GAME THEORETIC ANALYSIS OF DYNAMIC SPECTRUM LEASING IN LARGE SCALE WIRELESS NETWORKS

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DECLARATION

The candidate confirms that the work submitted is his/her own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others. It is to assert that the candidate has contributed solely to the technical part of the joint publication under the guidance of his academic supervisors. Detailed breakdown of the publications and the chapters which are constituted based on these is presented in the publications section of this thesis.

To the most wonderful people, whom I owe my success,

My great father,
— Abdul Hafeez Khokhar

My lovely mother,
— Nighat Hafeez

My dear husband,
— Ali

My precious daughter,
— Parisa

&

My loving sisters,
— Ghamza and Raana

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I am thankful to Allah Almighty for the strength He gave me to achieve everything I have in life. Words can never be enough to describe His blessings.

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ABSTRACT

In this thesis, the concept of dynamic spectrum leasing has been suggested as a solution to improve the spectral efficiency (SE) of wireless networks while reducing the energy expense. Wireless communication is facing the ever growing challenge of improving information transfer rate. This grand task is inherently coupled with managing the energy costs required to meet the capacity enhancement. To this end, the idea of cooperation among various networks which is inherently supported by the wireless medium, is utilized in this thesis. An intelligent division/sharing of resources between wireless terminals situated in close proximity of each other are the natural mechanisms, which when accurately modelled, can lead to commendable gains in the spectral utility of a wireless network. Cooperative use of existing resources in the network such that they are scheduled/divided in a way to fulfil the demands of each type of network can reap enhanced data transmission without additional energy sources. With this ideological foundation, the sharing of spectral resources between a legacy and a secondary network is suggested such that the quality of service of both types of networks is maintained/improved while improving the energy utilization of the network. Either access to the legacy spectrum is granted to the entire secondary network for a limited duration of time or a fixed term spectral access license is issued to a limited number of secondary users as a reward for their help in cooperating with the legacy network for its communication. An intelligent determination of this division of time/number of licenses, based on the geographical locations of all users is casted under the

framework of dynamic spectrum leasing (DSL). This unified approach suggesting dynamic spectrum leasing as an alternative to both uni-directional and bi-directional communication in legacy networks is the main contribution and novelty of this thesis. It is shown that DSL improves the energy efficiency (EE) of uni-directional communication by 10 times where the secondary network cooperates to relay the data while maintaining the desired quality of service of the legacy network. In doing so it improves the spectral efficiency of the secondary network and reserves up to 40% time for their activity. Otherwise, the secondary network does not have any right to carry out its own transmission in the frequencies owned by the primary user.

The thesis investigates how the techniques of network coding, beamforming and space time coding can help to harness better spectral and energy efficiency in DSL. It is shown that DSL with packet level network coding can help in bi-directional communication between a pair of primary nodes by improving the number of bits exchanged between them by 54%. The total energy cost in doing so is 10 times lower than direct bi-directional communication. DSL when applied with physical layer network coding and beamforming further enhances the SE gain by 17x or more. The thesis also studies how spectrum leasing can be applied to infrastructured cellular wireless networks. It is shown that if the cellular network of Leeds is considered, 12x improvement in the SE and 16x in the EE can be observed. Hence DSL can serve as a spectral and energy efficient alternative for wireless communication.

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ACRONYMS

5G Fifth Generation

ABS Almost Blank Subframe

AF Amplify and forward

AWGN Additive White Gaussian Noise

CAPEX Capital Expenditure

CCDF Complemetary Cumulative Distribution Function

CDMA Code Division Multiple Access

CR Cognitive Radio

CRN Cognitive Radio Network

DF Decode and forward

DSA Dynamic Spectrum Access

DSL Dynamic Spectrum Leasing

DSTC Distributed Space Time Coding

EC European Comission

EE Energy Efficiency

GPS Guided Positioning System

KKT Karush Kahn Tucker

LSA Licensed Shared Access

LTE Long Term Evolution

MABC Multiple Access Broadcast Channel

MBS Macro Base Station

MAC Medium Access Control

MANET Mobile Ad hoc Networks

MGF Moment Generating Function

MIMO Multiple Input and Multiple Output

MNO Mobile Network Operator

MRT Maximum Ratio Transmission

MU Mobile User

NE Nash Equilibrium

NC Network Coding

NRA National Research Authority

Ofcom Office for Communications

OPEX Operating Expense

OR Opportunistic Relaying

PGFL Probability Genearing Functional

PN Primary Network

PNC Physical Network Coding

PPP Poisson point process

PU Primary User

QoS Quality of Service

RSPG Radio Spectrum Policy Group

SDR Software Defined Radio

SE Spectral Efficiency

SINR Signal to Interference plus Noise Ratio

SN Secondary Network

SNR Signal to Noise Ratio

ST Space Time

TBP Time-Bandwidth Product

VoD Video on Demand

NOMENCLATURE

$\mathcal{B}^d$ Borel Sets of $\mathbb{R}^d$

α Path loss exponent

ϵ Radius of exclusion region

λ Mean number of points per unit volume..

λ_b MNO base station density

λ_s Density of small base stations

λ_u Density of cellular users

$\mathcal{A}$ Set of actions of players

$\mathcal{C}$ Set of consequences of the actions of players

$\mathcal{N}$ Set of players

$\mathcal{S}$ Strategy set of a player.

Φ_r Points within cooperation regions

Φ	Stationary Poisson Point Process.
σ^2	Variance of noise at the receiver front-end.
θ	Angle of the sector of cooperation
φ	Fraction of the density of selfish CRs
ζ_m	Fraction of offloading small cells
ζ_s	Fraction of LSA licensees
$l(r)$	Path loss function over a distance r
P_b	Transmit power of macro base station
P_s	Transmit power of secondary/small cell network
P_{rx}	Primary receiver
P_{tx}	Primary transmitter.
r_p	Radius of primary link
S_{rx}	Secondary receiver
S_{tx}	Secondary transmitter
β	Time-Bandwidth Product Ratio
$b_d(o,r)$	Ball of radius r centered at the origin o
$sec(\theta,r)$	Sector of radius r and angle θ
v_d	Lebesgue Measure.

1 INTRODUCTION

Wireless communication has witnessed an immense growth in recent years. Global trends of urbanization and socialization coupled with rising popularity of video-on-demand (VoD), multimedia rich social networking and high definition/quality content streaming are important factors leading to the inexorable spectrum demand [2]. According to various statistical reports, the global mobile traffic grew 69% only between 2013 and 2014 [3] where last year its size was 30 times more than that of the entire global internet in year 2000. It is forecasted that the world's total mobile data traffic is going to be nearly 25 exabytes in year 2020 with 9500 million mobile subscriptions [2]. The inexorable demand has made wireless spectrum a scarce resource, as now there is very limited spectrum available to be allocated for new wireless communication applications. The evolution of wireless services into a ubiquitous and pervasive paradigm of interconnectivity has significantly challenged the provision of wireless services for its providers and administrators. Wireless mobile networks are henceforth required to be more efficient, scalable, reliable and adaptive to the dynamics of the practical world. The spectrum occupancy measurements [4] suggest that the demand for spectrum varies across space and time leading to hotspots of spectral usage/demand e.g., in shopping malls, air-ports, rail-stations, universities etc. Consequently, future generation wireless networks will most likely consist of intelligent radio networks, capable of sensing the environment and successfully adjusting to the prevalent conditions and demands. It has been estimated that a capacity expansion by a factor of 1000 is needed in the next generation (5G) mobile networks [2].

In order to satisfy the sustained growth of mobile traffic, the development of more sophisticated and flexible radio networks is fundamental. This calls for additional spectral resources, planning/infrastructure deployment costs and energy requirements for the network operations. In the recent past, significant rise in the energy consumption of the communication networks has been recorded. Around 7.95% rise in the energy demand of Telecom Italia network was observed in 2007. At the same time, British Telecom contributed to about 0.7% of the total UK's energy consumption [5, 6]. It is predicted that in comparison to 2007, CO2 equivalent emissions of the communication network will increase by a factor of three by the year 2020. This corresponds to more than one third of the overall emissions in the UK [7, 8]. These alarming statistics and the rising costs have motivated the research and development community to target improving the energy efficiency of the mobile communication network by a factor of 1000 per transported bit for the emerging 5G networks [2].

Fundamentally, wireless communication is challenged due to the lossy nature of the wireless medium. A wireless signal suffers from path-loss, fading, scattering and shadowing during its propagation [9]. It also incurs harmful interference from any transmitters operating in the same channel. These phenomena inflict severe degradation to the direct point-to-point communication between two nodes. Apart from the ever growing traffic volume, mitigating the signal degradation causing outages under poor channel conditions has always been a challenge for network operators.

From the perspective of mobile network operators (MNOs), the goal is to serve the ever growing mobile traffic to earn revenue while minimizing the operational expenditure (OPEX) and capital expenditure (CAPEX). In order to meet the increase in the spectral resources, the MNOs require investment for buying additional spectrum and installing extended hardware to make use of the spectrum. Such measures result in a rise in the CAPEX. Similarly, for the MNOs to sustain their existing and new services, OPEX increases as a consequence due to the rise in the energy requirements to run the extended infrastructure. Under these demands/constraints, an ideal solution for the network operators would improve the spectral utility through better utilization of the current/new resources leading to better performance of the network with lower (if not zero) additional energy use. Such a solution enables an operator to realize network level throughput gains. Consequently, leading to growth in the revenue while keeping the increase in CAPEX and OPEX to the minimum. To realize such solutions, efforts are required on various fronts. These range from reconsidering spectrum allocation rights and policies and improving the current characterization by rigorously studying all degrees of freedom of the network. It is also essential to incorporate analytical tools that facilitate the modelling of the intelligent and adaptive behaviour of the network.

Traditionally, two approaches exist when it comes to acquiring access to the wireless spectrum:

1. *Property Rights Model*: According to this model, complete right to sell, rent, or lease spectrum should belong to the individual owners of the spectrum bands [10]. Such possession of exclusive rights ensures that the owner of the spectrum enjoys seamless connectivity with desired QoS at all times without any uncertainties. Spectrum allocation to the MNOs follows the same model where each of them pays for their license to access the spectrum and then enjoys its exclusive use. No other entity can use these frequencies even if they are temporarily not in use by the MNOs and their incumbent users. QoS guarantees are met under this

model, however, it leads to serious underutilization of the spectrum and creates scarcity for new users and services.

2. *Commons Model*. According to this approach, it is suggested that spectrum should be treated as a common resource and must be shared between all those who wish to benefit from it [11] e.g., Wi-Fi or Bluetooth. This model eliminates the rigid spectral boundaries and deprivation of spectrum for new users. On the other hand, it poses challenges of co-existence among the common users and meeting strict QoS objectives for incumbent applications.

There has been a lot of debate over these two models of spectrum access [12],[13]. For several years, the economic/engineering research and industrial community has been emphasizing the need for flexible spectrum access mechanisms. Property rights model of spectrum usage is still preferred by the MNOs due to the strict quality of service (QoS) guarantees it provides. However, the fast growth of mobile traffic has made it essential for the MNOs to consider dynamic spectrum access schemes which allow them to meet future spectral demands. *Given the interminable need for spectrum to meet the demands of the high volume of wireless data traffic, spectrum sharing has been perceived as an intermediate way between obtaining exclusive license and license exempted communication like WiFi and Bluetooth.* Sharing the spectrum with a secondary network can help an MNO manage its growing traffic by utilizing the additional resources of the secondary network e.g., cooperative relaying and offloading.

In spectrum occupancy reports, it has been shown that $13\% - 87\%$ of the allocated spectrum remains unused during various times of the day [14]. It can be seen from Fig. 1.1 that the licensed spectrum is used by its incumbent applications only sporadically. Due to the stringent spectrum access policies, these unused frequencies cannot be utilized by non incumbent users/applic-

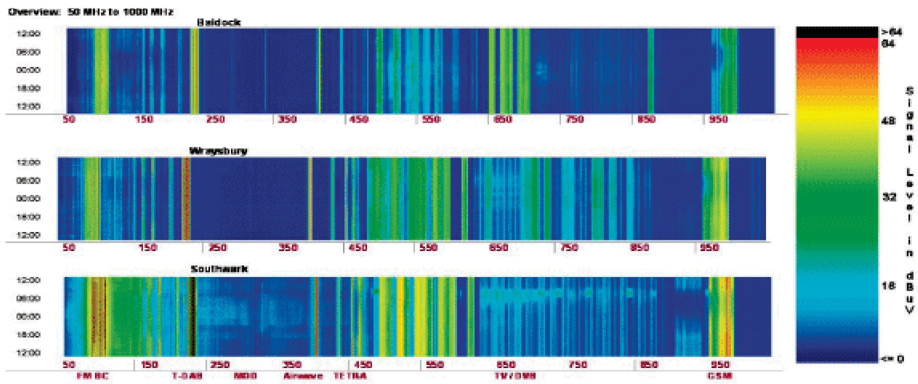


Figure 1.1: Spectrum Occupancy in the U.K.[1]

ations. Subsequently, for National Regulation Authorities (NRAs), spectrum sharing is one of the important candidates to address the issue of scarcity of un-allocated frequencies for new users.

The concepts of Software Defined Radio (SDR) and Cognitive Radio (CR) have been studied in the context of intelligent radios that can exploit the opportunities of spectral access within frequencies already allocated to different owners [15]. IEEE 802.22 wireless regional area network WRAN standard solicits CRs for exploiting white spaces in the TV frequency bands. In very recent times, the communications industry [16] has introduced the concept of licensed shared access (LSA) which attempts to bridge the gap between exclusive licensing and commonly sharing the spectrum. The Radio Spectrum Policy Group (RSPG) of the European Commission (EC) has laid out the framework of sharing opportunities between different types of wireless communication systems by developing the LSA concept [17]. Under regulatory constraints, LSA allows new spectrum users/licensees within a pre-allocated frequency band while ensuring that the QoS requirements of the legacy/incumbent users are always met. LSA has been defined as a complementary spectrum allocation mechanism which by definition is closer to the property rights model of individual licensing [18],[19].

CRs have received enormous attention by the research community and have produced fundamental academic research for almost 10 years now [20, 21]. The concept of LSA is, however, quite recent. Based on the incentive for adopting various dynamic spectrum sharing methods, existing literature/work can be characterized into three main types;

1. in which the incentive for sharing is based on monetary returns, [22]-[23],
2. where sharing is allowed as long as the interference from the non incumbents (e.g., CRs) is below an 'interference cap' [24]-[25],
3. where the incentive for sharing is based on service returns [26, 27, 28].

By sharing the spectrum with non incumbents, a spectrum owner's prime objective is to improve its own performance. Consequently, type 1 and 3 are preferable models because they also benefit the spectrum owner. Such sharing that comes in the form of monetary or service returns for the spectrum owner is referred to as *spectrum leasing*.

This thesis is based on studying dynamic spectrum leasing where the spectrum owner leases a part of its spectrum to another spectrum-less network when the latter helps to improve the performance of the incumbent network. The aim of the thesis is to study DSL as a unified model for the mutual benefit of the incumbent spectrum users (primary users (PUs), e.g., MNOs) and non incumbent networks (secondary users (SUs) e.g., CRs). The primary metrics of quantifying the benefits of DSL are the spectral and energy efficiency of the network. It explores how a network without having a pre-owned license to access the spectrum can help to improve the performance of a planned and deployed primary network (PN). It introduces a spectrum leasing framework that reconsiders spectrum allocation rights and policies, improves the current characterization by rigorously studying various degrees-of-freedom of the network, and also incorporates tools to enable the modelling of the intelligent and adaptive behaviour of such networks. It judiciously quantifies the service that a secondary network (SN) offers to get spectral access in return such that both primary and secondary network meet their own objectives of improved spectral utilization. It also studies how the proposed DSL schemes can help improve the energy efficiency of the primary network.

The evaluation of the DSL mechanism for wireless networks demands accurate characterization of all the performance determinants. The thesis aims to study the performance of various DSL schemes under two important considerations of; a) *the spatial distribution of nodes in the network;* and b) *the modelling of intelligent rational behaviour of these nodes*. An important factor that dictates the performance of the network is the spatial orientation and distribution of the nodes. The location and inter node distance directly relate to all the performance metrics like the capacity, throughput, outage etc. in the network. Recently, a branch of mathematics called *Stochastic Geometry* [29] has been applied to capture the spatial randomness of an ad hoc network [30]. It is an effective tool that formalizes the uncertainty in terms of inter-node distances, received power, signal to noise plus interference ratio (SINR) of the network. Secondly, the capability of intelligent and rational behaviour in the future wireless network needs to be analysed and mathematically modelled. Game theory serves as a strong tool to quantify and model such subjective properties [31]. Game theory, originally found its application in economics where it has been widely exploited to model the market interactions between rational entities like buyers and sellers. Market pricing and competition between various entities has also been studied using game theory. It has been also applied in the context of wireless networks to study various interactions between nodes in a network [32, 33]. These include negotiation in terms of relaying and sharing data, purchasing spectrum from amongst various operators, medium access etc., [34].

1.1 RESEARCH OBJECTIVES

In this thesis, the research objectives are:

1. To investigate how the spatial orientation of the network and the spatial distribution of the nodes relates to all the performance metrics like the capacity, throughput, outage and energy efficiency etc. of the DSL empowered CR networks (CRN). A realistic network topology for both primary and secondary networks for topological considerations and the efficient selection of the cooperation areas in DSL is aimed in the thesis.

2. To formulate and mathematically model an intelligent framework for sharing the spectrum within various degrees-of-freedom in a network. It is aimed to divide the resources like time and density of nodes in the network in a way that achieves mutual agreement and fairness in the network. An additional goal is to study that how this division affects the performance of DSL.

3. To formally carry out the modelling and analysis of the energy efficiency of leasing to measure its viability as an effective low power communication infrastructure for future wireless networks.

4. To study how point-to-point communication between two primary nodes can be made spectral and energy efficient by selecting geographically suitable nodes and sharing spectrum with them for a suitable time.

5. To study how bi-directional communication between two primary nodes can be improved in terms of its SE and EE by applying the concepts of network coding and beamforming.
6. To study how the performance of DSL is affected by the presence of selfish nodes in the network.
7. To study how DSL can be applied to future cellular communication for improving its SE and EE.
8. To investigate whether the proposed models present a viable solution to the coverage and capacity challenge in densely populated urban areas by studying the scheme under the key network parameters of a busy city square.

1.2 RESEARCH CONTRIBUTIONS

The following are the research contributions of the thesis.

1. A novel DSL scheme has been proposed for energy efficient uni-directional communication between two primary nodes. The SNs help to relay the data of the PN and in return get a fraction of time for exclusive spectrum access to primary owned spectrum. Bargaining based negotiations are devised to reach a mutual agreement on the fraction of time the SN relays the primary data and the remaining time when it enjoys exclusive spectrum access. The aim of the scheme is to meet the target data rate and save the energy of the PN. It has been shown that:

- DSL enables the PN to attain its required transmission rate. The agreement on the division of time enables the primary data to be relayed long enough to fulfil the throughput target of the PU.
- The SN which otherwise has no access to spectrum also benefits from the scheme. From 20 up to 50 percent of the total leasing time is reserved for the secondary activity during which it communicates with its own users.
- DSL with a sparse SN can be up to 10 times more energy efficient as compared to the PN.

2. DSL is modelled considering the random geometry of the CR nodes present in the network. A mathematical framework is proposed that captures the geographical locations of users in the network and the subsequent cooperation-area selection mechanism in the context of DSL enabled networks. These considerations help to demonstrate that successful data transmission can be increased while maintaining the QoS of the primary communication by leasing the spectrum to the secondary nodes that are at a spatially suitable location.

3. A *Nash Bargaining* based game theoretic framework is developed to divide the leasing time/density in a way where both PUs and SUs agree to their share of time. In previous studies, this division has been influenced more by the decision of the PU. The SU was required to observe the primary action and only decide in reaction to primary decision. Unlike previous studies, a mutual agreement based division is proposed in the work that provides proportional fairness.

4. A novel network coding (NC) based DSL technique for bi-directional communication is proposed in which the cognitive SUs cooperatively relay

the primary data for two way primary communication. A time division leasing based on Nash Bargaining is proposed. It is shown that:

- Under the considerations of geographical randomness in the network and employing NC, DSL can improve the number of bits that are successfully transmitted by 4x as compared to direct two way primary communication.
- Also the energy costs of the proposed DSL scheme are more than 10 times lower as compared to the direct two way communication.

5. A novel physical layer network coding (PNC) and distributed beamforming based DSL scheme is proposed for bi directional communication. This scheme also uses time division based spectrum sharing.
   - It is observed that DSL with PNC and beamforming is $2-5$ times more spectrally efficient as compared to DSL with NC.
   - It is shown that if selfish users are present in the network, the throughput of the primary communication can suffer 50% loss as compared to the case if all the secondary users were honest.
   - A monetary pricing strategy is suggested as a punishment for the selfish users. It is shown that the pricing enforced helps to discourage the selfish behaviour of the secondary nodes.
6. An LSA based spectrum sharing technique is proposed. Under the proposed scheme, the small cell network offers offloading services to improve the quality-of-service (QoS) of the macro network. In return, small cells are rewarded for their cooperation with a license to operate in the spectrum owned by the macro network. In this scheme, a density based

division is proposed in which the number of offloading small cells and those enjoying the license to access the spectrum is determined.

- The analysis based on the model shows that such spectrum sharing can lead to commendable gains in the spectral utilization of the network while assuring the desired QoS for both macro and small network.
- It is also shown that up to 90 % small cells get access to the spectrum at poor channel conditions.
- The scheme is evaluated using the network parameters of the city of Leeds. The results indicate that for the considered scenario, the spectral utility of Leeds can be improved by more than 12 times by adopting the proposed scheme. LSA offloading can be up to 16 times more energy efficient for Leeds.

1.3 PUBLICATIONS

The original contributions in this thesis are supported by the following publications:

P-1 Hafeez, M.; Elmirghani, J.M.H., "Analysis of the Energy Efficiency of Dynamic Spectrum Leasing for Cognitive Radio Networks", Energy Management of Wired and Wireless Ad-hoc Networks, Springer, 30^{th} April 2015. (Accepted with major revision)

P-2 Hafeez, M.; Elmirghani, J.M.H., "Analysis of Dynamic Spectrum Leasing for Coded Bi-Directional Communication," *IEEE Journal on Selected Areas in Communications (JSAC)*, vol.30, no.8, pp.1500-1512, September 2012.

P-3 Hafeez, M.; Elmirghani, J.M.H., "Dynamic Spectrum Leasing for Beamforming Cognitive Radio Networks using Network Coding," *IEEE International Conference on Communications (ICC)*, 2013, vol., no., pp.2840-2845, June 2013 [ICC 2013 Best paper award].

P-4 Hafeez, M.; Elmirghani, J.M.H., "Beamforming Empowered Spectrum Leasing for Bi-Directional Communication: Impact of Selfishness," *IEEE Transactions on Communications*, 30^{th} April 2015. (Accepted with major revision)

P-5 Hafeez, M.; Elmirghani, J.M.H., "Green Licensed Sharing Access", *IEEE Journal on Selected Areas in Communications (JSAC)*, 30^{th} March 2015. (Accepted with major revision)

Specifically, Ch. 3 develops the ideas reported in publication P-1. The work in Ch. 4 appeared in publication P-2. The work presented in publications P-3 and P-4 forms the core of the Ch. 5. Lastly, Ch. 6 develops the framework proposed in P-5.

1.4 THESIS ORGANIZATION

In this chapter, the motivation to use DSL to solve the problems of meeting growing traffic demands and increased energy consumption has been presented. In Ch. 2, the techniques borrowed from various domain of Mathematics and Communication theory are introduced and used later in the thesis to model DSL based solutions. In Ch. 3, the fundamental DSL model for uni-directional point-to-point communication is proposed and its analysis is presented. It is shown that DSL provides energy efficiency for the legacy network and improves the spectral efficiency of the secondary network. In Ch.

4, the DSL framework is extended for bi-directional communication between two primary nodes using NC. In Ch. 5, sophisticated physical layer techniques of beamforming and PNC have been used to improve the performance of bi-directional communication. It is shown that these techniques significantly improve the performance of DSL. In Ch. 6, the DSL framework is extended beyond point-to-point communication for future cellular networks using the concept of LSA. A case study of Leeds is presented for the implementation of LSA showing that commendable gains in terms of energy and spectral efficiency can be harnessed. In Ch. 7, a summary of the contributions of the thesis is provided. Ch. 7 also outlines a few possible directions of future research aligned with the work in this thesis. Fig. 1.2 presents a graphical illustration of the road-map of the thesis.

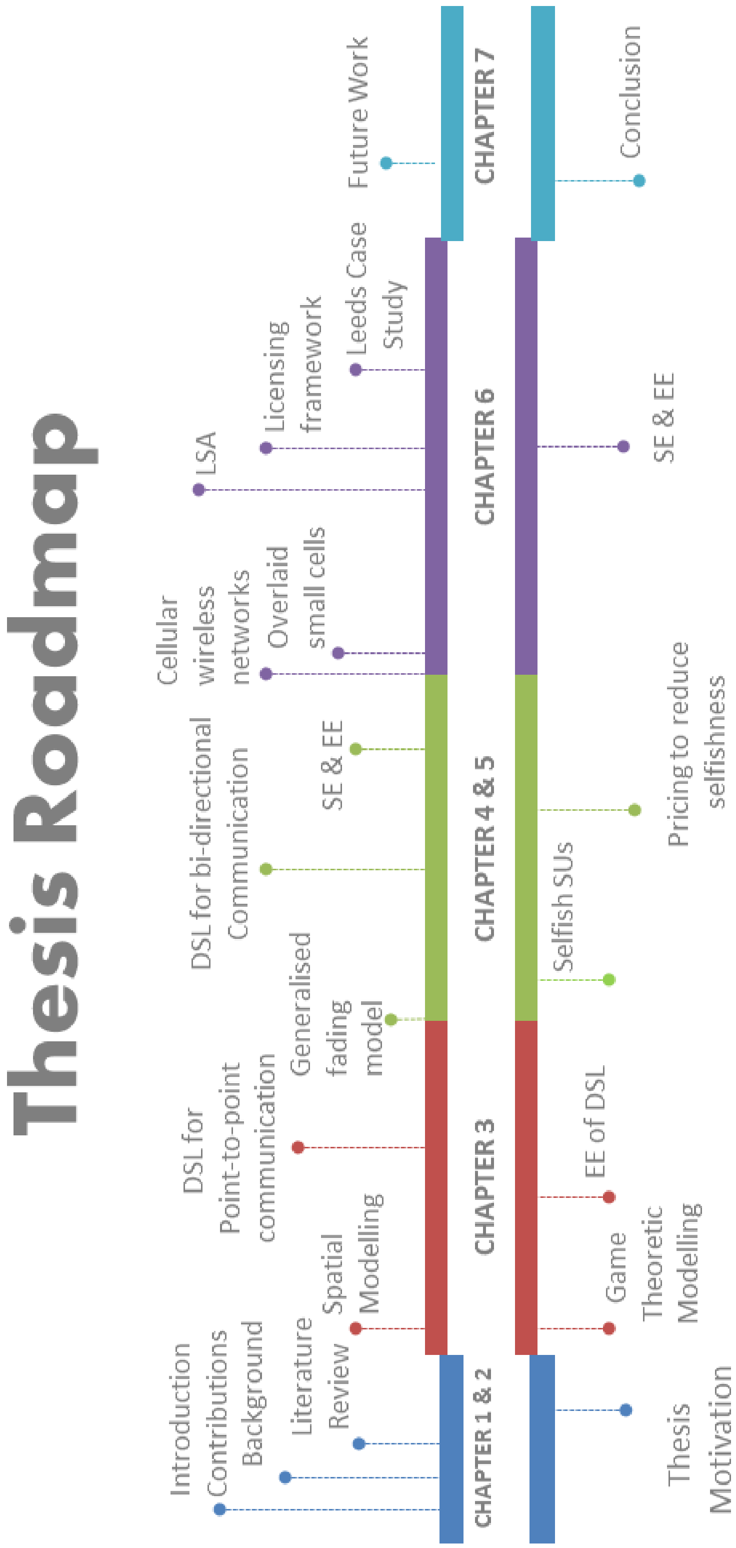


Figure 1.2: A road-map of the thesis.

2

BACKGROUND AND RELATED LITERATURE

In this chapter, a survey of the background work in the domain of wireless communication that is pertinent to the DSL empowered networks is presented. The aim is to study the fundamental techniques and work related to various modules of DSL so that subsequent treatment of the subject follows coherently. To this end, it is imperative to describe the baseline concepts that are borrowed from the communication and other theories that are used to model and analyse DSL as follows:

1: Cooperative Communication

Most forms of spectrum sharing inherit the idea of cooperation between wireless terminals. A discussion on cooperative communication and its methodologies is hence covered in this chapter. Cooperative communication techniques are a building block of DSL schemes that are studied in this thesis.

2: Stochastic Geometry

Stochastic geometry is an important tool rooted in mathematics. It is useful to model the spatial locations of wireless terminals particularly in spectrum sharing scenarios to explore the impact of the spatial locations on the performance of the network.

3: Game Theory

Game theory finds its application in modelling the rationality of the behaviour of the wireless terminals that are subject to interaction with each other. It is a remarkable tool developed in economics and mathematics but finds application in a wide variety of disciplines. In this thesis, the decision making ability and rationality in interaction between primary and secondary users is modelled using game theory.

4: Dynamic Spectrum Access (DSA)

A survey of existing spectrum sharing mechanisms is the most important component of this chapter. From its inception to standardization, it presents a classification of the approaches towards spectrum sharing/leasing and their benefits/drawbacks. It outlines the existing approaches towards DSA/DSL and highlights the open research ends addressed later on in the thesis.

The preliminaries provided in this chapter not only help to build up towards the research contributions of this thesis but also highlight the differentiation from the prior-art. To that end, a summary of relevant prior-literature is also presented in this chapter. It is noteworthy that the fundamental concepts of signal propagation and interference in wireless networks have been used in this thesis. These concepts are well known and are not the focus of the research presented in this work. For this reason, a survey of the signal propagation and interference techniques has not been included. However, interested readers are referred to [35, 30, 36] for a detailed discussion on the subject.

2.1 COOPERATIVE COMMUNICATION

Wireless communication experiences a fundamental challenge of fading and path-loss during signal propagation in the medium. Impairments like co-channel interference, shadowing and scattering can cause severe degradation in the quality of signal transmitted from a source to destination over a point-to-point link. Cooperative communication finds its roots in the efforts to exploit the presence of any intermediary terminals to relay the signal from the source to its intended destination [37]. An improvement in the received SINR can be attained due to both: (i) reduction in the path-loss; and (ii) diversity of available paths. Multiple or enhanced version of the source signal can be used at the destination to improve the decodability. The inherent broadcast nature of the wireless environment is the key enabler in exploiting the presence of other wireless terminals for relaying the source signal. The subject of cooperative communication has attracted great attention since its inception [38, 39, 40, 41]. The fundamental upper and lower bounds of the capacity of a three terminal relay channel were established in the work of Meulen [37, 42]. These fundamental bounds have been utilised by studies in cooperative communication to determine its performance.

2.2 RELAYING OPERATION

The operation of relaying is broadly based on two types of schemes. Fig. 2.1 shows the primitive relaying operation with three terminals.

2.2.1 *Repetition based Protocols*

The fundamental framework developed in [43, 40] studies methods described in the following where the signal from the source is repeated by the relay and received at the destination. These protocols are generally known as *repetition-based* protocols, mainly because the relays just repeat the signal sent by the source.

2.2.1.1 *Amplify-and-Forward (AF)*

In this scheme, the intermediate relays simply amplify the source signal with a scalar gain and forward it to the destination without any signal regeneration. In this process, noise also gets amplified and gets accumulated at the destination. AF strategy is shown to be optimal when the source-to-relay and relay-to-destination channels are comparable in terms of the propagation gains. Based on the nature of the scalar gain, AF is further divided into *variable gain* [44] and *fixed gain* AF [45]. In variable gain AF, full channel state information (CSI) of the first hop is required. While in fixed gain cooperative relaying, only the statistical distribution of fading between the relay and the first hop is sufficient for executing the amplification operation.

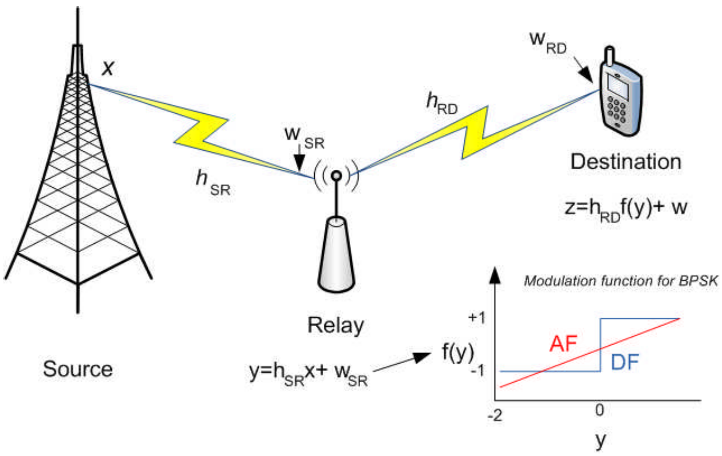


Figure 2.1: Three Terminal Relay Network

2.2.1.2 *Decode-and-Forward (DF)*

In DF method, a cooperating relay first decodes a signal received from the source and then relays or retransmits it. Therefore, error correction is done and noise is removed before the re-broadcasting of the message. The receiver at the destination uses information retransmitted from the relays and the source (when available) to decode the information.

DF is shown to be close to optimal under favourable channel conditions between the source and the relay [43]. It is shown in [44], that the performance of AF is comparable to that of the DF, whereas its complexity is lower than that of DF.

2.2.2 *Coded Cooperation*

Based on the work by Laneman et al. [46, 47], Hunter et al. [48, 49] proposed a parallel concept of coded cooperation where the relays transmit incremental redundancy instead of only replicating the received signal. A variety of channel coding techniques are used to attain the coding gain in the cooperation phase.

Besides channel coding, space time (ST) codes are also used to improve the performance of cooperation. For ST codes enabled cooperative transmissions, each relay is presumed to be a virtual antenna that forms the ST code in a distributed manner [46, 50, 51]. Consequently such space-time codes are also called distributed space time codes (DSTC).

Both repetition-based protocols (AF and DF) achieve full spatial diversity at the expense of a decrease in the spectral efficiency with the number of cooperating terminals, since each relay requires its own sub-channel for repetition. In order to overcome this drawback, Laneman *et. al* [46] introduced DSTC which allows all relays to transmit using the same sub-channel in the second phase of the transmission. In this way, the SE of the network is dramatically increased as compared to the orthogonal transmissions or repetition-based protocols.

In [52], the performance analysis of the DSTC is presented in conjunction with the AF and the DF relaying. It is shown in the paper that an ST code designed for a traditional multiple-input-multiple-output (MIMO) system can achieve full diversity but not necessarily maximize the coding gain when used with the DF protocol. This is due to the fact that not all the relays are able to correctly decode the source message. On the other hand, it was shown in [52] that ST code designed with AF protocol not only achieves full diversity but also maximizes the coding gain similar to the traditional MIMO systems.

2.2.3 *Selection Relaying*

Bletsas *et.al* introduced a new kind of protocol known as *opportunistic relaying* (OR) where the *best* relay is chosen from a set of available relays in the network to relay the data between the source and its desired destination [53]. The selection is made in terms of the instantaneous channel conditions (instantaneous

SNR). OR with DF is studied in [53] where it is shown to attain the global optimum outage performance. The analysis for AF relaying was presented in [54]. It was shown in [55], that OR with AF achieves diversity gain of the order of the number of relays in the network.

2.2.4 *Two way relaying*

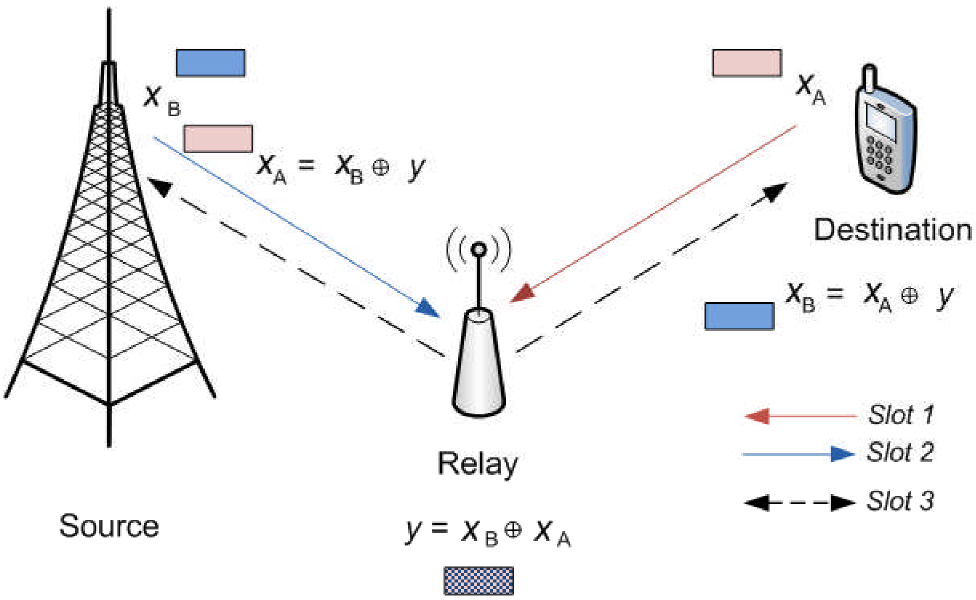
In conventional relaying, extra time slots are used up due to the need to send the information to the relays and then for the relays to resend the information to the destination. Despite the gains in signal quality and diversity, the number of time slots required for a communicating pair to exchange information can easily double by using relays. In the following, techniques that can be used to minimize the number of time slots used in relaying are discussed. By adopting these techniques, the spectral efficiency of the relaying networks can be improved.

2.2.4.1 *Network Coding (NC)*

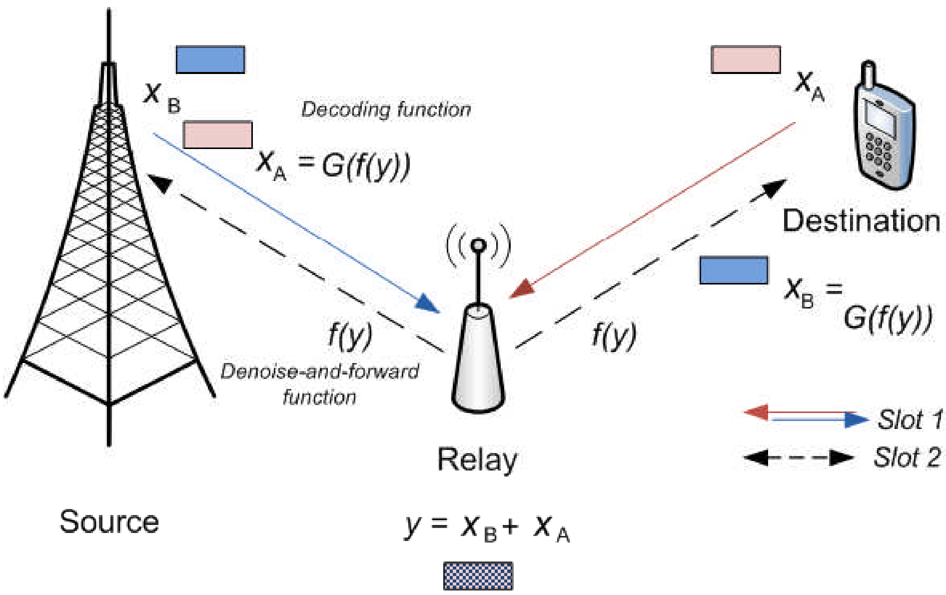
In order to reduce the number of time slots for the relaying operation, the concept of network coding proposes that the relays forward a linear combinations of the messages they receive [56, 57] from multiple destinations. Such schemes receive separately from the sources and then operate at packet level by applying a network coding scheme. At the receiver, a decoding algorithm is used to extract the message from the intended sender. Binary linear combination based network coding of the received packets at the relay were suggested by Xiao in [57]. A simple network coding scheme reduces the number of time slots required to exchange data between a pair of nodes using relays from 4 to 3. Thus 1 time slot is saved since the relay does not separately transmit packets to each terminal. Rather it broadcasts a linear combination of their packets in one time slot which is decoded at the receivers. Other schemes based on non binary coding were suggested in [58, 59]. The work in [60] is one of the fundamental works that introduced the concept and performance bounds of network coding. A detailed overview of various network coding based techniques can be found in [61]. These schemes are applicable to a wider variety of message structures, however, they differ in terms of the optimal decoding strategy and complexity. In Fig. 2.2a, NC for a three terminal network with the sequence of data transmission is shown.

2.2.4.2 *Physical Layer network coding*

Physical layer network coding (PNC) is an important technique that reduces the time required for multicast network communication. The concept of multiple access broadcast channel (MABC) is used in this scheme suggested primarily by [62]. In contrast to the conventional approaches of avoiding collision and interference, it exploits these phenomena to naturally combine packets from multiple sources to maximize the information flow across the network. This combined data can then be separated into intended information at the receivers using some algebraic techniques [63]. This scheme saves 2 time slots as compared to the packet level network coding explained above. Due to the multiple access strategy, two transmitters broadcast their data simultaneously. Also, the linear combination of the received data is broadcasted by the relay in one time slot. A detailed analysis of AF for PNC is provided in [64]. In Fig. 2.2b, PNC for a three terminal network with the sequence of data transmission is shown.



(a) Network coding at packet level



(b) Physical Layer Network Coding

Figure 2.2: Three Terminal Two way Relay Networks.

2.3 STOCHASTIC GEOMETRY

2.3.1 *Introduction*

Complex geometrical attributes of systems exist in almost all areas of science that require a statistical analysis. Common examples of subjects in which random patterns and their study are crucial include astronomy, forestry, material science, etc. Analysis and research in these areas requires a formal mathematical tool. Stochastic geometry is the branch of mathematics that provides the mathematical models and statistical methods to study complicated geometric patterns. Stochastic geometry deals with random geometric structures, ranging from simple points or line segments to arbitrary closed sets. Classical stochastic geometry finds its roots in geometric probability and integral geometry. The modern theory of random sets which studies more complicated random patterns and structures was developed by Stoyan et al. in [29]. Spatial Poisson point process play a central role in stochastic geometric models. A comprehensive treatment of Poisson point processes can be found in [65]. In context of wireless networks, the received SINR is a strong function of the network geometry. Thus, it is particularly important to model the spatial dynamics of the wireless networks to accurately analyse the behaviour of a real network. To this end, it is essential to introduce the following fundamental concepts of stochastic geometry that are useful specifically in the modelling of DSL enabled CRNs.

2.3.2 *Basic Concepts*

In order to explore the design space of large scale wireless network by employing stochastic geometry, it is important to first formalize the generic definitions and nomenclature. The definitions and notations followed here are adapted from [29].

Definition 2.1: $\sigma-$ additivity

If a set can be divided into countable union of subsets, then the volume of the set function on the whole set should be equal to the sum of the values of the set function on the subsets.

Definition 2.2: $\sigma-$ algebra

It is a system $\mathcal{X}$ of subsets of some ground set X satisfying three conditions;

1. $X \in \mathcal{X}$,
2. If $A \in \mathcal{X}$ then $A^c \in \mathcal{X}$,
3. If $A_1, A_2, \cdots \in \mathcal{X}$ then $\bigcup_{k=1}^{\infty} A_k \in \mathcal{X}$.

The following properties correspondingly become evident from the above;

$\emptyset \in \mathcal{X}$,

If $A_1, \cdots A_n \in \mathcal{X}$ then $A_1 \bigcap \cdots A_n \in \mathcal{X}$, $A_1 \bigcup \cdots \bigcup A_k \in \mathcal{X}$,

If $A, B \in \mathcal{X}$ then $A \backslash B \in \mathcal{X}$.

Here, $\backslash$ denotes the set exclusion operation. The most common example of σ-algebra is the power set of X.

Definition 2.3: Borel Set $\mathcal{B}^d$

The smallest σ-algebra on $\mathbb{R}^d$ that contains all open subsets of $\mathbb{R}^d$ is called the Borel set $\mathcal{B}^d$.

Definition 2.4: Measurable Space

The pair $[X, \mathcal{X}]$ formed by a set X and a σ-algebra of subsets of X, is called a measurable space.

A function $f : X \rightarrow \mathbb{R}$ is said to be $\mathcal{X}$- measurable if for all Borel sets $B \in \mathcal{B}^1$ the inverse image $f^{-1}(B) = \{x \in X : f(x) \in B\}$ belongs to the σ-algebra $\mathcal{X}$ associated with X. An important measure widely used in stochastic geometry is the Lebesgue measure. It is a way to standardize the length, area and volume of the subsets in the Euclidean space. For a family of Borel sets defined in the Euclidean space, it is defined over the measure space $[\mathbb{R}^d, \mathcal{B}^d]$ as;

$$v_d(Q) = (v_1 - u_1) \cdots (v_d - u_d),$$

where, $Q = [u_1, v_1] \times \ldots \times [u_d, v_d]$. Lebesgue measure for $d = 1$ corresponds to length, $d = 2$ to area and $d = 3$ to the volume measure. A ball with radius r is an important geometrical shape. The d-dim ball $(d \in \mathbb{N})$ with radius $r \in R$ is:

$$b_d(c, r) = x \in \mathbb{R}^d : \{|x - c \leq r|\}, \tag{2.1}$$

where c is the centre of the ball such that $c \in \mathbb{R}^d$. A d-dim ball has a Lebesgue measure

$$b_d(c, r) = c_d r^d, \tag{2.2}$$

where

$$c_d = \begin{cases} \frac{\pi^{\frac{d}{2}}}{\left(\frac{d}{2}\right)!} & \text{even } d \\ \frac{1}{(d)!}\pi^{\frac{d-1}{2}} 2^d \left(\frac{d-1}{2}\right)! & \text{odd } d \end{cases}. \quad (2.3)$$

A ball with $d = 2$, centred at the origin is denoted as $b_d\,(o, r)$.

2.3.2.1 *Point Process*

As discussed, the stochastic geometry deals with the random distributions and patterns of structures in space. Having defined the basic measures on space, distribution of objects in this space can be addressed. To start with, the stochastic spatial distribution of points is considered. Such a random distribution of points is named a point process. Mathematically,

Definition 2.5: Point Process

A point process on $\mathbb{R}^d$ is a random variable taking values in a measurable space $[X, \mathcal{X}]$, where $\mathcal{X}$ is the family of all sequences Φ of points of $\mathbb{R}^d$ satisfying two regularity conditions;

- that the sequence Φ is locally finite (each bounded subset of $\mathbb{R}^d$ must contain only a finite number of points of Φ).
- that the sequence is simple (no two pints coincide with each other).

The simplest point process can be considered to be of a single point. Consider a compact set $W \subset \mathbb{R}^d$ with a Borel set $\mathcal{B}$ contained in W. Then the probability that a random point Φ is uniformly distributed in W is given as;

$$\mathrm{P}(\Phi \in \mathcal{B}) = \frac{v_d(\mathcal{B})}{v_d(W)}.$$

2.3.2.2 *Binomial Point Process*

Definition 2.6: Binomial Point Process of n points

Consider a compact set $W \subset \mathbb{R}^d$ with Borel sets $\mathcal{B}_1, \ldots, \mathcal{B}_n$ contained in W. Then the probability that a pattern of n independent random points $\Phi_1, \ldots, \Phi_n$ are uniformly distributed in W is given as;

$$\begin{aligned} \mathrm{P}(\Phi_1 \in \mathcal{B}_1, \ldots, \Phi_n \in \mathcal{B}_n) &= \mathrm{P}(\Phi_1 \in \mathcal{B}_1), \ldots, \mathrm{P}(\Phi_n \in \mathcal{B}_n), \\ &= \frac{v_d(\mathcal{B}_1), \ldots, v_d(\mathcal{B}_n)}{v_d(W)^n}. \end{aligned}$$

The number of points falling in $\mathcal{B}_1, \ldots, \mathcal{B}_n$ is denoted as $\Phi_{W(n)}(\mathcal{B})$.

A slightly different scenario would be when n points fall in W. Unlike the previous case, now the probability $n_k \geq 1$ points fall in each Borel subset $\mathcal{B}_1, \ldots, \mathcal{B}_k$ of W where $\mathcal{B}_1, \ldots, \mathcal{B}_k$ are disjoint and satisfy; $\mathcal{B}_1 \bigcup \cdots \bigcup \mathcal{B}_k = W$ and $n_1 + \cdots + n_k = n$ is given as;

$$\mathrm{P}\left(\Phi_{W(n)}(\mathcal{B}_1) = n_1 \ldots, \Phi_{W(n)}(\mathcal{B}_k) = n_k\right) = \frac{n!}{n_1! \ldots n_k!} \cdot \frac{v_d(\mathcal{B}_1)^{n_1}, \ldots, v_d(\mathcal{B}_k)^{n_k}}{v_d(W)^n}.$$

2.3.2.3 *Poisson Point Process (PPP)*

Definition 2.7: Stationary Poisson Point Process

The Stationary Poisson Point Process (SPPP) Φ with mean $\lambda v_d(\mathcal{B}_i)$ can be defined as

$$\mathrm{P}\left(\Phi(\mathcal{B}_1) = n_1 \ldots, \Phi(\mathcal{B}_k) = n_k\right) = \prod_{i=1}^{k} \left(e^{-\lambda(\mathcal{B}_i) v_d(\mathcal{B}_i)} \frac{(\lambda v_d(\mathcal{B}_i))^{n_i}}{n_i!} \right),$$

where λ is a locally finite non-null measure on $\mathbb{R}^d$ and $\mathcal{B}_1, \ldots, \mathcal{B}_k$ are bounded, mutually disjoint and satisfy; $\mathcal{B}_1 \bigcup \cdots \bigcup \mathcal{B}_k = W$. λ is the mean number of points per unit volume.

Here, λ is called the intensity or density of the process Φ_i for a given Borel subset A_i. Mathematically,

$$\lambda v_d(A) = \mathbb{E}\left(\Phi(A)\right) = \Lambda(A),$$

where, Λ is called Radon measure of the PPP. This measure and its significance is discussed in later sections.

It can be seen that the distribution of the Stationary PPP follows asymptotically from the distribution of the Binomial point process of n points with each subset having more than one point. Hence the uniformly distributed pattern of points follows the Poisson distribution when the space containing them (i.e. W) and the number of points (i.e. n) forming the pattern both tend to infinity.

A typical example of a PPP is a random ad hoc wireless network where all the nodes are identical and independently distributed (i.i.d). It can be considered as a stationary Poisson point process where the number of nodes is Poisson distributed and their locations are uniformly distributed. Figure 2.3 shows Poisson distributed nodes in a network.

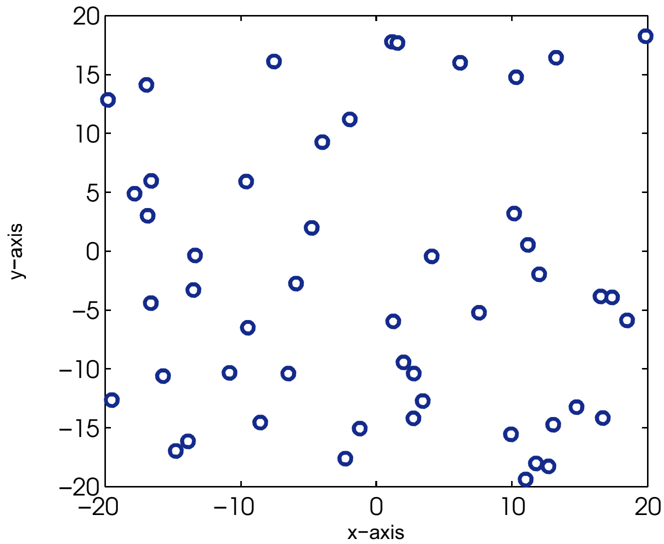


Figure 2.3: Poisson Network

PROPERTIES OF A STATIONARY PPP There are two important properties of a Stationary PPP

1. For every bounded closed set $\mathcal{B}_k$, the number of points n_k has a Poisson distribution with mean $\lambda(\mathcal{B}_i)v_d(\mathcal{B}_i)$.

2. If $\mathcal{B}_1, \ldots, \mathcal{B}_k$ are disjoint sets, then the number of points $\Phi_1, \ldots, \Phi_k$ are independent.

A point process $\Phi = \{x_n\}$ is *stationary*, when $\Phi_{tran} = \{x_n + x\}$ has the same distribution for all x in $\mathbb{R}^d$. The property of *isotropy* holds if the same is true for all rotated processes $\Phi_{rot} = \{\mathbf{r}x_n\}$. A PPP satisfying both the properties of stationarity and isotropy are called stationary PPP. It is important to mention that a stationary PPP is also *simple*.

GENERAL POISSON POINT PROCESS In general, a PPP can also be non-stationary. It implies that the mean number of points per unit area varies within the space over which the process is defined. In that case, the general Poisson point process is defined as follows.

Definition 2.8: General Poisson Point Process

The number of points in a bounded Borel set $\mathcal{B}$ has a Poisson distribution with mean $\Lambda(\mathcal{B})$ are given as

$$P\left(\Phi(\mathcal{B}) = n\right) = e^{-\Lambda(\mathcal{B})} \frac{\left(\Lambda(\mathcal{B}_i)\right)^n}{n!}.$$

The number of points in k disjoint Borel sets form k independent random variables.

The intensity of the process is defined as

$$\Lambda(A) = \int_A \lambda(x)dx,$$

where $\lambda(x)dx$ is the density function and its integral over the space in consideration defines the intensity of a general PPP.

As an example, consider a sensor network where the density of deployed sensor nodes increases in the direction of sink. The area far from the sink has fewer number of sensors. However, the total number of sensors is Poisson distributed. The deployed sensors in such a configuration form a general PPP in which the mean number of nodes varies with their location. The key motivation behind such deployment is that the nodes near sink relay the data from the sensors located further away. Consequently, these sensors deplete their batteries at much faster rates. Thus redundant deployment near sink when complemented with proper sleep scheduling ensures that network life time can be extended.

2.3.2.4 *Transformations on a Point Process*

New point processes can be defined by transforming already existing point processes. This helps in characterising the properties of the new process on the basis of known properties of the transformed process. Consider a known point process $\mathbf{X}$ in $\mathbb{R}^d$. The following are some types of point process transformations that can be applied to it.

MAPPING Let there be a fixed transformation $z : \mathbb{R}^d \rightarrow \mathbb{R}^d$, when applied to each point in $\mathbf{X}$, results in a process $\mathbf{Y}$ such that $\mathbf{Y} = \cup_{x \in \mathbf{X}} z(x)$. For example, for a mapping function $z(x) = a + x$, a constant a gets added to $\mathbf{Y}$.

THINNING Thinning deletes certain points from the process $\mathbf{X}$ and the retained points form the new process $\mathbf{Y}$. Formally, an indicator random variable $\mathbb{I}_x$ is associated with each point in $\mathbf{X}$. If the variable $\mathbb{I}_x$ assumes the value 1, the point is retained. If its value is 0, the point gets deleted.

SUPERPOSITION The union of all point in the processes $\mathbf{X}$ and $\mathbf{Y}$ consists of all the points in both the processes. If the number of points in $\mathbf{X}$ are say $n_{\mathbf{X}}$ and those in $\mathbf{Y}$ are $n_{\mathbf{Y}}$, the superposition of the two process is given as $n_{\mathbf{X} \cup \mathbf{Y}} = n_{\mathbf{X}} + n_{\mathbf{Y}}$.

CLUSTERING In clustering, each point $x \in \mathbf{X}$ is replaced by a random finite set of points Z_x, where Z_x is called the cluster associated with each x. The superposition of all clusters yields the process $\mathbf{Y} = \cup_{x \in \mathbf{X}} Z_x$.

2.3.3 *Some Useful functions on PPP*

Definition 2.9: Void Probability

For a PPP Φ containing points $\{x_i\}$,the probability that there are no points in a ball $b_d(o,r)$ given as

$$P\left(\Phi(b_d(o,r)) = \emptyset\right) = e^{-\lambda c_d r^d}, \tag{2.4}$$

is called the void probability. Here, c_d is as defined earlier in eq. (2.3).

Definition 2.10: Campbell's Theorem

Let P be a point process and consider a measurable function $f : X \to [0,\infty)$. Then $\int f(x)P(\mathrm{d}x)$ is a random variable and

$$\mathbb{E}\int f(x)P(\mathrm{d}x) = \int f(x)\Lambda(\mathrm{d}x), \tag{2.5}$$

where Λ is the intensity measure of P.

Definition 2.11: Generating functional of Homogeneous PPP [66]

Let $f : \mathbb{R}^2 \to [0,\infty)$ be a measurable function on a homogeneous PPP Φ, then the probability generating functional (PGFL) of Φ is given as

$$G(f(x)) = \mathbb{E}\left(\prod_{x\in\Phi} f(x)\right), \tag{2.6}$$

$$= \exp(-\int(1-f(x))\Lambda(x)\mathrm{d}x). \tag{2.7}$$

Here $\mathbb{E}(.)$ denotes the statistical expectation.

Definition 2.12: Thinning [66]

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. An indicator random variable $\mathbb{I}_x$ defines the thinning as

$$\mathbb{I}_x = \begin{cases} 1 & f(x) > c \\ 0 & f(x) \leq c \end{cases}, \tag{2.8}$$

where c is an arbitrary constant.

2.4 STOCHASTIC GEOMETRY IN WIRELESS NETWORKS

The research and development of wireless networks requires strategic and economic modelling, planing and analysis. For practical wireless networks, the spatial structures and orientation of the operational entities play a key role in their design and performance. To capture this spatial information, stochastic geometry is a widely applied tool in the context of wireless networks. Its application has been extended from infra-structured wireless networks like cellular to non-structured ad hoc networks like sensor networks.

Bettstetter in [67] considered a wireless multihop network and determined the isolation probability of any node. Following that, the probability distribution of k-connectivity of a wireless network was studied. In [68], the authors consider the problem of optimum placement of base stations in a cellular network so that demand points are covered in a deterministic way. The minimum number of base stations required to fully cover lattice grids with various range-length ratios are determined.

In [69],the author determine the density function of the distance to n-nearest neighbours of a homogeneous PPP. It is an important result which clarifies the distance distribution between say a transmitter and its n^{th} receiver in a wireless ad hoc network. Similarly, [70] has studied the connectivity of a Poisson network in a square grid.

Shannon in his seminal paper [71] characterized the capacity of communication channel in the presence of additive white Gaussian noise (AWGN). This statistical characterization has served as a benchmark for the design of communication networks over the last several decades. Since then, several information theoretic studies have investigated the design of optimal capacity achieving coding schemes. Convolutional codes, Turbo code, joint coding and modulation are important inventions which have enabled highly efficient utilization of the available bandwidth. Unfortunately, these capacity limits are only known for point-to-point communication. With the ever growing popularity of ubiquitous multimedia applications, development of large scale self-organizing ad hoc wireless networks has become a necessity. For such large scale networks with spatial characteristics, capacity regions cannot be characterized exactly. Hence the research community has developed alternative ways to characterize the performance of such wireless networks using the PPP theory. Gupta and Kumar in [72] derived capacity scaling laws for wireless ad hoc networks. They demonstrated that the capacity of wireless ad hoc network scales as $O(\sqrt{n})$, where n is the number of nodes in the network. This has provoked much interest in the research community to look at such asymptotic characterizations of capacity. In [73], Franchessti et al. adopted a physics based approach and derived the same result as Gupta and Kumar using Maxwell's equation. Recently, in [74], Ozgur and Leveque have characterized the throughput-delay trade-off in ad hoc wireless networks. Sousa et al. in [75], utilized the PPP theory to model uncertainties in transmission/reception distances in order to establish the optimum transmission range in a direct sequence spread spectrum multihop ad hoc packet radio network. All of these studies utilize PPP for characterizing the spatial distribution of wireless ad hoc networks. Stochastic geometry has been widely applied to model and analyse the impact of spatial orientation of terminals in a network. [30, 66, 76, 77, 78] and references therein provide a good resource to understand the application of stochastic geometry to wireless networks.

2.5 GAME THEORY

Game theory is a branch of applied mathematics that deals with situations of mutual interactions between multiple agents (players) resulting in decisions aiming to achieve (possibly) conflicting goals. It is a collection of analytical tools that enable one to comprehend the interactions of decision makers and the mutual consequences that arise at the end of the decision making process. The fundamentals of the theory have been comprehensively covered in many text books [79],[31]. Game theory and its applications is a well investigated subject in the field of Economics. It is primarily used to study the competition between different companies in a market and the strategies they adopt to maximize their monetary profit etc., [79]. Since game theory models real life behaviours and processes, its concepts have been widely extended to many disciplines of science like biology, socio-political science, communication engineering etc., [32],[80].

2.5.1 *Elements of a game*

As discussed, game theory revolves around the entities; their interests, behaviours, knowledge etc., and the consequences of a decision making process. Formally, a game consists of the following elements[1]:

1. A set $\mathcal{N}$ of players,
2. A set $\mathcal{A}$ of the actions for each of the players,
3. A set $\mathcal{C}$ of the consequences according to the actions chosen by the players,
4. A consequence function $g : \mathcal{A} \to \mathcal{C}$ that associates a consequence with each action,
5. Preferences $\succsim_i$ for each of the players i defined over all the possible outcomes $\mathcal{C}$.

In game theory, it is widely considered that the players are "rational" in their decisions. It implies that a player is aware of his alternatives and preferences. He forms expectations about any unknowns, and chooses his actions in the light of his preferences and knowledge. The actions are the alternative behaviours available to the players where the result of the entire interactive decision process is determined by the actions and the particular system in which the players are operating. Mathematically;

$$\mathcal{N} = \{n_1, n_2 \ldots n_n\},$$

$$\mathcal{A} = \{a_1, a_2 \ldots a_m\}$$

1 The notations and definitions used in this chapter are adapted from [31].

where it is assumed that any action/strategy from amongst the m strategies can be adopted by the n^{th} player. This selection of action is rational and is specified by the relative inclination of the player towards a particular result/outcome of the game with respect to all possible outcomes. It is formally denoted by the preference relation $\succsim$. Mathematically, $c^1 \succsim_i c^2$, that is, a player i prefers a consequence c^1 at least as much as c^2. A strict preference relation $\succ_i$ exists if c^1 is always preferred over c^2 by player i.

An n player game where each player has m actions, could reasonably have m^n different consequences and the resulting number of preference relations grows steeply. For a tractable analysis, utility/pay-off functions are used in which each player assigns a real number (called the pay-off or utility) to each consequence; $\mathcal{U} = \mathcal{C} \rightarrow \mathbb{R}$ such that for $c^1 \succsim_i c^2$, $u_i\left(c^1\right) \geq u_i\left(c^2\right)$.

Rationality implies that given any set $B \subset \mathcal{A}$ of actions that are feasible in some particular case, a rational player chooses an action $a^* \in B$ such that the consequence function g satisfies, $g(a^*) \succsim g(a)$ for all $a \in B$. It is similar to stating that the rational player attempts to maximize its utility by solving $\max_{a \in B} u_i\left(g(a)\right)$.

EXAMPLES Game theory has been widely studied in various domains of knowledge. Economic phenomena like duopoly, auctions, bargaining, etc. are typically modelled using game theory. Numerous other examples exist in diverse contexts. Some of these examples have become very famous due to their simplicity and large scale applicability to similar scenarios occurring in various fields. Here some of those are discussed;

Prisoner's Dilemma: Consider a scenario where two criminals have been arrested with crime charges. They are interrogated separately and each prisoner knows that if neither of them confesses the crime, they will be punished for lesser charges since there will be no strong proof of their crime. If this happens, each will get one year in prison. If both confess, both of them will be equally proven guilty and each will get 5 years in prison. If only one confesses and testifies against the other, the one who did not cooperate and lied to the police, will get a life sentence (10 years) and the one who did cooperate will get parole. If neither of them speaks out, they get minimal charges of 1 year imprisonment.

This situation is studied under a game theoretic setting where both the criminals are considered to be players of the game. Their individual pay-offs are represented as a matrix below according to the chosen actions where player 1 is the column player and player 2 is the row player.

	Confess	Do not Confess
Confess	(-5,-5)	(-10,0)
Do not Confess	(0,-10)	(-1,-1)

Table 2.1: Payoff Matrix for Prisoner's Dilemma

Battle of Sexes: Imagine another scenario where a couple that agreed to meet this evening, has to decide whether they will be attending the opera or a football match. The husband's most likely preference would be the football game. The wife obviously would like to go to the opera. However, both would prefer to enjoy each other's company and for that sake they also want to go to the same place rather than different ones. Given the situation where they have to make an independent decision without communicating with each other, now where should they go?

In this situation, a pleasure measure can be hypothetically assigned; 5 is the maximum that any one of them can achieve if he/she goes to his preferred place with his/her partner. In this case, the other person gets a slightly lower pleasure/pay-off (3) since the venue is not his/her favourite. If they go to their desired places alone, then both of them get the pay-off of 1. Their individual pay-offs can be represented as a matrix below according to the chosen actions.

	Football	Opera
Football	(5,3)	(1,1)
Opera	(0,0)	(3,5)

Table 2.2: Payoff Matrix for Battle of Sexes.

2.5.2 *Nash Equilibrium (NE)*

As discussed, players of a game are assumed to be rational utility maximizers. However, in game theoretic modelling, all players are considered to possess the same level of intellect hence all of them try to increase their pay-offs by choosing such actions that maximize their respective utilities. In doing so, the utilities of other players might suffer. NE defines that state of the game in which no player can unilaterally increase its pay-off without causing any harm to others. It was proposed and thus named after John Forbes Nash.

Definition 2.13: Nash Equilibrium

For player i, a strategy $a^* \in \mathcal{A}_i$ is a Nash equilibrium if,

$$u_i\left(a^*, a_{-i}\right) \geq u_i\left(a_i, a_{-i}\right) \forall i \in \mathcal{N}, a_i \in \mathcal{A}_i$$

where $\mathcal{A}_i$ is a vector of possible actions taken by player i .

In game theoretic formulations, efforts are dedicated to achieve the NE because once reached, there is no incentive for a player to change its strategy of action. In order to explain the concept briefly, the equilibria of the example games can be computed. In Prisoner's Dilemma, the case when both play conservatively and confess their crime, NE is achieved. It is so because if one remains silent, then the threat of betrayal from the other prisoner is so high that each of them prefers to confess his crime. Such an equilibrium is always in favour of the police because it gets easier for them to prove the crime.

In case of the battle of sexes, there are two NE from the table, i.e., both of them go together, be it football or opera. In any of the other states, the players have the incentive to unilaterally change their action and increase the pay-off.

2.5.3 *Pareto Optimality*

Pareto optimality deals with the choice of the best possible strategies of the players that lead to the most desired pay-off for each player. There are situations where one can have more than one Nash Equilibrium. Mostly in these cases, the Pareto-optimal is the best desired equilibrium. To define the concept, it is better to first define Pareto superiority:

Definition 2.14: Pareto Optimality

The strategy profile $a^{'}$ is superior to any other profile a, if for any player $i \in \mathcal{N}$,

$$u_i\left(a_i^{'}, a_{-i}\right) \geq u_i\left(a_i, a_{-i}\right) \forall i \in \mathcal{N}, a_i, a_i^{'} \in \mathcal{A}_i,$$

with strict inequality for at least one player. And a strategy profile a^{po} is Pareto optimal if there is not another profile that is Pareto superior to a^{po}.

2.5.4 *Classification of Games*

Games can be classified into different categories according to their properties.

2.5.4.1 *Non cooperative and cooperative games*

Games in which each player participates as an entity whose decisions are only dependent upon his/her own benefit are called non-cooperative games. Typical examples of non cooperative games are *Prisoner's Dilemma* and *Battle of Sexes.* In wireless communications, ALOHA based medium access control is a straightforward scenario that can be casted in the framework of non-cooperative games. Similarly, in cooperative games, the joint actions of players are analysed where they cooperate with each other to maximize a utility function (mostly global). Cooperative networks are fast emerging in wireless communications and the phenomenon of cooperative listening/relaying can be modelled using cooperative game theory.

2.5.4.2 *Strategic and extensive games*

In strategic or static games, the players make their decisions at the beginning of the game, once and for all. Once this decision is made, the players cannot change their actions. The *Prisoner's Dilemma* and *Battle of Sexes* are both strategic games. On the other hand, the model of an extensive game defines the case in which agents choose their actions sequentially. The players can make decisions knowing the previously taken actions. Extensive games can be finite or infinite. Situations of bargaining with alternative offers is one example of extensive games. The actions $\mathcal{A}$ in strategic and extensive games can be regarded as strategies $\mathcal{S} = \{s_1, s_2 \ldots s_m\}$. Most of the theoretic development of game theory prefers the term 'action', while in applied game theory, the term 'strategy' is used more frequently. Throughout the thesis, the terms actions and strategies and the notations are used interchangeably.

2.5.4.3 *Zero-sum games*

Games can be divided according to their pay-off structures. A game is called zero sum game, if the sum of the utilities is constant in every outcome. Whatever is gained by one player, is lost by the other players. Gambling is a typical zero-sum game. Neither of the example games are zero-sum games. Zero-sum games are also called strictly competitive games. In telecommunications, the games are usually not zero-sum games. However, if a simple scenario like, for example, accessing the bandwidth of a single link, is studied, the game may be a zero-sum game.

2.5.4.4 *Games with perfect and imperfect information*

If the players are fully informed about each other's moves, the game has perfect information. Games with simultaneous moves always have imperfect information, thus only extensive games can have perfect information. A game with imperfect information is a good framework in telecommunications, because the users of a network seldom know the exact actions of the other users. However, it is often more convenient to assume perfect information.

2.5.4.5 *Games with complete and incomplete information*

In games with complete information the preferences of the players are common knowledge, i.e. all the players know all the utility functions. In a game of incomplete information, in contrast, at least one player is uncertain about another player's preferences. A sealed-bid auction is a typical game with incomplete information.

2.6 RESOURCE PRODUCTION AND DIVISION GAMES

In this section some commonly used games to model the production and division of resources are introduced. These games directly find their application in wireless networks where resources like time, frequency, code etc., are acquired and divided among several candidates.

2.6.1 *Cournot Game*

The Cournot model is a one period game derived from economic game theory. In this game, two firms, i.e., $\mathcal{N} = 2$, produce an undifferentiated product with a known demand curve. It is important for each firm to determine the quantity of their production since they have another firm competing over the same product. The two firms compete by choosing their respective level of output simultaneously. Each firm assumes that the other firm has a fixed output. Based on this assumption, each firm chooses its output. Once each firm chooses its output quantity, the market determines the price at which it is sold. If there is an inverse demand function $f^{-1}(\mathcal{D})$, where $\mathcal{D}$ is the market demand of the commodity, the individual outputs say O_1 and O_2 for each firm respectively can be related to the market price $\mathcal{P}$ as

$$\mathcal{P} = (O_1 + O_2) f^{-1}(\mathcal{D}).$$

If the total cost function of each firm is $\mathcal{C}_1(O)$ and $\mathcal{C}_2(O)$ respectively, then each firm has a total revenue when the pair of outputs chosen by the firms is (O_1, O_2) is $O_1(O_1 + O_2)f^{-1}(\mathcal{D})$ and $O_2(O_1 + O_2)f^{-1}(\mathcal{D})$ receptively and the profit is

$$\mathcal{U} = \begin{cases} O_1\mathcal{P} - \mathcal{C}_1(O) & \text{firm 1} \\ O_2\mathcal{P} - \mathcal{C}_2(O) & \text{firm 2} \end{cases}.$$

Each firm tries to determine its output using the best response strategy whereby it tries to maximize its utility/profit. For every pair of best response functions, there is a unique Nash equilibrium where the two best responses meet.

2.6.2 *Stackleberg Game*

Stackelberg is a variant of Cournot game in which the action space is sequential. One firm/player decides the output quantity first and the second one decides after knowing the output of the first player. Relying on the assumption of the other firm's rationality, the first player chooses an output accordingly to maximize its own profit. Due to this sequential nature, first mover advantage is harnessed by firm 1. Consequently, Stackelberg game is often referred to as the *leader-follower* game. The second firm's output is usually limited since the first firm always chooses large output quantities. It leaves less margin for the second firm to produce an output. Like the Cournot game, for every pair of best response functions, there is a unique Nash equilibrium where the two best responses meet.

2.6.3 *Rubinstein Bargaining*

This classic game deals with the problem of dividing a pie between two players of the game. It follows the concept of sequential bargaining where various offers can be made, accepted or rejected to reach an agreement. It captures the natural desire of a rational player to get as much as possible share in the pie with least delay. In order to determine the split between two players, one and two, Rubinstein proposed a model in which both players take turns making offers about how to divide a pie of size one. Time runs from $t = [0, 1, 2, ...]$. At a time instant t_i, player 1 proposes a split $(x_0, 1 - x_0)$ where $x_0 \in [0, 1]$ which the player 2 can accept or reject. If player 2 accepts, the game ends and the pie is consumed. If player two rejects, the game continues to time t_{i+1}, when player 2 proposes a split $(y_1, 1 - y_1)$. At this point, player 1 decides to accept or reject the offer. In this way, as long as the two players do not mutually agree to a split, the game continues. Thus, if an agreement to split the pie $(x, 1 - x)$ is reached at time t_i, the pay-off for the player 1 is $\delta_1^t x$ and the pay-off for the player 2 is $\delta_2^t(1 - x)$, for some delay discounting factors δ_1, $\delta_2 \in (0, 1)$.

Definition 2.15: Equilibrium of Rubinstein Bargaining for finite horizon [81]

For a game played for two rounds of sequential bargaining game, the unique sub-game perfect equilibrium involves an immediate $(1 - \delta_2, \delta_2)$ split.

Definition 2.16: Equilibrium of Rubinstein Bargaining for in-finite horizon [81]

Player 1 suggests a split $(x, 1-x)$ with $x = (1-\delta_2)/(1-\delta_1\delta_2)$. Player two accepts any division giving her at least $1-x$. Player two suggests a split $(y, 1-y)$ with $y = \delta_1(1-\delta_2)/(1-\delta_1\delta_2)$. Player one accepts any division giving her at least y. Thus, bargaining ends immediately with a split$(x, 1-x)$ depending upon which player proposes first.

2.6.4 *Nash Bargaining, Axiomatic Approach*

Definition 2.17: Equilibrium of Nash Bargaining [82]

A bargaining problem is a pair $\{\mathcal{S}, d_i\}$, where $\mathcal{S} \subset \mathbb{R}^2$ is compact (i.e. closed and bounded) and convex, $d \in \mathcal{S}$, and there exists $\{s \in \mathcal{S}\}$ such that $s_i > d_i$ for i = 1, 2. The set of all bargaining problems is denoted $\mathcal{B}$. A bargaining solution is a function $f : \mathcal{B} \to \mathbb{R}^2$ that assigns to each bargaining problem $\{\mathcal{S}, d_i\} \in \mathcal{B}$ a unique element of $\mathcal{S}$.

Nash adopted an axiomatic approach to solve the bargaining problem. According to Nash, "one states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely". Nash 1953. Nash imposes four axioms on a bargaining solution $f : \mathcal{B} \to \mathbb{R}^2$

- ***Invariance to Equivalent Utility Representations***: Suppose that the bargaining problem $\{\mathcal{S}', d'\}$ is obtained from $\{\mathcal{S}, d\}$ by the transformations

$s_i \to \alpha_i s_i + \beta_i$ for $i = 1, 2$ where $_i > 0$. Then $f_i(\mathcal{S}', d') = \alpha_i f_i(\mathcal{S}, d) + \beta_i$ for $i = 1, 2$.

- ***Symmetry***: If the bargaining problem $\{\mathcal{S}, d\}$ is symmetric, then $f_1(\mathcal{S}, d) = f_2(\mathcal{S}, d)$.
- ***Independence of Irrelevant Alternatives***: If $\{\mathcal{S}, d\}$ and $\{\mathcal{K}, d\}$ are bargaining problems with $\mathcal{S} \subset \mathcal{K}$ and $f(\mathcal{K}, d) \in \mathcal{S}$, then $f(\mathcal{S}, d) = f(\mathcal{K}, d)$.
- ***Pareto Efficiency***: Suppose $\{\mathcal{S}, d\}$ is a bargaining problem, $s \in \mathcal{S}$, $t \in \mathcal{S}$, and $t_i > s_i$ for $i = 1, 2$. Then $f(\mathcal{S}, d) \neq s$.

2.7 GAME THEORETIC MODELLING OF WIRELESS AD-HOC NETWORKS

Wireless ad-hoc networks are infra-structureless decentralized networks with a wide variety of applications in practical networks. Each wireless node participates in data transmission, routing and forwarding data for other nodes, etc. These operations are mostly related to the dynamics of channel access and connectivity in the network. Major advantages of the these networks are the rapid deployment, robustness, flexibility and support for mobility, which are useful in a wide range of applications. Ad hoc networks are valuable when temporary networks are needed. They are also useful in areas, where natural disasters have destroyed existing infrastructure. The independence of infrastructure is also a great benefit in a battlefield environment. These networks form the basis of mobile ad hoc networks (MANETs), wireless mesh networks, wireless sensor networks etc.

Modelling and controlling the dynamics of a wireless ad hoc network are the biggest challenges in the study of these networks. Game theory becomes a useful tool in such situations due to its ability to capture real world interactions

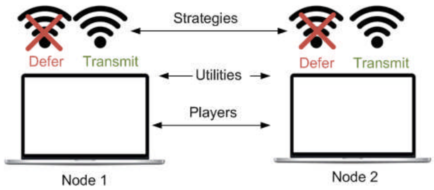


Figure 2.4: Mapping of wireless network elements to the components of a game

where the outcome of interaction depends upon the dynamic decisions of the players and current network state. Wireless nodes are rational and sometimes selfish entities that are participating in the network to maximize their utility. The utility can be measured in terms of the actual throughput, link quality, energy consumption, delay constraints etc., depending upon the nature of the network and its operations. In cooperatively communicating relaying, request by a source node and its acceptance by possible relays forms a game in which the utility of the source node depends upon the maximum number of relays and the end to end throughput whereas the relays tend to optimize their energy requirements and decide whether to participate in relaying or not. Such a problem is considered in [83] and its NE is studied.

Different elements of a wireless network map onto a certain component of a game when these networks are modelled using game theory. The following figure shows how wireless network nodes (laptops), can have various possible strategies (transmit/defer) resulting in an output/profit (SNR, throughput, delay etc). The wireless nodes become the players, having a strategy set and a set of possible outcomes (shown as the screen output of the nodes in figure 2.4).

A game $\mathcal{G}$ for the above scenario can be defined as

$$\mathcal{G} = \left(\mathcal{N}|\mathcal{N} \in \{1,2\}, \mathcal{S}|\mathcal{S}_{\mathcal{N}} \in \{\text{Transmit}, \text{Defer}\}, \mathcal{U}|\mathcal{U}_{\mathcal{N}} \in \{\text{Throughput}\}\right).$$

2.7.1 *Game Theoretic Modelling of Physical Layer in Wireless Networks*

Game theory has been applied to model physical layer phenomena such as power control and waveform adaptation. In [84], an algorithm for power control for wireless networks has been discussed. In their work, the goal of each player is to adjust the transmit power based on the SNR conditions at the receiver. Similar work has been done in [85], where various forms of power adaptations have been suggested for power control. Also an overview of various other possible applications of game theory on the physical and higher layers have been discussed.

Waveform adaptation is another technique which is used at the physical layer to improve the quality of communication. Interference avoidance is a technique suggested to improve the signal quality at the receiver. Game theory has been used for this purpose in [86]. In their work, Mackenzie et. al. considered a fixed uplink power based CDMA system in which adaptive code-on-pulse signatures are used. In [87] signature sequence adaptation in CDMA systems is modelled as a non-cooperative game and it is shown to achieve a Nash equilibrium when each players maximises its own utility.

2.7.2 *Game Theoretic Modelling of Medium Access Control (MAC) in Wireless Networks*

As discussed earlier, wireless ad hoc networks are decentralized where several nodes utilize the same medium for their communication. The channel access mechanism here is mostly random which results in contention and collisions among the data packets of different nodes during channel access [88]. Game theory has been applied in the following ways to model the MAC of wireless networks;

1. A non-cooperative group of nodes sharing a channel and using ALOHA and slotted ALOHA to access the channel. The nodes adjust their transmission probabilities to maximize the desired throughputs.

2. Cooperative models for selfish nodes employing slotted ALOHA based MAC.

3. Selfish users with perfect information operating in multi-packet reception with slotted ALOHA.

In [89], the behaviour of competing users of common wireless medium using ALOHA is considered. They consider a setup in which users advertise their transmission probabilities q_n to all the nodes in the networks and try to achieve their desired throughput. However, they keep this value, y_n, secret. The nodes play a repeated game with adjusting their transmission probabilities $q_i^0, q_i^1, q_i^2, \cdots$. The equilibria of this game are studied.

In [90], a one shot game of n selfish users is considered. These users compete for channel access and their success(utility) depends upon channel access and cost of transmission c. The equilibria of the game when the number of contenders tend to infinity is considered.

Both cooperative and non cooperative MAC models for slotted ALOHA are studied in [91]. A two state Markov Chain is considered where channel access probabilities are changed to maximize the network throughput. It is shown that cooperation is always useful and some throughput is attained when nodes cooperate with each other. However, when the nodes become selfish, the network throughput collapses and a Prisoner's Dilemma like solution is attained.

The above games are incomplete information games. In [92], a perfect information situation is studied. Equilibrium medium access probabilities are determined.There is a plethora of wireless communication models inspired by game theoretic concepts. [93, 94] and references therein provide a detailed survey of various techniques and models in wireless networks inspired by game theory.

2.8 DYNAMIC SPECTRUM ACCESS

Dynamic spectrum access (DSA) is defined as "a technique by which a radio system dynamically adapts to select the operating spectrum to use available spectrum holes with limited spectrum use rights" in IEEE standard draft 1900.1. DSA has been sought as a promising solution to alleviate the spectrum scarcity and improve the performance of legacy systems. Following the definition of DSA, a radio technology that is agile and flexible is best suited to exploit DSA. CRs are envisioned to be environment aware, intelligent and flexible in behaviour. Thereby, CRs are naturally a strong candidate to benefit from DSA schemes. In the hierarchy of dynamic spectrum access, a band of frequencies owned by a legacy/primary user can be accessed by secondary users. The PU holds exclusive rights to access the owned spectrum band. Beyond CRs, the SUs may have an ad hoc nature like sensor networks or a structured form like small cells etc. Hence DSA is an important concept with a wide applicability in diverse wireless systems.

DSA is usually studied in a non-cooperative setting where the primary user is unaware of the presence of the secondary users. As stated earlier, this approach is referred to as the commons model. Since this approach does not require any modification to the existing structures of primary license holders,

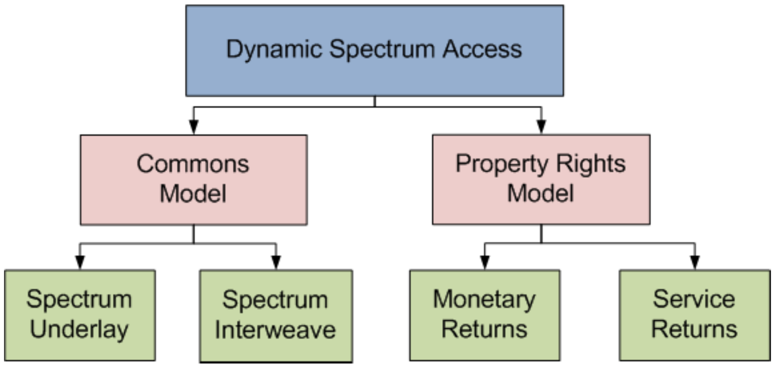


Figure 2.5: Hierarchical Structure of DSA.

commons model is widely studied as a beneficial spectrum sharing technique. Most of the literature on CRs also exploits this mode of spectrum exploitation. It provides ease of implementation, however,

- the secondary access to the spectrum is only opportunistic/intermittent depending upon the availability of spectrum,
- increasing interference at the primary receivers due to the operation of secondary users can degrade the performance of the primary network.

A cooperative model of DSA is regarded as the property rights model. According to this model, the access to the SUs is granted under the knowledge and agreement of the PU. It is a structured model that treats the spectrum as a trade commodity. This model supports the rigid spectrum allocation boundaries but provisions access of secondary users to it. The sharing of the spectrum usually follows an etiquette, thereby, provides QoS assurances for both PUs and SUs.

A hierarchical view of DSA and its types is shown in Fig. 2.5.

2.9 COMMONS MODEL

Commons model can further be classified into two types. Without the knowledge of their existence, SUs share the spectrum with the PU in underlay or interweave mode. Maintaining a low interference, in spectrum underlay SUs transmit along with primary transmissions. In interweave model, SUs exploit the opportunities of spectrum access by sensing the spectrum to determine when the primary is inactive. Fundamental limits of cognitive operation via theoretic analysis were determined in [95]. Also in [96], the information theoretic limits of the performance of CRs is studied assuming knowledge of primary's message at the CR.

Spectrum sensing has been extensively studied in the context of interweave model of spectrum sharing. These studies mainly differ in the signal processing techniques used and amount of knowledge about the primary signal. Energy detection is a simple way to sense the presence of the primary activity. Feature detection is suggested as a technique to sense the primary activity if partial knowledge of the primary signal is available [97]. Covariance based signal detection was proposed in [98] where there the primary signal characteristics are assumed to be unknown. A comprehensive study of various schemes is given in [99]. Along with independent sensing, cooperative sensing schemes are also studied in [20] and reference therein.

Spectrum sharing between multiple SUs is suggested in a variety of studies. Opportunistic sharing on the basis of peak power constraints is studied in [100]. Under maximum collusion on primary's packets constraint, spectrum sharing was suggested in [101]. Traffic models with queuing analysis was considered in [102]. A comprehensive survey of various spectrum assignment schemes is presented in [103].

2.10 PROPERTY RIGHTS MODEL

As stated before, the property rights model is driven by the economics of spectrum sharing in which spectrum is treated as a commodity owned by the legacy/incumbent network. The valuation of the commodity can be done by the owner, buyer/lessee or both. Afterwards, it is traded on the basis of the valuation. In this model, unlike the usual assumption of obliviousness to the spectrum owner, the owner is aware of the presence of non incumbent users and their access to its spectrum. Broadly, the literature on this model can be divided into two classes on the basis of the type of remuneration when spectrum is traded.

1. Monetary returns
2. Service return

The allocation of spectral resources takes place when an agreement on constraints/price put forwarded by either the primary or secondary user or both is reached. These are regarded as the etiquettes and the entire process of DSA is regarded as dynamic spectrum leasing (DSL).

2.10.1 *Monetary returns*

In [104], a competitive pricing model has been suggested in which multiple primary users offer their spectrum to the SUs. Authors suggest a non-cooperative game to determine a price to maximize the profit of a PU while maintaining its QoS. In [105], the authors present a comprehensive model for competitive spectrum pricing by PUs where the secondaries show an adaptive buying behaviour. In [106], an auction based mechanism is proposed where the SUs bid to buy spectrum bands from the primary spectrum owner who acts as the auctioneer, selling idle spectrum bands to make a profit. Under interference limiting constraints, tariff in dollars per bit is proposed as the price for sharing primary spectral resources with an SU in [107]. In [24], spectrum sharing in cognitive underlay networks is studied. The authors propose an auction based mechanism in which social welfare is maximized. In [108], a pricing based scheme is proposed to discourage the selfish behaviour of users in the network for collusion avoidance. In [109], the authors have introduced spectrum trading for heterogeneous cellular network with macro and femto base stations. A refunding based offloading scheme was introduced where MUs are allowed to access femto cell resources in exchange for an appropriate revenue refunding.

2.10.2 *Service Returns*

Under this model, the benefit of the PN for sharing its spectrum with SUs is in terms of the services they can provide to improve the performance of the primary system. It was explored by Simone et al. in [28] where an analytical study of service based DSL for one way communication is provided and cooperative diversity of the secondary relays has been exploited. The authors present a leasing framework in which the secondary users relay the data of the primary network to get a fixed time during which they can access the primary spectrum. A Stackelberg (leader-follower) game as explained in Sec. 2.6.2 has been suggested to determine the duration of leasing time to the secondary nodes for their cooperation. The framework is characterized by a hierarchical structure, where the primary optimizes the leased time and amount of cooperation based on the knowledge of the effects of its decision on the behaviour of the SUs. Further Simone et. al in [28] extend their framework from a single primary to multiple primary users where all of them compete to get relaying services from the secondary while reimbursing them with spectrum access time that assures minimal QoS for them. In [26], the same framework is carried forward and applied in an ARQ based model where a portion of the retransmission slot is leased by the legacy network to the relays for their traffic in exchange for cooperative retransmission by the relays. In the [110], secondary user decodes and forwards the interference signal to a primary to help it mitigate interference. In return, the secondary gets an opportunity to access the spectrum. Chiarotto et al. in [111] has studied bandwidth leasing when the secondary users help in opportunistic routing of the primary data over multiple hops. In [27], the authors consider an infrastructured hierarchical spectrum leasing approach. In their work, they consider multiple primary nodes that select their respective individual relays for cooperation. Karim et. al in [112] consider a multiple primary, multiple CR scenario where they all communicate to a common destination. They use a Lyapunov optimization function to determine the share in leasing time of the cooperating CRs. They study both long term and short term rewards. In [113], the authors study the two way relaying based on PNC. They study the optimal beamforming design to increase the achievable capacity. In [114], the authors investigate a time sharing based resource sharing scheme using PNC and beamforming. In [115], an incentive based approach for offloading is introduced where they studied a time division scheme for uplink communication in macro cells.

In some literature, the primary allows secondary nodes to share its spectrum as long as the QoS of primary is not affected. In [116, 25], the PUs get rewarded when the SUs access their spectrum as long as the interference is kept below the interference cap. In [24], spectrum auction is suggested where the total interference power at the primary is the constrained. SINR based charge and transmit power based charge for SUs is studied in the paper.

The DSL schemes in this thesis differ from the above in the following ways. Firstly, these studies abstract out the spatial geometry of the network. The impact of network geometry and provision of negotiation over the leasing time is not studied in these papers. In this thesis, however, DSL is investigated in a geometric framework where the capacity of direct and DSL based communication is studied in terms of the spatial characteristics of the secondary network. As discussed, Nash bargaining has been used for solving various problems of resource allocation in wireless networks. However, to the best of the author's knowledge, it has not been exploited to model leasing time division by any previous study. Prior studies have mostly relied upon the Stackelberg model of resource division where one of the players dictates its terms and acts as the leader while the other one is influenced by the leader's decision. In this thesis, Nash Bargaining framework is extended to enable the primary and secondary users to reach a mutual agreement over the leasing time. In this thesis, both point-to-point uni-directional and bi-directional communication is studied using DSL. Infrastructured cellular networks are also studied in later chapters. In previous studies usually time or frequency band sharing is considered. In the extensions to the DSL model in this thesis, bargaining over the number of cooperators is also studied. Hence, the thesis examines a variety of possible resources that can be shared in a wireless network resulting in improved spectral efficiency. Finally, the studies regarding the energy efficiency of CRNs mostly consider a generic scenario where spectrum sensing is employed. Particularly, the energy efficiency of DSL has not been studied before. The energy efficiency of DSL based communication for point-to-point and infrastructured large scale networks is derived and analysed in this thesis. Hence, the thesis provides a unified DSL framework for both the spectral and energy efficiency.

3

DSL FOR CRNS—MODELLING AND ANALYSIS

Summary of Contribution

The key objective of this chapter is to investigate the design space of a DSL empowered large scale CRN collocated with a point-to-point primary communication link. The ultimate design objective is to improve both the network level energy efficiency and the spectral efficiency through the exploitation of cooperation gains rendered by the proposed optimally dimensioned DSL mechanism.

3.1 INTRODUCTION

In contrast to passive spectrum sharing between a PN and a CRN (provisioned through hierarchical access mechanisms), DSL employs an active approach to improve the overall spectrum utilization through DSA [116]. More specifically, under DSL enabled DSA:

1. The PN has a certain incentive for rewarding the CRN with access to its licensed spectrum. Incentives can be either monetary or non-monetary in nature.
2. The PN can dynamically adapt the rewarding mechanism by observing changes in its incentive. In other words, the PN can actively control the

amount of spectral resources it is willing to share across various dimensions of the Hertzian medium. Note that the radio spectrum has a multi-dimensional nature, i.e., variations across time, frequency, polarity, space, etc., all determine the available spectral resources.

3. The PN can ensure that the required QoS constraint for its own users is guaranteed. Thus transparency in terms of the performance of the PN is an intrinsic feature of DSL.

While it is easy to argue that DSL enabled CRNs have the potential to maximize the spectral utilisation, it is not clear if the potential gains are harnessed at a cost of increased energy consumption. This leads to the following design question:

Design Issue 3.1

Is it possible to develop a DSL mechanism which maximizes the network level spectral efficiency (which is a function of individual spectral efficiencies of the PN and the CRN) while also ensuring an increase in the network wide energy efficiency?

Additionally, another related design issue stems from the fact that the existing literature on DSL, refrains from considering the impact of the network topology and propagation uncertainties on the promised potential gains. Specifically:

Design Issue 3.2

Does DSL successfully deliver its promised spectral/energy gains under realistic channel propagation conditions while considering the topological uncertainties due to varying spatial dynamics of the CRN?

3.2 RESEARCH OBJECTIVES & CONTRIBUTIONS

In this chapter, answers to the above-mentioned design issues/questions are investigated by developing a framework to quantify the performance of both the PN and the CRN under a proposed DSL mechanism. The aim of this chapter is to study how DSL can be used as an energy efficient alternative for PN communication while improving the spectral efficiency of the SN. The proposed DSL mechanism considers that in a dense CRN deployment, cooperation of the CRs with the PN can be traded for spectrum access opportunities. More specifically, the intrinsic distributed diversity gain provided by a cooperative relaying protocol and reduced propagation loss due to dense deployment can be treated as a resource which a CRN can offer to a PN to sustain its operations i.e., maintaining its QoS while reducing its energy expenditure. However, the improvement in the performance of the PN through cooperation comes at a cost paid by the CRs in terms of their energy consumption. Consequently, the CRs wish to trade the incurred cost for a spectrum access opportunity. Thus in a nutshell, the proposed DSL mechanism provides transmission opportunities to the CRs if they in return help in improving the energy utility of the PN through inter-network cooperation. Consequently by shrinking the transmission window of the PUs, during the remaining time the spectral resources are reserved to provide access to the cooperating CRs. Such a DSL approach for a CRN communication where the services of cooperative relaying by the CRs serve as an incentive for the PN spectrum leasing resulting in improved EE of the PN and better SE of the CRN is the focus in this chapter.

In the proposed DSL mechanism, the PN leases its spectrum to the SN, which in this chapter is a CRN, and forwards its data to the CRs for cooperative transmission. The SUs/CRs relay the PN data during some fraction of the leasing time. For the remaining leased time, the CRN exclusively uses the spectrum and carries out its own communication. The share in time and bandwidth for the secondary communication is the motivation for the CRs to cooperate with the PN. A PU is interested in maximizing the time for which the CR nodes relay its data. Greater negotiation power can help the PN to ensure that for most of the time the CRs relay its data. On the other hand, the CRs intend to schedule their own transmissions for most of the leasing time. The reward for the cooperative relaying of a few CRs can ripple across the entire CRN, enabling spectral access. However, the cooperating CRs need to negotiate with the PN to get a certain duration of leasing time so that the entire CR network can benefit from it. Such selfish yet rational behaviour of the PUs and SUs makes the appropriate division of the leasing time very important for successful DSL operation. An optimal division is described as a division which is mutually agreed upon and satisfies the demand of both networks. A greater negotiation power can help each network to procure more time for itself.

In this chapter, fundamental mathematical modelling and analysis of the proposed DSL mechanism for CRN is pursued. Despite the wide scale applicability and potential benefits of service based DSL, contributions in the existing literature are very limited as discussed in Chapter 2. The following aspects of DSL for the CRNs need to be addressed:

1. In order to analyse a DSL empowered CRN, it is important to consider a realistic network topology for both the PN and the CRN. The necessity of a realistic geometry based network model manifests itself not only in topological considerations but also in terms of the efficient selection of the cooperation areas under the DSL mechanism. Unfortunately, it is common practice to ignore the network geometry in order to simplify the analytical model. However, such simplifications come at the cost of limited insights.

2. In order to ensure fairness and mutual satisfaction, it is important to divide the leasing time in a way that both the PUs and the CRs agree to their share of time. In previous studies, as discussed in Ch. 2, this division has been influenced more by the decision of the primary network which needs to possess cross-network channel state information (CSI) (i.e., CSI of the secondary network) to make the bargaining decisions. The CRN needs to observe the primary action and only decides in reaction to primary decision.

3. It is important to quantify how the division between cooperating and leasing time is dictated by the negotiating power of the PN and the CRN. Unlike existing studies, it is important to develop a comprehensive model to capture the scenarios where one network exercises greater influence on the decision, yet attains mutual agreement over the division of the leasing time and vice versa.

4. As mentioned earlier, the energy requirements of the design of any communication system has become a key concern due to the rapid growth in energy consumption. This warrants a formal analysis of the energy efficiency of leasing to measure its viability.

In summary, the main contribution of this chapter is to address the above mentioned design issues for enabling DSL based spectrum sharing in large scale wireless networks. In previous studies based on obliviousness of the primary user regarding the presence of CRs, the secondary access to the spectrum is only opportunistic/intermittent depending upon the availability of spectrum with no QoS assurances. In the cooperative model of DSL under the property rights model proposed in this work, the access to the CRs is granted under the knowledge and agreement of the PU. It is a structured model that treats the spectrum as a trade commodity. This model supports the rigid spectrum allocation boundaries but provisions access of secondary users to it. The sharing of the spectrum follows specific etiquettes (as explained in later sections), thereby, provides QoS assurances for both PUs and CRs. The CRs have a guaranteed duration of spectrum access with QoS promise under the DSL model which is not the case under others models based on spectrum sensing. The CRs may rely on the leased spectrum under DSL in the primary owned frequencies. However, the CRs might have access to other frequencies under different spectrum allocation/access mechanisms, thereby, having a positive throughput even in the absense of a DSL agreement in certain frequencies.

3.3 KEY FINDINGS

In this chapter, tools from stochastic geometry and game theory are used to build a quantitative framework for investigating the introduced design issues. The developed framework explicitly incorporates the impact of randomness rendered by the channel impairment process and geometry of the cooperation region on the DSL mechanism. In turn, these considerations demonstrate that a desired data transmission rate with a certain reliability can be provisioned for the PU links by leasing the spectrum to the CR nodes occupying spatially suitable locations. As a reward for cooperation, the entire CRN obtains access to the spectrum and thus the CRs can schedule their transmissions at a reasonable rate among themselves. The provision of negotiation between the PN and the CRN, over the division of leased time (reserved for cooperation with the PN and for the CRN communication) is ensured using the Nash bargaining framework. Unlike existing literature, a mutual agreement based division is attained that ensures proportional fairness for both networks. Also, the PN is not required to have CSI knowledge of the CRN. The work quantifies how the individual bargaining powers of the PN and the CRN can influence the division of cooperation and leasing time. Furthermore, it is shown that for equal bargaining powers, out of the total DSL operational time, $20\% - 50\%$ of the time is reserved exclusively for the CRN which otherwise is dormant. It is demonstrated that the entire CRN can benefit from the leasing time that is procured by the cooperation of a few CRs with the PN. The variety of possible divisions of the leasing time ensures the flexibility and wide scale applicability of the considered DSL model. Moreover, the quantification of the energy requirements of the legacy and the DSL networks are established which to date was an open issue. The results indicate that DSL empowered networks can be more than 10x energy efficient as compared to traditional networks. It is shown that choosing a smaller cooperation area is more energy efficient for the PN. It is also shown that DSL is spectrally efficient at the network level where the CRN improves its spectral access considerably. At the same time, the QoS requirements for both the PN and CRN can be guaranteed.

3.4 SYSTEM AND NETWORK MODEL FOR DSL EMPOWERED CRN

3.4.1 *Network Geometric & Physical Layer Model*

A primary link operating in the presence of a geographically co-located secondary network is considered. For simplicity, it is assumed that the primary communication link (P_{tx}, P_{rx}) is formed by a primary receiver (P_{rx}) located at the origin and a primary transmitter (P_{tx}) located at a distance $r_p > 1$ from P_{rx}. A region of 'exclusion' with radius ϵ is centred at the P_{rx}. The secondary network is formed by the CR nodes, whose locations form a stationary PPP Φ of intensity λ as defined in Sec. 2.3.2.3. From the theory of PPP, the probability of finding $k \in \mathbb{N}$ CRs in an area $A \subset \mathbb{R}^2$ is given as

$$P_k = \Pr\{k \,\text{nodes in}\, A\} = \frac{(\lambda\,|A|)^k}{k!} \exp\left(-\lambda|A|\right), \tag{3.1}$$

The average number of the CRs in an arbitrary region A with area $|A|$ is quantified as $\lambda|A|$. Each CR transmitter S_{tx} communicates with an associated CR receiver S_{rx} when spectral access is granted by the PN. In this chapter, it is considered that the CR receivers are associated with their nearest CR transmitter. In other words, 'nearest neighbour association' is adopted for the transmitter-receiver pairing in the CRN. Notice that such association mechanism indeed captures many emerging CR deployment paradigms. Specifically, it captures overlaid cellular CRN where the CR transmitters may be data aggregators for machine type communication or small cells associated with MUs based on the average path-loss, etc.

Based on the relative distances from the P_{tx} and the P_{rx}, the nodes lying within a radius r_p between the two primary nodes are expected to best serve as the potential relays for the PN in cooperation mode under DSL operation[1]. It is considered that nodes within a radius ϵ from the P_{tx} or the P_{rx} are excluded from the cooperation phase of the DSL. This particular constraint reflects that only the nodes lying in the proximity of the half-way mark between the primary nodes can become cooperative relays. The key motivation behind such selection is to minimize the energy penalty, while balancing the average channel gain for the two hop communication. In other words, the condition where the average channel gain for the first hop is significantly larger than the second hop and vice versa are excluded. It is well known that an optimal relaying strategy can be devised by selecting relays which balance the average gains for both hops [117]. Consequently, the cooperation region, bounded by a sector, i.e., $\sec(\theta, r)$ of radius $r_p - 2\epsilon$ and an angle θ in radians, is considered to be the effective area of cooperation in DSL operation mode. Formally, it can be denoted as,

$$A_c\left(\theta, r_p, \epsilon\right) = \left\{(r,\theta) \in \mathbb{R}^2 : \epsilon < r \leq r_p - \epsilon \,\text{and}\, \theta \in [0, 2\pi]\right\},$$

where $\epsilon \geq 1$. The selected relays also form a PPP $\Phi_r \subset \Phi$ with an average number of CR relay nodes $k = \lambda \left|A_c\left(\theta, r_p, \epsilon\right)\right|$ in the region $A\left(\theta, r_p, \epsilon\right) \subset \mathbb{R}^2$.

It is assumed that the wireless channel suffers from path-loss and small-scale fading. For a distance r between any arbitrary pair of nodes, the channel between them can be expressed as $ahl\left(r\right)$ [35] where the fading power gain h is an independent and identically distributed (i.i.d.) exponential random variable with a unit mean, a is a frequency dependent constant and $l(r)$ is the distance dependent path-loss function. For the sake of simplicity, a is considered to be unity throughout the rest of the discussion. The power-law path-loss function $l(r) = \min(1, r^{-\alpha})$ is upper bounded by unity which corresponds to the refer-

1 Such a selection is inspired by the optimal forwarding area selection techniques [117, 118]

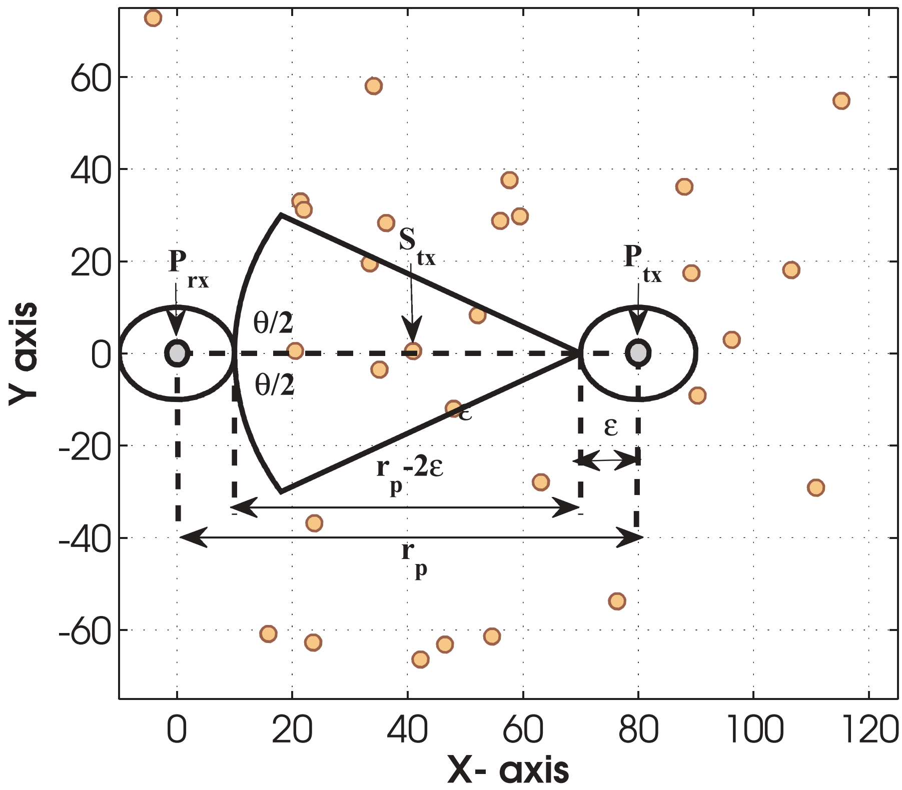


Figure 3.1: Geometric model of the network.

ence distance. Also, $\alpha > 2$ is the operational environment dependent path-loss exponent. The noise at the receiver front end, is considered to be additive white Gaussian noise (AWGN) with power σ^2. For a given transmit power P and link distance r, the SNR at a receiver is given as

$$\mathrm{SNR} = \frac{Phl\left(r\right)}{\sigma^2}. \tag{3.2}$$

Similarly, in the presence of co-channel interference, the received SINR is defined by adding the aggregate received interference power I in denominator of eq. (3.2).

3.4.2 *MAC Layer Model and Bargaining Game*

A primary system in which there is a certain rate demand (R_{dir}) for a sustainable link operation at a desired reliability ($\tilde{\rho} = 1 - \rho$) is considered. In other words, the QoS demand for the PUs is completely characterized by the desired rate R_{dir} and the percentage of time $\tilde{\rho}$ over which this rate can be guaranteed. To meet this demand, the PU has a choice between continuing its communication in the legacy mode through direct communication or through the cooperative relaying of CRs via spectrum leasing mechanism. In direct communication, the primary transmitter communicates with its corresponding primary receiver at a rate R_{dir} for a duration T. The duration T corresponds to the duration of a temporal spectral resource such as the length of a transmission frame.

Under DSL operational mode, the primary transmitter indicates its willingness to lease the spectrum for the same time duration T to the CR nodes inside a certain cooperation region $A_c\left(\theta, r_p, \epsilon\right)$. The choice of θ and willingness to lease are indicated over a dedicated control channel. The idea to lease the spectrum to CRs within a certain sector of angle θ is motivated by the average achievable rate in cooperative DSL mode. The primary network is able to estimate this rate based on the knowledge of the average density λ of CRs and the average channel gain in the network. By choosing an appropriate value of θ (which in turn determines the number of CR cooperators), the primary can maximize the throughput of DSL. It is to be noted that a large number of cooperators corresponds to increased energy expense of the CR network. In order to compensate for this, the primary network will have to reimburse the CRs accordingly in terms of the duration of exclusive spectrum access to the CRs. Also, higher CR density λ with small θ or a sparse CR deployment with a lager sector effectively result in comparable number of CRs participating in the relaying operation.

3.4.2.1 *Phases of DSL*

The process of dynamic spectrum leasing can be divided into three sub intervals:

Phase 1: Broadcast

The primary broadcasts its data to be relayed to the CR transmitters for a time $t_{ps} < T$.

Phase 2: Cooperate

During the second sub-interval, called the cooperation phase, k secondary nodes that are best suited for relaying on the basis of their geographical location, cooperatively relay the data of the P_{tx} to the P_{rx} for a time $t_{sp} < T$ by forming a distributed k-antenna array through ideal orthogonal distributed space time coding (DSTC) [46]. The details of DSTC codebook and operational parameters can be found in [46] and [28].

Phase 3: Reimburse

Out of the total leased time T, the last sub-interval is reserved for the S_{tx_i} to carry out their own transmission to their respective receivers, S_{rx_i}. In other words, it is a fare that the primary has to pay in return for the relaying services of the secondary. In this duration $t_{ss} = T - t_{sp} - t_{ps}$, the primary refrains from transmission and grants exclusive access to the secondary network.

If the PN decides to seek the help of the CRs, it broadcasts a leasing beacon over the control channel. This beacon contains the information of cooperation and exclusion region θ, ϵ and the demand of relaying co-operation duration t_{sp}[2]. The concept of an exclusion region is exploited for minimizing the interference to the PU and also enhancing the cooperative transmission rate by selecting nodes within the exclusion region that lie between the primary transmitter and its corresponding receiver as shown in Fig. 3.2. The CR nodes employ listening mechanism over control channel. Beacon enabled signalling is adopted for DSL to initiate and agree on the leasing parameters. Listening only on the control channel is an energy efficient way for the CRs to monitor the primary activity. In this approach, the CRs only listen to short control messages, whereas, if the control channel is not used, then the CRs have to monitor the entire PU activity to learn about possible spectrum availabilities. The CRs are assumed to be aware of their location with respect to the primary transmitter and receiver. Upon the reception of the leasing beacon, only those CRs that lie within the desired cooperation region participate in cooperation.

2 The PN is assumed to be aware of the average fading characteristics of its link with the secondary transmitters.

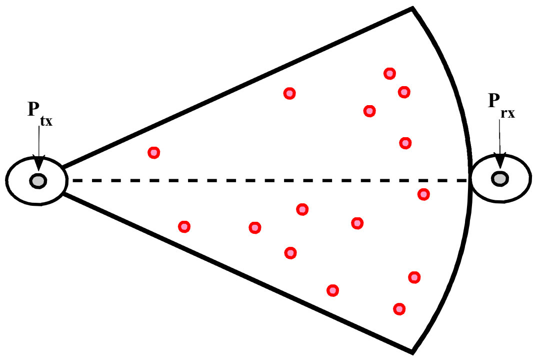


Figure 3.2: Secondary nodes in the cooperation region.

Based on the leasing information and the potential cooperation cost, the CRs also establish their reimbursement duration demand t_{ss}[3].

The process of bargaining over the demand of t_{sp}, and t_{ss} is executed and if the negotiations are successful, a leasing agreement is reached. In DSL, during the first interval t_{ps}, the primary transmitter broadcasts its data to be relayed to the CRs at a low power, $\bar{P}_p < P_p$, since only geographically close CR relays need to receive and relay the data. In the second interval, the CRs cooperatively relay the data to the P_{rx} using DSTC for a duration t_{sp}. As a result of the leasing agreement, the entire CRN gets access to the spectrum for a duration t_{ss}. All the secondary nodes transmit with the same power P_s during the cooperation and reimbursement phase. P_s is significantly lower than the transmit power of the primary $P_s \ll P_t$. This maintains low energy consumption in DSL and also ensures that in the last phase of secondary communication, the aggregate power of all the selected relay nodes does not increase excessively to avoid very high interference.

3 The primary is assumed to be aware of the average fading characteristics of its link with the secondary transmitters.

3.4.2.2 *Bargaining Game*

During the process of leasing, the most crucial factor is the division of leasing time between the above three phases. It is important that each operational element of the network gets enough share of time to meet its transmission throughput requirements. To ensure such a time division, a network level game is formulated where each of player, i.e., primary network (player 1) and the secondary network (player 2) engages itself in an arbitration for the time division over a control channel. As stated, the primary user initiates the leasing process. In response, the secondary users determine their demand and adopt a strategy according to the primary offer. If the offer is acceptable, the game is concluded and leasing is successful. If the CRs want to bargain further, another round of offer and respective response is played. In case the negotiations are unsuccessful, the game ends and the leasing is not done. It is further assumed that the CRs form a homogeneous network in terms of the hardware platform, leasing time demands and they do not show malicious or selfish behaviour.

During the process of leasing, the primary has a bargaining power Δp. The bargaining power of the primary determines the bias of the division of time in favour of the primary's demand. Similarly, the secondary CR network has a bargaining power Δs. The provision of variable bargaining powers in the model makes it flexible and adaptable to various real network settings. These include scenarios where the primary network has greater inherent power to determine the division of leasing time. For example, when the data traffic of the primary link is low or the channel conditions are favourable, the primary might have a greater bargaining power. Similarly scenarios where the CRs have a greater power can also be well studied using this model.

3.4.2.3 *Assumptions*

For simplicity and tractability of the analysis, it is assumed that the PN and the CRN are aware of the CSI within their respective networks. A practical implementation of such information exchange can be found in [119]. The CRs are aware of their location with respect to the primary transmitter and receiver. Moreover, the PU and the SUs are considered to be in perfect time synchronization with each other. Cost effective methodologies for implementing time synchronization in ad hoc networks have been suggested in [120] , hence encouraging the proposal of the time sharing based communication scheme introduced here. The control beacon signal by the PU to initiate spectrum leasing can also be used for synchronization between the primary and the secondary nodes.

It is important to note that the proposed DSL scheme inherently avoids any additional delay in the primary communication. This is ensured as the total duration of operation of direct and DSL mode is dimensioned to be the same, i.e., T. The primary communication is carried out in the first two phases of DSL; the duration of $t_{ps} + t_{sp} < T$. Hence, there is no added latency in DSL based communication.

3.5 ANALYSIS OF DSL

3.5.1 *Average Link Capacities R_{dir}, $\overline{R}_{ps}$, $\overline{R}_{sp}$ and $\overline{R}_{ss}$*

Under conventional operation, the legacy network continues its communication over the direct link with its respective receiver at a certain rate R_{QoS}. Due to the small scale fading, the communication link is subject to outage. Thus, enforcing a certain reliability constraint restricts the operational rates to a limited regime. In other words, if $\tilde{\rho} = 1 - \rho$ is the reliability constraint, then the maximum rate which can be sustained is given as

$$R_{\text{dir}} = \sup \left\{ R_{\text{QoS}} : \text{pout}(R_{\text{QoS}}) \leq \rho \right\}, \tag{3.3}$$

where $\text{pout}(R_{\text{QoS}})$ is the link outage probability at a particular desired rate R_{QoS}. The performance of the direct link (P_{tx}, P_{rx}) pre-dominantly is noise limited, since it is assumed that there is no interference caused by the CRN to the primary transmission. The instantaneous capacity R of this link can be defined as,

$$R = \log_2(1 + \text{SNR}), (\text{bits/sec}) \tag{3.4}$$

where SNR is as defined previously and h_p is the channel power gain between the source and the destination, P_p is the transmit power and $l(r_p)$ is the distance dependent path loss between the nodes. The ρ-outage rate R_{dir} is defined as the largest rate of transmission R such that the outage probability of the direct primary link is less than ρ. For a conventional operation mode it can be quantified as follows:

Proposition 3.1: Direct Communication Rate

The ρ-outage rate, R_{dir}, for the link (P_{tx}, P_{rx}) is given as,

$$R_{\text{dir}} = \log_2\left(1 - \left(\frac{P_p h_p l\left(r_p\right)}{\sigma^2}\right) \ln\left(1-\rho\right)\right), \qquad \left(\frac{bits}{sec}\right) \tag{3.5}$$

Proof. According to eq. (3.4), the instantaneous data transmission rate from P_{tx} to P_{rx} depends upon the channel gain h_p between the source and the destination. However, to maintain this rate above R_{QoS}, the outage probability needs to be quantified. Mathematically,

$$\begin{aligned} \text{pout}(R_{\text{QoS}}) &= \Pr\left\{\log_2(1+\text{SNR}) < R_{\text{QoS}}\right\}, \\ &= \Pr\left\{\frac{P_p h_p l\left(r_p\right)}{\sigma^2} < 2^{R_{\text{QoS}}} - 1\right\}, \\ &= \Pr\left\{h_p < \left(2^{R_{\text{QoS}}} - 1\right)\left(\frac{\sigma^2}{P_p l\left(r_p\right)}\right)\right\}, \end{aligned}$$

Using the exponential distribution of the channel power gain, the ρ-outage probability becomes

$$\rho = 1 - \exp\left(-\left(2^{R_{\text{QoS}}} - 1\right)\left(\frac{\sigma^2}{P_p l\left(r_p\right)}\right)\right).$$

The transmission rate achieved between a typical primary source-destination pair for a given quality of service constraint ρ is

$$R_{\text{dir}} = \log_2\left(1 - \left(\frac{P_p l\left(r_p\right)}{\sigma^2}\right) \ln\left(1-\rho\right)\right), \tag{3.6}$$

which is the rate that satisfies the primary user's desired QoS constraint. □

When the spectrum is leased to the SUs, the cooperative link performance is dictated by the attainable rate over the relay link, i.e., the cooperative channel capacity. The cooperative channel capacity depends upon both i) the transmission rate R_{ps} achieved between the primary transmitter and any selected relay during the first leasing sub-interval and ii) the rate R_{sp} between the selected relay nodes and the primary receiver assuming that DSTC cooperation is employed. Also, as mentioned earlier, nodes centred only in the effective area of communication, $A_c(\theta, r_p, \epsilon)$, are considered for cooperation.

Proposition 3.2: Attainable Rate between the PU and CR relay

The average transmission rate from the primary transmitter to secondary relay, $\overline{R}_{ps}$, is upper-bounded as,

$$\overline{R}_{ps} = \log_2\left(1 + \left(\frac{1-\exp\left(-\lambda\frac{\theta}{2}\left(r_p - 2\epsilon\right)^2\right)}{\left(r_p - 2\epsilon\right) - C}\right)^{\alpha}\frac{\bar{P}_p}{\sigma^2}\right), \qquad \left(\frac{bits}{sec}\right) \quad (3.7)$$

where $C = \sqrt{\frac{\pi}{2\lambda\theta}}\exp\left(-\lambda\frac{\theta}{2}\left(r_p - 2\epsilon\right)^2\right)\operatorname{erfi}\left(\left(r_p - 2\epsilon\right)\sqrt{\frac{\lambda\theta}{2}}\right)$ and $\operatorname{erfi}(x)$ is the imaginary error function such that $\operatorname{erfi}(x) = 2\sqrt{\pi}\int_{t=0}^{x}\exp\left(-t^2\right)dt$.

Proof. At any secondary relay i located at a distance r_i, the achieved communication rate is,

$$R_{ps_i} = \log_2\left(1 + \frac{P_p h_{ps_i} l\left(r_i\right)}{\sigma^2}\right), \quad (3.8)$$

where h_{ps_i} is the channel gain between P_{tx}, $\bar{P}_p < P_p$ and the arbitrary relay i and $l\left(r_i\right)$ is the path loss. The average value of the rate R_{ps_i} at any typical relay can be found using a similar approach as in Proposition 3.1. However, it is important to mention that the overall performance of the cooperative communication link will be bounded by the minimum transmission rate of all the relays. For simplicity, it is considered that the relay at the maximum distance from the primary transmitter will result in the worst rate performance. The rate is bounded by the worst case performance by considering the distance between P_{tx} and the relay node to be maximum. The average distance $\mathbb{E}[r]$[4] from the primary transmitter to its farthest neighbour within a sector with angle θ and radius r i.e., $\sec(\theta, r)$ in a 2-dimensional PPP can be found out on the same lines as in [69] to be

$$\mathbb{E}[r] = \frac{(r_p - 2\epsilon) - C}{1 - \exp\left(-\lambda\frac{\theta}{2}\left(r_p - 2\epsilon\right)^2\right)}, \tag{3.9}$$

where $C = \sqrt{\frac{\pi}{2\lambda\theta}} \exp\left(-\lambda\frac{\theta}{2}\left(r_p - 2\epsilon\right)^2\right) \operatorname{erfi}\left(\left(r_p - 2\epsilon\right)\sqrt{\frac{\lambda\theta}{2}}\right)$. Using the above relation, the rate of transmission from P_{tx} to the farthest neighbour within $A_c(\theta, r_p, \epsilon)$ can be given as

$$R_{ps} = \log_2\left(1 + \frac{h_{ps} l(r) \bar{P}_p}{\sigma^2}\right). \tag{3.10}$$

Using the Jensen's inequality and the fact that $\mathbb{E}\left[h_{ps}\right]$, the average rate takes the form;

$$\overline{R}_{ps} = \mathbb{E}\left[R_{ps}\right] \leq \log_2\left(1 + \frac{\mathbb{E}\left[l(r)\right]\bar{P}_p}{\sigma^2}\right). \tag{3.11}$$

4 Due to the stationarity of the point process, it can be safely assumed that the distance r_n can be measured while considering the primary transmitter to be at the origin.

From eq. (3.9), the average path loss can be calculated if the secondary node density and the area of cooperation are known. Hence the average rate of the furthest relay in the first phase is upper bounded by,

$$\overline{R}_{ps} = \log_2\left(1 + \left(\frac{1 - \exp\left(-\lambda\frac{\theta}{2}\left(r_p - 2\epsilon\right)^2\right)}{\left(r_p - 2\epsilon\right) - C}\right)^{\alpha}\frac{\bar{P}_p}{\sigma^2}\right), \tag{3.12}$$

and the other relays can achieve a better rate than eq. (3.12). □

In the second phase of cooperation, the selected secondary relays form a $k = \lambda A_c(\theta, r_p, \epsilon)$ antenna array and perform DSTC to send the data to the receiver with a rate R_{sp}. The rate of communication when DSTC is employed for multiple relay transmission to a common destination has been evaluated in [28, 121, 110, 46]. In the context of the geometric modelling of dynamic spectrum leasing, the DSTC communication rate is used and its mean value is determined considering the geometric parameters.

Proposition 3.3: Average CR to PU rate

The average transmission rate, $\overline{R}_{sp}$, when k secondary relays, i.e., $k \in |\Phi_r|$ form an antenna array, where secondary relay i is located at a distance r_i from P_{rx} is given by

$$\overline{R}_{sp} = \log_2\left(1 + \frac{\lambda\theta P_s}{\sigma^2}\left(\frac{(r_p - \epsilon)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha}\right)\right), \qquad \left(\tfrac{bits}{sec}\right) \tag{3.13}$$

where, the secondary transmits with a power P_s, the channel gain between S_{tx} and P_{rx} is h_{sp_i}.

Proof. The rate of the DSTC communication with k relay nodes in Φ_r is given as,

$$R_{sp} = \log_2 \left(1 + \sum_{i \in \Phi_r} \frac{P_s h_{sp_i} l\left(r_i\right)}{\sigma^2} \right).$$

Using the Jensen's inequality to find the average value of R_{sp} gives,

$$\overline{R}_{sp} = \mathbb{E}\left[R_{sp}\right] \leq \log_2 \left(1 + \frac{P_s}{\sigma^2} \mathbb{E}\left[\sum_{i \in \Phi_r} \frac{h_{sp_i}}{r_i^{\alpha}} \right] \right). \tag{3.14}$$

The aggregate contribution to the received power due to the channel gains and the distances of all the relays from P_{rx} can be calculated using Campbell's theorem for stationary PPP (see Sec. 2.3.3 Def. 2.10). Mathematically,

$$\mathbb{E}\left[\sum_{i \in \Phi_r} \frac{h_{sp_i}}{r_i^{\alpha}} \right] = \lambda \mathbb{E}\left[h_{sp}\right] \int_{\mathbb{R}^2 \cap \text{sec}(\theta, r)} r^{1-\alpha} \text{d}r\text{d}\theta,$$

where $\mathbb{E}\left[h_{sp}\right] = 1$ due to the assumption of Rayleigh distributed fading channel between the secondary transmitter and the primary receiver. In this case, the region of interest is the two dimensional area bounded by the sector of radius $r_p - 2\epsilon$ and angle θ in radians. Solving the above integral for this area, the expectation results in

$$\mathbb{E}\left[\sum_{i \in \Phi_r} \frac{h_{sp_i}}{r_i^{\alpha}} \right] = \lambda\theta \left(\frac{(r_p - \epsilon)^{2-\alpha} - \epsilon^{2-\alpha}}{2 - \alpha} \right). \tag{3.15}$$

Similarly, the variance, $\text{var}\left[\sum_{i \in \Phi_r} \frac{h_{sp_i}}{r_i^{\alpha}}\right]$, of the aggregate signal can also be found using the Campbell's theorem as

$$\text{var}\left[\sum_{i \in \Phi_r} \frac{h_{sp_i}}{r_i^{\alpha}} \right] = \lambda\theta \left(\frac{(r_p - \epsilon)^{2-2\alpha} - \epsilon^{2-2\alpha}}{2 - 2\alpha} \right). \tag{3.16}$$

Plugging the above value for $\mathbb{E}\left[\sum_{i\in\Phi_r}\frac{h_{sp_i}}{r_i^{\alpha}}\right]$ in eq. (3.14), the average rate comes out to be as in eq. (3.13). □

In the last phase of spectrum leasing, all the secondary transmitters communicate with their respective receivers. A nearest neighbour model of the CR source destination pairs is considered in this chapter where each transmitter only communicates with its nearest receiver [66] as shown in Fig. 3.2. It is of interest to know the average transmission capacity of the (S_{tx}, S_{rx}) link, $\overline{R}_{ss}$. In this case, all the secondary transmitters in the CR network simultaneously communicate with their receivers in order to utilize the leased bandwidth for their own transmission. In this phase, similar to the direct communication, a realistic situation is considered under which the secondary network also operates under a fixed QoS constraint R_{QoS_s}.

Proposition 3.4: Attainable Rate for CRN during Reimbursement

The average rate, $\overline{R}_{ss}$, for the link (S_{tx_i}, S_{rx_i}) where the channel power gain h_{ss_i} between the source i and its destination (nearest neighbour) is exponential, the transmit power is P_s, is given as,

$$\overline{R}_{ss} = \frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{\left(2^{R_{\mathrm{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}}\exp\left(\frac{\left(\pi\lambda\left(\tau\left(R_{\mathrm{QoS}_s}\right)+1\right)\right)^2}{4\left(2^{R_{\mathrm{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}\right) \tag{3.17}$$

$$\times\ Q\left(\frac{\pi\lambda\left(\tau\left(R_{\mathrm{QoS}_s}\right)+1\right)}{\sqrt{\left(2^{R_{\mathrm{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}}\right)\bar{R}_{\mathrm{th}}. \qquad \left(\frac{bits}{sec}\right) \tag{3.18}$$

where R_{Qos_s} is the desired threshold rate for secondary communication.

Proof. To model the communication, the probability that a CR can successfully communicate with its nearest receiver at a desired rate R_{Qos_s} is sought. The received SINR at the secondary receiver can be quantified as,

$$\text{SINR} = \frac{P_s h_{ss_i} l\left(r\right)}{\sigma^2 + \sum_{j\epsilon\Phi_r} P_s h_{ss_j} l\left(r_j\right)}, \tag{3.19}$$

where, h_{ss_j} is the channel gain between the secondary transmitter j causing interference at S_{rx_i} at a distance r_j. The link distance r is the distance of a secondary receiver S_{rx} from its nearest CR transmitter S_{tx}. The interference from the set of neighbouring CR transmitters is denoted as $I = \sum_{j\in\Phi_r} h_{ss_j} l\left(r_j\right)$. The link distance r follows the nearest neighbour distribution of a PPP given as

$$f_R(r) = 2\lambda\pi r \exp\left(-\lambda\pi r^2\right). \tag{3.20}$$

The probability of outage on the (S_{tx}, S_{rx}) communication link if the required transmission rate is R_{Qos_s} during this phase can be stated as

$$\begin{aligned}
p_{\text{out}} &= \Pr\left\{\log_2\left(1+\text{SINR}\right) < R_{\text{Qos}_s}\right\}, \\
1 - p_{\text{out}} &= \mathbb{E}_r\mathbb{E}_I\left[\mathbb{P}\left\{\frac{h_{ss}l(r)}{I + \sigma^2/P_s} \geq \left(2^{R_{\text{Qos}_s}} - 1\right)\right\}|r, I\right], \\
&= \int_0^\infty \mathcal{L}_{I_s}\left(\left(2^{R_{\text{Qos}_s}} - 1\right) I r^\alpha\right) \\
&\times \exp\left(-\left(2^{R_{\text{Qos}_s}} - 1\right) r^\alpha \sigma^2/P_s\right) f_R(r)\mathrm{d}r.
\end{aligned} \tag{3.21}$$

The expected value $\mathbb{E}_I\left[e^{-\left(2^{R_{\text{Qos}_s}}-1\right)r^\alpha I}\right]$ or $\mathbb{E}_I\left[e^{-sI}\right]$ where $s = \left(2^{R_{\text{Qos}_s}} - 1\right) r^\alpha$ can be regarded as the Laplace transform of interference $\mathcal{L}_I\left(\left(2^{R_{\text{Qos}_s}} - 1\right) r^\alpha I\right)$. This approach greatly simplifies the calculation of such integrals. Adopting the approach in [122], the Laplace integral can be written as

$$\mathbb{E}_I\left[e^{-\left(2^{R_{\text{QoS}_s}}-1\right)r^{\alpha}I}\right] = \mathbb{E}_{\theta,h_{ss_j}}\left[\exp\left(-s\sum_{j\in\theta}{}^{h_{ss_j}}/r_j^{\alpha}\right)\right], \quad (3.22)$$

$$= \mathbb{E}_{\theta}\left[\prod_{j\in\Phi^c}\mathbb{E}_h\left[\exp\left({}^{-sh_{ss_j}}/r_j^{\alpha}\right)\right]\right], \quad (3.23)$$

$$\overset{(a)}{=} \mathbb{E}_{\theta}\left[\prod_{j\in\Phi^c}\frac{1}{1+{}^{s}/r_j^{\alpha}}\right], \quad (3.24)$$

$$\overset{(b)}{=} \exp\left(-2\pi\lambda\int_r^{\infty}\left(1-\frac{1}{1+{}^{s}/u^{\alpha}}\right)u\,du\right), \quad (3.25)$$

where (a) follows from solving the expectation over the exponential channel gain and (b) from applying the probability generating functional (PGFL) of the PPP (see Sec. 2.3.3 Definition 2.11) where $u \to r_j$. Simplifying and substituting $s = \left(2^{R_{\text{QoS}_s}}-1\right)r^{\alpha}$ in eq. (3.25) results in

$$\mathcal{L}_I\left(\left(2^{R_{\text{QoS}_s}}-1\right)r^{\alpha}I\right) = \exp\left(-2\pi\lambda\int_r\frac{2^{R_{\text{QoS}_s}}-1}{\left(2^{R_{\text{QoS}_s}}-1\right)+\left(\frac{u}{r}\right)^{\alpha}}u\,du\right).(3.26)$$

From the geometry of the network, the minimum separation distance between a CR receiver in a network and an interfering neighbouring CR transmitter is greater than the distance from its nearest CR transmitter. To find out the Laplace transform of this interference, eq. (3.26) is used. Simplifying and substituting $v = \left({}^{u}/r\left(2^{R_{\text{QoS}_s}}-1\right)^{-4}\right)^2$ in eq. (3.25) results in

$$\mathcal{L}_I\left(\left(2^{R_{\text{QoS}_s}}-1\right)r^{4}I\right) = \exp\left[-\pi\lambda r^2\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)}\int_{\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)}}^{\infty}\frac{1}{1+v^{1/2}}\right]v\,dv,$$

$$= \exp\left[-\pi\lambda r^2\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)}\right. \quad (3.27)$$

$$\left.\times\left({}^{\pi}/2-\tan^{-1}\left({}^{1}/\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)}\right)\right)\right].$$

Using the Laplace transform to solve eq. (3.21),

$$\begin{aligned} 1 - p_{\text{out}} &= 2\pi\lambda \int_0^{\infty} r \exp\left(-\left(\pi\lambda\left(\tau\left(R_{\text{Qos}_s}\right)+1\right)\right) r^2\right) \\ &\times \exp\left(-\left(2^{R_{\text{Qos}_s}}-1\right) r^{\alpha}\sigma^2/P_s\right) \mathrm{d}r \end{aligned} \tag{3.28}$$

$$\begin{aligned} p_{\text{out}} &= 1 - \frac{\pi^{\frac{3}{2}}\lambda}{\sqrt{\left(2^{R_{\text{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}} \exp\left(\frac{\left(\pi\lambda\left(\tau\left(R_{\text{Qos}_s}\right)+1\right)\right)^2}{4\left(2^{R_{\text{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}\right) \\ &\times Q\left(\frac{\pi\lambda\left(\tau\left(R_{\text{Qos}_s}\right)+1\right)}{\sqrt{\left(2^{R_{\text{Qos}_s}}-1\right)/\frac{P_s}{\sigma^2}}}\right). \end{aligned} \tag{3.29}$$

Here $\tau\left(R_{\text{Qos}_s}\right) = \sqrt{\left(2^{R_{\text{Qos}_s}}-1\right)}\left(\pi/2 - \tan^{-1}\left(1/\sqrt{\left(2^{R_{\text{Qos}_s}}-1\right)}\right)\right).$ $\square$

After computing the individual link transmission rates, the aim is to know the overall transmission rate achieved in the DSL operational mode. It is assumed that a decode and forward type single hop relaying mechanism is used in the cooperation phase. The effective DSL capacity R_{DSL} is then given as,

$$R_{\text{DSL}} = \min\{\overline{R}_{ps}, \overline{R}_{sp}\}. \qquad \text{(bits/s)} \tag{3.30}$$

3.5.2 *Optimal Division of Leased Time for Cooperation and Secondary Activity*

The most critical factor in the operation of spectrum leasing is the optimal division of the total leased time T between the time t_{sp} reserved for cooperation with the primary at a cooperative rate R_{DSL} and the remaining time t_{ss} for the secondary activity at a rate $\overline{R}_{ss}$. The goal of the primary node is to ensure that its rate and quality of communication, R_{dir} and ρ-outage probability respectively, are maintained by maximizing the time t_{ps} and t_{sp}. The primary node can ensure that $\overline{R}_{ps}$ attains the QoS rate R_{dir} by a proper choice of t_{ps} such that $t_{ps}\overline{R}_{ps} = TR_{\text{dir}}$. However, the remaining time ($T' = T - t_{ps}$) needs to be divided between phase two and three to get t_{sp} and t_{ss}.

The CR nodes intend to increase their benefits in terms of their spectrum utility and throughput by having spectrum access for maximum time and compensating for the cost of cooperation in relaying primary data. A very small fraction of t_{ss} will discourage the secondary, impacting cooperation and the overall throughput of the system suffers. On the other hand, prolonged t_{ss} will degrade the performance of the legacy network in terms of its bandwidth efficiency which is not acceptable in any case. Hence an intelligent division of time is very crucial for the operation of the network. Also, the secondary network must cooperate in relaying primary data for a time t_{sp} long enough so that the primary network maintains its communication standards. Hence the problem boils down to an optimal division of leasing time T' between phases two and three of DSL.

An optimal time division can be conveniently casted in the framework of Nash Bargaining: a game theoretic tool to model the situations of bargaining interactions as explained in Ch. 2 Sec. 2.6.4 . The situation can be modelled as a two player game using the Nash bargaining framework from cooperative game theory [123]. In this case, the primary transmitter is the first player whose utility is directly dependent upon the cooperation time t_{sp} and increases as it increases. For simplicity, we define the utility of the primary and the secondary node as;

$$\mathcal{U}_1(t) = t_{sp}, \tag{3.31}$$

and

$$\mathcal{U}_2(t) = t_{ss}, \tag{3.32}$$

respectively, where $t_{sp} + t_{ss} = T'$.

Bargaining as a two player game is considered because every single secondary node is representative of the utility of all the remaining secondary nodes as only the average rate values and equal transmit powers for all CRs are considered. The Nash bargaining framework is employed to model a situation in which the players negotiate for their agreement on a particular point out of a set of joint feasible payoffs $\mathcal{G}$. In this particular case, $\mathcal{G} \equiv \{g = (g_1, g_2) : g_i = \mathcal{U}_i(\mathbf{S}),\ i = 1, 2;\ \mathbf{S} \in S1 \times S2\}$, where the functions $\mathcal{U}_i(.)$ in this case of DSL are given in equations 3.31 and 3.32. $\mathbf{S}$ is the strategy of the i^{th} player in terms of the time it demands i.e., t_{sp}/t_{ss} from the strategy profile S_i. In Nash Bargaining, in case the negotiations render unsuccessful, the outcome of the game becomes $\mathcal{G} = (g_{01}, g_{02})$. It is a fixed vector known as the disagreement vector. The whole bargaining problem can be described conveniently by the pair $(\mathcal{G}, g_0)$. A pair of payoffs (g_1^*, g_2^*) is a Nash Bargaining solution if it solves the following optimization problem

$$\max_{g_1, g_2} (g_1 - g_{01})^{\Delta_p} (g_2 - g_{02})^{\Delta_s} \tag{3.33}$$

$$\text{subject to } \begin{array}{ccc} (g_1, g_2) & \in & \mathcal{G} \\ (g_1, g_2) & \geq & \mathcal{G}_0 \end{array}.$$

If the set $\mathcal{G}$ is compact and convex, and there exists at least one $g \in \mathcal{G}$ such that $g > g_0$, then a unique solution to the bargaining problem $(\mathcal{G}, g_0)$ corresponds to the unique solution of the optimization problem [123, 34][5]. Here Δ_p and Δ_s

5 From eqs. 3.31 and 3.32, the compactness and convexity of $\mathcal{G}$ can be seen as defined in Def. 2.6.4.

defined as $\left\{\Delta_p, \Delta_s \in [0,1] \,|\, \Delta_p = 1 - \Delta_s\right\}$ correspond to the bargaining powers of the primary and secondary network. Greater values of Δ_p and Δ_s correspond to higher bargaining powers. Increasing the bargaining power of a player corresponds to greater weightage of its preferences over the preferences of the other player. Increasing the power of one player implies decreasing power of the other.

In this case, the fraction of leased time should be large enough to ensure that the time-rate product of cooperation time t_{sp} and cooperative rate $\overline{R}_{sp}$ is at least equal or greater than the direct communication time T and rate R_{dir} product. During the second sub-interval, a secondary node must have enough time to at least overcome its cooperation cost cP_s given its average transmission rate $\overline{R}_{ss}$. Here c measures the bits transmitted per unit of power consumed.

Proposition 3.5: Division of Leasing Time

The optimal proportion of time for cooperative relaying is

$$t_{sp} = \frac{\Delta_p T' + \Delta_s \left(\frac{R_{\text{dir}}}{\overline{R}_{sp}}\right) - \Delta_p \left(\frac{cP_s}{\overline{R}_{ss}}\right)}{\Delta_p + \Delta_s}, \tag{3.34}$$

where the disagreement vector is $(t_{0p}, t_{0s}) = \left(\frac{TR_{\text{dir}}}{\overline{R}_{sp}}, \frac{cP_s}{\overline{R}_{ss}}\right)$ and secondary activity time $t_{ss} = T' - t_{sp}$.

Proof. From the definition of Nash Bargaining solution, the time division problem for a 2-player game can be written as

$$\max \left(\Delta_p \log\left(\mathcal{U}_1(t) - g_{01}\right) + \Delta_s \log\left(\mathcal{U}_2(t) - g_{02}\right)\right), \tag{3.35}$$

$$\text{subject to } T' = t_{sp} + t_{ss}.$$

From the definition of Nash Bargaining solution, the time division problem for a 2-player game can be written in a logarithmic form as above. Such representation of the maximization problem ensures proportional fairness of the solution for both the players [124]. Here the minimum required time for both primary and secondary is given as $(g_{01}, g_{02}) = (t_{0sp}, t_{0ss}) = \left(\frac{TR_{\text{dir}}}{R_{sp}}, \frac{cP_s}{R_{ss}}\right)$, which is the least time required to meet the respective objectives of QoS and cooperation cost compensation. The corresponding Lagrangian for the above optimization problem can be written as,

$$\begin{aligned} L(t_{sp}, \lambda_1, \lambda_2) &= \Delta_p \log\left(t_{sp} - t_{0sp}\right) + \Delta_s \log\left(t_{ss} - t_{0ss}\right) \\ &- \lambda_1\left(T' - t_{sp} + -t_{ss}\right). \end{aligned}$$

The original maximization problem can be solved by replacing t_{ss} by $T' - t_{sp}$ and using the Karush-Kuhn-Tucker (KKT) first order necessary conditions,

$$\frac{\delta L}{\delta t_{sp}} = \frac{\Delta_p}{t_{sp} - t_{0sp}} + \frac{\Delta_s}{t_{sp} - T' - t_{0ss}} = 0. \tag{3.36}$$

This follows from the definition of the Nash Bargaining problem that there exists a vector **S** such that the optimal value of the optimization problem is strictly positive. Solving for eq. (3.36) by using simple algebra, the result can be obtained as,

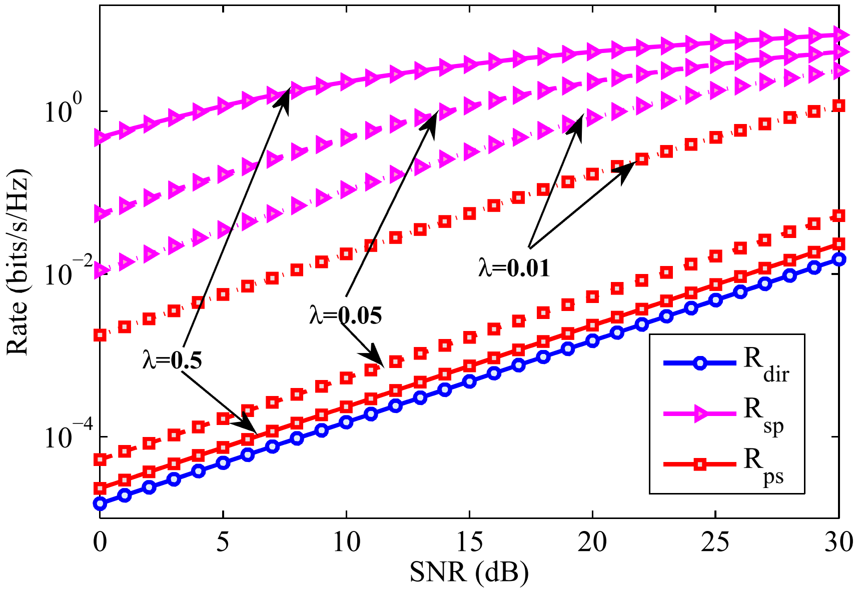
$$t_{sp} = \frac{\Delta_p T' + \Delta_s \left(\frac{R_{\text{dir}}}{R_{sp}}\right) - \Delta_p \left(\frac{cP_s}{R_{ss}}\right)}{\Delta_p + \Delta_s},$$

where the above equilibrium solution gives the optimal share of cooperation time t_{sp} out of the total leased time T that ensures a cooperative data transmission rate $\overline{R}_{sp} \geq R_{\text{dir}}$. It reserves the rest of the time for secondary user that at least allows the secondary to utilize the spectrum to compensate for their transmission cost during the cooperation phase. $\square$

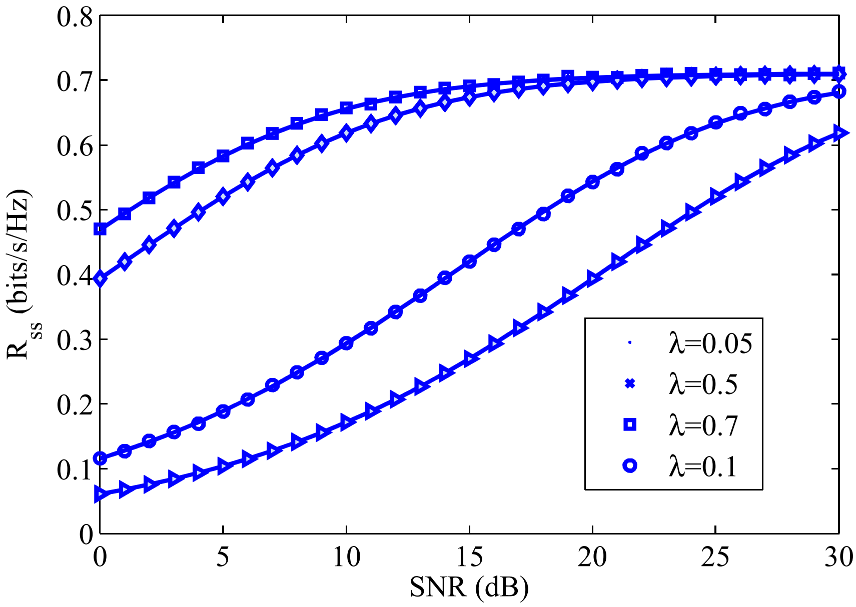
3.6 PERFORMANCE EVALUATION OF DSL

In this section, the design space of the DSL enabled CRN is investigated by employing the analytical model developed in the previous section. In order to verify the analysis and establish the validity of the assumptions made throughout, Monte Carlo simulations for the large scale DSL based CRN are performed. In order to simulate, a network radius of 200 meters in which secondary nodes are Poisson distributed with mean λ is considered. Direct communication under an outage constraint ρ at a transmit power P_p is simulated. Similarly, the operational phases of DSL are simulated. For each realization of the Poisson network, a Rayleigh distributed channel coefficient is generated. The transmission rate at the receiver for each spatial instance of the network is averaged for 10^4 different channel coefficients. This process is in turn repeated for 10^4 realizations of Poisson distributed CR network with intensity λ and the transmission rate is averaged. Secondary network communication under interference considerations is also studied in a similar fashion. All the simulations are carried out in MATLAB. Normalized values for transmit powers P_p and P_s are used. It is assumed that the secondary network operates at a low power profile i.e., $\sim \frac{1}{10^{\text{th}}}$ of P_p. Similar power profiles can be found for devices like HeNB in LTE rel. 12 and other examples in heterogeneous networks [125, 126]. These references also provide guidelines for the typical values of λ in the network.

Firstly, the average achievable transmission rates under both the normal and leasing mode of network operation are studied as shown in Fig. 3.3a. The rate



(a) Rates vs. SNR. $P_p = 1, r_p = 10$, $\bar{P}_p = P_s = 0.1$, $\epsilon = 1$.



(b) R_{ss} vs. SNR, $P_s = 0.1$.

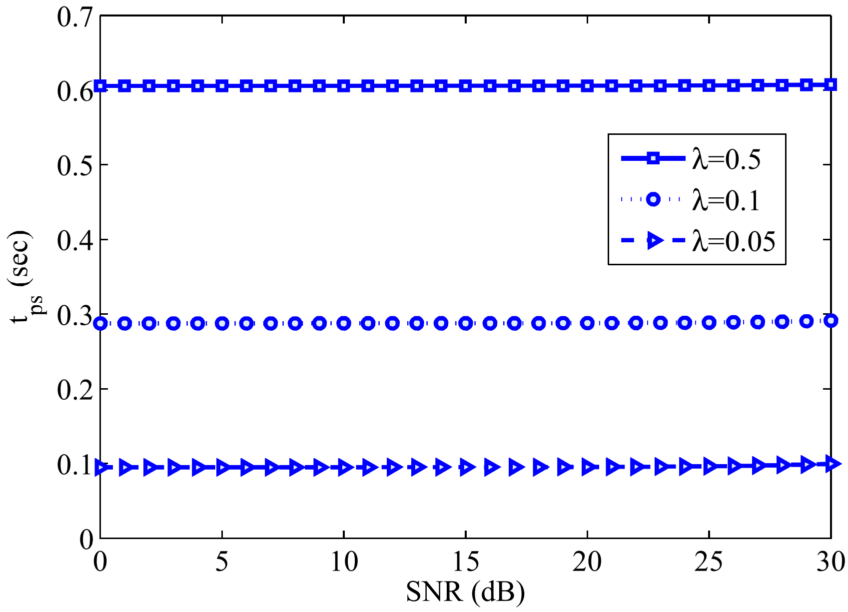
Figure 3.3: Achievable data rate of direct and DSL communication

under normal primary communication at a transmit power P_p increases with improving channel conditions. Here, the reliability in terms of the probability of success ($p_{\text{suc}} = 1 - \rho$) of direct communication is assumed to be 90%. The outage capacity, R_{dir}, defines the target capacity for communication in the primary network R_{th} for all operational modes i.e., direct and DSL. Under identical channel realizations, a demand for higher service quality (smaller ρ) straightforwardly results in lower R_{dir}.

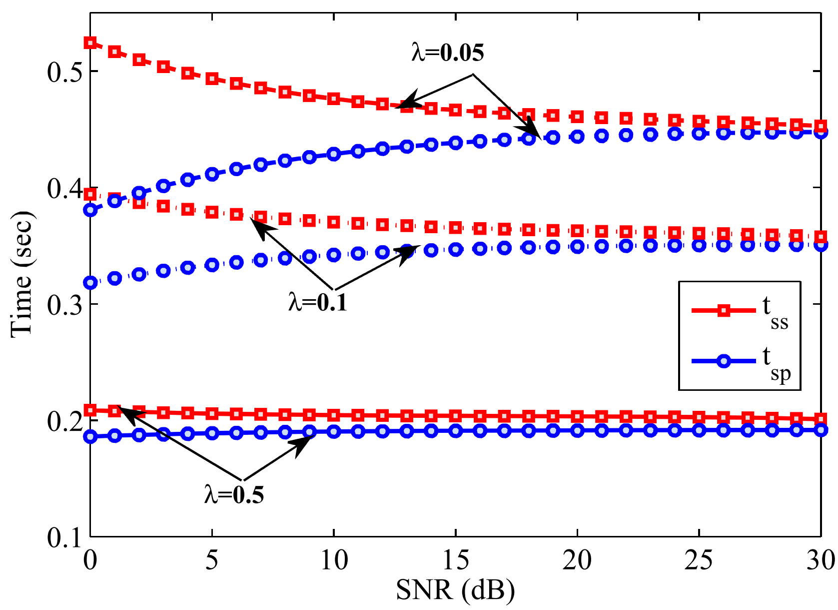
For the capacity analysis of DSL, the average achievable transmission rates in the three phases of leasing are studied. The capacity of the primary to secondary communication in the first phase is strongly dependent upon the number of secondary nodes present in the area of cooperation. As mentioned earlier, in this analysis, the lower bound to this rate is studied by considering the average transmission rate between the primary transmitter and the farthest relay. For very low secondary density, e.g., $\lambda < 0.01$, the probability of finding a neighbour in the region of cooperation is extremely low. For this reason, the capacity analysis for very sparse secondary network is not possible. For higher λ, it can be seen from Fig.3.3a that the average transmission rate $\overline{R}_{ps}$ is greater than R_{dir}. This phenomenon is a consequence of cooperation region selection such that relays are located in close proximity to both P_{tx} and P_{rx}. Hence greater rate is attained due to shorter distance between the relay and P_{tx}. However, if the number of secondary users increases in the cooperation region, the average distance between P_{tx} and the farthest node increases which follows from the average distance quantification in eq. (3.9). Hence $\overline{R}_{ps}$ decreases when λ increases (lower line in Fig.3.3a). However, the cooperative relaying rate $\overline{R}_{sp}$ increases with increasing relay density due to the diversity gain. Increasing λ increases the number of cooperating nodes, consequently, the rate $\overline{R}_{sp} \gg R_{\text{dir}}$ for increasing values of λ.

Along with the analytically drawn results, achievable transmission rates under a practical Poisson network are also shown in Fig. 3.3a. A PN with two nodes and a CRN for various λ are simulated in MATLAB. For each realization of the network, exponential distributed channel power gain is generated. The successful transmission probability at the rate $R_{\mathrm{QoS}}/R_{\mathrm{QoS}_s}$ at the receiver for each spatial instance of the network is averaged for 10^4 different channel coefficients. This process is in turn repeated for 10^4 network realizations. The practical simulation results are indicated by the lines running over the analytic results (analytic results are indicated with markers). It can be seen that the practical simulations closely match the analytic evaluation results. It validates the analytic formulation of DSL and the simplifying assumptions made for the simplification of the analysis. From Fig. 3.3a, it can be observed that the communication rates attained in all modes of primary communication, i.e., direct and DSL, are overall low (between $10^{-5}$and 1). It is merely a consequence of the long distance considered between the primary transmitter and receiver and the quantification of the rates per Hertz of bandwidth. If either higher transmit power or shorter primary link distance is considered, the direct communication rate can be improved for typical channel bandwidths, i.e., in MHz. In case of the DSL phase I, a lower bound on the rate is considered where most of the relays can achieve a better rate than the one shown in the figure. In phase II, higher rate is observed due to the cooperation gain. It can be further improved if higher transmission power of the CRs is considered. However, the primary link distance and transmit powers are kept constant in this study and the scenarios to follow for the sake of simplicity.

It is shown that the communication rate $\overline{R}_{ss}$ also increases with improving SNR values in Fig. 3.3b (here the desired QoS of the secondary network in terms of desired rate $\bar{R}_{\mathrm{th}}$ is 0.5 bits/sec). This is a consequence of the improved



(a) Time for first phase transmission R_{ps}.



(b) Time bargain for primary and secondary transmitters. $T = 1, c = 0.05, \lambda = 0.5$

Figure 3.4: Nash Bargaining Solution

signal strength at the receiver. As the density of the secondary nodes increases, the average transmission rate increases. However, $\overline{R}_{ss}$ tends to saturate with increasing SNR at higher values of λ. It is a consequence of the interference limited behaviour of the channel. Increasing interference due to increasing λ limits the increase in $\overline{R}_{ss}$. It is clear that increasing the desired threshold rate $\bar{R}_{\text{th}}$ causes the average rate to decrease because the decoding threshold at S_{rx} is raised. Hence, a graphical illustration of this result is intentionally skipped. The practical simulation results of $\overline{R}_{ss}$ are also shown in Fig. 3.3b which verifies the analytical derivations. It is to be noted that the rest of the results are based upon the communication rates of direct and DSL communication, which have been shown to be in a close agreement with each other. Therefore, the practical simulations of the remaining results can safely be assumed to be accurate and hence are skipped for the sake of brevity.

In order to intelligently exploit the diversity gains of DSL at low power, it is important to determine the appropriate operational time of each phase of DSL. The primary itself determines and communicates for time t_{ps} during the first phase such that $t_{ps}\overline{R}_{ps} = TR_{\text{dir}}$. In Fig. 3.4a, the time t_{ps} reserved for R_{ps} is shown. It can be seen that it increases with increase in the secondary network density. This behaviour follows from the lower transmission rate achieved with increasing λ as discussed earlier. Correspondingly, the time share of t_{sp} and t_{ss} i.e., T' decreases with increasing λ since a major portion of the time is reserved for primary to secondary transmission in the first slot.

The optimal relation for the division of the remaining leased time T' is found in eq. (3.34) and shown in Fig. 3.4b over a range of SNR values. At low CR densities, more time t_{sp} is required to harvest the gains from cooperative relaying. As the number of cooperating CRs increases due to increasing λ, the time required for cooperative relaying decreases. However, for higher λ, as discussed earlier, t_{ps} gets the major share of time. Since more help of secondary nodes is required when the channel conditions are not favourable, CRs are reimbursed more at low SNRs. As the SNRs increases, t_{ss} decreases. However, t_{ss} is always long enough to satisfy the minimum reimbursement required by the CRs. Overall, both t_{sp} and t_{ss} assume low values at higher λ due to greater t_{ps} requirement as explained previously.

Figure 3.5, studies the bargaining powers of the two players and its impact on the division of time. It can be seen that the player with higher bargaining power is able to procure more time to increase its utility. The primary can get up to $\sim$ 20% more time reserved for the cooperative relaying phase when its bargaining power is improved to 0.8 from 0.5. Similar increase in the CR bargaining power results in proportional increase in t_{ss}. The variety of possible divisions of the leasing time depicts the flexibility and wide scale applicability of the bargaining solutions. It can capture the scenarios where one player exercises greater influence on the decision.

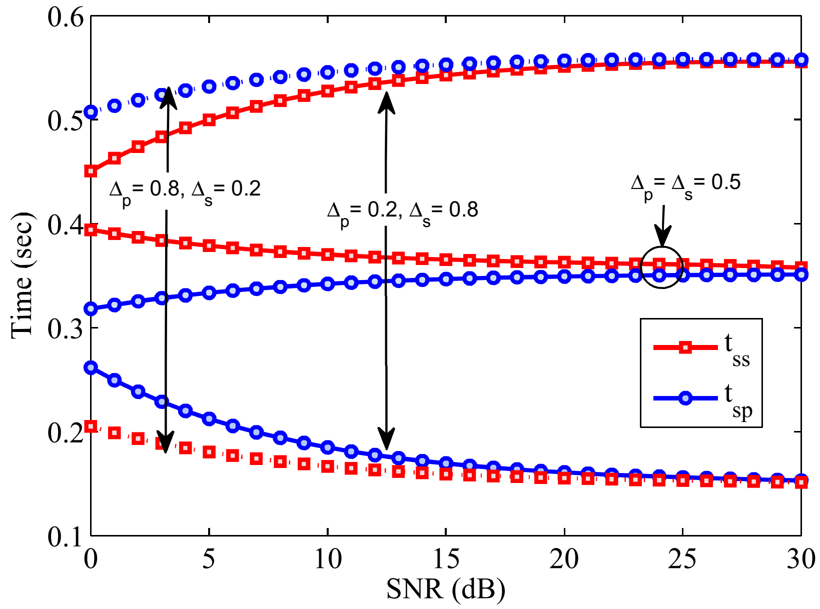


Figure 3.5: Impact of Bargaining Powers

3.7 ENERGY EFFICIENCY OF SPECTRUM LEASING MODEL

3.7.1 *Analytical quantification*

In this section, the energy efficiency (EE) of the spectrum leasing model for cognitive radio networks is defined and quantified. The energy efficiency is the number of bits transmitted successfully across the channel per unit of energy consumed, given as,

$$EE = \frac{n_B}{J}, \qquad \text{(bits/Joule)} \tag{3.37}$$

where n_B is the number of bits transmitted successfully and J is the energy consumed in Joules.

Theorem 1. *The energy efficiency of a licensed primary network employing direct communication EE_{dir} and while employing DSL, EE_{DSL} in terms of the number of successfully transmitted bits per unit energy can be given as*

$$EE_{\text{dir}} = \frac{n_{\text{dir}}}{TP_p}, \quad \text{and} \quad EE_{\text{DSL}} = \frac{n_{\text{DSL}}}{t_{ps}\bar{P}_p + t_{sp}P_s k}, \tag{3.38}$$

respectively, where n_{dir} is the number of successfully transmitted bits in direct communication, n_{DSL} are the successfully transmitted bits over the cooperative link.

Proof. The number of bits successfully transmitted in the transmission duration of the direct link n_{dir} is given as [9];

$$n_{\text{dir}} = R_{\text{dir}}T, \tag{3.39}$$

where R_{dir} follows from the result in lemma 1. In case the primary decides to lease the spectrum, the number of bits successfully transmitted in spectrum leasing is given as

$$n_{\text{DSL}} = \min\left(t_{ps}\overline{R}_{ps}, t_{sp}\overline{R}_{sp}\right), \tag{3.40}$$

where, $\overline{R}_{ps}$ and $\overline{R}_{sp}$ have been determined in eqs. 3.12 and 3.13, respectively. The total energy consumed during direct communication is TP_p and that during DSL based cooperation is $t_{ps}\bar{P}_p + t_{sp}P_s k$ where the first term accounts for the energy consumed in P_{tx} to S_{tx} communication and the later for the energy consumption when k secondary transmitters cooperatively relay the data to P_{rx} for a duration equal to the leased time t_{sp} and then transmit their own traffic for a time t_{ss}. Similarly, we also quantify the energy efficiency of secondary communication phase as

$$EE_{\text{sec}} = \frac{n_{\text{sec}}}{P_s t_{ss}}, \qquad \text{(bits/Joule)} \tag{3.41}$$

where $n_{\text{sec}} = t_{ss}\overline{R}_{ss}$. The total energy consumed when the secondary network communicates for a duration t_{ss} is $P_s t_{ss}$. □

3.7.2 *Analytic Results*

Following the analytical results for R_{dir}, $\overline{R}_{ps}$, $\overline{R}_{sp}$ and $\overline{R}_{ss}$, the energy efficiency is studied by observing both the direct link and DSL based communication over a variety of SNR values as shown in Fig. 3.6a. As in the discussion on the achievable capacity and time division, the EE of direct and DSL communication is studied on the basis of secondary network density. It is evident that the energy efficiency of DSL is significantly greater than that of the direct communication for smaller values of λ. This is because the transmit power of the primary and secondary in DSL mode is low. The selection of relays which are geographically closer to both P_{tx} and P_{rx} help in achieving the same transmission rate in shorter time and hence lower power. Also, the cooperative relaying based diversity benefits significantly increase the throughput at the primary receiver while maintaining a low transmit power. As λ increases, the EE of DSL decreases mainly due to two reasons;

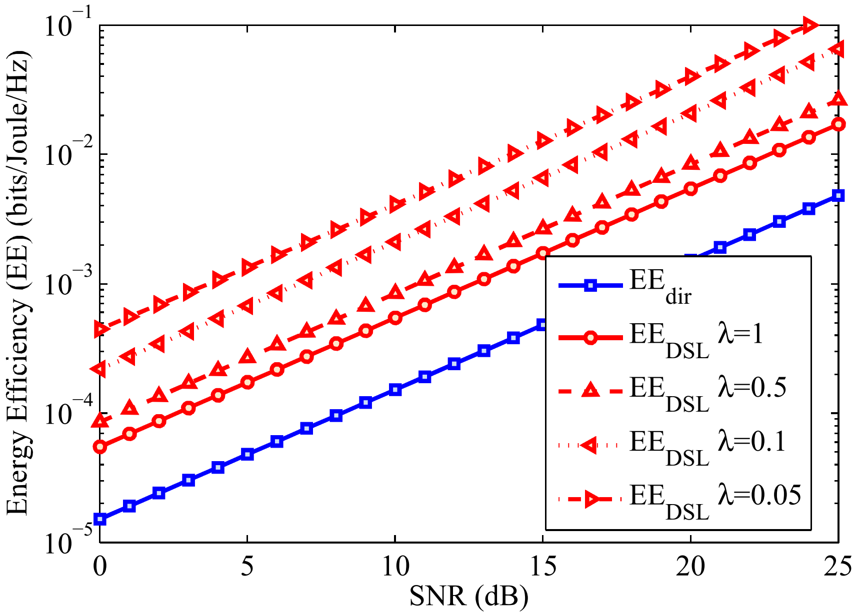
1. The throughput of the cooperative DSL communication decreases as the average primary to secondary rate $\overline{R}_{ps}$ decreases with increasing λ (see Fig. 3.3a). The energy consumed in the first phase of DSL grows as the primary to secondary link operation time t_{ps} increases.

2. Also, in the second phase of DSL, aggregate transmit energy is higher due to increased number of relays.

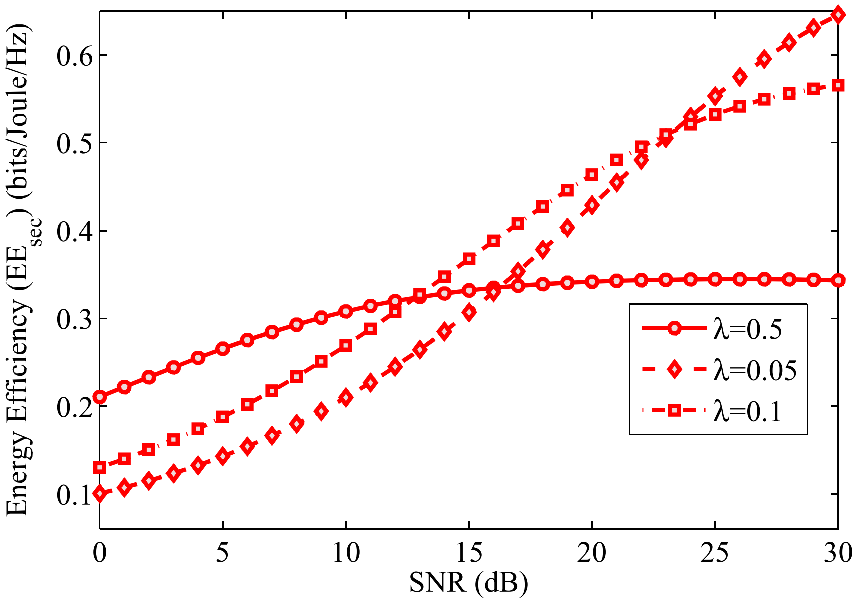
It can be seen that the bargaining based leasing time division results in significantly more energy efficient communication via DSL as compared to direct communication when the secondary network is relatively sparse (i.e., $\lambda \leq 0.05$).

In Fig. 3.6b, we study the EE of the secondary network in the third phase of leasing. During this phase, the energy efficiency of the network improves with increasing SNR. It attains a maximum value as R_{ss} converges to a constant rate. Moreover, if the number of secondary transmitters is increased, the aggregate energy consumption is increased by the presence of greater number of interferes. Hence, the EE of the secondary network EE_{sec} decreases for high λ when DSL is operational in the interference limited regime. Hence, DSL for sparse secondary network is the most energy efficient solution for both primary and secondary networks.

Further, the effect of the angle θ of the sector of cooperation on the EE of DSL is investigated. From Fig. 3.7, it can be seen that the EE of DSL degrades with increasing the area of cooperation. This happens because increasing θ increases the number of cooperators which in turn increases the aggregate transmit power used for cooperation. Also, the probability of finding a farther neighbour increases as the area of cooperation increases and hence limits $\overline{R}_{ps}$ in the first phase. For low values of θ, $\overline{R}_{sp}$ is low due to limited number of cooperators. However, low θ results in high $\overline{R}_{ps}$, thus, an improved energy efficiency is observed. For very low values of θ, DSL is not viable because the probability of finding even a single relay is infinitely low. As soon as the cooperation area is wide enough to find a few relays in it, DSL becomes viable and most energy efficient.



(a) Direct communication $\mathrm{EE_{dir}}$ vs DSL communication $\mathrm{EE_{DSL}}$, $P_p = 1$, $P_s = 0.1$, $T = 1, \theta = \frac{\pi}{4}$.



(b) Secondary Network $\mathrm{EE_{sec}}$, $P_s = 0.1$, time $= t_{ss}$.

Figure 3.6: Energy Efficiency

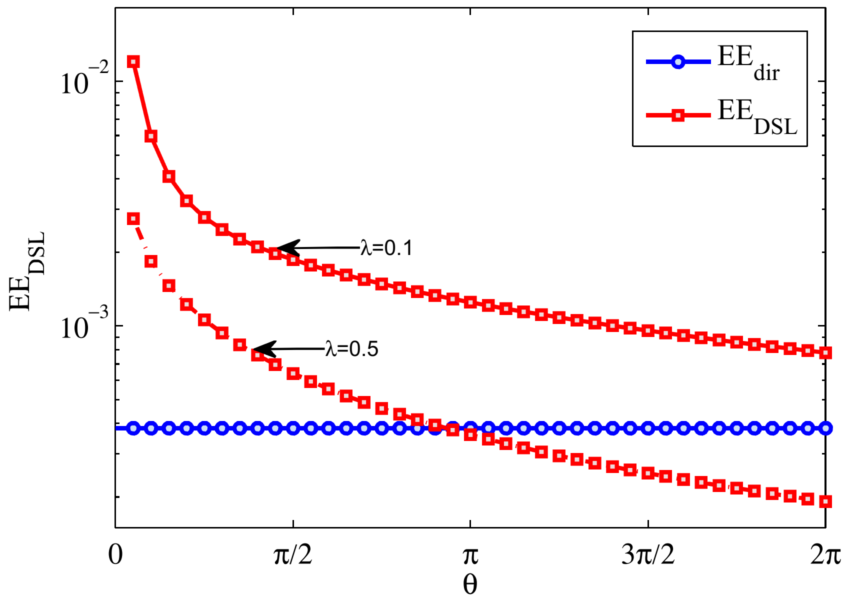


Figure 3.7: Impact of θ on EE of DSL.

Finally, the time rate product is analysed which determines the total number of bits that can be transmitted during both modes for a time T. The simulation results in Fig. 3.8 demonstrate the time-rate product of direct vs. DSL communication. It can be seen that DSL communication achieves exactly the same performance in terms of the effective number of bits delivered to the primary receiver as compared to the direct communication. It simply implies that by using DSL, the primary can transmit the same amount of data as with direct communication. However, as discussed earlier, this transmission is more energy efficient than direct communication when the CR network is sparse. This result further verifies the practical viability and attraction for the legacy network to operate in DSL mode.

It is important to mention that the EE and SE gains harnessed by the proposed DSL scheme may incur some underlying costs. These include the implementation of the actual arbitration mechanism for bargaining, time synchroniz-

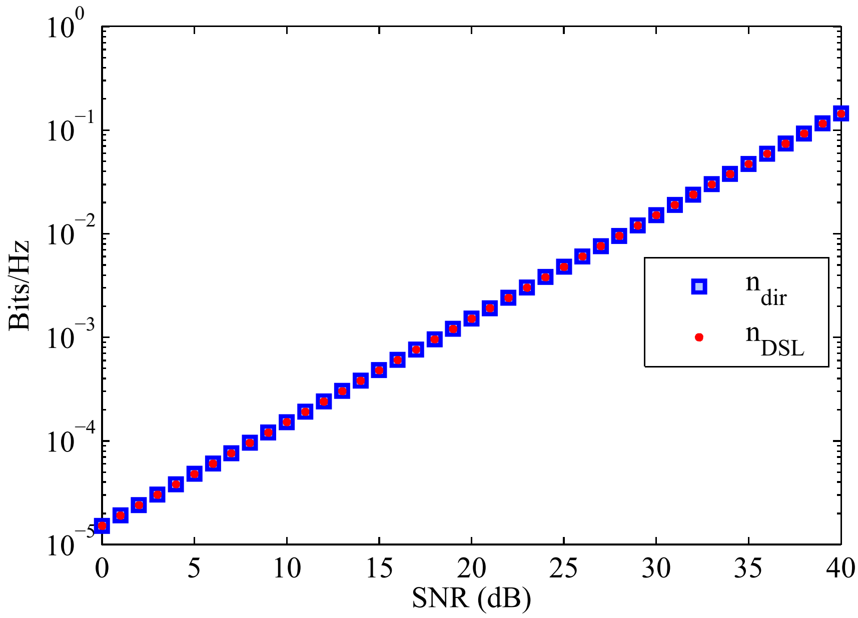


Figure 3.8: Time-Rate Product: Direct communication bits_{dir} vs. DSL bits_{DSL} $\lambda = 0.5$, $T = 1$

ation, DSTC implementation and control signalling costs. The bargaining negotiations can be carried out between the primary transmitter and a dedicated CR controller station that represents the entire CR network. Control signalling can be used to attain time synchronization and to exchange the DSTC codebook between the relays. Although these costs have not been explicitly catered for in the analysis, however, the significant gain of $10\times$ in EE using DSL provides a healthy margin to incorporate the above mentioned implementation costs while maintaining a notable gain of DSL.

The entire discussion can be summarized as follows. DSL based transmission serves as an energy efficient alternative to direct communication when the secondary network is sparse. For these low populated networks, the aggregate energy and time requirements for cooperation and secondary network activity are low. Hence an intelligent relay selection based on the spatial characteristics of the network and the optimal leasing time division can help in exploiting the diversity gain of cooperative relaying to enhance the performance of legacy communication. It also allows the otherwise deprived secondary network to utilize its share in the bandwidth therefore improving the overall spectral utilization of the network.

3.8 SUMMARY

In this chapter, an analytical study of dynamic spectrum leasing based on a geometrical framework was presented and the relative link performances in terms of the achieved capacities in DSL and direct communication were evaluated. A Nash bargaining based approach for the determination of the appropriate leasing time was introduced. It was demonstrated that the proposed algorithm results in a division of time that satisfies the requirements of both primary and secondary networks. Based on these operational features, the energy efficiency was quantified and investigated through simulations. The results indicate that DSL is more energy efficient in most of the practical SNR regimes, hence making DSL a viable option for energy efficient communication. Such energy efficient solution can be achieved only if a sparse CR network is considered with DSL operation at a low transmit power as compared to that of the transmit power of the direct communication. DSL is shown to be more than 10x energy efficient than direct communication when the CR density is low and/or the cooperation region is small. With only a few cooperating CR nodes, the entire CR network gets exclusive access to the spectrum. Hence DSL based communication enables the primary to communicate at its desired transmission rate and quality in an energy efficient manner and also enables the CR network to exploit the licensed spectrum for its own communication.

4

DSL FOR CODED BI-DIRECTIONAL COMMUNICATION

Summary of Contribution

Practical wireless networks commonly encounter scenarios of bi-directional communication between a set of two terminals. Examples include communication between mobile operator and user, over WiFi, Bluetooth and between other peer-to-peer networks. It is therefore important to quantify how DSL can be made useful for two way communication. Earlier in Chapter 3, the performance of DSL was quantified and analysed for uni-directional communication with special attention to its EE. In this chapter, the framework developed in the previous discussion is applied to bi-directional communication. The aim is to see how the Nash bargaining based time division applied to the communication of spatially co-existing CR and primary nodes can be used in two way communication. It is shown that using MAC layer techniques like network coding, spectral efficiency can also be attained for two way communication while maintaining the high EE gains of DSL that were observed for unidirectional case.

4.1 RESEARCH OBJECTIVES

In this chapter, the focus is to;

1. Extend the DSL framework to bi-directional communication by exploiting network coding at CRs.
2. Quantify the spectral efficiency of the DSL for bi-directional communication.
3. Analyse the EE of DSL for network coding enabled large scale CRN.

The specific design challenge which is addressed in this chapter is as follows:

Design Issue 4.1

Is it possible to employ DSL for spectrum sharing in bi-directional communication networks? How does networking, topological and channel dynamics impact the SE and the EE of DSL empowered bi-directional networks?

4.2 CONTRIBUTIONS

In this chapter, a primitive network coding and relaying technique is used to realize and enhance the two-way communication rate between two primary nodes present in the network with the help of relaying services of the CRs. Network coding (NC) was introduced in Chapter 2 in Sec. 2.2.4. A simple scheme used in this chapter, proposed in [127] is as follows:

NC Mechanism: Suppose two nodes N_1 and N_2 want to send packets D_1 and D_2 to each other respectively using a relay node. In relay assisted half duplex transmission, during the first time slot, N_1 transmits its data to the wireless relay. Similarly, N_2 also transmits its data to the relay in the second slot. The relay performs simple symbol level XOR operation to combine the two packets as follows:

$$D = D_1 \oplus D_2. \tag{4.1}$$

The combined packet is transmitted to both the nodes simultaneously in the third slot. Having knowledge of their own packet, each node can extract the packet sent by the other node. Without NC, the two packets take four time slots to get to their destinations using the relays. With NC, the same is achieved in three time slots. Here, this NC and relaying technique is extended for the DSL communication which has not been done so far in the existing studies. It is proposed that a set of cooperating secondary nodes perform NC to facilitate two-way communication between the two primary sources P_{tx_1} and P_{tx_2}. The time slot saved by using NC is used such that geographically suitable relays facilitate the communication between the primary nodes for a longer time hence attaining greater throughput. According to the DSL scheme, a fraction of time is allocated to the cooperating CRs as a reimbursement.

Due to the appropriate geometrical relay selection, intelligent division of leasing time and application of NC, DSL spectral efficiency can be improved. The analysis of DSL shows that there is more than $4\times$ improvement in the number of bits successfully exchanged between the two primary nodes during the same operational time as compared to direct two-way communication between the two primary nodes. Based on the proposed time division bargaining, out of the total DSL operational time, from $25\% - 35\%$ of the time is reserved for exclusive CRN communication which is otherwise dormant.

The results indicate that the proposed DSL scheme can be more than 10 times more energy efficient as compared to the dedicated primary link. In Chapter 3, it was shown that DSL could be more than 10 times energy efficient for unidirectional communication. It is a consequence of using NC that facilitates the two way communication in fewer transmission slots to improve the spectral efficiency while maintaining the EE of DSL. The analysis of the DSL using NC for

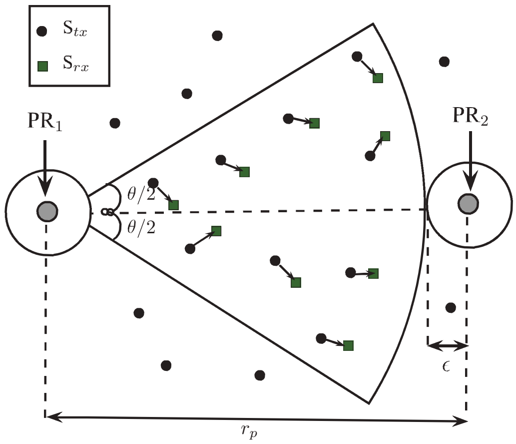


Figure 4.1: Network Geometry

two-way communication indicates its viability as an effective low power and spectrally efficient communication infrastructure for future wireless networks.

4.3 PHYSICAL AND MAC LAYER MODEL

In this chapter, two primary users P_{tx_1} and P_{tx_2} are considered to be communicating with each other with a link distance of r_p. Similar to the setup considered in Chapter 3, CRs form a spatially co-located PPP with density λ. Cooperative CR relays are selected in the sec $(\theta, r_p - 2\epsilon)$. Once the spectrum is leased to the secondary transmitters S_{tx} for their own activity, they transmit to their respective receivers S_{rx} located at a fixed distance r_0. As shown in [66], such network model for the CRN communication, called the 'bi-polar' model is simple and leads to tractable analysis. Also, it is shown that it is easily extendible to nearest neighbour model used in Ch. 3. The propagation channel model is considered to be the same as in Chapter 3.

4.3.1 *Phases of NC based DSL*

Beacon enabled signalling is adopted for DSL to initiate and agree on leasing parameters. Spectrum leasing for time $\mathcal{T} = 2T$ takes place in the following three phases (also see Fig. 4.2).

Phase 1

Each primary source transmits its data to be relayed to its receiver to the secondary nodes lying in the cooperation region for a time t_{p1s} and t_{p2s} at a rate R_{p1s} and R_{p2s} respectively.

Phase 2

The cooperating secondary nodes perform NC to combine the two primary signals as in eq. (4.1). The coded data D is then transmitted at the physical layer to both the primary nodes by forming a distributed antenna array employing Distributed Space Time Coding (DSTC). At the physical layer, this transmission is done at a rate R_{sp} and power $P_s < P_p$ for a duration t_{sp} such that

$$t_{p1s} + t_{p2s} + t_{sp} < \mathcal{T}.$$

Phase 3

Finally, for the remaining time, the secondary nodes gain an exclusive access to the channel. During this time t_{ss}, they transmit to their respective receiver at a rate R_{ss} and power P_s. This time is the fare that the primary network has to pay for the relaying services of the secondary nodes.

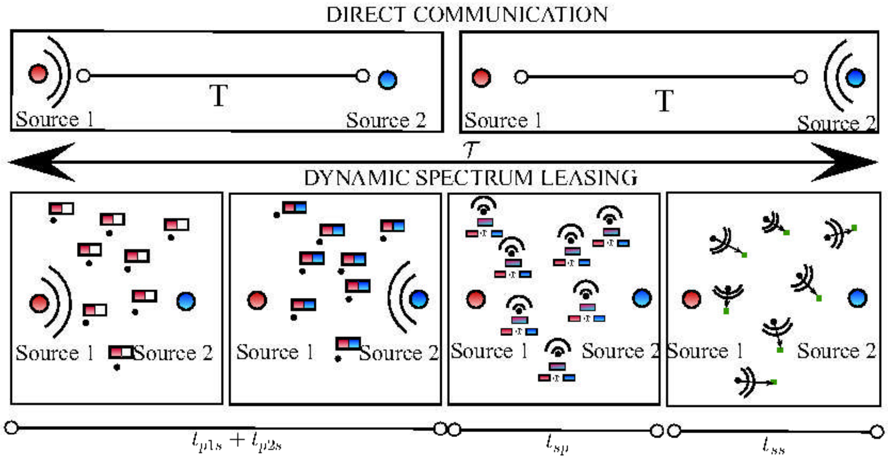


Figure 4.2: Direct communication and Dynamic Spectrum Leasing

It can be seen that DSL with NC mainly differs from the basic DSL model presented before in the following ways:

1. During phase I, both primary transmitters broadcast their data to the CRs.

2. In phase II, unlike previously, the CRs forward the coded data to the primary nodes in a single transmission.

3. If the bargaining over time is successful, only those CRs that participate in cooperating with the primary nodes get the reimbursement in terms of exclusive spectral access time. Previously, the entire CR network enjoyed the reimbursement. It is more realistic to consider that only the cooperating CRs enjoy the rewards.

4.4 ANALYSIS OF DYNAMIC SPECTRUM LEASING FOR CODED BI-DIRECTIONAL COMMUNICATION

The analysis of DSL with NC follows the geometric characterization and quantification on performance determinants like the transmit powers, link capacities, density of secondary networks etc., as in Sec. 3.5 In this section we adopt the same geometric framework to analyse the working and gains of DSL with NC. The SNR at the receiver is given as in eq. (3.2)

4.4.1 *Average Link Capacities R_{dir}, $\overline{R}_{ps}$, $\overline{R}_{sp}$ and $\overline{R}_{ss}$*

The ρ-outage capacity R_{dir} of the primary link (P_{tx_1}, P_{tx_2}) follows the same quantification as in Proposition 1 in Chapter 3. Note that the distance between the two nodes is fixed and the channel gain h_p is considered to be symmetric. Therefore, it is safely assumed that the communication rate from P_{tx_1} to P_{tx_2}, is the same as the transmission rate from P_{tx_2} to P_{tx_1}. Similarly, each P_{tx} broadcasts its data to the CR relays within the cooperation region A_c with reduced power $\bar{P}_p$. However, as mentioned before, the overall performance of the cooperative communication link is bounded by the minimum transmission rate of all the relays. For this reason, the rate is bounded by the worst case performance considering the distance between P_{tx_i}, $(i \in 1, 2)$, and the relay node to be maximum. The average distance $\mathbb{E}[r]$ from each primary transmitter to its farthest neighbour within a sector with angle θ and radius $r_p - 2\epsilon$ i.e., $\sec(\theta, r_p)$ in a 2-dimensional PPP is used as derived in Sec. 3.5.1. The average minimum rate $\overline{R}_{ps}$ experienced in P_{tx_i}-CR communication in phase I is given by eq. (3.12).

During the second phase, the secondary cooperating nodes combine the packets from both primary transmitters following a XOR operation as given in eq. (4.1). This network coded packet is then sent back to both the primary transmitters. Similar to phase I, the average rate $\overline{R}_{sp}$ in the 2nd phase where DSTC is used by the CRs to relay the primary data is the same as in eq. (3.13).

In the third phase of spectrum leasing, the secondary transmitters of the cooperation region communicate with their respective receivers. A bi-polar model of the secondary source-destination pairs [128] is considered as shown in Fig. 4.1. It is of interest to know the average transmission capacity of the (S_{tx}, S_{rx}) link, $\overline{R}_{SS}$. In this case, all the secondary transmitters in the cooperation region simultaneously communicate with their receivers in order to utilize the leased bandwidth for their own transmission. In this phase, similar to the direct communication, a realistic situation is considered under which the secondary network also operates under a fixed QoS constraint ρ_s.

Proposition 4.1: Reimbursement Rate

The average ρ_s-outage rate, $\overline{R}_{ss}$, for the link (S_{tx_i}, S_{rx_i}), where the channel gain between the source i and its destination h_{ss_i} is exponential, the transmit power is P_s and the distance between them is r_0 is given as

$$\overline{R}_{ss} \quad = \quad \log_2 \left[1 - \left(\frac{l(r_0)}{\frac{\sigma^2}{P_s} + \kappa_1} \right) \ln\left(1 - \rho_s\right) \right] \quad \left(\frac{bits}{sec}\right) , \tag{4.2}$$

where κ_1 is the average aggregate interference.

Proof. Similar to Chapter 3, the received SINR at the CR receiver can be quantified as SINR $= \frac{P_s h_{ss_i} l(r_0)}{\sigma^2 + \sum_{j \in \Phi_r} P_s h_{ss_j} l(r_j)}$, where, h_{ss_j} is the channel gain from the secondary transmitter j inflicting interference to S_{rx_i}, located at a distance r_j. The aggregate interference at an arbitrary CR receiver is denoted as

$I = \sum_{j\in\Phi_r} h_{ss_j} l\left(r_j\right)$. Followed by the proof of lemma 3.1, the transmission rate achieved between a typical secondary source-destination pair for a given quality of service constraint ρ_s can be stated. However, here we are interested in the average rate. From Jensen's inequality, the average ρ_s-outage rate $\overline{R}_{ss}$ is

$$\overline{R}_{ss} \geq \log_2\left[1-\left(\frac{l\left(r_0\right)}{\frac{\sigma^2}{P_s}+\mathbb{E}\left[I\right]}\right)\ln\left(1-\rho_s\right)\right]. \tag{4.3}$$

In order to find the average rate $\overline{R}_{ss}$, it can be noted that aggregate interference at a secondary receiver directly affects the average transmission rate. In the scenario under consideration where multiple transmitters gain access to the channel during the same time interval, interference between these simultaneous transmissions is a crucial issue. In order to determine the average rate $\overline{R}_{ss}$, it is essential to first quantify the average amount of interference,

$$\mathbb{E}\left[I\right] = \mathbb{E}\left[\sum_{j\in\Phi_r} h_{ss_j} l\left(r_j\right)\right].$$

Using the definition of Laplace Transform and taking expectation over both the point process and fading in the above integral,

$$\mathbb{E}\left[\exp^{-sI}\right] = \mathbb{E}_{\Phi}\left[\prod_{j\in\Phi_r}\mathbb{E}_h\left[\exp^{\left(-sh_j l\left(r_j\right)\right)}\right]\right].$$

From the definition of PGFL, the integral takes the form,

$$\mathbb{E}\left[\exp^{-sI}\right] = \exp^{\left(-\mathbb{E}_h\left[\int_{\mathbb{R}^d}\left(1-\exp^{\left(-shl(r)\right)}\right)\lambda \mathrm{d}x\right]\right)}. \tag{4.4}$$

Cumulants of a probability distribution can be defined in terms of the moment generation function (MGF)

$$\kappa_n = \frac{d^n}{ds^n}\ln\left(\mathbb{E}\left[\exp^{sI}\right]\right)|_{s=0} . \tag{4.5}$$

Eq. (4.4) and the definition in eq. (4.5) is used to find the cumulants of interference for d-dimensional network as

$$\kappa_n = -\lambda\mathbb{E}_h\left[h^n \int\limits_{\mathbb{R}^2 \cap \sec(\theta, r_p)} r^{-\alpha n}\exp^{\left(shr^{-\alpha}\right)} dr^{d-1}\mathrm{dr}\right] . \tag{4.6}$$

For any positive value of n, eq. (4.6) evaluated at $s = 0$ gives the n^{th} cumulant κ_n of the interference distribution. The first cumulant of the distribution for a two dimensional network, i.e., $d = 2$, can be calculated as,

$$\kappa_1 = \mathbb{E}\left[I\right] = \lambda\theta\left(\frac{\left(r_P - \epsilon\right)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha}\right) . \tag{4.7}$$

The average aggregate interference can be used to determine the ρ_s-outage rate following eq. (4.3). □

4.5 ANALYTICAL RESULTS OF DSL WITH NC

The direct, broadcast (phase I) and cooperative relaying (phase II) rates have been studied in the previous chapter. Here, the secondary-to-secondary communication rate is studied where the secondary network adopts bi-polar transmission model as discussed previously. The ρ_s-outage communication $\overline{R}_{ss}$ rate is shown in Fig. 3.3b which increases with improving SNR values. However, it is interference limited in higher SNR regions (here the desired QoS of the

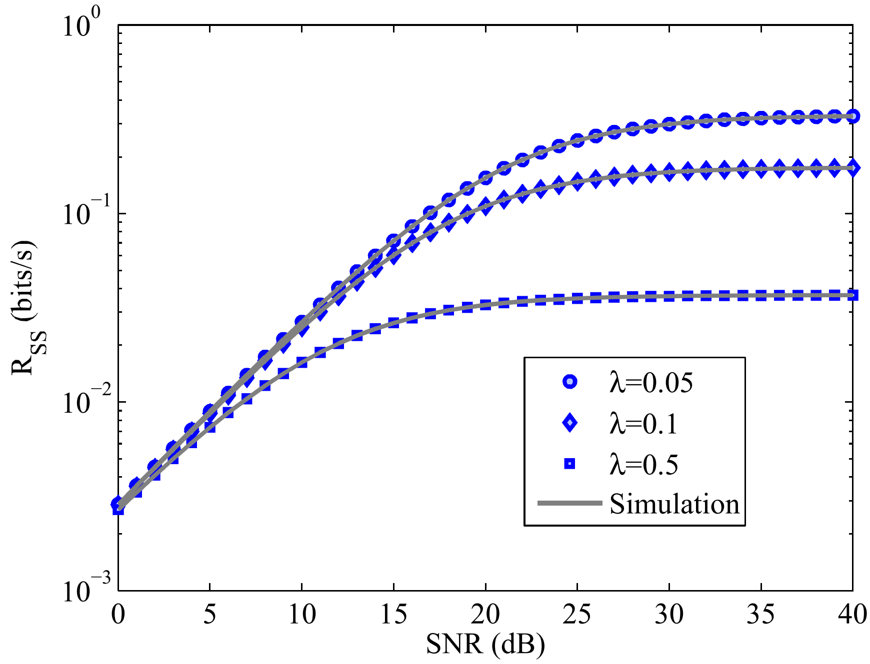


Figure 4.3: Communication rate during S_{tx}-S_{rx} transmission, $r_0 = 1$, $\rho_s = 0.1$.

secondary network in terms of the outage probability is 10^{-1}). This is a consequence of the improved signal strength at the receiver. However, as the density of the secondary nodes increases, the average transmission rate decreases due to the increased interference. It is clear that decreasing the outage constraint from 10^{-1} to 10^{-2} causes the average rate to decrease because the decoding threshold is raised. Hence, a graphical illustration of this result is intentionally skipped. The practical Monte Carlo simulation is shown by the solid lines running along the markers in the Figure.

4.5.1 *Division of Leasing Time*

Fig. 4.4. shows the proportion of time allocated for the first and last phase. It is important to mention that in this chapter, only the case where both PN and SN

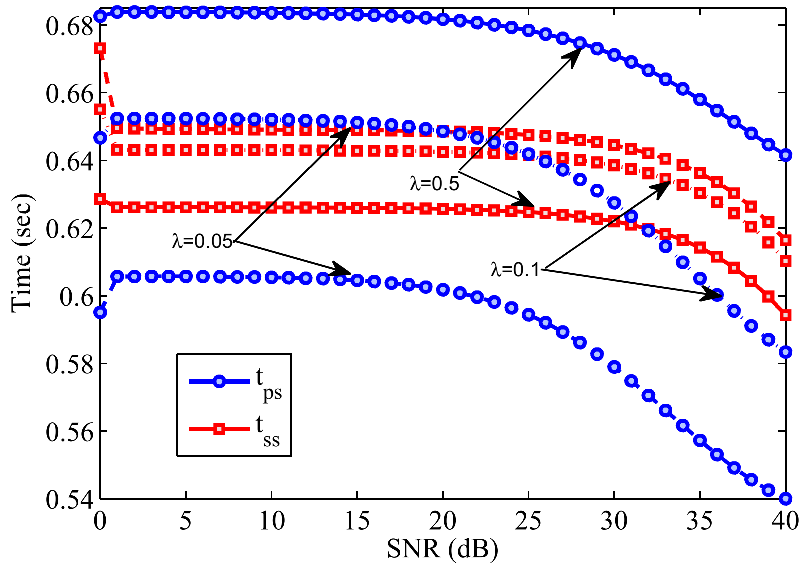


Figure 4.4: Nash Bargaining based division of $\mathcal{T} = 2T = 2$ between DSL phase I and III, $c = 1$, $\theta = \frac{\pi}{4}$.

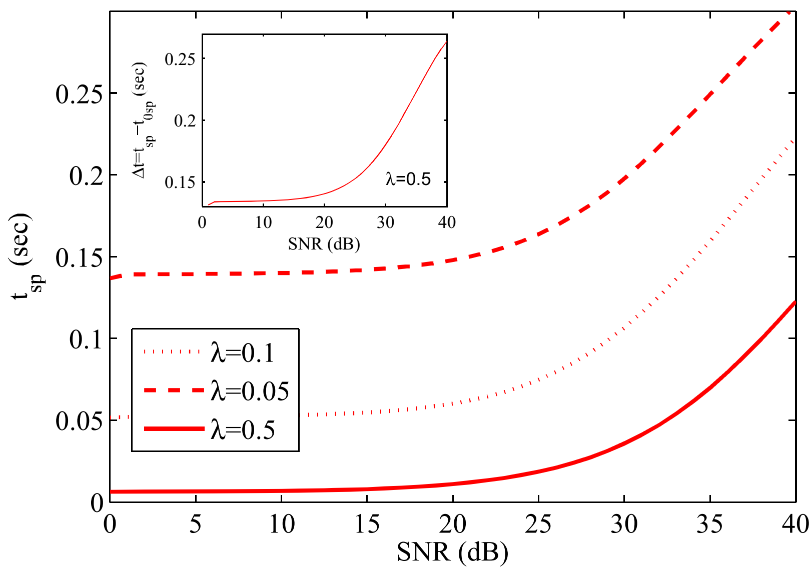


Figure 4.5: Reserved time for the second phase.

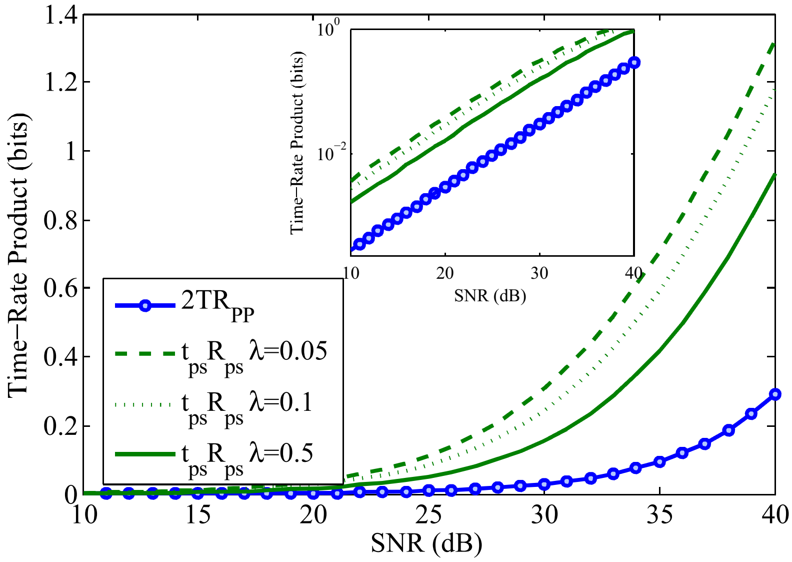


Figure 4.6: Time-Rate Product of Direct vs. NC based DSL based two way communication.

possess equal bargaining powers is considered. In order to maximize the gain in primary data transmission at high CR densities, as expected, t_{ps} is higher in order to meet the condition $t_{ps}\overline{R}_{ps} > TR_{\text{dir}}$. This takes place due to lower achievable transmission rate a high CR densities as discussed earlier. In total, $2t_{ps}$ time is spent in the first phase by both the primary sources in transmitting their data to the secondaries. On the other side, since $\overline{R}_{sp} > R_{\text{dir}}, \overline{R}_{ps}$ at high CR densities, the second phase is allocated shorter time. It can be seen in Fig. 4.5. However, the division of the time is such that $t_{ps}\overline{R}_{ps} = t_{sp}\overline{R}_{sp} > TR_{\text{dir}}$. For sparse CR network, the time t_{sp} is greater as the number of relays are fewer. The difference Δt between the minimum time required by the CRs in the second phase (t_{0sp}) and the time division output of the bargaining game (t_{sp}) is also shown in Fig. 4.5. Positive values of Δt show that the bargaining solution provides enough flexibility to incorporate the practical time required for network coding and distributed STC. In Fig. 4.4, a steep change can be observed in the division of time at 0dB. This can be assumed to be arising from the fact that the division of time is based on the quantification of the rate in DSL phases using a lower bound that results in a slightly disproportionate behaviour at this point.

The time reserved for secondary activity t_{ss} in the third phase is also shown (Fig. 4.4). To compensate for their energy costs in the second phase and deteriorated rate performance due to interference in the third phase, the CRs are given a reasonably high time for their activity specifically at low CR densities. It is important to emphasize again that at densities much lower than 0.05, the transmission rates become so low that successful bargaining cannot be established for a fair time division. Hence, analysis of lower λ values is not possible.

In Fig. 4.6, the increase in the time-rate product achieved by using DSL under a geometric and Nash Bargaining setup is shown. The results indicate that DSL provides significant gain in the number of bits that are successfully transmitted in DSL as compared to the number of bits $(2TR_{\text{dir}})$ in direct two way communication. This happens because the geometric vicinity and network coding services of the CR nodes provide higher transmission rates. Such enhanced performance is attained only when enough incentive is available for the secondary nodes to cooperate with the primary network. It can be seen that for very high CR node densities, the first phase rate $\overline{R}_{ps}$ is reduced to nearly equal to R_{dir}. As a result, the bargaining game only results in such division that the time demands of all the players are merely satisfied. However, when the secondary network is sparse, the gains in the time rate product are more than 4x times higher than the direct communication. Hence DSL under sparse secondary network maximizes the number of bits communicated successfully between the two primary networks.

4.6 ENERGY EFFICIENCY OF DYNAMIC SPECTRUM LEASING WITH NC

In this section, the EE of the spectrum leasing model for cognitive radio networks is studied using the same definition used in Chapter 3, Sec. 3.7.

Theorem 2. *The energy efficiency of a licensed primary network employing direct communication EE_{dir} and while employing DSL, EE_{DSL} in terms of the number of successfully transmitted bits per unit energy can be given as*

$$EE_{\text{dir}} = \frac{n_{\text{dir}}}{TP_p}, \quad \text{and} \quad EE_{DSL} = \frac{n_{\text{DSL}}}{2t_{ps}\overline{P}_p + P_s t_{sp} k}, \tag{4.8}$$

respectively, where n_{dir} is the number of successfully transmitted bits in direct communication, n_{DSL} are the successfully transmitted bits over the cooperative link.

Proof. The number of bits successfully transmitted in the transmission duration of the direct two way link n_{dir} is given as [9];

$$n_{\text{dir}} = 2R_{\text{dir}}T. \tag{4.9}$$

In case the primary decides to lease the spectrum, the number of bits successfully transmitted in spectrum leasing are given as

$$n_{\text{DSL}} = 2t_{ps}\overline{R}_{ps} = t_{sp}\overline{R}_{sp}, \tag{4.10}$$

where, $\overline{R}_{ps}$ and $\overline{R}_{sp}$ have been determined in eqs. 3.12 and 3.13, respectively. The total energy consumed during direct two way communication is $2TP_p$ and that during DSL based cooperation is $2t_{ps}P_p + P_s t_{sp} k$ where the first term accounts for the energy consumed in P_{tx_i} to S_{tx} communication during the first

DSL phase for a time t_{ps} and the latter for the energy consumption when k secondary transmitters cooperatively relay the data to P_{tx_i} for a duration equal to the leased time t_{sp}.

Now, the trends of the energy efficiency established above for direct and DSL communication are analysed. It is important to study whether the improved time rate products of DSL as seen in Fig. 4.6, come with high energy costs or DSL is also efficient on the energy front. It is clearly evident that the energy efficiency of DSL is greater than that of direct communication for smaller values of λ (see Fig. 4.7). This is because the transmit power of the secondary and transmission duration for the primary in DSL mode is low. The selection of relays which are geographically closer to both P_{tx_1} and P_{tx_2} helps in achieving the same transmission rate in shorter time duration and hence lower power. Also, the cooperative relaying based diversity benefits significantly increase the throughput at the primary receiver while maintaining a low transmit power. As λ increases, the throughput of the cooperative DSL communication decreases as the average primary to secondary rate $\overline{R}_{ps}$ decreases with increasing λ. Also, the energy consumed in the first phase of DSL grows as the primary to secondary link operation time t_{ps} increases. Hence the EE of DSL decreases. In the second phase of DSL, aggregate transmit energy is also higher due to increased number of relays. □

It can be seen that the bargaining based leasing time division results in significantly more energy efficient communication via DSL as compared to direct communication when the secondary network is relatively sparse (i.e., $\lambda = 0.05$). Moreover, the difference ΔE_{op} between the energy efficiency of direct and DSL based communication is also shown in the Figure. The positive values of ΔE_{op} indicate the available margin of miscellaneous circuitry and implementation energy costs. A future study that extends the DSL operation presented here by

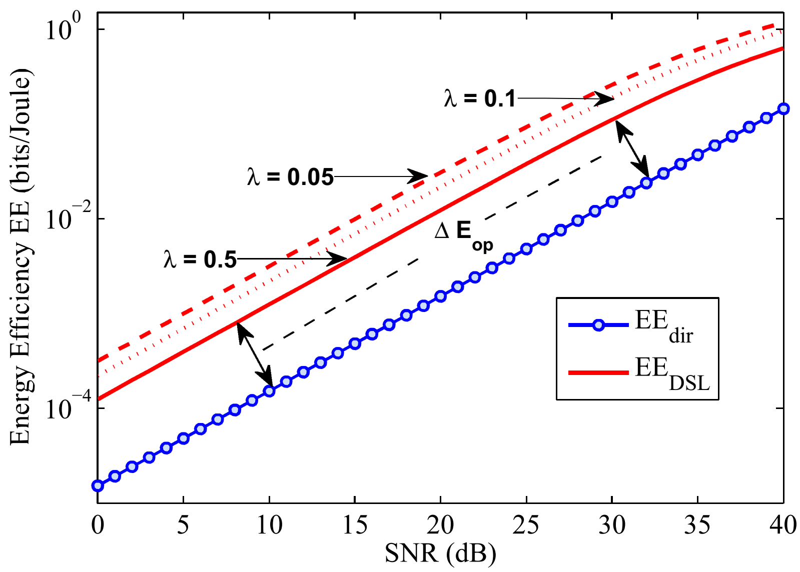


Figure 4.7: Energy Efficiency of Direct vs. DSL transmission, $P_p = 1$, $P_s = 0.1$, $\theta = \frac{\pi}{4}$.

discussing a specific hardware platform of the CRs and PUs may benefit from the indicated energy margins and compare their operational system energy efficiency against these results.

4.7 SUMMARY

In this chapter, a network coding based DSL approach was explored as a spectral and energy efficient alternative to two way direct communication. It was shown that geometry based selection of cooperating CR nodes and intelligent division of time enables greater than 4x bits to be successfully transmitted between the two primary sources at more than 10 times lower energy cost. These gains are attained only when the secondary network is relatively sparse. Hence network coding aided DSL under geometry and intelligent time division is a promising technique for future spectral and energy efficient two way wireless communication.

5

DSL FOR BI-DIRECTIONAL COMMUNICATION: IMPACT OF SELFISHNESS

Summary of Contribution

In this chapter, a beamforming based DSL technique is proposed to improve the spectral utility of the PN through the help of co-located SUs. Instead of mitigating the interference, it is proposed to exploit the interference in the wireless medium to perform physical layer network coding. In this chapter, the study of DSL is extended to a more practical scenario where a fraction of SUs can act selfishly by not helping the primary in relaying yet enjoy the reimbursement time. Another important design issue is the consideration of extended forms of fading and shadowing conditions in a network. In this chapter, the utility of the DSL scheme is measured in terms of a metric called time-bandwidth product (TBP) ratio. It is shown that the techniques of PNC and beamforming suggested in this chapter can increase the spectral efficiency of DSL from 2 up to 5 times as compared to DSL with NC only (previous chapter). However, in such a network, selfish behaviour can reduce the gain by more than a factor of 2. In this chapter, an iterative pricing based punishment algorithm to discourage the selfish behaviour of CRs is also suggested. It is shown that after about 8 rounds of pricing, the high TBP ratio of DSL based bi-directional communication is restored.

5.1 RESEARCH OBJECTIVES

In order to improve the spectral efficiency gains of bi-directional communication via DSL, in this work it is suggested to exploit the inherent wireless channel interference. PNC as introduced in [129] and discussed in Chapter. 2 exploits the interference and improves the spectral utility of bi-directional communication. To that end, for the CRs to cooperate with the primary, they exploit PNC to combine the data from two primary nodes and beamform it towards the intended primary destination. Each primary node is then able to extract its intended data from the PNC packet relayed by the CRs. Once the CRs have cooperated with the PN, they can enjoy the exclusive spectrum access during their reimbursement time. The cooperation and the reimbursement for the CRs takes place within the fixed time $\mathcal{T}$. Hence again, it is crucial to divide this time between cooperation and reimbursement phase so that a stable operation point is attained at which both the primary and the CRs benefit from leasing.

In reality, there is always a finite probability that some selfish CRs also exist in the network [130]. The presence of these selfish nodes can deteriorate the performance of all nodes in the network. These CRs enter the leasing agreement giving the impression that they will first cooperate with the PUs by relaying their data and then enjoy a proportional reimbursement time. However, while other honest CRs actually adhere to the terms of leasing, these selfish CRs not only enjoy the reimbursement time but also communicate with their respective receivers while they were supposed to cooperate by relaying for the PN. In this way they enjoy the entire cooperation time along with the reimbursement time for their own activity. As a result of this selfish behaviour, the primary endures a two fold loss; i) its performance is deteriorated since the selfish CRs interfere with the relayed primary communication and ii) the PN reimburses the CRs more in terms of the time allocated for CR operation.

In this chapter, the prime objectives are

1. To extend the DSL model to more generic fading and shadowing models;
2. To model the presence of selfish CRs in the network and quantify the impact of their selfish behaviour on the performance of DSL;
3. To model DSL for bi-directional communication with PNC and beamforming and quantify the gain in spectral efficiency of the primary network;
4. To devise a method to discourage the selfish behaviour of the CRs.

Design Issue 5.1

Is it possible to exploit the interference in the wireless medium and use beamforming to enhance the DSL gains for spectrum sharing in bi-directional communication networks? What are the impacts of selfish behaviour of CRs on the SE and the EE of DSL empowered bi-directional networks?

5.2 KEY RESULTS

The analysis and results show that DSL using PNC and distributed beamforming by the CRs can serve as a useful alternative to the primary network when the CRs act honestly and their density is low. It can increase the PN's throughput up to 17 times as compared to direct communication. This increase is measured in terms of *TBP ratio* denoted by β. These high gains can be realized due to; i) the accurate geometric modelling of the network; ii) appropriate selection of relays (i.e., harnessing the diversity gain); iii) exploiting PNC for distributed beamforming based relaying of CRs; and iv) the effective division of leasing time between the primary and secondary activities. Not only the PN, but the CRN also procures a considerable duration of exclusive spectrum access in DSL mode for a relatively sparse deployment. It is also shown how the presence of a few selfish CRs can reduce the utility of the PN by more than 50%. In order to discourage the selfish behaviour, an iterative pricing scheme is presented that aims to charge the selfish nodes a monetary price to minimize their utility while they act selfishly. It is shown that the proposed learning based pricing strategy helps to reduce the selfish behaviour in each round of pricing. The high performance offered by spectrum leasing can be re-gained after 8 rounds of pricing as a punishment for the selfish behaviour.

5.3 SYSTEM MODEL

A PN with bi-directional point-to-point communication (similar to Ch. 4) is considered. In this chapter, it is considered that the PPP of cooperating co-located CRs Φ_r can further be subdivided into sets of honest CR relays Φ_h and selfish relays Φ_s s.t. $\{\Phi_h, \Phi_s \subset \Phi_r\}$ and $\{\Phi_h \cup \Phi_s = \Phi_r\}$. If φ s.t. $\{0 \leq \varphi \leq 1\}$ is the fraction of the density of selfish users in the network, the intensity of the honest nodes in the network is $\lambda_h = (1-\varphi)\lambda$ and that of the selfish nodes is $\lambda_s = \varphi\lambda$. When there are no selfish nodes in the network, i.e., $\varphi = 0$, the point process takes the form $\{\Phi_s = \emptyset, \Phi_h = \Phi_r\}$.

The wireless signal propagation and its received quality is mainly dependent upon:

1. the number of different paths from which the signal arrives at the destination (fading);
2. multiple scattering of the signal that leads to variations in the local mean signal levels (shadowing);
3. distance dependent path loss.

The Nakagami-m distribution provides a comprehensive modelling of the fading conditions in the channel through different values of the fading parameter m_m whereas, shadowing is known to follow the Lognormal distribution. In recent studies [131, 132] however, Gamma distribution has been shown as a good fit to the experimental composite fading data. In this paper, we use the Gamma distribution of order m_s and mean power Ω_0 to model the small scale fading and shadowing. The SNR η at an arbitrary receiver located at a distance r from another node transmitting with a power P_p where the distance dependent path loss $l(r) = \min(1, r^{-\alpha})$ is upper bounded by unity for the case when the transmitter receiver separation distance is less than one. Here the channel gain h follows the Gamma distribution

$$f_{\mathcal{H}}(h) = \frac{1}{\Gamma(k)(\theta_0)^k} h^{(k-1)} \exp\left(-\frac{h}{\theta_0}\right), \tag{5.1}$$

where $k = \frac{1}{\frac{(m_m+1)(m_s+1)-1}{m_m m_s}}$ and $\theta_0 = \left(\left((m_m+1)(m_s+1)/m_s m_m\right) - 1\right)\Omega_0$ with moments $\mathbb{E}[h] = k\theta_0 = \Omega_0$, and $\mathrm{var}[h] = \frac{(m_m+1)(m_s+1)}{m_m m_s}\Omega_0^2$. The value of m_m is the Nakagami-m multipath fading parameter that determines the severity of fading for $\frac{1}{2} < m_m < \infty$. Lower values of m_m correspond to worse channel conditions. Similarly the order of Gamma function m_s allows varying the probability density function (PDF) of shadowing from Lognormal to Gaussian allowing flexibility. Hence, the Gamma distribution can be used to model different cases of multipath fading and shadowing by using the corresponding values of m_m and m_s respectively. In eq. (5.1), $h > 0, m_s > 0, m_m > 0$ and $\Omega_0 = 1$.

5.4 PROPOSED BEAMFORMING-DSL BASED MAC AND PHY

5.4.1 *Beamforming-DSL with Honest CRs*

Beamforming-DSL mode of communication is operational for a duration $\mathcal{T}$. This time is further divided into three phases;

Phase 1

Broadcast and PNC phase for time t_{ps} during which both P_{tx_1} and P_{tx_2} simultaneously transmit their data to the CR relays at a rate $\overline{R}_{ps}$.

The simultaneous transmission by P_{tx_1} and P_{tx_2} during the time t_{ps} allows a natural mixing of the data of both PUs in the air at the receiving CR relays leading to PNC. In contrast to the conventional approaches of avoiding collision and interference, PNC exploits these phenomena to naturally combine packets from multiple sources to maximize the information flow across the network. Algebraic techniques are used to separate the combined data into the intended information at the receivers. The received signal at the CRs takes the following form:

$$y_i = \sqrt{P_p l(r_{1i}) h_{1i}} s_1 + \sqrt{P_p l(r_{2i}) h_{2i}} s_2 + n_i, \tag{5.2}$$

where $\sqrt{h_{xi}}$ and $l(r_{xi})$ are the Gamma distributed channel and path gain from a primary transmitter P_{tx_x} to CR receiver i where $x \in \{1, 2\}$ and $i \in \Phi_r$. Also, s_x are the transmitted symbols from P_{tx_x} and n_i is AWGN at the CR receiver.

Phase 2

Denoise and Beamform phase for time t_{sp} during which the secondary relays divide themselves into two groups each of which denoises and beamforms the coded data towards its nearest primary receiver (P_{tx_1} or P_{tx_2}) at a rate $\overline{R}_{sp}$.

During t_{sp}, a group of CR nodes lying within a radius r_p cooperate with the primary and help to relay its data. The relays only attempt to remove the noise from the combined received signal by mapping the received signal to a denoise symbol d

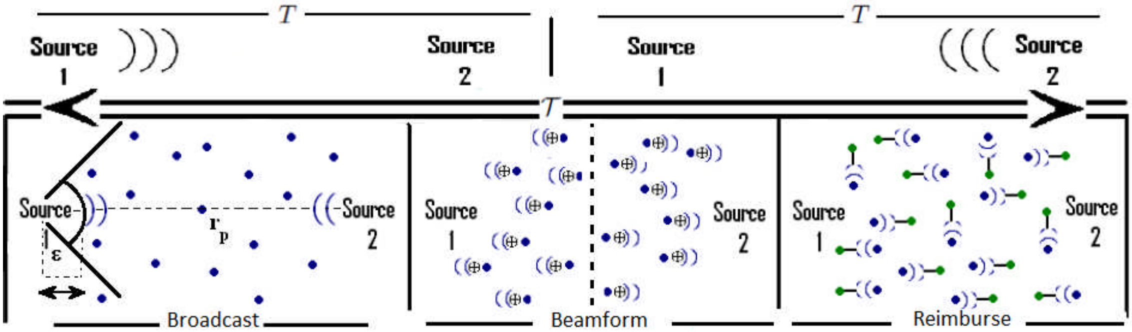
$$d \rightleftharpoons f_D(s_1, s_2), \tag{5.3}$$

where $f_D(s_1, s_2)$ is the denoising function used. In order to relay this information, the honest CRs divide into two groups forming a distributed multi antenna array while they forward the denoised data d to the closest of the two PUs. Here we exploit the channel knowledge present at the CR relays and propose Maximum Ratio Transmission (MRT) based beamforming at the secondary users.

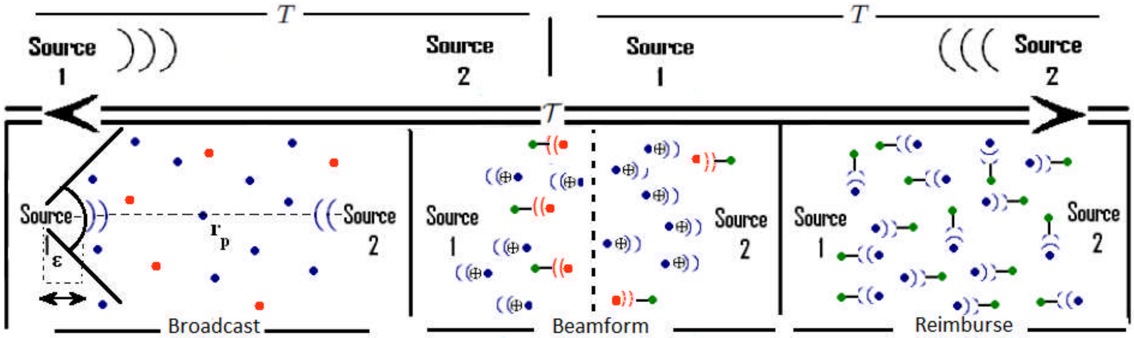
Phase 3

Reimburse phase for a duration t_{ss} where the spectrum is freely available to the CR relays to carry out their transmissions to their respective receivers at a rate $\overline{R}_{ss}$.

During the process of leasing, the primary assumes that all CRs are honest. The CRs demand a reimbursement based on the assumption that all CRs will



(a) DSL with honest CRs



(b) DSL with selfish CRs

Figure 5.1: DSL Operation

first honestly help the primary in relaying its data. Fig. 5.1a shows the honest operation of DSL.

The utility function for honest CRs in terms of the time-rate product attained during each phase is given as

$$\mathcal{U}_1(t) = \begin{cases} \overline{R}_{ps} t_{ps} & \text{Phase I} \\ \overline{R}_{sp} t_{sp} & \text{Phase II} \end{cases} \text{ and } \mathcal{U}_2(t) = \overline{R}_{ss} t_{ss} - c\lambda P_s \quad \text{Phase III.} \tag{5.4}$$

respectively, where $t_{sp} + t_{ps} + t_{ss} = \mathcal{T}$. Here the cost function in phase III is $c\lambda P_s$ where c measures the the bits transmitted per Watt of power per squared meter.

5.4.2 *Selfish CRs*

As stated, both the PN and the CRN inherently assume that all nodes are truthful and honest. However, in reality, some selfish nodes might want to increase their utility by not cooperating with the primary during the second DSL phase of denoising and beamforming. Instead, while the honest CRs relay the primary data, the selfish nodes carry out their own communication with their respective receivers during the entire duration of t_{sp}. In this way, they enjoy transmission during t_{sp} for their own communication at a rate $\overline{R}_{\text{selfish}}$ and in doing so they cause harmful interference to the cooperating honest CRs. Moreover, as per the DSL agreement, these nodes also enjoy transmission during the reimbursement time. So the total utility of a selfish CR becomes

$$\mathcal{U}_S(t) = t_{sp}\overline{R}_{\text{selfish}} + t_{ss}\overline{R}_{ss}. \tag{5.5}$$

In light of the discussion, it is important to carry out comprehensive modelling of the proposed scheme under physical channel characteristics i.e., node locations and densities, fading conditions and shadowing. A detailed discussion on the mechanisms of identifying the selfish nodes in the network is given later in Sec. 5.9.1. It is crucial to study the impact of selfish users that might be present in the network on the performance of the entire scheme.

5.5 ANALYTIC MODELLING OF BEAMFORMING-DSL

For a given SNR threshold γ^{th} and ρ-outage constraint, the data rate of direct communication can be expressed as;

$$p_{\text{out}} = \Pr\left\{\gamma < \gamma^{th}\right\} < \rho, \tag{5.6}$$

where γ^{th} is that threshold SNR above which ρ is less than p_{out}.

Proposition 5.1

The average ρ-outage rate, R_{dir}, for the link (P_{tx_1},P_{tx_2}), where the channel gain between the source and its destination h_p is Gamma distributed, the transmit power is P_p and the distance between them is r_p is given as

$$R_{\text{dir}} = \log_2 \left(1 + \frac{P_p l(r_p)}{\sigma^2}\Gamma^{-1}(\rho, k, \theta_0)\right)\left(\frac{bits}{sec}\right), \tag{5.7}$$

where $\Gamma^{-1}(\rho, k, \theta_0)$ is the inverse Gamma function.

Proof. The probability of success for direct communication can be written as,

$$1 - \rho = \Pr\left\{h_p > \frac{\gamma^{th}\sigma^2}{l(r_p)P_p}\right\}. \tag{5.8}$$

Using the cumulative density function (CDF) of Gamma distribution, the success probability can be written as.

$$\frac{Q\left(k, \frac{\gamma^{th}\sigma^2}{l(r_p)P_p}\theta_0^{-1}\right)}{\Gamma(k)} = \rho, \tag{5.9}$$

where $\frac{Q\left(k, \frac{x}{\theta_0}\right)}{\Gamma(k)}$ is the normalised lower incomplete Gamma function. Using the inverse Gamma function $\Gamma^{-1}(\rho, k, \theta_0)$ which can be calculated using standard

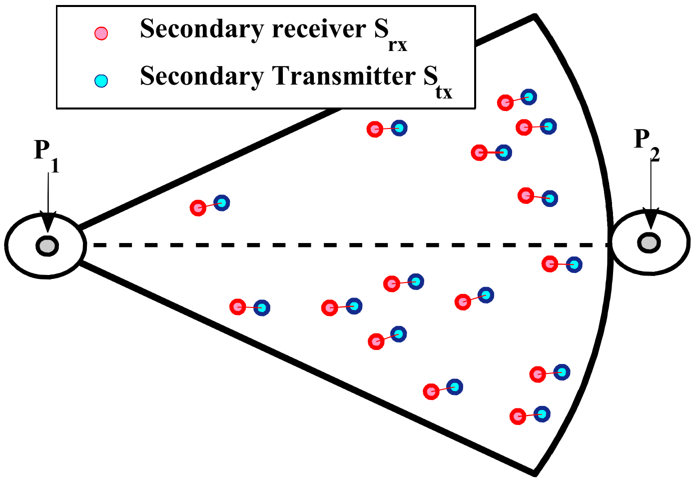


Figure 5.2: Secondary source-destination pairs in the cooperation region.

mathematical analytical tools like MATLAB, the outage capacity R_{dir} in eq. (5.7) can be stated as the least outage capacity that can be attained during direct communication. □

5.5.1 *Broadcast and PNC*

As explained earlier, in this phase both primary nodes simultaneously transmit their data to the cooperating CRs. This concurrent transmission allows PNC to take place in the air. The coded received signal at the CRs is given as in eq. (5.2). The outage rate $\overline{R}_{ps}$ at which this broadcast is received at the CRs is important since the effective data rate of CR cooperation is dependent upon the rate in both phases (i.e. $R_{DSL} = \min\left(\overline{R}_{ps}, \overline{R}_{sp}\right)$, eq.(3.30)). Also, the CRs get a better reimbursement if their offered cooperative rate to the PN is high.

Proposition 5.2

The worst average ρ-outage rate, $\overline{R}_{ps}$, for the link (P_{tx_i},S_{tx_n}), where the channel gain between the source and its farthest destination h_n is Gamma distributed and the transmit power is P_p is given as,

$$\overline{R}_{ps} \;\geq\; \log_2\left(1+\frac{P_p}{\sigma^2}\left(\left(\frac{\eta}{\lambda_h\theta/2}\right)+\left(r_p-\epsilon\right)^2\right)^{\frac{2}{\alpha}}\right), \tag{5.10}$$

where $\eta = \ln\left(1-\rho\left(1-\exp\left(-\lambda\frac{\theta}{2}\left(r_p-\epsilon\right)^2\right)\right)\right)$.

Proof. Different $\overline{R}_{ps}$ are experienced at individual cooperating relays due to their different and independent geographical locations and channel conditions. Since the minimum of these observed individual $\overline{R}_{ps}$ dictates the overall DSL performance, we only attempt to find the worst case $\overline{R}_{ps}$ of all the relays during this phase. Hence the outage capacity of the worst link in phase-I can be written as

$$p_{\text{out}} = \Pr\left\{\frac{h_n P_p r_n^{-\alpha}}{\sigma^2} < \gamma^{th}\right\}. \tag{5.11}$$

The above probability conditioned on the knowledge of the channel gain can be written as

$$\rho \;>\mathbb{E}_H \;\left[\Pr\left\{r_n > \left(\frac{h_n P_p}{\gamma^{th}\sigma^2}\right)^{\frac{1}{\alpha}} \mid h_n\right\}\right]. \tag{5.12}$$

The distance between the primary transmitter and its farthest relay in the sector (see Fig. 5.2) is the Complementary Cumulative Distribution Function

(CCDF) of finding at least one relay at a distance $r_n < R < r_p - \epsilon$. It is given in [69] as,

$$\Pr\{R > r_n\} = \frac{1 - \exp\left(-\lambda_h \frac{\theta}{2}\left((r_p - \epsilon)^2 - r_n^2\right)\right)}{1 - \exp\left(-\lambda \frac{\theta}{2}(r_p - \epsilon)^2\right)}, \tag{5.13}$$

where λ_h is the density of honestly cooperating CRs. From the above distribution, the outage probability becomes

$$\rho > \mathbb{E}_H\left[\frac{1 - \exp\left(-\lambda_h \frac{\theta}{2}\left((r_p - \epsilon)^2 - \left(\frac{h_n P_p}{\gamma^{th}\sigma^2}\right)^{\frac{2}{\alpha}}\right)\right)}{1 - \exp\left(-\lambda \frac{\theta}{2}(r_p - \epsilon)^2\right)}\right]. \tag{5.14}$$

Further, the above expression can be simplified to find out the outage threshold SNR by using the Jensen's inequality and the moment defined in Sec. 5.3. From the above expression, the lower bound on the final outage capacity is given as in eq. (5.10). It is notable that $\overline{R}_{ps}$ is the lower bound and other relays can achieve an average outage rate better than this. □

5.5.2 *Denoise and Beamform*

After denoising (as explained in Ch. 4), all the honest secondary nodes have the same information to transmit. In order to relay this information, the CRs divide into two groups forming a distributed multi antenna array while they forward the denoised data to the closest of the two primary users. Here, the channel knowledge present at the CR relay is exploited and Maximum Ratio Transmission (MRT) at the secondary users is proposed. In MRT, the channel knowledge at the relays is exploited to improve the signal strength as the receiver. This is done by assuming that the CRs are aware of the channel gains of the CR network to the primary nodes, each secondary i relay precodes its data according to the channel state information between itself and the closest primary transmitter as

$$o_p = \frac{d\sqrt{h_{xi}^*}}{\sqrt{\sum_{y\in\Phi_{hx}} \left|h_{xy}\right| l\left(r_{xy}\right)}}, \tag{5.15}$$

where Φ_{hx} consists of the set of honest nodes with intensity $\lambda_h = (1-\varphi)\lambda$,

precoding that data for its closest primary P_{tx_x} s.t. $\{x \in (1,2)\}$. In this way, all CRs transmit the same data and the added signal at the receiver attains a favorable SNR. The selfish CRs act greedily and instead of denoising and beamforming the primary data, they simply start communicating with their own receivers. By doing so, they reduce the number of CR cooperators for the primary network and also cause harmful interference I_g to the beamformed signal transmitted to the primary by the cooperating CRs. From the reciprocity of the channel gain on the reverse link, the received signal at P_{tx_x} is given as

$$y_{Ptx_x} = \sqrt{P_s}\sqrt{\sum_{y\in\Phi_{hx}} \left|h_{xy}\right| l(r_{xy})}d + n_{Ptx_x} + I_g. \tag{5.16}$$

The received SINR at PU is thus given by

$$\Gamma_p = \frac{\sum_{y\in\Phi_{hx}} \left|h_{xy}\right| l(r_{xy})}{\sigma^2 + I_g} P_s, \tag{5.17}$$

where P_s is the secondary transmit power, I_g is the interference from selfish CRs and σ^2 is noise power.

Proposition 5.3

The average ρ-outage rate, $\overline{R}_{sp}$, for the link (S_{tx_i},P_{tx_n}), where the channel gain between the source and its destination is h_i and the transmit power is P_s is given as,

$$\begin{aligned}\overline{R}_{sp} &= \frac{1}{2}\log_2\left[1+\left(\sigma^2+\left(\frac{\lambda_s\theta}{2}\right)\left(\frac{(r_p-\epsilon)^{2-\alpha}-\epsilon^{2-\alpha}}{2-\alpha}\right)\right)^{-1}\right. \\ &\times \left.\left(\sqrt{\frac{\kappa_2}{1-\rho}}+\kappa_1\right)P_s\right].\end{aligned} \tag{5.18}$$

which is the achievable rate by the beamforming CRs with density λ_h and selfish CRs with density λ_s.

Proof. In this scenario, the outage beamforming rate $\overline{R}_{sp}$ is half of the conventional rate i.e., $\overline{R}_{sp} = \frac{1}{2}\log_2\left(1+\gamma^{th}\right)$ since it is assumed that the two groups of CRs transmit simultaneously on two disjoint frequency bands. The outage capacity in this phase in the presence of selfish CRs can be worked out by considering the interference $I_g = \mathbb{E}\left[\sum_{y\in\Phi_{gx}}\left|H_{xy}\right| l(r_{xy})\right]$ caused by these selfish users where Φ_{gx} consists of the set of selfish nodes with intensity $\lambda_g = \varphi\lambda$. Outage is defined as the probability $p_{\text{out}} < \rho$ as

$$p_{\text{out}} = \Pr\left\{\frac{\sum_{y\in\Phi_{hx}}\left|h_{xy}\right| l(r_{xy})}{\sigma^2+I_g}P_s < \gamma^{th}\right\}. \tag{5.19}$$

For the sake of simplicity, we again apply Chebyshev's inequality as in eq. (5.9) to upper bound the outage probability to

$$1-\rho \quad \geq \quad \frac{\mathrm{var}\left[\sum_{y\in\Phi_{hx}}|h_{xy}|l(r_{xy})\right]}{\left(\frac{\gamma^{th}(\sigma^2+I_g)}{P_s}-\mathbb{E}\left[\sum_{y\in\Phi_{hx}}|h_{xy}|l(r_{xy})\right]\right)^2}\ . \tag{5.20}$$

Using PGFL of the point process, the cumulants of interference for 2-dimensional network can be given as,

$$\kappa_n \quad = \quad -\frac{\lambda}{2}\mathbb{E}_H\left[h_{xy}^n \int\limits_{\mathbb{R}^2\cap\mathrm{sec}(\theta,r_p)} r^{-\alpha n}\exp^{(sh_{xy}r^{-\alpha})}dr^{d-1}\mathrm{d}r\mathrm{d}\theta\right], \tag{5.21}$$

where κ_1 is the average of the aggregate composite channel gain and path loss.

$$\mathbb{E}\left[\sum_{y\in\Phi_{hx}}|h_{xy}|\,l(r_{xy})\right]=\kappa_1=\left(\frac{\lambda\theta}{2}\right)\left(\frac{(r_p-\epsilon)^{2-\alpha}-\epsilon^{2-\alpha}}{2-\alpha}\right). \tag{5.22}$$

The average interference $\mathbb{E}\left[I_g\right]=\mathbb{E}\left[\sum_{y\in\Phi_{hx}}|h_{xy}|\,l(r_{xy})\right]$ follows from κ_1. Similarly κ_2 is the variance of this aggregate signal

$$\begin{aligned}\mathrm{var}\left[\sum_{y\in\Phi_{hx}}|h_{xy}|\,l(r_{xy})\right] \quad &= \quad \kappa_2=\left(\frac{\lambda\theta}{2}\right)\left(\frac{(r_p-\epsilon)^{2-2\alpha}-\epsilon^{2-2\alpha}}{2-2\alpha}\right)\\ &\times \quad \frac{(m_m+1)\,(m_s+1)}{m_m m_s}.\end{aligned} \tag{5.23}$$

From eq. (5.22) and 5.23, the probability of outage in equation 5.20 can be completely characterized and the outage capacity $\overline{R}_{sp}$ takes the form as in eq. (5.18). It can be clearly seen that when the capacity in this phase with selfish CRs i.e., when $\varphi \neq 0 \rightleftharpoons \lambda_s \neq 0$, is lower compared to the the capacity when all CRs are honest. This is because of the interference they cause when the honest CRs beamform towards their respective primary node. □

5.5.2.1 *Outage capacity of Selfish CRs*

In order to quantify the utility of the selfish nodes, it is important to analyse the capacity $\overline{R}_{\text{selfish}}$ achieved by the selfish CRs. In terms of the SINR experienced at any CR receiver z, the probability of successful communication is given as

$$1 - \rho = \Pr\left\{ \frac{P_s h_{z0} l(r_0)}{\sum_{y \in \Phi_c} h_{zy} l\left(r_{zy}\right) + \sigma_z^2} > \gamma^{th} \right\}. \tag{5.24}$$

From eq. (5.21), the outage capacity can be expressed as

$$\begin{aligned} \overline{R}_{\text{selfish}} &= \log_2 \left[1 + \left(\sigma^2 + \left(\frac{\lambda\theta}{2} \right) \left(\frac{\left(r_p - \epsilon\right)^{2-\alpha} - \epsilon^{2-\alpha}}{2-\alpha} \right) \right)^{-1} \right. \\ &\times \left. \Gamma^{-1}\left(\rho, k, \theta_0\right) r_0^{-\alpha} P_s \right]. \end{aligned} \tag{5.25}$$

Remarks: It is important to note that both capacities $\overline{R}_{\text{selfish}}$ and $\overline{R}_{sp}$ strongly depend on the density of CR nodes. The aggregate interference experienced while beamforming towards the primary node increases with increasing the value of φ resulting in lesser $\overline{R}_{sp}$. However, $\overline{R}_{\text{selfish}}$ is dependent on the entire CR density λ since all nodes interfere with a selfishly operating node during the second phase of DSL.

5.5.3 *Reimburse*

Proposition 5.4

The average ρ_s-outage rate, $\overline{R}_{ss}$, for the link (S_{tx_i},S_{rx_i}), where the channel gain between the source and its destination is h_i and the transmit power is P_s is given as,

$$\begin{aligned}\overline{R}_{ss} &= \log_2\left[1+\left(\sigma^2+\left(\frac{\lambda\theta}{2}\right)\left(\frac{\left(r_p-\epsilon\right)^{2-\alpha}-\epsilon^{2-\alpha}}{2-\alpha}\right)\right)^{-1}\right. \\ &\times \left.\Gamma^{-1}\left(\rho,k,\theta_0\right)r_0^{-\alpha}P_s\right].\end{aligned} \tag{5.26}$$

which is the achievable rate by the CRs during their reimbursement.

Proof. Since all the CR nodes simultaneously transmit in this phase, each node experiences a certain interference coming from other communicating CRs. The same approach as in eq. (5.7) is adopted to find out the outage capacity $\overline{R}_{ss}$. By using the definition of PGFL and some mathematical simplifications, it comes out to be as in eq. (5.26). □

5.5.4 *Beamforming DSL for Rayleigh Fading–Special Case*

In case of Rayleigh fading channel without any shadowing, the rate $R_{\mathrm{dir}R}$ can be quantified using the same approach as in previous chapters. It is given as

$$R_{\mathrm{dir}R} = \log_2\left(1-\left(\frac{P_p l\left(r_p\right)}{\sigma^2}\right)\ln\left(1-\rho\right)\right). \tag{5.27}$$

Similarly, the rate for the three DSL phases are given as

$$\overline{R}_{psR} = \log_2\left(1+\left(\frac{\exp\left(-\lambda\frac{\theta}{2}\left(r_p-\epsilon\right)^2-1\right)}{\left(r_p-\epsilon\right)-C}\right)^{\alpha}\frac{P_p}{\sigma^2}\right), \tag{5.28}$$

$$\overline{R}_{spR} = \log_2\left(1+\frac{\lambda\theta P_s}{\sigma^2}\left(\frac{\left(r_p-\epsilon\right)^{2-\alpha}-\epsilon^{2-\alpha}}{2-\alpha}\right)\right), \tag{5.29}$$

and

$$\overline{R}_{ssR} = \log_2\left[1-\left(\frac{I(r_0)}{\frac{\sigma^2}{P_s}+\kappa_1}\right)\ln\left(1-\rho_s\right)\right]. \tag{5.30}$$

For the above equations for Rayleigh fading, it is considered that the CRs operate honestly.

5.5.5 *Game theoretic leasing time division*

The important question that still remains unanswered relates to the duration of time of each of the three phases (t_{ps}, t_{sp}, t_{ss}). As discussed earlier, for every primary and CR node, time division is crucial since a greater share in time leads to a greater throughput. It is important to mention that the game is played assuming that all players are honest and once an agreement is reached, all players abide by it i.e., $\varphi = 0$. A triplet of payoffs $\left(t^*_{ps}, t^*_{sp}, t^*_{ss}\right)$ is a Nash Bargaining solution if it solves the following optimization problem

$$\max\left(\log\left(t_{ps}-t_{0ps}\right)+\log\left(t_{sp}-t_{0sp}\right)+\log\left(t_{ss}-t_{0ss}\right)\right), \tag{5.31}$$

$$\text{subject to}\quad \begin{array}{rcl} t_{ps}\overline{R}_{ps} & = & t_{sp}\overline{R}_{sp} \\ \mathcal{T} & = & t_{sp}+t_{ps}+t_{ss} \end{array}.$$

where $t_{0ps} = {}^{\mathcal{T}R_{\text{dir}}}/_{\overline{R}_{ps}}$, $t_{0sp} = {}^{\mathcal{T}R_{\text{dir}}}/_{\overline{R}_{sp}}$ and $t_{0ss} = {}^{c\lambda P_s}/_{\overline{R}_{ss}}$. Using the Lagrangian dual of the above optimization problem, the time t_2 is the solution of the following quadratic equation

$$t_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{5.32}$$

where $a = \delta + 2\Upsilon_1$, $b = \Upsilon_1 \left(2\Upsilon_3 - t_{0sp} - \Upsilon_1 t_{0ps}\right) - \delta \left(t_{0sp} + t_{0ps}\right)$ and $c = \delta\Upsilon_1 t_{0sp} t_{0ps} - \Upsilon_1\Upsilon_3 t_{0sp} - \Upsilon_1^2\Upsilon_3 t_{0sp}$ with $\Upsilon_1 = {}^{\overline{R}_{ps}}/_{\overline{R}_{sp}}$, $\Upsilon_2 = \left(\Upsilon_1^{-1} + 1\right)^{-1}$, $\Upsilon_3 = \Upsilon_2 \left(t_{0ss} - \tau\right)$ and $\delta = \Upsilon_2 \left(\Upsilon_1 + 1\right)$. Also t_{ps} and t_{ss} can be found out using the constraints in eq. (5.31).

5.6 RESULTS

In this section, the behaviour of conventional bi-directional communication vs. the proposed beamforming based DSL is studied. We begin by analysing the capacity achieved while the primary nodes transmit i.e., R_{dir} and $\overline{R}_{ps}$ From Fig. 5.3 it can be seen that for a given link distance, the direct communication rate R_{dir} is significantly lower than $\overline{R}_{ps}$. As mentioned before, this phenomenon is a consequence of the fact that transmission in the DSL phase I encounters reduced path loss due the reduced link distance between the primary nodes and the relays. The capacity of the primary to secondary communication in the broadcast phase is strongly dependent upon the number of secondary nodes present in the area of cooperation. In Fig. 5.3, the rate from one primary source to the farthest CR node is shown. For a very low secondary density, e.g., $\lambda \lll 0.002$, the probability of finding a neighbour in the region of cooperation is extremely low. *For this reason, the capacity analysis for very sparse secondary network is not possible since the transmission rates from $P_{tx_{1,2}}$ to the CRs are nearly zero.*

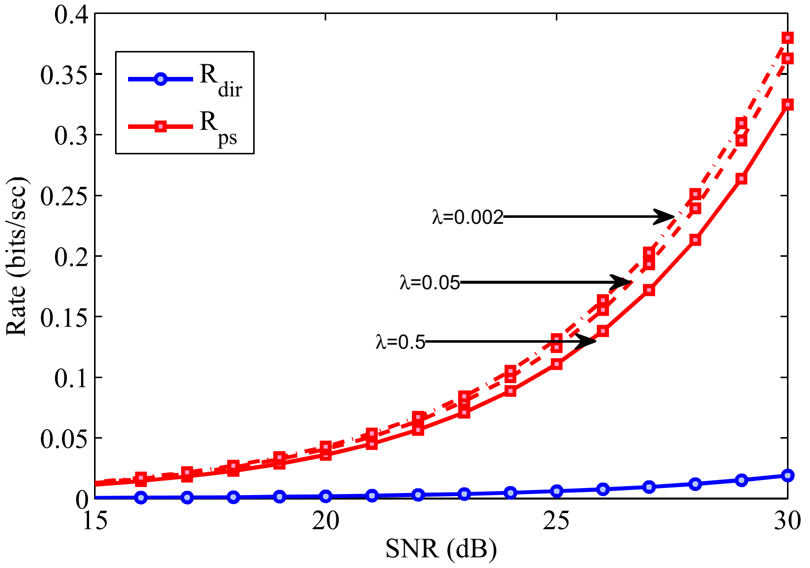


Figure 5.3: Data rate of the primary transmissions in direct communication and DSL phase I, $P_p = 1$, $P_s = 0.1$, $\rho = 0.1$, $r_p = 10$, $\epsilon = 1$.

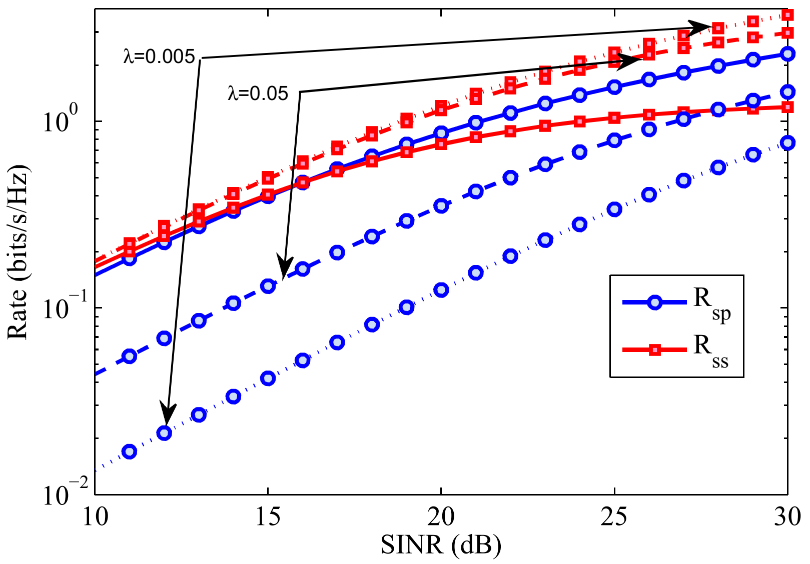


Figure 5.4: Data rate of the CR transmissions in DSL phase II and III, $\lambda = 0.5$.

For higher λ, it can be seen from Fig. 5.3 that the average transmission rate $\overline{R}_{ps}$ is greater than that of the direct communication. This is a consequence of such cooperation region selection where relays are located in a close proximity to both P_1 and P_2. However, if the number of secondary users increases in the cooperation region, the average distance between the transmitter $P_{tx_{1,2}}$ and the farthest node increases which follows from the average distance quantification in eq. (5.10). Hence $\overline{R}_{ps}$ decreases when λ increases. Also, both R_{dir} and $\overline{R}_{ps}$ increase with improving channel conditions.

The CR transmission in phase II and III i.e., $\overline{R}_{sp}$ and $\overline{R}_{ss}$ respectively is studied in Fig. 5.4. Phase II is simulated using the SINR relation given in eq. (5.17) which comes from the superposition of all the precoded signals (see eq. (5.15)) from the relays. It can be seen that $\overline{R}_{sp}$ increases with increasing density of the CRs. This happens because of the increase in the diversity due to multiple relays beamforming towards the primary nodes. $\overline{R}_{ss}$ on the other hand decreases as expected with increasing CR density. As discussed earlier, the aggregate interference due to multiple concurrent transmission poses a bottleneck and even improving channel conditions fail to improve $\overline{R}_{ss}$ proportionally. $\overline{R}_{ss}$ is interference limited in higher SNR regions.

In Figs. 5.5 and 5.6 the effect of severity of fading and shadowing conditions has been shown. Decreasing values of m_m and m_s in the analysis correspond to increasingly severe fading and shadowing. As shown in Figs. 5.5 and 5.6, the performance of the links improves with increasing m_s, m_m. It can be seen that smaller values of fading and shadowing coefficients have a greater impact in terms of capacity reduction in R_{dir} and $\overline{R}_{ss}$.

In Fig. 5.7 the amount of time reserved for DSL phase I and II is shown. It can be seen that at low values of λ, more time t_{sp} is reserved for phase II to increase the rate $\overline{R}_{sp}$. At lower CR densities, the time required by the CRs in

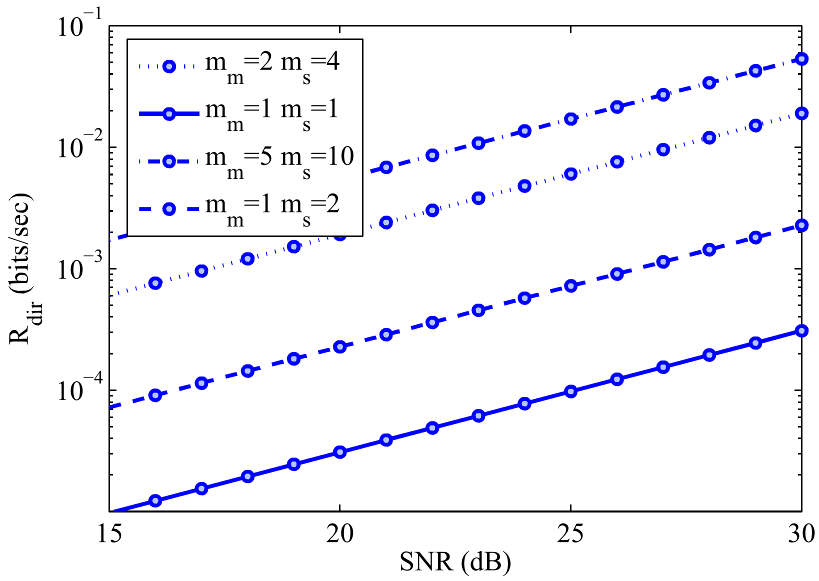


Figure 5.5: Data Rate of primary transmission i.e., direct communication and phase I of DSL.

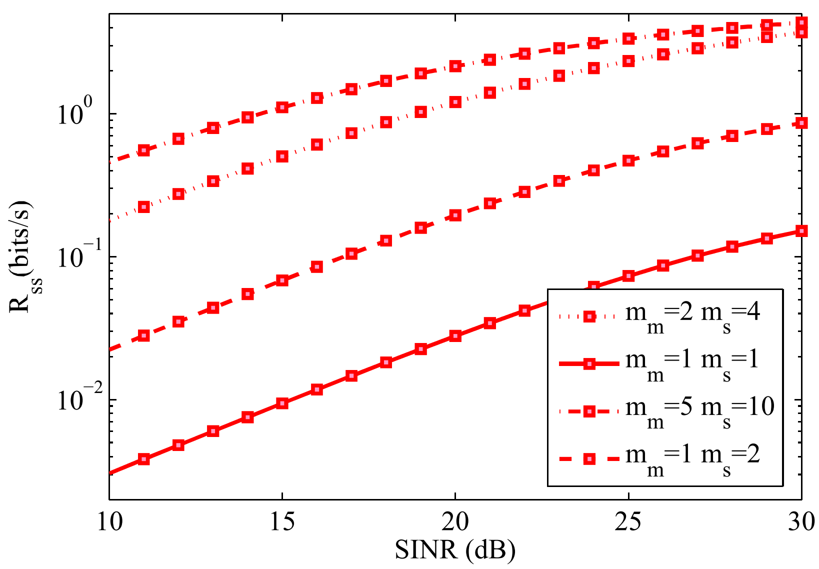


Figure 5.6: Data rate of the CR transmissions in DSL phase II and III.

the beamforming phase increases due to lower number of relays to beamform and hence lower achievable transmission rate. When $\overline{R}_{sp} > \max\{R_{\text{dir}}, \overline{R}_{ps}\}$, the second phase is allocated shorter time and vice verse. At higher values of λ, $\overline{R}_{ps}$ is the limiting factor and hence more time is reserved for it to enhance the capacity of this phase. This is because as seen in the previous discussion, $\overline{R}_{ps}$ is the lowest of all other DSL rates. In order to maximize the gain in primary data transmission, t_{ps} is higher in order to meet the condition, $t_{ps}\overline{R}_{ps} > \mathcal{T}R_{\text{dir}}$. Following eq. (3.30), the division of the time is such that $t_{ps}\overline{R}_{ps} = t_{sp}\overline{R}_{sp} > \mathcal{T}R_{\text{dir}}$.

The time reserved for secondary activity t_{ss} in the third phase is also shown. To compensate for their energy costs in the second phase and deteriorated rate performance due to interference in the third phase, the CRs are given a reasonably high time for their activity specifically at low CR densities. On the other hand, increasing λ decreases the time demand/share of the reimbursement phase. This is because of the increased interference due to higher λ as shown in Fig. 5.8. Improving SNR mostly improves the share of time t_{ss}.

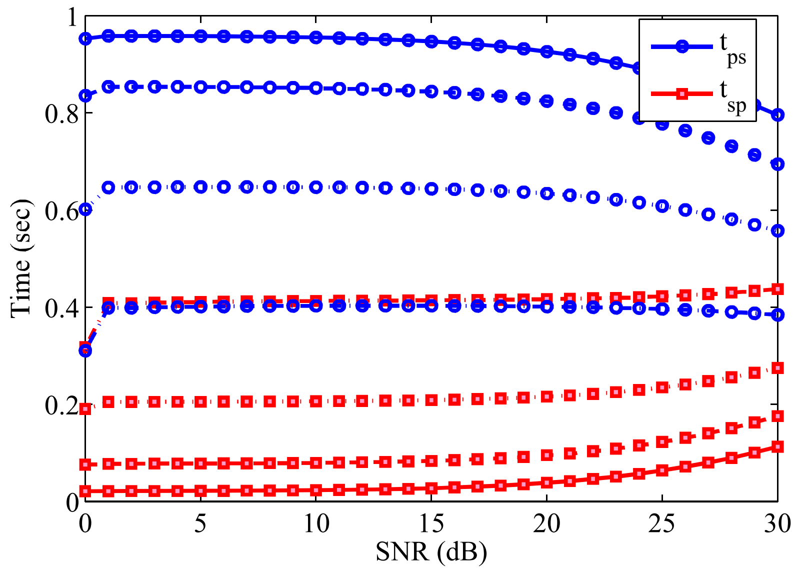


Figure 5.7: Division between DSL phase I and II, $\mathcal{T} = 2$, $\lambda = 0.5$ (solid), $\lambda = 0.05$ (dash), $\lambda = 0.005$ (dot), $\lambda = 0.002$ (dash-dot).

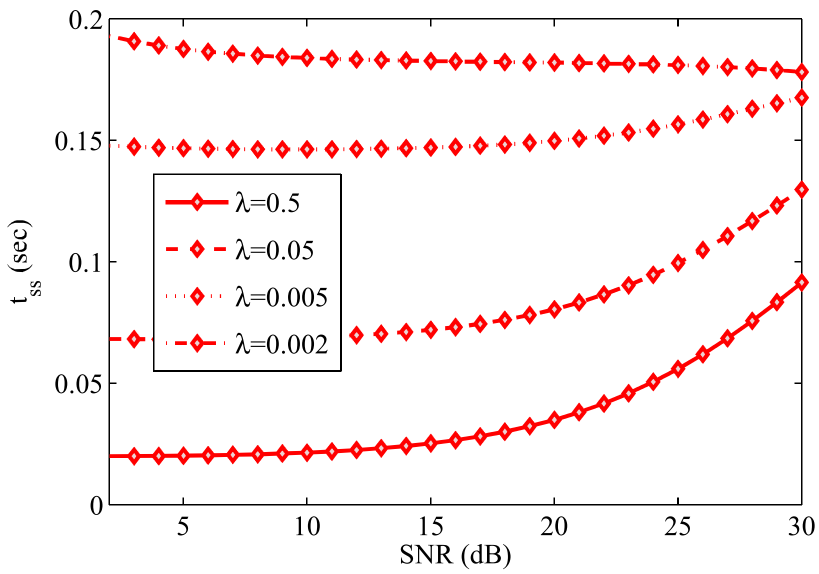


Figure 5.8: Time reserved for DSL phase III

5.7 TIME-BANDWIDTH GAIN

In this section, the potential benefit that the PN can get by leasing the spectrum are explored. This gain is measured in terms of TBP ratio denoted by β.

Theorem 3. *The ratio of the number of bits of primary data that are successfully transmitted in DSL based primary communication time to those transmitted via direct two-way transmission is defined as the TBP ratio. Mathematically,*

$$\beta = \frac{t_{ps}\overline{R}_{ps}}{\mathcal{T}R_{dir}}. \tag{5.33}$$

The alternative definition using the product $t_{sp}\overline{R}_{sp}$ can be equivalently used in eq. (5.33).

In Fig. 5.9, the increase in the TBP ratio achieved by using DSL under a geometric and Nash Bargaining setup is shown. The results indicate that DSL provides a significant gain in the number of bits that are successfully transmitted in DSL as compared to the number of bits ($\mathcal{T}R_{\text{dir}}$) in direct two way communication. This occurs because the geometric vicinity, PNC and beamforming services of the CR nodes provide higher transmission rates. Such enhanced performance is attained only when enough incentive is available for the secondary nodes to cooperate with the PN. It can be seen from the Figure that the TBP ratio increases with increase in the secondary density. It happens because $\overline{R}_{sp}$ improves with increasing λ. However, the ratio reaches a maximum ($\sim \lambda = 0.1$) after which further increase in the secondary density slightly degrades the TBP ratio. This can be credited to the fact that $\overline{R}_{ps}$ decreases at high densities and it becomes the limiting factor in any further increase in the TBP ratio. At very low densities, the TBP ratio is low. For increasing values of λ, maximum gain from DSL can be harvested. However, increasing the density beyond $\lambda = 0.1$

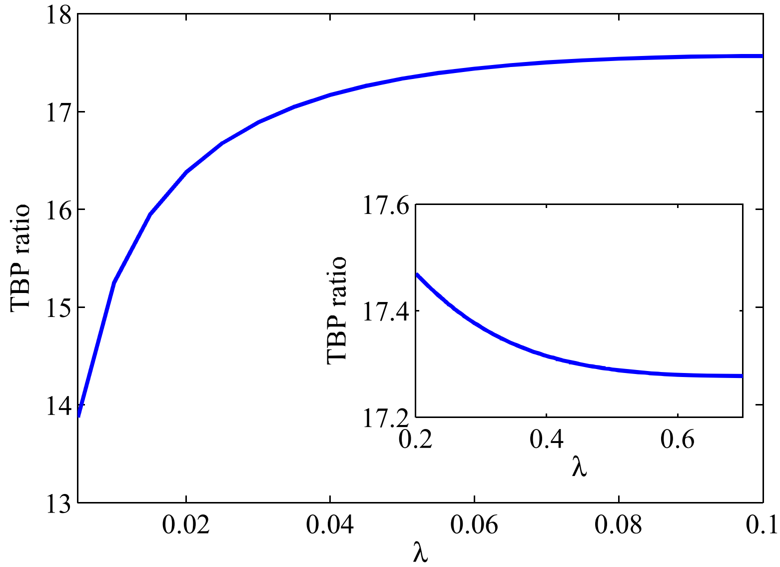


Figure 5.9: Time Bandwidth Product ratio $\mathcal{T} = 2$

degrades the DSL performance. Hence within the total direct communication time $\mathcal{T}$, DSL enables the PN to not only exchange greater amount of data but also improve the overall network utility by allocating a fraction of time to the CR users to communicate with each other. Fig. 5.9 shows that DSL with PNC and beamforming is most beneficial to the primary network at intermediary SN densities i.e., not very sparse and not too dense secondary network. DSL improves the performance if the PN by more than 17 times as shown in Fig. 5.9.

It is important to also quantify how much gain can be harvested by using PNC and beamforming like in this chapter as compared to DSTC and NC in Ch. 4. In order to do so, the comparison of the TBP ratio for DSL with PNC and beamforming and TBP ratio of NC DSL is shown here in Fig. 5.10. It can be seen that there is a remarkable gain in the TBP ratio from roughly about 2 up to 5 times when DSL uses PNC and beamforming. This significant improvement can be attributed to the efficient spectral utility by saving a time

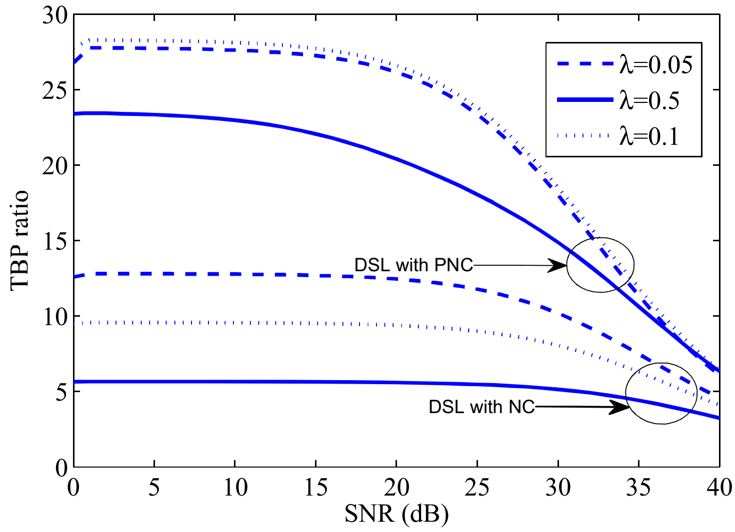


Figure 5.10: Comparison between DSL with NC and PNC (Rayleigh Fading).

slot using PNC. Also, the MRT beamforming used improves the channel gain at the receiver thereby improving the overall performance of DSL.

5.8 IMPACT OF SELFISH BEHAVIOUR

So far the performance of DSL assuming all nodes are honest i.e., $\varphi = 0$ is analysed. Now it is explored that how the selfish behaviour of some CR nodes effects the overall performance of DSL. Fig. 5.11 shows that increasing φ marginally improves $\overline{R}_{ps}$ during phase I. This can be explained as a consequence of the decrease in the effective number of CR relays that actually receive the primary data to relay later on. Since this number decreases, hence over all the distance of any primary node to the farthest honest CR relay decreases. However, the selfish behaviour largely deteriorates the capacity in phase II, where a decrease of more than 2x can be observed from Fig 5.11 when 70% of the CR nodes act selfishly. Such deterioration can lead to severe degradation in the

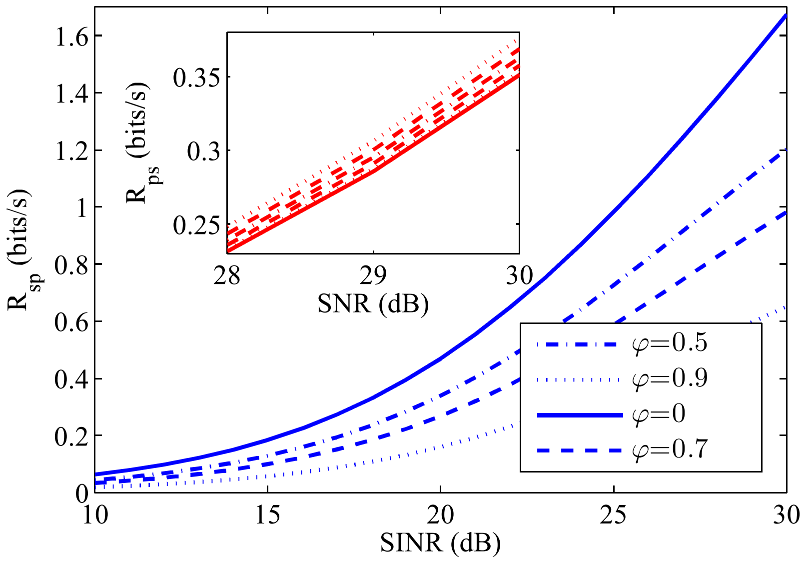


Figure 5.11: Effect of selfish behaviour on $\overline{R}_{ps}$ and $\overline{R}_{sp}$ where $\lambda = 0.1$.

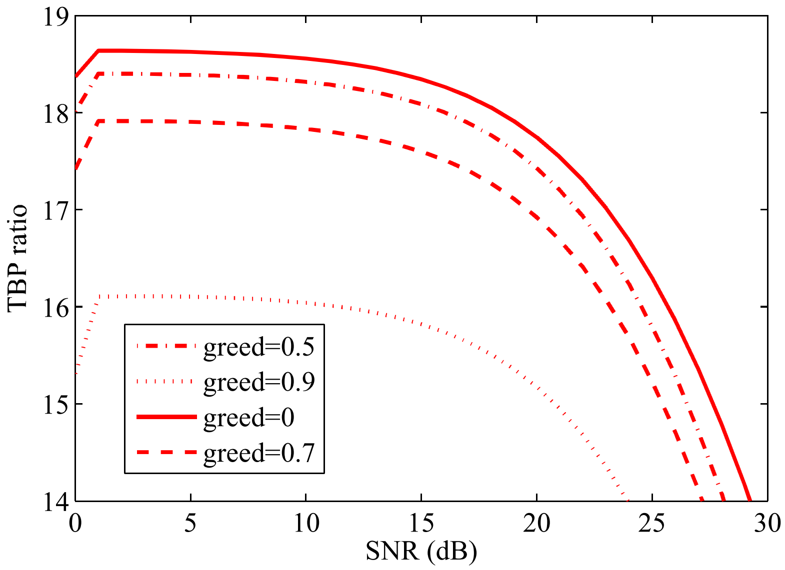


Figure 5.12: Effect of selfish behaviour on TBP where $\lambda = 0.1$.

quality of service of relaying promised by the CRs leading the primary network to incur loss in the expected $\overline{R}_{sp}$. The primary agrees to leasing time t_{ss} on the understanding that all CRs honestly cooperate. The selfish behaviour not only degrades the data rate of relaying of the primary data but also procures greater reimbursement as compared to the help offered to the primary network.

Fig. 5.12 shows the impact of selfish behaviour when DSL provides the highest gains. It was shown in Fig. 5.9 that a relatively sparse CR density of $\lambda = 0.1$ results in the highest DSL. It is most important to quantify the impact of selfishness of the CR network for this network density. It can be seen from Fig. 5.12 that the TBP ratio decreases sharply by increasing φ in the network. Therefore, the increasing selfish behaviour of CRs i.e., $\varphi = 0 \rightarrow 0.9$ can significantly degrade the performance of DSL. Here, the steep rise in TBP between 0 and 1 dB can again be attributed to the division of time in the bargaining phase. For a broader perspective, the impact of selfishness on a wide range of λs is shown in Fig. 5.13. In the Figure, the difference in TBP ratios between $\varphi = 0$ and $\varphi = 0.9$ is shown. It can be seen that increasing the CR causes less degradation in TBP ratio. For $\lambda = 0.1$, a steep decrease of more than $\frac{1}{2}$ in TBP can be observed. Hence, it is critical to study the impact of selfishness and devise a methodology to discourage it in the CR network.

5.9 MONETARY PRICING TO REDUCE SELFISH BEHAVIOUR

The bargaining process takes place under the inherent assumption that all nodes are honest. In order to recover from the loss incurred due to the selfish behaviour of the CRs, it is proposed that the primary network can impose monetary pricing on the nodes that exhibit selfish behaviour. This involves:

1. Identifying the selfish nodes;
2. Deciding an optimal price for selfish nodes.

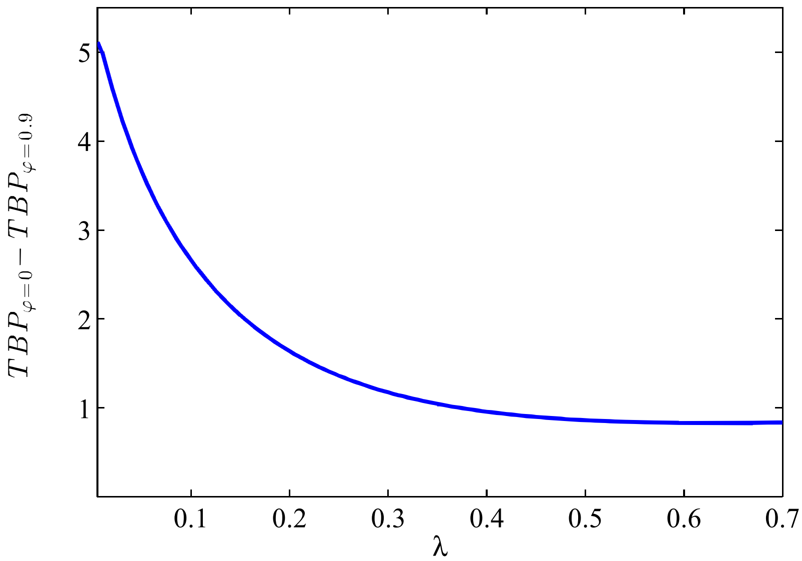


Figure 5.13: Effect of selfish behaviour on TBP vs. λ.

5.9.1 *Identifying selfish nodes*

There can be a number of ways to identify which nodes act selfishly and cause damage to the primary nodes. One possible way is to enforce a mechanism where every CR participating in the leasing process has to send an identification beacon to the primary in a mandated format. While decoding the message from CRs in the second DSL phase, the primary only needs to see the identities of the interfering nodes to find out the culprits. Another way to identify the selfish nodes is to use the RF fingerprints [133] of individual devices. This avoids the overhead of a mandated identification beacon. The concept of using taboo codes has been suggested in [134] claiming it to be highly efficient. For this study, the process of exchange of terms before a DSL agreement is reached makes it easy to use any of the above mentioned approaches to identify the selfish nodes.

5.9.2 *Optimal pricing for selfish nodes*

When the primary node observes loss in the expected R_{DSL}, it recognizes that all the CRs have not cooperated in relaying. In the presence of selfish nodes, it is important for the primary to enforce a mechanism to discourage such behaviour. It is proposed that the PN can charge the secondary nodes a monetary price for their selfish behaviour. A realistic price that the selfish nodes pay for violating the agreement terms so that the motivation for such violation in future is minimized is introduced as the optimal pricing. In order to do so, it is important for the primary to quantify the fraction φ, i.e., the fraction of nodes that act selfishly. However, there is no inherent way for the primary to know this fraction. To this end, it is proposed that a learning approach where a monetary cost p proportional to the duration t_{sp} while selfish behaviour is exhibited can be imposed. It can help discourage the CRs from being selfish in future. Mathematically, the new utility function of the selfish nodes becomes,

$$\mathcal{U}_S\left(t,p\right) = t_{sp}\overline{R}_{\text{selfish}} + t_{ss}\overline{R}_{ss} - \delta p, \tag{5.34}$$

where δ is a constant based on an estimate of the price of data transmission measured in $\frac{\text{bits}}{\text{£Hz}}$. The aim of the primary user is to make the utility of the selfish CRs in eq. (5.34) zero. In the absence of an estimate of $\overline{R}_{\text{selfish}}$, it is suggested that a simple quantification of the difference in the currently observed rate in second phase $\overline{R}_{sp}|_{\varphi_t}$ (φ_t denotes the fraction of selfish nodes at the current round/time t) and the rate promised by the CRs while bargaining assuming there are no selfish nodes i.e., $\overline{R}_{sp}|_{\varphi=0}$ can be useful. It is proposed that the primary begins charging the price p proportional to the difference $\overline{R}_{sp}|_{\varphi=0} - \overline{R}_{sp}|_{\varphi_t}$. Every time when the primary incurs a loss of capacity in the second phase, it learns that the price charged during the previous round was not sufficient to discourage the selfishness. In the next round, the primary charges a higher price in proportion to the loss incurred. Given that a fraction φ acts selfish to begin with, the behaviour of the selfish nodes during their next spectrum access depends upon the utility $\mathcal{U}_S\,(t+1, p+1)$. Any $p > 0$ leads to $\mathcal{U}_S\,(t+1, p+1) < \mathcal{U}_S\,(t, p)$. This reduced utility begins to discourage the CRs to again act selfishly next time. As a simple quantification, it is suggested that the number of selfish users for the next time φ_{t+1} depends upon how many of the selfish nodes stop being selfish due to the reduced utility $\mathcal{U}_S\,(t+1, p+1)$ because of the price they have to pay for their behaviour. Mathematically,

$$\varphi_{t+1} = \varphi_t - \varphi_t \mathcal{U}_S\,(t+1, p+1)\,. \tag{5.35}$$

In order to make the utility of the selfish nodes approach zero in the next round, we specify a price depending upon the loss in the rate in second phase of the current round i.e., $\overline{R}_{sp}|_{\varphi=0} - \overline{R}_{sp}|_{\varphi_t}$. The price during their next spectrum access for the selfish CRs can be set to

$$p(t+1) = t_{sp}\delta^{-1}\left(\overline{R}_{sp}|_{\varphi=0} - \overline{R}_{sp}|_{\varphi_{t+1}}\right), \tag{5.36}$$

such that it is aimed to minimize the difference between the agreed i.e., $t_{sp}\overline{R}_{sp}|_{\varphi=0}$ and the experienced $t_{sp}\overline{R}_{sp}|_{\varphi>0}$ TBP in phase II. Fig. 5.14 shows the performance of the given algorithm to discourage selfish behaviour for various densities of CRs. It can be seen that as the density of the CRs increases, the loss in primary's rate, i.e., $t_{sp}\overline{R}_{sp}|_{\varphi=0} - t_{sp}\overline{R}_{sp}|_{\varphi>0}$, due to selfish behaviour in phase II increases. It can be seen that more rounds of pricing are needed to discourage the CRs to act selfishly at high densities. Due to the large number of CRs present, even a small fraction of selfish nodes φ can cause a lot of difference in the promised rate while bargaining and the actual rate attained. The

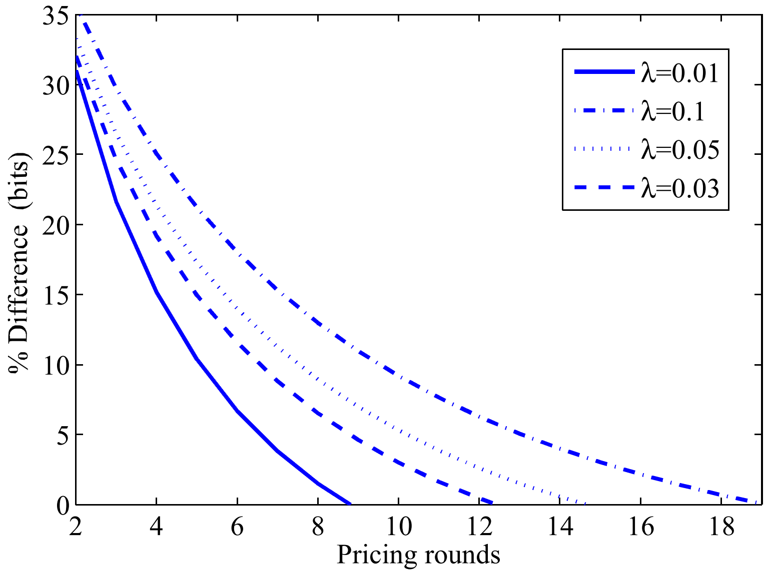


Figure 5.14: Number of pricing rounds vs. the % difference $t_2 C_{BF}|_{\varphi=0}$ and $t_2 C_{BF}|_{\varphi>0}$ for $\delta = 1$.

algorithm can be made to converge quicker by setting a higher price. However, the algorithm converges quickly for low densities of CR network which is the preferred network setting for both primary and CR networks.

5.10 CONCLUSIONS

In this chapter, the usefulness of DSL as a scheme which can improve the performance of the PN in terms of the number of bits that can be successfully transmitted between the primary sources was investigated. TBP ratio β indicated that DSL can improve the communication of the PN as compared to direct communication more than 17 times. Specifically, a relatively sparse deployment of the CRN is favourable for both the primary and CR network. For the PN, distributed beamforming and PNC with denoising are the key factors that result in enhanced cooperative relaying performance. Such high performance can be seen only when the secondary relay density is kept low. These performance determinants and their effect on the working of DSL can only be measured due to the detailed geometric modelling of the network. Hence denoise and beamforming based DSL provides an efficient alternative to direct two way communication for the primary network. Within the same available time and bandwidth, it allows the CR network to communicate with each other at an acceptable rate and QoS. The presence of selfish CRs in the network was studied and their utility was modelled. It is shown that the selfish behaviour by CRs can reduce the TBP ratio more to than 2x at low densities densities. An iterative pricing based punishment algorithm was devised to discourage the selfish CRs. The results indicate that after about 8 rounds of iterative pricing, the selfish behaviour of the CRs is reduced to zero and the high gains of DSL are restored.

6

GREEN LICENSED SHARED ACCESS

Summary of Contribution

In the previous chapters, the discussion has largely been focused at infrastructure-less networks where a pair of primary terminals communicate with each other. They are helped by co-located, secondary nodes that bargain over a limited spectral access time. In this chapter, spectrum sharing for future cellular communication is studied where unlike previous chapters, the *number of cooperators* is the resource to bargain over. The concept of hierarchical networks with the presence of a macro network and a single tier of small base stations serving their particular users is considered. The relatively new and emerging idea of licensed shared access (LSA) is studied as the mechanism of implementing dynamic spectrum leasing. LSA presents a more formal definition of spectrum leasing and is only in its inception phase. In this chapter, LSA spectrum sharing is analysed as an energy efficient way to increase the spectral utility of a network. The small cell network offers offloading services to the macro network to improve its quality of service. In return, the small cells are rewarded for their cooperation with a number of licenses to operate in the spectrum owned by the macro network. The short link distances and diversity offered by the small cell network to offload the macro user help to enhance the total spectral and energy utilization of the network. For effective functioning of the scheme, it is important to determine the division between the fraction of small cells that provide offloading services and those which obtain the licenses. In this chapter, a comprehensive mathematical model of this LSA offloading is presented and the optimal division between the offloading and licensing small cells is evaluated. The analysis shows that such spectrum sharing can lead to commendable gains in the spectral utilization of the network while assuring the desired QoS for both the macro and the small network. An intelligent determination of the fraction of the offloading small cell network results in efficient energy use while offloading. The scheme is also tested within the network parameters of the city of Leeds. The results indicate that for the considered scenario, the spectral utility of Leeds can be improved by more than 12x, and the energy efficiency by 16x by adopting to the proposed LSA scheme.

6.1 INTRODUCTION

As discussed previously, spectrum availability can be improved by changing the conventional exclusive spectrum licensing models where the license holders, e.g., MNOs, enjoy complete authority over a band of frequencies leased to it by the spectrum management authorities like Ofcom and FCC. In very recent times, the communications industry [16] has introduced the concept of LSA which attempts to bridge the gap between exclusive licensing and commonly sharing the spectrum. More recently, the Radio Spectrum Policy Group (RSPG) of the European Commission (EC) has laid out the framework of sharing opportunities between different types of wireless communication systems by developing the LSA concept [17]. The authors in [19] have described LSA as a complementary spectrum allocation mechanism which by definition is closer to the property rights model of individual licensing [18]. LSA allows new spectrum users/licensees within a pre-allocated frequency band, which require a certain level of guarantees in terms of spectrum access, while ensuring that the QoS requirements of the legacy/incumbent users are always met. In order to maintain a co-existence and meet the communication quality targets of both the incumbents and the small cells license holders, some regulatory constraints need to be observed. These constraints can be dictated by the owner of the spectrum and also by the the spectrum management and regulatory authority. The constraints may vary for each kind of service being provided. It is proposed that these technical and operational conditions can be stored in a repository so that the incumbent network, the new users and the spectrum regulatory authorities have access to them. Under these regulatory terms, the spectrum is licensed to new users (called licensees) for use in a way that complies with the defined constraints and guarantees for the incumbent users. Under LSA, both incumbents and the licensees have QoS guarantees and protection from harmful interference. Recently, studies [135] and [136] have suggested LSA to solve the problem of spectrum availability for primary and secondary users in a network. One of the very first trials of LSA have been carried out and have been shown to potentially increase the spectral utilization of a network [137]. However LSA based offloading leading to an energy efficient solution has not been considered before to the best of our knowledge.

Small cells on the other hand present a relatively mature concept that has made its way to practical deployment e.g., Sure Signal by Vodafone. Small cells are being deployed under the umbrella of heterogeneous networks to seamlessly extend the present capacity and coverage in cellular networks. Small cells are light-weight, short range usually low powered base stations that are mounted on low heights to serve users that are close to them. They help in enhancing the overall capacity of the network especially in busy urban areas. Small cells provide complementary services by offloading the traffic of the macro base stations [138]. In doing so, the QoS demands of the MNO can be met. According to [3], around 46% mobile traffic was offloaded to either Wi-Fi or small cells during 2014 showing the importance of offloading in coping with the growing mobile traffic. Their low power profile makes them an excellent candidate to implement energy efficient solutions. Such improved spectral utility attributed to shorter link distances also leads to energy efficiency [139]. However, current mobile communication networks are not designed to be trivially extended to integrate small cells within macro cells since small cells are bound to share the same spectral space as that of the macro networks. All mobile networks are bound to experience high volumes of traffic but it is not feasible for all operators to install their own dedicated small cells. Also they can be densely deployed by companies and individuals thereby not subject to as much radio planning as that of macro base stations. For this reason, the interference management between the small and macro networks has become a key challenge. The idea of heterogeneous networks allows a number of tiers of these smaller cells within a macro cell. Each tier may have its own traffic/dedicated services and may also be used to offload the macro cells in congested urban areas with high traffic. Under these circumstances, the problem of interference facing each tier of such a network becomes increasingly dominant and undermines the potential to provide increased capacity in a heterogeneous network. Also, small cells have largely been studied in the context of opportunistic access to the MNO's spectrum rendering their connectivity intermittent without any QoS assurance.

6.2 RESEARCH OBJECTIVES

In this chapter, the main question addressed are as following;

Design Issue 6.1

Can DSL improve the SE and EE of infrastructured wireless networks. Is it possible to use DSL in the form of LSA based offloading to help the MNO make use of existing small cell infrastructures saving the deployment and management overheads?

More specifically, the following objectives are outlined for this chapter;

1. How to provide the existing small cells an opportunity to operate and serve their own traffic in MNO licensed frequency bands;
2. How to assure the QoS demands of both MNO and small cell network;
3. How to improve the spectral and energy utilization of the MNO using LSA offloading.

6.3 CONTRIBUTIONS

In light of the potential benefits of spectrum sharing based offloading, a novel LSA mechanism for a two tiered network, i.e., 1st tier of macro cells and 2nd tier of small cells is considered in this chapter. Macro cell network does not own the small cells but it needs their offloading services to meet the high coverage and capacity demands. The small cells on the other hand do not have any transmission access in the frequencies owned by the macro network. Hence the small cells are obliged to refrain from any transmission to their receivers in these frequencies leading to spectral deprivation. To help both networks, an incentive based spectrum licensing scheme is introduced where the incumbent network shares its spectrum with small cells to get their offloading services. The small cells are encouraged to offload the traffic of the macro cell in order to get access to the spectrum as a reward for their offloading services. As a result, the macro network meets the high traffic demands by offloading without having to bear the planning and deployment costs of a small cell network. In the proposed model, a fraction of the total number of small cells cooperates with the macro network and offload its users. Meanwhile, the remaining fraction of small cells is granted a license to share the MNO spectrum and communicate with its own users. It is but natural for the macro network to desire that all small cells cooperate with it to offload its traffic to increase its performance and avoid any interference. On the other hand, all small cells want to obtain an LSA license to carry out their own communication. Consequently, it becomes critical to accurately determine the number of offloaders and licensees for the successful operation of the proposed model to attain the desired QoS for both tiers and maintain the interference at acceptable levels. The key idea is that interference due to spectrum sharing can be offset by cooperation gain when LSA based coordination/licensing strategy is in place. In this chapter, a mathematical framework is developed and used to determine the performance of the LSA based scheme while capturing the geometric and spatio temporal characteristics of base stations and the users in an urban network. The concept of bargaining is used to determine the optimal division between the densities of the offloading and spectrum sharing small base stations that lead to increased network performance. It is shown that mutual agreement on the number of offloaders and LSA licensees helps to control the interference between the two tiers of the network and therefore, the overall network capacity can be increased. An intelligent division between the number of offloaders also helps to restrict the energy usage for offloading. Geographic locations of the small cells play a vital role in the operation of LSA and the division between cooperators and licensees. Only a few offloaders at a spatially suitable location providing excellent received signal strength at the MNO user may suffice the requirement of the MNO. Such a situation would be ideal since more small cells can enjoy the licenses to access the spectrum for their own communication.

The important features of the novel LSA scheme based offloading in a two tiered network are:

1. It follows a *rational* approach to improve the capacity of both macro network (deprived of a small cell infrastructure of its own) and small cells (having no/limited access to frequency bands for transmission). It provides incentives to both of them to enter into a spectrum sharing agreement and provides them QoS guarantees.

2. It provides a *realistic* model capturing the geographical presence of users and their varying traffic demands. It uses analytical tools to build a mathematical framework to model the geometry and network occupancy at various times of the day in densely populated areas.

3. It exploits the density of small cell deployment as a network resource unlike traditional approaches of exploiting time/frequency.

4. It presents a *fair* division of the density of the small base stations into fractions of offloaders and licensees. Mutual agreement is attained between both macro and small network leaving both parties satisfied with their share.

5. It makes the network more spectrally *efficient* as more than 12x improvement is noticed in the network utility of spectral resources.

6. It makes the network up to 16x energy efficient.

It is claimed that the proposed model presents a viable solution to the coverage and capacity challenge in densely populated urban areas. Leeds City Center is one of the UK's busiest city squares and presents an ideal platform for the deployment of small cell based network with high capacity and coverage demands. Leeds City Center is analysed as a candidate for the deployment of the proposed model by studying the scheme under the key network parameters of Leeds. The results indicate that 12x or more improvement can be observed in the network capacity of Leeds city centre by employing the proposed LSA based offloading. Most importantly, 16x improvement in the energy efficiency of the network in Leeds is observed. These gains are attained with almost negligible additional infrastructure cost.

6.4 RELATED WORK

A conventional approach towards offloading to small cells assumes that the small cell infrastructure is owned by the same MNO hence making the deployment well planned based on rigorous cell planning. The spectrum access under offloading to small cells is traditionally casted under the paradigms of i) open; ii) closed; and iii) hybrid access models each referring to the number of MUs and ease of association to a particular small cell for MUs. These access models present a compromise between the number of MUs benefiting from the offloading services of small cells and the interference caused due to offloading. The work in [140], and references therein investigate various aspects of these models. Our LSA robust scheme presents a need based solution where both MNO and small network coordinate for mutual benefit. To the best of our knowledge, there is no work that provides strict QoS guarantees through a formal licensing procedure based on the user traffic profiles as proposed in our LSA based scheme.The studies in [141], [142] have done useful work in the modelling of heterogeneous networks. The studies in [143] and [144] investigated a coexistence technique according to which the small base stations are allowed to transmit as long as their interference is below a certain threshold. In [115], an incentive based approach for offloading is introduced and a time division scheme is studied for uplink communication in macro cells. In [109] a refunding based offloading scheme is introduced where MUs are allowed to access femto cell resources in exchange for appropriate revenue refunding. However, our work presents an LSA approach with a mutual agreement based rewarding unlike previous works where a leader follower approach is studied. LSA operates with predefined and agreed upon regulations and hence is a more promising way to increase the spectral utility while maintaining the QoS of the communication. A novel density based division for offloading is introduced. The bargaining based division of small cells for utility maximization is also one of our novel propositions. A thorough analysis based on the geometric and temporal characteristics of users proposing a game theoretic determination of the number of offloaders and licensees in the network are the contributions of this chapter. Small cells have been studied in the context of saving energy for the macro network by offloading the data to them in [145],[139] and references therein. However, to the best of the authors' knowledge, how LSA based licensing and offloading dictates the energy efficiency of the macro network remains unexplored. Validating the importance and timeliness of our work, [146] in Section 6 has stated the problem of offloading MNO traffic to small cells is an interesting open research issue. Based on our careful study, none of the previous studies have addressed LSA based spectrum sharing from these perspectives.

6.5 THE PROPOSED LSA BASED OFFLOADING MODEL

6.5.1 *Proposed LSA for small cells model*

An LSA based spectrum sharing technique is proposed in this chapter where randomly deployed small cells can share the spectrum of the macro network to serve their own as well as macro users. It is claimed that the LSA architecture with an intelligent determination of the number of offloaders in the network can provide strict QoS while attaining energy efficiency for the macro network.

6.5.1.1 *Spectral Utility Model*

A network of macro base stations (MBS) (incumbents) having a number of mobile users (MUs) within the network existing in an urban area is considered. This network operates in a particular band of frequencies for which it enjoys explicit individual rights of ownership. A random network of small base stations (SBS) overlaid on the same geographical area as that of the macro network is considered. These small base stations have a number of users called small users (SUs) to serve. Their performance is dictated by the aggregate capacity of all the users within the network i.e., MUs and SUs. Like in previous chapters, R_{QoS} and R_{QoS_s} $\frac{bits}{sec}$ are the target data rate in each network i.e., the incumbent and the small cells respectively want to attain when they communicate to their respective users. The reliability of this rate is dictated by the probability

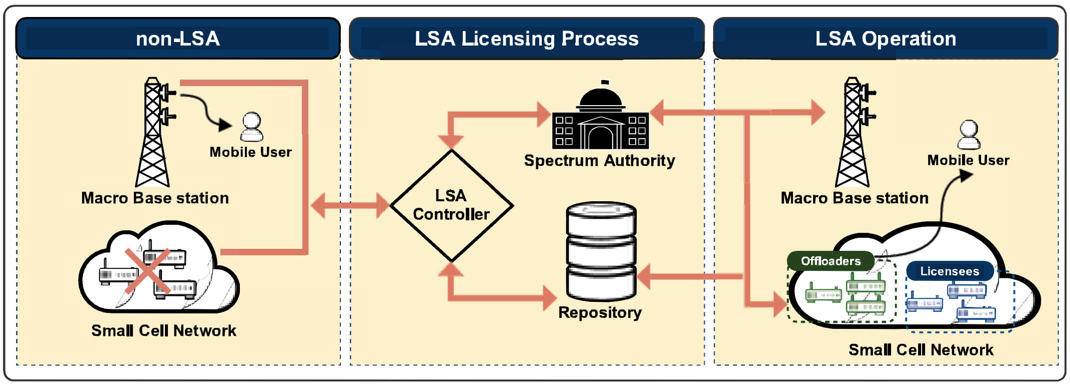


Figure 6.1: LSA schematic.

of attaining this rate successfully under the environmental channel conditions, power loss and interference experienced.

The network operates in two modes: direct mode and offloading mode. In the direct mode, the probability of success $p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u\left(t\right)\right)$ at a random MU communicating with a macro base station is dictated by the target data rate R_{QoS} of communication, the density $\lambda_u\left(t\right)$ i.e., the number of MUs present per squared meter at a specific time of the day t in the network and the density λ_b i.e., the number of macro base stations (MBSs) present per squared meter, some of which might interfere with it. The area throughput/ area spectral efficiency attained in this mode of communication is

$$R_T^{\mathrm{Direct}} = \lambda_b p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u(t)\right) R_{\mathrm{QoS}} \frac{\mathrm{b/s}}{\mathrm{m}^2}. \tag{6.1}$$

During this mode, the co-existing small base-stations (SBS) do not communicate with their users since they have no access rights to the spectrum, hence, their throughput is zero in direct mode. Fig. 6.1 gives a graphical view of the scheme. The left panel shows the non LSA i.e., direct mode of communication.

The macro network directly communicating with its MUs might decide to offload the MUs to small cells. In the offloading mode, it is proposed that the small base stations present in a network with a density λ_s can be divided into two groups based on the service quality they can offer to the offloaded MU.

The first group consists of $\zeta_m \lambda_s$ small base stations per unit area that can offer a service better than a certain threshold of required quality of service at the MU. These small base stations are referred to as the *offloaders*. The remaining SBSs form the second group of *LSA licensees* i.e., $\zeta_s \lambda_s$ small base stations per unit area where $\{\zeta_m, \zeta_s \in [0, 1] \text{ s.t. } \zeta_m + \zeta_s = 1\}$. The small base station offering the best service to the MU out of the first group then offloads the MU from the macro base station. This small base station serves the MU at rate R_{QoS} with a successful transmission probability of $p_{\text{suc}}^{\text{O}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u\left(t\right), \lambda_s\right)$. The remaining base stations in the first group then refrain from any transmission while the MU is being offloaded providing diversity gain to the MU. Given that one MU per macro cell is offloaded, effectively, $\zeta_m \lambda_s$ provide the diversity gain while the λ_b small base stations carry out actual transmission for offloading to MUs. Such setting is only possible when the small base stations are very densely deployed i.e., $\lambda_s \gg \lambda_b$. While the offloading mode of communication is adopted, the MBS does not attempt any direct communication with any user. It can be understood as related to the concept of *almost blank subframes* (ABS) provisioned by LTE rel-10 where the macro base stations assume silence to enable the small cells to operate without causing interference to them [147][1].

On the other hand, the second group of LSA licensees enjoys the rights to use the frequency band primarily owned by the macro network. The small base stations do not have any right to transmit in this frequency band otherwise. The incumbent MNO offers this license because it gets the benefit of offloading its traffic to the small base stations and improving its service to the MUs. The small base stations help the macro network by offloading its traffic but in turn

1 It is equivalent to consider that other macro base stations might transmit while an MU in a particular cell gets offloaded, however i) the frequency reuse plan; ii) the large distance between this MU and other base stations; and iii) the strong signal from nearby offloading small base station makes the SIR at the offloaded MU high and the interference from neighbouring macro base stations negligible.

acquire the license for some of the small base stations to carry out transmission in the macro's frequency band at rate R_{QoS_s} with a successful transmission probability of $p_{\mathrm{suc}}^{\mathrm{S}}\left(R_{\mathrm{QoS}_s}, \lambda_b, \lambda_u\left(t\right), \lambda_s\right)$. The mutual benefit of both the network tiers is the prime motivation for such spectrum sharing which is well casted in the framework of LSA. The right panel in Fig. 6.1 shows the operation of LSA.

For such spectrum sharing, the regulatory terms play the most critical role. These include the transmit power, frequency band, geographic region and licensing time, etc. The central panel of Fig. 6.1 depicts the regulatory and control mechanism of LSA. It consists of an LSA controller which is the decision making entity that decides the terms of LSA offloading between the macro and the small network. It also links to the spectrum administrative authority e.g., Ofcom in the U.K. LSA can be both long term or short term and needs authorization from the spectrum handling agency [135]. The repository as shown in the Figure contains the record of the frequency bands and their occupancy details. All records need to be updated in the repository for easy and accurate access to the particulars of spectrum usage.

Out of all the regulatory terms, the most important is the question of how many small base stations will become offloaders constituting the first group and how many will be those who benefit by becoming the licensees in the second group while keeping the interference down to an acceptable level. To answer this question, a game theoretic solution is proposed that divides the density of the small base stations into a fraction i.e., ζ_m that helps by offloading and the other $\zeta_s = 1 - \zeta_m$ which benefits by getting the LSA license. This division is attained via a bargaining game between two players i.e., macro network (player 1) and small network (player 2) that captures the rational utility maximization approach of both players [123]. Both macro and small network place an offer/demand of ζ_m/ζ_s. These offers are assessed at the controller that decides whether to accept or reject the current set of offers based on the resulting QoS for both networks and the regulatory constraints imposed by the spectrum management authority. If a mutual agreement between both players is reached on ζ_m/ζ_s, the LSA mode of operation is adopted. This information is updated in the repository once approved by the spectrum management authority. This division of density such that the MNO and the network of small cells both mutually benefit and provide services meeting a fixed QoS is the crucial component for the regulation of the entire scheme[2]. Figure 6.1 outlines the elements and their interactions in the network.

The aim of the chapter is to present the model of a spectrum sharing based offloading scheme that provides the required capacity and increases the overall network utility. The overall network spectral utility of the macro network R_T^{LSA} can be defined as

$$R_T^{\mathrm{LSA}} = p_{\mathrm{suc}}^{\mathrm{O}} \left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u(t), \lambda_s \right) \lambda_b R_{\mathrm{QoS}} \frac{\mathrm{b/s}}{\mathrm{m}^2}. \tag{6.2}$$

It is important to reiterate that the capacity of any type of communication in the network is dependent upon the target rate, the number of MUs, macro and small base stations and the time of the day. Hence LSA based offloading is a novel and important approach in which the performance of the overall network is quantified in terms of all the important realistic parameters and the utility of the proposed scheme is studied under assumptions of rationality of the entities under examination. Eq. (6.2), quantifies the aggregate communication rate in both tiers of the network in LSA based operation determining the overall utility of the spectral resources of the network.

2 Here we assume that such LSA based offloading is operable in a single LTE subframe.

6.5.1.2 *Energy Efficiency Model*

It is proposed that LSA based offloading provides better utility in terms of energy usage as compared to mere direct communication. In order to characterize the utility, the energy efficiency of the direct communication for unit bandwidth is quantified in terms of the number of bits transmitted per Joule of energy utilized. Formally,

$$EE^{\text{Direct}} = \frac{p_{\text{suc}}^{\text{M}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u(t)\right) R_{\text{QoS}}}{P_b} \frac{\text{bits}}{\text{Joule}} \tag{6.3}$$

where, P_b is the transmit power of the macro base station. Similarly, for LSA offloading, the energy efficiency for unit bandwidth is defined as the bits transmitted successfully when $\lambda_s \zeta_m$ small base stations offload for the macro network by using power P_s.

$$EE^{\text{LSA}} = \frac{p_{\text{suc}}^{\text{O}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u(t), \lambda_s\right) R_{\text{QoS}}}{P_s \lambda_s \zeta_m} \frac{\text{bits}}{\text{Joule}} \tag{6.4}$$

Like the spectral utility, the energy efficiency is dependant on performance factors like link distance, transmit power and number of offloading small base stations in the network. An accurate modelling and study of the LSA under these parameters dictates the performance of the scheme. An intelligent choice of the fraction ζ_m directly relates to better energy utilization of offloading as shown in eq. (6.4). Though only one MU per macro cell is offloaded by transmission from one small base station, however, the listening cost at all $\zeta_m \lambda_s$ small base stations is incurred and hence included in the energy efficiency quantification.

It is important to note that the LSA based offloading scheme ensures fairness for all MUs and SBS in terms of minimizing the likelihood that a particular SBS always has to offload an MU and never gets a chance to carry out its own trans-

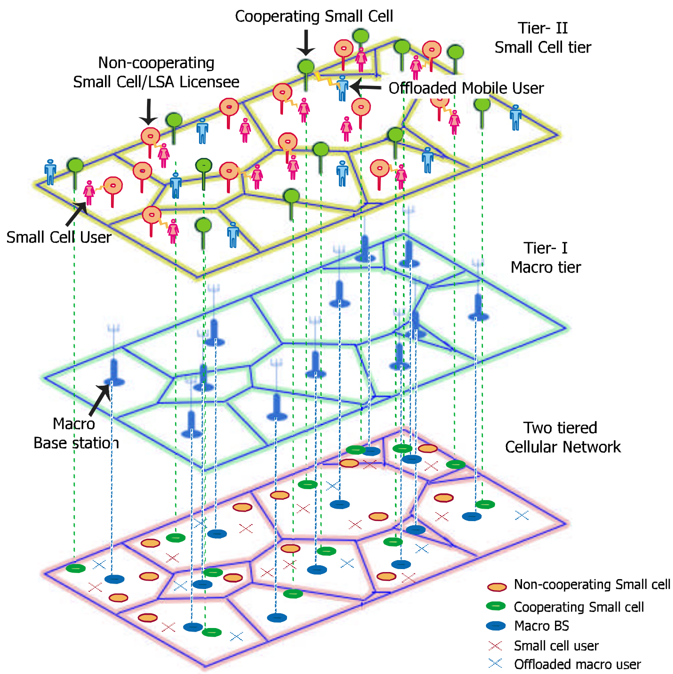


Figure 6.2: Network Model

mission. With considerations of fading channel conditions and mobile MUs, the probability that a particular small base station always has to offload is very low.

6.6 SYSTEM MODEL

A scenario is considered where the macro network, the ad hoc small base stations and the MUs are distributed in the same geographical space as shown in Fig. 6.2. The spatial distribution of the networks is captured by three independent stationary PPPs i.e., Φ_b of macro base stations with intensity λ_b, Φ_s of small base stations with intensity λ_s and Φ_u of MUs with an intensity λ_u. Specifically, at an arbitrary time instant the probability of finding n_i nodes belonging to these three networks i.e., $i \in$\{macro base station, MU, small base station\} follows the Poisson law with mean measure $\Lambda_i(A) = \lambda_i v_i(A)$. The mean measure is characterized by the average number of macro/small base stations and MUs per unit area and the Lebesgue measure $v(A) \int_A \mathrm{d}x$ on $\mathbb{R}^2$, where if A is a disc of radius r then $v(A) = r^2$ is the area of the disc (see Chapter 2, Sec. 2.3.2).

The propagation channel in the network suffers from path loss and fading; the latter is assumed to be Rayleigh distributed as described in Chap. 3 Sec. 3.4.1. According to the network parameters stated above, the signal to noise ratio (SNR) γ at an arbitrary receiver located at a distance r from another node transmitting with a power P is given as in eq. (3.2)

$$\gamma = \frac{hl(r)P}{\sigma^2} \tag{6.5}$$

where σ^2 is the power of the additive white Gaussian noise (AWGN) at the receiver front-end.

6.7 ANALYTICAL MODELLING OF LSA FOR SMALL CELLS

In this section a mathematical framework of our LSA model for small cells is developed as described in section 6.5. To start with, it is considered that a macro base station is serving its MU providing a data rate of R_{QoS} with probability $p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u(t)\right)$. It is assumed that the macro base station needs to offload its MU to the existing small cells in the network providing a data rate of $R_{\mathrm{QoS}} \frac{bits}{sec}$. The small cells are ready to offload the MU but they want to maximize the number of LSA licenses that they get in exchange for offloading to overcome their offloading cost and serve some of their own users in the licensed band at $R_{\mathrm{QoS}_s} \frac{bits}{sec}$. This demand is translated as the maximization of the fraction

ζ_s of λ_s small base stations per unit area attaining the LSA license. To meet the offloading QoS guarantee (i.e., offloading while maintaining a data rate of at least R_{QoS}), the macro base station also wishes to get a maximum fraction ζ_m of λ_s small base stations to cooperate and offload its MU. In the rest of this section, this scheme is modelled to attain the appropriate fractions ζ_s^* and ζ_m^* of λ_s so that R_{QoS} and R_{QoS_s} $\frac{bits}{sec}$ is guaranteed for the incumbents and licensees with probabilities $p_{\text{suc}}^{\text{O}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u(t), \lambda_s\right)$, and $p_{\text{suc}}^{\text{S}}\left(R_{\text{QoS}_s}, \lambda_b, \lambda_u(t), \lambda_s\right)$.

6.7.1 *How many MUs and macro base stations in the network?*

Realistic modelling of the density of users and base stations in a network is primitive to accurate analysis of any communication scheme. The density of MUs at any location $\lambda_u(t)$ largely depends upon the demographics of the area and the time of the day. The temporal details of the density of the users are captured by considering that during the down link operation of a macro cell in an urban area, the average traffic density varies as

$$\lambda_u(t) = \lambda_u\left(\omega_5 - \omega_6 \sin\left(\frac{\pi}{12}(t-\phi)\right)\right) \tag{6.6}$$

where ω_5 and ω_6 are traffic related constants and lie between $[0, 1]$. These parameters vary with each day of the week depending on voice/data traffic load, e.g., more traffic in busy hours etc. The sinusoidal variation of mean density has been derived from actual cellular networks in [148] and [149]. In this study, operation on a typical weekday is assumed by selecting the values of ω_5 and ω_6 as 0.5 and $\phi = -1$. These parameters produce well known characteristics associated with the user arrival such as decreasing behaviour for early hours, dip at around 4-5 a.m and attain a maxima around the evening time. However other values of ω_5, ω_6 and ϕ might be applicable in particular scenarios.

In our network setup, a base station can be active/inactive depending upon the presence of an MU within its Voronoi cell. From [150], the probability mass function of the number of MUs N in a Voronoi cell of a randomly chosen BS is given as,

$$\mathbb{P}\left[N=n\right] = \frac{3.5^{3.5}\Gamma(n+3.5)(\xi(t))^n}{\Gamma\left(3.5\right) n!(\xi(t)+3.5)^{3.5+n}}. \tag{6.7}$$

Correspondingly, the probability $p_b(t)$ of a base station being active is the complementary probability of a base station having no MU in its Voronoi cell, i.e.,

$$p_b(t) = 1 - \mathbb{P}\left[N=0\right] = 1 - \frac{1}{(\frac{\xi(t)}{3.5}+1)^{3.5}}, \tag{6.8}$$

where $\xi(t)$ is a function of MU density at a given time of the day as $\xi(t) = \frac{\lambda_u(t)}{\lambda_b}$. For the sake of simplicity, it is assumed that whenever the symbol $\bar{\lambda}_b$ is used, active base stations i.e., $\bar{\lambda}_b = p_b(t)\lambda_b$ are being referred to. Moreover, the probability that a particular MU will be selected for transmission in the downlink follows from the number of active base stations as,

$$p_m(t) = \frac{\bar{\lambda}_b}{\lambda_u\left(\omega_5 - \omega_6 \sin\left(\frac{\pi}{12}\left(t-\phi\right)\right)\right)}. \tag{6.9}$$

Remarks: The activation probability of a base station depends upon the number of active users within its cell. Increasing the number of active MUs increases the probability of more base stations being active. The probability an MU is selected for being served by its base stations varies across different times of the day. In peak hours, the MUs might suffer from outage due to high number of active MUs in the network. It is important to note that eq. (6.9) is only applicable when the active base station density is $\lambda_u(t) > \bar{\lambda}_b > 0$. Hence throughout our analysis, it is assumed that to be the case. Also, in eq. (6.9), the resource availability at the macro base station, the traffic arrival statistics and resource holding times are not included. This simplified model is adopted to develop the fundamental study for LSA based offloading.

6.7.2 *How many offloaders and licensees in LSA?*

The most important operational parameter of the scheme is the determination of the density of those small base stations that help the macro base stations by offloading their MUs. A division model needs to be devised ensuring that both macro and small base stations at least get the minimum share of the number of small base stations per unit area that they require to meet their quality of service while offloading and communication with their receivers respectively. A bargaining approach as proposed by Nash in [123] is used to model the agreement over the density division of the small base stations that help by cooperating (i.e., ζ_m^*) via offloading, and those who obtain the LSA license to carry out small cells communicating with their receivers (i.e., ζ_s^*). Here, the macro and small base stations determine the minimum gain/utility expectations from the entire process of offloading and attempt to bargain with each other to maximize this utility by maximizing their share in the density of small cells.

Theorem 4. *The Nash Bargaining solution of the density division leads to an equilibrium solution of* $(\zeta_m^*, \zeta_s^*) = \big((1+\underline{\zeta_m}-\underline{\zeta_s})/2, (1+\underline{\zeta_s}-\underline{\zeta_m})/2\big)$.

Proof. The Nash's Bargaining approach is followed to model the process of mutual agreement of macro and small cell networks on the division of the density of offloaders and LSA licensees. The scenario is modelled as a two player network level game. Both players i.e., macro and small cells wish to maximize their share in the division of the small cell density as offloading or licensee nodes respectively. S=(ζ_m^*, ζ_s^*) is the solution of the bargaining game $\mathcal{G} \equiv \{g = (g_1, g_2) : g_i = \mathcal{U}_i(\mathbf{S}),\ i = 1,2;\ \mathbf{S} \in S1 \times S2\}$, where the functions $\mathcal{U}_i(.)$ represents the individual utilities of the i^{th} player. Here both players bargain to increase their share beyond their least acceptable share $\underline{\zeta_m}$, $\underline{\zeta_s}$ from the set of joint feasible payoffs $\mathcal{G}$ as possible by selecting a strategy $\mathbf{S}$ from the strategy profile S_i. A share below the minimum leads to a failure of the bargaining process. The whole bargaining problem can be described conveniently by the pair $\left(\mathcal{G}, \underline{\zeta_{m,s}}\right)$. The least acceptable fraction of cooperators are defined for the macro as a function of the improvement offloading can offer in terms of the QoS. It can thus be expressed as

$$\underline{\zeta_m} = f\left(p_{\text{suc}}^{\text{M}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u\right), p_{\text{suc}}^{\text{O}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u, \lambda_s\right)\right). \tag{6.10}$$

The operation of the scheme is assumed over unit time and hence the time dependency of all variables is dropped, i.e., $\lambda_u(t) = \lambda_u$ etc. Similarly, the least acceptable fraction of small cells getting the LSA license depends upon the offloading cost C and the QoS guarantees for the small base stations.

$$\underline{\zeta_s} = f\left(p_{\text{suc}}^{\text{S}}\left(R_{\text{QoS}_s}, \lambda_b, \lambda_u, \lambda_s\right), C\right). \tag{6.11}$$

At every stage of the bargaining process, both macro and small network play the strategies (i.e., demand a density ζ_m, ζ_s) chosen from their respective strategy profiles. An agreement is not reached if the players are not satisfied by the outcome of the negotiations. Hence, the disagreement vector of the Nash

Bargaining game becomes $g_0 = \left(\underline{\zeta_m}, \underline{\zeta_s}\right)$. A set of payoffs (ζ_m^*, ζ_s^*) is a Nash Bargaining solution if it solves the following optimization problem

$$\max \left(\zeta_m - \underline{\zeta_m}\right) \left(\zeta_s - \underline{\zeta_s}\right), \tag{6.12}$$

$$\text{subject to } \zeta_m + \zeta_s = 1 \,.$$

It is easy to understand eq. (6.12) in terms of the level of satisfaction its solution offers to both players. Increasing the difference $\left(\zeta_m - \underline{\zeta_m}\right)$ results in greater number of small base stations reserved for providing offloading services to the macro network. This increase directly translates into enhanced performance of the macro network. Similarly increasing the difference $\left(\zeta_s - \underline{\zeta_s}\right)$ increases the fraction of licenses beyond the minimum, i.e., $\underline{\zeta_s}$, required by the small base stations to overcome their cooperation cost and meet their communication targets. Hence the joint maximization of the two differences is the objective here.

For proportional fairness, the logarithmic representation of the problem using the Lagrangian of the above optimization problem is solved leading to,

$$L(\zeta_m, \zeta_s, \mu) = \log\left(\zeta_m - \underline{\zeta_m}\right) + \log\left(\zeta_s - \underline{\zeta_s}\right) - \mu(1 - \zeta_m - \zeta_s) \,. \tag{6.13}$$

where μ is the Lagrange multiplier. The original maximization problem can be solved by replacing ζ_s by $1 - \zeta_m$. Solving the above Lagrangian using the first order conditions results in

$$(\zeta_m^*, \zeta_s^*) = \left((1+\underline{\zeta_m}-\underline{\zeta_s})/2, (1+\underline{\zeta_s}-\underline{\zeta_m})/2\right). \tag{6.14}$$

Hence, the solution can be reached in terms of the utility functions of both the macro and small base stations. Therefore, the analysis of the success probabilities $p^{\mathrm{M}}_{\mathrm{suc}}$, $p^{\mathrm{O}}_{\mathrm{suc}}$ and $p^{\mathrm{S}}_{\mathrm{suc}}$ are needed to determine the actual division of λ_s. □

6.7.3 *Probability of successful macro base station-MU communication* $p^{M}_{suc}\left(R_{QoS}, \lambda_b, \lambda_u\right)$

In cellular communication, multiple MUs are likely to be present within a single Voronoi cell competing for wireless services. To deal with multiple MUs, base stations select a particular MU from their respective Voronoi cells with probability p_m to be served within a designated time slot.

Proposition 6.1: Successful macro base station-MU communication

The probability of successful macro base station-MU communication is given as,

$$p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u\right) = \frac{\pi^{3/2}\lambda_b p_m}{\sqrt{\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}} \exp\left(\frac{\left(\lambda_b \pi\left(1+\tau\left(R_{\mathrm{QoS}}\right)\right)\right)^2}{4\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}\right) \tag{6.15}$$

$$\times \ \mathrm{Q}\left(\frac{\lambda_b \pi\left(1+\tau\left(R_{\mathrm{QoS}}\right)\right)}{\sqrt{2\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}}\right), \tag{6.16}$$

where $\tau\left(R\right) = \sqrt{2^R-1}\left(\pi/2 - \tan^{-1}\left(1/\sqrt{2^R-1}\right)\right)$ and $\mathrm{Q}\left(x\right)$ is the Q-function which by definition is the tail probability of the standard normal distribution defined as $\mathrm{Q}\left(x\right) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-\frac{u^2}{2}\right)\mathrm{d}u$.

Proof. The probability that the SINR at a selected MU while it is communicating to its nearest base station is greater than a desired threshold $2^{R_{\text{QoS}}} - 1$ is given as;

$$p_{\text{suc}}^{\text{M}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u\right) = p_m \mathbb{E}_r\left[\mathbb{P}\left\{\frac{hl(r)}{I_b + \sigma^2/P_b} \geq 2^{R_{\text{QoS}}} - 1\right\}|r\right], \tag{6.17}$$

where r is the distance of the MU from its nearest base station and I_b is the interference from the set of neighbouring base stations Φ_b^c consisting of all base stations in Φ_b except the base station serving the MU. Let o be an MU at the origin and $z \in \Phi_b$ be the serving base station i.e., $o \in C_z$, where C_x is defined as $C_z = \left\{x \in \mathbb{R}^2 | \, \|x - z\| < \|x - y\| \, \forall y \in \Phi_b \, \text{s.t.} \, y \neq z\right\}$. The link distance $r = \|z - o\|$ is distributed as

$$f_R(r) = 2\lambda_b \pi r \exp\left(-\lambda_b \pi r^2\right). \tag{6.18}$$

In this case, the aggregate interference at the MU can be expressed in terms of the sum of channel gains h_y and path losses $l(r)$ from every interfering base station i.e., $I_b = \sum_{y \in \Phi_b^c} \frac{h_y}{r_y^{\alpha}}$. From the fact that the channel power gain is assumed to be exponential in the analysis, the above probability becomes

$$\begin{aligned} p_{\text{suc}}^{\text{M}}\left(R_{\text{QoS}}, \lambda_b, \lambda_u\right) &= 2\pi\lambda_b p_m \int_r^{\infty} \mathbb{E}_I\left[\exp\left(-\left(2^{R_{\text{QoS}}} - 1\right) r^{\alpha}\left(I_b + \sigma^2/P_b\right)\right)\right] \quad (6.19)\\ &\times \; r \exp\left(-\lambda_{\bar{b}} \pi r^2\right) \mathrm{d}r, \\ &= 2\pi\lambda_b p_m \int_r^{\infty} r \exp\left(-\lambda_b \pi r^2\right) \\ &\times \; \exp\left(-\left(2^{R_{\text{QoS}}} - 1\right) r^{\alpha} \sigma^2/P_b\right) \mathcal{L}_{I_b}\left(\left(2^{R_{\text{QoS}}} - 1\right) r^{\alpha}\right) \mathrm{d}r. \quad (6.20) \end{aligned}$$

From the definition of Laplace transform i.e., $\mathcal{L}_{I_b}\left(\left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}\right)=\mathbb{E}\left[\exp^{-sI_b}\right]$ where $s=\left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}$. Using the probability generating functional, the above integral yields

$$\begin{aligned} p_{\mathrm{suc}}^{\mathrm{M}}\left(R_{\mathrm{QoS}},\lambda_b,\lambda_u\right) &= \frac{\pi^{3/2}\lambda_b p_m}{\sqrt{\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}}\exp\left(\frac{\left(\lambda_b\pi\left(1+\tau\left(R_{\mathrm{QoS}}\right)\right)\right)^2}{4\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}\right) \\ &\times \mathrm{Q}\left(\frac{\lambda_b\pi\left(1+\tau\left(R_{\mathrm{QoS}}\right)\right)}{\sqrt{2\left(2^{R_{\mathrm{QoS}}}-1\right)/\frac{P_b}{\sigma^2}}}\right), \end{aligned} \tag{6.21}$$

where $\tau\left(R\right)=\sqrt{2^R-1}\left(\pi/2-\tan^{-1}\left(1/\sqrt{2^R-1}\right)\right)$. The detailed derivation can be found in Appendix A Sec. A.2. □

Remarks: From eq. (6.21) it is evident that outage over the macro link is strongly dependant upon the number of MBS in the network. A greater number of MBS tends to minimize the outage. However, increasing the number of MBS involves huge planning, equipment and installation costs.

6.7.4 *Probability of successful offloading to small cells* $p_{suc}^{O}\left(R_{QoS},\lambda_b,\lambda_u,\lambda_s\right)$

Offloading the MU to one of the small cells is studied in this section. In offloading mode, it is proposed that the MU is offloaded to the small cell that offers the best received SINR at the MU. The set of offloading candidate base stations Φ_s^o is a stationary PPP within the PPP of the small base stations Φ_s. Those small base stations that acquire LSA licenses constitute Φ_s^l such that $\left\{\Phi_s^o+\Phi_s^l=\Phi_s\right\}$. During offloading, interference from small cells $I_s=\sum_{y\in\Phi_s^l}\frac{h_y}{r_y^{\alpha}}$ is received at the MU.

Proposition 6.2: Successful Offloading to small cells

The probability of successful offloading of the traffic of the macro base station is given as,

$$p^{\mathrm{O}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}},\lambda_b,\lambda_u,\lambda_s\right) = p_m\left[1-\exp\left[-\frac{\pi^{3/2}\lambda_s\zeta_m}{2\sqrt{b}}\exp\left(\frac{a^2}{4b}\right)\right.\right. \times\left.\left.\left(1-\operatorname{erf}\left(\frac{a}{2\sqrt{b}}\right)\right)\right]\right]. \quad (6.22)$$

Proof. In order to find the probability of success of offloading, it can be stated that

$$p^{\mathrm{O}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}},\lambda_b,\lambda_u,\lambda_s\right) = p_m\left(1-\mathbb{E}_{I_s}\left[\underbrace{\mathbb{P}\left\{\max_{j\in\Phi_s^c} h_j l\left(r_j\right)\leq\left(2^{R_{\mathrm{QoS}}}-1\right)\left(I_s+\sigma^2/P_s\right)\right\}}_{A_1}\right]\right)$$

The fraction $\zeta_m\lambda_s$ of SBS per unit area are considered to be providing the offloading services for the MUs. The small base station with best signal strength at the MU offloads the data providing a diversity gain to the MU. The term A_1 can be computed by constructing a Marked PPP by assigning the fading marks to each $j\in\Phi_s^l$ [3]. Additional Bernoulli or indicator marks are assigned to the PPP such that the intensity of modified process[4] can be expressed as

$$\lambda_s^c\left(h,r\right) = \lambda_s\zeta_m 2\pi r\mathbb{1}\left(hl(r)\geq\left(2^{R_{\mathrm{QoS}}}-1\right)\left(I_s+\sigma^2/P_s\right)\right)f_H(h).$$

3 A detailed discussion on the Marked PPP is beyond the scope of this chapter. Interested readers should refer to [29].

4 The modified intensity corresponds to the dependently thinned point process.

Now A_1 can be computed by the void probability of the modified point process as

$$A_1 = \exp\left(- \underbrace{\iint_0^\infty \lambda_s^c\,(h,r)\,\mathrm{d}r\mathrm{d}h}_{\Lambda_s} \right), \tag{6.23}$$

The mean measure Λ_s can be evaluated as

$$\begin{aligned}
\Lambda_s &= 2\pi\lambda_s\zeta_m \iint_0^\infty r\mathbb{1}\left(hl(r) \geq \left(2^{R_{\mathrm{QoS}}}-1\right)\left(I_s + \sigma^2/P_s\right)\right) f_H(h)\mathrm{d}r\mathrm{d}h, \\
&= 2\pi\lambda_s\zeta_m \int_0^\infty r\mathbb{P}\left\{h \geq \left(2^{R_{\mathrm{QoS}}}-1\right)\left(I_s + \sigma^2/P_s\right) r^\alpha\right\} \mathrm{d}r, \\
&= 2\pi\lambda_s\zeta_m \int_0^\infty r\exp\left(-\left(2^{R_{\mathrm{QoS}}}-1\right)\left(I_s + \sigma^2/P_s\right) r^\alpha\right) \mathrm{d}r, \\
&\overset{(b)}{=} 2\pi\lambda_s\zeta_m \int_0^\infty r\mathcal{L}_{I_s}\left(\left(2^{R_{\mathrm{QoS}}}-1\right) r^\alpha\right) && (6.24) \\
&\times \exp\left(-\left(2^{R_{\mathrm{QoS}}}-1\right) r^\alpha \sigma^2/P_s\right) \mathrm{d}r, && (6.25) \\
A_1 &\overset{(c)}{=} \exp\left(-\pi\lambda_s\zeta_m \int_0^\infty \exp\left(-wa + w^2 b\right)\mathrm{d}w\right), && (6.26)
\end{aligned}$$

where (b) follows from Appendix A, Sec. A.2 and A.3, (c) follows using the findings from the appendices where $a = \frac{\pi^2\lambda_s\zeta_s}{2}\sqrt{2^{R_{\mathrm{QoS}}}-1}$ and $b = \left(2^{R_{\mathrm{QoS}}}-1\right)\frac{\sigma^2}{P_s}$. The above integral can be solved and the probability of success $p_{\mathrm{suc}}^{\mathrm{O}}$ can be expressed in terms of the error function i.e., $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-t^2\right)\mathrm{d}t$, as

$$\begin{aligned}
p_{\mathrm{suc}}^{\mathrm{O}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u, \lambda_s\right) &= p_m\left[1 - \exp\left[-\frac{\pi^{3/2}\lambda_s\zeta_m}{2\sqrt{b}}\exp\left(\frac{a^2}{4b}\right)\right.\right. && (6.27) \\
&\times \left.\left.\left(1 - \mathrm{erf}\left(\frac{a}{2\sqrt{b}}\right)\right)\right]\right].
\end{aligned}$$

□

Remarks: From eq. (6.27) it can be seen that performance over the offloading link is driven by the fraction of helping SBS in the network. Increasing number of licensees interfering with the offloading services degrade the performance of this link. The dependence of the spectral utility of the macro network is strongly coupled with a greater number of offloading small base stations.

6.7.5 *Probability of successful communication within the small cells* $p^S_{suc}\left(R_{QoS_s}, \lambda_b, \lambda_u, \lambda_s\right)$

As stated, a fraction of small cells carry out their own operations as a reimbursement to the offloading services of the remaining small cells. To model the communication, the probability that a small base station can successfully communicate with its receiver with the probability of success being greater than a required threshold is sought.

Proposition 6.3: Successful small cell communication

The probability of successful small cell communication is given as,

$$p^{\mathrm{S}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}_s}, \lambda_b, \lambda_u, \lambda_s\right) = \frac{\pi^{\frac{3}{2}}\lambda_s\zeta_s}{\sqrt{\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}} \exp\left(\frac{\left(m_1+m_2\right)^2}{4\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}\right) \times \mathrm{Q}\left(\frac{m_1+m_2}{\sqrt{\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}}\right),$$

where, $m_1 = \pi\lambda_s\zeta_s\left(\tau\left(R_{\mathrm{QoS}_s}\right)+1\right)$ and $m_2 = \frac{\pi^2\lambda_b r^2\sqrt{2^{R_{\mathrm{QoS}_s}}-1}}{2}$.

Proof. The link distance r is the distance of the small cell user from its nearest base station and I_s is the interference from the set of neighbouring base stations Φ_s^c consisting of all base stations in Φ_s except the base station serving the small cell user that have obtained the license and are communicating to their respective SUs. Let o be a small cell user at the origin and $z \in \Phi_s$ be the serving base station i.e., $o \in C_z$, where C_z is defined as $C_z = \{x \in \mathbb{R}^2 |\, \|x-z\| < \|x-y\| \, \forall y \in \Phi_s \text{ s.t. } y \neq z\}$. The link distance $r = \|z-o\|$ follows from eq. (6.18). Here, I_o is the interference from the SBS which are offloading for the macro network.

$$
\begin{aligned}
p^{\mathrm{S}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}_s}, \lambda_b, \lambda_u, \lambda_s\right) &= \mathbb{E}_r \quad \mathbb{E}_{I_{s,b}}\left[\mathbb{P}\left\{\frac{hl(r)}{I_s + I_o + \sigma^2/P_s} \geq \left(2^{R_{\mathrm{QoS}_s}} - 1\right)\right\} | r, I_s, I_o\right], \\
&= \int_0^\infty \mathcal{L}_{I_s}\left(\left(2^{R_{\mathrm{QoS}_s}}-1\right) I_s r^\alpha\right) \mathcal{L}_{I_o}\left(\left(2^{R_{\mathrm{QoS}}}-1\right) I_o r^\alpha\right) \\
&\times \exp\left(-\left(2^{R_{\mathrm{QoS}_s}}-1\right) r^\alpha \sigma^2/P_s\right) f_R(r) \mathrm{d}r, \\
&\overset{(d)}{=} 2\pi\lambda_s\zeta_s \int_0^\infty r \exp\left(-\left(m_1+m_2\right) r^2\right) \\
&\times \exp\left(-\left(2^{R_{\mathrm{QoS}_s}} - 1\right) r^\alpha \sigma^2/P_s\right) \mathrm{d}r \\
&\overset{(e)}{=} \frac{\pi^{\frac{3}{2}}\lambda_s\zeta_s}{\sqrt{\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}} \exp\left(\frac{\left(m_1+m_2\right)^2}{4\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}\right) \\
&\times \mathrm{Q}\left(\frac{m_1+m_2}{\sqrt{\left(2^{R_{\mathrm{QoS}_s}}-1\right)/\frac{P_s}{\sigma^2}}}\right).
\end{aligned}
\tag{6.28}
$$

Here $\overset{(d)}{=}$ follows from the derivation in Appendix A where the density $\lambda_o \subseteq \zeta_m\lambda_s$ can be approximated to the density of the macro base stations, i.e., $\lambda_o \approx \lambda_b$. Also, $m_1 = \pi\lambda_s\zeta_s\left(\tau\left(R_{\mathrm{QoS}_s}\right)+1\right)$ and $m_2 = \frac{\pi^2\lambda_o r^2\sqrt{2^{R_{\mathrm{QoS}_s}}-1}}{2}$. □

Remarks: Conversely to eq. 6.27, here it can be seen that the success probability of the communication of the licensees is adversely affected by the concurrent transmission of the offloading SBS. The utility of the licensees hence increases with increasing ζ_s.

6.7.6 *Minimum required utility for successful bargaining*

The process of bargaining is crucially dependent upon the minimum acceptable utility of both networks. Since the determining factor is the density of cooperating and communicating nodes, the minimum utility in terms of the small base station density is quantified.

Proposition 6.4: Minimum utility of the MNO

The minimum utility for the MNO for which it agrees to share its spectrum with a fraction of small cells for their operation is more than the minimum fraction $\underline{\zeta_m}$ of λ_s such that $\underline{\zeta_m} = \frac{1.135\sqrt{2b}}{\pi\lambda_s} \frac{\ln\left(1-\frac{p_{\text{suc}}^{\text{M}}\left(R_{\text{QoS}},\lambda_b,\lambda_u\right)}{p_m}\right)^{-1}}{\left(1-\exp\left(-1.4\left(\frac{a}{\sqrt{2b}}\right)\right)\right)}$ where $\zeta_m^* - \underline{\zeta_m} > 0$.

Proof. The least expectation of the macro base station when its MU gets offloaded to small cells is that the probability of successful communication $p_{\text{suc}}^{\text{M}}$ at a rate R_{QoS} is at least equal to $p_{\text{suc}}^{\text{M}}$. From eq. (6.21) and (6.27), the minimum utility can be quantified as the minimum fraction $\underline{\zeta_m}$ of λ_s that satisfies the following:

$$
\begin{aligned}
p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u\right) &= p^{\mathrm{O}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u, \lambda_s\right) \\
\ln\left(1 - \frac{p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u\right)}{p_m}\right) &= \frac{\pi^{3/2}\lambda_s \underline{\zeta_m}}{\sqrt{b}} \exp\left(\frac{a^2}{4b}\right)\left(1 - \operatorname{erf}\left(\frac{a}{2\sqrt{b}}\right)\right), \\
&\overset{(d)}{\approx} \pi^{3/2}\lambda_s \underline{\zeta_m} \frac{\left(1 - \exp\left(-1.4\left(\frac{a}{\sqrt{2b}}\right)\right)\right)}{1.135\sqrt{2\pi b}\left(\frac{a}{\sqrt{2b}}\right)}, \\
\underline{\zeta_m} &= \frac{1.135\sqrt{2b}}{\pi\lambda_s} \frac{\ln\left(1 - \frac{p^{\mathrm{M}}_{\mathrm{suc}}\left(R_{\mathrm{QoS}}, \lambda_b, \lambda_u(t)\right)}{p_m}\right)}{\left(1 - \exp\left(-1.4\left(\frac{a}{\sqrt{2b}}\right)\right)\right)}. \quad (6.29)
\end{aligned}
$$

Here $\overset{(d)}{\approx}$ follows from Karagiannidis and Lioumpas approximation. □

Remarks: The least fraction of offloaders that a macro network demands decreases with increasing density of SBS in the network. This implies that under the forcasted dense small deployments in future, the help of a small number of offloaders can be helpful to improve the overall capacity of the network.

Proposition 6.5: Minimum utility of the small cells

The minimum utility for the small base stations for which they agree to cooperate and offload the MU is a fraction $\underline{\zeta_s}$ of small cells λ_s for their operation in the licensed spectrum such that $\zeta_s^* - \underline{\zeta_s} > 0$

Proof. The least expectation of the small base stations when they offload MU is that they get enough share in terms of the number of license holder small cells to overcome their cooperation cost C by successfully communicating with their respective receivers. The cost C defined as $C = \sqrt{cP_s\left(1-\underline{\zeta_s}\right)}$ is a decreasing function of the number of licensees (increasing function of the number of offloaders), operational cost c per Watt of transmitted power, transmit power of the SBS[5]. The minimum utility $\underline{\zeta_s}$ can be quantified as the fraction ζ_s of λ_s that satisfies the following

$$\begin{aligned} C &= p^S_{\text{suc}}\left(R_{\text{QoS}_s}, \lambda_b, \lambda_u, \lambda_s\right) \\ C &= \frac{\pi^{\frac{3}{2}}\lambda_s\underline{\zeta_s}}{\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)/\frac{P_b}{\sigma^2}}} \exp\left(\frac{(m_1+m_2)^2}{4\left(2^{R_{\text{QoS}_s}}-1\right)/\frac{P_b}{\sigma^2}}\right) \\ &\times \ \mathrm{Q}\left(\frac{m_1+m_2}{\sqrt{\left(2^{R_{\text{QoS}_s}}-1\right)/\frac{P_b}{\sigma^2}}}\right) \end{aligned} \tag{6.30}$$

Due to the nature of the expression eq. (6.30), the above can only be solved numerically. □

6.8 ANALYTICAL RESULTS

After having developed the expressions for the success probability and division of nodes for LSA based offloading, it is now important to see how our model performs in comparison to the baseline direct communication. To this end, we begin with validating the accuracy of our derived expressions for the success probabilities by doing Monte Carlo simulations. We simulate a 2 tier network setting in MATLAB. Macro base stations, small base stations and users with mean $\lambda_b = 10^{-3}$, $\lambda_s = 2.5\text{x}10^{-1}$ and $\lambda_u = 10^{-1}$ respectively are considered in a network. For each realization of the network, exponential distributed channel

5 The choice of cost function in terms of exponential functions is widely made (however not restricted to) in the literature [151].

power gain is generated. The successful transmission probability at the rate $R_{\text{QoS}}/R_{\text{QoS}_s}$ at the receiver for each spatial instance of the network is averaged for 10^4 different channel coefficients. This process is in turn repeated for 10^4 network realizations. To keep the Monte Carlo simulations simple, the values $p_b = \xi(t) = 1$ and $\zeta_m = 0.65$ are assumed. The transmit powers P_b and P_s are normalized and follow the typical values considered for LTE, e.g., $\sim 43 - 56$ dBm for macro base stations and $\sim 5 - 8$ dBm for Home eNode B (HeNB) [125, 126].

Fig. 6.3 shows the resulting probabilities $p_{\text{suc}}^{\text{M}}$, $p_{\text{suc}}^{\text{O}}$ and $p_{\text{suc}}^{\text{S}}$ through simulating the analytical expressions. Respective Monte Carlo simulation shown in solid lines follow each of the $p_{\text{suc}}^{\text{M}}$, $p_{\text{suc}}^{\text{O}}$ and $p_{\text{suc}}^{\text{S}}$ curves. It can be seen that both analytical and Monte Carlo based results are in close agreement with each other. It can hence be established that further analysis of density division and network utility based on these success probabilities also provides accurate results[6].

Fig. 6.3 shows that the probability of communicating successfully via offloading $p_{\text{suc}}^{\text{O}}$ is significantly higher than that of direct communication $p_{\text{suc}}^{\text{M}}$ when the small network is very dense. This happens because 1) the path loss is expected to be lower due to closer proximity of small base stations and 2) the diversity gain due to the selection of the small base station providing the best SINR at the MU. However, decreasing the density of the small network undermines both above mentioned sources of gains and hence $p_{\text{suc}}^{\text{O}}$ decreases. Small base stations transmit at a significantly low power as compared to the macro base stations. Here, a fixed division between offloaders and licensees has been considered since the focus is to understand the fundamental behaviour of offloading versus direct macro communication. The inherent advantage of low power

6 Having validated the fundamental results employed in the subsequent analysis, we do not explicitly show Monte Carlo results in the discussion to follow

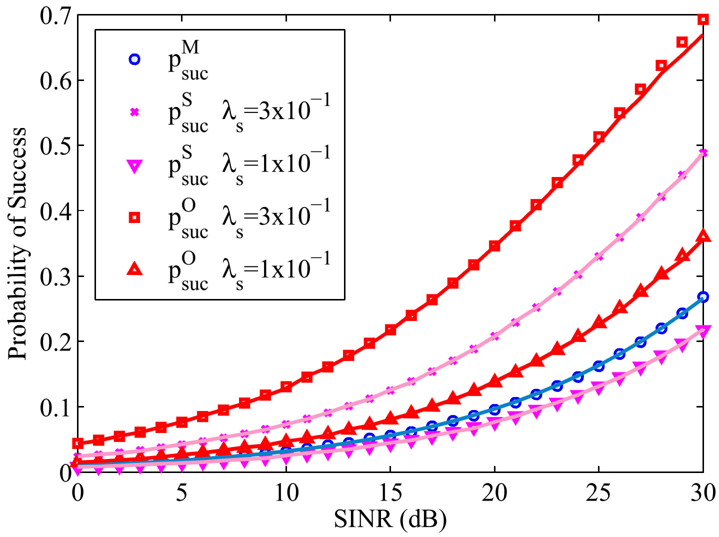


Figure 6.3: Success Probabilities with $P_b = 1$, $P_s = 5\text{x}10^{-4}$ and $R_{\text{QoS}} = R_{\text{QoS}_s} = 10^{-1} \frac{bits}{sec}$.

close-by small cells for offloading can hence be established. In Fig. 6.3, the impact of decreasing the density on the small cell communications is also shown. It can be seen that reducing the density of small base stations reduces the coverage within the small cells leading to lower $p^{\text{S}}_{\text{suc}}$. If $p^{\text{S}}_{\text{suc}}$ is compared to $p^{\text{M}}_{\text{suc}}$, the overall coverage of the two networks depends upon their respective base station densities. For denser networks, $p^{\text{S}}_{\text{suc}} > p^{\text{M}}_{\text{suc}}$ because of the extended coverage and low interference between small cells due to their low transmit power. In contrast, $p^{\text{S}}_{\text{suc}} < p^{\text{M}}_{\text{suc}}$ for a sparse network.

The QoS guaranteed by direct communication versus the proposed LSA offloading for different hours of the day is analysed in Fig. 6.4. The results indicate that direct communication can result in severe outage at busy hours of the day. It can be seen that around the peak hours i.e., around 15:00 hrs, the quality of service is seriously low. The variation in the user density throughout the day is shown in 6.5. $\lambda_u(t)$ reaches its maximum between 15:00 - 16:00 hrs. In early hours, the success probability is greater at higher SINR values. However,

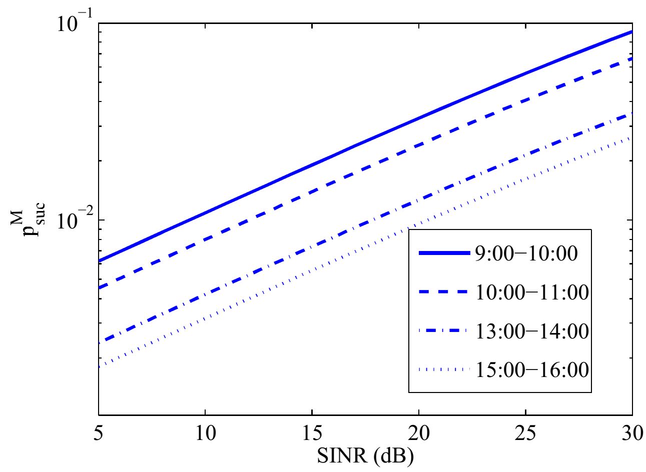


Figure 6.4: Probability of success of macro network across different hours of the day where $\lambda_b = 5\text{x}10^{-3}$,$\lambda_u = 10^{-1}$, $P_b = 1$.

at low SINRs, it remains significantly low mainly due to the reduced number of active MUs. During the evening, it assumes high values. It shows an overall decreasing behaviour as the evening progresses.

The division of λ_s small base stations per unit area into offloaders and licensees is shown in Figs. 6.6 and 6.7. In Fig. 6.6 it is shown that at low SINRs, larger fractions of small base stations acquire LSA licenses. This happens because $p^{\text{M}}_{\text{suc}}$ is very low at low SINRs. Only a few small cell offloaders can offer better QoS due to their geographically closer locations and diversity gain. At higher SINRs for the same set of parameters, a large number of small base stations help in offloading the MU as shown in Fig 6.7. Better channel conditions lead to better $p^{\text{M}}_{\text{suc}}$ requiring more diversity gain from small cells to improve the quality of communication. Hence the incentive for LSA is greater at low SINR values for both macro and small networks.

The impact of changing the densities of the base stations in the network, i.e., λ_b and λ_s is studied in Fig. 6.8. It can be seen from eqs. 6.21, 6.27 and 6.28 that increasing λ_b increases $p^{\text{M}}_{\text{suc}}$ since the number of MUs per MBS decreases.

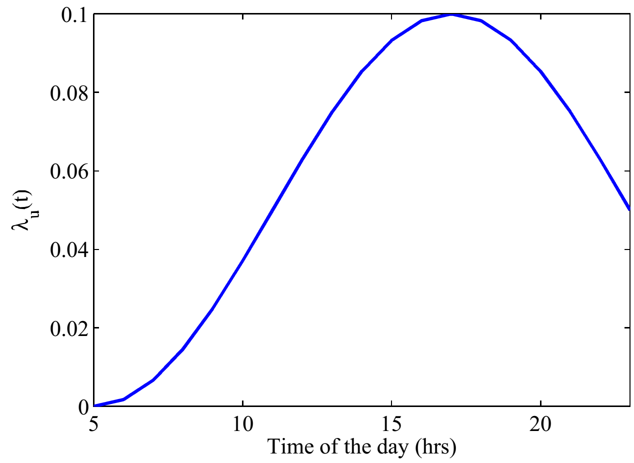


Figure 6.5: Variation in the density of MUs throughout the day, eq.(6.6) where $\lambda_u = 10^{-1}$.

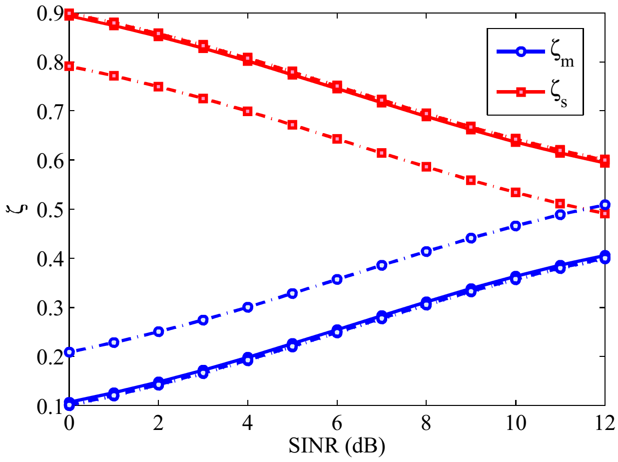


Figure 6.6: Fraction of offloaders and licensees at different times of the day in low SINR regime where $6:00-7:00$(dash dot), $9:00-10:00$(solid), $15:00-16:00$(dot), $22:00-23:00$(dash). The parameters assume following values: $\lambda_b = 10^{-3}$,$\lambda_u = 10^{-1}$, $\lambda_s = 10^{-1}$, $P_b = 1$,$P_s = 5\text{x}10^{-4}$ and $R_{\text{QoS}} = 1$.

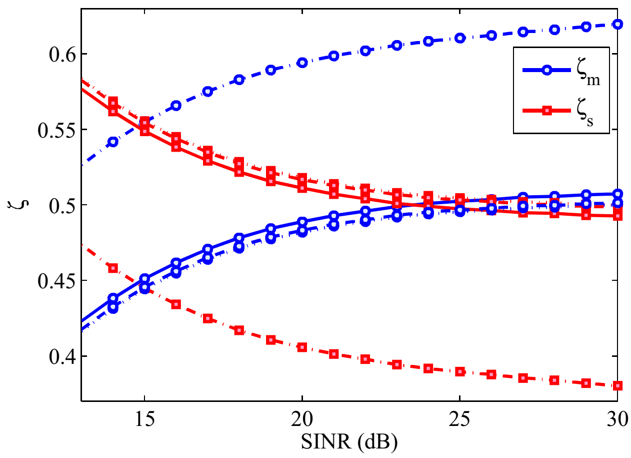


Figure 6.7: High SINR regime where $6:00-7:00$(dash dot), $9:00-10:00$(solid), $15:00-16:00$(dot), $22:00-23:00$(dash).

Hence from the perspective of network design, a greater number of macro base stations is useful especially during peak hours of the day since they provide better coverage. As expected, from Fig. 6.8 it can be seen that decreasing λ_b decreases $p_{\text{suc}}^{\text{M}}$. In order to understand the impact of changing the base station density on the performance of offloading with LSA, it is important to see the division of ζ_m and ζ_s while bargaining. In Fig. 6.9, it is shown that a greater number of offloaders is required if λ_b increases. It remains significantly high throughout the day as compared to $p_{\text{suc}}^{\text{M}}$. It can therefore be seen that LSA offloading provides higher and more stable success probability all along the day, as a result, better coverage for the macro network. Increasing λ_b increases $p_{\text{suc}}^{\text{O}}$ because the ζ_m is always negotiated and agreed upon only if $p_{\text{suc}}^{\text{O}} \geq p_{\text{suc}}^{\text{M}}$. Hence, the intelligent division of λ_s due to the proposed bargaining solution for LSA such that $p_{\text{suc}}^{\text{O}} \geq p_{\text{suc}}^{\text{M}}$ always improves $p_{\text{suc}}^{\text{O}}$. Therefore, greater λ_b improves the spectral utility for the macro network in both direct communication and LSA offloading as long as small cells can offer a better success probability

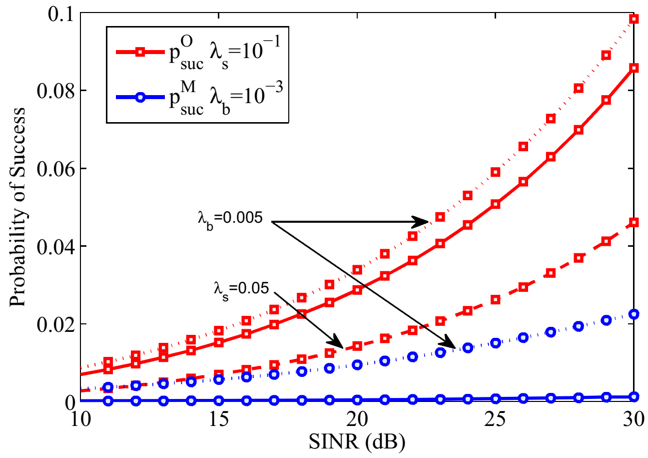


Figure 6.8: Effect of changing the density of λ_b, λ_s on LSA between 15:00-16:00 hrs

and a negotiation on ζ_m/ζ_s is successful. Also greater λ_b reduces the number of licenses that the macro network has to issue to the small cells.

For a given network setup with fixed number of base stations, it is favourable to have a larger number of small cells deployed to offload the traffic to during busy hours. In support of this argument, it is also shown in Fig. 6.8 that increasing λ_s increases $p^{\mathrm{O}}_{\mathrm{suc}}$ due to reduced link distances from the MUs while offloading and due to the increased diversity gain. With increasing λ_s, a few geographically closer small cells can provide a better offloading service while the remaining fraction of small cells can enjoy the license from the macro network.

In light of the discussion on how the division between offloaders and licensees is effected by changing λ_s, the probability of success for small cells when they acquire licenses to operate and communicate to their own users in their respective cells is studied. Fig. 6.10 shows that increasing λ_s increases $p^{\mathrm{S}}_{\mathrm{suc}}$ because of better coverage for the small cell users due to increase in the number of small base stations. Hence at low SINRs, increasing λ_s largely increases $p^{\mathrm{S}}_{\mathrm{suc}}$. Increasing interference due to increasing λ_s makes the system more in-

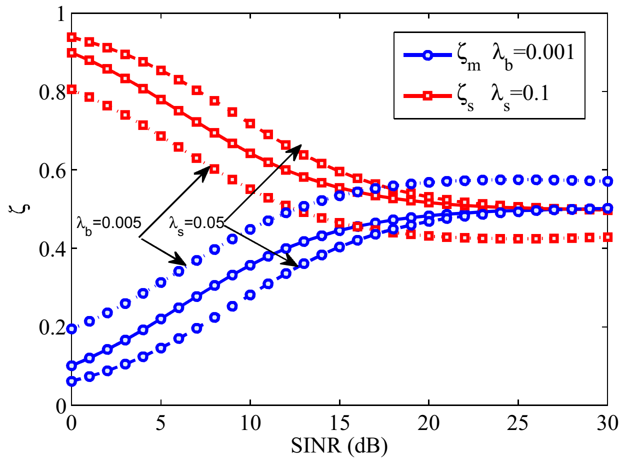


Figure 6.9: Effect of changing the density of λ_s,λ_b on the fraction of offloaders and licensees between 15:00-16:00 hrs

terference limited at higher SINR values. It can be deduced that increasing the density of small cells is beneficial for the performance of the small cells at low SINR values. Increasing λ_s at higher SINR values does not translate into proportional increase in $p_{\text{suc}}^{\text{S}}$ due to high interference.

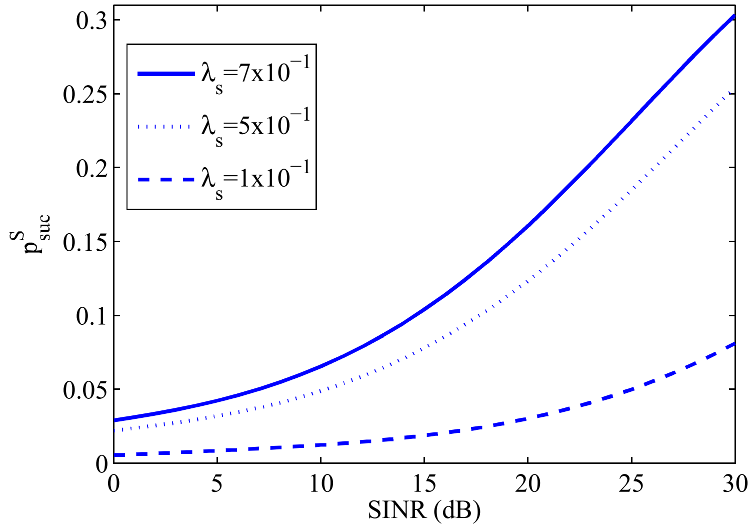


Figure 6.10: Probability of success of the small cells

6.9 CASE STUDY: LSA FOR SMALL CELLS IN LEEDS CITY CENTRE

A thorough analysis of the LSA model using real data from Leeds city centre which is one of the busiest city squares in the UK is carried out. An area of 1 km^2 in the heart of the city centre is considered. The demographics and MNO macro base station locations and their count within this area have been determined using the studies in [152],[153]. Fig. 6.11 shows the locations of base stations as per Ofcom [153]. As discussed earlier, the number of base stations and mobile users play a significant role in the accurate analysis of the performance of any scheme. The number of active base stations and MUs varies significantly throughout the day as in eq. (6.6).

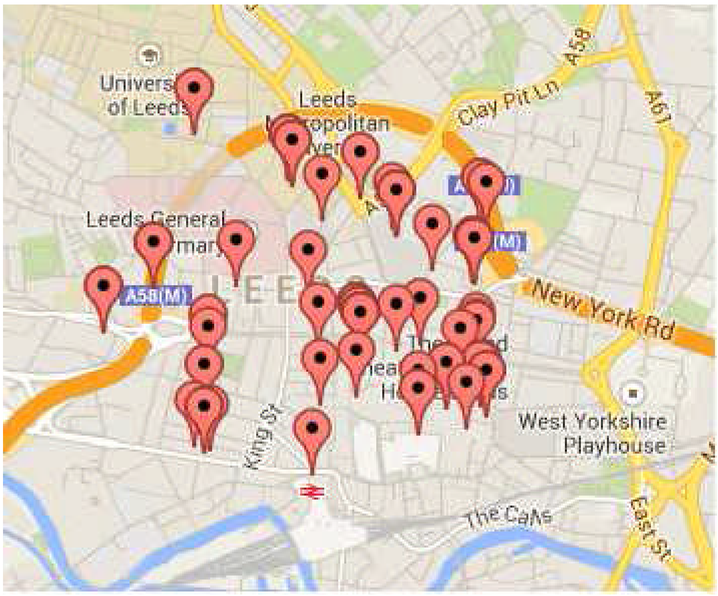


Figure 6.11: Locations of base stations in Leeds City Center

6.9.1 *Data Acquisition*

The first step in the process of data acquisition is to determine the area under consideration. For this purpose, Leeds city centre has been selected because it presents a typical example of a densely populated urban area where LSA offloading is attractive to operators. It is likely that small cells are deployed in such areas and LSA can be operational. The information regarding the base stations and their locations has been extracted from the Ofcom report [153]. These locations are available in terms of National Grid Reference (NGR). From these NGRs, the base stations falling in the area of consideration were shortlisted by translating the NGRs to Global Positioning System (GPS) coordinates. The locations of base stations in and around Leeds city centre are shown in Fig. 6.11. The information from [152] has been used to study the demographics of Leeds city centre to find out the average footfall in a day. The number and pattern of vehicles approaching the area on an average week day were also

Parameter	Value
Total area	4.62 km^2
Average footfall (7:00 - 10:00 am)	5000[152]
Inbound traffic flow (7:00 - 10:00 am)	93870 vehicles[152]
Mobile penetration	91% [154]
Number of base stations in 1 km^2	97 [153]
λ_u	12358.75 users/km^2[152]
λ_s	$2.36 \text{x} 10^{-1}$

Table 6.1: Leeds demographics

studied. It was observed that early hours of the day present low traffic. The traffic starts building up from 7:00 am to 10:00 am when the office and school hours begin. The traffic continues to grow till the peak pedestrian and vehicular traffic is observed around and after 15:00 hrs. It again goes low after 19:00 hours. There is a significant dip at late hours. It was found that the user traffic and its variation pattern in Leeds can be well modelled by eq. 6.6. The mobile penetration and urbanization trends in Leeds from were studied in [152, 154]. These factors indicate the typical percentage of population that actually uses a phone and contributes to the cellular traffic. These parameters also help in accurately determining the mobile user density at a particular time of the day. These statistics are summarised in table 6.1.

6.9.2 *Network Utility Gain in Leeds*

6.9.2.1 *Spectral Utility*

It is important to study how much aggregate capacity can be achieved using offloading with LSA as compared to mere direct communication. For that matter, R_T^{LSA} has been defined in Section 6.5 in eq. (6.2) as the total rate attained in the operational network. The gain can be defined as the difference in the capacity achieved by the direct communication mode and LSA offloading. From eq. (6.1) and (6.2),

$$R_G = \frac{R_T^{\text{LSA}}}{R_T^{\text{Direct}}}. \tag{6.31}$$

The gain during different times of the day at certain SINR conditions in Fig. 6.12. This gain is studied using the network parameters of λ_b, λ_u, λ_s from the data available for Leeds city centre as shown in Table 6.1 and MU density pattern shown in Fig. 6.5 from eq. (6.6). The values ω_5, $\omega_6 = 0.5$ and $\phi = -1$ have been used to model the typical user behaviour in Leeds city centre.

It can be seen that during peak hours, the entire LSA scheme with offloading gives a gain of 12x or more over mere direct communication at various channel conditions. For a given set of channel conditions, the gains are higher at peak hours of the day. This is a consequence of the fact that LSA continues to maintain its performance by negotiating over the number of licenses it gets for the small cells. In other words, small base stations charge the MNO more when the traffic load is higher in terms of the licenses they get. A dense deployment of small cells enable the small base stations to cope with high traffic loads and provide improved spectral efficiency.

6.9.2.2 *Energy efficiency*

Finally, the energy efficiency of the proposed LSA scheme is also studied. The number of bits successfully transmitted per Joule of energy is quantified as the energy efficiency of a scheme. From eq. (6.3) and 6.4, the gain in energy efficiency by using the LSA offloading scheme can be defined as

$$EE_G = \frac{EE^{\text{LSA}}}{EE^{\text{Direct}}}. \tag{6.32}$$

The plot of EE_G in Fig. 6.13 shows that LSA provides a remarkably energy efficient solution for the macro network as it improves the energy efficiency up to 16x for busy hours. As described earlier, the high density of small cells, shorter link distances and intelligent division between offloaders and licensees enables LSA to harvest these gains.

The analysis thus reveals that offloading with LSA provides a very promising solution to enhancing the spectral and energy utility of the macro network. It remarkably increases the services guarantees and overall network utility. In future, it is forecasted that the density of small cells will increase dramatically. However, increase in the density of macro base stations will not follow the same pattern due to high installation and planning costs. Such futuristic growth in the network parameters is an indicator that LSA based offloading is a promising candidate to meet the higher spectral demand in heterogeneous users in future.

In this chapter, the focus has been laid on the cost of attaining higher throughput in LSA while not considering any additional implementation costs it might incur. However, the huge gain in the EE as indicated in Fig. 6.13 suggests that any additional costs e.g., arbitration, hand-overs and miscellaneous circuitry etc., can be accommodated within the available margin. A future study that extends the LSA operation presented in this chapter by discussing a specific hardware platform of the small cells and macro base stations may benefit from the indicated energy margins and compare their operational system energy efficiency against these results.

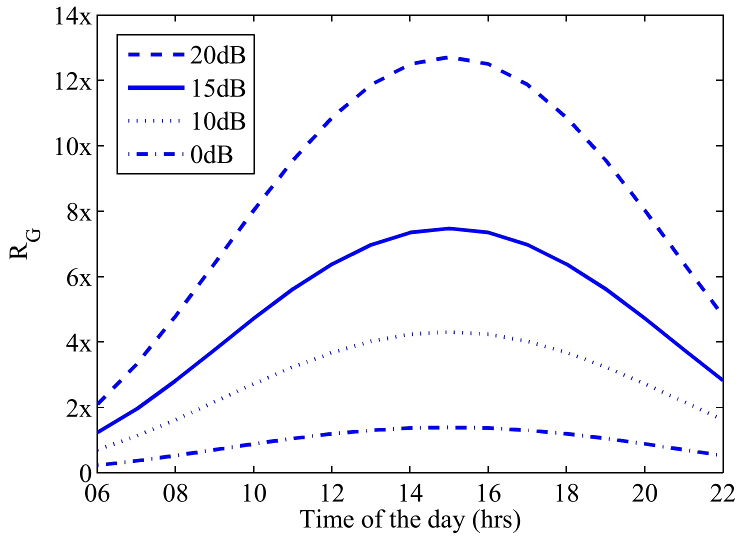


Figure 6.12: Spectral Utility Gain R_G.

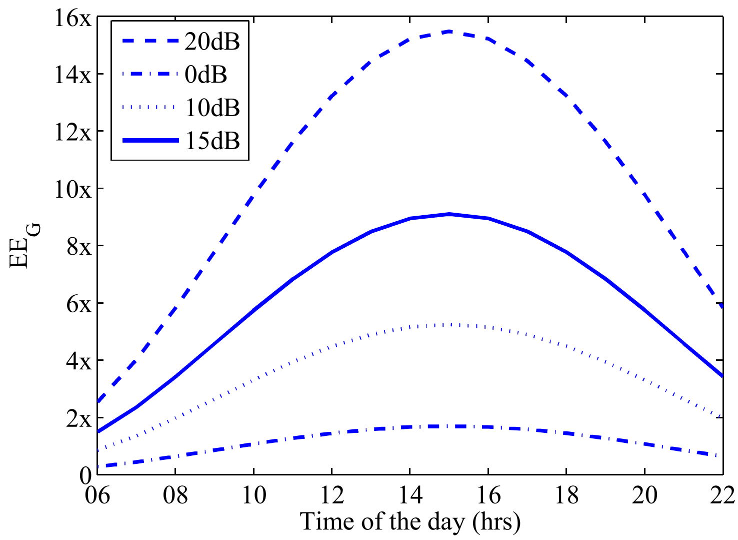


Figure 6.13: Energy Efficiency Gain EE_G.

6.10 CONCLUSIONS

In this chapter, a novel spectrum sharing scheme in which licenses to access the spectrum of a macro network are issued to existing small cells in the network has been presented and analysed. The small cell network offers offloading services to the MNO to improve its QoS and reduce its energy expense. Realistic parameters have been used like the geometry of the macro and small cell network and also the variation in the number of users across different times of the day in a densely populated zone. The scheme determines the number of offloading and spectrum sharing small cells on the basis of individual utilities of the macro and small network and results in improved utilisation of the spectral resources of the network. It has been demonstrated that LSA is a practical solution to increased coverage at reduced energy cost in urban areas by presenting the improvement in spectral and energy utility offered by the LSA scheme in Leeds city centre.

7

CONCLUSION & FUTURE WORK

In this chapter, a summary of the research findings of the thesis are presented. The main techniques used and the results obtained are outlined. To follow, the future research directions are also discussed.

7.1 SUMMARY OF CONTRIBUTION

In this thesis, a detailed analysis of DSL has been presented. The use of DSL as an alternative to conventional direct communication is strongly supported and shown to be more efficient in terms of its spectral and energy utility. Following are the research contributions of the thesis.

A novel DSL scheme for uni-directional communication between two primary nodes was introduced in Chapter 3. In the scheme, the secondary networks help relay the data of the primary network and in return it gets a fraction of time for exclusive spectrum access to the primary owned spectrum. DSL was modelled considering the random geometry of the CR nodes present in the network. A mathematical framework was proposed that captures the geographical locations of users in the network and the subsequent cooperation-area selection mechanism in the context of DSL enabled networks. Bargaining based negotiations were devised to reach a mutual agreement on the fraction of time the secondary network relays the primary data and the remaining time when it enjoys exclusive spectrum access. The aim of the scheme was to meet the target data rate and save energy of the primary network. It has been shown that DSL enables the PN to attain its required transmission rate. The agreement on the division of time enables the primary data to be relayed long enough to fulfil the throughput target of the primary user. The secondary network which otherwise has no access to spectrum also benefits from the scheme. Up to 50 percent of the total leasing time was observed to be reserved for the secondary activity during which it communicates with its own users. Also, DSL with a sparse secondary network was shown to be up to 10 times more energy efficient as compared to the PN.

Further, in Chapter 4, a novel NC based DSL technique for bi-directional communication was proposed in which the cognitive SUs cooperatively relay the primary data for two way primary communication. A time division leasing based on Nash Bargaining solution was proposed. It was shown that under the considerations of geographical randomness in the network and employing network coding, DSL can improve the number of bits that are successfully transmitted by 4x as compared to direct two way primary communication. Also the energy costs of the proposed DSL scheme are more than 10 times lower as compared to the direct two way communication.

A novel PNC and distributed beamforming based DSL scheme was proposed for bi directional communication in Chapter 5. This scheme also used time division based spectrum sharing. It was observed that DSL with PNC and beamforming is up to 5 times more spectrally efficient as compared to DSL with NC. It was shown that if selfish users are present in the network, the throughput of the primary communication can suffer 50% loss as compared to the case if all the secondary users were honest. A monetary pricing strategy was suggested as a punishment for the selfish users. It was shown that the pricing enforced helps to discourage the selfish behaviour of the secondary nodes.

An LSA based spectrum sharing technique as a more sophisticated and emerging DSL mechanism was proposed in Chapter 6. Under the proposed scheme, the small cell network offers offloading services to the macro network to improve its quality-of-service (QoS). In return, small cells are rewarded for their cooperation with a license to operate in the spectrum owned by the macro network. In this scheme, a density based division is proposed in which the number of offloading small cells and those enjoying the license to access the spectrum is determined. The analysis based on the model showed that such spectrum sharing can lead to commendable gains in the spectral utilization of the network while providing the desired QoS for both macro and small network. It was also shown that up to 90% small cells get access to the spectrum at poor channel conditions. The scheme was tested within the network parameters of the city of Leeds. The results indicated that for the considered scenario, the spectral utility of Leeds can be improved by more than 12 times by adapting to our proposed scheme. LSA offloading can be up to 16 times more energy efficient for Leeds.

7.2 FUTURE RESEARCH DIRECTIONS

DSL has been presented and analysed in the thesis. It is shown to be a promising technique to improve the performance of existing and future wireless networks. The research on DSL and its implementation still presents a lot of open issues and important research fronts that can be addressed to further enhance the knowledge and understanding of the applicability of DSL. Some of them are briefly discussed below.

7.2.1 *Further considerations in the proposed model*

The proposed DSL model can be extended to consider scenarios where each secondary and primary terminal is an autonomous entity. In the proposed work, only network level players (primary and secondary) were considered having only two utility functions belonging to each type of network. However, if all terminals are considered to be different from each other in terms of their hardware platform (e.g., battery levels, number of antenna, etc.), then they are bound to have their own utility functions. Such a setup would allow a refined study and facilitate the design of more robust solutions.

The discussion on the proposed DSL model can also be extended to further study the practical implementation of the arbitrator/central entity required to carry out the bargaining process. Methods to implement the synchronization required across the entire network for DSL to work are also an open issue. Secondary communication models beyond the bi-polar and nearest neighbor model studied in this thesis can be explored.

7.2.2 *Security of DSL*

DSL exploits the open nature of the wireless medium to use the *over-heard* information to help the PN. This help brings the cooperating SN a reimbursement in terms of spectral access. However, the same open nature of the medium makes it vulnerable to security breaches like eavesdropping, jamming, etc. The cooperation of co-located nodes being the primitive functional unit of DSL exposes the security of the spectrum owner's information to a high risk from malicious nodes. The security of wireless communications in general has attracted a lot of attention [155, 156]. The security of the wireless networks has been studied in terms of its integrity, authentication and secrecy. However, in terms of DSL, it is important to address the of security of the information of the

1. PUs during the first two phases of DSL. Since all transmission details e.g., timing, power etc., are exchanged and agreed up on while bargaining, it is very easy for any malicious nodes to exploit this information to inflict harm on the PUs.

2. SUs, since they share the same space and frequencies while they enjoy their reward.

The greedy behaviour of the secondary nodes is studied in Chapter 5. It can be seen that it can severely degrade the performance of DSL. On similar lines, it is important to quantify the impact of any malicious behaviour on the DSL performance. The relative geographical locations of terminals in the network also dictate which positions are favourable to breach the network communication. Hence the spatial locations are an important aspect to consider while analysing the security of DSL. According to [157], the relative secrecy of information transmission between two nodes s and r in the presence of an eavesdropper e can be defined in terms of *secrecy capacity* $R_{\text{secrecy}}(s, r_j)$as

$$R_{\text{secrecy}}(s, r) = \left[\log_2\left(1 + \text{SNR}_r\right) - \log_2\left(1 + \text{SNR}_e\right)\right]. \tag{7.1}$$

The terminal that maximizes the above secrecy rate is the best candidate for transmission. Here SNR at the eavesdropper is crucial in the achieved secrecy rate. For DSL leasing, intelligent cooperation area selection considering locations with most desirable characteristics in terms of security can be considered. According to the quantification above, the ideal cooperative area should consist of nodes offering maximum R_{secrecy} between the source (s), themselves (r_i) and the destination (d) as,

$$R_{\text{secrecy}}(s,d) = \forall_i \min\left(R_{\text{secrecy}}(s,r_i), R_{\text{secrecy}}(r_i,d)\right), \tag{7.2}$$

where DSL can be used to improve $R_{\text{secrecy}}(s,d)$ such that the cooperating nodes get some reimbursement while providing secure communication between a source and destination.

7.2.3 *Multi-operator Hetnet with the concept of virtualization/ Phantom cells*

In the context of future 5G networks, a key technical trend is to deploy dense *phantom cells*. According to this concept, a split between the control (C) plane and user (U) plane is made. The C-plane is provided by the macro user whereas the overlaid small cells provides the U-plane using different frequency bands. For better connectivity and mobility, the C-plane uses low frequencies and higher data rate is supported by the small cells in the U-plane. A dense deployment of these cells belonging to various operators is sought as a solution to the capacity enhancement challenge.

The splitting of the two planes greatly simplifies the control signalling and traffic servicing providing greater reliability and robustness. The splitting of C/U-plane provides a flexible platform for the implementation of DSL. With inherently separate C-plane, the DSL negotiations can be carried out in separate frequencies and simpler handovers to the U-plane for actual traffic transmission. Also, the resources belonging to various operators can be made available at a shared C/U-plane. Using the pool of shared resources, seamless operation of DSL can be made possible, serving the traffic of various operators. DSL within this concept of virtualization can provide a unified operation scheme working towards the spectral efficiency of the operators and the secondary nodes. However, it is an open issue to analyse how DSL operation in phantom cells improves the spectral utility. The respective utility functions of say two MNOs can be;

$$\mathcal{U}_1 = R_1 - f(S_1), \tag{7.3}$$

$$\mathcal{U}_2 = R_2 - f(S_2), \tag{7.4}$$

where $R_{1/2}$ are the achieved data rate by using DSL while $f(S_{1/2})$ is the cost incurred to the operator in terms of its resources being used. It is important to quantify its relative benefit to sole entities like an MNO or a secondary operator and also its network level gains. It is also crucial to study the energy costs involved in DSL for phantom cells.

7.2.4 *DSL for self organising networks (SON)*

SONs have been included as a part of LTE for a while a now. They were first introduced in LTE Release 8. Following that, Release 9 and 10 of LTE/LTE-A include use cases of SONs. SON helps to simplify and automate the provisioning, optimisation and maintenance of mobile networks. As the name suggests, the concept of SON relates to mobility and load balancing, coverage optimization, mobility robustness, interference coordination/reduction, automatic neighbour relation functions etc. It is a telecom management infrastructure that automates various functions of a network for spectral efficiency and energy saving.

DSL for SON is also an interesting research area. For SON, a variety of demand/utility functions need to be studied. This includes multivariate functions of time, location, neighbourhood, load, mobility etc. The bargaining/optimization for SON would require a joint treatment of all the network driving factors. A state based analysis of DSL with Markovian properties can be an initial research direction. Learning techniques can be incorporated to evolve the network in to an automatic functional unit. DSL for SON is specifically promising in terms of improving the energy utility of the network. An example of the utility function of an arbitrary player could be;

$$
\begin{aligned}
\mathcal{U}_i &= f\,(b, t, I) \\
&\text{s.t.} \begin{cases} b > & x\,\text{Hz.} \\ t \geq & y\,\text{sec.} \\ I \leq & z\,\text{Watt.} \end{cases}
\end{aligned}
$$

There could be multiple players with same or different utility functions. A joint/global optimum for the division of leased resources would possibly result in the most stable SON operation.

7.2.5 *Business Model of DSL*

For future networks with proposed DSL, it is important to study which business models are the most practical. One possibility could be to work with an end-user model where the user devices are programmable and can be set up into a network. This network can then be allowed to scale and transform and operate as a network with no purchased license. Another possibility is to develop and configure a mobile virtual network operator (MVNO) that unbundles retail services and spectrum licensing. These license holders can then operate and lease licenses to study the value chain of DSL. For either of the two models, a test bed can be set up with experimental DSL. Trials with various network settings over the test bed can help in the understanding of the business models giving further insights to the regulatory requirements.

A

APPENDIX FOR GREEN LSA

A.1 LAPLACE TRANSFORM OF INTERFERENCE

The expected value $\mathbb{E}_I\left[e^{-\left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}I}\right]$ or $\mathbb{E}_I\left[e^{-sI}\right]$ where $s = \left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}$ can be regarded as the Laplace transform of interference $\mathcal{L}_I\left(\left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}\right)$. This approach greatly simplifies the calculation of such integrals. From the definition of shot noise process and the approach in [122], the Laplace integral can be written as

$$
\begin{aligned}
\mathbb{E}_I\left[e^{-\mu r^{\alpha} I}\right] &= \mathbb{E}_{\theta,h_i}\left[\exp\left(-s\textstyle\sum_{i\in\theta} {}^{h_i}/_{d_i^{\alpha}}\right)\right], \\
&= \mathbb{E}_{\theta}\left[\prod_{i\in\theta}\mathbb{E}_h\left[\exp\left(-{}^{sh_i}/_{d_i^{\alpha}}\right)\right]\right], \\
&\overset{(a)}{=} \mathbb{E}_{\theta}\left[\prod_{i\in\theta}\frac{1}{1+{}^{s}/_{d^{\alpha}}}\right] \\
&\overset{(b)}{=} \exp\left(-2\pi\lambda_{\overline{b}}\int_r^{\infty}\left(1-\frac{1}{1+{}^{s}/\left(2^{R_{\mathrm{QoS}}}-1\right)^{\alpha}}\right)u\mathrm{d}u\right), \qquad \text{(A.1)}
\end{aligned}
$$

where (a) follows from solving the expectation over the exponential channel gain and (b) from applying the probability generating functional (PGFL) of the PPP where $u \to d$. Simplifying and substituting $s = \left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}$ in eq. A.1 results in

$$
\mathcal{L}_I\left(\left(2^{R_{\mathrm{QoS}}}-1\right)r^{\alpha}\right) = \exp\left(-2\pi\lambda_{\overline{b}}\int_r^{\infty}\frac{2^{R_{\mathrm{Qos}}}-1}{\left(2^{R_{\mathrm{Qos}}}-1\right)+\left(\frac{u}{r}\right)^{\alpha}}u\mathrm{d}u\right). \qquad \text{(A.2)}
$$

The result in eq. A.2 can be used to find the average interference experienced at a receiver from other transmitters in the network. In the following, we solve it for a few special cases where the interferers are at located in certain possible locations in our network setup. We consider the path loss exponent α to be 4.

A.2 INTERFERENCE AT THE MU FROM NEIGHBOURING BASE STATIONS

From the geometry of Voronoi cells, the minimum separation distance between an MU in a network and an interfering neighbouring base station is greater than the distance to the nearest base station. To determine the Laplace transform of this interference, we use eq. A.2 where only those neighbouring base stations interfere which are actively serving their own MUs. Simplifying and substituting $v = \left({}^{u}/_{r} \left(2^{R_{\mathrm{QoS}}} - 1 \right)^{-4} \right)^2$ in eq. A.1 results in

$$
\begin{aligned}
\mathcal{L}_I \left(\left(2^{R_{\mathrm{QoS}}} - 1 \right) r^4 \right) &= \exp \left(-\pi \lambda_{\bar{b}} r^2 \sqrt{\left(2^{R_{\mathrm{QoS}}} - 1 \right)} \int_{\sqrt{\left(2^{R_{\mathrm{QoS}}} - 1 \right)}}^{\infty} \frac{1}{1 + v^{1/2}} v \mathrm{d}v \right), \\
&= \exp \left[-\pi \lambda_{\bar{b}} r^2 \sqrt{\left(2^{R_{\mathrm{Qos}}} - 1 \right)} \right. \\
&\times \left. \left({}^{\pi}/_{2} - \tan^{-1} \left({}^{1}/\sqrt{\left(2^{R_{\mathrm{Qos}}} - 1 \right)} \right) \right) \right].
\end{aligned}
$$

where $\lambda_{\bar{b}} = p_b \lambda_b$ is the density of the base stations that are serving their own MUs in their respective Voronoi cells.

A.3 INTERFERENCE AT THE MU FROM SMALL BASE STATIONS

While the MU is offloaded to a cooperating small base station, the communication encounters interference from those small base stations that carry out their own transmission. The interfering small base stations can be found anywhere in the network unlike the interfering Voronoi base stations which are at least a given minimum distance apart. The small cells interfere with the MU and among themselves while they enjoy the shared spectral access. The Laplace transform of this interference I_s can be quantified by using eq. A.1

$$\begin{aligned}\mathcal{L}_{I_s}\left(\left(2^{R_{\mathrm{QoS}_s}}-1\right)r^4\right) &= \exp\left(-\pi\lambda_s\zeta_s r^2\sqrt{(2^{R_{\mathrm{QoS}_s}}-1)}\int_0^\infty \frac{1}{1+v^{1/2}}v\mathrm{d}v\right), \\ &= \exp\left(-\frac{\pi^2\lambda_s r^2\zeta_s\sqrt{(2^{R_{\mathrm{QoS}_s}}-1)}}{2}\right).\end{aligned}$$

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