# A QUANTITATIVE INVESTIGMTION OF THE HOMOGFNISING OF SRELL GLiSS RELTS BY STIRRING 

$\therefore$ Thesis Presented By
J AVED H $\therefore$ K K M
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## S U M M A R Y

The homogenising process in glass was studied by mixing two different glnsses, placed in a known initial distribution. The initial orientation was to have a horizontal interface between the two glnsses beoause usually the inhonogeneities lie in parallel horizontal leyers in a crucible containing glass unstirred by convection or a nechanical stirrer.

The homogeneity was measured quantitatively using an etohing end interferonetery technique developed in this work. The degree of honogeneity was defined in terns of the intensity of segregationy which varies fron one in a fully segregated system to zero in a fully nixed system. Mixing whs studied using three different simple stirrers.

The intensity of sogregation was plotted against numbor of revolutions of the stirrer. From these graphs it onn be seen that the value of intensity of segregation onloulated from the otohing dnta hns 0 quantitative significnnce for following the process of honogenisination.

The flow patterns observod in thin sections of the glass melta were completely different from those observed in room temperaturo nodel experinents. This wos because the cilasses mixed were of different viscosities and densitios, while the nodel had used liquids of equal visoosity and donsity.

The stirrers studied were not very offioiont mixing devioes but they were designed to compare results in glasses with the previous room temperature nodel experiments rather than to be nost efficient. The disc stirrer was a oonvenient standard for oomparison of the performanoe of other stirrors. The addition of a blade to the diso helped in
in breaking up the interiace between the two layers. The complex diso stirrer was only efficient when tho lower dise was above the Iiquid-Iiquid interface. IIthough no stirror was extremely efficient, all produced more homogeneous glass than were obtained from unstirrod melts.

$$
\begin{array}{lllllll}
\mathrm{C} & \mathrm{H} & \mathrm{~A} & \mathrm{P} & \mathrm{~T} & \mathrm{E} & \mathrm{R}
\end{array} \mathrm{I}
$$

$$
\begin{array}{lllllllllll}
I & N & T & O & D & U & C & T & I & 0 & N
\end{array}
$$

## 1 NATURE OF GLISS AND THE MELTING PROCESS

(i) Definition and composition

Most inorganic elments and compounds melt to form liquids having a viscosity about the same as that of water. When liquids of this type are oooled, rapid orystallisation occurs and the liquid can only be supercooled to a small extent. However, there are some materisls which melt to form very viscous liquids. If such a liquid is maintained for sometime at a temperature a little below the freezing point it will only crystnllise slowly, but if, instead of holding the temperature constant, tho liquid is oooled continuously, crystallisation may or may not occur depending on the rete of cooling. At a higher rate of cooling it is possible to reduce the temperature without orystallisation. The viscosity of the liquid increnses stoadily as the temparature is reduoed until aventually it is so high that the matorial is apparently solid. This solid material is onlled glass.

Most flasses are complex mixtures of several oxides. These oxides can be divided into two groups; simple glass forming oxidos nd molifying oxidos. The exides belonging to the former group, which can form glasses themsclves, are $\mathrm{SiO}_{2}, \mathrm{GoO}_{2}, \mathrm{~B}_{2} \mathrm{O}_{3}, \mathrm{P}_{2} \mathrm{O}_{5}$ and $\therefore \mathrm{S}_{2} \mathrm{O}_{3}$. The mojority of Elasses are composed of these glass formers togather with two or more modifiors suoh as $\mathrm{Na}_{2} \mathrm{O}, \mathrm{K}, \mathrm{O}, \mathrm{CaO}, \mathrm{MgO}$, $\mathrm{BaO}_{3} \mathrm{PbO}$ and $\mathrm{Fi}_{2} \mathrm{O}_{3}$. The complexity of the mixture is evident from the following examples of compositions of some comproial glasses.

| Glasses | $\mathrm{SiO}_{2}$ | $\mathrm{Al}_{2} \mathrm{O}_{3}$ | $\mathrm{~B}_{2} \mathrm{O}_{3}$ | $\mathrm{SO}_{3}$ | CaO | BaO | MgO | $\mathrm{Na}_{2} \mathrm{O}$ | $\mathrm{K}_{2} \mathrm{O}$ | Pb O |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Plato glass | 72.0 | 0.5 | - | 0.5 | 13.0 | - | - | 13.20 | - | - |
| Pyrex glass | 81.0 | 2.0 | 12.0 | - | - | - | - | 4.50 | - | - |
| Lead giass | 66.0 | 0.9 | 0.6 | - | 0.7 | 0.5 | - | 6.00 | 9.50 | 15.50 |

The process of "melting" is very complex and it depends upon the substances present in the batch. Different phenomena oocur in the "melting" prccess such as dehydration, decomposition, dissociation and chemical reactions. To illustrate the complexity of the "melting" process, consider a simple sodr-limu-silio glass in which the raw materials are soda ( $\mathrm{Na}_{2} \mathrm{Cos}_{3}$ ), limestone ( $\mathrm{CaCos}_{3}$ ), and sand ( $\mathrm{SiO}_{2}$ ). If heated separately to about $1400^{\circ} \mathrm{C} \mathrm{Na} \mathrm{Na}_{3}$ melts but does not decompose, $\mathrm{CaCo}_{3}$ docomposes to $\mathrm{CrO}+\mathrm{CO}_{2}$ without melting. The resultant CaO has a very high melting point of $2580^{\circ} \mathrm{C}$, a temperature which is never atteined in glass melting tank furnaces. Sand ( $\mathrm{SiO}_{2}$ ) is unchnnged except for the $\alpha-\beta$ quartz trangformation at $573^{\circ} \mathrm{C}$ and the frot that the quartz will very slowly bogin to change to its stable high tempernture form (cristobalite). Whon the mixture is hentod rapidly, there is some solid state ronotion which includes tho formation of a little $\mathrm{Na}_{2} \mathrm{O}-\mathrm{SiO}_{2}$ Iiquid uf eutectic oomposition, but the first inpurtant step is the melting of $\mathrm{Na}_{2} \mathrm{Co}_{3}$ at $851^{\circ} \mathrm{C}$ which flows around the other Ernins and begins to dissolve $\mathrm{CaCo}_{3}$ to form the double carbonato $\mathrm{Na}_{2} \mathrm{Co}_{3} \mathrm{CaCos}_{3}$ and to reaot with silica furming mostly a cumposition close to $\mathrm{Nag} \mathrm{O}_{\mathrm{o}} 2 \mathrm{SiO}_{2}$. Some of the lime also begins to decumpose and be incorporatoa into the Iuid Iiquid phose which tonds to rundow. The doublo carbonate with sodium carbonate melts and begins to react rapidly at $812^{C} \mathrm{C}$ so that the nelt formed by raaction with $\mathrm{Na}_{\mathrm{g}} \mathrm{Cos}_{3}$ at $851^{\circ} \mathrm{C}$ will also bogin to react with $\mathrm{SiO}_{\mathrm{a}}$ forming o sodn-lime-silica melt (not of the final
intended oomposition). it the same time the evolution of $\mathrm{Co}_{2}$ causes stirring and frothing of the whole mass. diter a time the very rapid gas evolution stops and a molt full of both gas bubbles and partly dissolved silica grains remains.
"Welting" is considered complete when all the silico has dissolved and the melt is batch-free but still contains gas bubbles. When the melt has become batch-free it is usually very inhonogeneous. During "melting", liquid richer in $\mathrm{Na}_{2} \mathrm{C}$ and CaO then the finally intended composition runs to the bottom and a sort of froth floatation process corries silion grains upwards to form silica rioh layer on the surface of the melt.

It is well known that glass formation involvos both solid and liquid state renctions. ilthough glass is not in equilibrium, studying the e:pproprinte phase dingrem of ten gives useful information about glass melting resctions. Morcy and Bowen ${ }^{\prime \prime}$ detormined the phase diagram of the part of $\mathrm{Na}_{2} \mathrm{O}-\mathrm{CaO}-\mathrm{SiO}_{2}$ system in whioh glasses are formed. The phase diagran ins shown in Figure 1. In this system the lowest eutectic is at $725^{\circ} \mathrm{C}$ and the chemical composition of that melt is $21 \% \mathrm{Na}_{2} \mathrm{O}, 5 \% \mathrm{CaO}, 74 \% \mathrm{SiO}_{2}$.

On slorily henting a mixture of $\mathrm{Nn}_{2} \mathrm{Cos}_{s}, \mathrm{CaCo}_{3}$ and $\mathrm{SiO}_{2}$ there would bo a considerable increase in renotion rete nt this tempernture. At higher temparatures other reactions nccur suoh as the formation of sodium netasilacate or sodiun disilicate or the formation of the euzeotic melt betwecn $\mathrm{Na} .2 \mathrm{O}_{2} 2 \mathrm{SiO}_{2}$ and $\mathrm{SiO}_{2}$ at $790^{\circ} \mathrm{C}$. The pears seen at various temperatures on differential thermal nalysis curves oan of ten bo correotly interpreted from the relevant phese diagrams.

Phaso diagrans cennot, however, give any information about the kinetics of these procossos, especinlly the dissolving of

FIG.I


PART OF THE SYSTEM
$\mathrm{NaO}-\mathrm{CaO}-\mathrm{SiO}_{2}$
residual sand grains which is a diffusion controlled process. Phase diagrams also give useful information about the probable products of crystallisation when glass melts are slowly cooled but discussion of nucleation and crystal growth is outside the scope of the present review.
(ii) Rerctions in the molten batch

It may be seen from the Figure i that many different compositicns in the sodium-calcium-silicate ternery system might be of interest and the reactions between the indiaidunl components upon henting could exhibit consideroble variety, depending upon the stability and the proportions in which the batch constituents are mixod.

The reactions in the batohes have beon studied exhaustively by Krbger ot $\mathrm{al}^{2}$, by dutermining the anount of Coa evolved thormogravimeterically. Other methods applied have beon olectrical mensurements and differentiol thermal analysis. Figure 2 shows the differential thermal analysis curve, for the mixture $85 \% \mathrm{SiO}_{2}$ and $15 \% \mathrm{No}_{2} \mathrm{Co}_{3}{ }^{2(\mathrm{a})}$. Both materials have grain size to pass 350 B.S.S. Peaks $A$ and $B$ indicnte the two polymorphic inversion in $\mathrm{Na}_{2} \mathrm{Co}_{2}$ at $340^{\circ} \mathrm{C}$ and $470^{\circ} \mathrm{C}$ respoctively. At about $550^{\circ} \mathrm{C}$ the curve shows an undothermic reaction due to the formation of a sodium meta silicate layer around the silion grains. As tho temperature rises, sodium disilicate is formed, show by the exothermio Peak $\underline{C}$ at $700^{\circ} \mathrm{C}$. Peak $\underline{D}$ indicates the melting of sodium disilicate as a euteftic with silica at $790^{\circ} \mathrm{C}$. A glassy matrix is formed immediately after the ondotherric peak ot $790^{\circ} \mathrm{C}$.

FIG. 2


FIG. 3


## (5)

Figure 3 shows the differontinl thermal analysis curve for the three componest system; 85 mole $\% \mathrm{SiO}_{2}, 14 \mathrm{~mole} \% \mathrm{Na}_{2} \mathrm{Co}_{3}$ and 2 mole \% NaF. In tiais diagram peak A and B include the orystalline inversion in $\mathrm{Na}_{2} \mathrm{Co}_{3}$ at 350 and $470^{\circ} \mathrm{C}$. Peak C at $540^{\circ} \mathrm{C}$ is due to the chemicel reaction resulting in the formation of sodiun meta silicate. The peak $\underline{D}$ indicates $\alpha-\beta$ quarts inversion $\left(573^{\circ} \mathrm{C}\right)$. The peak E at $680^{\circ} \mathrm{C}$ is due to chemical reaction between $\mathrm{N}_{2} \mathrm{Co}_{3}$ and $\mathrm{SiO}_{3}$. The exothermic penk at $710^{\circ} \mathrm{C}$ agroes with that in the two component system and is due to the formation of sodiun disilicate. It $760^{\circ} \mathrm{C}$ a glassy matrix is formed ${ }^{2(a)}$.

Differential thermal analysis and thermogravimeteric analysis lec.d to some understanding of the renctions which occur but give no direct evidence about the homogeneity of the melt or its subsequent refining.
ifter the vigoruus reactions are complete, sone send grains remein incomplotely dissolved. On dissolving, these will produoe regions or cords rich in silica. Preston and Turner ${ }^{3}$ have studied the rate of melting of soda-line-silice glass and found that at a constant tempersture it depends upon the grain size of the sand. Lit $1400^{\circ} \mathrm{C}$ about $90 \%$ of the total melting time is taken up in dissol: ing the residunl sond grains of ter the initinl vigorous reactions have subsided. The rato of so..ution of these grains is proportional to the surface aroa exposed to attack and proceeds at the approxinnte rate of 0.003 mar minute reduotion in average grain dinmoter (Figure 4). (The rate will vary with both glass composition and temperature.)


The observations of several workers support the notion that the dissolving of the residual snnd grains is a diffusion controlled process but only lireider and Cooper ${ }^{4}$ hovo so for reported experinents to confirm this, although a prper also confirming this has been read by Hlavac ${ }^{5}$ but not yet published.

## 2 ICCIDENTI INHOMOGINEITIES

It has already been mentioned that the heterogeneous reactions involved in glass melting lead to inhomogeneities. There are also some other importint sources of inhomogencities. ill commonly used refractories are attacked by glass and dissolve in it, proaucing cords rich in $\mathrm{il}_{2} \mathrm{O}_{3}, \mathrm{SiO}_{2}, \mathrm{ZnO}_{2}$; sometimes solid orystalline fragments (stones) break awny and do not dissolvo completaly. Stones can also occur when refractory materials in the latch do not dissolve completely. Cords produced by refrnctory oorrosion often lie on or very olose to the outer surface of the glass. More randomly oidented silica rich cords may be caused by drips from the crown of the furnoce.

Other specific types of inhomogeneity can be causad by evaporation or nbsorption at the top surface of the melt; alkali, boron, lead, fiuorine may easily be lost by volatilisation and water vapour or $\mathrm{SO}_{\mathrm{a}}$ absorbed from the furnace atmosphore. When an excessive amount of fines are usod in batch or sometimes, perhaps, batoh nay be charged in such a wry that there is a danger of batoh material being entrained by the flames and batch dust may be carried on to the rofrectories. The alkalies present oause fluxing of these refractories cousing drops of ocrrosion product to fall into the melt and resulting in aluminous or siliceous streaks in the glass.
$\therefore$ general inhomogeneity is of ten due to some carelossness in the batch mixing. When the batch is to be tronsferred some distanoe, dumixing of alreacy mixed batch can oocur if the dry batch is subjocted to vibration or poured from one container to another.

## (i) Segregation during melting

The importance of using o well mixed batch and thus by implication the seriousness of any inhomogoneity of the batch has been reported by Poole $e^{6}$ a . However, aren well mixed batch of ten tends to segregate during melting. The reason for this segregation is that during the early stagos of melting, wien batch is hoated rapidly, the liquid phase first formed is very fluid and can begin to drain downwards before it reacts. This leaves the upper part of the batch enriched in silica and other less rocctive constituents. Ilso the gases evolved during reaction stir the rencting mass. This might be expected to have a chiefly beneficisl effect but a froth floatation effect can also sometimes cause additional segregation by making partly dissolved silica grains rise nnd float on the surfince of the melt.
(ii) Cords formed during dissolving of refractories

Molten glass is very corrosive at high temperatures and so dissolves nluminous materinl from the surface of refraotory pieces in any part of the furnace. In the thront of a tank furnace or the doghouse corners, the effect of attack is much greater due to the relatively rapid movements of glass, giving forcod convootion conditions for corrosion, inother factor which profoundly influences the rate of solution of the refractories is the temperature of the glass at the interface. When the tank blocks corrode, there is usually enhanced attack at the top of the blocks near the glass surface. This onhnnced attack at the top of the block is known as fluxline attack. Many thoories heve been put forward to explain this profile of attrack. Some of these thaories are
based on the temperature gradiunts which exist in gloss melting tanks. It onn be shown that the rate of solution of refractories in molten glass is very much affected by the temperature and it can be seen from Figure 5 that change of tomperature from 1200 to $1500^{\circ} \mathrm{C}$ increases the rate twenty times ${ }^{6, h}$.

Kingery ${ }^{7}$ has reported that the kinetics of the solution of refractories can be deternined by twe processes; the rate of reaction at the interface and the diffusion of the solute from the reacting zone. This later effect can be expressed by the Nernst equation,

$$
n / i=K\left(C_{s}-C\right)=D / \operatorname{\delta eff}\left(C_{s}-C\right)
$$

where $\mathrm{n} / \mathrm{di}=$ rate of solution
$C=$ concentration in tho bulk fluid
$C_{s}=$ concentration of the soturated solution
$\mathrm{K}=$ rate constant
$D=$ diffusion constant
Geff $=$ effective filn thickness for diffusion
This equation shows that rate of solution depends directly on the difference in concentration between the bulk solution and a saturated solutior, on the diffusion constant and inversely on the effeotive boundary thickness.

If the clurinous ginss which is formed as a result of corrosion entors the min current of glass, it will give streaks or cords in the finish wrese. Usually much of the glass reciroulates between melting and working regions a number of times so that the inhomogoneitios are widoly distributed, greatly attenunted and onn be neglected. But if there is excessive solution of matorial near


EFFECT OF TEMPERATUR ON DISSOLUTION OF REFRACTORY BLOCK
the forming point, then thore is a real danger of developing streaks in the finished product. Cords are also formed when pieces of refractory materinl are dislodged as stones. The stone would be surrounded by a zone of glass which, inconsequence of its containing dissolved refractory, would be more viscous than the main mass of the glass end would appear as a sac or cord.

## (iii) Volatilization

It is a well established fact that loss of volatilo components from a melt, especially a laboratory melt of relatively small depth can produce serious inhomogeneity in glass. Kruithof et al ${ }^{8}$ have published a qualitative study of this process. Mass trensfer fron melt to atmosphere oan rosult in heterogenoous surface layers, bulk inhomogeneity and changes in nost of the chemical and physioal properties of at lonst the surface layers. Cords may be formed in borosilicate glasses by the volatilisation of $\mathrm{Na}_{2} \mathrm{O}, \mathrm{K}_{9} \mathrm{O}$ and $\mathrm{B}_{2} \mathrm{O}_{3}$ lonving a ailioa rich surface layer. It has been suggestod that it mny be best to leave this soum alone in order to retard further losses. Often the soum is skimmed off, leaving good quality glass, though in difficult oases heavy inhomogeneity may necessitate partial draining of the tank. Kruithof 's ${ }^{8}$ work seems to show that surface inhomogencities caused by volatilisation do not always disporso through the nelt but depending on composition, they may either sink or floot. is systomatic investigation of the kinetics of the proooss has beon mado by 0ldfiold and Tright ${ }^{9}$ with partioulor reforenoe to volatilisation and the formation of silica scum on borosilicate glass. They found the loss of alkali and $\mathrm{B}_{2} \mathrm{C}_{3}$ from
the glasses obeyed a linear relation with the square root of time indicating a diffusion oontrolled process through the oristobalite surface layer. The loss was acoounted for as metaborate in the ratio $5\left(\mathrm{Na}_{2} \mathrm{O}_{.} \mathrm{B}_{2} \mathrm{O}_{3}\right): 1\left(\mathrm{~K}_{2} \mathrm{O}_{.} \mathrm{B}_{2} \mathrm{O}_{3}\right)$. They also concluded that the two mechonism wore involved; evaporation from a free melt surface and diffusion through a SiOg-rioh layor whioh itself varjes with time. Volatilisation of alkalios from soda-lime-silica system tends to give siliceous cords which loads to higher surface tention and viscosity and lower refrnctive index. The specific evaporation of alkali is higher in the presence of water vapour, the rate of evaporation increases with the increase in water vapour content ${ }^{10}$. The water enters the melt, where it roacts with $\mathrm{Na}_{8} \mathrm{O}$ and is evaporated as Na OH. PbO is easily volatilised from Iead glasses giving a large decrease ir density wi.th correspondingly large changes in refractive index and viscosity.

## MECHENISK OF HOMOGENIZING

The final homogonizing of glass or any othor inhomogeneous viscous liquid depends upon the diffusion of any inhomogeneity into the bulk of the melt or liquid. The process of diffusion can be accelerated by increasing the surface area of the inhomogeneities and thus reducing the distance over which interdiffusion must take place or by increesing the diffusivity.

Since diffusivity is a physical property of the system, there are only two ways of influencing it; chonge in temperature or change in composition. Only change in temperature is possible for a particular glass but it will increese the refractory corrosion, losses by volatilisation and cost of running the furnace. It is, therefore, important to exanine the possibility of reduoing the distances over which diffusion must take place. This may be accomplished by attenuation of the inhomogeneities.

Sttenuation of inhomogeneitios by laminar flow was first discussed by Spencer and Wiley ${ }^{11}$. They reported that it can be considered as consisting of two processes, increasing the surface area of the interfece between the liquids being mixed and distributing that interface through the volume of the material. The first person to consider application of this to glass was Geffoken ${ }^{12}$ but the most detailed discussion of this process is due to Cooper ${ }^{13}$, who has also considered its application to homogenization of glass in tank furnaces ${ }^{14}$. He nas shown that the parameter that mensures the local mixing rate in a continuous Inminar flow systen is $D / \delta^{?} u_{x}$, where $D$ is the approprinte chemical diffusion coefficient, $\delta$ is the striation thickness and $u_{x}$ is the throughput velocity.

Cooper ${ }^{14}$ has suggested that the amount of mixing required ( H ) between two adjacent striations (i \& i +1 ) onn be defined ns

$$
\left.H_{i}=\int_{n_{i}}^{n}(i+1)-\bar{N}\right) d n=C_{1} \Delta N_{i} \delta_{i}
$$

where $C_{1}$ is a number depending on the details of concentration distribution, $\Delta N_{i}$ is the intensity of the individual strintions and $\delta_{i}$ the striation thiclmess. The rate of mixing is given by

$$
\frac{\partial H}{d t}=h=\frac{\delta(N-\bar{N})}{\delta t}
$$

which leads to

$$
\frac{d H}{d t}=C_{2} \frac{D \Delta N}{\delta}
$$

where $C_{2}$ accounts for the details of the concentration distribution and is of the order of $2 \pi, \Delta N$ is the intensity of segregation, $D$ is the chomical diffusion ooofficiont and $\delta$ is the striation thickness.

Cooper ${ }^{14}$ shows that for parallel sided inhomogeneities originally at angle $\alpha_{0}$ to the streamline, the offective striation thickness ( $\delta$ ) at 0 later tinc is given by
$\delta / \delta_{0}=\left\{1+2 t \operatorname{grad} u_{x} \cos \alpha_{0} \sin \alpha_{0}+\left(t \operatorname{arad} u_{x}\right)^{2}+\sin ^{2} \alpha_{0}\right\}^{-\frac{1}{2}}$
Figure 6 is a plot of this equation for various values of the total shear ( $t$ grad $u_{x}$ ). It is evident froin this figure that greater the total amount of shear, the groater the probability of attenuation for any value of $\alpha_{0}$.

Because of the complox flow patterns and temperature distributions difforent elements of glnss follow very different paths through the furnace and therefore are subject to very different anounts of attenuation and offoctive $D$ between entering

FIG. 6

defendence cf relative striation thickness on original orientation for several values of total. SHEAR.
(AFTER COOPER)
and leaving the furmoos. Cinoper ${ }^{13}$ has suggested that in this case a suitnble way to compare efficiency of processing of these different elements is to integrate the expression $D / \delta^{2} \underline{u}_{x}$ which is equivelent to $\frac{\mathrm{Dt}}{\delta^{x}}$, along the whole length of each streamline $\int_{x_{0}}^{x} \frac{D}{\delta^{2} u_{x}} d x$
and this expression should be maximized for ewery path. For diffusion alone in a parallel slab of thickness 26 it is a standnrd result that diffusion is approaching complecion when $\frac{D t}{\delta^{2}}>1$, so one would wish to have $\int \frac{D}{\delta^{2} u_{x}} d x>1$ for every path but if much greater than 1, this represents distinct over processing, even though its occurrence would not have the undesirable effocts of having $\int \frac{D}{\sigma^{x}} u_{x} \ll 1$.

4 REASON FOR TIVESTIGATION
It is generally assumed that despite minor evidence of slight heterogeneity, normal commercial glasses made in pots or tanks are substantinlly uniform in composition. This ossuruption is rarely tested although inhomogencities in glass orticles due to lncal composition variations may seriously affect its strength, thermal shock resistanoe, internal stress distribution and refrrotive index or other important properties.

Considoring the great importanse of homogeneity it is surprising that it has by no means exhaustively beon studied. Work previously published includes several methods of measurement but most of them give only a qualitative value of homogeneity. Very few attempts have boen made to mensure the homogeneity of glass quantitatively.

The common practice for obtaining homogeneous glass in the laboratory is to melt, crush and remelt repentedly until the sample is thought to be homogeneous. This is a very tedious process. If efficient stirrers wore developed, the homogenising process could be simplified. Although very little has been dono concerning homogenising of the glass melt, some work has been dono on mixing and homogeneity of batoh. The one exoeption is making of option glass where mechanioal stirrers are boing used beoause of the very good homogeneity required. The function of the stirrer is to improve homogenoity by attenuation of inhomogeneities, serving to reduce distance and inorease areas overwhich interdiffusion can operate. The concentration of the inhomogeneities as woll as tho details of the flow pattorn govern the offeotiveness of attenuation,

This work was done for several reasons. The most important are:-
(i) to show that homogeneity of glass can be measured quantitatively.
(ii) to follow the kinetics of mixing for several different simple stirrers.
(iii) to compare results obtained in slasses with trose obtained in room temperature models.

It was hoped that the results would incroase understanding of the process of mixing flasses or other visoous newtonian liquids and show what dimensionless parameters neod to be matohed to obtain reliable information from room temperature models. dlthough these experiments are not directly related to flow and homogenising in tank furnaces, botter understanding on laboratory scals may bo expoctod to help understand behaviow of ful. size pots or tank furnooes.

5 PRIVCIPIES OF ME.SURTNG HOMOGENEITY
A homogeneous body is one for which small samples taken from numerous different positions have exaotly the same chemical composition or the same value of some other property. To describe this quantitatively the values and accuracy of measurements of the property chosen and also the size of the samples taken must be stated. The minimum reliable description of homogeneity requires two parameters; the sonlo and the intensity of sagrogetion.
(i) Sonle of segregation

If the actual size of the inhomogeneity can be measured, this is the scole that really matters but it is not of ten accessible. Therefore, the size of the sample usualiy has to be accapted as the scale. If the somples are very much larger than the inhomogeneities the result will not indicate serious inhomogeneity, even if it exists on a smaller scale. The sample size thorefore needs to be very corefully chosen. It is inherent in any method of measurement that as the sample size increases, the variance of the results decreases. This is show in the figure 7, where $I$ is a function of the variance. The diagram shows how measured value of $I$ depends on sample size. So long as sample size is smaller than the actual size of the inhomogeneities $I$ has its maximum value (Region $\lambda$ ). When sample size is much bigger than the size of the inhomogeneities I is very low (Region C). The upper and lower limits of Region $B$, in which $I$ is sensitive to sample size roughly indicate the size range of the inhomogenoities. $0 x$ could be oonsidered as the sample size for this partioular case.

SCALE \＆INTENSITY
－INCREASING INTENSITY



## (ii.)

Intensity of segregation
This is a convenient monsure of the width of the distribution of the property measured. It is a statistical parameter describing the data such as variance or standard deviation; it is common to use the variance. Its value is a maximum with maximus segregation and goes to zero when mixing is perfeot. It is convenient to define the intensity so that its value is 1 when mixing is as bad as possible. Danckwerts ${ }^{15}$ has discussed this and aefines I as

$$
I=\frac{\sigma_{n}^{2}}{\bar{a}(1-\bar{a})}
$$

where $a$ is the concentration of ono component, $\vec{a}$ is the mean concentration of the same componert in the mixture and $\sigma^{2}$ is the variance of the concentration of the component obtained experimentally. Then even for perfeotly homogeneous body $\sigma 2$ is not zero because of errors in method of meesurement, it is bettor to define intensity of segregation as

$$
I=\frac{\sigma_{G}^{0}-\left(\sigma_{e_{C}}^{2}\right)_{F}}{a(T-M} .
$$

where $\left(\sigma_{i}^{2}\right.$.) F.M. is the variance of a fully mixed solution. Figure 8 shows qualitatively what these terms mean. The original unnixed comporients are represeated by black and white squares. Then perfectly mixed the body would be uniformly grey. The stristion thiokness $\delta$ is used as the mensure of soale of segregation. It is the minimum distance between adjacent intersections with the mean oonoentration. Very often the (nverage) intensity defined as suggested and average scalu are enough to desoribe the homogeneity in practice. However, this is not always true, for example, in
strength measurements maximum tensile stress is likely to be more important than the average value bocause it is the tensile stresses that weakens the article.

Before homogeneity con be properly described it is necessary to be able to neasure some property on a sample, small in relation to soale of inhomogeneity, to know the size of the sample and its position and to repeat the neasurements for samples taken from many position. The most important factors in mensuring the homogeneity are accuracy of the measurement, the size of the individual samples and the method of sampling the body to obtain them.

The size of the sample depends upon the size of the inhomogeneities prosent. If the sample is very much largor than the everage size of inhomogeneity, it will probably differ very little from the average in properties and yield little information. There will be no adventage using samples muoh smaller than the size of inhomogeneity, because the samples alroady give very close to the maximum possible intensity of sogregation. If the sample is very small, the less accurate will be the value of property measured. By decreasing the size of the samples trken one also reduces the probability of obtaining a representative result unless the number of samples is considerably increased.

This is illustrated by exmminntion of a glass systen with different layers (Figure 9). The thickness of each layer is " $x$ " units. If the thickness of the sample is a lot larger than "x" units, the measurod intensities will not differ much from the average and will not detect the actunl intensities of the layors.

FIG. 9

## EFFECT OF SAMPLE SIZE ON INTENSITY



VALUE OF INTENSITY:- CONSIDERABLY BELOW MAXIMUM


SLIGHTLY BELOW
MAXIMUM


MAXIMUM

Samples of the size equel to "x" will mostly comprise parts of two adjacent layers and the maximum deviation observed in intensity will be rather less thon the true spread. A sample size of about $x / 2$ will give very close to the true range of intensities and further decreasing samples size will not appreciably increase the rango.

For obtaining a representative sample it is very jifficult to lay down specific rules. It depends very much on the distribution of inhomogeneities through the body. In general a method of obtaining samples at random through the whole body is roquired. For practical reasons tho smallest somple giving useful results is to be preferred. The only why to be sure sampling is sufficiently representative is to check the reproducibility.

After deciding the method of mensurement, size of the individual semples, the nuraber and position of the individurl samples, the problen remains of the snmpling technique. The ideal sampling teohnique is that there should lo no disturbanoe in the body while removing the sample but this is often difficult to achieve. This requirement can ousily be met for glass, as the whole glass melt can be cooled in the crucible and removed without any disturbance. Then thin soctions may be out from this melt to investigate the homoreneity.

$$
C \quad \mathrm{H} P \mathrm{P} \quad \mathrm{~F} \quad \mathrm{R} \quad \mathrm{I}
$$



## 1 MEMHODS OF ¿SSESSIHG HOMOGSNEITY OF GL亡SS

(i) Containers

The dogree of homogeneity required varios according to the purpose for which the gless is to be used. The requircment of an opticiil Glass is that it should be of uniform refrective index throughout tho whole body. The containers glasses do not require uniform refractive index but they need to withstand some rough handling i.e. thoy must have reasonable strength. Bottles intonded for corboneted bovernges, beer and produots packed undor gas pressure must meet requirements of pressure resistanoe. Some highly carbonatod bevernges my develop pressures of $125 \mathrm{p} \cdot \mathrm{s} . \mathrm{i}_{\text {. }}$ during very warm weather. So the most important oritorion for the contciners is often strength because of rosistance to internsl pressure and long life of multi-trip bottles.

Thu most common method of irvostigating day to day homogeneity of contriners is the Swicker ${ }^{16}$ rating examinntion. In this method a ring section type of exnmination is used to determine and eviluate the degree of cordinoss of the glass containers. It is a feature of all blow glass ware that the cold is nanrly always found running parallel to the klown surfaces, forming concontric cyliniars nad the examinction of ring seotions is thus a useful test. Evaluation of the dogree of cordiness is based on the anount and positioning of the cords with respeot to the normal glass, together ith the mensuroments or estimation of the arnount of stress prosent. These stresses are usually the result of an inhomogencous condition in the glass. This is a quito
good method for dnily routine but it is essentinlly qualitative, depending on comparison with arbitrary standards.
din enormous amount of work has been done in past years, in stuaying the physicnl properties of the glass, to study closely the causes of inhomoceneity in glass. In the continuous tenk furnaces, the chenical composition of the glass may be controlled by routine chemical analysis or by the routine measurement of some physical property of the glass which is lnown to change wi.th variation in the chemical composition. Density, refractive index and softening point are three of the mare comon physical properties that are influenced by the chemioal composition and their meesurements on be used as a menns of routine control. The density of the glass is 0 function of its composition and since it can be monsured with very hish accuracy, in a relatively short tine, on large numbers of smples, it is of ton usod as a mons of operating control in furnace aperation. Table I shows the relativa quantitative effects of $\mathrm{CaO}, \mathrm{SiO}_{2}, \mathrm{Na}_{3} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}_{3}$. A change of about $0 * 5 \%$ in C 3 C has a preator effect on density than a corresponding change in $\mathrm{SiO}_{2}$, $\mathrm{Na}_{2} \mathrm{O}$, or $\mathrm{ill}_{2} \mathrm{O}_{3}$. in increase in lime content gradually raises the density, an increase in silica diminishos it. The relation betwoen density and composition in the systen $\mathrm{N}_{2} \mathrm{O}-\mathrm{CaO}-\mathrm{CiO}_{2}$ at room temperturo is shown in the Figure 10.
J. C. Turnbull ${ }^{17}$ was the first to sepnrate inhomogeneities in bottle elass by density difference. $\therefore$ quantitntive mothod of mensuring the homogeneity of glass is the density sproad and centrifuge of Turnbull and Ghering ${ }^{18}$. This consists essentially of separating the powdered glass into density ranges by centrifuging

in a suitrble dense liquid with accurately controllad temperature. The particles are first separated into light and heavy fractions which mey be removed and subdivided by further separation. This method has severol advantages over other methods. The chenical analysis of the separated fractions may be used to detarmine the materinls with which gloss is diluted or it may be usea to distinguish further among the various types of oords. The particle size of the sample is near to the true size of the inhomogeneitios, and the method of sampling the body is much better than the other methods. illso different eroin sizes from one somple may be used to deteruine the striation thickness of the cord. Disadvantages aro the fact that the method is a destructive one and no information can be obtained about the original position of the cord. The soparation of the fine particles is interfered with by change in density and flocculaiion produced by any absorbed materici. inother difficulty which also limits the aethod, arises from the comprescibility of the heavy orginic liquid. Also the experiment is too lenethy to be done on many samples in one dey.

Preston ${ }^{19}$ has described a simplified sink flont apparatus nnd method for meesuring the density of glass. The method involves the settling of lumps of gless in a mixture of henvy liquids. This is nccomplished in $n$ water bath by means of constont hentine of the liquid. The tomperetures are noted at which unknorm lurips and pieces of stinderd glosses settıo pest a reference lovel in a tube of heevy liquid. The densities of the test lumps cro then caiculated from those of the standerds by meens of the coofficient for the change of density with temperature for the liquid and for the glass. This method is
procise, simple, and easily operated. It is much more rapid thon the other nethods. The temperature difference corresponding to the test piece of maximum density and minimum density, taken from the sane bottle, can roadily be expressed as the oquivalent density spread.

In principle it is a good method but there are certain disadvantages. The comon practice is to take only five samples from one container once a day which can hardly be a representative sample. Scatter of densities indicates inhomogeneity but the method is not a sensitive indicator of inhonogeneity beonuse the sample size is much bigger ( $3 / 8$ to $5 / 8$ inch square) than the true inhomogeneities.
A. T. Coe ${ }^{20}$ tried to relate the density sprend detemaned by the Preston method to possible glass making troubles. He reported the control limits of $\pm 0.0030 \mathrm{gms} / \mathrm{co}$ on density as being satisfoctory in container glass production. In connection with trends in the density dita, he considered a trend of $0.0015 \mathrm{~g} / \mathrm{cc}$ in 24 hours to ceuse only slight inhomogeneity. If the trend continues, however, for nnother day then severe troubles would be likely to appear. $\operatorname{Cos}^{20}$ used donsity spreads as indications both of curront homogeneity and of impending trends.

Shelyubskii ${ }^{21}$ used the prinoiple of the christionsen filter and developed a method for determining the homogeneity of glass. It is based on the variation of scottoring of parallel beams of monoohromatio light by a layer of glass powder in an immersion Iiquid with small variations in temperature. On heating, the refractive index of the liquid is decreased and as the refractive
indioes of two phoses becomo closer to each othor, the scattering is docressed and so by raising the temperature of the suspension the tronsmission of light through it will be inoreased until the two voluos of refractive index are equal. On further heating the refroctive indices diverge and this again increases the scatterine. In other words the transmission reaches a anximum at the temperature at which the two refrnctive indices are equal. Thus the relation between tho intensity of the light travelling through the vessel and temperature will take the form of bell. shaped curves. Shelyubski $i^{21}$ plotted transmission agrinst the $\ell$ where $P=\left(n_{1}-n_{2}\right)($ Figure 11$)$. For a perfectly homogenoous glass, the peak formed is very narrow and showed a maximum transmittance of $100 \%$. The less the homogeneity of the glass, the nore comparatively wige and slanting will be the peak, as some grains will be metched at difforent temperatures. The amount by which the height of the peak is altered is dependent on the intensity and anount of cord and can therofore be used as a mensure of homogenuity.

Raman ${ }^{22}$ showed that the intensity of a parallel benn passing through the cell should bo

$$
I=I_{0} \exp \left[: \frac{I_{1}^{2} \Lambda_{2}^{2} \Delta}{\lambda^{2}}\left(n_{1}-n_{2}\right)^{2}\right]
$$

where $I_{0}$ is the incident intensity, $I$ the energent intensity, $K$ factor dopendini on proportion and shape of glass particles, $\left(n_{1}-n_{3}\right)$ the difference betreen refractive indices of homogeneous glass and liquid, $\Delta$ is tho size of glass particles, $z$ the length of the cell and $\lambda$ tho wavelongth on light.

Shelyubskii ${ }^{21}$ assuned thot there was a Gaussian distribution

FIG.II
PLOT OF TRANSMISSION, $\tau$ VS $P$

(AFTER SHELYUBSKII)

FIG.I2


SheLyubskil's method of calculating "a"
of refraotive index of tho glass so that

$$
N_{x}=N_{0} /(\sigma \sqrt{2 \pi}) \ldots \exp \left(\frac{-x^{2}}{2 \sigma^{\prime}}\right)
$$

The standard deviation $5^{2}$, which indicates the deviation of refractive index of small particles of the glass from its average, is used to measure inhomogeneity. Following Raman's ${ }^{22}$ treatment and introducing the parameter $a=\frac{K^{2} \pi^{2} \Delta_{z}}{\lambda^{2}}$ \& $P=\left(n_{1}-n_{a}\right)$ leads to

$$
I=I_{0} \exp -\underline{a}\left(\rho^{2}-\sigma^{2}\right)
$$

The coefficient a can be calculated from the value of $\rho$ at the half width of the peak (Figure 12).

$$
\begin{aligned}
& \operatorname{In}\left(\operatorname{In} / T^{\frac{1}{2}}\right)=\ln 2=a p^{2} \\
& \propto=\frac{\ln p}{p^{2}}
\end{aligned}
$$

No doubt the method is rapid and can be used for daily cheok but it is incopablo of derining the size and intensity of inhomogeneity as two separate parameters. The author assumed a gaussian distribution of refraotive index but it is highly unlikely that this is generalliy the case. The refroctive index at anximum transmission is due to the major component but it does not tell us about the amount of major componont peresent there. The experiment connot give precise information about the shape of the actual distribution of refractive index.

Budd and Blanchard ${ }^{23}$ havo shown one way in whioh this method may be usod for coloured glasses and this involves moking measurements at two wavelengths and taking the mean valuo of $\sigma_{0}$

Sohilling and Weiss ${ }^{24}$ also investigated the homogeneity of difforent glasses by Sholyubskii ${ }^{\prime} s^{21}$ method. The reproduoibility of the homogeneity coofficient is about $\pm 1 \%$. docording to them
the method is useful in optionl industry but not for blown glossware, where quality is more related to a uniform viscosity.

A method by which cord in glass can be identified without destruotion of the saraple is desoribed by Carr and Shoneberger ${ }^{15}$. This method is based on $a$ comparison of sample containing oord with a sample of glass, from the same object, free from cord. A glass sample is placed on the turntable of on m-ray fluurescen* apparatus and a beam of $x$-rays is directed into a small area of the surface. The elements in the saruple will absorb $x$-radiation, when the voltage of the primary radiation is inoreased above the exaitation voltage for each elenent. is the radiation is absorbed, the energy is released in the form of x-rays, the wavelengths of which are oharacteristio for the atom from which they originate. The sample must be cut in such a way that tho cord is exposed on the surface, as the owission of secondary radiation occur only within about 0.1 mm of the surfcoe. By comparing the oounts from the base glass and one containing cord, the composition of the cord can be determined. The method is only useful when the soale of inhomogeneity is not much sanller than the diameter of the $x$-ray beams ilso the method does not give any quantitative value of homogereity.

IHffler ${ }^{26}$ analysed the inhomogeneities in glass by a combination of etchin $n_{6}$ ard opticnl interference methods. The rate of solution in glasses is dependent on the composition of the glass and of the solvent. is cords rre due to variation in composition, etching a plant section through cordy glass produces contours which may be ovaluated by interference measurements. Ibffler ${ }^{26}$ examined the individunl inhonogeneities by this method.

He has demonstrated thet two otching experiments with different attacking solutions enoble the composition to be estimated more preoisely. Since this method gives all the information nocessary it can also be used to obtain a quantitative description of the homogeneity.

## (ii) Flat glass

The flat glass is usually viewed against a back ground of zebra stripes i.e. alternate black and white stripes. Inhomogeneities or surface irregularities cause the stripes to appear distortod. Viewing either by transmission or reflection may be used, and it is often helpful when examining flat glass to titt so as to obsorve at a glancing ancle.

Holliand and Turner ${ }^{27}$ used a photo optical method to determine the homogeneity of glass. The method is also known as the "Sohlieren" method and apparently is due to $\Lambda$. Topler ${ }^{28}$. It consists in determining the proportion of light lost by soattering from any oordiness present in a polished, parallel sided specimen. The apparatus is very simple, a reduced inage of a small souroe of light is formed at a diaphragm, an image of this diaphragm is then formed by a large, well correoted lens at a second "stop", behind this sop a photocell or comera is looctod. The test specimen is introduced between the large lens and the second stop. The light folling on the photocell is determined as a percentage of the incident light on the disc and the iniensity of this soattered light, expressed as a peroentage of the incilont light, can be used as a quantitative measure of honogeneity. However, it should be noted that no information about the scale of the inhomogeneities is given.

The sensitivity of the method depends upon the size of the image of the source, accurate alignment of the optios and the sensitivity of the cell used. The disadvantages are that the method is not truely quentitative. If the cords present are all in parallel layers, no distortion may be seen and if the surfinces are not flat and parallel or internal layers not parallel, distortiom will appear.

## 2 INVESTIGATION OF HOMOGENISIITION OF GLASS

A fairly comprehensive study of the factors, such as temperature, time, batch formulation, degree of mixing, segregation tendency in conveying and melting, was carried out by Tooley and Tiede ${ }^{29}$. They used the density spread techniquc to measure the homogenoity of the glass. The first series of glasses were melted in a platinum crucible at $1400^{\circ} \mathrm{C}$ undor conditions of minimum convective mixing. The melting tine varicd from 1 to 6't hours. The experimental results are shown graphicolly. Figure 13 indicates a rapid decrense in density spread during the first 16 hours of melting and a slower decrease up to 64 hours. The authors suggested that the portion from the top of the melt gavo freater density spread thon the bottom (Figure 14). This innomogeneity was due to the volatilisation from the surfree and a tendency of less dense particles to seek the surface.

Leter work showed the effect of tempernture on the homogeneity rate of glass. Glasses were melted at temperatures ranging from $1232^{8} \mathrm{C}$ to $1454^{\circ} \mathrm{C}$ for four hours under minimum conditions of convective mixing. The results showed a sharp drop in density in the range $1232^{\circ} \mathrm{C}$ to $1288^{\circ} \mathrm{C}$ and a minimum at about $1325^{\circ} \mathrm{C}$. Above this temperature a slowly increasing donsity sprend was observed (Figure 15). dgain the greater density spread was observed from the glnss concentroted in the top holf of the melt. Also the homogeneity was improved in the bottom layer as tile temperature was inorensed. is possible explanntion for the inorense density spread at higher temperature is that initial segregation at high tomperature is more intense due to the inorensed fluidity of the melt. It was observed that by repeatine cracking and remelting


FIG. 14

density distribution of bottom a top glass FROM SAME CRUCIBLE
A: TOP, DENSITY SPREAD $12.3 \dot{C}^{\circ}$
B:- BOTTOM, DENSITY SPREAD $2.3^{\circ}$ ] MELTING TIME $4 \frac{1}{2}$ HRS.
C- TOP, DENSITY SPREAD $9.6 \mathrm{C}^{\circ}$
D:- BOTTOM, DENSITY SPREAD $2.0{ }^{\circ} \mathrm{C}$ ] M. TIME $7 \frac{1}{2}$ HRS.


FIG. 16
Stationary crucible rotating crucible



D+60MESH. 日 60-80 MESH. 日 80-44O MESH .B14O-200 R-200 Effe.ct of batch mixing time \&GRain size on degree of
these glasses or by melting in a rotating tilted arucible, a markod effect in lowering the density sprecd can be obtained. The homogenoity was found to incrense with decrease in batch rain size, more so in the rotating crucible than in the stationary one and batch mixing time had relatively little influenco as indicated in Figure 16.

Bower ${ }^{30}$ used two different methods to measure the homogeneity of glass melts; Shelyubskii'a ${ }^{21}$ light scattoring method and Loffler's ${ }^{26}$ etohing and interferometery technique. He melted 50 gms of glass in a platinum - $40 \% \mathrm{Rh}$ crucible. The stirrer used was a platinum thimble on the end of a refractory tube. Two series of melts were made both of the same composition and melted at $1400^{\circ} \mathrm{C}$. One set was nelted for tro hours with a variable speed and the other stirred at a constant speed for different times. The tronsmission results are shown in the Figure 17. Bower ${ }^{30}$ reports that no doubt the light scattering method gives a numerical result but it does not eive a quantitative measurement of homozeneity. The etching and interferometery method is rather lengthier but is oapable of siving all necessary information. But he did not develop this method fully to get a quantitative value of homogencity. He also compared the maximum transmission with maximum scale and intensity (Figure 18). It does not indioate that the shape of transmission punk varies, and all the information that it provides is represented by the single parameter $\mathrm{P}_{\text {max }}$. Il.so it does not indicate the actual frequency of intensities in the sample as does the density sprend.

Plumnt et $0 I^{31}$ have studied the degree of homogeneity of molten glass by using a strioscopio apparntus. The apparatus

FIG.I 7


EFFECT OF STIRRING TIME (20 r.pm.)
(AFTER BOWER)


EFFECT OF SPEED (CONSTANT TIME)

## FIG.I8



CORRELATION OF Tinax WTH MAXIMUM INTENSITY AND MAXIMUM SCALE OF CORD. THE NUMBERS IN The figure are values of $T_{\text {max }}$
(AFTER BOWER)
consists of an optical system mounted above the melting furnace with arrangements to illuminate and photograph the melt from above. The glass is melted in an electrically fused alumina crucible. The bottom of the crucible is covered with molten lead to produce a clean, bright mirror-like surface. The path of any light refliected fron this surfnce depends upor the refractive index of the glass above it. If the glass is inhomogenecus, light and dark patches are seen where cords alter the refractive index in the light pait. Thus the progress of homogenisation can be followed by studying size, shope and distribution of these patches indicating inhomogeneity. The method has the advantage that the observations are taken at the melting temperatures. The disadvantages are that the patches may be due to the cord or temperature difference and it is difficult to drfferentiate between them. Also the method Eives the average over the whole depth of melt.

$$
C H A P T E R \quad I I I
$$

ROOM TEMPERATURE MODEL EXPERIMENTS Bradford ${ }^{32}$, Howse ${ }^{33}$, and Tunaley ${ }^{34}$ studied mixing of glycerine in a room temperature model of a crucible. The present work was. designed to see if their results had any relation to the behaviour of Glasses.

Complete modelling of mixing requires that both the diffusion and flow prooesses be matched. The impeller power should be a function of the geometery of the innellar and tank, the properties of the fluid, and the rotational force of the impeller. The most important similarities are geometrio, kinematio and dynamic. Geometric similarity requires that all oorresponding dimensions in systems of different size bear the same ratio. If inertial and viscous forces alone are important then the ratio of inertial to viscous force should be matched.

$$
\frac{\text { Inertial }}{\text { viscous }}=\frac{V L}{\eta} P
$$

which is called the Reynolds number (Re). Sinoe this ratio determines whether the flow is laminar or turbulent it is a aritionl group in correlation of power requirement. If surface waves are important, then the ratio of inertiel to gravitational force should be the same in both systens, whioh is the Fmoude number.

$$
\text { Fr }=\frac{\text { Inertial foroe }}{\text { Cravitational foroe }}=\frac{P V^{2} / L}{P G}=\frac{V^{2}}{g^{I}}
$$

The shape of the surfece and flow pattern in the vessel may be sufficiently affected by the gravitational field if the surface ia appreciably deformed frm the horizontal. Whem the temperature differenoes bring about differenoes in density, it is necessary to include buoyancy foroe. Thus we should matoh the Grashof number
(Gr) as well as the Re number.

$$
\frac{\text { Buoyancy force }}{\text { Inertial foroe }}=\frac{P g B \Delta T}{\rho V^{2} / L}=\frac{E B \Delta T L}{V^{2}}=\frac{\rho^{2}{ }_{g} B \Delta T L^{3}}{\eta^{2}} \cdot \frac{1}{R^{2}}=G r \frac{1}{R^{2}}
$$

so $\quad a r=\frac{\rho^{2} g B \Delta T L^{8}}{\eta^{2}}$
where $P$ is the denaity, $g$ is acceleration due to gravity, $\beta$ the coofficient of thermal expansion, $\Delta T$ the temperature differenoe, I the diameter of the stirrer, and $V$ is the speed of stirrere A model meets similarity oriteria for flow patterns when Re, Fr, Gr, have the same numericel values in both sjostems. If it is desired to matoh diffusion as well as flow it is necesanry to matoh Sohnidt number (So).

$$
\text { So }=\frac{\text { diffusivity of momentum }}{\text { diffusivity of mass }}=\frac{\nu}{D}=\frac{\eta}{\rho D}
$$

The general relationship for similarity may be writton as

$$
(\text { Re, Fr, Gr }, \text { Sc... })=0
$$

In practice one has to examine the process to try to determine what forces oontrol the behaviour and henoe what dimensionless mumbers it is most important to matoh.

In a complex system it may not be possible to match all the parameters that might be important. In such cases one has to matoh as well as possible those thought most important. Only experimenta. In boti systems can oonfirm whether matohing was good enough to use the model to prediot behaviour of the real system.

A ereat deal of experimental work, on mixing of visoous Ifquids, has been done. Spenoer and Wiley ${ }^{11}$ reported that as the turbulanco oan not play a part in mixing of visoous liquids, the mixer must continuously movs the Ifquid in a complex foshion to disperse the components to the greatest degree possible. The
mixing is most effioient when the stirrer induces flow in the liquid in direction at right ancles to the striation containing the inhomogeneity. If rapid mixing of viscous liquids is required, the method of deforming the material beoomes important. $A$ flow pattern is desirable that moves fluid readily into all parts of the vessel. If there is fluid in any part of the vessel whioh is not easily moved to the other parts of the vessel, then the fluid in that is not easily deformed and mixed with the rest of the solution. Such zones of slow movements are of ten found in a viscous liquid becnuse visoous drag quickly consumes the kinetio energy of a moving viscous liquid. Among the methods used atroom temperature to deter:zine the homogeneity of solutions are oolour, density, electrio conduotivity and ohomical analysis.

In their model work, Bradford ${ }^{32}$, Howse 33 and Tunaloy ${ }^{34}$ assumed that the only parameters that it was ossential to matoh would be Re and So numbers. But not knowing the diffusivity in glass, they corld not match So number and therefore matoher only Re number.

Bradford ${ }^{32}$ started with a known distribution of glycerine solutions and mixed these with a rod or disc stirrer. The mixing was studied by measuring the eleotrioal conductivity of the solution. He ooncluded that the diso stirrer was superior to the rod. This was because the disc induood much gronter radial and vertioal components of flow in the solution than the rod. Howse ${ }^{33}$ tried three different stirrers, a diso, a oombination stirrer and a crank stirrer. He concludod that oombination of the orank and diso stirrers proved to be a more officient mixer than
the disc as investigated by Bradford. He suggested that the idenl stirrer should induce tangentinl, vertical and radial shear in equal amounts into the solution. The disc stirrer certainly induces tangential and radial shear and some vertical shear. The effect of the combination stirrer is to disrupt the dead spot produced in the centre of the toroid resulting from the disc part of the stirrer but it does not inorease the vertical flow in the solution. Wo increase the vortionl flovi Tunaley ${ }^{34}$ used a helical screw stirrer. He reported that the screw satisfied the requiroment of well developad vortioal flow although use of a draught tube would incroase this aven further.

In all these experiments, done in this dopartment the homogeneity of the solution was expressed quentitatively in terms of two parameters, $I$, the intensity of segregntion and $\bar{\delta}$, the mean normalised clectrical conductivity for any number of revolutions of the stirring device used. The derivation of these val.ues involvod the use of a rindom sampling technique. The voluns for the intensity of sefregntion were shown to be a measure of heterogeneities within the main matrix, whilst values for the mean normalised conductivity gave information about the progress of diffusion in reduoirg the true intensities of the striations.
$\mathrm{C} H \mathrm{~A} \mathrm{P}$ T E R I
THE PRESENTMORK
Tho projoct was designed to study the homogenisation of glesses using different types of stimers. To compare the efficiency of different mechaniccl stirrers, some estimation of degree of homofencity is necessary. A qualitative assessment can bo obtained by observing the flow patterme induced by stirrers. Differenoe in flow patterns set up by various stirrers aid in explaining the obsorved differoncos in mixing time. In model experiments, flow patterns con be studiod by tnking photographs. Figure 19 shows some photogrophs of flow patterns induced by different stirrers in nodel experiments. Theso photographs wore taken of'ter equal number of rovolutions, using two glycerino solutions of the same viscosity and density, one of which is ooloured blue. Photograph No. 1 shows the simple f'low pattorns generatel by a rotating disc in the surface of the liquil. The layer below the diso rotates as the diso trensfers power to it by friction and is also moved away from under the diso by contrifugal foron. The motion is ccmpensated by liquid moving up under the contral part of the diso and taking the plnoe of the liquid thrown out to the sido. This means shat there exist three velooity components, in the radial dirootion, $r$, the ciroumforential dirootion, $\varnothing$, and tho axinl direction, 2 , which can be denoted respeotively by $u, v, w$. in axonomotric representation of this flow is show in the Figure 20. However, it should be mentioned that the flow pattern show here is for a diso rotating in an infinite body of liquid. When stirring

PLAIN DISC


VISCOSITY OF MODEL 7.9 Polse
REVOLUTIONS 200

FLOW DUE TO ROTATING LIQUID

| $\square$ |
| :--- |
| $\square$ |
|  |

FLOW DUE TO ROTATING DISC IN STATIONARY LIQUID


FIG. 22
FLOW GENERATED BY PLAIN DISC


PATH OF INDIVIDUAL PARTICLE

FIG. 23

## GEOMETRIC SIMILARITY



Ifquid in a vessel where the diameter and depth of the body of fluid are not very large compared with the diameter of the diso, the flow noor the disc will diffor from that show here. When a. body of fluid rotates over a flat plane a rather similar kind of flow pattern develops (see Figure 21). By considering these two flows, the condition of oontinuity and the boundnry conditions one con build up a qualitatively acaurate picture of the flow to be observed in a finite vessel stirred with a rotating diso inmersed in the top surface of the liquid. (sce Figure 22).

To study the mixing process in gloss, two different composition glasses, coloured and colvurless were mixed in 0 Rhft crucible. The size and shape of the crucible and of the model are given diagramatically in Figure 23.

## (i) Motching of Reynolds numbor

If the glosses have the same viscosity and density, Re oan be natohed as follows.

$$
\operatorname{Re}=\frac{D^{2} N P}{\eta}
$$

where $D$ is the diamotor of stirror, N the speed, $O$ the density of the liquid and 17 the viscosity.

$$
\text { For Elass, } \operatorname{Re}_{1}=\frac{(2 \cdot 6)^{2} \times N_{1} \times 2 \cdot 3}{100}=0.155 N_{1}
$$

For the model using Elycerine at room temperaturo,

$$
R_{\theta_{2}}=\frac{4^{2} \times N_{2} \times 1.25}{15}=1.33 \mathrm{~N}_{2}
$$

If $R e_{1}=\operatorname{Re}_{2}$ ono must hrve

$$
N_{1} / N_{2}=\frac{1.33}{0.155}=8.56
$$

In this oose, if the speed is 20 r.p.m. in the glass the speed of
stirrer in the model should be $2 \cdot 35 \mathrm{r} . \mathrm{p} . \mathrm{It}$.
If the two giasses havo viscosities $\eta_{1}$ and $\eta_{2}$ and densities $P_{1}$ and $P_{2}$, the position is more complicated. For $Q$, we can probably assume a linear relation betwoen $\rho$ and composition and therefore simply use the avorage density. It is rather unlikely that the relation between $\gamma$ and composition is linoar and in this case simply usine the average value may not be satisfactory.

Sc number could not be matched, beoause the diffusivity in glass was not known.

## (ii) Choice of experinental conditions

The initinl orientation was selected to be horizontal as in the room temperoture model, beoause usunlly tho inhomogeneities lic in parollol horizontel layers ir a cruoible of cinss unstirred by convection or meohenical stirrer. One layer of glass was put on the top of the other, so ns to start with the worst possibie distribution of the two phases. If botch is melted, the distribution of the inhonogeneities cannot be reproduced accurately from one experiment to another and it is very difficult to defino the situation for which $I=1$; for this reason a known distribution of previously melted giasses was preferred bocnuse this allows $I=1$ to be defined.

The flow patterns genorated were studied by taking shadow photographs and ordinary photographs of the thin sections of glass. From the ordinary photographs one can assess the mixing by observing the colour distribution. The shadow patterns give details of the flow of the glass induced by the atirrers beoouse of the tendency for inhonogeneitios to beoome aligned parallel to
the direotion of flow.
(iii) Determination of intensity of seererntion

To measure it quentitatively, the degree of homogeneity was defined in terms of $a$ dimensionless parameter, $I$, the intensity of sagregntion. To assess, I, for any particulor sample one needs information about any convonient property at a number of randon points. So in this investigation, it was decided to use chemical corrosion as the criterion.

Following Danckwerts ${ }^{15}$ the intensity of segregation, which varies from one in a fully sogregated solution to zero in a fully mixed solution, is defined as follows.

$$
I=\frac{S^{2}-S^{2} R}{S^{2} \max }=\frac{\sum(a-\bar{a})^{2}-\sum(a-\bar{a})^{2}}{N}
$$

where

$$
\begin{aligned}
& a=\text { volunc fraction of } A \text { in oach sample } \\
& \bar{a}=\text { mean value of } A \text { in ecoh set } \\
& n^{\prime}=\text { fraction of } A \text { in whole system } \\
& S^{2}=\text { measured variance of a } \\
& S_{R}^{2}=\text { variance of homogeneous solution } \\
& N=\text { number of observations } \\
& \text { In this investigation depth of etohing, } x, \text { is measured }
\end{aligned}
$$ instead of 凡. If wo assume a linoar rolation betwoon a and $x$, (which is permissible for these experiments)

$$
\begin{aligned}
& a=A^{\prime}+m\left(x-x^{\prime}\right) \quad \text { (see Figure 24) } \\
& m=\frac{d a}{d x}=\frac{1}{x_{A}-x_{B}}
\end{aligned}
$$

where $x_{1}=$ depth of otching of glass $\Lambda$

FIG. 24
ASSUMED RELATION BETWEEN COMPOSITION \& DEPTH OF ETCHING


DEPTH OF ETCHING, $X$

$$
x_{B}=\text { depth of etching of glass } B
$$

Unless mixing is very poor, cuusing sempling to be unrepresentative,

$$
\left(\bar{a}-a^{\prime}\right) \rightarrow 0
$$

so the.t we mey write.

$$
(a-\bar{a}) \equiv\left(a-a^{\prime}\right)=n\left(x-x^{\prime}\right)
$$

hence $I=n^{2}\left[\frac{\sum\left(x-x^{*}\right)^{2}-\sum\left(x-x^{\prime}\right)^{2} p}{N_{n}^{\prime}\left(1-a^{2}\right)}\right]$
or $\quad I=\frac{\sum\left(x-x^{\prime}\right)^{2}-\Sigma\left(x-x^{\prime}\right)^{2} R}{N\left(x_{A}-x_{B}\right)^{2} \cdot 0^{\prime}\left(1-\Omega^{\prime}\right)}$.
If the two glasses mixed together are oqual in proportion by volume, then $a^{\prime}=\frac{1}{2}$
$\therefore I=\frac{S^{2}-S^{2} R}{\frac{1}{4}\left(x_{A}-x_{B}\right)}$
(iv) Size of sample (sccle of exenination)

In the previous chapter it hos been alrendy mentionod that the best size of sample to take depends upon the sizes of the inhomofeneities present. It should not be much largor or much samnlior than the inhomogeneities. For the present investigation the sample size may bo estimated as follows.

The dopth, 0 , of the ctch pit is mensured in terms of the wovelength of sodium light by counting the number of fringes between $A$ (at the top edge) and $B$ (at the bottom) of the etch pit. (seo Figure 25). This distnnoe, b, obviously roprosonts the vertioal dinension of the somplo size and is obtained by mensurement between two points a horizontal distance e apart which ropresents another dinonsion.

Since intensity is defined in terms of volume fractions the sample size should also be defined as a volumo and this

FIG. 25

## SCALE OF EXAMINATION


requires definition of another distanoe o. Since the minimum distance apart at which it would generally be useful to make two different mensurenents would be one fringe, this gives a rinimum somple size, $c_{1}$. (see Figure 25). In fact, with a good interferometer mensurements to about $1 / 10$ fringe oan be made, so this represents a quite realistic sample size. It would be possible to mensure depth of otching by tracing the deviation of one particular fringe as it passes aoross the odge of the etch pit and in this case $c=c_{2}=n \times o_{1}$, where $n$ is the number of fringes. This gives a much larger sample size but this is not relavent here because this method of measuring dopth of otching was not used.

For a typical onso
$a=130$ microns
$b=16$ wavelengths $=n \lambda_{n}=9 \cdot 4 \mu$
$c_{1}=$ Fringe separation $=190 \mu$
$0_{2}=$ Total horizontal deviation of 1 fringe $=3040 \mu$
Thus the minirum sanple size is abc $1=130 \times 9 \cdot 4 \times 190=230000 \mu^{3}$ and the 1 neger sample size is $a b c_{2}=130 \times 9 \cdot 4 \times 3040=3714880 \mu$ We propose to assume that sample size is about $200,000 \mu^{3}$, which is the volume of a cube of side about $60 \mu$.

## (v) Choico of stirrers

The disc (Fieure 26 a) was chosen as the basio stirre: since the flow patterns generated by it in a liquid aro simple and symmetrical and the flow oan easily be interpreted, at least qualitatively, from dnta alrondy available. In addition, by covering a linge fraction of the surface of the melt, it can be

FIG. 26

## STIRRERS

(a)

## PLAIN DISC


(b)

DISC WITH BLADE

(c) COMPLEX DISC

expected to decronse volatilisation from the melt. Thus, all other stirrers were compared with the disc and in this work they were all based on the disc. The two chosen, after studying the resulta obtained by Howse ${ }^{33}$ and Tunaley ${ }^{34}$, were disc with blade and complex disc.

The disc with blade (Figure 26,b) would eppear to be useful in a stratified system such as tho horizontal distribution studied. This stirrer was originally designed to disturb the relotively stagnont ring exieting in tho flow developed by the simple disc. ilso, if the end of the blade is below the liquidliquid interfooe, it would be expected that mixing would be faster than for the disc beoause of the disturbanoe of the interface by the blade.

The complex disc (Figure 26.c) would only be suitable if the lowor disc lies above the liquid-liquid intorfaos. If the two disos are in two differont liquids moh of the mixing will be due to the disturbance of the liquid-liquid interface by the supports. Beocuse both discs are fixed relative to ench other the liquid between them will not be subjeot to the steop velocity Eradients necessary to attenuate the cords very much. In general, tho complex disc may only be suitable for mixing a thin layer of one liquid into the bulk of another.

The stirrors were made of $p t-R n$ alloy and were drivem by a small. motor fixed above the furnace.

$$
\mathrm{C} H \ldots \mathrm{P} \text { T E R V }
$$

THEMAINEXPERIMENTALWORK 1 APPAR.TUS AND METHOD (i) Design of the furnace

The furnace used was of the silit rod type normally used in the aepartment and shown diagrantionlly in the Figure 27. The inner tube was fitted with a refractory stopper, with a large hole in it, so that stirrer shaft could pass through it. The stirrers were driven by an olectric motor fitted on an adjustable platform, to change the depth of stirrer in the glass melt.

The whole apparatus is shown in the photograph ( $F H_{\text {gure }} 27 \mathrm{fa}$ ) The platinum rhodium cruoible was enclosed in a refractory shield fitted with a refractory lid with a large hole for the atirrers.

## (ii) Temperature measurement and oontrol

The furnace temperature wis recorded by one $\mathrm{Pt} / \mathrm{Pt}-\mathrm{Rh}$ thermocouple and controlled by another, both were projected through the rafraotory ring so as to reach near the level of the rofrootory shield. In the beginning the temperature was oontrolled by a cambridge regulator, cutting rosistance in and out of the oircuit after a fixed voltage had been tapped from a transformer, but later on this was roplaced by an Ether temperature oontrolline unit (Iype 1292, Ether Tryristor Unit Type 2620/50/250). The furnace temperature was set up by inserting a standard thermocouple through the stopper and lid into the centre of the refraotory shield. The e.m.f. of this thermocouple was measured by a potentiometer and the controller setting adjusted until the standard thermocouple indicated the roquired temperature (usualiy $1400^{\circ} \mathrm{C}$ ).

## FIG. 27

## CROSS SECTION OF FURNACE




FIG. $27 a$

Temperature distribution vertionlly down the f'urnace is show in the Figure 26. A slight variation of $2^{\circ} \mathrm{C}$ in temperature was found over the depth of the crucible.

## (iii) Glass composition

Since the determination of honogenoity by etoking with HF depends on the composition of the glass, two glasses were seleoted which gave a difference in the depth of etching of about 16 fringes. The relation betweun composition and the depth of etching acoording to Bower is shown in the Figure 29. These glasses also had different densitios and visoosities. In one of the glasses, Cobalt oxide was also added to identify it and also to observe the flow patterns generated by the stirrers. During the enrly strges, it was thought that differeroo in densities of these Elassos had a very inportant effect, so from the data alreddy available two other elasses expocted to have the same densities werc also tried. But later experiments showed that the densities were not equal and the data in the literature are therofore not roliable. The compositions of these glasses are given in the Table II.

## (iv) Bntch matorials

The main batoh matericls used were sodium carbonate, oaloium carbonate and sand. illl tho sand usod was taken irom a singlo batch and was ball milled, sieved, washed in acid and water, dried a.t $110^{\circ} \mathrm{C}$ and stored in screw oapped bottles. Two kilogran portions of sand were milled for about two hours and sieved through a set of cylindrical rotary sieves. The sieves available


and tho sizes of sand grains that was possible to obtain are Given in the Table III. The sand grain size of 0.15 mm was used in these investigations.

The other materials, $\mathrm{Na}_{2} \mathrm{Ca}_{3}$ and $\mathrm{CaCo}_{3}$ were analytionl reagents and were dried at $110^{\circ} \mathrm{C}$ before use. is they were considerably finer than the sand used, they were not sieved before use.
(v) Melting procedure

Each batch, to make 90 gms . of glass, was weighed out separately. The batch was mixed thoroughly with a spatula. dll the batch was not put into tho crucible at the same time but two fillings had to be used because of the risk of the melts boiling over. The bntches were meltad at $1450^{\circ} \mathrm{C}$ for four hours. One hour after putting the batch in the furnnce, the molts were stirred for ton minutes at $100 \mathrm{r} . \mathrm{p} . \mathrm{m}$. and then the stirrers were stopped. ifter another one hour again the melts were stirred but only for five minutes at the same speed. ifter four hours melting, the crucible wos withdrawn and the glass was taken out after cooling the cruciblo to room temperature, it was then annealed in a muffle furnaoe.

## 2 MIXING OF TWO GLISSES

After annealing, the glasses were cut horizuntally intu two equal portions. Then the two equal halves of the glasses $A$ and $B$ were put into the crucible. The initial orientation of the interface was horizontal in elll experiments but in some sets the coloured Class was on the top and in others it was at the bottom. The
crucible was then put in a pot furnace at about $650^{\circ} \mathrm{C}$ and the temperature was raised up to $1000^{\circ} \mathrm{C}$. The viscosity of the glasses was so high at this temperaturo that very little flow oould ocour during preheating. The crucible was then transferred to the main furnaoe end the glasses were stirred for the required time and speed. Ifter stirring the crucible was withdrawn as soon as possible (after about two minutes) and after cooling to room temperature, the giass was taken out in one piece withcut disturbing the flow patterns and annealed.

## 3 PRPPARUTION OF SimpLE

ifter annealing the glass was cut vortioally to obtain a thin sootion ( $2-3 \mathrm{~mm}$ thick) aoross one dianeter. The faces were then ground flat on a grindine wheei, using coarse carborundurs powder, followed by finer Erades. The surfaces were finely cround by rubbing the sample on a flat glass plate using sira as abrasive. Polishing was done on a polishing wheel using jowellers rouge. In this way a fint well polished surfa.oe of the sample was obtained. To etch the specimen at twenty randonly distributed points, a mask was prepared, as shown in the Flgure 30, using Kodnk neohanical orthofilm. A Kodnk photosensitive resist, KPCR, was coated on the surfice of the specimen thor allowed to stand at roon temperature for a few minutes to allow most of the solvont to evaporate. It was then dried in an oven for five minutes at about $60^{\circ} \mathrm{C}$. The photographic negntive mask, was plaood in contact with the resist conted on the sample and exposed to a
 at a distance of two feet. The purpose of exposure was to

FIG. 30
SAMPLING PATTERN

polymerise or harden the selected nreas of the resist layer not covered by opaque parts of the mask. These arons were insoluble in the developer and therefore remaned on the surfiace of the gloss, to forn the chemicallyresistont stencil. The unexposed areas of the resist were dissolved using $P C$ developer for about two ainutes at $20^{\circ} \mathrm{C}$ and washed in water. After development the resist image was dried in an oven at $80^{\circ} \mathrm{C}$ for five minutes. Now the specimen was ready for etching.

## 4 ETCHING AND INTERFEROMETERY

The surface wes etched in a $2 \% \mathrm{HF}$ solution at $20^{\circ} \mathrm{C}$ for ten minutes in a polythene tub containing $H \mathrm{H}$ and placed in a water bath. The temperature was adjusted automntically using a taulene thermoregrlator. The HF solution was stirred by a polythene stirrer. The upparatus is shown in the figure 3:. The etched surfina, of'ter washing and drying was silvered in a vapour plating appartus. The npparatus used to produce interforenoe fringes is also shown in the Figure 32. S diagramatio description of this apparatus is given in the FIgure 33. The monochromatic light souroe was $n$ sodium lamp mounted on an option benoh. $\Lambda$ glass strip one certimeter wide mounted on an adjustable stand at an angle of $45^{\circ}$ was used to defleot light on to the microscope stnge. This strip was muntod between the sampie and the microscope objective, so that part of the light beam, coming from sodium lamp, was reflected on to the sample. Part of the light reflected back from the samolo passed through the glass strip into the mioroscape objeotive. The glass semple was mountod on an adjustable microscope stage with otched surfaco facing the objective.



FIG. 33


SCHEMATIC DIAGRAM OF INTERFEROMETER

A microscope slide cover previously silvered to still transmit a rousonable proportion of light wns then placed over the area to be exarined. A specicl eye piece with orosswires and fitted with micrometer was used to count the number of fringes. Photographs of the interferenco fringes were also takon using a Pentax roflex camern nounted above the microscope eye-pieoe. The total magnification was 100, 10 times ench 'rom the objective and the eye-piece. The depth of etchine wos mensured in one place on three sides of every pit giving $a$ total of about sixty readings.

## 5 C.LCULATIONS OF RESULTS

Taking as a spocimen calculation the results obtained for 400 revolutions (elasses 1 and 2) of the disc stirrer, the following readings were obtained on the interferometer. Number of fringes $=15.15 .16,15.15 .15,19.17 .17,19.19 .18$, 19.17.17, $15.15 .15 .15,14.15 .16,15.15 .15 .15,14.14 .14,19.18 .20 .19,18.18 .16$, $20.20 .24,16.16 .16,13.13,10.10,12.13 .12,15.15,14.16 .17,20.19 .19$.

| $\pm$ | $\underline{x}$ | $\Delta x$ | fox | $(\Delta x)^{8}$ | $\underline{\Sigma g(\Delta x)^{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 10 | $-5$ | -10 | 25 | 50 |
| 2 | 12 | -3 | -6 | 9 | 18 |
| 3 | 13 | -2 | -6 | 4 | 12 |
| 5 | 14 | $-1$ | $-5$ | 1 | 5 |
| 16 | 15 | - | - | - | - |
| 9 | 16 | 1 | 9 | 1 | 9 |
| 6 | 17 | 2 | 12 | 4 | 24 |
| 4 | 18 | 3 | 12 | 9 | 36 |
| 10 | 19 | 4 | 40 | 16 | 160 |
| $\underline{4}$ | 20 | 5 | 20 | 25 | 100 |
| 61 |  |  | 66 |  | 414 |

True mean depth of etching $=15+\frac{66}{61}=16.08$
$S^{2}$, mariance $=\frac{414}{61}-\left(\frac{66}{61}\right)^{2}=5.52$
The meen depths of etching for the individual glossas were $x_{A}=26, x_{B}=10$, giving on expected mean of 18 fringes if the relation between composition and depth were exactly linear.

The intensity of seregation, I was therefore ooloulated as
$I=\frac{1}{\left(x_{A}-x_{B}\right)^{2} \cdot \frac{5}{4}}\left(S^{2}-S_{R}^{2}\right)$
where $S_{R}^{2}$ is the variance for fully mixed ginsses. These giasses were stirred at $100 \mathrm{r} . \mathrm{p} . \mathrm{m}$. for one hour. After one hour those glasses were again stirred for half an hour at 100 r.p.m. and the crucible was then taken out of the furnace and cooled to roon temperature. The variance $S_{R}^{2}$ was then calculated as usual, which was equal to 0.07 .

So intensity $I=\frac{1}{(26-10)^{2} \times \frac{1}{4}}(5.52-0.07)$
or $I=0.085$
Although the information was not usualyy usod, it may be noted that this experiment allows one to see how the inhomogeneities are distributed. For example, whether the major inhomageneity lay on the top or bottom of the sample.

TORQUF MELSUREMENTS AT ROOM TEMPERATURE
Torque measurenents were made in a simple apparatus used for viscosity measurement by tre Margules method. The stirrer was frmersed in golden syrup, a known weight applied to the scale pans and the time to oomplete six revclutions noted (Figure 34). The form of the relation betweon torque and angular velocity helped to interpret the observed flow patterns. This is disoussed lator.

FIG. 34
PLOT OF THE APPLIED WEIGHT VS THE RECIPROCAL OF TIME NEEDED TO COMPLETE 6 REVOLUTIONS


$$
\begin{aligned}
& \mathrm{C} \quad \mathrm{H} \& \quad \mathrm{P} \quad \mathrm{~T} \quad \mathrm{E} R \quad \mathrm{~V} \quad \mathrm{I} \\
& \text { RESULTSAND AISCUSSION }
\end{aligned}
$$

## ROTITING DISC

The disc is one of the simplest possible mechanical stirrer for which the flow can ensily be interpreted at least, qualitatively. The velocity components can be calculated for a disc rotating in the surface of an infinite fluid and for an infinite fluid rotating over a plane wall. In suitable oonditions these results would be expeoted to agree well with nctual flows in the centrol region of the crucible.

Consider a disc rotating in a large volume of a fluid. The layers near the disc are carried around through friction and are thrown outwards owing to the action of contrifugal forses. In the oase of a fluid rotating over a plane, there is a similar effect but its sign is reversed, the particles whioh rotate at a large distance from the plane are in equilibrium under the influence of the centrifugal force and the radial pressure gradient. The peripheral velocity of the particles near the plane is reduoed and so the centrifugnl force is decreased but the radial pressure Gradient remains the same and flow spiralling inwards towards the axis of rotation is produced. These cases of flow, a flow near a rotating disc and rotation near a plane, in large bodies of fluid were mentioned earlier and diagrams of both shown.(see Figures 20, 21). According to $L$. Prandti ${ }^{35}$ the boundnry layer thickness, defined as the distnnce from the plane or diso at which periphern velooity is $\%$ different from the velocity of the body of fluid or the disc, respectively, for these oc.ses is
(i) roteting diso $=\sigma_{1}=4 \sqrt{\frac{\nu}{\omega}}$
(ii) rotating fluid $=\sigma_{z}=8 \sqrt{\frac{\nu}{\omega}}$

The boundrry layer thickness in the lattor cass is twioe the thickness of the former case. Thus one would expeot the flow near the axis in a finite vessel to agree with this simple flow only if the depth of melt exoerded $\sigma_{1}+\delta_{2}$ and the dismeter of the vessel was much greater than dinmeter of the disc. It should also be pointed out that this analysis applies, in prinoiple only to a very large disc $(\mathrm{R} \rightarrow \infty)$ and in practice is likely to agree accurately with measurements only if $R>\delta_{1}$. If $R<\sigma_{1}$ it seems that the flow pattern will be similar but not fully developed. On putting in appropriate values for $\eta \& \rho$ for glass and a suitable volue for w one gets the totol boundary layer thickness $\left(\sigma_{1}+\sigma_{2}\right)$ equal to $47 \cdot 85 \mathrm{cms}$. The values used were $\mathrm{w}=\frac{2 \pi}{3}, \eta=80$ poise, So the dopth of melt and the radius of the disc should exceed $47 \cdot 85 \mathrm{cms}$, if very close agreement between experinent and theory is to be obtained near the axis of the vessel when depth and radius are much less, then radial and axiol flows will be grently suppressed oompored with rotational flow about the axis.

Evidence supportine this was obtained by measuring the dependence of angulnr velocity of torque npplied in room jemperature model if golden syrup having viscosity of 418 poise at $26^{\circ} \mathrm{C}$.

For the disc rotnting over an infinite fluid
$M=0.616 \pi \rho_{R^{4}}\left(\nu \omega^{3}\right)^{\frac{1}{2}}$
For creeping rotational flow (no radial or axial components)
$M=\frac{1}{2} \pi \eta \frac{\omega}{z}$.
where $z$ is the depth of the layer of liquid.

Figure 34 shows that the results agree with the second equation. However, observation of the flow did show the definite presence of radial and axial flows but these were, as suggestod, very much slower than the predictions from the diso rotating in an infinite fluid. One important implication of this is that the exact flow pattern might be sensitive to Re number. inother importent effect will be that the disk will not mix as efficiently as expocted from the theoretion model. One feature of the simple theory that the experiments do agree with is the position of the stagnation points on the vertical section about which the fluid rotates under axial and radial flows. The theoretioal modol predicts $\delta_{1}=\frac{1}{2} \delta_{2}$ and observation of room temperature models shows that the stagnation points do lie about $1 / 3$ the depth below the disk.

The results obcained for the model do not agree with the results obtained for the elass. One obvious reason for this is that Re number is not mntched in both systems. The proper matching of Re requires

or $\quad \frac{(2 \cdot 6)^{2} \times 20 \times 2 \cdot 3}{100}=\frac{(4)^{2} \times N_{2} \times 1 \cdot 25}{15}$
or $\quad N_{2}=2.35$ r.p.m.
This menns that room tomperature model to matoh glass should have been stirred at 2.35 r.p.m. The original model experiments used 60 r.p.m. and the radial and axiel flow was more developed. Nevertheless the differense observed between glass and room tempernture model sannot be explained only by difference in Re number.

This is becouse the model considered the mixing of equal volumes of liquid of the same viscosity and density. Thereas in the experiments with flass the donsity and viscosity of both components are different. In the earlier model experiments, no signifionnce was attached to interface stroility. Latcr work has shown that the interface stability is important and is a function of viscosity and height ratio.

Coments on the flows sean in the different experiments with flassos follow. (ill the results were obtained for the stirrers with glasses 1 and 2 at 20 r.p.m., unless otherwise stated.)

## 1 DISC STIRPER

(i) Denser erinss on top

The flow patterins and ordinary photographs ore show in the Figure 35.

50 revs:- Fron the shndow pattern it is clear that the denser gloss is folling to the botton under the influence of gravity although the axial symmetry suggests that stirrer is having some influenoe.

100 revs:- There is a sairly large region of denser (colourless) glass net bottom, which is not mixed nt ail. There is a very thin filn of coloured glass, which wes oricinally on the bottom, left undernenth it.

200 revs:- isfter 200 revolutions the flow pattern is daveloping fairly symmetrically, under the influence of the disc but still. with 0 lorge volume of unmixod densor glass at the bottom. The dark central "tail" of lishter (coloured) glass is rising right
$\qquad$
DISC STIPRPER

PHOTOGRAPH
"井
50
REVS.

SHADOW PATTERN
$\sqrt{(-18)+1}+$

from the bottom in the ontre as the thin layer of coloured glass is slowly boing drawn up into the main circulation. 400 revs:- The ordinary photograph shows little of original colourless or very dnrk elass. There is an indication of a ring of blue glass around the rin which was very marked at 200 revolutions. This is presumably the less dense glass adhering to the orucible w?lls rising towards the surfonce but why it takes up this porticular form is not clear. General flow pattern is sinilar to those of 200 revolutions but the central toil is less distinct.

800 revs:- ifter 800 revolutions vory symmetrical flow pattern ann be seen, showing two unexpected fentures; a diatinct ring of blue glass around the edge just above the contre ama a cish shaped ring of blue floss in the botton hif of the melt. The first of these looks as if it might reprosent in interfaco between tho two separato circulations, one above and one below. It also seems tanat this interfince could be the natural evolution of the rather similar fecture soen at 200 revs and 400 revs (left hand side).

Variation of depth of etching with position no longer shows distinct segregntion of denser glass at bottom.

1200 revs:- The shadori pattern shows very well developod rotation about the axis but vertionl circulation is not well
developed. There is a distinct sien of a rine around the odge already mentioned, though much less obvious than that seom at 200 and 800 revolutions. No sharply defined fine dotail is left (as compared with $50-400$ revolutions) and this shows that diffusion has rently reduced the true intensity of tho amall scole inhomogeneities.

a plot of intensity (I) against time (Figure 48) shows steady progress of homogenising end photographs also suggest the same trend. However the distinct decrense in intensity between 100 and 400 revolutions is not imnediately obvious by cye, showing the visusl obsorvation is not always reliable.

Flow patterns are not at s.ll like the room temperature model. It is becruse the Re matching is not exact and also due to the difference of density and viscosity. Possibly flow undar difference of donsities, before stirring began, may have had some influence (o.g. difference between 400 and 800 revolutions).
(ii) Denser class on bottom

To soe if density difference was importnnt the above series of experiments was repeated with the same glasses but the other way up. The shndow patterns and photographs are shown in Fisure 36.

50 revs:- There is hardly any mixing. The main effect of stirring is that a ring of densor glass is drawn up into the lighter near the odge of the disc.

200 revs:- i.gain there is very little mixing although the ring seen at 50 revolutions seems more disporsed and rising gas bubbles are also producing some mixdne. 400 revs:- The eeneral appearnnce is not very different from 200 revolutions but it begins to show some development of rotation about the nxis. A large volune of original colourless glass is still unnixed.

800 revs:- $\therefore$ distinot development of radial and vertion

FIG. 36 DISC STIRRER

## PHOTOGRAPH

SHADOW PATTERN


50
REVS.


400


800


1000

ciroulation can bo seon near the top but not in the bottom where there is still 2 large volune of colourless glass not mixed at all.

1000 revs:- It seems that flow is following the evolution of flow pattern seen at 800 revolutions. The vertionl ciroulation is fairly well developed just below the disc but still there is a large region at the bottom, where radial and vertical circulation is small.

1200 revs:- in unmixed lnyer at the bottom shows very little mixing but it is much thinner than soen at 800 and 1000 revolutions. There is a very curious flow pattern in the relatively well mixed region which does nct seen to be an obvious development or the flow pattern at 800 and 1000 revolutions.

Both photngraphs and the plot of intensity, I, against tine (see Figure 48) clenrly show that the density differenoe is very important. The mixing is much better when the denser glass was originally on top, producing more complex flow thon with denser on the bottom. The flow patterns ore very different from the room temperature model of equal density and viscosity in both cases.

Results at 1200 revolutions suggest further progress in mixing thon indionted by results at 800 and 1000 revolutions, this might be due to rither smaller depth of the melt or different depth of inmersion of the disc. The interface originally present betwean the two glasses seems to be stable but convection probably 0ooalarates mass transfer across it.

## 2 DISC STIRRER (GLLSSES 3 iND 4)

is it is evident from the previous results that density has on important effect, another pair of glasses was selected fron the scenty evidence availnble which were expected to differ much loss in density whilst still differing in viscosity. The flow patterns and photographs are given in Figure 37.

## (i) Denser on bottom

50 revs:- There is little mixing, a dome shaped region is drawn up undor the disc and there is a horizontal region between the layers in which some mixing has occurred but most of the denser Elass remans undisturbed.

200 revs:- The interfince between the Inyers is still stable and distinct. The mixinf above the interfnce has proceoded beyond that at 50 revolutions. There is $a$ slight indication of radial and vertion circulation above the interface.

1200 revs:- The interface betweon the layers is still stable across most of the seotion but it is disturbed beneath the stirrer and this region presumably represents the evolution of the central dome seen at 50 revolutions. Examinstion of the depth of etching rusults shows that, dospito the stability of the interfoce, the bottor layer has chenged appreoinbly in composition and this is confirmed by the oonsiderably reduction in $I$ (in this melt $I=0.039$ ).

The flow pattern at 200 revolutions looks rather similar to that at 200 revolutions in the previous experiment (glasses 1 and 2) but less disturbed by gas bubbles.

Tho interface between the layers still seems to be stable but flow patterns aro not idontical with previous exporiments. Mixing

FIG. 37 DISC STIRRER
DENSER ON BOTTOM


FIG. 38 DISC STIRRER
DENSER ON TOP
GLASSES $3 \& 4$

is faster in this pair of elasses than the other in the same arrangement (sce Figure 48).
(ii) Nerser on top (see figure 38)

100 revs:- This clearly shows irregular flow (eood for mixing) due to density differences. The denser glass is falling down under the influence of gravity and corresponding streams of lighter glass rising.

400 revs:- Flow system shows a layer of coloured (denser) glass on the botton but the bulk of the melt already contains a significnnt proportion of denser glass (becouse it is distinctly blue). The interface seems to be stablo and mixinis above it is fairly good (no fine detail in shadow pritern). 800 rovs:- $A$ thin lnyer on the bottom and around tho sides and the indication of a central column sugeest some vertical and rodial circulation.

Mixine again is faster than when the denser gloss is placed on botton (Figure 49) and is also fastor than for the other pair of glasses (giassos 1 and 2) with the same arrangement (densor on cop). The sucller density difference may aid mixing of glasses 3 and 4 but differences in viscosity and cffeotive diffusivity may have played some part.

## 3 DISC WITH BLUDE (GL.SSES 1 2ND 2)

(i) Denser on top

The shndow pattorns and photographs are shown in Figure 39. 200 revs:- The denser (colourless) glass has flowed towards bottom in rather irregular way but thore is an indiontion of upturnod ring

FIG. 49

around outside above half way down which might owe its existance to the action of the stirrer.

400 revs:- The shadow pattern looks like an obvious developnent of that at 200 revolutions. The photograph indiontes more mixing thnn at 200 revolutions and an unmixed layer is still on the bottom but considernbly thinner. Both melts at, 200 and 400 revolutions show that gas bubbles have attenuated sone inhomogeneities,

600 rovs:- in very thin lnyer of coloured glass still adheres to the bottom. The gas bubbles ore many fewer than at 400 revolutions. From the colour pattern it is evident that mixing is not good (very fine detail is still seen in the shodow pottern) but mixing is nevertheless obviously better than at 400 revolutions.

800 revs:- Nenrly all gns bubbles are gone there is an indication of stable interface on the shadow pattern exoept where it is disturbed by the tip of the blade. The verticol circulation is not well developed. The shodow pattorn does nut show as mach fine detail as is seen for 600 revolutions. Some almost colourless glass still lies near the bottom.

1200 revs:- The colourless glass is still not completely mixed but a thin layer can still be seen on the botton and around tho sides. There is some suggestion of development of vertical ciroulation.

1500 revs:- Mixing has obviously progressed further, the colour pattern now shows vury little difference in oolour. The shadow pattern was made more contrasty thn the others and did in foot show less detail than at 1200 rovolutions. The rathor curious

DISC WITH BLADE<br>DENSER ON TOP

PHOTOGRAPH


1200


FIG. 40

# DISC WITH BLADE 

DENSER ON BOTTOM

PHOTOGRAPH



pattern seen suggests further development of vertical and radial fiow.

The comparison of flow patterns, espocially at 400, 800 and 1200 sevolutions suggests rather different flow from that produced by plain disc, mixing bcing bettor in the regions trovelled by the blade.

Graphs of I against time (Figure 52) howover shows not much differenoe between stirrers in this cose. This may be because the blode suppresses further developmont of vertioal and radial flow.

## (ii) Denser on botton (see Figure 40)

100 rova:- The interfoce scems very little disturbed. There is very little indication of development of flow of stirror. 400 rovs:- The colourless glass is not completely segregated, the whole section being blue. in upturned ring onn be seen in the shodow pattorn, which is apparently a part of the original Interfnce. There is an indication of vertical and radial flow just below and outside the stirrer.

800 revs:- The colour patterns suggost that mixing is not better than for 400 rovolutions. The shadow patterns are of rather similar type but the layer below the interface is much thinner and the interfaco flattor. The central column drawn up is also smallor but intensity is nevertholess lower at 800 than at 400 revolutions (soe Fiture 50).

1000 revs:- The shadow pattern does not show any very fine dotril. The flow soems to be mainly rotation about the axis. $\therefore$ ring can be seen drawn up from the base near the corners.

1200 revs:- Very little dotail is visible in shedow pattern exoept near the centre at top. (disturbance on withdrawing the stirrer). A. ring is again rising from the base nenr the corners. The flow still scems to be almost entiruly rotation about the vertical axis. The relation between $\log$ I and time is very smooth with Iittle scatter (Figuro 50). Comparison with same instial distribution stirred by diso shows very marked differences. Mixing in this case is much better with blade, probably beoause its tip disturbs the interfnce. One result, that for 800 revolutions, does not exactly fit the general trend. This might be due to slightly different distance of tip of blade from the interface.

4 COMPLEX DISC
(i) Denser on botton (soe Figure 41)

100 rovs-- A thin layer of denser glass remains on the bottom. The flow is not fully developod. A oentral region of lighter (dark) glass on the bottom is vary distinct. Considerable attenuation by ís bubbles is seen.

400 revs:- $\therefore$ layer of denser gings is still on the bottom. is "tail" of denser glass is probably drawn up through the hole in lower disc and bent outward forming the shape like a handle. 800 revs:- Now the mixing of the bottom Inyor is beginning but flow is not well devcloped.

1000 revs:- The original interface has not completely disappenred but not puch of it remains. The rotational flow about the axis is beginning to look well dovelopad.

FIG. 4 I

## COMPLEX DISC STIRRER

DENSER ON BOTTOM


FIG. 51


1500 revs:- There is no distinct remnant of either layer on botton or original interfince. Verticrl and radial flow is teginning to develop but there is still a roughly lens shaped resion not well nixed, just nbove the position of lower disc.

Comparison with simple dise
The simple disc shows a strble interface at all times even up to 1200 revolutions for this initial arrongement. The complex disc stirrer disturbs and destroys the intorface giving much better mixing enrly in the process. But the rate of mixing with the complex disc soon decreases and extrapolation of results suggests that plain disc would give rather better mixing beyond about 1700 revolutions. (seo Figure 53).

This is probably dae to decreasod shen of the relatively large volume of liquid held in the spnoe between the two discs which tends to rotato between them with little axial or radiai flow.

Comparison with dise with blade
The disc with blade disturbs the interface somewhat, but less than oomplex disc atirrer. Up to about 500 revolutions ereater disturbence of the interfnoo with the complox diso gives better mixing. Beyond 500 revolutions the disu with blade is botter even though tho interfice is less disturbed. This presumably happens bootuse of larger volocity eradients and more attenuation with disc plus blado.

Tho comparison of flow patterns is shown in Figure 42.

# COMPARISON OF FLOW PATTERN 

DISC


DISC WITH BLADE



(i) Disc stirrer (denser on bottom)
A. comparison was nade between several melts stirred for the sane number of revolutions (1200) at several speeds. The shndow patterns and photocraphs aro shown in the Figura 43. 20 r.p.m. $(60$ min):- $\therefore$ thin layer at the bottom shows very little mixing but it is much thinner than the original layer. There is a very curious fllow pattern in the relatively well mixed region.
40 r.pol. $(30 \mathrm{~min}):-$ The interface is steblo but mixing is apprecicble.

60 r.p.m. $(20 \mathrm{~min})$ :- Tho interface is much more disturbed thouch it is not entirely destroyed. There is hardly any denser ginss which is not mixed at ell. The flow seems to be affected by gas bubbles.

100 r.pol. 12 min ):- The interfface still looks stablo but only a very thin Inyer or hardly mixad colourless elass is left. The time is too short for diffusion to reduce the actual intensity of mostettenuated cords. Cas bubbles agnin seom to be playing a part in attenunting cord. 120 r.p.m. $(10 \mathrm{~min})$ :- The interface is probnbly not stable. Only i very thin layer of denser elass is left on the bottom. Tho Eas $^{\text {as }}$ hubblos have distorted the loyers consilerably. Agnin the time is too short for much diffusion to occur and much fairly fine dotail is seon on the shadow pattorn.

The graph (soe Figuru 55) shows a stendy deorense in I as speod increasos sugeesting that incrensing attenintion because of
EFFECT OF SPEED

## DISC STIRRER



60


FIG. 55

EFFECT OF SPEED

$$
\begin{aligned}
\text { DISC } & \text { STIRRER } \\
& \text { (I2OO REVS.) }
\end{aligned}
$$

1) increasine disturbance of interface
2) the possibility that vertion and radinl flows become nore developod,
is the most importent feature in this experiment.
(ii) Disc with blado (denser on top)
inother similar experiment was made with the disc with blade stirrer, stirring for 2000 revolutions when the denser Elass was placed on top. The shadow pattern and photoeraphs are shown in Figure 44.

40 r.D.n. (50 min):- The flow pattern saen in the photograph is very curious. Some unmixed dense floss lies very near to the botton.

60 r.p.m. $(33 \mathrm{~min}):-$ The interfoce sэems to be stable and is drawn up towards the diso.

80r.p.m. ( 25 min ):- Central coluan is drawn up which could be further dovelopment of fllow seen for 60 and $40 \mathrm{r} \cdot \mathrm{p} \cdot \mathrm{m}$. 100 ropor. 20 min ):- The stable interfooce is drown up towards the disc but tho bottom Iayer is much thinner than seen for $60 \mathrm{r} . \mathrm{p} \cdot \mathrm{m}$.

Tho rosults do not foll into a simple intelligible pattorn and there aro two obvious roasons:-

1) The denser glasg put on top wight give different effectivo distributions at moment when stirrine began.
2) A possible effeot of depth of immersion of stirrer.

Unfortunately theso experiments were done before effect of density was fully ronlized and no further information likely to help intarprot them is available.




80


80


## (iii) For constrnt timo ( 60 minutes) (soe Figure 45)

40 r.p.me ( 2400 ruvs) :- is hole in the middle of layer is very curious. A thin dome shape is aeen drawn up towards the stirrer. 60 r.p.m. ( 3600 revs):- The vertionl and radinl oiroulation is beeinning to develop. A thiok layer of unmixed ginss is at the bottom. A very thin done shape was seen drawn up towards the stirrer at 40 r.p.m. and a similar feature ocours here but its upper port shows evidonce of vertioal and radial flow. 80 r.p.n. $(54,00$ revs $):-$ is central oolumn is drawn upwards. A very thin layer onn bo seen on the bottom.

100 r.p.m. $(6000$ revs $):-$ The flow pattern looks like further development of that seen at 60 r.p.m.

## 6 EFFECT OF STIRRING.

$\therefore$ fow flasses were melted from batch and stirred with disa at 20 r.p.al. for different lengths of time. The flow patterns and mixing aro compared with glosses not stirred, but melted for the same length of time. The flow pattorns of stirrod and unstirred melts are given in the Figure 46.

## (i) Unstirred melts

1 hour:- Vortionl nttonuntion of cords onn bo seon which is probably due to Ens bubblos.

2 hours:- A complex flow pattorn has been devoloped probably the flow is undor the influenoo of density difforenoes. If the flow were mainly coused by thermal convootion a more regular oiroulation would probably bo seen.

4hourg:- A alight bulk inhomogonoity can bo soen. A layer presumbly densar on tho botton is very distinct.

$$
\begin{aligned}
& \text { FIG. } 46 \\
& \text { COMPARISON OF STIRRED \& UNSTIRRED } \\
& \text { MELTS }
\end{aligned}
$$

UNSTIRRED




16

STIRRED


2


4


FIG. 57


8 hours:- The buik inhomogeneity is not serious. A fairly thick layer still lies on the bottom; it looks thicker but more diffuse than at 4 hours.

16 hours:- The lnyer on the bottom is still thero. This is relatively thick but becoming much more diffused. No cords are seen in the rust of the melt.
(ii) Stirred molts

1 hour:- This is much more homogeneaus than the unstirred melt at 1 hour. In interface has appenred half way down around the edge and there is evidence of well developed rutation about the axis but little sign of vertical or radiol flow.

2 hours:- From the shadow pattern this seems to show frirly good mixing. There is hardly ony fine detail visible but the features that are seen sugeest vertionl and radial flow as well as rotation sbout the axis.

4 hours:- The interface nenr the walls is unexpected. It might be a lator stage of that seen at one hour but does not seem to follow from the pattern seen at one hour.

8 hours:- The flow pattern is very like that at four hours but e distinot layer is seen on the bottom and this is considerably thicker thon the earlier results suggest.

16 hours:- Tho curious interfince soon nt 1,4 and 8 hours stirring is absent. A distinct layer thinner than that nt 8 hours is still seen. This rosult could be the natural succossor to that obtained at 8 hours. The stirred melts are תlways more homogeneous than the unstirred (soo Figuro 56). The differenoe is very obvious early in tho prooess on tho shadow patterns. The formation and

FIG. 56


TIME (HOURS)
persistenoo of tho thin layer on the botton with the simple disc was not cxpected. To removo this a better stirrer is needod. It is not clonr whether the presonce of the thin bottom layer in the stirred melts resulted from a slow sottling under gravity or to accidentrol differences in effective initisl distribution in the different melts.

## 7 EFFECT OF TIME ON MMOUNT OF DIFFUSION

$\therefore$ few experiments were done in which the melt was left in the furnnce for half an hour after stirring stopped. 4 comparison of theso results with these romoved from the furnace as soon as stirrine ceased is shown in Figure 47. Tho glasses used wore 1 and 2 with 0 verticol interface in the initial distribution. The stirrer was tho plain disc. 200 revs:- iffer 200 ruvolutions the flow pattern is not well doveloped. The shadow pattern of the melt removed at once shows very finc detail. Aftor leaving for half an hour a considereble amount of fine detail has disrpperred and the flow pattern appers to result from local differences in density. 400 revs:- Then removed as soon as stirring had stopped $a$ fairly well dovelopod flow pattern is seen.

After leaving for on additional half hour two importent effocts are seen.

1) There is a distinct tendenoy to flow givang horizontal layors but theso lajers aro deformod in some plaoes by rising gns bubbles.
2) Disappearance of much fine detail presumably beoause in half an hour $\frac{D t}{g^{2}}$ appronches or exceeds unity.

$$
\text { FIG. } 47
$$



THE EFFECT OF LE,$V I N G$ IN THE FURNN.CE SFTER STIRRING HCD STOPPED
Both pairs of shadow patterns show sienificant difference in general flow after lecving for half hour. This suggests that better mixing might possibly be obtained by several intermittent periods of stirring then by continuous stirring.

The disappearance of fine detail on laving half hour shows that this time is sufficient for diffusion to greatly raduoe the concentration differences between adjacent thin layars of cord. Considering a simple static model, a parallel sided slab of thickness $2 k$, the condition for diffusion to be almost completo is

$$
\frac{D t}{\delta^{2}}=1 \cdot 5
$$

This Eives reduction of maximum concentration difference, at the centro of the slab, to about $3 \%$ of the initial value.

Mensurements made on the shodow pattern of the melt stirred for 400 revs and removed at onee shows that the finer layers seen were actually about 0.08 cm thiok. It oan therefore be seen that these layers would be offectively removad by diffusion alone in another half hour if the diffusivity were given by

$$
D \geqslant \frac{1 \cdot 56^{2}}{t}=\frac{1 \cdot 5 \times 16 \times 10^{-4}}{1800}=1 \cdot 4 \times 10^{-6} \mathrm{~cm}^{2}{ }_{\mathrm{Soc}}-1
$$

No ocournte values of effective diffusivity of these glesses are available but Becker ${ }^{36}$ estimated a diffusivity of about $6 \times 10^{-8} \mathrm{~cm}^{2}$ yoe $^{-1}$ for the mixine of two sola-dine-silice glasses ut $1400^{\circ} \mathrm{C}$. If this diffusivity applied to the present glasses, half lour would give $\mathrm{Dt} / \mathrm{b}^{2} \simeq 6 \times 10^{-8} \times 1.8 \times 10^{3} / 1.6 \times 10^{-3}=0.07$ which would be just suffioient to affect the maximum concentration difference. If flow reduced $\delta$ by a faotor of two or moro after stirring
stopped; then half hour would clearly be sufficient time for the fine detaill to become considerably more diffuse on the shedow Fattern. If the true effective diffusivity were somewhat higher the complete disoppearance of fine detail in holf hour would be expected when some attenuation also happened.

USE OF THE ETCHING METHOD TO ME ISURE HOMOGENEITY
The figure 54 shows the results obtained for two different pairs of glasses, curves is and $B$ for one pair, $C$ and $D$ for the other. In curves $A$ and $C$ the denser glass was on the bottom and in ourves $B$ and $D$ the denser glass was on the top. The curves show a stendy improvement in homogeneity with time in overy case and that one can distinguish ensily the difference in the rate of homogensing between the two pairs of glasses and also the greater efficiency of homogeuisation when glasses flow under buoyanoy forces at the beginning. Fron these results one can see that values of I caloujated from the otching data have a quantitative signifionnoe in these experiments both for following improvement in honogeneity with time and for distinguishing different behaviaur.

Similarly the effioiency of different types of stirrers can be compared when they are used under identical conditions an shown in Figure 52.

When lograrithm of intensity of segregation, $I$, is plotted against tine the results nearly always lie on a smooth curve and there is a very small soatter, showing that the measurements are reasonably preaise. In sore onses duplicate experiments were performed. For example in one oase intensity, $I$, was oalculnted to be $1.1 \times 10^{-2}$ and $1.6 \times 10^{-2}$ in two different experiments.


Though the difference between these results is about $40 \%$, when $I$ is represented on a Iogarithmic scale the difference becomes very srall and this again confirms that the procedure used to calculate I gave sufficiently precise results, at least in the earlier stages of mixing.

This technique also gives the position of inhomogeneities and one can say, for example, whether the inhonogeneitios lie on the bottom or in the centre of the melt. This information is not used when une only considers how $I$ veries but it is always available for use in interpreting flow patterns or for any other purpose.

There are many methods which might be used to determine the homogeneity of any system inaluding a glass melt. The etohing method developed in this work has been found to be very oonvenient. for the derivation of quantitative results and investigation of efficienoy of stirrers, effect of time and so on. However, it may not alwnys be the best method to use with giass.

Signifionnce of intensity of serregation
In determining the intensity of segregation, $I$, it is necessary to speoify the variance before mixing has taken plooe. This is very easy in the mixing of cnly two different phases. In this work the mixing was started with a know initial distribution of two glasses with strintion thickness much greater than sample siae so that the varianoe before mixing began was a maximam and this value was assumed to be 1. The other important assumption made was that there is a linear relation between depth of etching and composition in the region between the two glasses used.

In melting of batch it is not possible to state the initial distribution because more than two components are involved and neither composition nor quantities nor spatial distribution can be specified. This means that it is by no means easy to specify intensity, I, for freshly melted glass. This was the reason for starting with previously melted and homogenised glasses. The difficulty of defining the maximum variance and hence defining I does not affect the present work but it does complioate the extension of studies of this kind to the melting of batoh. However, in this crse, comparison of actual varinnoe would be useful when studying differenoes between melts made from similar. batches which can be assumed to produce the same maximum (but unknown) variance.

Actual glasses are usually multioomponent and equal depths of otching are given by a range of compositions not one specifio composition. In the present work the glasses cauld be chosen to Ita on a line almost perpendicular to the contours of equal depth of etching, so this was of little importance. In other systems, where regions of considerably different composition but almost the same depth of etching could exist, moasurements of this kind would not give a grood evaluation of homogencity. Mass transfer and both definition and mensurement of affective diffusivity are oomplex and difficult to evaluate in multioomponent systems whioh makes interpretation of results diffioult.

IINE:R RELATITON BETHEEN COMPOSITION SND DEPTH OF ETCHING
$\therefore$ small dopartura from a linear relation is not serious
if the line jointre the glass composition is almost perpendicular
to lines of equal depth of etching. If this angle is very different departures from linearity or the curvature of the actunl diffusion path could be serious nnd might give misleading values of intensity, $I$.

In sodn-lime-silion glasses, the dopth of etching is predominantly controlled by the silion content of the gloss, so that depth of etching is a more sensitive neasurement of variation in silice than in soda or lime content. In the present work the difference in depth of etching of the original two glasses was 16 fringes and intermediate values in tho parily mixed melts were recorded only to the neorest fringe making an error of up to $\frac{1}{2}$ fringe possible. Sinco the difference in silica content whs $5 \cdot 5 \%$, $\frac{1}{2}$ fringe difference would be equivalent to about $5 \cdot 5 / 32=0.17 \mathrm{SiO}_{2}$. Many glasses require thet difference in composition be much smaller than this but it would be possible to measure depth of etching much more nocurately and doteot differenoo in $\mathrm{SiO}_{2}$ content at least 10 times smnller by using a. high quality interforometer. The present method therefore oould. be made more precise.

Ifffler ${ }^{26}$, in his oricinal work on etching of glasses, showed that other reagents (e-g $\mathrm{HPO}_{3}, 10 \% \mathrm{Na} \mathrm{OH}$ oto. ) gave a different relation between depth of etohing and composition. It is thorefore possible to devermine compositions more precisoly if two etching experiments are done with different otohing solutions and this would aid investigations mado in multicomponent systems.

## SIZE OF THE S.MPLE

It has already been mentioned that the size of the individual somples required to give a relioble estinate of intensity, I, deponds upon the size of the inhonogeneities present. The sample sizo should bo no bigeer than tho actual size of inhomogeneities and is best rather smaller. inother problem is that if the size of the melt is increased, how should the size and number of the individual samples be increased. To gret suffioiently precise results it is necessary that the sum of individual samples represents a certain proportion of the whole body. If the total sample is too small and therefore not representative, the result is inaccurnte and not reproducible; if the total semple is unneoessarily big, the result is accurnte but too much work is involved and time is watad. In general it is difficult to specify beforchend how larco a fraction of the whole body must be token and this is best found by checkine the reproducibility.

If it were desired to soale up the present work to investigate the mixine of lareger melts one would hove to consider how to change the size of the etch pits and the number of etch pits.

Initially the striation thickness was considerably greater than the size of the otch pit so that in nenrly all casos the sovern meesuroments made on each pit would neosssarily give the same result and 20 otch pits would reclly give on'y 20 indopendenti values. However, in most cases average strintion chickness soan became approcinbly smaller than the size of the etch pit and eoh of the measurerents mado on one pit becomes an independent observation. Then 3 mensuroments were made on each pit this

Eives 60 indopencent nbservations nnd gives a more representative sample. Using larger etch pits would not affect the true scale of examination but would nake the measurements at different places on the same pit more independent of each other and this could be an adventage.

In scoling up it seems likely that the etoh pits should be lorger but when it is not at present known whother striation thickness would remain the same or also be sonled up in approximately the same ratio of the linear dimensions of the system, it is not clear how much larger they should be. It mny be suggested thet, to give rather better sampling of the body, they should be scaled up rather more than the linear soale of the melt. It is also likely that a larger number of etch pits should be used becnuse the volume of the system increases with the cube of linear dimensions. To make the total sample the same volume fraction would mean making the etch pits very much larger unless the number is considernbly incrensed. No simple rule can be given about how to change both number and size of etch pits.

Goodness of sampling and precision of intensity of
segregation, $I$, should be tested by finding the reproducibility.

## COMP:RISON OF FLOF IN MELTS . LND ROOM TEMPERATURE MODELS

The flow patterns cbserved in the gloss melts are completely different from those observed in the roon temperature model. This nay be in part due to the difference in Re number of the two systems which differ by a factor of 10. But of prinary importance are the density and viscosity of the two phases to be mixed. The models, which mixed liquids of equal density and viscosity, oan bear

Iittle resemblanoe to the class melts, which mixed liquids of different density and viscosity. If the denser liquid is on the bottom mixing obviously nore difficult as was shown by experiments with glasses, see Figures 52 and 53. However, even if the densities ore identucal but the viscosities different, the two liquida may not mix. When the viscosities are difforent, the interface between the layers may be deformed but remin stable Givine very inefficient mixing. Yih ${ }^{37}$ has shown that even for a stratified liquid flowing over a plane surfince the interface may or may not be stable even at very low Re. He also showed that the ratios of the visoosities and the thicknesses of the two Inyers have a very important influence on stability of interface. Recent experiments by Moore ${ }^{38}$ have confirmed that density and viscosity differences affect mixing. In the ciosed system used, flow pattern is much affectod by speed of stirrer. Figure 58 shows how the stability of interfoce varies with speed of stirring for oparticular viscosity ratio. Theme is a partimar critical speed that is needed to make the liquid-liquid interface unstable, at lower speeds mixing is by diffusion aided by convection and at hioher speed by attenuntion and diffusion which is much nore rapid.

## EFFICIENCY OF STTRRERS

The efficiency of a stirrer ofton depends unon the intiol distribution. This is clearly shown by the dnta in Flguro 52 in wioh there is not much ifference been the two differnnt stirrers when the donser finss is plnced on the top but a big difference whon the dunser flins is on the bottom. The beat

## FIG. 58



VISCOSITY RATIO, $3 / 2$

TOTAL REVOLUTIONS $=100$ DISC STIRRER

stirrer to use in nny particular systom therefore must depend on the properties and initial distributiun. The stirrers studied were not very officient mixine devices but they wero originally chosen to produce simple flow patterns (in ax system of equal density and viscosity) which would aid understending of tho experimental results, rather than for maxinum efficiency. The stirrers used all left a thin layer of glass of different composition at the botton of the crucible but the nevessury oondition that velocity is zera in contaot with the metal wall mokes mixing of such a layer diffiouli in any case.

The diso is not an efficient mixer but it is a convenient standard for comparison of the performance of other stirrers.

The addition of $a$ blade to the diso breaks up the interface and inoreases the mixing efficiency particularly when the denser glass is on the bottom, but the homogeneity obtained is not of opticnl qunlity.

The complex disc stirrer is only efficient when the lower disc is above the liquid-Iiquid interfoce. This stirrer can be used to remove the thin layer on the bottom of the melt, rather quicker then either of the others but does not mix the bulk of the Iiquid so well.

## MEN DEPMH OF ETCHING

The originnl honogenised glasses 1 and 2 were otohed in the standard oonditions, that is for ten minutes at $20^{\circ} \mathrm{C}$, and gave dopth of etching 10 and 26 fringes respectively, ifter mixing the two glosses for difforant number of revolutions, for ono set
of experiments all the specimen were etched together at the same time. Differences in menn depth of etching in any set thorefore could not be due to differences in etching conditions. But ow the different sots were not etched together, they could differ slightly from oach other. This can happen with a very slieht differenco of experimental conditions, for examile a $\frac{1}{2}{ }^{4} C$ difference in temperature can vary the depth of etohine by three to four fringes, small differenoes in time of etching have much less effect.

The depth of etching onn also vary for individual samples if the photoresist is not properly washed away from the etch pits but care wos taken in preparine the seotions to ovoid this. Differences in morn depth of etching in any one set of experimenta night be due to smpling orrors or a non-lineer relation between depth of etching and composition. The former would be randon but the latter would probnbly change stendily as mixing improved.

To examine the way in which dopth of otching varied the results for each set of melts were troated as if mean depth wro to be used for a control chort and the probrbility of the various values being statistically significant estimated from the ranges. Sone of the results are shown in Toble XIV. Out of 44 results, 14 lie outside the linit that the probability of the differences is aignificant is $5 \%$ and 6 beyond the $0.1 \%$ probability lindt. Tho dita do not sugeest a general trend of mean depth with inoreased mixing and therefore do not suggest distinot non-linear relation betweon composition and depth of etohing. The number of observations beyond the $5 \%$ or $0.1 \%$ probnbility limits suggests
that the total semple taken in oach melt was not quite large onough to be truly reprosentative. The variation in grand means between sets of results shows some small difference in otching conditions.

## GONCLUSIONS

Most of the detailed conclusions drawn in the ass of exporinents with different stirrers, development of etching technique and different flow patterns have already been given at the approprinte places in the text. Figures 48 to 53 show the main results of the behaviour of different types of stirrers. However, sone inportont conclusions drawn during the present work con be sumnerised here.

1) The etchine and interferometery technique developed in this work gives a useful quantitative description of homogeneity, and information about the position and intensity of the cord. It is however, too lencthy a method to uso in industry for the daily or more frequent estimntion of the homogeneity of glass. However, any tochnique suitable for use when rates of production are 100 or more tons a day will be difficult to find.
2) The technique can be used to distinguish between systems that behove differently and to compare the effioiency of different types of stirrers.
3) The results obtained for intensity of segregntion and flow patterns showed that the behnviour of the glass melts was not sinilar to that of roow temparature systens. This might partiy be due to Reynolds number not being properly matched but was largely due to differences in viscosity and density of the two glasses used. 4) Of the three different stirrers used, the diso with blode is the best mixine device, especinlly when the denser gloss is put on the botton.
4) In molts that are not stirred the homogeneity iraprovas rapidly with tine up to $2-4$ hours melting and then more slowly
up to 8 hours. Beyond 8 hours, there is very IIttle further inprovement. Plots of I va tine were of rather similnr shape for melts which were stirred but the homogeneity of stirred melts was considerably better then of unstirrad melts.

## FURTHER TORRK

From the present work we know that density and viscosity have importent offects on the flow pettern and extensive work should be done on model nt room tempernture with different liquids of different properties as well as further experiments at higher temperature, to show that behaviour is sirailar when Reynolds number, visoosities and densities are properly matohed. To matoh these properly more accurnte values for density and viscosity of the flasses used will be necessary.

In this invastigation batoh was not sintored, but it is belleved that sintering the batch mitoricl before melting gives a nore homogerioous produot then one in which the raw botoh is melted dirootly and so this should be investigated in future.

In these experinents 90 of gloss was nelted. To see the effect of scoling up, experinents should be done with melts considerably larger scale, for exnnple about 1500 ems.

When very honogeneous elasses cre noeded the etching and interferometry method may not be sufficiently sensitive and other tochniques of measuring intensity of segregation may need to $\mathrm{b}_{\mathrm{e}}$ developed.

$$
T \therefore B L \mathbb{I}
$$

THE ETHECT OF CHLNGE IN GLASS COMPOSITION ON DEISITY OF A CONTAINER GLiSS

|  | Substituting one flass forming oxide for anothor |  |  |  | Change in Density |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{SiO}_{2}$ | $\mathrm{Na}_{2} \mathrm{O}$ | CaO | $\mathrm{AL}_{2} \mathrm{O}_{3}$ | g/ce |
| $+1 \% \mathrm{SiO}_{2}$ | - | -0.0075 | -0.00127 | -0.0043 | - 0.0085 |
| $+1 \% \mathrm{Na}_{2} \mathrm{O}$ | +0.0075 | - | -0.0053 | $+0.0033$ | +0.0065 |
| +1\% Cr 0 | $+0.0127$ | $+0.0053$ | - | $+0.0085$ | $+0.0120$ |
| $+1 \% \mathrm{SI}_{2} \mathrm{O}_{3}$ | $+0.0043$ | -0.0033 | -0.0085 | - | +0.0020 |


|  | TABLE I I |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | COAPOSITIONS OF GLASSES USED (wt. \%) |  |  |  |
| Glo.ss | $\mathrm{SiO}_{2}$ | $\mathrm{Na}_{2} \mathrm{O}$ | Ca 0 | CoO |
| 1 | $73 \cdot 50$ | $16 \cdot 50$ | $10 \cdot 0$ | $0 \cdot 025 \%$ of total batch |
| 2 | $68 \cdot 0$ | $17 \cdot 0$ | $15 \cdot 0$ | - |
| 3 | $70 \cdot 0$ | $10 \cdot 0$ | $20 \cdot 0$ | $0.025 \%$ of total brtoh |
| 4 | $72 \cdot 25$ | $15 \cdot 15$ | $12 \cdot 15$ | - |

$$
T \angle B L E \quad I I I
$$

B.S.Mush No. Aperture mand San size ma

| 25 | 0.599 | 0.510 |
| ---: | :--- | :--- |
| 36 | 0.422 | 0.337 |
| 60 | 0.251 | 0.220 |
| 85 | 0.178 | 0.150 |
| 120 | 0.124 | 0.114 |
| 150 | 0.104 | 0.196 |
| 170 | 0.089 | 0.071 |
| 240 | 0.066 |  |

FIG. 59
TYPICAL ETCH PITS


## $A \quad P \quad P \quad E \quad N \quad D \quad X$

## INDIVIDUALEXPERIMENTAL DATA

In each set of experiments, the mean depth of otching, variance and intensity of segregation was calculated separately. The deta for each set is siven in tables IV to XIII.

$$
\begin{aligned}
& \text { T } \angle \mathrm{BLE} \quad \mathrm{IV} \\
& \text { GLLSSES } 1 \text { AND } 2
\end{aligned}
$$

$$
\begin{gathered}
\text { Initicl Distribution - Denser gless on the top } \\
\text { Stirrer - Plain Disc } \\
\text { Speed - } 20 \text { r.p.n. }
\end{gathered}
$$

| Depth of etchin; in fringes | Trequency |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50 | 103 | 200 | 400 | 800 | 1200 |
|  | revs | revs | rovs | revs | revs | rovs |
| 10 | - | 5 | - | 2 | - | - |
| 11 | 7 | - | - | - | - | - |
| 12 | 15 | 7 | - | 2 | - | - |
| 13 | 3 | 3 | 6 | 3 | 2 | - |
| 14 | - | 7 | 5 | 5 | 7 | - |
| 15 | - | 5 | 10 | 16 | 16 | 33 |
| 16 | - | 5 | 7 | 9 | 11 | 13 |
| 17 | - | 2 | 5 | 6 | 13 | 7 |
| 18 | 1 | 1 | 2 | - | 7 | - |
| 19 | - | 2 | 5 | 10 | - | - |
| 20 | 1 | 7 | 6 | 4 | - | - |
| 21 | 1 | 3 | 1 | - | - | - |
| 22 | 2 | 1 | 4 | - | - | - |
| 23 | 3 | 1 | 2 | - | - | - |
| 24 | 7 | - | 3 | - | - | - |
| 25 | 2 | - | - | - | - | - |
| 26 | - | - | - | - | - | - |
| Menn Depth | $17 \cdot 05$ | $15 \cdot 54$ | $17 \cdot 0$ | $16 \cdot 08$ | $15 \cdot 84$ | $15 \cdot 50$ |
| Varianoe | $32 \cdot 1$ | $14 \cdot 40$ | $11 \cdot 44^{4}$ | $5 \cdot 52$ | $1 \cdot 8$ | 0.52 |
| Intensity | $0 \cdot 47$ | $0 \cdot 21$ | $0 \cdot 16$ | 0.079 | $0 \cdot 020$ | 0.0043 |


|  |  |  | L V |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | GLiLSS | 1 IND 2 |  |  |  |
| Init | 1. Dist | bution | enser | ass on | bottom |  |
|  |  | Stirrer | Plain Di |  |  |  |
|  |  | Speed | $0 \mathrm{r} . \mathrm{p}$. |  |  |  |
|  |  |  |  | quency |  |  |
| Dopth of etching in fringes | $\begin{aligned} & 50 \\ & \text { rovs } \end{aligned}$ | $\begin{aligned} & 200 \\ & \text { revs. } \end{aligned}$ | $\begin{aligned} & 400 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 800 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1000 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1200 \\ & \text { revs } \end{aligned}$ |
| 10 | 10 | - | - | - | - | - |
| 11 | 12 | - | - | - | - | - |
| 12 | 8 | 10 | - | - | - | - |
| 13 | 7 | 23 | 12 | - | - | $\cdots$ |
| 14 | - | 2 | 13 | 13 | 14 | - |
| 15 | - | - | - | - | - | 15 |
| 16 | - | 4 | 12 | 10 | 8 | - |
| 17 | - | - | - | 6 | 9 | - |
| 18 | - | - | - | 3 | 3 | 25 |
| 19 | - | - | 8 | - | - | - |
| 20 | - | - | 7 | 5 | 7 | 12. |
| 21 | - | - | - | 7 | 7 | - |
| 22 | - | 5. | - | - | - | - |
| 23 | 4 | - | - | - | - | - |
| 24 | 12 | - | - | - | - | 10 |
| 25 | 13 | 17 | 15 | 7 | 7 | - |
| 26 | - | - | - | - | - | $\cdots$ |
| Monn depth | $19 * 1$ | 16.5 | $17 \cdot 8$ | $17 \cdot 03$ | $18 \cdot 02$ | $18 \cdot 61$ |
| Voriance | 56.44 | $43 \cdot 23$ | $23 \cdot 97$ | $16 \cdot 67$ | $13 \cdot 7$ | $8 \cdot 7$ |
| Intensity | 0.85 | 0.65 | $0 \cdot 36$ | $0 \cdot 25$ | $0 \cdot 20$ | $0 \cdot 13$ |


|  |  | T A B | V |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | GLuisSES | $3 . N D$ |  |  |  |  |
| Ini | 1 Dis | bution - | nser | ss on top |  |  |  |
|  |  | Stirrer - | ain Di |  |  |  |  |
|  |  | Spoed - | $\mathrm{r} \cdot \mathrm{p}$. |  |  |  |  |
|  |  |  |  | Frequ |  |  |  |
| Depth of etching in fringes | $\begin{aligned} & 50 \\ & \text { rovs } \end{aligned}$ | $\begin{aligned} & 100 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 200 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 400 \\ & \text { revs } \end{aligned}$ | 800 <br> revs | $\begin{aligned} & 1000 \\ & \text { revs. } \end{aligned}$ | $\begin{aligned} & 1200 \\ & \text { revs. } \end{aligned}$ |
| 10 | - | - | - | - | - | - | - |
| 11 | - | 2 | - | - | - | - | - |
| 12 | - | 5 | - | 7 | 2 | - | - |
| 13 | - | 7 | - | 7. | - | - | - |
| 14 | - | 10 | - | 14 | 10 | - | - |
| 15 | - | 12 | - | 14 | 43 | - | - |
| 16 | - | 10 | - | 18 | 13 | - | - |
| 17 | - | 3 | - | 4 | - | - | - |
| 18 | - | 2 | - | 2 | - | - | - |
| 19 | - | 2 | - | - | - | - | - |
| 20 | - | 4 | - | - | $\cdots$ | - | - |
| 21. | - | 1 | - | - | - | - | - |
| Moan dopth | - | $16 \cdot 1$ | - | $14 \cdot 5$ | $15 \cdot 1$ | - | - |
| Varianoe | - | $5 \cdot 87$ | $\cdots$ | $2 \cdot 21$ | $0 \cdot 60$ | - | - |
| Intensity | - | 0.0835 | - | 0.029 | $0 \cdot 0053$ | $\cdots$ | - |

TABLEVII<br>GLISSES 3 IND 4<br>Initial Distribution - Denser glass on the bottom<br>Stirrer - Plain Disc<br>Speed - 20 r.p.m.

| Depth of etching in fringes | Frequenoy |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 50 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 100 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 200 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 400 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 800 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1000 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1200 \\ & \text { revs } \end{aligned}$ |
| 10 | 6 | - | 1 | - | - | - | - |
| 11 | 12 | - | 7 | - | $\cdots$ | - | - |
| 12 | 3 | - | 10 | - | - | - | 5 |
| 13 | 7 | - | 16 | - | - | - | 11 |
| 14 | 3 | - | 11 | - | - | - | 29 |
| 15 | - | - | 10 | - | - | - | 8 |
| 16 | 2 | - | 2 | - | - | - | 3 |
| 17 | 4 | - | 3 | - | - | - | 8 |
| 18 | 4 | - | 2. | - | - | - | 6 |
| 19 | 2 | - | 2 | - | - | - | 1 |
| 20 | 11 | - | - | - | - | - | - |
| 21 | 2 | - | - | - | - | - | - |
| 22 | 2 | - | - | - | - | - | - |
| Mern depth | $17 \cdot 0$ | - | $14 \cdot 1$ | - | - | - | $14 \cdot 7$ |
| Varianoe | $16 \cdot 60$ | - | $4 \cdot 0$ | - | - | - | $2 \cdot 86$ |
| Intensity | $0 \cdot 246$ | - | $0 \cdot 19$ | - | -- | - | 0.0394 |

## $T A B L E V I I I$

GLISSES 1 AND 2
Initial Distribution - Denser glass on the top

Stirrer - Disc with blado
Speod - 20 r.p.m.

| Depth of |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| etching in | 100 | 200 | 400 | 600 | 800 | 1000 | 1200 | 1500 |
| fringes | revs | revs | revs | revs | revs | revs | revs | revs |


| 10 | 2 | - | - | - | - | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 3 | - | - | - | - | - | - | - |
| 12 | 5 | 13 | - | - | - | - | - | - |
| 13 | 3 | - | 8 | - | - | - | - | - |
| 14 | - | 15 | 4 | 5 | - | - | - | - |
| 15 | - | 9 | 13 | 8 | 10 | - | 10 | - |
| 16 | 6 | - | 6 | 30 | 28 | - | 15 | 16 |
| 17 | 3 | 3 | 4 | 13 | 8 | - | 20 | 40 |
| 18 | 4 | 4 | 6 | 3 | 3 | 13 | 14 | - |
| 19 | - | 3 | 7 | 5 | 2 | - | - | - |
| 20 | 10 | 4 | 4 | - | 2 | 19 | - | - |
| 21 | 10 | 3 | - | - | - | 8 | - | - |
| 22 | - | - | 5 | - | - | 6 | - | - |
| 23 | - | - | - | - | - | - | - | - |
| 24 | - | - | - | - | - | - | - | - |
| 25 | - | - | - | - | - | - | - | - |
| 26 | - | - | - | - | - | - | - | - |

$\begin{array}{lllllllll}\text { Meon Depth } & 17 \cdot 13 & 15 \cdot 70 & 16 \cdot 66 & 16 \cdot 32 & 16 \cdot 36 & 19 \cdot 62 & 16 \cdot 71 & 16 \cdot 66\end{array}$ $\begin{array}{llllllllll}\text { Varianoe } & 13.78 & 13.31 & 7.31 & 1.74 & 1.47 & 1.39 & 1.04 & 0.23\end{array}$ $\begin{array}{llllllllll}\text { Intensity } & 0.204 & 0.179 & 0.107 & 0.026 & 0.020 & 0.018 & 0.012 & 0.0024\end{array}$

## Tin BLE IX <br> GLISSES 1 IND 2

Initial Distribution - Denser class on the bottom
Stirrer - Disc with blode
Speed - 20 r.j.m.

| Depth of etching in fringes | Frequency |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 100 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 400 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 800 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1000 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1200 \\ & \text { revs } \end{aligned}$ |
| 10 | - | - | - | - | - |
| 11 | 1 | - | - | - | - |
| 12 | 11 | - | - | - | - |
| 13 | 23 | 6 | - | - | - |
| 14 | 3 | 5 | 6 | - | - |
| 15 | - | 10 | 9 | - | - |
| 16 | 4 | 8 | 29 | - | - |
| 17 | - | 4 | 13 | - | 4 |
| 18 | 9 | 3 | 1 | 6 | 28 |
| 19 | - | 5 | - | 10 | 35 |
| 20 | 8 | 6 | - | 27 | - |
| 21 | - | - | - | 20 | - |
| 22 | - | 5 | - | - | - |
| 23 | - | 2 | - | - | - |
| 24 | - | 3 | - | - | - |
| 25 | 16 | - | - | - | - |
| 26 | - | - | - | - | - |
| Mean depth | $18 \cdot 19$ | $17 \cdot 0$ | $16 \cdot 80$ | $19 \cdot 97$ | $18 \cdot 46$ |
| Varienco | 43-1 | $11 \cdot 60$ | $1 \cdot 80$ | $0 \cdot 85$ | $0 \cdot 38$ |
| Intensity | $0 \cdot 62$ | $0 \cdot 17$ | $0 \cdot 027$ | 0.011 | 0.0046 |

## TABLEX

GLISSES 1 IND 2

> Initial Distribution - Denser glass on the bottom
> Stirrer - Complex Disc
> Speed - $20 \mathrm{r} \cdot \mathrm{p} \cdot \mathrm{m}$.

| Depth of etching in fringe | Frequency |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 100 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 400 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 800 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1000 \\ & \text { revs } \end{aligned}$ | $\begin{aligned} & 1500 \\ & \text { revs } \end{aligned}$ |
| 10 | 6 | $\cdots$ | - | - | $\cdots$ |
| 11 | 2 | - | - | - | - |
| 12 | 7 | - | - | - | - |
| 13 | 3 | - | - | - | $\cdots$ |
| 14 | - | - | - | - | - |
| 15 | - | - | - | - | - |
| 16 | - | - | - | - | - |
| 17 | 5 | 17 | 10 | 7 | 9 |
| 18 | 2 | 13 | 30 | 29 | 25 |
| 19 | 1 | 6 | - | - | - |
| 20 | 10 | 4 | 8 | 12 | 10 |
| 21 | 10 | 4 | - | - | - |
| 22 | 3 | - | - | - | - |
| 23 | 1 | - | - | - | $\cdots$ |
| 24 | - | 6 | 5 | - | - |
| 25 | - | 10 | 7 | 12 | 12 |
| 26 | - | - | - | - | - |
| Mean depth | 17•10 | $19 \cdot 71$ | $19 \cdot 50$ | 1970 | $19 \cdot 50$ |
| Variance | $13 \cdot 72$ | $9 \cdot 80$ | $7 \cdot 54$ | $7 \cdot 32$ | $7 \cdot 20$ |
| Intensity | $0 \cdot 20$ | $0 \cdot 14$ | $0 \cdot 11$ | 0.105 | C-10 |

> TABLE XI
> EPrFCCT OF SFEED
> (Constant Revolutions)

GLiASSES 1 IID 2
Initial distribution - Denser glass on the bottom
Total revolutions - 1200
Stirrer - Plain Disa

| Depth of etching in fringos | Frequency |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 20 \\ \text { r.p. } \end{gathered}$ | $\begin{gathered} 40 \\ \text { r.p.r. } \end{gathered}$ | $\begin{gathered} 60 \\ \text { r.p.m. } \end{gathered}$ | $\begin{aligned} & 100 \\ & \text { r.p.m. } \end{aligned}$ | $\begin{gathered} 120 \\ \text { r.p.m. } \end{gathered}$ |
| 10 | - | - | - | - | - |
| 11 | - | - | - | - | - |
| 12 | - | - | - | - | - |
| 13 | - | - | - | - | - |
| 14 | - | - | - | - | - |
| 15 | 15 | - | - | - | - |
| 16 | - | 5 | - | - | $\sim$ |
| 17 | - | 3 | 5 | - | - |
| 18 | 25 | 15 | 20 | - | - |
| 19 | - | 5 | 13 | - | - |
| 20 | 12 | 20 | 10 | 16 | 23 |
| 21 | - | - | - | 25 | 20 |
| 22 | - | - | 12 | 14 | 10 |
| 23 | - | - | - | - | - |
| 24 | 10 | 10 | - | - | - |
| 25 | - | - | - | 2 | - |
| 26 | - | - | * | - | - |
| Mean Depth | $18 \cdot 61$ | :9•7 | $19 \cdot 34$ | $2 \cdot 0$ | $20 \cdot 7$ |
| Vorianco | $8 \cdot 7$ | $5 \cdot 38$ | $2 \cdot 65$ | $1 \cdot 06$ | $0 \cdot 55$ |
| Intensity | $0 \cdot 13$ | $0 \cdot 080$ | $0 \cdot 040$ | $0 \cdot 015$ | $0 \cdot 0075$ |

```
    T&BLE XII
    EIFECT OF SPEID
    (constant tine)
    GL`SSES 1 AID 2
Initiol distribution - Denser glass on the top
            Stirror - Disc with blado
                Total time - one hour
```

| Depth of otchine | Frequenoy |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 20 | 40 | 60 | 80 | 100 |
| in fringes | r.p.n. | r.p.m. | r.p.rin. | r.p.m. | $r . p . m$. |
| 10 | - | - | - | - | $\cdots$ |
| 11 | - | - | - | - | - |
| 12 | - | - | - | $\cdots$ | - |
| 13 | - | - | - | - | - |
| 14 | - | - | 11 | - | - |
| 15 | 10 | - | 41 | - | - |
| 16 | 15 | 8 | - | 6 | 5 |
| 17 | 20 | 39 | - | 10 | 42 |
| 18 | 14 | 3 | - | 12 | 6 |
| 19 | - | - | - | 3 | - |
| 20 | - | - | - | - | - |
| 21 | - | - | - | - | - |
| 22 | - | 1 | - | - | - |
| 23 | - | - | - | - | - |
| 24 | - | - | - | - | - |
| 25 | - | 10 | 6 | - | - |
| 26 | - | - | - | - | - |
| Noan Depth | $16 \cdot 71$ | $18 \cdot 31$ | $15 \cdot 84$ | 17-38 | $17 \cdot 02$ |
| Varianoe | $1 \cdot 04$ | 9-37 | 9-48 | $0 \cdot 82$ | $0 \cdot 20$ |
| Intensity | $0 \cdot 014$ | $0 \cdot 140$ | $0 \cdot 141$ | 0.0112 | $0 \cdot 0019$ |

# TABLEXIII <br> EFTECT OF SFESD (Constant Rovolutions) 

GLISSES 1 .iND 2

> Initiol distribution - Denser glass on the top
> Stirrer - Diso with blade
> Total revolutions - 2000

| Depth of etohing in fringes | Frequency |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 20 | $40$ | $60$ | $80$ | $100$ |
|  | r.p.m. | r.p.m. | r.p.r. | r.p.m. |  |
| 10 | - | - | - | - | - |
| 11 | - | - | - | - | $\cdots$ |
| 12 | - | - | - | - | - |
| 13 | - | - | - | - | - |
| 14 | - | - | - | - | - |
| 15 | - | - | - | - | - |
| 16 | - | 8 | 7 | 10 | - |
| 17 | 50 | 39 | 37 | 42 | - |
| 18 | - | 3 | 11 | 2 | 5 |
| 19 | 8 | - | - | - | 30 |
| 20 | 6 | - | - | - | 20 |
| 21 | - | - | - | - | - |
| 22 | - | - | - | - | - |
| 23 | - | - | $\cdots$ | - | - |
| 24 | - | - | - | - | - |
| 25 | - | 11 | - | - | - |
| 26 | - | - | - | - | - |
| Mean Depth | $17 \cdot 48$ | $18 \cdot 36$ | $17 \cdot 07$ | $16 \cdot 89$ | $19 \cdot 27$ |
| Varianoe | 0.99 | 9•88 | $0 \cdot 32$ | $0 \cdot 18$ | $0 \cdot 40$ |
| Intensity | 0.0138 | $0 \cdot 147$ | $0 \cdot 0039$ | 0.0015 | $0 \cdot 0049$ |

## TABLEXIV

Revolutions | Nean Standard Group |
| :--- |
| Depth Deviation Mean |
| Standard Notea |
| Deviation |

| Ginsses - $1 \& 2$ | 50 | $17 \cdot 05$ | $5 \cdot 65$ |  |  | Outside 5\%Limit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Denser glass on | 100 | $15 \cdot 54$ | 3-94 | $16 \cdot 17$ | $2 \cdot 91$ |  |
| top | 200 | $17 \cdot 00$ | 3-51 |  |  | Outside 5\%_imit |
| Stirrer - | 400 | $16 \cdot 08$ | $2 \cdot 34$ |  |  | - |
| Ploin Disc | 800 | $15 \cdot 84$ | $1 \cdot 34$ |  |  |  |
|  | 1200 | $15 \cdot 50$ | $0 \cdot 72$ |  |  | Outsida 0-86init |
| Glasses - 1 d2 | 50 | $19 \cdot 10$ | 7-51 | $17 \cdot 84$ | 4-93 | Outside 5\% [imit |
| Denser glass on | 200 | 16.50 | $6 \cdot 50$ |  |  | Outside 5\% imit |
| botton | 400 | $17 \cdot 80$ | $4 \cdot 88$ |  |  | - |
| Stirror - | 800 | 17.03 | $4 \cdot 07$ |  |  | - |
| Plain Diso | 1000 | $18 \cdot 02$ | $3 \cdot 70$ |  |  | - |
|  | 1200 | $18 \cdot 61$ | $2 \cdot 94$ |  |  | - |
| Glasses - 1 \& 2 | 100 | 17-1.3 | 3-70 | $16 \cdot 89$ | $1 \cdot 98$ | - |
| Denser giass on | 200 | $15 \cdot 70$ | $3 \cdot 64$ |  |  | Outside Orilimit |
| top | 400 | $16 \cdot 66$ | $2 \cdot 70$ |  |  | - |
| Stirrer - Disawith blade | 600 | $16 \cdot 32$ | $1 \cdot 31$ |  |  | Jutside 5\%-imit |
|  | 800 | $16 \cdot 36$ | $1 \cdot 21$ |  |  | Outside 5\% linit |
|  | 1000 | $19 \cdot 62$ | $1 \cdot 79$ |  |  | Outside O 017 zimit |
|  | 1200 | $16 \cdot 71$ | $1 \cdot 01$ |  |  | - |
|  | 1500 | $16 \cdot 66$ | $0 \cdot 48$ |  |  | - |
| Glasses - 182 | 100 | $18 \cdot 19$ | $6 \cdot 56$ | $18 \cdot 10$ | $2 \cdot 57$ | - |
| Denser glass on | 400 | $17 \cdot 0$ | 3-40 |  |  | Ontsido 0.1\%]init |
| bottom | 800 | $16 \cdot 80$ | $1 \cdot 34$ |  |  | Outside Oothimit |
| Stirrer - Disa | 1000 | $19 \cdot 97$ | 0.92 |  |  | Outside 0.f\%isimit |
| with blade | 1200 | $18 \cdot 46$ | $0 \cdot 62$ |  |  | - |
| Glasses - 1 \& 2 | 100 | $17 \cdot 10$ | $3 \cdot 70$ | $19 \cdot 10$ | $2 \cdot 99$ | Outsido 5\% l imit |
| Denser on bottom | 400 | $19 \cdot 71$ | $3 \cdot 13$ |  |  | - |
| Complex Diso | 800 | $19 \cdot 50$ | $2 \cdot 74$ |  |  | - |
| stirrer | 1000 | $19 \cdot 70$ | $2 \cdot 70$ |  |  | - |
|  | 1500 | $19 \cdot 50$ | $2 \cdot 68$ |  |  | - |

$$
\begin{array}{llllllll}
\mathrm{R} & \mathrm{E} & \mathrm{E} & \mathrm{R} & \mathrm{~N} & \mathrm{C} & \mathrm{E}
\end{array}
$$

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