Stochastic process model of the evolution of providers and users in a shared on-demand ride service system

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DECLARATION

The candidate confirms that the work submitted is her own and that appropriate credit has been given where reference has been made to the work of others.

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ABSTRACT

Shared use of on-demand ride service can only reduce traffic congestion and its concomitant problem if its popularity exceeds a non-shared use which is often provided through the same platform and may dominate. Endogeneity among users (i.e. availability of sharing partners) and between users and providers (i.e. feedback loop between them) embeds uncertainty in the system. It complicates predicting what level of service users will experience. Several studies modelled day-to-day dynamics in an on-demand ride service system considering a feedback loop between providers and users, yet, it is often modelled with a deterministic approach. This thesis aims to develop a stochastic process model to investigate the impact of variability on the evolution of a system attribute to the feedback loop between users and providers and the endogeneity among users.

A stochastic process model represents the day-to-day learning and decision-making process of users and providers. Users (providers) reconsider a service to request (offer) every day by comparing their experienced utility (profit) to use (provide) each service with the collective average utility (profit) of unselected service between non-shared and shared service. The collective average utility (profit) is estimated every day as the weighted sum of the mean utility (profit) among today's users (drivers) and the collective average utility (profit) of the previous day. Service shift occurs for only a proportion of those who consider a change, which reflects a range of choice inertia and hesitation towards change. Changes in fleet size and demand give rise to a new experience for users and drivers on the next day. A queueing model is utilized to model an on-demand ride service system, which provides variables to estimate utility and profit. For a shared service, a fair cost split among users is modelled with a modified Shapley Value, which is newly proposed in this research.

With the developed model, the two types of numerical experiments have been conducted with different parameter settings; 1) those with a fixed fleet size and 2) those with variable fleet size. The former experiments aimed to understand the attributes of the proposed model. The results suggest that the service network geometry is the main determinants of the stationary distribution of mode share. The experiment with unfixed fleet size showed that the proposed stochastic process consists of three regimes, the pseudo stable, the pseudo periodic, and the swan regime. It is discovered that, depending on the parameter setting, the frequency and length for each regime changes, which results in changing the stationary distribution of mode share and the proportion of fleet.

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Chapter 1 Introduction

1.1 Background

It is widely recognised that the transport sector has contributed significantly to the current climate emergency by producing an enormous amount of greenhouse gas (GHG) emissions. For instance, in 2018, approximately 15% of GHG emissions came from the transport sector in the UK(Thomas, 2020). The fundamental problem arises from the number of vehicles on the road being too many, especially private cars with low car occupancy. According to UK Environmental Accounts:2020, approximately12% of GHG emissions were generated by domestic car use in 2018 in the UK (Thomas, 2020). Shifting travellers from car to mass transit is one solution to reduce GHG emissions by reducing traffic volume and vehicle kilometres travelled (VKT). It could also contribute to other concomitant problems such as traffic congestion which is economically and environmentally unfavourable. However, car-based mobility has set our expectations for well-connected "door-to-door" travel as the standard transport option. This level of service and convenience is challenging to achieve with mass transit.

Recently, transportation network companies (TNCs) (e.g. Uber, Lyft, and DiDi) are recording rapid growth worldwide. TNCs provide a platform for an on-demand online matching between those willing to take a ride and those willing to offer a ride. Though TNCs were founded only five to ten years ago (e.g. Uber in 2009, Ola in 2010, and Didi, Grab and Lyft in 2012), they have already reached 15 to 20 billion rides in 2018 (ITF, 2020). Initially, on-demand ride services provided by TNCs were expected to reduce the amount of traffic and, therefore, total VKT by solving the "last mile problem", which is often identified as the main barrier to use mass transit (Docherty et al., 2018). The "last mile problem" is the lack of adequate transport options from a transit station to the final destination (Wang, Hai and Odoni, 2016).

Docherty et al. (2018) pointed out that the rapid growth of TNCs and on-demand ride services are led by a technological sector that provides software and hardware. From the technological industry’s perspective, it is more beneficial to have a bigger market to maximise their returns on investment. Therefore, they seek to encourage more mobility, not less. Thus, there is a need for intervention from transport planners and policymakers to ensure that the introduction of on-demand ride services will bring the desired outcome, reducing VKT. It is, therefore, crucial to understand how an on-demand ride service evolves over the long term to deliver effective measures.

1.2 **Characteristics of on-demand ride services**

On-demand ride services are described as a transport service via an on-demand ride service platform, enabling app users to match with potential sharing partners and drivers. On-demand ride service is classified as one type of shared mobility, specifically within the category of shared ride services. Shared ride service is defined as two or more people share their trip simultaneously. Another type of shared mobility, car sharing, indicates two or more people share a vehicle over time, yet, not their ride, which is beyond the scope of this study.

Following the classification conducted by Shaheen and Cohen (2019), shared ride services can be divided into three categories: 1) core pooled services, 2) ridesharing, and 3) on-demand ride services. Core pooled services include a wide range of non-app-based passenger transport which involves pooling – simultaneous sharing of a vehicle journey between multiple travellers (e.g. bus and shuttle). Ridesharing indicates a non-profit shared ride between two or more travellers (e.g. carpooling and vanpooling).

The difference between ridesharing and on-demand ride service is in the drivers' motivation and the presence of drivers' primal origin and destination (OD). In on-demand ride services, drivers offer services to passengers (service users) to make a profit. Therefore, they do not have their own specific ODs to travel and, instead, drive to the origin of a trip requested by users, which produces "empty-miles". On the other hand, ridesharing drivers have their OD, and their motivation is often to share the cost of travelling between that specific OD. Therefore, a trip to pick up a passenger is treated as a detour rather than producing empty miles.

On-demand ride services could be further classified into two subcategories[1] based on if they are used as a non-shared (non-shared on-demand ride service) or shared service (shared on-demand ride service) (see Figure 1). A non-shared on-demand ride service implies that a vehicle accommodates only one trip request at the same time. It should be noted that a trip request could contain more than one user but contains only one OD pair. Ridesourcing and taxi/e-hail[2] services are classified into this category. A shared on-demand ride service indicates that one vehicle accommodates two or more requests at the same time. The example of this is

[1]Shaheen and Cohen (2019) did not classify an on-demand ride service in this manner. Hence, this subcategorization is inspired by them but original in this study

[2] In Shaheen and Cohen (2019), taxi and E-hail are not clearly defined if they are categorised as an on-demand ride service. However, in this study, both services are classified as an on-demand ride service

ridesplitting, taxi sharing and Demand Responsive Transport (DRT). Detailed descriptions of each service are summarised in Table 1.

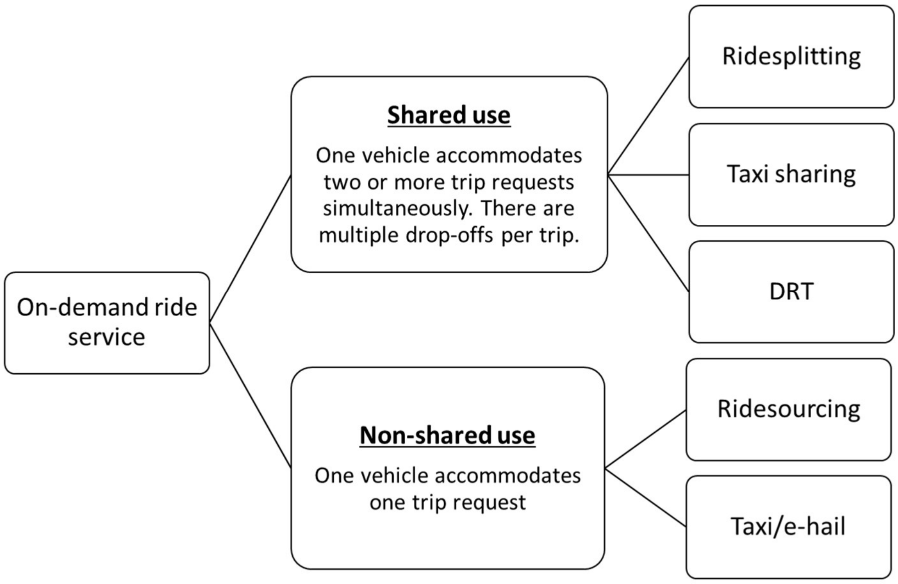

Figure 1 the subcategorisation of on-demand ride service and example of transport service belonging to each subcategory

Table 1 Description of different forms of on-demand services (Jin et al., 2018, Shaheen et al., 2016, and Shaheen and Cohen, 2018)

Service name	Definition
Ridesourcing	Ridesourcing refers to transport services that connect a driver with a private vehicle and a passenger through a mobile app provided by Ridesourcing company also known as Transport Network Companies (TNCs), mobility service providers and ride-hailing.
Ridesplitting	A variation of Ridesourcing where multiple passengers with different origin and destination but a similar route share their rides at a reduced fare.
Taxi sharing	Taxis pick up two or more unaffiliated individual or group with different origins or/and destinations. It is also referred to as taxi sharing, taxi splitting, shared ride taxis.
Taxi/E-hail	Traditional taxi picking up only one group of people with the same origin and destination. If it is operated with an app-based system, it is defined as an e-hail service.
DRT	Demand responsive transit service typically consists of a van or a minibus. It can have a flexible route or/and schedules depending on service. It is also referred to as Microtransit.

Table 2 compares shared on-demand ride services, non-shared on-demand ride services, and mass transit systems (e.g. buses and trams). A publicly operated mass transit (e.g. buses, trams) is intended to provide a low-cost service to many users. However, their routes and schedules are fixed, which requires users to adjust their travel plans accordingly and is often perceived as inconvenient. On the other hand, a non-shared on-demand ride service (e.g. taxi) can offer customised door-to-door service, yet, its cost is higher than mass transit, which could be perceived as unaffordable. Shared on-demand ride services can provide a semi-individualised service with a medium-ranged price; in other words, a service "somewhere between" bus and taxi (Ho et al., 2018). Hence, it is expected to perform as an affordable and convenient service and be the middle solution.

Table 2 Comparison among mass transit, shared and non-shared on-demand ride service in terms of various attributes (modified from Ho et al., 2018)

	Mass transit	Shared on-demand ride services	Non-shared on-demand ride service
Route	fixed	fixed/flexible	flexible
Schedule	fixed	semi-fixed/ by request	by request
Speed	slow	Medium	fast
Cost	low	medium	high
Mode	shared	shared	non-shared
Capacity	High	medium	low
Reservation	not needed	often needed	often not needed

1.3 Shared on-demand ride service: good or bad?

Several studies suggest the positive impact a shared on-demand ride service could bring. Using taxi trip data in New York City, Santi et al. (2014) estimated that replacing some compatible non-shared taxi trips with shared taxi trips could reduce the number of trips by approximately 40%, reducing carbon dioxide emissions by 423 grams per mile. Chen, X. et al. (2018) analysed trip data from a ridesplitting service, DiDi Express, and ridesharing platform, DiDi Hitch, in Hangzhou, China and conducted a questionnaire survey to service users. The survey results showed the potential of ridesplitting to reduce private car usage and vehicle kilometres travelled (VKT).

Despite the positive expectation towards shared on-demand ride service, it does not guarantee beneficial outcomes. As a result of a before-and-after assessment using real-world data, Erhardt et al. (2019) concluded that Transport Network Companies (TNCs), which could offer both non-shared and shared on-demand ride service, added congestion and VKT in San Francisco as well as worsening travel time reliability. Wu et al. (2018) suggested from a case study that the introduction of TNCs[3] in China has generated additional GHG emissions. Also, several studies discovered that on-demand ride services attract users from mass transit (e.g. buses and trains) as well as from active transport mode (e.g. bicycle and walking) (Schwieterman and Smith, 2018, Chen et al., 2018, Clewlow and Mishra, 2017, Tiranchini. A., 2020). The reduction in VKT becomes less when the modal shift occurs from mass transit or active transport mode more intensely than a private car (Chen et al., 2018).

As on-demand ride service could be a substitute and supplement of mass transit, coordination between shared on-demand ride service and mass transit is essential to offer a more flexible and affordable door-to-door mobility option without the need for a private vehicle. It is also the background idea of the new concept, "Mobility-as-a-Service (MaaS)" (Matyas and Kamargianni, 2017). Ultimately, unless the introduction of a trip-sharing mode induces a shift from low occupancy car to high occupancy car (and other) mode, the level of road traffic will not decrease. Besides, even if they are well-integrated, it does not necessarily mitigate negative externalities unless shared use of on-demand ride service becomes the standard option. Considering that on-demand ride services produce "empty miles", they could generate more VKT than private cars, especially if used as a non-shared service. Several simulation studies indicate that the mean number of passenger of on-demand ride service needs to be higher than 1 per vehicle to achieve optimal service use and reduce VKT (Alonso-Mora et al., 2017). However, the growth of a shared on-demand ride service tends to be much limited compared to a non-shared on-demand ride service (Tirachini et al., 2020).

Since both non-shared and shared on-demand ride services are often offered through the same platform, there are almost no physical or technical barriers for users to change between them. With the tendency of on-demand ride services to substitute mass transit, it is crucial to understand how mode shift between shared and non-shared on-demand services occurs to prevent the introduction of on-demand ride service from bringing negative impacts to the current transport system.

[3] Wu et al. (2018) use the term “online car hailing service” to describe services provided by TNCs such as Uber, Lyft, Didi.

1.4 The evolution of transport service system depends on feedback

The existence of a feedback loop between service providers and users has been well recognised and studied primarily in the context of mass transit, such as bus lines (Bar-Yosef et al., 2013). When there is a decline in bus service ridership, the service operator tends to cut operational costs to compensate for the revenue reduction instead of improving the level of service to attract more customers. Deterioration in the service level can further reduce the number of customers. This phenomenon is called a vicious cycle and is observed in real-world operations. In contrast, a virtuous cycle indicates the tendency of popular services to keep attracting more demand as they can afford to improve their service level (Bar-Yosef et al., 2013).

In mass transit, the strategic attributes of services such as routes, schedules and fares are updated periodically. Hence, service level does not vary dramatically on a daily basis unless some unexpected event occurs (e.g. accident, natural disaster). On the other hand, an on-demand ride service has a classic two-sided market in which changes occur in both the service user and provider (i.e. driver) side every day (Wang, Hai and Yang, 2019). As a freelancer, a service provider (i.e. driver) can decide whether to work on the platform every day as well as when, where, and how long to work[4]. Hence, the service capacity could change every day based on drivers' decisions. It implies a more tightly coupled feedback loop between service users and providers; changes can happen more intensely in the shorter term compared to conventional mass transit.

As mentioned in section 1.3, switching between shared and non-shared use of on-demand ride service does not have any physical or technical barrier. However, each service could impact the transport system in a very different way (e.g. decrease/increase in VKT and CO_2). Given the potential of a feedback loop to occur shorter than the conventional mass transit, the small shift between non-shared and shared use may ultimately result in a significant change in the impact that the on-demand ride service system brings. Thus, a feedback loop between users and providers of on-demand ride service is an essential aspect to consider to understand the long-term evolution of the system.

[4] It should be noted that some traditional taxi drivers do not have such flexibility as an employee of a taxi company.

1.5 Sources of uncertainty in an on-demand ride service system and their consequences

Uncertainty plays a significant role in determining the performance of the on-demand ride service system in the real world. There are three primary sources of uncertainty in the system; users' behaviour, providers' behaviour, and variation in the traffic system. In the real-world transport system, travel time varies between days and time of day due to random variation in traffic volume (Fu, 2002). Besides, the total number of service requests can vary every day for various reasons (Tang et al., 2018, Wang et al., 2019, and Zhang et al., 2016). Some studies also consider behaviour such as a user's delayed arrival at the pick-up location(Heilporn et al., 2011) and user's cancellation/no-show up to the pick-up location (Xiang et al., 2008). As summarised by (Wang, Hai and Yang, 2019), providers' decide each day whether, where, and how long to provide a service embed a variability to service capacity.

As there is no fixed route and schedule, the availability of an on-demand ride service at a given time and a place is influenced by past, current, and future users' decisions within a day. In the case of a shared on-demand service, the availability of other users who can and are willing to share a ride is an additional factor affecting service usage (Thaithatkul et al., 2019). Day-to-day changes in demand and supply of the service embed further uncertainty in users' and providers' experience. Users' mode choice and providers' working decisions ultimately rely on users/providers being satisfied with their experience in using/offering a service. As multiple sources of uncertainty influence those experiences, excluding such uncertainty would result in misrepresenting how users and providers will behave and, consequently, how such a system evolves in terms of its performance.

1.6 Day-to-day dynamics analysis in transport research

It is a commonly used approach to represent a transport system with a mathematical model to understand system characteristics and "predict" the possible impacts resulting from alternative measures. Day-to-day dynamics analysis is one approach that can represent the evolution of the traffic state or system state over time, and the benefit of this approach is well recognised (Li and Yang, 2016). Day-to-day dynamics analysis is one type of dynamic analysis representing an evolution of a system that occurs in discrete time from a given day to the next day (Cantarella et al., 2019). It investigates the evolution of the transport system through changes in user's behaviour and the adjustment of the user's behaviour caused by a system evolution (Smith et al., 2014, Li and Yang, 2016). Another type of dynamics analysis is within-day dynamics which occurs over continuous time within a whole or part of a given day. It is often used to investigate dynamic traffic assignment (DTA) (Cantarella et al., 2019)

A deterministic process (DP) model is one approach to investigate day-to-day dynamics. Horowitz (1984) first introduced DP models for day-to-day dynamics analysis in the transport context (Cantallera et al., 2019). Several studies successfully represented feedback loops between users and providers in passenger transport service with DP models and investigated long term evolution in bus systems (Bar-Yosef et al., 2013 and Cantallera et al., 2015) and on-demand ride services (Djavadian and Chow, 2016, Djavadian and Chow, 2017, and Thaitakulu et al., 2018). Analysis with DP models can examine under which conditions the system state might evolve towards an equilibrium state if it exists and, if not, whether the system will evolve towards some basin of attraction (Cantallera et al., 2019). Such insight cannot be obtained from equilibrium analysis since this only provides the end state and not the adjustment process to reach the endpoint.

A stochastic process (SP) model is another approach to conduct day-to-day dynamics analysis introduced into the transport context by Cascetta (1989). SP models include the uncertainty/variability in the system with random variables following probability theory, while DP models are described with deterministic variables. The inclusion of uncertainty changes the nature of analysis and the outputs of SP models when compared with DP models. Unlike DP models, SP models do not have one or more “fixed points”; instead, they could have a “unique distribution” that describes the likelihood for each state to occur (Watling, D. P., 1995). In DP models, parameter changes typically change the attractor to which a process converges if there is one. In SP models, changes to parameters could change the “shape” of the distribution; in other words, the probability of each state to occur.

1.7 Research motivation, aim and objectives

As discussed in section 1.3, on-demand ride services could easily negatively impact the performance of the transport system, which brings further impacts on wider society due to the empty mile problem and the potential to compete with more sustainable mass transit modes. Therefore, the implementation of on-demand ride service should be carefully navigated with a long-term and strategic point of view as well as a business or operational point of view. As discussed in section 1.1, intervention from transport planners is essential since the rapid growth of on-demand ride services is sought by the profit-motivated technology sector whose primary purpose is to expand the market, i.e. to induce more mobility.

There is existing research investigating how much impact on-demand ride services would bring (e.g. reduction in VKT) when on-demand ride services are used in a certain way, as summarised in section 1.3. However, as explained in section 1.4 and 1.5, there is no guarantee that the introduction of an on-demand ride service to a transport system will invoke a desirable transition, such as efficient collaborations between on-demand ride services and mass transit and the extensive use of shared services. Therefore, it is essential to investigate how the system may evolve and which of the potential end-points would guarantee a positive outcome from the scheme.

As explained in section 1.6, day-to-day dynamics analysis can be used to investigate the evolution of such a system. In particular, a stochastic process model is a powerful approach to represent dynamic changes considering uncertain components of the system. It could represent the day-to-day dynamic changes in users' and providers' choices within an on-demand ride service system, capturing the feedback loop between them. A stochastic process can also capture various sources of uncertainty by utilising random variables. It enables investigation of the influence of uncertainty on the evolution of an on-demand ride service system. Analysis of the day-to-day dynamics of on-demand ride services has so far received little attention in the research literature.

Thus, this thesis is motivated by understanding the long-term evolution of an on-demand ride service system using a mathematical model. It aims to develop a model that investigates the impact of variability on; (i) the evolution of system attributes (ii) the feedback loop between users and providers, and (iii) the interdependency among users. In order to include those three points effectively, A stochastic process approach is utilised which has never been done in the previous research, hence, this choice of modelling approach brings the originality to the research. The relevance to the state-of-the-art research is summarised in section 2.8 in Chapter 2. Objectives of this research are;

O1. to specify and develop a stochastic process model that represents the long-term evolution of an on-demand ride service system that provides non-shared and shared use.

O2. to extend the model in O1 to include the impact of the availability of both sharing partner and a vehicle to the users’ experience by simplifying the service supply process with a queueing representation.

O3. To propose a fair cost distribution strategy within the framework developed in O2, which captures the trade-off aspects of shared services, such as a reduction in monetary cost and increase in in-vehicle time.
O4. to understand the properties of the proposed models by conducting numerical experiments within a simple setting where the fleet size for each service is fixed.
O5. to investigate how the system evolves under different parameter settings through numerical experiments where both fleet size and mode share for each service change day-to-day.

1.8 **Outline of the thesis**

Chapter 2 (Literature review) provides the two types of literature review motivated by different objectives: 1) to position this research and 2) to understand the on-demand ride service system to identify essential aspects of services that should be included in the model. Based on the literature review, Chapter 2 discusses the gaps in existing literature in relation to objectives specified in the previous section.

Chapter 3 (Model specification) provides a detailed description of the proposed stochastic process model. The model is divided into two parts (i.e. learning and decision model and supply model) described separately in each subsection. The detailed assumptions and the reflections on them are also summarised. Objective O1, O2 and O3 are achieved in this Chapter.

Chapter 4 (Numerical experiments with fixed fleet size) provides the results of the numerical experiments with fixed fleet size. The experiments were designed to investigate how day-to-day evolution occurs and the key parameters that contribute to changes in the trend of the process within a given parameter setting. In addition, sensitivity analysis and scenario experiments are conducted to understand how changing some parameters would affect the model's behaviour. Objective O4 is delivered in this chapter.

Chapter 5 (Numerical experiment with variable fleet size) summarised the results of the numerical experiments with variable fleet size. As both drivers and users make their service choice every day, the system's evolution is less predictable than the fixed fleet size case. The main focus of the analysis was to investigate the attribute of three regimes appeared with variable fleet size case. The condition and frequency for each regime's occurrence were also investigated and are summarised. Objective O5 is delivered in this chapter.

Chapter 6 (Conclusions) draws together the main findings and the key contributions. Besides, the critical reflection on the proposed model is presented. In the same chapter, the future research needs are also identified and presented.

Chapter 2 Literature review

2.1 Introduction

This chapter provides the literature review conducted with two different motivations, which are;

1) To identify the gaps in the existing literature to set the scope of current research
2) To understand the nature of on-demand ride service systems to identify the key components to be included in the model and to be investigated through the numerical experiment

Section 2.2. to 2.4. provide a review with the first motivation. In section 2.2., a comprehensive review is provided on studies about day-to-day dynamics analysis in the context of on-demand ride services and other public transport modes (e.g. conventional buses and bike sharing). In section 2.3. the review on modelling on-demand ride services is conducted focusing on the simplified representation of the service system. Section 2.4. provides a brief summary of the pricing problem in the on-demand ride service system.

Section 2.5 to 2.7. provide a review with the second motivation. In section 2.5., the observed characteristics of trips made by on-demand ride services are summarised. Section 2.6. and 2.7. summarise the motivation and attitude of users and drivers towards using/providing shared and non-shared on-demand ride services. It is very recent that empirical evidence regarding behavioural aspects of on-demand ride service users and providers has become available. Hence, the quantity of publications regarding those points is not many; however, they offer interesting insights. In section 2.8, research gaps in existing literature reviewed in this chapter is identified and summarised in relation to each objective specified in section 1.7. Finally, this chapter is concluded with a summary provided in section 2.9.

2.2 Day-to-day dynamics analysis in a passenger transport service

This section summarises the existing studies which analysed day-to-day dynamics in the passenger transport system. In particular, an application of the deterministic process and the stochastic process is mainly discussed. The review focuses on studies on on-demand ride service; however, those focusing on other passenger transport service (e.g. conventional bus service) is also included as dynamics between users and providers for those services has a relevance.

A few research analyse the day-to-day dynamics in a conventional bus service (Bar-Yosef et al. 2013, Cantallera et al., 2015, and Li and Yang, 2016). Bar-Yosef et al. (2013) modelled a feedback loop in bus service (i.e. vicious and virtuous cycle). They assumed a circular bus line in which the number of buses determines the service frequency. The total number of buses is estimated by the total number of potential bus users in their model. Some proportions of potential users are captive users who have no other choice than to use the bus. Others are non-captive users who can choose if they use a bus or not based on their experience (i.e. waiting time).

With this simple representation, they analytically identified the threshold value for the total number of potential users and the proportion of captive users, leading to three states with; 1) single equilibrium point with low bus usage, 2) two equilibrium points, and 3) single equilibrium point with high bus usage. The vicious and virtuous cycle occurs in the second state. In that state, Bar-Yosef et al. identified the threshold fleet size of the bus, above/below which the system enters a vicious/virtuous cycle. Regarding policy implication, they suggested that, in the long-term, it would be beneficial for bus companies to increase the fleet size above the most profitable amount to enter the virtuous cycle, which results in higher bus ridership.

Cantallera et al. (2015) developed a day-to-day dynamics model in a bi-modal network comprising buses and private cars. They considered the impact of mode choice on the congestion level of the network, which affects the travel time, and hence mode choice the following day. They suggested the two utility functions for bus users with and without the disutility regarding a bus's crowdedness. Through numerical experiments, they observed multiple equilibria with both utility functions. They also investigated the impact of dispersion parameter, which control the degree of dispersion in the user mode choice behaviour. The low dispersion parameter indicates less variance in the user's mode choice, while the high dispersion parameter indicates the high variance in the user's mode choice.

There are two stable fixed points, "many-on-bus" and "all-on-car", when the dispersion parameters are low in the case with a utility function without the crowding disutility. However, the "many-on-bus" point moves towards "equal-modal-split", which then becomes a unique fixed point as the dispersion parameter increases. Therefore, as a policy recommendation, they suggested maintaining the dispersion parameter low to encourage the "many-on-bus" scenario. A unique "all-on-car" stable fixed point is observed when the dispersion parameter is low for the case with the crowding disutility. With the high dispersion parameter, an "equal-modal-split" stable fixed point appears as well as an "all-on-car" fixed point. Hence, the dispersion parameter needs to be kept high to avoid the “all-on-car” scenario. They also investigated the case where a bus operator changes the fleet size according to the service demand. Regardless of the dispersion parameter value, it always has a unique equilibrium point. As the dispersion parameter increases, they observed that the unique equilibrium point moves towards the “equal-modal-split” stable fixed point.

Li, X. and Yang (2016) also investigated day-to-day dynamics in the bi-modal network consisting of bus and private cars assuming heterogeneous users. They assumed the bus travel time is not influenced by the traffic volume, which implies that the bus runs on the reserved lane. In their model, the users make day-to-day mode choice while the bus operator changes the number of buses periodically. The numerical experiment is conducted to investigate the impact of travel demand, bus capacity, and service frequency.

The results of their numerical experiments show the existence of two domains of attraction, one of which leads to high auto-share and the other leads to the almost equal split of mode share. As the latter domain area is larger, the *vicious* cycle (in terms of bus ridership) is unlikely to happen. The only condition for the *vicious* cycle to occurs is when the initial auto-share is very high. They recommended subsidising bus lines to decrease fare or/and increase the service frequency to prevent the system from entering the vicious cycle with such condition, consistent with Bar-Yosef et al. (2013)’s recommendation. They also discovered that a smaller bus capacity (i.e. less than 60) causes the *virtuous* cycle. With a given travel demand, reducing bus capacity increases the frequency of service and reduces travel costs by bus, which results in increasing bus ridership.

In the context of the on-demand ride service system, Djavadian and Chow (2016 and 2017) proposed the first model which explicitly represents the day-to-day dynamics of an on-demand ride service system. Unlike conventional mass transit, on-demand ride services do not have a fixed route or schedule. As a result, it is often the case that modelling day-to-day dynamics in on-demand ride service requires a higher level of detail than modelling conventional public transport (Calderón and Miller, 2019). Few studies have focussed on the long-term evolution of the on-demand ride service system through day-to-day dynamics analysis. The following section presents a detailed review of such literature.

Djavadian and Chow (2016) proposed a framework for agent-based deterministic process models of day-to-day dynamics in Flexible Transport Service (FTS). FTS, in their context, implies the ridesourcing type of service where drivers can make a daily choice of whether they enter the market on that day. Their model is designed to almost surely converge an agent-based stochastic user equilibrium (SUE) through the Method of Successive Average (MSA). In their model, users update their strategy (i.e. mode and departure time) by maximising their perceived utility which is updated through their and others' past experience. Each vehicle's routes and schedules are determined, given a fixed fleet size based on each user's strategy. That will determine the arrival time to each user's destination, which influences the user's utility, hence, their strategy on the next day (see Figure 2). Their framework has the flexibility to assess the various policies such as pricing policies and vehicle routing and scheduling policies within the context of different type of FTS (e.g. ridesharing, taxis, DRT). A numerical experiment conducted using their proposed model demonstrated that the proposed model converges to the agent-based SUE. The capability of their model to capture the impact of different policies is also demonstrated.

Djavadian and Chow (2017) extended their previous work (Djavadian and Chow, 2016) and included the day-to-day dynamics of the provider side (see Figure 3). They developed the first model to analyse a two-sided flexible transport market using an agent-based day-to-day adjustment process. This model nicely captured and simplified the interaction between users' and FTS's provider's behaviour. The users' side day-to-day learning and decision process follows their previous work, Djavadian and Chow (2016). The drivers chose whether they provide a service on the day according to expected profits estimated by their experience. That will change the fleet size of each mode every day, which was fixed in their previous model proposed by Djavadian and Chow (2016). The drivers' earnings depend on users' strategies, while users' perceived travel cost also depends on the fleet size. The proposed model almost surely converges to the agent-based SUE, which is supported by computational experiments. They conducted a case study using real data in Oakville, Ontario, with the proposed model and the model developed by Djavadian and Chow (2016). The results indicate the importance of including the drivers' day-to-day adjustment process. For instance, they observed that the threshold of acceptable profit for drivers to stay in the service affects how the service demand evolves in the long term.

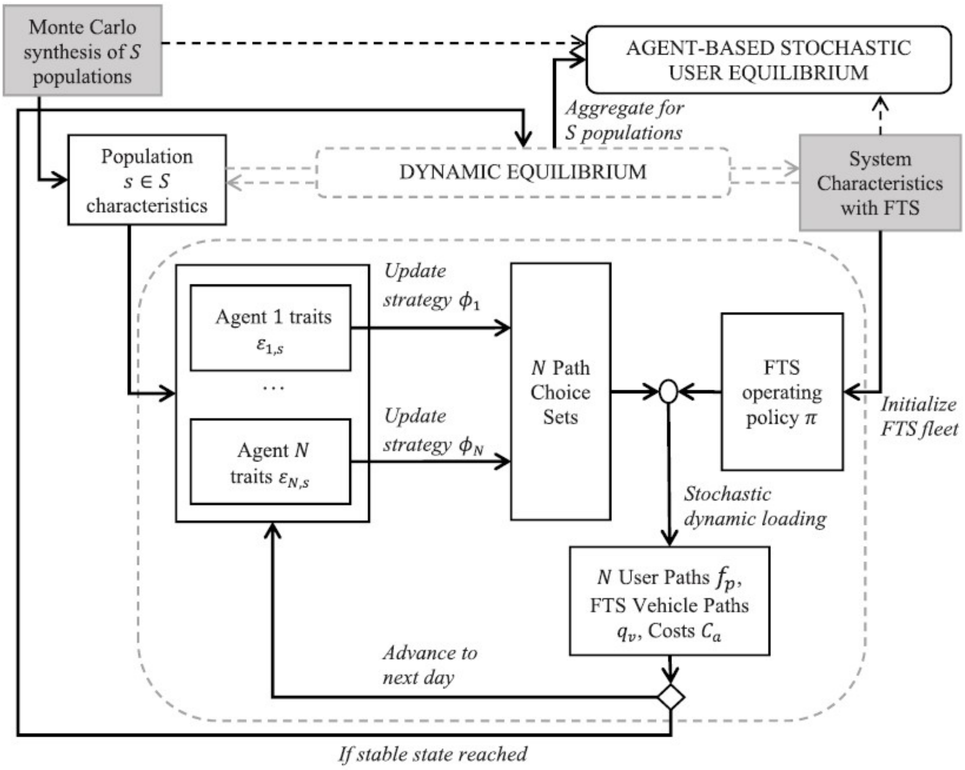

Figure 2 a conceptual diagram of the proposed model by Djavadian and Chow (2016) (Djavadaian and Chow, 2016, pp.287)

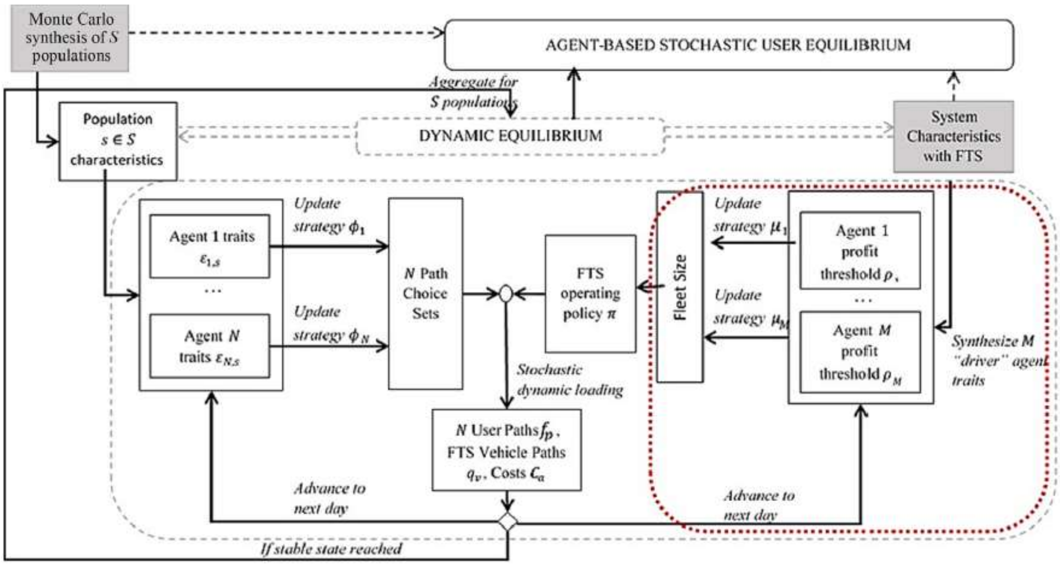

Figure 3 a diagram describing the components of the model proposed by Djavadian and Chow (2017) and their interactions (Djavadian and Chow, 2017, pp.42)

Thaithatkul et al. (2019) developed a behaviour-based dynamic ride sharing (DRS) model as a day-to-day deterministic process. They explicitly model the user-driven nature of DRS through users' mode choice and partner choice assuming their rational behaviour (i.e. utility maximisation). The day-to-day dynamics model represents how users' expectation towards the performance of DRS is updated through their experience and the collective experience of others. The within-day dynamic model represents the process of matching potential DRS users, which is influenced by and on users' mode choice (see Figure 4). It should be noted that DRS in their context indicates shared use of on-demand ride service (e.g. ride-splitting or taxi sharing) rather than "ridesharing"[5] with on-demand matching. Hence, users mode choice can be interpreted as the choice between shared and non-shared on-demand ride service. As their model focuses on the influence of partners' availability, they did not consider any impact of vehicle availability by which the user's mode choice could also be affected. They conducted a numerical experiment with the proposed model to investigate the effects of different user learning pattern and OD pattern. The results suggest that users will eventually stop choosing sharing option if the collective learning from the others' experience is excluded. They also identified the demand level corresponding to "critical mass" below which DRS cannot be sustained as the number of DRS users becomes zero eventually.

While the literature described above represent day-to-day dynamics in a deterministic process, Zhang and Schmöcker (2019) applied a stochastic process approach (i.e. Markov model) to represent the long-term change in demand, using panel data. The previously mentioned literature in this section focused on modelling supply and demand interaction and identifying if the process reaches (multiple) equilibria and, if so, what are the conditions for that. On the contrary, Zhang and Schmöcker (2019) focused on estimating the transition matrix for a stochastic process to achieve better demand prediction. They applied well-known ideas from marketing literature and introduced concepts as "customer life cycle", "potential demand", and "willingness to use" into their proposed model. The adaption of the "life cycle" concept allows distinguishing inactive state, i.e. temporal withdrawal from the service, and drop-out state, i.e. permanent exit from the service. They conducted a case study with the proposed model using panel data from Kyoto University's bicycle share system. By analysing panel data, they identified a gradual change in the usage pattern of the scheme. Also, there was no sudden drop-out observed from the frequent scheme users. Such attributes are those their proposed model is intended to take into account. The comparison between the simple Markov

[5] The term "ridesharing" is commonly used to describe the situation where a private car owner with predetermined OD finds a passenger(s) to share (part of) the trip with through a matching platform. Therefore, there are two type of users; 1) those who are willing to give a ride and 2) those who are willing to take a ride. The common motivation of type (1) user is to save their cost while the common motivation of type (2) user is to receive a door-to-door service with less cost than taxi or other private passenger service (Shaheen and Cohen 2019).

model and the proposed model using the panel data illustrates that the inclusion of the “life cycle” concept improves the model fitness.

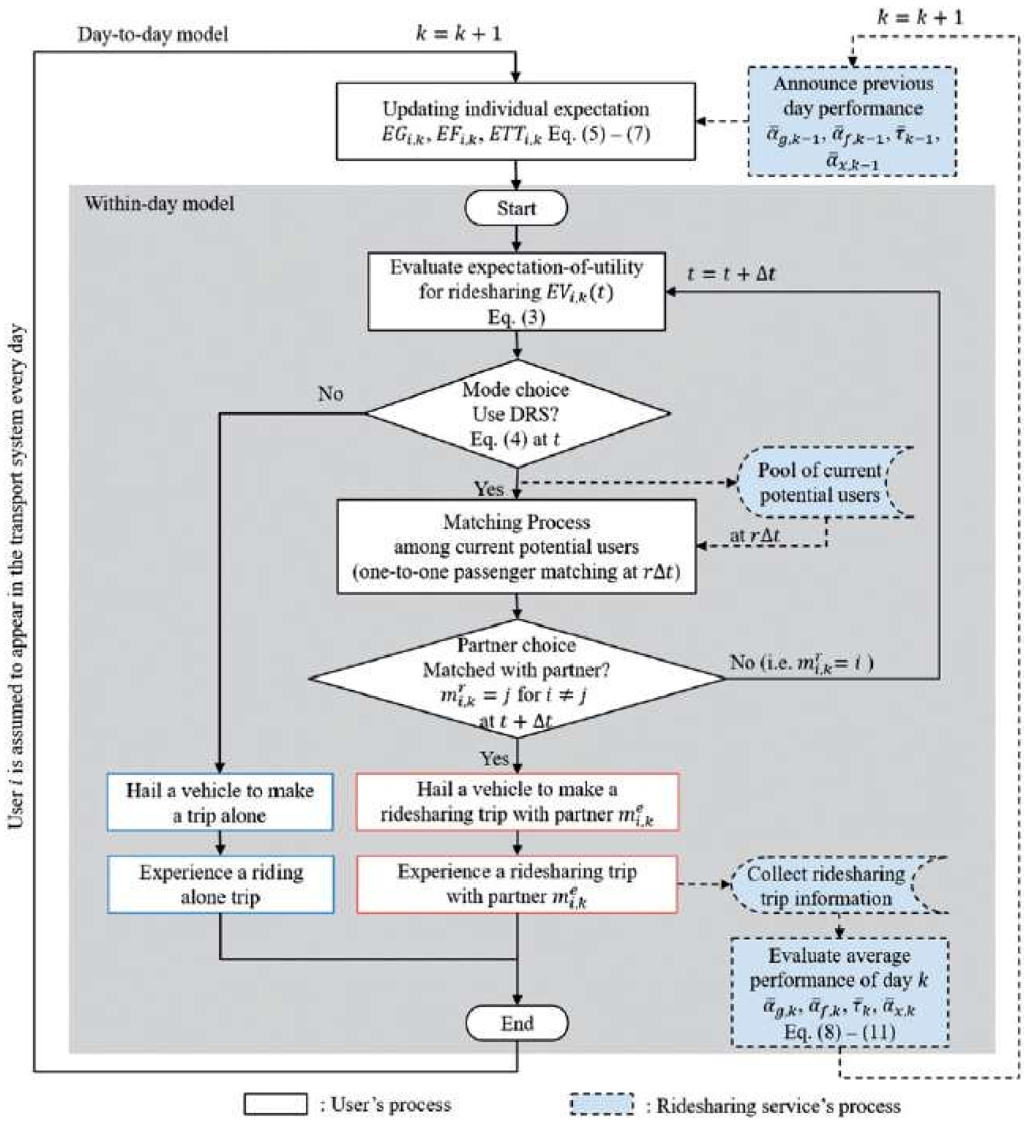

Figure 4 the diagram of the day-to-day dynamic model proposed by Thaithatkul et al. (2019) considering (Thaithatkul et al., 2019, pp.618)

2.3 **Modelling on-demand ride service**

In this section, models to represent the on-demand ride service system are summarised. To begin, the commonly used routing and scheduling problem, Dial-a-Ride problem (DARP), is summarised in subsection 2.3.1. DARP is not only applicable to on-demand ride services, it is used to determine routes and schedules for many other flexible passenger services (e.g. taxi, paratransit). Therefore, a large amount of literature has already been dedicated to investigating DARP with the motivation to provide better tools for operational planning (Ho et al., 2018 and Cordeau and Laporte, 2007). As this research is not aiming to develop a novel method to improve the existing solution algorithm or model framework for DARP, the review in subsection 2.3.1 is motivated by understanding an essential characteristic of DARP. In subsection 2.3.2, queueing representation of DARP is summarised, which is one of the common ways to simplify dynamic DARP and can include various sources of uncertainty.

2.3.1 **Dial-a-ride problem**

In order to operate on-demand ride services, it is required to match a trip request(s) with an available vehicle upon the arrival of a new request(s) and the route and schedule of the assigned vehicle changes accordingly. Planning routes and schedules for fleet mainly involves three stages; 1) clustering, 2) assignment and 3) routing(Cordeau and Laporte, 2003a). Clustering, which only occurs when the service is operated as a shared service, involves formulating a group with multiple trip requests. Assignment indicates the process of selecting a vehicle and single or grouped requests to combine. Routing consists of determining the order to visit pick-up and drop-off locations of each request and a route between stops. Depending on the nature of the system, and the operational strategy, namely, the algorithm they use, these stages could occur consecutively or interdependently (Cordeau and Laporte, 2003b)

When the fleet size is too large to operate manually, DARP is applied to determine the routes and schedules of a given fleet. DARP is one type of vehicle routing problem – a combinational optimisation and integer programming problem (Andreasson et al., 2016). It often formulates an on-demand ride service with sets of travellers and N vehicle with n seats for customers. Each traveller has a specific pick-up and drop-off time (windows) and locations (Cordeau and Laporte, 2007). Cordeau and Laporte (2007) further mention that what distinguishes DARP from the other routing problem is a consideration of human perspectives, which is usually introduced through constraints or/and as one of the objectives.

Ho et al. (2018) classify DARP based on if the service is *static* or *dynamic* and *deterministic* or *stochastic* (see Table 3). A problem is *static* if any modification in routing and scheduling is not allowed after the operation started. It implies that all information relevant to planning routes and schedules of all the fleet is known before the operation. Even if new relevant information is revealed as time passes, the “grand plan” will not be updated in the realm of the static problem. On the other hand, if the operator allows modifying the plan for routing and scheduling for a given fleet as new information revealed after operation starts, it is classified as a *dynamic* problem. According to Ho et al. (2018), a problem is *deterministic* if all the information available prior to the service operation comes with certainty. In other words, a problem is deterministic if the decision is made on the basis of *perfect* *information*. On the contrary, a problem is *stochastic* if imperfect information is assumed.

If the problem is *static* and *deterministic*, it means that all information listed below is known with certainty before the start of the operation:

1) The exact number of trip requests, their pick-up and drop-off locations and time
2) All users' exact behaviour includes delayed arrival (Heilporn et al., 2011), cancellation (Xiang et al., 2008), and any other possible changes users could request before and during their ride.
3) The exact time to complete each step of every operation (e.g. travel time between stops and time for users to get in/out of a vehicle).

If the problem is *dynamic* and *deterministic*, it implies that information regarding 1) reveals as the operation progresses. However, information regarding 2) and 3) about already existing users are perfectly known. If the problem is *static* and *stochastic*, all the routes and schedules for the whole service period is determined considering the uncertainty regarding 2) and 3) before the operation starts. If a problem is *dynamic* and *stochastic*, a route and schedule of the fleet are updated as a trip request appears considering the uncertainty of information regarding 2) and 3).

Table 3 Classification of Dial-a-Ride problem proposed by Ho et al. (2018) (Ho et al., 2018, pp.399)

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Information is known with certainty (at time of decision)</td></tr>
<tr><td>Yes</td><td>No</td></tr>
<tr><td rowspan="2">The decision can be modified in response to new information received after time 0</td><td>No</td><td>Static and deterministic</td><td>Static and stochastic</td></tr>
<tr><td>Yes</td><td>Dynamic and deterministic</td><td>Dynamic and stochastic</td></tr>
</table>

In the past, there were services with no predetermined routes and schedules. However, most of the requests were assumed to be arranged in well advance (e.g. one day before), such as door-to-door transportation services for elderly or disabled people (Madsen et al., 1995, and Toth and Vigo, 1997), airport shuttles (Reinhardt et al., 2013), and patient transport service (Hanne et al., 2009). Hence, the timely update was not necessary once the routes and schedules were fixed. In other words, it could be modelled and operated as a static problem. However, it could be a dynamic problem if operators decide to react to an unexpected event (Pillac et al., 2013). On the contrary, the current On-Demand Ride services, which this study focuses on, mostly receive trip requests shortly before the expected pick-up time. Hence, it requires constant adjustment of routes and schedules as a new request arrives. Therefore, it is, by nature, dynamic DARP.

In the real-world system, the service is always exposed to uncertainty from various sources. For instance, travel time varies between days and time-of-day due to random variation in traffic volume (Fu, 2002). Such variance in travel time would influence on predicting the vehicle arrival time to a pick-up or drop-off location. Also, users delayed arrival would cause a delay in future service. Especially for the shared service, a user's delayed arrival would delay the pick-up time of users who are scheduled to be picked up later. Such delay could result in deterioration in the service level and frustrating a driver resulting in discoursing drivers to provide shared service (Morris et al., 2020).

It is an operator's choice of how much uncertainty they would take into account. However, it should be pointed out that considering uncertainty also embeds the extra challenges (e.g. modelling an impact of stochastic component, the interpretation of outcome) and needs for higher computational power if a quick readjustment is expected, especially for a dynamic and stochastic problem (Ho et al., 2018).

2.3.2 **Queuing representation of DARP**

When the motivation to model an on-demand ride service attributes to the operational interests (e.g. developing a faster estimating algorithm), DARP needs to be precisely formulated. However, when the motivation of developing a model comes from a strategic perspective (e.g. understating long-term evolution of system), solving DARP could be too computationally demanding, especially in the case of a dynamic and stochastic problem. In addition, there is also the fact that solving DARP inevitably becomes highly case-specific. Therefore, abstracting the problem by simplifying DARP helps to understand more general characteristics of the system. Hence, it is common to utilise a simplified representation of DARP (e.g. Djavadian and Chow, 2017 and Thaithatkul et al., 2019).

Some research introduced a graph-theoretical approach to limit the combinations of route and schedules in order to conduct a large-scale simulation (Santi et al., 2014 and Alonso-Mola et al., 2017). Others applied a queuing theory as a heuristic solution algorithm for dynamic and stochastic DARP (Hyytiä et al., 2012) and as a simplified representation of DARP as a part of the other model (e.g. Djavadian and Chow, 2017 and Wang and Odoni, 2016).

Application of queueing theory has been studied as one of the approaches to represent dynamic routing problem by several authors, including Psaraftis (1988), Bertsimas and Ryzin (1991, 1993) and Swihart and Papastavrou (1999). One benefit of this simplification is that it can include the stochasticity of the system, which is often seen as the most computationally demanding form of DARP. The queueing model represents the real-life queueing phenomenon, in general, with the interest of investigating the queuing system's long-term performance for future planning and operation purposes (Bose, 2002). In particular, given a customer arrival process (e.g. arrival rate) and service mechanism (e.g. service time), two aspects, the number of people in the queue and their waiting time, are often investigated as the main focus of queuing analysis.

Originally, Psaraftis (1988) identifies the connection between the queueing problem and the dynamic routing problem when defining the dynamic travel salesman problem (DTSP). Psaraftis set the arrival process of customers following a Poisson process as with the standard queueing model. These demands are served by a “salesman” who travel from one node to another. A “salesman” spend a stochastic time at each node for serving. The objective of DTSP is to optimise the system according to some performance measure (e.g. the number of the served customer within the period or minimising the waiting time)

Inspired by Parafits’s work, Bertsimas and Ryzin (1991) and Bertsimas and Ryzin (1993) proposed a dynamic travel repairman problem (DTRP). DTRP intends to minimise user waiting time by minimising the average system time. The main difference between DTSP and DTRP is that DTRP is defined in the Euclidean plane, while DTSP is set to be on the graph of nodes. As with the case of DTSP, demand for service occurs randomly following Poisson distribution, the location of which independently and uniformly distributed in a convex service area. Swihart and Papastavrou (1999) extended DTRP into Dynamic Pick-up and Delivery problem (DPDP). Unlike DTRP, customers are not served on-site. Instead, a server picks up the customer at a pick-up location and travel with them to a delivery location. Pick-up and delivery location are distributed uniformly and independently of each other. They derived the lower bound for users' expected time to spend in the system with the light traffic and the heavy traffic cases. They applied a single server queue to investigate the case of a single-vehicle with unit-capacity where only one pick-up is allowed for one service and multi-capacity where multiple pick-ups are feasible within one service.

Hyttiä et al. (2012) proposed a non-myopic vehicle assignment and routing policy in dynamic DARP by applying the Markov Decision Process model (MDP). They suggested the framework to select the best action, which minimises the cost taking into account unknown future request. The cost is defined as the weighted sum of the system's effort (i.e. the vehicle travel distance) and the customer's interest (i.e. the mean passenger travel time). As there is an enormous number of possible combinations of routes and schedules in dynamic DARP, they simplify the system to deliver the cost function by introducing M/M/1 queue as a highly abstract representation of a vehicle in an on-demand ride service system.

They formulate a customer arrival with Poisson process and assume a service time as a required time to serve a newly requested trip which varies among queues. This feature well-reflects the real-world system where the expected pick-up time could vary among fleets depending on, for instance, the proximity of location between the pick-up spot of new request and drop-off spot of the last request. Nevertheless, it should be mentioned that spatial perspective is not directly considered in this representation but indirectly suggested into a random service time. M/M/1 queue serves only one customer simultaneously; hence, this representation can not express the pooling aspect.

Lees-Miller (2016) used queuing approach to study the lower boundary of mean passenger waiting time for the Personal Rapid Transit system (PRT). PRT is a computer-operated small pod that carries a single or small number of passengers between stations located on a network with guideways (Less-Miller et al., 2016). The purpose of their study is to deliver the reallocation strategies for an empty pod in PRT, which minimise the mean passenger waiting time. As the PRT operates on the guideways and choice of pick-up and drop-off locations is limited, a queuing model closely represents how PRT is operated in real life.

Wang and Odoni (2016) designed a hypothetical Last Mile Transport System (LMTS) as a solution for the last mile problem. The proposed LMTS is assumed to be operated under a dynamic and stochastic environment with batch demand. They simplified LMTS through a queueing model, then derived the approximate estimate of the system performance mainly focused on user waiting time as a function of system parameters such as fleet size. Though all the trip made by LMTS starts from the fixed location (i.e. station), drop-off points are not predetermined, unlike PRT studied by Less-Miller et al. (2016). Assuming that the users' destination is known prior to their arrival, users arriving in a batch are clustered into subgroups by solving VRP and assigned to each vehicle. Once a vehicle drops off all passengers at their destination following the route determined by VRP, it returns to the station to pick-up a new group. They assume a server as a vehicle and represent multiple vehicle operations by applying a multi-server queue.

As demonstrated above, the queueing model has been used as an abstract model of a vehicle in several different passenger transport system without a fixed route and/or schedule. In some special case, such as PRT, a queueing model could capture the system characteristic as it is (Less-Miller et al., 2016). The queueing application in on-demand ride service systems is often driven by the intention to reduce the enormous number of possibilities to include stochastic and dynamics in DARP when the study is motivated by operational application (Hyttiä et al., 2012 and Wang H., 2019). Nevertheless, as Wang and Odoni (2016) established, it could also be employed when a model is designed for strategic planning, where real-time operational planning is not the main focus.

2.4 **Pricing problem for on-demand ride service**

Pricing mechanisms vary among different service platforms. Usually, price is calculated based on travel time and travel distance (Li et al., 2019). Some Transport Network Companies (TNC), such as Uber and Lyft, utilise dynamic pricing, which is often called “surge pricing” (Shaheen and Cohen, 2019). Surge pricing refers to the system where the price changes according to the demand-supply balance Castillo et al. (2017). The price will increase during high demand hours then fall back to normal once demand becomes lower. It aims to balance the supply and demand of service by incentivising drivers to participate in the service at peak times and locations when the service platform does not directly control drivers' behaviour (Karamanis et al., 2020). From the user’s point of view, the monetary cost for travelling between the same locations could vary due to surge pricing.

Several studies investigated how surge pricing is changing in practice using real-world data. Henao and Marshall (2019) reported that they encountered surge pricing in only 7.2 % of trips with a range of 1.2 to 2.0 in Denver in the US. Cohen et al. (2016) found that the period when surge pricing was more than 1 in major cities in the US is less than 30%; 27.7% in Chicago, 25.1% in San Francisco, and 17.1 % in Los Angeles, 14.7% in New York. Chen et al. (2015) discovered that surge price was activated 57% of the time in San Francisco and 14% in Manhattan, mostly from 1.25 to 1.50. However, they observed that surge pricing could become higher up to quadruple prices. They also found that surge pricing would last not so long and less than 10 mins. The majority of the time, it was less than 5 min.

There is an additional pricing problem for shared services, which can be described as a “fair cost distribution” problem, in other words, how to split the cost among more than two passengers of shared service in a somewhat equitable way. It should be pointed out that this problem is also applicable to conventional taxi sharing, which does not use dynamic pricing. Karamanis et al. (2020) indicated the lack of consideration for “cost distribution” aspects in an on-demand ride service context, especially for dynamic pricing problem. On the other hand, a fair cost distribution problem has been investigated in the goods delivery context and modelled as Travelling Salesman Game (TSG) by Potters et al. (1992). Aziz et al. (2016) summarise this game as “a cooperative transferable utility game in which agents correspond to locations in a travelling salesperson problem (Aziz et al., 2016, pp.573). This game's solution method is often taken from cooperative game theory, especially Shapley value (Shapley, 1953).

Potters et al. (1992) stated three principles for travelling salesman games;

1) The contribution of the sponsors (passengers in this context) sums up the total cost (“efficiency”).
2) No sponsor pays more than the cost of a direct trip from the traveller’s home city (the origin of trips) to the sponsor’s residence (passengers destination) and back (“individual rationality”)
3) Each sponsor (passenger) pays at least his own marginal cost (“minimal obligation”)

Marginal cost in this context indicates the difference between the total cost and the cost when a passenger’s destination is skipped—for instance, assuming the case like Figure 5 where a service vehicle needs to travel from and to a black star (e.g. pick-up spot) visiting two drop-off points.

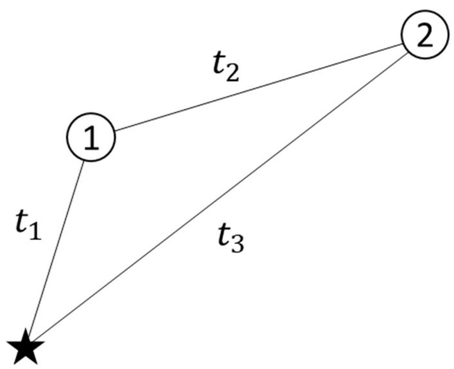

Figure 5 An example case with two drop-offs

In such a case, the marginal cost for user 1 and user 2 can be calculated as below;

$$mc(1) = t_1 + t_2 + t_3 - 2t_3 = t_1 + t_2 - t_3 \quad (2.1)$$

$$mc(2) = t_1 + t_2 + t_3 - 2t_1 = t_2 + t_3 - t_1 \quad (2.2)$$

where $mc(1)$ and $\mathrm{m}c(2)$ indicates a marginal cost for each user. Potters et al. (1992) stated that if the route has the minimum length, the sum of marginal cost does not exceed the total travel cost, which, in this case, means;

$$mc(1) + \mathrm{m}c(2) = 2t_2 < t_1 + t_2 + t_3 \quad (2.3)$$

The statement above is proven to be true following the formulation condition of a triangle, which is;

$$t_2 < t_1 + c \tag{2.4}$$

The common solution method for TSG, Shapley value, for this example, can be estimated as follows;

$$\tau(1) = t_1 - t_2 + t_3 + \frac{t_1 - t_2 + t_3}{2} \tag{2.5}$$

$$\tau(2) = -t_1 + t_2 + t_3 + \frac{t_1 - t_2 + t_3}{2} \tag{2.6}$$

where $\tau(1)$ and $\tau(2)$ are Shapley value for user 1 and 2. Following "individual rationality", the principle specified by Potters et al. (1992), the maximum willingness to pay for each user is assumed to be equivalent to when they use the exclusive service (i.e. $2a$ and $2c$). The total benefit of sharing scheme is:

$$\psi = 2t_1 + 2t_3 - (t_1 + t_2 + t_3) = t_1 + t_3 - t_2 \tag{2.7}$$

Application of TSG and Shapley value in on-demand ride service has been introduced by Levinger et al. (2019). However, as Levinger et al. (2019) suggested, Shapley value is rarely mentioned in the on-demand ride service context, possibly, because Shapley value is computationally demanding. Services involving dynamic ridesharing (DRS) such as ride-splitting and taxi sharing requires fast estimation. Hence, Shapley value may not be attractive enough, especially for an operational model. Besides, in some cases, pricing is considered in the process of clustering of users and assignment to drivers, for instance, as a combinational double auction model (Karamnis et al., 2020). In such a case, the analogue of TSG, determining the route before distributing the price, is not necessarily applicable.

2.5 Trip characteristics of on-demand ride service

This section summarises the trip characteristics of the on-demand ride service. Mainly, the study using real data are reviewed and summarised. It is essential to understand the basic attributes of trips made by on-demand ride service to establish reasonable assumptions for a proposed model and determine the threshold of various parameters for numerical experiments. In subsection 2.5.1., the general trip characteristics made by on-demand ride service is summarised. In the following subsection, the spatiotemporal features are mainly focused on and discussed.

2.5.1 Observed trip characteristics

When one platform is offering both shared service and non-shared service, the results of several studies consistently indicate that the non-shared service is used much more than the shared service (Li et al., 2019, Young et al., 2020, Shaheen and Cohen, 2019). For instance, Li et al. (2019) found that the percentage of shared ride orders among total orders for a resourcing company, DiDi, Chengdu, China, is 6.2 %. Young et al. (2020) found that 14.80% of trips made through a resourcing platform, Uber, was shared use in Toronto, Canada, between September 2016 and March 2017.

Li et al. (2019) discovered that 90.50% of shared rides are shared between 2 trip requests, whereas the trips shared among 3 and 4+ requests are minimal at 9.33 % and 0.17%. Li et al. (2019) pointed out that this is because DiDi allows drivers to accept a maximum of 2 shared ride requests at one time. For one request, a maximum of 2 people is allowed to be picked up from one location. Although this is a specific rule for DiDi, Li et al. mention that similar regulations are set by most resourcing companies (e.g. Uber and Lyft) to reserve a seat for future passengers.

Young et al. (2020) determined that 51.7 % of the trips made by pooled ride-hailing service[6] (i.e. UberPOOL) were unmatched in Toronto, Canada. “Unmatched” indicates that a requested pooled ride was completed without sharing the ride with other pooled ride requests. Schwieterman and Smith (2018) identified that 60% of trips were matched among their sample data collected in Chicago, USA. Young et al. (2020) also discovered that the likelihood of matching increases as the origin of a trip request becomes closer to the downtown area, where the demand for pooled ride-hailing service is higher.

Travel time distribution is found to be positively skewed for both non-shared and shared ride in different cities. (Li et al. 2019, Chen et al., 2017, and Haglund et al., 2019). In their context, travel time is the duration from the start to the end of a trip. The mean travel time for both non-shared and shared services varies depending on the area of the study. This heterogeneity is attributed to several factors, including the city structure, the purpose of trips (Schwieterman and Smith et al. 2018), and which transport mode an on-demand ride service has replaced.

Few studies compare the difference between non-shared and shared service offered by the same platform. Li et al. (2019) estimated the mean travel time as 21.53 min for a non-shared ride and 31.98 min for a shared ride among trips made by DiDi in Chengdu, China. As illustrated in Figure 6, Li et al. (2019) further analysed that the mean delay caused by sharing is approximately 9.86 min, accounting for 30% of travel time. In Toronto, Canada, Young et al. (2020) showed that the mean travel time is 21.82 min for a matched shared ride and 17.23 min for

[6] It is classified as one type of shared on-demand ride service in the context of this study.

an unmatched shared ride. It indicates that detour travel time accounts for 16.7% of total travel time for a shared ride.

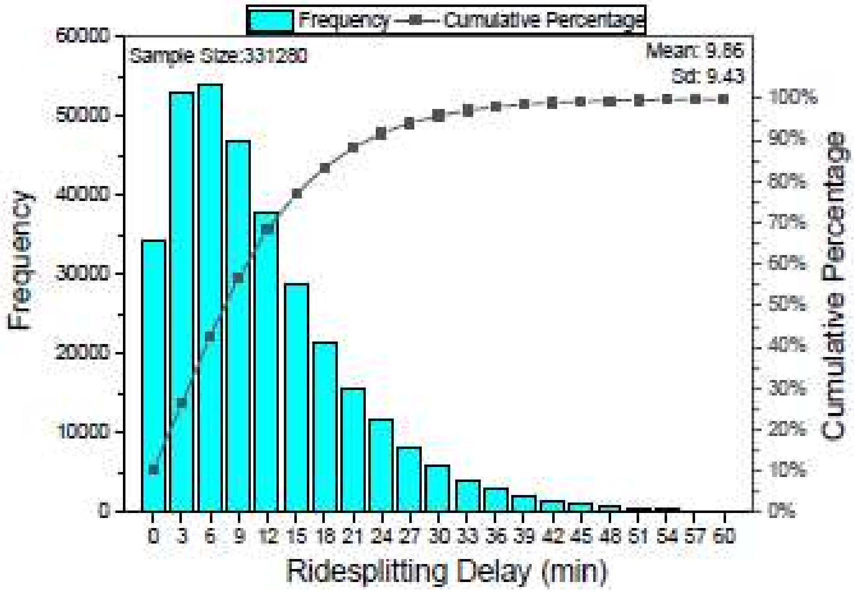

Figure 6 Distribution of delay caused by sharing a ride [min] (Li et al., 2019, p.345)

In terms of waiting time, Schwieterman and Smith (2018) estimated that the mean waiting time for UberPool was 7.3 min among their sample trips in Chicago in the US, where the mean travel time was 35.9 min. Chen et al. (2017) identified that waiting time is 6.13 min on weekdays for non-shared service in Hangzhou, China, where the mean travel time is 18.72 min for peak hour and 12.78 min for an off-peak hour.

Haglund et al. (2019) investigated an on-demand micro-transit pilot, the Kutsuplus[7], in Helsinki Metropolitan Region (HMR). They discovered that the average interval between acceptance of a trip request and user pick-up (i.e. waiting time) was 20.87 min for the Kutsuplus pilot, where the average trip duration was 16.98 min. However, the waiting time distribution was positively skewed, the few cases with extremely high-value impact on the mean. For this reason, Haglund et al. argue that users experienced a shorter waiting time than 20.87 min in most cases, and it was perceived as acceptable.

2.5.2 **Observed spatiotemporal characteristic**

There has been extensive research on understanding the spatial and temporal characteristics of demand for on-demand ride service. Studies are mostly motivated

[7] Kutsplus does not always require to share a ride. Their results showed that the mean number of passengers per ride was approximately 1.27.

by developing a demand prediction model to mitigate the demand-supply imbalance. The temporal and spatial distribution of trip requests is often influenced by users’ travel pattern, constraints given by the operators and land-use patterns.

Several studies show that the demand for on-demand ride service fluctuates throughout the day (e.g. Li et al., 2019, Chen et al., 2018, Dong et al., 2018). Li et al. (2019) compared the distribution of departure and arrival time for a non-shared and a shared ride for DiDi in Chengdu, China (see Figure 7). Their results suggest that the temporal distribution of non-shared trips is flatter than the one of shared service. Besides, all three peaks in the morning, noon and evening occur later for shared service than non-shared service.

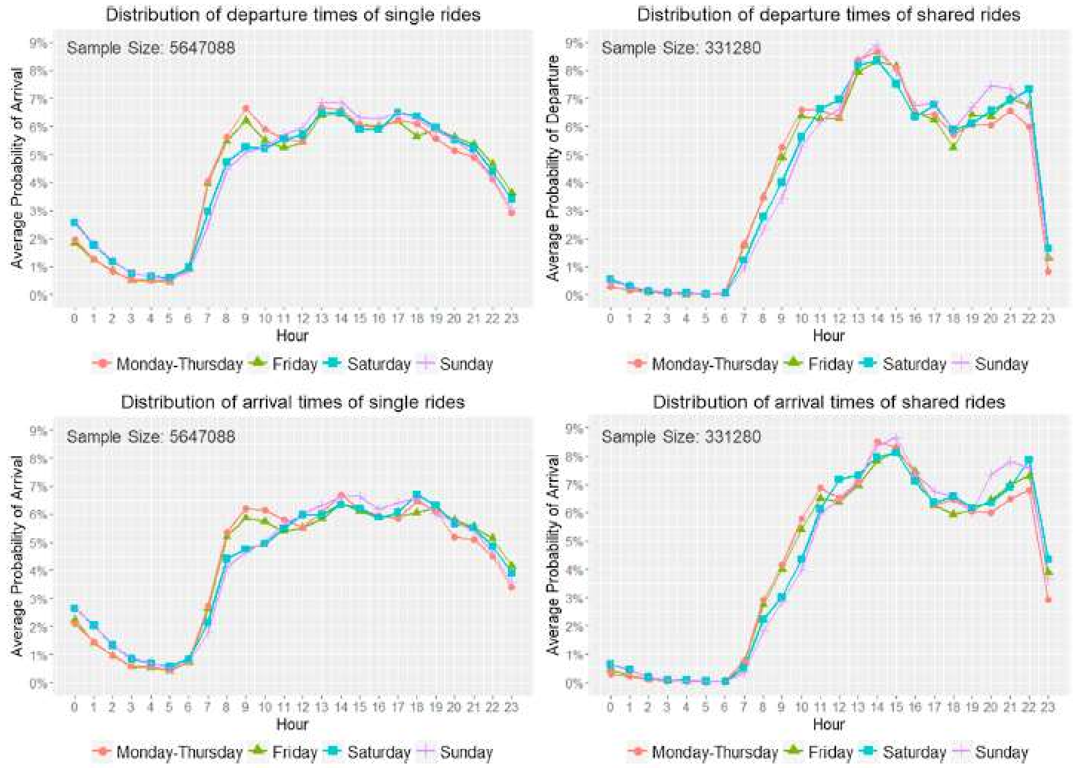

Figure 7 Distribution of departure and arrival time for a shared and a non-shared ride (Li et al., 2019, pp.342)

Chen et al. (2017) suggested that the non-shared use of DiDi in Hangzhou, China, has two peaks, AM peak (7:00-9:00) and PM peak (17:00-19:00). Haglund et al. (2019) also show that the hourly demand pattern for Kutsuplus had two peaks, one in the morning and one in the afternoon, which has a similar shape with fixed public transport systems operating in the Helsinki Metropolitan Region (HMR).

In terms of spatial distribution, many studies have been identified the existence of service demand hotspots for e-hailing (Dong et al., 2018), taxi (Chang et al., 2010, Yu et al., 2019 and Zhang et al., 2017), and ride-sourcing/ride-splitting (Faghih et a., 2018, Dong et al. 2018 and Chen et al.,2017). In general, service demand hotspots are associated with the land use pattern. In particular, residential zone, transportation hub, business districts, educational zone, health care facility, and commercial regions are often identified as hotspots (Yu et al., 2019, Dong et al., 2018, Zhang et al., 2016 and Wang et al., 2019).

Some studies focused on O.D. patterns in on-demand ride services more than just analysing the distribution patterns of pick-up and drop-off locations separately (Dong et al., 2018 and Liu et al., 2019). Liu et al. (2019) analysed data from the world largest ride-sourcing company, DiDi, in Chengdu, China. They discovered the general tendency throughout one targeted month that ride-sourcing users tend to use the service to travel away from the centre rather than towards the centre. (see Figure 8).

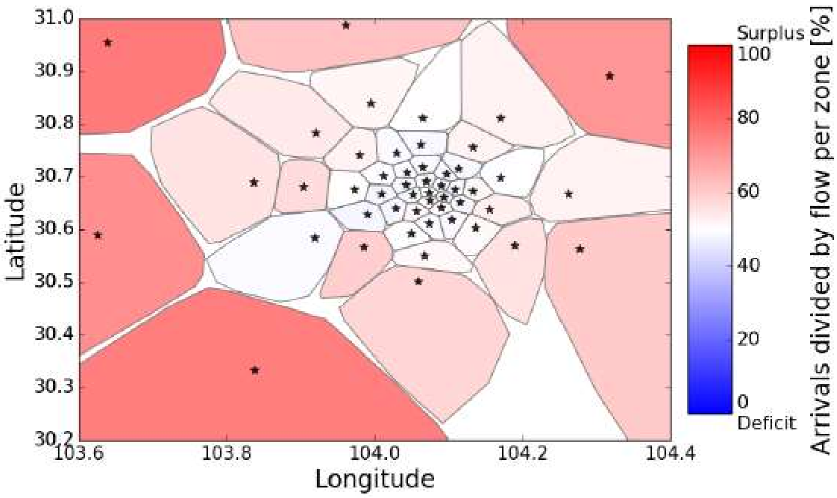

Figure 8 Proportion of arrivals in relation to total tripes made in each zone (Liu et al., 2019, pp. 6)

Dong et al. (2018) also investigated data from DiDi in Beijing, China, focusing on the spatial pattern at each time of the day. They visualised hotspots where pick-ups dominate drop-offs in morning rush hour and evening rush hour (see subfigure (a) and (c) in Figure 9). Hotspots, where drop-offs dominate pick-ups in the morning rush and evening rush hour, are also visualised (see subfigure (b) and (d) in Figure 9). Their results illustrate that users tend to travel to the central business district from the surrounded area, speculated as users residences, using ride-sourcing services in the morning rush hours. In the evening hour, the opposite tendency is observed. It indicates the existent of difference in the distribution of pick-up and drop-off hotspots. In particular, it is observed that pick-up hotspots are more concentrated and surrounded by drop-off spots or the other way around depending on the time of the day, as illustrated in Figure 9. Also, if the service is used as a mode to travel from home to work, it is often used as the return trip from work to home (Dong et al., 2018 and Yu et al., 2019)

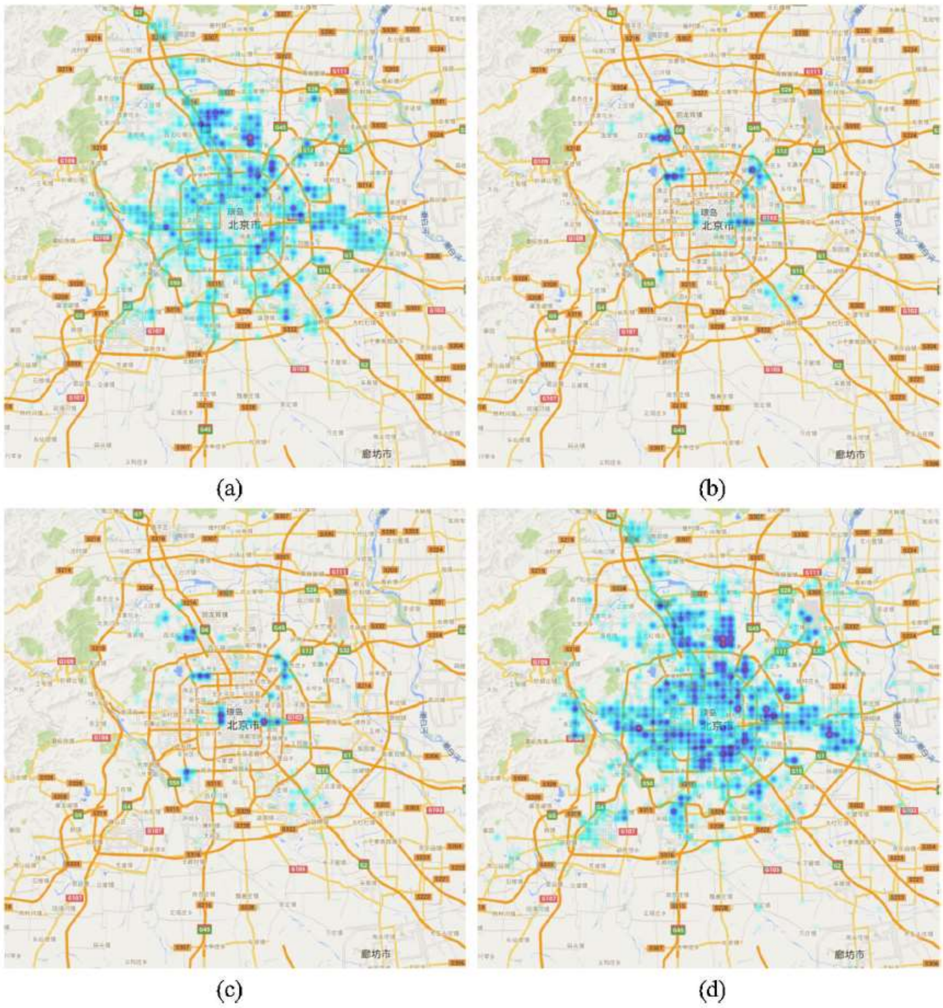

Figure 9 Hotspot visualisation for morning rush hour ((a) and (b)) and evening rush hour ((c) and (d)). Places where the number of pick-up places dominates the drop-off location are visualised in (a) and (c) and places the other way around in (b) and (d) (Dong et al., 2018, pp. 15)

Dong et al. (2018) found out that outside of rush hours, the distance of trips tends to be shorter, and they are concentrated in the central area in the CBD. Though they have not provided more microscopic analysis in the off-peak period, another study suggests the difference in pick-up and drop-off hotspots at the microscopic level. For instance, Li et al. (2012) investigated a traditional taxi where users need to catch a taxi on the road. They discovered that drop-off locations are more dispersed than pick-up locations as users often catch a taxi on the main road due to the easiness to find the taxi. In contrast, users chose drop-off locations based on their final destination.

It should be noted that the scale and the number of hotspots are determined by the interest of research as well as the nature of the mobility pattern. Chang et al. (2010) investigated real taxi request data provided by the taxi company in Taiwan to predict the location of future taxi requests better. Their results illustrate that the number of hotspots will vary based on the clustering technique (see Figure 10).

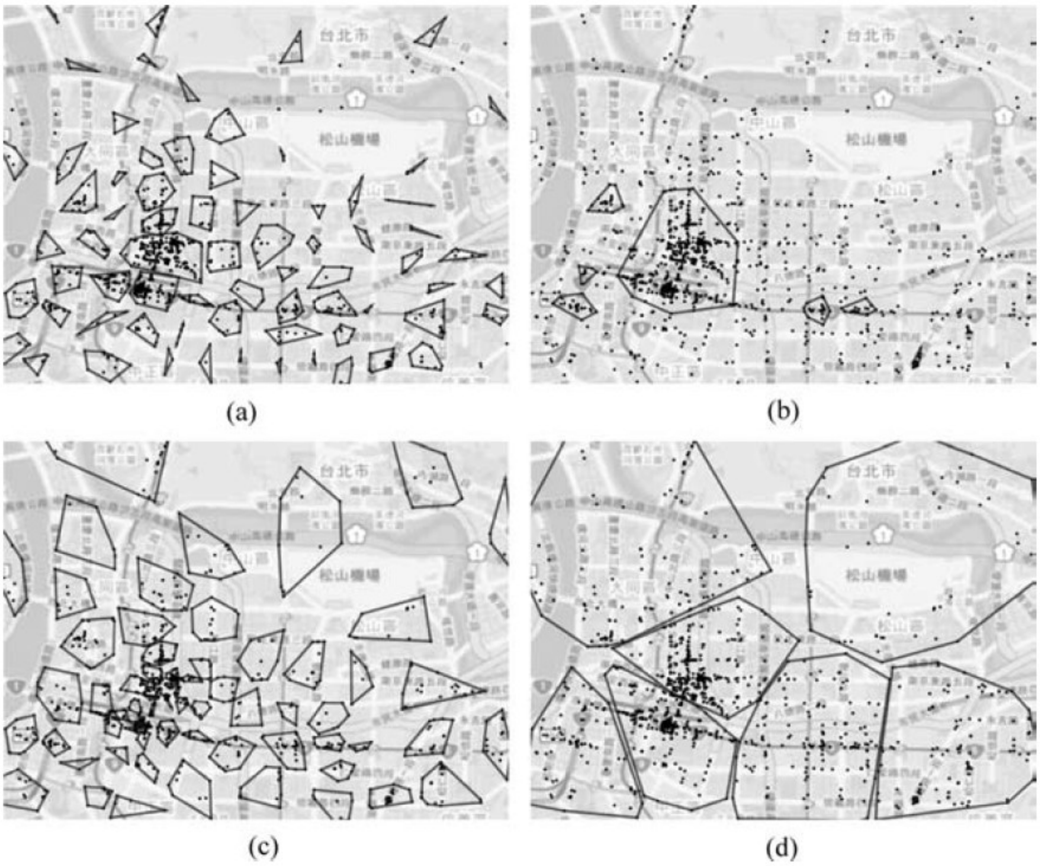

Figure 10 Clusters of taxi stopping locations generated by four different algorithms (Chang et al., 2010, pp,14)

2.6 On-demand ride service users' behaviour

This section summarises the literature about on-demand ride service user. The focus of this section is to understand on-demand ride service users' behaviour by reviewing the studies of real would system and by investigating how users' behaviour is modelled in other studies. This section consists of two subsections. In subsection 2.6.1, the motivation for choosing an on-demand ride service and choosing to share the service is described. In subsection 2.6.2, the value of travel time with on-demand ride service and willingness to share a ride are discussed.

2.6.1 Motivation to choose shared on-demand ride service

A number of studies investigated the reason why people uses on-demand ride services compared with other transport modes. Tiranchini (2019) conducted a comprehensive review of the motivation for the user to choose a ride-hailing service. The result presents a variety of reasons to use ride-hailing service. Among them, most of the paper determined trip cost, travel time, ease of payment, no need to drive after drinking, and waiting time are highly valued.

Lavieri et al. (2018) identified through their literature review that the easiness to use the service (i.e. payment and booking process), lower cost, and shorter waiting time are often mentioned as reasons to use ride-hailing service over conventional taxi service. Compared with mass transit, shorter travel time is the primary reason to choose a ride-hailing service. In terms of the trip purpose, social and recreational trips are the most common purpose when ride-hailing services are used (Tiranchini, 2019, Lavieri et al., 2018, and Li et al., 2019).

Note that the following discussion includes studies focusing on shared autonomous vehicles (SAV), where “shared” indicates that the *vehicle* is shared between multiple individuals. It may also be the case that multiple users ride-share in an SAV for a particular trip. Hence, a comparison between shared and non-shared use of SAV is relevant to the more general comparison between shared and non-shared use of on-demand ride service. It is worth acknowledging that the absence of a driver may influence an individual’s perception of using such a service in an autonomous vehicle (AV) or SAV.

Since the concept of sharing a ride appeared in the transport context, several operational and psychological aspects have been identified to encourage/discourage sharing. Psychological factors are often identified as barriers to ridesharing, such as a desire for a personal space and security concerns (Chan and Shaheen, 2012 and Lavieri et al., 2018). However, a green lifestyle propensity (GLP), i.e. individuals’ tendency to choose an environmentally friendly option, could positively influence on the decision to use a shared ride (Lavieri et al., 2018). Sarriera et al. (2017) discovered the asymmetry in the relationship between the experience of sharing a ride and the decision to share again. The negative social experience of sharing a ride would be more discouraging than the positive social experience would be an incentive.

Sarriera et al. (2017) and Middleton and Zhao (2019) investigated the discriminatory attitude between ride-splitting users (i.e. UberPool and Lyft Line) in the USA through an online survey. Both studies discovered that a substantial proportion of TNCs users have discriminatory attitudes towards passengers from different social class and race. Brown (2018) also found that users are less likely to share their ride in racially and ethnically diverse neighbourhoods. Furthermore, Sarriera et al. (2017) discovered that women tend to prefer to match with passengers of the same sex than men due to security-related concerns.

Moody et al. (2019) investigated the relationship between user’s willingness to share rides and their discriminatory attitudes the first time in the ride-splitting context through an online survey targeted to UberPool and Lyft Line users in the USA in 2016 and 2018. Their results suggest that rider-to-rider discriminatory attitudes are not significantly predictive of user's decisions on the first-time use of ride-splitting service. However, they are significantly predictive of the satisfaction level and lower frequency of usage and lower willingness to adapt to sharing in the future.

Though psychological and social factors would impact the mode choice, several studies concluded that they are not as significant as traditional factors such as time and cost (Sarriera et al. 2017 and Moody et al., 2019). However, Middleton and Zhao (2019) discussed that given the discriminatory attitudes discovered in their study, if, one day, some TNCs decide to implement a new feature to consider the preference of matching, it is reasonable to expect that some riders would take advantage of the feature to avoid potential fellow passenger based on their social class or/and race. Besides, such a feature could potentially be used to express the preference to share a ride with a same-sex passenger(s) to mitigate security concerns.

In terms of operational aspects, the trade-off between monetary cost and travel time is often mentioned in the literature (e.g. Chen et al., 2017, and Sarriera et al., 2017, Lavieri and Bhat, 2019). Chen et al. (2017) identified that in-vehicle time, monetary cost and waiting time are in the top 5 most important features influencing whether users chose to share or not. Sarriera et al. (2017) also discovered that the primary motivator to share an on-demand ride service is the cost being cheaper than a non-shared option. According to Shaeen and Cohen (2019), 25-60% of monetary cost saving is expected compared to the non-shared option.

As presented in the previous section (i.e. section 2.5.1), the trip length for shared service is longer than the non-shared service. It is easy to imagine that the increase in in-vehicle time could discourage users from choosing a shared service. Also, the uncertainty of trip length is indicated as a potential deterrent to using shared service by Sarriera et al. (2017) and Li et al. (2019).

2.6.2 Value of travel time and willingness to share a ride

The value of travel time (VoT), conventionally referred to as the value of travel time savings, reflects the amount of money a traveller is willing to pay to save travel time. It is defined as the ratio of the marginal utility of time and money and comprises the opportunity cost and actual disutility regarding time spent for travel (Wardman et al., 2004). The value of in-vehicle time (VoIVT) is known to vary according to a travel mode. Moreover, walking time to/from and waiting time at the station have the corresponding VoT in the case of public transport use (Wardman et al., 2004).

Krueger et al. (2016) estimated VoT for shared autonomous vehicle (SAV) with dynamic ridesharing (DRS) and SVA without DRS by conducting an online stated choice survey targeted to residents living in a major metropolitan area in Australia. Their results indicated that the value of VoIVT for SAV with DRS was higher than the VoIVT for SAV without DRS. Lavieri and Bhat (2019) measured phycological disutility to share a ride, such as hesitation to share a vehicle with strangers separately from an operational disutility such as an increase in in-vehicle time due to an insertion of extra stops. They introduced the willingness to pay not to share with an additional person (the willingness to share (WTS)) and discovered that WTS is a fixed cost and independent of travel time.

Lavieri and Bhat (2019) also identified that the trip purpose significantly affects the user’s sensitivity towards strangers' presence in a vehicle. People seem to be less sensitive towards a stranger's existence for a commuting trip than a leisure trip. On the other hand, people are more sensitive towards travel time for a commuting trip than a leisure trip. Al-Ayyash et al. (2016) discovered the impact of the number of co-riders on the willingness to adopt a shared trip. Their results indicate that individuals are 7-8% more willing to use the shared service if a trip is shared among a maximum of two additional passengers rather than up to five passengers

Alonso-González et al. (2020a) compared the different ratio of WTS against the value of IVT estimated by several studies, including theirs. They investigate the influence of the number of additional passengers on WTS by conducting an online survey targeted at individuals living in urban areas of the Netherlands. The results show that WTS is consistent when the number of additional passengers is 1-2 while WTS gets higher for the 4 co-rider scenario. Table 4 compares the ratio between WTS/VOT for the 1-2 co-rider case obtained by multiple research.

Table 4 the comparison of the ratio between WTS/VOT for 1-2 co-rider case (Alonso-Gonzalez et al., 2020a)

Authors	WTS/VOT	Mobility option	Country
Al-Ayyash et al. (2016)	0.1	Shared taxi	Lebanon
Lavieri and Bhat (2019)	0.05-0.1	pooled SAV	USA
Alonso-Gonzalez et al. (2020 a)	0.02-0.07	Shared on-demand ride service	Netherland

Conventionally, the ratio between the value of waiting time and in-vehicle time is estimated as 2 or above (Wardman, 2004). However, the study of Alonso-González et al. (2020b) in a Dutch urban area suggests a lower value in the range of 1-1.5 depending on the length of waiting time. Frei et al. (2017)‘s study showed the value of waiting time is lower than the value of in-vehicle time for commute trip with shared on-demand ride service in the USA. It possibly is because the commute trip's origin tends to be an individual’s home or office. It is reasonable to speculate that individuals would perceive waiting in a familiar indoor space as less uncomfortable than waiting outside.

2.7 On-demand ride service driver’s behaviour

This section summarises the characteristics of on-demand ride service drivers’ behaviour. Understanding the driver’s behaviour helps to establish reasonable assumptions for the driver side’s learning and decision process. This section consists of two subsections. In subsection 2.7.1, a drive’s service location choice behaviour is discussed, including surge pricing effects. Then, in subsection 2.7.2, the driver’s motivation to provide on-demand ride service is summarised, and their experience and attitude towards delivering non-shared service and shared service.

2.7.1 Driver’s choice on service location

Traditionally, the decision regarding if drivers should change their location or not has been seen as an interesting topic to discuss. It could depend on many factors such as drivers’ strategy, experience and knowledge, parking availability, the fuel efficiency of their car, etc. (Henao and Marshall, 2020). Naji et al. (2017) analysed the vacant taxi's temporal and spatial behavioural patterns using taxi data in Wuhan city, China. They categorised drivers into three categories, high, moderate, and low profitable classes, based on distance and duration of the chargeable trip (i.e. trip with a passenger) and income. They discovered that highly profitable drivers tend to cruise around crowded places such as railway stations and main economic area compared to moderate- and low-income drivers. They suggest that the high-income group may have known which area has a higher potential to get new users for each period of the day from their experience or some other sources. According to that information, they cruise or stop around such “demand hotspot”.

Chang et al. (2010) demonstrated the recommendation system for a vacant taxi, which endorses the nearest pick-up hotspots based on each taxi's current location. It suggests that a vehicle could come back to the pick-up hotspot once they complete a current service, depending on the service operation strategy. Besides, the geographical proximity of a hotspot may influence on the selection of a hotspot if there are multiple of them. The surge pricing could also play the same role as the recommendation system Change et al. (2010) suggested. With a surge pricing scheme, the operation area is divided into smaller zones, and a service platform periodically updates the surge multiplier in each zone. Drivers can usually access the “surge heat map” which shows the surge prices in the zone where they are at and surrounding zones. Relocation of drivers to high demand zone with higher surge price is one of the expectations to introduce surge pricing, and, in theory, it should work in that way (Guda and Subramanian, 2019).

However, Chen et al. (2015) observed the controversial results from their data collection in San Francisco and New York City in the US in 2015. They discovered that though surge pricing had effects on encouraging off-line drivers to be online, it induced drivers to move out from the high surge area from other places, which is the opposite of the expected impact of surge pricing. Also, even considering surge pricing, Henao and Marshall (2019) do not recommend the cruising behaviour of drivers unless there is a guarantee of reducing 30 % of deadheading time (i.e. time without serving passengers). Experienced drivers may learn such strategies from their experience, which could affect drivers' location choice behaviour.

2.7.2 Drivers' motivation, experience and attitude towards providing shared and non-shared on-demand ride service

In most cases, on-demand ride service drivers are not hired by a provider of the service platform. Instead, they are "partnering" with the service platform provider. Therefore, they can decide when, where, and how long they offer their service. Many studies identified such flexibility as the primary motivation for drivers to enter the platform (Hall and Krueger, 2018, Hong et al., 2020, and Fielbaum and Tiranchini, 2020). Hall and Krueger (2018) provided the first comprehensive analyses of Uber's "driver-partners" in the US using the survey data conducted in 2014 and 2015. Their results suggest that most Uber drivers have a full-time or part-time job aside from being a driver. Hence, they use Uber as a supplemental income source to earn more or smooth out their income fluctuations. On the other hand, Fielbaum and Tiranchini (2020) discovered that most drivers are working full time indefinitely from the online survey target drivers of the two biggest raid-hailing companies in Chile (i.e. Uber and Cabify) 2018. The difference in the results could be attributed to the different data collection time.

Depending on the motivation of partnering with an on-demand ride service platform, drivers' working hours and patterns vary. When drivers have other full-time and part-time jobs, their working hours are much shorter than traditional taxi drivers, and working patterns vary week by week (Hall and Krueger, 2018). On the contrary, when providing on-demand ride service is their full-time job, most drivers have a fixed routine, which makes working patterns not so different from a traditional taxi. However, the absence of a fixed working schedule is still appreciated by drivers (Fielbaum and Tiranchini, 2020).

Several studies declare that both profitability and drivers' earnings are questionable (Henao and Marshall, 2019 and Hong et al., 2020). Hall and Krueger (2018) concluded that drivers earn as much per hour or probably more than an average taxi driver and chauffeur. In particular, their estimation suggests that drivers' earnings are between $16.20 and $23.70 per hour, where operational cost is $2.90 and $6.50 per hour in the US. On the other hand, Fielbaum and Tirachini (2020) estimated that drivers' actual hourly earnings are between $5.10 and $6.50 per hour, which is higher than the earnings perceived by drivers (i.e. $4.20 per hour); however, less than half of the amount which Uber claims (i.e. $12.10 - $17.20 per hour without discounting operational cost).

Henao and Marshall (2019) estimated that on-demand ride service drivers' (i.e. Uber and Lyft) wage is between $5.70 and $10.50 per hour in Denver in the US, which is lower than the minimum wage in the state of Colorado. Hong et al. (2020)and Henao and Marshall (2019) pointed out that the price of on-demand ride service could be below the threshold to cover operational costs when demand is deficient and the number of active vehicles is saturated. Though the expected amount of earnings differs among drivers, Fielbaum and Tiranchini (2020) discovered that the insufficient amount of earnings is often mentioned as the reason to quit being a driver.

Not much literature is focusing on drivers' preference for providing shared and non-shared service. Results of a survey targeted 1,000 Uber and Lyft drivers in the US indicate that a higher percentage of drivers used negative sentences to explain the satisfaction level of providing shared service (i.e. UberPool and LyftShare) (Campbell, 2018). Morris et al. (2020) also conducted an online survey that targeted Uber and Lyft drivers in cities across the US. Their results also suggest that the average satisfaction level for providing shared service is lower than the non-shared option and service in general, with 0.1% significance level. Besides, two-thirds of former drivers mentioned providing a shared service as somewhat reason for them quitting.

According to Morris et al. (2020) and Pratt et al. (2019), drivers tend to ignore shared service requests. However, they pointed out that ignoring too many requests resulted in "time-out", during which drivers cannot accept a request[8]. In addition, Griswold (2017) reported that though Uber allowed drivers to turn off UberPool options, it automatically turned off Uber X which covered the majority of trip

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[8] Morris et al (2020) and Pratt et al (2019) refer the link to Uber and Lyft website accessed in October 2018. The link to Uber's webpage is not working anymore. Besides, Lyft website does not mention "time-out" anymore. Instead, they indicate the acceptance rate could affect to the accessibility to several services.

requests as of 2017[9]. 60% of drivers who participated in the survey conducted by Morris et al. (2020) stated that they would refuse the request for a shared ride if it were easier to do so.

Interestingly, the compensation was most frequently mentioned in both what drivers *like* and *dislike* about providing shared service (Morris et al., 2020). Drivers seem to like the fact that they can keep passengers in a vehicle longer time instead of cruising around to find another passenger, which results in more earnings. They also mention that they could get more tips by providing shared ride than non-shared service as there are more passengers per ride. However, some drivers perceive shared service users as bad tippers. It should be noted that Uber now gives $1 per pick-up for a shared service, while Lyft does not offer any rewards. Morris et al. (2020) discovered that approximately $3.40 per pick-up would make the average driver feel fairly compensated. Some drivers also mentioned that what they earn from providing one shared trip is not high enough, considering how long it takes to complete the trip. Nevertheless, Uber and other observers estimated that providing shared service results in better compensation, according to Morrison et al. (2020). Hence, they recommend a platform to provide better communication in terms of financial benefit with drivers.

2.8 Research gaps in the existing literature

In this section, the deficiencies of the models presented in reviewed studies are presented with respect to the motivation and objectives of this research. In particular, research gaps associated with objective O1 to O3 are summarised individually.

O1.to specify and develop a stochastic process model that represents the long-term evolution of an on-demand ride service system that provides non-shared and shared use.

Section 2.2 in Chapter 2 summarised existing research focusing on the day-to-day dynamics in on-demand ride service systems and other types of "shared" systems (e.g. bike-sharing, conventional bus service). In the context of conventional mass transit such as bus services, several studies modelled the feedback loop caused by users' mode choices and changes in the system capacity (Cantallera et al. 2015 and, Li and Yang, 2016). Periodic changes in the frequency of bus services (Li and Yang, 2016) and daily changes in travel time influenced by private car usage and

[9] Author did not find information regarding that is still applicable or not.

bus ridership (Cantallera et al., 2015) are introduced as causes of service capacity change. However, unlike an on-demand ride service, the changes in the system capacity are not related to (bus) drivers' decisions.

Djavadian and Chow (2016, 2017) proposed a framework to represent the dynamics between users and drivers of an on-demand ride service as an agent-based model. Their framework can capture the day-to-day learning and decision making of both users and drivers regarding which service they will use or provide. However, the nature of their model is deterministic. Research conducted by Zhan and Schmocker (2019) is the only one that utilised a stochastic process model in the "shared" service context. In particular, they applied a Markovian approach to develop a demand prediction model that can capture the long-term evolution of bike-sharing users' choices. The bike-sharing scheme is similar to an on-demand ride service in the sense that the service level is influenced by the supply-demand balance, which could change every day. However, bike-sharing is fundamentally different from on-demand ride service since no drivers are involved and there is no pooling of passengers.

Hence, a research gap remains in that there exists no stochastic process to represent the long-term evolution of on-demand ride services.

O2. to extend the model in O1 to include the impact of the availability of both sharing partner and a vehicle to the users' experience by simplifying the service supply process with a queueing representation

In the case of shared service, the dynamics among shared service users influence the service level. For instance, even if the number of drivers is large enough compared to the number of shared service users, the small number of shared service users may still prevent users from finding others to share a vehicle with. It leads to the deterioration of service level by increasing waiting time. Thaitakul et al. (2019) simplified the sharing aspect of on-demand ride service and investigated the impact of sharing partners' availability on the long-term evolution in DRS. The problem was reduced to matching multiple users based on their OD and travel time constraints by removing the discussion regarding vehicle availability.

Discussion regarding the availability of vehicles could be ignored if it is assumed that the fleet size is large enough for users to access the service anytime, anywhere. However, the fleet size is not often that large in the real-world system as it would be financially inefficient for the service providers, namely, drivers. As explained in the section 2.7, drivers have the flexibility to decide if they provide the service at certain time of the day. Therefore, when drivers discover the market is oversaturated and they cannot earn as much money as they would like, it is highly likely that they will stop providing the services. Therefore, it is very unlikely to see the situation where the fleet size is large enough to be able to ignore the availability of vehicle. Besides, interesting aspects of a shared ride cannot be represented if the impact of service capacity is excluded. One example of such elements is the balance between competition among users to access the limited resources (e.g. fleet) and dependency among users to find sharing partners.

Hence, the research gap remains where there is no research about day-to-day dynamics in on-demand ride service, including the dynamics between users and providers and the dynamics among shared service users.

In order to represent the dynamics between shared service users, the trip matching process needs to be included in the model. The trip matching process is challenging to represent in a simplified way, especially for shared service; too much simplification risks losing aspects of the interactions that are essential to the system's evolution and performance. On the other hand, an overly detailed representation would lose the transferability of results and/or would not provide a general understanding of the system. Section 2.3.2 in Chapter 2 reviewed existing research which uses the queuing representation of on-demand ride service and similar type of passenger services. In particular, studies conducted by Wang and Odoni (2016) and Less-Miller et al. (2016) are discussed.

A critical aspect of the queueing representation proposed by Wang and Odoni (2016) is that trip requests arrive in batches, and their destination is known prior to their arrival at the pick-up spot (i.e. a mass transit station). The pick-up location is assumed to be the same spot, a mass transit station, while drop-off locations are spread throughout the given service operation area. As they focused on Last-Mile Transport System (LMTS), this is a reasonable assumption; however, this assumption limits their model's application to the more general context of on-demand ride services.

As discussed in subsection 2.5.2, empirical research discovered from the real-world data that some "hotspots" for pick-up requests are observed (Dong et al., 2018). Besides, in some cities, it is observed that users travel with on-demand services from the city centre to the outer area (Liu et al., 2019). Hence, their spatial representation could also be applicable even when on-demand ride services are not used only as a LMTS. However, the arrival pattern of the trip requests or users is different if the on-demand ride service is not used as a LMTS. In specific, it is unlikely for trip requests to have a temporal pattern where batch requests repeatedly arrive with a short constant interval (e.g. 5-20 min) outside of the LMTS context. Therefore, Wang and Odoni's representation cannot be applied to model on-demand ride services outside of LMTS context.

Furthermore, the batch arrival assumption of Wang and Odoni cannot capture a trade-off between waiting time and in-vehicle time. One of the challenges to cluster the multiple trip requests for shared service is that it is uncertain when other trip requests which share a similar OD would arrive. The longer waiting time would increase the chances of such requests arriving and reduce total travel time by offering more efficient routing. However, a longer waiting time increases the time from a trip request entering the system to arrive at their destination. The availability of a sharing partner and the expected time until such requests arrive would change depending on the level of demand for the shared service. Hence, it is an important aspect to consider in order to capture the influence of variability in trip requests arrival time on users' experience.

Less-Miller et al. (2016) assumed that trip requests arrive individually. However, as their contexts are PRT which is operated on the guideway with fixed stations, destination choices are much limited compared to on-demand ride services. This could be seen as a simplified representation of shared on-demand ride service, where the pick-up and drop-off points are limited and predetermined. However, this representation also limits the capability to capture the trade-off between waiting time and in-vehicle time. Unlike Wang and Odoni et al. (2016), the Less-Miller representation can express variability in trip request arrival time. Nevertheless, it cannot express uncertainty regarding whether or not that trip request has an OD that can be shared with other requests.

Hence, a research gap remains where there is no queueing representation of on-demand ride service system outside of LMTS context and without pre-fixed OD choices.

O3. To propose a fair cost distribution strategy within the framework developed in O2, which captures the trade-off aspects of shared services, such as a reduction in monetary cost and increase in in-vehicle time

As summarised in section 2.6, monetary cost and in-vehicle time are essential for users' mode choice between non-shared and shared services. In Section 2.4, Shapley value is introduced as a possible method to achieve a fair cost distribution for shared service. As explained in section 2.4, the Shapley value can be interpreted as being that each user receives some proportion of "benefit" from sharing the vehicle. However, it does not consider the disutility of users staying longer in the vehicle. If the case illustrated in Figure 11 is assumed where both destinations are located at the same distance from the origin (a star in Figure 11), the Shapley value will be equal and $\varphi(1) = \varphi(2)$. However, as user 2's in-vehicle time is longer than the one for user 1, it would be perceived as unfair for user 2 to pay the same price as user 1. As TSG has often been applied in the logistics field, it is sensible that previous studies had no problem using Shapley Value as a solution method. Unless a fleet carries highly perishable goods, the length of in-vehicle time is not a primary concern for both a carrier and goods. Nevertheless, when moving people rather than goods, the disutility of longer in-vehicle time cannot be ignored as it is essential to capture a trade-off between monetary cost saving and the increase in travel time in on-demand shared ride service.

Hence, in this research, a modified Shapley value which includes the penalty of longer in-vehicle time, is proposed and investigated.

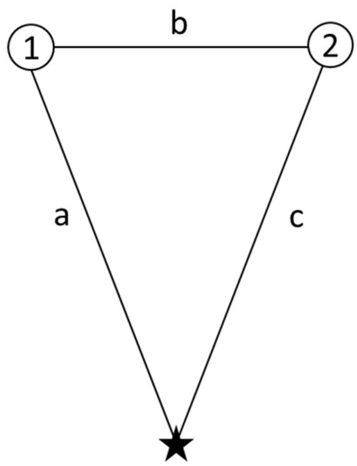

Figure 11 An example where two drop-offs are located the same distance from the pick-up point.

2.9 **Summary**

In this chapter, two types of the literature review were presented. Section 2.2 to section 2.4 presents a review conducted to identify the gaps in the existing literature. The remaining sections discussed the nature of on-demand ride service, mainly using studies through real data analysis.

Some research focused on the day-to-day dynamics model in the context of on-demand ride service, most of which utilised the deterministic process. Each research used a different approach to simplify the problem to investigate the long-term evolution of the system, which is presented in section 2.2.

Section 2.3. describes the queueing representation of DARP after introducing the summary of DARP in general. Subsection 2.3.1 focused on describing the different way to represent DARP. The following subsection summarises a queuing representation of DARP, which often used to reduce the size of the problem while capturing stochasticity and dynamic aspect.

Section 2.4 provides a brief review of the pricing problem regarding non-shared and shared service. Although dynamic pricing is often discussed as the unique feature of the pricing problem for on-demand ride service, the empirical evidence suggests that surge pricing remains deactivated most of the time (e.g. 75% of the day). Besides, the fair cost distribution problem is mentioned as a unique problem for shared on-demand ride service.

Section 2.5 summarises a basic characteristic of trips made by non-shared and shared on-demand ride service. Empirical evidence suggests that non-shared service is much more frequently used compared to shared service in several cities. Several studies analysed spatiotemporal characteristics of trips and identified the existence of demand hotspots for pick-ups and drop-offs.

Section 2.6 discusses the behavioural aspects of on-demand ride service users. A shorter waiting time and lower cost are mentioned as to why people chose on-demand ride service over other passenger services such as taxis. The influence of the phycological factor on the willingness to share an on-demand ride service is also identified. However, it does not seem to have a bigger impact than traditional factors (i.e. monetary cost and travel time).

Section 2.7 presents reviews regarding on-demand ride service drivers' behaviour. Drivers' relocation behaviour is observed when they are on the standby phase. Surge pricing seems to encourage off-line drivers to be online, although online drivers' tendency to move away from high surge zone is discovered. Also, several studies observed drivers' negative attitude towards providing a shared service. There is not much literature investigating a driver side's behaviour aspect yet. Hence, it is too early to conclude those are the universal trend. However, it seems to be the case that drivers perceive providing a non-shared and shared service as a different experience.

At last, in section 2.8, research gaps are identified and presented regarding several objectives stated in Chapter 1.

Chapter 3 Model specification

3.1 Introduction

As mentioned in Chapter 1, one of the objectives of this research is to develop a stochastic process model that represents the long-term evolution of an on-demand ride service system. This chapter provides a detailed description of the proposed stochastic process model that reflects three points specified with the objective O1, O2, and O3 in Chapter 1.

In particular, through a user's utility-based decision model and driver's profit-based decision model, the modelling framework is able to represent users' choice of whether to use a shared ~~and~~ or non-shared on-demand ride service, as well as drivers' choice of whether to provide such services. The long-term evolution of the system is captured by letting users and drivers collectively learn about experience regarding using and providing shared and non-shared service day by day (the objective O1). Day-to-day users' and drivers' service type choices are reflected in a supply model[10] and determine the number of users and drivers for each service on each day. The changes in the supply-demand balance, as a result, affect users' and drivers' experience of using and providing each service. The estimated experiences are fed back to the users' and drivers' learning and decision process the next day, with which the feedback loop is completed. Those processes are specified in section 3.3.

A queuing representation is utilised to achieve a simplified representation of non-shared and shared service, which is specified in section 3.4. In order to include the trade-off aspects of shared service, which is inevitably detailed while keeping the simplicity of the model, a particular service network representation is introduced (the objective O2). The fair splitting of monetary costs among shared service users is modelled by introducing the modified Shapley Value, which is newly introduced in this research (the objective O3). Section 3.4 also explains the simulation process of outputs from the supply model, such as user waiting time, round trip time, user's in-vehicle time, and fare.

The structure of this chapter is as follows. After summarising the list of notations, the learning and decision process model is described in section 3.3. In section 3.3, key assumptions are summarised in subsection 3.3.1; then, the user's and driver's learning and decision processes are separately explained in the following subsections. Section 3.4 is dedicated to explaining the supply model. After key

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[10] A stochastic process consists of three main models; 1) a learning model, 2) a decision model, and 3) a supply model. (Watling and Cantarella, 2015).

assumptions are listed, the general structure of the queueing representation of an on-demand ride service is described in subsection 3.4.2. The process to estimate key outputs used as inputs for the learning and decision process is explained with small numerical experiments. The chapter is concluded with the summary section in section 3.5.

3.2 The list of notation

$\mathbf{N}_\mathrm{d}$	The vector contains the number of non-shared and shared service users on day d which are N_{1d} and N_{2d}.
$\mathbf{X}_\mathrm{d}$	The vector contains the number of users who experienced unsatisfactory service using a non-shared service, X_{1d}, and a shared service, X_{2d}, on day d
φ	The total expected number of trip requests during the period of interest per day
α_u	The proportion of users who satisfied with the service they chose yet stated that they would change to the alternative service on the following day
β_u	The proportion of users who did not satisfy with the service they chose yet stated that they would stay in the same service on the following day
k	The type of service ($k = 1$ indicates a non-shared service while $k = 2$ indicates a shared service)
$u_{i,k,d}$	The utility of a user who made ith request for the service k on day d
$WT_{i,\mathrm{k},d}$	Waiting time of a user who made ith request for the service k on day d [min]
$IVT_{i,k,d}$	In-vehicle time of a user who made ith request for the service k on day d [min]
$c_{i,k,d}$	The fare charged for a user who made ith request for the service k on day d [monetary unit]
WTS	Users' willingness to pay to avoid sharing their ride with stranger(s) which is consistent among all users regardless of the number of strangers share their ride with other shared service users [monetary unit]

$\omega_{k,d}$	A dummy parameter to count the number of unsatisfied users using the service k on day d
γ_w	The value for user waiting time [monetary unit/min]
$\boldsymbol{\gamma}_v(k)$	A vector contains the value for in-vehicle time for non-shared service and shared service [monetary unit/min]
$PU_{k,d}$	The collective average utility of the service k on day d.
η_u	An updating filter which determines how much the average utility on day d influences on the collective average utility on day d $PU_{k,d}$. When, $\eta_u = 1$, the collective average utility on day d is equal to the average utility on day d
$\mathbf{S}_d$	The vector contains the number of drivers providing non-shared services, S_{1d} , and shared services, S_{2d}, on day d
$\mathbf{Y}_d$	The vector contains the number of drivers with unsatisfactory experience providing non-shared services, Y_{1d}, and shared services, Y_{2d}, on day d
χ	The expected number of total drivers per day
α_p	The proportion of drivers who satisfied with providing the service they chose yet stated that they would change to providing the alternative service on the following day
β_p	The proportion of drivers who did not satisfy with providing the service they chose yet stated that they would keep providing the same service on the following day
$\rho_{k,d}$	A dummy parameter to count the number of unsatisfactory drivers providing the service k on day d
γ_f	The parameter determines the fare charged for a minute of a round trip [monetary unit/min]
γ_c	The parameter represents the cost for drivers to make a round trip [monetary unit/min]
$r_{l,j,k,d}$	The duration of the lth round trip for a driver j providing the service k on day d [min]
$q_{j,k,d}$	The total number of trips made by a driver j providing the service k on day d
$p_{j,k,d}$	The profit made by a driver j providing the service k on day d

$PP_{k,d}$	The collective average profit for the service k on day d
η_p	An updating filter which determines how much the average profit on day d influences on the collective average profit on day d $PP_{k,d}$. When, $\eta_p = 1$, the collective average profit on day d is equal to the average profit on day d
$\mathbf{M}_d$	The vector contains the mean service rate of non-shared services, M_{1d}, and shared services, M_{2d}, on day d
$\mathbf{\Lambda}_d$	The vector contains the mean arrival rate of a trip request for the non-shared service, Λ_{1d}, and the shared service, Λ_{2d}, on day d
a	The minimum number of requests per cluster for shared service. A cluster indicates the set of trip requests for shared service simultaneously served by a vehicle.
b	The number of passenger seat per vehicle
$\delta(\boldsymbol{\zeta}, \kappa)$	The set of parameters determines the service network geometry. $\boldsymbol{\zeta} = (\zeta_c, \zeta_d)$ consists of the length of the corridor, ζ_c, and the side length of a drop-off area, ζ_d.
θ	The threshold waiting time till non-clustered requests to be clustered, beyond which trip requests become ready to be matched with any available vehicle providing shared service without forming a cluster
ϖ	The duration of interest [min]
$z_{l,j,k,d}$	The number of drop-offs in the lth round trip of a driver j providing the service k on day d. ($a < z_{l,j,k,d} < b$)
$g_{i,l,j,k,d}$	The order of a user i to be dropped off during the lth round trip of a driver j providing the service k on day d
h	The mean number of accompanied people per request
$\tau(\, g_{i,l,j,k,d}\,)$	The modified Shapley value for a user i who is dropped off at $g_{i,l,j,k,d}$th stop during the lth round trip of a driver j providing the service k on day d

3.3 The learning and decision model

This section describes the learning and decision process of the proposed model. In the first subsection, key assumptions of the learning and decision process are specified. Then, the user's learning and decision model is specified in subsection

3.3.2. At last, the driver side's learning and decision model is explained in subsection 3.3.3.

3.3.1 Key assumptions

The user's and driver's learning and decision model are formulated based on the assumptions specified below. Figure 12 presents the model framework for the user's and driver's learning and decision model.

A1. A trip can be made by an on-demand ride service only, which offers non-shared and shared services. A non-shared service implies that only one trip request is matched with a driver. A shared service suggests that two or more trip requests are matched with a driver. Hence, two unknown (groups of) user share (the part of) their rip.

A2. The service users and drivers can use the service by signing up for the service platform. Through an online application provided by the service platform, a user can make a trip request. Drivers enter the market by activating their status on the same application.

A3. The expected number of users and drivers during a period of interest is fixed every day. However, the total number of users and drivers who actually use/provide each service varies day by day.

A4. A user could travel with accompanied people who travel from the same origin and destination using an on-demand ride service. A user needs to specify the number of travellers when she/he requests a trip. The number of travellers per request does not correlate with any other attribute of trips. Hence, the number of service users on the day is not necessarily equivalent to the number of people travelling. However, it is equal to the number of trip requests made on that day.

A5. Service users and drivers choose the service they want to use/provide based on the review provided by the application. The review is updated and delivered at the end of the day every day.

A6. The service platform collects users' and drivers' experience as the service progress within a day. The collective average performance of using and providing the service is estimated for each period when all users who made a trip request during the period completed their service. A day is divided into several periods by the service platform based on the similarity in activities (i.e. peak and off-peak period)

A7. The collective average performance is estimated as the weighted sum of the user's and driver's average experience of the day and accumulated performance in the past. The service platform operator determines the weight.

A8. After the collective average performance of each service is released, users and driver asses if their experience was satisfactory compared to the alternative service's collective average performance. Based on their assessment, all users and drivers send feedback regarding if they would choose the different service on the next day or not through the application.

A9. Users and drivers do not always make a decision which is seemingly consistent with their experience. For instance, among those who were not satisfied with their experience, a certain proportion of them would send feedback as “staying with the same service on the next day”. Besides, a certain proportion of those who had a satisfactory experience would send feedback as “changing the service on the next day” for various reasons.

A10. Feedback is sent one per one trip request even if they travelled as a group. Hence, the total number of feedbacks from users is equivalent to the total number of trips on the day.

A11. The review of each service is updated every day before the beginning of a certain period, reflecting the previous day’s users and drivers' feedback.

The below section presents the reflection on some of the assumptions specified above.

Assumption A4

The number of passengers in a vehicle is not a sufficient indicator to define shared or non-shared service. For instance, if a group of 3 people requests a non-shared service, as illustrated in Figure 13, the number of passengers per vehicle is 3. However, it is still a non-shared service.

Assumptions A5 to A11

As illustrated in Figure 12 and specified with assumptions above, individual users and drivers do not remember their own experience in this model. Instead, individual experiences collectively update the overall reputation of each service. Individuals decide which service they would like to use/provide the next day by comparing their own experience of the current day and the overall reputation of the alternative service. Therefore, though the learning is done collectively, differences in individual experience would still be reflected through the decision process in this model.

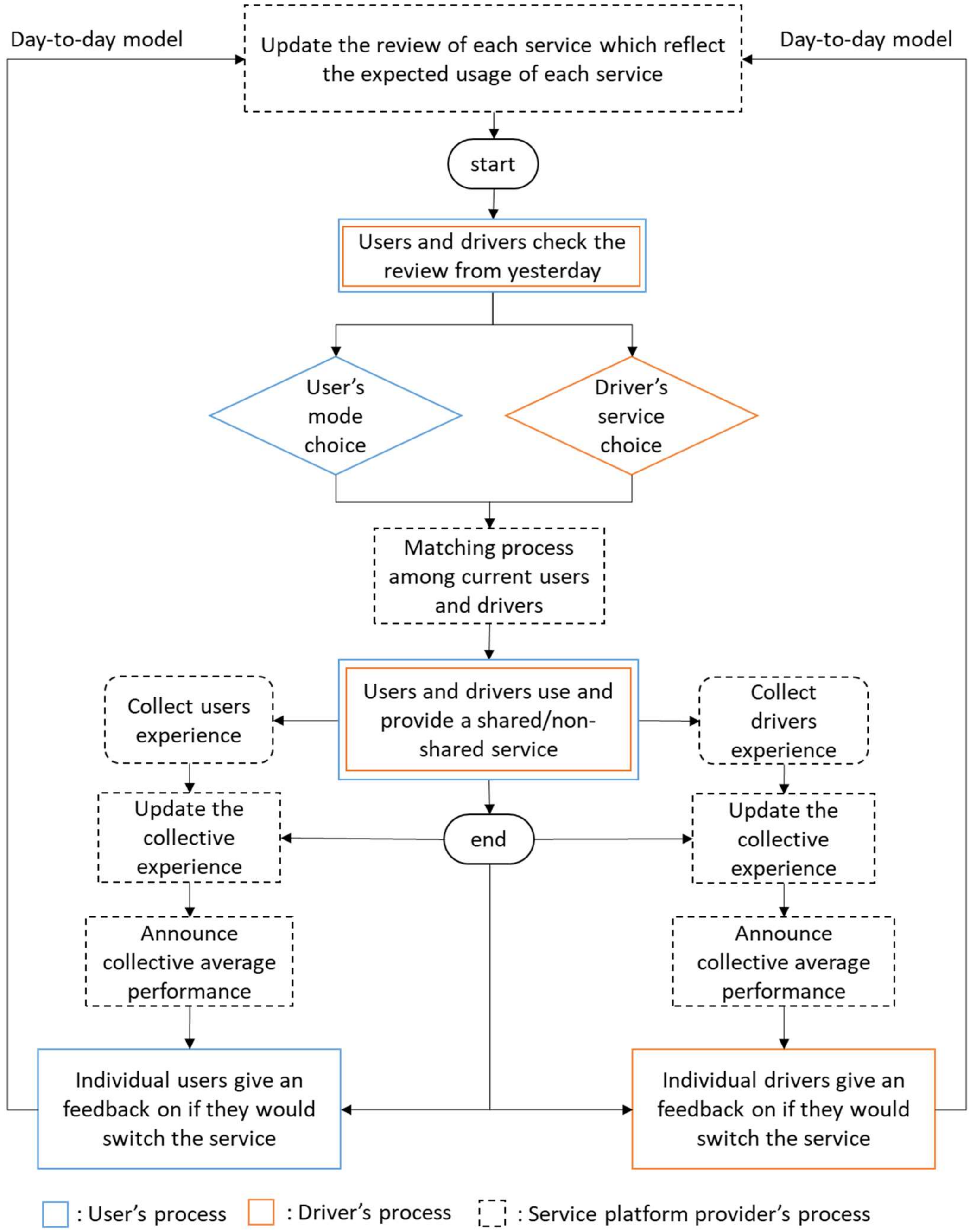

Figure 12 a diagram of the user's and driver's learning and decision process

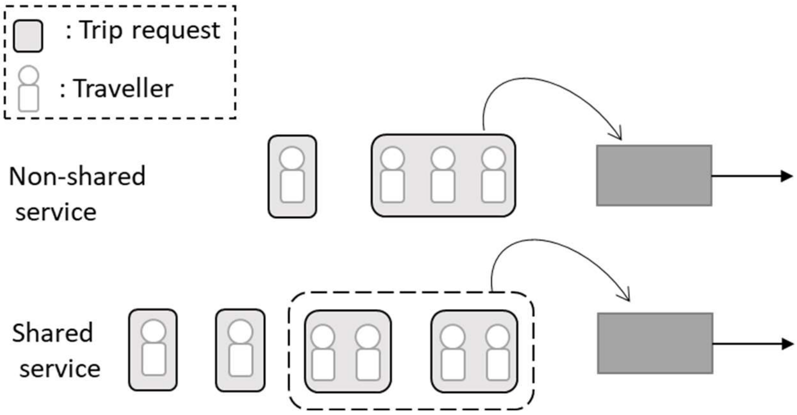

Figure 13 Illustrative example to clarify the number of users is not the determinant of the type of service.

Assumption A5

As illustrated in Figure 12, drivers are assumed to keep providing the same service selected at the beginning of the day. As discussed in section 2.7.2, the current major on-demand ride service platforms do not offer the feature for drivers to choose the type of service to provide. They can refuse the request of a specific type of service (e.g. shared service) based on their preference. However, some platforms penalise drivers if they refuse the requests too many times.

In the current real-world system, the demand for non-shared services is much greater than the shared service. Hence, it is reasonable to expect some drivers are able to and willing to keep providing the non-shared service by refusing the request for the shared service. That behaviour has already been observed according to the survey conducted by Morris et al. (2020) and Pratt et al. (2019) (see section 2.7.2. for details). On the other hand, it would be more difficult for drivers to keep providing shared services only by refusing non-shared service due to the low level of demand for shared service. Nevertheless, some drivers prefer to provide the shared service than non-shared service, as discussed in section 2.7.2.

Hence, it is plausible to assume that some drivers would intentionally choose to provide shared services when the trip requests for shared services were more than the non-shared services in the future. In addition, it is possible to have a service platform that incentives drivers to stay in one service to maintain the minimum level of capacity for each service in the future. In such a case, assumption A5 would give a capability to represent and investigate such a system.

Assumption 7

It should be mentioned that the weight could represent the differences in individuals' travel patterns. For instance, the weight could be assumed to reflect the proportion of the regular and occasional service users and providers. In such a case, when the weight for the accumulated performance is set to be higher it could be interpreted as the proportion of regular service users/providers is high. Conversely, when the weight of the average user's and driver's experience of the day is set to be higher, it could be interpreted as the proportion of occasional service users/providers is high.

In addition, assumption A7 indicates that only collective learning is considered, and individual learning is excluded. It is expected that if individual learning has been included, some dependency on departure time would be observed in users' service choice behaviour, given individuals have a fixed range of departure times. For instance, those who arrived earlier do not wait for a vehicle but may need to wait for sharing partners if they chose a shared service and the mode share for shared service is low. On the other hand, those who arrived when all vehicles were busy would select shared service as waiting time for shared service tends to be lower than non-shared service during the busy period. As described with the example above, the within-day dynamics in mode choice behaviour could be observed and day-to-day dynamics if the individual experiences are reflected.

3.3.2 **User's learning and decision model**

The process of uses' day-to-day service choice is modelled as a stochastic process. N_{1d} and N_{2d} represent the number of users who requested a non-shared service and a shared service on day d and contained in the vector $\mathbf{N}_d$ as shown below;

$$\mathbf{N}_d = (N_{1d}, N_{2d}) \tag{3.1}$$

X_{1d} and X_{2d} represent the number of users who experienced an unsatisfactory non-shared service and shared service on day $d,$ and contained in the vector $\mathbf{X}_d$ as displayed below;

$$\mathbf{X}_d = (X_{1d}, X_{2d}) \tag{3.2}$$

Given the (constant) expected total number of requests during the period of interest per day, φ, the number of requests for non-shared and shared service on day d is estimated with the following equations (3.3) to (3.6);

$$N_{k,d+1} | (\mathbf{N}_d = \mathbf{n}_d, \boldsymbol{X}_d = \mathbf{x}_d) \sim Poisson\big(\varphi \cdot f_k(\mathbf{n}_d, \mathbf{x}_d)\big) \qquad (\mathrm{k} = 1{,}2) \tag{3.3}$$

where

$$f_1(\mathbf{n}_d, \mathbf{x}_d) = \frac{n_{1,d} - \alpha_u \cdot \big(n_{1,d} - x_{1,d}\big) - (1-\beta_u) \cdot x_{1,d} + \alpha_u \cdot \big(n_{2,d} - x_{2,d}\big) + (1-\beta_u) \cdot x_{2,d}}{n_{1,d} + n_{2,d}}$$

(3.4)

$$f_2(\mathbf{n}_d, \mathbf{x}_d) = \frac{n_{2,d} - \alpha_u \cdot (n_{2,d} - x_{2,d}) - (1 - \beta_u) \cdot x_{2,d} + \alpha_u \cdot (n_{1,d} - x_{1,d}) + (1 - \beta_u) \cdot x_{1,d}}{n_{1,d} + n_{2,d}}$$

(3.5)

$$n_{1,d}, n_{2,d} \geq 1 \tag{3.6}$$

α_u indicates a certain proportion of those who *satisfied* with today's experience, yet, submitted the feedback as they would *change their service* on the next day. Conversely, $\beta_u$indicates a certain proportion of those who *were not satisfied* with today's experience, yet, submitted feedback as they *would stay in their current service* the next day. It is assumed that there is always one request made for each service.

The level of service is estimated by calculating the utility for each trip request. A utility-based approach has also been applied in the model of Djavadian and Chow (2016) and Thaithatkul et al. (2019). The level-of-service experienced by an individual or group of users who made the ith request on day d with the service k, $u_{i,k,d}$, is compared with the collective average utility for the alternative service, $PU_{k,d}$. Then, those which had lower utility than the collective average utility of the alternative service is defined as unsatisfied, the number of which is calculated for each service as follows;

$$x_{1d} = \sum \omega_{1d} = \begin{cases} 0 & if \quad u_{i1,d} \geq PU_{2,d} \\ 1 & if \quad u_{i1,d} < PU_{2,d} \end{cases} \tag{3.7}$$

$$x_{2d} = \sum \omega_{2d} = \begin{cases} 0 & if \quad u_{i2,d} \geq PU_{1,d} \\ 1 & if \quad u_{i2,d} < PU_{1,d} \end{cases} \tag{3.8}$$

The following equation indicates a utility function for an individual or group of users who made the ith request on day d with the service k, $u_{i,k,d}$.

$$u_{i,k,d} = \begin{cases} -\gamma_w \cdot WT_{i,k,d} - \boldsymbol{\gamma}_v(1) \cdot IVT_{i,k,d} - c_{i,k,d} & if \quad k = 1 \\ -\gamma_w \cdot WT_{i,k,d} - \boldsymbol{\gamma}_v(2) \cdot IVT_{i,k,d} - c_{i,k,d} + WTS & if \quad k = 2 \end{cases} \tag{3.9}$$

$$\boldsymbol{\gamma}_v(k) = \begin{cases} \gamma_{v1} & if \quad k = 1 \\ \gamma_{v2} & if \quad k = 2 \end{cases}$$

where

$$\gamma_w, \boldsymbol{\gamma}_{v(k)} > 0 \tag{3.10}$$

$WT_{i,k,d}$,$IVT_{i,k,d}$, and $c_{i,k,d}$ represent a waiting time, in-vehicle time, and the fare of ith request on day d with service k. WTS indicates the users' willingness to share their ride which is constant among all users regardless of the number of passengers to share the ride with. An estimation process of each variable is explained in subsection 3.4.3 to 3.4.6 in this chapter. It should be mentioned that the ith request, in this context, is not necessarily served by ith service. The parameters γ_w and $\boldsymbol{\gamma}_{v(k)}$, express the weight of each parameters' contribution to the overall utility and are positive and constant among all users.

By applying the exponential smoothing technique, the collective average utility for the service k is estimated as the weighted average of the mean utility among the service k users on day d and the collective average utility on day $d-1$. η_u is an updating filter and determines the weight of each value. $PU_{k,d}$ is estimated by an equation below;

$$PU_{k,d} = (1-\eta_u) \cdot PU_{k,d-1} + \eta_u \cdot \frac{\sum_{i=1}^{n_{k,d}} u_{i,k,d}}{n_{k,d}} \qquad (0 < \eta_u \leq 1) \tag{3.11}$$

3.3.3 The driver's learning and decision model

Service providers (drivers) may also react to their previous day's experience by changing the service type they offer. Hence, the fleet size for non-shared service S_{1d} and shared service S_{2d}fluctuate day by day. Fleet size is shown by the vector $\boldsymbol{S}_d$;

$$\mathbf{S}_d = (S_{1d}, S_{2d}) \tag{3.12}$$

The number of drivers having an unsatisfactory experience on day d for non-shared and shared service $Y_{1d}$and Y_{2d} is expressed with the vector $\boldsymbol{Y}_d$ as below;

$$\mathbf{Y}_d = (Y_{1d}, Y_{2d}) \tag{3.13}$$

Given the expected total fleet size χ, the fleet size for each service on day $d+1$ is represented with the random variable $\boldsymbol{S}_{k,d+1}$ which is generated by;

$$\mathbf{S}_{k,d+1} | (\mathbf{S}_d = \mathbf{s}_d, \mathbf{Y}_d = \mathbf{y}_d) \sim Poisson\big(\chi \cdot f_k(\mathbf{s}_d, \mathbf{y}_d)\big) \qquad (k = 1,2) \tag{3.14}$$

where

$$f_1(\mathbf{s}_d, \mathbf{y}_d) = \frac{s_{1d} - \alpha_p \cdot (s_{1d} - y_{1d}) - \left(1-\beta_p\right) \cdot y_{1d} + \alpha_p \cdot (s_{2d} - y_{2d}) + \left(1-\beta_p\right) \cdot y_{2d}}{s_{1d} + s_{2d}} \tag{3.15}$$

$$f_2(\mathbf{s}_d, \mathbf{y}_d) = \frac{s_{2d} - \alpha_p \cdot (s_{2d} - y_{2d}) - \left(1-\beta_p\right) \cdot y_{2d} + \alpha_p \cdot (s_{1d} - y_{1d}) + \left(1-\beta_p\right) \cdot y_{1d}}{s_{1d} + s_{2d}} \tag{3.16}$$

where

$$s_{1,d}, s_{2,d} \geq 1 \tag{3.17}$$

α_p indicates a certain proportion of those who *satisfied* with today's experience, yet, submitted the feedback as "changing their service" on the next day. Conversely, $\beta_p$indicates the certain proportion of those who *were not satisfied with* today's experience, yet, submitted the review as "staying in their current service" the next day. It is assumed that there is always one driver providing each service.

The drivers decide their experience by comparing the profit they gained on the day with the collective average profit for an alternative service, $PP_{k,d}$.Then, those which earned lower profit than the collective average profit for alternative service is defined as an unsatisfied, the number of which is calculated for each service as follows;

$$y_{1d} = \sum \rho_{1d} = \begin{cases} 0 & if \quad p_{j1,d} \geq PP_{2,d} \\ 1 & if \quad p_{j1,d} < PP_{2,d} \end{cases} \tag{3.18}$$

$$y_{2d} = \sum \rho_{2d} = \begin{cases} 0 & if \quad p_{j2,d} \geq PP_{1,d} \\ 1 & if \quad p_{j2,d} < PP_{1,d} \end{cases} \tag{3.19}$$

The profit earned by a driver j providing the service k on day d is estimated by subtracting the total cost of all trips they provided on day d (e.g. gas) from the total fare they collected on day d as following ;

$$p_{j,k,d} = (\gamma_f - \gamma_c) \cdot \sum_{l=1}^{q_{j,k,d}} r_{l,j,k,d} \qquad (\gamma_f > \gamma_c) \tag{3.20}$$

The total number of round trips made by driver j for service k on day d is $q_{j,k,d}$. γ_f and γ_c indicate the fare and cost per min. The cost and fare are determined by the total length of round trips made by a driver j during the period of interest on day d.

By applying the exponential smoothing technique, the collective average profit for the service k is estimated as the weighted average of the mean profit among service k drivers earned on day d and the collective average profit on day $d-1$. η_p is an updating filter and determines the weight of each value. $PP_{k,d}$ is estimated by the equation specified below;

$$PP_{k,d} = (1 - \eta_p) \cdot PP_{k,d-1} + \eta_p \cdot \frac{\sum_{j=1}^{s_{k,d}} p_{j,k,d}}{s_{k,d}} \qquad (0 < \eta_p \leq 1) \tag{3.21}$$

3.4 **The supply model specifications**

This section provides the supply model specification. The supply model outputs are used as input for the learning model specified in the previous section. In subsection 3.4.1, key assumptions regarding the supply model are described with a reflection on some of them. In subsection 3.4.2, the queueing representation of non-shared and shared service is explained. Then, the estimation process of user waiting time, round trip time, in-vehicle time and fare are specified in subsection 3.4.3 to 3.4.6.

3.4.1 **Key assumptions**

There are three types of assumptions described in this section regarding network properties, trip request arrival process, and service process. In each section, the assumptions are explained first. Then, the reflection of each assumption is discussed at the end of each subsection.

3.4.1.1 **Network properties**

This study uses a network illustrated in Figure 14 following assumptions listed below;

A12. The trip request is always served from the pick-up demand hotspot represented with a star in Figure 14.

A13. Individual drop-off locations are randomly spread within the service area (i.e. dashed square in Figure 14).

A14. The pick-up demand hotspot is only connected to the drop-off area via a corridor connected to the centre of the drop-off area.

A15. The travel time within the pick-up demand hotspot can be ignored.

A16. Travel time from the centre to anywhere in the drop-off area is supposed to be short enough itself or against travel time on the corridor; therefore, all users have the potential to share their trip in terms of geographical proximity.

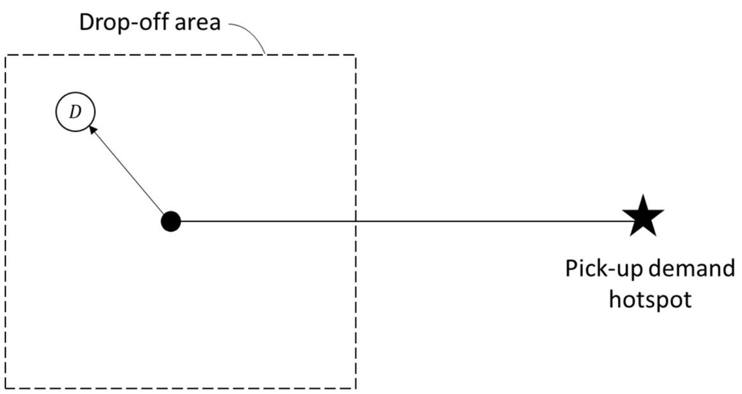

Figure 14: The structure of the service network assumed in this study

The assumption A12 and A16 are key to simplify the trip matching process for shared use of the on-demand ride service. By considering the demand hotspot as a node with assumption A12, the geographical proximity of pick-up locations can be ignored during a trip matching process between trip request(s) and available driver. Besides, assumptions A16 made all trip requests in the proposed network to be shareable from the spatial point of view. As a result, the component of a trip matching problem for shared service is reduced to the temporal problem only. Meanwhile, as the drop-off points could be anywhere in the drop-off area, it could express numerous combinations of different relative positions of drop-off points for shared services. Thus, it distinguishes this approach from the spatially more limited representation by Less-Miller et al. (2016) mentioned in Chapter 2.

The practical implication of this network could be a trip between a central business district (CBD) to a residential area where a neighbourhood area is connected to CBD with a motorway, as illustrated in Figure 15. It should be noted that the proportion of objects in Figure 15 does not follow the real-world scale. According to the literature review conducted in section 2.5. in Chapter 2, it is observed that the drop-off locations are more spread than the pick-up locations in some cases in the real-world system. Hence, the network representation proposed in this study is sensible, however, limited to the case which has the specific OD pattern.

The current network representation could be easily expanded to the case where users travel to multiple drop-off locations, as demonstrated in Figure 16. It should also be noted that the shape of the drop-off area is not limited to a square. Suppose the real-world data were utilised in the proposed model. In that case, the number of clusters should be determined depending on the nature of mobility patterns in the study area and which attribute would be mainly examined through the model regarding the influence on the evolution of a system.

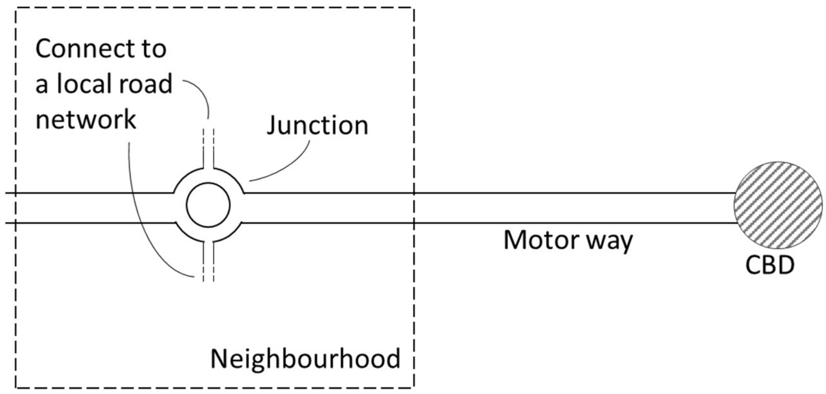

Figure 15 an example of the simplified representation of a service network assumed in this study where a neighbourhood area is connected to the central business district (CBD) by a motorway that connects to the local road network via a single junction

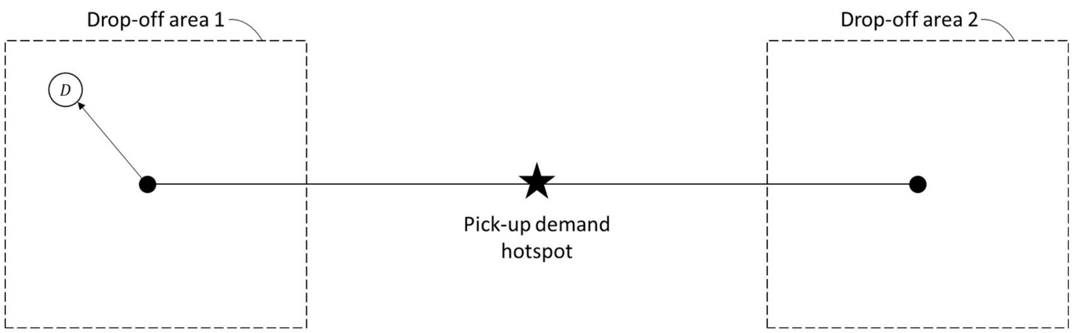

Figure 16 the potential expansion of the network with two separable destination area.

3.4.1.2 **The trip request arrival process**

The trip request arrival process is modelled with the assumptions listed below;

A17. Trip requests are made when the service users are willing to travel. In other words, there is no pre-booked trip request considered in this context.

A18. A trip request contains four information; trip request time, the number of travellers, the drop-off location and the type of service they are willing to use. That information is used to match the request with a driver and other trip requests, if applicable.

A19. Assuming that users are aware of a vehicle capacity, the number of travellers per request cannot exceed the maximum number of passenger seats per vehicle (i.e. vehicle capacity). When service users request a shared service with accompanied people, their group size needs to be smaller than the vehicle capacity so that (the part of) the trip could be shared with other requests. When the group size is the same as the maximum number of passengers per vehicle, they cannot request a shared service.

For instance, when a vehicle has 4 passenger seats, at maximum, a group of 4 people can request a non-shared service. However, assuming that the minimum number of trip requests per shared service is 2, only the group with three or less people can request a shared service.

A20. If there is no available vehicle for a selected service type when a trip is requested, a user will wait until she/he is served. In other words, once they made a request, they will not cancel their trip or switch the service type.

A21. A trip request is set to be arriving separately with the random interval.

A22. The arrival rate is influenced by the demand level of each service. It is assumed that the expected total travel demand for a certain period in a certain area is fixed. However, the actual total number of request varies day by day.

The below section presents the reflection of some assumptions specified above.

The assumption A19

The number of travellers per trip could limit their option to chose the shared service. Also, it could affect the probability of finding other trip requests to share a ride with for a shared service. For instance, assuming a vehicle has 4 seats for passengers, if the group of 3 people request a shared service, they can only share a ride with a trip request with a single traveller. On the other hand, if a single person requests a shared service, they could be matched with a trip request made by a group of 1 to 3 people. Therefore, the lower the number of travellers per request, the higher the probability to be matched with other trip requests for shared service.

3.4.1.3 **The service process**

In this model, two types of on-demand ride service are modelled, non-shared service and shared service. Both services are operated under the assumptions listed below;

A23. The capacity of a vehicle is uniform and fixed among all fleet.

A24. The fleet size for each service is constant during a period of interest on the given day.

A25. The fleet of on-demand ride service does not influence on the congestion level.

A26. A vehicle always makes a round trip from and to the pick-up demand hotspot, as displayed in Figure 17.

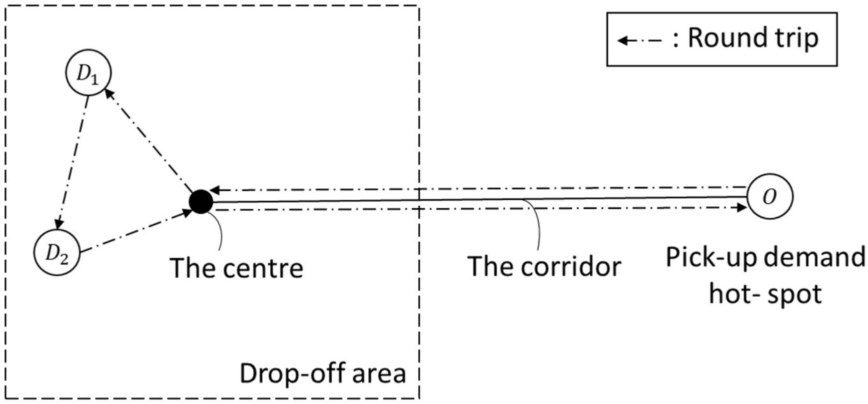

Figure 17 An example trajectory of a round trip with two trip requests (i.e. two different drop-off points D_1andD_2) from the pick-up hotspot (i.e. O). All round-trips initially follow the corridor (e.g. a motorway) to the centre of the drop-off area, within which drop-off points are randomly distributed.

A27. The loading/unloading time of each passenger is assumed to be small enough compared to an overall travel time and is not included in a round trip time.

A28. A vehicle does not formulate a physical queue when they are on standby and waiting to be matched with trip requests.

A29. When a trip request arrives, the service operation system matches trip request(s) and an available driver and decide the order to visit each drop-off location following the predetermined strategies

A30. The shortest path is selected as a route to follow between each drop-off, the centre of the drop-off area, the pick-up demand hotspot. In this model, it is represented as the straight line between two points.

A31. If there is more than one driver available, the trip request is randomly assigned to one of them.

A32. If there is no driver available, a trip request is stored at the end of the virtual queue in the system

A33. Once a round trip starts, there will be no change in schedules and routes all the way.

A34. Both drivers and users always follow the instruction provided by the service platform through the application.

The trip matching process of non-shared and shared service operates under different assumptions as a shared service involves an additional step.

In the case of non-shared service,

A35. The non-shared service is operated based on the first-come-first-served (FCFS) principle. Hence, the trip request that arrived first is assigned to an available driver (i.e. a driver in the standby phase) first. If there is no available driver, a request will be put at the end of the virtual queue in the system and wait until a driver becomes available (see Figure 18).

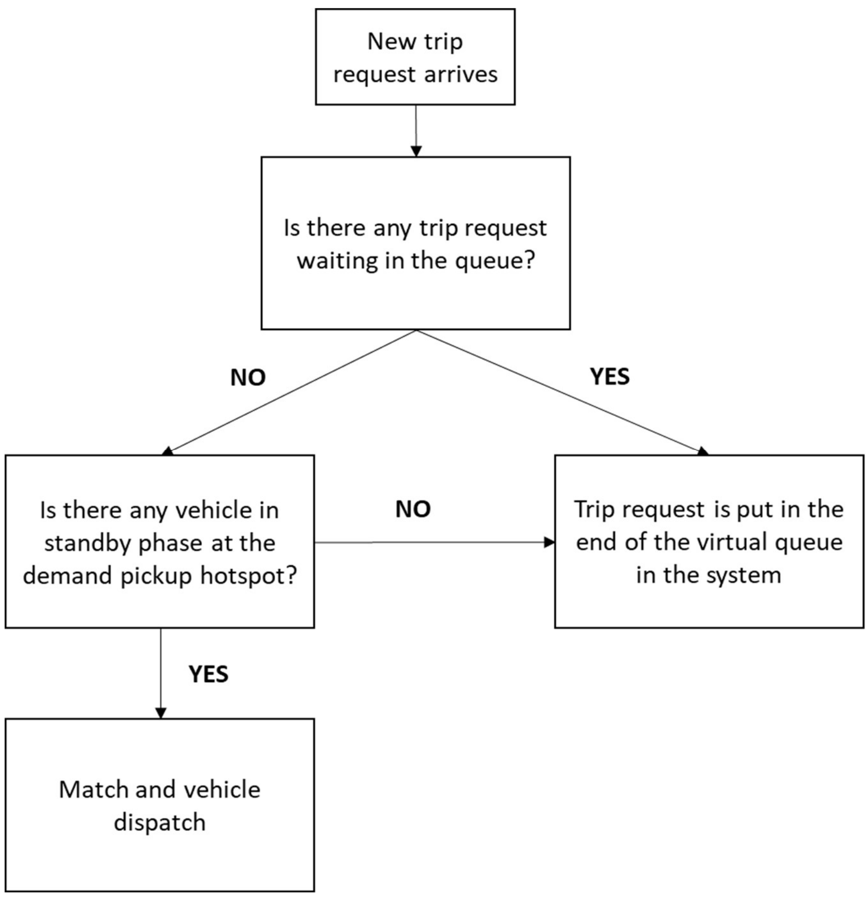

Figure 18 the flowchart of a trip matching process for a non-shared service

In the case of shared service, the service process follows the flowchart summarised in Figure 19. From the trip requests arrival to the vehicle dispatch, a trip request needs to go through two stages; clustering and assignment. The clustering indicates the process where the multiple trip requests are combined. The assignment implies the process where the clustered requests are matched with an available driver.

A36. The minimum number of trip requests per cluster is predetermined.

A37. The total number of users per clustered requests cannot exceed the number of passenger seats per vehicle.

A38. When a trip request for shared service arrives, it is first assessed if it can be combined with the existing *clustered* requests waiting to be matched with an available driver. The resulting action would be as follows;

1) If any clustered request waiting is compatible with a newly arrived request, the newly arrived request will be added to the cluster.

2) If there is no clustered request waiting or any of the waiting clustered requests are not compatible, the request is assessed if it can be combined with any *non-clustered* trip request.

If the newly arrived request could not find any other trips to form a cluster, it will be stored at the end of the virtual queue for non-clustered request.

A39. If a trip request waited longer than the threshold waiting time for a potential sharing partner to arrive, it would be ready to be matched with an available shared service driver without forming a cluster.

A40. The threshold waiting time for sharing partners is predetermined and fixed by the service platform operator.

A41. Both the clustering process and the assignment process follows the first-come-first-served (clustered) principle in general.

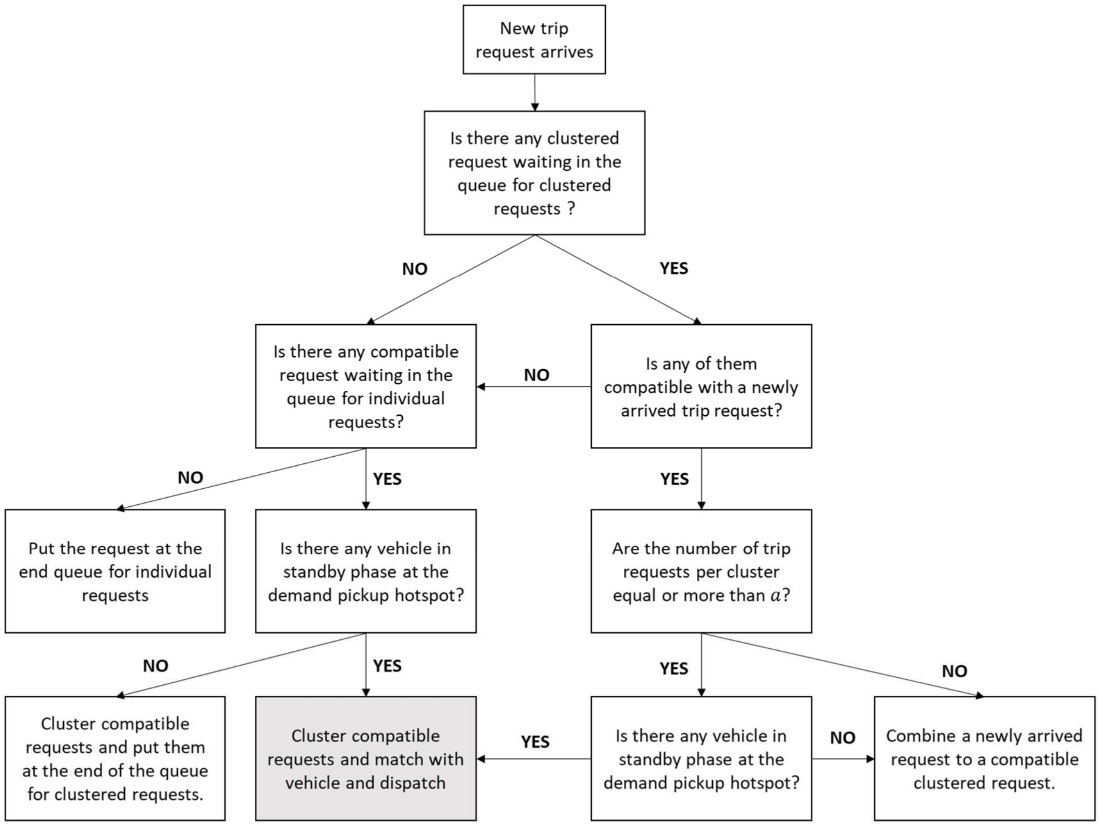

Figure 19 the flow chart explaining the process of a newly arrived request for shared service to find sharing partners or to join the queue for non-clustered trips

The below section presents the reflection of some assumptions specified above.

The assumption A26

A service process of an on-demand ride service consists of two parts of trips; a trip with passengers and a trip without any passenger. As illustrated in Figure 20, the order of those two parts can be different based on the model assumption. The left figure in Figure 20 shows the case where drivers stay at the drop-off location of the last request. Hence, when the new request arrives, they need to travel to the pick-up location first. As a result, the trip with no passenger comes first. It could be the case when there is no apparent demand hotspot. The right figure in Figure 20 illustrates the case assumed in the proposed model where drivers always come back to the demand hotspot represented as a star. As summarised in section 2.7 in Chapter 2, both assumptions could be observed in the real-world situation.

Assuming the driver making a round trip from and to the pick-up demand hotspot could simplify the matching process, there is no need to consider the proximity of the drivers idling location to the requested pickup location.

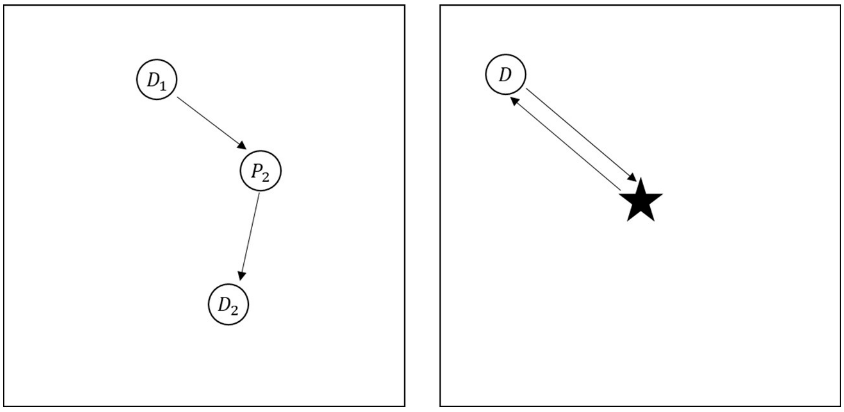

Figure 20 an example of two approaches for defining a driver's behaviour. The left figure represents the case where a driver stays at the last drop-off point. The right figure represents the case where a driver always returns to the pick-up demand hotspot illustrated with a star.

The assumption A28 and A32

In the real-world system, the situation where both users and drivers (vehicles) formulate a physical queue is somewhat limited. It is only possible when there is a dedicated point prepared for them to form a queue. It could be seen in some airports or big train or bus stations located in a busy area. However, even in that case, users or drivers could move for a short distance to avoid queueing. Besides, forming a physical queue is one way to reserve a "spot" to receive the service later, which is currently unavailable. In the current research, the application would reserve a "spot" for users and drivers; hence, there is no initiative for both users and drivers to form a physical queue by restricting their movement.

It should also be mentioned that an absence of a physical queue allows ignoring users' and drivers' feeling of unfairness when a trip matching process does not respect the drivers' or trip requests' arrival order. They do not have the mean to check that.

The assumption A38 and A39

Trip requests become ready to be matched with an available driver when;

1) they formed the cluster consists of the minimum number of requests per cluster

2) they waited longer than the threshold waiting time for sharing partner.

Until the conditions mentioned above are satisfied, trip requests are pending in the virtual queue. It could be defined as the waiting time for sharing partners during which they need to wait regardless of the driver's availability. In general, the stricter the condition for clustering becomes, the longer users need to wait. Hence, if users are only allowed to share with the people whose destination are very close to their own destination, they need to wait longer to find those who satisfied this condition. As a result, an additional waiting time may end up negating the benefit of in-vehicle time saving. The service platform operator needs to consider such a trade-off when they decide the clustering conditions.

Also, it should be noted that a trip request which waited for sharing partners longer than the threshold time is not always served immediately without forming the cluster. Even if they become ready to be matched with a driver, a user still needs to wait if no driver is available. Meanwhile, the new request could arrive and be clustered with it.

Assumption A41

Unlike non-shared service, shared service would not always follow the FCFS principle from the individual trip request point of view. The exception is when the trip request always contains one traveller, namely if they will never bring an accompanied person. On the other hand, clustered requests are always served following the FCFS principle. Figure 21 illustrates the example of a case where the clustering process does not follow the FCFS principle. A trip request is added to the left side as it arrives in this example. Assuming that a vehicle can carry 4 passengers maximum, the first two requests from the right cannot be combined as they will exceed the vehicle capacity. Hence, the first and third request from the right would be clustered. Like this example, in only the case where the clustering is infeasible due to a predetermined constrain (e.g. the assumption A37), the FCFS principle is not strictly applied.

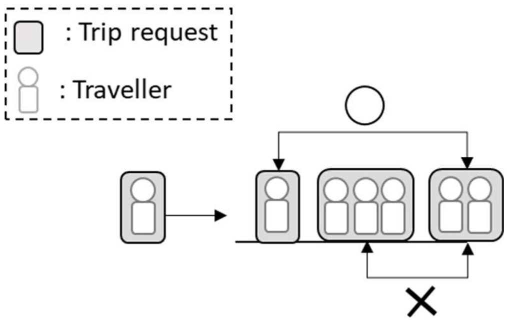

Figure 21 the example of a case where the first two requests cannot be combined because the total number of users per request exceeds the number of passengers seats per vehicle (i.e. 4).

3.4.1.4 **The summary of terminologies**

Table 5 the summary of terminologies used in this chapter

Terminology	Description

Trip request	Made by a user when they are willing to travel. It contains four information; the trip request time, the number of users, the drop-off location and the type of service they are willing to use
The number of travellers per trip request	Expresses how many people are travelling as a group between the same pick-up and drop-off location. It is 1 when a user travels alone. It could be more than 1 when a user travels with accompanied people. It could limit their service choice as specified with the assumption A19
Clustered request	It consists of multiple trip requests for shared service.
The number of requests per cluster	Express how many trip requests are combined to form a cluster. The minimum number of requests per cluster, a, is fixed and given by the service operator.
The number of travellers per cluster	Expresses the total number of travellers among clustered trip requests. The maximum number of travellers per cluster cannot exceed the number of passenger seats per vehicle, b.
Round trip	A trip from and to the pick-up demand hotspot
The number of drop-offs per round trip	It is equivalent to the number of requests per cluster. For non-shared service, it is always 1. For shared service, it is equal or higher than a. * *(unless the trip request waited for shared partner longer than threshold waiting time is served without forming a cluster)

3.4.2 The queuing representation of an on-demand ride service

Following the assumptions specified in subsection 3.4.1, an on-demand ride service is represented with a queueing model by positioning a "server" for a queue as each vehicle and service time as a round trip time. A simplified representation of an on-demand ride service with queueing theory is observed in the other literature as summarised in section 2.3.2 in Chapter 2. Queuing system can be decomposed into three parts; 1) arrival process, 2) service mechanism, and 3) characteristics of queues (Bose, 2002). There are several qualities to be checked for each part to define the characteristics of a queue. Table 6 summarises attributes for each part of the proposed model.

Table 6 the summary of attributes of a queuing model utilised for the model

	Category	**Attributes of a queueing model for this study**
Service process	Number of servers	Multi-server
	Number of queues	Single queue for *non-shared* service Multiple queues for *shared* service
	Serving type	Single service
Arrival process		Single arrival with different load per service
Characteristics of queue		No balking*, reneging**, jockeying*** allowed. The capacity of a queue is infinite.

* balking: customers deciding not to join the queue,** jockeying: customers switch between queues depending on the expected time to be served, *** reneging: customers leave the queue

A *non-shared service* is modelled with a single service multiple server queue. Figure 22 visualises the service process of the queue representing a non-shared service. A trip request arrived at the system is served one by one with one of the available servers (i.e. vehicle). A trip request forms a queue only when none of the servers is available. A *shared service* is also modelled with a single service multiple server queue. However, as illustrated in Figure 23, it has multiple queues, one for single (non-clustered) requests and one for clustered requests, which are vertically connected. A trip request for shared service arrives at the queue for single requests first. The transition to the queue for clustered request happens if one of the below criteria is satisfied. Three criteria are summarised below.

1) There is any compatible clustered request waiting in the queue for clustered request. A compatible clustered request indicates that when the newly arrived request joins it, the total number of travellers per cluster does not exceed b.

2) With trip requests waiting in the queue for single requests, a newly arrived request can form a cluster, the total number of which does not exceed b and the number of requests per which is equal or higher than a.

3) A trip request waited in the queue for single requests longer than the threshold waiting time

In the case of users always travelling alone, it works as a bulk service queue.

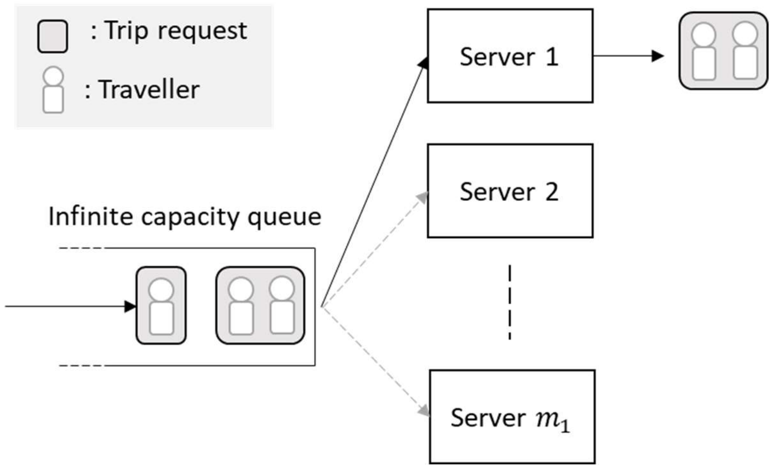

Figure 22 the diagram illustrating a queuing representation of non-shared service where each server is defined as a vehicle

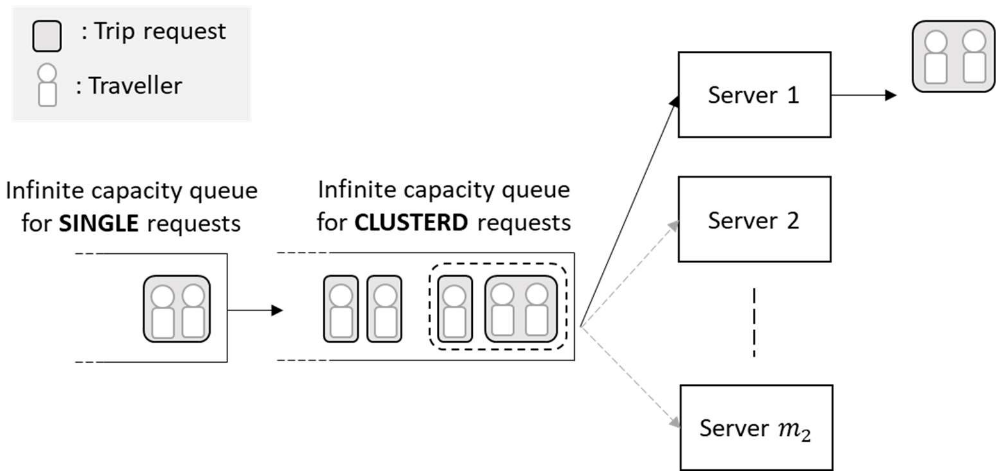

Figure 23 the diagram illustrating a queuing representation of shared service where each server is defined as a vehicle

3.4.3 User waiting time

A user waiting time for ith trip with the service k on day d, $WT_{i,k,d}$ is generated through the simulation and can be expressed as a function of the mean arrival rate and the mean service rate as displayed below,

$$WT_{i,\mathrm{k},d} \sim f(\mathbf{\Lambda}_d, \mathbf{M}_d) \tag{3.22}$$

where

$$\mathbf{\Lambda}_{\boldsymbol{d}} = (\Lambda_{1d}, \Lambda_{2d}) = \frac{\mathbf{N}_d}{\varpi} \tag{3.23}$$

$$\mathbf{M}_d = (\mathrm{M}_{1d}, \mathrm{M}_{2d}) \sim f(a, b, \delta(\boldsymbol{\zeta}, \kappa), \mathbf{S}_d, \theta, h) \quad (3.24)$$

$\boldsymbol{\Lambda}_d$ is the vector containing the mean arrival rate for non-shared and shared service (i.e. Λ_{1d} and Λ_{2d}) where ϖ expresses the duration of interest during which $\mathbf{N}_d$ trip requests arrive. The trip requests are modelled to randomly arrive following the Poisson distribution with the given mean arrival rate, which is $\boldsymbol{N}_d/\varpi$. The number of users per trip request is determined by randomly generating the number of accompanied people per request from the Poisson distribution with the fixed mean, h.

The service rate for non-shared service, M_{1d}, and shared service, M_{2d}, are defined as the function of the fleet size for both services, $\mathbf{S}_d$, and few parameters. a indicate the minimum number of requests per cluster, while b implies the maximum number of passengers per vehicle. $\delta(\mathrm{v}, \mathrm{s})$ is the mean round trip duration for non-shared service, which consists of the vector that specifies the service network geography, $\boldsymbol{\zeta} = (\zeta_c, \zeta_d)$, and the mean speed of the vehicle, η. ζ_c indicates the length of corridor while ζ_d indicates the side length of the drop-off area. θ indicates the threshold waiting time for sharing partner(s).

3.4.4 **Round trip time inference**

A travel time for each round trip is randomly sampled from a corresponding round trip time distribution. The round trip time distribution varies according to the number of drop-offs per round trip, $z_{l,j,k,d}$, and network property specified with $\boldsymbol{\zeta}$ and κ. Hence, travel time for the lth round trip made by a driver j providing the service k on day d can be expressed as below;

$$r_{l,j,k,d} \sim f(z_{l,j,k,d}, \boldsymbol{\zeta}, \kappa,) \quad (3.25)$$

Figure 24 illustrates the flow chart for a round trip time inference. The round trip time distribution is inferred in advance with given multiple critical attributes of the system through the simulation in this research, yet, it could be estimated from the empirical data if it is available. It should be emphasised that this inference process is intended to bring the effect from the operational environment to an on-demand ride service while keeping the generality in the model. The embedded randomness in sampling a round trip is intended to capture the variation in travel time regarding geographical differences among destinations and some other factors.

A whole round trip from and to a pick-up hotspot can be divided into two parts, a trip on the corridor connecting pickup location and the centre of the drop-off area (i.e. solid line in Figure 17) and a trip from the centre of the drop-off area to the final destination of each trip (i.e. dashed line in Figure 17). As the length of the corridor is fixed, a round trip in the drop-off area is the only source of variance in terms of travel distance and time. The round trip length distribution pattern in the drop-off area differs based on five components, a network structure, the size and shape of the drop-off area, the distribution pattern of the final destinations, and the number of drop-offs per round trip.

The trip length distribution in the drop-off area is estimated through a simulation by repeatedly solving Travelling Salesman Problem (TSP) for each number of drop-offs per round trip. The distribution of round trip length is determined by adding the doubled length of the corridor. The service time distribution can be estimated by dividing it by the mean speed of a vehicle in the network, which is influenced by the congestion level. The effects to and of congestions level and the loading/unloading time of each passenger are not considered as defined by the assumption A25 and A27.

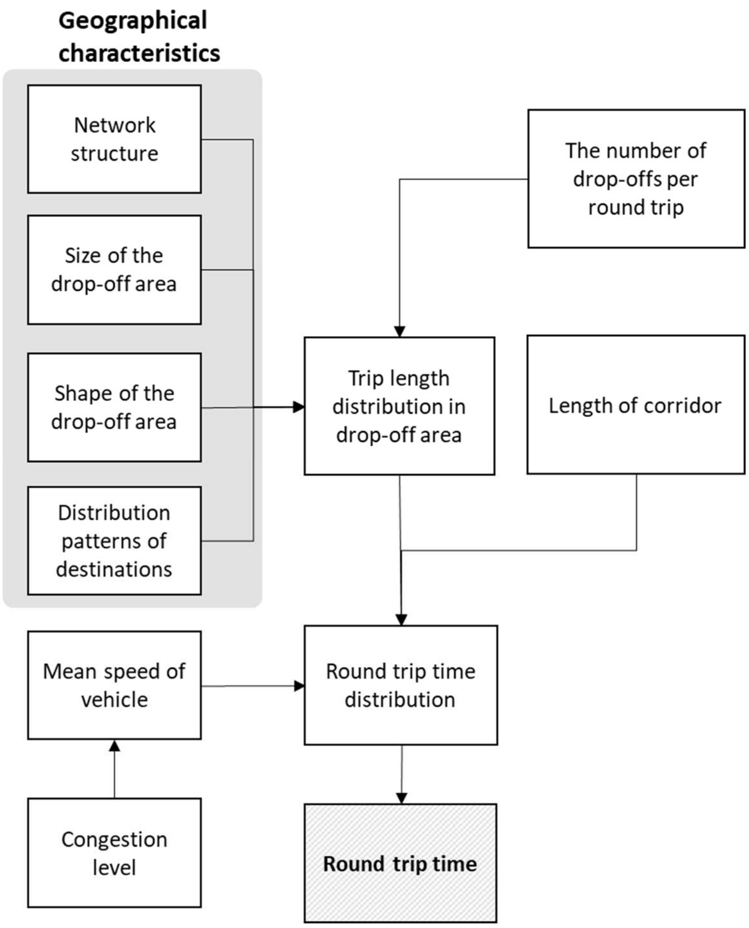

Figure 24 the diagram of a round trip time inference process

3.4.4.1 **Example**

This section shows how the round trip time distribution would differ with a different number of drop-offs (NoDs) per round trip with a given condition assuming a test network as illustrated in Figure 25. It consists of the square drop-off area with an 4.39 km side length and a 10 km corridor connecting the pick-up demand hotspot and the centre of the drop-off area. Drop-off locations are uniformly distributed in the area, and a vehicle moves 20 km/h on average inside. On the other hand, the average speed of a vehicle is set to be 60km/h on the corridor, assuming this as the main road, for instance, connecting the city centre and a neighbourhood area.

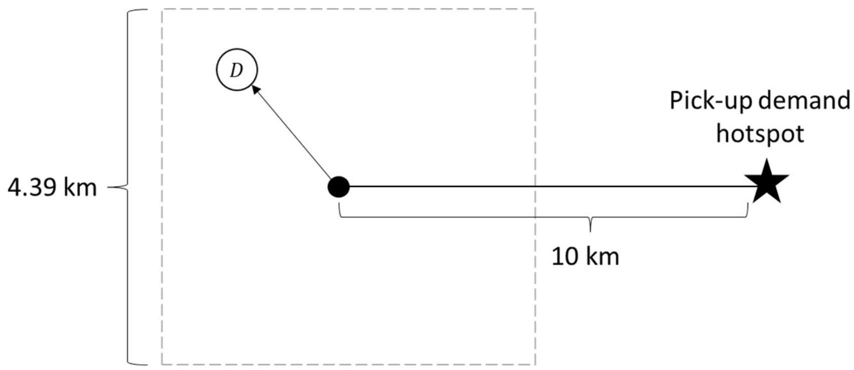

Figure 25 the simple example of a service network

Figure 26 illustrates simulated round time distributions for different NoDs per round trip, z. A round trip time is repeatedly simulated 10,000 times for each z ($1 \leq z \leq 10$) with the conditions specified above, from which a round trip time distribution is estimated. For each iteration, a drop-off location for each trip request is randomly generated within the drop-off area. The shortest route from the centre of the area is estimated by solving Travelling Salesman Problem (TSP). Figure 27 displays the mean round trip time for each z in the range of $1 \leq z \leq 10$. As shown in Figure 27, the marginal increase of the mean round trip time decreases as NoDs per round trip increases. It is reasonable as the mean distance between drop-off locations decreases as the drop-off locations increases.

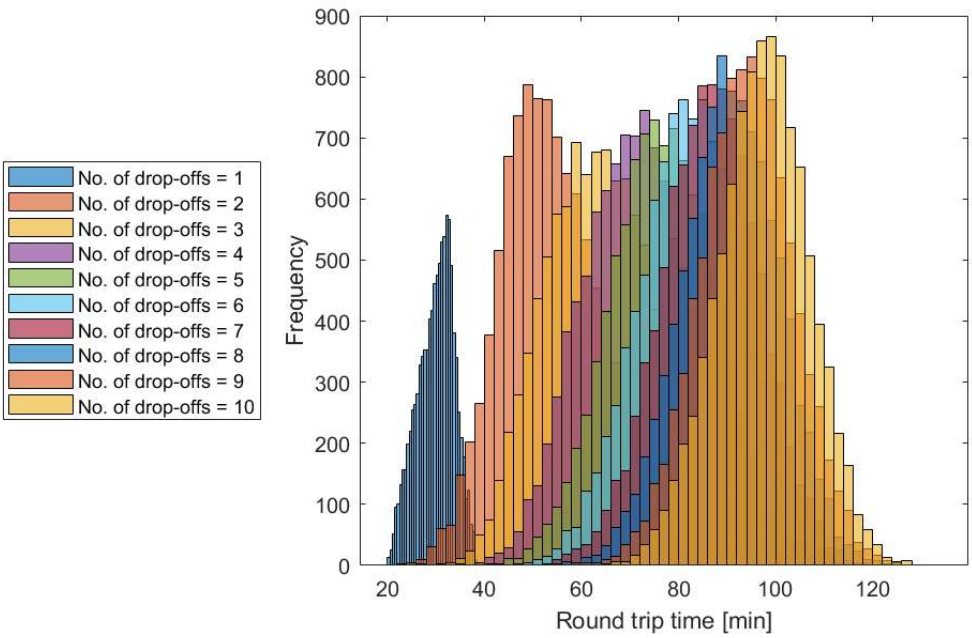

Figure 26 round trip time distributions at the service network specified in Figure 25 with the NoDs per round trip 1 (i.e. non-shared service) to 10.

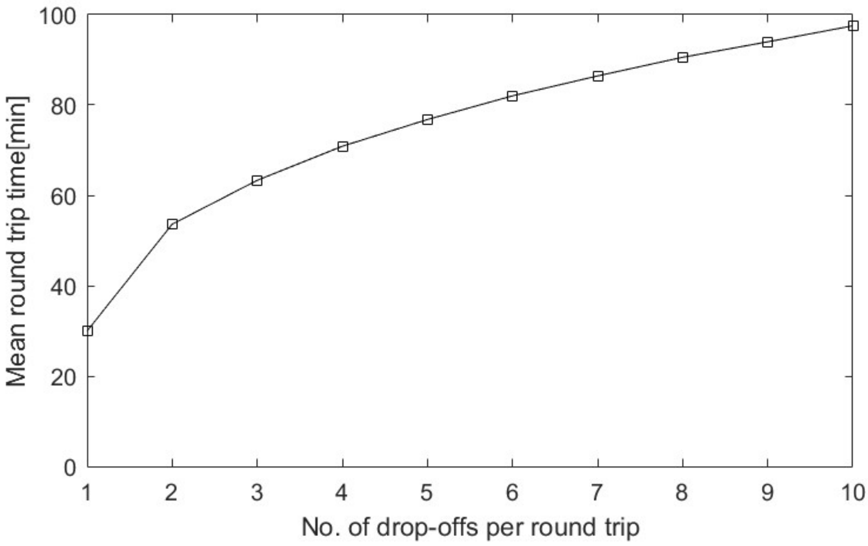

Figure 27 the mean round trip time at the service network specified in Figure 25, with NoDs per round trip 1 (i.e. exclusive service) to 10.

3.4.5 **User's in-vehicle time**

This section explains the process to estimate an in-vehicle time for each user from a round trip time. An in-vehicle time for user i included in the lth round trip of a driver j providing the service k on day d, $t_{i,k,d}$, can be expressed with the function of a round trip time, $r_{l,j,k,d}$, and the order of user i to be dropped off in the lth round trip of driver j, $g_{i,l,j,k,d}$, as follows;

$$IVT_{i,k,d} \sim (r_{l,j,k,d}, g_{i,l,j,k,d}) \tag{3.26}$$

where

$$1 \leq g_{i,l,j,k,d} < z_{l,j,k,d}$$

In the case of non-shared services, a driver visits only one drop-off location requested by a user, hence, $g_{i,l,j,k,d} = z_{l,j,k,d} = 1$. As specified by assumption A30, a driver moves the straight line between two locations (e.g. a drop-off location, a pick-up location, the centre of the drop-off area). Hence, the in-vehicle time for a non-shared service user is equal to half of the round trip time as presented below;

$$IVT_{i,1,d} = r_{l,j,1,d}/2 \tag{3.27}$$

The estimation process for in-vehicle time with a shared service involves a few more steps than the one for a non-shared service. As all clustered trip requests for a round trip share their journey on the corridor, only a trip in the drop-off area needs to be modelled.

A round trip time is generated from a specific round trip distribution for NoDs per round trip. Therefore, a relative position of drop-off points against the centre of the drop-off area is generated. Then, in-vehicle time for each drop-off points is calculated, which is later randomly assigned to each request involved in a round trip. It implies that the order to visit final destinations does not correlate with trip requests' arrival order. In the real-world system, the order of arrival of the request could influence on the order of delivery to drop-off locations if the passengers time window is uniform and very strict. However, the relative proximity between a drop-off and a pick-up location among shared partners would anyway not correlate with the order of arrival.

3.4.5.1 **Example**

This section illustrates an example of how to estimate in-vehicle time when NoDs per round trip is 3. Three drop-off points are randomly generated in a unit square space, which determines the relative positioning of drop-offs and the centre of the square. By solving the TSP, the optimised route to visit 3 points are determined. To simplify the example, three points (A, B, and C) and the centre of the square (the star) are forming the square shape the side length of which is 0.25 (see Figure 28). The last thing that needs to be determined is the direction of a round trip. In this example, it needs to be decided if a driver visits point A first or point C first. In this model, a driver will visit the one closest to the centre of the drop-off area, which will minimise the total in-vehicle time of all users.

In this example, both clockwise and counter-clockwise would give the same total in-vehicle time; therefore, the clockwise is selected randomly. Assuming that the round trip time in the drop-off area is 20 min, the travel time on each leg of the square would be 5 min each. Hence, an in-vehicle time from the centre to the first stop, A, the second stop, B, and the last stops, C, in this unit square are estimated as 5 min, 10 min, and 15 min.

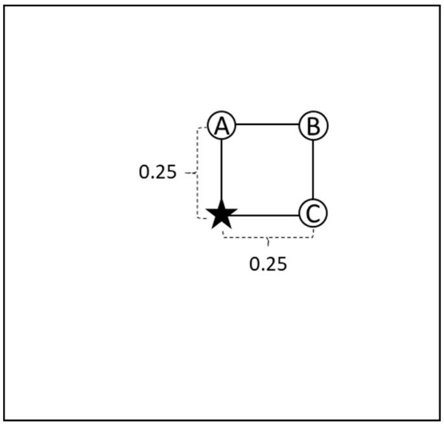

Figure 28 an example of randomly generated three drop-offs

3.4.6 **Fare**

The fare for ith user of the service k on day d is estimated based on the round trip time required by a vehicle to serve requested trip(s) and can be expressed as the function specified below;

$$c_{i,k,d} \sim f\left(r_{l,j,k,d}, \gamma_f, g_{i,l,j,k,d}, z_{l,j,k,d}\right) \tag{3.28}$$

where $r_{l,j,k,d}$ is the travel time of the lth round trip of a driver j providing the service k, with which ith user was travelling on day d. γ_f is a parameter deciding a fare/min. $g_{i,l,j,k,d}$ specifies the ith user's order to be dropped off during the lth round trip of a driver j providing the service k. $z_{l,j,k,d}$ is NoDs during the lth round trip of a driver j.

In this model, a round trip time multiplied by γ_f is paid to a driver as a driver made each round trip. Hence, in the case of non-shared service, one user is responsible for paying all of them as a fare as specified with the below equation;

$$c_{i,1,d} = r_{l,j,1,d} \times \gamma_f \tag{3.29}$$

In shared service, the total fare charged for a round trip is divided among all users who made a request assigned to the round trip. Hence, drivers earn the same amount of money regardless of which service they provide if a round trip time is the same. On the other hand, if a round trip time is the same, a shared service user always pays less fare than a non-shared service user does as they split the fare among multiple people. The fare for ith shared service user who is assigned to the lth round trip of a driver j on day d is estimated as follows;

$$c_{i,2,d} = \gamma_f \cdot \left(\frac{\tau(g_{i,l,j,2,d})}{\sum_{v=1}^{z_{l,j,2,d}} \tau(v)} + \frac{2\zeta_c}{z_{l,j,2,d}} \right) \tag{3.30}$$

where

$$\sum_{v=1}^{z_{l,j,2,d}} \tau(v) = r_{l,j,2,d} - 2\zeta_c \tag{3.31}$$

$$c_{i,2,d} \le c_{i,2,d}^{ex} \tag{3.32}$$

$\tau(g_{i,l,j,2,d})$ indicates the modified Shapley value for a user who is travelling to $g_{i,l,j,2,d}$th drop-off of the lth round trip of a shared service driver j on day d. It estimates the length of trip user i is responsible for paying during the trip in the drop-off area. The fare charged for the travel on the corridor, $2\zeta_c$, is equally split among users. $\tau(g_{i,l,j,2,d})$ is calculated with the following equation;

$$\tau(g_{i,l,j,2,d}) = mc_{g_{i,2,d}} + \left(r_{l,j,2,d} - 2\zeta_c - \sum_{v=1}^{z_{l,j,2,d}} mc_v \right) \cdot \frac{w_{g_{i,l,j,2,d}}}{\sum_{v=1}^{z_{l,j,2,d}} w_v} \tag{3.33}$$

Similar to the conventional Shapley value, it follows three principles of travelling salesman games (i.e. efficiency, individual rationality, and minimal obligation) which are specified in section 2.4 in Chapter 2. Also, the process to estimate the marginal cost for each user included in a round trip, $mc_{g_{i,l,j,2,d}}$, is the same with the conventional Shapley value. However, while the rest of the cost is allocated equally among travellers in the conventional Shapley value, the proposed method distributes it disproportional to in-vehicle time by estimating some "weight", $w_{g_{i,l,j,2,d}}$. Specifically, the weight is designed to consider users' disutility of staying in a vehicle longer to the distribution process of monetary cost. $w_{g_{i,l,j,2,d}}$ is given by;

$$w_{g_{i.l.j.2,d}} = \frac{IVT_{i,2,d}^{ex}}{IVT_{i,2,d}} \tag{3.34}$$

where $IVT_{i,2,d}^{ex}$ indicates the expected in-vehicle time if user i selected a non-shared service.

It should be mentioned that the fare is estimated only based on travel time though travel distance could also be considered to estimate fare in the real-world system. If travel distance is considered in the network assumed in Figure 25, shared service drivers will earn less than non-shared service drivers. This is because shared service drivers travel longer in the drop-off area and shorter on the corridor than non-shared service drivers. Hence, the distance per minute would be shorter for shared service. As a result, the fare would be cheaper for shared service. It could be a motivation for users to choose a shared service. At the same time, it could also discourage drivers from choosing to provide a shared service.

3.4.6.1 **Example**

Assuming that the same round trip used in section 2.4 specified with Figure 5, this section demonstrates how to estimate a modified Shapley value for the user dropped off at first and for the user dropped off at the second. As explained in the previous section, the marginal cost estimation process follows equation (2.1) and (2.2).

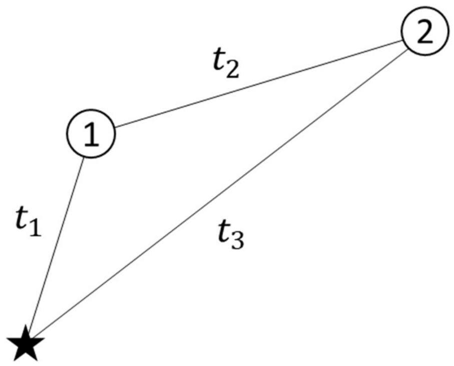

Figure 29 an example of a round trip that consists of two drop-off locations

Then, the weight for the user travelling to the 1st drop-off, w_1, and the 2nd drop-off, w_2, is estimated as follows,

$$w_1 = \frac{IVT_1^{ex}}{IVT_1} = \frac{t_1}{t_1} \tag{3.35}$$

$$w_2 = \frac{IVT_2^{ex}}{IVT_2} = \frac{t_3}{\mathrm{t}_1 + \mathrm{t}_2} \tag{3.36}$$

Then, the modified Shapley value for each user $\tau(1)$ and $\tau(2)$ are estimated by distributing the remaining cost according to weights as follows,

$$\begin{aligned}\tau(1) &= t_1 + t_2 - t_3 + (t_1 - t_2 + t_3) \cdot \frac{w_1}{w_1 + w_2} \\ &= t_1 + t_2 - t_3 + (t_1 + t_3 - t_2) \cdot \frac{t_1 + t_2}{t_1 + t_2 + t_3}\end{aligned} \tag{3.37}$$

$$\begin{aligned}\tau(2) &= t_2 + t_3 - t_1 + (t_1 - t_2 + t_3) \cdot \frac{w_2}{w_1 + w_2} \\ &= t_2 + t_3 - t_1 + (t_1 + t_3 - t_2) \cdot \frac{t_3}{t_1 + t_2 + t_3}\end{aligned} \tag{3.38}$$

When there are only 2 drop-offs (i.e. $z_{l,j,k,d} = 2$), individual rationality specified with equation (4.32) is always satisfied. Equation (4.32) can be rewritten for users travelling to the 1st drop-off location as follows;

$$t_1 + t_2 - t_3 + \frac{(t_1 + t_3 - t_2)(t_1 + t_2)}{t_1 + t_2 + t_3} \leq 2t_1 \tag{3.39}$$

and for users travelling to the 2nd drop-off location as follows;

$$t_2 + t_3 - t_1 + \frac{(t_1 + t_3 - t_2) \cdot t_3}{(t_1 + t_2 + t_3)} \leq 2t_3 \tag{3.40}$$

Both equations can be expressed as follows;

$$(3.39) \Leftrightarrow \frac{t_1 + t_2}{t_1 + t_2 + t_3} \leq 1 \tag{3.41}$$

$$(3.40) \Leftrightarrow \frac{t_3}{t_1 + t_2 + t_3} \leq 1 \tag{3.42}$$

since all leg length between two points in Figure 29 is positive (i.e. $t_1, t_2, t_3 > 0$), equation (3.41) and (3.42) are always true. However, the results of a simulation indicate that when the drop-off locations are more than 2 (i.e. $z_{l,j,k,d} > 2$), the estimated costs could exceed the maximum amount that users are willing to pay for some cases. In such cases, the excess cost is distributed proportionally to a weight estimated with an equation (3.34) among those who still have an allowance to pay.

3.4.6.2 **The numerical experiment: Discount vs additional in-vehicle time**

In this section, a numerical experiment is conducted to assess the attribute of the modified Shapley value. In particular, it is examined how much discount each user can get regarding an additional time to spend in a vehicle. It is assumed that NoDs per round trip and a user's order to be dropped off would be the primary contributor to the trade-off between additional in-vehicle time and the fare discount. Hence, the four cases with the different NoDs per round-trip (i.e. $z_{l,j,k,d} = 2\ to\ 5$) are generated and compared. The service network is assumed to be the same as the one specified in Figure 25.

The 10,000 instances are generated for each case with different NoDs per round trip. In each case, the drop-off points are uniformly randomly generated, based on which in-vehicle time [min] and "the allocated cost to cover [min]" are estimated. The allocated cost to cover implies the length of a trip that a user is responsible for covering and estimated as the modified Shapley value. The fare is estimated by multiplying it with γ_f, which is assumed to be $\gamma_f = 1$ [monetary unit/min] in this example. Also, the user's VoT is assumed to be 1 [min/monetary unit]. Hence, users expect to get 1 min worth of discount when in-vehicle time extended 1 min compared to the expected fare and in-vehicle time when they travel between the same OD with non-shared service.

Figure 30 compared the fare discount distribution among 4 cases with NoDs per round trip being 2 to 5. The discount is estimated by subtracting the actual fare of using a shared service from the expected fare for travelling by a non-shared service. In general, the mean amount of discount increases as NoDs increases. At the same time, the variance of discount increases as NoDs increases. As Figure 31 indicates, the additional in-vehicle time also increases as NoDs increases. The additional in-vehicle time is estimated by subtracting the expected in-vehicle time of travelling by a non-shared service from in-vehicle time of travelling with a shared service. The variance in the additional in-vehicle time also increases as NoDs increases.

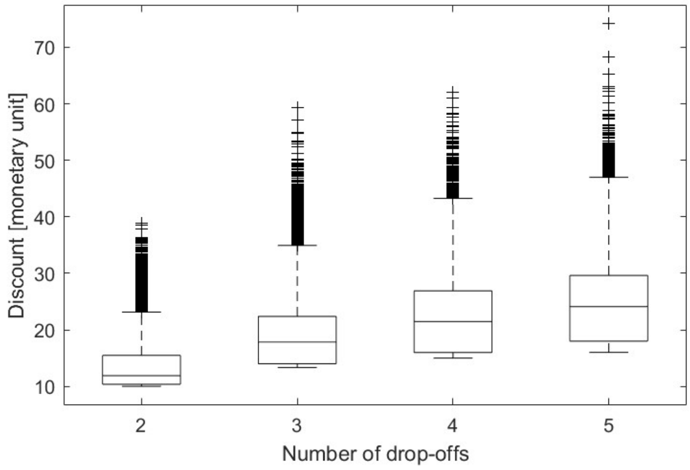

Figure 30 The comparisons of the distribution of discount in allocated minutes for each trip requests to cover among 4 cases with different NoD (i.e. NoD = 2,3,4,5)

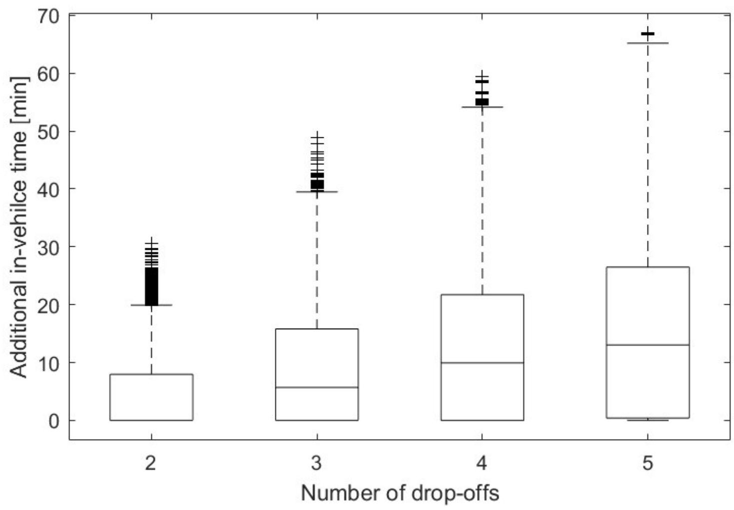

Figure 31 The comparisons of the distribution of additional in-vehicle time among 4 cases with different NoD (i.e. NoD = 2, 3, 4, 5)

The tendencies described above are consistent with the expectations. However, there is still an unanswered question: if the discount is enough to compensate for the additional in-vehicle time. In this example, users expect to receive 1 monetary unit discount for a minute of additional in-vehicle time, as explained above. Otherwise, users are supposed to feel that they experienced an uncompensated trip. Table 7 compares 2 types of values estimated with a modified and conventional method which are 1) the proportion of those who have not received enough discount for each NoDs per round trip and 2) the proportion of those who have not received enough discount summarised for each order to be dropped off. In both cases, according to Table 7, the proportion of uncompensated trips increases as the NoDs. Also, the later the order to be dropped off becomes, the higher the proportion of uncompensated trips becomes for both cases.

It can also be observed from Table 7 that the proportion of the uncompensated trips is higher when the conventional Shapley value is applied. The proportion of the uncompensated trips is higher for modified Shapley value only in the cases for those dropped off at the 2nd place and only when NoDs is 4 and 5. It is because the amount of discount for those dropped off at 4th or 5th places is high, most of which is distributed among those who are dropped off earlier and experienced shorter in-vehicle time. It indicates that the modified Shapley value managed to reflect the disutility of longer in-vehicle time. It should also be mentioned that the proportion of the uncompensated trips depends on VoT. In this scenario, VoT is set as 1 [min/monetary unit]. However, if it were higher (lower), the proportion of uncompensated trips would be higher (lower). Nevertheless, even in the case of an uncompensated trip, a user always pays less than the amount she/he would have paid using a non-shared service.

Table 7 Comparison of the proportion of the uncompensated trips for the order to be dropped off and in total with different NoDs estimated with a modified (top) and conventional method (bottom).

NoDs	Type of Shapley Value	Total	The order to be dropped off				
			1	2	3	4	5
2	Modified	0.132	0	0.263	-	-	-
	Conventional	0.148	0	0.295	-	-	-
3	Modified	0.174	0	0.029	0.494	-	-
	Conventional	0.195	0	0.030	0.554	-	-
4	Modified	0.205	0	0.022	0.121	0.678	-
	Conventional	0.226	0	0.018	0.149	0.737	-
5	Modified	0.230	0	0.009	0.077	0.264	0.800
	Conventional	0.257	0	0.007	0.087	0.331	0.859

In particular, when drop-off locations are horizontally spread, it tends to be an uncompensated trip compared to the cases where drop-off locations are vertically spread. In other words, when two lines are drawn between the first/last drop-off location and the centre of the area, as illustrated in Figure 32, the angle between the two lines of an uncompensated trip tends to be wide. In other words, the wider the angle becomes, the higher the proportion of uncompensated trip becomes, as illustrated in Figure 33. Figure 33 plots the proportion of uncompensated trips against the maximum angle between two lines for four cases with different NoDs (i.e. NoDs = 2 to 5).

For instance, when NoDs per round trip is 3, the proportion of uncompensated trips are 0.031 when the angle is less than 90°. On the other hand, the proportion is 0.175 when the angle is less than 180°. From this observation, it could be concluded that this “angle” could be used as a measure to express the constraints for geographical proximity of drop-offs when the multiple trip requests are clustered in this model. However, the stronger the limitation in angle becomes, the less probable it is to find a sharing partner, meaning more waiting time. Therefore, introducing such restriction would not always increase the users' experience. In addition, it should be noted that it depends on the VoT and fare/min if users feel the additional in-vehicle time is compensated with the discount or not.

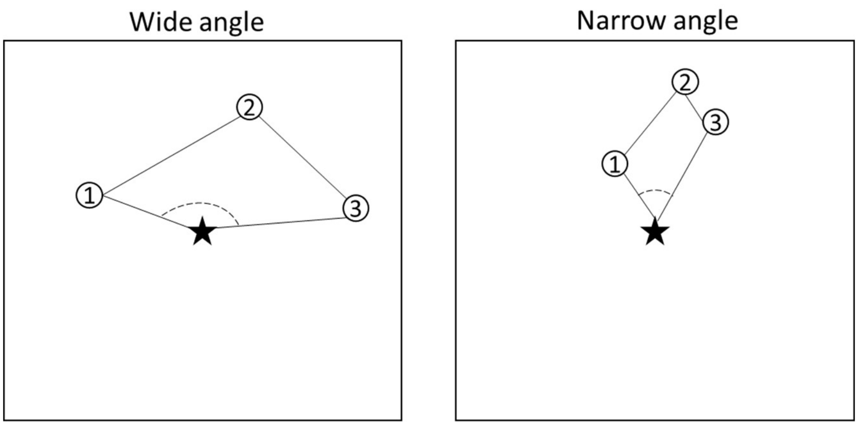

Figure 32 the example of a round trip with the wide angle (left side) and the narrow angle (right side)

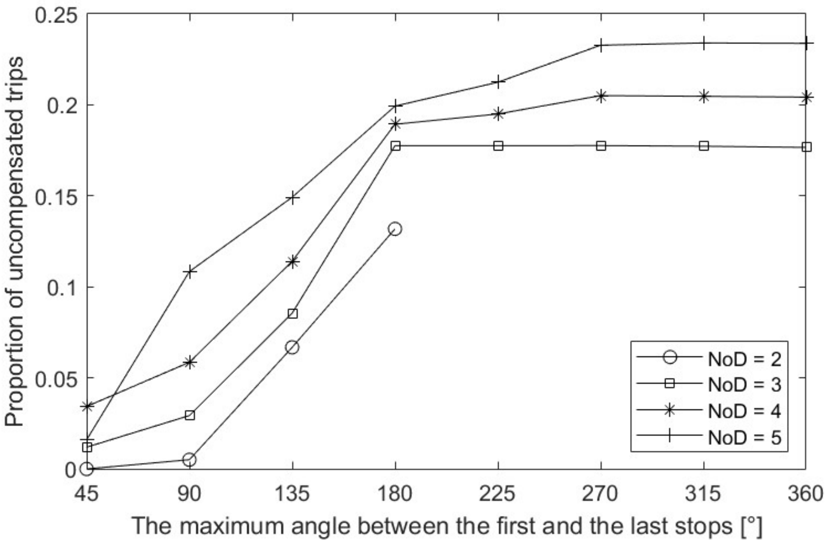

Figure 33 the proportion of uncompensated trips with the different maximum angle between the first and the last stops for 4 cases with different NoDs (i.e. NoDs = 2 to 5).

3.5 **Summary**

This chapter provided a detailed explanation of the proposed model, which includes three attributes specified with objective O1, O2, and O3 in Chapter 1. In section 3.3, the user's and driver's learning and decision model are specified. The key assumptions of this representation are the existence and the role of the third party, namely, the service platform operator. Both users and drivers request and offer a trip through the online application provided by the service platform operator. Collective learning and individual decision making are also conducted through the application.

Section 3.4. is dedicated to describing the supply model specifications. In order to represent enough details to capture essential trade-offs for shared service, a certain network structure and a driver's behaviour are assumed though both of them are observed in the real-world system as described in Chapter 2. To represent the process to split the fare among those who share a ride, the modified Shapley Value is proposed, which includes the disutility of users staying longer in a vehicle.

In the next chapter, the numerical experiment is conducted to understand the property of the proposed model.

Chapter 4 Numerical experiment with the fixed fleet size

4.1 Introduction

A detailed description of the proposed stochastic process model and assumptions are given in Chapter 3. This chapter presents some numerical results with fixed fleet size in the hypothetical service network. The numerical experiments have been designed to deliver two objectives specified below, which are;

1) to investigate how day-to-day evolution occurs within a given parameter setting
2) to understand how changing some parameters would affect the model behaviour.

The analysis has been conducted from four dimensions to deliver the second objective; 1) the sets of parameters in the utility function, 2) parameters to specify users' irregular behaviour, 3) the fleet size for each service, and 4) the service network geometry. Assuming that people often choose the option they can expect to gain the higher utility, the word "irregular" is selected to describe the opposite behaviour. The first two dimensions were assessed by conducting sensitivity tests. The last two dimensions were investigated by comparing results generated with different scenario settings.

The rest of the chapter is structured as follows. In subsection 4.2, the default parameter settings for experiments are specified. In subsection 4.3, a base scenario analysis is presented, aiming to achieve the first objectives stated above. Besides, the results from sensitivity analysis against several parameters are presented. In subsections 4.4, 4.5, and 4.6, the impact of the fleet size, ~~and~~ the service network geometry and the impact of modified Shapley Value are assessed. Finally, this chapter is concluded with the summary in section 4.7.

4.2 Default parameter setting

The service network is specified as presented in Figure 34. The length of the corridor is set to be 10 km on which the vehicle moves with the fixed average speed of 60 km/h. The drop-off area is assumed to be a square with the side length being 8.71 km, in which the vehicle moves with the fixed average speed of 20k m/h. The side length was specified for the mean round trip time with one drop-off point to 30 min. The mean duration of a round-trip for each number of drop-offs is summarised in Table 8.

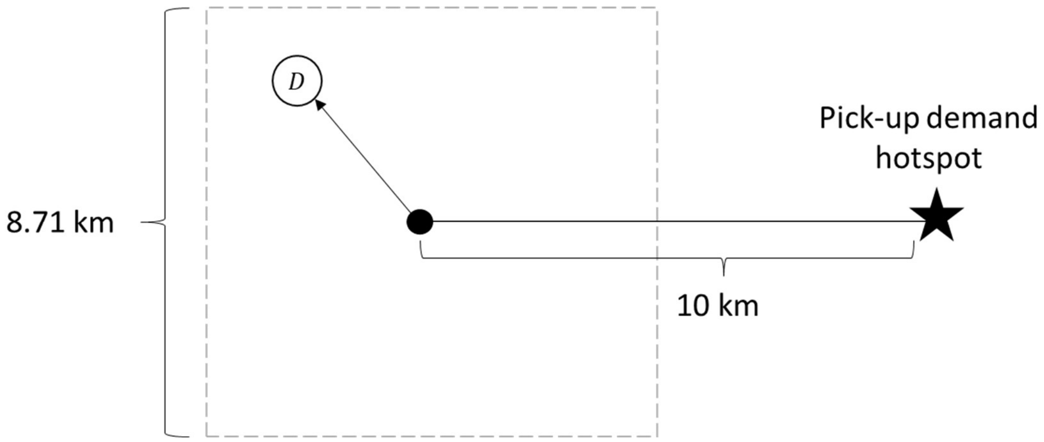

Figure 34 the specification of the default service network geometry for this experiment

Table 8 The mean duration of a round trip for different number of drop-offs per round trip

No. of drop-offs per round-trip	1	2	3	4
The mean duration of round trip [min]	30.01	53.70	63.56	70.77

The expected number of total trip requests was set to be 120 requests per hour. All trip requests were assigned to arrive within 120 min. Hence the total of 240 trip requests was expected to arrive within the given period. It should be noted that the total time to serve all requests could take longer than 120 min depending on the balance between service demand and the capacity. The initial mode share for each service was set to be 0.5. The minimum number of requests per round trip and the vehicle capacity was defined as 2 and 4, respectively. All trips were assumed to be homogeneous, indicating that everyone would accept to share a ride with anyone.

The mean number of accompanied users (NoA) per request was set to be 0.3. Table 9 summarises the proportion of trip requests with a certain number of users (NoU) when the mean NoA is 0.3 among 100,000 samples. It should be noted that a trip request with four or more users cannot choose a shared service since the vehicle capacity is 4 and the minimum number of requests per cluster is 2. However, as Table 9 shows, the proportion of requests with 4+ users is small enough for this constraint to bring a significant impact.

Table 9 the proportion of requests with the certain NoU among 100,000 samples when the mean NoA = 0.3.

NoU	1	2	3	4	5
Proportion	0.740	0.223	0.033	0.004	0.00

The maximum clustering waiting time (MCWT) is set to be 3 min. It is expected that MCWT primarily affects users' waiting time in scenarios with few trip requests and a high fleet size. For instance, if the trip request arrival rate is 240 per hour, the expected inter-arrival time of requests is 0.25 min. Therefore, users are highly likely to find another request to share the ride in less than 3 min. On the other hand, if the trip request arrival rate is 15 per hour, the expected inter-arrival time of requests is 4 min. Hence, many requests will be sent to the dispatching queue without forming a cluster. However, suppose the fleet size is small compared to the trip request arrival rate. In that case, users will need to wait for a vehicle to become available and, while waiting in the dispatching queue, have the opportunity to find a sharing partner and thereby form a cluster.

In this experiment, fare/min is defined as 1 monetary unit per minute. The value of in-vehicle time (VoIVT) is defined as 2 monetary unity/min for both non-shared and shared service (i.e. $\gamma_{v1} = \gamma_{v2} = -2$). It is based on the scenario setting and estimated VoT by the study of Alonso-Gonzalez et al. (2020) that targeted Dutch urban individuals. The ratio between fare/min and VoIVT varies within the research literature, motivating the sensitivity analysis conducted here; relevant results will be discussed in section 4.3.3.4.

Following the research of Alonso-Gonzalez et al. (2020 transportation) and Lavieri and Bhat (2019), the willingness to pay to avoid sharing (i.e. willingness to share (WTS)) is set to be independent of the length of travel time. The ratio between the willingness to pay to avoid sharing (i.e. willingness to share (WTS)) and VoIVT is set to be 0.05, hence, $WTS = -0.1$. Besides, the value of waiting time (VoWT) is defined 1.25 times as much as VoIVT, therefore $\gamma_w = -2.5$. The sensitivity tests against those two parameters have also been conducted, and the results will be summarised in the later section (i.e. Section 4.3.3.4)

It is assumed that with 50% of chance, users do not change their service on the next day even if they do not satisfy with the experience of the current service. On the other hand, it is assumed that people will always stay with their current option if they satisfy with their service. An updated filter is set to be 0.5. The simulation represents the continuous decision of 200 days.

Table 10 the list of parameters for sensitivity test and their default value

Parameters	Default value
The total expected number of trip requests per hour	120
The fleet size for the non-shared service	36
The fleet size for the shared service	28
The initial mode share for non-shared service	0.5

The mean number of accompanied persons per request (NoA)	0.3
The minimum number of requests per cluster	2
The maximum number of users per cluster, i.e. a vehicle capacity	4
The maximum clustering waiting time (MCWT)	3 min
The side length of the square drop-ff area	8.71 km
The length of the corridor	10 km
The period of interest	120 min
The mean speed of the vehicle in the drop-off area	20 km/h
The mean speed of the vehicle on the corridor	60km/h
Fare/min	1
The value of in-vehicle travel time (VoIVT)	2
The value of waiting time (VoWT)	2.5
The willingness to share (WTS)	0.1
α_u : the proportion of those who stay in the service even if the experienced profit is lower than the expected profit of the alternative service.	0
β_u: the proportion of users who did not satisfy with the service they chose yet stated that they would stay in the same service on the following day.	0.5
η_u: an updating filter which determines how much the average utility on day d influences on the collective average utility on day d, $PU_{k,d}$.	0.5

4.3 The analysis of the base scenario

This section summarises the analysis of the base scenario where a parameter setting was set as a default value specified in the previous section.

4.3.1 Key findings

Key findings from the analysis of the base scenario are;

- Within a given parameter setting, mode share for non-shared services is higher than shared services in most of the days.

- The randomness in the total number of trip requests combined with the mode share causes a significant change in a mode share from one day to the next day. However, that change would not last permanently.

Key findings from the sensitivity analysis are;

- This process is ergodic as the stationary distribution is not affected by the initial condition.

- The parameters expressing the users' irregular behaviour and the updating filters are observed to change a variance of the stationary distribution. However, they do not influence the mean value of the stationary distribution.

- When the value of waiting time (VoWT) is within the realistic range (i.e. higher than VoIVT), changes in VoWT did not impact the unique stationary distribution of mode share

- As the Value of In-Vehicle time (VoIVT) increases, the mean value for the stationary distribution of mode share for non-shared (shared) service increases (decreases).

- Increasing the fare per minute for non-shared service has more impact on the stationary distribution of mode share than decreasing the fare per minute for shared service.

- Willingness to share (WTS) is too small compared to the difference in non-shared and shared service's utility. Hence, in this experiment, WTS does not significantly influence on the stationary distribution of mode share.

The detailed analysis and numerical evidence supporting the above findings are presented in the following subsection 4.3.2 and 4.3.3.

4.3.2 **Results**

Figure 35 illustrates the evolution of mode share for 200 days and the distribution of mode share for non-shared service after the warming-up period (i.e. from day 51 and 200). The results of sensitivity analysis against initial condition indicated that this process is ergodic, hence, it is a stationary distribution. The details of sensitivity analysis are summarised in section 4.3.1. With the visual observation, it is assumed that the warming-up period of the process has ended sometime before day 51. As shown in Figure 35, the mode share for non-shared service is generally dominant throughout the period. However, some days, the mode share for the shared services becomes higher than the non-shared services. It always switches back to non-shared services being dominant on the next day.

As described in Chapter 3, users' service choice is made by comparing their actual experiences of using the service they chose on the day with the collective average utility of the alternative service. Hence, the switching from the non-shared service dominance to the shared service dominance would be associated with either the decline in the utility of using non-shared services or increases in the utility of using shared services.

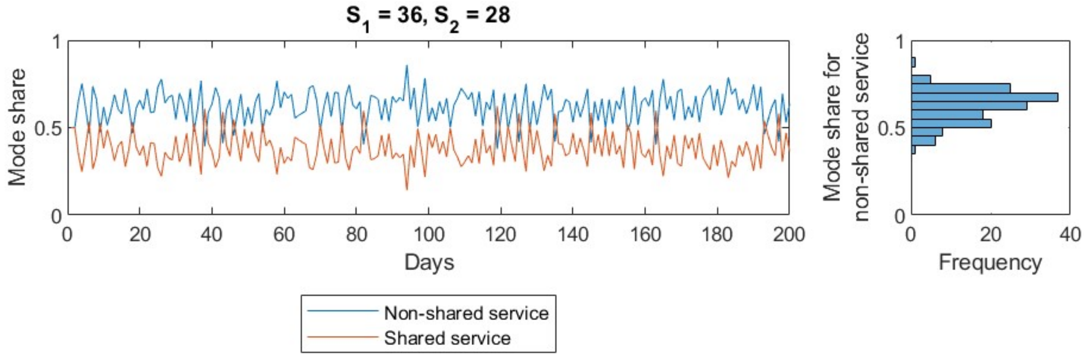

Figure 35 The evolution of mode share for non-shared and shared service for 200 days (left) and the stationary distribution of mode share for non-shared service from day 51 to 200 (right).

Figure 36 presented the evolution of four values from day 105 to day 120, which are; 1) the total number of trip requests, 2) the mode share for non-shared services, and 3) the mean experienced utility for non-shared service and the collective average utility for shared services. During those 15 days, the mode share for non-shared service becomes less than 0.5 on day 119. On the previous day (i.e. day 118), the mean utility for non-shared service significantly dropped while the collective average utility for shared service being constant (see the bottom figure in Figure 36). The significant drop in the mean utility for non-shared service on day 118 caused the sharp decline in the mode share for non-shared service on the next day, namely, day 119. Though the mode share for non-shared service on day 118 is higher than the other days (i.e. 0.717), it is lower than day 109 (i.e. 0.724). As the mode share did not drop sharply on day 110, it can be concluded that the mode share is not the only cause for the sharp drop in the mode share for non-shared service.

Comparing day 109 and day 118, the number of total trip requests has a big difference according to the top figure in Figure 36. On day 109, it is 229, while it is

283 on day 118. Even if the mode share is the same, if the total number of trip requests is different, the number of requests for each service is different. As the fleet size is fixed, a higher number of trip requests increases waiting time and decreases the utility. Hence, it can be concluded that the randomness in the number of total trip requests as well as the mode share contribute to the switch of the dominant service though it would not affect the long-term trend.

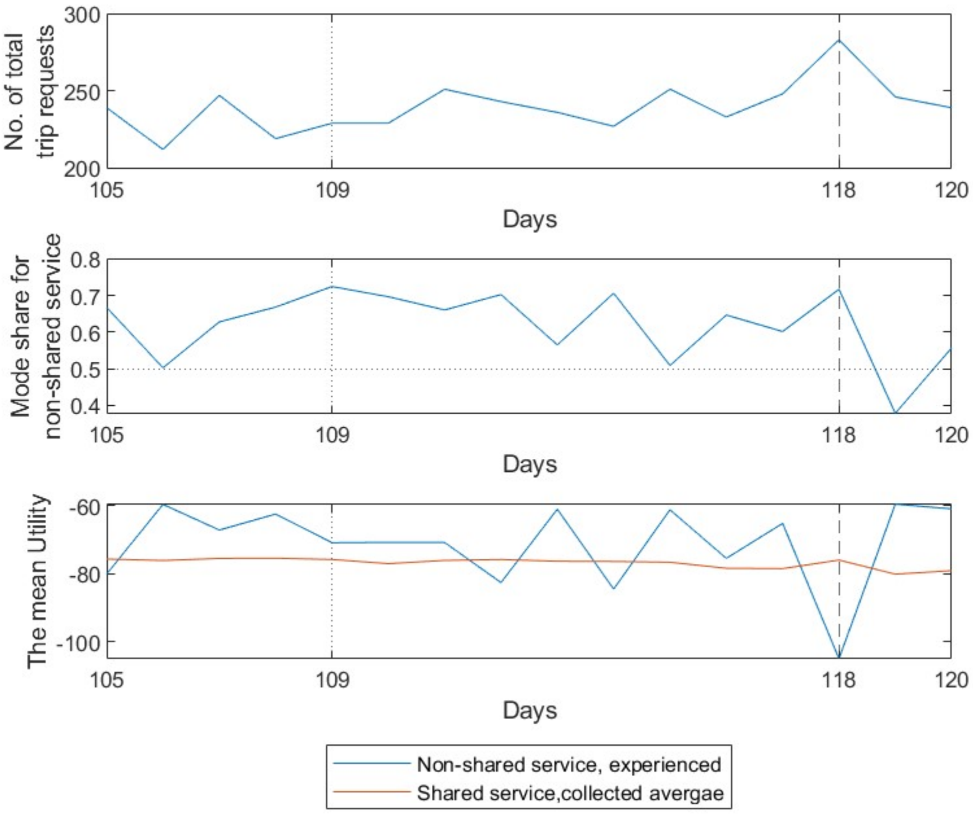

Figure 36 the number of trip requests (top), the mode share for non-shared service (middle) and the mean utility of non-shared and shared service (bottom) from day 105 to day 120 when $S_1 = 36$ and $S_2 = 28$.

4.3.3 **Sensitivity analysis**

This subsection consists of summarising results for sensitivity analysis against 4 different parameters, which are; 1) initial conditions, hesitation parameters, updating filters, and parameters in the utility function.

4.3.3.1 **Initial condition**

A stochastic process is called ergodic if a stationary distribution is independent of an initial state once the stationary period is reached. Moreover, it guarantees the uniqueness of stationary distribution, which indicates that one pseudo-realisation of the process can provide all statistical description of the system state (Watling and

Cantarella, 2013). Sensitivity analysis against an initial condition (i.e. initial mode share for shared and non-shared service) is conducted to examine if the proposed stochastic process has an ergodic property. The results of the test are summarised in this section.

Figure 37 compared a stationary distribution of mode share for non-shared service with 11 different initial mode share for non-shared service from 0 to 1. Table 11 summarise the mean and variance of each distribution. According to Table 11 and Figure 37, it can be observed that the stationary distribution does not change based on the initial condition. In addition, a two-sample t-test has been conducted for all 110 combinations of 10 samples. In all cases, the null hypothesis is not rejected (at 1% level), indicating that all 11 stationary distributions are indifferent. Hence, the stochastic process proposed in this study is ergodic.

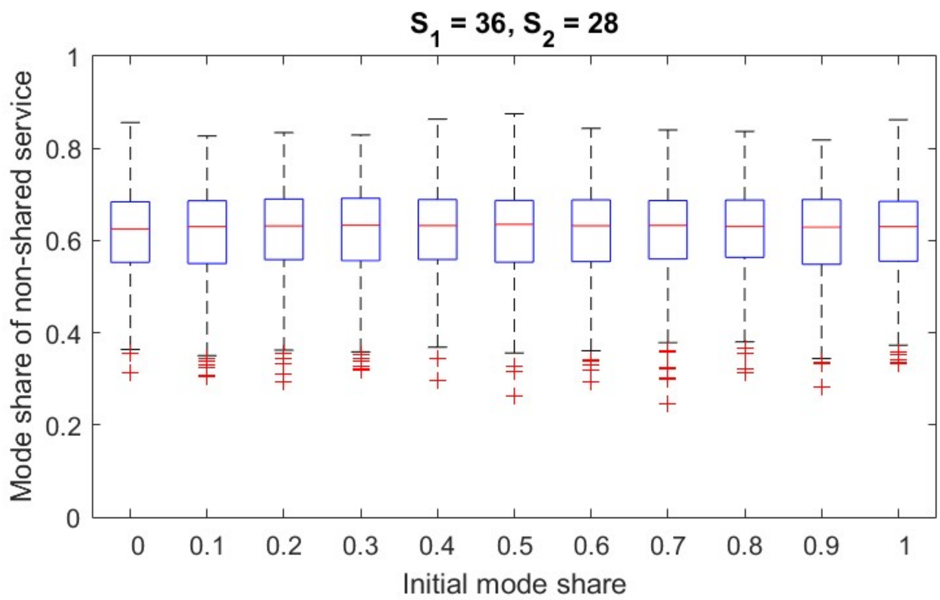

Figure 37 the distribution of mode share of non-shared service when initial mode share is 0 to 1 when $S_1 = 36$ and $S_2 = 28$.

Table 11 the mean and standard deviation for distribution of mode share of non-shared service when initial mode share is 0 to 1

	Initial mode share										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mean	0.61	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.61	0.62
Standard deviation	0.09	0.10	0.10	0.10	0.09	0.10	0.10	0.09	0.09	0.10	0.09

4.3.3.2 The impact parameters for users' irregular behaviour

Parameter α and β are introduced to represent the irregular behaviours of users. α represents the proportion of users who declare to change the service for the next day even though the experienced utility of their current service is higher than the collective average utility of the alternative service. β indicates the proportion of those who stay in the service even if the experienced utility of their current mode is lower than the collective average utility of the alternative service. Assuming that people often choose the option they can expect to gain the higher utility, the word “irregular” is selected to describe the opposite behaviour.

When $\alpha = 0$, everyone will stay their service if their experienced utility is higher than the collective average utility of alternative. Hence, to see the influence of β on the output, the sensitivity analysis has been conducted for $\beta = 0$ to $\beta = 1$ with $\alpha = 0$. All the other parameters are kept as default value described in section 4.2. Figure 38Figure 38 illustrates how the stationary distribution of mode share for non-shared service changes as β changes. It should be noted that when $\alpha = \beta = 1$ nobody will make any change in their mode choice. Hence, it stays at 0.5, which is the initial mode share for non-shared and shared service.

As observed in Figure 38, the distribution becomes less dispersed as β increases. Besides, as Table 12 summarises, there has been no distinctive change observed in the mean value. Higher β indicates that people are more reluctant to change their mode even if the alternative mode is expected to have a higher level of service. Hence, there would be a less dramatic change in mode share from one day to the next as β increases as shown in Figure 39.

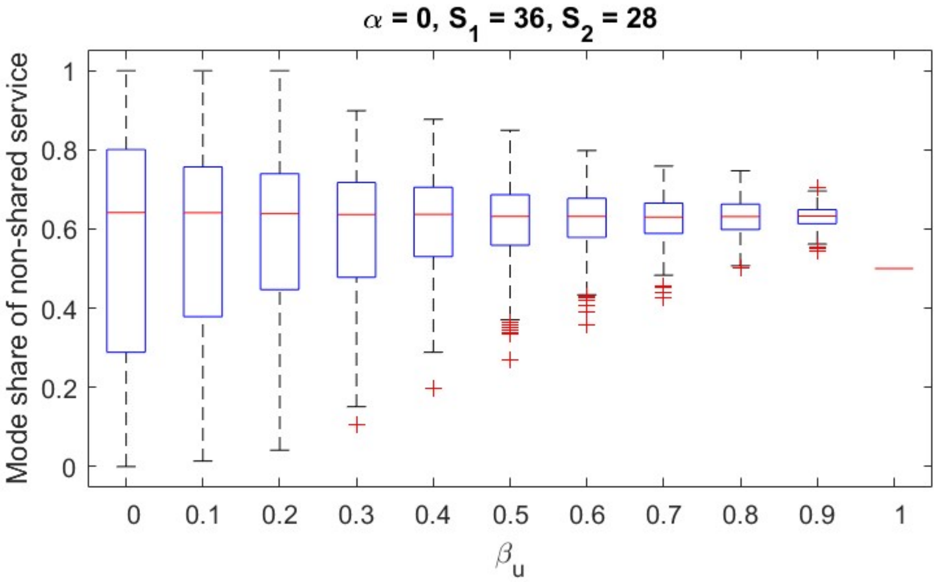

Figure 38 Stationary distribution of mode share for non-shared service (i.e. the distribution of mode share for non-shared mode from day 51 to 200) when $\beta = 0$ to $\beta = 1$ and $\alpha = 0$.

Table 12 the mean mode share of non-shared service with 10 different β from $\beta = 0$ to $\beta = 0.9$

	β									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mean	0.56	0.58	0.59	0.60	0.61	0.62	0.62	0.62	0.63	0.63
Standard deviation	0.29	0.22	0.19	0.15	0.12	0.10	0.08	0.06	0.04	0.03

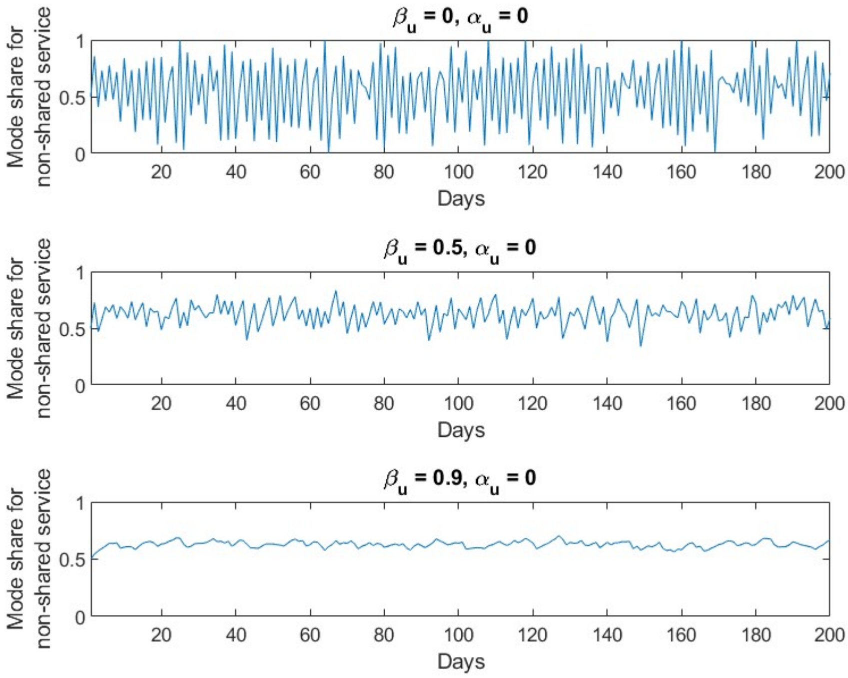

Figure 39 the evolution of mode share for non-shared service for 200 days with different β (i.e. $\beta = 0.1, 0.5, and\ 0.9$).

Figure 40 displays the change in the distribution of mode share for non-shared service with different α (i.e. $0 \leq \alpha \leq 1$) when $\beta = 1$ and the other parameters are set to be a default value. $\beta = 1$ implies that any of those whose experienced utility of current service is lower than the collective average utility of alternative mode will not change their service on the next day. Hence, when $\alpha > 0$, mode change happened only among those who experienced higher utility than the collective average utility of the alternative service. According to Figure 40, as α increases, in other words, as people are more willing to try the alternative service even if they are happy with the current service, the distribution of mode share becomes more dispersed. It is because as α increases, the variance of mode share for non-shared service increases as displayed in Figure 41.

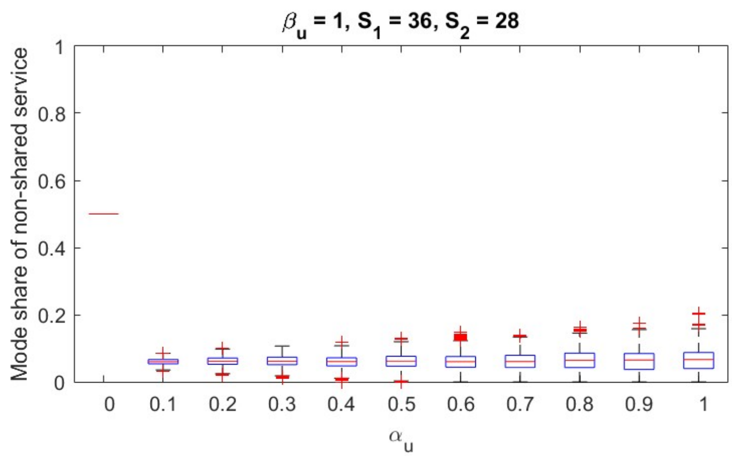

Figure 40 the distribution of mode share for non-shared service for different values of α.

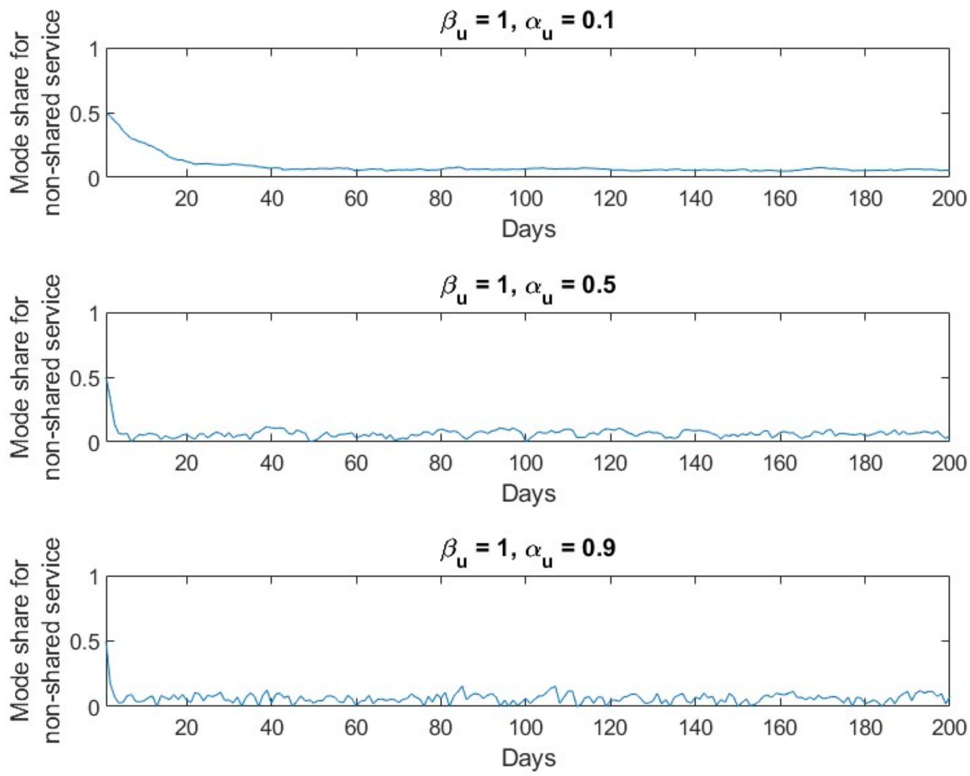

Figure 41 the evolution of mode share for non-shared service for 200 days with different α (i.e. $\alpha = 0.1, 0.5, and\ 0.9$).

4.3.3.3 The impact of updating filters

An updating filter, η_u, decides how the collective average utility is updated every day. In particular, it determines how much the mean utility on day d contributed to estimate the collective average utility on day d, $PU_{k,d}$. When $\eta_u = 1$, the collective average utility is estimated only based on the mean utility among service user on day d. On the other hand, when $\eta_u \approx 0$, the contribution of the mean utility on day d

to $PU_{k,d}$ is almost zero. In other words, the amount of change in $PU_{k,d}$ from $PU_{k,d-1}$ is highly limited.

In order to investigate the impact of η_u, the sensitivity test was conducted with 10 different η_u from $\eta_u = 0.1$ to $\eta_u = 1$ where the other parameters are set as default. The key results from the test are summarised below. Figure 42 compares the distribution of mode share for non-shared services with different η_u. As it can be observed, there is no significant difference in the mean mode share associated with changes in η_u. As illustrated shown in Figure 43, the standard deviation increases as η_u increases. Figure 43 displays the mean value for 100 standard deviations of mode share for non-shared service generated for different η_u from 0.1 to 1. The error bar in the figure indicates the standard deviation among 100 samples. It is consistent with the expectation as higher η_u makes $PU_{k,d}$ to be influenced by one day's experience more, therefore, less stable. As a result, the number of users who are satisfied/unsatisfied with the service fluctuates more every day so as mode share.

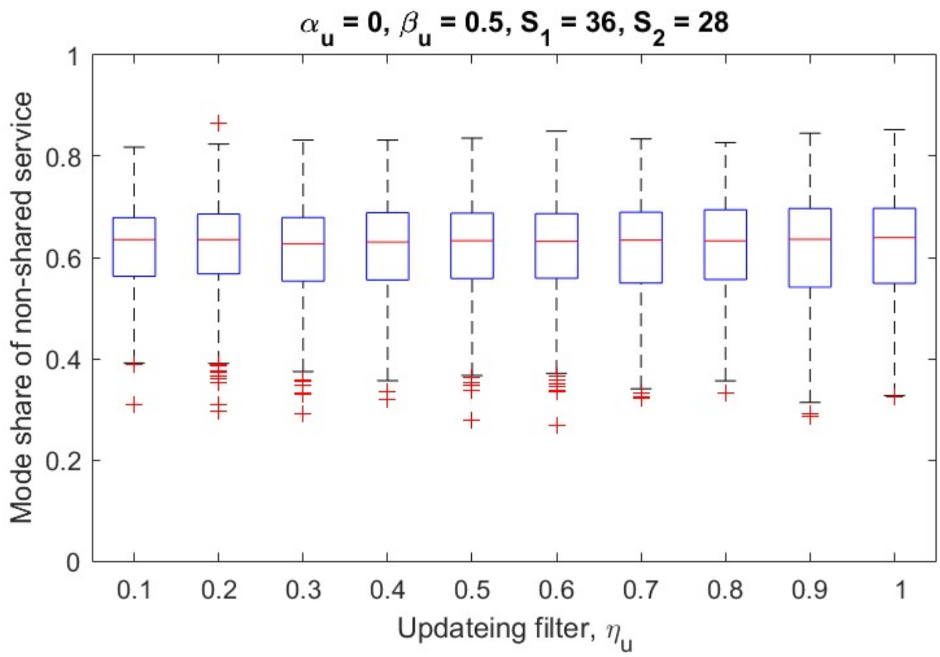

Figure 42 the distribution of mode share for non-shared service with different values of η_u from 0.1 to 1.

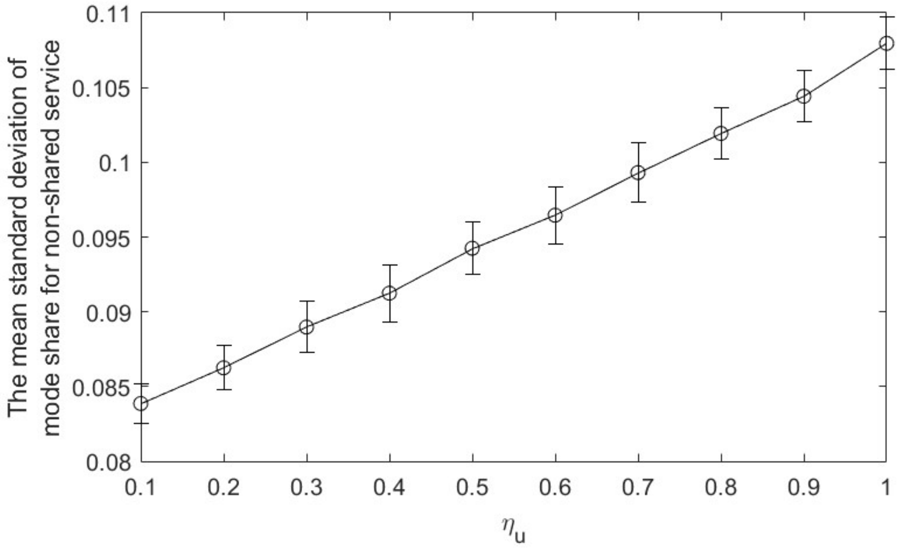

Figure 43 the mean of standard deviation of mode share for non-shared service among 100 realisations generated for different η_u from 0.1 to 1.

4.3.3.4 The impact of parameters in the utility function

This section summarises the sensitivity analysis against the coefficients in the utility function. At first, changes in mean utility regarding the expected number of requests for each service are investigated and presented in Figure 44. In specific, the mean utility is estimated for non-shared service with the expected number of trip requests for non-shared service from 10 to 120 per hour. The same process was repeated for shared service, and results were compared. The other parameters are set as default value specified in section 4.2. According to Figure 44, it can be observed that the mean utility exponentially deteriorates as the number of trip requests increases. It is associated with how waiting time changes as the number of trip requests increase.

Figure 45 compare how the mean utility regarding waiting time, in-vehicle time, and fare changes as the number of trip requests changes. Hence, the y-axis represents a variable multiplied by the coefficient for each variable, such as $-\gamma_{v1} \cdot IVT_{i,1,d}$, $-\gamma_w \cdot WT_{i,1,d}$ and $-c_{i,1,d}$ for non-shared service (i.e. left figure) and $-\gamma_{v2} \cdot IVT_{i,2,d}$ and $-\gamma_w \cdot WT_{i,2,d}$ and $-c_{i,2,d}$ for shared service (i.e. right figure) (see equation (3.9) in Chapter 4 as a reference for the utility function). In the right figure, WTS is excluded as it is constant, and the value is minimal compared to other variables in the utility function for shared services. WTS is lower than the other variables as this is the only parameter that does not change based on travel time. Hence, in order to add more impact of WTS, it is possible to change WTS to be travel time dependent. However, it should be remembered that the results of several studies suggest that WTS is independent of the travel time and distance, as explained in section 2.6 in Chapter 2.

In Figure 45, the mean utility attributes to in-vehicle time and fare are both constant and -30 for non-shared service. In the case of shared service, the mean utility attributes to fare is just above -30 to -20, while the mean utility attributes to in-vehicle time is -40 to -60. Therefore, the summation of mean utility attributes to in-vehicle time and fare is from approximately -70 to -80 for a shared service while it is constantly -60 for a non-shared service. This difference results in the constant difference between the mean utility of non-shared and shared service observed in Figure 44 when the number of trip request is low.

When the number of trip request is above approximately 60, the utility regarding waiting time started decreasing, according to both figures in Figure 45. That is the leading cause of the deterioration in the mean utility observed in Figure 44 for both services. As the shared service is more resilient when the demand reaches or exceeds the capacity of the service, the decreasing rate of the mean utility regarding the waiting time is slower for shared service than non-shared service. Hence, the mean utility for shared service becomes higher than the one for non-shared service when the number of trip requests is higher than 98 (see Figure 44).

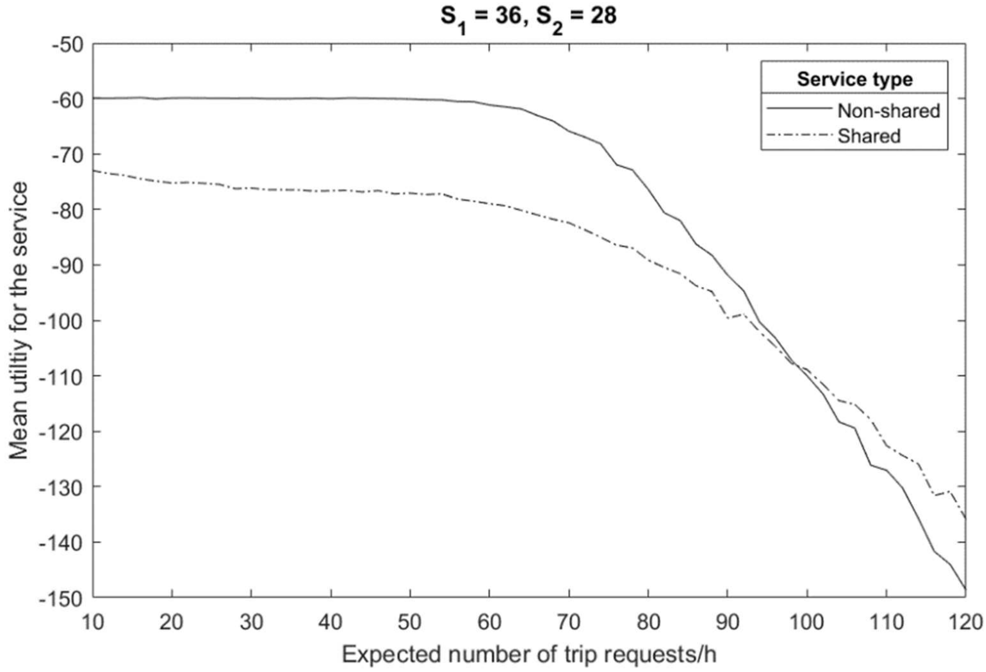

Figure 44 The mean utility for shared (dash-dotted line) and non-shared service (solid line) as the number of trip requests for the service is changed (i.e. from 10 to 120 request/h).

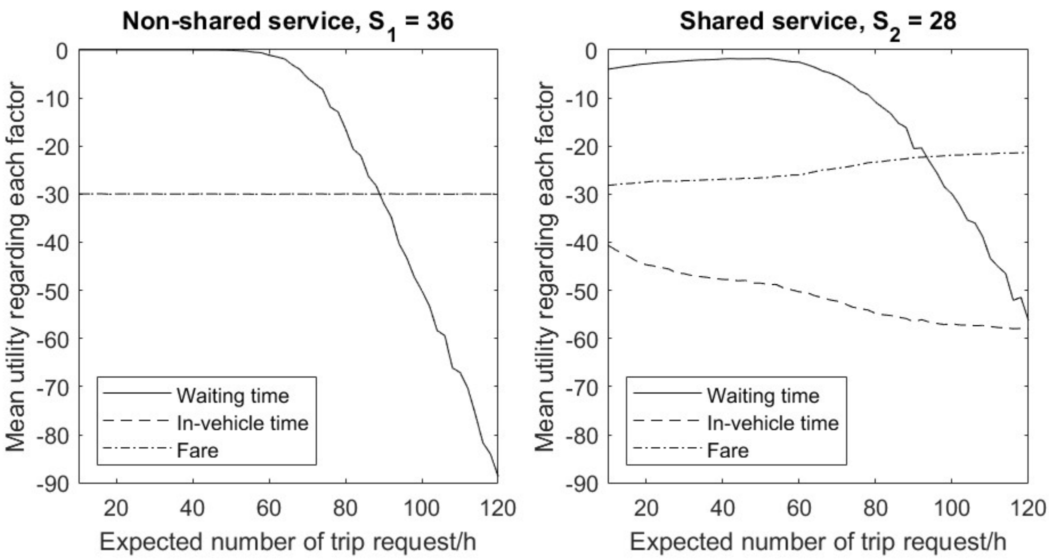

Figure 45 the mean utility regarding each factor, waiting time, in-vehicle time, and fare for non-shared service when the fleet size is $M_1 = 36$ (the left) and for shared service when the fleet size is $M_2 = 28$

(1) The Value of Waiting Time

The mean utility for both services has been estimated with VoWT from 0.1 to 3. The other parameters were set as default. For each VoWT, 56 cases with different mode share for non-shard service were generated from 0.08 to 1, namely, the expected number of trip requests for non-shared service from 10 to 120 per hour. As there was no distinctive difference observed when VoWT becomes higher than 2, only the results with VoWT between 0.1 to 2 are presented.

Figure 46 displays six cases with the different VoWT. It is observed that changing VoWT have a little effect on the mean utility for both non-shared and shared service when the expected number of trip requests is low. In Figure 46, when the x-value is 120, namely, the expected number of trip requests for non-shared service is highest in the current setting, the mean utility for non-shared service visibly changes as VoWT changes. It is because the waiting time is much longer compared to the other components when the x-value is 120, as illustrated in Figure 45. The same tendency is observed in the case of shared service—the lowest mean utility for shared service when x-value is the lowest decreases as VoWT decreases.

Figure 47 shows the distribution of mode share for non-shared service with different VoWT. As the VoWT decreases, the stationary distribution of mode share for non-shared service takes higher values. It is consistent with what can be observed in Figure 46. It should be pointed out that VoIVT was kept constant at the default value for this sensitivity test. Hence, VoWT lower than 2 means VoWT is lower than

VoIVT. In general, VoWT is estimated as a higher value than VoIVT. However, as mentioned above, there were not distinctive changes observed after VoWT becomes higher than 2. Therefore, it could be stated that within the realistic range of VoWT (i.e. higher than 2), VoWT does not have a significant influence on the behaviour of the model.

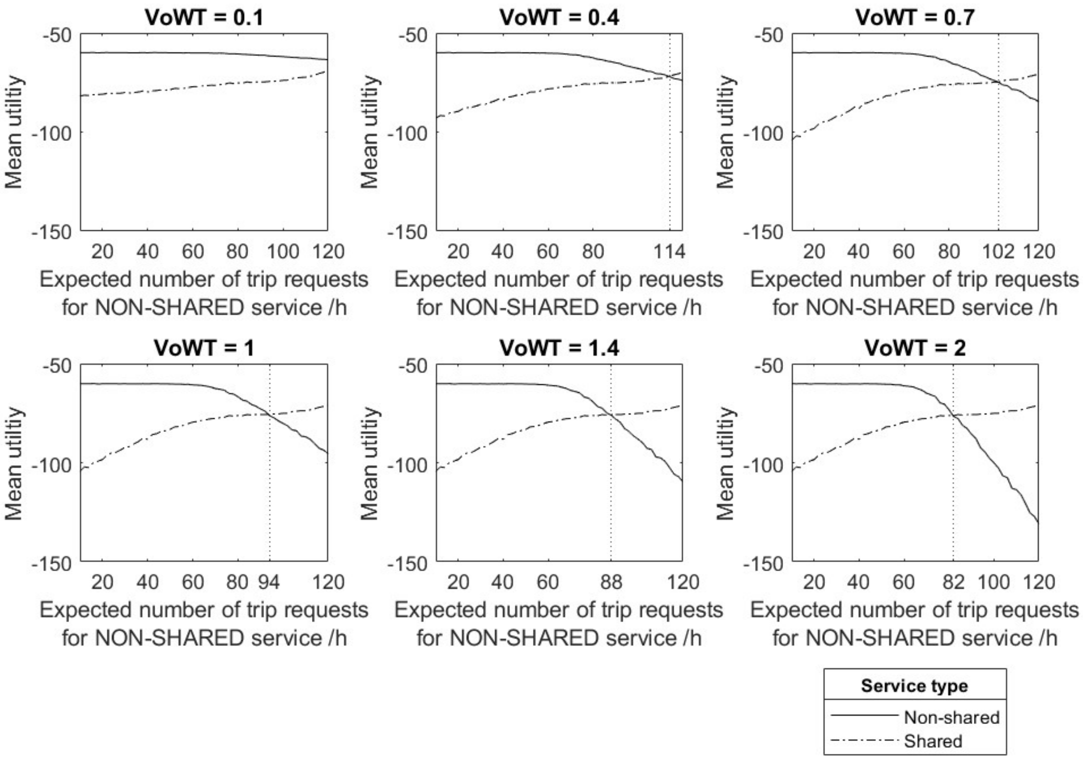

Figure 46 the mean utility of non-shared and shared service with a different expected number of trip requests for non-shared service from 10 to 120 with 6 different VoWT

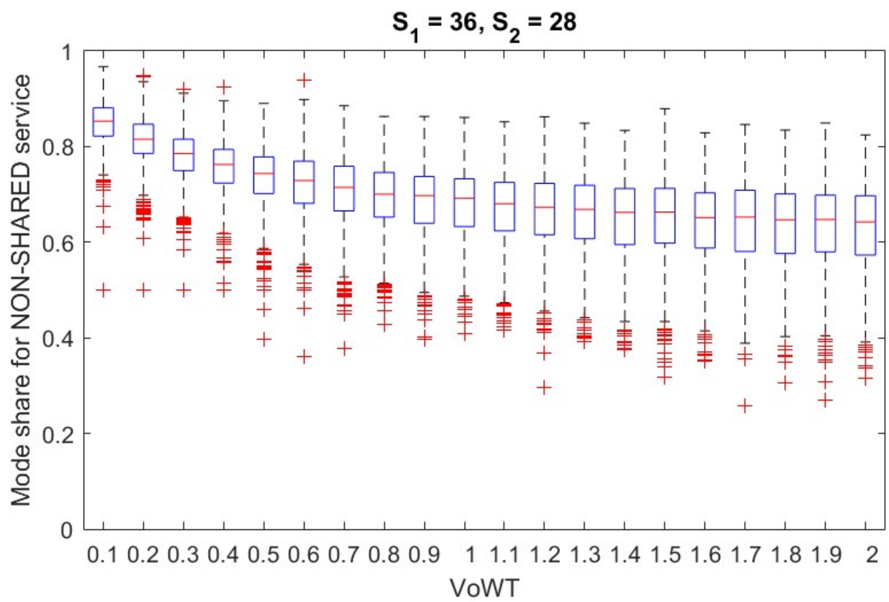

Figure 47 The distribution of mode share for non-shared service with different VoWT from 0.1 to 2

(2) The Value of In-Vehicle Time

The mean utility for both services is estimated with VoIVT from 0.1 to 3. The other parameters are set to be the default value as specified in Table 10. For each VoIVT, 56 cases with different mode share for non-shard service are generated, which is from 0.08 to 1, namely, the expected number of trip requests for non-shared service from 10 to 120 per hour. Figure 48 displays 6 cases with different VoIVT.

When the expected number of trip request for *non-share*d service is low (i.e. lower x value), the mean utility of non-shared service is higher than the shared service. As the expected number of trip request for non-shared service increases, the mean utility for non-shared service (shared service) decreases (increases). Then, at some point, the mean utility of shared service exceeds the mean utility of non-shared service. According to Figure 48, the expected number of trip requests for non-shared service with which such switching occurs becomes higher as the VoIVT increases. Consequently, the mode share for non-shared service increases as VoIVT increases as the non-shared services gains more “capacity” to provide the same service level to the larger number of users (see Figure 49). In other words, the low VoIVT always leads to the higher mode share in shared service.

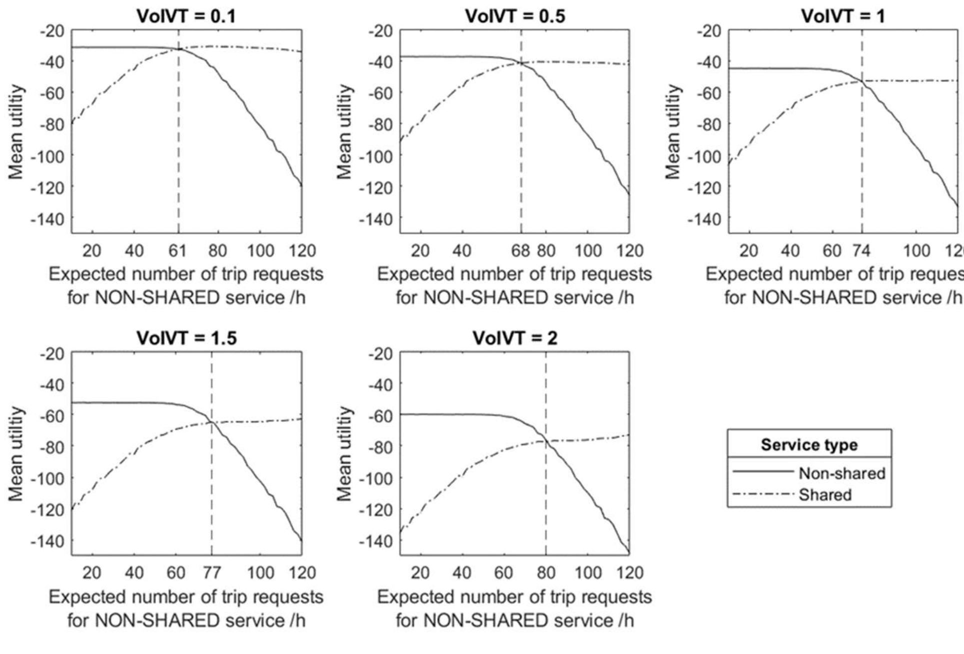

Figure 48 the mean utility of non-shared service and shared service with the different expected number of trip requests for non-shared service from 10 to 120 with 6 different VoIVT.

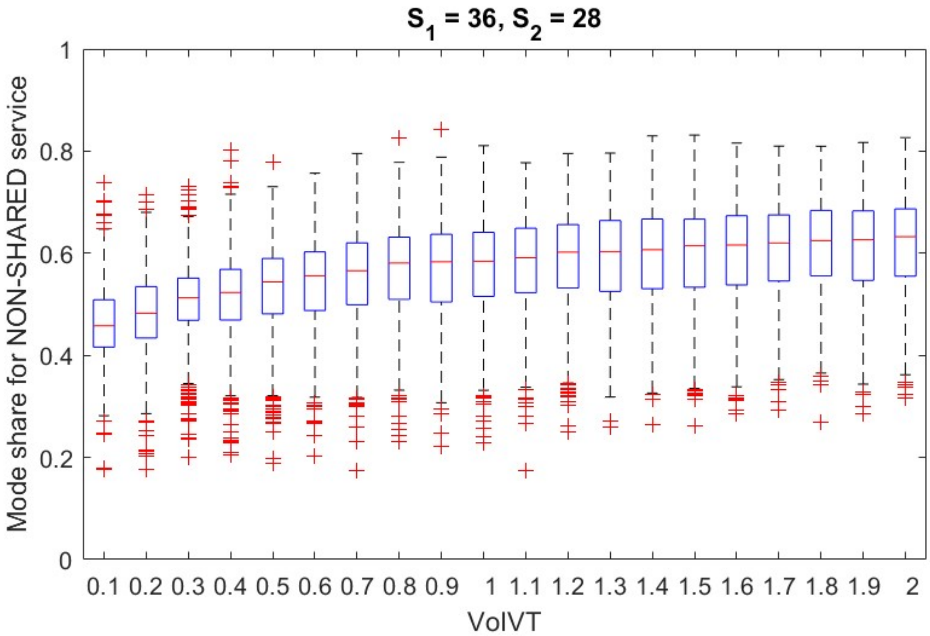

Figure 49 The distribution of mode share for non-shared service with different VoIVT from 0.1 to 2

(3) The fare/min for non-shared and shared service

Figure 50 summarise how the mean utility of non-shared service changes as the fare/min for non-shared service increases. On the contrary, Figure 51 illustrates how the mean utility for shared service changes as the fare/min for shared service decreases. By default, fare/min is set as 1 monetary unit per min for both services. In Figure 50, the mean utility for non-shared service where the fare for non-shared service is 1, 1.5, and 2 monetary units per min are presented while keeping the fare/min for shared service as default value. According to Figure 50, the higher fare/min for non-shared service becomes, the mean utility for non-shared service decreases. As a result, the mean utility for shared service becomes higher than the one for non-shared service with a lower x-value. Similarly, as fare/min for shared service decreases from 1 to 0.8 and 0.5, the mean utility for shared service increases, as displayed in Figure 51. Consequently, the mean utility for shared service becomes higher than the mean utility of non-shared service with the lower x-value as fare/min for shared service decreases.

In Figure 52, the distribution of mode share for non-shared service is summarised with the different fare for *non-shared service* per minute. In Figure 53, the distribution of mode share for non-shared service is summarised with the different fare for *shared service* per minute. In both figures, the higher fare/min for non-shared service becomes comparing to shared service, the lower the mode shared for non-shared service becomes. Figure 19 and Figure 20 show that the amount of change in the distribution of non-shared service is larger when the non-shared service fare is increased compared to when the shared service fare is discounted. It is because the amount of deterioration in the mean utility for non-shared service caused by an increase in fare for non-shared service is higher than the amount of increase in the mean utility for shared service caused by a discount in fare for shared service (see Figure 50 and Figure 51). It implies that, with the current setting, it is more effective to increases the fare for non-shared service than giving a discount to the fare for shared service to encourage the use of shared service.

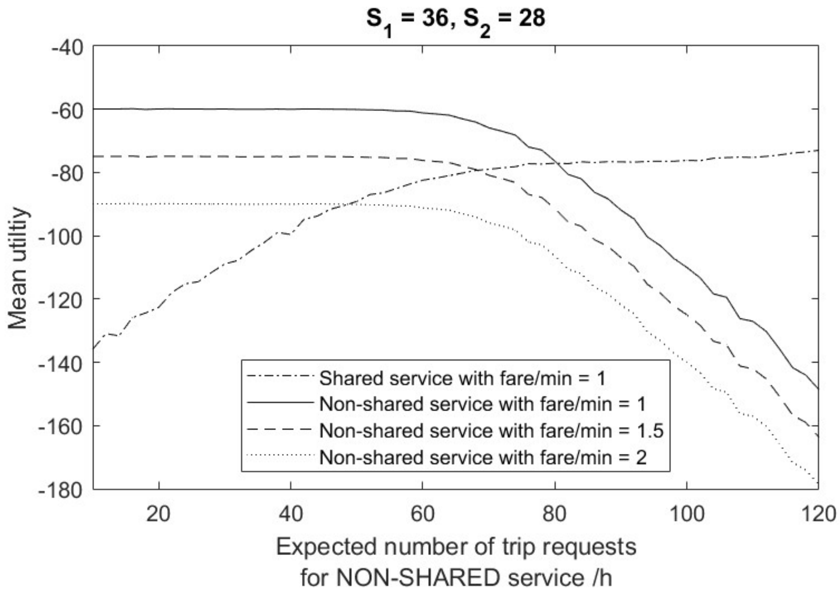

Figure 50 the mean utility for shared service with default fare value and the mean utility for non-shared service with 3 different fare value (i.e. 1, 1.5, and 2 monetary unit/min) for the different expected number of trip requests for non-shared service when $S_1 = 36$, and $S_2 = 28$

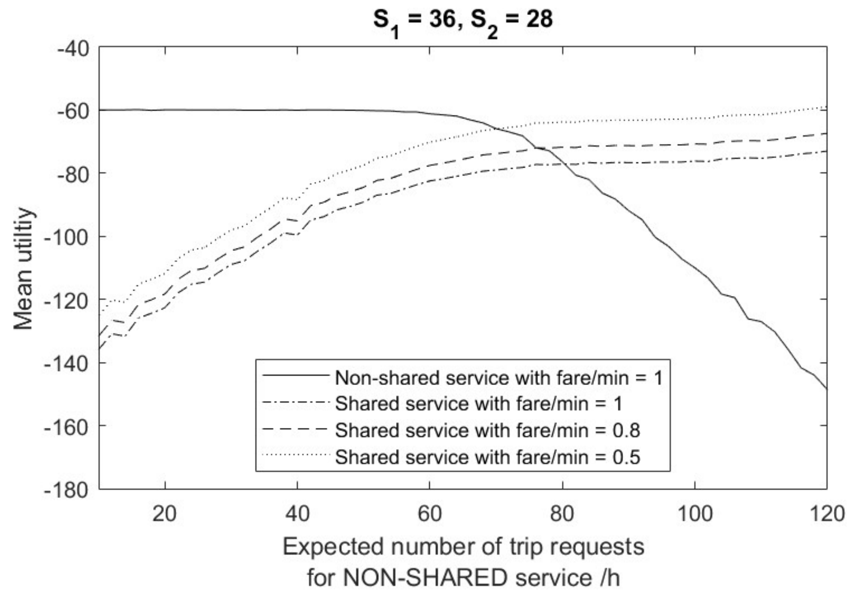

Figure 51 the mean utility for non-shared service with default fare value and the mean utility for shared service with 3 different fare value (i.e. 0.5, 0.8, and 1 monetary unit/min) for the different expected number of trip requests for non-shared service when $S_1 = 36$, and $S_2 = 28$

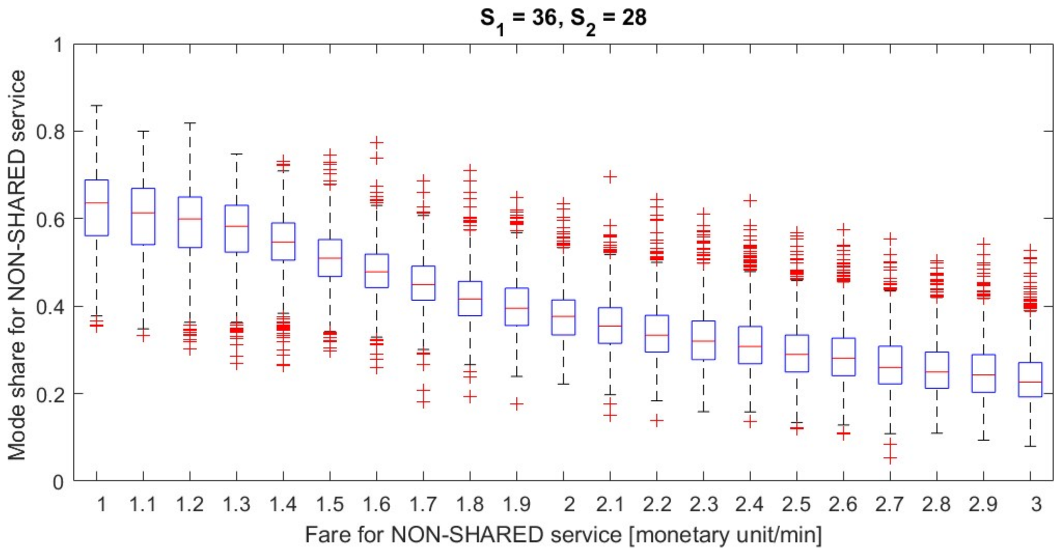

Figure 52 The distribution of mode share for non-shared service with a different fare/min for non-shared service from 1 to 3.

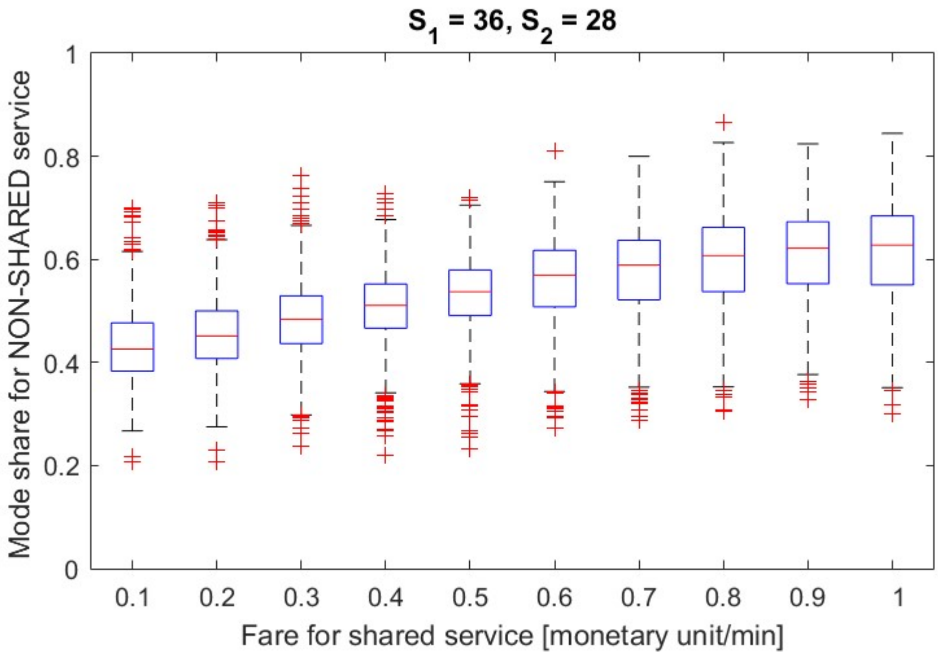

Figure 53 The distribution of mode share for non-shared service with a different fare/min for shared service from 0.1 to 1

4.4 The impact of the fleet size

4.4.1 Scenario settings

The simulation was conducted with 441 combinations of fleet size for non-shared service, S_1, and shared service, S_2. The range of fleet size for non-shared service is from $S_1 = 36$ to $S_2 = 76$ increased in increments of 2, while the range of fleet size for non-shared service is from $S_2 = 28$ to $S_2 = 68$. For each combination of fleet sizes, 200 days were simulated. As this stochastic process is ergodic, as proved in section 4.3.3.14.3.3.1, the stationary distribution of mode share is estimated using the value from day 51 to 200. The results are summarised in the following section.

4.4.2 **Key findings**

Key findings from this numerical experiment are listed below;

- With a given setting, the stationary distribution of mode share changes according to the change in the fleet size for non-shared service, regardless of shared service fleet size

- It is because the maximum mean utility of non-shared service is higher than the maximum mean utility of shared service. Hence, the mean utility of shared service becomes higher only by the reduction in the mean utility of non-shared service. Therefore, the changes in fleet size for shared service does not affect the distribution of mode share.

- As the fleet size for non-shared service, S_1, increase, the distribution of mode share for non-shared service takes higher value. No significant changes in the distribution of mode share are observed after the fleet size for non-shared service reaches $S_1 > 60$.

The detailed analysis and numerical evidence supporting the above findings are presented in the following subsection.

4.4.3 **Results**

Figure 54 displays the distribution of mode share for non-shared service with different fleet size for shared service (i.e. $S_2 = 28$ to $S_2 = 68$) when the fleet size for non-shared service is $S_1 = 36$ (the top figure) and $S_1 = 76$ (the bottom figure). According to Figure 54, there is no visible change in the distribution of mode share in response to the changes in fleet size for shared service for both cases. Figure 55 illustrates how the distribution of mode share for non-shared service changes as S_1 changes from 36 to 76 when $S_2 = 28$. In Figure 55, it is observed that as S_1 increases, so does the mode share for non-shared services. This increase in mode share stops above approximately $S_1 = 60$. In other words, the increase in S_1 does not have any influence on distribution of mode share as of $S_1 = 60$.

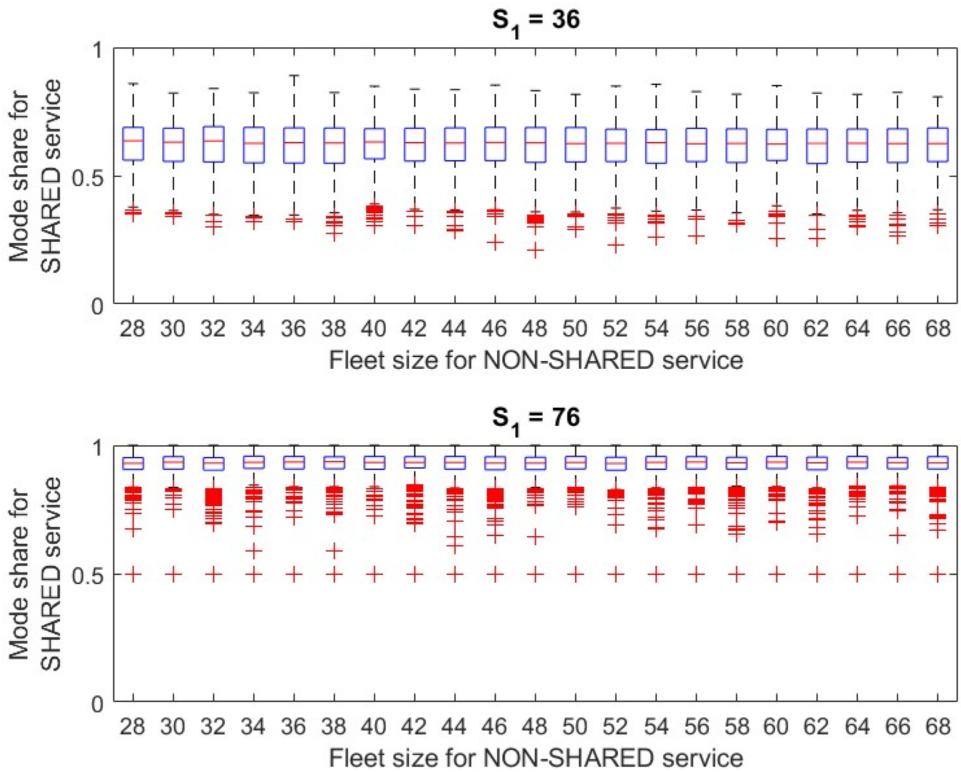

Figure 54 The distribution of mode share for non-shared service with different fleet size for shared service (i.e. $S_2 = 28$ to $S_2 = 68$) with the fleet size for non-shared service $S_1 = 36$ (upper figure) and $S_1 = 76$ (lower figure)

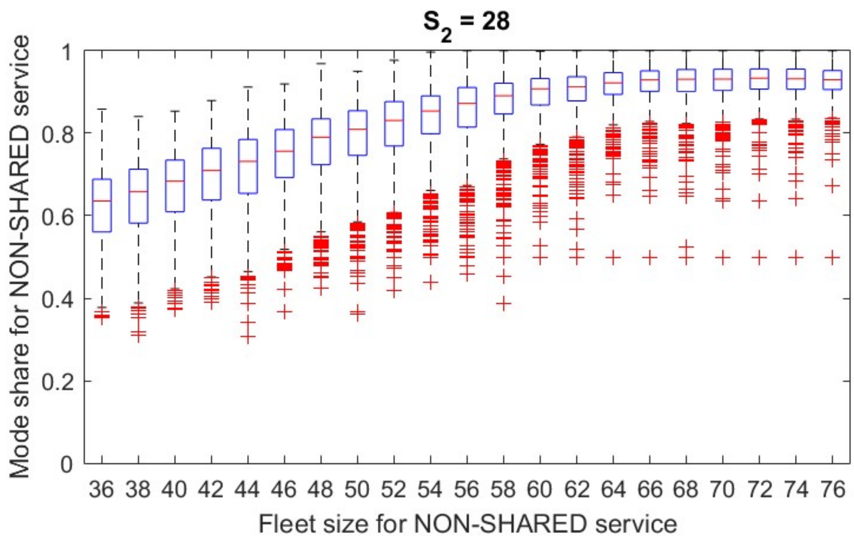

Figure 55 the distribution of mode share for non-shared service with different fleet size for non-shared service (i.e. $S_1 = 36$ to $S_1 = 76$) with the fleet size for shared service $S_2 = 28$.

Figure 56 compares the evolution of mode share and the mean utility for non-shared and shared services when $S_1 = 36$, $S_1 = 46$, and $S_1 = 60$. According to Figure 56, the fluctuation of the mean utility for non-shared service becomes less variable as the fleet size increases. It is because the number of trip requests which triggers the sharp decline in the mean utility increases as S_1 and therefore the service capacity increases (see Figure 57).

The same analysis has been conducted against two different fleet sizes for shared service (i.e. $S_2 = 48\ and\ 68$) with $S_1 = 36$. However, no significant difference has been observed compared to the case with $S_1 = 36$ and $S_2 = 28$. Therefore, the results are not displayed in this section. It could be because the fleet size, $S_2 = 28$, is high enough for the current setting so that the increase in the fleet size does not improve the user's experience. Alternatively, with the current setting, the fleet size of shared service may not impact the service capacity. The investigation has been conducted on how utility changes as fleet size and the number of trip requests for non-shared and shared service changes. The results are summarised below.

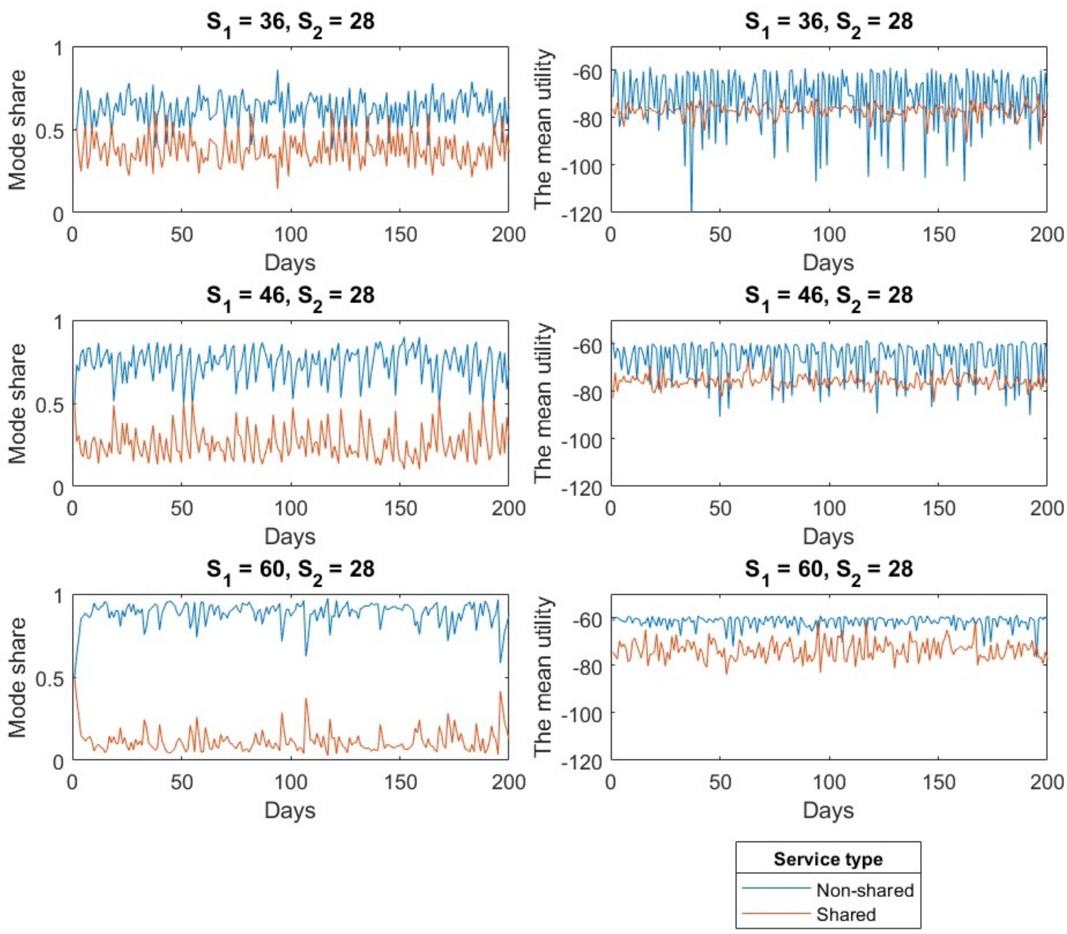

Figure 56 Comparison of the evolution of mode share and changes in the mean utility for non-shared and shared service for 200 days when $M_1 = 36$ (the top row), $M_1 = 46$(the middle row) and $M_1 = 60$ (the bottom row)

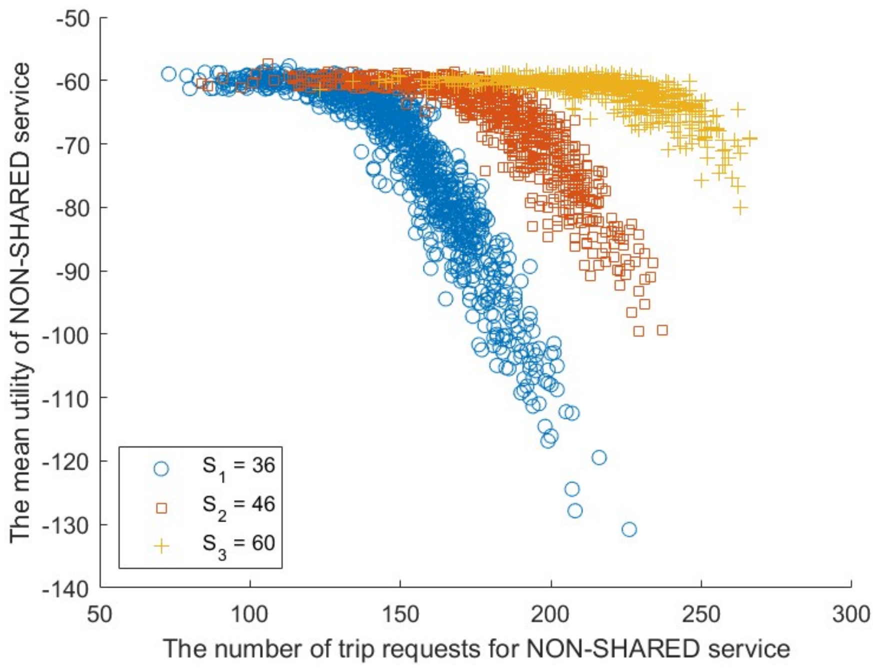

Figure 57 the mean utility of non-shared service against the expected number of trip requests for non-shared service when $M_1 = 36, 46, and\ 60$.

Figure 58 compares changes in mean utility against the expected number of trip requests for non-shared service. In particular, the mean utility for non-shared service when $S_1 = 36$ is compared with the mean utility for shared service when $S_1 = 28$ and $S_2 = 68$. As the total expected number of trip requests per hour is 120, the expected number of trip requests for shared service is calculated by subtracting the x-value from 120. As observed in Figure 58, the maximum mean utility of non-shared service is higher than the maximum mean utility of shared service. Hence, the mean utility of shared service becomes higher only by the reduction in the mean utility of non-shared service. Therefore, the unique distribution of mode share did not change as the fleet size changes for shared service, as presented in Figure 54.

Hence, it is concluded that, with a given parameter setting described in section 4.2, only the fleet size of non-shared service impacts the distribution of mode share. However, there are several other factors that can influence the utility; hence, the distribution of mode shares, such as the service network property and the coefficient in the utility function. The impact of coefficients in the utility function is already summarised in section 4.3.3.4. The impact of the service network geometry will be presented in the following section.

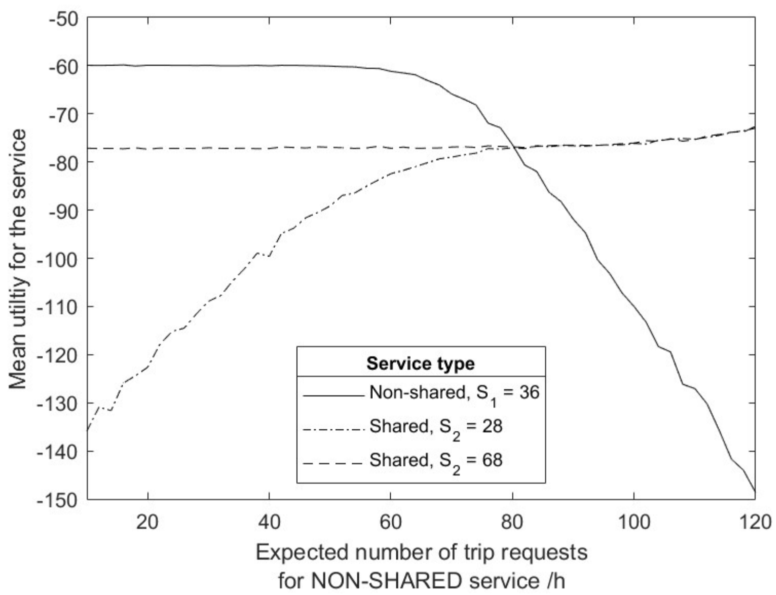

Figure 58 the comparison of the mean utility of non-share service with $S_1 = 36$ and the mean utiltiy of shared service for 2 different cases when $S_2 = 28$ and $S_2 = 68$.

4.5 The impact of service network geometry

4.5.1 Scenario settings

This section examines the influence of the service network property on the unique distribution of mode share. The simulation has been conducted with 9 different service networks, and results were compared and analysed below. As described in Figure 34, the service network consists of two parts, a corridor and a square-shaped drop-off area. Table 13 summarised 9 scenarios with different combinations of corridor length and the side length of the drop-off area where scenario 1 is the default setting.

For all cases, a drop-off point is randomly and uniformly distributed in the squared drop-off area. The mean travel speed is 20 km/h in the drop-off area and 60 km/h in the corridor. For each scenario, the simulation was conducted with 441 combinations of fleet size for non-shared service, S_1, and shared service, S_2. The range of fleet size for non-shared service is from $S_1 = 36$ to $S_2 = 76$ increased in increments of 2, while the range of fleet size for non-shared service is from $S_2 = 28$ to $S_2 = 68$.

Table 13 9 cases with different combination of corridor length and side length

Scenario	1	2	3	4	5	6	7	8	9
Side length (km)	8.71			4.39			2.18		
Corridor (km)	15	10	5	15	10	5	15	10	5

When the composition of the service network geometry changes, the distribution of round-trip time also changes. Figure 59 illustrates the mean round trip time with the different number of drop-offs (NoDs) per round trip when the corridor length is 10 km, and the side length of drop-off area is 8.71 km (i.e. default), 4.39 km and 2.18 km. It can be observed that the bigger the drop-off area becomes, the more the mean round trip time increases as NoDs per round trip increases. As the travel time on the corridor is constant among three cases, increases in the round-trip time only contribute to the increase in travel time in the drop-off area.

When the length of the corridor is increased, as illustrated in Figure 60, the amount of increase in the mean round trip time corresponding to the increase in the NoDs is constant. As only the travel time on the corridor increases, the mean travel time equally increases regardless of NoDs per round trip. It should be pointed out that, as the length of the corridor changes, the proportion of trips shared among all passengers also changes in the case of shared service. After a vehicle dispatches from the pick-up hot-spot, a trip would be shared by all passengers until a vehicle arrives at the first drop-off points. The longer proportion of that part becomes, the longer proportion of their trip can be shared among the highest number of passengers. In the case of non-shared service, the increase in the corridor just indicates the increase in the mean in-vehicle time from the users' point of view.

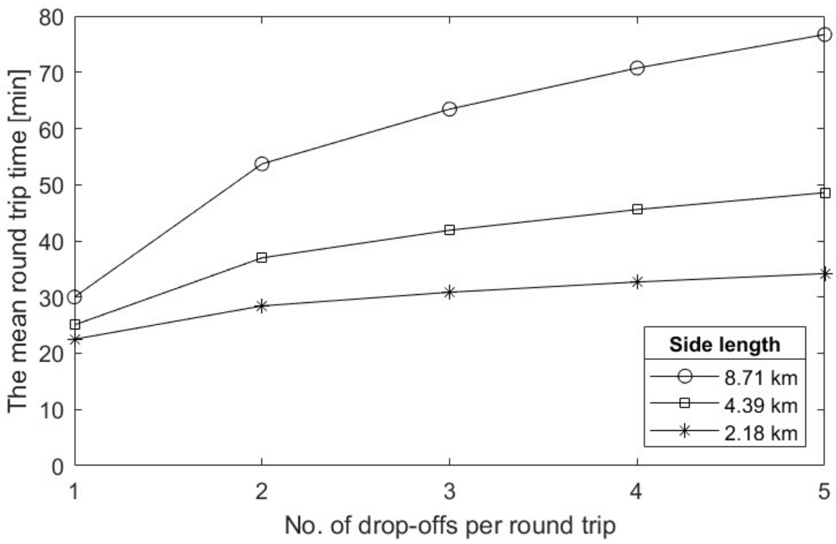

Figure 59 the comparison of the mean round trip time when the corridor length is 10 km/h with the different side length of drop-off area (i.e. 8.71 km, 4. 39 km, and 2.18 km)

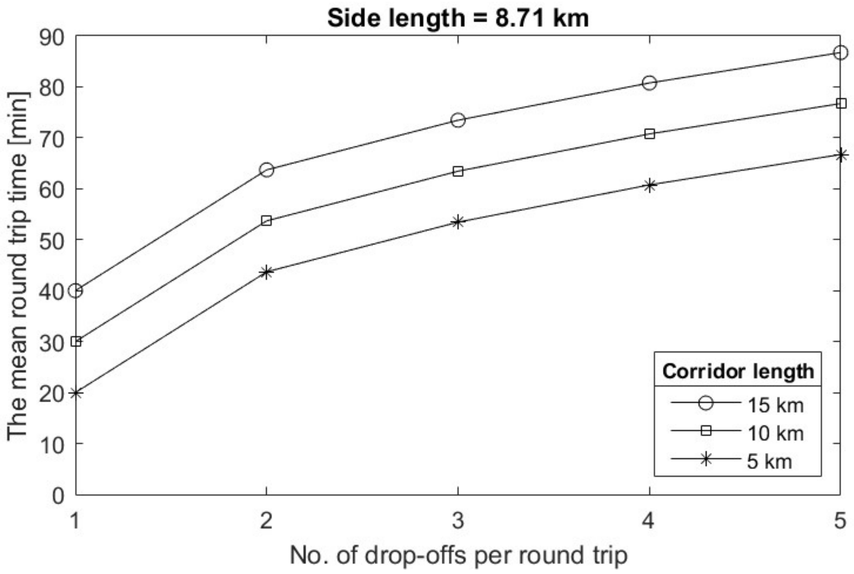

Figure 60 the comparison of the mean round trip time when the side length of the drop-off area is 8.71 km with different corridor length (i.e. 15 km, 10km and 5 km)

4.5.2 Key findings

The key findings from this numerical experiment are summarised below.

- As the proportion of a trip on the corridor becomes longer in a round trip, the mean utility of shared service becomes higher than the non-shared service—the dominant service changes as the service network geometry changes.

- The impact of fleet size for shared services becomes the determinant of the stationary distribution of mode share when the proportion of a trip in the corridor is high and the mean round trip time is not too short.

- When the mean round trip time is too short, the stationary distribution of mode share is not affected by both fleet sizes. It is because the minimum fleet size in this numerical experiment already guarantees a large enough capacity for the given expected number of total trip requests for those cases.

- The service network geometry determines the boundary of the stationary distribution of mode share.

The detailed analysis and numerical evidence supporting the above findings are presented in the following subsection.

4.5.3 **Results**

Figure 61, Figure 63, and Figure 65 show how the distribution of mode share for non-shared service changes as the fleet size of shared (non-shared) services changes when $S_1 = 36$ ($S_2 = 28$) with a different service network geometry. Figure 61 compares the scenario 1 to 3 where the side length of the drop-off area is 8.71 km. Figure 63 compares scenarios 4 to 6 where the side length of the drop-off area is 4.39 km. Figure 65 compares scenarios 7 to 9 where the side length of the drop-off area is 2.18 km.

In the case of scenario 1 to 3 where the side length of the drop-off area is 8.71km, the distribution of mode share is only influenced by the fleet size of non-shared service, S_1 for all 3 cases with different corridor length (see Figure 61). For all cases, the mean mode share for non-shared service is higher than 0.5. Figure 62 compares how the mean utility of shared service and non-shared service changes when the expected number of trip requests for non-shared service is 10 to 120 for scenario 1 to 3. From Figure 62, it can be observed that, as the corridor length increases, the mean utility for shared service becomes higher than non-shared service with the lower expected number of trip request for non-shared service. However, in all three cases, the maximum mean utility of non-shared service is higher than the maximum mean utility of shared service. Hence, the mean utility of shared service becomes higher only by the reduction in the mean utility of non-shared service. Consequently, only the fleet size of non-shared service affects to the distribution of mode share.

In the case of scenario 4 to 6 where the side length of the drop-off area is 4.39 km, the distribution of mode share changes differently as the fleet size changes depending on the length of the corridor. When the corridor length is 15 km, the mean mode share for non-shared service slightly decreases as the fleet size for shared service increases (see the top left figure in Figure 63) and slightly increases as the fleet size for non-shared service increases. In both cases, the mean mode share for non-shared service stays below 0.5. This happens as the maximum utility of non-shared and shared service is almost the same (i.e. -70.0 and -68.2), as illustrated in Figure 64. Figure 64 displays the mean utility for non-shared and shared service for the different expected number of trip requests for non-shared service from 10 to 120 for scenario 4 to 6

When the corridor length is 10 km, only the changes in fleet size for non-shared service impact the distribution of mode share. The mean mode share for non-shard service stays above 0.5. When the corridor length is 5 km, neither changes in fleet size for non-shared service nor shared service impact the distribution of mode share. The mean mode share for non-shared service is approximately 0.95 and constant. It is because the mean utility of non-shared service is higher than shared service regardless of the number of trip requests per hour, as presented in the right figure in Figure 64.

In the case of scenario 7 to 9 where the side length of the drop-off area is 2.18 km, the fleet size of non-shared service does not influence on the distribution of mode-share in any case (see Figure 65). When the corridor length is 15 km, the mode share for non-shared service decreases as the fleet size for shared service increases (see the left top figure in Figure 65). The mean mode share for non-shared service is constantly less than 0.5. It is because the mean utility for shared service is constantly higher than the non-shared service except for the trip requests for non-shared service (shared service) is 10 (110), as illustrated in Figure 66.

When the side length of the drop-off area is 10km and 5 km, neither changes in fleet size for non-shared service and shared service influence the distribution of mode share (see middle and bottom two figures in Figure 65). In the case of the corridor length being 10 km, it is because the mean utility for a shared service is constantly higher than the mean utility for a non-shared service, according to the middle figure in Figure 66. Hence, the mean mode share for non-shared service is less than 0.5 and approximately 0.26. In the case of the corridor length being 5 km, the mean mode share for non-shared service is constantly about 0.96. It is because the mean utility for non-shared service is higher than the shared service (see the left figure in Figure 66).

Overall, it can be concluded that as the proportion of trip on the corridor becomes longer in a round trip, the mean utility of shared service becomes higher than the non-shared service. Hence, the impact of fleet size for shared services becomes the determinant of the distribution of mode share. In addition, when the mean round trip time is too short (i.e. scenario 6 and 9), the distribution of mode share is not affected by both fleet size. It is because the minimum fleet size for this experiment ($S_1 = 36$ and $S_2 = 28$) is already high enough in those scenarios setting so that the additional fleet does not improve users experience as it is hence no effects to the distribution of mode share. Also, it is discovered that the service network geometry determines the boundary of the stationary distribution of mode share can take regardless of the fleet size.

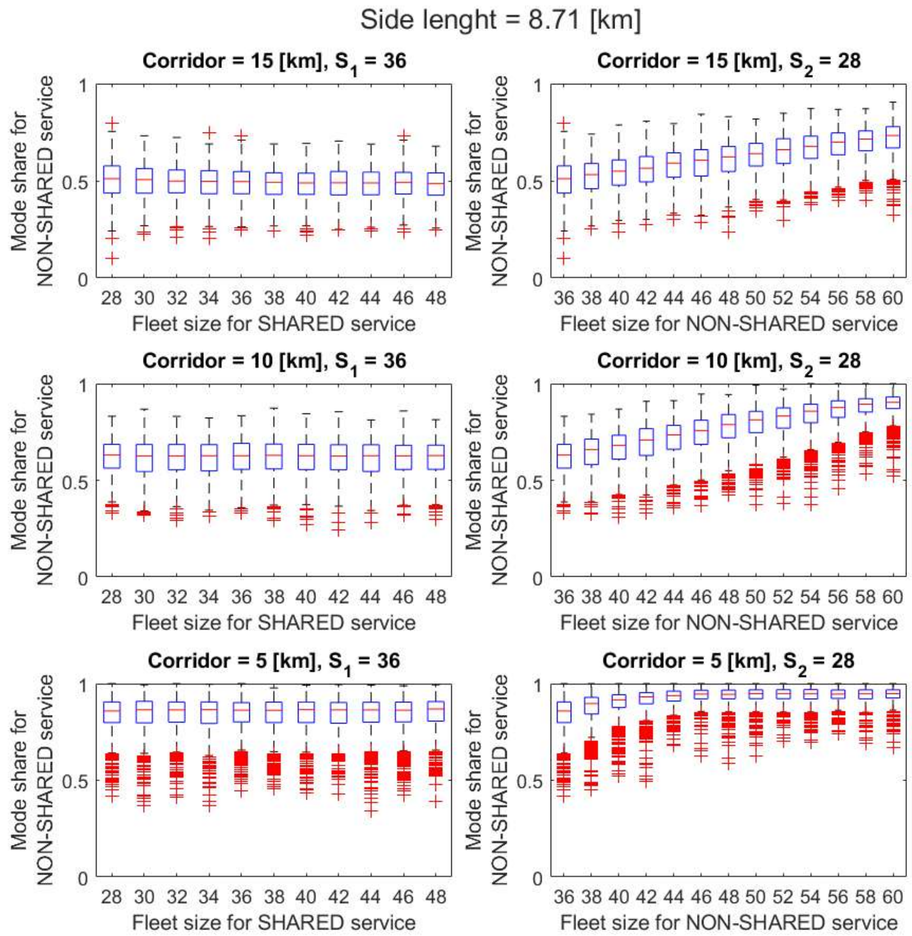

Figure 61 the distribution of mode share for non-shared service with a different fleet size of shared service (the left column) and non-shared service (the right column) with different length of the corridor (i.e. 15 km, 10km, and 5 km) when the side length of drop-off area is 8.71km, $M_1 = 36$, and $M_2 = 28$.

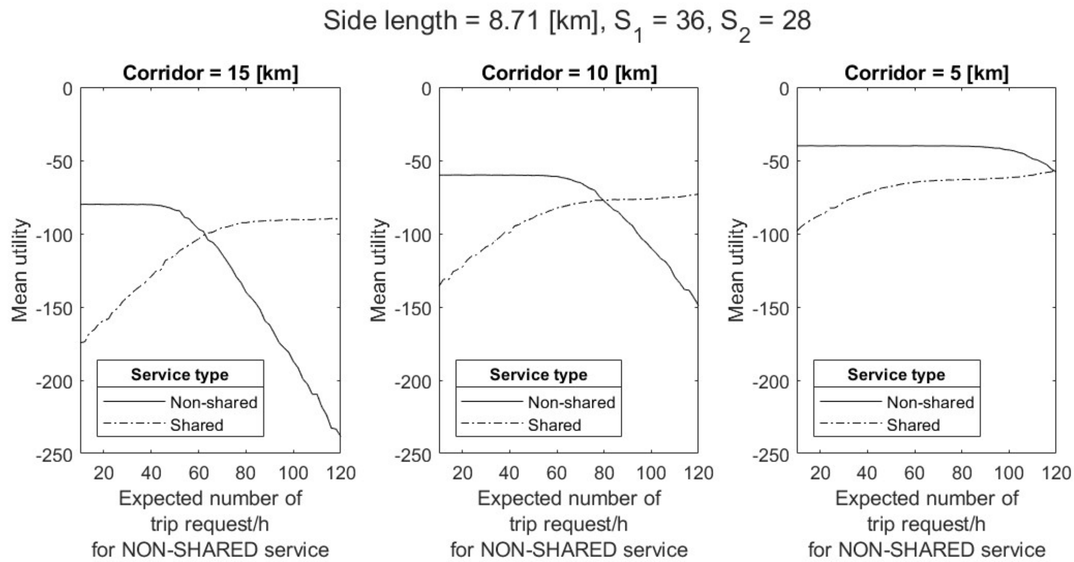

Figure 62 the mean utility of non-shared and shared service with different expected number of trip request for non-shared service from 10 to 120 per hour with different corridor length, 15km, 10km, and 5 km when the side length of the drop-off area is 8.71km, $S_1 = 36$, and $S_2 = 28$

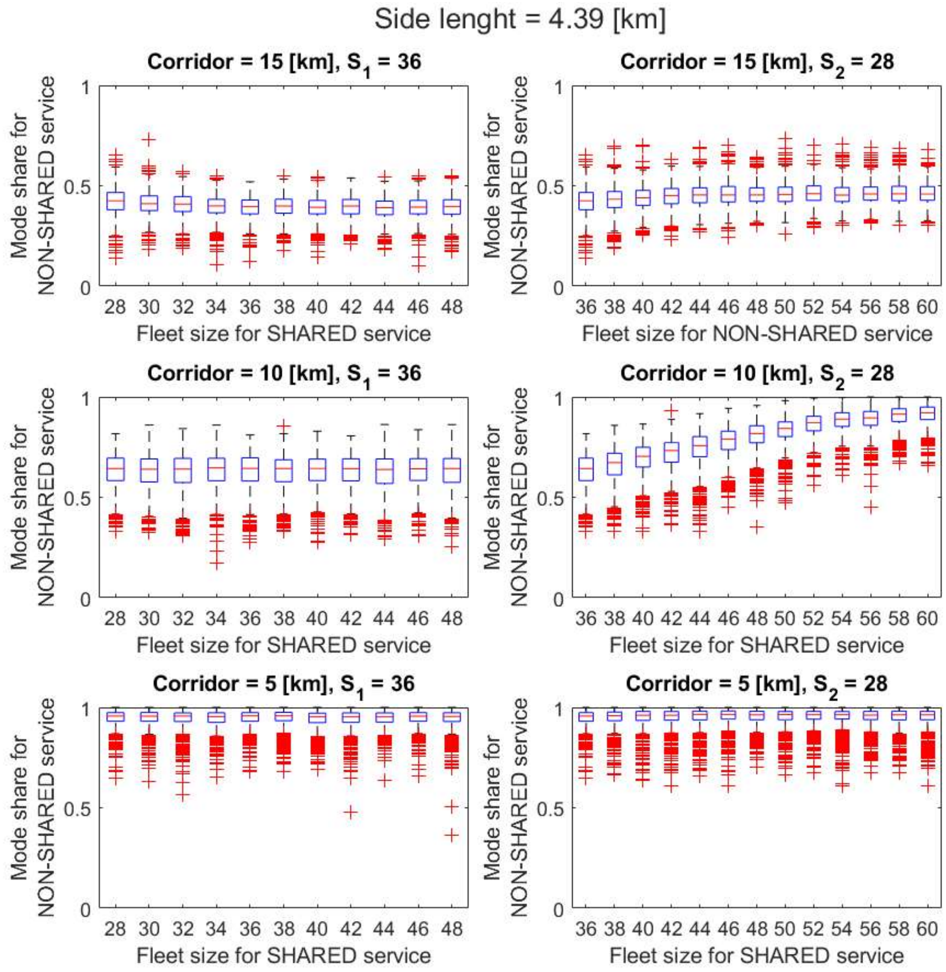

Figure 63 the distribution of mode share for non-shared service with a different fleet size of shared service (the left column) and non-shared service (the right column) with different length of the corridor (i.e. 15 km, 10km, and 5 km) when the side length of drop-off area is 4.39 km.

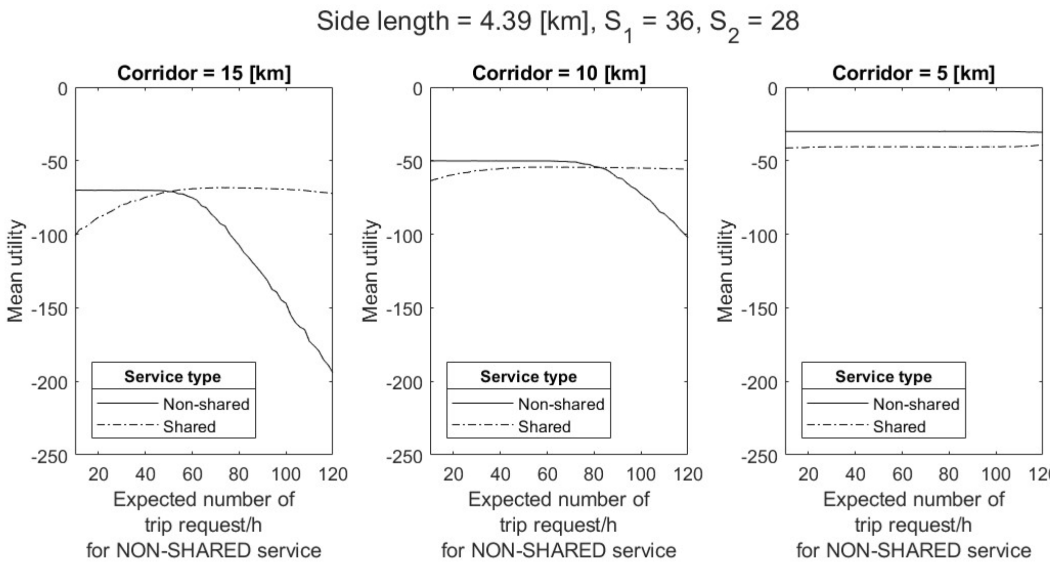

Figure 64 the mean utility of non-shared and shared service with the different expected number of trip request for non-shared service from 10 to 120 per hour with different corridor length, 15km, 10km, and 5 km when the side length of the drop-off area is 4.39 km, $S_1 = 36$, and $S_2 = 28$

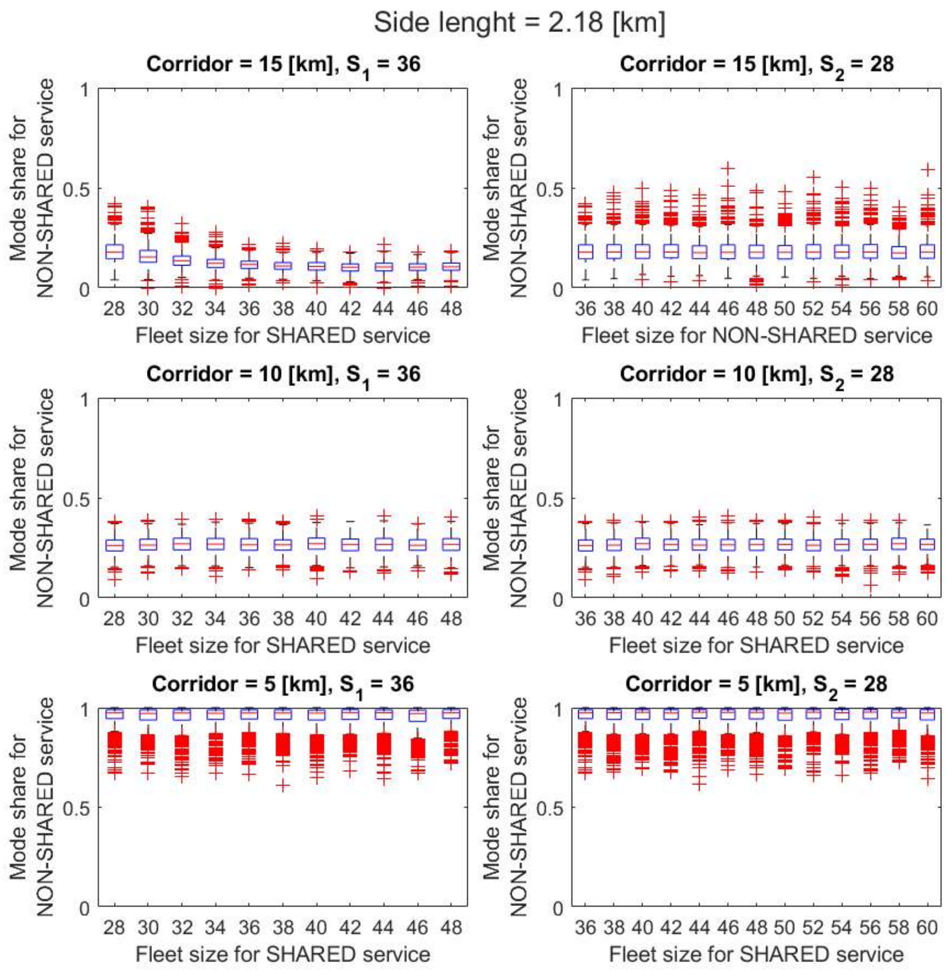

Figure 65 the distribution of mode share for non-shared service with a different fleet size of shared service (the left column) and non-shared service (the right column) with different length of the corridor (i.e. 15 km, 10km, and 5 km) when the side length of drop-off area is 2.18 km.

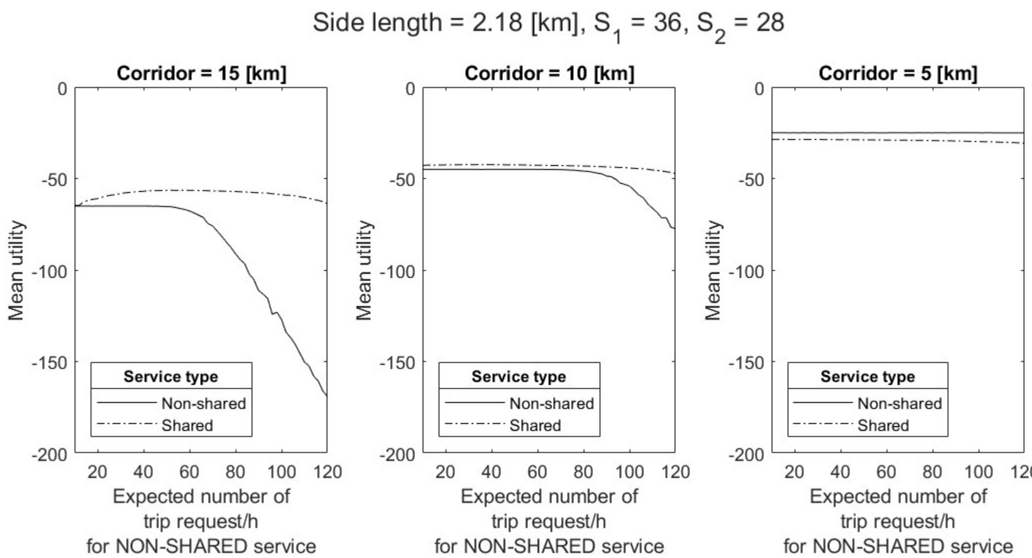

Figure 66 the mean utility of non-shared and shared service with the different expected number of trip request for non-shared service from 10 to 120 per hour with different corridor length, 15km, 10km, and 5 km when the side length of the drop-off area is 2.18 km, $S_1 = 36$, and $S_2 = 28$

4.6 The impact of modified Shapley value

4.6.1 Scenario settings

This section investigates the impact of the fare distribution method for shared service on the system evolution. The simulation was conducted to compare the impact of traditional Shapley value and modified Shapley value. As demonstrated in section 3.4.6, using modified Shapley value increases the proportion of those who received enough discount in fare, which compensates the additional in-vehicle time caused by sharing their ride. However, it is expected that the difference in the fare distribution method would only impact the case where the shared service is dominant or at least when the *non-shared* service is *not* dominant.

Hence, at first, two scenarios with different service network geometry are generated and compared. One is the default service network specified in Figure 34 (scenario 1) and the other is the service network with the corridor length 15 km and the side length of drop-off area 2.18 km (scenario 2). As discussed in section 4.5, the default service network leads the system to be a high mode share for non-shared service, while the service network used for scenario 2 leads the system to have a high mode share for shared service. For each scenario, the fleet size for non-shared service is set as constant at 36 while 4 different fleet sizes for shared service are tested from 28 to 46 in increment of 6.

Additional experiments are conducted to see the effect of VoIVT in the same service network geometry with scenario 2. The fleet size for non-shared service is set as 36 while the fleet size for shared service is set as 46. Three cases with different VoIVT (i.e. 0.5, 1, and 2) are tested, and the results are summarised in the following section. The other parameters are set as default. The parameter settings for all 3 scenarios are summarised in Table 14. Besides, to minimise the effect of randomness when the results with traditional and modified Shapley value are compared, a random seed is set to be the same for the simulation with the traditional and with the modified Shapely value.

Table 14 The parameter settings for the 3 scenarios.

<table>
<tr><th rowspan="2">Scenario</th><th colspan="2">Service network geometry</th><th rowspan="2">VoIVT [min/monetary unit]</th><th rowspan="2">Fleet size for Shared service</th></tr>
<tr><th>Corridor length [km]</th><th>Side length [km]</th></tr>
<tr><td>1</td><td rowspan="3">15</td><td>8.71</td><td rowspan="2">1</td><td rowspan="2">28 to 46</td></tr>
<tr><td>2</td><td rowspan="2">2.18</td></tr>
<tr><td>3</td><td>[0.5, 1, 2]</td><td>46</td></tr>
</table>

4.6.2 **Key findings**

- The differences between traditional and modified Shapley value can only be observed with the specific combination of service network geometry and the fleet size where the non-shared service is *not* highly dominant.

- The modified Shapley value leads the mean mode share for shared service (non-shared service) to be higher (lower) compared to the case with the traditional Shapley value.

- The difference in the mean mode share between the case with traditional and modified Shapley value increases as the VoIVT decreases.

4.6.3 **Results**

Figure 67 compares the mean mode share for non-shared service from day 51 to 500 between the case generated with the traditional Shapely value and the case generated with the modified Shapley value for different fleet sizes for scenario 1. The error bar in the figure indicates the standard deviation of mode share for non-shared service from day 51 to 500. Following the different experiments summarised in Chapter 4, the process is assumed to leave from the warming up period as of day 50. In Figure 67, there is no distinctive difference observed when the fleet size for shared service is 28 and 34. However, when the fleet size for shared service is 40 and 46, the mean mode share for non-shared service is lower in the case with modified Shapley value than the case with traditional Shapley value. As the range of the error bar also becomes lower for the modified Shapley case, it can be said that this difference is not the result of randomness.

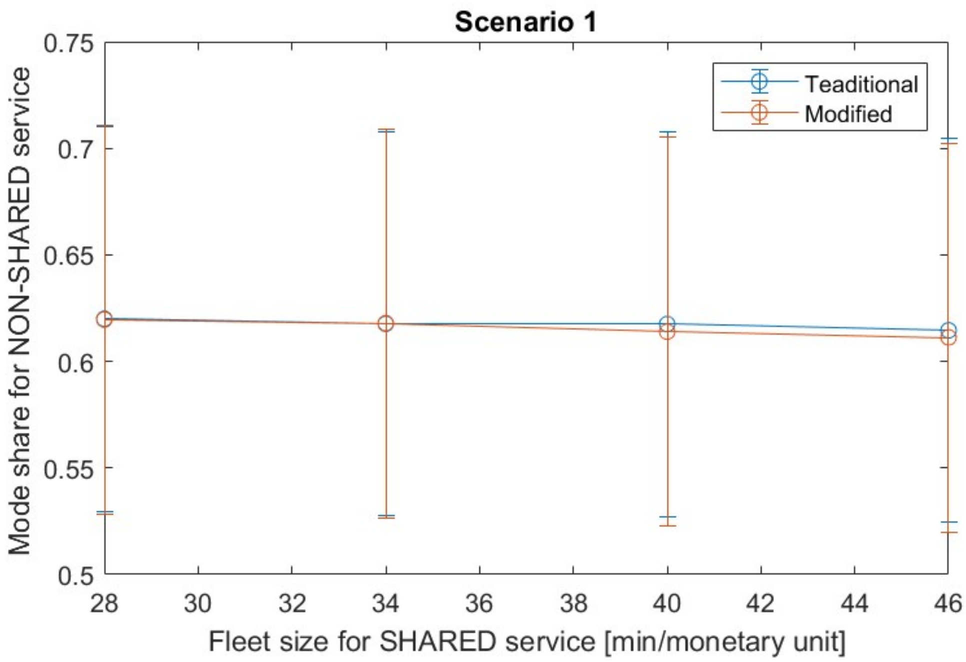

Figure 67 the comparison of mean mode share for non-shared service from day 51 to 500 between the case generated with the traditional Shapely value and the case generated with the modified Shapley value for the different fleet sizes from 28 to 46 for scenario 1. The error bar indicates the standard deviation.

Figure 68 summarises the same values as Figure 67 but for scenario 2. As shown in Figure 65, the service network geometry assumed in scenario 2 allows shared service to provide a higher level of service than the non-shared service. Therefore, the mode share for shared service is much higher than the non-shared service. Following the expectation mentioned at the beginning of this section (i.e. section 4.6), the difference in the mean mode share for non-shared service regarding the fare distribution method is revealed more distinctively than scenario 1. According to Figure 68, The mean mode share for non-shared service is lower for the modified Shapley value for all cases with different fleet sizes from 28 to 46. In addition, it is observed that the difference in the mean value increases as the fleet size increases. Therefore, it can be concluded that as the parameter settings become more advantageous for shared services to provide a high level of service, the impact of the fare distribution method on the system evolution increases.

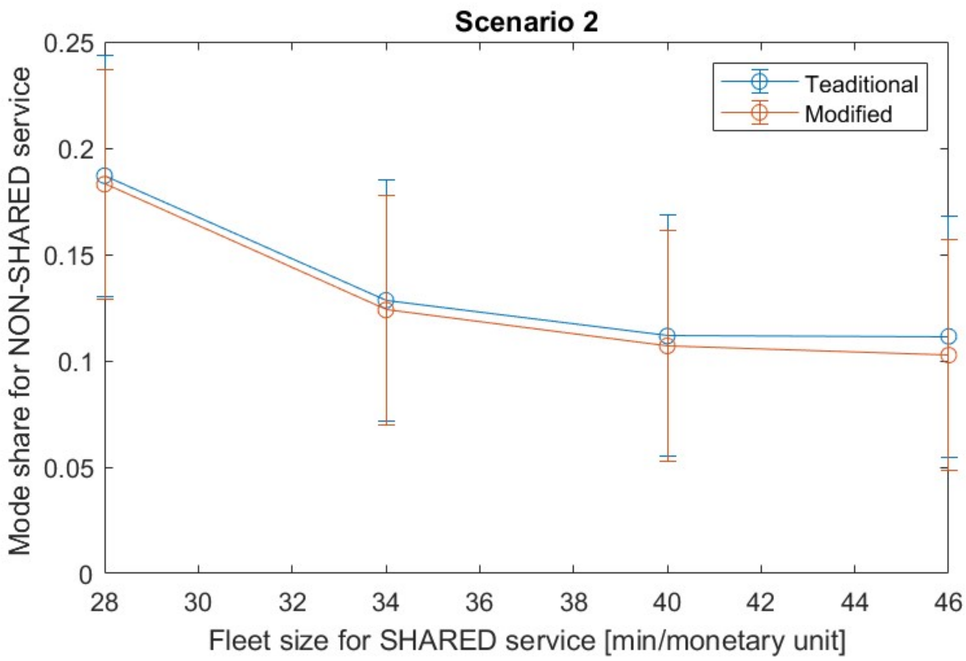

Figure 68 the comparison of mean mode share for non-shared service from day 51 to 500 between the case generated with the traditional Shapely value and the case generated with the modified Shapley value for the different fleet sizes from 28 to 46 for scenario 2. The error bar indicates the standard deviation.

Figure 69 summarises the results for scenario 3 and compares the mean mode share for non-shared services generated with three different VoIVT (i.e. 0.5, 1, 2). According to Figure 69, for all three cases with different VoIVT, the mean mode share for non-shared service is lower for the case with modified Shapley value than the case with traditional Shapley value. In addition, it is observed that as the VoIVT decrease, the difference between the two fare distribution methods increases. It is because, as discussed in subsection 3.4.6, the proportion of uncompensated trip requests decreases as VoIVT decreases. It should be remembered that the uncompensated trip requests in this context imply the situation where trip requests did not receive enough discount in fare to compensate for the additional in-vehicle time caused by sharing their trip. The higher proportion of the compensated trip requests implies that more shared service users experienced higher service levels. Thus, the mode share for shared service (non-shared service) increases (decreases) more for the case with modified Shapley value when VoIVT is smaller.

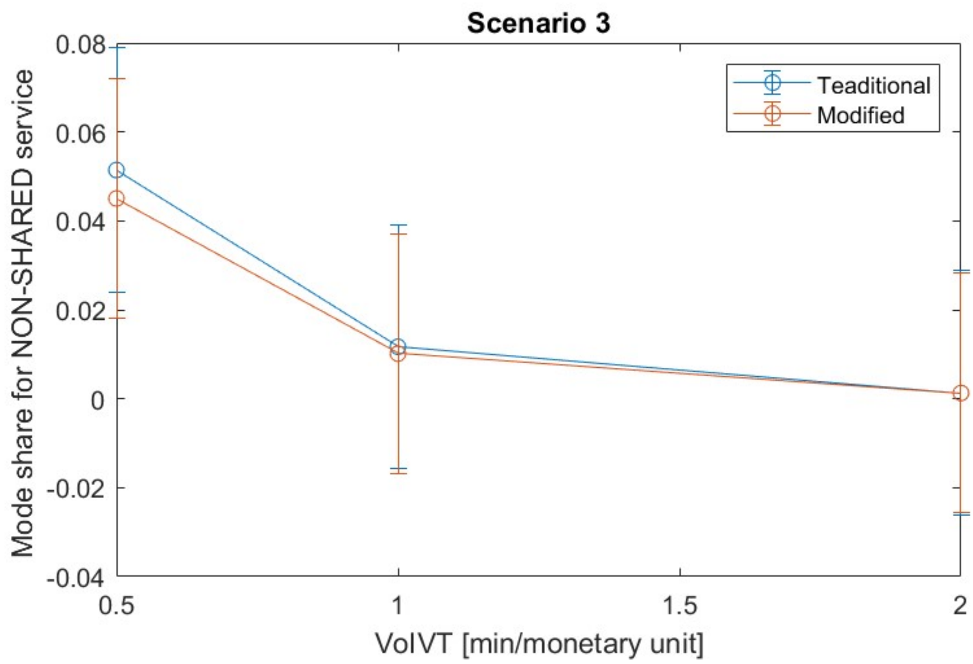

Figure 69 the comparison of mean mode share for non-shared service from day 51 to 500 between the case generated with the traditional Shapely value and the case generated with the modified Shapley value for the different VoIVT for scenario 3. The error bar indicates the standard deviation

4.7 Summary

This chapter summarised the results of a numerical experiment with the fixed fleet size and the model's sensitivity to different parameters. In section 4.3, the analysis of the base scenario is presented and the sensitivity analysis against several parameters. The analysis results suggested that within the given parameter settings specified as the default parameters, the mode share for the non-shared service was higher than the shared service for most of the days. Nevertheless, the mode share for shared service became higher than the non-shared service from time to time. It happens when the mode share and the total number of trip request are high on the same day. However, the mode share for non-shared service always became dominant again on the next day.

The sensitivity analysis was conducted against the initial condition, the parameters determining the user's irregular behaviour, the updating filter, and several parameters in the utility function. The sensitivity analysis against the initial conditions (i.e. initial mode share) concluded that the initial condition did not influence the stationary distribution. Therefore, the stochastic process is ergodic with the fixed fleet size case. The parameters controlling the user's irregular behaviour and the updating filters change a variance of the stationary distribution. However, they do not influence the mean mode share

VoWT, VoIVT, and fare per minute for non-shared and shared service are selected for the sensitivity analysis. As willingness to share (WTS) was too small compared to other variables in the utility function, the sensitivity analysis was not conducted against it. When VoWT was within the realistic range (i.e. higher than VoIVT), changes in VoWT was observed not to bring any impact to the stationary distribution of mode share. Besides, it was discovered that, as VoIVT increases, the mean value for the stationary distribution of mode share for non-shared service increases. Moreover, increasing the fare per minute for non-shared service was observed to have more impact on the stationary distribution of mode share than decreasing the fare per minute for shared service.

In section 4.4, the impact of fleet size for non-shared and shared service was examined. With a given setting, the results suggest that the stationary distribution of mode share changed according to the change in the fleet size for non-shared service, regardless of shared service fleet size. It was because the maximum mean utility of non-shared service was higher than the maximum mean utility of shared service. Hence, the mean utility of shared service became higher only by the reduction in the mean utility of non-shared service. Therefore, the changes in fleet size for shared service did not affect the distribution of mode share. Furthermore, as the fleet size for non-shared service, S_1, increased, the distribution of mode share for non-shared service took higher value. Besides, no significant changes in the distribution of mode share were observed after the fleet size for non-shared service reaches $S_1 > 60$.

In section 4.5, the impact of the service network geometry is investigated. The results suggest that the mean utility of shared service becomes higher than the non-shared service as the proportion of a trip on the corridor becomes longer in a round trip. Besides, the service network geometry was discovered to determine the boundary of the stationary distribution of mode share. Hence, the dominant service changes as the service network geometry changes. Moreover, the impact of fleet size for shared services becomes the determinant of the stationary distribution of mode share when the proportion of a trip in the corridor is high and the mean round trip time was not too short. When the mean round trip time is too short, the stationary distribution of mode share was not affected by both fleet sizes as the minimum fleet size in this numerical experiment was already high for those cases.

In section 4.6, the impact of modified Shapley value on the system evolution is demonstrated. With the hypothesis that the difference between modified and traditional Shapley value is influential only when the non-shared service is not dominant, the simulation is conducted with two different service network geometry, one leading to the significantly high mode share for non-shared service and one leading to the significantly high mode share for shared service, with four different fleet sizes. The results suggest that even with the case where non-shared service is dominant, the fare distribution method impacts the system evolution when the fleet size for shared service is larger. Besides, the difference between modified and traditional Shapley values increases as the fleet size for shared service increases and as the service network geometry becomes more advantageous for shared service to provide a higher level of service with a given fleet size for non-shared service. In particular, the mean mode share for shared service (non-shared) service becomes higher (lower) for the case of modified Shapley value. In addition, experiments with different VoIVT showed that as VoIVT decreases, the difference between the modified and traditional Shapley value increases with a given fleet size and service network geometry.

Chapter 5 Numerical experiment with the variable fleet size

5.1 Introduction

This chapter summarised the results of numerical experiments with variable fleet size. It means that drivers also learn and would change their service to provide day by day. Unlike the case with fixed fleet size, it was difficult to predict how the process would behave before running an experiment because of the complexity of the model. Running a few simulations revealed that the proposed stochastic process had three regimes; 1) the PS regime, 2) the swan regime, and 3) the PP regime. The definition and attribute of each regime is analysed and presented in section 5.4 in this chapter. Besides, the detailed analysis of how the transition occurs between each regime was conducted and summarised. In particular, the main focus of the analysis was to identify if the series of a random event caused the transition or not. In addition, sensitivity analysis was conducted to investigate how the occurrence pattern of three regimes changes as parameters change, hence, the stationary distribution of mode share.

The structure of this chapter is as follows. In section 5.2. default parameter settings for numerical experiments are provided. In section 5.3, the comparison of results with a given parameter setting between the fixed fleet size case and the variable fleet size case is summarised. In section 5.4, the definition and attributes of each regime are specified. In section 5.5, a detailed analysis of the transition process between each regime are provided. In section 5.6, the sensitivity analysis against the initial conditions is summarised for two scenarios with different parameter settings. In section 5.7, the occurrence pattern of each regime is discussed with the result of the sensitivity analysis. In section 5.8, the condition to change the system from demand-driven to supply-driven is discussed. In section 5.9, how changing the updating function affects to the nature of the process is discussed. The chapter is concluded by providing the summary in section 5.10.

5.2 Default parameter setting

For this experiment, the same service network assumed in Chapter 4 is also assumed (see Figure 34 in Chapter 4). As with Chapter 4, the length of the corridor is set to be 10 km on which the vehicle moves with the fixed average speed of 60 km/h. The drop-off area is assumed to be a square with the side length being 8.71 km, in which vehicles move with the fixed average speed of 20 km/h. Other parameters settings also follow in Chapter 4, which is summarised in Table 15. The default setting for the new parameters introduced in this section is summarised in

Table 16. Newly introduced parameters are related to the drivers' learning and decision-making process. Also, as the fleet size is not fixed anymore, the expected total number of drivers (fleet) is set as 100 instead of individual setting the number of drivers. The initial proportion of fleet size for each service is set to be 0.5. In addition, the cost for drives' making a round trip is set as 0.5 monetary unit/min.

Table 15 default parameter setting

Parameters	**Default value**
The expected total number of trip request per hour	120 /h
The initial mode share for non-shared service	0.5
The mean number of accompanied persons per request (NoA)	0.3
The minimum number of requests per cluster	2
The maximum number of users per cluster, i.e. a vehicle capacity	4
The maximum clustering waiting time (MCWT)	3 min
The side length of the square drop-ff area	8.71 km
The length of the corridor	10 km
The period of interest	120 min
The mean speed of the vehicle in the drop-off area	20 km/h
The mean speed of the vehicle on the corridor	60km/h
Fare/min	1
The value of in-vehicle travel time (VoITT)	2
The value of waiting time (VoWT)	2.5
The willingness to share (WTS)	0.1
α_u: The proportion of those who stay in the service even if the experienced profit is lower than the expected profit of the alternative service	0
β_u: The proportion of users who did not satisfy with the service they chose yet stated that they would stay in the same service on the following day	0.5
η_u: An updating filter which determines how much the average utility on day d influences on the collective average utility on day d $PU_{k,d}$.	0.3

Table 16 default values for the newly introduced parameters

Parameters	Default value
α_p: The proportion of drivers who satisfied with providing the service they chose yet stated that they would change to providing the alternative service on the following day	0
β_p: The proportion of drivers who did not satisfy with providing the service they chose yet stated that they would keep providing the same service on the following day	0.5
η_p: An updating filter which determines how much the average profit on day d influences on the collective average profit on day d An updating filter which determines how much the average profit on day d influences on the collective average profit on day d	0.3
The cost to provide the service per minute	0.5
The expected total fleet size	100
The initial proportion of fleet for non-shared service	0.5

5.3 Comparison with the fixed fleet size case

In this section, the simulation results with the fixed fleet size and the variable fleet size are compared to see the effect of introducing drivers' service choice behaviour. The parameter setting follows the default value for both cases specified in section 4.2 for the case of fixed fleet size and section 5.2 for the case of variable fleet size. 500 consecutive days are generated for both cases. For variable fleet size case, the initial proportion of fleet is 0.5 for both services where the expected total fleet size is 100. Hence, the fleet size for non-shared and shared service is set as 50 and 50 for the case of fixed fleet size. As the actual fleet size is randomly generated with the mean of 100 for variable fleet size case, the fleet size of the first day is not necessarily 50 and 50. However, the initial proportion of fleet for both cases are still consistent and 0.5 for non-shared service and shared service.

Figure 70 shows the evolution of mode share and the stationary distribution of modes share for non-shared service for the fixed fleet size case. It is assumed that the warming-up period of the process ends before day 50 from the visual observation. Hence, the stationary distribution is generated from the mode share on day 51 to day 500. Figure 71 displays the evolution of mode share (the upper left figure) and the evolution of the proportion of fleet size (the lower left figure) for variable fleet size case. The stationary distribution of mode share for non-shared service and the proportion of fleet for non-shared service are also illustrated in the right column.

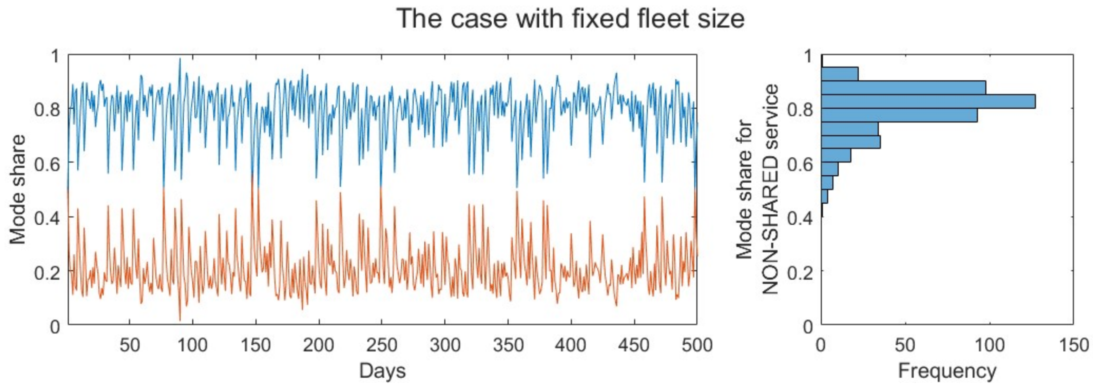

Figure 70 the evolution of mode share (left) and the unique stationary distribution of mode share for non-shared service (right) for fixed fleet size case generated with the default parameter settings.

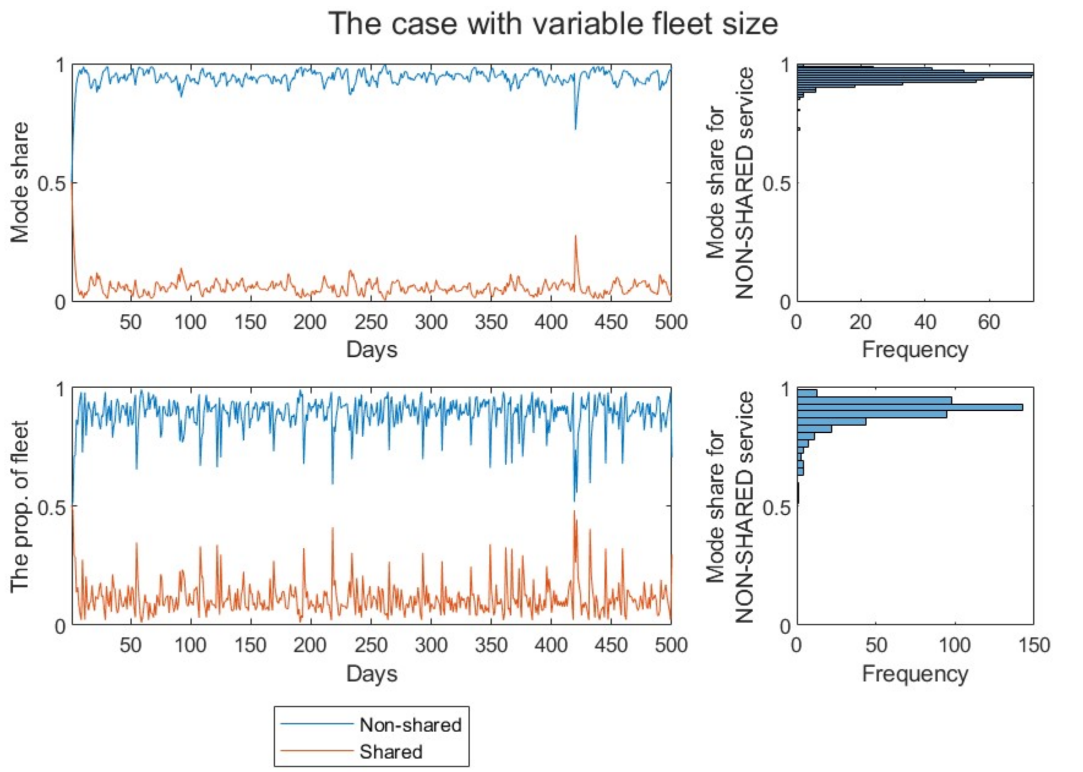

Figure 71 the evolution of mode share (upper left) and the proportion of fleet (lower left) and the unique stationary distribution of mode share for non-shared service (upper right) and the proportion of fleet (lower right) for the variable fleet size case generated with the default parameter settings.

It can be observed that the mode share for non-shared service takes the higher value for the case with variable fleet size than the case with fixed fleet size, according to Figure 70 and Figure 71. In addition, the fluctuation in a mode share is smaller in Figure 71 than in Figure 70. As the lower figure in Figure 71 shows, the proportion of fleet for non-shared service increases as the mode share for non-shared service increases during the first several days. The larger fleet size enables the service to serve more people without increase the number of queues. Hence, users experience higher utility which results in a higher mode share.

Figure 72 compares the evolution of the mean utility for non-shared and shared service for the case with the fixed fleet size (the upper row) and with the variable fleet size (the lower row). Comparing the upper-left figure and the lower-left figure, the mean utility for non-shared service is higher for the case with *variable* fleet size than the *fixed* fleet size case in most of the days. Besides, the mean utility for non-shared service fluctuates less in the variable fleet size case. On the other hand, the mean utility of shared service is lower in the variable fleet size case than the fixed fleet size case. Hence, fleet size for shared service is smaller in the variable fleet size case, as observed in Figure 71. At the same time, the mean utility for shared service fluctuates more in the variable fleet size case than the fixed fleet service case (see the right figures in Figure 72).

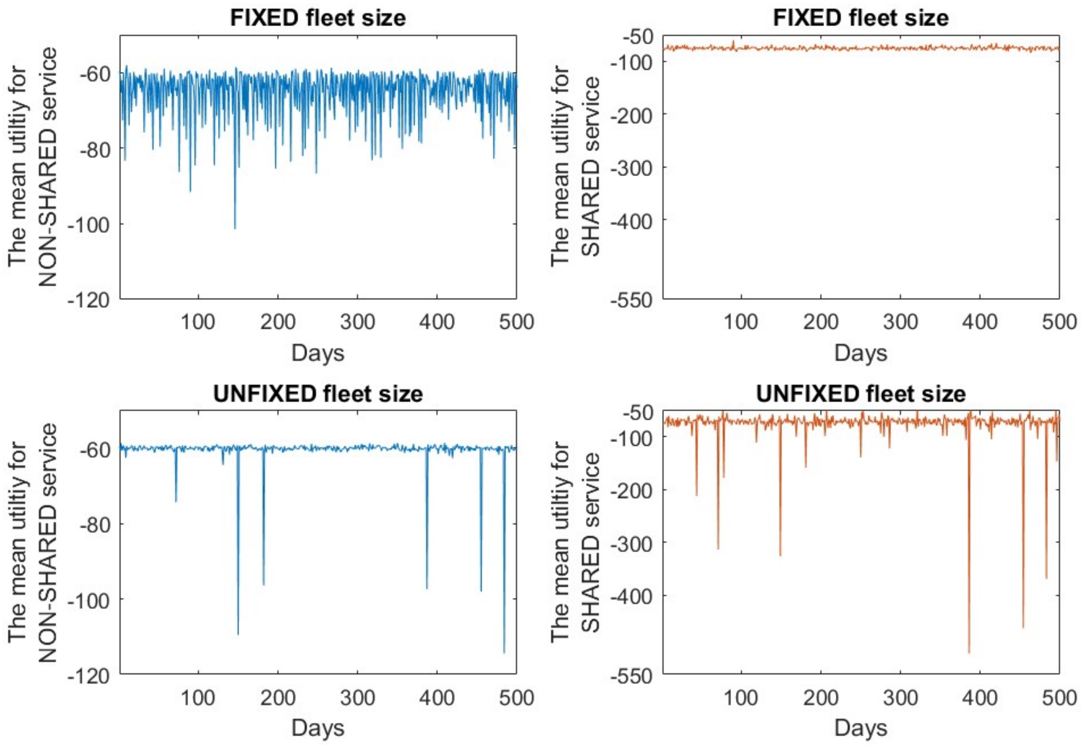

Figure 72 the mean utility for non-shared and shared service for the case with fixed fleet size and variable fleet size where

5.4 **The three regimes**

This section summarises the definition and attributes of three regimes identified through the numerical experiment with a default parameter setting specified in section 5.2. They are named as; 1) The pseudo stable regime (the PS regime), 2) the swan regime, and 3) the pseudo periodic regime (the PP regime). In Figure 73, the one realisation is presented as an example case since it contains three regimes at one realisation. The updating filter and the proportion of hesitant users and drivers are set as $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ for the case presented in Figure 73. The PS regime is emphasised with the dashed square labelled as (1) in the figure. One of the swan regimes is highlighted with the dashed square marked as (2) in the figure. One of the PP regimes is emphasised with the dashed square labelled as (3) in the figure. It should be noted that depending on the updating filters and hesitation parameters, some realisations contains only one or two regimes.

The below section consists of 5 subsections. In subsection 5.4.1, the quantitative definition of each regime is presented. In subsection 5.4.2 to 5.4.3, the qualitative description of each regime is summarised. The case with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ is used as an example to describe the attribute of each regime that was generated for 100,000 days.

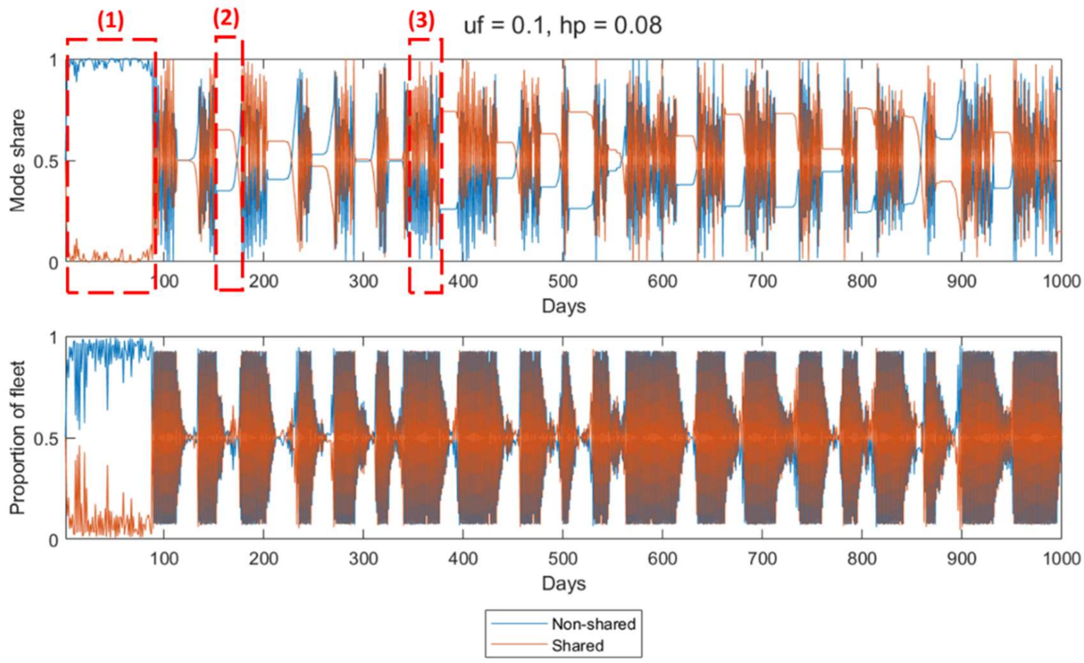

Figure 73 one realisation of the process with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ where all three regimes are observed

5.4.1 The definition of three regimes

The quantitative definition of each regime is summarised below.

1) The pseudo stable regime

The pseudo stable regime could be defined as the mean experienced utility and the mean experienced profit for the dominant service is always higher than the collective average utility for the other service. For instance, in the case of default parameter settings, the non-shared service is dominant. Hence, the mean experienced utility and profit is consistently higher than the collective average utility of shared service.

(2) The swan regime

If the following three conditions are satisfied, it is the swan regime;

- the mode share is constant during the regime
- the mean experienced utility is lower than the collective average utility for the other service for both services (i.e. $\bar{u}_{1d} > PU_{2d}$ and $\bar{u}_{2d} > PU_{1d}$).
- The difference between the two values, $|\bar{u}_{1d} - PU_{2d}|$ and $|\bar{u}_{2d} - PU_{1d}|$, constantly decreases during the regime.

(3) The pseudo periodic regime

The pseudo periodic regime can be defined with the autocorrelation in the proportion of fleet. In specific, the autocorrelation with one day lag, two-day lag, three-day lag, four-day lag, and five-day lag is always negative, positive, negative, positive, and negative, as summarised in Table 17.

Table 17 the sign for autocorrelation in the proportion of fleet when the lag length is from 1 to 5 days

Lag in days	1	2	3	4	5
sign	−	+	−	+	−

Figure 74 shows the example of the autocorrelation in the PP regime when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. Depending on the value of updating filters and the hesitation parameters, the absolute values of the autocorrelation change. However, in the majority of cases, it is above 0.9 or below -0.9.

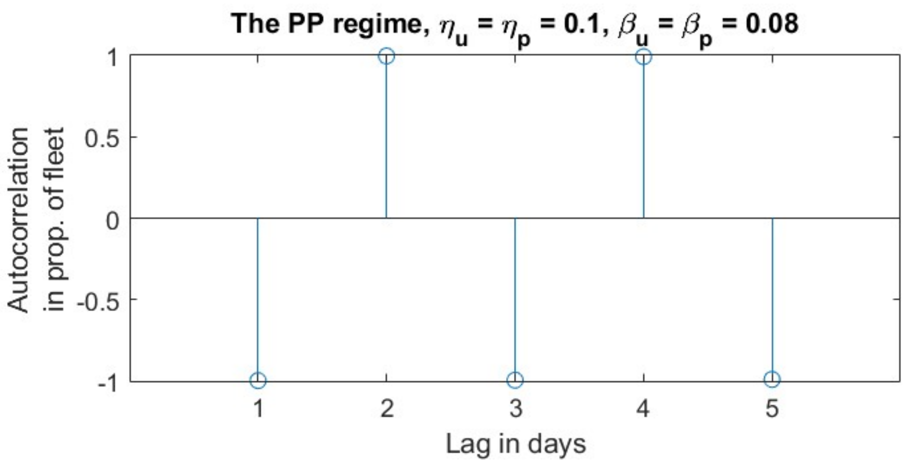

Figure 74 an example of autocorrelation in the proportion of fleet when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$

5.4.2 The pseudo-stable regime

The pseudo stable (PS) regime is the most stable regime as represented by its name. First, both mode share and proportion of fleet do not fluctuate intensely compared to the pseudo periodic (PP) regime during the regime, as observed in Figure 73. Also, the dominant mode is consistent and non-shared service during the PS regime. Figure 75 shows the histogram of mode share and the proportion of fleet for non-shared service when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ as an example. The variance of those value changes as the value for updating parameter and hesitation parameter changes. The length of the regime also varies among one realisation. In this example, the PS regime appeared 4 times, the length of which is summarised in Table 18. The occurrence frequency of the PS regime and the distribution of regime length also changes as the value for updating parameter and hesitation parameter changes, which are summarised in section 5.7.

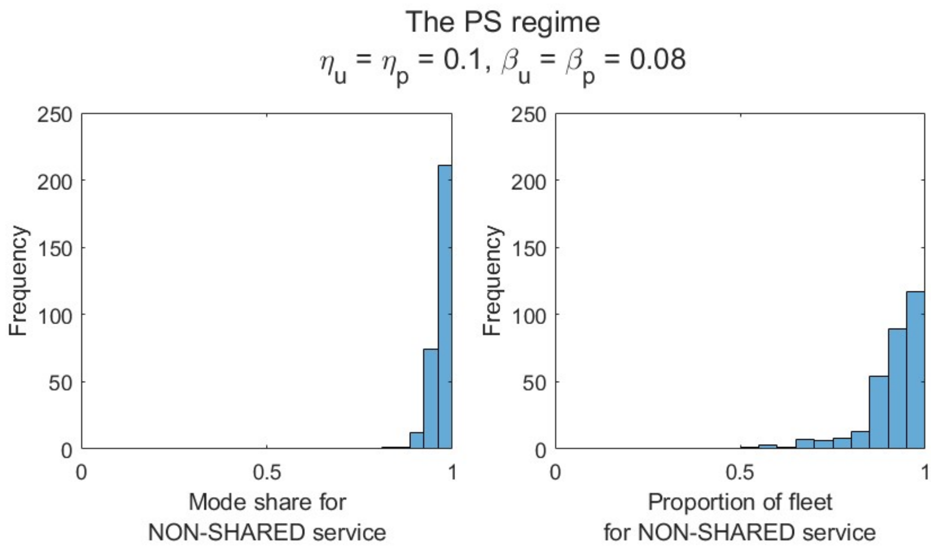

Figure 75 the histogram for the mode share (left) and the proportion of fleet (right) for non-shared service during all the PS regime when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$.

Table 18 the length of 4 PS regimes occurred in one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$

The length of the PS regime	117	105	51	21

Figure 76 presents the evolution of key values related to non-shared services during one PS regime. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for non-shared service. The middle left figure compares the mean utility of *non-shared service* users, $\bar{u}_{1d}$, and the collective average utility of *shared services,* PU_{2d}. Those parameters influence on the service choice for the *non-shared service* user of the day. The middle right figure compares the mean profit of *non-shared service* drivers, $\bar{p}_{1d}$, and the collective average profit of *shared services,* PP_{2d}. Those parameters influence on the service choice for the *shared service* user of the day. The bottom figures display the proportion of non-shared service users (left) and drivers (right) who change the service each day.

From the middle left figure in Figure 76, it is observed that the mean utility for non-shared users is higher than the collective average utility of shared service during the PS regime (i.e. $\bar{u}_{1d} > PU_{2d}$). Similarly, the mean profit among non-shared service driver is constantly higher than the collective average profit of shared service (i.e. $\bar{p}_{1d} > PP_{2d}$) Therefore, the proportion of non-shared service users and drivers who change the service is constantly low. As a result, the mode share and the proportion of fleet are stable during the PS regime.

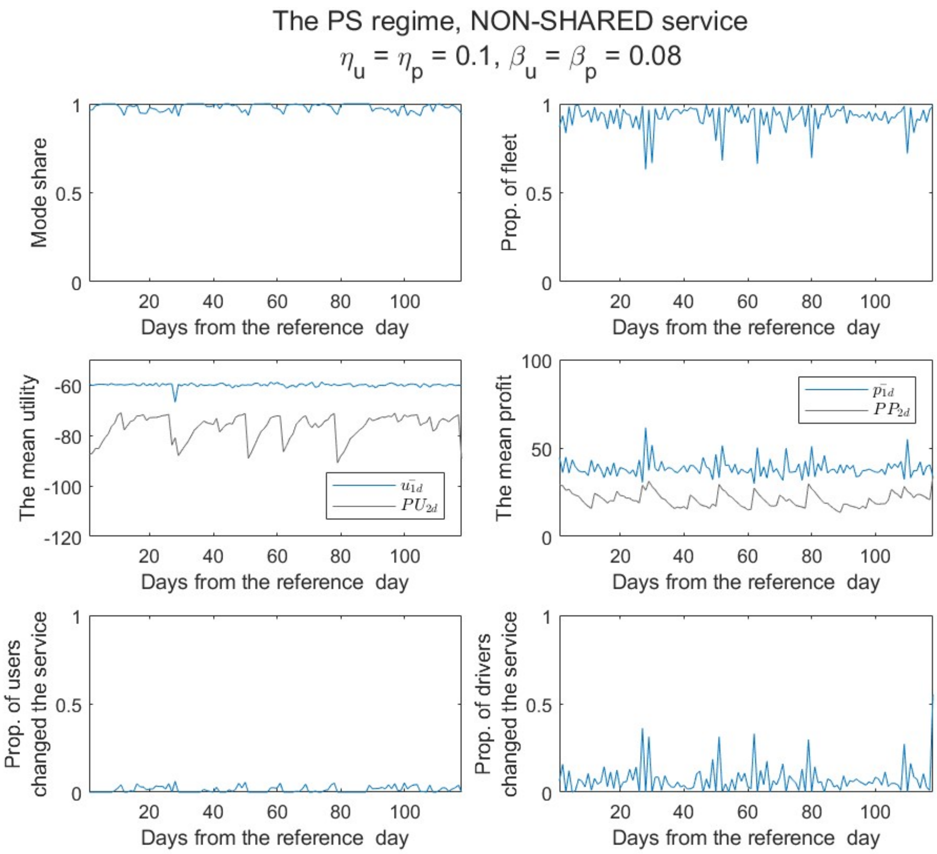

Figure 76 an example of one pseudo stable regime lasted for 117 days when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day.

Figure 77 displays the same variables as Figure 76 but for shared service users and drivers. As the number of shared service users and drivers is constantly low during PS regime, the small shift in users and drivers could greatly change the utility and profit for shared service users and drivers. Therefore, compared to the middle two figures in Figure 76, the mean utility and profit fluctuate greater in Figure 77. Though it changes periodically, most of the days, the mean utility of *shared service* is lower than the collective average utility of *non-shared service* (i.e. $\bar{u}_{2d} < PU_{1d}$). On those days, high proportion of users and drivers change their service to non-shared service. Nevertheless, as the number of shared service users and drivers are small, such a shift do not always cause a significant change in the system during the PS regime. Hence, the PS regime remains stable.

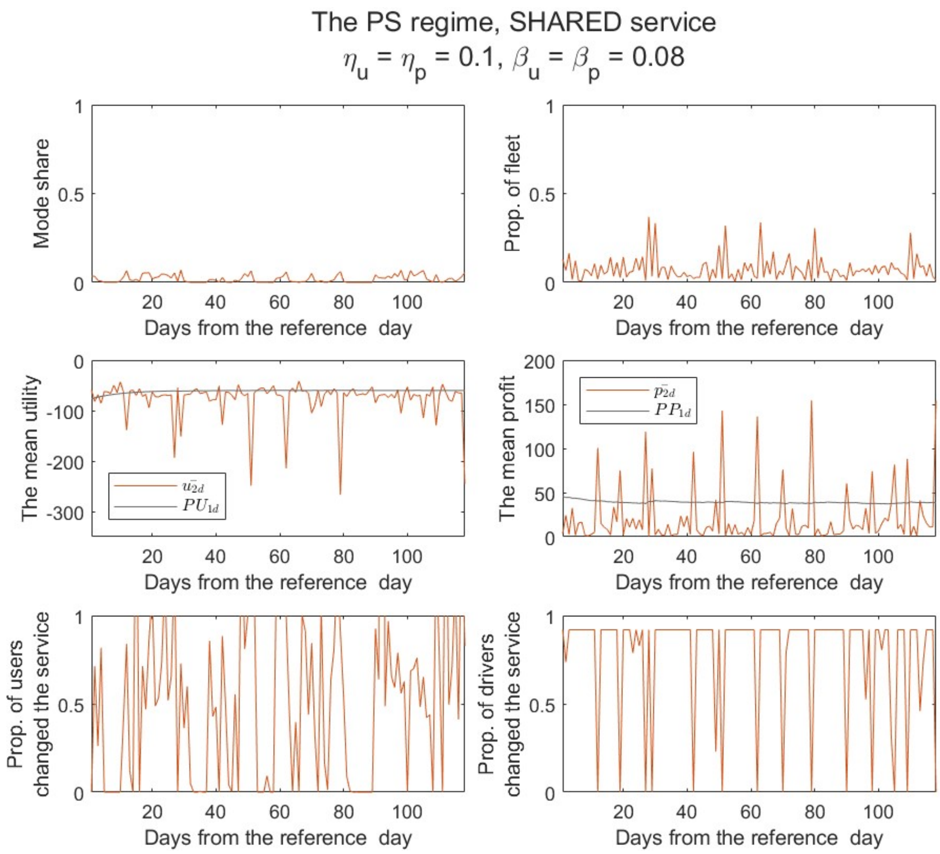

Figure 77 an example of one pseudo stable regime lasted for 117 days when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

5.4.3 **The swan regime**

The swan regime has two distinctive attributes. During this regime, a process looks stable from the users' point of view. However, almost all drivers are changing the service every day; hence, it is unstable from the drivers' point of view. The swan regime having two contradicting attributes, stable and unstable, from different points of views, reminds swans. They look smoothly swimming if only seen above the water. On the other hand, many movements are constantly happening under the water as they continually move their pedals (see Figure 78). Hence, this regime is named “the swan regime”.

Figure 78 a drawing of the swan metaphor representing two distinctive states observed in one system that is smooth and elegant on the water and hectic under the water.

Figure 79 and Figure 80 illustrate the same variables presented in Figure 76 and Figure 77 but for the swan regime. From the middle left figure in Figure 79 and Figure 80, it is observed that the mean utility of non-shared (shared) service is greater than the collective average utility of shared (non-shared) service (i.e. $\bar{u}_{1d} > PU_{2d}$ and $\bar{u}_{2d} > PU_{1d}$) Therefore, the proportion of users who shift from both services is 0; hence the mode share stays constant during the swan regime (see the middle and bottom left figure in Figure 79 and Figure 80).

From the driver side summarised in the right column in Figure 79 and Figure 80, it is observed that the mean profit among non-shared (shared) service drivers is constantly lower than the collective average profit of the alternative service (i.e. $\bar{p}_{1d} < PP_{2d}$ and $\bar{p}_{2d} < PP_{1d}$) according to the right middle figures in Figure 79 and Figure 80. Therefore, the proportion of drivers who changes the service is constant and 0.92, which is the highest value it can possibly take (see the bottom right figures). It implies that all drivers are unsatisfied with their experience every day, and 92% of them (i.e. $1 - \beta_p$) actually decided to switch the service. However, as the proportion of fleet for each service fluctuate around 0.5 during the swan regime, even if almost everyone each service shifts the service every day, the proportion of fleet stay within the similar range. Hence, some equilibrium is maintained.

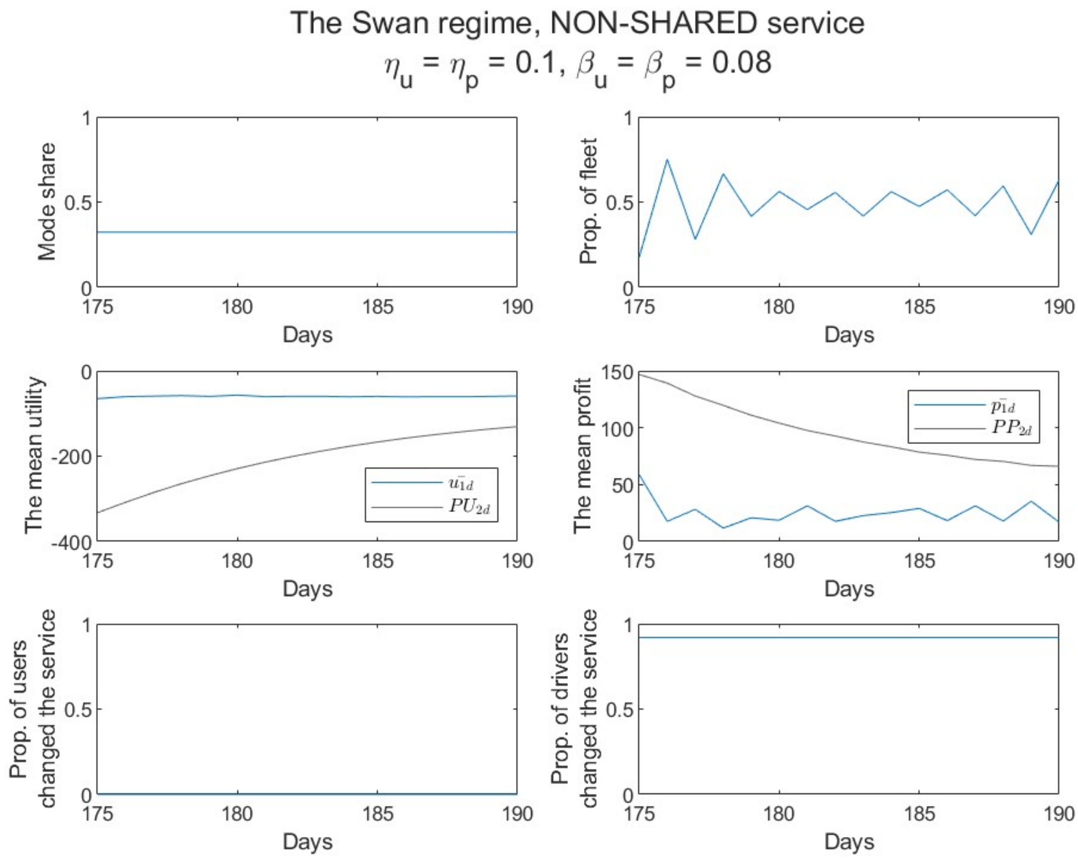

Figure 79 an example of the pseudo stable regime from day 175 to day 190 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed a service on each day.

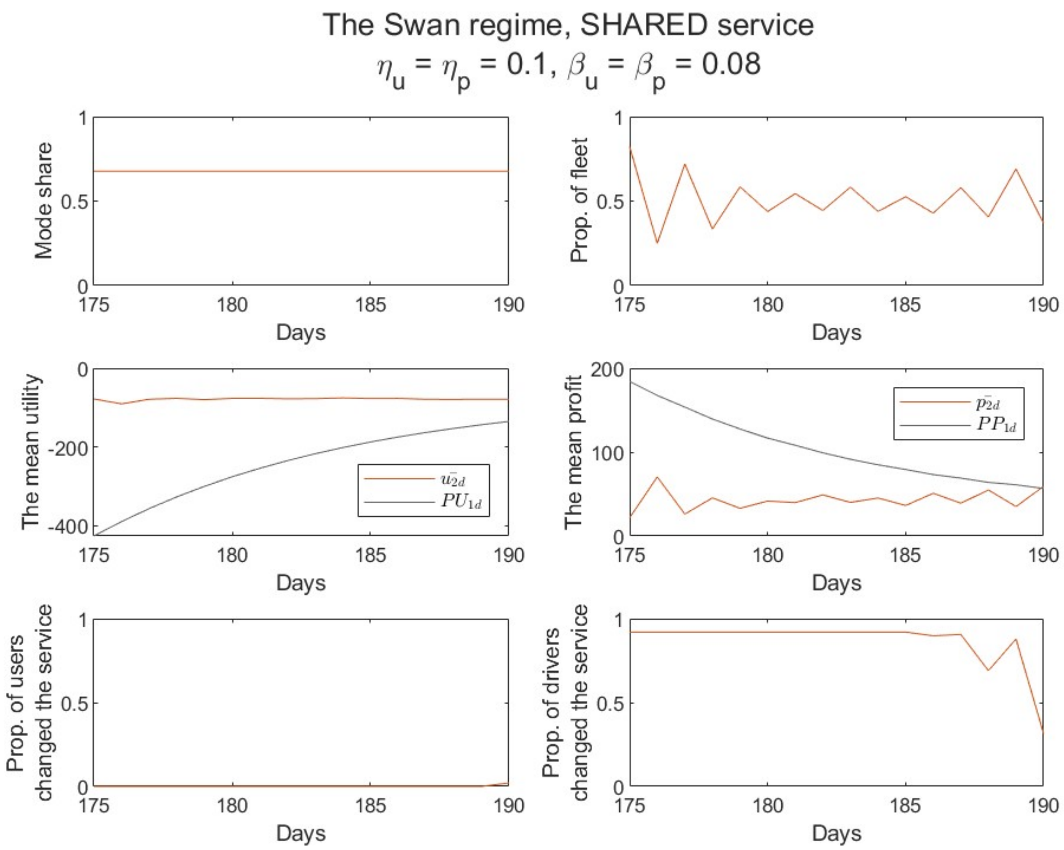

Figure 80 an example of the pseudo stable regime from day 175 to day 190 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

As shown in Figure 4, the mode share and regime length during the swan regime vary. The below section summarises the attribute of the swan regime in one realisation with $\beta_u = \beta_p = 0.08$ and $\eta_u = \eta_p = 0.1$ which was simulated for 100,000 days. During that time, the swan regime appeared 2,173 times. The maximum length of the swan regime was 35 days. The distribution of the swan regime length is displayed in Figure 81. Compared to the maximum length of the PS regime in the same realisation (i.e. 117), it can be observed that the swan regime is less stable than the PS regime.

According to the middle figures in Figure 79 and Figure 80, the difference in the mean utility and the collective average utility is decreasing as the day passes during the swan regime. It is because while the mean utility is constant during the swan regime, the collective average utility is updated every day. The same tendency is observed in the mean profit and the collective average profit for both services.

When the difference between mean utility (profit) and the collective average utility (profit) becomes small enough, the process transitions to a different regime. Hence, the swan regime cannot stay as long as the other two regimes. The length of the regime depending on how great the difference between mean utility (profit) and the collective average utility (profit) is at the beginning of the regime, which is randomly determined. Also, the updating filters, η_u and η_p, would impact on the regime length as it determines the rate of reduction in the difference between two values.

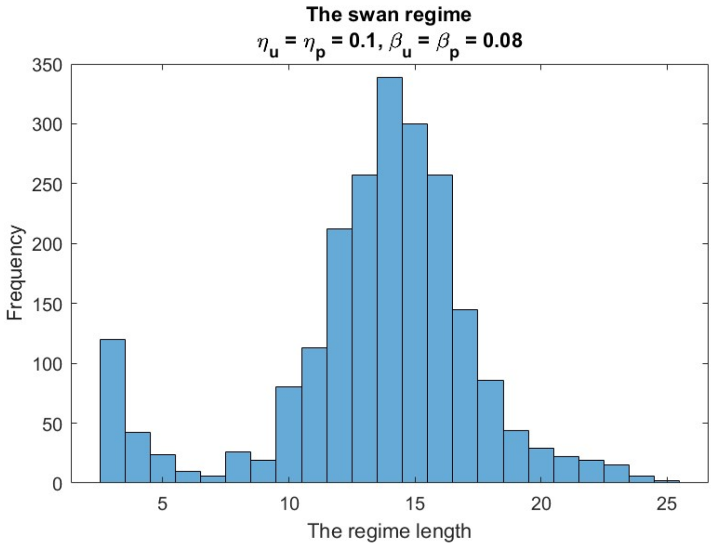

Figure 81 histogram for the swan regime length appeared during 100,000 days of one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$

Figure 82 summarised the mode share (left) and the mean proportion of fleet (right) for non-shared service during the swan regime. According to the left figure, unlike the PS regime, the mode share for shared service tends to be dominant during the swan regime. The mean mode share for non-shared service among 2,173 swan regimes is 0.362. As the proportion of fleet fluctuates during the swan regime, the right figure presents the histogram for the mean proportion of fleet for non-shared service. It is discovered that the proportion of fleet fluctuates around 0.5 in the above section. Hence, the mean proportion of the fleet takes approximately 0.5 most of the time. It implies that, unlike the mode share, the proportion of fleet shows a consistent tendency among 2,173 regimes.

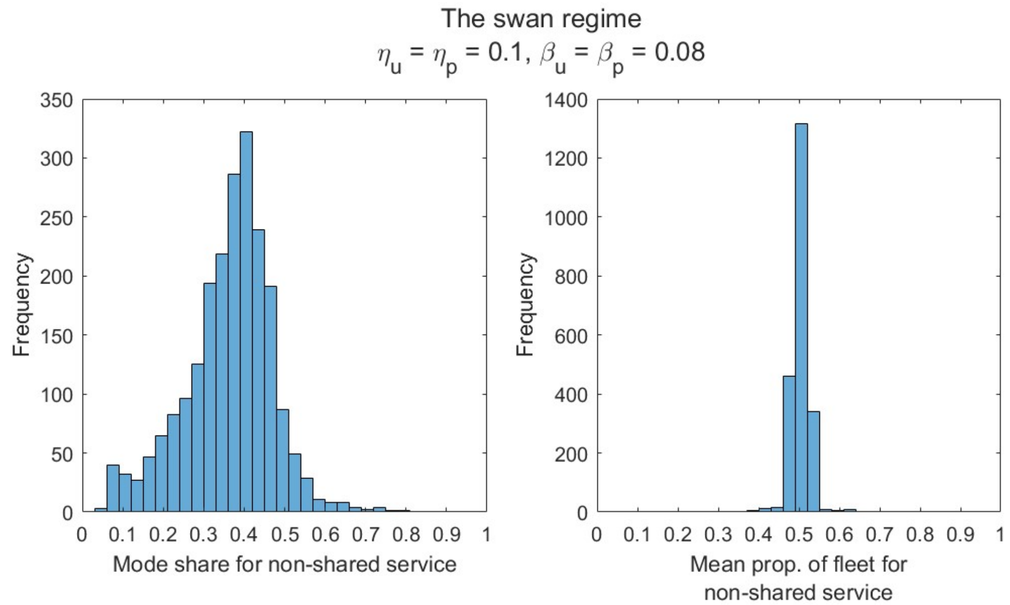

Figure 82 histogram of mode share (left) and the mean proportion of fleet (right) for NON-SHARED service during the swan regime, which appeared during 100,000 days of one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$

5.4.4 **The pseudo-periodic regime**

In the pseudo-periodic (PP) regime, both the mode share and the proportion of fleet show the “flip-flop” behaviour as observed in top figures in Figure 83 and Figure 84. Figure 83 and Figure 84 summarised the same set of variables with Figure 76 and Figure 77 but for the PP regime instead of the PS regime. The flip-flop behaviour is observed more distinctively in the proportion of the fleet than in the mode share. Besides, as described in section 5.4.1, the autocorrelation in the proportion of fleet with one day lag is always a negative value.

The middle left figures in Figure 83 and Figure 84 shows that the greater value between the mean utility among non-shared (shared) users and the collective average utility of the alternative service is switching every day. Responding to such changes, the proportion of users who changed the service also switches every day between 0 and approximately 0.7. The same observation can be made in the driver sides according to the middle right figure in Figure 83 and Figure 84. During the PP regime, the fleet size is always overly large or small compared to the number of users.

When the fleet size is overly large compared to the number of users, the mean profit of drivers becomes much lower than the collective average profit of the alternative service. When the fleet size is overly small compared to the number of users, drivers' mean profit becomes much higher than the other two regimes (see the middle right figure in Figure 76, Figure 77, Figure 79, and Figure 80). As a result of the extreme shift between the mode share and especially the proportion of fleet for each service, the difference between the mean utility and profit of one service and the collective average utility and profit of the alternative service is kept large. Therefore, the 'all-or-nothing' type of behaviour continues during the PP regime.

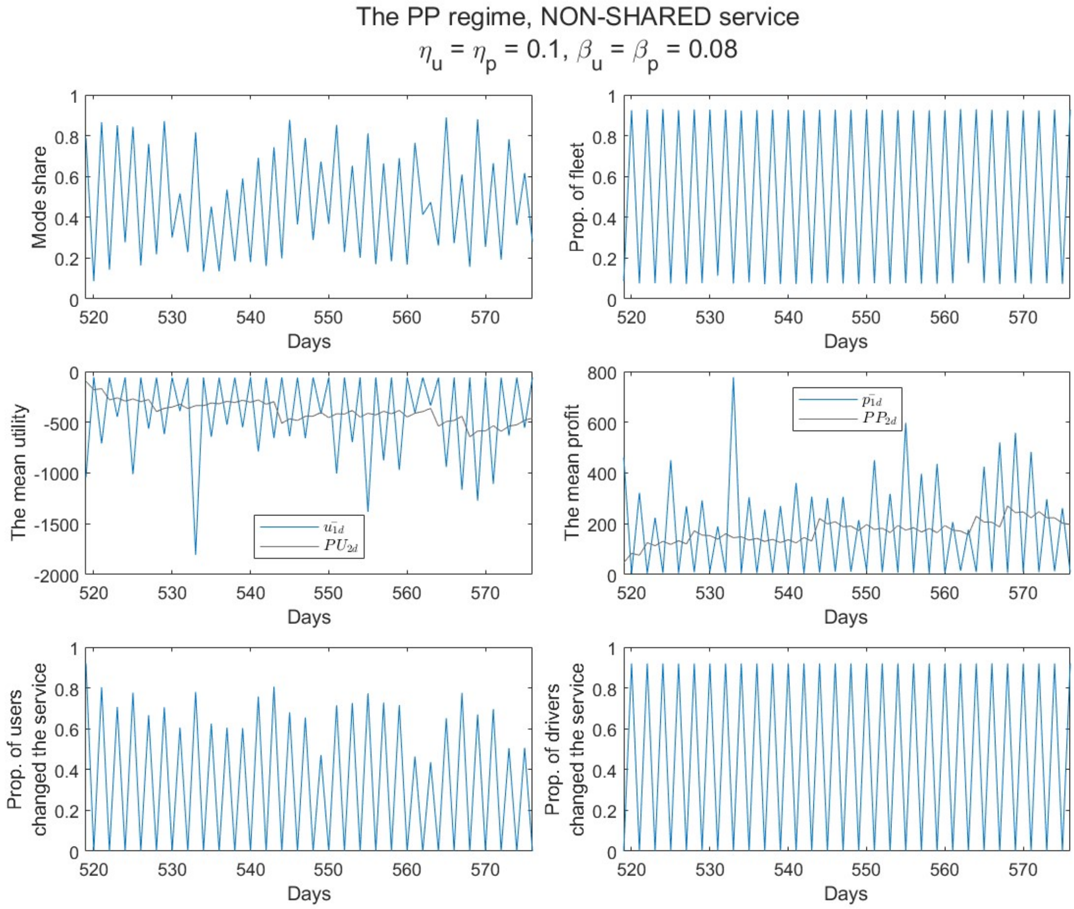

Figure 83 an example of the pseudo periodic regime from day 519 to 576 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day.

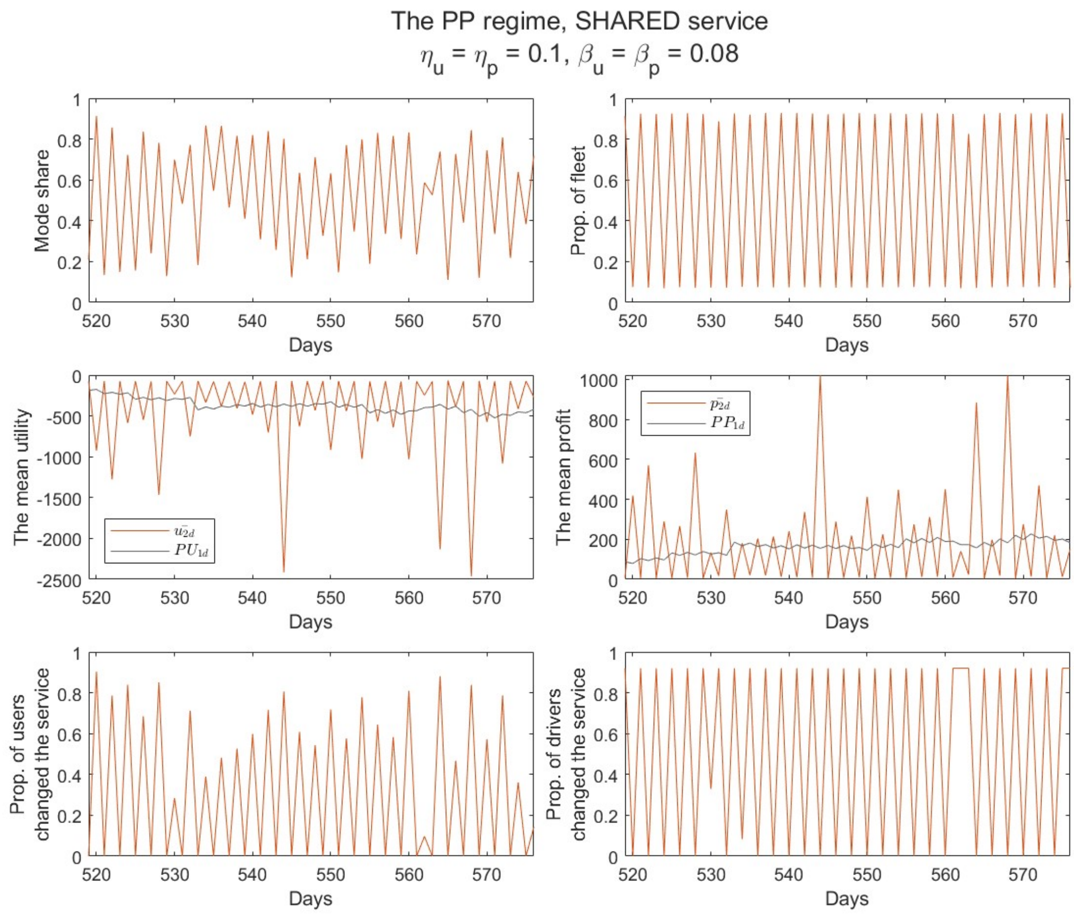

Figure 84 an example of the pseudo periodic regime from day 519 to 576 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

The below section summarises the attribute of the PP regime in one realisation with $\beta_u = \beta_p = 0.08$ and $\eta_u = \eta_p = 0.1$, which was simulated for 100,000 days. During that time, the PP regime appeared 2,002 times. Figure 85 illustrates the distribution of the PP regime length. The mean length of the PP regime was 30.01 days, and the maximum length was 136 days. Figure 86 shows the distribution of mode share (left) and the proportion of fleet (right) for non-shared service during one PP regime from day 99,230 to 99,325. As a result of continuous "flip-flop" changes in the mode share and the proportion of fleet, the distribution of mode share and proportion of fleet showed two peaks (see Figure 86). This tendency was consistent with the other PP regime observed during the same realisation.

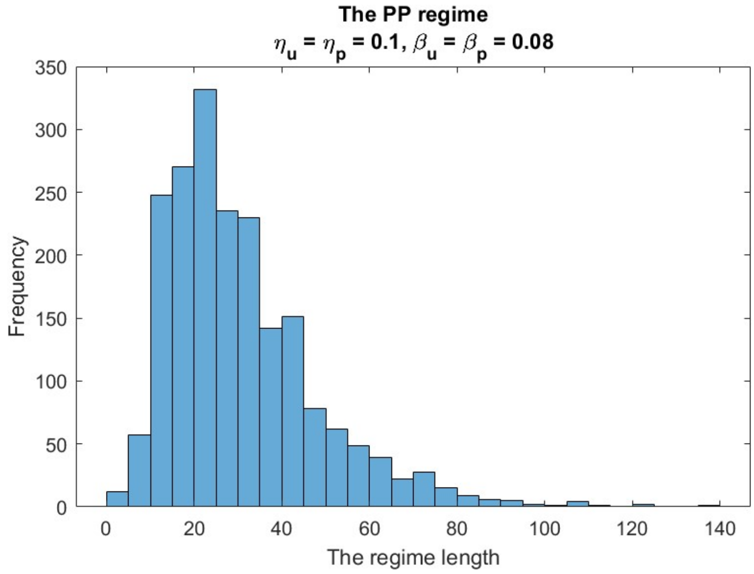

Figure 85 the histogram of the PP regime length appeared during 100,000 days of one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$

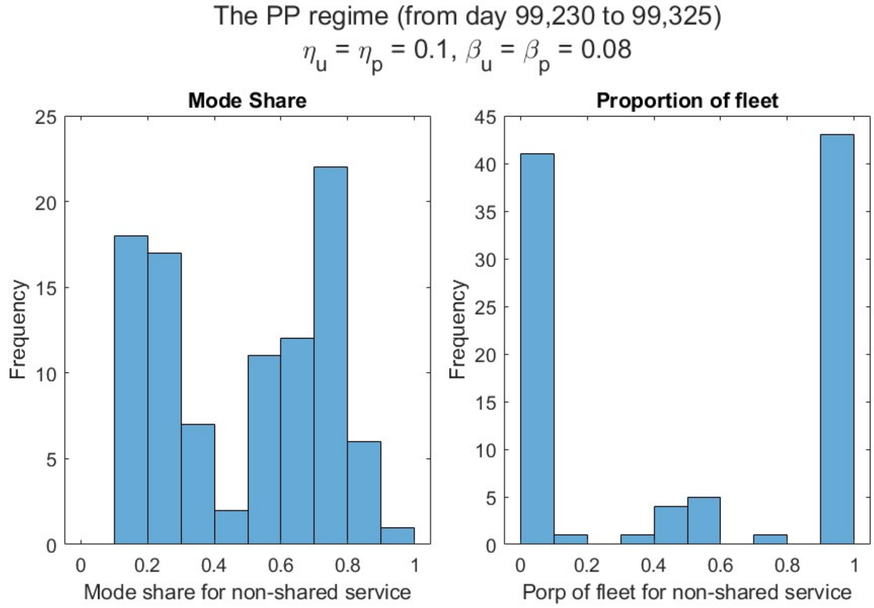

Figure 86 the histogram for the mode share (left) and the proportion of fleet (right) for non-shared service during one PP regime from day 99,230 to 99,325 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$.

5.5 The transition between each regime

In this section, how the process moves from one regime to the other regime is investigated. In particular, the mean experienced utility of users and drivers and the collective average utility of users and drivers for both services are analysed during the transition. Mainly, the one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ is used to show the transition, which was also used to show the example in the previous section. However, there is a specific transition that is not observed with those parameter settings. In that case, one realisation with different combinations of η_u, η_p, β_u and β_p are presented. The following sections are divided into three subsections which summarise 1) the transition between the pseudo periodic (PP) regime and the swan regime, 2) the transition between the swan regime and the pseudo stable (PS) regime, and 3) the transition between the PP regime and the PS regime.

5.5.1 **The transition between the pseudo periodic regime and the swan regime**

Figure 87 and Figure 88 summarise changes in several variables related to non-sharing service and shared service during the transition from/to the PP regime to/from the swan regime. Those variables are the same as those summarised in Figure 76 and Figure 77 presented in section 5.4. Hence, for the description of each variable in Figure 87 and Figure 88, the description of variables in Figure 7 and Figure 8 should be referred which is presented in subsection 5.4.2.

Comparing Figure 87 and Figure 88, it can be observed that the relation between the mean utility of one service and the collective average utility of the alternative service is always opposite during the PP regime. In specific, when the mean utility of *non-shared services* is higher than the collective average utility of *shared services* (i.e. $\bar{u}_{1,d} > PU_{2,d}$), it is the other way around for *shared service* (i.e. $\bar{u}_{2,d} < PU_{1,d}$) and vice versa. The same logic applys for the relationship between the mean profit of one service and the collective average profit of the alternative service.

The transition from the PP regime to the swan regime occurs when the mean utility of both services becomes higher than the collective average of the alternative service on the same day. According to the middle figures in Figure 87 and Figure 88, those conditions had met on day 240 for this case. On that day, the mean utility and the collective average utility would have been $\bar{u}_{1,d} > PU_{2,d}$ and $\bar{u}_{2,d} < PU_{1,d}$ if it were still in the PP regime. However, on the day 235, the mean utility of non-shared service experienced a sharp decline (see the left middle figure in Figure 87). Therefore, the collective average utility for non-shared also decreased by a great amount (see the black line in the left middle figure in Figure 88). Following another sharp decline in the mean utility of non-shared service on day 237, the mean utility of shared service exceeded the collective average utility, $\bar{u}_{2,d} > PU_{1,d}$, on day 240.

The sharp decline in the mean utility for non-shared service on day 235 is the consequence of random events. As shown in Table 19, the proportion of fleet for non-shared service was lower than the other 10 days in the table. However, the mode share for non-shared service was not so high compared to the other days.

However, on day 235, the randomly generated total number of users (i.e. 253) was higher than the expected value (i.e.240), and the randomly generated total fleet size (i.e. 93) was lower than the expected value (i.e. 100) at the same time. As a result, the number of users for non-shared service (i.e. 143) became too high compared to the fleet size for non-shared service (i.e. 2). Then, the mean utility for non-shared services dropped significantly. The changes in the process resulting from the random event were also observed in the fixed fleet size case presented in Chapter 4 (see subsection 4.3.2 in Chapter 4). However, the fixed capacity of the service prevented a process from changing its behaviour. Thus, it is highly likely that the different regime emerged as the limitation in the service capacity is relaxed with variable fleet size.

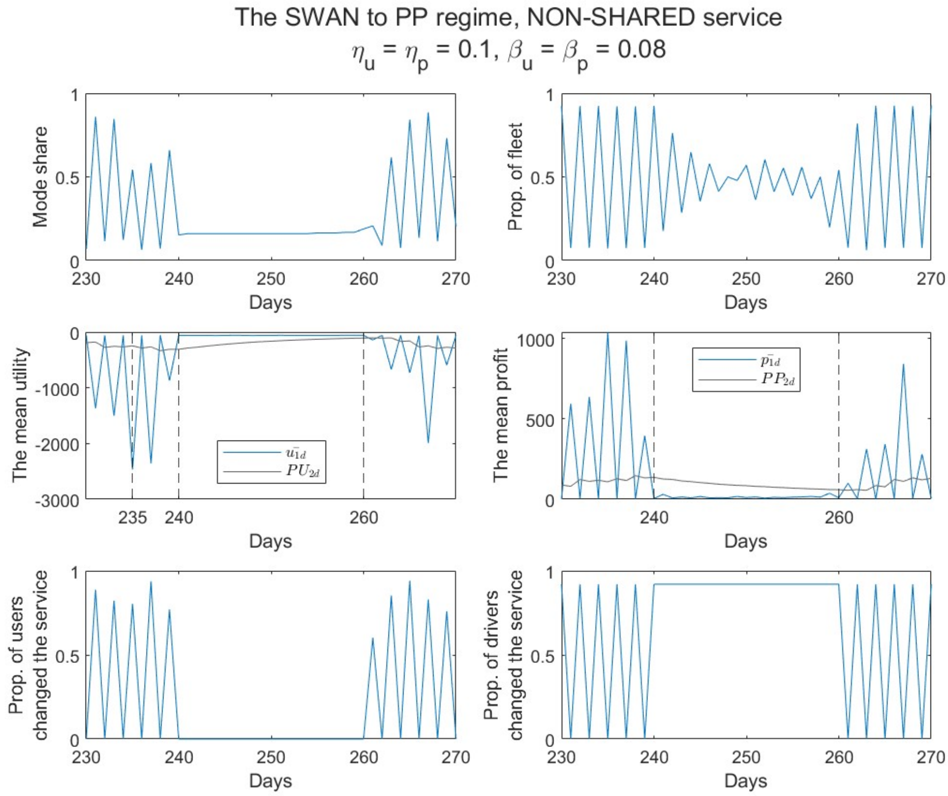

Figure 87 an example of the transition between the pp regime and the swan regime from day 230 to day 270 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed a service on each day.

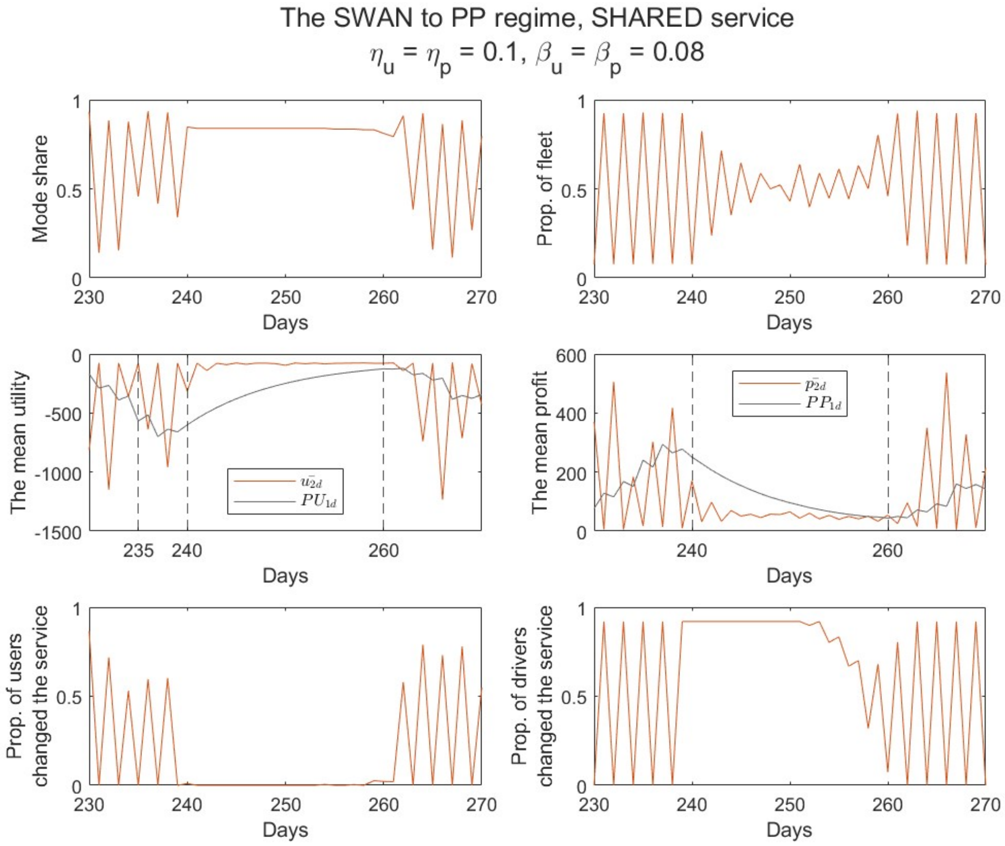

Figure 88 an example of the transition between the pp regime and the swan regime from day 230 to day 270 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

Table 19 the changes in the total number of users and total fleet size as well as mode share, the proportion of fleet, No. of users, fleet size for NON-SHARED service from day 230 to 240.

Days	Total no. of users	Total fleet size	Non-shared service			
			Mode share	Prop. of fleet	No.of users	Fleet size
230	233	98	0.0687	0.9241	14	92
231	240	96	0.8590	0.0751	201	5
232	274	97	0.1159	0.9242	30	92
233	249	102	0.8459	0.0759	211	5
234	209	102	0.1230	0.9239	19	91
235	253	93	0.5434	0.0714	143	2
236	226	96	0.0644	0.9217	17	89
237	227	94	0.5833	0.0742	131	2
238	245	100	0.0712	0.9217	10	94
239	223	102	0.6603	0.0752	158	6
240	227	104	0.1528	0.9247	31	92

As described in subsection 5.4.3, the difference between the mean utility (profit) and the collective average utility (profit) decreases during the swan regime. Hence, the transition from the swan regime to the other regime always occur. In this example, it shifted to the PP regime most of the time, which was consistent with a realisation with the different combination of updating filters and hesitation parameters. The transition from the swan regime to the PP regime follows the two steps.

First, the swan regime ends and the mode share for non-shared service begins to increase. It is because the shared service users start to change the service to the non-shared service as the difference between the mean utility among shared service users and the collective average utility of non-shared service (i.e. $|\bar{u}_{2,d} - PU_{1,d}|$) becomes small enough. As Figure 89 presents, the difference between $\bar{u}_{1,d}$ and $PU_{2,d}$ is smaller than the difference between $\bar{u}_{2,d}$ and $PU_{1,d}$ at the beginning of the swan regime. However, as the mean utility for non-shared service is constantly higher than the shared service as presented in Figure 90, the collective average utility for *non-shared service* increase at a higher rate than the collective average utility for *shared service.* Consequently, $PU_{1,d}$ beomces too close to $\bar{u}_{2,d}$ at some point before $PU_{2,d}$ becomes too close to $\bar{u}_{1,d}$ which results in some users for shared service to change their service to non-shred service. Hence, the swan regime always ends with an increase in the mode share for non-shared service.

Responding to changes in mode share, the proportion of the fleet starts to fluctuate. While the transition from the PP to the swan regime was triggered by the changes in the relationship between the mean utility and the collective average utility, the transition from the swan to the PP regime was led by the changes in the relationship between the mean profit and the collective average profits. As presented in the middle right figures in Figure 87 and Figure 88, the mean profit of non-shared service exceeded the collective average profit of shared service (i.e. $\bar{p}_{1,d} > PP_{2,d}$) on day 260 and when $\bar{p}_{2,d} < PP_{1,d}$. On the next day, the relationship becomes the other way around, namely, $\bar{p}_{1,d} < PP_{2,d}$ and $\bar{p}_{2,d} < PP_{1,d}$. While this flip-floping continues, it became $\bar{u}_{1,d} < PU_{2,d}$ and $\bar{u}_{2,d} > PU_{1,d}$ on 261. Then, the process entered to the PP regime.

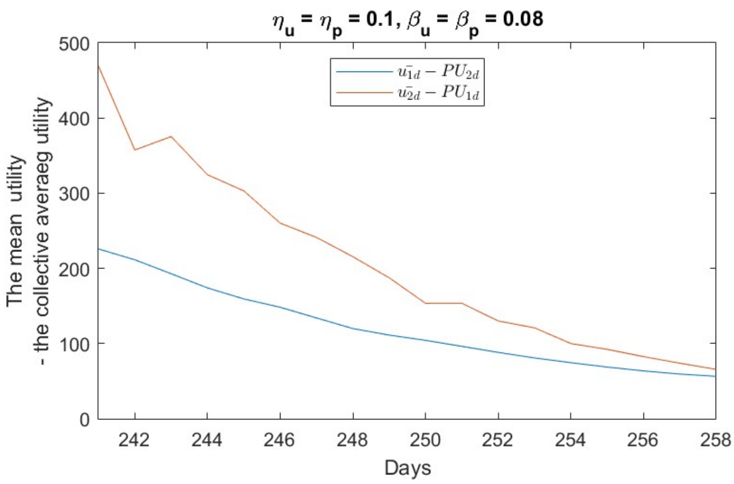

Figure 89 the changes in the difference between the mean utility for non-shared (shared) service and the collective average utility for the alternative service.

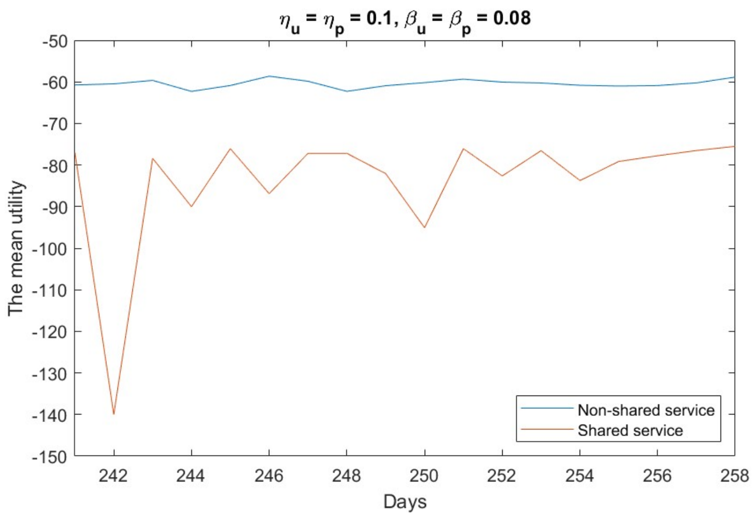

Figure 90 the difference between the mean utility for non-shared service and shared service during the swan regime

5.5.2 **The transition between the swan regime and the pseudo stable regime**

The PS regime was observed 4 times in an example realisation. For all cases, the transition occurs from the swan regime to the PS regime. Figure 91 and Figure 91Figure 92 present the changes in the same set of variables with Figure 87 and Figure 88 but from day 16,950 to day 17,000, which includes one of four transitions from the swan regime to the PS regime. In this realisation, the swan regime was observed 2,173 times, among which the swan regime shifted to the PS regime only 4 times and to the PP regime for the rest of the cases. Those 4 times are the only times the PS regime appeared in this realisation, and the case shown in Figure 91 and Figure 92 is the first PS regime that appeared in this realisation.

It should be noted that though the PS regime first occurred on day 16,970 in this realisation, it could and would appear on much earlier days (e.g. day 200) for some of the other realisations. As mentioned in the previous paragraph, the process does not enter the PS regime as often as the PP regime and swan regime with the current parameter settings. In particular, the transition from the swan to the PP regime occurred 543.25 times as much as the transition from the swan to PS regime in this realisation. As the PS regime occurs very rarely, the first appearance of the regime happened to be very late in this realisation. The likelihood of the PS regime's occurrence would not vary among multiple realisations with the same parameter settings. However, it would change as the parameter settings changes, which were investigated and are summarised in section 5.7.

When the transition from the swan regime to the PP regime occurs, the relationship between the mean profit of non-shared (shared) service and the alternative service changed to $\bar{p}_{1,d} > PP_{2,d}$ and $\bar{p}_{2,d} < PP_{1,d}$ from $\bar{p}_{1,d} < PP_{2,d}$ and $\bar{p}_{2,d} < PP_{1,d}$. The same phenomenon happened in the process of transition from the swan regime to the PS regime on day 16,970, as presented in the middle left figures in Figure 91 and Figure 92. However, in the following days, those relationships remained the same instead of shifted to the other way around. As a result, the proportion of fleet for non-shared service continued to increase as the mode share for non-shared service increases. Consequently, the process shifted to the PS regime.

On the other hand, the transition from the PS regime to the swan regime was not observed in this realisation and any other realisation. As described in subsection 5.4.3, the condition for the process to be in the swan regime is $\bar{u}_{1,d} > PU_{2,d}$ and $\bar{u}_{2,d} > PU_{1,d}$. During the PS regime, the mean utility for non-shared service is constantly higher than the collective average utility for shared service (i.e. $\bar{u}_{1,d} > PU_{2,d}$) as described in subsection 5.4.2. On the other hand, the mean utility for shared service is lower than the collective average utility for non-shared service most of the time (i.e. $\bar{u}_{2,d} < PU_{1,d}$).

In order for the PS reimage to enter the swan regime, the mean utility for *shared service* needs to be higher than the collective average utility for *non-shared service*. However, the improvement of the mean utility for shared service would not happen without the deterioration of the mean utility for non-shared service. As both mean utility needs to be higher than the collective average of the alternative service (i.e. $\bar{u}_{1,d} > PU_{2,d}$ and $\bar{u}_{2,d} > PU_{1,d}$). Thus, it is highly likely that the transition from the PS reimage to the swan regime would never occur with the current parameter settings.

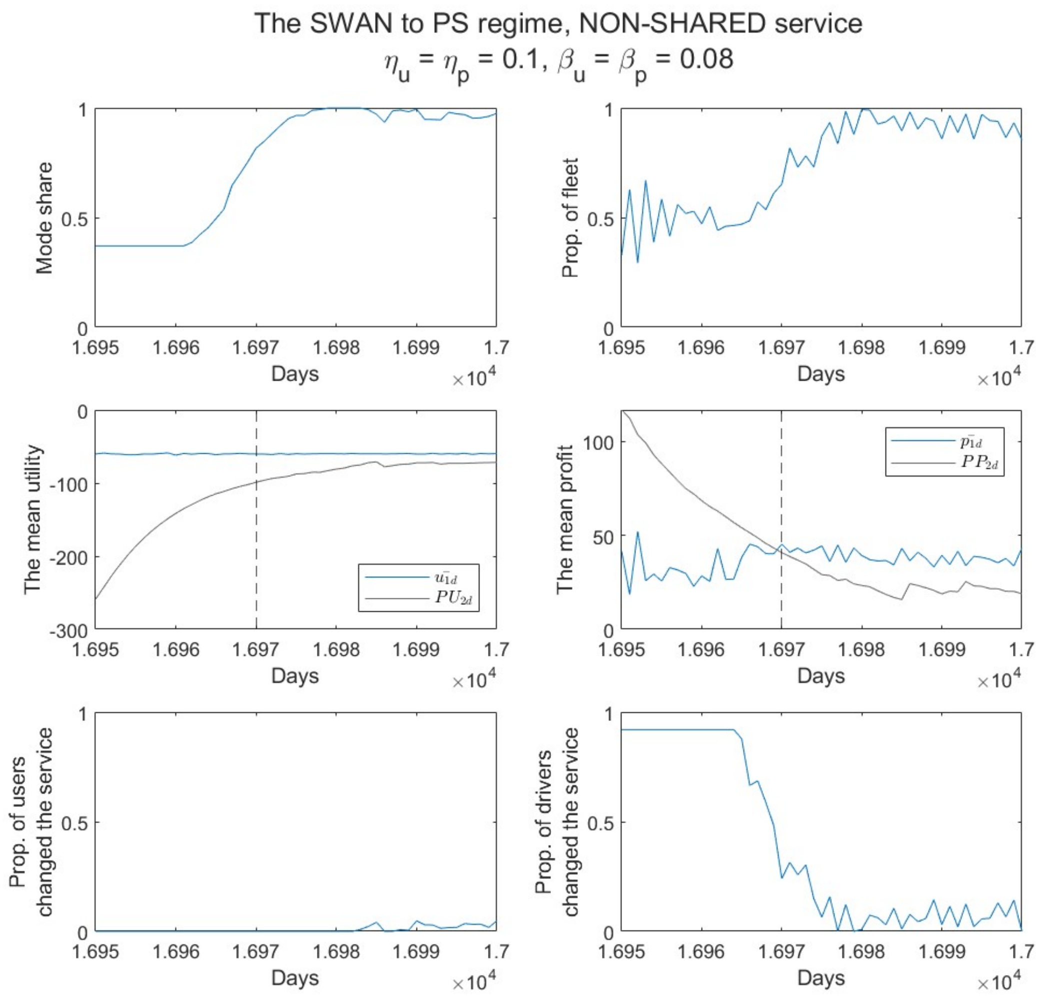

Figure 91 an example of the transition from the swan regime and the PS regime from day 16,950 to day 17,000 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed a service on each day.

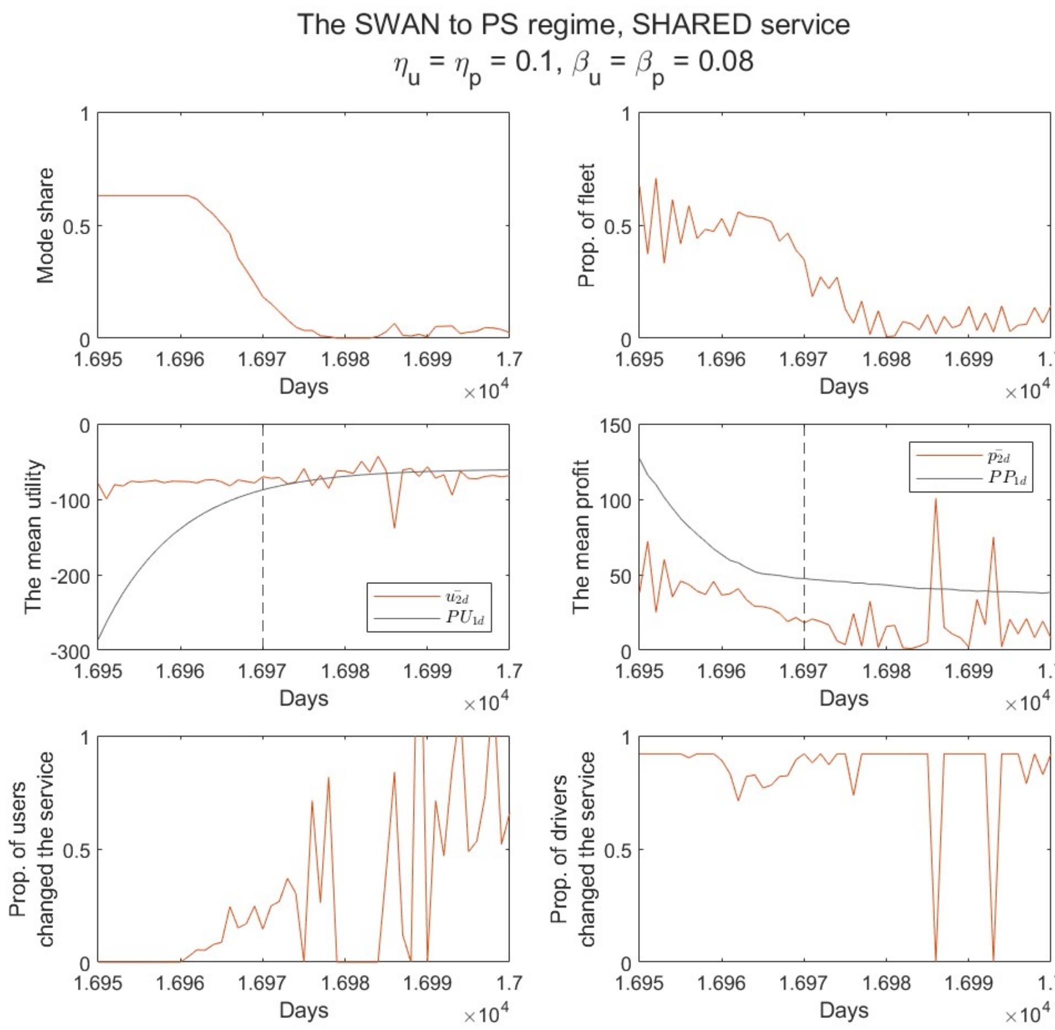

Figure 92 an example of the transition from the swan regime and the PS regime from day 16,950 to day 17,000 when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

5.5.3 The transition between the pseudo stable regime and the pseudo periodic regime

In subsection 5.5.1 and 5.5.2, the one realisation with $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$ are used to describe the transition between two regimes. However, the transition from PP to PS has not occurred during 100,000 days. Hence, this subsection uses a realisation with different parameter settings, which are $\eta_u = \eta_p = 0.2$ and $\beta_u = \beta_p = 0.3$. The evolution of mode share and the proportion of fleet for the realisation is presented in Figure 93 as well as the stationary distribution of mode share and the proportion of fleet for non-shared service. According to Figure 93, it is observed that this process consists of two regimes, the PS regime and the PP regime.

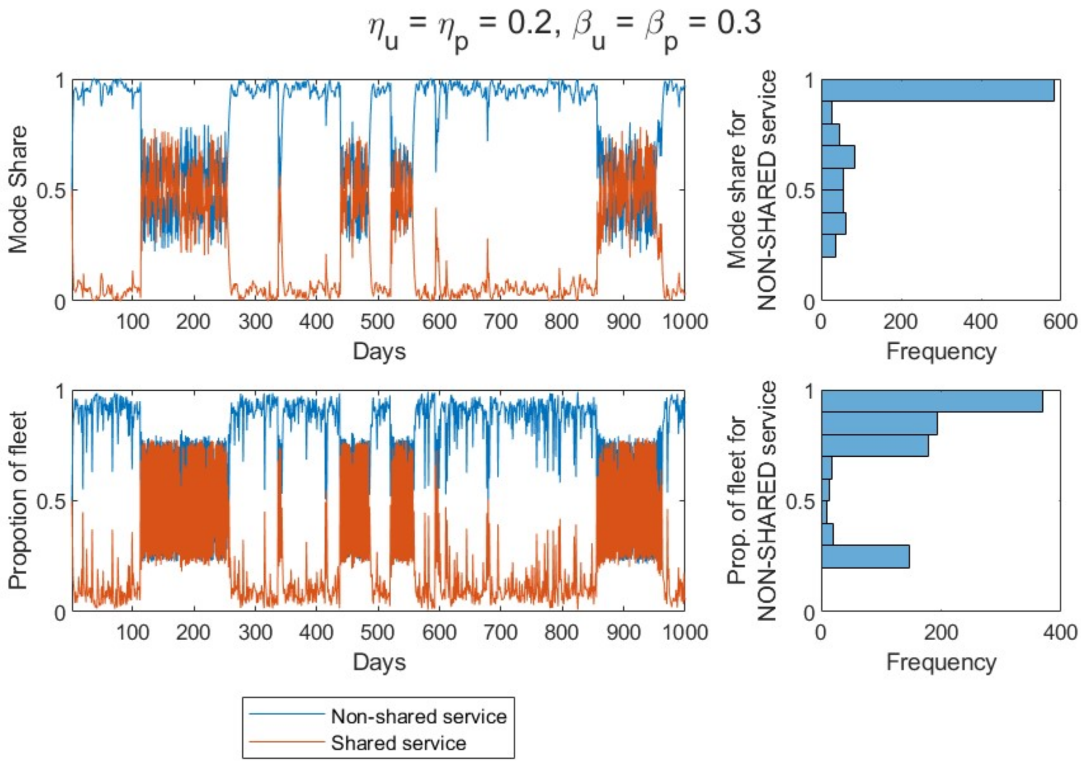

Figure 93 the evolution of mode share (top left), the proportion of fleet (bottom left), the stationary distribution of mode share (top right) and the proportion of fleet for non-shared service (bottom right) of one realisation with $\boldsymbol{\eta_u = \eta_p = 0.2}$ and $\boldsymbol{\beta_u = \beta_p = 0.3}$ for 1000 days

Figure 94 and Figure 95 summarise the same set of values with Figure 91 and Figure 92 from day 470 to 530 of the realisation presented in Figure 93, where the process moves from the PP regime to the PS regime to the PP regime. The transition from the PP regime to the PS regimes start with the relationship between the mean utility of non-shared and shared service and the collective average of the alternative service being $\bar{u}_{1,d} > PU_{2,d}$ and $\bar{u}_{2,d} > PU_{1,d}$ on the same day. In Figure 94 and Figure 95, it happened on day 481. The cause of this was the two sharp declines in the mean utility for non-shared service on day 478 and 480. It decreases the collective average utility for non-shared service on day 479 and 480. Consequently, the mean utility for shared service did not become lower than the collective average utility for shared service on day 481. If it were still in the PP regime, it would have been $\bar{u}_{2,d} < PU_{1,d}$.

After the abovementioned condition, $\bar{u}_{1,d} > PU_{2,d}$ and $\bar{u}_{2,d} > PU_{1,d}$, was achieved, the mode share for non-shared service started increasing continuously. The logic behind this phenomenon is equivalent to how the swan regime ends with the increases in the mode share for non-shared service, which is discussed in subsection 5.5.1. As $|\bar{u}_{1,d} - PU_{2,d}|$ is constantly greater than $|\bar{u}_{2,d} - PU_{1,d}|$, the more users unsatisfied with using shared services and, therefore, change to the non-shared service. At some point, the mean utility for shared service became lower than the collective average utility of non-shared service, $\bar{u}_{2,d} > PU_{1,d}$ while keeping the relationship in the non-shared service side as $\bar{u}_{1,d} > PU_{2,d}$. That satisfied the condition to be in the PS regime on day 491.

From the middle right figures in Figure 94 and Figure 95, it is observed that the driver side was reacting to the user side's change rather than leading the transition since there was no change in trend observed on day 481. However, their reaction to the user side accelerated the transition by creating a virtuous cycle for non-shared services and a vicious cycle for shared services. The increase in the mode share attracted more drivers in non-shared service, which created additional capacity to serve more users without deteriorating the service level. The opposite phenomenon was observed for shared services.

According to top figures in Figure 94 and Figure 95, the process shifted to the PP regime on day 520. On the day before (i.e. day 519), the demand exceeded the capacity for shared services which caused a sharp decline and a sharp increase in the mean utility and profit for shared service. The reaction of the following day was the users' shift to the non-shared service and drivers' shift to the shared service. As a result, the mean utility of non-shared service became lower than the collective average utility of the shared service, $\bar{u}_{1,d} < PU_{2,d}$, while the mean utility of shared service exceeded the collective average utility of non-shared service, $\bar{u}_{2,d} > PU_{1,d}$. Consequently, the process shifted to the PP regime.

As there was no particular change in trend during the PP regime and the PS regime, it can be concluded that the transition between the PP regime and the PS regimes is led by the series of random events described above.

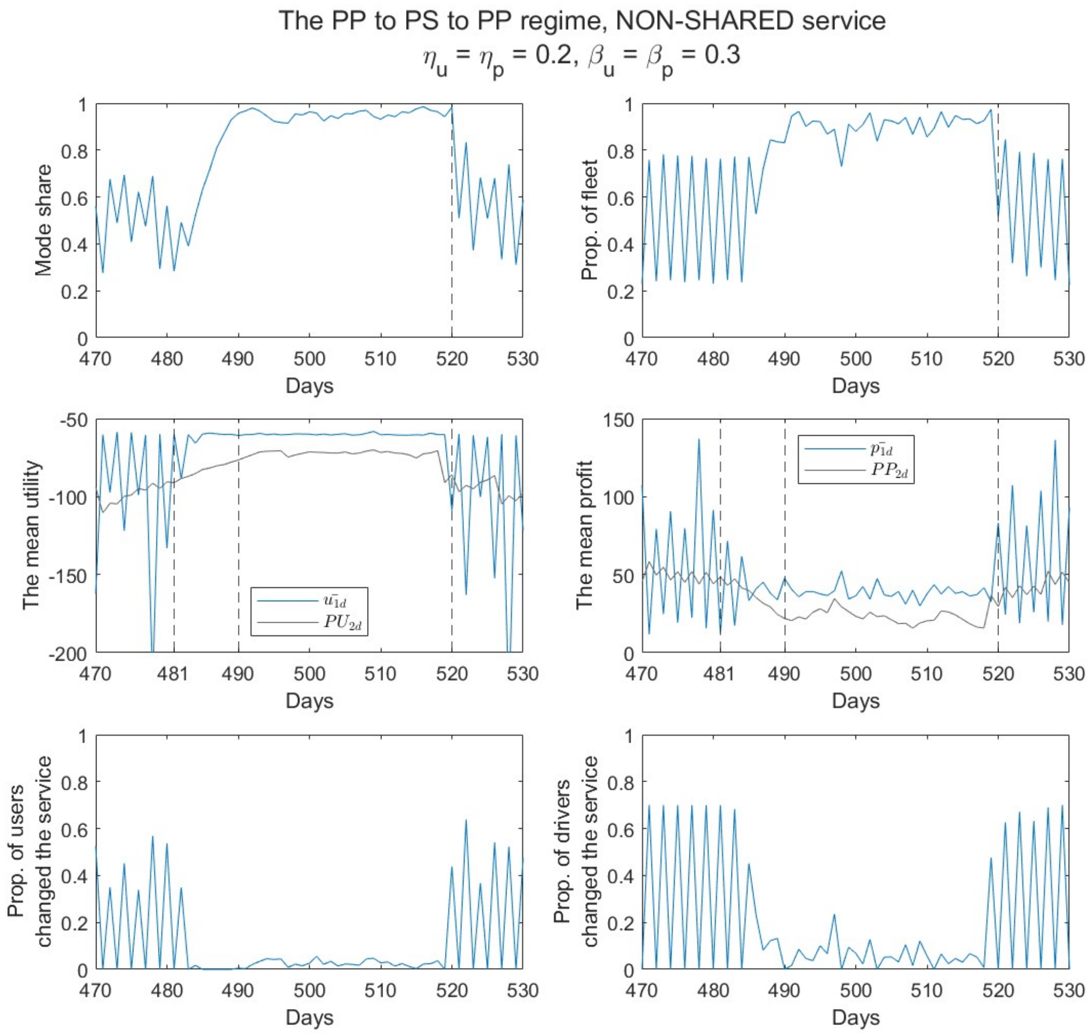

Figure 94 an example of the transition between the PP regime and the PS regime from day 450 to day 500 when $\eta_u = \eta_p = 0.2$ and $\beta_u = \beta_p = 0.3$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day.

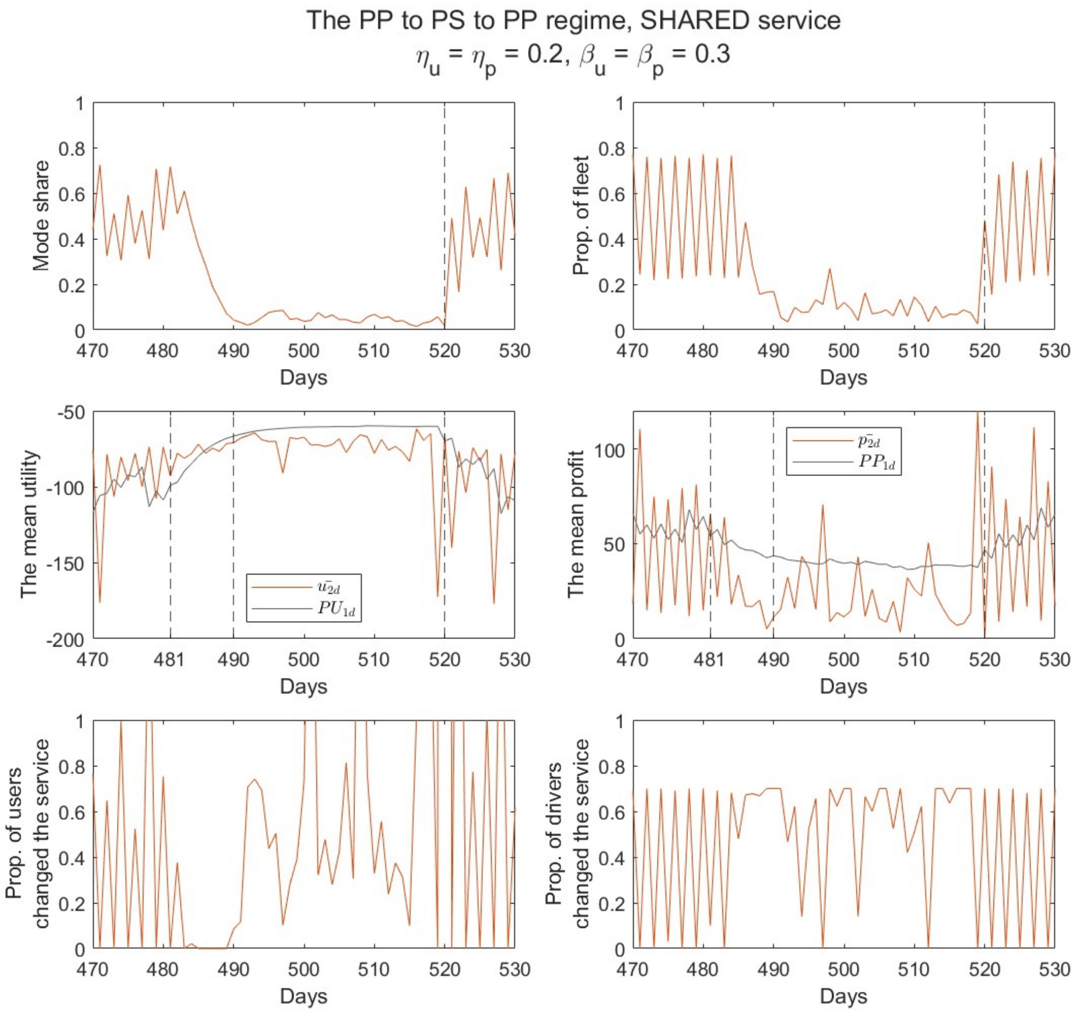

Figure 95 an example of the transition between the PP regime and the PS regime from day 450 to day 500 when $\eta_u = \eta_p = 0.2$ and $\beta_u = \beta_p = 0.3$. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

5.6 Impact of initial conditions

This subsection summarises the results of the sensitivity analysis against initial conditions. There are four initial conditions: the initial mode share for the non-shared service and shared service and the initial proportion of fleet for the non-shared service and shared service. In subsection 5.6.1, the scenario settings for the sensitivity analysis is summarised. In subsection 5.6.2, the results of the analysis are presented.

5.6.1 Scenario settings

There are two scenarios examined with the different parameter settings for the updating filters (i.e. η_u, η_p) and the hesitation parameters (i.e. β_u, β_p). Table 20 specifies the values for those parameters for each scenario. For each scenario, 110 cases with the different combinations of initial mode share and the proportion of fleet for non-shared service are generated where the initial mode share from 0 to 1 increased in increments of 0.1 and the initial proportion of fleet from 0.1 to 1 increased in increments of 0.1. 10,000 consecutive days are generated for each realisation of the process. The other parameters follow the default value specified in subsection 5.2.

Table 20 the values of η_u, η_p, β_u and β_p for each scenario

	$\eta_u = \eta_p$	$\beta_u = \beta_p$
Scenario 1	0.1	0.08
Scenario 2	0.3	0.5

5.6.2 **Results**

The results from scenario 1 suggested that there is no distinctive change observed in the distribution of mode share and the proportion of fleet in relation to the changes in the initial condition. Figure 96 and Figure 97 display the 11 cases with different initial mode share for non-shared service from 0 to 1 when the initial proportion of fleet for non-shared service is 1 for scenario 1. In Figure 96, three peaks consistently observed in the distribution of mode share; around the mode share with 0.15 to 0.13, 0.39 to 0.42, and around 0.75 to 0.78.

As it can be observed in Figure 73, when the updating filters and hesitation parameters are set as $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$, a realisatioan of the process could contain three regimes. Nevertheless, it mainly consists of the swan regime and the PP regime, and the PS regime barely occurs. Therefore, stationary distributions for mode share for non-shared service summarised in Figure 95 are mainly a mixture of two distributions during the swan regime (see Figure 82) and during the PP regime (see Figure 86). The same observation can be applied for the stationary distributions of proportion of fleet for non-shared service presented in Figure 96.

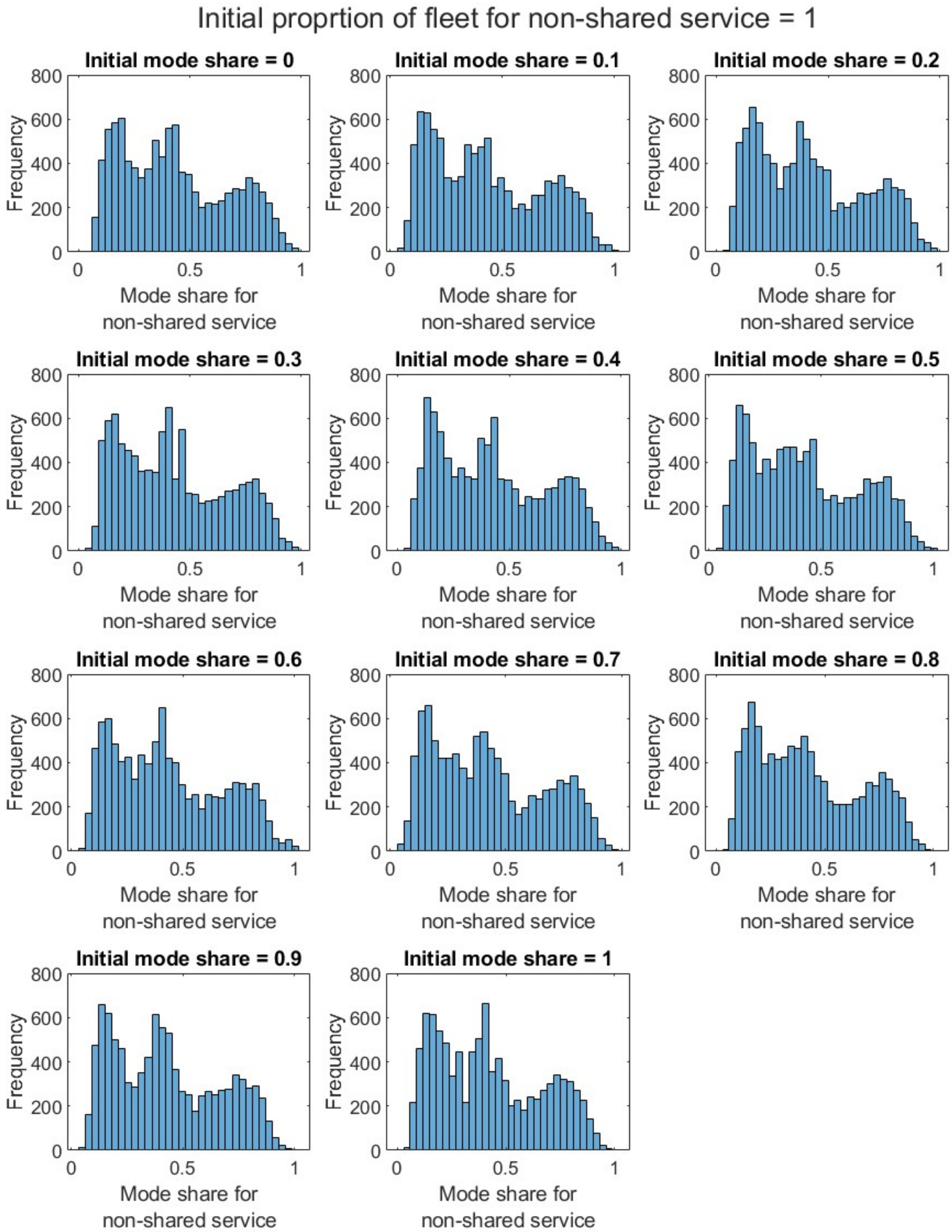

Figure 96 distribution of mode share for non-shared service for 11 cases with different initial mode share for non-shared service from 0 to 1 when the initial proportion of fleet for non-shared service is 1 for scenario 1

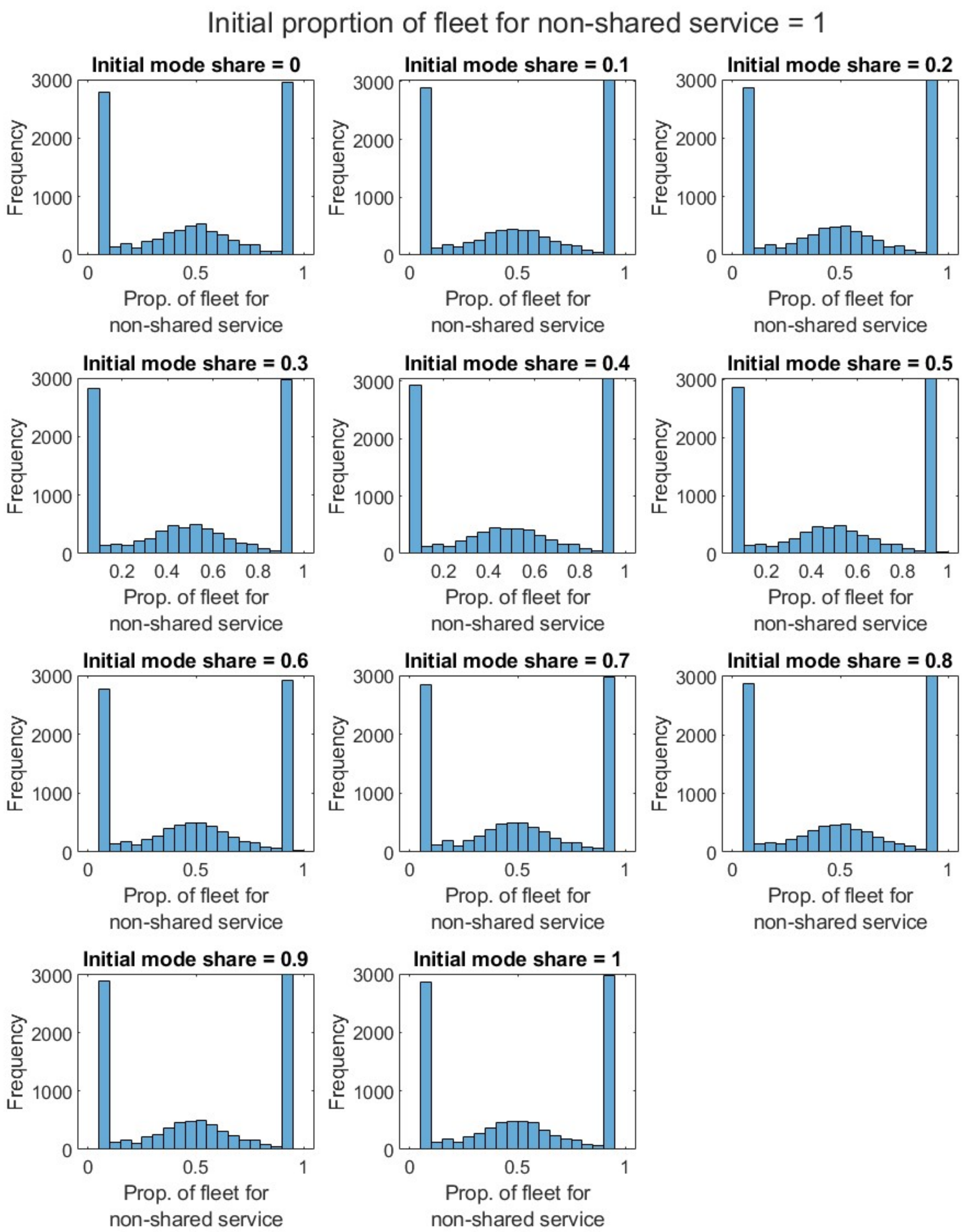

Figure 97 distributions of the proportion of fleet for non-shared service for 11 cases with different initial mode share for non-shared service from 0 to 1 when the initial proportion of fleet for non-shared service is 1 for scenario 1.

For all 110 cases, the mean value for the stationary distribution of mode share and proportion of fleet was estimated, and the results are summarised in Figure 98. As shown in Figure 98, the mean mode share varied about 0.2 during 110 cases though the mean proportion of fleet varied only around 0.08. There was no correlation observed between the mean value for the stationary distribution of mode share and the proportion of fleet with the initial mode share or proportion of fleet.

Instead, a clear positive (negative) correlation was observed between the proportion of days in the PS regime (the swan regime) and the mean mode share (see Figure 99). It should be reminded that, as the proportion of days in the PS regime increases, the proportion of days in the swan regime decreases and vice versa. The distribution of the regime length and mode share during the swan regime do not change as the initial condition changes. The same thing could be applied to the PP regime. Hence, it is highly likely that the mean mode share of one realisation of the process varies only based on the total length of each regime and not based on the initial condition.

In order to check the above hypothesis, 110 realisations were generated with $\eta_u = \eta_p = 0.3$ and $\beta_u = \beta_p = 0.05$ with a different set of initial conditions for scenario 2. With these parameter settings, there is only the PS regime appears as presented in Figure 71. It indicates that the stationary distribution of mode share and the proportion of fleet would not be affected by the proportion of total days for each regime. Figure 100 shows the histogram of the mean mode share (left) and the mean proportion of fleet (right) for non-shared service estimated for 110 cases with different initial condition.

Compared to Figure 98, the mean mode share for non-shared service is observed to be less variable for scenario 2 than scenario 1. Table 21 summarises the mean and standard deviation of 110 sets of mean mode share and proportion of fleet with a different initial condition for scenario 1 and scenario 2. According to Table 21, the standard deviation of the mean mode share and the proportion of fleet for scenario 2 is approximately ten times lower than scenario 1. These results further support the hypothesis that initial conditions do not affect the stationary distribution of mode share and proportion of fleet.

As discussed in subsection 4.3.3.1 in Chapter 4, if a stochastic process has ergodicity, a stationarity distribution is not influenced by initial conditions. In the case of fixed fleet size, the sensitivity analysis against the initial condition showed strong evidence for independence of a stationarity distribution from the initial condition. The experiment above suggested that the variance in the stationary distribution of the mode share and the proportion of fleet attributes to the proportion of days for a process to be in each regime. Besides, any correlation between the initial conditions and the changes in a stationary distribution was not observed. Hence, in this study, it is concluded that a stochastic process with the variable fleet size also has ergodicity with given parameter settings.

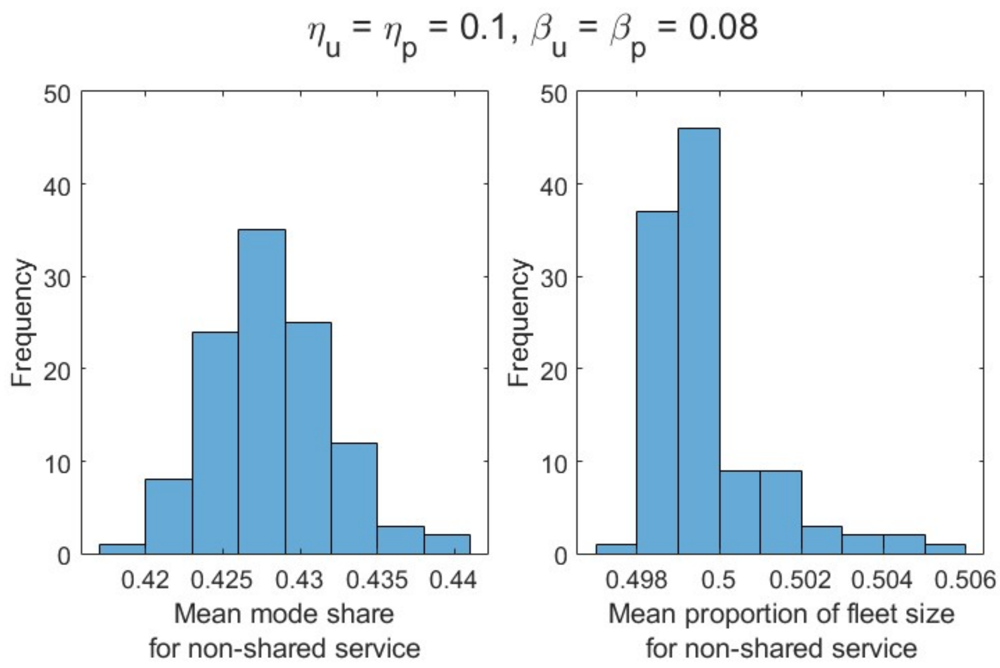

Figure 98 the histogram for the mean mode share for non-shared service from day 51 to 10,000for 110 cases with different initial conditions (the left figure) and the mean proportion of fleet for non-shared service from day 51 to 10,000 for 110 cases with different initial conditions (the right figure) when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$.

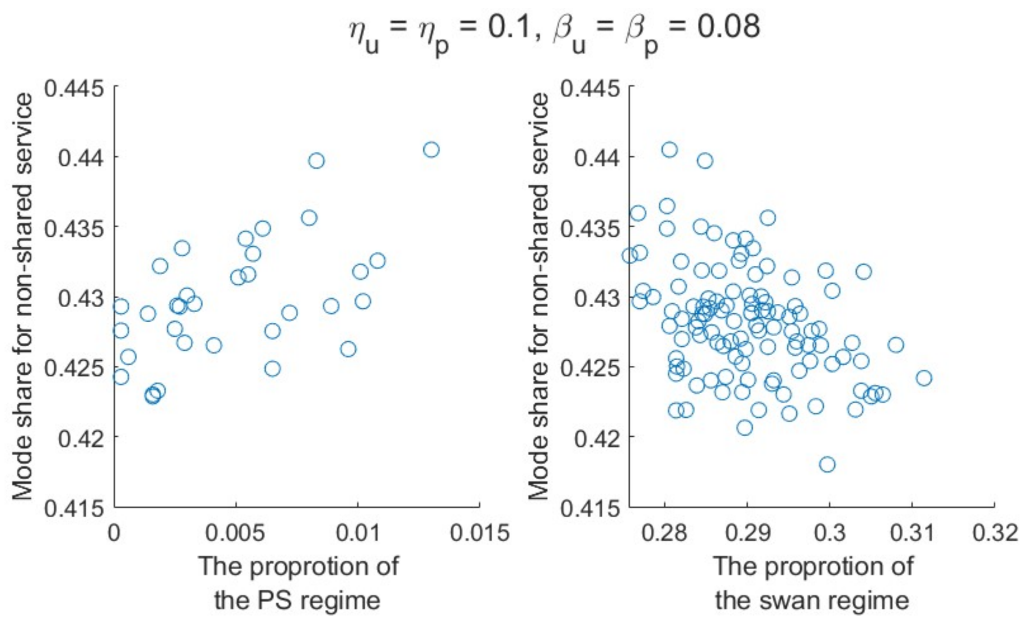

Figure 99 the mean mode share for non-shared service and the proportion of total days in the PS regime (left) and in the swan regime (right). In the left figure, only the realisation, including the PS regime, is presented (i.e.e 32 cases). In the right figure, all 110 cases are presented.

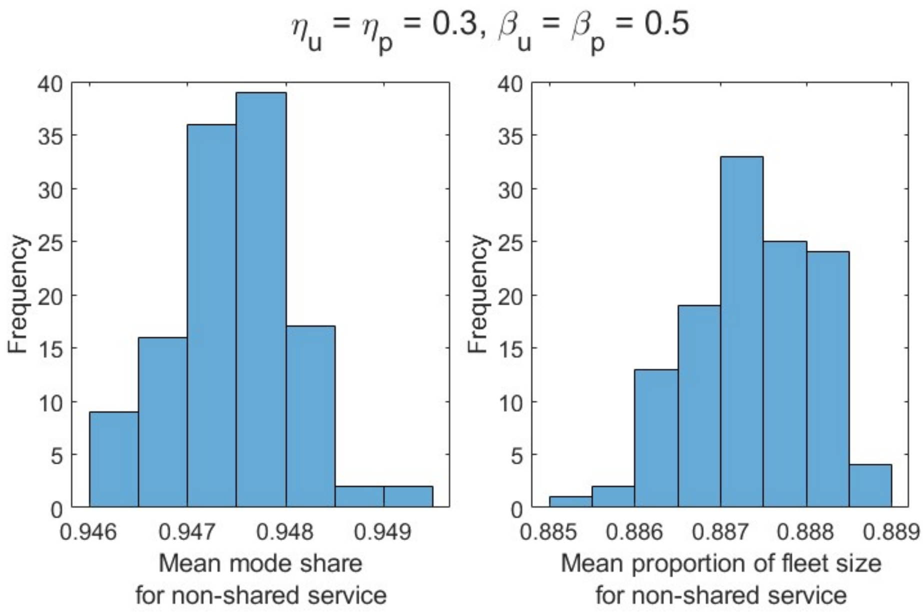

Figure 100 the histogram of the mean mode share for non-shared service from day 51 to day 10,000 for 110 cases with different initial conditions (the left figure) and the mean proportion of fleet for non-shared service from day 51 to day 10,000 for 110 cases with different initial conditions (the right figure) when $\eta_u = \eta_p = 0.3$ and $\beta_u = \beta_p = 0.5$.

Table 21 Comparison of the mean and standard deviation of the mean mode share and the proportion of fleet for non-shared service among 110 cases with the different initial conditions for scenario 1 and 2.

	Mean Mode share for non-shared service		Mean Proportion of fleet for non-shared service	
	Mean	Std	Mean	Std
Scenario 1	0.4281	0.0040	0.4997	0.0014
Scenario 2	0.9475	0.0006	0.8874	0.0007

5.7 Sensitivity analysis

This section summarises the results of the sensitivity analysis against updating filters (i.e. η_u, η_p) and hesitation parameters (i.e. β_u, β_p). 100 realisations of the process with different combinations of those parameters were generated for 1000 days. Updating filters for the collective average utility, η_u, and the collective average profit, η_p, are set to be equivalent. 10 values for $\eta_u = \eta_p$ from 0.1 to 1 increased in increments of 0.1 are investigated. It should be reminded that updating filter controls how much the mean utility (profit) among users (drivers) of the current day contributes to updating the collective average utility (profit). Hence, $\eta_u = \eta_p = 0$

indicates that the collective average utility and profit are not updated and are not included.

Hesitation parameters for users and drivers are set to be equal. It indicates that the same proportion of users and drivers do not change their service the next day even if they did not satisfy with their experience on the current day. 10 values for $\beta_u = \beta_p$ from 0 to 0.9 increased in increments of 0.1 are assessed. $\beta_u = \beta_p = 1$ indicates that no one will change their service regardless of their experience; hence, it is not included. Other parameters are set as default value specified in subsection 5.2.

For the analysis of the swan regime, the additional simulations were conducted with the smaller interval between values. In specific, 1260 combinations of $\eta_u = \eta_p$ and $\beta_u = \beta_p$ are tested from 0.1 to 0.6 increased in increments of 0.01 and from 0 to 0.2 increases in increments of 0.01, respectively. Table 22 summarises the parameter settings for $\eta_u = \eta_p$ and $\beta_u = \beta_p$. Unlike the PP regime and the PS regime, the swan regime occurs only in the limited values for $\eta_u = \eta_p$ and $\beta_u = \beta_p$. Thus, the further analysis within the limited range was conducted. The below sections summarise the results of the analysis.

Table 22 the parameter settings for $\boldsymbol{\eta_u = \eta_p}$ and $\boldsymbol{\beta_u = \beta_p}$.

	$\eta_u = \eta_p$	$\beta_u = \beta_p$
For all three regimes	0.1 to 1 (up by 0.1)	0 to 0.9 (up by 0.1)
Additional for the swan regime	0.1 to 0.6 (up by 0.01)	0 to 0.2 (up by 0.01)

5.7.1 **Results**

Figure 101 compares the proportion of total days for a process to be in the PS regime (top left), the swan regime (top right) and the PP regime (bottom left). The x-axis shows the hesitation parameter (i.e. $\beta_u = \beta_p$) while the y-axix shows the updating filter (i.e. $\eta_u = \eta_p$). As Figure 101 is intended to compare the area for each regime to appeared among three regimes, the value for each level is not displayed. Figure 102, Figure 103, and Figure 104 show the contour plot for each regime separately with the value for each level. Through the sensitivity analysis, 5 cases with different combinations of three regimes are observed to occur, which are summarised in Table 23

According to Figure 101, Figure 102, and Figure 104, it is observed that the proportion of total days in the PS regime and the PP regime has a negative correlation. With a given updating filter, the proportion of total days in the PS regime increases as the hesitation parameter increases. On the other hand, the proportion of total days in the PP regime decreases as the hesitation parameter increases. The higher hesitation parameter means that more users and drivers do not change the service on the next day even if they are not satisfied with their experience on the current day. Hence, a process becomes more stable with the higher hesitation parameter. Therefore, it is reasonable that the proportion of total days in the PS regime (the PP regime) increases (decreases) as the hesitation parameter increases.

Changes in η_u and η_p influence the proportion of total days in the PP regime when the hesitation parameter is $0.1 < \beta_u = \beta_p < 0.5$. In the case of the PS regime, η_u and η_p effects the proportion of total days when the hesitation parameter is $0.2 < \beta_u = \beta_p < 0.7$. It indicates that the changes in the proportion of total days in the PP regime correlate with the proportion of total days in the PS regime when the hesitation parameter is between $0.2 < \beta_u = \beta_p < 0.7$. With a given hesitation parameter, for instance, when $\beta_u = \beta_p = 0.3$, the proportion of days in the PS regime (the PP regime) decreases (increases) as the updating filter increases. It is because as the updating filter becomes closer to 1, the weight of the mean utility and profit of the current day becomes bigger when the collective average of utility and profit is updated. Hence, the process becomes less stable with the higher updating filter.

As observed in Figure 101 and Figure 103, the swan regime started to appearing when $0 < \beta_u = \beta_p < 0.2$ and $0.1 < \eta_u = \eta_p < 0.6$. Therefore, the changes in the proportion of total days in the PP regime is mainly affected by the proportion of total days in the swan regime when $0.2 > \beta_u = \beta_p$. Unlike the PP regime and the PS regime, the swan regime cannot occur alone. In addition, any case with the proportion of the total days in the swan regime exceeded 0.5 was not observed, unlike the PP regime and the PS regime. It would be because the swan regime cannot last as much as the PP regime and the PS regime, as explained in subsection 5.4.3 and subsection 5.5.

Figure 105, Figure 106, and Figure 107 summarise the mean length of each regime against a different combination of the updating filter and the hesitation parameter. According to Figure 102 and Figure 105, the PS regime's mean length increases as the hesitation parameter becomes closer to 1 after the proportion of days for the PS regime exceeds 0.9. The PP regime's mean length increases as the hesitation parameter become closer to 0 after the proportion of days for the PP regime exceeds 0.9, according to Figure 103 and Figure 106. On the other hand, the mean length of the swan regime does not change dramatically like the PS and PP regime, even if the proportion of days in the swan regime increases (see Figure 104 and

Figure 107). That is another proof that the swan regime is fundamentally unstable compared to the PP and the PS regime regardless of the parameter setting.

Table 23 the summary of 5 cases with the different combinations of three regimes.

	The PS regime	The Swan regime	The PP regime
Case 1	✓	✓	✓
Case 2	✓	✓	
Case 3	✓		✓
Case 4	✓		
Case 5			✓

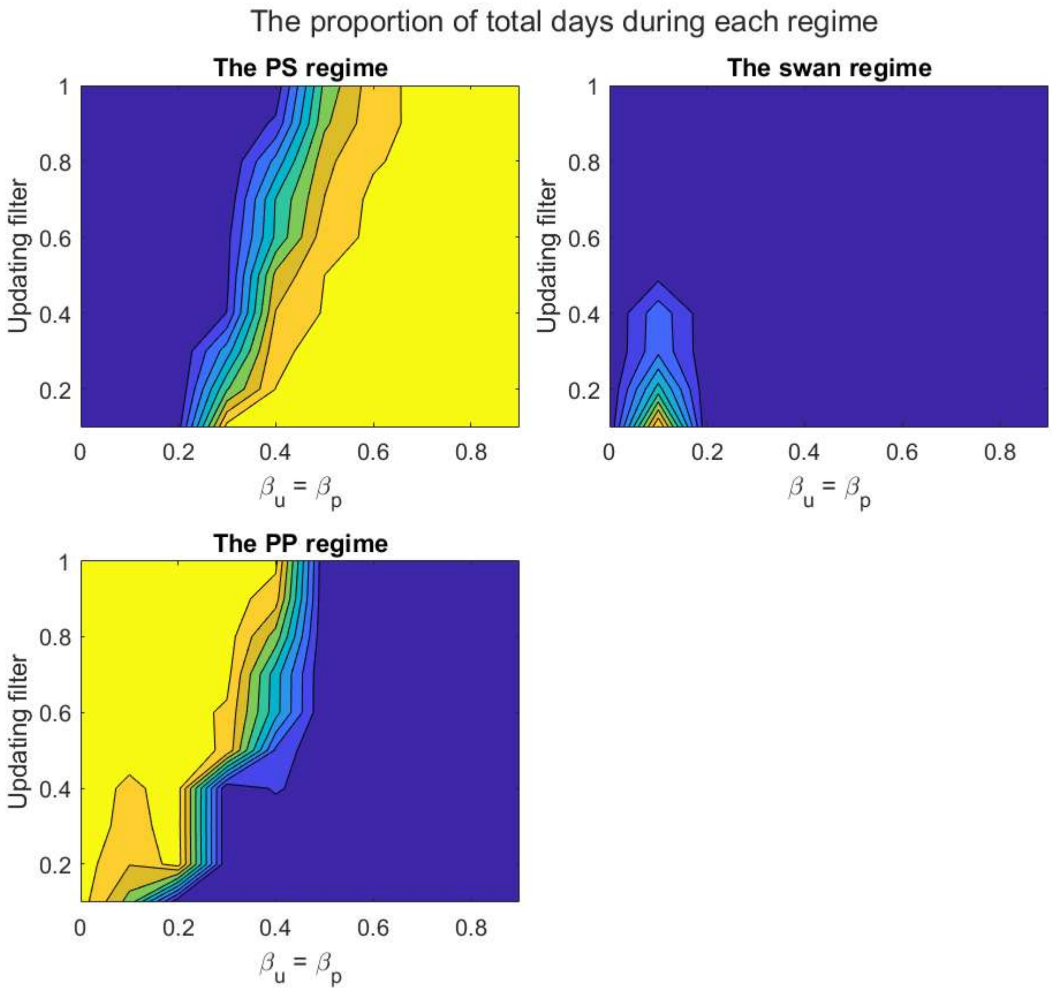

Figure 101 the contour plot for the proportion of total days in the PS regime (top left), in the swan regime (top right) and the PP regime (bottom left) with different updating filters and hesitation parameters.

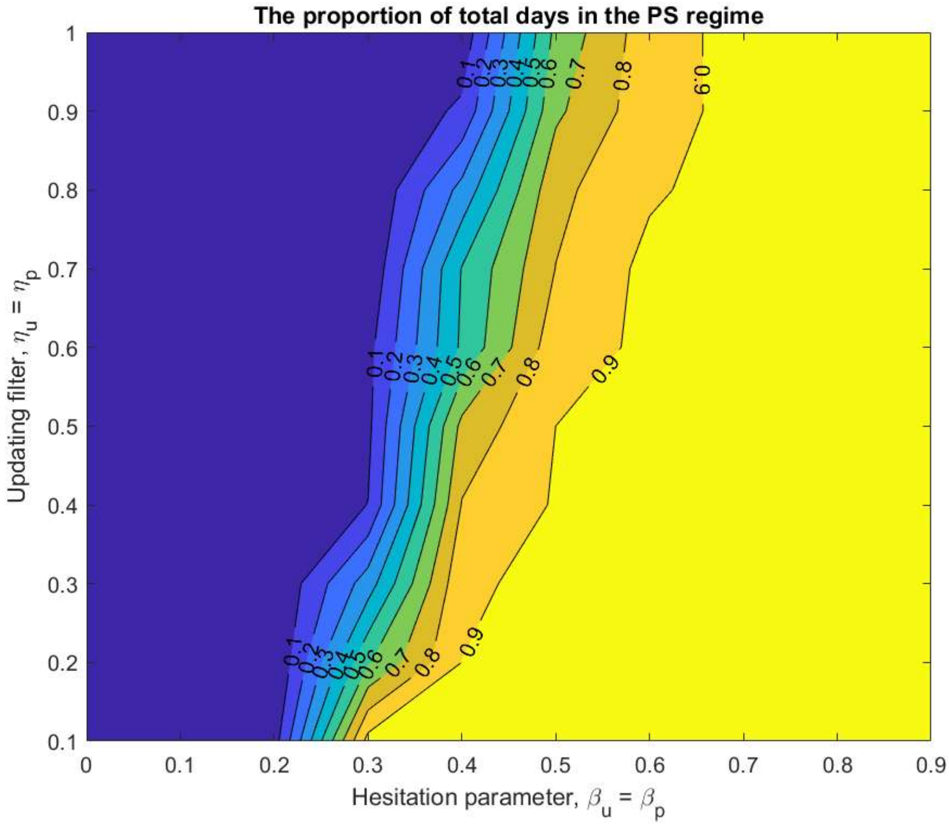

Figure 102 the contour plot for the proportion of total days in the PS regime with different combination of updating filters and hesitation parameters.

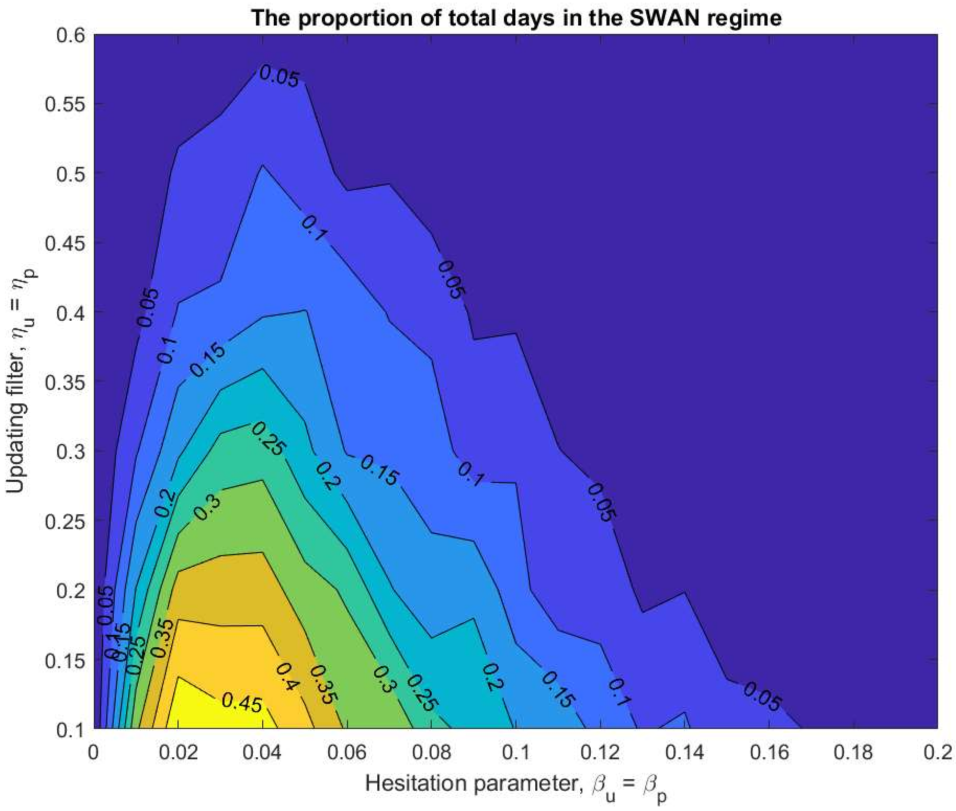

Figure 103 the contour plot for the proportion of total days in the swan regime with different combination of updating filters and hesitation parameters

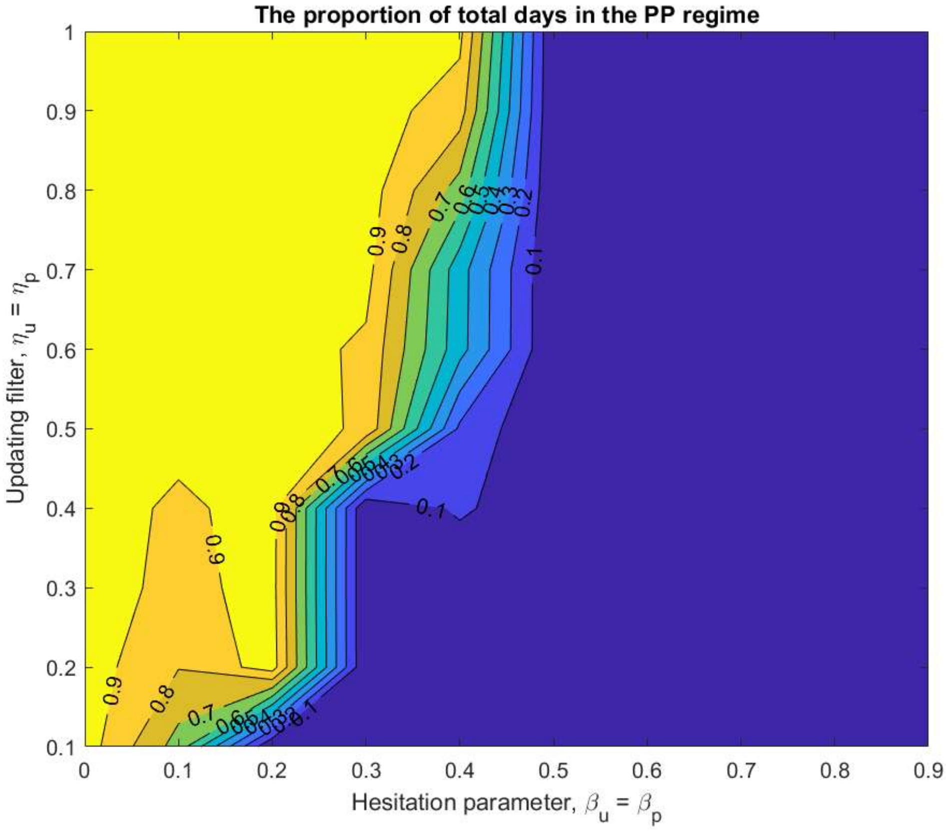

Figure 104 the contour plot for the proportion of total days in the PP regime with different combination of updating filters and hesitation parameters

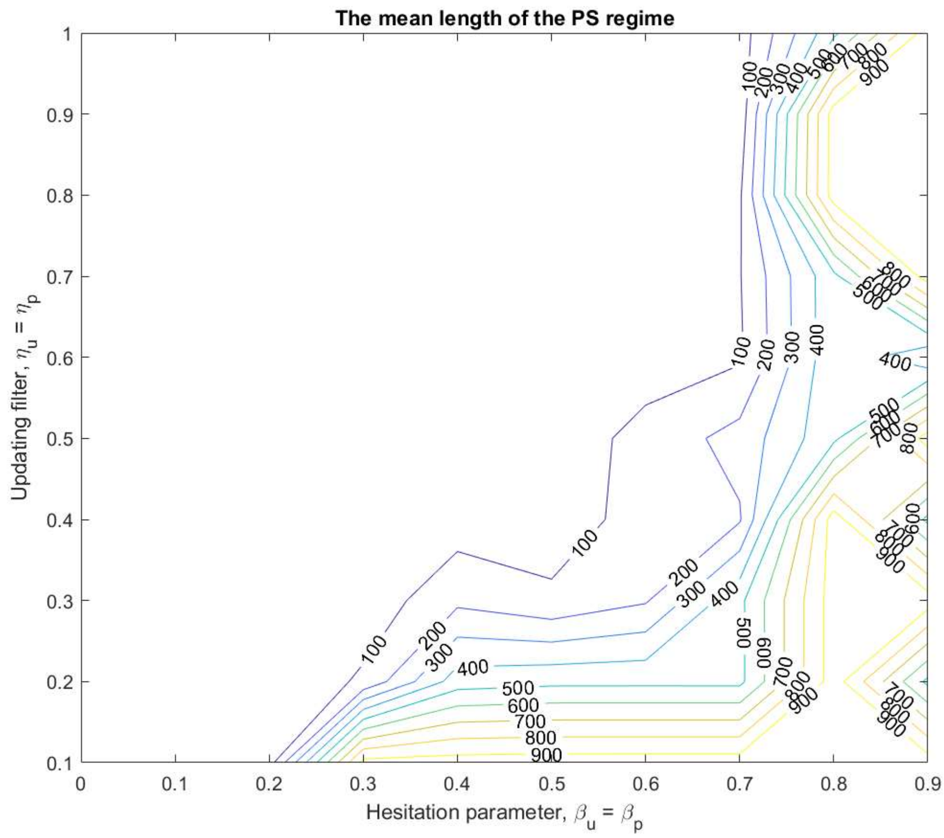

Figure 105 the contour plot for the mean length of the PS regime with different combination of updating filters and hesitation parameters

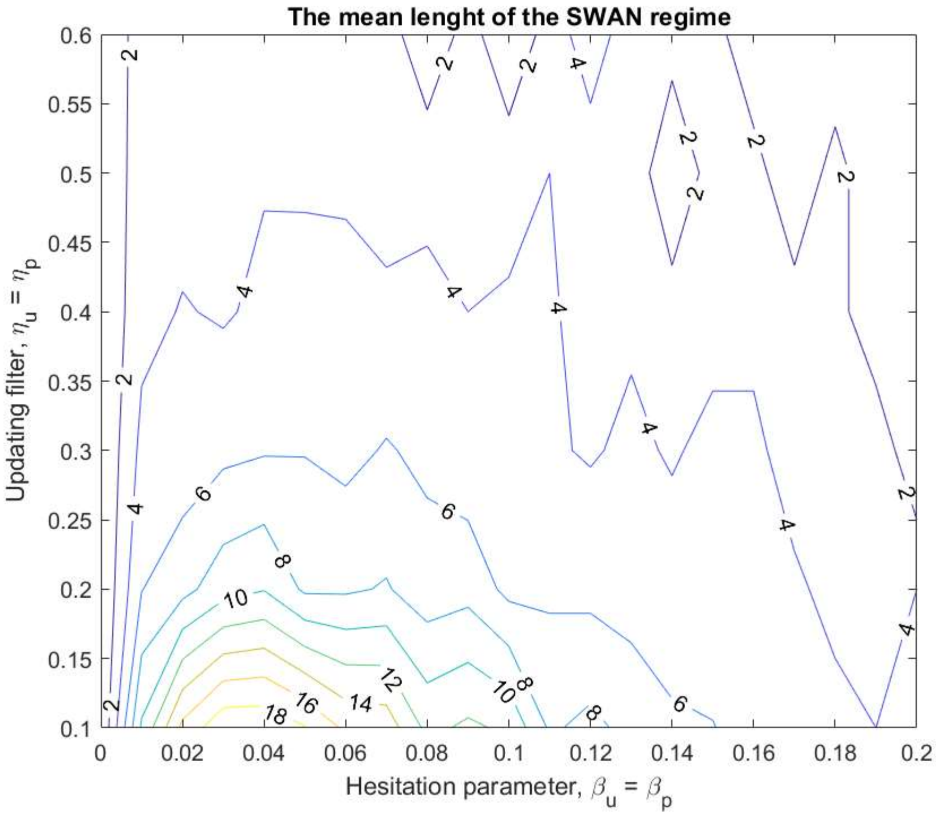

Figure 106 the contour plot for the mean length of the swan regime with different combination of updating filters and hesitation parameters

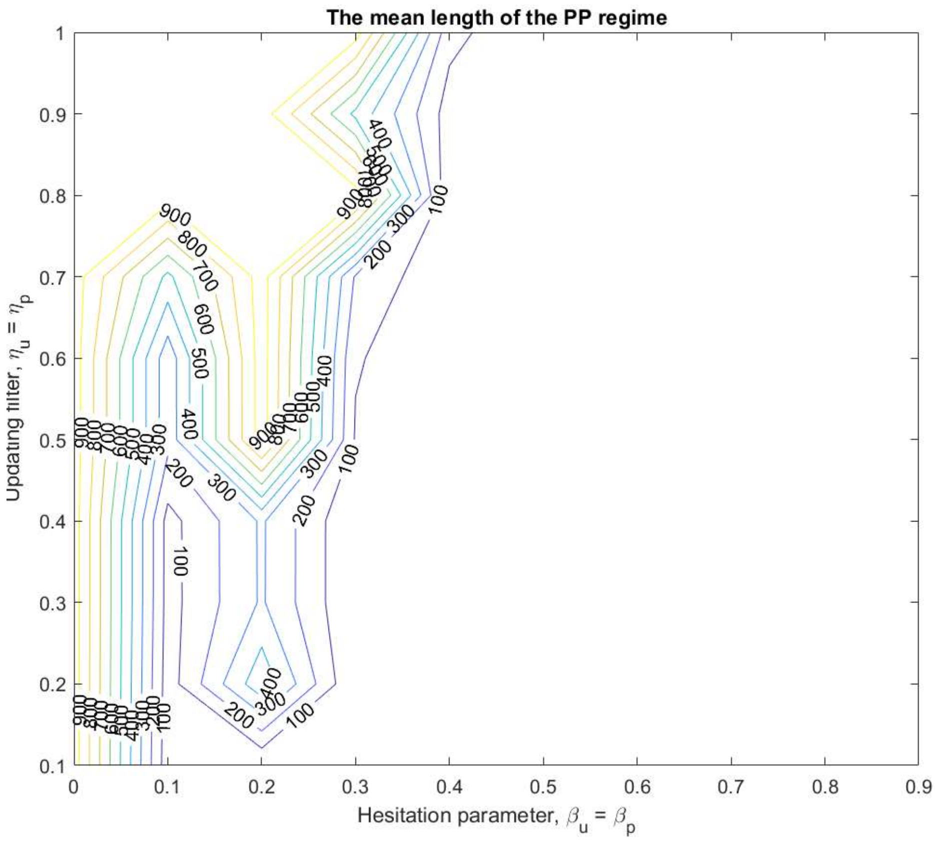

Figure 107 the contour plot for the mean length of the PP regime with different combination of updating filters and hesitation parameters

5.8 From the demand-driven to the supply-driven system

This section summarises the results of two experiments conducted to investigate how to transfer the system from demand-driven to supply-driven. The results of experiments presented in sections 5.3 to 5.6 imply that with the current parameter settings this system is demand-driven, especially during the PS regime. As the on-demand ride services are demand-responsive, it is natural to observe that the system is led by the changes in demand.

As summarised in section 5.5, the transition between the three regimes occurred responding to the changes in the demand side rather than the supply side. The transition from the swan regime to the PS regime and the PP regime was triggered by the changes in the supply side (i.e. the relationship between the collective average profit of one service and the mean experienced profit of the alternative service). However, those triggering events occur due to constant changes in the demand side (i.e. the monotonic decrease in the difference between the collective average utility of one service and the mean utility of the other service). Hence, even those transitions are a result of changes in the demand side.

From the policy makers perspective, it is easier to control the system evolution if the system is supply-driven. For instance, they could limit the number of licenses to issue for each service to restrict the fleet size. Therefore, it is worth investigating which parameter could change the process to be supply-driven from demand-driven. There are two hypothesises specified below;

1) Changing the total fleet size would shift the system into the supply-driven system
2) Changing the minimum number of drivers for each service would shift the system into the supply-driven system

The scenario settings for two experiments to investigate those two hypothesises are summarised in the following subsection 5.8.1. The results are summarised and presented in subsections 5.8.2 and 5.8.3.

5.8.1 Scenario settings

There are two experiments conducted to investigate the above two hypothesises. For the first experiment, 9 different cases with different total fleet sizes are tested, as specified in Table 24 and Table 25. The set of fleet sizes summarised in Table 24 is selected to examine the behaviour of the process when the total fleet size is decreased compared to the cases specified in section 5.3. On the other hand, the set of fleet sizes displayed in Table 25 is selected to see the effect of increasing the total fleet size from the cases specified in section 5.3. For each scenario, 500 consecutive days are generated. The other parameters are set as default.

Table 24 the set of the fleet used to see the impact of decreasing the total fleet size from the case specified in section 5.3.

Total fleet size	50	60	70	80	90	100

Table 25 the set of the fleet used to see the impact of increasing the total fleet size from the case specified in section 5.3.

Total fleet size	400	800	1200

For the second experiment, 4 scenarios are examined with the different minimum number of fleets for non-shared service and shared service as summarised in Table 26 while keeping the other parameters as default which is specified in Table 15. The flexible fleet specified in the last row of the table implies the number of fleets with a driver who can choose the service to provide day to day by comparing their own experience and the collective average experience. For each scenario, 500 consecutive days are generated. The other parameters are set as default.

Table 26 specification of the minimum number of fleets for each service in each scenario

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
The min fleet size for NON-SHARED service	10	70	5	15
The min fleet size for SHARED service	70	10	15	5
Flexible fleet	20	20	80	80

5.8.2 **Results (1): the impact of total fleet size**

Figure 108 to Figure 111 show the evolution of mode share and the proportion of fleet for each service and the distribution of mode share and the proportion of fleet for non-shared service from day 51 to day 500 when the total fleet size is 50, 70, 400 and 1200. Instead of presenting all the cases summarised in Table 24Table 24 and Table 25, a few interesting cases are selected and presented. According to Figure 108 and Figure 109, decreasing the total fleet size makes the process more unstable and increases the PP regime's occurrence frequency. As the sensitivity analysis results summarised in section 5.7.1 show, the proportion of days where the process is in the PP regime is less than 0.1 when the total fleet size is 100. However, as the total fleet size decreases, the proportion of the days where a process is in the PP regime increases according to Figure 108 and Figure 109. It is because the total fleet size is not enough to provide a satisfactory service level above the certain proportion of mode share. It is true, with a given expected total number of trip requests, even if all the fleet provides one service. Therefore, when the mode share for non-shared service reaches a certain value, the utility for non-shared service drops dramatically, which results in the mode shift to the shared service. As the same phenomenon occurs for the shared service, the process becomes unstable. Thus, it can be concluded that the reduction in the total fleet size makes the system unstable but does not make it a supply-driven system.

As the fleet size increases, variabilities in both the evolution of mode share and the proportion of fleet decrease, according to Figure 110 and Figure 111. At the same time, the difference between the proportion of fleet for shared and non-shared service decreases as the total fleet size increases. It is because the total fleet size is much larger compared to the expected total number of trip requests. Therefore, the satisfactory service level for non-shared service can be provided with the lower proportion of fleet for non-shared service. Besides, compared to the case with the total fleet size being 100 presented in Figure 71, the variability in the proportion of fleet decreases when the total fleet size is 400 and 1200, which supports the conclusion of the previous paragraph.

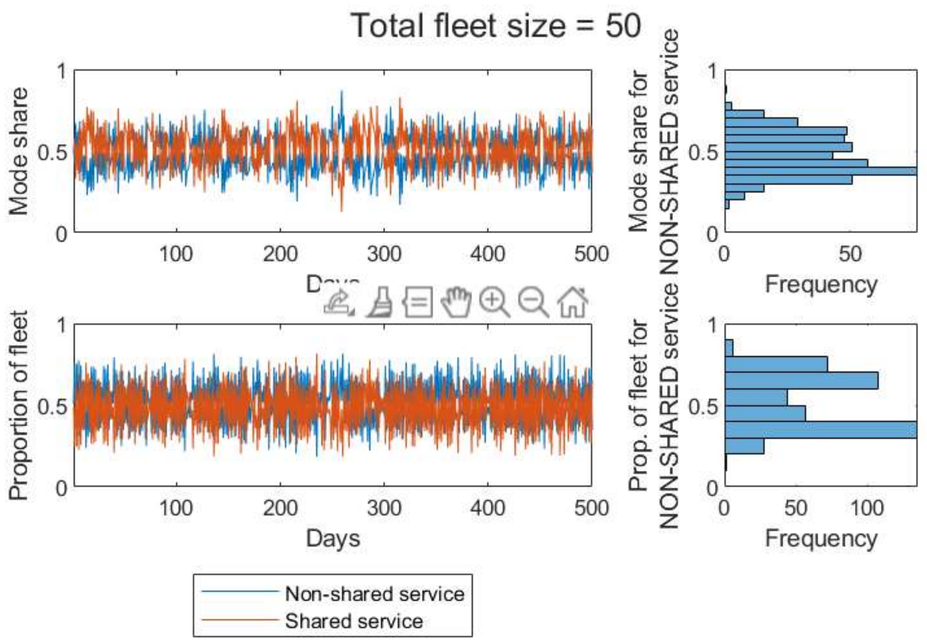

Figure 108 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) when the total fleet size is 50.

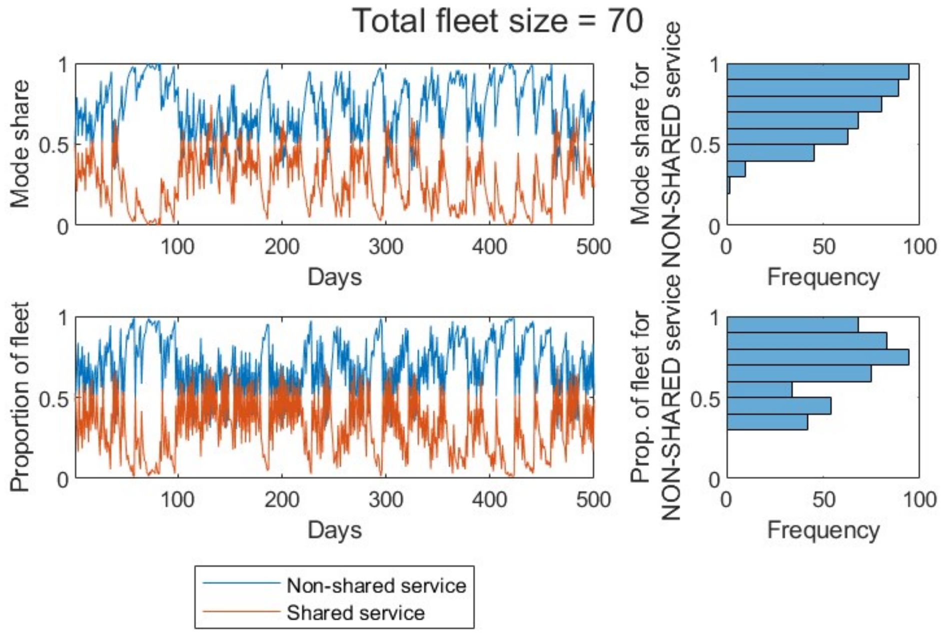

Figure 109 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) when the total fleet size is 70.

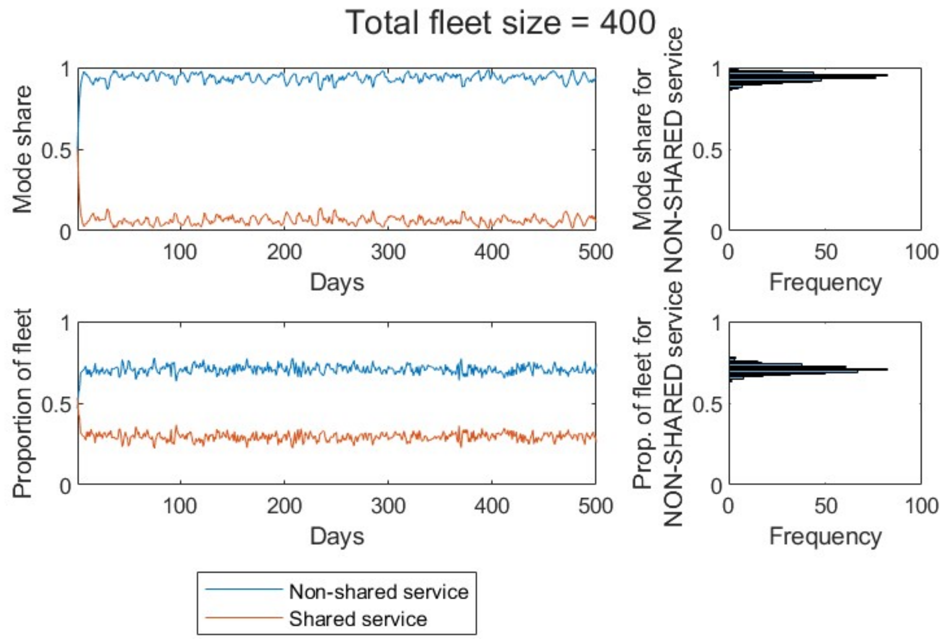

Figure 110 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) when the total fleet size is 400.

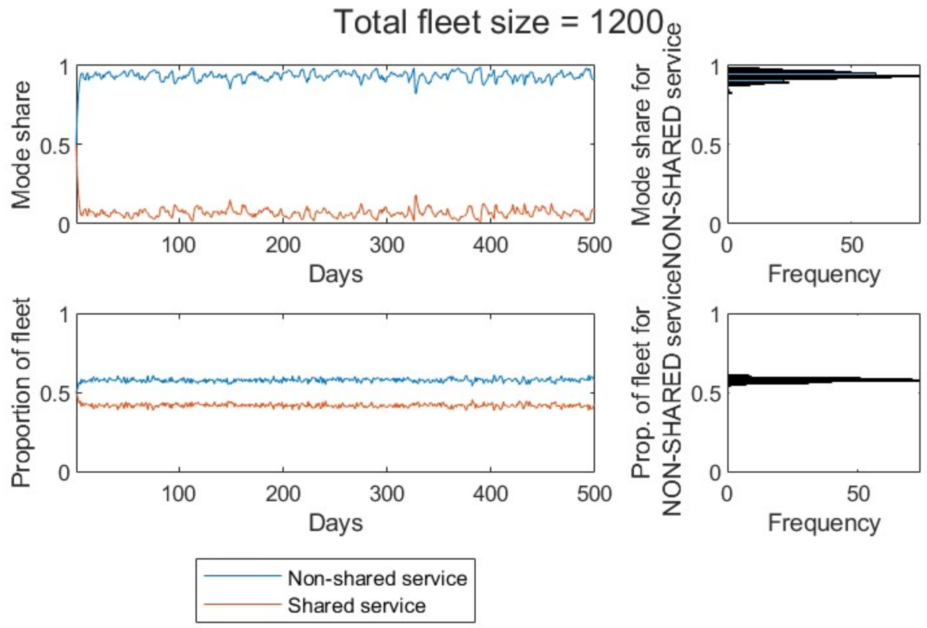

Figure 111 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) when the total fleet size is 1200.

5.8.3 **Results (2): the impact of the minimum fleet size for each service**

This subsection demonstrates how securing a certain number of drivers for each service impacts the system evolution. As specified by equation 3.17, it is assumed that there is always at least one driver providing each service. If the minimum number of drivers is increased, the minimum service capacity also increases. Hence, regardless of the number of trip requests for each service, a certain service level is secured. Figure 112 to Figure 115 presents the evolution of mode share and proportion of fleet for scenarios 1 to 4. The distributions of mode share and the proportion of fleet for non-shared service are also presented for each scenario. In section 5.4.2, it is mentioned that the distribution of mode share and proportion of fleet changes as the parameter setting changes. The four figures from Figure 112 to Figure 115 exemplify such changes regarding parameter settings. In particular, Figure 112 is a distinctive example showing that the dominant service during the PS regime would change depending on the parameter settings.

In scenario 1 presented in Figure 112, there is only 20 fleet choosing their service option every day. In addition, the minimum fleet size for non-shared service is set as 10. As a result, the fleet size for non-shared service becomes 30 in maximum even if it is more profitable to provide non-shared service as presented in the middle-left figure in Figure 117Figure 116. 30 vehicles is not enough to provide satisfactory service when the mode share for non-shared service is over approximately 0.45. Therefore, the mode share for shared service is dominant most in this case as presented in Figure 112. At the same time, the mode share for shared service is not continuously increasing as the proportion of fleet for shared service does not monotonically increase. It is because the profit of providing the shared service is lower than the collective average profit of non-shared service (see the middle-left figure in Figure 117). Unlike the case presented in Figure 71 in subsection 5.3, where the minimum fleet size for both services is 1, the behaviour of the process is restricted by the supply side's conditions and behaviour in this case. Therefore, it can be concluded that changing the minimum fleet size would make the process from a demand-driven to a supply-driven system.

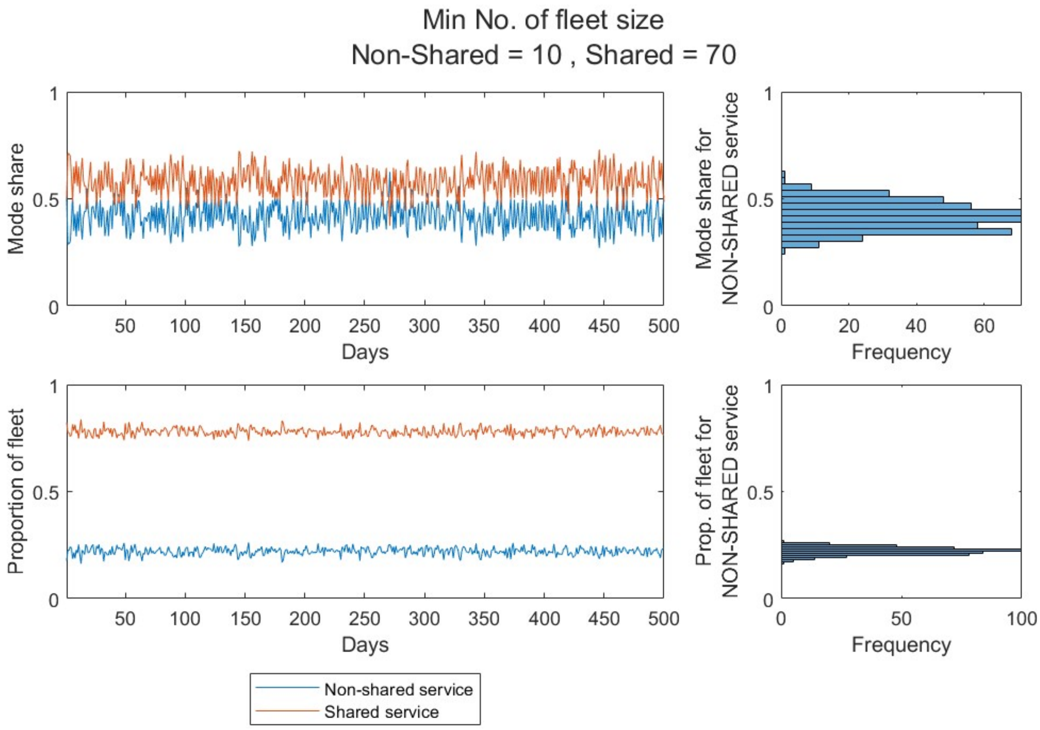

Figure 112 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) for case 1.

For both cases presented in Figure 113 and Figure 114, it can be said that the variability in the proportion of fleet is smaller than the one observed in Figure 115 and Figure 116. It is because scenarios 1 and 2 have only 20 fleets who can choose their service option every day, while 80 fleets can choose their service option for the case of Figure 115 and Figure 116. Compared to Figure 115 and Figure 116, the proportion of fleet for shared service is constantly higher in the case presented in Figure 115 as the minimum fleet size for shared service is higher (i.e. 15) for that scenario.

Besides, compared to the case where the minimum fleet size for both services is 1 presented in Figure 71, the variability in the proportion for fleet size is smaller for scenarios 3 and 4 (see Figure 114 and Figure 115). As it is observed from Figure 71, the mode share and the proportion of fleet for non-shared service are much higher than the shared service except for the day with a sharp drop. Hence, in general, the fleet size for SHARED service is generally very low, around 10 or less. When the fleet size for SHARED service is especially small such as 2 or 3, the profit for the driver could become really high even if the number of users is much lower than the non-shared service. As a result, the collective average profit for shared service sometimes becomes higher than the actual profit experienced by all non-shared service drivers. Consequently, the large shift in the proportion of fleet size occurs on the next day. However, for scenarios 3 and 4, the minimum fleet size for shared service is set as 15 and 5. That prevents the extreme case where the fleet size is 1,2, or 3, which cause the big shift in the proportion of the fleet. That is the reason why scenarios 3 and 4 presented in Figure 114 and Figure 115 have less variability in the proportion of fleet while keeping other aspects very similar to Figure 71 (e.g. the mean proportion of fleet and mode share).

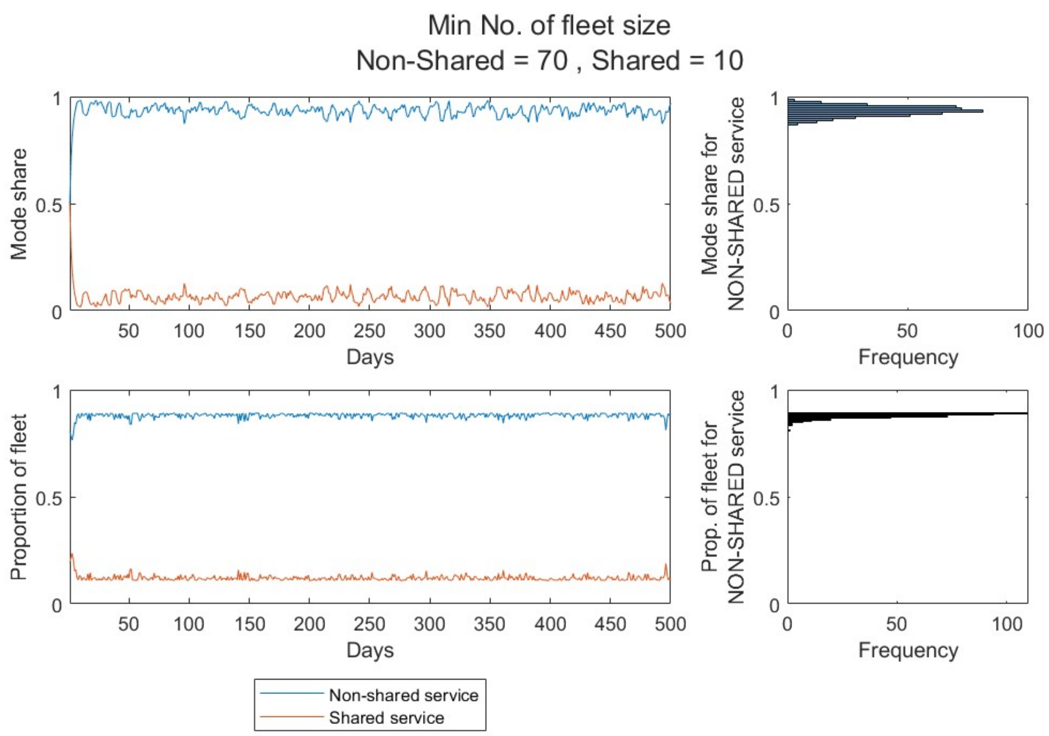

Figure 113 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) for case 2.

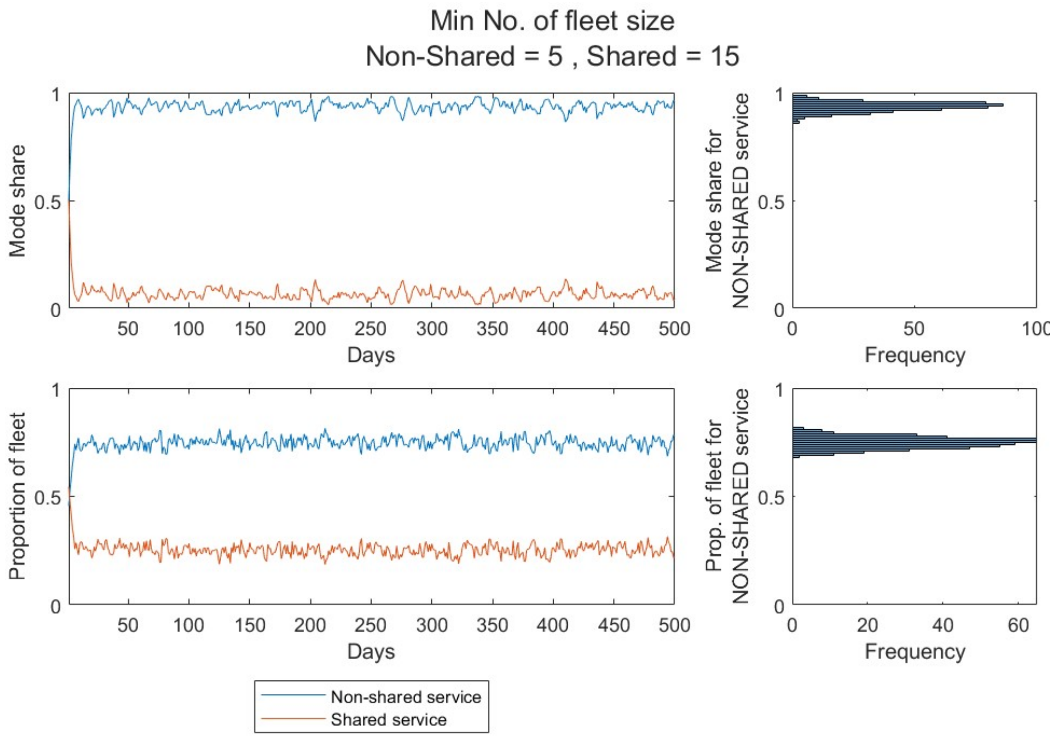

Figure 114 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) for case 3.

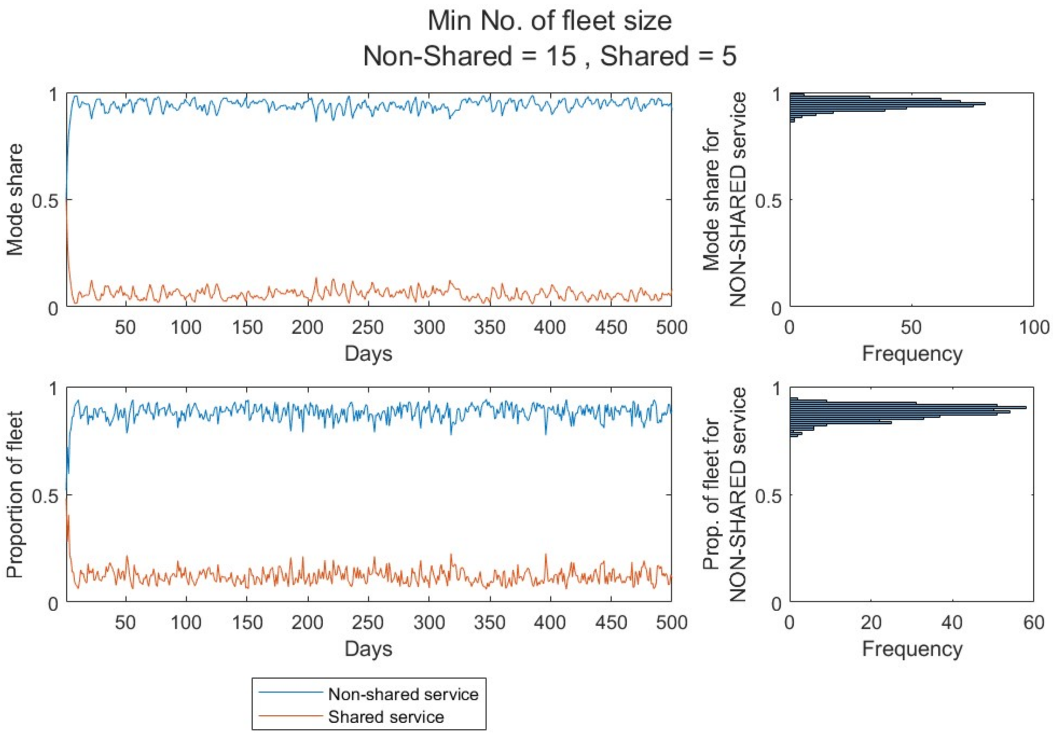

Figure 115 the evolution of mode share (top left) and the proportion of fleet (bottom right), the distribution of mode share for non-shared service from day 51 to day 500 (top right) and the distribution of the proportion of fleet for non-shared service from day 51 to day 500 (bottom right) for case 4.

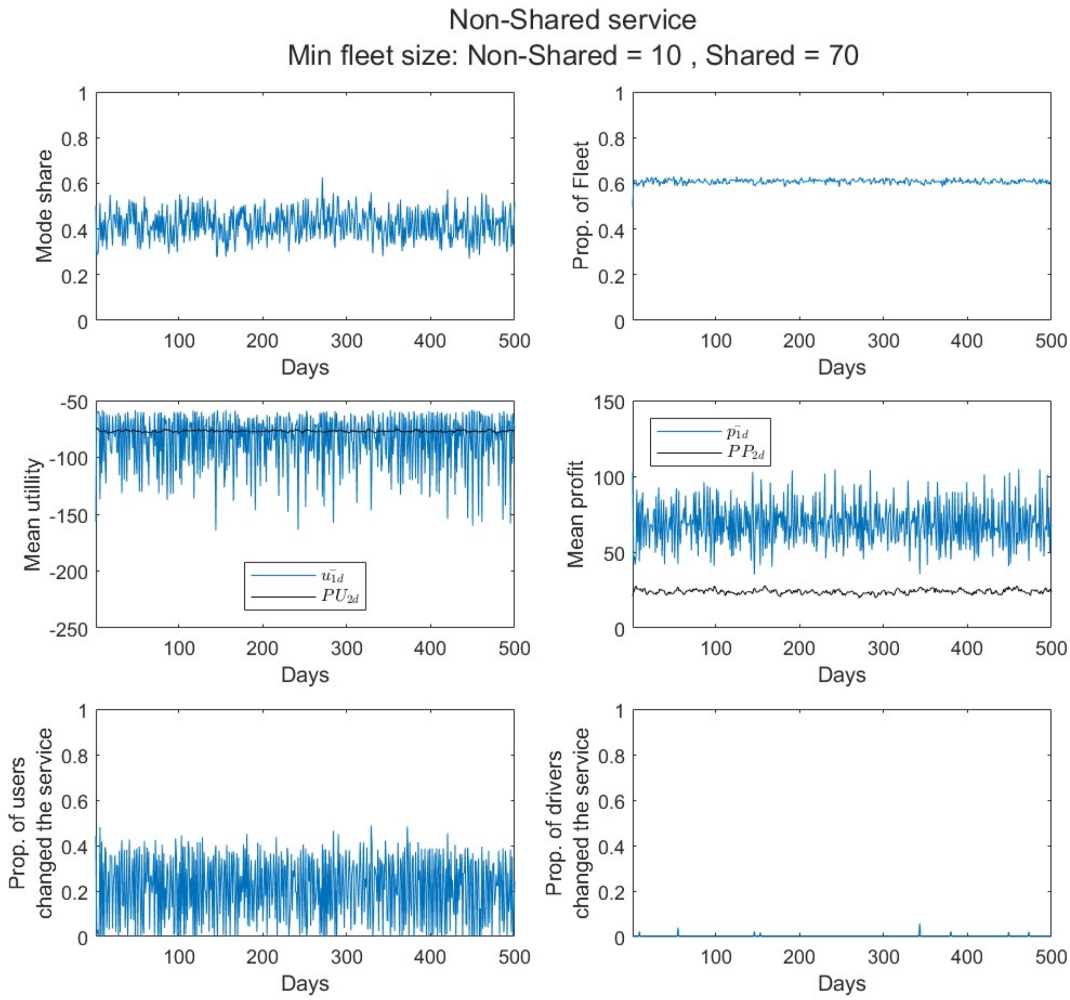

Figure 116 Summary of 6 values related to non-shared service for scenario 1. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for NON-SHARED service during the period. The middle figures compare the mean utility among NON-SHARED service users and the collective average utility of SHARED service (left) and the mean profit among NON-SHARED service drivers and the collective average profit of SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day.

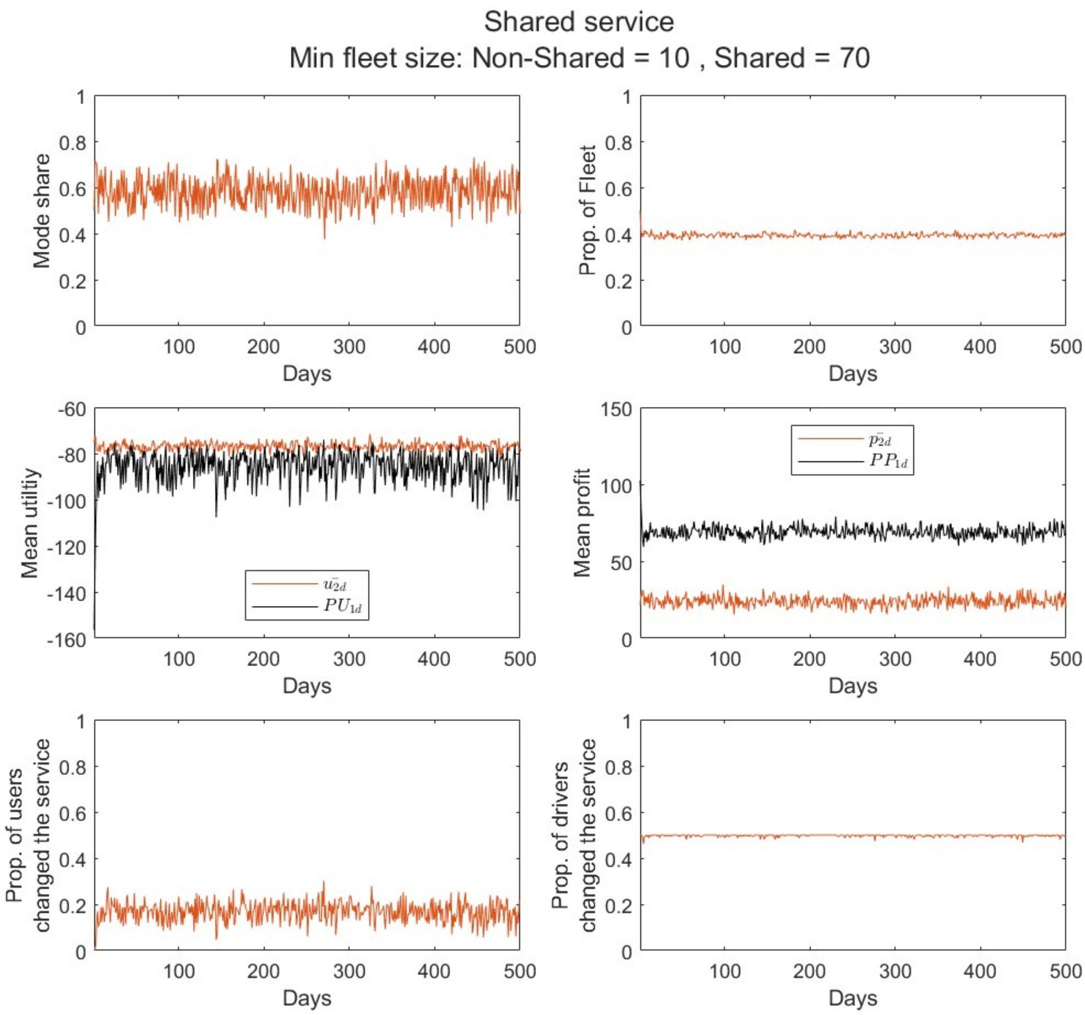

Figure 117 Summary of 6 values related shared service for scenario 1. The top figures show the evolution of mode share (left) and the proportion of fleet (right) for SHARED service during the period. The middle figures compare the mean utility among SHARED service users and the collective average utility of NON-SHARED service (left) and the mean profit among SHARED service drivers and the collective average profit of NON-SHARED service (right) who changed service on each day. The bottom figures show the proportion of users (left) and drivers (right) who changed service on each day

5.9 **Impact of updating filters**

This section demonstrates how changing the updating filters, η_u and η_p, impacts on the behaviour of the process. The updating filters are parameters in the updating functions specified by equations 3.11 and 3.21 and express the weight of each value, the collective average utility/profit of the day before and the mean experienced utility/profit of the day, in the updating function. The new updating filters and scenario settings are described in subsection 5.9.1. The results of numerical experiments to demonstrate the impact of different updating filters is summarised in subsection 5.9.2.

5.9.1 **Scenario settings**

As specified in equations 3.11 and 3.21, the updating filters are set as a given number between $0 < \eta_u \leq 1$ and $0 < \eta_p \leq 1$ by default. Instead, for this experiment, the updating filter is set as specified by equations 5.1 and 5.2, where d implies the day from the beginning of the simulation. Hence, as the day passes, updating filters becomes smaller. As a result, the weight of the mean experienced utility (profit) of the day becomes smaller, and, therefore, the weight of the collective average utility (profit) of the day before becomes bigger when the collective average utility (profit) is estimated (see equations 3.11 and 3.21).

$$\eta_u = \frac{1}{d} \tag{5.1}$$

$$\eta_p = \frac{1}{d} \tag{5.2}$$

While setting other parameters as default, 10 scenarios are testes with different β_u and β_p from 0.1 to 1 increased by the increment of 0.1. For each experiment 500 consecutive days are generated, and the results are summarised in the following subsection. Only for the case of $\beta_u = \beta_p = 0.1$, the simulation is continued to day 10,000 to understand the pattern more deeply. For simplicity, the updating filters specified by equations 5.1 and 5.2 are mentioned as the variable updating filter, while those specified by equations 3.11 and 3.21 are cited as the constant updating filter.

5.9.2 **Results**

Figure 118 and Figure 120 display the evolution of mode share and proportion of fleet for the case with the variable updating filter (see equations 5.1 and 5.2) when β_u and β_p are 0.1 and 0.3. As those cases illustrated the particularly interesting results, only those cases are presented instead of showing all generated cases. Figure 119 and Figure 121 shows the same values, but for the cases with the constant updating filters (see equations 3.11 and 3.21) when β_u and β_p are 0.1 and 0.3 and $\eta_u = \eta_p = 0.3$ following the defulat parameter settings.

When the hesitation parameter is set as $\beta_u = \beta_p = 0.1$, the distinctive differences observed between the cases with the constant updating filter and the variable updating filter are the length of the swan regime. According to Figure 106, the maximum mean length of the swan regime observed by the sensitivity analysis conducted in section 5.7 is approximately 20 days. Figure 119 also suggests that the length of the swan regime is short (i.e. max 8 days). However, in the case of Figure 118, the length of the swan regime started around day 50 continues over

100 days. According to the analysis conducted in subsections 5.4.3, the collective average utility of the alternative service is always smaller than the mean average utility of one service during the swan regime. However, the collective average utility keeps increasing during the swan regime. Eventually, it becomes close enough to the mean utility, leading to the shift of the process to the other regimes as analysed in subsection 5.5.1. For the case with the variable updating filters, η_u and η_p decreases as time passes, therefore, the collective average utility and profit are updated the smaller amount. Hence, a longer time is needed for the swan regime to end. It can be observed in the bottom figure of Figure 122, where the slope of the collective average utility for non-shared service becomes less steep during the swan regime as time passes and the length of the swan regime become longer. At some point, the collective average utility will be almost constant. As a result, the process is expected to continuously stay in the swan regime for a very long time.

When $\beta_u = \beta_p = 0.3$, the process is stable in the PS regime in the case with variable updating filters, as illustrated in Figure 120. On the other hand, for the case with the constant updating filters, the process consists of the PP regime and the PS reimage when $\beta_u = \beta_p = 0.3$ (see Figure 121). It is because the η_u and η_p becomes closer to 0 as time passes for variable updating filters, which makes the system more stable. It is consistent with the results of sensitivity analysis summarized in Figure 102 and Figure 104. According to those figures, as η_u and η_p decreases, the occurrence probability of the PS regime increases and the occurrence probability of the PP regime decreases. When it is set as $\beta_u = \beta_p \geq 0.4$, the process consists of the PS regime for most of the time with the constant updating filters. Hence, the distinctive difference was not brought by applying the variable updating filters except for the reduction of variabilities in the proportion of the fleet

According to the analysis summarized above, it can be concluded that the process will eventually be locked into either the PS regime or the swan regime when the updating filters are changed from the constant to the variable updating filters. It is consistent with the tendency observed through the sensitivity analysis summarised in section 5.7 that the occurrence probability of the PP regime decreases as η_u and η_p decreases when the updating filters are constant. It should be noted that there are some parameter settings where all three regimes are observed in one realisation with the constant updating filter (e.g. $\beta_u = \beta_p = 0.08$ and $\eta_u = \eta_p = 0.1$). In such a case, when the variable updating filter is utilised, the process could stay in the PS regime or the swan regime, depending on when each regime occurs. However, the occurrence probability of the PS regime is much lower than the swan regime. Hence, it is more likely for the process to stay in the swan regime rather than the PS regime. Besides, while the mean mode share during the PS regime would not vary within a given parameter setting, the mode share during the swan regime varies, as explained in subsections 5.5.1 and 5.5.2. Hence, it is expected that the mode share during the swan regime varies when the process is locked in.

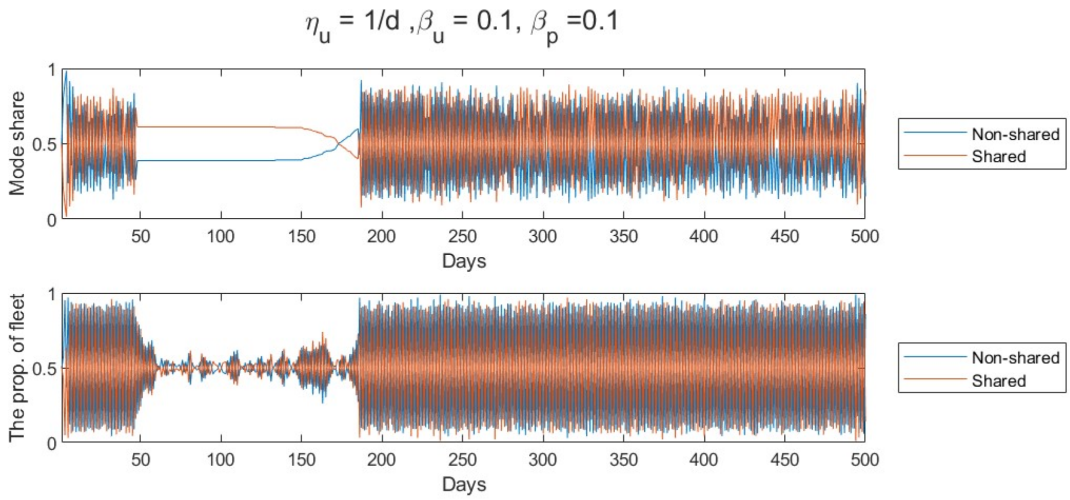

Figure 118 the evolution of mode share (top) and the proportion of fleet (bottom) when $\beta_u = \beta_p = 0.1$ for the case of variable updating filters

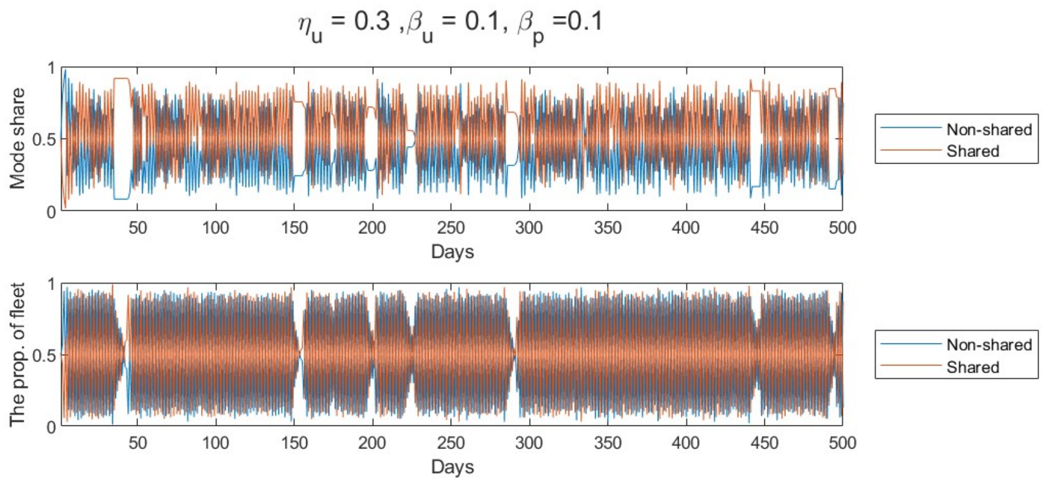

Figure 119 the evolution of mode share (top) and the proportion of fleet (bottom) when $\beta_u = \beta_p = 0.1$ for the case of constant updating filters where $\eta_u = 0.3$

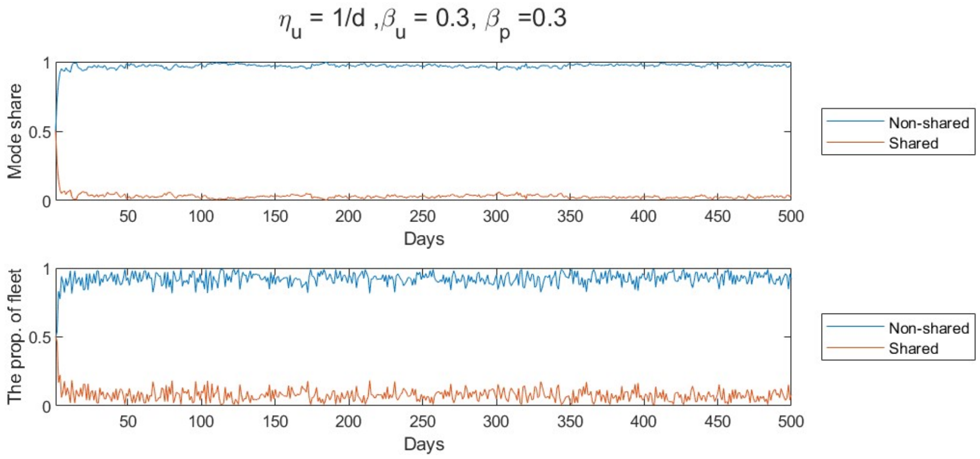

Figure 120 the evolution of mode share (top) and the proportion of fleet (bottom) when $\beta_u = \beta_p = 0.3$ for the case of variable updating filters

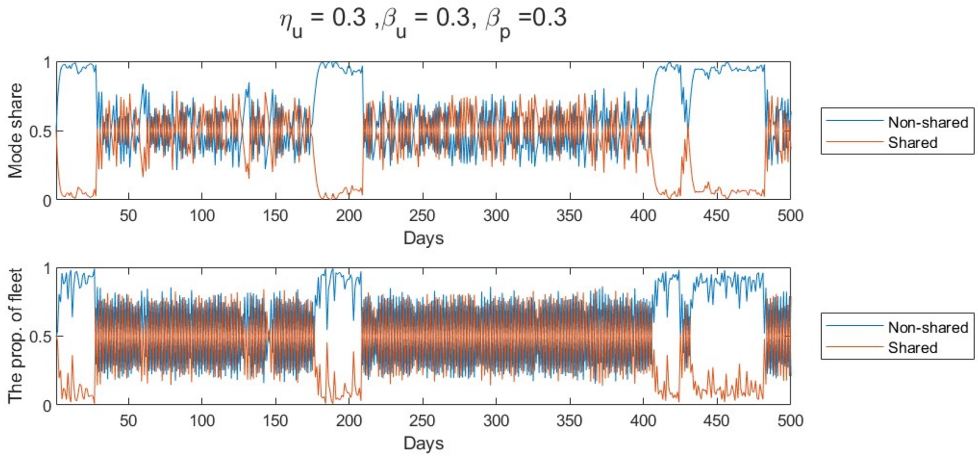

Figure 121 the evolution of mode share (top) and the proportion of fleet (bottom) when $\beta_u = \beta_p = 0.3$ for the case of constant updating filters where $\eta_u = 0.3$

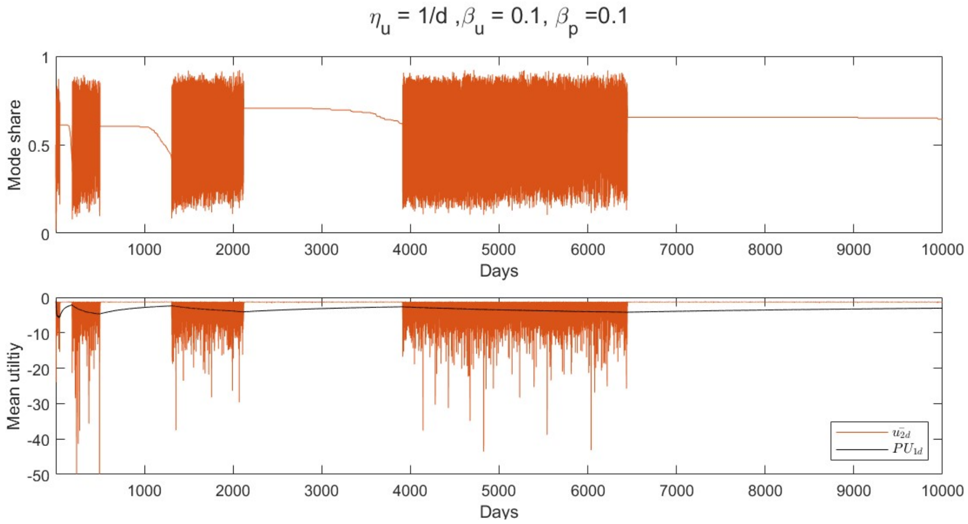

Figure 122 the evolution of mode share for SHARED service (top) and the comparison of the mean utility for SHARED service, $\overline{u_{2d}}$, and the collective average utility for NON-SHARED service, PU_{1d} for 10,000 days when $\beta_u = \beta_p = 0.1$ for the case of variable updating filters

5.10 **Summary**

This section summarised the results of numerical experiments with variable fleet size.

In subsection 5.3, by comparing the case with fixed fleet size and the case with variable fleet size, it is demonstrated that the introduction of the driver's service choice changes the stationary distribution of mode share.

In subsection 5.4, the definition of the three regimes, the PS regime, the swan regime, and the PP regime, are provided. In addition, the attributes of each regime are described using an example of a case with $\beta_u = \beta_p = 0.08$ and $\eta_u = \eta_p = 0.1$. It is discovered that the relationship between the mean utility (profit) and the collective average utility (profit) determined which regime the process was in. There was no change in the relationship between the mean utility (profit) and the collective average utility (profit) during the PS regime and the PP regime. However, during the swan regime, it is observed that the difference between the mean utility (profit) and the collective average utility (profit) decreases. It indicates that the swan regime will definitely end at some point.

In subsection 5.5, a detailed analysis of the transition between each regime was provided. It is discovered that the transition between PS and PP regime and the transition to the swan regime occurred due to the vicious and virtuous cycle triggered by radome events. On the other hand, it is observed that the transition from the swan regime happens due to the changes in the relationship between the mean utility (profit) and the collective average utility (profit). It is also discovered that when the three regimes occur in one realisation, the transition to the PS regime always occurs from the swan regime. On the contrary, it was concluded that the transition from the PS regime to the swan regime would not happen as the relationship between the mean utility(profit) and the collective average utility(profit) cannot transfer from the one in the PS regime to the one in the swan regime.

In subsection 5.6, the sensitivity analysis against initial conditions is presented. The sensitivity analysis was conducted against two scenarios with a different combination of the hesitation parameter and the updating filter. It was identified that the variance in the mean value for the stationary distribution of mode share and the fleet size is not ignorable when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. However, it was discovered that the changes in the mean mode share were correlated with the proportion of days in each regime rather than the initial condition. In addition, the variance in the mean mode share and the proportion of fleet was smaller in scenario 2, where there was only one regime (i.e. PS regime) appeared in the process. Therefore, it was concluded that there would be no dependency between the initial condition and the stationary distribution of mode share and the proportion of fleet. It also demonstrated that the proposed stochastic process was ergodic.

In subsection 5.7, the results of the sensitivity analysis are presented. The sensitivity analysis was conducted against two parameters, the hesitation parameter and the updating filter. It was discovered that there could be 5 cases with different combination of three regimes. Besides, there was a correlation between the proportion of total days in the PS regime and the PP regime as well as between the swan regime and the PP regime. As the updating filter becomes closer to 0 and the hesitation parameter becomes closer to 1, the PS regime appears more often and lasts longer. As the updating filter becomes closer to 1 and the hesitation parameter becomes closer to 0, the PP regime appears more often and lasts longer. The swan regime only appeared with very limited conditions where $0 < \beta_u = \beta_p < 0.2$ and $0.1 < \eta_u = \eta_p < 0.6$ and lasted a much shorter period than the PS regime and the PP regime.

In subsection 5.8, it is examined what parameter would change the process to be supply-driven from demand-driven. The impact of the total fleet size and the minimum fleet size for each service is examined with the hypothesis that both or one of them would impact shifting the system to be supply-driven. The results of numerical experiments show that changing the total fleet size impacts how stable the process can be. However, it would not determine if the system is demand-driven or supply-driven. On the other hand, the minimum fleet size is proved to determine if the system to be demand-driven to supply-driven. Especially when the total minimum number of fleet size for both services is set high compared to the total fleet size and the minimum fleet size for the service which can perform better in the given service network is set as low.

In subsection 5.9, the impact of updating filters is investigated and summarised. By changing the updating filters from constant value to the variable value depending on the days in the simulation, it is observed that the process would highly likely be locked in some state (e.g. the PS regime or the swan regime) with the given parameter settings regardless of the hesitation parameters (e.g. β_u, β_p). When β_u and β_p are high and at least larger than 0.3, the process will be locked in the PS regime with given parameter settings. When β_u and β_p are lower than 0.3, it is likely to be locked in the swan regime. However, the process would take much longer to be stable in the swan regime than in the PS regime. Besides, when the process is locked into the PS regime, the mean mode share will be the same, while when the process is locked into the swan regime, the mode share will vary among multiple realisations.

Chapter 6 Conclusions

6.1 Introduction

This chapter summarises the findings of this study in relation to the stated aims and research objectives and the original contribution of the current study. Critical reflections on the proposed model are also presented. Finally, this thesis concludes with the researcher's thoughts on this study.

This study was motivated by understanding the long-term evolution of an on-demand ride service system using a mathematical model. The aim of the study was to develop a model that investigated; (1) the impact of variability on the evolution of system attributes, (2) the feedback loop between users and providers, and (3) the interdependency among users. Six objectives are specified in Chapter 1 in order to achieve the research aim.

The model was formulated as a stochastic process model to capture both the variability and day-to-day dynamics of the system. Users and drivers were assumed to learn from their experiences of using and providing each service collectively through the online application provided by the service platform provider. The on-demand ride service was modelled with a queueing model. The modified Shapley value was proposed to achieve fair cost distribution, including users' disutility for staying a long time in a vehicle (Chapter 3).

By conducting numerical experiments with the fleet size fixed, attributes of the proposed model were investigated. Within the parameter setting of the experiment, it was discovered that the service network geometry had a significant influence on how the mode share of each service evolved (Chapter 4). Numerical experiments were also conducted with variable fleet size. In that case, drivers select the service they provide every day while users select the service to use. It was discovered that the system could exist in three regimes, 1) the pseudo stable regime, 2) the swan regime, and 3) the pseudo periodic regime. The conditions for occurrence and the frequency for each regime were also investigated (Chapter 5).

The rest of this chapter is organised as follows. In section 6.2, it is summarised how each objective is delivered. In section 6.3, research conclusions are presented. In section 6.4, critical reflections on the proposed model are discussed. In section 6.5, policy implications drawn from the results of numerical experiments summarised in chapter 4 and 5 are presented. In section 6.6, several future research directions are suggested. Finally, this thesis is concluded with the researcher's reflections on delivering this thesis.

6.2 Addressing objectives of this research

In this research, a wide range of aspects regarding the model formulation for an on-demand ride service system and users' and drivers' behaviour were identified through the literature review in Chapter 2. A summary of how the proposed model has addressed those issues is presented below in relation to each objective.

O1. to specify and develop a stochastic process model that represents the long-term evolution of an on-demand ride service system that provides non-shared and shared use.

In order to deliver objective O1, three attributes were formulated as specified below.

- Users' day-to-day service choice was formulated based on utility maximisation. In the model, users compare their experienced utility of using the service on the day with the collective average utility for the alternative service. While only the user's experience of one day is used as a reference to estimate the utility of the selected service, the collective average utility is calculated as a weighted average of the mean utility among all users on that day and the collective average utility estimated on the day before. This assumes a means to gather the relevant information, e.g. via an application provided by the service platform operator. Hence, learning is collectively done among all users. Nevertheless, decision making reflects the different experiences of each user. Drivers' day-to-day service choices follow the exact same process as that for users. However, in place of utility, the total profit per day determines if they are satisfied with their service provisions on the day or not.

- The users' utility and the drivers' profit were estimated by the supply model. Users' service choice on the previous day impacts drivers' profit on the current day and their service choice for the next day. Simultaneously, drivers' service choice on the previous day affects users' utility on the current day and their service choice for the next day. In this way, the feedback loops between users and drivers were represented in the proposed model.

- The competition between non-shared and shared service are represented by fixing the expected total number of users and drivers per day.

The detailed specification of the above three points is summarised in Section 3.3 in Chapter 3.

O2. to extend the model in O1 to include the impact of the availability of both sharing partner and a vehicle to the users' experience by simplifying the service supply process with a queueing representation.

With the specification of network geometry, a matching process among shared service users was reduced from a spatial and temporal problem to only a temporal problem. In particular;

- All trips were assumed to start from the pick-up demand hotspot. In addition, drop-off locations were assumed to be randomly uniformly spread in the drop-off area. The drop-off area was assumed to be sufficiently small that any trip request with a destination in that area can be matched.

- A driver was assumed to only make round trips from and returning to the pick-up hotspot. With these assumptions, the proximity between a driver's location and the requested pick-up location can be ignored during the matching process between trip requests and a driver.

By setting these assumptions regarding a driver's behaviour and the service network geometry, both matching processes between shared service users and between shared service users and drivers were included while maintaining the model's simplicity as much as possible. As a result, the availability of both sharing partner and a vehicle was included as one of the determinants of shared service users' experience. In addition, different shapes of the drop-off area (i.e. non-square shape) and other distribution patterns of drop-off points can easily be implanted in this approach. Such flexibility can be listed as one strong point of the proposed model.

The multiple server single service queue was used to represent both non-shared and shared on-demand ride service in the proposed model. In order to include several aspects regarding a shared ride, the model was formulated as specified below.

- For shared service, two queues were connected, one queue contains non-clustered trip requests, and the other queue is for clustered requests. Transfer from the non-clustered requests queue to the clustered requests queue was determined by several criteria (e.g. the minimum number of requests per cluster and the maximum number of people per vehicle). With those criteria, a matching process between users was established and integrated into the queuing representation.

- For each round trip of the shared service, the relative positions among multiple drop-off points were randomly generated. This allowed the representation of the difference in in-vehicle time experienced by different users and created the basis to estimate the difference in monetary cost based on their in-vehicle time.

The detailed specifications are summarised in Section 3.4 in Chapter 3

O3. To propose a fair cost distribution strategy within the framework developed in O2, which captures the trade-off aspects of shared services, such as a reduction in monetary cost and increase in in-vehicle time

The modified Shapley value was proposed, which considered the disutility of users staying in a vehicle for a long time. In the proposed model, the round-trip time of each service was randomly generated following the given round trip time distribution based on the number of drop-offs (NoDs) per round trip. The driver was assumed to earn their fare in proportion to the round-trip time regardless of NoDs per round trip. Hence, the total fare charged for a round trip was divided among all users involved in that trip. The modified Shapley value estimation process follows three principles of the travelling salesman game (i.e. efficiency, individual rationality, and minimal obligation). In addition, the marginal cost for each user is estimated in the same way as the conventional Shapley value.

For the modified Shapley value, the remaining cost when the minimal obligation is subtracted is allocated inversely proportional to the in-vehicle time for each user. Hence, those who dropped off later are assigned less amount of the remaining cost. This differs from the conventional Shapley value, which, if applied, would have split the remaining cost equally among users. Compared to the conventional method, the modified Shapley value proposed in this research takes into account the disutility regarding a long in-vehicle time. As a result, the trade-off between reduction in monetary cost and increase in in-vehicle time is captured.

A detailed description of the modified Shapley value is provided in section 3.4 in Chapter 3.

O4. to understand the properties of the proposed models by conducting numerical experiments within a simple setting where the fleet size for each service is fixed.

With the proposed model, a set of numerical experiments were conducted with the fleet size being fixed to understand the properties of the model. At first, the experiment was conducted with the fixed parameter in order to investigate how the mode share evolves in a detailed way. In addition, the sensitivity analysis was conducted against the initial condition, parameters controlling users' irregular behaviour, updating filter, and the parameters in the utility functions. Besides, scenario experiments with different fleet size and network geometry were also conducted. The total expected number of users and other parameters were fixed throughout the experiments.

The key findings from the experiments are summarised below.

- The results of sensitivity analysis against the initial condition suggested that the proposed stochastic process was ergodic, which guaranteed the existence of a unique stationary distribution.

- With the given expected total number of users, the fleet size of each service influenced the stationary distribution of mode share in a different way depending on the service network geometry. Specifically, when the length of the corridor was greater compared to the side length of the drop-off area, the fleet size of the shared service primarily determined the stationary distribution of mode share. On the other hand, when the length of the corridor was shorter than the side length of the drop-off area, the fleet size of non-shared service was a primary determinant.

- In addition, it was discovered that the service network geometry set the limitation for the range in the distribution of mode share regardless of the fleet size of both services.

- The sensitivity test against components of utility function suggests that the value of waiting would affect to the distribution of mode share if it is set as lower than the value of in-vehicle time, which is against the real-world evidence of the value of in-vehicle time being larger than the value of waiting time.

- The value of in-vehicle time could influence the distribution of mode share when it is set to be lower than the fare/min. However, it is questionable if users would use the service when the fare/min is more expensive than the value of time-saving.

- With a given setting, the increasing fare for non-shared service changes the distribution of mode share more than decreasing the shared service fare.

- The pricing method for shared service impacts the system evolution more when the service network geometry works better for shared service or the fleet size for shared service is higher.

- The modified Shapley value results in the higher mode share for shared service than the traditional Shapley value as the higher proportion of users receive enough discount in fare to compensate for the additional in-vehicle time

- As VoIVT decreases, the difference between the modified and traditional Shapley value increases with a given fleet size and service network geometry.

A detailed description of the results is summarised in Chapter 4.

O5.to investigate how the system evolves under different parameter settings through numerical experiments where both fleet size and mode share for each service change day-to-day.

With the proposed model, the numerical experiments with variable fleet size were conducted. The detailed analysis was conducted mainly using one realisation from the fixed-parameter settings with hesitation parameters and updating filters being $\beta_u = \beta_p = 0.08$ and $\eta_u = \eta_p = 0.1$. In addition, the sensitivity analysis against the initial conditions was also conducted and summarised against two scenarios with different combination of updating filters and hesitation parameters. In order to investigate the occurrence pattern of each regime attributed to those two parameters, the sensitivity analysis was also conducted.

Key findings from numerical experiments are summarised below.

- Three regimes were identified, which are 1) the pseudo stable (PS) regime, 2) the swan regime, and 3) the pseudo periodic (PP) regime). The mean utility (profit) and the collective average utility (profit) were identified as the determinant for which regime the process was in.

- During the PS regime, the mean mode share for non-shared service is higher than 0.8, and the mean proportion of fleet for non-shared service is higher than 0.4 during the regime.

- During the swan regime, the mode share is always constant. The mode share would vary among different swan regimes.

- During the PP regime, the proportion of the fleet demonstrates "all-or-nothing" behaviour. Hence, it is defined as the autocorrelation with one day lag, two days lag, three days lag, four days lag, and five days lag as negative, positive, negative, positive, negative.

- The transition between PS and PP regime and the transition to the swan regime occurred as a result of vicious and virtuous cycle triggered by the random event. On the other hand, the transition from the swan regime happened dues to the changes in the relationship between the mean utility (profit) and the collective average utility (profit) during the swan regime.

- When the updating filters and the hesitation parameters are set for three regimes to occur in one realisation, the transition to the PS regime was from the swan regime.

- The transition from the PS regime to the swan regime was not observed and concluded that it would not occur since the relationship between the mean utility(profit) and the collective average utility(profit) cannot transfer from the one in PS reimage to the one in the swan regime.

The sensitivity analysis was conducted against two scenarios with a different combination of the hesitation parameter and the updating filter. Key findings from it are summarised below;

- It was identified that the variance in the mean value for the stationary distribution of mode share and the fleet size vary is not ignorable when $\eta_u = \eta_p = 0.1$ and $\beta_u = \beta_p = 0.08$. However, it was discovered that the changes in the mean mode share correlated with the proportion of days in each regime rather than the initial condition.

- The variance in the mean mode share and the proportion of fleet was much smaller when there was only one regime (i.e. PS regime) appears in the process.

- Therefore, it was concluded that there would be no dependency between the initial condition and the stationary distribution of mode share and the proportion of fleet. It also demonstrated that the proposed stochastic process was ergodic.

Key findings from the sensitivity analysis against updating filters and hesitation parameters are summarised below.

- As the updating filter becomes closer to 0 and the hesitation parameter becomes closer to 1, the PS regime was discovered to appear more often and last longer.

- As the updating filter becomes closer to 1 and the hesitation parameter becomes closer to 0, the PP regime was discovered to appear more often and last longer.

- When the proportion of days in PS (PP) regime becomes higher than 0.9, the mean length of the regime increases as the hesitation parameter increases (decreases).

- The swan regime only appeared with very limited conditions where $0 < \beta_u = \beta_p < 0.2$ and $0.1 < \eta_u = \eta_p < 0.6$ and lasted a much shorter period than the PS regime and the PP regime.

Key findings from the analysis regarding how to shift the process to be supply-driven are summarised below.

- Changing the total fleet size impacts how stable the process can be. However, it would not determine if the system is demand-driven or supply-driven.

- The minimum fleet size determines if the system to be demand-driven to supply-driven. Especially when the total minimum number of fleet size for both services is set high compared to the total fleet size and the minimum fleet size for the service which can perform better in the given service network is set as low.

Key findings from the analysis regarding the impact of updating filters are summarised below.

- By changing the updating filters from constant value to the variable value depending on the days in the simulation, it is observed that the process would highly likely be locked in some state (e.g. the PS regime or the swan regime) with the given parameter settings regardless of the hesitation parameters (e.g. β_u, β_p).

- When β_u and β_p are high and at least larger than 0.3, the process will be locked in the PS regime with given parameter settings. When β_u and β_p are lower than 0.3, it is likely to be locked in the swan regime.

- It would take much longer until the process is stable in the swan regime than the PS regime.

- When the process is locked into the PS regime, the mean mode share will be the same, while when the process is locked into the swan regime, the mode share will vary among multiple realisations.

A detailed description of the results is summarised in Chapter 5.

6.3 Research contributions

The key contributions of this research arise from the model formulation and through numerical experiments with the proposed model. The contributions of this research are summarised below;

C1: Proposed a stochastic process model representing the competition between shared and non-shared use of on-demand ride service and how the system would evolve in the long term under the existence of a feedback loop.

C2: Provided a simplified representation of on-demand shared ride service by combining a graph theoretical reduction of a problem and queuing representation of DARP, which can capture the impact of variability in trip request arrival time and their drop-off locations on the availability of sharing partner.

C3: Included the trade-off aspects of reduction in monetary cost and increase in in-vehicle time by proposing modified Shapley Value

C4: Through numerical experiments with *fixed fleet size*, discovered that the specification of service network geometry determined how the fleet size of non-shared and shared services affected the distribution of mode share.

C5: Through numerical experiments with *variable fleet size*, identified the three regimes, which are the pseudo stable regime, the swan regime, and the pseudo periodic regime and the conditions under which they occur.

Table 27 the summary of which contributions are related to different objectives and which chapters are mentioned.

Objectives	Contributions	Chapters
O1	C1	CH3
O2	C2	CH3
O3	C3	CH3
O4	C4	CH4
O5	C5	CH5

6.4 Critical reflection on research method

In this section, critical reflections on the proposed model are summarised. In Chapter 3, detailed reflections of key assumptions were discussed. Therefore, this section focuses on more general assumptions regarding this model. In particular, the choices of representation method regarding three components in this model are discussed. One of the challenges in this research was to find the appropriate level of detail to represent the real-world phenomena within the model. Initially, this research aimed to make the model as simple as possible to gain more general insight into on-demand ride service systems. However, as Calderón and Miller (2020) stated, the nature of new shared modes does not allow as much simplification as can be achieved with conventional transport models. Considering those points, this section argues why each aspect of the model was not represented in a more simple or more detailed way. This section is intended to be beneficial for anyone interested in creating simple models for shared on-demand ride services in order to assess the bigger system in which shared on-demand ride services are included (e.g. Mobility-as-a-Service).

Three key questions needed to be answered in the process of developing the proposed model. The reflection is summarised for each question in the following section.

1) How to model on-demand ride services?

If a model is developed with the operational motivation, solving a DARP is often used to model the trip matching process between users and drivers. Such a representation is the most detailed representation and the most computationally demanding. Moreover, the performance of the service becomes case-specific. Hence, to understand the general service performance, multiple simulations with different sample populations would be needed.

The queuing representation of the on-demand ride service has already been applied in a few research studies as reviewed in Chapter 2. It is used as a heuristic solution method for DARP or a simplified representation of on-demand ride service as a part of a bigger model. It is helpful to understand the average performance of on-demand ride services. Hence, unlike solving a DARP, repeated simulations with a different sample is not necessary to gain a general understanding of the system. Also, it is computationally less demanding. I am not aware of a simpler representation of an on-demand ride service than the queueing representation.

Overall, if the model were intended to be used for a case study or a model's focus were within-day dynamics, solving DARP could have be useful. However, neither of them was consistent with the research objectives specified in Chapter 1.

2) To what extent are individuals specified?

In this model, the day-to-day evolution of the system was expressed by updating the collective value of the system (i.e. mode share, the proportion of fleet for each service, the collective perception of using and providing each service). The simulations represented and updated the populations of drivers and of users rather than following specific individuals. Alternatively, an agent-based model could have been utilised. In that case, some or all of the agents who were generated at the beginning of the simulation repeatedly used or provided each service every day. With an agent-based model, more detailed information could have been specified for each agent. Also, an individual's past experience could have been included when users' perception of each service was determined.

However, the more information is specified, the more case-specific it becomes. Therefore, to gain a general insight into the system, the repeated tests should be conducted. At the same time, generating more information becomes more computationally demanding. As this model was not intended to apply a real-world system operation, there was no strict time budget to generate the results. Nevertheless, if the differences in individuals' experience are not the main focus, such details would be not necessary.

In order to assess how beneficial it is to represent such great detail, an agent-based approach could be utilised and compared with the results obtained from the current model in the future.

3) How to specify the service network?

As a simper representation of the service network, the node-based representation could have been selected. In that case, the representation of an on-demand ride service would have been very similar to the one suggested by Less-Miller et al. (2016). It could have been utilised if the non-shared use of on-demand ride service were investigated in this research. However, it could have been too simple to represent shared on-demand ride services, as indifference in pick-up and drop-off locations among users was one of the key attributes which make sharing challenging.

Also, instead of a pick-up hotspot, a drop-off hotspot could have been assumed as a node with a pick-up area. In such a case, the driver's cruising behaviour should be modified from the current model. OD pattern where pick-up locations are more spread than the drop-off locations is observed from the real-world data, especially in the morning peak period. Therefore, both of those two representations could be investigated as future research, which could be compared with the results provided with the current model.

6.5 **Policy implications**

This section summarises policy implications drawn from numerical experiments discussed in Chapter 4 and Chapter 5.

- The experiments summarized in 4.3.3.4 indicate that lower VoIVT leads to the high mode share for shared service in the fixed fleet size case. Therefore, it implies that implementing a shared on-demand ride service scheme would work more effectively if the trip with lower VoIVT is targeted. In other words, it isn't easy to encourage the active use of shared service when VoIVT is high. Hence, some policy intervention or restrictions should be introduced if the regulators want to promote the intense use of shared service where VoIVT is high.

- The results of experiments summarised in section 4.6 imply that the difference in pricing method for shared service only matters when the non-shared service is dominant. Therefore, from the regulators' perspective, it would be more effective to work on different aspects rather than improving the pricing strategy for shared service when; 1) the service network geometry works better for non-shared service than shared service to provide higher service level, 2) when the fleet size for shared service is small, and/or 3) when the VoIVT is high.

- According to section 5.8, changing the total fleet size would impact the system's stability while not deciding the system to be a demand- or supply-driven system. Instead, setting the minimum fleet size for each service has the potential to make the system supply-driven. For instance, to make a shared service dominant in the given service networks and demand pattern, the below conditions should be achieved;

  1) to set the minimum fleet size for shared service higher than the one for non-shared service
  2) to limit the fleet sizes which can choose their service option every day

  However, it should be considered how the drivers would react to such regulation or if such intervention could be feasible depending on the system used by service platform operators.

- The numerical experiments summarised in section 5.9 indicate that the system could be locked in either the PS regime or the swan regime if the updating filters change as the day passes. Such variable updating filters imply that users and drivers start to believe the collective average experience reflected all the experiences from day 1 more than the mean experience of drivers and users on one day as time passes. It could be interpreted as some common impression is attached to each service as time passes. Then, as bias from those common impressions becomes stronger, the probability of the system staying in one state increases. From the regulator's point of view, it would be beneficial if the "locked-in" state was preferable for them. Otherwise, it indicates that policy intervention should be conducted before "the bias" becomes too strong. When the proportion of users and drivers who hesitate to change the service even if they are unsatisfied is very low, the system needs a longer time to be "locked into" one state. It implies that the regulator has more time budget to implement some policy interventions. Nevertheless, the mode share for the system to stay is randomly determined; hence, it is difficult to control the system to be a certain state.

6.6 **Future research directions**

This research provided several ideas for future studies. In general, there can be classified into three categories; 1) further numerical experiments with the current model, 2) extensions of the proposed models and 3) the application of real-world data. The suggestions are presented in the following section.

At first, the idea for further numerical experiments with the proposed model is summarised.

i. Conduct additional sensitivity tests against currently fixed parameters such as the vehicle capacity, the minimum number of requests per cluster, the number of captive users and drivers for each service.

ii. Conduct the numerical experiment with the different distribution patterns of drop-off locations. For instance, the density of drop-off points could be set to become higher as it gets closer to the centre of the drop-off area.

iii. Conduct the numerical experiment with the different trip request arrival patterns.

iv. Conduct scenario experiments in order to find the optimal strategy to navigate the long-term evolution of the system under different objectives. The current research investigates how the on-demand ride service system would evolve with different parameter settings. Based on the obtained results, specific strategies to navigate the on-demand ride service system towards the desirable direction could be investigated.

There are several ways to extend the proposed model, which is to;

v. Introduce the dependency in the willingness to share and the number of strangers per vehicle.

vi. Introduce the concept of in-group and out-group sharing. For instance, assuming that users are classified into two groups and some do not accept out-group sharing. In such a situation, it is expected that those in a minor group would face difficulty finding a sharing partner depending on the proportion of those who accept out-group sharing in their group and in the other group. Then, it can be investigated how the willingness to out-group sharing would influence the long-term evolution of mode share in each group and in the general system.

vii. Introduce a heterogeneous fleet with different capacities That would allow assessing the impact of the composition of the fleet with different capacity to the service level. For instance, using a vehicle with a bigger capacity could reduce the users' waiting time if the demand for shared service is high. However, it may result in increasing the mean in-vehicle time for users. Investigating such trade-off aspects would be interesting and could be useful for the service operators.

viii. Introduce more variables to determine a driver's experience, which is currently determined by the total monetary cost and the total fare. One idea is to consider a driver's willingness/hesitation to provide non-shared service and shared service. The literature review in Chapter 2 shows that some drivers show hesitation to provide a shared service for various reasons. Therefore, this aspect would affect the long-term evolution of on-demand ride services in reality and so is worth investigating.

ix. Introduce other modes such as buses and private cars. It will allow exploring the competition between on-demand ride services and other modes such as buses and private cars in addition to the competition between non-shared and shared service.

x. Change the service network geometry to a different shape. For instance, a network could be assumed with multiple pick-up hotspots and/or drop-off areas. A service network geometry could be represented with a node-based network. In addition, the pick-up area and the drop-off hotspot could be assumed instead of the pick-up hotspot and the drop-off area.

Finally, some idea to apply the real-world data is proposed, which are to;

xi. Utilise the arrival pattern extracted from the real-world data to the model and conduct the simulation.

xii. Specify a network structure based on the real-world OD pattern.

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