STRESSES IN YARN PACKAGES

A thesis presented to the University of Leeds

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Doctor of Philosophy

by

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being an account of work carried out in the Department of Textile Industries under the direction of Dr. K. Hepworth

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CHAPTER I

INTRODUCTION

The winding of yarn is an essential part of preparing the yarn for the weaving process. The object of winding a package such as cheese, cone etc. is to produce a package which is suited to the requirements of the next process such as warping, dyeing, pirn winding, weaving etc. A flangeless self-supporting cross wound package is better in many respects than a flanged package. For unwinding the yarn at a high speed for the next process the overend withdrawal of yarn is essential. Also the flangeless core is cheaper and lighter. These factors make a crosswound package an economical proposition as compared to a flanged package and it is therefore preferred.

For the package to be self supporting, it is essential that the yarn be wound at an angle to a plane perpendicular to the axis of the package; this angle is called the wind angle. Because of the wind angle the tension in the thread has an axial component which holds the package together. The greater is the wind angle the greater is this effect; but if the wind angle is too large the yarn will slip off away from the ends of the cheese during winding so that the wind angle which can be used is limited by the friction of the yarn. Because of the tendency of the yarn to slip off from the ends of the package it is difficult to make a cross wound package from a smooth yarn with low friction. (The friction of the yarn also helps to prevent the sloughing off of the yarn from the package during unwinding.)

A package meant for package dyeing should have low and uniform pressure inside it to facilitate the circulation of dye liquor and to obtain uniformity of dyeing. (A package should be stable enough to go through normal handling and storing. A uniform residual tension in the yarn inside the package is desirable, particularly with visco-elastic yarns where different levels of residual tension in yarn can give rise to bars in the fabrics woven from it.) During unwinding of the package at high speeds the shape of the package and friction of the yarn are important as they affect the ease of unwinding and sloughing off of the yarn.

A cross wound cheese may be produced by random winding or precision winding. In a random wound package the traverse rate is related to the surface speed so that the number of winds, which is equal to the number of wraps of yarn wound on the package for one traverse of the yarn from one end of the package to the other, decreases and the traverse per wind increases as the radius of the package increases but the wind angle remains the same. There is no control on the relative positions of the successive wraps of yarn over the cheese.

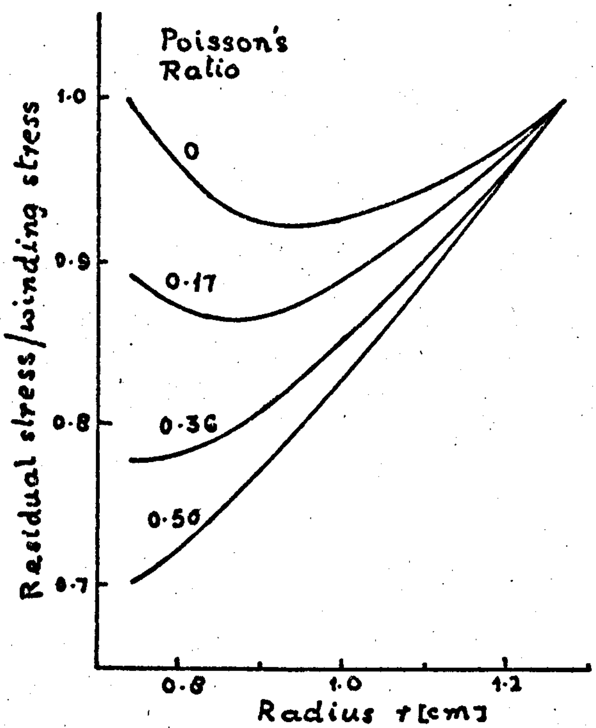
In precision winding the traverse rate is related to the angular speed so that each wrap of yarn is positioned precisely relative to the previous one and the traverse per wind remains constant but the wind angle reduces as the radius of the cheese increases. This is the basic difference between the precision winding and the random winding. In the precision wound packages the number of threads in a given axial length remains constant regardless of the winding radius and the threads are therefore laid with a slightly increasing gap as the wind angle reduces due to increasing winding radius. The present work is intended as a first approach to assess the way the pressure and the tension in the yarn are likely to change in a package and its scope is limited to a precision wound cheese. This type of cheese was selected as it was thought to be easiest to solve. An attempt is made to determine theoretically the residual tension in the yarn, the pressure and the compression within a precision wound cheese.

In actual winding for one stroke of the traverse guide from left to right only a few wraps of yarn equal to the number of winds are laid on the cheese. On the return stroke from right to left the same number of wraps are laid again but the wraps are in the opposite direction. The two series of wraps cross each other at several points, the number of such crossing points is equal to twice the number of winds. The second stroke from left to right places the next wrap of yarn adjacent to the first wrap of the first stroke displaced by an axial distance (here termed 'spacing') equal to slightly more than one diameter of the yarn or more. The spacing of the adjacent wraps is precisely controlled. One series of wraps would be complete when the point at which the yarn crosses a given generator of the package surface has progressed the width of one wind. Similarly another series would be complete when the crossing point of the return wrap has moved the width of the other wind from the other end. Thus the two series of wraps are laid simultaneously and each thread of the first series crosses the thread of the other series and the two series get interlaced. The points of interlacing lie on planes perpendicular to the axis of the package. The two series of wraps of yarn are defined here as two layers of yarn though strictly speaking the yarn is not in distinct layers because of the interlacing.

The cheese is made by adding layers of yarn which is wound on under tension. The addition of a layer at the outer radius of the cheese imposes pressure on the cheese beneath it as a result of which the cheese deforms radially. The imposition of the pressure on the cheese causes an increase in the pressure inside the cheese as a result of which, in general, any element of the cheese moves inwards a little towards the centre of the cheese. This results in a smaller circumference of that wrap permitting the yarn to contract and its tension to reduce. As more layers are added to increase the outer radius of the cheese the pressure, the compression and the change in tension in the yarn inside the cheese continue to increase. It is possible that the yarn, due to sufficient compression, loses its tension completely at some radii inside the cheese.

In a cross wound package the changes in circumference may be accommodated either by yarn contraction or by distortion of the layers because reduction of circumference is accompanied by increase in axial length, i.e. the wind angle may change slightly within the package. Thus the reduction in the length of the yarn may not change proportionally to the change in the circumference of the cross wound layer due to the inward movement of the layer as the axial length of the layer may change also. Therefore the change in the tension of the yarn may not be as much as expected in a parallel wound package under similar forces. As all the layers are not subjected to the same inward movement the axial movements which are necessary to equalise the tensions would be different at different radii, i.e. there is a tendency for the layers of yarn to move axially relative to each other. This can only take place if slipping or rolling occurs and this will depend on the shear force and the pressure between the layers.

The process has similarities with but is more complex than that for parallel winding which has been studied in some detail. Catlow and Walls[1] have derived equations for determining the stress distribution in the pirns formed by parallel winding. In the solution the pirn is assumed to be made up of layers of yarn of infinitesimal thickness and it is also assumed to be isotropic and homogenous. Free axial expansion is allowed - which is reasonable because of the way in which a pirn is built. At first the effect of adding a layer at the outer radius of the pirn is considered. This results in second order differential equations involving the incremental radial and circumferential stresses. The boundary conditions for the solution of these equations are that the compression of the pirn at the core radius is zero and the pressure imposed by the added layer at the outer radius of the pirn is known from the tension in the yarn and the thickness of that layer. (The yarn at the core cannot reduce its length as the core is assumed not to deform, but it is possible for this yarn to lose its tension due to Poisson's effect by which increasing pressure can reduce lengthwise tension). The equations are integrated analytically to give the expressions of incremental radial and circumferential stresses at any radius of the pirn for each layer added at the outside. The total changes at any radius for the final outer radius of the pirn are obtained by integrating the



Variation of the ratio of residual stress to winding stress with Poisson's Ratio, winding stress being constant. (core rad. = 0·74 cm, outer rad. = 1·27 cm)

FIG. 1.1

contributions of all the layers added.

They have calculated the values of the ratio of the residual circumferential stress to the winding stress at different radii of the pirn for different types of pirns. Fig. 1.1 shows the ratio of the residual circumferential stress to the winding stress in a pirn of core radius 0.74 cm and outer radius 1.27 cm along the radius of the pirn for the different values of Poisson's ratio. This shows how the tension reduction due to compression which is greatest at middle values combines with that due to Poisson's effect, which being a function of pressure, is greatest near the core. The minimum is most pronounced when Poisson's ratio is zero, the residual tension at the core then being equal to the winding tension. With the increase in the value of Poisson's ratio the minimum tension in the yarn occurs at points closer to the core and for the value of 0.5 the minimum of the curve has disappeared and the lowest value is at the core.

By making the pirn sufficiently large it would be possible for the yarn to lose all its tension at some radius of the pirn. The other similarly wound packages like straight wound cheese, warp beam etc. are likely to show this effect. The calculation based on the formula of Catlow and Walls shows that in the above pirn when the value of Poisson's ratio is zero the yarn at the radius of 1.55 cm has lost its tension completely when the outer radius is about 6.3 cm, giving the ratio of outer radius to the core radius of about 8.5. In the case when the value of Poisson's ratio is 0.5 the yarn at the core loses all its tension when the outer radius is about 3.8 cm giving the ratio of outer radius to the core radius of about 5. Any further increase in the outer radius would cause the yarn tension at these radii to become negative, and might give rise to the buckling sometimes seen on rolls of tape etc.

This approach is the one used previously for analysing the winding of wire to strengthen the barrels of guns - a very similar process but in which Poisson's effect is not always included.

One restriction of this analysis is its assumption that the material is homogenous. Beddoe[2] has used the same approach in establishing a theory of winding anisotropic elastic yarn on a thick flanged tube in order to predict pressures on the tube and its flanges. He found that the modulus ratio, which he has defined as the ratio of Young's Modulus in the circumferential direction to that for radial direction and axial direction in the wound beam, could be as high as 20. Due to this anisotropy the package compresses easily in the radial direction as the value of Young's Modulus in that direction is low and therefore results in a higher compression of the beam for a given pressure imposed by the added layer. This results in a large change of circumferential strain due to which the change in the tension of the yarn is also large. The loss in yarn tension is further enhanced due to the higher value of Young's Modulus in the circumferential direction. He has calculated the effect of the variation of the modulus ratio on the pressure imposed by the wound yarn on the tube. The pressure imposed by the wound yarn in the isotropic case is twice as much as when the modulus ratio is 20. Also the ratio of the maximum pressure at the tube to the minimum pressure (zero) at the outside of the completed beam is also twice as much in the isotropic case. Another important finding is the radial distribution of the residual circumferential stress or the tension in the yarn of the beam. In the isotropic case the ratio of the residual circumferential stress to the winding stress varies linearly from a minimum value of about 0.48 at the tube radius of 1 to the maximum value of 1 at the outer radius 2 of the beam. The calculation of the value of the same ratio by Catlow and Walls inside a pirn does not show this linearity with the radius of the pirn for the value of Poisson's ratio of 0.36 as shown in Fig. 1.1. This difference is probably due to the material of the pirn being able to expand freely in the axial direction whereas on the beam the flanges constrain it and axial pressures enter into the equations. In the case when the modulus ratio is 20 the value of the ratio of the residual circumferential stress to the winding stress varies from about 0.12 to 1. The curve of the ratio with respect to radius has a minimum of

0.12 and occurs at the radius of 1.3. The value of the ratio at the tube radius is about 0.75. The value of Poisson's ratio chosen for these calculations is 0.3. The yarn at the tube radius loses its tension only due to Poisson's effect as the tube cannot be radially deformed and due to the higher pressure at the tube radius in the isotropic case the loss in the tension of the yarn is higher.

Wegner and Schubert[3] give expressions for determining the pressure on the cores of precision wound and random wound cheeses. Their approach is as follows. They consider the pressure of an element of a layer at some intermediate radius of the cheese and obtain an expression for this pressure. The tension in the yarn at that radius is assumed to be the same with which the yarn of the element was wound, that is the winding tension in the yarn. This implies that the tension in the yarn of the particular element of the layer considered did not change due to the subsequent addition of the layers above it. However this would only be possible if there was no compression (deformation) of the cheese at that radius as the cheese was completed. Now in order to obtain the pressure at the core the expression so obtained is integrated between the limits of the core radius and the final outer radius of the cheese.

This method precludes the changes in the pressure due to the deformation of the cheese and the values of the pressure obtained would be correct only if the yarn was incompressible. But this is not the case in practice. The deformation of the cheese has considerable effect on the build up of the pressure at any radius even though that deformation may not be large as the present work will show. This is particularly true of the cheese made of anisotropic yarn which has a higher value of Young's Modulus in the circumferential direction than in the radial direction. These expressions could give a rough idea of the pressures in the case of isotropic cheeses which are made out of hard material such as metal wire.

The object of the present work is to combine the approach of Catlow and Walls with the geometry of the cross wound package. The model of the cheese solved theoretically differs from the actual cheese. In the model the cheese is assumed to be made up of layers of small thickness, which is composed of wraps of yarn in tension laid side by side with a shift equal to the spacing between the adjacent wraps. Each layer is supported by a similarly made layer beneath it with the wraps of yarn in the opposite direction. Interlacing of the two layers is ignored and the contact between the two layers is established at several crossing points made by the threads of the two layers thus forming a cylindrical trellis. In the actual cheese the direction of the thread reverses at the two ends due to a change in the direction of the traverse and therefore the two ends of the cheese are different from the central part of the cheese. The present theoretical solution is aimed at the central part of the cheese to start with and therefore the theoretical model is treated as if it were of infinite axial length to exclude the effect of the reversal of the yarn at the two ends of the cheese and slipping which might occur there.

Also it is initially assumed that the friction between adjacent layers and between yarn and core is sufficient under all conditions to prevent any slip taking place. This assumption eliminates the axial deformation of the cheese model and makes it different from a real cheese which is known to have axial deformation generally. Nevertheless this assumption simplifies the analysis enough to enable a restricted solution to be obtained and also enables useful information to be obtained about the build up of the cheese prior to the axial deformation of the cheese. In fact this case must be solved before the more realistic one can be attempted. Then a criterion might be applied on the basis of the coefficient of friction between the adjacent layers and the core and the cheese for which the cheese would not undergo any axial deformation.

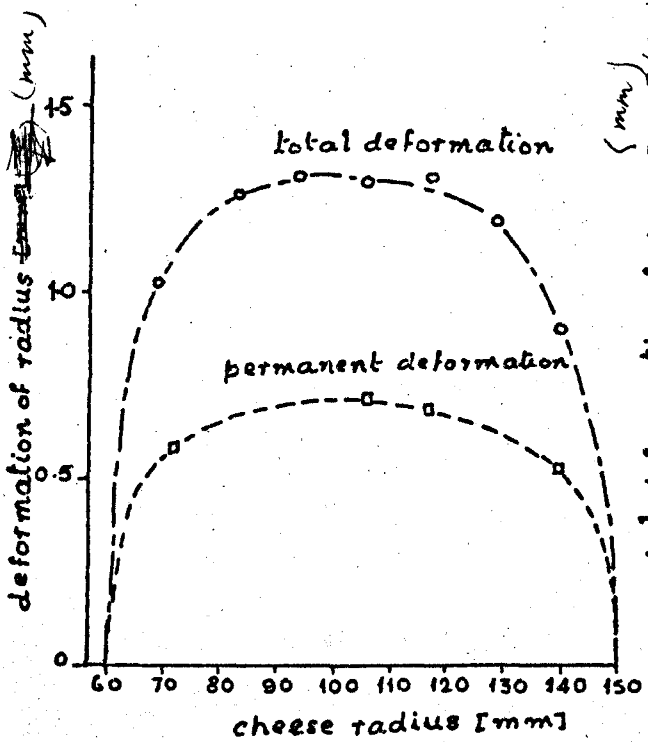
The value of Poisson's ratio of the model cheese would probably be small because the space between the adjacent wraps of yarn allows the yarn to expand, the more so as the radius of the cheese increases. Therefore the value of Poisson's ratio is assumed to be negligible and this also simplifies the analysis. The core of the cheese is assumed to be incapable of being deformed enough either axially or radially by the shear force or the pressure developed on it due to the building of the cheese on it to have any effect on the results. The Modulus of Compression of the cheese and the Elasticity of yarn in Extension are assumed to be constant for the first solution developed in Chapter 3 but in the later solution these are treated as variable and their values depend on the pressure between the layers and the tension in the yarn respectively. This solution is developed in Chapter 4.

The problems of measuring the changes which take place during winding of a real package are considerable. Deformations are in general fairly small and the material is fairly elastic. The package solved theoretically is acted on by no external forces except those imposed by the core and any attempt to hold a real package in such a way as to measure it accurately might deform it more than the actual winding process would. The insertion of any measuring device during winding is liable to disturb the normal winding process. Thus it is not surprising that indirect methods have sometimes had to be used.

de Ruig[4] in describing a new type of a traverse pattern called 'Megaphone Traverse Pattern' for winding drawtwist packages gives details of measuring the deformation of the package. "The degree of deformation is measured experimentally by measuring the diameter of a cylindrical part of a package of polyamide fibre yarn - at different package weights - during drawing and winding and by measuring these diameters again two days after the completion of the winding and after reeling the yarn off to the corresponding package weights". His findings show that the change in the diameter of the package amount to a maximum of about 2.3% and this change occurs nearer the inside of the package. This method is useful for measuring the permanent deformation of the package made of visco-elastic yarn. It would not show accurately the deformation of the package at a given radius as the package is further built up from that radius. Also it would fail to show the elastic deformation of the cheese completely.

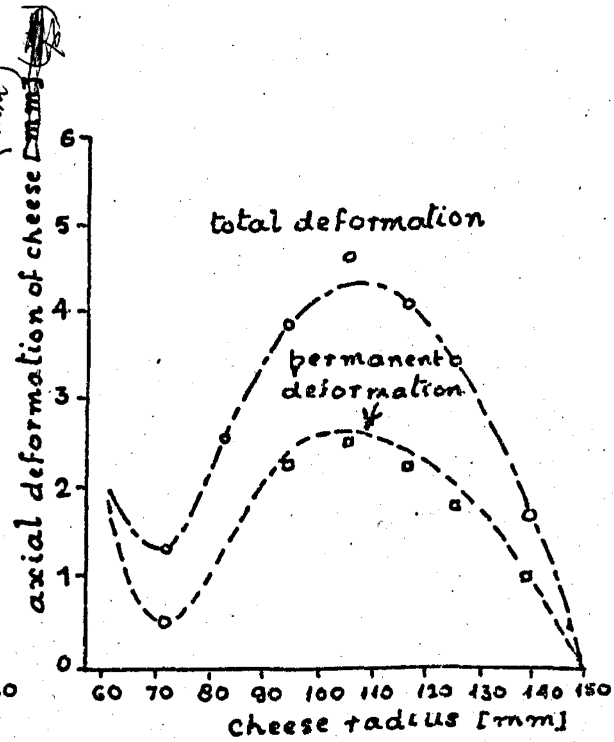
Wegner and Schubert[5] have devised methods for measuring radial and axial deformations of a cross wound cheese. This work was published when the present study was almost complete and uses successfully a method which had been considered here and rejected as unlikely to be accurate. For measuring the radial deformation of a cheese they insert, one by one, pieces of metal foil with a lead attached to one end of each foil piece at various radii as the cheese is built up to a given final outer radius. The leads protrude out of the cheese at one end. The position of each foil piece when inserted is indicated on a fixed scale by a probing needle carried on a vernier on the fixed scale. This gives the first series of measurements of the position of the foils at the original radii. During these measurements the cheese is placed on a fixed spindle under the probing needle. The contact between the needle and the foil is indicated by a signal lamp. When the cheese is completed the position of each foil is again measured by the probing needle; this time the needle forces its way down to the foil through the yarn of the package. This gives the second series of measurements. The third series of measurements is taken during unwinding when each foil is uncovered. The difference between the first and the second series of measurements show the total deformation of the cheese, the difference between the first and the third series of measurements show the permanent deformation of the cheese and the difference between the second and the third series of measurements show the elastic deformation of the cheese at various radii. The dangers of this method would seem to be that the inserted foil might disturb the transmission of shear between layers and that the insertion of the probe might produce local deformation at the very point where the measurement is being made.

For measuring the axial deformation of the cheese they have devised a semi-cylindrical cap which can be placed on the core of the completed cheese. The cap has pins parallel to the core arranged radially at the sides of the cap which can slide towards the cheese and contact it. The position of the needles when in contact with the cheese at various radii indicate the axial length of the cheese



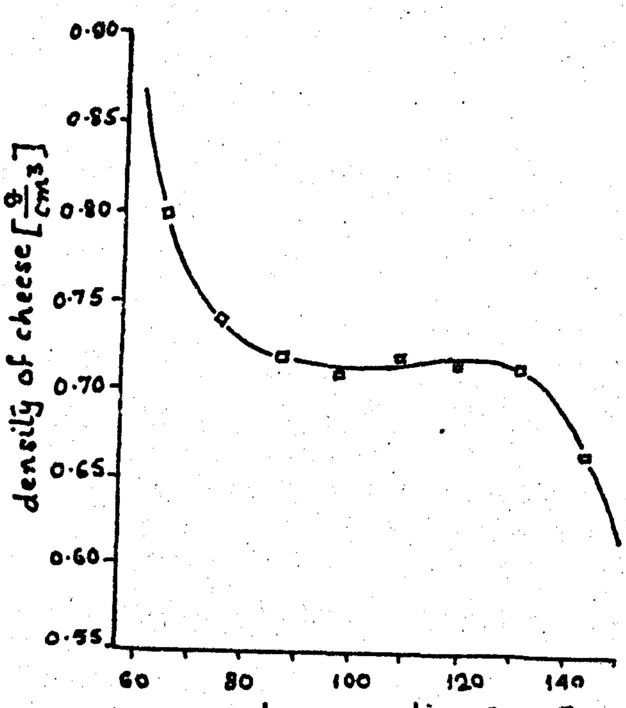
radial deformation of the cheese

FIG. 1.2



axial deformation of the cheese

FIG. 1.3



variation of the density of cheese with radius

FIG. 1.4

at those radii. The original axial length of the cheese at different radii is estimated by a formula developed by them. The difference between the estimate of the original axial length and the actual measured axial length after the completion of the cheese gives the total deformation of the cheese. Also the axial length at each radius is measured at each radius during unwinding as the cheese is reduced to that radius. The difference between this measurement and the estimate of the original axial length gives the permanent axial deformation of the cheese and the difference between the two types of deformations gives the elastic deformation of the cheese.

Figs. 1.2, 1.3 and 1.4 give the results for a cheese of Wegner and Schübert made out of a polyamide yarn of outer radius of 15 cm and a core radius of 6 cm. Fig. 1.2 shows the radial deformation of the cheese plotted against the radius of the cheese. The maximum total radial deformation of about 1.4% occurs at a radius of about 9 cm. The permanent deformation of about 0.7% at the radius of 9 cm is more than half of the total deformation. The axial deformation of the cheese is shown in Fig. 1.3. The maximum axial deformation of about 4.5 mm is at the radius of about 11 cm and is of the order of 3.1% of the original length of about 14.7 cm. The permanent deformation is about 60% of the total deformation. The curve has a minimum near the core.

Fig. 1.4 shows the density of the cheese along the radius of the completed cheese. This shows a sharp reduction in the density of the spool at the core and after the initial fall there is a very gradual increase in the density of the cheese with the increasing radius and finally the density falls again sharply near the final outer radius of the cheese. The weight of yarn between two radii of the cheese must correspond to the pressure inside the cheese and hence the curve is representative of the pressure inside the cheese. They give no information about the behaviour of the deformation, either radial or axial, at a given radius of the cheese as the cheese was built up from that radius to the outer radius.

In the present work the radial deformation of the cheese is indicated by an electrical resistance strain gauge. The gauge is prepared by fixing a wrap of strain gauge wire at some intermediate radius of the cheese on a suitable paper base and then measuring the change in the resistance of the wire as the cheese is built up by continuing the winding. This method needed some development. However as the gauge could not be calibrated the assessment of the radial deformation of the cheese is qualitative, nevertheless, the behaviour of the radial deformation can be reasonably assessed.

An attempt was made to measure the axial deformation of the cheese at any intermediate radius by a similarly made electrical resistance strain gauge. The axial deformation of the cheese is fairly large and because of it the gauge wire snapped. Therefore this method had to be abandoned. However a very simple alternative mechanical method, made possible by the large axial deformation, was devised to measure the axial deformation.

The experimental methods of measuring the deformation of the cheese are described in Chapter II along with the details of construction of the gauges and their development. The gauges for measuring the radial deformation and the axial deformation are treated in separate sections. The results of the winding tests are discussed. The observations, calculations and tabulated results are given in Appendix B.

The theoretical equations for determining the radial deformation and the forces within a completed cheese which does not expand axially assuming Modulus of Compression of cheese and Elasticity of yarn in Extension as constant are derived in Chapter III. The numerical method of integrating the equation is outlined. It also gives an estimation of the error in calculating the value of the compression of the cheese as the cheese is built up. The computer program in KDF9 Algol with explanations and flow diagram is given in Appendix A. The theoretical results obtained by solving the cheese model for different values of the variables are also discussed in this chapter and these results are compared with the results obtained experimentally. The criterion for which the cheese would not deform axially is examined and the need for the use of varying values of Modulus of Compression of cheese and Elasticity of yarn in Extension with pressure in the cheese and the tension in the yarn respectively is indicated. The tabulated results are given in Appendix C.

The behaviour of the model depends very much on the elastic modulii. These are known not to be constant as assumed in the theory - in particular the values vary a lot at low loads. As it appears that low values of tension in the yarn occur in the package and low values of pressure occur near the surface some attempt should be made to allow for the variation in modulii.

The measurement of these, their expression as functions of pressure and tension and the use of these functions in the analysis are dealt with in Chapter IV. The computer program in KDF9 Algol with flow diagram is given in Appendix A. The tabulated results appear in Appendix D.

One aspect not yet referred to is the way the forces imposed by winding at speed might affect the behaviour. The effect of centrifugal forces has in fact been considered and shown not to be important in the present context. This study appears in Appendix F.

All the theoretical work is based on the assumption that axial deformation does not take place and this should be remembered when considering the results. This restricted and artificial case was a necessary first step in obtaining more realistic solution; the difficulties in obtaining a numerical solution of even this case prevented more complex problems being solved. The discussion is largely concerned with the interpretation of these results in conjunction with the practical work, in considering to what extent they are relevant and how this work might be continued to allow more useful calculations to be made.

CHAPTER II

EXPERIMENTAL

2.1 Introduction

An attempt is made to devise a method to measure radial and axial deformation of the cheese at some radius as the cheese is further built up. A preliminary test showed that the radial deformation of the cheese can be indicated by a resistance strain gauge wire. The wire placed on the circumference of the cheese shortens along with the circumference of the cheese at that radius as the cheese is further built up. The change in the length of the wire, i.e. in the circumference of the cheese at that radius, is estimated by the change in the resistance of the wire. A gauge was prepared to measure the radial deformation of the cheese. This method needed some development which has been described later in some detail.

An attempt was made to measure the axial deformation of the cheese also by the resistance wire. But the axial change was found to be large causing the breakage of the wire. Therefore a much simpler method, made possible by the large axial deformation, was developed and is also described.

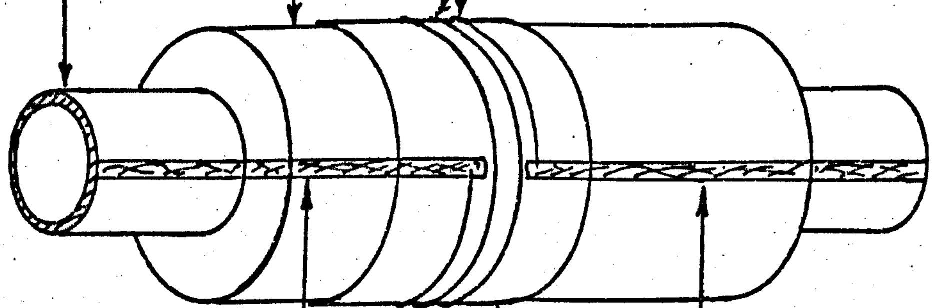
The strain measurements were made both during winding and unwinding by stopping the winding or unwinding. The unwinding was done overend and the strain during unwinding was initially continuously recorded by a level recorder and from the chart of the recorder the strain and the radius of the cheese were estimated. But stoppage of the unwinding for the measurement of the strain had no effect on the strain and therefore the method of continuous recording was discontinued.

A Leesona Style 50 Precision Winder was used for preparing the cheeses. For measuring the change in the resistance of the wire a Strain Gauge Apparatus Type 1516 by Brüel and Kjœr was used. A Brüel and Kjœr Level Recorder was initially used to obtain a record of the change in the resistance of the gauge during unwinding. A Zivy hand tensiometer was used to measure the winding tension. The observations and calculations for all the winding tests are given in Appendix B.

2.2 Preliminary Test

For this test the cheese was built on a wooden core of 0.5 in. radius on a Leesona Style 50 Precision winder. The machine has a spindle speed of 895 r.p.m. and a traverse of 5 in. with three winds per stroke. The yarn used was $2/22^s$ cotton and the cheese was just built up to a radius of 0.75 in.

At this diameter the cheese was wrapped with a rectangle of thin paper covering about the middle 4 in. of the length of the



Strain gauge to indicate radial deformation.

Preliminary Test

FIG. 2.1

cheese. The two ends of the paper overlapped and were gummed together to form a complete wrap. Care was taken that the paper was not unduly slack over the cheese and that it did not compress the cheese. A small spot was gummed to the cheese to prevent any rotation of the paper with respect to the cheese.

Two thin copper foil strips about 0.25 in. wide and 3 in. long cleaned thoroughly with emery paper and alcohol were cemented with Durofix as shown in Fig. 2.1. Then a nickel chrome wire of 0.001 in. diameter was wound over the cheese in a close helix of three turns in between the strips. The two ends of the wire were soldered to the copper strips. The wire was cemented to the paper with Durofix. The gauge was left overnight for the cement to harden. Another similar gauge was prepared under the same conditions on a second cheese. The winding tension was kept the same for preparation of the two gauges by keeping the machine adjustments the same. The lengths of wire in the two gauges were kept nearly the same to obtain similar resistances.

The resistances of the two gauges were measured very accurately in the standards section of Electrical Engineering Department after conditioning the cheeses in the conditioning room for three days. Then the winding of the cheeses was continued to different diameters under similar conditions. The resistance of the gauges were measured again as above. The results are given below.

Table 2.1

	Gauge No.1	Gauge No.2
Gauge radius	0.75 in.	0.75 in.
Resistance at 20°C before winding	696.25 ohm	695.18 ohm
Radius of the finished cheese	1.25 in.	1.7 in.
Resistance at 20°C after winding	694.31 ohm	689.77 ohm
Fall in resistance	1.94 ohm	5.41 ohm

The preliminary test showed that the fall in resistances were of the order of 0.28% and 0.78% for the first and second gauges respectively; also the change in resistance in either case was large enough to be conveniently measured and increased as winding proceeded. This test suggested that this method was capable of being developed for measuring the compression of the inner layers of the cheese.

2.3 Experimental Set-up

2.3.1 The Winder

A Leesona style 50 Precision winder was used to wind the cheeses during tests. The machine was different from the one used for the preliminary test. It has a spindle of $5/8$ in. diameter and rotates at 900 r.p.m. and gives approximately $2\frac{1}{2}$ winds per stroke. The machine is fitted with a belt gainer mechanism. The amount of gain or space between two adjacent wraps of yarn is adjusted by a hand wheel at the rear end of the gainer mechanism. At the start of the winding the gainer mechanism was set to give a shift of slightly more than one diameter of yarn between adjacent wraps of yarn, i.e. the adjacents wraps of yarn touch each other. The tension in the yarn and the pressure on the cheese during winding were applied by weights through a system of levers. As the diameter of the cheese increases these levers turn on their pivots resulting in reduced moments, thereby trying to maintain the tension in the yarn and the pressure on the cheese at a roughly constant level. Small weights were used on the pressure lever and the pressure roller was kept in contact with the cheese by the tension in the yarn.

2.3.2 Measurement of Winding Tension

A Zivy hand tensiometer was used for measuring the winding tension. This consists of a guide roller and a tension roller carried on ball bearings. The guide roller is connected to a handle at the top end and when the handle is pressed down the guide roller moves past the tension roller. The tension roller is spring loaded. For measuring the tension in the thread, the thread is placed in between the rollers and the handle is pressed down. This causes the tension roller to be supported by the thread. The tension roller moves against its spring and its movement is proportional to the tension in the thread. The movement of the roller causes a pointer to move on a graduated dial to give the tension on the thread in g. The movement of the pointer on the dial is checked by reading the tension in a thread caused by suspending a known weight from the thread. During winding, the winding tension in the thread fluctuated and the mean position of the pointer was taken as indicating the winding tension.

2.3.3 Measurement of Change in Resistance

For measuring the change in the resistance of the gauge due to further winding a Brüel and Kjoer Strain Gauge Apparatus Type 1516 was used. This instrument uses an A.C. Bridge Circuit energised by a 3 KHz supply to measure the change in gauge resistance and can indicate directly strains of the order of 1×10^{-6}.

The bridge supply can be set at three voltages, namely, 3 volt, 1 volt and 0.3 volt. The instrument can be worked at five different sensitivity settings. There is also a 'gauge factor' setting which is a continuous sensitivity control permitting the sensitivity to be set so that when the gauge factor (sensitivity) is known the dial can be made to indicate strain directly. For balancing the active and the dummy gauges the adjustment switch is turned to balance position. This switches out the phase-sensitive rectifier of the instrument to allow small capacity as well as resistance out of balance to be sensed. The meter reading is brought to zero by balancing capacity and by balancing resistance alternately. If the resistances of active and dummy gauges are not much different then it should be possible to bring the needle to zero position. Finer balance is obtained at a higher sensitivity setting. The adjustment switch is then put in the operation position which brings the 'phase-sensitive rectifier' into play making it less sensitive to stray capacity changes.

Calibration of the pointer movement on dial of the strain gauge apparatus is done by a resistor of known resistance. This resistance R_c, which is large as compared to gauge resistance R_o, when connected in parallel with the gauge changes its effective resistance by $R_o \times 100 / R_c$ per cent. This change in gauge resistance causes the pointer to move by 'n' divisions on the dial. Therefore one division of the dial represents $R_o \times 100 / (R_c \times n)$ per cent change in gauge resistance. The linearity of the pointer movement is checked by connecting the calibrating resistance in parallel with the dummy gauge. An equal movement of the pointer in the opposite direction indicates that the pointer movement is probably linear. This simple test was performed at the start of each working session.

2.4 Strain Gauge for Radial Compression

2.4.1 The Base for the Gauge

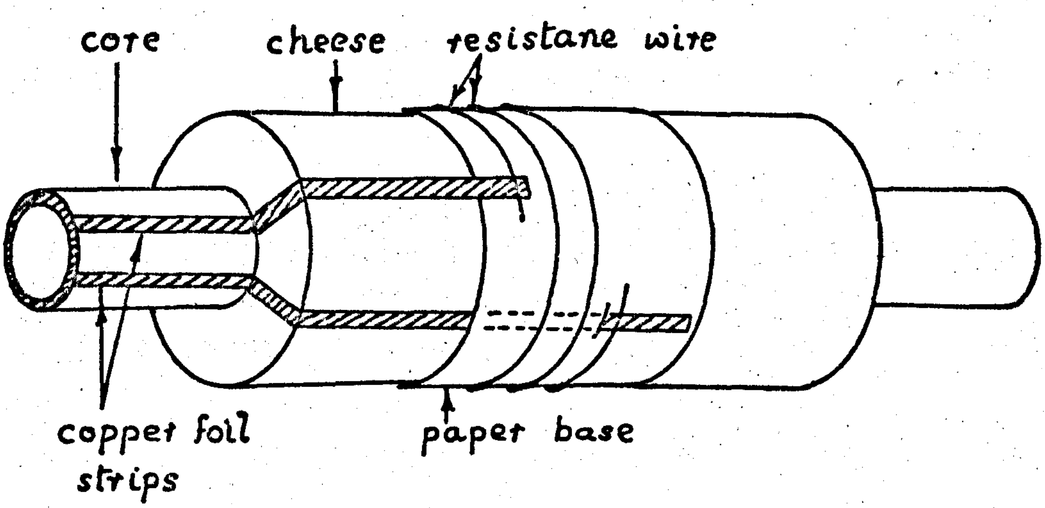
For preparing a gauge, first a base was prepared by winding a cheese up to the gauge radius on a core. Before the start of the winding a few parallel wraps of yarn under tension were wound at each end to prevent the cross wound coils of yarn of regular cheese from slipping towards the centre of the package. The machine was adjusted to lay the adjacent wraps of yarn touching each other at the core. The winding tension, which was recorded, during the formation of the base was normally the same as would be used for the subsequent winding.

The diameter of the cheese was measured correct to the nearest mm with a pair of callipers and a scale with mm divisions. The Zivy hand tensiometer was used for measuring the winding tension at intervals.

2.4.2 The Gauge

A rectangular piece of paper about 4 cm wide and slightly longer than the circumference of the cheese was used for making the strain gauge. Two strips of copper foil 5 mm wide and about 10 cm long were cemented to the paper with Durofix as shown in Fig. 2.2. The first strip was cemented over the paper for a length of 1 cm. The second strip was fixed to the paper from below for most of the width of the paper, but at about 1 cm from the other edge of the paper this strip was taken over the paper through a slit cut in the paper and the end of this strip was also cemented to the paper.

The paper was then wrapped over the cheese keeping it in the centre of the cheese. The paper was neither slack nor tight over the



Strain gauge for radial deformation with plain paper base.

FIG.2.2

cheese. The two ends of the paper overlapped and were gummed together so that the paper enclosed the cheese underneath it. The free ends of the copper foil strips were bent twice at right angles as shown in Fig. 2.2, conforming to the shape of the cheese. The strips were then fixed to the core with Durofix and any extra length of the strips beyond the core was cut off. This also prevented any movement of strips with respect to the core and also prevents their flying off during winding. Both strips were kept to one end of the cheese to facilitate overend withdrawal of yarn from the other end during unwinding of the cheese.

The strain gauge wire was cemented on either side of the first copper foil strip with Durofix as shown in Fig. 2.2. Then the cheese was turned through a small angle, a further portion of wire was placed in position with some tension and was again cemented to the paper. This process was continued until about $2\frac{1}{4}$ to $2\frac{1}{2}$ turns of wire was fixed to the paper. The placing of the wire was such that it gave a neat helix with the other end of the wire falling on the other copper foil strip. This end was also fixed at both sides of the strip with Durofix. Then the wire at both ends was soldered to the strips. For a good joint it was essential that the strips were clean. The resistance of the gauge was measured by a resistance meter of 5% accuracy. This incidentally also checked the soldering of the wire. Then a thin layer of Durofix was applied to the edges of the strips to cover them up. This was done to prevent the yarn finding its way underneath any uncovered corner of the strip and resulted in plucking of the strips during unwinding. This could give wrong results and could also result in the breakage of the delicate wire. The axial distance between the two ends of the wire was about 1.5 cm. The gauge was then left overnight at room temperature. After the drying period was over the resistance of the gauge was again measured. The gauge was now ready for use.

Another gauge was also prepared under similar conditions to be used as a dummy gauge for temperature compensation. Care was taken to keep the resistance of the dummy gauge close to the resistance of the active gauge to facilitate balancing of the bridge.

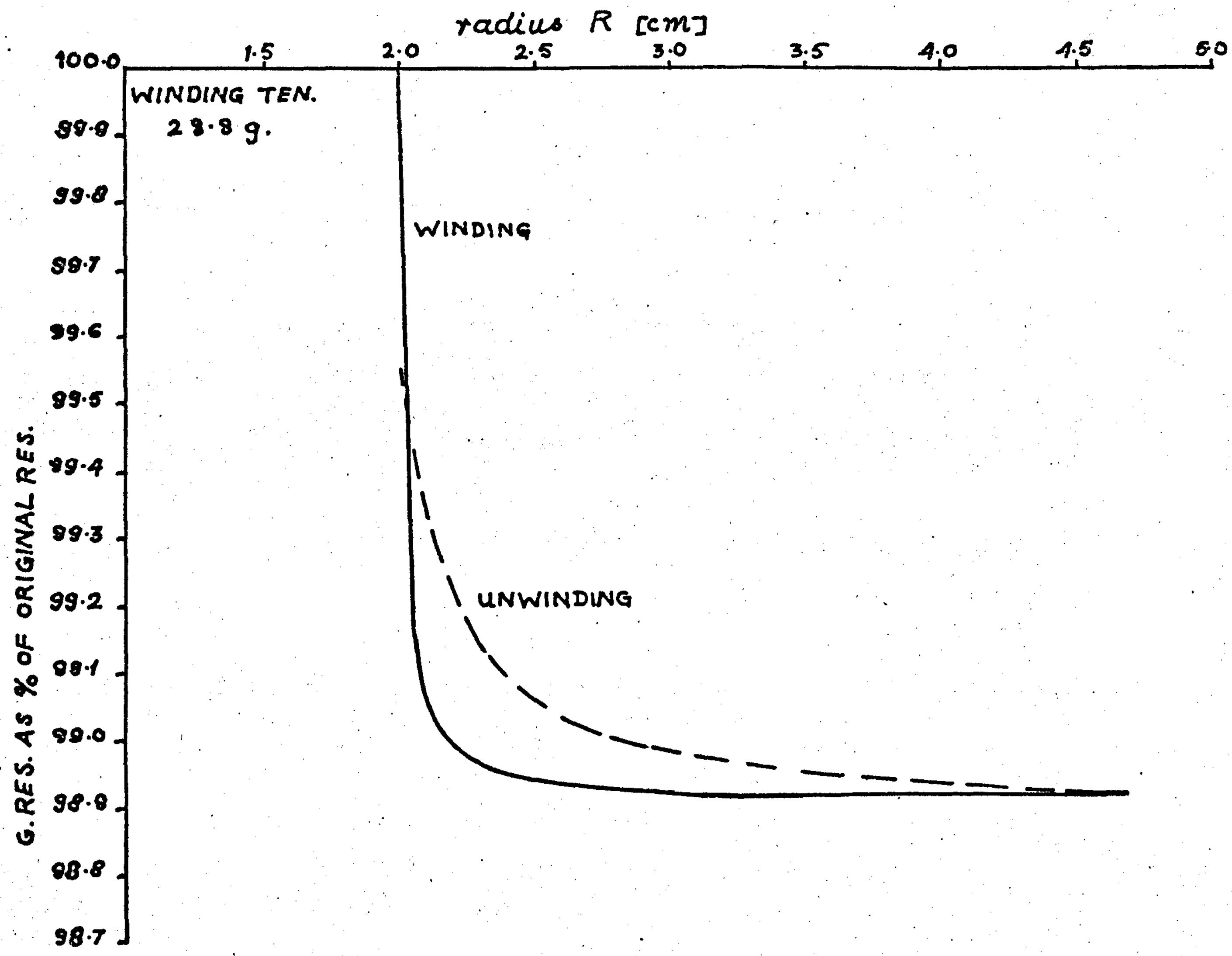
2.4.3 Test Procedure

The base of the gauge was prepared and the winding tension during its preparation was recorded at intervals. Then the gauge was prepared and its resistance measured. A similar dummy gauge was also prepared. The two gauges were connected to the strain gauge apparatus in their respective positions by spring loaded crocodile jaws with leads soldered to the jaws. This method may seem rather crude but it was convenient and as a large number of repetitions of the same measurement gave consistent results seemed justified.

After the strain gauge apparatus was balanced and set the reading on the dial was recorded and the adjustment switch was put to

TEST. 1

Winding tension for the base = 29g.



RADIAL DEFORMATION OF THE CHEESE

FIG. 2.3

the zero position and the gauge was disconnected. Then some winding was done over the gauge. Initial winding was done carefully to avoid any damage occurring to the gauge. The winding was stopped after recording the winding tension, the cheese diameter was measured and the gauge was again connected to the apparatus. The adjustment switch was put in the operation position. The reading on the dial was noted. This process of recording and winding was repeated several times until the cheese was built up to the required radius. The same process was repeated at intervals during unwinding. The gauge factor of the gauge was not known in the absence of the calibration of the gauge which was difficult to arrange, the winding test shows the radial deformation of the cheese only qualitatively.

2.4.4 The First Test

Fig. 2.3 shows the per cent change in the gauge resistance at the gauge radius of 2 cm plotted against the outer radius R of the cheese. The compression U of the cheese at the gauge radius is given by the expression

$$U = \left(\frac{\Delta R}{R} \times 100\right) \times r \,/\, (F \times 100) \;;$$

where $\left(\frac{\Delta R}{R} \times 100\right)$ is the per cent change of the gauge resistance, r is the gauge radius of the cheese and F is the 'gauge factor'. F is expected to be constant over a reasonable range of strain but is dependent on the construction of the gauge. Therefore the compression U of the cheese is directly proportional to the per cent change in the gauge resistance and can be represented qualitatively by the change in the gauge resistance. The figure shows a large initial compression of the cheese at the gauge radius and little compression later on. An increase in the outer radius after about 3.2 cm does not appear to cause any further compression at the gauge radius.

The behaviour of the gauge resistance during unwinding is also shown in the figure. The change in resistance as the cheese is unwound is similar. The recovery of the resistance after unwinding is not complete; the resistance returns to only 99.55% of the original resistance. The loss of nearly half of the total change in resistance indicates an unrecoverable part or non-elastic compression of the cheese.

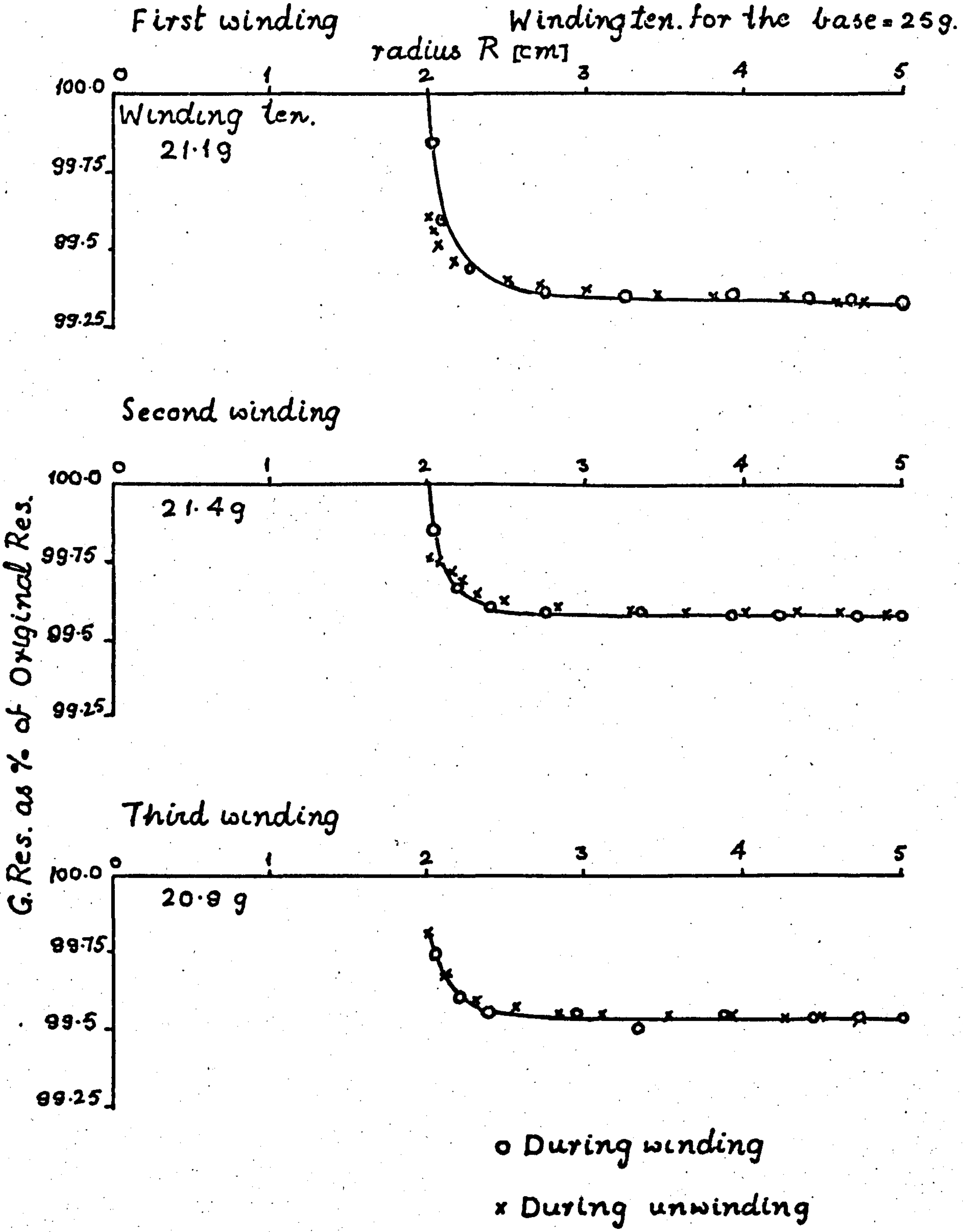
2.5 Development of the Gauge

The result of the first test was surprising in that the deformation stopped when only a few layers of yarn had been added. It was therefore necessary to find out whether this was a correct picture of the behaviour of the package or whether there was some fault in the method of measurement.

2.5.1 Tests 2.A and 2.B

These tests were devised to ascertain the reproducibility

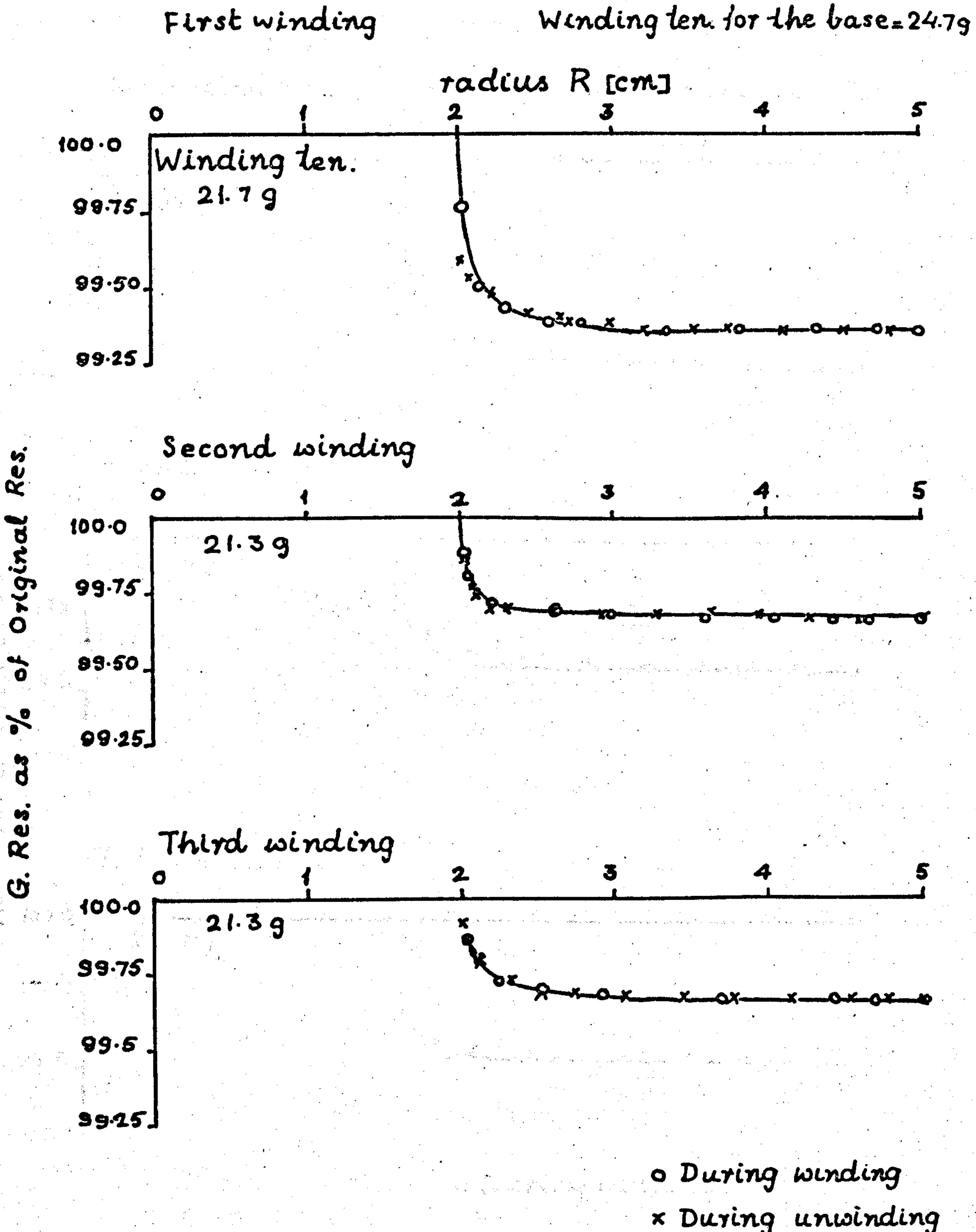
TEST. 2.A



Radial deformation of the cheese

FIG. 2.4

TEST. 2.B



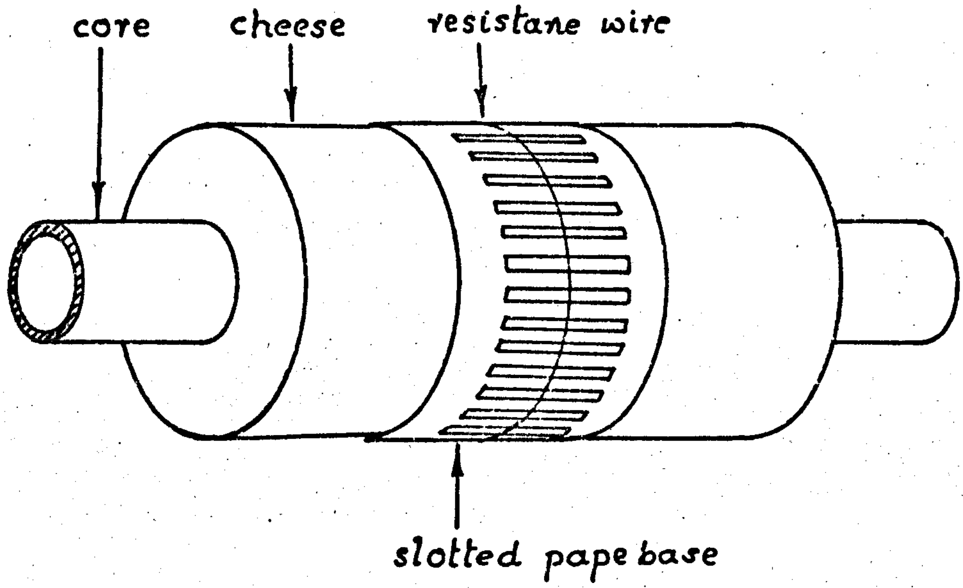
Radial deformation of the cheese

FIG. 2.5

of the results and the effect of repeated winding on the compression of the cheese. To achieve this two similar gauges, 2.A and 2.B were prepared under similar conditions and were used as active and dummy gauges. The cheeses were unwound to a radius of 2.05 cm during the second unwinding.

Figs. 2.4 and 2.5 show the results of tests 2.A and 2.B respectively. These figures show the resistance of the gauges during winding and unwinding as the per cent of the resistance before each winding. The compressional behaviour of both the cheeses at the gauge radii, which is represented qualitatively by the resistance of the gauges, is similar. The changes in gauge resistances for first winding are 0.66% and 0.65% for 2.A and 2.B respectively for winding tensions of 21.1g and 21.7g. The non-recoverable part of the change in the resistances are 0.4% and 0.41% respectively. As the results for these tests are very close to each other, both in behaviour and magnitude it may be reasonably concluded that the results are reproducible.

Gauges 2.A and 2.B show higher changes in the gauge resistances for first winding as compared to second and third windings. The values for these changes are 0.66%, 0.42% and 0.45% for 2.A, and 0.65%, 0.33% and 0.34% for 2.B for the first, second and the third winding respectively for nearly equal values of the winding tension. This shows that the radial deformation of the cheese reduces for second and subsequent windings probably because after the first winding the cheese



Slotted paper base for radial strain gauge

FIG.2.6

suffers a large amount of permanent deformation and as a result of which it is more resistant to the later windings.

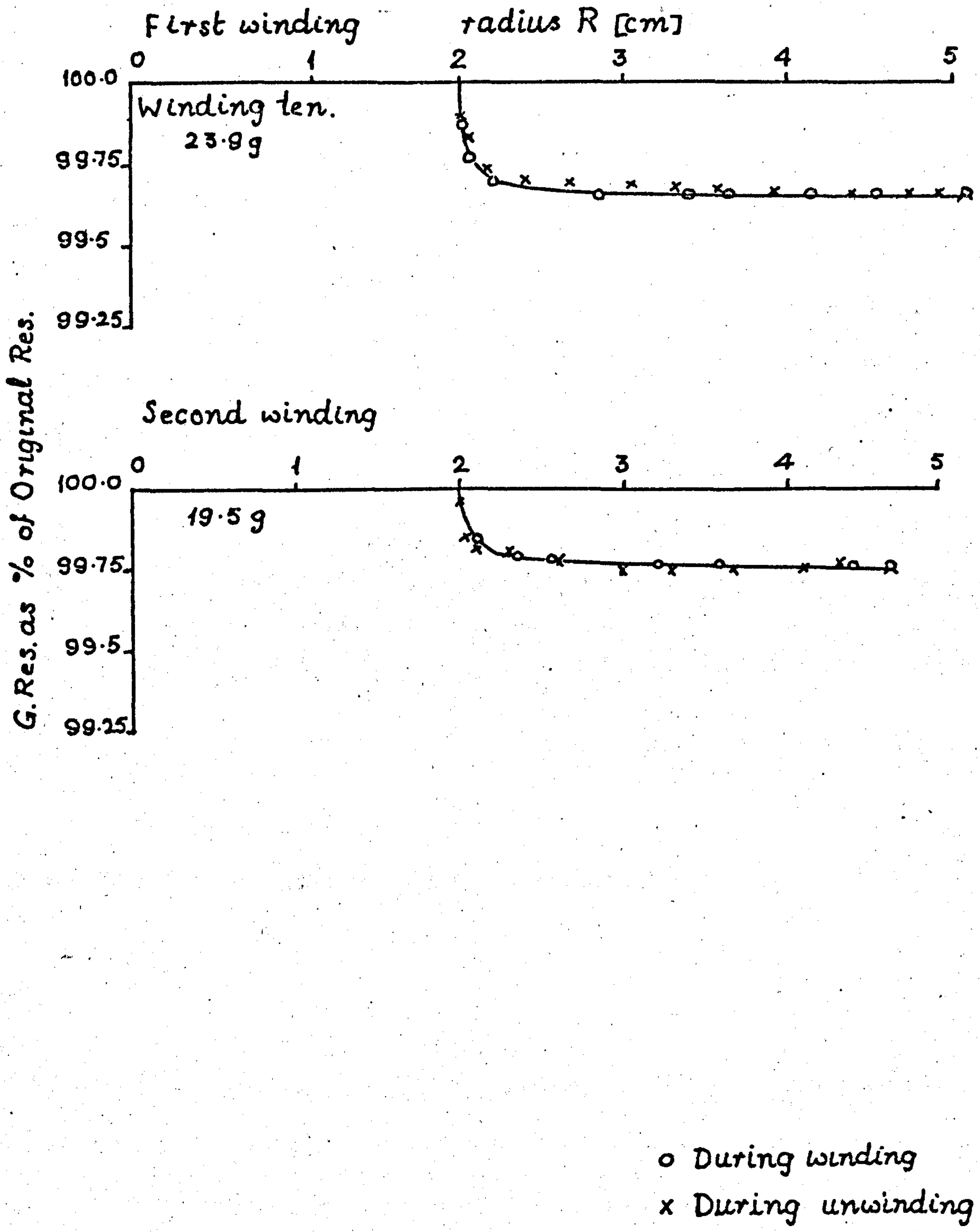
A comparison with Test 1 shows that the results of the tests 2.A and 2.B are similar to the results of Test 1. The curves of the tests 2.A and 2.B also show a quick large initial fall in the gauge resistance and then practically no change. The magnitude of the resistance change in Test 1 is higher, namely, 1.08% for Test 1 to 0.66% for Test 2.A, and 0.65% for Test 2.B. This difference is possibly due to a higher winding tension used in Test 1, i.e. 28.8g as compared to 21.1g for Test 2.A, and 21.7g for Test 2.B.

2.5.2 <u>The Slotted Paper Base. Test 3</u>.

An examination of the paper base for the previous tests showed it to be crimped after unwinding. The crimping of the paper base was considered to be undesirable because it shows that the paper is not able to follow the change in the circumference of the cheese surface as it shrinks under the pressure applied by the subsequent winding and instead of shrinking the paper base crimps and the reduction in the length of the wire may be less than it should be. In this test an attempt is made to eliminate the crimping of the paper base by adopting a slotted construction of the paper base for fixing the wire. This type of paper base did not show crimping after the unwinding. This is shown in Fig. 2.6. The strips of paper alternated with the blank spaces created by cutting alternate strips with a razor

TEST. 3.

Winding ten. for the base = 25 g.



Radial deformation of the cheese

FIG. 2. 7

blade and removing them. The wire was fixed to the strips of paper with Durofix and over the blank spaces the wire was free. Now as the cheese contracted circumferentially at this radius due to further winding the strips moved closer to each other and any crimping of the strips was not likely. The strips in moving closer to each other would allow the wire to shrink in the gaps. The paper was gummed to the cheese at two spots near the edge to prevent any movement of the paper with respect to the cheese but each individual strip was not gummed to the cheese.

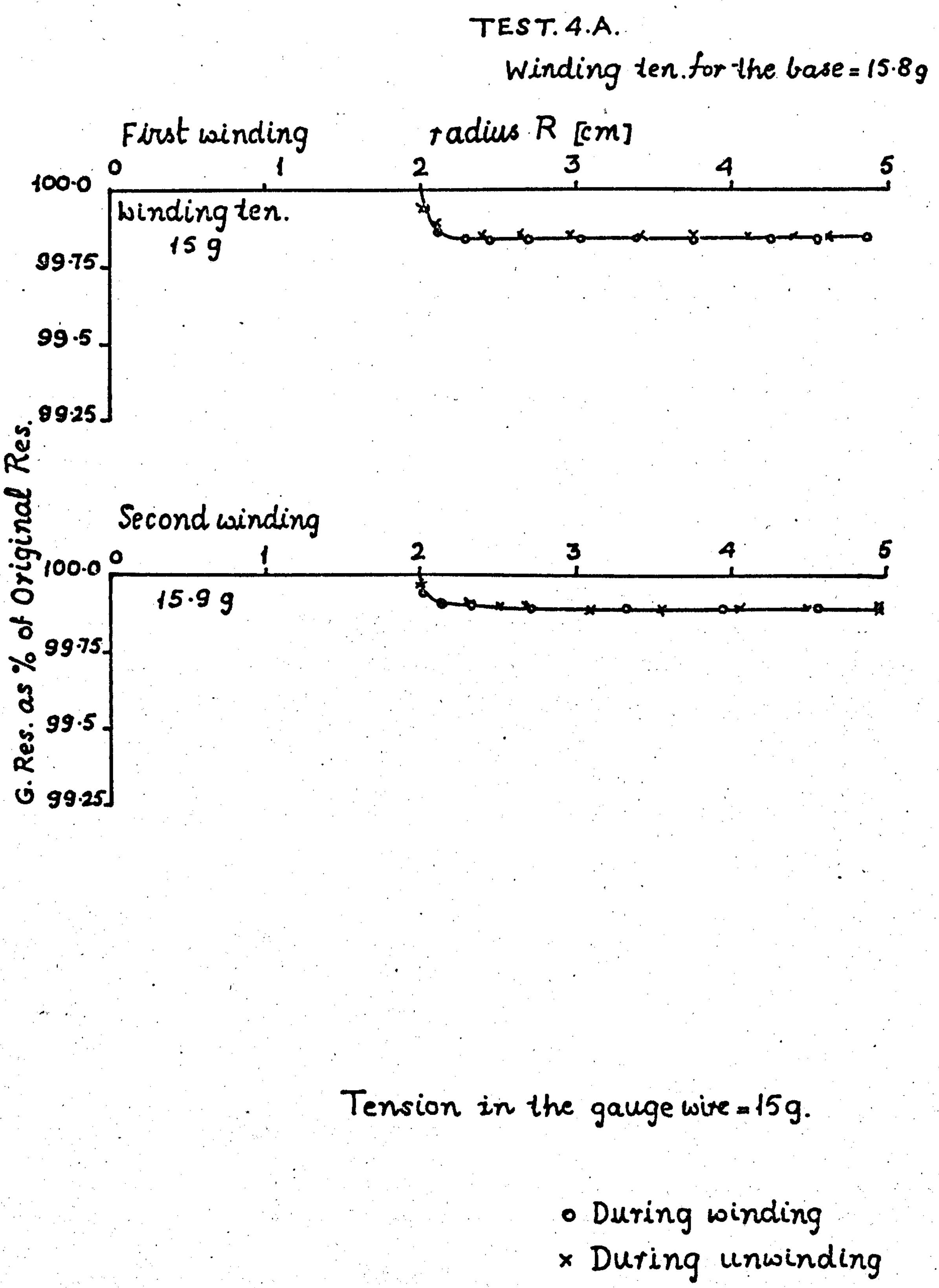
Fig. 2.7 shows the result of Test 3. Again the graph is of a similar shape. The magnitude of the change in the resistance of the gauge is 0.35% for the winding tension of 23.9g. The non-recoverable part of the change in the gauge resistance due to the permanent deformation of the cheese is about 28.5%. These values are different from the values of the previous tests and this difference is probably due to the changed value of the gauge factor because of the different type of paper base.

During second winding the winding tension is reduced to 19.5g from 23.9g and consequently the change in the gauge resistance is 0.25% as compared to 0.35%. The non-recoverable part of the change in the gauge resistance is about 20%.

2.5.3 Pre-strained Gauge Wire. Test 4

In Test 3, as the cheese compresses the strips at the gauge radius move closer to each other and while these strips move the wire, if not sufficiently prestrained while fixing, may become slack between the gaps. If this happens then the wire would cease to contract and after some initial change no further change in the resistance of the gauge would possibly be shown. This test is devised to check such a possibility. This is tested by preparing two gauges one with a high tension in the gauge wire and the other with a low tension in the gauge wire. Now if any slackening of the gauge wire due to the compression of the cheese should occur it should occur first in the gauge with the low tension in the wire. This gauge should show a smaller fall in the gauge resistance and should cease to show the change in the gauge resistance earlier as compared to the other gauge with the high tension in the wire. In order to ensure the comparability of the results the gauges are made on the same paper base.

In this test two gauges, 4.A and 4.B, were made by fixing two gauge wires on the same slotted paper base. Before fixing the wire of gauge 4.A was strained by a weight of 15g freely suspended from it. The wire of the gauge 4.B was similarly strained by a weight of 5g. Only $1\frac{1}{2}$ turns of wire was used in each case keeping the length of the wire used approximately equal to obtain nearly equal resistances for the two gauges. The two wires crossed each other at

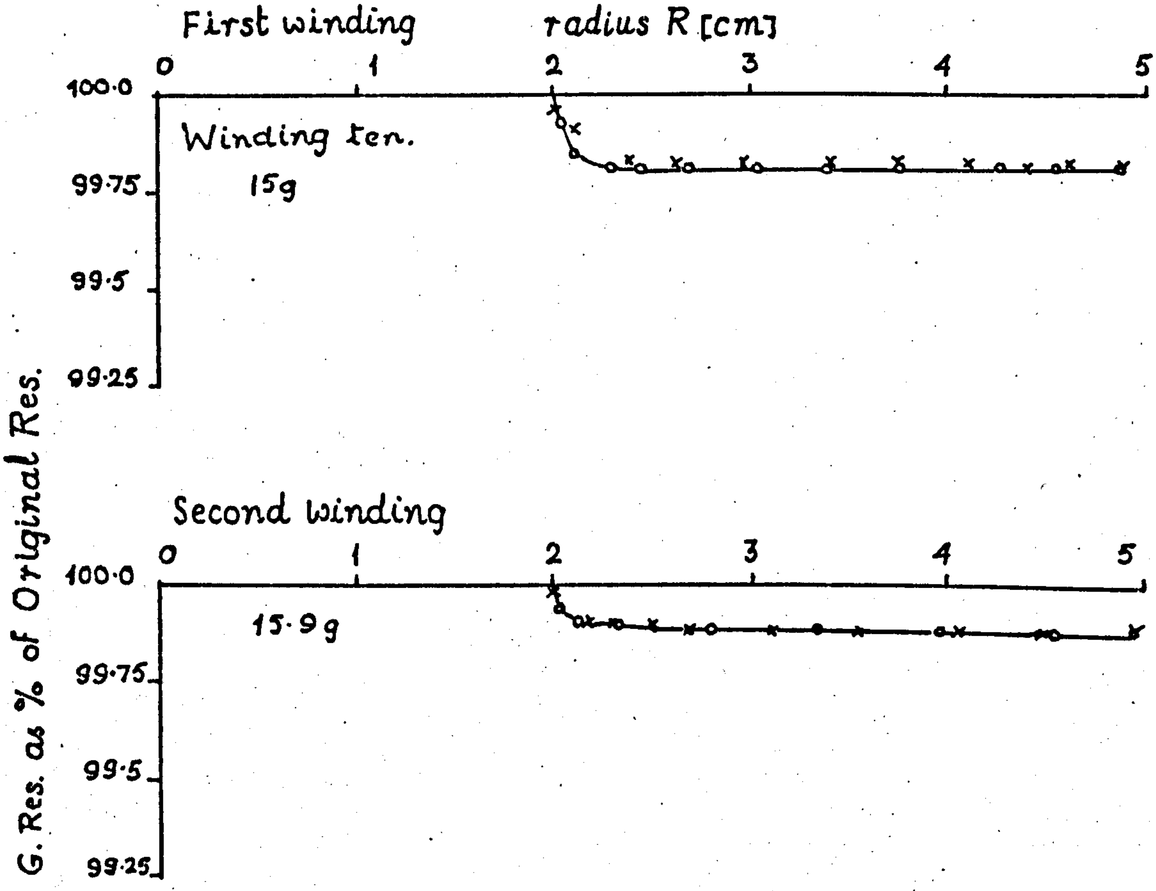


Radial deformation of the cheese

FIG.2.8

TEST.4.B

Winding ten. for the base = 15·8g.



Tension in the gauge wire = 5g

o During winding
x During unwinding

Radial deformation of the cheese

FIG. 2.9

one point and they were kept separate by a small piece of paper gummed over the wire of the first gauge. The copper foil strips were marked to indicate which leads were for which gauge.

The two gauges, which had nearly equal resistances, were balanced by a dummy gauge of nearly same resistance. The position of the pointer on the dial of the strain gauge apparatus was noted in each case at the start of the winding. The per cent change in the resistance of the gauge represented by one division of the dial was found out separately for each gauge. The test was carried out as before except that the changes in this case were read off on the dial for both the gauges by connecting each of them turn by turn to the strain gauge apparatus.

The results of this test for the gauges 4.A and 4.B are shown in Figs. 2.8 and 2.9 respectively. The compressional behaviour, represented by the changes in the gauge resistances, of the two gauges is similar. The magnitudes of the changes in the gauge resistances are also close, namely, 0.17% for 4.A and 0.18% for 4.B. This, incidentally, further shows that the results are reproducible as the behaviour of the compression in this test is similar to that of the previous tests.

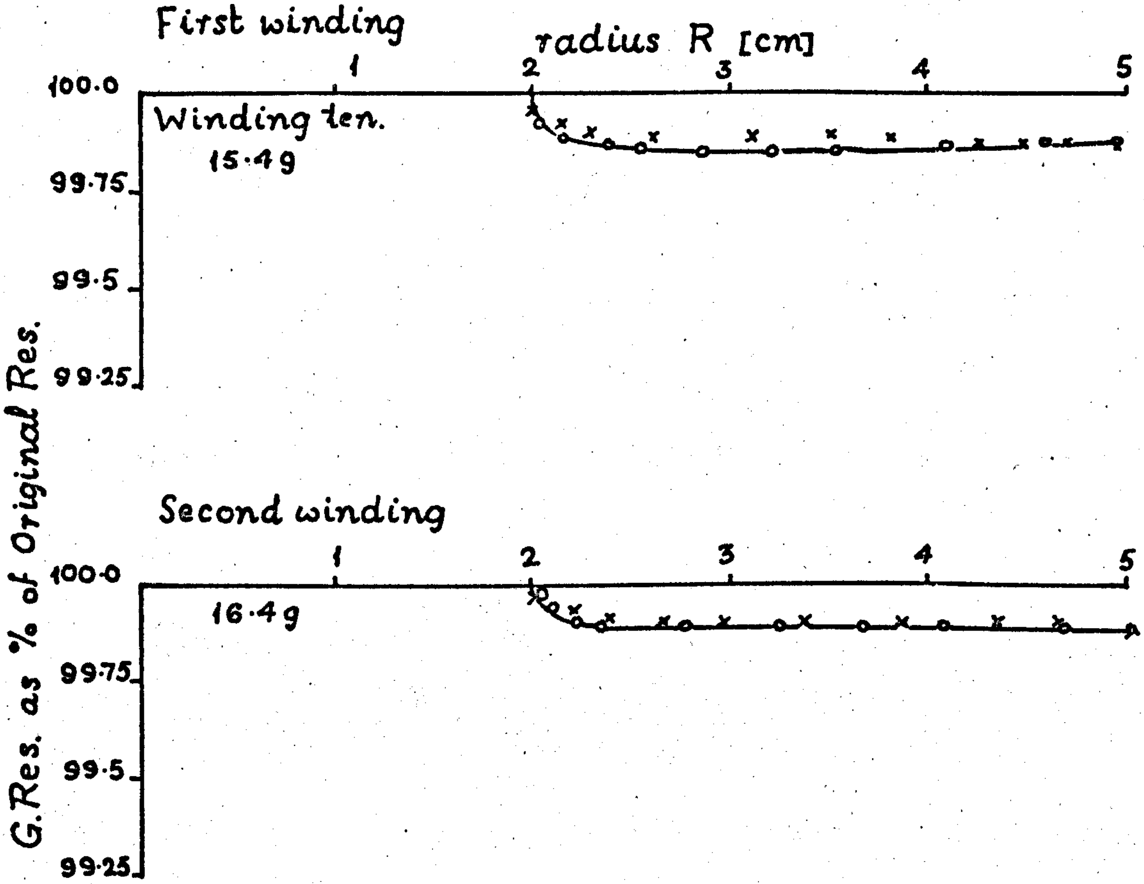
Gauge 4.B with the low tension of 5g in the gauge wire when fixed show a slightly higher fall of 0.18% in the gauge resistance as compared to that of 0.17% for Gauge 4.A with the high tension of 15g in the wire when fixed. The values for the second winding are 0.11% for Gauge 4.B and 0.1% for Gauge 4.A. This shows that the tensioning of the gauge wire differently while fixing has no apparent effect on the results with the winding tensions used during the windings and a tension of 5g in gauge wire when fixing it stretches the wire sufficiently to be able to measure the changes caused due to the compression of the cheese. However, a tension of 10g in the gauge wire is adopted as standard as this tension in the gauge wire will stretch the wire sufficiently to measure the compression of the cheese even with high winding tensions. A higher tension can result in breaking the wire if not handled very carefully.

2.5.4 Rubber Base Gauge. Test 5

The compressional behaviour of the cheese was further checked by making two gauges, 5.A and 5.B, on the same cheese base. In the gauge 5.B a rubber base was used for fixing the gauge wire. For preparing this gauge a band of rubber was placed on the cheese in a stretched condition with the gauge wire fixed to the rubber band firmly at all points. The band would contract as soon as the cheese compresses due to further winding and the wire which was fixed to the band would be forced to contract as well along with the band. In this case the possibility of the compression of the cheese not indicated by the change in the gauge resistance due to the slackening of the wire was considerably

TEST. 5.A.

Winding tension for the base = 15·4g



Tension in the gauge wire = 10g

o During winding

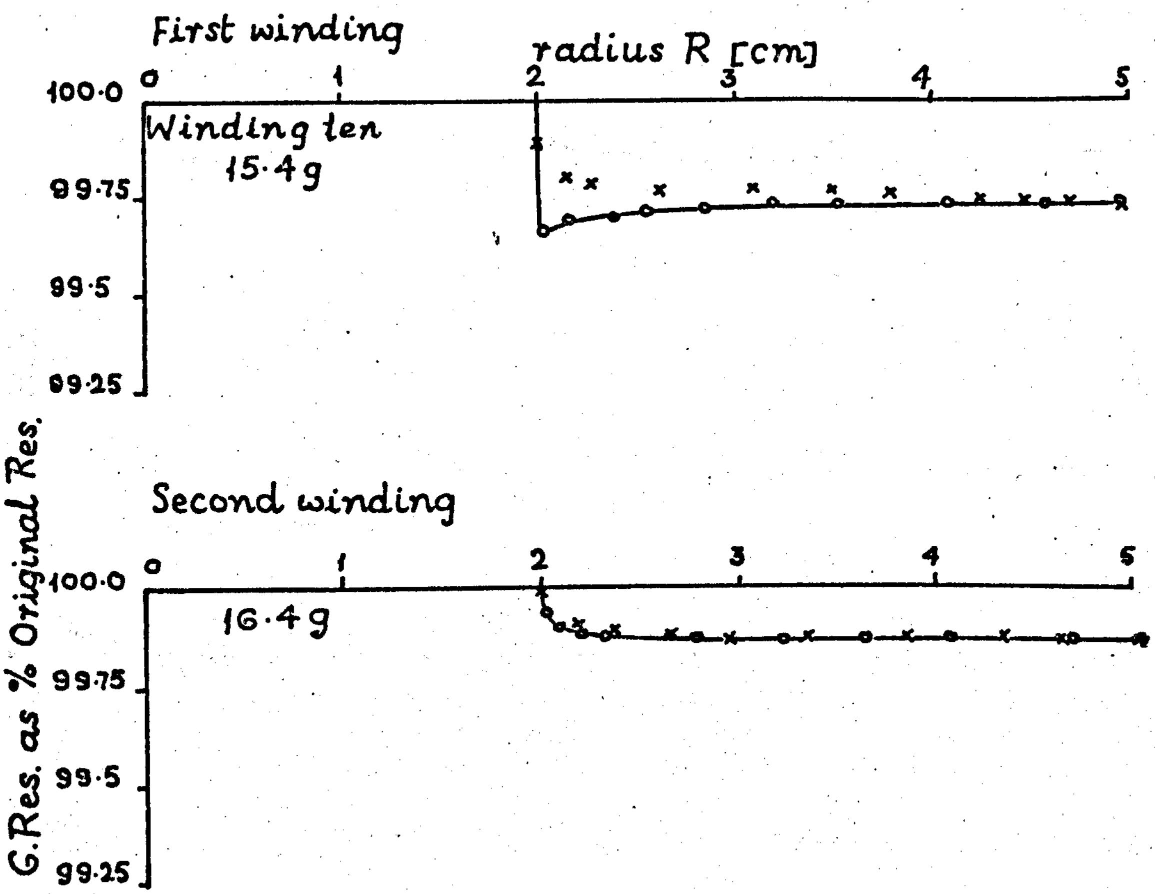
x During unwinding

Radial deformation of the cheese - Paper base

FIG. 2.10

TEST. 5.B.

Winding tension for the base=15·4g



Tension in the gauge wire=10g

o During winding

x During unwinding

Radial deformation of the cheese - Rubber base

FIG. 2.11

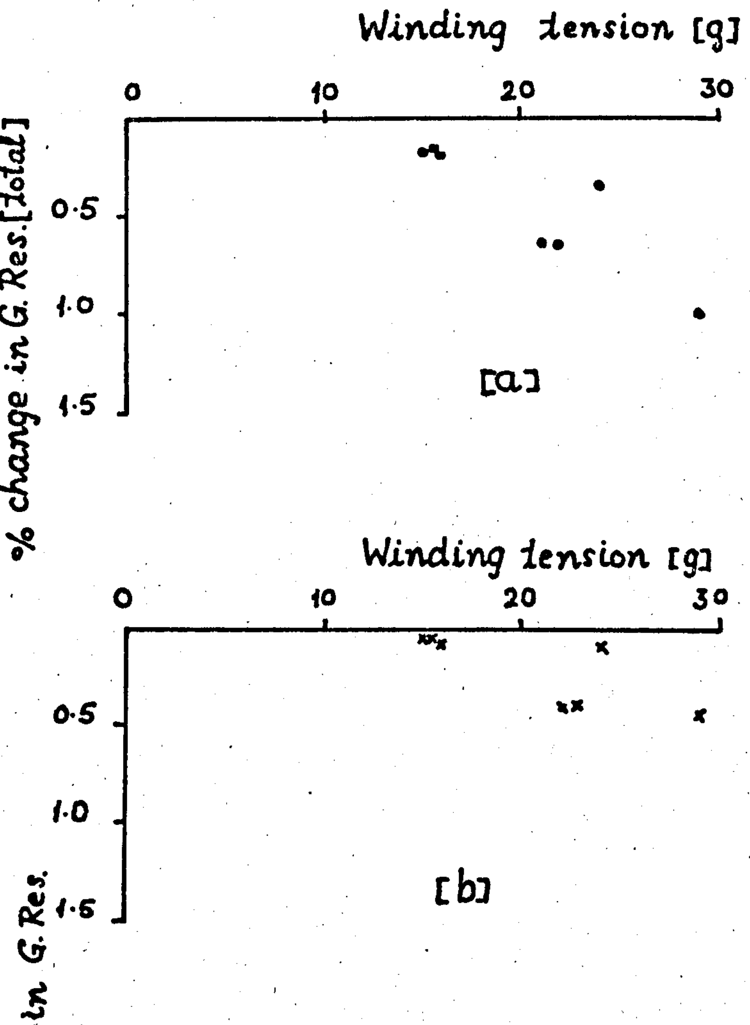
reduced. For comparing the results a slotted paper base gauge 5.A was also made on the same cheese along with the rubber base gauge 5.B having leads at the other end.

The construction of the rubber base gauge was similar to the paper base gauge. A band of rubber of about 2 cm wide was cut from a balloon and was placed on the cheese base of 2 cm radius. The rubber sleeve was in a slightly stretched condition and caused some compression of the cheese underneath it. The rubber band was not fixed to the cheese as it was tightly held over the cheese by its own tension and was not likely to slip over the cheese. Bostik no. 1 was used for fixing the wire and the copper foil strip leads to the rubber band. The resistances of the two gauges were kept close to each other and were balanced by the same dummy gauge. The conduction of the test was similar to the conduction of the previous test.

Figs. 2.10 and 2.11 show the results of the test for the gauges 5.A and 5.B respectively. The compressional behaviour indicated by the rubber base gauge 5.B, is nearly similar to that shown by the paper base gauge 5.A but is greater in magnitude. For the second winding also the changes in the resistances of the gauges are similar.

2.5.5 Conclusion

The results of these tests indicate that the method developed seems to be capable of showing the compression of the cheese qualitatively. Although it seems strange that after a few layers deformation stops a variety of different gauges confirm this behaviour. For a quantitative measurement of the compression it is necessary to calibrate the gauge. However, the great difficulty in calibrating the gauge is that there is no guarantee that the same calibration will hold for subsequent windings. Each time the winding is done the gauge factor is likely to alter as the paper on which the wire is fixed is not likely to keep exactly the same relative position with respect to the cheese. Also even slight crimping of the paper can alter the gauge factor. The calibration method is likely to involve imposition of pressure on the cheese and due to this gauge factor is likely to be incorrect even for the first winding. The imposition of the pressure on the cheese can deform it permanently which is undesirable. If the paper is firmly fixed on the cheese by some adhesive to avoid slip of the paper then that adhesive may alter locally the compressional property of the cheese. This makes it necessary to calibrate the gauge for each winding and the gauge once calibrated cannot be used repeatedly. Also as the cheese suffers a permanent deformation after each winding the results of two or more windings are not comparable. Moreover for each different test a new gauge is necessary. This is because the cheese base on which the gauge is built must be prepared with the same winding tension in the yarn as would be used for further



Radial deformation for different values of winding tension.

FIG. 2.12

winding. This rules out the use of the same gauge to measure the effects of different winding tensions. Also it is not possible to construct two gauges on exactly similar cheese bases and, therefore, it cannot be presumed that one of the gauges would have the same gauge factor as the other gauge which can be used for calibration only.

All these objections make accurate calibration of the gauge impossible and attention must be directed to the shape of the curve, i.e. the way the deformation changes rather to absolute values.

The results of the winding tests show qualitatively the compression of the cheese. There is a quick, large, initial compression of the cheese at any radius for some initial winding at that radius and as the winding continues further there is little increase in the value of the compression at that radius. This behaviour is consistently shown by all the winding tests.

The curve (a) of Fig. 2.12 shows the per cent change in the resistance of the gauge at the radius of 2 cm plotted against the winding tension in the yarn. This curve suggests the radial deformation of the cheese is probably related to the winding tension in the yarn. The curve (b) shows the permanent change in the gauge resistance after the unwinding is completed plotted against the winding tension in the yarn. This shows that the permanent radial deformation of the cheese at any radius is nearly half of the total radial deformation of the cheese at that radius as the cheese was built up to a given outer radius. This is shown by all the winding tests.

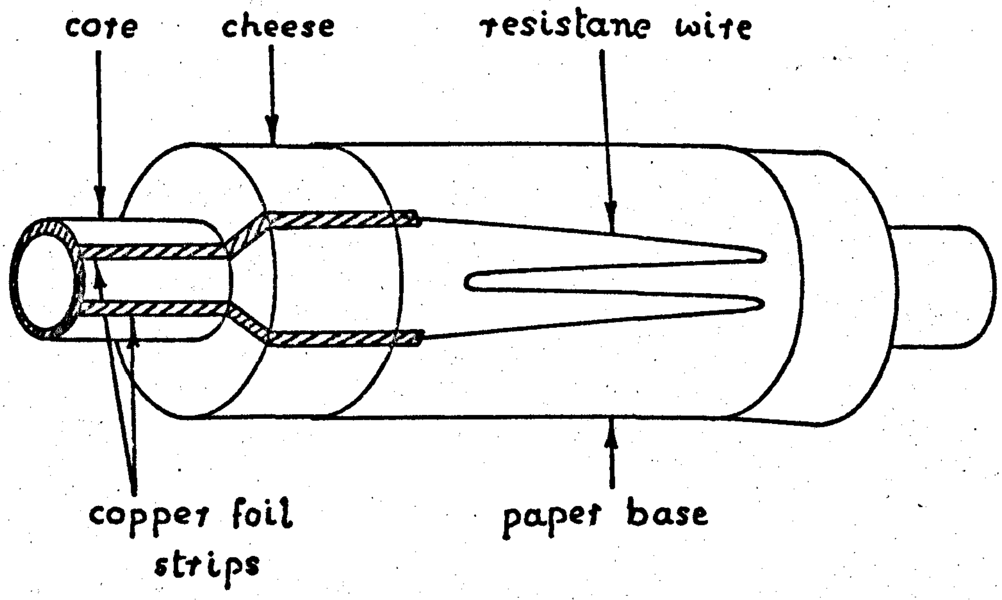
2.6 Measurement of Axial Strain

2.6.1 Introduction

An approach similar to the one used for measuring the radial deformation of the cheese was used to measure the axial deformation of the cheese by fixing the gauge wire nearly parallel to the axis of the cheese on a rectangular piece of paper wrapped over the cheese and a large change in the resistance of the wire was shown. However when the gauge wire was fixed on a helically wound strip of paper on the cheese to avoid the slip of the paper the wire snapped after showing a much larger change in the resistance. Hence this method was abandoned.

An alternative, simple, mechanical gauge is developed to measure the axial deformation of the cheese and consists of two stiff cardboard tags at the two ends of the cheese held in position by two strips of paper gummed to the cheese. The axial deformation of the cheese at the gauge radius is given by the change in the distance between the tags measured by a vernier calliper.

Winding tests were conducted to measure the effect of different winding tensions on the axial deformation of the cheese, the axial deformation of the cheese at different radii and the effect of repeated



Strain gauge for indicating axial deformation

FIG. 2.13

windings on the axial deformation of the cheese at the same radius. Another set of tests was done with a different spacing between the adjacent wraps of the yarn.

2.6.2 Electrical Resistance Method

(a) Preliminary Test. Test 6

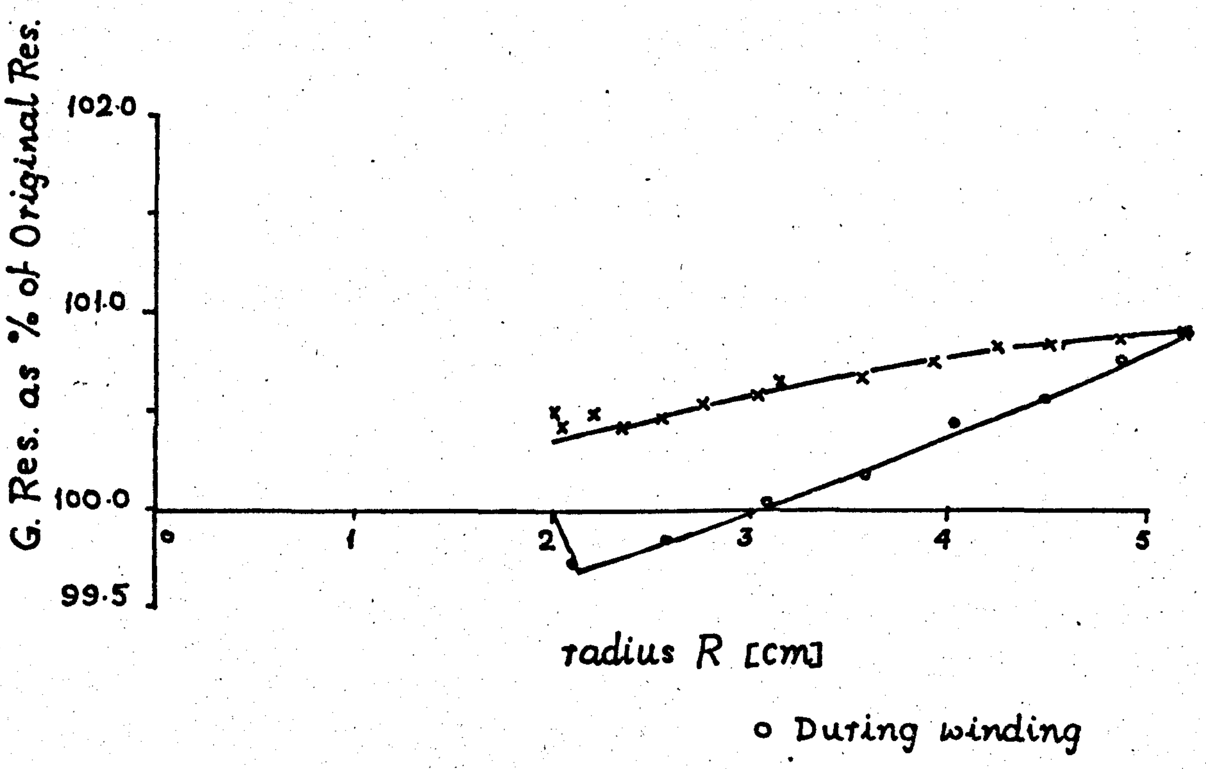
The construction of this gauge was similar to the one used for measuring the radial deformation of the cheese and is shown in Fig. 2.13. It consisted of a rectangular piece of thin paper encircling the cheese. The two ends of the paper which overlapped each other were gummed together. Two copper foil strips 0.5 cm wide were fixed to one end of the paper with Durofix and acted as leads for the gauge. The resistance wire was fixed to the paper with Durofix keeping it at a small angle to the axis of the package as shown in the figure. The two ends of the wire were soldered to the respective copper foil strips. The gauge was prepared at a radius of 2 cm. A similar dummy gauge with nearly same resistance was also made.

The measurement of the change in the gauge resistance was done by using a Brüel and Kjœr Strain Gauge Apparatus. The procedure was the same as was used for measuring the radial strain. In this case also the per cent change in the gauge resistance was proportional to the axial strain of the cheese at the gauge radius.

TEST. 6

Winding tension for the base = 20·1g

Winding tension = 25·5g



Axial deformation of the cheese.

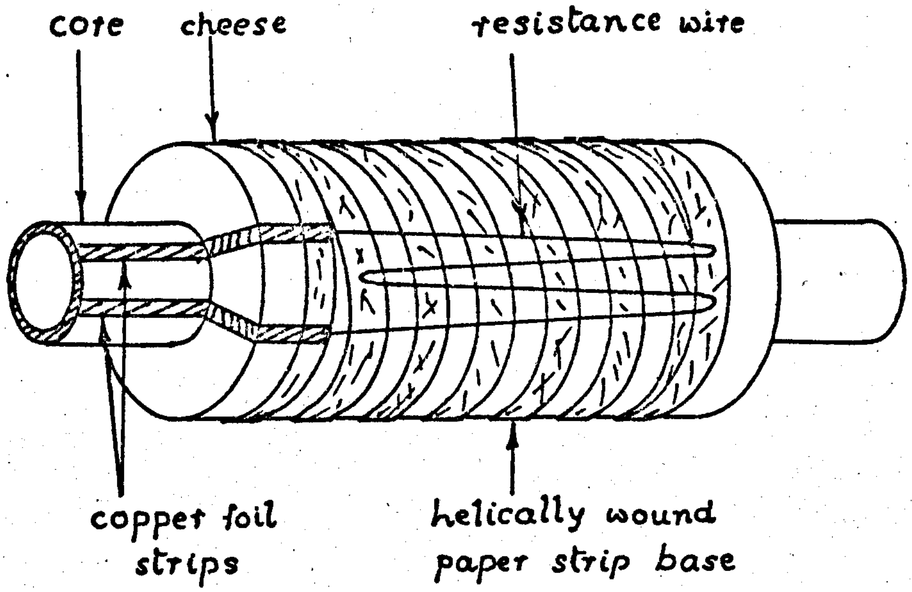
FIG. 2.14

Fig. 2.14 shows the per cent change in the gauge resistance, which is representative of the axial strain at the gauge radius of 2 cm, plotted against the outer radius of the cheese. The curve shows that the resistance of the gauge and hence the axial length of the package falls initially with an increase in the outer radius of the cheese and then starts increasing as the winding continues. The change in the gauge resistance is about 0.9% when the outer radius is 5.2 cm. The return curve shows the axial strain during unwinding. Initially the rate of fall of the axial strain is slow but increases later as the unwinding proceeds towards the gauge radius. A permanent strain is indicated by the permanent change in the resistance of about 0.5%.

This test was used as a preliminary test to ascertain the possibility of measuring the axial strain of the cheese. However for this method to be useful it has to be developed further. The main objection to this type of gauge is the likelihood of the paper carrying the resistance wire slipping over the cheese as the paper is not fixed to the cheese and in such an eventuality the gauge would fail to show the full axial deformation. Also as the paper covers a large part of the cheese it could affect the transmission of shear force between the layers and thus disturb the behaviour of the whole cheese.

(b) <u>Development of the Gauge. Test. 7</u>

In this case the paper for mounting the resistance wire was



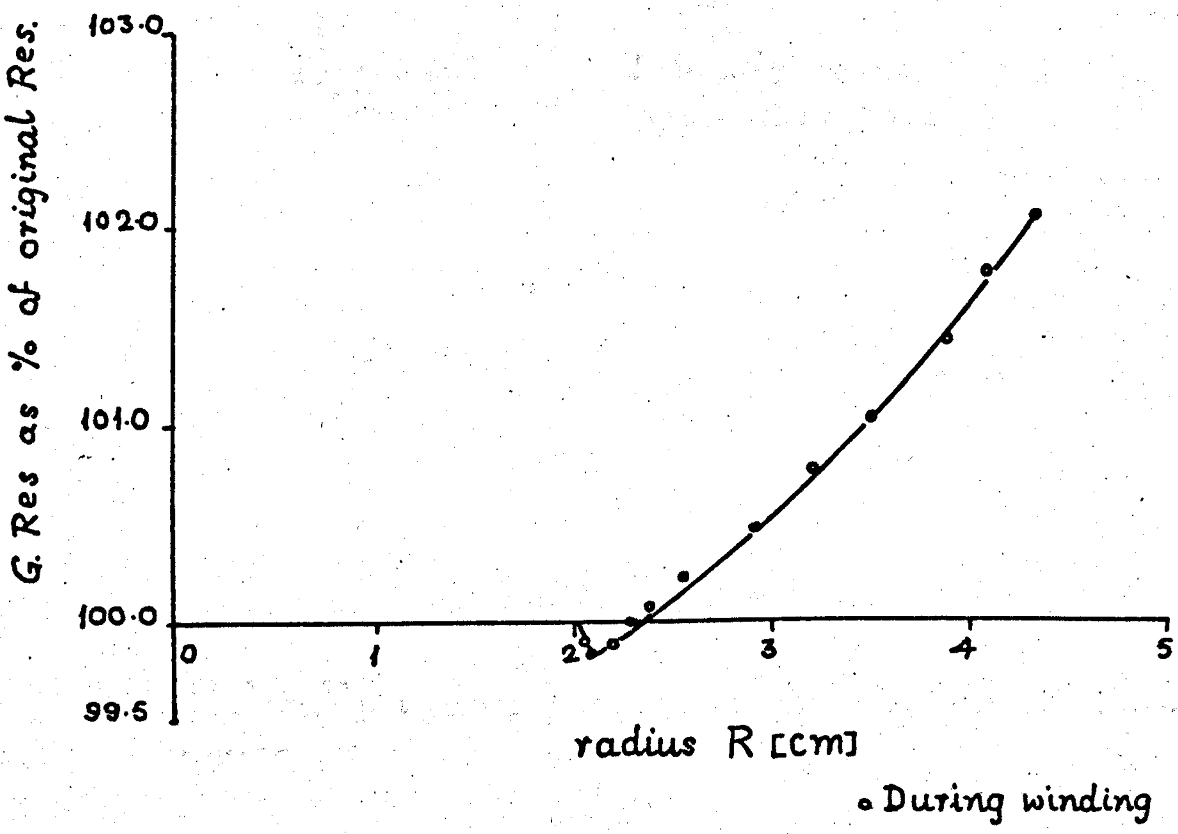
Helically wound paper strip base for axial strain gauge

FIG. 2.15

TEST. 7.

Winding ten. for the base = 24.5g

Winding tension = 24.5 g



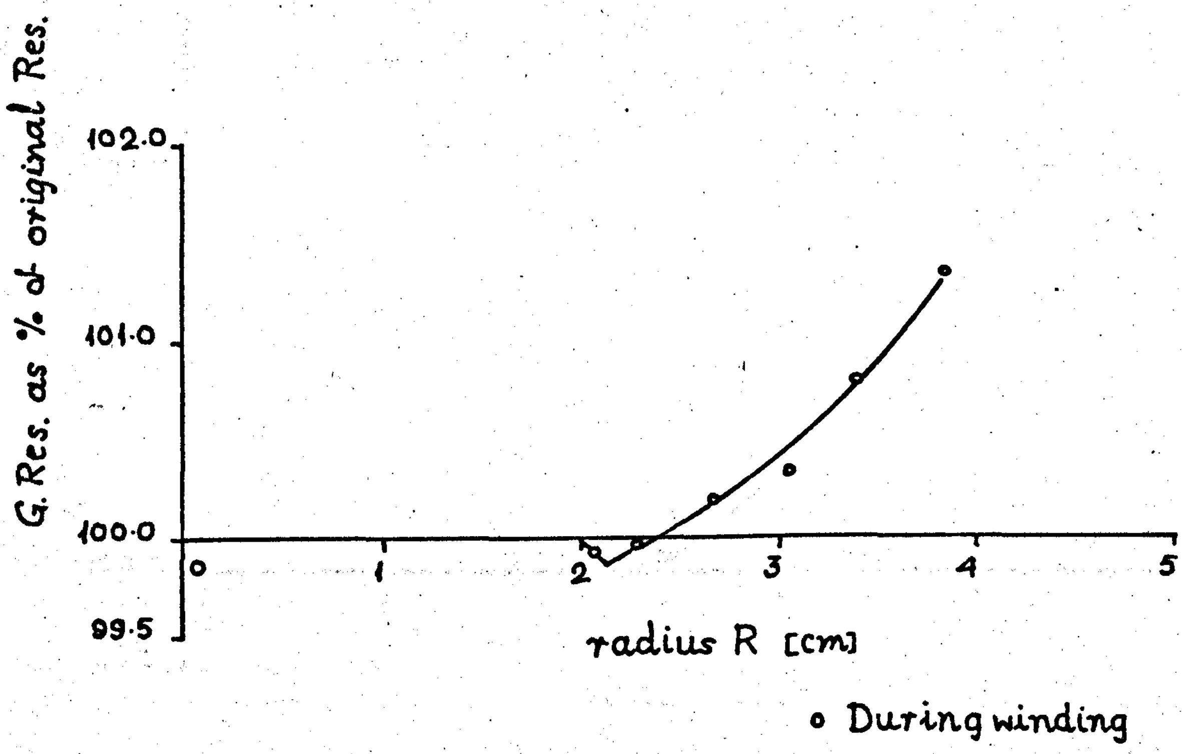
Axial deformation of the cheese

FIG. 2.16

TEST.8.

Winding ten. for the base = 23.7g

Winding tension = 25g



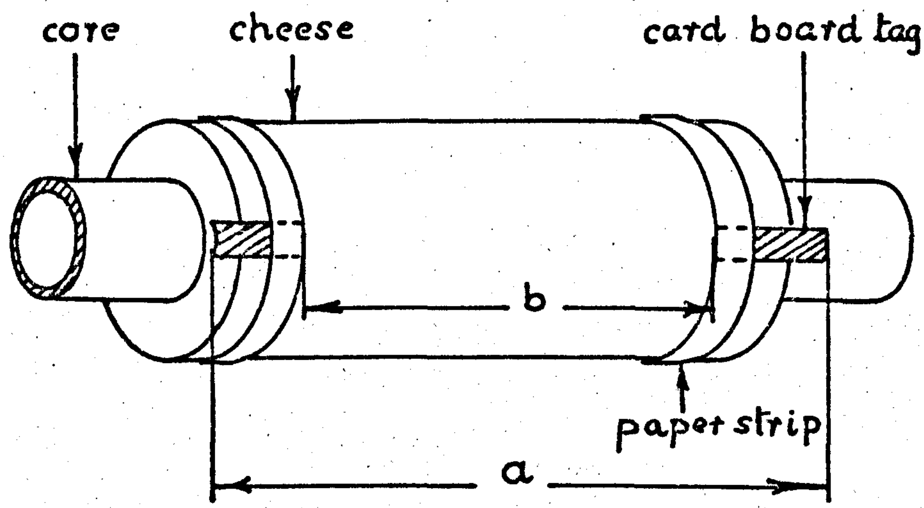
Axial deformation of the cheese

FIG. 2.17

replaced by a long strip of paper 0.5 cm wide helically wound over the width of the cheese of radius 2 cm with a gap of about 0.2 cm between the successive wraps of the strip. The two ends of the helically wound strip were gummed to two other strips at the end of the cheese. Each of the end strips was about 0.5 cm wide and was gummed to the cheese at about 1 cm from the edge of the cheese keeping it parallel to the edge. The resistance wire was fixed to the strip as shown in Fig. 2.15 with its ends soldered to the two copper foil strips which acted as leads for the gauge. A similar dummy gauge was also prepared. This type of gauge is shown in Fig. 2.15.

In this type of construction it is unlikely that the paper strip slips over the cheese as it does not resist axial expansion, also it permits contact between the two adjacent layers containing the gauge.

Figs. 2.16 and 2.17 show the resistance of the gauge as per cent of the original resistance at the radius of 2 cm as the cheese was built up further in Test 7 and Test 8. As before these show a slight initial fall in the gauge resistance for the immediate winding and then an increase in the resistance for the subsequent winding. The magnitude of the change in the gauge resistance in Test 7 is nearly 3.7 times to that in the previous test when the outer radius is 4.35 cm. But possibly due to high extension the gauge wire snapped when the outer radius was 4.35 cm and no further winding was done.



Gauge for measuring axial deformation of cheese.

FIG. 2.18

The test was repeated again as Test 8 with a new gauge but this time also the gauge wire snapped presumably due to high extension and because of this, this method was abandoned.

This it seems established that there is axial expansion of the cheese greater than either paper or wire will follow. The one is strong and slips while the other is weak and breaks. In view of the 2% change of resistance before the wire broke an extension of at least 1% of the length might be occurring. If so this should be measurable with reasonable accuracy by simpler methods.

2.6.3 Mechanical Gauge for Measuring Axial Deformation

(a) Construction of the Gauge

Fig. 2.18 shows the construction of this type of gauge. It consisted of two stiff cardboard tags each 0.5 cm wide and 2 cm long. Each of the tags was gummed to a separate thin paper strip about 1 cm wide. These strips along with their tags were gummed round at each end of the cheese encircling it. The paper strip was at about 0.5 cm from the edge of the cheese and was parallel to it. With this arrangement the tags project out from the cheese.

The distances between the far ends and the near ends of the tags were measured by a vernier calliper capable of measuring 0.001 in and were marked as 'a' and 'b' respectively. The gauge length was taken as 'b'. The strain was measured by measuring the length 'a' at intervals by stopping the winding and the change in the distance 'a' was expressed as the per cent change of the gauge length 'b'. This was done on the assumption that the tags themselves were not strained and the movement of the tags with respect to each other in the axial direction represented the deformation of the axial length 'b'. In fact it probably corresponds to a length a little greater than 'b'.

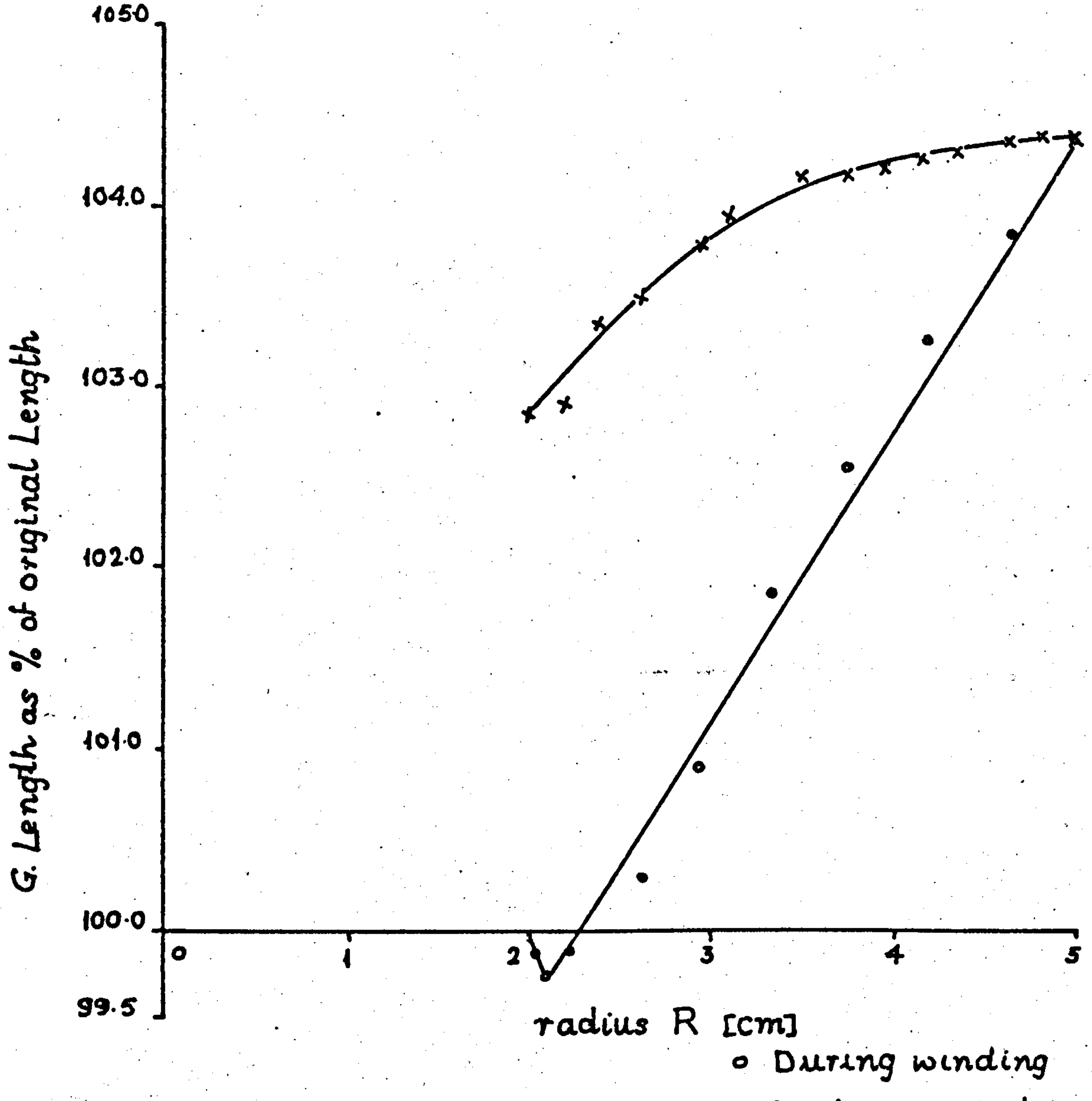
(b) Procedure

After preparing the cheese base of the required radius two strips of thin paper, each 1 cm wide and long enough to encircle the cheese were cut out and on these stiff cardboard tags of 2 cm x 0.5 cm size were gummed. Then each of these strips was gummed to each end of the cheese at about 0.5 cm from the edge of the cheese taking care that the long edges of the tags were in the same line parallel to the axis of the package. The tags came in between the paper strip and the cheese. The cheese was left for some time to allow the gum to dry. The tags were firmly fixed to the cheese. Before the commencement of the winding the distances 'a' and 'b' were measured. The distance 'a' was measured at intervals by stopping the winding. Similar measurements were taken during the unwinding of the cheese. These were tabulated and the axial strain of the length was expressed as the per cent change in the gauge length 'b'. The observations and calculations for winding tests are given in Appendix B.

TEST. 9.

Winding tension for the base = 24.1g

Winding tension = 24.6 g



Axial deformation of the cheese

FIG. 2.19.

This type of gauge was simpler to make and use and took less time to make it as compared to electrical resistance strain gauge. Also no calibration was required. However its success was due to the large strain of the length and the effect of slight inaccuracy in measuring the strain was negligible.

2.6.4 Preliminary Test. Test 9

Fig. 2.19 shows the gauge length at the radius of 2cm as per cent of the original gauge length with the outer radius R during winding and unwinding. The cheese surface at the radius of 2 cm contracts axially for a slight initial increase in the outer radius. But as the outer radius R increases further from about 2.25 cm the cheese surface expands axially at a steady rate. During unwinding the cheese surface contracts slowly at first and rapidly later. The expansion is large and is about 4.35% when the outer radius is 5 cm. It also shows a fairly large permanent strain of about 2.8% when the unwinding is complete.

The behaviour of the axial strain in this test is similar to that of Test 7. The magnitude of axial strain, when R is about 4.25 cm, is 3.15% as compared to that of 1% of Test 7, presuming a gauge factor of 2 for that gauge. For this gauge to show equally high strain the gauge factor should be low - about 0.6 instead of 2 usual for made up gauges. Yet the similar behaviour of the two types of gauges suggests that the former method is capable of measuring the axial strain up to a certain level.

2.6.5 <u>Winding Tests</u>

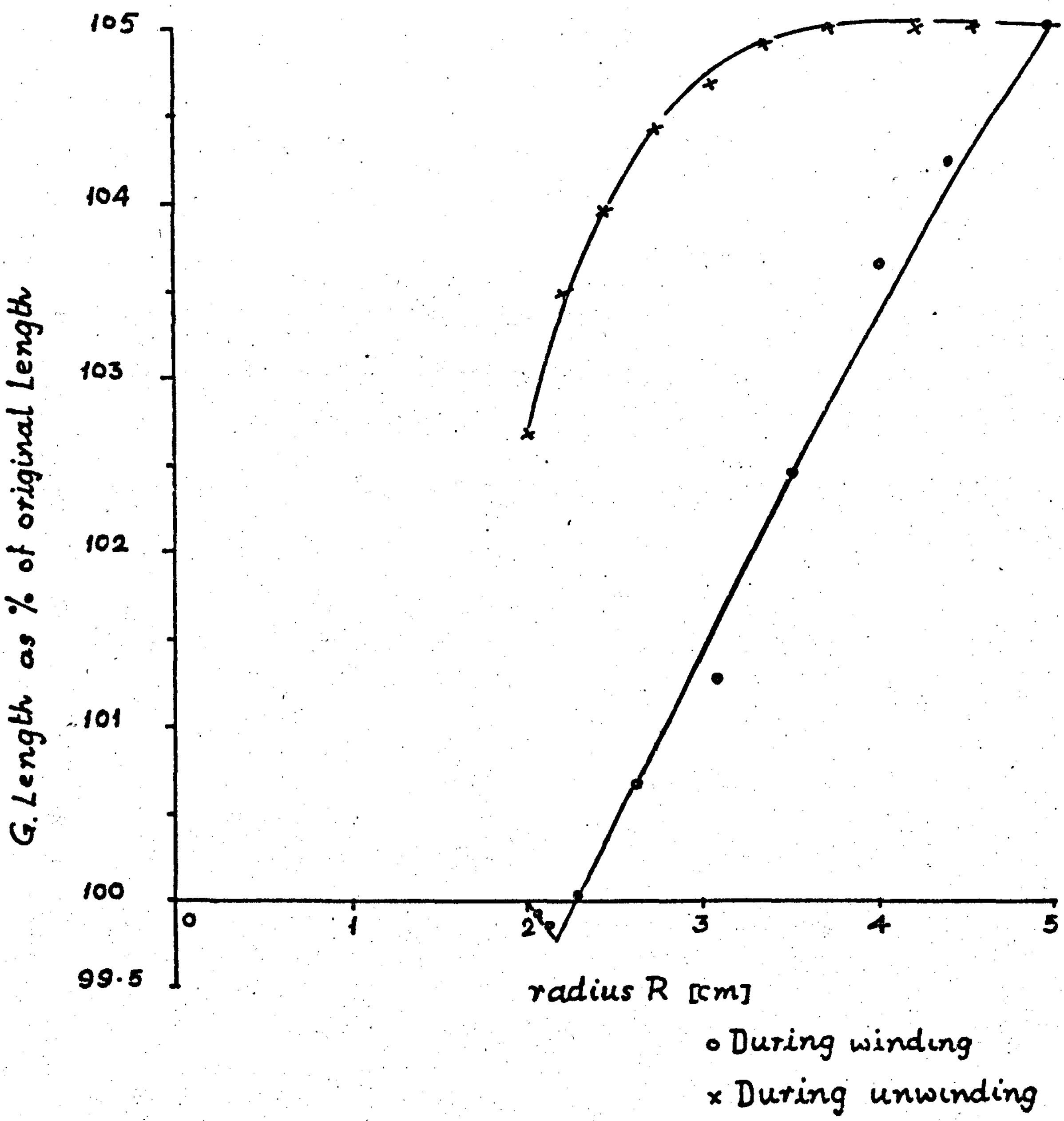
The winding tests for measuring the axial strain of the cheese were divided into two groups according to the space between the adjacent wraps of yarn in the cheese. In the first group of tests there were 20 threads per cm at the core. The effect of varying the winding tension was mea sured on a base prepared at the same winding tension at which the subsequent winding was done. Also the effect of repeated windings on the same base was observed. This concluded by measuring the axial deformation of the same cheese at different radii. In the second group of tests the above tests were repeated with 10 threads per cm at the core instead of 20.

Another test was conducted to observe the effect of varying the winding tension in the yarn on the shear force causing the axial deformation at the gauge radius. In this test the cheese bases for different windings were prepared at the same winding tension in the yarn. In such a case the shear force at the gauge radius should be proportional to the axial strain of the cheese at that radius while the material resisting deformation is the same.

TEST. 10. i.

Winding ten. for the base = 8g

Winding tension = 8·7g



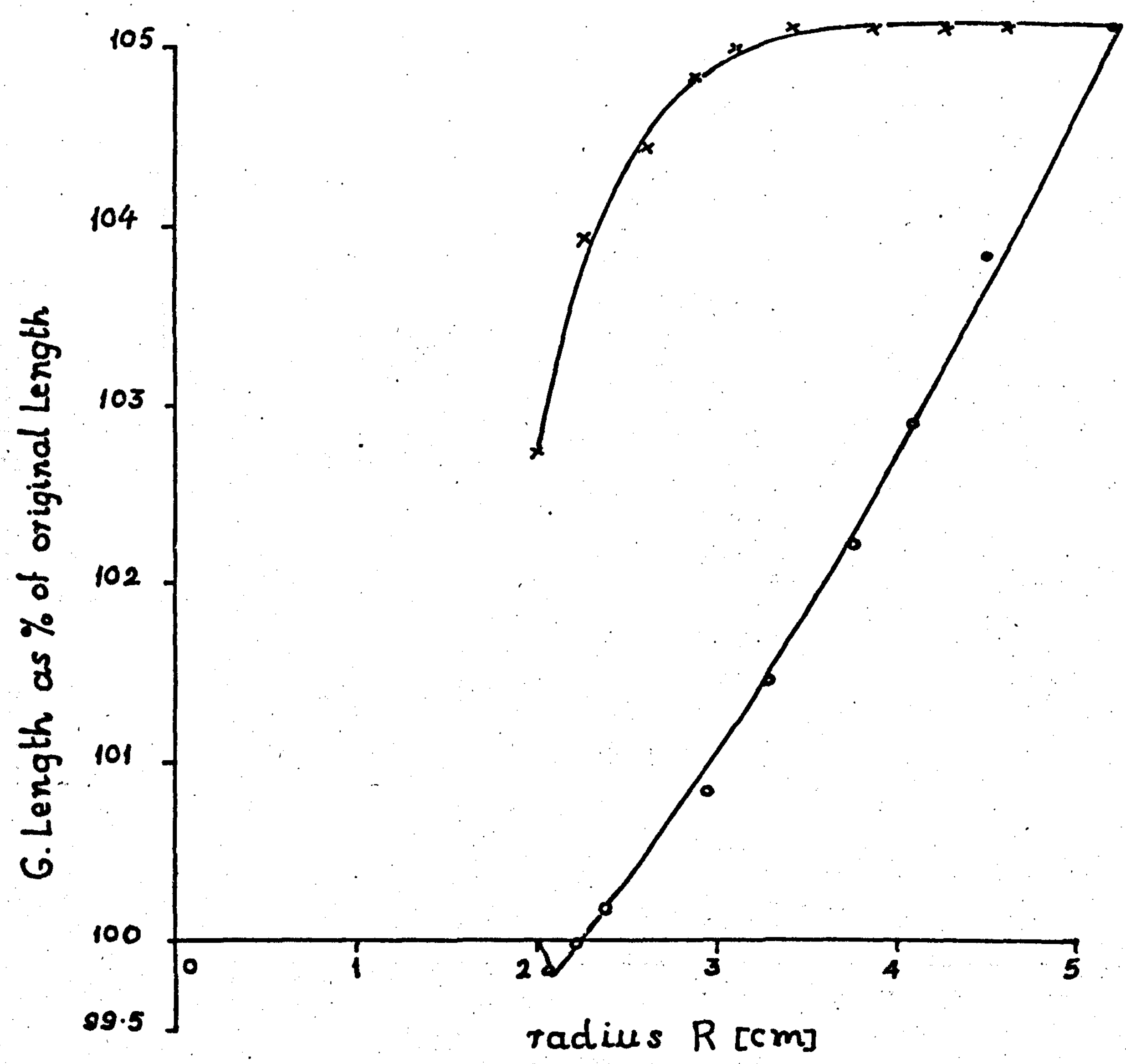
Axial deformation of the cheese

FIG. 2.20

TEST. 10. ii.

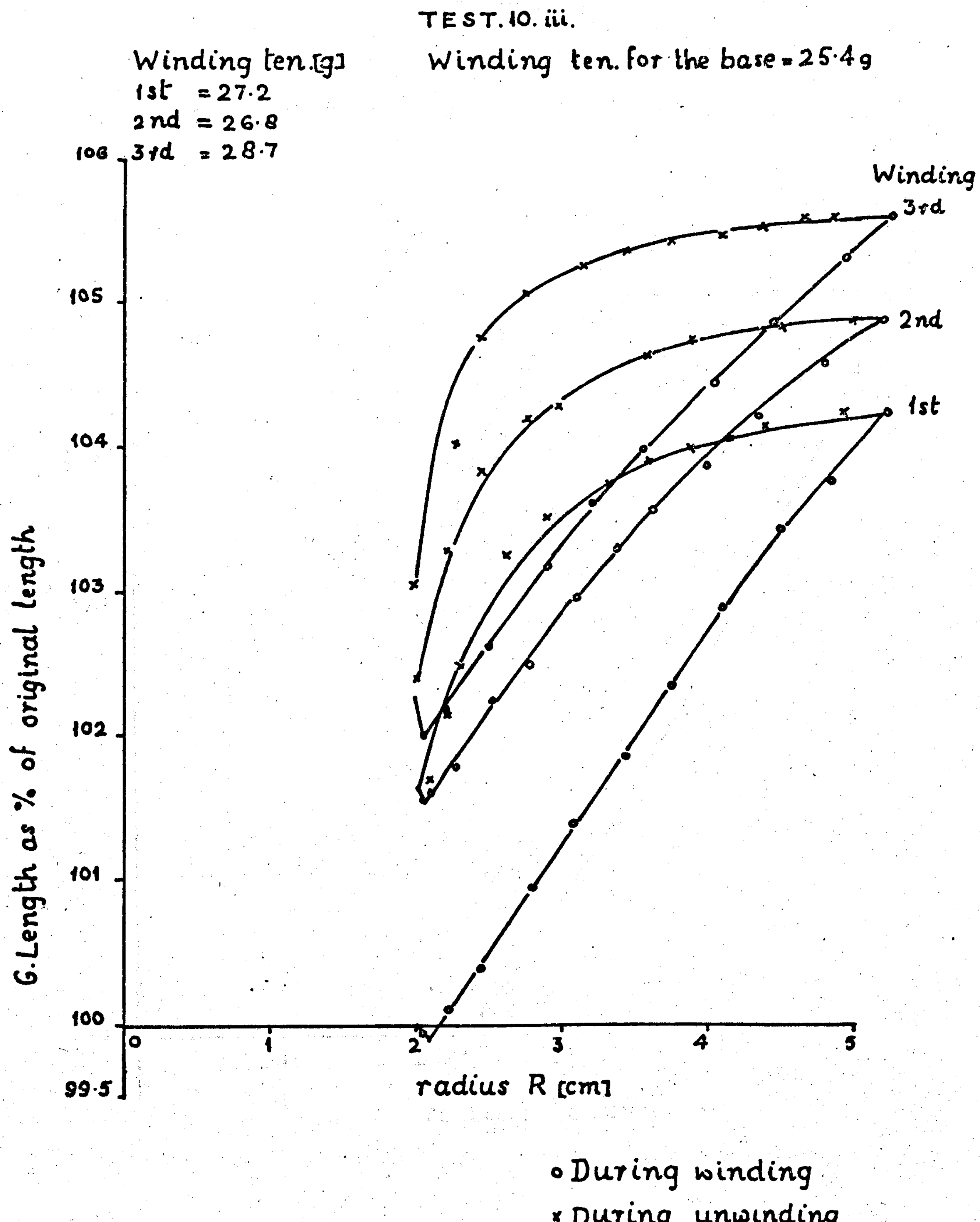
Winding ten. for the base = 20·6 g

Winding tension = 20·3 g



Axial deformation of the cheese

FIG. 2.21



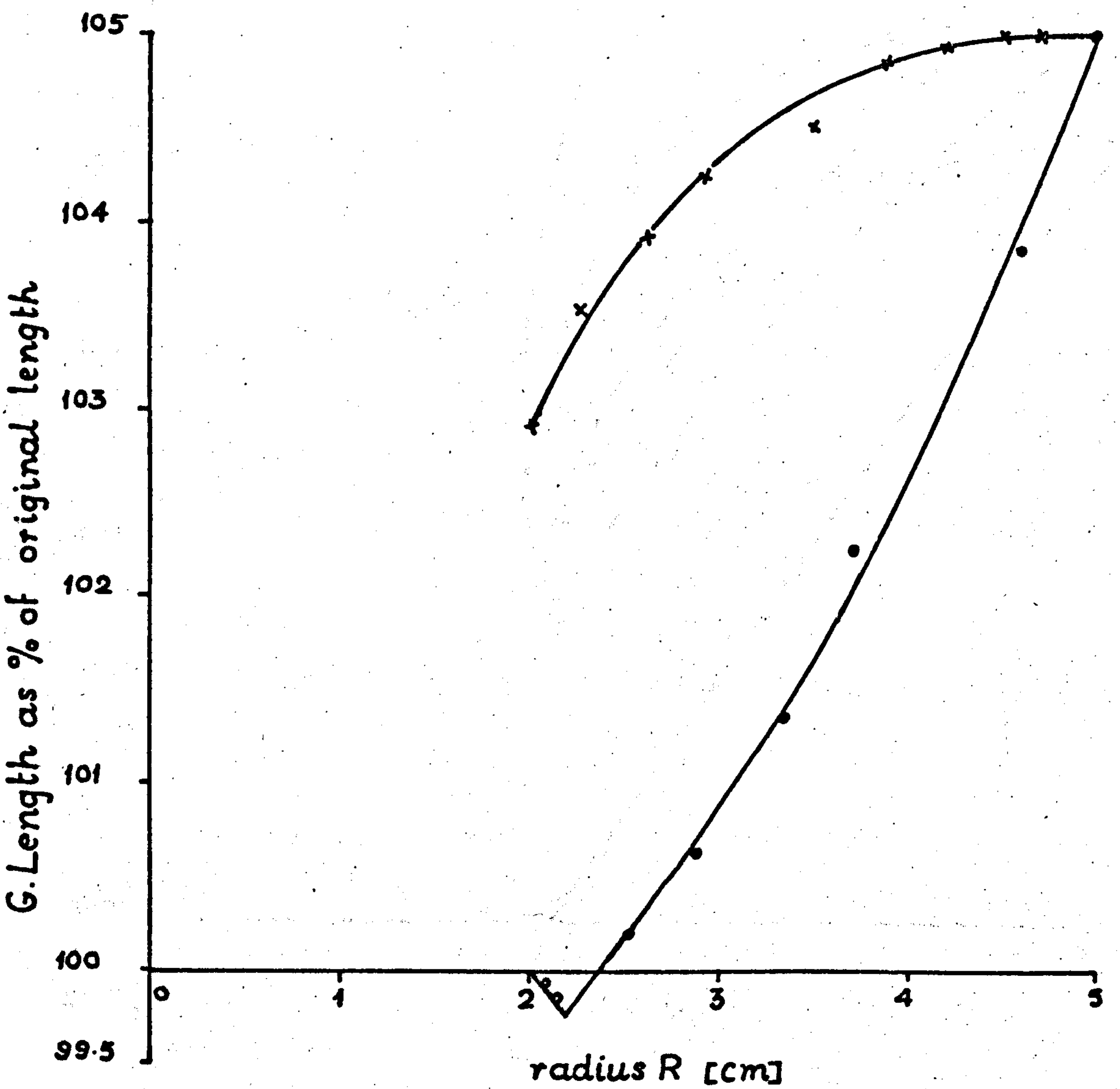
Axial deformation of the cheese

FIG. 2.22

TEST. 10.iv.

Winding ten. for the base = 26·2g

Winding tension = 27·3g



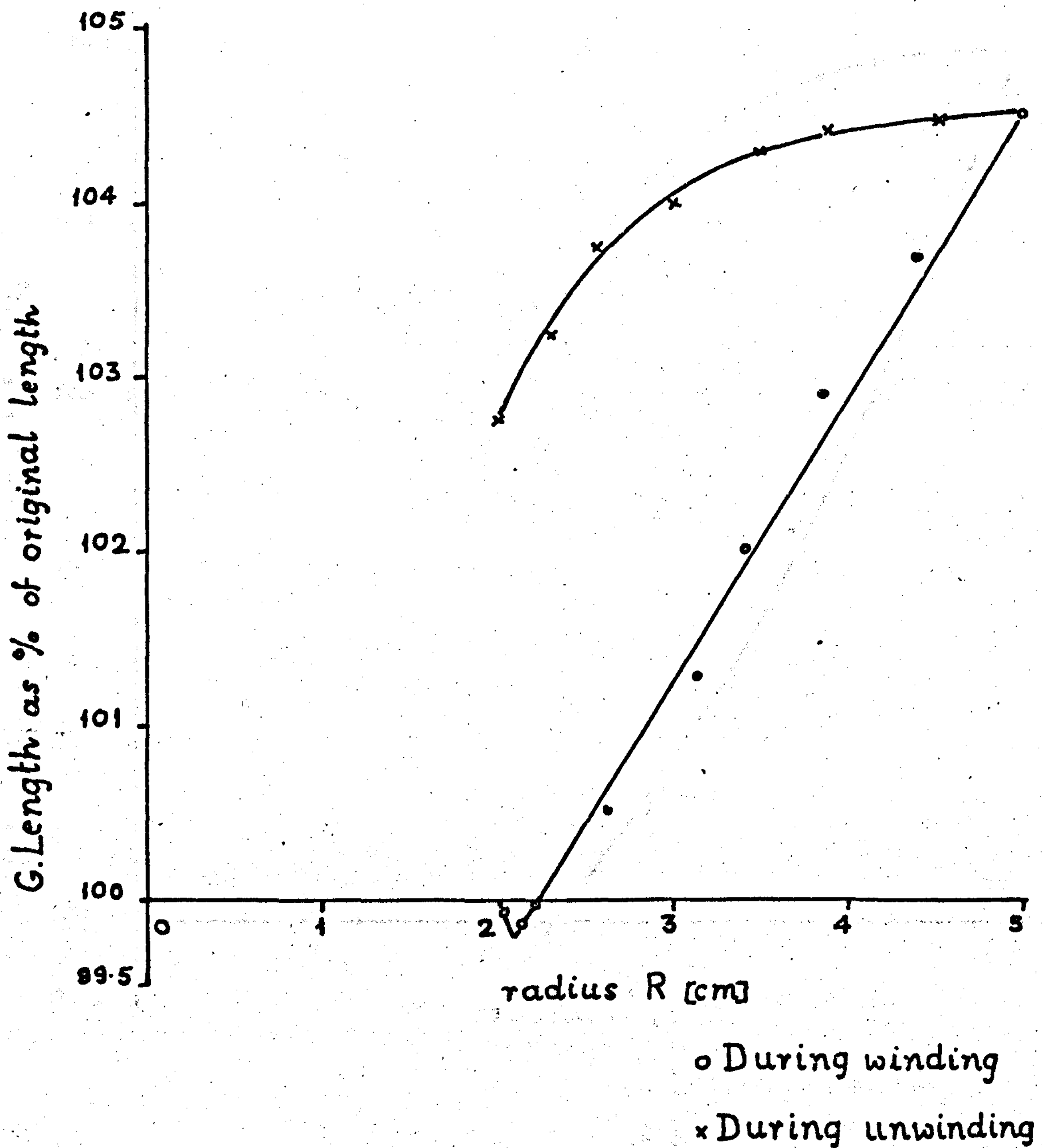
Axial deformation of the cheese

FIG.2.23

TEST. 10.v.

Winding tension for the base = 33·2 g

Winding tension = 33·1 g



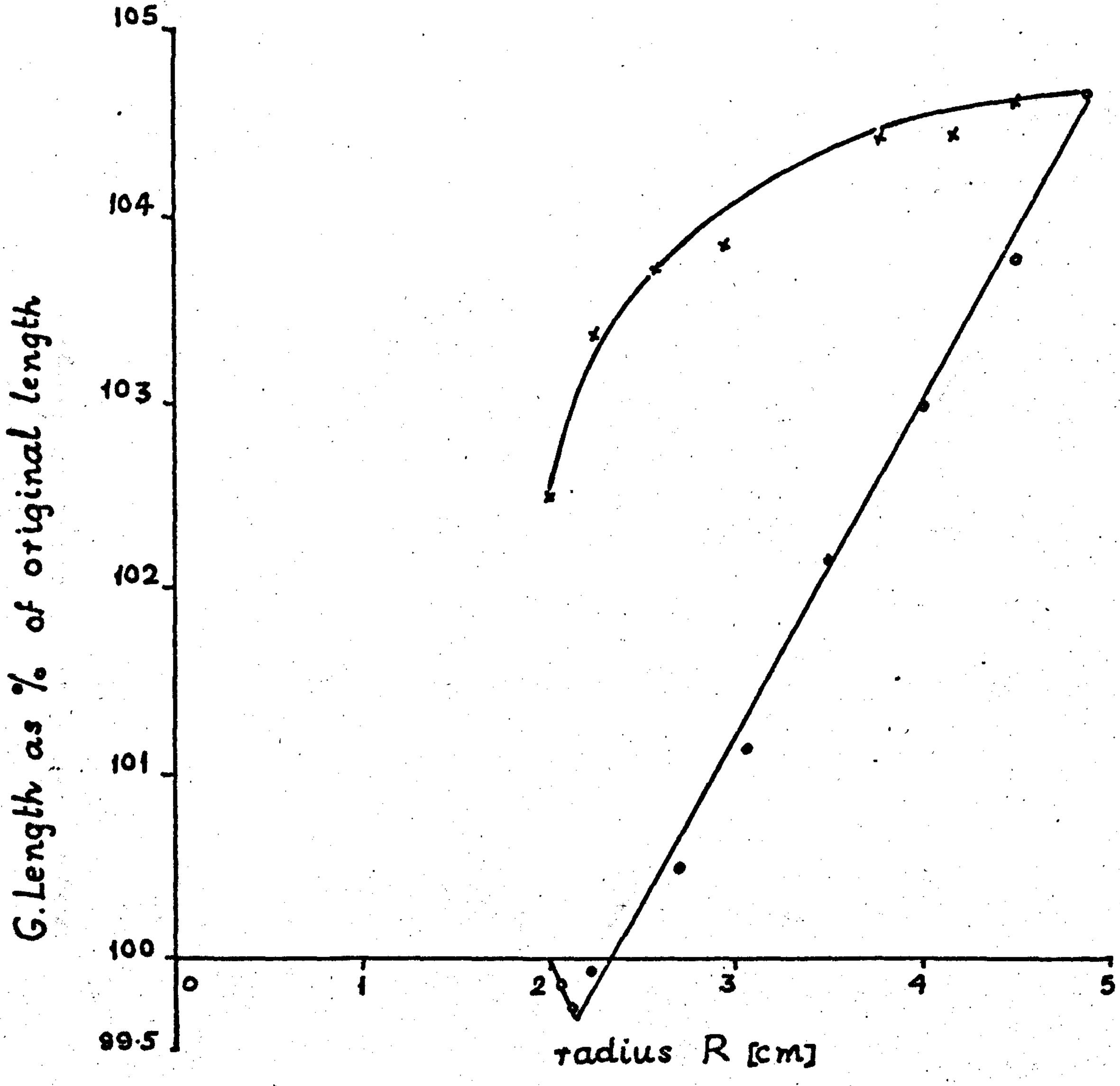
Axial deformation of the cheese

FIG. 2.24

TEST. 10. vi.

Winding tension for the base = 40·6

Winding tension = 40·3g



Axial deformation of the cheese

FIG. 2.25

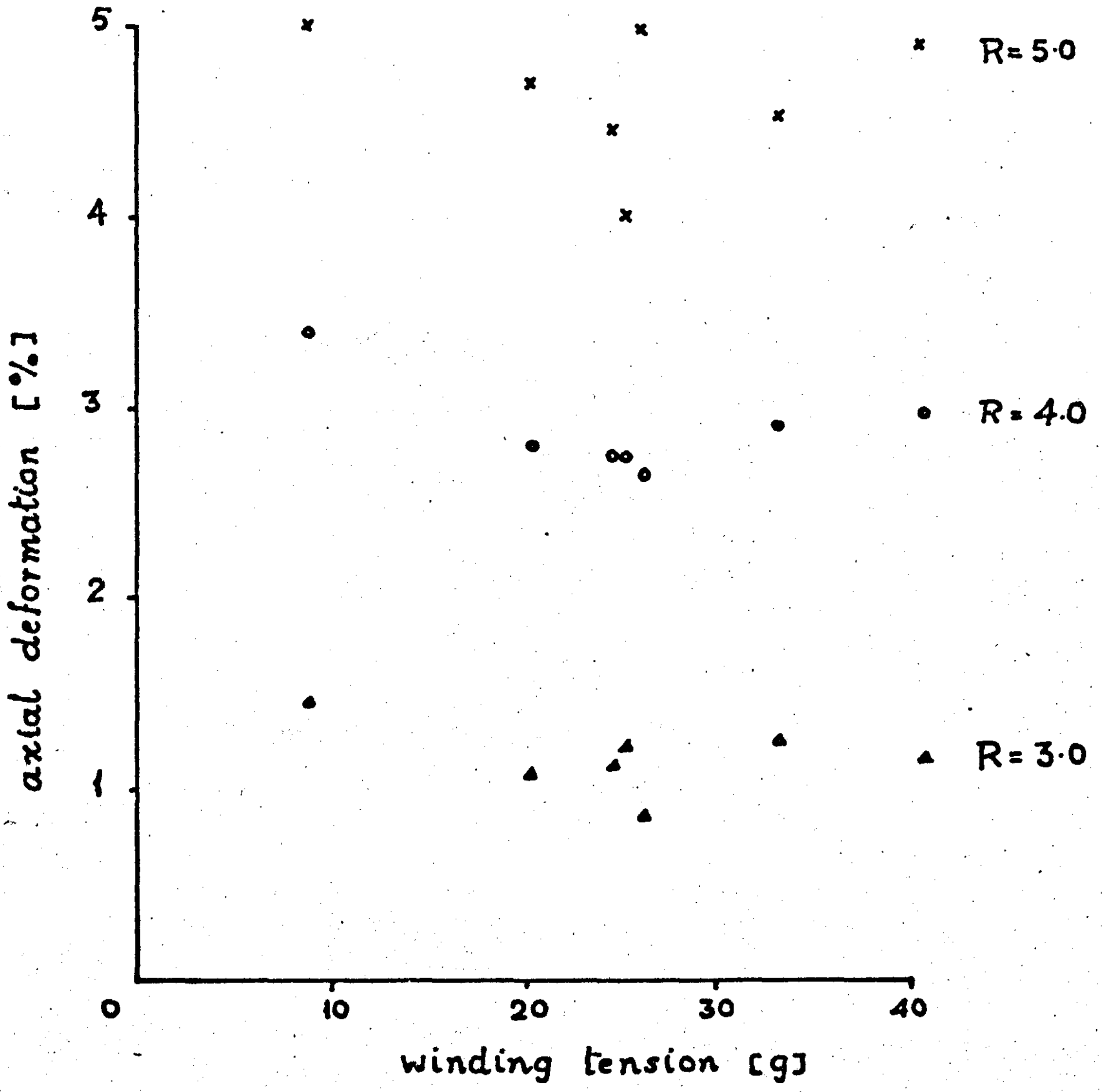
2.6.6 The First Group of Tests

In this group of tests the spacing between the adjacent wraps of yarn at the core was 0.05 cm, i.e. nearly one diameter of the yarn.

(a) Effect of Varying the Winding Tension in the Yarn. Test 10

In this test the effect of varying the winding tension in the yarn on the axial deformation of the cheese at the radius of 2 cm was measured. The range of winding tension varies from about 8g to about 40g. A number of windings were done one for each winding tension of the range. During these windings the cheese base for the gauge was prepared at the same winding tension in the yarn at which the subsequent winding was done. The results of the present test are shown in figures from 2.20 to 2.26.

These figures show that the behaviour of the axial deformation of the cheese at the gauge radius is similar to that of the previous test, namely, there is a slight reduction in the gauge length for some initial winding and subsequently the gauge length increases at a fairly steady rate as the winding continues. During unwinding the change in gauge length is very slow to start with but becomes faster as the gauge radius is approached. The permanent deformation of the cheese at the gauge radius after the unwinding is complete is high and is generally more than half of the total



Effect of winding ten. on axial def. of the cheese

FIG. 2.26

deformation of the cheese at that radius.

Fig. 2.26 shows the per cent axial deformation of the cheese at the radius of 2 cm plotted against the winding tension in the yarn for different outer radii of the cheese. This figure shows that there is practically no difference in the magnitude of the axial deformation of the cheese as the winding tension in the yarn is varied.

(b) Repeated Winding Test. Test 10

The results of the repeated winding on the cheese are shown in Fig. 2.22 which shows the gauge length at the radius of 2 cm as per cent of the original gauge length during repeated winding and unwinding at the winding tensions of 27.2g, 26.8g and 28.7g. The values of the axial deformation for the outer radius of 5 cm and the permanent deformation after the unwinding is over in the three cases are 4% and 1.65%, 3.1% and 0.8% and 3.4% and 0.7% respectively. The behaviour of the strains in the three cases are similar but the magnitudes of the total strain and the permanent strain are higher for the first winding as compared to those of the subsequent windings.

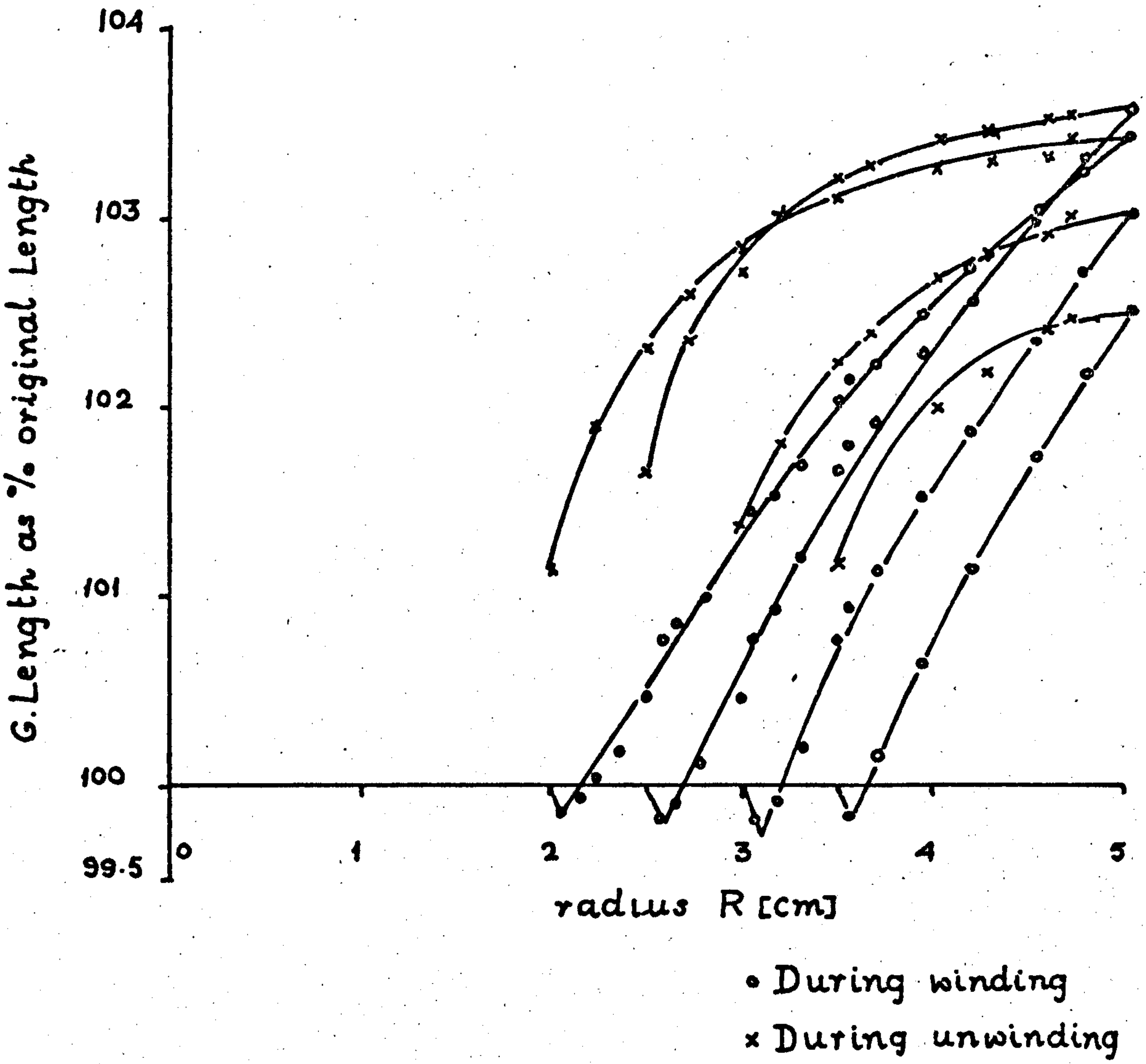
(c) Axial Deformation of the Cheese at Different Radii. Test 11

Test 11 was devised to measure the deformation of the cheese at different radii. Gauges were inserted at radii of 2 cm, 2.5 cm, 3 cm and 3.5 cm. Each time winding was stopped the lengths of all gauges were measured.

TEST. 11.

Winding tension for the base = 24·5 g.

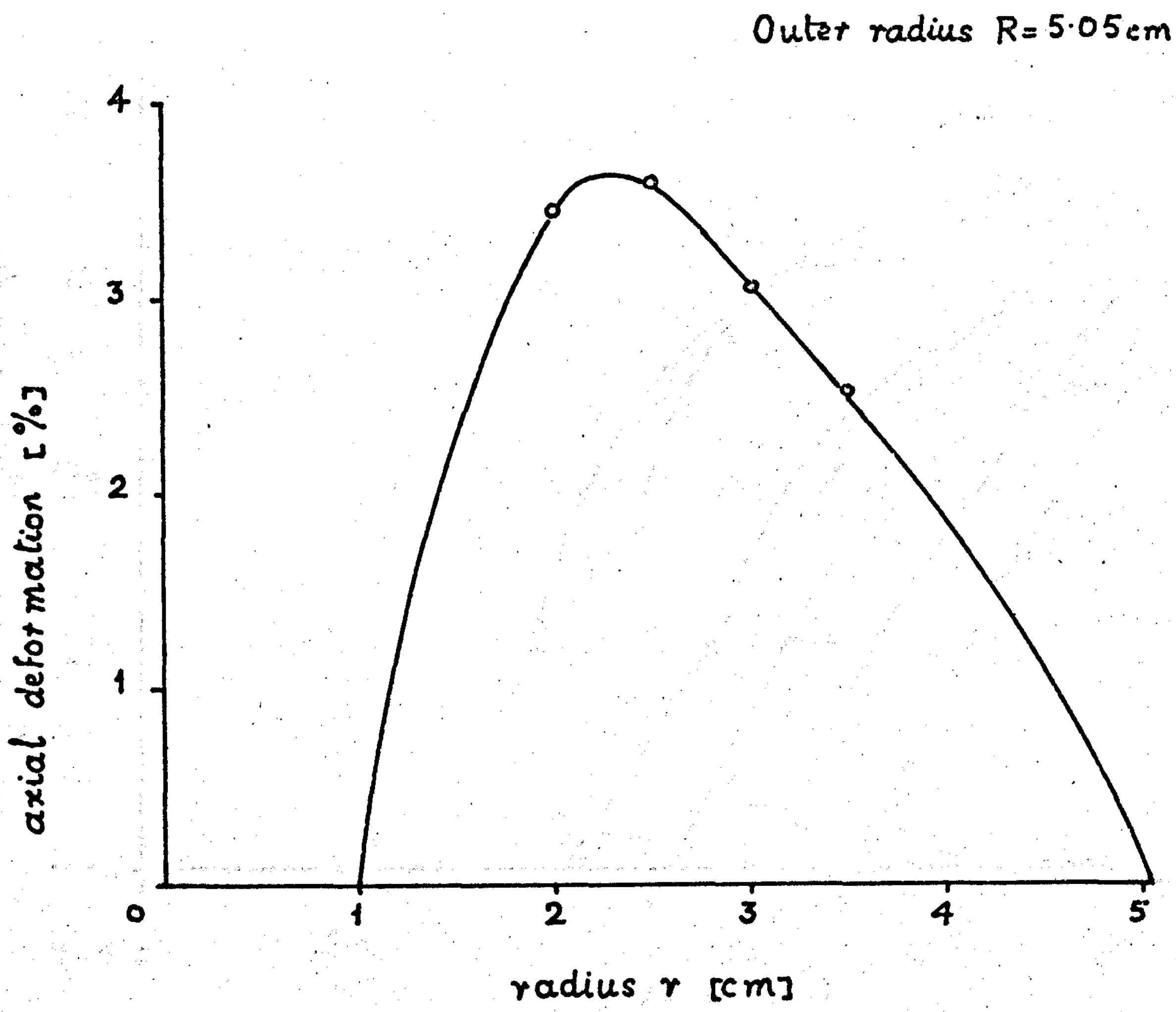
Winding tension = 25·5 g



Axial deformation of the cheese at different radii

FIG. 2. 27

TEST. 11.



Axial deformation of the cheese at different radii
FIG. 2.28

During unwinding also the deformation was measured at intervals at all the gauge points, each gauge being removed as it was released.

The results of the test are given in Fig. 2.27 which shows the gauge length as per cent of the original gauge length at the radii of 2 cm, 2.5 cm, 3 cm and 3.5 cm as the cheese is built up to a radius of 5.1 cm. It shows that the behaviour of the deformation at all the four radii is similar showing an initial fall in the gauge length for some initial winding and then a steady increase in the gauge length. The magnitude of the deformations at different radii are different. Fig. 2.28 shows the per cent axial deformation of the cheese with the radius of the cheese for the outer radius of 5.1 cm of the cheese. This shows that the maximum axial deformation is at the radius of about 2.5 cm.

2.6.7 Package with Increased Thread Spacing

In these tests the spacing between the adjacent wraps of yarn at the core was 0.1 cm giving 10 threads per cm.

(a) Effect of Varying the Winding Tension in Yarn. Test 12

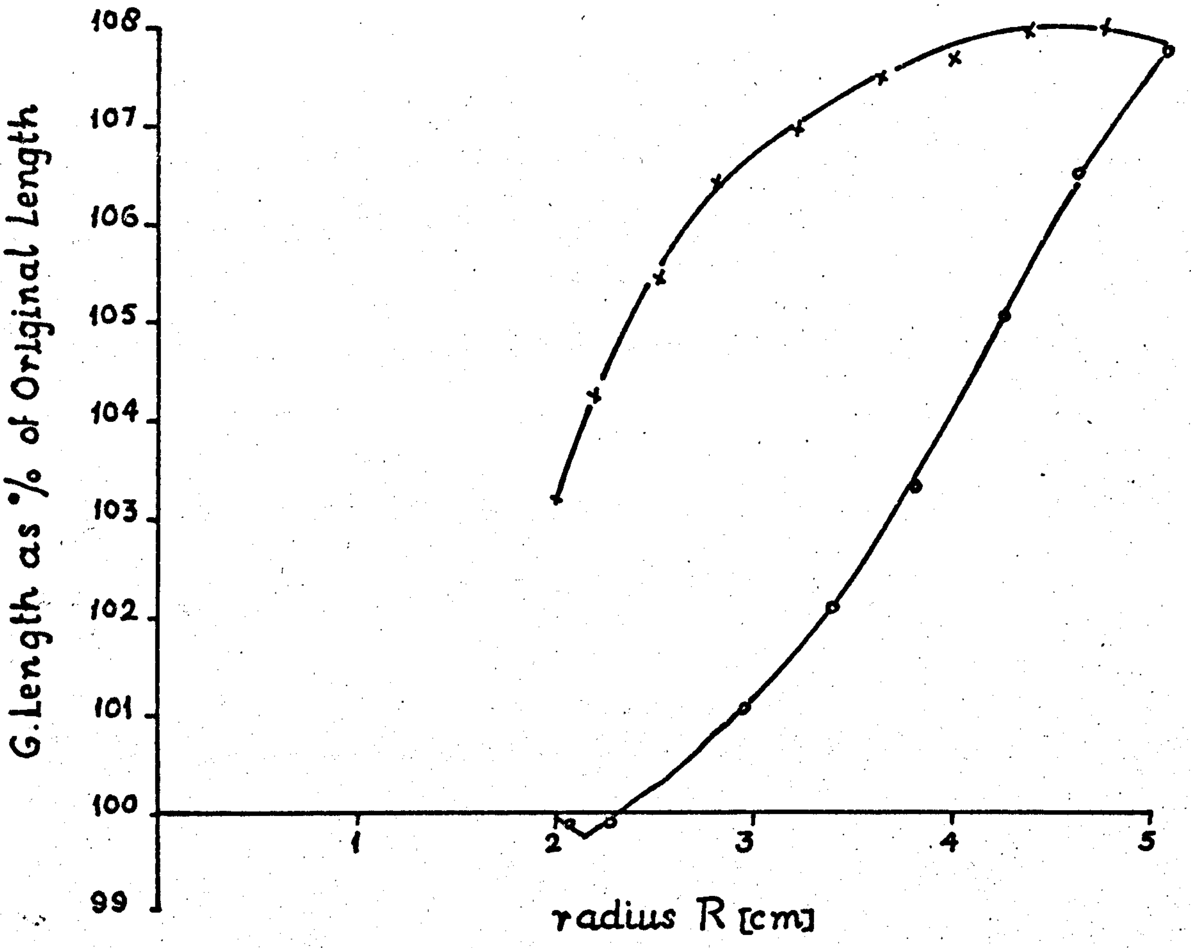
In this test the effect of varying the winding tension in the yarn on the axial deformation of the cheese at the radius of 2 cm is measured. The range of winding tension varies from about 6g to about 42 g. This test is similar to Test 10 described before. The

TEST. 12. i.

Winding tension for the base = 8·3g.

Winding tension = 6·3g.

Space = 2D.



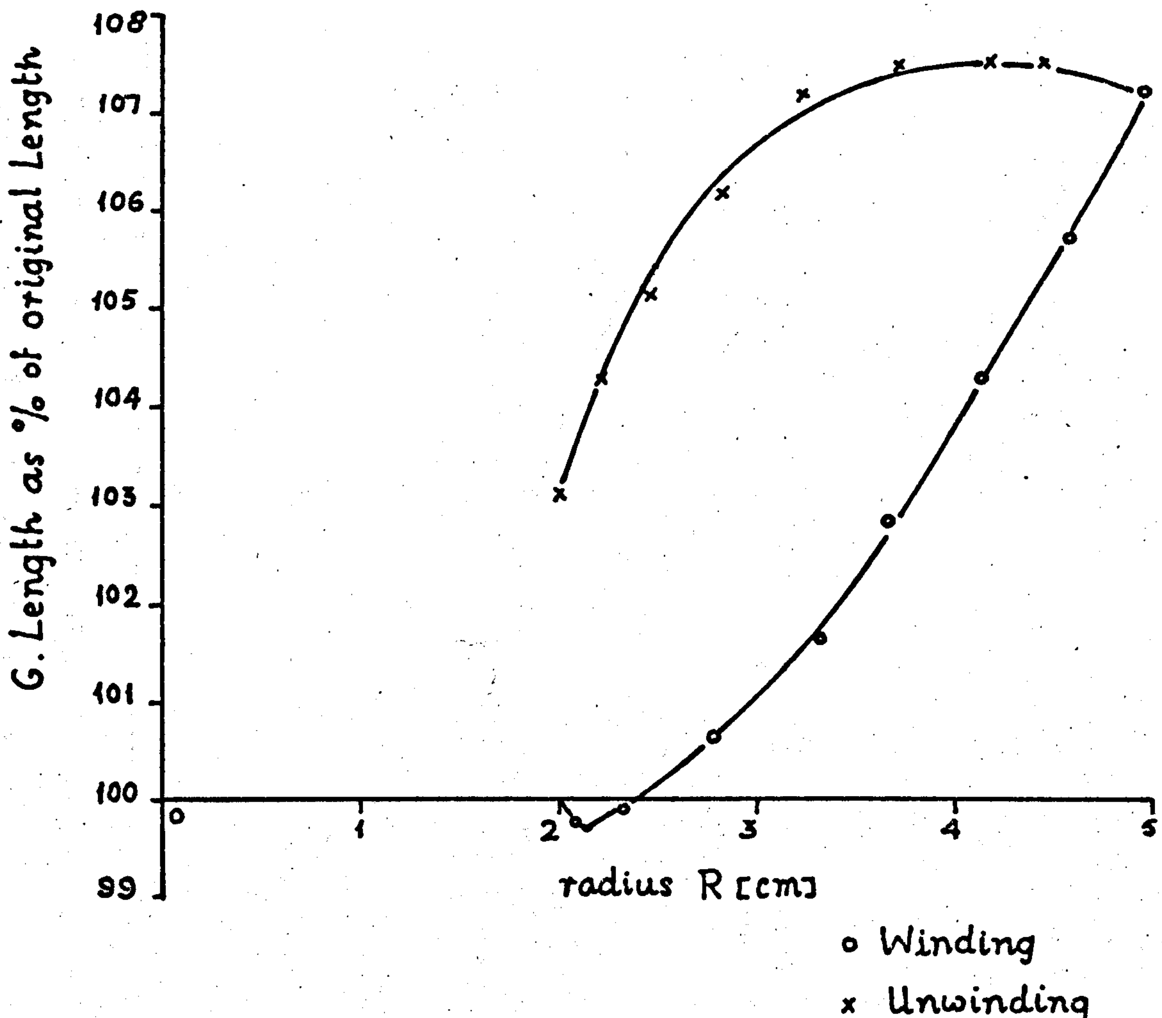
Axial deformation of the cheese

FIG. 2.29

TEST. 12. ii.

Winding ten. for the base = 12·2 g.

Winding tension = 12·2 g.
Space = 2D.



Axial deformation of the cheese

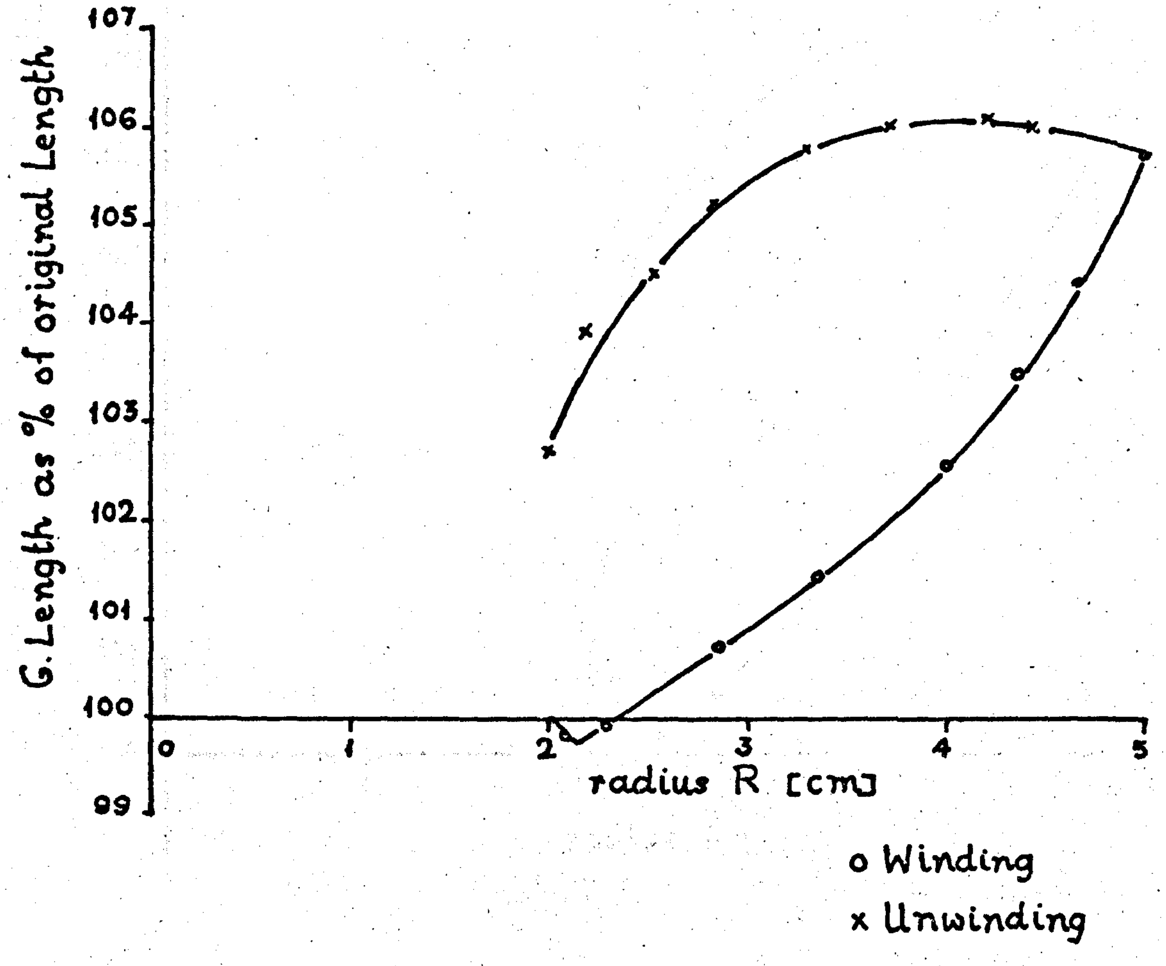
FIG. 2.30

TEST.12. iii.

Winding ten. for the base = 18·2g.

Winding tension = 18·9g.

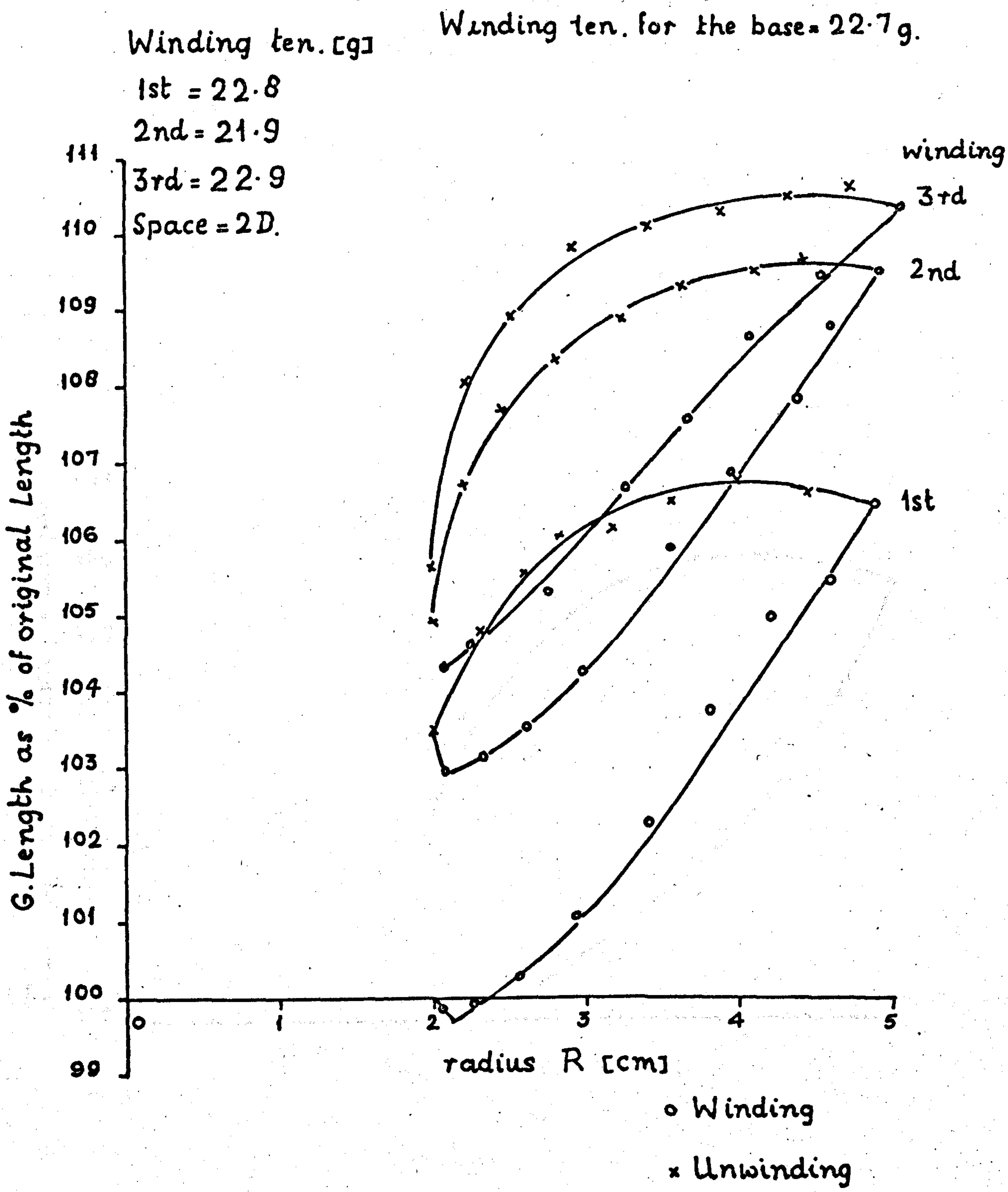
Space = 2D.



Axial deformation of the cheese

FIG. 2.31

TEST. 12. IV.



Axial deformation of the cheese

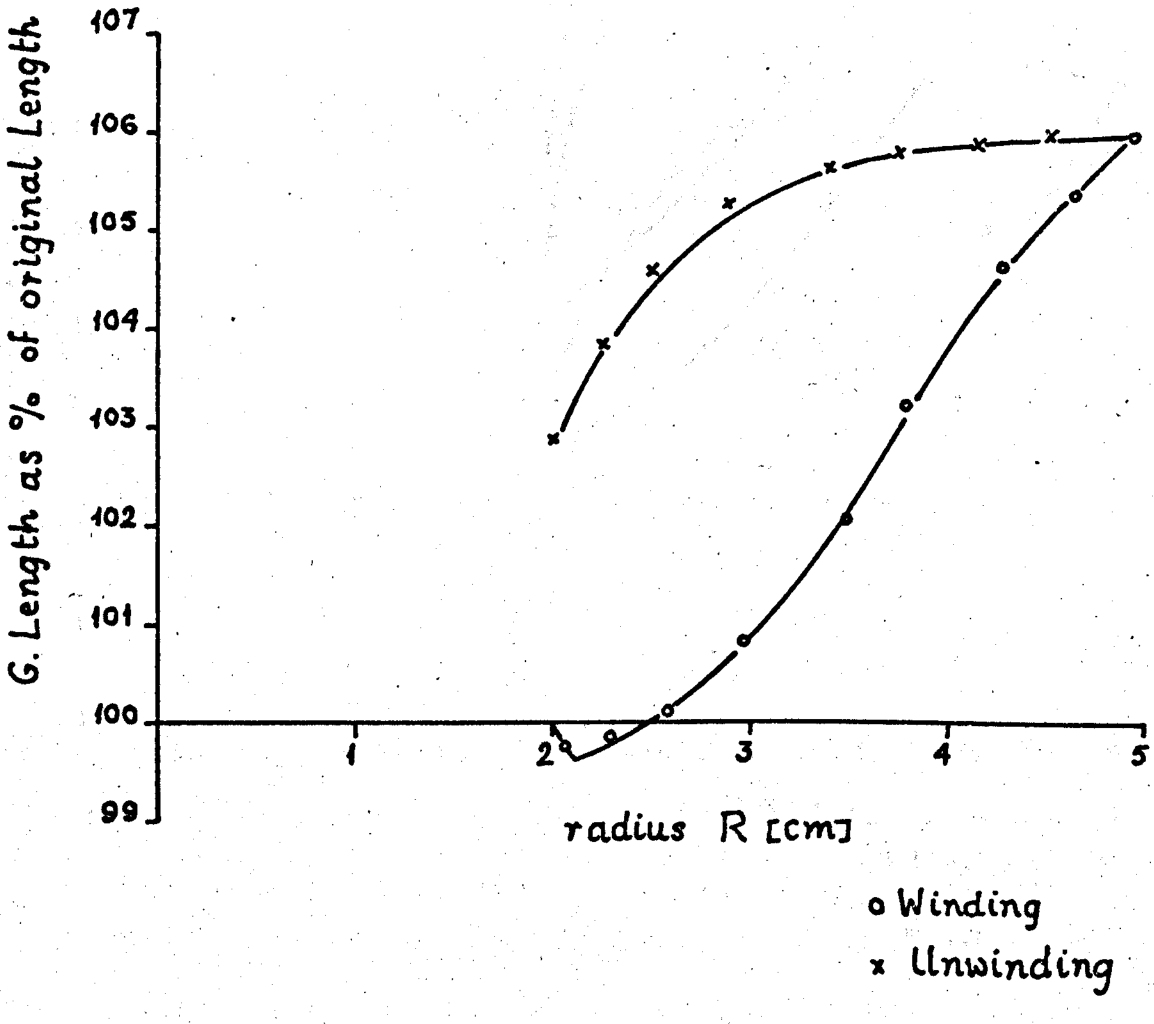
FIG. 2.32

TEST.12.v.

Winding ten. for the base = 27.4 g

Winding ten = 27g

Space = 2D



Axial deformation of the cheese

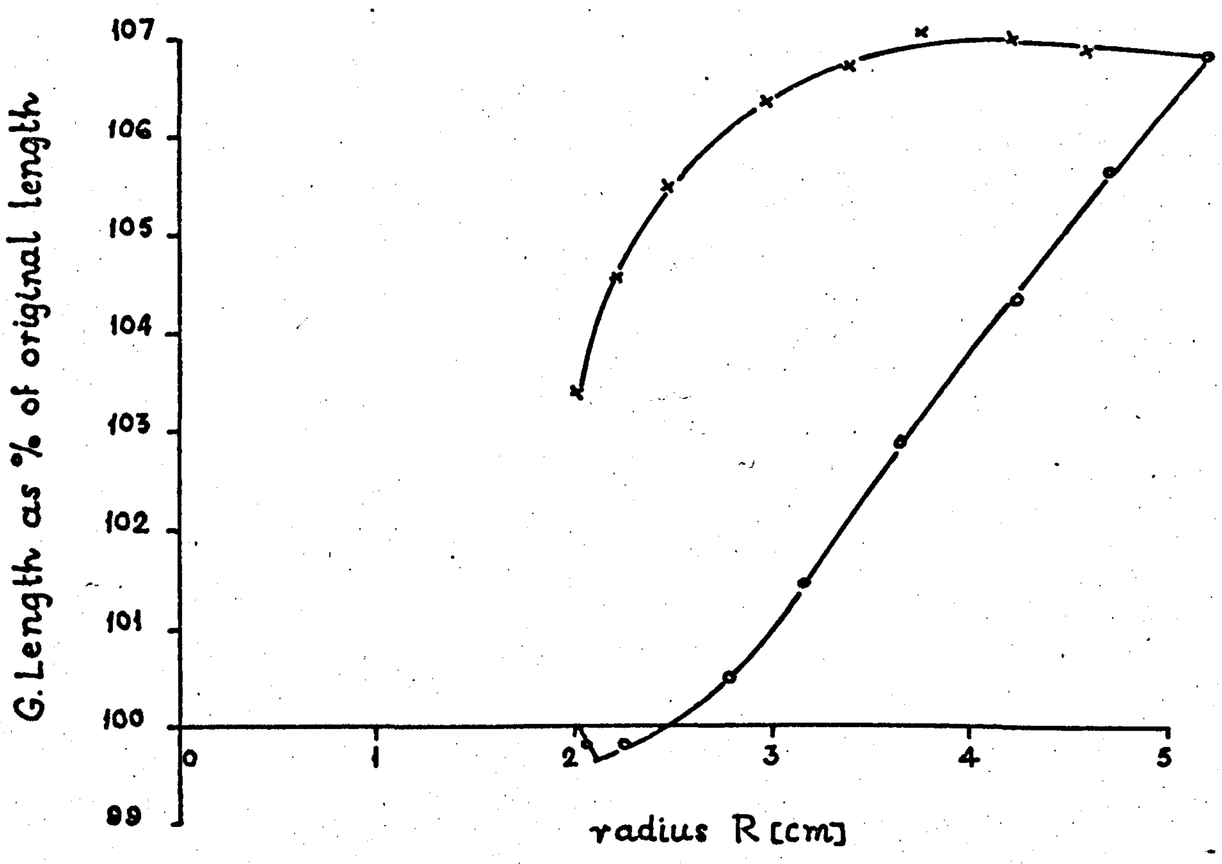
FIG.2.33

TEST. 12.vi

Winding ten. for the base = 32·3g

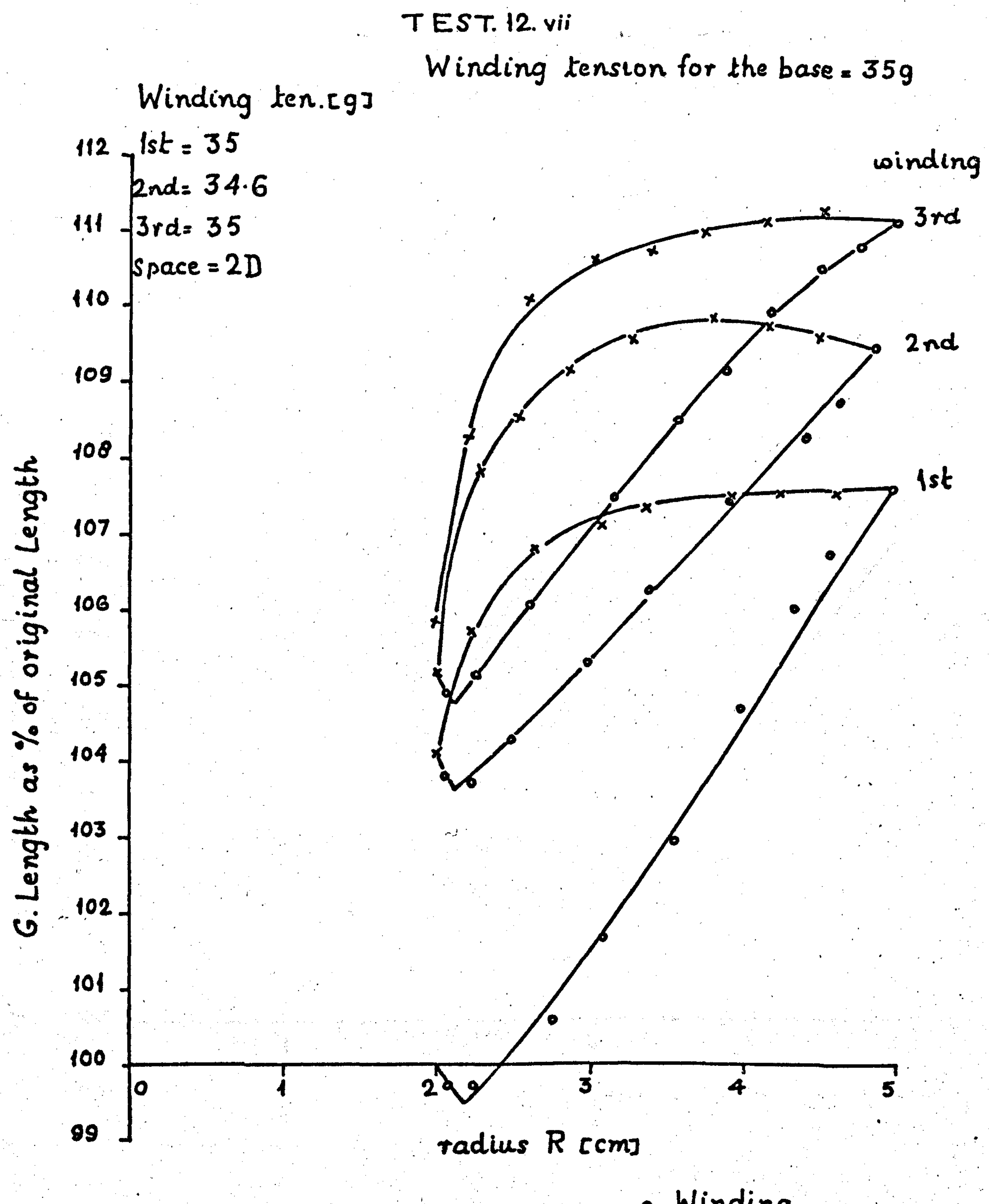
Winding tension = 32·4 g

Space = 2D



Axial deformation of the cheese

FIG.2.34



Axial deformation of the cheese

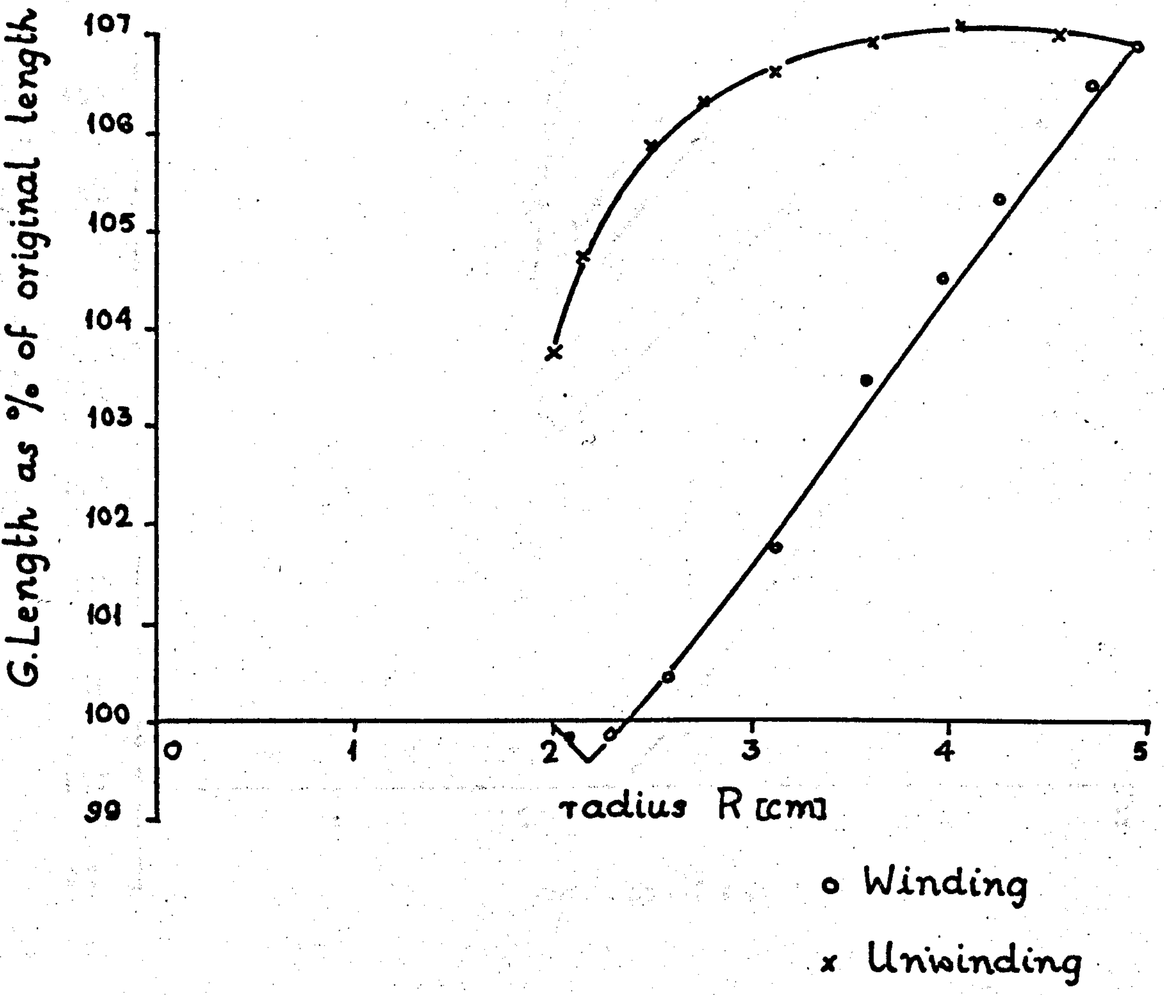
FIG.2.35

TEST. 12.viii.

Winding ten. for the base = 41.6 g

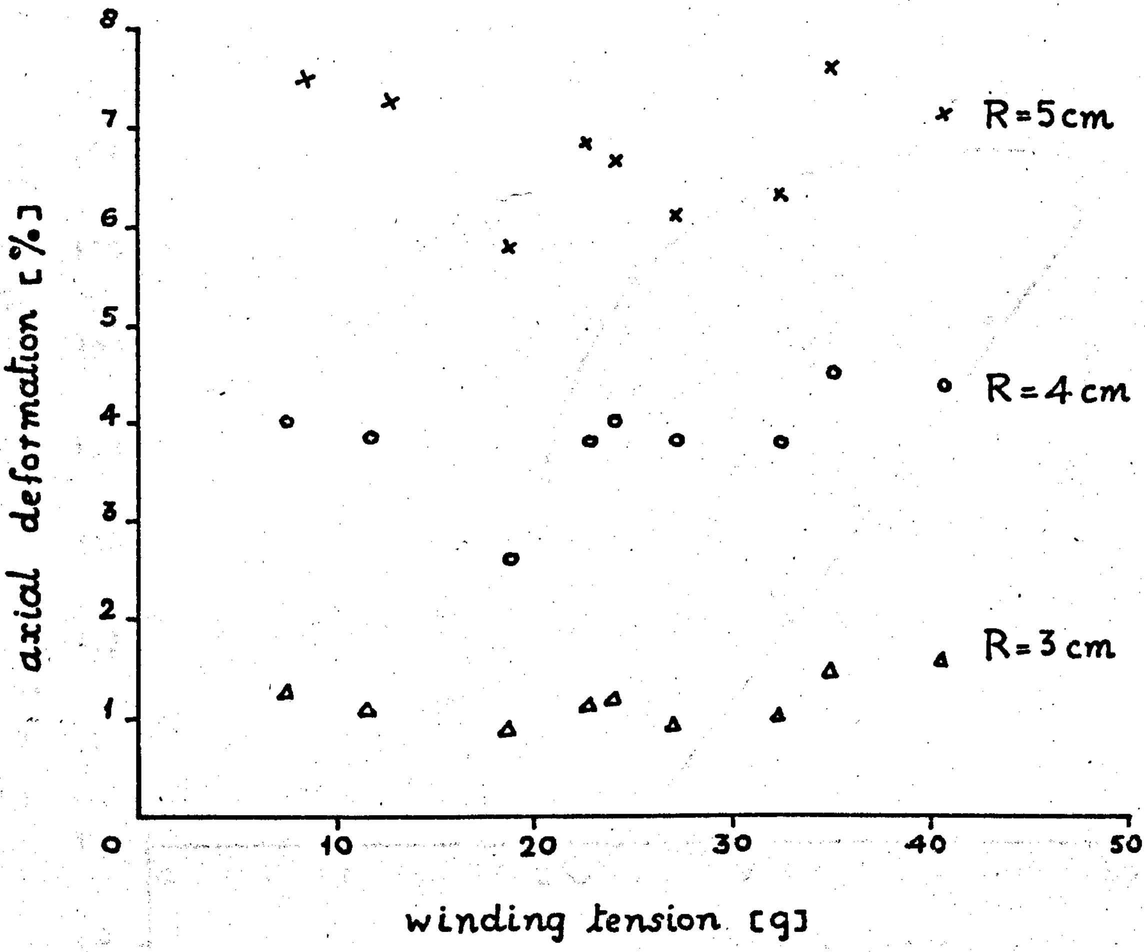
Winding tension = 40.8 g

Space = 2D



Axial deformation of the cheese

FIG. 2.36



Effect of winding tension on axial def. of the cheese

FIG.2.37

results of this test are shown in figures from 2.29 to 2.37.

These figures show that the behaviour of the axial deformation at the gauge radius is similar to that of the previous tests. The magnitude of the axial deformation at the gauge radius for the outer radius of 5 cm is higher, namely about 6.8% as compared to that of about 4.7% of Test 10. During unwinding the change in length towards original undeformed length is slower initially and in some cases a slight increase in the deformation is indicated on initial unwinding. The permanent axial deformation at the gauge radius after the unwinding is complete is also high and is generally about half of the total axial deformation at that radius.

Fig. 2.37 shows the per cent axial deformation of the cheese and the radius of 2 cm plotted against the winding tension in the yarn for the outer radii of 3 cm, 4 cm and 5 cm of the cheese. In this case also the magnitude of the axial deformation of the cheese appears to be independent of the winding tension in the yarn.

(b) Effect of Repeated Winding

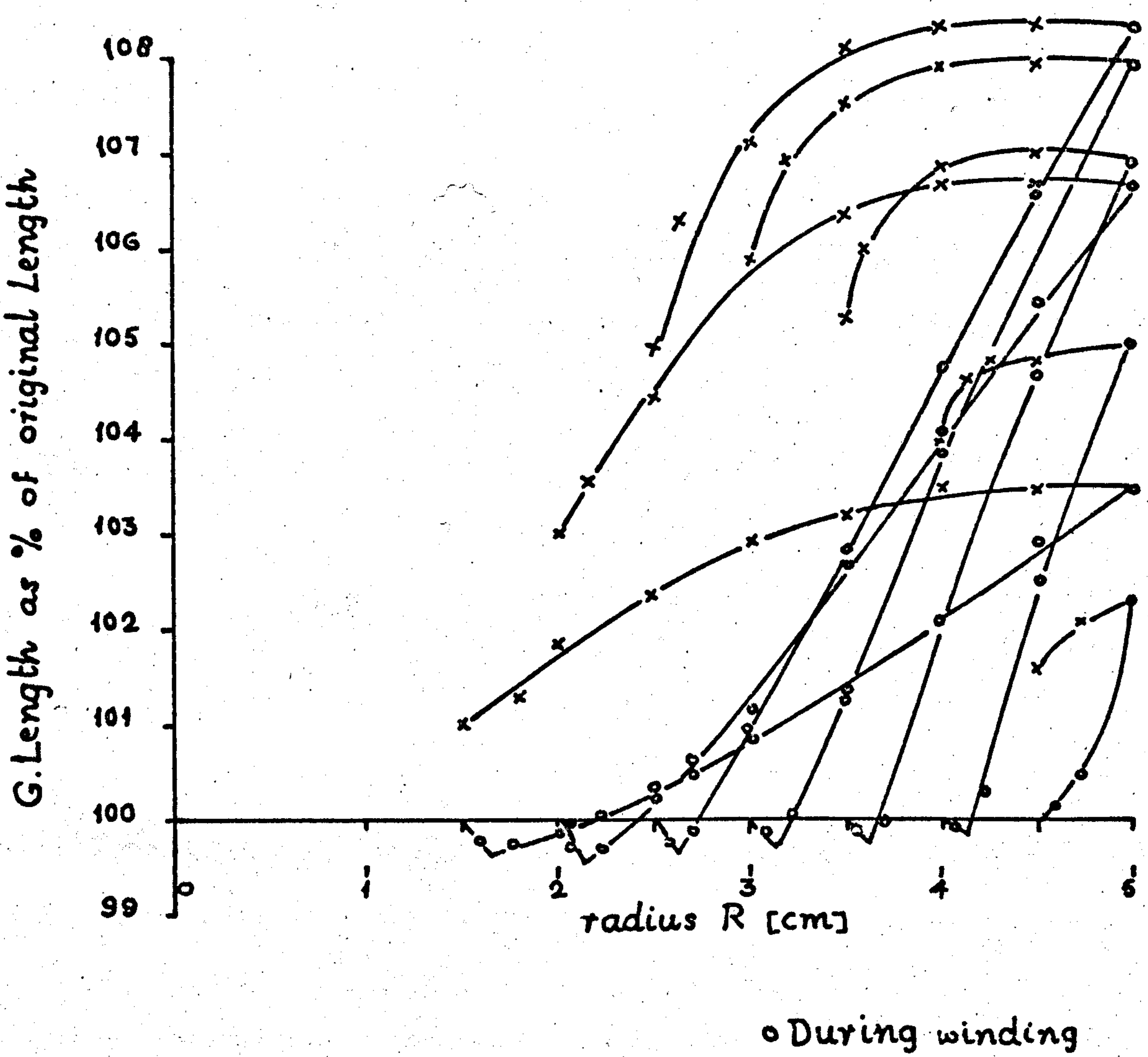
Figs. 2.32 and 2.35 show the effect of repeated windings on the axial deformation of the cheese at the radius of 2 cm at two different winding tensions in the yarn, namely about 23g and about 35g respectively. It shows that the axial deformation of the cheese at the radius of 2 cm is greater due to the first winding as to those of the subsequent windings especially in the second case of higher winding

TEST. 13.

Winding tension for the base = 23.4g

Winding tension = 24 g

Space = 2D



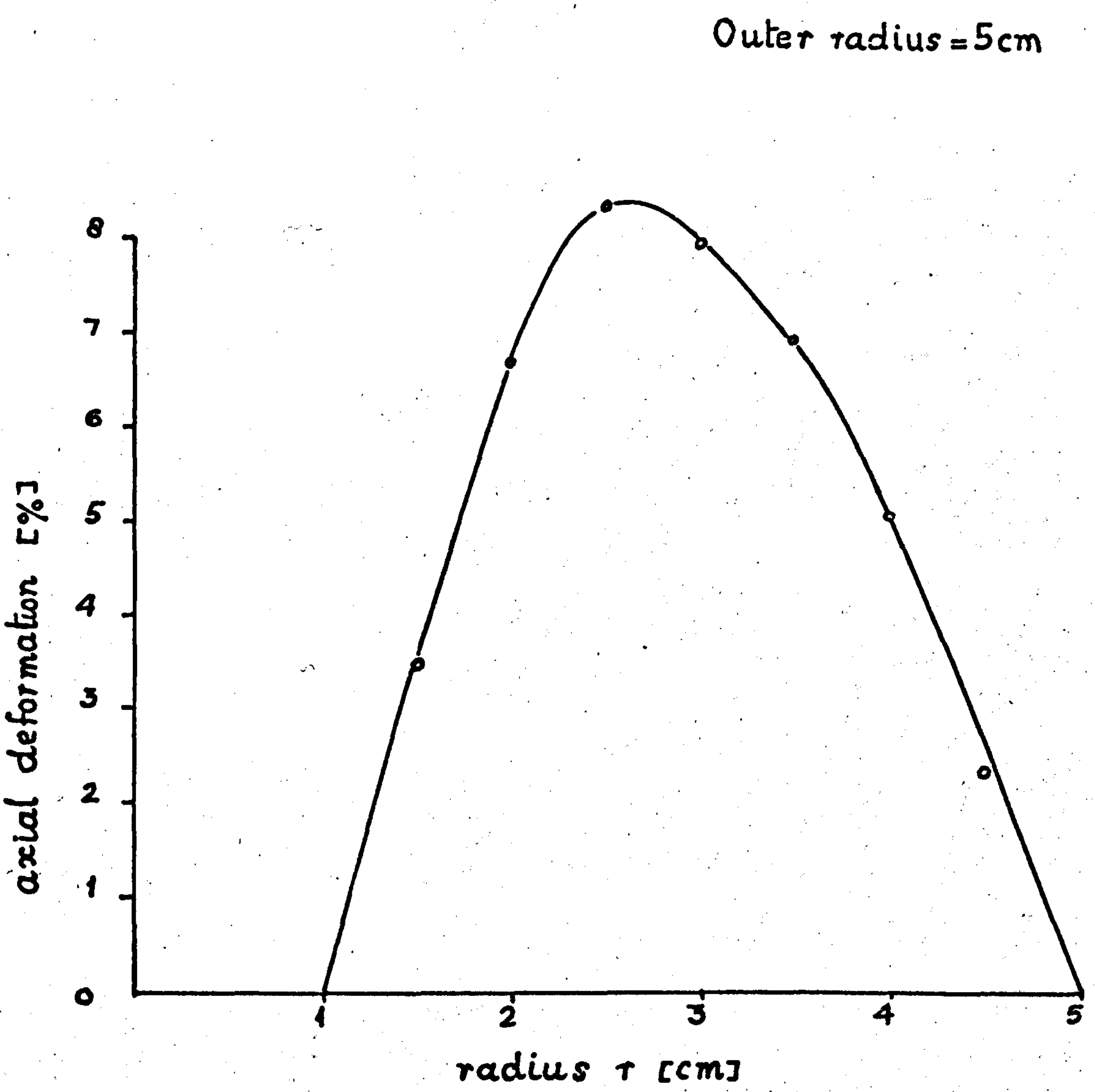
Axial deformation of the cheese at different radii

FIG. 2.38

TEST. 13.

Winding tension = 24 g

Space = 2D;

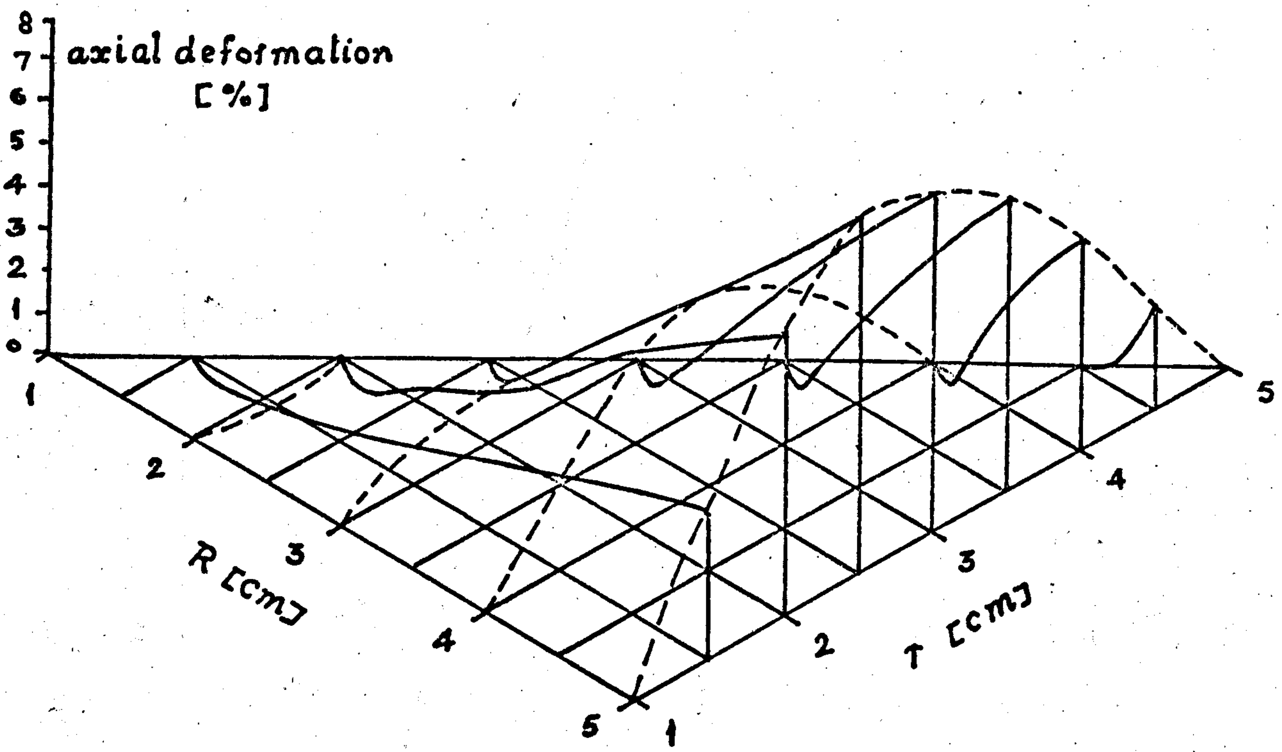


Axial deformation of the cheese at different radii

FIG.2.39

TEST. 13.

Winding tension = 24 g
Space = 2D



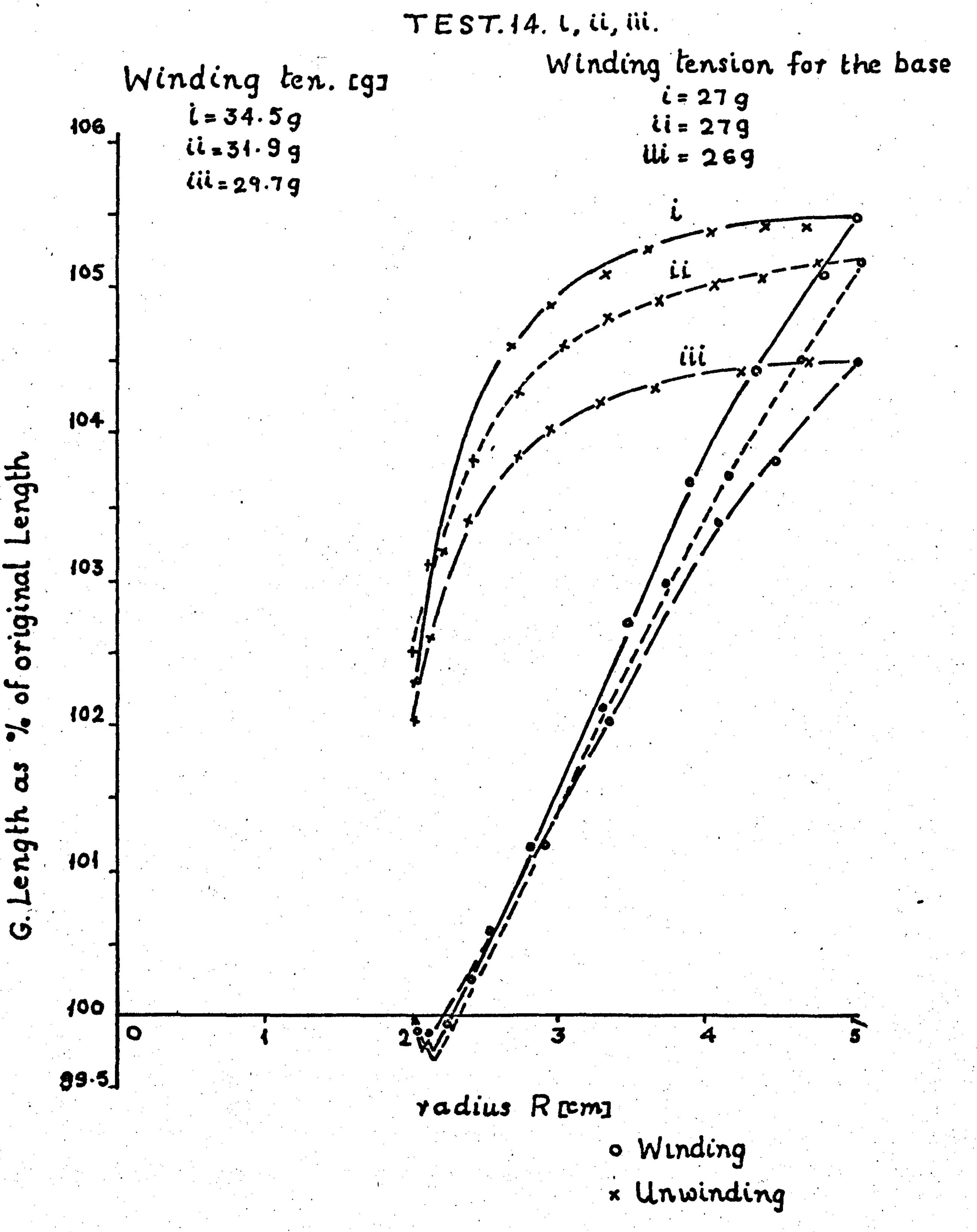
Axial deformation of the cheese for all values of T and R

FIG. 2.40.

tension in the yarn. Also the permanent axial deformation of the cheese after first unwinding is greater than that of the subsequent windings. The pattern of the axial deformation is similar in all the windings of both the cases.

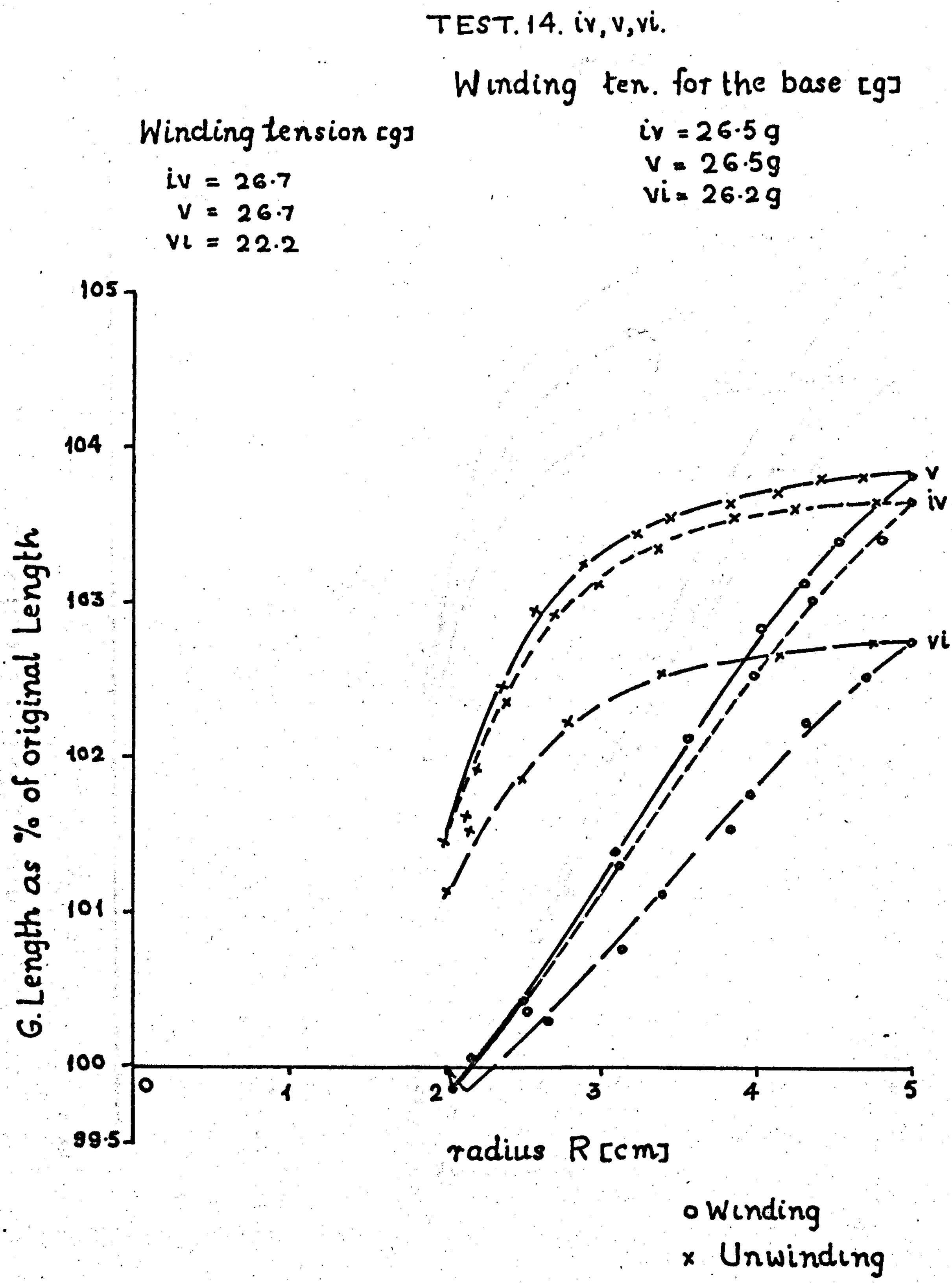
(c) Axial Deformation of Cheese at Different Radii. Test 13

The method of measuring the axial deformation of the cheese at the radii of 1.5 cm, 2 cm, 3 cm, 3.5 cm, 4 cm and 4.5 cm in this test was similar to that of Test 11. The results of this test are shown in Figs. 2.38, 2.39 and 2.40. Fig. 2.38 shows the gauge length as per cent of the original gauge length at various radii as the cheese is built up to the outer radius of 5 cm. The magnitude of the initial reduction in the gauge length due to some initial winding reduces as the radius of the cheese at which the measurement is done increases. At the radius of $4\frac{1}{2}$ cm there is no reduction in the axial gauge length. Fig. 2.39 shows the per cent axial deformation of the cheese plotted against the radius of the cheese for the outer radius of 10 cm.? The maximum axial deformation of the cheese occurs at the radius of about 2.5 cm. Fig. 2.40 gives a three dimensional view of the axial deformation of the cheese. It shows the axial deformation at any radius r of the cheese for any outer radius R of the cheese.



Axial deformation of a standard base

FIG. 2.41



Axial deformation of a standard base

FIG. 2.42

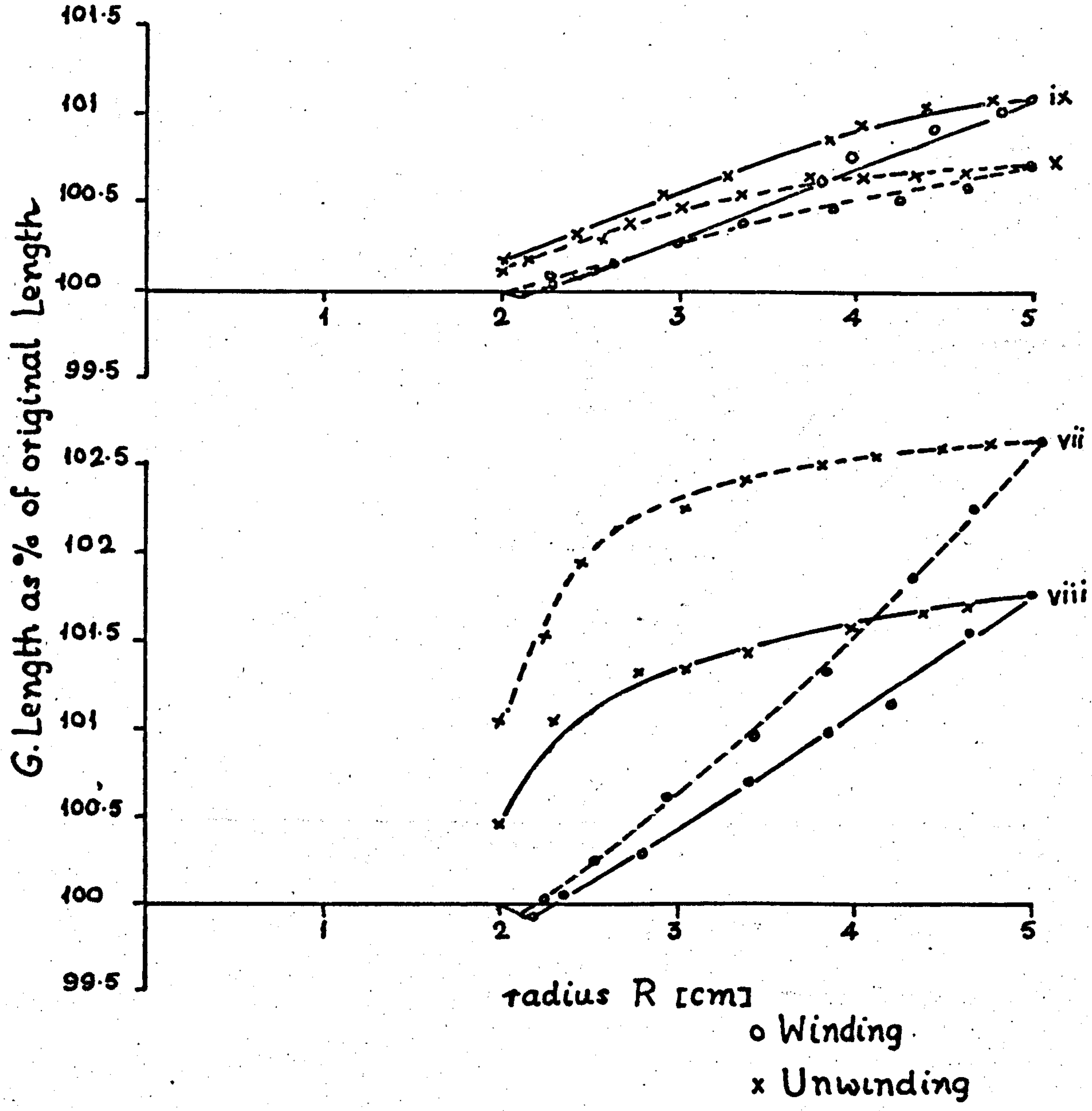
TEST. 14. vii, viii, ix, x.

Winding ten for the base [g]

Winding tension [g]

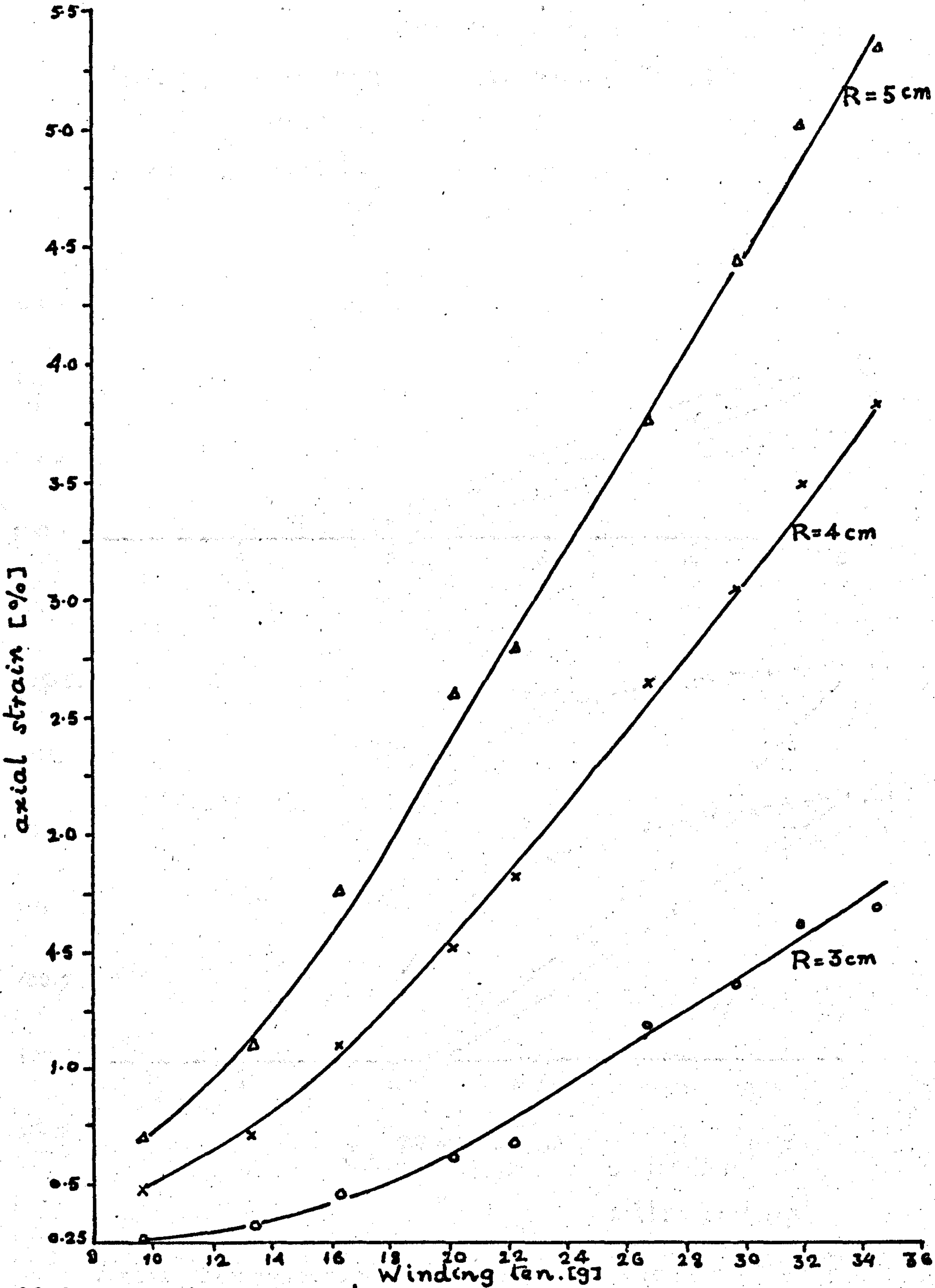
vii = 20·1; viii = 16·3; ix = 13·3; x = 9·6;

vii = 26·3; viii = 27·3; ix = 26·5; x = 26·2;



Axial deformation of a standard base

FIG. 2.43



Effect of win. ten on axial def. of a standard cheese base

FIG. 2.44

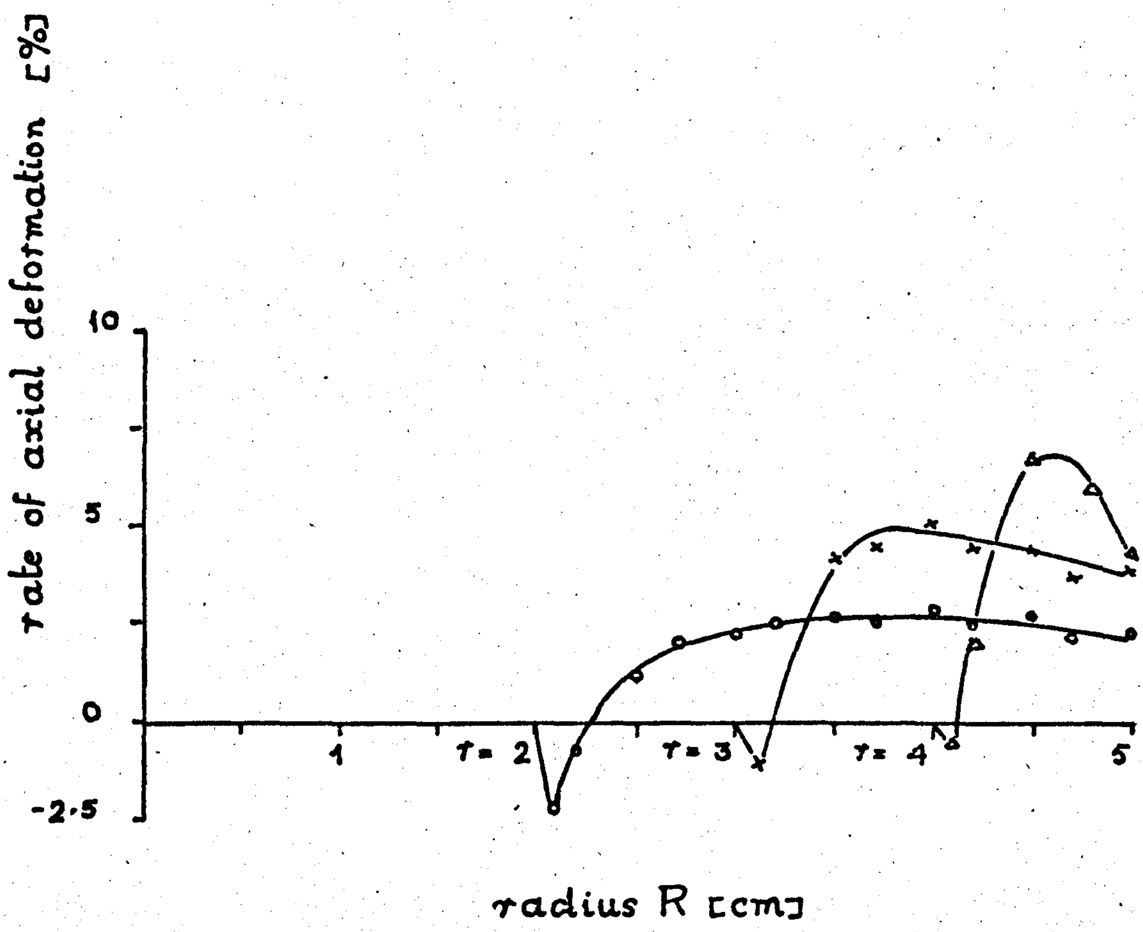
2.6.8 Effect of Winding Tension on Axial Deformation of a Uniform Cheese Base. Test 14

This test was conducted to assess the effect of varying the winding tension in the yarn on the axial extension of a cheese base prepared at the same winding tension, namely about 26.5g, for each winding. This did not correspond to a real cheese but was included in case it was useful in exploring the behaviour of the cheese. For this a range of winding tensions varying from about 10g to 35g was chosen and a separate winding was done for each value of the winding tension. The results of the test are given in Figs. from 2.41 to 2.43 which show the gauge length at the radius of 2 cm as per cent of the original gauge length during winding and unwinding.

The behaviour of the axial deformation is similar in all the cases but the magnitude of axial deformation is different for each winding and is proportional to the winding tension in the yarn. The initial fall in the gauge length is also proportional to the winding tension in the yarn. At the low value of 9.6g of the winding tension in the yarn there is practically no initial shortening of the gauge length.

Fig. 2.44 shows the per cent axial deformation of the cheese at the radius of 2 cm plotted against the winding tension in the yarn for the outer radii of 3 cm, 4 cm and 5 cm. This figure shows a steady increase in the amount of axial strain with the increase in the winding tension of the yarn.

TEST. 13



Rate of axial deformation of cheese with R at constant τ.

FIG. 2. 45

2.6.9 Discussion of Results

The axial deformation is fairly large - up to about 5% or more. This makes its measurement easy and eliminates the problem of calibration. The results of the various tests agree with each other in general and are reliable. The results can be summarised as follows. Each layer of yarn contracts slightly while about 3 mm or so thickness of material is wound over it; then it begins to expand and when the outer radius has increased by another 3 mm or so it is back to its original length. Then it continues to expand at a rate more or less proportional to the growth of the outside radius. The rate of expansion for layers near the outside is greater than for those near the core. This is shown by Fig. 2.45 based on the results of Test 13.

This expansion is not all recoverable when the package is unwound - rather more than half of it seems permanent. There is a large hysteresis in unwinding but on rewinding the behaviour is almost the same as on the first winding; the extension rising to higher level and almost half of the additional extension being recoverable.

The winding tension hardly affects the behaviour - it must by making a harder package beneath the gauge radius affect the resistance to deformation in the same way as it increases the deforming force. If however the tension is different in winding over the gauge from that used in winding the inner part the axial deformation is changed a great deal - for instance in a package of 5 cm radius the strain at 2 cm radius increases by about 0.2% for each gramme increase in winding tension over the gauge. This shows that all the results are likely to be affected quite a lot by fluctuations in tension during winding.

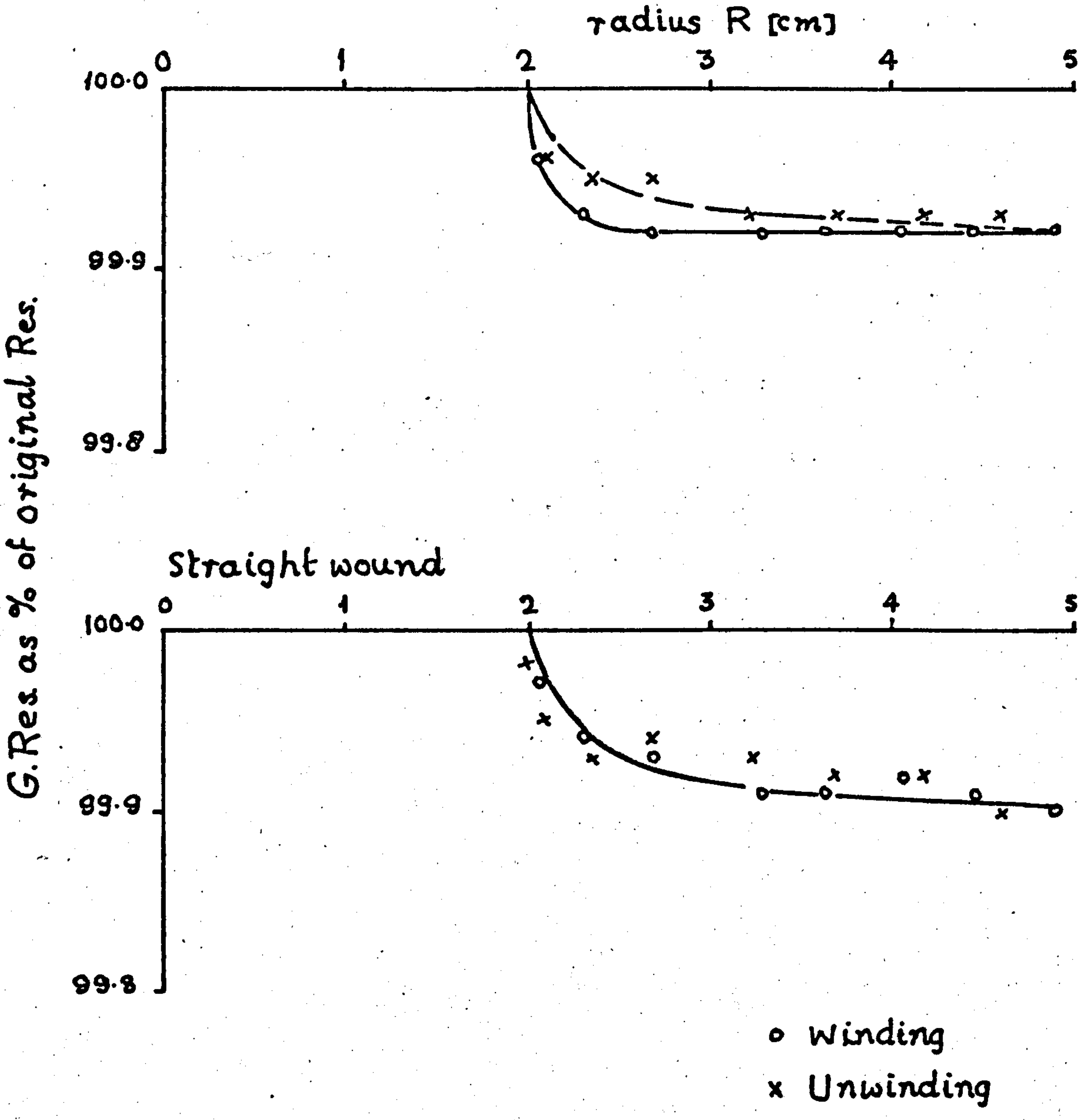
The spacing of the threads has some effect on the behaviour; a wider spacing producing rather greater deformation - it would be expected to reduce the elastic modulus of the package rather more than it reduces the deforming forces.

The rather large axial deformation throws some slight doubt on the validity of the measurement of radial deformation. The helical winding used in those tests (to accommodate leads and prevent wire crossing) means that if these gauges responded correctly the sensitivity to axial strain of those gauges (with $2\frac{1}{2}$ turns of wire and 1.5 cm axial distance between the two ends of the wire) would be about 1/20 of that to radial; the large axial strain thus could produce errors in the measurement of the smaller radial strain. If the gauge was fully sensitive to the axial strain a 5% axial extension could reduce the change in the gauge resistance of about 0.08% per cm increase of the outer radius. In fact, as was shown in the first axial tests, the gauges do not seem to respond fully to axial extension because of slipping of the paper base and the

TEST. 15

Winding ten. for the base = 25·3 g

Winding tension = 25·1g
Space = 2D
Helically wound



Effect of axial def. on the indication of radial def.

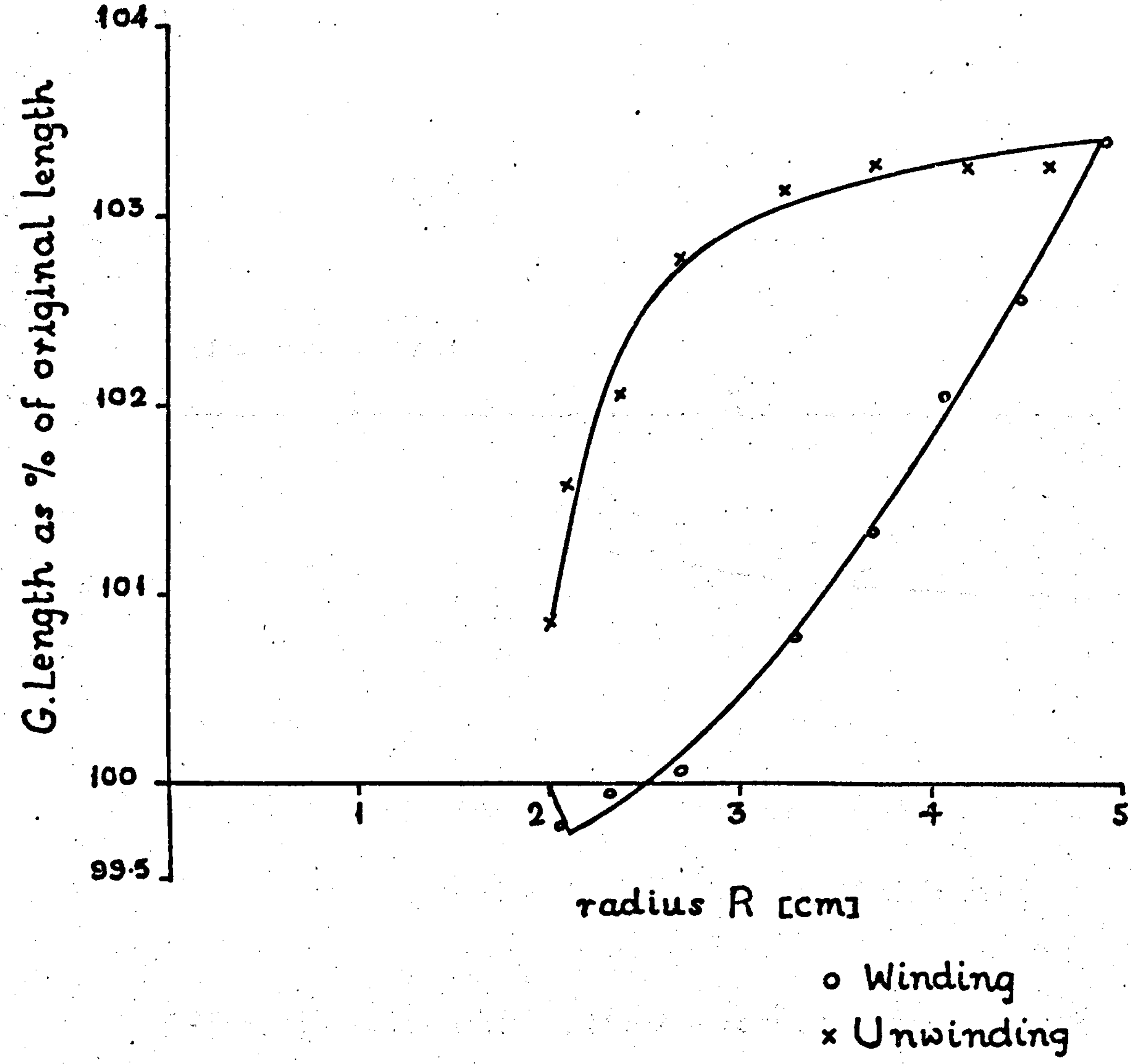
FIG. 2.46

TEST. 15.

Winding ten. for the base = 25·3g

Winding ten. = 25·1g

Space = 2D



Axial deformation of the cheese

FIG. 2.47

error would be much less than might be expected. This prediction was tested by winding a straight-wound gauge wire along with a previously used helically-wound type of gauge on the same base. In this gauge the ratio of axial distance between the two ends of the wire to the length of the wire was 1/8.5. The results are shown in Figs. 2.46 and 2.47. The maximum apparent reduction of the change in the gauge resistance if the gauge was fully sensitive to the axial extension of 3.4% measured by the axial gauge would have been 0.1% per cm increase of the outer radius. However the actual effect as shown by the difference between the changes in the resistances of the two gauges is only 0.007% per cm increase of the outer radius. This is apparently due to the large slip of the paper base which might have occurred, particularly as the width of the base was much smaller than that of the first axial test and was also slotted. The results suggest that the slow compression of the cheese does in fact occur rather more than was previously suspected and the qualitative conclusions about the radial deformation should thus be modified. The whole method however did not seem sufficiently accurate to justify a further prolonged series of tests.

Further discussion of the results is deferred until the theoretical approach to the problem has been presented.

CHAPTER III

THEORETICAL

3.1 Introduction

The approach to the theoretical solution of the cheese model is basically the same as has been used by Catlow and Walls and others. However there are important differences due to the fact that in the present case the model is a cross-wound cheese and not a parallel wound pirn. In the case of a pirn the axial component of the tension in the yarn is fairly small and was in fact considered to be zero and the circumferential stress of the added layer at the outer radius is constant regardless of the radius. The material was assumed to be homogenous and isotropic. In the cheese model the tension in the yarn, due to the wind angle, has two components the circumferential and the axial. Also due to the change in the wind angle with the outer radius the circumferential and the axial components of the tension are functions of the radius of the cheese at which the winding takes place. The cheese is not considered to be homogenous. The top layer of the cheese contacts the lower layer at a number of crossing points which bear the pressure imposed by the top layer. Also the cheese is not considered to be isotropic because the behaviour of these crossing points under compression is not likely to be the same as that of the whole yarn under longitudinal forces. Poisson's ratio of the cheese model is assumed to be negligible because the yarn is not close-packed as is likely in parallel winding and in general the yarn which is compressed radially can expand sideways, i.e. deform in cross section. Poisson's effect is likely to be much smaller in this case and its omission simplifies the analysis considerably.

Another difference appears in the analysis; corresponding elements of the cheese at different radii (i.e. those subtending the same angle at the axis and of the same axial length) contain the same number of yarn elements and crossing points. Although the size of the element of the cheese increases with radius the load bearing crossing points do not. Therefore it is more convenient to consider forces on elements rather than stresses (these correspond in fact to stresses in the individual yarns). One unfortunate effect of this is that it is not possible to check the theory against that of Catlow and Walls by simply letting the wind angle tend to zero, which would otherwise be possible.

In the solution the yarn is treated as elastic. The tension in the yarn is the only force acting on the yarn and acts along a tangent to the cheese; aerodynamic and centrifugal forces in the package are neglected. The effect of centrifugal force on the package while winding was considered and was found to be small. It is assumed that the tension is known in the yarn when it takes its place on the outside of the package.

The approach to the theoretical solution of the cheese model is as follows. A layer is added at the outer radius of the cheese. This layer imposes a known pressure on the cheese and causes it to deform. A second order differential equation is developed to give the radial deformation of the cheese due to the pressure imposed by the added layer. The equation is integrated numerically by Euler's modified method to give radial deformation at any radius of the cheese from the core radius to the outer radius. The boundary conditions for the solution of the equation are that the compression at the core is zero as the core is assumed to be incompressible and that the pressure imposed at the outer radius by the added layer is known.

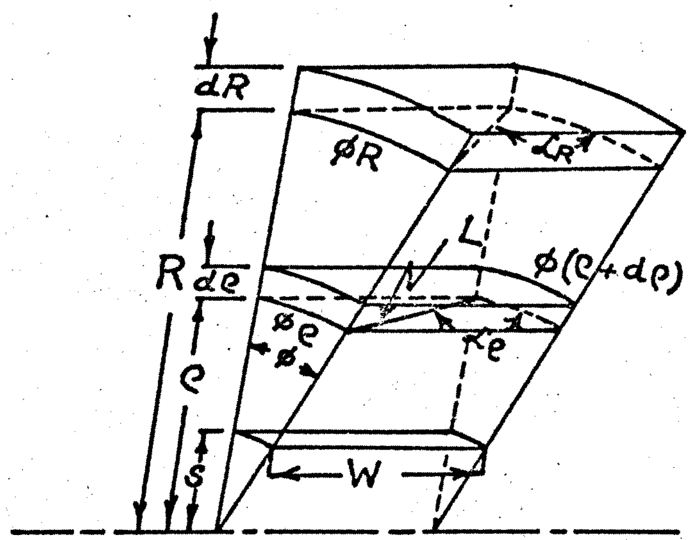
Now the next layer is added and the differential equation due to the addition of this layer is again solved, but the outer radius and the radius of the cheese have increased by the thickness of the layer added previously. The pressure imposed by the second layer added depends on the outer radius at which it is added and is therefore different from the previous layer. The total compression of the cheese at any radius is equal to the combined compression of the cheese at that radius due to the addition of both the layers. This procedure is continued till the cheese is solved up to the desired value of the outer radius of the cheese. The total value of the compression of the cheese at a given radius is obtained by the addition of the successive incremental compressions caused by the addition of the successive layers to make the cheese of required radius. The values of other variables are also similarly obtained. The solution is rather complicated because the coefficients of the differential equation describing the effect of adding one layer are themselves functions of the total effects of previous layers.

When a trellis-like layer reduces in radius it can do so by reducing the length of - i.e. the tension in - its elements or by changing the trellis angle slightly so it increases in axial length. Conversely axial forces may produce radial deformations. Changes in axial length of adjacent layers will tend to be different and it is necessary to examine how they can move relative to each other. Some small elastic deformation in the nature of a shear within the layer probably occurs but there is also the possibility of much larger movements occurring by either slipping of the layers or perhaps by rolling of the yarns of one layer over those of the other - this probably depends on some function of shear force and radial pressure. Initially it will be assumed that there is no such relative movement in order to discover what the forces tending to produce such movements would be.

Because the differential equation involves the incremental compression and other quantities as functions of the cheese radius and its solution gives these quantities at every radius of the cheese due to the addition of a layer at the outside of the cheese these quantities at each radius grow as the outer radius of the cheese increases and thus are functions of two variables, namely their present radius - or, better, that at which they were wound on, and the outer radius of the cheese.

The differential equation giving the incremental compression of the cheese due to the addition of a layer at the outer radius of the cheese involves the compression of the cheese prior to the addition of the layer in its coefficients. The cheese therefore cannot be solved for any value of the outer radius until solutions for previous values are known. Therefore the cheese is solved up to the desired outer radius by starting from the core and by increasing the outer radius layer by layer and simultaneously calculating the new values of the dependent variables at every radius of the cheese. A computer program in KDF 9 Algol is written to solve the cheese up to the desired value of the outer radius in this manner and is given in Appendix A.

In the present chapter the equations are developed to give the values of the incremental compression, the circumferential, and the axial components of the force through the face of an element, the change of tension in the yarn, the shear force and the pressure on an element at any radius of the cheese. The numerical method of integrating the equation is described which also includes the calcula-



Element at any radius ρ

FIG. 3.1

tion of total values of the compression, pressure, etc. at any radius of the cheese. An estimate of the error in calculating the value of the compression is given. The chapter concludes with the presentation and discussion of the computed results.

3.2 The Equation for the Compression of the Cheese

3.2.1 The Element at Any Radius ρ

Consider an element at any radius ρ in a cheese of outer radius R. This element was wound originally at r and as the cheese was subsequently built from r to R the radius r deformed by U such that $\rho = r + U$. This element is formed of a number of layers of small thickness composed of a number of threads in tension laid side by side with a gap which depends on the radius at which the element is situated. In such an arrangement the radially adjacent layers contact each other. The element subtends an angle ϕ at the centre of the cheese. The number of threads in one layer of an element in either axial or circumferential direction remain constant regardless of the radius ρ. The dimensions of the element are $\phi\rho$, $d\rho$ and W in the circumferential, radial and axial directions respectively and they are chosen so that a diagonal lies in the yarn direction. The diagonal length of the element at ρ is L and the threads are at an angle α to a plane perpendicular to the axis of the package. In the element

$$\cos\alpha = \phi\rho/L = \phi\rho/\sqrt{(\phi\rho)^2 + W^2} = \rho/\sqrt{\rho^2 + a^2}, \quad \ldots \quad \ldots \quad \text{(i)}$$

where $a = W/\phi$; and

$$\sin\alpha = W/L = W/\sqrt{(\phi\rho)^2 + W^2} = a/\sqrt{\rho^2 + a^2}, \quad \ldots \quad \ldots \quad \text{(ii)}$$

$$\tan\alpha = W/\phi\rho = a/\rho \quad \ldots \quad \ldots \quad \text{(iii)}$$

and $$L = \sqrt{W^2 + (\phi\rho)^2} = \phi\sqrt{\rho^2 + a^2} \quad \ldots \quad \ldots \quad \text{(iv)}$$

Such an element is shown in Fig. 3.1.

The element is so chosen that if ϕ became 2π, W would become 'x'; where x is the traverse per wind. In such an element the number of ends in the axial and circumferential directions are the same because in one full circle in the circumferential direction the same number of ends will appear as will appear in one traverse. $x/2\pi$, denoted by 'a', is therefore a constant for a given cheese the value of which depends on machine setting, namely, the traverse per wind. This is termed as 'machine constant'.

$$x/2\pi = a = W/\phi \quad \ldots \quad \ldots \quad (3.1)$$

From the construction of the element it is evident that a layer is supported by the layer beneath at a number of crossing points. Whatever pressure which may be imposed by any layer is carried by the next lower layer on these crossing points. As the number of threads in the element remain constant regardless of the radius of the cheese the total number of crossing points will also remain constant at any radius ϱ of the cheese. Let $K\phi$ be the number of threads in a layer through a boundary face of the element (the same through each, because of the proportions of the element chosen), then the total number of crossing points in the element to support the upper layer are

$$2(K\phi)^2 \qquad \ldots \quad \ldots \quad (v)$$

If the winding at the core radius 's' is started by laying threads touching each other and 'D' be the diameter of the yarn then

$$K\phi = W.\cos\alpha_{os}/D;$$

where α_{os} is the wind angle at the core radius s. Substituting the value of $\cos\alpha_{os}$

$$K = \frac{a.s}{D.\sqrt{s^2 + a^2}} \qquad \ldots \quad \ldots \quad (vi)$$

To enable the spacing of the adjacent threads to be altered the relation is written as

$$K = a.s.\ \text{space} / (\sqrt{s^2 + a^2} \times D) \qquad \ldots \quad \ldots \quad (3.2)$$

where by assigning different values to 'space' the spacing of the adjacent threads can be altered. The maximum value of space = 1 gives the spacing of one diameter and a value of less than 1 gives a wider spacing of the adjacent threads.

The number of ends per layer per face of the element are $k\phi$ and the total number of the ends through the face of the element in either the circumferential or the axial direction is $(K.\phi.d\rho/D_p)$ where $d\rho/D_p$* is the number of layers in the element. This number is equal to

$K.\phi.dr/D;$

where D is the original diameter of the yarn and r is the original radius at which the element was added. If Q and Z be the nett circumferential and axial components of the force through the face of the element, then

$$Q = \frac{K\phi.dr}{D}.T.\cos\alpha, \qquad \ldots \quad \ldots \quad (3.3)$$

and $$Z = \frac{K\phi.dr}{D}.T.\sin\alpha. \qquad \ldots \quad \ldots \quad (3.4)$$

Note that in any single layer there are shear forces along the face of the element but those in successive layers are in the opposite direction so that the nett effect is zero.

3.2.2 <u>The Addition of a Layer at R</u>

Now consider that a layer of thickness dR is added at the outer radius R of the cheese. This imposes pressure over the cheese

* D_p is the present value of yarn diameter

as a result of which the element at ϱ deforms. The relations between small deformations of the element are obtained by differentiating the relations (i) and (iii) with respect to R. Differentiating (i) with respect to R

$$-\sin\alpha.\theta = \phi.u/L - \phi.\varrho.1/L^2$$

$$\text{or} \qquad 1/L = \frac{u}{\varrho} + \tan\alpha.\theta \qquad \ldots \quad \ldots \quad \text{(vii)}$$

where $1 = \frac{\partial L}{\partial R}.dR$, $u = \frac{\partial U}{\partial R}.dR$ and $\theta = \frac{\partial \alpha}{\partial R}.dR$. Differentiating (iii) with respect to R

$$\sec^2\alpha.\theta = \frac{w}{\phi\varrho} - \frac{u}{(\phi\varrho)^2}.W$$

$$\text{or} \qquad \frac{u}{\varrho} = \frac{w}{W} - \theta/(\sin\alpha.\cos\alpha) \qquad \ldots \quad \ldots \quad \text{(viii)}$$

where $w = \frac{\partial W}{\partial R}.dR$. From (viii)

$$\theta = \left(\frac{w}{W} - \frac{u}{\varrho}\right).\sin\alpha.\cos\alpha \qquad \ldots \quad \ldots \quad \text{(ix)}$$

If the change in axial strain, that is $w/W = 0$, then the above relations reduce to

$$u/\varrho = 1/L - \theta.\tan\alpha\ , \qquad \ldots \quad \ldots \quad (3.5)$$

$$1/L = u/\varrho + \theta.\tan\alpha \qquad \ldots \quad \ldots \quad (3.6)$$

$$\text{and} \qquad \theta = -u.\sin\alpha.\cos\alpha/\varrho. \qquad \ldots \quad \ldots \quad (3.7)$$

Due to the addition of the layer of thickness dR at the outer radius R the tension in the thread has changed by t $(= \frac{\partial T}{\partial R} . dR)$ from T to (T+t) and the angle of the thread has changed from α to $(\alpha+\theta)$. The total number of threads through the face of the element is $K.\phi.dr/D$. The changes in the circumferential and the axial components of the force through the face of the element are $q(= \frac{\partial Q}{\partial R}.dR)$ and $z(= \frac{\partial Z}{\partial R}.dR)$ respectively. The change in the circumferential component of the force through the face of the element, i.e. Q, is given by the expression

$$q = \frac{K.\phi.dr}{D} \left((T+t) \cos(\alpha+\theta) - T.\cos\alpha \right)$$

or

$$q = \frac{K.\phi.dr}{D} (T.\cos\alpha.\cos\theta + t.\cos\alpha.\cos\theta - T.\sin\alpha.\sin\theta - t.\sin\alpha.\sin\theta - T.\cos\alpha)$$

As θ is small, therefore $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1$. Therefore

$$q = \frac{K.\phi.dr}{D} (t.\cos\alpha - T.\sin\alpha.\theta - t.\sin\alpha.\theta) \quad \ldots \quad (3.8)$$

Similarly the change z in the axial component of the force through the face of the element, i.e. Z, is given by the expression

$$z = \frac{K.\phi.dr}{D} (t.\sin\alpha + T.\cos\alpha.\theta + t.\cos\alpha.\theta) \quad \ldots \quad (3.9)$$

The element is under equilibrium and by resolving the changes in the forces along the axis of the symmetry of the element

$$p + \frac{\partial p}{\partial r}.dr - p = q\,\phi$$

or $$\frac{\partial p}{\partial r} = \frac{q\phi}{dr} \qquad \ldots \quad \ldots \qquad (3.10)$$

The change in the circumferential strain of the element is u/r where u is $\frac{\partial U}{\partial R}.dR$ and the change in the radial strain of the element is $\partial u/\partial r$. Eliminating q from (3.8) and (3.10)

$$\frac{\partial p}{\partial r} = \frac{K.\phi^2}{D}(t.\cos\alpha - T.\sin\alpha.\theta - t.\sin\alpha.\theta) \qquad \ldots \qquad (x)$$

The change in the radial strain of the element, i.e. $\frac{\partial u}{\partial r}$, is related to the change of pressure per crossing point, therefore

$$p = \frac{\partial u}{\partial r}.E.2(K\phi)^2; \qquad \ldots \quad \ldots \qquad (3.11)$$

where $2(K\phi)^2$ is the total number of the crossing points supporting the element and E is the Modulus of Compression of the cheese and is defined as $1/2(K.\phi)^2$ times the force required to produce unit strain in the thickness of the element. The value of E is not necessarily constant but in the present analysis it is assumed as constant. Differentiating (3.11) with respect to r

$$\frac{\partial p}{\partial r} = 2.(K.\phi)^2.E.\frac{\partial^2 u}{\partial r^2} \qquad \ldots \quad \ldots \qquad (xi)$$

Eliminating $\frac{\partial p}{\partial r}$ from (x) and (xi)

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{2.K.D.E}(t.\cos\alpha - T.\sin\alpha.\theta - t.\sin\alpha.\theta) \quad \ldots \qquad (3.12)$$

Now 't' is the change in the tension T in the yarn due to change l in the length L of the thread due to the addition of the layer at the outer radius R, therefore,

$$t = EY.l/L \qquad \ldots \quad \ldots \quad (xii)$$

where EY is the Elasticity of the yarn in Extension and is defined as the force in the yarn required to produce unit strain in the length of the yarn. Substituting the value of l/L from equation (3.6)

$$t = EY.\left(\frac{u}{\varrho} + \theta.\tan\alpha\right).$$

Substituting the value of θ from equation (3.7)

$$t = EY.\left(\frac{u}{\varrho} - \frac{u}{\varrho}.\sin^2\alpha\right)$$

or

$$t = EY.\frac{u}{\varrho}.\cos^2\alpha \qquad \ldots \quad \ldots \quad (3.13)$$

Substituting the value of t from equation (3.13) and of θ from equation (3.7) in equation (3.12)

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{2.K.D.E}\left(EY.\frac{u}{\varrho}.\cos^3\alpha + T.\frac{u}{\varrho}.\sin^2\alpha.\cos\alpha + EY\left(\frac{u}{\varrho}\right)^2.\cos^3\alpha.\sin^2\alpha\right)$$

or

$$\frac{\partial^2 u}{\partial r^2} = \frac{EY}{2.K.D.E}.\frac{u}{\varrho}.\cos^3\alpha\left(1 + \frac{T}{EY}\tan^2\alpha + \frac{u}{\varrho}\sin^2\alpha\right) \quad \ldots \quad (xiii)$$

3.2.3 Values of ρ, T and α

The values of ρ, α and T in equation (xiii) are current values. Now

$$\rho = r + \int_r^R \frac{\partial U}{\partial R}.dR = r + U;$$

where U is the compression of the cheese at the radius r for the outer radius R of the cheese and r is the original radius at which the element was wound. This compression U of the cheese at r is due to the successive additions of the layers of thickness dR as the cheese was built up from R = r to R = R. The value of U at r is given by the integral

$$\int_r^R \frac{\partial U}{\partial R}.dR \quad \text{where} \quad \frac{\partial U}{\partial R}.dR = u,$$

and its numerical value is obtained as the commulative total of u's at r produced by the addition of the successive layers at the outer radius as it increases from R = r to R = R. Similarly,

$$\alpha = \alpha_o + \int_r^R \frac{\partial \alpha}{\partial R}.dR \text{ and } T = T_o + \int_r^R \frac{\partial T}{\partial R}.dR;$$

where α_o and T_o are the wind angle and the tension in the yarn with which the element containing the yarn was wound at r. The differences between ρ and r and α and α_o are small and might be neglected, though in numerical solution they can be taken account of. That between T and T_o is not small because EY is involved and in fact

T_o and

$\int_r^R \frac{\partial T}{\partial R}.dR$ might be of the same order. Now

$$T = T_o + \int_r^R \frac{\partial T}{\partial R}.dR \qquad \ldots \qquad \ldots \qquad (xiv)$$

and substituting the value of $\frac{\partial T}{\partial R}.dR$ (=t) from equation (3.13) and using the original values r and α_o in the above relation

$$T = T_o + \int_r^R EY.(\frac{\partial U}{\partial R}.dR).\cos^2\alpha_o/r$$

or $$T = T_o + EY.\cos^2\alpha_o.U/r \qquad \ldots \qquad \ldots \qquad (xv)$$

Substituting the values of T and replacing α and ρ by α_o and r respectively in equation (xiii)

$$\frac{\partial^2 u}{\partial r^2} = \frac{EY}{2.K.D.E}.\frac{u}{r}.\cos^3\alpha_o \ (1 + \frac{T_o}{EY}\tan^2\alpha_o + \frac{\sin^2\alpha_o}{r}(u+U))..(xvi)$$

Substituting the values of $\cos\alpha_o$, $\tan\alpha_o$ and $\sin\alpha_o$ in terms of r

$$\frac{\partial^2 u}{\partial r^2} = \frac{EY}{2.K.D.E}.\frac{u.r^2}{(r^2+a^2)^{3/2}}(1+\frac{T_o}{EY}.\frac{a^2}{r^2} + \frac{a^2}{r.(r^2+a^2)}.(u+U)) \ .. \ (3.14)$$

3.2.4 <u>Values of q, z, p, Z and Q</u>

From equation (3.9)

$$z = \frac{K.\phi.dr}{D}(t.\sin\alpha + T.\cos\alpha.\theta + t.\cos\alpha.\theta)$$

Replacing ρ and α by the original values r and α_o respectively and substituting the values of t, T and θ in the above equation

$$z = \frac{K.\phi.dr}{D}.(EY.\frac{u}{r}.\cos^2\alpha_o.\sin\alpha_o + T_o.\cos\alpha_o(-\frac{u}{r}.\sin\alpha_o.\cos\alpha_o) + EY.\frac{U}{r}.\cos^2\alpha_o.\cos\alpha_o.(-\frac{u}{r}.\sin\alpha_o.\cos\alpha_o) + EY.\frac{u}{r}.\cos^2\alpha_o.\cos\alpha_o.(-\frac{u}{r}.\sin\alpha_o.\cos\alpha_o))$$

or

$$z = \frac{K.\phi.dr}{D}.EY.\frac{u.r.a}{(r^2+a^2)^{3/2}}\left(1 - \frac{T_o}{EY} - \frac{r}{(r^2+a^2)}.(u+U)\right) \quad \ldots \ (3.15)$$

Substituting the original values of ϱ, $d\varrho$ and α and substituting the values of t, T and θ in equation (3.8) and simplifying

$$q = \frac{K.\phi.dr}{D}.EY.\frac{u.r^2}{(r^2+a^2)^{3/2}}\left(1 + \frac{T_o}{EY}.\frac{a^2}{r^2} + \frac{a^2}{r.(r^2+a^2)}.(u+U)\right). \ (3.16)$$

Substituting the value of ϕ as W/a in equation (3.11)

$$p = \frac{2K^2.W^2}{a^2}.E.\frac{\partial u}{\partial r} \quad \ldots \quad \ldots \quad (3.17)$$

When the element itself was wound at r, i.e. when R = r, the axial component of the force Z_{or} through the face of the element is given by the following equation

$$\underset{(R=r)}{Z_{or}} = \frac{K.\phi.dr}{D}.T_o.\cos\alpha_{or}$$

However as the cheese is built up from this radius to any other radius R, the axial component of the force through the face of the element has changed. If $\frac{\partial Z}{\partial R}.dR$ (=z) is the change in Z due to the addition of a layer at the outer radius then the total change in Z_{or} when the outer radius is R is given by the integral $\int_r^R \frac{\partial Z}{\partial R}.dR$. Therefore the current or the residual axial component of the force through the face of the element is given by the expression

$$Z = Z_{or} + \int_r^R \frac{\partial Z}{\partial R}.dR \qquad \ldots \quad \ldots \quad (3.18)$$

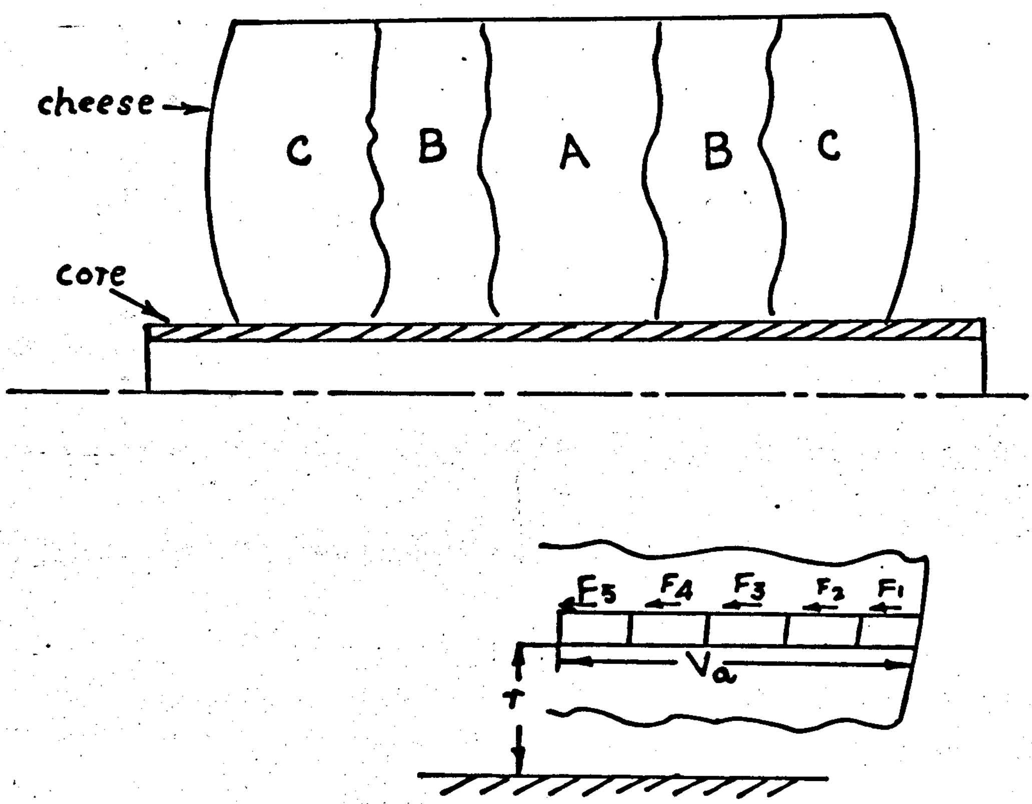
Similarly the residual circumferential component Q of the force through the face of the element is given by the expression

$$Q = Q_{or} + \int_r^R \frac{\partial Q}{\partial R}.dR \qquad \ldots \quad \ldots \quad (3.19)$$

3.2.5 Axial Forces

During the winding of the cheese the thread changes its direction at the ends and therefore the axial component of the tension in the yarn at the ends is zero. Therefore in a more realistic model the axial force of a layer would vary from a value of zero at the package end by increments of shear force caused by friction between layers so that

$$\frac{\partial Z}{\partial V}.W + \frac{\partial F}{\partial r}.dr = 0 ; \qquad \ldots \quad \ldots \quad \text{(xvii)}$$



A - no axial deformation.

B - axial deformation by shear within layers.

C - axial deformation by shear and slip between layers.

$$\sum F \text{ at } (r, V_a) = \int_r^R Z \, . dr \, ,$$

Regions of the cheese

FIG. 3.2

where V is the axial distance of the element from the end of the package and F is a friction force and acts only where there is a tendency to slip and where, in the absence of such a force, there would be relative movement between the layers. In the region where this tendency to slip is absent F is zero and does not change with V. Thus there might be three regions near the end of the package - A where there is no tendency to slip, B where there is shear, and C where slip might occur before the shear stabilises.

In the solution, which is based on the assumption that the axial deformation of the package is zero, the tension and its axial component in the yarn in an element change with r but not with the axial position of the element, i.e. V. The solution derived here would apply to that region where there is no tendency for the layers to slip, i.e. A. This is shown in Fig. 3.2.

Apart from slipping this end region would be expected to deform elastically, each element deforming by shearing through an angle proportional to the change in the value of F each time a layer of thickness dR is added. In fact the deformation must be compatible with both shear and extension of these elements and can only be calculated by a full solution of this complicated system. If in this region the resistance to extension in the layer were ignored the deformation due to shear would be proportional to $\int_s^r F.dr$ for each layer of thickness dR added. The total F at any r is the sum of the calculated Z's outside that r but the axial distribution of F is not known. If it could be assumed uniform and the axial length of this region also did not vary with r then the deformation would vary with r as:-

$$\int_s^r \left(\int_r^R Z.dr \right).dr$$

On the other hand, if extension only determined the deformation it would vary as Z, i.e. as $\int_r^R z.dR$. The tendency for layers to slip over each other at any point will depend on the ratio F/P which again cannot be known without a full solution but it seems reasonable to expect this to be linked with the value of the sum of F's in the axial direction, i.e. $\int_r^R Z.dr$ and the value of P in the central region. The value of $\int_r^R Z.dr$ is given by the following expression

$$\int_r^R Z.dr = \int_r^R Zo.dr + \int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr \quad \ldots \quad \ldots \quad (3.20)$$

These functions have been evaluated in the program - not because they represent actual behaviour but because they are involved in the boundary condition of any complete solution. If they vary in a similar way with external conditions it seems likely that the deformation varies in that way. If they behave in different ways no conclusion can safely be drawn.

3.2.6 Change of Wind Angle

The value of wind angle depends on 'x' which is defined as traverse length per wind. A change of wind angle is accomplished by altering the value of x. In the equation derived the element is so chosen that when W, the axial length of the element, becomes x, ϕ becomes 2π. This gives the relation $x/2\pi = W/\phi =$ 'a' – a constant for a given cheese termed as a machine constant. A change in wind angle due to a change in x would either change W or ϕ and 'a'. A change in W or ϕ alters the size of the element and in order that the forces are comparable in the cases of cheeses with different wind angles these are worked out on a standard size of the element, one which is of constant length, thickness and subtends a constant angle at the axis. The standard size of the element adopted is that when x = 5 cm, W = 1 cm; this gives $\phi = 2\pi.W/x$ and the thickness chosen is 0.1 cm. The computer program is arranged to output the results of the solution of the problem in this form.

3.3 Integration of the Differential Equation

3.3.1 Boundary Conditions

The differential equation (3.14) is

$$\frac{\partial^2 u}{\partial r^2} = \frac{EY}{2.K.D.E} \; \frac{u.r^2}{(r^2+a^2)^{3/2}} \left(1 + \frac{T_o}{EY}.\frac{a^2}{r^2} + \frac{a^2}{r.(r^2+a^2)} (u+U) \right) = \text{say, } F(u,r).$$

This is a second order differential equation in u and r and hence two boundary conditions are necessary to solve the equation. An analytical solution is not likely to be possible and, therefore, the equation is to be integrated numerically. The necessary boundary conditions are as follows. (In this case the more strictly correct version in which the rs on R.H.S. are replaced by ϱs can be used).

1. At the core radius 's' the deformation in r due to the addition of a layer of thickness dR at R is zero, as the core is assumed to be incompressible. This means that U, the total compression of the cheese, at core radius 's' is also zero. Therefore the first boundary condition is that when

$$r = s \quad u = o, \quad \text{also} \quad U = o. \qquad \ldots \quad \ldots \quad (3.21)$$

2. At the outer radius R, the change in the pressure of the element caused by the winding of a layer is the pressure imposed by the element of the layer added. This pressure is given by the expression

$$P_{OR} = -\frac{K.\phi^2.dR}{D}.\ T_o.\frac{R}{\sqrt{R^2+a^2}} \qquad \ldots \quad \ldots \quad (3.22)$$

Therefore at r = R,

$$\frac{\partial u}{\partial r} = -\frac{T_o}{2.K.D.E}\cdot\frac{R}{\sqrt{R^2+a^2}}.dR \qquad \ldots \quad \ldots \quad (3.23)$$

Above equation gives the second boundary condition.

3.3.2 Procedure for Solving the Equation

The numerical solution can be obtained by adding layers of yarn at the outside of the package and adding their effects to get the total changes necessary to evaluate the coefficients. Alternatively the effects of adding smaller layers at intervals of R can be calculated and then integrating with respect to R to get the coefficients.

The second method gives a more complex program but should be more accurate. The first method will probably be sufficiently accurate if the added layers are sufficiently small; it is necessary to compromise between a more complex program and a longer number of iterations for the best use of the computer.

As the necessary boundary conditions are established the calculations for solving the equation can be started. The start can be made either at the core radius 's' or at the outer radius R. For the first solutions the start was made at the core, although for solving the second case with varying values of E and EY, as explained later, it is advantageous to start the calculation at the outer radius R.

From the first boundary condition u and U are zero at core radius 's', this gives $\frac{\partial^2 u}{\partial r^2}$ as zero at the core radius. A guess of the value of $\frac{\partial u}{\partial r}$ is made at $r = s$. The equation is integrated numerically using the modified method of Euler for integration. The calculation is carried up to the radius when $r = R$. The calculated value of $\frac{\partial u}{\partial r}$ at R is compared with the correct value of $\frac{\partial u}{\partial r}$ at $r = R$ obtainable from equation (3.23), i.e. from the second boundary condition. If the two values are different a fresh guess of the value of $\frac{\partial u}{\partial r}$ at $r = s$ is made and the entire calculation is repeated. The second value of $\frac{\partial u}{\partial r}$ at $r = R$ is again compared with the correct value of $\frac{\partial u}{\partial r}$ at $r = R$. If different again the entire calculation is repeated with yet another value of $\frac{\partial u}{\partial r}$ at $r = s$. This value of $\frac{\partial u}{\partial r}$ is obtained by linear interpolation from the previous two sets of values of $\frac{\partial u}{\partial r}$ at $r = s$ and at $r = R$. The process is repeated again and again till the value of $\frac{\partial u}{\partial r}$ at $r = R$ is sufficiently close to the correct value of $\frac{\partial u}{\partial r}$ at $r = R$. The last calculation is the solution of the equation. This gives the value of u at any radius r such that $s \leq r \leq R$ for the addition of a layer of thickness dR at the outer radius R.

3.3.3 Interpolation of the Value of $\partial u/\partial r$ at the Core Radius

The formula for linear interpolation of the next value of $\frac{\partial u}{\partial r}$ from the two previous sets of values of $\frac{\partial u}{\partial r}$ at $r = s$ and at $r = R$ is

$$ndu = sdu + \frac{(sdu - C)}{(du - D1)} \times (cdu - du) \quad \ldots \quad \ldots \quad (3.24)$$

where ndu - is the new value of $\partial u/\partial r$ at $r = s$;

C - is the first guess of the value of $\partial u/\partial r$ at $r = s$;

D1 - is the first calculated value of $\partial u/\partial r$ at $r = R$ obtained from C;

sdu - is the second guess of the value of $\partial u/\partial r$ at $r = s$;

du - is the second calculated value of $\partial u/\partial r$ at $r = R$ obtained from sdu;

cdu - is the correct value of $\partial u/\partial r$ at $r = R$ obtained from the second boundary condition.

3.3.4 Euler's Modified Method for Integrating the Equation

For integrating the equation the cheese is divided into a number of layers. Starting from the known values at the core, i.e. at the beginning of the first layer the values at the beginning of the second layer are calculated. This is repeated for the next layer and continued until the cheese is solved up to the given radius when $r = R$. The method is as follows.

Let tu, tdu and td2u represent the values of u, $\frac{\partial u}{\partial r}$ and $\frac{\partial^2 u}{\partial r^2}$ respectively at the radius denoted by [k], i.e. at r[k] . Let dr be the thickness of the layer, i.e. the step length. Then at (r + dr), i.e. at r[k + 1]

u = tu + tdu.dr;

du = tdu + td2u.dr;

d2u = F(u, r[k + 1]);

The values obtained are the projected values; these values are averaged and the new values are calculated and suffixed by 'a'.

dua = tdu + (td2u + d2u).dr/2;

ua = tu + (tdu + dua).dr/2;

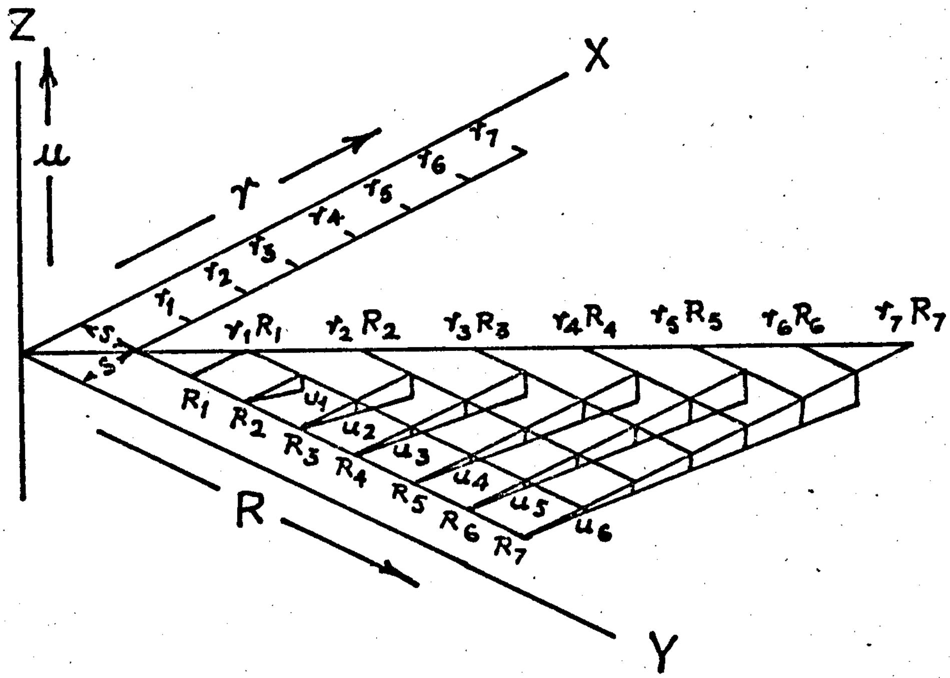
d2ua = F(ua, r[k + 1]);

Now u and ua are compared and if different the calculation is repeated again using the new values, and the process continued until two successive values are sufficiently close to each other.

The values of u, du and d2u obtained from the last calculation are the values of u, $\frac{\partial u}{\partial r}$ and $\frac{\partial^2 u}{\partial r^2}$ at r[k + 1]. For calculating the values of u, $\frac{\partial u}{\partial r}$ and $\frac{\partial^2 u}{\partial r^2}$ at the next step of the radius, i.e. at r[k + 2] the values at r[k + 1] are used as the starting values and the calculation is repeated as above. The process is continued till the whole range of radius from r = s to r = R is exhausted.

3.3.5 Calculation of the Value of U

The equation (3.14) contains the term $U \left(= \int_r^R \frac{\partial U}{\partial R}.dR\right)$. This is the total deformation of r as the cheese was built up from R = r to R = R before the addition of the layer of thickness dR at the outer radius R. The integration of the equation at any R



$$U \text{ at } r_1 = u_1 + u_2 + u_3 + u_4 + u_5 + u_6$$

Calculation of the value of U

FIG.3.3

requires the prior knowledge of U at every step of r for the outer radius R. Therefore the calculation has to start right from the beginning (i.e. from the core) and proceed for $s \leqslant r \leqslant R$ every time a layer is added at the outer radius R. Then as the cheese is built up layer by layer the values of u and U are calculated at each step of r for the addition of each layer at the outer radius R. The values of U at each step of r obtained from the previous solution are used in the next solution for the addition of the next layer at R. This type of calculation is made possible by the fact that for the first layer added U is zero and makes it possible to solve the equation for the addition of the second layer.

In Fig. 3.3 x-axis represents the radius r, y-axis represents the outer radius R and the z-axis represents the deformation of r due to the addition of a layer at the outer radius R, i.e. u $(= \frac{\partial U}{\partial R} . dR)$. Consider the point r_1R_1. This represents the values after the addition of the first layer. Since this is the only layer added u and U are numerically zero and there is no curve for u with respect to r. Consider the point r_2R_2. This represents the values after the addition of the second layer over the first layer. The outer radius is R_2 and the radius of the cheese is r_1. The curve for u, i.e. u_1, with respect to r is drawn. Likewise the picture is completed for many points r_3R_3, r_4R_4, r_5R_5, etc. These represent the addition of many more layers. The curves u_2, u_3, u_4, etc. are drawn. From these curves the values of U can be calculated at any point, for example

$$U \text{ at } r_2 = \text{value of } u \text{ at } r_2 \text{ from curve } u_2$$
$$+ \text{ value of } u \text{ at } r_2 \text{ from curve } u_3$$
$$+ \text{ value of } u \text{ at } r_3 \text{ from curve } u_4$$

- -

However any later curve of u, say u_2 when $R = R_3$ can only be drawn when the value of U at all steps of r, i.e. r_1 and r_2, when $R = R_2$ is known.

It may be observed that if the size of the layer added be doubled the value of u for the addition of each layer too would be nearly doubled, but the number of layers added to make the given outer radius would be halved. The value of U at any radius r would remain nearly the same, as now the number of u's to be added to give U would be halved though the value of each u would be almost twice.

3.3.6 Error in the Values of R and U at r

In the solution the cheese is built up from r to the final radius RO by adding a number of layers, say 'n', each of thickness dR, i.e.

$$r + n \times dR = RO$$

However, while the layer is being added at R it causes compression u of the cheese at r (= R) and the outer radius would be short of (R + dR) by nearly u; and after the addition of 'n' layers the outer radius would be short of RO by $\sum^{n} u$. The values of $u_1, u_2, \ldots, u_n$ depend on R, the radius at which the layer is added. The effect of this error can be reduced by adding extra layer so that the total does in fact reach RO. The value of $\sum^{n} u$ is small and is approximately 0.16 cm in a cheese with the values of 0.05 cm, 100g, 5000g, 5 cm, 1D, 0.1 cm and 20g for the diameter of the yarn, E, EY, x, spacing, dR and T respectively as compared to the value of RO of 5 cm.

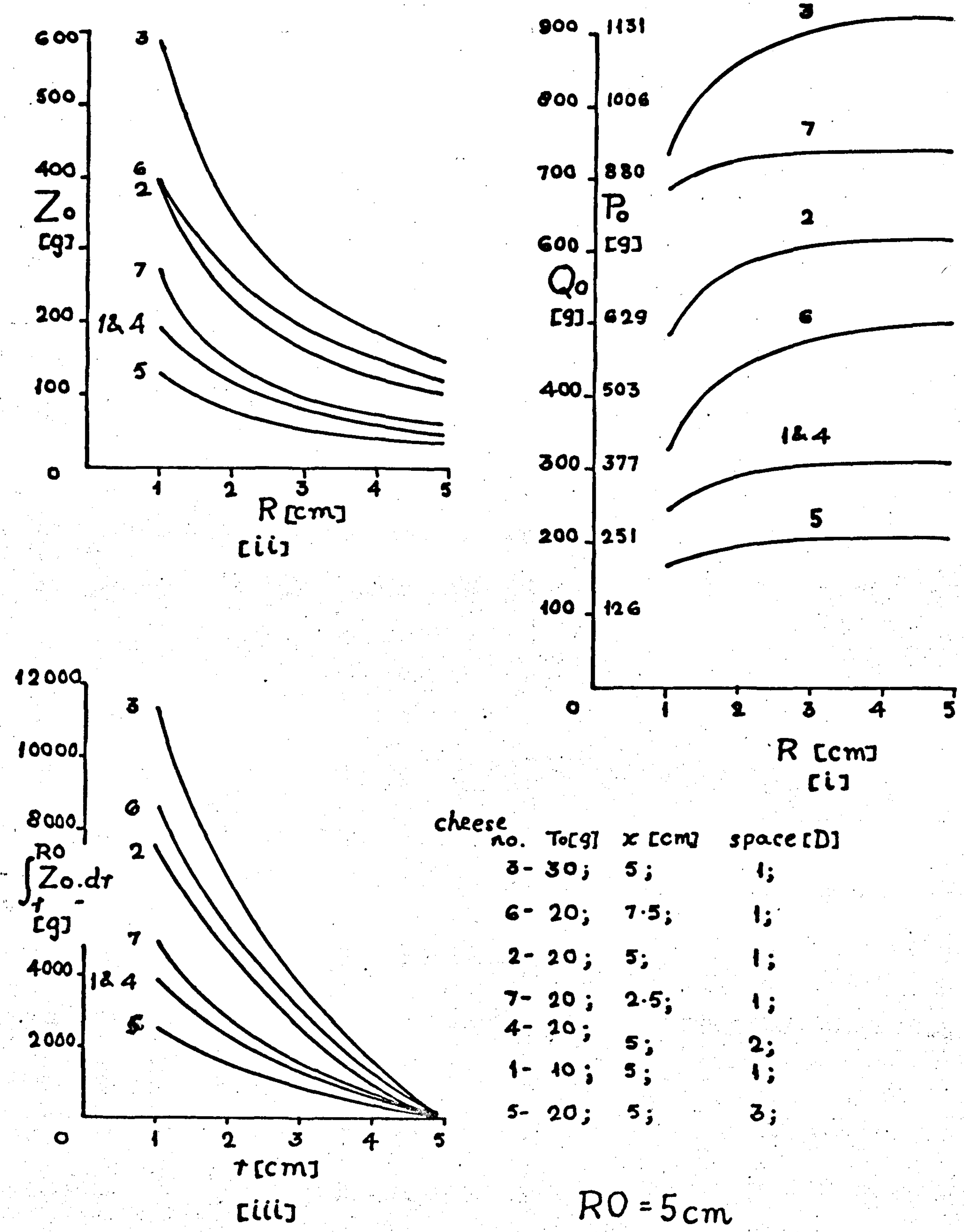
The addition of a layer at the outer radius causes the radius r of the cheese to shorten by u, the deformation in r. When the next layer is added the value of u should then be read off at (r + u) and not r to get the correct value U at r. However for keeping the computer program simple the value of u is read at r and not at (r + u) in evaluating the value of U. The error introduced at the end of the completed cheese would approximately be equal to the difference of the values of u at r and at $(r + \sum^{n} u)$. The difference is small and in the above cheese is approximately equal to the difference in the values of u at r (= 2.9 cm) and $(r + \sum^{n} u)$ (= (2.9 - 0.0153) cm) when the radius under consideration is 2.9 cm. The u is caused by the increase of 0.1 cm in the outer radius to make it 5 cm. If, for softer material the deformation in r was larger, the radii could be adjusted in the program to correct this.

3.4 Theoretical Results with Discussion

3.4.1 Values of the Variables

The tensions, pressures, etc. within a precision wound cheese have been calculated by program 15 using a range of values of yarn modulii which include those applicable to the yarn used in the experiments of Chapter 2 to measure the deformation of a precision cheese. Similarly the dimensions of the cheese are the same, namely a core radius of 1 cm and a final outer radius of 5 cm. The diameter of the yarn is 0.05 cm as 20 wraps of yarn when laid side by side touching each other on the experimental cheese make 1 cm.

In order to study effects of varying winding tension, spacing of the adjacent wraps of yarn, traverse per wind i.e. wind angle, Modulus of Compression of cheese and Elasticity of yarn in Extension on the tensions, pressures etc. in the cheeses a number of cheeses with different combinations of the values of these variables are solved. Also the cheeses with isotropic yarn and with near parallel winding are solved. These cheeses are numbered from 1 to 14 and their details are given in Table 3.1. The detailed results of each cheese are given in Appendix C and the forces are converted to facilitate comparisons to total forces acting through the faces of a standard size of element of length 1 cm, thickness 0.1 cm and subtending an angle of $2\pi/5$ at the axis of the cheese.



cheese no.	To[g]	x [cm]	space [D]
3-	30;	5;	1;
6-	20;	7·5;	1;
2-	20;	5;	1;
7-	20;	2·5;	1;
4-	20;	5;	2;
1-	10;	5;	1;
5-	20;	5;	3;

RO = 5cm

FIG.3.4

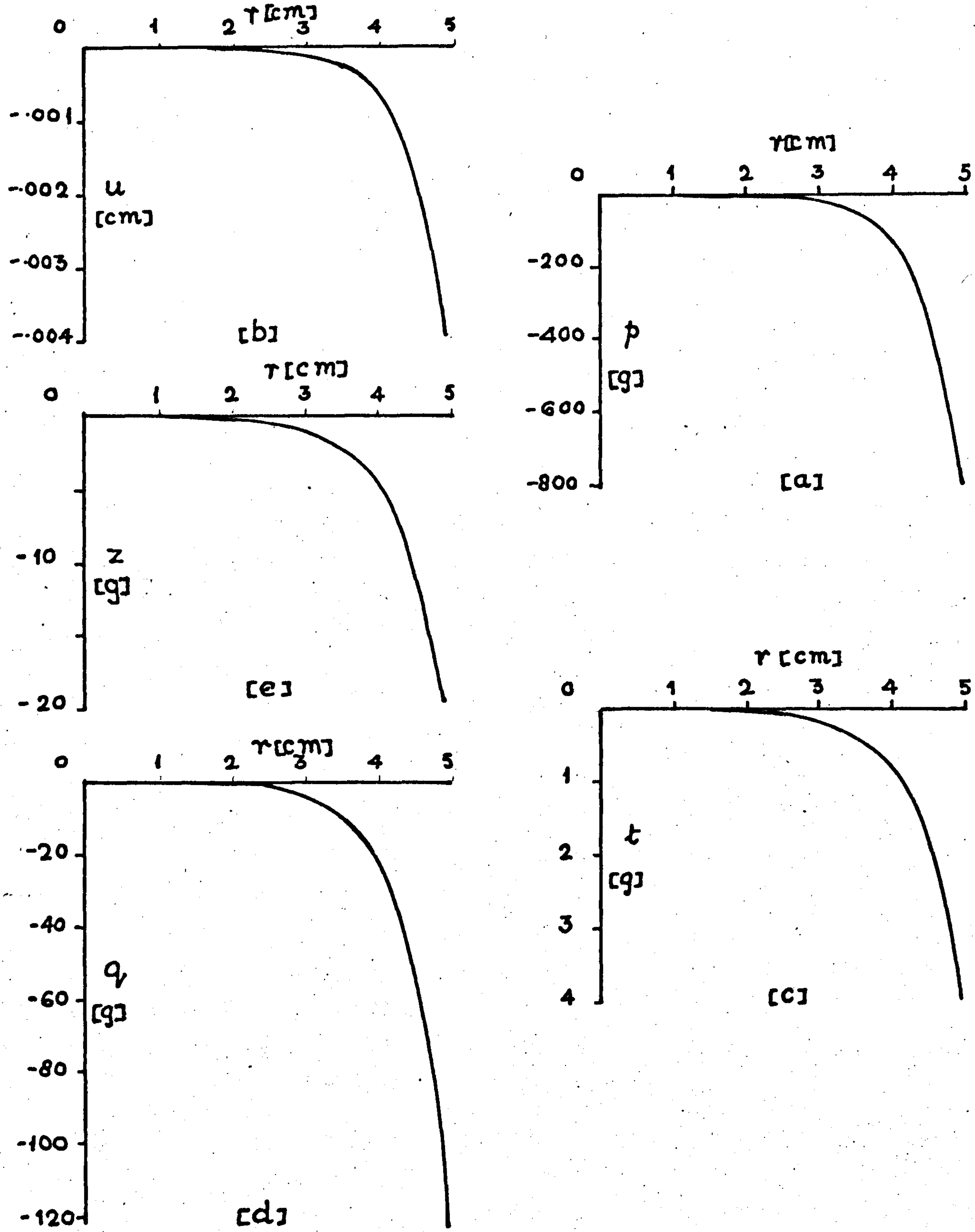
Table 3.1

cheese no	winding tension g	spacing D	traverse per wind cm	E g	EY g
1	20	1	5	200	5000
2	30	1	5	200	5000
3	10	1	5	200	5000
4	20	2	5	200	2000
5	20	3	5	200	2000
6	20	1	7.5	200	5000
7	20	1	2.5	200	5000
8	20	1	5	100	5000
9	20	1	5	400	5000
10	20	1	5	200	2000
11	20	1	5	200	8000
12	20	1	5	1000	1000
13	20	1	0.05	1000	1000
14	20	1	5	600	600

3.4.2 Forces Through the Face of the Added Element

Because the wind angle changes with winding-on radius the forces in the added layer vary with radius. Q_o, P_o and Z_o of the element added are shown by curves (i) and (ii) of Fig. 3.4. The curves (iii) show $\int_r^R Zo.dr$ with r in a completed cheese of 5 cm radius. The number on the curve shows the cheese to which that curve refers.

Q_o and P_o increase and Z_o reduces with R. This is due to the reduction in the wind angle with R. The values of Q_o, P_o, Z_o and $\int_r^R Zo.dr$ also depend on the winding tension T in the yarn and



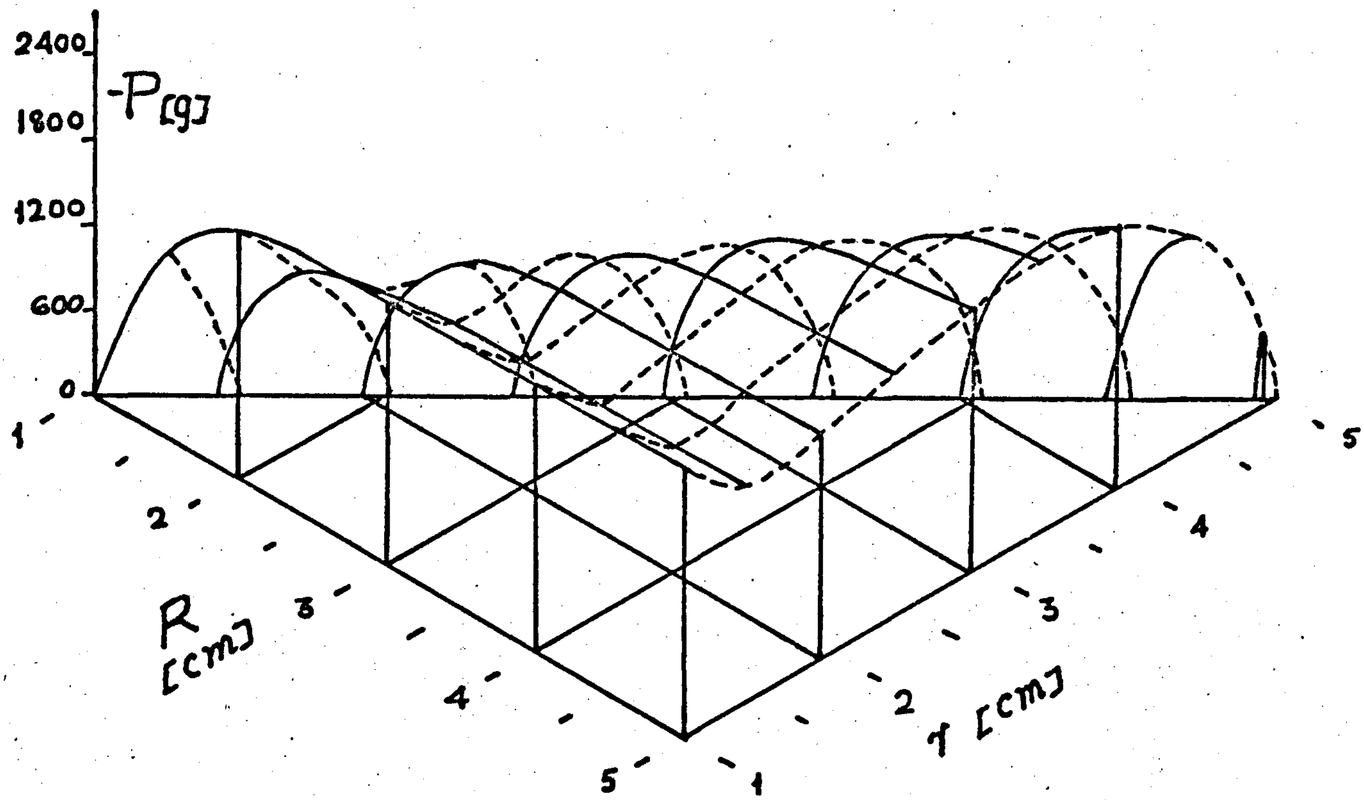
RO=5cm; To=20g; traverse=5cm; space=1D; E=200g; EY=5000g;

FIG.3.5

increase proportionally to the increase in T as shown by curves 1, 2 and 3. An increase in the spacing of the adjacent wraps of yarn from 1D to 2D and 3D reduces the number of threads through the face of the element added to $\frac{1}{2}$ and $\frac{1}{3}$ respectively and therefore Q_o, P_o, Z_o and $\int_r^R Z_o.dr$ also reduce to $\frac{1}{2}$ and $\frac{1}{3}$ as shown by curves 2, 4 and 5. A decrease in traverse per wind reduces the wind angle and as a consequence Q_o and P_o increase and Z_o and $\int_r^R Z_o.dr$ reduce. This is shown by curves 7, 2 and 6.

3.4.3 <u>Changes Due to the Addition of a Layer at R</u>

Fig. 3.5 shows the effect on pressure, tension, etc. within a cheese with a realistic value of 25 of the ratio of EY/E when a layer of thickness dR (= 0.1 cm) is added at R (= 4.9 cm). The value of p $(= \frac{\partial P}{\partial R}.dR)$, shown by curve (a), shows that it reduces very rapidly initially with r and there is no significant change in p at r below 3 cm. The most of the pressure imposed by the added layer is supported by a few layers at the outer part of the cheese because of the value of the ratio EY/E (= 25) which allows the yarn to lose its tension quickly for small value of U and the cheese not being allowed to deform axially (the effects of a smaller ratio will be seen later). The incremental compression u $(= \frac{\partial U}{\partial R}.dR)$, shown by curve (b), depend on 'p' and is, therefore, similar. The change in tension i.e. $t = (\frac{\partial T}{\partial R}.dR)$, shown by curve (c), depends on 'u' and the changes 'q' and 'z' in Q and Z, shown by curves (d) and



To=20g; x=5cm; spacing=1D; E=200g; EY=5000g; RO=5cm;

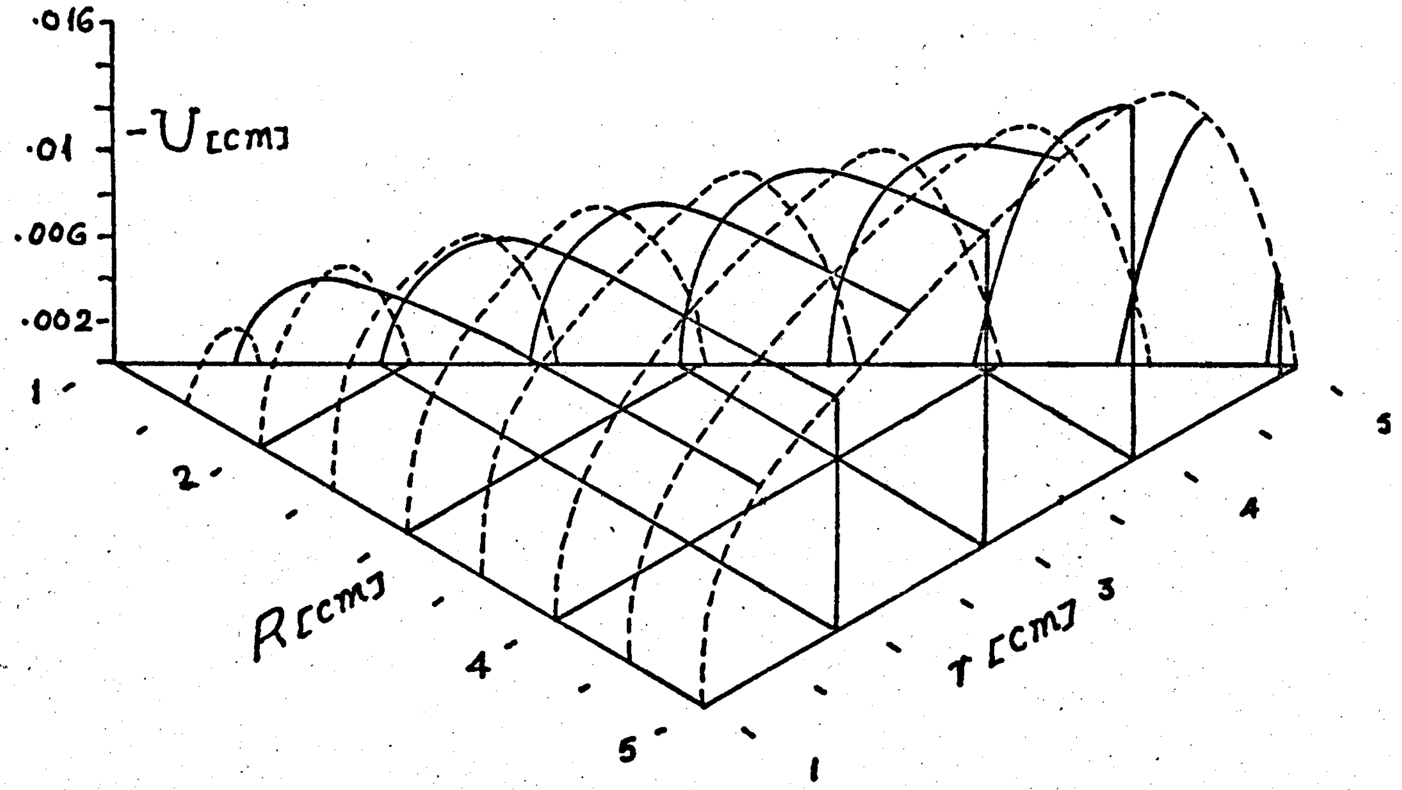
FIG.3.6

(e) respectively, depend on 't' and these changes are also similar to 'p'.

3.4.4 Results in a Completed Cheese with Typical Yarn Properties

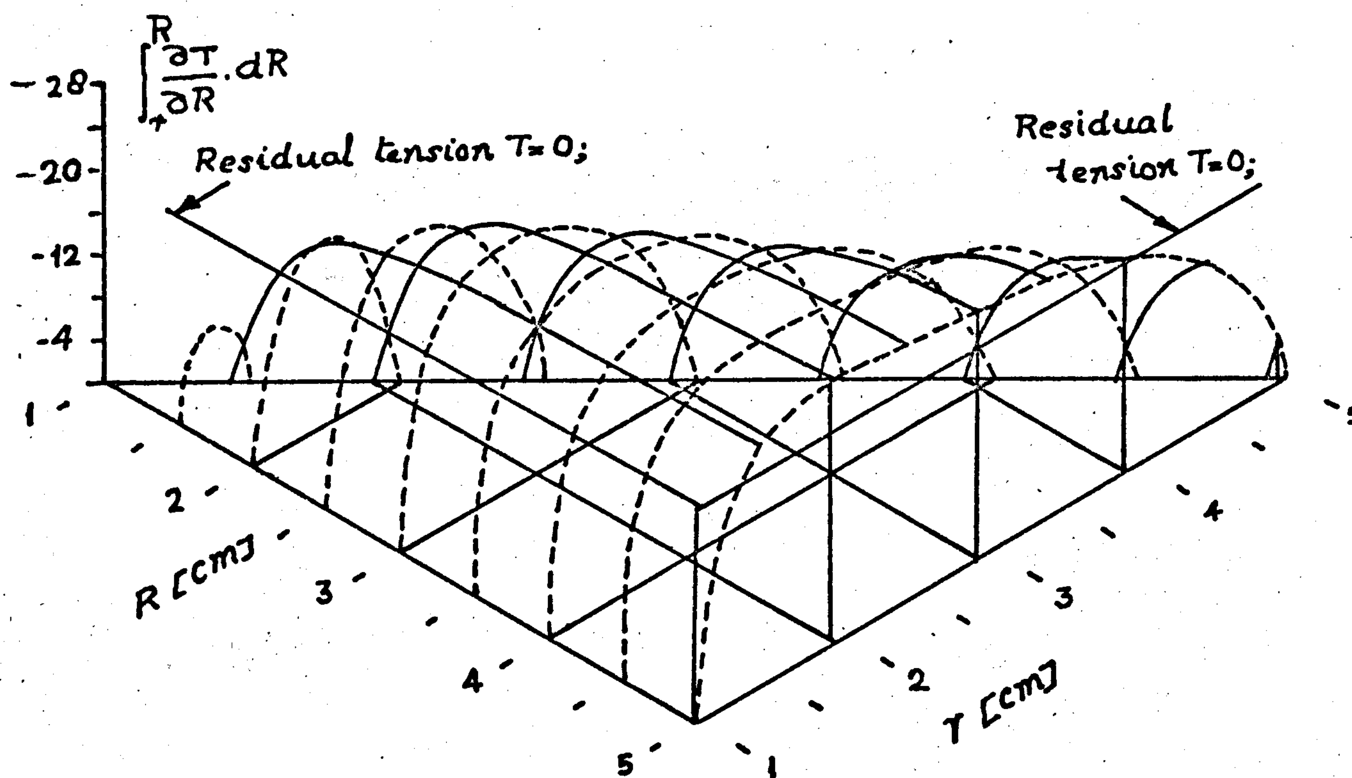
Figs. 3.6 to 3.11 show the distribution of pressure, compression etc. within a completed cheese (no. 2) of radius of 5 cm. Fig. 3.6 shows P at all values of r and R. The pressure at any given r increases rapidly for some initial increase in R from r but after the addition of few layers there is no significant increase in P at r. This is shown most obviously by the curve at r = 1.4 cm. This indicates that most of P_o, the pressure imposed by the layer added, is supported by a few layers at the outer part of the cheese and the inner part is left comparatively unaffected. This is due to a high value of the ratio EY/E which allows the yarn in a few layers beneath the added layer at R to lose all its tension for a small value of U in these layers and any further increase in U uses up P_o, again, due to the high value of the ratio EY/E.

In the completed cheese the maximum pressure occurs at the core and the pressure curve shows a minimum at the radius of 1.4 cm. The pressure then increases gradually with the radius and falls rapidly again after the radius of 3.9 cm. The built up of the pressure at the core is due to the core being incompressible so that the layers just above it do not lose their tension and contribute



To = 20 g; x = 5 cm; spacing = 1D; E = 200; EY = 5000; RO = 5 cm;

FIG.3.7



$$T = T_o + \int_r^R \frac{\partial T}{\partial R} \cdot dR$$

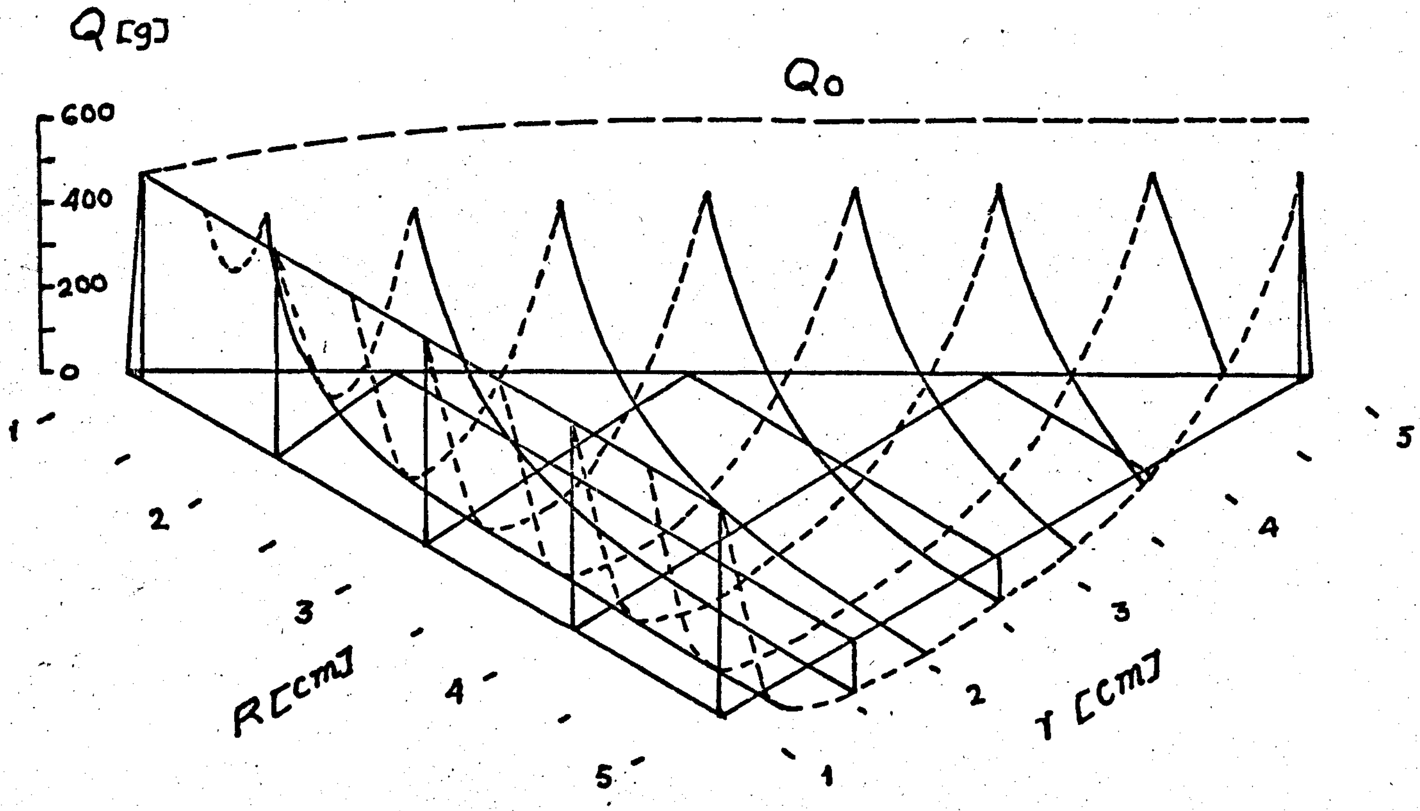
To=20g; x=5cm; spacing=1D; E=200g; EY=5000; R0=5cm;

FIG.3.8

to the pressure at the core. The later gradual increase in the pressure with r is probably due to the increasing value of P_o with R due to the changing wind angle. Finally the pressure falls off towards the end of r as the number of layers above r affecting the pressure at r reduce.

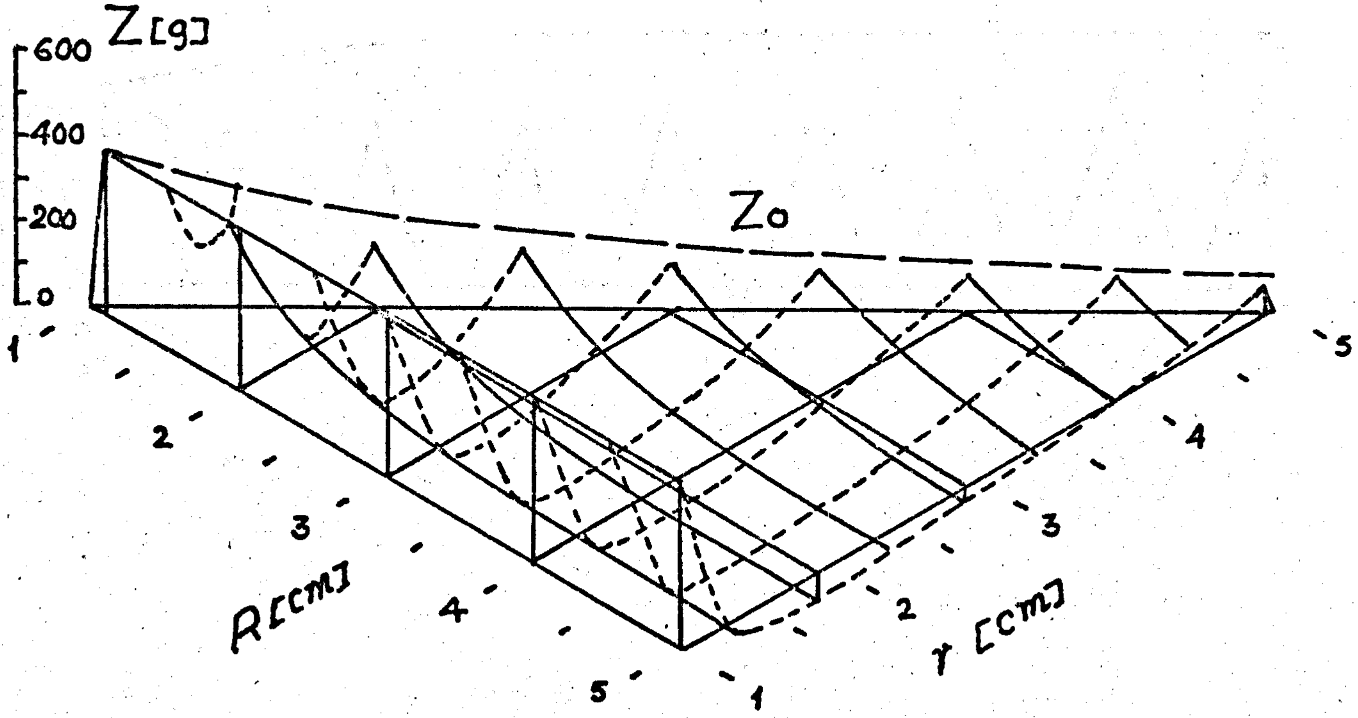
Fig. 3.7 shows U at all values of r and R in a completed cheese of 5 cm radius. U at any radius depends on P and on the thickness of elastic material beneath that radius. Thus the curves of U against R for constant r are similar in shape to those of P, but the curves of U against r at constant R increase with r so that the slopes of these curves is greater than those of the curves for P. The maximum value of U occurs at $r \simeq 3.9$ cm and represents a relatively small deformation of 0.017 cm or about 0.42% of the radius. If the modulus-ratio was changed so that the deformation was affected by a greater number of layers the maximum would occur at a smaller value of r.

The change in the tension of the yarn, i.e. $t \ (= \frac{\partial T}{\partial R}.dR)$ and the corresponding residual tension T in the yarn at any r for any R in the completed cheese of radius 5 cm is shown by Fig. 3.8. The value of t depends on U/r and is zero at the core radius as U is zero and T in the yarn is equal to T_o, the winding tension in the yarn. At other points the curves are roughly what one would expect to get by dividing those of Fig. 3.7 by r - as at any radius the



To=20g; x=5 cm; spacing=1D; E=200g; EY=5000g; RO=5 cm;

FIG.3.9



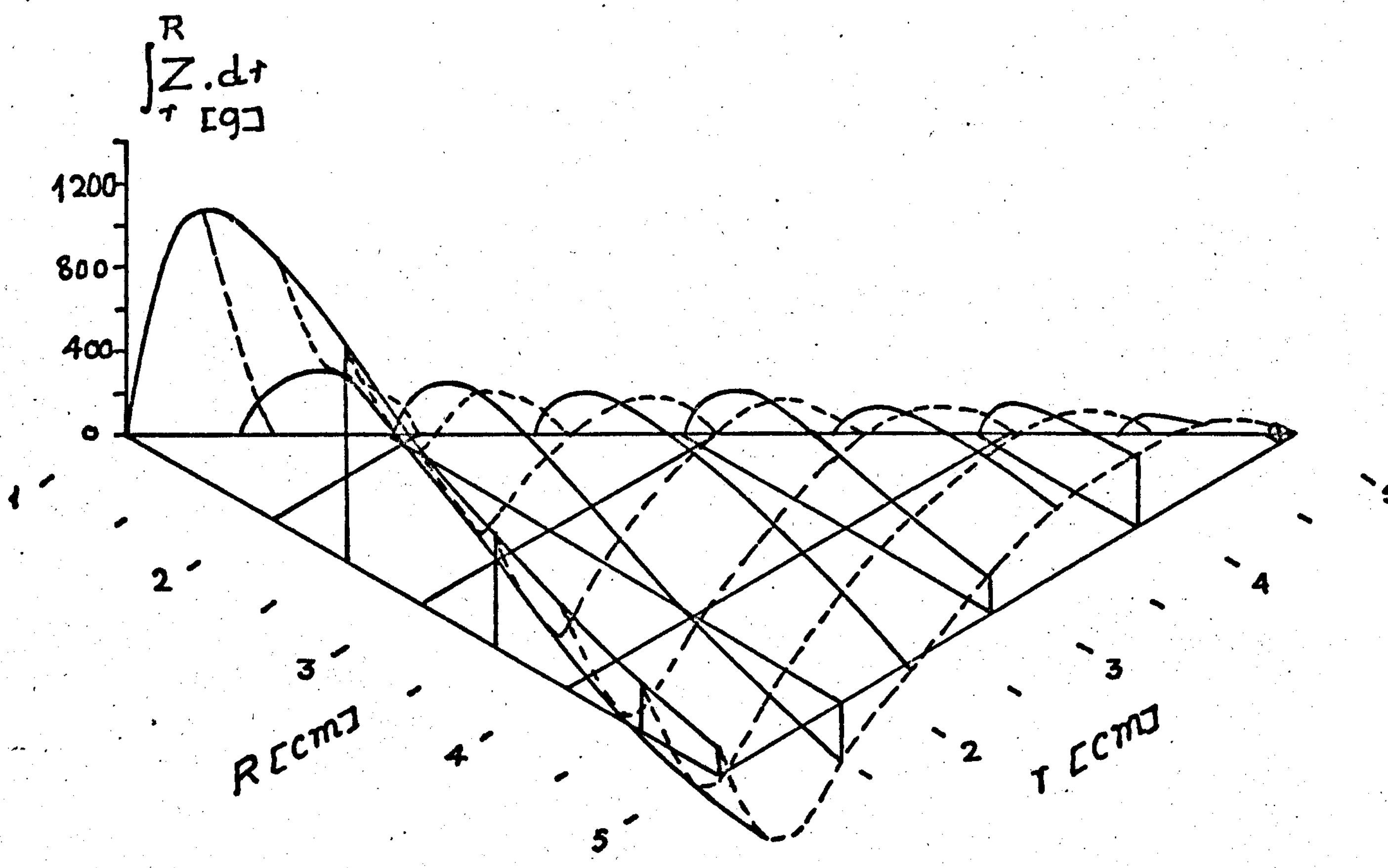
To = 20g; x = 5cm; spacing = 1D; E = 200g; EY = 5000g; RO = 5cm;

FIG. 3.10

change of length approximately proportional to U is occurring on a basic length approximately proportional to r. Over the middle region the change of tension is greater than the original winding tension and the yarn is therefore in compression. The behaviour of the total change in 't', i.e. $\int_r^R \frac{\partial T}{\partial R}.dR$, at any r as R increases from R = r to RO is similar to that of U as shown by the curve of $\int_r^R \frac{\partial T}{\partial R}.dR$ at r = 1.4 cm; T becoming negative. In the completed cheese the maximum value of the total change in tension of 24.4g or the minimum residual tension of -4.4g is at the radius of about 1.9cm.

Figs. 3.9 and 3.10 show respectively Q and Z at any r for any R in the completed cheese of 5 cm radius. The value of Q and Z depend on T and α but the changes in Q and Z, i.e. $\int_r^R \frac{\partial Q}{\partial R}.dR$ and $\int_r^R \frac{\partial Z}{\partial R}.dR$, at a given r as R increases from R = r to RO mainly depend on the change in tension T of the yarn as the change in α is small.

The changes in Q and Z at a given r, like the change in the tension in the yarn, become insignificant after the addition of a few layers at r. This is clearly shown by the curves of Q and Z at r = 1.4 cm becoming parallel to the R-axis. Also Q and Z change sign when T becomes negative indicating that Z is now trying to expand the element axially. The value of Q_o increases and that of Z_o reduces with R and the effect of this is evident in the curves of Q and Z.



T=20g; x=5cm; spacing=1D; E=200g; EY=5000g; RO=5cm;

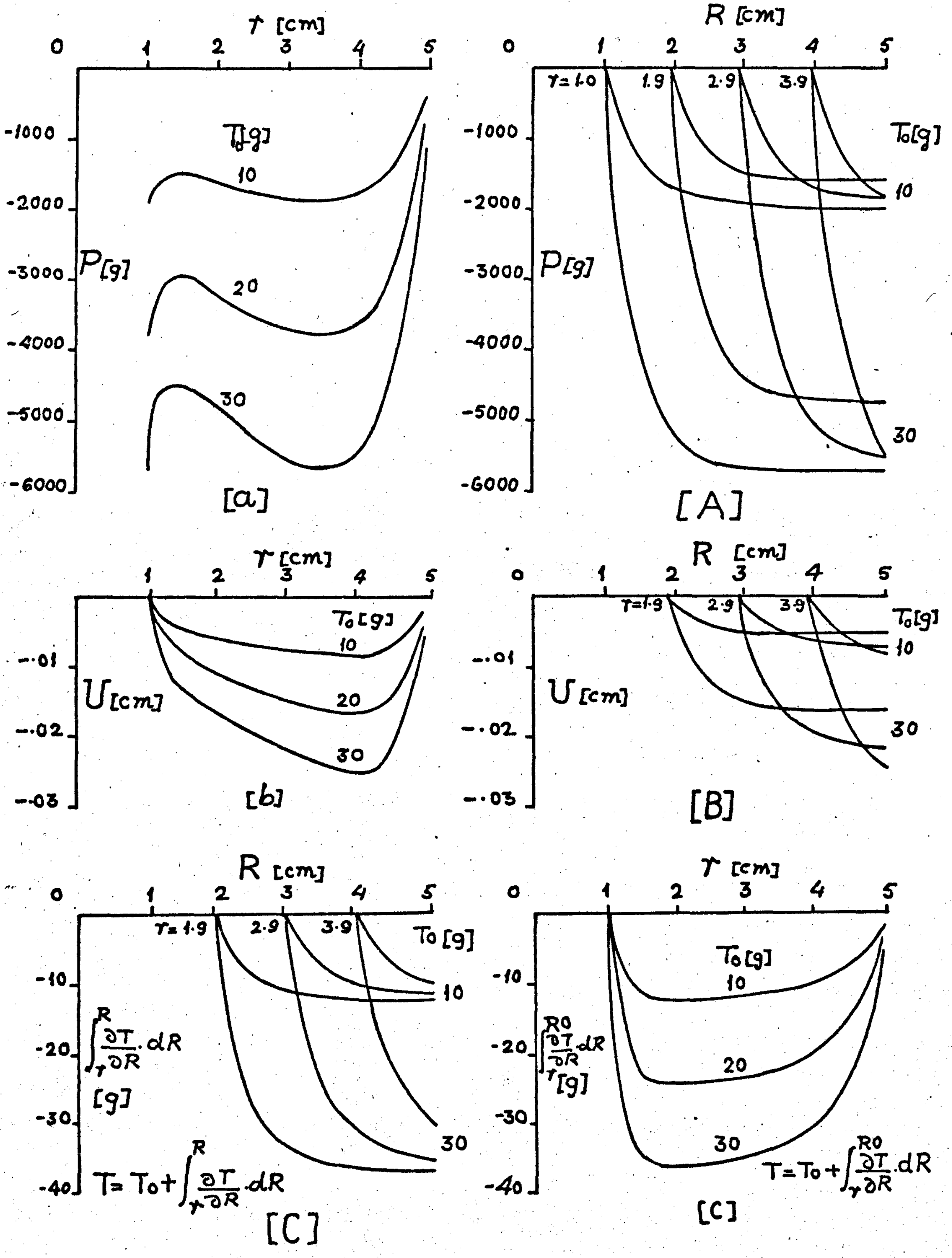
FIG. 3.11

Fig. 3.11 shows the shear force on an element at all values of r and R in the completed cheese. A positive value of the shear force shows the tendency of the outer layers of the cheese to contract relative to the inner layers and vice versa. The shear force at any r at first quickly builds up to a maximum positive value and after attaining its maximum value starts reducing steadily as R increases from R = r to RO. The shear force at r is at first positive and increases with R because the tension in the yarn in the layers added above r is positive but at a certain value of R the yarn in the layer at r loses all its tension. The shear force at r, then, has a maximum value and any further increase in R causes the yarn at r to compress and acquire a negative tension and as more layers are added the yarn in the layers above r, similarly, acquire negative tension. This starts a decline in the shear force on the element at r. The shear force, i.e. $\int_r^R Z.dr$, at r is maximum when T at r is zero. This is shown, for example, by the curve of T at r = 1.4 cm in Fig. 3.8; T at this radius is zero when R = 2 cm for which value of R $\int_r^R Z.dr$ at r = 1.4 cm is also maximum. Addition of each layer after R = 2 cm causes an addition of positive shear force at r but it also causes a loss in it due to radial deformation of the cheese and the result is a nett loss in the shear force and therefore the shear force at r reduces steadily as R increases. This effect is further enhanced due to a reducing value of Z_0 and an increasing value of Q_0 with R; the latter causing a higher compression of the cheese and therefore a greater loss in the shear force at r.

As before the incompressibility of the core causes a slightly different behaviour of the shear force in the region near the core from that in the rest of the package and this results in a minimum of the shear force near the core. In this example of Fig. 3.11 the reduction and going negative of the shear force has reduced the shear at the core to almost zero - the Z in the inner layers almost balancing that in the outer layers.

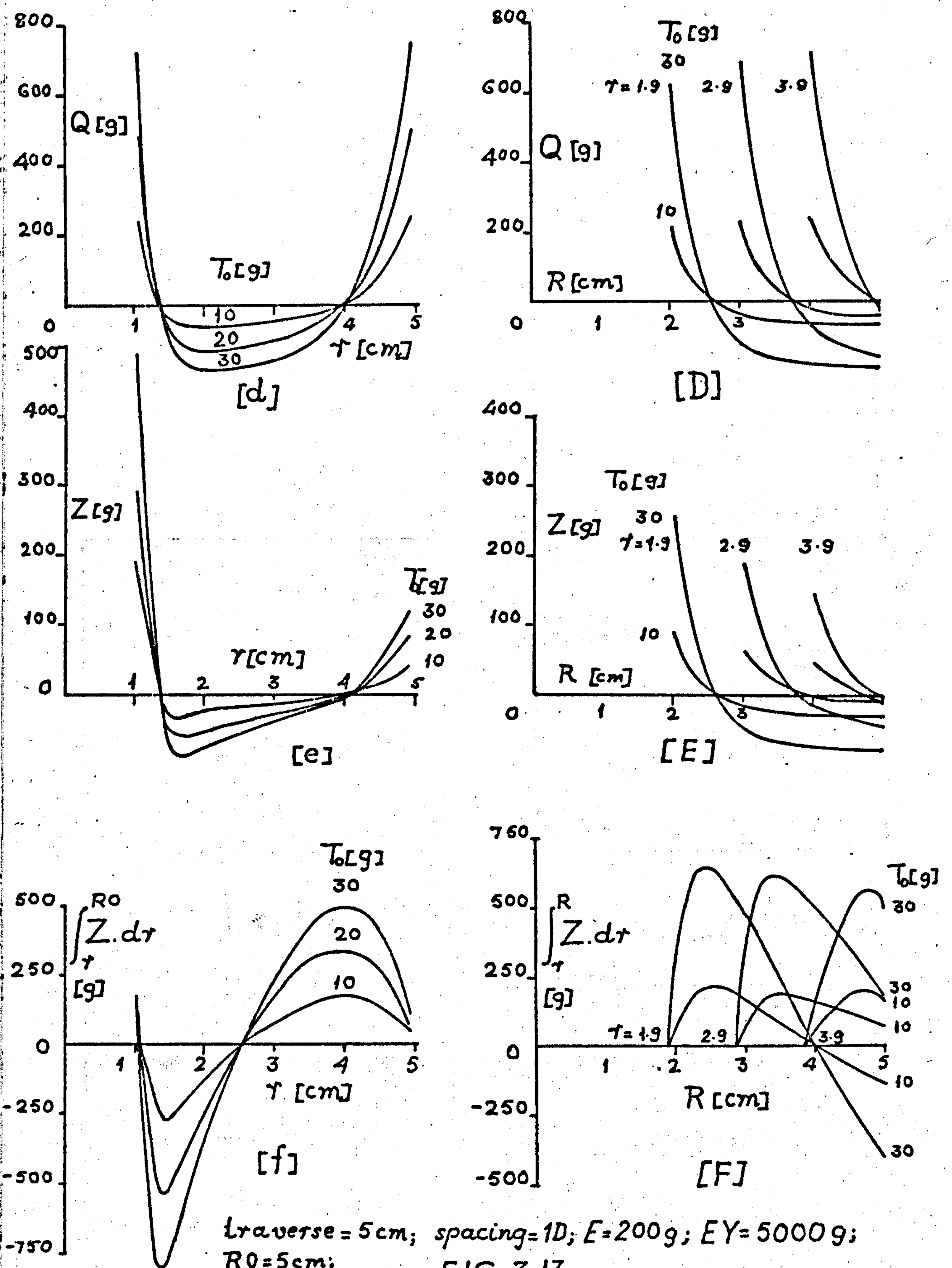
3.5 Compression of Cheese Under Different Winding Conditions

This section gives effects of varying the variables like winding tension, E, etc. on the pressure, compression etc. in a cheese. This is done by computing the results in a cheese for three different values of one variable keeping the other variables constant. The variables are the winding tension in the yarn, the spacing of the adjacent wraps of thread, the traverse per wind, the Modulus of Compression of the cheese and the Elasticity of yarn in Extension. Effects of varying the value of each variable is shown by a set of two figures each figure having 6 sets of curves. The curves marked as (a), (b), (c) of the first figure and the curves marked as (d), (e) and (f) of the second figure show P, U,



Traverse = 5cm; spacing = 1D; E = 200g; EY = 5000g; R0 = 5cm;

FIG.3.12



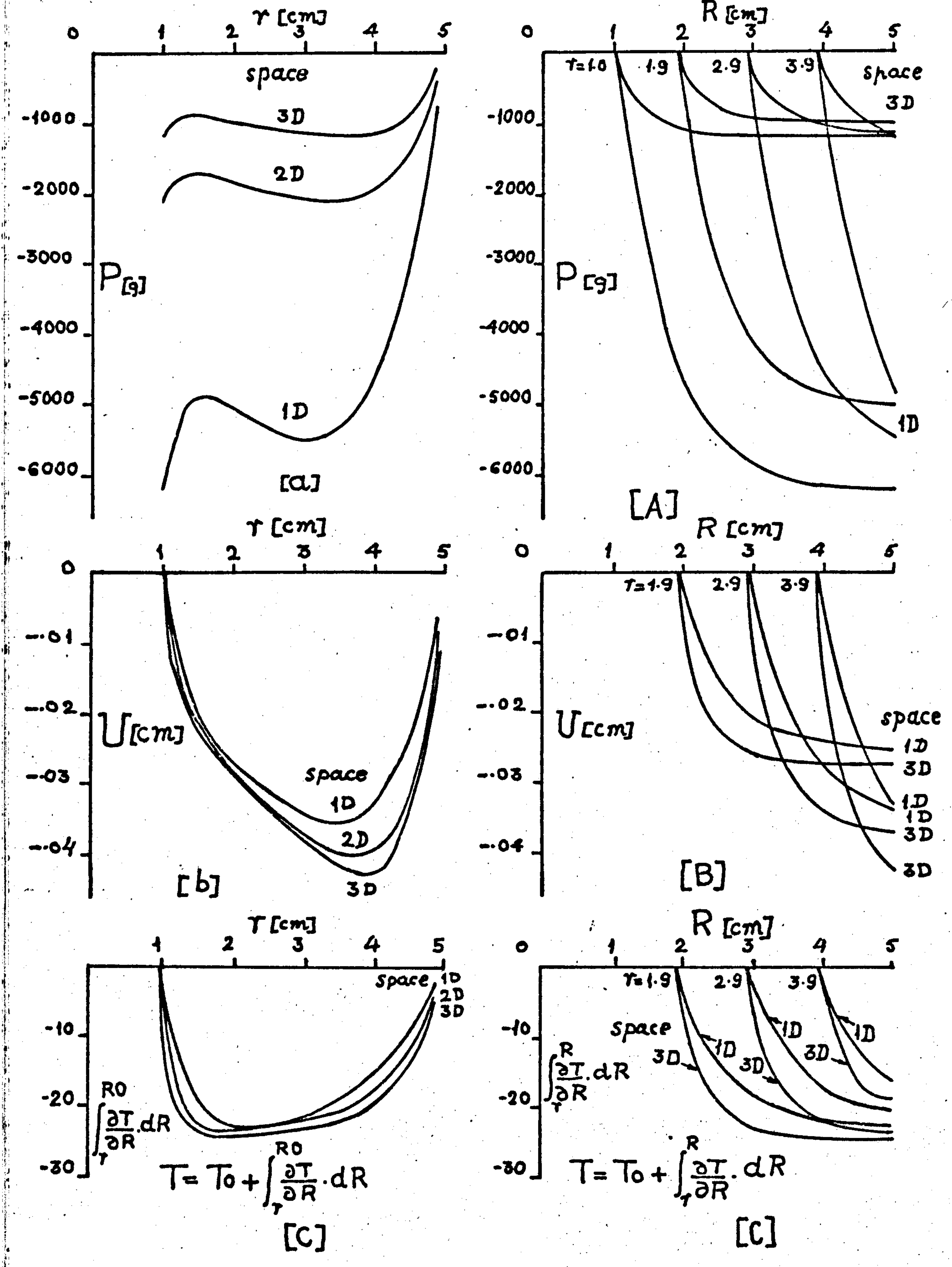
traverse = 5 cm; spacing = 1D; E = 200 g; EY = 5000 g; R0 = 5 cm;

FIG. 3.13.

$\int_r^R \frac{\partial T}{\partial R}.dR$ and T, Q, Z and $\int_r^R Z.dr$ respectively at any radius of the cheese for three values of the variable for the completed cheese of 5 cm radius. The curves marked as (A), (B), (C) of the first figure and (D), (E) and (F) of the second figure show respectively P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$ and T, Q, Z and $\int_r^R Z.dr$ at different values of r, namely 1.0 cm, 1.9 cm, 2.9 cm and 3.9 cm, as the cheese is built up from these radii to the final outer radius of 5 cm for two extreme values of the variable. This representation of P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$ and T, Q, Z and $\int_r^R Z.dr$ is uniform for showing effects of the variation of each variable on the cheese.

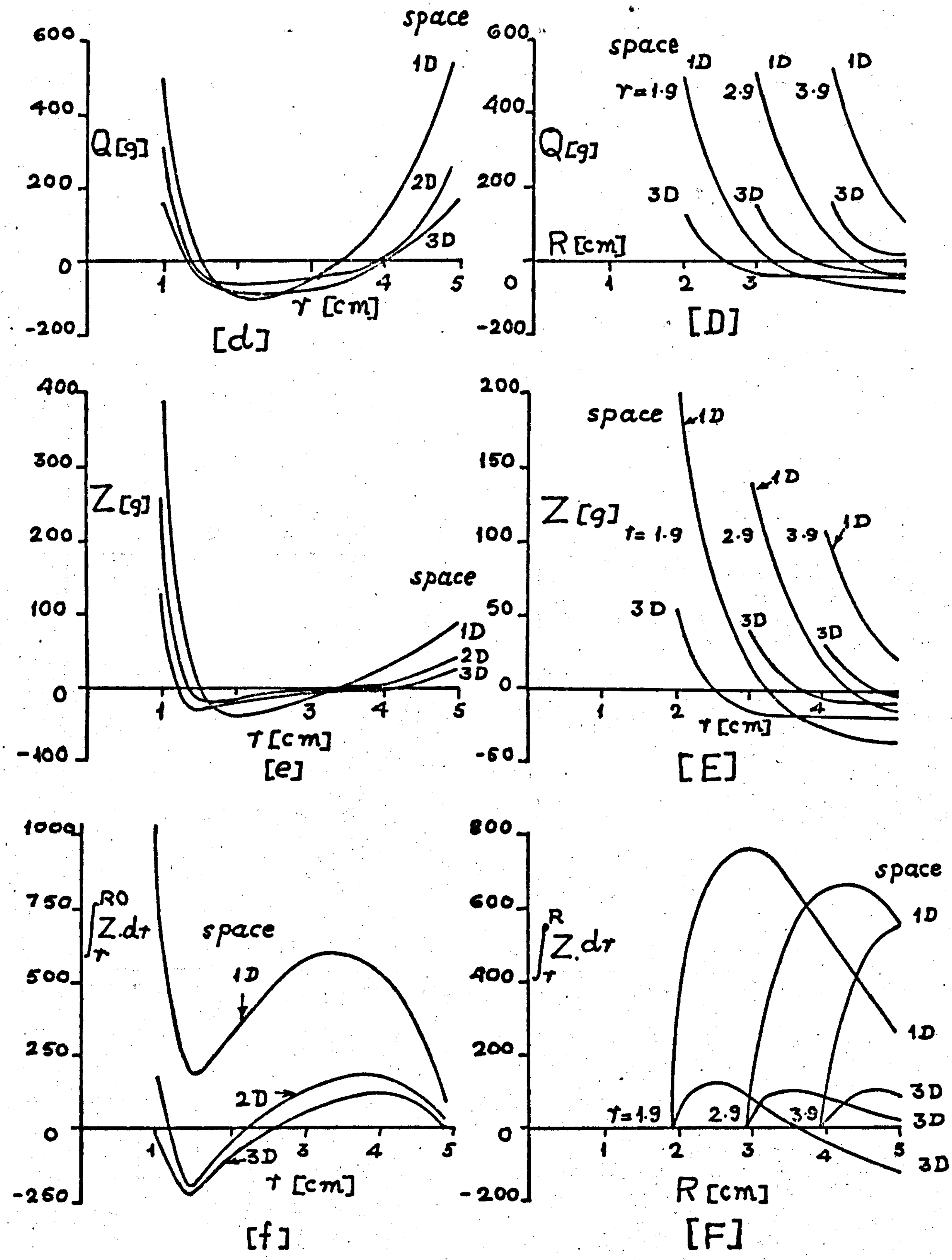
3.5.1 Effects of Varying the Winding Tension in the Yarn

An increase in the winding tension of the yarn from 10g to 20g and 30g increases the pressure and the shear force imposed by the element added at the outer radius of the cheese proportionately and causes a proportional increase in the values of P. U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. The per cent changes in T, Z and Q are the same for all the three values of the winding tensions in the yarn. This is also suggested by the equation (3.14) in which the term, $T_o.a^2/(EY.r^2)$, containing T_o is small as compared to the other terms due to the large value of EY and therefore a change in the value of the tension has little effect on the value of the compression of the cheese except the linear change caused by the change in the boundary



$T_0 = 20g$; traverse = 5cm; $E = 200g$; $EY = 2000g$; $R0 = 5cm$;

FIG.3.14



T_0= 20g; traverse = 5cm; E = 200g; EY = 2000g; R0 = 5cm;

FIG. 3.15

condition, i.e. the change in the pressure imposed by the added layer. The results are shown by Fig. 3.12 and Fig. 3.13.

3.5.2 <u>Effects of Varying the Space Between Adjacent Wraps of Yarn</u>

To study effects of the spacing of the adjacent coils of yarn three values of the space between the adjacent wraps of yarn, namely 1D, 2D and 3D, are chosen. The results are given in Fig. 3.14 and Fig. 3.15.

An increase in the spacing of the adjacent wraps of yarn from 1D to 2D or 3D reduces the number of threads in the element and therefore the force through the face of the element added at R to $\frac{1}{2}$ or $\frac{1}{3}$ and the number of crossing points in the element to $\frac{1}{4}$ or $1/9$, i.e. from 490 to 122.5 or 54.5. This shows that pressure of the element reduces but the pressure per point increases and consequently U is higher in the cheese with the spacing of 3D as compared to that of the cheese with a spacing of 1D. This is shown by curves (b) of Fig. 3.14. Similarly curves (c) show a higher value of $\int_r^R \frac{\partial T}{\partial R} . dR$ in the former cheese. In both cases however the change with spacing is fairly small.

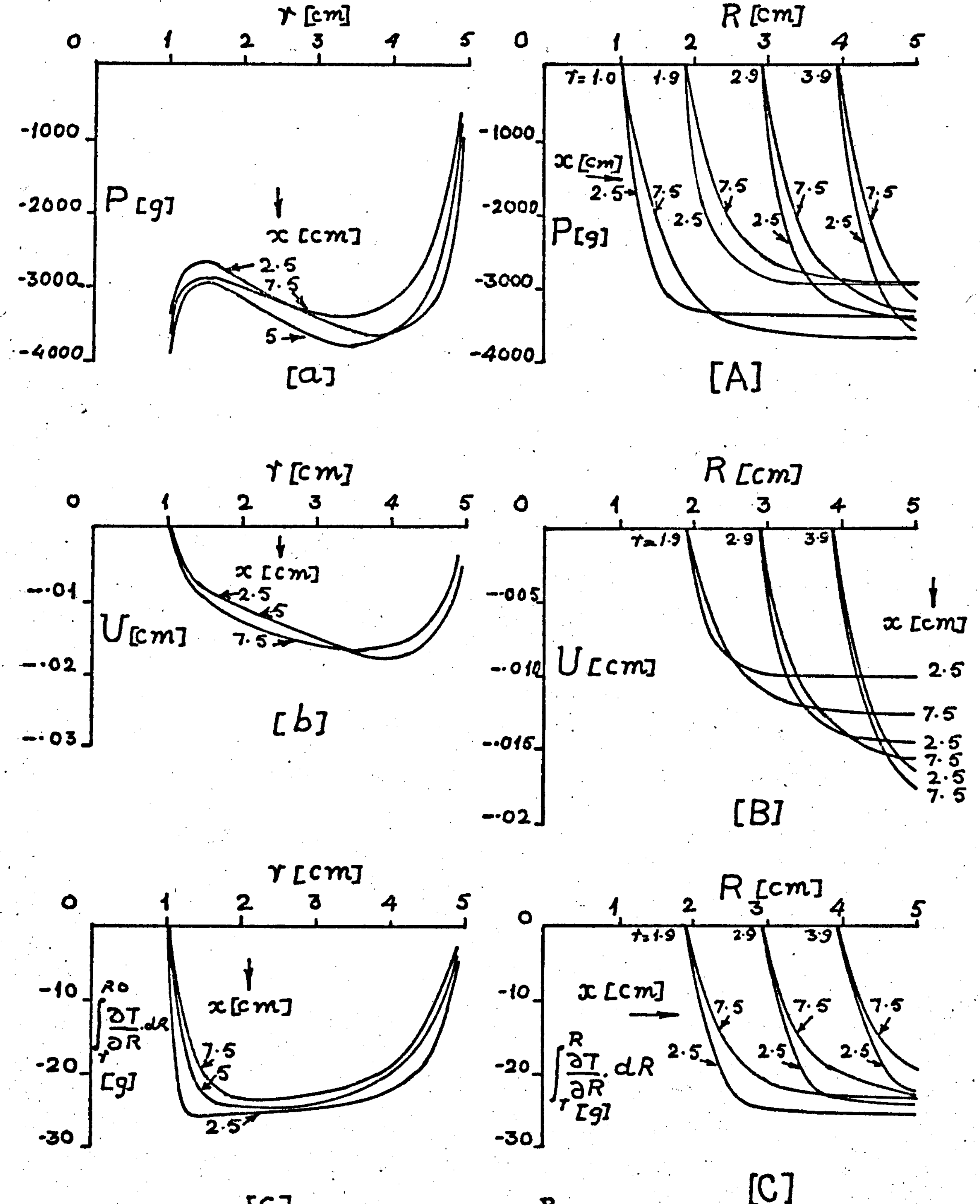
P is higher in the cheese with the spacing of 1D but the pressure/point is higher in the other cheese and is 16.5g per point at r = 1.4 cm in the later cheese as compared to 10.3g per point of the former cheese at the same radius. This probably is due mainly to the assumption of no axial extension which makes the change in α very small.

The curves (D) and (E) of Fig. 3.15 for Q and Z are similar to the pressure curves (A) of Fig. 3.14. The curves (A), (B), (C), (D) and (E) for P, U, $\int_r^R \frac{\partial T}{\partial R} . dR$ and T, Q and Z show a slightly greater tendency to flatten for the cheese with a spacing of 3D between adjacent wraps of yarn and this is due to the cheese being comparatively softer because of the less number of crossing points giving in effect a change in the ratio of tangential to radial elastic modulii.

The curves (f) show a progressive increase in the magnitude of the negative shear force tending to expand the cheese axially and acts on a larger part of the cheese as the spacing between the adjacent wraps of yarn increases and this is due to a progressive increase in the compression of the cheese. The cheese with a spacing of 1D between adjacent coils of yarn does not show any tendency to axial expansion as shear force is never negative. The curves (F) show a similar situation with a marked tendency for the cheese with a spacing of 3D between adjacent wraps of yarn to expand axially at the radius of 1.9 cm.

3.5.3 Effects of Varying the Traverse per Wind

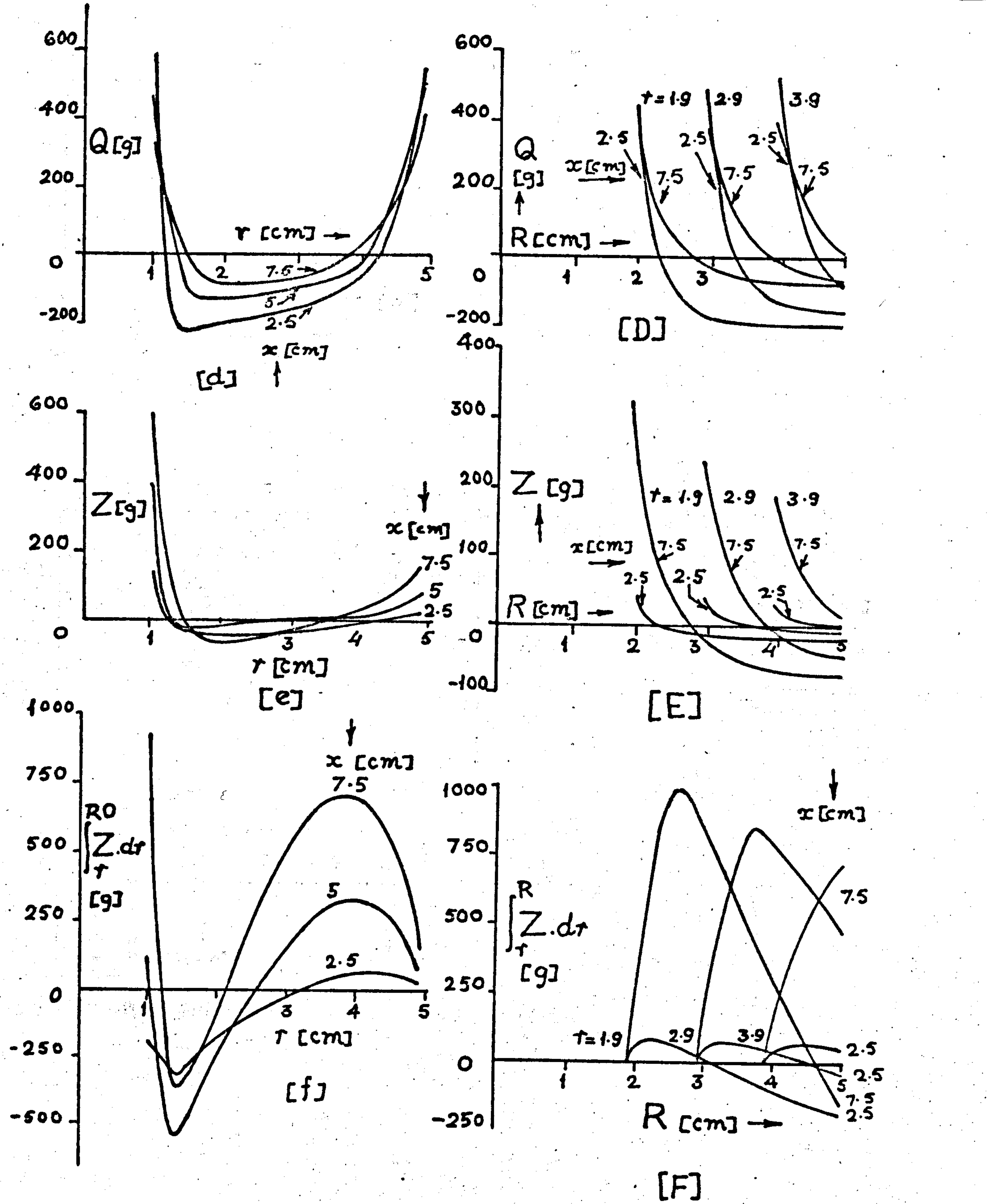
The values of x, the traverse per wind, chosen for this study are 7.5 cm, 5 cm, and 2.5 cm and the results are shown in



$$T = T_0 + \int_r^R \frac{\partial T}{\partial R} \cdot dR$$

R0 = 5 cm; T_0 = 20 g; spacing = 1D; E = 200 g; EY = 5000 g;

FIG.3.16



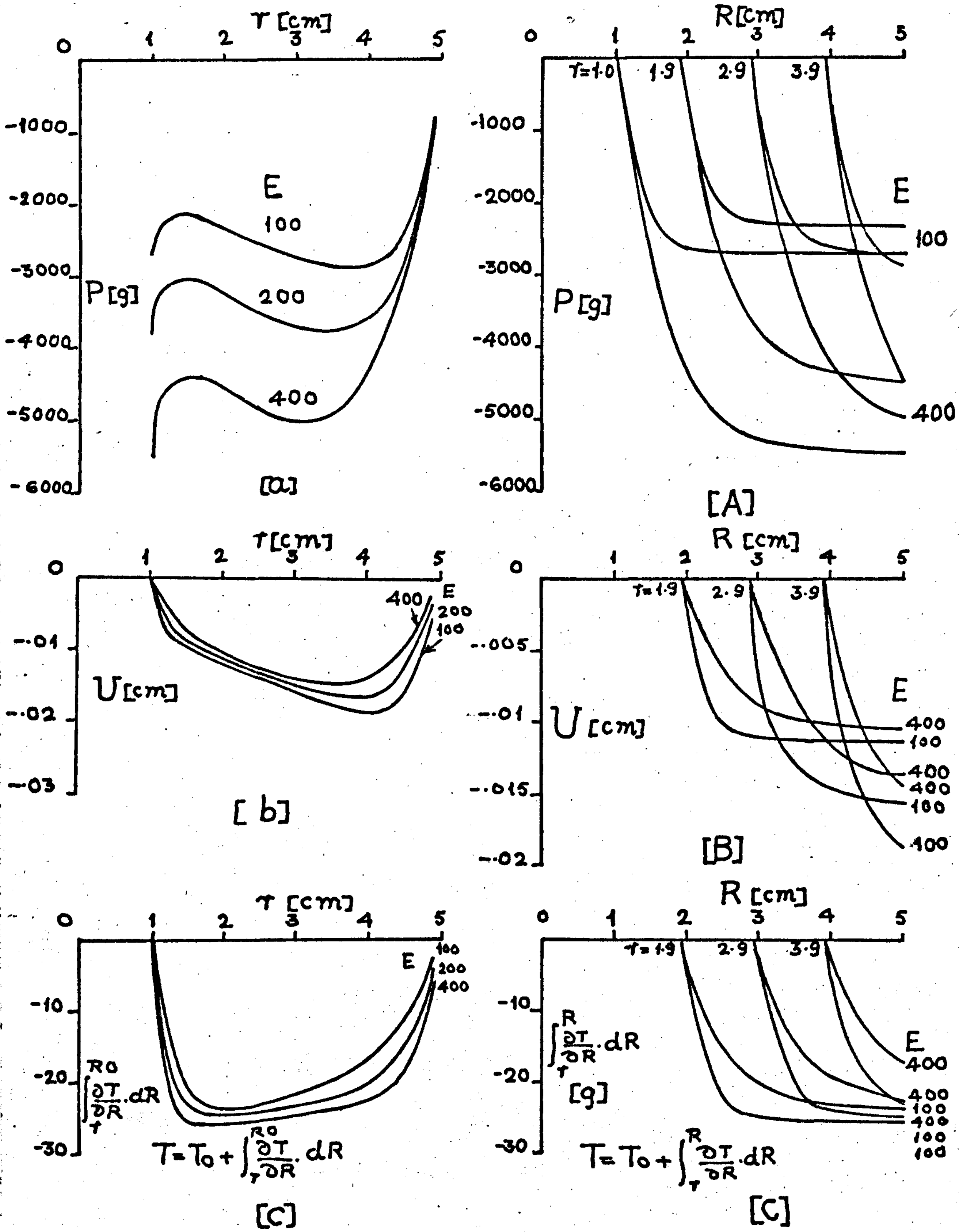
To = 20g; space = 1D; E = 200g; EY = 5000g; RO = 5cm;

FIG. 3.17

Fig. 3.16 and Fig. 3.17.

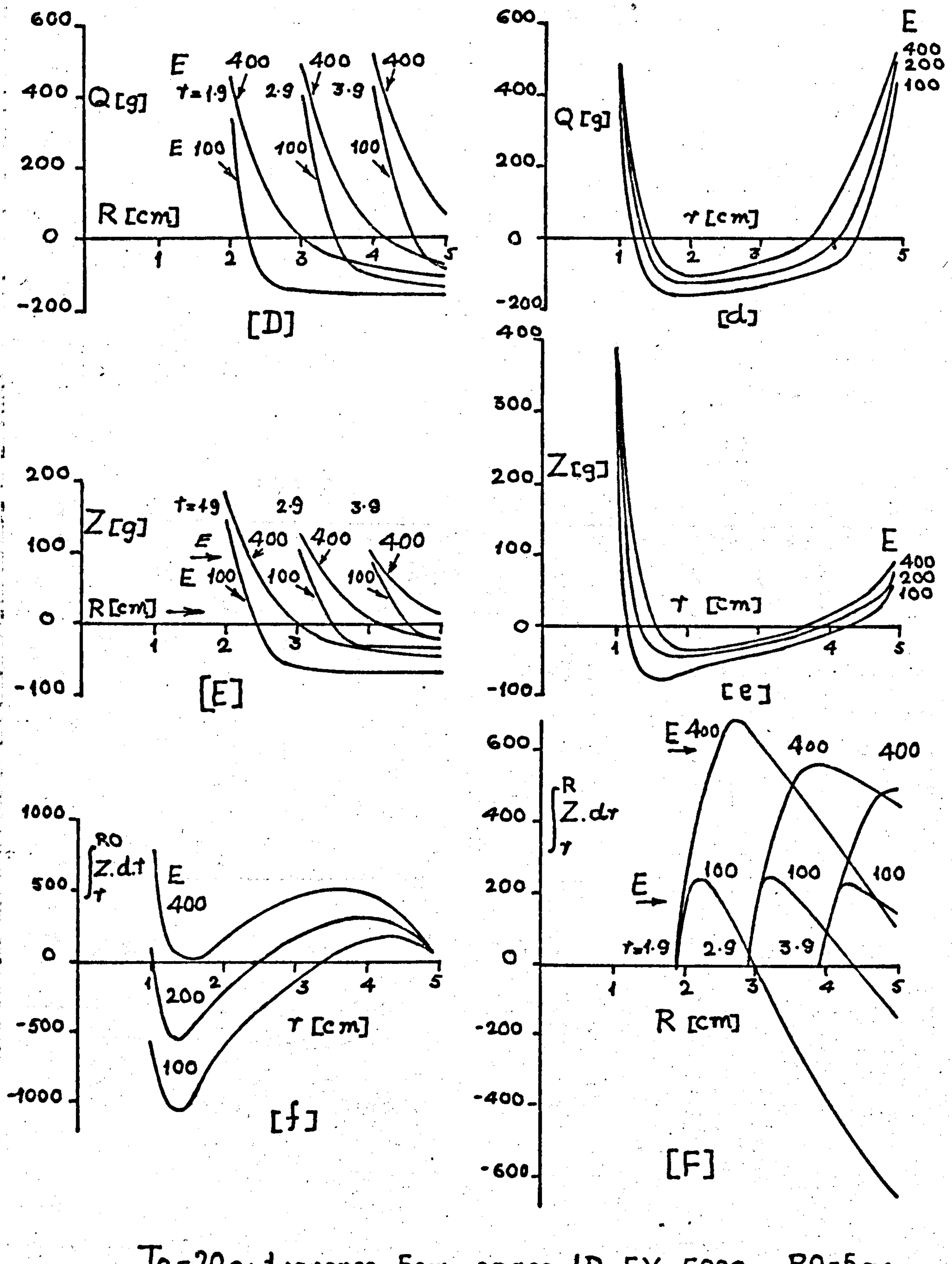
A reduction in the value of x, the traverse per wind, reduces the wind angle, and therefore the number of threads in a given axial width W, which varies with (D.secα), increases. This is shown by the value of K, when W = 1 cm, changing from 14.78 to 12.45 and 10.22 as x changes from 2.5 cm to 5 cm and 7.5 cm for fixed spacing at the core. The value of ϕ when x = 5 cm and W = 1 cm is taken as standard and kept the same in different solutions and therefore the axial width W of the element, according to the definition of the element in the analysis, must change and this is altered by the same factor, called angle, by which x is altered. However, to facilitate direct comparison of the results, the results are expressed on the standard size of the element which has been defined already. The alteration in the value of K due to a change of x and hence α alters the number of crossing points in an element of standard size and this number is 345.2, 489.5 and 494.9 when x = 2.5 cm, 5 cm and 7.5 cm respectively. Incidentally the maximum number of crossing points in an element of standard size would occur when $\alpha = 45^{o}$ and theoretically this type of cheese would offer maximum resistance to radial deformation.

A change in the value of x from 7.5 cm to 2.5 cm reduces the number of pressure bearing crossing points in the cheese giving



RO=5cm; To=20g; traverse=5cm; space=1D; EY=5000g;

FIG.3.18



To = 20g; traverse = 5cm; space = 1D; EY = 5000g; RO = 5cm;

FIG. 3.19

in effect a higher value of the ratio of EY to E and therefore the pressure of the added layer affects comparatively a fewer number of layers underneath it and vice versa. This is shown by the curves of U, P, etc. flattening earlier and quicker when x = 2.5 cm. This causes P when x = 7.5 cm to be nearly equal to P when x = 2.5 cm though the value of P_o is higher in the latter case due to a higher value of K and a smaller value of α. Similarly U and $\int_r^R \frac{\partial T}{\partial R}.dR$ are nearly equal. Also the maximum values of U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. occur comparatively nearer the centre of the cheese when x = 7.5 cm.

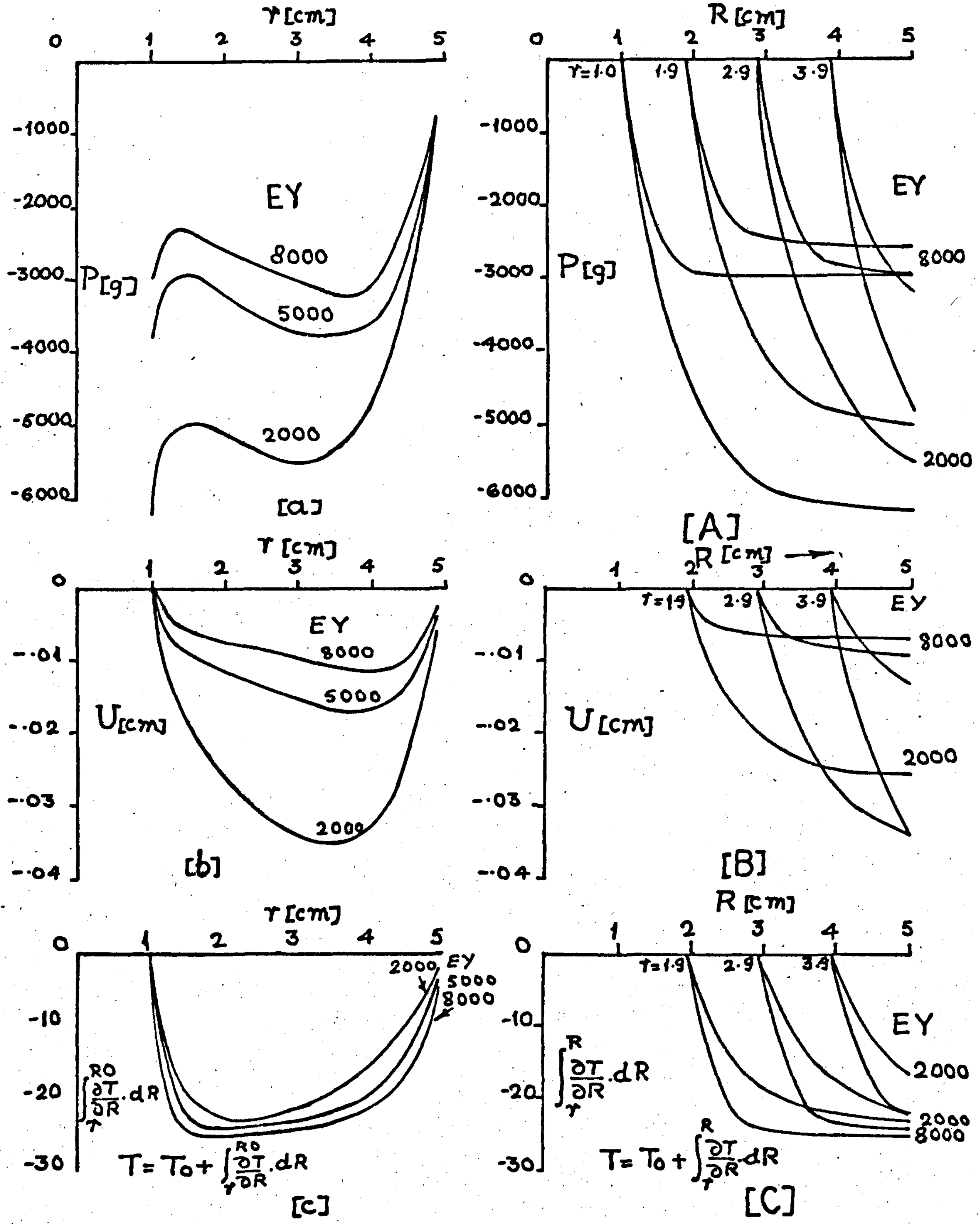
Due to higher value of Z_o when x = 7.5 cm, the shear force is higher in this cheese. The changes in the direction of the shear force roughly occur at the same radii in both the cases as shown by curves (f) of Fig. 3.17, but the radius at which shear force is zero is larger in the cheese with x = 2.5 cm showing that a greater part of the cheese is subjected to a tendency to expand axially. Also the shear force at r = 1.9 cm, as shown by curve (F) of Fig. 3.17, has a smaller maximum and becomes negative earlier.

3.5.4 <u>Effects of Varying the Modulus of Compression of Cheese</u>

Three values of E, namely 100g, 200g and 400g giving three values of the ratio of EY to E of 50, 25 and 12.5 respectively, are chosen to study effects of varying the value of E on the cheese. The results are shown in Fig. 3.18 and Fig. 3.19.

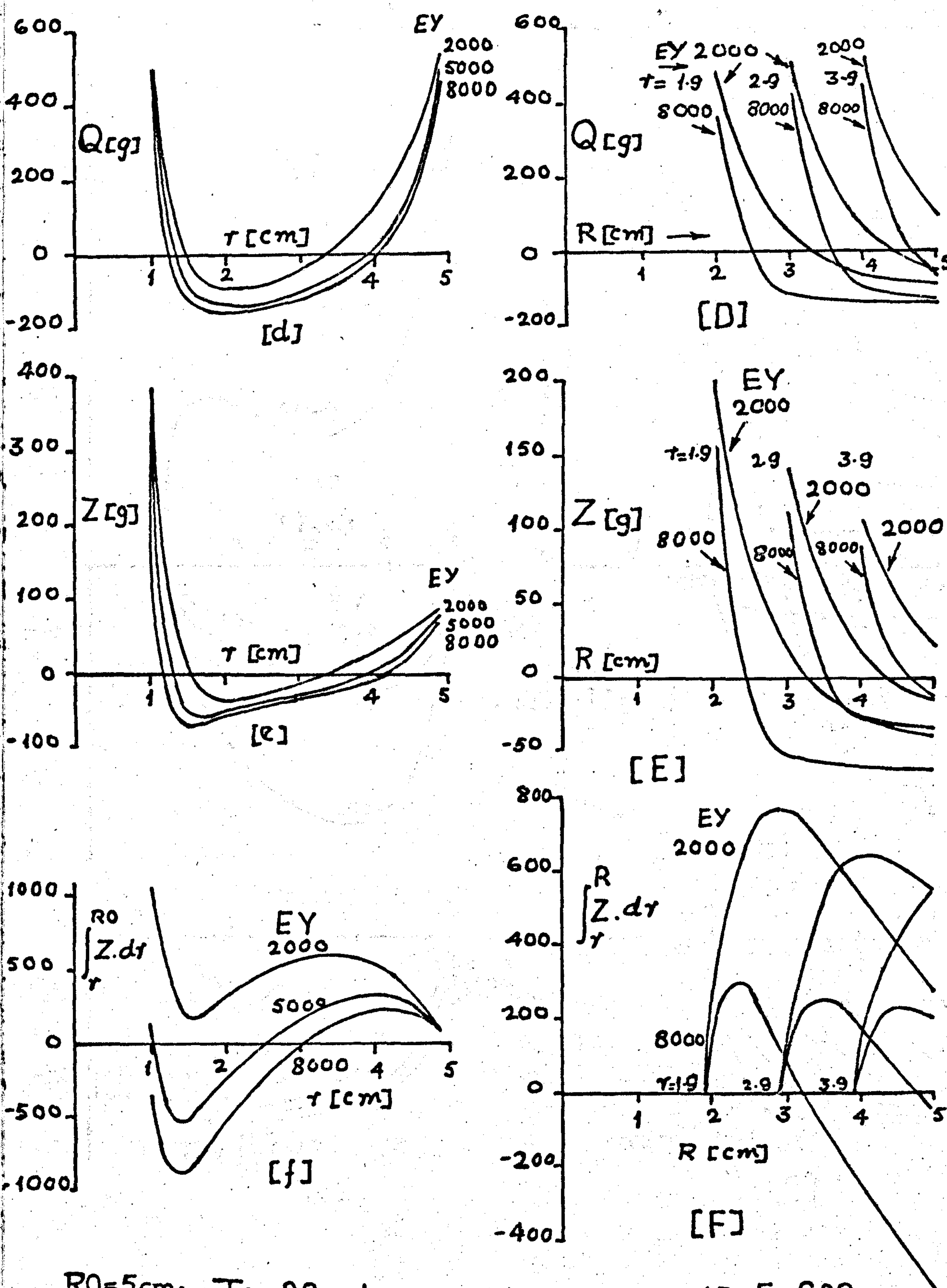
The pressure curve (a) of Fig. 3.18 shows that the pressure inside the cheese is smaller when E is smaller, i.e. 100g. This is due to the higher value of the ratio of EY to E, i.e. modulus ratio. Beddoe[2] also gives similar results, namely when the modulus ratio is changed from 1 to 20 the pressure at the tube of a yarn beam reduces to nearly half. Because of the lower value of E = 100g the value of U is higher as the material is easier to compress radially; this is shown by the curves (b) and (B) of U. Also the initial compression of the cheese at any r is easy and a greater value of U occurs for some initial increase in R from when R = r; allowing the yarn at r to lose all its tension. Any further increase in U at r is difficult because of the high value of EY and the cheese being not allowed to expand axially. This condition is reached gradually when the value of E is higher, i.e. when E = 400g. This means that the cheese, when E = 400g, is affected by the addition of comparatively a greater number of layers at that radius. This is shown by comparatively gradual flattening of the curves of U, P, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. at constant r with R when E = 400g as compared to when E = 100g. This also results in higher value of P at any r.

The curves (f) for the shear force show considerable difference as the value of E is changed. When the value of E is 100g the shear force is negative for a greater part of the cheese trying



RO = 5cm; To = 20g; traverse = 5cm; space = 1D; E = 200;

FIG.3.20



R0=5cm; To=20g; traverse=5cm; space=1D; E=200g;

FIG.3.21

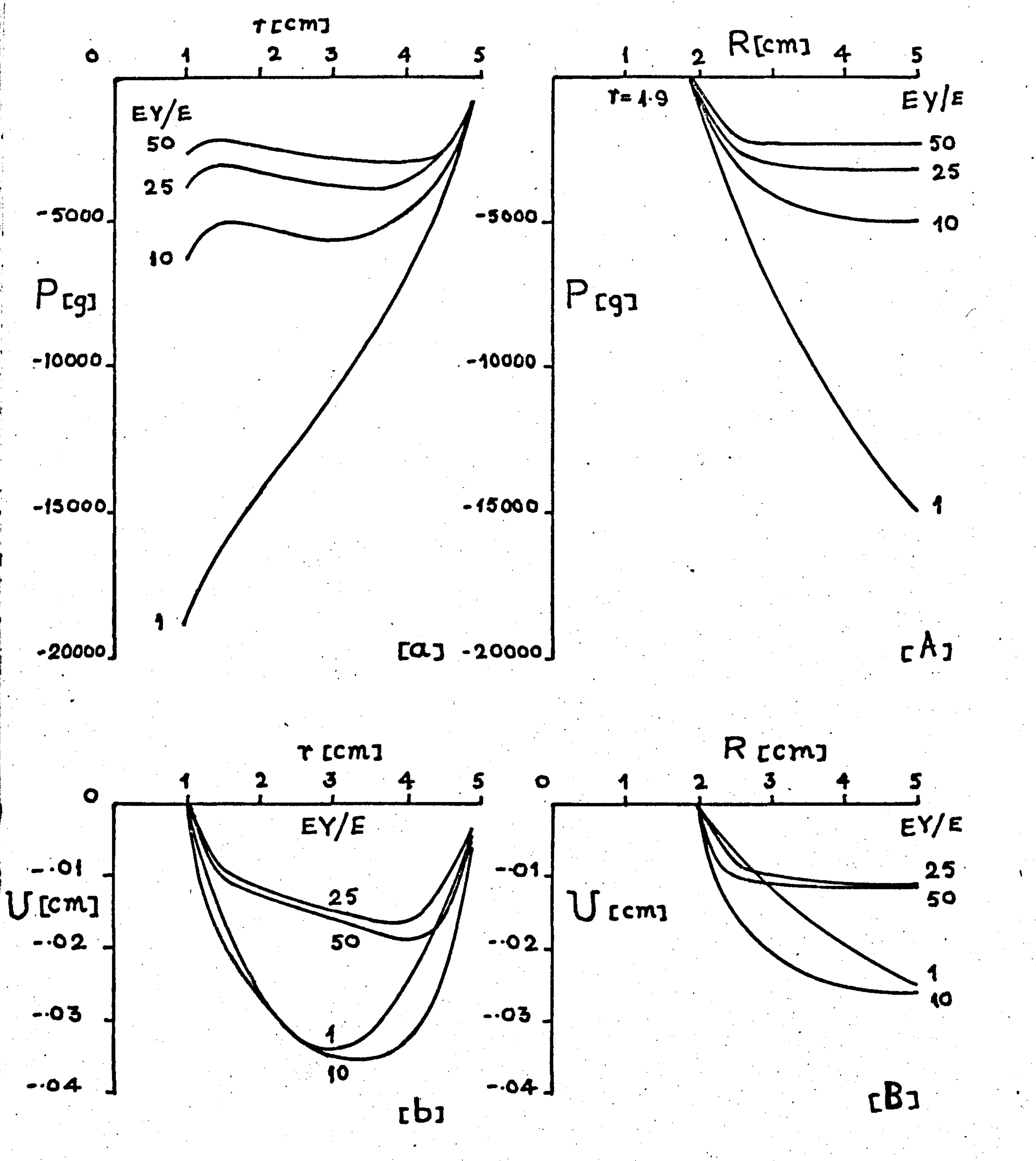
to expand that part of the cheese axially and the magnitude of the shear force is also higher as to that when E = 400g. In the later case the shear force is positive throughout failing to show any tendency for the axial expansion of the cheese. The build up of the shear force at constant r with R is also different in the two cases as shown by the curves (F) of Fig. 3.19. In the case when E is 100g the shear force at r = 1.9 cm reverses its direction earlier with a much smaller maximum and continues to increase in the other direction steadily attaining a fairly large negative value, whereas in the other case the maximum of the shear force is much higher and it is never negative. It appears that the value of E has considerable influence on U, P and $\int_r^R Z.dr$.

3.5.5 <u>Effects of Varying the Elasticity of Yarn in Extension</u>

Effects of varying the value of the Elasticity of yarn in Extension on the compression of the cheese is studied by using three values for it, namely 2000g, 5000g and 8000g. The results are shown in Fig. 3.20 and Fig. 3.21. The values of modulus ratio are 10, 20 and 40 respectively.

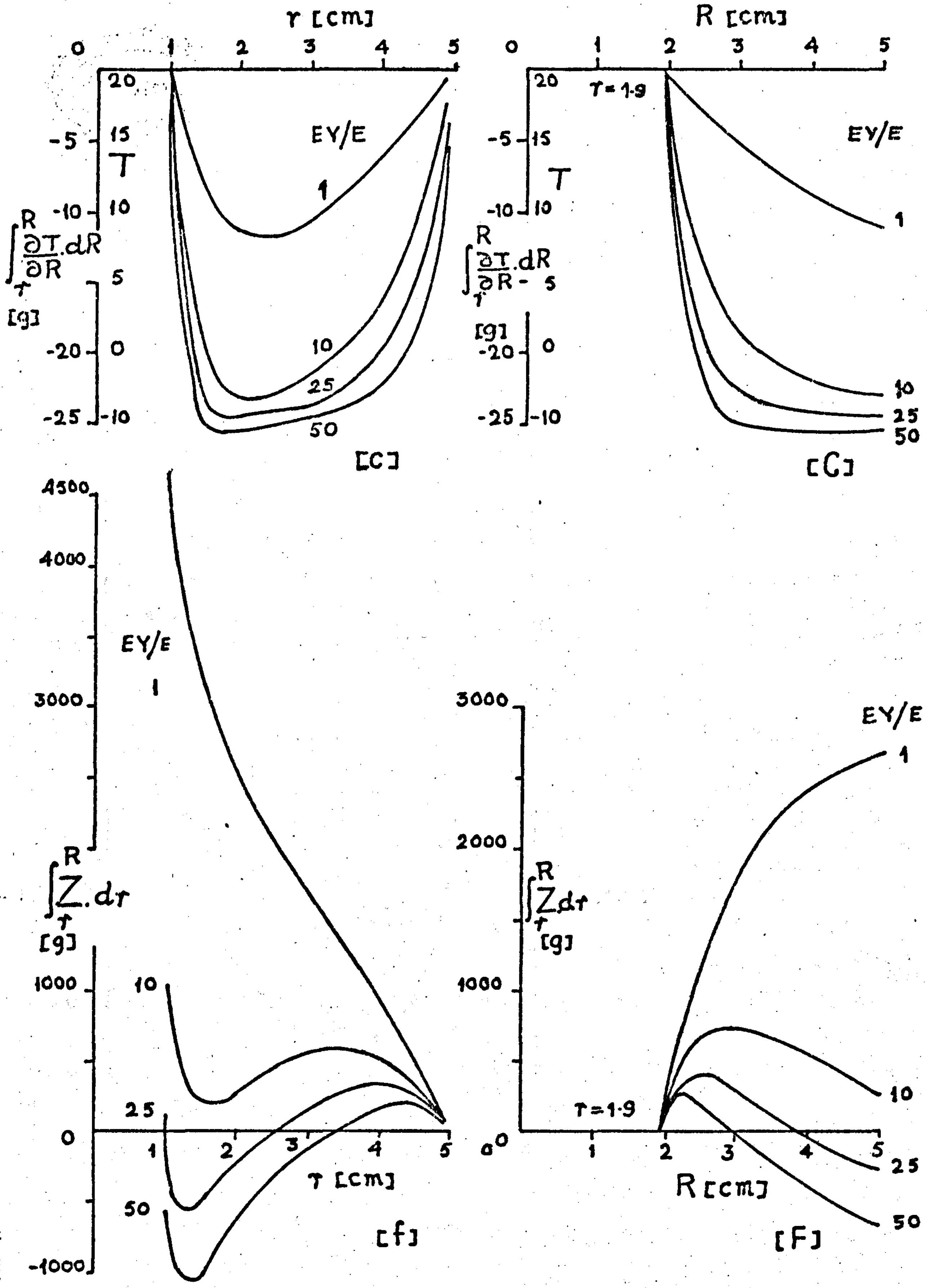
A high value of EY causes the yarn to lose all its tension quickly even for a slight compression of the cheese which results in the shortening of the length of the yarn and once the tension is lost in the yarn it is difficult to compress the cheese further as it would require further shortening of the yarn to a state of compressive strain which is difficult, again, due to the high value of EY. Therefore U, with the high value of EY, is much smaller and is shown by curves (b) of Fig. 3.20. The pressure of the added layer, once the yarn in a few layers immediately below the added layers has lost its tension completely due to the initial compression, is quickly utilised in converting the yarn to a state of compressive strain in these layers and the pressure of the added layer fails to reach beyond these few layers. This is shown by the quicker flattening of the curves of P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. at constant r as R increases from R = r. As only a few layers are affected by the addition of a layer at R when EY is high, P is much less as compared to when EY is low. This is shown by the pressure curves (a) and (A). This also results in lopsided curves like (b) and (c) of Fig. 3.20 with the maximum values occurring nearer the ends of the cheese rather than at the mid radius.

The effect of increasing the value of EY on the shear force inside the cheese is similar to that of lowering the value of E as both these changes cause an increase in the value of modulus ratio. The behaviour of the shear force in this case is similar to its behaviour in the previous case when E is varied. It appears that like E, EY also has considerable influence on P, U and $\int_r^R$ Z.dr in



T_0= 20g; traverse=5cm; space=1D; RO=5cm;

FIG.3.22



$T_0 = 20$; traverse = 5 cm; space = 1D; RO = 5 cm;

FIG 323

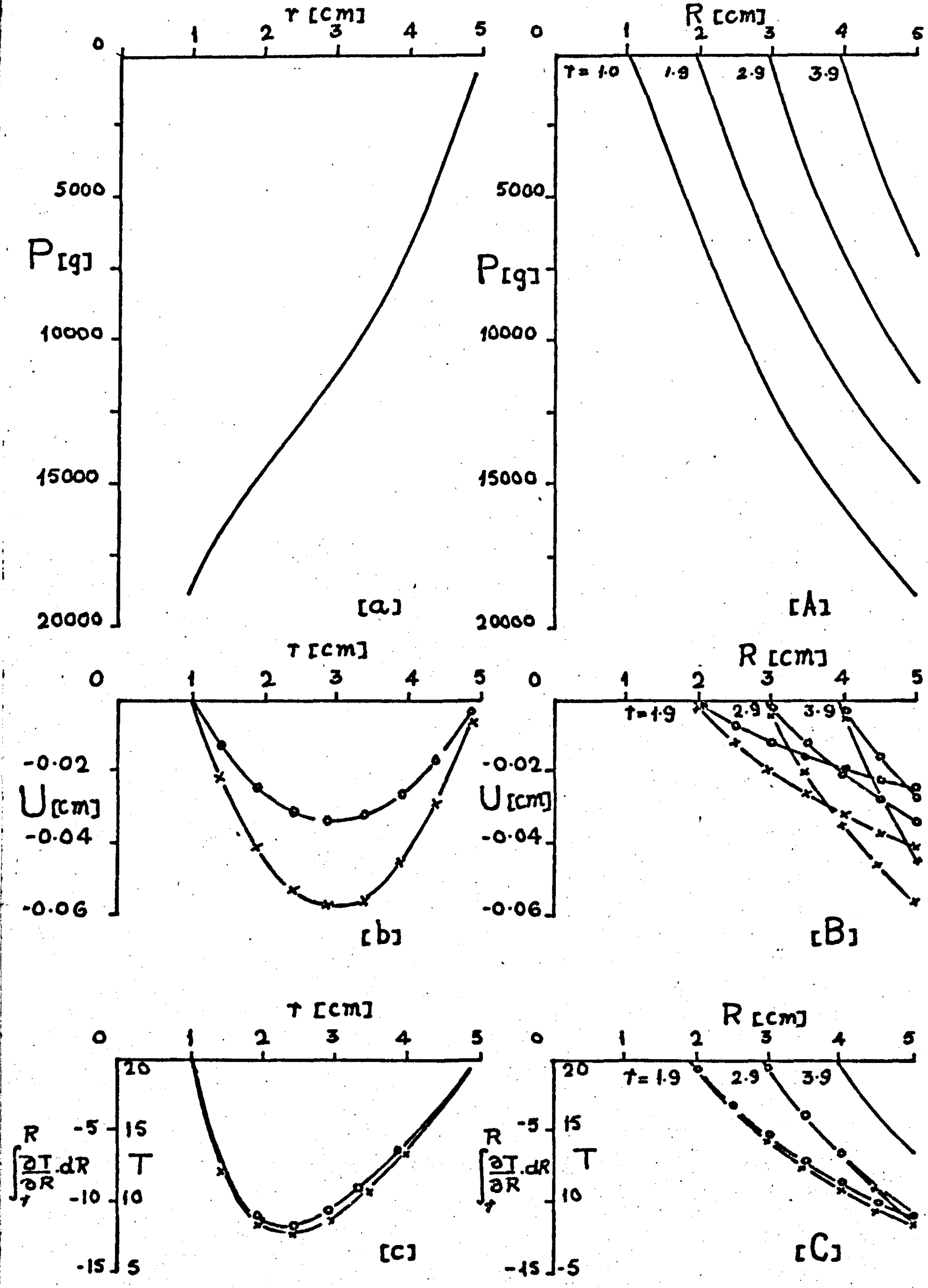
the cheese.

It may be observed that the changes of all the variables except U with EY are very much like the changes with E - as would be expected because the ratio EY/E is the main cause. The difference between the values of U in the two cases is because a low value of E allows easy compression and the yarn loses all its tension only when the value of U is comparatively high whereas a high value of EY permits the yarn to all its tension even for a small value of U and in both cases the value of U does not change much once the tension in the yarn is lost - due to high modulus ratio.

It appears that the value of modulus ratio, which is a measure of the anisotropy of the yarn, is of considerable importance as the behaviour of the cheese depends on its value. This is in agreement with the results of Beddoe, and follow from the form of the equation.

3.5.6 Effects of Varying the Modulus Ratio

The effects of varying E or EY, i.e. in effect varying the modulus ratio of yarn, influences the behaviour of the cheese considerably. Therefore in order to study effects of varying modulus ratio four values of modulus ratio are chosen to cover a wider range of it than when E or EY were varied separately. These are 1(= 1000/1000), 10(= 2000/200), 25(= 5000/200) and 50(= 5000/100). The results are shown in Figs. 3.22 and 3.23.



x - cheese No. 14. E = 600; EY = 600;
o - cheese no. 12. E = 1000; EY = 1000;

traverse = 5cm; space = 1D; T_o = 20g;
RO = 5cm;

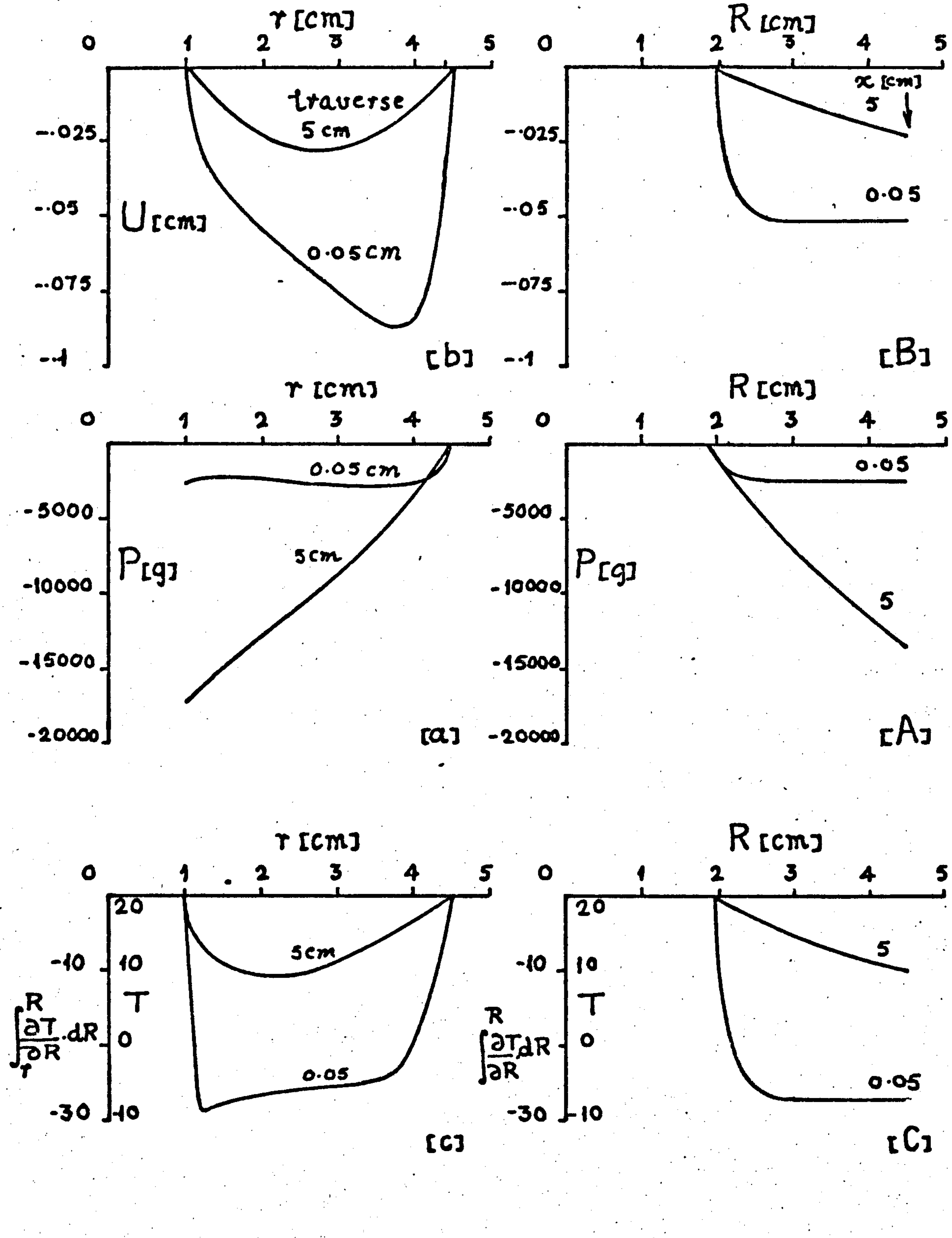
FIG. 3.24

The results confirm the comments already made. However the cheese made with the isotropic yarn show a very different behaviour and this difference increases with the increase of the modulus ratio. The cheese shows high values of P and low values of U and $\int_r^R \frac{\partial T}{\partial R}.dR$. The tension in the yarn is always positive. A cheese made with an isotropic yarn is always likely to show a tendency to contract axially.

3.5.7 Cheese with an Isotropic Yarn

The yarn can be made isotropic with the value of modulus ratio as unity by assigning equal values to EY and E. Two values of EY (or E), namely 600g and 1000g are chosen. The results are shown in Fig. 3.24. The curves (a), (b) and (c) show P, U and $\int_r^R \frac{\partial T}{\partial R}.dR$ and T respectively with r for R of 5 cm and the curves (A), (B) and (C) show P, U and $\int_r^R \frac{\partial T}{\partial R}.dR$ and T at constant r of 1.9 cm as R increases from 1.9 cm to 5 cm.

The results show that P and $\int_r^R \frac{\partial T}{\partial R}.dR$ remain virtually unaffected by a change in the value of E (or EY) from 600g to 1000g whereas U is higher when EY (or E) is lower. This is as would be expected, e.g. if E (= EY) was very large then there would be virtually no U because the smallest deformation would cause the yarn to lose its tension and it would then add nothing to the pressure, whereas the opposite case of a very low E (= EY) would permit deformation but not increase the pressure etc. in proportion.



RO = 4.5 cm; T_0 = 20g; space = 1D; E = 1000g; EY = 1000g;

Fig. 3.25

3.5.8 Parallel Wound Cheese with Isotropic Yarn

A parallel wound cheese is obtained by reducing the value of x, i.e. traverse per wind, equal to the diameter of the yarn, namely 0.05 cm. By this the axial component of the tension in the yarn becomes small and the circumferential component of the tension in the yarn becomes nearly equal to the tension itself. Because of this type of element construction the number of crossing points in the element is greatly reduced and is 8 in an element of standard size as against 490 when x = 5 cm; in effect increasing the value of modulus ratio considerably. The pressure imposed by the added layer is increased due to a higher value of K and a reduced value of α. The results expected would therefore be similar to that of a cheese with high modulus ratio.

Fig. 3.25 shows the results of two cheese with the values of x of 0.05 cm and 5 cm. Curves (a), (b) and (c) show P, U and $\int_r^R \frac{\partial T}{\partial R}.dR$ and T for both the cheeses with r when RO is 4.5 cm and curves (A), (B) and (C) show the same at r = 1.9 cm as R increases from 1.9 cm to 4.5 cm. The cheese with parallel winding show larger values of U and $\int_r^R \frac{\partial T}{\partial R}.dR$ but a smaller value of P. Also the values of U, P and $\int_r^R \frac{\partial T}{\partial R}.dR$ cease to grow with R after some increase in R from when R = r. These results are similar to the results of the cheese made with a yarn of high modulus ratio shown before.

However the present analysis is not suitable for a parallel wound cheese as the contact between the yarns of the adjacent layers is no longer theoretically a point contact but is a line contact and the construction of the element for a parallel wound cheese would be different. In all practical cases this will be an area of contact which will tend to a line for α = zero but will be changing little for other values of α. This effect will be discussed later.

3.6 Summary of Results

The values of U, P, $\int_r^R \frac{\partial T}{\partial R}.dR$, $\int_r^R Z.dr$, etc. are proportional to the winding tension in the yarn and change according to it.

The modulus ratio, i.e. the ratio of EY to E, is important and has considerable effect on the behaviour of the cheese. A high value of the ratio results in smaller values of the pressure, greater changes in the tension of the yarn and the yarn acquires negative tension. Q and Z also become negative in those layers in which the yarn has a negative tension. Due to this the shear force changes sign and generates a tendency for the cheese to expand axially. The maximum of the shear force at a given r occurs when tension in the yarn at r is zero. The magnitude of this tendency and the part of the cheese subjected to this tendency increase with the increase in the value of the modulus ratio. The value of U, i.e. the amount of the compression, is influenced by the individual values of E and EY and is high when E or EY is low and vice versa. Another influence is that the pressure of the added layer at the outer radius R affects only a few layers immediately below R and the rest of the layers remain virtually unaffected. This is shown by the curves of P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. becoming flat very soon. These curves show large initial changes at r due to some increase in R from when R = r and little changes for subsequent increase in R. This also results in a pressure distribution in the cheese which does not vary much with r except near the outer part of the cheese where pressure falls rapidly.

The behaviour of the cheese made with isotropic yarn is quite different. In this cheese the yarn is not likely to acquire negative tension and the cheese probably would not show any tendency to expand axially though it might show a strong tendency to contract axially. The pressure within the cheese increases steadily towards the core unlike the cheese made with a yarn of high modulus ratio.

A change in traverse per wind, i.e. wind angle, affects the circumferential and axial components of the tension in the yarn and also the number of pressure bearing points in the cheese. Maximum number of crossing points are available when $\alpha = 45^{\circ}$ and theoretically the resistance of this type of the cheese to radial deformation would be maximum. A change of wind angle from 45°

reduces the number of crossing points which in effect is equivalent to lowering the value of E and the results are therefore similar to those obtained with a higher value of the modulus ratio. Near parallel winding can be obtained by reducing the traverse per wind equal to the diameter of the yarn. But the analysis based on this type of element construction is not applicable to that case as the contact between the adjacent layers in that case is theoretically no longer a point contact.

An increase in the spacing of the adjacent wraps of yarn from the minimum value of one diameter of the yarn reduces the number of ends in the element proportionally but the number of crossing points reduces as the square of the reduction in the number of the ends and this in effect is again equivalent to the lowering of the value of E and the results are therefore again similar to those obtained with a higher value of the modulus ratio.

CHAPTER IV

THE MODIFIED THEORY

4.1 Introduction

In the theoretical analysis of the previous chapter E, the Modulus of Compression of cheese, and EY, the Elasticity of yarn in Extension are assumed to have constant values under all conditions. However the values of E and EY are not constant particularly when the loads are small, though at high loads the variation in them may be small. The results of the previous chapter shows that small values of the pressure in the cheese and of tension in the yarn do occur; the former occur beneath the added layers at the outer radius of the cheese and the latter may occur at any radius after the addition of a few layers beyond that radius. The tension in the yarn may even show a negative value. Further the results of the previous chapter show that the behaviour of the cheese is sensitive to the value of modulus ratio and that it alters considerably as modulus ratio is changed. Now with the changing values of E and EY with the pressure in the cheese and the tension in the yarn, modulus ratio within the cheese changes constantly as the cheese is built up and this could alter the behaviour of the cheese considerably. Therefore the provision should be made in the theoretical analysis to take into account the variation in the values of E and EY due to the changing values of the pressure in the cheese and the tension in the yarn as the cheese is built up. To allow this provision in the theoretical analysis it is necessary to determine the relations between E and P, the pressure in the cheese and between EY and T, the tension in the yarn.

This chapter describes the methods to determine approximately the above relations. In developing the analysis which incorporates these relations it was found advantageous to integrate the differential equation numerically from outside to the core and not from core to the outside as was done in the previous chapter. The Runge-Kutta method is used to integrate the equation instead of Euler's modified method. A new computer program is written to solve the equation and is given in Appendix A with flow diagram. The chapter ends with the presentation and discussion of the results calculated by the program.

4.2 Relation Between EY and T

4.2.1 Tension Strain Curve

EY, the Elasticity of yarn in Extension, is defined as the force or tension required in the yarn to produce unit strain in the length of the yarn. It is expressed in g. This is determined by conducting a load-elongation test on the yarn on the Instron Tester. The particulars for the test were: cross head speed = 0.5 in per sec; chart speed = 20 in. per sec; length of the yarn tested = 10 in. The trace of the load extension curve was obtained on a paper with

Tension-strain curve of the yarn

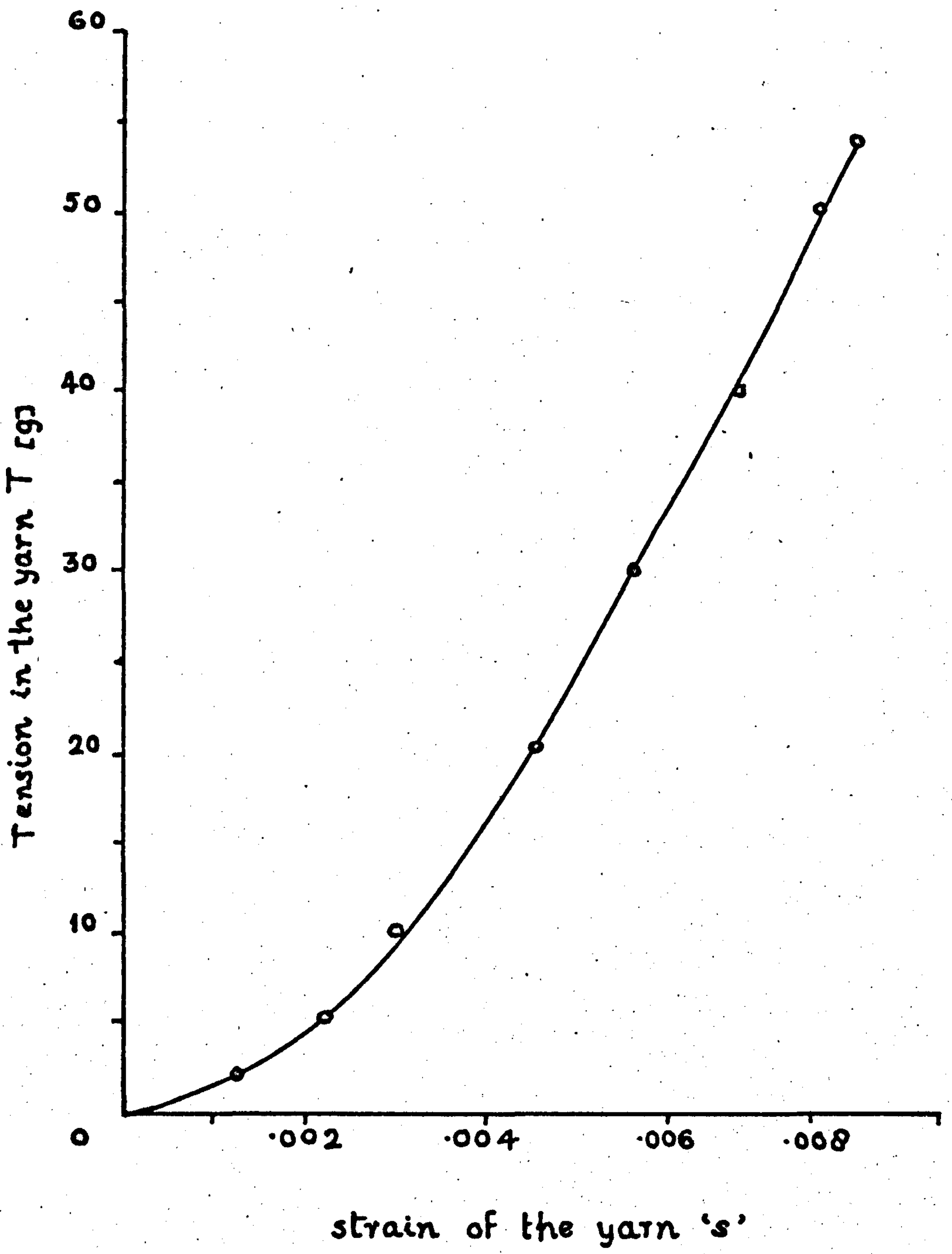


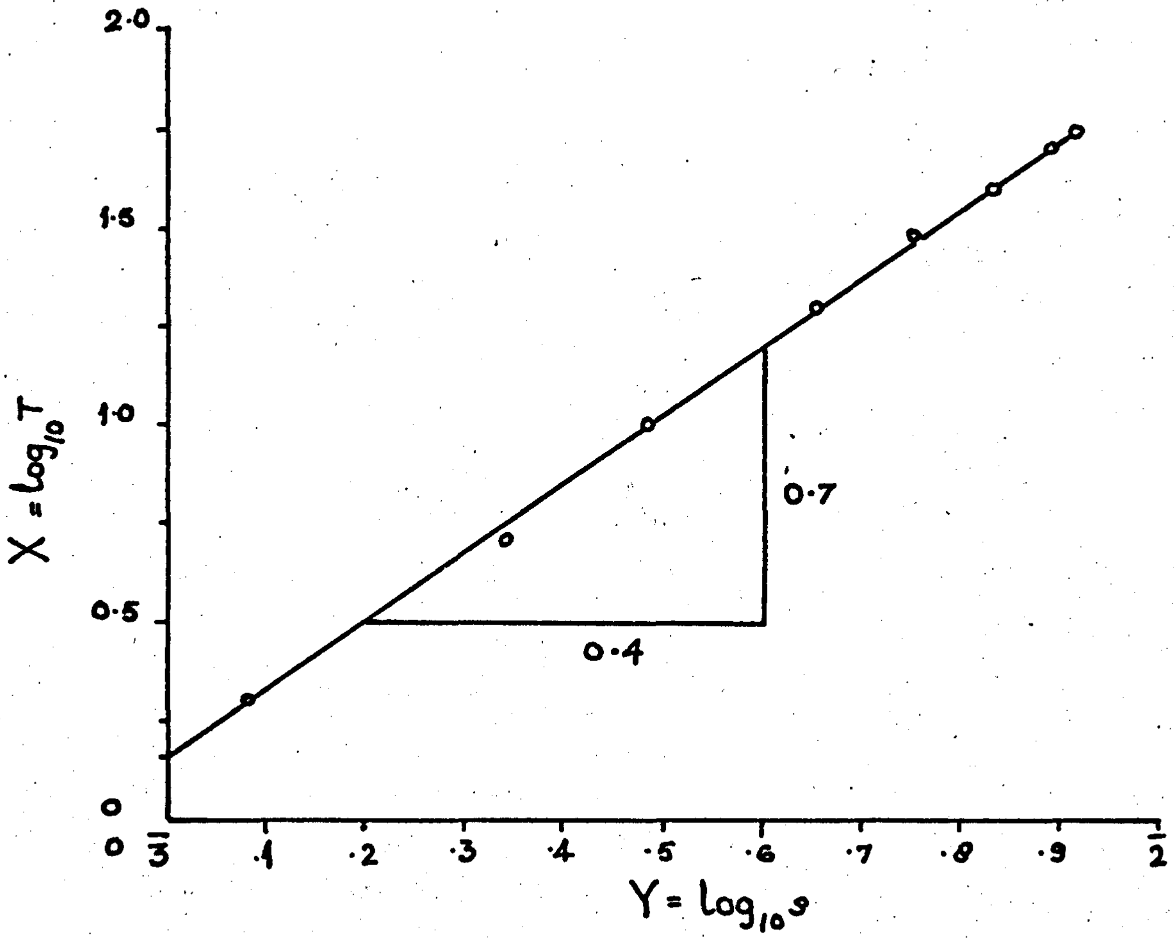
FIG. 4.1

cm markings. Therefore one cm of paper represents a strain of (0.5 / (20 x 10 x 2.54)) and is approximately equal to 0.001.

Four tests were conducted and in each test the tensioning of the yarn was repeated four times. The maximum load used in tensioning was 55 g. The chart movements in cm were read off from each of the sixteen plots against tension values of 2g, 5g, 10g, 20g, 30g, 40g, 50g and 54g and these are tabulated in Table 4.1. These sets of values are then averaged to obtain the values of strain in the yarn at different values of the tension in the yarn. Fig. 4.1 shows the tension T in the yarn plotted against the strain 's' in the yarn.

Table 4.1

tension in the yarn (g)		2	5	10	20	30	40	50	54
chart movement (cm) 1st Test	1	1.2	2.6	3.3	5.0	6.4	7.5	8.4	8.8
	2	1.3	2.4	3.2	5.0	6.2	7.2	8.0	8.4
	3	1.4	2.4	3.3	4.8	6.1	7.1	8.0	8.3
	4	1.3	2.3	3.2	4.7	5.9	6.9	7.8	8.3
2nd Test	1	2.4	3.8	5.1	7.4	9.6	11.4	12.8	13.6
	2	1.8	2.9	4.0	5.8	6.4	8.7	10.0	10.6
	3	1.3	2.5	3.5	5.3	6.4	8.1	9.2	10.0
	4	1.3	2.4	3.4	5.2	6.6	7.9	9.0	9.4
3rd Test	1	1.2	2.0	2.7	4.0	5.0	5.9	6.7	7.0
	2	0.8	1.5	2.2	3.5	4.4	5.3	6.0	6.3
	3	1.0	1.7	2.4	3.5	4.5	5.4	6.1	6.4
	4	1.0	1.6	2.3	3.4	4.4	5.3	6.0	6.3
4th Test	1	1.3	2.1	2.8	4.0	5.0	5.8	6.6	6.9
	2	1.0	1.7	2.4	3.5	4.5	5.3	6.1	6.3
	3	1.0	1.7	2.3	3.4	4.4	5.2	5.9	6.2
	4	0.9	1.6	2.3	3.4	4.3	5.1	5.8	6.1
Total		20.2	35.2	48.4	71.9	90.1	108.1	122.4	128.9
Average		1.26	2.2	3.03	4.49	5.63	6.75	7.65	8.05
Strain		0.0013	0.0022	0.0030	0.0045	0.0056	0.0068	0.0077	0.0081



Determination of the value of EY

FIG.4.2

4.2.2 Relation Between EY and T

The tension-strain curve of Fig. 4.1 is roughly hyperbolic. This curve is redrawn in Fig. 4.2 on a log scale such that $\log_{10}T = X$ and $\log_{10}s = Y$. The curve of Fig. 4.2 is a straight line and its slope as measured from the figure is 7/4. Now from the graph

$$\log_{10}T = 0.168 + \frac{7}{4}.(\log_{10}s + 3)$$

or $$\frac{4}{7}.\log_{10}T = 0.96 + \log_{10}s + 3$$

or $$T^{4/7} = 1247.s \qquad \ldots \quad \ldots \quad (i)$$

The value of EY at any instant is the slope of the tension-strain curve at that instant, i.e. dT/ds. Differentiating the above relation with respect to s

$$\frac{dT}{ds}.\frac{4}{7}.T^{-3/7} = 1247.$$

Therefore

$$EY = \frac{dT}{ds} = 2184.T^{3/7}$$

or $$EY = \text{tencon}.T^{y} \qquad \ldots \quad \ldots \quad (4.1)$$

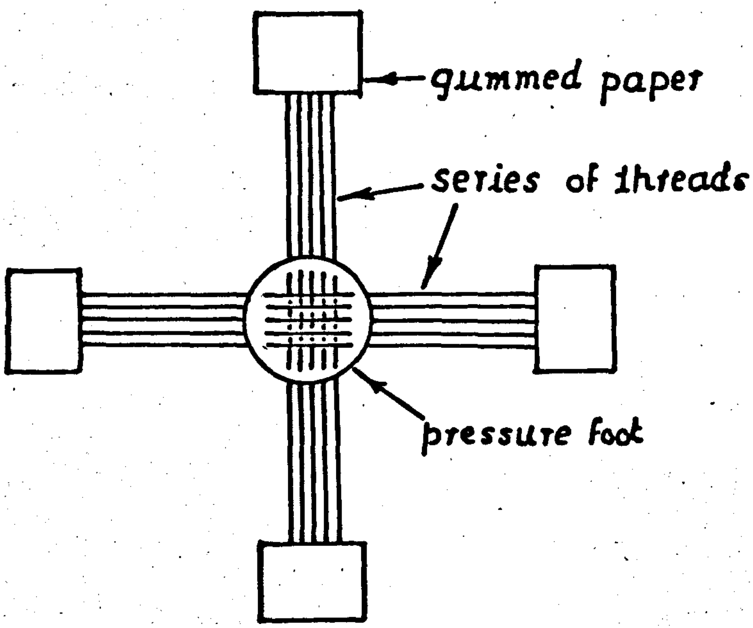
where the values of tencon and y depend on the extensional behaviour of the yarn. The equation (4.1) gives the required relation between EY and T.

4.3 Relation Between E and P

In the cheese a layer at any radius of the cheese is supported by a similar layer beneath it. The contact between the two layers is made at a number of crossing points and the pressure of the upper layer is applied to the layer beneath through these points. The yarn deforms at these points of contact. The area occupied by a given number of crossing points increases with the radius making the cheese non-homogeneous. Therefore the Modulus of Compression of cheese (in the radial direction), i.e. E, if defined on the basis of the area would not be constant but will vary with the radius. To avoid this E is defined on the basis of a crossing point and as the number of crossing points remain constant with the radius E also remains constant. Therefore E is defined as the force required to produce unit strain in the thickness of a crossing point and is expressed in g.

An experiment is devised, which attempts to simulate the conditions of the pressure application on the layer of the cheese, to determine the value of E approximately. From the results of the experiment the approximate relation between E and P is obtained. The method is as follows.

A cloth thickness tester is used to measure the deformation of crossed threads. Two series of threads are laid on the anvil of the tester, one series lying transverse to the other series thereby



No. of crossing points = 5 × 5 = 25.

Measurement of the value of E.

FIG.4.3

giving a number of crossing points as shown in Fig. 4.3. Only minimum tension is applied to the series of the threads to keep them straight. The thickness of the crossing points is read off from the dial of the tester under different pressures of the pressure foot. From these the value of E is computed.

4.3.1 The Cloth Thickness Tester

This consists of an engineer's dial gauge from which the return spring has been removed and to which has been added a loading pan for gravity return of the main spindle. The base of the spindle carries a circular pressure foot with a diameter of 0.375 in. The pressure foot and gauge is mounted above a steel anvil set in a wooden table.

The pressure during test is applied by the foot and may be varied in steps of 1 lb/sq.in. from 1 lb/sq.in. to 10 lbs/sq.in. by using different weights or different combinations of weights.

4.3.2 The Preparation of the Sample

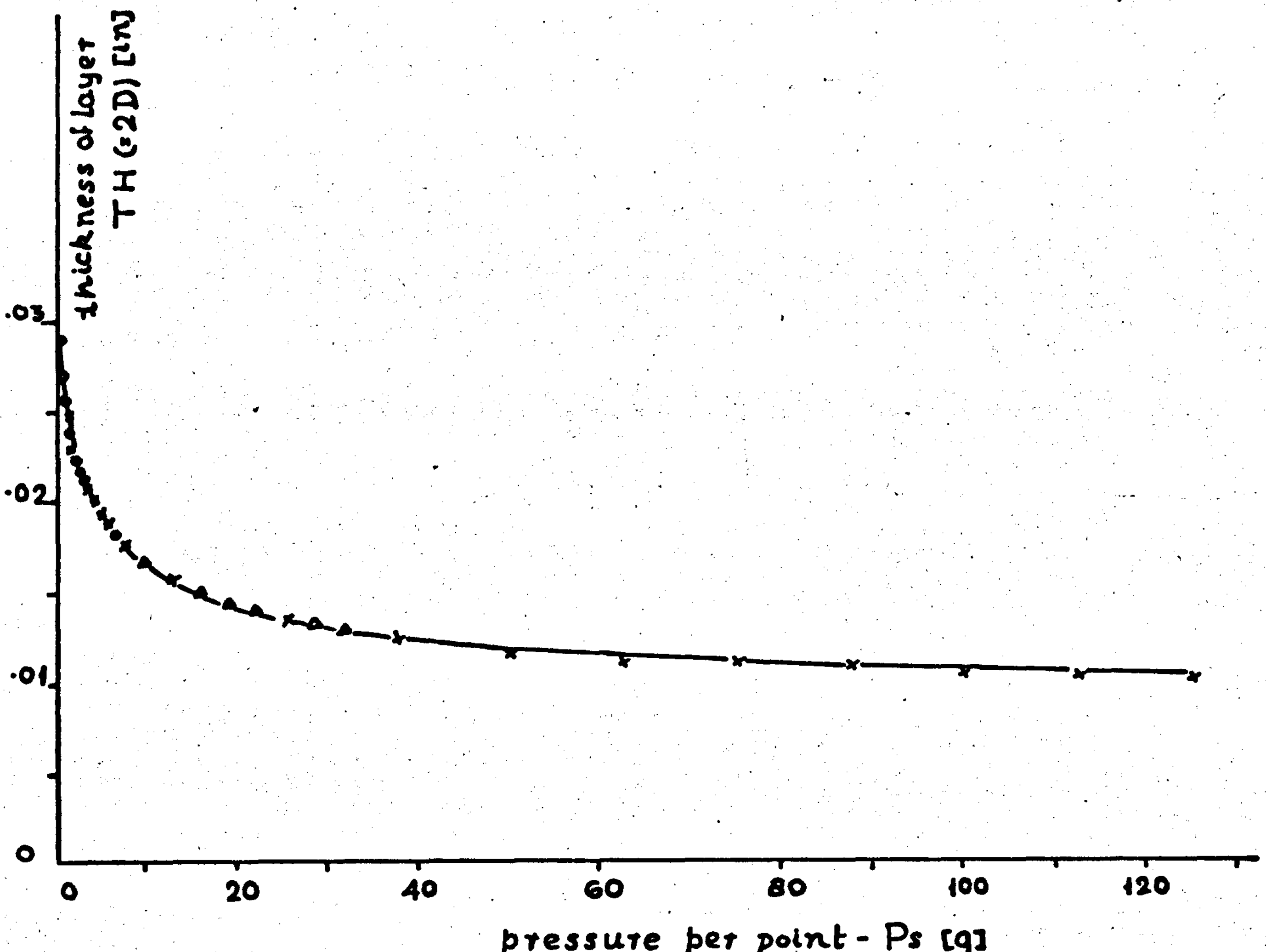
A series of threads was prepared by laying the desired number of threads for which E is to be measured so that the ends of the threads fell on a gummed paper strip. The threads were carefully laid side by side almost touching each other with only enough tension in the threads to keep them straight. The ends were then stuck to the gummed tape by moistening the paper. Another piece of gummed paper tape was stuck to the threads from the top thereby enclosing the threads between the paper strips. The length of the exposed threads was about 15 cm. Another series containing the same number of threads was also prepared. This series was laid transverse to the first series during the test thereby giving a number of crossing points.

Four sets of threads with two series in each set containing 2, 4, 8 and 14 threads were prepared for the first test thereby giving 4, 16, 64 and 196 crossing points respectively. For the second experiment four sets of threads with 6 layers in each set containing 6, 8, 10 and 12 threads were prepared thereby giving 36, 64, 100 and 144 crossing points respectively.

4.3.3 The Experiment

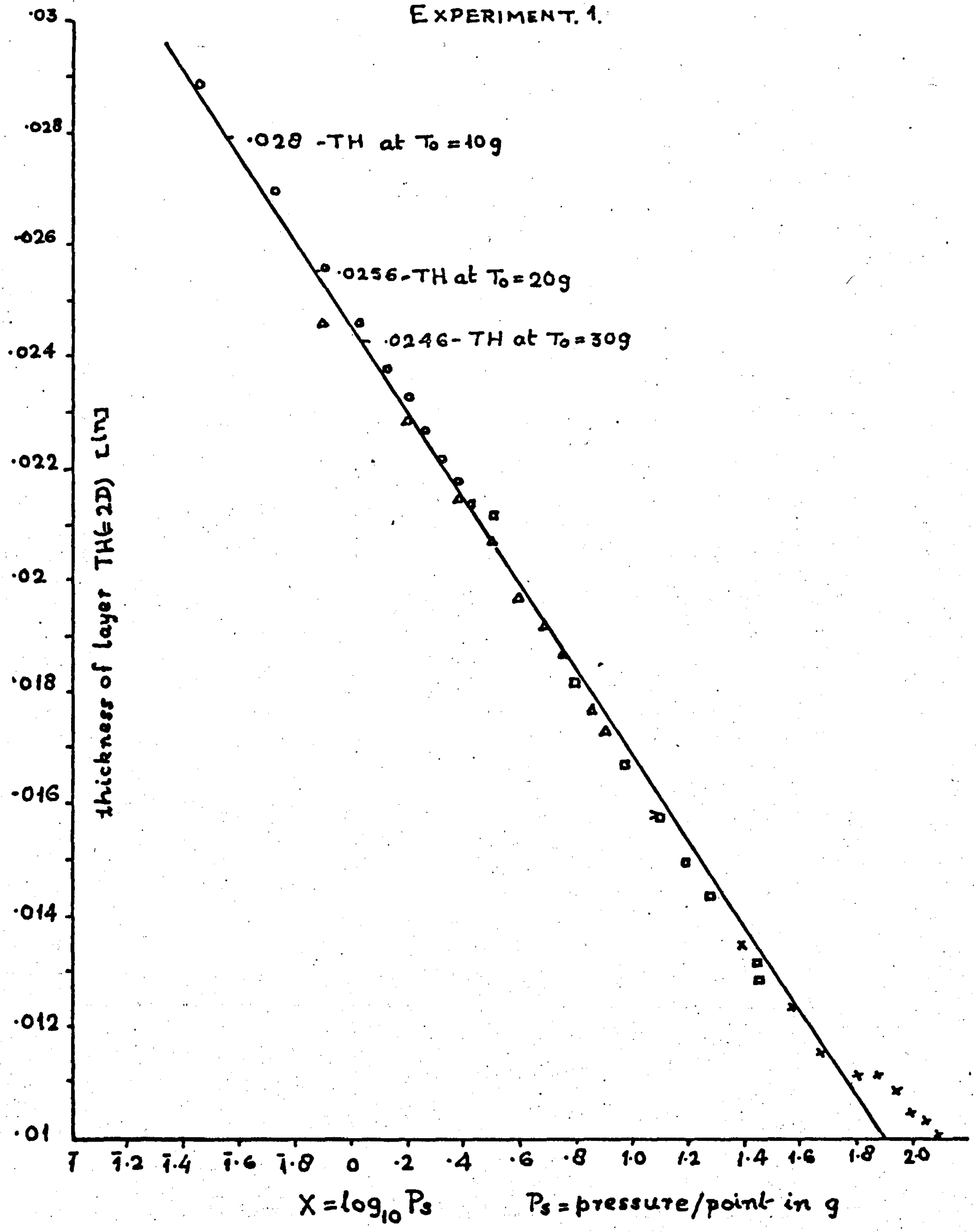
The first series of threads was placed on the anvil under the pressure foot of the tester. This was kept in position by a cellotape fixed to the paper strip of the threads and the board of the tester. Again minimum tension was used to keep the threads straight. The other series of threads was fixed over the first keeping the threads of the second series approximately at right angles to those of the first. Care was taken to ensure that no crossing point was out of the pressure foot. For the second test the alternate series of threads were kept parallel with the remaining series of threads lying transverse to the first set of series thus

Experiment. 1



Deformation of the layer under pressure

FIG.4.4

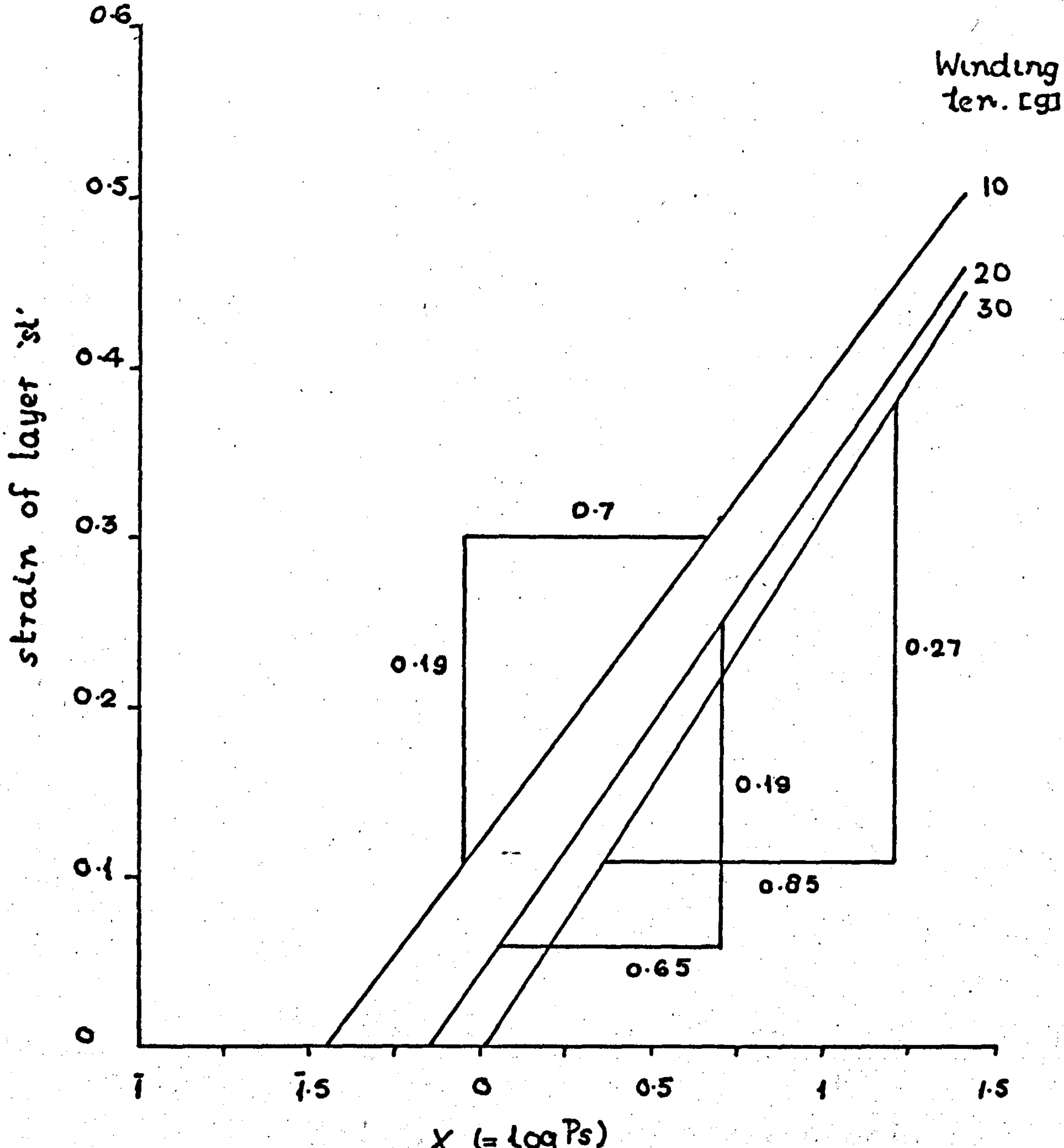


EXPERIMENT. 1.
·03
·028
·026
·024
·022
·02
·018
·016
·014
·012
·01
·028 -TH at T_o = 10g
·0256-TH at T_o = 20g
·0246-TH at T_o = 30g
thickness of layer TH(=2D) [in]
$\bar{1}$
$\bar{1}$·2
$\bar{1}$·4
$\bar{1}$·6
$\bar{1}$·8
0
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
2·0
X = $\log_{10} P_s$
P_s = pressure/point in g

FIG. 4.5

Experiment. 1.

T [g]	
10	$E = \frac{0.7}{0.19} \times 2.3025 \times P_s = 8.5 . P_s$
20	$E = \frac{0.65}{0.19} \times 2.3025 \times P_s = 7.87 . P_s$
30	$E = \frac{0.85}{0.27} \times 2.3025 \times P_s = 7.25 P_s$



Determination of the value of E

FIG. 4.6

forming six layers. The threads of one series were kept approximately above the threads of the series below parallel to it. The two parallel series were, however, separated by the threads of the transverse series.

Before starting the experiment the anvil and the pressure foot were cleaned by withdrawing a paper sheet through them under pressure. The dial position was also set to zero. The pressure foot was lowered very gently to avoid any impact. In all the observations the pressure was increased from 1 lb/sq.in. to 10 lbs/sq.in. in steps of 1 lb/sq.in. Also when adding weights the pressure was taken off the threads by lifting the pressure foot by the platform lever. The pressure per point, i.e. P_s, is given by the expression

$$P_s = A \times p \times 453.6/n;$$

where A is the area of the pressure foot, p is the pressure in lbs/sq.in. and n is the number of crossing points. The observations and calculations for both the experiments are given in Appendix D.

4.3.4 Results of the Experiments

The results of the first experiment are shown in Figs. 4.4 to 4.6. The thickness of the layer in in., i.e. TH which consists of two diameters of yarn, under different values of the pressure per crossing point in g, i.e. Ps is shown in Fig. 4.4. Fig. 4.5 shows TH plotted against $\log_{10}$Ps. The curve is a straight line.

In calculating the strain of the crossing point it is necessary to know the original thickness of the crossing point at no load. However the thickness of the crossing point, i.e. of the layer, under no load, according to the curve of Fig. 4.5, would be infinite and therefore it would be necessary to take the original thickness of the crossing point at some small value of the load. The value of the thickness of the crossing point changes very rapidly in this region and therefore a slight variation in the load can cause large errors in the calculated values of the strain. To avoid this the thickness of the crossing point at a given pressure, Ps, the value of which depends on the cheese conditions, is taken as the original thickness of the layer to obtain the values of the strain under different loads. In the cheese the pressure at the outside is zero before the addition of the layer and is P_o after the addition of the layer. Therefore the thickness of the crossing point at an equivalent pressure of $P_o/2$ is taken as its original thickness to calculate strain. P_o for a given cheese varies with R but this variation is not much. In a cheese with x = 5 cm, D = 0.05 cm, dR = 0.05 cm, spacing = 1D the value of $P_o/2$ at the core radius of 1 cm is 461g and at the final radius of 4.9 cm of the cheese at which the last layer is added is 585g when the winding tension in the yarn is 30g. The average of these two values is taken as the approximate value for determining the value of Ps. These values of Ps are 0.35g, 0.7g and 1.05 g when the winding tensions in the yarn are 10g, 20g and 30g respectively for the cheeses with the specification above.

The value of TH at the values of Ps of 0.35g, 0.7g and 1.05g are read off from the curve of Fig. 4.5. These are .028 in., .0256 in. and .0246 in. respectively. These values are used to draw the stress strain curves of Fig. 4.6 which shows strain 'st' of the crossing point plotted against stress, i.e. $\log_{10}Ps$. There are three curves for three different values of the thickness 'TH' corresponding to the three values of the winding tension in the yarn as shown in the figure. The relation between E and Ps is obtained from these curves.

4.3.5 Relation Between E and P

The value of E is given by the expression

$$E = \frac{dPs}{dst} \qquad \ldots \quad \ldots \quad (ii)$$

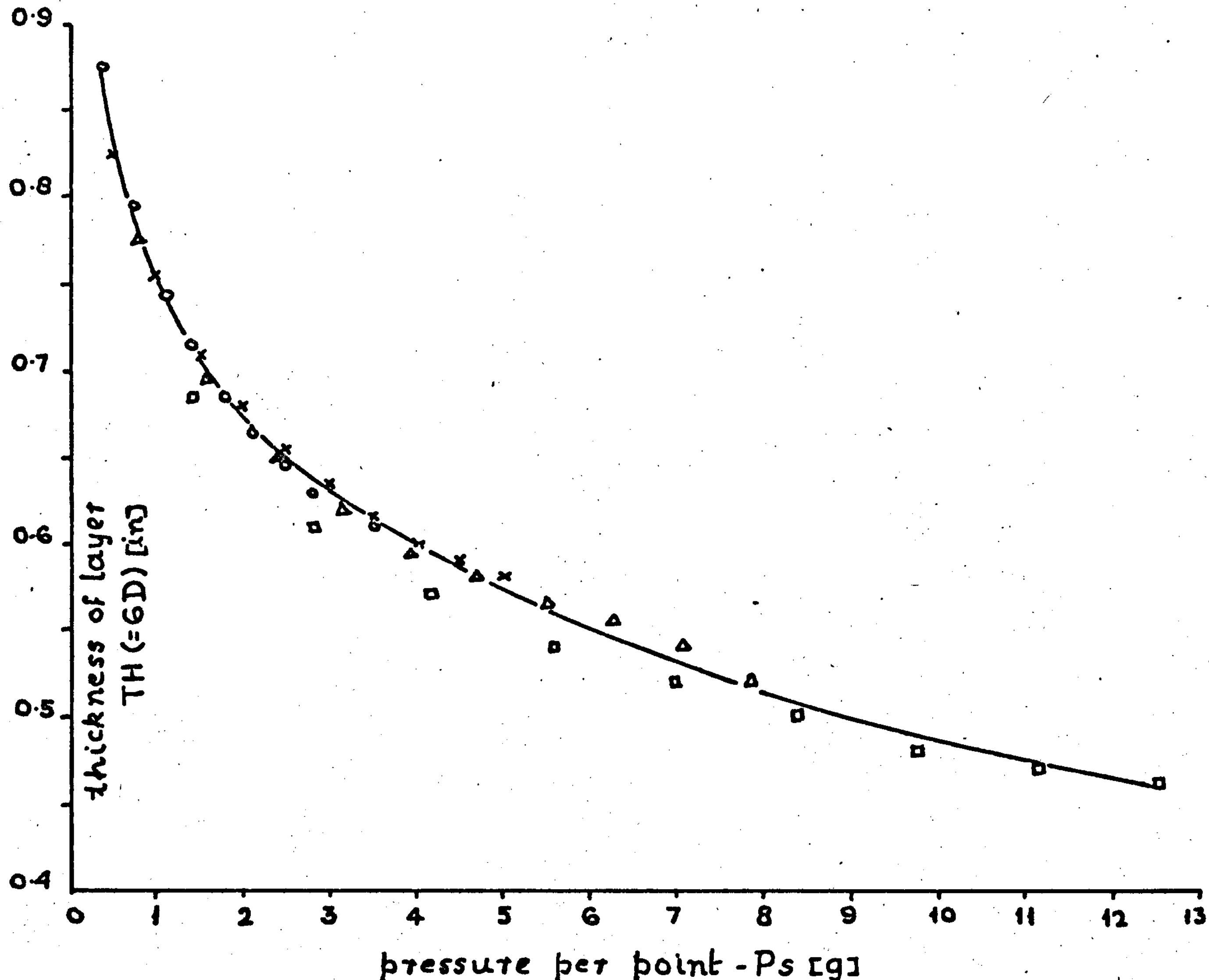
Let $$X = \log_{10}Ps \qquad \ldots \quad \ldots \quad (iii)$$

Differentiating (iii) with respect to X

$$\frac{dPs}{dX} = Ps.\log_e 10 \qquad \ldots \quad \ldots \quad (iv)$$

Substituting the value of dPs from (iv) in (ii)

Experiment. 2



Deformation of layer under pressure

FIG. 4.7

Experiment. 2.

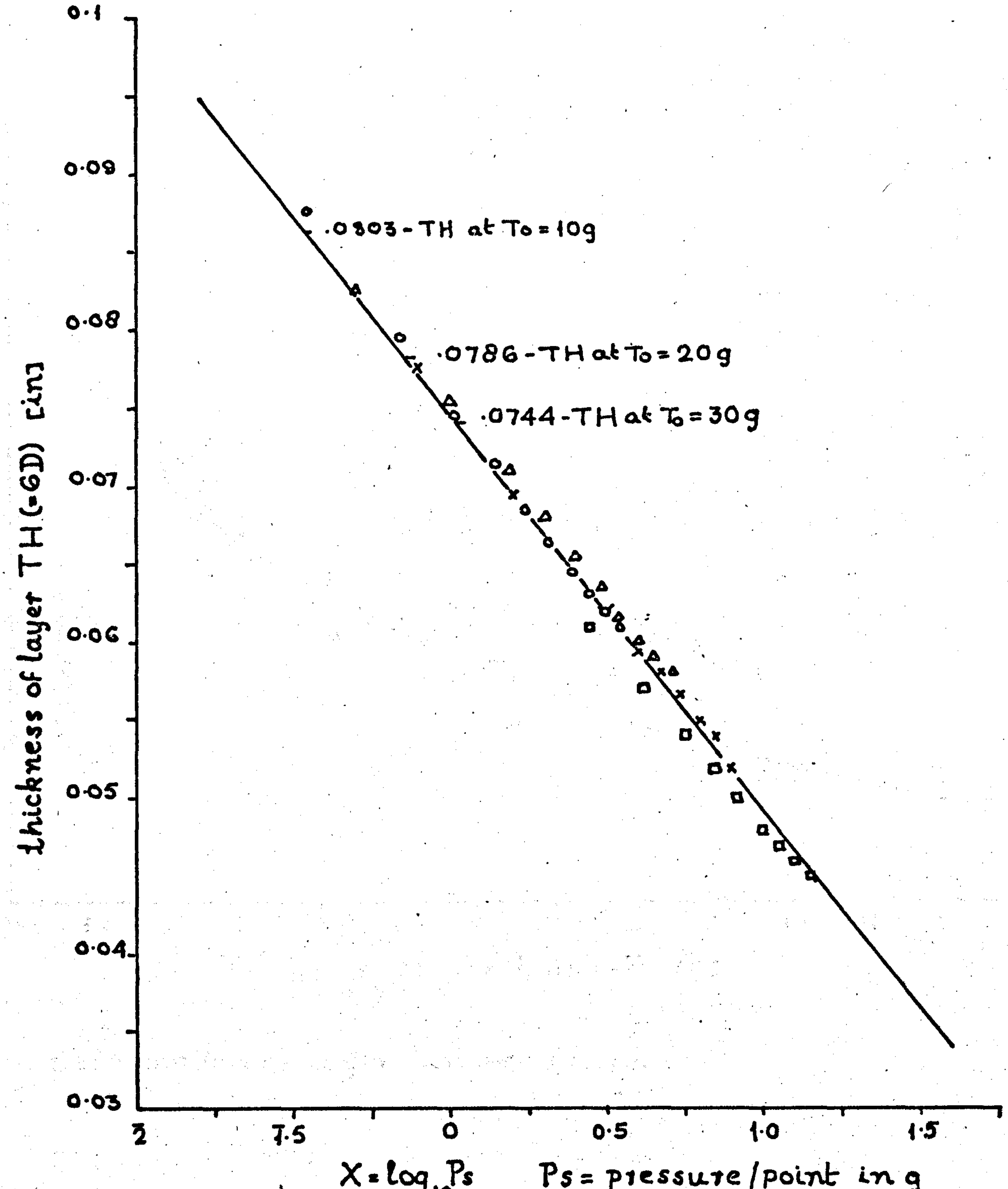
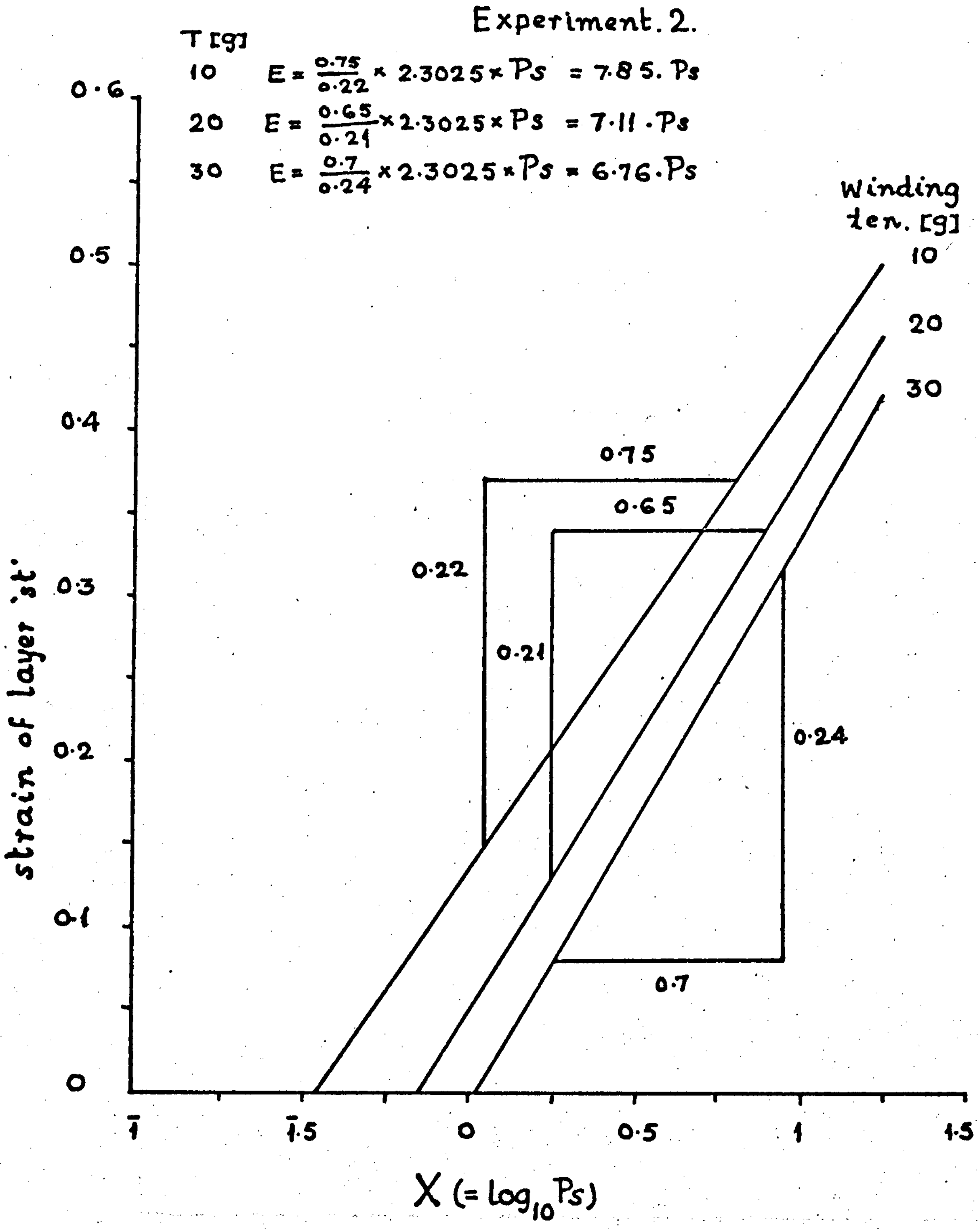


FIG. 4.8



Determination of the value of E

FIG.4.9

$$E = Ps. \log_e 10. \frac{dX}{dst}$$

or $$E = prcon. Ps \qquad \dots \quad \dots \quad (4.2)$$

where prcon is constant for a given cheese and is equal to ($\log_e 10. \frac{dX}{dst}$). $\frac{dst}{dX}$ is the slope of the curve of Fig. 4.6 and is constant for a cheese wound with a given winding tension in the yarn. The value of $\frac{dst}{dX}$ and hence of 'prcon' depends on the winding tension in the yarn and the compressional behaviour of the yarn with which the given cheese is wound.

The results of the second experiment are given in figures from 4.7 to 4.9. These results have been calculated in a way similar to those of the first experiment. In this experiment the layer was formed by six series of threads and therefore the thickness of the layer was equal to six diameters of the yarn. The following table gives the summary of the results.

Table 4.2

	E			D - dia. of yarn in cm.		
Winding tension in yarn	10g	20g	30g	10g	20g	30g
First Experiment	8.5 x Ps	7.87 x Ps	7.25 x Ps	.036	.033	.031
Second Experiment	7.85 x Ps	7.11 x Ps	6.76 x Ps	.037	.033	.031

The second experiment gives consistently lower values of the modulus than the first. As it reproduces conditions in the cheese more realistically than the first these values are to be preferred. For the present calculations which are necessarily only an approximation of the real behaviour of the package a figure of 7.5 for the constant 'prcon' would be sufficiently accurate.

4.4 The Value of EY As a Function of T

The need for the variation of the value of EY according to the tension T in the yarn is clearly demonstrated by Fig. 4.1. The curve shows that the value of EY varies considerably with T, particularly for small values of T and small values of T, as shown by the results of Chapter 3, do occur inside the cheese. Therefore a provision to vary the value of EY with T according to the equation (4.1), namely,

$$EY = tencon.T^{y}$$

should be provided in the integration of the equation (3.14). However an equation different in form from equation (3.14) is used because of convenience in writing the program.

The equation (3.12), namely,

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{2.K.E.D} (t.\cos\alpha - T\sin\alpha.\theta - t.\sin\alpha.\theta)$$

is due to the addition of a layer of thickness dR at the outer radius R of the cheese. Substituting the values of $\cos\alpha$, $\sin\alpha$ and θ from equation (3.7) in terms of r and u the above equation becomes

$$\frac{\partial^2 u}{\partial r^2} = \left(t.\frac{r}{(r^2+a^2)^{\frac{1}{2}}} + \frac{u}{r}.\frac{a^2}{(r^2+a^2)}.\frac{r}{(r^2+a^2)^{\frac{1}{2}}}.(T+t)\right).\frac{1}{2.K.E.D}$$

or

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{2.K.E.D} \cdot \frac{r}{(r^2+a^2)^{\frac{1}{2}}}\left(t + \frac{u.a^2.(T+t)}{r.(r^2+a^2)}\right) \quad \ldots \quad \ldots \quad (4.3)$$

The equation (4.3) is used to solve the cheese. The above equation by substituting the values of t and T in terms of u, r, U, and T_o can be reduced to the former equation (3.14).

The integration of the equation (4.3) is similar to the integration of equation (3.14). In this case a knowledge of the values of T at each step of r, instead of U, before the addition of the layer of thickness dR at R is required. The value of t, i.e. the change in T as R increases by dR is obtained by the equation (3.13), namely

$$t = EY.\frac{u}{r}.\cos^2\alpha.$$

The value of EY used in the above equation is the value at r when outer radius was R and u is the change in U as R increases by dR and is obtained by extrapolation from the values of u, $\frac{\partial u}{\partial r}$,

$\frac{\partial^2 u}{\partial r^2}$, etc. at the previous step of r, i.e. at (r - dr). From this the new values of T and EY, when outer radius is (R + dR), are calculated by the following relations

$$T_{R+dR} = T_R + t$$

and $$EY_{R+dR} = \text{tencon. abs}(T_{R+dR})^y$$

The use of absolute value of (T_{R+dR}) keeps EY positive according to its definition though T might become negative due to large U. This is repeated at each step of r, i.e. the radius of the cheese.

4.5 Modification of the Equation (4.3) to Allow E to Vary with P

4.5.1 Values of E, e, $\partial E/\partial r$ and $\partial e/\partial r$

The influence of the pressure on the value of the Modulus of Compression of the cheese is demonstrated by Figs. 4.4 and 4.7. As in the case of EY this is of greater importance at low values of P which occur in the cheese just underneath the added layer. The value of E is given by the relation (4.2) and in the cheese it is given by the equation

$$E = \frac{-\text{prcon.}P}{2.(K.\phi)^2} \qquad \ldots \quad \ldots \quad (4.4)$$

where $P/(2K\phi)^2$ is the pressure per crossing point and the negative sign ensures that the value E is always positive (pressure being negative in the present convention). Now E, like P, is a function of both r and R. The addition of a layer at the outer radius R causes a change p in the pressure P and therefore the value of E which depends on the pressure also changes, say by e. Here $e = \frac{\partial E}{\partial R}.dR$. Then

$$e = -\frac{p.prcon}{2.(K.\phi)^2} \qquad \ldots \quad \ldots \qquad (4.5)$$

Differentiating (4.5) with respect to r

$$\frac{\partial e}{\partial r} = -\frac{\partial p}{\partial r}.\frac{prcon}{2.(K.\phi)^2}$$

Substituting the value of $\frac{\partial p}{\partial r}$ from equation (3.10)

$$\frac{\partial e}{\partial r} = -q.\frac{\phi.prcon}{dr.2(K.\phi)^2}$$

or

$$\frac{\partial e}{\partial r} = -\frac{prcon.a}{2.K^2.dr.W}.q \qquad \ldots \quad \ldots \qquad (4.6)$$

Similarly by differentiating (4.4) with respect to r

$$\frac{\partial E}{\partial r} = -\frac{prcon.a}{2.K^2.dr.W}.Q \qquad \ldots \quad \ldots \qquad (4.7)$$

4.5.2 Modification of Equation (4.3)

The equation (3.11) is

$$\frac{p}{2.(K.\phi)^2} = \frac{\partial u}{\partial r}.E$$

In this equation p is the change in P as the outer radius of the cheese increases from R to (R + dR). E is the current value of the Modulus of Compression of the cheese when outer radius is (R + dR). Differentiating this relation with respect to r

$$\frac{\partial p}{\partial r} = 2.(K.\phi)^2.\left(\frac{\partial^2 u}{\partial r^2}.E + \frac{\partial u}{\partial r}.\frac{\partial E}{\partial r}\right)$$

Substituting the value of $\frac{\partial p}{\partial r}$ from equation (3.10)

$$q.\phi = 2.(K.\phi)^2 \left(\frac{\partial^2 u}{\partial r^2}.E + \frac{\partial u}{\partial r}.\frac{\partial E}{\partial r}\right).dr.$$

Substituting the value of q from equation (3.8)

$$2.(K.\phi)^2.\left(\frac{\partial^2 u}{\partial r^2}.E + \frac{\partial u}{\partial r}.\frac{\partial E}{\partial r}\right) = \frac{K\phi^2.dr}{dr.D}(t.\cos\alpha - \sin\alpha\ \theta\ (t+T)).$$

Substituting the value of θ, cosα and sinα

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{2.K.D.E}.\frac{r}{(r^2+a^2)^{\frac{1}{2}}}.\left(t + \frac{u.a^2.(T+t)}{r.(r^2+a^2)}\right) - \frac{\partial u}{\partial r}.\frac{\partial E}{\partial r} / E. \quad \ldots \quad \ldots \quad (4.8)$$

In the above equation T is the value of the tension in the yarn when outer radius was R and t and u are the changes in T and U respectively due to an increase in the outer radius from R to (R + dR). E and $\frac{\partial E}{\partial r}$ are the values when the outer radius is (R + dR).

Now $$E = (E_R + e) \qquad \ldots \quad \ldots \quad (4.9)$$

and $$\frac{\partial E}{\partial r} = \left(\frac{\partial E}{\partial r}_R + \frac{\partial e}{\partial r}\right) \qquad \ldots \quad \ldots \quad (4.10)$$

where E_R and $\frac{\partial E}{\partial r}_R$ are the values when the outer radius was R and $e\left(= \frac{\partial E}{\partial R}.dR\right)$ and $\frac{\partial e}{\partial r}\left(= \frac{\partial\left(\frac{\partial E}{\partial R}.dR\right)}{\partial r}\right)$ are the changes in E and $\frac{\partial E}{\partial r}$ as the outer radius increased from R to (R + dR).

4.5.3 <u>Relation Between E, $\partial u/\partial r$ and e</u>

From equation (4.5)

$$p = -\frac{2.(K.\phi)^2}{prcon}.e\ .$$

Substituting the value p from equation (3.11)

$$2.(K.\phi)^2.\frac{\partial u}{\partial r}.E = -\frac{2.(K.\phi)^2}{prcon}.e\ .$$

Substituting the value of E from (4.9)

$$\frac{\partial u}{\partial r}.(E_R + e) = - \frac{e}{\text{prcon}}$$

or $$e = - E_R.\text{prcon}.\frac{\partial u}{\partial r} / (1 + \text{prcon}.\frac{\partial u}{\partial r}) \quad \ldots \quad \ldots \quad (4.11)$$

4.6 Integration of the Equation (4.8)

4.6.1 Boundary Conditions

The equation (4.8) is a second order differential equation with two variables, namely, u and E and therefore requires four boundary conditions for its solution. At the core radius s, u and U are zero as the core is assumed to be incompressible. Therefore q at the core radius s given by equation (3.8) is also zero and therefore $Q_o = Q_{os}$, irrespective of the value of R. Now from equation (4.7)

$$\frac{\partial E}{\partial r} \text{ at } s = - \frac{\text{prcon.a}}{2.K^2.\text{dr.W}}. Q_{os}$$

and $\frac{\partial E}{\partial r}$ at s is constant irrespective of the outer radius R.

The pressure imposed by the element of the layer added at R is P_{oR} and at R, P_{oR} is equal to the change in P, i.e. p. By equation (3.22)

$$P_{oR} = - \frac{K.\phi^2.dR}{D}. T_o \frac{R}{\sqrt{R^2+a^2}} .$$

The Modulus of Compression of cheese is related to P by equation (4.4), therefore at R

$$E = -\frac{prcon.P_{oR}}{2.(K.\phi)^2} = \frac{prcon}{2.K.D} . dR . T_o \frac{R}{\sqrt{R^2+a^2}}$$

The change in pressure p and E are related by the equation (3.11) and substituting the values of E and p in this equation (3.11) when r = R

$$\frac{P_{oR}}{2.(K.\phi)^2} = -\frac{\partial u}{\partial r} . \frac{prcon.P_{oR}}{2.(K.\phi)^2}$$

or $$\frac{\partial u}{\partial r} = -1/prcon.$$

The above relation appears in this form because at r = R pressure P is equal to p, previous value of P being zero.

Therefore the four boundary conditions are that when r = s

(1) $u = o$ and $U = o$; ... ... (4.12)

(2) $\frac{\partial E}{\partial r} = -\frac{prcon.a}{2.K^2.dr.W} . Q_{os}$ (a constant quantity irrespective of R); ... (4.13)

and when r = R

(3) $E = \frac{prcon}{2.K.D}.dR.T_o. \frac{R}{\sqrt{R^2+a^2}}$ ... ... (4.14)

(4) $\frac{\partial u}{\partial r} = -1/prcon$ ... ... (4.15)

4.6.2 Procedure for Integrating Equation (4.8)

Applying a method, similar to the one used before for solving Equation (3.14), to start the solution of equation (4.8) at the core radius the values to be assumed are those of $\frac{\partial u}{\partial r}$ and E at the core radius. The values known at the core radius from the boundary conditions are those of u, U and $\frac{\partial E}{\partial R}$. For the correct solution the calculated values of E and $\frac{\partial u}{\partial r}$ must agree with those of the boundary conditions at r = R. The method uses E and $\frac{\partial u}{\partial r}$ as separate variables and the iterative solution requires one loop with in another in the computer program. This can be avoided by using equation (4.11).

The equation (4.11) is

$$e = - E_R \ .\text{prcon} \cdot \frac{\partial u}{\partial r} / \left(1 + \frac{u}{r}.\text{prcon}\right).$$

With the help of this equation the value e, i.e. the change in the value of E at any r, due to the increase in the outer radius from R to (R + dR) can be directly calculated. This requires the knowledge of E at each step of r when the outer radius was R and is similar to U in this respect. This, however, is known from the previous solution. The relations enables the value of E when the outer radius is (R + dR) to be determined uniquely from the values of E_R and $\frac{\partial u}{\partial r}$ instead of its being extrapolated from the values of E and $\frac{\partial E}{\partial R}$ at the previous step of r, i.e. at (r - dr). Therefore the boundary conditions for

E or $\frac{\partial E}{\partial \tau}$ are no longer necessary and in the computer program only one loop is required thus effecting a considerable saving in the computer time.

Now if the equation is being integrated from core radius s to the outer radius R and as the correct solution is approached this relation fails to give the value of e at R; because at this radius the previous value of E, i.e. E_R, is zero as there was no pressure before the addition of the layer. Also the denominator of the relation for the correct solution is zero because $\frac{\partial u}{\partial r}$ for the correct solution is equal to (– 1/prcon) and this makes the denominator zero. The relation then takes the indeterminate form of o/o. For the correct solution the value of e at r = R is known from the boundary condition and can be used instead of getting it from the above relation. But for a trial solution which is near the correct solution an approximate value of e equal to the value of e for the correct solution obtainable from the boundary condition has to be used. This makes the program difficult and will affect the accuracy of the results due to the approximation.

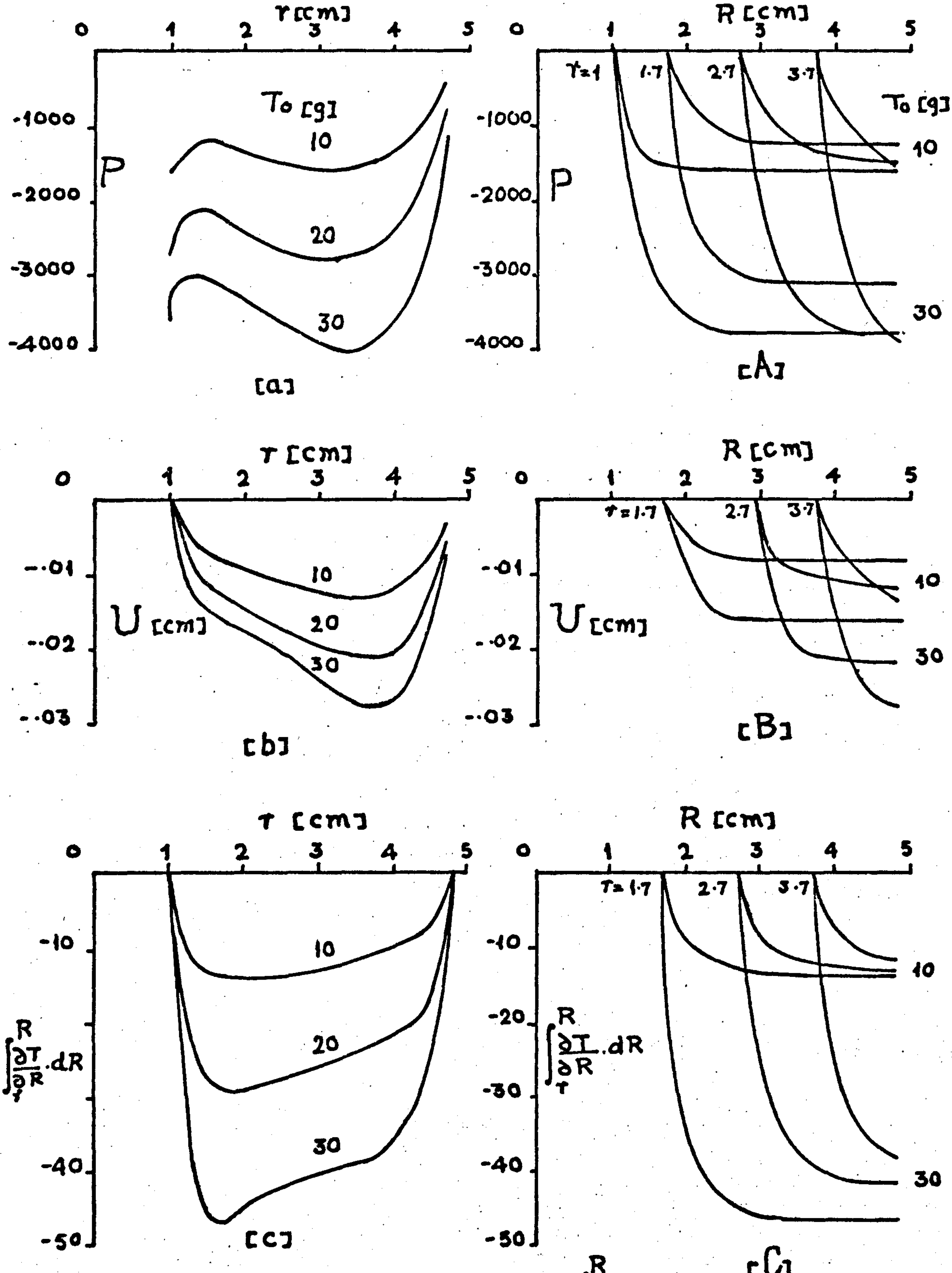
This difficulty is avoided by integrating the equation from outer radius R to the core radius s. At starting radius r = R both $\frac{\partial u}{\partial r}$ and e are known by the boundary condition (equation (4.14)). At this radius E and e are equal as P and p are both equal to P_{oR}

and there is no need to use this relation at $r = R$.

The solution starts by assuming a value of u at $r = R$. The calculated value of u at the core radius is compared to the value of u available from the boundary condition, i.e. zero. The calculation continues to repeat, each time with a fresh value of u at $r = R$, till the value of u at core is close to o. In this case the Runge-Kutta[6] method is used to integrate the equation with r as Euler's modified method, used in the previous case, gave divergent successive values of u. To make the program efficient the fresh value of u at $r = R$ at first, is obtained by quadratic interpolation; the method is given by Beckett and Hart[7]. In the event of the quadratic equation having imaginery roots the value of u is automatically interpolated linearly.

4.7 <u>The Results</u>

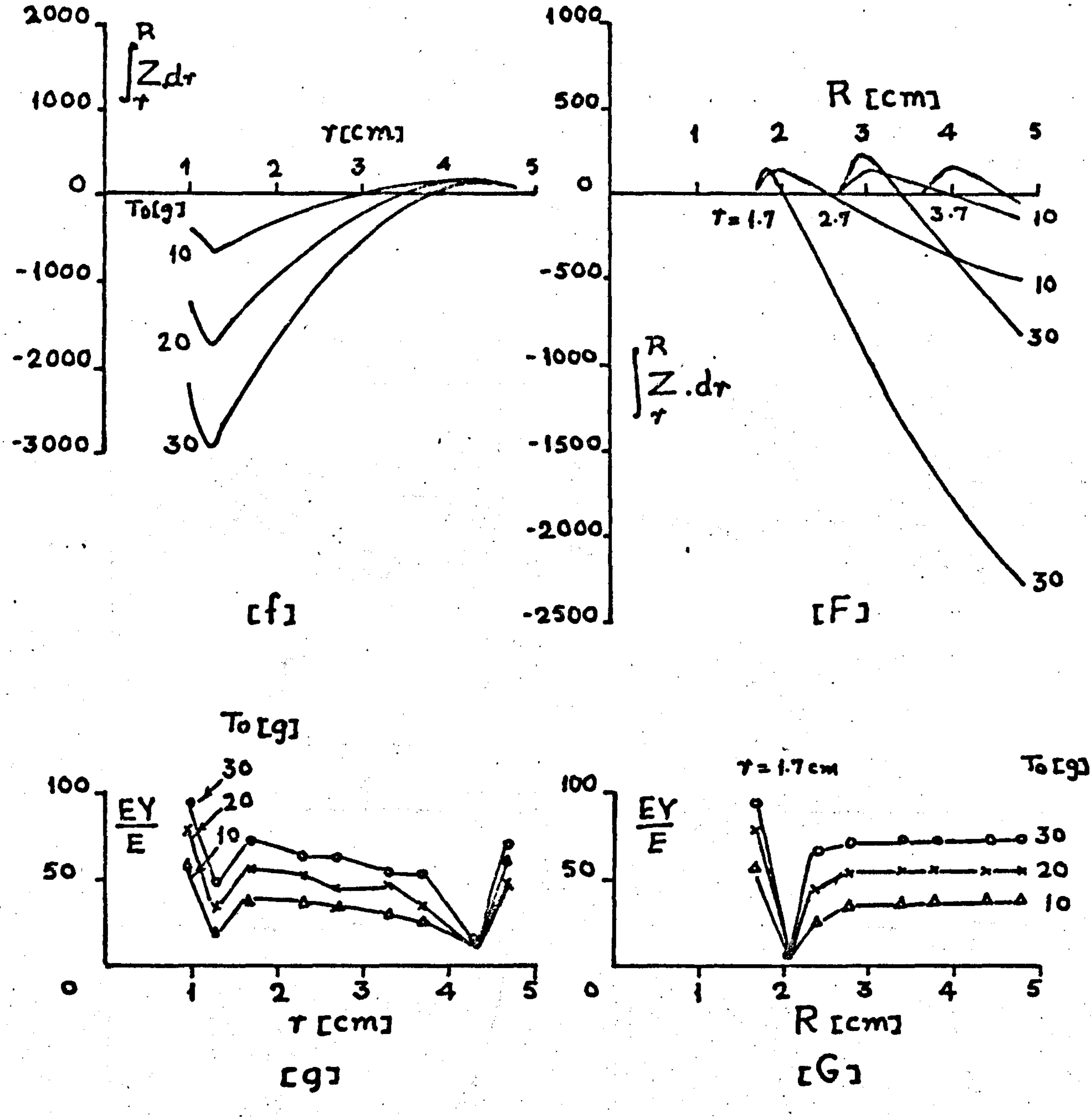
Program 36 written in KDF 9 Algol to solve the cheese model according to the modified version of the theory is given in Appendix A. The program incorporates a provision by which the cheese can be solved either with constant values of E and EY or with values of E and EY entirely dependent on P and T respectively or with some constant initial values of E and EY which remain as the constant part and the further changes in the values of E and EY depend on P and T as before. The three types of solutions are obtained by



$$T = T_0 + \int_r^R \frac{\partial T}{\partial R}\,dR$$

$x = 5$ cm; space = 1D; E = 100 g; $EY = 2185.T^{3/7}$; RO = 4.8 cm.

FIG.4.10



$x = 5$ cm; space = 1D; E = 100g; $EY = 2185.T^{3/7}$; RO = 4·8;

FIG. 4.11

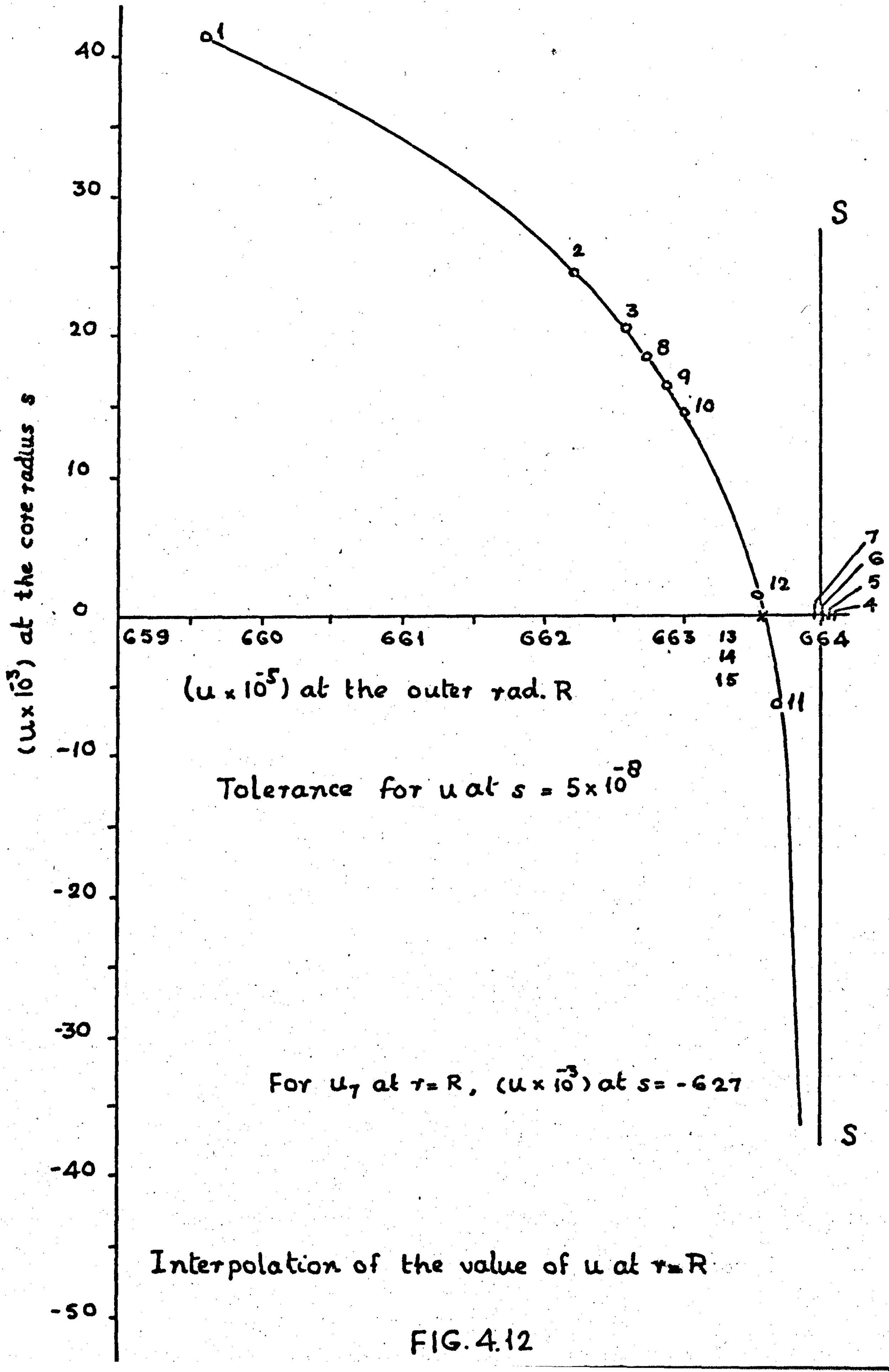
assigning zero values to 'prcon' and 'tencon' and constant values to IE and IEY or by assigning zero values of IE and IEY and required values to 'prcon' and 'tencon' or by assigning required values to prcon, IE, tencon and IEY respectively.

The results are divided into three sections, each section containing three solutions for three values of the winding tension. In the first section a constant value of E of 100g is assumed and the value of EY is allowed to vary according to the relation (4.1). This is done to study the effect of the variation of the value of EY on the behaviour of the cheese. The starting value of EY in the layer added, depends on the value of the winding tension in the yarn.

4.7.1 E Constant. EY Varies with T

The results of the solutions in which E has a constant value of 100g and EY varies with T according to the relation (4.1) are shown in Figs. 4.10 and 4.11. In the solutions the starting values of the modulus ratio are high, namely about 94, 79 and 59 when the winding tensions are 30g, 20g and 10g respectively. The values of the modulus ratio at a given r reduces as R increases from r and reaches its minimum value as the tension in the yarn continues to fall. Then the modulus ratio at r increases for some increase in R due to the yarn acquiring negative tension as the compression of the cheese at r continues with R. Finally the modulus ratio becomes constant as further increase in R does not affect the cheese at r. This is shown by the curves (G) of Fig. 4.11. The modulus ratio acquires low values before it starts increasing again and apparently due to this a slightly greater number of layers are affected by the addition of a layer at R as compared to the cheese with a constant modulus ratio of 50 (5000/100). This is shown by the curves of P, U and $\int_r^R \frac{\partial T}{\partial R}.dR$ at constant r showing comparatively gradual flattening with R. These curves when T_0 is 10g show a lesser tendency to flatten possibly due to a lower starting value of the modulus ratio. The low values of modulus ratio also allow comparatively greater values of U and $\int_r^R \frac{\partial T}{\partial R}.dR$. Also the pressure inside the cheese is higher than might be expected.

An increase in winding tension from 10g to 30g does not increase P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. proportionately because the modulus ratio changes. Trebling the winding tension increases the modulus ratio by about 1.6 times. However the total change in tension is more than three times and is probably due to a higher value of EY, when T_0 is high, which gives a greater change of tension for the same amount of compression. The shear force $\int_r^R Z.dr$ depends on tension and shows, similarly, more than proportional increase with the increase in the winding tension. This increase in shear force is accompanied by a sharper and quicker reversal in its direction. This is shown by curves (g) and (G) of Fig. 4.11. The shear force has a larger negative value and a greater part of the cheese is



Interpolation of the value of u at $r = R$

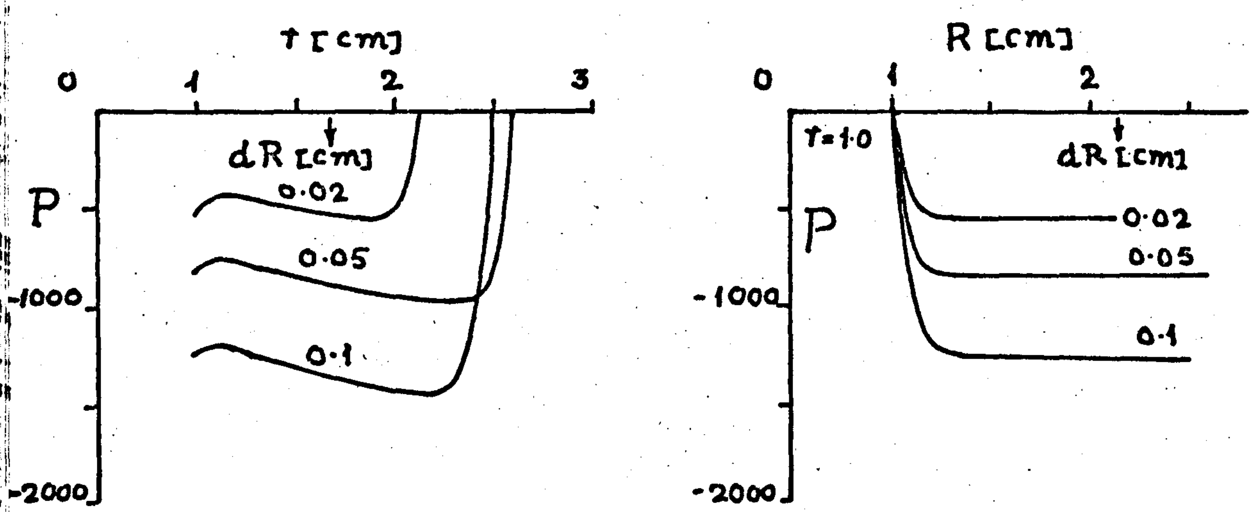
FIG. 4.12

subjected to negative shear as compared to that of the cheese with a constant value of modulus ratio of 50 (5000/100). Also the change in the direction of the shear at any r is sharper and quicker.

4.7.2 E and EY Vary with P and T

The program solves very slowly the cheese in which E and EY vary with P and T respectively. After long runs it is able to solve the cheese for small values of R. The main reason for the slow running is that many trial solutions are required. The value of u at r = R converges slowly to the correct value. The substitution of quadratic interpolation instead of linear interpolation for the value of u at r = R gave slight improvement. Another reason for the slow progress is that many trial solutions fail due to the value of E becoming very small or the value of u becoming very large. Consequently checks are inserted in the program which cause the program to abandon the solution likely to fail and attempt the solution with a fresh reduced value of u at r = R. This further increased the number of trial solutions. The maximum starting value of u at r = R is limited for which the solution does not fail. This limits the range of starting values of u at r = R and makes the interpolation of the correct value of u at r = R slower.

Fig. 4.12 shows a plot of the values of u at the core radius s (y-axis) against the starting values u at R for the addition of a layer at R. This took 15 solutions to obtain the correct



$x = 5$ cm; space $= 1D$; $E = 0.0144.P$; $EY = 2185.T^{3/7}$.

Effect of varying the thickness of the layer added

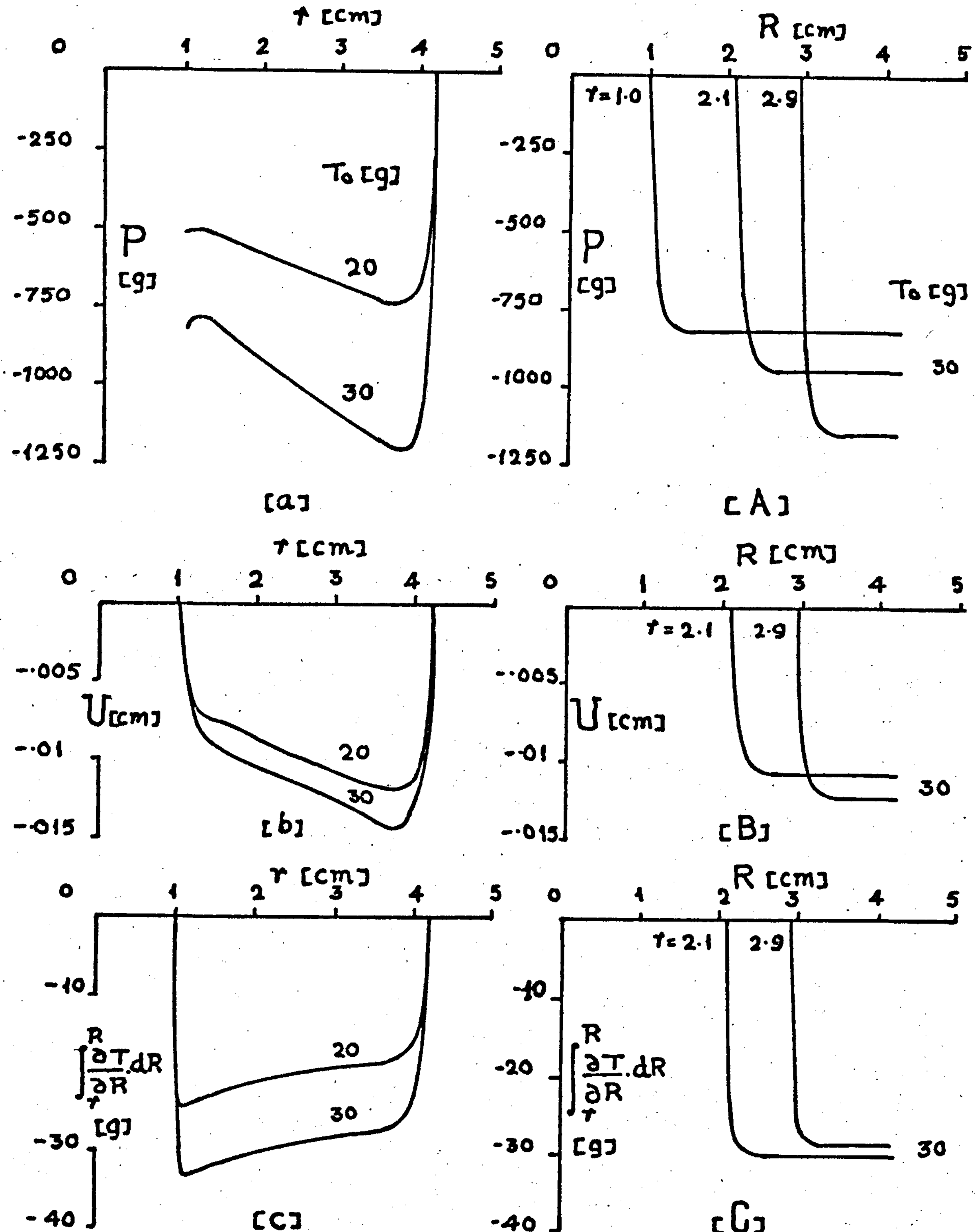
FIG. 4.13

solution. The line SS shows the rough limit of the starting value of u at r = R beyond which no solution is available because E becomes zero. The solution failed for values of u at r = R marked as 4, 5 and 6. In the event of a solution failing the next value of u at r = R is the average of the previous two values; these are (a) the one for which solution is available and the second (b) for which the solution fails. If the solution fails for the new averaged value too then (b) is changed, (a) remains the same. This continues and (b) gets closer to (a), till the solution is available for (b). In the solution shown in Fig. 4.12 the value of u at r = R was reduced by small steps instead of by averaging.

An attempt to improve the solution by reducing the thickness dR of the added layer to 0.05 cm and less from 0.1cm showed some saving of the time due to the reduced number of trial solutions. However the thickness of 0.05 cm gave best results and further reduction in the thickness of the layer increased the number of solutions required for a given value of R without a corresponding reduction in the time of each solution.

Unfortunately the change in the value of dR gave a different solution to the problem as shown by Fig. 4.13. This figure shows P for three values of dR, namely 0.1 cm, 0.05 cm and 0.02 cm, with r for given values of R and P at r = 1 cm as R increases from 1 cm. The winding tension in the yarn is 30g. E is proportional to P and is therefore shown qualitatively by the curves of P. Usually a solution which is very dependent on step size indicates an error in the numerical analysis. In this case however the reason appears to be a physical one rather than a mathematical one.

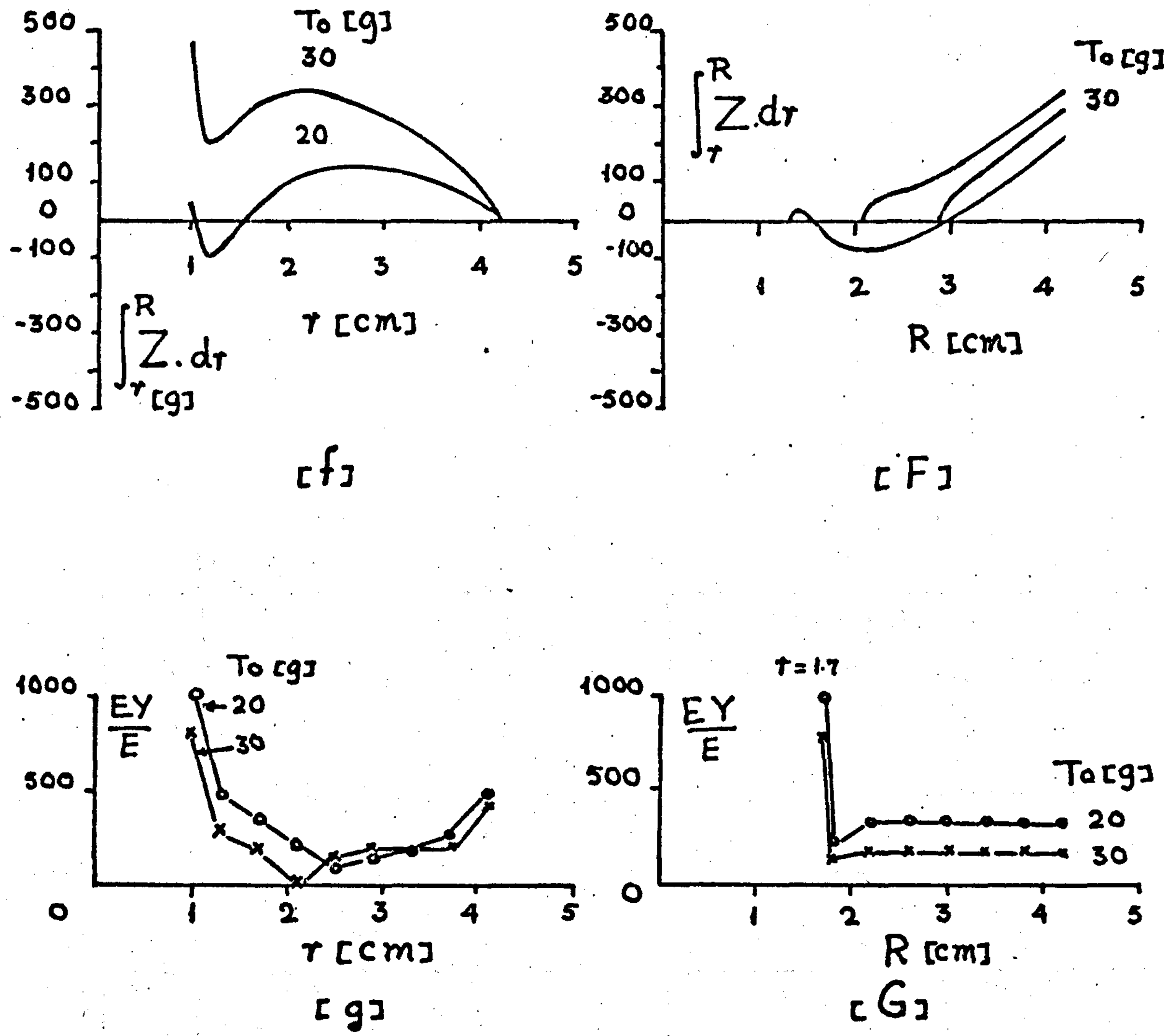
The value of E depends on pressure and therefore it would vary within the layer added at R being maximum at the lower end of the layer and zero at the upper end of the layer. The added layer dR is excluded from the solution - it merely provides a boundary condition for it. With E in its present form some of the tension would be lost within layer and use of thick layer may lead to large errors. However use of a thin layer with low pressures and low values of E at $r = R$ leads to a loss of yarn tension very close to the surface and to a slow convergence of the solution - the situation is approaching that of a zero force acting on zero resistance. These difficulties can be avoided by setting a lower limit to E in the program. In fact this seems to correspond to the practical situation where a lower limit to P and hence to E is set by use of a pressure roller. The diameter of the yarn in the layer added is proportional to the pressure and therefore would differ slightly with the winding tension in the yarn. In making a comparative study this slight difference in the diameter due to different values of T_0 is ignored as this would not affect the comparison and the great amount of time



$$T = T_0 + \int_r^R \frac{\partial T}{\partial R} . dR$$

$x = 5\,cm$; space = 1D; E = prcon.P; $EY = 2184.T^{3/7}$; $RO = 4.2\,cm$;

FIG.4.14



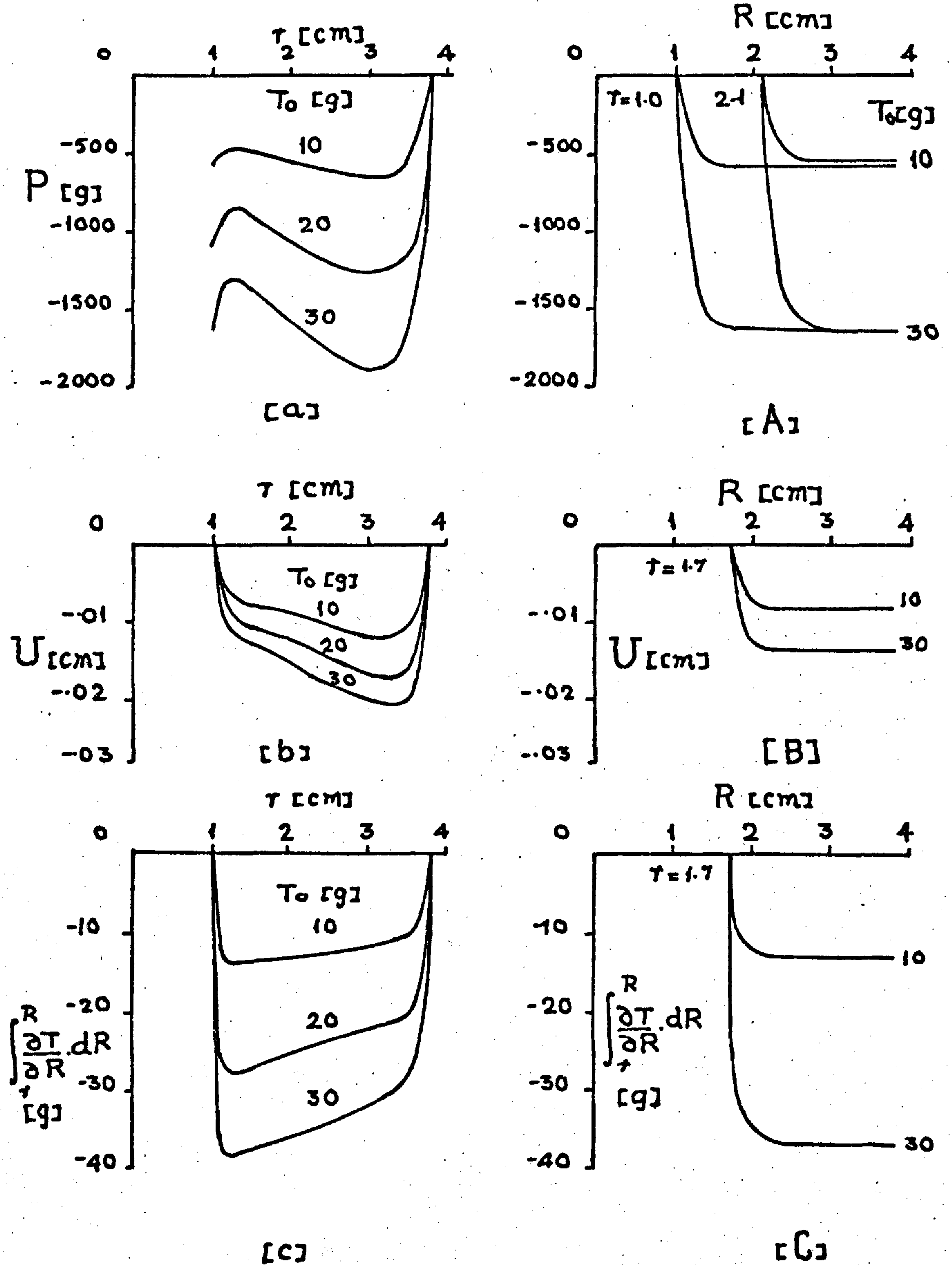
x = 5 cm; space = 1D; E = prcon. P; $EY = 2184.T^{3/7}$; RO = 4·2 cm;

FIG. 4.15

which this program takes to solve the cheese for even small values of R is saved by not re-running the program.

The results of the available solutions in which E and EY vary with P and T respectively are shown in Figs. 4.14 and 4.15. These results apply only for the value of dR used in this program. In effect this was such as to limit the lowest value of E to the values stated in the figures. The starting values of the modulus ratio are very high, namely 1001 and 793 when the values of T_0 are 20g and 30g respectively and due to this the effect of the added layer is limited to even fewer layers immediately beneath the added layer. This is shown by the curves of P, U and $\int_r^R \frac{\partial T}{\partial R} . dR$ of Fig. 4.14 becoming flat very sharply. This prevents the build up of higher value of P, E and U inside the cheese. Due to small U the tension in the yarn does not become negative at higher radii. This affects the shear force. The change in the sign of shear force from positive to negative does not occur at higher radii and at smaller radii the shear force changes its sign a second time becoming positive again. This is due to the tension not becoming negative and the addition of a layer at R, when R is large, results in a nett addition of the positive shear.

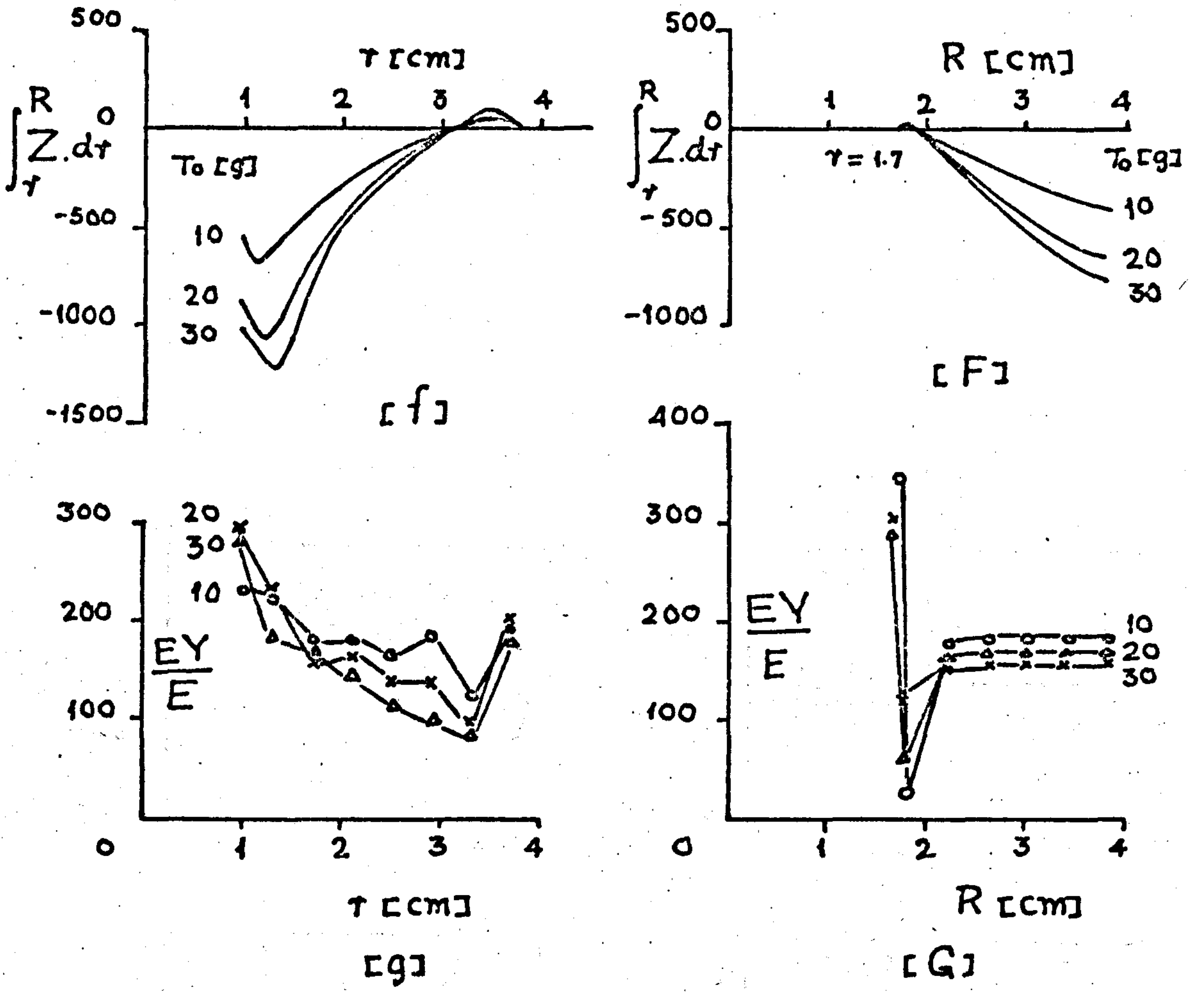
The modulus ratio at a given r with R falls sharply in the beginning due to the fall in the value of EY as the tension falls. The value of E changes little from the starting value and as the



$$T = T_0 + \int_r^R \frac{\partial T}{\partial R} \cdot dR$$

$x = 5cm$; space = 1D; $E = (prcon.P + 10)g$; $EY = 2184.T^{3/7}$; $R0 = 3.8cm$;

FIG. 4.16



$x = 5cm$; space = 1D; $E = (prcon.P + 10)g$; $EY = 2184.T^{3/7}$; $RO = 3.8cm$;

FIG.4.17

subsequent increase in R, after the initial increase in R from R = r, does not affect the layer at r the value of the modulus ratio remains low and constant. The value of the modulus ratio is higher when winding tension is lower, i.e. 10g, because of the comparatively low increase in the value of EY with the increase in the winding tension.

4.7.3 <u>Solutions With Initial Value of E</u>

In actual winding it is usual to apply pressure to the cheese during winding in order to make a firm hard cheese by some external means like a pressure roller apart from the pressure produced by the winding tension in the yarn. This is equivalent to raising the starting values of E and P and lowering the starting value of the modulus ratio and by avoiding the very small values of E giving results not so critically dependent on the arbitrary choice of step length. This is done in the theoretical solution by assigning some value to IE which is then equivalent to the increase in the starting value of E due to the pressure of the pressure roller. The value of IE chosen for this solution is 10g which is roughly equivalent to a pressure of 1.3g per crossing point. The results of the solutions for three values of 10g, 20g and 30g of winding tension in the yarn are given in Figs. 4.16 and 4.17.

The initial value of E reduces the starting values of the modulus ratio and the results are accordingly modified. Now the added layer affects a greater number of layers underneath it as compared to the previous case and therefore the growth of P, E, U, etc. at any radius r with R is higher. The tension in the yarn at a given r becomes negative as the cheese is further built up. The shear force at a given r changes its sign quickly and sharply and its negative value increases steadily with R.

Due to higher starting values of E the starting values of the modulus ratio are lower as compared to the previous case and the initial fall in their values due to the fall in the values of EY for initial increase in R are also smaller. In this case the increase in EY as R increases is accompanied by an increase in E and therefore the subsequent increase in the values of the modulus ratio after the initial fall are similar but greater in magnitude to those of the previous case.

4.8 Summary of Results

The value of modulus ratio at a given r falls sharply for slight initial increase in R from R = r and then increases as T becomes negative but the high starting values of the modulus ratio prevents further change in the value of the modulus ratio as R increases further. This is shown by all the three cases particularly so by the second case in which the values of the modulus ratio are highest. In this case the shear force is generally with a positive sign as the tension in the yarn at higher radii remains positive due to comparatively low values of U.

In all the cases the values of P, U, $\int_r^R \frac{\partial T}{\partial R}.dR$, etc. are not proportional to the winding tension in the yarn and is due to the change in the modulus ratio with the increase in the winding tension.

An increase in the starting value of E, which is taken as the value of E due to the pressure of the pressure roller on the cheese during winding, gives different results. The changes in the modulus ratio are similar to those of the other cases but are different in magnitude. The shear force shows a different behaviour and has generally a higher negative value with a sharper and quicker reversal in its direction.

CHAPTER V

CONCLUSIONS

5.1 Mechanics of Deformation of Cheese

The first object of any discussion of the results which have been presented must be to resolve the apparent contradiction between (a) the measured axial and radial deformation of the cheese, (b) the fact that measured and calculated behaviour regarding radial deformation agree so well when the calculations are for a very restricted solution with no axial expansion whereas in reality there is considerable axial expansion.

The experimental indication of the radial deformation of the cheese was itself rather unexpected. The results show that the cheese at a given radius shows fairly large radial compression for some initial winding at that radius, but after this initial compression there is no further radial deformation as the winding continues. In the absence of the calibration of the gauge the radial deformation of the cheese is known only qualitatively but this behaviour is shown consistently by different types of gauges and with different winding tensions in the yarn. Any effect of the axial expansion on the experimental indication of the radial deformation is small and the cheese at the most would show very slight continuous compression with the outer radius after the large initial compression.

However the axial deformation of the cheese behaves differently.

The cheese at r shows slight initial axial contraction for some slight increase in R from r = R and then a steady axial expansion at r with R. The initial increase in R which causes the axial contraction of the cheese is roughly the same which causes the large initial radial contraction of the cheese. At higher values of r = R the initial axial contraction of the cheese is not shown.

This is surprising for if there was no compression of the cheese at r then axial expansion of the trellis-like layer of the cheese at r would only be possible by extending the length of the yarn, i.e. by extending the length of each member. In view of the large axial deformation of about 5% - which would need correspondingly large extension of the yarn - this type of extension does not seem possible even if allowance were made for Poisson's effect, so far neglected. This is because of the large value of EY; a 5% extension of yarn would require a tension of over 1000g and the thread would rather break than extend. Therefore this type of extension could only be very small and the axial extension of the order of 5% could only be possible due to change of the angle of the thread along with the compression of the cheese of about 1%.

Therefore a continuous axial expansion of the cheese at r with increasing R only seems possible when there is a continuing radial compression of the cheese with a continuous build up of the pressure and the shear force at r with R. Also the initial large compression of the cheese should imply an axial extension of the cheese instead of the axial contraction as shown by the experimental results.

The compression of the cheese calculated theoretically is qualitatively very similar to the one indicated experimentally; even the magnitudes in the two cases seem to be of the same order though this cannot be confirmed in the absence of the calibration of the gauge used in the experimental method. This was very surprising because the theoretical analysis is of a rather artificial package in which no axial deformation is allowed.

Thus in discussing the results of the work the object must be to explain (1) the apparently conflicting requirements of axial and radial deformations, (2) why the theory for the artificially restricted model fits so well with the experimental results for radial deformation - which themselves showed a rather odd behaviour.

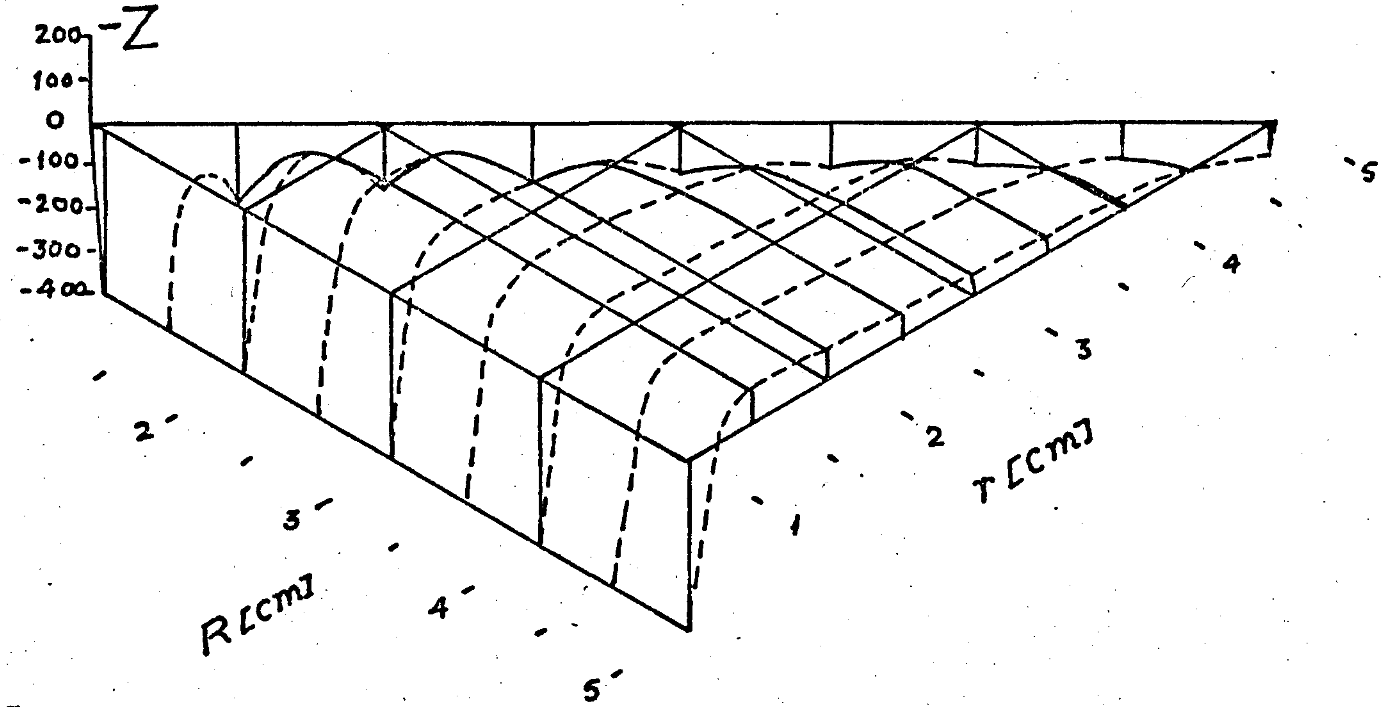
To do this it seems necessary to take a broader view of the deformation process. It is clear that when the yarn in tension is wound on the package there is in general a component - positive or negative - of axial tension in each layer of the package. The axial force must however diminish to zero at the ends of the package because there is no force applied externally. Moving in from the end the axial force builds up by the shear force between layers; this arises mainly from friction between layers. Ultimately, if the package is long enough a region will be reached in which the axial tension in adjacent layers have reached the values for which there is no tendency for relative movement to occur.

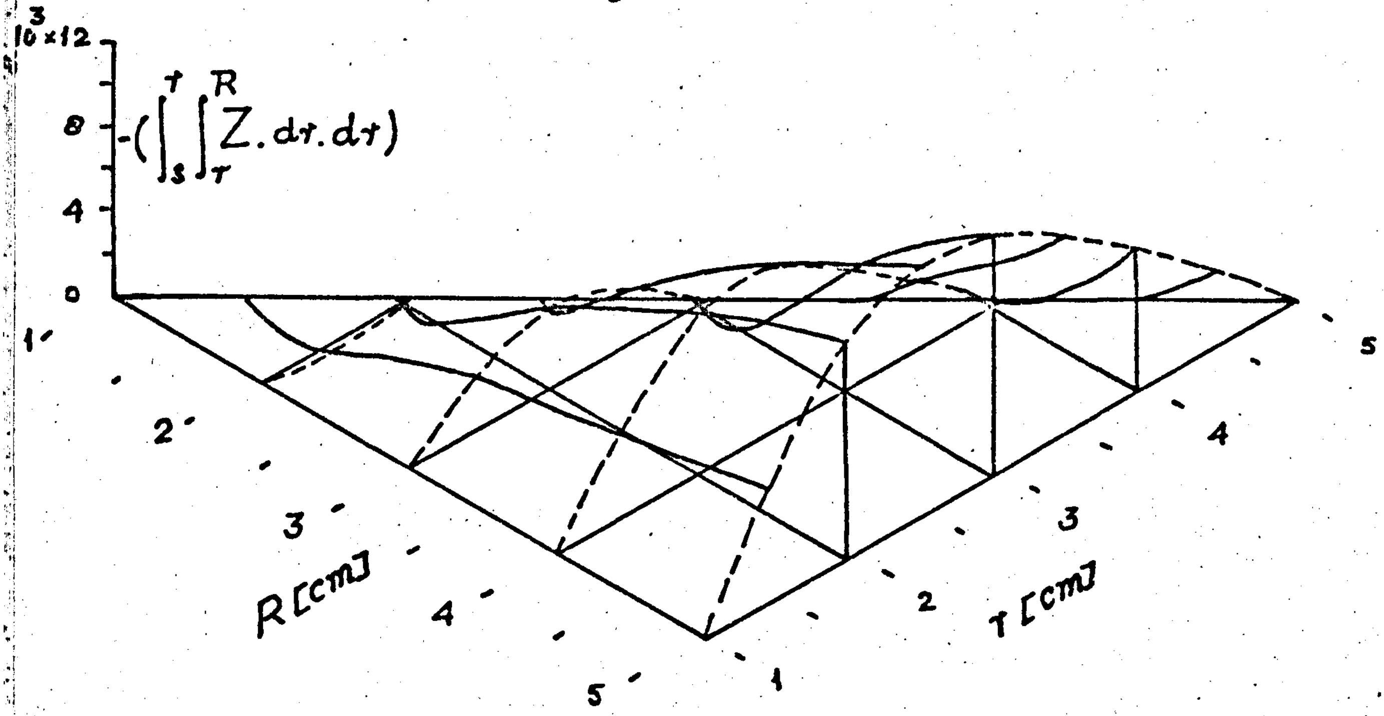
In practice this situation might not arise. If it does, and if the friction at the core is included in this argument, then when this does arise the problem will fit the restricted solution developed here. Moving closer (axially) to the centre of the package will show no further shear forces, no changes in the axial component of the tension, and no axial deformation. Until this stage is reached, i.e. in the end region of the package, there will be shear forces, axial component of the tensions varying from zero at one end of the region to the value calculated here at the other end. Within this region the behaviour will be so complicated that it seems almost impossible to predict it. The introduction of shear force introduces another dimension 'V' (axial co-ordinate) to the whole solution; also at the package ends the wind angle varies; and there is possibility also that slipping will occur between layers if the equilibrium demands a too high value of the shear force.

This view of the deformation process does suggest that the solution derived here is applicable to the central region of the cheese (where the radial deformation was measured) and it also suggests that the axial deformation takes place largely in a different part so that the contradiction between the two sets of experiments no longer exist.

If this explanation holds it implies that the axial extension all takes place in the end regions and therefore this, as a percentage, is greater than the values given. It also implies that radial compression to permit axial expansion takes place in these end regions. This is not apparent, because the package diameter tends to be larger towards the ends because of the change in wind angle due to reversal of the traverse. Another implication of this view of the mechanism of deformation is that the axial deformation should be nearly independent of the length of the package, other things being equal; provided that it is long enough for the central region to exist at all radii. The amount of the axial deformation would be difficult to estimate without carrying out a full two-dimensional solution - that is one in which all variables are functions of three parameters r, R and V. Allowance would also have to be made for slip between the layers where this was indicated.

In the absence of a full-scale solution some slight indication of the sort of axial deformation to be expected might be obtained by looking at three functions as mentioned briefly in Chapter 3. The axial force Z in any layer reduces from the calculated value in the central region to zero at that end. The way in which it reduces is not known nor is the length over which it reduces but it would be expected that the greater the length of the deforming region the greater would be the movement of the package-end. Another indication of elastic extension will be deformation by shear. The shearing effect on any element depends on the value of the shear force F. The total F at any radius is the sum of the Z's in the inner (axially) region outside that radius. The angle of shear will be dependent on the distribution of F in the deforming region - and the greater the length over which the total F is distributed the smaller would be the angle of the





To=20g; x=5cm; spacing=1D; E=100; EY=5000; RO=5cm.

FIG. 5.1

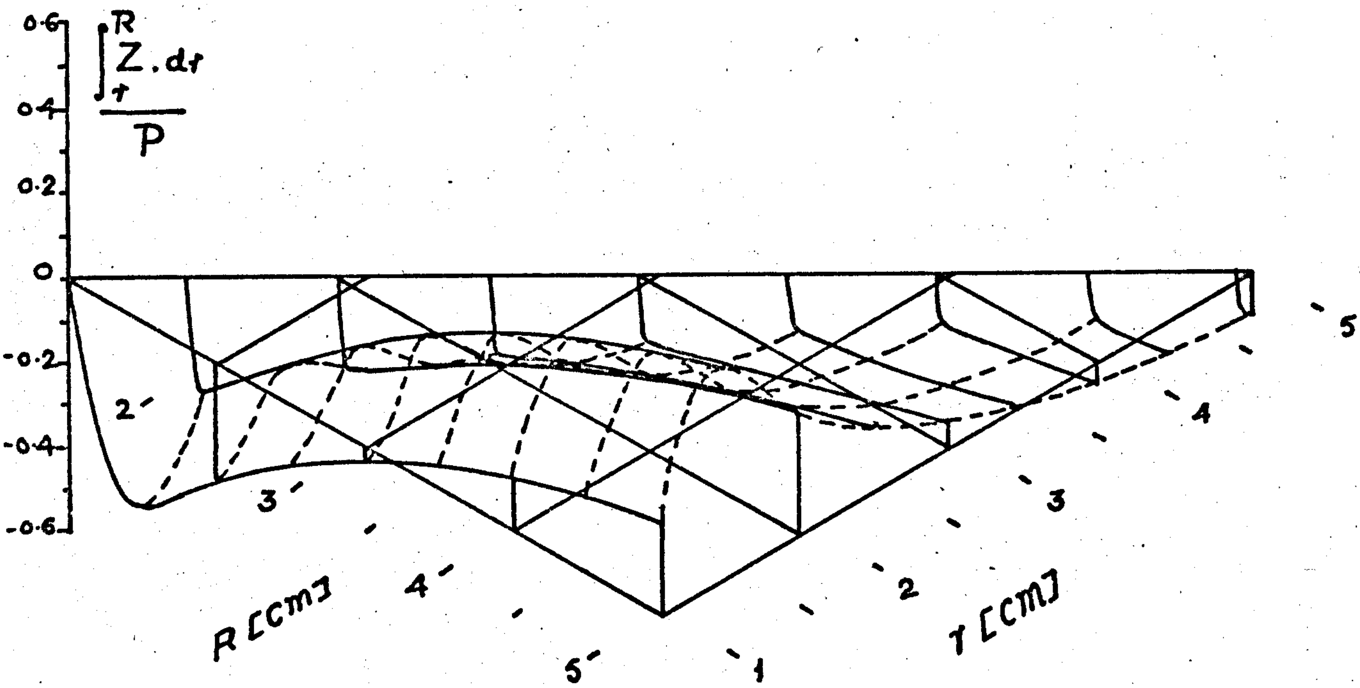
deforming element. The only case for which this effect can be evaluated is that in which F is distributed uniformly over the same length at any r. Then the angle of shear would be expected to be proportional to $\int_r^R Z.dr$. The deformation at any layer due to this would be the total deformation from the core outwards, i.e. $\int_s^r \int_r^R Z.dr.dr$. This however includes some deformation of the inner layers which would have taken place before the layer at r was added, i.e. in the practical case before the gauge was inserted. This has to be subtracted. Finally there is the extension due to slip; the tendency to slip will depend on the ratio of F/P where the total F at any r is $\int_{r=r}^{r=R} Z.dr$; thus the value of $\left(\int_r^R Z.dr\right) / P$ may give some indication of the possibility of extension occurring by slip.

None of these expressions by itself means anything at all - but the extent to which their values vary in a similar manner with r and R suggests the general form of the distribution of axial extension.

Fig. 5.1 shows the value of the integral

$$\int_s^r \int_r^R Z.dr.dr$$

when modulus ratio is 50 for the central region of the cheese for all values of r and R. The curves of this figure are similar to the curves of Fig. 2.40 which shows the measured axial deformation of the cheese. The similarity between the two sets of curves suggests that the above integral gives an approximate picture of the axial deformation of the cheese. The axial component of the force through the face of each



RO=5cm;To=20g; traverse=5cm; space=1D; E=100g; EY=5000g.

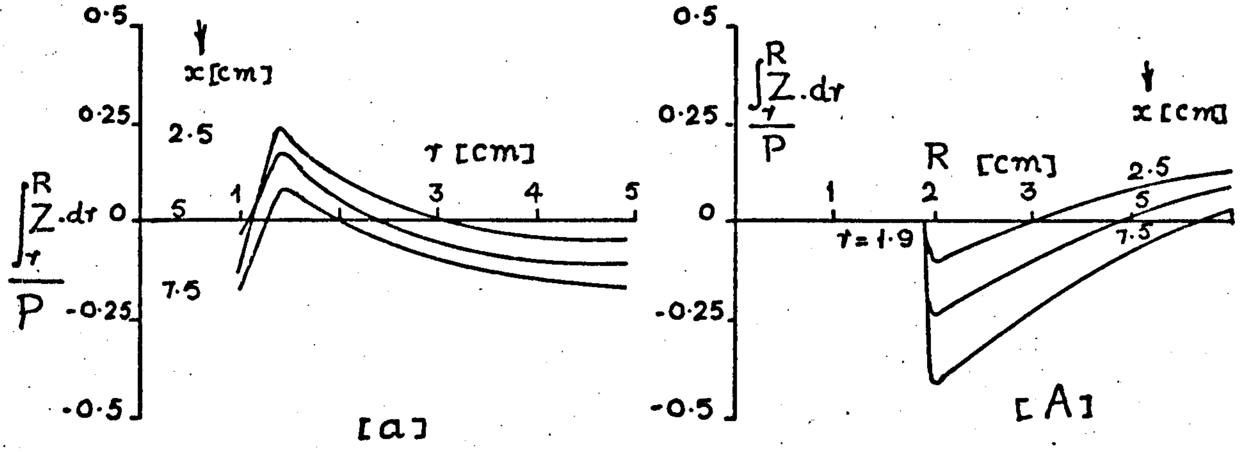
FIG.5.2

element, i.e. Z is also given in Fig. 5.1. This curve shows that the axial force in most of the layers except near the core and near the outside of the package is negative because of negative tension in the yarn and tries to expand the layer.

The tendency for the layer to slip (initially at least - which is probably a reason for pressure transmission through the layers to cause continual compression of the cheese with R) is probably greatest when $\left(\int_r^R Z.dr\right)/P$ is greatest. Fig. 5.2 shows the value of this ratio for all values of r and R. A negative value of the ratio indicates that if slip takes place it would probably result in axial contraction of the cheese and vice-versa. The figure shows higher negative values of the ratio at smaller radii of the cheese which corresponds to the axial contraction of the cheese shown by the value of the integral and by the measured results of axial deformation. Also the value of the ratio at a given radius r is initially negative and high when r = R for small values of R but this value reduces as R increases. This also corresponds to the diminishing initial contraction of the cheese at r = R with R. For larger values of R the value of the ratio with r is generally positive and would indicate axial expansion of the cheese by slip if any. The behaviour of this ratio also seems to be compatible with the measured axial expansion of the cheese and the deforming force which is probably related to the value of the integral.

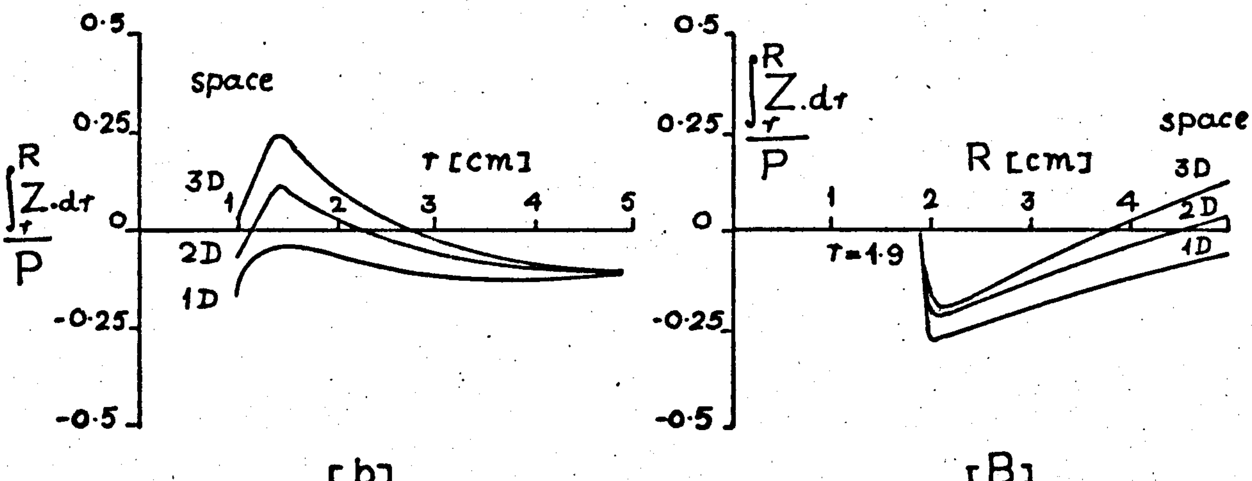
These three factors which show separately various aspects of the tendency of the package to deform axially agree to the extent

Variation of traverse/wind-x



space = 1D.

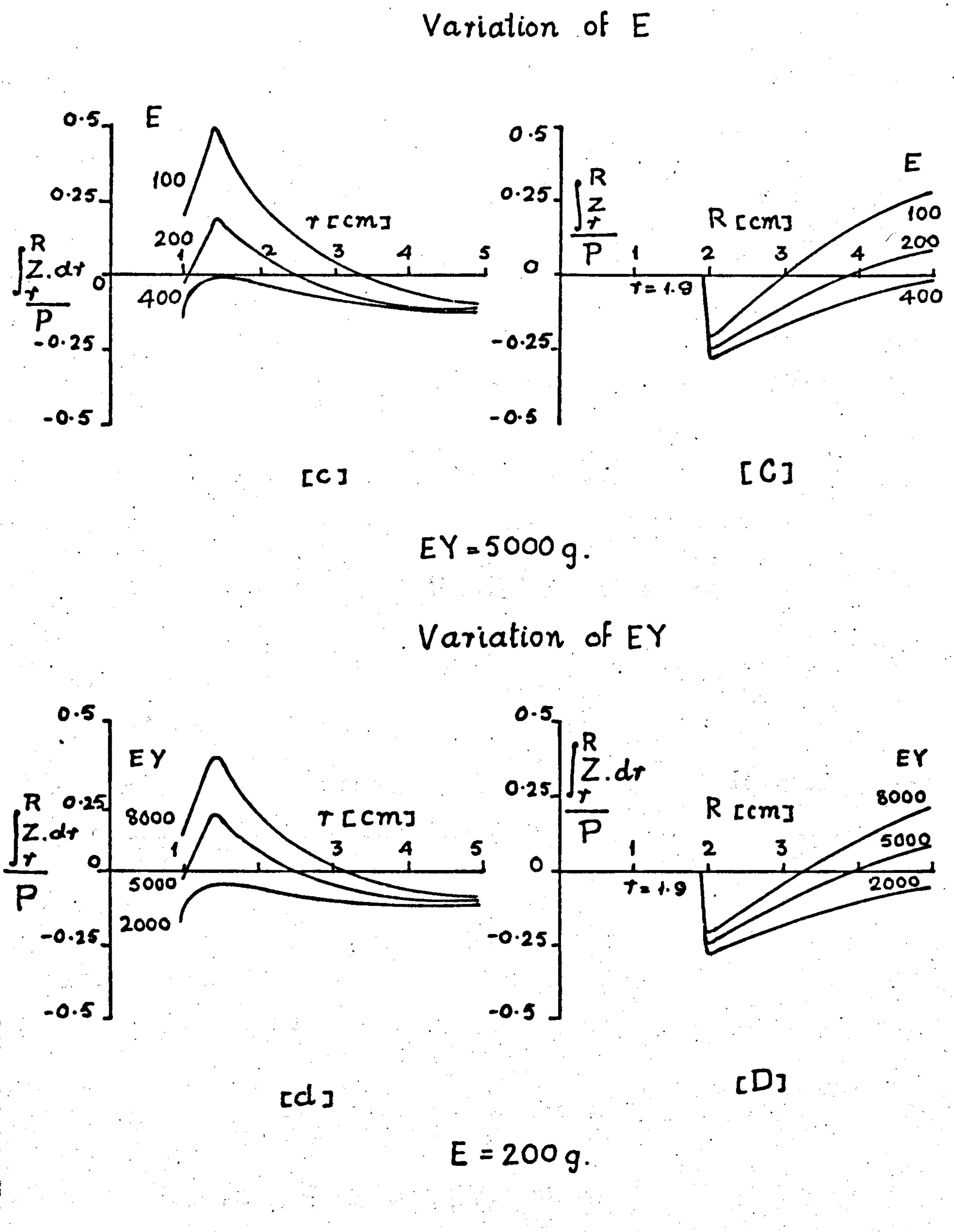
Variation of space



traverse/wind = 5 cm.

$T_o = 20\,g;\ E = 200\,g;\ EY = 5000\,g.$

Fig. 5.3



T_0 = 20 g; traverse = 5 cm; space = 1D; R0 = 5 cm.

FIG. 5.4

that they all suggest axial expansion reaching a maximum at about mid-values of radius and they all suggest a small initial contraction. While none of the expression evaluated has any direct physical meaning because in practice each influences the others, the fact they agree to this extent not only with each other but with the measured deformations supports the picture of the nature of the deformation which has been given. The ratio of total shear force to pressure per element has been evaluated for a number of values of the winding parameters and the results are shown in Figs. 5.3 and 5.4 and in tables of Appendix E. Obviously because of the definition of this term its absolute values are not of interest (and should not be compared with friction coefficients) but its variation with r and R is .

A change in the winding tension in the yarn does not change the value of the ratio as both $\int_r^R Z.dr$ and P change proportionately with the tension when E and EY have constant values. The results suggest that as x and E decrease and as 'space' and EY increase layers in a greater part of the cheese would probably tend to slip due to an effective increase in the value of the modulus ratio. Layers in the other cheese with low values of 'space' and EY and high values of x and E would probably have a higher initial tendency to slip at r as winding proceeds from that radius. This seems to be particularly so when x = 7.5 cm. Another feature of note is that the values of this ratio at a constant r reaches its minimum value very quickly in all cases for a small increase in R from R = r. This probably suggests that the added layer has a strong tendency to slip very soon after its addition. It also suggests that the difficulties of winding due to slip of the added layer would probably increase with the increase in modulus ratio, traverse per wind - i.e. wind angle and the closer spacing of the adjacent wraps of yarn. Another possibility which emerges is that if the cheese could be kept intact at the winding radius by some external means like the pressure of a pressure roller then the cheese would perhaps remain stable as shear to pressure ratio would probably decrease with R.

The value of R for which slip may occur at r would probably depend on the rate of build up of shear and pressure at r with R. This is much influenced by the modulus ratio. The following tables gives the value of the ratio at r = 1.4 cm with R for different values of modulus ratio.

Table 5.1 : $\left(\int_r^R Z.dr \right) / P$ at $r = 1.4$ cm

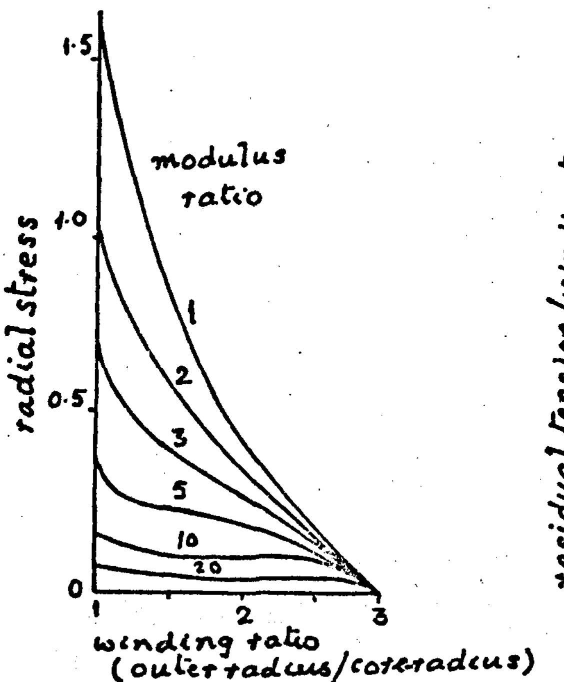
Modulus ratio	R(cm) 1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
50	-0.269	-0.104	0.041	0.179	0.281	0.365	0.434	0.491
25	-0.337	-0.225	-0.125	-0.039	0.031	0.089	0.137	0.178
10	-0.397	-0.315	-0.249	-0.186	-0.145	-0.105	-0.071	-0.042
1	-0.531	-0.448	-0.392	-0.350	-0.317	-0.290	-0.268	-0.249

The value of the ratio changes from -0.531 to -0.249 and from -0.269 to 0.491 with R when the modulus ratio is 1 and 50 respectively.

This suggests that the layers in a cheese of a low modulus ratio yarn would probably have a greater tendency to slip off by contracting axially nearer the outside of the cheese (more likely if slip occurs the very first layer added would probably slip off) and the layers in a cheese of a high modulus ratio yarn would probably tend to slip off by expanding axially at some radius inside the cheese. This seems to be supported by practical experience. The harder types of yarn with lower values of modulus ratio, which are likely to be smoother as well, tend to slip off as they are being wound and the cheese can only be wound with some difficulty; but the cheese once made shows little or no trouble afterwards. The cheese made with softer spun yarns like woollen, worsted, etc., which are likely to have high values of modulus ratio, tend to become unstable when large in diameter.

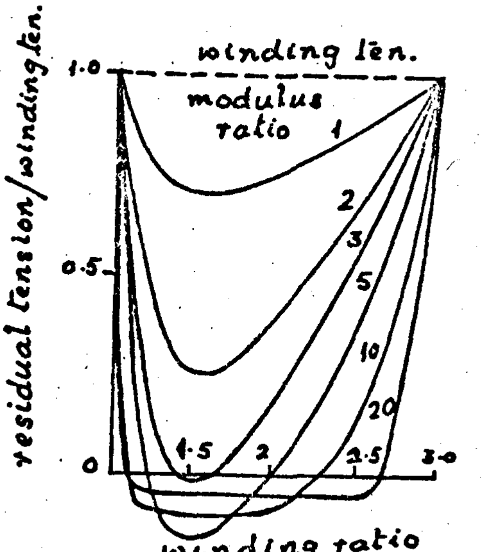
5.2 Central Region of the Cheese

Having suggested how the measured and calculated results can be combined to give a picture of the behaviour of a real package, it remains to discuss in greater detail the meaning of the different results actually obtained. The indication of the radial deformation in the various curves of Chapter 2 are not only supported by the calculations of Chapter 3 but they are also strongly supported by the results of Nakashima[8,9] and others while analysing the tension, pressure, etc. in a warp beam. Their results are reproduced in Fig. 5.5. The curves (a) show the radial distribution of pressure calculated



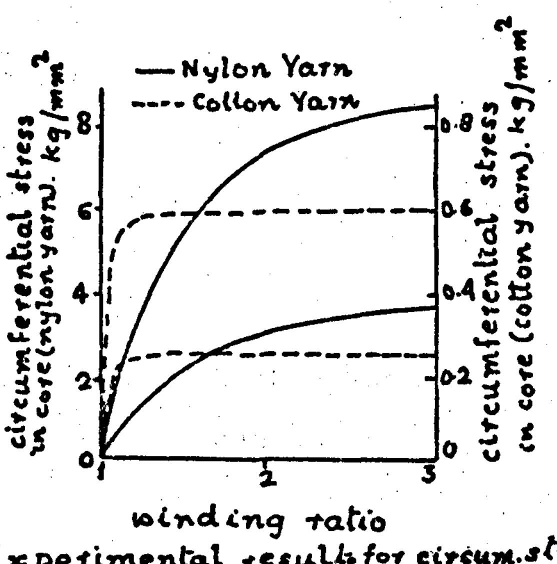
Distribution of radial stress in yarn

[a]



Distribution of tension in the yarn

[b]



Experimental results for circum. stress in core (winding with constant ten.)

[c]

Results of Nakashima and others for a yarn beam.

FIG. 5.5

theoretically in the yarn wound with constant winding tension on a beam of outer radius three times the radius of the core for different values of the modulus ratio. It is evident that with the increase in the modulus ratio the pressure of the added layer is supported by the yarn layers immediately beneath it. These are strikingly similar to the results obtained in the present work and shown by the curves (a) of Fig. 3.22. The results of the residual tension distribution shown by curves (b) of Fig. 5.5 are similar to those shown by curves (c) of Fig. 3.23. However the magnitudes of the differences in pressures with different modulus ratio are larger in their case and is evidently due to the difference in the structure of a parallel wound beam and a cross wound package.

The curves (c) of Fig. 5.5 show the values of circumferential stress in the core determined experimentally by them. Their results for cotton yarns are qualitatively similar to our results of radial deformation of the cheese. However when nylon yarn is wound the results are different; the curves then do not become flat so early and sharply and the circumferential stress in the core continues to increase with the outer radius. They estimate the value of modulus ratio of cotton and nylon yarn as 33 and 6 when winding tension in the yarn is 30g and 29 and 5 when winding tension in the yarn is 50g. The peculiar behaviour of the circumferential stress in the core with the cotton yarn is due to the high value of the modulus ratio of the cotton yarn. A similar behaviour can be expected from a cheese wound with cotton yarn and is indeed shown by our results.

The measured results of radial deformation give resistance changes of between 0.3% to 0.4% at the gauge radius of 2 cm for winding tensions of about 20g. The computed results for the compression of the cheese at the same radius in which the modulus ratio is a function of the pressure in the cheese, the pressure of the pressure roller and tension in the yarn is about 0.6%. A similar value of U is also available when the modulus ratio has a constant value of 50 (5000/100). This suggests a gauge factor of between 0.5 to 0.7. A similar value of the gauge factor was suggested by the test to measure the axial deformation of the cheese (page 48).

The reason for this type of behaviour, namely the curve of U becoming flat with R, would be apparent by considering a trellis like layer which is prevented from expanding sideways. If the trellis is not allowed to expand sideways then the pressure on it would have to shorten the length of the members - which is difficult as the members are difficult to compress longitudinally (due to high value of EY). Therefore the pressure would be supported by the trellis and the pressure would affect only slightly the trellis (or layer below). The resistance to the expansion of the trellis is provided by the frictional forces. Now as the cheese is further built up the pressure does not reach the layer (or trellis) at r as it is supported by the layers (or trellises) above. Therefore the compression of the package at r stops which corresponds to the flattening of the curve. If the yarn was of a low modulus ratio then the picture of radial deformation would probably be different and the compression of the package at r would continue with R.

Considering that the analysis is applicable to the central region of the cheese some conclusions can be drawn about the behaviour of the central part. One factor which influences the package most is the modulus ratio of the yarn, i.e. the ratio of the longitudinal modulus of the yarn to its lateral modulus. A package made with a yarn of high modulus ratio like cotton, woollen, worsted, etc. would behave very differently from the package made with a yarn of low modulus ratio like filament nylon. In the former the pressure inside the package would be low and except at the very outside where the pressure falls off rapidly the pressure variation within the package would be small. The yarn is likely to acquire negative tension and the compression at a given radius is caused by a small increase in the outer radius of the package from that radius. In the latter type of cheese the pressure in the cheese continues to build up with the outer radius of the package and its value at the core is considerably higher than in the former kind. The pressure within the package falls continuously from the core to the outside and the difference is considerable. The tension will not reduce so much as in the former case and the yarn is not likely to acquire negative tension. The amount of the compression of the package depends on the individual values of E and EY and reduces with the higher values of both. In both the cases the compression is likely to be small.

The residual tension in the yarn, like pressure, depends considerably on the modulus ratio of the yarn. With a high modulus ratio of yarn the residual tension in the yarn falls sharply with r near the core and quickly becomes negative showing the yarn in compression. After the initial fall it changes little with r till it rises again near the outside of the package. The tension at a constant r is affected only by a small increase in R from R = r. With a low modulus ratio of yarn it falls gradually with r till it starts rising from near mid radius of the cheese. Total changes in the tension are comparatively smaller and is not likely to become negative. The tension in the yarn at r shows a continual progressive decrease with R.

Pressure, compression, etc. tend to increase with the winding tension in the yarn but the axial deformation does not seem to show any increase. With yarns of high modulus ratio the increases are proportionately less due to increase in the working value of E because of higher pressures the yarn becoming harder to compress. This effect would be less marked with yarns of low modulus ratio, particularly those which have high value of E, because the value of E in those cases would be less sensitive to pressure.

The greater spacing between the adjacent wraps of yarn reduce the number of pressure bearing crossing points more than the number of ends in the element and this effectively reduces the value of E and the cheese consequently show higher values of radial and axial deformation.

A wind angle of 45^{o} would give maximum number of crossing points and this type of cheese should show maximum resistance to radial deformation. A reduction in the value of wind angle would result in a fewer crossing points and therefore in a lower effective value of E and the cheese would show a higher compression. The effect is only slight for reasonable changes in the wind angle. For a large wind angle the added layer would tend to show a strong tendency to slip.

5.3 Effects of Pressure Roller

For obtaining a solution by the modified theory of Chapter 4 in which E is proportional to P it was found essential to use some initial value of E in order to obtain a solution not critically dependent on the thickness of the added layer at R, i.e. to add a constant to the value of E caused by the pressure of the added layer at R. As thickness of added layer reduces its pressure also reduces and therefore the value of E at r = R reduces and that of modulus ratio of yarn increases. This results in very rapid changes in tension and it is lost very quickly. These changes are confined to the very outside of the package leaving its inside unaffected. Due to these rapid changes the working of the program becomes difficult. This situation is then similar to the difficulty experienced in actual winding of yarns (particularly of high modulus ratio). To ease the winding process some additional pressure is applied by some external means usually by a pressure roller. The pressure of the roller by pressing the yarn at the time of winding - which is in effect equal to raising the value of E and thereby lowering the value of modulus ratio of the yarn which has already passed the highly compressible stage when put on the cheese - helps the yarn to retain tension longer and thus facilitates winding. This pressure of the roller appears in the computer solution as initial value of E and likewise makes the solution easier and it also sets the lower limit of E. This is a positive feed back process which suggests a critical pressure below which package cannot be wound with stability - and a critical layer thickness below which the computer solution will not converge rapidly.

The effect of this is twofold; firstly it lowers the modulus ratio of the yarn and prevents very quick loss of tension in the yarn and secondly it increases the outer part of the cheese affected by the added layer and which in turn promotes greater growth of pressure inside the cheese and therefore lower values of modulus ratio. This seems to be what actually happens in the cheese when wound under the pressure of the pressure roller.

When winding yarns of low modulus ratio, which tend to slip off near the outer radius of the cheese, the additional pressure would in effect reduce the shear to pressure ratio and keep the cheese stable at the winding radius till the cheese itself becomes stable at that radius due to the fall in the value of the ratio at that radius with R as shown before. The value of the ratio at r = R decreases as R increases and in a completed cheese of average radius there should not be any tendency to slip at the outside of the cheese.

From the slipping point of view also the additional pressure helps in the formation of the cheese of a yarn of high modulus ratio by reducing the effective value of the modulus ratio. As stated in the discussion of radial deformation the modulus ratio of these types of yarn is comparatively more sensitive to pressure and the higher starting value of E due to additional pressure permits greater values of E and P inside the cheese. The overall effect would be to reduce the value of shear/pressure ratio and probably the cheese would remain stable when large. Increasing the pressure this way should be more effective and better than by increasing the pressure by increasing the winding tension in the yarn. A high winding tension in the yarn may strain the yarn, cause excessive breakage of the yarn and the increase in the value of E would, to some extent, be compensated by the increase in the value of EY due to higher tension in the yarn.

5.4 General Comments

The theoretical analysis developed in the present work is not applicable to a random wound cheese because of the basic inherent differences in the two types of packages. In the random wound cheese the wind angle remains constant with r but the number of threads in a given axial width changes with r, therefore the number of crossing points in the element - which does not vary with r for the precision wound cheese - does vary in this case. Also the space between the adjacent wraps of yarn varies considerably with the radius and the formation of the layer is not there. Also the theoretical analysis developed is not applicable to a conical package, precision wound or random wound, because of the different geometrical shape.

The theory in the form set out here is also not applicable to a parallel wound package. In that case the contact between the wraps of yarn of radially adjacent layers in an undeformed state is a line contact and changes to a surface contact in the deformed state. This surface would vary with the radius of the cheese. In the present case the contact between the wraps of yarn of radially adjacent layers is nearly a point contact in an undeformed state and changes to a surface contact surrounding the point in the deformed state. However the area of contact would still be nearly independent of the radius but would depend on the number of crossing points which is independent of r. The analysis is based on the independence of the number of crossing points of radius and therefore it is not possible to derive the equation for parallel winding as a particular case of the present analysis. Strictly in this analysis there should be a small allowance for the change of modulus of crossing point with angle of crossing; the error introduced by omitting this is likely to be very small for the range of angles met in practice. For very small angles it would increase.

The mechanism for the deformation of the cheese put forward is only a plausible one from the results obtained so far. The problem turned out to be much more complex than it was thought to be at the start of the project. The present analysis of a very restricted type of cheese was considered to be as a necessary inevitable step to be able to solve the much more complex realistic case. In fact it was not even hoped that the results of this case might be directly applicable to the central region of the cheese which they seem to be.

In summarizing the work it should be kept in view that the theoretical solution attempted is that of a very restricted type of cheese, namely in which no axial deformation - elastic or by slip of layers - is allowed. In practice this type of cheese is not likely to be met. The values of the physical properties of the yarn like diameter, E, etc. are only approximate and therefore the results would also be only approximate, but the difference from results with exact values of the yarn properties would only be small and qualitatively these results would remain the same in both the cases. However, the results strongly suggest that this restricted solution is probably applicable to the central region of the cheese. The confirmation of the mechanism of deformation of the cheese suggested should be possible by a series of practical tests to measure the radial deformation of the cheese at different places corresponding to different regions of the cheese. The method developed and used for the indication of the radial deformation seems to be capable of showing the radial deformation qualitatively but in the absence of the calibration of the gauge, which appears to be impossible, quantitative measurements of the radial deformation of the cheese cannot be made; and in the absence of quantitative results the element of doubt in the difference of the magnitudes of radial deformation at various regions and radii would always remain. Therefore it would be advisable to develop a new direct method for the measurement of the radial deformation before these tests are attempted.

Another check can be made by measuring the axial deformation of the packages of different lengths under the same winding conditions. If the central region exists in the packages of different lengths and the end regions, which are supposed to contribute to the axial deformation, are of the same length in each of them then the axial deformation of the packages of different lengths would be of the same order because the winding conditions are the same. However these results would be based on the assumption stated above and also that the end regions are not affected by the change in the length of the packages. If they are then the results would not be comparable. Also it could be possible that the central region may not exist in packages of smaller widths than the one tested in the present project.

The object of measuring the value of E was to determine the nature of the relationship between E and the pressure and hence this relation is only approximate. However it would be useful to determine the value of E of the yarn under different tensions to see how the value of E is affected not only by the pressure but by the tension in the yarn. For this the method for the measuring of the value of E would need modification. The method of Nakashima[9] is not useful here as that method of measuring the value of E restrains the yarn to expand sideways which restraint is not there in the cheese. Their value of E would be higher because of this restraint and consequently the value of the modulus ratio would be lower. The increasing gap between the adjacent wraps of yarn with radius due to a reduction in wind angle might cause slightly smaller value of E with r because due to wider gap bending of yarn, i.e. crimping, would be easier. However the effect would only be small.

Though the cheese has not been solved completely useful results have been obtained. From the winding point of view it emerges that the pressure roller, helpful in winding all types of yarn, has its effect not directly through the pressure it applies but through the effect of this on modulus ratio. It is to be preferred to a high winding tension for making a firm and a stable package. High winding tensions could result in a denser package but not necessarily one in which the yarn layers did not slip. The slip of the yarn at the winding radius can be eliminated by reducing the wind angle. Again the increase of winding tension in the yarn is not likely to help in this case.

The other useful aspect is the information regarding pressure and tension inside the package and its variations, which are important in the subsequent processes like weaving and package dyeing.

√The tension variations in pirns were important enough to provoke the study of Catlow and Walls. In cheeses and cones the radius ratios are greater and because of the nature of contact between yarn layers the effective modulus ratios are probably greater. Also the increasing use of these packages for direct weft supply means that the variations in the package are more likely to result in variations in the cloth. The tension in the yarn in packages of cotton, woollen, worsted yarn, etc. would be, except at the core and near the outside, fairly even (though the yarn may be in compression) and therefore these packages could be comparatively of large diameters with high ratios of outer radius to core radius without introducing much variation in the ratio of residual to winding tension. On the other hand packages of filament yarns like nylon should not be large because of large tension variation which increases with the outer radius of the package. Also a great amount of pressure would be exerted on the core in the case of these materials.

A similar situation also exists as regards pressure inside the packages which would probably matter in package dyeing. Again the packages of softer spun yarn like cotton, etc. would have low pressures inside with little variations throughout except near the outside of the package. On the other hand a package of filament yarn like nylon would have a great pressure variation inside the package with comparatively much higher values of the pressure inside the package and would probably be much more difficult to dye successfully.

Finally one aspect of such studies which has not been mentioned but might become increasingly important would be the winding of reinforcing yarns in the making of cylindrical shells of plastics materials.

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APPENDIX A

COMPUTER PROGRAMS

A.1 Computer Program 15 for Solving the Equation (3.14)

Program 15 is written in KDF 9 Algol to solve the equation developed in Chapter 3 using constant values of E and EY. The symbols used in the equations cannot possibly be used in the program and hence an equivalent notation is used for the program. The program is accompanied by its notation, necessary explanations and flow diagram.

A.1.1 Notation

The symbols are given in the order in which they occur in Program 15. Only those symbols are given in the notation which are either additional or have been changed for the program. The remaining symbols which correspond to the theoretical notation are used as such.

T - is To, the winding tension in the yarn.

sr - is the radius at which the calculation starts. It is normally the core radius.

b - = a x a.

R - = (R + dR).

m - = $(r^2 + a^2)$

K1, K2, K3, K4, K5, K6 - are constants used in the calculations for solving the equations and are evaluated before the calculations start.

cdu - = $\frac{\partial u}{\partial r}$ at R. Its value is available from the boundary condition.

sdu - is the guess or the estimation of the value of $\frac{\partial u}{\partial r}$ at the core radius s.

du - = $\frac{\partial u}{r}$.

d2u = $\frac{\partial^2 u}{\partial r^2}$.

au, asu - are the values of u and U at the mid radius of a layer and are obtained by averaging the values of these at the two end radii of the layer.

Su - = U.

c - is a small number to specify the permissible difference between two successive values of u at a given r.

d - is a small number to specify the permissible difference between the values of $\frac{\partial u}{\partial r}$ at r = R and cdu.

SZ - is $\int_s^R (-\frac{Z}{R}.dR).dr = \int_s^R z.dr$

z - = W.

Z - = $\frac{\partial Z}{\partial R}.dR$ = z.

g - is a small number used to vary the estimated value of $\frac{\partial u}{\partial r}$ at s.

C, C1, D1 - are used in the formula to interpolate the value of sdu from the values of $\frac{\partial u}{\partial r}$ at r = R and r = s of the last two solutions.

angle - is used to change the wind angle by changing x.

space - is used to alter the spacing between the adjacent wraps of yarn.

F1, F2, F3, F4, F5, F6, F7, F8 - are integers to denote the formats used in the program.

k - is a counter to number the steps of r or layers with r.

p - is a counter used in conjunction with 'h' or 'k' or both to call the value of a variable at any r.

ct - is a counter to count the number of repititions of calculation for one step or r.

h - is a counter to count and number the steps of R.

l - is a counter to count the number of times the calculation is repeated to solve the equation for the addition of a layer at R.

Arrays, preceded by letter T or P before the symbol of the variable, have been used to retain the values of the respective variable at all values of r.

$$SZR - = \int_r^R Zo.dr$$

ZR - $= Zo.$

QR - $= Qo.$

pr - $= \frac{\partial P}{\partial R}.dR = p.$

SAZ - $= \int_r^R \frac{\partial Z}{\partial R}.dR.$

ZZ - = Z, this appears only in the write statements of the program.

SSAZ- $= \int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr.$

SZZ - $= \int_r^R Z\,dr,$ this appears only in the write statements of the program.

Sq - $= \int_r^R \frac{\partial Q}{\partial R}.dR.$

St - $= \int_r^R \frac{\partial T}{\partial R}.dR.$

Spr - $= \int_r^R \frac{\partial P}{\partial R}.dR$

A.1.2 Structure of the Program

The program can be conveniently divided into four parts. The first part consists of declarations in which real, integer and array variables, procedure 'trapezium', format declarations are made. The procedure trapezium is used to solve the equation from the core radius to the outer radius for one solution due to the addition of a layer at R. This procedure is explained later. The data from the data tape is also read in the program.

In the second part of the program the values of constants, e.g. a, b, K, K1, K2, etc., used later in the calculations are worked out. These values appear in the output in a tabulated form.

In the third part the first layer is added to the core. As the core is incompressible u and U at the core radius s are zero. The values of ZR[1], QR[1], pr[1], Spr[1] and cdu are worked out and appear in the output.

The fourth part of the program is the main part and it appears under a loop under <u>for</u> statement. This loop causes a layer of thickness dR to be added at the outer radius R of the cheese. The equation is solved for the addition of this layer, then the next layer is added and the equation due to the addition of this layer is also solved. The process is repeated till the cheese is built up to the required value of R, i.e. RO. This part can be divided into five sub-parts.

The first subpart calculates the values of ZR[h], QR[h], SZR[h], cdu, etc. after advancing the counter 'h' by 1. The second subpart under the label "diff press" finds out the correct solution of the equation due to the addition of the layer at R. This uses the procedure "trapezium" to solve the equation from the core radius s to the outer radius R and the later portion of this subpart interpolates the value of du at the core.

The procedure trapezium once called integrates the equation due to the addition of a layer at R with an assumed value of du (guessed or interpolated from the results of previous trial solutions) at s. From the values of u, du, d2u at the core (or any other step of r[k]) the values at the next step of the radius, i.e. r[k + 1], are interpolated by Euler's modified method. The values are then averaged repeatedly by the loop labelled 'correction' and the integration moves to the next step of r only when the difference between the two successive values of u at the same radius r[k + 1] is small and less than the limit set for it. The final values of u, du, etc. at r[k + 1] are assigned to T arrays by k, which had moved up by 1 before the assignment. This process of solving the cheese layer by layer is controlled by the loop labelled 'layer' and allows the calculation to proceed up to r = R. During the integration the value of Su at each step of r is called with the help of the counter 'k' which denotes the step of r at which the calculation is done.

The loop 'diff press' ensures the correct solution of the equation due to the addition of a layer at R. As the program goes out of the procedure trapezium the value of du at R is compared to cdu, the correct value of du at r = R according to the boundary condition. If the two values differ more than the permissible difference the program goes back to 'diff press' loop with a new value of du at s. This new value of du at s, i.e. sdu, is for the first time another guess of the value of du at s. For the second and the subsequent times the value is an interpolated one obtained from the results of the last two solutions. Exit from the 'diff press' loop is only possible when the condition of the difference of du at r = R and cdu is satisfied. The counter 'l' counts the number of the solutions; i.e. the number of times the program enters the diff press loop to obtain the correct solution of the equation due to the addition of a layer at R.

In the third subpart the values of the variables at each step of r available from the final correct solution are transferred from T arrays to P arrays in order to retain them and T arrays are then free and available for the next solution. In the next part the values of the remaining variables are calculated and these are assigned to respective arrays. The final subpart causes the values of the required variables at every step of r for every step of R to be output in a tabulated form.

A.1.3 Features of the Program

(a) Use of Arrays

The cheese has been divided into a number of layers. During the calculations and during the output of the results it is necessary to call the value of any desired variable at any step of

r. To enable this the values of the variables are assigned to arrays; one array for one set of the values of one variable. There are two types of arrays preceded by letters T and P. T arrays are intermediate arrays and are used during trial solutions. P arrays are final arrays to retain the values permanently after each final integration of the equation due to the addition of a layer at R. T arrays assign the values of the variables from the final correct solution to P arrays. These values are then retained by P arrays and T arrays are then released to be used again for the next solution due to the addition of the next layer at R.

(b) Commulative Totals

For any commulative total of a variable which is the integration of its value at a given r as R increases from r = R, e.g. $\int_r^R \frac{\partial Z}{\partial R}.dR$ (= SAZ), the following type of statement is used

Sq[p] := Sq[p] + q[p] ;

here 'p' refers to a particular layer (or step of r, i.e. r[p]). Sq[p] on the right hand side was the value of Sq at r[p] before the addition of the layer at R and q[p] is the change in Q at r[p] due to the addition of the layer at R to be added to it to give the new value of Sq[p] after the addition of the said layer.

For any commulative total of a variable which is the integration of the variable with respect to r for a given R, e.g. $\int_r^R$ Zo.dr (= SZR), the following type of statement is used.

SZR[h] := SZR [h - 1] + ZR [h] ;

here 'h' refers to the layer added. SZR[h - 1] is the previous commulative total before the addition of the layer to which ZR[h], the value of Zo of the layer added at R, is to be added.

(c) Calling a Particular Value

Any change in the value of any variable at a given r due to the addition of layer at R as also its integral with respect to R can be called directly by its respective array having a counter of the same value as layer (or the step of r) whose value is to be called. Variables r, u, du, d2u, Z, q, t, pr, Su, SAZ, Sq, St, Spr, Q and ZZ come in this group.

Any value of the integral of a variable with respect to r at any r for a given R is called by the following type of statement

SZR[h] - SZR[p - 1] ;

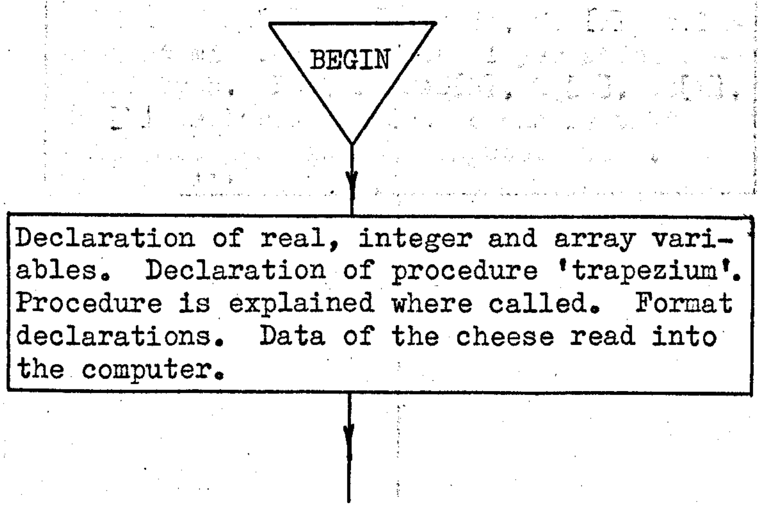
here 'h' is the number of the outermost layer and SZR[h] = $\int_s^R$ Zo.dr, 'p' is the number of the layer (or the step of r) at which the value of SZR, i.e. $\int_{r[p]}^R$ Zo.dr, is to be called and SZR[p - 1] = $\int_s^{R = r[p]}$ Zo.dr. Variables SZR, SSAZ and SZZ come in this group.

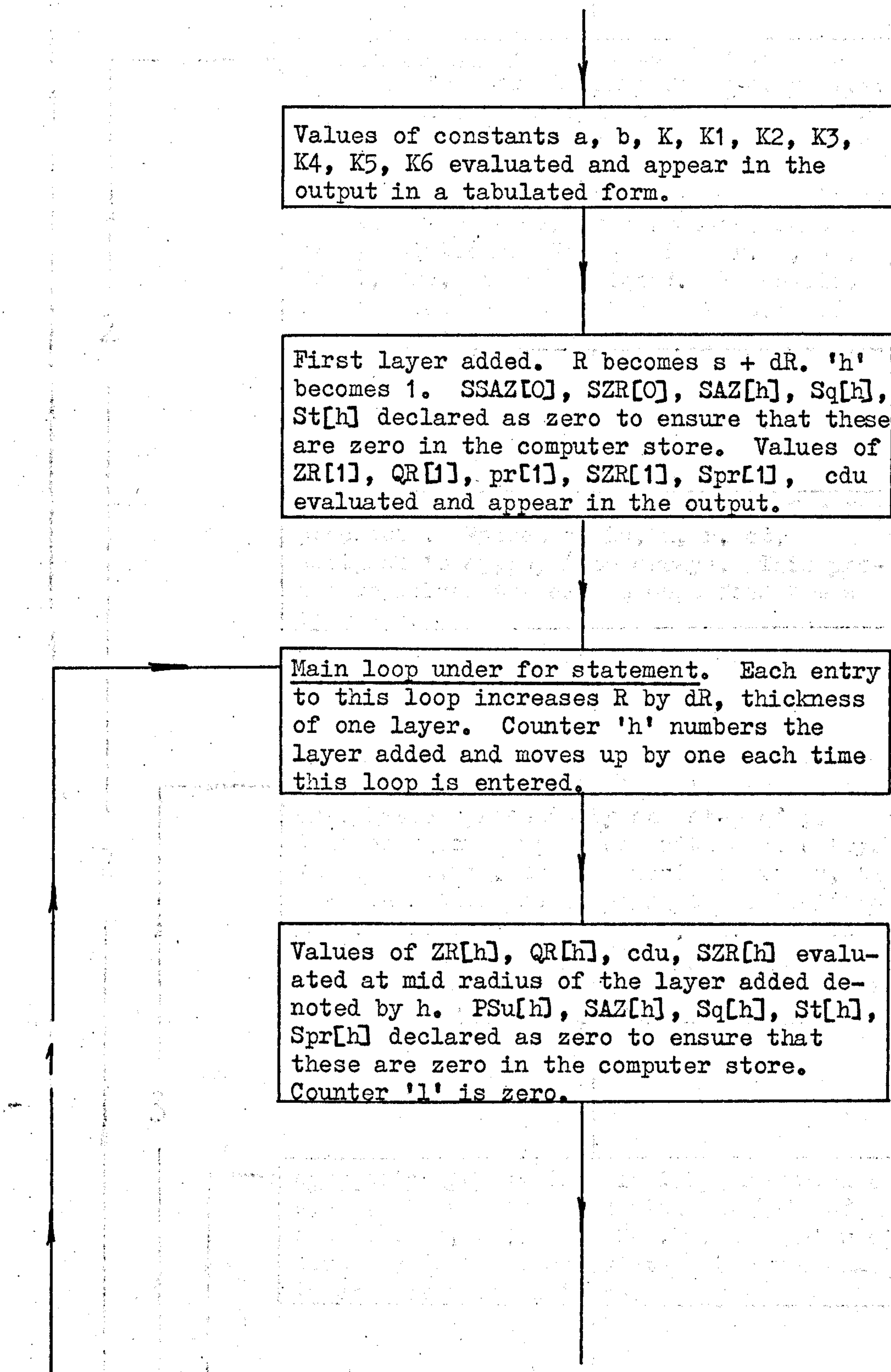
(d) Change of Wind Angle and Space Between Adjacent Wraps of Yarn

A change in the wind angle is accomplished by assigning a different value to 'angle' which changes the values of 'x' and z(=W) read in the program from the data tape. The value of x and z read in the program from the data tape are those of a standard size of element for which $\phi = 2\pi/5$. A value of 1 of 'angle' gives the standard size of the element. After the completion of the calculations Q and Spr (= P) are multiplied by angle in the write statements and are then for an element of standard size.

The maximum value of 'space' of 1 gives a spacing of one diameter between the adjacent wraps of yarn. To increase this spacing the value of 'space' is reduced, e.g. a value of $\frac{1}{2}$ gives spacing of two diameters between the adjacent wraps of yarn.

A.1.4 Flow Diagram for Program 15



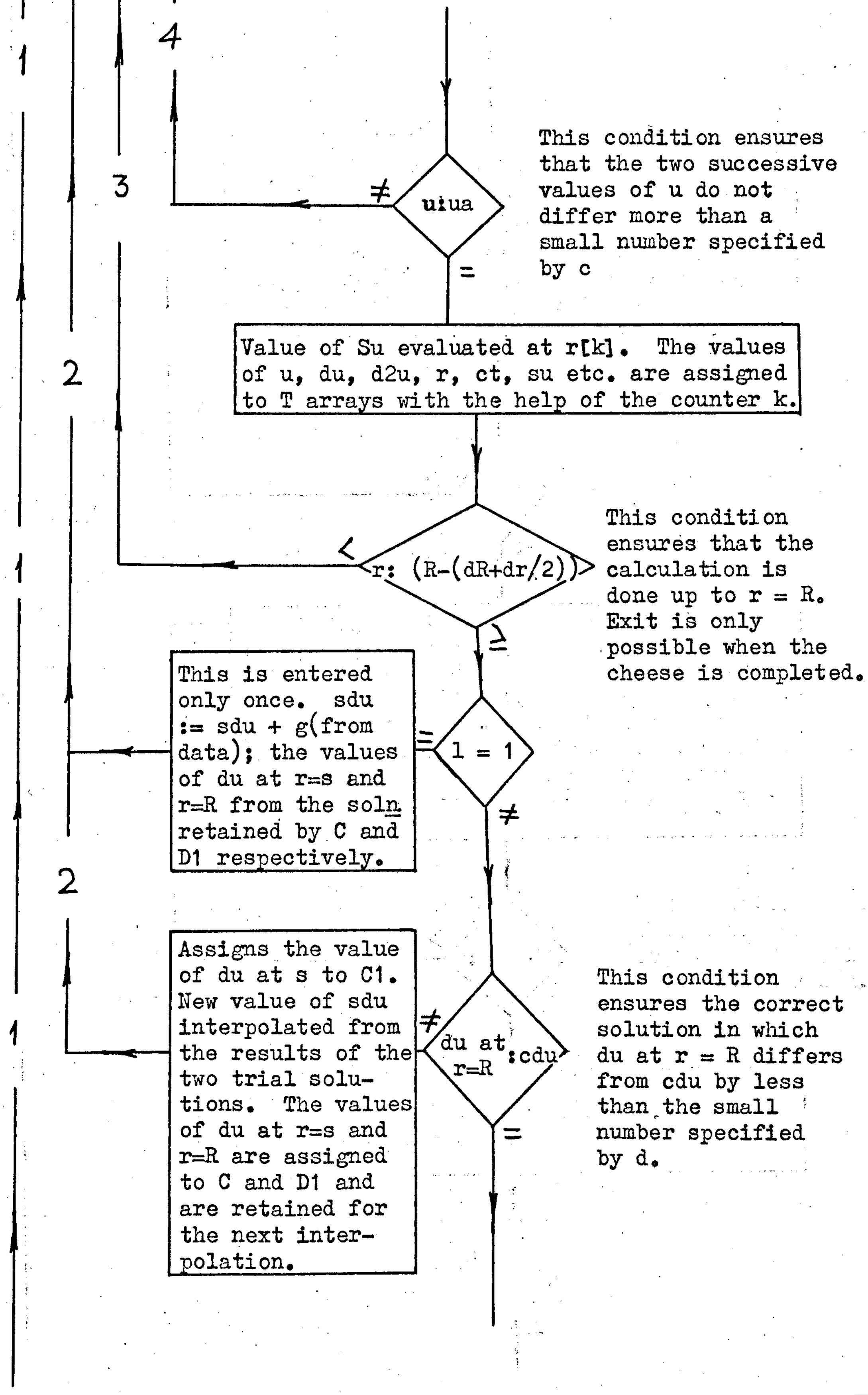

Values of constants a, b, K, K1, K2, K3, K4, K5, K6 evaluated and appear in the output in a tabulated form.
First layer added. R becomes s + dR. 'h' becomes 1. SSAZ[O], SZR[O], SAZ[h], Sq[h], St[h] declared as zero to ensure that these are zero in the computer store. Values of ZR[1], QR[1], pr[1], SZR[1], Spr[1], cdu evaluated and appear in the output.
Main loop under for statement. Each entry to this loop increases R by dR, thickness of one layer. Counter 'h' numbers the layer added and moves up by one each time this loop is entered.
Values of ZR[h], QR[h], cdu, SZR[h] evaluated at mid radius of the layer added denoted by h. PSu[h], SAZ[h], Sq[h], St[h], Spr[h] declared as zero to ensure that these are zero in the computer store. Counter 'l' is zero.

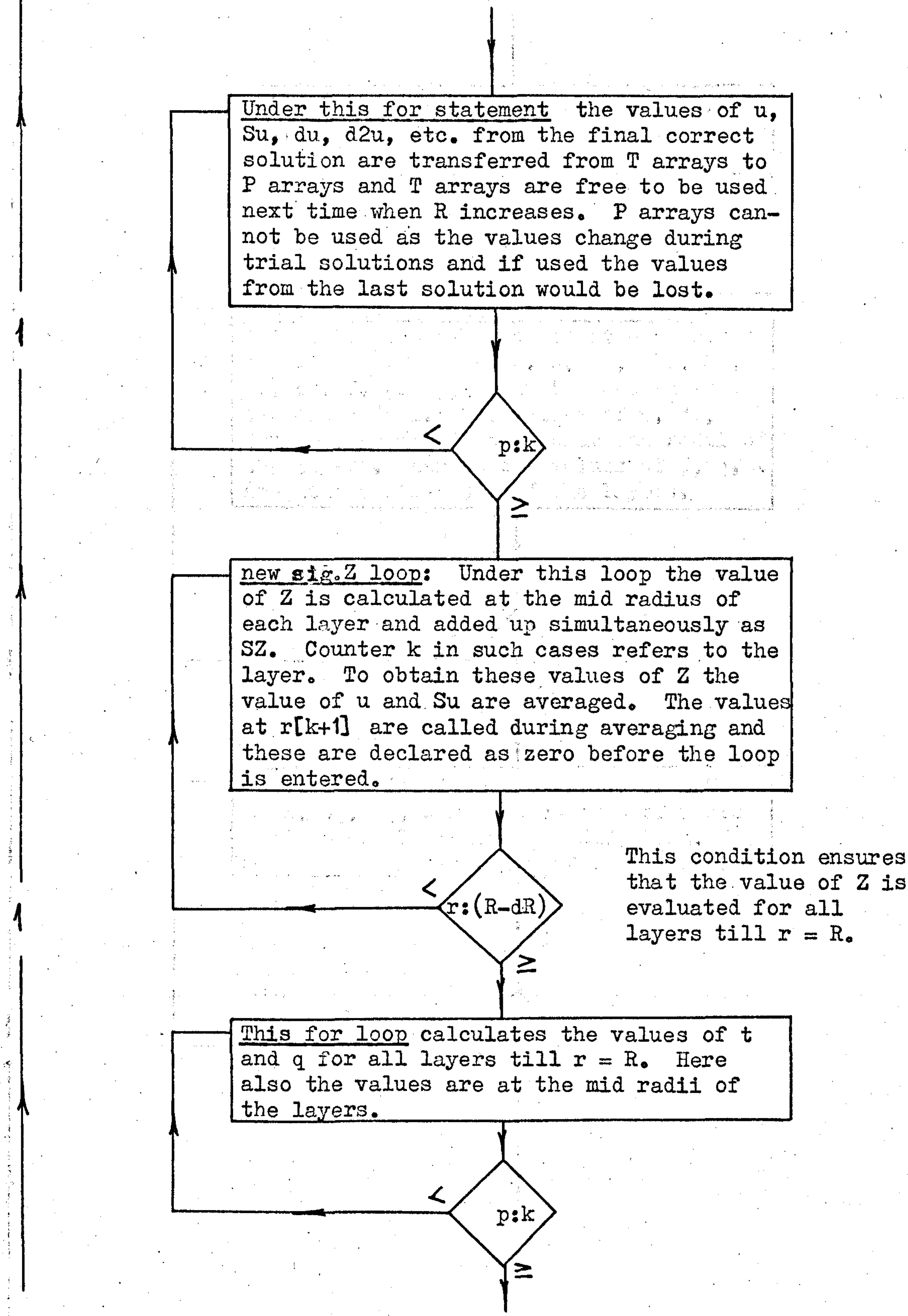
<u>diff press loop</u>: Entry to this loop is always with a fresh value of du at s, i.e. sdu, and the counter l moves up by 1 for each entry. sdu has a guess of the value of du at s for the first two entries and for the subsequent entries it has an interpolated value of du at s from the last two trial solutions. Values of k, r, u, Su, m, du, l, d2u, ct are assigned. These also include the values of the parameters of the procedure.

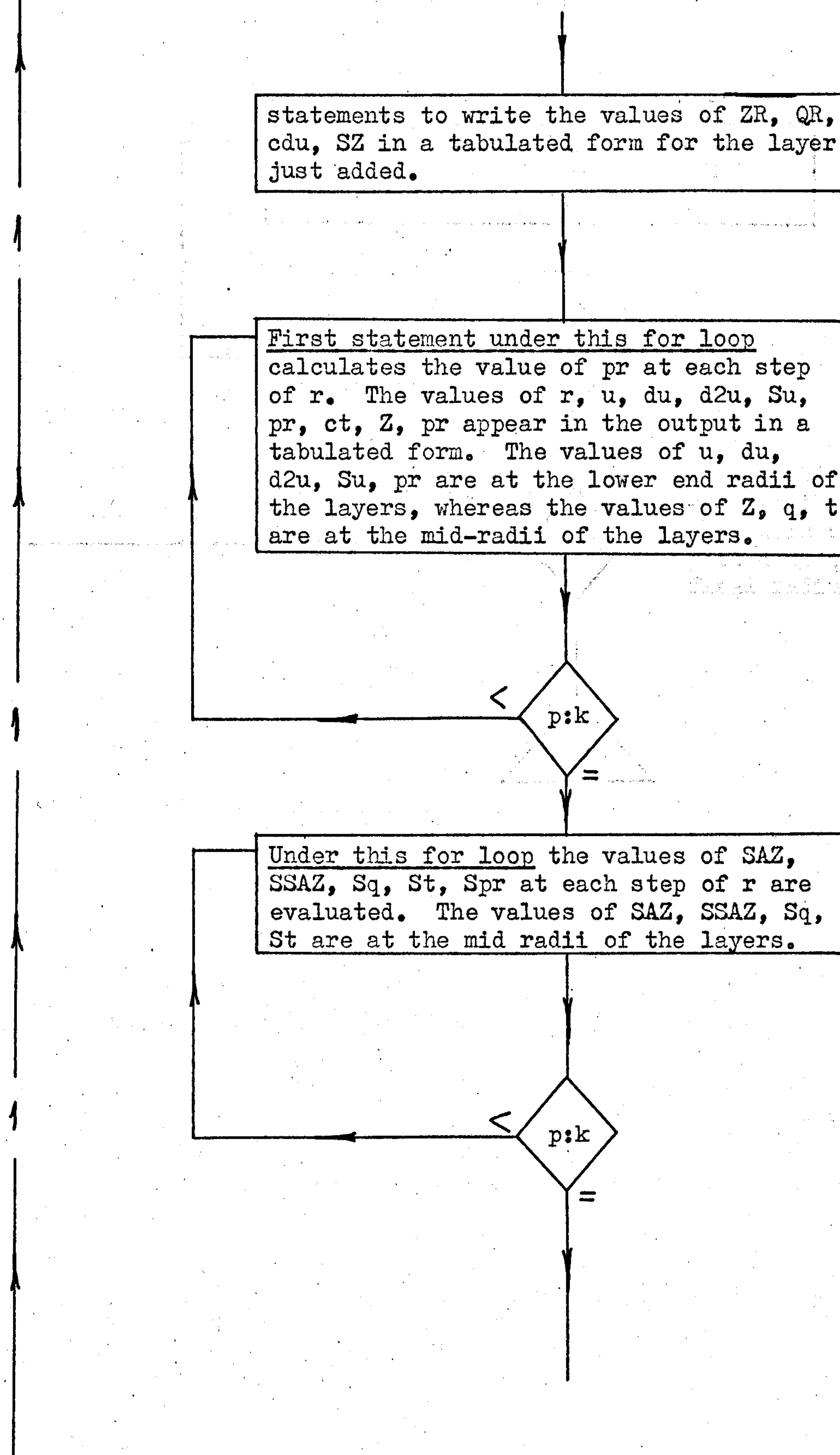
<u>procedure trapezium</u>: Declarations for the procedure. Values of Su, u, r, ct, assigned to appropriate arrays. This procedure solves the cheese once from r = s to r = R.

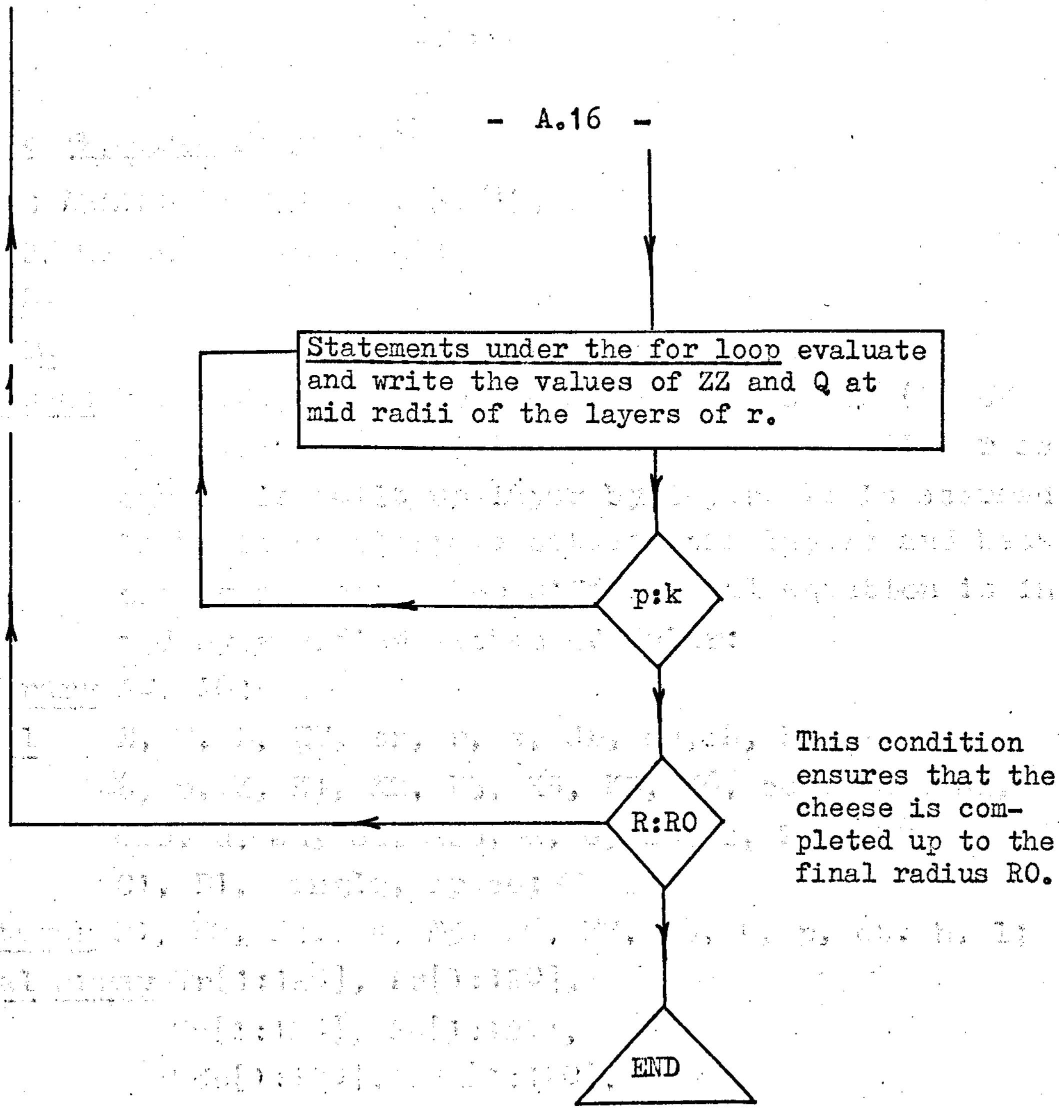
<u>layer loop</u>: Once this loop is entered the calculation proceeds by one step of r. Counter k moves up by one with each entry. Value of u, du, d2u are assigned to tu, tdu, td2u to retain these during the calculation for one layer. Values of u, du, d2u are extrapolated for the next layer. During calculations the value of Su at r k obtainable from previous solution is called.

<u>correction loop</u>: In this loop the value of u, du, d2u are averaged with the help of identifiers, ua, dua, d2ua. The counter ct moves by 2 as the calculation for the same layer goes through twice.


4
3
2
1
u:ua
≠
=
This condition ensures that the two successive values of u do not differ more than a small number specified by c
Value of Su evaluated at r[k]. The values of u, du, d2u, r, ct, su etc. are assigned to T arrays with the help of the counter k.
<
r: (R-(dR+dr/2))
≥
This condition ensures that the calculation is done up to r = R. Exit is only possible when the cheese is completed.
This is entered only once. sdu := sdu + g(from data); the values of du at r=s and r=R from the soln retained by C and D1 respectively.
1 = 1
=
≠
2
Assigns the value of du at s to C1. New value of sdu interpolated from the results of the two trial solutions. The values of du at r=s and r=R are assigned to C and D1 and are retained for the next interpolation.
du at r=R : cdu
≠
=
This condition ensures the correct solution in which du at r = R differs from cdu by less than the small number specified by d.
1


Under this for statement the values of u, Su, du, d2u, etc. from the final correct solution are transferred from T arrays to P arrays and T arrays are free to be used next time when R increases. P arrays cannot be used as the values change during trial solutions and if used the values from the last solution would be lost.
<
p:k
≥
new sig.Z loop: Under this loop the value of Z is calculated at the mid radius of each layer and added up simultaneously as SZ. Counter k in such cases refers to the layer. To obtain these values of Z the value of u and Su are averaged. The values at r[k+1] are called during averaging and these are declared as zero before the loop is entered.
<
r:(R-dR)
≥
This condition ensures that the value of Z is evaluated for all layers till r = R.
This for loop calculates the values of t and q for all layers till r = R. Here also the values are at the mid radii of the layers.
<
p:k
≥


statements to write the values of ZR, QR, cdu, SZ in a tabulated form for the layer just added.
First statement under this for loop calculates the value of pr at each step of r. The values of r, u, du, d2u, Su, pr, ct, Z, pr appear in the output in a tabulated form. The values of u, du, d2u, Su, pr are at the lower end radii of the layers, whereas the values of Z, q, t are at the mid-radii of the layers.
<
p:k
=
Under this for loop the values of SAZ, SSAZ, Sq, St, Spr at each step of r are evaluated. The values of SAZ, SSAZ, Sq, St are at the mid radii of the layers.
<
p:k
=


Statements under the for loop evaluate and write the values of ZZ and Q at mid radii of the layers of r.
p:k
R:RO
This condition ensures that the cheese is completed up to the final radius RO.
END

A.1.5 Computer Program 15

```
→ESTABLISH EBTSJ1500APU+T/15;
COMP. OF CHEESE. w ZERO.;
D/PL→
begin
comment This program calculates the compression (U) of
        the package (precision wound) at any radius r as the
        cheese is built up layer by layer. It is assumed that
        there is no slippage between the layers and between
        cheese and core. The differential equation is integra
        ted by modified method of Euler;
library A0, A6;
real    E, T, D, EY, sr, r, s, dr, dR, R, R0, x, a,
        b, m, K, K1, K2, K3, K4, K5, K6, cdu, sdu, du,
        d2u, u, au, Su, asu, c, d, SZ, z, Z, g, C,
        C1, D1, angle, space;
integer F1, F2, F3, F4, F5, F6, F7, F8, k, p, ct, h, l;
real array Tr[1:120], Pr[1:120],
           Tu[1:120], Pu[1:120],
           Tdu[1:120], Pdu[1:120],
           Td2u[1:120], Pd2u[1:120],
           TSu[1:120], PSu[1:120],
           Tct[1:120], Pct[1:120],
           PZ[1:120], SZR[0:120],
           ZR[1:120], QR[1:120],
           pr[1:120], q[1:120], t[1:120],
           SAZ[1:120], SSAZ[0:120],
           Sq[1:120], St[1:120],
           Spr[1:120];
procedure trapezium (tu, tdu, td2u);
        value tu, tdu, td2u;
        real tu, tdu, td2u;
        comment This procedure is used to solve the cheese from
        the core radius s to the outer radius R. It
```

```
              also approximates the value of U as the sum of
              the previous value of U and the present value of u;
begin         real ua, dua, d2ua;
              TSu[k]:=Su; Tu[k]:=u; Tr[k]:=r;
              Tdu[k]:=du; Td2u[k]:=d2u; Tct[k]:=ct;
layer:        tu:=u; tdu:=du; td2u:=d2u;
              ct:=1; k:=k+1; r:=r+dr; m:=r×r+b;
              u:= tu+tdu×dr;
              Su:=PSu[k];
              du:=tdu+td2u×dr;
              d2u:=K1×u×r×r×(1+(u+Su)×b/(r×m)
              +T×b/(EY×r×r))/(m↑1.5);
correction:   ct:=ct+2;
              dua:=tdu+(td2u+d2u)×dr/2;
              ua:=tu+(tdu+dua)×dr/2;
              d2ua:=K1×ua×r×r×(1+(ua+Su)×b/(r×m)
                       +T×b/(EY×r×r))/(m↑1.5);
                       du:= tdu+(td2u+d2ua)×dr/2;
                       u:=tu+(tdu+du)×dr/2;
              d2u:=K1×u×r×r×(1+(u+Su)×b/(r×m)
              +T×b/(EY×r×r))/(m↑1.5);
              if abs(u-ua) > c then goto correction else
                       TSu[k]:=PSu[k]+u;    Tu[k]:=u;    Tr[k]:=r;
                       Tdu[k]:=du; Td2u[k]:=d2u; Tct[k]:=ct;
              if r < (R-(dR+dr/2)) then goto layer else
end           end of procedure;
              F1:=format([2sd.dd]);
              F2:=format([2s-d.dddddddd]);
              F3:=format([2snd]);
              F4:=format([2s-nddd.ddddddc]);
              F5:=format([2s-ndd.dddddd]);
              F6:=format([2s-nddd.dddddd]);
              F7:=format([2s-ndddd.ddddd]);
```

```
        F8:=format([2s-ndddd.dddddc]);
        open(20);  open(70);
        E:=read(20);  T:=read(20);  EY:=read(20);
        D:=read(20);  sr:=read(20);   s:=read(20);
        dr:=read(20);  RD:=read(20);  z:=read(20);
        dR:=read(20); sdu:=read(20);  x:=read(20);
        c:=read(20);  d:=read(20);  g:=read(20);
        angle:=read(20); space:=read(20);
        close(20);
        x:=x/angle;  z:=z/angle;
        a:=x/6.28318;     b:=a×a;
        K:=a×s×space/(D×(s×s+b)↑0.5);
        K1:=EY/(2×K×D×E); K2:=T/(2×K×Γ×E); K3:=T×K×dR×z/(Γ×a);
        K4:=K×dr×z×EY/(D×a); K5:=2×K×K×z×z×E/b;
        K6:=2×K×K×dr×z×E/a;
write text(70,[[7s]a[15s]b[13s]K[12s]K1[12s]K2[c]]);
                 write(70,F6,a);
                 write(70,F6,b);
                 write(70,F6,K);
                 write(70,F6,K1);
                 write(70,F4,K2);
write text(70,[[7s]K3[13s]K4[9s]K5/1000[7s]K6/1000[c]]);
                 write(70,F6,K3);
                 write(70,F7,K4);
                 write(70,F6,(K5/1000));
                 write(70,F4,(K6/1000));
        R:=s+dR;     SSAZ[0]:=0;  SZR[0]:=0;
        ZR[1]:=K3×a/(((R-dR/2)↑2+b)↑0.5);  SZR[1]:=ZR[1];
        QR[1]:=K3×(R-dR/2)/(((R-dR/2)↑2+b)↑0.5);
                 write(70,F4,ZR[1]);
                 write(70,F4,QR[1]×angle);
        pr[1]:=-K×z×z×dR×T×(R-dR/2)/((((R-dR/2)↑2+b)↑0.5)×Γ×b);
        Spr[1]:=pr[1];
                 write(70,F4,pr[1]×angle);
```

```
        cdu:=-K2×dR×(R-dR/2)/((((R-dR/2)↑2+b)↑0.5));
              write(70,F4,cdu);
        h:=1; SAZ[h]:=0; Sq[h]:=0; St[h]:=0;
for R:=(R+dR) step dR until R0 do
begin   h:=h+1;
        ZR[h]:=K3×a/(((R-dR/2)↑2+b)↑0.5);
        QR[h]:=K3×(R-dR/2)/(((R-dR/2)↑2+b)↑0.5);
        cdu:=-K2×dR×(R-dR/2)/(((R-dR/2)↑2+b)↑0.5);
        SZR[h]:=SZR[h-1]+ZR[h];
        PSu[h]:=0; SAZ[h]:=0; Sq[h]:=0; St[h]:=0; Spr[h]:=0;
        l:=0;
diff press:   k:=1; r:=sr; u:=0; Su:=0;    m:=r×r+b;
              du:=sdu; l:=l+1; d2u:=0;  ct:=0;
              trapezium(u,du,d2u);
if l=1 then
        begin   D1:=du; C:=sdu; sdu:=sdu+g;
                goto diff press
        end;
if abs(cdu-du) > d then
        begin   C1:=sdu;
                sdu:=sdu+(sdu-C)×(cdu-du)/(du-D1);
                D1:= du; C:=C1;
                goto diff press
        end;
                for p:=1 step 1 until k do
                begin PSu[p]:= TSu[p];    Pu[p]:=Tu[p];
                      Pr[p]:=Tr[p];  Pdu[p]:=Tdu[p];
                      Pd2u[p]:=Td2u[p]; Pct[p]:=Tct[p];
                end;
        Pu[k+1]:=Pd2u[k+1]:=PSu[k+1]:=0;
        r:=sr-dr/2; SZ:=0; k:=0;
new sig z: r:=r+dr; k:=k+1; m:=r×r+b;
        au:=(Pu[k+1]+Pu[k])/2;
```

```
        asu:=(PSu[k+1]+PSu[k])/2;
        Z:=K4xauxrxax(1-T/EY-rx(au+asu)/m)/m↑1.5;
        SZ:=SZ+Z;    PZ[k]:=Z;
if r < (R-dr) then goto new sig z;
        r:=sr-dr/2;
        for p:= 1 step 1 until k do
        begin r:=r+dr; m:=rxr+b; au:=(Pu[p+1]+Pu[p])/2;
              t[p]:=EYxauxr/m;
              q[p]:=K6x(Pd2u[p+1]+Pd2u[p])/2;
        end ;
write text(70,[[6s]ZR[12s]QR[12s]cdu[12s]SZ[c]]);
              write(70,F6,ZR[h]);
              write(70,F6,QR[h]xangle);
              write(70,F6,cdu);
              write(70,F4,SZ);
write text(70,[[3s]r[10s]u[12s]du[12s]d2u[12s]Su[6s]ct[7s]Z
              [12s]q[12s]t[11s]pr[c]]);
        for p:= 1 step 1 until k do
        begin pr[p]:=K5xPdu[p];
              write(70,F1,Pr[p]);
              write(70,F2,Pu[p]);
              write(70,F2,Pdu[p]);
              write(70,F2,Pd2u[p]);
              write(70,F2,PSu[p]);
              write(70,F3,Pct[p]);
              write(70,F5,PZ[p]);
              write(70,F5,q[p]xangle);
              write(70,F5,t[p]);
              write(70,F4,pr[p]xangle);
        end;
write text (70,[[3s]r[8s]SAZ[12s]Sq[12s]St[12s]Spr[11s]
              SZR[11s]SSAZ[12s]SZZ[c]]);
        for p:= 1  step 1 until k do
        begin SAZ[p]:=SAZ[p]+PZ[p];
              SSAZ[p]:=SSAZ[p-1]+SAZ[p];
```

```
            Sq[p]:=Sq[p]+q[p];
            St[p]:=St[p]+t[p];
            Spr[p]:=Spr[p]+pr[p];
      end;
      for p:= 1 step 1 until k do
      begin   write(70,F1,Pr[p]);
              write(70,F6,SAZ[p]);
              write(70,F6,Sq[p]xangle);
              write(70,F5,St[p]);
              write(70,F7,Spr[p]xangle);
              write(70,F7,(SZR[h]-SZR[p-1]));
              write(70,F7,(SSAZ[h]-SSAZ[p-1]));
              write(70,F8,((SZR[h]-SZR[p-1])+(SSAZ[h]-
                                     SSAZ[p-1])));
      end;
write text(70,[[3s]r[10s]ZZ[12s]Q[c]]);
      for p:= 1step 1until k do
      begin   write (70,F1,Pr[p]);
              write(70,F6,(ZR[p]+SAZ[p]));
              write(70,F4,(QR[p]+Sq[p])xangle);
      end;
end;
      close(70);
end→
→→
```

A.2 Computer Program 36 for Solving the Equation (4.8)

A.2.1 Introduction

A new program, namely 36, is written in KDF 9 Algol to solve the equation (4.8) which uses varying values of E and EY depending on the pressure in the cheese and the tension in the yarn. This necessitates the use of many more symbols. These are given in the additional notation. This program is a development of the previous program and the structure of the program is basically the same as outlined in § A.1.2 on page A.4. This program also uses the features given in § A.1.3 on page A.7. It incorporates a provision in it to run it with constant values of E and EY; i.e. in that case it reduces to the previous program.

This program integrates the equation from the outer radius R to the core radius s using the Rungekutta method unlike the previous program which integrated the equation from s to R using Euler's modified method. The reason for these differences are given in Chapter 4. The results obtained from the two programs with the same data were the same (up to five significant figures). The starting value of u at r = R (in the previous case it was the value of du at r = s) is obtained by quadratic interpolation. In the event of quadratic equation having imaginery roots the value of u at r = R is obtained by linear interpolation. This replaces the linear interpolation of the previous program. The procedure called 'trapezium' to solve the cheese from $r = R$ to $r = s$ is also different. These differences can be observed in the flow diagram of this program. The output of the results in this program has been modified to obtain the results at specified intervals of r for specified intervals of R; in the previous program the output was obtained at every step of r for every step of R. The provision effects a considerable saving of computer time.

A.2.2 Additional Notation

The symbols are given in the order in which they appear in the program and only those are included which are additional to this program and do not correspond to the theoretical notation.

CE - is the value of E at $r = R$.

dE - $= \partial E/\partial r$.

RT - is the residual tension in the yarn, i.e. T.

IEY - is the value of EY read in the program from the data tape.

t - $= \frac{\partial T}{\partial R}.dR$.

K1, K2, K3, K4, K5, K6 - are constants used in the calculations for solving the equations. The values of these are different from those of the previous program.

IE - is the value of E read in the program from the data tape.

Su - is the guessed or the interpolated value of u at R.

d - is a small number to specify the tolerance between the value of u at s and zero.

e, mm - are used in guessing the value of u at R. Inside the procedure $e = \frac{\partial E}{\partial R}.dR.$

f - is used in the procedure to control the output of the detailed calculations.

g,n - are to specify the interval of R for the output of results.

u1, u2, u3 and su1, su2, su3 - are used to retain the values of u at s and u at R respectively from the trial solutions for interpolating the next value of u at R.

L1, D1, G1, C1, L - are used in the interpolation of the next value of u at R.

ee - is a number used in the procedure to prevent the value of u becoming large and causing program failure.

ccc - is a small number used in the procedure to prevent the value of E becoming small and causing program failure.

c,cc- specify the interval of r for the output of results.

nn - specifies the size of the arrays and is equal to the number of layers into which the cheese is divided for the solution.

SQR - $= \int_r^R Qo.dr.$

SPR - $= \int_r^R Po.dr.$

Additional symbols declared in the procedure 'trapezium'.

11, 12, 13, 14 - each one is used to retain the value of (d2u x dr), i.e. du, at different stages during the calculations of the values for the next step of r.

e - $= \frac{\partial E}{\partial R}.dR$. It is the change in E at r as R increases by dR.

de - $= \frac{\partial e}{\partial r} = \frac{\partial(\frac{\partial E}{\partial R}.dR)}{\partial r}$.

tr - is used to retain the value of r during the calculations of the values for the next step of r.

A.2.3 Additional Features of Program 36

(a) Solution With Constant Values of E and EY

The values of E and EY at the start of the solution, i.e. at r = R, are given by the following statements respectively,

PE[h] := PR[h] x prcon / KP + IE and

EY := tencon x RT ↑ y + IEY

Now if 'prcon' and 'tencon' are made zero then this program reduces to Program 15 with constant values of E and EY equal to IE and IEY respectively read in the program from the data tape. If IE and IEY are made zero then the values of E and EY are entirely dependent on P and T. Any other combination of the values of E or EY can also be used.

(b) Output of Results

For getting the output of results at specified intervals of r the for statement which causes the output is modified as follows

for p := 1, cc step c until k do.

Now by assigning appropriate values to cc and c the results can be output at specified intervals of r. For example, if cc = c = 5, dr = 0.1 cm and s = 1 cm then the output will appear only when r = 1.0 cm, 1.4 cm, 1.9 cm etc.

For getting the output of the results at specified intervals of R the following condition is inserted in the program before the output statements.

if h/n = g then begin g := g + 1; output statements; end;

where h is the number of the last layer added, n is the size of the interval and g controls the entry to the output statements. For example if n = 5 and g = 1 then the output statements would be entered for the first time when h = 5 giving h/n equal to g, i.e. 1. As it enters the output statements g becomes 2 and the next time the entry to the output statements would only be possible when h = 10. Now if the step length is 0.1 cm then output will appear only when R = 1.5 cm, 2.0 cm, 2.5 cm, etc.

A.2.4 Flow Diagram for Program 36

BEGIN

Declarations of real and integer variables. Data for the cheese read into the program. Value of the integer nn determines the size of the arrays. The array variables, procedure 'trapezium' and formats are declared. The procedure is explained where called.

Values of constants used in later calculations are evaluated and appear in the output in a tabulated form.

First layer added. R becomes (s+dR) and h has a value of 1. SSAZ[0], SZR[0], SAZ[1], PSq[1], PSt[1], Pq[1], PSu[1] are declared as zero to ensure that these exist in the computer store as such before the start of the calculations. Values of ZR[1], SZR[1], QR[1], pr[1], Spr[1], PE[1], PEY[1], cdu are evaluated and appear in the output in a tabulated form. PdE[1] is also evaluated.

1 →

<u>Main loop under for statement:</u> Each time this loop is entered h goes up by one and R by dR. Values of ZR[h], QR[h], pr[h], CE, PEY[h], cdu, SZR[h], PE[h], PdE[h] are

evaluated. PSu[h], SAZ[h], PSq[h], PSt[h], Spr[h] are declared as zero to ensure that these exist in the computer store as such before the start of the calculation. Counter 1 has a value 1. This counter counts the no. of trial solutions to obtain the correct solution.

2

<u>diff press loop</u>: This loop is entered each time with a new value of u at R. u at R has a guessed value for the first 2 entries and has an interpolated one from the results of the previous two or three solutions for the subsequent entries. The starting values of the variables including the parameters of the procedure, namely, u, du and d2u, are evaluated. Here R includes the added layer as well and the solution required is up to (R - dR)

2

<u>'procedure' trapezium</u> is called with actual parameters. Declarations of the procedure and the starting values of the variables assigned to the appropriate members of T arrays.

h:f

≤ a

>

2

Write statements for r, u, d2u, E, e, EY and dE. By assigning some value to f from the data tape this output appears only when h, i.e. the number of layers added, > f. This is useful in the event of program failure as the output can be obtained after any value of 'h' (for which the program does not fail) for detailed scrutiny to spot the reason for failure. However calling this output is very wasteful of time and paper.

a

2

1

3

layer loop: Each time this loop is entered the calculation advances by one step of r. This is done in stages. The values of r, u, du, d2u for the layer entered are retained by tr, tu, tdu, td2u. The Runge-Kutta method is used for the extrapolation of the values at the next step.

3

First stage. Values of u, du extrapolated at the mid radius of next layer and from these values t, RT, q, de, dE, e, E, are evaluated. During these calculations appropriate values of PEY, PdE, PE from the last solution are called by K. In this case these values are the average of the two end values of the layer as this is at the mid radius.

1

2

Comment. In the trial soln. E may become very small and cause program failure. Therefore E is not allowed to have a lower value than ccc.

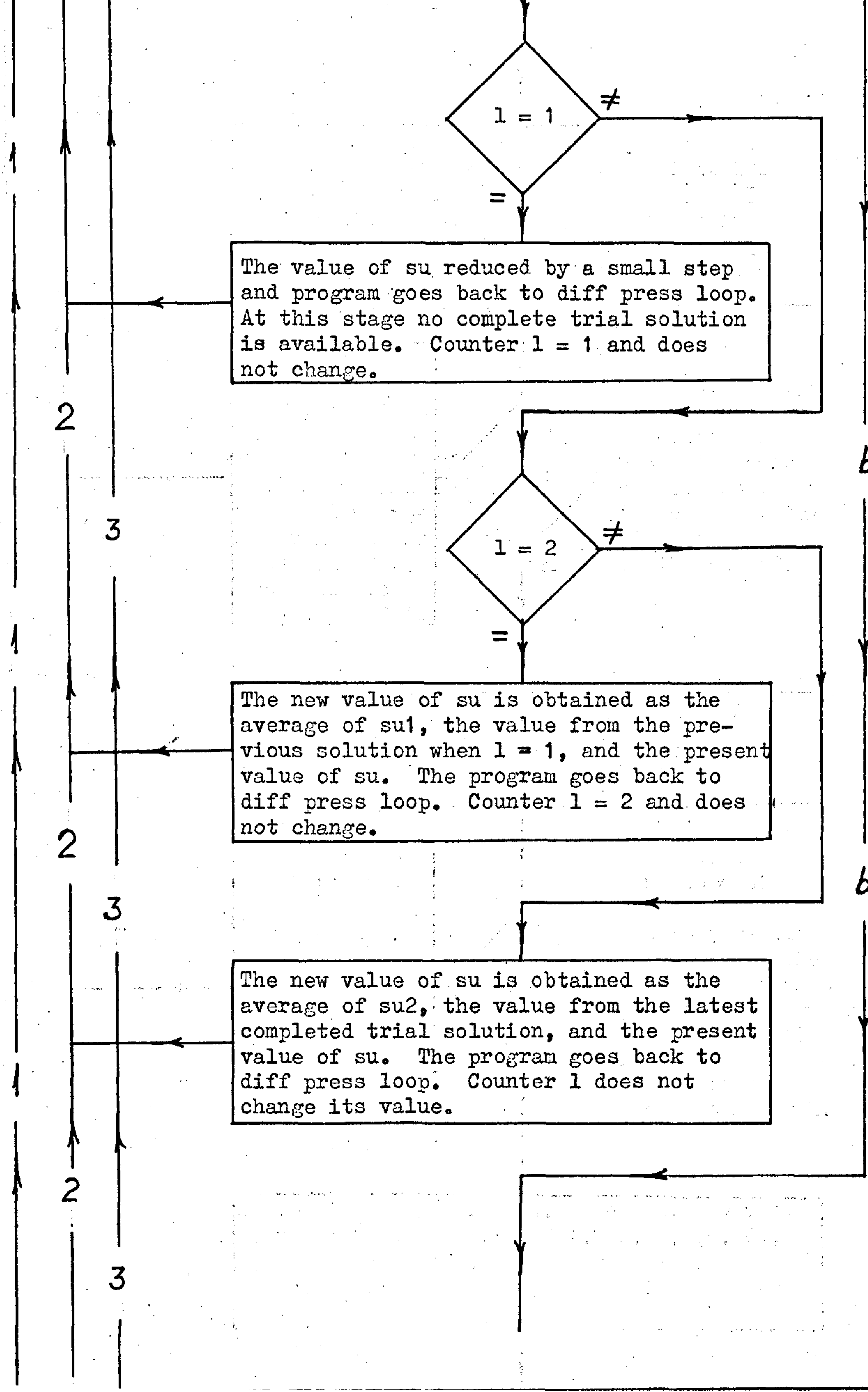
abs(E):ccc

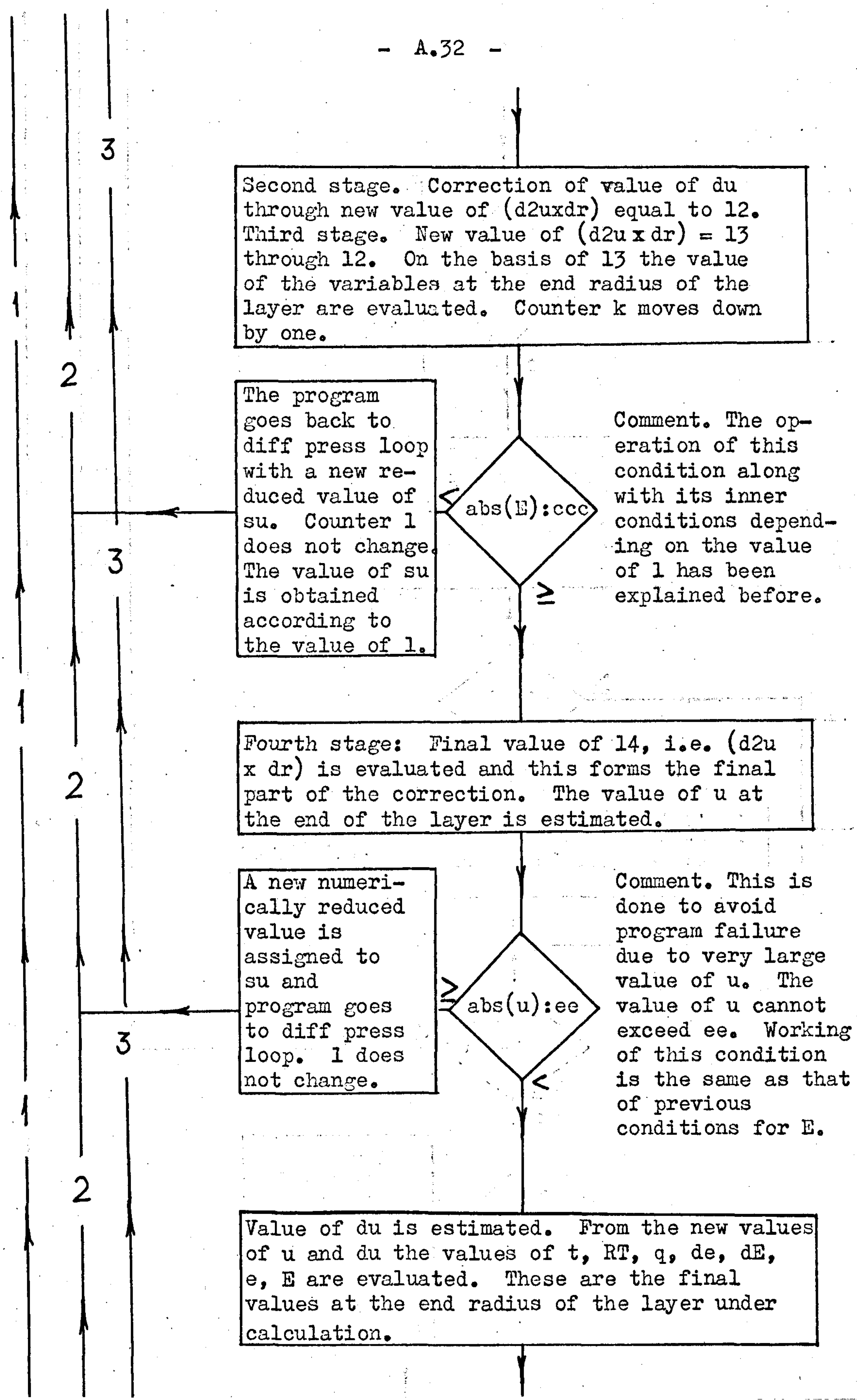
≥

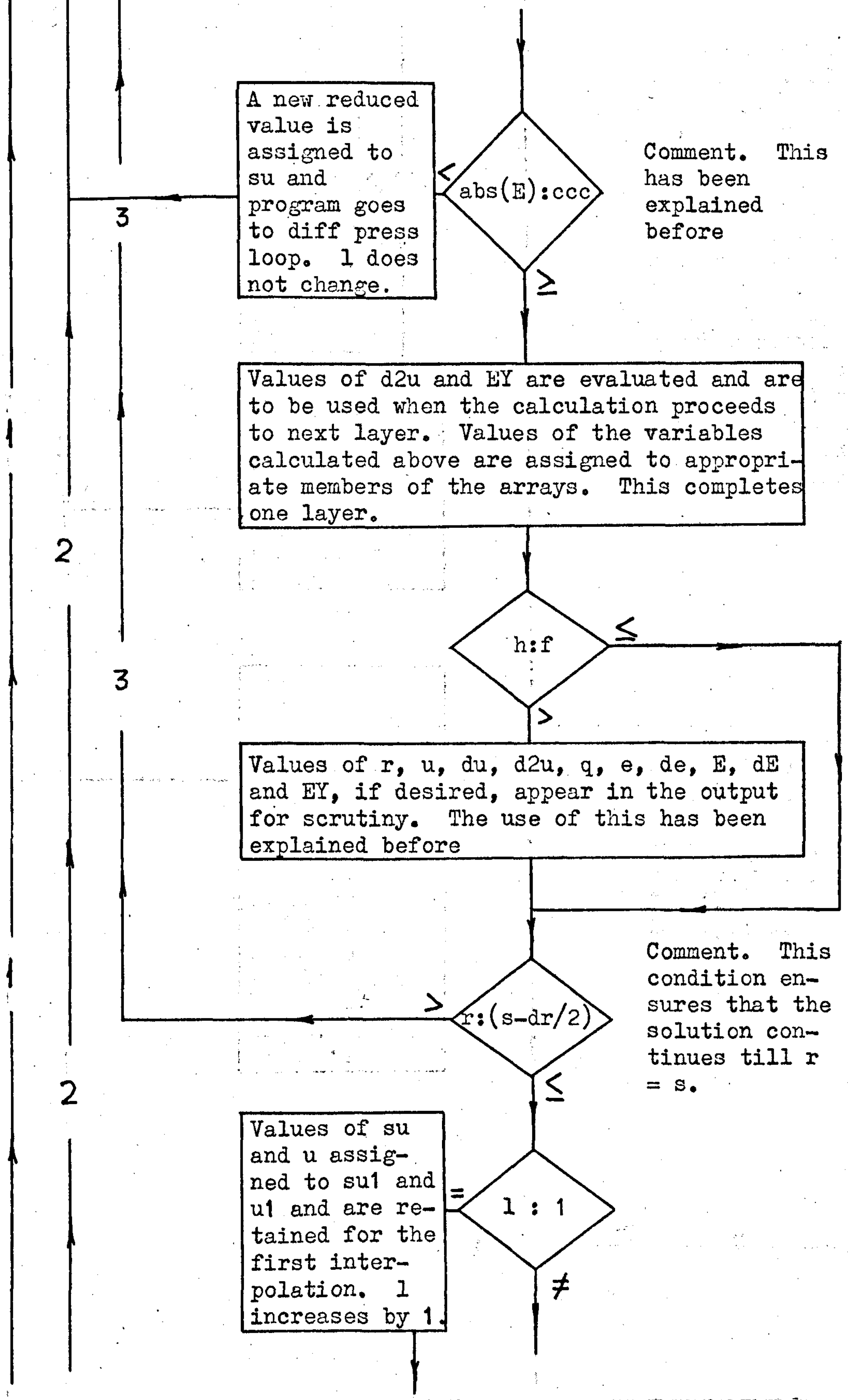
<

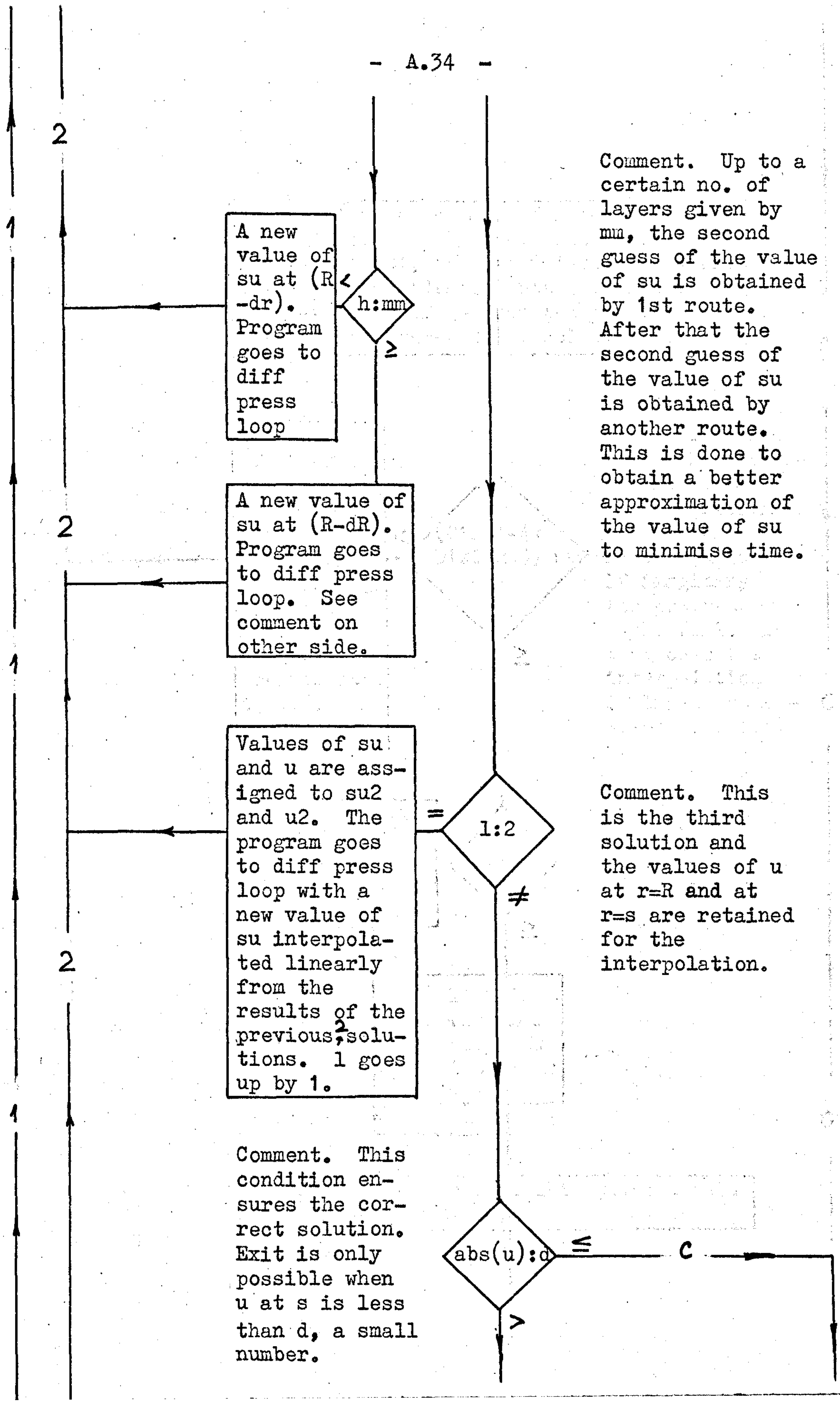
3

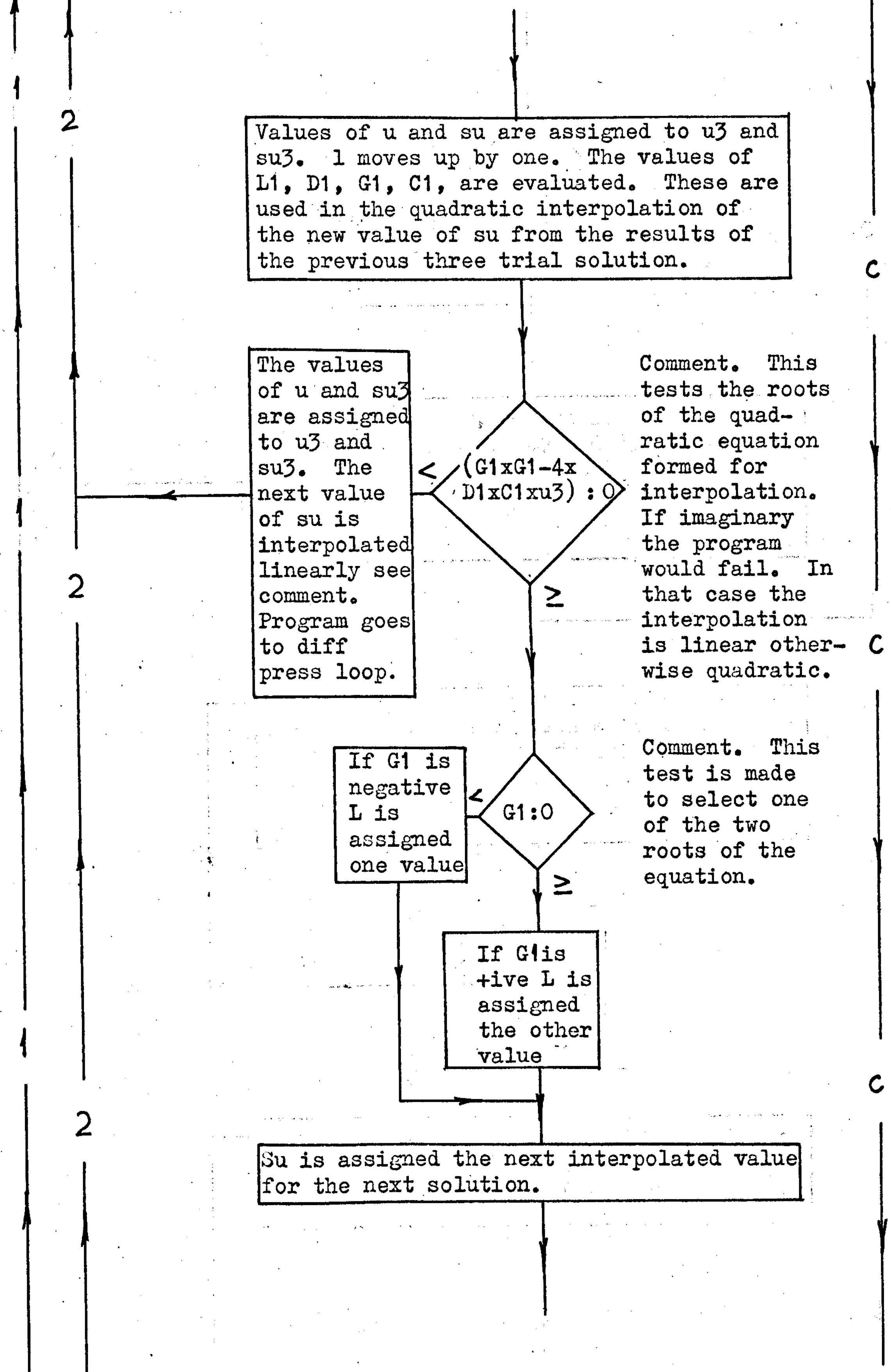
b

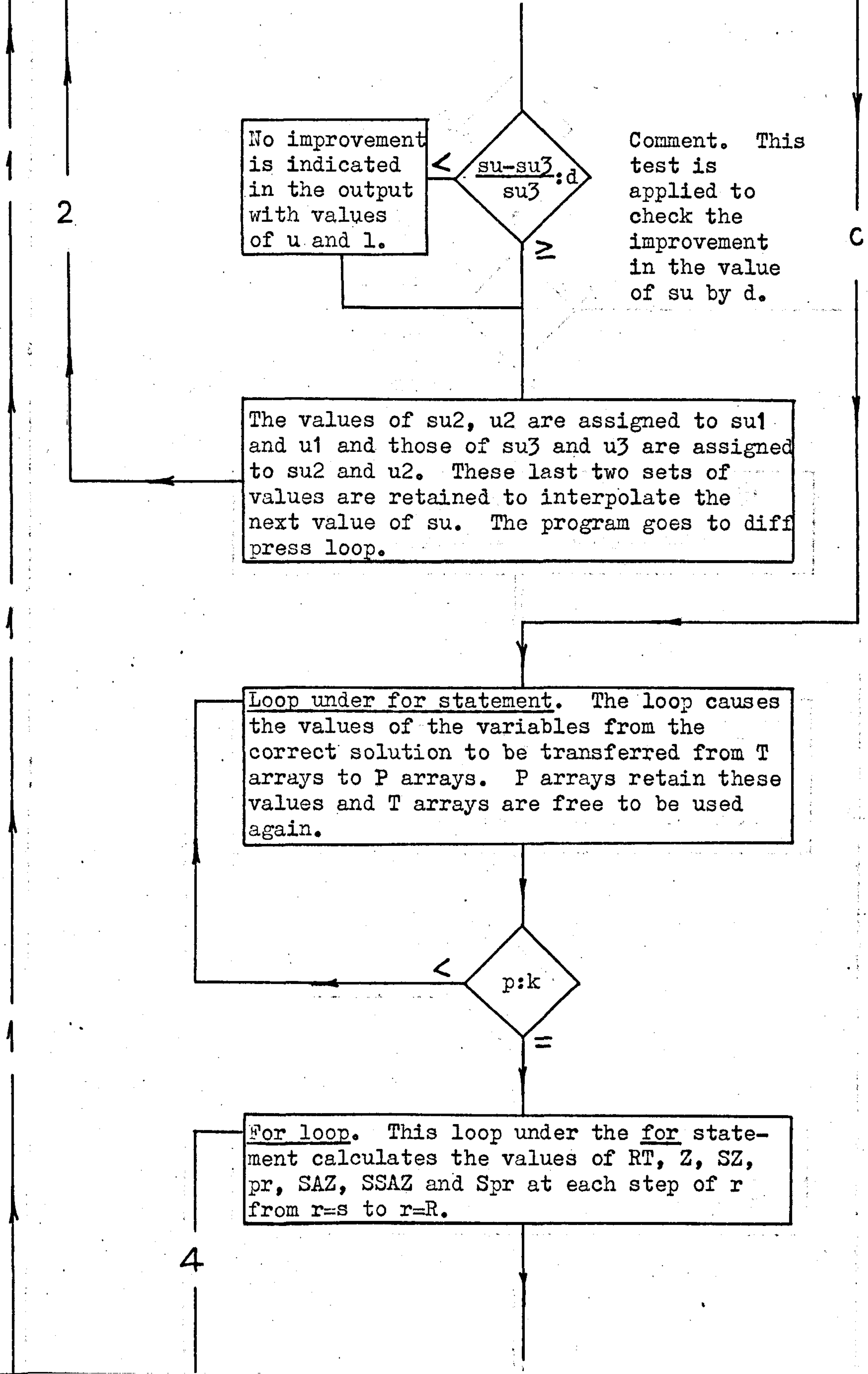

l = 1
≠
=
The value of su reduced by a small step and program goes back to diff press loop. At this stage no complete trial solution is available. Counter l = 1 and does not change.
2
3
b
l = 2
≠
=
The new value of su is obtained as the average of su1, the value from the previous solution when l = 1, and the present value of su. The program goes back to diff press loop. Counter l = 2 and does not change.
2
3
b
The new value of su is obtained as the average of su2, the value from the latest completed trial solution, and the present value of su. The program goes back to diff press loop. Counter l does not change its value.
2
3

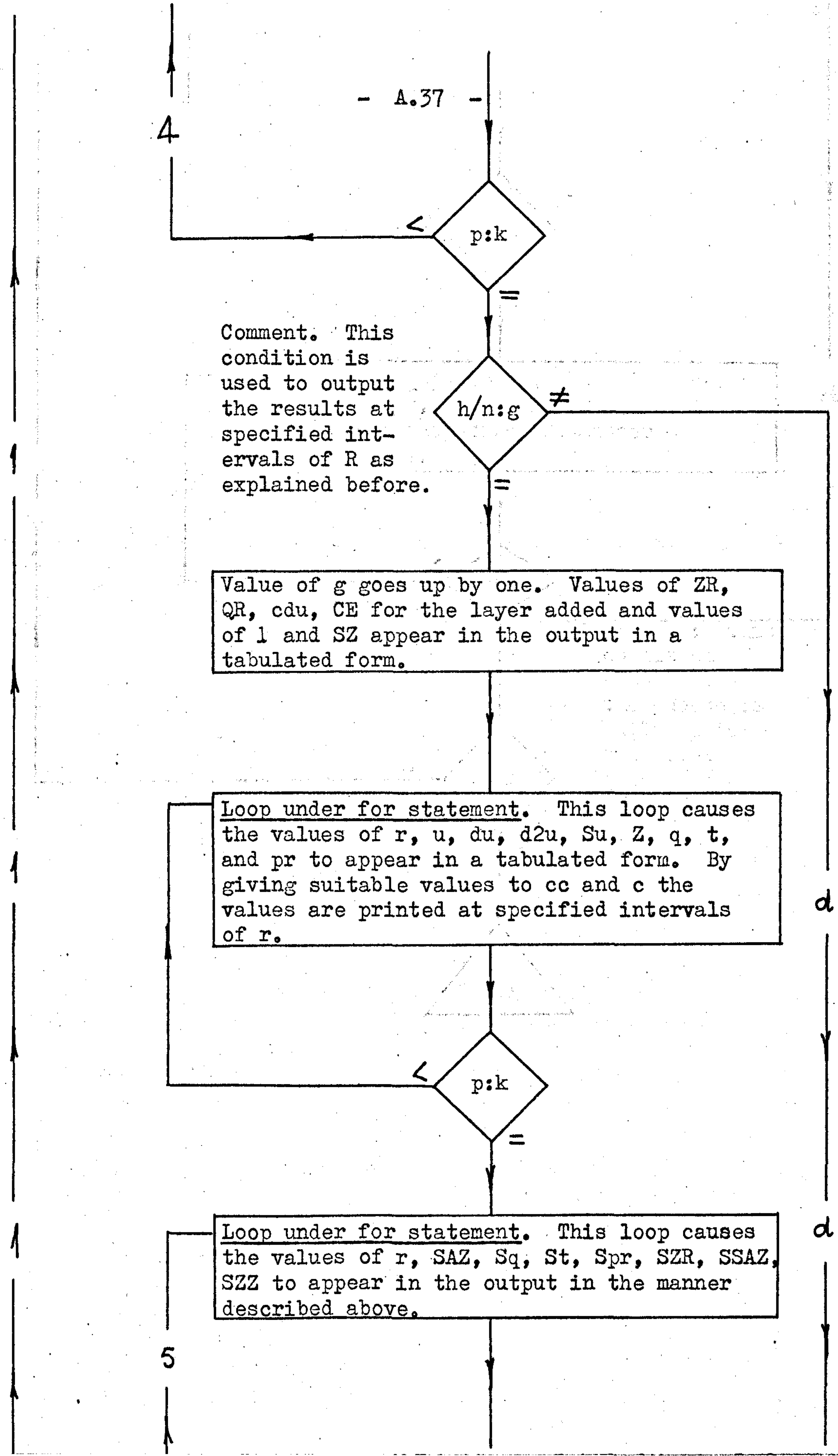

3
2
3
2
3
2
Second stage. Correction of value of du through new value of (d2uxdr) equal to 12. Third stage. New value of (d2u x dr) = 13 through 12. On the basis of 13 the value of the variables at the end radius of the layer are evaluated. Counter k moves down by one.
The program goes back to diff press loop with a new reduced value of su. Counter l does not change. The value of su is obtained according to the value of l.
<
abs(E):ccc
≥
Comment. The operation of this condition along with its inner conditions depending on the value of l has been explained before.
Fourth stage: Final value of 14, i.e. (d2u x dr) is evaluated and this forms the final part of the correction. The value of u at the end of the layer is estimated.
A new numerically reduced value is assigned to su and program goes to diff press loop. l does not change.
≥
abs(u):ee
<
Comment. This is done to avoid program failure due to very large value of u. The value of u cannot exceed ee. Working of this condition is the same as that of previous conditions for E.
Value of du is estimated. From the new values of u and du the values of t, RT, q, de, dE, e, E are evaluated. These are the final values at the end radius of the layer under calculation.

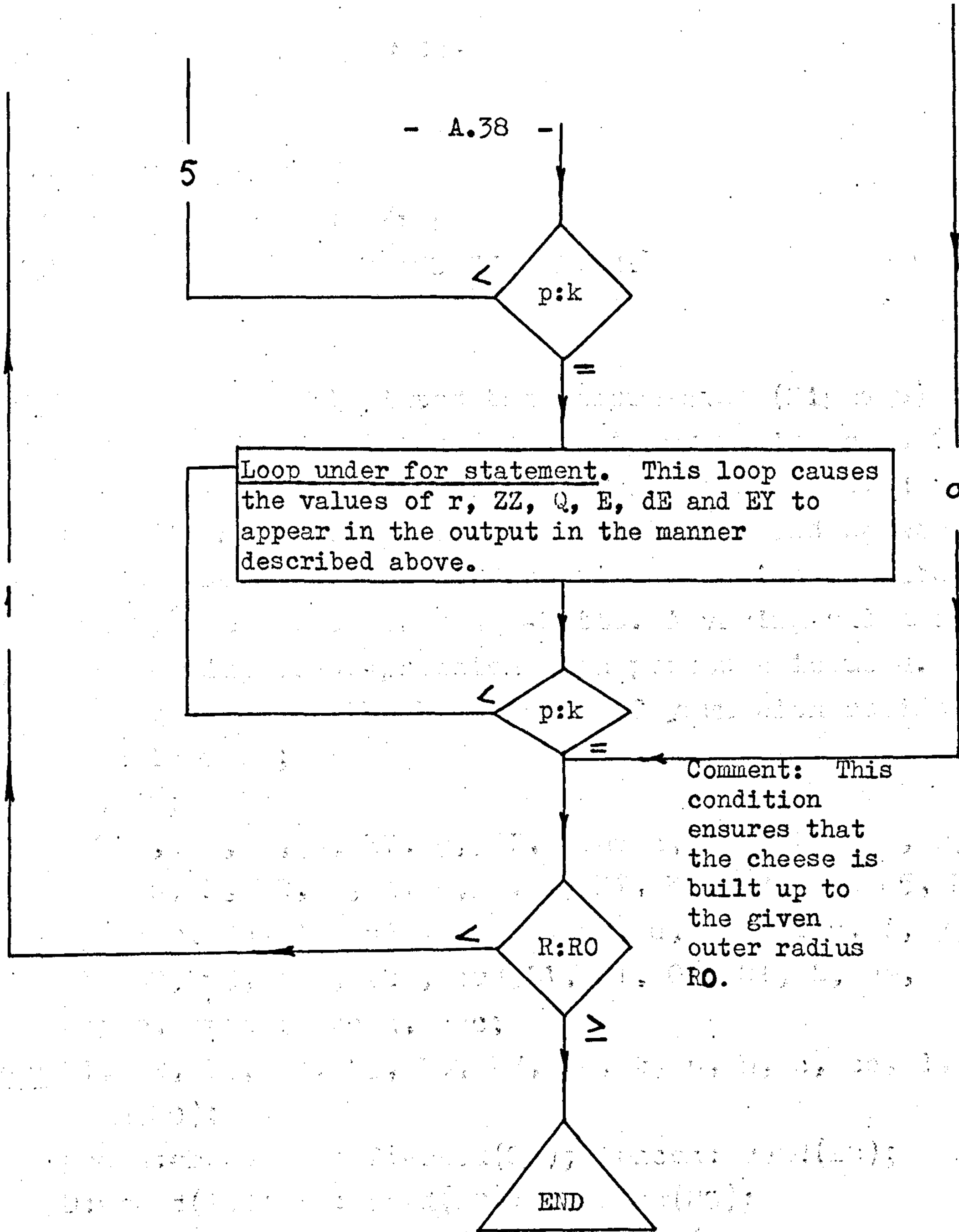

A new reduced value is assigned to su and program goes to diff press loop. 1 does not change.
<
abs(E):ccc
≥
Comment. This has been explained before
3
Values of d2u and EY are evaluated and are to be used when the calculation proceeds to next layer. Values of the variables calculated above are assigned to appropriate members of the arrays. This completes one layer.
2
h:f
≤
>
3
Values of r, u, du, d2u, q, e, de, E, dE and EY, if desired, appear in the output for scrutiny. The use of this has been explained before
Comment. This condition ensures that the solution continues till r = s.
>
r:(s-dr/2)
≤
2
Values of su and u assigned to su1 and u1 and are retained for the first interpolation. 1 increases by 1.
=
1 : 1
≠


2
1
2
1
2
1
A new value of su at (R -dr). Program goes to diff press loop
h:mm
<
≥
Comment. Up to a certain no. of layers given by mm, the second guess of the value of su is obtained by 1st route. After that the second guess of the value of su is obtained by another route. This is done to obtain a better approximation of the value of su to minimise time.
A new value of su at (R-dR). Program goes to diff press loop. See comment on other side.
Values of su and u are ass-igned to su2 and u2. The program goes to diff press loop with a new value of su interpola-ted linearly from the results of the previous 2 solu-tions. 1 goes up by 1.
=
1:2
≠
Comment. This is the third solution and the values of u at r=R and at r=s are retained for the interpolation.
Comment. This condition en-sures the cor-rect solution. Exit is only possible when u at s is less than d, a small number.
abs(u):d
≤
C
>


Values of u and su are assigned to u3 and su3. l moves up by one. The values of L1, D1, G1, C1, are evaluated. These are used in the quadratic interpolation of the new value of su from the results of the previous three trial solution.
(G1xG1-4x D1xC1xu3) : 0
<
The values of u and su3 are assigned to u3 and su3. The next value of su is interpolated linearly see comment. Program goes to diff press loop.
≥
Comment. This tests the roots of the quadratic equation formed for interpolation. If imaginary the program would fail. In that case the interpolation is linear otherwise quadratic.
G1:0
<
If G1 is negative L is assigned one value
≥
If G1 is +ive L is assigned the other value
Comment. This test is made to select one of the two roots of the equation.
Su is assigned the next interpolated value for the next solution.
2
2
2
C
C
C


No improvement is indicated in the output with values of u and l.
<
su-su3 / su3 : d
≥
Comment. This test is applied to check the improvement in the value of su by d.
2
C
The values of su2, u2 are assigned to su1 and u1 and those of su3 and u3 are assigned to su2 and u2. These last two sets of values are retained to interpolate the next value of su. The program goes to diff press loop.
Loop under for statement. The loop causes the values of the variables from the correct solution to be transferred from T arrays to P arrays. P arrays retain these values and T arrays are free to be used again.
<
p:k
=
For loop. This loop under the for statement calculates the values of RT, Z, SZ, pr, SAZ, SSAZ and Spr at each step of r from r=s to r=R.
4


4
p:k
<
=
Comment. This condition is used to output the results at specified intervals of R as explained before.
h/n:g
≠
=
Value of g goes up by one. Values of ZR, QR, cdu, CE for the layer added and values of l and SZ appear in the output in a tabulated form.
Loop under for statement. This loop causes the values of r, u, du, d2u, Su, Z, q, t, and pr to appear in a tabulated form. By giving suitable values to cc and c the values are printed at specified intervals of r.
d
p:k
<
=
Loop under for statement. This loop causes the values of r, SAZ, Sq, St, Spr, SZR, SSAZ, SZZ to appear in the output in the manner described above.
d
5


5
p:k
<
=
Loop under for statement. This loop causes the values of r, ZZ, Q, E, dE and EY to appear in the output in the manner described above.
d
p:k
<
=
Comment: This condition ensures that the cheese is built up to the given outer radius RO.
R:RO
<
≥
END

A.2.5 Computer Program 36

```
→→ESTABLISH EBTSJ3600APU+T/15;
COMP. OF CHEESE. W ZERO. E AND EY VARING;
O/PL→
begin
comment This program calculates the compression (Sigma u) of
        the package (precision wound) at any radius r as the
        cheese is built up layer by layer. It is assumed that
        there is no slippage between the layers and between
        cheese and core. The differential equation is integra
        ted by the method of Runge-Kutta. A varing value of
        the Modulus of Compression with pressure is used. Also a
        varing value of the Elasticity of yarn with residual ten-
        sion is used;
library AO, A6;
real    E, CE, dE, T, D, EY, y, RT, tencon, IEY, t, sr, r, s,
        dr, dR, R, RO, x, a, b, m, K, K1, K2, K3, K4, K5, K6,
        KP, cdu, du, IE, d2u, u, su, Su, d, e, SZ, z, f, g, mm,
        u1, u2, u3, su1, su2, su3,L1, D1, G1, C1, L, ee,
        angle, space, prcon, ccc;
integer F1, F2, F3, F4, F5, F6, F7, F8, k, p, h, c, cc, 1, nn, n;
        open(20);
        prcon:=read(20); T:=read(20); tencon:=read(20);
        D:=read(20); sr:=read(20); s:=read(20);
        dr:=read(20); RO:=read(20); z:=read(20);
        dR:=read(20); su:=read(20); x:=read(20);
        cc:=read(20); c:=read(20); d:=read(20);
        e:=read(20); g:=read(20); f:=read(20);
        angle:=read(20); space:=read(20); nn:=read(20);
        IE:=read(20); IEY:=read(20); y:=read(20);
        n:=read(20); mm:=read(20); ccc:=read(20);
        ee:=read(20);
        close(20);
```

```
begin
real array Tr, Pr, Tu, Pu, Tdu, Pdu, Td2u, Pd2u, TSu, PSu, TE,
        TEY, PEY, PE, TdE, PdE, Tq, Pq, TSq, PSq, Z, ZR, QR,
        PR, pr, Tt, Pt, SAZ, TSt, PSt, Spr[1:nn], SZR, SQR,
        SPR, SSAZ[0:nn];
procedure trapezium (tu, tdu, td2u, press);
        value tu, tdu, td2u;
real    tu, tdu, td2u;  label press;
        comment This procedure is used to solve the cheese
                from core to the outer radius R. It also
                calculates the values of Su, E, dE, q and
                Sq at every step of the inner radius r;
begin   real l1,l2,l3, l4, de, e, tr, q;
                Tt[k]:=t;  TSu[k]:=Su+u;  Tu[k]:=u;  Tr[k]:=r;
                Tdu[k]:=du;  Td2u[k]:=d2u;  TEY[k]:=EY;
                TE[k]:=E;  TdE[k]:=dE;  TSq[k]:=PSq[k]+Tq[k];
                TSt[k]:=PSt[k]+t;  e:=E;
                if h > f then begin
                write(70,F1,r);
                write(70,F6,u);
                write(70,F7,d2u);
                write(70,F7,E);
                write(70,F7,e);
                write(70,F8,EY);
                write(70,F4,dE);
                end;
layer:  tr:=r;  tu:=u;  tdu:=du;  td2u:=d2u;
                l1:=td2u×dr;
                r:=tr+dr/2;
                m:=r×r+b;  u:=tu+tdu×dr/2+dr×l1/8;
                du:=tdu+l1/2;
                t:=(PEY[k]+PEY[k-1])×u×r/(2×m);
                RT:=T+(PSt[k]+PSt[k-1])/2+t;
        q:=K4×r×(t+RT×u×b/(r×m))/m↑0.5;
```

```
        de:=-K6×q;
        dE:=(PdE[k]+PdE[k-1])/2+de;
        e:=-(PE[k]+PE[k-1])×prcon×du/(2×(1+prcon×du));
        E:=(PE[k]+PE[k-1])/2+e;
if abs(E) < ccc then
begin if l=1 then su:=su-su/(10×h×h) else if l=2 then
su:=(su+su1)/2 else su:=(su+su2)/2;
goto press
end;
l2:=(K1×r×(t+RT×u×b/(r×m))/(m↑0.5×E)-dE×du/E)×dr;
        du:=tdu+l2/2;
l3:=(K1×r×(t+RT×u×b/(r×m))/(m↑0.5×E)-dE×du/E)×dr;
        k:=k-1; r:=tr+dr; m:=r×r+b;
        u:=tu+dr×tdu+l3×dr/2; du:=tdu+l3;
        t:=PEY[k]×u×r/m; RT:=T+PSt[k]+t;
q:=K4×r×(t+RT×u×b/(r×m))/m↑0.5;
        de:=-K6×q;
        dE:=PdE[k]+de;
        e:=-PE[k] × prcon × du/(1+prcon×du);
        E:=PE[k]+e;
if abs (E) < ccc then
begin if l=1 then su:=su-su/(10×h×h) else if l=2 then
su:=(su+su1)/2 else su:=(su+su2)/2;
goto press
end;
l4:=(K1×r×(t+RT×u×b/(r×m))/(m↑0.5×E)-dE×du/E)×dr;
        u:=tu+(tdu+(l1+l2+l3)/6)×dr;
if abs(u) > ee then
begin if l=1 then su:=su-su/(10×h×h) else if l=2 then
su:=(su+su1)/2 else su:=(su+su2)/2;
goto press
end;
        du:=tdu+(l1+2×l2+2×l3+l4)/6;
```

```
          t:=PEY[k]×u×r/m;   RT:=T+PSt[k]+t;
      q:=K4×r×(t+RT×u×b/(r×m))/m↑0.5;
          de:=-K6×q;
          dE:=PdE[k]+de;
          e:=-PE[k]×prcon×du/(1+prcon×du);
          E:=PE[k]+e;
      if abs (E) < (ccc×0.001) then
      begin if l=1 then su:=su-su/(10×h×h) else if l=2 then
      su:=(su+su1)/2 else su:=(su+su2)/2;
      goto press
      end;
      d2u:=K1×r×(t+RT×u×b/(r×m))/(m↑0.5×E)-dE×du/E;
          RT:=abs(RT);  EY:=tencon×RT↑y+IEY;
          TSu[k]:=PSu[k]+u;  Tu[k]:=u;  Tr[k]:=r;
          Tdu[k]:=du; Td2u[k]:=d2u; TE[k]:=E;  TdE[k]:=dE;
          TSq[k]:=PSq[k]+q;  TEY[k]:=EY;
          TSt[k]:=PSt[k]+t;  Tt[k]:=t; Tq[k]:=q;
          if h > f then begin
          write(70,F1,r);
          write(70,F7,u);
          write(70,F7,du);
          write(70,F7,d2u);
          write(70,F7,q);
          write(70,F7,e);
          write(70,F7,de);
          write(70,F7,E);
          write(70,F5,dE);
          write(70,F5,EY);
          end;
      if r > (s-dr/2) then goto layer else
end       end of procedure;
      F1:=format([d.dd]);
      F2:=format([2s-nd.ddddddd]);
```

```
        F3:=format([2snd]);
        F4:=format([2s-ndd.dddddddc]);
        F5:=format([2s-nddddd.ddddc]);
        F6:=format([2s-ndd.ddddddd]);
        F7:=format([2s-nddd.dddddd]);
        F8:=format([2s-ndddd.ddddd]);
        open(70);
        x:=x/angle;    z:=z/angle;
        a:=x/6.28318;    b:=a×a;
        K:=a×s×space/(D×(s×s+b)↑0.5);    KP:=2×K×K×z×z/b;
        K1:=1/(2×K×D);  K2:=T/(2×K×D);  K3:=T×K×dR×z/(D×a);
        K4:=-K×dr×z/(D×a);  K5:=2×K×K×z×z/b;
        K6:=-prcon×z/(KP×dr×a);
write text(70,[[9s]a[13s]b[13s]K[12s]KP[12s]K1[12s]K2[c]]);
          write(70,F7,a);
          write(70,F7,b);
          write(70,F7,K);
          write(70,F7,KP);
          write(70,F7,K1);
          write(70,F4,K2);
write text(70,[[7s]K3[13s]K4/100[8s]K6[11s]K5/100[c]]);
          write(70,F7,K3);
          write(70,F8,K4/100);
          write(70,F7,K6);
          write(70,F5,K5/100);
        R:=s+dR;    SSAZ[0]:=0;    SZR[0]:=0;
        ZR[1]:=K3×a/(((R-dR)↑2+b)↑0.5);    SZR[1]:=ZR[1];
        QR[1]:=K3×(R-dR)/(((R-dR)↑2+b)↑0.5);
        pr[1]:=-K×z×z×dR×T×(R-dR)/((((R-dR)↑2+b)↑0.5)×D×b);
        Spr[1]:=pr[1];
        PE[1]:=-prcon×pr[1]/KP+IE;    PEY[1]:=tencon×T↑y+IEY;
        cdu:=-K2×dR×(R-dR)/((((R-dR)↑2+b)↑0.5)×PE[1]);
write text(70,[[8s]ZR[12s]QR[12s]E[12s]EY[12s]cdu[12s]pr[c]]);
```

```
                    write(70,F7,ZR[1]);
                    write(70,F7,QR[1]×angle);
                    write(70,F7,PE[1]);
                    write(70,F8,PEY[1]);
                    write(70,F2,cdu);
                    write(70,F5,pr[1]×angle);
          h:=1;  SAZ[h]:=0; PSq[h]:=0; PSt[h]:=0; Pq[h]:=0;
          PSu[h]:=0;  PdE[1]:=-K6×QR[h];
for R:=(R+dR) step dR until RO do
begin     h:=h+1;
          ZR[h]:=K3×a/(((R-dR)↑2+b)↑0.5);
          QR[h]:=K3×(R-dR)/(((R-dR)↑2+b)↑0.5);
          pr[h]:=-K×z×z×dR×T×(R-dR)/((((R-dR)↑2+b)↑0.5)×D×b);
          CE:=-prcon×pr[h]/KP+IE;   PEY[h]:=tencon×T↑y+IEY;
          cdu:=-K2×dR×(R-dR)/((((R-dR)↑2+b)↑0.5)×CE);
          SZR[h]:=SZR[h-1]+ZR[h];
          l:=1;     PE[h]:=CE;   PdE[h]:=-K6×QR[h];
          PSu[h]:=0; SAZ[h]:=0; PSq[h]:=0; PSt[h]:=0; Spr[h]:=0;
diff press:   du:=cdu;  E:=CE;
          k:=h;  r:=R-dR;  u:=su;  Su:=0; m:=r×r+b;
          dE:=-(QR[h]+PSq[h])×K6;
          t:=PEY[k]×u×r/m;  RT:=T+PSt[k]+t;
          Tq[k]:=K4×r×(t+RT×u×b/(r×m))/m↑0.5;
          d2u:=K1×r×(t+RT×u×b/(r×m))/(m↑0.5×E)-dE×du/E;
          RT:=abs(RT);  EY:=tencon×RT↑y+IEY;
          trapezium(u,du,d2u, diff press);
if l=1 then
          begin  l:=l+1;  u1:=u;  su1:=su;  if h < mm
                 then su:=su+e×su/(h×h) else  begin
                  e:=e+1; su:=su+e×su/(h×h);  end;
                 goto diff press
          end;
if l=2 then
          begin  l:=l+1;   u2:=u;    su2:=su;
```

```
                     su:=su2+(su2-su1)×(0-u2)/(u2-u1);
                     goto diff press
              end;
      if abs(u) > d then
              begin  l:=l+1;   u3:= u;  su3:= su;
                     L1:=(su3-su2)/(su2-su1);
                     D1:=(su3-su1)/(su2-su1);
                     G1:= L1×L1×u1-D1×D1×u2+(L1+D1)×u3;
                     C1:=L1×(L1×u1-D1×u2+u3);
                     if (G1×G1-4×D1×C1×u3) < 0 then
                     begin  su:=su+(su3-su2)×(0-u3)/(u3-u2);
                            su1:=su2; su2:=su3;
                            u1:=u2;  u2:= u3;
                            write(70,F4,l);
                            goto diff press
                     end;
                     if G1 > 0 then
                     L:=-2×D1×u3/(G1+(G1×G1-4×D1×C1×u3)↑0.5) else
                     L:=-2×D1×u3/(G1-(G1×G1-4×D1×C1×u3)↑0.5);
                     su:=su3+L×(su3-su2);
                     if abs((su-su3)/su3) < d then
                     begin write text(70,[[12s] condition not
                       satisfied[cc]]);
                       write(70,F2,u);
                       write(70,F4,l);
                     end;
                     su1:=su2;  su2:=su3;
                     u1:= u2;   u2:= u3;
                     goto diff press
              end;
                     k:=h;
                     for p:= 1 step 1 until k do
```

```
                begin  PSu[p]:= TSu[p];     Pu[p]:=Tu[p];
                       Pr[p]:=Tr[p];  Pdu[p]:=Tdu[p];
                       Pd2u[p]:=Td2u[p]; PE[p]:=TE[p];
                       PdE[p]:=TdE[p]; Pq[p]:=Tq[p];
                       PSq[p]:=TSq[p];  PEY[p]:=TEY[p];
                       PSt[p]:=TSt[p];  Pt[p]:=Tt[p];
                end;
         SZ:=0;    r:=sr+dr;
for p:=1 step 1 until k do
begin    r:=r-dr;  m:=r×r+b;  RT:=T+PSt[p];
         Z[p]:=K4×a×(Pt[p]-RT×u×r/m)/m↑0.5;
         SZ:=SZ+Z[p];
         pr[p]:=K5×PE[p]×Pdu[p];
         SAZ[p]:=SAZ[p]+Z[p];
         SSAZ[p]:=SSAZ[p-1]+SAZ[p];
         Spr[p]:=Spr[p]+pr[p];
end;
if  h/n = g then
begin g:=g+1;
write text(70,[[7s]ZR[12s]QR[12s]cdu[12s]CE[7s]1[6s]SZ[c]]);
                write(70,F7,ZR[h]);
                write(70,F7,QR[h]×angle);
                write(70,F2,cdu);
                write(70,F6,CE);
                write(70,F3,1);
                write(70,F4,SZ);
write text(70,[[1s]r[10s]u[12s]du[12s]d2u[12s]Su[11s]Z
                [12s]q[13s]t[13s]pr[c]]);
         for p:= 1, cc step c until k do
         begin  write(70,F1,Pr[p]);
                write(70,F2,Pu[p]);
                write(70,F2,Pdu[p]);
                write(70,F2,Pd2u[p]);
                write(70,F2,PSu[p]);
```

```
                write(70,F7,Z[p]);
                write(70,F7,Pq[p]xangle);
                write(70,F6,Pt[p]);
                write(70,F5,pr[p]xangle);
        end;
 write text (70,[[1s]r[8s]SAZ[12s]Sq[12s]St[12s]Spr[11s]
                SZR[11s]SSAZ[11s]SZZ[c]]);
        for p:= 1, cc step c until k do
        begin   write(70,F1,Pr[p]);
                write(70,F7,SAZ[p]);
                write(70,F7,PSq[p]xangle);
                write(70,F6,PSt[p]);
                write(70,F8,Spr[p]xangle);
                write(70,F8,(SZR[h]-SZR[p-1]));
                write(70,F8,(SSAZ[h]-SSAZ[p-1]));
                write(70,F5,((SZR[h]-SZR[p-1])+(SSAZ[h]-
                                SSAZ[p-1])));
        end;
 write text(70,[[1s]r[10s]ZZ[12s]Q[13s]E[14s]dE[13s]EY[c]]);
        for p:= 1, cc step c until k do
        begin   write (70,F1,Pr[p]);
                write(70,F7,(ZR[p]+SAZ[p]));
                write(70,F7,(QR[p]+PSq[p])xangle);
                write(70,F7,PE[p]);
                write(70,F8,PdE[p]);
                write(70,F5,PEY[p]);
        end;
        end;
 end;
        close(70);
 end;
 end→
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APPENDIX B

TEST 1

Average winding tension for preparing the base for the gauge = 29 g; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G.Res. = 660 ohm.

During first winding.

Calibrating resistance = 470,000 ohm; movement on dial = 1.2 div.;

1 div. of dial = 0.0002128 x 660/12 = 0.117% change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500; potentiometer db range = 10; paper speed = 0.3 mm/sec.; calibrating resistance = 470,000 ohm; movement of pen = 4 div.;

1 div. of paper chart = 0.0002128 x 660/4 = 0.0345% change in G.Res.

Table B.1

No	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	-	0.00	9.35	0	1.08
2	4.05	32	3.1	0.36	9.1	0	1.08
3	4.1	35	7.1	0.83	8.8	0.6	1.06
4	4.2	30	7.8	0.91	8.45	0.6	1.06
5	4.35	29	8.5	1	8.15	0.6	1.06
6	4.85	27	9.0	1.05	7.7	0.9	1.05
7	5.65	27	9.1	1.06	7.4	0.9	1.05
8	6.45	26	9.2	1.08	7	1.4	1.03
9	7.3	28	9.2	1.08	6.6	2	1.01
10	8.25	27	9.2	1.08	6.2	2.3	1
11	9.35	27	9.2	1.08	5.7	2.6	0.99
12					5.2	3.2	0.97
13					4.6	8.1	0.8
14					4	18.2	0.45

Average winding tension = 28.8 g.

TEST 2.A

Average winding tension for preparing the base for the gauge = 25 g; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G.Res. = 670 ohm.

During first winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.8 div.;

1 div. of dial = 0.0002128 x 670/3.8 = 0.0375% change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

Calibrating resistance = 470,000 ohm; movement of pen = 4.5 div.;

1 div. of paper chart = 0.0002128 x 670/4.5 = 0.0313% change in G.Res.

Table B.2

No	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	-	0.0	10	0	0.67
2	4.05	23	7.1	0.27	9.7	0	0.67
3	4.15	21	11.8	0.44	9.15	0	0.67
4	4.5	21	15.1	0.57	8.5	1	0.64
5	5.5	23	17	0.64	7.6	1	0.64
6	6.5	21	17.1	0.64	6.9	1.3	0.63
7	7.85	21	17.2	0.65	6	1.3	0.63
8	8.8	20	17.3	0.65	5.4	1.6	0.62
9	9.35	20	17.5	0.66	5	2.2	0.6
10	10	20	17.8	0.67	4.55	3.2	0.57
11					4.3	4.2	0.54
12					4.1	6.1	0.48
13					4.05	7.7	0.43
14					4	8.6	0.4

Average winding tension = 21.1 g.

During second winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.8 div.;

1 div. of dial = 0.0002128 x 670/3.8 = 0.0375% change in G.Res.

During unwinding.

Lower limiting frequency = 80 cs/sec; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 11.5 div.;

1 div. of paper chart = 0.0002128 x 670/11.5 = 0.0123% change in G.Res.

Table B.3

No.	During winding: Dia. of cheese cm	During winding: winding tension g	During winding: Movement on dial div.	During winding: % change in G. Res.	During unwinding: Dia. of cheese cm	During unwinding: movement of pen. div.	During unwinding: % change in G. Res.
1	4	-	-	0.0	10	0	0.42
2	4.05	-	3.8	0.13	9.8	0.8	0.41
3	4.15	24	7.1	0.27	9.2	0.8	0.41
4	4.35	22	8.7	0.33	8.65	0.8	0.41
5	4.8	22	10	0.38	8	0.8	0.41
6	5.55	22	10.5	0.39	7.25	0.8	0.41
7	6.7	22	10.5	0.39	6.55	2.4	0.39
8	7.85	21	10.7	0.41	5.65	3.3	0.38
9	8.45	20	10.9	0.41	4.95	4.1	0.37
10	9.45	20	11.2	0.42	4.65	5.7	0.35
11	10	20	11.2	0.42	4.45	8.9	0.31
12					4.3	12.2	0.27
13					4.15	13.8	0.25
14					4.1	13.8	0.25

Average winding tension = 21.4 g.

The cheese was unwound up to a radius of 2.05 cm.

During third winding. The winding started at a radius of 2.05 cm.

Calibrating resistance = 470,000 ohm; movement on dial = 12.2 div.;

1 div. of dial = 0.0002128 x 670/12.2 = 0.0117% change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 9 div.;

1 div. of paper chart = 0.0002128 x 670/9 = 0.0158% change in G.Res.

Table B.4

	During winding				During unwinding		
No.	Dia. of cheese cm	winding tension g	Movement on dial div.	% change in G. Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4.1	-	-	0.25	10	0	0.48
2	4.4	22	11	0.38	9.45	0.6	0.47
3	4.75	22	15.5	0.43	8.95	1.3	0.46
4	5.85	22	18	0.46	8.5	0.6	0.47
5	6.65	21	17.2	0.45	7.85	1.3	0.45
6	7.75	21	18.5	0.46	7.05	0.6	0.47
7	8.9	18	19.1	0.47	6.2	1.3	0.46
8	9.45	21	19.1	0.47	5.65	1.9	0.45
9	10	20	19.7	0.48	5.1	2.5	0.44
10					4.6	4.4	0.41
11					4.2	8.9	0.34
12					4.1	10.8	0.31
13					4	18.4	0.19

Average winding tension = 20.9 g.

TEST 2.B

Average winding tension for preparing the base for the gauge = 24.7 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e.G.Res. = 675 ohm.

During first winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.8 div.;

1 div. of dial = 0.0002128 x 675/3.8 = 0.0378% change in G.Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 4.5 div.;

1 div. of paper chart = 0.0002128 x 675/4.5 = 0.032 % change in G.Res.

Table B.5

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	10	0	0.65
2	4.05	24	7.4	0.28	9.6	0	0.65
3	4.25	23	13	0.49	9	0	0.65
4	4.6	19	15	0.57	8.2	0	0.65
5	5.15	22	16.5	0.62	7.05	0.5	0.64
6	5.6	25	16.5	0.62	6.4	0.5	0.64
7	6.7	24	16.8	0.64	5.45	0.8	0.62
8	7.65	20	16.8	0.64	4.9	2	0.59
9	8.7	20	16.8	0.64	4.55	3	0.55
10	9.45	20	17	0.64	4.4	4	0.52
11	10	20	17.2	0.65	4.15	5	0.49
12					4	7.5	0.41

Average winding tension = 21.7 g.

During second winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.5 div.;

1 div. of dial = 0.0002128 x 675/3.5 = 0.0411 % change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec;

calibrating resistance = 470,000 ohm; movement of pen = 12 div.;

1 div. of paper chart = 0.0002128 x 675/12 = 0.012 % change in G. Res.

Table B.6

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	10	0	0.37
2	4.05	25	3	0.12	9.2	0	0.37
3	4.1	25	4.8	0.20	8.55	0	0.37
4	4.4	24	6.8	0.28	7.9	0.5	0.32
5	5.2	19	7.8	0.31	7.25	1.0	0.32
6	5.95	21	7.9	0.32	6.55	1.2	0.31
7	7.2	20	7.9	0.32	5.9	1.2	0.31
8	8.1	20	7.9	0.32	5.25	1.5	0.31
9	8.85	19	7.9	0.32	4.6	2.5	0.3
10	9.3	20	7.9	0.32	4.35	3	0.29
11	10	20	8	0.33	4.2	6.5	0.25
12					4.15	12.5	0.18
13					4.05	13.5	0.13
14							

Average winding tension = 21.3 g.

The cheese was unwound up to a diameter of 4.05 cm.

During third winding. The winding started at a diameter of 4.05 cm.

Calibrating resistance = 470,000 ohm; movement on dial = 11.2 div.;

1 div. of dial = 0.0002128 x 675/11.2 = 0.0128 % change in G.Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 11 div.;

1 div. of paper chart = 0.0002128 x 675/11 = 0.013% change in G.Res.

Table B.7

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4.05	-	0	0.13	10	0	0.34
2	4.2	22	5	0.19	9.55	0	0.34
3	4.45	22	11	0.27	9.05	0.5	0.33
4	5.0	24	13.4	0.3	8.25	0.5	0.33
5	5.8	22	15.1	0.32	7.5	0.5	0.33
6	7.35	22	15.8	0.33	6.85	1.5	0.32
7	8.85	20	16.1	0.34	6.1	1.5	0.32
8	9.35	19	16	0.34	5.45	2	0.31
9	10	20	16.2	0.34	5	3	0.3
10					4.6	5.5	0.27
11					4.2	10.5	0.2
12					4	19.5	0.08

Average winding tension = 21.3 g.

TEST 3

Average winding tension for preparing the base for the gauge = 25 g; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G.Res = 630 ohm

During first winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.5 div.;

1 div. of dial = 0.0002128 x 630/3.5 = 0.0393 % change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500; potentiometer db range = 10; paper speed = 0.3 mm/sec.; calibrating resistance = 470,000 ohm; movement of pen = 8 div.;

1 div. of paper chart = 0.0002128 x 630/8 = 0.0167 % change in G.Res.

Table B.8

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	10.2	0	0.35
2	4.05	24	3.3	0.13	9.85	0.5	0.34
3	4.1	25	6	0.23	9.5	0	0.35
4	4.4	24	8.2	0.31	8.8	0.5	0.34
5	5.7	25	9.2	0.35	7.85	1	0.33
6	6.8	24	9	0.35	7.15	1	0.33
7	7.3	25	9	0.35	6.65	1.5	0.32
8	8.3	21	9	0.35	6.1	1.5	0.32
9	9.1	25	9	0.35	5.35	2	0.32
10	10.2	22	9	0.35	4.8	2.5	0.31
11					4.35	5	0.27
12					4.1	10	0.18
13					4	14	0.12

Average winding tension = 23.9 g.

During second winding.

Calibrating resistance = 470,000 ohm; movement on dial = 9.2 div.;

1 div. of dial = 0.0002128 x 630/9.2 = 0.0145% change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 10 div.;

1 div. of paper chart = 0.0002128 x 630/10 = 0.0134 % change in G.Res.

Table B.9

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial div.	% change in G. Res.	Dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	9.25	0	0.24
2	4.05	23	6.9	0.11	8.65	1	0.23
3	4.1	24	11	0.16	8.2	0	0.24
4	4.7	23	14.4	0.21	7.35	- 0.5	0.25
5	5.1	22	15.2	0.22	6.6	- 0.5	0.25
6	6.4	20	16.5	0.24	6	- 0.5	0.25
7	7.15	19	16.5	0.24	5.2	2	0.21
8	8.8	5	16.2	0.24	4.6	4	0.19
9	9.25	20	16.6	0.24	4.2	5.5	0.18
10					4.1	7	0.15
11					4	15	0.04

Average winding tension = 19.5 g.

TEST 4.A

Average winding tension for preparing the base for the gauge = 15.8 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G.Res. = 341 ohm; tension in the gauge wire = 15 g.

During first winding.

Calibrating resistance = 88,000 ohm; movement on dial = 17.8 div.;

1 div. of dial = 341 x 100 / (88,000 x 17.8) = 0.0218 % change in G.Res.

Table B.10

No.	During winding: Dia. of cheese cm	During winding: winding tension g	During winding: movement on dial div.	During winding: % change in G. Res.	During unwinding: dia. of cheese cm	During unwinding: movement of pen. div.	During unwinding: % change in G. Res.
1	4	-	-	0.0	9.75	7.7	0.17
2	4.05	15	3.1	0.07	9.25	7.5	0.16
3	4.2	15	6.5	0.14	8.8	7.5	0.16
4	4.55	16	7.3	0.17	8.2	7.5	0.16
5	4.85	15	7.7	0.17	7.5	7.2	0.16
6	5.35	15	7.7	0.17	6.8	7.5	0.16
7	6.05	15	7.7	0.17	5.9	7.2	0.16
8	6.75	15	7.7	0.17	5.25	6.9	0.15
9	7.5	15	7.7	0.17	4.75	6.7	0.15
10	8.5	14	7.7	0.17	4.2	5.5	0.12
11	9.1	15	7.7	0.17	4	3.2	0.07
12	9.75	15	7.7	0.17			

Average winding tension = 15 g.

During second winding.

Calibrating resistance = 88,000 ohm; movement on dial = 36.8 div.;

1 div. of dial = 341 x 100/(88,000 x 36.8) = 0.0105 % change in G. Res.

Table B.11

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial. div.	% change in G. Res.	dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0	9.9	9.3	0.1
2	4.05	16	5	0.05	8.95	9.5	0.1
3	4.25	16	7.5	0.08	8.1	9.5	0.1
4	4.05	17	8.7	0.09	7.1	9.4	0.1
5	5.6	15	9.3	0.1	6.2	9.3	0.1
6	6.65	15	9.3	0.1	5.35	9.1	0.1
7	7.9	16	9.3	0.1	5	8.6	0.9
8	9.1	16	9.3	0.1	4.6	8.2	0.09
9	9.9	16	9.3	0.1	4.3	7.4	0.08
10					4	2.5	0.03

Average winding tension = 15.9 g.

TEST 4.B

Average winding tension for preparing the base for the gauge = 15.8 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G. Res. = 341 ohm; tension in the gauge wire = 5 g.

During first winding.

Calibrating resistance = 88,000 ohm; movement on dial = 17.8 div.;

1 div. of dial = 341 x 100/(88,000 x 17.8) = 0.0218 % change in G. Res.

Table B.12

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g	movement on dial. div.	% change in G. Res.	dia. of cheese cm	movement of pen div.	% change in G. Res.
1	4	-	-	0.0	9.75	8.3	0.18
2	4.05	15	3.1	0.07	9.25	8	0.17
3	4.2	15	7	0.15	8.8	8.1	0.18
4	4.55	16	8.1	0.18	8.2	8	0.17
5	4.85	15	8.3	0.18	7.5	7.8	0.17
6	5.35	15	8.3	0.18	6.8	7.8	0.17
7	6.05	15	8.3	0.18	5.9	7.8	0.17
8	6.75	15	8.3	0.18	5.25	7.6	0.17
9	7.5	15	8.3	0.18	4.75	7.3	0.16
10	8.5	14	8.3	0.18	4.2	6	0.13
11	9.1	15	8.3	0.18	4	1.8	0.04
12	9.75	15	8.3	0.18			

Average winding tension = 15 g.

During second winding.

Calibrating resistance = 88,000 ohm; movement on dial = 37.2 div.;

1 div. of dial = 341 x 100/(88,000 x 37.2) = 0.0104 % change in G. Res.

Table B.13

	During winding				During unwinding		
No.	Dia. of cheese cm	winding tension g.	movement on dial div.	% change in G. Res.	dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	9.9	10.7	0.11
2	4.05	16	6	0.06	8.95	10.8	0.11
3	4.25	17	8.6	0.09	8.1	10.8	0.11
4	4.65	17	10	0.10	7.1	10.6	0.11
5	5.6	15	10.5	0.11	6.2	10.5	0.11
6	6.65	15	10.7	0.11	5.35	10.3	0.11
7	7.9	15	10.7	0.11	5	9.8	0.1
8	9.1	15	10.7	0.11	4.6	9.3	0.1
9	9.9	17	10.7	0.11	4.3	8.7	0.09
10					4	1.5	0.02

Average winding tension = 15.9 g.

TEST 5.A

Average winding tension for preparing the base for the gauge = 15.6 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G. Res. = 386 ohm; tension in the gauge wire = 10 g.

During first winding.

Calibrating resistance = 22,000 ohm; movement on dial = 26 div.;

1 div. of dial = 386 x 100/(22,000 x 26) = 0.0675 % change in G. Res.

Table B.14

No.	During winding: Dia. of cheese cm	winding tension g.	movement on dial. div.	% change in G. Res.	During unwinding: dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	16	0	0.0	9.9	2	0.13
2	4.05	16	1.2	0.08	9.4	2	0.13
3	4.3	15	1.8	0.12	8.95	2	0.13
4	4.75	15	2	0.13	8.5	1.9	0.13
5	5.1	16	2.2	0.15	7.6	1.8	0.12
6	5.7	16	2.4	0.16	7	1.8	0.12
7	6.4	16	2.4	0.16	6.2	1.7	0.11
8	7.05	15	2.2	0.15	5.25	1.7	0.11
9	8.15	15	2.1	0.14	4.55	1.5	0.1
10	9.15	15	2	0.13	4.3	1.2	0.08
11	9.9	14	2	0.13	4	0.7	0.05

Average winding tension = 15.4 g.

During second winding.

Calibrating resistance = 44,000 ohm; movement on dial = 41 div.;

1 div. of dial = 386 x 100/(44,000 x 41) = 0.0214 % change in G. Res.

Table B.15

No.	During winding: Dia. of cheese cm	winding tension g.	movement on dial. div.	% change in G. Res.	During unwinding: dia. of cheese cm	movement of pen div.	% change in G. Res.
1	4	-	0	0.0	10.05	5	0.11
2	4.05	16	1	0.02	9.3	4.9	0.1
3	4.15	16	3	0.06	8.7	4.8	0.1
4	4.4	17	4	0.09	7.7	4.7	0.1
5	4.65	16	5	0.11	6.7	4.6	0.1
6	5.55	17	5	0.11	5.9	4.5	0.1
7	6.45	17	5	0.11	5.3	4.5	0.1
8	7.3	15	5	0.11	4.75	3.9	0.08
9	8.1	18	5	0.11	4.4	3.3	0.07
10	9.35	16	5	0.11	4	1.2	0.03
11	10.05	16	5	0.11			

Average winding tension = 16.4 g.

TEST 5.B

Average winding tension for preparing the base for the gauge = 15.6 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G. Res. = 385 ohm; tension in the gauge wire = 10 g.

During first winding.

Calibrating resistance = 22,000 ohm; movement on dial = 25.8 div.;

1 div. of dial = 385 x 100/(22,000x 25.8) = 0.0678 % change in G. Res.

Table B.16

No.	During winding: Dia. of cheese cm	winding tension g	movement on dial. div.	% change in G. Res.	During unwinding: dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4		0	0.00	9.9	4	0.27
2	4.05	16	4.8	0.33	9.4	3.8	0.26
3	4.3	15	4.5	0.31	8.95	3.8	0.26
4	4.75	16	4.4	0.30	8.5	3.7	0.25
5	5.1	16	4.2	0.28	7.6	3.6	0.24
6	5.7	16	4.1	0.28	7	3.6	0.24
7	6.4	15	4	0.27	6.2	3.6	0.24
8	7.05	15	4	0.27	5.25	3.5	0.24
9	8.15	16	4	0.27	4.55	3.3	0.22
10	9.15	14	4	0.27	4.3	3	0.2
11	9.9	15	4	0.27	4	1.6	0.11

Average winding tension = 15.4 g.

During second winding.

Calibrating resistance = 44,000 ohm;. movement on dial = 41 div.;

1 div. of dial = 385 x 100/(44,000 x 41) = 0.0213 % change in G. Res.

Table B.17

No.	During winding				During unwinding		
	Dia. of cheese cm	winding tension g.	movement on dial. div.	% change in G. Res.	dia. of cheese cm	movement of pen. div.	% change in G. Res.
1	4	-	0	0.0	10.05	6.2	0.13
2	4.05	16	2.9	0.06	9.3	6.1	0.13
3	4.15	16	4.7	0.1	8.7	6	0.13
4	4.4	16	5	0.11	7.7	6	0.13
5	4.65	17	5.7	0.12	6.7	6	0.13
6	5.55	16	6	0.13	5.9	5.9	0.13
7	6.45	17	6.2	0.13	5.3	5.8	0.12
8	7.3	17	6.2	0.13	4.75	5.3	0.11
9	8.1	16	6.2	0.13	4.4	4.3	0.09
10	9.35	17	6.2	0.13	4	0	0.0
11	10.05	16	6.2	0.13			

Average winding tension = 16.4 g.

TEST 6

Average winding tension for preparing the base for the gauge = 20.1 g.; rad. of the base = 2 cm; app. resistance of the gauge, i.e. G.Res. = 740 ohm.

During first winding.

Calibrating resistance = 470,000 ohm; movement on dial = 3.9 div.;

1 div. of dial = 740 x 100/(470,000 x 3.9) = 0.0404 % change in G. Res.

During unwinding.

Lower limiting frequency = 80 cs/sec.; writing speed = 500;

potentiometer db range = 10; paper speed = 0.3 mm/sec.;

calibrating resistance = 470,000 ohm; movement of pen = 7 div.;

1 div. of paper chart = 740 x 100/(470,000 x 7) = 0.0225 % change in G.Res.

Table B.18

	During winding				During unwinding		
No.	Dia. of cheese cm	winding tension g	movement on dial. div.	% change in G. Res.	dia. of cheese cm	movement of pen div.	% change in G. Res.
1	4	-	0	0	10.4	0	0.91
2	4.2	25	- 7	- 0.28	9.75	1	0.89
3	5.15	25	- 4.1	- 0.17	9.05	3	0.84
4	6.15	25	0.7	0.03	8.5	4	0.82
5	7.15	30	4.7	0.19	7.85	7	0.75
6	8.05	28	11.2	0.45	7.1	10	0.68
7	8.95	25	14.3	0.59	6.3	11.5	0.65
8	9.75	24	19.4	0.78	6.05	14	0.59
9	10.4	22	22.4	0.91	5.5	16.5	0.54
10					5.1	19	0.48
11					4.7	22	0.41
12					4.4	18.5	0.49
13					4.1	21.5	0.43
14					4	18	0.5

Average winding tension = 25.5 g.

TEST 7

Average winding tension for winding the cheese base = 24.5 g.;

dia. of cheese base = 4 cm; app. resistance of the gauge = 790 ohm;

calibrating resistance = 470 K.ohm; movement on dial = 1.3 div.;

1 div. of dial = 790 x 100/(470,000 x 1.3) = 0.129 % change in G. Res.

During winding.

Table B.19

No.	cheese dia. cm	winding tension g	movement on dial div.	% change in G. Res.
1	4	-	0	0.0
2	4.05	26	- 0.9	- 0.12
3	4.15	26	- 1.2	- 0.15
4	4.35	25	- 0.9	- 0.12
5	4.55	27	0	0.0
6	4.75	25	0.7	0.09
7	5.1	25	1.7	0.22
8	5.8	25	3.7	0.48
9	6.4	25	5.2	0.67
10	7	25	8	1.03
11	7.75	24	11	1.42
12	8.2	22	13.8	1.78
13	8.7	23	16	2.06
14	9.5	gauge wire broken		

Average winding tension = 24.8 g.

TEST 8.

Average winding tension for winding the cheese base = 23.7 g;

dia. of cheese base = 4 cm;

app. resistance of the gauge = 780 ohm;

calibrating resistance = 470 K.ohm; movement on dial = 1.8 div.;

1 div. of dial = 780 x 100/(470,000 x 1.8) = 0.092 % change in G.Res.

During winding.

Table B.20

No.	cheese dia. cm	winding tension g	movement on dial div.	% change in G. Res.
1	4		0	0.0
2	4.05	25	- 0.2	- 0.02
3	4.1	26	- 0.7	- 0.06
4	4.55	25	- 0.2	- 0.02
5	5.35	25	2.2	0.2
6	6.1	25	3.7	0.34
7	6.8	25	8.8	0.81
8	7.7	24	14.9	1.37
9	8.4	gauge wire broken		

Average winding tension = 25 g.

TEST 9

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 24.1 g;

axial gauge length = 3.779 in.

Table B.21

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4		0	100.00	10	166	104.38
2	4.05	26	- 5	99.87	9.65	166	104.38
3	4.2	25	- 14	99.63	9.35	165	104.36
4	4.45	25	- 8	99.79	8.7	163	104.3
5	5.25	25	11	100.29	8.3	161	104.25
6	5.85	24	34	100.9	7.9	159	104.2
7	6.65	25	70	101.85	7.5	158	104.17
8	7.5	25	97	102.56	7	157	104.15
9	8.35	24	124	103.27	6.2	149	103.93
10	9.3	24	145	103.83	5.9	139	103.67
11	10	23	166	104.38	5.2	132	103.48
12					4.75	127	103.35
13					4.4	109	102.88
14					4	107	102.82

Average winding tension = 24.6 g.

TEST 10.i.

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 8 g.

axial gauge length = 3.785 in.

Table B.22

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g.length .001 in	g.length as % of original	cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0	-	0	100.00	9.95	190	105.1
2	4.1	8	- 3	99.92	9.10	190	105.1
3	4.25	7	- 5	99.87	8.45	190	105.1
4	4.55	8	6	100.16	7.75	190	105.1
5	5.25	9	25	100.66	6.75	186	104.91
6	6.15	9	48	101.27	6.10	177	104.67
7	7.0	9	93	102.46	5.5	167	104.41
8	8.0	9	138	103.64	4.9	150	103.96
9	8.8	9	161	104.25	4.4	132	103.48
10	9.95	10	190	105.01	4.0	101	102.67

Average winding tension = 8.7 g.

TEST 10.ii.

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 206 g.

axial gauge length = 3.796 in.

Table B.23

No	during winding: cheese dia. cm	winding tension g	change in g.length .001 in	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0	-	0	100.00	10.4	194	105.11
2	4.1	20	- 6	100.16	9.25	194	105.11
3	4.4	19	0	100.00	8.55	194	105.11
4	4.95	20	7	100.18	7.75	194	105.11
5	5.9	22	32	100.84	6.9	194	105.11
6	6.55	20	56	101.47	6.2	189	104.98
7	7.5	22	84	102.21	5.75	183	104.82
8	8.15	19	110	102.90	5.25	169	104.45
9	9.1	21	146	103.84	4.55	150	102.95
10	10.4	20	194	105.11	4.0	104	102.74

Average winding tension = 20.3 g.

TEST 10.iii

Dia. of cheese base = 4 cm;

average winding tensions for preparing the cheese base = 25.4 g;

axial gauge length = 4.133 in;

First winding and unwinding

Table B.24

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0	-	0	100.00	10.5	174	104.21
2	4.1	28	- 3	99.93	9.85	174	104.21
3	4.45	29	+ 4	100.10	8.8	171	104.14
4	4.9	29	16	100.39	8.3	168	104.07
5	5.6	29	39	100.94	7.75	165	103.99
6	6.15	27	57	101.38	7.2	161	103.90
7	6.85	28	77	101.86	6.55	154	103.73
8	7.5	29	96	102.32	5.8	145	103.51
9	8.2	27	119	102.88	5.25	135	103.27
10	9.0	25	141	103.41	4.6	103	102.49
11	9.7	25	155	103.75	4.4	88	102.13
12	10.5	24	174	104.21	4.2	70	101.69
13					4.0	68	101.65

Average winding tension = 27.2 g.

Second winding and unwinding

Table B.25

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4	-	68	101.65	10.6	201	104.86
2	4.1	27	65	101.57	10.0	201	104.86
3	4.15	27	66	101.60	9.0	199	104.82
4	4.55	28	74	101.79	7.85	195	104.72
5	5.05	28	92	102.23	7.15	191	104.62
6	5.55	29	103	102.49	5.95	177	104.28
7	6.2	28	122	102.96	5.5	173	104.19
8	6.75	27	136	103.29	4.9	158	103.82
9	7.25	27	147	103.56	4.45	136	103.29
10	8.0	27	159	103.85	4.0	99	102.40
11	8.7	25	174	104.21			
12	9.6	25	189	104.58			
13	10.6	24	201	104.86			

Average winding tension = 26.8 g.

Third winding and unwinding

Table B.26

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0	-	93	102.25	10.5	231	105.59
2	4.1	33	82	101.98	9.75	231	105.59
3	4.4	33	88	102.13	9.3	231	105.59
4	5.0	32	108	102.61	8.8	228	105.52
5	5.8	30	131	103.17	8.2	226	105.47
6	6.4	30	149	103.61	7.5	224	105.42
7	7.1	30	164	103.97	6.9	222	105.37
8	8.1	27	187	104.53	6.3	217	105.25
9	8.9	25	200	104.84	5.5	209	105.06
10	9.9	24	219	105.3	4.9	196	104.74
11	10.5	23	231	105.59	4.55	167	104.04
12					4.0	127	103.07

Average winding tension = 28.7 g.

TEST 10.iv

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.2 g;

axial gauge length = 3.766 in.

Table B.27

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0	-	0	100.00	10.0	188	104.99
2	4.15	26	- 3	99.92	9.4	188	104.99
3	4.3	28	- 6	99.84	9.0	188	104.99
4	5.05	27	7	100.19	8.35	185	104.92
5	5.75	27	23	100.61	7.75	182	104.84
6	6.65	25	50	101.33	6.95	169	104.49
7	7.4	25	84	102.23	5.85	160	104.25
8	9.2	26	145	103.85	5.15	148	103.93
9	10.0	25	188	104.99	4.55	135	103.54
10					4.0	109	102.90

Average winding tension = 27.3 g.

TEST 10.v

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 33.2 g.

axial gauge length = 3.788 in.

Table B.28

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0		0	100.00	10.0	171	104.51
2	4.05	32	- 3	99.92	9.05	170	104.49
3	4.15	34	- 6	99.84	7.75	167	104.41
4	4.4	33	- 1	99.97	6.95	163	104.30
5	5.25	35	19	100.19	6.0	154	104.06
6	6.15	35	49	101.29	5.1	143	103.77
7	6.8	32	76	102.01	4.6	124	103.27
8	7.7	35	110	102.90	4.0	95	102.77
9	8.75	32	140	103.69			
10	10.0	30	171	104.51			

Average winding tension = 33.1 g.

TEST 10.vi

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 40.6 g;

axial gauge length = 3.800 in.

Table B.29

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in	g.length as % of original
1	4.0		0	100.00	9.75	177	104.66
2	4.1	40	- 3	99.82	8.95	176	104.63
3	4.15	39	- 10	99.74	8.30	168	104.42
4	4.45	41	- 3	99.82	7.5	168	104.42
5	5.4	40	18	100.47	5.85	146	103.84
6	6.1	40	42	101.11	5.15	141	103.71
7	7.0	40	81	102.13	4.5	128	103.37
8	8.0	40	113	102.97	4.0	95	102.50
9	8.95	40	147	103.87			
10	9.75	43	177	104.66			

Average winding tension = 40.3 g.

TEST 11

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 24.5 g;

gauge dia. d1 = 4 cm; axial g. length b1 = 4.29 in;

gauge dia. d2 = 5 cm; axial g. length b2 = 4.25 in;

gauge dia. d3 = 6 cm; axial g. length b3 = 4.33 in;

gauge dia. d4 = 7 cm; axial g. length b4 = 4.33 in.

During winding

Table B.30

No	cheese dia. cm	winding tension g	A1 g.length change .001 in	A1 g.length as % of original	A2 g.length change .001 in	A2 g.length as % of original
1	4	-	0	100		
2	4.1	26	- 6	99.86		
3	4.3	26	- 3	99.93		
4	4.45	27	2	100.05		
5	4.7	28	8	100.19		
6	5	28	21	100.49	0	100
7	5.15	26	34	100.79	- 7	99.84
8	5.3	27	37	100.86	- 4	99.91
9	5.55	26	43	101	5	100.12
10	6	26	58	101.35	21	100.49
11	6.1	27	62	101.45	34	100.8
12	6.35	26	66	101.54	40	100.94
13	6.6	26	73	101.7	52	101.22
14	7	25	88	102.05	72	101.69
15	7.1	25	93	102.17	77	101.81
16	7.4	25	97	102.26	82	101.93
17	7.9	25	107	102.5	98	102.3
18	8.4	24	117	102.73	110	102.58
19	9.1	24	131	103.05	128	103.01
20	9.6	22	140	103.26	141	103.31
21	10.1	21	148	103.45	153	103.6

Average winding tension = 25.5 g.

Table B.30 (continued)

No.	cheese dia. cm	winding tension g	A3 g.length change .001 in.	A3 g.length as % of original	A4 g.length change .001 in.	A4 g.length as % of original
10	6	26	0	100		
11	6.1	27	- 8	99.82		
12	6.35	26	- 3	99.93		
13	6.6	26	9	100.21		
14	7	25	34	100.79	0	100
15	7.1	25	42	100.97	- 7	99.84
16	7.4	25	49	101.13	8	100.19
17	7.9	25	66	101.52	30	100.69
18	8.4	24	82	101.89	50	101.16
19	9.1	24	103	102.38	75	101.73
20	9.6	22	119	102.75	95	102.2
21	10.1	21	132	103.05	109	102.52

Average winding tension = 25.5 g.

During unwinding

Table B.31

No	cheese dia. cm	A1 g. length change .001 in	A1 g. length as % of original	A2 g. length change .001 in	A2 g. length as % of original	A3 g. length change .001 in	A3 g. length as % of original	A4 g. length change .001 in	A4 g. length as % of original
1	10.1	148	103.45	153	103.6	132	103.05	109	102.52
2	9.45	147	103.43	152	103.57	131	103.03	108	102.5
3	9.2	143	103.33	151	103.55	127	102.93	105	102.43
4	8.6	142	103.31	148	103.48	122	102.82	95	102.2
5	8.05	141	103.29	146	103.43	117	102.7	87	102.01
6	7.35	138	103.22	140	103.29	104	102.4	80	101.85
7	7	134	103.12	137	103.22	98	102.26	51	101.18
8	6.4	129	103.01	128	103.01	74	101.71		
9	6	123	102.87	116	102.73	59	101.36		
10	5.45	112	102.61	101	102.37				
11	5	100	102.33	71	101.67				
12	4.45	82	101.91						
13	4	49	101.14						

TEST 12.i

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D; average winding tension for preparing the cheese base = 6.3 g; axial gauge length = 3.87 in.

Table B.32

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g. length /.001 in.	g.length as % of original	cheese dia. cm	change in g. length /.001 in.	g.length as % of original
1	4	-	0	100	10.2	301	107.77
2	4.15	7	- 3	99.92	9.55	310	108.01
3	4.55	8	- 3	99.92	8.8	308	107.96
4	5.9	9	42	101.08	8.05	295	107.62
5	6.8	9	82	102.12	7.3	290	107.49
6	7.65	9	129	103.33	6.45	269	106.95
7	8.55	8	196	105.06	5.65	248	106.41
8	9.3	8	251	106.49	5.05	211	105.45
9	10.2	8	301	107.77	4.4	163	104.21
10					4	124	103.2

Average winding tension = 8.3 g.

TEST 12.ii

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D; average winding tension for preparing the cheese base = 122 g; axial gauge length = 3.829 in.

2

Table B.33

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	9.9	276	107.21
2	4.2	11	- 8	99.79	8.9	288	107.52
3	4.65	12	- 1	99.97	8.35	288	107.52
4	5.55	13	25	100.65	7.45	286	107.46
5	6.65	14	62	101.62	6.45	275	107.18
6	7.3	13	108	102.82	5.65	236	106.16
7	8.25	13	162	104.23	4.95	196	105.12
8	9.15	13	219	105.72	4.45	164	104.28
9	9.9	13	276	107.21	4	120	103.14

Average winding tension = 12.8 g.

TEST 12.iii

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D; average winding tension for preparing the cheese base = 18.2 g; axial gauge length = 3.8 in;

Table B.34

No	during winding: cheese dia. cm	winding tension g	change in g.length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	9,95	219	105.76
2	4.15	19	- 6	99.84	8.85	228	106
3	4.55	19	- 4	99.89	8.4	233	106.13
4	5.7	20	28	100.74	7.4	228	106
5	6.7	19	54	101.42	6.55	220	105.79
6	8	19	98	102.58	5.65	198	105.21
7	8.7	19	132	103.48	5.05	171	104.5
8	9.3	19	168	104.42	4.35	150	103.95
9	9.95	17	219	105.76	4	103	102.71

Average winding tension = 18.9 g.

TEST 12.iv

Space between adjacent wraps = 2D; dia. of cheese base = 4 cm; average winding tension for preparing the cheese base = 22.7 g; axial gauge length = 3.78 in;

First winding and unwinding.

Table B.35

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	9.75	247	106.54
2	4.1	25	- 6	99.84	8.9	252	106.66
3	4.5	23	- 3	99.92	7.95	257	106.8
4	5.1	22	10	100.26	7.1	247	106.54
5	5.85	22	40	101.06	6.35	234	106.18
6	6.8	24	87	102.3	5.65	229	106.06
7	7.6	25	143	103.78	5.2	211	105.58
8	8.4	20	190	105.02	4.6	182	104.82
9	9.15	22	208	105.5	4	131	103.46
10	9.75	22	247	106.54			

Average winding tension = 22.8 g.

Second winding and unwinding

Table B.36

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g.length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	131	103.46	9.85	362	109.56
2	4.15	22	112	102.96	8.85	367	109.7
3	4.65	22	120	103.17	8.2	362	109.56
4	5.2	22	134	103.54	7.25	354	109.36
5	5.95	22	160	104.23	6.45	337	108.91
6	7.1	20	225	105.95	5.6	315	108.34
7	7.9	22	261	106.9	4.9	291	107.7
8	8.75	21	298	107.88	4.4	254	106.72
9	9.2	24	333	108.81	4	187	104.94
10	9.85	22	362	109.56			

Average winding tension = 21.9 g.

Third winding and unwinding

Table B.37

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		187	104.94	10.15	392	110.38
2	4.15	24	162	104.28	9.45	403	110.68
3	4.5	21	174	104.6	8.65	399	110.56
4	5.5	25	202	105.34	7.75	392	110.38
5	6.5	22	252	106.66	6.8	384	110.16
6	7.3	22	287	107.6	5.8	372	109.84
7	8.15	22	326	108.68	5	337	108.92
8	9.1	22	360	109.52	4.4	304	108.04
9	10.15	25	392	110.38	4	212	105.61

Average winding tension = 22.9 g.

TEST 12.v

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D; average winding tension for preparing the cheese base = 27.4 g; axial gauge length = 3.85 in.

Table B.38

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	9.9	232	106.03
2	4.1	27	- 9	99.77	9.05	232	106.03
3	4.55	27	- 6	99.84	8.3	227	105.9
4	5.15	27	5	100.13	7.5	224	105.82
5	5.95	28	34	100.84	6.8	217	105.64
6	6.95	27	80	102.08	5.85	202	105.25
7	7.55	26	124	103.22	5	177	104.6
8	8.55	26	179	104.65	4.3	148	103.85
9	9.3	29	206	105.41	4	110	102.86
10	9.9	26	232	106.03			

Average winding tension = 27 g.

TEST 12.vi

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D; average winding tension for preparing the cheese base = 32.3 g; axial gauge length = 3.8 in.

Table B.39

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	10.4	260	106.84
2	4.1	33	- 7	99.82	9.15	263	106.92
3	4.5	32	- 7	99.82	8.4	266	107
4	5.55	32	19	100.5	7.5	267	107.03
5	6.3	32	56	101.47	6.75	255	106.72
6	7.35	33	113	102.88	5.9	235	106.19
7	8.45	33	165	104.34	4.95	209	105.5
8	9.4	32	215	105.66	4.4	174	104.58
9	10.4	32	260	106.84	4	129	103.4

Average winding tension = 32.4 g.

TEST 12.vii

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D;

average winding tension for preparing the cheese base = 35 g.

axial gauge length = 3.805 in.

First winding and unwinding

Table B.40

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	10	289	107.6
2	4.15	35	- 10	99.74	9.25	288	107.56
3	4.45	40	- 10	99.74	8.7	287	107.54
4	5.5	36	22	100.58	7.85	286	107.51
5	6.15	34	63	101.65	6.75	280	107.35
6	7.1	34	112	102.94	6.15	271	107.12
7	7.95	34	178	104.67	5.25	259	106.8
8	8.7	35	229	106.02	4.45	219	105.75
9	9.15	32	257	106.75	4	155	104.07
10	10	34	289	107.6			

Average winding tension = 35 g.

Second winding and unwinding

Table B.41

No	during winding: cheese dia. cm	winding tension g	change in g. length /.001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length /.001 in.	g.length as % of original
1	4	-	155	104.07	9.75	359	109.44
2	4.1	35	143	103.76	9	366	109.6
3	4.45	35	140	103.68	8.35	371	109.75
4	4.95	35	161	104.23	7.6	375	109.85
5	5.95	35	201	105.26	6.55	363	109.55
6	6.75	35	237	106.22	5.7	348	109.14
7	7.85	34	285	107.49	5.05	326	108.56
8	8.85	34	314	108.25	4.55	297	107.8
9	9.3	34	331	108.7	4	197	105.17
10	9.75	34	359	109.44			

Average winding tension = 34.6 g.

Third winding and unwinding

Table B.42

No	during winding: cheese dia. cm	winding tension g	change in g. length /.001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	197	105.17	10.05	423	111.12
2	4.15	35	185	104.86	9.1	427	111.23
3	4.5	35	195	105.12	8.3	424	111.13
4	5.2	35	230	106.04	7.5	418	110.99
5	6.3	35	283	107.44	6.8	409	110.73
6	7.15	35	323	108.48	6.05	405	110.62
7	7.8	35	348	109.14	5.2	383	110.06
8	8.35	35	378	109.93	4.4	333	109.24
9	9.05	35	401	110.52	4	222	105.83
10	9.55	35	410	110.78			
11	10.05	35	423	111.12			

Average winding tension = 35 g.

TEST 12.viii

Dia. of cheese base = 4 cm; space between adjacent wraps = 2D;

average winding tension for preparing the cheese base = 41.6 g;

axial gauge length = 3.86 in.

Table B.43

	during winding				during unwinding		
No	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100	9.9	264	106.84
2	4.15	44	- 7	99.82	9.1	270	107
3	4.55	42	- 6	99.84	8.1	273	107.08
4	5.15	42	17	100.44	7.2	266	106.89
5	6.25	40	68	101.76	6.2	255	106.61
6	7.15	40	134	103.47	5.5	243	106.3
7	7.9	39	174	104.51	4.95	225	105.83
8	8.5	40	207	105.36	4.3	183	104.74
9	9.45	40	251	106.5	4	145	103.76
10	9.9	40	264	106.84			

Average winding tension = 40.8 g.

TEST 13

Dia. of cheese base = 3 cm; space between adjacent wraps = 2D;

average winding tension for preparing the cheese base = 23.4 g;

gauge dia. d1 = 3 cm; axial g. length b1 = 3.8 in;

gauge dia. d2 = 4 cm; axial g. length b2 = 3.93 in;

gauge dia. d3 = 5 cm; axial g. length b3 = 3.99 in;

gauge dia. d4 = 6 cm; axial g. length b4 = 3.96 in;

gauge dia. d5 = 7 cm; axial g. length b5 = 4 in;

gauge dia. d6 = 8 cm; axial g. length b6 = 4 in;

gauge dia. d7 = 9 cm; axial g. length b7 = 3.99 in.

During winding

Table B.44

No	cheese dia. cm	winding tension g	A1 g.length change /.001 in.	A1 g.length as % of original	A2 g.length change /.001 in.	A2 g.length as % of original
1	3	-	0	100		
2	3.2	23	- 8	99.79		
3	3.5	23	- 10	99.74		
4	4	24	- 7	99.82	0	100
5	4.1	24	- 2	99.95	- 12	99.69
6	4.45	24	0	100	- 13	99.67
7	5	25	12	100.32	10	100.25
8	5.15	24	14	100.37	14	100.36
9	5.4	24	17	100.45	24	100.61
10	6	24	31	100.82	45	101.14
11	6.15	25				
12	6.45	24				
13	7	24	59	101.55	104	102.64
14	7.1	24				
15	7.4	24				
16	8	24	131	103.44	262	106.66
17	8.15	25				
18	8.45	24				
19	9	24	110	102.89	214	105.44
20	9.15	24				
21	9.45	23				
22	10	23	131	103.44	262	106.66

Average winding tension = 24 g.

During winding

Table B.44 (continued)

No	cheese dia. cm	winding tension g	A3 g.length change .001 in.	A3 g.length as % of original	A4 g.length change .001 in.	A4 g.length as % of original
7	5	25	0	100		
8	5.15	24	- 10	99.75		
9	5.4	24	- 1	99.97		
10	6	24	38	100.95	0	100
11	6.15	25			- 6	99.85
12	6.45	24			2	100.05
13	7	24	111	102.78	58	101.45
14	7.1	24				
15	7.4	24				
16	8	24	332	108.32	317	107.92
17	8.15	25				
18	8.45	24				
19	9	24	263	106.59	234	105.85
20	9.15	24				
21	9.45	23				
22	10	23	332	108.32	317	107.92

Average winding tension = 24 g.

Table B.45

No	cheese dia. cm	winding tension g	A5 g.length change .001 in.	A5 g.length as % of original	A6 g.length change .001 in.	A6 g.length as % of original	A7 g.length change .001 in.	A7 g.length as % of original
13	7	24	0	100				
14	7.1	24	- 5	99.87				
15	7.4	24	- 2	99.95				
16	8	24	83	102.08	0	100		
17	8.15	25			- 3	99.92		
18	8.45	24			11	100.28		
19	9	24	187	104.67	100	102.5	0	100
20	9.15	24					4	100.1
21	9.45	23					17	100.43
22	10	23	277	106.92	200	105	92	102.3

During unwinding

Table B.46

No	cheese dia. cm	A1 g. length change .001in	A1 g. length as % of original	A2 g. length change .001in	A2 g. length as % of original	A3 g. length change .001in	A3 g. length as % of original	A4 g. length change .001in	A4 g. length as % of original
1	10	131	103.44	262	106.66	332	108.32	317	107.92
2	9.45								
3	9	131	103.44	262	106.66	333	108.35	317	107.92
4	8.25								
5	8	131	103.44	262	106.66	333	108.35	317	107.92
6	7.2								
7	7	122	103.21	251	106.38	323	108.1	300	107.5
8	6.4							275	106.9
9	6	110	102.89	229	105.83	283	107.09	235	105.88
10	5.25					251	106.29		
11	5	90	102.37	174	104.42	197	104.94		
13	4.3			141	103.58				
14	4	70	101.84	118	103.01				
15	3.55	50	101.32						
16	3	41	101.08						

Table B.47

No	cheese dia. cm	A5 g.length change .001 in.	A5 g.length as % of original	A6 g.length change .001 in.	A6 g.length as % of original	A7 g.length change .001 in.	A7 g.length as % of original
1	10	277	106.92	200	105	92	102.3
2	9.45					81	102.07
3	9	280	107	193	104.82	64	101.6
4	8.25			185	104.62		
5	8	274	106.85	170	104.25		
6	7.2	240	106				
7	7	210	105.25				

TEST 14.i

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 27 g;

axial gauge length = 4.228 in.

Table B.48

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length /.001 in.	g.length as % of original	cheese dia. cm	change in g.length /.001 in.	g.length as % of original
1	4		0	100	10	226	105.42
2	4.05	37	- 4	99.9	9.3	224	105.37
3	4.20	37	- 6	99.86	8.75	224	105.37
4	4.45	38	- 3	99.93	8	222	105.33
5	4.8	37	9	100.22	7.15	217	105.21
6	5.6	37	51	101.22	6.6	210	105.04
7	6.9	35	111	102.66	5.85	201	104.82
8	7.7	33	151	103.62	5.3	189	104.54
9	8.6	32	183	104.39	4.4	132	103.17
10	9.55	30	209	105.02	4	94	102.26
11	10	29	226	105.42			

Average winding tension = 34.5 g.

TEST 14.ii

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 27 g;

axial gauge length = 4.163 in.

Table B.49

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		0	100	10.05	213	105.11
2	4.1	34	- 6	99.86	9.45	213	105.11
3	4.15	34	- 8	99.81	8.7	209	105.02
4	4.5	34	- 2	99.95	8.05	205	104.92
5	5.6	35	51	101.22	7.3	203	104.87
6	6.5	33	87	102.09	6.6	197	104.73
7	7.35	31	122	102.93	6	189	104.54
8	8.25	30	153	103.68	5.4	176	104.22
9	9.25	29	185	104.44	4.8	158	103.79
10	10.05	27	213	105.11	4.2	128	103.07
11					4	103	102.47

Average winding tension = 31.9 g.

TEST 14.iii

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26 g;

axial gauge length = 4.225 in.

Table B.50

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		0	100	10	188	104.44
2	4.1	31	- 9	99.75	9.3	188	104.44
3	4.2	32	- 8	99.81	8.4	186	104.39
4	4.4	33	- 1	99.98	7.8	184	104.35
5	5.05	32	24	100.57	7.25	181	104.28
6	5.75	30	48	101.13	6.5	176	104.16
7	6.6	30	84	101.99	5.85	169	103.99
8	7.3	29	116	102.74	5.4	161	103.8
9	8.1	27	142	103.36	4.7	142	103.36
10	8.9	27	160	103.78	4.2	109	102.58
11	10	26	188	104.44	4	84	101.99

Average winding tension = 29.7 g.

TEST 14.iv

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.5 g;

axial gauge length = 4.176 in.

Table B.51

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		0	100	10	153	103.67
2	4.1	27	- 4	99.9	9.55	153	103.67
3	4.25	28	- 1	99.98	9.15	153	103.67
4	4.95	29	17	100.41	8.5	151	103.62
5	6.2	27	54	101.3	7.7	149	103.58
6	6.75	27	71	101.7	6.75	140	103.36
7	7.95	27	105	102.52	6	130	103.12
8	8.7	25	126	103.02	5.4	122	102.93
9	9.65	25	142	103.41	4.8	99	102.38
10	10	25	153	103.67	4.45	80	101.92
11					4	64	101.54

Average winding tension = 26.7 g.

TEST 14.v

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.5 g;

axial gauge length = 4.2 in.

Table B.52

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		0	100.00	10	160	103.81
2	4.1	27	- 6	99.86	9.4	160	103.81
3	4.3	28	3	100.07	8.85	160	103.81
4	5	29	16	100.38	8.3	156	103.71
5	6.2	28	58	101.38	7.7	153	103.64
6	7.1	27	89	102.12	6.95	149	103.55
7	8.05	26	119	102.83	6.5	146	103.47
8	8.6	25	131	103.12	5.8	137	103.26
9	9.05	25	143	103.4	5.15	125	102.98
10	10	25	160	103.81	4.75	102	102.43
11					4.3	68	101.62
12					4	60	101.43

Average winding tension = 26.7 g.

TEST 14.vi

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.2 g;

axial gauge length = 4.235 in.

Table B.53

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4		0	100.00	10	118	102.79
2	4.15	24	- 5	99.88	9.5	117	102.76
3	4.4	24	- 4	99.91	8.8	115	102.71
4	5.3	24	11	100.3	8.3	114	102.69
5	6.15	23	32	100.76	7.55	111	102.62
6	6.75	20	47	101.11	6.8	108	102.55
7	7.65	19	65	101.53	5.95	98	102.31
8	7.9	25	75	101.77	5.55	95	102.24
9	8.6	22	94	102.22	5	79	101.86
10	9.4	21	107	103.01	4.3	65	101.53
11	10	20	118	102.75	4	48	101.13

Average winding tension = 22.2 g.

TEST 14.vii

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.3 g;

axial gauge length = 4.148 in.

Table B.54

No	during winding: cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	during unwinding: cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100.00	10.1	110	102.65
2	4.1	20	0	100.00	9.5	109	102.62
3	4.2	20	- 3	99.93	9	108	102.6
4	4.5	21	1	100.02	8.25	106	102.55
5	5.05	21	10	100.24	7.6	104	102.5
6	5.85	21	25	100.6	6.85	100	102.41
7	6.85	20	40	100.96	6.1	94	102.27
8	7.65	20	55	101.32	5.6	91	102.19
9	8.65	19	77	101.85	4.9	81	101.95
10	9.35	19	94	102.26	4.5	63	101.52
11	10.1	20	110	102.65	4	43	101.03

Average winding tension = 20.1 g.

TEST 14.viii

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 27.3 g;

axial gauge length = 4.17 in.

Table B.55

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g.length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100.00	10	73	101.75
2	4.1	17	- 1	99.98	9.3	71	101.7
3	4.35	17	- 2	99.95	8.8	69	101.66
4	4.7	17	2	100.05	8	65	101.56
5	5.6	17	12	100.29	7.55	64	101.54
6	6.8	17	29	100.7	6.8	60	101.44
7	7.7	16	41	100.98	6.1	56	101.34
8	8.4	16	53	101.27	5.65	55	101.32
9	9.3	15	64	101.54	4.6	44	101.05
10	10	15	73	101.75	4.2	24	100.58
11					4	19	100.46

Average winding tension = 16.3 g.

TEST 14.ix

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.5 g;

axial gauge length = 4.163 in.

Table B.56

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length /.001 in.	g.length as % of original	cheese dia. cm	change in g. length /.001 in.	g.length as % of original
1	4	-	0	100.00	10	45	101.08
2	4.1	13	0	100.00	9.55	45	101.08
3	4.25	14	- 1	99.98	8.8	43	101.03
4	4.55	14	1	100.02	8.05	38	100.91
5	5.25	14	7	100.17	7.7	35	100.84
6	6	14	11	100.26	7	30	100.72
7	6.7	14	16	100.38	6.45	27	100.65
8	7.95	13	31	100.74	5.8	22	100.53
9	8.9	13	38	100.91	5.15	12	100.29
10	9.65	12	42	101.01	4.4	9	100.22
11	10	12	45	101.08	4	7	100.17

Average winding tension = 13.3 g.

TEST 14.x

Dia. of cheese base = 4 cm;

average winding tension for preparing the cheese base = 26.2 g;

axial gauge length = 4.142 in.

Table B.57

No	during winding				during unwinding		
	cheese dia. cm	winding tension g	change in g. length .001 in.	g.length as % of original	cheese dia. cm	change in g. length .001 in.	g.length as % of original
1	4	-	0	100.00	10	30	100.72
2	4.1	9	0	100.00	9.3	27	100.65
3	4.25	9	1	100.02	8.7	27	100.65
4	4.55	9	3	100.07	8.1	26	100.63
5	5.55	10	8	100.19	7.5	26	100.63
6	6.9	10	15	100.36	6.75	22	100.53
7	7.75	10	19	100.46	6.05	19	100.46
8	8.5	10	21	100.51	5.45	16	100.39
9	9.25	10	24	100.58	4.85	13	100.31
10	10	9	30	100.72	4.3	8	100.19
11					4	5	100.12

Average winding tension = 9.6 g.

TEST 15

Average winding tension for preparing the base for the gauge = 1.5g;

rad. of the base = 2 cm; tension in the gauge wire = 10;

axial distance between the two ends of the wire = 1.6 cm;

length of the wire = 13.6;

resistance of the helically-wound gauge = 296 ohm;

resistance of the straight-wound gauge = 248 ohm.

For helically-wound gauge.

Calibrating resistance = 200,000 ohm; movement on dial = 14 div.;

1 div. of dial = 296 x 100/(200,000 x 14) = 0.0106% change in G.Res.

For straight-wound gauge.

Calibrating resistance = 200,000 ohm; movement on dial = 11.6 div.;

1 div. of dial = 248 x 100/(200,000 x 11.6) = 0.0107% change in G.Res.

Axial gauge length = 3.732 in.

During second winding

Table B.58

No	cheese dia. cm	winding tension g	change in g. length .001 in	g.length as % of original	helically wound		straight wound	
					movement on dial div	% change in G.Res.	movement on dial div.	% change in G.Res.
1	4	-	0	100.00	0	100.00	0	100.00
2	4.1	15	- 8	99.79	3.4	99.96	2.8	99.97
3	4.6	15	- 2	99.95	6.8	99.93	5.6	99.94
4	5.35	15	2	100.05	7.5	99.92	6.1	99.93
5	6.55	15	29	100.78	8	99.92	8.3	99.91
6	7.25	15	49	101.31	7.3	99.92	8.3	99.91
7	8.1	15	76	102.04	7.3	99.92	7.8	99.92
8	8.9	15	96	102.57	7.2	99.92	8	99.91
9	9.8	16	127	103.4	7.5	99.92	9.6	99

Average winding tension = 15.1g.

During unwinding

Table B.59

No	cheese dia. cm	change in g. length .001 in.	g.length as % of original	helically wound		straight wound	
				movement on dial div.	% change in G.Res.	movement on dial div.	% change in G.Res.
1	9.8	127	103.4	7.5	99.92	9.6	99.9
2	9.2	122	103.27	7	99.93	9.3	99.9
3	8.35	122	103.27	6.9	99.93	7.9	99.92
4	7.4	122	103.27	6.7	99.93	7.6	99.92
5	6.45	117	103.14	6.3	99.93	6.3	99.93
6	5.35	104	102.79	5	99.95	5.2	99.94
7	4.7	77	102.06	5	99.95	6.8	99.93
8	4.2	59	101.58	3.5	99.96	4.3	99.95
9	4	32	100.86	0	100.00	2.3	99.98

APPENDIX C

Table C.1

Assumption w = 0; std. size of the element; RO = 5 cm; dR = 0.1 cm;

T = 20g; spacing = 1D;

r or R	Zo			Qo			$\int_r^{RO}$ Zo.dr		
traverse cm	2.5	5	7.5	2.5	5	7.5	2.5	5	7.5
1.0	137.4	389.8	590.7	690.7	489.8	329.9	2382	7548	12881
1.4	101.6	309.3	500.1	715.0	544.2	390.9	1893	6118	10658
1.9	76.2	241.8	410.1	727.5	577.4	435.0	1441	4718	8349
2.4	60.8	197.0	388.2	733.3	594.2	460.0	1094	3606	6431
2.9	50.5	165.6	343.2	736.4	603.7	475.1	812	2687	4830
3.4	43.2	142.7	293.3	738.3	609.5	484.7	575	1908	3443
3.9	37.7	125.2	225.6	739.5	613.3	491.3	370	1231	2229
4.4	33.5	111.4	201.8	740.3	616.0	495.8	191	634	1151
4.9	30.1	100.3	182.4	740.9	619.9	499.1	30	100	183

The values for winding tensions of 10g and 30g are 0.5 and 1.5 times the above values respectively. The values for spacing of 2D and 3D between adjacent wraps of the yarn are $\frac{1}{2}$ and $\frac{1}{3}$ of the above values respectively.

Table C.2

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

r	$\frac{\partial U}{\partial R}.dR$	$\frac{\partial P}{\partial R}.dR$	$\frac{\partial T}{\partial R}.dR$	$\frac{\partial Q}{\partial R}.dR$	$\frac{\partial Z}{\partial R}.dR$
1.0	- 0.000000	- 0.1	- 0.0	0.00	0.00
1.4	- 0.000000	- 0.2	- 0.001	- 0.00	- 0.02
1.9	- 0.000003	- 0.8	- 0.006	- 0.2	- 0.07
2.4	- 0.000011	- 3.0	- 0.021	- 0.6	- 0.20
2.9	- 0.000041	- 10.4	- 0.067	- 2.0	- 0.55
3.4	- 0.000143	- 33.6	- 0.2	- 6.1	- 1.42
3.9	- 0.000458	- 101.4	- 0.564	- 17.3	- 3.53
4.4	- 0.00137	- 287.9	- 1.512	- 46.6	- 8.42
4.9	- 0.0039	- 776.5	- 3.875	-119.7	- 19.39

Table C.3

Cheese No.1

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

-P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2592	684							
2.0	3484	2354	726						
2.5	3716	2789	2473	747					
3.0	3781	2910	2958	2619	759				
3.5	3800	2946	3104	3182	2737	766			
4.0	3807	2958	3152	3365	3377	2834	771		
4.5	3809	2962	3168	3427	3597	3546	2916	774	
5.0	3809	2963	3174	3450	3677	3805	3696	2988	776

$-U \times 10^3$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	1.91							
2.0	0000	6.56	2.49						
2.5	0000	7.78	8.50	2.77					
3.0	0000	8.11	10.16	9.72	3.02				
3.5	0000	8.22	10.67	11.82	10.91	3.26			
4.0	0000	8.25	10.83	12.50	13.46	12.06	3.48		
4.5	0000	8.26	10.88	12.73	14.33	15.09	13.18	3.70	
5.0	0000	8.26	10.91	12.81	14.66	16.19	16.71	14.26	3.90

$$- \int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	5.15							
2.0	0000	17.72	5.58						
2.5	0000	20.99	19.02	5.20					
3.0	0000	21.90	22.76	18.25	4.84				
3.5	0000	22.18	23.88	22.18	17.49	4.54			
4.0	0000	22.26	24.25	23.45	21.58	16.82	4.29		
4.5	0000	22.29	24.37	23.89	22.99	21.04	16.22	4.07	
5.0	0000	22.31	24.42	24.05	23.50	22.58	20.56	15.70	3.87

Table C.4

Cheese no.1

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	489.8	404.0							
2.0	489.8	61.9	416.2						
2.5	489.8	-27.3	28.1	439.5					
3.0	489.8	-52.0	-79.8	51.8	457.3				
3.5	489.8	-59.5	-112.2	-65.0	75.6	471.0			
4.0	489.8	-61.9	-122.7	-102.7	-47.8	96.9	481.8		
4.5	489.8	-62.7	-126.3	-115.7	-90.3	-31.9	115.7	490.7	
5.0	489.8	-63.0	-127.6	-120.4	-105.7	-78.7	-17.3	132.5	498.2

Z

1.0	0000								
1.5	389.8	229.8							
2.0	389.8	35.7	174.5						
2.5	389.8	-15.0	12.0	145.8					
3.0	389.8	-29.1	-33.1	17.4	125.6				
3.5	389.8	-33.3	-46.7	-21.3	20.9	110.3			
4.0	389.8	-34.7	-51.1	-33.8	-12.9	22.9	98.4		
4.5	389.8	-35.2	-52.7	-38.2	-24.6	- 7.3	23.8	88.8	
5.0	389.8	-35.3	-53.2	-39.7	-28.8	-18.2	- 3.4	24.1	81.0

$$\int_r^R Z.dr$$

1.0	0000								
1.5	1266	230							
2.0	1275	530	174						
2.5	1017	348	428	146					
3.0	762	114	315	394	126				
3.5	550	- 92	146	332	365	110			
4.0	376	-264	- 15	207	337	340	98		
4.5	232	-407	-154	80	244	335	318	89	
5.0	111	-528	-273	- 35	141	265	329	298	81

Table C.5

Cheese no. 1

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R} . dR$$

R\r	10	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-140.2							
2.0	0000	-482.3	-161.2						
2.5	0000	-571.5	-549.3	-154.7					
3.0	0000	-596.2	-657.4	-542.4	-146.4				
3.5	0000	-603.7	-689.6	-659.1	-528.0	-138.5			
4.0	0000	-606.1	-700.1	-696.9	-651.5	-512.6	-131.5		
4.5	0000	-606.9	-703.7	-709.9	-694.0	-641.4	-497.6	-125.3	
5.0	0000	-607.2	-705.0	-714.6	-709.4	-688.2	-630.6	- 48	

$$\int_r^R \frac{\partial Z}{\partial R} . dR$$

R\r	10	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 79.4							
2.0	0000	-273.7	- 67.4						
2.5	0000	-324.3	-229.8	- 51.2					
3.0	0000	-338.4	-275.0	-179.6	- 40.1				
3.5	0000	-342.7	-288.6	-218.3	-144.7	- 32.3			
4.0	0000	-344.0	-293.0	-230.9	-178.6	-119.8	- 26.8		
4.5	0000	-344.5	-294.5	-235.2	-190.2	-149.9	-101.4	- 22.6	
5.0	0000	-344.7	-295.0	-236.7	-194.5	-160.9	-128.5	- 87.3	- 19.4

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr$$

R\r	10	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 472	- 79							
2.0	-1796	-1112	- 67						
2.5	-3122	-2362	- 882	- 51					
3.0	-4263	-3482	-1881	- 690	- 40				
3.5	-5233	-4445	-2808	-1508	- 557	- 32			
4.0	-6066	-5276	-3627	- 2292	-1244	- 462	- 27		
4.5	-6793	-6003	-4349	-3003	-1921	-1050	- 390	- 23	
5.0	-7437	-6646	-4992	-3641	-2546	-1643	- 902	- 336	- 19

Table C.6

Cheese no. 2

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 30g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	3887	1026							
2.0	5225	3530	1088						
2.5	5574	4183	3709	1120					
3.0	5670	4364	4436	3928	1138				
3.5	5699	4419	4656	4773	4105	1149			
4.0	5709	4437	4727	5047	5065	4251	1156		
4.5	5712	4443	4752	5141	5396	5319	4375	1161	
5.0	5713	4445	4760	5175	5516	5707	5544	4481	1165

$- U \times 10^3$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	2.86							
2.0	0000	9.84	3.74						
2.5	0000	+11.66	12.74	4.16					
3.0	0000	12.17	15.25	14.87	4.54				
3.5	0000	12.32	16.00	17.73	16.36	4.89			
4.0	0000	12.37	16.24	18.74	20.19	18.09	5.23		
4.5	0000	12.39	16.33	19.09	21.51	22.64	19.77	5.54	
5.0	0000	12.39	16.36	19.22	21.98	24.29	25.06	21.40	5.85

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 7.72							
2.0	0000	-26.59	- 8.37						
2.5	0000	-31.48	-28.53	- 7.81					
3.0	0000	-32.85	-34.13	-27.38	- 7.27				
3.5	0000	-33.26	-35.82	-33.27	-26.24	- 6.82			
4.0	0000	-33.39	-36.37	-35.18	-32.37	-25.23	- 6.43		
4.5	0000	-33.44	-36.55	-35.84	-34.48	-31.57	-24.34	- 6.09	
5.0	0000	-33.45	-36.62	-36.07	-35.25	-33.87	-30.84	-23.54	- 5.81

Table C.7

Cheese no. 2

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 30g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	734.7	606.0							
2.0	734.7	92.7	624.2						
2.5	734.7	41.0	42.1	659.2					
3.0	734.7	- 78.1	-119.7	77.6	685.9				
3.5	734.7	- 89.3	-168.4	- 97.5	113.4	706.5			
4.0	734.7	- 92.9	-184.1	-154.1	- 71.7	145.3	722.8		
4.5	734.7	- 94.2	-189.5	-173.6	-135.5	- 47.8	173.6	736.1	
5.0	734.7	- 94.6	-191.5	-180.7	-158.6	-118.0	- 25.9	198.8	747.2

Z

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	000								
1.5	584.7	345.0							
2.0	584.7	53.8	261.8						
2.5	584.7	-22.2	18.3	218.8					
3.0	584.7	-43.3	-49.5	26.3	188.4				
3.5	584.7	-49.7	-69.9	-31.8	31.6	165.5			
4.0	584.7	-51.7	-76.5	-50.6	-19.2	34.4	147.6		
4.5	584.7	-52.5	-78.8	-57.1	-36.8	-10.8	35.8	133.3	
5.0	584.7	-52.7	-79.7	-59.4	-43.1	-27.2	- 4.9	36.3	121.5

$\int_r^R Z.dr$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	1901	345							
2.0	1915	796	262						
2.5	1529	524	642	219					
3.0	1148	174	475	592	188				
3.5	829	- 134	221	500	548	166			
4.0	569	- 392	- 19	313	507	511	148		
4.5	353	- 606	- 228	122	367	504	477	133	
5.0	172	- 787	- 406	- 50	214	398	495	448	121

Table C.8

Cheese no. 2

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 30g; traverse = 5 cm; space = 1D; E = 200g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-210.3							
2.0	0000	-723.6	-241.9						
2.5	0000	-857.3	-824.0	-232.0					
3.0	0000	-894.4	-985.7	-813.6	-219.6				
3.5	0000	-905.6	-1034.4	-988.7	-792.1	-207.8			
4.0	0000	-909.2	-1050.2	-1045	-977.2	-769.0	-197.3		
4.5	0000	-910.5	-1056	-1065	-1041	-962.1	-746.4	-187.9	
5.0	0000	-910.9	-1058	-1072	-1064	-1032	-945.9	-725.2	-179.6

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

1.0	0000								
1.5	0000	-119.0							
2.0	0000	-410.2	-101.0						
2.5	0000	-486.2	-344.5	- 76.7					
3.0	0000	-507.3	-412.3	-269.2	- 60.0				
3.5	0000	-513.7	-432.7	-327.3	-216.9	- 48.7			
4.0	0000	-515.7	-439.3	-346.1	-2.677	-179.6	- 40.1		
4.5	0000	-516.5	-441.6	-352.6	-285.2	-224.8	-152.0	- 33.9	
5.0	0000	-516.7	-442.4	-354.0	-291.6	-241.2	-192.7	-130.8	- 29.1

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr$$

1.0	0000								
1.5	- 708	- 119							
2.0	- 2692	-1666	- 101						
2.5	- 4680	-3541	-1322	- 77					
3.0	- 6391	-5220	-2820	-1035	- 60				
3.5	- 7844	-6664	-4209	-2261	- 835	- 48			
4.0	- 9093	-7910	-5437	-3436	-1864	- 692	- 40		
4.5	-10184	-8999	-6521	-4502	-2879	-1573	- 585	- 34	
5.0	-11150	-9965	-7484	-5458	-3817	-2463	-1353	- 503	- 29

Table C.9

Cheese no.3

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 10g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	1296	342							
2.0	1742	1177	363						
2.5	1858	1395	1236	373					
3.0	1890	1455	1479	1309	379				
3.5	1901	1473	1552	1591	1368	383			
4.0	1904	1479	1576	1682	1688	1417	385		
4.5	1905	1481	1584	1713	1799	1773	1458	387	
5.0	1905	1482	1587	1725	1839	1903	1848	1494	388

$- U \times 10^3$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	0.95							
2.0	0000	3.28	1.25						
2.5	0000	3.89	4.25	1.39					
3.0	0000	4.06	5.08	4.86	1.51				
3.5	0000	4.11	5.33	5.91	5.45	1.63			
4.0	0000	4.12	5.42	6.25	6.73	6.03	1.74		
4.5	0000	4.13	5.44	6.36	7.17	7.54	6.59	1.85	
5.0	0000	4.13	5.45	6.41	7.33	8.10	8.35	7.13	1.95

$$\int_r^R \frac{\partial T}{\partial R} \cdot dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-2.57							
2.0	0000	-8.86	-2.79						
2.5	0000	-10.50	-9.51	-2.60					
3.0	0000	-10.95	-11.38	-9.13	-2.42				
3.5	0000	-11.09	-11.94	-11.09	-8.75	-2.27			
4.0	0000	-11.13	-12.12	-11.73	-10.79	-8.41	-2.14		
4.5	0000	-11.15	-12.19	-11.95	-11.50	-10.52	-8.11	-2.03	
5.0	0000	-11.15	-12.21	-12.03	-11.75	-11.29	-10.28	-7.84	-1.94

Table C.10

Cheese no. 3

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 10g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	244.9	202.0							
2.0	244.9	30.9	208.1						
2.5	244.9	-13.6	14.1	219.8					
3.0	244.9	-26.0	-40.0	25.9	228.7				
3.5	244.9	-29.7	-56.1	-32.5	37.8	235.5			
4.0	244.9	-30.9	-61.3	-51.3	-23.9	48.4	240.9		
4.5	244.9	-31.3	-63.2	-57.8	-45.1	-15.9	57.9	245.4	
5.0	244.9	-31.5	-63.8	-60.2	-52.9	-39.3	- 8.6	66.3	249.1

Z

1.0	0000								
1.5	194.9	114.9							
2.0	194.9	17.7	87.2						
2.5	194.9	- 7.6	6.0	72.9					
3.0	194.9	-14.6	-16.6	8.6	62.8				
3.5	194.9	-16.8	-23.4	-10.7	10.4	55.1			
4.0	194.9	-17.5	-25.6	-17.0	- 6.5	11.4	49.2		
4.5	194.9	-17.7	-26.4	-19.1	-12.3	- 3.7	11.8	44.4	
5.0	194.9	-17.8	-26.7	-19.9	-14.5	- 9.2	- 1.7	12.0	40.5

$\int_r^R Z.dr$

1.0	0000								
1.5	632.9	114.9							
2.0	636.8	264.6	87.2						
2.5	507.3	173.2	213.5	72.9					
3.0	379.9	56.2	157.1	196.5	62.8				
3.5	273.4	- 47.1	72.0	165.7	182.4	55.1			
4.0	186.1	-133.3	- 8.3	102.9	168.3	169.9	49.2		
4.5	113.9	-205.1	- 78.1	39.1	121.3	167.2	158.8	44.4	
5.0	53.2	-265.7	-138.0	- 18.5	69.7	131.7	164.1	149.0	40.5

Table C.11

Cheese no. 3

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 10g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 70.1							
2.0	0000	-241.1	- 80.6						
2.5	0000	-285.7	-274.6	- 77.3					
3.0	0000	-298.1	-328.6	-271.2	- 73.2				
3.5	0000	-301.8	-344.8	-329.6	-264.0	- 69.3			
4.0	0000	-303.0	-350.0	-348.4	-325.7	-256.3	- 65.7		
4.5	0000	-303.4	-351.9	-354.9	-347.0	-320.7	-248.8	- 62.6	
5.0	0000	-303.6	-352.5	-357.3	-354.7	-344.1	-315.3	-241.2	- 59.9

$$\int_r^R \frac{\partial Z}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 39.8							
2.0	0000	-136.9	- 33.7						
2.5	0000	-162.3	-115.0	- 25.6					
3.0	0000	-169.3	-137.5	- 89.9	- 20.1				
3.5	0000	-171.4	-144.3	-109.2	- 72.4	- 16.1			
4.0	0000	-172.1	-146.5	-115.5	- 89.3	- 59.9	- 13.4		
4.5	0000	-172.4	-147.3	-117.6	- 95.2	- 75.0	- 50.7	- 11.3	
5.0	0000	-172.4	-147.6	-118.4	- 97.3	- 80.5	- 64.3	- 43.7	- 9.7

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	-236.5	- 39.8							
2.0	-898.8	-556.3	- 33.7						
2.5	-1562	-1182	-441	- 25.6					
3.0	-2133	-1742	-941	- 345	- 20.1				
3.5	-2617	-2224	-1405	- 755	- 279	- 16.2			
4.0	-3035	-2639	-1814	-1147	- 622	- 231	- 13.4		
4.5	-3398	-3003	-2176	-1502	- 961	- 525	- 195	- 11.3	
5.0	-3721	-3325	-2497	-1821	-1274	- 822	- 452	- 168	- 9.7

Table C.12

Cheese no. 4

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 2D; E = 200g; EY = 2000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	1345	342							
2.0	1889	1257	363						
2.5	2056	1536	1305	373					
3.0	2109	1626	1608	1377	379				
3.5	2128	1657	1711	1722	1436	383			
4.0	2134	1688	1749	1849	1823	1484	386		
4.5	2137	1673	1764	1897	1972	1910	1524	387	
5.0	2138	1674	1770	1917	2033	2083	1986	1559	388

$- U \times 10^3$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	4.05							
2.0	0000	14.89	5.52						
2.5	0000	18.20	19.88	6.18					
3.0	0000	19.27	24.49	22.79	6.74				
3.5	0000	19.63	26.07	28.50	25.51	7.27			
4.0	0000	19.76	26.64	30.58	32.38	28.15	7.64		
4.5	0000	19.81	26.87	31.40	35.04	36.23	30.71	8.24	
5.0	0000	19.84	26.96	31.73	36.12	39.51	40.02	33.18	8.69

$\int_r^R \frac{\partial T}{\partial R} . dR$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 4.37							
2.0	0000	-16.07	- 4.95						
2.5	0000	-19.65	-17.81	- 4.64					
3.0	0000	-20.80	-21.93	-17.11	- 4.32				
3.5	0000	-21.20	-23.34	-21.40	-16.36	- 4.05			
4.0	0000	-21.34	-23.86	-22.97	-20.77	-15.70	- 3.82		
4.5	0000	-21.40	-24.06	-23.57	-22.48	-20.21	-15.12	- 3.63	
5.0	0000	-21.42	-24.15	-23.82	-23.17	-22.04	-19.70	-14.60	- 3.46

Table C.13

Cheese no. 4

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 2D; E = 200g; EY = 2000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	244.9	212.5							
2.0	244.9	53.0	217.2						
2.5	244.9	4.3	31.5	288.1					
3.0	244.9	-11.3	-28.0	42.8	236.6				
3.5	244.9	-16.7	-48.5	-20.9	54.8	242.9			
4.0	244.9	-18.6	-55.9	-44.2	-11.7	65.4	248.0		
4.5	244.9	-19.4	-58.9	-53.2	-37.5	- 3.2	74.8	252.1	
5.0	244.9	-19.7	-60.0	-56.9	-47.9	-31.1	4.5	83.1	255.6

Z

1.0	0000								
1.5	194.9	121.0							
2.0	194.9	30.8	91.2						
2.5	194.9	3.1	13.6	75.8					
3.0	194.9	- 5.7	-11.3	14.5	65.0				
3.5	194.9	- 8.8	-19.9	- 6.6	15.3	57.0			
4.0	194.9	- 9.9	-23.1	-14.3	- 2.9	15.5	50.7		
4.5	194.9	-10.3	-24.3	-17.3	-10.0	- 0.5	15.5	45.7	
5.0	194.9	-10.5	-24.8	-18.6	-12.9	- 7.0	1.1	15.2	41.6

$\int_r^R Z.dr$

1.0	0000								
1.5	670.7	121.0							
2.0	729.7	319.2	91.2						
2.5	621.7	253.9	249.3	75.8					
3.0	499.7	145.5	212.8	224.5	65.0				
3.5	393.2	43.7	135.7	211.3	205.4	57.0			
4.0	304.5	- 43.2	57.8	156.7	207.2	189.3	50.7		
4.5	230.7	-116.3	- 11.9	96.1	168.4	201.1	175.5	45.7	
5.0	168.6	-178.3	- 72.4	39.3	120.5	173.7	194.0	163.6	41.6

Table C.14

Cheese no. 4

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 2D; E = 200g; EY = 2000g;

$$\int_r^R \frac{\partial Q}{\partial R} \cdot dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 59.7							
2.0	0000	-219.1	- 71.5						
2.5	0000	-267.8	-257.2	- 68.9					
3.0	0000	-283.4	-316.7	-254.3	- 65.3				
3.5	0000	-288.8	-337.2	-318.0	-247.0	- 61.8			
4.0	0000	-290.7	-344.7	-341.3	-313.6	-239.3	- 58.6		
4.5	0000	-291.5	-347.6	-350.3	-339.3	-308.0	-231.9	- 55.9	
5.0	0000	-291.5	-348.7	-354.0	-349.7	-335.8	-302.2	-224.9	- 53.4

$$\int_r^R \frac{\partial Z}{\partial R} \cdot dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 33.6							
2.0	0000	-123.8	- 29.8						
2.5	0000	-151.5	-107.3	- 22.7					
3.0	0000	-160.4	-132.3	- 84.0	- 17.8				
3.5	0000	-163.5	-140.8	-105.1	- 67.5	- 14.4			
4.0	0000	-164.6	-144.0	-112.9	- 85.8	- 55.8	- 11.9		
4.5	0000	-165.0	-145.2	-115.8	- 92.8	- 71.8	- 47.1	- 10.0	
5.0	0000	-165.2	-145.6	-117.1	- 95.7	- 78.4	- 61.4	- 40.5	- 8.6

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} \cdot dR \cdot dr$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 199	- 34							
2.0	- 805	- 502	- 30						
2.5	-1448	-1101	- 406	- 23					
3.0	-2013	-1653	- 885	- 317	- 18				
3.5	-2498	-2133	-1341	- 709	- 256	- 14			
4.0	-2916	-2549	-1748	-1093	- 583	- 212	- 12		
4.5	-3282	-2914	-2110	-1445	- 914	- 491	- 179	- 10	
5.0	-3605	-3237	-2432	-1763	-1223	- 780	- 422	- 154	- 9

Table C.15

Cheese no.5

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 3D; E = 200g; EY = 2000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	834	228							
2.0	1082	742	242						
2.5	1138	858	786	249					
3.0	1151	886	919	835	253				
3.5	1155	894	955	992	874	255			
4.0	1156	896	965	1037	1054	907	257		
4.5	1156	897	968	1051	1110	1109	935	258	
5.0	1157	897	969	1056	1128	1176	1158	959	259

- U x 10^3

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	5.41							
2.0	0000	17.61	6.86						
2.5	0000	20.36	22.31	7.61					
3.0	0000	21.04	26.08	25.54	8.30				
3.5	0000	21.22	27.09	30.33	28.71	8.95			
4.0	0000	21.27	27.39	31.72	34.61	31.80	9.56		
4.5	0000	21.29	27.48	32.15	36.44	38.88	34.80	10.14	
5.0	0000	21.29	27.51	32.29	37.04	41.21	43.10	37.71	10.70

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 5.84							
2.0	0000	-19.01	- 6.14						
2.5	0000	-21.99	-19.98	- 5.71					
3.0	0000	-22.72	-23.36	-19.18	- 5.32				
3.5	0000	-22.91	-24.26	-22.74	-18.41	- 4.99			
4.0	0000	-22.97	-24.52	-23.81	-22.20	-17.74	- 4.7		
4.5	0000	-22.98	-24.61	-24.14	-23.37	-21.68	-17.14	- 4.46	
5.0	0000	-22.98	-24.63	-24.24	-23.76	-22.98	-21.22	-16.60	- 4.25

Table C.16

Cheese no. 5

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 3D; E = 200g; EY = 2000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	163.3	128.3							
2.0	163.3	8.7	133.3						
2.5	163.3	-18.3	0.1	141.4					
3.0	163.3	-24.9	-32.5	8.1	147.6				
3.5	163.3	-26.6	-41.2	-27.6	15.9	152.5			
4.0	163.3	-27.1	-43.6	-37.9	-22.2	22.9	156.3		
4.5	163.3	-27.3	-44.4	-41.1	-34.0	-17.2	29.2	159.5	
5.0	163.3	-27.3	-44.7	-42.1	-37.9	-30.3	-12.5	34.9	162.1

Z

1.0	0000								
1.5	129.9	73.1							
2.0	129.9	5.3	55.9						
2.5	129.9	-10.0	0.2	47.0					
3.0	129.9	-13.8	-13.4	2.8	40.6				
3.5	129.9	-14.8	-17.1	- 9.0	4.5	35.7			
4.0	129.9	-15.1	-18.1	-12.4	- 5.9	5.5	32.0		
4.5	129.9	-15.2	-18.4	-13.5	- 9.2	- 3.9	6.1	28.9	
5.0	129.9	-15.2	-18.5	-13.8	-10.3	- 7.0	- 2.4	6.4	26.4

$$\int_r^R Z.dr$$

1.0	0000								
1.5	400.6	73.1							
2.0	377.0	148.4	55.9						
2.5	282.4	76.2	123.8	50.1					
3.0	196.2	- 4.5	77.3	116.4	40.6				
3.5	126.1	-73.2	17.8	87.9	109.6	35.7			
4.0	69.1	-129.7	- 36.1	42.9	92.9	103.1	32.0		
4.5	22.2	-176.6	- 82.1	- 0.3	58.5	94.7	97.2	28.9	
5.0	-17.3	-216.0	-121.3	-38.5	23.1	67.9	94.7	91.8	26.4

Table C.17

Cheese no. 5

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 3D; E = 200g; EY = 2000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-53.1							
2.0	0000	-172.7	-59.2						
2.5	0000	-199.7	-192.4	-56.6					
3.0	0000	-206.3	-224.9	-190.0	-53.6				
3.5	0000	-208.0	-233.6	-225.6	-185.3	-50.7			
4.0	0000	-208.5	-236.1	-235.9	-223.4	-180.2	-48.1		
4.5	0000	-208.7	-236.9	-239.1	-235.2	-220.3	-175.2	-45.9	
5.0	0000	-208.7	-237.2	-240.2	-239.1	-233.5	-217.0	-170.4	-43.8

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

1.0	0000								
1.5	0000	-30.0							
2.0	0000	-97.8	-24.7						
2.5	0000	-113.1	-80.4	-18.7					
3.0	0000	-116.9	-94.0	-62.8	-14.6				
3.5	0000	-117.9	-97.7	-74.6	-50.7	-11.8			
4.0	0000	-118.2	-98.7	-78.1	-61.2	-42.0	-9.8		
4.5	0000	-118.3	-99.0	-79.1	-64.4	-51.4	-35.6	-8.2	
5.0	0000	-118.3	-99.2	-79.5	-65.5	-54.5	-44.1	-30.7	-7.1

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr$$

1.0	0000								
1.5	-179	-30							
2.0	-647	-399	-25						
2.5	-1097	-827	-313	-19					
3.0	-1479	-1203	-655	-245	-15				
3.5	-1801	-1524	-967	-526	-198	-12			
4.0	-2078	-1800	-1240	-790	-434	-164	-8		
4.5	-2319	-2042	-1481	-1028	-663	-367	-139	-8	
5.0	-2533	-2255	-1694	-1240	-873	-568	-316	-120	-7

Table C.18

Cheese no. 6

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 7.5 cm; spacing = 1D; E = 200g; EY = 5000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	1993	491							
2.0	3060	2008	547						
2.5	3449	2562	2073	578					
3.0	3587	2758	2613	2201	597				
3.5	3638	2829	2810	2794	2308	609			
4.0	3657	2857	2885	3021	2961	2395	617		
4.5	3664	2867	2915	3111	3221	3106	2467	623	
5.0	3668	2872	2928	3149	3329	3400	3233	2528	627

$U \times 10^{-3}$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	1.58							
2.0	0000	6.47	2.34						
2.5	0000	8.25	8.86	2.60					
3.0	0000	8.89	11.17	9.88	2.80				
3.5	0000	9.12	12.02	12.55	10.82	2.99			
4.0	0000	9.20	12.34	13.57	13.88	11.77	3.18		
4.5	0000	9.24	12.47	13.97	15.10	15.26	12.72	3.36	
5.0	0000	9.25	12.52	14.12	15.61	16.71	16.66	13.65	3.54

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 3.27							
2.0	0000	-13.38	- 4.41						
2.5	0000	-17.07	-16.73	- 4.34					
3.0	0000	-18.38	-21.08	-16.51	- 4.13				
3.5	0000	-18.85	-22.68	-20.96	-15.95	- 3.92			
4.0	0000	-19.04	-23.28	-22.66	-20.47	-15.41	- 3.73		
4.5	0000	-19.11	-23.53	-23.34	-22.27	-19.98	-14.91	- 3.56	
5.0	0000	-19.14	-23.63	-23.62	-23.01	-21.88	-19.53	-14.45	- 3.41

Table C.19

Cheese no. 6

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 7.5 cm; spacing = 1D; E = 200g; EY = 5000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	329.9	326.8							
2.0	329.9	128.9	339.0						
2.5	329.9	56.7	70.9	360.2					
3.0	329.9	31.2	-23.8	80.1	377.0				
3.5	329.9	21.9	-58.5	-22.3	96.0	389.7			
4.0	329.9	18.3	-71.7	-61.4	-11.3	111.2	399.6		
4.5	329.9	16.9	-77.0	-77.0	-54.0	0.3	125.0	407.5	
5.0	329.9	16.3	-79.2	-83.4	-71.7	-45.6	11.4	137.6	414.0

Z

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	590.7	418.5							
2.0	590.7	166.2	319.8						
2.5	590.7	84.1	67.8	269.3					
3.0	590.7	41.4	-21.9	60.6	233.1				
3.5	590.7	29.4	-54.6	-16.2	59.7	205.5			
4.0	590.7	24.9	-66.9	-45.3	- 6.6	59.1	183.6		
4.5	590.7	23.1	-72.0	-56.9	-33.0	0.6	57.9	165.9	
5.0	590.7	21.8	-73.9	-61.7	-43.8	-23.6	5.6	56.3	151.4

$\int_r^R Z.dr$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2280	420							
2.0	2770	1287	321						
2.5	2496	1170	945	270					
3.0	2120	822	858	831	234				
3.5	1745	318	597	795	756	207			
4.0	1424	153	321	621	777	696	183		
4.5	1154	-117	66	405	642	747	642	168	
5.0	924	-345	-156	198	471	651	717	600	153

Table C.20

Cheese no. 6

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 7.5 cm; spacing = 1D; E = 200g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 64.1							
2.0	0000	-262.1	- 96.0						
2.5	0000	-334.2	-364.1	- 99.8					
3.0	0000	-359.7	-458.8	-379.9	- 98.1				
3.5	0000	-369.1	-493.5	-482.3	-379.1	- 95.0			
4.0	0000	-372.6	-506.7	-521.4	-486.4	-373.5	- 91.6		
4.5	0000	-374.0	-512.0	-537.0	-529.0	-484.4	-366.2	- 88.3	
5.0	0000	-374.6	-514.2	-543.4	-546.8	-530.4	-479.8	-358.2	- 85.1

$$\int_r^R \frac{\partial Z}{\partial R} . dR$$

1.0	0000								
1.5	0000	- 81.6							
2.0	0000	-333.9	- 90.6						
2.5	0000	-426.0	-342.3	- 74.1					
3.0	0000	-458.7	-431.7	-282.9	- 60.3				
3.5	0000	-470.7	-464.4	-359.4	-233.7	- 49.5			
4.0	0000	-475.2	-477.0	-388.5	-300.0	-196.5	- 42.0		
4.5	0000	-477.0	-481.8	-400.2	-326.4	-264.7	-168.0	- 36.0	
5.0	0000	-477.9	-483.9	-405.0	-337.2	-279.0	-219.9	-135.5	- 31.2

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr$$

1.0	0000								
1.5	- 441	- 81							
2.0	- 2172	- 1434	- 90						
2.5	- 4269	- 3393	-1308	- 75					
3.0	- 6225	- 5301	-2955	-1074	- 60				
3.5	- 7947	- 7005	-4563	-2445	- 888	- 51			
4.0	- 9453	- 8502	-6027	-3816	-2049	- 747	- 42		
4.5	-10779	- 9825	-7335	-5088	-3240	-1749	- 639	- 36	
5.0	-11958	-11004	-8505	-6243	-4362	-2799	-1515	- 552	- 33

Table C.21

Cheese no. 7

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 2.5 cm; spacing = 1D; E = 200g; EY = 5000g;

- P

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2859	899							
2.0	3285	2401	914						
2.5	3343	2608	2587	922					
3.0	3353	2642	2865	2749	925				
3.5	3355	2649	2919	3102	2883	928			
4.0	3355	2650	2930	3179	3308	2996	929		
4.5	3355	2651	2933	3197	3410	3491	3094	930	
5.0	3355	2651	2934	3202	3437	3620	3655	3180	931

$- U \times 10^3$

1.0	0000								
1.5	0000	2.62							
2.0	0000	7.00	3.15						
2.5	0000	7.61	8.91	3.53					
3.0	0000	7.71	9.86	10.54	3.88				
3.5	0000	7.73	10.05	11.89	12.09	4.20			
4.0	0000	7.73	10.09	12.18	13.89	13.58	4.51		
4.5	0000	7.73	10.10	12.25	14.31	15.82	15.00	4.79	
5.0	0000	7.73	10.10	12.27	14.42	16.41	17.73	16.37	5.06

$\int_r^R \frac{\partial T}{\partial R} . dR$

1.0	0000								
1.5	0000	- 8.66							
2.0	0000	-23.15	- 7.93						
2.5	0000	-25.14	-22.45	- 7.16					
3.0	0000	-25.47	-24.86	-21.37	- 6.57				
3.5	0000	-25.53	-25.33	-24.11	-20.47	- 6.10			
4.0	0000	-25.55	-25.43	-24.71	-23.49	-19.70	- 5.72		
4.5	0000	-25.55	-25.46	-24.85	-24.21	-22.96	-19.04	- 5.40	
5.0	0000	-25.55	-25.46	-24.88	-24.40	-23.80	-22.49	-18.45	- 5.13

Table C.22

Cheese no. 7

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 2.5 cm; spacing = 1D; E = 200g; EY = 5000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	690.6	405.3							
2.0	690.6	-112.6	438.9						
2.5	690.6	-183.8	- 89.2	470.7					
3.0	690.6	-195.6	-176.9	- 50.1	494.5				
3.5	690.6	-197.9	-193.9	-150.7	- 17.2	513.1			
4.0	690.6	-198.4	-197.6	-172.5	-128.4	11.0	528.1		
4.5	690.6	-198.5	-198.5	-177.8	-154.9	-109.1	35.6	540.5	
5.0	690.6	-198.5	-198.7	-179.1	-161.9	-140.3	- 92.1	57.2	550.9

Z

1.0	0000								
1.5	137.4	57.7							
2.0	137.4	-16.0	46.0						
2.5	137.4	-26.1	- 9.3	39.1					
3.0	137.4	-27.8	-18.5	- 4.1	34.0				
3.5	137.4	-28.2	-20.3	-12.5	- 1.2	30.1			
4.0	137.4	-28.2	-20.7	-14.3	- 8.8	0.7	27.0		
4.5	137.4	-28.2	-20.8	-14.7	-10.6	- 6.4	1.9	24.5	
5.0	137.4	-28.2	-20.8	-14.9	-11.1	- 8.2	- 4.7	2.6	22.4

$\int_r^R Z.dr$

1.0	0000								
1.5	304	58							
2.0	204	55	46						
2.5	97	- 39	67	39					
3.0	14	-119	5	72	34				
3.5	- 52	-184	- 56	30	73	30			
4.0	-104	-236	-107	- 17	44	72	27		
4.5	-146	-278	-150	- 59	6	51	70	25	
5.0	-182	-314	-186	- 95	- 29	21	56	67	23

Table C.23

Cheese no. 7

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 2.5 cm; spacing = 1D; E = 200g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R} . dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-309.7							
2.0	0000	-827.6	-288.6						
2.5	0000	-898.8	-816.7	-262.6					
3.0	0000	-910.6	-904.5	-783.5	-241.9				
3.5	0000	-912.9	-921.5	-884.0	-753.6	-225.2			
4.0	0000	-913.4	-925.1	-905.9	-864.8	-727.3	-211.4		
4.5	0000	-913.5	-926.0	-911.1	-891.4	-847.4	-703.9	-199.8	
5.0	0000	-913.6	-926.3	-912.4	-898.3	-878.6	-831.6	-683.1	-189.9

$$\int_r^R \frac{\partial Z}{\partial R} . dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 44.0							
2.0	0000	-117.6	-30.2						
2.5	0000	-127.3	-85.5	-21.8					
3.0	0000	-129.4	-94.7	-64.9	-16.6				
3.5	0000	-129.8	-96.5	-73.3	-51.6	-13.2			
4.0	0000	-129.8	-96.8	-75.0	-59.3	-42.5	-10.8		
4.5	0000	-129.8	-96.9	-75.5	-61.1	-49.6	-35.9	- 9.0	
5.0	0000	-129.8	-96.9	-75.6	-61.6	-51.4	-42.4	-30.9	- 7.7

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 287	- 44							
2.0	- 814	- 476	- 30						
2.5	-1253	- 898	- 342	- 22					
3.0	-1608	-1251	- 675	- 261	- 17				
3.5	-1902	-1545	- 966	- 533	- 208	- 13			
4.0	-2153	-1796	-1216	- 774	- 436	- 172	- 11		
4.5	-2371	-2014	-1434	- 996	- 649	- 367	- 144	- 9	
5.0	-2564	-2206	-1626	-1188	- 840	- 554	- 315	- 124	- 8

Table C.24

Cheese no. 8

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 100g; EY = 5000g;

- P

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2224	683							
2.0	2630	1890	726						
2.5	2687	2061	2046	747					
3.0	2697	2088	2256	2187	759				
3.5	2698	2092	2293	2444	2302	766			
4.0	2698	2093	2301	2496	2609	2400	771		
4.5	2699	2094	2302	2507	2676	2756	2485	774	
5.0	2699	2094	2303	2510	2692	2839	2888	2559	776

$- U \times 10^3$

1.0	0000								
1.5	0000	3.01							
2.0	0000	8.31	3.57						
2.5	0000	9.06	10.08	3.95					
3.0	0000	9.18	11.11	11.58	4.31				
3.5	0000	9.20	11.30	12.94	13.08	4.65			
4.0	0000	9.20	11.33	13.21	14.83	14.56	4.96		
4.5	0000	9.20	11.34	13.27	15.21	16.72	16.00	5.27	
5.0	0000	9.20	11.34	13.29	15.29	17.23	18.60	17.40	5.55

$$\int_r^R \frac{\partial T}{\partial R} \cdot dR$$

1.0	0000								
1.5	0000	- 8.11							
2.0	0000	-22.44	- 8.00						
2.5	0000	-24.45	-22.56	- 7.42					
3.0	0000	-24.77	-24.87	-21.73	- 6.91				
3.5	0000	-24.83	-25.29	-24.29	-20.98	- 6.48			
4.0	0000	-24.84	-25.37	-24.80	-23.77	-20.30	- 6.11		
4.5	0000	-24.85	-25.39	-24.91	-24.38	-23.31	-19.70	- 5.79	
5.0	0000	-24.85	-25.39	-24.94	-24.52	-24.02	-22.90	-19.15	- 5.52

Table C.25

Cheese no. 8

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 100g; EY = 5000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	489.8	323.2							
2.0	489.8	- 66.6	346.3						
2.5	489.8	-121.4	- 74.1	373.7					
3.0	489.8	-130.1	-140.8	- 51.5	395.0				
3.5	489.8	-131.7	-152.8	-127.7	- 29.5	412.0			
4.0	489.8	-132.0	-155.2	-142.8	-113.9	- 9.2	425.9		
4.5	489.8	-132.1	-155.7	-146.1	-132.3	-100.9	9.2	437.5	
5.0	489.8	-132.1	-155.8	-146.9	-136.6	-122.5	- 88.8	26.0	447.4

Z

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	389.8	183.8							
2.0	389.8	-37.7	145.1						
2.5	389.8	-68.9	-30.9	123.9					
3.0	389.8	-73.8	-58.9	-17.1	108.4				
3.5	389.8	-74.7	-63.9	-42.3	- 8.1	-96.4			
4.0	389.8	-74.9	-64.9	-47.3	-31.2	- 2.1	86.9		
4.5	389.8	-74.9	-65.1	-48.4	-36.3	-23.6	1.9	79.1	
5.0	389.8	-74.9	-65.2	-48.7	-37.5	-28.7	-18.1	4.7	72.7

$\int_r^R Z.dr$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	981	184							
2.0	703	196	145						
2.5	359	- 106	199	124					
3.0	85	- 374	- 13	211	108				
3.5	-131	- 589	-217	59	214	96			
4.0	-305	- 763	-390	-102	103	212	87		
4.5	-449	- 907	-534	-244	- 27	131	207	79	
5.0	-571	-1029	-655	-365	-146	24	148	202	73

Table C.26

Cheese no. 8

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 100g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-221.0							
2.0	0000	-610.8	-231.1						
2.5	0000	-665.6	-651.5	-220.5					
3.0	0000	-674.4	-718.2	-645.7	-208.7				
3.5	0000	-675.9	-730.2	-721.8	-633.2	-197.5			
4.0	0000	-676.2	-732.5	-737.0	-717.6	-618.7	-187.4		
4.5	0000	-676.3	-733.1	-740.2	-735.9	-710.4	-604.1	-178.5	
5.0	0000	-676.3	-733.2	-741.0	-740.3	-732.0	-702.2	-589.9	-170.5

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

1.0	0000								
1.5	0000	-125.5							
2.0	0000	-347.0	- 96.7						
2.5	0000	-378.2	-272.8	- 73.1					
3.0	0000	-383.2	-300.8	-214.1	- 57.2				
3.5	0000	-384.1	-305.8	-239.3	-173.7	- 46.2			
4.0	0000	-384.2	-306.8	-244.3	-196.9	-144.8	- 38.2		
4.5	0000	-384.3	-307.0	-245.4	-202.0	-166.3	-123.3	- 32.3	
5.0	0000	-384.3	-307.0	-245.6	-203.1	-171.3	-143.3	-106.7	- 27.7

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr$$

1.0	0000								
1.5	- 758	- 126							
2.0	-2369	-1446	- 97						
2.5	-3779	-2815	-1110	- 73					
3.0	-4940	-3970	-2209	- 873	- 57				
3.5	-5913	-4942	-3171	-1781	- 708	- 46			
4.0	-6747	-5775	-4002	-2602	-1478	- 590	- 38		
4.5	-7474	-6502	-4729	-3327	-2192	-1254	- 501	- 32	
5.0	-8119	-7147	-5374	-3971	-2833	-1884	-1083	- 433	- 28

Table C.27

Cheese no. 9

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 400g; EY = 5000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2867	684							
2.0	4407	2854	726						
2.5	5037	3742	2901	747					
3.0	5298	4109	3801	3031	759				
3.5	5411	4269	4192	4024	3143	766			
4.0	5463	4342	4370	4476	4230	3234	771		
4.5	5487	4377	4455	4691	4747	4407	3309	774	
5.0	5499	4394	4497	4798	5002	4986	4561	3373	776

$- U \times 10^3$

1.0	0000								
1.5	0000	1.12							
2.0	0000	4.69	1.69						
2.5	0000	6.15	6.77	1.93					
3.0	0000	6.75	8.87	7.84	2.11				
3.5	0000	7.01	9.78	10.40	8.75	2.28			
4.0	0000	7.13	10.20	11.57	11.78	9.62	2.44		
4.5	0000	7.19	10.40	12.13	13.22	13.11	10.46	2.59	
5.0	0000	7.22	10.49	12.41	13.93	14.84	14.42	11.26	2.73

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

1.0	0000								
1.5	0000	- 3.03							
2.0	0000	-12.65	- 3.79						
2.5	0000	-16.59	-15.15	- 3.62					
3.0	0000	-18.22	-19.85	-14.71	- 3.39				
3.5	0000	-18.93	-21.90	-19.53	-14.03	- 3.18			
4.0	0000	-19.25	-22.83	-21.73	-18.89	-13.42	- 3.00		
4.5	0000	-19.40	-23.27	-22.77	-21.19	-18.28	-12.87	- 2.84	
5.0	0000	-19.48	-23.49	-23.29	-22.33	-20.69	-17.74	-12.40	- 2.71

Table C.28

Cheese no. 9

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 400g; EY = 5000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	000								
1.5	489.8	461.6							
2.0	489.8	199.6	467.9						
2.5	489.8	92.5	139.8	486.5					
3.0	489.8	48.1	4.0	157.0	501.4				
3.5	489.8	28.8	-55.0	13.9	180.0	512.7			
4.0	489.8	20.1	-81.8	-51.4	33.5	200.6	521.4		
4.5	489.8	15.9	-94.7	-82.4	-36.1	52.2	218.5	528.4	
5.0	489.8	13.8	-101.0	-97.7	-70.5	-20.9	69.1	234.2	534.1

Z

1.0	0000								
1.5	389.8	262.4							
2.0	389.8	113.6	196.0						
2.5	389.8	52.7	58.6	161.3					
3.0	389.8	27.5	1.8	52.1	137.6				
3.5	389.8	16.6	-22.9	4.6	49.4	120.0			
4.0	389.8	11.6	-34.2	-17.0	9.2	47.0	106.4		
4.5	389.8	9.2	-39.6	-27.3	- 9.9	12.2	44.6	95.6	
5.0	389.8	8.0	-42.2	-32.5	-19.3	- 4.8	14.1	42.4	86.7

$$\int_r^R Z.dr$$

1.0	0000								
1.5	1465	262							
2.0	1811	849	196						
2.5	1704	841	635	161					
3.0	1493	670	649	552	138				
3.5	1279	474	532	600	495	120			
4.0	1088	291	386	528	564	449	106		
4.5	923	130	243	420	523	531	411	96	
5.0	782	- 9	112	307	443	512	501	379	87

Table C.29

Cheese no. 9

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 400g; EY = 5000g;

$$\int_r^R \frac{\partial Q}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 82.6							
2.0	0000	-344.6	-109.5						
2.5	0000	-451.7	-437.6	-107.7					
3.0	0000	-496.1	-573.4	-437.1	-102.3				
3.5	0000	-515.4	-632.4	-580.3	-423.7	- 96.9			
4.0	0000	-524.1	-659.3	-645.5	-570.2	-408.9	- 92.0		
4.5	0000	-528.3	-672.0	-676.6	-639.8	-557.2	-394.8	- 87.6	
5.0	0000	-530.4	-678.4	-691.9	-674.2	-630.5	-544.2	-381.8	- 83.8

$$\int_r^R \frac{\partial Z}{\partial R} . dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 46.9							
2.0	0000	-195.7	- 45.8						
2.5	0000	-256.6	-183.2	- 35.7					
3.0	0000	-281.8	-240.1	-144.9	- 28.1				
3.5	0000	-292.7	-264.8	-192.4	-116.2	- 22.7			
4.0	0000	-297.7	-276.0	-214.0	-156.4	- 95.7	- 18.8		
4.5	0000	-300.1	-281.4	-224.3	-175.5	-130.4	- 80.6	- 15.8	
5.0	0000	-301.3	-284.0	-229.4	-185.0	-147.5	-111.0	- 69.0	- 13.6

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 274	- 47							
2.0	-1261	- 793	- 46						
2.5	-2435	-1869	- 675	- 36					
3.0	-3533	-2926	-1548	- 532	- 28				
3.5	-4504	-3879	-2421	-1241	- 428	- 23			
4.0	-5354	-4721	-3226	-1971	-1017	- 353	- 19		
4.5	-6102	-5465	-3953	-2662	-1641	- 854	- 298	- 16	
5.0	-6765	-6127	-4606	-3298	-2244	-1396	- 730	- 255	- 14

Table C.30

Cheese no. 10

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 2000g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2933	684							
2.0	4691	3011	726						
2.5	5501	4083	3035	747					
3.0	5873	4575	4095	3156	759				
3.5	6050	4810	4601	4305	3263	766			
4.0	6139	4927	4852	4877	4509	3351	771		
4.5	6185	4988	4982	5173	5152	4685	3423	774	
5.0	6209	5020	5052	5330	5495	5397	4837	3484	776

$- U \times 10^3$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	2.33							
2.0	0000	10.27	3.70						
2.5	0000	13.93	15.47	4.28					
3.0	0000	15.61	20.87	18.08	4.70				
3.5	0000	16.42	23.44	24.67	20.20	5.07			
4.0	0000	16.81	24.73	27.95	27.92	22.19	5.42		
4.5	0000	17.02	25.39	29.64	31.90	31.02	24.08	5.76	
5.0	0000	17.13	25.74	30.55	34.03	35.73	34.03	25.91	6.07

$\int_r^R \frac{\partial T}{\partial R} \cdot dR$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 2.52							
2.0	0000	-11.09	- 3.31						
2.5	0000	-15.05	-13.85	- 3.21					
3.0	0000	-16.86	-18.69	-13.58	- 3.01				
3.5	0000	-17.72	-21.00	-18.52	-12.96	- 2.83			
4.0	0000	-18.15	-22.14	-20.99	-17.91	-12.37	- 2.67		
4.5	0000	-18.38	-22.74	-22.26	-20.46	-17.30	-11.86	- 2.53	
5.0	0000	-18.50	-23.05	-22.93	-21.83	-19.93	-16.76	-11.40	- 2.42

Table C.31

Cheese no.10

Assumption w = 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 2000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	489.8	475.5							
2.0	489.8	241.7	481.7						
2.5	489.8	134.1	177.1	498.7					
3.0	489.8	84.7	37.5	190.6	512.7				
3.5	489.8	61.1	-29.2	43.6	212.3	523.3			
4.0	489.8	49.4	-62.3	29.5	62.9	232.3	531.4		
4.5	489.8	43.4	-79.5	-67.3	-14.2	82.1	249.6	537.9	
5.0	489.8	40.1	-88.6	-87.5	-55.3	2.0	99.4	264.7	543.2

Z

1.0	0000								
1.5	389.8	270.7							
2.0	389.8	138.7	202.1						
2.5	389.8	77.8	75.1	165.6					
3.0	389.8	49.8	16.7	63.9	140.9				
3.5	389.8	36.4	-11.2	15.2	58.9	122.6			
4.0	389.8	29.8	-25.1	- 9.0	17.9	54.9	108.6		
4.5	389.8	26.3	-32.3	-21.5	- 3.2	19.7	51.4	97.4	
5.0	389.8	24.5	-36.1	-28.2	-14.5	1.0	20.8	48.3	88.3

$\int_r^R Z.dr$

1.0	0000								
1.5	1514	271							
2.0	1976	949	202						
2.5	1944	1016	701	166					
3.0	1764	883	767	602	141				
3.5	1560	700	679	691	534	122			
4.0	1368	519	545	646	640	482	109		
4.5	1198	355	405	552	624	596	439	97	
5.0	1050	210	273	443	558	601	558	404	88

Table C.32

Cheese no.10

Assumption w= 0; std. size of the element; R0 = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 2000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 68.8							
2.0	0000	-302.5	- 95.7						
2.5	0000	-410.2	-400.3	- 95.5					
3.0	0000	-459.5	-539.9	-403.6	- 90.9				
3.5	0000	-483.1	-606.6	-550.6	-391.3	- 86.2			
4.0	0000	-494.8	-639.7	-623.7	-540.7	-377.2	- 81.9		
4.5	0000	-500.9	-656.8	-661.5	-617.8	-527.4	-363.7	- 78.1	
5.0	0000	-504.1	-666.0	-681.6	-659.0	-607.5	-514.0	-351.3	- 74.6

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 38.7							
2.0	0000	-170.1	- 39.8						
2.5	0000	-231.5	-166.7	- 31.4					
3.0	0000	-259.5	-225.1	-133.1	- 24.8				
3.5	0000	-272.9	-253.0	-181.8	-106.8	- 20.0			
4.0	0000	-279.6	-266.9	-206.0	-147.7	- 87.8	- 16.6		
4.5	0000	-282.9	-274.1	-218.6	-168.9	-122.9	- 73.7	- 14.0	
5.0	0000	-284.8	-277.9	-225.2	-180.2	-141.6	-104.4	- 63.2	- 12.0

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 225	- 39							
2.0	-1095	- 693	- 40						
2.5	-2196	-1694	- 608	- 31					
3.0	-3261	-2714	-1430	- 482	- 24				
3.5	-4222	-3653	-2274	-1149	- 388	- 20			
4.0	-5073	-4493	-3067	-1853	- 941	- 320	- 17		
4.5	-5827	-5240	-3790	-2531	-1540	- 788	- 269	- 14	
5.0	-6498	-5908	-4445	-3162	-2129	-1307	- 672	- 231	- 12

Table C.33

Cheese no. 11

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 8000g;

- P

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.00	0000								
1.5	2351	684							
2.0	2890	2033	726						
2.5	2984	2269	2182	747					
3.0	3003	2315	2465	2326	759				
3.5	3006	2325	2525	2667	2443	766			
4.0	3008	2327	2540	2748	2842	2542	771		
4.5	3008	2328	2544	2768	2945	2998	2626	774	
5.0	3008	2328	2545	2774	2973	3123	3137	2700	776

$- U \times 10^3$

1.0	0000								
1.5	0000	1.64							
2.0	0000	4.86	1.99						
2.5	0000	5.43	5.99	2.21					
3.0	0000	5.54	6.77	6.87	2.40				
3.5	0000	5.56	6.93	7.88	7.74	2.59			
4.0	0000	5.57	6.97	8.11	9.01	8.60	2.77		
4.5	0000	5.57	6.98	8.18	9.33	10.14	9.44	2.94	
5.0	0000	5.57	6.99	8.19	9.42	10.57	11.27	10.25	3.10

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

1.0	0000								
1.5	0000	- 7.06							
2.0	0000	-21.00	- 7.14						
2.5	0000	-23.45	-21.46	- 6.62					
3.0	0000	-23.92	-24.24	-20.63	- 6.12				
3.5	0000	-24.02	-24.84	-23.65	-19.86	- 5.78			
4.0	0000	-24.05	-24.98	-24.37	-23.11	-19.19	- 5.45		
4.5	0000	-24.05	-25.02	-24.55	-23.94	-22.63	-18.58	- 5.17	
5.0	0000	-24.05	-25.03	-24.61	-24.17	-23.58	-22.20	-18.04	- 4.93

Table C.34

Cheese no.11

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 8000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	489.8	351.9							
2.0	489.8	- 27.4	371.3						
2.5	489.8	- 93.9	- 42.2	397.4					
3.0	489.8	-106.8	-122.5	- 18.7	417.5				
3.5	489.8	-109.6	-139.9	-108.6	4.0	433.3			
4.0	489.8	-110.3	-144.0	-129.9	- 93.9	24.8	446.1		
4.5	489.8	-110.4	-145.0	-135.4	-119.1	- 80.1	43.5	456.7	
5.0	489.8	-110.5	-145.3	-136.9	-126.0	-109.0	- 67.4	60.4	465.7

Z

1.0	000								
1.5	389.8	200.2							
2.0	389.8	-15.4	155.6						
2.5	389.8	-53.2	-17.6	131.8					
3.0	389.8	-60.6	-51.2	- 6.1	114.6				
3.5	389.8	-62.1	-58.5	-35.9	1.2	101.5			
4.0	389.8	-62.5	-60.2	-43.0	-25.7	5.9	91.1		
4.5	389.8	-62.6	-60.7	-44.8	-32.6	-18.7	9.0	82.7	
5.0	389.8	-62.6	-60.8	-45.3	-34.5	-25.4	-13.7	11.0	75.7

$\int_r^R Z.dr$

1.0	0000								
1.5	1082	200							
2.0	888	304	156						
2.5	568	36	275	132					
3.0	301	-221	93	272	115				
3.5	88	-431	-101	147	265	101			
4.0	- 83	-602	-268	- 3	179	255	91		
4.5	-225	-744	-409	-139	60	197	245	83	
5.0	-344	-863	-528	-257	- 54	101	207	234	76

Table C.35

Cheese no.11

Assumption w = o; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 8000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-192.3							
2.0	0000	-571.6	-206.1						
2.5	0000	-638.1	-619.6	-196.8					
3.0	0000	-651.1	-699.9	-612.9	-186.2				
3.5	0000	-653.8	-717.2	-702.8	-599.6	-176.2			
4.0	0000	-654.5	-721.4	-724.1	-697.6	-584.7	-167.2		
4.5	0000	-654.7	-722.4	-729.5	-722.8	-689.7	-569.9	-159.3	
5.0	0000	-654.7	-722.7	-731.0	-729.7	-718.5	-680.7	-555.6	-152.2

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-109.2							
2.0	0000	-324.7	- 86.2						
2.5	0000	-362.5	-259.3	- 65.2					
3.0	0000	-369.9	-293.0	-203.1	- 51.0				
3.5	0000	-371.5	-300.3	-232.9	-164.5	- 41.2			
4.0	0000	-371.5	-302.0	-240.0	-191.3	-136.8	- 34.1		
4.5	0000	-371.9	-302.5	-241.8	-198.3	-161.4	-116.2	- 28.8	
5.0	0000	-371.9	-302.6	-242.3	-200.2	-168.1	-138.8	-100.4	- 24.7

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}dR$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 657	- 109							
2.0	-2183	-1338	- 86						
2.5	-3571	-2673	-1034	- 65					
3.0	-4725	-3817	-2103	- 812	- 51				
3.5	-5694	-4784	-3054	-1693	- 657	- 41			
4.0	-6525	-5614	-3880	-2502	-1402	- 546	- 34		
4.5	-7250	-6339	-4604	-3222	-2105	-1188	- 464	- 29	
5.0	-7892	-6981	-5246	-3862	-2741	-1807	-1025	- 400	- 25

Table C.36

Cheese no.12

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 1000g; EY = 1000g;

- P

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	3225	684							
2.0	6472	4033	726						
2.5	9413	7064	4106	747					
3.0	11951	9682	7023	4150	759				
3.5	14099	11896	9492	7031	4188	766			
4.0	15906	13759	11569	9454	7072	4221	771		
4.5	17426	15327	13316	11492	9497	7126	4249	774	
5.0	18708	16648	14789	13211	11543	9576	7183	4275	776

$- U \times 10^3$

1.0	0000								
1.5	0000	0.54							
2.0	0000	3.23	1.22						
2.5	0000	5.65	6.90	1.78					
3.0	0000	7.75	11.80	9.89	2.23				
3.5	0000	9.50	15.95	16.76	12.31	2.59			
4.0	0000	11.01	19.44	22.54	20.78	14.30	2.90		
4.5	0000	12.27	22.37	27.39	27.91	24.14	15.99	3.17	
5.0	0000	13.32	24.85	31.49	33.92	32.44	27.03	17.49	3.40

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

1.0	0000								
1.5	0000	- 0.30							
2.0	0000	- 1.74	- 0.55						
2.5	0000	- 3.05	- 3.09	- 0.67					
3.0	0000	- 4.18	- 5.28	- 3.71	- 0.71				
3.5	0000	- 5.14	- 7.14	- 6.29	- 3.95	- 0.72			
4.0	0000	- 5.94	- 8.70	- 8.46	- 6.66	- 3.99	- 0.71		
4.5	0000	- 6.62	-10.02	-10.28	- 8.95	- 6.73	- 3.94	- 0.70	
5.0	0000	- 7.19	-11.13	-11.82	-10.88	- 9.05	- 6.65	- 3.85	- 0.68

Table C.37

Cheese no.12

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 1000g; EY = 1000g;

Q

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	489.8	536.1							
2.0	489.8	496.5	561.6						
2.5	489.8	460.7	487.9	574.3					
3.0	489.8	429.7	424.4	483.6	582.1				
3.5	489.8	403.6	370.7	406.9	484.4	587.4			
4.0	489.8	381.6	325.4	342.4	403.3	487.9	591.4		
4.5	489.8	363.1	287.4	288.2	333.2	404.2	492.5	594.5	
5.0	489.8	347.5	255.4	242.4	275.0	333.6	409.1	497.4	596.9

Z

1.0	0000								
1.5	389.8	304.9							
2.0	389.8	282.9	235.4						
2.5	389.8	262.9	205.1	190.6					
3.0	389.8	245.8	179.0	161.1	159.8				
3.5	389.8	231.2	156.8	136.0	133.5	137.6			
4.0	389.8	218.9	138.2	114.9	111.3	114.7	120.8		
4.5	389.8	208.6	122.4	97.1	92.6	95.4	100.9	107.6	
5.0	389.8	199.9	109.2	82.1	76.8	79.1	84.2	90.3	97.0

$\int_r^R Z.dr$

1.0	0000								
1.5	1713	305							
2.0	2901	1531	235						
2.5	3685	2349	1178	191					
3.0	4178	2872	1807	959	160				
3.5	4467	3187	2214	1483	812	138			
4.0	4618	3359	2462	1831	1268	706	121		
4.5	4673	3432	2600	2052	1580	1113	625	108	
5.0	4663	3436	2659	2182	1787	1400	994	562	97

Table C.38

Cheese no.12

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5 cm; spacing = 1D; E = 1000g; EY = 1000g;

$$\int_r^R \frac{\partial Q}{\partial R}.dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 8.09							
2.0	0000	- 47.7	- 15.8						
2.5	0000	- 83.5	- 89.4	- 19.9					
3.0	0000	-114.5	-153.0	-110.5	- 21.6				
3.5	0000	-140.6	-206.7	-187.3	-119.3	- 22.1			
4.0	0000	-162.6	-251.9	-251.8	-201.4	-121.6	- 21.9		
4.5	0000	-181.1	-290.0	-306.0	-270.5	-205.3	-120.8	- 21.5	
5.0	0000	-196.7	-322.0	-351.8	-328.7	-275.9	-204.2	-118.6	- 20.9

$$\int_r^R \frac{\partial Z}{\partial R}.dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	- 4.48							
2.0	0000	- 26.4	- 6.5						
2.5	0000	- 46.4	- 36.7	- 6.5					
3.0	0000	- 63.6	- 62.8	- 35.9	- 5.8				
3.5	0000	- 78.1	- 85.0	- 61.0	- 32.1	- 5.1			
4.0	0000	- 90.4	-103.7	- 82.1	- 54.3	- 27.9	- 4.4		
4.5	0000	-100.7	-119.4	- 99.9	- 73.0	- 47.3	- 24.2	- 3.8	
5.0	0000	-109.5	-132.7	-114.9	- 88.9	- 63.6	- 41.0	- 21.1	- 3.3

$$\int_r^R \int_r^R \frac{\partial Z}{\partial R}.dR.dr$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	- 25.4	- 4.5							
2.0	-169.8	-110.7	- 6.5						
2.5	-454.1	-360.5	-132.0	- 6.5					
3.0	-848.4	-724.9	-389.0	-124.9	- 5.8				
3.5	-1315	-1166	-739.3	-357.6	-110.4	- 5.1			
4.0	-1823	-1653	-1150	- 669	-313.7	- 95.9	- 4.4		
4.5	-2351	-2164	-1595	-1031	- 585	-272.2	- 83.1	- 3.8	
5.0	-2885	-2682	-2059	-1424	- 901	-508.1	-236.6	- 72.4	- 3.3

Table C.39

Cheese no.13

Assumption w = 0; std. size of the element; RO = 4.5 cm;

T = 20g; traverse = 0.05 cm; spacing = 1D; E = 1000g; EY = 1000g;

- P

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	2545	1005							
2.0	2674	2116	1005						
2.5	2682	2180	2306	1005					
3.0	2682	2185	2410	2466	1005				
3.5	2682	2185	2420	2612	2601	1005			
4.0	2682	2185	2421	2629	2792	2718	1005		
4.5	2682	2185	2421	2632	2818	2955	2822	1005	
5.0									

$- U \times 10^3$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	18.13							
2.0	0000	38.15	21.34						
2.5	0000	39.31	48.95	24.05					
3.0	0000	39.40	51.14	58.98	26.49				
3.5	0000	39.41	51.36	62.49	68.54	28.72			
4.0	0000	39.41	51.39	62.90	73.57	77.68	30.80		
4.5	0000	39.41	51.39	62.96	74.26	84.43	86.47	32.75	
5.0									

$$\int_r^R \frac{\partial T}{\partial R} \cdot dr$$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-12.95							
2.0	0000	-27.25	-11.23						
2.5	0000	-28.08	-25.76	-10.02					
3.0	0000	-28.14	-26.92	-24.58	- 9.13				
3.5	0000	-28.15	-27.03	-26.04	-23.63	- 8.45			
4.0	0000	-28.15	-27.05	-26.21	-25.37	-22.85	- 7.90		
4.5	0000	-28.15	-27.05	-26.23	-25.61	-24.83	-22.17	- 7.44	
5.0									

Table C.40

Cheese no.13

Assumption w = 0; std. size of the element; RO = 4.5 cm;

T = 20g; traverse = 0.05 cm; spacing = 1D; E = 1000g; EY = 1000g;

Q

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	799.9	282.0							
2.0	799.9	-290.1	350.8						
2.5	799.9	-323.0	-230.5	399.2					
3.0	799.9	-325.7	-276.7	-183.1	434.6				
3.5	799.9	-325.9	-281.3	-241.4	-145.3	462.1			
4.0	799.9	-325.9	-281.9	-248.4	-214.7	-113.9	484.1		
4.5	799.9	-326.0	-282.0	-249.4	-224.3	-193.3	- 86.9	502.3	
5.0									

$Z \times 10^2$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	6.366	1.586							
2.0	6.366	-1.689	1.465						
2.5	6.365	-1.878	-0.982	1.324					
3.0	6.365	-1.893	-1.177	-0.614	1.194				
3.5	6.366	-1.894	-1.197	-0.808	-0.401	1.084			
4.0	6.366	-1.895	-1.999	-0.832	-0.592	-0.266	0.990		
4.5	6.365	-1.895	-1.999	-0.835	-0.618	-0.452	-0.175	0.911	
5.0									

$\left(\int_r^R Z.dr \right) \times 10^2$

R\r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	7.336	1.586							
2.0	-0.259	-2.659	1.465						
2.5	-5.852	-8.059	-0.459	1.324					
3.0	-9.937	-12.13	-4.252	0.582	1.194				
3.5	-13.07	-15.26	-7.355	-2.216	1.117	1.084			
4.0	-15.57	-17.76	-9.849	-4.673	-1.014	1.407	0.991		
4.5	-17.62	-19.81	-11.90	-6.716	-3.012	-0.253	1.566	0.911	
5.0									

Table C.41

Cheese no.13

Assumption w = 0; std. size of the element; RO = 4.5 cm;

r = 20g; traverse = 0.05cm; spacing = 1D; E = 1000g; EY = 1000g;

$$\int_r^R \frac{\partial Q_o}{\partial R} dR$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-517.9							
2.0	0000	-1090	-449.2						
2.5	0000	-1123	-1030	-400.8					
3.0	0000	-1126	-1077	-983.0	-365.3				
3.5	0000	-1126	-1081	-1041	-945.3	-337.9			
4.0	0000	-1126	-1082	-1048	-1015	-914.0	-315.9		
4.5	0000	-1126	-1082	-1049	-1024	-993.2	-886.9	-297.7	
5.0									

$$\left(\int_r^R \frac{\partial Z}{\partial R} . dR \right) \times 10^2$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	0000	-2.961							
2.0	0000	-6.236	-1.886						
2.5	0000	-6.425	-4.333	-1.329					
3.0	0000	-6.440	-4.527	-3.266	-1.001				
3.5	0000	-6.441	-4.547	-3.461	-2.596	-0.788			
4.0	0000	-6.442	-4.549	-3.484	-2.787	-2.138	-0.642		
4.5	0000	-6.442	-4.550	-3.487	-2.814	-2.325	-1.808	-0.536	
5.0									

$$\left(\int_r^R \int_r^R \frac{\partial Z}{\partial R} . dR . dr \right) \times 10^2$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	-19.57	- 2.96							
2.0	-46.02	-26.06	- 1.89						
2.5	-66.14	-45.99	-18.34	- 1.33					
3.0	-82.04	-61.88	-33.95	-13.89	- 1.001				
3.5	-95.14	-74.98	-47.02	-26.66	-11.04	- 0.79			
4.0	-106.3	-86.09	-58.13	-37.73	021.79	- 9.08	- 0.64		
4.5	-115.9	-95.73	-67.77	-47.36	-31.38	-18.33	- 7.65	- 0.54	
5.0									

Table C.42

Cheese no.14

Assumption w = 0; std. size of the element; RO = 5 cm;

T = 20g; traverse = 5cm; spacing = 1D; E = 600g; EY = 600g;

- P

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0000								
1.5	3266	690							
2.0	6531	4056	728						
2.5	9480	7096	4117	748					
3.0	12022	9717	7039	4157	759				
3.5	14172	11934	9510	7040	4192	766			
4.0	15980	13798	11589	9465	7077	4223	771		
4.5	17501	15367	13336	11503	9504	7130	4251	774	
5.0	18784	16689	14810	13223	11550	9581	7186	4276	777

$- U \times 10^3$

1.0	0000								
1.5	0000	0.92							
2.0	0000	5.41	2.04						
2.5	0000	9.47	11.53	2.97					
3.0	0000	12.96	19.12	16.52	3.72				
3.5	0000	15.92	26.64	27.98	20.53	4.32			
4.0	0000	18.41	32.46	37.61	34.67	23.84	4.84		
4.5	0000	20.50	37.38	45.72	46.56	38.13	26.66	5.28	
5.0	0000	22.27	41.49	52.55	56.58	54.10	45.08	29.15	5.67

$$\int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.05	1.45	1.95	2.45	2.95	3.45	3.95	4.45	4.95
1.00									
1.5	-0.16	-0.29							
2.0	-0.36	-1.90	- 0.54						
2.5	-0.54	-3.36	- 3.20	- 0.66					
3.0	-0.70	-4.61	- 5.49	- 3.79	- 0.70				
3.5	-0.82	-5.67	- 7.42	- 6.44	- 4.00	- 0.71			
4.0	-0.95	-6.57	- 9.06	- 8.67	- 6.78	- 4.04	- 0.71		
4.5	-1.04	-7.32	-10.43	-10.54	- 9.11	- 6.83	- 3.98	- 0.69	
5.0	-1.12	-7.95	-11.58	-12.12	-11.08	- 9.19	- 6.74	- 3.89	- 0.67

APPENDIX D

D.1 Relation between EY and T

Table D.1

No.	T (g)	$X = \log_{10}T$	S	$Y = \log_{10}S$
1	2	0.301	0.0012	$\bar{3}.0792$
2	5	0.699	0.0022	$\bar{3}.3424$
3	10	1	0.003	$\bar{3}.4771$
4	20	1.301	0.0045	$\bar{3}.6532$
5	30	1.4771	0.0056	$\bar{3}.7482$
6	40	1.6021	0.0068	$\bar{3}.8325$
7	50	1.699	0.0077	$\bar{3}.8865$
8	54	1.732	0.0081	$\bar{3}.9085$

D.2 Measurement of the value of E

D.2.1 Observations

Table D.2

pressure lbs/sq. in.	Experiment 1. Thickness of layer = 2D. Reading on dial. The number of crossing points				Experiment 2. Thickness of layer = 6D. Reading on dial. The number of crossing points			
	196	64	16	4	144	100	64	36
1	28.9	24.6	21.2	15.8	1-37.5	1-32.5	1-27.5	18.5
2	27	22.9	18.2	13.5	1-29.5	1-25.5	1-19.5	11
3	25.6	21.5	16.7	12.4	1-24.5	1-21	1-15	7
4	24.6	20.7	15.2	11.6	1-21.5	1-18	1-12	4
5	23.8	19.7	15	11.2	1-18.5	1-15.5	1- 9.5	2
6	23.3	19.2	14.4	11.2	1-16.5	1-13.5	1- 8	0
7	22.7	18.7	14	10.9	1-14.5	1-11.5	1- 6.5	48
8	22.2	18.2	13.5	10.5	1-13	1-10	1- 5	47
9	21.8	17.7	13.2	10.4	1-12	1- 9	1- 4	46
10	21.4	17.3	12.9	10.1	1-11	1- 8	1- 2	45

D.2.2 Calculations

Experiment 1

Table D.3

press. lbs/sq in.	196 points: thickness of layer in in. i.e. TH	196 points: pressure/ crossing pt. in g i.e. Ps	196 points: $\log_{10}Ps$	64 points: thickness of layer in in. i.e. TH	64 points: pressure/ crossing pt. in g i.e. Ps	64 points: $\log_{10}Ps$
1	.0189	0.256	$\bar{1}$.4082	.0246	0.783	$\bar{1}$.8938
2	.027	0.512	$\bar{1}$.7093	.0229	1.566	0.1948
3	.0256	0.768	1.8854	.0215	2.349	0.3709
4	.0246	1.024	0.0103	.0207	3.132	0.4958
5	.0238	1.28	0.1072	.0197	3.915	0.5928
6	.0233	1.536	0.1864	.0192	4.698	0.6719
7	.0227	1.792	0.2531	.0187	5.481	0.7389
8	.0222	2.048	0.3113	.0182	6.264	0.7969
9	.0218	2.304	0.3624	.0177	7.047	0.848
10	.0214	2.56	0.4082	.0173	7.83	0.8938

Table D.3 (continued)

press. lbs/sq in	16 points: thickness of layer in in. i.e. TH	16 points: pressure/ crossing pt. in g i.e. Ps	16 points: $\log_{10}Ps$	4 points: thickness of layer in in. i.e. TH	4 points: pressure /crossing pt. in g i.e. Ps	4 points: $\log_{10}Ps$
1	.0212	3.132	0.4956	.0158	12.52	1.0976
2	.0182	6.262	0.7967	.0135	25.04	1.3987
3	.0167	9.363	0.9728	.0124	37.56	1.5747
4	.0158	12.53	1.0978	.0116	50.08	1.6997
5	.015	15.66	1.1947	.0112	62.60	1.7966
6	.0144	18.79	1.2739	.0112	75.12	1.8757
7	.014	21.92	1.3408	.0109	87.64	1.9427
8	.0135	25.05	1.3988	.0105	100.2	2.0008
9	.0132	28.18	1.4499	.0104	112.7	2.0519
10	.0129	31.13	1.4956	.0101	125.2	2.0976

Experiment 2

Table D.4

press. lbs/sq in	144 points			100 points		
	thickness of layer in in. i.e. TH	pressure/ crossing pt. in g i.e. Ps	$\log_{10}$Ps	thickness of layer in in. i.e. TH	pressure/ crossing pt. in g i.e. Ps	$\log_{10}$Ps
1	.0875	0.348	$\bar{1}.5416$	.0825	0.501	$\bar{1}.6998$
2	.0795	0.696	$\bar{1}.8426$	.0755	1.002	0.0008
3	.0745	1.044	0.0187	.071	1.503	0.1769
4	.0715	1.392	0.1436	.068	2.004	0.3018
5	.0685	1.74	0.2405	.0655	2.505	0.3988
6	.0665	2.088	0.3198	.0635	3.006	0.4779
7	.0645	2.436	0.3867	.0615	3.507	0.5449
8	.063	2.784	0.4446	.06	4.008	0.603
9	.062	3.132	0.4958	.059	4.509	0.6541
10	.061	3.48	0.5416	.058	5.01	0.6998

Table D.4 (continued)

press. lbs/sq in	64 points			36 points		
	thickness of layer in in. i.e. TH	pressure/ crossing pt. in g i.e. Ps	$\log_{10}$Ps	thickness of layer in in. i.e. TH	pressure/ crossing pt. in g i.e. Ps	$\log_{10}$Ps
1	0.0775	0.783	$\bar{1}.8937$	.0685	1.391	0.1433
2	0.0695	1.566	0.1949	.061	2.782	0.4443
3	0.065	2.349	0.3709	.057	4.173	0.6204
4	0.062	3.132	0.4958	.054	5.564	0.7554
5	0.0595	3.915	0.5928	.052	6.955	0.8423
6	0.058	4.698	0.6719	.05	8.346	0.9215
7	0.0565	5.481	0.7389	.048	9.737	0.9984
8	0.055	6.264	0.7969	.047	11.13	1.0465
9	0.054	7.047	0.848	.046	12.52	1.0976
10	0.052	7.83	0.8938	.045	13.91	1.1433

Table D.5

Assumption w = 0; std. size of the element; R0 = 4.8 cm;

T = 10g; traverse = 5 cm; spacing = 1D; E = 100g; EY = 2185. $T^{3/7}$g;

- P

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	981	335							
1.8	1317	918	356						
2.4	1533	1222	1049	372					
2.8	1566	1266	1185	981	377				
3.4	1578	1283	1245	1342	1169	382			
3.8	1580	1286	1254	1414	1368	1044	384		
4.4	1581	1287	1258	1449	1476	1456	1234	387	
4.8	1581	1287	1259	1456	1498	1566	1474	1098	388

$- U \times 10^3$

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	0000	1.32							
1.8	0000	3.90	1.83						
2.8	0000	5.68	7.80	7.03	2.37				
3.4	0000	5.77	8.15	9.58	9.22	2.60			
3.8	0000	5.78	8.21	10.03	10.71	8.58	2.73		
4.4	0000	5.79	8.24	10.24	11.46	12.31	11.12	2.94	
4.8	0000	5.79	8.24	10.28	11.60	13.15	13.35	9.99	3.05

$$- \int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	0000	4.35							
1.8	0000	9.50	5.17						
2.4	0000	10.44	11.63	5.06					
2.8	0000	10.65	12.87	10.32	4.72				
3.4	0000	10.75	13.47	12.38	10.80	4.37			
3.8	0000	10.76	13.57	12.95	11.98	10.02	4.13		
4.4	0000	10.77	13.62	13.24	12.77	11.40	10.31	3.87	
4.8	0000	10.77	13.62	13.30	12.94	12.04	11.32	9.64	3.70

Table D.6

Assumption w = 0; std. size of the element; RO = 4.8 cm;

T = 10g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 2185. $T_g^{3/7}$

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	460.1	92.4							
1.8	405.3	123.3	64.1						
2.4	209.6	- 36.4	28.8	50.5					
2.8	69.6	-172.6	- 75.6	74.0	46.7				
3.4	-108.2	-349.0	-237.8	- 1.3	62.8	41.3			
3.8	-203.8	-444.4	-330.9	- 77.4	16.3	75.3	38.6		
4.4	-322.6	-563.2	-448.6	-187.0	- 76.8	44.1	70.7	34.9	
4.8	-388.6	-629.2	-514.4	-251.2	-137.6	- 0.1	49.0	73.4	32.9

EY/E

1.0	0000								
1.4	58.62	45.91							
1.8	58.62	16.22	42.92						
2.4	58.62	15.39	26.93	43.32					
2.8	58.62	18.20	34.32	13.39	44.56				
3.4	58.62	19.26	37.24	31.70	19.83	45.83			
3.8	58.62	19.43	38.09	34.76	29.29	4.36	46.65		
4.4	58.62	19.50	38.49	36.18	33.09	20.05	10.28	47.52	
4.8	58.62	19.52	37.94	36.45	34.61	29.69	24.65	14.18	48.08

Table D.7

Assumption w = 0; std. size of the element; RO = 4.8 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 2185. $T_g^{3/7}$;

- P

R\r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	1847	671							
1.8	2419	1721	712						
2.4	2688	2108	1977	743					
2.8	2712	2144	2159	1892	754				
3.4	2719	2156	2223	2505	2271	765			
3.8	2720	2158	2231	2589	2580	1993	769		
4.4	2721	2158	2233	2617	2695	2683	2360	773	
4.8	2721	2158	2234	2622	2713	2818	2746	2077	776

$- U \times 10^3$

1.0	0000								
1.4	0000	2.42							
1.8	0000	6.95	3.35						
2.4	0000	8.91	11.09	3.82					
2.8	0000	9.08	11.98	11.66	4.19				
3.4	0000	9.14	12.28	15.15	15.94	4.72			
3.8	0000	9.14	12.31	15.58	17.81	14.72	4.76		
4.4	0000	9.15	12.33	15.73	18.47	19.45	18.16	5.14	
4.8	0000	9.15	12.33	15.75	18.57	20.27	21.00	16.77	5.32

$$- \int_r^R \frac{\partial T}{\partial R} . dR$$

1.0	0000								
1.4	0000	10.68							
1.8	0000	20.30	12.74						
2.4	0000	22.56	25.89	11.71					
2.8	0000	22.88	28.01	21.61	11.26				
3.4	0000	22.99	28.79	26.43	21.90	10.67			
3.8	0000	23.00	28.89	27.26	24.01	21.18	9.69		
4.4	0000	23.01	28.92	27.55	24.93	25.59	21.51	9.11	
4.8	0000	23.01	28.92	27.59	25.09	26.70	23.76	20.24	8.68

Table D.8

Assumption w = 0; std. size of the element; RO = 4.8 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 2185. $T^{3/7}$g;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	000								
1.4	812	152							
1.8	594	103	96						
2.4	77	- 374	- 87	85					
2.8	- 241	- 689	- 352	82	77				
3.4	- 636	-1083	- 729	-146	54	68			
3.8	- 842	-1289	- 933	-330	- 79	98	68		
4.4	-1093	-1540	-1183	-574	-303	- 19	89	62	
4.8	-1233	-1680	-1323	-713	-439	-135	16	108	59

EY/E

1.0	0000								
1.4	78.89	56.88							
1.8	78.89	13.09	51.11						
2.4	78.89	32.66	46.72	54.09					
2.8	78.89	34.38	53.28	26.81	55.34				
3.4	78.89	34.93	55.47	48.51	28.77	56.90			
3.8	78.89	35.00	55.72	51.10	39.61	23.49	59.38		
4.4	78.89	35.01	55.80	51.96	44.85	45.69	26.07	60.78	
4.8	78.89	35.02	55.82	52.09	43.88	49.37	34.29	11.82	61.81

Table D.9

Assumption w = 0; std. size of the element; RO = 4.8 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 2185. $T^{3/7}$g;

- P

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	2667	1006							
1.8	3461	2480	1069						
2.4	3748	2943	2846	1115					
2.8	3765	2974	3040	2763	1132				
3.4	3769	2980	3088	3402	3260	1147			
3.8	3770	2982	3094	3484	3646	2921	1154		
4.4	3770	2982	3095	3510	3778	3880	3409	1160	
4.8	3770	2982	3096	3513	3793	4012	3868	3041	1163

$- U \times 10^3$

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	0000	3.43							
1.8	0000	9.80	4.77						
2.4	0000	11.96	14.38	5.41					
2.8	0000	12.09	15.99	15.38	5.66				
3.4	0000	12.12	16.20	18.55	19.13	6.32			
3.8	0000	12.12	16.22	18.93	21.14	20.19	6.86		
4.4	0000	12.12	16.23	19.05	21.80	26.35	25.14	7.08	
4.8	0000	12.12	16.23	19.06	21.87	27.11	28.08	22.47	7.40

$$- \int_r^R \frac{\partial T}{\partial R} . dR$$

R \ r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	0000	18.01							
1.8	0000	31.95	21.60						
2.4	0000	36.32	42.93	19.72					
2.8	0000	36.68	45.70	35.13	18.12				
3.4	0000	36.77	46.42	41.43	36.46	16.99			
3.8	0000	36.78	46.50	42.35	40.05	30.91	16.63		
4.4	0000	36.78	46.53	42.65	41.40	36.56	34.64	14.93	
4.8	0000	36.78	46.53	42.68	41.55	37.65	38.15	30.37	14.37

Table D.10

Assumption w = 0; std. size of the element; RO = 4.8 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 2185. $T^{3/7}$g;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.3	2.7	3.3	3.7	4.3	4.7
1.0	0000								
1.4	1111	196							
1.8	690	8	111						
2.4	- 218	- 848	- 300	105					
2.8	- 722	01348	0 748	45	105				
3.4	-1318	-1944	-1331	- 384	1	95			
3.8	-1645	-2270	-1656	- 691	-241	109	88		
4.4	-2049	-2674	-2059	-1088	-614	-110	65	86	
4.8	-2268	-2893	-2278	-1306	-831	-304	- 79	133	82

EY/E

1.0	0000								
1.4	93.87	63.36							
1.8	93.87	29.10	54.39						
2.4	93.87	48.16	65.44	59.33					
2.8	93.87	49.31	71.13	44.05	63.12				
3.4	93.87	49.58	72.50	62.06	48.62	65.61			
3.8	93.87	49.61	72.65	64.16	58.74	20.94	66.39		
4.4	93.87	49.62	72.70	64.83	62.00	48.92	42.18	69.87	
4.8	93.87	49.62	72.71	64.90	62.36	52.25	53.71	14.19	70.98

Table D.11

Assumption w = 0; std. size of the element; RO = 4.2 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = - 7.49.Pg;

EY = 2185. $T^{3/7}$g; dr = dR = 0.05 cm;

- P

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	000								
1.4	510	433							
1.8	510	508	468						
2.2	510	508	555	489					
2.6	510	508	555	601	513				
3.0	510	508	555	602	638	532			
3.4	510	508	555	602	640	672	548		
3.8	510	508	555	602	640	673	709	561	
4.2	510	508	555	602	640	673	711	745	573

$- U \times 10^3$

1.0	000								
1.4	000	6.17							
1.8	000	7.21	6.50						
2.2	000	7.21	7.76	7.18					
2.6	000	7.21	7.76	9.06	7.60				
3.0	000	7.21	7.77	9.07	9.87	8.02			
3.4	000	7.21	7.77	9.07	9.89	10.37	8.45		
3.8	000	7.21	7.77	9.07	9.90	10.40	11.01	8.90	
4.2	000	7.21	7.77	9.07	9.90	10.40	11.04	11.78	9.35

$$- \int_r^R \frac{\partial T}{\partial R}.dR$$

1.0	0000								
1.4	0000	21.71							
1.8	0000	23.47	20.53						
2.2	0000	23.47	21.76	19.93					
2.6	0000	23.47	21.77	20.74	18.83				
3.0	0000	23.47	21.77	20.75	20.10	17.82			
3.4	0000	23.47	21.77	20.75	20.10	19.68	16.95		
3.8	0000	23.47	21.77	20.75	20.10	19.69	19.16	16.23	
4.2	0000	23.47	21.77	20.75	20.10	19.69	19.17	18.72	15.60

Table D.12

Assumption w = 0; std. size of the element; RO = 4.2 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = - 7.4g.Pg;

EY = 2185. $T^{3/7}$g; dr = dR = 0.05 cm;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	000								
1.4	115.6	- 13.4							
1.8	7.6	-113.1	8.6						
2.2	- 46.9	-167.6	-37.7	17.1					
2.6	- 59.7	-180.4	-50.5	11.0	26.5				
3.0	- 50.1	-171.7	-41.8	19.8	39.8	32.0			
3.4	- 29.4	-150.1	-20.2	41.4	61.5	59.5	35.0		
3.8	1.2	-119.5	-10.4	72.0	92.2	90.2	73.2	36.3	
4.2	38.0	- 82.7	47.1	108.7	128.8	126.9	110.0	81.5	36.8

EY/E

1.0	0000								
1.4	1001	416							
1.8	1001	479	233						
2.2	1001	479	328	96					
2.6	1001	479	329	209	298				
3.0	1001	479	329	210	83	375			
3.4	1001	479	329	211	86	131	421		
3.8	1001	479	329	211	87	129	187	449	
4.2	1001	479	329	211	87	129	186	239	470

Table D.13

Assumption w = 0; std. size of the element; RO = 4.2 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = - 7.05 Pg;

EY = 2185. $T^{3/7}$g; dr = dR = 0.05 cm;

- P

R \ r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	0000								
1.4	821	663							
1.8	822	790	715						
2.2	822	791	881	760					
2.6	822	791	882	949	796				
3.0	822	791	882	951	1016	824			
3.4	822	791	882	951	1018	1082	848		
3.8	822	791	882	951	1018	1086	1146	868	
4.2	822	791	882	951	1018	1086	1151	1209	886

- U x 10^3

R \ r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	0000								
1.4	0000	7.02							
1.8	0000	8.45	7.72						
2.2	0000	8.45	9.75	8.19					
2.6	0000	8.45	9.76	10.66	8.70				
3.0	0000	8.45	9.76	10.69	11.29	9.24			
3.4	0000	8.45	9.76	10.69	11.32	12.16	9.80		
3.8	0000	8.45	9.76	10.69	11.32	12.20	13.14	10.34	
4.2	0000	8.45	9.76	10.69	11.32	12.20	13.19	14.17	10.87

$- \int_r^R \frac{\partial T}{\partial R} . dR$

R \ r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	0000								
1.4	0000	30.70							
1.8	0000	32.54	29.66						
2.2	0000	32.55	31.15	27.83					
2.6	0000	32.55	31.17	30.00	26.19				
3.0	0000	32.55	31.17	30.00	29.21	24.82			
3.4	0000	32.55	31.17	30.00	29.23	28.38	23.67		
3.8	0000	32.55	31.17	30.00	29.23	28.42	27.70	22.67	
4.2	0000	32.55	31.17	30.00	29.23	28.42	27.75	27.13	21.78

Table D.14

Assumption w = 0; std. size of the element; RO = 4.2 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = - 7.05 Pg;

$EY = 2185.T^{3/7}g$; dr = dR = 0.05 cm;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7	4.1
1.0	0000								
1.4	288.9	18.9							
1.8	202.0	-53.0	34.8						
2.2	187.6	-67.3	32.9	51.4					
2.6	212.2	-42.8	57.5	84.0	60.4				
3.0	259.2	4.2	104.5	131.1	119.7	64.5			
3.4	321.1	66.1	166.4	193.0	181.7	141.1	65.9		
3.8	392.1	137.2	237.5	264.1	252.8	212.4	153.1	65.8	
4.2	468.4	213.4	313.7	340.3	329.0	288.6	229.6	159.4	64.9

EY/E

1.0	000								
1.4	793	197							
1.8	793	286	133						
2.2	793	286	184	279					
2.6	793	286	184	2.77	338				
3.0	793	286	184	6.74	135	372			
3.4	793	286	184	6.8	133	172	395		
3.8	793	286	184	6.8	133	171	189	411	
4.2	793	286	184	6.8	133	171	187	199	424

Table D.15

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 10g; traverse = 5 cm; spacing = 1 D; E = (-8.18.P + 10)g;

EY = $2185.T^{3/7}$g; dr = dR = 0.05 cm;

- P

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	540	254						
1.8	576	438	273					
2.2	578	445	511	287				
2.6	578	446	522	533	297			
3.0	578	446	523	551	576	302		
3.4	578	446	523	553	606	517	310	
3.8	578	446	523	553	608	630	627	314

- U x 10^3

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	000	3.78						
1.8	000	6.61	4.27					
2.2	000	6.70	7.94	4.57				
2.6	000	6.71	8.08	8.45	4.86			
3.0	000	6.71	8.09	8.69	9.78	5.36		
3.4	000	6.71	8.09	8.70	10.12	11.40	5.51	
3.8	000	6.71	8.09	8.70	10.23	11.92	11.79	5.88

$$- \int_r^R \frac{\partial T}{\partial R} . dR$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	000	10.21						
1.8	000	13.56	10.01					
2.2	000	13.76	12.57	9.47				
2.6	000	13.76	12.79	12.67	8.94			
3.0	000	13.76	12.80	13.00	12.15	8.74		
3.4	000	13.76	12.81	13.03	12.63	11.38	8.16	
3.8	000	13.76	12.81	13.03	12.66	11.82	11.43	7.90

Table D.16

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 10g; traverse = 5 cm; spacing = 1 D; E = (-8.18.P + 10)g;

EY = $2185.T^{3/7}$g; dr = dR = 0.05 cm;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	118.4	16.3						
1.8	- 72.2	-126.4	17.4					
2.2	-226.4	-278.5	- 67.5	21.0				
2.6	-342.3	-394.4	-180.4	- 39.2	22.7			
3.0	-433.2	-485.3	-271.1	-126.0	- 15.0	21.7		
3.4	-505.2	-557.3	-343.0	-197.7	- 81.2	- 2.4	23.2	
3.8	-563.7	-615.8	-401.6	-256.2	-139.4	- 55.4	9.7	22.4

EY/E

1.0	000							
1.4	345	78						
1.8	299	217	247					
2.2	299	221	176	113				
2.6	299	221	181	171	151			
3.0	229	221	181	182	154	160		
3.4	229	221	181	183	165	125	187	
3.8	229	221	181	183	165	227	124	197

Table D.17

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = (-7.49.P + 10)g;

EY = 2185.$T^{3/7}$g; dr = dR = 0.05 cm;

- P

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	0000							
1.4	1043	503						
1.8	1103	833	545					
2.2	1104	846	985	576				
2.6	1104	846	1005	1043	593			
3.0	1104	846	1006	1080	1141	606		
3.4	1105	846	1006	1082	1194	1160	622	
3.8	1105	846	1006	1082	1199	1235	1221	631

- U x 10^3

1.0	0000							
1.4	0000	5.78						
1.8	0000	9.56	6.33					
2.2	0000	9.68	11.32	6.69				
2.6	0000	9.68	11.50	12.05	7.23			
3.0	0000	9.68	11.51	12.39	14.25	7.91		
3.4	0000	9.68	11.51	12.41	14.78	15.53	8.12	
3.8	0000	9.68	11.51	12.42	14.82	16.32	17.30	8.64

$$- \int_r^R \frac{\partial T}{\partial R}.dR$$

1.0	0000							
1.4	0000	20.78						
1.8	0000	27.50	20.01					
2.2	0000	27.84	23.89	18.73				
2.6	0000	27.86	24.23	24.35	17.89			
3.0	0000	27.86	24.25	24.94	23.31	17.40		
3.4	0000	27.86	24.25	24.98	24.03	23.17	16.21	
3.8	0000	27.86	24.25	24.99	24.10	24.13	21.87	15.67

Table D.18

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 20g; traverse = 5 cm; spacing = 1 D; E = (- 7.49.P + 10)g;

EY = $2185.T^{3/7}$g; dr = dR = 0.05 cm;

$$\int_r^R Z.dr$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	231.2	24.3						
1.8	- 97.3	-232.7	33.4					
2.2	-352.3	-484.7	-107.9	2.7				
2.6	-544.0	-676.3	-294.6	- 55.3	44.6			
3.0	-688.5	-820.8	-438.9	-192.7	- 8.6	43.4		
3.4	-801.4	-933.7	-551.8	-305.1	-112.6	7.3	46.6	
3.8	-891.2	-1024	-641.6	-394.9	-201.6	- 71.8	28.2	45.2

EY/E

1.0	000							
1.4	303	111						
1.8	294	228	119					
2.2	294	231	156	129				
2.6	294	231	160	158	161			
3.0	294	231	160	164	132	171		
3.4	294	231	160	164	141	131	198	
3.8	294	231	160	164	141	139	96	208

Table D.19

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = (-7.05.P + 10)g;

EY = 2185.$T^{3/7}$g; dr = dR = 0.05 cm;

- P

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	1532	761						
1.8	1619	1274	819					
2.2	1622	1293	1422	866				
2.6	1622	1294	1454	1570	890			
3.0	1622	1294	1456	1632	1653	920		
3.4	1622	1294	1456	1636	1731	1769	938	
3.8	1622	1294	1456	1636	1739	1882	1822	955

- U x 10^3

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	0000							
1.4	0000	6.99						
1.8	0000	11.70	7.85					
2.2	0000	11.84	13.37	8.20				
2.6	0000	11.84	13.61	14.66	9.06			
3.0	0000	11.84	13.62	15.41	17.64	9.41		
3.4	0000	11.84	13.63	15.44	18.30	18.76	10.04	
3.8	0000	11.84	13.63	15.44	18.36	19.75	20.42	10.48

$$- \int_r^R \frac{\partial T}{\partial R}.dR$$

R\r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	0000							
1.4	0000	30.33						
1.8	0000	37.78	29.70					
2.2	0000	38.18	36.37	27.51				
2.6	0000	38.20	36.92	35.24	26.72			
3.0	0000	38.20	36.96	36.15	33.29	24.91		
3.4	0000	38.20	36.96	36.22	34.19	32.37	23.93	
3.8	0000	38.20	36.96	36.22	34.29	33.46	32.52	22.74

Table D.20

Assumption w = 0; std. size of the element; RO = 3.8 cm;

T = 30g; traverse = 5 cm; spacing = 1 D; E = (-7.05.P + 10)g;

EY = $2185.T^{3/7}$g; dr = dR = 0.05 cm.

$$\int_r^R Z.dr$$

R \ r	1.0	1.3	1.7	2.1	2.5	2.9	3.3	3.7
1.0	000							
1.4	366.4	50.0						
1.8	- 50.9	-256.0	54.0					
2.2	-374.0	-574.9	-131.6	69.1				
2.6	-61.08	-811.6	-361.3	- 38.7	67.5			
3.0	-783.0	-983.8	-533.0	-199.3	6.2	73.2		
3.4	-918.1	-1119	-668.0	-333.6	-116.9	47.0	71.9	
3.8	-1025	-1226	-774.9	-440.3	-222.5	- 44.3	62.2	71.7

EY/E

1.0	000							
1.4	293	64.8						
1.8	282	186	60					
2.2	282	187	159	144				
2.6	282	187	166	136	159			
3.0	282	187	167	142	108	189		
3.4	282	187	167	143	116	89	201	
3.8	282	187	167	143	116	100	90	215

APPENDIX E

Table E.1

Cheese No.8

Assumption w = 0; std. size of the element; R0 = 5.0 cm;
T = 20g; traverse = 5 cm; spacing = 1 D; E = 100g; EY = 5000g;

$$\int_s^r \int_r^R Z.dr.dr$$

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	0								
1.5	0	2664							
2.0	0	2772	3807						
2.5	0	1464	2314	3207					
3.0	0	300	- 73	862	1702				
3.5	0	- 420	-1900	-1847	- 819	- 279			
4.0	0	-1360	-3760	-4585	-4222	-3164	-2424		
4.5	0	-1880	-4930	-6398	-6633	-5988	-4790	-4032	
5.0	0	-2476	-6271	-8479	-9437	-9445	-8745	-7667	-6994

$$\left(\int_r^R Z.dr \right) / P$$

1.0	-								
1.5	-0.442	-0.269							
2.0	-0.267	-0.104	-0.199						
2.5	-0.133	-0.041	-0.098	-0.166					
3.0	-0.032	0.179	-0.006	-0.096	-0.142				
3.5	0.049	0.281	0.095	-0.024	-0.093	-0.125			
4.0	0.113	0.365	0.170	0.041	-0.040	-0.088	-0.113		
4.5	0.166	0.434	0.232	0.097	-0.010	-0.048	-0.083	-0.102	
5.0	0.211	0.491	0.284	0.145	0.053	-0.008	-0.051	-0.079	-0.094

Table E.2

Assumption w = 0; std. size of the element; RO = 5 cm;

$$\left(\int_r^R Z.dr\right) / P$$

Cheese no. 1

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 5000g;

R \ r	1.0	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9
1.0	-								
1.5	-0.489	-0.337							
2.0	-0.366	-0.225	-0.239						
2.5	-0.274	-0.125	-0.173	-0.196					
3.0	-0.201	-0.039	-0.106	-0.150	-0.166				
3.5	-0.145	0.031	-0.047	-0.107	-0.133	-0.144			
4.0	-0.099	0.089	0.005	-0.062	-0.100	-0.120	-0.127		
4.5	-0.061	0.137	0.049	-0.023	-0.068	-0.095	-0.109	-0.115	
5.0	-0.029	0.178	0.086	-0.010	-0.038	-0.070	-0.089	-0.100	-0.104

Cheese no. 10

T = 20g; traverse = 5 cm; spacing = 1D; E = 200g; EY = 2000g;

1.0	-								
1.5	-0.517	-0.397							
2.0	-0.421	-0.315	-0.268						
2.5	-0.354	-0.249	-0.231	-0.222					
3.0	-0.301	-0.186	-0.187	-0.191	-0.186				
3.5	-0.258	-0.145	-0.148	-0.161	-0.163	-0.158			
4.0	-0.223	-0.105	-0.112	-0.132	-0.142	-0.144	-0.136		
4.5	-0.194	-0.071	-0.081	-0.107	-0.121	-0.127	-0.129	-0.125	
5.0	-0.169	-0.042	-0.054	-0.083	-0.101	-0.111	-0.115	-0.116	-0.113

Cheese no. 13

T = 20g; traverse = 5 cm; spacing = 1D; E = 1000g; EY = 1000g;

1.0	-								
1.5	-0.531	-0.446							
2.0	-0.448	-0.380	-0.324						
2.5	-0.392	-0.333	-0.287	-0.256					
3.0	-0.350	-0.297	-0.257	-0.231	-0.211				
3.5	-0.317	-0.268	-0.234	-0.211	-0.194	-0.180			
4.0	-0.290	-0.242	-0.211	-0.194	-0.179	-0.167	-0.157		
4.5	-0.268	-0.224	-0.195	-0.178	-0.166	-0.156	-0.147	-0.140	
5.0	-0.249	-0.206	-0.175	-0.165	-0.155	-0.146	-0.138	-0.131	-0.125

APPENDIX F

CONSIDERATION OF CENTRIFUGAL FORCE

F.1 Effect of Centrifugal Force

In a rotating package an element at any radius r will generate centrifugal force thereby reducing the pressure it exerts on the layer beneath it while the package is rotating. Similarly the element at the outer radius R will generate centrifugal force as soon as it is added to the rotating package. The centrifugal force so developed will reduce the effective pressure imposed by the element on the package beneath it during rotation.

For a given rotating package, the magnitude of the centrifugal force develbped in the element will depend on the mass of the element and on the circumferential speed of the element, which itself depends on the angular velocity and the radius of the element.

In some cases of winding, when spindle speed is very high and the diameter of the package is large, it may be possible that the centrifugal force developed in the element just added to the package is large. This force may reduce the effective pressure imposed by the element beneath it considerably and thus affect the compression of the package significantly. It may also be possible, in an extreme case, that the centrifugal force developed in the element overcomes its pressure altogether rendering the winding impossible. It shall be useful to determine the magnitude of centrifugal force developed and its effect on the compression of the package.

However, the centrifugal force to reduce the pressure of the element exists as long as the package is rotating. As soon as the package is stationary the full pressure of the element is available and is imposed over the cheese beneath it. But if the centrifugal force is large, it might affect the formation of the package, specially when there is a possibility of slip between core and cheese or between layers.

F.2 Centrifugal Force of the Element

The centrifugal force of the element is given by the expression

$$\frac{m.\ (wv)^2.\ r}{gr} \qquad \ldots \quad \ldots \quad (F.1)$$

where m is the mass of the element in g, wv is the angular velocity of the element in radians per sec., r is the radius in cm at which the element is rotating and gr is the acceleration due to gravity in cms per sec. per sec.

F.2.1 Mass of the Element

The total number of threads in the element are

$$\frac{2.K.\phi.dr}{D}$$

The diagonal length of the element at r is 'L'. The average length is L/2, where

$$L = \phi.\sqrt{r^2 + a^2}$$

therefore, the total length of yarn in the element is

$$\frac{2.K.\phi.dr}{D} \cdot \frac{L}{2} = \frac{K.\phi^2.dr}{D} \cdot \sqrt{r^2 + a^2}$$

If 'mass' be the mass per unit length (in g per cm) of the yarn and 'm' be the total mass of the element then

$$m = \frac{K.\phi^2.dr}{D} \cdot \sqrt{r^2 + a^2} \cdot \text{mass} \qquad \ldots \quad \ldots \quad (i)$$

Now if 'count' is the count of unstretched yarn in the cotton system then the 'mass' of the yarn in g per cm is given by the expression

$$\text{mass} = \frac{453.6}{\text{count} \times 840 \times 91.44}$$

The yarn wound on the cheese is in a stretched condition, stretched by the winding tension in the yarn. Therefore the apparent count at the time of winding is count.(1 + T/EY); EY being the Elasticity of yarn in Extension. Therefore

$$\text{mass} = \frac{453.6}{\text{count} \times (1 + T/EY) \times 840 \times 91.44} \qquad \ldots \quad \ldots \quad (ii)$$

F.2.2 Angular Velocity of the Element

If 'rev' be the revolutions per minute of the spindle, then angular velocity 'wv' in radians per sec. of the element is

$$wv = \text{rev} \times 2 \times 3.14159 \,/\, 60 \qquad \ldots \quad \ldots \quad (iii)$$

F.3 Consideration of the Centrifugal Force

The centrifugal force of the element at r in a rotating package was present before the addition of the layer at R and is also present after the addition of a layer. The change in the centrifugal force of the element can occur only due to the deformation u in r; mass and angular velocity of the element remaining constant.

In the equation (3.11) 'p' represents the change in the pressure of the element due to the addition of the said layer. This change of pressure p will also include change of pressure of the element because of any change in the centrifugal force. Therefore the main equation (3.14) remains unaltered.

The centrifugal force of the element added reduces the pressure imposed by the element beneath it given by the equation (3.22). The nett pressure imposed is given by the expression

$$P_{oR} = -\frac{K.\phi^2.dR}{D}.T_o.\cos\alpha_{oR} + \frac{(wv)^2.m.R}{gr}.$$

Therefore the boundary condition at r = R is given by the following expression

$$\frac{\partial u}{\partial r} \text{ at } (r=R) = -\frac{T_o.dR.\cos\alpha_{oR}}{2.K.D.E} + \frac{m.(wv)^2.R}{2(K.\phi)^2.E.gr} \qquad \ldots \qquad (F.2)$$

The solution of equation (3.14) with the above boundary condition at r = R gives the value of u in a rotating package.

F.4 Changes in Computer Program 15

Program 15 is changed as follows to do calculations for a rotating package. Only change in the program is in the boundary condition at the outer radius R. This condition in the program is given by the following equation

$$\frac{\partial u}{\partial r} \text{ (at } r=R) = \frac{T_o}{2.K.D.E} .dR. \frac{R}{\sqrt{R^2 + a^2}} + \text{cfcdu}$$

where cfcdu represents the second part of the equation (F.2) namely

$$\frac{dR}{2.K.D.E}\sqrt{R^2 + a^2} \cdot \frac{\text{mass.}(wv)^2.R}{gr};$$

where mass = 453.6/(840 x 91.44 x count x (1+T/EY));

and wv = rev x 2 x 3.14159/60.

The values of count, rev and gr are read in the program through the data tape. The value of 'cfcdu' appears in the output. By giving a value of zero to 'rev' the program reduces to 15 giving results for a stationary package.

F.5 Results and Conclusions

The values chosen for count of yarn in cotton system, revolutions per minute of the spindle and acceleration due to gravity in cms/sec./sec. are 11, 900 and 981 respectively. These values are representative of the cheese used in the practical work. The results are tabulated in the table F.1.

Table F.1

No.	outer radius R cm	% reduction in P_o due to centri-fugal force	radius r cm	U rev = 0 a	U rev = 900 b	$\frac{a-b}{a}$ x 100
1	1.1	0.093	1.0	0000	0000	00
2	1.5	0.131	1.5	0.01342	0.01335	0.522
3	2.0	0.212	2.0	0.02215	0.02203	0.542
4	2.5	0.317	2.5	0.02695	0.02677	0.668
5	3.0	0.446	3.0	0.0283	0.02808	0.777
6	3.5	0.6	3.5	0.02641	0.02618	0.87
7	4.0	0.776	4.0	0.02123	0.02102	0.989
8	4.5	0.976	4.5	0.01254	0.01240	1.129
9	5.0	1.202	4.9	0.00284	0.0028	1.409

The above table is compiled with the values of x, angle and space as 5, 1 and 1 respectively. The results show that the reduction in the pressure imposed by the added layer due to the centrifugal force of the layer is maximum at the outermost radius and is 1.2% of the pressure which would be imposed if the cheese was stationary. Also the maximum difference in the compression of a rotating and a stationary cheese is 1.41% of the compression of a stationary cheese. This occurs at the radius of 4.9 cm. At the other radii differences are smaller. Hence the effect of centrifugal force of the cheese, in the present case, is small.

NOTATION

r - is the radius at which an annular element was wound.

dr - is the original radial thickness of the element.

ρ - is the current radius of the element.

$d\rho$ - is the current radial thickness of the element

R - is the present outer radius on to which the yarn is wound.

dR - is the radial thickness of the element being wound at outer radius R.

V - is the axial distance of the element from the end of the cheese.

RO - is the final radius of the cheese to which it is built up.

ϕ - is the angle subtended by the element at the axis.

K - is a constant related to the number of ends in the element.

$K\phi$ - is the number of ends in the element in either axial or circumferential direction (see definition of element).

D - is the diameter of the yarn.

E - is the Modulus of Compression of the cheese. It is defined as the force required to produce unit radial strain at one crossing point. It is expressed in g.

e - is the change in E due to the addition of a layer at R. It is equal to $\left(\frac{\partial E}{\partial R}.dR\right)$.

EY - is the Elasticity of yarn in Extension. It is defined as the force or tension in the yarn required to produce unit strain in the length of the yarn. It is also expressed in g.

s - is the radius of the core or former on which the package is built.

x - is the traverse per wind. The value of x depends on machine setting.

a - is equal to $x/2\pi = W/\phi$.

W - is the axial length of the element.

w - is the change in W due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial W}{\partial R}.dR$.

α - is the angle which the threads in the element make with a plane perpendicular to the axis of the cheese.

θ - is the change in α due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial \alpha}{\partial R}.dR$.

U - is the total radial distance moved by the element in deforming and is equal to $(\rho - r)$.

u - is the incremental value of U and is equal to $\frac{\partial U}{\partial R}.dR$.

T - is the tension in the yarn.

t - is the change in T due to the addition of a layer at R. It is equal to $\frac{\partial T}{\partial R}.dR$.

P - is the radial pressure acting on the element.

p - is the change in P due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial P}{\partial R}.dR$.

Q - is the circumferential component of the force through the face of the element.

q - is the change in Q due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial Q}{\partial R}.dR$.

Z - is the axial component of the force through the end face of the element.

z - is the change in Z due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial Z}{\partial R}.dR$.

L - is the diagonal length of the element at ρ.

ℓ - is the change in L due to the addition of a layer at the outer radius R. It is equal to $\frac{\partial L}{\partial R}.dR$.

prcon - is a constant used in the relation of E and P. Its value depends on the compressional behaviour of the yarn.

tencon - is a constant used in the relation of EY and T. Its value depends on the extensional behaviour of the yarn.

The suffix 'o' at the bottom of the quantity represents the value of the quantity at the time of winding, e.g. T_o, represents the winding tension in the yarn. A suffix representing any of the radii at the bottom of a quantity show the radius at which the suffixed quantity is being considered, e.g. α_{oR} represents the wind angle at the outer radius R or T_r represents the tension in the yarn at radius r.