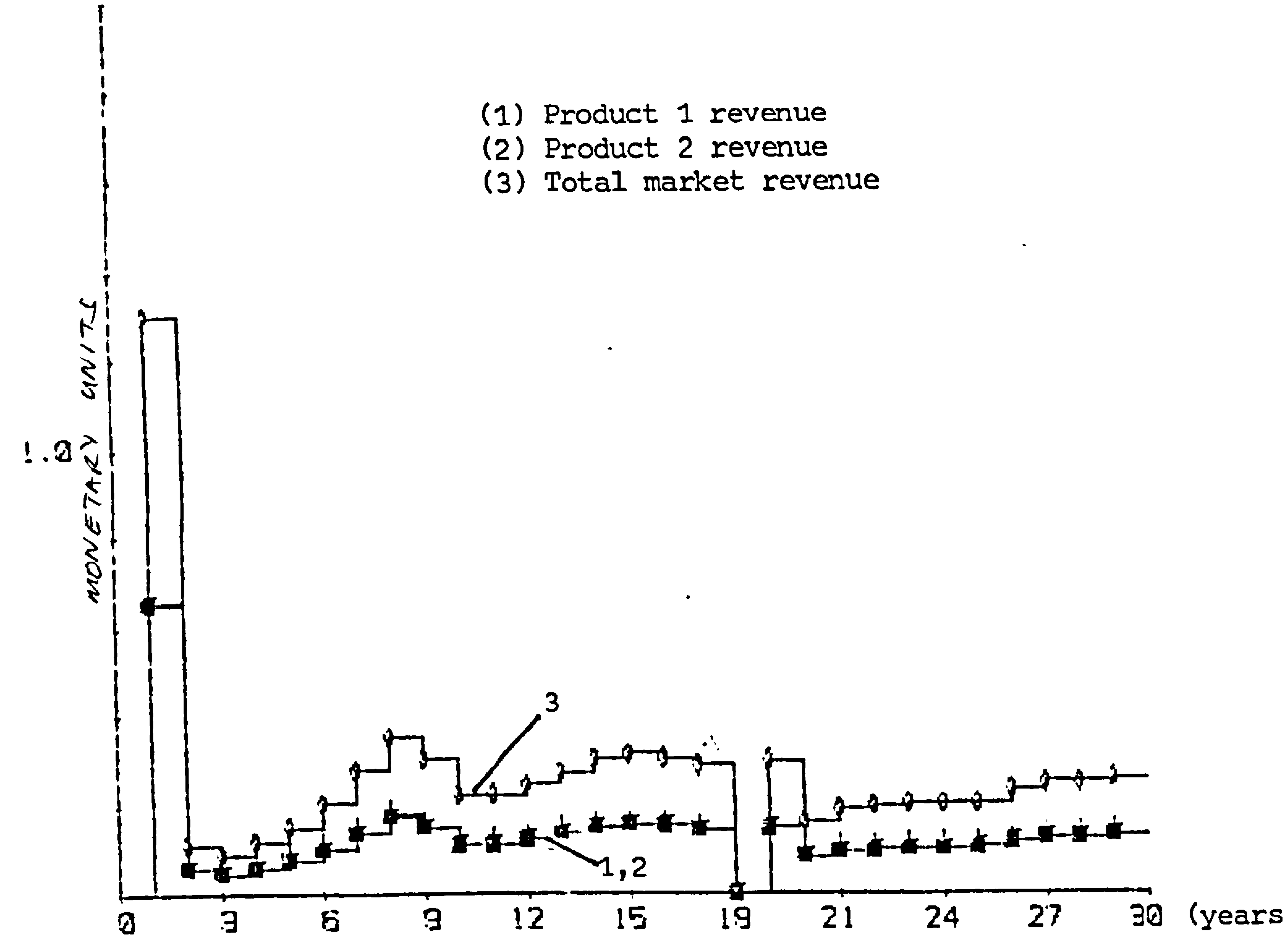

FIGURE 5.25(b) Revenue-Time Curves: Equal Parameter Simulation

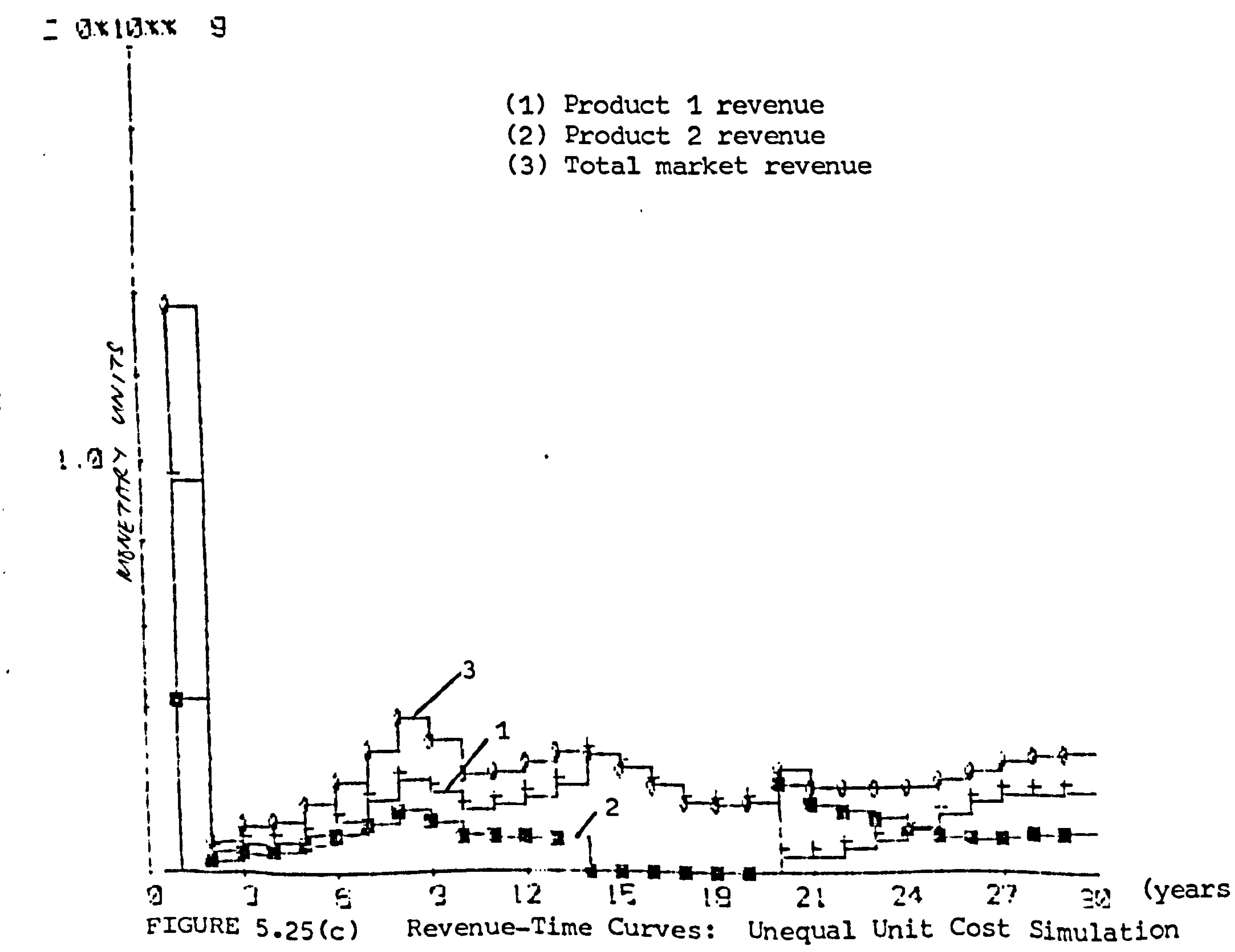

FIGURE 5.25(c) Revenue-Time Curves: Unequal Unit Cost Simulation

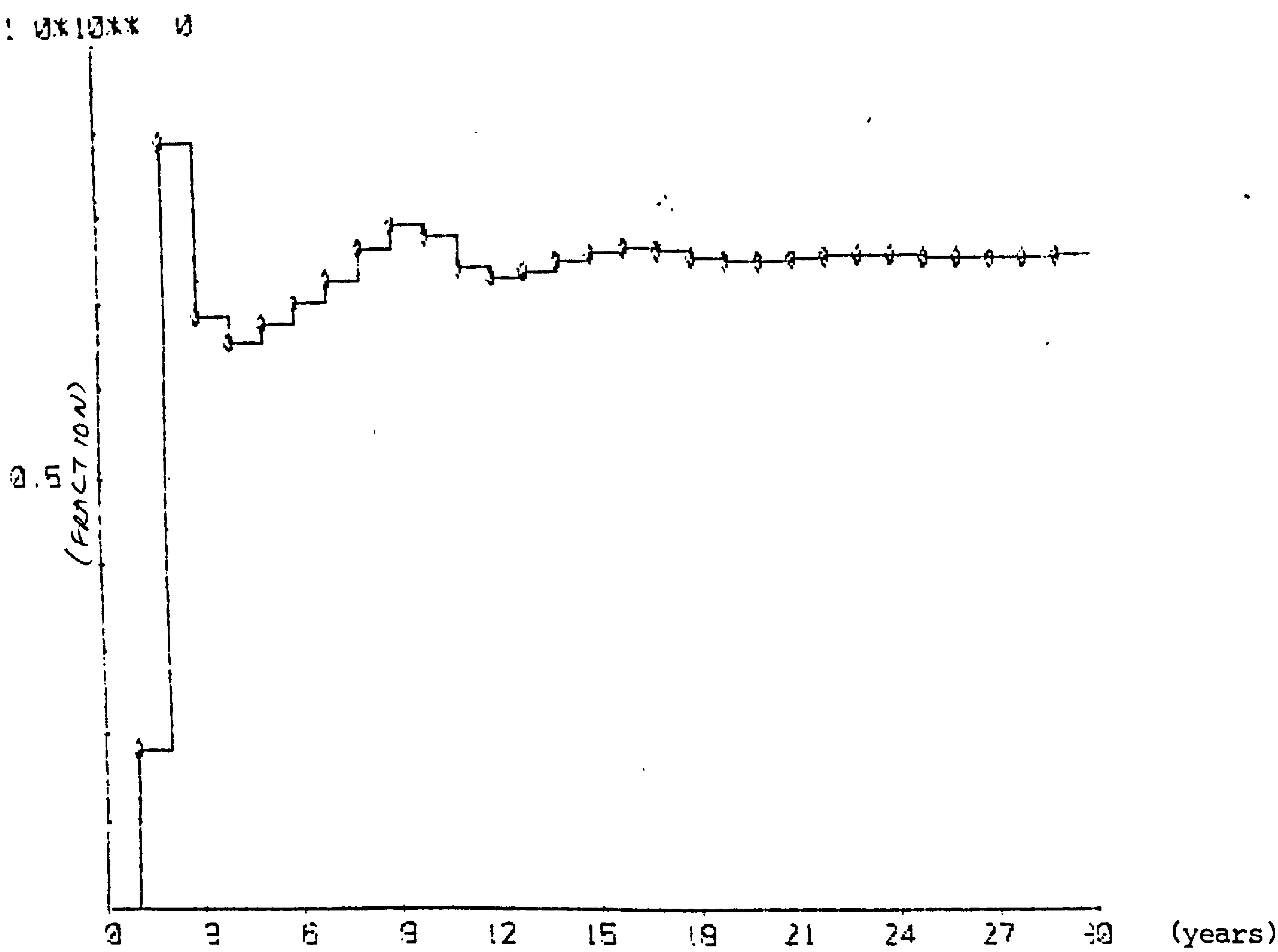

FIGURE 5.26(a) Market Penetration-Time Curve: Monopoly Simulation

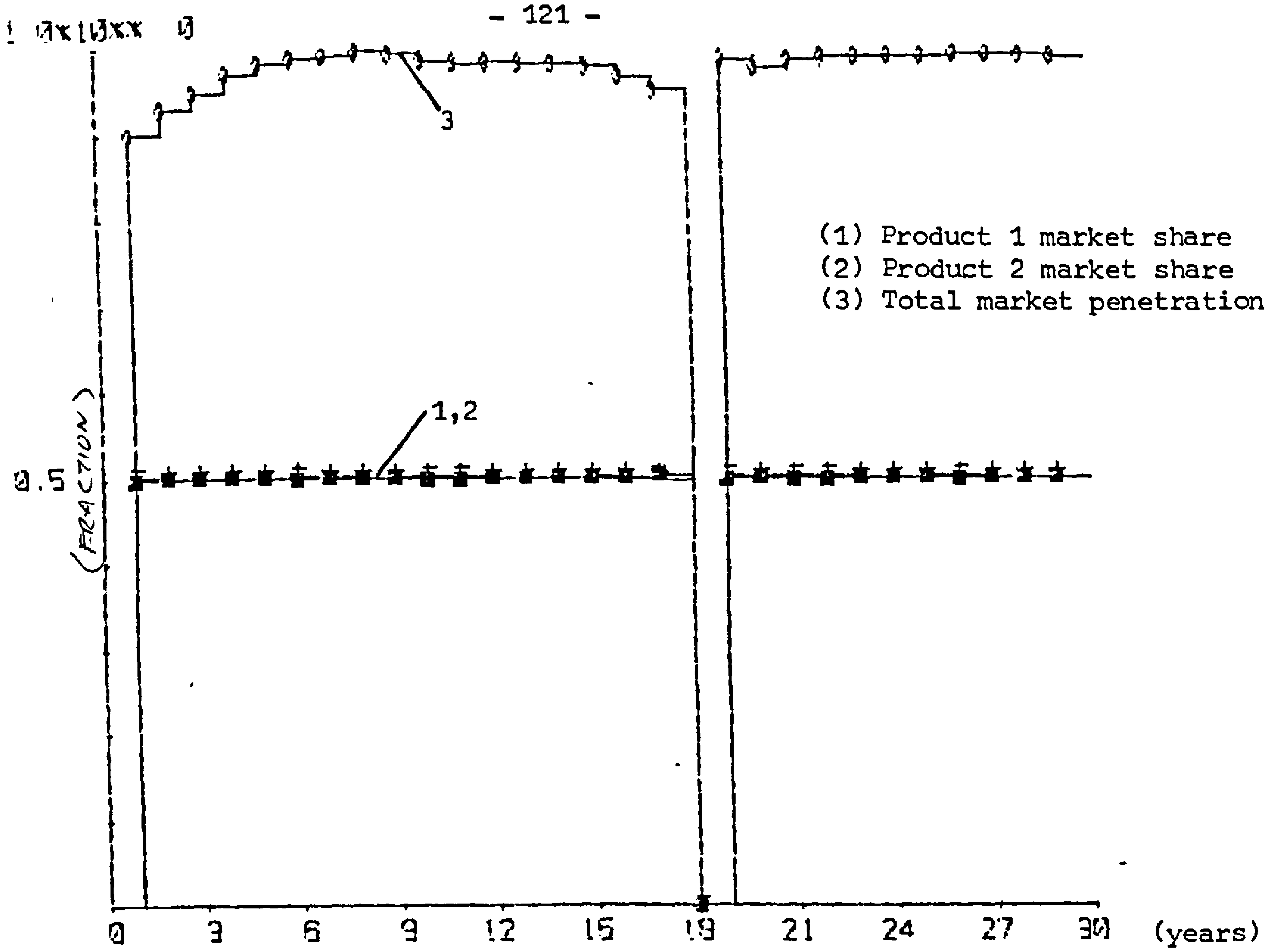

FIGURE 5.26(b) Market Penetration/Share-Time Curves: Equal Parameter Simulation

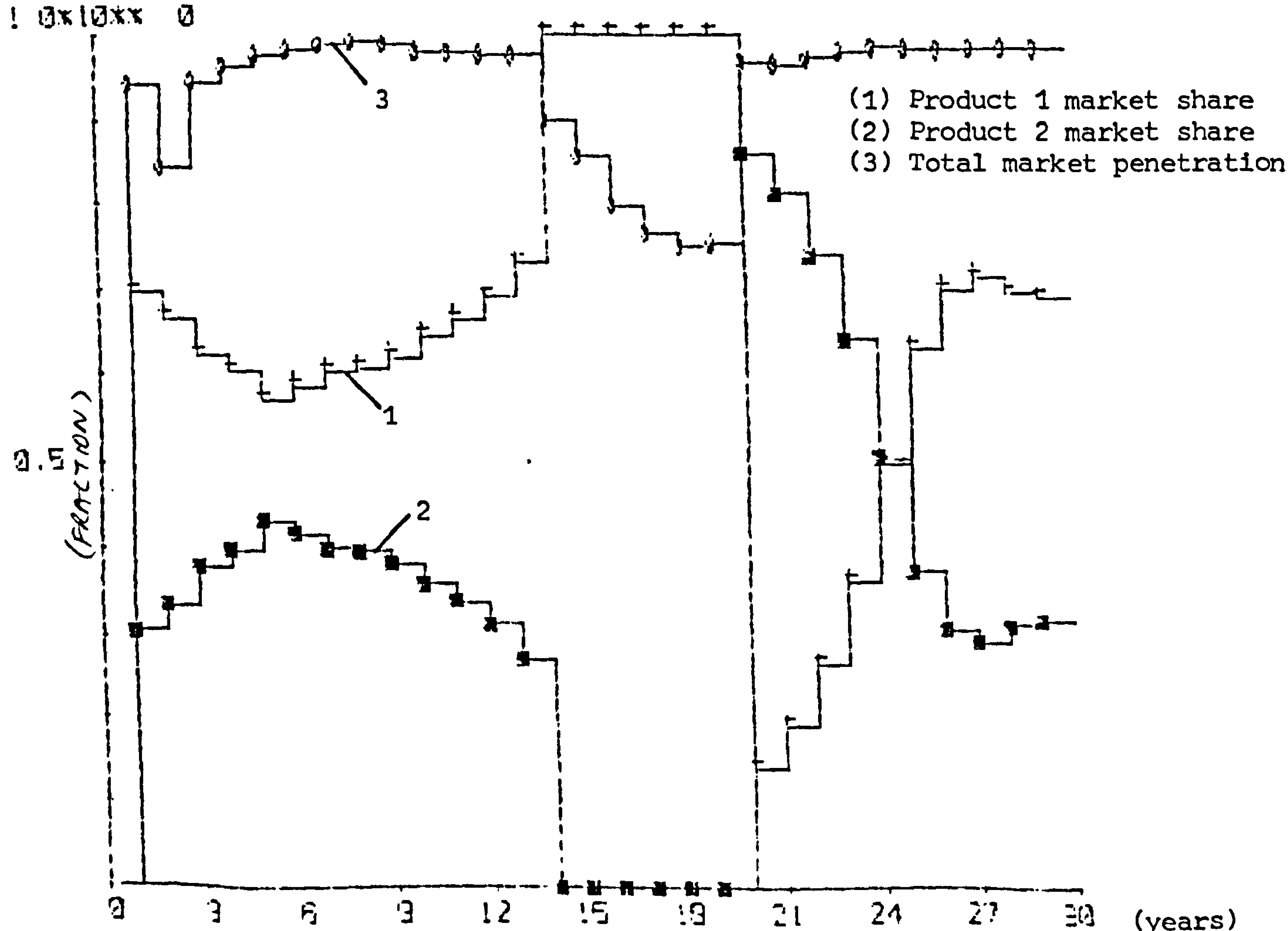

FIGURE 5.26(c) Market Penetration/Shares-Time Curves: Unequal Unit Cost Simulation

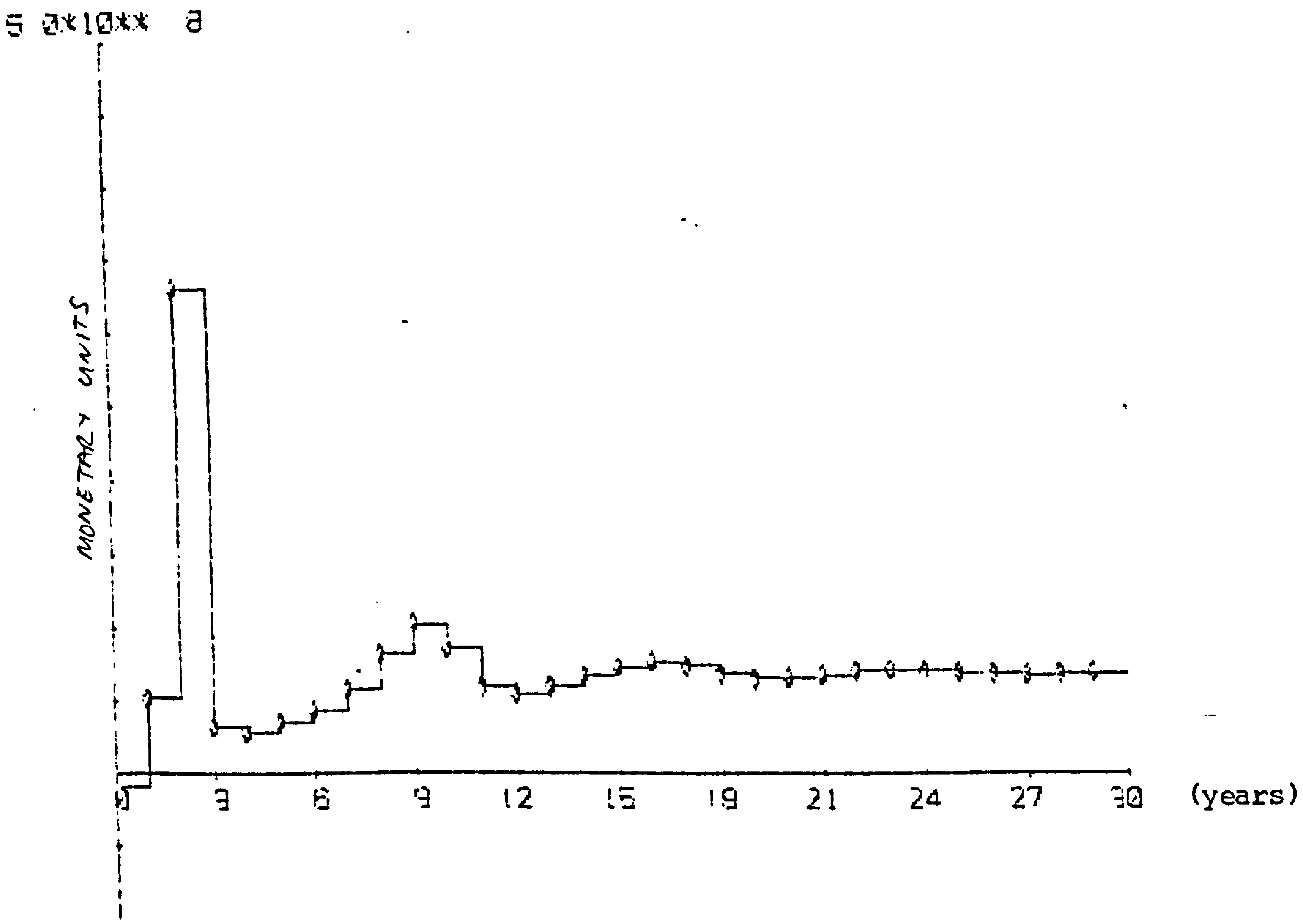

FIGURE 5.27(a) Profit-Time Curve: Monopoly Simulation

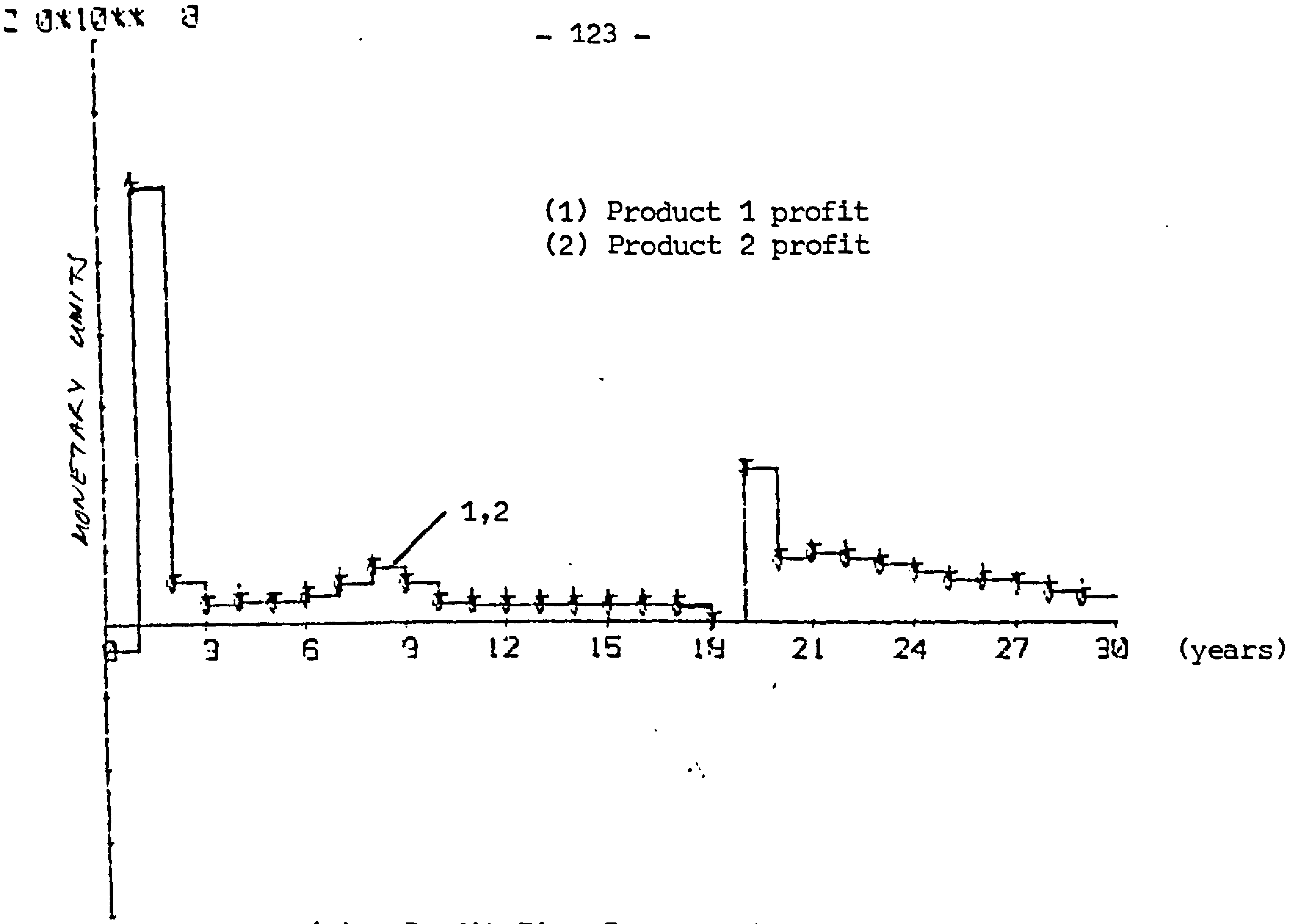

FIGURE 5.27(b) Profit-Time Curves: Equal Parameter Simulation

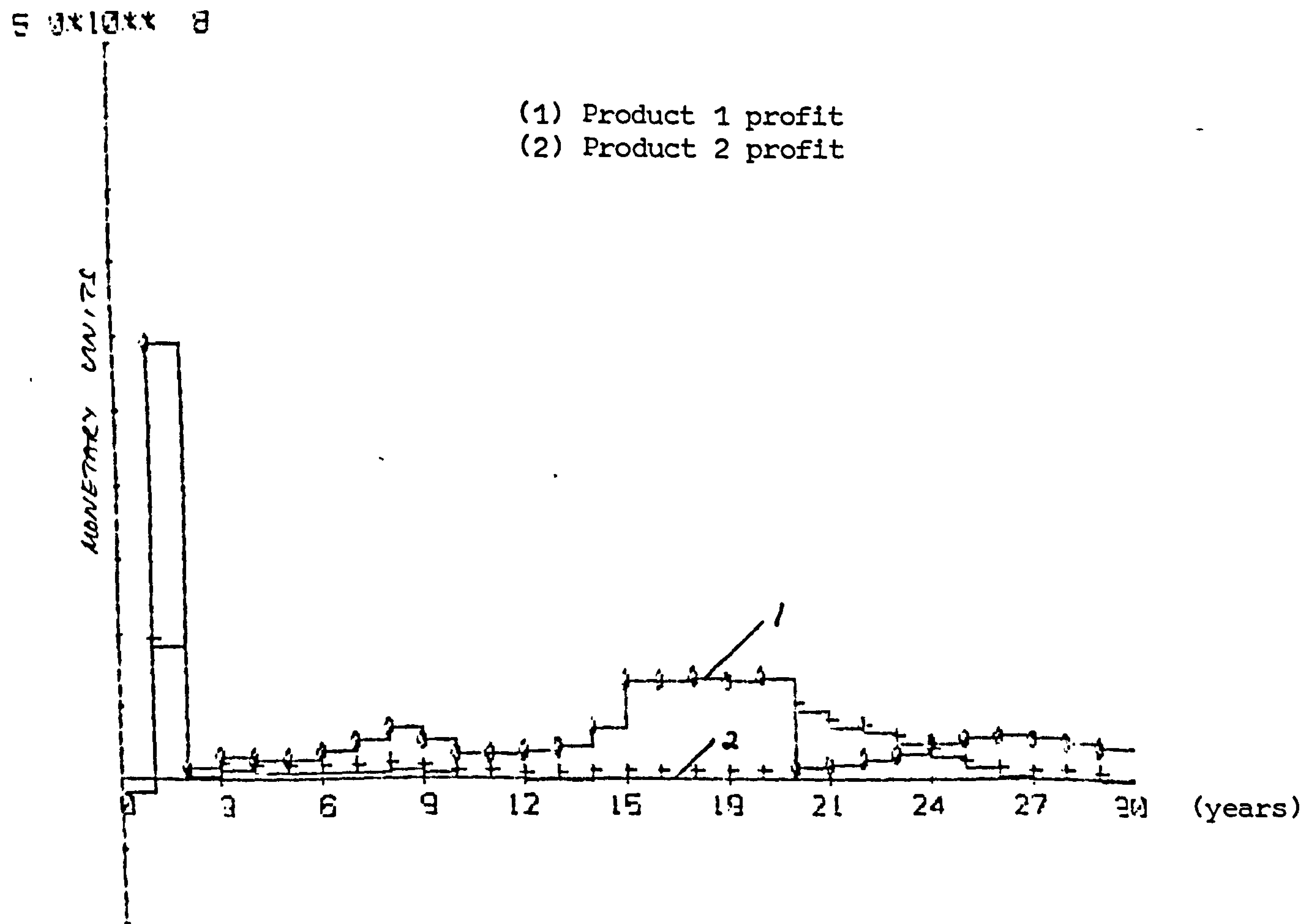

FIGURE 5.27(c) Profit-Time Curves: Unequal Unit Cost Simulation

product 1 over product 2 (400 against 450) was sufficient to place product 2 in a very unfavourable position, so much so that it was forced out of the market completely for 6 periods out of the 30 period simulation horizon. The market share curves (1,2) of Figure 5.26(b) and (c) and the profit curves of Figure 5.27(b) and (c) highlight this.

5.3 SIMULATION OF A MULTI-PRODUCT, VARIABLE DECISION, MARKETING SYSTEM WITH FINITE PRODUCTION CAPACITY

In the marketing system simulations of sections 5.1 and 5.2 it had been assumed that there were no limitations to the supply of product to be marketed. In effect therefore, we had simulated the dynamic behaviour of the marketing aspect of a production-marketing system (PMS) with infinite production capacity and with constant unit production costs. In this section we concentrate on the production aspect of the PMS. Our desire is to simulate the medium-term dynamic behaviour of a finite and constant capacity, multiple final-product production system bearing in mind that it is part of a PMS whose marketing system is described by the multi-product, variable-decision marketing system model of 4.3.1 above.

The production system model is described in Chapter 3 and we note that the dynamic behaviour of the production system reflects the dynamics of external demand* for the output of the production system, the dynamics of its decision making system and the dynamics of the production process itself. The decisions that influence the medium-term production system behaviour are summarized as follows:

1) allocation of resources to the marketing system to manipulate external demand;

2) adjustment of the size of the workforce by hiring/laying-off workers in response to fluctuations in external demand;

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*The word 'demand' as used in this section is synonymous with 'sales' and <u>not</u> market potential as in Chapter 4.

3) adjustment of production rate by working overtime or undertime with the same workforce;

4) adjustment of inventory levels, backlogging of unfulfilled demand or allowance for lost sales; and

5) variation in the amount of subcontracting to absorb fluctuations in external demand.

The marketing model is a long-term model while the production model is a medium-term one. To couple the two together, certain assumptions must be made concerning the relationship between medium-term external demand and long-term sales rate as there is no medium-term marketing model to generate this output directly. These assumptions are described in detail later. From Chapter 3, we note that there is assumed a 1 medium-term period delay between taking a hiring/laying-off decision and effecting it and a 3 medium-term period delay between placing an order for raw materials or for subcontracting the manufacture of work-in-process and taking delivery of same. It is the effects of these delays, medium-term demand dynamics and process dynamics (production lead-time, etc.) on the production system dynamic behaviour that is simulated.

5.3.1 Production-Marketing System Data

The PMS under consideration (hereafter referred to as Seller 1) produces and markets two product items, respectively F1 and F2, in its product line and is in competition with Seller 2 in the marketplace represented by a single aggregate product. The marketing system parameters are identical with those described in 5.2.1 above except for slight modifications to the product quality rating cost functions whose parameters are given as follows:

1) $Y_1 = 0.875 \times 10^5$ for F1

2) $Y_1 = 0.750 \times 10^5$ for F2

3) $Y_1 = 0.750 \times 10^5$ for Seller 2's aggregate product and

$Y_2 = 2.500 \times 10^5$ for all three products.

Unit production cost for Seller 2 is constant at 450 while it is variable (depending on production levels) for Seller 1's product line. As in the 2-product, variable-decision marketing system simulation of 5.2 above, Seller 2's decision-making system (DMS) selects the values of the product's marketing variables (α, p, q and k) in each of the four long-term periods in its decision horizon so as to maximize expected profit. Seller 1's DMS also selects the values of these variables and an additional one, the long-term production level for each of its product items in each period of its four long-term period decision horizon such that expected profit over this horizon from both product items is maximized. Unit production costs for F1 and F2 vary from a minimum when the product items are produced without using sub-contracted work-in-process (WP) materials and without overtime (in effect the unit cost of producing the first unit of product) to a maximum when operating at full capacity, i.e., only the last stage of manufacture of F1 and F2 carried out at the factory, all input inventories to the last stage being purchased from outside and full overtime hours utilized. A quasi-linearization of the unit cost/production level relation for each product item is carried out using a third known point in the relation - the previous long-term period's actual unit production cost and production level - resulting in two linear portions; 1) the portion connecting the minimum unit cost point to last long-term period's actual unit production cost and 2) the portion from the latter point to the maximum unit cost point. The production level for each product item is of course limited by the respective capacity of the last stage of manufacture of the item over the long-term period; the expression for period cost also includes an opportunity cost element when available supply (beginning stock plus production level) is less than anticipated sales rate.

Having obtained the long-term marketing decisions for Seller 1 and hence the estimated sales rates for F1 and F2 in each long-term period of the decision horizon, the medium-term demands for F1 and F2 can be calculated.

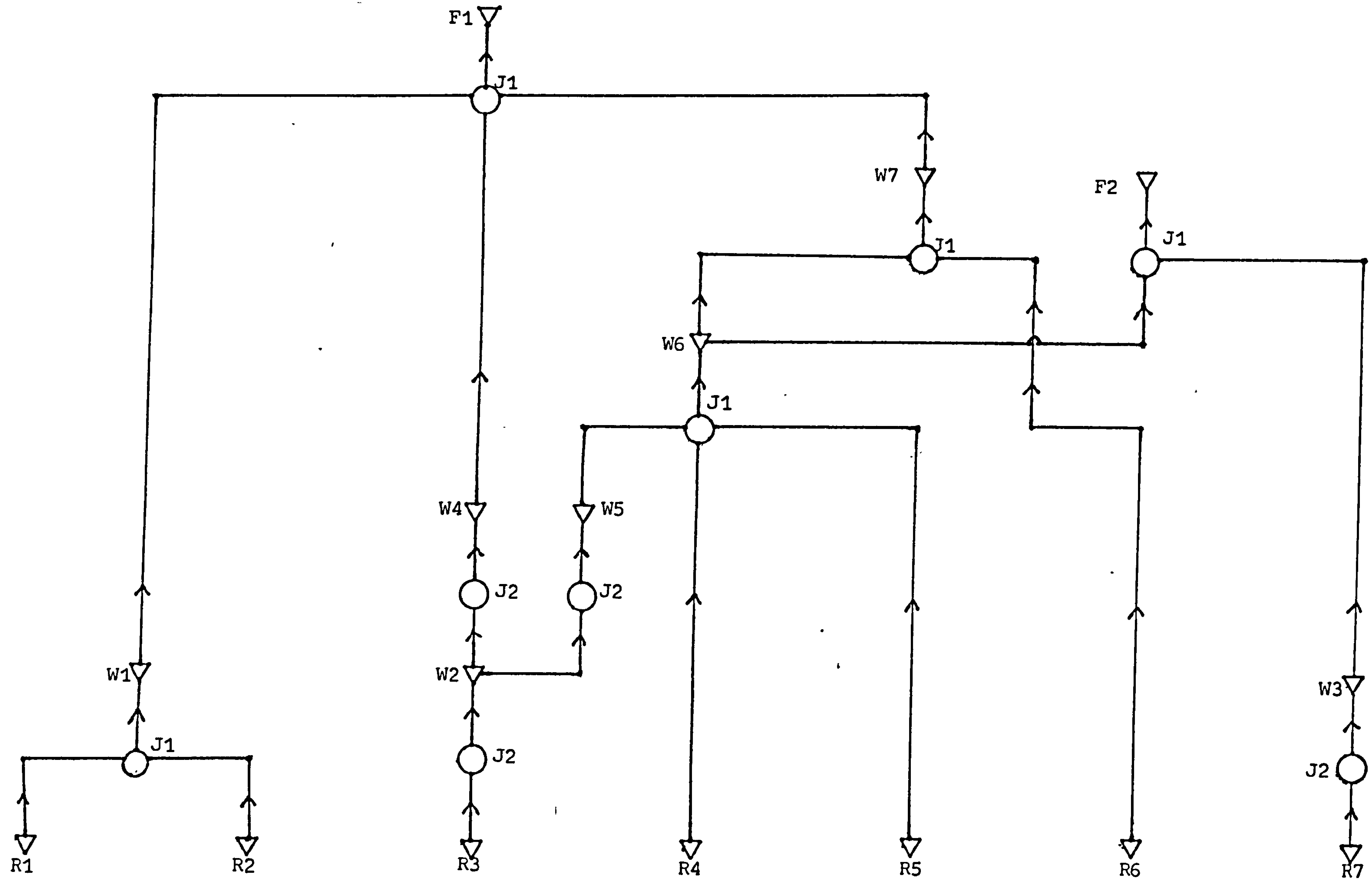
There are twelve medium-term periods in each long-term period (MINT=12) with each medium- and long-term period corresponding respectively to one month and one year of real-time. It is assumed that there are no seasonal variations (variations occurring wholly within each long-term period) in the demand for F1 hence each medium-term period's demand for F1 is 1/12 of the current long-term period's sales rate. F2 however is assumed to exhibit seasonal variations and its demand in the Mth medium-term period is 1/12 of its current long-term period's sales rate multiplied by a seasonal factor given by

$$1 + 0.9 \sin(2\pi \times (m-1)/12)$$

The possibility that the total volume of product shipped (sold) to customers in each long-term period (i.e., the sum of the volume of product shipped to customers in each of the twelve medium-term periods in the long-term period) differs from the predicted sales rate requires that at the end of each long-term period the respective distribution effort factor for each product item or at least the aggregate distribution effort factor for Seller 1 be corrected to allow for this discrepancy.

The production process itself is depicted in Figure 5.28. It is a multi-stage non-linear assembly tree process with two finished goods (FG) items, F1 and F2, seven work-in-process (WP) items, W1 to W7, and seven raw material (RM) items, R1 to R7, to make a total of sixteen inventory items ordered as described. There are five production levels in all; F1 is producted at the fifth level and F2 at the fourth. There are two work centres, J1 and J2, requiring respectively the skills S1 and S2 for their operation. Assembly operations are carried out on J1 while unit operations are carried out on J2. There are fifteen workstations in J1 each requiring one worker for its operation and thirty-five workstations in J2 also requiring one worker each for their operation. The number of workhours per worker per medium-term period in each workcentre is 168 (corresponding to a 40 hour week); overtime hours per worker per medium-term period cannot

FIGURE 5.28 The Production Process



exceed 25% of normal workhours; and a maximum of three shifts of workers can be scheduled at each workcentre. Further details of production process parameters are given in Appendix 4.

The production DMS considered here is identical with that described in Chapter 3 except for the following:

1) we no longer set limits to the amount of financial resources that can be invested in production activities in any period or over any interval;

2) we consider now an 18 medium-term period decision horizon to ensure that production decisions in the 12 medium-term periods of the current long-term period anticipate the requirements of the next long-term period, i.e., current long-term period ending inventory stock levels, and workforce levels and purchased inventory orders which take effect in the up coming long-term period are adequately catered for;

3) it is now assumed that no backorders are lost in the current long-term period, however backorders at the end of this period are assumed lost as far as the production system is concerned. The marketing system acknowledges that part of this unfulfilled demand is converted to sales by Seller 2 and the rest is included in the revised sales rate estimate for the upcoming long-term period simply by correcting Seller 1's aggregate distribution effort factor for the current long-term period at the end of the period.

Finally, the production and marketing systems are assumed to start from cold, i.e., initially there are zero demand-generating market populations, zero inventory stock levels, zero workforce levels, zero backorders, etc. A four long-term period (48 medium-term period) simulation interval is utilized.

5.3.2 <u>Production-Marketing System Simulation</u>

The simulation results were obtained using the program PRDMRK.FTN. The aggregate marketing system results (with Seller 1's product line regarded as a single aggregate product) are displayed in Figures 5.29 to 5.36 and are respectively Price-Time, Advertizing Effort Factor-Time, Quality Rating-

Time, Distribution Effort Factor-Time, Sales-Time, Revenue-Time, Market Penetration/Share-Time, and Profit-Time curves. These figures correspond respectively to Figures 5.20 to 5.27 of Section 5.3 above. As in Section 5.3, curve 1 refers to Seller 1's aggregate product, curve 2 refers to Seller 2's aggregate product and curve 3, where it exists, refers to the combined result for both sellers. Time is measured in long-term periods (years).

The Price-Time curves of Figure 5.29 show that as a result of production capacity constraints, market prices are higher than expected. This is even more significant when it is noted that Seller 1's unit production cost is less than half of his competitor's (see Figure 5.41 below) and also when Figure 5.29 is compared with Figure 5.20(c) above, where the latter figure shows the results of the simulation in which Seller 1 has unlimited production capacity and his unit production costs are 89% of his competitor. We note that Seller 1's advertizing effort factors are slightly superior and his quality ratings are slightly inferior to those of his competitor, however Seller 1's distribution effort factors are very much inferior to Seller 2's although the degree of inferiority reduces with time. Obviously the poor distribution effort factors are a direct result of Seller 1's limited production capacity. The Sales-Time, Revenue-Time, Market Share/Penetration-Time and Profit-Time curves show that initially Seller 1's market performance is poor, but as the market contracts, his relative capacity (production capacity relative to market potential) improves and soon his sales, revenue, market share and profits exceed those of his competitor.

The details of market behaviour of items F1 and F2 that make up Seller 1's product line are shown in Figures 5.37 to 5.42 which are respectively the Price-Time, Quality Rating-Time, Distribution Effort Factor-Time, Production Rate-Time, Unit Production Cost-Time and Stock Level-Time curves. Time is in long-term periods (years). Curves 1 and 2 in each figure refer respectively to items F1 and F2 while curve 3 in Figures 5.37, 5.38 5.39 and 5.42 refer to the aggregate or total value for F1 and F2. In Figures 5.40 and 5.41 curve 3 refers to the equivalent Seller 2 parameter. From these figures and Figures 5.33 and 5.34

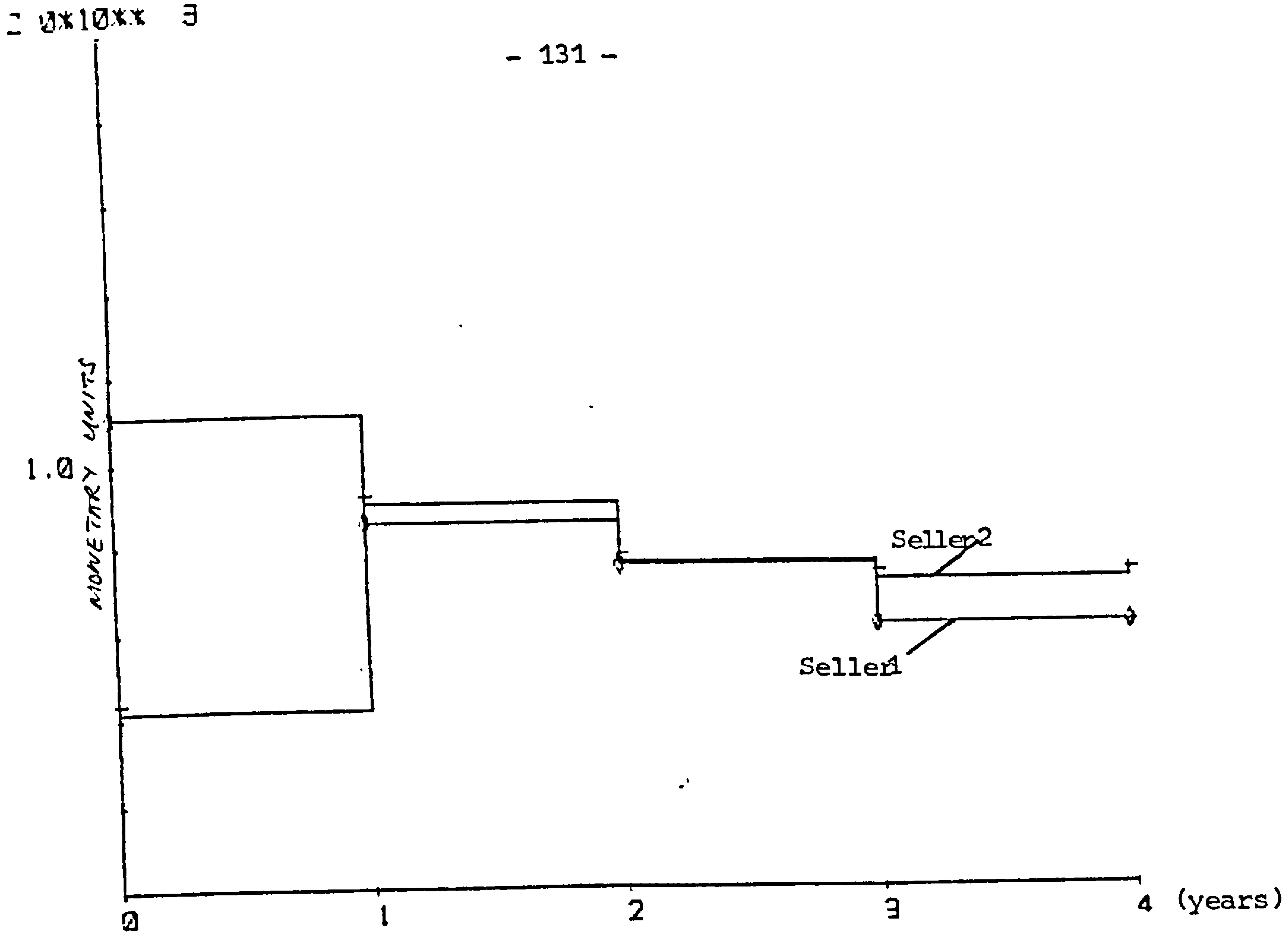


FIGURE 5.29 Price-Time Curves

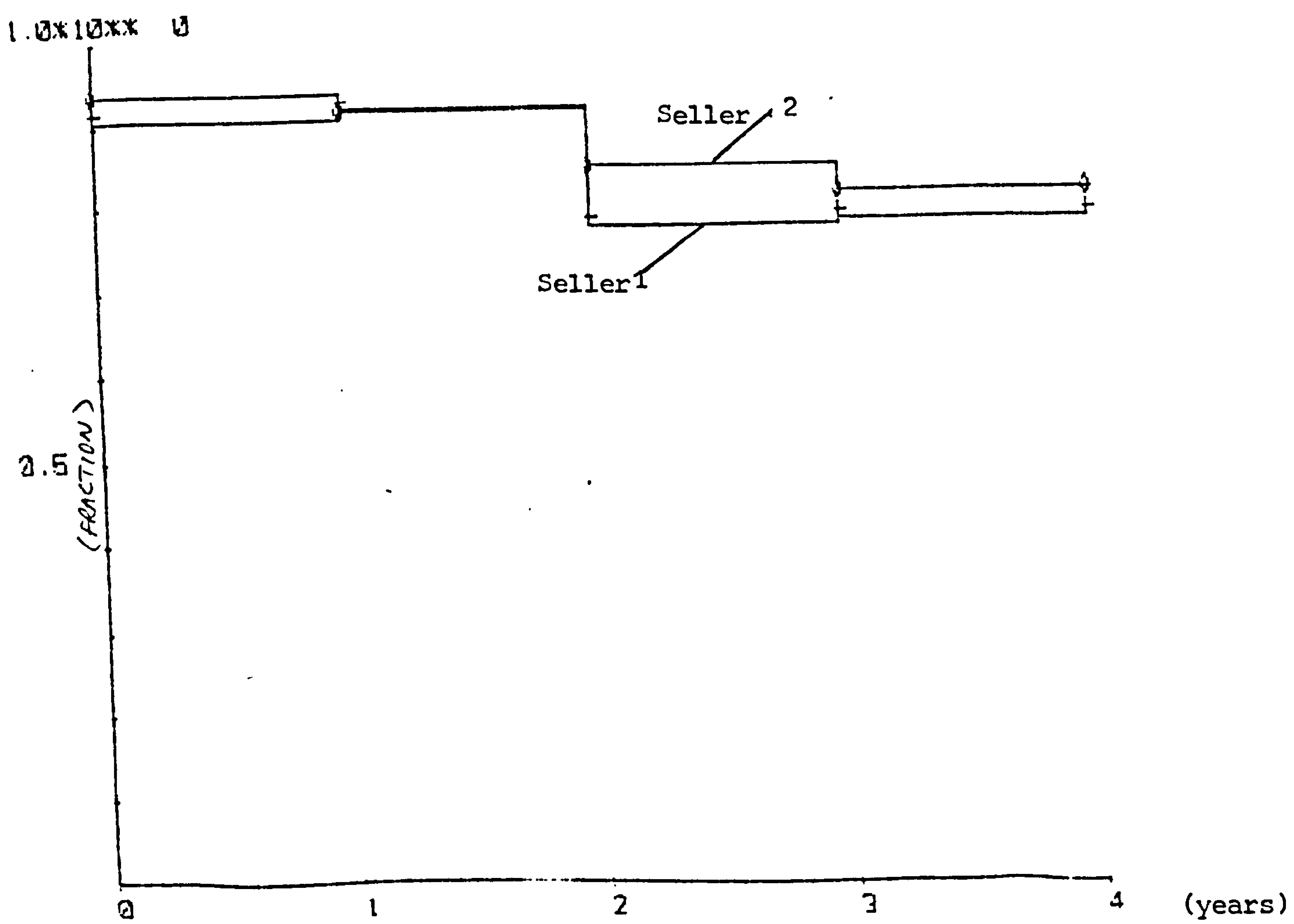


FIGURE 5.30 Advertizing Effort-Time Curves

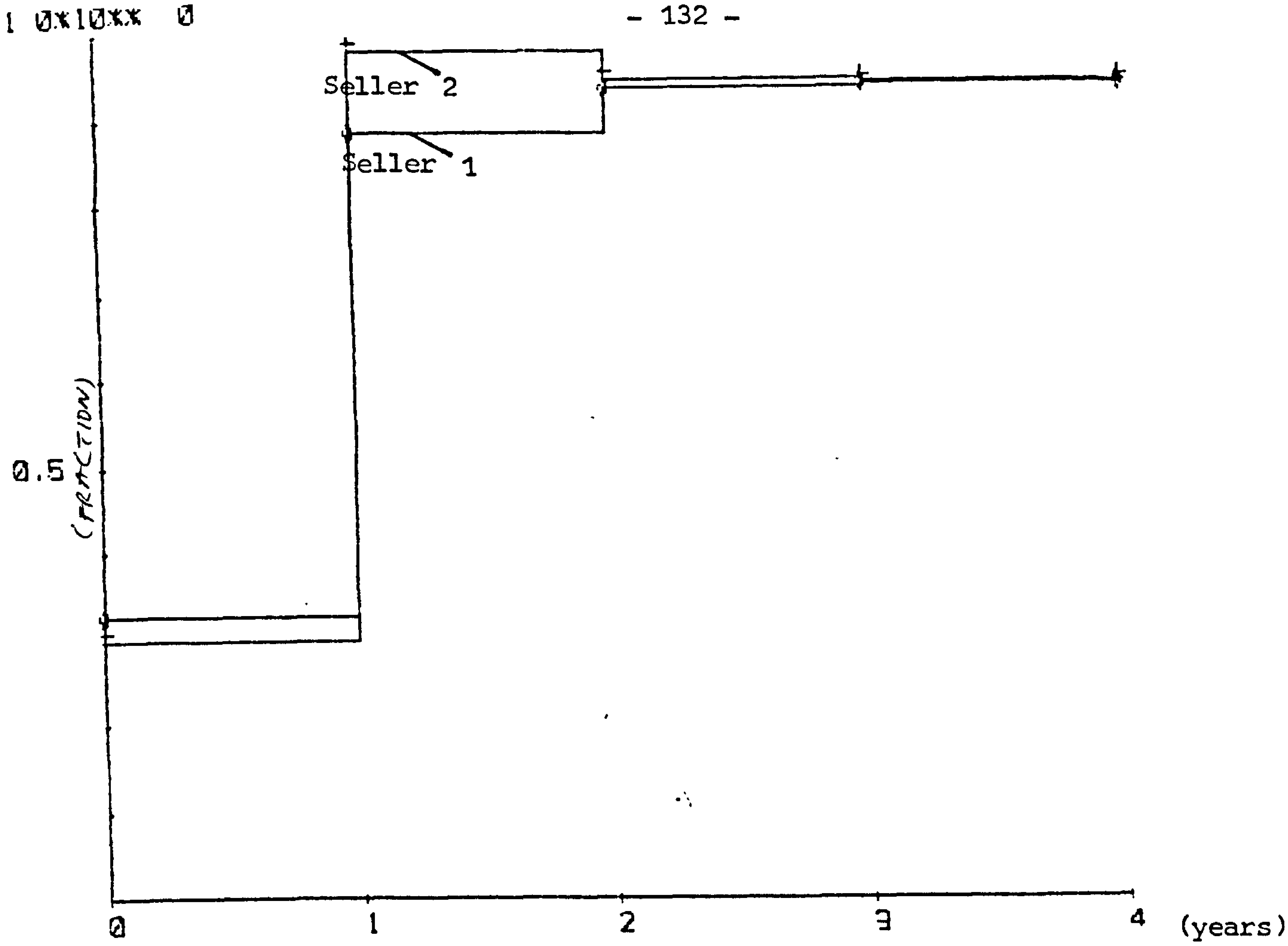


FIGURE 5.31 Quality Rating-Time Curves

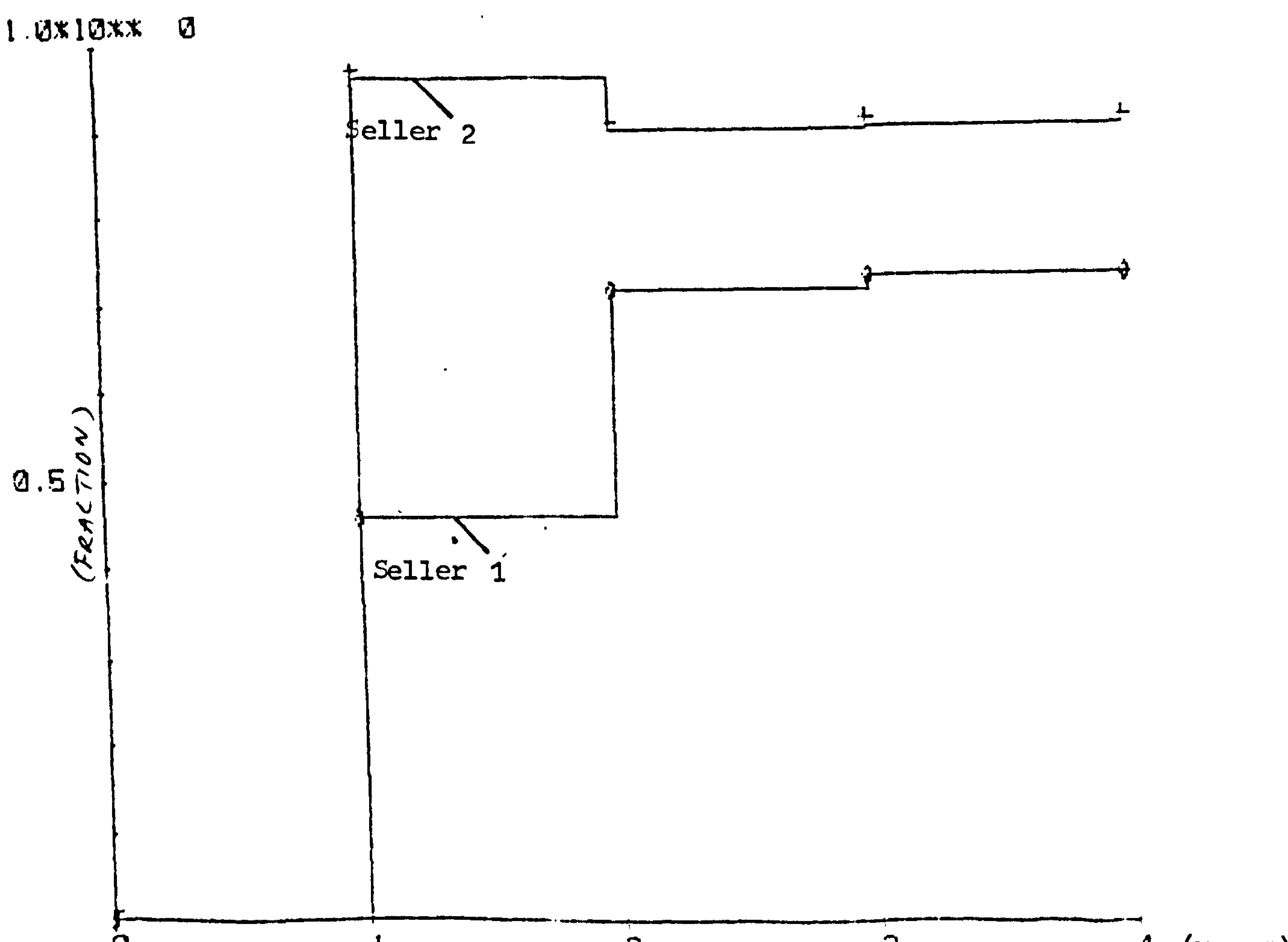


FIGURE 5.32 Distribution Effort Factor-Time Curves

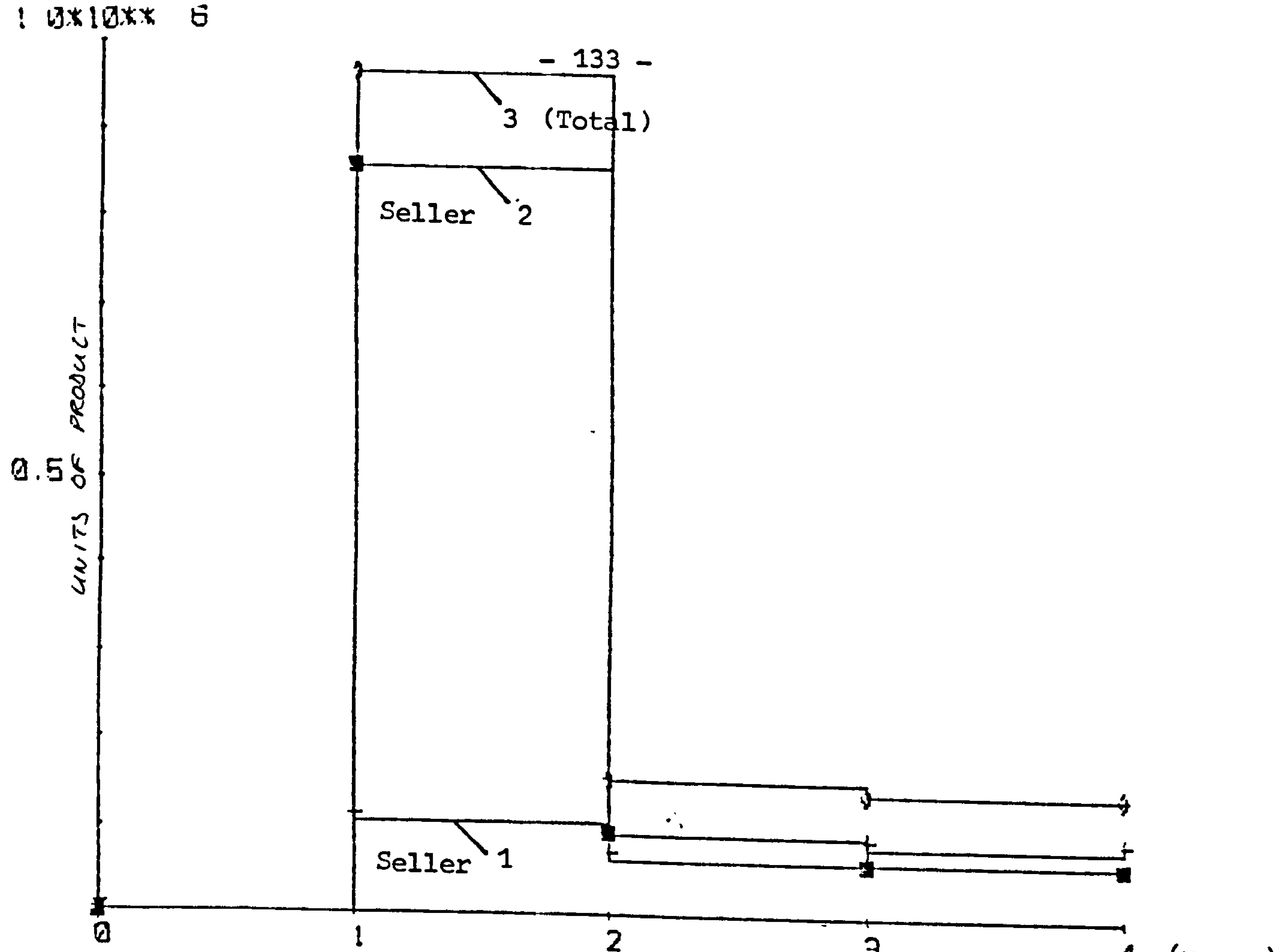


FIGURE 5.33 Sales-Time Curves

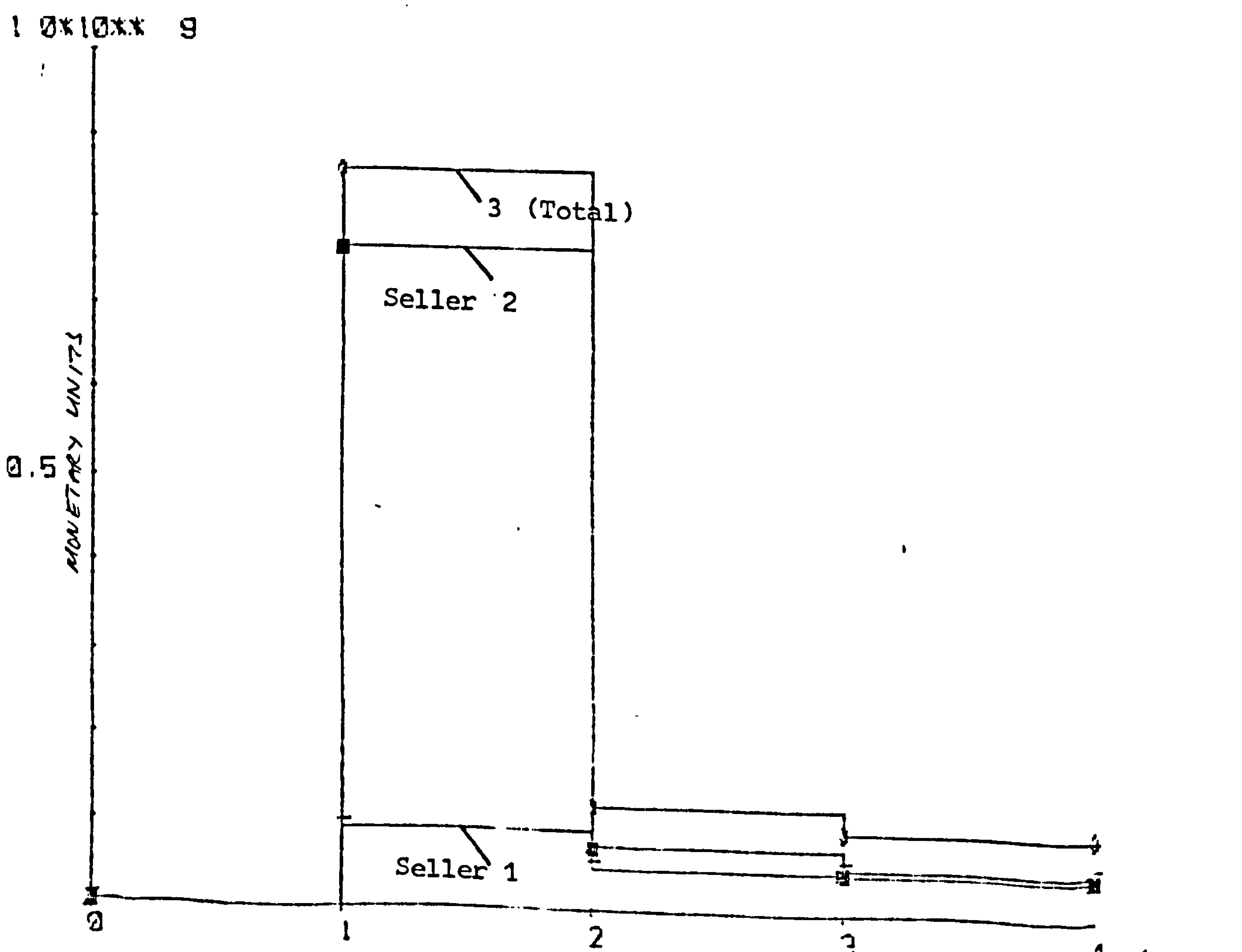


FIGURE 5.34 Revenue-Time Curves

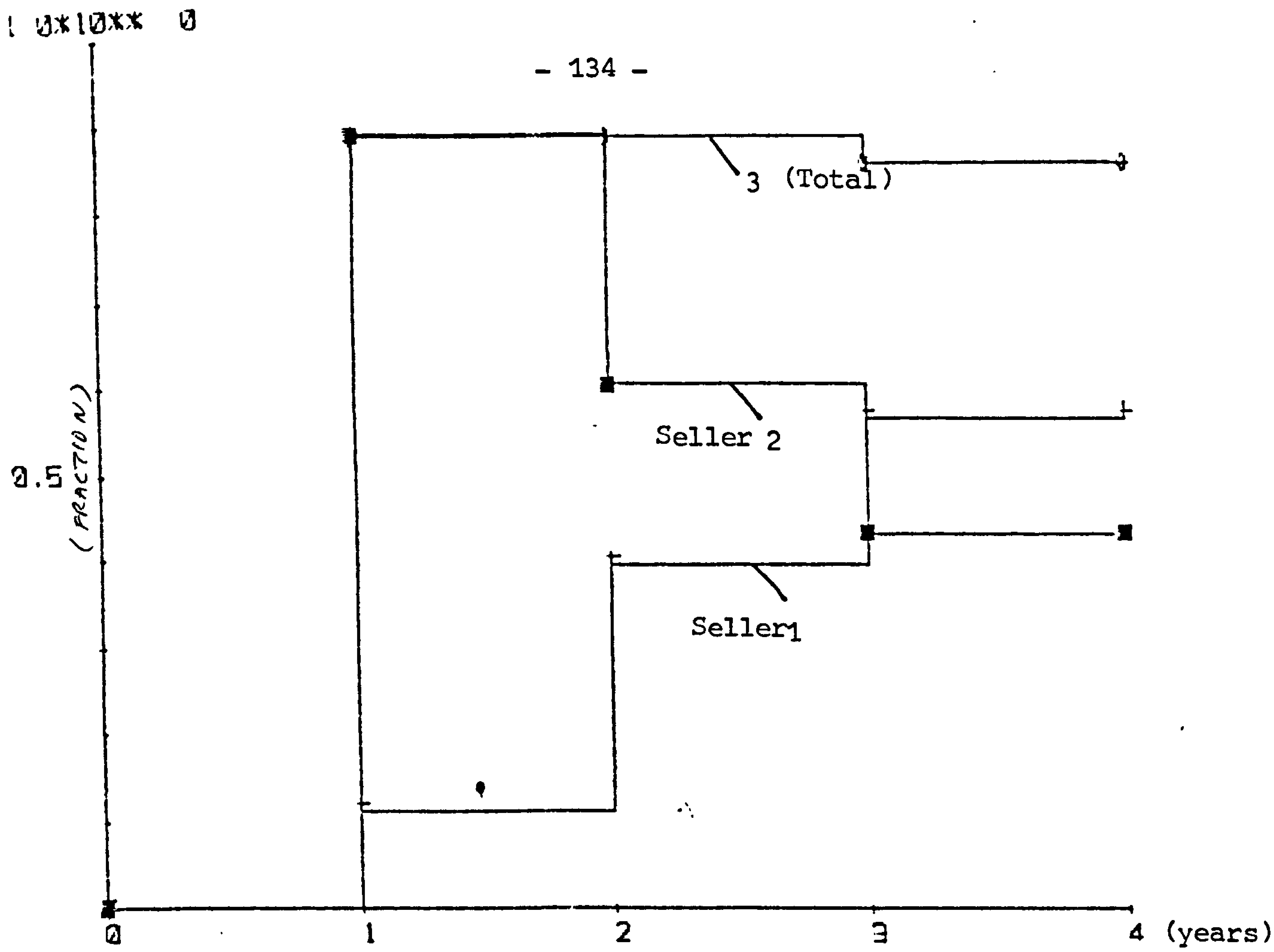


FIGURE 5.35 Market Penetration/Share-Time Curves

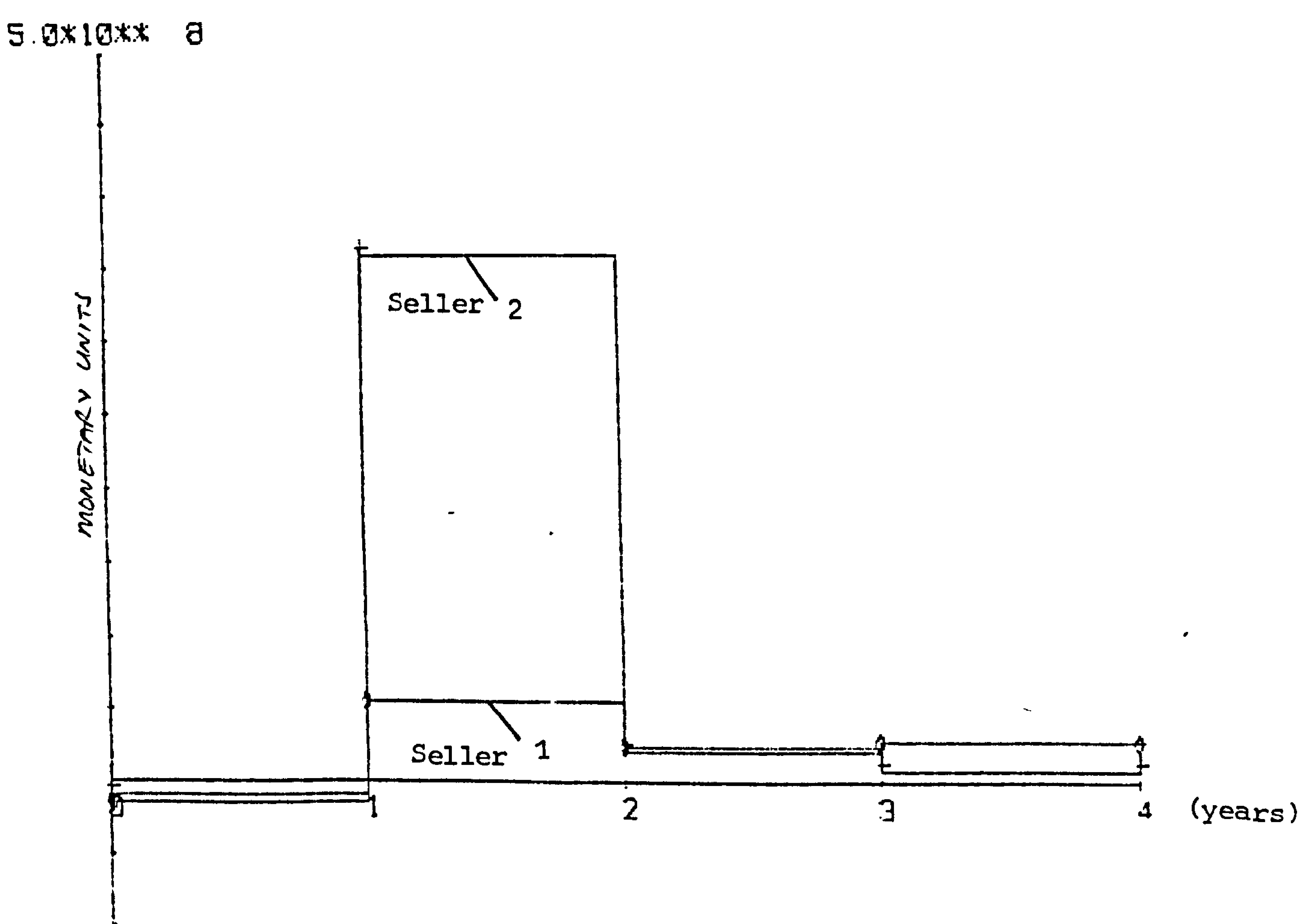


FIGURE 5.36 Profit-Time Curves

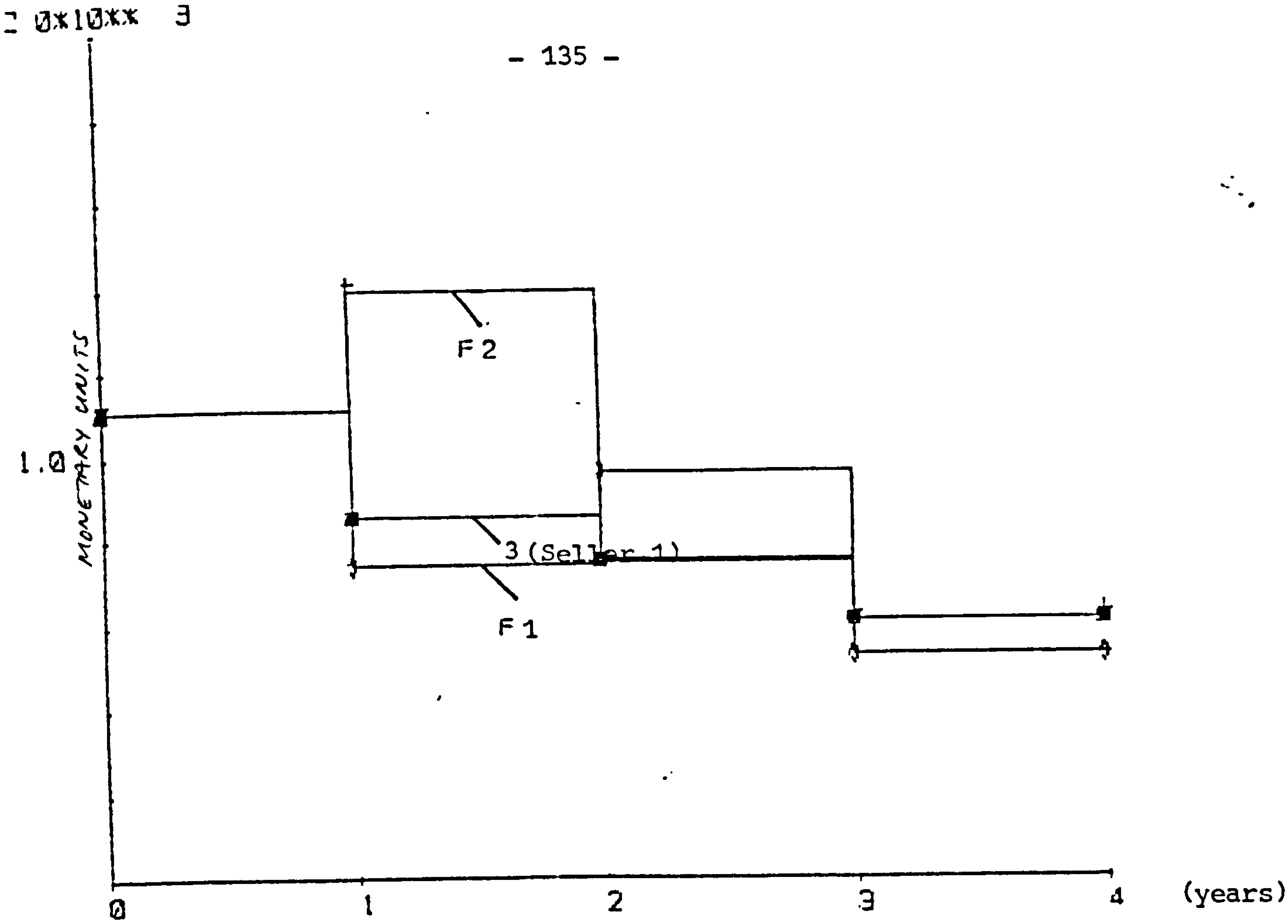


FIGURE 5.37 Price-Time Curves

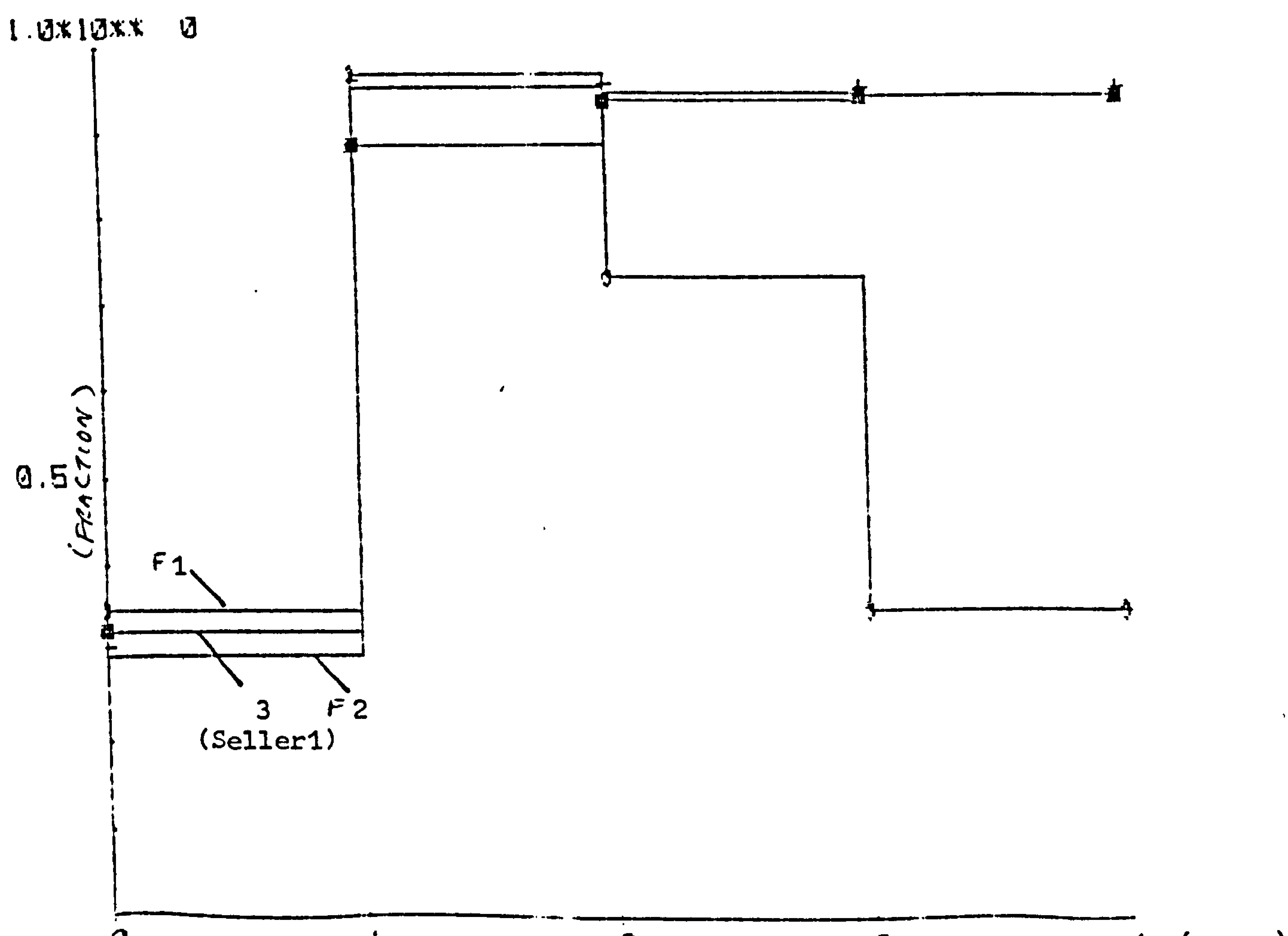


FIGURE 5.38 Quality Rating-Time Curves

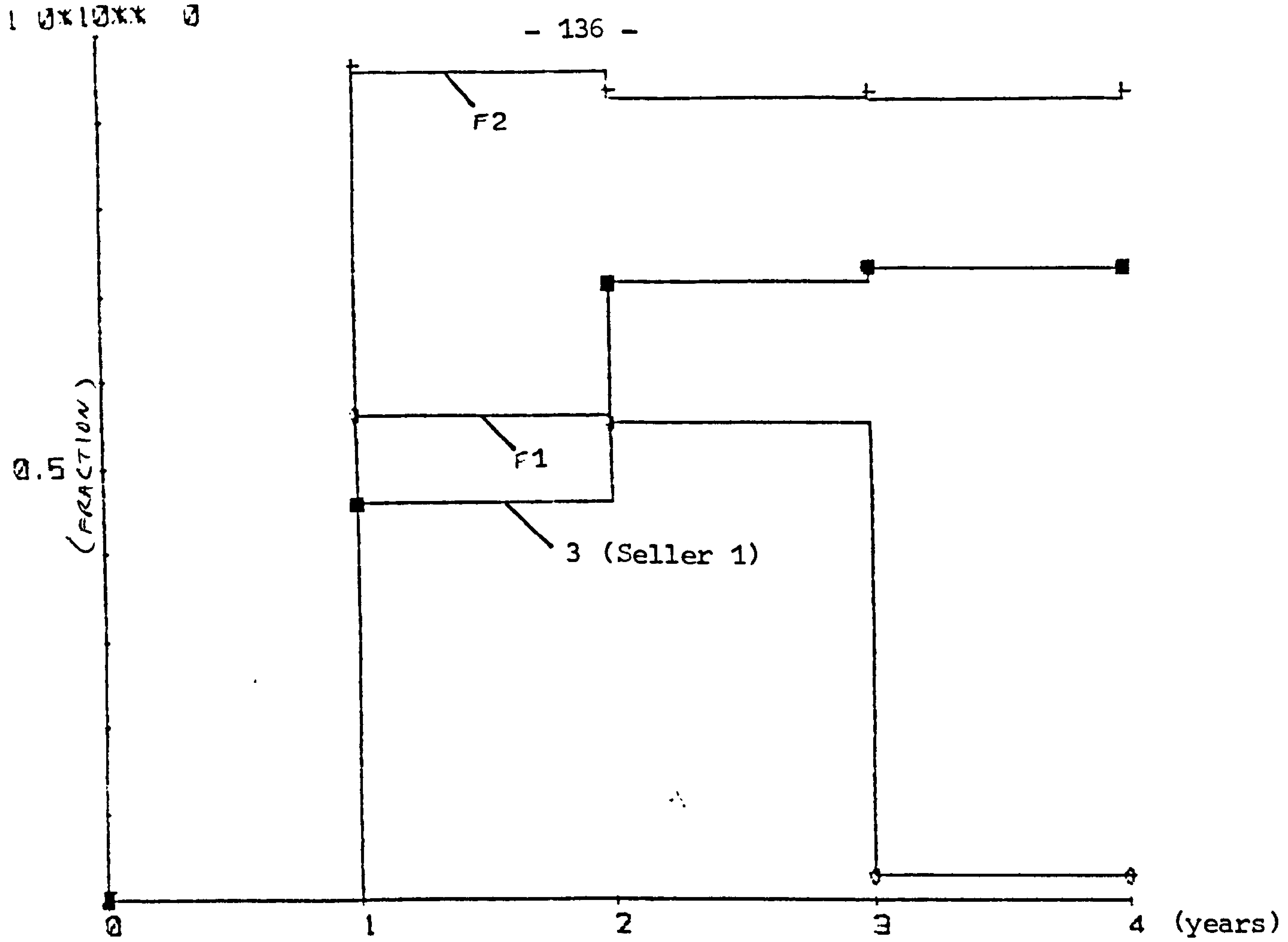


FIGURE 5.39 Distribution Effort Factor-Time Curves

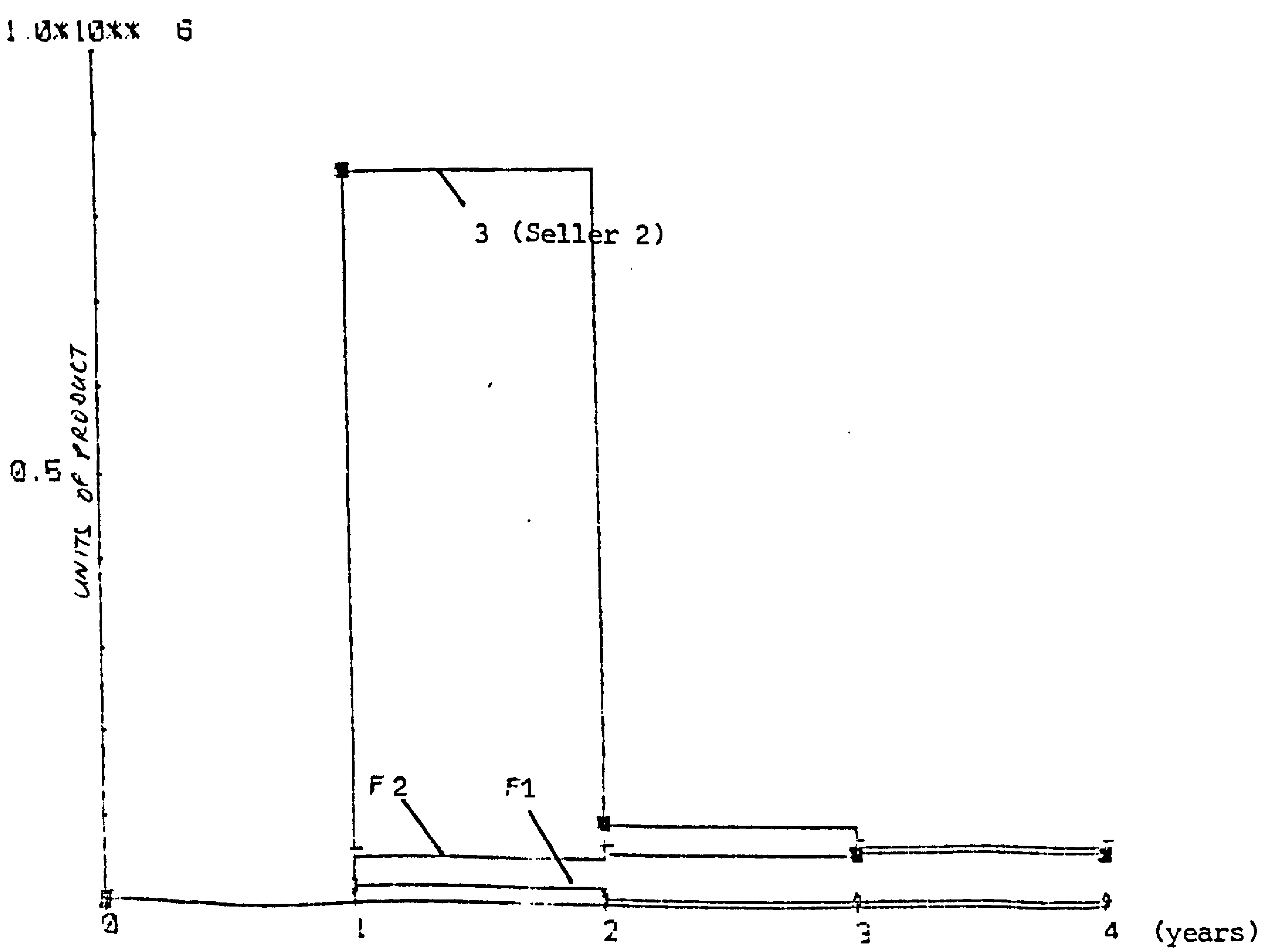


FIGURE 5.40 Production Rate (units)-Time Curves

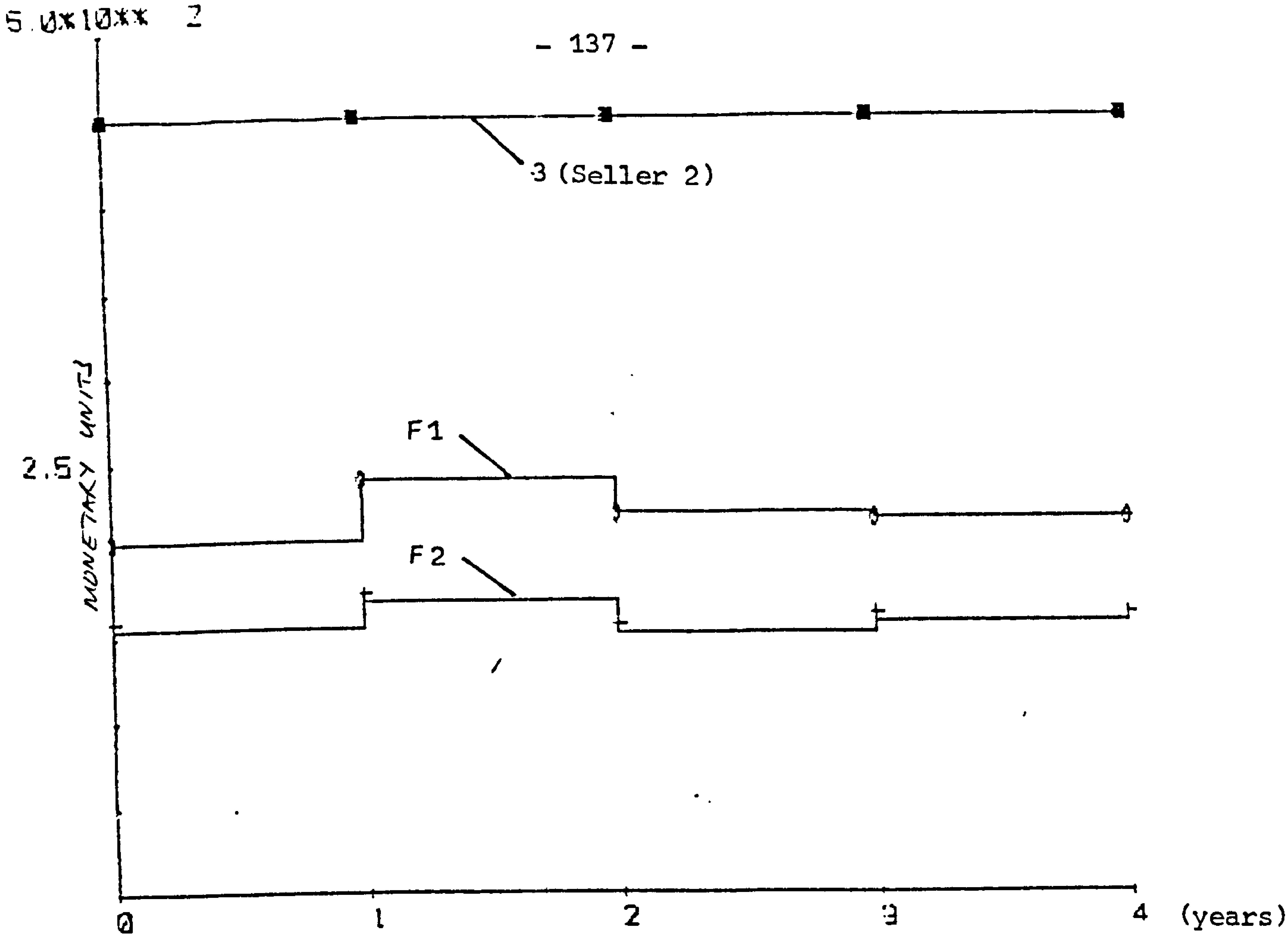


FIGURE 5.41 Unit Production Cost-Time Curves

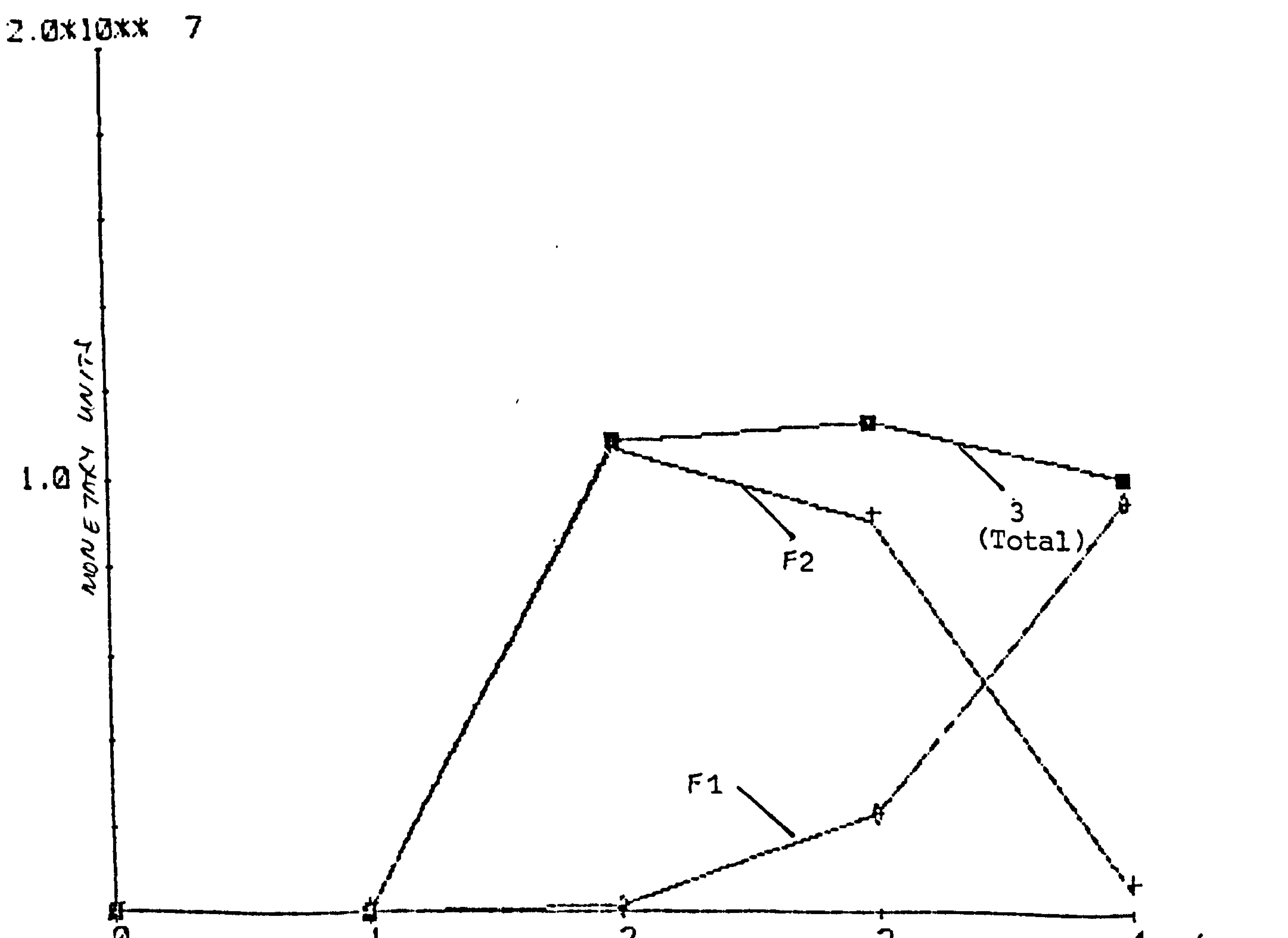


FIGURE 5.42 Stock Level-Time Curves (valued at current selling prices)

we note that item F2 accounts for the majority of Seller 1's sales and revenue. This is as expected since its average unit cost is 150 over the 4 long-term periods, which is less than that of F1 (225) and much less than that of Seller 2 (450). We also note that by the 4th period, production of F2 already exceeds that of Seller 2's product.

The medium-term timescale production system results are displayed in Figures 5.43 to 5.49. Figure 5.43 shows the Backorder-Time curve, Figure 5.44 the Demand-Time and Shipping Rate-Time curves, Figure 5.45 the Shipping Rate-Time and Production Rate (finished goods items)-Time curves, Figure 5.46 the finished goods (FG), work-in-process (WP) and raw material (RM) stock levels over time, and Figure 5.47 the WP Production Rate-Time, WP Subcontracting Rate-Time and RM Purchase Rate-Time curves. The inventory data shown in Figures 5.43 to 5.47 are all in monetary units, i.e., FG inventory is valued at current selling price, WP inventory is valued at subcontract price and RM inventory is valued at cost price. Figure 5.48 shows the total number of workers in each period and Figure 5.49 shows the
Time is measured in medium-term units (months)
Revenue-Time and Cost (shown as a negative quantity for clarity)-Time curves.

In Figure 5.43 we note the high levels of backorders over time. Two factors contribute to this.

1) Seller 1's actual long-term sales rates, resulting from the application of Seller 2's actual decisions, are higher than the estimated sales rates that would have obtained had Seller 1's predictions of his competitor's decisions been applied; the production schedules derived were based on the actual rather than the estimated sales rates. The structure of the simulation programme, PRDMRK.FTN, is such that Seller 1 derives its marketing decisions on the basis of predicted decisions for Seller 2 and vice versa. If the predicted decisions differ from the actual ones, the actual sales rates for Seller 1 (on which the production schedules are based) may not only be in excess of the predicted ones but may also exceed the capacity of the production systems (the predicted sales rates are always within the

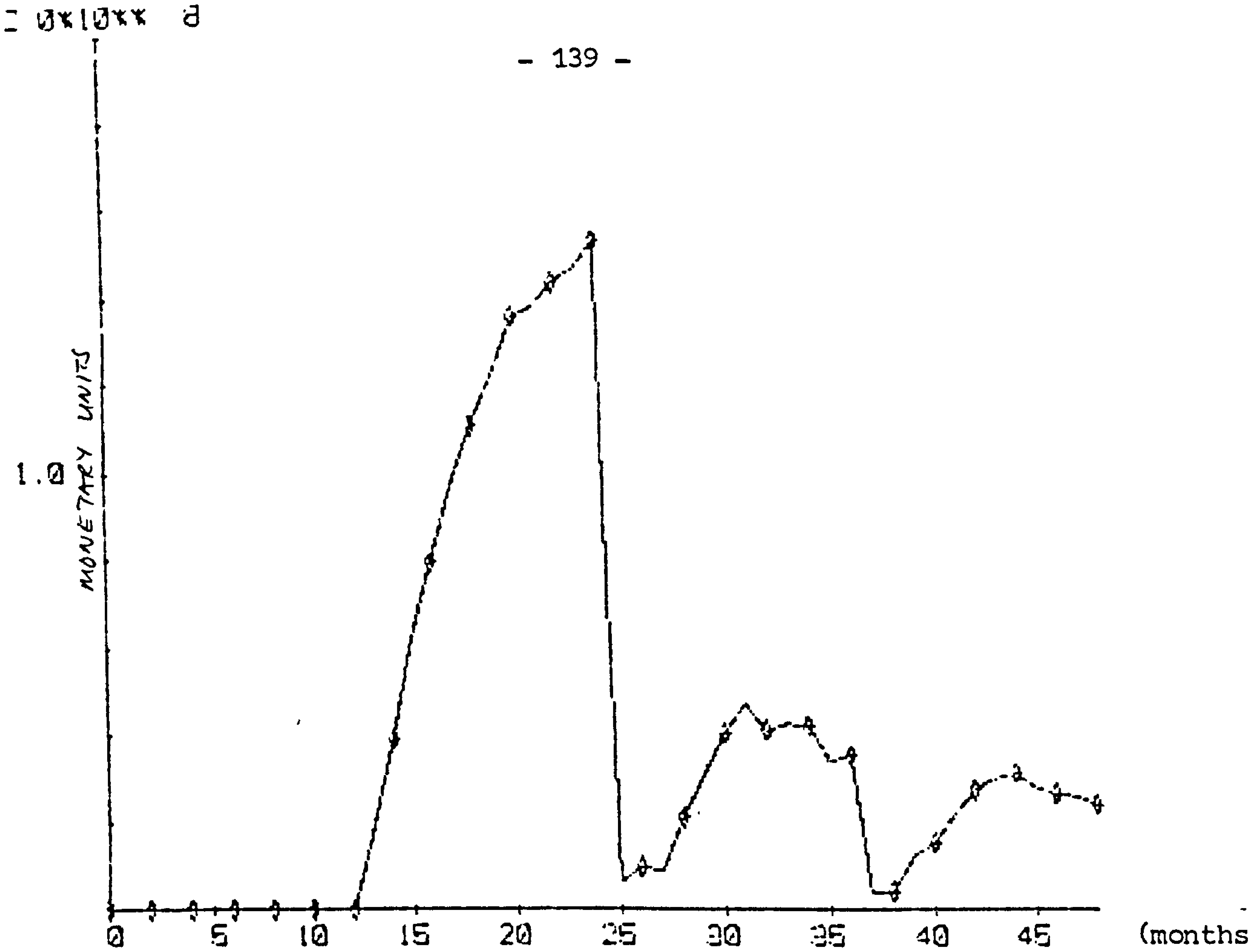


FIGURE 5.43 Backorder-Time Curve (valued at current selling prices)

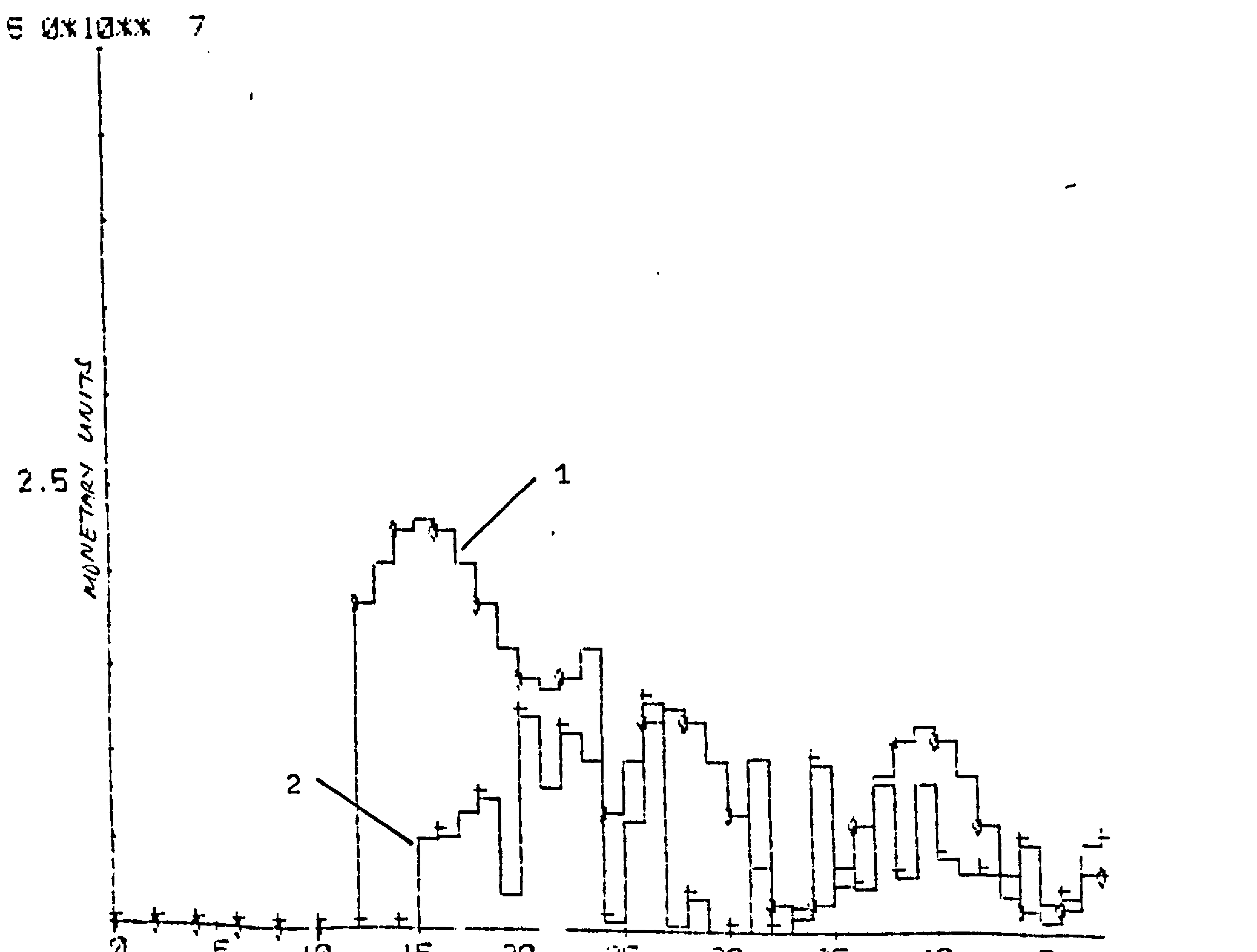


FIGURE 5.44 (1) Demand and (2) Shipping Rate-Time Curves (Valued at current selling prices)

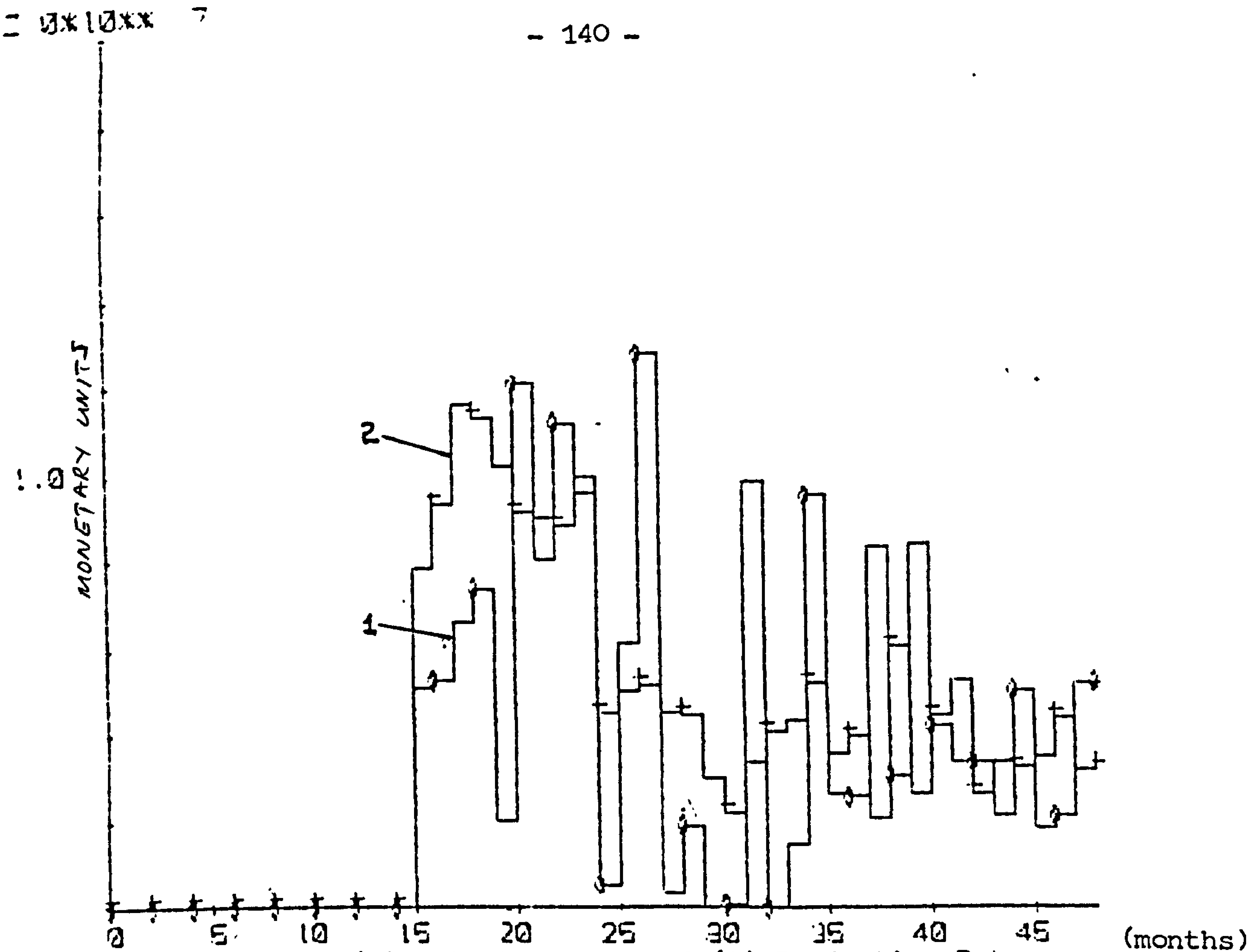


FIGURE 5.45 (1) Shipping Rate and (2) Production Rate (FG items)-Time Curves (valued at current selling prices)

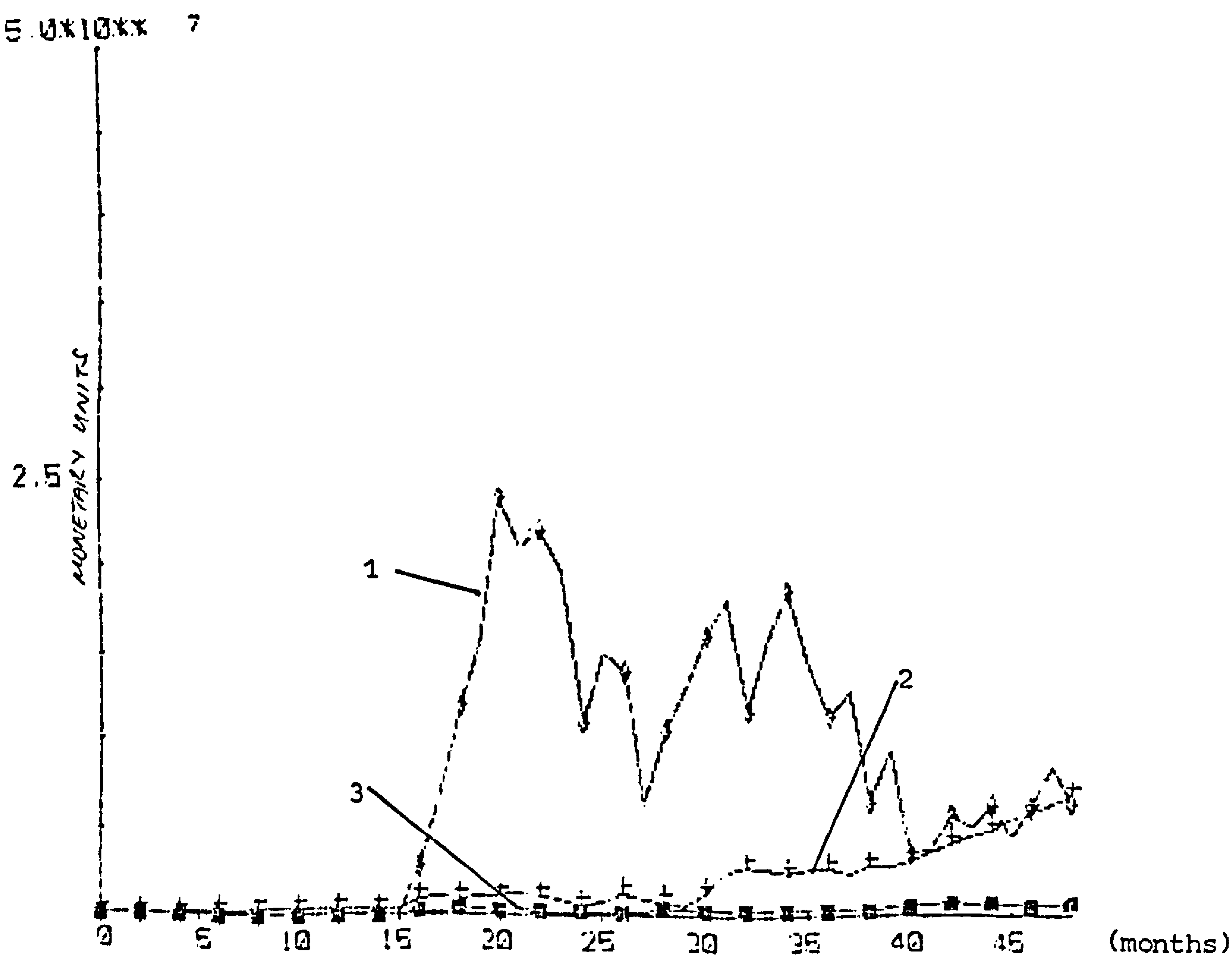


FIGURE 5.46 (1) Finished Goods (at current selling prices), (2) Work-in-Process (at sub-contract cost) and (3) Raw Material (at purchase cost) Stock Level-Time Curves

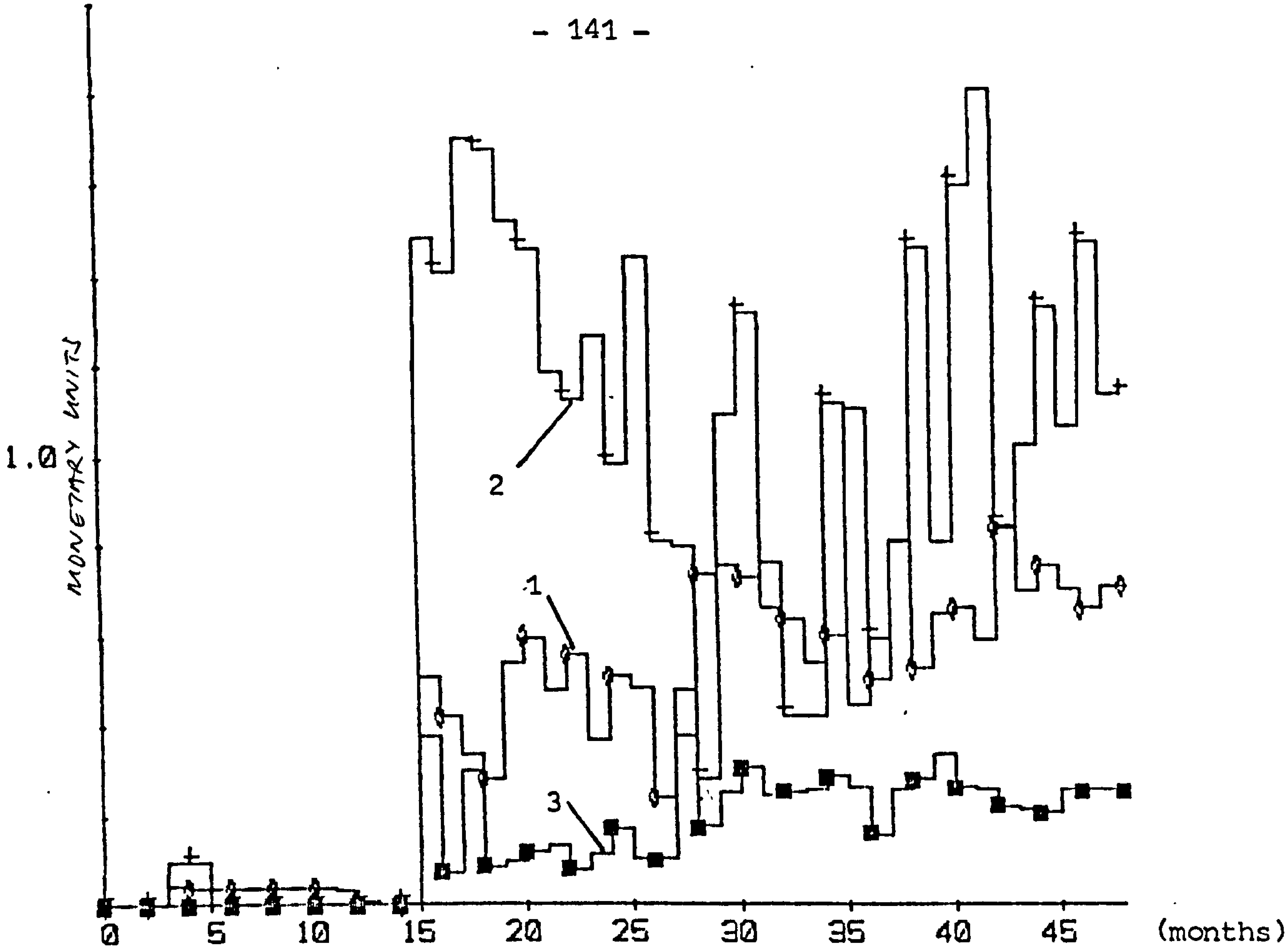


FIGURE 5.47 (1) WP Production Rate (at sub-contract cost)
(2) WP Sub-contract Rate (at sub-contract cost)
(3) RM Purchase Rate (at Purchase cost)
-Time Curves

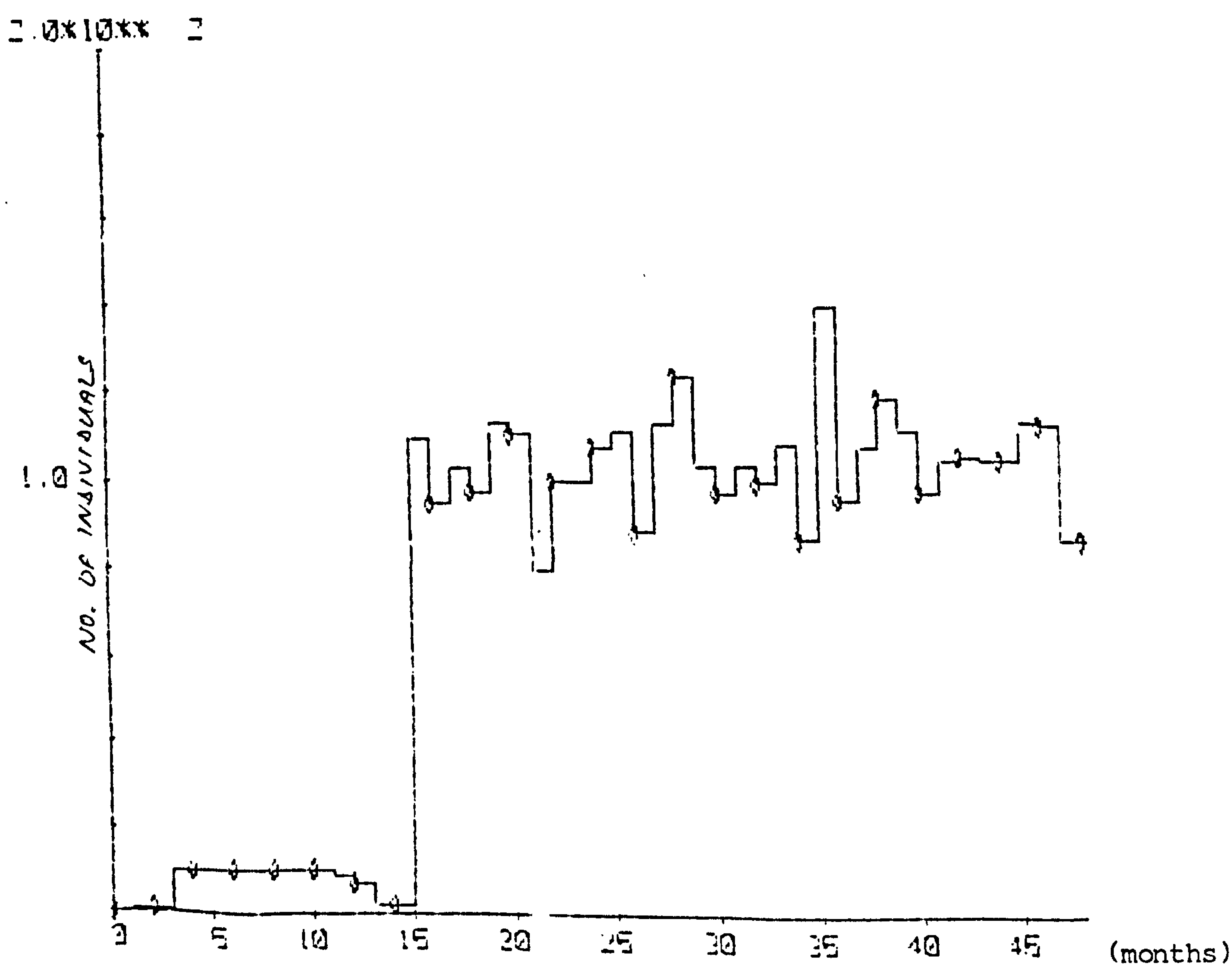


FIGURE 5.48 Total number of workers-Time Curve

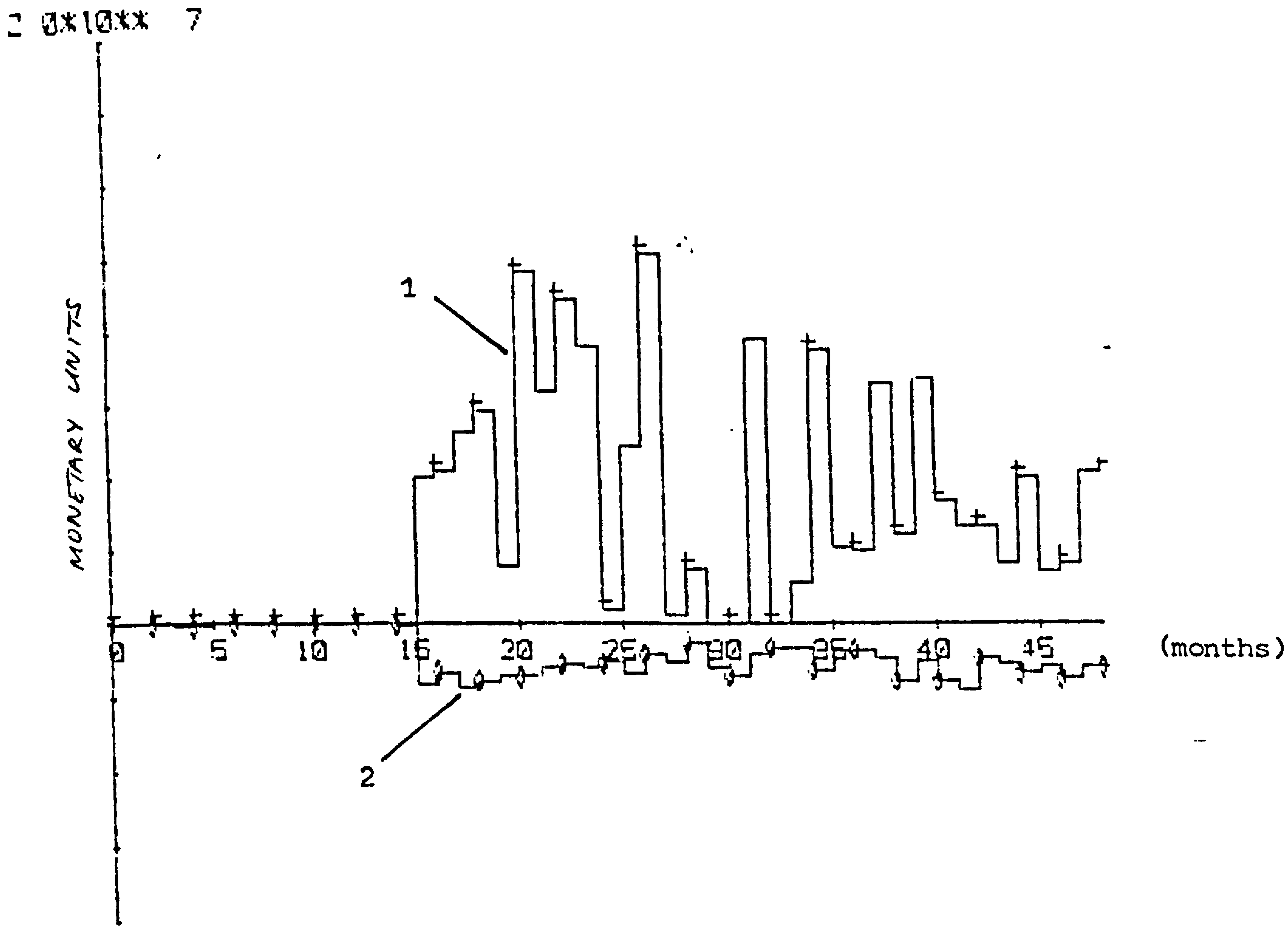


FIGURE 5.49 (1) Revenue and (2) Production System Cost-Time Curves

capacity of the production system).

2) The production decisions themselves are not optimal. Had the production decisions been optimal there would not be non-zero FG inventory stock levels (Figures 5.42 and 5.46) and FG shipping rate would not be less than FG production rate in any period (Figure 5.45) whilst non-zero backorder levels existed (Figure 5.43).

Notwithstanding the non-optimality of the production decisions, the decisions do represent feasible production decision making system responses to marketing system activities and Figures 5.43 to 5.49 legitimately describe production system behaviour over the simulation horizon. As expected, FG shipping rates are less than external demand over most of the simulation horizon (Figure 5.44) and the production of most of the WP inventories was subcontracted rather than produced within the production system (Figure 5.47). On average, from the 16th period onwards, there are 100 workers in the workforce, with actual numbers varying by not more than 20% from this value (Figure 5.48). In the main, from the 16th period onwards, the production system records a net profit (Figure 5.49). Figures 5.44 to 5.49 show that in spite of the strong seasonal variation in the demand for item F2, which, as shown in Figure 5.40, accounts for the bulk of Seller 1's production output, the production decisions and outputs appear not to be affected by seasonality of demand. Obviously this is due to the fact that demand is greater than capacity in each long-term period, thus seasonality of demand makes little difference to the operation of the production system.

5.4 SUMMARY

The results of computer simulations of marketing systems described by the models developed in Chapter 4 and utilizing assumed values of marketing parameters/data were described and commented upon.

In Section 5.1, simulation results pertaining to a constant decision marketing system were presented and discussed. Three cases were considered in this regard.

1) The single (monopoly) product simulation. Only two marketing decisions, advertizing and distribution effort factors influence marketing system behaviour. It was shown that in markets where the product of the advertizing and distribution effort factors were high (possibly aided by a large word-of-mouth factor) age-dependent product repurchasability has a destabilizing effect on market dynamics.

2) The 2-product (homogeneous) simulation. Both products have identical prices and quality ratings and thus market behaviour is influenced by advertizing and distribution effort for each product only. Here the effects of competition were to expand the market relative to the corresponding monopoly product situation, and, because of the increased overall market system advertizing and distribution effort factors product, accentuate the marketing system oscillations.

3) The 2-product (non-homogeneous) simulation. The effects of the full complement of constant market decisions (advertizing and distribution effort factors, price and quality ratings) on market behaviour were analyzed. It was found that

a) when both products have the same value-for-money rating, the market revenues and the steady state market penetration and shares are identical to that obtaining in the analogous homogeneous case;

b) when a given product possesses a higher value-for-money rating than its competitor, the steady-state market penetration, the given product's market share and sales rates and the overall sales rates increase relative to the case where both products sell at the same price (that of the given product) but enjoy the same value-for-money rating (that of the competitor);

c) it may be more advantageous to combat competition with combined increase in price and quality of own product so that own product value-for-money rating is improved rather than to improve value-for-money rating by reducing product price.

Section 5.2 considers the simulation results derived from a 2-product, variable-decision, non-homogeneous marketing system. To simulate the variable decisions for each product, the decisions were obtained from the maximization of a given objective function (revenue less costs) over a 4-period moving horizon. Each product decision-making system was capable of predicting its competitor's decisions in each of the periods in the horizon and they operated under the assumption that there were no limits to the available marketing resources, i.e., there were no limits to the supply of either product or of financial resources to acquire the products and effect their sale. Three simulation runs were described; a monopoly product simulation, a 2-product equal parameter simulation and a 2-product unequal unit production cost simulation. In the monopoly product simulation the market system oscillations were clearly observable; in the 2-product simulations, market prices were lower and expenditures to affect other market decisions were much greater than those for the monopoly simulation resulting in an expanded market, more revenue but much less profits for the 2-product simulations. A basic feature of 2-product market behaviour is that of successive price-cutting. This resulted in both products being withdrawn from the market for one period in the equal parameter simulation and the product with the higher unit product cost was withdrawn for six periods in the unequal unit cost simulation.

In Section 5.3, the results of the simulation of a multi-product, variable decision marketing system with finite production capacity was described and analyzed. The marketing system model operated on a long-term timescale while the production system model operated on a medium-term timescale. The coordination between the medium-term production outputs (units shipped to customers per medium-term period) and long-term marketing outputs (sales rate in each long-term period) lay in the updating of the distribution effort factor at the end of each long-term period to account for any discrepancy between these two quantities over each long-term period. The simulation showed that the distribution effort factor for the given product with finite production capacity was significantly lower than its competitor's even though the given product's unit production costs were less than half those of its competitors thus reflecting its limited production capacity. The given product's prices were also higher than we might have expected after consideration of the 2-product unequal unit cost simulation of Section 5.3. Again this reflects the influence of limited production capacity. It was found that the production system decisions did not reflect the strong seasonal variations in the demand for one of the produced items, due to the fact that the plant was operated at near full-capacity. The high levels of backorders led to the conclusion that the predicted long-term sales rate for given product was less than the actual long-term sales rate experienced by the production system and that the production decisions, while feasible, were not optimal and thus did not accurately reflect the goals of the decision making system as embodied in the objective function.

CHAPTER VI

CONCLUSION

In Chapter 1 three basic problems involved in the study of a production-marketing system (PMS) were described and summarized as

1) Model complexity,

2) Decision-making complexity, and

3) Behavioural complexity.

The aims of the work described in the thesis were stated as

1) the review and assessment of the efficacy of control theoretic and other techniques to the solutions of these problems and

2) contribution to the understanding of the dynamic behaviour of a PMS (an aspect of problem (3) above) by the development of an interactive computer simulation package.

In the rest of the chapter, the work reported in this thesis is reviewed and assessed to see how far the stated aims were achieved and to identify possible areas in which the work could be profitably extended.

The marketing and production system models developed in the thesis were described by sets of deterministic difference equations. This was a most convenient representation because, aside from its ready implementation on a digital computer, it reflected key attributes of an actual PMS, for instance:

1) an actual PMS is operated as a discrete time-based dynamic system;

2) decisions are made with the knowledge that they may be changed at a future date - thus it is reasonable to presume that full information concerning the present states of the PMS and its environment is available at the outset of the decision-making process with the knowledge that errors arising from such a presumption could be corrected at a later decision-making session.

Consequent to 2) above, an actual PMS is operated with a significant amount of slack to provide a buffer against these errors. A decision of crucial importance in the operation of a PMS is that of how much slack is to be allowed for, recognizing that slack represents the tying-down of resources that may be profitably utilized elsewhere. If the errors are such that there is a priori knowledge of its statistical properties and their variation over time, then a stochastic discrete time representation of the PMS would help significantly in arriving at this decision.

Model simplification techniques (2.2.2 above) were utilized in deriving the models of Chapters 3 and 4. For instance, non-linear aggregation methods were used in forming the 'aggregate product' and its marketing parameters and decisions from its component set of products and their parameters and decisions (4.3.1 above), while linear aggregation methods were used in forming the aggregate 'workcentre' from its component workstations. Singular perturbation ideas were used in the correction of the distribution effort factor to allow for the interaction between the medium-term variable, FG shipping rate, and the long-term variable, sales rate (5.3 above) and both aggregation and singular perturbation ideas were used in the estimation of unit production cost for each product in each long-term period (5.3). A significant drawback of available model simplification techniques is that they are not easily applied to non-linear models. To do so in their present format requires the linearization of the model which may be unacceptable in many instances.

From the decision-making viewpoint, the PMS simulated in 5.3 is a disaggregated decision-making system (2.3.3), in particular it is a multi-layer (hierarchical) decision making system (DMS) with a single criterion DMS on each level. As analyzed in 2.3.3, such a decision-making structure induces and is induced by a hierarchy of decision models (multi-strata hierarchy); in our case, the long-term marketing model and the medium-term production models. We note from 2.2.3 that this multi-strata hierarchy is based on a temporal description of the PMS and is not the only type of description possible. Indeed an actual PMS can also be described in terms of the goals and functions of the agents charged with making and carrying out decisions (goal-functional description) or even in terms of the attributes of the markets in which the PMS is active (attribute-based description). A useful extension of the modelling work is to consider these other descriptions and the multi-layer hierarchies induced by them, include the possibility of multiple criteria decision making on each level, and find means of coordinating decision strategies based on each of these descriptions.

Another dimension to the decision-making viewpoint is provided by the variable-decision simulations of 5.2 and 5.3. In these simulations, each seller had to possess a prediction of his competitors decisions over the decision horizon in order to be able to arrive at his own. The PMS simulation of 5.3 demonstrates the scope for error inherent in such an approach especially if the decisions are not re-assessed until the end of each long-term period. In an actual PMS, in the course of each long-term period, the prediction of the competitor's decisions and hence calculation of own decisions may be upgraded several times depending on the magnitude of perceived errors. This obviously brings to attention the need for a medium-term marketing model as well as the measurement dynamics of the long-term marketing DMS. Further work done here would greatly enhance the adaptation potential of the multi-layered PMS structure. On the other hand, instead of this sequential updating of predictions and decisions ('second guessing') with its potential for destabilization, advantage may be taken of the fact that marketing activities of a given seller affect and are affected by his marketing environment - an environment that obviously includes his competitor - and his marketing decisions formulated as the solution to a 2-person non-zero-sum, differential game.

A third dimension is provided by the production system simulation of 5.3. There it was stated that the production decisions were feasible but not optimal. The PMS simulation run used up 4.5 hours of CPU time on the departmental Perkin-Elmer 3220 computer compared to the 29 minutes (average) required for each of the simulation runs described in 5.2. On average the non-linear optimization routine, SDRMIN, required 9820 evaluations of the objective function, each evaluation calling for the simulation of the production system over the 18 medium-term period decision horizon, to arrive at the decisions at the beginning of each long-term period out of which the first 12-period decisions were used in the actual simulation recorded. Obviously there is considerable scope for reduction of decision processing requirements here. Some approaches to improvement are described below.

1) Use of more powerful optimization routines: In an attempt to improve SDRMIN efficiency penalty functions were introduced to signal the infeasibility of a solution and speed up convergence to a feasible one but the sheer number of decision variables, 478 in all over the decision horizon, made SDRMIN convergence very slow indeed. The use of a linear programming formulation proved unviable because the storage space requirements were beyond the capacity of the computer used. Obviously the dimensionality of the decision problem would be a difficulty for any optimization routine thus approaches that included the reduction of dimensionality would have to be given serious consideration.

2) Decomposition of the optimization problem: A hierarchical (multi-layer or multi-echelon) decomposition of the optimization problem [23,30, 71,75,82-88] may be employed to permit the reduction of problem dimensionality and thereby reduce the processing requirements (since computation time usually rises at a greater than linear rate with problem size). However, as has been observed by Sandell et al [23], in order to arrive at the optimal solution, the need to iterate and coordinate implies that any computational savings associated with hierarchical optimization techniques are problem-dependent. Thus there is no a priori guarantee that the desired improvement in processing requirements would result.

3) Decomposition of the Production DMS: If some degree of suboptimal performance is acceptable, then a multi-layer or multi-echelon decomposition of the production DMS itself may be the answer. We note that by so doing, we induce a multi-strata hierarchy of medium-term production system models if a multi-layer structure is chosen or a SDMP-type MCDM situation (2.3.2) if a multi-echelon hierarchy is used. The production DMS and its processing requirements are certainly simplified, however this is achieved at the cost of overall suboptimal performance.

4) Use of Heuristic decision algorithms: If suboptimal performance is acceptable then another approach is the utilization of heuristic decision algorithms. This approach bypasses the need to decompose the Production DMS. Even if the Production DMS is decomposed as in 3) above, heuristic decision algorithms can further reduce the processing requirements of component decision-making units. We note however that in both 3) and 4) there is no way of knowing before hand the degree of suboptimality that may result.

The marketing system models developed in Chapter 4 have been shown to be superior in several respects to the available analytical models. In the Chapter 4 models we have been able to incorporate in a simple 2-product framework the effects of multiple products/sellers, and for each product/seller, the four marketing variables, advertizing and distribution effort, price and quality rating, in an explicit manner. The marketing system dynamics have been shown to be influenced by these decisions as well as the age-dependent product repurchasability factor, i.e., the dynamics of repeat purchases. In the brief survey of available analytical models in 2.4, no model surveyed had been able to consider these marketing factors altogether; thus none of the models surveyed could simultaneously account for the effects of introduction of new products, determination of product (item, line or class) life-cycle, determination of market shares, effects of the four marketing variables for each product, repeat purchase dynamics and consumers - their number, income and tastes, as could the models developed in Chapter 4 - especially that of 4.3.1.

Obviously, the superiority of the marketing model utilized is one of the strengths of the simulations reported in 5.2 and 5.3. In Forrester's industrial dynamics study [5], the marketing model utilized considered only the effect of advertizing on a pool of repeat purchasers, i.e., shortening the average time for repurchase, and the rate of inflow of repeat purchasers into this pool was the independently specified test input [5; Chapter 16]. The marketing model used in the MATE simulation [98,99] regarded sales rate as a multiplicative function involving a growth term, a seasonal term, a marketing effort term (price and advertizing expenditure) and an aggregate consumer income term. The effects of the firm's two other marketing variables and his competitor's marketing activities were not explicitly considered. On the other hand, the marketing model used in the TOMES simulation [95-97] was more appropriate to a short-term or medium-term market analysis rather than to a long-term one as the level of disaggregation did not permit direct evaluation of the effects of the four long-term marketing variables or those of the competition on sales rate. It is therefore difficult to see how the 'price-war' phenomenon and its effect, and that of superior unit production costs, on product marketability (i.e., how long and how profitably can the product be marketed in the face of competition) could be demonstrated in these other simulations as was done in 5.2 and 5.3 above.

Another strength of the PMS simulation of 5.3 is its use of the production model of Chapter 3 which is that of a multiple final product, multistage, non-linear assembly tree process where backlogging of demand is permissible, there are non-zero manufacturing and purchasing lead times, processing routes for final products share common machinery and facilities, production capacity is limited, there are more than one labour shifts, and subcontracting of production of intermediate products (WP inventory) is permissible with non-zero purchasing lead-time. This is one of the most general production system models in use and is superior to that used in

Forrester's study [5] and most aggregate production planning studies [114]. Its generality implies that its decision-processing requirements would be enormous but, as has been discussed in an earlier paragraph, these processing requirements problems are by no means insurmountable.

A further advantage of the simulations of 5.2 and 5.3 is the open-loop, optimization-type, decision making process utilized. This differs radically from that used in the systems dynamics simulations of Forrester and others [5,6,95-97,107]. In the latter simulations, the system is forced to behave as a closed-loop feedback regulated system which is allowed to respond to a very few (usually one) external inputs. Thus the very important effects of variations in costs attached to the use of resources in the PMS on decision policies, and hence their scope for use in the adaptation of these policies to current or future conditions cannot be allowed for. Obviously, Forrester's approach simplifies the simulation task considerably and may be quite suitable for some applications (a 'small-signal' simulation study, centred around well-defined operating conditions, comes to mind) but for the type of application considered in the thesis, involving the tracing of the product life-cycle, the use of Forrester's methodology is suspect. An open-loop simulation methodology that derives decisions on the basis of an optimization routine over a limited decision horizon, is a more accurate representation of the actual decision-making process in an industrial enterprise than a collection of ad hoc feedback rules which have no scope for amendment as operating conditions vary.

Lastly, the simulations of 5.2 and 5.3 have shown that the same simulation programme could be used for three different purposes with only slight modifications.

1) actual control of a PMS,
2) simulating future behaviour of the PMS under various assumptions of future environmental conditions, and
3) simulating alternative behavioural modes consequent to changes in decision-making objectives.

In conclusion, it has been shown that many of the techniques of modern control and decision theories are applicable to the resolution of complexities in a PMS. Further work has to be done to develop non-linear versions of what are predominantly linear systems - based techniques before they can receive wide-spread adoption. However, the basic ideas behind these techniques are already in use albeit in a mostly intuitive manner. The PMS models developed in Chapters 3 and 4 have been shown to be an improvement on the PMS models extant though they could perhaps benefit from the introduction of stochastic elements. The temporal hierarchy inherent in the PMS models is not the only one possible; a goal-functional or attribute-based description could provide further understanding of the behaviour of an actual PMS by providing, in coordination with the temporal description, more insightful models of the PMS. The open-loop decision-making methodology used in 5.2 and 5.3 is demonstrably superior to the closed-loop methodology fundamental to 'Industrial dynamics' - type simulations but at the cost of possible increase in simulation complexity. By utilizing hierarchical or heuristic decision-making techniques the problems attendant to increased complexity may be significantly obviated. The use of game-theoretic based decision-strategies for the marketing system may also prove helpful. From the results of 5.2 and 5.3, it is evident that the simulation programme PRDMRK.FTN provides the nucleus of a PMS simulation and control package. But to fully realise its potential other features need to be included, for instance

1) a long-term production model, to take cognizance of the fact that production capacity is variable in the long-term, and to account for the 'learning effect' phenomenon (increase in labour proficiency as hands-on experience is gained);

2) a medium-term marketing model to generate the medium-term demand inputs to the production system, and account for the dynamics of distribution logistics;

3) short-term production model (for detailed manpower, machine and operations scheduling within each workcentre) and marketing model (order taking and filling);

4) an interface package that would relate model parameters to real-world data (and vice versa) contained in an appropriately designed data-base, and finally

5) the provision of routines in each DMS in the package that would simulate and evaluate the effects of measurement costs and delays on PMS behaviour. This last point is non-trivial as it lies at the heart of the difference between the PMS and the type of systems usually encountered in control theory.

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APPENDIX 1 : FLOW DESCRIPTION OF SIMULATION PROGRAMME M22.FTN

1. INITIALIZE MARKET SYSTEM PARAMETERS:

SIMULATION PERIOD	NINT = 30
MARKET POPULATION	PNO = 10^6
WORD OF MOUTH FACTOR	CLAMB = 0.4×10^{-6}

ADVERTIZING EFFORT FACTOR PRODUCT 1 (AEF1),
2 VALUES (NPA = 2) PA(1) = 0.2, PA(2) = 0.8
DISTRIBUTION EFFORT FACTOR PRODUCT 1 (DEF1),
2 VALUES (NPB = 2), PB(1) = 0.3, PB(2) = 0.6

AEF2	PC = 0.5
DEF2	PD = 0.3

2. READ SIMULATION TYPE:
SINGLE PRODUCT, CONSTANT DECISION SIMULATION
MS = 1, NSRPC = 6
2-PRODUCT, HOMOGENEOUS, CONSTANT DECISION SIMULATION
MS = 2, NSRPC = 2

2.1 IF NO SIMULATION TYPE CHOSEN, EXIT PROGRAMME

3. INITIALIZE AGE-DEPENDENT REPURCHASABILITY FACTORS:
G(1) = 0.054, G(2) = 0.072, G(3) = 0.102, G(4) = 0.156
G(5) = 0.278, G(6) = 0.532, G(7) = 0.776, G(8) = 0.878
G(9) = 0.922, G(10) = 0.945

3.1 IF SINGLE PRODUCT SIMULATION (MS=1), AND EXTREME S-TYPE AGE DISTRIBUTION OF REPURCHASABILITY FACTORS DESIRED, THEN READ STARTING NON-ZERO COMPONENT, MEX
I.E. G(1),...,G(MEX-1) = 0 ; G(MEX),...,G(10) = 1.

4. FOR EACH VALUE OF AEF1 (KA = 1, NPA):
4.1 FOR EACH VALUE OF DEF1 (KK = 1, NPB):
4.1.1 FOR EACH SIMULATION RUN (I = 1, NSRPC):
4.1.1.1 OBTAIN SIMULATION DEPENDENT PARAMETERS
WORD OF MOUTH DYNAMICS, CLAM = CLAMB; ELSE CLAM = 0.
REPEAT PURCHASE DYNAMICS,
ZERO REPURCHASABILITY, IJG = 0
AGE-DEPENDENT REPURCHASABILITY, IJG = 1
AGE-INDEPENDENT REPURCHASABILITY, IJG = 2
(SUBROUTINE EQGAM CALCULATES THE AGE-INDEPENDENT FACTOR)
4.1.1.2 INITIALIZE SYSTEM STATES
4.1.1.3 RUN SIMULATION (L = 1, NINT + 1)
4.1.1.3.1 OBTAIN OUTPUTS
4.1.1.3.2 OBTAIN NEXT STATES
4.1.1.3.3 STORE DATA
4.1.2 DISPLAY DATA (SUBROUTINE MGRAPH SCALES AND OUTPUTS SEVERAL CURVES IN ONE GRAPHICAL DISPLAY)

5. IF SINGLE PRODUCT SIMULATION, DISPLAY MARKET PENETRATION DATA

6. GO TO 2

APPENDIX 2 : FLOW DESCRIPTION OF SIMULATION PROGRAMME MARK2.FTN

```
1.    INITIALIZE MARKET SYSTEM PARAMETERS:
           SIMULATION PERIOD,                          NINT = 30
           MARKET POPULATION,                           PNO = 10^6
           REPURCHASABILITY FACTORS, G(1) = 0.054, G(2) = 0.072, G(3) = 0.102
              G(4) = 0.156, G(5) - 0.278, G(6) = 0.532, G(7) = 0.776,
              G(8) = 0.878, G(9) = 0.922, G(10) = 0.945
      AEF1;  2 VALUES, NPA = 2, PA(1) = 0.2, PA(2) = 0.8
      DEF1;  2 VALUES, NPB = 2, PA(3) = 0.3, PA(4) = 0.6
      PRICE1;2 VALUES, NPC = 2, PA(5) = 825.0, PA(6) = 495.0
      QUALITY1;  2 VALUES, NPD = 2, PA(7) = 0.55, PA(8) = 0.33
      AEF2, AL2 = 0.50
      DEF2, C2Ø = 0.30
      PRICE2, P2 = 600.0
      QUALITY2, Q2 = 0.4
      MARKET SEGMENT INCOME CONSTANT I', SINC = 0.9 x 10^6
      'r' CONSTANT, R = 52.5
      WORD OF MOUTH DYNAMICS, CLAM = 0.
 1.1  OBTAIN INTRINSIC REPURCHASABILITY FACTORS GG(I) WHERE
           (1 - G(I))  =  (1 - RxQ1/P2)(1 - GG(I))


 2.   FOR EACH VALUE AEF1  (MA = 1, NPA):
 2.1  FOR EACH VALUE DEF1  (MB = 1, NPB):
 2.1.1  FOR EACH VALUE PRICE1  (MC = 1, NPC):
 2.1.1.1  FOR EACH VALUE QUALITY1 (MD = 1, NPD):
 2.1.1.1.1  INITIALIZE SYSTEM STATES
 2.1.1.1.2  RUN SIMULATION (L = 1, NINT+1)
 2.1.1.1.2.1  OBTAIN OUTPUTS
 2.1.1.1.2.2  OBTAIN NEXT STATES
 2.1.1.1.2.3  STORE DATA
 2.1.1.1.3  DISPLAY DATA


 3.   EXIT PROGRAMME
```

APPENDIX 3 : FLOW DESCRIPTION OF SIMULATION PROGRAMME MTEST2.FTN

1. INITIALIZE MARKET SYSTEM PARAMETERS
(DETAILS ON NEXT PAGE)

2. RUN SIMULATION (LT = 1, LINT):

2.1 OBTAIN CURRENT MARKETING DECISIONS FOR BOTH SELLERS/PRODUCTS (CALL MK2DEC)

2.2 OBTAIN CURRENT MARKETING OUTPUTS AND OBTAIN NEXT SYSTEM STATES (CALL MK2OUT)

3. STORE SIMULATION RESULTS IN DATA FILE

4. EXIT PROGRAMME

MARKET SYSTEM PARAMETERS

```
C
C-----DATA FOR /TIME/ BLOCK
C
      LINK=30         SIMULATION TIME (PERIODS)
      LINT=LINT+1     SIMULATION TIME (TIME INSTANTS)
C
C-----DATA FOR /MCONT/ BLOCK
C
C-----TOTAL MARKET POPULATION
      PNO=1.0E6
C-----CLAM CHOSEN SUCH THAT MAX CONTRIBUTION IS 0.15
C     I.E. WHEN POP(1)=POP(2)=POP(3)=PNO
      CLAM=0.15/PNO
C-----SINC CHOSEN SUCH THAT FOR Q=0.4 and P=600
C     DEL=SINC*(Q/P)/P=1
      SINC=900000
C-----R CHOSEN SUCH THAT FOR Q=0.4 and P=600
C     R*(Q/P)=0.035
      R=52.5
C-----ELEMENTS OF GO(10) CHOSEN SUCH AS TO APPROACH UNITY
C     IN AN S-CURVE MANNER (INTRINSIC REPURCHASABILITY FACTORS)
      GO(1)=0.0197
      GO(2)=0.0383
      GO(3)=0.0694
      GO(4)=0.1254
      GO(5)=0.2518
      GO(6)=0.5158
      GO(7)=0.7679
      GO(8)=0.8736
      GO(9)=0.9192
      GO(10)=0.9430
C-----IT IS ASSUMED THAT BOTH COMPETITORS HAVE IDENTICAL COST
C     STRUCTURES FOR ADVERTIZING AND DISTRIBUTION (PRESUMABLY
C     THEY USE SAME ADVERTIZING AND DISTRIBUTION AGENTS) BUT
C     DIFFERENT COST STRUCTURES FOR QUALITY
C     BOTH ADVERTIZING AND SELLING EFFORT FACTORS ARE ZERO
C     FOR ZERO EXPENDITURE,QUALITY RATING IS NON-ZERO AT ZERO
C     EXPENDITURE (INTRINSIC PRODUCT QUALITY)
C     PCON(1) IS SUCH THAT FOR EXPENDITURE OF 1E5,AEF=0.1
      PCON(1)=oE5
C     PCON(2) IS SUCH THAT FOR EXPENDITURE OF 1E4,DEF=0.05
      PCON(2)=1.9E5
C     PCON(3),PCON(4),PCON(5) CHOSEN SUCH THAT Q1=0.35 AND
C     Q2=0.30 AT ZERO EXPENDITURE,AND AT 1E6 EXPENDITURE
C     Q=0.8*(1.-QO)+QO
      PCON(3)=((1.-0.8)/0.8)*1E6
      PCON(4)=0.30*PCON(3)
      PCON(5)=0.30*PCON(3)
C
C---- DATA FOR /MDECL/ BLOCK
C
      AL1=0
      AL2=0.2
      P1=600.
      P2=600.
      Q1=0.40
      Q2=0.40
      C10=0
      C20=0.5
```

```
C
C-----DATA FOR /MSTAL/ BLOCK (INITIAL VALUE OF STATES)
C
      DO 25 J=1,10
      POP(J)=0.
      POPST(J)=0.
      DO 25 I=1,3
      B(I,J)=0.
25    BST(I,J)=0.
      RTPFT1=0.
      RTPFT2=0.
C
C-----DATA FOR /DECIDE/ BLOCK
C
C-----READ IN MAXTRY,NHOR,DFACT
      MAXTRY=1600      NUMBER OF OBJECTIVE FUNCTION EVALUATIONS
      NHOR=4           LENGTH OF DECISION HORIZON
      DFACT=0.15       DISCOUNTING FACTOR
C
      DO 45 I=1,2
      DO 45 J=1,5
      DO 45 K=1,5
45    DMST(I,J,K)=0.
      DO 50 NT=1,NHOR
      PRED(2,1,NT)=AL1
      PRED(2,2,NT)=P1
      PRED(2,3,NT)=Q1
      PRED(2,4,NT)=C10
      PRED(4,1,NT)=AL2
      PRED(4,2,NT)=P2


      PRED(4,3,NT)=Q2
      PRED(4,4,NT)=C20
50    CONTINUE
C
C-----DATA FOR /PDMKL/ BLOCK
C
      CP1=450.         UNIT PURCHASE/PRODUCTION COST PRODUCT 1
      CP2=450.         UNIT PURCHASE/PRODUCTION COST PRODUCT 2
      JT=1
```

FLOW DESCRIPTION OF SUBROUTINE MK2DEC

1. STORE CURRENT STATES

2. SAVE COMPETITOR'S LAST PERIOD'S DECISIONS

3. PREDICT COMPETITOR'S DECISIONS OVER CURRENT DECISION HORIZON (I = 1,2):

3.1 IF CURRENT TIME INDICATOR, LT, IS LESS THAN PRODUCT 1 LAUNCH TIME, JT: PREDICT DECISIONS OF SELLER 2'S COMPETITOR ONLY

3.2 IF JT = LT:

3.2.1 INITIALIZE PREDICTED DECISIONS OF SELLER 1'S COMPETITOR

3.2.2 REVISE PREDICTED DECISIONS OF SELLER 2'S COMPETITOR

3.2.3 INITIALIZE SELLER 1'S OWN CURRENT DECISIONS AND STORE

3.3 PREDICTION OF COMPETITOR'S DECISIONS OVER CURRENT DECISION HORIZON CALL PF2 AND CORRECT RESULTS TO ENSURE THEY ARE FEASIBLE. (SUBROUTINE PF2 EXECUTES AN EXPONENTIALLY SMOOTHED PREDICTION ALGORITHM WITH ALLOWANCE FOR TREND EFFECTS - SEE [113])

4. OBTAIN EACH SELLER'S OWN DECISIONS OVER DECISION HORIZON (J = 1,2):

4.1 IF JT $\leq$ LT OBTAIN SELLER 2'S DECISIONS ONLY

4.2 INITIALIZE DECISIONS

4.3 OBTAIN DECISIONS - EXECUTE SEARCH ROUTINE CODE AND CALL MK2OBJ FOR OBJECTIVE FUNCTION EVALUATION (SEARCH ROUTINE CODE IS A SLIGHTLY MODIFIED VERSION OF SUBROUTINE SDRMIN DESCRIBED IN DETAIL IN REF [113])

4.4 SAVE DECISIONS

5. EXTRACT CURRENT PERIOD'S DECISIONS AND RESTORE STATES

6. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE MK2OBJ

1. RESTORE CURRENT STATES

2. FOR EACH PERIOD IN DECISION HORIZON (NT = 1,NHOR):
2.1 EXTRACT CURRENT DECISIONS
2.2 OBTAIN OBJECTIVE FUNCTION VALUE, TRIALV
2.2.1 CALL MK2OUT (THIS SUBROUTINE EVALUATES THE OBJECTIVE FUNCTION IN EACH PERIOD AS A SYSTEM OUTPUT)
2.2.2 DISCOUNT FUTURE PROFITS USING DFACT

3. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE MK2OUT(LDEC)

1. OBTAIN CURRENT OUTPUTS

2. OBTAIN OPERATING PROFIT/LOSS

3. IF CALLING PROGRAMME IS MTEST2.FTN IE LDEC=0, STORE DATA

4. OBTAIN NEXT STATES

5. RETURN TO CALLING PROGRAMME

APPENDIX 4 : FLOW DESCRIPTION OF SIMULATION PROGRAMME PRDMRK.FTN

1. INITIALIZE MARKET AND PRODUCTION SYSTEM PARAMETERS
(DETAILS ON NEXT PAGE)

2. OBTAIN OTHER PRODUCTION PARAMETERS
2.1 OBTAIN INVENTORY LEVEL VECTOR
2.2 OBTAIN INITIAL UNIT COST OF WP INVENTORIES

3. RUN SIMULATION (LT = 1,LINT):
3.1 OBTAIN CURRENT MARKETING DECISIONS
(CALL LMPDEC)
3.2 OBTAIN CURRENT OUTPUTS
(CALL LMPOUT)

4. STORE SIMULATION RESULTS IN DATA FILE

5. EXIT PROGRAMME

PRODUCTION AND MARKETING SYSTEM PARAMETERS

```
      LINT=4       SIMULATION PERIOD (YEARS)
      MINT=12      NUMBER OF MEDIUM TERM PERIODS (MONTHS) IN EACH LONG TERM PERIOD
      JMT=1        TIME INSTANT IN WHICH SELLER 1 LAUNCHES HIS PRODUCTS
C
C-----DATA FOR /MCONT/ BLOCK
C
C-----TOTAL MARKET POPULATION
      PNO=1.0E6
C-----CLAM CHOSEN SUCH THAT MAX CONTRIBUTION IS 0.15
C     I.E. WHEN POP(1)=POP(2)=POP(3)=PNO
      CLAM=0.15/PNO
C-----SINC CHOSEN SUCH THAT FOR Q=0.4 and P=600.
C     DEL=SINC*(Q/P)/P=1.
      SINC=900000.
C-----R CHOSEN SUCH THAT FOR Q=0.4 and P=600.
C     R*(Q/P)=0.035
      R=52.5
C-----ELEMENTS OF GO(10) CHOSEN SUCH AS TO APPROACH UNITY
C     IN AN S-CURVE MANNER
      GO(1)=0.0197
      GO(2)=0.0383
      GO(3)=0.0694
      GO(4)=0.1254
      GO(5)=0.2518
      GO(6)=0.5158
      GO(7)=0.7679
      GO(8)=0.8736
      GO(9)=0.9192
      GO(10)=0.9430
C-----IT IS ASSUMED THAT BOTH COMPETITORS HAVE IDENTICAL COST
C     STRUCTURES FOR ADVERTIZING AND DISTRIBUTION (PRESUMABLY
C     THEY USE SAME ADVERTIZING AND DISTRIBUTION AGENTS) BUT
C     DIFFERENT COST STRUCTURES FOR QUALITY.
C     BOTH ADVERTIZING AND SELLING EFFORT FACTORS ARE ZERO
C     FOR ZERO EXPENDITURE,QUALITY RATING IS NON-ZERO AT ZERO
C     EXPENDITURE (INTRINSIC PRODUCT QUALITY)
C     PCON(1) IS SUCH THAT FOR EXPENDITURE OF 1E5,AEF=0.1
      PCON(1)=9E5
C     PCON(2) IS SUCH THAT FOR EXPENDITURE OF 1E4,SEF=0.05
      PCON(2)=1.9E5
C     PCON(3),PCON(4),PCON(5) CHOSEN SUCH THAT Q1=0.35 AND
C     Q2=0.30 AT ZERO EXPENDITURE,AND AT 1E6 EXPENDITURE
C     Q=0.8*(1.-Q0)+Q0
      PCON(3)=(1.-0.8)/0.8)*1E6
      PCON(4)=0.35*PCON(3)
      PCON(5)=0.30*PCON(3)
C
      PCON4(1)=PCON(4)
      PCON4(2)=PCON(5)
C
```

```
C
C-----DATA FOR /MSTATE/ BLOCK
C
      DO 25 J=1,10
      POP(J)=0.
      POPST(J)=0.
      DO 25 I=1,3
      B(I,J)=0.
25    BST(I,J)=0.
      RTPFT1=0.
      RTPFT2=0.
C
      DO 30 I=1,10
      STOCK(I)=0.
30    CPSL(I)=0.
C
C-----DATA FOR /MDECL/ BLOCK
C
      AL1=0.
      AL2=0.5
      P1=600.
      P2=600.
      Q1=0.50
      Q2=0.50
      C10=0.
      C20=0.5
C
C     DATA FOR /DECIDE/ BLOCK
C
      NHOR=4
      KTMAX=18
      DFACT=0.15
C
      DO 35 I=1,2
      DO 35 J=1,5
      DO 35 K=1,5
35    DMST(I,J,K)=0.
      DO 40 NT=1,NHOR
      PRED(2,1,NT)=AL1
      PRED(2,2,NT)=P1
      PRED(2,3,NT)=Q1
      PRED(2,4,NT)=C10
      PRED(4,1,NT)=AL2
      PRED(4,2,NT)=P2
      PRED(4,3,NT)=Q2
      PRED(4,4,NT)=C20
40
C
C     /DMOUT/
C
      DO 45 I=1,30
        DO 45 J=1,150
        DML(I,J)=0.
45    DML1(I,J)=0.
C
```

```
C      /PCONST/
C
       JWKC=2              NUMBER OF WORKCENTRES
       IFG=2               NUMBER OF FINISHED GOODS ITEMS
       IWP=7               NUMBER OF WORK IN PROCESS ITEMS

       IRM=7               NUMBER OF RAW MATERIAL ITEMS
       KSHIFT=3            MINIMUM NUMBER OF SHIFTS
C
       DO 50 I=1,IFG+IWP
       DO50 N=1,IWP+IRM          ELEMENTS OF UTILIZATION MATRIX
 50    UM(I,N)=0.
       UM(1,1)=1.
       UM(1,4)=1.
       UM(1,7)=1.
       UM(2,6)=1.
       UM(2,3)=1.
       UM(3,8)=1.
       UM(3.9)=1.
       UM(4,10)=1.
       UM(5,14)=1.
       UM(6,2)=1.
       UM(7,2)=1.
       UM(8,5)=1.
       UM(8,11)=1.
       UM(8,12)=1.
       UM(9,6)=1.
       UM(9,13)=1.
C
       MPV(1)=1            ELEMENTS OF WORKCENTRE PRODUCTION VECTOR
       MPV(2)=1
       MPV(3)=1
       MPV(4)=2
       MPV(5)=2
       MPV(6)=2
       MPV(7)=2
       MPV(8)=1
       MPV(9)=1
C
       WAV(1)=0.95         ELEMENTS OF WORKCENTRE AVAILABILITY VECTOR
       WAV(2)=0.92
C
       EFV(1)=1.0          ELEMENTS OF EFFICIENCY VECTOR
       EFV(2)=1.4
       EFV(3)=2.0
       EFV(4)=0.5
       EFV(5)=0.4
       EFV(6)=0.4
       EFV(7)=0.3
       EFV(8)=1.2
       EFV(9)=1.6
C
       MMM(1,1)=1          ELEMENTS OF WORKCENTRE MANPOWER MATRIX
       MMM(1,2)=1
       MMM(2,1)=15
       MMM(2,2)=35
 300
```

```
C
      DO 55 I=1,IFG+IWP
35    STV(I)=2                 INVENTORY SET-UP VECTOR
C
      DO 60 J=1,JWKC
      THETA(J)=0.25      MAX OVERTIME HOURS AS A FRACTION OF NORMAL WORK HOURS
60    NLBH(J)=168        NORMAL WORK HOURS IN EACH MEDIUM-TERM PERIOD
C
C     /PDMDTA/
C
      MFG=IFG
      CP2=450.
      DO 65 J=1,JWKC
65    CMM(J)=WAV(J)*12.*KSHIFT*(1.+THETA(J))*NLBH(J)*MMM(1,J)*
     *MMM(2,J)
      DO 70 I=1,IFG
        PRLO(I)=0.
70    PRHI(I)=EFV(I)*CMM(MPV(I))
C
C     /PINV/
C
      DO 75 I=1,IFG+IWP+IRM
        IF(I.LE.IFG)BORD(I)=0.
75    SLVL(I)=0.
C
C     /PCOST/
C
      COST1(1)=0.33          HOLDING COST
      COST1(2)=0.25
      COST1(3)=0.028
      COST1(4)=0.022
      COST1(5)=0.038
      COST1(6)=0.038
      COST1(7)=0.044
      COST1(8)=0.084
      COST1(9)=0.096
      COST1(10)=0.014
      COST1(11)=0.013
      COST1(12)=0.0083
      COST1(13)=0.013
      COST1(14)=0.025
      COST1(15)=0.010
      COST1(16)=0.022
C
      DO 85 I=1,IFG+IWP      SET-UP COST
      COST2(I)=15.
85    IF(MPV(I).EQ.2)COST2(I)=25.
C
      DO 90 I=1,IWP+IRM      ORDERING COST
90    COST3(I)=20.
C
      COST4(1)=44.           PURCHASE COST
      COST4(2)=34.
      COST4(3)=60.
      COST4(4)=60.
      COST4(5)=69.
      COST4(6)=131.
      COST4(7)=150.
      COST4(8)=17.
      COST4(9)=15.
```

```
      COST4(10)=10.
      COST4(11)=15.
      COST4(12)=30.
      COST4(13)=12.
      COST4(14)=26.
C
      DO 95 I=1,IWP+IRM
95    COST5(I)=0.                    SHIPPING COST
C
      COST6(1)=60.
      COST6(2)=245.
C
      COST7(1)=1.1                   ENERGY AND MAINTENANCE
      COST7(2)=3.0                   COSTS
C
      COST8(1)=0.
      COST8(2)=0.
C
      COST9(1)=3.                    REGULAR TIME PAYROLL COST
      COST9(2)=5.
C
      COST10(1)=120.                 HIRING COST
      COST10(2)=200.
C
      COST11(1)=120.                 LAY-OFF COST
      COST11(2)=200.
C
      COST12(1)=2000.                SHIFT START-UP COST
      COST12(2)=2000.
C
      COST13(1)=2000.                SHIFT SHUT-DOWN COST
      COST13(2)=2000.
C
      COST14(1)=4.5                  OVERTIME COST
      COST14(2)=7.5
```

FLOW DESCRIPTION OF SUBROUTINE LMPDEC AS FOR SUBROUTINE MK2DEC
EXCEPT SEARCH ROUTINE CODE CALLS SUBROUTINE LMPOBJ

FLOW DESCRIPTION OF SUBROUTINE LMPOBJ AS FOR SUBROUTINE MK2OUT
EXCEPT LMPOBJ CALLS LMPOUT

<u>FLOW DESCRIPTION OF SUBROUTINE LMPOUT</u>

1. OBTAIN CURRENT OUTPUTS
1.1 IND=0
1.2 OBTAIN AGGREGATE DECISIONS FOR SELLER 1 IF SUBROUTINE CALLED BY PRDMRK.FTN OR OBJECTIVE FUNCTION FOR SELLER 1 IS BEING CALCULATED (IE. LDEC=0 OR LDEC=1)
1.3 OBTAIN OTHER AGGREGATE OUTPUTS
1.3.1 OBTAIN ESTIMATED SALES RATES FOR INDIVIDUAL PRODUCT ITEMS IN SELLER 1'S PRODUCT LINE IF LDEC=0 OR LDEC=1 AND IND=0
1.4 OBTAIN CURRENT OPERATING PROFIT/LOSS
1.4.1 IF SELLER 1'S PRODUCT ITEMS ARE YET TO BE LAUNCHED (JMT<LT) OR SELLER 2'S OBJECTIVE FUNCTION IS BEING CALCULATED (LDEC=2), GO TO 1.4.9
1.4.2 IF SELLER 1'S ACTUAL OR CORRECTED (TO ALLOW FOR CAPACITY CONSTRAINTS) INDIVIDUAL PRODUCT ITEM SALES RATES HAVE BEEN OBTAINED IE IND≠0 : GO TO 1.4.8
1.4.3 IF LDEC=0 :
1.4.3.1 OBTAIN MEDIUM-TERM PRODUCTION OUTPUTS AND ANNUAL PRODUCTION COSTS, IE. CALL PRDM
1.4.3.2 UPDATE 3 KNOWN POINTS OF SELLER 1'S LONG TERM PRODUCTION UNIT COST CURVE
1.4.4 IF LDEC=1 : ESTIMATE UNIT PRODUCTION COSTS PRDM 3 KNOWN POINTS OF SELLER 1'S LONG TERM PRODUCTION UNIT COST CURVE
1.4.5 CHECK FOR UNDERSUPPLY OF GOODS IF LDEC=0 OR SUPPLY LESS THAN ESTIMATED SALES RATE IF LDEC=1
1.4.5.1 IF UNDERSUPPLY EXISTS OR SUPPLY LESS THAN ESTIMATED SALES RATE, SET IND=1
1.4.6 <u>IF IND=0, GO TO 1.4.8</u>
1.4.7 OBTAIN CORRECTED DISTRIBUTION EFFORT FACTOR FOR SELLER 1 AND <u>GO TO 1.2 TO RESTART</u>
1.4.8 OBTAIN COST OF SELLER 1'S GOODS SOLD
1.4.9 OBTAIN OTHER COSTS, AND PROFIT/LOSS

2. STORE DATA

3. OBTAIN NEXT SYSTEM STATES

4. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE PRDM

1. OBTAIN MEDIUM TERM PRODUCTION DECISIONS OVER CURRENT DECISION HORIZON, CALL PD2DEC

2. OBTAIN ACTUAL COSTS AND OUTPUTS OVER THE MINT MEDIUM TERM PERIODS IN THE CURRENT LONG TERM PERIOD

2.1 INITIALIZE COST ANALYSIS DATA

2.2 OBTAIN ACTUAL COSTS AND OUTPUTS IN EACH OF THE MINT MEDIUM TERM PERIODS,
FOR MT=1,MINT:

2.2.1 CALL PD2OUT

2.3 COST ANALYSIS

3. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE PD2DEC

1. INITIALIZATION

1.1 STORE CURRENT STATES

1.2 OBTAIN INDIVIDUAL PRODUCT DEMANDS OVER DECISION HORIZON

1.3 INITIALIZE DECISIONS

2. DECISION MAKING. EXECUTE SEARCH ROUTINE CODE CALLING PD2OBJ FOR OBJECTIVE FUNCTION EVALUATION

3. SAVE DECISIONS

4. RESTORE STATES

5. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE PD2OBJ

1. INITIALIZATION
1.1 INITIALIZE DECISIONS
1.2 INITIALIZE COSTS
1.3 INITIALIZE STATES

2. OBTAIN OBJECTIVE FUNCTION VALUE
2.1 FOR KT=1, KTMAX :
CALL PD2OUT (OBTAINS OUTPUTS, UPDATES STATES, OBTAINS OBJECTIVE FUNCTION VALUE AND CORRECTS DECISIONS TO ENSURE FEASIBILITY)
2.2 OBTAIN PENALTY COST OF FG SHIPMENT OVER KTMAX PERIOD BEING LESS THAN DEMAND OVER SAME PERIOD

3. SAVE AMENDED DECISIONS

4. RETURN TO CALLING PROGRAMME

FLOW DESCRIPTION OF SUBROUTINE PD2OUT

1. INITIALIZATION
1.1 INITIALIZE COST ELEMENTS
1.2 INITIALIZE TIME POINTERS

2. CHECK FEASIBILITY OF DECISIONS
2.1 IF PD2OUT CALLED BY PRDM (NN=O) GO TO 3
2.2 CHECK LABOUR DECISIONS - AMEND IF NECESSARY
2.3 CHECK INVENTORY DECISIONS - AMEND IF NECESSARY

3. OBTAIN INVENTORY OUTPUTS (I=1,INV)
3.1 OBTAIN UTILIZATION RATES OF WP AND RM INVENTORIES
3.2 IF PD2OUT CALLED BY PD2OBJ (NN≠O), ADD CURRENT PERIOD SHIPPED FG INVENTORIES TO TOTAL
3.3 IF PD2OUT CALLED BY PRDM (NN=O) OBTAIN LONG TERM PRODUCTION, SHIPPING, PURCHASING AND UTILIZATION OUTPUTS
3.4 OBTAIN PERIOD REVENUE
3.5 OBTAIN INVENTORY COSTS
3.6 IF NN=O, STORE INVENTORY DATA AND OBTAIN LONG TERM INVENTORY COSTS

4. OBTAIN LABOUR AND CAPACITY OUTPUTS (J=1,JWKC)
4.1 OBTAIN WORKSTATION COSTS; IF NN=O, STORE LONG TERM COSTS
4.2 OBTAIN LABOUR COSTS; IF NN=O, STORE LONG TERM COSTS
4.3 OBTAIN OPPORTUNITY COST OF UNDERTIME
4.4 IF NN=O, STORE LABOUR DATA

5. SUMMARIZE COST AND OBJECTIVE FUNCTION INFORMATION
5.1 OBTAIN TOTAL PERIOD COSTS
5.2 OBTAIN TOTAL PERIOD CONTRIBUTION
5.3 IF NN=O, SAVE COST INFORMATION

6. OBTAIN NEXT STATES

7. RETURN TO CALLING PROGRAMME

APPENDIX 5 : Correction to account for discrepancy between medium-term production output and long-term sales rate.

From Section 4.3.1 above, Seller 1's current sales rate is given by

$$S_1 = [k_1N_1 + m_1k_{12}N_{12}]\delta_1$$

(where the time arguments have been omitted for brevity) and the sales rate of the ith product item in the product line is given by

$$S_{1i} = \frac{m_{1i}\delta_{1i}}{\delta_1} S_1$$

$$= [k_1N_1 + m_1k_{12}N_{12}]m_{1i}\delta_{1i}$$

Let the actual or measured sales of the ith item in the product line be given by S^m_{1i} (the superscript m denotes measured quantity or value based on measured quantity) then

$$m^m_{1i} = \frac{S^m_{1i}/\delta_{1i}}{\sum_{j=1}^{PL} S^m_{1j}/\delta_{1j}}$$

where there are PL items in the product line.

$$v^m_1 = \sum_{i=1}^{PL} v_{1i}\, m^m_{1i}$$

$$\delta^m_1 = \sum_{i=1}^{PL} \delta_{1i}\, m^m_{1i}$$

$$p^m_1 = \frac{v^m_1\, I'}{\delta^m_1}$$

$$S^m_1 = \delta^m_1 \sum_{i=1}^{PL} \frac{S^m_{1i}}{\delta^m_{1i}}$$