A Contextual Deterministic Stochastic Model For Quantum Mechanics

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Abstract

Since its conception, Quantum mechanics has provided description for the behaviour of the smallest parts of the universe, but has also introduced new, paradoxical, and unintuitive features to the scheme of physics. While quantum wave mechanics is well understood descriptively, the actual physical nature behind it has long been debated, and numerous different interpretations exist. In this thesis, we argue the merits of a contextual realist design over the traditional Copenhagen interpretation and demonstrate with a novel numerical model that the assumptions of the Copenhagen interpretation are unnecessary when quantum effects can be modelled through deterministic ensembles.

This model uses the general-purpose programming language Python to simulate a collection of particles which act deterministically while demonstrating a characteristic quantum interference effect. This is achieved by allowing deterministic interaction between each particle and the 'apparatus' of the virtual experiment. In this way, it is demonstrated that deterministic contextual behaviour is sufficient to explain certain quantum effects, and that it is unnecessary to assign indeterminacy or a dual-nature to individual objects.

Chapters 1 and 2 review some of the history and existing interpretations of quantum mechanics, and highlight which stems of thought can be adapted into a modern contextual quantum theory. Chapter 3 explores a novel numerical result related to Bell-test experiments, which are a cornerstone in examinations of quantum foundations. Chapters 4 and 5 more explicitly connect existing contextual ideas to those used in this thesis' new model, which is described in chapter 6. Finally chapter 7 compares the new model with

the known ‘Feynman Little Arrow’ notation.

Contents

Chapter 1

The Theory of Quantum Mechanics

1.1 Introduction: The History and Problems of Quantum Mechanics

For decades now quantum mechanics has been considered a successful theory. The application of its methods has been put to use in experimental description and prediction, technologies have emerged and continue to be developed [2–10] that make fundamental use of quantum mechanical ideas only made possible by the introduction of codified quantum physics in the early 20th century. Moreover, difficult ideas such as ‘wave-particle duality’ even begin to make their way into the realms of popular science common vocabulary [11, 12].

While the succesful application of quantum mechanics has achieved much, what shall be of interest to this thesis are the fundamental concepts and beliefs that underpin its formulation. There are many ways in which quantum mechanics can be taught, theorised, and applied [10, 13–22]. Each manages to describe experimental reality to good degrees of accuracy. However, the guiding ideas behind each method can vary significantly. As such there are many papers discussing the choice and theory of the foundations applied

[23–26], and still great discussion occurs over these ideas [27–31]. Chapter 2 contains an overview of the main branches of differing interpretations.

Every current working theory of quantum mechanics gives only probabilistic results; there is no fully deterministic theory that captures all the features of quantum mechanics that we observe by experiment. This probabilistic nature is taken axiomatically in most teachings [32, 33], and is sufficient for the description of experiments on large ensembles of particles. However, the non-deterministic, non-classical, and unintuitive results have been used as grounds to question the validity of quantum theory since the earliest publications in the field. In their famous 1935 paper [23], Albert Einstein, Boris Podolsky, and Nathan Rosen put forward their reasonable expectation:

> A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. [23, p. 1]

And went on to show that even simple quantum-entangled systems fall short of fulfilling this description of "objective reality" — concluding that existing quantum theory, including its wavefunctions and probabilities, must be an *incomplete* description of reality.

Their arguments lead to further theories which question whether any traditional-style theory involving determinism and some hidden variables could be possible in regards to quantum mechanics. John Bell's work produced statistical inequalities which evaluate the feasibility of hidden-variable theories [34, 35], and recent years have shown experimental testing of such theorems [36, 37]. However, such works still leave open possibilities of non-local, or non-real-valued hidden variable theories. Additionally, works upon which this thesis will draw open further questions as to what can be a suitable definition for 'element of physical reality' as Einstein and company desired, and whether the metaphysical foundations upon which we begin must be adjusted in light of the emergence of quantum mechanics in modern physics [14, 38]. Chapter 4 discusses the details of the ideas of *Contextual Objectivity*.

Furthermore, the apparent faster-than-light transfer of information observed as the phenomenon of 'entanglement' is another facet of quantum theory that defies our intuition. We take it as true that information cannot be transferred faster than light — a concept dating back to Einstein's famous work on relativity [39], yet quantum interactions seem to violate this law. We may ask whether it is the axioms of special relativity or those of quantum mechanics that need questioning under these circumstances — both have experimental backing. One way in which relativity holds firmer is that it is based upon a *simple physical understanding* — that c is an absolute speed limit for the physical universe, and upon this a mathematical theory forms. What existing foundations of quantum mechanics lack is some similar physical truism that naturally describes its observed consequences.

This absence of an intuitive core idea may be a distasteful but non-fatal flaw of quantum mechanics; practicalists may point to the accurate descriptions of physical events and the existing developments and technologies that have been achieved as evidence of quantum theory's validity and value. However, as David Deutsch's text [40] elaborates, *understanding*, more than mere *description*, is key to a structure of knowledge. The ability of quantum mechanics to describe the universe accurately, but not intuitively, may be paralleled with an astronomical example: Ptolemaic epicycles can accurately describe the motion of planets in the solar system — but with great complexity and lacking a simple physical intuition; introducing the core idea of the force of gravity between massive objects leads to the now favoured Copernican heliocentric model — which provides a better *understanding* and deeper grasp of the physical universe involved, and such deeper comprehension lead to much further and greater developments.

Finally, we must highlight the emergence of commercial quantum technologies. For decades practical applications of quantum computing technology has been promised [41, 42], yet now we finally step upon the threshold of such technology seeing real use [5–9]. With such applications being put to such important uses as modern cyber-security, we must re-examine our grounding to be sure of the validity of claims such as 'secure key distribution'. In particular, this thesis will highlight how quantum effects necessarily emerge from large collections of particles acting in an ensemble, in contrast to the existing quantum technologies using only a small number of qubits.

1.2 Key Ideas: Mutuality, Contextuality, and Stochasticity

This thesis hopes to tackle some of the problems of the interpretations of quantum mechanics by holding to some basic premises, and adapting some of the methods found in early, more forgotten, literature, along with some modern viewpoints.

The scheme of the modern scientific method is to apply laws which are homogeneous and isotropic — that identical experimental actions produce identical results at any other time or place; that is to say, that physics can be applied in a *universal* sense. The first difficulty of quantum mechanics is that of the quantum-classical division. The separation between the totality of objects for which we must consider wavefunction collapse and measurements operators (in usual quantum theory), compared to those for which we may use classical models, remains ill-defined. The division of areas where $\hbar$ is 'significant' is an unclear one, and apparently depends on a subjective observer. Hugh Everett's well-known Many Worlds Interpretation (MWI) [19, 20] began exploring ideas of a 'universal wavefunction' as a continuous universal application of quantum laws; details can be see in section 2.3.

Furthermore, it should be remembered that when measuring or experimenting with quantum objects, the operator cannot be excluded from the model. The wavefunction collapse of the usual Copenhagen Interpretation is often taught as axiomatic, and while it can be 'understood' to the degree that the student may calculate (probabilistic) experimental results, the concept is central to the unintuitive and anti-classical elements of quantum theory. This collapse occurs when a disparate hand-of-God measurement operator instantaneously affects the state of some quantum matter before returning to the aether. However we must remember that these measurement actions are in fact physical *inter*actions — it is only by the means of multiple physical systems affecting each other that we may learn of or change these systems. And if quantum theory is to be a universal one, then these observer/apparatus/environment objects must also follow some quantum dynamics in their action.

This idea of a *mutually* effective action between observed and observer is carried through this work in examples wherein both quantum objects, and the apparatus with which they interact, are modelled with methods including wavefunctions and complex phases.

The continuation of this reasoning leads to the inclusion of *contextuality* — quantum models where any state or result must be given as *in relation to* other systems; that is to say, *context* is an essential part. Chapter 4 contains further discussion on contextuality.

Finally, the nature of the wavefunction must be tackled. It is the wavelike nature of certain quantum results that leads to the tricky ideas of instantaneous collapse, and duality, and even superposition of mutually exclusive states. However, it must be remembered that while some interpretations view the wavefunction as a complete description of an individual object, it is fact only in *ensembles* that we see wavelike properties such as interference. Even in a demonstration of 'single-photon' interference it is the conglomeration of *many* collected test runs that shows periodic (wavelike) results — the wave-character is only begat by statistical ensembles. The necessity to view ensembles of particles for wave-like properties to emerge is related to the idea of context being a crucial part of quantum effects.

The appearances of the ideas of mutual interaction, necessary context, and stochastic waveforms shall be explored in subsequent chapters; and a new novel model built upon a selection of those methods from the literature is demonstrated in chapter 6.

1.3 Goals: A Deterministic Contextual Stochastic Model

The major result of this thesis shall be a demonstration of a toy-model which uses deterministic stochastic rules and shows a quantum-like result. The demonstration (seen in chapter 6) uses the Python 3.6 programming language to simulate a single-photon interference experiment, as would be as seen in, for example, [33]. The results shall be as if like the detectors of a physical apparatus measured such an experiment, but the simulated action of the photons will obey a set of deterministic rules. The idea is to show that it is possible to have a model of deterministic corpuscular photons which still shows quantum (wavelike) results by the inclusion of mutuality, contextuality, and stochasticity.

The specific ideas amalgamated include Louis de Broglie's early notion of an *internal periodic phenomenon* in quantum objects [13], Richard Feynman's 'little arrow' notation [18], and David Bohm and Jeffrey Bub's 'double solution' work [43]. Chapter 5 expands on these ideas.

The given model is not intended to be a total description of the processes of physical reality, it holds only the modest goals:

1. To present an explicit implementation of a mutual two-wavefunction contextual model.
2. To demonstrate that the new model successfully replicates quantum behaviour through deterministic particles.

Similar theoretical models can be seen in [44, 45], and by physical experiment [46].

1.4 Summary

The established goal of this work is to present a model based upon mutuality, contextuality, and stochasticity, which shows deterministic particles exhibiting quantum (wavelike) behaviour in chapter 6.

Leading up to that, detail of the ideas included are discussed in chapter 5; including the foundational notion of contextuality in chapter 4.

Details on a selection of current quantum foundational approaches are provided in chapter 2, followed by some original examination of Bell-test experiments, which crucially link to hidden-variable interpretations of quantum mechanics, in chapter 3.

Appendix A includes the Python script used.

Chapter 2

Foundations and Interpretations

2.1 On the Reinterpretation of Quantum Foundations

Quantum mechanics has for more than a century provided us with adequate descriptions of physical phenomena. We have a set of postulates and Schrödinger's celebrated equation. Yet there is still discussion as to how this branch of physics relates to the real world, or rather what the quantum phenomena that we observe can tell us about what it means to have 'reality'.

The probabilistic nature of quantum mechanics was an initial concern for the founding fathers of the theory — famously causing Einstein to question God's gambling habits [47, p. 91]. Furthermore, Einstein and his contemporaries even went so far as to criticise quantum mechanics as an 'incomplete' theory [23], arguing that any reasonable description of reality must satisfy some assumptions on repeatability and predictability. Bell's theorem [34] threw more fuel onto this fire when he apparently denied the possibility of any description of reality as Einstein desired. However with the recent [36, 37] experimental verification of Bell's theorem, we have not given up on finding some complete description of reality, we have instead begun asking more careful questions of what it means to 'describe reality'. See for example Philippe Grangier's papers on contextualisation of quantum mechanics [21] which assert that 'reality' (something with what we can call 'physical realness') can only exist as a conjunction of a 'thing' and its 'context'.

2.2 A Description of Reality

As previously discussed, the problems of whether quantum mechanics is, or can be, a successful description of reality dates back to the founding fathers of the theory; Einstein's primary definition of reality in the physical sciences (page 2) being known to be incompatible with the results of quantum experiments [23]. Much is discussed about what may constitute 'objective reality' in quantum mechanics [21, 48]. The usual Copenhagen interpretation asserts that a particle's wavefunction is a complete descriptor of its physical state, and a straightforward definition of reality would be of some property that persists unchanged (or evolving in a known way) until read-out by a measurement; in which case any particle in a superposition or un-measured state must *not have* any physical reality until a measurement operation occurs and it collapses to a single defined result. This problem of reality not existing until some ill-defined 'observer' has recorded it is another concern on which Einstein is known to have spoken[1].

The theories which hope to assign objective measurable reality to physical objects are known as Hidden Variable (HV) theories; developed on the idea that the uncertainty and probabilisticness of quantum mechanics is due to some unknown underlying governing properties yet to be discovered — a methodology carried over from classical mechanics. The works of John von Neuman and John Bell explored the fundamental possibility of such a solution for quantum mechanics and developed experimentally testable statistical

[1] 'We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it. The rest of this walk was devoted to a discussion of what a physicist should mean by the term "to exist." ' [49, p. 907]

values which may verify or contradict the possibilty of a classical local hidden variable being responsible for certain quantum behaviours. These experiments are discussed more fully in chapter 3.

Aside from the search for a hidden variable solution, many other works have proposed alternatives to the Copenhagen approach — reinterpretations of the fundamental ideas behind wavefunctions and quantum behaviour which hope to better explain reality and provide further understanding of quantum physics, while still matching the verifiable experimental predictions recorded by lab-work. A selection of some of the main camps of interpretations are described below.

2.3 A Selection of Foundational Approaches

Here we will summarise some key branches of foundational theories and highlight which ideas might be useful to us in this thesis.

While every author takes their own personal perspective on quantum interpretations, several distinct branches stand out and are outlined here. We also highlight where their ideas link to those of this thesis.

1. **Bohmians** Ultimately favouring a hidden variable solution, Louis de Broglie and David Bohm introduce a pilot-wave and a 'guiding equation' [13, 14, 50, 51]. It asserts that the Schrödinger equation, along with one other guiding equation, fully describes quantum systems of multiple particles without recourse to introduce further axioms on observers or measurement — apparent 'collapse' and probabilities as the squares of amplitudes emerge as consequences.

   Bohm's 1952 method wrote the wavefunction as $\psi = R\exp(iS/\hbar)$ and arrived at a continuity equation along with a modified Hamilton-Jacobi equation which included a term of *quantum potential*. The two equations are accurate as a description and are similar enough to classical mechanics (with a quantum addendum) to be comfortable, but the quantum potential term, even to Bohm himself, seemed a useful and necessary but ungrounded addition.

The earliest attempts by de Broglie at this method were largely ignored by the newly emerging quantum physics community in favour of the Copenhagen interpretation; it wasn't until Bohm's expansion in 1952 [14, 50] that neatened up the ideas that 'pilot wave theory' began to reach recognition.

The de Broglie-Bohm pilot-wave interpretation of quantum mechanics restores determinism to the calculations and behaviour of quantum objects, without denying the probabilistic results seen. That is, there is no need for an axiomatic spontaneous probabilistic change. Each physical interaction in the pilot wave model is deterministic and continuous, yet still produces the random and discrete results verified by experiment.

2. **Consistent Historians** The Consistent Histories approach aims to tackle problems of the Copenhagen interpretation such as Schrödinger's cat and the claim that properties do not exist until measured.

   First published through a series of papers from 1984 by Robert Griffiths, Roland Omnès and others [52–54]. It proposes that while solving a Hamiltonian represents the deterministic time evolution of a physical system in *classical* mechanics, the same *cannot* be said of the Schrödinger equation and quantum mechanics, though it is treated as such in the orthodox interpretation, and it was Schrödinger's intention. Treating the Schrödinger equation as such is what leads to untenable macroscopic superpositions such as the famous cat.

   Conversely, the consistent historians' approach is that quantum mechanics is fundamentally probabilistic — it is unnecessary to introduce hidden variables or to suppose that probabilistic interpretations imply that we have an incomplete theory. In this view the Schrödinger equation is not tasked with producing deterministic predictions of quantum objects' trajectories, but to assign probabilities to 'quantum histories' — sequences of quantum events at a succession of times. In this way deterministic histories are only a special case where a given sequence of events has probability 1. With this approach a 50/50 beamsplitter experiment would no longer predict (as in the Copenhagen interpretation) a superposition of states $(|0\rangle + |1\rangle)/\sqrt{2}$ but merely a prediction of *either* state $|0\rangle$ or $|1\rangle$ with probability each $1/2$.

This approach ascribes its 'element of reality' to consistent quantum histories (sequences of events). A quantum event can be any wavefunction.

Consistent histories forbids states (or histories) such as the pair $(S_x = -1/2)$ *and* $(S_z = -1/2)$ as meaningless to physical theories since they can never be experimentally tested. The combinations by $\{$*and*, *or*$\}$ of quantum states is carefully restricted by an idea of 'quantum incompatibility'.

This method of exclusions applied to a two beamsplitter experiments gives that the history "a photon travels one definite path between beamsplitters and emerges at a given definite detector" is *not* a consistent one — *i.e.* is not a valid description of physical reality. However it is valid to say that "the photon travels one definite path between beamsplitters and emerges in a superposition state of the two detector outcomes".

The key idea is that sets of mutually-exclusive consistent histories with assigned probabilities are the element of physical reality; and that the Born rule of absolute squares defines the probabilities as in familiar quantum mechanics methods.

Classical mechanics, in this interpretation can be recovered by 'coarse graining' restriction on families of histories.

3. **Transactionalists** John Cramer's 1986 and 1988 works [55,56] apply the 'transactional' title to work building on Wheeler-Feynman "absorber theory". This posits

that $\exp(+i\omega t)$ and $\exp(-i\omega t)$ are valid both mathematically *and* physically in describing quantum systems [57, 58]. The positive part being an 'advanced wave', and the negative a 'retarded wave'; the two interacting through a medium of apparatus (absorbers), in which the advanced wave has a causal effect but is *post hoc* cancelled or erased, leaving the observer with the usual experimental results.

The advanced and retarded waves can be thought of as complements of a single event propagating simultaneously forwards and backwards through spacetime, their 'handshake' transaction being the element wherein physical reality exists and gives experimental observables. The idea that the single quantum wavefunction is not the unilateral dictator that creates for us an observable element of reality is common among hidden variable interpretations and is used in a way in this thesis, in a form like the second wavefunction proposed in [43].

4. **Spontaneous Collapseans** In examining the position of the statevector in quantum mechanics, GianCarlo Ghirardi and Philip Pearle developed ideas that a wavefunction collapse — the measurement of definite outcomes — is a consequence of *strong localisation* of a wavefunction. In [59] they argue that an equal superposition of states is inherently unphysical:

> If we take the point of view that what we see around us is real, what occurs in reality is one of the two following evolutions:
>
> $$\text{cat alive} \;\rightarrow\; \text{cat alive} \;\; \text{OR} \;\; \text{cat alive} \;\rightarrow\; \text{cat dead} \qquad (1.1\text{a, b})$$
>
> [...]
>
> According to Schrödinger's equation, the evolution of the statevector describing this situation is
>
> $$|\text{cat alive}\rangle \rightarrow \frac{1}{\sqrt{2}}\,|\text{cat alive}\rangle + \frac{1}{\sqrt{2}}\,|\text{cat dead}\rangle \qquad (1.2)$$

> The right hand side of Eq. (1.2) does not correspond to either the reality on the right side of (1.1a) nor to the reality on the right hand side of (1.1b). [59, p. 2]

Instead, Ghirardi and Pearle's Continuous Spontaneous Localization (CSL) theory modifies the Schrödinger equation so that such uncertain states instead evolve into *one of*:

$$|\text{cat alive}\rangle \rightarrow 0.99\ldots|\text{cat alive}\rangle + 0.00\ldots|\text{cat dead}\rangle$$
$$|\text{cat alive}\rangle \rightarrow 0.00\ldots|\text{cat alive}\rangle + 0.99\ldots|\text{cat dead}\rangle\,.$$

Non-zero but small 'tails' are included in a way the exhibits the strong localisation needed to define distinct experimental results.

In this model, physical interactions favour the collapse of superpositions to the more clearly defined almost-one/almost-zero end states, and larger, more complex, physical systems (that can be considered *macroscopic*) favour collapse so strongly that they appear as the entirely localised points of classical mechanics.

The modified Schrödinger equation describes wavefunction evolution including these tails, while fulfilling other desirable properties such as agreeing with experiment, denying superluminal communication, and naturally deriving the Born rule of probabilities. A random background fluctuation fits in the model as a kind of non-local hidden variable which leads to the random but continuous evolution of superpositions to eigenstates.

The idea of spontaneous quantum collapse being replaced with a rapid but continuous evolution, governed by a kind of randomised hidden variable, is similar to that explored by Bohm and Bub [43].

5. **Einselectionists** 'Environment-induced superselection' aims to describe quantum systems and their environments in relation to decoherence converting quantum en-

tanglement into classical correlations. The relation between states (such as observer memory and quantum state) is *co-related* to relations with environment which allow prediction of quantum states without disturbance.

Wojciech Zurek [60] favours a view that experimental measurement of physical properties is no more than induced correlations between one physical system (the observed/measured object) and another (the measurement read-out). Through such actions, environments are made to correlate (or rather, become entangled) with the observables of a system in a way which monitors them. This correlation between states and monitors is a part of "reality", similar to the ideas of contextual objectivity properly detailed later (Ch. 4, §4.2).

This approach argues that the criteria of 'reality' are to be some state or property that can be both (i) correctly identified, and (ii) unchanged by the action of learning. Consequent to this, quantum states cannot be said to have 'objective reality', as condition (ii) cannot necessarily be satisfied. Zurek goes on to favour the definition 'relatively objective existence', to emphasise that the greatest degree of certainty of physical states is in relation to those other systems (environments) that act as witness. These ideas that the influence of environment and context are not just experimentally immanent, but conceptually essential, are followed through in this thesis.

6. **Contextual Objectivists** Philippe Grangier's examinations of quantum foundations [21, 48] argue in favour of objective reality, but with a caveat similar to the Einselectionists' approach — that an objective existence is still only in relation to some *context*.

   Contextual Objectivity argues that the conceptual problems of quantum theory: the EPR paradox and Bell's results on local realism, arise from an *a priori* assumption that an isolated system or physical property is enough to define an element of reality.

   Grangier's series of publications [21, 48, 61, 62] (latterly in collaboration with

Alexia Auffèves) continue the argument that experimental results in fact only describe a property in relation to a greater context, and develop a formulation which recovers the familiar predictions of quantum mechanics without need of any measurement/collapse postulate. The key principles of this formulation being the relation between context, system, and modalities, this scheme is abbreviated CSM. Further details are explored in chapter 4, §4.2.

The assumptions of the contextually objective view: that objectively real results exist *only* in relation to measurement and environment, underpins this thesis's work. We model disjoint but interacting hidden phenomena to govern the outcome of experimental results; and appropriate the idea that individual objective reality does not exist (may be predicted) from within only one system.

7. **Everettics** Commonly known as the Many Worlds Interpretation (MWI). Hugh Everett's 1957 thesis [20] identified two ideas within existing quantum theory that lead to the main paradoxes and problems faced by physicists. Firstly, the assumption that the observer has some special place in the formulation, independent and objective from an observed system; secondly, that we have a continuous evolution given by the Schrödinger equation for isolated systems, but an instantaneous discrete collapse of the wavefunction in the moment of a measurement operation. Everett hoped that a theory with fewer unintuitive or paradoxical problems could be arrived at by paring down these assumptions to merely 'quantum mechanics evolves according to unitary evolutions'.

   It was Bryce DeWitt's later work [63, 64] that properly developed the ideas and popularised it as the 'many-worlds' understanding. DeWitt argues that when system and observer are treated in equal measure subject to unitary evolutions, then any quantum observation is in fact a pairing of the system-observer states, which evolves into a superposition of the discrete quantum outcomes, paired with states in which the observer has recorded such an outcome. For example:

$$|\psi_1\rangle = |s_1\rangle\,|M_1\rangle + |s_2\rangle\,|M_2\rangle + |s_3\rangle\,|M_3\rangle + \ldots$$

where s_i are the possible quantum states of the system, and M_i are the states of an observer (or apparatus) which has recorded state s_i. In this view, rather than an evolution in which the quantum state collapses to a single eigenvector result, each and every result remains. Since the observer themself is necessarily described by one of the M_i states, which is paired with only a single one of the s_i states, the observer's experience is that of a single definitive outcome, and the others that remain in the equation are of an equally real nature, but in an un-interactable 'other world'. The idea that both object and observer be treated as equals in the physical system is similar to others discussed in this section, and forms a part of the model presented later in this thesis.

David Deutsch's later text [40] holds the optimistic goal of unifying the theories of evolution, computability, quantum mechanics, and epistemology into a master universal theory. He follows the viewpoint of DeWitt that the multitude of outcomes of a quantum measurement are all equally real though separate, and goes further to propose that quantum interference effects are the result of interactions between objects of separate universes.

This thesis does not explore the philosophical or physical possibilities of many worlds; however, Everett's removal of the observer as a special-operator is incorporated. Furthermore, DeWitt's formulation [63] assigns a kind of apparatus memory to the paired system-observer state, we do not here use his language but the use of apparatus memory is key to the demonstration in chapter 6.

2.4 The Hopes of Reinterpretations

The proliferation of competing interpretational ideas suggests that progress is still to be made in achieving a total understanding of quantum mechanics. As has been highlighted, while current quantum theories provide sufficient mechanics to accurately describe experiments, we still face unintuitive or seemingly paradoxical results. It could be hoped that there is some formulation of quantum mechanics which assuages some of these difficulties, without contradicting the certified mathematical results already established. Christopher Fuchs compared modern difficulties with quantum mechanics to Einstein's breakthrough in special relativity [65, 66]: the Lorentz transformations were known pre-Einstein and were mathematically sound and empirically adequate, but offered no conceptual piece of mind; Einstein supplied "simple, crisp *physical* statements" [66, p. 2]:

- The speed of light is constant, and
- Physics is the same in all reference frames,

which naturally derive the Lorentz transformations. This more fundamental understanding lead to the greater insights of general relativity, and became accepted as a 'true' description of the universe thanks to its simplicity and beauty, in addition to agreement with existing empirical formulation.

As Fuchs highlights, we currently we have the quantum mechanical axioms:

- Hilbert space.
- Projection operators.
- *Somehow* collapse to eigenvectors.
- Schrödinger equation.

In attempting to re-interpret or re-formalise the bases of quantum theory, it is hoped that some easily grasped grounding concepts can build to quantum theory as we know it, as Einstein's physically-grounded insight did in the twentieth century.

It would be a necessary criterion for acceptance that any re-formulation of quantum mechanics must agree with current experiments (and the mathematics behind them). The ideas developed in this thesis and detailed primarily in chapter 5 are mainly adjustments of thought and association of existing ideas from the literature, such as the rôle of context, non-local elements of reality, and the removal of the observer as a special object. The model described in chapter 6 demonstrates a way in which known quantum effects can emerge from application of a simple physical interaction rule. Through this, we hope to present evidence that models of quantum mechanics which maintain a deterministic intuition while still matching experimental results are possible.

Chapter 3

Local-Realist Theories and Tests of Bell Inequalities

Early hopes for quantum theory included a desire for it to be explicable by some *local-realist* theory. Einstein, Podolsky, and Rosen famously used the fact that quantum mechanics had no local-realist explanation to argue that the theory was incomplete [23].

3.1 Hidden Variables and Bell Tests

The testing of Bell inequalities is the favoured method of investigating the relationship between local-realism and quantum mechanics. Significant results have been achieved in this area, but further work is still ongoing.

Realism is the assumption that the physical universe *exists* (has some 'element of reality') independent of any measurement or observation. Locality is the assumption that physical influences cannot travel faster than the speed of light — the cornerstone of special relativity. In their 1935 paper [23], Einstein, Podolsky, and Rosen noted the disparity arising between the assumptions of local-realism and the behaviour of entangled quantum objects. John Bell's later work on that idea [34] lead to experimentally testable results, now known as Bell-inequalities, that can test whether certain quantum mechanical systems obey the ideas of local-realism.

A commonly examined type of Bell-test experiment involves two entangled qubits being measured under space-like separation sufficient to exclude relativistic interaction and noting the correlation of the two sets of results. The correlations become a test value which, under the assumption of local-realism, has a theoretical bound. Hence such experiments can verify or contradict any local-realist theory for the behaviour of quantum objects.

The testable theoretical bound arises from consideration of a 'correlation coefficient'. If a is a measurement setting examining the state of the first qubit, and likewise b for the second, then the correlation coefficient for the joint outcome ab is

$$E_{ab} = probability\ of\ correlation - probability\ of\ anti\text{-}correlation$$
$$\text{realised experimentally as: } \frac{n(\text{corr})_{ab} - n(\text{anti-corr})_{ab}}{n(ab\text{ trials})},$$

where $n((\text{anti-})\text{corr})_{ab}$ denotes the number of (anti-)correlated results of tests performed with settings ab, and $n(ab\text{ trials})$ is the total number of tests performed.

Bell's work showed that if there exists some local hidden variable which acts as the element of reality behind these results, then the inequality:

$$E_{ac} - E_{ba} - E_{bc} \leqslant 1 \tag{3.1.1}$$

should hold (in a set-up with perfect anti-correlation).

Bell's original inequalities were formulated for measurements with perfect anti-correlation — the case where result a at detector 1 *guarantees* result $\neg a$ at detector 2. However, more workable data can be acquired from experiments where result a at detector 1 implies result a *or* $\neg a$ with some known probabilities at detector 2. It was the later work of Clauser, Horne, Shimony, and Holt [35] that generalised Bell's method to allow for non-perfect anti-correlation in what is now known as the CHSH inequality. With two measurements prepared for each qubit, labelled 0 and 1, chosen such that measurement 1 is precisely the complement of measurement 0 (*e.g.* measures of spin in perpendicular directions); if there is some local hidden variable, then

$$E_{00} + E_{01} + E_{10} - E_{11} \leqslant 2. \tag{3.1.2}$$

Recent work by Hensen *et al.* [36, 37] have shown statistically significant violation of the CHSH-inequality, suggesting to us that there is no local-realist theory that completely describes quantum mechanics.

3.2 A Bell-Test Experiment using CHSH

Practical Bell-test experiments must be measured by two criteria: (i) violation of the CHSH inequality must be statistically significant, and (ii) the set-up must guarantee that no subluminal communication could be responsible for the correlation or results. Some results on ways in which these criteria may be examined are presented in the following sections.

A general Bell-test experiment produces results with *eight* independent variables: N the total number of trials; a, b, c the number of trials with setting configurations $00, 01, 10$, respectively; and the number of correlated results under each setting $(n_{00}, n_{01}, n_{10}, n_{11})$ (See Table 3.1).

Giving test value S, defined to be:

$$S = E_{00} + E_{01} + E_{10} - E_{11} = 2\left(\frac{n_{00}}{a} + \frac{n_{01}}{b} + \frac{n_{10}}{c} - \frac{n_{11}}{d} - 1\right). \tag{3.2.3}$$

Table 3.1: General results from a CHSH experiment.

Setting	00	01	10	11
No. of Trials	a	b	c	$d = N - a - b - c$
No. of Correlated results	n_{00}	n_{01}	n_{10}	n_{11}
No. of Anti-correlated results	$a - n_{00}$	$b - n_{01}$	$c - n_{10}$	$d - n_{11}$
$p(corr)$	n_{00}/a	n_{01}/b	n_{10}/c	n_{11}/d
$p(anti\text{-}corr)$	$1 - n_{00}/a$	$1 - n_{01}/b$	$1 - n_{10}/c$	$1 - n_{11}/d$
$E = p(corr) - p(anti\text{-}corr)$	$(2n_{00}/a) - 1$	$(2n_{01}/b) - 1$	$(2n_{10}/c) - 1$	$(2n_{11}/d) - 1$

Local-realism predicts $S \leqslant 2$, so an experiment disproving local-realist explanations of quantum entanglement will violate the CHSH-inequality — showing $S > 2$.

3.3 The Significance of These Probabilities' Skew

Suppose that the probabilities of correlation under each experimental setting are equal (to P say), then $S = 2\,(2P - 1)$; in which case $S > 2 \Leftrightarrow P > 1$. This means that it is not possible to observe the quantum result $S > 2$ for four equal probabilities — the behaviour of the quantum system is dependent on these probabilities being somewhat skewed away from equality.

Similarly, we may examine Bell's original inequality (3.1.1). Let n_{ac} denote the number of correlated results under setting ac; N_{ac} the total number of trials under setting ac; n_{ba} the number of correlated results under measurement setting ba *etc*. Then correlation values take the form $E_{ac} = (n_{ac} - (N_{ac} - n_{ac}))\,/N_{ac} = 2n_{ac}/N_{ac} - 1$ and Bell's inequality asserts that a local-realist theory implies the following inequality:

$$\frac{n_{ac}}{N_{ac}} - \frac{n_{ba}}{N_{ba}} - \frac{n_{bc}}{N_{bc}} \leqslant 0\,. \tag{3.3.4}$$

Again, if these three fractions (each equal to the probability of correlation under a given setting) are all equal, then the inequality will always be satisfied.

3.4 Approximating this Experiment for Large Numbers of Particles

Since the experimental settings are determined randomly, we would expect that the number of trials under each setting would tend to a uniform distribution for suitably large N. Supposing this is the case, let $a = b = c = N/4$, then we get

$$S = \frac{8}{N}\left(n_{00} + n_{01} + n_{10} - n_{11} - \frac{N}{4}\right) . \tag{3.4.5}$$

Let us denote the range of correlation counts κ.

$$\kappa := \text{range}\left\{n_{00}, n_{01}, n_{10}, n_{11}\right\} = n_{\max} - n_{\min} \tag{3.4.6}$$

Where $n_{\max} = \max\left\{n_{00}, n_{01}, n_{10}, n_{11}\right\}$ and $n_{\min} = \min\left\{n_{00}, n_{01}, n_{10}, n_{11}\right\}$. Let us only consider $\kappa \geqslant 1$ (for if $n_{\max} - n_{\min} = 0$, then we will always get $S < 2$). Additionally, we must have that $n_{00} + n_{01} + n_{10} + n_{11} \leqslant N$; consequently, it is clear that we must have

$$n_{\min} \leqslant N/4. \tag{3.4.7}$$

Let us denote $(n_{00} + n_{01} + n_{10} - n_{11})$ by S'. Then we will observe violation of the CHSH inequality ($S > 2$) if and only if $S' > N/2$. Note also that S' is bounded above and below by $S'_{\max} = 3n_{\max} - n_{\min} = 2n_{\min} + 3\kappa$ and $S'_{\min} = 3n_{\min} - n_{\max} = 2n_{\min} - \kappa$.

Violation of the CHSH inequality is only possible if $S'_{\max} > N/2$, that is to say: $2n_{\min} + 3\kappa > N/2$ is *necessary* (but not sufficient) for violation. If we desire, for some level of experimental certainty, violation of a certain magnitude δ, then we may say that

$$S'_{\max} > \frac{N}{2} + \delta \tag{3.4.8}$$

$$\Leftrightarrow 2n_{\min} + 3\kappa > \frac{N}{2} + \delta \tag{3.4.9}$$

is necessary. From this (3.4.7, 3.4.9) we may see that for a violation of magnitude δ, the skewness of the probabilities (range of correlation counts) κ must be strictly greater than $\delta/3$:

$$\left.\begin{array}{r} n_{\min} \leqslant \frac{N}{4} \\ \kappa > \frac{N}{6} + \frac{\delta}{3} - \frac{2}{3}n_{\min} \end{array}\right\} \quad \Rightarrow \kappa > \frac{\delta}{3}\,. \tag{3.4.10}$$

If we desire $S > 2 + \Delta$, then we are in fact asking that $S' > N/2 + (N\Delta)/8$, so $\delta = (N\Delta)/8$; hence $\kappa > (N\Delta)/24$ is necessary.

3.5 The No-Signalling Problem

Bell-tests of this type try to establish whether two spatially separated systems are capable of showing physically correlated properties (measurement results) without *a priori* local hidden variables (elements of reality) *or* by some means of classical communication. It is assumed that any communication must occur slower than the speed of light (in accordance with special relativity), so the spatially separated systems are measured rapidly in small time windows — getting as close to simultaneous measurement events as possible. If the two events are (very close to) simultaneous, and are sufficiently far apart (the aforementioned Hensen experiments achieved a separation of 1.3 kilometres) then no signal slower than light may pass between the two measurement events.

While the experimental procedure including large distances and small measurement time windows is arranged so as to preclude the possibility of signals, their exclusion can be verified by careful analysis of the experimental data, as has been done by Andrei Khrennikov on the first Hensen experiments [38], and similarly by Adam Bednorz [67].

The method involves looking at the marginal probabilities of results. That is to say, with a fixed measurement setting for the first qubit s_1, if all correlation probabilities are equal regardless of the measurement setting on the second qubit s_2, then we certainly have independent results.

For example, with $s_1 = 0$, independent results would show:

$$p(corr|s_1 = 0) = p(corr|s_1 = 0 \wedge s_2 = 0) \tag{3.5.11}$$

$$= p(corr|s_1 = 0 \wedge s_2 = 1)\ , \tag{3.5.12}$$

and similarly for other settings. Experimentally we are unlikely to observe equality of these probabilities, so instead consider the differences $p(corr|s_1 = 0) - p(corr|s_1 = 0 \wedge s_2 = 0)$. Here small (nearly zero) differences suggest to us independence.

Using the variables established in table 3.1 all such probability differences look like

$$p(corr|0s_b) - p(corr|00) = \frac{n_{00}+n_{01}}{a+b} - \frac{n_{00}}{a} = \frac{an_{01}-bn_{00}}{a(a+b)}. \tag{3.5.13}$$

The family of all these equations can be described by $\left(\alpha n_{\bar{\beta}} - \beta n_{\bar{\alpha}}\right) / \left(\alpha(\alpha + \beta)\right)$ for experimental settings $\bar{\alpha}, \bar{\beta} \in \{00, 10, 10, 11\}$ and numbers of tests $\alpha, \beta \in \{a, b, c, d\}$.

Experiments will show a good no-signalling accuracy if all these probability differences are small. All such probabilities will be 'small' (to some ε), if

$$\frac{|\alpha n_\beta - \beta n_\alpha|}{\alpha\,(\alpha + \beta)} < \varepsilon. \tag{3.5.14}$$

In the case of uniform experimental test settings considered above ($a = b = c = N/4$), this condition becomes: $|n_\alpha - n_\beta| < (\varepsilon N)/2 \,\forall \alpha, \beta$. This is satisfied if

$$n_{\max} - n_{\min} < \frac{\varepsilon N}{2} \,. \tag{3.5.15}$$

So $\kappa < (\varepsilon N)/2$ is necessary to achieve results with a good no-signalling accuracy. Knowing also that $\kappa \geqslant 1$ is necessary for a CHSH-inequality violation, we see a useful result that provides a guideline for all experiments of this type:

> $N > 2/\varepsilon$ trials must be performed if an experiment violating the CHSH-inequality is to achieve no-signalling tolerance of accuracy ε.

For example, to see a no-signalling accuracy of order $\varepsilon \sim 10^{-3}$ implies the need for the total number of measurements N to be $\sim 10^3$.

Combining this with our knowledge that $\kappa > (N\Delta)/24$ is necessary for violation of the CHSH inequality to magnitude Δ, we conclude that

$$\varepsilon > \frac{\Delta}{12} \tag{3.5.16}$$

is a limit on our no-signalling tolerance — independent of the number of trials we perform.

Consequently, any further experiments that aim to reduce the no-signalling problem will be limited by this bound. Practically, we must have $\Delta \sim 10^{-1}$, so this bound will be $\sim 10^{-2}$. In fact, if we recorded the $S = 2\sqrt{2}$ predicted by quantum theory, then we would record a smallest possible no-signalling check of $\left(2\sqrt{2} - 2\right)/12 \approx 0.07$.

While the ongoing experimental improvements of practical Bell-tests may gather further results which suggest that quantum mechanics is not governed by some as-yet unknown local hidden variable theory, the results here show that the statistical significance of such experiments may always be limited by the skew of probabilities that is necessary for the effect being tested to emerge at all.

Chapter 4

Contextual Approaches

As has been discussed in earlier chapters, several modern viewpoints on quantum mechanics believe that the greater *context* of an experiment or system should be considered a significant part of the physical theory at work. Here we wish to highlight some of the ideas of the contextual viewpoints. In the following chapters we then explore how contextuality fits with the other ideas we developed to produce the novel model in chapter 6.

4.1 Khrennikov and the Växjö Interpretation

The ongoing work of Andrei Khrennikov and the Växjö conferences on quantum foundations [22, 28, 29, 31, 44, 68, 69] have continued to re-examine the assumptions of quantum mechanics and to publish work including contextual models that hope to improve upon the current understanding of quantum theory.

The earliest iterations of the ideas that would become the Växjö interpretation began with an approach to quantum mechanics from a statistical perspective. Khrennikov's commentary on existing quantum theories was that attempts were made to explain the observation of non-classical probabilities by introducing special quantum probabilities — and consequently the ideas of wave-like particles and superposition. Khrennikov's approach was to begin with a statistical framework that naturally gives rise to the interference terms characteristic of quantum behaviour. Papers in 2001 [44, 70] demonstrate the characteristic sum of probabilities with interference, $P_{12} = P_1 + P_2 + 2\sqrt{P_1 P_2}\,cos(t)$, first from statistical methods where the context of events is intrinsically a part of its probability; and secondly from a numerical gedankenexperiment. Similar numerical approaches are seen in [45, 71] and in this thesis.

The expansion of these ideas developed the idea of *prespace* in quantum probabilities as a consequence of the context-immanent probabilistic approach. A prespace of what could be called physical attributes of objects, but which can not be interacted with directly, provides a framework for a kind of hidden variable theory.

Khrennikov's results, namely: (1) it is possible for interference-like effects to arise from only classical statistics, and (2) such a model can be linked to a kind of contextual hidden variable, are used as a basis for the attempts in this thesis to describe quantum interactions through a deterministic stochastic model.

Through a series of papers, Khrennikov provided justification for the inclusion of context as a crucial part of quantum probabilities. In [70] the assumptions of Kolmogovarian statistics is examined and the importance of context highlighted. Khrennikov supposes two contexts, S and S', ('context' here defined to mean a *complex of physical conditions*), which both partition into subcomplexes:

$$S = S_1 \cup S_2, \text{ and } S_1 \cap S_2 = \emptyset,$$
$$S' = S'_1 \cup S'_2, \text{ and } S'_1 \cap S'_2 = \emptyset;$$

where there is some transition or correspondence between contexts:

$$S_1 \sim S'_1, \quad S_2 \sim S'_2$$

(such as different slits being opened or closed). Standard probability theory for some event B gives us $P(B|S) = P(B|S_1)+P(B|S_2)$, but we must not assume that $P(B|S) = P(B|S_1') + P(B|S_2')$. In fact,

$$P(B|S) = P(B|S_1') + P(B|S_2') + \delta(S, S'),$$

where

$$\delta(S, S') = [P(B|S_1) - P(B|S_1')] + [P(B|S_2) - P(B|S_2')] \,.$$

The δ term, under some assumptions, can be given a form like

$$2\sqrt{P(B|S_1')P(B|S_2')}\cos\theta(S, S').$$

Hence giving rise to the characteristic periodic interference term of quantum probabilities, without having to introduce any new or axiomatic quantum probability rules. The difference between quantum and classical effects is simply the state of being 'context stable' — wherein $\delta(S, S') = 0$ and we recover classical statistics.

Similarly, in [44], Khrennikov produced a numerical model of a typical double-slit experiment, using only the familiar equations of moving charged particles with a careful consideration of context. Using a digital simulation quantum-like behaviour was again seen from a contextual application of classical methods. Similar methods are seen in [45, 71], and this thesis.

In this way it is demonstrated that the experimental results of quantum mechanics can be described within the purview of classical statistics, provided that context is remembered and included with care.

Prespace

In successive papers [70, 72, 73] Khrennikov establishes contextual probabilities showing interference effects can be related to a preimage space (prespace) wherein every real vector corresponding to physical space can be mapped from many prespace points. This approach is similar to Willem De Muynck *et al.*'s examination of hidden variable theories [74] wherein the preparation of apparatus *and* environment was in correspondence to some hidden variable λ in a phase space Λ.

Khrennikov suggests a prespace Ω of all possible contexts which maps to both classical space $X_{cl} = \mathbb{R}^3$ and quantum space X_q (a Hilbert space). Since such maps are (or may be) many-to-one, a 'compression of information' is observed. A superposition of positions (of physical particles) can be understood as two points x_1, x_2 in X_q which have preimages $B_{x_1}, B_{x_2} \in \Omega$ which overlap ($B_{x_1} \cap B_{x_2} \neq \emptyset$).

This approach is both in comparison and contrast to other hidden variable theories. Khrennikov takes time to address how previous no-go theorems forbidding hidden variable quantum theories must be carefully considered in the context of their own expectations. Rather: realist theories that include a kind of hidden information *are* permissible.

Växjö Interpretation

Formalised and updated in [22, 68], the Växjö interpretation is a Statistical Contextual Realist Interpretation. It argues that the key quantum feature of *complementarity* is being misused when supposed to relate to individual particles — it is fundamentally a property of *statistical* ensembles, which are only understood through interaction with measurement apparatus (*context*), which does not imply the behaviour of individual particles cannot have objective properties, hence *realist*.

The interpretation describes the rôle of the wavefunction in quantum theory. In contrast to the orthodox (Copenhagen) interpretation wherein the wavefunction is a complete descriptor of an individual quantum system, or the ensemble (statistical) interpretation wherein the wavefunction describes the statistics of groups of similarly prepared quantum systems; the Växjö stance is that the wavefunction is a descriptor of correlations in the quantum prespace discussed above.

4.2 Grangier and Contextual Objectivity

It was the early days of quantum mechanical theory when the problem of independent objective reality was highlighted by Einstein, Podolsky, and Rosen [23] with their famed thought experiment. They supposed that individual quantum objects possessed independent and objective properties which could be read out by measurement, and found paradoxical results. This result was assimilated into the literature as a rejection of determinism in quantum mechanics, and lead to other well known difficult ideas such as Schrödinger's simultaneously live-and-dead feline. We take for this thesis instead ideas like that of Philippe Grangier [21, 48, 61] that the assumption of individual objective reality is flawed, and that instead a state of reality is only sensible when it exists within a given context — there can be no greater level of objective reality than *contextual objectivity*.

Grangier's first papers [21, 48] begin by arguing that observer independent predictions of experimental results can only exist in a conditional sense. For example, once a photon has been observed to have a given polarisation in a $0°$-aligned filter, we cannot with certainty predict its polarisation from a $45°$-aligned filter, only a *conditional* probability like $P(\text{transmission}, 45° \mid \text{transmission}, 0°)$. In general we see these as $P(a_i, E|b_j, E')$, where a_i, b_j are elements of finite sets of discrete outcomes ('modalities' in Grangier's terminology), and E, E' are the contexts under which those modalities may be recorded. Since our predictions, and any certainty of physical properties must intrinsically be in relation to some context, Grangier argues that it would be incorrect to presume any greater level of objective reality than that which is context-dependent.

Continuing in [48, 61, 62], Grangier (latterly in collaboration with Alexia Auffèves), ascribes operators and a Hilbert space structure to this interpretation, recovering the extant quantum theory.

Chapter 5

A Synthesis of Ideas

5.1 Contextuality

The model put forward in this thesis is contextual in nature — the effects of environment on the quantum systems is fundamental and crucial. The inclusion of context in quantum theory throughout the decades has been sporadic. David Bohm and Jeffrey Bub's 1966 method [43] explicitly proposed a hidden variable related to environment or apparatus, but with the proliferation of the Copenhagen interpretation attempts to include context were pushed to the sides.

In this thesis we wish to stress the relevance of context within a physical model when dealing with quantum scale problems. All experimental physical measurements at some level are a result of physical processes. For example a potential difference may charge some circuits which lifts a needle on a voltmeter, the mass of some object may compress a piezoelectric sensor which lights a digital display to read weight, or a photon strikes a detector which passes signals to some microprocessor that eventually displays pixels on a screen for us to read. At many levels, these sequences of physical effects may be ignored or thought of as simplified to some instantaneous event which produces information read out by the experimenter. However, at the quantum scale, where our objects under scrutiny are similar or identical to the physically responsive 'detector' parts, we must consider that the electrons which move one way to trigger our computer readouts may also significantly push back on the 'measured' objects. In this way the apparatus which physically constitutes the measurement act as to become *a part of* the result which we read out — the 'objective reality' is created from both object and measurement apparatus. Indeed, it has long been acknowledged that the act of measurement changes the properties of quantum objects; the small change of interpretation here is to say that the 'property' exists only as a result of the interaction of object and measurement action, rather than the 'property' exists objectively and independently and that the entirely separate 'measurement' changes it.

We may consider a macro-scale analogy: if we were only able to measure the position of a football by throwing (similarly scaled) tennis balls at it, then it would seem natural that both football and tennis balls react in some way to the operation and that it would be insensible to consider the reality of the football independent of the rest of the system.

Furthermore, the statistical element of quantum mechanics must be remembered as a core point. While it is possible to prepare and execute experiments with individual quantum objects, the random and discrete nature of the results mean that all meaningful laws or properties must be read from experiments performed over ensembles of many particles. That is, the properties and probabilities which we take as physically meaningful come from proportions of a set of particles giving certain results. Quantisation and randomness shall be taken as intrinsic, but not axiomatic in this thesis. The following chapters shall describe a model by which random quantised results are gleaned from a basic set of deterministic interaction rules on quantum objects.

5.2 A Deterministic Internal Periodic Phenomenon

Louis de Broglie's 1923 note "Ondes et quanta" [13] first introduced the idea of an 'internal periodic phenomenon' to quantum mechanics, which set the ground for later work in quantum wave ideas, and it was the later work of Bohm that fully developed this into the pilot-wave theory [14,50]. While Bohm developed the internal periodic phenomenon into a more external guiding wave, we shall instead interpret the idea of an internal periodic phenomenon as a kind of complex hidden variable. We develop the idea that the wave-like nature of quantum experiments is a consequence of the interaction of these periodic variables acting on each other through a large ensemble of similar objects.

The initial publication by de Broglie took the simple and familiar equations for the energy of a particle: m_0c^2 from relativity, and hv_0 from quantum principles, where v_0 is some periodic frequency; and showed that by their juxtaposition, and including relativistic principles, one may derive a sinusoidal wavefunction that agrees with the internal periodic phenomenon over all of the particle's trajectory.

While Bohm's later work on these ideas developed the pilot-wave as an external 'guide' to the dynamics of the particle, here we instead focus on the idea of the wavelike nature as an internal periodic property.

The idea to take an internal property and allow it its own deterministic continuous evolution was seen in Bohm and Bub's work.

5.3 Bohm and Bub's 'Double Solution'

Proposed in a 1966 paper [43], the joint work of Bohm and Bub proposed a hidden variable interpretation of quantum mechanics that utilised the mutual interaction between *two* wavefunctions to explain quantum measurement effects without any axiomatic waveform collapse.

Bohm and Bub postulate that in addition to the familiar wavefunction of a quantum object given in Hilbert space:

$$|\Psi\rangle = \psi_1 |S_1\rangle + \psi_2 |S_2\rangle \tag{5.3.1}$$

(as a two-dimensional example); there exists a dual Hilbert space containing dual vector:

$$\langle\Xi| = \xi_1 \langle S_1| + \xi_2 \langle S_2| \,. \tag{5.3.2}$$

This second vector is associated with an apparatus or external environment and affects the evolution of the $|\Psi\rangle$ wavefunction during interaction events.

In addition to the usual continuous evolution under the Schrödinger equation, Bohm and Bub postulate that during measurement (interaction with an apparatus), the $|\Psi\rangle$ vector changes according to another continuous evolution dependent on $\langle\Xi|$:

$$\frac{d\psi_1}{dt} = \gamma(R_1 - R_2)\psi_1 J_2 \tag{5.3.3}$$

$$\frac{d\psi_2}{dt} = \gamma(R_2 - R_1)\psi_2 J_1 \tag{5.3.4}$$

$$\tag{5.3.5}$$

where

$$R_1 = \frac{|\psi_1|^2}{|\xi_1|^2} = \frac{J_1}{|\xi_1|^2}, \text{ and } R_2 = \frac{|\psi_2|^2}{|\xi_2|^2} = \frac{J_2}{|\xi_2|^2}. \tag{5.3.6}$$

More generally, The entire quantum evolution can be neatly described for an N dimensional system by

$$\psi_i = \gamma\psi_i \sum_j J_j(R_i - R_j) - \frac{i}{\hbar}\sum_j H_{ij}\psi_j \tag{5.3.7}$$

where the γ variable does the job of being almost zero in free-space (recovering the usual Schrödinger equation), and being large enough to be far more significant than the Hamiltonian in the vicinity of an interacting measurement apparatus.

The continuous deterministic interaction of these two wavefunctions satisfy the desires of mutuality and contextuality as discussed in chapter 1. This formulation does not have a separate and immutable measurement operator causing wavefunction collapse, and the context of the environment and arrangement of interacting elements is crucial. As discussed in §5.1, we are seeing here simply physical *inter*actions which result in some recorded information or property acting as de facto measurements. The probabilisticness is still retained however, thanks to the 'hidden' $\langle\Xi|$ component.

We see that this set of interaction rules naturally results in $|\Psi\rangle = \psi_1 |S_1\rangle + \psi_2 |S_2\rangle$ evolving to one of $|S_1\rangle$ or $|S_2\rangle$: if $R_1 > R_2$ then $dJ_1/dt > 0$ and $dJ_2/dt < 0$, since $J_1 + J_2 = 1$ and $d(J_1 + J_2)/dt = 0$; this means that $\psi_1 \to 1$ and $\psi_2 \to 0$, *i.e.* $|\Psi\rangle \to |S_1\rangle$.

Similarly, $R_1 < R_2$ gives the result $|\Psi\rangle \to |S_2\rangle$, and we see that after a measurement interaction the $|\Psi\rangle$ system has 'collapsed' to a single eigenstate, though within this scheme the collapse is neither axiomatic nor instantaneous.

The probabilistic nature of quantum results emerges as a consequence of $\langle\Xi|$ being a *hidden* variable — its exact state is unknown prior to measurement. By assuming that over an ensemble of tests the value of $\langle\Xi|$ is uniformly distributed on the sphere $|\xi_1|^2 + |\xi_2|^2 = 1$, it can be demonstrated that the probability of $|\Psi\rangle$ showing result $|S_i\rangle$ is equal to $|\psi_i|^2$ — the expected Born rule.

We see that while a focus on contextuality in quantum foundations is a relatively recent emergence [21, 22, 28, 29, 31, 44, 48, 61, 68–70, 72, 73], these early authors were in fact skirting similar ideas. Furthermore, these methods of assigning a *complex* hidden variable sidestep various no-go theorems.

The key idea of assigning a realist but unknown wavefunction to describe environmental effects and a deterministic evolution during measurement are utilised in the new model presented in this thesis.

5.4 Feynman's Path Integral Formulation

In physics, Richard Feynman is remembered as a singularly talented communicator and teacher. So it was when he simplified the Hilbert spaces, phase differences, and complex probability amplitudes of quantum mechanics into a 'little arrow' formulation [17, 18].

Taking de Broglie's conceptualisation of the wave-like nature of quantum mechanics as an internal periodic phenomenon, Feynman neatly provided an intuitive view of complex phases as a rapidly rotating single-handed clock associated with each quantum object; the hands of these clocks being simply described as 'little arrows'.

The description of certain phenomena related to the reflection of light, the effects of interference can be conceptualised as simply adding together these 'little arrows'. That is to say, calculating the complex probability amplitudes of the situation can be seen as taking a vector sum of little arrows from each contributing element, the resultant of which is squared to give a positive real-valued of probability. The problem of calculating probability amplitudes is reduced to finding little arrows whose square represents a probability.

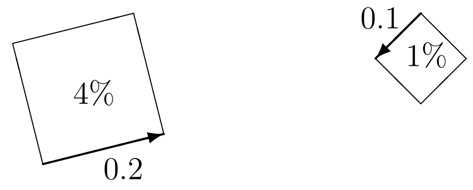
The simplest example of such a method, given in Feynman's light text *QED* [18], is the partial reflection of light on glass. We begin with some simplifications: given a single-photon source and detector arranged near a flat sheet of glass, 4% of emitted photons will be reflected and picked up by the detector, which is positioned so as to receive photons reflected off of the top surface of the glass only.

The quantum effect which makes this experiment notable occurs when we factor in the reflection of photons from the bottom surface of the glass - correctly positioning the detector so as to pick up these photons also. It can be assumed that the bottom surface also has a 4% reflection rate, and that then the detector will register in total eight out of every one-hundred photons emitted. In fact, the detection rate varies periodically between 0% and 16% as the thickness of the glass is altered. Introduced to a neophyte of quantum mechanics, it should seem quite peculiar that the inclusion of *more* photon reflection (from the bottom surface) somehow cancels out the reflection of photons from the top surface. Of course, thinking of light in waves poses no conceptual problems — waves can constructively or destructively interfere. However, when such an experiment is performed with very low energy photon sources it does not show *weaker* detections (as waves would be reduced), instead it registers *fewer* full-strength clicks — confirming that there must in fact be individual particles reflected or not reflected at each interaction.

In exploring this, Feynman stuck to a practicalist's approach of explaining how to calculate the probability of reflection (proportion of photons detected). This was done by the introduction of the 'little arrows'. The situation of 4% photon reflection on one surface corresponds to a 'little arrow' of length 0.2. Squaring the length of the arrow gives the probability of detection as in fig. 5.1. In the case where we include top-surface and bottom-surface reflection, we draw one arrow for each possibility — that is, a 0.2 length arrow representing top reflection, and a 0.2 length arrow representing bottom reflection. As we 'draw' these arrows their orientation is significant.

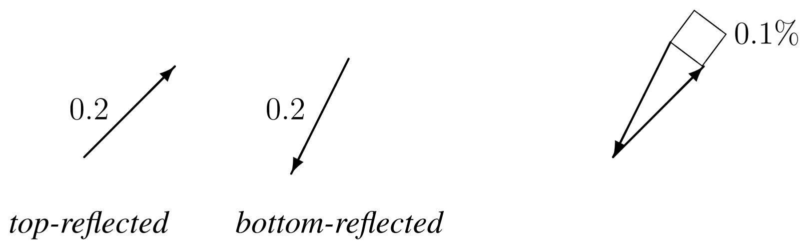
Feynman imagined having a rapidly rotating single-armed stopwatch, which spins at a rate proportional to the frequency of light used, and which starts and stops as the photon leaves the sources and arrives at the detector. The orientation of our little arrows corresponds to the arm of this stopwatch. Obviously the bottom-reflected photons travel a longer period, and hence bottom-reflected arrows show a different orientation after de-

Figure 5.1: probability squares



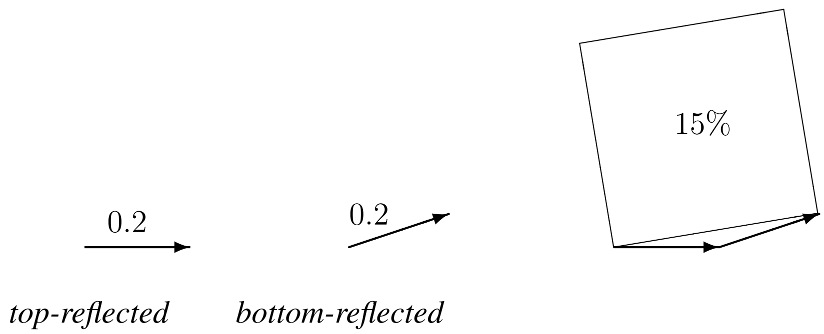
tection. The only additional caveat being that top-reflected arrows are in the matching direction to the stopwatch arm, bottom-reflected arrows are in the opposite. In calculating the probabilities for the two-surface arrangement, we combine the two little arrows in a vector sum. Taking the square of the length of the *resultant* vector shall give the probability of detection for top and bottom reflection from glass of that given thickness. One should begin to see that if the bottom-reflected photons have time to rotate a further half-turn than the top-reflected photons, then the resultant vector-summed arrow will be zero (or very small), whereas a bottom-reflected photon that completes any number of full rotations will add its little arrow constructively to the top-reflected arrow — giving a square (probability) up to quadruple that of a single photon/arrow. See figs. 5.2, 5.3.

Figure 5.2: little arrows summed destructively



A slight modification we make to Feynman's concepts is to imagine the rapidly rotat-

Figure 5.3: little arrows summed constructively



ing stopwatches as moving with each individual photon. This is more representative of de Broglie's idea of quantum objects possessing an 'internal periodic phenomenon', and more easily allows us to conceptualise each individual photon interacting with its environment or apparatus.

Chapter 6

A Deterministic Stochastic Model of Single-Photon Interference

6.1 Single-Photon Interference

The well-known double-slit experiment has long been used to demonstrate the wave-like interference of photons. Indeed, when modelling light as a wave, such a demonstration poses no conceptual problems — amplitudes sum constructively or destructively and the intensity (received energy) at the detector screen shows peaks and troughs where the waves reach maxima and minima. However, with the advent of finely tuned photon sources which can be calibrated to emit only single quanta of light at a time, and detectors precise enough to register such energy readings, it became possible to simultaneously observe the corpuscular and wavelike properties of light. This type of single-photon interference experiment is typical in physics texts [33, §2.A.1] and will be used here to demonstrate a new framework emerging from the combination of ideas previously discussed in this thesis.

Toy models describing contextual deterministic models of double-slit experiments have already been published [44, 75]. Here we shall describe a two-beamsplitter experiment, which similarly demonstrates a wavelike interference pattern from an ensemble of discrete photon paths.

Typically, a single-photon source is aligned with a beamsplitter which is known to split light in a fifty-fifty distribution, creating two mutually exclusive paths. Both paths are later recombined by a second beamsplitter, with one of the paths modified so that the light passing along it has an altered phase compared to the alternative path. A detector or detectors are set up so as to read the intensity of the light (photon count) from the recombined paths. Lower intensities are read when the phase alteration produces destructive interference, and higher intensities when the two paths' phases complement each other. Recording the set phase-alteration and the measured intensities it is possible to graph the results to clearly see the wavelike periodicity. Figure 6.1 represents a typical arrangement.

In a wavelike framework of light, it is simple to consider that the incoming wave is halved by the first beamsplitter, and it is the two halved-waves simultaneously travelling the two paths and recombining at the second beamsplitter, with the emergent wave created from their sum reaching the detector. However, with a single-photon emitter, and only one quantum of light travelling the apparatus at a time, it is harder to consider that single corpuscle is split, or that the single object travels the two paths simultaneously. These thoughts make it necessary to introduce a wave-particle duality to the model of light.

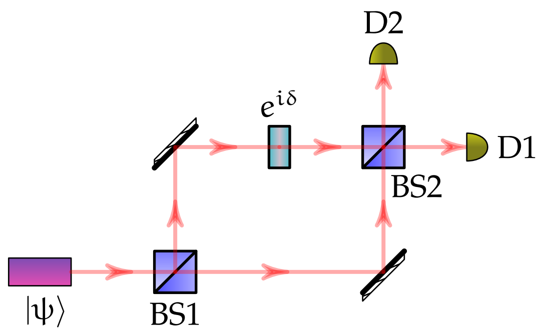


Figure 6.1: The standard two beam splitters arrangement in Mach-Zehnder interferometer. [1]

With the mind of the statistical and contextual schools previously discussed (Ch. 5), it must be highlighted that while single photons do travel the apparatus individually, it is the collective count of them at the final detector(s) which shows a wavelike nature — an individual photon and its single detector trigger is merely a discrete count, it is only through the ensemble of many repeated photons that anything wavelike is seen. This is a major idea from the quantum theory orthodoxy that we wish to challenge — the emergence of wavelike behaviour from an *ensemble* of objects does not necessarily imply that each object must be wavelike.

Furthermore, photons being emitted and detected individually does not guarantee that they act independently — they each pass through the same physical objects making up the apparatus, and may by some means leave evidence of their passage which affects subsequent photons, a kind of 'apparatus memory'. While in experiments such as those discussed in chapter 3 it is easy to ensure the space-like separation needed to ensure no-signalling; the time-like separation that ensures only single photon events are tracked in this type of experiment necessarily means that particles may 'signal' each other through the ensemble or environment. Shan-Liang Liu [71] has explored a similar kind of apparatus memory demonstrating interference patterns.

We seek here to describe a deterministic particle model showing interference effects without the introduction of any dual nature. We do this by considering the internal periodic phenomenon which formed the beginnings of de Broglie's work on waves and quanta, discussed in detail in chapter 5. We model the internal periodic phenomenon as a complex phase with some given frequency ν and initial value ϕ_0:

$$\exp\left(i(\nu t + \phi_0)\right).$$

This is a unit complex vector — a physical property carried with the particle, which we think of as rotating as the time variable t increases. We highlight that de Broglie introduced this periodic property simply by appealing to the quantum principle for a particle's rest energy: $E_0 = h\nu_0$ [13]. For this model however we will not include de Broglie's inclusion of relativistic factors. In the experiment we lay out here, we assume that our source produces particles in phase with each other, in which case the ϕ_0 can be ignored; and that they are emitted at random time intervals, but the same distance from the rest of the apparatus, in which case the specifics of ν and t may be ignored in favour of the single random instantaneous phase detailed in section 6.2.

In this two-beamsplitter arrangement, the wave model tells us that *two* waves meet and interact at the second beam splitter; we wish to instead describe each photon as having travelled a single definite path. To do this we allow for the interaction between the phases of each particle. While the particles do not directly physically interact with each other, they may still affect the rest of the ensemble via the medium of the apparatus — we are here introducing the contextualists' idea that the physical environment plays a part in the realisation of particles' properties. We consider that the apparatus has its own internal periodic phenomenon — as argued in chapter 1, the quantum formulation must be applicable to even the parts which we consider macroscopic. Rather to any individual molecule or atom, we will assign a complex phase to a beamsplitter as whole. The internal phenomena of both particle and apparatus shall interact by some deterministic rule.

Effectively the wave phenomenon of the apparatus serves as some 'memory' of the phase of particles with which it interacted before. Then, subsequent particles passing the apparatus are affected by their predecessors. Also, the interaction of particles with subsequent emissions clearly falls within the realms of locality, giving hope that such models may help to explain more adequately the effects typically attributed to instantaneous wave-function collapse.

In the following sections, the general-purpose programming language Python is used to run a numerical simulation of single-photon interference within the contextual deterministic framework discussed. It is based on the following assumptions:

- Individual photons are emitted from a coherent source at random time intervals.
- The parts of the apparatus (beamsplitters) with which the photons interact have their own phase and frequency, comparable to the $\langle\xi|$ vector of Bohm and Bub's 'double solution'.
- The corpuscular photons have an instantaneous interaction effect with beam splitters.
- At the instant of interaction, the beamsplitter *reflects* (sending the photon down path 1) if and only if the *phase difference* between the particle and beam splitter is greater than π. Otherwise, the particle is *transmitted* (taking path 2).
- If a reflection occurs, the phases of both photon and beamsplitter take new values, proportional to the phases at the moment of interaction. If a particle passed through without reflections then both phases are unchanged.

In other words, the post-interaction phase change acts as the 'apparatus memory'. Note that if the two paths available are of different lengths, and a particle has taken a path *different* to its predecessor, then there will be a difference between particles' phases and the cumulative 'memory' phase of the second beam splitter. Note that the given reflection rule follows the guide of being a *simple, crisp, and physical* statement, as discussed in section 2.4.

As described in chapter 1, this model does not claim to be a full descriptor of physical reality, its goals are to:

1. Present an explicit implementation of the proposed two wavefunctions contextual model.
2. Demonstrate that the new model successfully replicates quantum behaviour through deterministic particles, similarly to early theoretical models [44, 45] and physical experiments with droplets [46].

In a sense, this numerical model is a contemporary version of a typical 'thought experiment' familiar to many physicists.

6.2 Details of this Model

Appendix A contains the full code used here, and it is also available from

`https://github.com/DaleRHodgson/simulated_wavelike_behaviour`.

The Python model begins (lines `13–24`) by assigning random float (real decimal) values to variables `xi_1` and `xi_2` representing the phases assigned to the two beamsplitters. These can be labelled $\langle \xi_1 |$, $\langle \xi_2 |$ to follow a notation similar to the previously discussed 'double solution' [43]. While these would typically represent a complex wavefunction like $\exp(i\pi\theta)$, it is sufficient here to record only the $\theta \in \mathbb{R}$ element. For simplicity, this experiment will neglect the factor of π and record phases in the range $(0, 2) \subset \mathbb{R}$. A final float variable `ratio` is also introduced to be used in later functions.

Continuing (lines `26–81`), the script defines methods (functions) which transform given variables (here representing phases) according to simple deterministic rules. The first method (`beamsplitter`) describes the effect of a photon interacting with a single beamsplitter, returning the transformed phases of the photon and the apparatus, as well as a boolean result `reflection` recording whether that interaction event showed a reflection (`1`) or transmission (`0`) of the photon. The method `photon_in_one_bs` can be run repeatedly with randomly generated phases to simulate many photons passing through a single beamplitter, while `photon_in_two_bs` is similarly used to simulate many photons passing through a two-beamsplitter arrangement.

6.3 Verifying a Single Beamsplitter Result

Lines `87–102` verify that the previously defined operations act as expected. The `beamsplitter` method is defined to take two (float) variables representing the phase of an incoming photon and that of the first beamsplitter, and begins by calculating their phase difference $\Delta = (\varphi - \xi) \mod 2$ (line `31`). Here φ represents `phase` of the photon, and ξ the `Xi` phase of the beamsplitter; the $\mod 2$ is used to keep to the $(0,2) \subset \mathbb{R}$ scale used. It then applies the rule that reflection occurs if and only if the angle between the two phases is obtuse (line `32`). If we have a reflection event, then the two phases of the photon and beamsplitter are mutually transformed by the interaction (lines `36–42`):

$$\begin{aligned} \varphi &\mapsto \frac{\varphi + \frac{1}{2} - \Delta \mathtt{r}}{1 + \mathtt{r}} \mod 2, \\ \xi &\mapsto \frac{\xi + \Delta}{1 + \mathtt{r}} \mod 2. \end{aligned}$$

Where `r` is the `ratio` variable defined earlier, representing a weight factor of how much a photon phase is altered compared to the apparatus phase. The essential conclusions of this model hold true for weight factors of orders $\sim 10^{-1}$ to 10^5.

The `photon_in_one_bs` method is defined to take as input some phase (of an incident photon) and apply the `beamsplitter` action for that photon and the persistent `global xi_1` variable. This method is used (lines `87–102`) in a loop of `N = 100000` to simulate 10^5 photons, each with a new randomly assigned phase `2.0*random()`$\in (0,2) \subset \mathbb{R}$ interacting with a single beamsplitter, and incrementing the count `reflection_count1` whenever a reflection event occurs. It then prints out a reading of the percentage reflection of single photons:

```
Percentage of reflections in one beam splitter 0.50267
```

This test consistently shows a result very close to fifty percent — demonstrating that this model accurately simulates a single fifty-fifty beamsplitter. The script continues to use the same structure to model two beamsplitters as in the described experiment.

6.4 Simulating Two Beamsplitters

With the behaviour of the `beamsplitter` method verified, the script goes on to run another `N = 100000` simulation with the `photon_in_two_bs` method. `photon_in_two_bs` is defined (lines `58–81`) to again take a randomly generated photon `phase`, and to interact it with the persistent `global xi_1` and `global xi_2` variables, returning a boolean value `1` if a photon is reflected out of the second beamsplitter, and `0` otherwise. In the case that the photon reflects from the first beamsplitter, its phase is altered by some amount `optical_difference` (lines `70–72`) as was discussed in section 6.1.

The script sets up variables `steps = 50`, to allow the test to run across a range of offsets for `optical_difference`; and `reflection_percentages = []`, an empty list to be populated with the percentage reflections for each given `optical_difference` (lines `107–111`). For `j` taking the values `0` to `50` set by `steps = 50`, the script resets the reflection count to zero, and reinitialises the ξ variables for the beamsplitter phases to some random values (lines `113–123`). It then runs `N = 100000` tests of `photon_in_two_bs` of randomly assigned photon phases and an `optical_difference` equal to `2.0*j/steps`, so that `optical_difference` runs through values `0.0` to `1.96` in steps of `0.04` (lines `125–130`). The list `reflection_percentages` records the percentage of reflections out of the final beamsplitter for that `optical_difference`, and an immediate readout of the value is printed to the screen (lines `133–135`). The loop then continues until complete, and a graph of the percentage reflection (as `reflection_count2 / N`) for each given `optical_difference` is shown (lines `137–140`).

6.5 A Wavelike Result from Deterministic Stochastic Processes

This proposed model demonstrates the following features:

1. The fifty-fifty distribution of reflection/transmission seen for a stream of photons interacting with a single beamsplitter.
2. A variation from even distribution for two beam splitters, as per constructive and destructive interference. The achieved maximal deviation is around 50%–75%.
3. The final reflection rate varying periodically with the variation of path length between paths 1 and 2.

A sample graph of results from this script is show in Fig. 6.2.

This numerical simulation shows that single-photon interference is compatible with a deterministic particle model of light — we do not need to say that photons travel two paths at once, or that they are simultaneously particle-like and wavelike. This is achieved by an ensemble of particles self-interacting through the medium of the surrounding context. This picture is a blend of the de Broglie–Bohm pilot wave theory [14, 15, 76, 77] with the contextual interpretation of quantum mechanics [21, 22, 68]. The realisation of discrete particles showing wavelike effects in contextual ensembles is supported by physical evidence [46, 78].

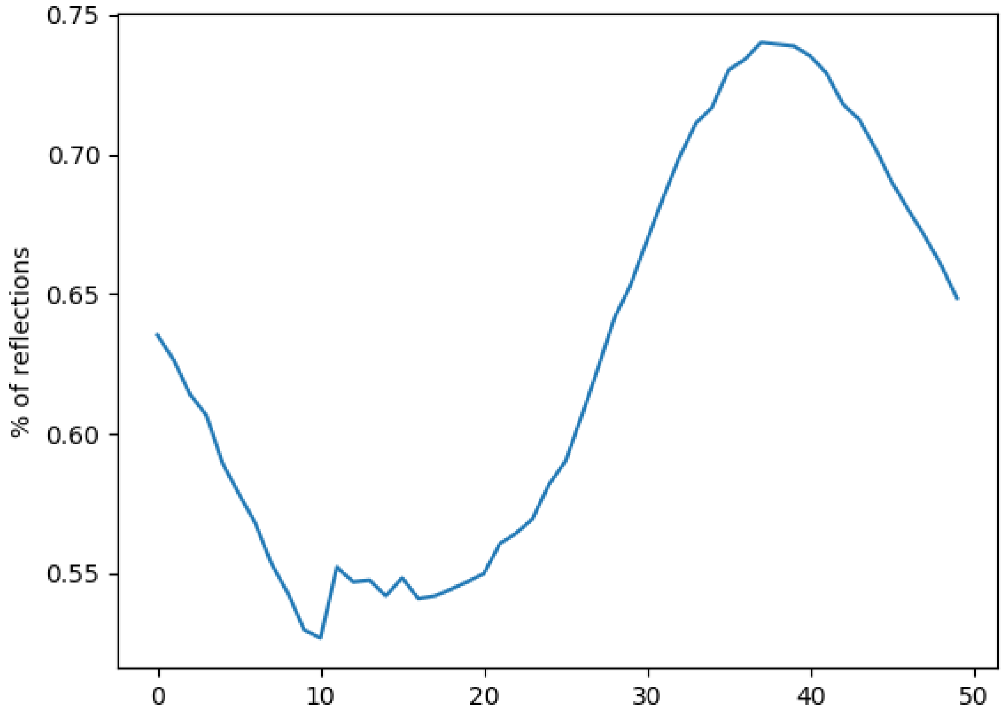


Figure 6.2: Results of the numerical experiment: there is sine-like dependence of percentage of reflections from the optical paths difference.

Chapter 7

A Notation of our Python Model

Feynman's 'little arrow' formulation outlined in [18], and here in chapter 5 is similar enough to be suitable to describe the model used in the previous chapter.

7.1 A Basic 'Little Arrow' Formulation of this Experiment

We associate with the beam of photons a complex phase $|\psi\rangle = R\exp(i\varphi)$, which represents a probability *amplitude* — meaning that placing a detector at any point along the path of the photons, we may take the absolute square of the complex phase to obtain a real positive value (less than 1) which represents the *probability* of detection at that point, *i.e.* the proportion of photons out of the entire ensemble which will be detected there. For example, a detector placed directly in front of the photon source would see probability of detection $P = |\exp(i\varphi)|^2 = 1$, as all photons are detected. Such a complex phase may of course be represented by a vector on the complex plane, so may be thought of as 'little arrows'.

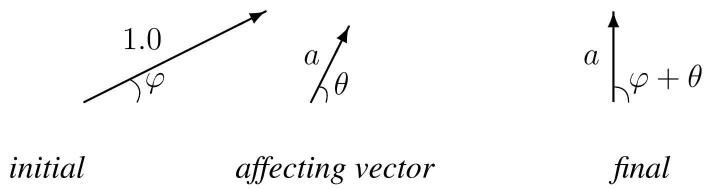
Furthermore, these complex amplitudes may be modified in two ways: when some event along the path affects the photon beam, the little arrow may 'turn and shrink' [18, p. 57]; or when photons may be detected from two different paths (with different complex amplitudes), the vectors will be summed.

'Turn and Shrink' Events

Demonstrated by partial reflection on mirrored surfaces in [18], the 'event' for which we must account here is the partial reflection by beamsplitter. The shrinking and turning of a complex vector is modelled mathematically by multiplication with another complex vector. For example, amplitude $\exp(i\varphi)$ shrunk by $|a| < 1$ and turned by θ is seen as:

$$\exp(i\varphi) \ \cdot \ a\exp(i\theta) = a\exp(i(\varphi + \theta)).$$

Figure 7.1: Turn and shrink



In this beamsplitter experiment, interaction with a beamsplitter demonstrates shrinking by a factor of $1/\sqrt{2}$, and in the event of reflection, a turn of $\pi/2$. That is, for amplitude $\exp(i\varphi)$:

$$\begin{aligned} \text{in transmission } \exp(i\varphi) &\mapsto \frac{1}{\sqrt{2}}\exp(i\varphi), \text{ and} \\ \text{in reflection } \exp(i\varphi) &\mapsto \frac{1}{\sqrt{2}}\exp(i\varphi)\exp\left(i\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\exp\left(i\left(\varphi + \frac{\pi}{2}\right)\right). \end{aligned}$$

The values $1/\sqrt{2}$ and $\pi/2$ are found empirically. With this, we see that a detector collecting either transmitted ($|T\rangle$) or reflected ($|R\rangle$) photons sees a 50% detection rate, since $\left|\exp(i\varphi)/\sqrt{2}\right|^2 = \left|\exp(i(\varphi+\theta))/\sqrt{2}\right|^2 = 1/2$.

Following this, a photon which is transmitted by the first beamsplitter, and reflected by the second ($|TR\rangle$) is shrunk twice, and turned once, leaving:

$$\begin{aligned}|TR\rangle &= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\exp(i\varphi)\right)\exp\left(i\frac{\pi}{2}\right)\\ &= \frac{1}{2}\exp\left(i\left(\varphi+\frac{\pi}{2}\right)\right).\end{aligned}$$

Similarly, for reflected-then-transmitted photons, we have

$$\begin{aligned}|RT\rangle &= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\exp\left(i\left(\varphi+\frac{\pi}{2}\right)\right)\exp(i\delta)\right)\\ &= \frac{1}{2}\exp\left(i\left(\varphi+\frac{\pi}{2}+\delta\right)\right).\end{aligned}$$

Where the additional $\exp(i\delta)$ term represents the addition path optical difference introduced to photons reflected out of the first beamsplitter.

Logical Conjunction

As mentioned, the other action to be applied to these 'little arrows' is the case when we seek probabilities including a logical 'OR', for example, detection from path 1 *or* from path 2. Within Feynman's scheme what we must do is sum the two (or more) complex amplitudes, and then take the absolute square. It is this case which gives rise to the distinctively non-classical probabilities of quantum mechanics.

We have within classical probability that for two events A and B, the probability of their union (either event or both events)

$$P(A \vee B) = P(A) + P(B).$$

This is not the case for quantum probabilities, where instead we see

$$P(A \vee B) = P(A) + P(B) + I(A, B),$$

where $I(A, B)$ is a (positive or negative) interference term. When using the Feynman scheme of probability amplitudes, and sum of complex phases for conjunction, this interference term arises naturally from the mathematics. For two events A, B with probability amplitudes $a \exp(it)$, and $b \exp(is)$, we have:

$$\begin{aligned} P(A \vee B) &= |a \exp(it) + a \exp(is)|^2 \\ &= a^2 + b^2 + 2ab \cos(t - s) \\ &= |a \exp(it)|^2 + |b \exp(is)|^2 + 2ab \cos(t - s) \\ &= P(A) + P(B) + I(A, B). \end{aligned}$$

This is the case of constructive and destructive interference by little arrows seen in section 5.4.

Final probabilities by Feynman's scheme

With these two ideas we can formalise the detection probabilities for detector D_1 of our arrangement (see fig. 6.1); we must sum the amplitudes of photons that took paths either Transmitted-Reflected by the two beamsplitters, or Reflected-Transmitted:

$$\begin{aligned} P(\text{detection at } D_1) &= ||TR\rangle + |RT\rangle|^2, \\ \text{where } |TR\rangle &= \frac{1}{2} \exp\left(i\left(\varphi + \frac{\pi}{2}\right)\right), \\ \text{and } |RT\rangle &= \frac{1}{2} \exp\left(i\left(\varphi + \frac{\pi}{2} + \delta\right)\right). \end{aligned}$$

Giving

$$\begin{aligned}
P(\text{detection at } D_1) &= \left|\frac{1}{2}\left(\exp\left(i\left(\varphi+\frac{\pi}{2}\right)\right)+\exp\left(i\left(\varphi+\frac{\pi}{2}+\delta\right)\right)\right)\right|^2 \\
&= \left|\frac{1}{2}\right|^2\left|\exp\left(i\left(\varphi+\frac{\pi}{2}\right)\right)\right|^2\left|1+\exp\left(i\delta\right)\right|^2 \\
&= \frac{1}{4}\left|1+\exp(i\delta)\right|^2 \\
&= \frac{1}{2}+\frac{1}{2}\cos\delta.
\end{aligned}$$

And a matching

$$\begin{aligned}
P(\text{detection at } D_2) &= \left||TT\rangle+|RR\rangle\right|^2 \\
&= \frac{1}{2}-\frac{1}{2}\cos\delta.
\end{aligned}$$

7.2 Comparison With Our Model

Consideration of de Broglie's Internal Periodic Phenomena as a kind-of internal clock or arrow was a key idea in making the model presented in this thesis (see chapter 5). Feynman's model ascribing a complex vector to the beam of photons to represent a probability amplitude, where a detector placed at any point will record a probability equal to the absolute square of the complex vector, translates well to our model.

Feynman's 'turn and shrink' action which, as described above, can be seen as like a multiplication by another complex vector, is much like where we transform photon phases φ by interaction with apparatus phase ξ (see section 6.3). Whereas Feynman's model uses a simple multiplication of complex vectors, we have written a kind-of 'weighted phase average' to the interaction event, and added a mutual change to the apparatus phase ξ. It is through this mutual change that a kind-of 'apparatus memory' evolves to allow self-interaction of the ensemble through time.

We compare:

1. Feynman's 'turn and shrink' action at one beamsplitter:

$$\text{Photons' amplitude } \exp(i\varphi) \;\mapsto\; \exp(i\varphi) \;\cdot\; \frac{1}{\sqrt{2}}\exp\left(i\widehat{R}\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\exp\left(i\left(\varphi + \widehat{R}\frac{\pi}{2}\right)\right).$$

   Where $\widehat{R} = 1$ for reflected photons, and 0 for those transmitted. As modulus-phase, this is:

$$(1, \varphi) \;\mapsto\; \left(\frac{1}{\sqrt{2}}\,,\; \varphi + \widehat{R}\frac{\pi}{2}\right).$$

2. Our Python 'weighted phase average':

$$\text{Photon and apparatus phases}$$

$$\begin{aligned} \varphi &\mapsto \frac{\mathtt{r}}{1+\mathtt{r}}\varphi + \frac{1}{1+\mathtt{r}}\xi + \frac{\pi}{2} \\ \xi &\mapsto \frac{\mathtt{r}}{1+\mathtt{r}}\xi + \frac{1}{1+\mathtt{r}}\varphi \\ &\qquad \text{when reflected,} \\ \varphi &\mapsto \varphi \\ \xi &\mapsto \xi \\ &\qquad \text{when transmitted.} \end{aligned}$$

In the Python model, it is only the real-valued phase which is altered (as if a 'turn-only' action). However, Feynman's application of a second complex vector to alter the photon beam's amplitude ties to both Bohm and Bub's double solution [43] (discussed in section 5.3), and the mutual interaction written in this Python model.

As Feynman's model shows its final detection probability to depend periodically on the optical path difference, δ, applied to one photon path,

$$P(\text{detection at } D_1) = \frac{1}{2} + \frac{1}{2}\cos\delta,$$

so to does our numerical simulation demonstrate such a periodic dependency (figure 6.2).

Chapter 8

Conclusion

Through this thesis we have highlighted how the common Copenhagen Interpretation of quantum mechanics may be suitable to describe quantum mechanics, but falls short of adequately *explaining* the theory. Various authors have proposed alternative schemes over the years, and we have collected together viewpoints which tackle the problems by careful consideration of *context*.

We have presented arguments as to why it is desirable to reconsider the foundations of quantum mechanics; and we have discussed why contextuality is both a suitable and necessary inclusion. The model presented in chapter 6 shows that it is possible to record interference-like effects characteristic of quantum mechanics using deterministic actions acting in a contextual way on a stochastic ensemble of particles. With this we have shown that it is *not necessary* to describe individual particles as waves with non-deterministic behaviour in order to model quantum mechanics.

Appendices

Appendix A

Python Model

Available at

https://github.com/DaleRHodgson/simulated_wavelike_behaviour.

```
from math import *
from cmath import *
import matplotlib.pyplot as plt
from random import *

# Initialise the random number generator
seed()

#################################################
# Functions and preliminaries
#################################################

# Phases are normalised to the interval [0,2]
# for simplicity. Variable phase for first
# beamsplitter, xi_1, starts from a random value.
```

```
xi_1 = 2.0 * random()

# Variable phase for second beamsplitter, xi_2,
# starts from a random value.
xi_2 = 2.0 * random()

# Measure of how beamsplitter atom is heavier
# than photon.
ratio = 20.0

# Function describing the effect of a beamsplitter.
def beamsplitter(phase, xi):

    # Difference between photon phase (phase), and
    # beamsplitter phase (xi).
    delta = (phase - xi) % 2
    reflection = ( abs(delta - 1.0) < 0.5)

    if reflection:  # Reflection condition.

        # Both phases are transformed by the interaction:

        # Beamsplitter phase is changed.
        xi = (xi + delta / (1.0 + ratio)) % 2

        # Photon phase is changed, with a 0.5 offset.
        phase = (phase + 0.5 - delta * ratio / (1.0 + ratio)) %2

    return phase, xi, reflection
```

```
# Function to check the distribution in a single beamsplitter.
def photon_in_one_bs(phase):

    global xi_1

    _, xi_1, reflection = beamsplitter(phase, xi_1)

    if reflection:
        return 1
    else:
        return 0

# Function to simulate two consecutive beamsplitters by
# running the beamsplitter() procedure twice, introducing
# an optical difference if the simulated photon is
# reflected by the first beamsplitter.
def photon_in_two_bs(phase, optical_difference):

    global xi_1
    global xi_2

    # First beamsplitter.
    phase, xi_1, reflection = beamsplitter(phase, xi_1)

    # Add the optical path difference for reflected photon.
    if reflection:
        phase = (phase + optical_difference) % 2

```

```
    # The same rules are applied to the second beamsplitter
    # but the outgoing phase is not needed.
    _, xi_2, reflection = beamsplitter(phase, xi_2)

    if reflection:
        return 1
    else:
        return 0

##################################################
# Simulated experiment
##################################################

# Number of photons in each experiment.
N = 100000

# Count of reflections.
reflection_count1 = 0.0

# Using photon_in_one_bs() to check the distribution of
# single photons under these rules. Simulate a sequence
# of N photons interacting with a single beamsplitter:
for i in  range(N):

    # Count of photons leaving 'reflected' output
    # of beamsplitter.
    reflection_count1 = reflection_count1 +
       photon_in_one_bs(2.0 * random())

```

```
print('Percentage␣of␣reflections␣in␣one␣beamsplitter',
    reflection_count1 / N)

# Using photon_in_two_bs() to simulate the two beamsplitter
# experiment for a range of optical path differences.

# Subdivisions of the optical path difference.
steps = 50

# Array of results.
reflection_percentages = []

# Run through all possible optical path differences:
for j in  range(steps):

    # Count of reflections.
    reflection_count2 = 0.0

    # Randomly re-initialise phase xi_1 for first beamsplitter.
    xi_1 = 2.0 * random()

    # Randomly re-initialise phase xi_2 for second beamsplitter.
    xi_2 = 2.0 * random()

    # Simulate a sequence of N photons:
    for i in  range(N):

        # Count of photons leaving given output of
        # second beamsplitter.
```

```
        reflection_count2 = reflection_count2 +
            photon_in_two_bs(2.0 * random(), 2.0 * j / steps)

    # Log reflection rate for given optical path difference
    reflection_percentages.append(reflection_count2 / N)

    print(j, reflection_count2 / N)

# Plotting the results of two beamsplitter simulation.
plt.plot(reflection_percentages)
plt.ylabel('% of reflections')
plt.show()
```

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